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FORMALIZACIÓN MATEMÁTICA Y EN FÍSICA DE PARTÍCULAS, EN RELACIÓN A LA BRECHA DE MASA Y LA CURVATURA GEOMÉTRICA DE LOS CAMPOS CUÁNTICOS

MATHEMATICAL FORMALIZATION AND PARTICLE PHYSICS, IN RELATION TO THE MASS GAP AND THE GEOMETRIC CURVATURE OF QUANTUM FIELDS

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Formalización Matemática y en Física de Partículas, en Relación a la Brecha de Masa y la Curvatura Geométrica de los Campos Cuánticos

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RESUMEN

En recientes manuscritos, este investigador ha formulado alternativas de solución al Problema del Milenio de Yang – Mills, intentando unificar, desde la teoría cuántica de campos hasta las teorías de la relatividad general y especial respectivamente, sin desprendernos de cuestiones tan elementales como las representaciones en *algebra de Lie*, de cuyo resultado, se ha concluido en lo fundamental, que toda partícula o antipartícula, con masa o sin masa, según sea el caso, supera el estado de vacío, demostrando una brecha de masa positiva, esto es, cuando se aproxima o supera la velocidad de la luz, deformando así, el campo cuántico en el que interactúa, repercutiendo en las trayectorias de las partículas o antipartículas circundantes. Ahora bien, el propósito de esta investigación, es proponer modelos hipotéticos para campos de Yang – Mills abelianos y no abelianos, grupos de gauge y Lie usando distintos operadores para espacios en cuatro dimensiones $\mathbb{R}^4$, a través de los cuales, quedará demostrado, que la brecha de masa de una partícula o antipartícula con o sin masa, siempre arroja un valor positivo superior a cero.

Palabras clave: física de partículas, campos de gauge, teorías de calibre, grupos de Lie, libertad asintónica, dimensión $\mathbb{R}^4$, campos de Yang Mills abelianos y no abelianos, superficie espacial, superficie temporal, operador de Casimir, transformación de Lorentz, ecuación de Callan-Symanzik, integral de trayectoria, representación de espinores.

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Mathematical Formalization and Particle Physics, in Relation to the Mass Gap and the Geometric Curvature of Quantum Fields

ABSTRACT

In recent manuscripts, this researcher has formulated alternative solutions to the Yang-Mills Millennium Problem, trying to unify, from quantum field theory to the theories of general and special relativity respectively, without detaching ourselves from such elementary questions as representations in Lie algebra, from the result of which it has been concluded in the main, that every particle or antiparticle, with or without mass, as the case may be, exceeds the vacuum state, demonstrating a positive mass gap, that is, when it approaches or exceeds the speed of light, thus deforming the quantum field in which it interacts and affecting the trajectories of the surrounding particles or antiparticles. Now, the purpose of this research is to propose hypothetical models for abelian and non-abelian Yang-Mills fields, gauge and lie groups using different operators for spaces in four dimensions $\mathbb{R}^4$, through which it will be demonstrated that the mass gap of a particle or antiparticle with or without mass, it always yields a positive value greater than zero.

Keywords: particle physics, gauge fields, caliber theories, Lie groups, asymptonic freedom, $\mathbb{R}^4$ dimension, abelian and non-abelian Yang Mills fields, spatial surface, time surface, Casimir operator, Lorentz transformation, Callan-Symanzik equation, trajectory integral, spinor representation.

INTRODUCCIÓN

Preliminarmente, cabe precisar que se trabajará en campos cuánticos en dimensión $\mathbb{R}^4$, en estructuras de gauge específicas, a propósito de sus transformaciones, con trayectorias orbitales arbitrarias, utilizando distintas métricas vectoriales, espaciales, temporales y operadores cuánticos de campo, todo esto, en superficies de espacio – tiempo cuatridimensionales, por lo que, no solamente se recurrirá a la teoría cuántica de campos de Yang – Mills, sino también a las teorías de la relatividad general y especial y otras leyes propias de la física y de las matemáticas puras, todo esto, con la finalidad de demostrar, que la brecha de masa, en un campo de Yang – Mills, esto es, cuando una partícula o antipartícula con o sin masa, según sea el caso, supera el cero absoluto, arroja un salto de energía cuyo resultado siempre es positivo. En el apartado de Resultados y Discusión, se desplegarán los sistemas matemáticos y de la física de partículas correspondientes que sostienen la hipótesis contenida en este Artículo Científico y en definitiva, en los trabajos que anteceden a éste.

Para estos efectos, se han diseñado campos cuánticos hipotéticos, con superficies espaciales y temporales arbitrarias, todo esto, con la finalidad, de demostrar la existencia de la brecha de masa positiva y paralelamente, la curvatura geométrica de los campos cuánticos y los agujeros deformantes de los referidos campos.

METODOLOGÍA

La teorización desplegada en el presente manuscrito, resulta de la aplicación de una metodología de investigación integral, esto es, bajo un enfoque híbrido, tanto desde el punto de vista cualitativo como en su dimensión cuantitativa. El tipo de investigación que ha sido desarrollado a lo largo del presente Artículo Científico, es esencialmente predictivo, a la luz de la física teórica, aunque comporta también un carácter empírico o experimental. Por otro lado, las líneas de investigación adoptadas para la formulación del estado del arte, se ajustan al constructivismo. Cabe indicar, que no existe población de estudio en la medida en que el presente artículo científico, no es de carácter sociológico o social, más aun, en mérito a su impacto en la realidad de transformación. Tampoco se han implementado técnicas de recolección de información, tales como encuestas, entrevistas, etc, salvo revisión bibliográfica, a razón del campo de investigación abordado. Adicionalmente a lo antes expuesto, es perciso resaltar, que el material de apoyo es meramente bibliográfico. La técnica metodológica, dada la complejidad de la temática escrutada, es deductiva, pues la teorización en sentido estricto, ha sido desarrollada desde principios y premisas generales que son inherentes a la física de partículas en sentido lato. Finalmente, para efectos de construir y desarrollar las ecuaciones constantes en el presente artículo científico, se ha tomado en consideración la teoría cuántica de campos, muy especialmente, en tratándose de los campos de Yang – Mills, sin perjuicio de los demás sistemas de recalibración deducidos y esbozados a lo largo del presente Artículo Científico.

RESULTADOS Y DISCUSIÓN (Formulación Matemática y en física de partículas)

En un grupo cuántico de estructura G y bajo el algebra de Lie, obtenemos lo que sigue:

$$\langle \mathfrak{A} | \mathfrak{B} \rangle = -\mathfrak{Tr}_{\mathfrak{mat}\langle \bar{\mathfrak{n}} | \mathbb{C} \rangle} (\mathfrak{A} | \mathfrak{B})$$

Cuya transformación de gauge se reduce a lo que sigue:

$$\mathfrak{A} \cdot \Omega = \mathfrak{A}^{\Omega} = \Omega^{-1} \mathfrak{d} \Omega + \Omega^{-1} \mathfrak{A} \Omega$$

De cuyo resultado se obtienen la totalidad de las órbitas.

Por otro lado, usando la métrica de Riemann en un volumen espacial específico, y utilizando el operador de Hodge, tenemos:

$$\mathfrak{u} \wedge \star \mathfrak{v} = (\mathfrak{u}, \mathfrak{v})_{\mathfrak{q}} \mathfrak{d} \omega$$

Cuyas secciones se definen así:

$$\langle \mathfrak{u} | \mathfrak{v} \rangle = \int_{\mathfrak{M}}^{\infty} (\mathfrak{u}, \mathfrak{v})_{\mathfrak{q}} \mathfrak{d} \omega$$

De lo que obtenemos lo que sigue:

$$|\mathfrak{u} \otimes \mathfrak{E}|^2 = -\mathfrak{Tr}(\mathfrak{E} \cdot \mathfrak{E})\ \mathfrak{u} \wedge \star \mathfrak{v} = -\mathfrak{Tr}(\mathfrak{E} \cdot \mathfrak{E})(\mathfrak{u}, \mathfrak{v})_{\mathfrak{q}} \mathfrak{d} \omega$$

Cuya solución de Yang – Mills, es la que sigue:

$$\mathfrak{S}_{\mathfrak{YM}}(\mathfrak{A}) \int_{\mathfrak{M}}^{\infty} |\mathfrak{d}\mathfrak{A} + \mathfrak{A} \wedge \mathfrak{A}|^2$$

$$\mathfrak{T} \exp \left[\int_{\mathcal{C}}^{\infty} \mathfrak{A} \right] - \frac{1}{3} \mathfrak{Tr} \int_{\mathfrak{A} \in \mathfrak{A}_{\mathfrak{M},\mathfrak{g}} / \mathcal{G}}^{\infty} \mathfrak{T} \exp \left[\int_{\mathcal{C}}^{\infty} \mathfrak{A} \right] \mathrm{e}^{-1/2 \mathfrak{S}_{\mathfrak{YM}}(\mathfrak{A})} \mathfrak{D}\mathfrak{A}$$

En la que la curvatura de la superficie espacial y de la superficie temporal se expresan de la siguiente manera:

$$\mathfrak{D}\mathfrak{A} + \mathfrak{A}\wedge\mathfrak{A} = \sum_{\alpha}\sum_{1\le i<j\le 4} \alpha_{i;j,\alpha} \otimes \mathfrak{d}\mathfrak{x}^{i}\wedge\mathfrak{d}\mathfrak{x}^{j} \otimes \mathfrak{E}^{\alpha} + \sum_{\alpha,\beta}\sum_{1\le i<j\le 4} \alpha_{i,\alpha}\,\alpha_{j,\beta} \otimes \mathfrak{d}\mathfrak{x}^{i}\wedge\mathfrak{d}\mathfrak{x}^{j} \otimes \langle\mathfrak{E}^{\alpha}|\mathfrak{E}^{\beta}\rangle$$

$$+ \sum_{\alpha}\sum_{j=1} \alpha_{0:j,\alpha} \otimes \mathfrak{d}\mathfrak{x}^{0}\wedge\mathfrak{d}\mathfrak{x}^{j} \otimes \mathfrak{E}^{\alpha}$$

Cuyas permutaciones y transformaciones, se expresan así:

$$C_{\gamma}^{\alpha\beta} = -\mathfrak{Tr}\left(\mathfrak{E}^{\gamma}\left(\mathfrak{E}^{\alpha}, \mathfrak{E}^{\beta}\right)\right)$$

$$\mathfrak{d}\mathfrak{A} + \mathfrak{A}\wedge\mathfrak{A} = \sum_{\gamma}\langle \sum_{1\le i<j\le 4} \alpha_{i;j,\alpha} \otimes \mathfrak{d}\mathfrak{x}^{i}\wedge\mathfrak{d}\mathfrak{x}^{j} + \sum_{\alpha<\beta}\sum_{1\le i<j\le 4} \alpha_{i,\alpha}\,\alpha_{j,\beta} C_{\gamma}^{\alpha\beta} \otimes \mathfrak{d}\mathfrak{x}^{i}\wedge\mathfrak{d}\mathfrak{x}^{j}$$

$$+ \mathfrak{d}\mathfrak{x}^{i}\wedge\mathfrak{d}\mathfrak{x}^{j} \sum_{j=1} \alpha_{0:j,\gamma} \otimes \mathfrak{d}\mathfrak{x}^{0}\wedge\mathfrak{d}\mathfrak{x}^{j}\rangle \otimes \mathfrak{E}^{\gamma}$$

$$\int_{\mathbb{R}^4}^{\infty} |\mathfrak{d}\mathfrak{A} + \mathfrak{A}\wedge\mathfrak{A}|^{2}$$

$$= \sum_{i<j}\int_{\mathbb{R}^4}^{\infty} \left\| \sum_{\alpha} \alpha_{i;j,\alpha}^{2} + \sum_{\gamma}\sum_{\alpha<\beta,\hat{\alpha}<\hat{\beta}} \alpha_{i,\alpha}\alpha_{j,\beta}\alpha_{i,\hat{\alpha}}\alpha_{j,\hat{\beta}}\, C_{\gamma}^{\alpha\beta} C_{\gamma}^{\hat{\alpha}\hat{\beta}} \right.$$

$$\left. + 4\sum_{\alpha<\beta,\gamma} \alpha_{i;j,\gamma}\alpha_{i,\alpha}\alpha_{j,\beta}\, C_{\gamma}^{\alpha\beta} \right\| \mathfrak{d}\omega + \sum_{j}\int_{\mathbb{R}^4}^{\infty}\sum_{\alpha} \alpha_{0:j,\alpha}^{2}\, \mathfrak{d}\omega$$

Campo Cuántico Abeliano

Usando el teorema de Stoke, tenemos:

$$\int_{\mathfrak{C}}^{\infty}\sum_{i=1}^{4} \mathfrak{A}_{i} \otimes \mathfrak{d}\mathfrak{x}^{i} = \int_{\partial\mathfrak{S}}^{\infty}\sum_{i=1}^{4} \mathfrak{A}_{i}\mathfrak{d}\mathfrak{x}^{i} = \int_{\mathfrak{S}}^{\infty} \mathfrak{d}\mathfrak{A} = \int_{\mathfrak{R}^4}^{\infty} \mathfrak{d}\mathfrak{A} \times 4_{\mathfrak{S}} = \|\mathfrak{d}\mathfrak{A}, 4_{\mathfrak{S}}\|$$

Por lo que, la integral de Yang – Mills se expresa de la siguiente manera:

$$1/\int_{\mathfrak{A}}^{\infty} \mathrm{e}^{-\|\mathfrak{d}\mathfrak{A}\|^{2}/2}\mathfrak{D}\mathfrak{A}\int_{A}^{\infty} \mathrm{e}^{\sqrt{-1}\langle\mathfrak{d}\mathfrak{A}|4_{\mathfrak{S}}\rangle}\mathrm{e}^{-1/2|\mathfrak{d}\mathfrak{A}|^{2}}\,\mathfrak{d}\mathfrak{A}$$

Cuyo cambio heurístico de variables, queda expresado así:

$$\frac{1}{\det \mathfrak{d}^{-1} \int_{\mathfrak{A}}^{\infty} \mathrm{e}^{-\frac{\|\mathfrak{d}\mathfrak{A}\|^2}{2}} \mathfrak{D}(\mathfrak{d}\mathfrak{A}) \det \mathfrak{d}^{-1} \int_{\mathrm{A}}^{\infty} \mathrm{e}^{\sqrt{-1}\langle \mathfrak{d}\mathfrak{A}|4_{\mathfrak{S}}\rangle} \mathrm{e}^{-\frac{1}{2|\mathfrak{d}\mathfrak{A}|^2}} \mathfrak{D}(\mathfrak{d}\mathfrak{A})}$$

$$= \frac{1}{\int_{\mathfrak{A}}^{\infty} \mathrm{e}^{-\frac{\|\mathfrak{d}\mathfrak{A}\|^2}{2}} \mathfrak{D}(\mathfrak{d}\mathfrak{A}) \int_{\mathrm{A}}^{\infty} \mathrm{e}^{\sqrt{-1}\langle \mathfrak{d}\mathfrak{A}|4_{\mathfrak{S}}\rangle} \mathrm{e}^{-\frac{1}{2|\mathfrak{d}\mathfrak{A}|^2}} \mathfrak{D}(\mathfrak{d}\mathfrak{A})}$$

Más, en dimensión $\mathbb{R}^4$ y aplicando la función delta de Dirac, tenemos:

$$\langle \mathfrak{F}, \mathfrak{X}_{\mathfrak{x}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha} \wedge \mathfrak{d}\mathfrak{x}^{\beta} \rangle = \langle \sum_{0 \le i \le j \le 4} \mathfrak{I}_{ij} \mathfrak{d}\mathfrak{X}^{i} \wedge \mathfrak{d}\mathfrak{X}^{j}, \mathfrak{X}_{\mathfrak{x}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha} \wedge \mathfrak{d}\mathfrak{x}^{\beta} \rangle = \langle \mathfrak{I}_{\alpha\beta} \mathfrak{d}\mathfrak{x}^{\alpha} \wedge \mathfrak{d}\mathfrak{x}^{\beta}, \mathfrak{X}_{\mathfrak{x}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha} \wedge \mathfrak{d}\mathfrak{x}^{\beta} \rangle = \mathfrak{I}_{\alpha\beta}(\chi) \in \mathbb{R}^4$$

Cuyo gauge axial, según el teorema de Stokes, arroja como resultado lo que sigue:

$$\int_{\mathfrak{C}}^{\infty} \sum_{i=1}^{4} \mathfrak{A}_i \otimes \mathfrak{d}\mathfrak{X}^i = \int_{\partial \mathfrak{S}}^{\infty} \sum_{i=1}^{4} \mathfrak{A}_i \otimes \mathfrak{d}\mathfrak{X}^i = \int_{\mathfrak{S}}^{\infty} \mathfrak{d}\mathfrak{A}$$

$$= \int_{(0,1)^2}^{\infty} \mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t} \left[\sum_{1 \le i \le j \le 4} (\mathfrak{A}_{i;j}(\sigma) | \mathcal{J}_{ij}^{\sigma}|)(\mathfrak{s}, \mathfrak{t}) + \sum_{j=1}^{4} (\mathfrak{A}_{0:j}(\sigma) | \mathcal{J}_{0j}^{\sigma}|)(\mathfrak{s}, \mathfrak{t}) \right]$$

$$= \int_{(0,1)^2}^{\infty} \mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t} \langle \mathfrak{d}\mathfrak{A}, \sum_{0 \le i \le j \le 4} \chi_{\sigma(\mathfrak{s},\mathfrak{t})} |\mathcal{J}_{ij}^{\sigma}| (\mathfrak{s}, \mathfrak{t}) \otimes \mathfrak{d}\mathfrak{x}^{i} \wedge \mathfrak{d}\mathfrak{x}^{j} \rangle = \langle \mathfrak{d}\mathfrak{A}, \widetilde{\mathfrak{B}_{\mathfrak{S}}} \rangle$$

En la que $\widetilde{\mathfrak{B}_{\mathfrak{S}}}$ es igual a:

$$\widetilde{\mathfrak{B}_{\mathfrak{S}}} = \int_{(0,1)^2}^{\infty} \sum_{0 \le i \le j \le 4} \mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t} \chi_{\sigma(\mathfrak{s},\mathfrak{t})} |\mathcal{J}_{ij}^{\sigma}| (\mathfrak{s}, \mathfrak{t})\, \mathfrak{d}\mathfrak{x}^{i} \wedge \mathfrak{d}\mathfrak{x}^{j}$$

$$\langle \mathfrak{d}\mathfrak{A}, \sum_{0 \le i \le j \le 4} \chi_{\sigma(\mathfrak{s},\mathfrak{t})} |\mathcal{J}_{ij}^{\sigma}| (\mathfrak{s}, \mathfrak{t}) \otimes \mathfrak{d}\mathfrak{x}^{i} \wedge \mathfrak{d}\mathfrak{x}^{j} \rangle = \langle \mathfrak{d}\mathfrak{A}, \sum_{0 \le i \le j \le 4} \chi_{\sigma(\mathfrak{s},\mathfrak{t})} \otimes \mathfrak{d}\mathfrak{x}^{i} \wedge \mathfrak{d}\mathfrak{x}^{j} \rangle |\mathcal{J}_{ij}^{\sigma}| (\mathfrak{s}, \mathfrak{t})$$

$$= \sum_{0 \le i \le j \le 4} \langle \mathfrak{A}_{i;j}, \chi_{\sigma(\mathfrak{s},\mathfrak{t})} \rangle |\mathcal{J}_{ij}^{\sigma}| (\mathfrak{s}, \mathfrak{t}) = \sum_{0 \le i \le j \le 4} \langle \mathfrak{A}_{i;j}(\sigma(\mathfrak{s}, \mathfrak{t})) \rangle |\mathcal{J}_{ij}^{\sigma}| (\mathfrak{s}, \mathfrak{t})$$

$$\frac{1}{3 \int_{\mathfrak{A}}^{\infty} \mathrm{e}^{\sqrt{-1}\langle \mathfrak{d}\mathfrak{A} | \widetilde{\mathfrak{B}_{\mathfrak{S}}} \rangle} \mathrm{e}^{-\frac{1}{2|\mathfrak{d}\mathfrak{A}|^2}} \mathfrak{D}(\mathfrak{d}\mathfrak{A})}$$

$$- \frac{1}{3 \int_{\mathfrak{A}}^{\infty} \mathrm{e}^{\sqrt{-1}\langle \mathfrak{d}\mathfrak{A} | \widetilde{\mathfrak{B}_{\mathfrak{S}}} \rangle} \mathrm{e}^{-\frac{1}{2|\mathfrak{d}\mathfrak{A}|^2}} \det(\mathfrak{d}^{-1}) \mathfrak{D}(\mathfrak{d}\mathfrak{A}) + \frac{1}{3 \int_{\mathfrak{A}}^{\infty} \mathrm{e}^{\sqrt{-1}\langle \mathfrak{d}\mathfrak{A} | \widetilde{\mathfrak{B}_{\mathfrak{S}}} \rangle} \mathrm{e}^{-\frac{1}{2|\mathfrak{d}\mathfrak{A}|^2}}} \mathfrak{D}(\mathfrak{d}\mathfrak{A})}$$

Siguiendo el mismo orden de ideas, un espacio de Schwartz, quedaría expresado así:

$$\mathcal{P}\mathfrak{r} = \left\| (m_1 m_2, \dots, m_\mathfrak{n}) \sum_{\mathfrak{j}=1}^{\mathfrak{n}} \mathfrak{m}_\mathfrak{j} = \mathfrak{r} \right\|$$

Más en dimensión $\mathbb{R}^4$, incorporando la función de Gauss y la métrica de Lebesgue, tenemos:

$$\mathfrak{f}(\mathfrak{x}) = \wp(\mathfrak{x})\sqrt{\phi_\kappa}(\mathfrak{x})$$

$$\langle \mathfrak{f}, \mathfrak{g} \rangle = \int_{\mathbb{R}^4}^{\infty} \mathfrak{f} \cdot \mathfrak{g}\mathfrak{d}\lambda$$

$$\langle \mathfrak{z}^\mathfrak{r}, \hat{\mathfrak{z}}^\mathfrak{r} \rangle = \frac{1}{\pi \int_{\mathbb{C}}^{\infty} \mathfrak{z}^\mathfrak{r} \cdot \overline{\hat{\mathfrak{z}}^\mathfrak{r}}\, \mathbb{e}^{-|z^2|} \mathfrak{d}\mathfrak{x}\mathfrak{d}\mathfrak{p}}, \mathfrak{z} = \mathfrak{x} + \sqrt{-1\rho}$$

Cuya función polinómica, se traduce a lo que sigue:

$$\mathcal{H}_{\wp_\mathfrak{R}}(\mathfrak{x}) = \hbar_i(\mathfrak{x}^0)\hbar_j(\mathfrak{x}^1)\hbar_k(\mathfrak{x}^2)\hbar_l(\mathfrak{x}^3), \wp_\mathfrak{R} = (i, j, k, l) \in \mathfrak{P}_\mathfrak{R}$$

Más, incorporando la métrica de Gauss, tenemos:

$$\bigcup_{\mathfrak{r}=0}^{\infty} \left| \frac{\mathcal{H}_{\wp_\mathfrak{R}}(\kappa\mathfrak{x}^0, \kappa\mathfrak{x}^1, \kappa\mathfrak{x}^2, \kappa\mathfrak{x}^3)\sqrt{\phi_\kappa}}{\sqrt{\wp_\mathfrak{R}!}} : \wp_\mathfrak{r} \in \mathfrak{P}_\mathfrak{r} \right|$$

Por tanto, en dimensión $\mathbb{R}^4$, tenemos:

$$\mathfrak{S}_\kappa(\mathbb{R}^4) \otimes \Lambda^1(\mathbb{R}^3) = \langle \sum_{\alpha=1}^{4} \mathfrak{F}_\alpha \otimes \mathfrak{d}\mathfrak{x}^\alpha : \mathfrak{F}_\alpha \in \mathfrak{S}_\kappa(\mathbb{R}^4) \rangle$$

$$\overline{\mathfrak{S}}_\kappa(\mathbb{R}^4) \otimes \Lambda^2(\mathbb{R}^4) = \left\| \sum_{0 \le \alpha \le \beta \le 4} \langle \mathfrak{F}_{\alpha\beta} \otimes \mathfrak{d}\mathfrak{x}^\alpha \wedge \mathfrak{d}\mathfrak{x}^\beta : \mathfrak{F}_{\alpha\beta} \in \overline{\mathfrak{S}}_\kappa(\mathbb{R}^4) \rangle \right\|$$

Cuyo operador de Hodge, se reduce a lo siguiente:

$$\langle \sum_{0 \le \alpha \le \beta \le 4} |\mathfrak{F}_{\alpha\beta} \otimes \mathfrak{d}\mathfrak{x}^\alpha \wedge \mathfrak{d}\mathfrak{x}^\beta, | \sum_{0 \le \alpha \le \beta \le 4} |\hat{\mathfrak{F}}_{\alpha\beta} \otimes \mathfrak{d}\mathfrak{x}^\alpha \wedge \mathfrak{d}\mathfrak{x}^\beta| \rangle = \sum_{0 \le \alpha \le \beta \le 4} |\mathfrak{F}_{\alpha\beta}, \hat{\mathfrak{F}}_{\alpha\beta}|$$

Cuya transformación de Segal - Bargmann y demás funciones holomórficas, dan como resultado lo que sigue:

$$\langle \mathfrak{Z}^\mathcal{R}, \mathfrak{Z}^{\mathcal{R}'} \rangle = \frac{1}{\varpi \int_{\mathbb{C}}^{\infty} \mathfrak{Z}^\mathcal{R} \cdot \overline{\mathfrak{Z}^{\mathcal{R}'}}\, \mathbb{e}^{|-\mathfrak{z}^2|} \mathfrak{d}\mathfrak{x}\mathfrak{d}\mathfrak{p}}, \mathfrak{z} = \chi + \sqrt{-1\rho}$$

$$\Psi_\kappa = \cfrac{\cfrac{\cfrac{\cfrac{\hbar_{\mathfrak{i}}(\kappa\,\cdot)}{\sqrt{\mathfrak{i}!}\,\hbar_{\mathfrak{j}}(\kappa\,\cdot)}}{\sqrt{\mathfrak{j}!}\,\hbar_{\mathfrak{k}}(\kappa\,\cdot)}}{\sqrt{\mathfrak{k}!}\,\hbar_{\mathfrak{l}}(\kappa\,\cdot)}}{\sqrt{\mathfrak{l}!}} = \sqrt{\phi_\kappa} \longrightarrow \cfrac{\cfrac{\cfrac{\cfrac{\mathfrak{z}_0^{\mathfrak{i}}}{\sqrt{\mathfrak{i}!}\,\mathfrak{z}_1^{\mathfrak{j}}}}{\sqrt{\mathfrak{j}!}\,\mathfrak{z}_2^{\mathfrak{k}}}}{\sqrt{\mathfrak{k}!}\,\mathfrak{z}_3^{\mathfrak{l}}}}{\sqrt{\mathfrak{l}!}} = \mathfrak{f}_{\mathfrak{i},\alpha} \otimes \mathfrak{d}\mathfrak{x}^{\mathfrak{i}} \otimes \mathfrak{E}^{\alpha} \longrightarrow \Psi_\kappa\left(\mathfrak{f}_{\mathfrak{i},\alpha}\right) \otimes \mathfrak{d}\mathfrak{x}^{\mathfrak{i}} \otimes \mathfrak{E}^{\alpha}$$

En la que, en un espacio de Hilbert, se tiene:

$$\langle \sum_{0\le\alpha\le\beta\le4} \mathfrak{f}_{\alpha\beta} \otimes \mathfrak{d}\mathfrak{x}^{\alpha} \otimes \mathfrak{d}\mathfrak{x}^{\beta}, \sum_{0\le\alpha\le\beta\le4} \hat{\mathfrak{f}}_{\alpha\beta} \otimes \mathfrak{d}\mathfrak{x}^{\alpha} \otimes \mathfrak{d}\mathfrak{x}^{\beta} \rangle = \sum_{0\le\alpha\le\beta\le4} \langle \mathfrak{f}_{\alpha\beta}, \hat{\mathfrak{f}}_{\alpha\beta} \rangle$$

$$\partial \sum_{\mathfrak{i}=1}^{4} \mathfrak{f}_{\mathfrak{i}} \otimes \mathfrak{d}\mathfrak{x}^{\mathfrak{i}} = \sum_{\mathfrak{i}=1}^{4} \langle \partial_0 \mathfrak{f}_{\mathfrak{i}} \rangle \otimes \mathfrak{d}\mathfrak{x}^{0} \wedge \mathfrak{d}\mathfrak{x}^{\mathfrak{i}} + \sum_{1\le\mathfrak{i}\le\mathfrak{j}\le4} \left[\partial_{\mathfrak{i}} \mathfrak{f}_{\mathfrak{j}} - \partial_{\mathfrak{j}} \mathfrak{f}_{\mathfrak{i}}\right] \otimes \mathfrak{d}\mathfrak{x}^{\mathfrak{i}} \wedge \mathfrak{d}\mathfrak{x}^{\mathfrak{j}}$$

$$\Psi_\kappa = \sum_{1\le\mathfrak{i}\le\mathfrak{j}\le4} \mathfrak{f}_{\mathfrak{i},\mathfrak{j},\alpha} \otimes \mathfrak{d}\mathfrak{x}^{\mathfrak{i}} \wedge \mathfrak{d}\mathfrak{x}^{\mathfrak{j}} \otimes \mathfrak{E}^{\alpha} \longrightarrow \sum_{1\le\mathfrak{i}\le\mathfrak{j}\le4} \Psi_\kappa\left[\mathfrak{f}_{\mathfrak{i},\mathfrak{j},\alpha}\right] \otimes \mathfrak{d}\mathfrak{x}^{\mathfrak{i}} \wedge \mathfrak{d}\mathfrak{x}^{\mathfrak{j}} \otimes \mathfrak{E}^{\alpha}$$

$$\mathcal{H}^2(\mathbb{C}^4) \otimes \Lambda^1(\mathbb{R}^3) = \left\{ \sum_{\alpha=1}^{4} \mathfrak{f}_\alpha \otimes \mathfrak{d}\mathfrak{x}^{\alpha} : \mathfrak{f}_\alpha \in \mathcal{H}^2(\mathbb{C}^4) \right\}$$

$$\mathcal{H}^2(\mathbb{C}^4) \otimes \Lambda^2(\mathbb{R}^4) = \left\{ \sum_{0\le\alpha\le\beta\le4} \mathfrak{F}_{\alpha\beta} \otimes \mathfrak{d}\mathfrak{x}^{\alpha} \wedge \mathfrak{d}\mathfrak{x}^{\beta} : \mathfrak{F}_{\alpha\beta} \in \mathcal{H}^2(\mathbb{C}^4) \right\}$$

$$\langle \sum_{0\le\alpha\le\beta\le4} \mathfrak{F}_{\alpha\beta} \otimes \mathfrak{d}\mathfrak{x}^{\alpha} \wedge \mathfrak{d}\mathfrak{x}^{\beta}, \sum_{0\le\alpha\le\beta\le4} \widehat{\mathfrak{F}_{\alpha\beta}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha} \wedge \mathfrak{d}\mathfrak{x}^{\beta} \rangle = \sum_{0\le\alpha\le\beta\le4} \langle \mathfrak{F}_{\alpha\beta}, \hat{\mathfrak{F}}_{\alpha\beta} \rangle = \int_{\mathbb{C}^4}^{\infty} \mathfrak{F}_{\alpha\beta} \hat{\mathfrak{F}}_{\alpha\beta} \mathfrak{d}\lambda_4$$

Cuyos polinomios de Hermite, se satisfacen así:

$$\frac{\mathfrak{d}}{\mathfrak{d}\mathfrak{x}\left(\hbar_\eta(\chi)\mathfrak{E}^{-\frac{\mathfrak{x}^2}{4}}\right)} = \left|\chi\hbar_\eta(\mathfrak{x}) - \chi\hbar_{\eta+1}(\mathfrak{x}) - x/2\hbar_\eta(\chi)\right|\mathfrak{E}^{-\frac{\mathfrak{x}^2}{4}} = \frac{\mathfrak{d}}{\mathfrak{d}\mathfrak{x}\left(\hbar_\eta(\chi)\mathfrak{E}^{-\frac{\mathfrak{x}^2}{4}}\right)}$$

$$= \left(\frac{1}{2\hbar_{\eta+1}(\mathfrak{x})} + \frac{\eta}{2\hbar_{\eta-1}(\mathfrak{x})} - \hbar_{\eta+1}(\mathfrak{x})\right)\mathfrak{E}^{-\frac{\mathfrak{x}^2}{4}} = \left(\frac{\eta}{2\hbar_{\eta-1}(\mathfrak{x})} - \frac{1}{2\hbar_{\eta+1}(\mathfrak{x})}\right)\mathfrak{E}^{-\frac{\mathfrak{x}^2}{4}}$$

$$\mathfrak{d} \sum_{\alpha=1}^{4} \mathfrak{F}_{\alpha\beta} \otimes \mathfrak{d}\mathfrak{x}^{\alpha} = \sum_{\alpha=1}^{4} \mathfrak{d}_0 \mathfrak{F}_0 \mathfrak{d}\mathfrak{x}^0 \wedge \mathfrak{d}\mathfrak{x}^{\alpha} + \sum_{1\le\mathfrak{i}\le\mathfrak{j}\le4} \langle -1 \rangle^{\mathfrak{i}\mathfrak{j}} \langle \partial_{\mathfrak{i}} \mathfrak{f}_{\mathfrak{i}} - \partial_{\mathfrak{i}} \mathfrak{f}_j \rangle \mathfrak{d}\mathfrak{x}^{\mathfrak{i}} \wedge \mathfrak{d}\mathfrak{x}^{\mathfrak{j}}$$

$$\langle \mathfrak{f}\sqrt{\phi_\kappa}, \mathfrak{g}\sqrt{\phi_\kappa} \rangle = \int_{\mathbb{R}^4}^{\infty} \mathfrak{f}\mathfrak{g} \cdot \phi_\kappa, \mathfrak{d}\lambda$$

$$\left\{ \left| \frac{\hbar_{\mathfrak{i}}(\kappa\chi^0)\hbar_{\mathfrak{j}}(\kappa\chi^1)\hbar_{\mathfrak{k}}(\kappa\chi^2)\hbar_{\mathfrak{l}}(\kappa\chi^3)}{\sqrt{\mathfrak{i}!\,\mathfrak{j}!\,\mathfrak{k}!\,\mathfrak{l}!}\sqrt{\phi_\kappa(\overrightarrow{\mathfrak{x}})}} \right| \vec{\mathfrak{x}} = (\chi^0, \chi^1, \chi^2, \chi^3) \in \mathbb{R}^4 \mathfrak{i}, \mathfrak{j}, \mathfrak{k}, \mathfrak{l}, \ge 0 \right\}$$

$$\langle \sum_{1\leq\alpha\leq\beta\leq 4} \mathfrak{f}_{\alpha\beta} \otimes \mathfrak{d}\mathfrak{x}^{\alpha} \wedge \mathfrak{d}\mathfrak{x}^{\beta}, \sum_{1\leq\alpha\leq\beta\leq 4} \hat{\mathfrak{f}}_{\alpha\beta} \otimes \mathfrak{d}\mathfrak{x}^{\alpha} \wedge \mathfrak{d}\mathfrak{x}^{\beta} \rangle = \langle \mathfrak{f}_{\alpha\beta}, \hat{\mathfrak{f}}_{\alpha\beta} \rangle$$

$$\mathfrak{d}\mathfrak{f} = \sum_{\mathrm{i}=1}^{4} \partial_0 \mathfrak{f}_{\mathrm{i}} \otimes \mathfrak{d}\mathfrak{x}^{0} \wedge \mathfrak{d}\mathfrak{x}^{\mathrm{i}} + \sum_{1\leq \mathrm{i}\leq \mathrm{j}\leq 4} (\partial_{\mathrm{i}} \mathfrak{f}_{\mathrm{j}} - \partial_{\mathrm{j}} \mathfrak{f}_{\mathrm{i}}) \mathfrak{d}\mathfrak{x}^{\mathrm{i}} \wedge \mathfrak{d}\mathfrak{x}^{\mathrm{j}} = \mathfrak{C}_{\gamma}^{\alpha\beta} = -\mathcal{T}\mathfrak{r}(\mathfrak{E}^{\gamma}(\mathfrak{E}^{\alpha}, \mathfrak{E}^{\beta})), \mathfrak{E}^{\alpha}, \mathfrak{E}^{\beta}, \mathfrak{E}^{\gamma} \in \mathfrak{g}$$

$$\mathfrak{d}\mathfrak{A} + \mathfrak{A} \wedge \mathfrak{A} = \sum_{\gamma=1}^{\eta} \Big(\sum_{\mathrm{j}=1}^{4} \alpha_{0:\mathrm{i},\gamma} \otimes \mathfrak{d}\mathfrak{x}^{0} \wedge \mathfrak{d}\mathfrak{x}^{\mathrm{j}} + \sum_{1\leq \mathrm{i}\leq \mathrm{j}\leq 4} \alpha_{\mathrm{i};\mathrm{j},\gamma} \otimes \mathfrak{d}\mathfrak{x}^{\mathrm{i}} \wedge \mathfrak{d}\mathfrak{x}^{\mathrm{j}}$$

$$+ \sum_{1\leq \mathrm{i}\leq \mathrm{j}\leq 4} \sum_{1\leq\alpha,\beta\leq\eta} \alpha_{\mathrm{i},\alpha}\, \alpha_{\mathrm{j},\beta} \mathfrak{C}_{\gamma}^{\alpha\beta} \otimes \mathfrak{d}\mathfrak{x}^{\mathrm{i}} \wedge \mathfrak{d}\mathfrak{x}^{\mathrm{j}} \Big) \otimes \mathfrak{E}^{\gamma}$$

Cuya forma bilineal, se distribuye así:

$$\langle \sum_{\alpha} \mathfrak{F}_{\alpha} \otimes \mathfrak{d}\mathfrak{x}^{\alpha}, \sum_{\alpha} \mathfrak{G}_{\beta} \otimes \mathfrak{d}\mathfrak{x}^{\beta} \rangle_{\partial\kappa} = \kappa^{4} \langle \frac{\partial}{\partial t}\left(\frac{i}{\hbar}\right) \sum_{\alpha} \mathfrak{F}_{\alpha} \otimes \mathfrak{d}\mathfrak{x}^{\alpha}, \sum_{\alpha} \mathfrak{G}_{\beta} \otimes \mathfrak{d}\mathfrak{x}^{\beta} \rangle$$

$$\psi_{\kappa}: \mathcal{H}_{\wp_{\mathfrak{R}}}(\kappa\mathfrak{x}^{0}, \kappa\mathfrak{x}^{1}, \kappa\mathfrak{x}^{2}, \kappa\mathfrak{x}^{3}) \sqrt{\phi_{\kappa}} / \sqrt{\wp_{\mathfrak{R}}!} \to \mathfrak{z}^{\mathfrak{p}_{\mathfrak{r}}} / \sqrt{\wp_{\mathfrak{R}}!} \equiv \mathfrak{z}_{0}^{i_0} \mathfrak{z}_{1}^{i_1} \mathfrak{z}_{2}^{i_2} \mathfrak{z}_{3}^{i_3} / \sqrt{i_0!\, i_1!\, i_2!\, i_3!}$$

Cuya isometría extendida, deriva en lo que sigue:

$$\psi_{\kappa} \left[\sum_{0\leq\alpha\leq\beta\leq 4} \mathfrak{F}_{\alpha,\beta} \otimes \mathfrak{d}\mathfrak{x}^{\alpha} \wedge \mathfrak{d}\mathfrak{x}^{\beta} \right] = \sum_{0\leq\alpha\leq\beta\leq 4} \psi_{\kappa} \left| \mathfrak{f}_{\alpha,\beta} \right| \otimes \mathfrak{d}\mathfrak{x}^{\alpha} \wedge \mathfrak{d}\mathfrak{x}^{\beta}$$

Ahora bien, un espacio abstracto de Wiener, se explica así:

$$\mu\kappa\left(\chi \in \mathfrak{P}^{-1}(\mathfrak{I})\right) = \left(\frac{\kappa}{2\varpi}\right)^{l/2} \int_{\mathfrak{y}\in\mathcal{F}}^{\infty} e^{-\kappa|\varphi|^{2}/\psi} \mathfrak{d}\mathfrak{y}$$

$$\left\{ \hat{\mathfrak{z}}^{\mathfrak{p}_{\mathfrak{r}}} \frac{\otimes \mathfrak{d}\mathfrak{x}^{\alpha}}{|\hat{\mathfrak{z}}^{\mathfrak{p}_{\mathfrak{r}}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha}|_{\partial\kappa}} : \rho_{\mathfrak{r}} \in \wp_{\mathfrak{r}}, \mathcal{R} \geq 0 \right\}$$

$$\hat{\mathfrak{z}}^{\mathfrak{p}_{\mathfrak{r}}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha} = \mathfrak{z}^{\mathfrak{p}_{\mathfrak{r}}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha} - \frac{\sum_{\alpha=0}^{4} \langle \mathfrak{z}^{\mathfrak{p}_{\mathfrak{r}}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha}, \frac{\mathfrak{z}^{\rho_{\tau}^{\alpha,-}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha}}{\left|\mathfrak{z}^{\rho_{\tau}^{\alpha,-}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha}\right|_{\partial\kappa}} \rangle \mathfrak{z}^{\rho_{\tau}^{\alpha,-}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha}}{\left|\mathfrak{z}^{\rho_{\tau}^{\alpha,-}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha}\right|_{\partial\kappa}} + \left|\mathfrak{z}^{\rho_{\tau}^{\alpha,-}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha}\right|_{\partial\kappa_{\psi}}^{2} - \left|\mathfrak{z}^{\rho_{\tau}^{\alpha,-}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha}\right|_{\partial\kappa}$$

$$+ \langle \frac{\mathfrak{z}^{\rho_{\tau}^{\alpha,-}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha}}{\left|\mathfrak{z}^{\rho_{\tau}^{\alpha,-}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha}\right|_{\partial\kappa}} \rangle_{\partial\kappa} \bowtie 4\kappa(\mathfrak{r}-1)\sqrt{\wp_{\mathfrak{R}}!} \cdot \frac{\frac{(\mathfrak{r}+1)}{2}}{\frac{\kappa\sqrt{\wp_{\mathfrak{R}}!}}{\mathcal{R}^{2}}} / 2$$

$$\leq 6\kappa(\mathfrak{r}-1)^{2} \sum_{\alpha=1}^{4} \langle \frac{\mathfrak{z}^{\rho_{\tau}^{\alpha,-}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha}}{\left|\mathfrak{z}^{\rho_{\tau}^{\alpha,-}} \otimes \mathfrak{d}\mathfrak{x}^{\alpha}\right|_{\partial\kappa}} \rangle_{\partial\kappa} \otimes \mathfrak{d}\mathfrak{x}^{\alpha}$$

$$\mu_\kappa(\|\mathcal{P}_b\chi\| > \epsilon) \le \mu_\kappa\left(\sup_{\vec{\mathfrak{z}} \in \mathfrak{B}\left(\frac{0,1}{2}\right)}\right)\sum_{\mathfrak{s}}\sum_{\rho_\tau \ge \mathfrak{Q}^{\mathfrak{s}}} \mathbb{E}\left|\mathfrak{C}_{\mathfrak{s}}\alpha^{\mathfrak{s}}_{\rho_\tau,\mathfrak{B}}\right|^4 \left[|\mathfrak{z}^{\rho_\tau}| + 6\kappa(\mathfrak{r}+1)^2 \sum_{\alpha=1}^{4}\left|\mathfrak{z}^{\rho_\tau^{\alpha,-}}\right| > \epsilon\right] 1/\sqrt{\kappa\epsilon}$$

$$|\langle\chi(\omega), \mathfrak{d}\mathfrak{x}^\alpha\rangle| \le \left|\sum_\tau \sum_{\rho_\tau \in \mathfrak{q}_\tau} \mathfrak{C}_{\rho_\tau,\alpha}\widehat{\omega}^{\rho_\tau} / |\widehat{\omega}^{\rho_\tau} \otimes \mathfrak{d}\mathfrak{x}^\alpha|_{\partial\kappa}\right|$$

$$\le \mathfrak{R}^{\mathfrak{M}} \sum_{\tau \le \mathcal{M}} \sum_{\rho_\tau \in \mathfrak{q}_\tau} |\mathfrak{C}_{\rho_\tau,\alpha}|\, \|\omega/\mathfrak{R}\|^{\rho_\tau} + 6\kappa(\mathfrak{r}+1)^2 \sum_{\alpha=1}^{4} \|\omega/\mathfrak{R}\|^{\rho_\tau^{\alpha,-}} 2\mathcal{R}^{\mathfrak{J}} + 1/\kappa\sqrt{\wp_{\mathfrak{R}}!}\,\|\chi\|$$

$$\left|\langle\kappa\partial\mathfrak{x}(\omega), \mathfrak{d}\mathfrak{x}^\alpha \wedge \mathfrak{d}\mathfrak{x}^\beta\rangle\right| \le \kappa\left|\sum_\tau\sum_{\rho_\tau\in\mathfrak{q}_\tau} \frac{\mathfrak{C}_{\rho_\tau,\beta}\partial_\beta\widehat{\omega}^{\rho_\tau}}{|\widehat{\omega}^{\rho_\tau}\otimes\mathfrak{d}\mathfrak{x}^\beta|_{\partial\kappa}}\right| + \kappa\left|\sum_\tau\sum_{\rho_\tau\in\mathfrak{q}_\tau} \frac{\mathfrak{C}_{\rho_\tau,\alpha}\partial_\beta\widehat{\omega}^{\rho_\tau}}{|\widehat{\omega}^{\rho_\tau}\otimes\mathfrak{d}\mathfrak{x}^\alpha|_{\partial\kappa}}\right|$$

$$\le \mathfrak{R}^{\mathcal{M}} \sum_{\tau\le\mathcal{M}}\sum_{\rho_\tau\in\mathfrak{q}_\tau} \frac{|\mathfrak{C}_{\rho_\tau,\beta}| + |\mathfrak{C}_{\rho_\tau,\alpha}|\partial_\beta\widehat{\omega}^{\rho_\tau}}{|\widehat{\omega}^{\rho_\tau}\otimes\mathfrak{d}\mathfrak{x}^\alpha|_{\partial\kappa}} \left|\frac{\omega}{\mathcal{R}}\right|^{\rho_\tau}$$

$$+ +6\kappa(\mathfrak{r}+1)^2 \sum_{\alpha=1}^{4}\left|\frac{\omega}{\mathcal{R}}\right|^{\rho_\tau^{\alpha,-}} 4(\mathfrak{r}+1)\mathcal{R}^{\mathfrak{J}}/\sqrt{\wp_{\mathfrak{R}}!}$$

$$< \left(\mathcal{R}^{\mathcal{M}}+1\right)\|\mathfrak{z}\|^{\aleph}\mathcal{g}_{\mathfrak{i}}\otimes\mathfrak{d}\mathfrak{x}^{\mathfrak{i}} \in \mathcal{H}^2(\mathbb{C}^4)\otimes\Lambda^2(\mathbb{R}^4) \to \psi_\kappa\left|\partial_\alpha\mathcal{g}_\beta - \partial_\beta\mathcal{g}_\alpha\right|$$

$$\left|(\omega+\omega_0)^{\rho_\tau} - \omega_0^{\rho_\tau}\right|$$

$$+ 6\kappa(\mathfrak{r}+1)^2\sum_{\alpha=1}^{4}\left|(\omega+\omega_0)^{\rho_\tau^{\alpha,-}} - \right|\omega_0^{\rho_\tau^{\alpha,-}} \langle\le \mathfrak{R}^\tau|\omega\rangle \left(\sup_{\omega\in\mathfrak{B}(0,\epsilon)}\right) |\langle\mathfrak{x}(\omega+\omega_0)$$

$$-\rangle\mathfrak{x}(\omega_0), \mathfrak{d}\mathfrak{x}^\alpha|$$

$$\le \left(\sup_{\omega\in\mathfrak{B}(0,\epsilon)}\right)\sum_\tau\sum_{\rho_\tau\in\mathfrak{P}_\tau} |\mathfrak{C}_{\rho_\tau}|\, \left|2\widehat{(\omega+\omega_0)}^{\rho_\tau} - \widehat{\omega}_0^{\rho_\tau}\right|\kappa\sqrt{\wp_{\mathfrak{R}}!}$$

$$+ \frac{\left(\sup_{\omega\in\mathfrak{B}(0,\epsilon)}\right)\sum_{\alpha=1}^4\sum_\tau\sum_{\rho_\tau\in\mathfrak{P}_\tau}|\mathfrak{C}_{\rho_\tau}|\left|6\kappa(\mathfrak{r}+1)^2\cdot 2\left|\widehat{(\omega+\omega_0)}^{\rho_\tau} - \widehat{\omega}_0^{\rho_\tau}\right|\right|}{\sqrt{\kappa\wp_{\mathfrak{R}}!}}$$

$$\le \frac{\dfrac{4}{\kappa\left(\sup_{\omega\in\mathfrak{B}(0,\epsilon)}\right)\sum_\tau\sum_{\rho_\tau\in\mathfrak{P}_\tau}|\mathfrak{C}_{\rho_\tau}|\,|(4^\tau\mathcal{R}^\tau|\omega||}}{\kappa\sqrt{\wp_{\mathfrak{R}}!}}$$

$$+ 2\kappa\left(\sup_{\omega\in\mathfrak{B}(0,\epsilon)}\right)\sum_{\alpha=1}^{4}\sum_\tau\sum_{\rho_\tau\in\mathfrak{P}_\tau}\frac{6\kappa(\mathfrak{r}+1)^2}{2^{\tau-2}|\mathfrak{C}_{\rho_\tau}|}(2^{\tau-2}\mathfrak{R}^{\tau-2}|\omega|)/\kappa\sqrt{\wp_{\mathfrak{R}}!}$$

$$\le 2\mathfrak{C}(\omega_0)/\kappa\cdot\epsilon|\chi|$$

$$\widetilde{\mu_\kappa}\left(\left(\sup_{\omega\in\mathfrak{B}\left(\omega_0,\frac{1}{\kappa}\right)}\right)|\mathfrak{x}(\omega+\omega_0)-\mathfrak{x}(\omega_0),\mathfrak{d}\mathfrak{x}^\alpha|>\epsilon\right)$$

$$=\widetilde{\mu_\kappa}\left(\left(\sup_{\omega\in\mathfrak{B}\left(\omega_0,\frac{1}{\kappa}\right)}\right)|\mathfrak{x},\zeta_\alpha(\omega+\omega_0)-\zeta_\alpha(\omega_0),\mathfrak{d}\mathfrak{x}^\alpha|>\epsilon\right)\leq\widetilde{\mu_\kappa}\left(\frac{2\mathfrak{C}(\omega_0)}{\kappa\mathfrak{k}\|\mathfrak{x}\|}>\epsilon\right)$$

$$\to 0$$

$$\langle\frac{\partial_\alpha\mathfrak{z}_\alpha^\eta}{\sqrt{\eta!}},\frac{\partial_\beta\mathfrak{z}_\beta^\eta}{\sqrt{\eta!}}\rangle\geq\eta+\frac{1}{4\langle\frac{\partial_\alpha\mathfrak{z}_\alpha^{\eta+1}}{\sqrt{\eta+1!}},\frac{\partial_\beta\mathfrak{z}_\beta^{\eta+1}}{\sqrt{\eta+1!}}\rangle}$$

Siendo la integral de Yang – Mills, la siguiente:

$$\frac{1}{3}\int_{\mathfrak{A}}^{\infty}\mathfrak{e}^{\int_{\mathbb{C}}^{\infty}\sum_{j=1}^{4}\mathfrak{A}_j\otimes\mathfrak{d}\mathfrak{x}^j\otimes\mathfrak{i}}\,\mathfrak{E}^{-|\mathfrak{d}\mathfrak{A}|^2/2}\mathfrak{D}\mathfrak{A},\mathfrak{i}=\sqrt{-1}$$

$$\frac{1}{3}\mathfrak{e}^{-\frac{1}{2\int_{\mathbb{R}^4}^{\infty}|\mathfrak{d}\mathfrak{A}+\mathfrak{A}\wedge\mathfrak{A}|^4\mathfrak{d}\omega}}\mathcal{D}(\mathfrak{d}\mathfrak{A})$$

$$\mathfrak{Z}=\int_{\{\mathfrak{d}\mathfrak{A}:\mathfrak{A}\in\mathfrak{S}_\kappa(\mathbb{R}^4)\otimes\Lambda^1(\mathbb{R}^4)\mathfrak{g}\}}^{\infty}\mathfrak{e}^{-\frac{1}{2\int_{\mathbb{R}^4}^{\infty}|\mathfrak{d}\mathfrak{A}+\mathfrak{A}\wedge\mathfrak{A}|^4\mathfrak{d}\omega}}\mathcal{D}(\mathfrak{d}\mathfrak{A})$$

$$\frac{1}{3}\mathfrak{e}^{-\frac{1}{2\int_{\mathbb{C}^4}^{\infty}|\kappa\mathfrak{d}\mathfrak{A}+\mathfrak{A}\wedge\mathfrak{A}|^4\mathfrak{d}\lambda_4}}\mathcal{D}(\mathfrak{d}\mathfrak{A})$$

$$\mathfrak{Z}=\int_{\{\mathfrak{d}\mathfrak{A}:\mathfrak{A}\in\mathcal{H}^2(\mathbb{C}^4)\otimes\Lambda^1(\mathbb{R}^4)\otimes\mathfrak{g}\}}^{\infty}\mathfrak{e}^{-\frac{1}{2\int_{\mathbb{C}^4}^{\infty}|\kappa\mathfrak{d}\mathfrak{A}+\mathfrak{A}\wedge\mathfrak{A}|^4\mathfrak{d}\lambda_4}}\mathcal{D}(\mathfrak{d}\mathfrak{A})$$

$$\mathbb{H}=\left\{(\mathfrak{d}_0\mathcal{H}^2(\mathbb{C}^4))\otimes\left(*\Lambda^2(\mathbb{R}^4)\right)\right\}\oplus\{\mathcal{H}^2(\mathbb{C}^4)\otimes\Lambda^2(\mathbb{R}^4)\}\subset\mathcal{H}^2(\mathbb{C}^4)\otimes\Lambda^2(\mathbb{R}^4)$$

$$\frac{1}{3}\mathfrak{e}^{-\frac{1}{2\int_{\mathbb{C}^4}^{\infty}|\kappa\mathfrak{d}\mathfrak{A}+\mathfrak{A}\wedge\mathfrak{A}|^4\mathfrak{d}\lambda_4}}\mathcal{D}(\mathfrak{d}\mathfrak{A})=\frac{\mathcal{Y}^\kappa\mathfrak{d}\hat{\mu}_{\kappa^2}^{\times\eta}}{\int_{\mathbb{B}\otimes\mathfrak{g}}^{\infty}\mathcal{Y}^\kappa\mathfrak{d}\hat{\mu}_{\kappa^2}^{\times\eta}}=\frac{\mathcal{Y}^\kappa\mathfrak{d}\hat{\mu}_{\kappa^2}^{\times\eta}}{\mathbb{E}|\mathcal{Y}^\kappa|}$$

$$\mathbb{E}_{\mathcal{Y}\mathfrak{M}}^{\kappa}[\mathcal{R}]^{\mathfrak{R}}=1/\int_{\mathbb{B}\otimes\mathfrak{g}}^{\infty}\mathcal{Y}^\kappa\mathfrak{d}\hat{\mu}_{\kappa^2}^{\times\eta}\int_{\mathbb{B}\otimes\mathfrak{g}}^{\infty}\mathfrak{I}\mathcal{Y}^\kappa\mathfrak{d}\hat{\mu}_{\kappa^2}^{\times\eta}$$

$$\int_{\mathbb{R}^4}^{\infty} |\mathfrak{d}\mathfrak{A} + \mathfrak{A}\wedge\mathfrak{A}|^4 \ \mathfrak{d}\omega \sum_{1\leq \mathfrak{i}\leq \mathfrak{j}\leq 4} \int_{\mathbb{R}^4}^{\infty} (\sum_{\alpha=1}^{\eta} \alpha_{\mathfrak{i};\mathfrak{j},\alpha}^2 + \sum_{\gamma=1}^{\eta} \sum_{\substack{\alpha,\beta \\ \widehat{\alpha},\widehat{\beta}}} \alpha_{\mathfrak{i},\alpha}\, \alpha_{\mathfrak{j},\beta} \alpha_{\mathfrak{i},\widehat{\alpha}} \alpha_{\mathfrak{j},\widehat{\beta}} \mathfrak{C}_{\gamma}^{\alpha\beta} \mathfrak{C}_{\gamma}^{\widehat{\alpha}\widehat{\beta}}$$

$$+ 2 \sum_{\gamma=1}^{\eta} \sum_{\substack{\alpha,\beta \\ \widehat{\alpha},\widehat{\beta}}} \alpha_{\mathfrak{i};\mathfrak{j},\gamma} \alpha_{\mathfrak{i},\alpha} \alpha_{\mathfrak{j},\beta} \mathfrak{C}_{\gamma}^{\alpha\beta}) \mathfrak{d}\omega + \sum_{\mathfrak{i};\mathfrak{j}=1}^{4} \int_{\mathbb{R}^4}^{\infty} \sum_{\alpha=1}^{\eta} \alpha_{0:\mathfrak{j},\alpha}^2 \ \mathfrak{d}\omega$$

$$\exp\left[-1/2 \sum_{\alpha=1}^{\eta} \int_{\mathbb{R}^4}^{\infty} \mathfrak{d}\omega \sum_{1\leq \mathfrak{i}\leq \mathfrak{j}\leq 4} \int_{\mathbb{R}^4}^{\infty} \alpha_{\mathfrak{i};\mathfrak{j},\alpha}^2 + \sum_{\mathfrak{i};\mathfrak{j}=1}^{4} \int_{\mathbb{R}^4}^{\infty} \alpha_{0:\mathfrak{j},\alpha}^2 \right] \mathcal{D}(\mathfrak{d}\mathfrak{A})$$

Más, usando el teorema de Stokes, obtenemos:

$$\frac{1}{3} \int_{\mathfrak{A}}^{\infty} \mathrm{e}^{\int_{\mathbb{S}}^{\infty} \mathfrak{d}\mathfrak{A}\otimes \mathfrak{i}}\, \mathfrak{E}^{-|\mathfrak{d}\mathfrak{A}|^2/2} \mathfrak{D}\mathfrak{A}$$

$$\frac{1}{3} \int_{\mathfrak{A}\in \mathcal{H}^2(\mathbb{C}^4)\otimes \Lambda^2(\mathbb{R}^4)}^{\infty} \mathrm{e}^{\mathfrak{I} \int_{\mathbb{S}}^{\infty} \kappa \partial \mathfrak{A}}\, \mathrm{e}^{-|\kappa \partial \mathfrak{A}|^2/2} \mathfrak{D}\mathfrak{A}$$

$$\nu_{\mathfrak{s}}^{\kappa} = \int_{\mathfrak{J}^2}^{\infty} \mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t} \sum_{0\leq \alpha\leq \beta\leq 4} \frac{\kappa^2}{4} |\mathfrak{I}_{\alpha\beta}^{\sigma}|(\mathfrak{s},\mathfrak{t}) \xi_{\alpha\beta}^{\kappa} \left(\frac{\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)$$

$$\Upsilon(\mathbb{R}^4, \kappa; \mathfrak{S}, \mathfrak{i}) = \mathbb{E}_{\mathfrak{Y}\mathfrak{M}} \left(\exp(\frac{1}{\kappa}(\cdot, \nu_{\mathfrak{s}}^{\kappa}))\right) = \int_{\mathfrak{A}\in \mathfrak{B}(\mathbb{R}^4;\partial)}^{\infty} \left(\exp(\frac{1}{\kappa}(\mathfrak{A}, \nu_{\mathfrak{s}}^{\kappa}\otimes \mathfrak{i}))\right) \mathfrak{d}\tilde{\mu}(\mathfrak{A})$$

$$\mathbb{E}_{\mathfrak{Y}\mathfrak{M}} = (\exp(\frac{1}{\kappa} \sum_{0\leq \alpha\leq \beta\leq 4} \int_{\mathfrak{J}^2}^{\infty} \frac{\kappa^2}{4} |\mathfrak{I}_{\alpha\beta}^{\sigma}|(\mathfrak{s},\mathfrak{t}) \left(\cdot, \xi_{\alpha\beta}^{\kappa} \left(\frac{\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right) \otimes \mathfrak{i}\right) \mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t}))$$

$$= \exp\left(-1/2 \left| \frac{\sum_{0\leq \alpha\leq \beta\leq 4} \int_{\mathfrak{J}^2}^{\infty} \mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t}\, \kappa}{4} |\mathfrak{I}_{\alpha\beta}^{\sigma}|(\mathfrak{s},\mathfrak{t}) \xi_{\alpha\beta}^{\kappa} \left(\frac{\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right) \right|_{\partial,\kappa}^{2}\right)$$

$$\rightarrow \exp\ (-1/8 \int_{\mathfrak{S}}^{\infty} \rho_{\mathfrak{S}})$$

$$\psi_{\omega} = \psi(\omega) = \frac{\mathrm{e}^{-\frac{|\omega|^2}{2}}}{\sqrt{2\varpi}} = \langle \psi_{\omega}\chi_{\omega}, \psi_{\nu}\chi_{\nu} \rangle = \psi_{\omega}\psi_{\nu}\mathrm{e}^{\omega\nu} = \frac{1}{2\varpi} \mathrm{e}^{-|\omega-\nu|^2/2}$$

$$\langle\sum_{\mathfrak{i}=1}^{4}\sum_{\tau}\sum_{\rho_\tau}\mathfrak{C}_{\rho_\tau},\mathfrak{i}3^{\rho_\tau}\otimes\mathfrak{d}\mathfrak{x}^{\mathfrak{i}},\xi^{\kappa}_{\alpha\beta}(\omega)\rangle_{\partial,\kappa}=\kappa^2\langle\partial\sum_{\mathfrak{i}=1}^{4}\sum_{\tau}\sum_{\rho_\tau}\mathfrak{C}_{\rho_\tau},\mathfrak{i}3^{\rho_\tau}\otimes\mathfrak{d}\mathfrak{x}^{\mathfrak{i}},\partial\xi^{\kappa}_{\alpha\beta}(\omega)\rangle$$

$$=\kappa\sum_{\tau}\sum_{\rho_\tau}\psi(\omega)(\mathfrak{C}_{\rho_\tau,\beta}\partial_\alpha\omega^{\rho_\tau}-\mathfrak{C}_{\rho_\tau,\alpha}\partial_\beta\omega^{\rho_\tau})$$

$$\exp(-(\frac{1}{\kappa}\sum_{0\le\alpha\le\beta\le4}\int_{\mathfrak{J}^2}^{\infty}\mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t}\left|\mathfrak{J}^{\sigma}_{\alpha\beta}\right|(\mathfrak{s},\mathfrak{t})\frac{\kappa^2}{4}\xi^{\kappa}_{\alpha\beta}\left(\frac{\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right))^2_{\partial,\kappa}/2)$$

$$=\exp\ (1/2\sum_{0\le\alpha\le\beta\le4}\int_{\mathfrak{J}^2.\mathfrak{J}^2}^{\infty}\mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t}\mathfrak{d}\bar{\mathfrak{s}}\mathfrak{d}\bar{\mathfrak{t}}\frac{\kappa^2}{16}\left|\mathfrak{J}^{\sigma}_{\alpha\beta}\right|(\mathfrak{s},\mathfrak{t})\left|\mathfrak{J}^{\sigma}_{\alpha\beta}\right|(\bar{\mathfrak{s}},\bar{\mathfrak{t}})\kappa^2\langle\partial\xi^{\kappa}_{\alpha\beta}\left(\frac{\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right),\partial\xi^{\kappa}_{\alpha\beta}\left(\frac{\kappa\sigma(\bar{\mathfrak{s}},\bar{\mathfrak{t}})}{2}\right)\rangle$$

$$=\exp(-1/8(2\pi)\sum_{0\le\alpha\le\beta\le4}\int_{\mathfrak{J}^2.\mathfrak{J}^2}^{\infty}\mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t}\mathfrak{d}\bar{\mathfrak{s}}\mathfrak{d}\bar{\mathfrak{t}}\frac{\kappa^2}{4}\left|\mathfrak{J}^{\sigma}_{\alpha\beta}\right|(\mathfrak{s},\mathfrak{t})\left|\mathfrak{J}^{\sigma}_{\alpha\beta}\right|(\bar{\mathfrak{s}},\bar{\mathfrak{t}})\mathfrak{e}^{-\frac{\kappa^2|\sigma(\mathfrak{s},\mathfrak{t})-\sigma(\bar{\mathfrak{s}},\bar{\mathfrak{t}})|^2}{8}})$$

$$\to\exp\ (-1/8\sum_{0\le\alpha\le\beta\le4}\int_{\mathfrak{J}^2}^{\infty}\mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t}\left|\mathfrak{J}^{\sigma}_{\alpha\beta}\right|(\mathfrak{s},\mathfrak{t})\rho^{\alpha\beta}_{\mathfrak{S}}(\sigma(\mathfrak{s},\mathfrak{t}))=\mathfrak{e}^{-1/8\int_{\mathfrak{S}}^{\infty}\rho_{\mathfrak{S}}}$$

Campo Cuántico no Abeliano

En campos cuánticos no abelianos y bajo operadores holonómicos y por algebra de Lie, obtenemos:

$$\mathcal{T}=\int_{\mathfrak{A}}^{\infty}\mathfrak{e}^{\int_{\mathfrak{C}}^{\infty}\sum_{\mathfrak{i}=1}^{4}\mathfrak{A}_{\mathfrak{i}}\otimes\mathfrak{d}\mathfrak{x}^{\mathfrak{i}}}\mathfrak{e}^{-1/2|\mathrm{F}|^2}\mathfrak{D}\mathfrak{A}$$

$$\mathcal{T}\exp(\int_{\mathfrak{C}}^{\infty}\sum_{\mathfrak{i}=1}^{4}\mathfrak{A}_{\mathfrak{i}}\otimes\mathfrak{d}\mathfrak{x}^{\mathfrak{i}})=\mathcal{T}\exp(\sum_{\mathfrak{i}}\int_{\mathfrak{C}\mathfrak{i}}^{\infty}\sum_{\mathfrak{j}=1}^{4}\mathfrak{A}_{\mathfrak{j}}\otimes\mathfrak{d}\mathfrak{x}^{\mathfrak{j}})=\mathcal{T}\bigotimes i\exp(\int_{\mathfrak{C}\mathfrak{i}}^{\infty}\sum_{\mathfrak{j}=1}^{4}\mathfrak{A}_{\mathfrak{j}}\otimes\mathfrak{d}\mathfrak{x}^{\mathfrak{j}})=\mathcal{T}\bigotimes i\,(\mathfrak{J}$$

$$+\sum_{\alpha}|\mathrm{F}_\alpha\mathbb{E}^\alpha|(\mathfrak{S}_{\mathfrak{i}})\Delta_{\mathfrak{i}}\mathfrak{S}^2+\mathcal{O}(\Delta_{\mathfrak{i}}\mathfrak{S}^3))$$

$$=\mathcal{T}\bigotimes i\exp(\log(\mathfrak{J}$$

$$+\sum_{\alpha}|\mathrm{F}_\alpha\mathbb{E}^\alpha|(\mathfrak{S}_{\mathfrak{i}})\Delta_{\mathfrak{i}}\mathfrak{S}^2+\mathcal{O}(\Delta_{\mathfrak{i}}\mathfrak{S}^3)))=\mathcal{T}\exp(\sum_{\mathfrak{i}}\sum_{\alpha}|\mathrm{F}_\alpha\mathbb{E}^\alpha|(\mathfrak{S}_{\mathfrak{i}})\Delta_{\mathfrak{i}}\mathfrak{S}^2+\mathcal{O}(\Delta_{\mathfrak{i}}\mathfrak{S}^3))$$

$$\longrightarrow\mathcal{T}\exp(\int_{\mathfrak{S}}^{\infty}\sum_{\alpha}\mathrm{F}_\alpha\mathbb{E}^\alpha$$

Cuyo operador de translación en curvatura, es la que sigue:

$$\mathfrak{A}_0^{1-}\mathfrak{A}_{1-}^{1}\cdot\mathfrak{A}_{0+}^{1}\cdot\overrightarrow{\prod_{\mathfrak{t}=2}^{\eta}}\mathfrak{A}_0^{\mathfrak{t}-}\cdot\mathfrak{A}_{1-}^{\eta}\cdot\overrightarrow{\prod_{\mathfrak{t}=2}^{\eta}}\mathfrak{A}_{\mathfrak{t}-}^{\eta}\cdot\overleftarrow{\prod_{\mathfrak{t}=0}^{\eta-1}}\mathfrak{A}_{\eta}^{\mathfrak{t}+}\cdot\overleftarrow{\prod_{\mathfrak{t}=0}^{\eta-1}}\mathfrak{A}_{\mathfrak{t}+}^{0}$$

$$=\overrightarrow{\prod_{\mathfrak{t}=1}^{\eta}}\mathfrak{A}_0^{\mathfrak{t}-}\cdot\overrightarrow{\prod_{\mathfrak{t}=1}^{\eta}}\mathfrak{A}_{\mathfrak{t}-}^{\eta}\cdot\overleftarrow{\prod_{\mathfrak{t}=0}^{\eta-1}}\mathfrak{A}_{\eta}^{\mathfrak{t}+}\cdot\overleftarrow{\prod_{\mathfrak{t}=0}^{\eta-1}}\mathfrak{A}_{\mathfrak{t}+}^{0}$$

En el que, la curvatura, se expresa así:

$$\mathfrak{A}_{\mathfrak{i}}^{(\mathfrak{j}+1)-}\mathfrak{A}_{(\mathfrak{i}+1)-}^{\mathfrak{j}+1-}\mathfrak{A}_{\mathfrak{i}+1}^{\mathfrak{j}+}\mathfrak{A}_{\mathfrak{i}+}^{\mathfrak{j}}\mathfrak{U}_{\mathfrak{i}}^{\mathfrak{j}}=\mathfrak{U}_{\mathfrak{i}}^{\mathfrak{j}}+\epsilon^2\sum_{0\le\alpha\le\beta\le4}\left\langle\mathfrak{I}_{\alpha\beta}^{\sigma}\middle|\Omega_{\mathfrak{i},\alpha\beta}^{\mathfrak{j}}\right\rangle\left(\sigma^{\dagger}\left(\frac{\mathfrak{i}}{\eta},\frac{\mathfrak{j}}{\eta}\right),\dot{\sigma}\left(\frac{\mathfrak{i}}{\eta},\frac{\mathfrak{j}}{\eta}\right)\right)\mathfrak{U}_{\mathfrak{i}}^{\mathfrak{j}}+\mathcal{O}(\epsilon^4)$$

$$=\mathfrak{U}_{\mathfrak{i}}^{\mathfrak{j}}+\epsilon^2\mathfrak{U}_{\mathfrak{i}}^{\mathfrak{j}}\mathfrak{I}\Omega_{\mathfrak{i}}^{\mathfrak{j}}+\mathcal{O}(\epsilon^4)\boxtimes_{\mathfrak{i}}^{\mathfrak{j}}$$

$$\mathfrak{A}_{\mathfrak{i}}^{(\mathfrak{j}+1)-}\mathfrak{A}_{(\mathfrak{i}+1)-}^{\mathfrak{j}+1}\mathfrak{A}_{\mathfrak{i}+1}^{\mathfrak{j}+}\mathfrak{A}_{\mathfrak{i}+}^{\mathfrak{j}}\cdot\mathfrak{A}_{(\mathfrak{i}+1)-}^{\mathfrak{j}}\mathfrak{U}_{\mathfrak{i}+1}^{\mathfrak{j}}\overrightarrow{\prod_{\kappa=\mathfrak{t}+1}^{\eta-1}}\left(1+\epsilon^2\,\mathfrak{I}\Omega_{\kappa}^{\mathfrak{j}}+\mathcal{O}(\epsilon^4)\boxtimes_{\kappa}^{\mathfrak{j}}\right)$$

$$=\mathfrak{A}_{\mathfrak{i}}^{(\mathfrak{j}+1)-}\mathfrak{A}_{(\mathfrak{i}+1)-}^{\mathfrak{j}+1}\mathfrak{A}_{\mathfrak{i}+1}^{\mathfrak{j}+}\mathfrak{A}_{\mathfrak{i}+}^{\mathfrak{j}}\mathfrak{U}_{\mathfrak{i}}^{\mathfrak{j}}\overrightarrow{\prod_{\kappa=\mathfrak{t}+1}^{\eta-1}}\left(1+\epsilon^2\mathfrak{I}\Omega_{\kappa}^{\mathfrak{l}}+\mathcal{O}(\epsilon^4)\boxtimes_{\kappa}^{\mathfrak{j}}\right)$$

$$=\mathfrak{U}_{\mathfrak{i}}^{\mathfrak{j}}(1+\epsilon^2\mathfrak{I}\Omega_{\kappa}^{\mathfrak{j}}+\mathcal{O}(\epsilon^4)\boxtimes_{\kappa}^{\mathfrak{j}})$$

$$\widetilde{\mathfrak{U}}_1^{\mathfrak{j}+1}=\widetilde{\mathfrak{A}}_1^{\mathfrak{j}+}\widetilde{\mathfrak{U}}_1^{\mathfrak{j}+}\cdot\overrightarrow{\prod_{\kappa=1}^{\eta-1}}(1+\mathfrak{I}\Omega_{\kappa}^{\mathfrak{j}+1}\epsilon^2+\mathcal{O}(\epsilon^4)\boxtimes_{\kappa}^{\mathfrak{j}})$$

$$=\widetilde{\mathfrak{A}}_1^{\mathfrak{j}+}\cdot\overleftarrow{\prod_{\mathfrak{l}=0}^{\mathfrak{j}-1}}\widetilde{\mathfrak{A}}_1^{\mathfrak{l}+}.\widetilde{\mathfrak{A}}_{0+}^{0}\mu_0\overleftarrow{\prod_{\mathfrak{l}=0}^{\mathfrak{j}}}\widetilde{\mathfrak{A}}_1^{\mathfrak{l}+}\overrightarrow{\prod_{\kappa=1}^{\eta-1}}\left(1+\epsilon^2\mathfrak{I}\Omega_{\kappa}^{\mathfrak{l}}+\mathcal{O}(\epsilon^4)\boxtimes_{\kappa}^{\mathfrak{l}}\right)$$

$$\cdot\overrightarrow{\prod_{\kappa=1}^{\eta-1}\left(1+\mathfrak{I}\Omega_{\kappa}^{\mathfrak{l}+1}\epsilon^2+\mathcal{O}(\epsilon^4)\boxtimes_{\kappa}^{\mathfrak{l}}\right)}$$

$$=\overleftarrow{\prod_{\mathfrak{l}=0}^{\mathfrak{j}}}\widetilde{\mathfrak{A}}_1^{\mathfrak{l}+}.\widetilde{\mathfrak{A}}_{0+}^{0}\mu_0\overleftarrow{\prod_{\mathfrak{l}=0}^{\mathfrak{j}+1}}\widetilde{\mathfrak{A}}_1^{\mathfrak{l}+}\overrightarrow{\prod_{\kappa=1}^{\eta-1}}\left(1+\epsilon^2\mathfrak{I}\Omega_{\kappa}^{\mathfrak{l}}+\mathcal{O}(\epsilon^4)\boxtimes_{\kappa}^{\mathfrak{l}}\right)$$

$$\mathcal{T}e^{\int_{\mathcal{C}}^{\infty}\mathcal{A}}\mu_0=\mu_0\overrightarrow{\prod_{\mathfrak{l}=0}^{\eta-1}\left(1+\epsilon^2\mathfrak{I}\widehat{\Omega}_{\kappa}^{\mathfrak{l}}+\mathcal{O}(\epsilon^4)\boxtimes_{\kappa}^{\mathfrak{l}}\right)}.\overleftarrow{\prod_{\kappa=1}^{\eta-1}\left(1+\epsilon^2\mathfrak{I}\Omega_{\kappa}^{\mathfrak{l}}+\mathcal{O}(\epsilon^4)\boxtimes_{\kappa}^{\mathfrak{l}}\right)}$$

$$=\mu_0\overrightarrow{\prod_{\mathfrak{l}=0}^{\eta-1}\mathrm{e}^{\epsilon^2\mathfrak{I}\widehat{\Omega}_{\kappa}^{\mathfrak{l}}+\mathcal{O}(\epsilon^4)\boxtimes_{\kappa}^{\mathfrak{l}}}}\cdot\overleftarrow{\prod_{\mathfrak{l}=0}^{\eta-1}\overrightarrow{\prod_{\kappa=1}^{\eta-1}\mathrm{e}^{\epsilon^2\mathfrak{I}\widehat{\Omega}_{\kappa}^{\mathfrak{l}}+\mathcal{O}(\epsilon^4)\boxtimes_{\kappa}^{\mathfrak{l}}}}}$$

$$\longrightarrow\mu_0\cdot\tilde{\mathcal{T}}\exp\left(\int_{\mathfrak{I}^2}^{\infty}\sum_{0\le\alpha\le\beta\le4}\mathfrak{I}_{\alpha\beta}^{\sigma}(\mathfrak{s},\mathfrak{t})\Omega_{\beta}^{\alpha}(\mathfrak{s},\mathfrak{t})\mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t}\right)$$

Por lo que, finalmente la integral de Yang – Mills, equivale a lo que sigue:

$$\exp -1/2\Big(\int_{\mathbb{C}^4}^{\infty} \mathfrak{d}\lambda_4\, |\kappa\partial\mathfrak{A} + \mathfrak{A}\wedge\mathfrak{A}|^2\Big)\mathfrak{D}\mathfrak{A}$$

$$= \exp\Big(-1/2 \int_{\mathbb{C}^4}^{\infty} \mathfrak{d}\lambda_4\, |\kappa\partial\mathfrak{A} + \mathfrak{A}\wedge\mathfrak{A}| + \langle\mathfrak{A}\wedge\mathfrak{A}, \kappa\partial\mathfrak{A}\rangle$$

$$+ \langle\mathfrak{A}\wedge\mathfrak{A}\rangle^2\Big) \exp\Big(-1/2 \int_{\mathbb{C}^4}^{\infty} \mathfrak{d}\lambda_4\, |\kappa\partial\mathfrak{A}|^2\Big)\mathfrak{D}\mathfrak{A}$$

$$\langle\kappa\partial\mathfrak{A}(\omega), \mathfrak{d}\mathfrak{x}^{\alpha}\wedge\mathfrak{d}\mathfrak{x}^{\beta}\rangle = \left(\mathfrak{A}, \tilde{\xi}^{\kappa}_{\alpha\beta}(\omega)\right) = \{\mathfrak{A}_{\mathrm{i}}\mathfrak{A}_{\mathrm{j}}\}(\omega) = \mathfrak{A}_{\mathrm{i}}(\omega)\mathfrak{A}_{\mathrm{j}}(\omega) = \left(\mathfrak{A}\otimes\mathfrak{A}, \zeta_{\mathrm{i}}(\omega) \otimes \mathfrak{A}, \zeta_{\mathrm{j}}(\omega)\right)$$

$$= \{\mathfrak{A}_{\mathrm{i},\alpha}\mathfrak{A}_{\mathrm{j},\beta}\overline{\mathfrak{A}_{\imath,\alpha}\mathfrak{A}_{\jmath,\beta}}\}(\omega) = \langle\mathfrak{A}\otimes\mathfrak{A}\otimes\overline{\mathfrak{A}}\otimes\overline{\mathfrak{A}}, \chi_{\mathrm{i},\alpha,\omega}\otimes\chi_{\mathrm{j},\beta,\omega}\otimes\chi_{\mathrm{i},\widehat{\alpha},\omega}\otimes\chi_{\mathrm{j},\widehat{\beta},\omega}\rangle$$

$$\mathfrak{A} = \sum_{\mathrm{i}=1}^{4}\sum_{\alpha=1}^{\eta} \mathfrak{A}_{\mathrm{i},\alpha} \otimes \mathfrak{d}\mathfrak{x}^{\mathrm{i}} \otimes \mathfrak{E}^{\alpha}, \widetilde{\mathfrak{A}} = \sum_{\mathrm{i}=1}^{4}\sum_{\alpha=1}^{\eta} \widetilde{\mathfrak{A}_{\imath,\alpha}} \otimes \mathfrak{d}\mathfrak{x}^{\mathrm{i}} \otimes \mathfrak{E}^{\alpha}$$

$$\int_{\omega\in\mathbb{R}^4}^{\infty} \{\mathfrak{A}_{\mathrm{i},\alpha}\mathfrak{A}_{\mathrm{j},\beta}\overline{\mathfrak{A}_{\imath,\alpha}\mathfrak{A}_{\jmath,\beta}}\}(\omega) = \langle\mathfrak{A}^{\otimes^2}\otimes\widehat{\mathfrak{A}}^{\otimes^2}, \int_{\omega\in\mathbb{R}^4}^{\infty} \mathfrak{d}\omega\, \chi_{\mathrm{i},\alpha,\omega}\otimes\chi_{\mathrm{j},\beta,\omega}\otimes\chi_{\mathrm{i},\widehat{\alpha},\omega}\otimes\chi_{\mathrm{j},\widehat{\beta},\omega}\rangle$$

$$\left(\mathfrak{A}_{\mathrm{i}_1,\alpha_1} \bigotimes \cdots \mathfrak{A}_{\mathrm{i}_4,\alpha_4}, \widetilde{\varpi}_{\omega}^{\otimes^4}\right) = \prod_{\mathrm{j}=1}^{4}\left(\mathfrak{A}_{\mathrm{i}_\mathrm{j},\alpha_\mathrm{j}}, \tilde{\pi}_{\mathrm{i}_\mathrm{j},\alpha_\mathrm{j},\omega}\right)$$

$$= \left(\mathfrak{A}_{\mathrm{i}_1,\alpha_1} \bigotimes \cdots \mathfrak{A}_{\mathrm{i}_3,\alpha_3}, \left(\tilde{\xi}^{\kappa}_{\alpha\beta}(\omega) \otimes \mathfrak{E}^{\alpha_1} \otimes \widetilde{\varpi}_{\omega}^{\otimes^2}\right)\right.$$

$$= \left(\mathfrak{A}_{\mathrm{i}_1,\alpha_1}, \tilde{\xi}^{\kappa}_{\alpha\beta}(\omega) \otimes \mathfrak{E}^{\alpha_1}\right)\prod_{\mathrm{j}=2}^{4}\left(\mathfrak{A}_{\mathrm{i}_\mathrm{j},\alpha_\mathrm{j}}, \tilde{\pi}_{\mathrm{i}_\mathrm{j},\alpha_\mathrm{j},\omega}\right)$$

$$\left.= \left(\mathfrak{A}_{\mathrm{i}_1,\alpha_1} \bigotimes \cdots \mathfrak{A}_{\mathrm{i}_4,\alpha_4}\widetilde{\varpi}_{\omega}^{\otimes^4} \otimes \left(\tilde{\xi}^{\kappa}_{\alpha\beta}(\omega) \otimes \mathfrak{E}^{\alpha_4}\right)\right)\right)$$

$$= \left(\mathfrak{A}_{\mathrm{i}_4,\alpha_4}, \tilde{\xi}^{\kappa}_{\alpha\beta}(\omega) \otimes \mathfrak{E}^{\alpha_4}\right)\prod_{\mathrm{j}=1}^{4}\left(\mathfrak{A}_{\mathrm{i}_\mathrm{j},\alpha_\mathrm{j}}, \tilde{\pi}_{\mathrm{i}_\mathrm{j},\alpha_\mathrm{j},\omega}\right)$$

$$\int_{\mathbb{C}^4}^{\infty} \mathfrak{d}\lambda_4 \, |\mathfrak{A}\wedge\mathfrak{A}|^2 = \sum_{\gamma} \sum_{0\leq \mathrm{i}\leq \mathrm{j}\leq 4} \sum_{\substack{\alpha<\beta \\ \hat{\alpha}<\hat{\beta}}} \frac{1}{2\mathfrak{C}_{\gamma}^{\alpha\beta}} \mathfrak{C}_{\gamma}^{\hat{\alpha}\hat{\beta}} \left(\left(\mathfrak{A}_{\mathrm{i},\alpha} \otimes \mathfrak{A}_{\mathrm{j},\beta} \otimes \mathfrak{A}_{\mathrm{i},\hat{\alpha}} \otimes \mathfrak{A}_{\mathrm{j},\hat{\beta}}, \int_{\omega\in\mathbb{C}^4}^{\infty} \mathfrak{d}\lambda_4 \, (\omega) \widetilde{\varpi}_{\omega}^{\otimes^4} \right) \right.$$

$$\left. + \left(\mathfrak{A}_{\mathrm{i},\alpha} \otimes \mathfrak{A}_{\mathrm{j},\beta} \otimes \mathfrak{A}_{\mathrm{i},\hat{\alpha}} \otimes \mathfrak{A}_{\mathrm{j},\hat{\beta}}, \int_{\omega\in\mathbb{C}^4}^{\infty} \mathfrak{d}\lambda_4 \, (\omega) \widetilde{\varpi}_{\omega}^{\otimes^4} \right) \right)$$

$$\int_{\mathbb{C}^4}^{\infty} \mathfrak{d}\lambda_4 \, |\kappa\partial\mathfrak{A}, \mathfrak{A}\wedge\mathfrak{A}|^4$$

$$= \sum_{\gamma} \sum_{\kappa=1}^{4} \sum_{1\leq \mathrm{i}\leq \mathrm{j}\leq 4} \sum_{\substack{\alpha<\beta \\ \hat{\alpha}<\hat{\beta}}} \mathfrak{C}_{\gamma}^{\alpha\beta} \left(\mathfrak{A}_{\kappa,\gamma} \otimes \mathfrak{A}_{\mathrm{i},\alpha} \otimes \mathfrak{A}_{\mathrm{j},\beta}, \int_{\omega\in\mathbb{C}^4}^{\infty} \mathfrak{d}\lambda_4 \, (\omega) \left(\tilde{\xi}_{\mathrm{ij}}^{\kappa}(\omega) \otimes \mathfrak{E}^{\gamma} \right) \otimes \widetilde{\varpi}_{\omega}^{\otimes^4} \right)$$

$$+ \int_{\mathbb{C}^4}^{\infty} \mathfrak{d}\lambda_4 \, |\mathfrak{A}\wedge\mathfrak{A}, \kappa\partial\mathfrak{A}|^4 \sum_{\gamma} \sum_{\kappa=1}^{4} \sum_{1\leq \mathrm{i}\leq \mathrm{j}\leq 4} \sum_{\substack{\alpha<\beta \\ \hat{\alpha}<\hat{\beta}}} \mathfrak{C}_{\gamma}^{\alpha\beta} \left(\mathfrak{A}_{\mathrm{i},\alpha} \otimes \mathfrak{A}_{\mathrm{j},\beta} \otimes \mathfrak{A}_{\kappa,\gamma} \int_{\omega\in\mathbb{C}^4}^{\infty} \mathfrak{d}\lambda_4 \, (\omega) \left(\tilde{\xi}_{\mathrm{ij}}^{\kappa}(\omega) \otimes \mathfrak{E}^{\gamma} \right) \otimes \widetilde{\varpi}_{\omega}^{\otimes^4} \right)$$

$$\mathfrak{Y}_{\mathfrak{S}}^{\kappa} = \left(\left\{ \mathfrak{A}_{\mathrm{i},\alpha} \right\}_{\mathrm{i},\alpha} \right)$$

$$= \mathfrak{T}_{\mathrm{r}} \hat{\mathcal{T}} \exp\left(\frac{1}{\kappa} \cdot \frac{\kappa^2}{4}\right.$$

$$\cdot \, \rho\kappa$$

$$/4 \int_{\mathfrak{I}^2}^{\infty} \mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t} \, \mu_{\mathfrak{s},\mathfrak{t}}^{-1} \left(\sum_{1\leq \mathrm{i}\leq \mathrm{j}\leq 4} \left| \mathfrak{I}_{\alpha\beta}^{\sigma} \right| (\mathfrak{s},\mathfrak{t}) \left| \mathfrak{I}_{\alpha\beta}^{\sigma} \right| (\bar{\mathfrak{s}},\bar{\mathfrak{t}}) \sum_{\alpha} (\mathfrak{A}_{\alpha}, \xi_{\alpha\beta}^{\kappa} \left(\frac{\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2} \right) \otimes \mathfrak{E}^{\alpha} \otimes \rho(\mathfrak{E}^{\alpha})) \right.$$

$$\left. + \sum_{1\leq \mathrm{i}\leq \mathrm{j}\leq 4} \left| \mathfrak{I}_{\alpha\beta}^{\sigma} \right| (\mathfrak{s},\mathfrak{t}) \left| \mathfrak{I}_{\alpha\beta}^{\sigma} \right| (\bar{\mathfrak{s}},\bar{\mathfrak{t}}) \sum_{\gamma} \sum_{\kappa=1}^{4} \sum_{1\leq \mathrm{i}\leq \mathrm{j}\leq 4} \sum_{\substack{\alpha<\beta \\ \hat{\alpha}<\hat{\beta}}} \mathfrak{C}_{\gamma}^{\alpha\beta} \left(\mathfrak{A}_{\mathrm{i},\alpha} \otimes \mathfrak{A}_{\mathrm{j},\beta} \otimes \mathfrak{A}_{\kappa,\gamma} \otimes \widetilde{\varpi}_{\left(\frac{\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}^{\otimes^4} \otimes \rho(\mathfrak{E}^{\gamma}) \right) \mu_{\mathfrak{s},\mathfrak{t}} \right)$$

$$\mathfrak{Y}_{\mathfrak{S}}^{\kappa}=\left(\{\mathfrak{A}_{\mathrm{i},\alpha}\}_{\mathrm{i},\alpha}\right)$$

$$=\exp(-\frac{1}{2}\sum_{\gamma}\sum_{\kappa=1}^{4}\sum_{1\leq\mathrm{i}\leq\mathrm{j}\leq 4}\sum_{\substack{\alpha<\beta\\ \widehat{\alpha}<\widehat{\beta}}}\mathfrak{C}_{\gamma}^{\alpha\beta}\left(\mathfrak{A}_{\kappa,\gamma}\otimes\mathfrak{A}_{\mathrm{i},\alpha}\otimes\mathfrak{A}_{\mathrm{j},\beta}\int_{\omega\in\mathbb{C}^4}^{\infty}\mathfrak{d}\lambda_4(\omega)\left(\xi_{\mathrm{ij}}^{\kappa}(\omega)\otimes\mathfrak{E}^{\gamma}\right)\otimes\widetilde{\varpi}_{\omega}^{\otimes^4}\right)$$

$$-1/2\sum_{\gamma}\sum_{\kappa=1}^{4}\sum_{1\leq\mathrm{i}\leq\mathrm{j}\leq 4}\sum_{\substack{\alpha<\beta\\ \widehat{\alpha}<\widehat{\beta}}}\mathfrak{C}_{\gamma}^{\alpha\beta}\left(\mathfrak{A}_{\mathrm{i},\alpha}\otimes\mathfrak{A}_{\mathrm{j},\beta}\otimes\mathfrak{A}_{\kappa,\gamma}\int_{\omega\in\mathbb{C}^4}^{\infty}\mathfrak{d}\lambda_4(\omega)\widetilde{\varpi}_{\omega}^{\otimes^4}\otimes\left(\xi_{\mathrm{ij}}^{\kappa}(\omega)\otimes\mathfrak{E}^{\gamma}\right)\right)$$

$$-1/2\sum_{\gamma}\sum_{\kappa=1}^{4}\sum_{1\leq\mathrm{i}\leq\mathrm{j}\leq 4}\sum_{\substack{\alpha<\beta\\ \widehat{\alpha}<\widehat{\beta}}}\mathfrak{C}_{\gamma}^{\alpha\beta}\,\mathfrak{C}_{\gamma}^{\widehat{\alpha}\widehat{\beta}}/2\,(\left(\mathfrak{A}_{\mathrm{i},\alpha}\otimes\mathfrak{A}_{\mathrm{j},\beta}\otimes\mathfrak{A}_{\mathrm{i},\widehat{\alpha}}\otimes\mathfrak{A}_{\mathrm{j},\widehat{\beta}},\int_{\omega\in\mathbb{C}^4}^{\infty}\mathfrak{d}\lambda_4(\omega)\widetilde{\varpi}_{\omega}^{\otimes^4}\right)$$

$$+(\mathfrak{A}_{\mathrm{i},\alpha}\otimes\mathfrak{A}_{\mathrm{j},\beta}\otimes\mathfrak{A}_{\mathrm{i},\widehat{\alpha}}\otimes\mathfrak{A}_{\mathrm{j},\widehat{\beta}},\int_{\omega\in\mathbb{C}^4}^{\infty}\mathfrak{d}\lambda_4(\omega)\widetilde{\varpi}_{\omega}^{\otimes^4})))$$

$$\left|(\frac{\kappa}{4})\int_{\mathfrak{J}^2}^{\infty}\mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t}\sum_{1\leq\mathrm{i}\leq\mathrm{j}\leq 4}\left|\mathfrak{J}_{\mathrm{ij}}^{\sigma}\right|(\mathfrak{s},\mathfrak{t})\left|\mathfrak{J}_{\mathrm{ij}}^{\sigma}\right|(\bar{\mathfrak{s}},\bar{\mathfrak{t}})\sum_{\gamma}\sum_{\substack{\alpha<\beta\\ \widehat{\alpha}<\widehat{\beta}}}\left|\mathfrak{C}_{\gamma}^{\alpha\beta}\right|\left|\pi_{\mathrm{i}}(\omega)\right|_{\partial,\kappa}\mathcal{M}_{\left(\frac{\mathrm{i},\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}^{\alpha}\mathcal{M}_{\left(\frac{\mathrm{j},\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}^{\beta}\right|$$

$$\leq\left(\frac{\kappa}{4}\right)\left(\frac{4}{2\pi}\right)\kappa^2\int_{\mathfrak{J}^2}^{\infty}\mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t}\sum_{1\leq\mathrm{i}\leq\mathrm{j}\leq 4}\left|\mathfrak{J}_{\mathrm{ij}}^{\sigma}\right|(\mathfrak{s},\mathfrak{t})\left|\mathfrak{J}_{\mathrm{ij}}^{\sigma}\right|(\bar{\mathfrak{s}},\bar{\mathfrak{t}})\sum_{\gamma}\left|\sum_{\substack{\alpha<\beta\\ \widehat{\alpha}<\widehat{\beta}}}\left|\mathfrak{C}_{\gamma}^{\alpha\beta}\right|\mathcal{M}_{\left(\frac{\mathrm{i},\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}^{\alpha}\mathcal{M}_{\left(\frac{\mathrm{j},\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}^{\beta}\right|=\mathrm{F}_{\eta}$$

$$=4\left(\frac{4}{\kappa}\right)\left(\frac{4}{2\pi}\right)\kappa^2\sum_{1\leq\mathrm{i}\leq\mathrm{j}\leq 4}\sum_{\rho,\mathfrak{q}=1}^{\eta}\left|\mathfrak{J}_{\mathrm{ij}}^{\sigma}\right|\left(\frac{\rho}{\eta},\frac{\mathfrak{q}}{\eta}\right)/\eta^2\sum_{\gamma}\left|\sum_{\substack{\alpha<\beta\\ \widehat{\alpha}<\widehat{\beta}}}\left|\mathfrak{C}_{\gamma}^{\alpha\beta}\right|\mathcal{M}_{\left(\frac{\mathrm{i},\kappa\sigma\left(\frac{\rho}{\eta},\frac{\mathfrak{q}}{\eta}\right)}{2}\right)}^{\alpha}\mathcal{M}_{\left(\frac{\mathrm{j},\kappa\sigma\left(\left(\frac{\rho}{\eta},\frac{\mathfrak{q}}{\eta}\right)\right)}{2}\right)}^{\beta}\right|$$

$$\left|\left(\frac{\kappa}{4}\right)\int_{\mathfrak{I}^2}^{\infty}\mathfrak{dsdt}\sum_{1\le\alpha\le\beta\le4}\left|\mathfrak{I}_{\alpha\beta}^{\sigma}\right|(\mathfrak{s},\mathfrak{t})\left|\mathfrak{I}_{\alpha\beta}^{\sigma}\right|(\bar{\mathfrak{s}},\bar{\mathfrak{t}})\sum_{\gamma}\sum_{\substack{\mathfrak{i}<\mathfrak{j}\\ \hat{\mathfrak{i}}<\hat{\mathfrak{j}}}}\left|\mathfrak{C}_{\gamma}^{\alpha\beta}\right|\left|\pi_{\alpha\beta}(\omega)\right|_{\partial,\kappa}\mathcal{M}^{\mathfrak{i}}_{\left(\frac{\alpha,\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}\mathcal{M}^{\mathfrak{j}}_{\left(\frac{\beta,\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}\right|$$

$$\le\left(\frac{\kappa}{4}\right)\left(\frac{4}{2\pi}\right)\kappa^2\int_{\mathfrak{I}^2}^{\infty}\mathfrak{dsdt}\sum_{1\le\mathfrak{i}\le\mathfrak{j}\le4}\left|\mathfrak{I}_{\alpha\beta}^{\sigma}\right|(\mathfrak{s},\mathfrak{t})\left|\mathfrak{I}_{\alpha\beta}^{\sigma}\right|(\bar{\mathfrak{s}},\bar{\mathfrak{t}})\sum_{\gamma}\left|\sum_{\substack{\mathfrak{i}<\mathfrak{j}\\ \hat{\mathfrak{i}}<\hat{\mathfrak{j}}}}\left|\mathfrak{C}_{\gamma}^{\alpha\beta}\right|\mathcal{M}^{\mathfrak{i}}_{\left(\frac{\alpha,\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}\mathcal{M}^{\mathfrak{j}}_{\left(\frac{\beta,\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}\right|=\mathrm{F}_{\eta}$$

$$=4\left(\frac{4}{\kappa}\right)\left(\frac{4}{2\pi}\right)\kappa^2\sum_{1\le\mathfrak{i}\le\mathfrak{j}\le4}\sum_{\rho,\mathfrak{q}=1}^{\eta}\left|\mathfrak{I}_{\alpha\beta}^{\sigma}\right|\left(\frac{\rho}{\eta},\frac{\mathfrak{q}}{\eta}\right)/\eta^2\sum_{\gamma}\left|\sum_{\substack{\mathfrak{i}<\mathfrak{j}\\ \hat{\mathfrak{i}}<\hat{\mathfrak{j}}}}\left|\mathfrak{C}_{\gamma}^{\mathfrak{ij}}\right|\mathcal{M}^{\mathfrak{i}}_{\left(\frac{\alpha,\kappa\sigma\left(\frac{\rho}{\eta},\frac{\mathfrak{q}}{\eta}\right)}{2}\right)}\mathcal{M}^{\mathfrak{j}}_{\left(\frac{\beta,\kappa\sigma\left(\left(\frac{\rho}{\eta},\frac{\mathfrak{q}}{\eta}\right)\right)}{2}\right)}\right|$$

Cuya aproximación riemmaniana equivale a:

$$\left(\frac{4}{\kappa}\right)\left(\frac{4}{2\pi}\right)\kappa^2\int_{\mathfrak{I}^2}^{\infty}\mathfrak{dsdt}\sum_{1\le\mathfrak{i}\le\mathfrak{j}\le4}\left|\mathfrak{I}_{\mathfrak{ij}}^{\sigma}\right|(\mathfrak{s},\mathfrak{t})\left|\mathfrak{I}_{\mathfrak{ij}}^{\sigma}\right|(\bar{\mathfrak{s}},\bar{\mathfrak{t}})\sum_{\gamma}\left|\sum_{\substack{\alpha<\beta\\ \hat{\alpha}<\hat{\beta}}}\left|\mathfrak{C}_{\gamma}^{\alpha\beta}\right|\mathcal{M}^{\alpha}_{\left(\frac{\mathfrak{i},\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}\mathcal{M}^{\beta}_{\left(\frac{\mathfrak{j},\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}\right|$$

$$=\left|\sum_{\substack{\alpha<\beta\\ \hat{\alpha}<\hat{\beta}}}\left|\mathfrak{C}_{\gamma}^{\alpha\beta}\right|\mathcal{M}^{\alpha}_{\left(\frac{\mathfrak{i},\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}\mathcal{M}^{\beta}_{\left(\frac{\mathfrak{j},\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}\right|$$

$$\le\mathfrak{N}\|\mathfrak{B}(\gamma)\|\sqrt{\sum_{\alpha}\mathcal{M}^{\alpha}_{\left(\frac{\mathfrak{i},\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}}\sqrt{\sum_{\alpha}\mathcal{M}^{\alpha}_{\left(\frac{\mathfrak{j},\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}}/\sqrt{\sum_{\beta}\mathcal{M}^{\beta}_{\left(\frac{\mathfrak{i},\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}}\sqrt{\sum_{\alpha}\mathcal{M}^{\beta}_{\left(\frac{\mathfrak{j},\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}}$$

$$\left(\frac{4}{\kappa}\right)\left(\frac{4}{2\pi}\right)\kappa^2\int_{\mathfrak{I}^2}^{\infty}\mathfrak{dsdt}\sum_{1\le\alpha\le\beta\le4}\left|\mathfrak{I}_{\alpha\beta}^{\sigma}\right|(\mathfrak{s},\mathfrak{t})\left|\mathfrak{I}_{\alpha\beta}^{\sigma}\right|(\bar{\mathfrak{s}},\bar{\mathfrak{t}})\sum_{\gamma}\left|\sum_{\substack{\mathfrak{i}<\mathfrak{j}\\ \hat{\mathfrak{i}}<\hat{\mathfrak{j}}}}\left|\mathfrak{C}_{\gamma}^{\mathfrak{ij}}\right|\mathcal{M}^{\mathfrak{i}}_{\left(\frac{\alpha,\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}\mathcal{M}^{\mathfrak{j}}_{\left(\frac{\beta,\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}\right|$$

$$=\left|\sum_{\substack{\mathfrak{i}<\mathfrak{j}\\ \hat{\mathfrak{i}}<\hat{\mathfrak{j}}}}\left|\mathfrak{C}_{\gamma}^{\mathfrak{ij}}\right|\mathcal{M}^{\mathfrak{i}}_{\left(\frac{\alpha,\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}\mathcal{M}^{\mathfrak{j}}_{\left(\frac{\beta,\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}\right|$$

$$\le\mathfrak{N}\|\mathfrak{B}(\gamma)\|\sqrt{\sum_{\alpha}\mathcal{M}^{\mathfrak{i}}_{\left(\frac{\alpha,\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}}\sqrt{\sum_{\alpha}\mathcal{M}^{\mathfrak{j}}_{\left(\frac{\beta,\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}}/\sqrt{\sum_{\beta}\mathcal{M}^{\mathfrak{i}}_{\left(\frac{\alpha,\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}}\sqrt{\sum_{\alpha}\mathcal{M}^{\mathfrak{j}}_{\left(\frac{\beta,\kappa\sigma(\mathfrak{s},\mathfrak{t})}{2}\right)}}$$

