



Ciencia Latina
Internacional

Ciencia Latina Revista Científica Multidisciplinar, Ciudad de México, México.
ISSN 2707-2207 / ISSN 2707-2215 (en línea), marzo-abril 2024,
Volumen 8, Número 2.

https://doi.org/10.37811/cl_rcm.v8i2

**TEORÍA DE CAMPOS: REFORZAMIENTO
TEÓRICO – MATEMÁTICO AL MODELO
ESTÁNDAR DE PARTÍCULAS, BAJO LA
ESTRUCTURA ECUACIONAL DE YANG – MILLS**

**FIELD THEORY: THEORETICAL – MATHEMATICAL
REINFORCEMENT TO THE STANDARD PARTICLE MODEL,
UNDER THE YANG – MILLS EQUATIONAL STRUCTURE**

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DOI: https://doi.org/10.37811/cl_rcm.v8i2.10737

Teoría de Campos: Reforzamiento Teórico – Matemático al Modelo Estándar de Partículas, bajo la estructura ecuacional de Yang – Mills

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RESUMEN

El presente artículo científico, tiene como propósito, demostrar, la mecánica de partículas (física de partículas elementales) en un campo determinado, sea cual fuere la fuerza fundamental involucrada, bajo la teoría de campo de Yang – Mills, esto es, bajo estándares generales y uniformemente aplicables, es decir, sin perjuicio del campo de que se trate y en consecuencia, el conjunto de partículas susceptibles de interacción, para lo cual, se optimizan los sistemas de referenciación aquí desglosados (verbigracia, desde la óptica del sistema lagrangiano, etc), desde una perspectiva einsteniana, desde el ángulo de percepción de las teorías de gauge y de la estructura de campo de Higgs, así como del modelo estándar de física de partículas, etc. Asimismo, este artículo científico, procura, reforzar la propuesta de solución formulada por este investigador, bajo la siguiente tríada de premisas: **(i)** la conjetura de que las excitaciones más bajas de una teoría pura de Yang-Mills (es decir, sin campos de materia) tienen una brecha de masa finita con respecto al estado de vacío; **(ii)** la propiedad de confinamiento en presencia de partículas adicionales; y, **(iii)** que, para un campo de Yang-Mills no abeliano, existe un valor positivo mínimo de la energía.

Palabras clave: física de partículas, escala subatómica, campos de yang-mills, teorías de gauge, ecuación de Higgs

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Field Theory: Theoretical – Mathematical Reinforcement To The Standard Particle Model, Under The Yang – Mills Equational Structure

ABSTRACT

The purpose of this scientific article is to demonstrate particle mechanics (elementary particle physics) in a given field, whatever the fundamental force involved, under the Yang-Mills field theory, that is, under general and uniformly applicable standards, that is, without prejudice to the field in question and consequently, the set of particles susceptible to interaction, for which the referential systems broken down here are optimized (e.g., from the perspective of the Lagrangian system, etc.), from an Einsteinian perspective, from the angle of perception of the theories of gauge and the Higgs field structure, as well as the standard model of particle physics, etc. Likewise, this scientific article seeks to reinforce the proposed solution formulated by this researcher, under the following triad of premises: (i) the conjecture that the lowest excitations of a pure Yang-Mills theory (i.e., without matter fields) have a finite mass gap with respect to the vacuum state; (ii) the property of confinement in the presence of additional particles; and, (iii) that, for a non-abelian Yang-Mills field, there is a minimum positive value of energy.

Keywords: particle physics, subatomic scale, Yang-Mills fields, gauge theories, Higgs equation



INTRODUCCION

En la física cuántica, la posición y la velocidad de una partícula se tienen como operadores no conmutadores que interactúan en un espacio de Hilbert. Es así, donde muchos aspectos de la naturaleza se describen en forma de campos. Dado que los campos interactúan con las partículas, deviene en indispensable, incorporar conceptos cuánticos tanto para describir campos como para describir partículas. En los campos convencionales, existe una partícula y por regla general, una antipartícula, con la misma masa y carga, pero opuesta, verbigracia, el campo cuantizado de los electrones.

Siguiendo este mismo orden de cosas, se tiene que, las teorías de gauge (teorías cuánticas de campos [QFT]), es una de las más importantes en cuanto a física de partículas se refiere. Un ejemplo claro de ello, es la teoría del electromagnetismo de Maxwell que comporta un grupo de simetría gauge en un grupo abeliano U(1). Sin embargo, la teoría de Yang – Mills, en este contexto, califica una teoría gauge no abeliana.

La ecuación clásica y variacional central del lagrangiano Yang-Mills, se escribe así:

$$L = \frac{1}{4g^2} \int \text{Tr } F \wedge *F,$$

donde Tr denota una forma cuadrática invariante en el álgebra de Lie de G. Las ecuaciones de Yang-Mills no son lineales, por lo que, no existen soluciones exactas de la ecuación clásica antes referida, y es lo que se propone resolver este trabajo a través de un riguroso cálculo matemático. En consecuencia, este trabajo, pretende demostrar, que la teoría gauge no abeliana de Yang – Mills, describe otras fuerzas en la naturaleza, especialmente la fuerza débil (responsable, entre otras cosas, de ciertas formas de radiactividad) y la fuerza fuerte o nuclear (responsable, entre otras cosas, de la unión de protones y neutrones en núcleos), pero sin perder las premisas esenciales de la teoría de campos de Yang – Mills, esto es, por fuera de la teoría electrodébil de Glashow-Salam-Weinberg o la teoría del “campo de Higgs”. Si bien es cierto, constituyese en una propiedad notable de la teoría cuántica de Yang-Mills, la nominada “libertad asintótica”, la misma que supone, que a distancias cortas, el campo muestra un comportamiento cuántico muy similar a su comportamiento clásico; sin embargo, a largas distancias, la teoría de Yang – Mills, fracasa en la descripción del campo. Por tanto, el presente trabajo, tiene como finalidad,

comprobar que: **(i)** existe una "*brecha de masa*" $\Delta > \text{constante}$, tal que cada excitación del vacío tiene energía de al menos Δ ; **(ii)** existe un confinamiento de quarks, partiendo de la premisa de que, los estados físicos de las partículas, como el protón, el neutrón y el pión, son invariantes en SU(3); y, **(iii)** existe una "*ruptura de simetría quiral*", lo que significa que el vacío es potencialmente invariante solo bajo un cierto subgrupo de simetría completa que actúa sobre los campos de quarks.

METODOLOGÍA

La teorización desplegada en el presente manuscrito, resulta de la aplicación de una metodología de investigación integral, esto es, bajo un enfoque híbrido, tanto desde el punto de vista cualitativo como en su dimensión cuantitativa. El tipo de investigación que ha sido desarrollado a lo largo del presente Artículo Científico, es esencialmente predictivo, a la luz de la física teórica, más no, acusa carácter empírico o experimental. Por otro lado, las líneas de investigación adoptadas para la formulación del estado del arte, se ajustan al constructivismo. Cabe indicar, que no existe población de estudio en la medida en que el presente artículo científico, no es de carácter sociológico o social, más aun, en mérito a su impacto en la realidad de transformación. Tampoco se han implementado técnicas de recolección de información, tales como encuestas, entrevistas, etc, salvo revisión bibliográfica, a razón del campo de investigación abordado. Adicionalmente a lo antes expuesto, es preciso resaltar, que el material de apoyo es meramente bibliográfico. La técnica metodológica, dada la complejidad de la temática escrutada, es deductiva, pues la teorización en sentido estricto, ha sido desarrollada desde principios y premisas generales que son inherentes a la física de partículas en sentido lato. Finalmente, para efectos de construir y desarrollar las ecuaciones constantes en el presente artículo científico, se ha tomado en consideración el Modelo Estándar de Física de Partículas, muy especialmente, en tratándose de los campos de Yang – Mills, sin perjuicio de los demás sistemas de recalibración deducidos y esbozados a lo largo del presente Artículo Científico.

RESULTADOS Y DISCUSIÓN

Análisis Único de Movimiento de Partículas en Campos de Yang – Mills

$$\begin{aligned}
 \mathcal{L} &= -\frac{1}{4\pi F^{\mu\nu}(x)F_{\nu\mu}(x)F_{\nu\mu}^{\mu\nu}}(x)F_{\mu\nu}^{\nu\mu}(x) \neq \mathcal{L} = -\frac{1}{4\pi F^{\nu\mu}(x)F_{\mu\nu}(x)F_{\mu\nu}^{\nu\mu}}(x)F_{\nu\mu}^{\mu\nu}(x) \neq \mathcal{L} \\
 &= -\frac{1}{4\pi F^{\mu\nu}(x)F_{\mu\nu}(x)F_{\nu\mu}^{\mu\nu}}(x)F_{\mu\nu}^{\nu\mu}(x) \neq \mathcal{L} = -\frac{1}{4\pi F^{\nu\mu}(x)F_{\nu\mu}(x)F_{\mu\nu}^{\nu\mu}}(x)F_{\nu\mu}^{\mu\nu}(x) \\
 \mathcal{L} &= -\frac{1}{4\pi F^{\mu\nu}(y)F_{\nu\mu}(y)F_{\nu\mu}^{\mu\nu}}(y)F_{\mu\nu}^{\nu\mu}(y) \neq \mathcal{L} = -\frac{1}{4\pi F^{\nu\mu}(y)F_{\mu\nu}(y)F_{\mu\nu}^{\nu\mu}}(y)F_{\nu\mu}^{\mu\nu}(y) \neq \mathcal{L} \\
 &= -\frac{1}{4\pi F^{\mu\nu}(y)F_{\mu\nu}(y)F_{\nu\mu}^{\mu\nu}}(y)F_{\mu\nu}^{\nu\mu}(y) \neq \mathcal{L} = -\frac{1}{4\pi F^{\nu\mu}(y)F_{\nu\mu}(y)F_{\mu\nu}^{\nu\mu}}(y)F_{\nu\mu}^{\mu\nu}(y) \\
 \mathcal{L} &= -\frac{1}{4\pi F^{\mu\nu}(z)F_{\nu\mu}(z)F_{\nu\mu}^{\mu\nu}}(z)F_{\mu\nu}^{\nu\mu}(z) \neq \mathcal{L} = -\frac{1}{4\pi F^{\nu\mu}(z)F_{\mu\nu}(z)F_{\mu\nu}^{\nu\mu}}(z)F_{\nu\mu}^{\mu\nu}(z) \neq \mathcal{L} \\
 &= -\frac{1}{4\pi F^{\mu\nu}(z)F_{\mu\nu}(z)F_{\nu\mu}^{\mu\nu}}(z)F_{\mu\nu}^{\nu\mu}(z) \neq \mathcal{L} = -\frac{1}{4\pi F^{\nu\mu}(z)F_{\nu\mu}(z)F_{\mu\nu}^{\nu\mu}}(z)F_{\nu\mu}^{\mu\nu}(z)
 \end{aligned}$$

$$\begin{aligned}
 &F^{\mu\nu}(x,t)F_{\nu\mu}(x,t)F_{\mu\nu}(x,t)F^{\nu\mu}(x,t)F^{\nu\mu}(x,t)F_{\mu\nu}(x,t)F_{\nu\mu}(x,t)F^{\mu\nu}(x,t) \\
 &+ F^{\mu\nu}(y,t)F_{\nu\mu}(y,t)F_{\mu\nu}(y,t)F^{\nu\mu}(y,t)F^{\nu\mu}(y,t)F_{\mu\nu}(y,t)F_{\nu\mu}(y,t)F^{\mu\nu}(y,t) \\
 &+ F^{\mu\nu}(z,t)F_{\nu\mu}(z,t)F_{\mu\nu}(z,t)F^{\nu\mu}(z,t)F^{\nu\mu}(z,t)F_{\mu\nu}(z,t)F_{\nu\mu}(z,t)F^{\mu\nu}(z,t) \\
 &+ F^{\mu\nu}(x)F_{\nu\mu}(x)F_{\mu\nu}(x)F^{\nu\mu}(x)F^{\nu\mu}(x)F_{\mu\nu}(x)F_{\nu\mu}(x)F^{\mu\nu}(x) \\
 &+ F^{\mu\nu}(y)F_{\nu\mu}(y)F_{\mu\nu}(y)F^{\nu\mu}(y)F^{\nu\mu}(y)F_{\mu\nu}(y)F_{\nu\mu}(y)F^{\mu\nu}(y) \\
 &+ F^{\mu\nu}(z)F_{\nu\mu}(z)F_{\mu\nu}(z)F^{\nu\mu}(z)F^{\nu\mu}(z)F_{\mu\nu}(z)F_{\nu\mu}(z)F^{\mu\nu}(z) \\
 &= \partial^\mu A_\nu(x,t) - \partial^\nu A_\mu(x,t) + \partial^\mu A_\nu(y,t) - \partial^\nu A_\mu(y,t) + \partial^\mu A_\nu(z,t) - \partial^\nu A_\mu(z,t) \\
 &= \partial^\mu A_\nu(x) - \partial^\nu A_\mu(x) + \partial^\mu A_\nu(y) - \partial^\nu A_\mu(y) + \partial^\mu A_\nu(z) - \partial^\nu A_\mu(z)
 \end{aligned}$$

$$\begin{aligned}
 &F_{ij}(x,t), F^{ji}(x,t), F_j^i F_i^j(x,t), F_{ij}(y,t), F^{ji}(y,t), F_j^i F_i^j(y,t), F_{ij}(z,t), F^{ji}(z,t), F_j^i F_i^j(z,t) \\
 &= -\epsilon^{ijk} \epsilon_{ijk} B^k B_k(x,t) - \epsilon^{ijk} \epsilon_{ijk} B^k B_k(y,t) - \epsilon^{ijk} \epsilon_{ijk} B^k B_k(z,t)
 \end{aligned}$$

$$\begin{aligned}
 &A^\mu A_\mu A^\nu A_\nu A^\mu A_\nu A^\nu A_\mu A^{\mu\nu} A_{\nu\mu} A^{\nu\mu} A_{\mu\nu}(x) \rightarrow A^\mu A_\mu A^\nu A_\nu A^\mu A_\nu A^\nu A_\mu A^{\mu\nu} A_{\nu\mu} A^{\nu\mu} A_{\mu\nu}(y) \\
 &\rightarrow A^\mu A_\mu A^\nu A_\nu A^\mu A_\nu A^\nu A_\mu A^{\mu\nu} A_{\nu\mu} A^{\nu\mu} A_{\mu\nu}(z) \rightarrow A'_\mu A'_\nu A'_\nu A'_\mu(x) \rightarrow A'_\mu A'_\nu A'_\nu A'_\mu(y) \\
 &\rightarrow A'_\mu A'_\nu A'_\nu A'_\mu(z) = A^\mu A_\mu A^\nu A_\nu A^\mu A_\nu A^\nu A_\mu A^{\mu\nu} A_{\nu\mu} A^{\nu\mu} A_{\mu\nu}(x) \\
 &\rightarrow A^\mu A_\mu A^\nu A_\nu A^\mu A_\nu A^\nu A_\mu A^{\mu\nu} A_{\nu\mu} A^{\nu\mu} A_{\mu\nu}(y) \\
 &\rightarrow A^\mu A_\mu A^\nu A_\nu A^\mu A_\nu A^\nu A_\mu A^{\mu\nu} A_{\nu\mu} A^{\nu\mu} A_{\mu\nu}(z) + \partial^\mu \partial_\nu \partial^\nu \partial_\mu \partial^{\mu\nu} \partial_{\nu\mu} \partial^{\nu\mu} \partial_{\mu\nu} \alpha(x) \\
 &+ \partial^\mu \partial_\nu \partial^\nu \partial_\mu \partial^{\mu\nu} \partial_{\nu\mu} \partial^{\nu\mu} \partial_{\mu\nu}(y) + \partial^\mu \partial_\nu \partial^\nu \partial_\mu \partial^{\mu\nu} \partial_{\nu\mu} \partial^{\nu\mu} \partial_{\mu\nu}(z)
 \end{aligned}$$

$$\begin{aligned}
F^{\mu\nu}F_{\mu\nu}F^{\nu\mu}F_{\nu\mu}F^{\mu\nu}F_{\nu\mu}F^{\nu\mu}F_{\mu\nu}(x) &\rightarrow F^{\mu\nu}F_{\mu\nu}F^{\nu\mu}F_{\nu\mu}F^{\mu\nu}F_{\nu\mu}F^{\nu\mu}F_{\mu\nu}(y) \\
&\rightarrow F^{\mu\nu}F_{\mu\nu}F^{\nu\mu}F_{\nu\mu}F^{\mu\nu}F_{\nu\mu}F^{\nu\mu}F_{\mu\nu}(z) \rightarrow F'_{\mu\nu}F'_{\nu\mu}(x) \rightarrow F'_{\mu\nu}F'_{\nu\mu}(y) \rightarrow F'_{\mu\nu}F'_{\nu\mu}(z) \\
&= \partial_\mu \left(A^\mu A_\nu A^\nu A_\mu A^{\mu\nu} A_{\nu\mu} A^{\nu\mu} A_{\mu\nu}(x) + \partial^\mu \partial_\nu \partial^\nu \partial_\mu \partial^{\mu\nu} \partial_{\nu\mu} \partial^{\nu\mu} \partial_{\mu\nu} \alpha(x) \right) \\
&\quad - \partial_\nu \left(A^\nu A_\mu A^\mu A_\nu A^{\nu\mu} A_{\mu\nu} A^{\mu\nu} A_{\nu\mu}(x) + \partial^\nu \partial_\mu \partial^\mu \partial_\nu \partial^{\nu\mu} \partial_{\mu\nu} \partial^{\mu\nu} \partial_{\nu\mu} \alpha(x) \right) \\
&= \partial^\mu \partial^\nu A_\mu A_\nu \partial^\nu \partial^\mu A_\nu A_\mu \partial^{\mu\nu} \partial^{\nu\mu} A_{\mu\nu} A_{\nu\mu} \partial^{\nu\mu} \partial^{\mu\nu} A_{\nu\mu} A_{\mu\nu}(x) \\
&\quad + \partial^\mu \partial^\nu \partial_\mu \partial_\nu \partial^\nu \partial^\mu \partial_\nu A_\mu \partial^{\mu\nu} \partial^{\nu\mu} \partial_{\mu\nu} \partial_{\nu\mu} \partial^{\nu\mu} \partial^{\mu\nu} \partial_{\nu\mu} \alpha(x) \\
&= \partial_\mu \left(A^\mu A_\nu A^\nu A_\mu A^{\mu\nu} A_{\nu\mu} A^{\nu\mu} A_{\mu\nu}(y) + \partial^\mu \partial_\nu \partial^\nu \partial_\mu \partial^{\mu\nu} \partial_{\nu\mu} \partial^{\nu\mu} \partial_{\mu\nu} \alpha(y) \right) \\
&\quad - \partial_\nu \left(A^\nu A_\mu A^\mu A_\nu A^{\nu\mu} A_{\mu\nu} A^{\mu\nu} A_{\nu\mu}(y) + \partial^\nu \partial_\mu \partial^\mu \partial_\nu \partial^{\nu\mu} \partial_{\mu\nu} \partial^{\mu\nu} \partial_{\nu\mu} \alpha(y) \right) \\
&= \partial^\mu \partial^\nu A_\mu A_\nu \partial^\nu \partial^\mu A_\nu A_\mu \partial^{\mu\nu} \partial^{\nu\mu} A_{\mu\nu} A_{\nu\mu} \partial^{\nu\mu} \partial^{\mu\nu} A_{\nu\mu} A_{\mu\nu}(y) \\
&\quad + \partial^\mu \partial^\nu \partial_\mu \partial_\nu \partial^\nu \partial^\mu \partial_\nu A_\mu \partial^{\mu\nu} \partial^{\nu\mu} \partial_{\mu\nu} \partial_{\nu\mu} \partial^{\nu\mu} \partial^{\mu\nu} \partial_{\nu\mu} \alpha(y) \\
&= \partial_\mu \left(A^\mu A_\nu A^\nu A_\mu A^{\mu\nu} A_{\nu\mu} A^{\nu\mu} A_{\mu\nu}(z) + \partial^\mu \partial_\nu \partial^\nu \partial_\mu \partial^{\mu\nu} \partial_{\nu\mu} \partial^{\nu\mu} \partial_{\mu\nu} \alpha(z) \right) \\
&\quad - \partial_\nu \left(A^\nu A_\mu A^\mu A_\nu A^{\nu\mu} A_{\mu\nu} A^{\mu\nu} A_{\nu\mu}(z) + \partial^\nu \partial_\mu \partial^\mu \partial_\nu \partial^{\nu\mu} \partial_{\mu\nu} \partial^{\mu\nu} \partial_{\nu\mu} \alpha(z) \right) \\
&= \partial^\mu \partial^\nu A_\mu A_\nu \partial^\nu \partial^\mu A_\nu A_\mu \partial^{\mu\nu} \partial^{\nu\mu} A_{\mu\nu} A_{\nu\mu} \partial^{\nu\mu} \partial^{\mu\nu} A_{\nu\mu} A_{\mu\nu}(z) \\
&\quad + \partial^\mu \partial^\nu \partial_\mu \partial_\nu \partial^\nu \partial^\mu \partial_\nu A_\mu \partial^{\mu\nu} \partial^{\nu\mu} \partial_{\mu\nu} \partial_{\nu\mu} \partial^{\nu\mu} \partial^{\mu\nu} \partial_{\nu\mu} \alpha(z)
\end{aligned}$$

$$F'_{\mu\nu}F'_{\nu\mu}(x) = \partial^\mu A^\nu \partial_\nu A_\mu \partial^{\mu\nu} A^{\nu\mu} \partial_{\nu\mu} A_{\mu\nu}(x) - \partial^\nu A^\mu \partial_\mu A_\nu \partial^{\nu\mu} A^{\mu\nu} \partial_{\mu\nu} A_{\nu\mu}(x) = F^{\mu\nu}F_{\nu\mu}(x)$$

$$F'_{\mu\nu}F'_{\nu\mu}(y) = \partial^\mu A^\nu \partial_\nu A_\mu \partial^{\mu\nu} A^{\nu\mu} \partial_{\nu\mu} A_{\mu\nu}(y) - \partial^\nu A^\mu \partial_\mu A_\nu \partial^{\nu\mu} A^{\mu\nu} \partial_{\mu\nu} A_{\nu\mu}(y) = F^{\mu\nu}F_{\nu\mu}(y)$$

$$F'_{\mu\nu}F'_{\nu\mu}(z) = \partial^\mu A^\nu \partial_\nu A_\mu \partial^{\mu\nu} A^{\nu\mu} \partial_{\nu\mu} A_{\mu\nu}(z) - \partial^\nu A^\mu \partial_\mu A_\nu \partial^{\nu\mu} A^{\mu\nu} \partial_{\mu\nu} A_{\nu\mu}(z) = F^{\mu\nu}F_{\nu\mu}(z)$$

$$\begin{aligned}
\mathcal{A}[A^\mu \partial_\nu A^\nu \partial_\mu] &= \iiint_{\nu\mu}^{\mu\nu} \mu\nu\nu\mu d^4\chi \mathcal{L}[A^\mu \partial_\nu A^\nu \partial_\mu] + \delta \mathcal{A}[A^\mu \partial_\nu A^\nu \partial_\mu] = \delta \iiint_{\nu\mu}^{\mu\nu} \mu\nu\nu\mu d^4\chi \mathcal{L}[A^\mu \partial_\nu A^\nu \partial_\mu] \\
&= \iiint_{\nu\mu}^{\mu\nu} \mu\nu\nu\mu d^4\chi \delta \mathcal{L}[A^\mu \partial_\nu A^\nu \partial_\mu]
\end{aligned}$$

$$\delta \mathcal{L}[A^\mu \partial_\nu A^\nu \partial_\mu] = \frac{\partial \mathcal{L}}{\partial A^\mu A_\nu A^\nu A_\mu} \delta A^\mu_\nu A^\nu_\mu + \frac{\partial \mathcal{L}}{\partial (\partial^\mu A_\nu \partial^\nu A_\mu)} \delta (\partial^\mu A_\nu \partial^\nu A_\mu)$$

$$\delta \mathcal{A}[A^\mu \partial_\nu A^\nu \partial_\mu] = \delta \iiint_{\nu\mu}^{\mu\nu} \mu\nu\nu\mu d^4\chi \left[\frac{\partial \mathcal{L}}{\partial A^\mu A_\nu A^\nu A_\mu} \delta A^\mu_\nu A^\nu_\mu + \frac{\partial \mathcal{L}}{\partial (\partial^\mu A_\nu \partial^\nu A_\mu)} \delta (\partial^\mu A_\nu \partial^\nu A_\mu) \right]$$

$$\delta [A^\mu \partial_\nu A^\nu \partial_\mu] = \delta \frac{\partial A^\mu A_\nu A^\nu A_\mu}{\partial \chi^\mu \chi_\nu \chi^\nu \chi_\mu} = \frac{\partial}{\partial \chi^\mu \chi_\nu \chi^\nu \chi_\mu} \delta A^\mu_\nu A^\nu_\mu = \partial^\mu \partial_\nu (\delta A^\mu_\nu A^\nu_\mu)$$



$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial(\partial^\mu A_\nu \partial^\nu \partial_\mu)} \delta(\partial^\mu A_\nu \partial^\nu \partial_\mu) &= \frac{\partial \mathcal{L}}{\partial(\partial^\mu A_\nu \partial^\nu A_\mu)} \partial^\mu \partial_\nu (\delta A_\nu^\mu A_\mu^\nu) \\ &= \partial^\mu \partial_\nu \partial^\mu \partial_\nu \left[\frac{\partial \mathcal{L}}{\partial(\partial^\mu A_\nu \partial^\nu A_\mu)} \delta(\partial^\mu A_\nu \partial^\nu A_\mu) \right] - \frac{\partial \mathcal{L}}{\partial(\partial^\mu A_\nu \partial^\nu A_\mu)} \delta(\partial^\mu A_\nu \partial^\nu A_\mu) \end{aligned}$$

$$\begin{aligned} \delta \mathcal{A}[A^\mu \partial_\nu A^\nu \partial_\mu] &= \delta \iiint_{\nu\mu}^{\mu\nu} \mu\nu\nu\mu d^4\chi \left[\frac{\partial \mathcal{L}}{\partial A^\mu A_\nu A^\mu A_\mu} \delta A_\nu^\mu A_\mu^\nu \right. \\ &\quad \left. - \partial^\mu \partial_\nu \partial^\nu \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial^\mu A_\nu \partial^\nu A_\mu)} \delta(\partial^\nu A_\mu \partial^\mu A_\nu) \delta A_\nu^\mu A_\mu^\nu \right. \\ &\quad \left. + \delta \iiint_{\nu\mu}^{\mu\nu} \mu\nu\nu\mu d^4\chi \partial^\mu \partial_\nu \partial^\nu \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial^\mu A_\nu \partial^\nu A_\mu)} \delta(\partial^\nu A_\mu \partial^\mu A_\nu) \delta A_\nu^\mu A_\mu^\nu \right] \right] \end{aligned}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial A^\mu A_\nu A_\nu^\mu A_\mu^\nu} &= - \frac{1}{4\pi \frac{\partial}{\partial A^\mu A_\nu A_\nu^\mu A_\mu^\nu} [F^\mu F_\nu F_\nu^\mu F_\mu^\nu]} \\ &= -1 \\ &\quad / 4\pi \frac{\partial}{\partial A^\mu A_\nu A_\nu^\mu A_\mu^\nu} \left((\partial^\mu A_\nu(x) - \partial^\nu A_\mu(x)) (\partial^\nu A_\mu(x) - \partial^\mu A_\nu(x)) (\partial^\mu A^\nu(x) \right. \\ &\quad \left. - \partial^\nu A^\mu(x)) (\partial^\nu A^\mu(x) - \partial^\mu A^\nu(x)) (\partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)) (\partial_\nu^\mu A_\mu^\nu(x) \right. \\ &\quad \left. - \partial_\mu^\nu A_\nu^\mu(x)) (\partial_\nu^\mu \partial_\mu^\nu A(x) - \partial_\mu^\nu \partial_\nu^\mu A(x)) \right) \\ &\quad + -1 \\ &\quad / 4\pi \frac{\partial}{\partial A^\mu A_\nu A_\nu^\mu A_\mu^\nu} \left((\partial^\mu A_\nu(y) - \partial^\nu A_\mu(y)) (\partial^\nu A_\mu(y) - \partial^\mu A_\nu(y)) (\partial^\mu A^\nu(y) \right. \\ &\quad \left. - \partial^\nu A^\mu(y)) (\partial^\nu A^\mu(y) - \partial^\mu A^\nu(y)) (\partial_\mu A_\nu(y) - \partial_\nu A_\mu(y)) (\partial_\nu^\mu A_\mu^\nu(y) \right. \\ &\quad \left. - \partial_\mu^\nu A_\nu^\mu(y)) (\partial_\nu^\mu \partial_\mu^\nu A(y) - \partial_\mu^\nu \partial_\nu^\mu A(y)) \right) \\ &\quad + -1 \\ &\quad / 4\pi \frac{\partial}{\partial A^\mu A_\nu A_\nu^\mu A_\mu^\nu} \left((\partial^\mu A_\nu(z) - \partial^\nu A_\mu(z)) (\partial^\nu A_\mu(z) - \partial^\mu A_\nu(z)) (\partial^\mu A^\nu(z) \right. \\ &\quad \left. - \partial^\nu A^\mu(z)) (\partial^\nu A^\mu(z) - \partial^\mu A^\nu(z)) (\partial_\mu A_\nu(z) - \partial_\nu A_\mu(z)) (\partial_\nu^\mu A_\mu^\nu(z) \right. \\ &\quad \left. - \partial_\mu^\nu A_\nu^\mu(z)) (\partial_\nu^\mu \partial_\mu^\nu A(z) - \partial_\mu^\nu \partial_\nu^\mu A(z)) \right) \end{aligned}$$

$$\begin{aligned}
& \partial_i \partial^j \partial_j \partial^i F^{\mu\nu\phi} F_{\nu\mu\omega}(x) \\
&= \frac{\frac{\partial^\theta \partial_\theta F_\sigma^\rho \gamma \beta}{\varepsilon \epsilon \partial \pi}}{\frac{\Delta \nabla}{\tau}} + \prod_v^\mu \lambda \prod_\mu^v \lambda H_{iggs} \\
& - W^\mu W_\nu W^\nu W_\mu W_\nu^\mu W_\mu^\nu W_\nu^\mu W_\mu^\nu - \eta^\theta \eta_\beta \eta_{\phi\nu}^{\sigma\mu} \alpha_\eta / \mathbb{R}^4
\end{aligned}$$

En la que la constante H_{iggs} es igual a:

$$\begin{aligned}
\mathcal{L}_{SM} &= \frac{1}{2\partial_\nu g_\mu^a \partial_\mu g_\nu^b} - g_s f^{abc} \partial_\mu g_\nu^a g_\nu^b g_\nu^c \partial_\nu g_\mu^a g_\mu^b g_\mu^c - \frac{1}{4g_s^2 f^{abc} f^{ade} g_\mu^b g_\mu^c g_\mu^d g_\mu^d g_\nu^b g_\nu^c g_\nu^d g_\nu^d} \\
& - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^- - M^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- - \frac{1}{2\partial_\nu Z_\mu^0 \partial_\mu Z_\nu^0} - \frac{1}{2c_\omega^2 M^2 Z_\mu^0 Z_\nu^0} \\
& - \frac{1}{2\partial_\mu \mathcal{A}_\nu \partial_\nu \mathcal{A}_\mu} \\
& - igc_\omega (\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+) - Z_\mu^0 Z_\nu^0 (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-)) \\
& - ig s_\omega (\partial_\mu \mathcal{A}_\nu \partial_\nu \mathcal{A}_\mu (W_\mu^+ W_\mu^- W_\nu^+ W_\nu^-)) \\
& - \mathcal{A}_\mu (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-) - \mathcal{A}_\nu (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-) \\
& - \frac{1}{2g^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+} + g^2 c_\omega^2 (Z_\mu^0 W_\mu^+ W_\mu^- Z_\nu^0 W_\nu^+ W_\nu^-) \\
& + g^2 s_\omega^2 (\mathcal{A}_\mu W_\mu^+ W_\mu^- \mathcal{A}_\nu W_\nu^+ W_\nu^-) \\
& + g^2 c_\omega s_\omega (\mathcal{A}_\mu Z_\mu^0 (W_\mu^+ W_\mu^-) \mathcal{A}_\nu Z_\nu^0 (W_\nu^+ W_\nu^-) - 2\mathcal{A}_\mu W_\mu^+ W_\mu^- Z_\mu^0 \mathcal{A}_\nu W_\nu^+ W_\nu^- Z_\nu^0) \\
& - \frac{1}{2\partial_\mu \mathcal{H} \partial_\nu \mathcal{H}} - 2M^2 \alpha_h \mathcal{H}^2 - \partial_\mu \phi^+ \partial_\nu \phi^- \partial_\nu \phi^+ \partial_\mu \phi^- - \frac{1}{2\partial_\mu \phi^0 \partial_\nu \phi^0 \partial_\nu \phi^0 \partial_\mu \phi^0}
\end{aligned}$$



$$\begin{aligned}
& (\varphi\partial + m_d^\lambda)d_j^\lambda \\
& + ig s_\omega \mathcal{A}_\mu \left(-\left(\overrightarrow{e^\lambda} \varphi^\mu e^\lambda\right) + \frac{2}{3\left(\overrightarrow{\mu_j^\lambda} \varphi^\mu \mu_j^\lambda\right)} - \frac{1}{3\left(\overrightarrow{d^\lambda} \varphi^\mu d^\lambda\right)} + \frac{ig}{4c_\omega Z_\mu^0 \left(\overrightarrow{v^\lambda} \varphi^\mu (1 + \varphi^5) v^\lambda\right)} \right. \\
& + \left(\overrightarrow{e^\lambda} \varphi^\mu (4s_\omega^2 - 1 - \varphi^5) e^\lambda\right) + \left(\overrightarrow{d_j^\lambda} \varphi^\mu \left(\frac{4}{3s_\omega^2} - 1 - \varphi^5\right) d_j^\lambda\right) + \left(\overrightarrow{\mu_j^\lambda} \varphi^\mu \left(1 - \frac{8}{3s_\omega^2} + \varphi^5\right) \mu_j^\lambda\right) \\
& \left. + \left(\frac{ig}{\sqrt{2}W_\mu^+ \left(\overrightarrow{v^\lambda} \varphi^\mu (1 + \varphi^5) U^{lep^k}_\xi e^k\right)}\right) + \left(\overrightarrow{\mu_j^\lambda} \varphi^\mu (1 + \varphi^5) C_{k\lambda} d_j^k\right) \right) \\
& + \frac{ig}{\sqrt{2}W_\mu^- \left(\overrightarrow{e^k} U^{lep^+} \frac{v \psi \lambda}{\xi \rho \varpi} - \triangle \Pi_{\oplus \oplus \tau}^{\oplus \oplus \gamma} \otimes \frac{\prod_v^\mu ijk}{\otimes \sigma} \Omega \Psi \Phi \Delta (1 + \varphi^5) v^\lambda + \overrightarrow{d_j^k} C_{*k}^{\dagger \lambda} \varphi_{v\eta}^{\mu \zeta \eta} (1 + \varphi^5) \mu_j^\lambda\right)} \\
& + \frac{ig}{2M^2 \sqrt{2} \phi^+ \left(-m_c^\kappa \left(\overrightarrow{v^\lambda} U^{lep^k}_\xi e^k (1 - \varphi^5) \epsilon^\kappa\right) + m_\mu^\lambda \overrightarrow{\mu_j^\lambda} U^{lep^+} (1 + \varphi^5) \epsilon^\kappa\right)} \\
& + \frac{ig}{2M^2 \sqrt{2} \phi^- \left(m_c^\kappa \left(\overrightarrow{v^\lambda} U^{lep^k}_\xi e^k (1 - \varphi^5) \epsilon^\kappa\right) \pm m_\mu^\lambda \overrightarrow{\mu_j^\lambda} U^{lep^+} (1 + \varphi^5) \epsilon^\kappa\right)} - \frac{g}{M} \mathcal{H} \left(\overrightarrow{v^\lambda} v^\lambda\right)
\end{aligned}$$

$$\begin{aligned}
& -\frac{\frac{g}{2m_c^\lambda} \frac{M\mathcal{H}(\vec{\rightarrow} e^\lambda)}{e^\lambda}}{1} + \frac{ig}{2m_v^\lambda} - \frac{ig}{2m_c^\lambda} - \frac{1}{4\vec{\rightarrow} M_{\lambda\kappa}^R(1-\gamma_5)v^\kappa} \\
& + \frac{ig}{2M^2\sqrt{2}\phi^+ \left(-m_d^\kappa \left(\vec{\rightarrow} C_{\lambda\kappa}(1-\varphi^5)d_j^\kappa \right) + m_d^\kappa \left(\vec{\rightarrow} C_{\lambda\kappa}(1-\varphi^5)d_j^\kappa \right) \right)} \\
& + \frac{ig}{2M^2\sqrt{2}\phi^- \left(m_d^\lambda \left(\vec{\rightarrow} C_{\lambda\kappa}^\dagger \wedge_{\theta^*}(1+\varphi^5)\mu_j^\kappa \right) \pm m_d^\lambda \left(\vec{\rightarrow} C_{\lambda\kappa}^\dagger \wedge_{\nabla^*}(1+\varphi^5)\mu_j^\kappa \right) \right)} \\
& - \frac{\frac{g}{2m_\mu^\lambda}}{M\mathcal{H}(\vec{\rightarrow} \mu_j^\lambda)} - \frac{g}{M} \mathcal{H}(\vec{\rightarrow} d_j^\lambda) + \frac{ig}{M} \phi^0 \left(\vec{\rightarrow} \gamma^5 \mu_j^\lambda \right) - \frac{ig}{M} \phi^0 \left(\vec{\rightarrow} \gamma^5 d_j^\lambda \right) \\
& + \vec{\rightarrow} \partial^2 G^a + g_s f^{abc} \partial_\mu \vec{\rightarrow} G^b g_\mu^c + \vec{\rightarrow} (\partial^2 - M^2)\alpha^+ + \vec{\rightarrow} (\partial^2 - M^2)\alpha^- \\
& + \vec{\rightarrow} \alpha^0 \left(\partial^2 - \frac{M^2}{c_\omega^2} \right) \alpha^0 + \vec{\rightarrow} \partial^2 b + igc_\omega W_\mu^+ \left(\partial_\mu \vec{\rightarrow} \alpha^- - \partial_\mu \vec{\rightarrow} \alpha^0 \right) \\
& + ig s_w W_\mu^+ \left(\partial_\mu \vec{\rightarrow} \alpha^- - \partial_\mu \vec{\rightarrow} b^- \right) + igc_\omega W_\mu^- \left(\partial_\mu \vec{\rightarrow} \alpha^+ - \partial_\mu \vec{\rightarrow} \alpha^0 \right) \\
& + ig s_w W_\mu^- \left(\partial_\mu \vec{\rightarrow} \alpha^+ - \partial_\mu \vec{\rightarrow} \alpha^0 \right) + igc_\omega Z_\mu^0 \left(\partial_\mu \vec{\rightarrow} \alpha^+ - \partial_\mu \vec{\rightarrow} \alpha^0 \right) \\
& + ig s_\omega \mathcal{A}_\mu \left(\partial_\mu \vec{\rightarrow} \alpha^+ - \partial_\mu \vec{\rightarrow} \alpha^0 \right) - 1/2gM \left(\frac{\vec{\rightarrow} \alpha^+ \mathcal{H} \hbar \mathbb{R}^4}{h} + \vec{\rightarrow} \alpha^- \mathcal{H} + 1 \right) \\
& - \frac{2c_\omega^2}{2c_\omega igM \left(\vec{\rightarrow} \alpha^0 \phi^+ - \vec{\rightarrow} \alpha^0 \phi^- \right)} + \frac{1}{2c_\omega igM \left(\vec{\rightarrow} \alpha^- \phi^+ - \vec{\rightarrow} \alpha^- \phi^- \right)} \\
& + igMs_\omega \left(\vec{\rightarrow} \alpha^- \phi^+ - \vec{\rightarrow} \alpha^- \phi^- \right) + 1/2igM \left(\vec{\rightarrow} \alpha^+ \phi^0 - \vec{\rightarrow} \alpha^- \phi^0 \right)
\end{aligned}$$

$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

$$\mathcal{L}_{SBS} = (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) - V(\Phi)$$

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$



$$|\Phi|^2 = \Phi^\dagger \Phi = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}$$

$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \longrightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Phi(x) = \frac{1}{\sqrt{2}} e^{i\frac{\vec{\xi}(x)\cdot\vec{\tau}}{v}} \begin{pmatrix} 0 \\ v + \mathbf{h}(x) \end{pmatrix}$$

$$U(\xi) = e^{-i\frac{\vec{\xi}(x)\cdot\vec{\tau}}{v}}$$

$$\begin{aligned} \Phi' &= U(\xi)\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \mathbf{h}(x) \end{pmatrix} \\ \left(\frac{\vec{\tau}\cdot\vec{W}'_\mu}{2}\right) &= U(\xi) \left(\frac{\vec{\tau}\cdot\vec{W}_\mu}{2}\right) U^{-1}(\xi) - \frac{i}{g}(\partial_\mu U(\xi))U^{-1}(\xi) \\ B'_\mu &= B_\mu \end{aligned}$$

$$\mathcal{L} = \mathcal{L}_{bos.} + \mathcal{L}_{ferm.} + \mathcal{L}_{SBS}$$

$$(\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) = \frac{v^2}{8} [g^2 (W_{1\mu}^2 + W_{2\mu}^2) + (gW_{3\mu} - g'B_\mu)^2]$$

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}} (W_\mu^1 \mp W_\mu^2) \\ Z_\mu &= \cos \theta_W W_\mu^3 - \sin \theta_W B_\mu \\ A_\mu &= \sin \theta_W W_\mu^3 + \cos \theta_W B_\mu \end{aligned}$$

$$\tan \theta_W \equiv \frac{g'}{g}$$

$$M_W = \frac{1}{2} g v$$

$$M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$$



$$\mathbf{g} = \frac{e}{\sin \theta_W}$$

$$\mathbf{g}' = \frac{e}{\cos \theta_W}$$

$$m_H^2 = 2\lambda v^2$$

$$\mu \rightarrow \nu_\mu \bar{\nu}_e e$$

$$v = (\sqrt{2}G_F)^{-\frac{1}{2}}$$

$$\mathcal{L}_{YW} = \lambda_e \bar{\ell}_L \Phi \mathbf{e}_R + \lambda_u \bar{q}_L \tilde{\Phi} \mathbf{u}_R + \lambda_d \bar{q}_L \Phi \mathbf{d}_R + \text{h.c.}$$

$$\ell_L = \begin{pmatrix} e \\ \nu_e \end{pmatrix}_L, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$\ell'_L = U(\xi)\ell_L; \quad \mathbf{e}'_R = \mathbf{e}_R$$

$$q'_L = U(\xi)q_L; \quad \mathbf{u}'_R = \mathbf{u}_R; \quad \mathbf{d}'_R = \mathbf{d}$$

$$m_e = \lambda_e \frac{v}{\sqrt{2}}$$

$$m_u = \lambda_u \frac{v}{\sqrt{2}}$$

$$m_d = \lambda_d \frac{v}{\sqrt{2}}$$

O es igual a:

$$\mathcal{L}_{Higgs} = \overline{\left([\partial_\mu + \frac{1}{2}ig_1 B_\mu + \frac{1}{2}ig_2 \mathbf{W}_\mu] \phi \right)} \left([\partial_\mu + \frac{1}{2}ig_1 B_\mu + \frac{1}{2}ig_2 \mathbf{W}_\mu] \phi \right) - \frac{m_H^2 \left(\bar{\phi} \phi - \frac{v^2}{2} \right)^2}{2v^2}$$



$$\begin{aligned}
& \mathcal{L}_{SM}(x) \\
&= -\frac{1}{2\pi\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2\partial_\nu^2g_\mu^ag_\nu^bg_\nu^c} - g_s f^{ab} f_{ab}\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2g_\mu^ag_\nu^bg_\nu^c - \frac{1}{4\pi g_s^2 f^{cd} f_{cd}\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2g_\mu^cg_\nu^dg_\nu^e} \\
&- \partial^\mu W_\mu\partial^\nu W_\nu - M^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ - \frac{1}{2\pi\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2Z_\mu^0Z_\nu^0Z_\mu^0Z_\nu^0} - \frac{1}{2c_m^2 M^2 Z_\mu^0Z_\nu^0Z_\mu^0Z_\nu^0} - \frac{1}{2\partial^\mu A_\nu\partial^\nu A_\mu} \\
&- igc_w (\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2Z_\mu^0Z_\nu^0Z_\mu^0Z_\nu^0 (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+)) - Z_\mu^0 (\partial^\mu\partial_\nu W_\mu^+ W_\nu^- W_\mu^+ W_\nu^-) + Z_\nu^0 (\partial^\nu\partial_\mu W_\nu^+ W_\mu^- W_\nu^+ W_\mu^-) \\
&- igS_w (\partial^\mu A_\nu\partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+)) Z_\mu^0 Z_\nu^0 Z_\mu^0 Z_\nu^0 - A_\mu (\partial^\mu\partial_\nu W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- Z_\mu^0 Z_\nu^0) \\
&+ A_\nu (\partial^\nu\partial_\mu W_\nu^+ W_\mu^- W_\nu^+ W_\mu^- Z_\mu^0 Z_\nu^0) - \frac{1}{2g^2 (\partial^\mu A_\nu\partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ Z_\mu^0 Z_\nu^0 Z_\mu^0 Z_\nu^0))} \\
&+ g^2 c_w^2 (\partial^\mu A_\nu\partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ Z_\mu^0 Z_\nu^0 Z_\mu^0 Z_\nu^0)) \\
&+ g^2 S_w^2 (\partial^\mu A_\nu\partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ Z_\mu^0 Z_\nu^0 Z_\mu^0 Z_\nu^0)) \\
&- g^2 c_w S_w (\partial^\mu A_\nu\partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ Z_\mu^0 Z_\nu^0 Z_\mu^0 Z_\nu^0)) \\
&- \frac{1}{2\pi (\partial H^\mu A_\nu H^\nu H A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ Z_\mu^0 Z_\nu^0 Z_\mu^0 Z_\nu^0 H^\mu H_\nu H^\mu H_\nu))} + \frac{\frac{1}{2\pi(2M^2 H^2 H^3)}}{\frac{d^>em^c\gamma}{GUM_{scw}^2}} - \frac{2g_c^2 M_s^2}{\Psi\Phi\zeta} - \lambda\delta \\
&\frac{\omega}{\Delta\nabla\theta} \\
&\otimes \frac{\prod_{\underline{a}}^{\infty} \prod_{\underline{j}}^i k \left(\frac{\phi_\mu^+ \phi_\nu^- \phi_\mu^- \phi_\nu^+}{\phi_\mu^+ \phi_\nu^+ \phi_\mu^- \phi_\nu^-} \right) \left(\varphi\psi\omega\lambda_\mu^+ \varphi\psi\omega\lambda_\nu^- \varphi\psi\omega\lambda_\mu^- \varphi\psi\omega\lambda_\nu^+ \frac{2\varphi\psi\omega\lambda_\mu^+}{\varphi\psi\omega\lambda_\mu^+} \varphi\psi\omega\lambda_\nu^- \varphi\psi\omega\lambda_\mu^- \varphi\psi\omega\lambda_\nu^+ \frac{1}{\varphi\psi\omega\lambda_\mu} \frac{2\pi\varphi\psi\omega\lambda}{\varphi\psi\omega\lambda} \varphi\psi\omega\lambda_\nu^0 \varphi\psi\omega\lambda_\mu^0 \varphi\psi\omega\lambda_\nu^0 \right)}{2M \sqrt{\frac{2\xi\eta}{\zeta\varepsilon\varepsilon} \frac{\delta\alpha}{\sigma\rho} \frac{1}{\Psi\Omega}} \mathcal{U}} \\
&= \mathcal{L}_{Higgs} \\
&= \left(\partial^\mu\partial_\nu\partial^\nu\partial_\mu + \frac{1}{2ig_1 B^\mu B_\nu B^\nu B_\mu} + \frac{1}{2ig_2 B^\mu B_\nu B^\nu B_\mu} + \frac{1}{2ig_1 W^\mu W_\nu W^\nu W_\mu} + \frac{1}{2ig_2 W^\mu W_\nu W^\nu W_\mu} \right) - m_H^2 \phi' \phi - v^2 / 2v^2 / \tau^2
\end{aligned}$$

$$\partial_i \partial^j \partial_j \partial^i F^{\mu\nu\varphi} F_{\nu\mu\omega}(y)$$

$$\begin{aligned}
& \frac{\partial^\theta \partial_\theta F_\sigma^\rho \gamma \beta}{\varepsilon \varepsilon \partial \pi} \\
&= -\frac{\Delta \nabla}{\tau} + \prod_{\nu}^{\mu} \lambda \prod_{\mu}^{\nu} \lambda H_{iggs} \\
&- W^\mu W_\nu W^\nu W_\mu W_\nu^\mu W_\mu^\nu W_\nu^\mu W_\mu^\nu W - \eta^\theta \eta_\beta \eta_\mu^\sigma \eta_\nu^\alpha \eta / \mathbb{R}^4
\end{aligned}$$

En la que la constante H_{iggs} es igual a:

$$\begin{aligned}
\mathcal{L}_{SM} = & \frac{1}{2\partial_\nu g_\mu^a \partial_\mu g_\nu^b} - g_s f^{abc} \partial_\mu g_\nu^a g_\nu^b g_\nu^c \partial_\nu g_\mu^a g_\mu^b g_\mu^c - \frac{1}{4g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e g_\nu^f g_\nu^g} - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^- - M^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\nu^- \\
& - \frac{1}{2\partial_\nu Z_\mu^0 \partial_\mu Z_\nu^0} - \frac{1}{2c_m^2 M^2 Z_\mu^0 Z_\nu^0} - \frac{1}{2\partial_\mu A_\nu \partial_\nu A_\mu}
\end{aligned}$$



$$\begin{aligned}
& -igc_\omega \left(\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+) - Z_\mu^0 Z_\nu^0 (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-) \right) \\
& \quad - ig s_\omega \left(\partial_\mu \mathcal{A}_\nu \partial_\nu \mathcal{A}_\mu (W_\mu^+ W_\nu^- W_\nu^+ W_\mu^-) - \mathcal{A}_\mu (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-) - \mathcal{A}_\nu (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-) \right) \\
& \quad - \frac{1}{2g^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+} + g^2 c_\omega^2 (Z_\mu^0 W_\mu^+ W_\mu^- Z_\nu^0 W_\nu^+ W_\nu^-) \\
& + g^2 s_\omega^2 (\mathcal{A}_\mu W_\mu^+ W_\mu^- \mathcal{A}_\nu W_\nu^+ W_\nu^-) + g^2 c_\omega s_\omega (\mathcal{A}_\mu Z_\mu^0 (W_\mu^+ W_\mu^-) \mathcal{A}_\nu Z_\nu^0 (W_\nu^+ W_\nu^-) - 2\mathcal{A}_\mu W_\mu^+ W_\mu^- Z_\mu^0 \mathcal{A}_\nu W_\nu^+ W_\nu^- Z_\nu^0) - \frac{1}{2\partial_\mu \mathcal{H} \partial_\nu \mathcal{H}} - 2M^2 \alpha_h \mathcal{H}^2 \\
& \quad - \partial_\mu \phi^+ \partial_\nu \phi^- \partial_\nu \phi^+ \partial_\nu \phi^- - \frac{1}{2\partial_\mu \phi^0 \partial_\nu \phi^0 \partial_\nu \phi^0 \partial_\nu \phi^0} \\
& -\beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g\mathcal{H}} + \frac{1}{2} (\mathcal{H}^2 + \phi^0 \phi^+ \phi^0 \phi^- + 2\phi^0 \phi^+ \phi^0 \phi^-) \right) + \frac{2M^4}{g^2 \alpha_h} - g \alpha_h M (\mathcal{H}^3 + \mathcal{H} \phi^0 \phi^0 + 2\mathcal{H} \phi^+ \phi^-) - 1/8g^2 \alpha_h (\mathcal{H}^4 + (\phi^0)^4) \\
& \quad + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4\mathcal{H}^2 \phi^+ \phi^- + 2(\phi^0)^2 \mathcal{H}^2) - gM W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- \mathcal{H} - \frac{1}{c_\omega^2 Z_\mu^0 Z_\nu^0 \mathcal{H}} \\
& \quad - \frac{1}{2ig} (W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0) - W_\nu^+ (\phi^0 \partial_\nu \phi^- - \phi^- \partial_\nu \phi^0) \\
& \quad - W_\nu^- (\phi^0 \partial_\nu \phi^+ - \phi^+ \partial_\nu \phi^0)) \\
& \quad - \frac{1}{2g} \frac{1}{(W_\mu^+ (\mathcal{H} \partial_\mu \phi^- - \phi^- \partial_\mu \mathcal{H}) W_\mu^- (\mathcal{H} \partial_\mu \phi^+ - \phi^+ \partial_\mu \mathcal{H}) W_\nu^+ (\mathcal{H} \partial_\nu \phi^- - \phi^- \partial_\nu \mathcal{H}) W_\nu^- (\mathcal{H} \partial_\nu \phi^+ - \phi^+ \partial_\nu \mathcal{H}))} \\
& \quad + \frac{1}{c_\omega} \frac{1}{(Z_\mu^0 (\mathcal{H} \partial_\mu \phi^0 - \phi^0 \partial_\mu \mathcal{H}) Z_\nu^0 (\mathcal{H} \partial_\nu \phi^0 - \phi^0 \partial_\nu \mathcal{H}))} + M \left(\frac{1}{c_\omega Z_\mu^0 \partial_\mu \phi^0} + W_\mu^+ \partial_\mu \phi^- + W_\mu^- \partial_\mu \phi^+ \right) \\
& \quad - \frac{igs_\omega^2}{c_\omega M Z_\mu^0 (W^+ \phi^- - W^- \phi^+) ig s_\omega M \mathcal{A}_\mu (W_\mu^+ \phi^- + W_\mu^- \phi^+)} - ig1 - \frac{2c_\omega^2}{2c_\omega Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+)} \\
& \quad + ig s_\omega \mathcal{A}_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - 1/4g^2 W_\mu^+ W_\mu^- (\mathcal{H}^2 + (\phi^0)^2 + 2\phi^+ \phi^-)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{8g^2 1} \\
& \frac{1}{c_\omega^2 Z_\mu^0 Z_\nu^0} \\
& \mathcal{H}^2 + \\
& \phi^0)^2 + 2 \\
& \left(\left(\left(\frac{2s_\omega^2 - 1}{2g^2 s_\omega \mathcal{A}_\mu \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+)} + \frac{1}{2ig^2 s_\omega \mathcal{A}_\mu \mathcal{H} (W_\mu^+ \phi^- + W_\mu^- \phi^+)} - \frac{1}{c_\omega (2c_\omega^2 - 1) Z_\mu^0 \mathcal{A}_\mu \phi^+ \phi^-} - g^2 s_\omega^2 \mathcal{A}_\mu \mathcal{A}_\nu \phi^+ \phi^- + \frac{1}{2ig_s \lambda_{ij}^\alpha \left(\begin{smallmatrix} \rightarrow \sigma^1 & \varphi^\mu & \rightarrow \\ q\sigma^1 & & q\sigma^1 \end{smallmatrix} \right)} g_\mu^\alpha \right) e^\lambda \begin{smallmatrix} \rightarrow \\ \nu \end{smallmatrix} (\varphi \partial + m_\nu^\lambda) \right) v^\lambda \begin{smallmatrix} \rightarrow \\ \mu_j^\lambda \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \nu^\lambda \end{smallmatrix} (\varphi \partial + m_\mu^\lambda) \right) - (\mu_j^\lambda) \right) + \\
& \left(\left(\left(\frac{1}{2g^2 s_\omega \mathcal{A}_\mu \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+)} + \frac{1}{2ig^2 s_\omega \mathcal{A}_\mu \mathcal{H} (W_\mu^+ \phi^- + W_\mu^- \phi^+)} - \frac{1}{c_\omega (2c_\omega^2 - 1) Z_\mu^0 \mathcal{A}_\mu \phi^+ \phi^-} - g^2 s_\omega^2 \mathcal{A}_\mu \mathcal{A}_\nu \phi^+ \phi^- + \frac{1}{2ig_s \lambda_{ij}^\alpha \left(\begin{smallmatrix} \rightarrow \sigma^1 & \varphi^\mu & \rightarrow \\ q\sigma^1 & & q\sigma^1 \end{smallmatrix} \right)} g_\mu^\alpha \right) e^\lambda \begin{smallmatrix} \rightarrow \\ \nu \end{smallmatrix} (\varphi \partial + m_\nu^\lambda) \right) v^\lambda \begin{smallmatrix} \rightarrow \\ \mu_j^\lambda \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \nu^\lambda \end{smallmatrix} (\varphi \partial + m_\mu^\lambda) \right) - (\mu_j^\lambda) \right) +
\end{aligned}$$



$$\begin{aligned}
& (\varphi\partial + m_d^\lambda)d_j^\lambda + ig_{s_\omega}\mathcal{A}_\mu \left(-\left(\overrightarrow{e^\lambda} \varphi^\mu e^\lambda\right) + \frac{2}{3\left(\overrightarrow{\mu_j^\lambda} \varphi^\mu \mu_j^\lambda\right)} - \frac{1}{3\left(\overrightarrow{d_j^\lambda} \varphi^\mu d_j^\lambda\right)} + \frac{ig}{4c_\omega Z_\mu^0\left(\overrightarrow{v^\lambda} \varphi^\mu(1 + \varphi^5)v^\lambda\right)} \right. \\
& + \left(\overrightarrow{e^\lambda} \varphi^\mu(4s_\omega^2 - 1 - \varphi^5)e^\lambda\right) + \left(\overrightarrow{d_j^\lambda} \varphi^\mu\left(\frac{4}{3s_\omega^2} - 1 - \varphi^5\right)d_j^\lambda\right) + \left(\overrightarrow{\mu_j^\lambda} \varphi^\mu\left(1 - \frac{8}{3s_\omega^2} + \varphi^5\right)\mu_j^\lambda\right) \\
& \left. + \left(\frac{ig}{\sqrt[2]{2W_\mu^+}\left(\overrightarrow{v^\lambda} \varphi^\mu(1 + \varphi^5)\right)U^{lep^k}_\xi e^k}\right) + \left(\overrightarrow{\mu_j^\lambda} \varphi^\mu(1 + \varphi^5)c_{k\lambda}d_j^k\right) \right) \\
& + \frac{ig}{\sqrt[2]{2W_\mu^-}\left(\overrightarrow{e^\lambda} U^{lep^\dagger} \frac{\kappa\varphi}{\rho\varpi} - \triangle \Pi \oplus \oplus \gamma \otimes \frac{\prod_{ij}^{\mu} ijk}{\otimes \sigma} \Omega\Psi\Phi\Delta(1 + \varphi^5)v^\lambda + \overrightarrow{d_j^k} C_{\kappa}^{\dagger\lambda} \varphi_{v\eta}^{\mu\kappa\eta}(1 + \varphi^5)\mu_j^\lambda\right)} \\
& + \frac{ig}{2M\sqrt[2]{2\phi^+}\left(-m_c^k\left(\overrightarrow{v^\lambda} U^{lep^k}_\xi e^k(1 - \varphi^5)\epsilon^\kappa\right) + m_\mu^\lambda \overrightarrow{U^{lep^\dagger}}(1 + \varphi^5)\epsilon^\kappa\right)} \\
& + \frac{ig}{2M\sqrt[2]{2\phi^-}\left(m_c^k\left(\overrightarrow{v^\lambda} U^{lep^k}_\xi e^k(1 - \varphi^5)\epsilon^\kappa\right) \pm m_\mu^\lambda \overrightarrow{U^{lep^\dagger}}(1 + \varphi^5)\epsilon^\kappa\right)} - \frac{g}{M} \mathcal{H}\left(\overrightarrow{v^\lambda}\right) \\
& - \frac{\frac{g}{2m_c^\lambda}}{\frac{M\mathcal{H}\left(\overrightarrow{e^\lambda}\right)}{1}} + \frac{\frac{ig}{2m_v^\lambda}}{4\overrightarrow{v^\lambda} M_{\lambda\kappa}^R(1 - \gamma_5)v^\kappa} + \frac{\frac{ig}{2m_c^\lambda}}{M\phi^0\left(\overrightarrow{v^\lambda} \gamma^5 v^\lambda\right)} - \frac{\frac{ig}{2m_c^\lambda}}{M\phi^0\left(\overrightarrow{e^\lambda} \gamma^5 e^\lambda\right)} - \frac{1}{4\overrightarrow{v^\kappa} M_{\lambda\kappa}^R(1 - \gamma_5)\overrightarrow{v^\kappa}} \\
& + \frac{ig}{2M\sqrt[2]{2\phi^+}\left(-m_d^k\left(\overrightarrow{\mu_j^\lambda} C_{\lambda\kappa}(1 - \varphi^5)d_j^k\right) + m_d^k\left(\overrightarrow{\mu_j^\lambda} C_{\lambda\kappa}(1 - \varphi^5)d_j^k\right)\right)} \\
& + \frac{ig}{2M\sqrt[2]{2\phi^-}\left(m_d^\lambda\left(\overrightarrow{d_j^\lambda} C_{\lambda\kappa}^\dagger \wedge_{\theta^*}(1 + \varphi^5)\mu_j^\kappa\right) \pm m_d^\lambda\left(\overrightarrow{d_j^\lambda} C_{\lambda\kappa}^\dagger \wedge_{v^*}(1 + \varphi^5)\mu_j^\kappa\right)\right)} - \frac{g}{2m_\mu^\lambda} \mathcal{M}\mathcal{H}\left(\overrightarrow{\mu_j^\lambda}\right) \\
& - \frac{g}{2m_d^\lambda} \mathcal{H}\left(\overrightarrow{d_j^\lambda}\right) + \frac{ig}{M} \phi^0\left(\overrightarrow{\mu_j^\lambda} \gamma^5 \mu_j^\lambda\right) - \frac{ig}{M} \phi^0\left(\overrightarrow{d_j^\lambda} \gamma^5 d_j^\lambda\right) + \overrightarrow{\partial^2} G^a + g_s f^{abc} \partial_\mu \overrightarrow{G^b} G_\mu^c \\
& + \overrightarrow{\alpha^+} (\partial^2 - M^2)\alpha^+ + \overrightarrow{\alpha^-} (\partial^2 - M^2)\alpha^- + \overrightarrow{\alpha^0} \left(\partial^2 - \frac{M^2}{c_\omega^2}\right)\alpha^0 + \overrightarrow{\partial^2} b + igc_\omega W_\mu^+ \left(\partial_\mu \overrightarrow{\alpha^+} \overrightarrow{\alpha^-} - \partial_\mu \overrightarrow{\alpha^-} \overrightarrow{\alpha^+}\right) \\
& + ig_s W_\mu^+ \left(\partial_\mu \overrightarrow{\alpha^-} \overrightarrow{b} - \partial_\mu \overrightarrow{b} \overrightarrow{\alpha^-}\right) + igc_\omega W_\mu^- \left(\partial_\mu \overrightarrow{\alpha^+} \overrightarrow{\alpha^0} - \partial_\mu \overrightarrow{\alpha^0} \overrightarrow{\alpha^+}\right) + ig_s W_\mu^- \left(\partial_\mu \overrightarrow{\alpha^+} \overrightarrow{\alpha^0} - \partial_\mu \overrightarrow{\alpha^0} \overrightarrow{\alpha^+}\right) \\
& + igc_\omega Z_\mu^0 \left(\partial_\mu \overrightarrow{\alpha^+} \overrightarrow{\alpha^-} - \partial_\mu \overrightarrow{\alpha^-} \overrightarrow{\alpha^+}\right) + ig_s \mathcal{A}_\mu \left(\partial_\mu \overrightarrow{\alpha^+} \overrightarrow{\alpha^-} - \partial_\mu \overrightarrow{\alpha^-} \overrightarrow{\alpha^+}\right) - 1/2gM \left(\frac{\overrightarrow{\alpha^+} \mathcal{H} \hbar \mathbb{R}^4}{h} + \overrightarrow{\alpha^-} \mathcal{H}\right) \\
& + 1 - \frac{2c_\omega^2}{2c_\omega igM \left(\overrightarrow{\alpha^+} \phi^+ - \overrightarrow{\alpha^-} \phi^-\right)} + \frac{1}{2c_\omega igM \left(\overrightarrow{\alpha^0} \phi^+ - \overrightarrow{\alpha^+} \phi^-\right)} \\
& + igMs_\omega \left(\overrightarrow{\alpha^0} \phi^+ - \overrightarrow{\alpha^+} \phi^-\right) + 1/2igM \left(\overrightarrow{\alpha^+} \phi^0 - \overrightarrow{\alpha^-} \phi^0\right)
\end{aligned}$$