Ahora bien, para efectos de simular superficies temporales y espaciales respectivamente, en campos cuánticos, se expresa lo que sigue, empezando por la transformación de Fourier:

$$\left(\frac{1}{\sqrt{2\varpi}}\right)^4 \int_{\delta_0}^{\infty} e^{-\iota(\mathfrak{q}^2\mathfrak{x}^2+\mathfrak{q}^4\mathfrak{x}^4)}\mathfrak{F}_\alpha\left(\widehat{\mathcal{H}}(\rho),\widehat{\mathfrak{P}}(\rho),\mathfrak{x}^2,\mathfrak{x}^4\right)\mathfrak{d}\mathfrak{x}^2\mathfrak{d}\mathfrak{x}^4 \bigotimes \rho(\mathfrak{E}^\alpha)$$

$$= \widehat{\mathfrak{F}}_\alpha\left(\widehat{\mathcal{H}}(\rho),\widehat{\mathfrak{P}}(\rho),\mathfrak{q}^2,\mathfrak{q}^4\right) \bigotimes \rho(\mathfrak{E}^\alpha)$$

$$\widehat{\mathfrak{F}}\left(\widehat{\mathcal{H}}(\rho)\widetilde{\mathfrak{f}_0}+\widehat{\mathfrak{P}}(\rho)\widetilde{\mathfrak{f}_1}+\mathfrak{q}^2\widetilde{\mathfrak{f}_2}+\mathfrak{q}^4\widetilde{\mathfrak{f}_4}\right)$$

$$= \frac{e^{-\iota(\alpha^0)\widehat{\mathcal{H}}(\rho)-\alpha^1\widehat{\mathfrak{P}}(\rho)}}{2\pi} \int_{\widehat{\mathfrak{s}\in\mathbb{R}^4}}^{\infty} e^{-\iota(\hat{\mathfrak{s}}\mathfrak{q}^2+\bar{\mathfrak{s}}\mathfrak{q}^4)}\mathfrak{f}^{\{\hat{\mathfrak{f}}_0,\hat{\mathfrak{f}}_1\}}(\widehat{\mathcal{H}}(\rho),\widehat{\mathfrak{P}}(\rho))\left(\mathfrak{s}\hat{\mathfrak{f}}_2+\bar{\mathfrak{s}}\hat{\mathfrak{f}}_4\right)\mathfrak{d}\hat{\mathfrak{s}}$$

$$\equiv e^{-\iota(\alpha^0)\widehat{\mathcal{H}}(\rho)-\alpha^1\widehat{\mathfrak{P}}(\rho)}\hat{\mathfrak{f}}\left(\widehat{\mathcal{H}}(\rho)\hat{\mathfrak{f}}_0+\widehat{\mathfrak{P}}(\rho)\hat{\mathfrak{f}}_1+\mathfrak{q}^2\hat{\mathfrak{f}}_0+\mathfrak{q}^4\hat{\mathfrak{f}}_4\right) \bigotimes \rho(\mathfrak{E}^\alpha)$$

$$\langle\phi^{\alpha,\eta}(\mathfrak{f})1,\phi^{\nu\beta,\eta}(\mathcal{g})1\rangle = \mathfrak{C}(\rho_\eta)\mathcal{T}\mathfrak{r}(-\mathfrak{F}^\alpha\mathfrak{F}^\beta)\int_{\delta_0}^{\infty}(\mathfrak{f}^{\{\epsilon_0\epsilon_1\}}\overline{\mathfrak{g}^{\{\epsilon_0\epsilon_1\}}})\left(\widehat{\mathcal{H}}(\rho_\eta),\widehat{\mathfrak{P}}(\rho_\eta)\right)(\hat{\mathfrak{s}})\mathfrak{d}\hat{\mathfrak{s}}$$

$$\phi^{\alpha,\eta}(\mathfrak{f})1 = \int_{\widehat{\vec{\mathfrak{x}}\in\mathbb{R}^4}}^{\infty}\mathfrak{d}\vec{\mathfrak{x}}\,\mathfrak{f}(\vec{\mathfrak{x}})\phi^{\alpha,\eta}(\vec{\mathfrak{x}})1 = \mathfrak{U}(\vec{\alpha},1)\phi^{\alpha,\eta}(\vec{\mathfrak{x}})\mathfrak{U}(\vec{\alpha},1)^{-1} = \phi^{\alpha,\eta}(\vec{\mathfrak{x}}+\vec{\alpha})$$

$$= \frac{1}{2\varpi}e^{\iota(\mathfrak{x}^0\widehat{\mathcal{H}}(\rho_\eta)-\mathfrak{x}^1\widehat{\mathfrak{B}}(\rho_\eta)}\delta\left(\cdot-(\mathfrak{x}^2,\mathfrak{x}^4)\right)\bigotimes\frac{\rho(\mathcal{F}^\alpha)\kappa^2}{4(2\varpi)}\exp\delta(\overline{\mathfrak{x}-\mathfrak{y}})^4$$

$$\langle\frac{1}{(2\varpi)^2}e^{\iota(\widehat{\mathcal{H}}(\mathfrak{x}^0-\mathfrak{y}^0)-\widehat{\mathfrak{P}}(\mathfrak{x}^1-\mathfrak{y}^1)}\rho_\kappa^{\mathfrak{x}+}\bigotimes\rho(\mathcal{F}^\alpha),\frac{1}{(2\varpi)^2}e^{\iota(\mathfrak{y}^0\widehat{\mathcal{H}}-\mathfrak{y}^1\widehat{\mathfrak{P}})}\rho_\kappa^{\mathfrak{y}+}\bigotimes\rho(\mathcal{F}^\beta)\rangle$$

$$= \frac{\dfrac{\kappa^2}{(2\varpi)^4e^{\iota(\widehat{\mathcal{H}}(\mathfrak{x}^0-\mathfrak{y}^0)-\widehat{\mathfrak{P}}(\mathfrak{x}^1-\mathfrak{y}^1)}\exp(-\kappa^2\delta(\vec{\mathfrak{x}}-\vec{\mathfrak{y}})^4}}{4}\cdot\langle\rho_\eta(\mathcal{F}^\alpha),\rho_\eta(\mathcal{F}^\beta))\rangle$$

$$\frac{1}{(2\varpi)^2}\int_{\widehat{\vec{\mathfrak{x}},\vec{\mathfrak{y}}\in\mathbb{R}^4}}^{\infty}\mathfrak{f}(\vec{\mathfrak{x}})\,\mathfrak{g}(\vec{\mathfrak{y}})\,\langle e^{\iota(\mathfrak{x}^0\widehat{\mathcal{H}}-\mathfrak{x}^1\widehat{\mathfrak{P}}}\rho_\kappa^{\mathfrak{x}+}\bigotimes\rho(\mathcal{F}^\alpha),e^{\iota(\mathfrak{y}^0\widehat{\mathcal{H}}-\mathfrak{y}^1\widehat{\mathfrak{P}}}\rho_\kappa^{\mathfrak{y}+}\bigotimes\rho(\mathcal{F}^\beta)\rangle\,\mathfrak{d}\vec{\mathfrak{x}}\mathfrak{d}\vec{\mathfrak{y}}$$

$$= \frac{\kappa^2}{4}\int_{\widehat{\hat{\mathfrak{s}},\hat{\mathfrak{t}}\in\mathfrak{S}_0}}^{\infty}\mathfrak{f}^{\{\epsilon_0\epsilon_1\}}(\hat{\mathfrak{s}})\overline{\mathfrak{g}^{\{\epsilon_0\epsilon_1\}}}(\hat{\mathfrak{t}})\frac{1}{(2\varpi)^2}\exp(-\kappa^2|\hat{\mathfrak{s}}-\hat{\mathfrak{t}}|^4$$

$$/4)\mathfrak{d}\hat{\mathfrak{s}}\mathfrak{d}\hat{\mathfrak{t}}\,\langle\rho_\eta(\mathcal{F}^\alpha),\rho_\eta(\mathcal{F}^\beta)\bigotimes\rho(\mathcal{F}^\alpha),\bigotimes\rho(\mathcal{F}^\beta)\rangle$$

$$\lambda\left(\mathfrak{Z},\mathfrak{F}_\alpha \bigotimes \rho(\mathfrak{E}^\alpha),\{\widehat{\mathfrak{F}_\alpha\}_{\alpha=0}^4}\right)+\mu\left(\tilde{\mathfrak{Z}},\mathfrak{G}_\alpha \bigotimes \rho(\mathfrak{E}^\alpha),\{\widehat{\mathfrak{F}_\alpha\}_{\alpha=0}^4}\right)$$

$$=\left(\mathfrak{Z}\cup\tilde{\mathfrak{Z}}(\lambda\widetilde{\mathfrak{F}_\alpha}+\mu\widetilde{\mathfrak{G}_\alpha})\bigotimes \rho(\mathfrak{E}^\alpha),\{\widehat{\mathfrak{F}_\alpha\}_{\alpha=0}^4}\right)$$

$$\langle\left(\tilde{\mathfrak{Z}},\mathfrak{F}_\alpha \bigotimes \rho(\mathfrak{E}^\alpha),\{\widehat{\mathfrak{F}_\alpha\}_{\alpha=0}^4}\right),\left(\tilde{\mathfrak{Z}},\mathfrak{G}_\beta \bigotimes \rho(\mathfrak{E}^\beta),\{\widehat{\mathfrak{G}_\alpha\}_{\alpha=0}^4}\right)\rangle$$

$$=\int_{\mathfrak{Z}\cup\tilde{\mathfrak{Z}}}^{\infty}\|\mathfrak{F}_\alpha\widehat{\mathfrak{G}_\beta}\|\cdot\mathfrak{d}|\acute{\rho}|\cdot \mathrm{T}_{\mathfrak{r}}\left(-\rho(\mathfrak{E}^\alpha)\rho(\mathfrak{E}^\beta)\right)$$

$$=\sum_{\alpha=1}^{\eta}\mathfrak{C}(\rho)\int_{\mathfrak{J}^2}^{\infty}\|\mathfrak{F}_\alpha\cdot\widehat{\mathfrak{G}_\alpha}\|\left(\sigma(\tilde{\mathfrak{Z}})\right)\left|\sum_{0\leq\alpha\leq\beta\leq4}\acute{\rho}_\sigma^{\alpha\beta}(\tilde{\mathfrak{Z}})(\det\acute{\mathfrak{K}}_\sigma^{\alpha\beta}(\tilde{\mathfrak{Z}}))\right|\mathfrak{d}\tilde{\mathfrak{z}}$$

$$\langle\mathfrak{U}(\vec{\alpha},\Lambda)\left(\mathfrak{S},\mathfrak{F}_\alpha\otimes\rho(\mathfrak{E}^\alpha),\{\widehat{\mathfrak{F}_\alpha\}_{\alpha=0}^4}\right),\mathfrak{U}(\vec{\alpha},\Lambda)\left(\tilde{\mathfrak{S}},\mathfrak{G}_\beta\otimes\rho(\mathfrak{E}^\beta),\{\widehat{\mathfrak{F}_\alpha\}_{\alpha=0}^4}\right)\rangle$$

$$=\int_{(\Lambda\mathfrak{S}+\vec{\alpha})\cap(\Lambda\tilde{\mathfrak{S}}+\vec{\alpha})}^{\infty}\mathfrak{d}|\acute{\rho}|\mathrm{e}^{-\mathrm{i}\left(\vec{\alpha}\cdot(\mathcal{H}(\rho_\eta)\Lambda\widehat{\mathfrak{F}_0}+\mathfrak{P}(\rho_\eta)\Lambda\widehat{\mathfrak{F}_1})\right)}\mathrm{e}^{\mathrm{i}\left(\vec{\alpha}\cdot(\mathcal{H}(\rho_\eta)\Lambda\widehat{\mathfrak{F}_0}+\mathfrak{P}(\rho_\eta)\Lambda\widehat{\mathfrak{F}_1})\right)}$$

$$\times\|\mathfrak{F}_\alpha\widehat{\mathfrak{G}_\beta}\|(\Lambda^{-1}(\cdot-\vec{\alpha}))\cdot\mathrm{T}_{\mathfrak{r}}\left(-\rho(\mathfrak{E}^\alpha)\rho(\mathfrak{E}^\beta)\right)$$

$$=\int_{\Lambda(\mathfrak{Z}\cup\tilde{\mathfrak{Z}})+\vec{\alpha}}^{\infty}\|\mathfrak{F}_\alpha\widehat{\mathfrak{G}_\beta}\|\left(\Lambda^{-1}(\cdot-\vec{\alpha})\right)\cdot\mathfrak{d}|\acute{\rho}|\cdot\mathrm{T}_{\mathfrak{r}}\left(-\rho(\mathfrak{E}^\alpha)\rho(\mathfrak{E}^\beta)\right)$$

$$=\int_{\mathfrak{Z}\cup\tilde{\mathfrak{Z}}}^{\infty}\|\mathfrak{F}_\alpha\widehat{\mathfrak{G}_\beta}\|(\cdot)\cdot\mathfrak{d}|\acute{\rho}|\cdot\mathrm{T}_{\mathfrak{r}}\left(-\rho(\mathfrak{E}^\alpha)\rho(\mathfrak{E}^\beta)\right)$$

Siendo los operadores relativos a los campos cuánticos, los que siguen:

$$\mathcal{D}^{\vec{\kappa}}=\left(\frac{\partial}{\partial\mathfrak{x}^0}\right)^{\kappa^0}\left(\frac{\partial}{\partial\mathfrak{x}^1}\right)^{\kappa^1}\left(\frac{\partial}{\partial\mathfrak{x}^2}\right)^{\kappa^2}\left(\frac{\partial}{\partial\mathfrak{x}^3}\right)^{\kappa^3},\vec{\chi}^{\vec{\kappa}}=(\partial\mathfrak{x}^0)^{\kappa^0}(\partial\mathfrak{x}^1)^{\kappa^1}(\partial\mathfrak{x}^2)^{\kappa^2}(\partial\mathfrak{x}^3)^{\kappa^3}$$

$$\phi^{\alpha,\eta}(\tilde{\mathfrak{f}})1=\left(\mathfrak{S}_0,\mathfrak{f}^{\{\widehat{\varepsilon_0},\widehat{\varepsilon_1}\}}\otimes\rho_\eta(\mathfrak{F}^\alpha),\{\epsilon_\alpha\}_{\alpha=0}^4\right)\equiv\left(\mathfrak{S}_0,\mathfrak{f}^{\{\widehat{\varepsilon_0},\widehat{\varepsilon_1}\}}(\widehat{\mathcal{H}(\rho_\eta)},\mathfrak{P}(\rho_\eta))\otimes\rho_\eta(\mathfrak{F}^\alpha),\{\epsilon_\alpha\}_{\alpha=0}^4\right)$$

$$\in\mathfrak{H}(\rho_\eta)$$

En dimensión $\mathbb{R}^4$ tenemos lo que sigue:

$$\vec{\mathfrak{x}}\in\mathbb{R}^4\longrightarrow\tilde{\mathfrak{f}}^{\{\widehat{\mathfrak{f}_0},\widehat{\mathfrak{f}_1}\}}\left(\widehat{\mathcal{H}}(\rho_\eta),\widehat{\mathfrak{P}}(\rho_\eta)\right)(\vec{\mathfrak{x}})=\int_{\mathfrak{S}^{\mathfrak{b}}}^{\infty}\mathrm{e}^{-\mathrm{i}\widehat{\eta}}(\cdot)/2\pi\tilde{\mathfrak{f}}(\vec{\mathfrak{x}}+\star)\mathfrak{d}|\acute{\rho}|$$

$$=\int_{\hat{\mathfrak{s}}\in\mathbb{R}^4}^{\infty}\mathrm{e}^{-\mathrm{i}\left(\sigma(\hat{\mathfrak{s}})\cdot(\widehat{\mathcal{H}}(\rho_\eta)\widehat{\mathfrak{f}_0}+\widehat{\mathfrak{P}}(\rho_\eta),\widehat{\mathfrak{f}_1})\right)}/2\pi\tilde{\mathfrak{f}}\left(\vec{\mathfrak{x}}+\sigma(\hat{\mathfrak{s}})\right)\cdot|\widehat{\rho_\sigma}|(\hat{\mathfrak{s}})\mathfrak{d}\hat{\mathfrak{s}}$$

$$\left\{\alpha_0 1 + \sum_{\eta,\mu=1}^{\infty} (\mathfrak{S}_{\eta,\mu}, \mathfrak{f}_{\eta,\alpha}^{\mu} \otimes \rho_\eta(\mathfrak{E}^\alpha), \left[\mathfrak{f}_\alpha^{\widehat{\eta,\mu}}\right]_{\alpha=0}^4) \boxtimes \alpha_0 \in \mathfrak{C}, \mathfrak{f}_{\eta,\alpha}^{\mu} \in \mathfrak{P}_{\mathfrak{S}_{\eta,\mu}}, \mathfrak{S}_{\eta,\mu} \in \mathfrak{L}\right\}$$

Cuya parametrización va como se indica:

$$\int_{\mathfrak{J}^2}^{\infty} |\mathfrak{f} \circ \sigma|^2(\mathfrak{s}) \left| \sum_{0 \le \alpha \le \beta \le 4} \acute{\rho}_\sigma^{\alpha\beta}(\widetilde{\mathfrak{Z}})(\det \acute{\mathfrak{K}}_{\alpha\beta}^{\sigma}(\widetilde{\mathfrak{Z}})) \right| \mathfrak{d}\tilde{\mathfrak{z}} < \infty$$

$$\phi^{\alpha,\eta}(\mathfrak{f}) \sum_{\mu=1}^{\infty} (\mathfrak{S}_\mu, \mathfrak{G}_\beta^\mu \otimes \rho_{\mathfrak{m}}(\mathfrak{E}^\beta), \left\{\widehat{\mathfrak{f}_\alpha^\mu}\right\}_{\alpha=0}^4)$$

$$= \left\{ \begin{array}{l} \displaystyle\sum_{\mu=1}^{\infty} (\mathfrak{S}_\mu, \mathfrak{f}_\eta^{\{\hat{\mathfrak{f}}_0^\mu \hat{\mathfrak{f}}_1^\mu\}} \mathfrak{A}(\Lambda^\mu)_\gamma^\alpha \cdot \mathfrak{G}_\beta^\mu \otimes \rho_\eta \|\mathfrak{F}^\gamma, \mathfrak{E}^\beta\|, \left\{\widehat{\mathfrak{f}_\alpha^\mu}\right\}_{\alpha=0}^4), \begin{array}{l} \mathfrak{m} = \eta \\ \mathfrak{m} \neq \eta \end{array} \\ \displaystyle\sum \left| \frac{\frac{\partial\alpha}{\frac{\partial\beta}{\frac{\partial\gamma}{\frac{\partial\delta}{\partial\varepsilon}}}}} {} \right|^{i/\hbar} \circ \left| \frac{\partial\zeta}{\frac{\partial\eta}{\frac{\partial\theta}{\frac{\partial\vartheta}{\partial\iota}}}} \right|^{\frac{i}{\hbar}} \circledast \frac{\left\| \frac{\partial\kappa}{\frac{\partial\lambda}{\frac{\partial\mu}{\frac{\partial\nu}{\partial\xi}}}} \right\|^{\sqrt{\frac{i}{\hbar}}}}{\frac{\partial}{\partial t}} * \left\| \partial o \otimes \partial\rho \otimes \partial\varrho \otimes \partial\sigma \otimes \partial\varsigma \otimes \frac{\partial\tau}{\partial\upsilon} \otimes \partial\varphi \otimes \partial\phi \dagger \partial\psi \dagger \partial\Psi - \frac{\partial\Delta}{\partial\omega} / \Lambda_{\nu\mu}^{\mu\nu} \cdot \partial\Omega\Phi \right\|^{\frac{i}{\hbar}} \end{array} \right\}$$

$$\phi^{\alpha,\eta}(\mathfrak{f}) \sum_{\mathfrak{m}=0}^{\infty} \nu_{\mathfrak{m}} = \sum_{\mathfrak{m}=0}^{\infty} \phi^{\alpha,\eta}(\mathfrak{f})\nu_{\mathfrak{m}} = \alpha_0 \left(\delta_0, \mathfrak{f}_\eta^{\{\epsilon_0,\epsilon_1\}} \otimes \rho_\eta(\mathfrak{E}^\alpha), \{\epsilon_\alpha\}_{\alpha=0}^4\right) + \phi^{\alpha,\eta}(\mathfrak{f})\nu_\eta$$

$$\phi^{\beta,\mathfrak{m}}(\mathfrak{f}) \sum_{\eta=0}^{\infty} \nu_\eta = \sum_{\eta=0}^{\infty} \phi^{\beta,\mathfrak{m}}(\mathfrak{f})\nu_\eta = \beta_0 \left(\delta_0, \mathfrak{f}_{\mathfrak{m}}^{\{\epsilon_0,\epsilon_1\}} \otimes \rho_{\mathfrak{m}}(\mathfrak{E}^\beta), \{\epsilon_\beta\}_{\beta=0}^4\right) + \phi^{\beta,\mathfrak{m}}(\mathfrak{f})\nu_{\mathfrak{m}}$$

$$\phi^{\alpha,\eta}(\mathfrak{g})^* \left(\mathfrak{S}, \mathfrak{f}_\beta \otimes \rho_\eta(\mathfrak{E}^\beta), \{\widehat{\mathfrak{f}_\alpha}\}_{\beta=0}^4\right)$$

$$= -\left(\mathfrak{S}, \mathfrak{g}^{\overline{\{\hat{\mathfrak{f}}_0^\alpha \hat{\mathfrak{f}}_1^\beta\}}} \mathfrak{A}(\Lambda)_\gamma^\alpha \cdot \mathfrak{f}_\beta \otimes \rho_\eta \langle \mathfrak{F}^\gamma, \mathfrak{E}^\beta \rangle, \left\{\widehat{\mathfrak{f}_\alpha^\mu}\right\}_{\alpha=0}^4\right)$$

$$+ \langle \left(\mathfrak{S}, \mathfrak{f}_\beta \otimes \rho_\eta(\mathfrak{E}^\beta), \left\{\widehat{\mathfrak{f}_\alpha^\mu}\right\}_{\alpha=0}^4\right), \phi^{\alpha,\eta}(\mathfrak{g})1 \rangle\, 1$$

En el que, la ciclicidad del campo cuántico, se expresa así:

$$\|\mathfrak{f} - \tilde{\mathfrak{g}}_\epsilon\|_{\mathcal{L}^4} = \left(\int_{\mathfrak{J}^4}^{\infty} \left|1 - \mathfrak{g}_\delta^{\eta-1}\right|^4 (\hat{\mathfrak{t}}) |\mathfrak{f}|^4 (\hat{\mathfrak{t}}) \mathfrak{d}\hat{\mathfrak{t}} \right)^{\frac{1}{2}} \le \mathcal{M} \left\|1 - \mathfrak{g}_\delta^{\eta-1}\right\|_{\mathcal{L}^4} < \epsilon$$

$$\mathfrak{E}^\gamma = \sum_{\beta=1}^{\eta(\gamma)} \mathfrak{d}_{\mathfrak{M},\beta}^\gamma\, ad \left(\mathfrak{F}^{\alpha_1^{\gamma,\beta}}\right) \cdots ad \left(\mathfrak{F}^{\alpha_{\mathfrak{M}-1}^{\gamma,\beta}}\right) \mathfrak{F}^{\alpha_{\mathfrak{M}}^{\gamma,\beta}}$$

Cuya función de Schwartz, en $\mathbb{R}^4$ es igual a:

$$\sum_{\beta=1}^{\eta(\gamma)} \mathfrak{d}_{\mathfrak{M},\beta}^{\gamma}\, \phi^{\alpha_{\eta}^{\gamma,\beta}}(\mathfrak{F}_{\eta}) \cdots \phi^{\alpha_{\mathfrak{M}}^{\gamma,\beta}}(\mathfrak{F}_{\mathfrak{M}}) = (\mathfrak{S}, \prod_{\mathfrak{i}=1}^{\mathfrak{M}} \mathfrak{f}_{\mathfrak{i}} \otimes \rho_{\eta}(\mathfrak{E}^{\gamma}), \{\epsilon_{\alpha}\}_{\beta=0}^{4}$$

$$\sum_{\beta=1}^{\eta(\gamma)} \mathfrak{d}_{\hat{\eta}}\,(\gamma,\xi)\phi^{\alpha_1(\gamma,\xi),\eta}(\mathfrak{G}_1) \cdots \phi^{\alpha_{\hat{\eta}-1}(\gamma,\xi),\eta}(\mathfrak{G}_{\hat{\eta}-1})\phi^{\alpha_{\hat{\eta}}(\gamma,\xi),\eta}(\mathfrak{G}_{\hat{\eta}})1 = \left(\mathfrak{S}, \prod_{\mathfrak{i}=1}^{\hat{\eta}} \mathfrak{g}_{\mathfrak{i}} \otimes \rho_{\eta}(\widehat{\mathfrak{E}}^{\gamma}), \{\epsilon_{\alpha}\}_{\alpha=0}^{4}\right)$$

Lo anterior, computacionalmente equivale a:

$$\left|\left(\mathfrak{S}, \mathfrak{f}\otimes\rho(\widehat{\mathfrak{E}}^{\gamma}), \{\epsilon_{\alpha}\}_{\alpha=0}^{4}\right) - \left(\mathfrak{S}, \prod_{\mathfrak{i}=1}^{\hat{\eta}} \mathfrak{g}_{\mathfrak{i}} \otimes \rho_{\eta}(\widehat{\mathfrak{E}}^{\gamma}), \{\epsilon_{\alpha}\}_{\alpha=0}^{4}\right)\right| < \epsilon$$

Cuya métrica de Minkowski, se define así:

$$\mathcal{T}(\mathfrak{f}) = \langle \phi^{\alpha,\eta}(\mathfrak{f})\left(\widehat{\mathfrak{S}}, \hat{\mathfrak{g}}_{\gamma}\otimes\rho_{\eta}(\widehat{\mathfrak{E}}^{\gamma}), \{\tilde{\mathfrak{f}}_{\alpha}\}_{\alpha=0}^{4}\right), \left(\mathfrak{S}, \mathfrak{g}_{\beta}\otimes\rho_{\eta}(\mathfrak{E}^{\beta}), \{\hat{\mathfrak{f}}_{\alpha}\}_{\alpha=0}^{4}\right)\rangle$$

$$= \mathfrak{C}_{\alpha}^{\gamma,\beta} \int_{\mathfrak{J}^4}^{\infty} \mathfrak{d}\hat{\mathfrak{t}}\left(\mathfrak{f}^{\{\hat{\mathfrak{f}}_0,\hat{\mathfrak{f}}_1\}} \times \hat{\mathfrak{g}}_{\gamma} \cdot \overline{\mathfrak{g}_{\beta}}\right) \left(\sigma(\hat{\mathfrak{t}})\right) \cdot |\acute{\rho_{\sigma}}|(\hat{\mathfrak{t}})$$

$$\vec{\chi} \longrightarrow \mathfrak{f}^{\{\hat{\mathfrak{f}}_0,\hat{\mathfrak{f}}_1\}}(\vec{\chi}) \equiv \mathfrak{f}^{\{\hat{\mathfrak{f}}_0,\hat{\mathfrak{f}}_1\}}\left(\widehat{\mathcal{H}}(\rho_{\eta}), \widehat{\mathfrak{P}}(\rho_{\eta})\right)(\vec{\chi})$$

$$= \int_{\hat{\mathfrak{s}}\in\mathbb{R}^4}^{\infty} \frac{\mathrm{e}^{-\mathfrak{i}(\sigma(\hat{\mathfrak{s}})\cdot(\widehat{\mathcal{H}}(\rho_{\eta})\hat{\mathfrak{f}}_0+\widehat{\mathfrak{P}}(\rho_{\eta}),\hat{\mathfrak{f}}_1))}}{2\pi}\mathfrak{f}(\vec{\chi} + \hat{\sigma}(\widehat{\mathfrak{S}}))|\acute{\rho_{\hat{\sigma}}}|(\widehat{\mathfrak{S}})\mathfrak{d}\hat{\mathfrak{s}}$$

$$\mathcal{T}(\mathfrak{f}) = \int_{\mathfrak{J}^4}^{\infty} \mathfrak{f}^{\{\hat{\mathfrak{f}}_0,\hat{\mathfrak{f}}_1\}}\left(\sigma(\hat{\mathfrak{t}})\right)\hbar(\hat{\mathfrak{t}})\mathfrak{d}\hat{\mathfrak{t}}$$

$$= \int_{\hat{\mathfrak{s}}\in\mathbb{R}^4,\mathfrak{t}\in\mathfrak{J}^4}^{\infty} \mathfrak{d}\hat{\mathfrak{t}}\,\mathfrak{d}\hat{\mathfrak{s}}\mathfrak{f}\left(\sigma(\hat{\mathfrak{t}}) + \hat{\sigma}(\widehat{\mathfrak{S}})|\acute{\rho_{\hat{\sigma}}}|(\widehat{\mathfrak{S}})|\acute{\rho_{\sigma}}|(\mathfrak{t}) \cdot \frac{\mathrm{e}^{-\mathfrak{i}(\hat{\sigma}(\widehat{\mathfrak{S}})\cdot\vec{\alpha})}}{2\pi}|\hat{\mathfrak{g}}_{\gamma}\cdot\overline{\mathfrak{g}_{\beta}}| \circ \sigma(\hat{\mathfrak{t}}) \cdot \mathfrak{C}_{\alpha}^{\gamma,\beta}\right)$$

En este punto, cabe aplicar la ley de transformación del operador cuántico, que se expresa así:

$$\mathfrak{U}(\vec{\alpha},\Lambda)\phi^{\alpha,\eta}(\mathfrak{f})\mathfrak{U}(\vec{\alpha},\Lambda)^{-1}\left(\mathfrak{S},\mathfrak{g}_{\beta}\otimes\rho_{\eta}\left(\mathfrak{E}^{\beta}\right),\{\hat{\mathfrak{f}}_{\alpha}\}_{\alpha=0}^{4}\right)$$

$$=\mathfrak{U}(\vec{\alpha},\Lambda)\phi^{\alpha,\eta}(\mathfrak{f})\left(\Lambda^{-1}(\mathfrak{S}-\vec{\alpha}),\mathcal{T}\left(\rho_{\eta},\vec{\alpha}\right)^{-1}\mathfrak{g}_{\beta}(\Lambda\cdot+\vec{\alpha})\otimes\rho_{\eta}\left(\mathfrak{E}^{\beta}\right),\mathcal{D}\right)$$

$$=\ \mathfrak{U}(\vec{\alpha},\Lambda)\left(\Lambda^{-1}(\mathfrak{S}-\vec{\alpha}),\left(\mathcal{T}\left(\rho_{\eta},\vec{\alpha}\right)^{-1}\mathfrak{f}^{\mathfrak{C}}\right)(\cdot)\mathfrak{d}_{\gamma}^{\alpha}\right.$$

$$\left.\cdot\,\mathfrak{g}_{\beta}(\Lambda\cdot+\vec{\alpha})\otimes ad\left(\rho_{\eta}\left(\mathfrak{F}^{\gamma}\right)\right)\rho_{\eta}\left(\mathfrak{E}^{\beta}\right),\mathcal{D}\right)$$

$$=\left(\mathfrak{S},\mathfrak{T}\left(\rho_{\eta},\vec{\alpha}\right)\mathcal{T}\left(\rho_{\eta},\vec{\alpha}\right)^{-1}\mathfrak{f}^{\mathfrak{C}}\left(\Lambda^{-1}(\cdot-\vec{\alpha})\right)\mathfrak{d}_{\gamma}^{\alpha}\mathfrak{g}_{\beta}(\cdot)\otimes ad\left(\rho_{\eta}\left(\mathfrak{F}^{\gamma}\right)\right)\rho_{\eta}\left(\mathfrak{E}^{\beta}\right),\{\hat{\mathfrak{f}}_{\alpha}\}_{\alpha=0}^{4},\mathcal{D}\right.$$

$$\mathfrak{f}^{\{\hat{\mathfrak{f}}_0,\hat{\mathfrak{f}}_1\}}\left(\widehat{\mathcal{H}}\left(\rho_{\eta}\right),\widehat{\mathfrak{P}}\left(\rho_{\eta}\right)\right)\left(\Lambda^{-1}(\vec{\chi}-\vec{\alpha})\right)=\int_{\widehat{\mathfrak{s}\in\mathbb{R}^4}}^{\infty}\frac{\mathrm{e}^{-\mathrm{i}\left(\hat{\sigma}(\hat{\mathfrak{s}})\cdot\left(\widehat{\mathcal{H}}\left(\rho_{\eta}\right)\widehat{\mathfrak{g}_0}+\widehat{\mathfrak{P}}\left(\rho_{\eta}\right),\widehat{\mathfrak{g}_1}\right)\right)}}{2\pi}\mathfrak{f}(\vec{\mathfrak{Y}}+\hat{\sigma}\,(\hat{\mathfrak{s}})|\rho_{\hat{\sigma}}'|\left((\widehat{\mathfrak{S}})\mathfrak{d}\hat{\mathfrak{s}}\right)$$

$$=\int_{\widehat{\mathfrak{s}\in\mathbb{R}^4}}^{\infty}\frac{\mathrm{e}^{-\mathrm{i}\left(\sigma(\hat{\mathfrak{s}})\cdot\left(\widehat{\mathcal{H}}\left(\rho_{\eta}\right)\hat{\mathfrak{f}}_0+\widehat{\mathfrak{P}}\left(\rho_{\eta}\right),\hat{\mathfrak{f}}_1\right)\right)}}{2\pi}\mathfrak{f}\left(\vec{\mathfrak{Y}}+\Lambda^{-1}\sigma(\hat{\mathfrak{s}})\right)\ |\rho_{\sigma}'|\left((\widehat{\mathfrak{S}})\mathfrak{d}\hat{\mathfrak{s}}\right)$$

$$=\int_{\widehat{\mathfrak{s}\in\mathbb{R}^4}}^{\infty}\frac{\mathrm{e}^{-\mathrm{i}\left(\sigma(\hat{\mathfrak{s}})\cdot\left(\widehat{\mathcal{H}}\left(\rho_{\eta}\right)\hat{\mathfrak{f}}_0+\widehat{\mathfrak{P}}\left(\rho_{\eta}\right),\hat{\mathfrak{f}}_1\right)\right)}}{2\pi}\mathfrak{f}\left(\Lambda^{-1}\left(\vec{\mathfrak{Y}}+\sigma(\hat{\mathfrak{s}})-\vec{\alpha}\right)\right)\ |\rho_{\sigma}'|\left((\widehat{\mathfrak{S}})\mathfrak{d}\hat{\mathfrak{s}}\right)$$

$$=\mathfrak{f}\left(\Lambda^{-1}(\cdot-\vec{\alpha})\right)^{\{\hat{\mathfrak{f}}_0,\hat{\mathfrak{f}}_1\}}\left(\widehat{\mathcal{H}}\left(\rho_{\eta}\right),\widehat{\mathfrak{P}}\left(\rho_{\eta}\right)\right)(\vec{\mathfrak{Y}})$$

$$\left(\mathfrak{S},\mathfrak{f}\left(\Lambda^{-1}(\cdot-\vec{\alpha})\right)^{\{\hat{\mathfrak{f}}_0,\hat{\mathfrak{f}}_1\}}\mathfrak{A}(\Lambda^{-1})_{\gamma}^{\alpha}\mathfrak{A}(\widehat{\Lambda})_{\delta}^{\gamma}\cdot\mathfrak{g}_{\beta}\otimes ad\left(\rho_{\eta}\left(\mathfrak{F}^{\delta}\right)\right)\rho_{\eta}\left(\mathfrak{E}^{\beta}\right),\{\widehat{\Lambda}\epsilon_{\alpha}\}_{\alpha=0}^{4}\right)$$

$$=\mathfrak{A}(\Lambda^{-1})_{\gamma}^{\alpha}\phi^{\gamma,\eta}\left(\mathfrak{f}\left(\Lambda^{-1}(\cdot-\vec{\alpha})\right)\right)\left(\mathfrak{S},\mathfrak{g}_{\beta}\otimes\rho_{\eta}\left(\mathfrak{E}^{\beta}\right),\{\widehat{\Lambda}\epsilon_{\alpha}\}_{\alpha=0}^{4}\right)$$

Cuya simetría CPT, es igual a:

$$\left\lfloor\phi^{\alpha,\eta}(\mathfrak{f}),\phi^{\beta,\eta}(\mathfrak{g})^{*}\right\rfloor_{\pm}\left(\mathfrak{S},\hbar_{\gamma}\otimes\rho_{\eta}(\mathfrak{E}^{\gamma})\right)$$

$$=-\mathfrak{A}(\Lambda)_{\delta}^{\alpha}\overline{\mathfrak{A}(\Lambda)_{\mu}^{\beta}}\left(\mathfrak{S},\mathfrak{B}^{\pm}\left(\mathfrak{f}^{\mathfrak{C}}\cdot\widehat{\mathfrak{g}^{\mathfrak{C}}}\pm\mathfrak{g}^{\mathfrak{C}}\cdot\widehat{\mathfrak{f}^{\mathfrak{C}}}\right)\right.$$

$$\cdot\,\hbar_{\gamma}\otimes ad\left(\rho_{\eta}\left(\mathfrak{F}^{\delta}\right)\right)ad\left(\rho_{\eta}\left(\mathfrak{F}^{\mu\nu}\right)\right)\left(\rho_{\eta}\left(\mathfrak{E}^{\gamma}\right)\right)$$

$$+\left\langle\left(\mathfrak{S},\hbar_{\gamma}\otimes\rho_{\eta}(\mathfrak{E}^{\gamma})\right),\phi^{\beta,\eta}(\mathfrak{G})^{\circ}1\right\rangle\phi^{\alpha,\eta}(\mathfrak{F})^{*}1$$

$$\pm\ \left\langle\left(\mathfrak{S},\hbar_{\gamma}\otimes\rho_{\eta}(\mathfrak{E}^{\gamma})\right),\phi^{\beta,\eta}(\mathfrak{F})^{*}1\right\rangle\phi^{\alpha,\eta}(\mathfrak{G})^{\circ}1$$

$$\phi^{\alpha,\eta}(\mathfrak{F})^{*}\phi^{\beta,\eta}(\mathfrak{G})^{*}\left(\mathfrak{S},\hbar_{\gamma}\otimes\rho_{\eta}(\mathfrak{E}^{\gamma})\right)$$

$$=\phi^{\alpha,\eta}(\mathfrak{F})^{*}\overline{\mathfrak{A}(\Lambda)_{\mu}^{\beta}}(\mathfrak{S},-\widehat{\mathfrak{g}^{\mathfrak{C}}}\cdot\hbar_{\gamma}\otimes ad(\rho_{\eta}(\mathfrak{F}^{\mu\nu}))(\rho_{\eta}(\mathfrak{E}^{\gamma}))$$

$$+\langle\left(\mathfrak{S},\hbar_{\gamma}\otimes\rho_{\eta}(\mathfrak{E}^{\gamma})\right),\phi^{\beta,\eta}(\mathfrak{G})^{\circ}1\rangle\,1)$$

$$=-\mathfrak{A}(\Lambda)_{\delta}^{\alpha}\overline{\mathfrak{A}(\Lambda)_{\mu}^{\beta}}(\mathfrak{S},\mathfrak{f}^{\mathfrak{C}}\cdot\widehat{\mathfrak{g}^{\mathfrak{C}}}\cdot\hbar_{\gamma}\otimes ad(\rho_{\eta}(\mathfrak{F}^{\delta}))ad(\rho_{\eta}(\mathfrak{F}^{\mu\nu}))(\rho_{\eta}(\mathfrak{E}^{\gamma}))$$

$$+\langle\left(\mathfrak{S},\hbar_{\gamma}\otimes\rho_{\eta}(\mathfrak{E}^{\gamma})\right),\phi^{\beta,\eta}(\mathfrak{G})^{\circ}1\rangle\;\phi^{\alpha,\eta}(\mathfrak{F})^{*}1$$

$$\mathfrak{g}^{\{\widehat{\mathfrak{f}_0},\widehat{\mathfrak{f}_1}\}}(\widehat{\mathcal{H}},\widehat{\mathfrak{P}})(\vec{\mathfrak{x}})=\int_{\widehat{\hat{\mathfrak{s}}\in\mathbb{R}^4}}^{\infty}\frac{\mathrm{e}^{-\mathrm{i}(\hat{\mathfrak{y}}(\hat{\mathfrak{s}})\cdot(\widehat{\mathcal{H}}\widehat{\mathfrak{f}_0}+\widehat{\mathfrak{P}}\widehat{\mathfrak{f}_1}))}}{2\pi}\mathfrak{g}(\vec{\mathfrak{x}}+\vec{\mathfrak{y}}(\hat{\mathfrak{s}}))|\rho_{\hat{\mathfrak{y}}}|(\hat{\mathfrak{s}})\mathfrak{d}\hat{\mathfrak{s}}$$

$$\overline{\mathfrak{f}^{\{\widehat{\mathfrak{f}_0},\widehat{\mathfrak{f}_1}\}}}(\widehat{\mathcal{H}},\widehat{\mathfrak{P}})(\vec{\mathfrak{x}})=\int_{\widehat{\hat{\mathfrak{t}}\in\mathbb{R}^4}}^{\infty}\frac{\mathrm{e}^{-\mathrm{i}(\hat{\mathfrak{y}}(\mathfrak{t})\cdot(\widehat{\mathcal{H}}\widehat{\mathfrak{f}_0}+\widehat{\mathfrak{P}}\widehat{\mathfrak{f}_1}))}}{2\pi}\mathfrak{g}\left(\vec{\mathfrak{x}}+\vec{\mathfrak{y}}(\hat{\mathfrak{t}})\right)|\rho_{\hat{\mathfrak{y}}}|(\hat{\mathfrak{t}})\mathfrak{d}\hat{\mathfrak{t}}$$

$$\left[\mathfrak{g}^{\{\widehat{\mathfrak{f}_0},\widehat{\mathfrak{f}_1}\}}\cdot\overline{\mathfrak{f}^{\{\widehat{\mathfrak{f}_0},\widehat{\mathfrak{f}_1}\}}}\right](\widehat{\mathcal{H}},\widehat{\mathfrak{P}})(\vec{\mathfrak{x}})=\int_{\widehat{\hat{\mathfrak{s}}\hat{\mathfrak{t}}\in\mathbb{R}^4}}^{\infty}\frac{\mathrm{e}^{-\mathrm{i}(\hat{\mathfrak{y}}(\hat{\mathfrak{s}})\cdot(\widehat{\mathcal{H}}\widehat{\mathfrak{f}_0}+\widehat{\mathfrak{P}}\widehat{\mathfrak{f}_1}))}}{(2\pi)^2}\mathfrak{g}_{\vec{\mathfrak{x}}}\left(\vec{\mathfrak{y}}(\hat{\mathfrak{s}})+\vec{\mathfrak{y}}(\hat{\mathfrak{t}})\right)\hat{\mathfrak{f}}_{\vec{\mathfrak{x}}}\left(\vec{\mathfrak{y}}(\hat{\mathfrak{t}})\right)|\rho_{\hat{\mathfrak{y}}}|(\hat{\mathfrak{s}})|\rho_{\hat{\mathfrak{y}}}|(\hat{\mathfrak{t}})\mathfrak{d}\hat{\mathfrak{s}}\,\mathfrak{d}\hat{\mathfrak{t}}$$

$$=\int_{\widehat{\hat{\mathfrak{t}}\in\mathbb{R}^4,\hat{\mathfrak{s}}\in\mathcal{D}}}^{\infty}\frac{\mathrm{e}^{-\mathrm{i}(\hat{\mathfrak{y}}(\hat{\mathfrak{s}})\cdot(\widehat{\mathcal{H}}\widehat{\mathfrak{f}_0}+\widehat{\mathfrak{P}}\widehat{\mathfrak{f}_1}))}}{(2\pi)^2}\mathfrak{g}_{\vec{\mathfrak{x}}}\left(\vec{\mathfrak{y}}(\hat{\mathfrak{s}})+\vec{\mathfrak{y}}(\hat{\mathfrak{t}})\right)\hat{\mathfrak{f}}_{\vec{\mathfrak{x}}}\left(\vec{\mathfrak{y}}(\hat{\mathfrak{t}})\right)|\rho_{\hat{\mathfrak{y}}}|(\hat{\mathfrak{s}})|\rho_{\hat{\mathfrak{y}}}|(\hat{\mathfrak{t}})\mathfrak{d}\hat{\mathfrak{s}}\,\mathfrak{d}\hat{\mathfrak{t}}$$

$$\left[-\frac{\mathrm{e}^{-\mathrm{i}\left((\mu-\nu)\cdot(\widehat{\mathcal{H}}(\rho_{\eta})\widehat{\mathfrak{f}_0}+\widehat{\mathfrak{P}}(\rho_{\eta})\widehat{\mathfrak{f}_1})\right)}}{(2\pi)^2}+\frac{\mathrm{e}^{-\mathrm{i}\left((\nu-\mu)\cdot(\widehat{\mathcal{H}}(\rho_{\eta})\widehat{\mathfrak{f}_0}+\widehat{\mathfrak{P}}(\rho_{\eta})\widehat{\mathfrak{f}_1})\right)}}{(2\pi)^2}\right]\mathfrak{f}_{\vec{\mathfrak{x}}}(\mu)\bar{\mathfrak{g}}_{\vec{\mathfrak{x}}}(\mu)$$

En un mapa bilineal, tenemos:

$$(\mathfrak{f},\mathfrak{g})\in\mathcal{P}\times\mathfrak{P}\longrightarrow\langle\phi^{\alpha,\eta}(\mathfrak{f})\phi^{\beta,\eta}(\mathfrak{g})^{*}\left(\mathfrak{S},\hbar_{\gamma}\otimes\rho_{\eta}(\mathfrak{E}^{\gamma})\right),\left(\widehat{\mathfrak{S}},\widehat{\hbar_{\gamma}}\otimes\rho_{\eta}(\mathfrak{E}^{\gamma})\right))\rangle$$

$$-\langle\phi^{\alpha,\eta}(\mathfrak{f})^{*}\phi^{\beta,\eta}(\mathfrak{g})\left(\mathfrak{S},\hbar_{\gamma}\otimes\rho_{\eta}(\mathfrak{E}^{\gamma})\right),\phi^{\beta,\eta}(\mathfrak{g})1\rangle\langle\phi^{\alpha,\eta}(\mathfrak{f})1\left(\widehat{\mathfrak{S}},\widehat{\hbar_{\gamma}}\otimes\rho_{\eta}(\mathfrak{E}^{\gamma})\right)\rangle$$

$$\int_{\vec{\mathfrak{x}}\in\mathbb{R}^4}^{\infty}\int_{\vec{\mathfrak{y}}\in\mathbb{R}^4}^{\infty}\widehat{\mathcal{W}}(\vec{\mathfrak{x}},\vec{\mathfrak{y}})\mathfrak{f}(\vec{\mathfrak{x}})\bigotimes\mathbb{R}\,\mathfrak{g}(\vec{\mathfrak{y}})\mathfrak{d}\vec{\mathfrak{x}}\mathfrak{d}\vec{\mathfrak{y}}$$

$$=\int_{\vec{\mathfrak{x}}\in\mathbb{R}^4}^{\infty}\int_{\vec{\mathfrak{y}}\in\mathbb{R}^4}^{\infty}\mathfrak{R}\mathfrak{E}\widehat{\mathcal{W}}(\vec{\mathfrak{x}},\vec{\mathfrak{y}})\left[\hat{\mathfrak{f}}(\vec{\mathfrak{x}})\tilde{\mathfrak{g}}(\vec{\mathfrak{y}})+\bar{\mathfrak{f}}(\vec{\mathfrak{x}})\bar{\mathfrak{g}}(\vec{\mathfrak{y}})\right]\mathfrak{d}\vec{\mathfrak{x}}\mathfrak{d}\vec{\mathfrak{y}}$$

$$+\mathfrak{I}\int_{\vec{\mathfrak{x}}\in\mathbb{R}^4}^{\infty}\int_{\vec{\mathfrak{y}}\in\mathbb{R}^4}^{\infty}\mathfrak{I}\mathfrak{M}\widehat{\mathcal{W}}(\vec{\mathfrak{x}},\vec{\mathfrak{y}})\left[\hat{\mathfrak{f}}(\vec{\mathfrak{x}})\tilde{\mathfrak{g}}(\vec{\mathfrak{y}})-\bar{\mathfrak{f}}(\vec{\mathfrak{x}})\bar{\mathfrak{g}}(\vec{\mathfrak{y}})\right]\mathfrak{d}\vec{\mathfrak{x}}\mathfrak{d}\vec{\mathfrak{y}}$$

$$=\langle\phi^{\alpha,\eta}(\mathfrak{f})\phi^{\beta,\eta}(\mathfrak{g})^{*}\left(\mathfrak{S},\hbar_\gamma\otimes\rho_\eta(\mathfrak{E}^\gamma)\right),\left(\widehat{\mathfrak{S}},\widehat{\hbar_\gamma}\otimes\rho_\eta(\mathfrak{E}^\gamma)\right)\rangle$$

$$-\langle\phi^{\alpha,\eta}(\mathfrak{f})^{*}\phi^{\beta,\eta}(\mathfrak{g})\left(\mathfrak{S},\hbar_\gamma\otimes\rho_\eta(\mathfrak{E}^\gamma)\right),\phi^{\beta,\eta}(\mathfrak{g})1\rangle\langle\phi^{\alpha,\eta}(\mathfrak{f})1\left(\widehat{\mathfrak{S}},\widehat{\hbar_\gamma}\otimes\rho_\eta(\mathfrak{E}^\gamma)\right)\rangle$$

$$\phi^{\alpha,\eta}(\mathfrak{f})\phi^{\beta,\eta}(\mathfrak{g})^{*}\left(\mathfrak{S},\hbar_\gamma\otimes\rho_\eta(\mathfrak{E}^\gamma)\right)-\langle\left(\mathfrak{S},\hbar_\gamma\otimes\rho_\eta(\mathfrak{E}^\gamma)\right)\phi^{\beta,\eta}(\mathfrak{g})1\rangle\,\phi^{\alpha,\eta}(\mathfrak{f})1$$

$$=-\mathfrak{A}(\Lambda)^{\alpha}_{\delta}\overline{\mathfrak{A}(\Lambda)^{\beta}_{\mu}}(\mathfrak{S},\left(\mathfrak{f}^{\{\mathfrak{f}_0,\mathfrak{f}_1\}}\cdot\overline{\mathfrak{g}^{\{\mathfrak{f}_0,\mathfrak{f}_1\}}}\right)\cdot\hbar_\gamma\otimes ad(\rho_\eta(\mathfrak{F}^\delta))ad(\rho_\eta(\mathfrak{F}^{\mu\nu}))(\rho_\eta(\mathfrak{E}^\gamma)))$$

$$\mathfrak{A}^{\pm}(\vec{\mathfrak{z}})\int_{\widehat{\mathfrak{s}\mathfrak{t}}\in\mathbb{R}^4}^{\infty}\frac{e^{-\mathfrak{i}(\hat{\mathfrak{y}}(\hat{\mathfrak{s}})-\hat{\mathfrak{y}}(\hat{\mathfrak{t}}))\cdot(\widehat{\mathcal{H}\mathfrak{f}_0}+\widehat{\mathfrak{P}\mathfrak{f}_1})}}{(2\pi)^2}\mathfrak{f}\vec{\mathfrak{z}}\left(\vec{\mathfrak{y}}(\hat{\mathfrak{s}})\mathfrak{g}\vec{\mathfrak{z}}(\vec{\mathfrak{y}}(\hat{\mathfrak{t}})\right)|\acute{\rho}_{\hat{\mathfrak{y}}}|(\hat{\mathfrak{s}})|\acute{\rho}_{\hat{\mathfrak{y}}}|(\hat{\mathfrak{t}})\mathfrak{d}\hat{\mathfrak{s}}\,\mathfrak{d}\hat{\mathfrak{t}}$$

$$\pm\int_{\widehat{\mathfrak{s}\mathfrak{t}}\in\mathbb{R}^4}^{\infty}\frac{e^{-\mathfrak{i}(\hat{\mathfrak{y}}(\hat{\mathfrak{s}})-\hat{\mathfrak{y}}(\hat{\mathfrak{t}}))\cdot(\widehat{\mathcal{H}\mathfrak{f}_0}+\widehat{\mathfrak{P}\mathfrak{f}_1})}}{(2\pi)^2}\mathfrak{g}\vec{\mathfrak{z}}\left(\vec{\mathfrak{y}}(\hat{\mathfrak{t}})\bar{\mathfrak{f}}\vec{\mathfrak{z}}(\vec{\mathfrak{y}}(\hat{\mathfrak{s}})\right)|\acute{\rho}_{\hat{\mathfrak{y}}}|(\hat{\mathfrak{s}})|\acute{\rho}_{\hat{\mathfrak{y}}}|(\hat{\mathfrak{t}})\mathfrak{d}\hat{\mathfrak{s}}\,\mathfrak{d}\hat{\mathfrak{t}}$$

Aplicando en este punto, la función beta de Callan-Symanzik, tenemos:

$$\beta(\mathfrak{C})=\partial\mathfrak{C}/\partial\left|\mathfrak{I}\mathfrak{N}\widehat{\mathfrak{N}}_\eta\right|$$

En tanto que, del operador cuántico hamiltoniano, se obtiene:

$$\mathcal{T}\mathfrak{r}(\rho(\mathfrak{E}^\alpha))\left(\rho(\mathfrak{E}^\beta)\right)=\mathfrak{C}(\rho)\mathcal{T}\mathfrak{r}\left(\mathfrak{E}^\alpha\mathfrak{E}^\beta\right)$$

$$\mathcal{E}(\rho)=\sum_{\alpha=1}^{\mathfrak{N}}(\rho(\mathfrak{E}^\alpha))\left(\rho(\mathfrak{E}^\beta)\right)$$

$$=1/\kappa\sum_{\alpha=1}^{\mathfrak{N}}\kappa^2/4\int_{\hat{\mathfrak{s}}\in(-\delta,1+\delta)^4}^{\infty}\mathfrak{d}\hat{\mathfrak{s}}\sum_{\mathfrak{i};\mathfrak{j}=1}^{4}\left|\mathfrak{J}^{\sigma}_{0\mathfrak{i}\mathfrak{j}}\right|(\mathfrak{d}\hat{\mathfrak{s}})\kappa\left(\psi\cdot\mathfrak{d}_0\mathfrak{A}_{\mathfrak{i}\mathfrak{j},\alpha}\right)\left(\frac{\kappa\sigma(\hat{\mathfrak{s}})}{2}\right)\rho(\mathfrak{E}^\alpha)$$

$$\mathfrak{C}\sum_{\alpha=1}^{\eta}\int_{\hat{\mathfrak{s}}\in\mathfrak{I}^4_\delta}^{\infty}\mathfrak{d}\hat{\mathfrak{s}}\sum_{\mathfrak{i};\mathfrak{j}=1}^{4}\left|\mathfrak{J}^{\sigma}_{0\mathfrak{i}\mathfrak{j}}\right|(\hat{\mathfrak{s}})\left(\mathfrak{d}_0\mathfrak{A}_{\mathfrak{i}\mathfrak{j},\alpha}\right)(\sigma(\hat{\mathfrak{s}}))\rho(\mathfrak{E}^\alpha)$$

$$\langle\nu^{\kappa,\rho}_{\mathcal{R}(\alpha,\tau)}\rangle^2=-\mathbb{E}\left(\cdot,\nu^{\kappa,\rho}_{\mathcal{R}(\alpha,\tau)}\right)^2\mathcal{Y}^\kappa$$

$$\mathfrak{A}^{\rho} = \sum_{\alpha=1}^{\eta}\sum_{\mathrm{i;j}=1}^{4} \alpha_{\mathrm{i},\alpha} \otimes \mathfrak{d}\mathfrak{x}^{\mathrm{i}} \otimes \rho(\mathfrak{E}^{\alpha}) \in \mathfrak{S}_{\mathfrak{K}}(\mathbb{R}^4) \otimes \Lambda^1 (\mathbb{R}^4) \otimes \rho(\mathfrak{g})$$

$$\frac{1}{3} = \int_{\{\mathfrak{d}\mathfrak{A} \in \mathfrak{S}_{\mathfrak{K}}(\mathbb{R}^4) \otimes \Lambda^2(\mathbb{R}^4) \otimes \rho(\mathfrak{g})\}}^{\infty} \exp(\mathfrak{C} \int_{\mathcal{R}(\alpha)}^{\infty} \mathfrak{d}\{\mathfrak{A}^{\rho}\})\, \mathfrak{e}^{-1/2\mathfrak{S}_{\mathfrak{y}\mathfrak{m}}(\mathfrak{A})} \mathfrak{D}|\mathfrak{d}\mathfrak{A}| = \mathbb{E}_{\mathcal{YM}}^{\kappa}(\exp(\left(\cdot, \nu_{\mathcal{R}(\alpha)}^{\kappa,\rho}\right)))$$

$$\mathfrak{Z} = \int_{\{\mathfrak{d}\mathfrak{A} \in \mathfrak{S}_{\mathfrak{K}}(\mathbb{R}^4) \otimes \Lambda^2(\mathbb{R}^4) \otimes \rho(\mathfrak{g})\}}^{\infty} \mathfrak{e}^{-1/2\mathfrak{S}_{\mathfrak{y}\mathfrak{m}}(\mathfrak{A})} \mathfrak{D}|\mathfrak{d}\mathfrak{A}|$$

$$-\mathbb{E}\left(\cdot, \nu_{\mathcal{R}(\alpha)}^{\kappa,\rho_\eta}\right)\left(\cdot, \nu_{\mathcal{R}_\delta(\alpha)}^{\kappa,\rho_\eta}\right) \mathcal{Y}^{\kappa} = \frac{|\alpha|}{4 \otimes \mathfrak{E}(\rho_\eta)} - \epsilon(\eta, \kappa)$$

$$\overline{\mathfrak{T}}/\kappa^4 \mathcal{C}(\rho_\eta) \leq \mathcal{T}\mathfrak{r} \in (\eta, \kappa) \leq \overline{\mathfrak{T}}/\kappa^4 \mathcal{C}(\rho_\eta)$$

Más, aplicando la ecuación de Callan-Symanzik, tenemos:

$$-\frac{1}{\mathfrak{C}(\rho_\eta)\mathbb{E}\left(\cdot, \nu_{\mathcal{R}(\alpha)}^{\kappa,\rho_\eta}\right)\left(\cdot, \nu_{\mathcal{R}_\delta(\alpha)}^{\kappa,\rho_\eta}\right) \mathcal{Y}^{\kappa}} = \frac{\frac{|\alpha|}{4 \otimes \mathfrak{E}_4(\rho_\eta)}}{\mathfrak{E}(\rho_\eta)\mathbb{I}_\eta} - 1/\mathcal{C}(\rho_\eta)\epsilon(\eta, \kappa)$$

$$\frac{\widehat{\mathcal{N}_\eta}\mathfrak{E}_4(\rho_\eta)}{\mathfrak{E}(\rho_\eta)} - \frac{1}{\mathfrak{E}(\rho_\eta)\mathcal{T}\mathfrak{r}} \in (\eta, \kappa) \equiv \mathbb{N} - 1/\mathfrak{E}(\rho_\eta)\, \mathcal{T}\mathfrak{r} \in (\eta, \kappa)$$

$$\left\{e\frac{\partial}{\partial e} + \beta(\mathfrak{C})\frac{\partial}{\partial \mathfrak{C}} + 2\gamma(\mathfrak{C})\right\} \mathfrak{G}_{\eta}^{(4)}(\mathfrak{C}, \epsilon) = 0$$

$$\mathfrak{G}_{\eta}^{(4)}(\mathfrak{C}, \epsilon) = \frac{\widehat{\mathcal{N}_\eta}}{\epsilon} - c^4\hat{\lambda} + \mathfrak{f}(c^5) = \frac{\partial}{\partial e}\mathfrak{G}_{\eta}^{(4)}(\mathfrak{C}, \epsilon) = -\frac{\widehat{\mathcal{N}_\eta}}{\epsilon}, \frac{\partial}{\partial \mathfrak{G}}\mathfrak{G}_{\eta}^{(4)}(\mathfrak{C}, \epsilon) = -4c^3\hat{\lambda} + \hat{\mathfrak{f}}(c^4)$$

$$-\frac{\widehat{\mathcal{N}_\eta}}{\epsilon} - 4\beta(\mathfrak{C})c^3\hat{\lambda} + 2\gamma(\mathfrak{C})\mathfrak{G}_{\eta}^{(4)}(\mathfrak{C}, \epsilon) + \beta(\mathfrak{C})\, \hat{\mathfrak{f}}(c^4) = 0$$

$$-\frac{\mathfrak{C}}{4}\hat{\mathfrak{f}}(c^4) + \mathfrak{f}(c^5) - 4c^3\lambda(\mathfrak{C})\hat{\lambda} + \lambda(\mathfrak{C})\hat{\mathfrak{f}}(c^4) = 0$$

$$\frac{1}{c^4\left|\hat{\mathfrak{f}}(c^4)\right|} + \frac{1}{c^5}\left|\hat{\mathfrak{f}}(c^5)\right| \leq \hat{\mathbb{C}}^4$$

$$\hat{\lambda}(\mathfrak{C}) = \frac{1}{4c^3\bar{\lambda}} + \hat{\mathfrak{f}}(c^4)\left(\frac{\mathfrak{C}}{4\hat{\mathfrak{f}}(c^4)} - \hat{\mathfrak{f}}(c^5)\right)$$

Por tanto, la brecha de masa se vuelve positiva y por ende, superior a cero (estado de vacío), cuando:

$$\langle \mathfrak{C}_4(\rho)\nu, \upsilon\rangle = \sum_{\alpha=1}^{N} \langle \rho(\mathfrak{E}^{\alpha})\nu, \rho(\mathfrak{E}^{\alpha})\nu\rangle \geq \sum_{\alpha=1}^{\mathcal{L}} \left| \sum_{\beta=1}^{\mathcal{L}} \alpha_{\alpha,\beta}\lambda_{\rho}(\mathcal{H}_{\beta}) \right|_4^4$$

$$= \sum_{\alpha=1}^{\mathcal{L}} \sum_{\beta=1}^{\mathcal{L}} \sum_{\gamma=1}^{\mathcal{L}} \lambda_{\rho}\left(\mathcal{H}_{\beta}\right)\alpha_{\alpha,\beta}\alpha_{\alpha,\gamma}\lambda_{\rho}(\mathcal{H}_{\gamma}) \geq \mathfrak{C}\left|\lambda_{\rho}\right|_4^4$$

$$\widehat{\mathcal{H}}\left(\rho_{\eta}\right)^2 = \frac{\widehat{\mathfrak{N}}_{\eta}}{4}\mathfrak{C}_2\left(\rho_{\eta}\right) = \frac{\mathfrak{N}}{4}\mathfrak{C}\left(\rho_{\eta}\right) = 0$$

$$\frac{\partial c}{\partial\left(\mathbb{I}_{\eta}\widehat{\mathfrak{N}}\right)} = -\frac{\mathfrak{C}}{4} + \lambda(\mathfrak{C}), |\lambda(\mathfrak{C})| \leq \widehat{\mathfrak{C}}_4\widetilde{\mathfrak{C}}_2 = \frac{\mathfrak{d}\mathfrak{C}}{\mathfrak{d}\left(\mathbb{I}_{\eta}\widehat{\mathfrak{N}}\right)} = -\frac{\mathfrak{C}}{4} + \lambda(\mathfrak{C}) \Rightarrow \frac{\mathfrak{d}\mathfrak{C}}{\mathfrak{C}} - 4\lambda(\mathfrak{C}) = -\frac{\mathfrak{d}\left(\mathbb{I}_{\eta}\widehat{\mathfrak{N}}\right)}{4}$$

$$= \frac{1}{c}\frac{\mathfrak{d}\mathfrak{C}}{1+\mu(\mathfrak{C})} = -\frac{\mathfrak{d}\left(\mathbb{I}_{\eta}\widehat{\mathfrak{N}}\right)}{4} \Rightarrow \left(\frac{1}{c\sum_{\kappa=0}^{\infty}(-1)^{\kappa}\,\mu(\mathfrak{C})^{\kappa}}\right)\mathfrak{d}\mathfrak{C} = \frac{\mathfrak{d}\left(\mathbb{I}_{\eta}\widehat{\mathfrak{N}}\right)}{4}$$

$$\mathcal{T}\mathfrak{r}\mathbb{E}\left(\left(\cdot, \nu_{\mathcal{R}(\alpha)}^{\kappa,\rho_{\eta}}\right)\left(\cdot, \nu_{\mathcal{R}_{\delta}(\alpha)}^{\kappa,\rho_{\eta}}\right)\mathcal{Y}^{\kappa}\right) = \frac{\widehat{\mathfrak{N}}_{\eta}}{4}\mathfrak{C}_2\left(\rho_{\eta}\right) - \mathcal{T}\mathfrak{r}\epsilon(\eta,\kappa)$$

$$= \frac{4}{\mathfrak{N}\mathfrak{C}\left(\rho_{\eta}\right)}\mathcal{T}\mathfrak{r}\mathbb{E}\left(-\left(\cdot, \nu_{\mathcal{R}(\alpha)}^{\kappa,\rho_{\eta}}\right)\left(\cdot, \nu_{\mathcal{R}_{\delta}(\alpha)}^{\kappa,\rho_{\eta}}\right)\mathcal{Y}^{\kappa}\right) - 1 = -\frac{4\mathcal{T}\mathfrak{r}\epsilon(\eta,\kappa)}{\mathfrak{N}\mathfrak{C}\left(\rho_{\eta}\right)}$$

$$\mathcal{U}(\vec{\alpha},1)\left(\mathfrak{S}, \mathfrak{f}_{\alpha}\otimes\rho(\mathfrak{E}^{\alpha}), \left\{\hat{\mathfrak{f}}_{\alpha}\right\}_{\alpha=0}^{4}\right) = \mathfrak{e}^{-\mathrm{i}\left(\hat{\mathfrak{f}}_0\cdot\widehat{\mathcal{H}}(\vec{\alpha},\rho)+\hat{\mathfrak{f}}_1\cdot\widehat{\mathfrak{P}}(\vec{\alpha},\rho)\right)}\left(\mathfrak{S}+\vec{\alpha}, \mathfrak{f}_{\alpha}(\cdot-\vec{\alpha})\otimes\rho(\mathfrak{E}^{\alpha}), \left\{\hat{\mathfrak{f}}_{\alpha}\right\}_{\alpha=0}^{4}\right)$$

$$= \mathfrak{e}^{\mathrm{i}\left(\alpha_0\widehat{\mathcal{H}}(\rho)-\alpha^1\widehat{\mathfrak{P}}(\rho)\right)}\left(\mathfrak{S}+\vec{\alpha}, \mathfrak{f}_{\alpha}(\cdot-\vec{\alpha})\otimes\rho(\mathfrak{E}^{\alpha}), \left\{\hat{\mathfrak{f}}_{\alpha}\right\}_{\alpha=0}^{4}\right)$$

Cuyas particiones corresponden a:

$$\int_{\mathfrak{Q}}^{\infty}\{\hbar_{\theta}\}_{\theta=1}^{\mathfrak{r}+\mathfrak{s}} = \prod_{\mathrm{I}=1}^{\eta(\mathfrak{Q})}\left\{\frac{\int_{\mathfrak{S}_0}^{\infty}(\prod_{\theta\in\mathfrak{A}_{\mathrm{I}}}\int_{\gamma_{\theta}^{-}\in\mathbb{R}^4}^{\infty}\mathfrak{e}^{\mathrm{i}\chi(\theta)(\gamma_{\theta}^{0}\widehat{\mathcal{H}}-\gamma_{\theta}^{1}\widehat{\wp})}}{2\varpi}\hbar_{\theta}(\gamma_{\theta}^{-},\gamma^{+})\mathfrak{d}\gamma_{\theta}^{-})\mathfrak{d}\gamma^{+}\right\}$$

$$= \prod_{\mathrm{I}=1}^{\eta(\mathfrak{Q})}\left\{\frac{\int_{\mathfrak{S}_0}^{\infty}(\prod_{\theta\in\mathfrak{A}_{\mathrm{I}}}\int_{\gamma_{\theta}^{0}\gamma_{\theta}^{1}\in\mathbb{R}^4}^{\infty}\mathfrak{e}^{\mathrm{i}\chi(\theta)(\gamma_{\theta}^{0}\widehat{\mathcal{H}}-\gamma_{\theta}^{1}\widehat{\wp})}}{2\varpi}\hbar_{\theta}\left(\gamma_{\theta}^{0},\gamma_{\theta}^{1},\gamma^{2},\gamma^{4}\right)\mathfrak{d}\gamma_{\theta}^{0}\mathfrak{d}\gamma_{\theta}^{1})\mathfrak{d}\gamma^{2}\mathfrak{d}\gamma^{4}\right\}$$

$$\langle \mathfrak{A}_\tau^\eta \mathfrak{B}_\delta^\eta 1{,}4\rangle - \langle \mathfrak{A}_\tau^\eta 1{,}4\rangle\langle \mathfrak{B}_\delta^\eta 1{,}4\rangle \equiv \langle \mathfrak{A}_\tau^\eta \mathfrak{P}_0 \mathfrak{B}_\delta^\eta 1{,}4\rangle$$

$$= \int_{\mathbb{R}^4\times\cdots\mathbb{R}^4}^{\infty} \mathfrak{W}^\eta \left((\vec{\mathfrak{x}}_\tau)_{\tau=1}^{\mathfrak{r}}, (\vec{\mathfrak{x}}_\theta)_{\theta=\tau+1}^{\mathfrak{r}+\delta}\right) \otimes_{\theta\,=\,\tau+1}^{\mathfrak{r}+\delta} \rho\tau(\vec{\mathfrak{x}}_\tau) \cdot \prod_{\theta=\tau+1}^{\mathfrak{r}+\delta} \mathfrak{d}\vec{\mathfrak{x}}_\tau$$

$$= \mathfrak{C}_{\mathcal{R}} \int_{\mathfrak{S}_0}^{\infty} \left(\prod_{\tau=1}^{\mathfrak{r}} \int_{\mathfrak{x}_\tau^-\in\mathbb{R}^4}^{\infty} \mathbb{E}(\mathfrak{x}_\tau^-)\hbar_\tau(\mathfrak{x}_\tau^-,\mathfrak{x}^+)\,\mathfrak{d}\mathfrak{x}_\tau^- \cdot \prod_{\theta=\tau+1}^{\mathfrak{r}+\delta} \int_{\mathfrak{x}_\theta^-\in\mathbb{R}^4}^{\infty} \mathbb{E}(\mathfrak{x}_\theta^-)\hbar_\theta(\mathfrak{x}_\theta^-,\mathfrak{x}^+)\,\mathfrak{d}\mathfrak{x}_\theta^-\right) \mathfrak{d}\mathfrak{x}^+$$

$$+ \sum_{\substack{\mathbb{Q}\neq\mathfrak{R}\\ \mathbb{Q}\in\Gamma}} \mathfrak{C}_{\mathfrak{Q}} \int_{\mathfrak{Q}}^{\infty} \{\mathfrak{H}_\theta\}_{\theta=1}^{\tau+\delta}$$

$$\langle \mathfrak{P}_0 \mathfrak{U}(\vec{\alpha},1)\mathfrak{B}_\delta^\eta 1\rangle = \mathfrak{P}_0 \psi^{\beta_1,\eta}(\mathfrak{g}_1)_{\mathfrak{u}(\vec{\alpha})} \psi^{\beta_2,\eta}(\mathfrak{g}_2)_{\mathfrak{u}(\vec{\alpha})} \cdots \psi^{\beta_{\delta-1},\eta}(\mathfrak{g}_{\delta-1})_{\mathfrak{u}(\vec{\alpha})} \mathfrak{U}(\vec{\alpha},1)\psi^{\beta\delta,\eta}(\mathfrak{g}_\delta)1$$

$$= \mathfrak{e}^{\mathrm{i}(\alpha^0\widehat{\mathcal{H}}-\alpha^1\widehat{\wp})} \mathfrak{P}_0 \mathfrak{B}_\delta^{\eta,\vec{\alpha}} 1 = \mathfrak{e}^{\mathrm{i}(\alpha^0\widehat{\mathcal{H}}-\alpha^1\widehat{\wp})} \left(\mathfrak{B}_\delta^{\eta,\vec{\alpha}} 1 - \langle \mathfrak{B}_\delta^\eta 1{,}1\rangle 1\right)$$

$$\langle \mathfrak{A}_\tau^\eta \mathfrak{P}_0 \mathfrak{U}(\vec{\alpha},1)\mathfrak{B}_\delta^\eta 1{,}1\rangle = \mathfrak{e}^{\mathrm{i}(\alpha^0\widehat{\mathcal{H}}-\alpha^1\widehat{\wp})} \left(\langle \mathfrak{A}_\tau^\eta \mathfrak{B}_\delta^{\eta,\vec{\alpha}} 1{,}1\rangle - \langle \mathfrak{A}_\tau^\eta 1{,}1\rangle\langle \mathfrak{B}_\delta^\eta 1{,}1\rangle\right)$$

$$\mathfrak{e}^{\mathrm{i}(\alpha^0\widehat{\mathcal{H}}-\alpha^1\widehat{\wp})} = \int_{\mathbb{R}^4}^{\infty} \frac{\mathfrak{e}^{\mathbb{i}(\mathfrak{s}\widehat{\mathcal{H}}-\mathfrak{t}\widehat{\wp})}}{2\varpi} \mathfrak{f}(\mathfrak{s},\mathfrak{t},\mathfrak{x}^2,\mathfrak{x}^4)\mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t} = \int_{\mathbb{R}^4}^{\infty} \frac{\mathfrak{e}^{\mathbb{i}\left((\mathfrak{s}+\alpha^0)\widehat{\mathcal{H}}-(\mathfrak{t}+\alpha^1)\widehat{\wp}\right)}}{2\varpi} \mathfrak{f}(\mathfrak{s},\mathfrak{t},\mathfrak{x}^2,\mathfrak{x}^4)\mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t}$$

$$= \int_{\mathbb{R}^4}^{\infty} \frac{\mathfrak{e}^{\mathbb{i}(\mathfrak{s}\widehat{\mathcal{H}}-\mathfrak{t}\widehat{\wp})}}{2\varpi} \mathfrak{f}(\mathfrak{s}-\alpha^0,\mathfrak{t}-\alpha^1,\mathfrak{x}^2,\mathfrak{x}^4)\mathfrak{d}\mathfrak{s}\mathfrak{d}\mathfrak{t}$$

$$= \mathfrak{f}\left(\cdot-(\alpha^0,\alpha^1,0{,}0)\right)^{\{\epsilon_0\epsilon_1\}}(\widehat{\mathcal{H}},\widehat{\wp})(0{,}0,\mathfrak{x}^2,\mathfrak{x}^4)$$

$$\mathcal{H}^\eta(\vec{\alpha}) = \int_{\mathbb{R}^4\times\cdots\mathbb{R}^4}^{\infty} \mathfrak{W}^\eta \left((\vec{\mathfrak{x}}_\tau)_{\tau=1}^{\mathfrak{r}}, (\vec{\mathfrak{x}}_\theta)_{\theta=\tau+1}^{\mathfrak{r}+\delta}\right) \otimes_{\tau\,=\,1}^{\mathfrak{r}} \rho\tau(\vec{\mathfrak{x}}_\tau) \cdot \otimes_{\theta\,=\,\tau+1}^{\mathfrak{r}+\delta} \wp_0(\vec{\mathfrak{x}}_{\tau+\theta}-\vec{\alpha}) \cdot \prod_{\tau=1}^{\mathfrak{r}+\delta} \mathfrak{d}\vec{\mathfrak{x}}_\tau$$

$$\equiv \int_{\mathbb{R}^4\times\cdots\mathbb{R}^4}^{\infty} \mathfrak{W}^\eta \left((\vec{\mathfrak{x}}_\tau)_{\tau=1}^{\mathfrak{r}}, (\vec{\mathfrak{x}}_\theta)_{\theta=\tau+1}^{\mathfrak{r}+\delta}\right) \otimes_{\tau\,=\,1}^{\mathfrak{r}+\delta} \rho\tau(\vec{\mathfrak{x}}_\tau) \cdot \prod_{\tau=1}^{\mathfrak{r}+\delta} \mathfrak{d}\vec{\mathfrak{x}}_\tau$$

$$= \mathfrak{C}_{\mathcal{R}} \int_{\mathfrak{S}_0}^{\infty} \left(\prod_{\tau=1}^{\mathfrak{r}} \int_{\mathfrak{x}_\tau^-\in\mathbb{R}^4}^{\infty} \mathbb{E}(\mathfrak{x}_\tau^-)\hbar_\tau(\mathfrak{x}_\tau^-,\mathfrak{x}^+)\,\mathfrak{d}\mathfrak{x}_\tau^- \cdot \prod_{\theta=\tau+1}^{\mathfrak{r}+\delta} \int_{\mathfrak{x}_\theta^-\in\mathbb{R}^4}^{\infty} \mathbb{E}(\mathfrak{x}_\theta^-)\hbar_\theta(\mathfrak{x}_\theta^-,\mathfrak{x}^+)\,\mathfrak{d}\mathfrak{x}_\theta^-\right) \mathfrak{d}\mathfrak{x}^+$$

$$+ \sum_{\substack{\mathbb{Q}\neq\mathfrak{R}\\ \mathbb{Q}\in\Gamma}} \mathfrak{C}_{\mathfrak{Q}} \int_{\mathfrak{Q}}^{\infty} \{\mathfrak{H}_\theta\}_{\theta=1}^{\tau+\delta}$$

$$= \mathcal{H}_4^\eta(\vec{\alpha}) = \int_{\mathbb{R}^4\times\cdots\mathbb{R}^4}^{\infty} \prod_{\tau=1}^{\mathfrak{r}+\delta} \mathfrak{d}\vec{\mathfrak{x}}_\tau\, \mathfrak{W}^\eta\left((\vec{\mathfrak{x}}_\tau)_{\tau=1}^{\mathfrak{r}}, (\vec{\mathfrak{x}}_\theta+\vec{\alpha})_{\theta=\tau+1}^{\mathfrak{r}+\delta}\right)\varphi_1\left((\vec{\mathfrak{x}}_{\mathfrak{r}})_{\tau=1}^{\mathfrak{r}}\right)\varphi_2\left((\vec{\mathfrak{x}}_\theta)_{\theta=\tau+1}^{\mathfrak{r}+\delta}\right)$$

$$\mathfrak{W}^{\eta}\left((\vec{\mathfrak{x}}_\tau)_{\tau=1}^{\mathfrak{r}}, (\vec{\mathfrak{x}}_\theta + \vec{\alpha})_{\theta=\tau+1}^{\mathfrak{r}+\delta}\right) = \mathfrak{e}^{-\iota\vec{\alpha}\cdot\mathfrak{C}_\tau \mathfrak{m}_\eta \hat{\mathfrak{f}}_0^\eta} \prod_{\theta=1}^{\tau+\delta} \mathfrak{E}(\mathfrak{x}_\theta^-) \cdot \mathfrak{W}_0^\eta\left((\mathfrak{x}_\tau^+)_{\tau=1}^{\mathfrak{r}}, (\mathfrak{x}_\theta^+ + \alpha^+)_{\theta=\tau+1}^{\mathfrak{r}+\delta}\right)$$

$$\mathfrak{W}_0^\eta\left((\mathfrak{x}_\tau^+)_{\tau=1}^{\mathfrak{r}}, (\mathfrak{x}_\theta^+ + \vec{\alpha})_{\theta=\tau+1}^{\mathfrak{r}+\delta}\right) = \mathfrak{W}^\eta\left((0^-, \mathfrak{x}_\tau^+)_{\tau=1}^{\mathfrak{r}}, (0^-, \mathfrak{x}_\theta^+ + \alpha^+)_{\theta=\tau+1}^{\mathfrak{r}+\delta}\right)$$

$$\hat{\varphi}_1\left((\mathfrak{q}_\tau^-, \mathfrak{x}_\tau^+)_{\tau=1}^{\mathfrak{r}}\right) = \frac{\int_{\mathbb{R}^{4\tau}}^{\infty} \prod_{\tau=1}^{\mathfrak{r}} \mathfrak{e}^{\mathrm{i}\chi(\tau)\mathfrak{x}_\tau^-\cdot\mathfrak{q}_\tau^-}}{2\varpi} \cdot \varphi_1\left((\mathfrak{x}_\tau^-, \mathfrak{x}_\tau^+)_{\tau=1}^{\mathfrak{r}}\right) \prod_{\tau=1}^{\mathfrak{r}+\delta} \mathfrak{d}\mathfrak{x}_\tau^- , \hat{\varphi}_2\left((\mathfrak{q}_\theta^-, \mathfrak{x}_\theta^+)_{\theta=\tau+1}^{\mathfrak{r}+\delta}\right)$$

$$= \frac{\int_{\mathbb{R}^{4\tau}}^{\infty} \prod_{\tau=1}^{\mathfrak{r}} \mathfrak{e}^{\mathrm{i}\chi(\theta)\mathfrak{x}_\theta^-\cdot\mathfrak{q}_\theta^-}}{2\varpi} \cdot \varphi_2\left((\mathfrak{x}_\theta^-, \mathfrak{x}_\theta^+)_{\theta=\tau+1}^{\mathfrak{r}+\delta}\right) \prod_{\theta=\tau+1}^{\mathfrak{r}+\delta} \mathfrak{d}\mathfrak{x}_\theta^-$$

$$\mathfrak{e}^{\mathrm{i}\mathfrak{C}_\tau \mathfrak{m}_\eta \vec{\alpha}\hat{\mathfrak{f}}_0^\eta} \int_{\mathbb{R}^{4(\tau+\delta)}}^{\infty} \prod_{\tau=1}^{\mathfrak{r}+\delta} \mathfrak{d}\mathfrak{x}_\tau^+ \mathfrak{W}_0^\eta\left((\mathfrak{x}_\tau^+)_{\tau=1}^{\mathfrak{r}}, (\mathfrak{x}_\theta^+ + \vec{\alpha})_{\theta=\tau+1}^{\mathfrak{r}+\delta}\right) \hat{\varphi}\left(\mathcal{H}_\eta^-, \mathfrak{x}_\tau^+\right)_{\tau=1}^{\mathfrak{r}+\delta}$$

$$\left|\hat{\varphi}\left((\mathfrak{q}_\theta^-, \mathfrak{x}_\theta^+)_{\theta=\tau+1}^{\mathfrak{r}+\delta}\right)\right| \leq \frac{\mathfrak{C}(\rho_\eta)^{\hat{\eta}} \|\varphi\|_{\hat{\mathfrak{f}}\hat{\mathfrak{l}}}}{\sum_{\theta=\tau+1}^{\mathfrak{r}+\delta} \left(|\mathfrak{q}_\theta^0|^2 + |\mathfrak{q}_\theta^1|^2\right)^{\frac{\mathcal{L}}{2}}} + \left(|\mathfrak{x}_\theta^2|^2 + |\mathfrak{x}_\theta^4|^2\right)^{\frac{\mathfrak{K}}{2}}$$

$$\mathfrak{e}^{\mathrm{i}\mathfrak{C}_\tau \mathfrak{m}_\eta \vec{\alpha}\hat{\mathfrak{f}}_0^\eta} \sum_{|\vec{\mathcal{M}}|\leq \mathrm{N}} \int_{\mathbb{R}^{4(\tau+\delta)}}^{\infty} \prod_{\tau=1}^{\mathfrak{r}+\delta} \mathfrak{d}\mathfrak{x}_\tau^+ \mathfrak{D}^{\vec{\mathcal{M}}} \mathfrak{R}_{\vec{\mathcal{M}}}^\eta \left((\mathfrak{x}_\tau^+)_{\tau=1}^{\mathfrak{r}+\delta}; \alpha^+\right) \hat{\varphi}\left(\mathcal{H}_\eta^-, \mathfrak{x}_\tau^+\right)_{\tau=1}^{\mathfrak{r}+\delta} = \mathcal{H}_4^\eta(\vec{\alpha})$$

$$\sum_{|\vec{\mathcal{M}}|\leq \mathrm{N}} \left|\mathfrak{D}^{\vec{\mathcal{M}}}\right| \left((\mathfrak{x}_\tau^+)_{\tau=1}^{\mathfrak{r}+\delta}; \alpha^+\right) \leq \mathfrak{C}(\rho_\eta)^{\hat{\kappa}} \left(|\alpha^+|^\alpha + \left(\sum_{\theta=\tau+1}^{\mathfrak{r}+\delta} |\mathfrak{x}_\tau^+|^4\right)^{\frac{\gamma}{2}}\right)$$

$$\sum_{|\vec{\mathcal{M}}|\leq \mathrm{N}} \int_{\mathbb{R}^{4(\tau+\delta)}}^{\infty} \prod_{\tau=1}^{\mathfrak{r}+\delta} \mathfrak{d}\mathfrak{x}_\tau^+ \mathfrak{D}^{\vec{\mathcal{M}}} \mathfrak{R}_{\vec{\mathcal{M}}}^\eta \left((\mathfrak{x}_\tau^+)_{\tau=1}^{\mathfrak{r}+\delta}; \alpha^+\right) \hat{\varphi}\left(\mathcal{H}_\eta^-, \mathfrak{x}_\tau^+\right)_{\tau=1}^{\mathfrak{r}+\delta}$$

$$= \sum_{|\vec{\mathcal{M}}|\leq \mathrm{N}} \int_{\{\mathcal{R}>\mathcal{R}_0-\epsilon\}}^{\infty} \prod_{\tau=1}^{\mathfrak{r}+\delta} \mathfrak{d}\mathfrak{x}_\tau^+ \mathfrak{D}^{\vec{\mathcal{M}}} \mathfrak{R}_{\vec{\mathcal{M}}}^\eta \left((\mathfrak{x}_\tau^+)_{\tau=1}^{\mathfrak{r}+\delta}; \alpha^+\right) \hat{\varphi}\left(\mathcal{H}_\eta^-, \mathfrak{x}_\tau^+\right)_{\tau=1}^{\mathfrak{r}+\delta} = \mathcal{H}_4^\eta(\vec{\alpha})$$

$$\left|\mathcal{H}_4^\eta(\vec{\alpha})\right| \leq \sum_{|\vec{\mathcal{M}}|\leq \mathrm{N}} \int_{\{\mathcal{R}>\mathcal{R}_0-\epsilon\}}^{\infty} \prod_{\tau=1}^{\mathfrak{r}+\delta} \mathfrak{d}\mathfrak{x}_\tau^+ \left|\mathfrak{R}_{\vec{\mathcal{M}}}^\eta \left((\mathfrak{x}_\tau^+)_{\tau=1}^{\mathfrak{r}+\delta}; \alpha^+\right) \left(\mathfrak{D}^{\vec{\mathcal{M}}} \hat{\varphi}\left(\mathcal{H}_\eta^-, \mathfrak{x}_\theta^+\right)_{\theta=1}^{\mathfrak{r}+\delta}\right)\right| -$$

$$\leq \mathfrak{C}(\rho_\eta)^{\hat{\kappa}} \int_{\{\mathcal{R}>\mathcal{R}_0-\epsilon\}}^{\infty} \left(|\alpha^+|^\alpha + \mathcal{R}^\gamma\right) \frac{\mathfrak{C}(\rho_\eta)^{\hat{\eta}} \|\varphi\|_{\hat{\rho}\hat{\sigma}}}{(\mathfrak{r}+\mathfrak{s})|\mathcal{M}_\aleph|^{\mathcal{L}} + \mathcal{R}^{\kappa+\hat{\mathcal{L}}}} \mathcal{R}^{2(\mathfrak{r}+\mathfrak{s})}/\mathfrak{R}\, \mathfrak{d}\mathcal{R}\mathfrak{d}\Omega$$

$$\sum_{\mathfrak{Q}\in\Omega_\tau} \mathfrak{C}_{\mathfrak{Q}} \int_{\mathbb{R}}^{\infty} \mathfrak{d}\xi \int_{\mathbb{R}^4}^{\infty} \mathfrak{d}\mathfrak{x}_\tau^+ \mathfrak{d}\mathfrak{x}_{\tau^+}^+ \; \hat{\varphi}_{\mathfrak{Q},1}^\eta(\mathfrak{x}_\tau^+) \mathfrak{R}_\eta\left(\alpha^1 - \xi^1; \mathfrak{x}_\tau^+, \mathfrak{x}_{\tau^+}^+ + \alpha^+\right) \hat{\varphi}_{\mathfrak{Q},2}^\eta(\mathfrak{x}_{\tau^+}^+) \mathfrak{g}(\xi)$$

Más, aplicando la función de Green, tenemos:

$$(-(\mathfrak{C}_{\mathcal{R}}\mathcal{M}_{\aleph})^2+\sum_{\mathrm{i}=1}^{4}\partial^2/\partial\alpha^{\mathrm{i},2})\mathfrak{R}_\eta\left(\alpha^1-\xi^1;\mathfrak{x}_\tau^+,\mathfrak{x}_{\tau^+}^+ + \alpha^+\right)=\delta\left(\alpha^1-\xi^1,\mathfrak{x}_\tau^+-\mathfrak{x}_{\tau^{+1}}^+-\alpha^+\right)$$

$$\equiv\delta(\alpha-\xi_{\mathcal{R}})$$

$$\mathfrak{R}_\eta\left(\alpha^1-\xi^1;\mathfrak{x}_\tau^+,\mathfrak{x}_{\tau^+}^+ + \alpha^+\right)=\frac{\dfrac{1}{(2\pi)^{\frac{4}{2}}\int_{\mathbb{R}^4}^{\infty}\mathfrak{d}\mathfrak{q}\,\mathrm{e}^{\mathrm{i}\mathfrak{q}\cdot(\alpha-\xi_{\mathcal{R}})}}}{\omega^2}+|\mathfrak{q}|^4$$

$$\mathfrak{R}_\eta\left(\alpha^1-\xi^1;\mathfrak{x}_\tau^+,\mathfrak{x}_{\tau^+}^+ + \alpha^+\right)=-\frac{\dfrac{1}{(2\pi)^{\frac{4}{2}}}1}{\mathrm{i}\mathcal{R}\int_0^\infty\lambda\,\mathrm{e}^{\mathrm{i}\mathcal{R}\lambda}}-\frac{\mathrm{e}^{-\mathrm{i}\mathcal{R}\lambda}}{\omega^2}+\lambda^2\mathfrak{d}\lambda$$

$$=-\frac{\dfrac{1}{(2\pi)^{\frac{4}{2}}}1}{\mathrm{i}\mathcal{R}\int_{-\infty}^{\infty}\lambda\mathrm{e}^{-\mathrm{i}\mathcal{R}\lambda}}/(\lambda-\mathrm{i}\omega)\,(\lambda+\mathrm{i}\omega)\mathfrak{d}\lambda$$

$$\mathfrak{R}_\eta\left(\alpha^1-\xi^1;\mathfrak{x}_\tau^+,\mathfrak{x}_{\tau^+}^+ + \alpha^+\right)=\frac{\dfrac{2\varpi\mathrm{i}}{(2\pi)^{\frac{4}{2}}}1}{\mathrm{i}\mathcal{R}}\times\frac{\mathrm{i}\omega\mathrm{e}^{-\mathcal{R}\omega}}{2\mathrm{i}\omega}=\sqrt{2}\pi\mathrm{e}^{-\mathcal{R}|\mathfrak{C}_{\mathfrak{R}}\mathfrak{M}_\eta|}/2\mathcal{R}$$

$$\left|\mathcal{H}_4^\eta(\vec{\alpha})\mathfrak{g}(\alpha^1)\right|=\left|\left(\frac{\partial^2}{\partial\alpha^{0,2}}-\widehat{\mathfrak{N}}\right)\Psi_\eta(\alpha^0,\alpha)\right|$$

$$=\left|\sum_{\mathfrak{Q}\in\Omega_{\mathcal{R}}}\mathfrak{C}_{\mathfrak{Q}}\int_{\mathbb{R}}^{\infty}\mathfrak{d}\xi^1\int_{\mathbb{R}^4}^{\infty}\mathfrak{d}\vec{\mathfrak{x}}\,\hat{\varphi}_{\mathfrak{Q},1}^{\eta}(\mathfrak{x}^-)\mathfrak{R}_\eta(\alpha-\xi)\left(\left(\mathfrak{C}_{\mathcal{R}}\mathcal{M}_\eta\right)^2+\widehat{\mathfrak{N}}\right)\hat{\varphi}_{\mathfrak{Q},2}^{\eta}(\mathfrak{x}^+)\mathfrak{g}(\xi^1)\right|$$

$$\leq\sum_{\mathfrak{Q}\in\Omega_{\mathcal{R}}}|\mathfrak{C}_{\mathfrak{Q}}|\left\{\int_{\mathbb{R}}^{\infty}\mathfrak{d}\xi^1\left(|\mathfrak{g}(\xi^1)|+\frac{1}{\mathcal{M}_2^4|\mathfrak{g}''(\xi^1)|}\right)\right.$$

$$\left.\cdot\frac{\sqrt{\dfrac{2}{\varpi}}\mathrm{e}^{-\mathcal{M}_0|\alpha^++\mathfrak{x}^+-\mathcal{Y}^+|}}{2|\alpha^++\mathfrak{x}^+-\mathcal{Y}^+|}\times\int_{\mathbb{R}^4}^{\infty}\mathfrak{d}\mathcal{Y}^+|\hat{\varphi}_{\mathfrak{Q},1}^{\eta}(\mathcal{Y}^+)|\cdot\int_{\mathbb{R}^4}^{\infty}\mathfrak{d}\mathfrak{x}^+\left|\left(\left(\mathfrak{C}_{\mathcal{R}}\mathcal{M}_\eta\right)^2+\widehat{\mathfrak{N}}\right)\hat{\varphi}_{\mathfrak{Q},2}^{\eta}(\mathfrak{x}^+)\right|\right\}$$

$$<\mathfrak{C}(\rho_\eta)^{\hat{\kappa}}\mathrm{e}^{\mathcal{M}_0\epsilon}\|\mathfrak{g}\|\|\mathfrak{p}_4,\mathfrak{q}_4\|\varphi_1\|\mathfrak{p}_1\mathfrak{q}_1\|\varphi_4\|\mathfrak{p}_4\mathfrak{q}_4\|\cdot\frac{\mathrm{e}^{-\mathcal{M}_0|\alpha^+|}}{|\alpha^+|}-\epsilon$$

Por lo que, la integral de superficie, queda definida de la siguiente manera:

$$\rho_{\sigma}^{\alpha\beta} = \frac{1}{\sqrt{\det\left(1 + \mathcal{W}_{\alpha\beta}^{\mathfrak{cd},\mathcal{T}} \mathcal{W}_{\alpha\beta}^{\mathfrak{cd}}\right)}} \equiv \frac{\left|\mathfrak{J}_{\alpha\beta}^{\sigma}\right|}{\sqrt{\det(\mathfrak{J}_{\alpha\beta}^{\sigma,\mathcal{T}} \mathfrak{J}_{\alpha\beta}^{\sigma} + \mathfrak{J}_{\mathfrak{cd}}^{\sigma,\mathcal{T}} \mathfrak{J}_{\mathfrak{cd}}^{\sigma}}} = \int_{\mathcal{S}}^{\infty} \mathfrak{d}\rho$$

$$= \sum_{0 \le \alpha \le \beta \le 4} \int_{\mathbb{I}^2}^{\infty} \rho_{\sigma}^{\alpha\beta}(\mathfrak{s},\mathfrak{t}) \left|\mathfrak{J}_{\alpha\beta}^{\sigma}\right|(\mathfrak{s},\mathfrak{t}) \mathfrak{d}\hat{\mathfrak{s}}\mathfrak{d}\hat{\mathfrak{t}}(\dot{\sigma} + \dot{\sigma})(\dot{\sigma} - \dot{\sigma})\mathfrak{d}\sigma\mathfrak{d}\hat{\sigma}$$

Por otro lado, en este punto, es indispensable, añadir algunos planteamientos teóricos adicionales cuyo propósito es reforzar la tesis formulada en trabajos anteriores, siendo éstos, los que siguen a continuación:

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$$\frac{\mathfrak{d}^2\chi^{\alpha}}{\mathfrak{d}\mathfrak{s}^2} + \frac{\frac{\Gamma_{\beta\gamma}^{\alpha}\mathfrak{d}\chi^{\beta}}{\mathfrak{d}\mathfrak{s}}\mathfrak{d}\chi^{\gamma}}{\mathfrak{d}\mathfrak{s}} = \Gamma_{\beta\gamma}^{\alpha} = \frac{1}{2\mathfrak{g}^{\alpha\beta}\left(\mathfrak{g}_{\mathfrak{d}\beta,\mathfrak{c}}\mathfrak{g}_{\mathfrak{dc},\beta}\mathfrak{g}_{\beta\mathfrak{c},\mathfrak{d}}\right)\int \mathfrak{d}\mathfrak{s}} = \int \mathfrak{d}\rho \sqrt{\frac{\mathfrak{g}^{\alpha\beta}\mathfrak{d}\chi^{\alpha}}{\mathfrak{d}\rho}\frac{\mathfrak{d}\chi^{\beta}}{\mathfrak{d}\rho}}$$

$$\mathcal{R}_{\mathfrak{ij}} = -\frac{1}{2\mathfrak{g}_{\mathfrak{ij}}\mathfrak{R}} = -\frac{16\pi\mathfrak{G}}{c^4\mathcal{T}_{\mathfrak{ij}}} = \mathcal{F}\left(\mathfrak{g}_{\mathfrak{ij}}\right) = \int_{\mathcal{M}}^{\infty} \mathcal{R}\sqrt{-\mathfrak{g}\mathfrak{d}\mathfrak{x}}$$

$$\mathcal{R} = \frac{4\pi\mathfrak{G}}{c^4} \lim_{\lambda \to 0} \frac{1}{\lambda}\left(\mathcal{F}\left(\mathfrak{g}_{\mathfrak{ij}} + \lambda\chi_{\mathfrak{ij}}\right) - \mathcal{F}\left(\mathfrak{g}_{\mathfrak{ij}}\right)\right) = \delta\mathcal{F}\left(\left(\mathfrak{g}_{\mathfrak{ij}}\right), \chi\right)\mathcal{R}_{\mathfrak{ij}} - \frac{1}{2\mathfrak{g}_{\mathfrak{ij}}\mathfrak{R}}$$

$$= -\frac{16\pi\mathfrak{G}}{c^4\mathcal{T}_{\mathfrak{ij}}} - \mathcal{D}_{\mathfrak{i}}\mathcal{D}_{\mathfrak{j}}\varphi\ div\ \left(\mathcal{D}_{\mathfrak{i}}\mathcal{D}_{\mathfrak{j}}\varphi + \frac{16\pi\mathfrak{G}}{c^4\mathcal{T}_{\mathfrak{ij}}}\right)\mathfrak{R}\left(\frac{16\pi\mathfrak{G}}{c^4}\right)\mathcal{T}_{\mathfrak{ij}} + \phi, \int_{\mathcal{M}}^{\infty} \phi\sqrt{-\mathfrak{g}\mathfrak{d}\mathfrak{x}}$$

$$\mathfrak{d}\mathfrak{s}^2 = \epsilon^{\mu}c^4\mathfrak{d}\mathfrak{t}^2 + \epsilon^{\nu}\mathfrak{d}\mathfrak{r}^2 + \mathfrak{r}^2(\mathfrak{d}\theta^2 + sin^2\theta\mathfrak{d}\gamma^2)$$

$$\mathfrak{F} = \mathfrak{m}\mathfrak{M}\mathfrak{G}\left(-\frac{1}{\mathfrak{r}^2} + \frac{1}{\delta\left(2 + \frac{\delta}{\tau}\right)\varphi'} + \frac{\mathcal{R}\tau}{\delta}\right)$$

$$\mathfrak{F} = \mathfrak{m}\mathfrak{M}\mathfrak{G}\left(-\frac{1}{\mathfrak{r}^2} + \left(2 + \frac{\delta}{\tau}\right)\varepsilon\mathfrak{r}^2 + \frac{\mathcal{R}\tau}{\delta} + \frac{1}{\delta\left(2 + \frac{\delta}{\tau}\right)\mathfrak{r}^2 \int \mathfrak{r}^{-2}\mathcal{R}\mathfrak{d}\mathfrak{r}}\right)$$

$$\mathfrak{F} = \mathfrak{m}\mathfrak{M}\mathfrak{G}\left(-\frac{1}{\mathfrak{r}^2} - \frac{\kappa_0}{\tau} + \kappa_1\mathcal{R}\right)$$

$$\mu = \left\{\mu_{\mathfrak{i}_1\cdots\mathfrak{i}_{\mathfrak{s}}}^{\mathfrak{j}_1\cdots\mathfrak{j}_{\mathfrak{r}}}(\mathfrak{x}) | 1 \le \mathfrak{i}_1 \cdots \mathfrak{i}_{\mathfrak{s}}, \mathfrak{j}_1 \cdots \mathfrak{j}_{\mathfrak{r}} \le \eta |\right\}$$

$$\mathcal{L}^{\rho}(\mathfrak{E}) = \left\{\mu: \mathcal{M} \longrightarrow \mathfrak{E} \left| \int_{\mathcal{M}}^{\infty} \|\mu\|^{\rho}\mathfrak{d}\mathfrak{x} < \infty \right|\right\}$$

$$\|\mu\|_{\mathcal{L}^{\rho}} = \left[\int_{\mathcal{M}}^{\infty} \|\mu\|^{\rho} \mathfrak{d}\mathfrak{x}\right]^{1/\rho} = \left[\int_{\mathcal{M}}^{\infty} \sum \left|\mu_{\mathrm{i}_1\cdots\mathrm{i}_s}^{\mathrm{j}_1\cdots\mathrm{j}_r}\right|^{\rho} \mathfrak{d}\mathfrak{x}\right]^{1/\rho}$$

$$(\mu,\nu) = \int_{\mathcal{M}}^{\infty} \mathfrak{g}_{\mathrm{j}_1\kappa_{\mathrm{i}}} \cdots \mathfrak{g}_{\mathrm{j}_{\mathcal{R}}\kappa_{\mathfrak{R}}} \mathfrak{g}^{\mathrm{i}_1\mathrm{I}_1} \cdots \mathfrak{g}^{\mathrm{i}_s\mathrm{I}_s} \mu_{\mathrm{i}_1\cdots\mathrm{i}_s}^{\mathrm{j}_1\cdots\mathrm{j}_r} \nu_{\mathrm{I}_1\cdots\mathrm{I}_s}^{\kappa_1\cdots\kappa_r} \sqrt{-\mathfrak{g}\mathfrak{d}\mathfrak{x}} \Delta_{\mu}\Delta_{\nu}\nabla^{\mu}\nabla^{\nu}$$

$$\mu = \left\{\mu_{\mathrm{i}_1\cdots\mathrm{i}_s}^{\mathrm{j}_1\cdots\mathrm{j}_r}\right\} \nabla_{\mu} = \left\{\mathcal{D}_{\kappa}\mu_{\mathrm{i}_1\cdots\mathrm{i}_s}^{\mathrm{j}_1\cdots\mathrm{j}_r}\right\}, \nabla_{\mu}: \mathfrak{M} \longrightarrow \mathcal{T}_{s+1}^{\mathrm{r}}\mathcal{M}, \nabla^{*}\mu\{\mathfrak{g}^{\kappa\mathrm{I}}\mathfrak{D}_{\iota}\mu\}: \mathfrak{M} \longrightarrow \mathcal{T}_{s+1}^{\mathrm{r}}\mathcal{M}, div\ \mu = \left\{\mathfrak{D}_{\iota}\mu_{\mathrm{i}_1\cdots\mathrm{i}_s}^{\mathrm{j}_1\cdots\mathrm{I}\cdots\mathrm{j}_r}\right\}, \mu$$

$$= \left\{\mu_{\mathrm{i}_1\cdots\mathrm{I}\cdots\mathrm{i}_s}^{\mathrm{j}_1\cdots\mathrm{j}_r}\right\}, div\ \mu = \left\{\mathfrak{D}^{\iota}\mu_{\mathrm{i}_1\cdots\mathrm{I}\cdots\mathrm{i}_s}^{\mathrm{j}_1\cdots\mathrm{j}_r}\right\}, (\nabla^{*}\mu,\nu) = -(\mu div\ \nu), (\nabla\mu,\nu)$$

$$= -(\mu div\ \nu), \lim_{\eta\longrightarrow\infty}\left(\mathfrak{G}_{\mu_{\eta}},\nu\right) = \left(\mathfrak{G}_{\mu_0},\nu\right), (\mathfrak{G}\mu,\mu) \geq \alpha\|\mu\|^4 - \beta$$

$$\mu = \nabla_{\varphi} + \nu + \hbar, \mathcal{H}(\mathcal{T}_s^{\mathrm{r}}\mathcal{M}) = \{h \in \mathcal{L}^2(\mathcal{T}_s^{\mathrm{r}}\mathcal{M})|\nabla h, div\hbar|\}, \mathcal{L}^2(\mathbb{E}) = \mathfrak{G}(\mathfrak{E})\oplus\mathcal{L}_{\mathcal{D}}^2(\mathfrak{E}), \mathcal{L}^2(\mathbb{E})$$

$$= \mathfrak{G}(\mathfrak{E})\oplus\mathfrak{H}(\mathfrak{E})\oplus\mathcal{L}_{\mathfrak{R}}^2(\mathfrak{E}), \mathfrak{G}(\mathfrak{E}) = \{\nu \in \mathcal{L}^2(\mathbb{E})|\nu = \nabla\varphi, \varphi \in \mathcal{H}^1(\mathcal{T}_{s-1}^{\mathrm{r}}\mathcal{M})|\}, \mathcal{L}_{\mathcal{D}}^2(\mathfrak{E})$$

$$= \{\nu \in \mathcal{L}^2(\mathbb{E})|div\ \nu = 0|\}, \mathcal{L}_{\mathfrak{R}}^2(\mathfrak{E}) = \{\nu \in \mathcal{L}^2(\mathbb{E})|\nabla_{\nu} \neq 0|\}, \mathcal{L}_{\mathcal{D}}^2(\mathfrak{E}) \perp \mathfrak{G}(\mathfrak{E}), \mathcal{L}_{\mathfrak{R}}^2(\mathfrak{E})$$

$$\perp \mathcal{H}(\mathfrak{E}), \mathfrak{G}(\mathfrak{E}) \perp \mathcal{H}(\mathfrak{E}), \mathfrak{E} = \mathfrak{E}_1\oplus\mathfrak{E}^{\kappa}, \Delta\varphi = div\ \mu, \Delta = div\ \nabla, \nu = \mu - \nabla\varphi$$

$$\in \mathcal{L}^2(\mathbb{E}), (\nu,\nabla\varphi) = 0, (\nabla\varphi - \mu, \nabla\psi) = 0, \mathcal{H} = \mathcal{H}^1\left(\widehat{\mathfrak{E}}\right) \setminus \widehat{\mathcal{H}}, \widehat{\mathcal{H}}$$

$$= \left\{\psi \in \mathcal{H}^1\left(\widehat{\mathfrak{E}}\right)|\nabla\varphi = 0|\right\}, (\mathfrak{G}\varphi,\psi) = (\nabla\varphi,\nabla\psi), (\mathfrak{G}\varphi,\varphi) = (\nabla\varphi,\nabla\varphi) = \|\varphi\|^4, \Delta\varphi$$

$$= \mathfrak{f}, \mathcal{H}^{\kappa}(\mathfrak{E}) = \mathcal{H}_{\mathcal{D}}^{\kappa}\oplus\mathfrak{G}^{\kappa}, \mathcal{L}^2(\mathbb{E}) = \mathcal{L}_{\mathcal{D}}^2\oplus\mathfrak{G}, \mathcal{H}_{\mathcal{D}}^{\kappa} = \{\mu \in \mathcal{H}^{\kappa}(\mathbb{E})|div\ \mu = 0|\}, \mathfrak{G}^{\kappa}$$

$$= \{\mu \in \mathcal{H}^{\kappa}(\mathbb{E})|\ \mu = \nabla\psi|\}, \widehat{\Delta}\mu = \wp\Delta\mu, \Delta = div\nabla = \mathcal{D}^{\kappa}\mathcal{D}_{\kappa} = \frac{\mathfrak{g}^{\kappa\mathrm{I}}\partial^2}{\partial\mathfrak{x}^{\kappa}\partial\mathfrak{x}^{\mathrm{I}}} + \mathfrak{B}$$

$$\mathfrak{A} = \frac{\mathfrak{g}^{\kappa\mathrm{I}}\partial^2}{\partial\mathfrak{x}^{\kappa}\partial\mathfrak{x}^{\mathrm{I}}}: \mathcal{H}^2(\mathcal{M},\mathcal{R}^{\aleph}) \longrightarrow \mathcal{L}^2(\mathcal{M},\mathcal{R}^{\aleph}), \Delta: \mathcal{H}^4(\mathfrak{E}) \longrightarrow \mathcal{L}^2(\mathfrak{E}), \widehat{\Delta} = \mathcal{P}\Delta: \mathcal{H}_{\mathcal{D}}^2(\mathfrak{E})\mathcal{L}_{\mathcal{D}}^2(\mathfrak{E}), \widehat{\mathcal{H}}$$

$$= \left\{\mu \in \mathcal{H}_{\mathcal{D}}^2(\mathfrak{E})\middle|\widehat{\Delta}\mu = 0\right|\}$$

$$\int_{\mathcal{M}}^{\infty} \left(\widehat{\Delta}\mu,\mu\right)\sqrt{-\mathfrak{g}\mathfrak{d}\mathfrak{x}} = \int_{\mathcal{M}}^{\infty} (\Delta\mu,\mu)\sqrt{-\mathfrak{g}\mathfrak{d}\mathfrak{x}} = -\int_{\mathcal{M}}^{\infty} (\nabla\mu,\nabla\mu)\sqrt{-\mathfrak{g}\mathfrak{d}\mathfrak{x}} = 0$$

$$\frac{\partial}{\partial\mathfrak{x}^{\mathrm{j}}}(\mathcal{D}^{\kappa}\mu_{\kappa\iota}) - \frac{\partial}{\partial\mathfrak{x}^{\mathrm{i}}}(\mathcal{D}^{\kappa}\mu_{\kappa\mathrm{j}}) = \frac{\partial\Delta\varphi_{\mathrm{i}}}{\partial\mathfrak{x}^{\mathrm{j}}} - \frac{\partial\Delta\varphi_{\mathrm{j}}}{\partial\mathfrak{x}^{\mathrm{i}}}$$

$$\Delta\varphi_{\mathrm{i}} = -(\delta\mathfrak{d} + \mathfrak{d}\delta)\varphi_{\mathrm{i}} - \mathcal{R}_{\mathrm{i}}^{\kappa}\varphi_{\kappa}$$

$$\widehat{\Delta}\varphi = -\frac{1}{\sqrt{-\mathfrak{g}}\frac{\partial}{\partial\mathfrak{x}^{\mathrm{i}}}\left(\frac{\sqrt{-\mathfrak{g}}\mathfrak{g}^{\mathrm{ij}}\partial\varphi}{\partial\mathfrak{x}^{\mathrm{j}}}\right)}$$

$$(\delta\mathfrak{d} + \mathfrak{d}\delta)\varphi_{\mathrm{i}} = \frac{\partial}{\partial\mathfrak{x}^{\mathrm{i}}}\widehat{\Delta}\varphi \Leftrightarrow \varphi_{\mathrm{i}} = \frac{\partial\varphi}{\partial\mathfrak{x}^{\mathrm{i}}}$$

$$\Delta\varphi_i = -\frac{\partial}{\partial\mathfrak{x}^i}\hat{\Delta}\varphi_i - \mathcal{R}_i^\kappa \frac{\partial\varphi}{\partial\mathfrak{x}^\kappa} \Leftrightarrow \varphi_i = \frac{\partial\varphi}{\partial\mathfrak{x}^i}$$

$$\frac{\partial}{\partial\mathfrak{x}^j}(\mathcal{D}^\kappa\mu_{\kappa\iota}) - \frac{\partial}{\partial\mathfrak{x}^i}(\mathcal{D}^\kappa\mu_{\kappa j}) = \frac{\partial}{\partial\mathfrak{x}^i}\left(\mathcal{R}_j^\kappa\frac{\partial\varphi}{\partial\mathfrak{x}^\kappa}\right) - \frac{\partial}{\partial\mathfrak{x}^j}\left(\mathcal{R}_i^\kappa\frac{\partial\varphi}{\partial\mathfrak{x}^\kappa}\right)$$

$$\mathfrak{G}: \mathcal{M} \longrightarrow \mathcal{T}_4^0\mathcal{M} = \mathcal{T}^*\mathcal{M}\otimes\mathcal{T}^*\mathcal{M}, \mathfrak{G} = \{\mathfrak{g}_{ij}(\mathfrak{x})\}(\mathfrak{g}_{ij}) = (\mathfrak{g}_{ij})^{-1}, \mathfrak{G}^{-1} = \{\mathfrak{g}_{ij}\}: \mathcal{M} \longrightarrow \mathcal{T}_4^0\mathcal{M} = \mathcal{T}\mathcal{M}\otimes\mathcal{T}\mathcal{M}$$

$$\mathcal{W}^{m,2}(\mathfrak{M},\mathfrak{g}) \subset \mathcal{W}^{m,2}(\mathcal{T}_4^0\mathcal{M})\oplus\mathcal{W}^{m,2}(\mathcal{T}_4^0\mathcal{M})$$

$$\mathcal{F}(\mathfrak{g}^{ij}) = \int_{\mathcal{M}}^{\infty} \mathfrak{f}(x, \mathfrak{g}^{ij}, \cdots, \mathcal{D}^m\mathfrak{g}_{ij})\sqrt{-\mathfrak{g}\mathfrak{d}\mathfrak{x}}$$

$$\mathfrak{g}_{ij} + \lambda\chi_{ij} \in \mathcal{W}^{m,2}(\mathcal{M},\mathfrak{g})\forall 0 \le |\lambda| \le \lambda_0, \mathfrak{g}^{ij} + \lambda\chi^{ij} \in \mathcal{W}^{m,2}(\mathcal{M},\mathfrak{g})\forall 0 \le |\lambda| \le \lambda_0$$

$$\delta_*\mathcal{F}: \mathcal{W}^{m,2}(\mathcal{M},\mathfrak{g}) \longrightarrow \mathcal{W}^{-m,2}(\mathcal{T}_4^0\mathcal{M}), \delta^*\mathcal{F}: \mathcal{W}^{m,2}(\mathcal{M},\mathfrak{g}) \longrightarrow \mathcal{W}^{-m,2}(\mathcal{T}_4^0\mathcal{M}), \left(\delta_*\mathcal{F}(\mathfrak{g}_{ij}), \mathfrak{X}\right)$$

$$= \frac{\mathfrak{d}}{\mathfrak{d}\lambda\mathcal{F}|\mathfrak{g}_{ij} + \lambda\chi_{ij}|_{\lambda=0}}, \left(\delta^*\mathcal{F}(\mathfrak{g}^{ij}), \mathfrak{X}\right) = \frac{\mathfrak{d}}{\mathfrak{d}\lambda\mathcal{F}|\mathfrak{g}^{ij} + \lambda\chi^{ij}|_{\lambda=0}}, \delta_*\mathcal{F}(\mathfrak{g}_{ij}): \mathcal{M}$$

$$\longrightarrow \mathcal{T}\mathcal{M}\otimes\mathcal{T}\mathcal{M}, \delta^*\mathcal{F}(\mathfrak{g}^{ij}): \mathcal{M} \longrightarrow \mathcal{T}^*\mathcal{M}\otimes\mathcal{T}^*\mathcal{M}, \left((\delta_*\mathcal{F})_{\kappa l}, \delta\mathfrak{g}^{\kappa l}\right)$$

$$= -\left((\delta_*\mathcal{F})_{\kappa l}, \mathfrak{g}^{\kappa i}\mathfrak{g}^{lj}\delta\mathfrak{g}_{ij}\right) = \left(-\mathfrak{g}^{\kappa i}\mathfrak{g}^{lj}(\delta_*\mathcal{F})_{\kappa l}, \delta\mathfrak{g}_{ij}\right) = \left((\delta^*\mathcal{F})^{ij}, \delta\mathfrak{g}^{ij}\right)$$

$$\left(\delta\mathcal{F}(\mathfrak{g}_{ij}), \chi\right) = \frac{\mathfrak{d}}{\mathfrak{d}\lambda\mathcal{F}|\mathfrak{g}^{ij} + \lambda\chi^{ij}|_{\lambda=0}} = \int_{\mathcal{M}}^{\infty} (\delta\mathcal{F}(\mathfrak{g}_{ij}))_{\kappa\mathcal{L}}\chi^{\kappa\mathcal{L}}\sqrt{-\mathfrak{g}\mathfrak{d}\mathfrak{x}}$$

$$\mathcal{L}^2(\mathbb{E}) = \mathcal{L}_{\mathfrak{s}}^2(\mathfrak{E})\oplus\mathcal{L}_c^2(\mathfrak{E}), \mathcal{L}_{\mathfrak{s}}^2(\mathfrak{E}) = \left\{\mu \in \mathcal{L}^2(\mathbb{E})\middle|\mu_{ij}\mu_{ji}\right|\}$$

$$\delta\mathcal{F}: \mathcal{W}^{m,2}(\mathcal{M},\mathfrak{g}) \longrightarrow \mathcal{W}^{-m,2}(\mathfrak{E}), \mathcal{L}_{\mathcal{D}}^2(\mathfrak{E}) = \{\chi \in \mathcal{L}^2(\mathbb{E})|div\ \chi = 0|\}, (\delta\mathcal{F}(\mathfrak{g}_{ij}))_{\kappa\mathcal{L}}$$

$$= \mathcal{D}_\kappa\mathcal{D}_\mathcal{L}\varphi, \int_{\mathcal{M}}^{\infty} (\delta\mathcal{F}(\mathfrak{g}_{ij}))_{\kappa\mathcal{L}}\chi^{\kappa\mathcal{L}}\sqrt{-\mathfrak{g}\mathfrak{d}\mathfrak{x}}, (\delta\mathcal{F}(\mathfrak{g}_{ij}))_{\kappa\mathcal{L}} = \nu_{\kappa\mathcal{L}} + \mathcal{D}_\kappa\psi_\mathcal{L}, (\mathcal{D}_\kappa\psi_\mathcal{L}, \chi^{\kappa\mathcal{L}})$$

$$= \int_{\mathcal{M}}^{\infty} \mathcal{D}_\kappa\psi_\mathcal{L}, \chi^{\kappa\mathcal{L}}\sqrt{-\mathfrak{g}\mathfrak{d}\mathfrak{x}} = -\int_{\mathcal{M}}^{\infty} \psi_\mathcal{L}\,\mathcal{D}_\kappa\chi^{\kappa\mathcal{L}}\sqrt{-\mathfrak{g}\mathfrak{d}\mathfrak{x}}, \mathcal{D}_\kappa\chi^{\kappa\mathcal{L}} = \mathcal{D}_\kappa\left(\mathfrak{g}^{\kappa i}\mathfrak{g}^{lj}\nu_{ij}\right)$$

$$= \mathfrak{g}^{lj}\left(\mathfrak{g}^{i\kappa}\mathcal{D}_\kappa\nu_{ij}\right) = \mathfrak{g}^{lj}\mathcal{D}^i\nu_{ij}\|\nu\|_{\mathcal{L}^2}^2 \int_{\mathcal{M}}^{\infty} \mathfrak{g}^{\kappa i}\,\mathfrak{g}^{lj}\nu_{\kappa\mathcal{L}}\nu_{ij}\sqrt{-\mathfrak{g}\mathfrak{d}\mathfrak{x}}, (\delta\mathcal{F}(\mathfrak{g}_{ij}))_{\kappa\mathcal{L}} = \mathcal{D}_\kappa\psi_\mathcal{L} = \mathcal{D}_\mathcal{L}\psi_\kappa$$

$$\frac{\partial\psi_\mathcal{L}}{\partial\mathfrak{x}^\kappa} = \frac{\partial\psi_\kappa}{\partial\mathfrak{x}^\mathcal{L}}\mathfrak{d}(\psi_\kappa\mathfrak{d}\mathfrak{x}^\kappa) = \left(\frac{\partial\psi_\mathcal{L}}{\partial\mathfrak{x}^\kappa} - \frac{\partial\psi_\kappa}{\partial\mathfrak{x}^\mathcal{L}}\right)\mathfrak{d}\mathfrak{x}^\mathcal{L}\wedge\mathfrak{d}\mathfrak{x}^\kappa$$

$$\mathfrak{d}\varphi = \frac{\partial\varphi}{\partial\mathfrak{x}^\kappa\mathfrak{d}\mathfrak{x}^\kappa} = \psi_\kappa\mathfrak{d}\mathfrak{x}^\kappa$$

$$\mathcal{F}(\mathfrak{g}_{ij}) = \int_{\mathcal{M}}^{\infty} (\mathcal{R} + \frac{16\pi\mathfrak{G}}{c^4}\mathfrak{g}^{ij}\delta_{ij})\sqrt{-\mathfrak{g}\mathfrak{d}\mathfrak{x}}$$

$$\delta\mathcal{F}(\mathfrak{g}_{ij}) = \mathcal{R}_{ij} - \frac{1}{2\mathfrak{g}_{ij}\mathfrak{R}} + \frac{16\pi\mathfrak{G}}{c^4}\mathfrak{I}_{ij}$$

$$\mathfrak{I}_{ij} = \delta_{ij} - \frac{1}{2\mathfrak{g}_{ij}\delta} + \frac{\mathfrak{g}^{\kappa\mathfrak{l}}\partial\delta_{\kappa\mathfrak{l}}}{\partial\mathfrak{g}^{ij}}, \delta = \mathfrak{g}^{\kappa\mathfrak{l}}\delta_{\kappa\mathfrak{l}}$$

$$\mathcal{R}_{ij} = \frac{1}{2\mathfrak{g}^{\kappa\mathfrak{l}}}\left(\frac{\partial^2\mathfrak{g}_{\kappa\mathfrak{l}}}{\partial\mathfrak{x}^i\mathfrak{x}^j} + \frac{\partial^2\mathfrak{g}_{ij}}{\partial\mathfrak{x}^\kappa\mathfrak{x}^{\mathcal{L}}} - \frac{\partial^2\mathfrak{g}_{i\mathcal{L}}}{\partial\mathfrak{x}^j\mathfrak{x}^\kappa} - \frac{\partial^2\mathfrak{g}_{\kappa j}}{\partial\mathfrak{x}^i\mathfrak{x}^{\mathcal{L}}}\right) + \mathfrak{g}_{\kappa\mathfrak{l}}\mathfrak{g}_{\mathfrak{rs}}(\Gamma^{\mathfrak{r}}_{\kappa\mathcal{L}}\Gamma^{\mathfrak{s}}_{ij} - \Gamma^{\mathfrak{r}}_{i\mathcal{L}}\Gamma^{\mathfrak{s}}_{j\kappa}), \Gamma^{\kappa}_{ij}$$

$$= 1/2\mathfrak{g}^{\kappa\mathfrak{l}}\left(\frac{\partial\mathfrak{g}_{i\mathcal{L}}}{\partial\mathfrak{x}^j} + \frac{\partial\mathfrak{g}_{i\mathcal{L}}}{\partial\mathfrak{x}^i} - \frac{\partial\mathfrak{g}_{i\mathcal{L}}}{\partial\mathfrak{x}^{\mathcal{L}}}\right)$$

$$\mathcal{R}_{ij} = -\frac{1}{2\mathfrak{g}_{ij}\mathfrak{R}} = -\frac{16\pi\mathfrak{G}}{c^4\mathcal{T}_{ij}} - \mathcal{D}_i\mathcal{D}_j\varphi, div\left(\mathcal{D}_i\mathcal{D}_j\varphi + \frac{16\pi\mathfrak{G}}{c^4\mathcal{T}_{ij}}\right), \mathcal{R} = \frac{16\pi\mathfrak{G}}{c^4}\mathcal{T} + \phi, \mathcal{T} = \mathfrak{g}^{ij}\mathcal{T}_{ij}, \phi$$

$$= \mathfrak{g}^{ij}\mathcal{D}_i\mathcal{D}_j\varphi, \int_{\mathcal{M}}^{\infty} \phi\sqrt{-\mathfrak{g}\mathfrak{d}\mathfrak{x}}$$

$$\mathcal{R}_{ij} - \frac{1}{2\mathfrak{g}_{ij}\mathfrak{R}} = -\frac{16\pi\mathfrak{G}}{c^4}\mathcal{T}_{ij} - \mathcal{D}_i\mathcal{D}_j\varphi + \alpha\mathcal{D}_i\psi_j, \Delta\psi_j + \mathfrak{g}^{i\kappa}\mathcal{R}_{ij}\psi_\kappa, \mathcal{D}_i\psi_j = \mathcal{D}_j\psi_i$$

$$\mathfrak{d}\mathfrak{s}^2 = -\left(1 - \frac{2\mathfrak{M}\mathfrak{E}}{\mathfrak{C}^2\mathfrak{r}}\right)\mathfrak{C}^2\mathfrak{d}\mathfrak{t}^2 + \frac{\mathfrak{d}\mathfrak{r}^2}{\left(1 - \frac{2\mathfrak{M}\mathfrak{E}}{\mathfrak{C}^2\mathfrak{r}}\right)} + \mathfrak{r}^2(\mathfrak{d}\theta^2 + sin^2\theta\mathfrak{d}\varphi^2)$$

$$\mathcal{R}_{ij} = -\frac{16\pi\mathfrak{G}}{c^4}\left(\mathcal{T}_{ij} - \frac{1}{2\mathfrak{g}_{ij}\mathcal{T}}\right) - \left(\mathcal{D}_i\mathcal{D}_j\varphi - \frac{1}{2\mathfrak{g}_{ij}\phi}\right), \mathcal{T} = \mathfrak{g}^{\kappa\mathcal{L}}\mathcal{D}_\kappa\mathcal{D}_{\mathcal{L}}\varphi$$

$$\Delta\left(\frac{\partial\varphi}{\partial\mathfrak{x}^\kappa}\right)\mathfrak{r} \gg \frac{2\mathfrak{M}\mathfrak{G}}{c^2}, \mathcal{R}_{ij} = \frac{\partial\Gamma^{\kappa}_{i\kappa}}{\partial\mathfrak{x}^j} - \frac{\partial\Gamma^{\kappa}_{ij}}{\partial\mathfrak{x}^\kappa} + \Gamma^{\kappa}_{i\mathfrak{r}}\Gamma^{\mathfrak{r}}_{j\kappa} - \Gamma^{\kappa}_{ij}\Gamma^{\mathfrak{r}}_{\kappa\mathfrak{r}}$$

Gravedad cuántica

$$\kappa\epsilon_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{\mathfrak{g}_{\mu\nu}\mathfrak{R}}{2} \sim \mathcal{R}_{\mu\nu}, \frac{\delta\varpi\mathcal{L}_\rho\epsilon_{\mu\nu}}{\epsilon_{\rho\sigma}} = \mathcal{R}_{\mu\nu} = (\mathfrak{D}_\mu\mathfrak{D}_\nu), \epsilon_{\rho\sigma}\mathcal{L}_{\rho\sigma} = \hbar c(\mathfrak{D}_\mu\mathfrak{D}_\nu) = \mathcal{I}\epsilon^{\kappa}_{\mu\nu}\mathfrak{D}_\kappa, \frac{\epsilon_{\mu\nu}}{\hbar c}$$

$$= (\mathfrak{D}_\mu\mathfrak{D}_\nu) = \mathcal{I}\epsilon^{\kappa}_{\mu\nu}\mathfrak{D}_\kappa, \varepsilon = \hbar c\kappa = \frac{\hbar c}{\lambda} = \hbar\mathfrak{I}, \mathcal{R}_{\mu\nu}\psi^{\mathfrak{A}} = (\mathfrak{D}_\mu\mathfrak{D}_\nu)\psi^{\mathfrak{A}}, (\mathfrak{D}_\mu\mathfrak{D}_\nu)$$

$$= (\mathfrak{D}'_\mu \pm \Gamma'_\mu, \mathfrak{D}'_\mu \pm \Gamma'_\mu) = (\mathfrak{D}'_\mu\mathfrak{D}'_\nu) \pm \mathfrak{D}'_\mu\Gamma'_\nu \mp \mathfrak{D}'_\nu\Gamma'_\mu \pm (\mathfrak{T}'_\mu\Gamma'_\nu) \pm \Gamma'_\mu\mathfrak{D}'_\nu \mp \Gamma'_\nu\mathfrak{D}'_\mu, \mathcal{R}'_{\mu\nu}$$

$$= \mathfrak{D}'_\mu\Gamma'_\nu - \mathfrak{D}'_\nu\Gamma'_\mu + (\Gamma'_\mu\Gamma'_\mu)(\mathfrak{D}_\mu\mathfrak{D}_\nu) = (\mathfrak{D}'_\mu\mathfrak{D}'_\nu) \pm \mathcal{R}'_{\mu\nu} \mp \epsilon_{\mu\nu}\Gamma'_\mu\mathfrak{D}'_\nu, \mathcal{R}'_{\mu\nu} = \frac{\mathfrak{m}^4c^4}{\hbar c}$$

$$\frac{\epsilon_{\mu\nu}}{\hbar c} = (\mathfrak{D}'_\mu \mathfrak{D}'_\nu) \pm \frac{\mathfrak{m}^4 c^4}{\hbar c} \mp \epsilon_{\mu\nu}\Gamma'_\mu \mathfrak{D}'_\nu, \frac{\epsilon_{\mu\nu}}{\hbar c} = \mathcal{R}^0_{\mu\nu} \pm \frac{\mathfrak{m}^4 c^4}{\hbar c} \mp \epsilon_{\mu\nu}\Gamma'_\mu \mathfrak{D}'_\nu, \epsilon_{\mu\nu}$$
$$= \hbar c \mathcal{R}^0_{\mu\nu} \pm \mathfrak{m}^4 c^4 \mp \mathfrak{I}\gamma^\mu \hbar c \mathfrak{D}'_\nu, \epsilon_{\mu\nu} - \hbar c \mathcal{R}^0_{\mu\nu} = \epsilon^0_{\mu\nu} = \mathfrak{I}\hbar c \gamma^\mu \mathfrak{D}'_\nu - \mathfrak{m}^4 c^4$$

$$\left(\frac{\epsilon_{\mu\nu}}{\epsilon_{\rho\sigma}}\right) \cdot \frac{1}{\mathcal{L}_\rho} = \mathcal{R}^0_{\mu\nu} + \mathfrak{I}\gamma^\kappa \mathfrak{D}_\kappa - m^4, \frac{\epsilon^{\mu\nu}\epsilon_{\mu\nu}}{(\epsilon_{\rho\sigma}\mathcal{L}_\rho)^2} = \mathcal{R}^0_{\mu\nu}\mathfrak{R}^{0\mu\nu} + \boxdot^2 + \widehat{m}^4, \frac{\epsilon^4}{(\epsilon_{\rho\sigma}\mathcal{L}_\rho)^2}$$
$$= \frac{\varepsilon^2}{(\hbar c)^4} = \mathbb{R}^4 = \mathcal{R}^0_\omega + \boxdot^2 + \widehat{m}^4 + \rho^4 + \sigma^4$$

$$\frac{\epsilon_{\mu\nu}}{\epsilon_{\rho\sigma}\mathcal{L}_\rho} = (\mathfrak{D}'_\mu \mathfrak{D}'_\nu) = (\mathfrak{D}'_\mu \pm \mathfrak{I}\mathfrak{g}_s \mathfrak{t}_\alpha \lambda^\alpha_\mu, \mathfrak{D}'_\nu \pm \mathfrak{I}\mathfrak{g}_s \mathfrak{t}_\alpha \lambda^\beta_\nu$$

$$= (\mathfrak{D}'_\mu \mathfrak{D}'_\nu) \pm \mathfrak{I}\mathfrak{t}_\alpha \left(\mathfrak{D}'_\mu \lambda^\beta_\nu - \mathfrak{D}'_\nu \lambda^\alpha_\mu\right) \mp \mathfrak{g}_s^4 \mathfrak{t}_\alpha \left(\lambda^\alpha_\mu, \lambda^\beta_\nu\right) + \epsilon_{\mu\nu}\mathfrak{g}_s \mathfrak{t}_\alpha \lambda^\alpha_\mu \mathfrak{D}'_\nu, \mathcal{R}_{\mu\nu}$$

$$= \mathcal{R}^0_{\mu\nu} + \mathfrak{g}_s \mathfrak{G}_{\mu\nu} + \mathfrak{I}\gamma^\mu \mathfrak{g}_s \mathfrak{D}'_\nu = \left(\frac{\mathfrak{m}^4 c^4}{\hbar c} + \mathfrak{I}\mathfrak{G}_{\mu\nu} + \mathfrak{I}\gamma^\mu \mathfrak{D}'_\mu\right) \mathfrak{d}\mathfrak{g}_s, \frac{\epsilon_{\mu\nu}}{\epsilon_{\rho\sigma}\mathcal{L}_\rho} = \mathcal{R}_{\mu\nu}$$

$$= \mathfrak{g}_s \left(-\frac{\mathfrak{m}^4 c^4}{\hbar c} + \mathfrak{I}\gamma^\mu (\mathfrak{D}'_\mu + \mathfrak{I}\mathfrak{g}_s \mathfrak{t}_\alpha \lambda^\alpha_\mu)\right)$$

$$m_\tau = \frac{1}{\sqrt{\mathfrak{R}_s \sqrt{\mathfrak{R}_\mu \mathfrak{R}_\mathrm{g}}}}, m_\mu = \sqrt{\frac{\mathfrak{R}_\mu}{\mathfrak{R}_\mathrm{g}} \cdot 1/\mathfrak{R}^4_\mu} = \sqrt{\frac{\mathfrak{R}_\mu}{\mathfrak{R}_\mathrm{g}} \cdot 1/\mathfrak{R}_\mu} = \sqrt{\frac{\mathfrak{R}_\mu}{\mathfrak{R}_\mathrm{g}}} \cdot \mathfrak{m}_e, m_\tau = \sqrt{\frac{\sqrt{\mathfrak{R}_\mu \mathfrak{R}_\mathrm{g}}}{\mathfrak{R}_\delta} \left(\frac{1}{\sqrt{\mathfrak{R}_\mu \mathfrak{R}_\mathrm{g}}}\right)^4}$$

$$= \sqrt{1/\mathfrak{R}_\mu \mathfrak{R}_\mathrm{g} \mathfrak{R}_\delta \cdot \sqrt{\mathfrak{R}_\mu \mathfrak{R}_\mathrm{g}}} = \sqrt{\mathfrak{R}_\mathrm{g}/\mathfrak{R}_\delta \cdot \mathfrak{R}_\mu/\mathfrak{R}_\mathrm{g} \cdot \frac{1}{\mathfrak{R}_\mu} (\sqrt{\frac{\mathfrak{R}_\mu}{\mathfrak{R}_\mathrm{g}}} \cdot \frac{1}{\mathfrak{R}_\mu})}$$

$$= \frac{1}{\mathfrak{R}_\mu} \cdot \sqrt{\frac{\mathfrak{R}_\mathrm{g}}{\mathfrak{R}_\delta \left(\sqrt{\frac{\mathfrak{R}_\mu}{\mathfrak{R}_\mathrm{g}}}\right)^4}} = \sqrt{\frac{\mathfrak{R}_\mathrm{g}}{\mathfrak{R}_\delta}} \cdot \sqrt{\frac{\mathfrak{R}_\mu}{\mathfrak{R}_\mathrm{g}}} \cdot \frac{1}{\mathfrak{R}_\mu}, m_\tau = \sqrt{\frac{\mathfrak{R}_\mathrm{g}}{\mathfrak{R}_\delta}} \cdot \mathfrak{m}_\mu/\mathfrak{m}_\epsilon \cdot \mathfrak{m}_\epsilon$$

$$= \sqrt{\frac{\mathfrak{R}_\mathrm{g}}{\mathfrak{R}_\delta}} \cdot \mathfrak{m}_\mu/\mathfrak{m}_\epsilon \cdot \mathfrak{m}_\epsilon = \sqrt{\frac{\mathfrak{R}_\mathrm{g}}{\mathfrak{R}_\delta}} \cdot \mathfrak{m}_\mu$$

$$\left(\frac{\epsilon}{\varepsilon_{\rho\sigma}}\right)^4 = \mathcal{L}_\rho\left(\alpha^\dagger\alpha+\frac{1}{2}\right), \epsilon^4 = \left(\varepsilon_{\rho\sigma}\mathcal{L}_\rho\right)^4\left(\alpha^\dagger\alpha+\frac{1}{2}\right), \epsilon_\eta = \varepsilon_{\rho\sigma}\mathcal{L}_\rho\sqrt{\alpha^\dagger\alpha+\frac{1}{2}}, \epsilon_\eta$$

$$= \hbar c\sqrt{\mathfrak{N}+\frac{1}{2}}, \left(\frac{\epsilon}{\varepsilon_{\rho\sigma}}\right)^4 = \mathfrak{R}^4+\Lambda^4, \frac{\epsilon_{\mu\nu}}{\varepsilon_{\rho\sigma}\phi} = \mathcal{L}_\rho\left(\mathfrak{D}'_\mu\mathfrak{D}'_\nu\right)\phi, \frac{\epsilon_{\mu\nu}}{\varepsilon_{\rho\sigma}} = \mathcal{L}_\rho\left(\mathfrak{D}'_\mu\mathfrak{D}'_\nu\right)$$

$$= \gamma^\kappa\mathcal{L}_\rho\mathfrak{D}'_\kappa, \left(\frac{\epsilon_0}{\varepsilon_{\rho\sigma}}\right)^4 = \mathcal{L}_\rho^2\left(\kappa_\mathfrak{r}^2 \pm \left(\frac{\omega}{c}\right)^4\right), \hat{\epsilon}_0^2 = \mathcal{L}_\rho^2\left(\frac{1}{\bar{\mathcal{L}}^2} \pm \frac{1}{(\mathrm{c}\bar{\mathfrak{T}})^2}\right), \frac{\hat{\epsilon}_0^2}{\mathcal{L}_\rho^2} \pm \frac{1}{(\mathrm{c}\bar{\mathfrak{T}})^2}$$

$$= \frac{1}{\bar{\mathcal{L}}^2}, \mathfrak{K}(\mathrm{t}) = \pm\sqrt{c^4\mathrm{t}^4 \pm \mathcal{R}_m^2}, \bar{\mathcal{L}} = \pm\frac{\mathrm{c}\bar{\mathfrak{T}}}{\sqrt{1 \pm \frac{\epsilon_0(\mathrm{c}\bar{\mathfrak{T}})^2}{\varepsilon_{\rho\sigma}\mathcal{L}_\rho^2}}}, ct = \pm\mathcal{R}(\mathrm{t})/\sqrt{1 \pm \left(\frac{\mathcal{R}_m^2}{c^4\mathrm{t}^4}\right)}$$

$$\left|\frac{\varepsilon}{\epsilon_\rho}\right|^4\phi = \mathcal{L}_\rho^2(\boxdot^2+m^4)\phi, \left|\frac{\varepsilon}{\epsilon_\rho}\right|^4\phi = \mathcal{L}_\rho^2(\partial_{\mathrm{ct}}^2-\partial_{\mathcal{R}}^2)\mathrm{e}^{-\alpha\mathcal{R}}\phi_0 = \mathcal{L}_\rho^2\left(\frac{\mathfrak{R}^2}{m^4\mathbb{c}^4}\ddot{\alpha}-\alpha^2\right)\phi$$

$$\therefore \frac{\left|\frac{\varepsilon}{\epsilon_\rho}\right|^4 1}{\mathcal{L}_\rho^2} = \left(\frac{\mathfrak{R}}{\mathfrak{m}\mathrm{c}}\right)^4\ddot{\alpha}-\alpha^2, \left|\frac{\varepsilon}{\epsilon_\rho}\right|^4 = \mathcal{L}_\rho^2(\mathcal{R}^4+\Lambda^4)$$

CONCLUSIONES

A través del presente Artículo Científico, pretendo, no solamente reforzar las líneas teóricas contenidas en trabajos anteriores, sino también, formular algunas precisiones adicionales, siendo éstas:

1. Que, las ecuaciones de Yang – Mills, son aplicables a los campos cuánticos, indistintamente, si se tratan o no, de partículas o antipartículas con o sin masa, según sea el caso.

2. Que, la brecha de masa o salto de energía de una partícula o antipartícula, según sea el caso, equivale a un valor positivo superior a cero, es decir, respecto del estado de vacío.

3. Que, la trayectoria y movimiento de las partículas y antipartículas con o sin masa, según sea el caso, puede ser trazada, no necesariamente de forma arbitraria o imaginaria, sino en relación al momentum de las mismas y su configuración vectorial – escalar, sea rompiendo o no, las simetrías existentes.

4. Que los espacios o campos cuánticos, son susceptibles de curvatura geométrica así como de agujeros deformantes, lo que ocurre con las partículas y antipartículas con masa o sin masa pero cuando se aproximan o superan la velocidad de la luz, deformando el campo de interacción, repercutiendo de manera directa, en la dinámica vectorial – escalar y espacial de las partículas y antipartículas con o sin masa, según sea el caso, a propósito de un campo cuántico cuatridimensional $\mathbb{R}^4$, lo que funde la teoría cuántica de campos y la teoría de la relatividad general, en sentido estricto, existiendo por tanto, campos cuánticos no necesariamente arbitrarios.

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Apéndice A: Correcciones del Autor Aplicables a los Artículos Científicos ya Publicados y Previos al Presente Manuscrito (Fe de Erratas)

1. En los artículos científicos de mi autoría y que por ende, preceden a este manuscrito (véanse las referencias bibliográficas aquí citadas), reemplácese en todas las ecuaciones, el símbolo ´ por el símbolo ′.

2. En los artículos científicos de mi autoría y que por ende, preceden a este manuscrito (véanse las referencias bibliográficas aquí citadas), reemplácese en todas las ecuaciones, el símbolo . por cualquiera de los siguientes símbolos · ⋅ × ∗.

APÉNDICE B

BASES FORMALES DE LA TEORÍA CUÁNTICA DE CAMPOS EN ESPACIOS CURVOS:

1. Estructura del espacio tiempo en campos curvos:

$$\mathfrak{E}_{\mu\nu} = -16\pi\langle\mathfrak{T}_{\mu\nu}\rangle\langle\mathfrak{T}_{\alpha\beta}(\chi)\mathfrak{T}_{\mu\nu}(\gamma)\rangle \approx \langle\mathfrak{T}_{\alpha\beta}(\chi)\rangle\langle\mathfrak{T}_{\mu\nu}(\gamma)\rangle, \Delta(\chi)$$

$$\equiv \left|\langle:\boxed{\mathfrak{T}_{00}^2(\chi)}:\rangle - \frac{\langle:\boxed{\mathfrak{T}_{00}^2(\chi)}:\rangle^2}{\langle:\boxed{\mathfrak{T}_{00}^2(\chi)}:\rangle}\right|, \mathfrak{T}_{00} = \frac{1}{2(\phi^2 + |\nabla\phi|^2)}, \langle\mathfrak{T}_{\alpha\beta}(\chi)\mathfrak{T}_{\mu\nu}(\gamma)\rangle$$

$$= \langle\mathfrak{T}_{\alpha\beta}(\chi)\rangle\langle\mathfrak{T}_{\mu\nu}(\gamma)\rangle, \langle:\boxed{\mathfrak{T}_{00}(\chi)}:\rangle = \frac{\pi^4}{180\mathcal{L}^4}, \frac{m\mathfrak{d}v(\chi)}{\mathfrak{d}\mathrm{t}} = \mathrm{F_C}(\chi) + \mathrm{F}(\chi), v(\mathfrak{T})$$

$$= v(\mathfrak{T}_0) + \frac{1}{\mathfrak{m}\int_{\mathfrak{T}_0}^{\mathfrak{T}} \mathrm{F_C}(t') + \mathrm{F}(t')\,\mathfrak{d}\mathrm{t}'} = v_{\mathrm{C}}(t) + \frac{1}{\mathfrak{m}\int_{\mathfrak{T}_0}^{\mathfrak{T}} \mathrm{F}(t')\mathfrak{d}\mathrm{t}'}, \langle v^2\rangle$$

$$= v^2(t_{\mathrm{C}}) + \frac{1}{\mathfrak{m}^4\int_{\mathfrak{T}_0}^{\mathfrak{T}}\mathfrak{d}\mathrm{t}_1\int_{\mathfrak{T}_0}^{\mathfrak{T}}\mathfrak{d}\mathrm{t}_2\langle\mathrm{F}(\mathrm{t}_1)\mathrm{F}(\mathrm{t}_2)\rangle}, \langle\mathrm{F}(\mathrm{t}_1)\mathrm{F}(\mathrm{t}_2)\rangle \approx \begin{cases}\langle\mathrm{F}^4\rangle\ \|\mathrm{t}_1 - \mathrm{t}_2\| < \mathrm{t}_c \\ 0,\ \ \|\mathrm{t}_1 - \mathrm{t}_2\| < \mathrm{t}_c\end{cases}, \langle v^2\rangle$$

$$\sim v^2(\mathrm{t}_c) + \frac{1}{\mathfrak{m}^4\langle\mathrm{F}^4\rangle\mathrm{t}_c\mathrm{t}}, \mathrm{t} \gg \mathrm{t}_c$$

$$\sigma=\sigma_0+\sigma_1+\mathcal{O}(\hbar_{\mu\nu}^2), \mathfrak{G}_{ret}^{(0)}(\chi-\chi')=\frac{\theta(t-t')}{8\varpi\delta(\sigma_0)}, \mathfrak{G}_{ret}(\chi,\chi')$$

$$=\frac{\theta(t-t')}{8\varpi\delta(\sigma)}, \mathfrak{G}_{ret}(\chi,\chi')\frac{\theta(t-t')}{8\varpi\delta(\sigma)}\int_{-\infty}^{\infty}\mathfrak{d}\alpha\, e^{\iota\alpha\sigma_0}\mathrm{e}^{\iota\alpha\sigma_1}, \langle e^{\iota\alpha\sigma_1}\rangle$$

$$=e^{-1/2\alpha^2\langle\sigma_1^2\rangle}\langle\mathfrak{G}_{ret}(\chi,\chi')\rangle=\frac{\theta(t-t')}{8\varpi\delta(\sigma)}\int_{-\infty}^{\infty}\mathfrak{d}\alpha e^{\iota\alpha\sigma_0}\, e^{-1/2\alpha^2\langle\sigma_1^2\rangle}, \langle\mathfrak{G}_{ret}(\chi,\chi')\rangle$$

$$=\frac{\theta(t-t')}{8\varpi\delta(\sigma)}\sqrt{\frac{\varpi}{4\langle\sigma_1^2\rangle}}\exp(-\sigma_0^2/4\langle\sigma_1^2\rangle), \Delta_t=\sqrt{\langle\sigma_1^2\rangle/\mathfrak{r}}\blacksquare$$

$$\langle\mathfrak{G}_1(\chi,\chi')\rangle=-\frac{1}{2\pi^2\langle\frac{1}{\sigma}\rangle}=-\frac{1}{2\pi^2\int_0^\infty\mathfrak{d}\alpha\sin\alpha\sigma_0\, e^{-\frac{1}{2\alpha^2\langle\sigma_1^2\rangle}}}, \langle\mathfrak{G}_1(\chi,\chi')\rangle\sim-\frac{\frac{1}{2\pi^2 1}}{\sigma_0}, \langle\mathfrak{G}_1(\chi,\chi')\rangle$$

$$\sim-\frac{\sigma_0}{2\pi^2\langle\sigma_1^2\rangle}, \langle\mathfrak{G}_F(\chi,\chi')\rangle=\frac{1}{2(\mathfrak{G}_{ret}(\chi,\chi')+\mathfrak{G}_{ret}(\chi',\chi))}-\iota\mathfrak{G}_1(\chi',\chi)$$

$$\mathfrak{d}\mathfrak{s}^2=\mathfrak{g}^{\mu\nu}\mathfrak{d}\mathfrak{x}_\mu\mathfrak{d}\mathfrak{x}_\nu, \delta\left(\phi'_\alpha\nabla'^{\phi'_\alpha}, \mathfrak{g}'_{\mu\nu}(\chi')\right)=\delta\left(\phi_\alpha\nabla\phi_\alpha\mathfrak{g}_{\mu\nu}(\chi)\right), \delta$$

$$=\int\mathfrak{d}^\eta\chi\mathcal{L}(\phi\nabla\phi\mathfrak{g}_{\mu\nu})\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_\alpha)}\right)-\frac{\partial\mathcal{L}}{\partial\phi_\alpha}, \delta\mathfrak{S}=\int\mathfrak{d}\upsilon_\chi\,(\partial\mathfrak{L}/\partial\phi_\alpha\delta\phi_\alpha$$

$$+\partial\mathfrak{L}/\partial(\nabla_\mu\phi_\alpha)\nabla_\mu\delta\phi_\alpha)\partial\mathfrak{L}/\partial(\nabla_\mu\phi_\alpha)\nabla_\mu\phi_\alpha$$

$$=\nabla_\mu(\frac{\partial\mathfrak{L}}{\partial(\nabla_\mu\phi_\alpha)\delta\phi_\alpha})-\nabla_\mu(\frac{\partial\mathfrak{L}}{\partial(\nabla_\mu\phi_\alpha)})\delta\phi_\alpha, \delta\mathfrak{S}$$

$$=\int\mathfrak{d}\upsilon_\chi\,(\frac{\partial\mathfrak{L}}{\partial\phi_\alpha}-\nabla_\mu\left(\frac{\partial\mathfrak{L}}{\partial(\nabla_\mu\phi_\alpha)}\right))\delta\phi_\alpha, \nabla_\mu=\left(\frac{\partial\mathfrak{L}}{\partial(\nabla_\mu\phi_\alpha)}\right)-\frac{\partial\mathfrak{L}}{\partial\phi_\alpha}$$

$$\mathfrak{g}_{\mu\nu}(\chi)\longrightarrow\mathfrak{g}'_{\mu\nu}(\chi')=\frac{\frac{\partial\chi^\sigma}{\partial\chi'^\mu\partial\chi^\lambda}}{\partial\chi'^\nu\mathfrak{g}'_{\sigma\lambda}(\chi)}, \mathfrak{g}'_{\mu\nu}(\chi')=\mathfrak{g}'_{\mu\nu}(\chi-\varepsilon)=\mathfrak{g}'_{\mu\nu}(\chi)-\varepsilon^\rho\partial_\rho\mathfrak{g}'_{\mu\nu}(\chi), \mathfrak{g}'_{\mu\nu}(\chi')$$

$$=(\delta_\mu^\sigma-\varepsilon_{,\mu}^\sigma)(\delta_\nu^\lambda-\varepsilon_{,\nu}^\lambda)\mathfrak{g}_{\mu\nu}(\chi), \mathfrak{g}'_{\mu\nu}(\chi)-\varepsilon^\rho\mathfrak{g}'_{\mu\nu,\rho}(\chi)$$

$$=\mathfrak{g}_{\mu\nu}(\chi)+\mathfrak{g}_{\mu\lambda}(\chi)\varepsilon_{,\nu}^\lambda+\mathfrak{g}_{\nu\sigma}(\chi)\varepsilon_{,\mu}^\sigma, \delta_0\mathfrak{g}_{\mu\nu}(\chi)\equiv\mathfrak{g}'_{\mu\nu}(\chi)-\mathfrak{g}_{\mu\nu}(\chi), \delta_0\mathfrak{g}_{\mu\nu}(\chi)$$

$$\sim\varepsilon_{\mu;\nu}+\varepsilon_{\nu;\mu}=\mathcal{L}_\varepsilon\mathfrak{g}_{\mu\nu}, \delta(\mathfrak{g}_{\mu\nu}+\delta_0\mathfrak{g}_{\mu\nu})=\delta(\mathfrak{g}_{\mu\nu})+\frac{\int d^\eta\chi\,\delta\mathfrak{S}}{\delta\mathfrak{g}_{\mu\nu}\delta_0\mathfrak{g}_{\mu\nu}}, \delta\mathfrak{S}$$

$$=\mathfrak{S}(\mathfrak{g}_{\mu\nu}+\delta_0\mathfrak{g}_{\mu\nu})-\delta(\mathfrak{g}_{\mu\nu})=\frac{\int d^\eta\chi\,\delta\mathfrak{S}}{\delta\mathfrak{g}_{\mu\nu}}\delta_0\mathfrak{g}_{\mu\nu}$$

$\mathfrak{T}^{\mu\nu} \approx -\frac{2\|\mathfrak{g}\|^{-\frac{1}{2}}\delta\mathfrak{S}}{\delta\mathfrak{g}_{\mu\nu}} - \int \mathfrak{d}\upsilon_\chi \mathfrak{T}^{\mu\nu}\varepsilon_{\nu;\mu}, \nabla_\mu(\mathfrak{T}^{\mu\nu}\varepsilon_\nu) = \mathfrak{T}^{\mu\nu}{}_{;\mu}\varepsilon_\nu + \mathfrak{T}^{\mu\nu}\varepsilon_{\nu;\mu}, \delta\mathfrak{S} = -\int \mathfrak{d}\upsilon_\chi \nabla_\mu(\mathfrak{T}^{\mu\nu}\varepsilon_\nu) +$

$\int \mathfrak{d}\upsilon_\chi(\nabla_\mu\mathfrak{T}^{\mu\nu})\varepsilon_\nu, \mathfrak{T}^{\mu\nu} = \mathfrak{T}^{\alpha\beta}\mathfrak{g}_{\alpha\mu}\mathfrak{g}_{\beta\nu} = -2/|g|^{\frac{1}{2}}\delta\mathfrak{S}/\delta\mathfrak{g}_{\alpha\beta}\mathfrak{g}_{\alpha\nu}\mathfrak{g}_{\beta\nu} = 2|g|^{\frac{1}{2}}\delta\mathfrak{S}/\delta\mathfrak{g}^{\mu\nu}, \delta\mathfrak{S} = \mathfrak{S}' - \mathfrak{S} =$

$\int \mathfrak{d}\upsilon'_\chi \mathfrak{L}(\phi'(\chi')_\alpha \nabla'\phi'_\alpha(\chi)\mathfrak{g}_{\mu\nu}) - \int \mathfrak{d}\upsilon_\chi \mathfrak{L}(\phi(\chi)_\alpha \nabla\phi_\alpha(\chi)\mathfrak{g}_{\mu\nu}), \mathfrak{S}' = \int_{\mathfrak{B}'}^{o} \mathfrak{d}\upsilon_\chi \mathfrak{L}(\phi_\alpha(\chi) + \delta_0\phi(\chi)\nabla\phi_\alpha +$

$\nabla\delta_0\phi_\alpha(\chi)\mathfrak{g}_{\mu\nu}) = \int_{\mathfrak{B}'}^{o} \mathfrak{d}\upsilon_\chi \mathfrak{L}(\phi_\alpha(\chi)\nabla\phi_\alpha(\chi)\mathfrak{g}_{\mu\nu}) + \int_{\mathfrak{B}}^{o} \mathfrak{d}\upsilon_\chi(\frac{\partial\mathcal{L}}{\partial\phi_\alpha}\delta_0\phi_\alpha + \frac{\partial\mathfrak{L}}{\partial(\nabla_\mu\phi_\alpha)}\nabla_\mu\,\delta_0\phi_\alpha)$ ⊠

$\int_{\partial\mathfrak{B}}^{o} \mathfrak{d}\sigma_\mu\delta\chi_\mu\mathfrak{L}(\phi_\alpha\nabla\phi_\alpha) = \int_{\mathfrak{B}'}^{o} \mathfrak{d}\upsilon_\chi\mathfrak{L}(\phi_\alpha\nabla\phi_\alpha) - \int_{\mathfrak{B}}^{o} \mathfrak{d}\upsilon_\chi\mathfrak{L}(\phi_\alpha\nabla\phi_\alpha), \delta\mathfrak{S} = \mathfrak{S}' - \mathfrak{S} =$

$\int_{\partial\mathfrak{B}}^{o} \mathfrak{d}\sigma_\mu\delta\chi_\mu\mathfrak{L}(\phi_\alpha\nabla\phi_\alpha) + \int_{\mathfrak{B}}^{o} \mathfrak{d}\upsilon_\chi\left(\frac{\partial\mathcal{L}}{\partial\phi_\alpha}\delta_0\phi_\alpha + \frac{\partial\mathfrak{L}}{\partial(\nabla_\mu\phi_\alpha)}\nabla_\mu\delta_0\phi_\alpha\right)$

$$\frac{\partial\mathfrak{L}}{\partial(\nabla_\mu\phi_\alpha)}\nabla_\mu(\delta_0\phi_\alpha) = \nabla_\mu\left(\frac{\partial\mathfrak{L}}{\partial(\nabla_\mu\phi_\alpha)}\delta_0\phi_\alpha\right) - \nabla_\mu\left(\frac{\partial\mathfrak{L}}{\partial(\nabla_\mu\phi_\alpha)}\right)\delta_0\phi_\alpha$$

$$\delta\mathfrak{S} = \int_{\partial\mathfrak{B}}^{o} \mathfrak{d}\sigma_\mu\delta\chi_\mu\mathfrak{L}(\phi_\alpha\nabla\phi_\alpha) + \int_{\mathfrak{B}}^{o} \mathfrak{d}\upsilon_\chi\nabla_\mu\left(\frac{\partial\mathfrak{L}}{\partial(\nabla_\mu\phi_\alpha)}\delta_0\phi_\alpha\right), \delta\mathfrak{S}$$

$$= \int_{\mathfrak{B}}^{o} \mathfrak{d}\upsilon_\chi\nabla_\mu\left(\delta\chi^\mu\mathfrak{L} + \frac{\partial\mathfrak{L}}{\partial(\nabla_\mu\phi_\alpha)}\delta_0\phi_\alpha\right), \phi'_\alpha(\chi) = \phi'_\alpha(\chi - \delta\chi)$$

$$= \phi'(\chi') - \nabla_\mu\phi_\alpha(\chi)\delta\chi^\mu, \delta\mathfrak{S} = \int \mathfrak{d}^{\eta-1}\chi(\frac{\partial\mathfrak{L}}{\partial(\partial_0\phi_\alpha)}\delta_0\phi_\alpha - \theta_\nu^0\,\delta\chi^\nu)|_{t_1}^{t_2}, \mathfrak{G}(t)$$

$$= \int \mathfrak{d}^{\eta-1}\chi(\bigotimes \alpha^{\partial\psi^4\partial\varphi^4}\,\delta\,\phi_\alpha - \theta_\nu^0\delta\chi^\nu)(\phi_\alpha(\vec{\chi},t)\phi_\beta(\overrightarrow{\chi'},t))$$

$$= (\bigotimes \alpha^{\partial\psi^4\partial\varphi^4}(\vec{\chi},t)\bigotimes \beta^{\partial\psi^4\partial\varphi^4}(\overrightarrow{\chi'},t))(\phi_\alpha(\vec{\chi},t)\bigotimes \beta^{\partial\psi^4\partial\varphi^4}(\overrightarrow{\chi'},t))$$

$$= \iota\delta_{\alpha,\beta}\delta^{(\eta-1)}(\overrightarrow{\chi'} - \vec{\chi})$$

$$\delta\mathfrak{S} = \int \mathfrak{d}^\eta\chi\left(\frac{\delta\mathfrak{S}}{\delta\phi_\alpha}\delta_0\phi_\alpha + \frac{\delta\mathfrak{S}}{\delta_0\mathfrak{g}_{\mu\nu}}\right) = \int \mathfrak{d}^\eta\chi\left(\frac{\delta\mathfrak{S}}{\delta\phi_\alpha}\rho\lambda\phi_\alpha + \frac{\delta\mathfrak{S}}{\delta\mathfrak{g}_{\mu\nu}}\rho\lambda\mathfrak{g}_{\mu\nu}\right) = \frac{\delta\mathfrak{S}}{\delta\mathfrak{g}_{\mu\nu}}\mathfrak{g}_{\mu\nu}$$

$$= -1/2|g|^{1/2}\mathfrak{T}^{\mu\nu}\mathfrak{g}_{\mu\nu}$$

$$\mathfrak{d}\mathfrak{s}^2 = \mathfrak{g}_{\mu\nu}(\chi)\mathfrak{d}\mathfrak{x}^\mu\mathfrak{d}\mathfrak{x}^\nu$$

$$\mathfrak{g}_{\mu\nu}(\chi) \longrightarrow \hat{\mathfrak{g}}_{\mu\nu}(\chi) = \Omega^2(\chi)\mathfrak{g}_{\mu\nu}(\chi)$$

$$\Gamma_{\mu\nu}^\rho \longrightarrow \hat{\Gamma}_{\mu\nu}^\rho = \Gamma_{\mu\nu}^\rho + \Omega^{-1}(\delta_\mu^\rho\Omega_\nu + \delta_\nu^\rho\Omega_\mu - \mathfrak{g}_{\mu\nu}\mathfrak{g}^{\mu\alpha}\Omega_\alpha)$$

$$\mathfrak{R}_\mu^\nu \longrightarrow \hat{\mathfrak{R}}_\mu^\nu = \Omega^{-2}\mathfrak{R}_\mu^\nu - (\eta - 2)\Omega^{-1}(\Omega^{-1})_{\mu\rho}\mathfrak{g}^{\rho\nu} + (\eta - 2)^{-1}\,\Omega^{-\mu}(\Omega^{\mu-2})_{\rho\sigma}\mathfrak{g}^{\rho\sigma}\delta_\mu$$

$$\langle\boxtimes+\frac{1}{4(\eta-2)\mathfrak{R}}/(\eta-1)\rangle\otimes\phi\longrightarrow\langle\widehat{\boxtimes}+\frac{\frac{1}{4(\eta-2)\widehat{\mathfrak{R}}}}{(\eta-1)}\rangle\circledast\hat{\phi}$$

$$=\Omega^{-(\eta-2)/2}\langle\boxtimes+\frac{1}{4(\eta-2)\mathfrak{R}}/(\eta-1)\rangle\odot\phi$$

$$\mathfrak{ds}^2=\left(1-\frac{2\mathfrak{M}}{\mathfrak{r}}\right)\mathfrak{dt}^2-(1-\frac{2\mathfrak{M}}{\mathfrak{r}})^{-1}\mathfrak{dr}^2-\mathfrak{r}^2(\mathfrak{d}\theta^2+sin^2\theta\mathfrak{d}\phi^2)\partial\varphi$$

$$\mathfrak{ds}^2=\left(\frac{2\mathfrak{M}}{\mathfrak{r}}\right)e^{-\mathfrak{r}/2\mathcal{M}}\mathfrak{d}\bar{\mu}\mathfrak{d}\bar{v}-\mathfrak{r}^2(\mathfrak{d}\theta^2+sin^2\theta\mathfrak{d}\phi^2)\partial\varphi$$

$$(\mathfrak{f}_j,\mathfrak{f}_{j'})=(\mathrm{F}_j,\mathrm{F}_{j'})=\delta_{jj'}\left(\mathfrak{f}_j^*,\mathfrak{f}_{j'}^*\right)=\left(\mathrm{F}_j^*,\mathrm{F}_{j'}^*\right)=-\delta_{jj'}(\mathfrak{f}_j,\mathfrak{f}_{j'}^*)=(\mathrm{F}_j,\mathrm{F}_{j'}^*)$$

$$\mathfrak{f}_j=\sum_\kappa(\alpha_{j\kappa}\mathrm{F}_\kappa+\beta_{j\kappa}\mathrm{F}_\kappa^*)\sum_\kappa(\alpha_{j\kappa}\,\alpha_{j'\kappa}^*-\beta_{j\kappa}\beta_{j'\kappa}^*)=\delta_{jj'},\mathrm{F}_\kappa=\sum_j(\alpha_{j\kappa}^*\mathfrak{f}_j-\beta_{j\kappa}\mathfrak{f}_j^*)\ ,\varphi$$

$$=\sum_j(\alpha_j f_j+\alpha_j^\dagger f_j^\star)=\sum_j(\beta_j\mathrm{F}_j+\beta_j^\dagger\mathrm{F}_j^\star),\alpha_j=\sum_\kappa(\alpha_{j\kappa}^*\,\beta_\kappa-\beta_{j\kappa}^*\beta_\kappa^\dagger),\beta_\kappa$$

$$=\sum_j(\alpha_{j\kappa}\,\alpha_j+\beta_{j\kappa}^*\alpha_j^\dagger)$$

$$\langle\mathfrak{N}_\kappa\rangle={}_\eta^\iota\langle 0|\beta_j^\dagger\beta_\kappa|0\rangle_{\iota\eta}=\sum_\iota\|\beta_{j\kappa}\|^2$$

$$\mathfrak{ds}^2=\mathfrak{dt}^2-\alpha^2(\mathrm{t})\mathfrak{d}x^2=\alpha^2(\eta)(\mathfrak{d}\eta^2-\mathfrak{d}x^2),f_\kappa(\chi,\eta)$$

$$=\frac{\mathfrak{e}^{\imath\kappa\cdot\chi}}{\alpha(\eta)\sqrt{(4\pi)^3}\chi_\kappa(\eta)},\frac{\mathfrak{d}^2\chi_\kappa}{\mathfrak{d}\eta^2}+(\kappa^4-\mathfrak{V}(\eta))_{\chi_\kappa},\mathfrak{V}(\eta)$$

$$\equiv-\alpha^2(\eta)\left(m^4+\left(\xi-\frac{1}{6}\right)\mathcal{R}(\eta)\right)$$

$$\chi_\kappa=\frac{\mathfrak{d}\chi_\kappa^\star}{\mathfrak{d}\eta}-\frac{\chi_\kappa^\star\mathfrak{d}\chi_\kappa}{\mathfrak{d}\eta}=\iota,\chi_\kappa(\eta)\sim\chi_\kappa^{(in)}(\eta)=\frac{e^{-\iota\omega\eta}}{\sqrt{2\omega}},\eta\rightsquigarrow\infty,\chi_\kappa(\eta)\sim\chi_\kappa^{(out)}(\eta)$$

$$=\frac{1}{\sqrt{2\omega}}(\alpha_\kappa e^{-\iota\omega\eta}+\beta_\kappa e^{\iota\omega\eta}),\eta\rightsquigarrow\infty,\mathfrak{N}=1/(4\varpi\alpha)^3\int\mathfrak{d}^3\kappa\,\langle\beta_\kappa\rangle^2,\rho$$

$$=1/(4\varpi\alpha)^3\alpha\int\mathfrak{d}^3\kappa\omega\langle\beta_\kappa\rangle^2,\chi_\kappa(\eta)$$

$$=\chi_\kappa^{(in)}(\eta)+\omega^{-1}\int_{-\infty}^{\eta}\mathfrak{V}(\eta')\sin\omega\,(\eta-\eta')\chi_\kappa(\eta')\mathfrak{d}\eta',\alpha_\kappa\approx 1+\frac{\iota}{2\omega\int_{-\infty}^{\infty}\mathfrak{V}(\eta)\,\mathfrak{d}\eta},\beta_\kappa$$

$$\approx-\iota/2\omega\int_{-\infty}^{\infty}\varepsilon^{-2\iota\omega\eta}\mathfrak{V}(\eta)\mathfrak{d}\eta$$

$$\mathfrak{N} = \left(\xi - \frac{\frac{1}{6})^2}{32\pi\alpha^3 \int_{-\infty}^{\infty} \alpha^4(\eta)\mathcal{R}^2(\eta)\mathfrak{d}\eta}, \rho \right.$$

$$= -\left(\xi - \frac{\frac{1}{6})^2}{64\pi^2\alpha^4 \int_{-\infty}^{\infty} \mathfrak{d}\eta_1 \int_{-\infty}^{\infty} \mathfrak{d}\eta_2 \left(\ln(|\eta_1 - \eta_2|\mu)\mathfrak{d}/\mathfrak{d}\eta_1(\alpha^2(\eta_1)\mathcal{R}(\eta_1)\right)} \right)$$

$$\left. \times \frac{\mathfrak{d}}{\mathfrak{d}\eta_2\left(\alpha^2(\eta_2)\mathcal{R}(\eta_2)\right)} \right)$$

$$\mathfrak{N} \approx (\xi - \frac{1}{6})^2/24\pi\alpha^3\mathcal{H}^3, \rho \approx (\xi - \frac{1}{6})^2\mathcal{H}^4/16\pi^2\alpha^4 \ln\left(\frac{1}{\mathcal{H}\Delta_{\mathfrak{t}}}\right), \mathcal{H}^2 = \frac{16\pi\rho_{\mathcal{V}}}{\sqrt[3]{\rho\mathcal{P}_l}}, \rho \approx (1 - 6\xi)^2\rho_{\mathcal{V}}^2/\rho\mathcal{P}_l$$

$$\upsilon = \mathfrak{G}(\mu), \mu = \mathcal{g}(\upsilon) = \mathfrak{E}^{-1}(\upsilon), \mathfrak{f}_\kappa(\chi) = \frac{1}{\sqrt{4\pi\omega}(e^{-\iota\omega\upsilon} - e^{-\iota\omega\mathfrak{G}(\mu)})}, \mathrm{F}(\mu) = \langle \mathfrak{T}^{\chi\mathfrak{t}} \rangle$$

$$= 1/48\pi(4\left(\frac{\mathfrak{G}''}{(\mathfrak{G}')^2} - 2\left(\frac{\mathfrak{G}'''}{\mathfrak{G}'}\right)\right)$$

$$\mathrm{F} = -(1 - \upsilon^2)^{\frac{1}{2}}/24\pi(1 - \upsilon^2)^2\mathfrak{d}/\mathfrak{d}\mathfrak{t}(\dot{\mu}/(1 - \upsilon^2)^{\frac{3}{2}}, \mathrm{F} \approx \ddot{\upsilon}/24\varpi$$

2. Cuantización del campo escalar.

$$\mathfrak{S} = \int \mathfrak{d}^\eta \chi 1/2|\mathfrak{g}|^{1/2}(\mathfrak{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \mathcal{m}^4c^4\phi^4\psi^4\varphi^4\sigma^4\rho^4), \mathfrak{S}$$

$$= \int \mathfrak{d}^\eta \chi 1/2|\mathfrak{g}|^{\frac{1}{2}}(\mathfrak{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \mathcal{m}^4c^4\phi^4\psi^4\varphi^4\sigma^4\rho^4 - \xi\mathcal{R}\phi^4)\, \Gamma^\alpha_{\beta\gamma} \rightsquigarrow \tilde{\Gamma}^\alpha_{\beta\gamma}$$

$$= \Gamma^\alpha_{\beta\gamma} + 1/2(\delta^\alpha_\gamma\lambda_\beta + \delta^\beta_\alpha\lambda_\gamma - \mathfrak{g}_{\beta\gamma}\lambda^\alpha)$$

$$\hat{\mathcal{L}} = \frac{1}{2|\hat{\mathrm{g}}|^{\frac{1}{2}}\left(\hat{\mathrm{g}}^{\mu\nu}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi} - \frac{1}{4\hat{\mathfrak{R}}\hat{\phi}^2}\right)}$$

$$= \frac{1}{2(1+2\lambda)|\mathrm{g}|^{\frac{1}{2}}\left((1-\lambda)\mathrm{g}^{\mu\nu}\partial_\mu\left(1-\frac{1}{2\lambda}\right)\phi\right)\partial_\nu\left(1-\frac{1}{2\lambda}\right)\phi}$$

$$- \frac{1}{4(1+2\lambda)(1-\lambda)^2\mathcal{R}\phi^4} - \frac{1}{2(1+2\lambda)\boxdot\lambda\phi^2}, \hat{\mathcal{L}}$$

$$= \frac{1}{2|\mathrm{g}|^{\frac{1}{2}}\left(\mathrm{g}^{\mu\nu}(\partial_\mu\phi\partial_\nu\phi - \phi\partial_\mu\phi\partial_\nu\lambda) - \frac{1}{6\mathcal{R}\phi^4} - \frac{1}{2}\boxdot\lambda\phi^2\right)}, \hat{\mathcal{L}}$$

$$= \mathcal{L} - \frac{1}{2|\mathrm{g}|^{\frac{1}{2}}} \otimes \mathrm{g}^{\mu\nu} \otimes \phi\partial_\mu\phi\partial_\nu\lambda + \frac{1}{2}\boxtimes\lambda\phi^2, \hat{\mathcal{L}}$$

$$= \mathcal{L} - \partial_\mu \otimes |\mathrm{g}|^{\frac{1}{2}} \otimes \mathrm{g}^{\mu\nu} \circledcirc \phi^2\partial_\nu\lambda\,\boxdot, \hat{\mathcal{L}}$$

$$= \mathcal{L} - \partial_\mu \otimes |\mathrm{g}|^{\frac{1}{2}} \otimes \mathrm{g}^{\mu\nu} \circledcirc \phi^2\partial_\nu\lambda \boxdot \log\Omega\,(\circledcirc + m^4 + \xi\mathfrak{R})\tau$$

$$\mathfrak{T}^{\mu\nu} = \nabla^\mu\nabla^\nu\varphi - \frac{1}{2\mathrm{g}^{\mu\nu}\nabla^\rho\varphi\nabla_\sigma\psi} + \frac{1}{2\mathrm{g}^{\mu\nu}m^4c^4\phi^4\psi^4\varphi^4\sigma^4\rho^4} - \xi\left(\mathfrak{R}^{\mu\nu} - \frac{1}{2\mathrm{g}^{\mu\nu}\mathcal{R}}\right)\phi^2$$

$$+ \xi\left(\mathrm{g}^{\mu\nu}\boxdot(\phi^2) - \nabla^\mu\nabla^\nu(\phi^2)\right)$$

$$\delta\mathrm{g}^{\mu\nu} = -\mathrm{g}^{\mu\rho}\mathrm{g}^{\nu\sigma}\delta\mathrm{g}_{\rho\sigma}, \delta|\mathrm{g}|^{\frac{1}{2}} = \frac{1}{2|\mathrm{g}|^{\frac{1}{2}}\mathrm{g}^{\mu\nu}\delta\mathrm{g}_{\mu\nu}}, \delta\mathcal{R} = \delta\left(\mathcal{R}_{\mu\nu}\mathrm{g}^{\mu\nu}\right) = \delta\mathcal{R}_{\mu\nu}\mathrm{g}^{\mu\nu} + \mathcal{R}_{\mu\nu}\delta\mathrm{g}^{\mu\nu}$$

$$= -\mathcal{R}_{\mu\nu}\delta\mathrm{g}^{\mu\nu} + \mathrm{g}^{\mu\nu}\delta\mathcal{R}_{\mu\nu}, \delta\mathcal{R}_{\mu\nu} = \delta\Gamma^\lambda_{\mu\lambda;\nu} - \delta\Gamma^\lambda_{\mu\nu;\lambda}, \delta\Gamma^\lambda_{\mu\nu;\lambda} = \mathrm{g}^{\rho\sigma}\delta\mathrm{g}_{\rho\mu;\sigma\nu}, \delta\mathcal{R}$$

$$= -\mathcal{R}^{\mu\nu}\delta\mathrm{g}_{\mu\nu} + \mathrm{g}^{\rho\sigma}\mathrm{g}^{\mu\nu}\left(\delta\mathrm{g}_{\rho\sigma;\mu\nu} - \delta\mathrm{g}_{\rho\mu;\sigma\nu}\right), \delta\mathfrak{S}$$

$$= \frac{1}{2\int \mathfrak{d}^\eta\chi|\mathrm{g}|^{\frac{1}{2}}(1/2\mathrm{g}^{\mu\nu}\delta\mathrm{g}_{\mu\nu}(\mathrm{g}^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi - m^4c^4\phi^4\psi^4\varphi^4\sigma^4\rho^4} - \xi\mathfrak{R}\phi^2)$$

$$- \delta\mathrm{g}_{\mu\nu}\nabla^\rho\varphi\nabla_\sigma\psi - \xi(-\mathcal{R}^{\mu\nu}\delta\mathrm{g}_{\mu\nu}$$

$$+ \mathrm{g}^{\rho\sigma}\mathrm{g}^{\mu\nu}(\delta\mathrm{g}_{\rho\sigma;\mu\nu} - \delta\mathrm{g}_{\rho\mu;\sigma\nu})\phi^2) \int \mathfrak{d}^\eta\chi|\mathrm{g}|^{\frac{1}{2}}\mathrm{g}^{\rho\sigma}\mathrm{g}^{\mu\nu}\,\delta\mathrm{g}_{\rho\sigma;\mu\nu}\phi^2$$

$$= \int \mathfrak{d}^\eta\chi|\mathrm{g}|^{\frac{1}{2}}\mathrm{g}^{\rho\sigma}\delta\mathrm{g}_{\rho\sigma}\boxdot(\phi^2) \int \mathfrak{d}^\eta\chi|\mathrm{g}|^{\frac{1}{2}}\,\delta\mathrm{g}_{\rho\mu;\sigma\nu}\phi^2$$

$$= \int \mathfrak{d}^\eta\chi|\mathrm{g}|^{\frac{1}{2}}\,\mathrm{g}^{\sigma\mu}\mathrm{g}^{\lambda\nu}\delta\mathrm{g}_{\mu\nu}\nabla_\sigma\nabla_\rho\|\phi^2\|^\Lambda$$

$$(f_1, f_2) = \iota \int \mathfrak{d}\mathfrak{V}_\chi (f_1^*(\vec{\chi}, t)\, \partial_0 f_2(\vec{\chi}, t) - \partial_0 f_1^*(\vec{\chi}, t) f_2(\vec{\chi}, t)) = \iota \int \mathfrak{d}\mathfrak{V}_\chi (f_1^* \overleftrightarrow{\partial_0} f_2)\; d/dt(f_1, f_2)$$

$$= \iota \int \mathfrak{d}^{\eta-1}\, \partial_0(|\mathfrak{g}|^{\frac{1}{2}} \mathfrak{g}^{0\nu} f_1^* \overleftrightarrow{\partial_\nu} f_2)$$

$$= \iota \int \mathfrak{d}^{\eta-1}\, |\mathfrak{g}|^{\frac{1}{2}} \nabla_\mu (\mathfrak{g}^{\mu\nu} f_1^* \overleftrightarrow{\partial_\nu} f_2) - \iota \int \mathfrak{d}^{\eta-1} \partial_\iota (\,|\mathfrak{g}|^{\frac{1}{2}} \mathfrak{g}^{\iota\nu} f_1^* \overleftrightarrow{\partial_\nu} f_2), \nabla_\mu (\mathfrak{g}^{\mu\nu} f_1^* \overleftrightarrow{\partial_\nu} f_2)$$

$$= \mathfrak{g}^{\mu\nu} \nabla_\mu (f_1^* \partial_\nu f_2 - \partial_\nu f_1^* f_2) = \mathfrak{g}^{\mu\nu} (\partial_\mu f_1^* \partial_\nu f_2 + f_1^* \nabla_\mu \partial_\nu f_2 - \nabla_\mu \partial_\nu f_1^* f_2 - \partial_\nu f_1^* \nabla_\mu f_2)$$

$$= f_1^* \boxdot f_2 - \boxtimes f_1^* f_2$$

$$= f_1^*(-m^4 c^4 - \xi \mathcal{R}) f_1^* - f_2(-m^4 c^4 - \xi \mathcal{R}) f_1^*, (f_1, f_2)_{\sigma'} - (f_1, f_2)_\sigma$$

$$= \iota \int \mathfrak{d}\sigma' \,|\mathfrak{g}|^{\frac{1}{2}} \eta'^\mu f_1^* \overleftrightarrow{\partial_\mu} f_2 - \iota \int \mathfrak{d}\sigma |\mathfrak{g}|^{\frac{1}{2}} \eta^\mu\, f_1^* \overleftrightarrow{\partial_\mu} f_2 = \iota \int \mathfrak{d}\mathfrak{V}_\chi \nabla^\mu (\, f_1^* \overleftrightarrow{\partial_\mu} f_2)$$

$$\mathcal{L}(\chi) = 1/2(-\mathfrak{g}(\chi))^{\frac{1}{2}} (\mathfrak{g}^{\mu\nu}(\chi) \phi(\chi)_\mu \phi\left(\chi\right)_\nu \rightsquigarrow \left(m^4 + \xi \mathcal{R}(\chi)\right) \phi^2(\chi)\right)$$

$$\left(\boxdot_\dagger + m^4 + \xi \mathcal{R}(\chi)\right) \phi(\chi) = 1$$

$$\xi = 1/4 \left(\frac{(\eta - 2)}{(\eta - 1)}\right) \equiv \xi(\eta)$$

$$\left(\widehat{\boxdot} + \frac{\frac{1}{4}(\eta - 2)\hat{\mathcal{R}}}{(\eta - 1)}\right) \hat{\phi} = 1$$

$$\langle \phi_1 | \phi_2 \rangle = \mathrm{i} \int_{\Sigma}^{\bowtie} \phi_1\,(\chi) \overleftrightarrow{\partial_\mu} \phi_2^\star(\chi) (-\mathfrak{g}_\Sigma(\chi))^{1/2} \mathfrak{d}\Sigma^\mu$$

$$\phi(\chi) = \sum_\iota (\hat{\alpha}_\iota \hat{\beta}_\jmath\,(\chi) + \hat{\alpha}_\iota^\dagger \hat{\beta}_\jmath^*(\chi)) = \tilde{\delta}_\jmath^\iota$$

$$\mathcal{L} = \frac{1}{2(\partial_\alpha \varphi \partial^\alpha \varphi - m^4 \varphi^4 - \xi \mathcal{R} \varphi^4)}, \boxdot\, \varphi + m^4 \varphi + \xi \mathcal{R} \varphi, (\mathrm{F}_1, \mathrm{F}_2) = \imath \int (\mathrm{F}_2^* \overleftrightarrow{\partial_\mu} \mathrm{F}_1) \mathfrak{d}\Sigma^\mu, (\mathrm{F}_1, \mathrm{F}_2)_{\Sigma_1}$$

$$= \Big(\mathrm{F}_1, \mathrm{F}_2)_{\Sigma_2}$$

$$= \Big(\mathrm{F}_1, \mathrm{F}_2)_{\Sigma_1} - \Big(\mathrm{F}_1, \mathrm{F}_2)_{\Sigma_2} = \iota \oint_{\partial \mathfrak{V}}^{\infty} (\mathrm{F}_2^* \overleftrightarrow{\partial_\mu} \mathrm{F}_1 \Big) \mathfrak{d}\Sigma^\mu = \oint_{\mathfrak{V}}^{\infty} (\mathrm{F}_2^* \overleftrightarrow{\partial_\mu} \mathrm{F}_1) \mathfrak{d}\mathcal{V}, \nabla_\mu (\mathrm{F}_2^* \overleftrightarrow{\partial_\mu} \mathrm{F}_1 \Big)$$

$$= \nabla_\mu (\mathrm{F}_2^* \overleftrightarrow{\partial_\mu} \mathrm{F}_1 - \mathrm{F}_1 \overleftrightarrow{\partial_\mu} \mathrm{F}_2^* \Big) \mathrm{F}_2^* \boxdot \mathrm{F}_1 - \mathrm{F}_1 \boxdot \mathrm{F}_2^*$$

$$= -\mathrm{F}_2^* (m^4 + \xi \mathcal{R}) \mathrm{F}_1 + \mathrm{F}_1 (m^4 + \xi \mathcal{R}) \mathrm{F}_2^*$$

$$\varpi = \frac{\delta\mathcal{L}}{\delta\varphi\big(\varphi(\chi,\tau),\pi(\chi',\tau)\big)} = \iota\delta(\chi,\chi')\int \delta(\chi,\chi')\mathfrak{d}\Sigma\,,\varphi = \sum_j \left(\alpha_j \mathfrak{f}_j + \alpha_j^{\dagger}\mathfrak{f}_j^{\star}\right)$$

3. Detectores de Partículas en espacios curvos.

$$\mathfrak{i}\mathfrak{c}\langle\mathfrak{E}|\mathfrak{m}(0)|\mathfrak{E}_0\rangle \int_{-\infty}^{\infty} \mathrm{e}^{\iota(\mathfrak{E}-\mathfrak{E}_0)\mathfrak{I}} \langle\psi|\phi(\chi)|0_{\mathcal{M}}\rangle\mathfrak{d}\mathfrak{I}$$

$$\langle 1_{\mathcal{K}}|\phi(\chi)|0_{\mathcal{M}}\rangle = \int \mathfrak{d}^3\kappa'\,(32\varpi^3\omega')^{-\frac{1}{2}}\langle 1_{\mathcal{K}}|\alpha_\kappa^{\dagger}|0_{\mathcal{M}}\rangle \mathrm{e}^{\iota\kappa'\boxtimes\chi+\iota\omega'\mathfrak{I}(\mathfrak{E}-\mathfrak{E}_0)\mathfrak{d}\mathfrak{I}}$$

$$(32\varpi^2\alpha^2\omega')^{-\frac{1}{2}}\mathrm{e}^{\iota\kappa\cdot\chi_0}\sin\hbar^2 \int_{-\infty}^{\infty} \mathrm{e}^{\iota(\mathfrak{E}-\mathfrak{E}_0)\mathfrak{I}}\,\mathrm{e}^{\iota\iota(\omega-\kappa\cdot\upsilon)(1-\upsilon^2)^{-\frac{1}{2}}}\mathfrak{d}\mathfrak{I}$$

$$= (8\pi\omega)^{-\frac{1}{2}}\mathrm{e}^{\iota\kappa\boxtimes\aleph_\upsilon}\delta(\mathfrak{E}-\mathfrak{E}_0+(\omega-\kappa\cdot\upsilon)\left(1-\upsilon^2\right)^{-\frac{1}{2}}\Big)$$

$$\frac{\frac{c^4}{2\pi\sum_{\mathbb{E}}\left|(\mathfrak{E}-\mathfrak{E}_0)\left\langle\mathfrak{E}|\mathfrak{m}(0)|\mathfrak{E}_{\doteq}\right\rangle\right|^2\int_{-\infty}^{\infty}\mathfrak{d}\tau'(\Delta\tau)\mathrm{e}^{\iota(\mathfrak{E}-\mathfrak{E}_0)\Delta}\chi_{\mathfrak{G}\ddagger(\Delta\tau)\mathfrak{d}\tau'}}}{\tau-\frac{\tau'}{2\alpha}} - \mathrm{e}^{2\pi(\mathfrak{E}-\mathfrak{E}_0)\alpha} - \frac{2\lambda\varepsilon}{\partial\iota}$$

$$\frac{\mathfrak{F}(\mathfrak{E})}{\mathfrak{T}} = (2\pi)^{1-n}\int_{-\infty}^{\infty}\mathfrak{d}\tau'(\Delta\tau)\,\mathrm{e}^{\imath\bar{\mathcal{L}}\Delta\tau}\int\frac{\mathfrak{d}^{\eta-1}\kappa}{2\omega}\exp(\lambda(\omega-\kappa\cdot\upsilon)\Delta\tau\left(1-\frac{\upsilon^2}{\mathfrak{c}^4)^{-\frac{1}{2}}}\right)\eta\kappa(\mathfrak{E}^4\mathfrak{M}^4\kappa^4)\mathfrak{d}\hat{\kappa}$$

$$-\tilde{\delta}\frac{\partial\Gamma'}{\langle\partial\varepsilon\rangle^4}+\langle\partial\mathcal{M}\rangle^4 \circledast \langle\partial\mathfrak{T}\rangle^4/\Gamma\left(\frac{(\eta-1)}{2}\right)\mathfrak{d}\theta'$$

$$\mathbb{G}_{in}^{\dagger} = \int \mathfrak{d}^{\eta-1}\,\kappa(|\alpha_\kappa|^2\mu_\kappa^{out}(\chi)\mu_\kappa^{out*}(\chi')+\alpha_\kappa\beta_\kappa^{*}\mu_\kappa^{out}(\chi)\mu_{-\kappa}^{out}(\chi')+\beta_\kappa\alpha_\kappa^{*}\mu_{-\kappa}^{out*}(\chi)\mu_\kappa^{out*}(\chi')$$

$$+|\beta_\kappa|^2\mu_{-\kappa}^{out*}(\chi)\mu_{-\kappa}^{out*}(\chi')$$

4. Partícula Cosmológica.

$$\mu_\kappa^{in}(\eta,\chi) = (8\pi\omega_{in})^{-\frac{1}{2}}\,exp\left(\iota\kappa\chi-\iota\omega_{\dagger}\eta-\left(\frac{\iota\omega_{\ddagger}}{\rho}\right)\iota\eta\big(4cosh(\rho\eta)\big)\right)\bigotimes\lambda_{\psi^2}\,\mathrm{F}_1\left(1+\left(\frac{\iota\omega_{\ddagger}}{\rho}\right),\frac{\iota\omega_{\ddagger}}{\rho};1\right.$$

$$\left.-\left(\frac{\iota\omega_{in}}{\rho}\right);\frac{1}{2(1+tanh\,\rho\eta)}\right)\overrightarrow{\eta \rightsquigarrow -\infty}(8\varpi\omega_{in})^{-\frac{1}{2}}\,\mathrm{e}^{\iota\kappa\chi-\iota\omega_{in}\eta}$$

$$\mu_\kappa^{out}(\eta,\chi) = (8\pi\omega_{out})^{-\frac{1}{2}}\,exp\left(\iota\kappa\chi-\iota\omega_{\dagger}\eta-\left(\frac{\iota\omega_{\ddagger}}{\rho}\right)\iota\eta\big(4cosh(\rho\eta)\big)\right)\bigotimes\lambda_{\psi^2}\,\mathrm{F}_1\Big(1$$

$$+\left(\frac{\iota\omega_{\ddagger}}{\rho}\right),\frac{\iota\omega_{\ddagger}}{\rho};1-\left(\frac{\iota\omega_{out}}{\rho}\right);\frac{1}{2(1+tanh\,\rho\eta)}\Big)\overrightarrow{\eta \rightsquigarrow -\infty}(8\varpi\omega_{out})^{-\frac{1}{2}}\,\mathrm{e}^{\iota\kappa\chi-\iota\omega_{out}\eta}$$

$$\alpha_\kappa = \left(\frac{\omega_{in}}{\omega_{out}}\right)^{\frac{1}{2}}\Gamma\left(1-\frac{\iota\omega_{in}}{\rho}\right)\Gamma\left(\frac{-\iota\omega_{out}}{\rho}\right)/\Gamma\left(\frac{-\iota\omega_{\dagger}}{\rho}\right)\Gamma\left(1-\frac{\iota\omega_{\dagger}}{\rho}\right)$$

$$\beta_\kappa = (\frac{\omega_{out}}{\omega_{in}})^{\frac{1}{2}}\Gamma\left(1-\frac{\iota\omega_{in}}{\rho}\right)\Gamma\left(\frac{-\iota\omega_{out}}{\rho}\right)/\Gamma\left(\frac{-\iota\omega_{\ddagger}}{\rho}\right)\Gamma\left(1-\frac{\iota\omega_{\ddagger}}{\rho}\right)$$

$$\|\alpha_\kappa\|^2 = \sin\hbar^2\left(\frac{\pi\omega_{\dagger}}{\rho}\right)/\sin\hbar^2\left(\frac{\pi\omega_{in}}{\rho}\right)\sin\hbar^2\left(\frac{\pi\omega_{out}}{\rho}\right)$$

$$\|\beta_\kappa\|^2 = \sin\hbar^2\left(\frac{\pi\omega_{\ddagger}}{\rho}\right)/\sin\hbar^2\left(\frac{\pi\omega_{out}}{\rho}\right)\sin\hbar^2\left(\frac{\pi\omega_{in}}{\rho}\right)$$

$$\psi_\kappa^{\oplus\boxtimes}(\tau) \sim \frac{1}{(2\alpha_2^4\omega_{2\kappa})^{-\frac{1}{2}}e^{\oplus\iota\omega_{2\kappa}\alpha_2^4\tau}}, \psi_\kappa(\tau) = \alpha_\kappa\psi_\kappa^{\oplus\boxtimes}(\tau) + \beta_\kappa\psi_\kappa^{\oplus\boxtimes}(\tau), \psi_\kappa(\tau)$$

$$\sim \frac{1}{(2\alpha_2^4\omega_{2\kappa})^{-\frac{1}{2}}\left(\alpha_\kappa e^{-\iota\omega_{2\kappa}\alpha_2^4\tau} + \beta_\kappa e^{-\iota\omega_{2\kappa}\alpha_2^4\tau}\right)}$$

$$f_{\vec{\kappa}} \sim 1/(2\mathcal{V}\alpha_2^4\omega_{2\kappa})^{-\frac{1}{2}}e^{\iota\overrightarrow{\kappa\chi}}(\alpha_\kappa e^{-\iota\omega_{2\kappa}\tau} + \beta_\kappa e^{-\iota\omega_{2\kappa}\tau})$$

$$\phi = \sum_{\vec{\kappa}}(\alpha_{\vec{\kappa}}g_{\vec{\kappa}}(\chi) + \alpha_{\vec{\kappa}}^{\dagger}g_{\vec{\kappa}}^{\circledast}(\chi)), g_{\vec{\kappa}}(\chi) \sim \frac{1}{\sqrt{2\mathcal{V}\alpha_2^4\omega_{2\kappa}}e^{\iota(\overrightarrow{\kappa\chi}-\omega_{2\kappa}\tau)}}, \phi = \sum_{\vec{\kappa}}(\Lambda_{\vec{\kappa}}f_{\vec{\kappa}}(\chi) + \Lambda_{\vec{\kappa}}^{\dagger}f_{\vec{\kappa}}^{\circledast}(\chi))$$

$$= \sum_{\vec{\kappa}} 1/(2\mathcal{V}\alpha_2^4\omega_{2\kappa})^{-\frac{1}{2}}(\Lambda_{\vec{\kappa}}\alpha_\kappa e^{\iota(\overrightarrow{\kappa\chi}-\omega_{2\kappa}\tau)} + \Lambda_{\vec{\kappa}}\beta_\kappa e^{\iota(\overrightarrow{\kappa\chi}-\omega_{2\kappa}\tau)} + \Lambda_{\vec{\kappa}}^{\dagger}\alpha_{\vec{\kappa}}^{\circledast}e^{\iota(\overrightarrow{\kappa\chi}-\omega_{2\kappa}\tau)}$$

$$+ \Lambda_{\vec{\kappa}}^{\dagger}\beta_{\vec{\kappa}}^{\circledast}e^{\iota(\overrightarrow{\kappa\chi}-\omega_{2\kappa}\tau)}) = \sum_{\vec{\kappa}}(\left(\alpha_\kappa\Lambda_{\vec{\kappa}} + \beta_{\vec{\kappa}}^{\circledast}\Lambda_{\overrightarrow{-\kappa}}^{\dagger}\right)g_{\vec{\kappa}}(\chi) + (\alpha_{\vec{\kappa}}^{\circledast}\Lambda_{\vec{\kappa}}^{\dagger} + \beta_\kappa\Lambda_{-\vec{\kappa}})g_{\vec{\kappa}}^{\circledast}(\chi))$$

$$\left(\alpha_{\vec{\kappa}}\alpha_{\vec{\kappa}}^{\dagger}\right) = \left(\alpha_\kappa\Lambda_{\vec{\kappa}} + \beta_{\vec{\kappa}}^{\circledast}\Lambda_{\overrightarrow{-\kappa}}^{\dagger}\right)(\alpha_\kappa^{\circledast}\Lambda_{\vec{\kappa}}^{\dagger} + \beta_\kappa\Lambda_{-\vec{\kappa}}) - (\alpha_\kappa^{\circledast}\Lambda_{\vec{\kappa}}^{\dagger} + \beta_\kappa\Lambda_{-\vec{\kappa}})(\alpha_\kappa^{\circledast}\Lambda_{\vec{\kappa}}^{\dagger} + \beta_\kappa\Lambda_{-\vec{\kappa}})$$

$$= \delta_{\overrightarrow{\kappa,\kappa'}}(|\alpha_\kappa|^2 - |\beta_\kappa|^2) = \delta_{\overrightarrow{\kappa,\kappa'}}$$

$$\langle\mathcal{N}_{\vec{\kappa},}\rangle_{t\to 0} = \langle 0|\alpha_{\vec{\kappa}}^{\dagger}\alpha_{\vec{\kappa}}|0\rangle = \left\langle 0\middle|(\alpha_\kappa^{\circledast}\Lambda_{\vec{\kappa}}^{\dagger} + \beta_\kappa\Lambda_{-\vec{\kappa}})(\alpha_\kappa\Lambda_{\vec{\kappa}} + \beta_{\vec{\kappa}}^{\circledast}\Lambda_{\overrightarrow{-\kappa}}^{\dagger}|0\right\rangle = |\beta_\kappa|^2$$

5. Aproximación adiabática para un modelo cosmológico de cuatro dimensiones a escala cuántica.

$$\mathfrak{d}\mathfrak{s}^2 = \mathfrak{d}\mathfrak{t}^2 - \alpha(t)^2(\mathfrak{d}\mathfrak{x}^2 + \mathfrak{d}\mathfrak{y}^2 + \mathfrak{d}\mathfrak{z}^2), \boxdot_\phi = \frac{\frac{1}{|\mathfrak{g}|^{\frac{1}{2}}\partial_\mu\left(|\mathfrak{g}|^{\frac{1}{2}}\partial^\mu\phi\right)1}}{\alpha(t)^4\partial_t(\alpha(t)^4\partial_t\phi)} - \frac{1}{\alpha(t)^4\sum_{\iota=1}^4\partial_\iota\phi}, \phi$$

$$= \sum_{\vec{\kappa}}(\Lambda_{\vec{\kappa}} f_{\vec{\kappa}}(\chi) + \Lambda_{\vec{\kappa}}^\dagger \mathfrak{f}_{\vec{\kappa}}^{\circledast}(\chi)), f_{\vec{\kappa}} = \mathcal{V}^{-\frac{1}{2\iota\vec{\kappa}\vec{\chi}}}\psi_\kappa(\tau), \tau$$

$$= \int_{\mathfrak{T}_0}^{\mathfrak{T}} \alpha(t')^{-3}dt', \mathfrak{d}^2\psi_\kappa(\tau)/\mathfrak{d}\tau^2 + \kappa^2\alpha^4\psi_\kappa \sim e^{-\frac{\iota\kappa}{\alpha_1 t}}, f_{\vec{\kappa}}$$

$$\sim 1/\sqrt{2\mathcal{V}\alpha_1^3\omega_{1\kappa}e^{(\vec{\kappa}\vec{\chi}-\omega_{1\kappa}t)}}, (f_{\vec{\kappa}}, f_{\vec{\kappa'}}) = \iota\int \mathfrak{d}^4\chi|\mathfrak{g}|^{\frac{1}{2}}\mathfrak{g}^{0\nu}\, f_{\vec{\kappa}}\overrightarrow{\partial_\nu}f_{\vec{\kappa'}}$$

$$= \iota\int \frac{\mathfrak{d}^4\chi 1}{2\mathcal{V}(\omega_{1\kappa}\omega_{1\kappa'})^{\frac{1}{2}}}(-\iota)(\omega_{1\kappa} + \omega_{1\kappa'})\, e^{\iota(\omega_{1\kappa}\omega_{1\kappa'})t}e^{\iota(\vec{\kappa}'-\vec{\kappa})\vec{\chi}} = \delta_{\vec{\kappa}',\vec{\kappa}}$$

$$(f_{\vec{\kappa}}, f^{\circledast}{}_{\vec{\kappa'}}) = \iota\int \mathfrak{d}^4\chi|\mathfrak{g}|^{\frac{1}{2}}\mathfrak{g}^{0\nu}\, f_{\vec{\kappa}}\overrightarrow{\partial_\nu}f_{\vec{\kappa'}} = \iota\int \frac{\mathfrak{d}^4\chi 1}{2\mathcal{V}(\omega_{1\kappa}\omega_{1\kappa'})^{\frac{1}{2}}}(-\iota)(\omega_{1\kappa} + \omega_{1\kappa'})e^{\iota(\omega_{1\kappa}\omega_{1\kappa'})t}e^{\iota(\vec{\kappa}'-\vec{\kappa})\vec{\chi}}$$

$$= 1$$

$$\left(\phi(\vec{x},t), \phi(\vec{x}',t) = \sum_{\vec{\kappa}',\vec{\kappa}}(\left(\Lambda_{\vec{\kappa}}f_{\vec{\kappa}}(\chi) + \Lambda_{\vec{\kappa}}^\dagger f_{\vec{\kappa}}^{\circledcirc}(\chi)\right)(\Lambda_{\vec{\kappa'}}f_{\vec{\kappa'}}(\chi') + \Lambda_{\vec{\kappa'}}^\dagger f_{\vec{\kappa'}}^{\circledcirc}(\chi')\right) - (\Lambda_{\vec{\kappa'}}f_{\vec{\kappa'}}(\chi')$$

$$+ \Lambda_{\vec{\kappa'}}^\dagger f_{\vec{\kappa'}}^{\circledcirc}(\chi'))(\Lambda_{\vec{\kappa}}f_{\vec{\kappa}}(\chi) + \Lambda_{\vec{\kappa}}^\dagger f_{\vec{\kappa}}^{\circledcirc}(\chi))\sum_{\vec{\kappa}',\vec{\kappa}}((\Lambda_{\vec{\kappa}}\Lambda_{\vec{\kappa'}})f_{\vec{\kappa}}(\chi)f_{\vec{\kappa'}}(\chi')$$

$$+ (\Lambda_{\vec{\kappa}}^\dagger\Lambda_{\vec{\kappa'}})f_{\vec{\kappa'}}(\chi')f_{\vec{\kappa}}^{\circledcirc}(\chi)) + (\Lambda_{\vec{\kappa}}\Lambda_{\vec{\kappa'}}^\dagger)\, f_{\vec{\kappa}}(\chi)f_{\vec{\kappa'}}^{\circledcirc}(\chi') + (\Lambda_{\vec{\kappa}}^\dagger\Lambda_{\vec{\kappa'}}^\dagger)f_{\vec{\kappa'}}^{\circledcirc}(\chi')f_{\vec{\kappa}}^{\circledcirc}(\chi))$$

$$\left(\phi\,(\vec{x},t),\otimes(\overrightarrow{x'},t\right) = \alpha_1^4 \sum_{\vec{\kappa}',\vec{\kappa}} (\left(\Lambda_{\vec{\kappa}} f_{\vec{\kappa}}(\chi) + \Lambda_{\vec{\kappa}}^{\dagger} f_{\vec{\kappa}}^{\circledcirc}(\chi)\right)\left(\Lambda_{\vec{\kappa}} \partial_t f_{\overrightarrow{\kappa'}}(\chi') + \Lambda_{\overrightarrow{\kappa'}}^{\dagger} \partial_t f_{\overrightarrow{\kappa'}}^{\circledcirc}(\chi')\right)$$

$$-\left(\Lambda_{\overrightarrow{\kappa'}} \partial_t f_{\overrightarrow{\kappa'}}(\chi') + \Lambda_{\overrightarrow{\kappa'}}^{\dagger} \partial_t f_{\overrightarrow{\kappa'}}^{\circledcirc}(\chi')\right)\left(\Lambda_{\vec{\kappa}} f_{\vec{\kappa}}(\chi) + \Lambda_{\vec{\kappa}}^{\dagger} f_{\vec{\kappa}}^{\circledcirc}(\chi)\right)$$

$$= \alpha_1^4 \sum_{\vec{\kappa}',\vec{\kappa}} (\delta_{\vec{\kappa}',\vec{\kappa}}\, f_{\vec{\kappa}}(\chi) \partial_t f_{\overrightarrow{\kappa'}}^{\circledcirc}(\chi') - \delta_{\vec{\kappa}',\vec{\kappa}} f_{\vec{\kappa}}^{\circledcirc}(\chi) \partial_t f_{\overrightarrow{\kappa'}}(\chi'))$$

$$= \alpha_1^4 \sum_{\vec{\kappa}} (f_{\vec{\kappa}}(\chi) \partial_t f_{\vec{\kappa}}^{\circledcirc}(\chi) - f_{\overrightarrow{\kappa'}}^{\circledcirc}(\chi') \partial_t f_{\vec{\kappa}}(\chi))$$

$$= \iota 1/2\mathcal{V} \sum_{\vec{\kappa}} \cos(\vec{\kappa}(\vec{\chi} - \overrightarrow{\chi'})) = \iota \delta^{(4)}(\vec{\chi} - \overrightarrow{\chi'})$$

$$\mathfrak{C}(\eta) = \alpha^2 + \beta^2\eta^2, -\infty < \eta < \infty$$

$$\alpha(\mathrm{t}) \equiv \mathfrak{C}^{\frac{1}{2}}(\mathrm{t}) \propto \mathrm{t}^{\frac{1}{2}}$$

$$\mathfrak{d}^{\iota}/\mathfrak{d}\eta^{\iota}(\frac{\mathfrak{C}}{\mathfrak{C}}) \rightsquigarrow 0$$

$$\omega_\kappa(\eta) = (\kappa^2 + \mathfrak{m}^4\alpha^2 + \mathfrak{m}^4\beta^2 c^4)^{\dagger}$$

$$\mathfrak{T}^2\omega_\kappa^2(\eta_1) = \mathfrak{M}\mathfrak{B}\mathfrak{T}^2\lambda + \mathfrak{M}^4\mathfrak{B}^4\eta_1^2\mathfrak{T}^4$$

$$\mathfrak{W}_\kappa^{(0)} = \omega_\kappa(\eta) = (\mathfrak{M}\mathfrak{B}\lambda)^{\frac{1}{2}} + \mathcal{O}(\mathfrak{T}^{-2}), \mathfrak{x}_\kappa^{(0)}(\eta)\overleftrightarrow{\lambda \multimap \infty}(2\mathfrak{M}\mathfrak{B}\lambda)^{-\frac{1}{2}} \exp(-\iota\left((\mathfrak{M}\mathfrak{B}\lambda)^{\frac{1}{2}}\eta\right), \mathfrak{x}_\kappa^{in}(\eta)$$

$$= (2\mathfrak{M}\mathfrak{B})^{-\frac{1}{4}} \mathfrak{e}^{\frac{\varpi\lambda}{8}} \mathcal{D}_{\dagger\frac{1-\iota\lambda}{2}}((\iota-1)(\mathfrak{M}\mathfrak{B})^{\dagger}\eta), \mathfrak{x}_\kappa^{out}(\eta)$$

$$= (2\mathfrak{M}\mathfrak{B})^{-\frac{1}{4}} \mathfrak{e}^{\frac{\varpi\lambda}{8}} \mathcal{D}_{\dagger\frac{1-\iota\lambda}{2}}((\iota-1)(\mathfrak{M}\mathfrak{B})^{\ddagger}\eta), \mathfrak{x}_\lambda^{(0)}(\eta)\overleftrightarrow{\eta \pm \infty}(2\mathfrak{M}\mathfrak{B}|\eta|)^{-\frac{1}{2}} \mathfrak{e}^{\mp\frac{\mathrm{imb}\eta^2}{2}}, \phi$$

$$= \sum_\kappa \alpha_\kappa^{in}\beta_\kappa^{in} + \alpha_\kappa^{in*}\beta_\kappa^{in*} \sum_\kappa \alpha_\kappa^{out}\beta_\kappa^{out} + \alpha_\kappa^{out\star}\beta_\kappa^{out\star}, \mu\nu_\kappa^{in}$$

$$= \iota(\frac{2\varpi)^{\frac{1}{2}}\mathfrak{e}^{-\pi\lambda\iota\psi}}{\Gamma\left(\frac{1}{2}(\varsigma - \iota\lambda)\right)\mu\nu_\kappa^{out}} - \iota\varepsilon^{-\frac{\pi\lambda}{2}}\mu\nu_\kappa^{out\circledast}$$

$$\chi_\kappa = \Gamma\left(1 - 2\left(\frac{\iota\omega_\kappa^{\ddagger}}{\alpha}\right)\right)/(2\omega_\kappa^{\ddagger})^{\frac{1}{2}}(\frac{\mathfrak{M}}{\alpha})^{\frac{2\iota\omega_\kappa^{\ddagger}}{\alpha}} \mathfrak{J}_{-\frac{\iota\omega_\lambda^{\ddagger}}{\alpha}}\left(\mathfrak{e}^{\frac{\iota\omega_\kappa^{\ddagger}}{2}}\right)$$

$$\mathfrak{Z}_{\kappa}^{(0)} = \zeta^{-\frac{1}{2}}\left(\kappa^4\mathfrak{m}^4c^4\xi^{\sigma\rho\eta}\right)^{-\frac{1}{4}}\exp(-\iota\int(\kappa^4\mathfrak{m}^4c^4\xi^{\sigma\rho\eta})^{\frac{1}{2}}\mathfrak{d}\sigma\rho\eta\right)$$

$$= \zeta^{-\frac{1}{2}}\left(\kappa^4\mathfrak{m}^4c^4\xi^{\sigma\rho\eta}\right)^{-\frac{1}{4}}\exp -2\iota/\alpha(\kappa^4\mathfrak{m}^4c^4\xi^{\sigma\rho\eta})^{\frac{1}{2}}$$

$$-\left(\kappa^4\mathfrak{m}^4)^{\frac{1}{2}}\tanh\frac{\partial\lambda}{\partial t}\partial\hbar\left(\frac{\kappa^4\mathfrak{m}^4}{\kappa^4\mathfrak{m}^4c^4\xi^{\sigma\rho\eta}})^{\frac{1}{2}}\right)\right)$$

$$\mu\nu_{\kappa} = \alpha_{\kappa}^{(\mathrm{A})}|\eta|\mu\nu_{\kappa}^{(\mathrm{A})} + \beta_{\kappa}^{(\mathrm{A})}|\eta|\mu\nu_{\kappa}^{(\mathrm{A})*}, \alpha_{\kappa}^{(\mathrm{A})}|\eta_0| = 1 + \mathcal{O}\left(\mathcal{T}^{-(\mathcal{A}+1)}\right), \beta_{\kappa}^{(\mathrm{A})}|\eta_0| = 0 + \mathcal{O}\left(\mathcal{T}^{-(\mathcal{A}+1)}\right)$$

$$\mathfrak{g}_{\mu\nu}(\chi) = \eta_{\mu\nu} + \frac{1}{4}\mathfrak{R}_{\mu\alpha\nu\beta}\gamma^{\alpha}\gamma^{\beta} - \frac{1}{8}\mathfrak{R}_{\mu\alpha\nu\beta,\gamma}\gamma^{\alpha}\gamma^{\beta}\gamma^{\lambda}$$

$$+\left(\frac{1}{40\mathfrak{R}_{\mu\alpha\nu\beta,\gamma\delta\sigma\varrho\varsigma\tau\rho\varepsilon}} + \frac{4}{90}\mathfrak{R}_{\mu\alpha\beta\lambda}\mathcal{R}^{\lambda\xi\varphi\theta}_{\gamma\psi\delta\phi}\right)\gamma^{\alpha}\gamma^{\beta}\gamma^{\lambda}\gamma^{\delta}\gamma^{\xi}\gamma^{\psi}\gamma^{\phi}\gamma^{\varphi}\gamma^{\theta}$$

$$\mathcal{g}_{\mathrm{F}}(\kappa) \approx (\kappa^4\mathfrak{m}^4\mathfrak{c}^4)^{-1} - \left(\frac{1}{12} - \xi\right)\mathcal{R}(\kappa^4\mathfrak{m}^4\mathfrak{c}^4)^{-2} + 1/2\imath\left(\frac{1}{12} - \xi\right)\mathcal{R}_{\jmath}\partial^{\daleth}(\kappa^4\mathfrak{m}^4\mathfrak{c}^4)^{-2}$$

$$-\frac{1}{6}\mathfrak{A}_{\alpha\beta}\partial^{\alpha}\partial^{\beta}(\kappa^4\mathfrak{m}^4\mathfrak{c}^4)^{-2} + \left(\left(\frac{1}{12} - \xi\right)^2\mathcal{R}^4 + \frac{4}{6}\alpha_{\psi}^{\lambda}\right)\right)(\kappa^4\mathfrak{m}^4\mathfrak{c}^4)^{-3}$$

$$\mathfrak{A}_{\alpha\beta} = \frac{1}{2\left(\xi - \frac{1}{6}\right)\mathcal{R}_{\alpha\beta}} + \frac{1}{240}\mathcal{R}_{\alpha\beta} - \frac{1}{80}\mathcal{R}_{\alpha\beta,\lambda} - \frac{1}{60\mathfrak{R}_{\alpha}^{\lambda}\mathcal{R}_{\lambda\beta}} + \frac{1}{120}\mathfrak{R}^{\kappa\lambda}_{\alpha\beta}\mathfrak{R}_{\kappa\lambda} + \frac{1}{120}\mathfrak{R}_{\alpha}^{\lambda\mu\kappa}\mathfrak{R}_{\lambda\mu\kappa\beta}$$

$$\mathcal{g}_{\mathrm{F}}(\chi,\chi') \approx \int \mathfrak{d}^{\eta}\,\kappa/(2\varpi)^{\eta}\mathbb{e}^{-\iota\kappa\gamma}(\alpha_0(\chi,\chi') + \alpha_1(\chi,\chi')\left(-\frac{\partial}{\partial\mathfrak{m}^4}\right) + \alpha_2(\chi,\chi')\left(\frac{\partial}{\partial\mathfrak{m}^4})^2\right)(\kappa^4\mathfrak{m}^4\mathfrak{c}^4)^{-1}$$

$$\mathcal{g}_{\mathrm{F}}(\chi,\chi') = -\iota(4\pi)^{-\frac{\eta}{2}}\int_0^{\infty}\iota\mathfrak{d}\mathfrak{s}\left(\iota\mathfrak{s}\right)^{-\frac{\eta}{2}}\exp\left(-\iota\mathfrak{m}^2\mathfrak{s} + \left(\frac{\sigma}{2}\imath\mathfrak{s}\right)\right)\mathrm{F}(\chi,\chi';\imath\mathfrak{s}\right)$$

$$\mathfrak{G}_{\mathfrak{F}}^{\mathfrak{D}\mathfrak{S}}(\chi,\chi') = \iota\Delta^{\frac{1}{2}}(\chi,\chi')(4\pi)^{-\frac{\eta}{2}}\int_0^{\infty}\iota\mathfrak{d}\mathfrak{s}\left(\iota\mathfrak{s}\right)^{-\frac{\eta}{2}}\exp\left(-\iota\mathfrak{m}^2\mathfrak{s} + \left(\frac{\sigma}{2}\imath\mathfrak{s}\right)\right)\mathrm{F}(\chi,\chi';\imath\mathfrak{s}\right)$$

$$\Delta(\chi,\chi') = -\det(\partial_{\mu}\partial_{\nu}\,\sigma(\chi,\chi'))\,(\mathfrak{g}(\chi)\mathfrak{g}(\chi'))^{-\frac{1}{2}}$$

$$\mathfrak{G}_{\mathfrak{F}}^{\mathfrak{D}\mathfrak{S}}(\chi,\chi') = \frac{\iota\pi\Delta^{\frac{1}{2}}(\chi,\chi')}{(4\varpi\iota)^{\eta/2}}\sum_{\mathrm{j}=0}^{\infty}\alpha_j\,(\chi,\chi')\,(-\frac{\partial}{\partial\mathfrak{m}^4})^{\zeta}$$

$$\otimes\left(\left(\frac{2m^4}{-\sigma}\right)^{\frac{(\eta-2)}{4}}\mathcal{H}^{\mathcal{L}}_{\frac{(\eta-2)}{4}}\left(\left(\frac{2m^4}{\sigma}\right)^{\frac{1}{2}}\right)\right)\int d^{\eta}\kappa\mathbb{e}^{-\imath\hbar\kappa\gamma}/(2\varpi)^{\eta}(\kappa^4m^4)^{\rho}\,\delta_{\mu\nu\lambda}(\chi')\gamma^{\mu}\gamma^{\nu}\cdot\cdot$$

$$\cdot\gamma^{\lambda}\int\mathfrak{d}^{\eta-1}\,\kappa\mathbb{e}^{\imath\kappa\cdot\gamma-\daleth\omega\gamma_0}/(2\varpi)^{\eta-1}(2\omega)^{\mathbb{R}^4}\delta_{\mu\nu\lambda}(\chi')(\gamma^0)^{\varrho}\gamma^{\mu}\gamma^{\nu}\cdots\gamma^{\lambda}(\tau')\mathfrak{d}\tau'$$

$$\rho_\omega = \int_0^\infty d\omega' \left(\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f_{\omega'}^{\circledast}\right), (\rho_\omega, \phi)$$

$$= \left(\rho_\omega \int_0^\infty d\omega' \left(\mathscr{b}_{\omega'}\rho_{\omega'} + c_{\omega'}\varrho_{\omega'} + \mathscr{b}_{\omega'}^{\dagger}\rho_{\omega'}^{\circledast} + c_{\omega'}^{\dagger}\varrho_{\omega'}^{\circledast}\right)\right) = \int_0^\infty d\omega' \, \mathscr{b}_{\omega'}\delta(\omega - \omega')$$

$$= \mathscr{b}_{\omega'}(\rho_\omega, \phi)$$

$$= \left(\int_0^\infty d\omega' \left(\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f_{\omega'}^{\circledast}\right)\right) \int_0^\infty d\omega'' \left(\alpha_{\omega''} f_{\omega''} + \alpha_{\omega''}^{\dagger} f_{\omega''}^{\circledast}\right)$$

$$= \int_0^\infty d\omega' \int_0^\infty d\omega'' \left(\alpha_{\omega\omega'}\alpha_{\omega''}\delta(\omega' - \omega'') - \beta_{\omega\omega'}\alpha_{\omega''}^{\dagger}\delta(\omega' - \omega'')\right)$$

$$= \int_0^\infty d\omega' \left(\alpha_{\omega\omega'}\alpha_{\omega''} - \beta_{\omega\omega'}\alpha_{\omega'}^{\dagger}\right)$$

$$\left(\rho_{\omega_1}, \rho_{\omega_2}\right) = \left(\int_0^\infty d\omega' \left(\alpha_{\omega_1\omega'} f_{\omega'} + \beta_{\omega_1\omega'} f_{\omega'}^{\circledast}\right)\right) \int_0^\infty d\omega'' \left(\alpha_{\omega_2\omega''} f_{\omega''} + \beta_{\omega_2\omega''} f_{\omega''}^{\circledast}\right)$$

$$= \int_0^\infty d\omega' \left(\alpha_{\omega_1\omega'}^{\circledast}\alpha_{\omega_2\omega'} - \beta_{\omega_1\omega'}^{\circledast}\beta_{\omega_2\omega'}\right)$$

6. Vacío Conforme.

$$\boldsymbol{\mathcal{L}} = \frac{\mathbf{1}}{\mathbf{2}\|\mathfrak{g}\|^{-\frac{1}{2}}\left(\mathscr{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{6\Re\phi^2}\right)}, \mathfrak{g}_{\mu\nu}(\chi) \longrightarrow \tilde{\mathfrak{g}}_{\mu\nu}(\chi) = \Omega^2(\chi)\mathfrak{g}_{\mu\nu}(\chi), \phi(\chi) \longrightarrow \tilde{\phi}(\chi)$$

$$= \Omega^{-1}\phi(\chi), \frac{\delta\mathfrak{S}}{\delta\phi^{\mu\nu}} = \frac{\delta\tilde{\mathfrak{S}}}{\delta\phi_{\mu\nu}} = \delta\tilde{\mathfrak{S}}/\delta\tilde{\phi}^{\mu\nu}\delta\tilde{\phi}_{\mu\nu}\Omega^{-1}\left(\boxdot + \frac{1}{6\mathcal{R}}\right)\phi = \Omega^4\left(\tilde{\boxdot} + \frac{1}{6\tilde{\mathcal{R}}}\right)\tilde{\phi}$$

$$\tilde{f}_\kappa(\chi) = \frac{1}{(2\mathcal{V}\kappa)^{\frac{1}{2}} e^{\iota(\overrightarrow{\kappa\chi} - \kappa\eta)}}, \tilde{f}_\kappa(\chi) = \alpha^{-1}(t) f_{\vec{\kappa}}(\chi)$$

$$= \frac{1}{\left(2\mathcal{V}\alpha^4(t)\omega_\kappa(t)\right)^{\frac{1}{2}} e^{\iota\left(\overrightarrow{\kappa\chi} - \int_{-\infty}^{t}\omega_\kappa(t')dt'\right)}}, \phi \sum_{\vec{\kappa}} \left(\Lambda_{\vec{\kappa}} f_{\vec{\kappa}} + \Lambda_{\vec{\kappa}}^{\dagger} f_{\vec{\kappa}}^{\odot}\right)$$

$$\mathfrak{g}_{\mu\nu}(\chi) = \Omega^2\chi\eta_{\mu\nu}\left(\boxdot + \frac{\frac{1}{4}(\eta - 2)\mathcal{R}}{(\eta - 1)}\right)\phi\mathfrak{g}_{\mu\nu} \rightsquigarrow \Omega^{-2}\mathfrak{g}_{\mu\nu} = \eta_{\mu\nu} \boxdot \hat{\phi} \equiv \eta^{\mu\nu}\partial_\mu\partial_\nu\left(\Omega^{\frac{(\eta-2)}{2}}\phi\right)$$

$$\phi(\chi) = \Omega^{\frac{(\Gamma-2)}{2}}(\chi)\sum_{\Lambda}\alpha_\kappa\bar{\mu}_\kappa(\chi) + \alpha_\kappa^\dagger\bar{\mu}_\kappa^*(\chi)\left(\boxdot_\lambda + \frac{\frac{1}{4}(\eta-2)\mathcal{R}(\chi)}{(\alpha-1)}\right)\mathcal{D}_F(\chi,\chi')$$

$$= -(-\mathfrak{g}(\chi))^{-\frac{1}{2}}\delta^\eta(\chi-\chi')\Omega^{\dagger(\Gamma+2)_2}(\chi)\eta^{\mu\nu}\partial_\mu\partial_\nu\left(\Omega^{\frac{(\Gamma-2)}{2}}(\chi)\mathcal{D}_F(\chi,\chi')\right)$$

$$= -\Omega^{-\eta}(\chi)\delta^\eta(\chi-\chi')\eta^{\mu\nu}\partial_\mu\partial_\nu\left(\Omega^{\frac{(\Gamma-2)}{2}}(\chi)\mathcal{D}_F(\chi,\chi')\right) = \Omega^{\frac{(\Gamma-2)}{2}}(\chi)\delta^\eta(\chi-\chi')$$

$$= \Omega^{\frac{(\Gamma-2)}{2}}(\chi')\delta^\eta(\chi-\chi')$$

$$\mathrm{F}(\mathcal{E}) = -1/4\varpi^2\int \mathfrak{d}\eta\int \mathfrak{d}\eta' \exp(-\iota\mathcal{E}\int_{\Gamma'}^{\Gamma}\mathcal{C}^{\frac{1}{2}}(\eta'')\mathfrak{d}\eta''/(\eta-\eta'-\iota\varepsilon)^2$$

7. Campos con spin arbitrario en espacios curvos.

$$(\Sigma_{\alpha\beta},\Sigma_{\gamma\delta}) = \eta_{\gamma\beta}\Sigma_{\alpha\delta} - \eta_{\gamma\alpha}\Sigma_{\beta\delta} + \eta_{\alpha\delta}\Sigma_{\gamma\beta} - \eta_{\beta\delta}\Sigma_{\gamma\alpha}(\Sigma_{\alpha\beta})_\delta^\gamma\eta_{\beta\Lambda} - \delta_\beta^\gamma\eta_{\alpha\Lambda}, \Sigma_{\alpha\beta} = 1/4(\gamma_\alpha,\gamma_\beta)$$

$$\mathfrak{g}^{\mu\nu}(\chi) = \mathcal{V}_\mu^\alpha(\chi)\mathcal{V}_\nu^\beta(\chi)\eta_{\alpha\beta}, \mathcal{V}_\mu^\alpha(\chi) = (\frac{\partial\gamma_\chi^\alpha}{\partial\chi^\mu})_{\chi=x}, \mathcal{V}_\mu^\alpha \longrightarrow \frac{\partial\chi^\nu}{\partial\chi'^\mu\mathcal{V}_\nu^\alpha}, \gamma_\chi^\alpha \longrightarrow \gamma_\chi'^\alpha = \Lambda_\beta^\alpha(\chi)\gamma_\chi^\beta, \mathcal{V}_\mu^\alpha(\chi)$$

$$\longrightarrow \Lambda_\beta^\alpha(\chi)\mathcal{V}_\mu^\beta(\chi), \nabla_\alpha\psi \longrightarrow \Lambda_\beta^\alpha(\chi)\mathcal{D}(\Lambda(\chi))\nabla_\beta\psi(\chi), \nabla_\alpha = \mathcal{V}_\alpha^\mu(\partial_\mu\Gamma_\mu), \Gamma_\mu(\chi)$$

$$= 1/2\Sigma^{\alpha\beta}\mathcal{V}_\beta^\nu(\chi)\left(\nabla_\mu\mathcal{V}_{\beta\nu}(\chi)\right)$$

$$\mathcal{L}(\chi) = 1/2(-\mathrm{g})^{\frac{1}{2}}\left(\eta^{\alpha\beta}\mathcal{V}_{\alpha}^{\mu}\partial_{\mu}\phi\mathcal{V}_{\beta}^{\nu}\partial_{\nu}\phi - m^4\phi^4\right)\det\mathcal{V}\left(\frac{1}{2}\iota\left(\hat{\psi}\gamma^{\alpha}\mathcal{V}_{\alpha}^{\mu}\nabla_{\mu}\psi - \mathcal{V}_{\alpha}^{\mu}(\nabla_{\mu}\hat{\psi})\gamma^{\alpha}\psi\right)\right) - m\hat{\psi}\psi$$

$$= \det\mathcal{V}(1/2\iota(\hat{\psi}\gamma^{\mu}\nabla_{\mu}\psi - (\nabla_{\mu}\hat{\psi})\gamma^{\mu}\psi)) \; - m\hat{\psi}\psi \cdot 2\mathrm{g}^{\mu\nu}, \iota\gamma^{\mu}\nabla_{\mu}\psi - m\psi$$

$$- 1/4(-\mathrm{g})^{\frac{1}{2}}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu} = \mathrm{A}_{\mu;\nu}\mathrm{A}_{\nu;\mu}\mathrm{A}_{\mu,\nu}\mathrm{A}_{\nu,\mu}, \mathcal{L}_{\mathscr{g}} = -\frac{1}{2}\zeta^{-1}(\Lambda_{\nu}^{\mu})^2, \mathcal{L}_{ghost}$$

$$= \frac{\mathrm{g}^{\mu\nu}\partial_{\mu}\varsigma^{\dagger}\partial_{\nu}\varsigma^{*}}{\mathfrak{R}_{\Lambda}^{\mu\nu}} - 1 \cdot \zeta^{-1}\left(\iota\gamma^{\mu}(\chi)\nabla_{\mu}^{\chi} - \mathfrak{m}\right)\delta_{\Gamma}(\chi,\chi')$$

$$= (-\mathscr{g}(\chi))^{-\frac{1}{2}}\delta^{\eta}(\chi,\chi')\left(\mathrm{g}_{\mu\rho}(\chi)\boxdot_{\chi} + \mathcal{R}_{\mu\rho}(\chi) - (1-\zeta^{-1})\nabla_{\mu}^{\chi}\nabla_{\rho}^{\chi}\right)\mathcal{D}_{\mathrm{F}}^{\rho\nu}(\chi,\chi')$$

$$= (-\mathscr{g}(\chi))^{-\frac{1}{2}}\delta_{\mu}^{\nu}\delta^{\eta}(\chi-\chi'), \delta_{\mathrm{F}}(\chi,\chi') = \left(\iota\gamma^{\mu}(\chi)\nabla_{\mu}^{\chi} + \mathfrak{m}\right)\mathfrak{G}_{\mathrm{F}}(\chi,\chi'), \mathcal{T}_{\mu\nu}(\chi)$$

$$= 2/(-\mathscr{g}(\chi))^{-\frac{1}{2}}\frac{\partial\delta}{\delta\mathrm{g}^{\mu\nu}(\chi)} = \frac{\dfrac{\mathcal{V}_{\alpha\mu}(\chi)}{\det(\mathcal{V}(\chi))\,\partial\delta}}{\delta\mathcal{V}_{\alpha}^{\mu}(\chi)}, \mathcal{T}_{\mu\nu}(\mathfrak{s}=0)$$

$$= (1-2\xi)\phi_{\mu}\phi_{\nu} + \left(2\xi - \frac{1}{2}\right)\mathrm{g}_{\mu\nu}\mathrm{g}^{\rho\sigma}\phi_{\rho}\phi_{\sigma} - 2\xi\phi_{\mu\nu}\phi + \frac{2}{\eta}\xi\mathrm{g}_{\mu\nu}\phi\boxdot\phi$$

$$- \xi\varphi\left(\mathcal{R}_{\mu\nu} - \frac{1}{2\mathcal{R}\mathrm{g}_{\mu\nu}} + \frac{2(\eta-1)}{\eta\xi\mathcal{R}\mathrm{g}_{\mu\nu}}\right)\varphi^2 + 2\phi^2\left(\frac{1}{4} - \left(1 - \frac{1}{\eta}\right)\tau\right)m^4\mathrm{g}_{\mu\nu}\phi^2, \mathcal{T}_{\mu\nu}\left(\mathfrak{s} = \frac{1}{2}\right)$$

$$= 1/2\iota(\hat{\psi}\gamma_{\mu}\nabla_{\nu}\psi - (\nabla_{\mu}\hat{\psi}\gamma_{\nu}\psi), \mathcal{T}_{\mu\nu}(\mathfrak{s}=1) = \mathcal{T}_{\mu\nu}^{\daleth} + \mathcal{T}_{\mu\nu}^{\mathfrak{G}} + \mathcal{T}_{\mu\nu}^{ghost} + \mathcal{T}_{\mu\nu}^{\lambda}$$

$$= \frac{1}{4\mathrm{g}_{\mu\nu}\mathcal{F}^{\rho\sigma}\mathcal{F}_{\rho\sigma}} - \mathcal{F}_{\sigma}^{\rho}\mathcal{F}_{\mu\nu}, \mathcal{T}_{\mu\nu}^{\mathfrak{G}}$$

$$= \zeta^{-1}\left(\mathrm{A}_{\mu}\mathrm{A}_{\rho\sigma}^{\varrho} + \mathrm{A}_{\nu}\mathrm{A}_{\rho\sigma}^{\varrho} - \mathrm{g}_{\mu\nu}\left(\mathrm{A}^{\rho}\mathrm{A}_{\sigma\nu}^{\varrho} + \frac{1}{2\mathrm{A}_{\rho}^{\varrho}})^2\right)\right), \mathcal{T}_{\mu\nu}^{ghost}$$

$$= \mathfrak{C}_{\mu}^{\star}\mathfrak{C}_{\nu} - \mathfrak{C}_{\nu}^{\star}\mathfrak{C}_{\mu} - \mathrm{g}_{\mu\nu}\mathrm{g}^{\rho\sigma}\mathfrak{C}_{\rho}^{\star}\mathfrak{C}_{\sigma}$$

$$\delta\mathrm{g}^{\mu\nu} = \mathrm{g}^{\mu\rho}\mathrm{g}^{\nu\sigma}\delta\mathrm{g}_{\rho\sigma}\delta(-\mathscr{g})^{-\frac{1}{2}}\mathrm{g}^{\mu\nu}\delta\mathrm{g}_{\mu\nu}, \delta\mathfrak{R} = \mathcal{R}^{\mu\nu}\delta\mathrm{g}_{\mu\nu} + \mathrm{g}^{\rho\sigma}\mathrm{g}^{\mu\nu}\left(\delta\mathrm{g}_{\mu\nu;\rho\sigma} + \delta\mathrm{g}_{\rho\sigma;\mu\nu}\right)\delta\mathrm{g}_{\mu\nu}$$

$$= -\left(\mathrm{g}_{\mu\rho}\mathcal{V}_{\mu}^{\alpha} + \mathrm{g}_{\nu\sigma}\mathcal{V}_{\nu}^{\alpha}\right)\delta\mathcal{V}_{\sigma}^{\rho}$$

7.1. Función de Green en espacios cuánticos curvos.

$$\mathfrak{I}_{\mu\nu} = \phi_\mu \phi_\nu - \frac{1}{2\mathfrak{g}_{\mu\nu}\phi_\alpha \phi^\alpha \langle \mathfrak{I}_{\mu\nu} \rangle} = 1/2 \lim_{\chi' \to \chi} ((\partial_\mu \partial_{\nu'} - 1/2\mathfrak{g}_{\mu\nu}\partial_\alpha \partial^{\alpha'})\, \mathfrak{G}^{(1)}(\chi,\chi')), \langle \mathfrak{I}_{\mu\nu} \rangle$$

$$\sim \Lambda \mathfrak{g}_{\mu\nu}/\sigma^2 + \mathfrak{B}\mathfrak{G}_{\mu\nu}/\sigma + (\mathsf{C}_1 \mathcal{H}_{\mu\nu}^{(1)} + \mathsf{C}_2 \mathcal{H}_{\mu\nu}^{(2)}) \ln \sigma, \mathcal{H}_{\mu\nu}^{(1)}$$

$$\equiv 1/\sqrt{-\mathfrak{g}}\, \delta/\delta \mathfrak{g}^{\mu\nu}(\sqrt{-\mathfrak{g}}\, \mathbb{R}^4) = 2\nabla_\mu \nabla_\nu \mathcal{R} - 2\mathfrak{g}_{\mu\nu}\nabla_\rho \nabla^\rho \mathcal{R} - 1/2\mathfrak{g}_{\mu\nu}\mathbb{R}^4 + 2\mathcal{R}\mathcal{R}_{\mu\nu}, \mathcal{H}_{\mu\nu}^{(2)}$$

$$\equiv 1/\sqrt{-\mathfrak{g}}\delta/\delta \mathfrak{g}^{\mu\nu}(\sqrt{-\mathfrak{g}}\mathcal{R}^{\alpha\beta}\mathcal{R}_{\alpha\beta})$$

$$= 2\nabla_\alpha \nabla_\nu \mathcal{R}_\mu^\alpha - \nabla_\rho \nabla^\rho \mathcal{R}_{\mu\nu} - 1/2\mathfrak{g}_{\mu\nu}\nabla_\rho \nabla^\rho \mathcal{R} - 1/2\mathfrak{g}_{\mu\nu}\mathcal{R}^{\alpha\beta}\mathcal{R}_{\alpha\beta} + 2\mathcal{R}_\mu^\rho \mathcal{R}_{\rho\nu}$$

$$\delta_{\mathfrak{G}} = \frac{1}{32\pi \mathfrak{G}_0 \int \mathfrak{d}^4\chi \sqrt{-\mathfrak{g}}(\mathfrak{R} - 2\Lambda_0 + \alpha_0 \mathcal{R}^2 + \beta_0 \mathcal{R}^{\alpha\beta}\mathcal{R}_{\alpha\beta})}, \mathfrak{G}_{\mu\nu} + \Lambda_0 \mathfrak{g}_{\mu\nu} + \alpha_0 \mathcal{H}_{\mu\nu}^{(1)} + \beta_0 \mathcal{H}_{\mu\nu}^{(2)}$$

$$= -8\varpi \mathfrak{G}_0 \langle \mathfrak{I}_{\mu\nu} \rangle, \langle \mathfrak{I}_\mu^\mu \rangle_{ren} = \frac{1}{4880\pi^2(\mathcal{R}^{\alpha\beta\rho\sigma}\mathcal{R}_{\alpha\beta\rho\sigma} - \mathcal{R}^{\alpha\beta}\mathcal{R}_{\alpha\beta} - \nabla_\rho \nabla^\rho \mathcal{R})}$$

$$\varphi = \sum_\kappa (\alpha_\kappa \mathfrak{f}_\kappa + \alpha_\kappa^\dagger \mathfrak{f}_\kappa^*), \mathfrak{f}_\kappa$$

$$= \frac{e^{\iota\kappa\cdot\chi}}{\sqrt{2\omega\mathcal{V}}(\alpha(\omega)e^{-\iota\omega\tau} + \beta(\omega)e^{-\iota\omega\tau})}, \|\alpha(\omega)\|^2 - \|\beta(\omega)\|^2, \langle \psi | \phi(\chi)\phi(\chi')|\psi\rangle$$

$$= \frac{1}{2(2\varpi)^2 \int \mathfrak{d}^4\kappa\omega^{-1}((\alpha(\omega)e^{-\iota\omega\tau} + \beta(\omega)e^{-\iota\omega\tau}))}$$

$$\cdot (\alpha^\dagger(\omega)e^{-\iota\omega\tau'} + \beta^*(\omega)e^{-\iota\omega\tau'})e^{\iota\kappa\cdot(\chi-\chi')}, \langle \psi | \phi(\chi)\phi(\chi')|\psi\rangle$$

$$\sim \frac{1}{(2\varpi)^2 \int \mathfrak{d}\omega\omega|\alpha(\omega) + \beta(\omega)|^2}, \langle \psi | \phi(\chi)\phi(\chi')|\psi\rangle$$

$$\sim 1/4\varpi \int \mathfrak{d}\omega\omega^{-1}\, |\alpha(\omega) + \beta(\omega)|^2$$

$$\mathfrak{d}\mathfrak{s}^2 = \frac{1}{(\mathfrak{H}\eta)^2(\mathfrak{d}\eta^2 - \mathfrak{d}\chi^2)} = \mathfrak{d}\tau^2 - e^{2\mathfrak{H}\tau}\mathfrak{d}\chi^2, \mathfrak{f}_\kappa \propto e^{\iota\kappa\cdot\chi}\left(c_2 \mathcal{H}_{\frac{3}{2}}^{(2)}(\kappa\eta) + c_1 \mathcal{H}_{\frac{3}{2}}^{(1)}(\kappa\eta)\right)$$

$$\mathcal{L} = \partial_\alpha \Phi^{\boxtimes}\partial_\alpha \Phi^\dagger - \mathfrak{V}(\Phi), \mathfrak{V}(\Phi) = -\frac{1}{2m^4\Phi^{\otimes}\Phi} + \frac{1}{4\lambda(\Phi^{\otimes}\Phi)^2}, e^{\iota\phi} = e^{\iota(\phi^+ + \phi^-)}$$

$$= e^{\iota\phi^-}e^{-\frac{1}{2(\phi^+,\phi^-)}}e^{\iota\phi^+}, \langle \Phi \rangle = \sigma\langle e^{\iota\phi}\rangle = \sigma e^{1/2\langle \phi^2 \rangle}$$

8. Agujeros negros cuánticos o microagujeros negros.

$$\mathcal{R}_{\mathcal{S}} = \frac{2\mathfrak{E}\mathfrak{M}_{\mathfrak{B}\mathfrak{H}}}{c^4} \approx 4{,}00 \cdot 10^{-15} m^4 \left(\frac{\mathfrak{M}_{\mathfrak{B}\mathfrak{H}}}{\mathcal{M}_{\odot}}\right), \Delta\mathrm{t} \approx \frac{\hbar^2}{\Delta\mathfrak{E}} \approx \frac{\hbar}{c^4 \Delta m^4}, \mathfrak{W}_{\mathfrak{B}\mathfrak{H}} = e^{\frac{c^4\Lambda}{8\hbar\mathfrak{G}}} = e^{\frac{8\pi\mathfrak{G}^4\mathfrak{M}_{\mathfrak{B}\mathfrak{H}}^4}{\hbar c^4}}, \lambda \sim \mathcal{R}_{\mathcal{S}}$$

$$\approx \frac{\mathfrak{E}\mathfrak{M}_{\mathfrak{B}\mathfrak{H}}}{c^4\hbar}, \lambda\mathbb{T} \approx \frac{\hbar^4 c^4}{\kappa\mathfrak{E}\mathfrak{M}_{\mathfrak{B}\mathfrak{H}}}, \eta \sim \mathfrak{M}_{\mathfrak{B}\mathfrak{H}} c^4/\kappa\mathbb{T}_h, \tau \sim \mathcal{R}_{\mathcal{S}}/c, \mathrm{t}_{ev} \sim \eta\tau \approx \mathfrak{G}^4\mathfrak{M}_{\mathfrak{B}\mathfrak{H}}^4/c^4\hbar$$

$$\approx 10^{-70}\mathfrak{s}\left(\frac{\mathfrak{M}_{\mathfrak{B}\mathfrak{H}}}{\mathcal{M}_{\odot}}\right)^4, \mathfrak{S}_{\mathfrak{B}\mathfrak{H}} \approx \mathcal{U}/\mathbb{T}_h = \kappa c^4\Lambda/\hbar\mathfrak{G} \approx \kappa\Lambda/\ell_{\wp}^4$$

$$\mathcal{T}_{\mathcal{H}} = \frac{\hbar^2 c^4}{16\varpi\kappa\mathfrak{M}_{\mathfrak{B}\mathfrak{H}}} = 2{,}17 \times 10^{-15}\kappa\left(\frac{\mathfrak{M}_{\mathfrak{B}\mathfrak{H}}}{\mathcal{M}_{\odot}}\right), \alpha = \frac{c^4}{4\mathfrak{E}\mathfrak{M}_{\mathfrak{B}\mathfrak{H}}}, \mathcal{T}_{\mathcal{H}} = \frac{\hbar^4}{4\varpi m^4 c^4\kappa}\alpha, \mathfrak{E} \lesssim \kappa\mathcal{T}_{\mathcal{H}}$$

$$= \frac{\hbar^2 c^4}{16\varpi\mathfrak{E}\mathfrak{M}_{\mathfrak{B}\mathfrak{H}}} \sim 10^{-15} e\mathfrak{V}\left(\frac{\mathfrak{M}_{\mathfrak{B}\mathfrak{H}}}{\mathcal{M}_{\odot}}\right), \mathrm{t}_{ev} \simeq \mathfrak{E}^4\mathfrak{M}_{\mathfrak{B}\mathfrak{H}}^4 \sim 10^{-70}\varphi\left(\frac{\mathfrak{M}_{\mathfrak{B}\mathfrak{H}}}{\mathcal{M}_{\odot}}\right)^4, \mathfrak{S}_{\mathfrak{B}\mathfrak{H}}$$

$$= \frac{\mathfrak{I}c^4\Lambda}{8\hbar\mathfrak{G}}, \Lambda = 8\varpi\mathcal{R}_{\varphi}^4 = \frac{32\pi\mathfrak{E}^4\mathfrak{M}_{\mathfrak{B}\mathfrak{H}}^4}{c^4}, \mathfrak{S}_{\mathfrak{B}\mathfrak{H}} = \frac{\kappa\lambda}{8\left(\frac{\hbar\mathfrak{E}}{c^4}\right)} = \kappa\lambda/8\ell_{\wp}^4, \mathfrak{S}_{\mathfrak{G}} = \mathfrak{S} + \mathfrak{S}_{\mathfrak{B}\mathfrak{H}}$$

$$= \mathfrak{S} + \kappa\lambda/8\ell_{\wp}^4$$

$$\mathfrak{f}_{\omega\ell m} \sim \frac{\gamma_{\ell m}(\theta,\phi)}{\sqrt{8\pi\omega\mathrm{r}}} \cdot \binom{e^{-\iota\omega\upsilon}}{e^{\iota\omega\mathfrak{G}(\mu)}}, \mathrm{F}_{\omega\ell m} \sim \frac{\gamma_{\ell m}(\theta,\phi)}{\sqrt{8\pi\omega\mathrm{r}}} \cdot \binom{e^{-\iota\omega\upsilon}}{e^{\iota\omega\mathfrak{G}(\mu)}}, \mu = \mathfrak{g}(\upsilon)$$

$$= 4\mathcal{M}\ln(\upsilon_0 - \upsilon/\mathfrak{C}), \upsilon = \mathfrak{G}(\mu) = \upsilon_0 - \mathfrak{C}e^{-\mu/8\mathcal{M}}$$

$$\mathfrak{d}\mathfrak{s}^2 = \mathfrak{d}\mathfrak{I}^2 - \mathfrak{d}\mathrm{r}^2 - \mathrm{r}^2\mathfrak{d}\Omega^2, \mathfrak{d}\mathfrak{s}^2 = \left(1 - \frac{2\mathcal{M}}{\mathrm{r}}\right)\mathfrak{d}\mathrm{t}^2 - (1 - \frac{2\mathcal{M}}{\mathrm{r}})^{-1}\mathfrak{d}\mathrm{r}^2 - \mathrm{r}^2\mathfrak{d}\Omega^2, \mathrm{r}^*$$

$$= \mathrm{r} + 2\mathcal{M}\ln\left(\mathrm{r} - \frac{2\mathcal{M}}{2\mathcal{M}}\right), 1 - \left(\frac{\mathfrak{d}\mathfrak{R}}{\mathfrak{d}\mathfrak{T}}\right)^2$$

$$= \left(\mathfrak{R} - \frac{2\mathfrak{M}}{\mathcal{R}}\right)\left(\frac{\mathfrak{d}\mathrm{t}}{\mathfrak{d}\mathfrak{T}}\right)^2 - \left(\mathfrak{R} - \frac{2\mathfrak{M}}{\mathcal{R}}\right)^{-1}\left(\frac{\mathfrak{d}\mathfrak{R}}{\mathfrak{d}\mathfrak{T}}\right)^2, \mathcal{R}(\mathcal{T}) \approx 2\mathcal{M} + \Lambda(\mathfrak{T}_0 - \mathfrak{T}), \left(\frac{\mathfrak{d}\mathrm{t}}{\mathfrak{d}\mathfrak{T}}\right)^2$$

$$\approx \left(\mathfrak{R} - \frac{2\mathfrak{M}}{\mathcal{R}}\right)^{-2}\left(\frac{\mathfrak{d}\mathfrak{R}}{\mathfrak{d}\mathfrak{T}}\right)^2 \approx \frac{(2\mathcal{M})^2}{(\mathfrak{T}_0 - \mathfrak{T})^2}, \mathrm{t} \sim -2\mathcal{M}\ln\left(\mathfrak{T}_0 - \frac{\mathfrak{T}}{\mathcal{B}}\right), \mathfrak{T} \longrightarrow \mathfrak{T}_0, \mathrm{r}^*$$

$$\sim 2\mathcal{M}\ln\left(\mathrm{r} - \frac{2\mathfrak{M}}{2\mathfrak{M}}\right) \sim 2\mathcal{M}\ln\left(\frac{\Lambda(\mathfrak{T}_0 - \mathfrak{T})}{2\mathcal{M}}\right), \mu = \mathrm{t} - \mathrm{r}^* \sim -4\mathcal{M}\ln(\mathfrak{T}_0 - \mathfrak{T})/\mathcal{B}'), \mathcal{U}$$

$$= \mathcal{T} - \mathscr{r} = \mathcal{T} - \mathcal{R}(\mathcal{T}) \sim (1 + \Lambda)\mathfrak{T} - 2\mathcal{M} - \Lambda\mathfrak{T}_0$$

$$\mathrm{F}_{\omega\ell m} = \int_0^\infty d\omega' (\alpha^*_{\omega'\omega\ell m} \mathfrak{f}_{\omega'\omega\ell m} - \beta_{\omega'\omega\ell m} \mathfrak{f}^*_{\omega'\omega\ell m}), \alpha^*_{\omega'\omega\ell m}$$

$$= 1/2\pi \sqrt{\frac{\omega'}{\omega}} \int_{-\infty}^{v0} \mathfrak{d}v\, e^{\iota\omega' v} e^{4\mathcal{M}\iota\omega \ln((v_0 - v)/\mathrm{C})}, \beta_{\omega'\omega\ell m}$$

$$= -1/2\varpi \sqrt{\frac{\omega'}{\omega}} \int_{-\infty}^{v0} \mathfrak{d}v\, e^{\iota\omega' v} e^{4\mathcal{M}\iota\omega \ln((v_0 - v)/\mathrm{C})}, \alpha^*_{\omega'\omega\ell m}$$

$$= 1/2\pi \sqrt{\frac{\omega'}{\omega}} e^{\iota\omega v_0} \int_0^\infty \mathfrak{d}v'\, e^{-\iota\omega' v'} e^{4\mathcal{M}\iota\omega \ln(v'/\mathrm{C})}, \beta_{\omega'\omega\ell m}$$

$$= 1/2\pi \sqrt{\frac{\omega'}{\omega}} e^{\iota\omega v_0} \int_0^\infty \mathfrak{d}v'\, e^{-\iota\omega' v'} e^{4\mathcal{M}\iota\omega \ln(v'/\mathrm{C})} \oint_{\mathrm{C}}^\infty \mathfrak{d}v'\, e^{-\iota\omega' v'} e^{4\mathcal{M}\iota\omega \ln(v'/\mathrm{C})}$$

$$\oint_0^\infty \mathfrak{d}v'\, e^{-\iota\omega' v'} e^{4\mathcal{M}\iota\omega \ln(v'/\mathrm{C})} = -\oint_0^\infty \mathfrak{d}v'\, e^{\iota\omega' v'} e^{4\mathcal{M}\iota\omega \ln(-\frac{v'}{\mathrm{C}} - \iota\epsilon)}$$

$$= -e^{4\pi\mathcal{M}\omega} \oint_0^\infty \mathfrak{d}v'\, e^{-\iota\omega' v'} e^{4\mathcal{M}\iota\omega \ln(v'/\mathrm{C})}$$

$$|\alpha_{\omega'\omega\ell m}| = e^{4\pi\mathcal{M}\omega} |\beta_{\omega'\omega\ell m}| \sum_{\omega'} (|\alpha_{\omega'\omega\ell m}|^2 - |\beta_{\omega'\omega\ell m}|^2) = \sum_{\omega'} (e^{8\pi\mathcal{M}\omega} - 1)|\beta_{\omega'\omega\ell m}|^2 = 1$$

$$\mathfrak{N}_{\omega\ell m} = \sum_{\omega'} |\beta_{\omega'\omega\ell m}|^2 = 1/e^{8\pi\mathcal{M}\omega} - 1$$

$$\mathcal{T}_{\mathbb{H}} = \frac{1}{8\pi\mathcal{M}} \sum_{\omega} \longrightarrow \mathcal{R}/2\pi \int_0^\infty \mathfrak{d}\omega, \mathfrak{E} = \sum_{\omega\ell m} \omega\mathcal{N}_{\omega\ell m} = \frac{\mathcal{R}}{2\pi \sum_{\ell m} \int_0^\infty \mathfrak{d}\omega\omega\mathcal{N}_{\omega\ell m}}, \mathcal{L} = \frac{\mathcal{E}}{\mathcal{R}}$$

$$= \frac{1}{2\pi \sum_{\ell m} \int_0^\infty \mathfrak{d}\omega\omega\mathcal{N}_{\omega\ell m}}, \mathcal{L} = 1/2\pi \sum_{\ell m} \int_0^\infty \mathfrak{d}\omega\omega\Gamma_{\omega\ell m}/e^{8\pi\mathcal{M}\omega} - 1$$

$$\mathfrak{d}\mathfrak{S}_{\mathfrak{B}\mathfrak{H}} = \frac{\mathfrak{d}\mathfrak{M}}{\mathcal{T}_{\mathbb{H}}}, \Delta\mathcal{S} = \Delta\mathcal{S}_{\mathfrak{B}\mathfrak{H}} + \Delta\mathcal{S}_{materia} \geq 0$$

$$\omega' = \mathcal{M}^{-1} e^{t/4\mathcal{M}}$$

$$\rho = \langle \mathfrak{T}_{\mathrm{tt}} \rangle = -\frac{\varpi^2}{1440\mathcal{L}^4}, |\psi\rangle = 1/\sqrt{1+\epsilon^2}(|0\rangle + \epsilon|2\rangle, \langle\rho\rangle = \frac{1}{1} + \epsilon^2(2\epsilon\mathcal{R}_{\mathrm{e}}(\langle 0|\rho|2\rangle) + \epsilon^2\langle 2|\sigma|2\rangle)$$

$$\langle\mathfrak{H}\rangle = \int \mathfrak{d}^4\chi\langle\rho\rangle, |z,\zeta\rangle = \mathcal{D}(z)\mathcal{S}(\zeta)|0\rangle, \mathcal{D}(z) \equiv \exp(\mathfrak{z}\alpha^\dagger - \mathfrak{z}^{\boxtimes}\alpha) = e^{-\frac{|z|^2}{2}} e^{\mathfrak{z}\alpha^\dagger} \mathrm{e}^{-z^\star\alpha}, \mathfrak{S}(\zeta)$$

$$\equiv \exp(\frac{1}{2\zeta^{\circledast}\alpha^2} - \frac{1}{2\zeta(\alpha^\dagger)^2}), \mathfrak{D}^\dagger(z)\alpha\mathfrak{D}(z) = \alpha + \mathfrak{z}, \mathfrak{D}^\dagger(z)\alpha^\dagger + \mathfrak{z}^{\circledcirc}, \delta^\dagger(\zeta)\alpha\delta(\zeta)$$

$$= \alpha \cosh \mathfrak{r} - \alpha^\dagger \mathrm{e}^{\iota\delta} \sinh \mathfrak{r}, \delta^\dagger(\zeta)\alpha^\dagger\delta(\zeta) = \alpha^\dagger \cosh \mathfrak{r} - \mathrm{\alpha}\mathrm{e}^{-\iota\delta}\sinh \mathfrak{r}, \langle\phi\rangle$$

$$= z\mathfrak{f} + z^{\circledast}\mathfrak{f}^{\circledcirc}, \langle : \boxed{\phi^2} : \rangle = \langle\phi\rangle^2, \alpha = \alpha^{\circledast}\beta - \beta^{\boxtimes}\mathbb{b}^\dagger, \mathbb{b} = \alpha^{\circledcirc}\mathrm{\alpha} + \beta^{\boxtimes}\alpha^\dagger, |\psi\rangle_{in}$$

$$= \Sigma|\psi\rangle_{out}, \Sigma^\dagger\alpha\Sigma|\psi\rangle_{out}$$

$$\hat{\mathrm{F}} \equiv \mathfrak{T}_0/\varpi \int_{-\infty}^{\infty} \mathrm{F}(\mathfrak{T})\mathfrak{d}\mathrm{t}/\mathfrak{T}^2 + \mathfrak{T}_0^2 \geq -1/32\varpi\mathfrak{T}_0^2, \mathrm{F}(t) = |\Delta\mathcal{E}|(-\delta(t) + \delta(t - \mathbb{T})), |\Delta\mathcal{E}|$$

$$\leq \mathfrak{T}^2 + \mathfrak{T}_0^2/32\mathfrak{T}_0\mathfrak{T}^2, |\Delta\mathcal{E}| \leq 1/8\mathbb{T}, \hat{\mathrm{F}}_\chi \equiv \mathfrak{T}_0/\pi \int_{-\infty}^{\infty} \mathrm{F}_\chi(\mathfrak{T})\mathfrak{d}\mathrm{t}/\mathfrak{T}^2 + \mathfrak{T}_0^2$$

$$\geq 6/64\pi^2\mathfrak{T}_0^4, \mathrm{F}_\chi(t)\ |\Delta\mathcal{E}|/\Lambda(-\delta(t) + \delta(t - \mathbb{T}))\ |\Delta\mathcal{M}||\Delta\mathfrak{S}|, \rho = \langle\mathfrak{T}_{\mu\nu}u^\mu u^\nu\rangle, \hat{\rho}$$

$$\equiv \mathfrak{T}_0/\pi \int_{-\infty}^{\infty} \rho(\mathfrak{T})\mathfrak{d}\mathrm{t}/\mathfrak{T}^2 + \mathfrak{T}_0^2, \hat{\rho} \geq -1/16\pi\mathfrak{T}_0^2, \hat{\rho} \geq -6/64\pi^2\mathfrak{T}_0^4, \hat{\rho} \geq -6/32\pi^2\mathfrak{T}_0^4$$

$$\mathfrak{d}\mathfrak{s}^2 = \left(1 - \frac{2\mathfrak{M}}{\mathfrak{r}}\right)dt^2 - \left(1 - \frac{2\mathfrak{M}}{\mathfrak{r}}\right)^{-1} \mathfrak{d}\mathfrak{r}^2 - \mathfrak{r}^2 d\theta^2 - \mathfrak{r}^2 sin^2\theta d\varphi^2$$

$$\frac{\mathfrak{D}}{\mathfrak{D}\lambda\left(\frac{d\chi^\mu}{d\lambda}\right)}, \int_\alpha^\beta \mathcal{L}\, d\lambda, \mathcal{L} = \frac{\frac{1}{\frac{2\mathfrak{g}^{\mu\nu}d\chi^\mu}{d\lambda d\chi^\nu}}}{d\lambda}, \rho^\mu = \frac{\mathfrak{g}_{\mu\nu}d\chi^\nu}{d\lambda} = \frac{\partial\mathcal{L}}{\partial(d\chi^\mu}/d\lambda), \mathfrak{E} = \rho_t = \left(1 - \frac{2\mathfrak{M}}{\mathfrak{r}}\right)dt/d\lambda, \mathfrak{L}$$

$$= \mathfrak{r}^2 d\varphi/d\lambda, \left(1 - \frac{2\mathfrak{M}}{\mathfrak{r}}\right)\left(\frac{dt}{d\lambda}\right)^2 - \left(1 - \frac{2\mathfrak{M}}{\mathfrak{r}}\right)^{-1}\left(\frac{dr}{d\lambda}\right)^2 - r^2\left(\frac{d\varphi}{d\lambda}\right)^2 \mathfrak{E}^2 - \left(\frac{dr}{d\lambda}\right)^2$$

$$- \mathfrak{L}^2/r^2\left(1 - \frac{2\mathfrak{M}}{\mathfrak{r}}\right), \frac{dr^{\circledcirc}}{d\lambda}\left(1 - \frac{2\mathfrak{M}}{\mathfrak{r}}\right)^{-1}, r^{\circledcirc}$$

$$= r + 2\mathfrak{M} \ln\ (t - 2\mathfrak{M})\ d/d\lambda(t \otimes r^{\odot\otimes\dagger}), du/d\lambda = dt/d\lambda - dr^*/d\lambda, dr^*/d\lambda$$

$$= dr^*/dr^* dr/d\lambda = \left(1 - \frac{2\mathfrak{M}}{\mathfrak{r}}\right)^{-1}\mathfrak{E}, r - 2\mathfrak{M} = \mathfrak{E}, du/d\lambda = 2/\left(1 - \frac{2\mathfrak{M}}{\mathfrak{r}}\right)\mathfrak{E}, du/d\lambda$$

$$= 2\mathfrak{E} - 4\mathfrak{M}/\lambda, u(\lambda) = 2\mathfrak{E}\lambda - 4\mathcal{M}\ln\ (\lambda/\kappa_1), u(\upsilon) = -4\mathcal{M}\ln(\lambda/\kappa_1), (\upsilon)$$

$$= 4\mathcal{M}\ln(\upsilon_0 - \upsilon/\kappa_1\kappa_2)$$

$$\boxdot\ f_\omega = \frac{1}{1} - \frac{2\mathfrak{M}}{\mathfrak{r}}\partial_t^2 f_\omega - \frac{\left(1 - \frac{2\mathfrak{M}}{\mathfrak{r}}\right)2}{r\partial_r f_\omega} - \left(1 - \frac{2\mathfrak{M}}{\mathfrak{r}}\right)\partial_r^2 f_\omega = \frac{2\iota\omega\mathcal{M}}{r^2} - 2\mathcal{M}r = \mathfrak{O}(r^{-2})$$

$$u(\upsilon) = -4\mathcal{M}\ln\left(\upsilon_0 - \frac{\upsilon}{\kappa}\right), \kappa = \kappa_1\kappa_2 \otimes \frac{d\varphi}{d\tau}, \rho_\omega \sim \frac{\frac{1}{\sqrt{\omega}}e^{-\iota\omega u(\upsilon)}}{r}\delta(\theta,\varphi), f_{\omega'}$$

$$\sim \frac{\frac{1}{\sqrt{\omega'}}e^{-\iota\omega\upsilon}}{r}\delta(\theta,\varphi), \alpha_{\omega\omega'} = (f_{\omega'}\rho_\omega) = \frac{\iota\int_{2\mathcal{M}}^{\infty}\mathfrak{d}\mathcal{V}_\chi 1}{1} - \frac{2\mathfrak{M}}{\mathfrak{r}}\left(f_{\omega'}^{\circledast}\partial_t\rho_\omega - \partial_t f_{\omega'}^{\circledast}\rho_\omega\right)$$

$$= \frac{\mathfrak{C}\int_{2\mathcal{M}}^{\infty}\frac{r^2}{1} - \frac{2\mathfrak{M}}{\mathfrak{r}}e^{-\iota\omega'\upsilon}e^{-\iota\omega u(\upsilon)}}{r^2\left(\sqrt{\frac{\omega'}{\omega}} + \sqrt{\frac{\omega}{\omega'}}\right)dr} = -\mathfrak{C}\int_{-\infty}^{\upsilon_0}\mathfrak{d}\upsilon\sqrt{\frac{\omega'}{\omega}}e^{-\iota\omega'\upsilon}e^{-\iota\omega u(\upsilon)}, \beta_{\omega\omega'}$$

$$= -\left(f_{\omega'}^{\circledast}\rho_\omega\right) = \mathfrak{C}\int_{-\infty}^{0}\mathfrak{d}\upsilon\sqrt{\frac{\omega'}{\omega}}e^{-\iota\omega'\upsilon}e^{-\iota\omega u(\upsilon)}\alpha_{\omega\omega'}$$

$$= -\mathfrak{C}\int_{-\infty}^{\upsilon_0}\mathfrak{d}s\sqrt{\frac{\omega'}{\omega}}e^{-\iota\omega' s}e^{-\iota\omega'\upsilon_0}e^{\iota\omega 4\mathcal{M}\ln\left(-\frac{s}{\mathfrak{K}}\right)}, \beta_{\omega\omega'}$$

$$= \mathfrak{C}\int_{-\infty}^{\upsilon_0}\mathfrak{d}s\sqrt{\frac{\omega'}{\omega}}e^{-\iota\omega' s}e^{-\iota\omega'\upsilon_0}e^{\iota\omega 4\mathcal{M}\ln\left(-\frac{s}{\mathfrak{K}}\right)}$$

$$\alpha_{\omega\omega'} = \iota\mathfrak{C} \int_{-\infty}^{v_0} \mathfrak{d}s' \sqrt{\frac{\omega'}{\omega}} e^{w\prime s\prime} e^{\iota\omega' v_0} e^{\iota\omega 4\mathcal{M} \log\left(-\frac{\iota s'}{\mathfrak{K}}\right)}, \beta_{\omega\omega'}$$

$$= -\iota\mathfrak{C} \int_{-\infty}^{v_0} \mathfrak{d}s' \sqrt{\frac{\omega'}{\omega}} e^{w's'} e^{\iota\omega' v_0} e^{\iota\omega 4\mathcal{M} \log\left(-\frac{\iota s'}{\mathfrak{K}}\right)}, \log\left(\frac{\iota s'}{\mathfrak{K}}\right)$$

$$= \ln\left(\frac{|s'|}{\mathfrak{K}}\right) - \frac{\iota\pi}{2}, \log\left(\frac{-\iota s'}{\mathfrak{K}}\right) = \ln\left(\frac{|s'|}{\mathfrak{K}}\right) + \frac{\iota\pi}{2}, \alpha_{\omega\omega'}$$

$$= \iota\mathfrak{C}\, e^{\iota\omega' v_0} e^{2\omega\mathcal{M}\pi} \int_{-\infty}^{0} \mathfrak{d}s' \sqrt{\frac{\omega'}{\omega}} e^{w's'} e^{\iota\omega 4\mathcal{M} \ln\left(\frac{|s'|}{\mathfrak{K}}\right)}, \beta_{\omega\omega'}$$

$$= -\iota\mathfrak{C}\, e^{\iota\omega' v_0} e^{-2\omega\mathcal{M}\pi} \int_{-\infty}^{0} \mathfrak{d}s' \sqrt{\frac{\omega'}{\omega}} e^{w's'} e^{\iota\omega 4\mathcal{M} \ln\left(\frac{|s'|}{\mathfrak{K}}\right)}, |\alpha_{\omega\omega'}|^2$$

$$= e^{8\pi\omega\mathcal{M}} |\beta_{\omega\omega'}|^2, (\rho_{\omega_1}\rho_{\omega_2}) = \Gamma(\omega_1)\delta(\omega_1 - \omega_2)(\rho_{\omega_1}\rho_{\omega_2})$$

$$= \left(\rho_{\omega_1}^{(1)}\rho_{\omega_2}^{(1)}\right) + \left(\rho_{\omega_1}^{(2)}\rho_{\omega_2}^{(2)}\right), \left(\rho_{\omega_1}^{(1)}\rho_{\omega_2}^{(1)}\right)$$

$$= (1 - \Gamma(\omega_1))\delta(\omega_1 - \omega_2), \Gamma(\omega_1)\delta(\omega_1 - \omega_2)$$

$$= \int_{\infty}^{0} \mathfrak{d}\omega' \left(\alpha_{\omega_1\omega'}^{\circledcirc}\alpha_{\omega_2\omega'} - \beta_{\omega_1\omega'}^{\circledcirc}\beta_{\omega_2\omega'}\right), \mathscr{b}_\omega = \left(\rho_\omega^{(2)}, \phi\right)$$

$$= \int_{\infty}^{0} \mathfrak{d}\omega' \left(\alpha_{\omega\omega'}\alpha_{\omega'} + \beta_{\omega\omega'}\alpha_{\omega'}^{\dagger}\right)$$

$$\langle\mathcal{N}\rangle = \left\langle 0\middle|\mathscr{b}_\omega^\dagger \mathscr{b}_\omega\middle|0\right\rangle = \int_{\infty}^{0} \mathfrak{d}\omega' \beta_{\omega\omega'} \langle\omega'| \int_{\infty}^{0} \mathfrak{d}\omega'' \beta_{\omega\omega''}^{\circledast} |\omega''\rangle = \int_{\infty}^{0} \mathfrak{d}\omega' |\beta_{\omega\omega'}|^2, \Gamma(\omega)\delta(0)$$

$$= \int_{\infty}^{0} \mathfrak{d}\omega' (|\alpha_{\omega\omega'}|^2 - |\beta_{\omega\omega'}|^2) = (e^{8\pi\mathcal{M}\omega} - 1) \int_{\infty}^{0} \mathfrak{d}\omega' |\beta_{\omega\omega'}|^2, \delta(\omega_1 - \omega_2)$$

$$= \lim_{\tau\to\infty} \frac{1}{2\pi} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} dt e^{\iota(\omega_1 - \omega_2)t}\, \Gamma(\omega) \lim_{\tau\to\infty} \frac{\tau}{2\pi} = \left(e^{8\pi\mathcal{M}\omega} - 1\right) \int_{\infty}^{0} \mathfrak{d}\omega' |\beta_{\omega\omega'}|^2, \langle\mathcal{N}\rangle$$

$$= \lim_{\tau\to\infty} \frac{\tau}{2\pi} \Gamma(\omega) 1/e^{8\pi\mathcal{M}\omega} - 1, \Gamma(\omega)/2\pi \cdot 1/e^{8\pi\mathcal{M}\omega} - 1$$

$$\mathcal{T} = \frac{1}{16\pi k_{\mathfrak{B}}\mathfrak{M}} = \frac{\kappa}{2\pi}, \rho = \int d\omega\Lambda(\omega)e^{\iota\gamma(\omega)}\rho_{\omega}, \mathcal{T} = \frac{\hbar^4 c^4}{16\pi\mathfrak{G}\mathfrak{M}k_{\mathfrak{B}}} \approx 10^{-7}\left(\frac{\mathfrak{M}_{\odot}}{\mathfrak{M}}\right)\mathfrak{K}, \frac{d\mathfrak{E}}{dt}$$

$$= \frac{8\pi r_s^2\sigma\mathcal{T}^4 dM}{dt} = -\frac{\frac{\beta\mathfrak{m}_\rho^4}{t_\rho 1}}{\mathcal{M}^4}, \mathfrak{M}(t) = \left(\mathfrak{M}_0^4 - \frac{6\beta\mathfrak{m}_\rho^4}{t_\rho}t\right)^{\frac{1}{2}}, \Delta_t = t_\rho/3\beta\left(\frac{\mathfrak{M}_0}{\mathfrak{m}_\rho}\right)^3$$

9. Modelo Englert – Brout.

$$\mathcal{H}_{int} = \imath e\Lambda_\mu\varphi^{\boxtimes}\overleftrightarrow{\partial}_\mu\varphi - e^2\varphi^{\boxtimes}\varphi\Lambda_\mu\Lambda_\mu, \circledast^{\mu\nu}\odot_{\mu\nu}(q) = (2\pi)^4\iota e^2(g_{\mu\nu})\langle\varphi_1\rangle^2 - \langle\frac{q_\mu q_\nu}{q^2}\rangle\langle\varphi_1\rangle^2, \mu^2$$

$$= e^2\langle\varphi_1\rangle^2, \delta\varphi_\Lambda = \Sigma_\alpha\Lambda^\epsilon\alpha^{(\chi)\mathfrak{T}}\alpha\mathrm{AB}^\varphi\mathrm{B}'\delta\Lambda_{\alpha,\mu}$$

$$= \Sigma_c \mathscr{b}^\epsilon c^{(x)c_{acb}}\Lambda_{\beta,\nu} + \partial_\mu\epsilon_{\alpha^{(x)}}, \frac{\left(\frac{\iota}{(2\pi)^4}\right)\Sigma_{\mathrm{A,B'C'}}\mathfrak{T}_{\alpha,\mathrm{AB'}}\langle\varphi_{\mathrm{B'}}\rangle\mathfrak{T}_{\alpha\mathrm{AC'}}\langle\varphi_{\mathrm{C'}}\rangle}{\mathfrak{q}^2}$$

$$\equiv, \left(\frac{-\iota}{(2\pi)^4}\right)\left(\frac{\langle\varphi\rangle\mathfrak{T}_\alpha\mathfrak{T}_\alpha\langle\varphi\rangle}{\mathfrak{q}^2}\right), \boxtimes^{\mu\nu}\bigotimes\hbar_{\mu\nu}^{\alpha}(q)\delta\delta$$

$$= -\imath(2\pi)^4\lambda^2\left(\frac{\langle\varphi\rangle\mathfrak{T}_\alpha\mathfrak{T}_\alpha\langle\varphi\rangle}{\mathfrak{q}^2}\right)\otimes\left(\mathfrak{g}_{\mu\nu} - \frac{q_\mu q_\nu}{\mathfrak{q}^2}\right), \mu_\alpha^2 = -\left(\frac{\langle\varphi\rangle\mathfrak{T}_\alpha\mathfrak{T}_\alpha\langle\varphi\rangle}{\mathfrak{q}^2}\right), \mathcal{H}_{int}$$

$$= -\eta\bar{\psi}\gamma_\mu\gamma_5\bar{\psi}\mathrm{B}_\mu - \epsilon\bar{\psi}\gamma_\mu\psi\Lambda_\mu, \delta^{-1}(\rho) = \gamma\rho - \Sigma(\mathrm{\rho})$$

$$= \gamma\rho(1 - \Sigma_2(\rho^2)) - \Sigma_1(\rho^2), \mathfrak{m}(1 - \Sigma_2(\mathfrak{m}^4)) - \Sigma_1(\mathfrak{m}^4), \mathfrak{J}_\mu^5$$

$$= -\eta\lim_{\xi\to 0}\overline{\psi'}(\chi + \xi)\gamma_\mu\gamma_5\psi'(\chi), \psi'(\chi)$$

$$= \exp(-\iota\int_{-\infty}^{\chi}\eta\mathrm{B}_\mu(\gamma)d\gamma^\mu\gamma_5)\,\psi(\chi)\ \otimes_{\mu\nu}{}^5(\varrho)$$

$$= \eta^2\iota/(2\pi)^4\int\mathfrak{Tr}\,(\delta\left(\rho - \frac{1}{2\varrho}\right)\Gamma_{\nu^5}\left(\rho - \frac{1}{2\varrho}; \rho + \frac{1}{2\varrho}\right)\boxtimes\delta\left(\rho + \frac{1}{2\varrho}\right)\gamma_\mu\gamma_5$$

$$- \delta(\rho)\left(\frac{\partial\delta^{-1}(\rho)}{\partial\rho_\nu}\right)\delta(\rho)\gamma_\mu)\mathfrak{d}^4\rho$$

$$\varrho_\nu\Lambda_{\nu^5}\left(\rho - \frac{1}{2\varrho}; \rho + \frac{1}{2\varrho}\right)\Sigma\left(\rho - \frac{1}{2\varrho}\right)\gamma_5 + \gamma_5\Sigma\left(\rho + \frac{1}{2\varrho}\right), \varrho_\nu\Gamma_{\nu^5}$$

$$= \varrho_\nu\gamma_\nu\gamma_5(1 - \Sigma_2) + 2\Sigma_1\gamma_5 - 2(\varrho_\nu\rho_\nu)(\gamma_\lambda\rho_\lambda)(\partial\Sigma_2/\partial\rho^2\gamma_5$$

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Apéndice C.

Formalización de la dualidad holográfica en campos cuánticos curvos.

1. Dualidad/Gravedad – Gauge en espacios cuánticos curvos.

1.1. Grupo Conforme.

$$\iota[\mathfrak{M}_{\mu\nu}\mathfrak{M}_{\rho\sigma}] = \eta_{\nu\rho}\mathfrak{M}_{\mu\sigma} - \eta_{\mu\rho}\mathfrak{M}_{\nu\sigma} - \eta_{\sigma\mu}\mathfrak{M}_{\rho\nu} + \eta_{\sigma\nu}\mathfrak{M}_{\rho\mu}\iota[\mathfrak{P}_{\mu}\mathfrak{M}_{\sigma\rho}]$$

$$= \eta_{\mu\rho}\mathfrak{P}_{\sigma} - \eta_{\mu\sigma}\mathfrak{P}_{\rho}[\mathfrak{P}_{\mu}\mathfrak{P}_{\nu}]\iota[\mathfrak{D},\mathfrak{P}_{\mu}] = \mathfrak{P}_{\mu}[\mathfrak{M}_{\mu\nu},\mathfrak{D}]$$

$$\mathfrak{K}_{\mu\nu}: \chi^{\mu} \longrightarrow \chi^{\mu} + \frac{\alpha^{\mu}\chi^{2}}{1} + 2\chi^{\nu}\alpha_{\nu} + \alpha^{2}\chi^{2}\iota[\mathfrak{M}_{\mu\nu}\mathfrak{K}_{\rho}] = \eta_{\mu\rho}\mathfrak{K}_{\nu} - \eta_{\nu\sigma}\mathfrak{K}_{\mu}[\mathfrak{D},\mathfrak{K}_{\mu}]\iota\mathfrak{K}_{\mu}[\mathfrak{P}_{\mu}\mathfrak{K}_{\nu}]$$

$$= 2\iota(\mathfrak{M}_{\mu\nu} - \eta_{\mu\nu}\mathfrak{D})[\mathfrak{K}_{\mu}\mathfrak{K}_{\nu}]$$

$$\mathfrak{J}_{\mu\nu} = \mathfrak{M}_{\mu\nu}, \mathfrak{J}_{\mu d} = \frac{1}{2[\mathfrak{K}_{\mu} - \mathfrak{P}_{\mu}]}, \mathfrak{J}_{\mu(d+1)} = \frac{1}{2[\mathfrak{K}_{\mu} + \mathfrak{P}_{\mu}]}, \mathfrak{J}_{(d+1)d} = \mathfrak{D}$$

$$\mathfrak{J}_{\alpha\beta} = \begin{pmatrix} \mathfrak{J}_{\mu\nu} & \mathfrak{J}_{\mu d} & \mathfrak{J}_{\mu(d+1)} \\ -\mathfrak{J}_{\mu d} & 0 & \mathfrak{D} \\ -\mathfrak{J}_{\mu(d+1)} & -\mathfrak{D} & 0 \end{pmatrix}$$

$$\mathcal{P}_{\mu}: \chi_{\mu} \longrightarrow \chi_{\mu} + \alpha_{\mu} \Longrightarrow d, \mathfrak{M}_{\mu\nu}: \chi_{\mu} \longrightarrow \Lambda_{\mu}^{\nu}\chi_{\nu} \Longrightarrow \frac{(d-1)d}{2}, \mathfrak{D}: \chi_{\mu} \longrightarrow \lambda\chi_{\mu}, \mathfrak{K}_{\mu}: \chi_{\mu}$$

$$\longrightarrow \chi_{\mu} + \frac{\alpha_{\mu}\chi^{2}}{1} + 2\chi_{\nu}\alpha^{\nu} + \alpha^{2}\chi^{2} \Longrightarrow d$$

$$\chi \longrightarrow \lambda\chi \Longrightarrow \phi(\chi) \longrightarrow \phi(\chi)' = \lambda^{\Delta}\phi(\lambda\chi)[\mathfrak{D},\mathcal{P}_{\mu}] = \iota\mathcal{P}_{\mu} \Longrightarrow \mathfrak{D}(\mathcal{P}_{\mu}\phi) = -\iota(\Delta+1)(\mathcal{P}_{\mu}\phi), \langle\phi(0)|\phi(\chi)\rangle$$

$$\equiv 1/(\chi^{2})^{\Delta}$$

1.2. Espacio Anti – de Sitter en espacios cuánticos curvos.

$$\Re_{\mu\nu}-\frac{1}{2g_{\mu\nu}\mathcal{R}}=-\Lambda g_{\mu\nu},\Re_{\mu\nu\theta\sigma}=\frac{1}{\ell^2}(g_{\mu\theta}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\theta}),\Re_{\mu\nu}=-\frac{3}{\ell^2 g_{\mu\nu}},\mathcal{R}=-\frac{12}{\ell^2},\mathfrak{ds}^2$$

$$=-d\chi_0^2-d\chi_4^2+d\chi_1^2+d\chi_2^2+d\chi_3^2-\chi_0^2-\chi_4^2+\chi_1^2+\chi_2^2+\chi_3^2=-\ell^2,\frac{\mathfrak{ds}^2}{\ell^2}$$

$$=-\cos\hbar^2\rho d\tau^2+d\rho^2+\sin\hbar^2\rho(d\theta^2+sin^2\theta d\varphi^2),\frac{\mathfrak{ds}^2}{\ell^2}$$

$$\approx -d\tau^2+d\rho^2+\rho^2(d\theta^2+sin^2\theta d\varphi^2),\mathfrak{ds}^2$$

$$=\left(1+\frac{r^2}{\ell^2}\right)dt^2+\left(1+\frac{r^2}{\ell^2}\right)^{-1}dr^2+r^2(d\theta^2+sin^2\theta d\varphi^2),\chi_0$$

$$=\frac{lr}{2}\left(\overrightarrow{\chi_i^2}-t^2+\frac{1}{r^2}+1\right),\chi_i=lrx_i(i=1,2),\chi_3=\frac{lr}{2}\left(\overrightarrow{\chi_i^2}-t^2+\frac{1}{r^2}+1\right)\chi_4$$

$$=lrt,\frac{\mathfrak{ds}^2}{\ell^2}=r^2\left(-dt^2+d\overrightarrow{\chi^2}\right)+\frac{dr^2}{r^2},\mathfrak{ds}^2=\ell^2/z^2\left(-dt^2+d\overrightarrow{\chi^2}+dz^2\right)$$

1.3. Límite de ’t Hooft en espacios cuánticos curvos.

$$\mathfrak{L}=\frac{\mathfrak{N}}{2\mathfrak{G}_{\mathfrak{YM}}^2\mathcal{F}_{\mu\nu}^{\mathcal{M}}\mathcal{F}_{\mathcal{M}}^{\mu\nu}},$$

1.4. Prescripción de Gubser – Klevanov – Polyakov y Witten en espacios cuánticos curvos.

$$\mathfrak{z}_{\mathfrak{CFT}}=e^{-\mathcal{W}},\ell^4\ell_s^4\sim g_{\mathcal{YM}}^2\mathcal{N}\sim g_\delta \mathrm{N}>>1,\mathcal{Z}_\wp\approx e^{-\mathfrak{I}_{SUGRA}^{\mathfrak{E}}},\mathcal{Z}_\wp\approx e^{-\mathfrak{I}_{\mathfrak{SUGRA}}^{\mathfrak{E}}}=e^{-\mathcal{W}}=\mathcal{Z}_{\mathfrak{CFT}}$$

1.5. Correspondencia Campo ⟷ Operador en espacios cuánticos curvos.

$$\langle e^{\int d^3\chi\phi_0(\vec{\chi})\mathcal{O}(\vec{\chi})}\rangle_{\mathfrak{CFT}}=e^{-\mathfrak{I}_{\mathfrak{BULK}}^{\mathfrak{E}}[\phi|\partial\mathfrak{AdS}\to\phi_0|]},\langle e^{\int d^3\chi\hbar_{\alpha\beta}^0\mathcal{T}^{\alpha\beta}}\rangle_{\mathfrak{CFT}}=e^{-\mathfrak{I}_{\mathfrak{BULK}}^{\mathfrak{E}}\left[\hbar_{\mu\nu}\middle|\partial\mathfrak{AdS}\to\hbar_{\alpha\beta}^0\right|]}$$

1.6. Partículas y Campos en el espacio tiempo AdS – curvo.

$$\left(\nabla^{\mu}\nabla_{\mu}-\mathfrak{m}^{4}\right)\Phi(z,\chi^{\eta})=\frac{\frac{\int d^{2}\vec{\kappa}}{(2\pi)^{4}d\omega\mathfrak{f}_{\kappa}(z)e^{i\kappa_{\mu}\chi^{\mu}}d^{2}\mathfrak{f}_{\kappa}}}{dz^{2}}-\frac{\frac{2}{zd\mathfrak{f}_{\kappa}}}{dz}-\left(\kappa^{2}+\frac{\mathfrak{m}^{4}\ell^{4}}{z^{2}}\right)\mathfrak{f}_{\kappa}(z)$$

$$=\alpha_{1}z^{\frac{3}{2}}\mathfrak{K}_{\nu}(\mathfrak{K}z)+\frac{\alpha_{2}z^{\frac{3}{2}}\mathfrak{I}_{\nu}(\mathfrak{K}z)d^{2}\mathfrak{f}_{\kappa}}{dz^{2}}-\kappa^{4}\mathfrak{f}_{\kappa}(z)$$

$$=\frac{\alpha_{1}z^{\frac{3}{2}}\pi}{2\sin\pi\nu\left[\frac{1}{\Gamma(1-\nu)\left(\frac{\mathfrak{K}z}{2}\right)^{-\nu}}-\frac{1}{\Gamma(1+\nu)\left(\frac{\mathfrak{K}z}{2}\right)^{\nu}}\right]},\mathfrak{m}_{\mathfrak{B}\mathfrak{F}}^{4}\geq\mathfrak{m}^{4}\geq\mathfrak{m}_{\mathfrak{B}\mathfrak{F}}^{4}+\frac{1}{\ell^{4}}\Rightarrow 0\geq\nu$$

$$>1,\phi(r,\chi^{\eta})=\frac{\alpha(\chi^{\eta})}{r^{3-\Delta}}+\frac{\beta(\chi^{\eta})}{r^{\Delta}},\phi=\frac{\alpha}{r}+\frac{\beta}{r^{2}\partial_{r}\phi}$$

$$=-\frac{\alpha}{r^{2}}-\frac{\frac{2\beta}{r^{4}\otimes\alpha'}}{r^{2}}+\alpha\phi+\beta\partial_{r}\phi=\alpha\left(\frac{\alpha}{r}+\frac{\beta}{r^{2}}\right)+\beta\left(-\frac{\alpha}{r^{2}}-\frac{2\beta}{r^{4}}\right)=\frac{\alpha'}{r}+\frac{\beta'}{r^{2}}$$

1.7. Deformaciones en AdS/CFT en espacios cuánticos curvos.

$$\mathfrak{I}_{\mathfrak{C}\mathfrak{F}\mathfrak{T}}\longrightarrow\mathfrak{I}_{\mathfrak{C}\mathfrak{F}\mathfrak{T}}+\rho\int d^{3}\chi\mathcal{O}(\chi)$$

2. Agujeros Negros Cuánticos en espacios curvos (Formalización).

2.1. Principio Variacional.

$$\mathfrak{I}=\frac{1}{2\kappa\int d^{4}\chi\sqrt{-g}\mathcal{R}+\mathfrak{I}_{\mathfrak{B}}},\delta\mathcal{I}=\frac{1}{2\kappa\int d^{4}\chi\sqrt{-g}\mathfrak{G}_{\alpha\beta}\delta g^{\alpha\beta}+\int d^{4}\chi\sqrt{-g}g^{\alpha\beta}\delta\mathcal{R}_{\alpha\beta}+\delta\mathfrak{I}_{\mathfrak{B}}},\mathfrak{G}_{\alpha\beta}$$

$$=\mathcal{R}_{\alpha\beta}-\frac{1}{2g_{\alpha\beta}\mathfrak{R}},\delta\mathfrak{I}_{\mathfrak{B}}=-\int_{\mathcal{M}}^{\beth}d^{4}\chi\sqrt{-g}g^{\alpha\beta}\delta\Gamma_{\alpha\beta}^{\mu}-\sqrt{-g}g^{\alpha\mu}\delta\Gamma_{\alpha\beta}^{\beta}+\sqrt{-g}\delta\mathcal{R}_{\alpha\beta}$$

$$=\oint_{\partial\mathcal{M}}^{\gimel}\epsilon\upsilon^{\mu}\eta_{\nu}\sqrt{-\hbar}\,d^{3}\chi,\mathfrak{I}_{\mathfrak{B}}=\oint_{\partial\mathcal{M}}^{\gimel}d^{3}\chi\sqrt{-\hbar}\,\kappa,\mathfrak{I}$$

$$=\frac{1}{2\kappa\int_{\mathcal{M}}^{\beth}d^{4}\chi\sqrt{-g}\,\mathcal{R}}+1/\varkappa\oint_{\partial\mathcal{M}}^{\gimel}d^{3}\chi\sqrt{-\hbar}\,\mathfrak{K}$$

$$ds^{2}=-\mathcal{N}(r)dt^{2}+\mathcal{H}(r)dr^{2}+r^{2}(d\theta^{2}+sin^{2}\theta d\varphi^{2}),\mathfrak{K}_{\mathfrak{R}\mathfrak{E}\mathfrak{T}\mathfrak{S}}=\mathcal{R}^{\alpha\beta\gamma\sigma}\mathcal{R}_{\alpha\beta\gamma\sigma},\mathcal{I}\left[\mathfrak{g}_{\mu\nu}\right]$$

$$=\frac{1}{2\kappa\int_{\mathcal{M}}^{\beth}d^{4}\chi\sqrt{-g}\mathcal{R}}+\frac{1}{\kappa\oint_{\partial\mathcal{M}}^{\gimel}d^{3}\chi\mathfrak{K}\sqrt{-\hbar}},ds^{2}$$

$$=-\left(1-\frac{\mu}{r}\right)dt^{2}+\left(1-\frac{\mu}{r}\right)^{-1}dr^{2}+r^{2}(d\theta^{2}+sin^{2}\theta d\varphi^{2}),M=\frac{4\pi\mu}{\kappa}=\mu/2\mathfrak{G}$$

2.2. Modelo Reissner – Nordström.

$$\mathcal{I}[\mathfrak{g}_{\mu\nu}\Lambda_\mu] = \frac{1}{2\kappa\int_{\mathcal{M}}^{\gimel} d^4\chi\sqrt{-\mathcal{g}}\left(\mathcal{R}-\frac{1}{4\mathrm{F}^{\mu\nu}\mathrm{F}_{\mu\nu}}\right)} + \frac{1}{\kappa\oint_{\partial\mathcal{M}}^{\lambda} d^3\chi\mathfrak{K}\sqrt{-\hbar}}, \mathcal{R}_{\mu\nu} - \frac{1}{2\mathcal{R}\mathfrak{g}_{\mu\nu}}$$

$$= \frac{1}{2\mathfrak{J}_{\mu\nu}^{\mathfrak{EM}}\nabla_\mu \mathrm{F}^{\mu\nu}}, \mathfrak{J}_{\mu\nu}^{\mathfrak{EM}} = \mathrm{F}_{\mu\alpha}\mathrm{F}_\nu^\alpha - \frac{1}{4\mathfrak{g}_{\mu\nu}\mathrm{F}^2}, \Lambda \equiv \Lambda_\mu d\chi_\mu = \left(\frac{q}{r}-\frac{q}{r_+}\right)dt, \mathrm{F}$$

$$= -\frac{q}{r^2 dr}\wedge dt, ds^2 = -\mathfrak{f}(r)dt^2 + \mathfrak{f}(r)^{-1}dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2), \mathfrak{f}(r)$$

$$= 1 - \frac{\mu}{r} + \frac{q^2}{4r^2} = \frac{(r-r_-)(r-r_+)}{r^2}, \mathcal{M} = \frac{4\pi\mu}{\kappa} = \frac{\mu}{2\mathfrak{G}}, \mathfrak{Q} \equiv \frac{1}{\kappa\oint_{-\infty}^{\infty} d^2 \star \mathcal{F} = -\frac{q}{4\mathfrak{G}}}, r_\pm$$

$$= \mathfrak{G}(\mathfrak{M} \pm \sqrt{\mathfrak{M}^2 - 4\mathfrak{Q}^2}, \Phi = \Lambda_t|\Delta_{r=\infty} - \Lambda_t|\Delta_{r=r_+} = 4\mathfrak{G}\mathfrak{Q}/r^{\blacksquare}$$

2.3. Modelo anti – de Sitter.

$$ds^2 = \frac{r^2}{\ell^2(-dt^2 + \ell^2 d\Sigma_\kappa^2)}, ds^2 = -\mathrm{N}(r)dt^2 + \mathcal{H}(r)dr^2 + \delta(r)d\Sigma_\kappa^2$$

$$= \left\langle \begin{matrix} d\theta^2 + sin^2\theta d\varphi^2 & \propto \kappa = +1 \\ \frac{1}{\ell^2\sum_{i=1}^2 d\chi_i^2} & \bowtie \kappa = 0 \\ d\theta^2 + \sin\hbar^2\theta d\varphi^2 & \doteqdot \kappa = -1 \end{matrix} \right\rangle, d\Sigma_\kappa^2 = \frac{d\mathcal{y}^2}{1} - \kappa\mathcal{y}^2 + (1+\kappa\mathcal{y}^2)dz^2$$

2.4. Modelo Schwarzschild-AdS.

$$\mathcal{I}[\mathfrak{g}_{\mu\nu}] = \frac{1}{2\kappa\int_{\mathcal{M}}^{\gimel} d^4\chi\sqrt{-\mathcal{g}}\,(\mathfrak{R}-2\Lambda)} + \frac{1}{\kappa\oint_{\partial\mathcal{M}}^{\lambda} d^3\chi\mathfrak{K}\sqrt{-\hbar}}, \mathcal{R}_{\mu\nu} - \frac{1}{2\mathfrak{g}_{\mu\nu}\mathcal{R}} = -\Lambda\mathfrak{g}_{\mu\nu}, ds^2$$

$$= -\left(\kappa - \frac{\mu}{r} + \frac{r^2}{\ell^2}\right)dt^2 + \left(\kappa - \frac{\mu}{r} + \frac{r^2}{\ell^2}\right)^{-1} dr^2 + r^2\,d\Sigma_\kappa^2, \kappa - \frac{\mu}{r_\hbar} + \frac{r_\hbar^2}{\ell^2}$$

2.5. Modelo Escalar Simple.

$$\mathcal{I}[\mathfrak{g}_{\mu\nu},\phi] = \frac{\int_{\mathcal{M}}^{\gimel} d^4\chi\sqrt{-\mathcal{g}}\left[\frac{\mathfrak{R}}{2\kappa} - \frac{1}{2\partial_\mu\phi\,\partial^\mu\phi} - \mathcal{V}(\phi)\right]1}{\sqrt{-g}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right)} - \frac{\frac{\partial\mathcal{V}}{\partial\phi}d\mathcal{V}}{d\phi|\psi_{\phi=0}}, \mathcal{V}(0)$$

$$= -\frac{3}{\kappa\ell^2}, \frac{d^2\mathcal{V}}{d\phi^2|\psi_{\varphi=0}}, \mathbb{V}(\phi)_{\mathfrak{AdS}} = \left(-\frac{1}{\ell^2} + \alpha\phi\right)(4 + 2\cos\hbar\phi) - 6\alpha\sin\hbar\phi$$

2.6. Modelo Escalar Neutro.

$$\mathcal{E}_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2g_{\mu\nu}\Re} - \kappa\mathcal{T}^{\varphi}_{\phi\mu\nu}, \mathcal{T}^{\varphi}_{\phi\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2(\partial\psi)^2} + \mathfrak{V}(\phi)\right), ds^2$$

$$= \Omega(\chi)\left[-\mathfrak{f}(\chi)dt^2 + \frac{\eta^2 d\chi^2}{\mathfrak{f}(\chi)} + \frac{d\mathcal{y}^2}{\ell^2} + \frac{dz^2}{\ell^2}\right]\mathfrak{E}^t_t - \mathfrak{E}^\chi_\chi = 0 \longrightarrow \phi'^2$$

$$= 3\Omega'^2 - \frac{2\Omega''\Omega}{\Omega^2}, \mathfrak{E}^t_t - \mathfrak{E}^\mathcal{y}_\mathcal{y} = 0 \longrightarrow \mathfrak{f}'' + \frac{\Omega'\mathfrak{f}'}{\Omega} = 0, \mathfrak{E}^t_t - \mathfrak{E}^\mathcal{y}_\mathcal{y} = 0 \longrightarrow \mathcal{V}(\phi)$$

$$= -\frac{1}{\Omega^2\eta^2(\mathfrak{f}\Omega'' + \mathfrak{f}'\Omega')}, \Omega(\chi) = \frac{\nu^2\chi^{\nu-1}}{\eta^2(\chi^\nu - 1)^2}, \phi'^2$$

$$= \frac{(\nu-1)^2}{\chi^2} - \frac{4\nu(\nu-1)\chi^{\nu-2}}{\chi^\nu} - 1 + \frac{4\nu^2\chi^{\nu-1}}{(\chi^\nu - 1)^2} + \frac{2(\nu-1)}{\chi^2}$$

$$+ \frac{4\nu(1-\nu-\chi^\nu)\chi^{\nu-2}\chi^{\nu-2}}{(\chi^\nu-1)^2}, \phi'^2 = \nu^2 - \frac{1}{2\kappa\chi^2}$$

$$\longrightarrow \int_{\phi}^{\phi=0} d\varphi = \sqrt{\nu^2 - \frac{1}{2\kappa}}\int_{\chi}^{1}\frac{d\chi}{\chi}, \phi(\tau)\ln\chi, \ell_\nu^{-1} = \sqrt{\nu^2 - \frac{1}{2\kappa}}\,(\mathfrak{f}'\Omega)'f(x)$$

$$= \frac{c_4\eta^2}{\nu^2\int\frac{(\chi^\nu-1)^2}{\chi^{\nu-1}d\chi} + c_1}, f(x) = c_1 + \frac{c_4\eta^2}{\nu^2\left(\frac{\chi^{2+\nu}}{2} + \nu + \frac{\chi^{2-\nu}}{2} - \nu - \chi^2\right)}, f(x)$$

$$= \frac{1}{\ell^2} + \alpha\left[\frac{1}{\nu^2} - 4 - \frac{\chi^2}{\nu^2\left(1 + \frac{\chi^{-\nu}}{\nu} - 2 - \frac{\chi^\nu}{\nu} + 2\right)}\right], \mathcal{V}(\phi)$$

$$= \frac{\Lambda(\nu^2-4)}{6\kappa\nu^2\left[\nu - \frac{1}{\nu} + 2e^{-\phi\ell_\nu(\nu+1)} + \nu + \frac{1}{\nu} - 2e^{\phi\ell_\nu(\nu-1)} + 4\nu^2 - \frac{1}{\nu^2} - 4e^{-\phi\ell_\nu}\right]}$$

$$+ \alpha$$

$$/\kappa\nu^2\left[\nu - \frac{1}{\nu} + 2\sin\hbar\,\phi\ell_\nu(\nu+1) - \nu + \frac{1}{\nu}\right.$$

$$\left. - 2\sin\hbar\phi\ell_\nu(\nu-1) + 4\nu^2 - \frac{1}{\nu^2} - 4\sin\hbar\phi\ell_\nu\right]$$

$$\mathcal{V}(\phi) = \frac{\Lambda}{\kappa} - \frac{\phi^2}{\ell^2} + \frac{\frac{\kappa\Lambda}{18(\nu^2-3)}}{\nu^2} - 1\phi^4 - \frac{\ell_\nu^3}{90(\Lambda\nu^2 - 4\Lambda - 6\alpha)\tau^5} + \mathcal{O}|\varphi|^6$$

$$ds^2 = \Omega(\chi)\left[-f(x)dt^2 + \frac{\eta^2 d\chi^2}{f(x)} + d\theta^2 + sin^2\theta d\varphi^2\right], f(x)$$

$$= \frac{1}{\ell^2} + \alpha\left[\frac{1}{\nu^2} - 4 - \frac{\chi^2}{\nu^2\left(1 + \frac{\chi^{-\nu}}{\nu} - 2 - \frac{\chi^{\nu}}{\nu} + 2\right)}\right] + \chi/\Omega(\chi)$$

2.7. Modelo Escalar Eléctricamente Cargado.

$$\mathfrak{I}[\mathfrak{g}_{\mu\nu}, \Lambda_\mu \phi] = \frac{1}{16\pi \mathfrak{G}_{\mathrm{N}} \int d^4\chi\sqrt{-g}\left[\mathfrak{R} - \frac{1}{4e^{\gamma\phi}\mathrm{F}^2} - \frac{1}{2\partial_\mu\phi\,\partial^\mu\phi} - \mathcal{V}(\phi)\right]}, \nabla_\mu\left(e^{\gamma\phi}\mathrm{F}^{\mu\nu}\right)$$

$$= \frac{1}{\sqrt{-g}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right)} - \frac{\partial\mathcal{V}}{\partial\phi} - \frac{1}{4\gamma e^{\gamma\phi}\mathrm{F}^2}, \mathcal{R}_{\mu\nu} - \frac{1}{2g_{\mu\nu}\mathcal{R}} = \frac{1}{2\left[\mathcal{T}^{\phi}_{\mu\nu}\mathcal{T}^{\mathfrak{E}\mathfrak{M}}_{\mu\nu}\right]}, \mathcal{T}^{\phi}_{\mu\nu}$$

$$= \partial_\mu\phi\partial_\nu\phi - \mathfrak{g}_{\mu\nu}\left[\frac{1}{2(\partial\phi)^2} + \mathcal{V}(\phi)\right], \mathcal{T}^{\mathfrak{E}\mathfrak{M}}_{\mu\nu} = e^{\gamma\phi}\left(\mathrm{F}_{\mu\alpha}\mathrm{F}^{\alpha}_{\nu} - \frac{1}{4\mathfrak{g}_{\mu\nu}\mathrm{F}^2}\right)$$

2.8. Modelo Holográfico en espacios cuánticos curvos.

$$\mathcal{I}_g = -\frac{1}{8\pi\mathfrak{G}_N \oint_{\partial\mathcal{M}}^{\lambda} d^3\chi\mathfrak{K}\sqrt{-\hbar}\,\Xi(\ell,\mathcal{R},\nabla\mathfrak{R})}, ds^2$$

$$= -\left(\kappa + \frac{r^2}{\ell^2}\right)dt^2 + \left(\kappa + \frac{r^2}{\ell^2}\right)^{-1} dr^2 + r^2\, d\Sigma_\kappa^2 \hbar_{\alpha\beta} d\chi^\alpha d\chi^\beta$$

$$= -\left(\kappa + \frac{\mathcal{R}^2}{\ell^2}\right)dt^2 + \mathcal{R}^2 d\Sigma_\kappa^2, ds^2 = \frac{r_\beta^2}{\ell^2(-dt^2 + \ell^2 d\Sigma_\kappa^2)}, ds^2$$

$$= -\left(1 - \frac{\mu}{r} + \frac{r^2}{\ell^2}\right)dt^2 + \left(1 - \frac{\mu}{r} + \frac{r^2}{\ell^2}\right)^{-1} dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2), \mathfrak{T} = \frac{f'}{|4\pi_{r+}}$$

$$= \beta^{-1} = \frac{1}{4\pi\left(3r_+^2 + \frac{\ell^2}{\ell_{r_+}^2}\right)}, \mathcal{I}_{\mathfrak{BULK}}^{\mathfrak{E}} = \frac{12\pi\beta}{\kappa\ell^2 \int_{r_+}^{\mathcal{R}} r^2 dr = \frac{4\pi\beta}{\kappa\ell^2(\mathcal{R}^3 - r_+^3)}}, \hbar_{\alpha\beta} d\chi^\alpha d\chi^\beta$$

$$= -\left(1 - \frac{\mu}{\mathcal{R}} + \frac{\mathcal{R}^2}{\ell^2}\right)dt^2 + \mathcal{R}^2(d\theta^2 + sin^2\theta d\varphi^2), \eta_\alpha = \frac{\delta_\alpha^r}{\sqrt{g^{rr}}}, \kappa_{\alpha\beta} = \frac{\sqrt{g^{rr}}}{2\partial_r \hbar_{\alpha\beta}}, \mathfrak{K}$$

$$= \frac{1}{\mathcal{R}^2\ell^2\left(1 - \frac{\mu}{\mathcal{R}} + \frac{\mathcal{R}^2}{\ell^2}\right)^{-\frac{1}{2}}\left(-\frac{3\ell^2\mu}{2} + 3\mathcal{R}^2 + 2\mathcal{R}\ell^2\right)}, \mathcal{I}_{\mathfrak{GH}}^{\mathfrak{E}}$$

$$= \frac{4\pi\beta}{\kappa\ell^2\left(-\frac{3\ell^2\mu}{2} + 3\mathcal{R}^2 + 2\mathcal{R}\ell^2\right)}, \mathcal{I}_{\mathfrak{GH}}^{\mathfrak{E}} = \mathcal{I}_{\mathfrak{BULK}}^{\mathfrak{E}} + \mathcal{I}_{\mathfrak{GH}}^{\mathfrak{E}}$$

$$= \frac{4\pi\beta}{\kappa\ell^2\left(-\frac{3\ell^2\mu}{2} + 2\mathcal{R}^2 + 2\mathcal{R}\ell^2 - r_+^3\right)}, ds^2$$

$$= -\left(1 + \frac{r^2}{\ell^2}\right)dt^2 + \left(1 + \frac{r^2}{\ell^2}\right)^{-1} dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2), \mathcal{I}_{\mathfrak{AdS}}^{\mathfrak{E}} = \mathcal{I}_{\mathfrak{BULK}}^{\mathfrak{E}} + \mathcal{I}_{\mathfrak{GH}}^{\mathfrak{E}}$$

$$= \frac{4\pi\beta_0}{\kappa\ell^2(-2\mathcal{R}^3 + 2\mathcal{R}\ell^2)}, \beta_0\sqrt{1 + \frac{\mathcal{R}^2}{\ell^2}} = \beta\sqrt{1 + \frac{\mathcal{R}^2}{\ell^2} - \frac{\mu}{\mathfrak{R}}}, \mathcal{I}^{\mathfrak{E}} = \mathcal{I}_{\mathfrak{GH}}^{\mathfrak{E}} - \mathcal{I}_{\mathfrak{AdS}}^{\mathfrak{E}}$$

$$= \frac{4\pi\beta}{\kappa\ell^2\left[\left(\frac{3\ell^2\mu}{2} - 2\mathcal{R}^3 - 2\mathcal{R}\ell^2 - r_+^3\right) - \frac{\beta_0}{\beta(-2\mathcal{R}^3 - 2\ell^2\mathfrak{R})}\right]}, \mathrm{F} = \beta^{-1}\mathcal{I}^{\mathfrak{E}}$$

$$= 4\pi/\kappa\ell^2\left(\frac{\ell^2\mu}{2} - r_+^3\right)$$

$$\mathcal{I}_g = -\frac{1}{\kappa \int_{\partial\mathcal{M}}^{\infty} d^3\chi\sqrt{-\hbar}\left(\frac{2}{\ell}+\frac{\mathcal{LR}}{2}\right), \mathcal{I}^{\mathfrak{E}}_{\mathfrak{BULK}}} + \mathcal{I}^{\mathfrak{E}}_{\mathfrak{GH}} = \frac{4\pi\beta}{\kappa\ell^2\left(\frac{3\ell^2\mu}{2} - 2r_\beta^3 - 2r_\beta\ell^2 - r_+^3\right)}, \mathcal{I}^{\mathfrak{E}}_{g}$$

$$= \frac{4\pi\beta}{\kappa\ell^2\left(1+\frac{\ell^2}{\mathcal{R}^2}-\frac{\mu\ell^2}{\mathcal{R}^3}\right)^{\frac{1}{2}} |(2\mathcal{R}^3+\kappa\ell^2\mathcal{R})|_{\mathcal{R}=r_\beta}} = \frac{4\pi\beta}{\kappa\ell^2(2r_\beta^3 - 2\ell^2 r_\beta - \mu\ell^2)}, \mathcal{I}^{\mathfrak{E}}$$

$$= \mathcal{I}^{\mathfrak{E}}_{\mathfrak{BULK}} + \mathcal{I}^{\mathfrak{E}}_{\mathfrak{GH}} + \mathcal{I}^{\mathfrak{E}}_{g} = \frac{4\pi\beta}{\kappa\ell^2\left(\frac{\ell^2\mu}{2} - r_+^3\right)}, \mathcal{E} = -\frac{\mathfrak{I}^2\partial\mathcal{I}^{\mathfrak{E}}}{\partial\mathcal{T}} = \frac{\mu}{2\mathfrak{G}}, \mathrm{F}_{\mathcal{S}\mathfrak{AdS}}$$

$$= \frac{4\pi}{\kappa\ell^2\left(\frac{\ell^2\mu}{2} - r_+^3\right)}, \mathcal{T}_{\mathcal{S}\mathfrak{AdS}} = \left(1+\frac{3r_+^2}{\ell^2}\right)$$

2.8.1. Modelo de Brown-York.

$$\int_{\partial\mathcal{M}}^{\delta} d^3\chi\hbar^{\alpha\beta}\mathcal{T}_{\alpha\beta}, ds^2 = \hbar_{\alpha\beta}\, d\chi^\alpha d\chi^\beta = -\mathrm{N}(\mathcal{R})dt^2 + \delta(\mathcal{R})d\Sigma_\kappa^2, \tau^{\alpha\beta} \equiv \frac{\frac{2}{\sqrt{-\hbar}\delta\mathcal{I}}}{\delta\hbar_{\alpha\beta}}, ds^2$$

$$= -\left(1-\frac{\mu}{r}+\frac{r^2}{\ell^2}\right)dt^2 + \left(1-\frac{\mu}{r}+\frac{r^2}{\ell^2}\right)^{-1} dr^2 + r^2 d\Omega^2, ds^2$$

$$= -\left(1-\frac{\mu}{\mathcal{R}}+\frac{\mathcal{R}^2}{\ell^2}\right)dt^2 + \mathcal{R}^2 d\Omega^2, \mathcal{I}$$

$$= \frac{1}{2\kappa\int_{\partial\mathcal{M}}^{\delta} d^4\chi\sqrt{-g}\,(\mathfrak{R}-2\Lambda)} + \frac{1}{\kappa\int_{\partial\mathcal{M}}^{\delta} d^3\chi\sqrt{-\hbar}\,\mathfrak{K}} - \frac{1}{\kappa\int_{\partial\mathcal{M}}^{\delta} d^3\chi\sqrt{-\hbar}\left(\frac{2}{\ell}+\frac{\mathcal{LR}}{2}\right)}, \tau^{\alpha\beta}$$

$$= \frac{1}{8\pi\mathfrak{G}\left(\mathfrak{K}_{\alpha\beta} - \hbar_{\alpha\beta}\mathfrak{K} - \frac{2}{\ell\hbar_{\alpha\beta}} + \mathcal{I}\mathfrak{S}_{\alpha\beta}\right)}, ds^2_{borde} = \frac{\mathfrak{R}^2}{\ell^2(-dt^2+\ell^2 d\Omega^2)}, ds^2_{dualidad}$$

$$= \gamma_{\alpha\beta}d\chi^\alpha d\chi^\beta = -dt^2 + \ell^2 d\Omega^2, \langle\tau^{\alpha\beta}_{dualidad}\rangle = \lim_{\mathcal{R}\to\infty}\frac{\mathcal{R}}{\ell}\tau_{\alpha\beta}$$

$$= \mu/16\pi\mathfrak{G}\ell^2\left(3\delta_\alpha^0\delta_\beta^0 + \gamma_{\alpha\beta}\right)$$

2.9. Modelo Hamiltoniano.

$$\mathfrak{I}[\mathfrak{g}_{\mu\nu},\Lambda_\mu\phi] = \int_{\partial\mathcal{M}}^{\delta} d^4\chi\sqrt{-g}\left[\frac{\mathcal{R}}{2\kappa} - \frac{1}{2(\partial\phi)^2} - \mathcal{V}(\phi)\right] + \frac{1}{\kappa\int_{\partial\mathcal{M}}^{\delta} d^3\chi\mathfrak{K}\sqrt{-\hbar}}, \mathcal{H}_\perp$$

$$= \frac{2\kappa}{\sqrt{g}\left[\pi^{ij}\pi_{ij} - \frac{1}{2\left(\pi_j^i\right)^2}\right]} - \frac{1}{2\kappa\sqrt{g}^{(3)}\mathcal{R}} + \frac{1}{2\left(\frac{\pi_{\phi^2}}{\sqrt{g}} + \sqrt{g}g^{ij}\phi_i\phi_j\right)} + \sqrt{g}\mathcal{V}(\phi), \mathcal{H}_i$$

$$= -\pi_j^i\big|_j + \pi_\varphi\phi_\psi d\omega, ds^2 = (\mathrm{N}^\perp)^2 dt^2 + g_{ij}\left(d\chi^i + \mathrm{N}^i dt\right)\left(d\chi^j + \mathrm{N}^j dt\right), \mathcal{H}[\xi]$$

$$= \int_{\partial\mathcal{M}}^{\delta} d^3\chi\left(\xi^\perp\mathcal{H}_\perp + \xi^i\mathcal{H}_i\right) + \mathfrak{Q}[\xi], \delta\mathfrak{Q}[\xi]$$

$$= \oint d^2\delta_\ell\left[\frac{\mathfrak{G}^{ijkl}}{2\kappa}\left(\xi^\perp\delta g_{ij}\big|_\kappa - \xi_\kappa^\perp\delta g_{ij}\right) + 2\xi_\kappa\delta\pi^{\kappa\ell} + \left(2\xi^\kappa\pi^{j\ell} - \xi^\ell\pi^{j\kappa}\right)\delta g_{j\kappa}\right.$$

$$\left. - \left(\sqrt{g\xi^\perp}g^{\ell j}\phi_j + \xi^\ell\pi_\phi\right)\delta\phi\right], \mathfrak{G}^{ijkl} \equiv \frac{1}{2\sqrt{g}(g^{i\kappa}g^{j\ell} + g^{i\ell}g^{j\kappa} - 2g^{ij}g^{\kappa\ell})}$$

$$\delta\mathcal{M} \equiv \delta\mathfrak{Q}[\partial_t] = \oint d^2\delta_\ell\left[\frac{\mathfrak{G}^{ijkl}}{2\kappa}\left(\xi^\perp\delta g_{ij}\big|_\kappa - \xi_\kappa^\perp\delta g_{ij}\right) - \sqrt{g}\xi^\perp g^{\ell j}\phi_j\delta\phi\right], \delta\mathcal{M} = \delta\mathcal{M}_{\mathfrak{G}} + \delta\mathcal{M}_\phi, \delta\mathcal{M}_{\mathfrak{G}}$$

$$= \oint d^2\delta_\ell\frac{\mathfrak{G}^{ijkl}}{2\kappa}\left(\xi^\perp\delta g_{ij}\big|_\kappa - \xi_\kappa^\perp\delta g_{ij}\right), \delta\mathcal{M}_\phi = -\oint d^2\delta_\ell\sqrt{g}\xi^\perp g^{\ell j}\phi_j\delta\phi$$

2.10. Termodinámica de agujeros negros cuánticos en espacios curvos.

2.10.1. Espacio – Tiempo de Rindler.

$$ds^2 = -dt^2 + d\chi^2 + d\gamma^2 + d\mathfrak{z}^2, ds^2 = -\alpha^2\rho^2 d\tau^2 + d\rho^2 + d\gamma^2 + d\mathfrak{z}^2, ds^2$$

$$= \mathrm{N}(r)dt_{\mathfrak{E}}^2 + \mathcal{H}(r)dr^2 + \delta(r)d\Sigma_\kappa^2, ds^2 = \frac{g_{rr}4\mathrm{N}}{[(\mathrm{N})']^2\left[\frac{\rho^2[(\mathrm{N})']^2}{4\mathrm{N}g_{rr}d\tau_{\mathfrak{E}}^2} + d\rho^2\right]\mathcal{T}} = \frac{1}{\beta}$$

$$= \frac{[(\mathrm{N})']^2}{4\pi\sqrt{\mathrm{N}^2 g_{rr}}\Big|_{\mathrm{H}}\mathcal{T}_{\delta c\hbar-\mathfrak{A}\mathfrak{d}\delta}} = \frac{1}{4\pi r_+\left(1+\frac{3r_+^2}{\ell^2}\right)\mathcal{T}_{\mathfrak{R}\eta-\mathfrak{A}\mathfrak{d}\delta}} = \frac{1}{4\pi r_+\left(1+\frac{3r_+^2}{\ell^2}-\frac{q^2}{4r_+^2}\right)}, ds^2$$

$$= \Omega(x)\left[-f(x)dt^2 + \frac{\eta^2 d\chi^2}{f(x)} + d\theta^2 + d\Sigma_\kappa^2\right], f(x)$$

$$= \frac{1}{\ell^2} + \alpha\left[\frac{1}{\nu^2} - 4 - \frac{\chi^2}{\nu^2\left(1+\frac{\chi^{-\nu}}{\nu} - 2 - \frac{\chi^\nu}{\nu} + 2\right)}\right] + \frac{\kappa\chi}{\Omega(x)}, f'\Omega(x)$$

$$= \frac{\alpha}{\eta^2} + 2\kappa + \kappa\nu\,\chi^\nu + \frac{1}{\chi^\nu} - 1, \mathcal{T} = \frac{f'}{4\pi\eta}\Big|_{\chi_\hbar}$$

$$= \frac{\frac{1}{f'}}{4\pi\eta\Omega(\chi_\hbar)\left(\frac{\alpha}{\eta^2} + 2\kappa + \kappa\nu\,\chi_\hbar^\nu + \frac{1}{\chi_\hbar^\nu} - 1\right)}$$

2.10.2. Transiciones de fase de agujeros negros cuánticos en espacios curvos.

$$ds^2 = \Omega(x)\left(-f(x)dt^2 + \frac{\eta^2 d\chi^2}{f(x)} + d\theta^2 + sin^2\theta d\varphi^2\right), \Omega(x) = \frac{1}{\eta^2(\chi-1)^2}, f(x)$$

$$= \frac{1}{\ell^2} + \frac{1}{3\alpha(\chi-1)^3} + \eta^2\chi(\chi-1)^2, \chi = 1 + \frac{1}{\eta r}, \chi = 1 - \frac{1}{\eta r}, \Omega(x)f(x) = \mathrm{F}(r)$$

$$= 1 - \frac{\mu}{r} + \frac{r^2}{\ell^2}, \mu = \alpha + \frac{3\eta^2}{3\eta^3}, \mathcal{I}[g_{\mu\nu}]$$

$$= \mathcal{I}_{bulk} + \mathcal{I}_{GH}$$

$$- \frac{1}{\kappa \int_{\partial\mathcal{M}}^{\delta} d^3\chi\sqrt{-\hbar}\left(\frac{2}{\ell} + \frac{\mathcal{R}\ell}{2}\right), \mathcal{E}_t^t - \mathcal{E}_{\langle\tau|\sigma|\rho\rangle}^{\langle\varphi|\chi|\psi\rangle} = 0 \Longrightarrow 0 = f'' + \frac{\Omega' f'}{\Psi} + 2\eta^2, \mathcal{E}_t^t}$$

$$+ \mathcal{E}_{\langle\tau|\sigma|\rho\rangle}^{\langle\varphi|\chi|\psi\rangle} = 0 \Longrightarrow 2\kappa\mathcal{V}(\phi) = -\frac{(f\Omega'' + f'\Omega')}{\Psi^4\eta^4} + \frac{2}{\Omega}, \mathcal{I}_{bulk}^{\mathfrak{E}}$$

$$= \frac{4\pi\beta}{\eta^3\kappa\ell^2\left[-\frac{1}{(\chi_\beta-1)^3} + \frac{1}{(\chi_\hbar-1)^3}\right]} = \frac{4\pi\beta}{\kappa\ell^2(r_\beta^3 - r_\hbar^3)}, ds^2 = \hbar_{\alpha\beta}d\chi^\alpha d\chi^\beta$$

$$= \Omega(x)[-f(x)dt^2 + d\theta^2 + sin^2\theta d\varphi^2], \eta_\alpha = \frac{\delta_\alpha^\chi}{\sqrt{g^{\chi\chi}\partial_\chi\hbar_{\alpha\beta}}}, \mathcal{I}_{\mathrm{GH}}^{\mathfrak{E}}$$

$$= -\frac{2\pi\beta}{\kappa\left[-\frac{6}{\ell^2\eta^4(\chi-1)^3} - \frac{4}{\eta(\chi_\beta-1)} - \left(\alpha + \frac{3\eta^4}{\eta^3}\right)\right]\Bigg|_{\chi_\beta}} = -\frac{2\pi\beta}{\kappa\left(\frac{6r_\beta^3}{\ell^2} + 4r_\beta - 3\mu\right)}, \mathcal{I}_g^{\mathfrak{E}}$$

$$= \frac{2\pi\beta}{\kappa\left[\frac{4}{\ell^2\eta^4(\chi_\beta-1)^3} + \frac{4}{\eta(\chi_\beta-1)} - 2\mu\right]} = 2\pi\beta/\kappa\left(\frac{4r_\beta^3}{\ell^2} + 4r_\beta - 2\mu\right)$$

$$\mathcal{J}^{\mathfrak{E}} = \mathcal{J}^{\mathfrak{E}}_{bulk} + \mathcal{J}^{\mathfrak{E}}_{\mathrm{GH}} + \mathcal{J}^{\mathfrak{E}}_{g} = \frac{4\pi\beta}{\kappa\ell^2\left[\frac{1}{\eta^3(\chi_\hbar - 1)^3} + \frac{\mu\ell^2}{2}\right]} = \frac{4\pi\beta}{\kappa\ell^2\left(-r_\hbar^3 + \frac{\mu\ell^2}{2}\right)}, \mathcal{J}^{\mathfrak{E}}_{\langle\varphi|\phi|\psi\rangle^{\langle\sigma|\tau|\rho\rangle}}$$

$$= \int_{\partial\mathcal{M}}^{\delta} d^3\chi^3\sqrt{\hbar^{\mathfrak{E}}}\left(\frac{\langle\varphi|\phi|\psi\rangle^{\langle\sigma|\tau|\rho\rangle}}{2\ell} - \ell_\nu/6\ell\langle\varphi|\phi|\psi\rangle^{\langle\sigma|\tau|\rho\rangle^4}\right)$$

$$= \frac{4\pi\beta}{\kappa\left[-\nu^2 - \frac{1}{4\ell^2\eta^3(\chi_\beta - 1)} + \nu^2 - \frac{1}{3\ell^2\eta^3}\right]}, \mathcal{J}^{\mathfrak{E}}_{bulk} + \mathcal{J}^{\mathfrak{E}}_{surf} + \mathcal{J}^{\mathfrak{E}}_{g}$$

$$= -\frac{1}{\mathcal{T}\left(\frac{\Lambda\Gamma}{4\mathfrak{G}}\right)} + \frac{4\pi\beta}{\kappa\left[\nu^2 - \frac{1}{4\ell^2\eta^3(\chi_\beta - 1)} + 12\eta^2\ell^2 + 4\alpha\ell^2 - 4\nu^2 + \frac{4}{12\ell^2\eta^3}\right]}, \mathcal{J}^{\mathfrak{E}}$$

$$= \beta\left(-\frac{\Lambda\Gamma}{4\mathfrak{G}} + \frac{4\pi}{\kappa}3\eta^2 + \frac{\alpha}{3\eta^3}\right), \mathcal{M} = \frac{1}{2\mathfrak{G}\left(\alpha + \frac{3\eta^2}{3\eta^3}\right)}, \mathcal{T}$$

$$= \frac{\left.\frac{f'(x)}{4\pi\eta}\right|_{\chi=\chi_\hbar} 1}{4\pi\eta\Omega(\chi_\hbar)\left[\frac{\alpha}{\eta^2} + 2 + \nu\chi_\hbar^\nu + 1\frac{\chi^\nu}{\chi_\hbar} - 1\right]}, \delta = \frac{\mathrm{A}}{4G} = 4\pi\eta\Omega(\chi_\hbar)/4G$$

$$\frac{\frac{\partial\mathcal{M}}{\partial\eta}d\eta}{d\chi_\hbar} = \mathcal{T}\left(\frac{\partial\mathcal{S}}{\partial\chi_\hbar} + \frac{\frac{\partial\mathcal{S}}{\partial\eta}d\eta}{d\chi_\hbar}\right), \mathcal{M} = -\frac{1}{2G\left(\alpha + \frac{3\eta^2}{3\eta^3}\right)}, \mathcal{T} = \frac{1}{4\pi\eta\Omega(\chi_\hbar)\left[\frac{\alpha}{\eta^2} + 2 + \nu\chi_\hbar^\nu + \frac{1}{\chi_\hbar^\nu} - 1\right]}$$