$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

$$\mathcal{L}_{SBS} = (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) - V(\Phi)$$

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$|\Phi|^2 = \Phi^\dagger \Phi = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}$$

$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \longrightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Phi(x) = \frac{1}{\sqrt{2}} e^{i \frac{\vec{\xi}(x) \cdot \vec{\tau}}{v}} \begin{pmatrix} 0 \\ v + \mathbf{h}(x) \end{pmatrix}$$

$$U(\xi) = e^{-i \frac{\vec{\xi}(x) \cdot \vec{\tau}}{v}}$$

$$\begin{aligned} \Phi' &= U(\xi) \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \mathbf{h}(x) \end{pmatrix} \\ \left(\frac{\vec{\tau} \cdot \vec{W}'_\mu}{2} \right) &= U(\xi) \left(\frac{\vec{\tau} \cdot \vec{W}_\mu}{2} \right) U^{-1}(\xi) - \frac{i}{g} (\partial_\mu U(\xi)) U^{-1}(\xi) \\ B'_\mu &= B_\mu \end{aligned}$$

$$\mathcal{L} = \mathcal{L}_{bos.} + \mathcal{L}_{ferm.} + \mathcal{L}_{SBS}$$

$$(\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) = \frac{v^2}{8} [g^2 (W_{1\mu}^2 + W_{2\mu}^2) + (gW_{3\mu} - g'B_\mu)^2]$$

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}} (W_\mu^1 \mp W_\mu^2) \\ Z_\mu &= \cos \theta_W W_\mu^3 - \sin \theta_W B_\mu \\ A_\mu &= \sin \theta_W W_\mu^3 + \cos \theta_W B_\mu \end{aligned}$$

$$\tan \theta_W \equiv \frac{g'}{g}$$



$$M_W = \frac{1}{2} g v$$

$$M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$$

$$g = \frac{e}{\sin \theta_W}$$

$$g' = \frac{e}{\cos \theta_W}$$

$$m_H^2 = 2\lambda v^2$$

$$\mu \rightarrow \nu_\mu \bar{\nu}_e e$$

$$v = (\sqrt{2} G_F)^{-\frac{1}{2}}$$

$$\mathcal{L}_{YW} = \lambda_e \bar{\ell}_L \Phi e_R + \lambda_u \bar{q}_L \tilde{\Phi} u_R + \lambda_d \bar{q}_L \Phi d_R + \text{h.c.}$$

$$\ell_L = \begin{pmatrix} e \\ \nu_e \end{pmatrix}_L, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$\ell'_L = U(\xi) \ell_L; \quad e'_R = e_R$$

$$q'_L = U(\xi) q_L; \quad u'_R = u_R; \quad d'_R = d$$

$$m_e = \lambda_e \frac{v}{\sqrt{2}}$$

$$m_u = \lambda_u \frac{v}{\sqrt{2}}$$

$$m_d = \lambda_d \frac{v}{\sqrt{2}}$$

O es igual a:

$$\mathcal{L}_{Higgs} = \overline{\left([\partial_\mu + \frac{1}{2} i g_1 B_\mu + \frac{1}{2} i g_2 \mathbf{W}_\mu] \phi \right)} \left([\partial_\mu + \frac{1}{2} i g_1 B_\mu + \frac{1}{2} i g_2 \mathbf{W}_\mu] \phi \right) - \frac{m_H^2 \left(\bar{\phi} \phi - \frac{v^2}{2} \right)^2}{2v^2}$$



$$\begin{aligned}
& \mathcal{L}_{SM}(y) \\
&= -\frac{1}{2\pi\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2g_\mu^ag_\nu^bg_\nu^c} - g_s f^{ab} f_{ab} \partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2g_\mu^ag_\nu^bg_\nu^c - \frac{1}{4\pi g_s^2 f^{cd} f_{cd} \partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2g_\mu^ag_\nu^bg_\nu^c} - \partial^\mu W_\mu \partial^\nu W_\nu \\
&- M^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ - \frac{1}{2\pi\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2Z_\mu^0Z_\nu^0Z_\nu^0} - \frac{1}{2c_m^2 M^2 Z_\mu^0 Z_\nu^0 Z_\nu^0} - \frac{1}{2\partial^\mu A_\nu \partial^\nu A_\mu} \\
&- igc_w (\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2Z_\mu^0Z_\nu^0Z_\nu^0(W_\mu^+W_\nu^-W_\mu^-W_\nu^+)) - Z_\mu^0(\partial^\mu\partial_\nu W_\mu^+W_\nu^-W_\mu^+W_\nu^-) + Z_\nu^0(\partial^\nu\partial_\mu W_\nu^+W_\mu^-W_\nu^+W_\mu^-) \\
&- igS_w(\partial^\mu A_\nu\partial^\nu A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+))Z_\mu^0Z_\nu^0Z_\nu^0 - A_\mu(\partial^\mu\partial_\nu W_\mu^+W_\nu^-W_\mu^+W_\nu^-Z_\mu^0Z_\nu^0) + A_\nu(\partial^\nu\partial_\mu W_\nu^+W_\mu^-W_\nu^+W_\mu^-Z_\nu^0Z_\mu^0) \\
&- \frac{1}{2g^2(\partial^\mu A_\nu\partial^\nu A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+Z_\mu^0Z_\nu^0Z_\nu^0))} + g^2c_w^2(\partial^\mu A_\nu\partial^\nu A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+Z_\mu^0Z_\nu^0Z_\nu^0)) \\
&+ g^2S_w^2(\partial^\mu A_\nu\partial^\nu A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+Z_\mu^0Z_\nu^0Z_\nu^0)) - g^2c_wS_w(\partial^\mu A_\nu\partial^\nu A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+Z_\mu^0Z_\nu^0Z_\nu^0)) \\
&- \frac{1}{2\pi(\partial H^\mu A H_\nu H^\nu H A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+Z_\mu^0Z_\nu^0Z_\nu^0 H^\mu H_\nu H_\nu H_\mu))} + \frac{\frac{1}{2\pi(2M^2H^2H^3)}}{\frac{d^>em^c\gamma}{GUM_{scw}^2}} - \frac{2g_e^2M_s^2}{\Psi\Phi\zeta} - \lambda\theta \\
&\otimes \frac{\omega}{\Delta\nabla\theta} \\
&/\prod_{\pm}^{\dagger} \prod_{\pm}^{\dagger} \infty \prod_{\pm}^{\dagger} k \left(\frac{\phi_{\mu}^{\pm}\phi_{\nu}^{\mp}\phi_{\mu}^{\mp}\phi_{\nu}^{\pm}}{\phi_{\mu}^{\pm}\phi_{\nu}^{\pm}\phi_{\mu}^{\pm}\phi_{\nu}^{\pm}} \right) (\varphi\psi\omega\lambda^+\varphi\psi\omega\lambda^-\varphi\psi\omega\lambda^-\varphi\psi\omega\lambda^+ \frac{2\varphi\psi\omega\lambda^{\mu}}{\varphi\psi\omega\lambda^+} \varphi\psi\omega\lambda^{\nu} \varphi\psi\omega\lambda^{\mu} \varphi\psi\omega\lambda^{\nu} \frac{1/2\pi\varphi\psi\omega\lambda^0}{\varphi\psi\omega\lambda^{\mu}} \varphi\psi\omega\lambda^0 \varphi\psi\omega\lambda^0 \varphi\psi\omega\lambda^0) \\
&/2M \sqrt{\frac{2\xi\eta}{\xi\varepsilon\varepsilon}} \frac{\delta\alpha}{\alpha\sigma\rho} / \Psi\Omega U = \mathcal{L}_{Higgs} = \left(\partial^\mu\partial_\nu\partial^\nu\partial_\mu + \frac{1}{2ig_1B^\mu B_\nu B^\nu B_\mu} + \frac{1}{2jg_2B^\mu B_\nu B^\nu B_\mu} + \frac{1}{2ig_1W^\mu W_\nu W^\nu W_\mu} + \frac{1}{2jg_2W^\mu W_\nu W^\nu W_\mu} \right) - m_H^2\phi^+\phi^- - v^2/2v^2 \\
&/\tau^2
\end{aligned}$$

$$\begin{aligned}
& \partial_i\partial^j\partial_j\partial^i F^{\mu\nu\varphi} F_{\nu\mu\omega}(z) \\
&= \frac{\partial^\theta\partial_\theta F_\sigma^\rho\gamma\beta}{\frac{\varepsilon\varepsilon\vartheta\pi}{\Delta\nabla}} + \prod_{\nu}^{\mu} \lambda \prod_{\mu}^{\nu} \lambda H_{igg_s} \\
&- W^\mu W_\nu W^\nu W_\mu W_\nu^\mu W_\mu^\nu W_\mu^\nu W - \eta^\theta \eta_\beta \eta_{\phi\nu}^{\sigma\mu} \alpha_\Omega \eta / \mathbb{R}^4
\end{aligned}$$

En la que la constante H_{igg_s} es igual a:

$$\begin{aligned}
\mathcal{L}_{SM} &= \frac{1}{2\partial_\nu g_\mu^a \partial_\mu g_\nu^b} - g_s f^{abc} \partial_\mu g_\nu^a g_\nu^b g_\nu^c \partial_\nu g_\mu^a g_\mu^b g_\mu^c - \frac{1}{4g_s^2 f^{abc} f^{ade} g_\mu^b g_\mu^c g_\mu^d g_\mu^e g_\nu^b g_\nu^c g_\nu^d g_\nu^e} - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^- - M^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- \\
&- \frac{1}{2\partial_\nu Z_\mu^0 \partial_\mu Z_\nu^0} - \frac{1}{2c_m^2 M^2 Z_\mu^0 Z_\nu^0} - \frac{1}{2\partial_\mu \mathcal{A}_\nu \partial_\nu \mathcal{A}_\mu} \\
&- igc_w (\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+) - Z_\mu^0 Z_\nu^0 (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-)) \\
&- igS_w (\partial_\mu \mathcal{A}_\nu \partial_\nu \mathcal{A}_\mu (W_\mu^+ W_\nu^- W_\nu^+ W_\mu^-) - \mathcal{A}_\mu (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-) - \mathcal{A}_\nu (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-)) \\
&- \frac{1}{2g^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+} + g^2 c_w^2 (Z_\mu^0 W_\mu^+ W_\mu^- Z_\nu^0 W_\nu^+ W_\nu^-) \\
&+ g^2 S_w^2 (\mathcal{A}_\mu W_\mu^+ W_\mu^- \mathcal{A}_\nu W_\nu^+ W_\nu^-) + g^2 c_w S_w (\mathcal{A}_\mu Z_\mu^0 (W_\mu^+ W_\mu^-) \mathcal{A}_\nu Z_\nu^0 (W_\nu^+ W_\nu^-) - 2\mathcal{A}_\mu W_\mu^+ W_\mu^- Z_\mu^0 \mathcal{A}_\nu W_\nu^+ W_\nu^- Z_\nu^0) - \frac{1}{2\partial_\mu \mathcal{H} \partial_\nu \mathcal{H}} - 2M^2 \alpha_h \mathcal{H}^2 \\
&- \partial_\mu \phi^+ \partial_\nu \phi^- \partial_\nu \phi^+ \partial_\mu \phi^- - \frac{1}{2\partial_\mu \phi^0 \partial_\nu \phi^0 \partial_\nu \phi^0 \partial_\mu \phi^0}
\end{aligned}$$



$$\begin{aligned}
& -\beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g\mathcal{H}} + \frac{1}{2}(\mathcal{H}^2 + \phi^0\phi^+\phi^0\phi^- + 2\phi^0\phi^+\phi^0\phi^-) \right) + \frac{2M^4}{g^2\alpha_h} - g\alpha_h M(\mathcal{H}^3 + \mathcal{H}\phi^0\phi^0 + 2\mathcal{H}\phi^+\phi^-) - 1/8g^2\alpha_h(\mathcal{H}^4 + (\phi^0)^4 \\
& + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4\mathcal{H}^2\phi^+\phi^- + 2(\phi^0)^2\mathcal{H}^2) - gMW_\mu^+W_\mu^-W_v^+W_v^-\mathcal{H} - \frac{1}{c_\omega^2Z_\mu^0Z_v^0\mathcal{H}} \\
& - \frac{1}{2ig} (W_\mu^+(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W_\mu^-(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0) - W_v^+(\phi^0\partial_v\phi^- - \phi^-\partial_v\phi^0) \\
& - W_v^-(\phi^0\partial_v\phi^+ - \phi^+\partial_v\phi^0)) \\
& - \frac{1}{2g} (W_\mu^+(\mathcal{H}\partial_\mu\phi^- - \phi^-\partial_\mu\mathcal{H})W_\mu^-(\mathcal{H}\partial_\mu\phi^+ - \phi^+\partial_\mu\mathcal{H})W_v^+(\mathcal{H}\partial_v\phi^- - \phi^-\partial_v\mathcal{H})W_v^-(\mathcal{H}\partial_v\phi^+ - \phi^+\partial_v\mathcal{H})) \\
& + \frac{1}{c_\omega} \left(\frac{1}{Z_\mu^0(\mathcal{H}\partial_\mu\phi^0 - \phi^0\partial_\mu\mathcal{H})Z_v^0(\mathcal{H}\partial_v\phi^0 - \phi^0\partial_v\mathcal{H})} + M \left(\frac{1}{c_\omega Z_\mu^0\partial_\mu\phi^0} + W_\mu^+\partial_\mu\phi^- + W_\mu^-\partial_\mu\phi^+ \right) \right) \\
& - \frac{igs_\omega^2}{c_\omega MZ_\mu^0(W^+\phi^- - W^-\phi^+)ig s_\omega M\mathcal{A}_\mu(W_\mu^+\phi^- + W_\mu^-\phi^+) - ig1 - \frac{2c_\omega^2}{2c_\omega Z_\mu^0(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+)}} \\
& + ig s_\omega \mathcal{A}_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - 1/4g^2W_\mu^+W_\mu^-(\mathcal{H}^2 + (\phi^0)^2 + 2\phi^+\phi^-) \\
& - \frac{1}{8g^2\mathbb{1}} \\
& - \frac{1}{c_\omega^2Z_\mu^0Z_v^0} \\
& - \frac{1}{c_\omega^2Z_\mu^0Z_v^0} \left(\left(\left(\left(\left(\left(\frac{2s_\omega^2 - 1}{2g^2s_\omega\mathcal{A}_\mu\phi^0(W_\mu^+\phi^- + W_\mu^-\phi^+)} + \frac{1}{2ig^2s_\omega\mathcal{A}_\mu\mathcal{H}(W_\mu^+\phi^- + W_\mu^-\phi^+)} - \frac{1}{c_\omega(2c_\omega^2 - 1)Z_\mu^0\mathcal{A}_\mu\phi^+\phi^-} - g^2s_\omega^2\mathcal{A}_\mu\mathcal{A}_\nu\phi^+\phi^- + \frac{1}{2ig_s\lambda_{ij}^a} \left(\frac{1}{q^\sigma} \varphi^\mu \right)_{q^\sigma} \right) g_\mu^a \right) \right) \right) \right) e^\lambda \rightarrow_{v^\lambda} (\varphi\partial + m_\mu^\lambda) v^\lambda \rightarrow_{\mu_j^\lambda} (\varphi\partial + m_\mu^\lambda) \right) - (\mu_j^\lambda) \\
& - \frac{1}{d_j^\lambda} + \\
& (\varphi\partial + m_d^\lambda)d_j^\lambda + ig s_\omega \mathcal{A}_\mu \left(- \left(\frac{1}{e^\lambda} \varphi^\mu e^\lambda \right) + \frac{2}{3 \left(\frac{1}{\mu_j^\lambda} \varphi^\mu \mu_j^\lambda \right)} - \frac{1}{3 \left(\frac{1}{d^\lambda} \varphi^\mu d^\lambda \right)} + \frac{ig}{4c_\omega Z_\mu^0 \left(\frac{1}{v^\lambda} \varphi^\mu (1 + \varphi^5) v^\lambda \right)} \right) \\
& + \left(\frac{1}{e^\lambda} \varphi^\mu (4s_\omega^2 - 1 - \varphi^5) e^\lambda \right) + \left(\frac{1}{d_j^\lambda} \varphi^\mu \left(\frac{4}{3s_\omega^2} - 1 - \varphi^5 \right) d_j^\lambda \right) + \left(\frac{1}{\mu_j^\lambda} \varphi^\mu \left(1 - \frac{8}{3s_\omega^2} + \varphi^5 \right) \mu_j^\lambda \right) \\
& + \left(\frac{ig}{\sqrt{2}W_\mu^+ \left(\frac{1}{v^\lambda} \varphi^\mu (1 + \varphi^5) \right) U^{lep^k} \xi^k} \right) + \left(\frac{1}{\mu_j^\lambda} \varphi^\mu (1 + \varphi^5) c_{k\lambda} d_j^k \right) \\
& + \frac{ig}{\sqrt{2}W_\mu^- \left(\frac{1}{e^\kappa} U^{lep^\dagger} \frac{\kappa\rho}{\xi} - \Pi_{\ominus\oplus\oplus\ominus}^{\oplus\ominus\oplus\ominus} \otimes \frac{\prod_{\nu}^{\mu} ij k}{\otimes_{\sigma}} \Omega\Psi\Phi\Delta(1 + \varphi^5)v^\lambda + \frac{1}{d_j^\kappa} C_{\kappa}^{\lambda} \varphi_{\nu\eta}^{\mu\zeta\eta} (1 + \varphi^5) \mu_j^\lambda \right)} \\
& + \frac{ig}{2M\sqrt{2}\phi^+ \left(-m_c^\kappa \left(\frac{1}{v^\lambda} U^{lep^k} \xi^k (1 - \varphi^5) \epsilon^\kappa \right) + m_\mu^\lambda \rightarrow_{\mu_j^\lambda} U^{lep^\dagger} (1 + \varphi^5) \epsilon^\kappa \right)} \\
& + \frac{ig}{2M\sqrt{2}\phi^- \left(m_c^\kappa \left(\frac{1}{v^\lambda} U^{lep^k} \xi^k (1 - \varphi^5) \epsilon^\kappa \right) \pm m_\mu^\lambda \rightarrow_{\mu_j^\lambda} U^{lep^\dagger} (1 + \varphi^5) \epsilon^\kappa \right)} - \frac{g}{M} \mathcal{H} \left(\frac{1}{v^\lambda} v^\lambda \right)
\end{aligned}$$



$$\begin{aligned}
& - \frac{\frac{g}{2m_c^\lambda}}{\frac{M\mathcal{H}\left(\begin{smallmatrix} \rightarrow e^\lambda \\ e^\lambda \end{smallmatrix}\right)}{1}} + \frac{\frac{ig}{2m_v^\lambda}}{M\phi^0\left(\begin{smallmatrix} \rightarrow \gamma^5 v^\lambda \\ v^\lambda \end{smallmatrix}\right)} - \frac{\frac{ig}{2m_c^\lambda}}{M\phi^0\left(\begin{smallmatrix} \rightarrow \gamma^5 e^\lambda \\ e^\lambda \end{smallmatrix}\right)} - \frac{1}{4\begin{smallmatrix} \rightarrow M_{\lambda\kappa}^R(1-\gamma_5) \\ v^\kappa \end{smallmatrix}} \rightarrow \\
& + \frac{2M^2\sqrt{2}\phi^+ \left(-m_d^\kappa \left(\begin{smallmatrix} \rightarrow C_{\lambda\kappa}(1-\varphi^5)d_j^\kappa \\ \mu_j^\lambda \end{smallmatrix} \right) + m_d^\kappa \left(\begin{smallmatrix} \rightarrow C_{\lambda\kappa}(1-\varphi^5)d_j^\kappa \\ \mu_j^\lambda \end{smallmatrix} \right) \right)}{ig} \\
& + \frac{2M^2\sqrt{2}\phi^- \left(m_d^\lambda \left(\begin{smallmatrix} \rightarrow C_{\lambda\kappa}^\dagger \wedge_{\theta^*}(1+\varphi^5)\mu_j^\kappa \\ d_j^\dagger \end{smallmatrix} \right) \pm m_d^\lambda \left(\begin{smallmatrix} \rightarrow C_{\lambda\kappa}^\dagger \wedge_{\nu^*}(1+\varphi^5)\mu_j^\kappa \\ d_j^\dagger \end{smallmatrix} \right) \right)}{ig} - \frac{\frac{g}{2m_\mu^\lambda}}{M\mathcal{H}\left(\begin{smallmatrix} \rightarrow \mu_j^\lambda \\ \mu_j^\lambda \end{smallmatrix}\right)} \\
& - \frac{\frac{g}{2m_d^\lambda}}{M} \mathcal{H}\left(\begin{smallmatrix} \rightarrow d_j^\lambda \\ d_j^\lambda \end{smallmatrix}\right) + \frac{\frac{ig}{2m_\mu^\lambda}}{M} \phi^0\left(\begin{smallmatrix} \rightarrow \gamma^5 \mu_j^\lambda \\ \mu_j^\lambda \end{smallmatrix}\right) - \frac{\frac{ig}{2m_d^\lambda}}{M} \phi^0\left(\begin{smallmatrix} \rightarrow \gamma^5 d_j^\lambda \\ d_j^\lambda \end{smallmatrix}\right) + \begin{smallmatrix} \rightarrow \partial^2 G^a + g_s f^{abc} \partial_\mu \\ c^a \end{smallmatrix} \rightarrow G^b g_\mu^c \\
& + \begin{smallmatrix} \rightarrow (\partial^2 - M^2)\alpha^+ \\ \alpha^+ \end{smallmatrix} + \begin{smallmatrix} \rightarrow (\partial^2 - M^2)\alpha^- \\ \alpha^- \end{smallmatrix} + \begin{smallmatrix} \rightarrow \left(\partial^2 - \frac{M^2}{c_\omega^2}\right)\alpha^0 \\ \alpha^0 \end{smallmatrix} + \begin{smallmatrix} \rightarrow \partial^2 b \\ b \end{smallmatrix} + igc_\omega W_\mu^+ \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^0 \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^- \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^- \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^0 \end{smallmatrix} \right) \\
& + ig s_\omega W_\mu^+ \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ b \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^- \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ a \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ b^- \end{smallmatrix} \right) + ig c_\omega W_\mu^- \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^+ \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^0 \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^0 \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^+ \end{smallmatrix} \right) + ig s_\omega W_\mu^- \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^+ \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^0 \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^0 \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^+ \end{smallmatrix} \right) \\
& + ig c_\omega Z_\mu^0 \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^+ \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^- \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^- \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^+ \end{smallmatrix} \right) + ig s_\omega \mathcal{A}_\mu \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^+ \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^- \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^- \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^+ \end{smallmatrix} \right) - 1/2gM \left(\frac{\begin{smallmatrix} \rightarrow \alpha^+ \mathcal{H} \hbar \mathbb{R}^4 \\ \alpha^+ \end{smallmatrix}}{h} + \begin{smallmatrix} \rightarrow \alpha^- \mathcal{H} \\ \alpha^- \end{smallmatrix} \right) \\
& + 1 - \frac{2c_3^2}{2c_\omega igM \left(\begin{smallmatrix} \rightarrow a^0 \phi^+ \\ \alpha^+ \end{smallmatrix} - \begin{smallmatrix} \rightarrow a^0 \phi^- \\ \alpha^- \end{smallmatrix} \right)} + \frac{1}{2c_\omega igM \left(\begin{smallmatrix} \rightarrow a^- \phi^+ \\ \alpha^0 \end{smallmatrix} - \begin{smallmatrix} \rightarrow a^+ \phi^- \\ \alpha^0 \end{smallmatrix} \right)} \\
& + igM s_\omega \left(\begin{smallmatrix} \rightarrow a^- \phi^+ \\ \alpha^0 \end{smallmatrix} - \begin{smallmatrix} \rightarrow a^+ \phi^- \\ \alpha^0 \end{smallmatrix} \right) + 1/2igM \left(\begin{smallmatrix} \rightarrow a^+ \phi^0 \\ \alpha^+ \end{smallmatrix} - \begin{smallmatrix} \rightarrow a^- \phi^0 \\ \alpha^- \end{smallmatrix} \right)
\end{aligned}$$

$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

$$\mathcal{L}_{SBS} = (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) - V(\Phi)$$

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$|\Phi|^2 = \Phi^\dagger \Phi = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}$$

$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \longrightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Phi(x) = \frac{1}{\sqrt{2}} e^{i\frac{\vec{\xi}(x)\cdot\vec{\tau}}{v}} \begin{pmatrix} 0 \\ v + \mathbf{h}(x) \end{pmatrix}$$



$$U(\xi) = e^{-i \frac{\vec{\xi}(x) \cdot \vec{\tau}}{v}}$$

$$\begin{aligned} \Phi' &= U(\xi) \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \mathbf{h}(x) \end{pmatrix} \\ \left(\frac{\vec{\tau} \vec{W}'_\mu}{2} \right) &= U(\xi) \left(\frac{\vec{\tau} \vec{W}_\mu}{2} \right) U^{-1}(\xi) - \frac{i}{g} (\partial_\mu U(\xi)) U^{-1}(\xi) \\ B'_\mu &= B_\mu \end{aligned}$$

$$\mathcal{L} = \mathcal{L}_{bos.} + \mathcal{L}_{ferm.} + \mathcal{L}_{SBS}$$

$$(\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) = \frac{v^2}{8} [g^2 (W_{1\mu}^2 + W_{2\mu}^2) + (gW_{3\mu} - g' B_\mu)^2]$$

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}} (W_\mu^1 \mp W_\mu^2) \\ Z_\mu &= \cos \theta_W W_\mu^3 - \sin \theta_W B_\mu \\ A_\mu &= \sin \theta_W W_\mu^3 + \cos \theta_W B_\mu \end{aligned}$$

$$\tan \theta_W \equiv \frac{g'}{g}$$

$$M_W = \frac{1}{2} g v$$

$$M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$$

$$g = \frac{e}{\sin \theta_W}$$

$$g' = \frac{e}{\cos \theta_W}$$

$$m_H^2 = 2\lambda v^2$$

$$\mu \rightarrow \nu_\mu \bar{\nu}_e e$$

$$v = (\sqrt{2} G_F)^{-\frac{1}{2}}$$

$$\mathcal{L}_{YW} = \lambda_e \bar{\ell}_L \Phi e_R + \lambda_u \bar{q}_L \tilde{\Phi} u_R + \lambda_d \bar{q}_L \Phi d_R + \text{h.c.}$$



$$\ell_L = \begin{pmatrix} e \\ \nu_e \end{pmatrix}_L, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$\begin{aligned} \ell'_L &= U(\xi)\ell_L; & e'_R &= e_R \\ q'_L &= U(\xi)q_L; & u'_R &= u_R; d'_R = d \end{aligned}$$

$$m_e = \lambda_e \frac{v}{\sqrt{2}}$$

$$m_u = \lambda_u \frac{v}{\sqrt{2}}$$

$$m_d = \lambda_d \frac{v}{\sqrt{2}}$$

O es igual a:

$$\mathcal{L}_{Higgs} = \overline{\left([\partial_\mu + \frac{1}{2}ig_1 B_\mu + \frac{1}{2}ig_2 \mathbf{W}_\mu] \phi \right)} \left([\partial_\mu + \frac{1}{2}ig_1 B_\mu + \frac{1}{2}ig_2 \mathbf{W}_\mu] \phi \right) - \frac{m_H^2 \left(\bar{\phi} \phi - \frac{v^2}{2} \right)^2}{2v^2}$$



$$\begin{aligned}
& \mathcal{L}_{SM}(z) \\
&= -\frac{1}{2\pi\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu\partial_\mu^2\partial_\nu^2g_\mu^ag_\nu^bg_\nu^c} - g_s f^{ab} f_{ab} \partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2g_\mu^ag_\nu^bg_\nu^c - \frac{1}{4\pi g_s^2 f^{cd} f_{cd} \partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2g_\mu^cg_\nu^dg_\nu^e} \\
&- \partial^\mu W_\mu \partial^\nu W_\nu - M^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_+^\mu W_-^\nu W_-^\mu W_+^\nu - \frac{1}{2\pi\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2Z_\mu^0Z_\nu^0Z_0^\mu Z_0^\nu} - \frac{1}{2c_m^2 M^2 Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu} - \frac{1}{2\partial^\mu A_\nu \partial^\nu A_\mu} \\
&- igc_w (\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2Z_\mu^0Z_\nu^0Z_0^\mu Z_0^\nu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+)) - Z_\mu^0 (\partial^\mu\partial_\nu W_\mu^+ W_\nu^- W_+^\mu W_-^\nu) + Z_\nu^0 (\partial^\nu\partial_\mu W_\nu^+ W_\mu^- W_+^\nu W_-^\mu) \\
&- igS_w (\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_+^\mu W_-^\nu W_-^\mu W_+^\nu) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu) - A_\mu (\partial^\mu\partial_\nu W_\mu^+ W_\nu^- W_+^\mu W_-^\nu Z_\mu^0 Z_\nu^0) \\
&+ A_\nu (\partial^\nu\partial_\mu W_\nu^+ W_\mu^- W_+^\nu W_-^\mu Z_\nu^0 Z_0^\nu) - \frac{1}{2g^2 (\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_+^\mu W_-^\nu W_-^\mu W_+^\nu) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu)} \\
&+ g^2 c_w^2 (\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_+^\mu W_-^\nu W_-^\mu W_+^\nu) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu) \\
&+ g^2 S_w^2 (\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_+^\mu W_-^\nu W_-^\mu W_+^\nu) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu) \\
&- g^2 c_w S_w (\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_+^\mu W_-^\nu W_-^\mu W_+^\nu) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu) \\
&- \frac{1}{2\pi (\partial H^\mu A_\nu H \partial^\nu H A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_+^\mu W_-^\nu W_-^\mu W_+^\nu) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu H^\mu H_\nu H_\nu^\mu H_\nu^\nu)} + \frac{\frac{1}{2\pi(2M^2 H^2 H^3)}}{\frac{d^x em^c \gamma}{GUM_{scw}^2}} - \frac{2g_c^2 M_s^2}{\Psi\Phi\zeta} - \lambda\delta \\
&\frac{\beta_\eta^\xi}{\prod_\sigma^\rho \frac{\hbar^4}{\hbar^2}}
\end{aligned}$$

$$\begin{aligned}
& \otimes \frac{\omega}{\Delta\nabla\theta} \\
&/ \prod_{\underline{\equiv}}^{\dagger} \infty \prod_j^i k \left(\frac{\phi_\mu^+ \phi_\nu^- \phi_\mu^- \phi_\nu^+}{\phi_+^\mu \phi_-^\nu \phi_0^\mu \phi_0^\nu} \right) (\varphi\psi\omega\lambda_+^+ \varphi\psi\omega\lambda_-^- \varphi\psi\omega\lambda_\mu^- \varphi\psi\omega\lambda_\nu^+ \frac{2\varphi\psi\omega\lambda^\mu}{\varphi\psi\omega\lambda_+} \varphi\psi\omega\lambda_-^\nu \varphi\psi\omega\lambda_\mu^\mu \varphi\psi\omega\lambda_\nu^+ \frac{1/2\pi\varphi\psi\omega\lambda^0}{\varphi\psi\omega\lambda_\mu} \varphi\psi\omega\lambda_\nu^0 \varphi\psi\omega\lambda_0^\mu \varphi\psi\omega\lambda_0^\nu) \\
&/ 2M \sqrt{\frac{\frac{2\xi\eta}{\zeta\epsilon\epsilon}}{\frac{\delta\alpha}{o\sigma\rho}}} / \Psi\Omega\Upsilon = \mathcal{L}_{Higgs} \\
&= \left(\partial^\mu\partial_\nu\partial^\nu\partial_\mu + \frac{1}{2ig_1 B^\mu B_\nu B^\nu B_\mu} + \frac{1}{2jg_2 B^\mu B_\nu B^\nu B_\mu} + \frac{1}{2ig_1 W^\mu W_\nu W^\nu W_\mu} + \frac{1}{2jg_2 W^\mu W_\nu W^\nu W_\mu} \right) - m_H^2 \phi' \phi - v^2 / 2v^2 / \tau^2
\end{aligned}$$

$$\begin{aligned}
\mathcal{H}_c &\equiv \frac{1}{2\pi \prod_i^k(x) + \prod_k^i(x) \partial^i \partial_k A^k A_i(x) + \frac{1}{4\pi F^{ki}(x) F_{ik}(x)}}} \\
&= H_c \prod_i^k d^3\chi \left[\frac{1}{2\pi \prod_i^k(x) + \prod_k^i(x) \partial^i \partial_k A^k A_i(x) + \frac{1}{4\pi F^{ki}(x) F_{ik}(x)}}} \right] \\
&= H^\rho H_c H^c H_\rho H_c^\rho H_\rho^c \equiv \prod_i^k \frac{d^3\chi\lambda}{\hbar} \mathcal{U}\Omega\mathbb{R}^4 / G_\epsilon R_e \\
&\quad [\lambda\Phi \underline{\equiv}]
\end{aligned}$$

Donde:



$$G_{\varepsilon} = \mathbf{G}_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_e =$$

$$\begin{aligned} \mathcal{H}_c &\equiv \frac{1}{2\pi \prod_i^k(\mathbf{y}) + \prod_k^i(\mathbf{y}) \partial^i \partial_k A^k A_i(\mathbf{y}) + \frac{1}{4F^{ki}(\mathbf{y}) F_{ik}(\mathbf{y})}} \\ &= H_c \prod_i^k d^3\chi \left[\frac{1}{2\pi \prod_i^k(\mathbf{y}) + \prod_k^i(\mathbf{y}) \partial^i \partial_k A^k A_i(\mathbf{y}) + \frac{1}{4\pi F^{ki}(\mathbf{y}) F_{ik}(\mathbf{y})}} \right] \\ &= H^\rho H_c H^c H_\rho H_c^\rho H_\rho^\rho \varrho \equiv \prod_i^k \frac{d^3\chi\lambda}{\hbar} \mathcal{U}\Omega\mathbb{R}^4 / G_\varepsilon R_e \\ &\quad [\lambda\Phi \triangleq] \end{aligned}$$

Donde:

$$G_{\varepsilon} = \mathbf{G}_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_e =$$



$$\begin{aligned}
\mathcal{H}_c &\equiv \frac{1}{2\pi \prod_i^k(z) + \prod_k^i(z) \partial^i \partial_k A^k A_i(z) + \frac{1}{4\pi F^{ki}(z) F_{ik}(z)}} \\
&= H_c \prod_i^k d^3 \chi \left[\frac{1}{2\pi \prod_i^k(z) + \prod_k^i(z) \partial^i \partial_k A^k A_i(z) + \frac{1}{4\pi F^{ki}(z) F_{ik}(z)}} \right] \\
&= H^\rho H_c H^c H_\rho H_c^\rho H_\rho^c \equiv \prod_i^k \frac{d^3 \chi \lambda}{\hbar} \mathcal{U} \Omega \mathbb{R}^4 / G_\varepsilon R_e \\
&\quad [\lambda \Phi \triangleq]
\end{aligned}$$

Donde:

$$G_\varepsilon = \mathbf{G}_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_e =$$

$$\{B(x, t), C(x, t)\} / \Phi \Psi \kappa \varphi \theta$$

$$\begin{aligned}
&= \prod_v^\mu (x, t) \lambda \phi \frac{\oint_\sigma^\varphi d^3 z [\delta_v^\mu B(x, t) \lambda \phi / \delta_v^\mu A_{\mu\nu}(x, t) \lambda \phi]}{\delta_v^\mu C(x, t) \lambda \phi} \\
&/\delta \prod_\mu^v (x, t) \lambda \phi - \frac{\oint_\sigma^\varphi d^3 z [\delta_\mu^v B(x, t) \lambda \phi / \delta_\mu^v C(z, t) \lambda \phi]}{\delta_\mu^v A_{v\mu}(x, t) \lambda \phi} / \delta \prod_{v\mu}^{\mu\nu} (x, t) \lambda \phi
\end{aligned}$$

$$\{B(y, t), C(y, t)\} / \Phi \Psi \kappa \varphi \theta$$

$$\begin{aligned}
&= \prod_v^\mu (y, t) \lambda \phi \frac{\oint_\sigma^\varphi d^3 z [\delta_v^\mu B(y, t) \lambda \phi / \delta_v^\mu A_{\mu\nu}(y, t) \lambda \phi]}{\delta_v^\mu C(y, t) \lambda \phi} \\
&/\delta \prod_\mu^v (y, t) \lambda \phi - \frac{\oint_\sigma^\varphi d^3 z [\delta_\mu^v B(y, t) \lambda \phi / \delta_\mu^v C(y, t) \lambda \phi]}{\delta_\mu^v A_{v\mu}(y, t) \lambda \phi} / \delta \prod_{v\mu}^{\mu\nu} (y, t) \lambda \phi
\end{aligned}$$



$$\{B(z, t), C(z, t)\} / \Phi \Psi \kappa \varphi \theta$$

$$= \prod_v^\mu (z, t) \lambda \phi \frac{\oint_\sigma^\varphi d^3 z [\delta_v^\mu B(z, t) \lambda \phi / \delta_v^\mu A_{\mu\nu}(z, t) \lambda \phi]}{\delta_v^\mu C(z, t) \lambda \phi}$$

$$/ \delta \prod_\mu^v (z, t) \lambda \phi - \frac{\oint_\sigma^\varphi d^3 z [\delta_\mu^v B(z, t) \lambda \phi / \delta_\mu^v C(z, t) \lambda \phi]}{\delta_\mu^v A_{v\mu}(z, t) \lambda \phi} / \delta \prod_{v\mu}^{\mu\nu} (z, t) \lambda \phi$$

$$\{F(x), G(x)\}_{D\infty}$$

$$= * \{F(x), G(x)\} \oplus$$

$$- \prod_\varphi^\gamma \psi \prod_\gamma^\varphi \lambda$$

$$\approx \frac{\oint_v^\mu \oint_\beta^\zeta d^3 \mu \nu^3 \mu \nu_3 \mu \nu^d \mu \nu_d \nu \mu^3 \nu \mu_3 \nu \mu^d \nu \mu_d \phi^\mu \phi_\nu \phi^v \phi_\mu \varphi^\nu \varphi_\mu \varphi^\mu \varphi_\nu \phi^{\mu\nu} \phi_{\nu\mu} \phi^{v\mu} \phi_{\mu\nu} \varphi^{\mu\nu} \varphi_{\nu\mu} \varphi^{v\mu} \varphi_{\mu\nu} C_{\mu\nu\nu c}^{-1\pi} e^{-i\omega t} m c \hbar^4}{\alpha \beta / h \Omega \oint \frac{\dagger}{\pi} / \Delta \nabla \otimes \boxtimes \ltimes \times \sphericalangle \sphericalangle}$$

$$\{F(y), G(y)\}_{D\infty}$$

$$= * \{F(y), G(y)\} \oplus$$

$$- \prod_\varphi^\gamma \psi \prod_\gamma^\varphi \lambda$$

$$\approx \frac{\oint_v^\mu \oint_\beta^\zeta d^3 \mu \nu^3 \mu \nu_3 \mu \nu^d \mu \nu_d \nu \mu^3 \nu \mu_3 \nu \mu^d \nu \mu_d \phi^\mu \phi_\nu \phi^v \phi_\mu \varphi^\nu \varphi_\mu \varphi^\mu \varphi_\nu \phi^{\mu\nu} \phi_{\nu\mu} \phi^{v\mu} \phi_{\mu\nu} \varphi^{\mu\nu} \varphi_{\nu\mu} \varphi^{v\mu} \varphi_{\mu\nu} C_{\mu\nu\nu c}^{-1\pi} e^{-i\omega t} m c \hbar^4}{\alpha \beta / h \Omega \oint \frac{\dagger}{\pi} / \Delta \nabla \otimes \boxtimes \ltimes \times \sphericalangle \sphericalangle}$$

$$\{F(z), G(z)\}_{D\infty}$$

$$= * \{F(z), G(z)\} \oplus$$

$$- \prod_\varphi^\gamma \psi \prod_\gamma^\varphi \lambda$$

$$\approx \frac{\oint_v^\mu \oint_\beta^\zeta d^3 \mu \nu^3 \mu \nu_3 \mu \nu^d \mu \nu_d \nu \mu^3 \nu \mu_3 \nu \mu^d \nu \mu_d \phi^\mu \phi_\nu \phi^v \phi_\mu \varphi^\nu \varphi_\mu \varphi^\mu \varphi_\nu \phi^{\mu\nu} \phi_{\nu\mu} \phi^{v\mu} \phi_{\mu\nu} \varphi^{\mu\nu} \varphi_{\nu\mu} \varphi^{v\mu} \varphi_{\mu\nu} C_{\mu\nu\nu c}^{-1\pi} e^{-i\omega t} m c \hbar^4}{\alpha \beta / h \Omega \oint \frac{\dagger}{\pi} / \Delta \nabla \otimes \boxtimes \ltimes \times \sphericalangle \sphericalangle}$$

$$\mathcal{L} = - \frac{1}{4\pi f^{ab}(x) t_{ab}(x) f_{ab} t^{ab} f_{ba}} (x) t_{ba}^{ab}(x) f_{ab}^{ba}(x) t_{ab}^{ba}(x) \neq \mathcal{L}$$

$$= - \frac{1}{4\pi f^{ba}(x) t_{ba}(x) f_{ba} t^{ba} f_{ab}} (x) t_{ab}^{ba}(x) f_{ba}^{ab}(x) t_{ba}^{ab}(x)$$



$$\begin{aligned} \mathcal{L} &= -\frac{1}{4\pi f^{ab}(y)t_{ab}(y)f_{ab}t^{ab}f_{ba}^{ab}}(y)t_{ba}^{ab}(y)f_{ab}^{ba}(y)t_{ab}^{ba}(y) \neq \mathcal{L} \\ &= -\frac{1}{4\pi f^{ba}(y)t_{ba}(y)f_{ba}t^{ba}f_{ab}^{ba}}(y)t_{ab}^{ba}(y)f_{ba}^{ab}(y)t_{ba}^{ab}(y) \\ \mathcal{L} &= -\frac{1}{4\pi f^{ab}(z)t_{ab}(z)f_{ab}t^{ab}f_{ba}^{ab}}(z)t_{ba}^{ab}(z)f_{ab}^{ba}(z)t_{ab}^{ba}(z) \neq \mathcal{L} \\ &= -\frac{1}{4\pi f^{ba}(z)t_{ba}(z)f_{ba}t^{ba}f_{ab}^{ba}}(z)t_{ab}^{ba}(z)f_{ba}^{ab}(z)t_{ba}^{ab}(z) \end{aligned}$$

$$\begin{aligned} &f^{ab}(x,t)t_{ba}(x,t)f_{ab}(x,t)t^{ba}(x,t)f^{ba}(x,t)t_{ab}(x,t)f_{ba}(x,t)t^{ab}(x,t) \\ &+ f^{ab}(y,t)t_{ba}(y,t)f_{ab}(y,t)t^{ba}(y,t)f^{ba}(y,t)t_{ab}(y,t)f_{ba}(y,t)t^{ab}(y,t) \\ &+ f^{ab}(x)t_{ba}(x)f_{ab}(x)t^{ba}(x)f^{ba}(x)t_{ab}(x)f_{ba}(x)t^{ab}(x) \\ &+ f^{ab}(y)t_{ba}(y)f_{ab}(y)t^{ba}(y)f^{ba}(y)t_{ab}(y)f_{ba}(y)t^{ab}(y) \\ &+ f^{ab}(z)t_{ba}(z)f_{ab}(z)t^{ba}(z)f^{ba}(z)t_{ab}(z)f_{ba}(z)t^{ab}(z) \\ &= \partial^a A_b(x,t) - \partial^b A_a(x,t) + \partial^a A_b(y,t) - \partial^b A_a(y,t) + \partial^a A_b(z,t) - \partial^b A_a(z,t) \\ &= \partial^a A_b(x) - \partial^b A_a(x) + \partial^a A_b(y) - \partial^b A_a(y) + \partial^a A_b(z) - \partial^b A_a(z) \end{aligned}$$

$$\begin{aligned} &f_{ij}(x,k), t^{ji}(x,k), f^{ij}(x,k)t_{ji}(x,k), f_{ji}(x,k), t^{ij}(x,k), f^{ji}(x,k)t_{ij}(x,k) + f_j^i t_i^j(x,k), f_i^j t_j^i(x,k) \\ &+ f_{ij}(y,k), t^{ji}(y,k), f^{ij}(y,k)t_{ji}(y,k), f_{ji}(y,k), t^{ij}(y,k), f^{ji}(y,k)t_{ij}(y,k) + f_j^i t_i^j(y,k), f_i^j t_j^i(y,k) \\ &+ f_{ij}(z,k), t^{ji}(z,k), f^{ij}(z,k)t_{ji}(z,k), f_{ji}(z,k), t^{ij}(z,k), f^{ji}(z,k)t_{ij}(z,k) + f_j^i t_i^j(z,k), f_i^j t_j^i(z,k) \\ &= -\epsilon^{ijk}\epsilon_{ijk}B^k B_k(x,k) - \epsilon^{ijk}\epsilon_{ijk}B^k B_k(y,k) - \epsilon^{ijk}\epsilon_{ijk}B^k B_k(z,k) \end{aligned}$$

$$\begin{aligned} &A^a A_a A^b A_b A^a A_b A^b A_a A^{ab} A_{ba} A^{ba} A_{ab}(x) \rightarrow A^a A_a A^b A_b A^a A_b A^b A_a A^{ab} A_{ba} A^{ba} A_{ab}(y) \\ &\rightarrow A^a A_a A^b A_b A^a A_b A^b A_a A^{ab} A_{ba} A^{ba} A_{ab}(z) \rightarrow A'_a A'_b A'_b A'_a(x) \rightarrow A'_a A'_b A'_b A'_a(y) \\ &\rightarrow A'_a A'_b A'_b A'_a(z) = A^a A_a A^b A_b A^a A_b A^b A_a A^{ab} A_{ba} A^{ba} A_{ab}(x) \\ &\rightarrow A^a A_a A^b A_b A^a A_b A^b A_a A^{ab} A_{ba} A^{ba} A_{ab}(y) \\ &\rightarrow A^a A_a A^b A_b A^a A_b A^b A_a A^{ab} A_{ba} A^{ba} A_{ab}(z) + \partial^a \partial_b \partial^b \partial_a \partial^{ab} \partial_{ba} \partial^{ba} \partial_{ab} \alpha(x) \\ &+ \partial^a \partial_b \partial^b \partial_a \partial^{ab} \partial_{ba} \partial^{ba} \partial_{ab} \alpha(y) + \partial^a \partial_b \partial^b \partial_a \partial^{ab} \partial_{ba} \partial^{ba} \partial_{ab} \alpha(z) \end{aligned}$$



$$\begin{aligned}
& f^{ab}t_{ba}f^{ba}t_{ab}f^{ab}t_{ab}f^{ba}t_{ba}(x) \rightarrow f^{ab}t_{ba}f^{ba}t_{ab}f^{ab}t_{ab}f^{ba}t_{ba}(y) \\
& \rightarrow f^{ab}t_{ba}f^{ba}t_{ab}f^{ab}t_{ab}f^{ba}t_{ba}(z) \rightarrow f'_{ab}t'_{ba}f'_{ba}t'_{ab}(x) \rightarrow f'_{ab}t'_{ba}f'_{ba}t'_{ab}(y) \\
& \rightarrow f'_{ab}t'_{ba}f'_{ba}t'_{ab}(z) \\
& = \partial_a \left(A^a A_b A^b A_a A^{ab} A_{ba} A^{ba} A_{ab}(x) + \partial^a \partial_b \partial^b \partial_a \partial^{ab} \partial_{ba} \partial^{ba} \partial_{ab} \alpha(x) \right) \\
& - \partial_b \left(A^b A_a A^a A_b A^{ba} A_{ab} A^{ab} A_{ba}(x) + \partial^b \partial_a \partial^a \partial_b \partial^{ba} \partial_{ab} \partial^{ab} \partial_{ba} \alpha(x) \right) \\
& = \partial^a \partial^b A_a A_b \partial^b \partial^a A_b A_a \partial^{ab} \partial^{ba} A_{ab} A_{ba} \partial^{ba} \partial^{ab} A_{ba} A_{ab}(x) \\
& + \partial^a \partial^b \partial_a \partial_b \partial^b \partial^a \partial_b A_a \partial^{ab} \partial^{ba} \partial_{ab} \partial_{ba} \partial^{ba} \partial^{ab} \partial_{ba} \partial_{ab} \alpha(x) \\
& = \partial_a \left(A^a A_b A^b A_a A^{ab} A_{ba} A^{ba} A_{ab}(y) + \partial^a \partial_b \partial^b \partial_a \partial^{ab} \partial_{ba} \partial^{ba} \partial_{ab} \alpha(y) \right) \\
& - \partial_b \left(A^b A_a A^a A_b A^{ba} A_{ab} A^{ab} A_{ba}(y) + \partial^b \partial_a \partial^a \partial_b \partial^{ba} \partial_{ab} \partial^{ab} \partial_{ba} \alpha(y) \right) \\
& = \partial^a \partial^b A_a A_b \partial^b \partial^a A_b A_a \partial^{ab} \partial^{ba} A_{ab} A_{ba} \partial^{ba} \partial^{ab} A_{ba} A_{ab}(y) \\
& + \partial^a \partial^b \partial_a \partial_b \partial^b \partial^a \partial_b A_a \partial^{ab} \partial^{ba} \partial_{ab} \partial_{ba} \partial^{ba} \partial^{ab} \partial_{ba} \partial_{ab} \alpha(y) \\
& = \partial_a \left(A^a A_b A^b A_a A^{ab} A_{ba} A^{ba} A_{ab}(z) + \partial^a \partial_b \partial^b \partial_a \partial^{ab} \partial_{ba} \partial^{ba} \partial_{ab} \alpha(z) \right) \\
& - \partial_b \left(A^b A_a A^a A_b A^{ba} A_{ab} A^{ab} A_{ba}(z) + \partial^b \partial_a \partial^a \partial_b \partial^{ba} \partial_{ab} \partial^{ab} \partial_{ba} \alpha(z) \right) \\
& = \partial^a \partial^b A_a A_b \partial^b \partial^a A_b A_a \partial^{ab} \partial^{ba} A_{ab} A_{ba} \partial^{ba} \partial^{ab} A_{ba} A_{ab}(z) \\
& + \partial^a \partial^b \partial_a \partial_b \partial^b \partial^a \partial_b A_a \partial^{ab} \partial^{ba} \partial_{ab} \partial_{ba} \partial^{ba} \partial^{ab} \partial_{ba} \partial_{ab} \alpha(z)
\end{aligned}$$

$$\begin{aligned}
f'_{ab}t'_{ba}f'_{ba}t'_{ab}(x) &= \partial^a A^b \partial_b A_a \partial^{ab} A^{ba} \partial_{ba} A_{ab}(x) - \partial^b A^a \partial_a A_b \partial^{ba} A^{ab} \partial_{ab} A_{ba}(x) \\
&= f^{ab}t_{ba}f^{ba}t_{ab}(x)
\end{aligned}$$

$$\begin{aligned}
f'_{ab}t'_{ba}f'_{ba}t'_{ab}(y) &= \partial^a A^b \partial_b A_a \partial^{ab} A^{ba} \partial_{ba} A_{ab}(y) - \partial^b A^a \partial_a A_b \partial^{ba} A^{ab} \partial_{ab} A_{ba}(y) \\
&= f^{ab}t_{ba}f^{ba}t_{ab}(y)
\end{aligned}$$

$$\begin{aligned}
f'_{ab}t'_{ba}f'_{ba}t'_{ab}(z) &= \partial^a A^b \partial_b A_a \partial^{ab} A^{ba} \partial_{ba} A_{ab}(z) - \partial^b A^a \partial_a A_b \partial^{ba} A^{ab} \partial_{ab} A_{ba}(z) \\
&= f^{ab}t_{ba}f^{ba}t_{ab}(z)
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}[A^a \partial_b A^b \partial_a] &= \iiint_{ba}^{ab} abba d^4 \chi \mathcal{L}[A^a \partial_b A^b \partial_a] + \delta \iiint_{ba}^{ab} abba d^4 \chi \mathcal{L}[A^a \partial_b A^b \partial_a] \\
&= \iiint_{ba}^{ab} abba d^4 \chi \delta \mathcal{L}[A^a \partial_b A^b \partial_a]
\end{aligned}$$

$$\delta \mathcal{L}[A^a \partial_b A^b \partial_a] = \frac{\partial \mathcal{L}}{\partial A^a A_b A^b A_a} \delta A_b^a A_a^b + \frac{\partial \mathcal{L}}{\partial (\partial^a A_b \partial^b A_a)} \delta (\partial^a A_b \partial^b A_a)$$

$$\delta \mathcal{A}[A^a \partial_b A^b \partial_a] = \delta \iiint_{ba}^{ab} abba d^4 \chi \left[\frac{\partial \mathcal{L}}{\partial A^a A_b A^b A_a} \delta A_b^a A_a^b + \frac{\partial \mathcal{L}}{\partial (\partial^a A_b \partial^b A_a)} \delta (\partial^a A_b \partial^b A_a) \right]$$



$$\delta[A^a \partial_b A^b \partial_a] = \delta \frac{\partial A^a A_b A^b A_a}{\partial \chi^a \chi_b \chi^b \chi_a} = \frac{\partial}{\partial \chi^a \chi_b \chi^b \chi_a} \delta A_b^a A_a^b = \partial^a \partial_b (\delta A_b^a A_a^b)$$

$$\frac{\partial \mathcal{L}}{\partial (\partial^a A_b \partial^a \partial_b)} \delta (\partial^a A_b \partial^b \partial_a) = \frac{\partial \mathcal{L}}{\partial (\partial^a A_b \partial^b A_a)} \partial^a \partial_b (\delta A_b^a A_a^b)$$

$$= \partial^a \partial_b \partial^a \partial_b \left[\frac{\partial \mathcal{L}}{\partial (\partial^a A_b \partial^b A_a)} \delta (\partial^a A_b \partial^b A_a) \right] - \frac{\partial \mathcal{L}}{\partial (\partial^a A_b \partial^b A_a)} \delta (\partial^a A_b \partial^b A_a)$$

$$\delta \mathcal{A} [A^a \partial_b A^b \partial_a] = \delta \left(\prod_{ba}^{ab} \right) abba d^4 \chi \left[\frac{\partial \mathcal{L}}{\partial A^a A_b A^b A_a} \delta A_b^a A_a^b \right.$$

$$\left. - \partial^a \partial_b \partial^b \partial_a \frac{\partial \mathcal{L}}{\partial (\partial^a A_b \partial^b A_a) \delta (\partial^b A_a \partial^a A_b)} \delta A_b^a A_a^b \right.$$

$$\left. + \delta \left(\prod_{ba}^{ab} \right) abba d^4 \chi \partial^a \partial_b \partial^b \partial_a \left[\frac{\partial \mathcal{L}}{\partial (\partial^a A_b \partial^b A_a) \delta (\partial^b A_a \partial^a A_b)} \delta A_b^a A_a^b \right] \right]$$

$$\frac{\partial \mathcal{L}}{\partial A^a A_b A_b^a A_a^b} = - \frac{1}{4\pi \frac{\partial}{\partial A^a A_b A_b^a A_a^b} [f^a t_b f_b^a t^b t^a f_b t_b^a f_b^a]}$$

$$= -1$$

$$/4\pi \frac{\partial}{\partial A^a A_b A_b^a A_a^b} \left((\partial^a A_b(x) - \partial^b A_a(x)) (\partial^b A_a(x) - \partial^a A_b(x)) (\partial^a A^b(x) \right.$$

$$\left. - \partial^b A^a(x)) (\partial^b A^a(x) - \partial^a A^b(x)) (\partial_a A_b(x) - \partial_b A_a(x)) (\partial_b^a A_a^b(x) \right.$$

$$\left. - \partial_a^b A_b^a(x)) (\partial_b^a \partial_a^b A(x) - \partial_a^b \partial_b^a A(x)) \right)$$

$$+ -1$$

$$/4\pi \frac{\partial}{\partial A^a A_b A_b^a A_a^b} \left((\partial^a A_b(y) - \partial^b A_a(y)) (\partial^b A_a(y) - \partial^a A_b(y)) (\partial^a A^b(y) \right.$$

$$\left. - \partial^b A^a(y)) (\partial^b A^a(y) - \partial^a A^b(y)) (\partial_a A_b(y) - \partial_b A_a(y)) (\partial_b^a A_a^b(y) \right.$$

$$\left. - \partial_a^b A_b^a(y)) (\partial_b^a \partial_a^b A(y) - \partial_a^b \partial_b^a A(y)) \right)$$

$$+ -1$$

$$/4\pi \frac{\partial}{\partial A^a A_b A_b^a A_a^b} \left((\partial^a A_b(z) - \partial^b A_a(z)) (\partial^b A_a(z) - \partial^a A_b(z)) (\partial^a A^b(z) \right.$$

$$\left. - \partial^b A^a(z)) (\partial^b A^a(z) - \partial^a A^b(z)) (\partial_a A_b(z) - \partial_b A_a(z)) (\partial_b^a A_a^b(z) \right.$$

$$\left. - \partial_a^b A_b^a(z)) (\partial_b^a \partial_a^b A(z) - \partial_a^b \partial_b^a A(z)) \right)$$

$$\begin{aligned} & \partial_i \partial^j \partial_j \partial^i f^{ab\varphi} t_{ba\omega} t^{ab\varphi} f_{ba\omega}(x) \\ &= \frac{\frac{\partial^\theta \partial_\theta F_\sigma^\rho \gamma \beta}{\varepsilon \epsilon \vartheta \pi}}{\frac{\Delta \nabla}{\tau}} + \prod_b^a \lambda \prod_a^b \lambda H_{iggs} \\ & - W^a W_b W^b W_a W_b^a W_a^b W_a^b W - \eta^\theta \eta_\beta \eta_{\phi v}^{\sigma \mu} \alpha \eta / \mathbb{R}^4 \end{aligned}$$

En la que la constante H_{iggs} es igual a:

$$\begin{aligned} \mathcal{L}_{SM} = & \frac{1}{2\partial_v g_\mu^a \partial_\mu g_v^b} - g_s f^{abc} \partial_\mu g_v^a g_v^b g_v^c \partial_v g_\mu^a g_\mu^b g_\mu^c - \frac{1}{4g_s^2 f^{abc} f^{ade} g_\mu^b g_\mu^c g_\mu^d g_\mu^e g_v^b g_v^c g_v^d g_v^e} - \partial_v W_\mu^+ \partial_v W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^- - M^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- \\ & - \frac{1}{2\partial_v Z_\mu^0 \partial_\mu Z_\nu^0} - \frac{1}{2c_\omega^2 M^2 Z_\mu^0 Z_\nu^0} - \frac{1}{2\partial_\mu \mathcal{A}_v \partial_v \mathcal{A}_\mu} \\ & - igc_\omega (\partial_v Z_\mu^0 (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+) - Z_\mu^0 Z_\nu^0 (\partial_v W_\mu^+ \partial_v W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-)) \\ & - ig s_\omega (\partial_\mu \mathcal{A}_v \partial_v \mathcal{A}_\mu (W_\mu^+ W_\nu^- W_\nu^+ W_\mu^-) - \mathcal{A}_\mu (\partial_v W_\mu^+ \partial_v W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-) - \mathcal{A}_v (\partial_\mu W_\mu^+ \partial_v W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-)) \\ & - \frac{1}{2g^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+} + g^2 c_\omega^2 (Z_\mu^0 W_\mu^+ W_\nu^- Z_\nu^0 W_\nu^+ W_\mu^-) \\ & + g^2 s_\omega^2 (\mathcal{A}_\mu W_\mu^+ W_\nu^- \mathcal{A}_v W_\nu^+ W_\mu^-) + g^2 c_\omega s_\omega (\mathcal{A}_\mu Z_\mu^0 (W_\mu^+ W_\mu^-) \mathcal{A}_v Z_\nu^0 (W_\nu^+ W_\nu^-) - 2\mathcal{A}_\mu W_\mu^+ W_\nu^- Z_\mu^0 \mathcal{A}_v W_\nu^+ W_\mu^- Z_\nu^0) - \frac{1}{2\partial_\mu \mathcal{H} \partial_v \mathcal{H}} - 2M^2 \alpha_h \mathcal{H}^2 \\ & - \partial_\mu \phi^+ \partial_v \phi^- \partial_v \phi^+ \partial_v \phi^- - \frac{1}{2\partial_\mu \phi^0 \partial_v \phi^0 \partial_v \phi^0 \partial_v \phi^0} \\ & - \beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g\mathcal{H}} + \frac{1}{2} (\mathcal{H}^2 + \phi^0 \phi^+ \phi^0 \phi^- + 2\phi^0 \phi^+ \phi^0 \phi^-) \right) + \frac{2M^4}{g^2 \alpha_h} - g \alpha_h M (\mathcal{H}^3 + \mathcal{H} \phi^0 \phi^0 + 2\mathcal{H} \phi^+ \phi^-) - 1/8g^2 \alpha_h (\mathcal{H}^4 + (\phi^0)^4 \\ & + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4\mathcal{H}^2 \phi^+ \phi^- + 2(\phi^0)^2 \mathcal{H}^2) - g M W_\mu^+ W_\nu^- W_\nu^+ W_\mu^- \mathcal{H} - \frac{2gM}{c_\omega^2 Z_\mu^0 Z_\nu^0 \mathcal{H}} \\ & - \frac{1}{2ig} (W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0) - W_\nu^+ (\phi^0 \partial_v \phi^- - \phi^- \partial_v \phi^0) \\ & - W_\nu^- (\phi^0 \partial_v \phi^+ - \phi^+ \partial_v \phi^0)) \\ & - \frac{1}{2g (W_\mu^+ (\mathcal{H} \partial_\mu \phi^- - \phi^- \partial_\mu \mathcal{H}) W_\mu^- (\mathcal{H} \partial_\mu \phi^+ - \phi^+ \partial_\mu \mathcal{H}) W_\nu^+ (\mathcal{H} \partial_v \phi^- - \phi^- \partial_v \mathcal{H}) W_\nu^- (\mathcal{H} \partial_v \phi^+ - \phi^+ \partial_v \mathcal{H}))} \\ & + \frac{1}{c_\omega Z_\mu^0 (\mathcal{H} \partial_\mu \phi^0 - \phi^0 \partial_\mu \mathcal{H}) Z_\nu^0 (\mathcal{H} \partial_\nu \phi^0 - \phi^0 \partial_\nu \mathcal{H})} + M \left(\frac{1}{c_\omega Z_\mu^0 \partial_\mu \phi^0} + W_\mu^+ \partial_\mu \phi^- + W_\mu^- \partial_\mu \phi^+ \right) \\ & - \frac{igs_\omega^2}{c_\omega M Z_\mu^0 (W^+ \phi^- - W^- \phi^+) ig s_\omega M \mathcal{A}_\mu (W_\mu^+ \phi^- + W_\mu^- \phi^+)} - ig 1 - \frac{2c_\omega^2}{2c_\omega Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+)} \\ & + ig s_\omega \mathcal{A}_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - 1/4g^2 W_\mu^+ W_\nu^- (\mathcal{H}^2 + (\phi^0)^2 + 2\phi^+ \phi^-) \\ & - \frac{1}{c_\omega^2 Z_\mu^0 Z_\nu^0} \frac{1}{8g^2 1} \\ & \left(\left(\left(\left(\left(\left(\frac{2s_\omega^2 - 1}{2g^2 s_\omega \mathcal{A}_\mu \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+)} + \frac{1}{2ig^2 s_\omega \mathcal{A}_\mu \mathcal{H} (W_\mu^+ \phi^- + W_\mu^- \phi^+)} - \frac{g^2 s_\omega}{c_\omega (2c_\omega^2 - 1) Z_\mu^0 \mathcal{A}_\mu \phi^+ \phi^-} - g^2 s_\omega^2 \mathcal{A}_\mu \mathcal{A}_v \phi^+ \phi^- + \frac{1}{2ig s_\omega \lambda_{ij}^\alpha \left(\frac{\rightarrow}{q\sigma^t} \varphi^\mu \rightarrow \frac{\rightarrow}{q\sigma^t} \right) g_\mu^\alpha} \right) \right) \right) \right) \right) e^\lambda \rightarrow_{v^\lambda} (\varphi \partial + m_\nu^2) v^\lambda \rightarrow_{\mu_j^\lambda v^\lambda} (\varphi \partial + m_\mu^2) - (\mu_j^\lambda) \end{aligned}$$



$$\begin{aligned}
& (\varphi\partial + m_d^\lambda)d_j^\lambda + ig_{s_\omega}\mathcal{A}_\mu \left(-\left(\overrightarrow{e^\lambda} \varphi^\mu e^\lambda\right) + \frac{2}{3\left(\overrightarrow{\mu_j^\lambda} \varphi^\mu \mu_j^\lambda\right)} - \frac{1}{3\left(\overrightarrow{d^\lambda} \varphi^\mu d^\lambda\right)} + \frac{ig}{4c_\omega Z_\mu^0\left(\overrightarrow{v^\lambda} \varphi^\mu(1 + \varphi^5)v^\lambda\right)} \right. \\
& + \left(\overrightarrow{e^\lambda} \varphi^\mu(4s_\omega^2 - 1 - \varphi^5)e^\lambda\right) + \left(\overrightarrow{d_j^\lambda} \varphi^\mu\left(\frac{4}{3s_\omega^2} - 1 - \varphi^5\right)d_j^\lambda\right) + \left(\overrightarrow{\mu_j^\lambda} \varphi^\mu\left(1 - \frac{8}{3s_\omega^2} + \varphi^5\right)\mu_j^\lambda\right) \\
& \left. + \left(\frac{ig}{\sqrt[2]{2W_\mu^+}\left(\overrightarrow{v^\lambda} \varphi^\mu(1 + \varphi^5)\right)U^{lep^k}_\xi e^k}\right) + \left(\overrightarrow{\mu_j^\lambda} \varphi^\mu(1 + \varphi^5)c_{k\lambda}d_j^k\right) \right) \\
& + \frac{ig}{\sqrt[2]{2W_\mu^-}\left(\overrightarrow{e^\kappa} U^{lep^\dagger} \frac{\kappa\varphi}{\xi\rho\omega} - \triangle \Pi \oplus \oplus \gamma \otimes \frac{\prod_{ij}^{\mu} ijk}{\otimes \sigma} \Omega\Psi\Phi\Delta(1 + \varphi^5)v^\lambda + \overrightarrow{d_j^k} C_{\kappa}^{\dagger\lambda} \varphi_{v\eta}^{\mu\kappa\eta}(1 + \varphi^5)\mu_j^\lambda\right)} \\
& + \frac{ig}{2M\sqrt[2]{2\phi^+}\left(-m_c^\kappa\left(\overrightarrow{v^\lambda} U^{lep^k}_\xi e^k(1 - \varphi^5)\epsilon^\kappa\right) + m_\mu^\lambda \overrightarrow{U^{lep^\dagger}}(1 + \varphi^5)\epsilon^\kappa\right)} \\
& + \frac{ig}{2M\sqrt[2]{2\phi^-}\left(m_c^\kappa\left(\overrightarrow{v^\lambda} U^{lep^k}_\xi e^k(1 - \varphi^5)\epsilon^\kappa\right) \pm m_\mu^\lambda \overrightarrow{U^{lep^\dagger}}(1 + \varphi^5)\epsilon^\kappa\right)} - \frac{g}{M} \mathcal{H}\left(\overrightarrow{v^\lambda}\right) \\
& - \frac{\frac{g}{2m_c^\lambda}}{\frac{M\mathcal{H}\left(\overrightarrow{e^\lambda}\right)}{1}} + \frac{\frac{ig}{2m_v^\lambda}}{4\overrightarrow{v^\lambda} M_{\lambda\kappa}^R(1 - \gamma_5)v^\kappa} + \frac{\frac{ig}{2m_c^\lambda}}{M\phi^0\left(\overrightarrow{v^\lambda} \gamma^5 v^\lambda\right)} - \frac{\frac{ig}{2m_c^\lambda}}{M\phi^0\left(\overrightarrow{e^\lambda} \gamma^5 e^\lambda\right)} - \frac{1}{4\overrightarrow{v^\kappa} M_{\lambda\kappa}^R(1 - \gamma_5)\overrightarrow{v^\kappa}} \\
& + \frac{ig}{2M\sqrt[2]{2\phi^+}\left(-m_d^\kappa\left(\overrightarrow{\mu_j^\lambda} C_{\lambda\kappa}(1 - \varphi^5)d_j^k\right) + m_d^\kappa\left(\overrightarrow{\mu_j^\lambda} C_{\lambda\kappa}(1 - \varphi^5)d_j^k\right)\right)} \\
& + \frac{ig}{2M\sqrt[2]{2\phi^-}\left(m_d^\lambda\left(\overrightarrow{d_j^\lambda} C_{\lambda\kappa}^\dagger \wedge_{\theta^*}(1 + \varphi^5)\mu_j^\kappa\right) \pm m_d^\lambda\left(\overrightarrow{d_j^\lambda} C_{\lambda\kappa}^\dagger \wedge_{v^*}(1 + \varphi^5)\mu_j^\kappa\right)\right)} - \frac{g}{2m_\mu^\lambda} \mathcal{M}\mathcal{H}\left(\overrightarrow{\mu_j^\lambda}\right) \\
& - \frac{g}{2m_d^\lambda} \mathcal{H}\left(\overrightarrow{d_j^\lambda}\right) + \frac{ig}{M} \phi^0\left(\overrightarrow{\mu_j^\lambda} \gamma^5 \mu_j^\lambda\right) - \frac{ig}{M} \phi^0\left(\overrightarrow{d_j^\lambda} \gamma^5 d_j^\lambda\right) + \overrightarrow{\partial^2} G^a + g_s f^{abc} \partial_\mu \overrightarrow{G^b} G_\mu^c \\
& + \overrightarrow{\partial^2 - M^2} \alpha^+ + \overrightarrow{\partial^2 - M^2} \alpha^- + \overrightarrow{\partial^2 - \frac{M^2}{c_\omega^2}} \alpha^0 + \overrightarrow{\partial^2} b + igc_\omega W_\mu^+ \left(\partial_\mu \overrightarrow{\alpha^0} \overrightarrow{\alpha^-} - \partial_\mu \overrightarrow{\alpha^-} \overrightarrow{\alpha^0}\right) \\
& + ig_s W_\mu^+ \left(\partial_\mu \overrightarrow{\alpha^-} \overrightarrow{b} - \partial_\mu \overrightarrow{b} \overrightarrow{\alpha^-}\right) + igc_\omega W_\mu^- \left(\partial_\mu \overrightarrow{\alpha^+} \overrightarrow{\alpha^0} - \partial_\mu \overrightarrow{\alpha^0} \overrightarrow{\alpha^+}\right) + ig_s W_\mu^- \left(\partial_\mu \overrightarrow{\alpha^+} \overrightarrow{\alpha^0} - \partial_\mu \overrightarrow{\alpha^0} \overrightarrow{\alpha^+}\right) \\
& + igc_\omega Z_\mu^0 \left(\partial_\mu \overrightarrow{\alpha^+} \overrightarrow{\alpha^-} - \partial_\mu \overrightarrow{\alpha^-} \overrightarrow{\alpha^+}\right) + ig_s \mathcal{A}_\mu \left(\partial_\mu \overrightarrow{\alpha^+} \overrightarrow{\alpha^-} - \partial_\mu \overrightarrow{\alpha^-} \overrightarrow{\alpha^+}\right) - 1/2gM \left(\frac{\overrightarrow{\alpha^+} \mathcal{H} \mathbb{R}^4}{h} + \overrightarrow{\alpha^-} \mathcal{H}\right) \\
& + 1 - \frac{2c_\omega^2}{2c_\omega igM \left(\overrightarrow{\alpha^+} \alpha^0 \phi^+ - \overrightarrow{\alpha^-} \alpha^0 \phi^-\right)} + \frac{1}{2c_\omega igM \left(\overrightarrow{\alpha^0} \alpha^- \phi^+ - \overrightarrow{\alpha^0} \alpha^+ \phi^-\right)} \\
& + igMs_\omega \left(\overrightarrow{\alpha^0} \alpha^- \phi^+ - \overrightarrow{\alpha^0} \alpha^+ \phi^-\right) + 1/2igM \left(\overrightarrow{\alpha^+} \alpha^0 \phi^0 - \overrightarrow{\alpha^-} \alpha^0 \phi^0\right)
\end{aligned}$$

$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

$$\mathcal{L}_{SBS} = (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) - V(\Phi)$$

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$|\Phi|^2 = \Phi^\dagger \Phi = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}$$

$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \longrightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Phi(x) = \frac{1}{\sqrt{2}} e^{i \frac{\vec{\xi}(x) \cdot \vec{\tau}}{v}} \begin{pmatrix} 0 \\ v + \mathbf{h}(x) \end{pmatrix}$$

$$U(\xi) = e^{-i \frac{\vec{\xi}(x) \cdot \vec{\tau}}{v}}$$

$$\begin{aligned} \Phi' &= U(\xi) \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \mathbf{h}(x) \end{pmatrix} \\ \left(\frac{\vec{\tau} \cdot \vec{W}'_\mu}{2} \right) &= U(\xi) \left(\frac{\vec{\tau} \cdot \vec{W}_\mu}{2} \right) U^{-1}(\xi) - \frac{i}{g} (\partial_\mu U(\xi)) U^{-1}(\xi) \\ B'_\mu &= B_\mu \end{aligned}$$

$$\mathcal{L} = \mathcal{L}_{bos.} + \mathcal{L}_{ferm.} + \mathcal{L}_{SBS}$$

$$(\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) = \frac{v^2}{8} [g^2 (W_{1\mu}^2 + W_{2\mu}^2) + (gW_{3\mu} - g'B_\mu)^2]$$

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}} (W_\mu^1 \mp W_\mu^2) \\ Z_\mu &= \cos \theta_W W_\mu^3 - \sin \theta_W B_\mu \\ A_\mu &= \sin \theta_W W_\mu^3 + \cos \theta_W B_\mu \end{aligned}$$

$$\tan \theta_W \equiv \frac{g'}{g}$$



$$M_W = \frac{1}{2} g v$$

$$M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$$

$$g = \frac{e}{\sin \theta_W}$$

$$g' = \frac{e}{\cos \theta_W}$$

$$m_H^2 = 2\lambda v^2$$

$$\mu \rightarrow \nu_\mu \bar{\nu}_e e$$

$$v = (\sqrt{2} G_F)^{-\frac{1}{2}}$$

$$\mathcal{L}_{YW} = \lambda_e \bar{\ell}_L \Phi e_R + \lambda_u \bar{q}_L \tilde{\Phi} u_R + \lambda_d \bar{q}_L \Phi d_R + \text{h.c.}$$

$$\ell_L = \begin{pmatrix} e \\ \nu_e \end{pmatrix}_L, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$\begin{aligned} \ell'_L &= U(\xi) \ell_L; & e'_R &= e_R \\ q'_L &= U(\xi) q_L; & u'_R &= u_R; \quad d'_R = d \end{aligned}$$

$$m_e = \lambda_e \frac{v}{\sqrt{2}}$$

$$m_u = \lambda_u \frac{v}{\sqrt{2}}$$

$$m_d = \lambda_d \frac{v}{\sqrt{2}}$$

O es igual a:

$$\mathcal{L}_{Higgs} = \overline{\left([\partial_\mu + \frac{1}{2} i g_1 B_\mu + \frac{1}{2} i g_2 \mathbf{W}_\mu] \phi \right)} \left([\partial_\mu + \frac{1}{2} i g_1 B_\mu + \frac{1}{2} i g_2 \mathbf{W}_\mu] \phi \right) - \frac{m_H^2 \left(\bar{\phi} \phi - \frac{v^2}{2} \right)^2}{2v^2}$$



$$\mathcal{L}_{SM}(x) \equiv (a, b) \simeq (b, a)$$

$$\begin{aligned}
 &= -\frac{1}{2\pi\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2g_\mu^ag_\nu^bg_\nu^c} - g_s f^{ab} f_{ab} \partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2g_\mu^ag_\nu^bg_\nu^c - \frac{1}{4\pi g_s^2 f^{cd} f_{cd} \partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2g_\mu^cg_\nu^dg_\nu^d} - \partial^\mu W_\mu \partial^\nu W_\nu \\
 &- M^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ - \frac{1}{2\pi\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2Z_\nu^0Z_\nu^0Z_\nu^0} - \frac{1}{2c_m^2 M^2 Z_\nu^0Z_\nu^0Z_\nu^0} - \frac{1}{2\partial^\mu A_\nu \partial^\nu A_\mu} \\
 &- igc_w \left(\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2Z_\nu^0Z_\nu^0Z_\nu^0(W_\mu^+W_\nu^-W_\mu^-W_\nu^+) \right) - Z_\nu^0(\partial^\mu\partial_\nu W_\mu^+W_\nu^-W_\mu^+W_\nu^-) + Z_\nu^0(\partial^\nu\partial_\nu W_\nu^+W_\nu^-W_\nu^+W_\nu^-) \\
 &- igS_w(\partial^\mu A_\nu \partial^\nu A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+)Z_\nu^0Z_\nu^0Z_\nu^0) - A_\mu(\partial^\mu\partial_\nu W_\mu^+W_\nu^-W_\mu^+W_\nu^-Z_\nu^0Z_\nu^0) + A_\nu(\partial^\nu\partial_\nu W_\nu^+W_\nu^-W_\nu^+W_\nu^-Z_\nu^0Z_\nu^0) \\
 &- \frac{1}{2g^2(\partial^\mu A_\nu \partial^\nu A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+)Z_\nu^0Z_\nu^0Z_\nu^0)} + g^2 c_w^2(\partial^\mu A_\nu \partial^\nu A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+)Z_\nu^0Z_\nu^0Z_\nu^0) \\
 &+ g^2 S_w^2(\partial^\mu A_\nu \partial^\nu A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+)Z_\nu^0Z_\nu^0Z_\nu^0) - g^2 c_w S_w(\partial^\mu A_\nu \partial^\nu A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+)Z_\nu^0Z_\nu^0Z_\nu^0) \\
 &- \frac{1}{2\pi(\partial H^\mu A H_\nu H \partial^\nu H A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+)Z_\nu^0Z_\nu^0Z_\nu^0 H^\mu H_\nu H_\mu H_\nu)} + \frac{\frac{1}{2\pi(2M^2 H^2 H^3)}}{\frac{d^>em^c\gamma}{GUM_{scw}^2}} - \frac{2g_e^2 M_S^2}{\Psi\Phi\zeta} - \lambda\partial \\
 &\quad \frac{2M}{\frac{\beta_n^\xi}{\prod_\sigma \frac{h^4}{\hbar^2}}}
 \end{aligned}$$

$$\otimes \frac{\omega}{\Delta\nabla\theta}$$

$$\prod_{\pm}^+ \prod_{\pm}^\infty \prod_{\pm}^i \prod_{\pm}^j k \left(\frac{\phi_\mu^+ \phi_\nu^- \phi_\mu^- \phi_\nu^+}{\phi_\mu^+ \phi_\nu^+ \phi_\mu^+ \phi_\nu^+} \right) (\varphi\psi\omega\lambda^+ \varphi\psi\omega\lambda^- \varphi\psi\omega\lambda^- \varphi\psi\omega\lambda^+ \frac{2\varphi\psi\omega\lambda^\mu}{\varphi\psi\omega\lambda^+} \varphi\psi\omega\lambda^\nu \varphi\psi\omega\lambda^\mu \varphi\psi\omega\lambda^\nu \frac{1/2\pi\varphi\psi\omega\lambda^0}{\varphi\psi\omega\lambda^-} \varphi\psi\omega\lambda^0 \varphi\psi\omega\lambda^0 \varphi\psi\omega\lambda^0)$$

$$\begin{aligned}
 &\frac{\phi\varphi\lambda\kappa}{/2M} \sqrt{\frac{2\xi\eta}{\zeta\varepsilon\epsilon}} \frac{\delta\alpha}{o\sigma\rho} / \Psi\Omega U = \mathcal{L}_{Higgs} = \left(\partial^\mu\partial_\nu\partial^\nu\partial_\mu + \frac{1}{2ig_1 B^\mu B_\nu B^\nu B_\mu} + \frac{1}{2jg_2 B^\mu B_\nu B^\nu B_\mu} + \frac{1}{2ig_1 W^\mu W_\nu W^\nu W_\mu} + \frac{1}{2jg_2 W^\mu W_\nu W^\nu W_\mu} \right) - m_H^2 \phi^+ \phi^- - v^2 / 2v^2 \\
 &/\tau^2
 \end{aligned}$$

$$\begin{aligned}
 &\partial_i \partial^j \partial_j \partial^i f^{ab\varphi} t_{ba\omega} t^{ab\varphi} f_{ba\omega}(y) \\
 &= \frac{\partial^\theta \partial_\theta F_\sigma^\rho \gamma \beta}{\frac{\varepsilon\epsilon\theta\pi}{\Delta\nabla}} + \prod_b^a \lambda \prod_a^b \lambda H_{iggs} \\
 &- W^a W_b W^b W_a W_b^a W_a^b W_b^a W - \eta^\theta \eta_\beta \eta_{\phi\nu}^{\sigma\mu} \alpha_\eta / \mathbb{R}^4
 \end{aligned}$$

En la que la constante H_{iggs} es igual a:

$$\begin{aligned}
 \mathcal{L}_{SM} &= \frac{1}{2\partial_\nu g_\mu^a \partial_\mu g_\nu^b} - g_s f^{abc} \partial_\mu g_\nu^a g_\nu^b g_\nu^c \partial_\nu g_\mu^a g_\mu^b g_\mu^c - \frac{1}{4g_s^2 f^{abcd} f_{abcd} g_\mu^b g_\mu^c g_\mu^d g_\nu^b g_\nu^c g_\nu^d g_\nu^d} - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^- - M^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- \\
 &- \frac{1}{2\partial_\nu Z_\mu^0 \partial_\mu Z_\nu^0} - \frac{1}{2c_m^2 M^2 Z_\nu^0 Z_\nu^0} - \frac{1}{2\partial_\mu \mathcal{A}_\nu \partial^\nu \mathcal{A}_\mu} \\
 &- igc_w \left(\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+) - Z_\mu^0 Z_\nu^0 (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-) \right) \\
 &- igS_w \left(\partial_\mu \mathcal{A}_\nu \partial^\nu \mathcal{A}_\mu (W_\mu^+ W_\nu^- W_\nu^+ W_\nu^-) - \mathcal{A}_\mu (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-) - \mathcal{A}_\nu (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-) \right) \\
 &- \frac{1}{2g^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+} + g^2 c_w^2 (Z_\mu^0 W_\mu^+ W_\mu^- Z_\nu^0 W_\nu^+ W_\nu^-) \\
 &+ g^2 S_w^2 (\mathcal{A}_\mu W_\mu^+ W_\mu^- \mathcal{A}_\nu W_\nu^+ W_\nu^-) + g^2 c_w S_w (\mathcal{A}_\mu Z_\mu^0 (W_\mu^+ W_\mu^-) \mathcal{A}_\nu Z_\nu^0 (W_\nu^+ W_\nu^-) - 2\mathcal{A}_\mu W_\mu^+ W_\mu^- Z_\mu^0 \mathcal{A}_\nu W_\nu^+ W_\nu^- Z_\nu^0) - \frac{1}{2\partial_\mu \mathcal{H} \partial_\nu \mathcal{H}} - 2M^2 \alpha_h \mathcal{H}^2 \\
 &- \partial_\mu \phi^+ \partial_\nu \phi^- \partial_\nu \phi^+ \partial_\mu \phi^- - \frac{1}{2\partial_\mu \phi^0 \partial_\nu \phi^0 \partial_\nu \phi^0 \partial_\mu \phi^0}
 \end{aligned}$$



$$\begin{aligned}
& -\beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g\mathcal{H}} + \frac{1}{2}(\mathcal{H}^2 + \phi^0\phi^+\phi^0\phi^- + 2\phi^0\phi^+\phi^0\phi^-) \right) + \frac{2M^4}{g^2\alpha_h} - g\alpha_h M(\mathcal{H}^3 + \mathcal{H}\phi^0\phi^0 + 2\mathcal{H}\phi^+\phi^-) - 1/8g^2\alpha_h(\mathcal{H}^4 + (\phi^0)^4) \\
& + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4\mathcal{H}^2\phi^+\phi^- + 2(\phi^0)^2\mathcal{H}^2) - gMW_\mu^+W_\mu^-W_v^+W_v^-\mathcal{H} - \frac{1}{c_\omega^2Z_\mu^0Z_v^0\mathcal{H}} \\
& - \frac{1}{2ig} (W_\mu^+(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W_\mu^-(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0) - W_v^+(\phi^0\partial_v\phi^- - \phi^-\partial_v\phi^0) \\
& - W_v^-(\phi^0\partial_v\phi^+ - \phi^+\partial_v\phi^0)) \\
& - \frac{1}{2g} (W_\mu^+(\mathcal{H}\partial_\mu\phi^- - \phi^-\partial_\mu\mathcal{H})W_\mu^-(\mathcal{H}\partial_\mu\phi^+ - \phi^+\partial_\mu\mathcal{H})W_v^+(\mathcal{H}\partial_v\phi^- - \phi^-\partial_v\mathcal{H})W_v^-(\mathcal{H}\partial_v\phi^+ - \phi^+\partial_v\mathcal{H})) \\
& + \frac{1}{c_\omega} \left(\frac{1}{Z_\mu^0(\mathcal{H}\partial_\mu\phi^0 - \phi^0\partial_\mu\mathcal{H})Z_v^0(\mathcal{H}\partial_v\phi^0 - \phi^0\partial_v\mathcal{H})} + M \left(\frac{1}{c_\omega Z_\mu^0\partial_\mu\phi^0} + W_\mu^+\partial_\mu\phi^- + W_\mu^-\partial_\mu\phi^+ \right) \right) \\
& - \frac{igs_\omega^2}{c_\omega M Z_\mu^0(W^+\phi^- - W^-\phi^+)ig s_\omega M \mathcal{A}_\mu(W_\mu^+\phi^- + W_\mu^-\phi^+) - ig1 - \frac{2c_\omega^2}{2c_\omega Z_\mu^0(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+)}} \\
& + igs_\omega \mathcal{A}_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - 1/4g^2W_\mu^+W_\mu^-(\mathcal{H}^2 + (\phi^0)^2 + 2\phi^+\phi^-) \\
& - \frac{1}{8g^2\mathbb{1}} \frac{1}{c_\omega^2Z_\mu^0Z_v^0} \mathcal{H}^2 + \phi^{0^2} + 2 \left(\frac{1}{2s_\omega^2 - 1} \phi^+\phi^- - \frac{1}{c_\omega Z_\mu^0\phi^0(W_\mu^+\phi^- + W_\mu^-\phi^+)} - \frac{1}{c_\omega Z_\mu^0} \mathcal{H}(W_\mu^+\phi^- + W_\mu^-\phi^+) + \frac{1}{2ig^2s_\omega \mathcal{A}_\mu\phi^0(W_\mu^+\phi^- + W_\mu^-\phi^+)} + \frac{1}{2ig^2s_\omega \mathcal{A}_\mu\mathcal{H}(W_\mu^+\phi^- + W_\mu^-\phi^+)} - \frac{g^2s_\omega}{c_\omega(2c_\omega^2 - 1)Z_\mu^0\mathcal{A}_\mu\phi^+\phi^-} - g^2s_\omega^2\mathcal{A}_\mu\mathcal{A}_\nu\phi^+\phi^- + \frac{e^\lambda \rightarrow_{v^\lambda}(\varphi\partial + m_\mu^\lambda)}{2ig_s\lambda_{ij}^a(\rightarrow_{q^\sigma}\varphi^\mu \rightarrow_{q^\sigma})g_\mu^a} \rightarrow_{e^\lambda}(\varphi\partial + m_e^\lambda) \right) v^\lambda \rightarrow_{\mu_j^\lambda}(\varphi\partial + m_\mu^\lambda) - (\mu_j^\lambda) \\
& \rightarrow_{d_j^\lambda} + \\
& (\varphi\partial + m_d^\lambda)d_j^\lambda + ig s_\omega \mathcal{A}_\mu \left(- \left(\rightarrow_{e^\lambda} \varphi^\mu e^\lambda \right) + \frac{2}{3 \left(\rightarrow_{\mu_j^\lambda} \varphi^\mu \mu_j^\lambda \right)} - \frac{1}{3 \left(\rightarrow_{d^\lambda} \varphi^\mu d^\lambda \right)} + \frac{ig}{4c_\omega Z_\mu^0 \left(\rightarrow_{v^\lambda} \varphi^\mu (1 + \varphi^5) v^\lambda \right)} \right. \\
& + \left(\rightarrow_{e^\lambda} \varphi^\mu (4s_\omega^2 - 1 - \varphi^5) e^\lambda \right) + \left(\rightarrow_{d_j^\lambda} \varphi^\mu \left(\frac{4}{3s_\omega^2} - 1 - \varphi^5 \right) d_j^\lambda \right) + \left(\rightarrow_{\mu_j^\lambda} \varphi^\mu \left(1 - \frac{8}{3s_\omega^2} + \varphi^5 \right) \mu_j^\lambda \right) \\
& + \left. \frac{ig}{\sqrt{2}W_\mu^+ \left(\rightarrow_{v^\lambda} \varphi^\mu (1 + \varphi^5) \right) U^{lep^k} \xi e^k} \right) + \left(\rightarrow_{\mu_j^\lambda} \varphi^\mu (1 + \varphi^5) c_{k\lambda} d_j^k \right) \\
& + \frac{ig}{\sqrt{2}W_\mu^- \left(\rightarrow_{e^\kappa} U^{lep^\dagger} \frac{\kappa\psi\lambda}{\xi\rho\omega} - \triangle \prod_{\odot\oplus\tau}^{\oplus\ominus\gamma} \otimes \frac{\prod_{\otimes\sigma}^{\mu} ij k}{\otimes\sigma} \Omega\Psi\Phi\Delta(1 + \varphi^5)v^\lambda + \rightarrow_{d_j^\kappa} C_{\kappa}^{*\lambda} \varphi_{v\eta}^{\mu\zeta\eta} (1 + \varphi^5)\mu_j^\lambda \right)} \\
& + \frac{ig}{2M\sqrt{2}\phi^+ \left(-m_c^\kappa \left(\rightarrow_{v^\lambda} U^{lep^k} \xi e^k (1 - \varphi^5) \epsilon^\kappa \right) + m_\mu^\lambda \rightarrow_{\mu_j^\lambda} U^{lep^\dagger} (1 + \varphi^5) \epsilon^\kappa \right)} \\
& + \frac{ig}{2M\sqrt{2}\phi^- \left(m_c^\kappa \left(\rightarrow_{v^\lambda} U^{lep^k} \xi e^k (1 - \varphi^5) \epsilon^\kappa \right) \pm m_\mu^\lambda \rightarrow_{\mu_j^\lambda} U^{lep^\dagger} (1 + \varphi^5) \epsilon^\kappa \right)} - \frac{g}{M} \mathcal{H} \left(\rightarrow_{v^\lambda} v^\lambda \right)
\end{aligned}$$



$$\begin{aligned}
& - \frac{\frac{g}{2m_c^\lambda}}{\frac{M\mathcal{H}\left(\begin{smallmatrix} \rightarrow e^\lambda \\ e^\lambda \end{smallmatrix}\right)}{1}} + \frac{\frac{ig}{2m_v^\lambda}}{M\phi^0\left(\begin{smallmatrix} \rightarrow \gamma^5 v^\lambda \\ v^\lambda \end{smallmatrix}\right)} - \frac{\frac{ig}{2m_c^\lambda}}{M\phi^0\left(\begin{smallmatrix} \rightarrow \gamma^5 e^\lambda \\ e^\lambda \end{smallmatrix}\right)} - \frac{1}{4 \begin{smallmatrix} \rightarrow M_{\lambda\kappa}^R(1-\gamma_5) \\ v^\kappa \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ v^\kappa \end{smallmatrix}} \\
& + \frac{2M^2\sqrt{2}\phi^+ \left(-m_d^\kappa \left(\begin{smallmatrix} \rightarrow C_{\lambda\kappa}(1-\varphi^5)d_j^\kappa \\ \mu_j^\lambda \end{smallmatrix} \right) + m_d^\kappa \left(\begin{smallmatrix} \rightarrow C_{\lambda\kappa}(1-\varphi^5)d_j^\kappa \\ \mu_j^\lambda \end{smallmatrix} \right) \right)}{ig} \\
& + \frac{ig}{2M^2\sqrt{2}\phi^- \left(m_d^\lambda \left(\begin{smallmatrix} \rightarrow C_{\lambda\kappa}^\dagger \wedge_{\theta^*}(1+\varphi^5)\mu_j^\kappa \\ d_j^\dagger \end{smallmatrix} \right) \pm m_d^\lambda \left(\begin{smallmatrix} \rightarrow C_{\lambda\kappa}^\dagger \wedge_{\nu^*}(1+\varphi^5)\mu_j^\kappa \\ d_j^\dagger \end{smallmatrix} \right) \right)} - \frac{\frac{g}{2m_\mu^\lambda}}{M\mathcal{H}\left(\begin{smallmatrix} \rightarrow \mu_j^\lambda \\ \mu_j^\lambda \end{smallmatrix}\right)} \\
& - \frac{\frac{g}{2m_d^\lambda}}{M} \mathcal{H}\left(\begin{smallmatrix} \rightarrow d_j^\lambda \\ d_j^\lambda \end{smallmatrix}\right) + \frac{\frac{ig}{2m_\mu^\lambda}}{M} \phi^0\left(\begin{smallmatrix} \rightarrow \gamma^5 \mu_j^\lambda \\ \mu_j^\lambda \end{smallmatrix}\right) - \frac{\frac{ig}{2m_d^\lambda}}{M} \phi^0\left(\begin{smallmatrix} \rightarrow \gamma^5 d_j^\lambda \\ d_j^\lambda \end{smallmatrix}\right) + \begin{smallmatrix} \rightarrow \partial^2 G^a + g_s f^{abc} \partial_\mu \\ c^a \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ c^a \end{smallmatrix} G^b g_\mu^c \\
& + \begin{smallmatrix} \rightarrow (\partial^2 - M^2)\alpha^+ \\ \alpha^+ \end{smallmatrix} + \begin{smallmatrix} \rightarrow (\partial^2 - M^2)\alpha^- \\ \alpha^- \end{smallmatrix} + \begin{smallmatrix} \rightarrow \left(\partial^2 - \frac{M^2}{c_\omega^2}\right)\alpha^0 \\ \alpha^0 \end{smallmatrix} + \begin{smallmatrix} \rightarrow \partial^2 b \\ b \end{smallmatrix} + igc_\omega W_\mu^+ \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^0 \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^- \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^- \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^0 \end{smallmatrix} \right) \\
& + ig s_\omega W_\mu^+ \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ b \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^- \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ a \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ b^- \end{smallmatrix} \right) + ig c_\omega W_\mu^- \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^+ \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^0 \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^0 \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^+ \end{smallmatrix} \right) + ig s_\omega W_\mu^- \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^+ \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^0 \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^0 \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^+ \end{smallmatrix} \right) \\
& + ig c_\omega Z_\mu^0 \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^+ \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^- \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^- \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^+ \end{smallmatrix} \right) + ig s_\omega \mathcal{A}_\mu \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^+ \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^- \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^- \end{smallmatrix} \begin{smallmatrix} \rightarrow \\ \alpha^+ \end{smallmatrix} \right) - 1/2gM \left(\frac{\begin{smallmatrix} \rightarrow \alpha^+ \mathcal{H} \hbar \mathbb{R}^4 \\ \alpha^+ \end{smallmatrix}}{h} + \begin{smallmatrix} \rightarrow \\ \alpha^- \end{smallmatrix} \mathcal{H} \right) \\
& + 1 - \frac{2c_3^2}{2c_\omega igM \left(\begin{smallmatrix} \rightarrow a^0 \phi^+ \\ \alpha^+ \end{smallmatrix} - \begin{smallmatrix} \rightarrow a^0 \phi^- \\ \alpha^- \end{smallmatrix} \right)} + \frac{1}{2c_\omega igM \left(\begin{smallmatrix} \rightarrow a^- \phi^+ \\ \alpha^0 \end{smallmatrix} - \begin{smallmatrix} \rightarrow a^+ \phi^- \\ \alpha^0 \end{smallmatrix} \right)} \\
& + igM s_\omega \left(\begin{smallmatrix} \rightarrow a^- \phi^+ \\ \alpha^0 \end{smallmatrix} - \begin{smallmatrix} \rightarrow a^+ \phi^- \\ \alpha^0 \end{smallmatrix} \right) + 1/2igM \left(\begin{smallmatrix} \rightarrow a^+ \phi^0 \\ \alpha^+ \end{smallmatrix} - \begin{smallmatrix} \rightarrow a^- \phi^0 \\ \alpha^- \end{smallmatrix} \right)
\end{aligned}$$

$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

$$\mathcal{L}_{SBS} = (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) - V(\Phi)$$

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$|\Phi|^2 = \Phi^\dagger \Phi = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}$$

$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \longrightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Phi(x) = \frac{1}{\sqrt{2}} e^{i \frac{\vec{\xi}(x) \cdot \vec{\tau}}{v}} \begin{pmatrix} 0 \\ v + \mathbf{h}(x) \end{pmatrix}$$



$$U(\xi) = e^{-i \frac{\vec{\xi}(x) \cdot \vec{\tau}}{v}}$$

$$\begin{aligned} \Phi' &= U(\xi) \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \mathbf{h}(x) \end{pmatrix} \\ \left(\frac{\vec{\tau} \cdot \vec{W}'_\mu}{2} \right) &= U(\xi) \left(\frac{\vec{\tau} \cdot \vec{W}_\mu}{2} \right) U^{-1}(\xi) - \frac{i}{g} (\partial_\mu U(\xi)) U^{-1}(\xi) \\ B'_\mu &= B_\mu \end{aligned}$$

$$\mathcal{L} = \mathcal{L}_{bos.} + \mathcal{L}_{ferm.} + \mathcal{L}_{SBS}$$

$$(\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) = \frac{v^2}{8} [g^2 (W_{1\mu}^2 + W_{2\mu}^2) + (gW_{3\mu} - g' B_\mu)^2]$$

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}} (W_\mu^1 \mp W_\mu^2) \\ Z_\mu &= \cos \theta_W W_\mu^3 - \sin \theta_W B_\mu \\ A_\mu &= \sin \theta_W W_\mu^3 + \cos \theta_W B_\mu \end{aligned}$$

$$\tan \theta_W \equiv \frac{g'}{g}$$

$$M_W = \frac{1}{2} g v$$

$$M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$$

$$g = \frac{e}{\sin \theta_W}$$

$$g' = \frac{e}{\cos \theta_W}$$

$$m_H^2 = 2\lambda v^2$$

$$\mu \rightarrow \nu_\mu \bar{\nu}_e e$$

$$v = (\sqrt{2} G_F)^{-\frac{1}{2}}$$

$$\mathcal{L}_{YW} = \lambda_e \bar{\ell}_L \Phi e_R + \lambda_u \bar{q}_L \tilde{\Phi} u_R + \lambda_d \bar{q}_L \Phi d_R + \text{h.c.}$$



$$\ell_L = \begin{pmatrix} e \\ \nu_e \end{pmatrix}_L, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$\begin{aligned} \ell'_L &= U(\xi)\ell_L; & e'_R &= e_R \\ q'_L &= U(\xi)q_L; & u'_R &= u_R; d'_R = d \end{aligned}$$

$$m_e = \lambda_e \frac{v}{\sqrt{2}}$$

$$m_u = \lambda_u \frac{v}{\sqrt{2}}$$

$$m_d = \lambda_d \frac{v}{\sqrt{2}}$$

O es igual a:

$$\mathcal{L}_{Higgs} = \overline{\left([\partial_\mu + \frac{1}{2}ig_1 B_\mu + \frac{1}{2}ig_2 \mathbf{W}_\mu] \phi \right)} \left([\partial_\mu + \frac{1}{2}ig_1 B_\mu + \frac{1}{2}ig_2 \mathbf{W}_\mu] \phi \right) - \frac{m_H^2 \left(\bar{\phi}\phi - \frac{v^2}{2} \right)^2}{2v^2}$$

$$\begin{aligned}
\mathcal{L}_{SM}(y) &\equiv (a, b) \simeq (b, a) \\
&= -\frac{1}{2\pi\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2g_\mu^ag_\nu^bg_\nu^c} - g_s f^{ab} f_{ab} \partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2g_\mu^ag_\nu^bg_\nu^c - \frac{1}{4\pi g_s^2 f^{cd} f_{cd} \partial^\mu\partial_\nu\partial_\mu\partial_\nu^2\partial_\mu^2g_\mu^cg_\nu^dg_\nu^d} - \partial^\mu W_\mu \partial^\nu W_\nu \\
&- M^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ - \frac{1}{2\pi\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2Z_\nu^0Z_\nu^0Z_\nu^0} - \frac{1}{2c_m^2 M^2 Z_\mu^0 Z_\nu^0 Z_\mu^0 Z_\nu^0} - \frac{1}{2\partial^\mu A_\nu \partial^\nu A_\mu} \\
&- igc_w (\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2Z_\nu^0Z_\nu^0Z_\nu^0(W_\mu^+W_\nu^-W_\mu^-W_\nu^+)) - Z_\mu^0(\partial^\mu\partial_\nu W_\mu^+W_\nu^-W_\mu^+W_\nu^-) + Z_\nu^0(\partial^\nu\partial_\nu W_\nu^+W_\nu^-W_\nu^+W_\nu^-) \\
&- igS_w(\partial^\mu A_\nu\partial^\nu A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+))Z_\mu^0Z_\nu^0Z_\nu^0 - A_\mu(\partial^\mu\partial_\nu W_\mu^+W_\nu^-W_\mu^+W_\nu^-Z_\mu^0Z_\nu^0) + A_\nu(\partial^\nu\partial_\nu W_\nu^+W_\nu^-W_\nu^+W_\nu^-Z_\nu^0Z_\mu^0) \\
&- \frac{1}{2g^2(\partial^\mu A_\nu\partial^\nu A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+Z_\mu^0Z_\nu^0Z_\nu^0))} + g^2c_w^2(\partial^\mu A_\nu\partial^\nu A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+Z_\mu^0Z_\nu^0Z_\nu^0)) \\
&+ g^2S_w^2(\partial^\mu A_\nu\partial^\nu A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+Z_\mu^0Z_\nu^0Z_\nu^0)) - g^2c_wS_w(\partial^\mu A_\nu\partial^\nu A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+Z_\mu^0Z_\nu^0Z_\nu^0)) \\
&- \frac{1}{2\pi(\partial H^\mu A H_\nu H^\nu H A_\mu(W_\mu^+W_\nu^-W_\mu^-W_\nu^+W_\mu^+W_\nu^-W_\mu^-W_\nu^+Z_\mu^0Z_\nu^0Z_\nu^0 H^\mu H_\nu H^\nu H_\mu))} + \frac{\frac{1}{2\pi(2M^2H^2H^3)}}{\frac{d^>em^c\gamma}{GUM_{scw}^2}} - \frac{2g_e^2M_S^2}{\Psi\Phi\zeta} - \lambda\partial \\
&\quad \frac{\frac{2M}{\beta_n^\xi}}{\prod_\sigma \frac{\hbar^4}{\hbar^2}} \\
&\quad \frac{\omega}{\Delta\nabla\theta} \\
&\quad \Pi_\pm^\dagger \infty \prod_j^i k \left(\frac{\phi_\mu^+\phi_\nu^-\phi_\mu^-\phi_\nu^+}{\phi_\mu^+\phi_\nu^-\phi_\mu^-\phi_\nu^+} \right) \left(\varphi\psi\omega\lambda_\mu^+\varphi\psi\omega\lambda_\nu^-\varphi\psi\omega\lambda_\mu^-\varphi\psi\omega\lambda_\nu^+ \frac{2\varphi\psi\omega\lambda^\mu}{\varphi\psi\omega\lambda^+} \varphi\psi\omega\lambda^\nu \varphi\psi\omega\lambda^\mu \varphi\psi\omega\lambda^\nu \frac{1}{\varphi\psi\omega\lambda} \frac{1}{\mu} \varphi\psi\omega\lambda_\nu^0\varphi\psi\omega\lambda_\mu^0\varphi\psi\omega\lambda_\nu^0 \right) \\
&\otimes \frac{\phi\varphi\lambda\kappa}{\sqrt{\frac{2\xi\eta}{\zeta\varepsilon\varepsilon} \frac{\delta\alpha}{\Psi\Omega}}} \mathcal{U} = \mathcal{L}_{Higgs} = \left(\partial^\mu\partial_\nu\partial^\nu\partial_\mu + \frac{1}{2ig_1B^\mu B_\nu B^\nu B_\mu} + \frac{1}{2jg_2B^\mu B_\nu B^\nu B_\mu} + \frac{1}{2ig_1W^\mu W_\nu W^\nu W_\mu} + \frac{1}{2jg_2W^\mu W_\nu W^\nu W_\mu} \right) - m_H^2\phi^+\phi - \frac{v^2}{2v^2} \\
&/\tau^2
\end{aligned}$$

$$\begin{aligned}
&\partial_i\partial^j\partial_j\partial^i f^{ab\varphi} t_{ba\omega} t^{ab\varphi} f_{ba\omega}(z) \\
&= \frac{\frac{\partial^\theta\partial_\theta F_\sigma^\rho\gamma\beta}{\varepsilon\varepsilon\partial\pi}}{\tau} + \prod_b^a \lambda \prod_a^b \lambda H_{iggs} \\
&- W^a W_b W^b W_a W_b^a W_a^b W_a^b W - \eta^\theta \eta_\beta \eta_{\phi\nu}^{\sigma\mu} \alpha_\nu \eta / \mathbb{R}^4
\end{aligned}$$