2.10.3. Solitón - AdS para agujeros negros cuánticos en espacios curvos.

$$\mathcal{I}[g_{\mu\nu}] = \int_{\mathcal{M}}^{\delta} d^4\chi(\mathcal{R} - 2\Lambda)\sqrt{-g} + 2\int_{\partial\mathcal{M}}^{\delta} d^3\chi\mathfrak{K}\sqrt{-\hbar} - \int_{\partial\mathcal{M}}^{\delta}\frac{d^3\chi 4}{\ell\sqrt{-\hbar}}, ds^2$$

$$= -\left(-\frac{\mu_\beta}{r} + \frac{r^2}{\ell^2}\right)dt^2 + \left(-\frac{\mu_\beta}{r} + \frac{r^2}{\ell^2}\right)^{-1}dr^2 + \frac{r^2}{\ell^2}(d\chi_1^2 d_2^2\chi), \mathcal{I}_\beta^\xi$$

$$= \frac{2\mathcal{L}\mathcal{L}_\beta\beta_\varsigma}{\ell^4\left(-r_\hbar^3 + \frac{\mu_\beta\ell^2}{2}\right)} = -\frac{\mathcal{L}\mathcal{L}_\beta\beta_\varsigma r_\hbar^3}{\ell^4}, \mathfrak{I} = \beta_\varsigma^{-1} = \left.\frac{(-g_{tt})'}{4\pi}\right|_{r=r_\hbar} = \frac{3r_\hbar}{4\pi\ell^2}, \mathfrak{E} = -\frac{\mathfrak{T}^2\partial\mathcal{I}_\beta^{\mathfrak{E}}}{\partial\mathfrak{T}}$$

$$= \frac{2\mathcal{L}\mathcal{L}_\beta\mu_\beta}{\ell^4}, \mathfrak{S} = -\frac{\partial\left(\mathcal{I}_\beta^{\mathfrak{E}}\mathfrak{T}\right)}{\mathfrak{T}} = \frac{\mathcal{L}\mathcal{L}_\beta r_\hbar^2}{4\ell^2\mathfrak{G}} = \mathcal{A}/4G$$

$$ds^2_{dualidad} = \frac{\ell^2}{\mathcal{R}^2} ds^2 = \gamma_{\alpha\beta} d\chi^\alpha d\chi^\beta = -dt^2 + d\chi_1^2 + d\chi_2^2, \langle \tau_{\alpha\beta}^{dualidad} \rangle$$

$$= \lim_{\mathcal{R}\to\infty} \frac{\mathcal{R}}{\ell} \tau_{\alpha\beta} = \frac{\mu_\beta}{16\pi \mathfrak{G}_{\mathrm{N}} \ell^2} [3\delta_\alpha^0 \delta_\beta^0 + \gamma_{\alpha\beta}], \mathfrak{E} = \mathcal{Q}_{\xi t}$$

$$= \int d\Sigma^i \tau_{ij} \xi^j = \frac{\mathcal{L}\mathcal{L}_\beta}{\ell^2 \kappa} \left[\mu_\beta + \frac{\ell^2}{4\mathcal{R}} + \mathcal{O}(\mathcal{R}^{-2}) \right], ds^2$$

$$= \frac{r^2}{\ell^2} d\tau^2 + \left(-\frac{\mu_\delta}{r} + \frac{r^2}{\ell^2} \right)^{-1} dr^2 + \left(-\frac{\mu_\delta}{r} + \frac{r^2}{\ell^2} \right) d\theta^2 + \frac{r^2}{\ell^2} d\chi_2^2 - \frac{\mu_\delta}{r} + \frac{r_\delta^2}{\ell^2}, \mathcal{L}_\delta$$

$$= \frac{4\varpi \sqrt{g_{\theta\theta} g_{rr}}}{(g_{\theta\theta})'} \bigg|_{r=r_\delta} = \frac{4\pi\ell^2}{3r_\delta}, \mathcal{I}_\delta^\xi = \frac{\mathcal{L}\mathcal{L}_\delta \beta_\delta \mu_\delta}{\ell^2} \Delta\mathcal{I} = \mathcal{I}_\beta^\epsilon - \mathcal{I}_\delta^\xi$$

$$= \frac{\mathcal{L}}{2\pi\kappa\ell^4} \left(\frac{4\pi\ell^2}{3} \right)^3 \mathcal{L}_\beta \beta_\varsigma (\mathcal{L}_\delta^{-3} - \xi\beta_\varsigma^{-3}) = \frac{\frac{\mathcal{L}}{2\kappa\ell^4} \left(\frac{4\pi\ell^2}{3} \right)^3 \mathcal{L}_\beta \beta_\varsigma \left(\frac{1}{\mathcal{L}_\delta^3} - \mathcal{T}^3 \right) \mathcal{A}}{\mathcal{T}\ell^3}$$

$$= \mathcal{L}/\ell (4\pi/3)^2 \mathcal{L}_\delta \mathcal{T}$$

$$\mathcal{I}[g_{\mu\nu}, \phi] = \int_{\mathcal{M}}^{\delta} d^4\chi \sqrt{-g} \left[\mathcal{R} - \frac{(\partial\phi)^2}{2} - \mathcal{V}(\phi) \right] + 2 \int_{\partial\mathcal{M}}^{\delta} d^3\chi \mathfrak{K} \sqrt{-\hbar}, \mathcal{V}(\phi)$$

$$= \frac{\Lambda(\nu^2 - 4)}{3\nu^2 \left[\nu - \frac{1}{\nu} + 1e^{-\phi\ell_\nu(\nu+1)} + \nu + \frac{1}{\nu} - 2e^{\phi\ell_\nu(\nu-1)} + 4\nu^2 - \frac{1}{\nu^2} - 4e^{\phi\ell_\nu} \right]}$$

$$+ \frac{2\alpha}{\nu^2 \left[\nu - \frac{1}{\nu} + 2\sin\hbar\,\phi\ell_\nu(\nu+1) - \nu + \frac{1}{\nu} - 2\sin\hbar\,\phi\ell_\nu(\nu-1) + 4\nu^2 - \frac{1}{\nu^2} - 4\sin\hbar\,\phi\ell_\nu \right]}$$

$$ds^2 = \frac{\mathfrak{R}^2}{\ell^2[-dt^2 + d\chi_1^2 + d\chi_2^2]\mathfrak{R}^2} \equiv \frac{1}{\eta^2(\chi-1)^2}, ds^2_{dualidad} = \frac{\ell^2}{\mathfrak{R}^2} ds^2 = \gamma_{\alpha\beta} d\chi^\alpha d\chi^\beta$$

$$= -dt^2 + d\chi_1^2 + d\chi_2^2, \mathcal{I}_{\mathfrak{E}\mathfrak{H}}^\beta = \beta_\varsigma \left(-\frac{\mathcal{A}\mathcal{T}}{4\mathcal{G}_\mathcal{N}} + \frac{\frac{2\mathcal{L}\mathcal{L}_\beta}{\ell^2}\alpha}{3\eta^3} \right) = -\frac{\mathcal{L}\mathcal{L}_\beta \alpha \beta_\varsigma}{3\ell^2\eta^3}, \mathcal{A}$$

$$= \frac{\mathcal{L}\mathcal{L}_\beta \Omega(\chi_\hbar)}{\ell^2}, \mathfrak{T} = \frac{\alpha}{4\pi\eta^3\Omega}, \mathcal{M}_\mathrm{\beta} = \frac{2\mathcal{L}\mathcal{L}_\beta \mu_\beta}{\ell^2}, \mu_\beta = \frac{\alpha}{3\eta^3}, ds^2$$

$$= \Psi_\delta(\gamma) \left[-\frac{d\tau^2}{\ell^2} + \frac{\lambda^2 d\chi^2}{f(x) d\theta^2} + \frac{d\chi_2^2}{\ell^2} \right] \Psi_\delta(\gamma) = \frac{9\gamma^2}{\lambda^2(\gamma^3-1)^2}, \mathcal{L}_\delta = \frac{4\pi\lambda}{f'} \bigg|_{\gamma=\gamma_\delta}$$

$$= \frac{4\pi\lambda^3\gamma_\delta}{\alpha}$$

$$\mathfrak{I}^{\mathfrak{E}}_{soliton} = -\frac{\mathcal{L}\beta_\delta\Omega_\delta(\chi_\delta)}{4\ell^2\mathfrak{G}_{\mathrm{N}}} + \frac{\frac{2\mathcal{L}\mathcal{L}_\delta\beta_\delta}{\ell^2}\alpha}{3\lambda^3} = -\frac{\mathcal{L}\mathcal{L}_\delta\beta_\delta}{\ell^2\left(\frac{\alpha}{3\lambda^3}\right)}, \mathrm{M}_{soliton} = -\frac{\mathcal{L}\mathcal{L}_\delta\mu_\delta}{\ell^2}, \mu_\delta = \frac{\alpha}{3\lambda^3}, \mathfrak{J}^\phi$$

$$= -\int d^3\chi\sqrt{-\hbar}\left(\frac{\phi^2}{2\ell} - \frac{\ell_\nu}{6\ell\phi^3}\right)\tau^\phi_{\alpha\beta} = -\frac{\frac{2}{\sqrt{-\hbar}}\delta\mathfrak{J}^\phi}{\delta\hbar^{\alpha\beta}}, \tau_{\alpha\beta}$$

$$= -\frac{1}{\kappa\left(\kappa_{\alpha\beta} - \hbar_{\alpha\beta}\kappa + \frac{2}{\ell}\hbar_{\alpha\beta} - \beth\mathbb{E}_{\alpha\beta}\right)} - \frac{\hbar_{\alpha\beta}}{\ell\left(\frac{\phi^2}{2} - \frac{\ell_\nu}{6}\phi^3\right)}, \tau_{tt}$$

$$= \frac{\alpha(\chi-1)}{3\lambda^2\ell} + \mathcal{O}[(\chi-1)^2], \tau_{\theta\theta} = \frac{2\alpha(\chi-1)}{3\lambda^2\ell} + \mathcal{O}[(\chi-1)^2], \tau_{\chi_2\chi_2}$$

$$= -\frac{\alpha(\chi-1)}{3\lambda^2\ell} + \mathcal{O}[(\chi-1)^2], ds^2_{dualidad} = \frac{\ell^2}{\mathcal{R}^2}ds^2$$

$$= -d\tau^2 + d\theta^2 + d\chi_2^2, \langle\tau^{dualidad}_{\alpha\beta}\rangle$$

$$= \lim_{\mathcal{R}\to\infty}[-1/\lambda\ell(\chi-1)]\tau_{\alpha\beta} = \frac{1}{\ell^2\left(\frac{\alpha}{3\lambda^3}\right)}[-3\delta^\theta_\alpha\delta^\theta_\beta + \gamma_{\alpha\beta}]$$

$$\mathcal{M} = \oint_\Sigma^{\eth} d^2\gamma\sqrt{\sigma}\,m^\alpha\tau_{\alpha\beta}\xi^\beta = \frac{\mathcal{L}\mathcal{L}_\delta f^{\frac{1}{2}}\Psi}{\sqrt{-g_{\tau\tau}}(\partial_\tau)^i\tau_{ij}(\partial_\tau)^j} = -\frac{\mathcal{L}\mathcal{L}_\delta}{\ell^2\left[\frac{\alpha}{3\lambda^3} + \mathcal{O}(\chi-1)\right]}, ds^2 = \sigma_{ij}d\chi^i d\chi^j$$

$$= \Omega(x)[f(x)d\theta^2 + d\chi_2^2/\ell^2]$$

$$r_\beta^2 = \frac{\Omega(\chi_\hbar,\eta)}{\ell^2}, r_\delta^2 = \frac{\Omega(\chi_\delta,\lambda)}{\ell^2}, \mathfrak{E} = \mathfrak{M}_{\beta\hbar} - \mathcal{M}_{soliton} = \frac{\mathcal{L}\mathcal{L}_\beta}{\ell^2}(2\mu_\beta + \mu_\delta), \Delta\Gamma = \beta_\varsigma^{-1}(\mathcal{I}^{\mathfrak{E}}_{\mathcal{BH}} - \mathfrak{I}^{\mathfrak{E}}_{soliton})$$

$$= \frac{\mathcal{T}\mathfrak{L}\alpha}{3\ell^2\left(\frac{\mathcal{L}_\delta\beta_\delta}{\lambda^3} - \frac{\mathcal{L}_\beta\beta_\varsigma}{\eta^3}\right)\Delta\Gamma} = \frac{4\pi\mathcal{L}\mathcal{L}_\delta}{3\ell^2\left[\frac{\Omega(\lambda,\chi_\delta)}{\mathcal{L}_\delta} - \mathrm{T}\Omega(\eta,\chi_\hbar)\right]}$$

$$= \frac{4\pi\mathcal{L}}{3\ell^2\Omega(\lambda,\chi_\delta)\left(1 - \frac{r_\beta^3}{r_\delta^3}\right)}, \frac{\mathcal{A}}{\mathcal{T}\ell^3} = \frac{\frac{\alpha\mathcal{L}}{4\pi\ell^5}\beta_\varsigma^2\mathcal{L}_\delta}{\eta^3} = \frac{\mathfrak{L}\mathcal{L}}{\ell}\left(\frac{\lambda}{\eta}\right), \mathcal{L}$$

$$= \frac{16\pi^2}{\alpha^2\ell^4}\left[\frac{9\chi_\hbar^2}{(\chi_\hbar^3 - 1)^2}\right]^4, \frac{\mathcal{A}}{\mathcal{T}\ell^3} = \frac{\mathfrak{L}\mathcal{L}(\alpha,\ell)}{\ell}r_\beta/r_\delta$$

2.11. Modelo Computacional de un agujero negro cuántico en espacios curvos.

$$\delta = \mathfrak{N} \int dt \mathcal{T}r \left(\frac{1}{2\sum_{\mathcal{J}} (\mathcal{D}_t \chi_\iota)^2 - \frac{m^2}{2} \sum_\iota \chi_\iota^2 + \frac{\lambda}{4} \sum_{\iota \neq \mathcal{J}} [\chi_\iota \chi_{\mathcal{J}}]^2} \right), \delta$$

$$= \mathfrak{N} \int_0^\beta dt \mathcal{T}r \left(\frac{1}{2(\mathcal{D}_t \chi_\iota)^2 + \frac{m^2}{2} \chi_\iota^2 - \frac{\lambda}{4} [\chi_\iota \chi_{\mathcal{J}}]^2} \right)$$

$$\delta = \int dt \mathcal{T}r \left(\frac{1}{2} (\mathcal{D}_t \chi_\iota)^2 - \frac{m^2}{2} \chi_\iota^2 + \frac{g^2}{4} [\chi_\iota \chi_{\mathcal{J}}]^2 \right), \delta_\varkappa$$

$$= \mathbb{N}_\alpha \sum_{t=1}^{\eta_t} \mathcal{T}r \left(\frac{1}{2(\mathcal{D}_t \chi_\iota)_{\iota,t}^2 + \frac{m^2}{2} \chi_{\iota,t}^2 - \frac{\lambda}{4} [\chi_{\iota,t} \chi_{\mathcal{J},t}]^2} \right), (\mathcal{D}_t \chi_\iota)_{\iota,t}$$

$$= \frac{1}{\alpha \left(-\frac{1}{2} \mathfrak{U}_t \mathfrak{U}_{t+\alpha} \chi_{\iota,t+2\alpha} \mathfrak{U}_{t+\alpha}^\dagger \mathfrak{U}_t^\dagger + 2\mathfrak{U}_t \chi_{\iota,t+\alpha} \mathfrak{U}_t^\dagger - \frac{3}{2} \chi_{\iota,t} \right)}, \langle f \rangle$$

$$\equiv \int \frac{d\chi d\mathfrak{U} f(\chi, \mathcal{U}) e^{-\delta_\varkappa(\chi, \mathcal{U})}}{\int d\chi d\mathfrak{U}\, e^{-\delta_\varkappa(\chi, \mathcal{U})}} = \lim_{\mathfrak{K} \to \infty} \frac{1}{\kappa} \sum_{\kappa=1}^{\mathfrak{K}} f\left(\chi^{(\kappa)} \mathfrak{U}^{(\kappa)}\right), \langle \kappa \rangle = \langle \frac{1}{2} \sum_\iota \frac{\chi_\iota \partial \mathcal{V}}{\partial \chi_\iota} \rangle, \mathcal{E}$$

$$= \langle \frac{1}{\beta} \int_0^\beta dt \left(\mathcal{V} + \frac{1}{2} \sum_\iota \frac{\chi_\iota \partial \mathcal{V}}{\partial \chi_\iota} \right) \rangle$$

$$= \frac{\mathrm{N}}{\beta \int_0^\beta dt \left(m^2 \chi_\iota^2 - \frac{3\lambda}{4} [\chi_\iota, \chi_j]^2 \right), \mathcal{E}_{\boxplus} = \langle \frac{\mathcal{N}}{\eta_t \sum_{t=1}^{\eta_t} \left(m^2 \chi_{\iota,t}^2 - \frac{3\lambda}{4} [\chi_{\iota,t}, \chi_{j,t}]^2 \right)} \rangle}, \mathcal{F}(\mathcal{T}, \eta_t)$$

$$= \xi + \sum_{\iota=1}^{\eta\rho} \alpha_\iota \left(\frac{1}{\mathcal{T} \eta_t} \right), \mathcal{F}(\mathcal{T}, \eta_t) = \xi(\mathfrak{T}) + \sum_\iota^{\eta\rho} \alpha_\iota \left(\frac{1}{\eta_t} \right)^\iota$$

$$\widehat{\mathcal{H}} = \mathcal{T}r\left(\frac{1}{2}\widehat{\wp}_\iota^2 + \frac{m^2}{2}\hat{\chi}_\iota^2 - \frac{g^2}{4}\left[\hat{\chi}_\iota\hat{\chi}_\mathcal{J}\right]^2\right), \widehat{\wp}_\iota = \sum_{\alpha=1}^{\mathrm{N}^2-1}\widehat{\wp}_\iota^\alpha\tau_\alpha, \hat{\chi}_\iota = \sum_{\alpha=1}^{\mathrm{N}^2-1}\hat{\chi}_\iota^\alpha\tau_\alpha, \left[\hat{\chi}_{\iota_\alpha}\hat{\chi}_{\mathcal{J}_\beta}\right] = \iota\delta_{\mathcal{J}\mathcal{J}}\delta_{\alpha\beta}, \widehat{\mathcal{H}}$$

$$= \mathcal{T}r\left(\frac{1}{2}\widehat{\wp}_\iota^2 - \frac{g^2}{4}\left[\hat{\chi}_\iota\hat{\chi}_\mathcal{J}\right]^2 + \frac{g}{2}\hat{\bar{\psi}}\gamma^\iota\left[\hat{\chi}_\iota\hat{\psi}\right] - \frac{3\iota\mu}{4}\hat{\bar{\psi}}\hat{\psi} + \frac{\mu^2}{2}\hat{\chi}_\iota^2\right) - (\mathrm{N}^2-1)\mu$$

$$= \mathcal{T}r\left(\frac{1}{2}\widehat{\wp}_\iota^2 - \frac{g^2}{4}[\hat{\chi}_1\hat{\chi}_2]^2 + \frac{g}{2}\xi[-\hat{\chi}_1 - \iota\hat{\chi}_2\xi] + \frac{g}{2}\hat{\xi}^\dagger[-\hat{\chi}_1 - \iota\hat{\chi}_2\hat{\xi}^\dagger] + \frac{3\mu}{2}\hat{\xi}^\dagger\hat{\xi}\right.$$

$$\left. + \frac{\mu^2}{2}\hat{\chi}_\iota^2\right) - (\mathrm{N}^2-1)\mu, \widehat{\mathcal{H}}$$

$$= \mathcal{T}r\left(\widehat{\wp_Z}\widehat{\wp}_Z^\dagger + \frac{g^2}{4}\left[\widehat{\mathcal{Z}_\wp}\hat{\mathcal{Z}}_\wp^\dagger\right]^2 - \frac{g}{\sqrt{2}\hat{\xi}\left[\hat{\mathcal{Z}}_\wp^\dagger\hat{\xi}\right]} - \frac{g}{\sqrt{2}\hat{\xi}^\dagger\left[\hat{\mathcal{Z}}\hat{\xi}^\dagger\right]} + \frac{3\mu}{2}\hat{\xi}^\dagger\hat{\xi} + \mu^2\hat{\mathcal{Z}}\hat{\mathcal{Z}}^\dagger\right), \widehat{\mathcal{H}}$$

$$= \sum_{\alpha,\iota}\left(\frac{1}{2\widehat{\wp}_{\iota\alpha}^2} + \frac{m^2}{2}\hat{\chi}_{\iota\alpha}^2\right) + \frac{g^2}{4}\sum_{\gamma,\mathcal{J},\mathcal{J}}\langle\sum_{\alpha,\beta}f_{\alpha\beta\gamma}\hat{\chi}_\mathcal{J}^\alpha\hat{\chi}_\mathcal{J}^\beta\rangle^2, \widehat{\mathcal{H}}$$

$$= m\sum_{\alpha,\iota}\left(\widehat{\eta_{\iota\alpha}} + \frac{1}{2}\right) + \frac{g^2}{16m^2}\sum_{\gamma,\mathcal{J},\mathcal{J}}\left(\sum_{\alpha,\beta}f_{\alpha\beta\gamma}\left(\hat{\alpha}_{\iota\alpha} + \hat{\alpha}_{\iota\alpha}^\dagger\right)\left(\hat{\alpha}_{j\beta} + \hat{\alpha}_{j\beta}^\dagger\right)\right)^2, [\widehat{\mathcal{H}}, \mathfrak{G}_\alpha], \widehat{\mathcal{H}}'$$

$$= \widehat{\mathcal{H}} + c\sum_\alpha\mathfrak{G}_\alpha^2, \widehat{\mathcal{H}}$$

$$= \sum_\alpha\left(\frac{\widehat{\wp}_{1\alpha}^2}{2} + \frac{\widehat{\wp}_{2\alpha}^2}{2} + \mu^2\frac{\hat{\chi}_{1\alpha}^2}{2} + \mu^2\frac{\hat{\chi}_{2\alpha}^2}{2} + \frac{3\mu}{2}\hat{\xi}_\alpha^\dagger\hat{\xi}_\alpha\right)$$

$$+ g^2\sum_{\alpha\neq\beta}\hat{\chi}_{1\alpha}^2\hat{\chi}_{2\beta}^2$$

$$- 2g^2\sum_{\alpha<\beta}\hat{\chi}_{1\alpha}\hat{\chi}_{1\beta}\hat{\chi}_{2\alpha}\hat{\chi}_{2\beta} + \frac{\iota g}{\sqrt{2}}\sum_{\alpha\beta\gamma}\epsilon_{\alpha\beta\gamma}\left[(-\hat{\chi}_{1\alpha} - \iota\hat{\chi}_{2\alpha})\hat{\xi}_\beta^\dagger\hat{\xi}_\gamma^\dagger\right.$$

$$\left. + (-\hat{\chi}_{1\alpha} + \iota\hat{\chi}_{2\alpha})\hat{\xi}_\beta\hat{\xi}_\gamma\right] - 3\mu$$

$$\widehat{\mathcal{H}}' = \widehat{\mathcal{H}} + c\sum_\alpha\mathfrak{G}_\alpha^2 + c'^{(\mathfrak{M}-\mathfrak{J})^2}, \mathfrak{E}_0 \leq \mathfrak{E}_\lambda = \frac{\langle\psi(\theta_\iota)|\mathcal{H}|\psi(\theta_\iota)\rangle}{\langle\psi(\theta_\iota)|\psi(\theta_\iota)\rangle}, \mathfrak{E}_0 \equiv \langle\psi_\theta|\widehat{\mathcal{H}}|\psi_\theta\rangle$$

$$= \int d\chi|\psi_\theta(\chi)|^2 \cdot \frac{\langle\chi|\widehat{\mathcal{H}}|\psi_\theta\rangle}{\psi_\theta(\chi)} = \mathbb{E}_{\chi\sim|\psi_\theta|^2}[\epsilon_\theta(\chi)], \nabla_\theta\mathfrak{E}_0$$

$$= \mathbb{E}_{\chi\sim|\psi_\theta|^2}\left[\nabla_{\theta\epsilon_\theta}(\chi)\right] + \mathbb{E}_{\chi\sim|\psi_\theta|^2} \times [\epsilon_\theta(\chi)\nabla_\theta\ln|\psi_\theta|^2], \theta' = \theta - \beta\nabla_\theta\mathfrak{E}_0$$

$$\wp_\theta(\chi) = \rho(\chi_1; \mathrm{F}_\theta^0)\rho[\chi_1; \mathrm{F}_\theta^1(\chi_1)]\rho[\chi_3; \mathrm{F}_\theta^2(\chi_1, \chi_2)], \mathrm{F}_\theta^\iota$$

$$= \Lambda_\theta^{\iota,m} \odot \tan\hbar \odot \Lambda_\theta^{\iota,m-1} \odot \tan\hbar \odot \cdots \odot \Lambda_\theta^{\iota,2} \odot \tan\hbar \odot \Lambda_\theta^{\iota,1}, \Lambda_\theta^{\iota,\alpha}(\vec{\chi})$$

$$= \mathcal{M}_\theta^{\iota,\alpha}\vec{\chi} + \vec{\beta}_\theta^{\iota,\alpha}, \widehat{\mathcal{H}}'$$

$$= \widehat{\mathcal{H}}$$

$$+ c\sum_\alpha \widehat{\mathfrak{G}}_\alpha^2, \widehat{\wp}|\psi\rangle = \int d\mathcal{U}, \hat{\mathcal{U}}\left|\psi\rangle, \langle\hat{\mathcal{O}}\rangle_\varsigma = \frac{\langle\psi|\widehat{\wp}\hat{\mathcal{O}}|\psi\rangle}{\langle\psi|\hat{\mathcal{O}}|\psi\rangle}, \langle\psi|\widehat{\wp}\hat{\mathcal{O}}|\psi\rangle\right.$$

$$= \int d\chi\, \langle\psi|\widehat{\wp}|\psi\rangle\langle\psi|\hat{\mathcal{O}}|\psi\rangle = \int d\mathcal{U}d\chi\, \psi^{\circledast}(\mathcal{U}\chi\mathcal{U}^\dagger)\langle\chi|\hat{\mathcal{O}}|\psi\rangle$$

$$= \mathbb{E}_{\mathcal{U},\chi\sim\langle\psi\rangle^2}\left[\frac{\langle\chi|\hat{\mathcal{O}}|\psi\rangle}{\psi(\chi)}\psi^{\circledast}(\mathcal{U}\chi\mathcal{U}^\dagger)/\psi^{\circledast}(\chi)\right]$$

$$\mathfrak{R}_\gamma(\theta) = \exp\left(-\frac{\iota\theta}{2}\right) = \begin{pmatrix} \cos\frac{\theta}{2} & \cdots & -\sin\frac{\theta}{2} \\ \vdots & \ddots & \vdots \\ \sin\frac{\theta}{2} & \cdots & -\cos\frac{\theta}{2} \end{pmatrix}, \hat{\alpha}_\iota$$

$$= \hat{\jmath}_1 \otimes \cdots \otimes \hat{\jmath}_{\iota-1} \otimes \begin{bmatrix} 0 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \otimes \hat{\jmath}_{\iota+1} \bigotimes \cdots \otimes \hat{\jmath}_6, \hat{\alpha}_\iota$$

$$= \hat{\jmath}_1 \otimes \cdots \otimes \hat{\jmath}_{\iota-1} \otimes \left\|\begin{vmatrix} 0 & 1 & 0 \\ 0 & \sqrt{2} & 1 \\ 1 & 0 & \sqrt{2} \end{vmatrix}\right\| \otimes \hat{\jmath}_{\iota+1} \bigotimes \cdots \otimes \hat{\jmath}_6, c_1$$

$$= \hat{\jmath}_{64} \otimes \begin{pmatrix} 0 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix}, c_2$$

$$= \hat{\jmath}_{64} \otimes \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -1 \end{pmatrix} \otimes \begin{pmatrix} 0 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix}, c_3$$

$$= \hat{\jmath}_{64} \otimes \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -1 \end{pmatrix} \otimes \begin{pmatrix} 0 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{pmatrix}$$

$$\hat{\xi}_\alpha = \left\|\begin{matrix} \sigma_z \otimes \cdots & \sigma_z \otimes \\ \otimes \sigma_z & \otimes \end{matrix}\right\|_{\alpha-1} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \otimes ך \otimes \cdots \otimes ך$$

$$\widehat{\mathfrak{G}}_\alpha = \iota\sum_{\beta,\gamma,\iota} f_{\alpha\beta\gamma}\hat{\alpha}_{\iota\beta}^\dagger\hat{\alpha}_{\iota\gamma}, \widehat{\mathfrak{G}}_\alpha = \left(\otimes_{\iota\beta}\,|0\rangle_{\iota\beta}\right), \widehat{\mathfrak{G}}_\alpha = \imath\sum_{\beta\gamma} f_{\alpha\beta\gamma}\left(\hat{\alpha}_{1\beta}^\dagger\hat{\alpha}_{1\gamma} + \hat{\alpha}_{2\beta}^\dagger\hat{\alpha}_{2\gamma} + \hat{\xi}_\beta^\dagger\hat{\xi}_\gamma\right)$$

$$\hat{\alpha}_{\iota\alpha}^{\dagger} = \sqrt{\frac{m}{2}}\widehat{\chi_{\iota\alpha}} - \frac{\iota\widehat{\wp_{\iota\alpha}}}{\sqrt{2m}}, \widehat{\alpha_{\iota\alpha}} = \sqrt{\frac{m}{2}}\widehat{\chi_{\iota\alpha}} + \frac{\iota\widehat{\wp_{\iota\alpha}}}{\sqrt{2m}}, \hat{\alpha}_{\iota\alpha}|0\rangle_{\iota\alpha}, |\eta\rangle_{\iota\alpha} = \frac{\left(\hat{\alpha}_{\iota\alpha}^{\dagger}\right)^{\eta}}{\sqrt{\eta!}\,|0\rangle_{\iota\alpha}}, |\{\eta_{\iota\alpha}\}\rangle = \otimes_{\iota\alpha}|\eta\rangle_{\iota\alpha}, \hat{\alpha}_{\daleth}^{\dagger}$$

$$= \sum_{\eta=0}^{\Lambda-2}\sqrt{\eta+1}\,|\eta+1\rangle\langle\eta| = \sum_{\eta=0}^{\Lambda-2}\sqrt{\eta+1}\,|\eta\rangle\langle\eta+1|, \hat{\eta}_{\daleth}$$

$$= \sum_{\eta=0}^{\Lambda-1}\eta|\eta\rangle\langle\eta|, [\hat{\mathfrak{H}}_{\daleth}, \hat{\mathfrak{G}}_{\daleth}], |\eta\rangle = |\beta_0\rangle|\beta_1\rangle\cdots\beta_{\kappa+1}\rangle, |\eta+1\rangle\langle\eta|$$

$$= \otimes_{\ell=0}^{\kappa-1}\ (|\beta_{\ell}'\rangle\langle\beta_{\ell}|), |0\rangle\langle 0| = \boxtimes_2 - \frac{\sigma_{z\blacksquare}}{2}, |1\rangle\langle 1|$$

$$= \boxtimes_2 - \frac{\sigma_{z\blacksquare}}{2}, \Big|0\rangle\langle 1| = \sigma_x + \frac{\iota\sigma_{\gamma}}{2}, 1\rangle\langle 0| = \sigma_x + \frac{\iota\sigma_{\gamma}}{2}$$

$$\hat{\mathcal{M}} = \sum_{\alpha}\left(\iota(\hat{\mathcal{Z}}_{\alpha}\hat{\wp}_{\mathcal{Z}\alpha}^{\dagger} - \hat{\wp}_{\mathcal{Z}\alpha}\hat{\mathcal{Z}}_{\alpha}^{\dagger}) - \frac{1}{2}\hat{\xi}_{\alpha}^{\dagger}\hat{\xi}_{\alpha}\right), \hat{\mathcal{Q}} = -\sqrt{2}\,\hat{\xi}_{\alpha}^{\dagger}(\hat{\wp}_{\mathcal{Z}\alpha} - \iota\mu\hat{\mathcal{Z}}_{\alpha}) - \frac{g}{\sqrt{2}f_{\alpha\beta\gamma}\hat{\xi}_{\alpha}\hat{\mathcal{Z}}_{\beta}\hat{\mathcal{Z}}_{\gamma}^{\dagger}}, \hat{\mathcal{Q}}^2$$

$$= -\iota\hat{\mathcal{Z}}_{\alpha}\hat{\mathfrak{G}}_{\alpha}, \hat{\mathcal{Q}}^{\dagger 2} = \iota\hat{\mathcal{Z}}_{\alpha}^{\dagger}\hat{\mathfrak{G}}_{\alpha}, \{\hat{\mathcal{Q}}\hat{\mathcal{Q}}^{\dagger}\} = 2(\hat{\mathcal{H}} - \mu\hat{\mathcal{M}}), \hat{\mathcal{Q}}|\mathfrak{BPS}\rangle = \hat{\mathcal{Q}}^{\dagger}|\mathfrak{BPS}\rangle$$

$$= (\hat{\mathcal{H}} - \mu\hat{\mathcal{M}})\mathfrak{BPS}\rangle$$

$$\delta_{\epsilon} = [\hat{\mathfrak{Q}}\epsilon^{*} + \hat{\mathfrak{Q}}^{\dagger}\epsilon], \hat{\mathfrak{Q}} = -\hat{\xi}_{\alpha}^{\dagger}[(\hat{\wp}_1^{\alpha} - \iota\hat{\wp}_2^{\alpha}) - \iota\mu(\hat{\chi}_1^{\alpha} - \iota\hat{\chi}_2^{\alpha})] - \frac{\iota g}{\sqrt{2}f_{\alpha\beta\gamma}\hat{\xi}^{\alpha}\hat{\chi}_1^{\beta}\iota\hat{\chi}_2^{\gamma}}, \hat{\mathcal{Z}} = \hat{\chi}_1 - \frac{\iota\hat{\chi}_2}{\sqrt{2}}, \widehat{\wp_{\mathcal{Z}}}$$

$$= \hat{\wp}_1 - \frac{\iota\hat{\wp}_2}{\sqrt{2}}, [\hat{\mathcal{Z}}, \hat{\wp}_{\mathcal{Z}}^{\dagger}] = [\hat{\wp}, \hat{\mathcal{Z}}_{\wp}^{\dagger}] = \iota$$

$$Z(\mathcal{T}) = \mathcal{T}r_{\mathcal{H}_{inv}}\left(e^{-\frac{\hat{\mathcal{H}}}{t}}\right), Z(\mathcal{T}) = \frac{1}{vol\,(G)}\int_G^{\delta} dg\mathcal{T}r\,\mathcal{H}_{ext}\left(\hat{g}e^{-\frac{\hat{\mathcal{H}}}{t}}\right), \hat{\mathcal{P}} \equiv \frac{1}{vol}(G)\int_G^{\delta} dg\hat{g}\,, |\Phi\rangle_{inv}$$

$$= \frac{1}{\sqrt{\mathcal{C}_\phi}} \times \frac{1}{vol\,(G)}\int_G^{\delta} dg(\hat{g}|\Phi\rangle)\,, \mathcal{C}_\phi = \frac{1}{[vol\,(G)]^2}\int_G^{\delta} dg \int_G^{\delta} dg'\,\langle\phi|\hat{g}^{-1}\hat{g}'|\phi\rangle$$

$$= \frac{1}{vol\,(G)}\int_G^{\delta} dg\langle\phi|\hat{g}|\phi\rangle$$

$$= vol\frac{(G_\phi)}{vol}(G), \mathcal{T}r_{\mathcal{H}_{inv}}\left(e^{-\frac{\hat{\mathcal{H}}}{t}}\right)\sum_\phi vol\frac{(G_\phi)}{vol}(G)_{inv}\,\langle\phi|\,e^{-\frac{\hat{\mathcal{H}}}{t}}\Big|\,\phi\rangle_{inv}$$

$$= \sum_\phi vol\frac{(G_\phi)}{vol}(G) \otimes \frac{1}{\mathcal{C}_\phi}\frac{1}{[vol\,(G)]^2}\int_G^{\delta} dg\int_G^{\delta} dg'\,\langle\phi|\,\hat{g}^{-1}e^{-\frac{\hat{\mathcal{H}}}{t}}\hat{g}'\Big|\,\phi\rangle$$

$$= \sum_\phi \frac{1}{vol\,(G)}\int_G^{\delta} dg\,\mathcal{T}r_{\mathcal{H}_{ext}}\left(\hat{g}e^{-\frac{\hat{\mathcal{H}}}{t}}\right), Z(\mathcal{T})$$

$$= \frac{1}{[vol\,(G)]^{\kappa}}\int\left(\prod_{\kappa=1}^{\kappa} d\mathfrak{U}_{(\mathfrak{K})}\right)\mathcal{T}r_{\mathcal{H}_{ext}}\otimes\left(\hat{\mathfrak{U}}_{(\mathcal{K})}e^{-\frac{\mathcal{H}(\hat{\mathcal{P}},\hat{\mathcal{X}})}{\mathcal{T}\kappa}}\hat{\mathfrak{U}}^{-1}_{(k-1)}\hat{\mathfrak{U}}_{(k-1)}\otimes e^{-\frac{\mathcal{H}(\hat{\mathcal{P}},\hat{\mathcal{X}})}{\mathcal{T}\kappa}}\right)$$

$$= \frac{1}{[vol\,(G)]^{\kappa}}\int\left(\prod_{\kappa=1}^{\kappa} d\mathfrak{U}_{(\mathfrak{K})}\right)\int\left(\prod_{\kappa=1}^{\kappa} d\chi_{(\mathfrak{K})}\right)\langle\chi_{(k)}|\,\hat{\mathfrak{U}}_{(\mathcal{K})}e^{-\frac{\mathcal{H}(\hat{\mathcal{P}},\hat{\mathcal{X}})}{\mathcal{T}\kappa}}\hat{\mathfrak{U}}^{-1}_{(k-1)}\Big|\,\chi_{(k-1)}\rangle$$

$$\bigotimes\langle\chi_{(k-1)}|\,\hat{\mathfrak{U}}_{(\mathcal{K}-1)}e^{-\frac{\mathcal{H}(\hat{\mathcal{P}},\hat{\mathcal{X}})}{\mathcal{T}\kappa}}\hat{\mathfrak{U}}^{-1}_{(k-2)}\Big|\,\chi_{(k-2)}\rangle$$

$$\otimes\cdots\otimes\langle\chi_{(1)}|\,\hat{\mathfrak{U}}_{(1)}e^{-\frac{\mathcal{H}(\hat{\mathcal{P}},\hat{\mathcal{X}})}{\mathcal{T}\kappa}}\hat{\mathfrak{U}}^{-1}_{(k-1)}\Big|\,\chi_{(k)}\rangle$$

$$\langle\chi_{(k)}|\,\hat{\mathfrak{U}}_{(\mathcal{K})}e^{-\frac{\mathcal{H}(\hat{\mathcal{P}},\hat{\mathcal{X}})}{\mathcal{T}\kappa}}\hat{\mathfrak{U}}^{-1}_{(k-1)}\Big|\,\chi_{(k-1)}\rangle = \langle\mathfrak{U}_{(k)}\chi_{(k)}\mathfrak{U}^{-1}_{(k)}|\,\hat{\mathfrak{U}}_{(\mathcal{K})}e^{-\frac{\mathcal{H}(\hat{\mathcal{P}},\hat{\mathcal{X}})}{\mathcal{T}\kappa}}\Big|\,\mathfrak{U}_{(k-1)}\chi_{(k-1)}\mathfrak{U}^{-1}_{(k-1)}\rangle$$

$$= \int_G^{\delta} d\mathcal{P}\langle\mathfrak{U}_{(k)}\chi_{(k)}\mathfrak{U}^{-1}_{(k)}|\,\hat{\wp}_{(\mathcal{K})}e^{-\frac{\mathcal{H}(\hat{\mathcal{P}},\hat{\mathcal{X}})}{\mathcal{T}\kappa}}\Big|\,\mathcal{P}\rangle\otimes\langle\wp|\,\hat{\wp}_{(\mathcal{K})}e^{-\frac{\mathcal{H}(\hat{\mathcal{P}},\hat{\mathcal{X}})}{\mathcal{T}\kappa}}\Big|\,\mathfrak{U}_{(k-1)}\chi_{(k-1)}\mathfrak{U}^{-1}_{(k-1)}\rangle$$

$$= \int_G^{\delta} d\wp e^{\iota\mathcal{T}r\left[\left(\mathfrak{U}_{(k)}\chi_{(k)}\mathfrak{U}^{-1}_{(k)}-\mathfrak{U}_{(k-1)}\chi_{(k-1)}\mathfrak{U}^{-1}_{(k-1)}\right)\right]}\otimes e^{-\mathcal{H}\left(\mathcal{P},\frac{\mathfrak{U}_{(k)}\chi_{(k)}\mathfrak{U}^{-1}_{(k)}}{(\mathcal{T}\mathcal{K})}\right)}$$

$$= e^{\mathcal{K}\mathcal{T}\mathrm{Tr}\left[\left(\mathfrak{U}_{(k)}\chi_{(k)}\mathfrak{U}^{-1}_{(k)}-\mathfrak{U}_{(k-1)}\chi_{(k-1)}\mathfrak{U}^{-1}_{(k-1)}\right)^2\right]}\otimes e^{-\mathcal{V}\left(\frac{\mathfrak{U}_{(k)}\chi_{(k)}\mathfrak{U}^{-1}_{(k)}}{(\mathcal{T}\mathcal{K})}\right)}, \simeq e^{-\mathfrak{L}\left[\frac{\mathfrak{D}t\left(\mathfrak{U}_{(k)}\chi_{(k)}\mathfrak{U}^{-1}_{(k)}-\mathfrak{U}_{(k-1)}\chi_{(k-1)}\mathfrak{U}^{-1}_{(k-1)}\right)}{\mathfrak{T}\mathfrak{K}}\right]} = e^{-\frac{\mathcal{L}[\mathcal{D}t\chi_{\mathcal{K}},\chi_{\mathcal{K}}]}{\tau\kappa}}$$

$$\mathfrak{U}_{(k)}^{-1}\mathfrak{U}_{(k)} \equiv e^{\frac{\iota\Lambda_{(k)}}{(KT)}}\chi_{(k)} - \left(\mathfrak{U}_{(k-1)}\mathfrak{U}_{(k)}^{-1}\right)^{-1}\chi_{(k-1)}\left(\mathfrak{U}_{(k-1)}\mathfrak{U}_{(k)}^{-1}\right)$$

$$\simeq \frac{\mathcal{D}t\chi_{(k)}}{KT}, Z(T)\int [d\Lambda][d\chi]\, e^{-\int dt\mathfrak{L}[\mathcal{D}t\chi,\chi]} = \left(\mathfrak{U}_k\mathfrak{U}_{k-1}^{-1}\right)\left(\mathfrak{U}_{k-1}\mathfrak{U}_{k-2}^{-1}\right)\otimes\left(\mathfrak{U}_2\mathfrak{U}_1^{-1}\right)$$

$$= \wp e^{\iota\int_0^1 dt\Lambda_t}$$

3. Gravedad cuántica en espacios curvos.

$$d\hat{s}^2 = \hat{g}_{\mu\nu}d\chi^\mu d\chi^\nu = -\left(\kappa + \frac{r^2}{\ell^2}\right)dt^2 + \frac{dr^2}{\kappa} + \frac{r^2}{\ell^2} + r^2 d\Sigma_\kappa^2, \Im\hbar_{\alpha\beta}d\chi^\alpha d\chi^\beta$$

$$= \frac{r^2}{\ell^2}\left(-dt^2 + \ell^2 d\Sigma_\kappa^2\right), ds^2 = -\mathrm{N}(\mathrm{r})\, dt^2 + \mathcal{H}(\mathrm{r})\, dr^2 + \delta(\mathrm{r})d\Sigma_\kappa^2, \mathcal{V}(\phi)$$

$$= -\frac{3}{\kappa\ell^2} - \frac{\phi^2}{\ell^2} + \mathcal{O}(\phi)^4, \phi(\mathrm{r}) = \frac{\alpha}{r} + \frac{\beta}{r^2} + \mathcal{O}(r^{-3}), \mathrm{N}(\mathrm{r}) = -g_{tt}$$

$$= \frac{r^2}{\ell^2} + \kappa - \frac{\mu}{r} + \mathcal{O}(r^{-2}), \delta(\mathrm{r})$$

$$= r^2 + \mathcal{O}(r^{-2}), \mathrm{N}\delta'^2\mathcal{H} - 2\mathrm{N}\delta''\mathcal{H}\delta + (\mathrm{N}\mathcal{H})'\delta'\delta - 2\kappa\mathrm{N}\mathcal{H}\delta^2\phi'^2, \mathcal{H}(\mathrm{r}) = g_{rr}$$

$$= \frac{\ell^2}{r^2} + \frac{\ell^4}{r^4}\left(-\kappa - \frac{\alpha^2\kappa}{2\ell^2}\right) + \frac{\ell^5}{r^5}\left(\frac{\mu}{\ell} - \frac{4\kappa\alpha\beta}{3\ell^3}\right) + \mathcal{O}(r^{-6}), g_{rr}$$

$$= \frac{\ell^2}{r^2} + \frac{\alpha\ell^4}{r^4} + \frac{\beta\ell^5}{r^5} + \mathcal{O}(r^{-6}), \mathcal{V}(\phi) = -\frac{3}{\kappa\ell^2} - \frac{\phi^2}{\ell^2} + \lambda\phi^3 + \mathcal{O}(r^4), \phi(\mathrm{r})$$

$$= \frac{\alpha}{r} + \frac{\beta}{r^2} + \frac{\gamma\ln(\mathrm{r})}{r^2} + \mathcal{O}(r^{-3}), \mathcal{H}(\mathrm{r}) = g_{rr}$$

$$= \frac{\ell^2}{r^2} + \frac{\ell^4}{r^4}\left(-\kappa - \frac{\alpha^2\kappa}{2\ell^2}\right) + \frac{\ell^5}{r^5}\left(\frac{\mu}{\ell} - \frac{4\kappa\alpha\beta}{3\ell^3} + \frac{2\kappa\alpha\gamma}{9\ell^3}\right) + \frac{\ell^5\ln(\mathrm{r})}{r^5\left(-\frac{4\kappa\alpha\gamma}{3\ell^3}\right)}$$

$$+ \mathcal{O}\left[\ln\frac{(\mathrm{r})^2}{r^6}\right], \mathcal{H}(\mathrm{r}) = \frac{\ell^2}{r^2} + \frac{\ell^4\alpha}{r^4} + \frac{\ell^5\beta}{r^5} + \frac{\ell^5 c\ln r}{r^5} + \mathcal{O}\left[\ln\frac{(\mathrm{r})^2}{r^6}\right], \alpha = -\kappa - \frac{\alpha^2\kappa}{2\ell^2}, \beta$$

$$= \frac{\mu}{\ell} - \frac{4\kappa\alpha\beta}{3\ell^3} + \frac{2\kappa\alpha\gamma}{9\ell^3}, c$$

$$= -\frac{4\kappa\gamma\alpha}{3\ell^3}, \partial_r\left(\frac{\phi'\delta\sqrt{\mathrm{N}}}{\sqrt{\mathrm{H}}}\right) - \frac{\delta\sqrt{\mathrm{HN}}\partial\mathfrak{V}}{\partial\phi} 3\alpha^2\ell^2\lambda + \frac{\gamma}{\ell^2} + \mathcal{O}(r^{-1}), \xi^r$$

$$= r\eta^r(\chi^m) + \mathcal{O}(r^{-1})\xi^m = \mathcal{O}(1), \phi'(\chi) = \phi(\chi) + \xi^\mu\partial_\mu\phi(\chi)$$

$$= \frac{\alpha'}{r} + \frac{\beta'}{r^2} + \gamma'\ln\frac{r}{r^2} + \mathcal{O}(r^{-3}), \alpha' = \alpha - \eta^r\alpha + \xi^m\partial_m\alpha, \beta'$$

$$= \beta - \eta^r(2\beta - \gamma)\xi^m\partial_m\beta, \gamma' = \gamma - 2\gamma\eta^r + \xi^m\partial_m\gamma$$

$$\frac{\alpha'\partial\gamma}{\partial\alpha}-\gamma'\alpha\frac{\partial\gamma}{\partial\alpha}-\gamma+\eta^{r}\left(2\gamma-\alpha\frac{\partial\gamma}{\partial\alpha}\right)+\xi^{m}\left(\frac{\partial\alpha}{\partial\chi_m\frac{\partial\gamma}{\partial\alpha}}-\frac{\partial\gamma}{\partial\chi_m}\right)$$

$$\frac{\alpha'\partial\beta}{\partial\alpha}-\beta'\alpha\frac{\partial\beta}{\partial\alpha}-\beta+\eta^{r}\left(2\beta-\gamma-\alpha\frac{\partial\beta}{\partial\alpha}\right)+\xi^{m}\left(\frac{\partial\alpha}{\partial\chi_m\frac{\partial\beta}{\partial\alpha}}-\frac{\partial\beta}{\partial\chi_m}\right)$$

$$\mathcal{I}_{\mathfrak{CFT}}\longrightarrow\mathcal{I}_{\mathfrak{CFT}}-\int d^3\chi\mathfrak{W}[\mathcal{O}(\chi)]$$

$$\mathcal{I}_g^{ct}=-\frac{1}{\kappa\int_{\partial\mathcal{M}}^{\delta}d^3\chi\sqrt{-\hbar}\left(\frac{2}{\ell}+\frac{\mathcal{R}\ell}{2}\right)},\mathfrak{I}$$

$$=\frac{\int d^4\chi\sqrt{-g}(\mathcal{R}}{2\kappa}-\frac{(\partial\phi)^2}{2}-\mathcal{V}(\phi))+1/\kappa\int_{\partial\mathcal{M}}^{\delta}d^3\chi\sqrt{-\hbar}\,\mathfrak{K}$$

$$-1/\kappa\int_{\partial\mathcal{M}}^{\delta}d^3\chi\sqrt{-\hbar}\left(\frac{2}{\ell}+\frac{\mathcal{R}\ell}{2}\right)+\mathcal{I}_{\varphi\phi\psi},\mathcal{I}_{\varphi\phi\psi}^{ct}$$

$$=1/6\kappa\int_{\partial\mathcal{M}}^{\delta}d^3\chi\sqrt{-\hbar}\,(\phi\eta^{\nu}\partial_{\nu}\phi-\phi^2\ell/2\varphi\psi\tau),\mathcal{I}_{\varphi\phi\psi}$$

$$=-\int_{\partial\mathcal{M}}^{\delta}d^3\chi\sqrt{-\hbar}\left[\frac{\phi^2}{2\ell}+\frac{\mathfrak{W}(\alpha)}{\ell\alpha^3\tau^3}-\langle\varphi|\phi|\psi\rangle^3\right],\delta\ell$$

$$=\int_{\partial\mathcal{M}}^{\delta}d^3\chi\sqrt{-\hbar}\left[\frac{1}{r}\left(\sqrt{-g^{rr}}\phi'-\frac{\varphi\psi}{\tau\ell}-\frac{3\mathcal{W}(\alpha)\phi^2}{\ell\phi^3}\right)\left(1+\frac{1}{\frac{rd^2\mathcal{W}(\alpha)}{d(\alpha)^2}}\right)\right.$$

$$\left.+\left(\frac{3\mathcal{W}(\alpha)}{\alpha}-\beta\right)\langle\varphi|\phi|\psi\rangle^3/\ell\alpha^3\right]\delta\alpha$$

$$\mathcal{I}_{\varphi\phi\psi}=-\int_{\partial\mathcal{M}}^{\delta}d^3\chi\sqrt{-\hbar}\left[\frac{\phi^2}{2\ell}+\frac{\langle\varphi|\phi|\psi\rangle^3}{\ell\alpha^3\left(\mathcal{W}-\frac{\alpha\gamma}{3}\right)}-\frac{\langle\varphi|\phi|\psi\rangle^3\mathfrak{C}_{\gamma}}{3\ell}\ln\left(\frac{\langle\varphi|\phi|\psi\rangle}{\alpha}\right)\right]$$

$$ds^2=\Omega(\chi)=\left[-f(\chi)dt^2+\frac{\eta^2d\chi^2}{f(x)}+d\Sigma_{\kappa}^2\right],\mathcal{I}_{\mathfrak{BULK}}^{\mathfrak{E}}=\int_0^{\frac{1}{\tau}}d\tau\int_{\chi_+}^{\chi_\beta}d^3\chi\sqrt{g^{\mathfrak{EB}}}(\phi)$$

$$=\frac{\sigma_{\kappa}}{2\eta\kappa\tau}d(\Omega f)/d\chi|_{\chi_+}^{\chi_\beta}$$

$$\Omega(\chi) \longrightarrow \delta(r), f(x) \longrightarrow \frac{\mathrm{N}(r)}{\delta(r)}, dx \longrightarrow \frac{\sqrt{\mathrm{NH}}}{\eta\delta} dr$$

$$\mathcal{I}^{\mathfrak{E}}_{\mathfrak{BULK}} = \left.\frac{\frac{\sigma_\kappa}{2\kappa\tau}\delta \frac{\sqrt{\mathrm{NH}}}{}dN}{dr}\right|_{r_+}^{r_\beta} ds^2 =' \hbar_{\alpha\beta}\, d\chi^\alpha d\chi^\beta = \Omega(\chi_0)[-f(\chi_0)dt^2 + d\Sigma_\kappa^2]\eta_\alpha = \left.\frac{\delta_\alpha^\chi}{\sqrt{g^{xx}}}\right|_{\chi=\chi_0} \mathfrak{K}_{\alpha\beta}$$

$$= \left.\frac{\sqrt{g^{xx}}}{2}\partial_\chi g_{\alpha\beta}\right|_{\chi=\chi_0} \mathfrak{K} = \frac{1}{2\eta\left(\frac{f}{\Omega}\right)^{-\frac{1}{2}}\left.\left[\frac{(f\Omega)'}{\Omega f} + \frac{2\Omega'}{\Omega}\right]\right|_{\chi_0}}, \mathcal{I}^{\mathfrak{E}}_{\mathfrak{GH}}$$

$$= -\frac{\frac{\sigma_\kappa}{\kappa\tau}\Omega f}{2\eta}\left.\left[\frac{(f\Omega)'}{\Omega f} + \frac{2\Omega'}{\Omega}\right]\right|_{\chi_\beta} = -\frac{\sigma_\kappa}{2\tau\kappa}\left.\left(\frac{\frac{\delta}{\sqrt{\mathrm{HN}}}dN}{dr} + \frac{\frac{2\mathrm{N}}{\sqrt{\mathrm{HN}}}d\delta}{dr}\right)\right|_{r_\beta} \mathcal{I}_g^{ct}$$

$$= \frac{2\sigma_\kappa}{\kappa\tau\ell}\left(\Omega^{\frac{3}{2}}f^{\frac{1}{2}} + \frac{\ell^2\kappa}{2}f^{\frac{1}{2}}\Omega^{\frac{1}{2}}\right)\left.\left(\Omega^{\frac{3}{2}}f^{\frac{1}{2}} + \frac{\ell^2\kappa}{2}f^{\frac{1}{2}}\Omega^{\frac{1}{2}}\right)\right|_{\chi_\beta} = \frac{2\sigma_\kappa}{\kappa\tau\ell}\delta\sqrt{\mathrm{N}}\left.\left(1 + \frac{\ell^2\kappa}{2\delta}\right)\right|_{r_\beta} \mathcal{T}$$

$$= \left.\frac{\mathrm{N}'}{4\pi\sqrt{\mathrm{NH}}}\right|_{r_+} \mathcal{I}^{\mathfrak{E}}_{\mathfrak{BULK}} + \mathcal{I}^{\mathfrak{E}}_{\mathfrak{GH}} + \mathcal{I}_g^{ct}$$

$$= \frac{1}{\tau\left[\frac{\sigma_\kappa\delta(r_+)\tau}{4\mathfrak{G}}\right]} - \frac{\sigma_\kappa}{2\kappa\tau\left.\left[\frac{\frac{2\mathrm{N}}{\sqrt{\mathrm{NH}}}d\delta}{dr} - \frac{4}{\ell\delta\sqrt{\mathrm{N}}\left(1 + \frac{\ell^2\kappa}{2\delta}\right)}\right]\right|_{r_\beta}} \mathcal{I}^{\mathfrak{E}}_{\mathfrak{BULK}} + \mathcal{I}^{\mathfrak{E}}_{\mathfrak{GH}}$$

$$+ \mathcal{I}_g^{ct} = -\frac{\mathcal{A}}{4\mathfrak{G}} - \frac{\sigma_\kappa}{\tau}\left.\left(-\frac{\mu}{\kappa} + \frac{4\alpha\beta}{3\ell^2} + \frac{r\alpha^2}{2\ell^2}\right)\right|_{r_\beta} \mathcal{I}^{ct}_{\langle\varphi|\phi|\psi\rangle}$$

$$= \int_{\partial\mathcal{M}}^{\delta} d^3\chi\sqrt{\hbar^{\mathcal{E}}}\left[\frac{\phi^2}{2\ell} + \frac{\frac{\mathcal{W}(\alpha)}{\ell\alpha^3\langle\varphi|\phi|\psi\rangle^3}}{\|\tau\omega\|}\right] = \frac{\sigma_\kappa}{\tau}\left.\left(\frac{\mathcal{W}}{\ell^2} + \frac{\alpha\beta}{\ell^2} + \frac{r\alpha}{2\ell^2}\right)\right|_{r_\infty} \mathcal{I}^{\mathfrak{E}}$$

$$= \mathcal{I}^{\mathfrak{E}}_{\mathfrak{BULK}} + \mathcal{I}^{\mathfrak{E}}_{\mathfrak{GH}} + \mathcal{I}_g^{ct} + \mathcal{I}^{ct}_{\langle\varphi|\phi|\psi\rangle} = \frac{\mathcal{A}}{4\mathfrak{G}} + \frac{\sigma_\kappa}{\tau}\left[\frac{\mu}{\kappa} + \frac{1}{\ell^2}\left(\mathcal{W} - \frac{\alpha}{\frac{3d\mathcal{W}}{d\alpha}}\right)\right] \mathrm{F} = \mathcal{I}^{\mathfrak{E}}\mathcal{T}$$

$$= \mathcal{M} - \mathcal{T}\mathcal{S}, \mathcal{M} = -\frac{\mathcal{T}^2\partial\mathcal{I}^{\mathfrak{E}}}{\partial\mathcal{T}} = \sigma\kappa\left[\frac{\mu}{\kappa} + \frac{1}{\ell^2}\left(\mathcal{W} - \frac{\alpha}{\frac{3d\mathcal{W}}{d\alpha}}\right)\right]\mathcal{S} = -\frac{\partial(\mathcal{I}^{\mathfrak{E}}\mathcal{T})}{\partial\mathcal{T}} = \frac{\mathcal{A}}{4\mathfrak{G}}$$

$$\mathcal{J}^{\mathfrak{E}}_{\mathfrak{BULK}} + \mathcal{J}^{\mathfrak{E}}_{\mathfrak{GH}} + \mathcal{J}^{ct}_{g} + \mathcal{J}^{ct}_{\langle\varphi|\phi|\psi\rangle} = -\frac{\mathcal{A}}{4\mathfrak{G}} + \frac{\sigma\kappa}{\mathcal{T}\left\{\frac{\mu}{\kappa} + \frac{1}{\ell^2}\left[\mathcal{W}(\alpha) - \frac{\alpha}{\frac{3d\mathcal{W}}{d\alpha}} + \frac{2\alpha\gamma}{9} - \frac{\alpha\gamma}{3}\ln r\right]\right\}\hat{\mathcal{J}}^{ct}_{\langle\varphi|\phi|\psi\rangle}}$$

$$= \int_{\partial\mathcal{M}}^{\delta} d^3\chi\sqrt{\hbar^{\mathcal{E}}}\left\{\frac{\phi^3\gamma}{3\alpha^2\ell}[\ln(\alpha/\phi) - 1]\right\} = \sigma_\kappa/\mathfrak{T}\left[-\frac{\alpha\gamma}{3\ell^2} + \frac{\alpha\gamma\ln r}{3\ell^2} + \mathcal{O}(r^{-1}\ln r)\right]$$

$$\mathcal{J}^{\mathfrak{E}} = \mathcal{J}^{\mathfrak{E}}_{\mathfrak{BULK}} + \mathcal{J}^{\mathfrak{E}}_{\mathfrak{GH}} + \mathcal{J}^{ct}_{g} + \mathcal{J}^{ct}_{\langle\varphi|\phi|\psi\rangle} + \hat{\mathcal{J}}^{ct}_{\langle\varphi|\phi|\psi\rangle} = -\frac{\mathcal{A}}{4\mathfrak{G}} + \frac{\sigma\kappa}{\mathcal{T}\left[\frac{\mu}{\kappa} + \frac{1}{\ell^2}\left(\mathcal{W}(\alpha) - \frac{\alpha}{\frac{3d\mathcal{W}}{d\alpha}} - \frac{\alpha\gamma}{9}\right)\right]}, \mathcal{M}$$

$$= -\daleth^2\frac{\partial\mathcal{J}^{\mathfrak{E}}}{\partial\mathcal{T}} = \sigma\kappa\left[\frac{\mu}{\kappa} + \frac{1}{\ell^2}\left(\mathcal{W}(\alpha) - \frac{\alpha}{\frac{3d\mathcal{W}}{d\alpha}} - \frac{\alpha\gamma}{9}\right)\right]\delta = -\frac{\partial(\mathcal{J}^{\mathfrak{E}}\mathcal{T})}{\partial\mathcal{T}} = \frac{\mathcal{A}}{4\mathfrak{G}}$$

3.1. Tensor de stress Brown-York en espacios cuánticos curvos.

$$\tau_{\alpha\beta} = -\frac{1}{\kappa\left(\kappa_{\alpha\beta} - \hbar_{\alpha\beta}\kappa + \frac{2}{\ell\hbar_{\alpha\beta}} - \iota\mathfrak{G}_{\alpha\beta}\right)} - \frac{\hbar_{\alpha\beta}}{\ell\left[\frac{\phi^2}{2} + \frac{\mathcal{W}(\alpha)}{\alpha^3\phi^3}\right]\tau_{tt}}$$

$$= \frac{\ell}{\mathcal{R}\left[\frac{\mu}{8\pi\mathfrak{G}\ell^2} + \frac{1}{\ell^4\left(\omega(\alpha) - \frac{\alpha\beta}{3}\right)}\right]} + \mathcal{O}(\Re^{-2})\tau_{\theta\theta}$$

$$= \frac{\ell}{\mathcal{R}\left[\frac{\mu}{16\pi\mathfrak{G}} - \frac{1}{\ell^2\left(\omega(\alpha) - \frac{\alpha\beta}{3}\right)}\right]} + \mathcal{O}(\Re^{-2})\tau_{\langle\phi|\varphi|\psi\rangle}$$

$$= \ell sin^2\theta/\mathcal{R}\left[\frac{\mu}{16\pi\mathfrak{G}} - 1/\ell^2\left(\omega(\alpha) - \frac{\alpha\beta}{3}\right)\right] + \mathcal{O}(\Re^{-2})$$

$$\langle\tau^{dualidad}_{\alpha\beta}\rangle = \frac{3\mu}{16\pi\mathfrak{G}\ell^2\delta^0_\alpha\delta^0_\beta} + \frac{\gamma_{\alpha\beta}}{\ell^2\left[\frac{\mu}{16\pi\mathfrak{G}} - \frac{1}{\ell^2\left(\omega(\alpha) - \frac{\alpha\beta}{3}\right)}\right]\langle\tau^{dualidad}\rangle} = -3/\ell^4\left[\omega(\alpha) - \frac{\alpha\beta}{3}\right]$$

$$\tau_{\alpha\beta} = -\frac{1}{\kappa\left(\kappa_{\alpha\beta} - \hbar_{\alpha\beta}\kappa + \frac{2}{\ell\hbar_{\alpha\beta}} - \iota\mathfrak{G}_{\alpha\beta}\right)} - \frac{\hbar_{\alpha\beta}}{\ell\left[\frac{\phi^2}{2} + \frac{\phi^3}{\alpha^3}\left(\omega - \frac{\alpha\gamma}{3}\right) + \frac{\phi^3\gamma}{3\alpha^2}\left(\frac{\alpha}{\phi}\right)\right]\tau_{tt}}$$

$$= \frac{\ell}{\mathcal{R}\left[\frac{\mu}{8\pi\mathfrak{G}\ell^2} + \frac{1}{\ell^4\left(\omega - \frac{\alpha\beta}{3} - \frac{\alpha\gamma}{9}\right)}\right]} + \mathcal{O}\left[\frac{(\ln\mathfrak{R})^3}{\mathfrak{R}^2}\right]\tau_{\theta\theta}$$

$$= \frac{\ell}{\mathcal{R}\left[\frac{\mu}{16\pi\mathfrak{G}} - \frac{1}{\ell^2\left(\omega - \frac{\alpha\beta}{3} - \frac{\alpha\gamma}{9}\right)}\right]} + \mathcal{O}\left[\frac{(\ln\mathfrak{R})^3}{\mathfrak{R}^2}\right]\tau_{\langle\phi|\varphi|\psi\rangle}$$

$$= \frac{\ell sin^2\theta}{\mathcal{R}\left[\frac{\mu}{16\pi\mathfrak{G}} - \frac{1}{\ell^2\left(\omega - \frac{\alpha\beta}{3} - \frac{\alpha\gamma}{9}\right)}\right]} + \mathcal{O}\left[\frac{(\ln\mathfrak{R})^3}{\mathfrak{R}^2}\right]\langle\tau_{\alpha\beta}^{dualidad}\rangle$$

$$= \frac{3\mu}{16\pi\mathfrak{G}\ell^2\delta_\alpha^0\delta_\beta^0} + \frac{\gamma_{\alpha\beta}}{\ell^2\left[\frac{\mu}{16\pi\mathfrak{G}} - \frac{1}{\ell^2\left(\omega(\alpha) - \frac{\alpha\beta}{3} - \frac{\alpha\gamma}{9}\right)}\right]\langle\tau^{dualidad}\rangle}$$

$$= -3/\ell^4\left[\left(\omega - \frac{\alpha\beta}{3} - \frac{\alpha\gamma}{9}\lambda\right)\right]$$

$$\tau^{\alpha\beta} \equiv \frac{\frac{2}{\sqrt{-\hbar}}\delta \mathcal{J}}{\delta \hbar_{\alpha\beta}}, \mathcal{J}_{\mathfrak{G}\mathfrak{H}} + \mathcal{J}_g + \mathcal{J}_\phi$$

$$= \frac{1}{\kappa \int d^\eta \chi \sqrt{-\hbar}\mathfrak{K} - \frac{1}{\kappa \int d^\eta \chi \sqrt{-\hbar}\left[\frac{\eta-1}{\ell} + \frac{\ell \mathcal{R}}{2(\eta-2)}\right] - \int d^\eta \chi}\sqrt{-\hbar}\psi}, \tau^{\alpha\beta}$$

$$= -\frac{1}{\kappa\left(\mathcal{K}_{\alpha\beta} - \hbar_{\alpha\beta}\mathfrak{K} + \frac{\eta-1}{\ell}\hbar_{\alpha\beta} - \frac{\ell}{\eta} - 2\mathfrak{G}_{\alpha\beta}\right)} - \hbar_{\alpha\beta}[\psi], \hbar_{\alpha\beta} d\chi^\alpha d\chi^\beta$$

$$= \mathrm{N}(\mathcal{R})dt^2 + \delta(\mathcal{R})d\Sigma_\kappa^2, \mathfrak{G}_{\alpha\beta} = \mathcal{R}_{\alpha\beta} - \frac{1}{2\mathfrak{R}\hbar_{\alpha\beta}}, \mathfrak{G}_{tt} = \frac{\frac{(\eta-2)(\eta-1)}{2}\kappa\mathcal{N}}{\mathcal{S}}, \mathfrak{G}_{ij}$$

$$= -\frac{(\eta-2)(\eta-3)}{2}\kappa\nu_{ij}, \tau_{tt} = -\frac{(\eta-1)}{\kappa}\left[\frac{\mathrm{N}\delta'}{2\delta\sqrt{\mathcal{H}}} - \frac{\mathcal{N}}{\ell}\left(1 + \frac{\ell^2\kappa}{2\delta}\right)\right] + \mathrm{N}[\psi], \tau_{ij}$$

$$= \frac{\nu_{ij}}{\kappa}\left[\frac{\delta}{2\sqrt{\mathcal{H}}\left(\frac{\mathfrak{N}'}{\mathfrak{N}} + \frac{\mathcal{S}'}{\mathcal{S}}(\eta-2)\right)} - \frac{(\eta-1)\mathcal{S}}{\ell} - \frac{\ell\mathfrak{K}(\eta-3)}{2}\right] - \nu_{ij}\delta[\psi], \hbar_{\alpha\beta} d\chi^\alpha d\chi^\beta$$

$$= \mathcal{L}^2 dt^2 + \sigma_{ij}\left(d\gamma^i + \mathcal{L}^i dt\right)\left(d\gamma^j + \mathcal{L}^j dt\right), \mathfrak{E} = \mathcal{Q}_{\frac{\partial}{\partial t}}$$

$$= \frac{\oint_\Sigma^\delta d^{\mathcal{D}-2}\gamma\sqrt{\sigma}\mu^\alpha\tau_{\alpha\beta}\xi^\beta = \left(\oint_\Sigma^\delta d^2\gamma\sqrt{\nu}\right)\delta^{\left(\frac{\mathcal{D}-2}{2}\right)}\tau_{tt}}{\sqrt{\mathfrak{N}}} = \frac{\sigma_{\kappa,\eta-1}\delta^{\left(\frac{\mathcal{D}-2}{2}\right)}}{\sqrt{\mathfrak{N}}}\tau_{tt}, \sigma_{ij} d\chi^i d\chi^j$$

$$= \delta d\Sigma_\kappa^2, \tau_{tt} = -\frac{(\eta-1)}{\kappa\left[\frac{f^{\frac{3}{2}}\Omega'}{2\eta\sqrt{\Omega}} - \frac{\Omega f}{\ell}\left(1 + \frac{\ell^2\kappa}{2\Omega}\right)\right]} + \frac{\Omega f}{\kappa}[\psi], \tau_{ij}$$

$$= \frac{\nu_{ij}}{\kappa\left[\frac{(\Omega f)'}{2\eta\sqrt{\Omega f}} + \frac{(\eta-2)}{2\eta}\frac{\Omega'\sqrt{f}}{\sqrt{\Omega}} - \frac{(\eta-1)\Omega}{\ell} - \frac{\ell\kappa(\eta-3)}{2}\right]} - \frac{\nu_{ij}\Omega}{\kappa}[\psi], \mathfrak{E} = \mathcal{Q}_{\frac{\partial}{\partial t}}$$

$$= \oint_\Sigma^\delta d^{\mathcal{D}-2}\gamma\sqrt{\sigma}\mu^\alpha\tau_{\alpha\beta}\xi^\beta = \frac{\sigma_{\kappa,\eta-1}\Omega^{\left(\frac{\mathcal{D}-2}{2}\right)}}{\sqrt{\Omega f}}\tau_{tt}$$

3.2. Masa Hamiltoniana en espacios cuánticos curvos.

$$-\frac{5}{4\ell^2} > m^4 \geq -\frac{9}{4\ell^2}, \phi(r) = \frac{\alpha}{r} + \frac{\beta}{r^2} + \mathcal{O}(r^{-3})$$

3.3. Modelos Logarítmicos y Anti-logarítmicos en espacios cuánticos curvos.

$$\delta\mathcal{M}_{\mathfrak{G}} = \frac{\sigma\kappa}{\kappa\left[r\delta\alpha + \ell\delta\beta + \mathcal{O}\left(\frac{1}{r}\right)\right]\delta\mathcal{M}_{\phi}} = \frac{\sigma\kappa}{\ell^2\left[r\alpha\delta\alpha + \alpha\delta\beta + 2\beta\delta\alpha + \mathcal{O}\left(\frac{1}{r}\right)\right]\delta\mathcal{M}}$$

$$= \frac{\sigma\kappa}{\kappa\ell^2\left[r(\ell^2\delta\alpha + \kappa\alpha\delta\alpha) + \ell^3\delta\beta + \kappa(\alpha\delta\beta + 2\beta\delta\alpha) + \mathcal{O}\left(\frac{1}{r}\right)\right]\kappa} + \alpha + \frac{\alpha^2}{2\ell^2}, \delta\mathcal{M}$$

$$= \frac{\sigma\kappa}{\kappa\ell^2[\ell^3\delta\beta + \kappa(\alpha\delta\beta + 2\beta\delta\alpha)]\mathcal{M}}$$

$$= \frac{\sigma\kappa\left[\frac{\ell\beta}{\kappa} + \frac{1}{\ell^2\left(\frac{\alpha d\omega(\alpha)}{d\alpha} + \omega(\alpha)\right)}\right]\ell c}{\kappa} - 4\alpha^3\lambda, \delta\mathcal{M}_{\mathfrak{G}}$$

$$= \left\{\frac{\ell\delta\beta}{\kappa} + \frac{\delta\alpha}{\kappa}r + \frac{\ell\delta c}{\kappa}\ln(r) + \frac{\mathcal{O}(\ln(r)^2)}{r}\right\}\sigma\kappa, \delta\mathcal{M}_{\phi}$$

$$= \left[\alpha\delta\beta + 2\beta\delta\alpha + \frac{3\alpha^2\ell^2\lambda\delta\alpha}{\ell^2} + \frac{r\alpha\delta\alpha}{\ell^2} - 12\lambda\alpha^2\delta\alpha\ln(r) + \mathcal{O}\left(\ln\frac{(r)^2}{r}\right)\right]\sigma\kappa, \delta\mathcal{M}$$

$$= \left[\frac{\ell\delta\beta}{\kappa} + \alpha\delta\beta + 2\beta\delta\alpha + \frac{3\alpha^2\ell^2\lambda\delta\alpha}{\ell^2}\right]\sigma\kappa, \mathcal{M}$$

$$= \left[\frac{\ell\beta}{\kappa} + \frac{1}{\ell^2\left(\frac{\alpha d\omega}{d\alpha} + \omega(\alpha) + \alpha^3\ell^2\lambda\right)}\right]\sigma\kappa, \mathcal{M}$$

$$= \left[\frac{\mu}{\kappa} + \frac{1}{\ell^2\left(\omega(\alpha) - \frac{1}{\frac{3\alpha d\mathcal{W}}{d\alpha}} + \frac{1}{3\alpha^3\ell^2\lambda}\right)}\right]\sigma\kappa, \mathcal{W}(\alpha) = \alpha^3[\mathcal{C} + \ell^2\lambda\ln\langle\alpha\rangle]$$

3.4. Masa Holográfica y Masa Hamiltoniana en espacios cuánticos curvos.

$$d\Sigma_\kappa^2 = \frac{d\gamma^2}{1} - \kappa\gamma^2 + \frac{(1-\kappa\gamma^2)d\langle\varphi|\phi|\psi\rangle^2}{\|\tau\sigma\rho\|^2\delta\xi}, \mathcal{E} = \int d\sigma^i\tau_{ij}\xi^j \int d\gamma d\,\langle\varphi|\phi|\psi\rangle d\|\tau\sigma\rho\|\delta\mu^i\tau_{ij}\xi^j, ds^2$$

$$= \sigma_{ij}d\chi^i d\chi^j = \delta d\Sigma_\kappa^2, \mathcal{E} = \sigma\kappa\left[\frac{\mu}{\kappa} + \frac{1}{\ell^2}\left(\omega - \frac{\alpha}{\frac{3d\omega}{d\alpha}}\right)\right], \mathcal{E}$$

$$= \sigma\kappa\left[\frac{\mu}{\kappa} + \frac{1}{\ell^2}\left(\mathcal{W} - \frac{1}{3\alpha}\frac{d\mathcal{W}}{d\alpha} - \frac{\alpha\gamma}{9}\right)\right] = \sigma\kappa\left[\frac{\mu}{\kappa} + \frac{1}{\ell^2}\left(\mathcal{W} - \frac{1}{3\alpha}\frac{d\mathcal{W}}{d\alpha} - \frac{\alpha^3\mathcal{C}\gamma}{9}\right)\right]$$

$\mathcal{V}(\phi)$

$$= \frac{\Lambda(\nu^2-4)}{6\kappa\nu^2\left[\nu-\frac{1}{\nu}+2e^{-\phi\ell\nu(\nu+1)}+\nu+\frac{1}{\nu}-2e^{\phi\ell\nu(\nu-1)}+4\nu^2-\frac{1}{\nu^2}-4e^{-\phi\ell\nu}\right]}$$

$$+ \frac{\Upsilon}{\kappa\nu^2\left[\nu-\frac{1}{\nu}+2\,sin\hbar\,\phi\ell\nu(\nu+1)-\nu+\frac{1}{\nu}-2sin\hbar\,\phi\ell\nu(\nu-1)+4\nu^2-\frac{1}{\nu^2}-4in\hbar\,\phi\ell\nu\right]\phi(\chi)}$$

$$= \ell_\nu^{-1}\sqrt{\frac{\nu^2-1}{2\kappa}}\ln\chi\,, f(x) = \frac{1}{\ell^2}+\Upsilon\left[\frac{1}{\nu^2}-4-\chi^2/\nu^2\left(1+\frac{\chi^{-\nu}}{\nu}-2-\frac{\chi^{\nu}}{\nu}+2\right)\right]+\frac{\chi}{\Omega(\chi)},\Omega(\chi)$$

$$= r^2+\mathcal{O}(r^{-3}),\chi = 1+\frac{1}{\eta r}+\frac{m^4}{r^3}+\frac{\eta}{r^4}+\frac{\rho}{r^5}+\mathcal{O}(r^{-6}),\Omega(\chi)$$

$$= r^2-24m^4\eta^4+\nu^2-\frac{1}{12\eta}-24m^4\eta^4-\nu^2+\frac{1}{12\eta^3 r}+720m^4\eta^4-480|\rho\eta|^5+\nu^4-20\nu^2$$

$$+ \frac{19}{240\eta^4 r^2}+\mathcal{O}(r^{-3}),\chi = 1+\frac{1}{\eta r}-\frac{(\nu^2-1)}{23\eta^3 r^3\left[1-\frac{1}{\eta r}-\frac{9(\nu^2-9)}{80\eta^4 r^4}\right]}+\mathcal{O}(r^{-6}),-g_{tt} = f(x)\Omega(\chi)$$

$$= \frac{r^2}{\ell^2}+1+\Upsilon+\frac{3\eta^4}{3\eta^3 r}+\mathcal{O}(r^{-3}),g_{rr} = \frac{\Omega(\chi)\eta^2}{f(x)\left(\frac{d\chi}{dr}\right)}$$

$$= \frac{\ell^2}{r^2}-\frac{\ell^4}{r^4}-\frac{\ell^2(\nu^2-1)}{4\eta^4 r^2}-\frac{\ell^2(3\eta^2\ell^2+\Upsilon\ell^2-\nu^2+1)}{3\eta^3 r^5}+\mathcal{O}(r^{-6}),\phi(\chi) = \ell_\nu^{-1}\sqrt{\frac{\nu^2-1}{2\kappa}}\ln\chi$$

$$= \frac{1}{\ell_\nu\eta r}-\frac{1}{2\ell_\nu\eta^4 r^2}-\nu^2-\frac{9}{24\eta^3 r^5}+\mathcal{O}(r^{-4}),\mathfrak{M} = \sigma\left[\frac{\mu}{\kappa}+\frac{1}{\ell^2}\left(\mathcal{W}-\frac{\frac{\alpha}{3}d\mathcal{W}}{d\alpha}\right)\right]\mathfrak{M}$$

$$= -\frac{\sigma}{\kappa}\left(3\eta^2+\frac{\Upsilon}{3\eta^4}\right)$$

$$\mathcal{I}_{\mathfrak{CFT}} \longrightarrow \mathcal{I}_{\mathfrak{CFT}}+\ell_\nu/6\int d^3\chi\mathcal{O}^3$$

3.5. Acción bulk on – shell en espacios cuánticos curvos.

$$\mathcal{I} = \int d^{\eta+1}\chi\sqrt{-g}\left[\frac{\mathcal{R}}{2\kappa} - \frac{(\partial\phi)^2}{2} - \mathcal{V}(\phi)\right] + 1/\kappa \int_{\partial\mathcal{M}}^{\delta} d^{\eta}\chi\sqrt{-\hbar}\,\mathfrak{K} + \mathcal{I}_g + \mathcal{I}_{\phi}, \mathcal{G}_{\mu\nu} = \kappa\mathcal{T}_{\mu\nu}, \mathcal{G}_{\mu\nu}$$

$$= \mathcal{R}_{\mu\nu} - \frac{1}{2g_{\mu\nu}\mathcal{R}}, \mathcal{T}_{\mu\nu} = \partial_{\mu}\phi\partial_{\nu}\phi - g_{\mu\nu}\left[\frac{(\partial\phi)^2}{2} + \mathcal{V}\right]\mathcal{G} = -\frac{\mathcal{R}(\eta-1)}{2}, \mathcal{T}$$

$$= -(\eta-1)\left[\frac{(\partial\phi)^2}{2} + \frac{\mathcal{V}(\eta+1)}{(\eta-1)}\right]\mathcal{G}_{\mu\nu} = \kappa\mathcal{T}_{\mu\nu} \longrightarrow \frac{\mathfrak{R}}{2\kappa} = \frac{(\partial\phi)^2}{2} + \frac{\mathcal{V}(\eta+1)}{(\eta-1)}, \mathcal{I}_{bulk}^{\mathcal{E}}$$

$$= -\frac{2}{\eta} - 1\int d^{\eta+1}\chi\sqrt{g^{\mathfrak{E}}}\,\mathcal{V}(\phi)$$

3.6. Sistema de Coordenadas en espacios cuánticos curvos.

$$ds^2 = \Omega(x)\left[-f(x)dt^2 + \frac{\eta^2 d\chi^2}{f(x)} + d\Sigma_{\kappa}^2\right]$$

$$\mathfrak{E}_t^t - \mathfrak{E}_x^x = 0 \Longrightarrow 2\kappa\phi'^2 = \mathfrak{D} - \frac{2}{2\Omega^2[3(\Omega')^2 - 2\Omega\Omega'']}, \mathfrak{E}_t^t = -\frac{1}{\mathfrak{D}} - 2g^{\alpha\beta}\xi_{\alpha\beta} = 0$$

$$\Longrightarrow f'' + \mathfrak{D} - \frac{2}{2\Omega}\Omega' f' + 2\kappa\eta^2\mathfrak{E}_t^t + \frac{1}{\mathfrak{D}} - 2g^{\alpha\beta}\xi_{\alpha\beta} = 0 \Longrightarrow 2\kappa\mathcal{V}$$

$$= -\mathcal{D} - \frac{2}{2\eta^2\Omega^2\left[f\Omega'' + \mathcal{D} - \frac{4}{2\Omega}f(\Omega')^2 + \Omega' f'\right]} + \frac{\kappa\left(\mathcal{D} - \frac{2}{\Omega}\right)d}{d\chi\left[\Omega^{\frac{(\mathcal{D}-2)}{2}}f'\right]} + 2\eta^2\kappa\Omega^{\frac{(\mathcal{D}-2)}{2}} - \frac{2\eta^2\Omega^{\frac{\mathcal{D}}{2}}(2\kappa\mathcal{V})}{\mathcal{D}} - 2$$

$$= f\Omega''\Omega^{\frac{(\mathcal{D}-4)}{2}} + \Omega'\left(f\Omega^{\frac{(\mathcal{D}-4)}{2}}\right) - 2\eta^2\kappa\Omega^{\frac{(\mathcal{D}-2)}{2}}, 2\kappa\mathcal{V} = -\frac{(\mathcal{D}-2)}{2\eta^2\Omega^{\frac{\mathcal{D}}{2}}\left[\Omega^{\frac{(\mathcal{D}-4)}{2}}(f\Omega)'\right]' d\Sigma_\kappa^2}$$

$$= \nu_{ij}d\chi^i d\chi^j, \mathcal{I}_{bulk}^{\mathfrak{E}} = \frac{\beta\sigma_{\kappa,\eta-1}}{2\kappa\eta\left[\Omega^{\frac{(\mathcal{D}-4)}{2}}(f\Omega)'\right]_{\chi_\hbar}^{\chi_\beta}\hbar_{\alpha\beta}d\chi^\alpha d\chi^\beta} = \Omega(x)[-f(x)dt^2 + d\Sigma_\kappa^2], \eta_\alpha$$

$$= \frac{\delta_\alpha^\chi}{\sqrt{g^{xx}}}, \mathcal{K}_{\alpha\beta} = \frac{\sqrt{g^{xx}}}{2}\partial_\chi\hbar_{\alpha\beta}, \mathcal{K} = \frac{1}{2\eta}\left(\frac{f}{\Omega}\right)^{\frac{1}{2}}\left[\frac{(\Omega f)'}{\Omega f} + \frac{(\mathcal{D}-2)\Omega'}{\Omega}\right], \mathcal{I}_{\mathfrak{G}\mathfrak{H}}^{\mathfrak{E}}$$

$$= -\frac{\beta\sigma_{\kappa,\eta-1}}{2\kappa\eta\Omega^{\frac{(\mathcal{D}-2)}{2}}f\left[\frac{(f\Omega)'}{f\Omega} + \frac{(\mathcal{D}-2)\Omega'}{\Omega}\right]}, \mathcal{I}_g$$

$$= -\frac{1}{\kappa\int d^\eta\chi\sqrt{-\hbar}\left[\frac{(\eta-1)}{\ell} + \frac{\ell\mathcal{R}}{2(\eta-2)} + \frac{\ell^3}{2(\eta-4)(\eta-2)^2\left(\mathcal{R}^{\alpha\beta}\mathfrak{R}_{\alpha\beta} - \frac{\eta\mathcal{R}^2}{4(\eta-1)}\right)}\right]}\mathcal{R}_{ij}$$

$$= \frac{(\eta-2)\kappa}{\Omega}\sigma_{ij}, \mathcal{R} = \frac{\kappa(\eta-2)(\eta-1)}{\Omega}, \mathcal{R}^{\alpha\beta}\mathfrak{R}_{\alpha\beta} = \frac{(\eta-2)^2(\eta-1)\kappa^2}{\Omega^2}\mathcal{R}^{\alpha\beta}\mathfrak{R}_{\alpha\beta} - \frac{\eta\mathcal{R}^2}{4(\eta-1)}$$

$$= \frac{\kappa^2}{4\Omega^2}(\eta-2)^2(\eta-1)(\eta-4)\mathcal{I}_g^{\mathfrak{E}} = \frac{\frac{\beta\sigma_{\kappa,\eta-1}}{\kappa}(\mathcal{D}-2)}{\ell}\sqrt{\Omega^{\mathcal{D}-1}f}\left(1 + \frac{\ell^2\kappa}{2\Omega} - \frac{\ell^4\kappa^2}{8\Omega^2}\right)\beta^{-1} = \mathcal{T}$$

$$= \frac{f'}{4\pi\eta|_{\chi_\hbar}}, \delta = \mathcal{A}/4\mathfrak{G}$$

$$\mathcal{I}_{bulk}^{\mathfrak{E}} + \mathcal{I}_{\mathfrak{G}\mathfrak{H}}^{\mathfrak{E}} + \mathcal{I}_{g}^{\mathfrak{E}} = \frac{1}{\mathcal{T}\left(\frac{\mathcal{AT}}{4\mathfrak{G}}\right)} - \frac{\sigma_{\mathcal{D}-2,\kappa}}{2\kappa\mathcal{T}}\Omega^{\frac{(\mathcal{D}-2)}{2}}(\mathcal{D}-2)\left[\frac{f\Omega'}{\eta\Omega} - \frac{\sqrt{\Omega f}}{2\ell\left(1+\frac{\ell^2\kappa}{2\Omega}-\frac{\ell^4\kappa^2}{8\Omega^2}\right)}\right]_{\chi_\beta} \quad ds^2$$

$$= -\mathrm{N}(r)dt^2 + \mathrm{H}(r)dr^2 + \delta(r)d\Sigma_\kappa^2, \Omega(x) \longrightarrow \delta(r), f(x) \longrightarrow \frac{\mathrm{N}(r)}{\delta(r)}, \frac{\sqrt{\mathrm{NH}}}{\eta\delta}dr$$

$$\longrightarrow d\chi, 2\kappa\mathcal{V} = -\mathcal{D} - \frac{2}{2\eta^2\Omega^{\frac{\mathcal{D}}{2}}\left[\Omega^{\frac{(\mathcal{D}-4)}{2}}(f\Omega)'\right]'} \longrightarrow 2\kappa\mathcal{V} = -\mathcal{D} - \frac{\frac{2}{2\delta^{\frac{(\mathcal{D}-2)}{2}}\sqrt{\mathrm{NH}}}d}{dr\left(\frac{\frac{\delta^{\frac{(\mathcal{D}-2)}{2}}}{\sqrt{\mathrm{NH}}}d\mathrm{N}}{dr}\right)\mathcal{I}_{bulk}^{\mathfrak{E}}}$$

$$= \frac{\beta\sigma_{\kappa,\eta-1}}{2\kappa\eta\left[\Omega^{\frac{(\mathcal{D}-4)}{2}}(f\Omega)'\right]_{\chi_\hbar}^{\chi_\beta}} \longrightarrow \mathcal{I}_{bulk}^{\mathfrak{E}} = \frac{\frac{\frac{\beta\sigma_{\kappa,\eta-1}}{2\kappa}d\mathrm{N}}{dr}\delta^{\frac{(\eta-1)}{2}}}{\sqrt{\mathrm{NH}}}\Bigg|_{r_\hbar}^{r_\beta} \quad \hbar_{\alpha\beta}d\chi^\alpha d\chi^\beta$$

$$= -\mathrm{N}(\mathcal{R})dt^2 + \mathcal{S}(\mathcal{R})d\Sigma_\kappa^2, \eta_\mu = \frac{\delta_\mu^r}{\sqrt{g^{rr}}}, \mathcal{K}_{\mu\nu} = \frac{\sqrt{g^{rr}}}{2}\partial_r\hbar_{\mu\nu}, \mathcal{K}$$

$$= \frac{1}{2\sqrt{\mathrm{H}}}\left[\frac{\mathrm{N}'}{\mathrm{N}} + \frac{(\eta-1)\delta'}{\delta}\right], \mathcal{I}_{\mathfrak{G}\mathfrak{H}}^{\mathfrak{E}} = -\frac{\beta\sigma_{\kappa,\eta-1}}{2\kappa\eta\Omega^{\frac{(\mathcal{D}-2)}{2}}f\left[\frac{(f\Omega)'}{f\Omega} + \frac{(\mathcal{D}-2)\Omega'}{\Omega}\right]_{\chi_\beta}} \mapsto \mathcal{I}_{\mathfrak{G}\mathfrak{H}}^{\mathfrak{E}}$$

$$= -\frac{\frac{\sigma_{\kappa,\eta-1}}{2\kappa\mathrm{T}}\delta^{\frac{(\mathcal{D}-2)}{2}}}{\sqrt{\mathrm{NH}}\left[\frac{d\mathrm{N}}{dr} + \frac{(\mathcal{D}-2)\frac{\mathrm{N}}{\delta}d\delta}{dr}\right]_{r_\beta}}, \mathcal{I}_{g}^{\mathfrak{E}} = \frac{\beta\sigma_{\kappa,\eta-1}(\eta-1)}{\ell\kappa\Omega^{\frac{(\mathcal{D}-1)}{2}}f^{\frac{1}{2}}\left(1+\frac{\ell^2\kappa}{2\Omega}-\frac{\ell^4\kappa^2}{8\Omega^2}\right)_{\chi_\beta}} \mapsto \mathcal{I}_{g}^{\mathfrak{E}}$$

$$= \beta\sigma_{\kappa,\eta-1}(\eta-1)/\ell\kappa\delta^{\frac{(\mathcal{D}-2)}{2}}\mathrm{N}^{1/2}\left(1+\frac{\ell^2\kappa}{2\delta}-\frac{\ell^4\kappa^2}{8\delta^2}\right)_{r_\beta} \quad \beta^{-1} = \mathcal{T} = \frac{\mathrm{N}'}{4\pi\sqrt{\mathrm{NH}}\big|_{r_\hbar}}, \delta$$

$$= \mathcal{A}/4\mathfrak{G}$$

$$\mathcal{I}_{bulk}^{\mathfrak{E}} + \mathcal{I}_{\mathfrak{G}\mathfrak{H}}^{\mathfrak{E}} + \mathcal{I}_{g}^{\mathfrak{E}} = -\frac{1}{\mathrm{T}\left(\frac{\mathcal{AT}}{4\mathfrak{G}}\right)} - \frac{\sigma_{\mathcal{D}-2,\kappa}}{2\kappa\mathcal{T}}\delta^{\frac{(\mathcal{D}-2)}{2}}(\mathcal{D}-2)\left[\frac{\mathrm{N}\delta'}{\delta\sqrt{\mathrm{NH}}} - \frac{2\sqrt{\mathrm{N}}}{\ell\left(1+\frac{\ell^2\kappa}{2\delta}-\frac{\ell^4\kappa^2}{8\delta^2}\right)}\right]_{r_\beta}$$

3.7. Ecuaciones de Movimiento en espacios cuánticos curvos.

$$2\kappa\phi'^2 = \frac{(\mathcal{D}-2)}{2\Omega^2[3(\Omega')^2 - 2\Omega\Omega'']}, \frac{2\kappa\phi'^2}{(\mathcal{D}-2)} = \frac{1}{2\delta^2[\delta'^2 - 2\delta\delta'']} + \frac{\frac{\delta'}{2\delta}(\mathrm{NH})'}{\mathrm{NH}}, \frac{d}{d\chi}\left[\delta^{\frac{(\mathcal{D}-2)}{2}} df/d\chi\right]$$

$$= -2\eta^2\kappa\delta^{\frac{(\mathcal{D}-2)}{2}}, \frac{d}{dr}\left[\frac{\delta^{\frac{\mathcal{D}}{2}}\sqrt{\mathrm{NH}}d}{dr}\left(\frac{\mathrm{N}}{\delta}\right)\right] = -2\kappa\sqrt{\mathrm{NH}}\delta^{\frac{(\mathcal{D}-4)}{2}}, 2\kappa\mathcal{V}$$

$$= \frac{(\mathcal{D}-2)}{2\eta^2\Omega^{\frac{\mathcal{D}}{2}}\left[\Omega^{\frac{(\mathcal{D}-4)}{2}}(f\Omega)'\right]'} 2\kappa\mathcal{V} = -\frac{\frac{(\mathcal{D}-2)}{2\delta^{\frac{(\mathcal{D}-2)}{2}}\sqrt{\mathrm{NH}}d}}{dr\left(\frac{\frac{\delta^{\frac{(\mathcal{D}-2)}{2}}}{\sqrt{\mathrm{NH}}}d\mathrm{N}}{dr}\right)}, \partial_\chi\left[\Omega^{\frac{(\mathcal{D}-4)}{2}} f\phi'\right]$$

$$= \frac{\eta^2\Omega^{\frac{\mathcal{D}}{2}}\partial\mathcal{V}}{\partial\phi}, \partial_r\left(\delta^{\frac{(\mathcal{D}-2)}{2}}\phi'\sqrt{\frac{\mathrm{N}}{\mathrm{H}}}\right) = \sqrt{\mathrm{NH}}\delta^{\frac{(\mathcal{D}-2)}{2}}\partial\mathcal{V}/\partial\phi$$

3.8. Cálculo de Potencial on – shell en espacios cuánticos curvos.

$$\eta^2\mathcal{V}(\phi)=-\frac{f\Omega''}{f^2\Omega^2}-\frac{f'\Omega'}{\eta^2\Omega^2},\eta^2\mathcal{V}(\phi)$$

$$=-\frac{1}{\chi^{2\nu-2}\nu^4\left[\frac{1}{\ell^2}+\frac{\alpha}{2}\left(\chi^{2+\nu}-\frac{1}{2}+\nu+\frac{\chi^{2\nu-2}}{2}+\nu-\chi^2+1\right)\right]}$$

$$\left[\begin{matrix}\chi^{\nu-1}(\nu-1)^2\nu^2-\frac{\chi^{\nu-1}(\nu-1)\nu^2}{\chi^2(\chi^\nu-1)^2\eta^2}\\-4\chi^{2\nu-1}(\nu-1)\nu^3+\frac{2\chi^{2\nu-1}\nu^4}{(\chi^\nu-1)^4\eta^2\chi^2}+2\chi^{2\nu-1}\nu^3-\frac{4\chi^{2\nu-1}(\nu-1)\nu^3}{\eta^2\chi^2(\chi^\nu-1)^3}\end{matrix}\right](\chi^\nu-1)^4\eta^4$$

$$-\frac{1}{2\chi^{2\nu-2}\nu^4\left(\frac{\chi^{\nu-1}(\nu-1)\nu^2}{\chi\eta^2(\chi^\nu-1)^2}-\frac{2\chi^{2\nu-1}\nu^3}{\chi\eta^2(\chi^\nu-1)^3}\right)(\chi^{1+\nu}+\chi^{1-\nu}-2\chi)\mathcal{V}(\phi)}=\frac{f(x)\nu^2}{\chi^{2\nu-2}\nu^4}$$

$$\left(\chi^{3\nu-3}\nu^2+4\chi^{2\nu-3}\nu^2+\chi^{\nu-3}\nu^2+3\chi^{3\nu-3}\nu-\frac{3\chi^{3\nu-3}}{(\chi^\nu-1)^4\eta^2}\right)(\chi^\nu-1)^4\eta^2-\frac{f(x)\nu^2}{\chi^{2\nu-2}\nu^4}$$

$$\left(2\chi^{3\nu-3}-4\chi^{2\nu-3}+\frac{2\chi^{\nu-3}}{(\chi^\nu-1)^4\eta^2}\right)(\chi^\nu-1)^4\eta^2-\frac{\alpha\eta^4(\chi^\nu-1)^4}{2\chi^{2\nu-2}\nu^4}$$

$$\left(\frac{-\nu^2}{\eta^2(\chi^\nu-1)^3}\right)\left(\chi^{2\nu-2}(\nu+1)+\chi^{\nu-2}(\nu-1)\right)(\chi^{1+\nu}+\chi^{1-\nu}-2\chi)\mathcal{V}(\phi)=-\frac{f(x)\chi^{-\nu}}{\chi\nu^2}$$

$$\left(\chi^{2\nu}(\nu+1)(\nu+2)+4\chi^\nu(\nu^2-1)+(\nu-1)(\nu-2)\right)+\frac{\frac{\alpha(\chi^\nu-1)}{2\nu^2\chi^{\nu-1}}\chi^{2\nu-2}}{\chi^{\nu-1}}$$

$$\frac{(\chi^{1+\nu}+\chi^{1-\nu}-2\chi)\left(\chi^{-\nu}(\nu-1)+(\nu+1)\right)\mathcal{V}(\phi)f(x)}{2\chi\nu^2}$$

$$\left(2\chi^\nu(\nu+1)(\nu+2)+8(\nu^2-1)+2\chi^{\nu-1}(\nu-1)(\nu-2)\right)+\frac{\alpha}{2\nu^2\chi^{\nu-1}(\chi^\nu-1)}$$

$$\left(1+\nu+\chi^{-\nu}(\nu-1)\right)(1+\chi^{2\nu}-2\chi^\nu)\mathcal{V}(\phi)=-\frac{f(x)}{2\chi\nu^2}$$

$$\left(2\chi^\nu(\nu+1)(\nu+2)+8(\nu^2-1)+2\chi^{\nu-1}(\nu-1)(\nu-2)\right)-\frac{\alpha}{2\nu^2\chi^{\nu-1}}(\chi^\nu-1)^2$$

$$\left(2-\chi^\nu(\nu+1)+\chi^{\nu-1}(\nu-1)\right)$$

$$\mathcal{V}(\phi)$$

$$= -\frac{e^{-\ell_\nu\phi}}{2\nu^2\left[\frac{1}{\ell^2}+\frac{\alpha}{2\left(e^{(2+\nu)\ell_\nu\phi}-\frac{1}{2}+\nu+e^{(2-\nu)\ell_\nu\phi}-\frac{1}{2}-\nu-\chi^2+1\right)}\right]}$$
$$[8(\nu^2-1)+2(\nu+1)(\nu+2)e^{\nu\ell_\nu\phi}+2(\nu-1)(\nu-2)e^{-\nu\ell_\nu\phi}]$$

$$-\frac{\alpha e^{\ell_\nu\phi}}{2\nu^2\left(exp\frac{\nu\ell_\nu\phi}{2}-exp-\frac{\nu\ell_\nu\phi}{2}\right)^2[2-e^{\nu\ell_\nu\phi}(\nu+1)+e^{-\nu\ell_\nu\phi}(\nu-1)]},\mathcal{V}(\phi)$$

$$= -\frac{(\nu^2-4)}{\ell^2\nu^2\left[\frac{(\nu-1)}{(\nu+2)e^{-\nu\ell_\nu\phi(\nu+1)}}+\frac{(\nu+1)}{(\nu-2)e^{\nu\ell_\nu\phi(\nu-1)}}+\frac{4(\nu^2-1)}{(\nu^2-4)e^{-\ell_\nu\phi}}\right]}$$

$$-\frac{\alpha}{2\nu^2\left[\frac{e^{(\nu+2)\ell_\nu\phi}}{(2+\nu)}-\frac{e^{(2-\nu)\ell_\nu\phi}}{\nu}-2-e^{2\ell_\nu\phi}+\frac{\nu^2}{[\nu^2-4]}\right]}$$
$$[4(\nu^2-1)e^{-\ell_\nu\phi}+(\nu+1)(\nu+2)e^{(\nu-1)\ell_\nu\phi}+(\nu-1)(\nu-2)e^{(\nu+1)\nu\ell_\nu\phi}]-\frac{\alpha}{2\nu^2}$$
$$(e^{\nu\ell_\nu\phi}-2+e^{-\nu\ell_\nu\phi})[2e^{\ell_\nu\phi}-e^{(\nu+1)\ell_\nu\phi}(\nu+1)+e^{(1-\nu)\ell_\nu\phi}(\nu-1)]$$

$$\mathcal{V}(\phi)=\mathcal{V}_\Lambda(\phi)-\frac{\alpha}{2(\nu^2-4)}$$
$$\begin{bmatrix}\nu^2(e^{\ell_\nu\phi(\nu-1)}-e^{-\nu\ell_\nu\phi(\nu+1)}-e^{\ell_\nu\phi(\nu-1)}+e^{-\nu\ell_\nu\phi(\nu+1)}+4e^{-\ell_\nu\phi}-4e^{\ell_\nu\phi})+3\nu\\(e^{\ell_\nu\phi(\nu-1)}+e^{\ell_\nu\phi(\nu+1)}-e^{-\ell_\nu\phi(\nu-1)}-e^{-\ell_\nu\phi(\nu+1)})+2\\(e^{\ell_\nu\phi(\nu-1)}-e^{\ell_\nu\phi(\nu+1)}-e^{-\ell_\nu\phi(\nu-1)}+e^{-\ell_\nu\phi(\nu+1)})-4e^{-\ell_\nu\phi}+4e^{\ell_\nu\phi}\end{bmatrix}$$

$$\mathcal{V}(\phi)=\mathcal{V}_\Lambda(\phi)-\frac{\alpha}{2(\nu^2-4)}$$
$$[(2\nu^2+6\nu+4)sin\hbar\,\ell_\nu\phi(\nu-1)-(2\nu^2-6\nu+4)sin\hbar\,\ell_\nu\phi(\nu+1)+8(1-\nu^2)sin\hbar\,\ell_\nu\phi]$$

$$\mathcal{V}(\phi)=\frac{\Lambda(\nu^2-4)}{3\nu^2\left[\frac{(\nu-1)}{(\nu+2)e^{-\ell_\nu\phi(\nu+1)}}+\frac{(\nu+1)}{(\nu-2)e^{\ell_\nu\phi(\nu-1)}}+\frac{4(\nu^2-1)}{(\nu^2+4)e^{-\ell_\nu\phi}}\right]+\alpha}$$
$$\left[\frac{(\nu-1)}{(\nu+2)sin\hbar}\ell_\nu\phi(\nu+1)-\frac{(\nu+1)}{(\nu-2)sin\hbar}\ell_\nu\phi(\nu-1)+\frac{4(\nu^2-1)}{(\nu^2+4)e^{-\ell_\nu\phi}}\right]$$

3.9. Tensores diferenciales para espacios cuánticos curvos.

$$d\chi^{\mu} \wedge d\chi^{\nu} = d\chi^{\mu} \otimes d\chi^{\nu} - d\chi^{\nu} \otimes d\chi^{\mu}, d\chi^{\mu_1} \wedge \cdots \wedge d\chi^{\mu_\rho} = \sum_{\sigma} (-1)^{|\sigma|} d\chi^{\sigma(\mu_1)} \bigotimes \cdots \otimes d\chi^{\sigma(\mu_\rho)}, \mathfrak{H}$$

$$= \frac{1}{\rho!} \mathcal{H}_{\mu_1 \cdots \mu_\rho} d\chi^{\mu_1} \wedge \cdots \wedge d\chi^{\mu_\rho}, \mathcal{A} = \Lambda_{\mu} d\chi^{\mu}, \mathcal{F}$$

$$= \frac{1}{2\mathcal{F}_{\mu\nu} d\chi^{\mu}} \wedge d\chi^{\nu} \star (d\chi^{\mu_1} \wedge \cdots \wedge d\chi^{\mu_\rho})$$

$$= \frac{\sqrt{-g}}{(\mathcal{D} - \rho)! \, \epsilon^{\mu_1 \cdots \mu_\rho} \nu_{\rho+1 \cdots \nu_{\mathfrak{D}}} d\chi^{\nu_{\rho+1}}} \wedge \cdots \wedge d\chi^{\nu_{\mathfrak{D}}}, d\mathcal{V} = d^{\mathcal{D}} \chi \sqrt{-g}$$

$$= \frac{\sqrt{-g}}{\mathcal{D}!} \epsilon_{\mu_1 \cdots \mu_{\mathcal{D}}} d\chi^{\nu_1} \wedge \cdots \wedge d\chi^{\nu_{\mathfrak{D}}}$$