En la que la constante H_{iggs} es igual a:

$$\begin{aligned}
\mathcal{L}_{SM} &= \frac{1}{2\partial_\nu g_\mu^a \partial_\mu g_\nu^b} - g_s f^{abc} \partial_\mu g_\nu^a g_\nu^b g_\nu^c \partial_\nu g_\mu^a g_\mu^b g_\mu^c - \frac{1}{4g_s^2 f^{abcd} f_{abcd} g_\mu^b g_\nu^c g_\mu^d g_\nu^d g_\mu^b g_\nu^c g_\mu^d g_\nu^d} - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^- - M^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ \\
&- \frac{1}{2\partial_\nu Z_\mu^0 \partial_\mu Z_\nu^0} - \frac{1}{2c_m^2 M^2 Z_\mu^0 Z_\nu^0} - \frac{1}{2\partial_\mu \mathcal{A}_\nu \partial^\nu \mathcal{A}_\mu} \\
&- igc_w (\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+) - Z_\mu^0 Z_\nu^0 (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-)) \\
&- igS_w (\partial_\mu \mathcal{A}_\nu \partial^\nu \mathcal{A}_\mu (W_\mu^+ W_\nu^- W_\nu^+ W_\nu^-) - \mathcal{A}_\mu (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-) - \mathcal{A}_\nu (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-)) \\
&- \frac{1}{2g^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+} + g^2 c_w^2 (Z_\mu^0 W_\mu^+ W_\nu^- Z_\nu^0 W_\nu^+ W_\nu^-) \\
&+ g^2 S_w^2 (\mathcal{A}_\mu W_\mu^+ W_\nu^- \mathcal{A}_\nu W_\nu^+ W_\nu^-) + g^2 c_w S_w (\mathcal{A}_\mu Z_\mu^0 (W_\mu^+ W_\nu^-) \mathcal{A}_\nu Z_\nu^0 (W_\nu^+ W_\nu^-) - 2\mathcal{A}_\mu W_\mu^+ W_\nu^- Z_\mu^0 \mathcal{A}_\nu W_\nu^+ W_\nu^- Z_\nu^0) - \frac{1}{2\partial_\mu \mathcal{H} \partial_\nu \mathcal{H}} - 2M^2 \alpha_h \mathcal{H}^2 \\
&- \partial_\mu \phi^+ \partial_\nu \phi^- \partial_\nu \phi^+ \partial_\nu \phi^- - \frac{1}{2\partial_\mu \phi^0 \partial_\nu \phi^0 \partial_\nu \phi^0 \partial_\nu \phi^0}
\end{aligned}$$



$$\begin{aligned}
& -\beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g\mathcal{H}} + \frac{1}{2}(\mathcal{H}^2 + \phi^0\phi^+\phi^0\phi^- + 2\phi^0\phi^+\phi^0\phi^-) \right) + \frac{2M^4}{g^2\alpha_h} - g\alpha_h M(\mathcal{H}^3 + \mathcal{H}\phi^0\phi^0 + 2\mathcal{H}\phi^+\phi^-) - 1/8g^2\alpha_h(\mathcal{H}^4 + (\phi^0)^4) \\
& + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4\mathcal{H}^2\phi^+\phi^- + 2(\phi^0)^2\mathcal{H}^2) - gMW_\mu^+W_\mu^-W_v^+W_v^-\mathcal{H} - \frac{1}{c_\omega^2Z_\mu^0Z_v^0\mathcal{H}} \\
& - \frac{1}{2ig} (W_\mu^+(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W_\mu^-(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0) - W_v^+(\phi^0\partial_v\phi^- - \phi^-\partial_v\phi^0) \\
& - W_v^-(\phi^0\partial_v\phi^+ - \phi^+\partial_v\phi^0)) \\
& - \frac{1}{2g} (W_\mu^+(\mathcal{H}\partial_\mu\phi^- - \phi^-\partial_\mu\mathcal{H})W_\mu^-(\mathcal{H}\partial_\mu\phi^+ - \phi^+\partial_\mu\mathcal{H})W_v^+(\mathcal{H}\partial_v\phi^- - \phi^-\partial_v\mathcal{H})W_v^-(\mathcal{H}\partial_v\phi^+ - \phi^+\partial_v\mathcal{H})) \\
& + \frac{1}{c_\omega} \left(\frac{1}{Z_\mu^0(\mathcal{H}\partial_\mu\phi^0 - \phi^0\partial_\mu\mathcal{H})Z_v^0(\mathcal{H}\partial_v\phi^0 - \phi^0\partial_v\mathcal{H})} + M \left(\frac{1}{c_\omega Z_\mu^0\partial_\mu\phi^0} + W_\mu^+\partial_\mu\phi^- + W_\mu^-\partial_\mu\phi^+ \right) \right) \\
& - \frac{2c_\omega^2}{c_\omega M Z_\mu^0(W^+\phi^- - W^-\phi^+)ig s_\omega M \mathcal{A}_\mu(W_\mu^+\phi^- + W_\mu^-\phi^+) - ig1 - \frac{2c_\omega^2}{2c_\omega Z_\mu^0(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+)}} \\
& + ig s_\omega \mathcal{A}_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - 1/4g^2W_\mu^+W_\mu^-(\mathcal{H}^2 + (\phi^0)^2 + 2\phi^+\phi^-) \\
& - \frac{1}{8g^2\mathbb{1}} \frac{1}{c_\omega^2Z_\mu^0Z_v^0} \mathcal{H}^2 + \frac{\phi^0)^2 + 2}{2g^2s_\omega^2} \left(\frac{1}{c_\omega Z_\mu^0\phi^0(W_\mu^+\phi^- + W_\mu^-\phi^+)} - \frac{2ig^2s_\omega^2}{c_\omega Z_\mu^0} \mathcal{H}(W_\mu^+\phi^- + W_\mu^-\phi^+) + \right. \\
& \left. \frac{1}{2g^2s_\omega \mathcal{A}_\mu\phi^0(W_\mu^+\phi^- + W_\mu^-\phi^+)} + \frac{1}{2ig^2s_\omega \mathcal{A}_\mu\mathcal{H}(W_\mu^+\phi^- + W_\mu^-\phi^+)} - \frac{g^2s_\omega}{c_\omega(2c_\omega^2 - 1)Z_\mu^0\mathcal{A}_\mu\phi^+\phi^-} - g^2s_\omega^2\mathcal{A}_\mu\mathcal{A}_\nu\phi^+\phi^- + \right) e^\lambda \xrightarrow{v^\lambda} (\varphi\partial + m_\mu^\lambda) v^\lambda \xrightarrow{\mu_j^\lambda} (\varphi\partial + m_\mu^\lambda) - (\mu_j^\lambda) \\
& \xrightarrow{d_j^\lambda} + \left(\frac{1}{2ig_s\lambda_{ij}^a} \left(\xrightarrow{q^\sigma} \varphi^\mu \xrightarrow{q^\sigma} \right) g_\mu^a \xrightarrow{e^\lambda} (\varphi\partial + m_e^\lambda) \right) \\
& (\varphi\partial + m_d^\lambda)d_j^\lambda + ig s_\omega \mathcal{A}_\mu \left(- \left(\xrightarrow{e^\lambda} \varphi^\mu e^\lambda \right) + \frac{2}{3 \left(\xrightarrow{\mu_j^\lambda} \varphi^\mu \mu_j^\lambda \right)} - \frac{1}{3 \left(\xrightarrow{d^\lambda} \varphi^\mu d^\lambda \right)} + \frac{ig}{4c_\omega Z_\mu^0 \left(\xrightarrow{v^\lambda} \varphi^\mu (1 + \varphi^5) v^\lambda \right)} \right. \\
& + \left(\xrightarrow{e^\lambda} \varphi^\mu (4s_\omega^2 - 1 - \varphi^5) e^\lambda \right) + \left(\xrightarrow{d_j^\lambda} \varphi^\mu \left(\frac{4}{3s_\omega^2} - 1 - \varphi^5 \right) d_j^\lambda \right) + \left(\xrightarrow{\mu_j^\lambda} \varphi^\mu \left(1 - \frac{8}{3s_\omega^2} + \varphi^5 \right) \mu_j^\lambda \right) \\
& + \left. \frac{ig}{\sqrt{2}W_\mu^+ \left(\xrightarrow{v^\lambda} \varphi^\mu (1 + \varphi^5) \right) U^{lep^k} \xi^k} \right) + \left(\xrightarrow{\mu_j^\lambda} \varphi^\mu (1 + \varphi^5) c_{k\lambda} d_j^k \right) \\
& + \frac{ig}{\sqrt{2}W_\mu^- \left(\xrightarrow{e^\kappa} U^{lep^\dagger} \frac{\kappa\rho}{\xi} - \triangle \prod_{\odot \oplus \gamma}^{\oplus \ominus} \otimes \frac{\prod_{\otimes \sigma}^{\mu} ij k}{\otimes} \Omega \Psi \Phi \Delta (1 + \varphi^5) v^\lambda + \xrightarrow{d_j^\kappa} C_{\kappa}^{*\lambda} \varphi_{\nu\eta}^{\mu\zeta\eta} (1 + \varphi^5) \mu_j^\lambda \right)} \\
& + \frac{ig}{2M\sqrt{2}\phi^+ \left(-m_c^\kappa \left(\xrightarrow{v^\lambda} U^{lep^k} \xi^k (1 - \varphi^5) \epsilon^\kappa \right) + m_\mu^\lambda \xrightarrow{\mu_j^\lambda} U^{lep^\dagger} (1 + \varphi^5) \epsilon^\kappa \right)} \\
& + \frac{ig}{2M\sqrt{2}\phi^- \left(m_c^\kappa \left(\xrightarrow{v^\lambda} U^{lep^k} \xi^k (1 - \varphi^5) \epsilon^\kappa \right) \pm m_\mu^\lambda \xrightarrow{\mu_j^\lambda} U^{lep^\dagger} (1 + \varphi^5) \epsilon^\kappa \right)} - \frac{g}{M} \mathcal{H} \left(\xrightarrow{v^\lambda} v^\lambda \right)
\end{aligned}$$

$$\begin{aligned}
& - \frac{\frac{g}{2m_c^\lambda}}{\frac{M\mathcal{H}\left(\begin{smallmatrix} \rightarrow e^\lambda \\ e^\lambda \end{smallmatrix}\right)}{1}} + \frac{\frac{ig}{2m_v^\lambda}}{M\phi^0\left(\begin{smallmatrix} \rightarrow \gamma^5 v^\lambda \\ v^\lambda \end{smallmatrix}\right)} - \frac{\frac{ig}{2m_c^\lambda}}{M\phi^0\left(\begin{smallmatrix} \rightarrow \gamma^5 e^\lambda \\ e^\lambda \end{smallmatrix}\right)} - \frac{1}{4 \begin{smallmatrix} \rightarrow M_{\lambda\kappa}^R(1-\gamma_5) \\ v^\kappa \end{smallmatrix} \rightarrow v^\kappa} \\
& + \frac{2M^2\sqrt{2}\phi^+ \left(-m_d^\kappa \left(\begin{smallmatrix} \rightarrow C_{\lambda\kappa}(1-\varphi^5)d_j^\kappa \\ \mu_j^\lambda \end{smallmatrix} \right) + m_d^\kappa \left(\begin{smallmatrix} \rightarrow C_{\lambda\kappa}(1-\varphi^5)d_j^\kappa \\ \mu_j^\lambda \end{smallmatrix} \right) \right)}{ig} \\
& + \frac{2M^2\sqrt{2}\phi^- \left(m_d^\lambda \left(\begin{smallmatrix} \rightarrow C_{\lambda\kappa}^\dagger \wedge_{\theta^*}(1+\varphi^5)\mu_j^\kappa \\ d_j^\dagger \end{smallmatrix} \right) \pm m_d^\lambda \left(\begin{smallmatrix} \rightarrow C_{\lambda\kappa}^\dagger \wedge_{\nu^*}(1+\varphi^5)\mu_j^\kappa \\ d_j^\dagger \end{smallmatrix} \right) \right)}{ig} - \frac{\frac{g}{2m_\mu^\lambda}}{M\mathcal{H}\left(\begin{smallmatrix} \rightarrow \mu_j^\lambda \\ \mu_j^\lambda \end{smallmatrix}\right)} \\
& - \frac{\frac{g}{2m_d^\lambda}}{M} \mathcal{H}\left(\begin{smallmatrix} \rightarrow d_j^\lambda \\ d_j^\lambda \end{smallmatrix}\right) + \frac{\frac{ig}{2m_\mu^\lambda}}{M} \phi^0\left(\begin{smallmatrix} \rightarrow \gamma^5 \mu_j^\lambda \\ \mu_j^\lambda \end{smallmatrix}\right) - \frac{\frac{ig}{2m_d^\lambda}}{M} \phi^0\left(\begin{smallmatrix} \rightarrow \gamma^5 d_j^\lambda \\ d_j^\lambda \end{smallmatrix}\right) + \begin{smallmatrix} \rightarrow \partial^2 G^a + g_s f^{abc} \partial_\mu \\ G^a \end{smallmatrix} \begin{smallmatrix} \rightarrow G^b g_\mu^c \\ G^a \end{smallmatrix} \\
& + \begin{smallmatrix} \rightarrow (\partial^2 - M^2)\alpha^+ \\ \alpha^+ \end{smallmatrix} + \begin{smallmatrix} \rightarrow (\partial^2 - M^2)\alpha^- \\ \alpha^- \end{smallmatrix} + \begin{smallmatrix} \rightarrow \left(\partial^2 - \frac{M^2}{c_\omega^2}\right)\alpha^0 \\ \alpha^0 \end{smallmatrix} + \begin{smallmatrix} \rightarrow \partial^2 b \\ b \end{smallmatrix} + igc_\omega W_\mu^+ \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^0 \end{smallmatrix} \begin{smallmatrix} \rightarrow \rightarrow \\ \alpha^- \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^- \end{smallmatrix} \begin{smallmatrix} \rightarrow \rightarrow \\ \alpha^0 \end{smallmatrix} \right) \\
& + ig s_\omega W_\mu^+ \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ b \end{smallmatrix} \begin{smallmatrix} \rightarrow \alpha^- \\ \alpha^- \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^- \end{smallmatrix} \begin{smallmatrix} \rightarrow b^- \\ \alpha^- \end{smallmatrix} \right) + ig c_\omega W_\mu^- \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^+ \end{smallmatrix} \begin{smallmatrix} \rightarrow \rightarrow \\ \alpha^0 \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^0 \end{smallmatrix} \begin{smallmatrix} \rightarrow \rightarrow \\ \alpha^+ \end{smallmatrix} \right) + ig s_\omega W_\mu^- \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^+ \end{smallmatrix} \begin{smallmatrix} \rightarrow \rightarrow \\ \alpha^0 \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^0 \end{smallmatrix} \begin{smallmatrix} \rightarrow \rightarrow \\ \alpha^+ \end{smallmatrix} \right) \\
& + ig c_\omega Z_\mu^0 \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^+ \end{smallmatrix} \begin{smallmatrix} \rightarrow \rightarrow \\ \alpha^- \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^- \end{smallmatrix} \begin{smallmatrix} \rightarrow \rightarrow \\ \alpha^+ \end{smallmatrix} \right) + ig s_\omega \mathcal{A}_\mu \left(\begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^+ \end{smallmatrix} \begin{smallmatrix} \rightarrow \rightarrow \\ \alpha^- \end{smallmatrix} - \begin{smallmatrix} \rightarrow \partial_\mu \\ \alpha^- \end{smallmatrix} \begin{smallmatrix} \rightarrow \rightarrow \\ \alpha^+ \end{smallmatrix} \right) - 1/2gM \left(\frac{\begin{smallmatrix} \rightarrow \alpha^+ \mathcal{H} \hbar \mathbb{R}^4 \\ \alpha^+ \end{smallmatrix}}{h} + \begin{smallmatrix} \rightarrow \alpha^- \mathcal{H} \\ \alpha^- \end{smallmatrix} \right) \\
& + 1 - \frac{2c_3^2}{2c_\omega igM \left(\begin{smallmatrix} \rightarrow a^0 \phi^+ \\ \alpha^+ \end{smallmatrix} - \begin{smallmatrix} \rightarrow a^0 \phi^- \\ \alpha^- \end{smallmatrix} \right)} + \frac{1}{2c_\omega igM \left(\begin{smallmatrix} \rightarrow a^- \phi^+ \\ \alpha^0 \end{smallmatrix} - \begin{smallmatrix} \rightarrow a^+ \phi^- \\ \alpha^0 \end{smallmatrix} \right)} \\
& + igM s_\omega \left(\begin{smallmatrix} \rightarrow a^- \phi^+ \\ \alpha^0 \end{smallmatrix} - \begin{smallmatrix} \rightarrow a^+ \phi^- \\ \alpha^0 \end{smallmatrix} \right) + 1/2igM \left(\begin{smallmatrix} \rightarrow a^+ \phi^0 \\ \alpha^+ \end{smallmatrix} - \begin{smallmatrix} \rightarrow a^- \phi^0 \\ \alpha^- \end{smallmatrix} \right)
\end{aligned}$$

$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

$$\mathcal{L}_{SBS} = (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) - V(\Phi)$$

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$|\Phi|^2 = \Phi^\dagger \Phi = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}$$

$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \longrightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Phi(x) = \frac{1}{\sqrt{2}} e^{i \frac{\vec{\xi}(x) \cdot \vec{\tau}}{v}} \begin{pmatrix} 0 \\ v + \mathbf{h}(x) \end{pmatrix}$$



$$U(\xi) = e^{-i \frac{\vec{\xi}(x) \cdot \vec{\tau}}{v}}$$

$$\begin{aligned} \Phi' &= U(\xi) \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \mathbf{h}(x) \end{pmatrix} \\ \left(\frac{\vec{\tau} \cdot \vec{W}'_\mu}{2} \right) &= U(\xi) \left(\frac{\vec{\tau} \cdot \vec{W}_\mu}{2} \right) U^{-1}(\xi) - \frac{i}{g} (\partial_\mu U(\xi)) U^{-1}(\xi) \\ B'_\mu &= B_\mu \end{aligned}$$

$$\mathcal{L} = \mathcal{L}_{bos.} + \mathcal{L}_{ferm.} + \mathcal{L}_{SBS}$$

$$(\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) = \frac{v^2}{8} [g^2 (W_{1\mu}^2 + W_{2\mu}^2) + (gW_{3\mu} - g'B_\mu)^2]$$

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}} (W_\mu^1 \mp W_\mu^2) \\ Z_\mu &= \cos \theta_W W_\mu^3 - \sin \theta_W B_\mu \\ A_\mu &= \sin \theta_W W_\mu^3 + \cos \theta_W B_\mu \end{aligned}$$

$$\tan \theta_W \equiv \frac{g'}{g}$$

$$M_W = \frac{1}{2} g v$$

$$M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$$

$$g = \frac{e}{\sin \theta_W}$$

$$g' = \frac{e}{\cos \theta_W}$$

$$m_H^2 = 2\lambda v^2$$

$$\mu \rightarrow \nu_\mu \bar{\nu}_e e$$

$$v = (\sqrt{2} G_F)^{-\frac{1}{2}}$$

$$\mathcal{L}_{YW} = \lambda_e \bar{\ell}_L \Phi e_R + \lambda_u \bar{q}_L \tilde{\Phi} u_R + \lambda_d \bar{q}_L \Phi d_R + \text{h.c.}$$



$$\ell_L = \begin{pmatrix} e \\ \nu_e \end{pmatrix}_L, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$\begin{aligned} \ell'_L &= U(\xi)\ell_L; & e'_R &= e_R \\ q'_L &= U(\xi)q_L; & u'_R &= u_R; d'_R = d \end{aligned}$$

$$m_e = \lambda_e \frac{v}{\sqrt{2}}$$

$$m_u = \lambda_u \frac{v}{\sqrt{2}}$$

$$m_d = \lambda_d \frac{v}{\sqrt{2}}$$

O es igual a:

$$\mathcal{L}_{Higgs} = \overline{\left([\partial_\mu + \frac{1}{2}ig_1B_\mu + \frac{1}{2}ig_2\mathbf{W}_\mu]\phi \right)} \left([\partial_\mu + \frac{1}{2}ig_1B_\mu + \frac{1}{2}ig_2\mathbf{W}_\mu]\phi \right) - \frac{m_H^2 \left(\bar{\phi}\phi - \frac{v^2}{2} \right)^2}{2v^2}$$

$$\begin{aligned}
\mathcal{L}_{SM}(z) &\equiv (a, b) \simeq (b, a) \\
&= -\frac{1}{2\pi\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2\partial_\nu^2g_\mu^ag_\nu^bg_\nu^b} - g_s f^{ab} f_{ab} \partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2g_\mu^ag_\nu^bg_\nu^b - \frac{1}{4\pi g_s^2 f^{cd} f_{cd} \partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2g_\mu^cg_\nu^dg_\nu^d} \\
&- \partial^\mu W_\mu \partial^\nu W_\nu - M^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_+^\mu W_-^\nu W_-^\mu W_+^\nu - \frac{1}{2\pi\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2Z_\mu^0Z_\nu^0Z_0^\mu Z_0^\nu} - \frac{1}{2c_m^2 M^2 Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu} - \frac{1}{2\partial^\mu A_\nu \partial^\nu A_\mu} \\
&- igc_w (\partial^\mu\partial_\nu\partial^\nu\partial_\mu\partial_\nu^2\partial_\mu^2Z_\mu^0Z_\nu^0Z_0^\mu Z_0^\nu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+)) - Z_\mu^0 (\partial^\mu\partial_\nu W_\mu^+ W_\nu^- W_+^\mu W_-^\nu) + Z_\nu^0 (\partial^\nu\partial_\mu W_\nu^+ W_\mu^- W_+^\nu W_-^\mu) \\
&- igS_w (\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_+^\mu W_-^\nu W_-^\mu W_+^\nu) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu) - A_\mu (\partial^\mu\partial_\nu W_\mu^+ W_\nu^- W_+^\mu W_-^\nu Z_\mu^0 Z_\nu^0) \\
&+ A_\nu (\partial^\nu\partial_\mu W_\nu^+ W_\mu^- W_+^\nu W_-^\mu Z_\nu^0 Z_0^\nu) - \frac{1}{2g^2 (\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_+^\mu W_-^\nu W_-^\mu W_+^\nu) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu)} \\
&+ g^2 c_w^2 (\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_+^\mu W_-^\nu W_-^\mu W_+^\nu) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu) \\
&+ g^2 S_w^2 (\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_+^\mu W_-^\nu W_-^\mu W_+^\nu) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu) \\
&- g^2 c_w S_w (\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_+^\mu W_-^\nu W_-^\mu W_+^\nu) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu) \\
&- \frac{1}{2\pi (\partial H^\mu A_\nu H \partial^\nu H A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_+^\mu W_-^\nu W_-^\mu W_+^\nu) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu H^\mu H_\nu H_\mu^\mu H_\nu^\nu)} + \frac{\frac{1}{2\pi(2M^2 H^2 H^3)}}{\frac{d^\nu em^c \gamma}{GUM_{scw}^2}} - \frac{2g_c^2 M_s^2}{\Psi\Phi\zeta} - \lambda\delta \\
&\quad \frac{\beta_\eta^\xi}{\Pi_\sigma^\rho \frac{\hbar^4}{\hbar^2}}
\end{aligned}$$

$$\begin{aligned}
&\otimes \frac{\omega}{\Delta\nabla\theta} \\
&/ \prod_{\underline{\equiv}}^{\dagger} \infty \prod_j^i k \left(\frac{\phi_\mu^+ \phi_\nu^- \phi_\mu^- \phi_\nu^+}{\phi_\mu^0 \phi_\nu^0 \phi_0^\mu \phi_0^\nu} \right) (\varphi\psi\omega\lambda_\mu^+ \varphi\psi\omega\lambda_\nu^- \varphi\psi\omega\lambda_\mu^- \varphi\psi\omega\lambda_\nu^+ \frac{2\varphi\psi\omega\lambda^\mu}{\varphi\psi\omega\lambda} + \varphi\psi\omega\lambda_\nu^2 \varphi\psi\omega\lambda_\mu^2 \varphi\psi\omega\lambda_\nu^+ \frac{1/2\pi\varphi\psi\omega\lambda^0}{\varphi\psi\omega\lambda} \varphi\psi\omega\lambda_\nu^0 \varphi\psi\omega\lambda_0^\mu \varphi\psi\omega\lambda_\nu^0) \\
&/ 2M \sqrt{\frac{\frac{2\xi\eta}{\zeta\epsilon\epsilon}}{\frac{\delta\alpha}{\sigma\rho}}} / \Psi\Omega\mathcal{U} = \mathcal{L}_{Higgs} \\
&= \left(\partial^\mu\partial_\nu\partial^\nu\partial_\mu + \frac{1}{2ig_1 B^\mu B_\nu B^\nu B_\mu} + \frac{1}{2jg_2 B^\mu B_\nu B^\nu B_\mu} + \frac{1}{2ig_1 W^\mu W_\nu W^\nu W_\mu} + \frac{1}{2jg_2 W^\mu W_\nu W^\nu W_\mu} \right) - m_\hbar^2 \phi' \phi - \frac{v^2}{2v^2} / \tau^2
\end{aligned}$$

$$\begin{aligned}
\mathcal{H}_{ab} &\equiv \frac{1}{2\pi \prod_i^k(x) + \prod_k^i(x) \partial^i \partial_k A^k A_i(x) + \frac{1}{4\pi F^{ki}(x) F_{ik}(x)}} \\
&= H_{ab} \prod_i^k d^3\chi \left[\frac{1}{2\pi \prod_i^k(x) + \prod_k^i(x) \partial^i \partial_k A^k A_i(x) + \frac{1}{4\pi F^{ki}(x) F_{ik}(x)}} \right] \\
&= H^\rho H_c H^c H_\rho H_c^\rho H_\rho^\rho \varrho \equiv \prod_i^k \frac{d^3\chi\lambda}{\hbar} \mathcal{U}\Omega\mathbb{R}^4 / G_\epsilon R_e \\
&\quad [\lambda\Phi \underline{\equiv}]
\end{aligned}$$

Donde:



$$G_{\varepsilon} = \mathbf{G}_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_e =$$

$$\begin{aligned} \mathcal{H}_{ab} &\equiv \frac{1}{2\pi \prod_i^k(\gamma) + \prod_k^i(\gamma) \partial^i \partial_k A^k A_i(\gamma) + \frac{1}{4F^{ki}(\gamma) F_{ik}(\gamma)}} \\ &= H_{ab} \iiint_i^k d^3\chi \left[\frac{1}{2\pi \prod_i^k(\gamma) + \prod_k^i(\gamma) \partial^i \partial_k A^k A_i(\gamma) + \frac{1}{4\pi F^{ki}(\gamma) F_{ik}(\gamma)}} \right] \\ &= H^\rho H_c H^c H_\rho H_c^\rho H_\rho^\rho \equiv \iiint_i^k \frac{d^3\chi\lambda}{\hbar} \mathcal{U}\Omega\mathbb{R}^4 / G_{\varepsilon} R_e \\ &\quad [\lambda\Phi \triangleq] \end{aligned}$$

Donde:

$$G_{\varepsilon} = \mathbf{G}_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_e =$$



$$\begin{aligned}
\mathcal{H}_{ab} &\equiv \frac{1}{2\pi \prod_i^k(z) + \prod_k^i(z) \partial^i \partial_k A^k A_i(z) + \frac{1}{4\pi F^{ki}(z) F_{ik}(z)}} \\
&= H_{ab} \prod_i^k d^3 \chi \left[\frac{1}{2\pi \prod_i^k(z) + \prod_k^i(z) \partial^i \partial_k A^k A_i(z) + \frac{1}{4\pi F^{ki}(z) F_{ik}(z)}} \right] \\
&= H^\rho H_c H^c H_\rho H_c^\rho H_\rho^c \varrho \equiv \prod_i^k \frac{d^3 \chi \lambda}{\hbar} \mathcal{U} \Omega \mathbb{R}^4 / G_\varepsilon R_e \\
&\quad [\lambda \Phi \triangleq]
\end{aligned}$$

Donde:

$$G_\varepsilon = \mathbf{G}_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$R_e =$

$$\begin{aligned}
\mathcal{H}_{ba} &\equiv \frac{1}{2\pi \prod_i^k(x) + \prod_k^i(x) \partial^i \partial_k A^k A_i(x) + \frac{1}{4\pi F^{ki}(x) F_{ik}(x)}} \\
&= H_{ba} \prod_i^k d^3 \chi \left[\frac{1}{2\pi \prod_i^k(x) + \prod_k^i(x) \partial^i \partial_k A^k A_i(x) + \frac{1}{4\pi F^{ki}(x) F_{ik}(x)}} \right] \\
&= H^\rho H_c H^c H_\rho H_c^\rho H_\rho^c \varrho \equiv \prod_i^k \frac{d^3 \chi \lambda}{\hbar} \mathcal{U} \Omega \mathbb{R}^4 / G_\varepsilon R_e \\
&\quad [\lambda \Phi \triangleq]
\end{aligned}$$

Donde:

$$G_\varepsilon = \mathbf{G}_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_e =$$

$$\begin{aligned} \mathcal{H}_{ba} &\equiv \frac{1}{2\pi \prod_i^k(y) + \prod_k^i(y) \partial^i \partial_k A^k A_i(y) + \frac{1}{4F^{ki}(y)F_{ik}(y)}}} \\ &= H_{ba} \prod_i^k d^3\chi \left[\frac{1}{2\pi \prod_i^k(y) + \prod_k^i(y) \partial^i \partial_k A^k A_i(y) + \frac{1}{4\pi F^{ki}(y)F_{ik}(y)}}} \right] \\ &= H^\rho H_c H^c H_\rho H_c^\rho H_\rho^\rho \varrho \equiv \prod_i^k \frac{d^3\chi^\lambda}{\hbar} \mathcal{U}\Omega\mathbb{R}^4 / G_\varepsilon R_e \\ &\quad [\lambda\Phi \triangleq] \end{aligned}$$

Donde:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$G_\varepsilon =$$

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_e =$$

$$\begin{aligned} \mathcal{H}_{ba} &\equiv \frac{1}{2\pi \prod_i^k(z) + \prod_k^i(z) \partial^i \partial_k A^k A_i(z) + \frac{1}{4\pi F^{ki}(z)F_{ik}(z)}}} \\ &= H_{ba} \prod_i^k d^3\chi \left[\frac{1}{2\pi \prod_i^k(z) + \prod_k^i(z) \partial^i \partial_k A^k A_i(z) + \frac{1}{4\pi F^{ki}(z)F_{ik}(z)}}} \right] \\ &= H^\rho H_c H^c H_\rho H_c^\rho H_\rho^\rho \varrho \equiv \prod_i^k \frac{d^3\chi^\lambda}{\hbar} \mathcal{U}\Omega\mathbb{R}^4 / G_\varepsilon R_e \\ &\quad [\lambda\Phi \triangleq] \end{aligned}$$

Donde:



$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$G_{\varepsilon} =$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$R_{\varepsilon} =$

$\{B(x, k), C(x, k)/\Phi\Psi\kappa\varphi\theta$

$$= \delta \prod_b^a (x, k) \lambda \phi \frac{\oint_{\sigma}^{\varphi} d^3 z [\delta_b^a B(x, k) \lambda \phi / \delta_b^a A_{ab}(x, k) \lambda \phi]}{\delta_b^a C(x, k) \lambda \phi}$$

$$/ \delta \prod_a^b (x, k) \lambda \phi \frac{\oint_{\sigma}^{\varphi} d^3 z [\delta_a^b B(x, k) \lambda \phi / \delta_a^b C(x, k) \lambda \phi]}{\delta_a^b A_{ba}(x, k) \lambda \phi} / \delta \prod_{ba}^{ab} (x, k) \lambda \phi$$

$\{B(y, k), C(y, k)/\Phi\Psi\kappa\varphi\theta$

$$= \delta \prod_b^a (y, k) \lambda \phi \frac{\oint_{\sigma}^{\varphi} d^3 z [\delta_b^a B(y, k) \lambda \phi / \delta_b^a A_{ab}(y, k) \lambda \phi]}{\delta_b^a C(y, k) \lambda \phi}$$

$$/ \delta \prod_a^b (y, k) \lambda \phi \frac{\oint_{\sigma}^{\varphi} d^3 z [\delta_a^b B(y, k) \lambda \phi / \delta_a^b C(y, k) \lambda \phi]}{\delta_a^b A_{ba}(y, k) \lambda \phi} / \delta \prod_{ba}^{ab} (y, k) \lambda \phi$$

$\{B(z, k), C(z, k)/\Phi\Psi\kappa\varphi\theta$

$$= \delta \prod_b^a (z, k) \lambda \phi \frac{\oint_{\sigma}^{\varphi} d^3 z [\delta_b^a B(z, k) \lambda \phi / \delta_b^a A_{ab}(z, k) \lambda \phi]}{\delta_b^a C(z, k) \lambda \phi}$$

$$/ \delta \prod_a^b (z, k) \lambda \phi \frac{\oint_{\sigma}^{\varphi} d^3 z [\delta_a^b B(z, k) \lambda \phi / \delta_a^b C(z, k) \lambda \phi]}{\delta_a^b A_{ba}(z, k) \lambda \phi} / \delta \prod_{ba}^{ab} (z, k) \lambda \phi$$

$\{F(x), G(x)\}_{D_{\infty}} = * \{F(x), G(x)\} \oplus$

$$- \prod_{\varphi}^{\gamma} \psi \prod_{\gamma}^{\varphi} \lambda$$

$$\cong \frac{\oint_{\sigma}^{\varphi} d^3 z ab^3 ab_3 ab^d ab_d ba^3 ba^d ba_d \phi^a \phi_b \phi_a \varphi^b \phi_a \varphi^a \varphi_b \phi^{ab} \phi_{ba} \phi^{ba} \phi_{ab} \varphi^{ab} \varphi_{ba} \varphi^{ba} \varphi_{ab} C_{abba}^{-1\pi} e^{-i\omega t m c \hbar^2}}{\alpha \beta / h \Omega \phi \frac{1}{\pi} / \Delta \nabla \otimes \boxtimes \times \times \times \times}$$



En consecuencia, este trabajo, demuestra que la teoría gauge no abeliana de Yang – Mills, describe otras fuerzas en la naturaleza, especialmente la fuerza débil (responsable, entre otras cosas, de ciertas formas de radiactividad) y la fuerza fuerte o nuclear (responsable, entre otras cosas, de la unión de protones y neutrones en núcleos), sin perder las premisas esenciales de la teoría de campos de Yang – Mills, esto es, por fuera de la teoría electrodébil de Glashow-Salam-Weinberg o la teoría del “campo de Higgs”.

Si bien es cierto, constituyese en una propiedad notable de la teoría cuántica de Yang-Mills, la nominada "*libertad asintótica*", la misma que, permite determinar, que a distancias cortas el campo muestra un comportamiento cuántico muy similar a su comportamiento clásico; sin embargo, a largas distancias, la teoría de Yang – Mills, como queda demostrado, también aplica a largas distancias en el campo.

Finalmente, queda demostrado concluyentemente, que: **(i)** en los campos de Yang – Mills, existe una "brecha de masa", es decir, $\Delta > \text{constante}$, por lo que, cada excitación del vacío tiene energía de al menos Δ ; **(ii)** en los campos de Yang – Mills, existe un confinamiento de quarks, partiendo de la premisa de que, los estados físicos de las partículas, como el protón, el neutrón y el pión, son invariantes; y, **(iii)** en los campos de Yang – Mills, existe una ruptura de simetría quiral, lo que significa que el vacío es potencialmente invariante bajo un cierto subgrupo de simetría completa que actúa sobre los campos de quarks.

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APÉNDICE A.

Formalización matemática relativa a agujeros negros cuánticos en espacios curvos y teoría cuántica de campos curvos.

1. Teorema de Helmholtz - condición Hessiana - Galileón vectorial - tensor doble dual de Riemann.

$$\begin{aligned}
 A_\mu &= \mathcal{A}_\mu + \partial_\mu \varpi, \mathcal{H}^{\mu\nu} \equiv \frac{\partial^2 \mathcal{L}}{\partial(\partial_0 \mathcal{A}_\mu) \partial(\partial_0 \mathcal{A}_\nu)}, \delta \\
 &= \int \left[-\frac{1}{4} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} + \frac{1}{2} m^2 A^2 + \sum_{\eta=2}^6 \mathcal{L}_{N,A}^{Gal} \right] d^4 \chi, \mathcal{L}_{2,A}^{Gal} = f_2(A_\mu, \mathcal{F}_{\mu\nu}, \tilde{\mathcal{F}}_{\mu\nu}), \mathcal{L}_{3,A}^{Gal} \\
 &= f_2(A^2) \delta_\mu^\mu, \mathcal{L}_{4,A}^{Gal} = f_4(A^2) \left[(\delta_\mu^\mu)^2 - \delta_\rho^\sigma \delta_\sigma^\rho \right], \mathcal{L}_{5,A}^{Gal} \\
 &= f_5(A^2) \left[(\delta_\mu^\mu)^3 - 3 \delta_\mu^\mu \delta_\rho^\sigma \delta_\sigma^\rho + 2 \delta_\rho^\sigma \delta_\sigma^\rho \delta_\nu^\nu \right] + g_5(A^2) \tilde{\mathcal{F}}^{\alpha\mu} \tilde{\mathcal{F}}_\mu^\beta \delta_{\alpha\beta}, \mathcal{L}_{6,A}^{Gal} \\
 &= g_6(A^2) \tilde{\mathcal{F}}^{\alpha\beta} \tilde{\mathcal{F}}^{\mu\nu} \delta_{\alpha\mu} \delta_{\beta\nu}, \delta = \int d^4 \chi \sqrt{-g} \left(\mathcal{F} + \sum_{i=2}^6 \mathcal{L}_i \right), \mathcal{L}_2 \\
 &= \mathfrak{G}_2(\chi, \mathcal{F}_{\mu\nu}, \tilde{\mathcal{F}}_{\mu\nu}), \mathcal{L}_3 = \mathfrak{G}_3(\chi) \nabla_\mu A^\mu, \mathcal{L}_4 \\
 &= \mathfrak{G}_4(\chi) \mathcal{R} + \mathfrak{G}_{4,\chi}(\chi) \left[(\nabla_\mu A^\mu)^2 - \nabla_\mu A_\nu \nabla^\nu A^\mu \right], \mathcal{L}_5 \\
 &= \mathfrak{G}_5(\chi) \mathfrak{G}_{\mu\nu} \nabla^\nu A^\mu \\
 &\quad - \frac{1}{6} \mathfrak{G}_{5,\chi}(\chi) \left[(\nabla_\mu A^\mu)^3 - 3 \nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho + 2 \nabla_\rho A_\sigma \nabla^\nu A^\rho A^\sigma \nabla_\nu \right] \\
 &\quad + g_5(\chi) \tilde{\mathcal{F}}^{\alpha\mu} \tilde{\mathcal{F}}_\mu^\beta \nabla_\alpha A_\beta, \mathcal{L}_6 \\
 &= \mathfrak{G}_6(\chi) \mathcal{L}^{\mu\nu\alpha\beta} \nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{1}{2} \mathfrak{G}_{6,\chi}(\chi) \tilde{\mathcal{F}}^{\alpha\beta} \tilde{\mathcal{F}}^{\mu\nu} \nabla_\alpha A_\mu \nabla_\beta A_\nu, \mathcal{L}^{\mu\nu\alpha\beta} \\
 &\equiv \frac{1}{4} \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta} \mathcal{R}_{\rho\sigma\gamma\delta}
 \end{aligned}$$

1.2. Teoría de coordenadas.

$$g = -f(r) dt \otimes dt + \hbar^{-1}(r) dr \otimes dr + r^2 d\theta \otimes d\theta + r^2 \sin^2 \theta d\phi \otimes d\phi$$

1.3. Solución de Reissner-Nordström y acoplamientos.

$$\begin{aligned}
 \mathfrak{G}_4 &= \frac{m_\rho^2 \nabla_\beta \mathcal{F}^{\alpha\beta} m_\rho^2}{2} \mathfrak{G}_{\mu\nu}, \frac{1}{2} \left(\mathcal{F}_{\mu\alpha} \mathcal{F}_\nu^\alpha - \frac{1}{4} g_{\mu\nu} \mathcal{F}_{\alpha\beta} \mathcal{F}^{\alpha\beta} \right), \mathcal{F}_{01} = \partial_0 A_1 - \partial_1 A_0 = -A'_0 \\
 &= \mathcal{F}_{10} - \frac{\hbar}{fr} A'_0 + \frac{\hbar f A'_0}{4f^2} - \frac{\hbar A'_0}{4f} - \frac{\hbar A''_0}{2f} + \frac{m_\rho^2}{2r^2} f - \frac{m_\rho^2 f \hbar}{2r^2} - \frac{1}{4} \hbar A_0'^2 - \frac{m_\rho^2 f \hbar'}{2r} \\
 &\quad + \frac{m_\rho^2}{2r^2} - \frac{m_\rho^2}{2\hbar r^2} + \frac{A_0'^2}{4f} + \frac{m_\rho^2 f'}{2fr} - \frac{m_\rho^2 r \hbar'}{4} - \frac{\hbar r^2 A_0'^2}{4f} + \frac{m_\rho^2 \hbar f' r}{4f} - \frac{m_\rho^2 \hbar f'^2 r^2}{8f^2} \\
 &\quad + \frac{m_\rho^2 f' \hbar' r^2}{8f} + \frac{m_\rho^2 \hbar f'' r^2}{4f} \int \left(1 - \frac{2\mathcal{M}}{r} - \frac{Q^2}{2m_\rho^2 r^2} \right) d\hbar
 \end{aligned}$$



$$\begin{aligned}
& \frac{\mu^2 A^2 \nabla_\beta \mathcal{F}^{\alpha\beta} m_\rho^2}{2} \mathfrak{G}_{\mu\nu} \\
&= \frac{1}{2} \left(\frac{1}{4} g_{\mu\nu} \mathcal{F}_{\alpha\beta} \mathcal{F}^{\alpha\beta} - \mathcal{F}_{\mu\alpha} \mathcal{F}_\nu^\alpha \right) + \frac{1}{2} \mu^2 A_\mu A_\nu + \frac{\frac{1}{2} \mu^2 \chi g_{\mu\nu} \mu^2}{f} A_0 - \frac{\hbar}{rf} A'_0 \\
&- \frac{\hbar}{2f} A''_0 - \frac{\hbar'}{4f} A'_0 + \frac{\hbar f'}{4f^2 A'_0} - \frac{\mu^2 A_1 \hbar m_\rho^2 f}{2r^2} - \frac{m_\rho^2 f \hbar}{2r^2} - \frac{m_\rho^2 f \hbar'}{2r} - \frac{1}{4} \hbar A_0'^2 \\
&- \frac{1}{4} \mu^2 A_0^2 - \frac{1}{4} \mu^2 f \hbar A_1^2 - \frac{m_\rho^2 f}{2r^2} + \frac{m_\rho^2}{f \hbar} + \frac{m_\rho^2 f \hbar'}{2r} + \frac{1}{4} \hbar A_0'^2 - \frac{1}{4} \mu^2 A_0^2 - \frac{1}{2} \mu f \hbar A_1^2 \\
&+ \frac{m_\rho^2 \hbar f' r}{4f} - \frac{m_\rho^2 \hbar f'^2 r^2}{8f^2} + \frac{1}{4} m_\rho^2 \hbar' r + \frac{m_\rho^2 f' \hbar' r^2}{8f} - \frac{\hbar r^2}{4f} A_0'^2 + \frac{m_\rho^2 \hbar f'' r^2}{4f} \\
&- \frac{\mu^2 r^2}{4f} A_0^2 + \frac{1}{4} \mu^2 \hbar r^2 A_1^2 \\
&- \frac{1}{2} \mu^2 A_0 A_1 m_\rho^2 \left(f' \hbar - \frac{f \hbar'}{\hbar^2} \right) \\
&= \frac{\mu^2 A_0^2 r}{\hbar^2} \lim_{r \rightarrow \infty} \left(f' \hbar - \frac{f \hbar'}{\hbar^2} \right) \lim_{r \rightarrow \infty} \left(\frac{\mu^2 A_0^2 r(r)}{m_\rho^2 \hbar^2} \right) - \hbar \left(\frac{A_0''}{2} + \frac{A_0'}{r} \right) + \frac{\mu^2 A_0^2 A_0'}{4 m_\rho^2 f} \\
&+ c_1 \frac{\mu^2 A_0 e^{-\sqrt{2\mu}r}}{r} - c_2 \frac{\mu^2 A_1 e^{\sqrt{2\mu}r}}{r}
\end{aligned}$$

$$\begin{aligned}
\frac{m_\rho^2}{2} \mathfrak{G}_{\mu\nu} = & \frac{1}{2} \left[\mathcal{F}_{\mu\alpha} \mathcal{F}_\nu^\alpha - \frac{\mathcal{F}^2}{4} g_{\mu\nu} \right] \\
& - \frac{1}{4} \left[\frac{1}{2} g_{\mu\nu} (\nabla_\alpha A^\alpha)^2 - 2A_\mu \nabla_\nu \nabla_\alpha A^\alpha - 2\nabla_\alpha A_\mu \nabla_\nu A^\alpha + \frac{1}{2} g_{\mu\nu} \nabla_\alpha A_\beta \nabla^\beta A^\alpha \right. \\
& + \nabla_\alpha (A_\mu \nabla^\alpha A_\nu + A_\mu \nabla_\nu A^\alpha - A^\alpha \nabla_\mu A_\nu) + g_{\mu\nu} A_\beta \nabla^\beta \nabla^\alpha A_\alpha \\
& - \frac{1}{2} (A_\mu A_\nu \mathcal{R} - A^2 \mathfrak{G}_{\mu\nu} - g_{\mu\nu} \square A^2 + \nabla_\mu \nabla_\nu A^2) \left. \right] \nabla_\beta \mathcal{F}^{\alpha\beta} - \frac{1}{2} \mathfrak{G}^{\beta\alpha} A_\beta \\
& - 2A_0 f (-1 + \hbar + r\hbar') \\
& + r [A'_0 (\hbar r f' - f(4\hbar + r\hbar)) - 2f\hbar r A''_0] A_1 \hbar (f(\hbar - 1) + \hbar r f') - 2\hbar r^2 A_0'^2 \\
& + A_0^2 (-1 + \hbar + r\hbar') \\
& - f [A_1 \hbar^2 (A_1 + 4rA'_1) + 4m_\rho^2 (r\hbar' - 1) \\
& + \hbar (4m_\rho^2 + A_1^2 (1 + 3r\hbar'))] 4A_0 A'_0 f \hbar r + A_0^2 [f(\hbar - 1) - \hbar r f'] \\
& + f [f(-4m_\rho^2 + (4m_\rho^2 + A_1^2)\hbar + 3A_1^2 \hbar^2) \\
& + \hbar r (2rA_0'^2 + (4m_\rho^2 - 3A_1^2 \hbar) f')] 4A_0 f [-2\hbar f' r A'_0 \\
& + f(rA'_0 \hbar' + 2\hbar(A'_0 + rA_0''))] \\
& + A_0^2 [3\hbar f'^2 r + 2f^2 \hbar' - f(rf' \hbar' + 2\hbar(f' + rf''))] \\
& + f [4A_1 f \hbar^2 A_1' (2f + rf') \\
& + 4m_\rho^2 (-\hbar r f'^2 + 2f^2 \hbar' + f(rf' \hbar' + 2\hbar(f' + rf'')))] \\
& + A_1^2 \hbar [-\hbar r f'^2 + 6f^2 \hbar' + f(2rf' \hbar' + 2\hbar(f' + rf''))] A_0 A_1 [f(\hbar - 1) \\
& + \hbar r f'] \int \mathcal{C} - 2\mathcal{M}/r\hbar r^2 A_0'^2 - 4\mathfrak{G}_{4,\chi} (-1 + \hbar + r\hbar) \\
& + 4f [\mathfrak{G}_{4,\chi} A_1 \hbar^2 (A_1 + 2rA'_1) + \mathfrak{G}_4 (r\hbar' - 1) \\
& + \hbar (\mathfrak{G}_4 + 2\mathfrak{G}_{4,\chi} A_1^2 r \hbar')] \int 4f^2 [-\mathfrak{G}_4 + (\mathfrak{G}_4 - \mathfrak{G}_{4,\chi} A_1^2) \hbar + 2\mathfrak{G}_{4,\chi} A_1^2 \hbar^2] \\
& - 4\mathfrak{G}_{4,\chi} A_0^2 \hbar f' r + f \hbar r [8\mathfrak{G}_{4,\chi} A_0 A'_0 + rA_0'^2 + 4(\mathfrak{G}_4 + 2\mathfrak{G}_{4,\chi} A_1^2 \hbar) f'] \\
A_1 = & \pm \sqrt{\frac{r^2 (A_0 A'_0 f - f r A_0'^2 + A_0^2 f')}{f}} \sqrt{2\mathcal{P}(\mathcal{M}\mathcal{P} + \mathcal{Q})r + \frac{\mathcal{Q}^2}{r - 2\mathcal{M}}}
\end{aligned}$$

$$\begin{aligned} & \sqrt{-2\chi_c \left(1 - \frac{2\mathcal{M}}{r}\right) + \frac{\left(\mathcal{P} + \frac{Q}{r}\right)}{r} - 2\mathcal{M} \sqrt{Q^2 + \frac{2\mathcal{P}(\mathcal{M}\mathcal{P} + Q)r}{r} - 2\mathcal{M} \left\{ f(\mathfrak{G}_{4,\chi} - \mathfrak{G}_{4,\chi}\hbar \right.} \\ & \quad \left. + \mathfrak{G}_{4,\chi,\chi} A_1^2 \hbar^2) + 2\mathfrak{G}_{4,\chi,\chi} A_0 \frac{\hbar}{f} A_0' r \right\}} \\ & \quad - \frac{\hbar}{f} \left[\mathfrak{G}_{4,\chi,\chi} A_0^2 + f(\mathfrak{G}_{4,\chi} - \mathfrak{G}_{4,\chi,\chi} A_1^2 \hbar) \right] r f'} \pm 2m_\rho^2 r / 2m_\rho^2 (r^2 - 2\mathcal{M}r) \\ & \quad + Q^2 \left[m_\rho^2 (\mathcal{P}^2 - 2\chi_c) r^2 + 2m_\rho^2 (\mathcal{P}Q + 2\mathcal{M}\chi_c) r + Q^2 (m_\rho^2 - \chi_c) \right]^{\frac{1}{2}} \\ & \quad / 2f r^2 \{ -m_\rho^2 (-1 + r f' + f) - r^2 A_0'^2 \} \\ & \quad + \frac{1}{4r^2} \mathfrak{G}_{5,\chi,\chi} \alpha_1^2 \{ 2f [A_0 A_1 A_0' + (A_0^2 - A_1^2 f^2) A_1'] + A_1 (A_0^2 - A_1^2 f^2) f' \} f \\ & \quad / 2r \{ m_\rho^2 (-1 + r f' + f) + r^2 A_0'^2 \} \\ & \quad + 1/4r^2 \mathfrak{G}_{5,\chi,\chi} \alpha_1^3 f^2 [2A_0 A_0' f + (-A_0^2 - A_1^2 f^2) f'] - 2r A_0'^2 + m_\rho^2 (2f' + r f'') \end{aligned}$$

$$\begin{aligned} A_0 = & \mathcal{P} + \frac{Q}{r} \left\{ \mathfrak{G}_{4,\chi,\chi} A_1^2 \hbar^2 f' r + f \left[\mathfrak{G}_{4,\chi} + \hbar (-\mathfrak{G}_{4,\chi,\chi} + \mathfrak{G}_{4,\chi,\chi} A_1^2 \hbar) r \hbar' \right] \right\} \\ & + \frac{r}{8} \left[A_0'^2 (\hbar f' r - f(4\hbar + r \hbar')) - 2f \hbar r A_0' \right] A_0 r + \frac{1}{2} A_0'' r^2 \\ & + \mathfrak{G}_{5,\chi,\chi} A_0 A_1^2 (f A_1' + A_1 f') \\ & + \frac{1}{f} \mathfrak{G}_{5,\chi} A_0 (-(-1 + f) f A_1' + A_1 (1 - 2f) f') \mathfrak{G}_{5,\chi} \left(A_0^2 (f - 1) \frac{f'}{f^2} \right. \\ & \quad \left. - \frac{2A_0 (f - 1) A_0'}{f} + A_1^2 (1 - 3f) f' \right) \\ & + \mathfrak{G}_{5,\chi,\chi} A_1^2 (2A_0 A_0' f + (A_1^2 f^2 - A_0^2) f') (\chi_c + A_0^2 - 3\chi_c f) (A_0^2 - 2\chi_c f) (A_0 A_0' \\ & \quad - \chi_c f') \end{aligned}$$

$$\begin{aligned} f' = & -\frac{4f}{r} + \frac{Q^2}{\chi_c r^3} + \frac{2\mathcal{P}Q}{\chi_c r^2} + \frac{2\mathcal{P}^2}{\chi_c r} + 4f^3 + \frac{r^4 f' Q^2 r}{\chi_c} + \frac{2\mathcal{P}Q r^2}{\chi_c} + \frac{2\mathcal{P}^2 r^3 (r^4 f)' Q^2 r}{\chi_c} + \frac{2\mathcal{P}Q r^2}{\chi_c} \\ & + \frac{2\mathcal{P}^2 r^3}{\chi_c} r^4 f \frac{Q^2 r^2}{2\chi_c} + \frac{2\mathcal{P}Q r^2 \mathcal{P}^2 r^4}{3\chi_c} + \mathcal{C} f \frac{1}{\chi_c} \left(\mathcal{P} + \frac{Q}{r} \right)^2 + \frac{\mathcal{C}}{r^4} \end{aligned}$$

$$\begin{aligned} A_\eta = & \pm \frac{2m_\rho^2 \sqrt{2(2\mathcal{M}^2 m_\rho^2 - Q)r^2}}{Q [2m_\rho^2 r^2 (2\mathcal{M} - r) - Q^2]} \sqrt{\frac{r(\mathcal{P}^2 r + 4\mathcal{M}\chi_c - 2r\chi_c)}{r} - 2\mathcal{M} m_\rho^2 / 2 f (r f'' + 2f')} \\ & - \frac{\chi_c}{4} r f'^2 - \chi_c f f' f'' \mathfrak{G}_6 (r f'' + 2f') - \frac{r f'^2}{2r f} \\ & - \frac{1}{r} [2f f'' + (f'^2 - 2f f'')] + f'^2 / f \mathfrak{G}_6 + 2\chi_c (f - 1) f'^2 \mathfrak{G}_{6,\chi} \end{aligned}$$

1.4. Agujeros negros cuánticos EGB en espacios curvos.



$$\begin{aligned}
\delta &= \frac{1}{32\pi} \int d^4 \sqrt{-g} [\mathcal{R} + \alpha \mathcal{L}_{\mathfrak{GB}}], \mathcal{L}_{\mathfrak{GB}} \\
&= \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\alpha\beta}\mathcal{R}^{\mu\nu\alpha\beta} 2\alpha [\mathcal{R}\mathcal{R}_{\alpha\beta} - 2\mathcal{R}_{\alpha\sigma}\mathcal{R}_{\beta}^{\sigma} + \mathcal{R}_{\alpha}^{\sigma\delta\epsilon}\mathcal{R}_{\beta\sigma\delta\epsilon}] \\
&\quad - \frac{1}{2}\alpha g_{\alpha\beta}\mathcal{L}_{\mathfrak{GB}} + \mathcal{R}_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}\mathcal{R} = \mathcal{T}_{\alpha\beta}, \mathcal{T}_{\alpha}^{\beta} \\
&= \rho(r)\varphi \left[B(r)\delta_{\alpha}^{\beta} - \frac{\{1 + 3B(r)\}r_{\alpha}r^{\beta}}{r_{\eta}r^{\eta}} \right], \langle \mathcal{T}_{\alpha}^{\beta} \rangle = -\rho(r)\frac{\varphi}{3}, \delta_{\alpha}^{\beta} \\
&= -\rho(r)\delta_{\alpha}^{\beta}, \langle r_{\alpha}r^{\beta} \rangle \\
&= \frac{1}{3}\delta_{\alpha}^{\beta}r_{\eta}r^{\eta} \int \omega_{\varrho}\rho \int B(r) - 3\omega_{\varrho} + \frac{1}{6\omega_{\varrho}} \int \mathcal{T}_{\theta}^{\theta} \mathcal{T}_{\phi}^{\phi} - \frac{1}{2}\rho(3\omega_{\varrho} + 1)
\end{aligned}$$

$$\begin{aligned}
ds^2 &= -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \langle \mathcal{T}_r^r \rangle \\
&= \frac{f'}{f} + f - \frac{1}{r^2} - \alpha \left[2ff' - \frac{2f'}{r^3} - f^2 - 2f + \frac{1}{r^4} \right], \langle \mathcal{T}_{\theta}^{\theta} \mathcal{T}_{\phi}^{\phi} \rangle \\
&= \frac{f''}{2} + \frac{f'}{r} \\
&\quad - \frac{\alpha}{2} \left[2ff'' - 2f' + \frac{2f'^2}{r^2} + 4f' + 2f^2 - \frac{4ff'}{r^3} + 2f^2 - 4f + \frac{2}{r^4} \right], \rho \\
&= \frac{f'}{f} + f - \frac{1}{r^2} - \alpha \left[2ff' - \frac{2f'}{r^3} - f^2 - 2f + \frac{1}{r^4} \right] - \frac{\rho(3\omega_{\varrho} + 1)}{2} \\
&= \frac{f''}{2} + \frac{f'}{r} \\
&\quad - \frac{\alpha}{2} \left[2ff'' - 2f'' + \frac{2f'^2}{r^2} + 4f' + 2f^2 - \frac{4ff'}{r^3} + 2f^2 - 4f + \frac{2}{r^4} \right] [r^2 f']' \\
&\quad + (3\omega_{\varrho} + 1)[r(f - 1)]' \\
&\quad - \alpha \left\{ 2[(f - 1)f']' + (3\omega_{\varrho} - 1) \left(\frac{f^2}{r} \right)' - (6\omega_{\varrho} - 2) \left(\frac{f}{r} \right)' - 3\omega_{\varrho} - 1/r^2 \right\}
\end{aligned}$$

$$\begin{aligned}
f_{\pm}(r) &= 1 + \frac{r^2}{2\alpha} \left[1 \pm \sqrt{1 + \frac{16\alpha\mathcal{M}}{r^3} + 16\alpha\Gamma/r^{3\omega_{\varrho}+3}} \right] \int 1 - \frac{2\mathcal{M}}{r} + \frac{2\mathfrak{C}}{r^{3\omega_{\varrho}+1}} + \frac{r^2}{\alpha} \int 1 + \frac{2\mathcal{M}}{r} \\
&\quad - \frac{2\mathfrak{C}}{r^{3\omega_{\varrho}+1}} - \frac{r^2}{\alpha}
\end{aligned}$$

1.5. Horizonte de eventos de un agujero negro cuántico en espacios curvos.



$$\begin{aligned}
& 2\alpha\tau_{\hbar}^{3\omega_{\varrho}/2} + \tau_{\hbar}^{(3\omega_{\varrho}+4)/2} - \sqrt{\tau_{\hbar}^{3\omega_{\varrho}+4} + 16\alpha\tau_{\hbar}^{3\omega_{\varrho}+1} + 16\alpha\Gamma\tau_{\hbar}, \alpha(\Gamma, \tau_{\hbar})} \\
& = 2\tau_{\hbar} - \tau_{\hbar}^2 \\
& + 2\Gamma \\
& / \tau_{\hbar}^{3\omega_{\varrho}-1} \int dc c_e(\tau_{\hbar}) \left(1 - \tau_{\hbar}\tau_{\hbar}^{3\omega_{\varrho}}/3\omega_{\varrho} - 1\right)^{3\omega_{\varrho}} \int 18\omega_{\varrho}^2 - 1 \\
& \equiv c_c \alpha_c(c_c, \tau_e) \left[\tau_{\hbar}^{3\omega_{\varrho}}(\tau_{\hbar}^3 + 2\alpha) + 2c\alpha(1 - 3\omega_{\varrho}) \right. \\
& \left. - \tau_{\hbar}^{(3\omega_{\varrho}+3)/2} \sqrt{\tau_{\hbar}^{3\omega_{\varrho}}(\tau_{\hbar}^3 + 16\alpha) + 16c\alpha} \right] \otimes 1 \\
& / \alpha\tau_{\hbar}^{(3\omega_{\varrho}+1)/2} \sqrt{\tau_{\hbar}^{3\omega_{\varrho}}(\tau_{\hbar}^3 + 16\alpha) + 16c\alpha}
\end{aligned}$$

1.6. Perturbaciones escalares de un agujero negro cuántico EGB.

$$\begin{aligned}
& 1/\sqrt{-g}\partial_{\mu}(\sqrt{-g}g^{\mu\nu}\partial_{\nu})\phi \int \frac{e^{-i\omega\tau}\mathcal{Y}(\theta, \phi)\xi(r)}{r} dr_{\odot} \frac{dr}{f(r)} d^2\xi(r)/dr_{\odot}^2 \\
& + [\omega^2 - \mathcal{V}_{\mathbb{E}}(r)]\xi(r), \mathcal{V}_{\mathbb{E}}(r) \\
& = f(r) \left[\frac{\ell(\ell+1)}{r^2} + \frac{f'(r)}{r} \right] 1 \\
& / \sqrt{-g}\partial_{\mu}(\sqrt{-g}\mathfrak{F}_{\beta\gamma}g^{\beta\nu}g^{\gamma\mu})\Lambda_{\phi}\xi(r) \sin(\omega t) \sin\theta d\mathcal{P}_{\ell}(\cos\theta)/d\theta, \mathcal{V}_{\mathbb{EM}}(r) \\
& = f(r)[\ell(\ell+1)/r^2] \\
& \omega^2 = [\mathcal{V}_0 + (-2\mathcal{V}_0'')^{1/2}\Lambda] - i\hat{\alpha}(-2\mathcal{V}_0'')^{1/2}[1 + \bar{\Omega}] \\
& \Omega = \sqrt{f(r_0)/r_0^2}, \Lambda = \sqrt{f(r_0)/2r_0^2(2f(r_0) - r_0f''(r_0))}
\end{aligned}$$

1.7. Agujeros negros cuánticos en espacios curvos (métrica).

1.7.1. Modelo Ashtekar-Pawlowski-Singh.

$$ds_{\mathcal{A}PS}^2 = -d\tau^2 + \alpha(\tau)^2 d\tilde{r}^2 + \alpha(\tau)^2 \tilde{r}^2 d\Omega_2^2, \mathcal{H}^2 \triangleq \left(\frac{\dot{\alpha}}{\alpha}\right)^2 = \frac{16\pi\mathfrak{G}\rho}{3}(1 - \rho/\rho_c)$$

1.7.2. Modelo Oppenheimer-Snyder.

$$\begin{aligned}
ds_{qoS}^2 & = -(1 - \mathcal{F}(r))dt^2 + \frac{dr^2}{1} - \mathfrak{G}(r) + r^2 d\Omega_2^2, ds_{qoS}^2 \\
& = -\left(1 - \frac{2\mathcal{M}}{r} + \frac{\alpha\mathcal{M}^2}{r^4}\right) dt^2 + \left(1 - \frac{2\mathcal{M}}{r} + \frac{\alpha\mathcal{M}^2}{r^4}\right) dr^2 + r^2 d\Omega_2^2, r_{\beta} \\
& = \left(\frac{\alpha\mathcal{M}}{2}\right)^{\frac{1}{3}}, r_{\pm} = \zeta(1 \pm \sqrt{2\zeta - 1})\sqrt{\alpha}/\sqrt{(1 + \zeta)(1 + \zeta)^3}
\end{aligned}$$

1.7.3. Algoritmo Newman-Janis.



$$\begin{aligned}
ds_{eff}^2 &= -\Delta(r) - \frac{\alpha^2 \sin^2 \theta}{\rho^2} dt^2 + \frac{\rho^2}{\Delta(r)} dr^2 + \rho^2 d\theta^2 + (r^2 + \alpha^2) \sin^2 \theta - \frac{\Delta(r) \alpha^2 \sin^4 \theta}{\rho^2} d\phi^2 \\
&\quad - \frac{2\alpha \sin^2 \theta (\alpha^2 + r^2 - \Delta(r))}{\rho^2} dt d\phi, \Delta(r) = r^2 + \alpha^2 - 2\mathcal{M}r + \frac{\alpha \mathcal{M}^2}{r^2}, \rho^2 \\
&= r^2 + \alpha^2 \cos^2 \theta, \Delta(r) = \Delta_{Kerr}(r) + \frac{\alpha \mathcal{M}^2}{r^2}
\end{aligned}$$

1.7.4. Formalismo Jacobi - Hamilton y métrica de Kerr.

$$\begin{aligned}
2\partial_t \delta &= -g^{\mu\nu} \partial_{\chi^\mu} \delta \partial_{\chi^\nu} \delta, \delta = \frac{1}{2} m_\rho^2 \tau - \mathbb{E}t + \mathcal{L}\phi + A_r(r) + A_\theta(\theta), \rho^2 \dot{t} \\
&= r^2 + \frac{\alpha^2}{A_r(r)(\mathbb{E}(r^2 + \alpha^2) - \alpha\mathcal{L})} + \alpha(\mathcal{L} - \alpha\mathfrak{E} \sin^2 \theta), \rho^2 \dot{r} = \pm \sqrt{\mathcal{R}(r)}, \rho^2 \dot{\theta} \\
&= \pm \sqrt{\Theta(\theta)}, \rho^2 \dot{\phi} = \frac{\alpha}{\Delta(r)(\mathbb{E}(r^2 + \alpha^2) - \alpha\mathcal{L})} + (\mathcal{L} \csc^2 \theta - \alpha\mathfrak{E}), \mathcal{R}(r) \\
&= ((r^2 + \alpha^2)\mathbb{E} - \alpha\mathcal{L})^2 - \Delta(r)(\mathcal{Z} + (\mathcal{L} - \alpha\mathfrak{E})^2), \Theta(\theta) \\
&= \mathcal{Z} + \cos^2 \theta (\alpha^2 \mathbb{E}^2 - \mathcal{L}^2 \csc^2 \theta), \mathcal{R}(r) \\
&= \mathcal{R}_{Kerr}(r) - \frac{\alpha(\mathcal{Z} + (\mathcal{L} - \alpha\mathfrak{E})^2) \mathcal{M}^2}{r^2}, \mathcal{V}^{eff}(r) \\
&= \mathcal{V}_{Kerr}^{eff}(r) + \alpha(\mathcal{Z} + (\mathcal{L} - \alpha\mathfrak{E})^2) \mathcal{M}^2 / 2r^6
\end{aligned}$$

$$\begin{aligned}
\xi(r_\rho) &= \xi_{Kerr}(r_\rho) + \frac{2\alpha \mathcal{M}^2 r_\rho (\Delta_{Kerr}(r_\rho) + r_\rho \Gamma)}{\alpha \Gamma (\alpha \mathcal{M}^2 - r_\rho^3 \Gamma)}, \eta(r_\rho) \\
&= \eta_{Kerr}(r_\rho) \\
&\quad + 4\alpha \mathcal{M}^2 r_\rho^3 \\
&\quad / \Gamma^2 (\alpha \mathcal{M}^2 - \Gamma r_\rho^3)^2 [2\Gamma r_\rho^3 (\mathcal{M} - \Gamma) - \alpha \mathcal{M}^3 \\
&\quad + r_\rho^2 (\mathcal{M} - 2\Gamma) [\mathcal{M} (\alpha \mathcal{M} - 3r_\rho^2 \Gamma) + \Gamma r_\rho^3] / \alpha^2]
\end{aligned}$$

$$\chi(r_\rho) = - \lim_{\substack{r \rightarrow \infty \\ \theta \rightarrow \theta_0}} r^2 \sin \theta d\phi / dr, \gamma(r_\rho) = - \lim_{\substack{r \rightarrow \infty \\ \theta \rightarrow \theta_0}} r^2 d\theta / dr$$

$$\begin{aligned}
\chi(r_\rho) &= -\xi(r_\rho), \gamma(r_\rho) = \pm \sqrt{\eta(r_\rho)}, \chi(r_\rho) = \chi_{Kerr}(r_\rho) - \frac{2\alpha \mathcal{M}^2 r_\rho (\Delta_{Kerr}(r_\rho) + r_\rho \Gamma)}{\alpha \Gamma (\alpha \mathcal{M}^2 - r_\rho^3 \Gamma)}, \gamma(r_\rho) \\
&= \gamma_{Kerr}(r_\rho) \\
&\quad \pm \frac{2\alpha \mathcal{M}^2 r_\rho^3}{\Gamma^2 \sqrt{\eta_{Kerr}(r_\rho) (\alpha \mathcal{M}^2 - \Gamma r_\rho^3)^2}} \left[\frac{r_\rho^2 (\mathcal{M} - 2\Gamma) [\mathcal{M} (\alpha \mathcal{M} - 3r_\rho^2 \Gamma) + \Gamma r_\rho^3]}{\alpha^2} \right. \\
&\quad \left. + 2\Gamma r_\rho^3 (\mathcal{M} - \Gamma) - \alpha \mathcal{M}^3 \right]
\end{aligned}$$

1.7.5. Método Kumar-Ghosh.

$$A_{sh} = 2 \int_{r_-}^{r_+} d\chi^r \gamma(r) \partial_r \chi(r), \theta_d = \frac{2}{d} \sqrt{\frac{A_{sh}}{\pi}}$$

2. Campos cuánticos gravitacionales en espacios curvos (gravedad cuántica).



$$\begin{aligned}
\mathfrak{G} = & -\frac{1}{2}(g_{\mu\nu}\mathcal{F} + \mathcal{F}_{\mu\alpha}\mathcal{F}_\nu^\alpha) - \frac{1}{2}g_{\mu\nu}\mathfrak{G}_2 - \frac{1}{2}\mathfrak{G}_{2,\chi}A_\mu A_\nu - \frac{1}{2}\mathfrak{G}_{2,\mathcal{F}}\mathcal{F}_{\mu\alpha}\mathcal{F}_\nu^\alpha \\
& + \mathfrak{G}_{2,\mathcal{Y}}(2A^\sigma A_\mu \mathcal{F}_\alpha^\nu \mathcal{F}_{\sigma\alpha} - A^\alpha A^\beta \mathcal{F}_{\beta\nu} \mathcal{F}_{\mu\alpha}) \\
& + \mathfrak{G}_{3,\chi} \left[-\frac{1}{2A_\mu A_\nu \nabla_\alpha A^\alpha} + A^\alpha \left(A_\mu \nabla_\nu A_\alpha - \frac{1}{2}g_{\mu\nu} A^\beta \nabla_\alpha A_\beta \right) \right] + \mathfrak{G}_4 \mathfrak{G}_{\mu\nu} \\
& + \mathfrak{G}_{4,\chi} \left[\frac{1}{2}g_{\mu\nu} (\nabla_\alpha A^\alpha)^2 - \frac{1}{2}g_{\mu\nu} \nabla_\alpha A_\beta \nabla^\beta A^\alpha - 2A_\mu \nabla_\nu \nabla_\alpha A^\alpha + g_{\mu\nu} A_\beta \nabla^\beta \nabla^\alpha A_\alpha \right. \\
& + \nabla_\alpha (A_\mu \nabla^\alpha A_\nu + A_\mu \nabla_\nu A^\alpha - A^\alpha \nabla_\mu A_\nu) - 2\nabla_\alpha A_\mu \nabla_\nu A^\alpha \\
& \left. - \frac{1}{2}(\mathcal{R}A_\mu A_\nu + \nabla_\mu \nabla_\nu A^2 - g_{\mu\nu} \square A^2) \right] \\
& + \mathfrak{G}_{4,\chi,\chi} \left[\frac{1}{4}\nabla_\mu A^2 \nabla_\nu A^2 - \frac{1}{4}g_{\mu\nu} \nabla^\alpha A^2 \nabla_\alpha A^2 + A_\mu \nabla_\nu A^2 \nabla_\alpha A^\alpha \right. \\
& \left. - \frac{1}{2}g_{\mu\nu} A^\beta \nabla_\beta A^2 \nabla_\alpha A^\alpha - \frac{1}{2}\nabla_\alpha A^2 (A_\mu \nabla^\alpha A_\nu + A_\mu \nabla_\nu A^\alpha - A^\alpha \nabla_\mu A_\nu) \right]
\end{aligned}$$

$$\begin{aligned}
\mathfrak{G} = & \nabla_\beta \mathcal{F}^{\alpha\beta} - \mathfrak{G}_{2,\chi} A^\alpha + \nabla_\beta \mathfrak{G}_{2,\mathcal{F}} \mathcal{F}^{\beta\alpha} + \mathfrak{G}_{2,\mathcal{F}} \nabla_\beta \mathcal{F}^{\beta\alpha} - 4\nabla_\beta \mathfrak{G}_{2,\mathcal{Y}} A^\nu A^\beta \mathcal{F}_\nu^\alpha \\
& - 2\mathfrak{G}_{2,\mathcal{Y}} [A_\mu \mathcal{F}_{\mu\lambda} \mathcal{F}^{\alpha\lambda} + 2(\nabla_\beta A^\nu) A^\beta \mathcal{F}_\nu^\alpha - 2A^\nu \nabla_\beta A^\beta \mathcal{F}_\nu^\alpha - 2A^\nu A^\beta \nabla_\beta \mathcal{F}_\nu^\alpha] \\
& + \mathfrak{G}_{3,\chi} \left(\frac{1}{2} \nabla^\alpha A^2 - A^\alpha \nabla_\mu A^\mu \right) + 2\mathfrak{G}_{4,\chi} \mathfrak{G}^{\beta\alpha} A_\beta \\
& - \mathfrak{G}_{4,\chi,\chi} \left[A^\alpha (\nabla_\mu A^\mu)^2 - A^\alpha \nabla_\mu A_\nu \nabla^\nu A^\mu - \nabla_\beta A^2 (g^{\alpha\beta} \nabla_\mu A^\mu - \nabla^\alpha A^\beta) \right] \\
& + \mathfrak{G}_{5,\chi} \left[\frac{1}{2} \nabla_\beta A^2 \mathfrak{G}^{\beta\alpha} - A^\alpha \mathfrak{G}_{\mu\nu} \nabla^\mu A^\nu + \frac{1}{2} \nabla_\mu A^\mu \nabla^\alpha \nabla_\beta A^\beta - \nabla_\beta (\nabla_\mu A^\mu \nabla^\alpha A^\beta) \right. \\
& \left. - \frac{1}{2} \nabla^\alpha (\nabla_\rho A_\sigma \nabla^\sigma A^\rho) \right] \\
& + \mathfrak{G}_{5,\chi,\chi} \left\{ \frac{1}{6} A^\alpha [(\nabla_\mu A^\mu)^3 - 3\nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho + 2\nabla_\rho A_\sigma \nabla^\nu A^\rho \nabla^\sigma A_\nu] \right. \\
& \left. - \frac{1}{4} [\nabla^\alpha A^2 (\nabla_\mu A^\mu)^2 + \nabla^\alpha A^2 \nabla_\rho A_\sigma \nabla^\sigma A^\rho + 2\nabla_\beta A^2 \nabla_\mu A^\mu \nabla^\alpha A^\beta \right. \\
& \left. - 2\nabla_\beta A^2 \nabla^\nu A^\beta \nabla^\alpha A_\nu + 4A^2 \nabla_\beta (\nabla^\nu A^\beta \nabla^\alpha A_\nu) \right\} \\
& + g_5 [\nabla_\beta \tilde{\mathcal{F}}^{\beta\mu} \tilde{\mathcal{F}}_\mu^\alpha + \tilde{\mathcal{F}}^{\beta\mu} \nabla_\beta \tilde{\mathcal{F}}_\mu^\alpha - 2\nabla_\beta \tilde{\mathcal{F}}_\mu^\sigma \varepsilon^{\lambda\mu\alpha\beta} \nabla_\lambda A_\sigma - 2\tilde{\mathcal{F}}_\mu^\sigma \varepsilon^{\lambda\mu\alpha\beta} \nabla_\beta \nabla_\lambda A_\sigma] \\
& + g_{5,\chi} \left[A^\alpha \tilde{\mathcal{F}}^{\gamma\mu} \tilde{\mathcal{F}}_\mu^\beta \nabla_\lambda A_\beta + A_\beta \tilde{\mathcal{F}}_\mu^\sigma \varepsilon^{\lambda\mu\alpha\beta} \nabla_\lambda A_\sigma - \frac{1}{2} \nabla_\beta A^2 \tilde{\mathcal{F}}^{\beta\mu} \tilde{\mathcal{F}}_\mu^\alpha \right] \\
& - 2\mathfrak{G}_6 [\nabla_\beta \mathcal{L}^{\beta\alpha\gamma\sigma} \nabla_\gamma A_\sigma + \mathcal{L}^{\beta\alpha\gamma\sigma} \nabla_\beta \nabla_\gamma A_\sigma] \\
& + \mathfrak{G}_{6,\chi} [\nabla_\beta A^2 \mathcal{L}^{\beta\alpha\gamma\sigma} \nabla_\gamma A_\sigma - \varepsilon^{\gamma\sigma\beta\alpha} \nabla_\beta \tilde{\mathcal{F}}^{\mu\nu} \nabla_\gamma A_\mu \nabla_\sigma A_\nu \\
& - \varepsilon^{\gamma\sigma\beta\alpha} \tilde{\mathcal{F}}^{\mu\nu} (\nabla_\beta \nabla_\gamma A_\mu) \nabla_\sigma A_\nu - \varepsilon^{\gamma\sigma\beta\alpha} \nabla_\beta \tilde{\mathcal{F}}^{\mu\nu} \nabla_\gamma A_\mu \nabla_\beta \nabla_\sigma A_\nu \\
& - A^\alpha \mathcal{L}^{\mu\nu\gamma\beta} \nabla_\mu A_\nu \nabla_\gamma A_\beta - \nabla_\beta \tilde{\mathcal{F}}^{\mu\nu} \varepsilon^{\gamma\sigma\beta\alpha} \nabla_\mu A_\gamma \nabla_\nu A_\sigma - \tilde{\mathcal{F}}^{\mu\nu} \varepsilon^{\gamma\sigma\beta\alpha} (\nabla_\beta \nabla_\mu A_\gamma) \nabla_\nu A_\sigma \\
& - \tilde{\mathcal{F}}^{\mu\nu} \varepsilon^{\gamma\sigma\beta\alpha} \nabla_\mu A_\gamma \nabla_\beta \nabla_\nu A_\sigma - \nabla_\beta \tilde{\mathcal{F}}^{\gamma\beta} \tilde{\mathcal{F}}^\mu \nabla_\gamma A_\mu - \tilde{\mathcal{F}}^{\gamma\beta} \nabla_\beta \mathcal{F}^{\mu\alpha} \nabla_\gamma A_\mu \\
& - \tilde{\mathcal{F}}^{\gamma\beta} \tilde{\mathcal{F}}^{\mu\alpha} \nabla_\beta \nabla_\gamma A_\mu] \\
& + \frac{\mathfrak{G}_{6,\chi,\chi}}{2} [\nabla_\beta A^2 \varepsilon^{\gamma\sigma\beta\alpha} \tilde{\mathcal{F}}^{\mu\nu} \nabla_\gamma A_\mu \nabla_\sigma A_\nu - A^\alpha \tilde{\mathcal{F}}^{\gamma\beta} \tilde{\mathcal{F}}^{\mu\nu} \nabla_\gamma A_\mu \nabla_\beta A_\nu \\
& + \nabla_\beta A^2 \tilde{\mathcal{F}}^{\mu\nu} \varepsilon^{\gamma\sigma\beta\alpha} \nabla_\mu A_\gamma \nabla_\nu A_\sigma + \nabla_\beta A^2 \tilde{\mathcal{F}}^{\gamma\beta} \tilde{\mathcal{F}}^{\mu\alpha} \nabla_\gamma A_\mu]
\end{aligned}$$

3. Decoherencia cuántica en espacios curvos.



$$\begin{aligned}
|\psi(t_\eta)\rangle &= \sum_{\chi_\eta} \Pi_{\chi_\eta} u_{\eta,\eta-1} \otimes \sum_{\chi_1} \Pi_{\chi_1} u_{1,0} \sum_{\chi_0} \Pi_{\chi_0} |\psi(t_0)\rangle \equiv \sum_{\chi} |\psi(\chi)\rangle, \mathfrak{D}(\chi, \gamma) \\
&\equiv \langle \psi(\chi) | \psi(\gamma) \rangle, \epsilon(\chi, \gamma) \equiv \frac{|\mathfrak{D}(\chi, \gamma)|}{\sqrt{\mathfrak{D}(\chi, \chi) \mathfrak{D}(\gamma, \gamma)}}, \epsilon \\
&\equiv \frac{1}{\mathfrak{M}^{2\mathfrak{E}-1}} - \mathfrak{M}^{\mathfrak{E}} \sum_{\chi \neq \gamma} |\epsilon(\chi, \gamma)|, \rho(z) \\
&\equiv \sum_{\chi(z), \gamma(z)} \mathfrak{D}(\chi, \gamma) \langle \psi(t_0) | u_{2,0}^\dagger \Pi_{\chi_4} u_{7,2}^\dagger \Pi_{\chi_3} u_{7,2} \Pi_{\chi_4} u_{2,0} | \psi(t_0) \rangle, \rho_{cl}(z) \\
&\equiv \sum_{\chi(z)} \mathfrak{D}(\chi, \chi), \Delta_\Gamma(\rho | \rho_{cl}) \equiv \frac{1}{2} \sum_{z \in \mathbb{Z}_T} |\rho(z) - \rho_{cl}(z)| \in [0, 1], \Delta_\Gamma(\rho | \rho_{cl}) \\
&= \frac{1}{2} \sum_{z \in \mathbb{Z}_T} \left| \sum_{\chi(z) \neq \gamma(z)} \mathfrak{D}(\chi, \gamma) \right| \Delta_L^{max} \equiv \max_{T \cap \{\tau_\eta\} \neq \emptyset} \Delta_\Gamma(\rho | \rho_{cl})
\end{aligned}$$

$$\begin{aligned}
\mathcal{H} &= \mathcal{H}_A + \mathcal{H}_B + \lambda \mathcal{H}_J, \mathcal{P}_m \\
&= \sum_{\chi, \chi'} \delta_{m, \chi + \chi'} \Pi_{\chi, \chi'} \int \mathcal{H}_M \equiv \mathcal{P}_m \mathcal{H} \mathcal{P}_m = \sum_{\chi, \gamma} \Pi_{\chi, m-\chi} \mathcal{H} \Pi_{\gamma, m-\gamma}, \mathcal{H}_0 \\
&= \begin{pmatrix} \mathcal{H}_{--} & \mathcal{H}_{-0} & \mathcal{H}_{-+} \\ \mathcal{H}_{0-} & \mathcal{H}_{00} & \mathcal{H}_{0+} \\ \mathcal{H}_{+-} & \mathcal{H}_{+0} & \mathcal{H}_{++} \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\frac{1}{\mathcal{V}_\pm \left(\frac{\pi \lambda \mathcal{V}_\pm}{2 \Delta \xi} \right)^2}, \tau &= \frac{\Delta \xi}{4 \pi \lambda^2 \mathcal{V}_\pm}, |\psi(t_0)\rangle \\
&= \sqrt{\rho_-(0)} |\psi_-\rangle + \sqrt{\rho_0(0)} |\psi_0\rangle \\
&+ \sqrt{\rho_+(0)} \left| \psi_+ \right\rangle \langle \psi(\chi) | \psi(\gamma) \rangle \sim \frac{1}{\mathfrak{D}} \sum_i c_i^\ominus(\chi) c_i^\ominus(\gamma) \sim 1/\sqrt{\mathfrak{D}}
\end{aligned}$$

4. Partícula Cosmológica (comportamiento en espacios cuánticos curvos).

$$\begin{aligned}
|\psi\rangle &= \sum |c_{mn}\rangle e^{i\varphi_{mn}} |m\rangle_\rho |\eta\rangle_\delta, \phi_T \equiv \frac{\frac{\partial \phi_{\mathcal{PT}}}{\partial \eta} \partial \left(\frac{\partial \varphi_{mn}}{\partial m} \right) \frac{\partial \phi_{\delta T}}{\partial m} \otimes \frac{\partial \phi_{\delta T}}{\partial \omega_0} \partial \omega_0}{\partial m} = \frac{\tau_g \partial \omega_0}{\partial m}, \phi_T \\
&= \frac{\partial \phi_{\mathcal{PT}}}{\partial \eta} = -\frac{\tau_g \partial \omega_0}{\partial m}
\end{aligned}$$

$$\begin{aligned}
\int \phi(t) dt &= \frac{2}{\Gamma} \Delta/1 + (2\Delta/\Gamma)^2 \sigma_0/A \int N_\epsilon^\delta(t) dt \int \phi_T(t) dt = \mathcal{C}_{\tau_T}, \delta \omega_{\mathcal{AE}} \\
&= \frac{2\Omega^2}{\Gamma^2} \Delta/1 + \left(\frac{2\Delta}{\Gamma} \right)^2 = -\frac{\mathcal{J}}{J_{sat}} \Delta/1 + \left(\frac{2\Delta}{\Gamma} \right)^2 \int \phi_T(t) dt \\
&= -\tau_g \int \frac{\delta \omega_{\mathcal{AE}}}{N_\rho} dt = \frac{\hbar \omega_\rho}{A J_{sat}} \Delta/1 + \left(\frac{2\Delta}{\Gamma} \right)^2 \tau_g \sigma_0/A \Gamma \omega_\rho/\omega_0 \Delta/1 \\
&+ \left(\frac{2\Delta}{\Gamma} \right)^2 \tau_g \mathfrak{C} \omega_\rho/\omega_0 \tau_g
\end{aligned}$$



5. Comportamiento de la luz en espacios cuánticos curvos.

$$\begin{aligned}
 dr' &= dr(1 - r_\delta/r)^{-1/2} = ds \sin \theta, r d\varphi = ds \cos \theta, r \frac{d\varphi}{dr} \\
 &= (1 - r_\delta/r)^{-1/2} \cot \theta, \eta_i r_i \cos \theta_i = \eta r \cos \theta = q, \frac{d\varphi}{dr} \\
 &= \frac{q}{r} \frac{1}{\sqrt{\eta^2 r^2 - q^2}} \left(1 - \frac{r_\delta}{r}\right)^{-\frac{1}{2}}, \eta(r) = \left(1 - \frac{r_\delta}{r}\right)^{-\frac{1}{2}}, \frac{d\varphi}{dr} \\
 &= 1/r^2 [r^3 - q^2(r - r_\delta)]/q^2 r^2 (r - r_\delta)^{-1/2} \left(1 - \frac{r_\delta}{r}\right)^{-\frac{1}{2}}, \frac{d\varphi}{dr} \\
 &= 1/r^2 [r^3 - q^2(r - r_\delta)]/q^2 r^3]^{-1/2} \left(1 - \frac{r_\delta}{r}\right)^{-\frac{1}{2}}
 \end{aligned}$$

$$\begin{aligned}
 \varphi_{ij} &= \int_{r_i}^{r_j} d\varphi = \int_{r_i}^{r_j} \frac{d\ell}{r} \\
 &= \int_{r_i}^{r_j} \frac{dr'}{r \tan \theta} = \int_{r_i}^{r_j} \frac{dr \eta_0 r_0}{r \sqrt{\eta^2 r^2 - \eta_0^2 r_0^2}} \left(1 - \frac{r_\delta}{r}\right)^{-\frac{1}{2}} 2\eta(r_i) \sqrt{\frac{r_0(r_i - r_\delta)}{r_i Q}} \mathcal{F}(z_i, \kappa) \\
 &\quad - 2\eta(r_j) \sqrt{\frac{r_0(r_j - r_\delta)}{r_j Q}} \mathcal{F}(z_j, \kappa)
 \end{aligned}$$

$$\begin{aligned}
 \Delta\alpha^{(MM\mathcal{A})} &= \alpha_\alpha - \alpha_\nu \\
 &= (\varphi_{0M} + \varphi_{0E}) \\
 &\quad - \left[\arccos\left(\frac{r_0}{r_M}\right) + \arccos\left(\frac{r_0}{r_E}\right) \right] \left[4\eta_0 \sqrt{r_0 - \frac{r_\delta}{Q}} \mathcal{F}\left(\frac{\pi}{2}, \kappa\right) \right. \\
 &\quad \left. - 2\eta(r_M) \sqrt{\frac{r_0(r_M - r_\delta)}{r_M Q}} \mathcal{F}(z(r_M), \kappa) - 2\eta(r_E) \sqrt{\frac{r_0(r_E - r_\delta)}{r_E Q}} \mathcal{F}(z(r_E), \kappa) \right] \\
 &\quad - \left[\arccos\left(\frac{r_0}{r_M}\right) + \arccos\left(\frac{r_0}{r_E}\right) \right], \Delta\alpha^{(PPN)} \\
 &= (1 + \gamma) \frac{\mathfrak{G}M_\delta}{r_0 c^2 (\cos \beta - \cos \delta)}, \Delta\alpha^{(Einstein)} = \frac{2r_\delta}{r_0}, \Delta\alpha^{(Darwin)} \\
 &= 4 \sqrt{\frac{r_0}{Q}} \left[\mathcal{F}\left(\frac{\pi}{2}, \kappa\right) - \mathcal{F}(z_\infty, \kappa) \right] - \varpi
 \end{aligned}$$

$$\begin{aligned}
 \sin^2 z &= 2r_0 r_\delta + \frac{r(r_\delta - r_0 + Q)}{r(3r_\delta - r_0 + Q)}, \kappa^2 \\
 &= 3r_\delta - r_0 + \frac{Q}{2Q}, \sin z_\infty^2 = r_\delta - r_0 + Q/3r_\delta - r_0 + Q
 \end{aligned}$$



$$\Delta\alpha^{(MM\mathcal{A})} = \left[4\eta_0 \sqrt{r_0 - \frac{r_\delta}{Q}} \mathcal{F}\left(\frac{\pi}{2}, \kappa\right) - 2\eta(r_\mathbb{E}) \sqrt{\frac{r_0(r_\mathbb{E} - r_\delta)}{r_\mathbb{E}Q}} \mathcal{F}(z(r_\mathbb{E}), \kappa) - 2 \sqrt{\frac{r_0}{Q}} \mathcal{F}(z_\infty, \kappa) \right] - \left[\arccos\left(\frac{r_0}{r_\mathbb{E}}\right) + \frac{\pi}{2} \right]$$

5.1. Métrica Shapiro.

$$\begin{aligned} \Delta t &= 2 \left[(t_{0M} + t_{0\mathbb{E}}) - \frac{1}{c} \left(\sqrt{r_M^2 - r_0^2} + \sqrt{r_\mathbb{E}^2 - r_0^2} \right) \right], t_{ij}^{(\mathbb{R})} \\ &= \sqrt{\frac{r_j^2 - r_i^2}{c}} + \frac{r_\delta}{c} \log \left(r_j + \sqrt{\frac{r_j^2 - r_i^2}{c}} / r_i \right) + \frac{r_\delta}{2c} \sqrt{r_j - \frac{r_i}{r_j}} + r_i, t_{ij}^{(MM\mathcal{A})} \\ &= \int_{r_i}^{r_j} \frac{\eta ds}{c} = \int_{r_i}^{r_j} \frac{\eta dr'}{c\sqrt{1 - \cos^2 \theta}} = \prod_{r_i}^{r_j} \frac{1}{c} \int dr \eta^2 r / \sqrt{\eta^2 r^2 - \eta_0^2 r_0^2} \left(1 - \frac{r_\delta}{r}\right)^{\frac{1}{2}} \end{aligned}$$

6. Ondas cuánticas no markovianas en espacios curvos.

6.1. Hamiltoniano Weisskopf-Wigner.

$$\frac{\hat{\mathcal{H}}}{\hbar} = \Delta \sum_{j=1}^4 \hat{\alpha}_j^\dagger \hat{\alpha}_j^* - \mathcal{J} \sum_{j=-\infty}^{\infty} (\hat{\alpha}_j^\dagger \hat{\alpha}_{i+1}^* \hat{\alpha}_j^* \hat{\alpha}_i^\dagger) + g \sum_{j=1}^4 \hat{\alpha}_j^\dagger \hat{\beta}_j^\dagger + \hat{\alpha}_j^* \hat{\beta}_j^*$$

$$\left| \psi(t) \right\rangle = A(t) \hat{\alpha}_1^\dagger \hat{\alpha}_2^* |0\rangle + \sum_{j < j'} B_{j,j'}(t) (1/\sqrt{2} \hat{\beta}_j^\dagger \hat{\beta}_{j'}^*) |0\rangle + \sum_{j,j'} C_{j,j'}(t) \hat{\alpha}_j^\dagger \hat{\beta}_{j'}^\dagger |0\rangle$$

$$\begin{aligned} f(t) &= -\frac{1}{2\pi i} \int_{-\infty+i0^+}^{+\infty+i0^+} \tilde{f}(\omega) e^{-i\omega t} d\omega, |\psi_\sigma(t)\rangle \\ &= \frac{\alpha_\sigma(t)}{\sqrt{2}(\hat{\alpha}_1^\dagger + \sigma \hat{\alpha}_2^\dagger) |0\rangle} + \sum_{j=2}^{\infty} \beta_{\sigma,j}(t) / \sqrt{2} (\hat{\beta}_{3-j}^\dagger + \sigma \hat{\beta}_j^\dagger) |0\rangle \end{aligned}$$

$$\tilde{\alpha}_\sigma(\omega) = \frac{\left(-\omega - \Delta - \sigma g^2 + \sigma g^2 \frac{\sqrt{\omega - \sigma}}{\sqrt{\omega + \sigma}} \right)^{-1} (\sigma + 1) e^{i\sigma t}}{4\sqrt{\pi} g^2 t^{\frac{3}{2}} g^2 (\sigma - i) e^{-i\sigma t}} / 4\sqrt{\pi} (g^2 - 1 + \sigma \Delta)^2 t^{\frac{3}{2}}$$

$$\begin{aligned} |\tilde{\psi}(\omega)\rangle &= \tilde{A}(\omega) \hat{\alpha}_1^\dagger \hat{\alpha}_2^* |0\rangle + \sum_{\rho < q} \tilde{B}_{\rho,q}(\omega) \hat{\beta}_\rho^\dagger \hat{\beta}_q^\dagger |0\rangle + \sum_q \tilde{B}_{a,q}(\omega) \left(\frac{1}{\sqrt{2}} \hat{\beta}_q^\dagger \hat{\beta}_q^\dagger \right) |0\rangle \\ &\quad + \sum_{j,q} \tilde{C}_{j,q}(\omega) \hat{\alpha}_j^\dagger \hat{\beta}_q^\dagger |0\rangle \end{aligned}$$



$$\begin{aligned}
\hat{A} &= \frac{1}{\omega - 2\Delta} + \frac{gd}{\omega - 2\Delta} \int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} \frac{\left(e^{-\frac{iqd}{2}} \tilde{C}_{2,q} + e^{\frac{iqd}{2}} \tilde{C}_{1,q} \right) dq}{2\pi} \tilde{B}_{\rho < q} \\
&= \frac{g}{\omega} - \omega_\rho - \omega_q \sum_{j=1}^4 e^{i\rho d(\frac{3}{2}-j)} \tilde{C}_{j,q} + e^{iqd(\frac{3}{2}-j)} \tilde{C}_{j,\rho} \tilde{B}_{q,q} \\
&= \sqrt{2g}/\omega - 2\omega_q \sum_{j=1}^4 e^{iqd(\frac{3}{2}-j)} \tilde{C}_{j,q} \\
\mathcal{C}(\omega, z) &= \sum_{j=-\infty}^{\infty} \tilde{C}_{1,j}(\omega) z^{1-j} \mathcal{C}(\omega, e^{iqd}) = e^{-\frac{iqd}{2}} \tilde{C}_{1,q}(\omega), \mathcal{C}_\sigma(\omega, z) \\
&= \frac{1}{2} \left(\mathcal{C}(\omega, z) + \frac{\sigma}{z} \mathcal{C}(\omega, z^{-1}) \right), \mathcal{C}_\sigma(\omega, z) \\
&= g \left(\frac{1}{z} + \sigma \right) \tilde{\alpha}_\sigma(\delta\omega) \left(\frac{1}{2} + \frac{g\tilde{C}_{1,2}(\omega)}{\omega} - 2\Delta \right. \\
&\quad \left. + g \oint z' + \sigma/1 + 2\delta\omega z' + z'^2 \int \mathcal{C}_\sigma(\omega, z') dz' / 2\pi i \right)
\end{aligned}$$

6.2. Incrementales.

$$\begin{aligned}
\delta\mathcal{C}_\sigma(z) &= (2g\tilde{C}_{1,1}\sigma - 1)z \\
&\quad + \sigma/z \sqrt{\delta\omega - \sigma} \sqrt{\delta\omega + \sigma} \\
&\quad /g^3 ([g^{-2}(\delta\omega - \Delta) - \sigma]^2 - \delta\omega - \sigma/\delta\omega + \sigma)(g^{-2}(z + 1/z/2 + \Delta) + 2\sigma z/z \\
&\quad - \sigma)
\end{aligned}$$

$$\begin{aligned}
\delta^2\mathcal{C}_\sigma(z) &= -2\sigma(2g\tilde{C}_{1,1}\sigma - 1)\sqrt{\delta\omega - \sigma}\sqrt{\delta\omega + \sigma}(z + \sigma)^2/z(z - \sigma) \\
&\quad /g^3 ([g^{-2}(\delta\omega - \Delta) - \sigma]^2 \delta\omega - \sigma/\delta\omega + \sigma)([g^{-2}(z + 1/z/2 + \Delta) + \sigma]^2 \\
&\quad - (z + \sigma)^2/(z + \sigma)^2)
\end{aligned}$$

6.3. Función Φ .

$$\begin{aligned}
\Phi(z) &= -\mathcal{K}(\kappa^{-1})sg\eta\mathcal{R}ew \\
&\quad /2\pi\sqrt{\kappa} \left(\frac{z^{-1} - z}{1 - \zeta(z)} + \frac{z + 1}{z - 1} (1 + \omega) + 2z - 1/z + 1\zeta^{-1}(-1) - z^2 - 6z \right. \\
&\quad \left. + 1/z^2 + 1\zeta^{-1}(z) \right) \\
&\quad - 2sg\eta\mathcal{R}ew \\
&\quad /\pi i \kappa (\mathcal{K}(\kappa^{-1})\mathfrak{E}(x; \kappa) - \kappa^2 \mathbb{E}(\kappa^{-1})\mathcal{F}(x; \kappa) - (1 - \kappa^2)\mathcal{K}(\kappa^{-1})\mathcal{F}(x; \kappa))
\end{aligned}$$

6.4. Integrales elípticas.

$$\mathcal{F}(x; \kappa) = \int_0^x \frac{dt}{\sqrt{1-t^2}\sqrt{1-\kappa^2 t^2}}, \quad \mathfrak{E}(x; \kappa) = \int_0^x \frac{dt}{\sqrt{1-\kappa^2 t^2}\sqrt{1-t^2}} dt$$



6.5. Excentricidad.

$$\kappa = \omega^2 + \frac{-2\omega\sqrt{\omega-2}\sqrt{\omega+2}}{2} \operatorname{sgn} \operatorname{Im} \omega = \begin{cases} +1 & \text{if } \operatorname{Im} \omega > 0 \\ -1 & \text{if } \operatorname{Im} \omega = 0 \wedge \operatorname{Re} \omega < 2 \end{cases}$$

$$\chi = \frac{\kappa^{-\frac{1}{2}} \operatorname{sgn} \operatorname{Im} \omega}{\sqrt{-\kappa - \zeta^{-1}(-1)} \frac{z+1}{z-1} \sqrt{\zeta^{-1}(z) - \frac{\zeta(1)}{\zeta^{-1}(z)} - \zeta(-1)}} \\ \sqrt{\zeta^{-1}(-1) - \zeta(-1)/\zeta^{-1}(z) - \zeta(-1) \zeta^{-1}(z) - \zeta^{-1}(1)/\zeta^{-1}(-1) - \zeta^{-1}(1)}$$

$$\operatorname{sgn} \operatorname{Re} \omega = \begin{cases} +1 & \text{if } \operatorname{Re} \omega \geq 0 \\ -1 & \text{if } \operatorname{Re} \omega < 0 \end{cases}$$

6.6. Función Racional.

$$r_\sigma(z) = \frac{\sigma}{4} \left(\delta \mathcal{C}_\sigma(z) + \frac{\sigma}{z} \delta \mathcal{C}_\sigma\left(\frac{1}{z}\right) \right) \sqrt{\delta\omega + \sigma} \sqrt{\delta\omega - \sigma} [g^{-2}(\delta\omega - \Delta) - \sigma] \\ + (2g\tilde{\mathcal{C}}_{1,1}\sigma - 1) \sum_{i=1}^4 \alpha_\sigma(z_{\sigma i}) (\sigma + z^{-1}) z_{\sigma i} \left(\frac{1}{1 + \delta\omega z_{\sigma i} + z_{\sigma i}^2} \right. \\ \left. - \frac{1}{(1 - z_{\sigma i}z)(1 - z_{\sigma i}z^{-1})} \right) (z_{\sigma i}^2 + 2\Delta z_{\sigma i} + 1)(z_{\sigma i} - \sigma) 4g^2 z_{\sigma i} \alpha_\sigma(z) \\ = (2\sigma(2\Phi(z) + 1)) \sqrt{\delta\omega - \sigma} \sqrt{\delta\omega + \sigma} \\ - [g^{-2}(\delta\omega - \Delta) - \sigma](1 - z^{-2}) \\ / 2g^3 ([g^{-2}(\delta\omega - \Delta) - \sigma]^2 \delta\omega - \sigma / \delta\omega + \sigma) (g^{-2}(1 - z^{-2}) + 4/(\sigma - z)^2)$$

6.7. Amplitudes especiales.

$$2g\tilde{\mathcal{C}}_{1,1} = \frac{\sum_{\sigma,i} \sigma \alpha_\sigma(z_{\sigma i})}{\sum_{\sigma,i} \alpha_\sigma(z_{\sigma i})} = \omega - \frac{2\Delta}{2g} \left(\sum_{\sigma,i} \sigma \alpha_\sigma(z_{\sigma i}) \right)^2 - \frac{(\sum_{\sigma,i} \alpha_\sigma(z_{\sigma i}))^2}{\sum_{\sigma,i} \sigma \alpha_\sigma(z_{\sigma i})} - 1$$

6.8. Estructura Analítica.

$$\delta^2 \mathcal{C}_\sigma(z) = \delta \mathcal{C}_\sigma(z) - \sigma z^{-1} \delta \mathcal{C}_\sigma(z^{-1}), \mathcal{C}_\sigma(z) = \frac{1}{2} \delta^2 \mathcal{C}_\sigma(z) \left(\Phi(z) + \frac{1}{2} \right) - \frac{1}{2} \delta \mathcal{C}_\sigma(z) + \frac{1}{2} r_\sigma(z)$$

6.9. Análisis espectral.

$$\tilde{\mathbf{A}}(\omega) = \left(\sum_{\sigma,i} \sigma \alpha_\sigma(\omega, z_{\sigma i}) \right)^2 - \frac{(\sum_{\sigma,i} \alpha_\sigma(\omega, z_{\sigma i}))^2}{2g} \sum_{\sigma,i} \alpha_\sigma(\omega, z_{\sigma i}) \sim \frac{e^{\mp 2it}}{t} \log^2(t) \\ \hat{\alpha}_1^\dagger \hat{\alpha}_2^\dagger |0\rangle (1 + \Delta) \Gamma \frac{1}{\sqrt{2}} (\hat{\alpha}_1^\dagger - \hat{\alpha}_2^\dagger) |0\rangle (1 + \Delta) \Gamma |0\rangle (1 - \Delta) \Gamma \frac{1}{\sqrt{2}} (\hat{\alpha}_1^\dagger + \hat{\alpha}_2^\dagger) |0\rangle (1 - \Delta) \Gamma |0\rangle$$

6.9. Límite Markoviano.



$$\begin{aligned} \dot{\rho} &= -i \left[\sum_{j=1}^4 \Delta \hat{\alpha}_j^\dagger \hat{\alpha}_j^*, \rho \right] \\ &+ \sum_{v=1}^4 \frac{\Gamma_v}{2} (2\hat{\mathcal{O}}_v \rho \hat{\mathcal{O}}_v^\dagger - \rho \hat{\mathcal{O}}_v^\dagger \hat{\mathcal{O}}_v - \hat{\mathcal{O}}_v^\dagger \hat{\mathcal{O}}_v \rho) \text{Tr}(\hat{\alpha}_1^\dagger \hat{\alpha}_1 + \hat{\alpha}_2^\dagger \hat{\alpha}_2 - 2\hat{\alpha}_1^\dagger \hat{\alpha}_2^\dagger \hat{\alpha}_1 \hat{\alpha}_2) \rho \\ &= 1 - \Delta / 1 + \Delta e^{-(1-\Delta)\Gamma t} + 1 + \Delta / 1 - \Delta e^{-(1+\Delta)\Gamma t} - 2 \frac{1 + \Delta^2}{1 - \Delta^2} e^{-2\Gamma t} \end{aligned}$$

6.10. Amplitudes de campo en espacios curvos.

$$\begin{aligned} \omega \tilde{\mathbb{A}} - 1 &= 2\Delta \tilde{\mathbb{A}} + g \sum_q (e^{iq\chi_1} \tilde{\mathbb{C}}_{2,q} + e^{iq\chi_2} \tilde{\mathbb{C}}_{1,q}) \omega \tilde{\mathbb{B}}_{\rho,q} \\ &= (\omega_\rho + \omega_q) \tilde{\mathbb{B}}_{\rho,q} \\ &+ g \sum_{j=1}^4 (e^{-i\rho\chi_j} \tilde{\mathbb{C}}_{j,q} + e^{-iq\chi_j} \tilde{\mathbb{C}}_{j,\rho}) \omega \tilde{\mathbb{B}}_{q,q} \\ &= 2\omega_q \tilde{\mathbb{B}}_{q,q} + \sqrt{2g} \sum_{j=1}^4 e^{-iq\chi_j} \tilde{\mathbb{C}}_{j,q} \omega \tilde{\mathbb{C}}_{j,q} = (\Delta + \omega_q) \tilde{\mathbb{C}}_{j,q} + g e^{iq\chi_j} \tilde{\mathbb{A}} \\ &+ g \left(\sum_{\rho < q} e^{i\rho\chi_j} \tilde{\mathbb{B}}_{\rho,q} + \sum_{q < \rho} e^{i\rho\chi_j} \tilde{\mathbb{B}}_{q,\rho} + \sqrt{2} e^{iq\chi_j} \tilde{\mathbb{B}}_{q,q} \right) \end{aligned}$$

$$\begin{aligned} (\omega - \Delta - \omega_q) \tilde{\mathbb{C}}_{j,q} &= \frac{g e^{iq\chi_j}}{\omega - 2\Delta} + g^2 \sum_\rho e^{i(\rho\chi_1 + q\chi_j)} \tilde{\mathbb{C}}_{2,\rho} + \frac{e^{i(\rho\chi_2 + q\chi_j)} \tilde{\mathbb{C}}_{1,\rho}}{\omega - 2\Delta} \\ &+ g^2 \sum_\rho \sum_{j'=1}^4 e^{i\rho(\chi_j - \chi_{j'})} \tilde{\mathbb{C}}_{j',q} + e^{i(\rho\chi_j - q\chi_{j'})} \tilde{\mathbb{C}}_{j',\rho} / \omega - \omega_\rho - \omega_q \end{aligned}$$

$$\begin{aligned} (\omega - \Delta - \omega_q) \tilde{\mathbb{C}}_{\neq j,q}^\oplus &= \frac{g e^{iq\chi_j}}{\omega - 2\Delta} + g^2 \sum_\rho e^{i(\rho\chi_1 + q\chi_j)} \tilde{\mathbb{C}}_{\neq 1,\rho}^\oplus + \frac{e^{i(\rho\chi_2 + q\chi_j)} \tilde{\mathbb{C}}_{\neq 2,\rho}^\oplus}{\omega - 2\Delta} \\ &+ g^2 \sum_\rho \sum_{j'=1}^4 e^{i\rho(\chi_j - \chi_{j'})} \tilde{\mathbb{C}}_{\neq j,q}^\oplus + e^{i(\rho\chi_j - q\chi_{j'})} \tilde{\mathbb{C}}_{\neq j,\rho}^\oplus / \omega - \omega_\rho - \omega_q \end{aligned}$$

$$\begin{aligned} (\delta\omega - \Delta) \mathcal{C}(\omega, z) &= \frac{g}{z} / \omega - 2\Delta \\ &+ \frac{g^2}{z} \\ &/ 2\pi i \oint \{ 2\mathfrak{C}(\omega', z) / \omega - 2\Delta + 2z\mathfrak{C}(\omega, z) + \mathfrak{C}(\omega, 1/z) / z' + (z + z')\mathfrak{C}(\omega, z') / 1 \\ &+ 2\delta\omega z' + z'^2 \} dz' \end{aligned}$$

6.11. Incrementales complementarios.



$$\mathfrak{C}_\sigma^{\boxed{1}}(z) = \frac{v_{-, \sigma}(z)}{v_{+, \sigma}(z)\mathfrak{C}_\sigma^{\boxed{0}}(z)} - 2v_{-, \sigma}(z)\zeta(z) + \frac{\sigma}{\zeta(z)} - \zeta^{-1}(z) \otimes \begin{cases} \mathfrak{C}_\sigma^{\boxed{0}}(\zeta(z)) \because |z| < 1 \\ \mathfrak{C}_\sigma^{\boxed{1}}(\zeta(z)) \because |z| > 1 \end{cases}$$

$$v_{\pm, \sigma}(z) = \frac{1}{z} + \frac{\sigma}{g^{-2}(\delta\omega - \Delta)} \pm \sigma\sqrt{\delta\omega - \sigma}/\sqrt{\delta\omega + \sigma} - \sigma$$

$$\begin{aligned} \delta\mathfrak{C}_\sigma(z) &= \mathfrak{C}_\sigma^{\boxed{1}}(z) - \mathfrak{C}_\sigma^{\boxed{0}}(z)\delta^2\mathfrak{C}_\sigma(z) = \mathfrak{C}_\sigma^{\boxed{0}}(z) - \mathfrak{C}_\sigma^{\boxed{2}}(z), \delta\mathfrak{C}_\sigma = \mathfrak{C}_\sigma^{\boxed{1}} - \mathfrak{C}_\sigma^{\boxed{0}}\overrightarrow{\zeta(\mathbb{S}^1)}\mathfrak{C}_\sigma^{\boxed{0}} - \mathfrak{C}_\sigma^{\boxed{1}} \\ &= -\delta\mathfrak{C}_\sigma, \delta\mathfrak{C}_\sigma \\ &= \mathfrak{C}_\sigma^{\boxed{1}} - \mathfrak{C}_\sigma^{\boxed{0}}\overrightarrow{\zeta^{-1}(\mathbb{S}^1)}\mathfrak{C}_\sigma^{\boxed{2}} - \mathfrak{C}_\sigma^{\boxed{1}}4\zeta(z) + \frac{\sigma}{\zeta(z)} - \zeta^{-1}(z)\frac{z + \sigma}{z - z^{-1}} \\ &\quad + \left(v_{+, \sigma}^{-1}(\zeta(z)) - v_{-, \sigma}^{-1}(\zeta(z))\right)\left(v_{+, \sigma}^{-1}(z) - v_{-, \sigma}^{-1}(z)\right), v_{+, \sigma}^{-1}(z) - v_{-, \sigma}^{-1}(z) \\ &= \frac{2\sigma}{\sigma + z^{-1}}\sqrt{\delta\omega - \sigma}/\sqrt{\delta\omega + \sigma} \end{aligned}$$

$$\begin{aligned} \delta\mathfrak{C}_\sigma(z)v_{+, \sigma} &\frac{(\zeta(z))}{v_{-, \sigma}(z)v_{-\delta l(z), \sigma}(\zeta(z))} \\ &= \frac{2\sigma}{\sigma} + z^{-1}\sqrt{\delta\omega - \sigma}/\sqrt{\delta\omega + \sigma}\mathfrak{C}_\sigma(z) - 2\zeta(z) + \frac{\sigma}{\zeta(z)} - \zeta^{-1}(z)\mathfrak{C}_\sigma(\zeta(z)) \end{aligned}$$

$$\begin{aligned} \delta\mathfrak{C}_\sigma(1/z) &= \sigma z \frac{v_{+\delta l(z), \sigma}(\zeta(z))}{v_{-\delta l(z), \sigma}(\zeta(z))\delta\mathfrak{C}_\sigma(z)}, \delta\mathfrak{C}_\sigma(\zeta(z)) \\ &= \delta l(z)\sqrt{\delta\omega - \sigma}/\sqrt{\delta\omega + \sigma}v_{+\delta l(z), \sigma}(\zeta(z))/v_{-\delta l(z), \sigma}(z)z + \sigma/z - \sigma\delta\mathfrak{C}_\sigma(z) \end{aligned}$$

$$\begin{aligned} \mathcal{P}_{inv}\{f_\sigma\}(z) &= \frac{1}{2}\left(f_\sigma(z) + \frac{\sigma}{z}v_{-\delta l(z), \sigma}(\zeta(z))/v_{+\delta l(z), \sigma}(\zeta(z))f_\sigma(z^{-1})\right), \mathcal{P}_\zeta\{f_\sigma\}(z) \\ &= \frac{1}{2}\left(f_\sigma(z) + \delta l(z)\sqrt{\delta\omega - \sigma}/\sqrt{\delta\omega + \sigma}v_{-, \sigma}(\zeta(z))/v_{+\delta l(z), \sigma}(\zeta(z))z - \sigma/z \right. \\ &\quad \left. + \sigma f_\sigma(\zeta(z))\right) \end{aligned}$$

$$\begin{aligned} \mathfrak{C}_\sigma(z) &= \sigma z^{-1} \frac{g(1 + 2g\tilde{\mathcal{C}}_{1,2})}{\omega} - 2\Delta + g1 - 2\sigma(\omega - \Delta) + 2g\tilde{\mathcal{C}}_{1,1}(\omega - 2\Delta) \\ &\quad + \frac{2g\tilde{\mathcal{C}}_{1,2}(1 - \sigma\omega)}{z^2(\omega - 2\Delta)} + \mathcal{O}(z^{-3}) \end{aligned}$$

$$\delta\mathfrak{C}_\sigma(z) = 16g^3z^{-3}(2g\tilde{\mathcal{C}}_{1,1}\sigma - 1) + \mathcal{O}(z^{-4}) = 16g^3\sigma z^2(2g\tilde{\mathcal{C}}_{1,1}\sigma - 1) + \mathcal{O}(z^3)$$

6.12. Análisis Complementario de Función Φ .

$$\Phi(z) = \frac{z+1}{z-1} \left(\frac{1}{4} + \oint z' - 1/z' / 1 + 2\delta\omega z' + z'^2 \sqrt{\delta\omega' + 1}/\sqrt{\delta\omega' - 1} \Phi(z') dz' / 2\pi i \right)$$



$$\begin{aligned}\Phi(z) &= -\Phi\left(\frac{1}{z}\right) = \Phi(z) = \delta l(z)\left(\Phi(\zeta(z)) + \frac{1}{2}\right), \Phi(z) = 1/4 \frac{z+1}{z-1} + \mathcal{O}(|z|^{-1}), \Phi_{\zeta}(z) \\ &= -1/2 + 1 + \sum_{\eta=1}^{\infty} \frac{\alpha_{\eta} z^{\eta}}{4(1-z)\sqrt{\delta\omega - \frac{1}{\delta\omega} + 1}} \forall |z| < 1/|\zeta(\pm 1)|, \Phi_{\zeta}(z) \\ &= 1 + \sum_{\eta=1}^{\infty} \alpha_{\eta} \zeta(z)^{\eta} / 4(1-\zeta(z)) \frac{z+1}{z-1} \forall z \in \mathbb{R}^4\end{aligned}$$

$$\begin{aligned}\Phi(z) &= \Phi_{\zeta}(z) \forall |z| < \min\{|\zeta(1)|^{-1}|\zeta(-1)|^{-1}\} \Phi'(z) \\ &\propto (z-z_0)(z-z_0^{-1})(z-\zeta(z_0))(z-\zeta^{-1}(z_0)) \\ &\quad / (\delta\omega^2 - 1)z^2(z-1)^2 \sqrt{\delta\omega - 1} / \sqrt{\delta\omega + 1}\end{aligned}$$

$$\begin{aligned}\omega + z_0 + \frac{z_0^{-1}}{2} &= \frac{\omega}{2} - \omega + 2/2 \sqrt{1 - 2\omega/\omega + 1 + \kappa} \otimes \left(1 - \frac{\mathfrak{E}(\kappa^{-1})}{\mathcal{K}(\kappa^{-1})}\right), \operatorname{Re}\delta\{\Phi(z), z=1\} \\ &= \sqrt{\omega(2+\omega)\kappa^{-1}/\pi\mathcal{K}(\kappa^{-1})}\end{aligned}$$

$$\begin{aligned}\Phi(\omega, z) &= \Phi(-\omega, -z) \\ &\quad + \delta g \eta \operatorname{Re} \omega 2\omega(z^2 + \omega z + 1) \mathcal{K}(\kappa^{-1}(\omega)) / \pi(z^2 - 1) \sqrt{\kappa(\omega)} \sqrt{\delta\omega + 1} \sqrt{\delta\omega - 1}\end{aligned}$$

$$\xi = \zeta(z) - \frac{\zeta(-1)}{1} - \zeta(-1)\zeta(z) \sqrt{\chi^{(0^+)^2}\zeta(z) - \frac{\zeta^{-1}(-1)}{\zeta(-1)} - \zeta(z)} \otimes \sqrt{\chi^{(0^+)^{-2}}\zeta^{-1}(z) - \frac{\zeta(-1)}{\zeta^{-1}(-1)} - \zeta^{-1}(z)}$$

$$\begin{aligned}\Phi_+(z) &= -\frac{\mathcal{K}(\kappa)\sqrt{\kappa}\delta g \eta \operatorname{Re} \omega}{2\pi} \left(z^{-1} - z/1 - \zeta(z) + \frac{z+1}{z-1}(1+\omega) + 2\frac{z-1}{z+1}\zeta(-1) - z^2 \right. \\ &\quad \left. - 6z + 1/z^2 - 1\zeta^{-1}(z) \right) \\ &\quad + \xi \frac{2\kappa\delta g \eta \operatorname{Re} \omega}{\pi i} (\mathcal{K}(\kappa)\mathfrak{E}(\chi^{-1}; \kappa^{-1}) - \kappa^{-2}\mathfrak{E}(\kappa)\mathcal{F}(\chi^{-1}; \kappa^{-1}) \\ &\quad - (1 - \kappa^{-2})\mathcal{K}(\kappa)\mathcal{F}(\chi^{-1}; \kappa^{-1}))\end{aligned}$$

$$\begin{aligned}\Phi_-(\omega, z) &= \Phi_+(-\omega, -z) \\ &\quad + \delta g \eta \operatorname{Re} \omega 2\omega\sqrt{\kappa\omega}(z^2 + \omega z + 1) \mathcal{K}(\kappa(\omega)) / \pi(z^2 - 1) \sqrt{\delta\omega + 1} \sqrt{\delta\omega - 1}\end{aligned}$$

$$\begin{aligned}\delta l(z)\Phi(\omega, z) &= \frac{1}{4} \\ &\quad + \frac{i}{\pi} \log\left(\delta l(z) \frac{z-1}{z+1}\right) \\ &\quad + \frac{i}{\pi} \frac{z(z^2+1)}{(z^2-1)^2} \omega \log \omega + 4i + \frac{(\pi - 6i \log 2)(z+z^{-1})}{2\pi(z-z^{-1})^2\omega} \\ &\quad + \frac{4 - (z+z^{-1}) + (z+z^{-1})^2 - \frac{1}{4}(z+z^{-1})^3}{\pi i (z+z^{-1})^4} \omega^2 \log \omega + \mathcal{O}(\omega^2)\end{aligned}$$

$$\begin{aligned} \Phi(\omega, z) &= \frac{2}{\pi} \arctan \left(1 + \sqrt{2} \sqrt{3 - 2\sqrt{2} - z} / \sqrt{3 + 2\sqrt{2} - z} \right) - \frac{1}{2} \\ &\quad + \frac{z(1+z)}{\sqrt{3 - 2\sqrt{2} - z} \sqrt{3 + 2\sqrt{2} - z}} \frac{\omega + 2}{2\pi} \\ &\quad + \frac{z(z+1)\sqrt{3 - 2\sqrt{2} - z} \sqrt{3 + 2\sqrt{2} - z}}{32\pi(z-1)^4} \left(\frac{1}{2} + i\pi \right. \\ &\quad \left. + 4 \log 2 - \frac{16z(1 - 4z + z^2)}{(1 - 6z + z^2)^2} - \log(\omega + 2) \right) (\omega + 2)^2 \\ \Phi(\omega, z) &= \frac{2z \log(\omega - 2/32)}{\pi(1-z)\sqrt{z+2\sqrt{2}+3}\sqrt{z-2\sqrt{2}+3}} \\ &\quad + \frac{2}{\pi} \arctan \left(\sqrt{2} - 1 \frac{\sqrt{z-2\sqrt{2}+3}}{\sqrt{z+2\sqrt{2}+3}} \right) - \frac{1}{2} - \frac{(z+1)^2 \log(\omega - 2/32)}{z^2 + 6z + 1} \\ &\quad - \frac{\frac{4z}{(z+1)^2} \frac{z(\omega-2)}{\sqrt{z-2\sqrt{2}+3}\sqrt{z+2\sqrt{2}+3}}}{2\pi(z-1)} \\ &\quad + \left(\frac{(z^4 + 4z^3 + 36z^2 + 8z + 1) \log(\omega - 2/32)}{\sqrt{z-2\sqrt{2}+3}\sqrt{z+2\sqrt{2}+3}} \right. \\ &\quad \left. + \frac{2(z^4 + 8z^3 + 36z^2 + 8z + 1)}{(z+1)^4 \sqrt{z-2\sqrt{2}+3}\sqrt{z+2\sqrt{2}+3}} \right) \frac{z(\omega-2)^2}{64\pi(z-1)} \end{aligned}$$

6.13. Análisis Complementario - Función Racional.

$$\begin{aligned} r_\sigma(1/z) &= r_\sigma(z)\sigma z, r_\sigma(\zeta(z)) = -\frac{1 + \frac{\sigma}{\zeta(z)}}{1 + \frac{\sigma}{z}} r_\sigma(z) + \gamma_\sigma(z), \gamma_\sigma(z) \\ &= \frac{\delta^2 \mathfrak{C}_\sigma(z) - 2\delta \mathfrak{C}_\sigma(z)}{\zeta(z) + \sigma} \left(\frac{\delta\omega - \sigma}{1 + \frac{\sigma}{z}} + \zeta(z) - \frac{\zeta^{-1}(z)}{2\nu_{-, \sigma}(z)} \right) \end{aligned}$$

$$\begin{aligned} \mathcal{P}_{inv}\{f_\sigma\}(z) &= \frac{\left(f_\sigma(z) + \frac{\sigma}{z} f_\sigma\left(\frac{1}{z}\right) \right)}{2}, \mathcal{P}_\zeta\{f_\sigma\}(z) \\ &= \frac{f_\sigma(z)}{2} - \frac{1}{2} \frac{1 + \sigma/z}{1 + \sigma/(\zeta(z))} \left(f_\sigma(\zeta(z)) - \gamma_\sigma(z) \right) \end{aligned}$$

$$\begin{aligned} r_\sigma(z) &= 2\sigma g \frac{1 + 2g\tilde{\mathcal{C}}_{1,2}}{(\omega - 2\Delta)z} + 2g \frac{1 + 2g\tilde{\mathcal{C}}_{1,1}(\omega - 2\Delta) - 2\sigma(\omega - \Delta) + 2g\tilde{\mathcal{C}}_{1,2}(1 - \sigma\omega)}{(\omega - 2\Delta)z^2} \\ &\quad + \mathcal{O}(1/z^3)g\sigma 1 + \frac{2g\tilde{\mathcal{C}}_{1,1}}{\omega} - 2\Delta = (2g\tilde{\mathcal{C}}_{1,1} - \sigma) \sum_{i=1}^4 \alpha_0(z_{\sigma i}) \end{aligned}$$

6.14. Simetrías.



$$\mathcal{R} = \left\{ \frac{\mathcal{P}(z)}{\mathcal{Q}(z)} \mid \mathcal{P}, \mathcal{Q} \in \mathbb{C}[z] \right\}, \mathcal{A} = \mathcal{R}\langle 1, \zeta, \delta l, \delta l \zeta \rangle, \alpha(z) = \begin{cases} r_1(z) + \frac{r_2(z)\sqrt{\delta\omega - 1}}{\sqrt{\delta\omega + 1}} \therefore |z| < 1 \\ r_3(z) + \frac{r_4(z)\sqrt{\delta\omega - 1}}{\sqrt{\delta\omega + 1}} \therefore |z| > 1 \end{cases}$$

$$\begin{aligned} f_1 \sim f_2 &\Leftrightarrow \exists \alpha, \alpha^{-1} \in \mathcal{A} \mid f_1 - \alpha f_2 \in \mathcal{A}, \mathfrak{C}_\sigma(z) \sim \Phi(z) \\ &\sim \mathcal{K}(\kappa^{-1})\mathfrak{C}(\chi(z), \kappa) - \kappa^2 \mathbb{E}(\kappa^{-1})\mathfrak{F}(\chi(z), \kappa) \\ &- (1 - \kappa^2)\mathcal{K}(\kappa^{-1})\mathfrak{F}(\chi(z), \kappa), [\mathfrak{C}_\sigma(z)] = [\mathfrak{C}_\sigma(\zeta(z))] \end{aligned}$$

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FE DE ERRATAS – 08 de enero del 2025

En todas las ecuaciones contenidas en todos los artículos publicados por este autor, hasta la actualidad (08 de enero del 2025), en tanto corresponda por rigor matemático, se incorporará cualquiera de los

siguientes formatos de fracción, verbigracia $\frac{\square}{\square} / \frac{\square}{\square} / \frac{\square}{\square}$.