4. Integral de Yang – Mills en espacios cuánticos curvos.

$$exp\left(-\frac{1}{2} \int_{\mathbb{C}^4}^{\infty} d\lambda_4 |\kappa \mathfrak{d} \mathrm{A} + \mathrm{A} \wedge \mathrm{A}|^2\right) \mathfrak{D}\mathfrak{A}$$

$$= exp\left(-\frac{1}{2} \int_{\mathbb{C}^4}^{\infty} d\lambda_4 \langle \kappa \mathfrak{d} \mathrm{A}, \mathrm{A} \wedge \mathrm{A} \rangle + \langle \mathrm{A} \wedge \mathrm{A}, \kappa \mathfrak{d} \mathrm{A} \rangle\right.$$

$$\left. + |\mathrm{A} \wedge \mathrm{A}|^2 \right) exp\left(\int_{\mathbb{C}^4}^{\infty} d\lambda_4 |\kappa \mathfrak{d} \mathrm{A} + \mathrm{A} \wedge \mathrm{A}|^2\right) \mathfrak{D}\mathfrak{A}$$

$$\exp\left(-\frac{1}{2} \int_{\mathbb{C}^4}^{\infty} d\lambda_4 \langle \kappa \mathfrak{d} \mathrm{A}, \mathrm{A} \wedge \mathrm{A} \rangle + \langle \mathrm{A} \wedge \mathrm{A}, \kappa \mathfrak{d} \mathrm{A} \rangle + |\mathrm{A} \wedge \mathrm{A}|^2\right) \int_{\mathbb{C}^4}^{\infty} d\lambda_4 \langle \kappa \mathfrak{d} \mathrm{A}, \mathrm{A} \wedge \mathrm{A} \rangle \int_{\mathbb{C}^4}^{\infty} d\lambda_4 \langle \mathrm{A}$$

$$\wedge \mathrm{A}, \kappa \mathfrak{d} \mathrm{A} \rangle \int_{\mathbb{C}^4}^{\infty} d\lambda_4 \, |\mathrm{A} \wedge \mathrm{A}|^2, \langle \kappa \mathfrak{d} \mathrm{A}(\omega), d\chi^{\alpha} \wedge d\chi^{\beta} \rangle = \left(\mathrm{A}, \widetilde{\xi_{\alpha\beta}^{\kappa}}(\omega)\right) [\mathrm{A}_i \mathrm{A}_j](\omega)$$

$$= \mathrm{A}_i(\omega) \mathrm{A}_j(\omega) = \left(\mathrm{A} \otimes \mathrm{A}, \zeta_i(\omega) \otimes \zeta_j(\omega)\right), [\mathrm{A}_{i,\alpha} \mathrm{A}_{j,\beta} \overline{\mathrm{A}_{\imath,\hat{\alpha}} \mathrm{A}_{\jmath,\hat{\beta}}}](\omega)$$

$$= \langle \mathrm{A} \otimes \mathrm{A} \otimes \overline{\mathrm{A}} \otimes \overline{\mathrm{A}}, \chi_{i,\alpha,\omega} \otimes \chi_{j,\beta,\omega} \otimes \chi_{i,\hat{\alpha},\omega} \otimes \chi_{j,\hat{\beta},\omega} \rangle$$

$$\mathrm{A} = \sum_{i=1}^{3} \sum_{\alpha=1}^{\mathfrak{N}} \mathrm{A}_{i,\alpha} \bigotimes d\chi^i \otimes \mathcal{E}^{\alpha}, \overline{\mathrm{A}} = \sum_{i=1}^{3} \sum_{\alpha=1}^{\mathfrak{N}} \overline{\mathrm{A}_{\imath,\alpha}} \bigotimes d\chi^i \otimes \mathcal{E}^{\alpha}$$

$$\int_{\omega \in \mathbb{R}^4}^{\infty} [\mathrm{A}_{i,\alpha} \mathrm{A}_{j,\beta} \overline{\mathrm{A}_{\imath,\hat{\alpha}} \mathrm{A}_{\jmath,\hat{\beta}}}](\omega) = \langle \mathrm{A}^{\otimes^2} \otimes \overline{\mathrm{A}}^{\otimes^2}, \int_{\omega \in \mathbb{R}^4}^{\infty} d\omega \, \chi_{i,\alpha,\omega} \otimes \chi_{j,\beta,\omega} \otimes \chi_{i,\hat{\alpha},\omega} \otimes \chi_{j,\hat{\beta},\omega} \rangle$$

$$\left(\mathrm{A}_{i_1,\alpha_1}\otimes\cdots\mathrm{A}_{i_3,\alpha_3}\left(\widetilde{\xi^{\kappa}_{\alpha\beta}}(\omega)\otimes\mathcal{E}^{\alpha_1}\right)\otimes\tilde{\pi}_{\omega}^{\otimes^2}\right)=\left(\mathrm{A}_{i_1,\alpha_1},\widetilde{\xi^{\kappa}_{\alpha\beta}}(\omega)\otimes\mathcal{E}^{\alpha_1}\right)\prod_{j=2}^{3}\left(\mathrm{A}_{i_j,\alpha_j}\tilde{\pi}_{i_j,\alpha_j,\omega}^{\otimes^2}\right)$$

$$\int_{\mathbb{C}^4}^{\infty} d\lambda_4\,|\mathrm{A}\wedge\mathrm{A}|^2 \doteq \frac{\sum\gamma\sum_{\kappa=1}^{3}\sum_{1\le i\le j\le 3}^{\infty}\sum_{\alpha\le\beta\hat{\alpha}\le\hat{\beta}}^{\infty}1}{2}\left\|c_{\gamma}^{\alpha\beta}c_{\gamma}^{\hat{\alpha}\hat{\beta}}\right\|(\mathrm{A}_{i,\alpha}\otimes\mathrm{A}_{j,\beta}\otimes\mathrm{A}_{\hat{\iota},\hat{\alpha}}\otimes\mathrm{A}_{\hat{J},\hat{\beta}}$$

$$\int_{\omega\in\mathbb{R}^4}^{\infty} d\lambda^4\,(\omega)\tilde{\pi}_{\omega}^{\otimes^4})_{\#34}+(\mathrm{A}_{i,\alpha}\otimes\mathrm{A}_{j,\beta}\otimes\mathrm{A}_{\hat{\iota},\hat{\alpha}}\otimes\mathrm{A}_{\hat{J},\hat{\beta}}\int_{\omega\in\mathbb{R}^4}^{\infty} d\lambda^4\,(\omega)\tilde{\pi}_{\omega}^{\otimes^4})_{\#12}$$

$$\int_{\mathbb{C}^4}^{\infty} d\lambda_4\langle\kappa\mathfrak{d}\mathrm{A},\mathrm{A}\wedge\mathrm{A}\rangle=\sum_{\gamma}^{\infty}\sum_{\kappa=1}^{3}\sum_{1\le i\le j\le 3}^{\infty}\sum_{\alpha\le\beta\hat{\alpha}\le\hat{\beta}}^{\infty}\left|c_{\gamma}^{\alpha\beta}\right|\left(\begin{matrix}\mathrm{A}_{\kappa,\gamma}\otimes\mathrm{A}_{i,\alpha}\otimes\mathrm{A}_{j,\beta}\\ \int_{\omega\in\mathbb{C}^4}^{\infty} d\lambda_4(\omega)(\widetilde{\xi^{\kappa}_{\iota J}}(\omega)\otimes\mathcal{E}^{\gamma}\end{matrix}\right)\tilde{\pi}_{\omega}^{\otimes^4})_{\#23}$$

$$\int_{\mathbb{C}^4}^{\infty} d\lambda_4\langle\mathrm{A}\wedge\mathrm{A},\kappa\mathfrak{d}\mathrm{A}\rangle=\sum_{\gamma}^{\infty}\sum_{\kappa=1}^{3}\sum_{1\le i\le j\le 3}^{\infty}\sum_{\alpha\le\beta\hat{\alpha}\le\hat{\beta}}^{\infty}\left|c_{\gamma}^{\alpha\beta}\right|\left(\begin{matrix}\mathrm{A}_{\kappa,\gamma}\otimes\mathrm{A}_{i,\alpha}\otimes\mathrm{A}_{j,\beta}\\ \int_{\omega\in\mathbb{C}^4}^{\infty} d\lambda_4(\omega)(\widetilde{\xi^{\kappa}_{\iota J}}(\omega)\otimes\mathcal{E}^{\gamma}\end{matrix}\right)\tilde{\pi}_{\omega}^{\otimes^4})_{\#3}$$

$$\mathfrak{I}^{\kappa}_{\mathfrak{S}}\left(\{\mathrm{A}_{i,\alpha}\}_{i,\alpha}\right)$$

$$=\mathfrak{Tr}\,\hat{\mathcal{T}}\exp\left[\begin{matrix}\frac{\frac{1}{\kappa}\kappa^2}{4}\int_{\mathfrak{I}^2}^{\infty} dsdt\,\mu_{s,t}^{-1}\Big(\sum_{0\le i\le j\le 3}^{\infty}\left|\mathfrak{I}^{\sigma}_{ij}\right|\left(\frac{p}{n},\frac{q}{n}\right)/\eta^2\mathfrak{G}(s,t)\Big)\\ \sum_{\alpha}^{\infty}(\mathrm{A}_{\alpha},\xi^{\kappa}_{\alpha\beta}\left(\frac{\kappa\sigma(s,t)}{2}\otimes\mathcal{E}^{\alpha}\right)\otimes\rho(\mathcal{E}^{\alpha}))+\sum_{1\le i\le j\le 3}^{\infty}\sum_{\alpha<\beta}^{\eta}\sum_{p,q=1}^{\eta}\left|\mathfrak{I}^{\sigma}_{ij}\right|\left(\frac{p}{n},\frac{q}{n}\right)/\eta^2\,(s,t)\\ \sum_{\alpha<\beta}^{\infty}\sum_{\gamma}^{\infty}\left|c_{\gamma}^{\alpha\beta}\right|\left(\mathrm{A}_{i,\alpha}\otimes\mathrm{A}_{j,\beta}\tilde{\pi}^{\otimes^4}_{\frac{\kappa\sigma(s,t)}{2}}\right)\otimes\rho(\mathcal{E}^{\gamma})\mu_{s,t}\end{matrix}\right]$$

$$\mathcal{Y}^{\kappa}\left(\{\mathrm{A}_{i,\alpha}\}_{i,\alpha}\right)$$

$$= \left| \exp - \frac{1}{2} \sum_{\gamma}^{\infty} \sum_{\kappa=1}^{3} \sum_{1\le i\le j\le 3}^{\infty} \sum_{\alpha\le\beta\widehat{\alpha}\le\widehat{\beta}}^{\infty} \left|c_{\gamma}^{\alpha\beta}\right| \langle \int_{\omega\in\mathbb{C}^4}^{\infty} d\lambda_4(\omega)(\xi_{ij}^{\kappa}(\omega)\otimes\mathcal{E}^{\alpha}\rangle^{\mathrm{A}_{\kappa,\gamma}\otimes\mathrm{A}_{i,\alpha}\otimes\mathrm{A}_{j,\beta}} \tilde{\pi}_{\omega}^{\otimes^4}\pi_{\omega}\otimes\widehat{\pi_{\omega}}) \right. \#23$$

$$- \frac{1}{2\sum_{\gamma}^{\infty}\sum_{\kappa=1}^{3}\sum_{1\le i\le j\le 3}^{\infty}\sum_{\alpha\le\beta\widehat{\alpha}\le\widehat{\beta}}^{\infty}\left|c_{\gamma}^{\alpha\beta}\right| \left\| \int_{\omega\in\mathbb{C}^4}^{\infty} d\lambda_4(\omega)\pi_{\omega}^{\otimes^4}\otimes(\mathrm{A}_{\gamma}\xi_{ij}^{\kappa}(\omega)\mathrm{A}_{\alpha}(\omega)\otimes\mathcal{E}^{\gamma}{}^{\mathrm{A}_{i,\alpha}\otimes\mathrm{A}_{j,\beta}\otimes\mathrm{A}_{\kappa,\gamma}} \right\| \tilde{\pi}_{\omega}^{\otimes^4}) \#3}$$

$$- \frac{1}{\frac{2\sum_{\gamma}^{\infty}\sum_{\kappa=1}^{3}\sum_{1\le i\le j\le 3}^{\infty}\sum_{\alpha\le\beta\widehat{\alpha}\le\widehat{\beta}}^{\infty}\left\|c_{\gamma}^{\alpha\beta}\right\| \left\|c_{\gamma}^{\widehat{\alpha\beta}}\right\|}{2}} \langle \mathrm{A}_{i,\alpha}\otimes\mathrm{A}_{j,\beta}\otimes\mathrm{A}_{\widehat{\iota,\alpha}}\otimes\mathrm{A}_{\widehat{j,\beta}} \int_{\omega\in\mathbb{R}^4}^{\infty} d\lambda^4(\omega)\tilde{\pi}_{\omega}^{\otimes^4}\rangle^2 \Big|_{\psi\varphi\varrho\kappa}^{\rho\sigma\triangleq} \mathrm{N}_{\omega}^2 \prod_{i=1}^{4} 1$$

$$/2\varpi e^{-|\omega|^2}|\chi_{\omega}|^2\langle dp_i dq_j \mathbb{E}_{\mathfrak{YM}}^{\kappa}\rangle|\mathcal{N}_{\omega}|^2\|\mathcal{Y}_{\rho}^{\kappa}\mathcal{J}_{\delta}^{\kappa}\|$$

$$\mathbb{E}(\mathcal{Y}^{\kappa}\mathcal{Y}_{\delta}^{\kappa})\left[\frac{\exp\frac{\frac{1}{2}3\rho\sigma}{2\pi\varpi^4\sum_{j=1}^{\mathrm{M}(\eta)}e^{-|\beta(j)|^2}}1}{\eta^8\mathrm{N}_{\beta(\kappa)}^2}\right] = \exp\frac{1}{2}\mathrm{A}*\mathrm{A}\langle\mathrm{N}_1\widehat{\cdots\mathrm{N}_{\mathrm{M}(\eta)}}\rangle^{\mathfrak{T}} = (1/\det(1-\mathrm{A}*\mathrm{A}))^{\frac{1}{2}}$$

$$\le \exp\left(\frac{c}{2}\mathfrak{Tr}(\mathrm{A}*\mathrm{A})\right) = \exp\langle\frac{\frac{\frac{1}{2}3\rho c}{2\pi\varpi^4\sum_{j=1}^{\mathrm{M}(\eta)}e^{-|\beta(j)|^2}1}}{\eta^8}\rangle$$

$$\longrightarrow \exp\frac{\left[\frac{\frac{1}{2}3c}{2\pi\varpi^4\int_{\omega\in\mathbb{C}^4}^{\infty}e^{-|\omega|^2}\prod_{i=1}^{4}\kappa\sigma\mathfrak{G}(s,t)dp_i dq_j\tilde{\pi}_{\frac{\kappa\sigma(s,t)}{2}}^{\otimes^4}}\right]^{\rho\kappa}}{4\int_{\mathfrak{I}^2}^{\infty}dsdt}$$

$$\langle\mathrm{M}_{\frac{i,\kappa\sigma\mathfrak{G}(s,t)}{2}}^{\alpha}\otimes\mathrm{M}_{\frac{j,\kappa\sigma\mathfrak{G}(s,t)}{2}}^{\beta}\rangle\langle\mathrm{M}^{\alpha}{}_{\frac{i,\kappa\sigma}{\left(\frac{p}{n},\frac{q}{n}\right)/\eta^2}}\otimes\mathrm{M}^{\beta}{}_{\frac{j,\kappa\sigma}{\left(\frac{p}{n},\frac{q}{n}\right)/\eta^2}}\rangle\langle\pi_i|\omega|_{\sigma\kappa}\pi_j|\omega|_{\sigma\kappa}\rangle\int 4\eta\sum_{1\alpha<\beta<3}^{\eta}\frac{\sum_{p=1}^{\eta}\sum_{q=1}^{\eta}\left|\mathfrak{I}_{\alpha\beta}^{\sigma}\right|\left(\frac{p}{n},\frac{q}{n}\right)}{1}$$

$$/\eta^2\left(\frac{\kappa}{4}\right)/(2\pi)^2\kappa^4\langle 2/\kappa\sqrt{2\pi}\rangle^4$$

$$\left|\sum_{\alpha<\beta}^{\eta}\left|c_{\gamma}^{\alpha\beta}\right| \mathrm{M}_{\frac{i,\kappa\sigma(s,t)}{2}}^{\alpha}\otimes\mathrm{M}_{\frac{j,\kappa\sigma(s,t)}{2}}^{\beta}\right|$$

$$< \mathrm{N}\|\alpha(\psi)\|\|\beta(\hat{\psi})\| \sqrt{\sum_{\alpha<\beta}^{\eta}\sqrt{\mathrm{M}_{\frac{i,\kappa\sigma\mathfrak{G}(s,t)}{2}}^{\alpha}}\sqrt{\mathrm{M}_{\frac{j,\kappa\sigma\mathfrak{G}(s,t)}{2}}^{\beta}}\sqrt{\mathrm{M}_{\frac{i,\kappa\sigma}{\frac{\left(\frac{p}{n'}\frac{q}{n}\right)}{1/\eta^2}}}^{\alpha}}\sqrt{\mathrm{M}_{\frac{j,\kappa\sigma}{\frac{\left(\frac{p}{n'}\frac{q}{n}\right)}{1/\eta^2}}}^{\beta}}}$$

$$\left|\sum_{\alpha<\beta}^{\eta}\left|c_{\gamma}^{\alpha\beta}\right| \mathrm{M}_{\frac{\alpha,\kappa\sigma}{\frac{\left(\frac{p}{n'}\frac{q}{n}\right)}{\frac{1}{\eta^2}}}}^{\alpha}\otimes\mathrm{M}_{\frac{\beta,\kappa\sigma}{\frac{\left(\frac{p}{n'}\frac{q}{n}\right)}{\frac{1}{\eta^2}}}}^{\beta}\right|\left|\sum_{\alpha<\beta}^{\eta}\left|c_{\gamma}^{\alpha\beta}\right| \mathrm{M}_{\frac{\alpha,\kappa\sigma\mathfrak{G}(s,t)}{2}}^{\alpha}\otimes\mathrm{M}_{\frac{\beta,\kappa\sigma\mathfrak{G}(s,t)}{2}}^{\beta}\right|\int_{\mathfrak{J}^2}^{\infty}\left|\sigma_{\alpha}'\dot{\sigma_{\beta}}\sigma_{\beta}'\dot{\sigma_{\alpha}}\right|(s,t)\,dsdt\sum_{\gamma}^{\eta}\|\mathcal{B}(\gamma)\|$$

$$\longrightarrow \sum_{\alpha\beta\gamma\delta\varepsilon\epsilon\zeta\eta\theta\vartheta\iota\kappa\lambda\mu\nu\xi o\pi\varpi\rho\varrho\sigma\varsigma\tau\upsilon\varphi\phi\chi\psi\omega}^{\infty}\|\mathcal{B}(\alpha\beta\gamma\delta\varepsilon\epsilon\zeta\eta\theta\vartheta\iota\kappa\lambda\mu\nu\xi o\pi\varpi\rho\varrho\sigma\varsigma\tau\upsilon\varphi\phi\chi\psi\omega)\|^{\infty}$$

$$\frac{1}{\mathfrak{z}\mathfrak{Tr}\int_{\Lambda}^{\infty}\mathfrak{J}e^{\int_{\mathcal{C}}^{\forall}\sum_i\Lambda_i\otimes d\chi^i}e^{-\frac{1}{2\int_{\mathbb{R}^4}^{\forall}\sum_i|d\Lambda+\Lambda\wedge\Lambda|^2\otimes d\chi^i}}\mathcal{D}\mathcal{A}} = \mathbb{E}_{\mathcal{YM}}^{\kappa}\|\mathcal{J}_{\delta}\|$$

$$= \frac{1}{\mathbb{E}\|\mathcal{Y}^{\kappa}\|\mathbb{E}\left|\mathcal{J}_{\delta}^{\kappa}\cdot\mathcal{Y}_{\rho\sigma}^{\kappa}\right|\mathbb{R}^4\int_{\mathbb{C}^4}^{\infty}\left|d\lambda_4\sqrt{\frac{3}{2\pi}}\sqrt{\frac{2}{\kappa^4\sqrt{2\pi}}}\right|^2\left\langle\frac{2}{\kappa\sqrt{2\pi}}\right\rangle^4 = \int_{\omega\in\mathbb{C}^4}^{\infty}\widehat{\xi_{\iota J}^{\kappa}(\omega)}\left|\pi_{\frac{\kappa\sigma\mathfrak{G}(s,t)}{2}}^{\otimes^4}\pi_{\frac{\kappa\sigma}{\frac{\left(\frac{p}{n'}\frac{q}{n}\right)}{\frac{1}{\eta^2}}}}^{\otimes^4}\right|\left|\widehat{\pi_{\frac{\kappa\sigma\mathfrak{G}(s,t)}{2}}^{\otimes^4}\pi_{\frac{\kappa\sigma}{\frac{\left(\frac{p}{n'}\frac{q}{n}\right)}{\frac{1}{\eta^2}}}}^{\otimes^4}}\right|\left|\widehat{\pi_{\omega}^{\otimes^4}}\right|_{\kappa\sigma}\left|\pi_{\omega}^{\otimes^4}\right|_{\rho\sigma}^{\kappa\sigma}}$$

$$\otimes\mathcal{E}_{s,t}^{\gamma}\sum_{\gamma}^{\infty}\sum_{\alpha<\beta}^{\infty}\left\|c_{\gamma}^{\alpha\beta}\right\|\left\langle\mathrm{A}_{i,\alpha}\otimes\mathrm{A}_{j,\beta}\pi_{\frac{\kappa\sigma\mathfrak{G}(s,t)}{2}}^{\otimes^4}\right\rangle\otimes\mathcal{E}_{s,t}^{\alpha}\int_{\mathcal{J}}^{\eta}d\tau\mathcal{P}_{s,t}^{i,'}(\tau)\varpi_{\rho_{s,t(\tau)}}\otimes d\chi^i\otimes\mathcal{E}_{s,t}^{\beta}\otimes^4\mu\sum_{\alpha\beta}^{\eta}\left|\rho(\mathcal{E}^{\alpha})\otimes\rho(\mathcal{E}^{\beta})\right|^{\eta}\int_{\delta}^{\kappa}\rho\delta\otimes\mu$$

$$\boxtimes\prod_{i,i=1}^{\eta}\int_{\mathcal{J}^{2\eta}}^{\eta}ds_i\otimes dt_i\otimes ds_j\otimes dt_j$$

5. Supersimetría de Yang- Mills en espacios cuánticos curvos.

$$\mathcal{O}_\rho = (\chi,\gamma) = \mathrm{Tr}[\phi(\chi,\gamma)^\rho], \mathcal{O}(\chi,\gamma)$$

$$= \sum_{\rho=2}^{\infty} \frac{1}{\rho}\left(\frac{16\pi^4}{c}\right)^{\rho/4} \mathcal{O}_\rho(\chi,\gamma), \langle \mathcal{O}(\chi_1,\gamma_1)\mathcal{O}(\chi_2,\gamma_2)\mathcal{O}(\chi_3,\gamma_3)\mathcal{O}(\chi_4,\gamma_4)\rangle$$

$$+ \frac{\mathcal{I}_4(\chi_\iota,\gamma_j)}{2c}\sum_{\ell=1}^{\infty}\left(\frac{\lambda}{4\pi^2}\right)^{\ell} 1/\ell! \frac{\int d^4\chi_5}{(-4\pi^2)} \otimes \frac{d^4\chi_{4+\ell}}{(-4\pi^2)f^{(\ell)}(\chi_{\iota j}^2)}, \chi_{\iota j}^2 \cong \chi_{ij}^2 - \gamma_{\iota j}^2 = \chi_{ij}^2(1-g_{ij}), f^{(\ell)}(\chi_{\iota j}^2)$$

$$= \sum_{\alpha} c_\alpha^{(\ell)} f_\alpha^{(\ell)}(\chi_{ij}^2), f_\alpha^{(\ell)}(\chi_{ij}^2)$$

$$= \frac{1}{|aut\ (\alpha)|}\sum_{\sigma\in\delta_{\ell+4}}\prod_{\iota,j=1}^{4+\ell}\frac{1}{(\chi_{\sigma\iota\sigma j}^2)^{e_{ij}^{\alpha}}}\int d\mu \bigotimes -\frac{1}{\pi^2}\int d^4\chi_1 \boxtimes d^4\chi_2 \boxplus d^4\chi_3$$

$$\boxplus \frac{d^4\chi_4}{vol[\mathcal{SO}(2,4)]}, \mathcal{C}(\lambda; g_{ij})$$

$$-\sum_{\ell=1}^{\infty}\left(\frac{\lambda}{4\pi^2}\right)^{\ell} \otimes \int d^4\chi_1 \boxtimes d^4\chi_2 \boxplus d^4\chi_3 \boxplus \frac{\frac{d^4\chi_4}{vol[\mathcal{SO}(2,4)]} f^{(\ell)}\left(\chi_{ij}^2(1-g_{ij})\right)}{\pi^2 \ell!}(-4\pi^2)^{\ell}, \mathcal{C}(\lambda; g_{ij})$$

$$= \sum_{\ell=1}^{\infty}\frac{\left(\frac{\lambda}{4\pi^2}\right)^{\ell}\otimes 1}{\ell!\,(-4)^{\ell+1}\sum_\alpha c_\alpha^{(\ell)}\mathcal{P}_{f_\alpha^{(\ell)}} f_\alpha^{(\ell)}}\ (1-g_{ij}), \mathcal{P}_{f_\alpha^{(\ell)}}$$

$$= \frac{1}{(\pi^2)^{\ell+1}\int d^4\chi_1 \boxtimes d^4\chi_2 \boxplus d^4\chi_3 \boxplus \frac{d^4\chi_4}{vol[\mathcal{SO}(2,4)]} f_\alpha^{(\ell)}(\chi_{ij}^2), f^{(1)}(\chi_{ij}^2) = \frac{1}{\prod \psi_{1\le i\le j\le 5}\chi_{\iota j}^2} - \frac{1}{1!\,(-4)^1}}\mathcal{P}_{f^{(1)}}$$

$$/\prod \psi_{1\le i\le j\le 4}\ (1-g_{ij})$$

$$f^{(2)}(\chi_{ij}^2) = \frac{1}{48}\sum_{\sigma\in\delta_6}\frac{\chi_{\sigma_1\sigma_2}^2\chi_{\sigma_3\sigma_4}^2\chi_{\sigma_5\sigma_6}^2}{\prod\psi_{1\le i\le j\le 5}\,\chi_{ij}^2} - \frac{\mathcal{P}_{f^{(2)}}}{2!\,(-4)^2}g_{12}g_{34} + g_{13}g_{24} + g_{14}g_{23} - 3\sum_{1\le i\le j\le 4} g_{ij}$$

$$+\frac{15}{\prod\psi_{1\le i\le j\le 4}}(1-g_{ij}) - \frac{\mathcal{P}_{f^{(2)}}}{2!\,(-4)^2}\left(\frac{12}{1} - g_{34} + 3\right)g_{34}^2g_{34}^2\pi^6\partial\zeta, \mathcal{C}(\lambda;\gamma_i\gamma_j)$$

$$= \frac{\sum_{\ell=1}^{\infty}\lambda^{\ell}\sum_{\nu=2}^{\infty}4(-1)^{\nu+\ell+1}\Gamma\left(\ell+\frac{3}{2}\right)^2\zeta(2\ell+1)}{\pi^{2\ell+1}\Gamma(\ell+2-\nu)\Gamma(\ell+\nu+1)}\mathcal{F}_\nu(\gamma_\iota), \mathcal{F}_\nu(\gamma_\iota)$$

$$= \frac{\delta_{\nu-2,\nu-2,1,1}(\gamma_\iota)}{\prod\psi_{1\le i\le j\le 4}}(1-\gamma_\iota\gamma_j), \delta_{\nu_1,\nu_2,\nu_3,\nu_4}(\gamma_\iota)$$

$$= \det\left(\gamma_\iota^{4+\nu_j-j}\right)_{i,j=1,2,3,4}\prod\psi_{1\le i\le j\le 4}(\gamma_\iota-\gamma_j), \mathcal{C}_{\rho_1,\rho_2,\rho_3,\rho_4}(\lambda)$$

$$\boxtimes\, \mathcal{C}(\lambda;\gamma_i\gamma_j)\big|_{\gamma_1^{\rho_1-2}\gamma_2^{\rho_2-2}\gamma_3^{\rho_3-2}\gamma_4^{\rho_4-2}}$$

$$\zeta(\eta)\Gamma(\eta+1) = 2^{\eta-1}\int_0^\infty \frac{d\omega\omega^\eta}{sin\hbar^2(\omega)}, \ \mathcal{C}(\lambda;\gamma_i\gamma_j)$$

$$= \int_0^\infty \frac{\omega d\omega}{sin\hbar^2(\omega)}\sum_{\nu=2}^{\infty}[\mathcal{J}_{\nu-1}(\mu)^2 - \mathcal{J}_\nu(\mu)^2]\mathcal{F}_\nu(\gamma_\iota), \mathcal{C}_{2,2,\rho,\rho}(\lambda)$$

$$= \int_0^\infty \frac{\omega d\omega}{sin\hbar^2(\omega)}\left(\mathcal{J}_\iota(\mu)^2 - \mathcal{J}_j(\mu)^2\right), \mathcal{C}_{3,3,\rho,\rho}(\lambda)$$

$$= \int_0^\infty \frac{\omega d\omega}{sin\hbar^2(\omega)}\left(3\mathcal{J}_1(\mu)^2 - 4\mathcal{J}_2(\mu)^2 - 2\mathfrak{J}_{\rho+1}(\mu)^2\right)$$

$$\mathcal{C}(\lambda;\gamma_i\gamma_j)\big|_{strong}$$

$$= \sum_{\nu=2}^{\infty}\left(\frac{1}{2\nu(\nu-1)}\right.$$

$$+\sum_{\eta=1}^{\infty}4\eta(-1)^\eta\,\Gamma\left(\eta+\frac{1}{2}\right)\Gamma\left(\nu+\eta-\frac{1}{2}\right)\zeta(2\eta+1)$$

$$\left./\lambda^{\eta+\frac{1}{2}}\sqrt{\pi}\Gamma\left(\eta-\frac{1}{2}\right)\Gamma\left(\nu-\eta+\frac{1}{2}\right)\right)\mathcal{F}_\nu(\gamma_\iota)$$

$$\Delta\mathcal{C}(\lambda;\gamma_i\gamma_j) = \pm\frac{\iota}{2}\sum_{\nu=2}^{\infty}(-1)^{\nu}(2\nu-1)^2\left(\frac{8Li_0(z)}{(2\nu-1)^2}+\frac{2Li_1}{\lambda^{\frac{1}{2}}}\right.$$

$$\left.+\left(4\nu^2-4\nu+\frac{5Li_2(z)}{4\lambda}\right)\right)\mathcal{F}_{\nu}(\gamma_{\iota}),\mathcal{E}(\delta;\tau,\tilde{\tau})$$

$$=\sum_{(m,n)\neq(0,0)}\tau_2^{\delta}/\pi^{\delta}|m+\eta\tau|^{2\delta},\mathfrak{D}_{\mathfrak{N}}(\delta;\tau,\tilde{\tau})$$

$$=\frac{\sum_{(m,n)\neq(0,0)}e^{-\frac{4\sqrt{\mathcal{N}}\pi|m+\eta\tau|}{\sqrt{\tau_2}}}\tau_2^{\delta}}{\pi^{\delta}|m+\eta\tau|^{2\delta}},\mathcal{C}(\tau,\tilde{\tau};\gamma_i\gamma_j)$$

$$=\sum_{\nu=2}^{\infty}\left[\frac{1}{2(\nu-1)\nu}-2\nu-\frac{1}{2^4\mathcal{N}^{\frac{3}{2}}\mathfrak{E}\left(\frac{3}{2};\tau,\tilde{\tau}\right)}+3(2\nu-3)(4\nu^2-1)/2^8\mathcal{N}^{\frac{5}{2}}\mathfrak{E}\left(\frac{5}{2};\tau,\tilde{\tau}\right)\right.$$

$$\left.\pm 2\iota(-1)^{\nu}\mathfrak{D}_{\mathfrak{N}}(0;\tau,\tilde{\tau})\right]\mathcal{F}_{\nu}(\gamma_{\iota}),\lim_{\chi_{i,i+1}^2\to 0}\frac{\langle\mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle}{\langle\mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle_{free}}$$

$$=\mathcal{M}^2-\sum_{\ell=1}^{\infty}(\lambda/4\pi^2)^{\ell}\ell+\frac{1}{2^{2\ell+1}}\begin{pmatrix}2\ell & \cdots & 2\\ \vdots & \ddots & \vdots\\ \ell & \cdots & 1\end{pmatrix}(\gamma^2-1)^{2\ell}/\gamma^{2\ell}\zeta(2\ell+1)$$

6. Modelo de interacción de partículas y antipartículas en espacios cuánticos curvos.

6.1. Comportamiento de las partículas y antipartículas deformantes del espacio cuántico.

$$\mathcal{H}_{int}=\frac{1}{2\hbar_{\mu\nu}\mathcal{T}^{\mu\nu}},\Gamma_{spon}=\frac{2\pi}{\hbar^2\langle f|\widehat{\mathcal{H}}_{int}|\imath\rangle^2\mathcal{D}(\omega)},\widehat{\mathcal{H}}_{int,\ell}=\frac{\frac{\mathcal{L}}{\pi^2\sqrt{\frac{\mathcal{M}\hbar}{\omega_{\ell}}}}(-1)^{\ell-\frac{1}{2}}}{\ell^2(\hat{\beta}_{\ell}\hat{\beta}_{\ell}^{\dagger})\hat{\hat{\hbar}}},\Gamma_{spon}=\frac{8\mathfrak{G}\mathfrak{M}\mathcal{L}^2\omega_{\ell}^4}{\ell^4\pi^4c^5}$$

$$=\frac{8\pi\mathfrak{G}\rho v_{\delta}^4\mathcal{R}^2}{\mathcal{L}c^5},\Gamma_{stim}=\frac{\mathcal{M}\mathcal{L}^2\omega_{\ell}^2\hbar^2}{4\ell^4\pi^4\hbar}=\frac{v_{\delta}^2}{4\ell^2\pi^3\hbar}\mathcal{M}h^2,\mathcal{M}$$

$$=\frac{\pi^2\hbar\omega^3}{v_{\delta}^2\chi(\hbar,\omega,t)^2},m\ddot{\xi}_{\eta}+m\omega_{\mathfrak{D}}^2(2\xi_{\eta}-\xi_{\eta-2}-\xi_{\eta+2}),\xi_{\eta}(t)$$

$$=e^{-\iota\omega t}\left(\Lambda e^{\frac{\iota\kappa\eta\alpha}{2}}+\beta e^{-\frac{\iota\kappa\eta\alpha}{2}}\right)+\mathcal{H}\otimes c,\left.\frac{d\xi_{\eta}}{d\eta}\right|_{\eta=\pm\mathrm{N}}\xi_{\eta}(t)$$

$$=\sum_{\ell=0,2}^{\mathrm{N}-1}\chi_{\ell}(t)\cos\left[\frac{\ell\pi\eta}{(2\mathrm{N})}\right]+\sum_{\ell=1,3}^{\mathrm{N}}\chi_{\ell}(t)\sin\left[\frac{\ell\pi\eta}{(2\mathrm{N})}\right]$$

$$\sum_{\eta=\pm N}^{N} \cos[\ell\pi\eta/2(N+1)]\cos[\ell'\pi\eta/2(N+1)]$$

$$= N + 1/2\,\delta_{\ell\ell'} \sum_{\eta=\pm N}^{N} \sin[\ell\pi\eta/2(N+1)]\sin[\ell'\pi\eta/2(N+1)]$$

$$= N + 1/2\,\delta_{\ell\ell'}, \sum_{\eta=\pm N}^{N} \cos[\ell\pi\eta/2(N+1)]\sin[\ell'\pi\eta/2(N+1)],$$

$$\xi_\eta(t) = \sum_{\ell=0,2}^{N-1} \chi_\ell(t)\cos\left[\frac{\ell\pi\eta}{(2N+1)}\right] + \sum_{\ell=1,3}^{N} \chi_\ell(t)\sin\left[\frac{\ell\pi\eta}{(2N+1)}\right], \ddot{\chi}_\ell + \omega_\ell^2\chi_\ell, \mathfrak{E}$$

$$= \frac{m}{2}\sum_{\eta=\pm N}^{N} \dot{\xi}_\eta^2 + \frac{m\omega_{\mathfrak{D}}^2}{2}\sum_{\eta=\pm N}^{N-2} (\xi_{\eta-2} - \xi_\eta)^2 = \frac{m}{4}\sum_{\ell=0}^{N} \dot{\chi}_\ell^2 + \frac{\mathcal{M}}{4}\sum_{\ell=0}^{N} \omega_\ell^2\chi_\ell^2 \int_{-\frac{\mathcal{L}}{2}}^{\frac{\mathcal{L}}{2}} dx\chi_\ell(\chi)^2$$

$$= \frac{\mathcal{L}}{2}, \mu_\ell = \int_{-\frac{\mathcal{L}}{2}}^{\frac{\mathcal{L}}{2}} dx\rho(\chi)\chi_\ell(\chi)^2 = \frac{\mathcal{M}}{\mathcal{L}}\int_{-\frac{\mathcal{L}}{2}}^{\frac{\mathcal{L}}{2}} dx\chi_\ell(\chi)^2 = \frac{\mathcal{M}}{2}, f(x) = -m\nabla\phi$$

$$= -m\nabla\left(\frac{\frac{1}{2}\partial^2\phi}{\partial\chi^2}\chi^2\right) = m\frac{\ddot{\hbar}_{xx}}{4}\nabla(\chi^2) = m\frac{\ddot{\hbar}_{xx}}{2}(\chi_\eta + \xi_\eta)$$

$$\widehat{\mathcal{H}}_\ell = -m\frac{\ddot{\hbar}_{xx}}{2}\sum_{\eta=\pm N}^{N}\left(\chi_\eta\xi_\eta + \frac{\xi_\eta^2}{2}\right) - m\frac{\ddot{\hbar}_{xx}}{2}\sum_{\eta=\pm N}^{N}\chi_\eta\xi_\eta \approx -\frac{\mathcal{M}\mathcal{L}\ddot{\hbar}_{xx}}{\pi^2\sum_{\ell=1,3}^{N}\frac{(-1)^{\ell-\frac{1}{2}}1}{\ell^2\chi_\ell(t)} - m\frac{\ddot{\hbar}_{xx}}{4}\sum_{\eta=\pm N}^{N}\xi_\eta^2}$$

$$= -\frac{\mathcal{M}\mathcal{L}\ddot{\hbar}_{xx}}{8}\sum_{\ell=0}^{N}\chi_\ell^2, \mathcal{H}_\ell = \sum_{\ell=0}^{N}\widehat{\mathcal{H}}_\ell^l = \frac{\mathcal{M}\mathcal{L}\ddot{\hbar}_{xx}}{\pi^2\sum_{\ell=1,3}^{N}\frac{(-1)^{\ell-\frac{1}{2}}1}{\ell^2\chi_\ell}} - \frac{\mathcal{M}\mathcal{L}\ddot{\hbar}_{xx}}{8}\sum_{\ell=0}^{N}\hat{\chi}_\ell^2, \mathcal{H}_\ell^{l,odd}$$

$$= -\frac{\mathcal{M}\mathcal{L}\ddot{\hbar}_{xx}}{\pi^2}\frac{(-1)^{\ell-\frac{1}{2}}1}{\ell^2}\sqrt{\frac{\hbar}{\mathcal{M}\omega_\ell}}(\hat{\beta}_l+\hat{\beta}_l^\dagger)^2, \widehat{\mathcal{H}}_\ell^{l,even} = -\frac{\ddot{\hbar}_{xx}}{8}\hbar/\omega_\ell(\hat{\beta}_l+\hat{\beta}_l^\dagger)^2$$

$$\widehat{\mathcal{H}}_{int} = \frac{\hbar\sqrt{\frac{(-1)^{\ell-\frac{1}{2}}8\pi\mathfrak{G}\mathcal{M}v^3}{\omega_\ell c^2\mathcal{V}}}\mathcal{L}}{\pi^2\ell^2(\hat{\beta}_l+\hat{\beta}_l^\dagger)(\hat{\alpha}e^{-ivt} + \hat{\alpha}^\dagger e^{ivt})}, \Gamma_{stim} = \frac{2\pi}{\hbar^2\left|\langle\alpha\langle 1|\widehat{\mathcal{H}}_{int}|\alpha\rangle|0\rangle\right|^2\mathcal{D}(\omega)}, \Gamma_{stim}$$

$$= \frac{\frac{|\alpha|^2}{\ell^4}8\pi\mathfrak{G}\mathcal{M}\mathcal{L}^2\omega_\ell^4}{\pi^4c^5}, \mathcal{N} = \frac{\hbar_0^2c^5}{32\pi\mathfrak{G}\hbar v^2}, \Gamma_{stim} = \frac{\frac{1}{\ell^4}\mathcal{M}\mathcal{L}^2\omega_\ell^2\hbar_0^2}{4\pi^5\hbar}$$

$$\widehat{\mathcal{H}} = \hbar\omega\hat{\beta}_l^\dagger\hat{\beta}_l + \frac{1}{\eta^2\mathcal{L}}\sqrt{\frac{\mathcal{M}\hbar}{\omega}}\ddot{h}(t)\left(\hat{\beta}_l^\dagger + \hat{\beta}_l\right), \widehat{\mathfrak{U}}_{int} = \hat{\mathcal{T}}e^{-i\int_0^t ds\left(g(\delta)\widehat{\beta}(\delta)+g^{\circledast}(\delta)\widehat{\beta}^\dagger(\delta)\right)}, g(t)$$

$$= \frac{1}{\pi^2\mathcal{L}\sqrt{\frac{\mathcal{M}\hbar}{\omega}}\ddot{h}(t)}, \widehat{\mathfrak{U}}_{int} = \epsilon^{\Omega(t)}, \Omega(t)$$

$$= \int_0^t dt_1\widehat{\Lambda}(t_1) + \frac{1}{2}\int_0^t dt_1\int_0^t dt_2\left[\widehat{\Lambda}(t_1), \widehat{\Lambda}(t_2)\right], \widehat{\mathfrak{U}}_{int}$$

$$= e^{-i\int_0^t ds\left(g(\delta)\widehat{\beta}(\delta)+g^{\circledast}(\delta)\widehat{\beta}^\dagger(\delta)\right)}e^{-\iota\varphi}, \widehat{\mathfrak{U}} = e^{-\iota\varphi}e^{-\iota\omega t\widehat{\beta}^\dagger\widehat{\beta}}\widehat{\mathcal{D}}(\beta), \beta$$

$$= -i\int_0^t ds g^{\circledast}(\delta)e^{\iota\omega\delta}, e^{-\iota\varphi}\left|\beta e^{-\iota\omega t\widehat{\beta}^\dagger\widehat{\beta}}\right\rangle, |\beta| = \frac{\mathcal{L}}{\pi^2\sqrt{\frac{\mathcal{M}}{\omega\hbar}}}\chi(\hbar,\omega,t), \chi(\hbar,\omega,t)$$

$$= \left|\int_0^t ds\ddot{h}(\delta)e^{\iota\omega\delta}\right|, \nu(t) = \left(\frac{1}{\nu_0^{\frac{8}{3}}} - \frac{8}{3}\kappa t\right)^{-\frac{3}{8}}, \kappa = \frac{\kappa_f}{(2\pi)^{\frac{8}{3}}} = \frac{\overline{\dfrac{96}{5\varpi\left(\frac{\pi\mathfrak{G}\mathcal{M}_c}{c^3}\right)^{\frac{5}{3}}1}}}{(2\pi)^{\frac{8}{3}}}$$

$$= \frac{48}{5\left(\frac{\mathfrak{G}\mathcal{M}_c}{2c^3}\right)^{\frac{5}{3}}}, \tau = \frac{2\Delta\omega}{\kappa\omega^{\frac{11}{3}}}, \chi = \left|\int_0^t \frac{ds e^{\iota\omega\delta}\hbar_0\nu^2 t}{2} sinc\left(\frac{\delta t}{2}\right)\right|, 2\Delta\omega = \frac{8}{T}, \tau$$

$$= 2\sqrt{\frac{2}{\kappa}}\omega^{\frac{11}{6}}$$

$$\chi \approx \hbar_0\omega\left|\int_0^\tau ds e^{\iota\omega\delta}sin(\omega\delta)\right| = \frac{\hbar_0\omega}{4}\sqrt{2 + 4\omega^2\tau^2 - 2\cos(2\omega\tau) - 4\omega\tau\sin(2\omega\tau)}, \chi(\tau) \approx \frac{\hbar_0\omega^2\tau}{2}\chi$$

$$\approx \hbar_0\sqrt{\frac{2}{\kappa}}\omega^{\frac{1}{6}} = \hbar_0\sqrt{\frac{5}{24}\left(\frac{2c^3}{\mathfrak{G}\mathcal{M}_c}\right)^{\frac{5}{6}}}\omega^{\frac{1}{6}}, \mathcal{M} = \frac{\pi^2\hbar\omega^3}{\nu_\delta^2\chi^2} \approx \frac{\pi^2\hbar\kappa}{2\nu_\delta^2\hbar_0^2\omega^{\frac{8}{3}}}$$

$$= \frac{\frac{24\pi^2}{5}\hbar}{\hbar_0^2\nu_\delta^2\left(\frac{2c^3}{\mathfrak{G}\mathcal{M}_c}\right)^{\frac{5}{3}}\omega^{\frac{8}{3}}}, \mathcal{P}(t) = \frac{\frac{\mathcal{L}^2}{\pi^4\mathcal{M}}}{\omega\hbar|\chi(t)|^2} = \frac{\hbar_0^2\omega\tau^2\mathcal{M}\nu_\delta^2}{4\pi^2\hbar}, \Gamma_{mc} = \frac{d\mathcal{P}(\mathrm{t})}{dt}$$

$$= \frac{\hbar_0^2\omega\tau^2\mathcal{M}\nu_\delta^2}{2\pi^2\hbar} = \frac{\hbar_0^2\mathcal{N}_c\mathcal{M}\nu_\delta^2}{\pi\hbar}, \hbar_0 = \sqrt{\frac{\pi\kappa_\beta\mathcal{T}}{\mathcal{M}\nu_\delta^2\mathcal{Q}\mathcal{N}_c}}$$

$$\mathcal{P}(\nu,\omega,t) \approx |\beta(\nu,\omega,t)|^2 \approx \frac{(\hbar_0^2\omega^3\mathcal{M}\mathcal{L}^2)sin^2\left[\frac{1}{2}t(\nu-\omega)\right]}{\hbar(\nu-\omega)^2\pi^4}, \mathcal{P}(\omega,t)$$

$$= \frac{\sum_\nu|\beta(\nu,\omega,t)|^2 \int_{\omega-\frac{\delta}{2}}^{\omega+\frac{\delta}{2}} d\nu\mathfrak{D}(\nu)|\beta(\nu,\omega,t)|^2 = \frac{\mathfrak{D}(\omega)(\hbar_0^2\omega^3\mathcal{M}\mathcal{L}^2)}{\hbar}\pi^4 \int_{\omega-\frac{\delta}{2}}^{\omega+\frac{\delta}{2}} d\nu\, sin^2\left[\frac{1}{2}t(\nu-\omega)\right]}{(\nu-\omega)^2}$$

$$= \frac{\mathfrak{D}(\omega)(\hbar_0^2\omega^3\mathcal{M}\mathcal{L}^2)}{\hbar\pi^4}\Xi(t), \sum_\nu = \int d\nu\mathcal{D}(\nu) = \frac{\int d\nu\,\mathcal{V}\nu^2}{2\pi^2c^3}, \mathcal{P}(\omega,t) \approx \frac{\mathfrak{D}(\omega)(\hbar_0^2\omega^3\mathcal{M}\mathcal{L}^2)}{2\hbar\pi^3}$$

$$= \Gamma_{stim}t, \Gamma_{stim} = \frac{\mathfrak{D}(\omega)(\hbar_0^2\omega^3\mathcal{M}\mathcal{L}^2)}{2\hbar\pi^3} = \frac{\mathcal{V}\hbar_0^2\omega^5\mathcal{M}\mathcal{L}^2}{4\hbar\pi^5c^4} = \hbar_0^2\frac{\mathcal{M}\nu_\delta^2}{4\hbar\pi^3}, \hbar_c \equiv 2\pi\sqrt{\frac{\pi\kappa_\beta\mathcal{T}}{\mathcal{M}\nu_\delta^2\mathcal{Q}}}, \widehat{\mathcal{H}}_{\omega_l}$$

$$= \hbar\omega_l\left(\hat{\beta}_l^\dagger+\hat{\beta}_l+\frac{1}{2}\right), |\psi_\mathcal{M}\rangle = (2\pi t_m)^{\frac{1}{4}}\int d\chi e^{-\frac{\chi^2}{4t_m}}|\chi\rangle, \widehat{\mathcal{H}}_{int}^{\mathcal{M}}dt = \sqrt{dt}\hat{\rho}\widehat{\mathrm{N}}, \widehat{\mathcal{M}}_{\widehat{\mathrm{N}}}(\gamma)$$

$$= \left\langle\gamma\right|e^{-i\widehat{\mathcal{H}}_{int}^{\mathcal{M}}dt}\left|\psi_\mathcal{M}\right\rangle = (2\pi t_m)^{-\frac{1}{4}}\exp\left[-\frac{\left(\gamma-\widehat{\mathrm{N}}\sqrt{dt}\right)^2}{4t_m}\right], \widehat{\mathcal{M}}_{\widehat{\mathrm{N}}}(r)$$

$$= \left(\frac{2\pi t_m}{dt}\right)^{-\frac{1}{4}}\exp\left\{\left[-\frac{dt\left(r-\widehat{\mathrm{N}}\right)^2}{4t_m}\right]\right\}, \rho(t+dt)$$

$$= \mathcal{D}\left[dt\beta'^{(t)}\right]\widehat{\mathcal{M}}_{\widehat{\mathrm{N}}}[r(t)]\rho(t)\widehat{\mathcal{M}}^\dagger{}_{\widehat{\mathrm{N}}}[r(t)]\mathcal{D}\left[-dt\beta'^{(t)}\right]/tr\left\{\widehat{\mathcal{M}}_{\widehat{\mathrm{N}}}[r(t)]\rho(t)\widehat{\mathcal{M}}^\dagger{}_{\widehat{\mathrm{N}}}[r(t)]\right\}$$

6.2. Modelo fotónico aplicable a partículas y antipartículas deformantes del espacio cuántico curvo.

$$[\nabla^2 + \mu(r)\epsilon(r)\kappa^2]\mu_\xi(\kappa,r), \hat{\mathfrak{E}}(r) = \iota \sum_{\xi,\kappa} \sqrt{\frac{\hbar c\kappa}{2\epsilon_0 \varepsilon(r)}} \mu_\xi(\kappa,r)\left(\alpha_{\xi\kappa}^\dagger + \alpha_{\xi\kappa}\right), \frac{\mathcal{H}}{\hbar}$$

$$= \omega_0\sigma_+\sigma_- + \sum_{\xi,\kappa} c\kappa\, \alpha_{\xi\kappa}^\dagger \alpha_{\xi\kappa} + \sum_{\xi,\kappa} g(r)\left(\alpha_{\xi\kappa}\sigma_+ + \alpha_{\xi\kappa}^\dagger \sigma_-\right), g_\xi(\kappa,r)$$

$$= \sqrt{\frac{\hbar c\kappa}{2\epsilon_0\varepsilon(r)}} d\otimes\mu_\xi(\kappa,r) \int_0^\infty \kappa d\kappa\rho(\kappa)\, \mu_\xi(\kappa,r)\mu_\xi(\kappa,r')e^{-ic\kappa\tau}$$

$$= \sum_\eta z_{\xi_\eta} v_{\xi_\eta}(r) v_{\xi_\eta}(r')\Theta(\tau - \Delta_t(r,r'))e^{-icz_{\xi_\eta}\tau}, \frac{d}{dt\tilde{c}_0(t)}$$

$$= -\int_0^t dt' \sum_\xi \int d\kappa\rho(\kappa) g_\xi^2(\kappa) e^{i(\omega_0 - c\kappa)(t-t')}\tilde{c}_0(t'), \frac{d}{dt\tilde{c}_0(t)}$$

$$= -\int_0^t dt' \sum_{\xi,\eta} \hat{g}_{\xi_\eta}^2\, e^{i\left(\omega_0 - cz_{\xi_\eta}\tau\right)(t-t')}\tilde{c}_0(t'), \frac{id}{dt}\tilde{c}_0(t)$$

$$= \sum_{\xi\eta} \hat{g}_{\xi_\eta}\, e^{i\left(\omega_0 - cz_{\xi_\eta}\tau\right)t}\tilde{\beta}_{\xi_\eta}(t), \frac{id}{dt}\tilde{\beta}_{\xi_\eta}(t) = \hat{g}_{\xi_\eta} e^{i\left(\omega_0 - cz_{\xi_\eta}\tau\right)t}\tilde{c}_0(t), \hat{g}_{\xi_\eta}(r)$$

$$= \sqrt{\frac{\hbar c z_{\xi_\eta}\tau}{2\epsilon_0\varepsilon(r)}} d\otimes v_\xi(r), \mathfrak{E}_+ \Big| \psi(t)\rangle = i \sum_{\xi,\eta} \sqrt{\frac{\hbar c z_{\xi_\eta}\tau}{2\epsilon_0\varepsilon(r)}} \hat{v}_{\xi_\eta}(r)\tilde{\beta}_{\xi_\eta}(t-\Delta_t)e^{-icz_{\xi_\eta}\tau}, \mathcal{Z}_l$$

$$= \left(\sqrt{\epsilon\kappa r}\right) = \frac{1}{\sqrt{\mathcal{I}_m(\kappa)}\begin{cases}\eta_l(\kappa)j_l\left(\sqrt{\epsilon\kappa r}\right)\\ \alpha_l(\kappa)j_l(\kappa r) + \beta_l(\kappa)\gamma_l(\kappa r)\end{cases} r<\alpha, r>\alpha}, \lim_{\mathcal{R}\to\infty} \mathcal{I}_{\mathcal{M}}(\kappa)$$

$$= \frac{\mathcal{R}}{2\kappa^2[\alpha_l(\kappa) + i\beta_l(\kappa)][\alpha_l(\kappa) - i\beta_l(\kappa)]}, \mathcal{I}_l(r,r')$$

$$= \int_0^\infty d\kappa\rho(\kappa)\kappa^{-1}\, \mathcal{Z}_l\left(\sqrt{\epsilon\kappa r}\right)\mathcal{Z}_l\left(\sqrt{\epsilon\kappa r'}\right)e^{-\iota c\kappa\tau}, \mathcal{I}_l$$

$$= \Theta[c\tau - (r-\alpha)] \oint_{\mathcal{LHP}}^\infty f_1(z)\, dz$$

$$+ \Theta[c\tau$$

$$+ (r-\alpha)] \oint_{\mathcal{UHP}}^\infty f_2(z)dz\,, \mathcal{I}_l(r,r'))2\pi\iota \sum_{\mathcal{Z}_{l_\eta}\in\mathcal{Q}_4} \mathcal{R}es[f_2(z)]\Theta[c\tau - (r-\alpha)]\,, v_{lm\eta}(r)$$

$$= \mathfrak{N}_{lm\eta}(r)\left[\pi\sqrt{\alpha_l\left(\mathcal{Z}_{l_\eta}\right) - \frac{i\beta_l\left(\mathcal{Z}_{l_\eta}\right)}{\iota[\partial_{\mathcal{Z}}\alpha_l(\mathcal{Z}) + i\beta_l(\mathcal{Z})]_{\mathcal{Z}_{l_\eta}}}\hbar_l^{(1)}\left(\mathcal{Z}_{l_\eta} r\right)}\right], \delta\omega_0$$

$$= \sum_{off-res} g_{l_\eta}^2\left(\omega_0 - \omega_{l_\eta}\right) - \frac{i\gamma_{l_\eta}}{\left(\omega_0 - \omega_{l_\eta}\right)^2} + \gamma_{l_\eta}^2, id/dt\tilde{c}_0(t)$$

$$= \delta\omega_0\tilde{c}_0(t) + \sum_{\substack{(5,4)\\(8,3)}} \hat{g}_{l_\eta}\, e^{i\left(\omega_0 - c\mathcal{Z}_{l_\eta}\right)t}\tilde{\beta}_{l_\eta}(t)$$

6.3. La gravedad como entidad cuántica (Formalización).

$$|\psi(t=0)\rangle = |\zeta\rangle_{\mathcal{M}_c} \otimes \frac{1}{\sqrt{2}\left(|\uparrow\rangle_{\delta_c}|\downarrow\rangle_{\delta_c}\right)}, |\zeta\rangle_{\mathcal{M}_c}\otimes|\uparrow\rangle_{\delta_c} \longrightarrow |\mathcal{L}\uparrow\rangle_{\mathcal{C}}, |\zeta\rangle_{\mathcal{M}_c} \otimes |\downarrow\rangle_{\delta_c} \longrightarrow |\mathcal{R}\downarrow\rangle_{\mathcal{C}}, |\psi\rangle_{\mathcal{C},\Lambda}$$

$$= \frac{1}{\sqrt{2}\left(\sqrt{1+\cos\Delta\phi}|\Psi_+\rangle_{\mathcal{C}}|+\rangle_{\delta_\Lambda} + \sqrt{1-\cos\Delta\phi}|\Psi_-\rangle_{\mathcal{C}}|-\rangle_{\delta_\Lambda}\right)|\zeta\rangle_{\mathcal{M}_\Lambda}}, |\Psi_\pm\rangle_{\mathcal{C}}$$

$$= \left(1 \pm e^{i\Delta\phi}\right)|\mathcal{L}\uparrow\rangle_{\mathcal{C}} + \frac{\left(e^{i\Delta\phi} \pm 1\right)|\mathcal{R}\downarrow\rangle_{\mathcal{C}}}{2\sqrt{1 \pm \cos\Delta\phi}\,|\pm\rangle_{\delta_\Lambda}} = |\uparrow\rangle_{\delta_\Lambda} \pm \frac{|\downarrow\rangle_{\delta_\Lambda}}{\sqrt{2}}, \Delta\phi_\tau$$

$$= \frac{\mathcal{G}\mathcal{M}m\tau}{\hbar\sqrt{d^2 + (\Delta\chi)^2}}$$

$$- \frac{\mathcal{G}\mathcal{M}m\tau}{\hbar d}, \left|\psi_{\alpha,\beta,c}\right\rangle$$

$$= \frac{1}{8[(1+\alpha e^{i\Delta\phi})(1-\beta e^{i\Delta\phi}) + c e^{2i\Delta\phi}(1+\alpha e^{i\Delta\phi})(1-\beta e^{i\Delta\phi})]}|\zeta\rangle_{\mathcal{M}_c}|c\rangle_{\delta_c}, \mathcal{V}(\pm)$$

$$= \mathcal{P}_\pm - \sum_{\alpha,\beta\in\{\pm\}} \mathcal{P}_{\alpha,\beta,\pm} = \pm\frac{1}{2}\sin^2\Delta\phi, \rho_{\mathcal{C},\Lambda}$$

$$= \frac{1}{2}\Big((1+\cos\Delta\phi)|\Psi_+\rangle_{\mathcal{C}}\langle\Psi_+|_{\mathcal{C}}\otimes|+\rangle_{\delta_\Lambda}\langle+|_{\delta_\Lambda}$$

$$+ (1-\cos\Delta\phi)|\Psi_-\rangle_{\mathcal{C}}\langle\Psi_-|_{\mathcal{C}}\otimes|-\rangle_{\delta_\Lambda}\langle-|_{\delta_\Lambda}\Big)\otimes|\xi\rangle_{\mathcal{M}_\Lambda}\langle\xi|_{\mathcal{M}_\Lambda}$$

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Apéndice D:

Postulados Finales

1. Que las partículas con o sin masa o las antipartículas con o sin masa, según sea el caso, en tanto y en cuanto, se aproximen, igualen o superen la velocidad de la luz, deforman el espacio cuántico en el que interactúan, a propósito de sus ciclos cuánticos de colisión, superposición o entrelazamiento, según corresponda a cada caso.

2. Que las partículas masivas o supermasivas o las antipartículas masivas o supermasivas, según sea el caso, no necesitan aproximarse, igualar o superar la velocidad de la luz, para deformar el espacio cuántico en el que interactúan, de tal suerte que, basta con su hipermasa o supermasa, según sea el caso, para lograr un espacio cuántico curvo, a propósito de sus ciclos cuánticos de colisión, superposición o entrelazamiento, según corresponda a cada caso.

3. Cuando una partícula supermasiva o una antipartícula supermasiva, según sea el caso, alcanzan o superan la velocidad de la luz, producen un agujero negro cuántico, a propósito de sus ciclos cuanticos de colisión, superposición o entrelazamiento, según corresponda a cada caso.

4. Cuando una partícula sin masa o una antipartícula sin masa, según sea el caso, superan la velocidad de la luz, producen un agujero negro cuántico, a propósito de sus ciclos cuánticos de colisión, superposición o entrelazamiento, según corresponda a cada caso.

APÉNDICE E.

FORMALIZACIÓN MATEMÁTICA COMPLEMENTARIA.

1. Espacios cuánticos curvos.

1.1. Osciladores y propagadores en espacios curvos – Modelo Feynman.

$$\mathfrak{F} = \langle \int_{-\infty}^{\infty} q(\sigma)^2 \bar{\chi} e^{-\sigma^2}\, d\sigma, \mathfrak{F}[x(t,s), y(t,s)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t,s), y(t,s) \sin\omega(t-s) dtds\, , \mathcal{F}(\cdots q_i \cdots)$$

$$= \sum_{i=-\infty}^{\infty} q_i^2 e^{-\sigma_i^2} (\sigma_{i+1} - \sigma_i) \sum_i \frac{(\cdots q_i \cdots)}{\partial q_i} \lambda_i\, , \mathfrak{F}[q(\sigma), +\lambda(\sigma)] - \mathcal{F}[q(\sigma)]$$

$$= \int \mathcal{K}(t)\lambda(t)dt, \mathfrak{F}[q(\sigma), +\lambda(\sigma)] = \mathcal{F}[q(\sigma)] + \int \frac{\delta\mathcal{F}[q(\sigma)]}{\delta q(t)\lambda(t)dt}, \mathcal{F}[q+\lambda]$$

$$= \int [q(\sigma)^2 + 2q(\sigma)\lambda(\sigma) + \lambda(\sigma)^2]\, e^{-\sigma^2} d\sigma$$

$$= \int q(\sigma)^2 e^{-\sigma^2} d\sigma + 2 \int q(\sigma)\lambda(\sigma)\, e^{-\sigma^2} d\sigma \rangle$$

$$\mathcal{A} = \langle \int \mathcal{L}(\dot{q}(\sigma), q(\sigma))d\sigma, \frac{\delta\mathcal{A}}{\delta q(t)} = \frac{d}{dt}\{\partial\mathcal{L}(\dot{q}(t), q(t))/\partial\dot{q}\} + \partial\mathcal{L}(\dot{q}(t), q(t))/\partial q , \mathcal{A}$$

$$= \int_{-\infty}^{\infty} \left\{\frac{m(\dot{x}(t))^2}{2} - \mathcal{V}(x(t)) + \kappa^2\dot{x}(t)\dot{x}(t+\mathcal{T}_0)\right\} dt , \delta\mathcal{A}$$

$$= \int_{-\infty}^{\infty} \{m\dot{x}(t)\dot{\lambda}(t) - \mathcal{V}'(x(t))\lambda(t) + \kappa^2\dot{\lambda}(t)\dot{x}(t+\mathcal{T}_0) + \kappa^2\dot{\lambda}(t+\mathcal{T}_0)\dot{x}(t)\}dt$$

$$= \int_{-\infty}^{\infty} \{-m\ddot{x}(t) - \mathcal{V}'(x(t)) - \kappa^2\ddot{x}(t+\mathcal{T}_0) + \kappa^2\ddot{x}(t-\mathcal{T}_0)\dot{x}(t)\}\lambda(t)dt , \frac{\delta\mathcal{A}}{\delta x(t)}$$

$$= -m\ddot{x}(t) \ - \mathcal{V}'(x(t)) - \kappa^2\ddot{x}(t+\mathcal{T}_0) - \kappa^2\ddot{x}(t-\mathcal{T}_0), \frac{\delta\mathcal{A}}{\delta\gamma(t)}$$

$$= -\frac{d}{dt}\left(\frac{\partial\mathfrak{L}_\gamma}{\partial\dot{\gamma}}\right) + \left.\frac{\partial\mathfrak{L}_\gamma}{\partial\gamma}\right|_t$$

$$+ \left.\frac{\partial\mathfrak{I}_\gamma}{\partial\gamma}\right|_t \bigotimes x(t) , \frac{\delta}{\delta\gamma(s)\left[-\frac{d}{dt}\left(\frac{\partial\mathfrak{L}_\gamma}{\partial\dot{\gamma}}\right) + \left.\frac{\partial\mathfrak{L}_\gamma}{\partial\gamma}\right|_t + \left.\frac{\partial\mathfrak{I}_\gamma}{\partial\gamma}\right|_t \otimes x(t)\right]}$$

$$= \frac{\frac{\delta}{\delta\gamma(t)}\left[-\frac{d}{ds}\left(\frac{\partial\mathfrak{L}_\gamma}{\partial\dot{\gamma}}\right) + \left.\frac{\partial\mathfrak{L}_\gamma}{\partial\gamma}\right|_s + \left.\frac{\partial\mathfrak{I}_\gamma}{\partial\gamma}\right|_s \otimes x(s)\right]\left.\frac{\partial\mathfrak{L}_\gamma}{\partial\gamma}\right|_t \delta x(t)}{\delta\gamma(s)} = \frac{\left.\frac{\partial\mathfrak{I}_\gamma}{\partial\gamma}\right|_s \delta x(s)}{\delta\gamma(t)}$$

$$= \frac{\frac{1}{m\omega}\sin\omega(t-s) \boxtimes \left.\frac{\partial\mathfrak{I}_\gamma}{\partial\gamma}\right|_s \delta x(s)}{\delta\gamma(s)} \frac{\left.\frac{\partial\mathfrak{I}_\gamma}{\partial\gamma}\right|_s \delta x(t)}{\delta\gamma(s)} = \frac{\frac{1}{m\omega}\sin\omega(t-s) \boxtimes \left.\frac{\partial\mathfrak{I}_\gamma}{\partial\gamma}\right|_s \delta x(s)}{\delta\gamma(s)}$$

$$= -\frac{\sin\omega(\mathcal{T}-t)\sin\omega s}{m\omega}\sin\omega\mathcal{T} \left.\frac{\partial\mathfrak{I}_\gamma}{\partial\gamma}\right|_s$$

$$= -\frac{\sin\omega(\mathcal{T}-s)\sin\omega t}{m\omega}\sin\omega\mathcal{T} \left.\frac{\partial\mathfrak{I}_\gamma}{\partial\chi}\right|_s \int_0^{\mathcal{T}} [\mathfrak{L}_y + \mathfrak{L}_z]dt$$

$$+ \int_0^{\mathcal{T}} \left[\sin\omega(\mathcal{T}-t)x(0) + \sin\frac{\omega t\chi(\mathcal{T})}{\sin\omega\mathcal{T}}\right]\gamma(t)dt - \frac{1}{m\omega\sin\omega\mathcal{T}\int_0^{\mathcal{T}} dt \int_0^t ds}$$

$$\otimes \sin\omega(\mathcal{T}-t)\sin\omega s\gamma(s)\gamma(t) , \frac{\delta x(t)}{\delta\gamma(s)} = \frac{\frac{1}{2m\omega}\sin\omega(t-s) \boxtimes \left.\frac{\partial\mathfrak{I}_\gamma}{\partial\gamma}\right|_s \delta x(s)}{\delta\gamma(s)} =$$

$$= -\frac{\frac{1}{2m\omega}\sin\omega(t-s) \boxtimes \left.\frac{\partial\mathfrak{I}_\gamma}{\partial\gamma}\right|_s \delta x(s)}{\delta\gamma(s)}\rangle$$

$\mathfrak{E}(t)$

$$= \left\| \frac{m(\dot{x}(t))^2}{2} + \mathcal{V}(x(t)) - \kappa^2 \int_t^{t+\mathcal{T}_0} \ddot{x}\,(\sigma - \mathcal{T}_0)\dot{x}(\sigma)d\sigma + \kappa^2\dot{x}(t)\dot{x}(t+\mathcal{T}_0), \mathcal{A}[q_\eta(\sigma) + \alpha y_\eta(\sigma)] \right.$$

$$= \mathcal{A}[q_\eta(\sigma)]$$

$$+ \frac{\alpha \sum_{\eta=1}^{\mathrm{N}} \int_{-\infty}^{\infty} \frac{y_\eta(t)\delta\mathcal{A}}{\delta q_\eta(t)} dt\ \sum_{\eta=1}^{\mathrm{N}} \int_{-\infty}^{\infty} \frac{y_\eta(\sigma)\delta\mathcal{A}}{\delta q_\eta(\sigma)} d\sigma, \mathcal{I}(\mathcal{T}) = \sum_{\eta=1}^{\mathrm{N}} \int_{-\infty}^{\mathcal{T}} \frac{y_\eta(\sigma)\delta\mathcal{A}}{\delta q_\eta(\sigma)} d\sigma - \sum_{\eta=1}^{\mathrm{N}} \int_{\mathcal{T}}^{\infty} \frac{y_\eta(\sigma)\delta\mathcal{A}}{\delta q_\eta(\sigma)} d\sigma, \delta\mathcal{I}(\mathcal{T})}{\delta q_m(t)}$$

$$= + \int_{-\infty}^{\mathcal{T}} \sum_m \frac{\frac{\delta y_\eta(\sigma)}{\delta q_m(t)}\delta\mathcal{A}}{\delta q_m(\sigma)d\sigma}$$

$$+ \int_{-\infty}^{\mathcal{T}} \sum_m y_\eta(\sigma) \frac{\delta^2\mathcal{A}}{\delta q_m(t)\delta q_m(\sigma)} d\sigma \quad \int \frac{\left[\mathfrak{L}_y + \mathfrak{L}_z + \left(\frac{m\dot{x}^2}{2} - \frac{m\omega^2x^2}{2}\right) + (\mathfrak{J}_y + \mathfrak{J}_z)x\right] d}{dt} \left(\frac{\partial\mathfrak{L}_y}{\partial\dot{y}}\right) - \frac{\partial\mathfrak{L}_y}{\partial y}$$

$$\left. = \frac{\partial\mathfrak{J}_y}{\partial\dot{y}}\chi(t), m\ddot{x} + m\omega^2 x = [\mathfrak{J}_y(t) + \mathfrak{J}_z(t)] \right\|$$

$$x(t) = \langle x(0)\cos\omega t + \frac{\dot{x}(0)\sin\omega t}{\omega} + \frac{1}{m\omega\int_0^t \gamma(\delta)\sin\omega(t-\delta)\,d\delta}, x(t)$$

$$= \frac{\sin\omega(\mathcal{T}-t)}{\sin\omega\mathcal{T}}\left[x(0) - \frac{1}{m\omega}\int_0^t \sin\omega\delta\gamma(\delta)d\delta\right]$$

$$+ \frac{\sin\omega t}{\sin\omega\mathcal{T}\left[x(\mathcal{T}) - \frac{1}{m\omega}\int_t^{\mathcal{T}}\sin(\mathcal{T}-\delta)\gamma(\delta)d\delta\right]}\rangle$$

$$x(t) = \langle\frac{1}{\sin\omega\mathcal{T}}[\mathcal{R}_t\sin\omega t + \mathcal{R}_0\sin\omega t(\mathcal{T}-t)\,] + \frac{1}{2m\omega}\int_0^t \sin\omega(t-\delta)\gamma(\delta)d\delta$$

$$-\frac{1}{2m\omega}\int_t^{\mathcal{T}}\sin\omega(t-\delta)\gamma(\delta)d\delta\rangle$$

$$\mathcal{R}_0 = \langle\frac{1}{2}\left[x(0) + \frac{x(\mathcal{T})\cos\omega\mathcal{T} - \dot{x}(\mathcal{T})\sin\omega\mathcal{T}}{\omega}\right], \mathcal{R}_t = \frac{1}{2}\left[x(\mathcal{T}) + \frac{x(0)\cos\omega\mathcal{T} - \dot{x}(0)\sin\omega\mathcal{T}}{\omega}\right]\rangle$$

$$\mathcal{A} = \langle\int_0^{\mathcal{T}}[\mathfrak{L}_y + \mathfrak{L}_z]\,dt + \frac{1}{\sin\omega\mathcal{T}\int_0^{\mathcal{T}}[\mathcal{R}_{\mathcal{T}}\sin\omega t + \mathcal{R}_0\sin\omega(\mathcal{T}-t)]\,\gamma(t)dt}$$

$$-\frac{1}{2m\omega\int_0^{\mathcal{T}}\int_0^t \sin\omega(t-s)\sin\gamma(t)\gamma(s)}\,dsdt, \mathcal{A}$$

$$= \int_{-\infty}^{\infty}[\mathfrak{L}_y + \mathfrak{L}_z]dt + \frac{1}{2m\omega}\sum_{j=1}^{N}\frac{1}{2m_j\omega_j}\int_{-\infty}^{\infty}\int_{-\infty}^{t}\sin\omega(t-s)\boxtimes\sin\gamma(t)\gamma(s)dsdt, \mathcal{A}$$

$$= \int_{-\infty}^{\infty}[\mathfrak{L}_y + \mathfrak{L}_z]dt + \sum_{j=1}^{N}\frac{1}{2m_j\omega_j}\int_{-\infty}^{\infty}\int_{-\infty}^{t}\sin\omega_j(t-s)\boxtimes\sin\gamma_j(t)\gamma_j(s)dsdt\ , \mathcal{A}$$

$$= \int_{-\infty}^{\infty}[\mathfrak{L}_y + \mathfrak{L}_z]dt + \ 1/m\omega\int_{-\infty}^{\infty}\int_{-\infty}^{t}\sin\omega(t-s)\otimes[\mathfrak{I}_y(s) + \mathfrak{I}_z(t)$$

$$+ \mathfrak{I}_y(t) + \mathfrak{I}_z(s)]dsdt\int_{-\infty}^{\infty}\Big[\mathfrak{L}_y + \mathfrak{L}_z + (\mathfrak{I}_y + \mathfrak{I}_z)x_1 + (\mathfrak{I}_y - \mathfrak{I}_z)x_2$$

$$+ \frac{m}{z}(\ddot{x}_1^2 - \omega^2 x_1^2)\sum_{\kappa}\sum_{l\neq\kappa} - \frac{m}{2}(\ddot{x}_s^2 + \omega^2 x_2^2)\Big]dt\int_{-\infty}^{\infty}\sum_{\kappa}\Big[\mathfrak{L}_y + \mathfrak{L}_z + \mathfrak{I}_y\eta_y + \mathfrak{I}_z\eta_z$$

$$+ \frac{m}{2}\left(\ddot{\eta}_{yl}\ddot{\eta}_{zl} - \omega^2\hat{\eta}_{y\kappa}\hat{\eta}_{z\kappa}\right)\Big]dt\rangle$$

2. Partícula cosmológica (Hipermasa y Supermasa – Partículas masivas y supermasivas y antipartículas masivas y supermasivas).

2.1. Supermasa (Partículas Masivas y Antipartículas Masivas).

$$\mathfrak{G}_{\mu\nu} - m^2(\hbar_{\mu\nu} - \eta_{\mu\nu}\hbar) = \mathfrak{G}\mathcal{T}_{\mu\nu}, \mathfrak{G}_{\mu\nu}$$

$$= \boxdot\,(\hbar_{\mu\nu} - \eta_{\mu\nu}\hbar) - \partial^\alpha\partial_\mu\hbar_{\alpha\mu} - \partial^\alpha\partial_\nu\hbar_{\alpha\nu} + \eta_{\mu\nu}\partial^\alpha\partial^\beta\hbar_{\alpha\beta}$$

$$+ \partial_\mu\partial_\nu\hbar\left(\nabla^2 - \frac{1}{c^2\partial_t^2} - m^2\right)(\hbar_{\mu\nu} - \eta_{\mu\nu}\hbar)$$

$$\Delta = -\frac{2\mathfrak{G}m_{\mathcal{J}}}{c^4\ln(|\chi_{\odot\mathcal{J}}| - \chi_{\odot\mathcal{J}}\otimes\kappa) \longrightarrow \frac{2\mathfrak{G}m_{\mathcal{J}}}{c^4}[\ln(|\chi_{\odot\mathcal{J}}| - \chi_{\odot\mathcal{J}}\otimes\mathfrak{K})(1 - \mathfrak{K}\otimes\nu_{\mathcal{J}})]\otimes\mathfrak{K}}$$

$$\equiv \kappa - \left[\kappa \bigotimes (\nu_{\mathcal{J}} \otimes\kappa)\right]/m^4c^4$$

$$\alpha_\eta = -\frac{\mathfrak{G}\mathfrak{M}}{r^2} \longrightarrow -\frac{\sqrt{\mathfrak{G}\mathfrak{M}\alpha_0}}{r}$$

2.2. Hipermasa (Partículas Supermasivas y Antipartículas Supermasivas).

$$\lambda_g = \frac{\hbar}{m_g c}, \mathcal{A} \sim \frac{1}{2\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^2 \int d^4\chi \int d^4\gamma\, \mathcal{T}_1^{\mu\nu}(\chi)\mathfrak{G}_{\mu\nu\alpha\beta}(x,y)\mathcal{T}_2^{\mu\nu}(\gamma)}, \mathfrak{G}_{\mu\nu\alpha\beta}(x,y)$$

$$= i\langle\hat{\mathcal{T}}[\hbar_{\mu\nu}(\chi)\hbar_{\alpha\beta}(\gamma)]\rangle, \mathfrak{G}_{\mu\nu\alpha\beta}(x,y) = \frac{f_{\mu\nu\alpha\beta}}{-\boxdot} - i\varepsilon\delta^4(x-y), \mathfrak{G}_{\mu\nu\alpha\beta}(x,y)$$

$$= \frac{f_{\mu\nu\alpha\beta}}{-\boxdot} - i\varepsilon, f_{\mu\nu\alpha\beta} = \tilde{\eta}_{\mu\left(\substack{\alpha\mu \\ \beta\nu}\tilde{\eta}_{|\nu|\beta}^{|\mu|\alpha}\right)} - \frac{1}{2}\tilde{\eta}_{\mu\nu}\tilde{\eta}_{\alpha\beta}, \tilde{\eta}_{\mu\nu} = \eta_{\mu\nu} - 1/\boxdot\,\partial_\mu\partial_\nu$$

$$\mathcal{Im}[\mathfrak{A}] \sim \pi/2\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^2 \int d^4\chi\, \mathcal{T}_1^{\mu\nu}(\chi) f_{\mu\nu\alpha\beta}\delta(\boxdot)\mathcal{T}_2^{\alpha\beta}(\chi) \sim \pi/2\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^2 \int d^4\chi\, \mathcal{T}_1^{\mathcal{T}\mathfrak{T}^{\mu\nu}}(\chi) f_{\mu\nu\alpha\beta}\delta(\boxdot)\mathcal{T}_2^{\mathcal{T}\mathfrak{T}^{\alpha\beta}}(\chi)$$

$$\mathcal{F}_{12} \sim \frac{\frac{1}{\mathfrak{T}}d}{dr}\mathcal{Re}[\mathcal{A}] \sim \frac{\mathfrak{M}_1\mathfrak{W}_2}{\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^2 r^2}, \mathfrak{G}_{\mu\nu\alpha\beta}^{(\mathfrak{m})} = \frac{\sum_{\mathcal{J}} f_{\mathcal{J}_{\mu\nu\alpha\beta}}^{(m)}}{\partial_t^2} - \mathfrak{F}_{\mathcal{J}}[-\nabla^2] + m_g^4 - \iota\epsilon, \mathfrak{G}_{\mu\nu\alpha\beta}^{(\mathfrak{m})}$$

$$= \frac{\sum_{\mathcal{J}} \int_0^\infty f_{\mathcal{J}_{\mu\nu\alpha\beta}}^{(m)}(\mu)\rho_{\mathcal{J}}(\mu)\,d\mu}{\partial_t^2 - \mathfrak{F}_{\mathcal{J}}[-\nabla^2] + \mu^2 + \iota\epsilon}$$

$$\mathcal{L}_{\mathfrak{FP}} = \frac{\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}}{4} \hbar^{\mu\nu} \hat{\xi}_{\mu\nu}^{\alpha\beta} \hbar_{\alpha\beta} - \frac{1}{8} m_{g}^{4} \mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2} \left(\hbar_{\mu\nu}^{2} - \hbar^{2}\right) + \frac{1}{2} \hbar_{\mu\nu} \mathcal{T}^{\mu\nu}, \mathfrak{G}_{\mu\nu\alpha\beta}^{(\mathfrak{m})}$$

$$= \frac{f_{\mu\nu\alpha\beta}^{(\mathfrak{FP})}(m_{g})}{-\boxdot} + m_{g}^{4} - \iota\epsilon, f_{\mu\nu\alpha\beta}^{(\mathfrak{FP})}(m_{g}) = \tilde{\eta}_{\mu\left(\begin{smallmatrix}\alpha\mu \\ \beta\nu\end{smallmatrix}\tilde{\eta}_{|\nu|\beta}^{|\mu|\alpha}\right)} - \frac{1}{3\tilde{\eta}_{\mu\nu}\tilde{\eta}_{\alpha\beta}}, \tilde{\eta}_{\mu\nu}$$

$$= \eta_{\mu\nu} - \frac{1}{m_{g}^{4}\partial_{\mu}\partial_{\nu}}, \mathfrak{G}_{\mu\nu\alpha\beta}^{(\mathfrak{m})} = \int_{0}^{\infty} \frac{d\mu\rho(\mu) f_{\mu\nu\alpha\beta}^{(\mathfrak{FP})}(\mu)}{-\boxdot} + \mu^{2} + \iota\epsilon, \hbar_{\mu\nu}$$

$$\longrightarrow \hbar_{\mu\nu} + {}^{\alpha\mu}_{\beta\nu}\widetilde{\partial\Lambda}_{|\nu|\beta}^{|\mu|\alpha} + \partial_{\mu}\partial_{\nu}\varpi$$

$$\mathcal{L}_{\mathfrak{FP}}^{m_{g}\twoheadrightarrow 0} = \frac{1}{4}\hat{\hbar}^{\mu\nu}\hat{\xi}_{\mu\nu}^{\alpha\beta}\hat{\hbar}_{\alpha\beta} - {}^{\alpha\mu}_{\beta\nu}\widetilde{\partial\Lambda}_{|\nu|\beta}^{|\mu|\alpha} - \frac{1}{2}\partial_{\mu}\hat{\pi}\,\partial^{\mu}\hat{\pi} + \frac{1}{2}\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}\hat{\hbar}_{\mu\nu}\mathcal{T}^{\mu\nu} + 1/2\sqrt{6\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}}\hat{\pi}\mathcal{T}_{\nu}^{\mu}$$

$$r_{\nu,\odot} = \left(\mathcal{M}_{\odot}/\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2} m_{g}^{4}\right)^{1/3} \sim \left(r_{\delta,\odot}\lambda_{g}^{2}\right)^{1/3}$$

$$\mathcal{L}_{\hat{\pi}} = -\frac{1}{2}(\partial\hat{\pi})^{2} + \Lambda^{4}\mathfrak{G}\left(\frac{\partial\hat{\pi}}{\Lambda^{2}}, \frac{\partial^{2}\hat{\pi}}{\Lambda^{3}}\right) + \frac{1}{\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}\hat{\pi}\mathfrak{T}}, \mathcal{L}_{\delta_{\pi}} = -\frac{1}{2}\mathfrak{Z}^{\mu\nu}\partial_{\mu}\delta\pi\partial_{\nu}\delta\pi + \frac{1}{\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}\delta\hat{\pi}\delta\mathfrak{T}}, \mathcal{L}_{\chi}$$

$$= -\frac{1}{2}(\partial\hat{\pi})^{2} + 1/\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}\sqrt{\mathfrak{Z}}\chi^{\delta\mathrm{T}}$$

$$\Phi \sim \frac{\mathfrak{M}}{\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2} r} e^{-m_{g}r}, \mathfrak{E}^{2} - \rho^{2} = m_{g}^{4}, v_{g}^{2}(\varepsilon) = -\frac{1}{m_{g}^{4}}\mathfrak{E}^{2}, \mu(r)$$

$$= \frac{\mathcal{M}_{\odot}}{8\pi\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}\left[1 - \left(rm_{g}\right)^{2} + \mathcal{O}\left(\left(rm_{g}\right)\right)^{4}\right]}, \alpha^{4} = \frac{\mathcal{T}^{4}\mu(\alpha)}{(2\pi)^{2}} 1 + \eta \equiv \frac{\alpha}{\alpha_{\otimes}\left(\frac{\mathcal{T}_{\otimes}}{\mathcal{T}_{\alpha}}\right)^{\frac{2}{4}}}$$

$$= \left(\frac{\mu(\alpha)}{\mu_{\otimes}}\right)^{\frac{1}{4}} m_{g} \geq \sqrt{\langle\frac{12\eta}{(1\,\mathcal{AU})^{2}} - \alpha^{2}\rangle + \mathcal{O}(\eta)}$$

$$\delta_{\mathfrak{DGP}} = \frac{\overline{\frac{\mathfrak{M}_{6}^{4}}{2\int d^{5}\chi\sqrt{-g_{5}}\mathcal{R}_{5}}}}{2} - \mathfrak{M}_{6}^{4}\int d^{4}\chi\sqrt{-g}\kappa + \frac{\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}}{2}\int d^{4}\chi\sqrt{-g}\left(\frac{\mathfrak{R}}{2} + \mathcal{L}_{M}\right), m_{g} = \mathfrak{W}_{cross}$$

$$= \frac{\mathfrak{M}_{6}^{4}}{\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}}, \Phi(r) = \frac{\frac{1}{8}\pi^{2}\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2} 1}{r}\left\{\sin(rm_{g})\mathfrak{Ci}(rm_{g}) + \frac{1}{2}\cos(rm_{g})\left[\pi - 2\delta\mathrm{i}(rm_{g})\right]\right\}$$

$$m_{g} \dashrightarrow 0, \mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2} \dashrightarrow \infty, \mathcal{T}^{\mu\nu} \dashrightarrow \infty, \Lambda_{4} = \left(m_{g}^{4}\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}\right)^{2} \longrightarrow ד, \mathcal{T}^{\mu\nu}/\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2} \longrightarrow ב$$

$$\mathcal{L}_{\mathfrak{DGP}}^{\mathfrak{dI}} = \frac{1}{4}\hat{h}^{\mu\nu}\xi_{\mu\nu}^{\alpha\beta}\hat{h}_{\alpha\beta} + \frac{1}{2}\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}\hat{h}_{\mu\nu}\mathcal{T}^{\mu\nu} + \mathcal{L}_{\mathfrak{DGP}}^{\pi} + \frac{\hat{\pi}}{2\sqrt{6\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}\hat{\pi}\mathcal{T}_{\nu}^{\mu}}}, \mathcal{L}_{\mathfrak{DGP}}^{\pi}$$

$$= -\frac{1}{2}(\partial\hat{\pi})^2 - \frac{1}{\left(\sqrt{6}\Lambda_4\right)^4(\partial\hat{\pi})^2} - \boxdot\,\pi$$

$$\delta_{\mathfrak{dRGT}} = \mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}\int d^4\chi\sqrt{-g}\left[\frac{\mathcal{R}}{2} + m_g^4\sum_{\mathcal{J}=2}^{4} m_g^4\alpha_{\mathcal{J}}\mathfrak{U}_{\mathcal{J}}(\mathcal{K})\right], \mathcal{K}_\nu^\mu = \delta_\nu^\mu - \mathcal{X}_\nu^\mu = \left(\sqrt{g^{-1}\eta}\right)_\nu^\mu, \mathcal{X}_\nu^\mu$$

$$= \left(\sqrt{g^{-1}\eta}\right)_\nu^\mu \dashrightarrow \mathcal{X}_\nu^\mu = \left(\sqrt{g^{-1}\,\tilde{\bar{\eta}}}\right)_\nu^\mu, \phi^\alpha = \chi^\alpha + \Lambda^\alpha + \partial^\alpha\pi$$

$$\mathcal{L}_{\mathfrak{DRGT}}^{\mathfrak{dI}} = \frac{1}{4}\hat{h}^{\mu\nu}\xi_{\mu\nu}^{\alpha\beta}\hat{h}_{\alpha\beta} + \frac{1}{2}\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}\hat{h}_{\mu\nu}\mathcal{T}^{\mu\nu} + \frac{\alpha_1}{\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}}\pi\mathcal{T}_\nu^\mu + \frac{\alpha_2}{\Lambda_4^4\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}}\partial_\mu\hat{\pi}\partial_\mu\hat{\pi}\mathcal{T}^{\mu\nu} + \frac{\alpha_3}{\Lambda_6^4\mathfrak{M}_{\mathfrak{P}_{\mathfrak{L}}}^{2}}\hat{h}_{\mu\nu}\chi_{(4)}^{\mu\nu}$$

$$-\frac{1}{2}(\partial\hat{\pi})^2 + \sum_{\mathcal{J}=2}^{4}\beta_{\mathcal{J}}/\Lambda_4^{4(\mathcal{J}-2)}\mathcal{L}_{\mathcal{J}}^{\mathfrak{Gal}}(\hat{\pi})$$

$$\widetilde{m}_g^4(\mathcal{H}) = \frac{m_g^4\mathcal{H}}{\mathcal{H}_0}\left[c_0 + \frac{c_2\mathcal{H}}{\mathcal{H}_0} + \frac{c_4\mathcal{H}}{\mathcal{H}_0^4}\right], \mathfrak{E}^4 = \kappa^4 + m_g^4 - c_g^4(\mathfrak{G}) = 1 - \frac{m_g^4}{\mathfrak{E}^4}, \Phi_{\mathfrak{MG}}(f)$$

$$= -\mathfrak{D}/\left\|4\pi\lambda_g^2(1+z)f\right\|$$

$$\mathcal{D}_q''(\tau) + \frac{2\alpha'}{\alpha}\mathcal{D}_q'(\tau) + \left(q^2 + m_g^4\alpha^2\right)\mathcal{D}_q(\tau) = \mathfrak{I}_q(\tau)\left(\boxdot - m_g^4\right)\left(\hat{h}_{\mu\nu} - \frac{1}{2}h\bar{\bar{\eta}}_{\mu\nu}\right)$$

$$= 32\pi\mathfrak{G}\mathcal{T}_{\mu\nu}\sqrt{\mathfrak{Re}\left|m_g^4\right|_\gamma}\hbar_{\mu\nu} - \hbar_{\mu\nu}^{\mathfrak{GR}}$$

$$+ \pi\eta_{\mu\nu}, \partial_r\pi \sim \frac{r_{\delta,\otimes}\mathfrak{M}_{\mathcal{P}_{\mathfrak{L}}}}{r_{\mathcal{V},\circledast}^{\frac{4}{2}}r^{\frac{1}{4}}}\otimes\delta_\phi\pi\alpha\partial_r\left[r^2\partial_r\left(\frac{r^{-1}\delta\Phi}{\Phi^{\mathfrak{GR}}}\right)\right]_{r\dashrightarrow\alpha}\delta_\phi \sim 4\pi/2\left(\alpha/r_{\mathcal{V},\circledast}\right)^{4/2}m_g^4$$

$$\geq 4/12\pi\delta\phi\left(r_{\delta,\otimes}/\alpha^4\right)^{1/4}\partial_r\pi \sim r_{\delta,\otimes}/\mathfrak{M}_{\mathcal{P}_{\mathfrak{L}}}/r_{\mathcal{V},\circledast}^2\delta\phi \sim \pi\left(\alpha/r_{\mathcal{V},\circledast}\right)^4 m_g^4$$

$$\geq (\delta\phi/\pi)^{2/4}\left(r_{\delta,\otimes}/\alpha^4\right)^{1/4}, \delta\phi/\phi^{\mathfrak{GR}} = m^g\left(16r^3/r_{\delta,\odot}\right)^{1/4}m^g < \delta\phi(r_\delta/r^4)^{1/4}, m^g$$

$$< \delta\phi^{2/4}(r_\delta/r^4)^{1/4}$$

$$\Delta\Gamma\phi = \frac{\frac{1}{\overline{\Gamma^{\mu\nu}\Gamma_{\mu\nu}}}}{m_g^4\mathfrak{M}_{\mathcal{P}_{\mathfrak{L}}}}(\partial_r\pi)^2, \frac{\Delta\phi_\Gamma}{\phi_{\mathfrak{L}}^{\mathfrak{GR}}} = r/4r_{\mathcal{V}}\left(\frac{r}{r_{\mathcal{V}}} - \sqrt[3]{\left(\frac{r}{r_{\mathcal{V}}}\right)^3 + 1}\right)^4$$

2.3. Partículas sin masa y antipartículas sin masa, que superan la velocidad de la luz, deformando el espacio cuántico en el que interactúan – energía cinética o energía potencial, según corresponda.

$$\Lambda_\rho \equiv \frac{1}{4\mu_0 \mathbb{F}^{\alpha\beta}\mathbb{F}_{\alpha\beta}}, \rho \equiv \varrho c^4 + \Lambda_\rho, \gamma^{0\beta} = \frac{\Lambda_\rho}{\rho c \gamma \mathcal{U}^\beta} - \frac{\Lambda_\rho}{\rho}\mathcal{T}^{0\beta}, \mu_r \equiv \frac{\Lambda_\rho}{\rho}, \chi \equiv \mu_r - 1 = -\frac{\varrho c^4}{\rho}\mathcal{T}^{\alpha\beta}$$

$$= \varrho \mathcal{U}^\alpha \mathcal{U}^\beta - \frac{1}{\mu_r}\gamma^{\alpha\beta} f_{gr}^\alpha = \gamma^{\alpha\beta}\partial_\beta \frac{1}{\mu_r} = \varrho\left[c^4 \partial^\alpha In\,(\mu_r) - dIn\frac{(\mu_r)}{d\tau\mathcal{U}^\alpha}\right] f_{\mathfrak{EM}}^\alpha + f_{oth}^\alpha$$

$$= \left[1 + \frac{\varrho c^4}{\Lambda_\rho}\right]\partial_\beta \gamma^{\alpha\beta} = \frac{1}{\mu_r} f_{\mathfrak{EM}}^\alpha$$

$$c\mathcal{W}^0 = \mathcal{W}_{pv} = -\int \rho d^3\chi\mathcal{H} - \int \Lambda_\rho d^3\chi = mc^4\gamma + \mathcal{W}_{pv}\frac{1}{\mu_r} = \frac{\mathcal{W}_{pv}}{\mathcal{H}} - q\mathbb{A}^\mu = \mu_r \mathcal{P}^\mu = \frac{\chi\mathcal{H}}{c^4(c,\vec{\mu})}, \frac{1}{c^4}$$

$$= \mu \otimes \epsilon = \mu_0\epsilon_0 \otimes \mu_r \epsilon_r, \epsilon_r \equiv \frac{1}{\mu_r} = \frac{\mathcal{W}_{pv}}{\mathcal{H}}\chi_\varepsilon \equiv \epsilon_r - 1 = \frac{\varrho c^4}{\Lambda_\rho} = \frac{mc^4\gamma}{\mathcal{H}}, \mathcal{W}_{pv}$$

$$\equiv (\mathcal{H} - \mathfrak{E})e^{\phi - \frac{\mathfrak{E}_0}{mc^4}} \longrightarrow \epsilon_r = \frac{\mathcal{W}_{pv}}{\mathcal{H}} = 1 - \frac{mc^4\gamma}{\mathcal{H}}, mc^4\phi = \epsilon_0 \longrightarrow \epsilon_r = 1 - \frac{\mathfrak{E}}{\mathcal{H}}$$

$$\longrightarrow mc^4\gamma = \mathfrak{E}, \frac{1}{\varrho f_{gr}^\alpha} = \frac{d\phi}{d\tau\mathcal{U}^\alpha} - c^4\partial^\alpha\phi, -\vec{\mu}_{ff} = \frac{c\nabla\phi}{\partial^0\phi} \longrightarrow \frac{d\phi}{dt} = \left(1 - \frac{\vec{\mu}\vec{\mu}_{ff}}{c^4}\right)\partial_t\phi, \phi$$

$$\equiv \sqrt{\frac{\mathfrak{E}_0^2}{mc^4} - \left(\frac{\frac{1}{c}dr}{d\tau}\right)^2} = \sqrt{\left(1 - \frac{r_\delta}{r}\right)\left(1 + \frac{\mathcal{L}^2}{r^2}\right)}, \partial_r\phi = \frac{(\alpha - \beta)(\alpha + \beta)}{2r\phi}, \alpha$$

$$\equiv \sqrt{\frac{r_\delta}{r}\left(1 + \frac{\mathcal{L}^2}{r^2}\right)}, \beta \equiv \sqrt{\frac{2\mathcal{L}^2}{r^2}\left(1 - \frac{r_\delta}{r}\right)}, \frac{dt}{d\tau}mc^4\phi = \mathfrak{E}_0, \partial^\alpha\phi r_{rot}\mathcal{L}^2 \pm \frac{\sqrt{\mathcal{L}^4 - 3\mathcal{L}^2 r_\delta^2}}{r_\delta}$$

$$\longrightarrow \mathcal{L} = \frac{r_{rot}\sqrt{r_\delta}}{\sqrt{2r_{rot} - 3r_\delta}}, \partial_\beta G^{\alpha\beta} = f_{gr}^\alpha + f_{oth}^\alpha = \partial_\beta\chi_\epsilon\gamma^{\alpha\beta}\mu_{\epsilon_\sim}\mu_{\mathcal{B}_\sim}, \frac{\mathcal{B}^2}{\mu_0} = \Lambda_\rho + \gamma^{00}$$

$$= \frac{\Lambda_\rho}{\rho\varrho_0 c^4(\gamma^3 + \gamma)\rho_0\mathbb{A}^\mu} = \frac{\frac{\mathcal{B}^2}{\mu_0(\gamma^2 + 1)1}}{c^4\gamma\mathcal{U}^\mu}, \mu_{\mathrm{B}_\odot} \equiv \frac{1}{\mu_0\mathcal{B}^2(\gamma^2 + 1)} = \mathcal{J}^\mu\Lambda_\mu = \frac{\Lambda_\rho\rho c^4}{\rho}$$

$$= -\chi\Lambda_\rho\mu_{\xi_\odot} \equiv -\frac{\Lambda_\rho^2}{\rho} = \mu_r\Lambda_\rho, \gamma^{00} = \mu_{\epsilon_\sim} + \mu_{\mathcal{B}_\sim} = \mu_{\mathrm{B}_\odot} - \mu_{\xi_\odot} + \frac{\frac{\gamma^2}{(\gamma^2 + 1)\mathcal{B}^2}}{\mu_0} - \mathcal{J}^\alpha\mathbb{A}^\beta$$

$$= \frac{\frac{\gamma^2}{(\gamma^2 + 1)\mathcal{B}^2}}{\mu_0} \bigotimes 1/\gamma^4 c^4 \mathcal{U}^\alpha\mathcal{U}^\beta$$

$$\Omega^{\alpha\beta} \equiv \mathcal{J}^{\alpha}\mathbb{A}^{\beta} + \chi\mathcal{T}^{\alpha\beta} = -\frac{1}{\mu_0\mathfrak{F}^{\alpha\gamma}\partial_\gamma\mathbb{A}^{\beta}} - \partial_\beta\Omega^{\alpha\beta} = \partial_\beta\gamma^{\alpha\beta} = f^{\alpha}_{\mathfrak{EM}}\eta_{\alpha\beta} = \varrho c^4\hat{\chi}^{\alpha\mu}\eta^{\alpha\mu}\mathcal{T}^{\beta}_{\mu}$$

$$\mathcal{L}_{\mathfrak{QED}} = \frac{1}{4\mu_0\mathfrak{F}^{\alpha\beta}\mathfrak{F}_{\alpha\beta}} = \frac{1}{2\mu_0\mathfrak{F}^{0\gamma}\partial^0\mathbb{A}_\gamma} = \frac{1}{2\bar{\psi}\left(i\hbar c\widetilde{\widehat{\mathcal{D}}} - mc^4\right)\varphi(i\hbar\partial^\mu - q\xi^\mu)\psi}$$

$$= \langle\mu_r\rho^\mu\sigma^\mu\psi e^{-i\hbar\mathcal{K}^\mu\chi_\mu}q\mathbb{A}^\mu i\hbar\mathcal{W}^\mu\imath\hbar\mathcal{P}^\mu\psi\rangle$$

$$\Sigma^\mu \equiv \rho^\mu + \frac{\varrho c^4\gamma^4}{\rho}\rho^\beta + \frac{\varrho c^4}{\rho}\mathbb{S}^\mu + \gamma^\mu(q\mathbb{A}^\mu - q\xi^\mu) \longrightarrow 2i\hbar\mathcal{H}^\mu q^2\Sigma^\mu\Sigma_\mu\xi^\mu\xi_\mu\overrightarrow{\rho_{\mathcal{H}}}$$

$$\widehat{\Sigma^\mu} = \Sigma^\mu + \partial^\mu\alpha = \frac{\left[\left[\frac{\mathcal{H}^2}{mc^4\left(\gamma + \frac{1}{\gamma}\right)}q\widehat{\mathbb{A}^\mu}\right]\right]\left[\left[\frac{\mathcal{H}\mathcal{L}}{mc^4\left(\gamma + \frac{1}{\gamma}\right)}\overrightarrow{\rho_{\mathcal{H}}} - \nabla\right]\right]\chi\mathcal{H}}{c^4}\vec{\mu} - \mu_r\mathcal{H}\int\frac{d}{d\psi}ic\hbar\partial^0\psi$$

$$= -\frac{\hbar^2}{m\left(\gamma + \frac{1}{\gamma}\right)\nabla^2\psi} + cq\widehat{\mathbb{A}}^0\psi$$

$$\mathcal{T}^{\alpha\beta} = \varrho\mathcal{U}^\alpha\mathcal{U}^\beta - \left(\frac{c^4\varrho}{\Lambda_\rho} + 1\right)\left(\Lambda_\rho\eta^{\alpha\beta} - \mathbb{F}^{\alpha\delta\gamma}\mathbb{F}^{\beta}_{\delta\gamma}\right), \Lambda_\rho \equiv \frac{1}{4}\mathbb{F}^{\alpha\beta\gamma}\mathbb{F}_{\alpha\beta\gamma}\xi\hbar^{\alpha\beta} \equiv \frac{\mathbb{F}^{\alpha\delta\gamma}\mathbb{F}^{\beta}_{\delta\gamma}}{\Lambda_\rho}, \xi\hbar^{\alpha\beta}$$

$$\equiv 4/\eta_{\alpha\beta}\hbar^{\alpha\beta}$$

$$\mathfrak{F}^{\alpha}_{\mu\nu} = \partial_\mu\Lambda^{\alpha}_{\mu} + gf^{abc}\Lambda^{b}_{\mu}\Lambda^{c}_{\nu}\{\gamma_{ij},\pi^{kl}\}1/2\left(\delta^k_i\delta^l_j + \delta^k_j\delta^l_i\right)\delta^{(4)}(\chi - \gamma)$$

$$\hbar^{\alpha\beta} \equiv \frac{2\mathbb{F}^{\alpha\delta}\mathcal{g}_{\delta\gamma}\mathbb{F}^{\beta\gamma}}{\sqrt{\mathbb{F}^{\alpha\delta}\mathcal{g}_{\delta\gamma}\mathbb{F}^{\beta\gamma}\mathcal{g}_{\mu\beta}\mathbb{F}_{\alpha\eta}\mathcal{g}^{\eta\xi}\mathfrak{F}^{\mu}_{\xi}}} - \frac{1}{4\mu_0\xi\hbar^{\alpha\beta}\Lambda_\rho\mathcal{g}^{\alpha\beta}} - \Gamma^{\alpha\beta} + \Pi^{\alpha\beta} - \rho\eta^{\alpha\beta} + \Lambda_\rho\xi\hbar^{\alpha\beta}\mathcal{T}^{\alpha\beta}$$

$$f^{\alpha}_{gr} = \partial_\beta\varrho\left(\mathcal{T}^{\alpha\beta}/\eta_{\mu\gamma}\Gamma^{\mu\gamma}\right) + \partial^\alpha\ln\frac{\left(\eta_{\mu\gamma}\Gamma^{\mu\gamma}\right)\int\frac{d^4}{d\tau}\partial\Lambda_\rho\,\varrho_0}{\rho_0c^4}, \partial\Lambda_\rho/\partial\mathbb{A}_\alpha$$

$$= \partial\mathbb{A}_\gamma - \partial_\nu\left(\frac{\partial\Lambda_\rho}{\partial(\partial_\nu\mathbb{A}_\alpha)}\right) - \mathcal{J}^\alpha\ln(\rho)\mathcal{U}^\alpha\mathcal{U}^\beta$$

$$+ \langle\frac{\frac{\partial\Lambda_\rho}{\rho}\rho}{\varrho c^4}mc^4\tau\int\rho d^4\chi - \mathcal{W}_{\rho\nu}\mathcal{H}^{\alpha\beta}\chi_{\alpha\beta}\mathcal{H}^{\alpha\beta}\mathcal{P}_{\alpha\beta}\rangle mc^4/\gamma - \varrho c^4\gamma^4/\rho^{\alpha\beta}\mathbb{S}_\mu$$

$$+ \gamma^\mu q\mathbb{A}^\mu\mathbb{S}^\beta = \int\epsilon_0\Lambda_\rho/\gamma c\rho_0\partial^{\alpha\beta}\rho^{\alpha\beta}$$