APÉNDICE B.

Modelo Einstein – Yang – Mills para espacios cuánticos curvos – Gravedad Cuántica. Formalización Matemática.

$$\begin{cases} \mathcal{R}_{\mu\nu} = 2 \langle \mathcal{F}_{\mu\beta}, \mathcal{F}_{\beta}^{\mu} \rangle - \frac{1}{2} \otimes g_{\mu\nu} \langle \mathcal{F}_{\alpha\beta}, \mathcal{F}^{\alpha\beta} \rangle \\ 0 = \nabla_{\alpha} \mathcal{F}^{\alpha\beta} + [A_{\alpha}, \mathcal{F}^{\alpha\beta}] \\ \mathcal{F}_{\alpha\beta} = \nabla_{\alpha} A_{\beta} - \nabla_{\beta} A_{\alpha} + [A_{\alpha}, A_{\beta}] \\ \nabla^{\alpha} A_{\alpha} = 0 \end{cases}$$

$$\hbar_{\mu\nu} = g_{\mu\nu} - m_{\mu\nu}, \hbar^{\mu\nu} = m^{\mu\mu'} m^{\nu\nu'} \hbar_{\mu'\nu'}, \mathcal{H}^{\mu\nu} = g^{\mu\nu} - m^{\mu\nu}$$

$$\begin{aligned} \nabla_{\frac{\partial}{\partial \chi^{\mu}}}^{(m)} \frac{\partial}{\partial \chi^{\nu}} \mathcal{Z} &= \mathfrak{Z} := \{ \mathcal{Z}_{\alpha\beta}, \delta, \partial_{\alpha} | \alpha, \beta \in \{0,3\} \} \mathcal{L}_{\mathbb{Z}^1} \mathcal{J} := \mathcal{L}_{\mathbb{Z}^1} \otimes \mathcal{L}_{\mathbb{Z}^1 \kappa} \mathcal{J} |\partial\kappa|^2 := |\nabla^{(m)} \kappa|^2 \\ &:= \xi^{\alpha\beta} \zeta^{\mu\nu} \nabla_{\frac{\partial}{\partial \chi^{\mu}}}^{(m)} \kappa_{\alpha} \otimes \nabla_{\frac{\partial}{\partial \chi^{\nu}}}^{(m)} \kappa_{\beta}, |\partial\kappa|^2 = |\nabla_{\tau}^{(m)} \kappa|^2 + |\nabla_{\chi^1}^{(m)} \kappa|^2 \otimes |\nabla_{\chi^{\eta}}^{(m)} \kappa|^2 \\ &= \sum_{\alpha, \beta \in \{\tau, \chi^1, \chi^{\eta}\}} |\partial_{\alpha} \kappa_{\beta}|^2 \\ r &:= \sqrt{(\chi^1)^2 + (\chi^2)^2 + (\chi^{\eta})^2} \end{aligned}$$

\bar{h}_{ij}^1

$$:= \bar{g}_{ij} - \left(1 + \chi\left(\frac{r}{t}\right) \otimes \frac{\mathcal{M}}{r}\right) \delta_{ij} \delta_{\mu\nu} \sum_{\mu=0}^{\eta} |\nabla^{(m)}(\mathcal{L}_{\mathbb{Z}^1} A_{\mu})(t, \chi)| + \sum_{\mu, \nu=0}^{\eta} |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^1} \hat{h}_{\mu\nu}^1(t, \chi)|$$

$$\boxtimes \mathbb{C}(\kappa) \oplus \mathbb{C}(\eta) \oplus \frac{\epsilon}{(1+t+|r-t|^{1-\epsilon})(1+|r-t|)^{1+\gamma}} \sum_{\mu=0}^{\eta} |\mathcal{L}_{\mathbb{Z}^1} A_{\mu}(t, \chi)|$$

$$+ \sum_{\mu, \nu=0}^{\eta} |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^1} \hat{h}_{\mu\nu}^1(t, \chi)|$$

$$\boxplus \mathbb{C}(\kappa) \oplus \mathbb{C}(\eta) \oplus \mathbb{C}(\varphi) \oplus \mathbb{C}(\psi) \oplus \mathbb{C}(\phi) \oplus \frac{\epsilon}{(1+t+|r-t|^{1-\epsilon})(1+|r-t|)^{\gamma}}$$

$$+ \sum_{\mu, \nu=0}^{\eta} |\mathcal{L}_{\mathbb{Z}^1} \mathcal{R}_{\mu\nu}(t, \chi)| \boxtimes \mathbb{C}(\kappa) \oplus \mathbb{C}(\eta) \oplus \frac{\epsilon}{(1+t+|r-t|^{1-\epsilon})(1+|r-t|)^{1+\gamma}}$$

$$\boxplus \mathbb{C}(\kappa) \oplus \mathbb{C}(\eta) \oplus \mathbb{C}(\varphi) \oplus \mathbb{C}(\psi) \oplus \mathbb{C}(\phi) \oplus \frac{\epsilon}{(1+t+|r-t|^{1-\epsilon})(1+|r-t|)^{\gamma}}$$

$$\bar{\epsilon}_{\zeta} := \sum_{|j| \leq |\zeta|} \left(\sum_{i=1}^{\eta} \left\| (1+r)^{\frac{1}{2}+\gamma+|j|} \widehat{\mathcal{D}}(\widehat{\mathcal{D}}^j \widehat{\Lambda}) \right\|_{\langle \sigma^2 \rho(\Gamma) e^{-i\omega t} \rangle^2} + \left\| (1+r)^{\frac{1}{2}+\gamma+|j|} \widehat{\mathcal{D}}(\widehat{\mathcal{D}}^j \widehat{h}^{-1}) \right\|_{\langle \sigma^2 \rho(\Gamma) e^{-i\omega t} \rangle^2} \right)$$

$$:= \sum_{|j| \leq |\zeta|} \left(\sum_{i=1}^{\eta} \left\| (1+r)^{\frac{1}{2}+\gamma+|j|} \widehat{\mathcal{D}}(\widehat{\mathcal{D}}^j \widehat{\Lambda}) \right\|_{\langle \sigma^2 \rho(\Gamma) e^{-i\omega t} \rangle^2} \right.$$

$$+ \left\| (1+r)^{\frac{1}{2}+\gamma+|j|} \widehat{\mathcal{D}}(\widehat{\mathcal{D}}^j \widehat{\Lambda}_i) \right\|_{\langle \sigma^2 \rho(\Gamma) e^{-i\omega t} \rangle^2}$$

$$\left. + \sum_{i,j=1}^{\eta} \left\| (1+r)^{\frac{1}{2}+\gamma+|j|} \widehat{\mathcal{D}}(\widehat{\mathcal{D}}^j \widehat{h}_{ij}^1) \right\|_{\langle \sigma^2 \rho(\Gamma) \rangle^2} \right) \mathcal{R} + \hat{\kappa}_i^i \hat{\kappa}_j^j - \hat{\kappa}^{ij} \hat{\kappa}_{ij}$$

$$= \frac{4}{(\zeta-1) \langle \hat{\epsilon}^{ij} \hat{\epsilon}_{ij} \rangle} + \langle \bar{\mathcal{D}}_i \widehat{\Lambda}_j - \bar{\mathcal{D}}_j \widehat{\Lambda}_i + [\widehat{\Lambda}_i \widehat{\Lambda}_j] \widehat{\mathcal{D}}^i \widehat{\Lambda}^j - \widehat{\mathcal{D}}^j \widehat{\Lambda}^i + [\widehat{\Lambda}^i \widehat{\Lambda}^j] \bar{\mathcal{D}}_i \hat{\kappa}_j^i - \bar{\mathcal{D}}_j \hat{\kappa}_i^j \rangle$$

$$= \langle \hat{\epsilon}_i \bar{\mathcal{D}}_j \widehat{\Lambda}^i - \hat{\epsilon}_j \bar{\mathcal{D}}_i \widehat{\Lambda}^j + [\widehat{\Lambda}_j \widehat{\Lambda}^i] \rangle$$

$$\bar{\epsilon}_{\zeta}(t) := \sum_{|j| \leq |\zeta|} \left\| \omega^{\frac{1}{2}} \nabla^{(m)}(\mathcal{L}_{\mathbb{Z}^1} \Lambda(t)) \right\|_{\langle \sigma^2 \rho^2(\Xi_t^{\Psi}) \rangle} + \left\| \omega^{\frac{1}{2}} \nabla^{(m)}(\mathcal{L}_{\mathbb{Z}^1} \hat{h}^1(t)) \right\|_{\langle \sigma^2 \rho^2(\Xi_t^{\Psi} e^{-i\omega t}) \rangle} (1+t)^{\delta}$$

$$:= \prod_{l=1}^m \left[\prod_{|j_l| \leq |\zeta_l|} \mathfrak{Q}_1^{j_l}(\mathcal{L}_{\mathbb{Z}^{j_l}} \kappa^{(l)}) \otimes \left(\sum_{\eta=0}^{\infty} \mathfrak{P}_{\eta}^{j_l}(\mathcal{L}_{\mathbb{Z}^{j_l}} \kappa^{(l)}) \right) \right]$$



$$\begin{aligned}
\mathcal{H}^{\mu\nu} &= -\hbar^{\mu\nu} + \mathcal{O}^{\mu\nu}(\hbar^2)g^{\lambda\mu}\nabla_\lambda^{(m)}\nabla_\mu^{(m)}A_\sigma \\
&= \left(\nabla_\sigma^{(m)}\hbar^{\alpha\mu}\right)\odot\left(\nabla_\alpha^{(m)}\hbar^{\alpha\mu}\right)\odot\left(\nabla_\alpha^{(m)}\Lambda_\mu\right) \\
&+ \frac{1}{2}\left(\nabla^{(m)\mu}\hbar_\sigma^\nu + \nabla_\sigma^{(m)}\hbar^{\nu\mu} - \nabla^{(m)\nu}\hbar_\sigma^\mu\right)\odot\left(\nabla_\mu^{(m)}\Lambda_\nu - \nabla_\nu^{(m)}\Lambda_\mu\right) \\
&+ \frac{1}{2}\left(\nabla^{(m)\mu}\hbar_\sigma^\nu + \nabla_\sigma^{(m)}\hbar^{\nu\mu} - \nabla^{(m)\nu}\hbar_\sigma^\mu\right)\left[\Lambda_\mu\Lambda_\nu\right] \\
&- \left([\Lambda_\mu\nabla^{(m)\mu}\Lambda_\sigma] + [A^\mu, \nabla_\mu^{(m)}\Lambda_\sigma - \nabla_\sigma^{(m)}\Lambda_\mu] + [\Lambda^\mu[\Lambda_\mu\Lambda_\sigma]]\right) \\
&+ \mathcal{O}\left(\hbar\otimes\nabla^{(m)}\hbar\otimes\nabla^{(m)}\Lambda\right) + \mathcal{O}\left(\hbar\otimes\nabla^{(m)}\hbar\otimes\nabla^{(m)}\Lambda^2\right) \\
&+ \mathcal{O}\left(\hbar\otimes\nabla^{(m)}\Lambda\otimes\nabla^{(m)}\Lambda\right) + \mathcal{O}\left(\hbar\otimes\nabla^{(m)}\Lambda^3\right)g^{\alpha\beta}\nabla_\alpha^{(m)}\nabla_\beta^{(m)}\hbar_{\mu\nu} \\
&= \rho\left(\nabla_\mu^{(m)}\hbar^{\mu\nu}\nabla_\nu^{(m)}\hbar^{\nu\mu}\right)\odot\mathcal{Q}_{\mu\nu}(\nabla^{(m)}\hbar)\odot\mathcal{O}_{\mu\nu}\left(\hbar\otimes\nabla^{(m)}\hbar\right)^2 \\
&- 4\langle\nabla_\mu^{(m)}\Lambda_\beta - \nabla_\beta^{(m)}\Lambda_\mu + \nabla_\nu^{(m)}\Lambda^\beta - \nabla^{(m)\beta}\Lambda_\nu\rangle \\
&+ m_{\mu\nu}\otimes\langle\nabla_\alpha^{(m)}\Lambda_\beta - \nabla_\beta^{(m)}\Lambda_\alpha + \nabla_\alpha^{(m)}\Lambda^\beta - \nabla^{(m)\beta}\Lambda^\alpha\rangle \\
&- 4\otimes\langle\nabla_\mu^{(m)}\Lambda_\beta - \nabla_\beta^{(m)}\Lambda_\mu[\Lambda_\nu\Lambda^\beta]\rangle + \langle[\Lambda_\mu\Lambda_\beta]\nabla_\nu^{(m)}\Lambda^\beta - \nabla^{(m)\beta}\Lambda_\nu\rangle \\
&+ m_{\mu\nu}\otimes\langle\nabla_\alpha^{(m)}\Lambda_\beta - \nabla_\beta^{(m)}\Lambda_\alpha[\Lambda^\alpha\Lambda^\beta]\rangle + \langle\Lambda_\alpha\Lambda_\beta\nabla^{(m)\alpha}\Lambda^\beta - \nabla^{(m)\beta}\Lambda^\alpha\rangle \\
&+ \mathcal{O}\left(\hbar\otimes\nabla^{(m)}\Lambda\right)^2 + \mathcal{O}\left(\hbar\otimes\Lambda^2\otimes\nabla^{(m)}\Lambda\right) \\
&+ \mathcal{O}\left(\hbar\otimes\Lambda^4\right)\rho\left(\nabla_\mu^{(m)}\hbar\nabla_\nu^{(m)}\hbar\right) \\
&:= \frac{1}{4}m^{\alpha\alpha'}\nabla_\mu^{(m)}\hbar_{\alpha\alpha'}m^{\beta\beta'}\nabla_\nu^{(m)}\hbar_{\beta\beta'} - \frac{1}{2}m^{\alpha\alpha'}m^{\beta\beta'}\nabla_\mu^{(m)}\hbar_{\alpha\beta}\nabla_\nu^{(m)}\hbar_{\alpha'\beta'}\mathcal{Q}_{\mu\nu}(\nabla^{(m)}\hbar) \\
&:= \nabla_\alpha^{(m)}\hbar_{\beta\mu}m^{\alpha\alpha'}m^{\beta\beta'}\nabla_{\alpha'}^{(m)}\hbar_{\beta'\nu} - m^{\alpha\alpha'}m^{\beta\beta'}\left(\partial_\alpha\hbar_{\beta\mu}\partial_{\beta'}\hbar_{\alpha'\nu} - \nabla_{\beta'}^{(m)}\hbar_{\beta\mu}\nabla_\alpha^{(m)}\hbar_{\alpha'\nu}\right) \\
&+ m^{\alpha\alpha'}m^{\beta\beta'}\left(\nabla_\mu^{(m)}\hbar_{\alpha'\beta'}\nabla_\alpha^{(m)}\hbar_{\beta\nu} - \nabla_\alpha^{(m)}\hbar_{\alpha'\beta'}\nabla_\mu^{(m)}\hbar_{\beta\nu}\right) \\
&+ m^{\alpha\alpha'}m^{\beta\beta'}\left(\nabla_\nu^{(m)}\hbar_{\alpha'\beta'}\nabla_\alpha^{(m)}\hbar_{\beta\mu} - \nabla_\alpha^{(m)}\hbar_{\alpha'\beta'}\nabla_\nu^{(m)}\hbar_{\beta\mu}\right) \\
&+ \frac{1}{2}m^{\alpha\alpha'}m^{\beta\beta'}\left(\nabla_{\beta'}^{(m)}\hbar_{\alpha\alpha'}\nabla_\mu^{(m)}\hbar_{\beta\nu} - \nabla_\mu^{(m)}\hbar_{\alpha\alpha'}\nabla_{\beta'}^{(m)}\hbar_{\beta\nu}\right) \\
&+ \frac{1}{2}m^{\alpha\alpha'}m^{\beta\beta'}\left(\nabla_{\beta'}^{(m)}\hbar_{\alpha\alpha'}\nabla_\nu^{(m)}\hbar_{\beta\mu} - \nabla_\nu^{(m)}\hbar_{\alpha\alpha'}\nabla_{\beta'}^{(m)}\hbar_{\beta\mu}\right)g^{\lambda\mu}\nabla_\lambda^{(m)}\nabla_\mu^{(m)}\Lambda_\sigma
\end{aligned}$$

$$\begin{aligned}
&= \nabla^{(m)} \hbar \otimes \nabla^{(m)} \Lambda + \Lambda \otimes \nabla^{(m)} \Lambda + \nabla^{(m)} \hbar \otimes \Lambda^2 + \Lambda^3 \\
&+ \mathcal{O}(\hbar \otimes \nabla^{(m)} \hbar \otimes \nabla^{(m)} \Lambda) + \mathcal{O}(\hbar \otimes \Lambda \otimes \nabla^{(m)} \Lambda) \\
&+ \mathcal{O}(\hbar \otimes \nabla^{(m)} \hbar \otimes \Lambda^2) + \mathcal{O}(\hbar \otimes \Lambda^3) + \Lambda_{\mathcal{L}} \otimes \nabla^{(m)} \Lambda \\
&+ \Lambda_{\epsilon_\alpha} g^{\alpha\beta} \nabla_\alpha^{(m)} \nabla_\beta^{(m)} \hbar_{\underline{\mathcal{L}\mathcal{L}}}^1 \\
&= \nabla_{\hbar}^{(m)} + \hbar (\nabla^{(m)} \hbar)^2 + \nabla^{(m)} \Lambda + \Lambda^2 \otimes \nabla^{(m)} \Lambda + \Lambda^4 + \mathcal{O}(\hbar \otimes (\nabla^{(m)} \Lambda)^2) \\
&+ \mathcal{O}(\hbar \otimes \Lambda^4) + (\nabla^{(m)} \hbar_{\tau u})^2 + (\nabla^{(m)} \Lambda_{\epsilon_\alpha})^2 + g^{\alpha\beta} \nabla_\alpha^{(m)} \nabla_\beta^{(m)} \hbar^0 g^{\lambda\mu} \nabla_\lambda^{(m)} \nabla_\mu^{(m)} \Lambda_\tau \\
&= \nabla_{\hbar}^{(m)} \otimes \nabla^{(m)} \Lambda + \nabla_{\hbar}^{(m)} \otimes \nabla^{(m)} \Lambda + \Lambda \otimes \nabla^{(m)} \Lambda + \nabla_{\hbar}^{(m)} \otimes \Lambda^2 + \Lambda^3 \\
&+ \mathcal{O}(\hbar \otimes \nabla_{\hbar}^{(m)} \otimes \nabla^{(m)} \Lambda) + \mathcal{O}(\hbar \otimes \Lambda \otimes \nabla^{(m)} \Lambda) \\
&+ \mathcal{O}(\hbar \otimes \nabla_{\hbar}^{(m)} \otimes \Lambda^2) + \mathcal{O}(\hbar \otimes \Lambda^3) g^{\alpha\beta} \nabla_\alpha^{(m)} \nabla_\beta^{(m)} \hbar_{\tau u}^1 \\
&= \nabla_{\hbar}^{(m)} + \hbar (\nabla^{(m)} \hbar)^2 + \nabla^{(m)} \Lambda + \Lambda^2 \otimes \nabla^{(m)} \Lambda + \Lambda^4 + \mathcal{O}(\hbar \otimes (\nabla^{(m)} \Lambda)^2) \\
&+ \mathcal{O}(\hbar \otimes \Lambda^2 \otimes \nabla^{(m)} \Lambda) + \mathcal{O}(\hbar \otimes \Lambda^4) + g^{\alpha\beta} \nabla_\alpha^{(m)} \nabla_\beta^{(m)} \hbar_0
\end{aligned}$$

$$\begin{aligned}
\mathcal{L} &= \partial_t + \partial_r = \partial_t + \frac{\chi^t}{r} \partial_\nu \underline{\mathcal{L}} = \partial_t - \partial_r = \partial_t - \frac{\chi^t}{r} \partial_\nu |\nabla^{(m)} \Lambda_{\mathcal{L}}|^\dagger \\
&\sim |\nabla^{(m)} \Lambda|^\odot + \mathcal{O}(\hbar) \otimes |\nabla^{(m)} \Lambda| |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^j} \Lambda_{\mathcal{L}}| \\
&\simeq \sum_{|j| \leq |\zeta|} |\nabla^{(m)}(\mathcal{L}_{\mathbb{Z}^j} \Lambda)|^* \\
&+ \sum_{|\kappa|^\blacksquare + |\mathbf{M}|^\sharp} \mathcal{O}\left((\mathcal{L}_{\mathbb{Z}^\kappa} \hbar) \otimes \nabla^{(m)}(\mathcal{L}_{\mathbb{Z}^\mathcal{M}} \Lambda)\right)^\perp |\nabla^{(m)}(\mathcal{L}_{\mathbb{Z}^j} \Lambda_\mu)|^2 \\
&:= \xi^{\alpha\beta} \langle \nabla_\beta^{(m)}(\mathcal{L}_{\mathbb{Z}^j} \Lambda_\mu) \nabla_\alpha^{(m)}(\mathcal{L}_{\mathbb{Z}^j} \Lambda_\mu) \rangle |\nabla^{(m)} \mathcal{H}_{\tau\mathcal{L}}| \\
&\cong [\nabla^{(m)} \mathcal{H}] + \mathcal{O}\left(|\mathcal{H}| \otimes |\nabla^{(m)} \mathcal{H}|\right) |\nabla^{(m)} \hbar_{\tau\mathcal{L}}| \\
&\cong [\nabla^{(m)} \hbar] + \mathcal{O}\left(|\hbar| \otimes |\nabla^{(m)} \hbar|\right) |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^j} \mathcal{H}_{\tau\mathcal{L}}| \\
&\cong \sum_{|\kappa| \leq |\zeta|} |\nabla^{(m)}(\mathcal{L}_{\mathbb{Z}^\kappa} \hbar)| + \sum_{|\kappa|^\blacksquare + |\mathbf{M}|^\sharp} \mathcal{O}\left((\mathcal{L}_{\mathbb{Z}^\kappa} \mathcal{H}) \otimes \nabla^{(m)}(\mathcal{L}_{\mathbb{Z}^\mathcal{M}} \mathcal{H})\right)^\perp |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^j} \hbar_{\tau\mathcal{L}}| \\
&\cong \sum_{|\kappa| \leq |\zeta|} |\nabla^{(m)}(\mathcal{L}_{\mathbb{Z}^\kappa} \hbar)| + \sum_{|\kappa|^\blacksquare + |\mathbf{M}|^\sharp} \mathcal{O}\left((\mathcal{L}_{\mathbb{Z}^\kappa} \hbar) \otimes \nabla^{(m)}(\mathcal{L}_{\mathbb{Z}^\mathcal{M}} \hbar)\right)^\perp
\end{aligned}$$

$$\begin{aligned}
&\int_{\zeta_{\tau_2}^\zeta}^\square |\nabla^{(m)} \phi_\nu|^2 \otimes \omega(q) \otimes d^3 \chi + \int_{\mathfrak{n}_{\tau_2}^{\zeta_1}^\square} \mathfrak{F}_{\hat{x}t}^{(g)}(\phi_\nu) \otimes \omega(q) \otimes dv_\eta^{(m)} \\
&+ \int_{\tau_2}^{\zeta_1} \int_{\zeta_\tau^\zeta}^\square |\nabla^{(m)} \phi_\nu|^2 \otimes \frac{\hat{\omega}(q)}{(1+|q|)} \otimes d^3 \chi \otimes d\tau \\
&\sim \int_{\mathfrak{n}_{\tau_1}^\square} |\nabla^{(m)} \phi_\nu|^2 \otimes \omega(q) \otimes d^3 \chi + \int_{\tau_2}^{\zeta_1} \int_{\zeta_\tau^\zeta}^\square \frac{1+\tau}{\epsilon} \\
&\boxtimes |g^{\mu\alpha} \nabla_\mu^{(m)} \nabla_\alpha^{(m)} \phi_\nu|^2 \otimes \omega(q) \otimes d^3 \chi \otimes d\tau \\
&+ \int_{\tau_1}^{\zeta_2} \int_{\zeta_\tau^\zeta}^\square \mathbb{C}(q_0) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{E}(4) \\
&\otimes \frac{\epsilon}{(1+\tau)} \otimes |\nabla^{(m)} \phi_\nu|^2 \otimes \omega(q) \otimes d^3 \chi \otimes d\tau
\end{aligned}$$



$$\begin{aligned}
& \int_{\zeta_{\tau_2}^{\zeta}}^{\square} |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^3} \Lambda_{\epsilon_\alpha}|^2 \otimes \omega(q) \otimes d^3 \chi \simeq \int_{\zeta_{\tau_1}^{\zeta}}^{\square} |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^3} \Lambda_{\epsilon_\alpha}|^2 \otimes \omega(q) \otimes d^3 \chi \\
& + \sum_{|j| \leq |\zeta|} \int_{\tau_1}^{\zeta_2} \left[\int_{\zeta_{\tau}^{\zeta}}^{\square} \left(\mathbb{C}(q_0) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \right) \right. \\
& \quad \left. \otimes \mathbb{C}(\mathcal{J}) \otimes \mathbb{E} \left(\left| \frac{|\mathbb{H}|}{2} \right| + \triangle \right) \otimes \frac{\epsilon \otimes |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^k} \hbar|^2}{(1 + \tau + |q|)} \right) \\
& + \mathcal{O} \left(\left(\mathbb{C}(q_0) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \right) \right. \\
& \quad \left. \otimes \mathbb{C}(\mathcal{J}) \otimes \mathbb{E} \left(\left| \frac{|\mathbb{H}|}{2} \right| + \triangle \right) \otimes \frac{\epsilon \otimes |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^k} \hbar|^2}{(1 + \tau + |q|)} \right) \otimes \omega(q) \otimes d^3 \chi \otimes dt \\
& + \int_{\tau_1}^{\zeta_2} \int_{\zeta_{\tau}^{\zeta}}^{\square} \left[\mathbb{C}(q_0) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \otimes \mathbb{E} \left(\left| \frac{|\mathbb{H}|}{2} \right| + \right. \right. \\
& \quad \left. \left. \frac{\epsilon}{(1 + \tau + |q|)^{1 - c(\gamma) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \otimes \epsilon \left(\left| \frac{|\mathbb{H}|}{2} \right| + \triangle \right) \otimes \mathbb{E} \left(\left| \frac{|\mathbb{H}|}{2} \right| + \triangle \right)} \right)} \otimes (1 + |q|)^2 \right. \\
& \quad \left. \otimes \left(\sum_{|\kappa|^{\sharp} + |\mathbb{M}|^{\sharp} - \mathbb{H}} |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^k} \Lambda|^2 \otimes \omega(q) \otimes d^3 \chi \otimes dt \right) \right] \\
& + \int_{\tau_1}^{\zeta_2} \int_{\zeta_{\tau}^{\zeta}}^{\square} \sum_{|j| \leq |\zeta|} \left[\left(\mathbb{C}(q_0) \otimes \mathbb{C}(\mathcal{J}) \otimes \mathbb{E} \left(\left| \frac{|\mathbb{H}|}{2} \right| + \right. \right. \right. \\
& \quad \left. \left. \left. \frac{\epsilon \otimes |\mathcal{L}_{\mathbb{Z}^k} \mathcal{H}_{\mathcal{L}\mathcal{L}}|^2}{(1 + \tau + |q|)^{1 - 2\delta} \otimes (1 + |q|)^{2\gamma - 4\delta}} \right) \right) \otimes \omega(q) \otimes d^3 \chi \otimes dt \right. \\
& \quad \left. + \mathbb{C}(q_0) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \otimes \mathbb{E} \left(\left| \frac{|\mathbb{H}|}{2} \right| + \right. \right. \right. \\
& \quad \left. \left. \left. \frac{\epsilon^4}{(1 + \tau + |q|)^{1 - c(\gamma) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \otimes \epsilon \left(\left| \frac{|\mathbb{H}|}{2} \right| + \triangle \right) \otimes \mathbb{E} \left(\left| \frac{|\mathbb{H}|}{2} \right| + \triangle \right)} \right)} \otimes (1 + |q|)^2 \right. \\
& \quad \left. \left. \left. \otimes \left(\sum_{|\kappa|^{\sharp} + |\mathbb{M}|^{\sharp} - \mathbb{H}} |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^k} \Lambda|^2 \otimes \omega(q) \otimes d^3 \chi \otimes dt \right) \right) \right]
\end{aligned}$$

$$|\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^j} \Lambda_{\epsilon_\alpha}|$$

$$\cong \frac{1}{(1 + \tau + |q|)(1 + |q|)^{2\gamma - 4\delta}} \otimes \left[\sum_{|j| \leq |k| + 2} \int_{\zeta_\tau^c}^{\square} |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^j} \Lambda_{\epsilon_\alpha}|^2 \otimes \omega(q) \otimes d^3 \chi \right]$$

$$+ \sum_{|k| \leq |l|} \int_{\tau_1}^{\tau} \left[\int_{\zeta_\tau^c}^{\square} \left[\mathcal{O} \left(\mathbb{C}(q_0) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \right) \otimes \mathbb{E} \left(\left| \frac{|l|}{2} \right| + \right. \right. \right]$$

$$\left. \left. \right) \otimes \frac{\epsilon \otimes |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^k} \Lambda|^2}{(1 + \tau + |q|)} \right]$$

$$+ \mathcal{O} \left(\mathbb{C}(q_0) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \right) \otimes \mathbb{E} \left(\left| \frac{|l|}{2} \right| + \right]$$

$$\left. \left. \right) \otimes \frac{\epsilon \otimes |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^k} \Lambda|^2}{(1 + \tau + |q|)} \right] \otimes \omega(q) \otimes d^3 \chi \otimes d\tau$$

$$+ \int_{\tau_1}^{\tau} \int_{\zeta_\tau^c}^{\square} \left[\mathbb{C}(q_0) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \right] \otimes \mathbb{E} \left(\left| \frac{|l|}{2} \right| + \right]$$

$$\left. \left. \right] \frac{\epsilon^4}{(1 + \tau + |q|)^{1 - c(\gamma) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \otimes \epsilon \left(\left| \frac{|l|}{2} \right| + \right) \otimes \mathbb{E} \otimes (1 + |q|)^2} \right]$$

$$\left. \left. \sum_{|k| \leq |l| + |M|^\sharp - 1} |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^k} \Lambda|^2 \otimes \omega(q) \otimes d^3 \chi \otimes d\tau \right] \right]$$

$$\begin{aligned}
& + \int_{\tau_1}^{\tau} \int_{\zeta_{\tau}^s}^{\square} \sum_{|\kappa| \leq |\Pi|} \left[\mathbb{C}(q_0) \otimes \mathbb{C}(\mathcal{J}) \otimes \mathbb{E} \left(\left| \frac{|\Pi|}{2} \right| + \right. \right. \\
& \stackrel{\triangle}{=} \left. \left. \otimes \frac{\epsilon \otimes |\mathcal{L}_{\mathbb{Z}^{\kappa}} \mathcal{H}_{\mathcal{L}\mathcal{L}}|^2}{(1 + \tau + |q|)^{1-2\delta} \otimes (1 + |q|)^{2\gamma-4\delta}} \right) \right] \otimes \omega(q) \otimes d^3 \chi \otimes d\tau \\
& + \mathbb{C}(q_0) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\mathcal{J}) \otimes \mathbb{E} \left(\left| \frac{|\Pi|}{2} \right| + \right.
\end{aligned}$$

$$\stackrel{\triangle}{=} \left(\otimes \left| \frac{\epsilon^4}{(1 + \tau + |q|)^{1-c(\gamma) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \otimes \epsilon \left(\left| \frac{|\Pi|}{2} \right| + \triangle \right) \otimes \mathbb{E} \otimes (1 + |q|)^{\frac{1}{2} + 2\delta}} \right| \right)^{\frac{1}{2}}$$

$$\left[\sum_{|\kappa|^{\square} + |\Pi|^{\sharp} - \Pi} |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^{\kappa}} \Lambda|^2 \otimes \omega(q) \otimes d^3 \chi \otimes d\tau \right]$$

$$\begin{aligned}
& |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^d} \hbar_{\epsilon_\alpha}| \\
& \cong \frac{1}{(1+\tau+|q|)(1+|q|)^{1+\gamma}} \otimes \left[\sum_{|j| \leq |k|+2} \int_{\zeta_{\tau_1}^c}^{\square} |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^d} \hbar_{\epsilon_\alpha}|^2 \otimes \omega(q) \otimes d^3 \chi \right] \\
& + \sum_{|k| \leq |l|} \int_{\tau_1}^{\tau} \left[\int_{\zeta_{\tau}^c}^{\square} \left[\mathcal{O} \left(\mathbb{C}(q_0) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \right) \mathbb{E} \left(\left| \frac{|l|}{2} \right| + \right. \right. \right. \\
& \left. \left. \left. \cong \right) \otimes \frac{\epsilon \otimes |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^d} \hbar|^2}{(1+\tau+|q|)} \right) \right. \\
& \left. + \mathcal{O} \left(\mathbb{C}(q_0) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \right) \mathbb{E} \left(\left| \frac{|l|}{2} \right| + \right. \right. \\
& \left. \left. \cong \right) \otimes \frac{\epsilon \otimes |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^d} \hbar|^2}{(1+\tau+|q|)} \right] \otimes \omega(q) \otimes d^3 \chi \otimes d\tau \\
& + \int_{\tau_1}^{\tau} \int_{\zeta_{\tau}^c}^{\square} \left[\mathbb{C}(q_0) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \right] \mathbb{E} \left(\left| \frac{|l|}{2} \right| + \right. \\
& \left. \left. \left. \cong \right) \otimes \frac{\epsilon^4}{(1+\tau+|q|)^{1-\mathbb{C}(\gamma) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \otimes \epsilon \left(\left| \frac{|l|}{2} \right| + \cong \right) \otimes \mathbb{E} \otimes (1+|q|)^2} \right. \right. \\
& \left. \left. \left. \sum_{|k| \leq |l|+|M|^\sharp - \mathbb{I}} |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^d} \hbar|^2 \otimes \omega(q) \otimes d^3 \chi \otimes d\tau \right] \right)
\end{aligned}$$

$$\begin{aligned}
& |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^j} \mathcal{H}_{\epsilon_\alpha}| \\
& \cong \frac{1}{(1+\tau+|q|)(1+|q|)^{1+\gamma}} \otimes \left[\sum_{|j| \leq |k|+2} \int_{\zeta_{\tau_1}^\zeta}^\square |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^j} \mathcal{H}_{\epsilon_\alpha}|^2 \otimes \omega(q) \otimes d^3 \chi \right] \\
& + \sum_{|k| \leq |\mathbb{I}|} \int_{\tau_1}^\tau \left[\int_{\zeta_\tau^\zeta}^\square \left[\mathcal{O} \left(\mathbb{C}(q_0) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \otimes \mathbb{E} \left(\left| \frac{|\mathbb{I}|}{2} \right| + \right. \right. \right. \right. \\
& \left. \left. \left. \left. \triangleq \right) \otimes \frac{\epsilon \otimes |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^k} \mathcal{H}|^2}{(1+\tau+|q|)} \right) \right. \right. \\
& \left. \left. + \mathcal{O} \left(\mathbb{C}(q_0) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \otimes \mathbb{E} \left(\left| \frac{|\mathbb{I}|}{2} \right| + \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \triangleq \right) \otimes \frac{\epsilon \otimes |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^k} \mathcal{H}|^2}{(1+\tau+|q|)} \right) \right] \otimes \omega(q) \otimes d^3 \chi \right] \otimes d\tau \\
& + \int_{\tau_1}^\tau \int_{\zeta_\tau^\zeta}^\square \left[\mathbb{C}(q_0) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \otimes \mathbb{E} \left(\left| \frac{|\mathbb{I}|}{2} \right| + \right. \right. \right. \\
& \left. \left. \left. \frac{\epsilon^4}{(1+\tau+|q|)^{1-\mathbb{C}(\gamma) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \otimes \epsilon \left(\left| \frac{|\mathbb{I}|}{2} \right| + \triangle \right) \otimes \mathbb{E} \otimes (1+|q|)^2} \right) \right. \right. \\
& \left. \left. \left. \sum_{|k|^\square + |\mathbb{M}|^\sharp - \mathbb{I}} |\nabla^{(m)} \mathcal{L}_{\mathbb{Z}^k} \mathcal{H}|^2 \otimes \omega(q) \otimes d^3 \chi \otimes d\tau \right] \right] \\
& \int_{\mathbb{S}^2} \int_{r=\mathcal{R}(\Omega)}^{r=\infty} \lim_{r \rightarrow \infty} \left(\frac{r^2}{(1+t+r)^\alpha \otimes (1+|q|)} \otimes \frac{\omega(q)}{(1+|q|)^2} \otimes |\partial_r \phi_\nu|^2 \otimes dr^2 \otimes d\sigma^2(t) \right) \mathbb{C}(\gamma) \otimes |\nabla^{(m)} \phi_\nu|^2 \\
& \left[\sum_{|k| \leq |\mathbb{I}|} \frac{\epsilon \otimes |\nabla^{(m)} \partial \mathcal{L}_{\mathbb{Z}^k} \Lambda_\mathcal{L}|^2}{(1+\tau+|q|)^{1-\mathbb{C}(\gamma) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \otimes \epsilon \left(\left| \frac{|\mathbb{I}|}{2} \right| + \triangle \right) \otimes \mathbb{E} \otimes (1+|q|)^{2\gamma-4\delta}} \right] \\
& \int \mathbb{C}(q_0) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \otimes \mathbb{E} \left(\left| \frac{|\mathbb{I}|}{2} \right| + \right. \\
& \left. \triangleq \right) \otimes \sum_{|k| \leq |\mathbb{I}|} \frac{\epsilon \otimes |\nabla^{(m)} \partial \mathcal{L}_{\mathbb{Z}^k} \Lambda|^2}{(1+\tau+|q|)^{1-\mathbb{C}(\gamma) \otimes \mathbb{C}(\delta) \otimes \mathbb{C}(\gamma) \otimes \mathbb{C}(\psi) \otimes \mathbb{C}(\varphi) \otimes \mathbb{C}(\phi) \otimes \mathbb{C}(\mathcal{J}) \otimes \epsilon \left(\left| \frac{|\mathbb{I}|}{2} \right| + \triangle \right) \otimes \mathbb{E} \otimes (1+|q|)^{2\gamma-4\delta}}
\end{aligned}$$

$$\bigotimes \omega(q) \bigotimes d^3\chi \bigotimes dt \frac{\epsilon}{(1 + \tau + |q|)^2}$$

$$\sum_{|\kappa| \leq |\mathbb{I}|} \left[|\nabla^{(m)} \partial \mathcal{L}_{Z^\kappa \Lambda}|^2 \right] \frac{\epsilon^4}{(1 + \tau)^{1 - c(\gamma) \otimes c(\delta) \otimes c(\gamma) \otimes c(\psi) \otimes c(\phi) \otimes c(\mathcal{J}) \otimes \epsilon \left(\left\lfloor \frac{|\mathbb{I}|}{2} \right\rfloor + \underline{\Delta} \right) \otimes \mathbb{E}} \otimes (1 + |q|)^{2\gamma - 4\delta}$$

Postulado Adicional: Cuando una partícula supermasiva o masiva, según el caso o cuando una antipartícula supermasiva o masiva, según el caso o cuando las partículas o antipartículas antes referidas, según sea el caso, se aproximan, alcanzan o superan la velocidad de la luz, o finalmente, cuando su estado de energía es infinitamente superior a cero, sin perjuicio de su carga, en cualquiera de estos casos, no solamente se deforma geoméricamente el espacio cuántico en el que interactúan, sino que también, la morfología inherente a las partículas o antipartículas antes referidas, se deforma.

La deformación de una partícula o una antipartícula a la que converjan las características anteriores, queda expresada así:

$$|\mathcal{Q}_t| = \left\langle \varphi \frac{\xi}{\sqrt{\zeta\pi}} \lambda_\epsilon (\delta_{\blacksquare})^2 \right\rangle |\Theta_\eta|$$

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APÉNDICE C.

(a) Formalización Matemática en relación a la Teoría Cuántica de Espacios Curvos, esto por la interacción de partículas supermasivas y masivas y antipartículas supermasivas y masivas respectivamente.

1. Cuantización Canónica y leyes de conservación.

$$\begin{aligned}
 ds^2 &= g_{\mu\nu}(\chi)d\chi^\mu d\chi^\nu, \delta[\phi'(\chi'), \nabla'\phi'(\chi'), g'_{\mu\nu}(\chi')] \\
 &= \delta[\phi(\chi), \nabla\phi(\chi), g_{\mu\nu}(\chi)] \int d^n\chi \mathcal{L}(\phi, \nabla\phi, g_{\mu\nu}) \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_\alpha)} \delta\phi_\alpha \right) - \frac{\partial \mathcal{L}}{\partial \phi_\alpha} g_{\mu\nu}(\chi) \\
 \rightarrow g'_{\mu\nu}(\chi') &= \frac{\partial \chi^\lambda}{\partial \chi'^\mu} \frac{\partial \chi^\sigma}{\partial \chi'^\nu} g_{\lambda\sigma}(\chi) \delta_0 g_{\mu\nu}(\chi) \equiv g'_{\mu\nu}(\chi') - g_{\mu\nu}(\chi) = \mathcal{L}_\epsilon g_{\mu\nu} \\
 &= \nabla_\mu \epsilon_\nu + \nabla_\nu \epsilon_\mu \delta \mathcal{S} \int d^n\chi \frac{\delta \mathcal{S}}{\delta g_{\mu\nu}(\chi)} \delta_0 g_{\mu\nu}(\chi) \int dv_\chi \mathcal{T}^{\mu\nu} \nabla_\mu \epsilon_\nu \\
 &\equiv -2|g|^{-\frac{1}{2}} \frac{\delta \mathcal{S}}{\delta g_{\mu\nu}(\chi)} \nabla_\mu (\mathcal{T}^{\mu\nu} \epsilon_\nu) = |g|^{-\frac{1}{2}} \partial_\mu \left(|g|^{-\frac{1}{2}} \mathcal{T}^{\mu\nu} \epsilon_\nu \right) \\
 &= (\nabla_\mu \mathcal{T}^{\mu\nu}) \epsilon_\nu + \mathcal{T}^{\mu\nu} \nabla_\mu \epsilon_\nu \int dv_\chi (\nabla_\mu \mathcal{T}^{\mu\nu}) \epsilon_\nu \mathcal{T}_{\mu\nu} \\
 &+ \frac{2}{|g|^{-\frac{1}{2}}} \frac{\delta \mathcal{S}}{\delta g^{\mu\nu}} \mathcal{G}(t) \int d^{\eta-1}\chi [\pi_\alpha \delta\phi_\alpha - \Theta_\nu^0 \delta\chi^\nu] = \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi_\alpha)} \{ \phi_\alpha(\vec{\chi}, t), \pi_\beta(\vec{\chi}, t) \} \\
 &= i\delta_{\alpha,\beta} \delta(\vec{\chi} - \vec{\chi}') \epsilon^\mu(\chi) \rho_\lambda \int d^{\eta-1}\chi \Theta_\lambda^0 \mathcal{L}_\epsilon g_{\mu\nu} \nabla_\mu \xi_\nu \\
 &+ \nabla_\nu \xi_\mu (\nabla_\mu \mathcal{T}^{\mu\nu} \xi_\nu) \partial_\mu \left(|g|^{-\frac{1}{2}} \mathcal{T}^{\mu\nu} \xi_\nu \right) \mathcal{P}_\xi \equiv \int d\mathcal{V}_\chi \mathcal{T}_\nu^0(\chi) \xi^\nu(\chi) \tilde{g}_{\mu\nu}(\chi) \\
 &= \Omega^{2\rho}(\chi) g_{\mu\nu}(\chi) 1 + |\lambda(\chi)| \delta_0 g_{\mu\nu}(\chi) = \tilde{g}_{\mu\nu}(\chi) - g_{\mu\nu}(\chi) = \lambda(\chi) g_{\mu\nu}(\chi) \delta_0 \phi(\chi) \\
 &= \tilde{\phi}(\chi) - \phi(\chi) \\
 &= \rho\lambda(\chi)\phi(\chi) \int d^n\chi \left\{ \frac{\delta \mathcal{S}}{\delta \phi} \delta_0 \phi + \frac{\delta \mathcal{S}}{\delta g_{\mu\nu}} \delta_0 g_{\mu\nu} \right\} \int d^n\chi \frac{\delta \mathcal{S}}{\delta g_{\mu\nu}(\chi)} g_{\mu\nu}(\chi) \lambda(\chi)
 \end{aligned}$$

2. Campo Escalar en espacios cuánticos curvos.



$$\delta = \int d^n \chi \mathcal{L} \frac{1}{2} |g|^{\frac{1}{2}} (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m^2 \phi^2 - \xi \mathcal{R} \phi^2) g_{\mu\nu}(\chi) \rightarrow \tilde{g}_{\mu\nu}(\chi) = 1 + |\lambda(\chi)| g_{\mu\nu}(\chi) \tilde{\phi}(\chi)$$

$$\rightarrow \phi(\chi) = \left(1 - \frac{1}{2} \lambda(\chi)\right) \phi(\chi) |g|^{\frac{1}{2}} \mapsto |\tilde{g}|^{\frac{1}{2}} = (1 - 2\lambda) |g|^{\frac{1}{2}} g^{\mu\nu} \rightarrow \tilde{g}^{\mu\nu}$$

$$= (1 - \lambda) g^{\mu\nu} \Gamma_{\nu\tau}^\mu \mapsto \tilde{\Gamma}_{\nu\tau}^\mu = \Gamma_{\nu\tau}^\mu + \frac{1}{2} (\delta_\tau^\mu \partial_\nu \lambda + \delta_\nu^\mu \partial_\tau \lambda - g^{\mu\sigma} g_{\nu\tau} \partial_\sigma \lambda) \mathcal{R} \mapsto \tilde{\mathcal{R}}$$

$$= (1 - \lambda) \mathcal{R} + 3 \square \lambda$$

$$\bar{\mathcal{L}} = \frac{1}{2} |\tilde{g}|^{\frac{1}{2}} \left(\tilde{g}^{\mu\nu} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} - \frac{1}{6} \tilde{\mathcal{R}} \tilde{\phi}^2 \right)$$

$$= \frac{1}{2} (1 + 2\lambda) |g|^{\frac{1}{2}} \left\{ (1 - \lambda) g^{\mu\nu} \partial_\mu \left[\left(1 - \frac{1}{2} \lambda\right) \phi \right] \partial_\nu \left[\left(1 - \frac{1}{2} \lambda\right) \phi \right] \right.$$

$$\left. - \frac{1}{6} [(1 - \lambda) \mathcal{R} + 3 \square \lambda] (1 - \lambda) \phi^2 \right\}$$

$$= \frac{1}{2} |g|^{\frac{1}{2}} \left\{ g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - g^{\mu\nu} \phi \partial_\mu \lambda \partial_\nu \phi - \frac{1}{6} \mathcal{R} \phi^2 - \frac{1}{2} (\square \lambda) \phi^2 \right\}$$

$$= -\frac{1}{2} |g|^{\frac{1}{2}} g^{\mu\nu} \phi \partial_\mu \lambda \partial_\nu \phi - \frac{1}{4} |g|^{\frac{1}{2}} (\square \lambda) \phi^2$$

$$- \partial_\mu \left(\frac{1}{4} |g|^{\frac{1}{2}} g^{\mu\nu} \phi^2 \partial_\nu \lambda \ln \Omega \right) \Omega^2 g_{\mu\nu} \Omega^{-1} \phi (\square + m^2 + \xi \mathcal{R}) \phi$$

$$\mathcal{T}^{\mu\nu} = \nabla^\mu \phi \nabla^\nu \phi - \frac{1}{2} g^{\mu\nu} \nabla^\rho \phi \nabla_\rho \phi + \frac{1}{2} g^{\mu\nu} m^2 \phi^2 - \xi \left(\mathcal{R}^{\mu\nu} - \frac{1}{2} g^{\mu\nu} \mathfrak{R} \right) \phi^2$$

$$+ \xi [g^{\mu\nu} \square (\phi^2) - \nabla^\mu \nabla^\nu (\phi^2)] \delta g^{\mu\nu} - g^{\mu\rho} g^{\nu\sigma} \delta g_{\rho\sigma} \delta |g|^{\frac{1}{2}}$$

$$= \frac{1}{2} |g|^{\frac{1}{2}} g^{\mu\nu} \delta g_{\mu\nu} \delta \mathcal{R} - \mathcal{R}^{\mu\nu} \delta g_{\mu\nu} + g^{\rho\sigma} g^{\mu\nu} (\delta g_{\rho\sigma;\mu\nu} - \delta g_{\rho\mu;\sigma\nu}) \delta \mathcal{S}$$

$$= \frac{1}{2} \int d^n \chi |g|^{\frac{1}{2}} \left\{ \frac{1}{2} g^{\mu\nu} \delta g_{\mu\nu} (g^{\rho\sigma} \partial_\rho \phi \partial_\sigma \phi - m^2 \phi^2 - \xi \mathcal{R} \phi^2) - \delta g_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi \right.$$

$$\left. - \xi [-\mathcal{R}^{\mu\nu} \delta g_{\mu\nu} + g^{\rho\sigma} g^{\mu\nu} (\delta g_{\rho\sigma;\mu\nu} - \delta g_{\rho\mu;\sigma\nu})] \phi^2 \right\} \int d^n \chi |g|^{\frac{1}{2}} g^{\rho\sigma} g^{\mu\nu} \delta g_{\rho\sigma;\mu\nu} \phi^2$$

$$= \int d^n \chi |g|^{\frac{1}{2}} g^{\rho\sigma} \delta g_{\rho\sigma} \square (\phi^2) \int d^n \chi |g|^{\frac{1}{2}} g^{\rho\sigma} g^{\mu\nu} \delta g_{\rho\mu;\sigma\nu} \phi^2$$

$$= \int d^n \chi |g|^{\frac{1}{2}} g^{\sigma\mu} g^{\lambda\nu} \delta g_{\mu\nu} \nabla_\sigma \nabla_\lambda (\phi^2)$$

$$\begin{aligned}
\frac{d}{dt}(f_1 f_2) &= i \int d^{\eta-1} \chi |g|^{\frac{1}{2}} \partial_0 g^{0\nu} f_1^*(\vec{\chi}, t) \vec{\partial}_\nu f_2(\vec{\chi}, t) \\
&= i \int d^{\eta-1} \chi |g|^{\frac{1}{2}} \nabla_\mu (g^{\mu\nu} f_1^* \vec{\partial}_\nu f_2) \\
&\quad - i \int d^{\eta-1} \chi \partial_i \left(|g|^{\frac{1}{2}} g^{i\nu} f_1^* \vec{\partial}_\nu f_2 \right) i \int_\sigma d\sigma |g|^{\frac{1}{2}} \eta^\nu f_1^* \vec{\partial}_\nu f_2 (f_1 f_2)_{,\sigma} - (f_1 f_2)_\sigma \\
&= i \int d^{\eta-1} \chi |g|^{\frac{1}{2}} \nabla^\mu (f_1^* \vec{\nabla}_\mu f_2)
\end{aligned}$$

3. Modelo Cosmológico en espacios cuánticos curvos.

$$\begin{aligned}
ds^2 &= dt^2 - \alpha^2(t)(d\chi^2 - d\gamma^2 - dz^2) \square \phi \alpha^{-3} \partial_t (\alpha^3 \partial_t \phi) \\
&\quad - \alpha^{-2} \sum_i \partial_i^2 \phi \sum_\kappa \{A_{\vec{\kappa}} f_{\vec{\kappa}}(\chi) + A_{\vec{\kappa}}^\dagger f_{\vec{\kappa}}^*\} \mathcal{V}^{-1} e^{i\vec{\kappa} \cdot \vec{\chi}} \psi_\kappa(\tau) \frac{d^2 \psi_\kappa}{d\tau^2} \\
&\quad + \kappa^2 \alpha^4 \psi_\kappa(\tau) \int (\mathcal{V}_{\alpha_1}^3)^{-\frac{1}{2}} (2\omega_{1\kappa})^{-\frac{1}{2}} \exp[i(\vec{\kappa} \cdot \vec{\chi} - \omega_{1\kappa} t)] dt' [\phi(\vec{\chi}, t), \pi(\vec{\chi}', t)] \\
&= \alpha^3(t) \sum_{\vec{\kappa}} f_{\vec{\kappa}}(\vec{\chi}, t) \partial_t f_{\vec{\kappa}}^*(\vec{\chi}', t) - f_{\vec{\kappa}}^*(\vec{\chi}, t) \partial_t f_{\vec{\kappa}}(\vec{\chi}', t)
\end{aligned}$$

$$\begin{aligned}
\hbar(\vec{\chi}, t) &= \sum_{\vec{\kappa}} \{f_{\vec{\kappa}}(\vec{\kappa}, t)(f_{\vec{\kappa}}, \hbar) - f_{\vec{\kappa}}^*(\vec{\chi}, t)(f_{\vec{\kappa}}^*, \hbar)\} \\
&= i \int d^3 \chi' \alpha^3(t) \sum_{\vec{\kappa}} \{f_{\vec{\kappa}}(\vec{\chi}, t) \partial_t f_{\vec{\kappa}}^*(\vec{\chi}', t) - f_{\vec{\kappa}}^*(\vec{\chi}, t) \partial_t f_{\vec{\kappa}}(\vec{\chi}', t)\} \hbar(\vec{\chi}', t) \\
&\quad + i \int d^3 \chi' \alpha^3(t) \sum_{\vec{\kappa}} \{f_{\vec{\kappa}}(\vec{\chi}, t) f_{\vec{\kappa}}^*(\vec{\chi}', t) - f_{\vec{\kappa}}^*(\vec{\chi}, t) \partial_t f_{\vec{\kappa}}(\vec{\chi}', t)\} \partial_t \hbar(\vec{\chi}', t) \\
&= i\delta(\vec{\chi} - \vec{\chi}')
\end{aligned}$$

4. Partícula Cosmológica en relación a espacios cuánticos curvos.



$$\begin{aligned}
\psi_{\kappa}^{(\pm)} &\sim (2\alpha^3\omega_{2\kappa})^{-\frac{1}{2}} \exp(\mp i\omega_{2\kappa}\alpha_2^3\tau)\psi_{\kappa}(t) = \alpha_{\kappa}\psi_{\kappa}^{(+)}(\tau) + \beta_{\kappa}\psi_{\kappa}^{(-)}(\tau) \\
&\sim [\alpha_{\kappa}e^{-i\alpha_2^3\omega_{2\kappa}\tau} + \beta_{\kappa}e^{i\alpha_2^3\omega_{2\kappa}\tau}]\psi_{\kappa}\partial_{\tau}\psi_{\kappa}^* - \psi_{\kappa}^*\partial_{\tau}\psi_{\kappa}|\alpha_{\kappa}|^2 - |\beta_{\kappa}|^2f_{\bar{\kappa}} \\
&\sim (\mathcal{V}\alpha_2^3)^{-\frac{1}{2}}(2\omega_{2\kappa})^{-\frac{1}{2}}e^{i\bar{\kappa}\cdot\vec{\chi}}[\alpha_{\kappa}e^{-i\omega_{2\kappa}\tau} \\
&\quad + \beta_{\kappa}e^{i\omega_{2\kappa}\tau}]\phi(\chi)\sum_{\bar{\kappa}}\{\alpha_{\bar{\kappa}}g_{\bar{\kappa}}(\chi) + \alpha_{\bar{\kappa}}^{\dagger}g_{\bar{\kappa}}^*(\chi)\} \\
&\sim (\mathcal{V}\alpha_2^3)^{-\frac{1}{2}}(2\omega_{2\kappa})^{-\frac{1}{2}}\exp[i(\bar{\kappa}\cdot\vec{\chi} - \omega_{2\kappa}\tau)]\{A_{\bar{\kappa}} + \beta_{\kappa}^*A_{-\bar{\kappa}}^{\dagger}[\alpha_{\bar{\kappa}}\alpha_{\bar{\kappa}'}^{\dagger}]\} \\
&= \delta_{\bar{\kappa}\bar{\kappa}'}(|\alpha_{\kappa}|^2 - |\beta_{\kappa}|^2) = \delta_{\bar{\kappa}\bar{\kappa}'}\langle N_{\bar{\kappa}} \rangle_{\tau\rightarrow\infty} \langle 0|\alpha_{\bar{\kappa}}^{\dagger}\alpha_{\bar{\kappa}}|0\rangle|\beta_{\kappa}|^2
\end{aligned}$$