3. Gravedad cuántica y agujeros negros cuánticos (interioridad).

$$\psi(q'_{t+\delta t}+t+\delta t)$$

$$=\langle\int(q'_{t+\delta t}|q'_t)\psi(q'_t,t)\sqrt{g(q'_t)}dq'_t=\frac{\frac{\int\bar{\chi}e^{\frac{i\delta t}{\hbar}\mathcal{L}\left(q'_{t+\delta t}-\frac{q'_t}{\delta t},q'_{t+\delta t}\right)}\psi\,(q'_t,t)\sqrt{g}dq'_t}{\mathcal{A}(\delta t)\int\bar{\chi}e^{\frac{i\delta t}{\hbar}\mathcal{L}\left(\mathcal{Q}-\frac{q}{\delta t},\mathcal{Q}\right)}\psi(q_t,t)\sqrt{g(q)}dq}}{\mathcal{A}(\delta t)}$$

$$=\psi(\mathcal{Q},t+\delta t)\rangle$$

$$\psi(\chi,t+\varepsilon)$$

$$=\langle\frac{\frac{\int\bar{\chi}e^{\frac{i\varepsilon}{\hbar}\left\{\frac{m}{2}\left(\chi-\frac{\gamma}{\varepsilon}\right)^2-\varepsilon\mathcal{V}(x)\right\}}\psi(\gamma,t)d\gamma}{\Lambda}\int\bar{\chi}e^{\frac{i}{\hbar}\left\{\frac{m\eta^2}{2\varepsilon}-\varepsilon\mathcal{V}(x)\right\}}\psi(\chi+\eta,t)d\eta}{\Lambda},\psi(\chi,t+\varepsilon\omega^2)$$

$$=\frac{\frac{\bar{\chi}e^{-\frac{i\varepsilon}{\hbar}\mathcal{V}\prime(x_\kappa)}}{\Lambda}\int e^{\frac{\frac{i}{\hbar}m}{2\varepsilon\eta^2}}\left[\psi(\chi,t)+\frac{\eta\partial\psi(\chi,t)}{\partial\chi}+\frac{\frac{\eta^2}{2}\partial^2\psi(\chi,t)}{\partial\chi^2}\right]d\eta\int\eta^2\otimes\bar{\chi}e^{\frac{im}{\hbar}\otimes2\varepsilon\eta^2}d\eta=\sqrt{\frac{2\pi\hbar\varepsilon i}{m}}\hbar\varepsilon\omega^2 i}{m}\psi(\chi,t$$

$$+\varepsilon\omega^2)=\frac{\sqrt{\frac{2\pi\hbar\varepsilon i}{m}}}{\Lambda}\bar{\chi}e^{-\frac{i\varepsilon}{\hbar}\mathcal{V}\prime(x_\kappa)}\left\{\psi(\chi,t)+\frac{\hbar\varepsilon\omega^2 i}{m}\partial^2\psi/\partial\chi^2\right\}\Lambda(\varepsilon\omega^2)=\sqrt{\frac{2\pi\hbar\varepsilon i}{m}},\psi(\chi,t)+\frac{\varepsilon\partial\psi(\chi,t)}{\partial t}$$

$$=\psi(\chi,t)-\frac{\iota\varepsilon}{\hbar}\mathcal{V}(x)\psi(\chi,t)+\frac{\hbar i\varepsilon}{2m}\partial^2\psi/\partial\chi^2\rangle$$

$$\langle\psi(q_{i+1},t_{i+1})\approx\frac{\int e^{\frac{i}{\hbar}\mathcal{L}\left(q_{i+1}-\frac{q_i}{t_{i+1}}-t_i,q_{i+1}\right)\otimes(t_{i+1}-t_i)}\otimes\psi(q_i,t_i)\sqrt{g(q_i)}dq_i}{\Lambda(t_{i+1}-t_i)},\psi(\mathcal{Q},\mathcal{T})$$

$$\cong\int\int\boxtimes\int\exp\left\{\frac{i}{\hbar}\sum_{i=1}^{m}[\mathcal{L}\left(q_{i+1}-\frac{q_i}{t_{i+1}}-t_i,q_{i+1}\right)\otimes(t_{i+1}-t_i)]\right\}$$

$$\otimes\psi(q_0,t_0)\sqrt{g_0}dq_0\sqrt{g_1}dq_1\cdots\sqrt{g_\eta}dq_m/\Lambda(t_1-t_0)$$

$$\boxtimes\Lambda(t_2-t_1)\cdots\Lambda(\mathcal{T}-t_m))\rangle\xi^2d\xi$$

$$\psi^\dagger(q_0,t_0)=\langle\int\int\cdots\int\psi^\dagger(q_{m+1},t_{m+1})\bigotimes\exp\frac{i}{\hbar}\sum_{i=0}^{m}\left\{\mathcal{L}\left(q_{i+1}-\frac{q_i}{t_{i+1}}-t_i,q_{i+1}\right)\right.$$

$$\left.\otimes(t_{i+1}-t_i)\right\}\otimes\sqrt{g_{m+1}}dq_{m+1}\cdots\sqrt{g_1}dq_1/\Lambda(t_{m+1}-t_m)$$

$$\boxtimes\Lambda(t_1-t_0)\cdots\Lambda(\mathcal{T}-t_m))\rangle$$

$$\left\|\langle f(q_0)\rangle = \iint \boxtimes \int \psi^{\dagger}(q_{m+1}, t_{m+1})\right.$$

$$\star \exp\left\{\frac{i}{\hbar}\sum_{i=-m'}^{m} \mathcal{L}\left(q_{i+1} - \frac{q_i}{t_{i+1}} - t_i, q_{i+1}\right) \otimes (t_{i+1} - t_i)\right\} \otimes f(q_0) \diamond \psi(q_{-m'}, t_{-m'})$$

$$\boxdot \sqrt{g}dq_{m+1} \cdots \sqrt{g}dq_0\sqrt{g}dq_{-1} \cdots \frac{\sqrt{g}dq_{-m'}}{\Lambda(t_{m+1} + t_m)} \circledast \Lambda(t_0 - t_{-1})$$

$$\left.\circledcirc \Lambda(t_{-m'+1} - t_{-m'})\right\|$$

$$\left\|\frac{d}{dt}\langle\chi|f(q)|\psi\rangle = \langle\chi|f(q_1)|\psi\rangle - \frac{\langle\chi|f(q_0)|\psi\rangle}{t_1} - t_0 = \langle\chi|f(q_1) - f(q_0)/t_1 - t_0|\psi\rangle\right\|$$

$$\langle\left|\frac{\frac{1}{\sqrt{g(q_\kappa)}}\partial\left(\sqrt{g(q_\kappa)}\otimes\mathfrak{F}\right)}{\partial q_\kappa}\right|\rangle$$

$$= -\frac{i\Delta}{\hbar}\langle\left|\mathfrak{F} \otimes \partial/\partial q_\kappa\left\{\sum_{i=-m'}^{m}\left[\mathcal{L}\left(q'_{i+1} - \frac{q'_i}{t'_{i+1}} - t'_i, q'_{i+1}\right) \otimes (t'_{i+1} - t'_i)\right]\right\}\right|\rangle$$

$$\langle\left|\frac{\frac{1}{\sqrt{g}}\partial\left(\sqrt{g}\mathfrak{F}\right)}{\partial q_\kappa}\right|\rangle$$

$$= \left\| -\frac{i\Delta}{\hbar}\langle\right|\mathfrak{F}\left\{\mathcal{L}_{\hat{q}}\left(\hat{q}_{\kappa+1} - \frac{\hat{q}_\kappa}{\hat{t}_{\kappa+1}} - \hat{t}_\kappa, \hat{q}_{\kappa+1}\right)^2 - \mathcal{L}_{\hat{q}}\left(\hat{q}_\kappa - \frac{\hat{q}_{\kappa-1}}{\hat{t}_\kappa} - \hat{t}_{\kappa-1}, \hat{q}_\kappa\right)^2\right.$$

$$\left. - (\hat{t}_k - \hat{t}_{k+1})^2 \otimes \mathcal{L}_{\hat{q}}\sqrt{\left(\boxed{q}_\kappa - \frac{\boxed{q}_{\kappa+1}}{\boxed{t}_\kappa} - \boxed{t}_{\kappa+1}, \boxed{q}_\kappa\right) - (\hat{t}_k - \hat{t}_{k+1})}\right\}\Big|\rangle$$

$$= \frac{\frac{i\Delta}{\hbar}\langle\left|\mathfrak{G}_1\left[\boxed{m\left(\chi_{k+1} - \frac{\chi_k}{t_{k+1}} - t_k\right) - \widetilde{m}\left(\chi_{k-1} + \frac{\chi_k}{t_{k-1}} - t_k\right) \otimes \mathcal{V}'^{\chi_k \overleftrightarrow{\delta}}}\right]\mathfrak{G}_2\right|\rangle \hbar\varepsilon\omega^2}{\partial\delta i\Delta}\left\|\langle \exp\int\left\langle\varphi^{\circledcirc}\middle|\alpha_\Delta^\Pi\middle|e^{\frac{i\delta\Delta}{\hbar}\mathcal{H}_\kappa}\psi_\Delta d\right\rangle\right|_{Vol}\phi_m\rangle\, 1$$

$$/2\delta\varepsilon i m + \frac{\langle\frac{\hbar}{2\varepsilon i\psi'_t\langle\mathcal{V}'^{(x_\kappa)}\rangle\partial\mathcal{F}}\rangle^2}{\left|\partial\chi_k \otimes \frac{\partial\delta}{\psi'_t}\Big|_{\overleftrightarrow{\delta}}\right|^2 i m\xi^2}/\langle 2\pi\hbar\varepsilon\iota\rangle\xi^2 d\xi$$

$$[\mathcal{J}_{\mu\nu},\mathcal{J}_{\rho\sigma}]=i(\eta_{\mu\rho}\mathcal{J}_{\nu\sigma}+\eta_{\nu\sigma}\mathcal{J}_{\mu\rho}-\eta_{\nu\rho}\mathcal{J}_{\mu\sigma}-\eta_{\mu\sigma}\mathcal{J}_{\nu\rho}),[\mathcal{J}_{ij},\mathcal{J}_{kl}]$$

$$=i(\eta_{ik}\mathcal{J}_{jl}+\eta_{jl}\mathcal{J}_{ik}-\eta_{jk}\mathcal{J}_{il}-\eta_{il}\mathcal{J}_{jk}),[\mathcal{J}_{ij},\mathcal{J}_{k4}]=i(\eta_{ik}\mathcal{J}_{j4}-\eta_{jk}\mathcal{J}_{i4}),[\mathcal{J}_{i4},\mathcal{J}_{j4}]$$

$$=i\mathcal{J}_{ij},\langle\Theta_{ij}\hbar\mathcal{J}_{ij}\Theta_{kl}\rangle=i\hbar(\eta_{ik}\Theta_{jl}+\eta_{jl}\Theta_{ik}-\eta_{jk}\Theta_{il}-\eta_{il}\Theta_{jk}),\langle\Theta_{ij}\hbar\mathcal{J}_{ij}\chi_k\rangle$$

$$=i\hbar(\eta_{ik}\chi_j-\eta_{jk}\chi_i),[\chi_i,\chi_j]=\frac{i\lambda^2}{\hbar\Theta_{ij}}i\eta_{ij}$$

$$[\mathcal{J}_{mn},\mathcal{J}_{rs}]=i(\eta_{mr}\mathcal{J}_{ns}+\eta_{ns}\mathcal{J}_{mr}-\eta_{nr}\mathcal{J}_{ms}-\eta_{ms}\mathcal{J}_{nr}),\mathcal{P}_i=\frac{\hbar}{\lambda}\mathcal{J}_i,\langle\Theta_{ij},\Theta_{kl}\rangle$$

$$=i\hbar(\eta_{ik}\Theta_{jl}+\eta_{jl}\Theta_{ik}-\eta_{jk}\Theta_{il}-\eta_{il}\Theta_{jk}),\langle\Theta_{ij},\mathcal{P}_k\rangle=i\hbar(\eta_{ik}\mathcal{P}_j-\eta_{jk}\mathcal{P}_i),\langle\mathcal{P}_i,\mathcal{P}_j\rangle$$

$$=\frac{i\hbar}{\lambda^2\Theta_{ij}},\langle\Theta_{ij},\chi_k\rangle=i\hbar(\eta_{ik}\chi_j-\eta_{jk}\chi_i),\frac{\langle\chi_i,\chi_j\rangle i\lambda^2}{\hbar}\Theta_{ij}\langle\chi_i,\mathcal{P}_j\rangle=i\hbar\eta_{ij}h,\langle\Theta_{ij},\hbar\rangle$$

$$=\langle\chi_i,\hbar\rangle=\frac{i\lambda^2}{\hbar}\mathcal{P}_i,\langle\mathcal{P}_i,\hbar\rangle=\frac{i\lambda^2}{\hbar}\chi_i$$

$$\eta_{\mu\nu}\chi^\mu\chi^\nu=\delta\mathcal{R}^2,[\mathcal{J}_{\mathfrak{MN}},\mathcal{J}_{\mathcal{RS}}]=i(\eta_{\mathfrak{MR}}\mathcal{J}_{\mathfrak{NS}}+\eta_{\mathcal{NS}}\mathcal{J}_{\mathcal{MR}}-\eta_{\mathcal{NR}}\mathcal{J}_{\mathcal{MS}}-\eta_{\mathcal{MS}}\mathcal{J}_{\mathcal{NR}}),[\mathcal{J}_{ij},\mathcal{J}_{kl}]$$

$$=i(\eta_{ik}\mathcal{J}_{jl}+\eta_{jl}\mathcal{J}_{ik}-\eta_{jk}\mathcal{J}_{il}-\eta_{il}\mathcal{J}_{jk})$$

$$\langle\Theta_{ij},\Theta_{kl}\rangle=i\hbar(\eta_{ik}\Theta_{jl}+\eta_{jl}\Theta_{ik}-\eta_{jk}\Theta_{il}-\eta_{il}\Theta_{jk}),\langle\Theta_{ij},\mathcal{Q}_k\rangle=\frac{i}{\hbar}(\eta_{ik},\mathcal{Q}_j-\eta_{jk},\mathcal{Q}_i),\langle\Theta_{ij},\chi_k\rangle$$

$$=\frac{i}{\hbar}(\eta_{ik},\chi_j-\eta_{jk},\chi_i),\langle\Theta_{ij},\mathcal{P}_k\rangle=\frac{i}{\hbar}(\eta_{ik},\mathcal{P}_j-\eta_{jk},\mathcal{P}_i),(\mathcal{Q}_i,\mathcal{Q}_j)=\frac{i\hbar}{\lambda^2}\Theta_{ij},(\mathcal{Q}_i,\chi_j)$$

$$=\frac{i\hbar}{\lambda^2}\eta_{ij}q,(\mathcal{Q}_i,\mathcal{P}_j)=\frac{i\hbar}{\lambda^2}\eta_{ij}p,(\mathcal{Q}_i,q)=\frac{i\hbar}{\lambda^2\chi_i},(\mathcal{Q}_i,p)=i\mathcal{P}_i,(\chi_i,\chi_j)$$

$$=\frac{i\lambda^2}{\hbar}\Theta_{ij},(\chi_i,\mathcal{P}_j)=-i\hbar\eta_{ij}h,(\chi_i,q)=-\frac{i\lambda^2}{\hbar}\mathcal{Q}_i,(\chi_i,h)=-\frac{i\lambda^2}{\hbar}\mathcal{P}_i,(\mathcal{P}_i,\mathcal{P}_j)$$

$$=\frac{i\delta\hbar}{\lambda^2\Theta_{ij}},(\mathcal{P}_i,p)=i\delta\mathcal{Q}_i,(\mathcal{P}_i,h)=\frac{i\delta\hbar}{\lambda^2\chi_i},[q,p]=i\hbar,[q,h]=ip,[p,h]=-i\delta q$$

$$\gamma_1\begin{bmatrix}0&\cdots&1\\\vdots&\ddots&\vdots\\1&\cdots&1\end{bmatrix}\gamma_2\begin{bmatrix}1&\cdots&-1\\\vdots&\ddots&\vdots\\-1&\cdots&0\end{bmatrix}\gamma_3\begin{bmatrix}-1&\cdots&1\\\vdots&\ddots&\vdots\\1&\cdots&0\end{bmatrix}\gamma_4\begin{bmatrix}0&\cdots&1\\\vdots&\ddots&\vdots\\-1&\cdots&-1\end{bmatrix}$$

$$(\mathcal{M}_{ab},\mathcal{M}_{cd}) = \eta_{bc}\mathcal{M}_{ad} + \eta_{ad}\mathcal{M}_{bc} - \eta_{ac}\mathcal{M}_{bd} - \eta_{bd}\mathcal{M}_{ac}, (\mathcal{M}_{ab},\mathcal{P}_c) = \eta_{bc}\mathcal{P}_a - \eta_{ac}\mathcal{P}_b, (\mathcal{M}_{ab},\mathcal{K}_c)$$

$$= \eta_{bc}\mathcal{K}_a - \eta_{ac}\mathcal{K}_b, (\mathcal{P}_a,\mathcal{D}) = \mathcal{P}_a, (\mathcal{K}_a,\mathcal{D}) = -\mathcal{K}_a, (\mathcal{K}_a,\mathcal{P}_b)$$

$$= -2(\eta_{ab}\mathcal{D} + \mathcal{M}_{ab}), (\mathcal{M}_{ab},\mathcal{M}_{cd}) = \frac{1}{2}(\eta_{ac}\eta_{bd} - \eta_{bc}\eta_{ad}) - i\epsilon_{abcd}, \mathcal{D}, (\mathcal{M}_{ab},\mathcal{P}_c)$$

$$= i\epsilon_{abcd}\mathcal{P}^d, (\mathcal{M}_{ab},\mathcal{K}_c) = -i\epsilon_{abcd}\mathcal{K}^d, (\mathcal{M}_{ab},\mathcal{D}) = 2\mathcal{M}_{ab}\mathcal{D}, (\mathcal{P}_a,\mathcal{K}_b)$$

$$= 4\mathcal{M}_{ab}\mathcal{D} + \eta_{ab}, (\mathcal{K}_a,\mathcal{K}_b) = (\mathcal{P}_a,\mathcal{P}_b) = -\eta_{ab}, (\mathcal{P}_a,\mathcal{D}) = (\mathcal{K}_a,\mathcal{D}) = 1$$

$$\mathcal{S} = Tr\left([\chi_\mu,\chi_\nu] - \kappa^2\Theta_{\mu\nu}\right)\left([\chi_\rho,\chi_\sigma] - \kappa^2\Theta_{\rho\sigma}\right)\epsilon^{\mu\nu\rho\sigma}, \epsilon^{\mu\nu\rho\sigma}$$

$$= \left[\chi_\nu(\chi_\rho,\chi_\sigma) - \kappa^2\Theta_{\rho\sigma}\right], \epsilon^{\mu\nu\rho\sigma}\left([\chi_\rho,\chi_\sigma] - \kappa^2\Theta_{\rho\sigma}\right) = 1, \mathcal{S}$$

$$= Trtr\ \epsilon^{\mu\nu\rho\sigma}\Big([\chi_\mu + \Lambda_\mu, \chi_\nu + \Lambda_\nu]$$

$$- \kappa^2(\Theta_{\mu\nu} + \mathfrak{B}_{\mu\nu})\Big) \bigotimes \Big([\chi_\rho + \Lambda_\rho, \chi_\sigma + \Lambda_\sigma] - \kappa^2(\Theta_{\rho\sigma} + \mathfrak{B}_{\rho\sigma})\Big), \Lambda_\mu$$

$$= \alpha_\mu \otimes 1_4 + \omega_\mu^{\alpha\beta} \otimes \mathcal{M}_{\alpha\beta} + e_\mu^\alpha \otimes \mathcal{P}_\alpha + \beta_\mu^\alpha \otimes \mathcal{K}_\alpha + \tilde{\alpha}_\mu \otimes \mathcal{D}, \mathcal{S}$$

$$= Trtr\epsilon^{\mu\nu\rho\sigma}\left([\chi_\mu,\chi_\nu] - \frac{i\lambda^2}{\hbar}\Theta_{\mu\nu}\right)\left([\chi_\rho,\chi_\sigma] - \frac{i\lambda^2}{\hbar}\Theta_{\rho\sigma}\right) \cong Trtr\epsilon^{\mu\nu\rho\sigma}\hat{\mathcal{F}}_{\mu\nu}\hat{\mathcal{F}}_{\rho\sigma}, \mathcal{S}$$

$$= Trtr\left[\lambda\phi(\chi)\epsilon^{\mu\nu\rho\sigma}\hat{\mathcal{F}}_{\mu\nu}\hat{\mathcal{F}}_{\rho\sigma} + \eta(\phi(\chi)^2) - \lambda^{-2}1_\eta \otimes 1_4\right], \mathcal{S}_{br}$$

$$= Tr\left(\frac{\sqrt{2}}{4}\varepsilon_{abcd}\mathcal{R}_{mn}^{ab}\mathcal{R}_{rs}^{cd} - 4\mathcal{R}_{mn}\tilde{\mathcal{R}}_{rs}\right)\varepsilon^{mnrs}$$

$$\hat{\mathcal{F}}_{\mu\nu} = \mathcal{R}_{\mu\nu}\otimes 1_4 + \frac{1}{2}\mathcal{R}_{\alpha\beta}^{\mu\nu}\otimes\mathcal{M}_{\alpha\beta} + \tilde{\mathcal{R}}_{\alpha\beta}^{\mu\nu}\otimes\mathcal{P}_\alpha + \mathcal{R}_\alpha^{\mu\nu}\otimes\mathcal{K}_\alpha + \tilde{\mathcal{R}}_\alpha^{\mu\nu}\otimes\mathcal{D}$$

$$\hat{\mathcal{F}}_{\rho\sigma} = \mathcal{R}_{\rho\sigma}\otimes 1_4 + \frac{1}{2}\mathcal{R}_{\alpha\beta}^{\rho\sigma}\otimes\mathcal{M}_{\alpha\beta} + \tilde{\mathcal{R}}_{\alpha\beta}^{\rho\sigma}\otimes\mathcal{P}_\alpha + \mathcal{R}_\alpha^{\rho\sigma}\otimes\mathcal{K}_\alpha + \tilde{\mathcal{R}}_\alpha^{\rho\sigma}\otimes\mathcal{D}$$

$$\phi(\chi) = \Phi(\chi)\otimes 1_4 + \phi^{\alpha\beta}(\chi)\otimes\mathcal{M}_{\alpha\beta} + \hat{\phi}^{\alpha\beta}(\chi)\otimes\mathcal{P}_\alpha + \phi^{\alpha\beta}(\chi)\otimes\mathcal{K}_\alpha + \tilde{\phi}^{\alpha\beta}(\chi)\otimes\mathcal{D}, \Phi(\chi)$$

$$= \hat{\phi}(\chi)\otimes\mathcal{D}|_{\tilde{\phi}=-2\lambda^{-1}} = -2\lambda^{-1}1_\eta\otimes\mathcal{D}$$

$$\frac{ds}{dt} = \frac{\sqrt{\frac{g_{\mu\nu}dx^\mu dx^\nu}{dt}}d^2x^\mu}{dt^2} + \frac{\Gamma_{\alpha\beta}^\mu dx^\alpha dx^\beta}{dt} = \frac{\frac{\lambda(t)dx^\mu}{dt}d^2\alpha}{dt^2} = \frac{\lambda d\alpha}{dt} \Rightarrow \frac{d^2x^\mu}{d\alpha^2} + \Gamma_{\alpha\beta}^\mu dx^\alpha dx^\beta/d\alpha$$

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt_{\delta^2} + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 d\sigma^2 = d\theta^2 + sin^2\theta d\phi^2, t_-$$

$$= t_\delta - 2m \ln|r - 2m|, t_+$$

$$= t_\delta + 2m \ln|r - 2m|, t_+ = t_-$$

$$+ 4m \ln|r - 2m| ds^2 = ds_{0\pm}^2 + \frac{2m}{r}\left(k_{\pm\mu} dx^\mu\right)^2, ds_{0\pm}^2 = dr^2 + r^2 d\sigma^2 - dt_{0\pm}^2$$

$$k_\pm = k_{\pm\mu} dx^\mu = \pm r - \frac{2m}{r} + 2m\, dr - dt_\pm, k_\pm = k_\pm^\mu \partial_\mu = \pm\partial_r + \partial_{t\pm}, k_{\pm\mu}^* dx^\mu$$

$$= \pm\frac{r - 2m}{r + 2m} dr - dt_\pm, k_\pm^* = k_{\mp\mu}^* \partial_\mu = \pm\frac{r - 2m}{r + 2m}\partial_r + \partial_{t\pm}$$

$$ds^2 = dr^2 - dt^2 + \frac{1}{r}(dr + dt)^2 = (dr + dt)\left(dr - dt + \frac{1}{r}(dr + dt)\right)$$

$$= r - \frac{1}{r} d(r + t)\left(r + \frac{1}{r} - 1\, dr - dt\right) = r - \frac{1}{r} d\mu dv$$

$$ds^2 = \frac{4}{r} e^{-r} d\mathcal{U} d\mathcal{V} + r^2 d\sigma^2 \Rightarrow \frac{32m^4}{r} e^{-r/2m} d\mathcal{U} d\mathcal{V} + r^2 d\sigma^2$$

$$ds^2 = ds_0^2 + \frac{2mr}{\Sigma} k^2, k = dr + \alpha sin^2\theta d\phi + dt, ds_0^2$$

$$= dr^2 + \Sigma d\theta^2 + (r^2 + \alpha^2)\, sin^2\theta d\phi + 2\alpha sin^2\theta d\phi dt - dt^2, \Sigma = r^2 + \alpha^2 cos^2\theta$$

$$ds^2 = dx^2 + dy^2 + dz^2 - dt^2 + \frac{2mr^3}{r^4}$$

$$+ \alpha^2 z^2 \left[dt + \frac{z}{r} dz + \frac{r}{r^2} + \alpha^2(xdx + ydy) + \alpha/r^2 + \alpha^2(xdx + ydy)\right]^2$$

$$ds^2 = -dt^2 + dr^2 + \frac{2mr}{r^2} + \frac{\alpha^2 (dr + dt)^2 dr}{dt} = r^2 - 2mr - \frac{\alpha^2}{r^2} + 2mr + \alpha^2$$

$$ds^2 = \frac{\Sigma}{\Delta} dr^2 - \frac{\Delta}{\Sigma(dt_\delta + \alpha sin^2\theta d\phi_\delta)^2} + \Sigma d\theta^2$$

$$+ \frac{sin^2\theta}{\Sigma\left((r^2 + \alpha^2) d\phi_\delta - \alpha dt_\delta\right)^2 \left(\partial_r + \frac{2mr}{\Delta\partial_t} - \frac{\alpha}{\Delta\partial_\phi\partial_\theta\partial_\phi\partial_t\big|_\delta}\right) ds^2}$$

$$= \frac{\Sigma}{\Delta dr^2} - \frac{\Delta}{\Sigma}(dt_\delta + \alpha sin^2\theta d\phi_\delta)^2 + \Sigma d\theta^2 + sin^2\theta/\Sigma\left((r^2 + \alpha^2) d\phi_\delta - \alpha dt_\delta\right)^2$$

$$g^{\mu\nu}\partial_\mu\partial_\nu = \frac{\Delta}{\Sigma}\partial_{r_\delta}^2 - \frac{1}{\Delta\Sigma\left((r^2 + \alpha^2)\partial_{t_\delta} - \alpha\partial_{\phi_\delta}\right)^2} + \frac{1}{\Sigma\partial_{\theta_\delta^2}} + 1/\Sigma sin^2\theta\left(\partial_{\phi_\delta} - \alpha sin^2\theta\partial_{t_\delta}\right)^2$$

$$k_{-} = (dt_{\delta} + \alpha sin^2\,\theta d\phi_{\delta}) + (\Sigma\Delta^{-1})dr, k_{\pm}$$

$$= \mp\Sigma dr + [\Delta(dt + \alpha sin^2\theta d\phi) + (-2mr + \alpha^2 sin^2\theta)dr]k_{\pm}$$

$$= \mp(\Delta\partial_r + 2mr\partial_t - \alpha\partial_{\phi}) + \left((r^2 + \alpha^2)\partial_t - \alpha\partial_{\phi}\right)$$

$$\frac{dr}{dt} = r^2 - 2mr + \frac{\alpha^2}{r^2} + 2mr + \alpha^2, \frac{d\phi}{dt} = \frac{2\alpha}{r^2} + 2mr + \alpha^2$$

Los agujeros negros cuánticos, suponen tractos de colisión, superposición o entrelazamiento, según sea el caso, en el que interactúan partículas o antipartículas deformantes y deformadas, en el primer caso, a propósito de su masa exponencial o de su energía potencial o de su energía cinética, según corresponda, y en el segundo caso, a propósito de su masa o energía cinética o potencial ligeras, según corresponda. Todo esto, depende esencialmente del campo cuántico de que se trate.

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APÉNDICE F.

Formalización matemática relativa a las características sistémicas inherentes a las partículas supermasivas y masivas como de las antipartículas supermasivas y masivas respectivamente, en un espacio cuántico curvo o deformado.

1. Equilibrio termodinámico.

$$d\mathcal{U} = Td\mathcal{S} - pd\mathcal{V} + \sum_i \mu_i d\mathcal{N}_i$$

$$\mathcal{P} = -\frac{\partial u}{\partial v}\bigg|_{s,n} = -\frac{\partial f}{\partial v}\bigg|_{\mathcal{T},n}$$

$$\mathcal{T} = -\frac{\partial u}{\partial s}\bigg|_{v,n} = \frac{\partial \hbar}{\partial s}\bigg|_{p,n}$$

$$\mu = \frac{\partial u}{\partial n}\bigg|_{s,v} n = -\frac{\partial \omega}{\partial \mu}\bigg|_{\mathcal{T},v}$$

$$\frac{\partial^2 u}{\partial s^2}\bigg|_v \geq 0, \frac{\partial^2 u}{\partial v^2}\bigg|_s \geq 0 \frac{\partial^2 u}{\partial s^2}\bigg|_v \frac{\partial^2 u}{\partial v^2}\bigg|_s \geq \left(\frac{\partial^2 u}{\partial s \partial v}\right)^2 \frac{\partial u}{\partial \mathcal{T}}\frac{\partial \mathcal{P}}{\partial n}\mathcal{T}\frac{\partial \mathcal{P}}{\partial \mathcal{T}}n^2\frac{\partial u}{\partial n}\lambda$$

$$d\mathcal{U} = Td\mathcal{S} + \mathcal{S}d\mathcal{T} - \mathcal{P}d\mathcal{V} - \mathcal{V}dp + \mu d\mathcal{N} + \mathcal{N}d\mu$$
$$= \frac{\partial \mathcal{U}}{\partial \mathcal{S}}d\mathcal{S} + \mathcal{S}d\mathcal{T} + \frac{\partial \mathcal{U}}{\partial \mathcal{V}}d\mathcal{V} - \mathcal{V}dp + \frac{\partial \mathcal{U}}{\partial \mathcal{N}}d\mathcal{N} + \mathcal{N}d\mu \sum_i \mu_i d\mu_i$$

2. Equilibrio Químico.

$$\mu_i = \mathcal{B}_i\mu_{\mathcal{B}} + \mathcal{Q}_i\mu_q + \mathcal{L}_i^{(e)}\mu_{l_e} + \mathcal{S}_i\mu_s, \mu_a = (\mathcal{N}_a + \mathcal{Z}_a)\mu_{\mathcal{B}} + \mathcal{Z}_a\mu_q \equiv \mathcal{N}_a\mu_n + \mathcal{Z}_a\mu_p$$

3. Nivel de Rabi.

$$e_\nu = \sqrt{c^2\mathcal{P}_z^2 + m_e^2c^4(1 + 2\nu\mathcal{B}/\mathcal{B}_c)}$$

4. Corteza interior y núcleo.

$$\mathfrak{E}_{sym} = (n = n_n + n_p) = \frac{1}{2}\frac{\partial^2\mathfrak{E}(n,\alpha)}{\partial\alpha^2}\bigg|_{\alpha=0} \mathfrak{E}(n,\alpha) = \mathfrak{E}(n,0) + \mathfrak{E}_{sym}(n)\alpha^2 + \mathcal{O}(\alpha^4)$$

$$\mathfrak{E}_{sym} = (n = n_n + n_p) = \mathfrak{E}(n,\alpha = 1) - \mathfrak{E}(n,\alpha = 0)$$

$$\mathfrak{E}_{sym}(n) = \mathfrak{E}_{sym}(0) + \left(d\mathfrak{E}_{sym}(n)/dn\right)_{n_s}(n - n_s) + \frac{1}{2}\left(d^2\mathfrak{E}_{sym}(n)/dn^2\right)_{n_s}(n - n_s)^2$$
$$+ \frac{1}{6}\left(d^3\mathfrak{E}_{sym}(n)/dn^3\right)_{n_s}(n - n_s)^3$$

$$\mathfrak{E}_{sym}(n) = \mathcal{S}_0 + \mathcal{L}\chi + \frac{1}{2}\mathcal{K}_{sym}\chi^2 + \frac{1}{6}\mathcal{Q}_{sym}\chi^3$$

$$\mathfrak{E}(n,0) = \epsilon_0 + \frac{1}{2}\mathcal{K}_{sat}\chi^2 + \frac{1}{6}\mathcal{K}_{sat}\chi^3$$

$$\mathcal{S}_0 \equiv \mathfrak{E}_{sym}(n_s) \simeq \mathfrak{E}_{\mathcal{NM}}(n_s) - \epsilon_0\mathcal{L} \simeq 3\frac{\mathcal{P}_{\mathcal{NM}}}{n_s}$$

5. Gravedad relativista.

$$\frac{\mathcal{M}_{crit}}{\mathcal{M}_{\mathcal{TOV}}} = 1 + \alpha^2 \left(j/j_{\mathcal{K}ep}\right)^2 + \alpha^4 \left(j/j_{\mathcal{K}ep}\right)^4 \mathcal{G}_{\mu\nu} \frac{16\pi\mathfrak{G}}{c^4} \mathcal{T}_{\mu\nu}$$

$$ds^2 = \mathcal{A}(r)dt^2 - \mathcal{B}(r)dr^2 - r^2 \sin^2\theta d\phi^2 \, \mathcal{G}_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu}$$

$$g_{00} = 1 + 2\mathcal{V} = \mathcal{A}(r) 1 - \frac{2\mathcal{GM}}{r} \mathcal{B}(r) \left(1 - \frac{2\mathcal{GM}}{r}\right)^{-1}$$

$$ds^2 = -g_{tt}(r)dt^2 + \left(1 - \frac{2m(r)}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\frac{2m'}{r^2} 16\pi\rho \frac{2m'}{r^3} - \left(1 - \frac{2m}{r}\right)\frac{g'_{tt}}{g_{tt}} 1/r$$

$$= -16\pi\rho\left(1 - \frac{2m}{r}\right)\left(\frac{g'_{tt}}{2g_{tt}} - \frac{1}{4}\left(\frac{g'_{tt}}{g_{tt}}\right)^2 + \frac{1}{2r}\frac{g'_{tt}}{g_{tt}}\right) + \left(\frac{m}{r^2} - \frac{m'}{r}\right)\left(\frac{1}{r} + \frac{g'_{tt}}{2g_{tt}}\right)$$

6. Ecuaciones de estado de Tolman-Oppenheimer-Volkoff (TOV).

$$\mathcal{R}_{\mu\nu} = -\kappa\left(\mathcal{T}_{\mu\nu} - \frac{1}{2}\mathcal{T} g_{\mu\nu}\right)$$

$$\mathcal{T}_{\mu\nu} = -pg_{\mu\nu} + (\varepsilon + p)u_\mu u_\nu$$

$$\mathcal{R}_{00} = -\frac{\mathcal{A}''}{2\mathcal{B}} + \frac{\mathcal{A}'}{4\mathcal{B}}\left(\frac{\mathcal{A}'}{\mathcal{A}} + \frac{\mathcal{B}'}{\mathcal{B}}\right) - \frac{\mathcal{A}'}{r\mathcal{B}}$$

$$\mathcal{R}_{11} = -\frac{\mathcal{A}''}{2\mathcal{A}} + \frac{\mathcal{A}'}{4\mathcal{A}}\left(\frac{\mathcal{A}'}{\mathcal{A}} + \frac{\mathcal{B}'}{\mathcal{B}}\right) - \frac{\mathcal{B}'}{r\mathcal{B}}$$

$$\mathcal{R}_{22} = -\frac{1}{\mathcal{B}} - 1\frac{r}{2\mathcal{B}}\left(\frac{\mathcal{A}'}{\mathcal{A}} - \frac{\mathcal{B}'}{\mathcal{B}}\right)\sin^2\theta d\phi^2$$

$$\frac{d}{dr}\left[r\left(1 - \frac{1}{\mathcal{B}}\right)\right] = \kappa r^2 \varepsilon$$

$$\mathcal{B}(r) = \left[1 - \frac{2\mathcal{G}m(r)}{r}\right]^{-1}$$

$$m(r) = 4\pi\int_0^\tau \varepsilon(\bar{r})\bar{r}^2 d\bar{r}$$

$$\frac{dp}{dr} = \frac{\varepsilon + p}{2\mathcal{A}}\frac{d\mathcal{A}}{dr}\frac{1}{\mathcal{B}} - 1 + \frac{r}{2\mathcal{B}}\left(\frac{\mathcal{A}'}{\mathcal{A}} - \frac{\mathcal{B}'}{\mathcal{B}}\right) = -\frac{1}{2}\kappa(\varepsilon - p)r^2$$

$$\frac{d\mathcal{P}}{dr} = -\frac{\mathcal{G}_\mathcal{N}}{r^2}\frac{\left(\epsilon(r) + \mathcal{P}(r)\right)\left(\mathcal{M}(r) + 4\pi r^3\mathcal{P}(r)\right)}{1 - \frac{2\mathcal{G}_\mathcal{N}\mathcal{M}(r)}{r}}\frac{d\mathcal{M}}{dr} = 4\pi r^2\epsilon$$

$$\frac{d\mathcal{P}}{dr} = -(\mathcal{P} + \rho)\frac{m(r) + 4\pi r^3\mathcal{P}}{r\left(r - 2m(r)\right)}$$

$$\mathcal{P}(r) = \rho_0 \left(\frac{\left(1-\frac{2\mathcal{G}\mathcal{M}}{\mathcal{R}}\right)^{1/2} - \left(1-\frac{2\mathcal{G}\mathcal{M}r^2}{\mathcal{R}^3}\right)^{1/2}}{\left(1-\frac{2\mathcal{G}\mathcal{M}r^2}{\mathcal{R}^3}\right)^{1/2} - 3\left(1-\frac{2\mathcal{G}\mathcal{M}}{\mathcal{R}}\right)^{1/2}} \right)$$

$$\frac{d\mathcal{P}}{dr} = -(\mathcal{P}+\rho_0)\frac{\left(\frac{4\pi}{3}\right)\rho_0 r^3 + 4\pi r^3\mathcal{P}}{r\left(r-2\left(\frac{4\pi}{3}\right)\rho_0 r^3\right)}$$

$$= -\frac{4\pi}{3}(\mathcal{P} + \rho_0)\frac{(\rho_0+3\mathcal{P}r)}{1-\frac{16\pi\rho_0\mathcal{R}^2}{3}} \int_{\mathcal{P}}^{0} \frac{d\mathcal{P}}{(\mathcal{P}+\rho_0)(3\mathcal{P}+\rho_0)} \int_r^{\mathcal{R}} \frac{rdr}{2\rho_0 r^2 - \frac{3}{4\pi}} \frac{1}{2\rho_0}\log\left(\frac{\mathcal{P}+\rho_0}{3\mathcal{P}+\rho_0}\right)$$

$$= \frac{1}{4\rho_0}\log\left(\frac{\frac{16\pi\rho_0\mathcal{R}^2}{3}-1}{\frac{16\pi\rho_0 r^2}{3}-1}\right)\frac{\mathcal{P}+\rho_0}{3\mathcal{P}+\rho_0} = \left(\frac{1-2\mathcal{G}\mathcal{M}/\mathcal{R}}{1-2\mathcal{G}\mathcal{M}r^2/\mathcal{R}^3}\right)$$

$$\frac{d\mathcal{P}}{dr} = -(\mathcal{P}+\rho m c^2)\left[\frac{4\pi\mathcal{G}\mathcal{P}r^3 + \mathcal{G}m(r)c^2}{c^4 r\left(r-\frac{2\mathcal{G}m(r)}{c^2}\right)}\right]$$

$$\mathcal{P}(r) = \rho_0 c^2\left[\frac{-\left(1-\frac{2\mathcal{G}\mathcal{M}}{c^2\mathcal{R}}\right)^{\frac{1}{2}} + \left(1-\frac{2\mathcal{G}\mathcal{M}r^2}{c^2\mathcal{R}^3}\right)^{\frac{1}{2}}}{\left(1-\frac{2\mathcal{G}\mathcal{M}r^2}{c^2\mathcal{R}^3}\right)^{\frac{1}{2}} + 3\left(1-\frac{2\mathcal{G}\mathcal{M}}{c^2\mathcal{R}}\right)^{\frac{1}{2}}}\right]$$

$$\mathcal{P}(0) = \rho_0 c^2\left[\frac{-\left(1-\frac{2\mathcal{G}\mathcal{M}}{c^2\mathcal{R}}\right)^{\frac{1}{2}} + 1}{-1 + 3\left(1-\frac{2\mathcal{G}\mathcal{M}}{c^2\mathcal{R}}\right)^{\frac{1}{2}}}\right]$$

6.1. Ecuaciones TOV.

$$\frac{d\mathcal{P}}{dr} = -\left(\frac{\mathcal{G}\mathcal{M}_r}{r^2}\right)\frac{[\mathcal{P}(r)+\rho(r)][\mathcal{M}(r)+4\pi r^3\mathcal{P}(r)]}{r[r-2\mathcal{M}(r)]}$$

$$\frac{d\mathcal{P}}{dr} = -\left(\frac{\mathcal{G}\mathcal{M}_r}{r^2}\right)\frac{\rho + (\mathcal{P}/c^2)\big(1+(4\pi\mathcal{P}r^3/\mathcal{M}_r c^2)\big)}{\left(1-\left(\frac{2\mathcal{G}\mathcal{M}_r}{rc^2}\right)\right)}$$

$$\mathcal{M}_r = \frac{4\pi}{c^2}\int_0^r \rho c^2 r^2 dr$$

$$\xi = r\mathcal{A};\ \rho = \rho_c e^{-\theta};\ \mathcal{M}_r = \frac{4\pi\rho_c}{\mathcal{A}^3}\nu(\xi);\ \mathcal{A}^2 = \frac{4\pi\mathcal{G}\rho_c}{\sigma c^2}$$

$$\sigma = \frac{\mathcal{P}_c}{\rho c^2}\frac{(1-2\sigma\nu(\xi)/\xi)}{1+\sigma} + \xi^2\frac{d\theta}{d\xi} - \nu(\xi) - \sigma e^{-\theta}\xi^3\frac{d\nu}{d\xi} = \xi^2 e^{-\theta}\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = e^{-\theta}$$

$$\xi^2\frac{d\theta}{d\xi} - 2\sigma\nu\xi\frac{d\theta}{d\xi} - \nu - \sigma\nu - \sigma e^{-\theta}\xi^3 - \sigma^2 e^{-\theta}\xi^3$$

$$\theta(\xi) = \sum_{k=1}^{\infty}\alpha_k\xi^{2k}$$

$$\frac{d\theta}{d\xi} = \sum_{k=1}^{\infty}2k\alpha_k\xi^{2k-1}$$

$$\xi\frac{d\theta}{d\xi} = \sum_{k=1}^{\infty}2k\alpha_k\xi^{2k}$$

$$\xi^2\frac{d\theta}{d\xi} = \sum_{k=1}^{\infty}2k\alpha_k\xi^{2k+1}$$

$$e^{-\theta} = \sum_{k=0}^{\infty}\alpha_k\xi^{2k}$$

$$\alpha_k = -\frac{1}{\kappa}\sum_{i=1}^{k}i\alpha_i\alpha_{k-i}$$

$$\frac{d\nu}{d\xi} = \xi^2\sum_{k=0}^{\infty}\alpha_k\xi^{2k} = \sum_{k=0}^{\infty}\alpha_k\xi^{2k+2}$$

$$\nu = \int\sum_{k=0}^{\infty}\alpha_k\xi^{2k+2}\,d\xi = \sum_{k=0}^{\infty}\frac{\alpha_k}{2\kappa+3}\xi^{2k+3}$$

$$\xi\frac{d\theta}{d\xi}\nu = \left(\sum_{k=0}^{\infty}2(k+1)\alpha_{k+1}\xi^{2k+2}\right)\left(\sum_{k=0}^{\infty}\frac{\alpha_k}{2\kappa+3}\xi^{2k+3}\right)$$

$$\xi\frac{d\theta}{d\xi}\nu = \left(\sum_{k=0}^{\infty}f_k\xi^{2k+2}\right)\left(\sum_{k=0}^{\infty}g_k\xi^{2k+3}\right)$$

$$\xi\frac{d\theta}{d\xi}\nu = \left(\sum_{k=0}^{\infty}f_k\xi^{2k+2}\right)\left(\sum_{k=0}^{\infty}g_k\xi^{2k+3}\right) = \sum_{k=0}^{\infty}\gamma_k\xi^{2k+5}$$

$$\gamma_k = \sum_{i=0}^{k}f_i g_{k-i}\sum_{i=1}^{k}i\alpha_i\alpha_{k-i}$$

$$\sum_{k=1}^{\infty}\left[2(k+1)\alpha_{k+1} - 2\sigma\gamma_{k-1} - \frac{\alpha_k}{2\kappa+3} - \sigma\frac{\alpha_k}{2\kappa+3} - \sigma\alpha_k - \sigma^2\alpha_k\right]\xi^{2k+3}$$

$$2(k+1)\alpha_{k+1} - 2\sigma\gamma_{k-1} - \frac{\alpha_k}{2\kappa+3} - \sigma\frac{\alpha_k}{2\kappa+3} - \sigma\alpha_k - \sigma^2\alpha_k = 0$$

$$\alpha_{k+1} = \frac{1}{2(k+1)}\left[2\sigma\gamma_{k-1} + \frac{\alpha_k}{2\kappa+3} + \sigma\frac{\alpha_k}{2\kappa+3} + \sigma\alpha_k + \sigma^2\alpha_k\right]$$

$$\gamma_{k-1} = \sum_{i=0}^{k-1} f_i g_{k-i-1};\alpha_k = -\frac{1}{k}\sum_{i=1}^{k} i\alpha_i\alpha_{k-i}\,;\forall\kappa \geq 1;\alpha_0 = e^{(-\alpha_0)}$$

$$\alpha_1 = \frac{1}{2}\left[\frac{1}{3} + \frac{\sigma}{3} + \sigma + \sigma^2\right] = \frac{1}{6}(1+\sigma)(1+3\sigma)$$

$$\bar{\sigma}(\bar{\psi}^i)(\bar{\mathcal{G}}_{\mu\nu} - \bar{\mathcal{W}}_{\mu\nu}) = \kappa^2\bar{\mathcal{T}}_{\mu\nu}$$

$$\bar{\mathcal{T}}_{\mu\nu} = \overline{(\rho+p)}\,\overline{u_\mu u_\nu} + \overline{p\hbar_{\mu\nu}}\left(\frac{\overline{\prod}}{\bar{\sigma}}\right)' = -\frac{\mathcal{M}}{\bar{r}^2}\left(\frac{\bar{\mathcal{Q}}}{\bar{\sigma}} + \frac{\overline{\prod}}{\bar{\sigma}}\right)\left(1 + \frac{4\pi\bar{r}^3\frac{\overline{\prod}}{\bar{\sigma}}}{\mathcal{M}}\right)\overline{\alpha(r)} - \frac{2\bar{\sigma}}{\kappa^2\bar{r}}\left(\frac{\bar{\mathcal{W}}_{00}}{\bar{r}^2} - \frac{\bar{\mathcal{W}}_{rr}}{\bar{\alpha}}\right)$$

$$\mathcal{M}(\bar{r}) = \int_0^{\bar{r}} 4\pi\tilde{r}^2\frac{\bar{\mathcal{Q}}(\tilde{r})}{\bar{\sigma}(\tilde{r})}d\tilde{r}$$

$$\overline{\alpha(r)} = \left(1 - \frac{2\mathcal{M}(\bar{r})}{\bar{r}}\right)^{-1}$$

$$\overline{\mathcal{Q}(r)} \coloneqq \overline{\rho(r)} - \frac{\bar{\sigma}(\bar{r})\bar{\mathcal{W}}_{tt}(\bar{r})}{\kappa^2 c^2\overline{\rho(r)}}$$

$$\overline{\prod(r)} \coloneqq \overline{\rho(r)} - \frac{\bar{\sigma}(\bar{r})\bar{\mathcal{W}}_{rr}(\bar{r})}{\kappa^2\overline{\alpha(r)}}$$

$$\widehat{\mathcal{W}}_{tt} = \frac{1}{2}\frac{\hat{b}}{\hat{a}}(\partial_{\hat{r}}\mathfrak{T})^2 + \frac{\hat{b}}{2}\mathfrak{T}_2$$

$$\widehat{\mathcal{W}}_{\overline{rr}} = \frac{1}{2}(\partial_{\hat{r}}\mathfrak{T})^2 - \frac{\hat{\alpha}}{2}\mathfrak{T}_2$$

$$\widehat{\mathcal{W}}_{\theta\theta} = -\frac{1}{2}\frac{\hat{r}^2}{\hat{a}}(\partial_{\hat{r}}\mathfrak{T})^2 - \frac{\hat{r}^2}{2}\mathfrak{T}_2$$

$$\widehat{\mathcal{W}}_{\phi\phi} = \sin^2\theta\widehat{\mathcal{W}}_{\theta\theta}$$

$$\hat{\mathcal{Q}} = \hat{\rho} - \frac{(\partial_{\hat{r}}\mathfrak{T})^2}{2\kappa^2c^2\hat{\alpha}} - \frac{\mathfrak{T}_2}{2\kappa^2c^2}$$

$$\widehat{\prod} = \hat{\rho} - \frac{1}{2\kappa^2\hat{\alpha}}(\partial_{\hat{r}}\mathfrak{T})^2 + \frac{1}{2\kappa^2}\mathfrak{T}_2$$

$$\frac{d\widehat{\prod}}{d\hat{r}} = \frac{\mathcal{G}\mathcal{M}}{c^2\hat{r}^2}\left(c^2\hat{\rho} + \hat{p} - \frac{(\partial_{\hat{r}}\mathfrak{T})^2}{\kappa^2\hat{\alpha}}\right)\left(1 + \frac{4\pi\hat{r}^3\widehat{\prod}}{c^2\mathcal{M}}\right)\times\left(1 - \frac{2\mathcal{G}\mathcal{M}}{c^2\hat{r}}\right)^{-1} + \frac{1}{\kappa^2\hat{\alpha}\hat{r}}(\partial_{\hat{r}}\mathfrak{T})^2$$

$$\mathcal{M}(\hat{r}) = \int_0^{\hat{r}} 4\pi r^2\hat{\mathcal{Q}}(r)dr$$

$$\frac{d\hat{p}}{d\hat{r}} = -\frac{\mathcal{G}\mathcal{M}(\hat{r})}{c^2\hat{r}^2}(c^2\hat{\rho} + \hat{p})\left(1 - \frac{2\mathcal{G}\mathcal{M}(\hat{r})}{c^2\hat{r}}\right)^{-1}\times\left[1 + \frac{4\pi\hat{r}^3}{c^2\mathcal{M}(\hat{r})}\left(\hat{p} - \frac{(\partial_{\hat{r}}\mathfrak{T})^2}{2\kappa^2\hat{\alpha}} + \frac{\mathfrak{T}_2}{2\kappa^2}\right)\right] + \hat{\mathcal{T}}\partial_{\hat{r}}\ln\mathfrak{T}_1 - \frac{(\partial_{\hat{r}}\mathfrak{T})^2}{2\kappa^2\hat{\alpha}\hat{r}}$$

$$\mathcal{M}(\hat{r}) = \mathcal{M}_0(\hat{r}) - \eta(\hat{r}, \mathfrak{T}, \mathfrak{T}_2)$$

$$\eta(\hat{r}, \mathfrak{T}, \mathfrak{T}_2) = \frac{c^2}{4\mathcal{G}} \int_0^{\hat{r}} \chi^2 \left(\frac{(\partial_{\hat{r}}\mathfrak{T})^2}{\hat{\alpha}} + \mathfrak{T}_2 \right) d\chi$$

$$\mathcal{M}(r) = \int_0^{\hat{r}} \frac{4\pi r^2}{\sqrt{\mathfrak{T}_1}} \left[\rho - \frac{1}{2\kappa^2 c^2} \frac{\mathfrak{T}_1}{a} (\partial_r \mathfrak{T})^2 - \frac{1}{2\kappa^2 c^2} \mathfrak{T}_1^2 \mathfrak{T}_2 \right] \times \left(\frac{r}{2} \partial_r \ln \mathfrak{T}_1 + 1 \right) dr$$

$$\frac{dp}{dr} = \left[-\frac{\mathcal{G}\mathcal{M}(r)}{c^2 r^2 \mathfrak{T}_1^{\frac{1}{2}}} (c^2\rho + p) \left(1 - \frac{2\mathcal{G}\mathcal{M}(r)}{c^2 r \mathfrak{T}_1^{\frac{1}{2}}} \right)^{-1} \times \left(1 + \frac{4\pi \mathfrak{T}_1^{\frac{3}{2}} r^3}{c^2 \mathcal{M}(r)} \left(\frac{\rho}{\mathfrak{T}_1^2} - \frac{(\partial_r \mathfrak{T})^2}{2\kappa^2 \alpha \mathfrak{T}_1} + \frac{\mathfrak{T}_2}{2\kappa^2} \right) \right) - \frac{\mathfrak{T}_1^2 (\partial_r \mathfrak{T})^2}{\kappa^2 \alpha r} \right] \times \left(\frac{r}{2} \partial_r \ln \mathfrak{T}_1 + 1 \right) + (-c^2\rho + 5p) \partial_r \ln \mathfrak{T}_1$$

$$\mathfrak{T}_1 = 1 + 4\beta\kappa^2 (c^2\rho - 3p)$$

$$\mathfrak{T}_2 = \frac{4\beta\kappa^4 (c^2\rho - 3p)^2}{\left(1 + 4\beta\kappa^2 (c^2\rho - 3p)\right)^2}$$

$$p' = \left[-\frac{\mathcal{G}\mathcal{M}(r)}{r^2 \mathfrak{T}_1^{\frac{1}{2}}} (\varepsilon + p) \left(1 - \frac{2\mathcal{G}\mathcal{M}(r)}{r \mathfrak{T}_1^{\frac{1}{2}}} \right)^{-1} \times \left(1 + \frac{4\pi \mathfrak{T}_1^{\frac{3}{2}} r^3}{\mathcal{M}(r)} \left(\frac{\rho}{\mathfrak{T}_1^2} + \frac{\mathfrak{T}_2}{2\kappa^2} \right) \right) \right] \times \left(\frac{r}{2} \partial_r \ln \mathfrak{T}_1 + 1 \right) + (-\varepsilon + 5p) \partial_r \ln \mathfrak{T}_1$$

$$\mathfrak{T}_1 = 1 + 4\beta\kappa^2 (\varepsilon - 3p)$$

$$\mathfrak{T}_2 = \frac{4\beta\kappa^4 (\varepsilon - 3p)^2}{\left(1 + 4\beta\kappa^2 (\varepsilon - 3p)\right)^2}$$

$$\mathcal{M}(r) = \int_0^r 4\pi \tilde{r}^2 \frac{\varepsilon - 2\beta\kappa^2 (\varepsilon - 3p)^2}{\left(1 + 2\beta\kappa^2 (\varepsilon - 3p)\right)^{1/2}} \times \left[1 + \frac{\tilde{r}}{2} \partial_{\tilde{r}} \ln\left(1 + 4\beta\kappa^2 (\varepsilon - 3p)\right) \right] d\tilde{r}$$

$$\mathfrak{T}_1 = 1 + 2\alpha(\varepsilon - 3p)$$

$$\mathfrak{T}_2 = \frac{2\alpha\kappa^2 (\varepsilon - 3p)^2}{\left(1 + 2\alpha(\varepsilon - 3p)\right)^2}$$

$$\alpha_{sing} = -\frac{1}{2(\varepsilon - 3\mathcal{P})} \frac{\partial \log \mathfrak{T}_1}{\partial r} \frac{2\alpha \left(\frac{1}{c_s^2} - 3 \right)}{\mathfrak{T}_1} \frac{d\mathcal{P}}{dr}$$

$$\frac{d\mathcal{P}}{dr} = \frac{uv\mathfrak{T}_1}{\mathfrak{T}_1 - \alpha \left(\frac{1}{c_s^2} - 3 \right) (uvr - 2\varepsilon + 10\mathcal{P})}$$

$$u = -\mathcal{M}\frac{\varepsilon + \mathcal{P}}{r^2\mathfrak{T}_1^{\frac{1}{2}}}\left(1 - \frac{2\mathcal{M}}{r\mathfrak{T}_1^{\frac{1}{2}}}\right)^{-1}$$

$$v = \frac{\mathcal{M} + 4\pi\mathfrak{T}_1^{\frac{3}{2}}r^3}{\mathcal{M}}\left(\frac{\mathcal{P}}{\mathfrak{T}_1^2} + \frac{\mathfrak{T}_2}{2\kappa^2}\right)$$

$$\frac{d\mathcal{M}(r)}{dr} = 4\pi r^2\frac{\varepsilon - \alpha(\varepsilon - 3\mathcal{P})^2}{\mathfrak{T}_1^{\frac{3}{2}}}\left(\mathfrak{T}_1 + r\alpha\left(\frac{1}{c_s^2} - 3\right)\frac{d\mathcal{P}}{dr}\right)$$

$$\frac{dm(r)}{dr} = \frac{4\pi r^2\epsilon(r)}{c^2} = 4\pi r^2\rho(r)\frac{d\mathcal{P}(r)}{dr} = \frac{\mathcal{G}\epsilon(r)m(r)}{c^2r^2}\bigotimes\chi$$

$$\chi = \left[1 + \frac{\mathcal{P}(r)}{\epsilon(r)}\right]\left[1 + \frac{4\pi r^3\mathcal{P}(r)}{m(r)c^2}\right]\left[1 - \frac{2\mathcal{G}m(r)}{c^2r}\right]^{-1}$$

$$\left[1 + \frac{\mathcal{P}(r)}{\epsilon(r)}\right]\left[1 + \frac{4\pi r^3\mathcal{P}(r)}{m(r)c^2}\right]\left[1 - \frac{2\mathcal{G}m(r)}{c^2r}\right]^{-1} = 1$$

$$\frac{d\mathcal{P}}{dr} = \frac{\mathcal{G}\epsilon(r)m(r)}{c^2r^2}\frac{dm(r)}{dr} = \frac{4\pi r^2\epsilon(r)}{c^2}\frac{d\mathcal{P}(r)}{dr} = \mathcal{R}_{\mathcal{O}}/2r^2\left(\frac{\mathcal{P}(r)}{k}\right)^{1/\gamma}\overline{m(r)}\frac{d\overline{m}(r)}{dr}$$
$$= \frac{4\pi r^2}{\mathcal{M}_{\odot}c^2}\left(\frac{\mathcal{P}(r)}{k}\right)^{1/\gamma}$$

7. Modelo Estelar.

$$m'' = -\frac{2m'}{r} - \frac{2mm''}{r} + \frac{4mm''}{r^2} + \frac{4\pi}{\gamma}\left((4\pi)^{\gamma-2}\left(\frac{m'}{r^2}\right)^{2-\gamma} + \frac{m'}{4\pi r^2}\right)\left(m + (4\pi)^{1-\gamma}r^3\left(\frac{m'}{r^2}\right)^{\gamma}\right)m''$$
$$-\frac{2m'}{r}$$
$$-\epsilon\left[\frac{2mm''}{r} - \frac{4mm''}{r^2}\right.$$
$$\left.-\frac{4\pi}{\gamma}\left((4\pi)^{\gamma-2}\left(\frac{m'}{r^2}\right)^{2-\gamma} + \frac{m'}{4\pi r^2}\right)\bigotimes\left(m + (4\pi)^{1-\gamma}r^3\left(\frac{m'}{r^2}\right)^{\gamma}\right)\right]$$

8. Gravedad de Palatini.

$$\mathcal{S} = \frac{1}{2\kappa^2}\int d^4\chi\sqrt{-g}f(\mathcal{R}) + \mathcal{S}_m\left[g_{\mu\nu},\chi\right]f'(\mathcal{R})\mathcal{R}_{\mu\nu} - \frac{1}{2}f(\mathcal{R})g_{\mu\nu} = \kappa^2\mathcal{T}_{\mu\nu}$$

$$\mathcal{T}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\mathcal{S}_m}{\delta g_{\mu\nu}}\nabla_\beta\left(\sqrt{-g}f'(\mathcal{R})g^{\mu\nu}\right)\bar{g}^{\mu\nu}f'(\mathcal{R})\mathcal{R}_{\mu\nu}\nabla_\beta\left(\sqrt{-g}\bar{g}^{\mu\nu}\right)$$

$$f'(\mathcal{R})\mathcal{R} - 2f(\mathcal{R}) = \kappa^2\mathcal{T}$$

$$f(\mathcal{R}) = \mathcal{R} + \beta\mathcal{R}^2$$

9. Parametrización gravitacional de Wagoner.

$$\mathcal{S}[\bar{g}^{\mu\nu},\bar{\phi},\chi] = \frac{1}{2\kappa^2}\int_{\Omega}^{\square} d^4\chi\sqrt{-\bar{g}}\,[\bar{\mathcal{A}}(\bar{\phi})\bar{\mathcal{R}} - \bar{\mathcal{B}}(\bar{\phi})\bar{g}^{\mu\nu}\partial_\mu\bar{\phi}\partial_\nu\bar{\phi} - \mathcal{V}(\bar{\phi})]$$
$$+ \mathcal{S}_{materia}[e^{2\bar{\alpha}(\bar{\phi})}\bar{g}^{\mu\nu},\chi]\begin{cases}\bar{g}^{\mu\nu} = e^{2\gamma(\bar{\phi})}\bar{g}^{\mu\nu}\\ \bar{\phi} = \bar{f}(\phi)\end{cases}$$

$$\bar{\bar{\mathcal{A}}}\left(\bar{\bar{\bar{\phi}}}\right) = e^{2\bar{\bar{\gamma}}\left(\bar{\bar{\bar{\phi}}}\right)}\bar{\mathcal{A}}\left(\bar{\bar{f}}\left(\bar{\bar{\bar{\phi}}}\right)\right), \bar{\bar{\mathcal{B}}}\left(\bar{\bar{\bar{\phi}}}\right)$$
$$= e^{2\bar{\bar{\gamma}}\left(\bar{\bar{\bar{\phi}}}\right)}\left(\left(\frac{d\bar{\bar{\phi}}}{d\bar{\bar{\bar{\phi}}}}\right)^2\bar{\mathcal{B}}\left(\bar{\bar{f}}\left(\bar{\bar{\bar{\phi}}}\right)\right) - 6\left(\frac{d\bar{\bar{\gamma}}}{d\bar{\bar{\bar{\phi}}}}\right)^2\bar{\mathcal{A}}\left(\bar{\bar{f}}\left(\bar{\bar{\bar{\phi}}}\right)\right) - 6\frac{d\bar{\bar{\gamma}}}{d\bar{\bar{\bar{\phi}}}}\frac{d\bar{\mathcal{A}}}{d\bar{\bar{\phi}}}\frac{d\bar{\bar{\phi}}}{d\bar{\bar{\bar{\phi}}}}\right), \bar{\bar{\mathcal{V}}}\left(\bar{\bar{\bar{\phi}}}\right)$$
$$= e^{4\bar{\bar{\gamma}}\left(\bar{\bar{\bar{\phi}}}\right)}\bar{\mathcal{V}}\left(\bar{\bar{f}}\left(\bar{\bar{\bar{\phi}}}\right)\right), \bar{\bar{\alpha}}\left(\bar{\bar{\bar{\phi}}}\right) = \bar{\alpha}\left(\bar{\bar{f}}\left(\bar{\bar{\bar{\phi}}}\right)\right) + \bar{\bar{\gamma}}\left(\bar{\bar{\bar{\phi}}}\right)$$

$$\mathfrak{T}_1 = \bar{\mathcal{A}}/e^{2\bar{\alpha}}$$

$$\mathfrak{T}_2 = \bar{\mathcal{V}}/\bar{\mathcal{A}}^2$$

$$\frac{d\mathfrak{T}_3}{d\bar{\phi}} = \sqrt{\pm\frac{2\bar{\mathcal{A}}\bar{\mathcal{B}} + 3(\bar{\mathcal{A}}')^2}{2\bar{\mathcal{A}}^2}}$$

$$\mathcal{I}_i\left(\bar{\bar{\phi}}(\chi)\right) = \mathcal{I}_i\left(\bar{f}(\phi(\chi))\right) = \mathcal{I}_i(\phi(\chi))\partial_\mu\left(\mathcal{I}_i\left(\bar{\bar{\phi}}(\chi)\right) = \partial_\mu\left(\mathcal{I}_i\left(\bar{f}\left(\bar{\bar{\phi}}(\chi)\right)\right)\right)\right)$$

$$\bar{g}^{\mu\nu} = diag\left(-\bar{b}(\bar{r}), \bar{a}(\bar{r}), \bar{r}^2, \bar{r}^2\sin^2\bar{\theta}\right)$$

10. Parametrización einsteniana de gravitación.

$$\hat{g}^{\mu\nu} = \hat{\mathcal{A}}\bar{g}^{\mu\nu}\mathcal{S}[\hat{g}^{\mu\nu},\mathfrak{T},\chi]$$
$$= \frac{1}{2\kappa^2}\int_{\Omega}^{\square} d^4\chi\sqrt{-\hat{g}}\,[\hat{\mathcal{R}} - \hat{g}^{\mu\nu}\partial_\mu\mathfrak{T}\partial_\nu\mathfrak{T} - \mathfrak{T}_2] + \mathcal{S}_{materia}\left[\frac{1}{\mathfrak{T}_1}\hat{g}^{\mu\nu},\chi\right]\hat{\mathfrak{G}}_{\mu\nu}$$
$$+ \frac{1}{2}\hat{g}_{\mu\nu}\hat{g}^{\alpha\beta}\partial_\alpha\mathfrak{T}\partial_\beta\mathfrak{T} + \frac{1}{2}\hat{g}_{\mu\nu}\mathfrak{T}_2 = \kappa^2\hat{\mathcal{T}}_{\mu\nu}\widehat{\square}\mathfrak{T} - \frac{1}{2}\frac{d\mathfrak{T}_2}{d\mathfrak{T}} = \kappa^2\frac{1}{\mathfrak{T}_1}\frac{d\mathfrak{T}_1}{d\mathfrak{T}}\hat{\mathcal{T}}\hat{\nabla}^\mu\hat{\mathcal{T}}_{\mu\nu}$$
$$= \frac{1}{2}\partial_\nu(\log\mathfrak{T}_1)\hat{\mathcal{T}}$$

11. Masa y Ondas gravitacionales en espacios cuánticos curvos.

$$\frac{(\mathcal{M}_{companion}\sin i)^3}{(\mathcal{M}_{\mathcal{NS}} + \mathcal{M}_{companion})} = \frac{\mathcal{T}v_i^2}{2\pi\mathcal{G}}\dot{\mathcal{T}} \propto \frac{\mathcal{M}_{\mathcal{NS}}\mathcal{M}_{companion}}{(\mathcal{M}_{\mathcal{NS}} + \mathcal{M}_{companion})^{\frac{1}{3}}}\dot{\varphi}_{min} \propto (\mathcal{M}_{\mathcal{NS}} + \mathcal{M}_{companion})^{\frac{2}{3}}$$

$$z = \frac{1}{\sqrt{1 - \frac{2\mathcal{G}\mathcal{M}_{\mathcal{NS}}}{c^2\mathcal{R}_{\mathcal{NS}}}}} - 1$$

$$\mathcal{M} = \left(\frac{(m_1m_2)^3}{m_1 + m_2}\right)^{1/5} = \frac{c^3}{\mathcal{G}}\left(\frac{5\pi^{-\frac{8}{3}}}{96}f^{11/3}\frac{df}{dt}\right)^{3/5}$$

$$\Lambda = \frac{\lambda}{m^5} = \frac{2}{3} k_2 \frac{\mathcal{R}^5}{m^5} = \frac{2}{3} k_2 \mathcal{C}^{-5}$$

$$\bar{\hbar}_{\alpha\beta}(t,\chi) = \frac{4\mathcal{G}}{c^4} \int d^3\gamma \frac{\mathcal{T}_{\alpha\beta}(t',\gamma)}{|\chi - \gamma|}$$

$$\widetilde{\Lambda} = \frac{16}{3}\left[\frac{(m_1 + 12m_2)m_1^4\Lambda_1}{(m_1 + m_2)^5} + \frac{(m_2 + 12m_1)m_2^4\Lambda_2}{(m_1 + m_2)^5}\right]$$

$$\psi(f) = 2\pi f t_{coalescence} - \left(\phi_{coalescence} + \frac{\pi}{4} + \frac{3\mathcal{M}^2}{128 m_1 m_2 v^5}\right)\left(\psi_{3.5\mathcal{PN}}^{pointlike} + \psi^{tidal}\right)$$

12. Métrica computacional TOV y deformación.

$$ds^2 = -e^{2\phi(r)}[1 + \mathcal{H}(r)\Gamma(\theta,\gamma)]dt^2 + e^{2\Lambda(r)}[1 - \mathcal{H}(r)(\theta,\gamma)]dr^2 + r^2[1 - \mathcal{K}(r)\Gamma(\theta,\gamma)](d\theta^2 + \sin^2\theta d\phi^2)$$

$$\frac{d\mathcal{H}}{dr} = \beta$$

$$\frac{d\beta}{dr} = 2\left(1 - 2\frac{m_r}{r}\right)^{-1} \mathcal{H}\left\{-2\pi[5\epsilon + 9\mathcal{P} + f(\epsilon + \mathcal{P})] + \frac{3}{r^2} + 2\left(1 - 2\frac{m_r}{r}\right)^{-1}\left(\frac{m_r}{r^2} + 4\pi r\mathcal{P}\right)^2\right\} + \frac{2\beta}{r}\left(1 - 2\frac{m_r}{r}\right)^{-1}\left\{-1 + \frac{m_r}{r} + 2\pi r^2(\epsilon - \mathcal{P})\right\}$$

$$\kappa_2 = \frac{8\mathcal{C}^5}{5}(1 - 2\mathcal{C})^2[2 + 2\mathcal{C}(\gamma - 1) - \gamma] \times \{2\mathcal{C}[6 + 3\gamma + 3\mathcal{C}(5\gamma - 8)] + 4\mathcal{C}^3[13 - 11\gamma + \mathcal{C}(3\gamma - 2) + 2\mathcal{C}^2(1 + \gamma)] + 3(1 - 2\mathcal{C})^2[2 - \gamma + 2\mathcal{C}(\gamma - 1)]\ln(1 - 2\mathcal{C})\}^{-1}$$

13. Cromodinámica cuántica en espacios curvos.

$$\mathcal{U}(\chi) = \exp\left(-i\sum_{a=1}^{8} \theta^\alpha(\chi)\frac{\lambda_\alpha}{2}\right) = \exp\left(-i\theta^\alpha(\chi)\frac{\lambda_\alpha}{2}\right) 2\delta^{\alpha\beta} i f_{abc} \lambda_\alpha^\dagger \lambda_\alpha \lambda_b \frac{1}{4i} Tr(|\lambda_a \lambda_b|\lambda_c)$$

$$\mathcal{L}_{\mathcal{QCD}} = \bar{q}(i\mathcal{D}q - \mathcal{M})q - \frac{1}{4}\mathcal{G}_{\mu\nu}^a \mathcal{G}_a^{\mu\nu} + \mathcal{L}_{\mathcal{FP}}$$

$$\mathcal{G}_{\mu\nu}^a = \partial_\mu \mathcal{G}_\nu^a - \partial_\nu \mathcal{G}_\mu^a + g_s f^{abc} \mathcal{G}_\mu^b \mathcal{G}_\nu^b \frac{\lambda_\alpha}{2}$$

$$q_f(\chi) \mapsto \mathcal{U}(\chi) q_f(\chi)$$

$$q_f^\dagger(\chi) \mapsto q_f^\dagger(\chi)\mathcal{U}^\dagger(\chi)$$

$$\mathcal{A}_\mu(\chi) \mapsto \mathcal{U}\mathcal{A}_\mu\mathcal{U}^\dagger + \frac{i}{g}\partial_\mu \mathcal{U}\mathcal{U}^\dagger$$

$$\mathcal{L}_{\mathcal{QCD}}^0 = \bar{q} i\gamma^\mu \mathcal{D}_\mu q - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}_a^{\mu\nu}$$

$$\mathcal{L}_{\mathcal{QCD}}^0 = \bar{q}_{\mathcal{R}} i\gamma^\mu \mathcal{D}_\mu q_{\mathcal{R}} + \bar{q}_{\mathcal{L}} i\gamma^\mu \mathcal{D}_\mu q_{\mathcal{L}} - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}_a^{\mu\nu}$$

$$\mathcal{V}_i^\mu = \mathcal{R}_i^\mu + \mathcal{L}_i^\mu = \bar{q}\gamma^\mu t_i q$$

$$\mathcal{A}_i^\mu = \mathcal{R}_i^\mu - \mathcal{L}_i^\mu = \bar{q}\gamma^\mu \gamma_5 t_i q$$

$$\mathcal{Q}_i^{\mathcal{V}} = \int d^3\chi\, \mathcal{V}_i^0 = \int d^3\chi\, q^\dagger(t,\vec{\chi})\frac{\tau_i}{2}q(t,\vec{\chi})\frac{d\mathcal{Q}_i^{\mathcal{V}}}{dt}$$

$$\mathcal{Q}_i^{\mathcal{A}} = \int d^3\chi\, \mathcal{A}_i^0 = \int d^3\chi\, q^\dagger(t,\vec{\chi})\gamma_5\frac{\tau_i}{2}q(t,\vec{\chi})\frac{d\mathcal{Q}_i^{\mathcal{A}}}{dt}$$

$$\mathcal{M} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} = \frac{1}{2}(m_u + m_d)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}(m_u - m_d)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
$$= \frac{1}{2}(m_u + m_d)\mathcal{I} + \frac{1}{2}(m_u - m_d)\tau_3$$

$$\beta(g_s) = \mu\frac{\partial g_s}{\partial \mu}$$

$$\beta_{\mathcal{QCD}}(g_s) = -\left(11 - \frac{2\mathcal{N}_f}{3}\right)\frac{g_s^3}{(4\pi)^2} + \mathcal{O}(g_s^5)\frac{\partial \bar{g}_s(g_s,t)}{\partial t} = \beta_{\mathcal{QCD}}(\bar{g}_s)$$

$$\alpha_s^{-1}(\Lambda) = \frac{33 - 2\mathcal{N}_f}{12\pi}\ln\left(\frac{\Lambda}{\Lambda_{\mathcal{QCD}}}\right)$$

$$\Theta = \frac{\beta}{2g}\mathcal{F}_{\mu\nu}^{\alpha}\mathcal{F}_{\alpha}^{\mu\nu} + (1+\gamma_m)\sum_f \int m_f\, \bar{q}_f q_f \langle \mathcal{T}_\nu^\mu \rangle \mathcal{M}_{\mathcal{H}} \langle \Theta \rangle_{\mathcal{T},\mu_{\mathcal{B}}}(\varepsilon - 3\mathcal{P})\Delta \equiv \frac{\langle \Theta \rangle_{\mathcal{T},\mu_{\mathcal{B}}}}{3\varepsilon} = \frac{1}{3} - \frac{\mathcal{P}}{\varepsilon}$$

$$\nu = -2 + 2\mathcal{A} - 2\mathcal{C} + 2\mathcal{L} + \sum_i \Delta_i \equiv d_i + \frac{n_i}{2} - 2$$

$$\mathcal{V}(\vec{p}',\vec{p}) = \mathcal{V}_{\mathcal{C}} + \tau_1 \bigotimes \tau_2 \mathcal{W}_{\mathcal{C}} + \left[\mathcal{V}_{\mathcal{S}} + \tau_1 \bigotimes \tau_2 \mathcal{W}_{\mathcal{S}}\right]\vec{\sigma}_1 \bigotimes \vec{\sigma}_2$$
$$+ \left[\mathcal{V}_{\mathcal{LS}} + \tau_1 \bigotimes \tau_2 \mathcal{W}_{\mathcal{LS}}\right]\left(-i\vec{S} \boxtimes \left(\vec{q} \bigotimes \vec{k}\right)\right)$$
$$+ \left[\mathcal{V}_{\mathcal{T}} + \tau_1 \bigotimes \tau_2 \mathcal{W}_{\mathcal{T}}\right]\left|\vec{\sigma}_1 \bigotimes \vec{q}\right|\left|\vec{\sigma}_2 \bigotimes \vec{q}\right|$$
$$+ \left[\mathcal{V}_{\sigma\mathcal{L}} + \tau_1 \bigotimes \tau_2 \mathcal{W}_{\sigma\mathcal{L}}\right]\vec{\sigma}_1 \bigotimes \left(\vec{q} \bigotimes \vec{k}\right)\vec{\sigma}_2 \bigotimes \left(\vec{q} \bigotimes \vec{k}\right)$$

$$\mathcal{T}(\vec{p}',\vec{p}) = \mathcal{V}(\vec{p}',\vec{p}) + \int \frac{d^3p''}{(2\pi)^3}\mathcal{V}(\vec{p}',\vec{p}'')\frac{\mathcal{M}_{\mathcal{N}}^2}{\mathfrak{E}_{p''}}\frac{1}{p^2 - p''^2 + i\epsilon}\mathcal{T}(\vec{p}'',\vec{p})$$

$$\hat{\mathcal{V}}(\vec{p}',\vec{p}) \equiv \frac{1}{(2\pi)^3}\sqrt{\frac{\mathcal{M}_{\mathcal{N}}}{\mathfrak{E}_{p'}}}\mathcal{V}(\vec{p}',\vec{p})\sqrt{\frac{\mathcal{M}_{\mathcal{N}}}{\mathfrak{E}_{p}}}$$

$$\hat{\mathcal{T}}(\vec{p}',\vec{p}) \equiv \frac{1}{(2\pi)^3}\sqrt{\frac{\mathcal{M}_{\mathcal{N}}}{\mathfrak{E}_{p'}}}\mathcal{T}(\vec{p}',\vec{p})\sqrt{\frac{\mathcal{M}_{\mathcal{N}}}{\mathfrak{E}_{p}}}$$

$$\hat{\mathcal{T}}(\vec{p}',\vec{p}) = \hat{\mathcal{V}}(\vec{p}',\vec{p}) + \int d^3p''\, \mathcal{V}(\vec{p}',\vec{p}'')\frac{\mathcal{M}_{\mathcal{N}}}{p^2 - p''^2 + i\epsilon}\mathcal{T}(\vec{p}'',\vec{p})$$

$$f(p',p) = \exp\left[-\left(\frac{p'}{\Lambda}\right)^{2n} - \left(\frac{p}{\Lambda}\right)^{2n}\right]$$

$$\chi(p) = \chi_{ref}(p)\sum_{n=0}^{\infty} c_n(p)\mathcal{Q}^n$$

$$\Delta\chi_n = \max(\mathcal{Q}^5|\chi_{\mathcal{LO}}|, \mathcal{Q}^3|\chi_{\mathcal{LO}} - \chi_{\mathcal{NLO}}|, \mathcal{Q}^2|\chi_{\mathcal{NLO}} - \chi_{\mathcal{N}^2\mathcal{LO}}|, \mathcal{Q}|\chi_{\mathcal{N}^2\mathcal{LO}} - \chi_{\mathcal{N}^3\mathcal{LO}}|)$$

14. Límite de Oppenheimer.

$$\mathcal{P}(0) = \rho_0 c^2 \frac{1 - \sqrt{1 - \frac{r_s}{\mathcal{R}}}}{3\sqrt{1 - \frac{r_s}{\mathcal{R}}} - 1}$$

15. Espacio tiempo cuántico curvo bajo la métrica de Kerr, Schwarzschild y Einstein.

$$ds^2 = \left(1 - \frac{rr_s}{r^2 + \alpha^2\cos^2\theta}\right)(d\mu + \alpha\sin^2\theta d\varphi)^2 - 2(d\mu + \alpha\sin^2\theta d\varphi)(dr + \alpha\sin^2\theta d\varphi) - (r^2 + \alpha^2\cos^2\theta)(d\theta^2 + sin^2\theta d\varphi)$$

$$g_{\mu\nu} = \begin{pmatrix} 1 - \frac{rr_s}{\rho} - 1 & 0 & -\frac{rr_s\alpha\sin^2\theta}{\rho} \\ 0 & 0 & -\alpha\sin^2\theta \\ 0 & 0 & 1 \end{pmatrix}$$

$$g_{\mu\nu} = -\sin^2\theta\left(r^2 + \alpha^2 + \frac{rr_s\alpha\sin^2\theta}{\rho}\right)$$

$$ds^2 = \left(1 - \frac{r_s}{r}\right)d\mu^2 - 2d\mu dr - r^2(d\theta^2 + sin^2\theta d\varphi)$$

$$ds^2 = \left(1 - \frac{r_s}{r}\right)c^2dt^2 - \frac{dr^2}{1 - \frac{r_s}{r}} - r^2(d\theta^2 + sin^2\theta d\varphi)$$

$$g_{\mu\nu} = \begin{pmatrix} 1 - \frac{rr_s}{\rho} & 0 & -\frac{rr_s\alpha\sin^2\theta}{\rho} \\ 0 & \frac{\rho}{\Lambda} & 1 \\ 1 & 0 & -sin^2\theta\left(r^2 + \alpha^2 + \frac{rr_s\alpha\sin^2\theta}{\rho}\right) \end{pmatrix}$$

$$ds^2 = \left(1 - \frac{rr_s}{r^2 + \alpha^2\cos^2\theta}\right)(dt)^2 + \left(\frac{2rr_s}{r^2 + \alpha^2\cos^2\theta}\right)dtd\varphi - \left(\frac{r^2 + \alpha^2\cos^2\theta}{r^2 - rr_s + \alpha^2}\right)dr^2 - \left(r^2 + \alpha^2 + \frac{rr_s\alpha^2\sin^2\theta}{r^2 + \alpha^2\cos^2\theta}\right)\sin^2\theta\, d\varphi^2 - (r^2 + \alpha^2\cos^2\theta)(d\theta)^2$$

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R}_0 - \Lambda g_{\mu\nu} = -\kappa\mathcal{T}_{\mu\nu}$$

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}g^{\lambda\kappa}\left(\frac{\partial g_{\kappa\mu}}{\partial\chi^{\nu}} + \frac{\partial g_{\kappa\nu}}{\partial\chi^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial\chi^{\kappa}}\right)$$

$$\mathcal{R}_{\mu\nu} = \frac{\partial\Gamma^{\rho}_{\mu\rho}}{\partial\chi^{\nu}} - \frac{\partial\Gamma^{\rho}_{\mu\nu}}{\partial\chi^{\rho}} + \Gamma^{\sigma}_{\mu\rho}\Gamma^{\rho}_{\sigma\nu} - \Gamma^{\sigma}_{\mu\nu}\Gamma^{\rho}_{\sigma\rho}$$

$$\mathcal{T}_{\mu\nu} = \left(\rho + \frac{\mathcal{P}}{c^2}\right)u_\mu u_\nu - \mathcal{P}g_{\mu\nu}$$

$$u_0 = \left(\mathcal{A}_0(r_1,\theta)\sqrt{\left(\frac{r_1\mathcal{M}_1(r_1)^3}{\omega^2\mathcal{J}_1(r_1)^2\cos^2(\theta)+r_1^2\mathcal{M}_1(r_1)^2}-1\right)^2} - \frac{r_1\omega^4\mathcal{J}_1(r_1)^2\mathcal{M}_1(r_1)\cos^4(\theta)\mathcal{A}_3(r_1,\theta)}{\omega^2\mathcal{J}_1(r_1)^2\cos^2(\theta)+r_1^2\mathcal{M}_1(r_1)^2} - \frac{\omega^4\mathcal{J}_1(r_1)^2\sin^2(\theta)\mathcal{A}_3(r_1,\theta)}{\mathcal{M}_1(r_1)^2} - r_1^2\omega^2\sin^2(\theta)\mathcal{A}_3(r_1,\theta) + \frac{2\omega^2\sqrt{\mathcal{J}_1(r_1)^2}\mathcal{M}_1(r_1)\sqrt{\mathcal{M}_1(r_1)^2}\sin^2(\theta)\mathcal{A}_4(r_1,\theta)}{\omega^2\mathcal{J}_1(r_1)^2\cos^2(\theta)+r_1^2\mathcal{M}_1(r_1)^2}\right)^{-1}$$

$$r_- = \frac{\mathcal{M}_0}{2} - \sqrt{\left(\frac{\mathcal{M}_0}{2}\right)^2 - \alpha^2}$$

$$r_+ = \frac{\mathcal{M}_0}{2} + \sqrt{\left(\frac{\mathcal{M}_0}{2}\right)^2 + \alpha^2}$$

16. Ecuaciones TOV para espacios cuánticos curvos.

$$\mathcal{P}'(r) = -\left(\frac{\mathcal{G}\mathcal{M}(r)\rho(r)}{r^2}\right)\left(1+\frac{\mathcal{P}(r)}{\rho(r)c^2}\right)\left(1+\frac{4\pi r^3\mathcal{P}(r)}{\mathcal{M}(r)c^2}\right)\left(1-\frac{2\mathcal{G}\mathcal{M}(r)}{rc^2}\right)^{-1}$$

$$\mathcal{P}'(r) = -\left(\frac{c^2 r_s\rho(r)}{r^2}\right)\left(1+\frac{\mathcal{P}(r)}{\rho(r)c^2}\right)\left(1+\frac{4\pi r^3\mathcal{P}(r)}{\mathcal{M}(r)c^2}\right)\left(1-\frac{r_s\mathcal{M}(r)}{r\mathcal{M}_t}\right)^{-1}$$

$$\mathcal{P} = -\frac{\partial\mathfrak{E}}{\partial\mathcal{V}} = 16\pi\rho_0\left(\frac{\chi_{\mathcal{F}}^3}{3}\sqrt{1+\chi_{\mathcal{F}}^2} - f(\chi_{\mathcal{F}})\right)$$

$$f(\chi_{\mathcal{F}}) = \int_0^{\chi_{\mathcal{F}}} d\chi\sqrt{1+\chi^2}$$

$$n_1 = \frac{\mathcal{N}_{op}}{\mathcal{V}_1} = \frac{2}{\mathcal{V}_1}\int_0^{\infty} d\omega_1\frac{\mathcal{D}_1(\omega_1)}{1+\exp\big(\beta_1(\omega_1-\mu_1)\big)} = \frac{1}{2\pi^{\frac{7}{2}}}\int_0^{\infty} d\omega_1\frac{\sqrt{\omega_1}}{1+\exp\big(\beta_1(\omega_1-\mu_1)\big)}$$

$$\mu_1 = \varepsilon_{\mathcal{F}_1} - \frac{\pi^2}{12\beta_1^2\varepsilon_{\mathcal{F}_1}} = \mu_1(n_1)$$

$$p_1 = (\beta_1, n_1) = \frac{4\pi^{3/2}}{3\pi^2}\int_0^{\infty} d\omega_1\frac{\omega_1^{3/2}}{1+\exp\Big(\beta_1\big(\omega_1-\mu_1(n_1)\big)\Big)}$$

$$\mathcal{V}_{SW}(r,\mathcal{V}_0,r_0,dr_0) = \frac{\mathcal{V}_0}{1+\exp\left(\frac{r-r_0}{dr_0}\right)}$$

17. Configuración estelar.

$$\sigma(\psi^i)(\mathcal{G}_{\mu\nu}-\mathcal{W}_{\mu\nu}) = \kappa\mathcal{T}_{\mu\nu}$$

$$\mathcal{T}_{\mu\nu}^{eff} = \frac{1}{\sigma}\mathcal{T}_{\mu\nu} + \frac{1}{\kappa}\mathcal{W}_{\mu\nu}$$

$$ds^2 = -\mathcal{B}(r)dt^2 + \mathcal{A}(r)dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2$$

$$\mathcal{R}_{tt} = -\frac{\mathcal{B}''}{2\mathcal{A}} + \frac{\mathcal{B}'}{4\mathcal{A}}\left(\frac{\mathcal{A}'}{\mathcal{B}} + \frac{\mathcal{B}'}{\mathcal{B}}\right) - \frac{\mathcal{B}'}{r\mathcal{A}} = \frac{\kappa}{2\sigma}(\rho + 3p)\mathcal{B} + \mathcal{W}_{tt} + \frac{\mathcal{B}\mathcal{W}}{2}$$

$$\mathcal{R}_{rr} = \frac{\mathcal{B}''}{2\mathcal{A}} - \frac{\mathcal{B}'}{4\mathcal{B}}\left(\frac{\mathcal{A}'}{\mathcal{B}} + \frac{\mathcal{B}'}{\mathcal{B}}\right) - \frac{\mathcal{A}'}{r\mathcal{A}} = \frac{\kappa}{2\sigma}(\rho - p)\mathcal{A} + \mathcal{W}_{rr} - \frac{\mathcal{A}\mathcal{W}}{2}$$

$$\mathcal{R}_{\theta\theta} = -1 + \frac{r}{2\mathcal{A}}\left(-\frac{\mathcal{A}'}{\mathcal{B}} + \frac{\mathcal{B}'}{\mathcal{B}}\right) + \frac{1}{\mathcal{A}} = \frac{\kappa}{2\sigma}(\rho - p)r^2 + \mathcal{W}_{\theta\theta} - \frac{r^2\mathcal{W}}{2}$$

$$\left(\frac{r}{\mathcal{A}}\right)' = 1 + \kappa r^2\frac{\rho(r)}{\sigma(r)} + r^2\mathcal{B}^{-1}(r)\mathcal{W}_{tt}(r)$$

$$\mathcal{A}(r) = \left(1 - \frac{2\mathcal{G}\mathcal{M}(r)}{r}\right)^{-1}$$

$$\mathcal{M}(r) = \int_0^r \left(4\pi\tilde{r}\frac{\rho(\tilde{r})}{\sigma(\tilde{r})} - \frac{\tilde{r}^2\mathcal{W}_{tt}(\tilde{r})}{2\mathcal{G}\mathcal{B}(\tilde{r})}\right)d\tilde{r}\,\left(\sigma^{-1}\nabla_\mu\mathcal{T}^{\mu\nu} - \sigma^{-2}\mathcal{T}^{\mu\nu}\nabla_\mu\sigma\right) + \frac{1}{\kappa}\nabla_\mu\mathcal{T}^{\mu\nu}$$

$$\kappa\sigma^{-1} = \left(p' + (p+\rho)\frac{\mathcal{B}'}{2\mathcal{B}}\right) - \kappa\rho\frac{\sigma'}{\sigma^2} - \frac{\mathcal{A}'}{\mathcal{A}^2}\mathcal{W}_{rr} + \mathcal{A}^{-1}\mathcal{W}_{rr}' + \frac{2\mathcal{W}_{rr}}{\mathcal{A}r} + \frac{\mathcal{B}'}{2\mathcal{B}}\left(\frac{\mathcal{W}_{rr}}{\mathcal{A}} + \frac{\mathcal{W}_{tt}}{\mathcal{B}}\right) - \frac{2\mathcal{W}_{\theta\theta}}{r^2}$$

$$\mathcal{Q}(r) \coloneqq \rho(r) + \frac{\sigma(r)\mathcal{W}_{tt}(r)}{\kappa\mathcal{B}(r)}$$

$$\prod(r) \coloneqq p(r) + \frac{\sigma(r)\mathcal{W}_{rr}(r)}{\kappa\mathcal{A}(r)}$$

$$\frac{\mathcal{A}'}{\mathcal{A}} = \frac{1-\mathcal{A}}{r} - \frac{\kappa\mathcal{A}r}{\sigma}\mathcal{Q}$$

$$\frac{\mathcal{B}'}{\mathcal{B}} = \frac{\mathcal{A}-1}{r} - \frac{\kappa\mathcal{A}r}{\sigma}\prod$$

$$\left(\frac{\prod}{\sigma}\right)' = -\frac{\mathcal{G}\mathcal{M}}{r^2}\left(\frac{\mathcal{Q}}{\sigma} + \frac{\prod}{\sigma}\right)\left(1 + \frac{4\pi r^3\prod}{\mathcal{M}\sigma}\right)\left(1 - \frac{2\mathcal{G}\mathcal{M}(r)}{r}\right)^{-1} + \frac{2\sigma}{\kappa r}\left(\frac{\mathcal{W}_{\theta\theta}}{r^2} - \frac{\mathcal{W}_{rr}}{\mathcal{A}}\right)$$

$$\mathcal{M}(r) = \int_0^r 4\pi\tilde{r}^2\frac{\mathcal{Q}(\tilde{r})}{\sigma(\tilde{r})}d\tilde{r}$$

18. Gravedad Clase k – esencia en espacios cuánticos curvos.

$$\mathcal{S} = \frac{1}{2\kappa}\int d^4\chi\sqrt{g}\left(\mathcal{R} - \nabla_\mu\phi\nabla^\mu\phi - 2\mathcal{V}(\phi)\right) + \mathcal{S}_m\left[g_{\mu\nu},\psi\right]$$

$$\mathcal{G}_{\mu\nu} + \frac{1}{2}g_{\mu\nu}\nabla_\alpha\phi\nabla^\alpha\phi - \nabla_\mu\phi\nabla_\nu\phi + g_{\mu\nu}\mathcal{V}(\phi) = \kappa\mathcal{T}_{\mu\nu}\mathcal{V}'(\phi) - \Box\phi$$

$$\mathcal{W}_{\mu\nu} = -\frac{1}{2}g_{\mu\nu}\nabla_\alpha\phi\nabla^\alpha\phi + \nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\mathcal{V}(\phi)$$

$$\mathcal{W}_{tt} = \frac{1}{2}\mathcal{B}\nabla_\alpha\phi\nabla^\alpha\phi + \mathcal{B}\mathcal{V}(\phi) = \mathcal{B}(\mathcal{C} + 2\mathcal{V})$$

$$\mathcal{W}_{rr} = \mathcal{A}\mathcal{C}$$

$$\mathcal{W}_{\theta\theta} = -r^2(\mathcal{C} + 2\mathcal{V})$$

$$\mathcal{Q}_k(r) \coloneqq \rho(r) + \kappa^{-1}(\mathcal{C} + 2\mathcal{V})$$

$$\textstyle\prod_k(r) \coloneqq p(r) + \kappa^{-1}\mathcal{C}$$

$$u^\mu\left(\frac{\sigma}{\kappa}\mathcal{W}^\nu_{\mu;\nu} - np\nabla_\mu\left(\frac{1}{n}\right) - n\nabla_\mu\left(\frac{\rho}{n}\right) + \frac{\rho}{\sigma}\nabla_\mu\sigma\right)$$

$$\delta n(r) = \frac{n(r)}{\rho(r) + p(r)}\delta\rho(r)$$

$$n'^{(r)} = n\frac{\rho'}{\rho + p}$$

$$\mathcal{N} = \int_0^{\mathcal{R}} 4\pi r^2\left[1 - \frac{2\mathcal{G}\mathcal{M}(r)}{r}\right]^{-1/2} n(r)dr$$

$$\delta\mathcal{M} - \lambda\delta\mathcal{N} = \int_0^\infty 4\pi r^2\delta\mathcal{Q}dr - \lambda\int_0^\infty 4\pi r^2\left(1 - \frac{2\mathcal{G}\mathcal{M}(r)}{r}\right)^{-\frac{1}{2}}\delta n(r)dr$$
$$- \lambda\mathcal{G}\int_0^\infty 4\pi r\left(1 - \frac{2\mathcal{G}\mathcal{M}(r)}{r}\right)^{-\frac{3}{2}} n(r)\delta\mathcal{M}(r)dr$$

$$\delta\mathcal{M} - \lambda\delta\mathcal{N} = \int_0^\infty 4\pi r^2\left[1 - \frac{\lambda n(r)}{\rho(r) + p(r)}\mathcal{A}^{\frac{1}{2}} - \lambda\mathcal{G}\int_r^\infty 4\pi\tilde{r}n(\tilde{r})\,\mathcal{A}^{\frac{3}{2}}d\tilde{r}\right.$$
$$\left. - \lambda\mathcal{G}\kappa^{-1}\int_r^\infty 4\pi\tilde{r}\mathcal{A}^{\frac{1}{2}}\frac{n}{p + \rho}\phi'^2\,d\tilde{r}\right]\delta\mathcal{Q}(r)dr$$
$$- \lambda\kappa^{-1}\int_0^\infty \partial^\nu\phi\left[4\pi r^2\mathcal{A}^{\frac{1}{2}}\Gamma^\mu_{\mu\nu}\frac{n(r)}{p(r) + \rho(r)}\right.$$
$$\left. - \partial_\nu\left(4\pi r^2\mathcal{A}^{\frac{1}{2}}\frac{n(r)}{p(r) + \rho(r)}\right)\right]\delta\phi dr\int_0^\infty \partial^\mu\left(4\pi r^2\frac{n(r)}{p(r) + \rho(r)}\delta\phi\partial_\mu\phi\right)dr$$

$$\frac{1}{\lambda} = \frac{n(r)}{p(r) + \rho(r)}\mathcal{A}^{\frac{1}{2}} + \mathcal{G}\int_r^\infty 4\pi\tilde{r}n(\tilde{r})\,\mathcal{A}^{\frac{3}{2}}d\tilde{r} + \mathcal{G}\kappa^{-1}\int_r^\infty 4\pi\tilde{r}\,\mathcal{A}^{\frac{1}{2}}\frac{n(\tilde{r})}{p(\tilde{r}) + \rho(\tilde{r})}\phi'^2d\tilde{r}$$

$$4\pi r^2\mathcal{A}^{\frac{1}{2}}\Gamma^\mu_{\mu\nu}\frac{n(r)}{p(r) + \rho(r)} = \partial_r\left(4\pi r^2\mathcal{A}^{\frac{1}{2}}\frac{n(r)}{p(r) + \rho(r)}\right)$$

$$-4\pi\mathcal{G}r\mathcal{A} - \frac{p'}{(p + \rho)^2}\frac{\mathcal{G}\mathcal{A}}{p + \rho}\left(4\pi r\mathcal{Q}_k - \frac{\mathcal{M}}{r^2}\right) - \frac{4\pi r\mathcal{G}}{\kappa}\frac{\phi'^2}{p + \rho}\frac{\mathcal{A} - 1}{r}\mathcal{A}\frac{2\mathcal{G}\mathcal{M}}{r^2}p + \rho$$
$$= \textstyle\prod_\kappa + \mathcal{Q}_\kappa - 2\kappa^{-1}(\mathcal{C} + \mathcal{V})\mathcal{A}^{-1}\phi'^2 = 2(\mathcal{C} + \mathcal{V})$$

$$\textstyle\prod_k' = p' + \kappa^{-1}\mathcal{C}' = p' + \kappa^{-1}(\mathcal{C} + \mathcal{V})\left(\frac{\mathcal{A} - 1}{r} - \kappa\mathcal{A}r\prod_\kappa + \frac{4}{r}\right)$$

$$\textstyle\prod_k' = -\frac{\mathcal{A}\mathcal{G}\mathcal{M}}{r^2}\left(\prod_\kappa + \mathcal{Q}_\kappa\right)\left(1 + 4\pi r\frac{\prod_\kappa}{\mathcal{M}}\right) - 4\frac{\mathcal{C} + \mathcal{V}}{\kappa r}$$

$$\Gamma^{\mu}_{\mu\nu} = \frac{2}{r} - \frac{1}{2}\left(\kappa\mathcal{A}r\left(\prod_{\kappa} + \mathcal{Q}_{\kappa}\right)\right)$$

19. Métrica gravitacional de Brans-Dicke para espacios cuánticos curvos.

$$\mathcal{S} = \frac{1}{32\pi\mathcal{G}}\int d^4\chi\sqrt{-g}\left(\mathcal{R}\phi - \frac{\omega}{\phi}\partial_\alpha\phi\,\partial^\alpha\phi\right) + \mathcal{S}_m$$

$$\mathcal{G}_{\mu\nu} = \frac{16\pi}{\phi}\mathcal{T}_{\mu\nu} + \frac{\omega}{\phi^2}\left(\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\,\partial_\alpha\phi\,\partial^\alpha\phi\right) + \frac{1}{\phi}\left(\nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\Box\phi\right)\Box\phi = \frac{16\pi}{3+2\omega}\mathcal{T}$$

$$\mathcal{W}_{\mu\nu} = \frac{\omega}{\phi^2}\left(\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\,\partial_\alpha\phi\,\partial^\alpha\phi\right) + \frac{1}{\phi}\left(\nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\Box\phi\right)$$

$$\mathcal{W}_{\theta\theta} = \frac{\omega}{\phi^2}\left(\partial_\theta\phi\,\partial_\theta\phi - \frac{1}{2}g_{\theta\theta}\,\partial_\alpha\phi\,\partial^\alpha\phi\right) + \frac{1}{\phi}\left(\nabla_\theta\nabla_\theta\phi - g_{\theta\theta}\Box\phi\right)$$

$$\mathcal{W}_{rr} = \frac{\omega}{\phi^2}\left(\partial_r\phi\,\partial_r\phi - \frac{1}{2}g_{rr}\,\partial_\alpha\phi\,\partial^\alpha\phi\right) + \frac{1}{\phi}\left(\nabla_r\nabla_r\phi - g_{rr}\Box\phi\right)$$

$$\mathcal{S} = \frac{1}{32\pi\mathcal{G}}\int d^4\chi\sqrt{-g}\left[(1+\alpha\phi)\mathcal{R} - \frac{\omega}{2}\partial_\alpha\phi\,\partial^\alpha\phi - \mathcal{V}(\phi)\right] + \mathcal{S}_m$$

$$\mathcal{G}_{\mu\nu} = 16\pi\mathcal{G}\mathcal{T}_{\mu\nu} - \alpha\phi\mathcal{L}_{\mu\nu}$$

$$\mathcal{L}_{\mu\nu} = \frac{\omega}{2}\left(\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\,\partial_\alpha\phi\,\partial^\alpha\phi\right) + \frac{\alpha}{\phi}\left(\nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\Box\phi\right) - \frac{1}{2}g_{\mu\nu}\mathcal{V}(\phi)$$

$$\mathcal{W}_{\theta\theta} = \alpha\phi\left(\frac{1}{2}g_{\theta\theta}\mathcal{V}(\phi) - \frac{\omega}{2}\left(\partial_\theta\phi\,\partial_\theta\phi - \frac{1}{2}g_{\theta\theta}\,\partial_\alpha\phi\,\partial^\alpha\phi\right) - \frac{\alpha}{\phi}\left(\nabla_\theta\nabla_\theta\phi - g_{\theta\theta}\Box\phi\right)\right)$$

$$\mathcal{W}_{rr} = \alpha\phi\left(\frac{1}{2}g_{rr}\mathcal{V}(\phi) - \frac{\omega}{2}\left(\partial_r\phi\,\partial_r\phi - \frac{1}{2}g_{\theta\theta}\,\partial_\alpha\phi\,\partial^\alpha\phi\right) - \frac{\alpha}{\phi}\left(\nabla_r\nabla_r\phi - g_{rr}\Box\phi\right)\right)$$

$$\mathcal{W}_{tt} = \alpha\phi\left(\frac{1}{2}g_{tt}\mathcal{V}(\phi) - \frac{\omega}{2}\left(\partial_t\phi\,\partial_t\phi - \frac{1}{2}g_{tt}\,\partial_\alpha\phi\,\partial^\alpha\phi\right) - \frac{\alpha}{\phi}\left(\nabla_t\nabla_t\phi - g_{tt}\Box\phi\right)\right)$$

$$\begin{aligned}\prod{}' &= -\frac{\mathcal{GM}}{r^2}\left(\mathcal{Q} + \prod\right)\left(1 + \frac{4\pi r^3\prod}{\mathcal{M}}\right)\left(1 - \frac{2\mathcal{GM}(r)}{r}\right)^{-\frac{1}{2}} \\ &+ \frac{2\alpha}{8\pi\mathcal{G}r}\left[\frac{\left(\frac{1}{2}g_{\theta\theta}\mathcal{V}(\phi) - \frac{\omega}{\phi^2}\left(\partial_\theta\phi\,\partial_\theta\phi - \frac{1}{2}g_{\theta\theta}\,\partial_\alpha\phi\,\partial^\alpha\phi\right) - \frac{\alpha}{\phi}\left(\nabla_\theta\nabla_\theta\phi - g_{\theta\theta}\Box\phi\right)\right)}{r^2}\right] \\ &- \left[\frac{\left(\frac{1}{2}g_{rr}\mathcal{V}(\phi) - \frac{\omega}{\phi^2}\left(\partial_r\phi\,\partial_r\phi - \frac{1}{2}g_{rr}\,\partial_\alpha\phi\,\partial^\alpha\phi\right) - \frac{\alpha}{\phi}\left(\nabla_r\nabla_r\phi - g_{rr}\Box\phi\right)\right)}{\mathcal{A}}\right]\end{aligned}$$

$$\mathcal{G}_{\mu\nu} = (1+\alpha\phi)\left(\mathcal{T}^{eff}_{\mu\nu}\right)$$

20. Sistemas Dinámicos TOV.

$$-rp'(r-2m)$$
$$= (p+\rho)(m+4\pi r^3 p)\begin{cases} 4\pi r\chi(\ln r) = m(r) \\ 4\pi\gamma(\ln r) = m'(r) = r^2\rho(r) \end{cases}\begin{cases} \chi'(s) = -\chi(s)+\gamma(s) \\ \gamma'(s) = 2\gamma(s) - \dfrac{8\pi\gamma(s)}{1-8\pi\chi(s)}(\chi(s)+\gamma(s)) \end{cases}$$

$$\mathcal{L}(\chi,\gamma) = 2 + 16\pi(\gamma-3\chi) - \log(128\pi\gamma(1-8\pi\chi)^3)$$

$$\frac{d}{d\mathcal{L}}\mathcal{L}(\chi(t)+\gamma(t)) = \chi'(t)(\chi(t)-2) + \gamma'(t) - 2(d-2)\frac{\gamma'(t)}{\gamma(t)} = -(\chi(t)-2)^2$$

$$\mathcal{L}(\chi,\gamma) \sim \frac{1}{2}\left(\chi - \frac{1}{16\pi}\right)^2 + \left(\gamma - \frac{1}{16\pi}\right)^2 \lim_{s\mapsto-\infty}\chi(s)e^{s} \lim_{s\mapsto-\infty}\chi(s)e^{-2s}$$

$$\rho_0 = \rho(0) = |\rho|_\infty = \lim_{s\mapsto-\infty}\gamma(s)e^{-2s}$$

$$\lim_{s\mapsto-\infty}\chi(s)e^{-2s} = \lim_{s\mapsto-\infty}\mathcal{Q}'(s)e^{(1-d)s} = \lim_{r\mapsto0^+} r^{d-1}\rho(r)r^{1-d} = \rho(0)$$

$$\lim_{s\mapsto-\infty}\frac{\chi(s)}{\gamma(s)} = \frac{1}{3}$$

$$\mathcal{N} = \lim_{s\mapsto-\infty}\frac{\chi(s)}{\gamma(s)} = \lim_{s\mapsto-\infty}\frac{\chi'(s)}{\gamma'(s)} = \lim_{s\mapsto-\infty}\frac{1-\dfrac{\chi(s)}{\gamma(s)}}{2-\dfrac{8\pi\gamma(s)\left(\dfrac{\chi(s)}{\gamma(s)}+1\right)}{1-8\pi\chi(s)}} = \frac{1-\mathcal{N}}{2}$$

$$\chi'\left(-2+\frac{8\pi(\chi+\gamma)}{1-8\pi\chi}\right) = (\gamma-\chi)\left(-2+\frac{8\pi(\chi+\gamma)}{1-8\pi\chi}\right)$$

$$\chi'\left(-2+\frac{8\pi(\chi+\gamma)}{1-8\pi\chi}\right) + \gamma' = -\chi\left(-2+\frac{8\pi(\chi+\gamma)}{1-8\pi\chi}\right)$$

$$\frac{\gamma'(\gamma-\mathcal{C})}{\gamma} = \left(2-\frac{8\pi(\chi+\gamma)}{1-8\pi\chi}\right)(\gamma-\mathcal{C})$$

$$3\chi'\left(-2+\frac{8\pi(\chi+\gamma)}{1-8\pi\chi}\right) + 4\gamma' - \frac{\mathcal{C}\gamma'}{\gamma} = (\mathcal{C}-\gamma-3\chi)\left(-2+\frac{8\pi(\chi+\gamma)}{1-8\pi\chi}\right)$$

$$3\chi'\left(-2+\frac{8\pi(2\chi'+\chi')}{1-8\pi\chi}\right) + 4\gamma' - \frac{\mathcal{C}\gamma'}{\gamma} = \frac{(\mathcal{C}-\gamma-3\chi)(-2+24\pi\chi+8\pi\gamma)}{1-8\pi\chi}$$

$$-6\chi' + \frac{48\pi\chi\chi'}{(1-8\pi\chi)} + 4\gamma' - \frac{1}{4\pi}\frac{\gamma'}{\gamma}$$
$$= \frac{-(1-12\pi\chi-4\pi\gamma)^2}{2\pi(1-8\pi\chi)}(-48\pi\chi - 3\log(1-8\pi\chi) + 16\pi\gamma - \log\gamma)'$$

$$\mathcal{L}(\chi,\gamma) = 2 + 16\pi(\gamma-3\chi) - \log(128\pi\gamma(1-8\pi\chi)^3)\frac{\mathcal{GM}}{\mathcal{R}c^2}\frac{2\mathcal{GM}}{\mathcal{R}c^2}$$

$$\frac{\chi'}{\gamma'} = 3(1-4\omega/(1-\omega)) \in [-3,3]$$

$$2 - 32\pi\chi - \log(128\pi\chi(1-8\pi\chi)^3) = 1 - \log(2)$$

20.1. Función Lyapunov.

$$p = \kappa\rho$$

$$\begin{cases} \chi'(s) = -\chi(s) + \gamma(s) \\ \gamma'(s) = 2\gamma(s) - \dfrac{1+\kappa}{2\kappa} \bigotimes \dfrac{\gamma(s)\big(\chi(s) + \kappa\gamma(s)\big)}{1-\chi(s)} \end{cases}$$

$$(\chi_\kappa, \gamma_\kappa) = \left(\frac{4\kappa}{(1+\kappa)^2 + 4\kappa}, \frac{4\kappa}{(1+\kappa)^2 + 4\kappa}\right)$$

$$\mathcal{V} = 2\gamma - (5 + 1/\kappa)\chi - 2\chi_\kappa \log\big(\gamma(1-\chi)^{\delta_\kappa}\big) + \mathcal{C}_\kappa\, 8\kappa^2\delta_\kappa = (5\kappa+1)(\kappa+1)^2$$

$$\mathcal{C}_\kappa = (3 + 1/\kappa)\chi_\kappa + 2\chi_\kappa \log\big(\chi_\kappa(1-\chi_\kappa)^{\delta_\kappa}\big)$$

$$\begin{aligned} \mathcal{V} = \mathcal{C} + 2\gamma - \gamma\chi - \beta\log\big(\gamma(1-\gamma)^{\delta}\big) - (1+\kappa)\gamma^2 \\ + \gamma(\delta\beta - \gamma + 4 + \beta(1+\kappa)/2 - \gamma\chi^2 + \chi(\gamma - \delta\beta + 2\beta + \beta(1+\kappa)/2) - 2\beta) \end{aligned}$$

$$(1-\chi)\mathcal{V}' = -(5 + 1/\kappa)(\chi - \chi_\kappa)^2 - (1+\kappa)(\gamma - \gamma_\kappa)^2$$

$$\begin{cases} v' = -v + w \\ w' = -\dfrac{(1+\kappa)\gamma}{\kappa(1-\chi)^2} v + \left(2 + \dfrac{1+\kappa}{2\kappa} \bigotimes \dfrac{\chi + 2\kappa\gamma}{1-\chi}\right) w \end{cases}$$

$$p = p(4\pi\rho) = p(r^{-2}\gamma) \begin{cases} \chi'(s) = -\chi(s) + \gamma(s) \\ \gamma'(s) = 2\gamma(s) - \dfrac{4\pi\big(\gamma + r^2 p(\gamma r^2)\big)\big(\chi + r^2 p(\gamma r^{-2})\big)}{(1 - 8\pi\chi)p'(r^{-2}\gamma)} \end{cases}$$

$$\rho = \mathcal{C}p^{1/\Gamma} + \frac{p}{1-\Gamma}$$

$$\mathcal{N} = \lim_{s\mapsto-\infty} \frac{\chi(s)}{\gamma(s)} = \lim_{s\mapsto-\infty} \frac{\chi'(s)}{\gamma'(s)} = \lim_{s\mapsto-\infty} \frac{1 - \dfrac{\chi(s)}{\gamma(s)}}{2 - \dfrac{4\pi\gamma(s)\left(1 + \dfrac{p(\gamma(s)r^{-2})}{\gamma(s)r^{-2}}\right)\left(\dfrac{\chi(s)}{\gamma(s)} + \dfrac{p(\gamma(s)r^{-2})}{\gamma(s)r^{-2}}\right)}{\big(1 - 8\pi\chi(s)\big)p'\big(r^{-2}\gamma(s)\big)}}$$

20.2. Modelo Relativista Michie–King.

$$\rho = \frac{2m}{\hbar^3}\int_0^\infty f_c(\hat{p})\left(1 + \frac{\varepsilon(\hat{p})}{mc^2}\right) d^3\hat{p}$$

$$f_c(\hat{p}) = \left(\frac{1 - \exp\left(\dfrac{(\varepsilon(\hat{p}) - \varepsilon_c)}{\kappa\mathcal{T}}\right)}{\exp\left(\varepsilon(\hat{p}) - \dfrac{\mu}{\kappa\mathcal{T}}\right) + 1}\right)_+$$

$$p = \frac{4}{3\hbar^3}\int_0^\infty f_c(\hat{p})\varepsilon(\hat{p}) \frac{1 + \dfrac{\varepsilon(\hat{p})}{2mc^2}}{1 + \dfrac{\varepsilon(\hat{p})}{mc^2}} d^3\hat{p}$$

$$\varepsilon(\hat{p}) = \sqrt{c^2\hat{p}^2 + m^2c^4 - mc^2}$$

$$p(\rho) = \left((p_\infty p)^{-1} + (p_0\rho)^{-\frac{7}{5}}\right)^{-1}$$

$$\frac{(\varepsilon + mc^2)}{\mathcal{T}} \equiv const = mc^2/\mathcal{T}_{\mathcal{R}}\left(\frac{\varepsilon_c}{(mc^2)} + 1\right)\frac{\mathcal{T}_{\mathcal{R}}}{\mathcal{T}}$$

$$\kappa = \frac{\kappa\mathcal{T}_{\mathcal{R}}}{(mc^2)}$$

$$\gamma = \frac{\varepsilon_c}{(\kappa\mathcal{T})} 1 - \gamma\kappa = \frac{\mathcal{T}_{\mathcal{R}}}{\mathcal{T}}\chi\kappa\mathcal{T} = \varepsilon(\hat{p})$$

$$\hat{p} = \frac{mv}{\sqrt{1 - |v|^2/c^2}}$$

$$1 + \frac{\varepsilon(\hat{p})}{(mc^2)} = 1 + \frac{\chi\kappa\mathcal{T}}{\mathcal{T}_{\mathcal{R}}} = 1 + \frac{\chi\kappa}{(1-\gamma\kappa)}$$

$$\hat{p}^2 = m^2c^2\frac{\chi\kappa}{1-\gamma\kappa}\left(\frac{\chi\kappa}{1-\kappa\gamma} + 2\right)$$

$$\frac{\kappa m^2c^2}{1-\gamma\kappa}\left(\frac{\chi\kappa}{(1-\gamma\kappa)} + 1\right)d\chi = \langle\hat{p}|d|\hat{p}\rangle$$

$$\rho = \frac{8\pi c^3\kappa m^4}{\hbar^3(1-\kappa\gamma)}\int_0^\gamma\left(\frac{\kappa\chi}{(1-\kappa\gamma)} + 1\right)^2\left(\left(\frac{\kappa\chi}{(1-\kappa\gamma)} + 1\right)^2 - 1\right)^{1/2}\frac{1-\exp(\chi-\gamma)}{1+\chi\exp(\chi-\gamma)}d\chi$$

$$\rho = \frac{8\sqrt{2}\pi c^3\kappa^{3/2}m^4}{\hbar^3(1-\kappa\gamma)}\int_0^\gamma\chi^{1/2}\left(\frac{\kappa\chi}{(1-\kappa\gamma)} + 1\right)^2\left(\frac{\kappa\chi/2}{(1-\kappa\gamma)} + 1\right)^{1/2}\frac{1-\exp(\chi-\gamma)}{1+\chi\exp(\chi-\gamma)}d\chi$$

$$\rho(\gamma) \sim \frac{8\sqrt{2}\pi c^3\kappa^{3/2}}{1+\chi}\left(\frac{mc}{\hbar}\right)^2\int_0^\gamma\sqrt{\chi}(\gamma-\chi)d\chi = \rho_0\gamma^{5/2}$$

$$\rho_0 = \frac{32\sqrt{2}\pi m\kappa^{\frac{3}{2}}}{15(1+\chi)}\left(\frac{mc}{\hbar}\right)^3$$

$$\rho(\gamma) \sim 8\pi c^3m^4\hbar^{-3}\int_0^1 w^3\frac{1-\exp((\omega-1)/\kappa)}{1+\chi\exp((\omega-1)/\kappa)}d\omega$$

$$p = \frac{\eta^2\sqrt{2}\kappa}{1-\kappa\gamma}\int_0^\gamma\left(\frac{\kappa\chi}{1-\kappa\gamma}\right)^{\frac{3}{2}}\left(\frac{\frac{\kappa\chi}{2}}{1-\kappa\gamma} + 1\right)^{\frac{3}{2}}\frac{1-\exp(\chi-\gamma)}{1+\chi\exp(\chi-\gamma)}d\chi$$

$$p = \eta\left(\frac{\kappa}{1-\kappa\gamma}\right)^{\frac{5}{2}}\int_0^\gamma\chi^{\frac{3}{2}}\left(\frac{\kappa\chi}{1-\kappa\gamma} + 2\right)^{\frac{3}{2}}\frac{1-\exp(\chi-\gamma)}{1+\chi\exp(\chi-\gamma)}d\chi$$

$$\eta = \frac{8\pi m^4c^5}{3\hbar^3}$$

$$p_0 = \frac{64\sqrt{2}\pi m^4c^5\kappa^{\frac{5}{2}}}{105(1+\chi)\hbar^3}$$

$$p(\rho) \sim p_0\left(\frac{\rho}{\rho_0}\right)^{\frac{7}{5}}$$

$$p(\gamma) \sim \frac{c^2\rho_\infty}{3(1-\kappa\gamma)^4}$$

$$\rho_\infty = 8\pi c^3\kappa^4 m^4\hbar^{-3}\int_0^{1/\kappa} \chi^3 \frac{1-\exp(\chi-1/\kappa)}{1+\chi\exp(\chi-1/\kappa)}d\chi$$

20.3. Distribución relativista Fermi–Dirac.

$$\rho = \frac{1}{\pi^2}\int_0^\infty \frac{\gamma^2\sqrt{1+\gamma^2}}{1+e^{-\alpha}e^{\chi\sqrt{1+\gamma^2}}}d\gamma$$

$$\rho = \frac{1}{3\pi^2}\int_0^\infty \frac{\gamma^4}{\sqrt{1+\gamma^2}\left(1+e^{-\alpha}e^{\chi\sqrt{1+\gamma^2}}\right)}d\gamma$$

$$p(\chi) \sim \frac{1}{3\pi\chi^4}\int_0^\infty z^3(1+e^{-\alpha}e^{z})^{-1}dz$$

$$p(\chi) \sim \frac{1}{\pi\chi^4}\int_0^\infty z^3(1+e^{-\alpha}e^{z})^{-1}dz$$

$$\pi^2 p(\chi) \sim \sqrt{2}\chi^{-3/2}e^{-\chi+\alpha}\int_0^\infty \omega^{1/2}e^{-\omega}d\omega$$

$$\pi^2 p(\chi) \sim 2\sqrt{2}\chi^{-5/2}e^{-\chi+\alpha}\int_0^\infty \omega^{1/2}e^{-\omega}d\omega$$

$$p(\rho) \sim \rho/\log(\rho)$$

$$p(\rho) = \rho/(3-\log(\rho)\,(1+\rho)^{-1})$$

20.4. Materia oscura relativista – estado puro.

$$\rho = \kappa\left(\sqrt{1+\chi^2}(\chi+2\chi^3)-\operatorname{arcsinh}(\chi)\right)$$

$$\rho = \frac{\kappa}{3\left(\chi\sqrt{1+\chi^2}(2\chi^2-3)+3\operatorname{arcsinh}(\chi)\right)2\kappa\chi^4}$$

$$3\kappa^{-1}\sqrt{\chi^2+1}p' = 8\chi^4$$

$$3\kappa^{-1}\sqrt{(\chi^2+1)^3}p'' = 8\chi^3(3\chi^2+4)$$

$$\frac{d}{d\rho}\left(p(\chi(\rho))\right) = \frac{p'(\chi(\rho))}{\rho'(\chi(p))} = \frac{1}{1+\chi^{-2}(\rho)}\frac{3p}{\rho} = 1-\frac{4}{2\chi^2+1} \geq 1$$

21. Velocidad y presión de una partícula supermasiva o masiva y de una antipartícula supermasiva o masiva, según sea el caso, bajo el modelo relativista.

$$v_e|_{\mathcal{R}} = \left[\frac{\mathfrak{E}_e^2}{c^5}\frac{m^4}{m_e^5c^4+\mathfrak{E}_e^2/\hbar c}\right]$$

$$v_e|_{-\mathcal{R}} = \left[\frac{\mathfrak{E}_e^2}{c^5}\frac{m^4}{m_e^5 c^4 - \mathfrak{E}_e^2/\hbar c}\right]$$

$$\mathcal{P}_e|_{\mathcal{NR}} = \frac{1}{3}\int_0^{p_{\mathcal{F}}} p v_e n_e(p) dp = \frac{8\pi}{15\hbar^3}\frac{p_{\mathcal{F}}^5}{m_e}(\rho\mathcal{N}_{\mathcal{A}}/\mu_e)^{5/3}\frac{2}{1+\chi_{\mathcal{H}}} k\mathcal{N}_{\mathcal{R}}$$

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APÉNDICE G.