5. Spin cero en espacios cuánticos curvos.

$$\begin{aligned}
&\square\phi + (m^2 + \xi\mathcal{R})\phi\mathfrak{R} \\
&= 6\left[\left(\frac{\dot{\alpha}}{\alpha}\right)^2 + \left(\frac{\ddot{\alpha}}{\alpha}\right)\right]\alpha^{-3}\partial_t(\alpha^3\partial_t\phi) \\
&\quad - \alpha^{-2}\sum_i\partial_i^2\phi(m^2 + \xi\mathcal{R})\phi\sum_{\bar{\kappa}}\{A_{\bar{\kappa}}f_{\bar{\kappa}}(\chi) - A_{\bar{\kappa}}^{\dagger}f_{\bar{\kappa}}^*(\chi)\} \\
&\sim (\mathcal{V}\alpha_1^3)^{-\frac{1}{2}}(2\omega_{1\kappa})^{-\frac{1}{2}}\exp[i(\bar{\kappa}\cdot\vec{\chi} - \omega_{1\kappa}t)]dt'\phi\sum_{\bar{\kappa}}\{\alpha_{\bar{\kappa}}g_{\bar{\kappa}}(\chi) + \alpha_{\bar{\kappa}}^{\dagger}g_{\bar{\kappa}}^*(\chi)\} \\
&\sim (\mathcal{V}\alpha_1^3)^{-\frac{1}{2}}(2\omega_{2\kappa})^{-\frac{1}{2}}\exp[i(\bar{\kappa}\cdot\vec{\chi} - \omega_{2\kappa}t)]\left[\left(\frac{\kappa}{\alpha^2}\right) + m^2\right]^{\frac{1}{2}}\alpha_{\bar{\kappa}}A_{\bar{\kappa}} + \beta_{\kappa}^*A_{-\bar{\kappa}}^{\dagger}[\alpha_{\bar{\kappa}}\alpha_{\bar{\kappa}'}]_{\pm} \\
&= \alpha_{\kappa}\beta_{\kappa'}^*[A_{\bar{\kappa}}A_{-\bar{\kappa}'}^{\dagger}]_{\pm} + \alpha_{\kappa'}\beta_{\kappa}^*[A_{-\bar{\kappa}}^{\dagger}A_{\bar{\kappa}'}]_{\pm} = (\alpha_{\kappa}\beta_{\kappa}^* \pm \alpha_{\kappa'}\beta_{\kappa'}^*)\delta_{\bar{\kappa}-\bar{\kappa}'}[\alpha_{\bar{\kappa}}\alpha_{\bar{\kappa}'}^{\dagger}]_{\pm} \\
&= (|\alpha_{\kappa}|^2 \pm |\beta_{\kappa}|^2)\delta_{\bar{\kappa},\bar{\kappa}'}
\end{aligned}$$

6. Invariante conforme.



$$\begin{aligned}
\mathcal{L} &= \frac{1}{2} |g|^{\frac{1}{2}} \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{6} \mathcal{R} \phi^2 \right) \tilde{g}_{\mu\nu}(\chi) = \Omega^2(\chi) g_{\mu\nu}(\chi) \tilde{\phi}(\chi) \\
&= \Omega^{-1}(\chi) \phi(\chi) \frac{\delta \mathcal{S}}{\delta \phi} \frac{\delta \tilde{\mathcal{S}}}{\delta \phi} \frac{\delta \tilde{\mathcal{S}}}{\delta \tilde{\phi}} \Omega^{-1} \left(\square + \frac{1}{6} \mathcal{R} \right) \phi \\
&= \Omega^3 \left(\widehat{\square} + \frac{1}{6} \mathcal{R} \right) \eta \int_{\square}^{\tau} \alpha^{-1}(t') dt' \int ds^2 \\
&= dt^2 - \alpha^2(t) (d\chi^2 - d\gamma^2 - dz^2) \tilde{f}_{\bar{\kappa}}(\chi) \mathcal{V}^{-1/2} (2\kappa)^{-1/2} \exp[i(\bar{\kappa} \cdot \tilde{\chi} - \kappa \eta)] f_{\bar{\kappa}}(\chi) \\
&= \alpha^{-1}(t) \tilde{f}_{\bar{\kappa}}(\chi) \\
&= (2\mathcal{V} \alpha^3(t) \omega_\kappa(\tau))^{-1/2} \exp \left[i \left(\bar{\kappa} \cdot \tilde{\chi} - \int_{\square}^{\tau} \omega_\kappa(\tau') dt' \right) \right] \phi \left[\sum_{\bar{\kappa}} \{ A_{\bar{\kappa}} f_{\bar{\kappa}}(\chi) + A_{\bar{\kappa}}^\dagger f_{\bar{\kappa}}^*(\chi) \} \right]
\end{aligned}$$

7. Dinámica de partículas en espacios cuánticos curvos.

$$\begin{aligned}
\alpha_{\bar{\kappa}} &= \alpha_{\bar{\kappa}} A_{\bar{\kappa}} + \beta_{\bar{\kappa}}^* A_{-\bar{\kappa}}^\dagger |\alpha_{\bar{\kappa}}|^2 - |\beta_{\bar{\kappa}}|^2 \eta(\bar{\kappa}) \eta(-\bar{\kappa}) |0\rangle = (\eta!)^{-1} \langle 0 | (\alpha_{-\bar{\kappa}})^\eta (\alpha_{\bar{\kappa}})^\eta |0\rangle \alpha_{\bar{\kappa}} |0\rangle = \beta_{\bar{\kappa}}^* A_{-\bar{\kappa}}^\dagger |0\rangle \\
&= \beta_{\bar{\kappa}}^* (\alpha_{\bar{\kappa}}^*)^{-1} \alpha_{-\bar{\kappa}}^\dagger |0\rangle \eta(\bar{\kappa}) (-\bar{\kappa}) |0\rangle = (\eta!)^{-1} \left| \frac{\beta_{\bar{\kappa}_j}^*}{\alpha_{\bar{\kappa}_j}^*} \right|^\eta \langle 0 | (\alpha_{-\bar{\kappa}})^\eta (\alpha_{-\bar{\kappa}}^\dagger)^\eta |0\rangle \\
&= (\eta!)^{-\frac{1}{2}} \left| \frac{\beta_{\bar{\kappa}_j}^*}{\alpha_{\bar{\kappa}_j}^*} \right|^\eta \langle \eta(-\bar{\kappa}) | (\alpha_{-\bar{\kappa}}^\dagger)^\eta |0\rangle | \eta(-\bar{\kappa}) \rangle \\
&= (\eta!)^{-\frac{1}{2}} (\alpha_{-\bar{\kappa}}^\dagger)^\eta |0\rangle \langle \{ \eta_j(\bar{\kappa}_j) \} |0\rangle \prod_j \left| \frac{\beta_{\bar{\kappa}_j}^*}{\alpha_{\bar{\kappa}_j}^*} \right|^{\eta_j} \langle 0 | 0 \rangle \sum_{\{ \eta_j(\bar{\kappa}_j) \}} \langle \{ \eta_j(\bar{\kappa}_j) \} | \{ \eta_j(\bar{\kappa}_j) \} |0\rangle
\end{aligned}$$

$$\begin{aligned}
&\sum_{\{ \eta_j(\bar{\kappa}_j) \}} | \langle \{ \eta_j(\bar{\kappa}_j) \} |0\rangle |^2 \\
&= \left(\sum_{\{ \eta_j(\bar{\kappa}_j) \}} \prod_j \left(\frac{\beta_{\bar{\kappa}_j}^*}{\alpha_{\bar{\kappa}_j}^*} \right)^{2\eta_j} \right) | \langle 0 | 0 \rangle |^2 \prod_j \sum_{\eta_j=0} \left| \frac{\beta_{\bar{\kappa}_j}^*}{\alpha_{\bar{\kappa}_j}^*} \right|^{2\eta_j} |\alpha_{\bar{\kappa}_j}|^2 \left(1 \right. \\
&\quad \left. - \left| \frac{\beta_{\bar{\kappa}_j}^*}{\alpha_{\bar{\kappa}_j}^*} \right|^2 |\alpha_{\bar{\kappa}_j}|^{-2} \right) | \langle \{ \eta_j(\bar{\kappa}_j) \} |0\rangle |^2 \mathcal{P}_\eta(\bar{\kappa}) = \left| \frac{\beta_{\bar{\kappa}}}{\alpha_{\bar{\kappa}}} \right|^{2\eta} |\alpha_{\bar{\kappa}}|^{-2} \langle N_{\bar{\kappa}} \rangle_{\tau \rightarrow \infty} = \sum_{\eta=0}^{\infty} \eta \mathcal{P}_\eta(\bar{\kappa}) \\
&= \sum_{\bar{\kappa}} |\beta_{\bar{\kappa}}|^2 = (2\pi^2 \alpha_2^3)^{-1} \int_0^\infty d\kappa \kappa^2 |\beta_{\bar{\kappa}}|^2
\end{aligned}$$

8. Partícula Cosmológica: Solución Exacta.



$\alpha(\tau)$

$$\begin{aligned}
 &= \left\{ \alpha_1^4 + e^\zeta [(\alpha_2^4 - \alpha_1^4)(e^\zeta + 1) + \beta] (e^\zeta + 1)^{-2} \right\}^{\frac{1}{4}} \left\{ \frac{\alpha_2^4 + \alpha_1^4}{2} + \frac{\alpha_2^4 - \alpha_1^4}{2} \tanh\left(\frac{\zeta}{2}\right) \right\}^{\frac{1}{4}} \alpha_1^4 \\
 &+ \frac{\beta}{4 \cosh^2\left(\frac{\zeta}{2}\right)} \left\}^{\frac{1}{4}} d \\
 &= \frac{1}{2} \left[1 - (1 + 4\kappa^2 \delta^2 \beta)^{\frac{1}{2}} \right] \psi_\kappa(1 + \mu)^d \mu^{-c_1} f(\mu) \mu(\mu + 1) f'' + [(2d - 2c_1 + 1)\mu + (1 - 2c_1)] f' \\
 &+ (d - c_1 + c_2) f \psi_\kappa = N_1 \mu^{-c_1} (1 + \mu)^d \mathcal{F}(d - c_1 + c_2, d - c_1 - c_2, 1 - 2c_1 - \mu) \\
 &= (\mu)^{-d+c_1-c_2} \mathcal{A}\mathcal{F}(d - c_1 + c_2, d + c_1 + c_2, 1 + 2c_1 - \mu^{-1}) \\
 &+ (\mu)^{-d+c_1+c_2} \mathcal{B}\mathcal{F}(d - c_1 - c_2, d + c_1 - c_2, 1 - 2c_1 \\
 &- \mu^{-1}) \left| \frac{\Gamma(1 - 2c_1)\Gamma(-2c_2)}{\Gamma(d - c_1 - c_2)\Gamma(1 - c_1 - c_2 - d)} \right| \left| \frac{\Gamma(1 - 2c_1)\Gamma(2c_2)}{\Gamma(d - c_1 + c_2)\Gamma(1 - c_1 + c_2 - d)} \right| (e^{\tau\delta^{-1}})^{-i\kappa\delta\alpha_2^2} \\
 &e^{-i\alpha_2^2\kappa\tau} e^{-i\alpha_2^3\omega_{2\kappa}\tau} \\
 &\left| \frac{\beta_\kappa}{\alpha_\kappa} \right|^2 = \frac{\Gamma(d - c_1 - c_2)\Gamma(1 + c_1 + c_2 - d)\Gamma(d + c_1 + c_2)\Gamma(1 - c_1 - c_2 - d)}{\Gamma(d - c_1 + c_2)\Gamma(1 + c_1 - c_2 - d)\Gamma(d + c_1 - c_2)\Gamma(1 - c_1 + c_2 - d)} \\
 &= \frac{\sin \pi(d - c_1 + c_2) \sin \pi(d + c_1 - c_2)}{\sin \pi(d - c_1 - c_2) \sin \pi(d + c_1 + c_2)}
 \end{aligned}$$

9. Distribución de cuerpo negro.

$$\begin{aligned}
 \mathcal{P}_\eta(\vec{\kappa}) &= \left| \frac{\beta_\kappa}{\alpha_\kappa} \right|^{2\eta} \left(1 - \left| \frac{\beta_\kappa}{\alpha_\kappa} \right|^{2\eta} \right) \approx \frac{\exp[2\pi\kappa\delta(\alpha_\>^2 - \alpha_\<^2)]}{\exp[2\pi\kappa\delta(\alpha_\>^2 + \alpha_\<^2)]} = \exp[-\eta\mu\kappa] \frac{\left| \frac{\beta_\kappa}{\alpha_\kappa} \right|^2}{1 - \left| \frac{\beta_\kappa}{\alpha_\kappa} \right|^2} \\
 &= \frac{1}{\exp(\mu\kappa) - 1} \langle N \rangle (2\pi^2 \alpha_2^3)^{-1} \int_0^\infty d\kappa \kappa^2 [\exp(\mu\kappa) - 1]^{-1}
 \end{aligned}$$

10. Espacio - tiempo de Sitter en espacios cuánticos curvos.



$$\begin{aligned}
ds^2 &= d\mathcal{T}^2 - d\mathcal{W}^2 - d\mathcal{X}^2 - d\mathcal{Y}^2 - d\mathcal{Z}^2, ds^2 = dt^2 - e^{2\mathcal{H}t}(d\chi^2 - d\gamma^2 - dz^2), ds^2 \\
&= (\mathcal{H}\eta)^{-2}(d\eta^2 - d\chi^2 - d\gamma^2 - dz^2)\partial_t^2\phi + 3\mathcal{H}\partial_t\phi \\
&- e^{-2\mathcal{H}t} \sum_i \partial_t^2\phi + \mathcal{M}^2\phi f_{\vec{\kappa}}(\vec{\chi}, t)(2\mathcal{V}e^{3\mathcal{H}t})^{-\frac{1}{2}}e^{i\vec{\kappa}\cdot\vec{\chi}}\hbar_{\kappa}v^2 \frac{d^2}{dv^2}\hbar_{\kappa} + v\frac{d}{dv}\hbar_{\kappa} + (v^2 - v^2)\hbar_{\kappa} \\
&\sim (\omega_{\kappa}(t))^{-\frac{1}{2}} \exp\left[-i \int_{\square}^{\tau} \omega_{\kappa}(t') dt'\right] \hbar_{\kappa}(\tau) \sqrt{\frac{\pi}{2\mathcal{H}}} \left\{ \mathfrak{E}(\kappa)\mathcal{H}_v^{(2)}(v) \right. \\
&+ \mathcal{F}(\kappa)\mathcal{H}_v^{(1)}(v) \left. \right\} \hbar_{\kappa'}(\tau') \sqrt{\frac{\pi}{2\mathcal{H}}} \left\{ \mathfrak{E}(\kappa')\mathcal{H}_v^{(2)}(v) \right. \\
&+ \mathcal{F}(\kappa')\mathcal{H}_v^{(1)}(v) \left. \right\} f_{\vec{\kappa}}(\vec{\chi}, t) \frac{1}{2} \sqrt{\frac{\pi}{\mathcal{H}}} \mathcal{V}^{-\frac{1}{2}} e^{-\frac{3}{2}\mathcal{H}t} \mathcal{H}_v^{(1)}(\kappa\mathcal{H}^{-1}e^{-\mathcal{H}t}) e^{i\vec{\kappa}\cdot\vec{\chi}} (\mathcal{V}e^{3\mathcal{H}t})^{-\frac{1}{2}} (2\kappa e^{-\mathcal{H}t})^{-\frac{1}{2}} \\
&\quad \exp(i\kappa\mathcal{H}^{-1}e^{-\mathcal{H}t}) e^{i\vec{\kappa}\cdot\vec{\chi}} \mathcal{H}_{1/2}^{(1)}(v) - i \sqrt{\frac{2}{\pi v}} e^{iv}
\end{aligned}$$

11. Expansiones perturbativas en espacios cuánticos curvos.



$$\begin{aligned}
\mathcal{L} &= \frac{1}{2} |g|^{\frac{1}{2}} \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2\mathcal{V}(\phi) \right) \int ds^2 = dt^2 - \alpha^2(t) (d\chi^2 - d\gamma^2 - dz^2) \alpha^{-3} \partial_t (\alpha^{-3} \partial_t \phi_0) \\
&+ \frac{d\mathcal{V}}{d\dot{\phi}_0} + 3\mathcal{H}(t) \dot{\phi}_0 + \mathcal{V}'(\phi_0) \mathcal{H}^2 \left(\frac{16\pi\mathfrak{G}}{3} \right) \rho \frac{\ddot{\alpha}}{\alpha} = \dot{\mathcal{H}} + \mathcal{H}^2 \\
&= - \left(\frac{8\pi\mathfrak{G}}{3} \right) \left(\frac{1}{2} (\dot{\phi}_0)^2 + \mathcal{V}(\phi_0) \right) (\rho + 3p) \rho \frac{1}{2} (\dot{\phi}_0)^2 + \mathcal{V}(\phi_0) \rho \frac{1}{2} (\dot{\phi}_0)^2 - \mathcal{V}(\phi_0) \dot{\mathcal{H}} \\
&= -4\pi\mathfrak{G} \dot{\phi}_0^2 \frac{1}{2} (\dot{\phi}_0)^2 \ll \mathcal{V}(\phi_0) \mathcal{H}^2 \approx \left(\frac{8\pi\mathfrak{G}}{3} \right) \mathcal{V}(\phi_0) |\mathcal{V}'(\phi_0) \dot{\phi}_0 \mathcal{H}^{-1}| \ll \mathcal{V}(\phi_0) \dot{\phi}_0 \\
&\approx - \frac{\mathcal{V}'(\phi_0)}{(3\mathcal{H})} \frac{1}{16\pi\mathfrak{G}} \left(\frac{\mathcal{V}'}{\mathcal{V}} \right)^2 \ll 1, \epsilon \\
&\equiv \frac{1}{16\pi\mathfrak{G}} \left(\frac{\mathcal{V}'}{\mathcal{V}} \right)^2 - \frac{1}{3\mathcal{H}} \mathcal{V}''(\phi_0) \dot{\phi}_0 + \frac{\dot{\mathcal{H}}}{3\mathcal{H}^2} \mathcal{V}'(\phi_0) |\mathcal{V}''(\phi_0) (\dot{\phi}_0)^2| \\
&\ll |\mathcal{H} \mathcal{V}'(\phi_0)| \left| \frac{1}{8\pi\mathfrak{G}} \frac{\mathcal{V}''(\phi_0)}{\mathcal{V}(\phi_0)} \right| \gg |\eta| \\
&\equiv \frac{1}{16\pi\mathfrak{G}} \frac{\mathcal{V}''(\phi_0)}{\mathcal{V}(\phi_0)} \partial_t^2 \delta\phi + 3\mathcal{H} \partial_t \delta\phi - e^{-2\mathcal{H}t} \sum_i \partial_i^2 \delta\phi \\
&+ \mathcal{V}''(\phi_0) \delta\phi \sum_{\vec{\kappa}} \{ A_{\vec{\kappa}} f_{\vec{\kappa}}(\chi) + A_{\vec{\kappa}}^\dagger f_{\vec{\kappa}}^*(\chi) \} (2\mathcal{V}\alpha^3(t))^{-\frac{1}{2}} e^{i\vec{\kappa} \cdot \vec{\chi}} \hbar_{\kappa}(\tau) \\
&\simeq (2\mathcal{V}\alpha^3(t))^{-\frac{1}{2}} \left(\frac{2\kappa}{\alpha(t)} \right)^{-\frac{1}{2}} \exp \left[i \left(\vec{\kappa} \cdot \vec{\chi} - \int_{\square}^{\tau} \frac{\kappa}{\alpha(t')} dt' \right) \phi \right]
\end{aligned}$$

$$\begin{aligned}
f_{\vec{\kappa}}(\vec{\chi}, t) &= \frac{1}{2} \sqrt{\frac{\pi}{\mathcal{H}}} \mathcal{V}^{-\frac{1}{2}} e^{-\frac{3}{2}\mathcal{H}t} \mathcal{H}_\nu^{(1)}(\kappa \mathcal{H}^{-1} e^{-\mathcal{H}t}) e^{i\vec{\kappa} \cdot \vec{\chi}} \langle 0 | \delta\phi(\vec{\chi}, t) | 0 \rangle \\
&= \mathcal{P}_\phi(\kappa, t) \kappa^{-1} d\kappa \langle 0 | \delta\phi(\vec{\chi}, t) \delta\phi(\vec{\chi}', t') | 0 \rangle = \sum_{\vec{\kappa}} f_{\vec{\kappa}}(\vec{\chi}, t) f_{\vec{\kappa}}^*(\vec{\chi}', t') \\
&= \frac{1}{2(2\pi)^3} [\alpha(t)\alpha(t')]^{-3/2} \int d^3\kappa e^{i\vec{\kappa} \cdot (\vec{\chi} - \vec{\chi}')} \hbar_\kappa(\tau) \hbar_\kappa^*(\tau') \langle 0 | \delta\phi(\vec{\chi}, t)^2 | 0 \rangle \\
&= (4\pi^2 \alpha^3(t))^{-1} \int_0^\infty d\kappa \kappa^2 |\hbar_\kappa(\tau)|^2 \mathcal{P}_\phi(\kappa, t) \\
&= (4\pi^2 \alpha^3(t))^{-1} \kappa^3 |\hbar_\kappa(\tau)|^2 \langle 0 | \delta\phi(\vec{\chi}_1, t) \delta\phi(\vec{\chi}_2, t) | 0 \rangle \\
&= \int_0^\infty \mathcal{P}_\phi(\kappa, t) \frac{\sin(\kappa |\vec{\chi}_1 - \vec{\chi}_2|)}{\kappa |\vec{\chi}_1 - \vec{\chi}_2|} \frac{d\kappa}{\kappa} \langle 0 | \delta\phi(\vec{\chi}, t)^2 | 0 \rangle_{phys} \\
&= (4\pi^2 \alpha^3(t))^{-1} \int_0^\infty d\kappa \kappa^2 \{ |\hbar_\kappa(\tau)|^2 - \omega_\kappa(t)^{-1} - (\mathcal{W}_\kappa(t)^{-1})^{(2)} \} - \frac{5\mathcal{H}^2 m^4}{8\omega_\kappa(t)^7} \\
&\quad + \frac{3\mathcal{H}^2 m^2}{4\omega_\kappa(t)^5} + \frac{\mathcal{H}^2}{\omega_\kappa(t)^3} \hbar_\kappa(\tau) \sqrt{\frac{\pi}{2\mathcal{H}}} \mathcal{H}_\nu^{(1)}(v)
\end{aligned}$$

12. Métrica Adiabática.



$$\begin{aligned}
ds^2 &= dt^2 - \alpha^2(t) d\vec{\chi}^2 \mathcal{L} \frac{1}{2} |g|^{\frac{1}{2}} (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m^2 \phi^2 - \xi \mathcal{R} \phi^2) (\square + m^2 + \xi \mathcal{R}) f_{\vec{\kappa}}(\chi) \\
&\sim (\mathcal{V} \alpha(t)^3)^{-\frac{1}{2}} (2\omega_\kappa(t))^{-\frac{1}{2}} \exp \left[i \left(\vec{\kappa} \cdot \vec{\chi} - \int_{\square}^{\tau} \omega_\kappa(\tau') dt' \right) \right] \phi_\kappa \sum_{\vec{\kappa}} \{ A_{\vec{\kappa}} f_{\vec{\kappa}}(\chi) \\
&+ A_{\vec{\kappa}}^\dagger f_{\vec{\kappa}}^*(\chi) \} (2\mathcal{V})^{-1} \alpha(t)^{-\frac{3}{2}} \hbar_\kappa(t) e^{i\vec{\kappa} \cdot \vec{\chi}} \frac{d^2}{dt^2} \hbar_\kappa \\
&+ \Omega_\kappa \omega_\kappa^2 \omega_\kappa(t) \left(\frac{\kappa^2}{\alpha(t)^2} + m^2 \right)^{\frac{1}{2}} \sigma(t) \left(6\xi - \frac{3}{4} \right) \left(\frac{\dot{\alpha}}{\alpha} \right)^2 \\
&+ \left(6\xi - \frac{3}{2} \right) \frac{\ddot{\alpha}}{\alpha} \mathcal{W}_\eta^{-\frac{1}{2}} \mathcal{W}(t)^{-\frac{1}{2}} \hbar \Omega^{-\frac{1}{2}} \exp \left(\pm i t \int_{\square}^{\tau} \Omega \mathcal{W}_\eta(t') dt' \right) \\
&+ \mathcal{O}(\mathcal{T}^{-\eta}) \Omega (1 + \epsilon)^{\frac{1}{2}} \frac{d^2 \hbar}{dt^2} + \left[\mathcal{W}^2 - \mathcal{W}^{\frac{1}{2}} \left(\frac{d^2}{dt^2} \mathcal{W}^{-\frac{1}{2}} \right) \right] 2\mathcal{W} \delta \hbar - \Omega^2 \omega^2 + \sigma \\
&+ \mathcal{W}^{\frac{1}{2}} \frac{d^2}{dt^2} \mathcal{W}^{-\frac{1}{2}} \langle 0 | \phi(\chi) \phi(\chi') | 0 \rangle \sum_{\vec{\kappa}} f_{\vec{\kappa}}(\chi) f_{\vec{\kappa}}^*(\chi) \\
&= \frac{1}{2(2\pi)^2} [\alpha(t) \alpha(t')]^{-\frac{3}{2}} \int d^3 \kappa e^{i\vec{\kappa} \cdot (\vec{\chi} - \vec{\chi}')} \hbar_\kappa(t) \hbar_\kappa^*(t') \langle 0 | \phi(\chi)^2 | 0 \rangle \\
&= (4\pi^2 \alpha^3(t))^{-1} \int_0^\infty d\kappa \kappa^2 |\hbar_\kappa(\tau)|^2 \langle 0 | \phi(\chi) \phi(\chi') | 0 \rangle \\
&\propto \int d^3 \kappa e^{i\vec{\kappa} \cdot (\vec{\chi} - \vec{\chi}')} \kappa^{-1} e^{-i\kappa \int_{t'}^t \alpha^{-1} dt'} \hbar_\kappa(t) \\
&\sim \mathcal{W}_\kappa^{-\frac{1}{2}}(t) \exp \left(-i \int_{\square}^{\tau} \omega_\kappa(t') dt' \right) \langle 0 | \phi(\chi)^2 | 0 \rangle \\
&\sim (4\pi^2 \alpha^3(t))^{-1} \int_0^\infty d\kappa \kappa^2 \omega_\kappa(t)^{-1} \langle 0 | \phi(\chi)^2 | 0 \rangle \\
&\sim (4\pi^2 \alpha^3(t))^{-1} \int_0^\infty d\kappa \kappa^2 \{ (\mathcal{W}_\kappa^{-1})^{(0)} + (\mathcal{W}_\kappa^{-1})^{(2)} + (\mathcal{W}_\kappa^{-1})^{(4)} \\
&+ \mathcal{O}(\mathcal{T}^{-\eta}) \} \ddot{\alpha}^2 \omega^3 \xi' \dot{\alpha}^2 \langle 0 | \phi(\chi)^2 | 0 \rangle_{phys} (4\pi^2 \alpha^3(t))^{-1} \int_0^\infty d\kappa \kappa^2 \{ |\hbar_\kappa(t)|^2 - \omega_\kappa(t)^{-1} \\
&- (\mathcal{W}_\kappa(t)^{-1})^{(2)} \}
\end{aligned}$$

$$\begin{aligned}
\langle 0|\mathcal{T}_\mu^\mu|0\rangle &= -\nabla^\mu\phi\nabla_\mu\phi + 2m^2\phi^2 + \frac{1}{6}\mathcal{R}\phi^2 + \frac{1}{2}\square(\phi^2)\lim_{\kappa\rightarrow\infty}\int_0^\kappa d\kappa\kappa^2\mathcal{W}_\kappa \\
&\sim \lim_{\kappa\rightarrow\infty}\{\kappa^4\mathcal{O}(\mathcal{T}^0) + \kappa^2\mathcal{O}(\mathcal{T}^{-2}) + \ln\kappa\mathcal{O}(\mathcal{T}^{-4})\}\langle 0|\mathcal{T}_\mu^\mu|0\rangle_{phys} \\
&= m^2(4\pi^2\alpha^3(t))^{-1}\int_0^\infty d\kappa\kappa^2\{|\hbar_\kappa(t)|^2 - \omega_\kappa(t)^{-1} - (\mathcal{W}_\kappa(t)^{-1})^{(2)} \\
&\quad - (\mathcal{W}_\kappa(t)^{-1})^{(4)}\}m^2\langle 0|\mathcal{T}_\mu^\mu|0\rangle_{phys} \\
&= m^2(4\pi^2\alpha^3(t))^{-1}\int_0^\infty d\kappa\kappa^2(\mathcal{W}_\kappa(t)^{-1})^{(4)}\langle 0|\mathcal{T}_\mu^\mu|0\rangle_{phys} \\
&= (4\pi^2\alpha^3(t))^{-1}\lim_{m\rightarrow\infty}m^2\int_0^\infty d\kappa\kappa^2(\mathcal{W}_\kappa(t)^{-1})^{(4)}\left[6\left(\frac{\ddot{\alpha}}{\alpha} + \frac{\dot{\alpha}^2}{\alpha^2} + \frac{\ddot{\alpha}}{\alpha} + \frac{\dot{\alpha}^2}{\alpha^2}\right.\right. \\
&\quad \left.\left. + \frac{\ddot{\alpha}^2\dot{\alpha}}{\alpha^3\dot{\alpha}^4}\right)\right]^2\left\{\square\mathcal{R} - \left(\mathcal{R}^{\mu\nu}\mathcal{R}_{\mu\nu} - \frac{1}{3}\mathcal{R}^2\right)\right\}
\end{aligned}$$

13. Tensor Energía – Momentum.

$$\begin{aligned}
\nabla_\mu\langle 0|\mathcal{T}_\mu^\mu|0\rangle_{phys}\langle 0|\mathcal{T}^{\mu\nu}|0\rangle &\sim \langle 0|\mathcal{T}^{\mu\nu}|0\rangle^{(0)} + \langle 0|\mathcal{T}^{\mu\nu}|0\rangle^{(2)}\mathcal{T}^{-2} + \langle 0|\mathcal{T}^{\mu\nu}|0\rangle^{(4)}\mathcal{T}^{-4} \\
&+ \mathcal{O}(\mathcal{T}^{-\eta})\xi^\mu\alpha(t)\delta_0^\mu\xi_\xi g_{\mu\nu}(\chi)\lambda(\chi)g_{\mu\nu}(\chi)2\dot{\alpha}(t)\xi_\xi g_{\mu\nu} \\
&= \xi^\alpha\partial_\alpha g_{\mu\nu} + g_{\mu\alpha}\partial_\nu\xi^\alpha + g_{\alpha\nu}\partial_\mu\xi^\alpha = 2\nabla_{(\frac{\mu}{\nu}\xi)}\nabla_\mu(\langle\mathcal{T}^{\mu\nu}\rangle\xi_\nu) \\
&= \frac{1}{2}\lambda\langle\mathcal{T}^{\mu\nu}\rangle g_{\mu\nu}\frac{1}{2}\int d^4\chi\sqrt{-g}\lambda\langle\mathcal{T}_\mu^\mu\rangle \\
&= \int_{\tau_2}^{\tau_1} d^3\chi\sqrt{-g}\langle\mathcal{T}^{\sigma\nu}\rangle\xi_\nu - \int_{\tau_1}^{\tau_2} d^3\chi\sqrt{-g}\langle\mathcal{T}^{\sigma\nu}\rangle\xi_\nu \int d^4\chi\alpha^3(t)\dot{\alpha}(t)\langle\mathcal{T}_\mu^\mu\rangle \\
&= \int_{\tau_2}^{\tau_1} d^3\chi - \alpha^4(t_2)\langle\mathcal{T}_{00}\rangle - \int_{\tau_1}^{\tau_2} d^3\chi - \alpha^4(t_1)\langle\mathcal{T}_{00}\rangle \int_{\tau_1}^{\tau_2} dt\alpha^3(t)\dot{\alpha}(t)\langle\mathcal{T}_\mu^\mu(t)\rangle \\
&= \alpha^4(t_2)\langle\mathcal{T}_{00}(t_2)\rangle - \alpha^4(t_1)\langle\mathcal{T}_{00}(t_1)\rangle dt\dot{\alpha}\frac{d}{dt}g(t_2) - g(t_1)g(t) + \mathfrak{E}
\end{aligned}$$



$$\mathcal{W} = \int_{\tau}^{\square} d^4\chi \sqrt{-g} \kappa_0 \mathcal{R}$$

$$+ \frac{\hbar}{64\pi^2} \int d^4\chi \sqrt{-g} \left[-m^4 \ln \left| \frac{\mathcal{M}^2}{m^2} \right| + m^2 \bar{\xi} \mathcal{R} \left(1 - 2 \ln \left| \frac{\mathcal{M}^2}{m^2} \right| \right) - 2\alpha_2 \ln \left| \frac{\mathcal{M}^2}{m^2} \right| \right. \\ \left. + \frac{3}{2} \bar{\xi}^2 \mathcal{R}^2 \right] + \frac{i}{64} \int d^4\chi \sqrt{-g} (\mathcal{M}^4 + 2\bar{\alpha}_2) \Theta(-\mathcal{M}^2)$$

14. Renormalización.

$$\begin{aligned}
ds^2 &= g_{\mu\nu}(\chi)d\chi^\mu d\chi^\nu (\square + m^2 + \xi\mathcal{R})\phi \sum_{\bar{j}} \{A_{\bar{j}}f_{\bar{j}}(\chi) + A_{\bar{j}}^\dagger f_{\bar{j}}^*(\chi)\} \mathfrak{G}_{\bar{j}}(\chi, \chi') \\
&= i\langle 0|\mathcal{T}\phi(\chi)\phi(\chi')|0\rangle (\square_\chi + m^2 + \xi\mathcal{R}(\chi)) - \delta(\chi, \chi') \equiv |g(\chi)|^{-\frac{1}{2}}\delta(\chi - \chi') \\
&\sim \Delta^{\frac{1}{2}}(\chi, \chi')(4\pi)^{-\frac{\eta}{2}} \int_0^\infty ds (i\delta)^{-\frac{\eta}{2}} \exp\left[im^2\delta + \frac{\sigma(\chi, \chi')}{2i\delta}\right] \Delta(\chi, \chi') \\
&= -|g(\chi)|^{-\frac{1}{2}} \det[-\partial_\mu\partial_{\nu'}\sigma(\chi, \chi')] |g(\chi')|^{-\frac{1}{2}} = \frac{1}{2}\tau(\chi, \chi')^2 \\
&\sim i\Delta^{\frac{1}{2}}(\chi, \chi')(4\pi)^{-\frac{\eta}{2}} \int_0^\infty ids (i\delta)^{-\frac{\eta}{2}} \\
&\times \exp\left[-im^2\delta + \frac{\sigma(\chi, \chi')}{2i\delta}\right] \mathfrak{F}(\chi, \chi'; i\delta) \langle 0|\phi(\chi)\phi(\chi')|0\rangle_{phys} \\
&= \lim_{\eta\rightarrow 4} \left\{ \langle 0|\phi(\chi)\phi(\chi')|0\rangle \right. \\
&+ i \times i\Delta^{\frac{1}{2}}(\chi, \chi')(4\pi)^{-\frac{\eta}{2}} \\
&\times \int_0^\infty ids (i\delta)^{-\frac{\eta}{2}} \exp\left[-im^2\delta + \frac{\sigma(\chi, \chi')}{2i\delta}\right] \times (1 + \alpha_1(\chi, \chi')(i\delta)) \left. \right\} \langle 0|\mathcal{T}_\mu^\mu(\chi)|0\rangle_{phys} \\
&= m^2\langle 0|\phi^2(\chi)|0\rangle \lim_{\substack{\eta\rightarrow 4 \\ m\rightarrow 0}} m^2 \left\{ \langle 0|\phi^2(\chi)|0\rangle \right. \\
&+ i \times i(4\pi)^{-\frac{\eta}{2}} \int_0^\infty ids (i\delta)^{-\frac{\eta}{2}} \exp[-im^2\delta] \times \left. \left((1 + \alpha_1(\chi)(i\delta)) + (1 + \alpha_1(\chi)(i\delta))^2 \right) \right\} \\
&= \lim_{\substack{\eta\rightarrow 4 \\ m\rightarrow 0}} m^2\langle 0|\phi^2(\chi)|0\rangle - \lim_{\substack{\eta\rightarrow 4 \\ m\rightarrow 0}} m^2(4\pi)^{-\frac{\eta}{2}} \int_0^\infty ids (i\delta)^{-\frac{\eta}{2}} \exp[-im^2\delta] \times \alpha_2(\chi)(i\delta)^2 \\
&= \lim_{\substack{\eta\rightarrow 4 \\ m\rightarrow 0}} \left\{ m^2(4\pi)^{-2} \int_0^\infty ids \exp[-im^2\delta] \right\} \alpha_2(\chi) \\
&= -(4\pi)^{-2}\alpha_2(\chi) \Big|_{\xi=\frac{1}{6}} \int_0^\infty ds \exp[-i(m^2 - i\epsilon)\delta] \\
&= \frac{1}{im^2} \frac{1}{360} \mathcal{R}_{\alpha\beta\gamma\delta} \mathcal{R}^{\alpha\beta\gamma\delta} - \frac{1}{360} \mathcal{R}_{\alpha\beta} \mathcal{R}^{\alpha\beta} - \frac{1}{360} \square \mathcal{R} \mathfrak{C}_{\alpha\beta\gamma\delta} \mathfrak{C}^{\alpha\beta\gamma\delta}
\end{aligned}$$

$$= \mathcal{R}_{\alpha\beta\gamma\delta}\mathcal{R}^{\alpha\beta\gamma\delta} - 2\mathcal{R}_{\alpha\beta}\mathcal{R}^{\alpha\beta} + \frac{1}{3}\mathcal{R}^2\langle 0|\mathcal{T}_\mu^\mu(\chi)|0\rangle_{phys} \left\{ \square\mathcal{R} - \left(\mathcal{R}_{\alpha\beta}\mathcal{R}^{\alpha\beta} - \frac{1}{3}\mathcal{R}^2 \right) \right\}$$

15. Aproximación de Gauss.

$$\begin{aligned} \mathfrak{G} &\equiv \mathcal{R}_{\alpha\beta\gamma\delta}\mathcal{R}^{\alpha\beta\gamma\delta} - 4\mathcal{R}_{\alpha\beta}\mathcal{R}^{\alpha\beta} + \mathcal{R}^2\langle 0|\mathcal{T}_\mu^\mu(\chi)|0\rangle_{phys} \\ &= \frac{1}{4\pi^2} \left\{ \frac{1}{240}\mathfrak{C}_{\alpha\beta\gamma\delta}\mathfrak{C}^{\alpha\beta\gamma\delta} + \mathfrak{B}_\delta\mathfrak{G} + \mathfrak{C}_\delta\square\mathfrak{R} \right\} \langle 0|\mathcal{T}_\mu^\mu(\chi)|0\rangle_{phys} \\ &= \frac{1}{4\pi^2} \left\{ \mathfrak{A}_\delta\mathfrak{C}_{\alpha\beta\gamma\delta}\mathfrak{C}^{\alpha\beta\gamma\delta} + \mathfrak{B}_\delta\mathfrak{G} + \mathfrak{C}_\delta\square\mathfrak{R} \right\} \\ \mathfrak{G}_{\xi}(\chi, \chi') &= i \int_0^\infty ds \exp[-im^2\delta] \langle \chi, \delta|\chi', 0\rangle i \frac{\partial}{\partial\delta} \langle \chi, \delta|\chi', 0\rangle = \left(\square_\chi + \xi\mathcal{R}(\chi) \right) \langle \chi, \delta|\chi', 0\rangle_{Gauss} \\ &= -i\Delta^{\frac{1}{2}}(\chi, \chi')(4\pi\delta)^{-\frac{\eta}{2}} \exp\left[\frac{\sigma(\chi, \chi')}{2i\delta} - i\left(\xi - \frac{1}{6} \right) \mathcal{R}_\delta \right] \end{aligned}$$

16. Regularización Hadamard.

$$\begin{aligned} \mathfrak{G}^{(1)}(\chi, \chi') &= \frac{\Delta^{-\frac{1}{2}}(\chi, \chi')}{(4\pi\delta)^{-\frac{\eta}{2}}} \left(\frac{2}{\sigma(\chi, \chi')} + v_\eta(\chi, \chi') \ln \sigma(\chi, \chi') + \omega_\eta(\chi, \chi') \right) \sigma_\eta v_0 + v_{0,\mu}\sigma^\mu \\ &= \left(\frac{1}{6} \right) \mathcal{R} - \Delta^{-\frac{1}{2}} \left(\Delta^{\frac{1}{2}} \right)_v^\mu v_\eta + \frac{v_{\eta,\mu}\sigma^\mu}{\eta + 1} \\ &= \frac{1}{2\eta(\eta + 1)} \left(\Delta^{-\frac{1}{2}} \left(\Delta^{\frac{1}{2}} v_{\eta-1} \right)_v^\mu - \left(\frac{1}{6} \right) \mathcal{R}_{v_{\eta-1}} \right) \omega_\eta + \frac{\omega_{\eta,\mu}\sigma^\mu}{\eta + 1} \\ &= \frac{1}{2\eta(\eta + 1)} \left(\Delta^{-\frac{1}{2}} \left(\Delta^{\frac{1}{2}} \omega_{\eta-1} \right)_v^\mu - \left(\frac{1}{6} \right) \mathcal{R}_{\omega_{\eta-1}} \right) - \frac{v_\eta}{\eta + 1} \\ &\quad - \frac{1}{2\eta^2(\eta + 1)} \left(\Delta^{-\frac{1}{2}} \left(\Delta^{\frac{1}{2}} v_{\eta-1} \right)_v^\mu - \left(\frac{1}{6} \right) \mathcal{R}_{v_{\eta-1}} \right) \\ \mathfrak{G}_{reg}^{(1)}(\chi, \chi') &\equiv \mathfrak{G}^{(1)}(\chi, \chi') - \mathfrak{G}^{\mathcal{L}}(\chi, \chi') \langle \phi^2(\chi) \rangle_{reg} \equiv \lim_{\chi' \rightarrow \chi} \mathfrak{G}_{reg}^{(1)}(\chi, \chi') \end{aligned}$$

$$\begin{aligned}
\mathcal{T}_{\mu\nu} &= \frac{2}{3} \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{6} g_{\mu\nu} \nabla^\alpha \phi \nabla_\alpha \phi - \frac{1}{3} \phi \nabla_\mu \nabla_\nu \phi + \frac{1}{3} g_{\mu\nu} \phi \nabla^\alpha \nabla_\alpha \phi - \frac{1}{6} \left(\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} \right) \phi^2 \langle \mathcal{T}_{\mu\nu}(\chi) \rangle \\
&= \lim_{\chi' \rightarrow \chi} \mathfrak{T}_{\mu\nu}(\chi, \chi') \\
&= \frac{1}{3} (\nabla_\mu \nabla_{\nu'} \mathfrak{G}^{(1)} + \nabla_{\mu'} \nabla_\nu \mathfrak{G}^{(1)}) - \frac{1}{6} g_{\mu\nu} \nabla_\alpha \phi \nabla^{\alpha'} \mathfrak{G}^{(1)} - \frac{1}{6} (\nabla_\mu \nabla_\nu \mathfrak{G}^{(1)} + \nabla_{\mu'} \nabla_{\nu'} \mathfrak{G}^{(1)}) \\
&\quad + \frac{1}{6} g_{\mu\nu} (\nabla_\alpha \nabla^{\alpha'} \mathfrak{G}^{(1)} + \nabla_{\alpha'} \nabla^\alpha \mathfrak{G}^{(1)}) - \frac{1}{6} \left(\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} \right) \mathfrak{G}^{(1)} \langle \mathcal{T}_{\mu\nu}(\chi) \rangle_{reg} \\
&\equiv \langle \mathcal{T}_{\mu\nu} \rangle_{reg}^0 - \frac{1}{2(4\pi)^2} g_{\mu\nu} \nu_1(\chi, \chi')
\end{aligned}$$

$$\begin{aligned}
\mathcal{J} &= \frac{\hbar \kappa}{2\pi \kappa_\beta \overline{\mathcal{R}}_{\mu\nu}} = \Lambda \overline{g}_{\mu\nu} = \Omega^{-2} \overline{g}_{\mu\nu} \overline{\mathfrak{G}}_{\mathfrak{F}}(\chi, \chi') \\
&= \Omega^{-1}(\chi) \mathfrak{G}(\chi, \chi') \Omega^{-1}(\chi') \mathfrak{G}_{\mathfrak{E}}(\chi, \chi') \int_0^\infty d\mu \kappa(\chi, \chi', \mu) \left(\frac{\partial}{\partial \mu} - \frac{\partial^2}{\partial \tau^2} - \nabla_i \nabla^i + \square \right. \\
&\quad \left. + \frac{1}{6} \mathcal{R} \right) \kappa(\chi, \chi', \mu)(\tau, \tau', \mu) \beta^{-1} \sum_{\eta=-\infty}^\infty \exp\left(-\frac{4\mu\pi^2 \eta^2}{\beta^2}\right) \\
&\quad - \frac{2\pi i \eta}{\beta} (\tau \\
&\quad - \tau') \mathfrak{G}_{Gauss}(\tau, \chi, 0, \chi') \frac{\Delta^{\frac{1}{2}}}{4\pi\beta r} \frac{\sinh(2\pi r/\beta)}{\cosh(2\pi r/\beta) - \cos(2\pi r/\beta)} \langle \hat{\phi}^2 \rangle_{Gauss} \left(\frac{4\pi^2}{\beta^2 \Omega^2} \right. \\
&\quad \left. - \frac{\square \Omega}{\Omega^3} \right) \mathfrak{R} - \Omega^2 \left(\frac{\Lambda}{4\pi^2} \beta_{loc}^{-2} - \beta_{acc}^{-2} \left(\frac{\alpha}{2\pi} \right)^2 \right) d\bar{s}^2 \\
&= e^{2\alpha\xi} (d\eta^2 - d\xi^2 - e^{-2\alpha\xi} d\gamma^2 - e^{-2\alpha\xi} dz^2) \delta_\nu^\mu \tilde{g}_{\alpha\beta}(\chi') \frac{\delta}{\delta \tilde{g}_{\alpha\beta}(\chi')} (|\tilde{g}|^{1/2} \mathcal{T}_\nu^\mu(\chi)) \\
&= \tilde{g}_{\nu\gamma}(\chi) \frac{\delta}{\delta \tilde{g}_{\mu\gamma}(\chi)} (|\tilde{g}|^{1/2} \mathcal{T}_\lambda^\lambda(\chi')) \bar{\mathcal{T}}_\nu^\mu \\
&= \Omega^{-4} \mathcal{T}_\nu^\mu - 8\alpha_\delta \Omega^{-4} \left\{ - \left(\mathfrak{C}_{\beta\nu}^{\alpha\mu} \ln \Omega \right)_\alpha^\beta + \frac{1}{2} \mathcal{R}_\beta^\alpha \mathfrak{C}_{\beta\nu}^{\alpha\mu} \ln \Omega \right\} \\
&\quad + \beta_\delta \left\{ \left(4\bar{\mathcal{R}}_\alpha^\beta \bar{\mathfrak{C}}_{\beta\nu}^{\alpha\mu} - 2\bar{\mathcal{H}}_\nu^\mu \right) - \Omega^{-4} \left(4\mathcal{R}_\alpha^\beta \mathfrak{C}_{\beta\nu}^{\alpha\mu} - 2\mathcal{H}_\nu^\mu \right) \right\} - \frac{1}{6} \gamma_\delta \left\{ \bar{\mathbb{I}}_\nu^\mu - \Omega^{-4} \mathbb{I}_\nu^\mu \right\} \\
\mathcal{H}_{\mu\nu} &= \mathcal{R}_\mu^\rho \mathcal{R}_{\rho\nu} - \frac{2}{3} \mathcal{R} \mathcal{R}_{\mu\nu} - \frac{1}{2} \mathcal{R}_{\rho\sigma} \mathcal{R}^{\rho\sigma} g_{\mu\nu} + \frac{1}{2} \mathcal{R}^2 g_{\mu\nu} \mathbb{I}_{\mu\nu} = 2\mathcal{R}_{\mu\nu} - 2g_{\mu\nu} \square \mathcal{R} - \frac{1}{2} g_{\mu\nu} \mathcal{R}^2 + 2\mathcal{R} \mathcal{R}_{\mu\nu}
\end{aligned}$$

$$d\bar{s}^2 = -\left(1 - \frac{\Lambda r^2}{3}\right) \times \left\{ d\tau^2 + \frac{dr^2}{\left(1 - \frac{\Lambda r^2}{3}\right)^2} + \frac{r^2}{\left(1 - \frac{\Lambda r^2}{3}\right)} (d\theta^2 + \sin^2\theta d\phi^2) \right\}$$

17. Coordenadas espaciales en espacios cuánticos curvos: Momentum normal.

$$\begin{aligned} \xi^\mu &= (d\chi^\mu/d\lambda)_Q \frac{d^2\gamma^\alpha}{d\lambda^2} \Gamma_{\beta\gamma}^\alpha(\gamma) \frac{d\gamma^\beta}{d\lambda} \frac{d\gamma^\gamma}{d\lambda} \\ &= \Gamma_{\beta\gamma}^\alpha(\gamma) \xi^\beta(\gamma) \xi^\gamma(\gamma) g_{\mu\nu}(Q) \eta_{\mu\nu\tau} \int_Q^P d\sigma \left(g_{\alpha\beta} \frac{d\chi^\alpha}{d\sigma} \frac{\partial\chi^\beta}{\partial\sigma} \right)^{1/2} g_{\alpha\beta}(\gamma) \\ &= \eta_{\alpha\beta} - \frac{1}{3} \mathcal{R}_{\alpha\mu\beta\lambda}(0) \gamma^\mu \gamma^\lambda - \frac{1}{3!} \mathcal{R}_{\alpha\gamma\beta\lambda,\mu}(0) \gamma^\lambda \gamma^\mu \gamma^\gamma + \frac{1}{5!} \left(-6\mathcal{R}_{\alpha\delta\beta\gamma,\lambda\mu} + \frac{16}{3} \mathcal{R}_{\lambda\beta\mu\rho} \right)_0 \gamma^\lambda \gamma^\mu \gamma^\gamma \gamma^\delta \\ &\quad - \Gamma_{(\alpha\beta)}^\nu [2\Gamma_{\sigma(\alpha}^\nu \Gamma_{\beta\gamma)}^\sigma - \partial_{(\alpha} \Gamma_{\beta\gamma)}^\nu]_{\chi_0} \gamma^\alpha \gamma^\beta \gamma^\gamma \left(\square_\chi + m^2 + \xi \mathcal{R}(\chi) \right) \mathfrak{G}(\chi, \chi') = -\delta(\chi - \chi') \\ &\equiv |g(\chi)|^{-\frac{1}{4}} \bar{\mathfrak{G}}(\chi, \chi') \delta(\gamma) \\ &= -\eta^{\mu\nu} \partial_\mu \partial_\nu \bar{\mathfrak{G}} - \left[m^2 + \left(\xi - \frac{1}{6} \right) \mathcal{R} \right] \bar{\mathfrak{G}} - \frac{1}{3} \mathcal{R}_\alpha^\nu \gamma^\alpha \partial_\nu \bar{\mathfrak{G}} + \frac{1}{3} \mathcal{R}_{\alpha\beta}^\mu \gamma^\alpha \gamma^\beta \partial_\mu \partial_\nu \bar{\mathfrak{G}} - \left(\xi - \frac{1}{6} \right) \mathcal{R}_\alpha \gamma^\alpha \bar{\mathfrak{G}} \\ &\quad + \left(-\frac{1}{3} \mathcal{R}_{\alpha\beta}^\nu + \frac{1}{6} \mathcal{R}_{\alpha\beta}^\nu \right) \gamma^\alpha \gamma^\beta \partial_\nu \bar{\mathfrak{G}} \\ &\quad + \frac{1}{6} \mathcal{R}_{\alpha\beta,\gamma}^\mu \gamma^\alpha \gamma^\beta \gamma^\gamma \partial_\mu \partial_\nu \bar{\mathfrak{G}} - \frac{1}{2} \left(\xi - \frac{1}{6} \right) \|\mathcal{R}_{\alpha\beta} \gamma^\alpha \gamma^\beta \bar{\mathfrak{G}}\| \langle \square \mathcal{R}_{\alpha\beta} \mathcal{R}_{\kappa\lambda} \mathcal{R}_\alpha^\lambda \mathcal{R}_{\lambda\beta} \mathcal{R}_\alpha^{\lambda\mu\kappa} \mathcal{R}_{\lambda\mu\kappa\beta} \mathcal{R}_{\alpha\beta}^\kappa \gamma^\nu \mathcal{R}_\kappa \gamma^\nu \partial_\nu \bar{\mathfrak{G}} \rangle \end{aligned}$$

$$\begin{aligned}
& \mathfrak{G}(\chi, \chi') \\
&= - \int \frac{d^\eta \kappa}{(2\pi)^\eta} e^{iky} \bar{\mathfrak{G}}(\kappa, \chi) (-\eta^{\mu\nu} \kappa_\mu \kappa_\nu + m^2) \bar{\mathfrak{G}}_2(\kappa, \chi) + \left(\xi - \frac{1}{6}\right) \mathcal{R} \bar{\mathfrak{G}}_0(\kappa, \chi) \\
&+ \frac{1}{3} \mathcal{R}_\alpha^\nu (i\kappa_\nu) \left(i \frac{\partial}{\partial \kappa_\alpha}\right) \bar{\mathfrak{G}}_0(\kappa, \chi) - \frac{1}{3} \mathcal{R}_{\alpha\beta}^{\mu\nu} (i\kappa_\mu) (i\kappa_\nu) \left(i \frac{\partial}{\partial \kappa_\alpha}\right) \left(i \frac{\partial}{\partial \kappa_\beta}\right) \bar{\mathfrak{G}}(\kappa) \left(\xi - \frac{1}{6}\right) \mathcal{R} (-\kappa^2 + m^2)^{-2} \\
&+ i \left(\xi - \frac{1}{6}\right) \mathcal{R}_\alpha (-\kappa^2 + m^2)^{-1} \frac{\partial}{\partial \kappa_\alpha} (-\kappa^2 + m^2)^{-1} + \left(\xi - \frac{1}{6}\right)^2 \mathcal{R}^2 (-\kappa^2 + m^2)^{-3} \\
&+ \alpha_{\alpha\beta} (-\kappa^2 + m^2)^{-1} \frac{\partial}{\partial \kappa_\alpha} \frac{\partial}{\partial \kappa_\beta} (-\kappa^2 + m^2)^{-1} \left\| \square \mathcal{R}_{\alpha\beta} \mathcal{R}_{\kappa\lambda} \mathcal{R}_\alpha^\lambda \mathcal{R}_{\lambda\beta} \mathcal{R}_\alpha^{\lambda\mu\kappa} \mathcal{R}_{\lambda\mu\kappa\beta} \mathcal{R}_{\alpha\beta}^{\kappa\nu} \mathcal{R}_\kappa \right\| \eta_{\alpha\beta} (-\kappa^2 \\
&+ m^2)^{-3} \frac{\partial}{\partial \kappa_\alpha} \frac{\partial}{\partial \kappa_\beta} (-\kappa^2 \\
&+ m^2)^{-2} \int \frac{d^4 \kappa}{(2\pi)^\eta} \frac{\partial}{\partial \kappa_\alpha} (-\kappa^2 + m^2)^{-2} \int \frac{d^4 \kappa}{(2\pi)^\eta} \frac{\partial}{\partial \kappa_\beta} (-\kappa^2 \\
&+ m^2)^{-3} \int \frac{d^\eta \kappa}{(2\pi)^\eta} \left(\frac{i\pi^2}{2m^2(2\pi)^4}\right) \left\| \square \mathcal{R}_{\alpha\beta} \mathcal{R}^{\alpha\beta} \mathcal{R}_{\alpha\beta\gamma\delta} \mathcal{R}^{\alpha\beta\gamma\delta} \right\| \int \frac{d^\eta \kappa}{(2\pi)^\eta} id\delta \exp[-i\delta(-\kappa^2 \\
&+ m^2)] \int \frac{d^\eta \kappa}{(2\pi)^\eta} \exp[-i\delta(-\kappa^2 + m^2) + i\kappa\gamma] \frac{i}{(4\pi)^\eta} (i\delta)^{-\eta/2} \exp[-im^2\delta \\
&+ \frac{\sigma(\chi, \chi')}{2i\delta}] \int_0^\infty \frac{id\delta}{(i\delta)^{-\eta/2}} \exp[-im^2\delta + \frac{\sigma}{2i\delta}]
\end{aligned}$$

(b) Agujeros negros cuánticos.

1. Cuantización General.

$$\begin{aligned}
& (-g)^{1/2} \partial_\mu [(-g)^{1/2} g^{\mu\nu} \partial_\nu \phi] \int d\omega' (\alpha_{\omega'} \alpha_\omega + \alpha_{\omega'}^\dagger \beta_{\omega'}^*) \int d\omega (\alpha_{\omega_1\omega}^\dagger \alpha_{\omega_2\omega}^* - \beta_{\omega_1\omega}^\dagger \beta_{\omega_1\omega}^*) \\
&+ c_{\omega\omega'}^\dagger q_{\omega\omega'}^* - c_{\omega_1\omega}^\dagger q_{\omega_1\omega}^* - c_{\omega_2\omega}^\dagger q_{\omega_2\omega}^* \delta(\omega_1 - \omega_2)
\end{aligned}$$

2. Métrica de Schwarzschild.



$$\begin{aligned}
ds^2 &= \left(1 - \frac{2\mathcal{M}}{r}\right) dt^2 - \left(1 - \frac{2\mathcal{M}}{r}\right) dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \left| \frac{\Delta}{\rho^2} \right| [dt - \alpha \sin^2\theta d\phi]^2 \\
&\quad - \frac{\sin^2\theta}{\rho^2} [(r^2 + \alpha^2)d\phi - \alpha dt]^2 - \frac{\rho^2}{\Delta} dr^2 - \rho^2 d\theta^2 \frac{d\phi}{dt} \Omega_{\mathcal{H}} \\
&\equiv \frac{\alpha}{\alpha^2 + r_{\pm}^2} \left(\frac{\mathcal{D}}{d\lambda}\right) \left(\frac{d\chi^\mu}{d\lambda}\right) \frac{1}{2} g_{\mu\nu} \frac{d\chi^\mu}{d\lambda} \frac{d\chi^\nu}{d\lambda} \rho_{\mu\nu} \equiv \frac{\partial \mathcal{L}}{\left(\frac{d\chi^\mu}{d\lambda}\right)} \\
&= g_{\mu\nu} \frac{d\chi^\nu}{d\lambda} \left(1 - \frac{2\mathcal{M}}{r}\right) \frac{dt}{d\lambda} \mathfrak{E} r^2 \frac{d\phi}{d\lambda} \\
&= \mathcal{L} \left(\frac{dr}{d\lambda}\right)^2 + \frac{\mathcal{L}^2}{r^2} \left(1 - \frac{2\mathcal{M}}{r}\right) \mathfrak{E}^2 \frac{dt}{d\lambda} \\
&\quad \mp \left(1 - \frac{2\mathcal{M}}{r}\right)^{-1} \frac{dr^*}{d\lambda} \frac{dv}{d\lambda} 2 \left(1 - \frac{2\mathcal{M}}{r}\right)^{-1} \mathfrak{E} \frac{dv}{d\lambda} 2 \mathfrak{E} \frac{4\mathcal{M}}{\mathfrak{E}\lambda} - 4\mathcal{M} \ln \frac{\lambda}{\kappa_1} \left(\frac{v - v_0}{\kappa_1 \kappa_2}\right)
\end{aligned}$$

$$\mu(v) = 4\mathcal{M} \ln \left(\frac{v_0 - v}{\mathcal{K}}\right) \rho_\omega \sim \omega^{\frac{1}{2}} r^{-1} \exp(-i\omega\mu(v)) \delta(\theta, \phi) \alpha_{\omega\omega'} =$$

$$\mathfrak{E} \int_{-\infty}^{v_0} dv \left(\frac{\omega'}{\omega}\right)^{1/2} e^{i\omega'v} e^{-i\omega\mu(v)} \beta_{\omega\omega'} = \mathfrak{E} \int_{-\infty}^{v_0} dv \left(\frac{\omega'}{\omega}\right)^{1/2} e^{-i\omega'v} e^{-i\omega\mu(v)} \alpha_{\omega\omega'} =$$

$$-\mathfrak{E} \int_{\infty}^{\mu\nu} ds' \left(\frac{\omega'}{\omega}\right)^{1/2} e^{i\omega's'} e^{i\omega'\mu\nu} \exp\left[i\omega 4\mathcal{M} \ln\left(\frac{is'}{\mathcal{K}}\right)\right] \beta_{\omega\omega'} =$$

$$\mathfrak{E} \int_{-\infty}^{\mu\nu} ds' \left(\frac{\omega'}{\omega}\right)^{1/2} e^{-i\omega's'} e^{-i\omega'\mu\nu} \exp\left[i\omega 4\mathcal{M} \ln\left(-\frac{is'}{\mathcal{K}}\right)\right]$$

$$\Gamma(\omega_1) \delta(\omega_1 - \omega_2)$$

$$= \int d\omega' \left(\alpha_{\omega_1\omega'}^* \alpha_{\omega_2\omega'}^*\right.$$

$$\left. - \beta_{\omega_1\omega'}^* \beta_{\omega_2\omega'}^*\right) \langle 0 | \alpha_{\omega'}^\dagger \beta_{\omega'}^* | 0 \rangle \int d\omega' \left| \alpha_{\omega_1\omega'}^\dagger \right|^2 - \left| \beta_{\omega_1\omega'}^* \right|^2 \lim_{T \rightarrow \infty} \frac{1}{2\pi} \int_{-T/2}^{T/2} dt \exp[i((\omega_1$$

$$- \omega_2))t] \int d\omega_1 \int d\omega_2 A(\omega_1) A(\omega_2) e^{i[\gamma(\omega_1) - \gamma(\omega_2)]} (\rho_{\omega_1}, \rho_{\omega_2}) \Gamma(\omega) \int d\omega' \langle |\alpha_{\rho\omega'}| |\beta_{\rho\omega'}| \rangle^2 \delta(\theta, \phi)$$

3. Métrica de Kerr.



$$\begin{aligned}
\rho^4 \left(\frac{dr}{d\lambda}\right)^2 &= [(r^2 + \alpha^2)\mathfrak{E} - \alpha\mathfrak{L}_z]^2 - \mathcal{K}_c \Delta \rho^4 \left(\frac{d\theta}{d\lambda}\right)^2 = -(\alpha\mathfrak{E} \sin \theta - \mathfrak{L}_z \operatorname{cosec} \theta)^2 + \mathcal{K}_c \rho^2 \left(\frac{dt}{d\lambda}\right) \\
&= \Delta^{-1} \{[(r^2 + \alpha^2)^2 - \Delta \alpha^2 \sin^2 \theta] \mathfrak{E} - 2\alpha \mathcal{M} r \mathfrak{L}_z\} \rho^2 \left(\frac{dt}{d\lambda}\right) \\
&= \Delta^{-1} [2\alpha \mathcal{M} r \mathfrak{E} + (\rho^2 - 2\mathcal{M} r) \mathfrak{L}_z \operatorname{cosec}^2 \theta] \frac{dt}{d\lambda} \mathfrak{E} / \Delta \frac{d\phi}{d\lambda} \alpha \mathfrak{E} / \Delta (r^2 + \alpha^2)^2 \\
&\quad - \Delta \alpha^2 \sin^2 \theta = \rho^2 (r^2 + \alpha^2)^2 + 2\alpha^2 \mathcal{M} r \sin^2 \theta \frac{dr^*}{d\lambda} \frac{r^2 + \alpha^2}{\Delta} \tilde{\phi}_+ \equiv \phi - \frac{\alpha}{r_+^2 + \alpha^2} t \\
&= \phi - \Omega_{\mathcal{H}} \tau \frac{d\tilde{\phi}_+}{d\lambda} = \frac{\alpha \mathfrak{E}}{\Delta} - \frac{\alpha}{r_+^2 + \alpha^2} \frac{r^2 + \alpha^2}{\Delta} \mathfrak{E} = -\Omega_{\mathcal{H}} \mathfrak{E} (r + r_+) / (r + r_-) \\
\frac{d\mu}{d\lambda} &= \frac{dt}{d\lambda} - \frac{dr^*}{dr} \frac{dr}{d\lambda} = 2\mathfrak{E} \frac{r^2 + \alpha^2}{\Delta} \frac{d\mu}{d\lambda} = 2 \frac{(r_+ - \mathfrak{E}\lambda)^2 + \alpha^2}{\mathfrak{E}\lambda[\mathfrak{E}\lambda - (r_+ - r_-)]} \mu \\
&= 2\mathfrak{E}\lambda - \left(\frac{1}{\alpha_+}\right) \ln \frac{\mathfrak{E}\lambda}{\kappa_1} + \left(\frac{1}{\alpha_-}\right) \ln \{[\mathfrak{E}\lambda - (r_+ - r_-)]/\kappa'_1\} \\
\mu(v) &\approx -\left(\frac{1}{\kappa}\right) \ln \left[\frac{(v - v_0)}{\kappa}\right] \alpha_+ \frac{1}{2} (r_+ - r_-) \\
&\quad / (r_+^2 + \alpha^2) |\alpha_{\omega\omega'}|^2 \exp[2\pi\kappa^{-1}(\omega - m\Omega_{\mathcal{H}})] |\beta_{\omega\omega'}|^2 \langle N_{\omega l m} \rangle \\
&= \Gamma_{lm}(\omega) \{ \exp[2\pi\kappa^{-1}(\omega - m\Omega_{\mathcal{H}})] - 1 \}^{-1}
\end{aligned}$$

4. Entropía.

$$\begin{aligned}
d\delta &= \frac{4\pi\mathcal{M}(\mathcal{M} + \sqrt{\mathcal{M}^2 - \alpha^2})}{\sqrt{\mathcal{M}^2 - \alpha^2}} \left\{ d\mathcal{M} - \left[\frac{\alpha}{2\mathcal{M}(\mathcal{M} + \sqrt{\mathcal{M}^2 - \alpha^2})} \right] d(\alpha\mathcal{M}) \right\} \\
&= \frac{4\pi\mathcal{M}^2}{\sqrt{\mathcal{M}^2 - \alpha^2}} d\mathcal{M} + 4\pi\mathcal{M} d\mathcal{M} - \frac{2\pi\alpha}{\sqrt{\mathcal{M}^2 - \alpha^2}} d(\alpha\mathcal{M}) \\
&= d \left(2\pi\mathcal{M}^2 + 2\pi\mathcal{M}\sqrt{\mathcal{M}^2 - \alpha^2} \right) \int_{r=r_+}^{\square} \sqrt{g_{\theta\theta}g_{\phi\phi}} d\theta d\phi 4\pi(r_+^2 + \alpha^2) \\
&= 8\pi\mathcal{M} \left(\mathcal{M} + \sqrt{\mathcal{M}^2 - \alpha^2} \right)
\end{aligned}$$

5. Tensor energía – momentum de Hawking.



$$\begin{aligned}
\mathfrak{G}(\chi, \chi') &= \frac{1}{2} \langle 0 | \phi(\chi) \phi(\chi') + \phi(\chi') \phi(\chi) | 0 \rangle \mathfrak{G}^u(\chi, \chi') - \mathfrak{G}^{\mathcal{H}\mathcal{H}}(\chi, \chi') \\
&= -(4\pi r r')^{-1} \int_0^\infty d\omega \omega^{-1} e^{-i\omega(t-t')} \frac{1}{(e^{8\pi\mathcal{M}\omega} - 1)^{-1}} \\
&\times \sum_{m, \eta} \Upsilon_\eta^m(\theta, \phi) \Upsilon_\eta^{m*}(\theta', \phi') \mathcal{R}_{\omega\eta}(r) \mathcal{R}_{\omega\eta}^*(r') + c \otimes c \frac{d^2 \mathcal{R}}{d\bar{r}^2} \\
&+ [\omega^2 - \eta(\eta + 1)r^{-3}(r - 2\mathcal{M})r^{-4}(r - 2\mathcal{M})] \mathfrak{R}\bar{r} \\
&+ 2\mathcal{M} \ln\left(\frac{r}{(2\mathcal{M})} - 1\right) \tau_{\omega\eta} e^{-i\omega\bar{r}} \sum_{\kappa=0}^\infty c_\kappa \left(\frac{r}{(2\mathcal{M})} - 1\right)^\kappa \frac{d\omega}{\omega} \\
&\times \sum_{\eta=0}^\infty \int \frac{d\omega}{\omega} (e^{8\pi\mathcal{M}\omega} - 1)^{-1} (2\eta + 1) \hbar_{\omega\eta}(r) g_{\omega\eta}(r) \frac{\mathcal{L}}{4\pi r(r - 2\mathcal{M})} f_{\omega\eta}(r) \\
f_{\omega\eta}(r) &= 2 \frac{(\gamma_2 - 3\delta)}{r(r - 2\mathcal{M})} + 2\gamma_1 \frac{(r - 3\mathcal{M})}{r^2(r - 2\mathcal{M})} + 2 \left[\eta(\eta + 1) \frac{r - 4\mathcal{M}}{r} - \omega^2 \frac{r^3}{r - 2\mathcal{M}} \right] \frac{\epsilon}{r^4} \\
g_{\omega\eta}(r) &= 2 \frac{\delta}{r(r - 2\mathcal{M})} - 2 \left[2\eta(\eta + 1) \frac{r - 2\mathcal{M}}{r} + \omega^2 \frac{r^3}{r - 2\mathcal{M}} \right] \frac{\epsilon}{r^4} \\
\hbar_{\omega\eta}(r) &= -2 \frac{\delta}{r(r - 2\mathcal{M})} + 2\gamma_1 \frac{(r - 3\mathcal{M})}{r^2(r - 2\mathcal{M})} \\
&- 2 \left[\eta(\eta + 1) + \frac{r - 4\mathcal{M}}{r} + 5\omega^2 \frac{r^3}{r - 2\mathcal{M}} \right] \frac{\epsilon}{r^4} \left| \frac{d\mathcal{R}_{\omega\eta}}{d\bar{r}} \right| \mathfrak{R}_{\omega\eta}^* \frac{d_i \mathcal{R}_{\omega\eta}}{d\bar{r}^i} + c \otimes c \\
ds^2 &= e^{2\psi} \left(1 - \frac{2\mathcal{M}(r, v)}{r} \right) dv^2 + 2e^\psi dv dr + r^2 d\Omega^2
\end{aligned}$$

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APÉNDICE D.

Formalización lagrangiana relativa a gravedad cuántica, morfología de las partículas y antipartículas supermasivas y de las hiperpartículas y agujeros negros cuánticos en espacios curvos.

$$\begin{aligned}
 \mathcal{L}_{SM}(x) &\equiv (a, b) \simeq (b, a) \\
 &= -\frac{1}{2\pi} \partial^\mu \partial_\nu \partial^\nu \partial_\mu \overline{\partial_\mu^v} \overline{\partial_\nu^u} g_\mu^a g_\nu^b g_b^v - g_s f^{ab} f_{ab} \partial^\mu \partial_\nu \partial^\nu \partial_\mu \overline{\partial_\mu^v} \overline{\partial_\nu^u} g_\mu^a g_\nu^b g_b^v - \frac{1}{4\pi} \overline{g_s^2 f^{cd} f_{cd} \partial^\mu \partial_\nu \partial_\mu \partial_\nu \overline{\partial_\mu^v} \overline{\partial_\nu^u} g_\mu^a g_\nu^b g_b^v} \\
 &- \partial^\mu W_\mu \partial^\nu W_\nu - M^2 W_\mu^+ W_\mu^- W_\mu^- W_\mu^+ W_\mu^+ W_\mu^- W_\mu^- W_\mu^+ - \frac{1}{2\pi} \partial^\mu \partial_\nu \partial^\nu \partial_\mu \partial_\nu^0 Z_\mu^0 Z_\nu^0 Z_\nu^0 - \frac{1}{2} \partial^\mu A_\nu \partial^\nu A_\mu - \frac{1}{2} c_m^2 M^2 Z_\mu^0 Z_\nu^0 Z_\nu^0 \\
 &- ig c_w \left(\partial^\mu \partial_\nu \partial^\nu \partial_\mu \partial_\nu^0 Z_\mu^0 Z_\nu^0 Z_\nu^0 (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+) \right) - Z_\nu^0 (\partial^\mu \partial_\mu W_\mu^+ W_\mu^- W_\mu^+ W_\mu^-) + Z_\nu^0 (\partial^\nu \partial_\nu W_\nu^+ W_\nu^- W_\nu^+ W_\nu^-) \\
 &- ig S_w (\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+) Z_\mu^0 Z_\nu^0 Z_\nu^0) - A_\mu (\partial^\mu \partial_\mu W_\mu^+ W_\mu^- W_\mu^+ W_\mu^- Z_\mu^0 Z_\mu^0) + A_\nu (\partial^\nu \partial_\nu W_\nu^+ W_\nu^- W_\nu^+ W_\nu^- Z_\nu^0 Z_\nu^0) \\
 &- \frac{1}{2g^2} \left(\overline{\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+) Z_\mu^0 Z_\nu^0 Z_\nu^0} \right) + g^2 c_w^2 \left(\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+) Z_\mu^0 Z_\nu^0 Z_\nu^0 \right) \\
 &+ g^2 S_w^2 \left(\frac{A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+) Z_\mu^0 Z_\nu^0 Z_\nu^0}{\partial^\mu A_\nu \partial^\nu} \right) - g^2 c_w S_w \left(\frac{(W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+) Z_\mu^0 Z_\nu^0 Z_\nu^0}{\partial^\mu A_\nu \partial^\nu A_\mu} \right) \\
 &- \frac{1}{2\pi} \left(\overline{\partial H^\mu A H_\nu H \partial^\nu H A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+) Z_\mu^0 Z_\nu^0 Z_\nu^0 H^\mu H_\nu H^\mu H_\nu} \right) + \left\| \frac{\frac{1}{2\pi} (2M^2 H^2 H^3)}{\frac{d^4 e m^c \gamma}{GUM_{Scw}^2}} \right\|^2 - \left\| \frac{2g_c^2 M_s^2}{\partial \Psi \Phi \zeta} \right\| - \langle \partial \lambda \rangle
 \end{aligned}$$

$$\begin{aligned}
 &\frac{\partial \omega}{\partial \Delta \nabla \theta} \\
 &\Pi_\pm^\dagger \infty \int_\partial \left| \frac{\phi_\mu^+ \phi_\nu^- \phi_\mu^- \phi_\nu^+}{\phi_\mu^+ \phi_\nu^- \phi_\mu^+ \phi_\nu^-} \right|^2 \\
 &\left(\varphi \psi \omega \lambda_\mu^+ \varphi \psi \omega \lambda_\nu^- \varphi \psi \omega \lambda_\mu^- \varphi \psi \omega \lambda_\nu^+ \left| \frac{2\varphi \psi \omega \lambda}{\varphi \psi \omega \lambda} \right|_+^\mu \varphi \psi \omega \lambda_\nu^- \varphi \psi \omega \lambda_\mu^\pm \varphi \psi \omega \lambda_\nu^\pm \left| \frac{1}{\varphi \psi \omega \lambda} \right|_0^\mu \varphi \psi \omega \lambda_\nu^0 \varphi \psi \omega \lambda_\mu^0 \varphi \psi \omega \lambda_\nu^0 \right) \\
 &\otimes \left\langle 2M \right. \\
 &\int_\partial \left\| \frac{2\xi\eta}{\zeta\varepsilon\varepsilon} \right\|^{1/2} \left\| \frac{\delta\alpha}{\sigma\rho\rho} \right\| \square = \mathcal{L}_{Higgs} = \\
 &\sqrt{\left(\partial^\mu \partial_\nu \partial^\nu \partial_\mu + \frac{1}{2i} g_1 B^\mu B_\nu B^\nu B_\mu + \frac{1}{2j} g_2 B^\mu B_\nu B^\nu B_\mu + \frac{1}{2i} g_1 W^\mu W_\nu W^\nu W_\mu + \frac{1}{2j} g_2 W^\mu W_\nu W^\nu W_\mu \right) - m_H^2 \phi^2 - \frac{v^2}{2v^2}}
 \end{aligned}$$



$$\partial_i \partial^j \partial_j \partial^i f^{ab\varphi} t_{ba\omega} t^{ab\varphi} f_{ba\omega}(x)$$

$$= \int_{\partial} \left\langle \frac{\frac{\partial^\theta \partial_\theta F_\sigma^\rho \gamma \beta}{\varepsilon \epsilon \theta \pi}}{\Delta \nabla} \right\rangle + \prod_b^a \lambda \prod_a^b \lambda H_{iggs}$$

$$- W^a W_b \widehat{W^b W_a} W_b^a W_a^b \widehat{W_a^b W} - \eta^\theta \widehat{\eta_\beta \eta} \frac{\sigma^\mu \alpha_\nu}{\phi \nu \Omega \eta} / \mathbb{R}^4 \left\langle d\tau^2 \right\rangle$$

$$\mathcal{L}_{SM}(y) \equiv (a, b) \simeq (b, a)$$

$$\begin{aligned} &= -\frac{1}{2\pi} \partial^\mu \partial_\nu \partial^\nu \partial_\mu \widehat{\partial_\mu^\mu g_a^\mu g_b^\mu g_b^\nu} - g_s f^{ab} f_{ab} \partial_\nu \partial^\nu \widehat{\partial_\mu^\mu g_a^\mu g_b^\mu g_b^\nu} - \frac{1}{4\pi} \overline{g_s^2 f^{cd} f_{cd} \partial^\mu \partial_\nu \partial_\mu \partial_\nu g_a^\mu g_c^\mu g_d^\mu} \\ &- \partial^\mu W_\mu \partial^\nu W_\nu - M^2 W_\mu^+ \widehat{W_\mu^-} W_\mu^- W_\nu^+ W_\nu^- W_\mu^+ W_\nu^- - \frac{1}{2\pi} \partial^\mu \partial_\nu \partial^\nu \partial_\mu \partial_\nu^0 Z_\mu^0 Z_\nu^0 Z_\nu^0 - \frac{1}{2} \partial^\mu A_\nu \partial^\nu A_\mu - \frac{1}{2} c_m^2 M^2 Z_\mu^0 Z_\nu^0 Z_\nu^0 \\ &- ig c_w (\partial^\mu \partial_\nu \partial^\nu \partial_\mu \partial_\nu^0 Z_\mu^0 Z_\nu^0 Z_\nu^0 (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+)) - Z_\mu^0 (\partial^\mu \partial_\nu W_\mu^+ W_\nu^- W_\mu^+ W_\nu^-) + Z_\nu^0 (\partial^\nu \partial_\mu W_\nu^+ W_\mu^- W_\nu^+ W_\mu^-) \\ &- ig S_w (\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+)) Z_\mu^0 Z_\nu^0 Z_\nu^0 - A_\mu (\partial^\mu \partial_\nu W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- Z_\mu^0 Z_\nu^0) + A_\nu (\partial^\nu \partial_\mu W_\nu^+ W_\mu^- W_\nu^+ W_\mu^- Z_\nu^0 Z_\mu^0) \\ &- \frac{1}{2g^2} (\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ Z_\mu^0 Z_\nu^0 Z_\nu^0)) + g^2 c_w^2 (\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ Z_\mu^0 Z_\nu^0 Z_\nu^0)) \\ &+ g^2 S_w^2 \left(\frac{A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ Z_\mu^0 Z_\nu^0 Z_\nu^0)}{\partial^\mu A_\nu \partial^\nu} \right) - g^2 c_w S_w \left(\frac{(W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ Z_\mu^0 Z_\nu^0 Z_\nu^0)}{\partial^\mu A_\nu \partial^\nu A_\mu} \right) \end{aligned}$$

$$- \frac{1}{2\pi} \left(\partial H^\mu A H_\nu H \partial^\nu H A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ Z_\mu^0 Z_\nu^0 Z_\nu^0 H^\mu H_\nu H_\nu^\mu H_\nu^\mu) \right) + \left\| \frac{\frac{1}{2\pi} (2M^2 H^2 H^3)}{\frac{d^> em^c \gamma}{GUM_{Scw}^2}} \right\|^2 - \left\| \frac{2g_c^2 M_s^2}{\partial \Psi \Phi \zeta} \right\| - \langle \partial \lambda \rangle$$

$$\otimes \left\langle 2M \right\rangle \left\{ \frac{\partial \omega}{\partial \Delta \nabla \theta} \prod_{\pm}^{\dagger} \prod_j^i k \int_{\partial} \left| \frac{\phi_\mu^+ \phi_\nu^- \phi_\mu^- \phi_\nu^+}{\phi_\mu^+ \phi_\nu^- \phi_\mu^+ \phi_\nu^-} \right|^2 \left(\varphi \psi \omega \lambda_\mu^+ \varphi \psi \omega \lambda_\nu^- \varphi \psi \omega \lambda_\mu^- \varphi \psi \omega \lambda_\nu^+ \left| \frac{2\varphi \psi \omega \lambda}{\varphi \psi \omega \lambda} \right|_+^\mu \varphi \psi \omega \lambda_\nu^- \varphi \psi \omega \lambda_\mu^\pm \varphi \psi \omega \lambda_\nu^\pm \left| \frac{1}{2\pi \varphi \psi \omega \lambda} \right|_\mu^0 \varphi \psi \omega \lambda_\nu^0 \varphi \psi \omega \lambda_\mu^0 \varphi \psi \omega \lambda_\nu^0 \right) \right\} \left\| \frac{2\xi \eta}{\zeta \varepsilon \varepsilon} \frac{\delta \alpha}{\partial \sigma \rho} \frac{\partial \sigma \rho}{\Psi \Omega} \right\|^{1/2} \int_{\partial} \square = \mathcal{L}_{iggs} = \sqrt{\left(\partial^\mu \partial_\nu \partial^\nu \partial_\mu + \frac{1}{2i} g_1 B^\mu B_\nu B^\nu B_\mu + \frac{1}{2j} g_2 B^\mu B_\nu B^\nu B_\mu + \frac{1}{2i} g_1 W^\mu W_\nu W^\nu W_\mu + \frac{1}{2j} g_2 W^\mu W_\nu W^\nu W_\mu \right) - m_H^2 \phi^\dagger \phi - \frac{v^2}{2v^2}}$$



$$\partial_i \partial^j \partial_j \partial^i f^{ab\varphi} t_{ba\omega} t^{ab\varphi} f_{ba\omega}(x)$$

$$= \int_{\partial} \left\langle \frac{\frac{\partial^\theta \partial_\theta F_\sigma^\rho \gamma \beta}{\varepsilon \epsilon \theta \pi}}{\Delta \nabla} \right\rangle + \prod_b^a \lambda \prod_a^b \lambda H_{iggs}$$

$$- W^a W_b \widehat{W^b W_a} W_b^a W_a^b \widehat{W_a^b W} - \eta^\theta \widehat{\eta_\beta \eta}^{\sigma\mu} \alpha_\nu \Omega \eta / \mathbb{R}^4 \left\langle d\tau^2 \right\rangle$$

$$\mathcal{L}_{SM}(z) \equiv (a, b) \simeq (b, a)$$

$$\begin{aligned} &= -\frac{1}{2\pi} \partial^\mu \partial_\nu \partial^\nu \partial_\mu \widehat{\partial_\nu^\mu g_a^\mu g_b^\nu g_b^\nu} - g_s f^{ab} f_{ab} \partial^\mu \partial_\nu \partial_\mu \widehat{\partial_\nu^\mu g_a^\mu g_b^\nu g_b^\nu} - \frac{1}{4\pi} \overline{g_s^2 f^{cd} f_{cd} \partial^\mu \partial_\nu \partial_\mu \partial_\nu^\mu g_a^\mu g_b^\nu g_b^\nu} \\ &- \partial^\mu W_\mu \partial^\nu W_\nu - M^2 W_\mu^+ \widehat{W_\mu^-} W_\mu^- W_\nu^+ W_\nu^- W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- - \frac{1}{2\pi} \partial^\mu \partial_\nu \partial^\nu \partial_\mu \partial_\nu^\mu Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu - \frac{1}{2} \partial^\mu A_\nu \partial^\nu A_\mu - \frac{1}{2} c_m^2 M^2 Z_\nu^0 Z_0^\mu Z_0^\nu \\ &- ig c_w (\partial^\mu \partial_\nu \partial^\nu \partial_\mu \partial_\nu^\mu Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+)) - Z_\mu^0 (\partial^\mu \partial_\mu W_\mu^+ W_\mu^- W_\mu^+ W_\mu^-) + Z_\nu^0 (\partial^\nu \partial_\nu W_\nu^+ W_\nu^- W_\nu^+ W_\nu^-) \\ &- ig S_w (\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^+ W_\nu^-) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu) - A_\mu (\partial^\mu \partial_\mu W_\mu^+ W_\mu^- W_\mu^+ W_\mu^- Z_\mu^0 Z_0^\mu) + A_\nu (\partial^\nu \partial_\nu W_\nu^+ W_\nu^- W_\nu^+ W_\nu^- Z_\nu^0 Z_0^\nu) \\ &- \frac{1}{2g^2} (\overline{\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^+ W_\nu^-) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu}) + g^2 c_w^2 (\partial^\mu A_\nu \partial^\nu A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^+ W_\nu^-) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu) \\ &+ g^2 S_w^2 \left(\frac{A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^+ W_\nu^-) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu}{\partial^\mu A_\nu \partial^\nu A_\mu} \right) - g^2 c_w S_w \left(\frac{(W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^+ W_\nu^-) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu}{\partial^\mu A_\nu \partial^\nu A_\mu} \right) \end{aligned}$$

$$- \frac{1}{2\pi} (\overline{\partial H^\mu A H_\nu H \partial^\nu H A_\mu (W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ W_\mu^+ W_\nu^- W_\mu^+ W_\nu^-) Z_\mu^0 Z_\nu^0 Z_0^\mu Z_0^\nu H^\mu H_\nu H_\nu H_\nu}) + \left\| \frac{\frac{1}{2\pi} (2M^2 H^2 H^3)}{\frac{d^{\gamma} em^{\epsilon} \gamma}{GUM_{scw}^2}} \right\|^2 - \left| \frac{2g_c^2 M_s^2}{\partial \Psi \Phi \zeta} \right| \langle \partial \lambda \rangle$$

$$\otimes \left(\frac{\frac{\partial \omega}{\partial \Delta \nabla \theta}}{\Pi_{\pm}^{\dagger} \infty \int_{\partial} \left| \frac{\phi_{\mu}^+ \phi_{\nu}^- \phi_{\mu}^+ \phi_{\nu}^+}{\phi_{\mu}^+ \phi_{\nu}^+ \phi_{\mu}^+ \phi_{\nu}^+} \right|^2} \right) \left(\varphi \psi \omega \lambda_{\mu}^+ \varphi \psi \omega \lambda_{\nu}^- \varphi \psi \omega \lambda_{\mu}^- \varphi \psi \omega \lambda_{\nu}^+ \left| \frac{2\varphi \psi \omega \lambda}{\varphi \psi \omega \lambda} \right|_{\mu}^{\mu} \varphi \psi \omega \lambda_{\nu}^{\nu} \varphi \psi \omega \lambda_{\mu}^{\mu} \varphi \psi \omega \lambda_{\nu}^{\nu} \left| \frac{1}{\varphi \psi \omega \lambda} \right|_{\mu}^0 \varphi \psi \omega \lambda_{\nu}^{\nu} \varphi \psi \omega \lambda_{\mu}^{\mu} \varphi \psi \omega \lambda_{\nu}^{\nu} \right)$$

$$\int_{\partial} \left\| \frac{\frac{2\xi \eta}{\zeta \epsilon \epsilon}}{\frac{\delta \alpha}{\sigma \sigma \rho}} \right\|^{1/2} \Psi \Omega \quad \square = \mathcal{L}_{Higgs} =$$

$$\sqrt{\left(\partial^\mu \partial_\nu \partial^\nu \partial_\mu + \frac{1}{2i} g_1 B^\mu B_\nu B^\nu B_\mu + \frac{1}{2j} g_2 B^\mu B_\nu B^\nu B_\mu + \frac{1}{2i} g_1 W^\mu W_\nu W^\nu W_\mu + \frac{1}{2j} g_2 W^\mu W_\nu W^\nu W_\mu \right) - m_H^2 \phi' \phi - \frac{v^2}{2v^2}}$$



$$\partial_i \partial^j \partial_j \partial^i f^{ab\varphi} t_{ba\omega} t^{ab\varphi} f_{ba\omega}(x)$$

$$= \int \left\langle \frac{\partial^\theta \partial_\theta F_\sigma^\rho \gamma \beta}{\frac{\varepsilon \in \theta \pi}{\Delta \nabla} \tau} \right\rangle + \prod_b^a \lambda \prod_a^b \lambda H_{igg_s}$$

$$- W^a W_b \widehat{W^b W_a} W_b^a W_a^b \widehat{W_a^b W_b^a} W - \widehat{\eta^\theta \eta_\beta \eta}^{\sigma\mu} \alpha_{\phi\nu} \Omega \eta / \mathbb{R}^4 \left\langle d\tau^2 \right\rangle$$

$$\begin{aligned} \langle \mathcal{L}_{SM} \rangle &= \frac{1}{2} \partial_\nu g_\mu^a \partial_\mu g_\nu^b - g_s f^{abc} \partial_\mu g_\nu^a \widehat{g_\nu^b g_\nu^c} \partial_\nu g_\mu^a g_\mu^b g_\mu^c - \frac{1}{4} g_s^2 f^{abc} f^{ade} g_\mu^b g_\mu^c g_\mu^d g_\nu^e g_\nu^f g_\nu^g - \overline{\partial_\nu W_\mu^+ \partial_\nu W_\mu^- \partial_\mu W_\nu^+ \partial_\mu W_\nu^-} - M^2 W_\mu^+ \widehat{W_\mu^-} W_\nu^+ W_\nu^- \\ &- \frac{1}{2} \partial_\nu Z_\mu^0 \partial_\mu Z_\nu^0 - \frac{1}{2} c_\omega^2 M^2 Z_\mu^0 Z_\nu^0 - \frac{1}{2} \partial_\mu \mathcal{A}_\nu \partial_\nu \mathcal{A}_\mu - ig c_\omega (\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+) - Z_\mu^0 Z_\nu^0 (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- - \partial_\mu W_\nu^+ \partial_\mu W_\nu^-)) \\ &- ig s_\omega (\partial_\mu \mathcal{A}_\nu \partial_\nu \mathcal{A}_\mu (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+) - \mathcal{A}_\mu (\partial_\nu W_\mu^+ \partial_\nu W_\mu^- - \partial_\mu W_\nu^+ \partial_\mu W_\nu^-) - \mathcal{A}_\nu (\partial_\mu W_\mu^+ \partial_\mu W_\mu^- - \partial_\mu W_\nu^+ \partial_\mu W_\nu^-)) - \frac{1}{2g^2} W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ \\ &+ g^2 c_\omega^2 (Z_\mu^0 W_\mu^+ W_\nu^- - Z_\nu^0 W_\nu^+ W_\mu^-) + g^2 s_\omega^2 (\mathcal{A}_\mu W_\mu^+ W_\nu^- - \mathcal{A}_\nu W_\nu^+ W_\mu^-) + g^2 c_\omega s_\omega (\mathcal{A}_\mu Z_\mu^0 (W_\mu^+ W_\nu^-) - \mathcal{A}_\nu Z_\nu^0 (W_\nu^+ W_\mu^-) - 2\mathcal{A}_\mu W_\mu^+ W_\nu^- Z_\mu^0 \mathcal{A}_\nu W_\nu^+ W_\mu^- Z_\nu^0) \\ &- \frac{1}{2} \partial_\mu \mathcal{H} \partial_\nu \mathcal{H} - 2M^2 \alpha_h \mathcal{H}^2 - \partial_\mu \phi^+ \partial_\nu \phi^- \partial_\nu \phi^+ \partial_\mu \phi^- - \frac{1}{2} \partial_\mu \phi^0 \partial_\nu \phi^0 \partial_\nu \phi^0 \partial_\mu \phi^0 \\ &- \beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g\mathcal{H}} + \frac{1}{2} (\mathcal{H}^2 + \phi^0 \phi^+ \phi^0 \phi^- + 2\phi^0 \phi^+ \phi^0 \phi^-) \right) + \frac{2M^4}{g^2 \alpha_h} - g \alpha_h M (\mathcal{H}^3 + \mathcal{H} \phi^0 \phi^0 + 2\mathcal{H} \phi^+ \phi^-) - 1/8 g^2 \alpha_h (\mathcal{H}^4 + (\phi^0)^4) \\ &+ 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4\mathcal{H}^2 \phi^+ \phi^- + 2(\phi^0)^2 \mathcal{H}^2 - g M W_\mu^+ W_\nu^- W_\nu^+ W_\mu^- \mathcal{H} - \frac{1}{2gM} c_\omega^2 Z_\mu^0 Z_\nu^0 \mathcal{H} \\ &- \frac{1}{2ig} (W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0) - W_\nu^+ (\phi^0 \partial_\nu \phi^- - \phi^- \partial_\nu \phi^0) - W_\nu^- (\phi^0 \partial_\nu \phi^+ - \phi^+ \partial_\nu \phi^0)) \\ &- \frac{1}{2g} (W_\mu^+ (\mathcal{H} \partial_\mu \phi^- - \phi^- \partial_\mu \mathcal{H}) - W_\mu^- (\mathcal{H} \partial_\mu \phi^+ - \phi^+ \partial_\mu \mathcal{H})) W_\nu^+ (\mathcal{H} \partial_\nu \phi^- - \phi^- \partial_\nu \mathcal{H}) - W_\nu^- (\mathcal{H} \partial_\nu \phi^+ - \phi^+ \partial_\nu \mathcal{H}) \\ &+ \frac{1}{2g} \frac{1}{c_\omega} (Z_\mu^0 (\mathcal{H} \partial_\mu \phi^0 - \phi^0 \partial_\mu \mathcal{H}) - Z_\nu^0 (\mathcal{H} \partial_\nu \phi^0 - \phi^0 \partial_\nu \mathcal{H})) + M \left(\frac{\partial^{\mu\nu} \phi^+ \partial_{\mu\nu} \phi^+}{c_\omega Z_\mu^0 \partial_\mu \phi^0} + W_\mu^+ \partial_\mu \phi^- + W_\mu^- \partial_\mu \phi^+ \right) \\ &- \frac{ig s_\omega^2}{c_\omega} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) ig s_\omega M \mathcal{A}_\mu (W_\mu^+ \phi^- + W_\mu^- \phi^+) - ig - \frac{d^4 c_\omega^2}{dc_\omega} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + ig s_\omega \mathcal{A}_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) \\ &- 1/4 g^2 W_\mu^+ W_\nu^- (\mathcal{H}^2 + (\phi^0)^2 + 2\phi^+ \phi^-) \end{aligned}$$

$$\frac{1}{8} g^2 \left(\frac{c_\omega^2 Z_\mu^0 Z_\nu^0}{(\mathcal{H}^2 + \phi^0)^2} - \frac{1}{d_j^2} \right)$$

$$\left((2s_\omega^2 - 1)^2 \phi^+ \phi^- - \frac{\partial}{c_\omega Z_\mu^0 \phi^0} (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \frac{\partial}{c_\omega Z_\mu^0} \mathcal{H} (W_\mu^+ \phi^- + W_\mu^- \phi^+) + \frac{1}{2g^2} s_\omega \mathcal{A}_\mu \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) + \frac{1}{2ig^2} s_\omega \mathcal{A}_\mu \mathcal{H} (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \frac{g^2 s_\omega}{c_\omega} (2c_\omega^2 - 1) Z_\mu^0 \mathcal{A}_\mu \phi^+ \phi^- - g^2 s_\omega^2 \mathcal{A}_\mu \mathcal{A}_\nu \phi^+ \phi^- + \frac{1}{2ig_s} \lambda_{ij}^a \left(\begin{matrix} \rightarrow \varphi^\mu \rightarrow \\ q\sigma^i \end{matrix} \right) g_\mu^a \begin{matrix} \rightarrow \\ e^\lambda \end{matrix} (\partial\varphi + m_\phi^2) e^\lambda \begin{matrix} \rightarrow \\ \nu^\lambda \end{matrix} \partial\varphi + m_\phi^2 \nu^\lambda \begin{matrix} \rightarrow \\ \mu^\nu \nu^\lambda \end{matrix} \right) \right) m_\mu^\lambda - \mu_j^\lambda$$



$$\begin{aligned}
& (\varphi\partial + m_d^\lambda)d_j^\lambda + ig_{s_\omega}\mathcal{A}_\mu \left(-\left(\overset{\rightarrow}{e^\lambda}\varphi^\mu e^\lambda\right) + \frac{2}{3\left(\overset{\rightarrow}{\mu_j^\lambda}\varphi^\mu\mu_j^\lambda\right)} - \frac{1}{3\left(\overset{\rightarrow}{d^\lambda}\varphi^\mu d^\lambda\right)} + \frac{ig}{4c_\omega Z_\mu^0\left(\overset{\rightarrow}{v^\lambda}\varphi^\mu(1+\varphi^5)v^\lambda\right)} \right. \\
& + \left(\overset{\rightarrow}{e^\lambda}\varphi^\mu(4s_\omega^2 - 1 - \varphi^5)e^\lambda\right) + \left(\overset{\rightarrow}{d_j^\lambda}\varphi^\mu\left(\frac{4}{3s_\omega^2} - 1 - \varphi^5\right)d_j^\lambda\right) + \left(\overset{\rightarrow}{\mu_j^\lambda}\varphi^\mu\left(1 - \frac{8}{3s_\omega^2} + \varphi^5\right)\mu_j^\lambda\right) \\
& \left. + \left(\frac{ig}{\sqrt[2]{2}W_\mu^+\left(\overset{\rightarrow}{v^\lambda}\varphi^\mu(1+\varphi^5)\right)U^{lep^k}_\xi e^k}\right) + \left(\overset{\rightarrow}{\mu_j^\lambda}\varphi^\mu(1+\varphi^5)c_{k\lambda}d_j^k\right) \right) \\
& + \frac{ig}{\sqrt[2]{2}W_\mu^-\left(\overset{\rightarrow}{e^k}U^{lep^\dagger}_\xi \frac{\kappa\varphi}{\rho\omega} - \triangle \Pi \oplus \oplus \oplus \gamma \otimes \frac{\prod_{\mu}^{ijk}}{\otimes \sigma} \Omega\Psi\Phi\Delta(1+\varphi^5)v^\lambda + \overset{\rightarrow}{d_j^k} C_{*\kappa}^{\dagger\lambda}\varphi^{\mu\kappa\eta}(1+\varphi^5)\mu_j^\lambda\right)} \\
& + \frac{ig}{2M^2\sqrt{2}\phi^+\left(-m_c^k\left(\overset{\rightarrow}{v^\lambda}U^{lep^k}_\xi e^k(1-\varphi^5)\epsilon^k\right) + m_\mu^\lambda \overset{\rightarrow}{U^{lep^\dagger}}(1+\varphi^5)\epsilon^k\right)} \\
& + \frac{ig}{2M^2\sqrt{2}\phi^-\left(m_c^k\left(\overset{\rightarrow}{v^\lambda}U^{lep^k}_\xi e^k(1-\varphi^5)\epsilon^k\right) \pm m_\mu^\lambda \overset{\rightarrow}{U^{lep^\dagger}}(1+\varphi^5)\epsilon^k\right)} - \frac{g}{2m_v^\lambda M}\left(\overset{\rightarrow}{v^\lambda}\right) \\
& - \frac{\frac{g}{2m_c^\lambda}}{\frac{M\mathcal{H}\left(\overset{\rightarrow}{e^\lambda}\right)}{1}} + \frac{\frac{ig}{2m_v^\lambda}}{M\phi^0\left(\overset{\rightarrow}{v^\lambda}\gamma^5 v^\lambda\right)} - \frac{\frac{ig}{2m_c^\lambda}}{M\phi^0\left(\overset{\rightarrow}{e^\lambda}\gamma^5 e^\lambda\right)} - \frac{1}{4\overset{\rightarrow}{v^\lambda}M_{\lambda\kappa}^R(1-\gamma_5)\overset{\rightarrow}{v^\kappa}} \\
& + \frac{ig}{2M^2\sqrt{2}\phi^+\left(-m_d^k\left(\overset{\rightarrow}{\mu_j^\lambda}c_{\lambda\kappa}(1-\varphi^5)d_j^k\right) + m_d^k\left(\overset{\rightarrow}{\mu_j^\lambda}c_{\lambda\kappa}(1-\varphi^5)d_j^k\right)\right)} \\
& + \frac{ig}{2M^2\sqrt{2}\phi^-\left(m_d^\lambda\left(\overset{\rightarrow}{d_j^\lambda}C_{\lambda\kappa}^\dagger\wedge_{\theta^*}(1+\varphi^5)\mu_j^k\right) \pm m_d^\lambda\left(\overset{\rightarrow}{d_j^\lambda}C_{\lambda\kappa}^\dagger\wedge_{\nu^*}(1+\varphi^5)\mu_j^k\right)\right)} - \frac{g}{2m_\mu^\lambda}M\mathcal{H}\left(\overset{\rightarrow}{\mu_j^\lambda}\right) \\
& - \frac{g}{2m_d^\lambda M}\left(\overset{\rightarrow}{d_j^\lambda}\right) + \frac{\frac{ig}{2m_\mu^\lambda}}{M}\phi^0\left(\overset{\rightarrow}{\mu_j^\lambda}\gamma^5\mu_j^\lambda\right) - \frac{\frac{ig}{2m_d^\lambda}}{M}\phi^0\left(\overset{\rightarrow}{d_j^\lambda}\gamma^5 d_j^\lambda\right) + \overset{\rightarrow}{G^a}\partial^2 G^a + g_s f^{abc}\partial_\mu \overset{\rightarrow}{G^b}g_\mu^c \\
& + \overset{\rightarrow}{\alpha^+}(\partial^2 - M^2)\alpha^+ + \overset{\rightarrow}{\alpha^-}(\partial^2 - M^2)\alpha^- + \overset{\rightarrow}{\alpha^0}\left(\partial^2 - \frac{M^2}{c_\omega^2}\right)\alpha^0 + \vec{b}\partial^2 b + igc_\omega W_\mu^+\left(\partial_\mu \overset{\rightarrow}{\alpha^0} \overset{\rightarrow}{\alpha^-} - \partial_\mu \overset{\rightarrow}{\alpha^-} \overset{\rightarrow}{\alpha^0}\right) \\
& + ig s_\omega W_\mu^+\left(\partial_\mu \overset{\rightarrow}{\alpha^-} \overset{\rightarrow}{\alpha^+} - \partial_\mu \overset{\rightarrow}{\alpha^+} \overset{\rightarrow}{\alpha^-}\right) + igc_\omega W_\mu^-\left(\partial_\mu \overset{\rightarrow}{\alpha^+} \overset{\rightarrow}{\alpha^0} - \partial_\mu \overset{\rightarrow}{\alpha^0} \overset{\rightarrow}{\alpha^+}\right) + ig s_\omega W_\mu^-\left(\partial_\mu \overset{\rightarrow}{\alpha^+} \overset{\rightarrow}{\alpha^0} - \partial_\mu \overset{\rightarrow}{\alpha^0} \overset{\rightarrow}{\alpha^+}\right) \\
& + igc_\omega Z_\mu^0\left(\partial_\mu \overset{\rightarrow}{\alpha^+} \overset{\rightarrow}{\alpha^-} - \partial_\mu \overset{\rightarrow}{\alpha^-} \overset{\rightarrow}{\alpha^+}\right) + ig s_\omega \mathcal{A}_\mu\left(\partial_\mu \overset{\rightarrow}{\alpha^+} \overset{\rightarrow}{\alpha^-} - \partial_\mu \overset{\rightarrow}{\alpha^-} \overset{\rightarrow}{\alpha^+}\right) \\
& - 1 \\
& /2gM\left\langle \frac{\overset{\rightarrow}{\alpha^+}\mathcal{H}\hbar\mathbb{R}^4}{h} + \overset{\rightarrow}{\alpha^-}\mathcal{H} + 1 - \frac{2c_\omega^2}{2c_\omega igM\left(\overset{\rightarrow}{\alpha^+}a^0\phi^+ - \overset{\rightarrow}{\alpha^-}a^0\phi^-\right)} + \frac{1}{2c_\omega igM\left(\overset{\rightarrow}{\alpha^0}a^-\phi^+ - \overset{\rightarrow}{\alpha^0}a^+\phi^-\right)} \right\rangle \\
& + igMs_\omega\left(\overset{\rightarrow}{\alpha^0}a^-\phi^+ - \overset{\rightarrow}{\alpha^0}a^+\phi^-\right) + 1/2igM\left(\overset{\rightarrow}{\alpha^+}a^+\phi^0 - \overset{\rightarrow}{\alpha^-}a^-\phi^0\right)
\end{aligned}$$



$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \quad \mathcal{L}_{SBS} = (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) - V(\Phi)$$

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \quad |\Phi|^2 = \Phi^\dagger \Phi = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}$$

$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Phi(x) = \frac{1}{\sqrt{2}} e^{i \frac{\vec{\xi}(x) \cdot \vec{\tau}}{v}} \begin{pmatrix} 0 \\ v + \mathbf{h}(x) \end{pmatrix}$$

$$U(\xi) = e^{-i \frac{\vec{\xi}(x) \cdot \vec{\tau}}{v}}$$

$$\Phi' = U(\xi) \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \mathbf{h}(x) \end{pmatrix}$$

$$\left(\frac{\vec{\tau} \vec{W}'_\mu}{2} \right) = U(\xi) \left(\frac{\vec{\tau} \vec{W}_\mu}{2} \right) U^{-1}(\xi) - \frac{i}{g} (\partial_\mu U(\xi)) U^{-1}(\xi)$$

$$\mathbf{B}'_\mu = \mathbf{B}_\mu$$

$$\mathcal{L} = \mathcal{L}_{bos.} + \mathcal{L}_{ferm.} + \mathcal{L}_{SBS}$$

$$(\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) = \frac{v^2}{8} [g^2 (W_{1\mu}^2 + W_{2\mu}^2) + (gW_{3\mu} - g'B_\mu)^2]$$



$$\begin{aligned}
W_\mu^\pm &= \frac{1}{\sqrt{2}}(W_\mu^1 \mp W_\mu^2) \\
Z_\mu &= \cos \theta_W W_\mu^3 - \sin \theta_W B_\mu \\
A_\mu &= \sin \theta_W W_\mu^3 + \cos \theta_W B_\mu \quad \tan \theta_W \equiv \frac{g'}{g}
\end{aligned}$$

$$M_W = \frac{1}{2} g v$$

$$M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$$

$$g = \frac{e}{\sin \theta_W}$$

$$g' = \frac{e}{\cos \theta_W} \quad m_H^2 = 2\lambda v^2 \quad \mu \rightarrow \nu_\mu \bar{\nu}_e e$$

$$v = (\sqrt{2} G_F)^{-\frac{1}{2}}$$

$$\mathcal{L}_{YW} = \lambda_e \bar{\ell}_L \Phi e_R + \lambda_u \bar{q}_L \tilde{\Phi} u_R + \lambda_d \bar{q}_L \Phi d_R + \text{h.c.}$$

$$\ell_L = \begin{pmatrix} e \\ \nu_e \end{pmatrix}_L, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$\ell'_L = U(\xi) \ell_L; \quad e'_R = e_R$$

$$q'_L = U(\xi) q_L; \quad u'_R = u_R; \quad d'_R = d$$

$$m_e = \lambda_e \frac{v}{\sqrt{2}}$$

$$m_u = \lambda_u \frac{v}{\sqrt{2}}$$

$$m_d = \lambda_d \frac{v}{\sqrt{2}}$$

$$\begin{aligned}
\mathcal{H}_c &\equiv \frac{1}{2\pi} \prod_i^k(x) + \prod_k^i(x) \partial^i \partial_k A^k A_i(x) + \frac{1}{4\pi} F^{ki}(x) F_{ik}(x) \\
&= H_c \left[\prod_i^k d^3 \chi \left[\frac{1}{2\pi} \prod_i^k(x) + \prod_k^i(x) \partial^i \partial_k A^k A_i(x) + \frac{1}{4\pi} F^{ki}(x) F_{ik}(x) \right] \right] \\
&= \overline{H^\rho H_c H^c H_\rho H_c^\rho H_\rho^c \varrho} \equiv \prod_i^k \frac{d^3 \chi^\lambda}{\partial \hbar} \partial \mathcal{U} \partial \Omega \partial^2 \mathbb{R}^4 / \partial G_\varepsilon \partial R_e \| d\lambda \triangleq d\phi \|
\end{aligned}$$

$$\begin{aligned}
\mathcal{H}_{ab} &\equiv \frac{1}{2\pi} \prod_i^k(x) + \prod_k^i(x) \partial^i \partial_k A^k A_i(x) + \frac{1}{4\pi} F^{ki}(x) F_{ik}(x) \\
&= H_c \left[\prod_i^k d^3 \chi \left[\frac{1}{2\pi} \prod_i^k(x) + \prod_k^i(x) \partial^i \partial_k A^k A_i(x) + \frac{1}{4\pi} F^{ki}(x) F_{ik}(x) \right] \right] \\
&= \overline{H^\rho H_c H^c H_\rho H_c^\rho H_\rho^c \varrho} \equiv \prod_i^k \frac{d^3 \chi^\lambda}{\partial \hbar} \partial \mathcal{U} \partial \Omega \partial^2 \mathbb{R}^4 / \partial G_\varepsilon \partial R_e \| d\lambda \triangleq d\phi \|
\end{aligned}$$

$$\begin{aligned}
\mathcal{H}_{bc} &\equiv \frac{1}{2\pi} \prod_i^k(x) + \prod_k^i(x) \partial^i \partial_k A^k A_i(x) + \frac{1}{4\pi} F^{ki}(x) F_{ik}(x) \\
&= H_c \left[\prod_i^k d^3 \chi \left[\frac{1}{2\pi} \prod_i^k(x) + \prod_k^i(x) \partial^i \partial_k A^k A_i(x) + \frac{1}{4\pi} F^{ki}(x) F_{ik}(x) \right] \right] \\
&= \overline{H^\rho H_c H^c H_\rho H_c^\rho H_\rho^c \varrho} \equiv \prod_i^k \frac{d^3 \chi^\lambda}{\partial \hbar} \partial \mathcal{U} \partial \Omega \partial^2 \mathbb{R}^4 / \partial G_\varepsilon \partial R_e \| d\lambda \triangleq d\phi \|
\end{aligned}$$

$$\begin{aligned}
\mathcal{H}_c &\equiv \frac{1}{2\pi} \prod_i^k(y) + \prod_k^i(y) \partial^i \partial_k A^k A_i(y) + \frac{1}{4\pi} F^{ki}(y) F_{ik}(y) \\
&= H_c \left[\prod_i^k d^3 \gamma \left[\frac{1}{2\pi} \prod_i^k(y) + \prod_k^i(y) \partial^i \partial_k A^k A_i(y) + \frac{1}{4\pi} F^{ki}(y) F_{ik}(y) \right] \right] \\
&= \overline{H^\rho H_c H^c H_\rho H_c^\rho H_\rho^c \varrho} \equiv \prod_i^k \frac{d^3 \gamma^\lambda}{\partial \hbar} \partial \mathcal{U} \partial \Omega \partial^2 \mathbb{R}^4 / \partial G_\varepsilon \partial R_e \| d\lambda \triangleq d\phi \|
\end{aligned}$$



$$\begin{aligned}
\mathcal{H}_{ab} &\equiv \frac{1}{2\pi} \prod_i^k (y) + \prod_k^i (y) \partial^i \partial_k A^k A_i(y) + \frac{1}{4\pi} F^{ki}(y) F_{ik}(y) \\
&= H_c \left(\prod_i^k d^3 \gamma \left[\frac{1}{2\pi} \prod_i^k (y) + \prod_k^i (y) \partial^i \partial_k A^k A_i(y) + \frac{1}{4\pi} F^{ki}(y) F_{ik}(y) \right] \right) \\
&= \overline{H^\rho H_c H^c H_\rho H_c^\rho H_\rho^c \varrho} \equiv \left(\prod_i^k \frac{d^3 \gamma^\lambda}{\partial \hbar} \partial \mathcal{U} \partial \Omega \partial^2 \mathbb{R}^4 / \partial G_\varepsilon \partial R_e \right) \| d\lambda \triangleq d\phi \|
\end{aligned}$$

$$\begin{aligned}
\mathcal{H}_{bc} &\equiv \frac{1}{2\pi} \prod_i^k (y) + \prod_k^i (y) \partial^i \partial_k A^k A_i(y) + \frac{1}{4\pi} F^{ki}(y) F_{ik}(y) \\
&= H_c \left(\prod_i^k d^3 \gamma \left[\frac{1}{2\pi} \prod_i^k (y) + \prod_k^i (y) \partial^i \partial_k A^k A_i(y) + \frac{1}{4\pi} F^{ki}(y) F_{ik}(y) \right] \right) \\
&= \overline{H^\rho H_c H^c H_\rho H_c^\rho H_\rho^c \varrho} \equiv \left(\prod_i^k \frac{d^3 \gamma^\lambda}{\partial \hbar} \partial \mathcal{U} \partial \Omega \partial^2 \mathbb{R}^4 / \partial G_\varepsilon \partial R_e \right) \| d\lambda \triangleq d\phi \|
\end{aligned}$$

$$\begin{aligned}
\mathcal{H}_c &\equiv \frac{1}{2\pi} \prod_i^k (z) + \prod_k^i (z) \partial^i \partial_k A^k A_i(z) + \frac{1}{4\pi} F^{ki}(z) F_{ik}(z) \\
&= H_c \left(\prod_i^k d^3 z \left[\frac{1}{2\pi} \prod_i^k (z) + \prod_k^i (z) \partial^i \partial_k A^k A_i(z) + \frac{1}{4\pi} F^{ki}(z) F_{ik}(z) \right] \right) \\
&= \overline{H^\rho H_c H^c H_\rho H_c^\rho H_\rho^c \varrho} \equiv \left(\prod_i^k \frac{d^3 z^\lambda}{\partial \hbar} \partial \mathcal{U} \partial \Omega \partial^2 \mathbb{R}^4 / \partial G_\varepsilon \partial R_e \right) \| d\lambda \triangleq d\phi \|
\end{aligned}$$

$$\begin{aligned}
\mathcal{H}_{ab} &\equiv \frac{1}{2\pi} \prod_i^k (z) + \prod_k^i (z) \partial^i \partial_k A^k A_i(z) + \frac{1}{4\pi} F^{ki}(z) F_{ik}(z) \\
&= H_c \left(\prod_i^k d^3 z \left[\frac{1}{2\pi} \prod_i^k (z) + \prod_k^i (z) \partial^i \partial_k A^k A_i(z) + \frac{1}{4\pi} F^{ki}(z) F_{ik}(z) \right] \right) \\
&= \overline{H^\rho H_c H^c H_\rho H_c^\rho H_\rho^c \varrho} \equiv \left(\prod_i^k \frac{d^3 z^\lambda}{\partial \hbar} \partial \mathcal{U} \partial \Omega \partial^2 \mathbb{R}^4 / \partial G_\varepsilon \partial R_e \right) \| d\lambda \triangleq d\phi \|
\end{aligned}$$

$$\begin{aligned}
& \left\| \{B(x, k), C(x, k)\} / \Phi \Psi \kappa \varphi \theta \right. \\
& = \delta \prod_b^a(x, k) \lambda \phi \int_{\sigma}^{\varphi} d^3 z [\delta_b^a B(x, k) \lambda \phi / \delta_b^a A_{ab}(x, k) \lambda \phi \\
& / \delta_b^a C(x, k) \lambda \phi \prod_a^b(x, k) \lambda \phi \int_{\sigma}^{\varphi} d^3 z [\delta_a^b B(x, k) \lambda \phi / \delta_a^b C(x, k) \lambda \phi \\
& / \delta_a^b A_{ba}(x, k) \lambda \phi \prod_{ba}^{ab}(x, k) \lambda \phi \left. \right\|^{-1/2}
\end{aligned}$$

$$\begin{aligned}
& \left\| \{B(y, k), C(y, k)\} / \Phi \Psi \kappa \varphi \theta \right. \\
& = \delta \prod_b^a(y, k) \lambda \phi \int_{\sigma}^{\varphi} d^3 z [\delta_b^a B(y, k) \lambda \phi / \delta_b^a A_{ab}(y, k) \lambda \phi \\
& / \delta_b^a C(y, k) \lambda \phi \prod_a^b(y, k) \lambda \phi \int_{\sigma}^{\varphi} d^3 z [\delta_a^b B(y, k) \lambda \phi / \delta_a^b C(y, k) \lambda \phi \\
& / \delta_a^b A_{ba}(y, k) \lambda \phi \prod_{ba}^{ab}(y, k) \lambda \phi \left. \right\|^{-1/2}
\end{aligned}$$

$$\begin{aligned}
& \left\| \{B(z, k), C(z, k)\} / \Phi \Psi \kappa \varphi \theta \right. \\
& = \delta \prod_b^a(z, k) \lambda \phi \int_{\sigma}^{\varphi} d^3 z [\delta_b^a B(z, k) \lambda \phi / \delta_b^a A_{ab}(z, k) \lambda \phi \\
& / \delta_b^a C(z, k) \lambda \phi \prod_a^b(z, k) \lambda \phi \int_{\sigma}^{\varphi} d^3 z [\delta_a^b B(z, k) \lambda \phi / \delta_a^b C(z, k) \lambda \phi \\
& / \delta_a^b A_{ba}(z, k) \lambda \phi \prod_{ba}^{ab}(z, k) \lambda \phi \left. \right\|^{-1/2}
\end{aligned}$$

$$\left\| \frac{u\sqrt{2}}{d\sqrt{2}} \right. = \left[\phi' M'^d d'_R + \phi M^u u_L + \phi' \widetilde{M'^u} u'_L + \phi M^d d_L + \phi' M'^d d'_L \overline{\left(\frac{u'_L}{d'_L} \right) \left(\frac{u_L}{d_L} \right) \left(\frac{u'_R}{d'_R} \right) \left(\frac{u_R}{d_R} \right)} \right]$$

$$/ \tau^2 = \xi_{\lambda\Omega\psi}^{\sigma\zeta} \sum \int \int \int \int \hbar \Phi \mathbb{K} \mathbb{Z} \mathbb{K} \mathbb{D} \mathbb{K} \mathbb{H} \mathbb{K} \mathbb{Z} \zeta \pi m c^{\mathbb{R}^4} \left\| (u'_L d'_L u'_R d'_R) \phi M^u u_R + \phi' M'^u u'_R + \phi M^d d_R \right.$$