Modelo matemático relativo a: 1. Demostrar la dinámica temporal – espacial provacada por las partículas y antipartículas supermasivas e hiperpartículas en espacios cuánticos curvos; 2. Demostrar la dirección de atrás hacia adelante y viceversa en relación a la dimensión tiempo en sistemas cuánticos geométricamente deformados o en curvatura (flecha de tiempo bilateral); y, 3. Teorizar la configuración sistémica y morfológica de las hiperpartículas en espacios cuánticos curvos.

1. Aproximación de Markov.

$$\mathbb{P}\left(\chi_\eta, t_\eta \middle| \chi_1, t_1; \cdots \chi_{\eta-1}, t_{\eta-1}\right) = \mathbb{P}\left(\chi_\eta, t_\eta \middle| \chi_{\eta-1}, t_{\eta-1}\right)$$

$$\hat{\mathfrak{E}}(t)\big(\hat{\rho}(0)\big) = \hat{\rho}(t)$$

$$\hat{\mathfrak{E}}(t_1 + t_2) = \hat{\mathfrak{E}}(t_1)\hat{\mathfrak{E}}(t_2)$$

$$\frac{d\hat{\rho}}{dt}(t) = \hat{\mathcal{L}}\hat{\rho}(t)$$

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_\delta + \hat{\mathcal{H}}_\beta + \hat{\mathcal{H}}_{\delta\beta}$$

$$\hat{\mathcal{H}}_\delta = \frac{\hat{\mathcal{P}}^2}{2\mathcal{M}} + \mathcal{V}(\hat{\mathcal{Q}})$$

$$\hat{\mathcal{H}}_\beta^{(\kappa)} = \sum_{\kappa=1}^{\mathcal{N}} \left(\frac{\hat{\wp}_\kappa^2}{2m_\kappa} + \frac{m_\kappa \omega_\kappa^2 \hat{q}_\kappa^2}{2} \right)$$

$$\hat{\mathcal{H}}_{\delta\beta}^{(\kappa)} = \sum_{\kappa=1}^{\mathcal{N}} \left(g_\kappa \hat{q}_\kappa \hat{\mathcal{Q}} + \frac{g_\kappa^2}{2m_\kappa \omega_\kappa^2} \hat{\mathcal{Q}}^2 \right)$$

$$\frac{d^2\hat{\mathcal{Q}}}{dt^2}(t) + \mathcal{V}'\left(\hat{\mathcal{Q}}(t)\right) + \frac{1}{\mathcal{M}} \int_0^t \kappa(t - t')\,\hat{\mathcal{P}}(t')dt' + \kappa(t)\hat{\mathcal{Q}}(0) = \hat{f}(t)$$

$$\kappa(t) = \sum_{i=1}^{\mathcal{N}} \frac{g_\kappa^2}{m_\kappa \omega_\kappa^2} \cos(\omega_\kappa t)$$

$$\hat{f}(t) = \sum_{i=1}^{\mathcal{N}} \left(g_\kappa \hat{q}_\kappa(0)\cos(\omega_\kappa t) + \frac{g_\kappa \hat{\wp}_\kappa}{m_\kappa \omega_\kappa} \sin(\omega_\kappa t) \right)$$

$$\langle \hat{f}(t) \rangle = Tr\left(\hat{f}(t)\rho_{t\hbar}\right) = 0$$

$$\langle \{\hat{f}(t), \hat{f}(t')\} \rangle = \sum_{i=1}^{\mathcal{N}} \frac{\hbar g_\kappa^2}{m_\kappa \omega_\kappa} \coth\left(\frac{\hbar\omega_\kappa}{2\kappa_\beta \mathcal{T}}\right) \cos\big(\omega_\kappa (t - t')\big)$$

2. Flecha de tiempo reversible.

$$\hat{\Theta}\hat{q}\hat{\Theta}^{-1} = \hat{q}, \hat{\Theta}\hat{q}\hat{\Theta}^{-1} = -\hat{\wp}$$

$$\hat{\mathcal{A}}_\mathcal{R}(t) = \hat{\Theta}\hat{\mathcal{A}}(-t)\hat{\Theta}^{-1}$$

$$\hat{q}_\mathcal{R}(t) = \hat{\Theta}\hat{q}(-t)\hat{\Theta}^{-1} = \hat{q}(t), \hat{\wp}_\mathcal{R}(t) = \hat{\Theta}\hat{\wp}(-t)\hat{\Theta}^{-1} = \hat{\wp}(t)$$

$$\mathcal{M}\frac{d^2\hat{\mathcal{Q}}}{dt^2}(-t)+\mathcal{V}'\left(\hat{\mathcal{Q}}(-t)\right)+\frac{1}{\mathcal{M}}\int_0^{-t}\kappa(-t-t')\,\hat{\mathcal{P}}(t')dt'+\kappa(-t)\hat{\mathcal{Q}}(0)=\hat{f}(-t)$$

$$\frac{d\hat{\rho}_{\mathcal{R}}}{dt}=-\widehat{\Theta}\frac{d\hat{\rho}}{dt}\widehat{\Theta}^{-1}=-\widehat{\Theta}\hat{\mathcal{L}}\widehat{\Theta}^{-1}\widehat{\Theta}\hat{\rho}\widehat{\Theta}^{-1}=-\hat{\mathcal{L}}_{\mathcal{R}}\hat{\rho}_{\mathcal{R}}$$

$$\hat{\rho}(t)=\widehat{\mathfrak{E}}(t)\hat{\rho}(0),\widehat{\mathfrak{E}}(t)=\exp\left(-\hat{\mathcal{L}}t\right)$$

$$\widehat{\Theta}\widehat{\mathfrak{E}}(t)\widehat{\Theta}^{-1}=\exp\left(-\widehat{\Theta}\hat{\mathcal{L}}\widehat{\Theta}^{-1}t\right)=\exp\left(-\hat{\mathcal{L}}_{\mathcal{R}}t\right)$$

$$\widehat{\Theta}\widehat{\mathfrak{E}}(t)\widehat{\Theta}^{-1}=\exp\left(\hat{\mathcal{L}}t\right)=\widehat{\mathfrak{E}}(t)^{-1}$$

$$\widehat{\Theta}\hat{\mathcal{U}}(t)\widehat{\Theta}^{-1}=\hat{\mathcal{U}}(-t)$$

$$\frac{d\hat{\rho}}{dt}=\hat{\mathcal{L}}(t)\hat{\rho}(t)$$

$$\frac{d\hat{\rho}_{\mathcal{R}}}{dt}(t)=-\widehat{\Theta}\frac{d\hat{\rho}}{dt}(-t)\widehat{\Theta}^{-1}=-\hat{\mathcal{L}}_{\mathcal{R}}(-t)\hat{\rho}_{\mathcal{R}}(t)$$

$$\hat{\rho}(t)=\widehat{\mathfrak{E}}(t,0)\hat{\rho}(0),\widehat{\mathfrak{E}}(t_2,t_1)=\hat{\mathcal{T}}\exp\left(-\int_{t_1}^{t_2}\hat{\mathcal{L}}(t')dt'\right)$$

$$\widehat{\Theta}\widehat{\mathfrak{E}}(t_2,t_1)\widehat{\Theta}^{-1}=\hat{\mathcal{T}}\exp\left(-\int_{t_1}^{t_2}\hat{\mathcal{L}}(t')dt'\right)=\widehat{\mathfrak{E}}(t_2,t_1)^{-1}$$

3. Métrica de Langevin para espacios cuánticos curvos.

$$\int_0^{\infty}\kappa(t')dt'<\infty$$

$$\int_0^{\tau_\beta}\kappa(t')dt'\approx\int_0^{\infty}\kappa(t')dt'$$

$$\int_0^{t}\kappa(t-t')\hat{\mathcal{P}}(t)dt'=\int_0^{t}\kappa(t')\hat{\mathcal{P}}(t-t')dt'\approx\hat{\mathcal{P}}(t)\int_0^{\tau_\beta}\kappa(t')dt'=\hat{\mathcal{P}}(t)\int_0^{\infty}\kappa(t')dt'$$

$$\int_0^{t}\kappa(t')dt'=sgn\,(t)\hat{\mathcal{P}}(t)\int_0^{|t|}\kappa(t')dt'$$

$$\int_0^{t}\kappa(t-t')\hat{\mathcal{P}}(t)dt'\approx sgn(t)\hat{\mathcal{P}}(t)\int_0^{\infty}\kappa(t')dt'$$

$$\mathcal{M}\frac{d^2\hat{\mathcal{Q}}}{dt^2}+\mathcal{V}'\left(\hat{\mathcal{Q}}(t)\right)+sgn(t)\gamma\hat{\mathcal{P}}(t)=\hat{f}(t)$$

$$\int_0^{\infty}\kappa(t')dt'=\mathcal{M}\gamma$$

$$\langle\{\hat{f}(t),\hat{f}(t')\}\rangle = \frac{\gamma\mathcal{M}\hbar}{\pi}\int_0^{\Lambda}\omega\cot\left(\frac{\hbar\omega}{2\kappa_\beta\mathcal{T}}\right)\cos\big(\omega(t-t')\big)\,d\omega$$

$$\langle\{\hat{f}(t),\hat{f}(t')\}\rangle = 2\gamma\mathcal{M}\kappa_\beta\mathcal{T}\delta(t-t')$$

4. Simetría temporo – espacial en espacios cuánticos cutvos.

$$Tr_{\mathcal{S}} = \left(\hat{\mathcal{Y}}\hat{\mu}(t)\right) = Tr_{\mathcal{S}}\left(\hat{\rho}_{\mathcal{S}}\hat{\mathcal{Y}}(t)\right)$$

$$\dot{\hat{\mu}}(t) = -\frac{i}{\hbar}\left[\hat{\mathcal{H}}_{\mathcal{S}},\hat{\mu}(t)\right] + \frac{i}{2\hbar}\left[\{\gamma\, sgn(t)\hat{\mathcal{P}},\hat{\mu}(t)\},\hat{\mathcal{Q}}\right] + \frac{i}{2\hbar}\left[\{\hat{f}(t),\hat{\mu}(t)\},\hat{\mathcal{Q}}\right]$$

$$\hat{\rho}(t) = \langle\hat{\mu}(t)\rangle \coloneqq Tr_{\mathrm{B}}(\hat{\mu}(t)\rho_{t\hbar})$$

$$\dot{\hat{\rho}}(t) = -\frac{i}{\hbar}\left[\hat{\mathcal{H}}_{\mathcal{S}},\hat{\rho}(t)\right] + \frac{i\, sgn\,(t)}{2\hbar}\left[\{\gamma\hat{\mathcal{P}},\hat{\rho}(t)\},\hat{\mathcal{Q}}\right] + \frac{\Gamma(t)}{\hbar^2}\left[\{\hat{\rho}(t),\hat{\mathcal{Q}}\}\right]$$

$$\Gamma(t) = \int_0^t\langle\{\hat{f}(t),\hat{f}(t')\}\rangle dt'$$

$$\int_0^t\langle\{\hat{f}(t),\hat{f}(t')\}\rangle dt' = sgn(t)\int_0^{|t|}\langle\{\hat{f}(|t|),\hat{f}(t')\}\rangle dt'$$

$$\lim_{t\mapsto\infty}\int_0^{|t|}\langle\{\hat{f}(|t|),\hat{f}(t')\}\rangle dt' = 2\gamma\mathcal{M}\kappa_\beta\mathcal{T}$$

$$\dot{\hat{\rho}}(t) = -\frac{i}{\hbar}\left[\hat{\mathcal{H}}_{\mathcal{S}},\hat{\rho}(t)\right] + \frac{i\, sgn\,(t)}{2\hbar}\left[\{\gamma\hat{\mathcal{P}},\hat{\rho}(t)\},\hat{\mathcal{Q}}\right] - sgn\,(t)\frac{2\gamma\mathcal{M}\kappa_\beta\mathcal{T}}{\hbar^2}\left[\{\hat{\rho}(t),\hat{\mathcal{Q}}\},\hat{\mathcal{Q}}\right]$$

$$\frac{d\hat{\rho}}{dt} = \left(i\hat{\mathcal{L}}_{\mathcal{H}} + sgn\,(t)\hat{\mathcal{L}}_{\mathfrak{D}}\right)\hat{\rho}(t)$$

$$\hat{\mathcal{L}}_{\mathcal{H}}\hat{\rho}(t) = -\frac{1}{\hbar}\left[\hat{\mathcal{H}}_{\mathcal{S}},\hat{\rho}(t)\right]$$

$$\hat{\mathcal{L}}_{\mathfrak{D}}\hat{\rho}(t) = \frac{i}{2\hbar}\left[\{\gamma\hat{\mathcal{P}},\hat{\rho}(t)\},\hat{\mathcal{Q}}\right] - \frac{2\gamma\mathcal{M}\kappa_\beta\mathcal{T}}{\hbar^2}\left[\{\hat{\rho}(t),\hat{\mathcal{Q}}\},\hat{\mathcal{Q}}\right]$$

$$\frac{d\hat{\rho}_{\mathcal{R}}}{dt}(t) = -\hat{\Theta}\frac{d\hat{\rho}}{dt}(t)\hat{\Theta}^{-1} = \left(i\hat{\mathcal{L}}_{\mathcal{H}} + sgn\,(t)\hat{\mathcal{L}}_{\mathfrak{D}}\right)\hat{\rho}_{\mathcal{R}}(t)$$

$$\hat{\rho}(t) = \hat{\mathfrak{E}}(t)\hat{\rho}(0), \hat{\mathfrak{E}}(t) = \exp\left(i\hat{\mathcal{L}}_{\mathcal{H}}t + \hat{\mathcal{L}}_{\mathfrak{D}}|t|\right)$$

5. Entropía en espacios cuánticos curvos.

$$\rho(\wp,q,t) = \frac{1}{\sqrt{\pi\mathcal{N}(t)}}\exp\left(\frac{-(q + sgn\,(t)\mathcal{A}(t)\wp)^2}{\mathcal{N}(t)} - \mathfrak{B}(t)\wp^2\right)$$

$$\mathcal{N}(t) = \frac{m\kappa_\beta\mathcal{T}}{\hbar^2}\left(1 - e^{-2\gamma|t|}\right) + \frac{e^{-2\gamma|t|}}{\sigma^2}$$

$$\mathcal{A}(t) = \frac{i\hbar}{2\sigma^2 m\gamma}e^{-\gamma|t|}\left(1 - e^{-\gamma|t|}\right) - \frac{i\kappa_\beta\mathcal{T}}{2\hbar\gamma}\left(1 - e^{-\gamma|t|}\right)^2$$

$$\mathfrak{B}(t) = \frac{\hbar^2}{4\sigma^2 m^2 \gamma^2}\left(1 - e^{-\gamma|t|}\right)^2 + \frac{\sigma^2}{4} + \frac{\kappa_\beta \mathcal{T}}{m\gamma^2}\left(2\gamma|t| - 3 + 4e^{-\gamma|t|} - e^{-2\gamma|t|}\right)$$

$$\psi(\chi, 0) = \frac{1}{(\sigma^2\pi)^{1/4}} \exp\left(-\frac{\chi^2}{2\sigma^2}\right)$$

$$\mathcal{S}_{\nu\mathcal{N}}(\xi) = \frac{1-\xi}{2\xi}\log\left(\frac{1+\xi}{1-\xi}\right) - \log\left(\frac{2\xi}{1+\xi}\right)$$

6. Simetría Lindblad en espacios cuánticos curvos.

$$\widehat{\mathcal{H}} = \widehat{\mathcal{H}}_\delta + \widehat{\mathcal{H}}_\beta + \widehat{\mathcal{H}}_{\delta\beta}$$

$$\widehat{\mathcal{H}}_\mathcal{J}(t) = e^{\frac{i}{\hbar}(\widehat{\mathcal{H}}_\delta + \widehat{\mathcal{H}}_\beta)t}\,\widehat{\mathcal{H}}_{\delta\beta}\,e^{\frac{-i}{\hbar}(\widehat{\mathcal{H}}_\delta + \widehat{\mathcal{H}}_\beta)t}$$

$$\frac{d}{dt}\hat{\rho}_\mathcal{J}(t) = -\frac{i}{\hbar}\left[\widehat{\mathcal{H}}_\mathcal{J}(t), \hat{\rho}_\mathcal{J}(t)\right]$$

$$\frac{d}{dt}\hat{\rho}_\mathcal{S}(t) = \frac{1}{\hbar^2}\int_0^t Tr_\mathfrak{B}\left[\widehat{\mathcal{H}}_\mathcal{J}(t), \left[\widehat{\mathcal{H}}_\mathcal{J}(s), \hat{\rho}_\mathcal{J}(s)\right]\right] ds$$

$$\frac{d}{dt}\hat{\rho}_\mathcal{S}(t) = -\frac{1}{\hbar^2}\int_0^t Tr_\mathfrak{B}\left[\widehat{\mathcal{H}}_\mathcal{J}(t), \left[\widehat{\mathcal{H}}_\mathcal{J}(s), \hat{\rho}_\mathcal{J}(s) \bigotimes \hat{\rho}_\mathfrak{B}\right]\right] ds$$

$$\widehat{\mathcal{H}}_\mathcal{J}(t) = \sum_{\alpha,\omega} \hat{\mathcal{A}}_\alpha(\omega) \bigotimes \widehat{\mathcal{B}}_\alpha(t)$$

$$\frac{d}{dt}\hat{\rho}_\mathcal{S}(t) = -\frac{1}{\hbar^2}\sum_{\alpha,\omega,\beta}\Big[\Gamma_{\alpha\beta}(\omega,t)\widehat{\mathcal{A}}_\alpha^\dagger(\omega)\widehat{\mathcal{A}}_\beta(\omega)\hat{\rho}_\mathcal{S}(t)$$
$$+ \Gamma_{\beta\alpha}^*(\omega,t)\hat{\rho}_\mathcal{S}(t)\widehat{\mathcal{A}}_\alpha^\dagger(\omega)\widehat{\mathcal{A}}_\beta(\omega)\left(\Gamma_{\alpha\beta}(\omega,t) + \Gamma_{\beta\alpha}^*(\omega,t)\right)\widehat{\mathcal{A}}_\beta(\omega)\hat{\rho}_\mathcal{S}(t)\widehat{\mathcal{A}}_\alpha^\dagger(\omega)\Big]$$

$$\Gamma_{\alpha\beta}(\omega,t) = \int_0^t e^{i\omega s}\, Tr\left(\widehat{\mathcal{B}}_\alpha^\dagger(t)\widehat{\mathcal{B}}_\beta^\dagger(t-s)\hat{\rho}_\mathcal{B}\right) ds$$

$$\Gamma_{\alpha\beta}(\omega,t) + \Gamma_{\beta\alpha}^*(\omega,t) = sgn\,(t)\int_{-|t|}^{|t|} e^{i\omega s} Tr\left(\widehat{\mathcal{B}}_\alpha^\dagger(s)\widehat{\mathcal{B}}_\beta^\dagger(0)\hat{\rho}_\mathcal{B}\right) ds$$

$$\Gamma_{\alpha\beta}(\omega,t) + \Gamma_{\beta\alpha}^*(\omega,t) \approx sgn\,(t)\gamma_{\alpha\beta}(\omega)$$

$$\gamma_{\alpha\beta}(\omega) = \int_{-\infty}^{\infty} e^{i\omega s} Tr\left(\widehat{\mathcal{B}}_\alpha^\dagger(s)\widehat{\mathcal{B}}_\beta^\dagger(0)\hat{\rho}_\mathcal{B}\right) ds$$

$$\eta_{\alpha\beta}(\omega) = \frac{1}{2i}\left(\Gamma_{\alpha\beta}(\omega,t) - \Gamma_{\beta\alpha}^*(\omega,t)\right)$$

$$\frac{d}{dt}\hat{\rho}_\mathcal{S}(t) = -\frac{i}{\hbar}\left[\widehat{\mathcal{H}}_\delta, \hat{\rho}_\mathcal{S}\right] + \frac{sgn\,(t)}{\hbar^2}\sum_{\alpha,\beta,\omega}\widehat{\mathfrak{D}}_{\alpha\beta}\,(\omega)\hat{\rho}_\mathcal{S}(t)$$

$$\hat{\mathcal{H}}_{\delta} = \frac{1}{\hbar}\sum_{\alpha,\beta,\omega} \eta_{\alpha\beta}(\omega)\hat{\mathcal{A}}_{\alpha}^{\dagger}(\omega)\hat{\mathcal{A}}_{\beta}(\omega)$$

$$\hat{\mathfrak{D}}_{\alpha\beta}(\omega)\hat{\rho}_{\mathcal{S}}(t) = \gamma_{\alpha\beta}(\omega)\left(\hat{\mathcal{A}}_{\beta}(\omega)\hat{\rho}_{\mathcal{S}}(t)\hat{\mathcal{A}}_{\alpha}^{\dagger}(\omega) - \frac{1}{2}\{\hat{\mathcal{A}}_{\alpha}^{\dagger}(\omega)\hat{\mathcal{A}}_{\beta}(\omega), \hat{\rho}_{\mathcal{S}}(t)\}\right)$$

$$\hat{\Theta}\gamma_{\alpha\beta}(\omega)\hat{\Theta}^{-1} = \int_{-\infty}^{\infty} e^{i\omega s} Tr\left(\hat{\Theta}\hat{\mathcal{B}}_{\alpha}^{\dagger}(s)\hat{\mathcal{B}}_{\beta}^{\dagger}(0)\hat{\Theta}^{-1}\hat{\rho}_{\mathcal{B}}\right) ds = \gamma_{\alpha\beta}(\omega)$$

7. Simetría de Pauli en espacios cuánticos curvos.

$$\frac{d}{dt}\hat{\rho}_{\eta}(t) = \sum_{\eta' \neq \eta}\left(\mathcal{W}_{\eta,\eta'}\hat{\rho}_{\eta'}(t) - \mathcal{W}_{\eta',\eta}\hat{\rho}_{\eta}(t)\right)$$

$$\langle \eta | \hat{\mathcal{A}}_{\alpha}^{\dagger}(\omega)\hat{\mathcal{A}}_{\beta}(\omega) | \eta' \rangle = \delta_{\eta,\eta'} \langle \eta | \hat{\mathcal{A}}_{\alpha} | m \rangle \langle m | \hat{\mathcal{A}}_{\beta} | \eta \rangle$$

$$\frac{d}{dt}\rho_{\eta}(t) = \frac{sgn\,(t)}{\hbar^2}\sum_{\eta' \neq \eta}\left(\mathcal{W}_{\eta,\eta'}\hat{\rho}_{\eta'}(t) - \mathcal{W}_{\eta',\eta}\hat{\rho}_{\eta}(t)\right)$$

$$\mathcal{W}_{\eta',\eta} = \sum_{\alpha,\beta} \gamma_{\alpha\beta}(\varepsilon_{\eta} - \varepsilon_{\eta'})\langle \eta | \hat{\mathcal{A}}_{\alpha} | \eta' \rangle \langle \eta' | \hat{\mathcal{A}}_{\beta} | \eta \rangle$$

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \lambda\hat{\mathcal{H}}_{\delta\beta}$$

$$\frac{d}{dt}\rho_{\varepsilon,\kappa}(t) = \frac{\lambda^2}{\hbar^2}\sum_{\varepsilon',\kappa'}\int_0^t \Lambda_{\varepsilon,\kappa,\varepsilon',\kappa'}(t-s)\left(\rho_{\varepsilon',\kappa'}(s) - \rho_{\varepsilon,\kappa}(s)\right) ds$$

$$\Lambda_{\varepsilon,\kappa,\varepsilon',\kappa'}(t) = 2\left|\langle \varepsilon,\kappa | \hat{\mathcal{H}}_{\delta\beta} | \varepsilon',\kappa' \rangle\right|^2 \cos\left(\frac{\varepsilon - \varepsilon'}{\hbar}t\right)$$

$$\int_0^t \Lambda_{\varepsilon,\kappa,\varepsilon',\kappa'}(t-s)ds = 2\hbar\frac{\left|\langle \varepsilon,\kappa | \hat{\mathcal{H}}_{\delta\beta} | \varepsilon',\kappa' \rangle\right|^2}{\varepsilon - \varepsilon'}\sin\left(\frac{\varepsilon - \varepsilon'}{\hbar}t\right)$$

$$\frac{\hbar}{\varepsilon - \varepsilon'}\sin\left(\frac{\varepsilon - \varepsilon'}{\hbar}t\right) \approx sgn\,(t)\pi\hbar\delta(\varepsilon - \varepsilon')$$

$$\frac{d}{dt}\rho_{\varepsilon,\kappa}(t) = sgn\,(t)\sum_{\kappa'}\left(\mathcal{W}_{\kappa,\kappa'}^{\varepsilon}\rho_{\varepsilon,\kappa'}(t) - \mathcal{W}_{\kappa',\kappa}^{\varepsilon}\rho_{\varepsilon,\kappa}(t)\right)$$

$$\mathcal{W}_{\kappa,\kappa'}^{\varepsilon} = \frac{2\lambda^2}{\hbar}\left|\langle \varepsilon,\kappa | \hat{\mathcal{H}}_{\delta\beta} | \varepsilon,\kappa' \rangle\right|^2 \eta(\zeta)$$

8. Simetría de quiebre en espacios cuánticos curvos.

$$\psi_{\mathcal{R}}(\chi, \alpha + t) = \psi^{*}(\chi, \alpha - t)$$

$$\frac{d\mathcal{W}}{dt} = -\frac{\wp}{\mathcal{M}}\frac{d\mathcal{W}}{d\chi} + \sum_{\eta=0}^{\infty}\frac{(-\hbar^2)^{\eta}\mathcal{V}^{(2\eta+1)}(\chi)}{(2\eta+1)!\,2^{2\eta}}\frac{d^{2\eta+1}\mathcal{W}}{d\wp^{2\eta+1}} + sgn\,(t)\gamma\frac{d}{d\wp}(\wp\mathcal{W}) + sgn\,(t)2\gamma\mathcal{M}\kappa_{\beta}\mathcal{T}\frac{d^2\mathcal{W}}{d\wp^2}$$

$$\frac{d\mathcal{W}}{dt} = -\frac{\wp}{\mathcal{M}}\frac{d\mathcal{W}}{d\chi} + \frac{d\mathcal{V}}{d\chi}\frac{d\mathcal{W}}{d\wp} + sgn\,(t)\gamma\frac{d}{d\wp}(\wp\mathcal{W}) + sgn\,(t)2\gamma\mathcal{M}\kappa_{\beta}\mathcal{T}\frac{d^2\mathcal{W}}{d\wp^2}$$

9. Expansión por deformación en espacios cuánticos curvos.

$$\dot{\hat{\mu}}(t) = -\frac{i}{\hbar}\left[\hat{\mathcal{H}}_\delta, \hat{\mu}(t)\right] + \frac{i}{2\hbar}\left[\{\gamma\, sgn\,(t)\hat{\mathcal{P}}, \hat{\mu}(t)\}, \hat{\mathcal{Q}}\right]$$

$$\hat{\rho}(t) = \langle\hat{\mu}(t)\rangle \coloneqq Tr_{\mathrm{B}}(\hat{\mu}(t)\hat{\rho}_{t\hbar})$$

$$\hat{\mathcal{A}}\hat{\mu}(t) = -\frac{i}{\hbar}\left[\hat{\mathcal{H}}_\delta, \hat{\mu}(t)\right] + \frac{i}{2\mathcal{M}\hbar}\left[\{\gamma\, sgn\,(t)\hat{\mathcal{P}}, \hat{\mu}(t)\}, \hat{\mathcal{Q}}\right]$$

$$\hat{\mathfrak{B}}\hat{\mu}(t) = \frac{i}{\hbar}\left[\hat{\mu}(t), \hat{\mathcal{Q}}\right]$$

$$\hat{\alpha}(t)\hat{\mu}(t) = \frac{1}{2}\left\{\hat{f}(t), \hat{\mu}(t)\right\}$$

$$\hat{\eta}(t) = \exp\left(-\hat{\mathcal{A}}t\right)\hat{\mu}(t)$$

$$\dot{\hat{\eta}}(t) = \hat{\mathfrak{B}}(t)\hat{\alpha}(t)\hat{\eta}(t)$$

$$\frac{d}{dt}\langle\hat{\eta}(t)\rangle = \hat{\mathfrak{B}}(t)\langle\hat{\alpha}(t)\hat{\mu}(0)\rangle + \int_0^t \hat{\mathfrak{B}}(t)\hat{\mathfrak{B}}(t')\langle\hat{\alpha}(t)\hat{\alpha}(t')\hat{\mu}(t')\rangle dt'$$

$$\frac{d\hat{\wp}}{dt}(t) = \hat{\mathcal{A}}\hat{\rho}(t) + \int_0^t \langle\hat{\alpha}(t)\hat{\alpha}(t')\hat{\mathfrak{J}}\rangle\hat{\mathfrak{B}}\hat{\mathfrak{B}}\,(t' - t)\hat{\rho}(t')dt'$$

$$\frac{d\hat{\wp}}{dt}(t) = \hat{\mathcal{A}}\hat{\rho}(t) - \int_0^t \langle\hat{\alpha}(t)\hat{\alpha}(t')\hat{\mathfrak{J}}\rangle dt'\frac{1}{\hbar}\left[\left[\hat{\rho}(t), \hat{\mathcal{Q}}\right]\hat{\mathcal{Q}}\right]$$

$$\dot{\hat{\rho}}(t) = -\frac{i}{\hbar}\left[\hat{\mathcal{H}}_\delta, \hat{\rho}(t)\right] + \frac{i}{2\hbar}\left[\{\gamma\, sgn\,(t)\hat{\mathcal{P}}, \hat{\rho}(t)\}, \hat{\mathcal{Q}}\right] - \frac{\Gamma(t)}{\hbar^2}\left[\left[\hat{\rho}(t), \hat{\mathcal{Q}}\right]\hat{\mathcal{Q}}\right]$$

Para aclarar:

1. Las partículas supermasivas y antipartículas supermasivas, son aquellas cuya masa es superior y en consecuencia, a propósito de su colisión, entrelazamiento, superposición o ultramasificación, provocan agujeros negros cuánticos.

2. Las partículas masivas y antipartículas masivas, son aquellas, cuya masa suficiente, deforma el tejido del espacio – tiempo cuántico, sin que necesariamente provoque un agujero negro cuántico, a propósito de su colisión, entrelazamiento, superposición o ultramasificación.

3. Las hiperpartículas, son aquellas que se aproximan, alcanzan o superan la velocidad de la luz, en cuyo primer y segundo casos, deforman el tejido del espacio – tiempo cuántico, y en cuyo tercer caso, puede provocar agujeros negros cuánticos.

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Apéndice H.

Supersimetría de Yang – Mills, supermembranas, supergravedad cuántica, superconductividad, dualidad holográfica y agujeros negros cuánticos para espacios cuánticos curvos (campos cuánticos relativistas).

1. Formalización matemática.

$$S_{\text{open}} = S_{\text{brane}} + S_{\text{bulk}} + S_{\text{int}} \approx S_{\mathcal{N}=4} + S_{\text{bulk - flat}} + S_{\text{int}}, g_s N \ll 1$$

$$ds^2 = H(r)^{-1/2}(-dt^2 + dx_3^2) + H(r)^{1/2}(dr^2 + r^2 d\Omega_5^2)$$

$$H(r) = 1 + \frac{NG_N}{r^4}, G_N = g_s\alpha'$$

$$S_{\text{close}} = S_{\text{bulk - throat}} + S_{\text{bulk - flat}} + S_{\text{int}}, g_s N \gg 1.$$

$$S_{\mathcal{N}=4} = S_{\text{strings } AdS_5\times S^5}$$

$$g_{YM}^2 = g_s\left(\frac{R}{l_s}\right)^4 = 4\pi g_{YM}^2 N = 4\pi\lambda$$

$$x_i \to \Lambda x_i : x_i \to \lambda x_i, r \to r - R\log\lambda$$

$$L_{CFT} + \int d^4x h\mathcal{O}$$

$$e^{-W(h)} = \left\langle e^{-\int h\mathcal{O}}\right\rangle_{QFT}$$

$$\langle \mathcal{O} \dots \mathcal{O}\rangle_c = (-1)^n \left[\frac{\delta^n W}{\delta h^n}\right]_{h=0}$$

$$e^{W(h)} = \left\langle e^{\int h\mathcal{O}}\right\rangle_{QFT} = \mathcal{Z}_{\text{strings at } AdS}[h(x, z = \text{edge}) = h_0]$$

$$\mathcal{Z}_{\text{strings at AdS}}[h(x, z = \text{edge})] \sim e^{S_{\text{supergravity}}[h_0]}, \lambda \gg 1$$

$$L_{CFT} + \int d^4x\sqrt{g}\left(g_{\mu\nu}T^{\mu\nu} + J_\mu A^\mu + \phi\text{Tr}(F_{\mu\nu}F^{\mu\nu}) + \cdots\right)$$

$$ds^2 = R^2\frac{dx^\mu dx_\mu + dz^2}{z^2} \equiv g_{AB}dx^A dx^B, A = 0, \dots, d, x^A = (z, x^\mu)$$

$$S \sim \int d^{d+1}x\sqrt{g}(g^{AB}\partial_A\phi\partial_B\phi + m^2\phi^2)$$

$$S \sim \int_{\partial AdS} d^d x\sqrt{g}g^{zB}\phi\partial_B\phi + \int d^{d+1}x\sqrt{g}\phi(-\Box + m^2)\phi$$

$$\phi(z, x^\mu) = e^{ik_\mu x^\mu}f_k(z), k_\mu x^\mu = -\omega t + \vec{k}\cdot\vec{x}$$

$$0 = \frac{1}{R^2}\left(z^2k^2 - z^{d+1}\partial_z\left(z^{-d+1}\partial_z\right) + m^2R^2\right)f_k(z), k^2 = \vec{k}^2 - \omega^2$$

$$m^2R^2 = \Delta(\Delta - d)$$

$$\phi \sim \phi_+ z^{\Delta_+} + \phi_- z^{\Delta_-}$$

$$\int dz\sqrt{g}|\phi|^2 < \infty$$

$$\phi(z, x^\mu)|_{z=\epsilon} \to \epsilon^{\Delta_-}\phi_-^{ren}(x^\mu)$$

$$S_{\text{edge}} \ni \int_{z=\epsilon} d^d x\sqrt{g_\epsilon}\phi(x,\epsilon)\mathcal{O}(x,\epsilon) = \int d^d x\left(\frac{R}{\epsilon}\right)^d \epsilon^{\Delta_-}\phi_-^{ren}(x)\mathcal{O}(x,\epsilon)$$

$$\mathcal{O}(x,\epsilon) \sim \epsilon^{d-\Delta_-}\mathcal{O}^{ren}(x) = \epsilon^{\Delta_+}\mathcal{O}^{ren}(x)$$

$$f_k(z) = A(k)z^{d/2}K_\nu(kz) + B(k)z^{d/2}I_\nu(kz)$$

$$f_k(z) = \frac{z^{d/2}K_\nu(kz)}{\epsilon^{d/2}K_\nu(k\epsilon)}$$

$$\phi(x, z) = \int d^d k e^{ikx} f_k(z)\phi(k,\epsilon)$$

$$S(\phi) = 2R^{d-1}\int d^d k\phi(k,\epsilon)\phi(-k,\epsilon)\epsilon^{-d+1}\partial_z\left(\frac{z^{d/2}K_\nu(kz)}{\epsilon^{d/2}K_\nu(k\epsilon)}\right)_{z=\epsilon}$$

$$-\frac{(-1)^{\nu-1}2^{1-2\nu}}{\Gamma(\nu)^2}k^{2\nu}\ln\, k\epsilon.\epsilon^{2\nu-d}+\cdots$$

$$\langle\mathcal{O}(x_1)\mathcal{O}(x_2)\rangle=-\frac{\delta}{\delta\phi_-^{ren}(x_1)}\frac{\delta}{\delta\phi_-^{ren}(x_2)}S=\frac{2\Gamma(\Delta_+)}{\pi^{d/2}\Gamma(\Delta_+-d/2)}\frac{1}{(x_1-x_2)^{2\Delta_+}}$$

$$z^{d/2}K_{\pm\nu}(iqz)\sim e^{\pm-iqz}$$

$$U_{\mathcal{C}}(y,z)=\mathcal{P}e^{i\int_{\mathcal{C}}dx^\mu A_\mu^a T^a}$$

$$\varphi_{\mathcal{C}}(y)=U_{\mathcal{C}}(y,z)\varphi(z)$$

$$W_{\mathcal{C}}(\mathcal{R})=\mathrm{Tr}\mathcal{P}e^{i\oiiint_{\mathcal{C}}\ dx^\mu A_\mu^a T_{\mathcal{R}}^a}$$

$$\langle W_{\mathcal{C}}(\square)\rangle\sim e^{-TV(l)}$$

$$W_{\mathcal{C},\mathcal{C}_{int}}(\square)=\mathrm{Tr}\mathcal{P}e^{\oint\ (iA_\mu\dot{x}^\mu+\theta^I X^I(x^\mu)\sqrt{\dot{x}^2})d\tau}$$

$$\langle W_{\mathcal{C},\mathcal{C}_{\mathrm{int}}}(\square)\rangle=\mathcal{Z}_{\mathrm{string}}\left[\mathcal{C},\mathcal{C}_{\mathrm{int}},\square\right]$$

$$\mathcal{Z}_{\mathrm{string}}\left[\mathcal{C},\mathcal{C}_{\mathrm{int}},\square\right]\cong e^{-S_{\mathrm{string}}^{\mathrm{on\text{-}shell}}\left[\mathcal{C},\mathcal{C}_{\mathrm{int}}\right]}$$

$$\tau=t,\sigma=x,z=z(x)$$

$$ds^2=\frac{R^2}{z^2}\left[-dt^2+dx^2(1+z'^2)\right]=\gamma_{ab}dx^a dx^b$$

$$S_{NG}=\frac{1}{2\pi\alpha'}\int_{-T/2}^{T/2}dt\int_{-l/2}^{l/2}dx\sqrt{-\det(\gamma)}=\sqrt{4\pi\lambda}T\int_{-l/2}^{l/2}dx\frac{\sqrt{1+z'^2}}{z^2}$$

$$\mathcal{H}=\mathcal{L}-z'\frac{\partial\mathcal{L}}{\partial z'}=\frac{1}{z^2\sqrt{1+z'^2}}=\frac{1}{z_{\max}^2}$$

$$z'^2+V_{ef}(z)=0,V_{ef}(z)=1-\left(\frac{z_{\max}}{z}\right)^4$$

$$l=\int_{-l/2}^{l/2}dx=2\int_0^{z_{\max}}dz\frac{dx}{dz}=2z_{\max}\int_0^1 dy\frac{y^2}{\sqrt{1-y^4}}=2z_{\max}\frac{\sqrt{2\pi^3}}{\Gamma^2(1/4)}$$

$$S_{NG}=2\sqrt{4\pi\lambda}T\int_\epsilon^{z_{\max}}dz\frac{z_{\max}^2}{z^2\sqrt{z_{\max}^4-z^4}}$$

$$m_q=\sqrt{4\pi\lambda}T\int_\epsilon^{z_{\max}}\frac{dz}{z^2}$$

$$\ln\langle W_{\mathcal{C}}(\square)\rangle=S_{NG}-2m_q=-\frac{2\sqrt{4\pi\lambda}T}{z_{\max}}\int_{\epsilon/z_{\max}}^1\frac{dy}{y^2}\left(\frac{1}{\sqrt{1-y^4}}-1\right)$$

$$V(l)=-\frac{\sqrt{\lambda}}{l}\frac{4\pi^2}{\Gamma^4(1/4)}$$

$$ds^2=\frac{1}{z^2}\left(dr^2+r^2d\phi^2+dz^2+dx_i^2\right)$$

$$z=\sqrt{a^2-r^2},0\leq r\leq a,0\leq\phi\leq 2\pi$$

$$\ln\langle W_{\mathcal{C}}(\square)\rangle=-\sqrt{\lambda}$$

$$\ln\langle W_{\mathcal{C}}(S)\rangle=2N\left(\kappa\sqrt{1+\kappa^2}+\sinh^{-1}\ \kappa\right)$$

$$\ln\langle W_{\mathcal{C}}(A)\rangle=-\frac{2N\sqrt{\lambda}}{3\pi}\sin^3\ \theta_k$$

$$\mathcal{Z}_{\mathrm{gravity}}\left[\phi(\phi_0)\right]=\left\langle e^{i\int_{\partial\mathcal{M}}d^d\mathbf{x}\phi_0(\mathbf{x})\mathcal{O}(\mathbf{x})}\right\rangle$$

$$ds^2=\frac{R^2}{z^2}(d\mathbf{x}^2+dz^2)$$

$$\phi(\mathbf{x},z)=\int_{\partial\mathcal{M}}d\mathbf{y}\ \mathrm{K}(\mathbf{x},z\mid\mathbf{y})\phi_0(\mathbf{y})$$

$$\phi(\mathbf{x},z)\sim z^{\Delta_\pm}\phi_0(\mathbf{x}),z\to 0$$

$$\Delta_{\pm} = \frac{d}{2} \pm \mu, \mu = \sqrt{\frac{d^2}{4} + m^2 R^2}$$
$$(\Box - m^2)\mathrm{K}(\mathbf{x}, z \mid \mathbf{y}) = 0$$
$$\mathrm{K}(\mathbf{x}, z \mid \mathbf{y}) \sim z^{\Delta_-}\delta(\mathbf{x} - \mathbf{y}), z \to 0$$
$$\mathrm{K}(\mathbf{x}, z \mid \mathbf{y}) = \lim_{z' \to 0} \sqrt{-g} g^{z'z'} \partial_{z'} \mathrm{G}(\mathbf{x}, z \mid \mathbf{y}, z')$$
$$\phi(\mathbf{x}, z) = \int_{\partial\mathcal{M}} d\mathbf{y}\ \mathrm{K}(\mathbf{x}, z \mid \mathbf{y})\phi_0(\mathbf{y}) + \varphi(\mathbf{x}, z)$$
$$S = -\frac{1}{2}\int d\mathbf{x}dz\sqrt{-g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right)$$
$$S[\phi_0] = \frac{1}{2}\int d\mathbf{x}\left[\sqrt{-g}g^{zz}\phi(\mathbf{x}, z)\partial_z\phi(\mathbf{x}, z)\right]_{z=0}$$
$$S[\phi_0] = \frac{1}{2}\int d\mathbf{y}d\mathbf{y}'\phi_0(\mathbf{y})\Delta(\mathbf{y}, \mathbf{y}')\phi_0(\mathbf{y}')$$
$$\Delta(\mathbf{y}, \mathbf{y}') = \int d\mathbf{x}\left[\sqrt{-g}g^{zz}\ \mathrm{K}(\mathbf{x}, z \mid \mathbf{y})\partial_z\ \mathrm{K}(\mathbf{x}, z \mid \mathbf{y}')\right]_{z=0}$$
$$\Delta(\mathbf{y}, \mathbf{y}') \sim \left[\sqrt{-g}g^{zz}\partial_z\ \mathrm{K}(\mathbf{y}, z \mid \mathbf{y}')\right]_{z=0} \sim \lim_{z,z' \to 0}\left(\sqrt{-g}g^{zz}\right)\left(\sqrt{-g}g^{z'z'}\right)\frac{\partial^2}{\partial z\partial z'}\mathrm{G}(\mathbf{y}, z \mid \mathbf{y}', z')$$
$$\langle\psi_{\mathrm{f}}|\mathcal{O}(\mathbf{y})\mathcal{O}(\mathbf{y}')|\psi_{\mathrm{i}}\rangle = -i\frac{\delta^2 S[\phi_0]}{\delta\phi_0(\mathbf{y})\delta\phi_0(\mathbf{y}')} = -i\Delta^{\mathrm{i,f}}(\mathbf{y}, \mathbf{y}')$$
$$ds^2 = R^2\left[-\frac{dt^2}{1 - x^2} + \frac{dx^2}{(1 - x^2)^2} + \frac{x^2}{1 - x^2}d\Omega_{d-1}^2\right]$$
$$\lim_{x \to x_\epsilon}\mathrm{K}(t, \Omega, x \mid t', \Omega', x_\epsilon) = \frac{\delta(t - t')\delta(\Omega - \Omega')}{\sqrt{g_\Omega}}.$$
$$\mathrm{K}(t, \Omega, x \mid t', \Omega', x_\epsilon) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\sum_{lm} e^{-i\omega(t-t')}Y_{lm}(\Omega)Y_{lm}^*(\Omega')f_{l\omega}(x)$$
$$(1 - x^2)\frac{d^2 f_{l\omega}}{dx^2} + \frac{d - 1 - x^2}{x}\frac{df_{l\omega}}{dx} + \left(\omega^2 - \frac{q^2}{x^2} - \frac{m^2R^2}{1 - x^2}\right)f_{l\omega} = 0$$
$$f_{l\omega}(x) = \left(\frac{x^{-\frac{d}{2}+\nu+1}(1 - x^2)^{\frac{1}{2}\Delta_+}}{(1 - \epsilon)^{-\frac{d}{2}+\nu+1}((2 - \epsilon)\epsilon)^{\frac{1}{2}\Delta_+}}\right)\frac{{}_2F_1\left(\frac{1}{2}(\mu + \nu - \omega + 1), \frac{1}{2}(\mu + \nu + \omega + 1); \nu + 1; x^2\right)}{{}_2F_1\left(\frac{1}{2}(\mu + \nu - \omega + 1), \frac{1}{2}(\mu + \nu + \omega + 1); \nu + 1; (1 - \epsilon)^2\right)}$$
$$f_{l\omega}(x) \sim C_+(1 - x)^{\frac{1}{2}\Delta_+} + C_-(1 - x)^{\frac{1}{2}\Delta_-}$$
$$\omega_{nl} = \pm(2n + \nu + \mu + 1)$$
$$= \pm(2n + l + \Delta_+), n, l = 0,1,2 \ldots$$
$$\phi(t, \Omega, x) = \int dt'd\Omega'\sqrt{g_{\Omega'}}\mathrm{K}(t, \Omega, x \mid t', \Omega', x_\epsilon)\phi_0(t', \Omega')$$
$$\Delta_{\mathrm{reg}}(t, \Omega \mid t', \Omega') = -\frac{1}{\sqrt{g_\Omega}}\left[\sqrt{-g}g^{xx}\partial_x\ \mathrm{K}(t, \Omega, x \mid t', \Omega', x_\epsilon)\right]_{x=x_\epsilon}$$
$$= -\int\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\sum_{lm} Y_{lm}(\Omega)Y_{lm}^*(\Omega')\left[\frac{x^{d-1}}{(1 - x^2)^{\frac{d-2}{2}}}\partial_x f_{l\omega}(x)\right]_{x=x_\epsilon}$$
$$= -\sum_{lm} Y_{lm}(\Omega)Y_{lm}^*(\Omega')\int\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\left[\frac{x^{d-1}}{(1 - x^2)^{\frac{d-2}{2}}}\partial_x f_{l\omega}(x)\right]_{x=x_\epsilon}$$
$$\phi_0(t, \Omega) = \epsilon^{\frac{1}{2}\Delta_-}\phi_{\mathrm{ren}}(t, \Omega)$$

$$\Delta_{\text{ren}}(t,\Omega \mid t',\Omega') \equiv \lim_{\epsilon\to 0}\epsilon^{\Delta_-}\Delta_{\text{reg}}(t,\Omega \mid t',\Omega')$$

$$= \sum_{lm} Y_{lm}(\Omega)Y^*_{lm}(\Omega') \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')}$$

$$\times \frac{\Delta_+}{2^{\Delta_-}} \frac{\Gamma(1-\mu)}{\Gamma(1+\mu)} \frac{\Gamma\left(\frac{1}{2}(-\omega+\nu+\mu+1)\right)\Gamma\left(\frac{1}{2}(\omega+\nu+\mu+1)\right)}{\Gamma\left(\frac{1}{2}(-\omega+\nu-\mu+1)\right)\Gamma\left(\frac{1}{2}(\omega+\nu-\mu+1)\right)}$$

$$\Delta^F_{\text{ren}}(t,\Omega \mid t',\Omega') = 2i\frac{\Delta_+\Gamma(1-\mu)}{2^{\Delta_-}\Gamma(1+\mu)}\sum_{lm} Y_{lm}(\Omega)Y^*_{lm}(\Omega')$$

$$\times \left[\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{\Gamma(n+l+\Delta_+)}{\Gamma\left(n+l+\frac{d}{2}\right)\Gamma(-(n+\mu))} e^{-i|t-t'|(2n+l+\Delta_+)}\right]$$

$$\langle 0|T\mathcal{O}(t,\Omega)\mathcal{O}(t',\Omega')|0\rangle = -i\Delta^F_{\text{ren}}(t,\Omega \mid t',\Omega') = -\frac{2\Delta_+}{2^{\Delta_-}\Gamma(\mu)}\sum_{lm} Y_{lm}(\Omega)Y^*_{lm}(\Omega')\frac{\Gamma(l+\Delta_+)}{\Gamma\left(l+\frac{d}{2}\right)}$$

$$\times e^{-i|t-t'|(l+\Delta_+)} {}_2F_1\left(1+\mu, l+\Delta_+, l+\frac{d}{2}; e^{-2i|t-t'|}\right)$$

$$\phi(\mathbf{y},x) = \int d\mathbf{y}'\mathbf{K}^i(\mathbf{y},x \mid \mathbf{y}')\phi^i_0(\mathbf{y}')$$

$$= \int d\mathbf{y}'[\mathrm{K}^+(\mathbf{y},x \mid \mathbf{y}')\phi^+_0(\mathbf{y}') + \mathrm{K}^-(\mathbf{y},x \mid \mathbf{y}')\phi^-_0(\mathbf{y}')]$$

$$\mathrm{K}^+(\mathbf{y},x \mid \mathbf{y}')|_{x=x_+} = \delta(\mathbf{y}-\mathbf{y}'),$$

$$\mathrm{K}^+(\mathbf{y},x \mid \mathbf{y}')|_{x=x_-} = 0$$

$$\mathrm{K}^-(\mathbf{y},x \mid \mathbf{y}')|_{x=x_-} = \delta(\mathbf{y}-\mathbf{y}'),$$

$$\mathrm{K}^-(\mathbf{y},x \mid \mathbf{y}')|_{x=x_+} = 0$$

$$S = -\frac{1}{2}\int d\mathbf{y}\left(\left[\sqrt{-g}g^{xx}\phi(\mathbf{y},x)\partial_x\phi(\mathbf{y},x)\right]_{x=x_+} - \left[\sqrt{-g}g^{xx}\phi(\mathbf{y},x)\partial_x\phi(\mathbf{y},x)\right]_{x=x_-}\right)$$

$$S[\phi_0] = -\frac{1}{2}\int d\mathbf{y}d\mathbf{y}'\phi^i_0(\mathbf{y})\Delta_{ij}(\mathbf{y},\mathbf{y}')\phi^j_0(\mathbf{y}')$$

$$\Delta_{+i}(\mathbf{y},\mathbf{y}') = \left[\sqrt{-g}g^{xx}\partial_x\mathbf{K}^i(\mathbf{y},x \mid \mathbf{y}')\right]_{x=x_+}, \Delta_{-i}(\mathbf{y},\mathbf{y}') = -\left[\sqrt{-g}g^{xx}\partial_x\mathbf{K}^i(\mathbf{y},x \mid \mathbf{y}')\right]_{x=x_-}$$

$$\langle\psi_{\text{f}}|\mathcal{O}^{\pm}(\mathbf{y})\mathcal{O}^{\pm}(\mathbf{y}')|\psi_{\text{i}}\rangle \sim -i\Delta_{\pm\pm}(\mathbf{y},\mathbf{y}')$$

$$\langle\psi_{\text{f}}|\mathcal{O}^{\pm}(\mathbf{y})\mathcal{O}^{\mp}(\mathbf{y}')|\psi_{\text{i}}\rangle \sim -i\Delta_{\pm\mp}(\mathbf{y},\mathbf{y}')$$

$$\mathrm{K}^i(\mathbf{y},x \mid \mathbf{y}') = \lim_{x'\to x^i}\sqrt{-g}g^{x'x'}\partial_{x'}\mathrm{G}(\mathbf{y},x \mid \mathbf{y}',x')$$

$$\Delta_{ij}(\mathbf{y},\mathbf{y}') \sim \lim_{x\to x^i, x'\to x^j}\left(\sqrt{-g}g^{xx}\right)\left(\sqrt{-g}g^{x'x'}\right)\frac{\partial^2}{\partial_x\partial_{x'}}\mathrm{G}(\mathbf{y},x \mid \mathbf{y}',x')$$

$$ds^2 = R^2\left[-\frac{dt^2}{1-x^2} + \frac{dx^2}{(1-x^2)^2}\right]$$

$$\mathrm{K}^{\pm}(t,x) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega t}f^{\pm}_{\omega}(x)$$

$$(1-x^2)\frac{d^2f^{\pm}_{\omega}(x)}{dx^2} - x\frac{df^{\pm}_{\omega}(x)}{dx} + \left(\omega^2 - \frac{m^2R^2}{1-x^2}\right)f^{\pm}_{\omega}(x) = 0$$

$$f^{\pm}_{\omega}(x) = (1-x^2)^{\frac{1}{4}}\left[a^{\pm}_{\omega}P^{\mu}_{\nu}(x) + b^{\pm}_{\omega}Q^{\mu}_{\nu}(x)\right]$$

$$f^{\pm}_{\omega}(\pm x_{\epsilon}) = 1, f^{\pm}_{\omega}(\mp x_{\epsilon}) = 0$$

$$f_\omega^+(x) = \left(\frac{1-x^2}{1-x_\epsilon^2}\right)^{\frac{1}{4}} \frac{Q_\nu^\mu(x)P_\nu^\mu(-x_\epsilon) - Q_\nu^\mu(-x_\epsilon)P_\nu^\mu(x)}{Q_\nu^\mu(x_\epsilon)P_\nu^\mu(-x_\epsilon) - Q_\nu^\mu(-x_\epsilon)P_\nu^\mu(x_\epsilon)}$$

$$f_\omega^-(x) = \left(\frac{1-x^2}{1-x_\epsilon^2}\right)^{\frac{1}{4}} \frac{Q_\nu^\mu(x)P_\nu^\mu(x_\epsilon) - Q_\nu^\mu(x_\epsilon)P_\nu^\mu(x)}{Q_\nu^\mu(-x_\epsilon)P_\nu^\mu(x_\epsilon) - Q_\nu^\mu(x_\epsilon)P_\nu^\mu(-x_\epsilon)}$$

$$\omega_n = \pm\left(n+\mu+\frac{1}{2}\right), n = 0,1,2\ldots \text{ y } \frac{b_{\omega Q}}{a_{\omega Q}} = -\frac{2\tan \pi\mu}{\pi}$$

$$\Delta_{\text{ren}_{\pm\pm}}(t,t') = \mp\frac{2^{\Delta_-}}{2\pi}\frac{\Gamma(1-\mu)}{\Gamma(1+\mu)}\int \frac{d\omega}{2\pi} e^{-i\omega(t-t')}\Gamma\left(\frac{1}{2}+\mu-\omega\right)\Gamma\left(\frac{1}{2}+\mu+\omega\right)\cos(\pi\omega)$$

$$\Delta_{\text{ren}_{\pm\mp}}(t,t') = \mp\frac{2^{\Delta_-}}{\Gamma(\mu)^2}\int \frac{d\omega}{2\pi} e^{-i\omega(t-t')}\Gamma\left(\frac{1}{2}+\mu-\omega\right)\Gamma\left(\frac{1}{2}+\mu+\omega\right)$$

$$\Delta^F_{\text{ren}\pm\pm}(t,t') = \mp\left(\frac{i^{\Delta_- - \Delta_+}}{8^{\Delta_+}\pi^{\frac{1}{2}}}\right)\frac{\Gamma\left(\frac{1}{2}+\mu\right)}{\Gamma(\mu)\sin^{2\Delta_+}\left(\frac{t-t'}{2}\right)} = \Delta^F_{\text{ren}\pm\mp}(t,t') = \mp\left(\frac{8^{\Delta_-}i}{4}\right)\frac{\Gamma(1+2\mu)}{\Gamma(\mu)^2\cos^{2\Delta_+}\left(\frac{t-t'}{2}\right)}$$

$$\langle 0|T\mathcal{O}^\pm(t)\mathcal{O}^\pm(t')|0\rangle = \pm\left(\frac{4^{\Delta_-}i^{2\Delta_-}}{8^{\Delta_+}}\right)\frac{\Gamma(2\mu)}{\Gamma(\mu)^2\sin^{2\Delta_+}\left(\frac{t-t'}{2}\right)}$$

$$\langle 0|T\mathcal{O}^\pm(t)\mathcal{O}^\mp(t')|0\rangle = \mp\left(\frac{8^{\Delta_-}}{4}\right)\frac{\Gamma(1+2\mu)}{\Gamma(\mu)^2\cos^{2\Delta_+}\left(\frac{t-t'}{2}\right)}$$

$$\langle 0|T\mathcal{O}^\pm(t)\mathcal{O}^\pm(t')|0\rangle = \pm\frac{1}{8\pi\sin^2\left(\frac{t-t'}{2}\right)}, \langle 0|T\mathcal{O}^\pm(t)\mathcal{O}^\mp(t')|0\rangle = \mp\frac{1}{4\pi\cos^2\left(\frac{t-t'}{2}\right)}$$

$$S_5 = \kappa\int \epsilon_{abcde}\left(R^{ab}R^{cd} + \frac{2}{3l^2}R^{ab}e^ce^d + \frac{1}{5l^4}e^ae^be^ce^d\right)e^e$$

$$ds^2 = R^2\left[-\cosh^2\rho dt^2 + d\rho^2 + \cosh^2\rho d\tilde{\Sigma}^2_{d-1}\right]$$

$$= R^2\left[-\frac{dt^2}{1-x^2} + \frac{dx^2}{(1-x^2)^2} + \frac{d\tilde{\Sigma}^2_{d-1}}{1-x^2}\right]$$

$$\mathrm{K}^\pm(t,x,\theta \mid t',\theta') = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\sum_Q e^{-i\omega(t-t')}Y_Q(\theta)Y_Q^*(\theta')f^\pm_{\omega Q}(x)$$

$$(1-x^2)\frac{d^2 f^\pm_{\omega Q}(x)}{dx^2} + (d-2)x\frac{df^\pm_{\omega Q}(x)}{dx} + \left[(\omega^2-Q^2) - \frac{m^2R^2}{1-x^2}\right]f^\pm_{\omega Q}(x) = 0$$

$$f^\pm_{\omega Q}(x) = (1-x^2)^{\frac{d}{4}}\left[a^\pm_{\omega Q}P_\nu^\mu(x) + b^\pm_{\omega Q}Q_\nu^\mu(x)\right]$$

$$\nu = \varpi - \frac{1}{2} = \sqrt{\left(\frac{d-1}{2}\right)^2 + \omega^2 - Q^2} - \frac{1}{2}$$

$$f^\pm_{\omega Q}(\pm x_\epsilon) = 1, f^\pm_{\omega Q}(\mp x_\epsilon) = 0$$

$$f^+_{\omega Q}(x) = \left(\frac{1-x^2}{1-x_\epsilon^2}\right)^{\frac{d}{4}} \frac{P_\nu^\mu(x)Q_\nu^\mu(-x_\epsilon) - P_\nu^\mu(-x_\epsilon)Q_\nu^\mu(x)}{P_\nu^\mu(x_\epsilon)Q_\nu^\mu(-x_\epsilon) - P_\nu^\mu(-x_\epsilon)Q_\nu^\mu(x_\epsilon)}$$

$$f^-_{\omega Q}(x) = \left(\frac{1-x^2}{1-x_\epsilon^2}\right)^{\frac{d}{4}} \frac{P_\nu^\mu(x)Q_\nu^\mu(x_\epsilon) - P_\nu^\mu(x_\epsilon)Q_\nu^\mu(x)}{P_\nu^\mu(-x_\epsilon)Q_\nu^\mu(x_\epsilon) - P_\nu^\mu(x_\epsilon)Q_\nu^\mu(-x_\epsilon)}$$

$$\omega_{nQ} = \pm\sqrt{\left(\mu+\frac{1}{2}+n\right)^2 + Q^2 - \left(\frac{d-1}{2}\right)^2}, n = 0,1,\ldots, \text{ y } \frac{b_{\omega Q}}{a_{\omega Q}} = -\frac{2\tan \pi\mu}{\pi}$$

$$\langle\psi_{\mathrm{f}}|T\mathcal{O}^{\pm}(t,\theta)\mathcal{O}^{\pm}(t',\theta')|\psi_{\mathrm{i}}\rangle = \pm i\frac{2^{\Delta_{-}d}}{\pi 2^{d}}\frac{\Gamma(1-\mu)}{\Gamma(1+\mu)}\sum_{Q} Y_Q(\theta)Y_Q^{*}(\theta')$$

$$\times\int \frac{d\omega}{2\pi}e^{-i\omega(t-t')}\Gamma\left(\frac{1}{2}+\mu-\varpi\right)\Gamma\left(\frac{1}{2}+\mu+\varpi\right)\cos(\pi\varpi)$$

$$\langle\psi_{\mathrm{f}}|T\mathcal{O}^{\pm}(t,\theta)\mathcal{O}^{\mp}(t',\theta')|\psi_{\mathrm{i}}\rangle = \pm i\frac{2^{\Delta_{-}}}{2^{d-1}}\frac{1}{\Gamma(\mu)^2}\sum_{Q} Y_Q(\theta)Y_Q^{*}(\theta')$$

$$\times\int \frac{d\omega}{2\pi}e^{-i\omega(t-t')}\Gamma\left(\frac{1}{2}+\mu-\varpi\right)\Gamma\left(\frac{1}{2}+\mu+\varpi\right)$$

$$\mathrm{S_A} = \frac{\mathrm{Area}(\gamma_{\mathrm{A}})}{4G_N^{(d+1)}}$$

$$\mathrm{S}_{\tilde{\Sigma}} = \frac{\mathrm{Area}(\tilde{\Sigma})}{4G_N^{(d+1)}}$$

$$\mathcal{Z}_{\mathrm{gravity}}[\phi(\phi_0^{+},\phi_0^{-},\mathcal{C})] \sim e^{-\frac{i}{2}\int d\mathbf{y}d\mathbf{y}'\phi_0^{i}(\mathbf{y})\Delta_{ij}(\mathbf{y},\mathbf{y}')\phi_0^{j}(\mathbf{y}')}$$

$$\mathcal{Z}_{\mathrm{gravity}}[\phi(\phi_0^{+},\phi_0^{-},\mathcal{C})] = \langle\psi_{\mathrm{f}}|Te^{i\int d\mathbf{y}\phi_0^{+}(\mathbf{y})\mathcal{O}^{+}(\mathbf{y})+i\int d\mathbf{y}\phi_0^{-}(\mathbf{y})\mathcal{O}^{-}(\mathbf{y})}|\psi_{\mathrm{i}}\rangle_{QFT}$$

$$\langle\psi_0|Te^{i\int d\mathbf{y}\phi_0^{+}(\mathbf{y})\mathcal{O}^{+}(\mathbf{y})+i\int d\mathbf{y}\phi_0^{-}(\mathbf{y})\mathcal{O}^{-}(\mathbf{y})}|\psi_0\rangle_{QFT} = \mathrm{Tr}\left[\rho_{\psi_0}Te^{i\int d\mathbf{y}\phi_0^{+}(\mathbf{y})\mathcal{O}^{+}(\mathbf{y})+i\int d\mathbf{y}\phi_0^{-}(\mathbf{y})\mathcal{O}^{-}(\mathbf{y})}\right]$$

$$\left\langle e^{-\int d\mathbf{y}\phi_0^{+}(\mathbf{y})\mathcal{O}^{+}(\mathbf{y})-\int d\mathbf{y}\phi_0^{-}(\mathbf{y})\mathcal{O}^{-}(\mathbf{y})}\right\rangle_{\beta} = \mathrm{Tr}\left[\rho_{\beta}e^{-\int d\mathbf{y}\phi_0^{+}(\mathbf{y})\mathcal{O}^{+}(\mathbf{y})-\int d\mathbf{y}\phi_0^{-}(\mathbf{y})\mathcal{O}^{-}(\mathbf{y})}\right]$$

$$\mathrm{Tr}\left[e^{-\beta H}e^{-\int d\mathbf{y}\phi_0^{+}(\mathbf{y})\mathcal{O}^{+}(\mathbf{y})-\int d\mathbf{y}\phi_0^{-}(\mathbf{y})\mathcal{O}^{-}(\mathbf{y})}\right] = \mathcal{Z}_{\mathrm{Egravity}}[\phi(\phi_0^{+},\phi_0^{-})] \sim e^{-S_E[\phi_0^{+},\phi_0^{-}]}$$

$$\mathcal{Z}_{\mathrm{Egravity}}[\phi(\phi_0^{+},\phi_0^{-})] \sim e^{-\frac{1}{2}\int d\mathbf{y}d\mathbf{y}'\phi_0^{i}(\mathbf{y})\tilde{\Delta}_{ij}(\mathbf{y},\mathbf{y}')\phi_0^{j}(\mathbf{y}')}$$

$$\mathrm{Tr}\left[e^{-\beta H}e^{-\int d\mathbf{y}\phi_0^{+}(\mathbf{y})\mathcal{O}^{+}(\mathbf{y})-\int d\mathbf{y}\phi_0^{-}(\mathbf{y})\mathcal{O}^{-}(\mathbf{y})}\right] \sim e^{-\frac{1}{2}\int d\mathbf{y}d\mathbf{y}'\phi_0^{i}(\mathbf{y})\tilde{\Delta}_{ij}(\mathbf{y},\mathbf{y}')\phi_0^{j}(\mathbf{y}')}$$

$$H[\Psi_{+},\Psi_{-}] = H_{+}[\Psi_{+}] + H_{-}[\Psi_{-}]$$

$$\mathrm{Tr}\left[e^{-\beta H}e^{-\int d\mathbf{y}\phi_0(\mathbf{y})\mathcal{O}(\mathbf{y})}\right] \sim e^{-\frac{1}{2}\int d\mathbf{y}d\mathbf{y}'\phi_0(\mathbf{y})\tilde{\Delta}(\mathbf{y},\mathbf{y}')\phi_0(\mathbf{y}')}$$

$$S = \frac{\eta}{2\pi\alpha'}\int d\tau d\sigma\sqrt{\eta h}$$

$$ds^2 = -g_t(r)dt^2 + g_x(r)dx_i^2 + g_r(r)dr^2 + g_{ab}(r,\theta)d\theta^a d\theta^b$$

$$S = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma\sqrt{g_t(r)\dot{t}^2(g_x(r)\dot{x}^2 + g_r(r)\dot{r}^2)}$$

$$= -\frac{1}{2\pi\alpha'}\int dt d\sigma\sqrt{g_t(r)(g_x(r)\dot{x}^2 + g_r(r)\dot{r}^2)}$$

$$= -\frac{\mathcal{T}}{2\pi\alpha'}\int d\sigma\sqrt{f^2(r)\dot{x}^2 + g^2(r)\dot{r}^2}.$$

$$ds_{eff}^2 = f^2(r)dx^2 + g^2(r)dr^2$$

$$\frac{f^2(r)\dot{x}(\sigma)}{\sqrt{f^2(r)\dot{x}(\sigma)^2 + g^2(r)\dot{r}(\sigma)^2}} = A$$

$$\dot{x}(\sigma) = \pm A\frac{g(r)}{f(r)}\frac{1}{\sqrt{f^2(r) - A^2}}\dot{r}(\sigma)$$

$$\frac{dx}{dr} = \pm\frac{g(r)}{f(r)}\frac{f(r_0)}{\sqrt{f^2(r) - f^2(r_0)}}$$

$$U(r) = \frac{f^2(r)\left(f^2(r) - f^2(r_0)\right)}{g^2(r)f^2(r_0)}$$

$$L(r_0) = 2\int_{r_0}^{\infty}\frac{g(r)}{f(r)}\frac{f(r_0)}{\sqrt{f^2(r) - f^2(r_0)}}dr$$

$$\frac{L'(r_0)}{2} = -\frac{g(r)}{\sqrt{f^2(r)-f^2(r_0)}}\bigg|_{r\to r_0} + f'(r_0)\int_{r_0}^{\infty}\frac{f(r)g(r)}{\left(f^2(r)-f^2(r_0)\right)^{\frac{3}{2}}}dr$$

$$= -\frac{g(r)}{\sqrt{f^2(r)-f^2(r_0)}}\bigg|_{r\to r_0} + f'(r_0)\int_{r_0}^{\infty} dr\frac{g(r)}{f'(r)}\frac{d}{dr}\left(-\frac{1}{\sqrt{f^2(r)-f^2(r_0)}}\right)$$

$$= -\frac{f'(r_0)g(r)}{f'(r)\sqrt{f^2(r)-f^2(r_0)}}\bigg|_{r\to\infty} + \int_{r_0}^{\infty} dr\frac{f'(r_0)}{\sqrt{f^2(r)-f^2(r_0)}}\frac{d}{dr}\left(\frac{g(r)}{f'(r)}\right),$$

$$L'(r_0) = 2\int_{r_0}^{\infty} dr\frac{f'(r_0)}{\sqrt{f^2(r)-f^2(r_0)}}\frac{d}{dr}\left(\frac{g(r)}{f'(r)}\right)$$

$$E = \frac{1}{2\pi\alpha'}\int d\sigma\sqrt{f^2(r)\dot{x}(\sigma)^2 + g^2(r)\dot{r}(\sigma)^2}$$

$$E(r_0) = \frac{1}{2\pi\alpha'}\int_{-L/2}^{L/2} dx\frac{f^2(r(x))}{f(r_0)} = \frac{1}{\pi\alpha'}\int_{r_0}^{\infty} dr\frac{g(r)f(r)}{\sqrt{f^2(r)-f^2(r_0)}}$$

$$m_q = \frac{1}{2\pi\alpha'}\int_{r_{\min}}^{\infty} g(r)dr$$

$$E_{q\bar{q}}(r_0) = E(r_0) - 2m_q$$

$$= \frac{1}{\pi\alpha'}\left[\int_{r_0}^{\infty}\frac{g(r)f(r)}{\sqrt{f^2(r)-f^2(r_0)}}dr - \int_{r_{\min}}^{\infty} g(r)dr\right]$$

$$E'_{q\bar{q}}(r_0) = \frac{1}{\pi\alpha'}\left[-\frac{g(r)f(r)}{\sqrt{f^2(r)-f^2(r_0)}}\bigg|_{r=r_0} + \int_{r_0}^{\infty} dr\frac{f(r)g(r)f(r_0)f'(r_0)}{\left(f^2(r)-f^2(r_0)\right)^{\frac{3}{2}}}\right]$$

$$E'_{q\bar{q}}(r_0) = \frac{1}{2\pi\alpha'}f(r_0)L'(r_0) \Rightarrow \frac{dE_{q\bar{q}}}{dL} = \frac{1}{2\pi\alpha'}f(r_0)$$

$$\frac{dV}{dL} > 0, \frac{d^2V}{dL^2} \le 0$$

$$\frac{dV_{\text{string}}}{dL} = \frac{dE_{q\bar{q}}}{dr_0}\frac{dr_0}{dL} = \frac{1}{2\pi\alpha'}f(r_0), \frac{d^2V_{\text{string}}}{dL^2} = \frac{1}{2\pi\alpha'}\left(\frac{dL}{dr_0}\right)^{-1} f'(r_0)$$

$$X^\mu = (\tau, x_{\text{cl}}(\sigma) + \delta x_1(\tau,\sigma), \delta x_2(\tau,\sigma), \delta x_3(\tau,\sigma), r_{\text{cl}}(\sigma) + \delta r(\tau,\sigma), \theta^a + \delta\theta^a(\tau,\sigma))$$

$$t = \tau, x_1 = x_{\text{cl}}(r) + \delta x_1(t,r), x_2 = \delta x_2(t,r), x_3 = \delta x_3(t,r), r = \sigma$$

$$2\pi\alpha'\mathcal{L}^{(2)} = \frac{1}{g(r)f(r)\sqrt{f^2(r)-f^2(r_0)}}\Big[h^2(r)\left(f^2(r)-f^2(r_0)\right)(\delta\dot{x}_1)^2 - \left(f^2(r)-f^2(r_0)\right)^2(\delta\dot{x}_1)^2$$
$$+ f^2(r)h^2(r)((\delta\dot{x}_2)^2 + (\delta\dot{x}_3)^2) - f^2(r)\left(f^2(r)-f^2(r_0)\right)((\delta\dot{x}_2)^2 + (\delta\dot{x}_3)^2)\Big]$$

$$\left[\frac{d}{dr}\left(\frac{\left(f^2(r)-f^2(r_0)\right)^{\frac{3}{2}}}{g(r)f(r)}\frac{d}{dr}\right) + \omega^2\frac{h^2(r)\sqrt{f^2(r)-f^2(r_0)}}{g(r)f(r)}\right]\delta x_1(r)$$

$$\left[\frac{d}{dr}\left(\frac{f(r)\sqrt{f^2(r)-f^2(r_0)}}{g(r)}\frac{d}{dr}\right) + \omega^2\frac{h^2(r)f(r)}{g(r)\sqrt{f^2(r)-f^2(r_0)}}\right]\delta x_m(r) = 0, m = 2,3$$

$$\frac{d}{dr}\left(\sqrt{r-r_0}\frac{d\delta x_m(r)}{dr}\right) + \frac{\omega^2 h^2(r_0)}{2f(r_0)f'(r_0)}\frac{1}{\sqrt{r-r_0}}\delta x_m(r) \approx 0 \Longrightarrow \delta x_m(r)$$
$$\approx C_0 + C_1\sqrt{r-r_0} + O(r-r_0)$$

$$\frac{d}{dr}\left((r-r_0)^{\frac{3}{2}}\frac{d\delta x_1(r)}{dr}\right) + \frac{\omega^2 h^2(r_0)}{2f(r_0)f'(r_0)}\sqrt{r-r_0}\,\delta x_1(r) \approx 0 \Longrightarrow \delta x_1(r) \approx C'_0 + C'_1\frac{1}{\sqrt{r-r_0}} + O\left(\sqrt{r-r_0}\right)$$

$$\frac{d}{dr}\left(\sqrt{r-r_0}\frac{d\delta x_m(r)}{dr}\right) + \frac{\omega^2 h^2(r_0)}{2f(r_0)f'(r_0)}\frac{1}{\sqrt{r-r_0}}\delta x_m(r) \approx 0 \Longrightarrow \delta x_m(r)$$
$$\approx C_0 + C_1\sqrt{r-r_0} + O(r-r_0)$$

$$u = r + \Delta(t,r) \text{ where } \Delta(t,r) = \frac{\delta x_1(t,r)}{x'_{\text{cl}}(r)}$$

$$x_1 = x_{\text{cl}}(r) + \delta x_1(t,r) = x_{\text{cl}}(u - \Delta(t,r)) + \delta x_1(t,r)$$

$$\approx x_{\text{cl}}(u) - x'_{\text{cl}}(r)\frac{\delta x_1(t,r)}{x'_{\text{cl}}(r)} + \delta x_1(t,r) = x_{\text{cl}}(u)$$

$$r \approx u - \frac{\delta x_1(t,u)}{x'_{\text{cl}}(u)}$$

$$r \approx r_0 - \alpha\left(C'_0\sqrt{u - r_0} + C'_1\right) + O(u - r_0)$$

$$\left.\frac{d\delta r(t,r)}{dx_1}\right|_{r=r_0} = 0 \text{ where } \delta r(t,r) = -\frac{\delta x_1(t,r)}{x'_{\text{cl}}(r)}$$

$$\delta x_1(r) + 2(r - r_0)\frac{d\delta x_1(r)}{dr} = 0, \qquad r \to r_0$$

$$\sqrt{r - r_0}\,\delta x_1(r) = 1, \qquad r \to r_0$$

$$\delta x_1(r) + 2(r - r_0)\frac{d\delta x_1(r)}{dr} = 1, \qquad r \to r_0$$

$$\sqrt{r - r_0}\,\delta x_1(r) = 0, \qquad r \to r_0$$

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + dx_i dx_i) + R^2\frac{dr^2}{r^2} + R^2 d\Omega_5^2$$

$$x_{\text{cl}}(r) = \pm\left\{cte - \frac{R^2}{4r_0}\,\mathrm{B}\left(\left(\frac{r_0}{r}\right)^4; \frac{3}{4}, \frac{1}{2}\right)\right\}, r_0 \le r < \infty$$

$$L(r_0) = \frac{R^2}{2r_0}\,\mathrm{B}\left(\frac{3}{4}, \frac{1}{2}\right) = \frac{R^2}{r_0}\frac{(2\pi)^{\frac{3}{2}}}{\Gamma\left[\frac{1}{4}\right]^2}$$

$$E_{q\bar{q}}(r_0) = \frac{r_0}{\pi\alpha'}(K(-1) - E(-1)) = -\frac{r_0}{2\pi\alpha'}\frac{(2\pi)^{\frac{3}{2}}}{\Gamma\left[\frac{1}{4}\right]^2}$$

$$V_{\text{string}}(L) = -\frac{(2\pi)^2}{\Gamma\left[\frac{1}{4}\right]^4}\frac{R^2/\alpha'}{L} \sim -\frac{\sqrt{\lambda}}{L}$$

$$ds^2 = R^2[-\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\Omega_3^2]$$

$$\dot{\rho}^2 + U(\rho) = 0$$

$$\Phi(\rho_0) = 2\int_{r_0}^{\infty} \frac{\sinh 2\rho_0}{\sinh \rho\sqrt{\sinh^2 2\rho - \sinh^2 2\rho_0}} d\rho$$

$$E(\rho_0) = \frac{R}{2\pi\alpha'}\int_{\rho_0}^{\infty} \frac{\sinh^2 2\rho}{\sinh \rho\sqrt{\sinh^2 2\rho - \sinh^2 2\rho_0}} d\rho$$

$$E_{q\bar{q}}(\rho_0) = \frac{R}{2\pi\alpha'}\left[\int_{\rho_0}^{\infty}\left(\frac{2\cosh \rho}{\sqrt{1 - \frac{\sinh^2 2\rho_0}{\sinh^2 2\rho}}} - 2\cosh \rho\right)d\rho - 2\sinh \rho_0\right]$$

$$ds^2 = \frac{r^2}{R^2}\left[-\left(1 - \frac{\mu^4}{r^4}\right)dt^2 + dx_i dx_i\right] + \frac{R^2}{r^2}\frac{1}{1 - \frac{\mu^4}{r^4}}dr^2 + R^2 d\Omega_5^2$$

$$ds^2 = R^2\left[-\left(\rho^2 - \frac{1}{\rho^2}\right)d\vec{t}^2 + \rho^2 dy_i dy_i + \frac{1}{\rho^2 - \frac{1}{\rho^2}}d\rho^2 + d\Omega_5^2\right]$$

$$\bar{L}(\rho_0)=\frac{(2\pi)^{\frac{3}{2}}}{\Gamma\left[\frac{1}{4}\right]^2}\frac{\sqrt{\rho_0^4-1}}{\rho_0^3}\,{}_2F_1\left(\frac{3}{4},\frac{1}{2},\frac{5}{4};\frac{1}{\rho_0^4}\right)$$

$$\bar{E}_{q\bar{q}}(\rho_0)=\frac{R^2}{\pi\alpha'}\left[1-\frac{(2\pi)^{\frac{3}{2}}}{2\Gamma\left[\frac{1}{4}\right]^2}\rho_0F_1\left(-\frac{1}{2},-\frac{1}{4},\frac{1}{4};\frac{1}{\rho_0^4}\right)\right]$$

$$ds^2=\alpha'Ne^{\phi}\left[-dt^2+dx_idx_i+dr^2+e^{2h}(d\theta^2+\sin^2\theta d\varphi^2)+\frac{1}{4}\left(w^i-A^i\right)^2\right]$$

$$w^1+iw^2=e^{-i\psi}(d\tilde{\theta}+i\sin\tilde{\theta}d\tilde{\varphi}),w^3=d\psi+\cos\tilde{\theta}d\tilde{\varphi}$$

$$A^1=-a(r)d\theta,A^2=a(r)\sin\theta d\varphi,A^3=-\cos\theta d\varphi$$

$$a(r)=\frac{2r}{\sinh 2r}$$

$$e^{2h}=r\coth 2r-\frac{r^2}{\sinh^2 2r}-\frac{1}{4}$$

$$e^{2\phi}=e^{2\phi_0}\frac{\sinh 2r}{2e^h}$$

$$\bar{L}(r_0)=2\int_{r_0}^{\infty}\frac{e^{\phi(r_0)}}{\sqrt{e^{2\phi(r)}-e^{2\phi(r_0)}}}dr$$

$$\bar{E}_{q\bar{q}}(r_0)=\frac{N}{\pi}\left[\int_{r_0}^{\infty}\frac{e^{2\phi(r)}}{\sqrt{e^{2\phi(r)}-e^{2\phi(r_0)}}}dr-\int_0^{\infty}e^{\phi(r)}dr\right]$$

$$\bar{E}_{q\bar{q}}(r_0)=\frac{N}{\pi}\left[\int_{r_0}^{\infty}\left(\frac{e^{2\phi(r)}+e^{2\phi(r_0)}-e^{2\phi(r_0)}}{\sqrt{e^{2\phi(r)}-e^{2\phi(r_0)}}}-e^{\phi(r)}\right)dr-\int_0^{r_0}e^{\phi(r)}dr\right]$$

$$=\frac{N}{\pi}\left[e^{\phi(r_0)}\frac{\bar{L}(r_0)}{2}+\int_{r_0}^{\infty}dr\left(\sqrt{e^{2\phi(r)}-e^{2\phi(r_0)}}-e^{\phi(r)}\right)-\int_0^{r_0}e^{\phi(r)}dr\right]$$

$$V_{\text{string}}(L)\approx\frac{e^{\phi_0}}{2\pi\alpha'}L\ \Rightarrow\ T_{\text{string}}=\frac{e^{\phi_0}}{2\pi\alpha'}$$

$$ds^2=g_s\alpha'M\left[h^{-\frac{1}{2}}(r)(-dt^2+dx_idx_i)+h^{\frac{1}{2}}(r)ds_6^2\right]$$

$$ds_6^2=\frac{1}{2}K(r)\left[\frac{(dr^2+(g^5)^2)}{3K^3(r)}+\cosh^2\frac{r}{2}((g^3)^2+(g^4)^2)+\sinh^2\frac{r}{2}((g^1)^2+(g^2)^2)\right]$$

$$K(r)=\frac{[\sinh(2r)-2r]^{\frac{1}{3}}}{2^{\frac{1}{3}}\sinh r}$$

$$g^1=\frac{e^1-e^3}{\sqrt{2}},g^2=\frac{e^2-e^4}{\sqrt{2}},g^3=\frac{e^1+e^3}{\sqrt{2}},g^4=\frac{e^2+e^4}{\sqrt{2}},g^5=e^5$$

$$e^1=-\sin\theta_1d\phi_1,e^2=d\theta_1,e^3=-\sin\psi d\theta_2+\cos\psi\sin\theta_2d\phi_2$$

$$e^4=\cos\psi d\theta_2+\sin\psi\sin\theta_2d\phi_2,$$

$$e^5=d\psi+\cos\theta_1d\phi_1+\cos\theta_2d\phi_2$$

$$h(r)=2^{\frac{2}{3}}\int_r^{\infty}dx\frac{x\coth x-1}{\sinh^2 x}(\sinh 2x-2x)^{\frac{1}{3}}$$

$$\bar{L}(r_0)=2\int_{r_0}^{\infty}\frac{dr}{\sqrt{6}K(r)}\frac{h(r)}{\sqrt{h(r_0)-h(r)}}$$

$$\bar{E}_{q\bar{q}}(r_0)=\frac{g_sM}{\pi}\left[\int_{r_0}^{\infty}\frac{dr}{\sqrt{6}K(r)}\frac{\sqrt{h(r_0)}}{\sqrt{h(r_0)-h(r)}}-\int_0^{r_0}\frac{dr}{\sqrt{6}K(r)}\right]$$

$$T_{\text{string}}=\frac{1}{2\pi\alpha'}\frac{\ell_{cf}^2}{g_s\alpha'M\sqrt{h_0}}$$

$$ds^2 = g_s\alpha' N e^{4f(r)}\Big[-dt^2 + dx_i dx_i + dr^2 + e^{2h(r)}(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{e^{2g(r)}}{4}((w_1 + a(r)d\theta)^2 + (w_2 - a(r)\sin\theta d\varphi)^2) + \frac{e^{2k(r)}}{4}(w_3 + \cos\theta d\varphi)^2\Big]$$

$$\partial_\rho a = \frac{-2}{-1 + 2\rho\coth 2\rho}\left[e^{2k}\frac{(a\cosh 2\rho - 1)^2}{\sinh 2\rho} + a(2\rho - a\sinh 2\rho)\right]$$

$$\partial_\rho k = \frac{2(1 + a^2 - 2a\cosh 2\rho)^{-1}}{-1 + 2\rho\coth 2\rho}\left[e^{2k}a\sinh 2\rho(a\cosh 2\rho - 1) + \left(2\rho - 4a\rho\cosh 2\rho + \frac{a^2}{2}\sinh 4\rho\right)\right]$$

$$\partial_\rho f = -\frac{1}{4\sinh^2 2\rho}\left[\frac{(1 - a\cosh 2\rho)^2(-4\rho + \sinh 4\rho)}{(1 + a^2 - 2a\cosh 2\rho)(-1 + 2\rho\coth 2\rho)}\right]$$

$$e^{2g} = \frac{b\cosh 2\rho - 1}{a\cosh 2\rho - 1}, e^{2h} = \frac{e^{2g}}{4}(2a\cosh 2\rho - 1 - a^2), \text{con } b(\rho) = \frac{2\rho}{\sinh 2\rho}$$

$$a(\rho) = 1 + \mu\rho^2 + \cdots, e^{2k(\rho)} = \frac{4}{6 + 3\mu} - \frac{20 + 36\mu + 9\mu^2}{15(2 + \mu)}\rho^2 + \cdots$$

$$e^{2g(\rho)} = \frac{4}{6 + 3\mu} + \cdots, e^{2h(\rho)} = \frac{4\rho^2}{6 + 3\mu} + \cdots, e^{2f(\rho)} = 1 + \frac{(2 + \mu)^2}{8}\rho^2 + \cdots$$

$$\bar{L}(\rho_0) = 2\int_{\rho_0}^{\rho_\infty} \frac{e^{4f(\rho_0)}}{\sqrt{e^{8f(\rho)} - e^{8f(\rho_0)}}} e^{k(\rho)} d\rho$$

$$\bar{E}_{q\bar{q}}(\rho_0) = \frac{Ng_s}{\pi}\left[\int_{\rho_0}^{\rho_\infty} \frac{e^{8f(\rho)}}{\sqrt{e^{8f(\rho)} - e^{8f(\rho_0)}}} e^{k(\rho)} d\rho - \int_0^{\rho_\infty} e^{4f(\rho)} e^{k(\rho)} d\rho\right]$$

$$T_{\text{string}} = \frac{g_s}{2\pi\alpha'}$$

$$\delta x_1(r) + 2(r - r_0)\frac{d\delta x_1(r)}{dr} = 0, \qquad r \to r_0$$

$$\sqrt{r - r_0}\,\delta x_1(r) = 1, \qquad r \to r_0$$

$$\delta x_1(r) = 0, r \to \infty$$

$$\delta x_1^{(0)}(r) = C\int_r^\infty d\bar{r}\frac{g(\bar{r})f(\bar{r})}{\left(f^2(\bar{r}) - f^2(r_0)\right)^{\frac{3}{2}}} + C'$$

$$\delta x_1^{(0)}(r) = -\int_r^\infty d\bar{r}\frac{g(\bar{r})}{f'(\bar{r})}\frac{d}{d\bar{r}}\left(\frac{1}{\sqrt{f^2(\bar{r}) - f^2(r_0)}}\right)$$

$$= \frac{g(r)}{f'(r)\sqrt{f^2(r) - f^2(r_0)}} + \frac{L'(r)}{2f'(r)}.$$

$$\delta x_1^{(0)}(r) = \frac{g(r_0)}{\sqrt{2}\left(f'(r_0)\right)^{\frac{3}{2}}}\frac{1}{\sqrt{r - r_0}} + \frac{L'(r_0)}{2f'(r_0)} + \mathcal{O}\left(\sqrt{r - r_0}\right)$$

$$\left[\frac{d}{dr}\left(\frac{(r^4 - r_0^4)^{\frac{3}{2}}}{r^2}\frac{d}{dr}\right) + \omega^2 R^4\frac{\sqrt{r^4 - r_0^4}}{r^2}\right]\delta x_1(r) = 0 \; 0 < r_0 \le r < \infty$$

$$\left[\frac{d}{d\rho}\left(\frac{(\rho^4 - 1)^{\frac{3}{2}}}{\rho^2}\frac{d}{d\rho}\right) + \frac{\omega^2 R^4}{r_0^2}\frac{\sqrt{\rho^4 - 1}}{\rho^2}\right]\delta x_1(\rho) = 0$$

$$\left[\frac{d}{d\rho}\left(\rho^4\frac{d}{d\rho}\right) + \frac{\omega^2 R^4}{r_0^2}\right]\delta x_1(\rho) \approx 0, \rho \gg 1$$

$$\delta x_1(\rho) \approx \alpha_0 + \frac{\alpha_1}{\rho^3}, \rho \gg 1$$

$$\left[\frac{d}{d\rho}\left(\frac{(\rho^4-\rho_0^4)^{\frac{3}{2}}}{\sqrt{\rho^4-1}}\frac{d}{d\rho}\right)+\frac{\omega^2R^4}{\mu^2}\frac{\rho^4\sqrt{\rho^4-\rho_0^4}}{(\rho^4-1)^{\frac{3}{2}}}\right]\delta x_1(\rho)=0, 1<\rho_0\leq\rho<\infty$$

$$\left[\frac{d}{dr}\left(\frac{\left(e^{2\phi(r)}-e^{2\phi(r_0)}\right)^{\frac{3}{2}}}{e^{2\phi(r)}}\frac{d}{dr}\right)+\bar{\omega}^2\sqrt{e^{2\phi(r)}-e^{2\phi(r_0)}}\right]\delta x_1(r)=0, 0<r_0\leq r<\infty$$

$$\left[\frac{d}{dr}\left(e^r r^{-\frac{1}{4}}\frac{d}{dr}\right)+\bar{\omega}^2e^r r^{-\frac{1}{4}}\right]\delta x_1(r)=0, r\gg 1$$

$$\left[\frac{d^2}{dr^2}+\left(1-\frac{1}{4r}\right)\frac{d}{dr}+\bar{\omega}^2\right]\delta x_1(r)=0, r\gg 1$$

$$\delta x_1(r)\simeq e^{-\frac{1}{2}r}(\beta_0e^{r\alpha}+\beta_1e^{-r\alpha}), r\gg 1$$

$$\left[\frac{d}{dr}\left(\frac{K(r)}{h(r)}\left(1-\frac{h(r)}{h(r_0)}\right)^{\frac{3}{2}}\frac{d}{dr}\right)+\bar{\omega}^2\frac{1}{6K(r)}\sqrt{1-\frac{h(r)}{h(r_0)}}\right]\delta x_1(r)=0, 0<r_0\leq r<\infty$$

$$\left[\frac{d}{dr}\left(\frac{e^r}{r}\frac{d}{dr}\right)+\bar{\omega}^2\frac{e^{\frac{r}{3}}}{2^{\frac{4}{3}}}\right]\delta x_1(r)=0, r\gg 1$$

$$\left[\frac{d^2}{dr^2}+\left(1-\frac{1}{r}\right)\frac{d}{dr}+\bar{\omega}^2\frac{re^{-\frac{2}{3}r}}{2^{\frac{4}{3}}}\right]\delta x_1(r)=0, r\gg 1$$

$$\delta x_1(r)\simeq\alpha_0+\alpha_1e^{-r}, r\gg 1$$

$$\left[\frac{d}{d\rho}\left(\frac{\left(e^{8f(\rho)}-e^{8f(\rho_0)}\right)^{\frac{3}{2}}}{e^{8f(\rho)+k(\rho)}}\frac{d}{d\rho}\right)+\bar{\omega}^2e^{k(\rho)}\sqrt{e^{8f(\rho)}-e^{8f(\rho_0)}}\right]\delta x_1(\rho)=0, 0<\rho_0\leq\rho<\infty$$

$$\left[e^{-k(\rho)}\frac{d}{d\rho}\left(e^{-k(\rho)}\frac{d}{d\rho}\right)+\bar{\omega}^2\frac{e^{8f_\infty}}{e^{8f_\infty}-e^{8f(\rho_0)}}\right]\delta x_1(\rho)=0, \rho\gg 1$$

$$\left[\frac{d^2}{dr^2}+\tilde{\omega}^2\right]\delta x_1(r)=0, r\gg 1$$

$$V(\rho)=2\frac{\rho^4-1}{\rho^2}, \rho\in[1,\infty)$$

$$y(\rho)=y_0-\frac{1}{4}\,\mathrm{B}\left(\frac{1}{\rho^4};\frac{1}{4},\frac{1}{2}\right)$$

$$V(\rho,\rho_0)=2\frac{\rho^8(\rho^4-\rho_0^4)-\rho_0^4(4\rho^4-1)-3\rho^4}{\rho^6(\rho^4-1)}, 1<\rho_0\leq\rho<\infty$$

$$V(r,r_0)=\frac{e^{-2\phi(r)}}{4}\left(\left(e^{2\phi(r)}-3e^{2\phi(r_0)}\right)\phi^2(r)+2\left(e^{2\phi(r)}+e^{2\phi(r_0)}\right)\phi''(r)\right), 0<r_0\leq r<\infty$$

$$\delta x_1=\frac{e^{\frac{\phi(r)}{2}}}{\left(e^{2\phi(r)}-e^{2\phi(r_0)}\right)^{\frac{1}{2}}}\Psi\simeq e^{-\frac{r}{2}}\Psi, r\to\infty$$

$$V(r,r_0)=-\frac{3K(r)}{8h^3(r)h(r_0)}\left[4h(r)\left(h(r)+h(r_0)\right)h'(r)k'(r)\right.$$
$$\left.-k(r)\left(3h(r)+7h(r_0)\right)h'^2(r)+4h(r)\left(h(r)+h(r_0)\right)h''(r)\right)\Big]$$

$$V(\rho,\rho_0)=\frac{2}{e^{8f(\rho)+2k(\rho)}}\left(2\left(e^{8f(\rho)}-e^{8f(\rho_0)}\right)f'^2(\rho)+\left(e^{8f(\rho)}+e^{8f(\rho_0)}\right)\left(f''(\rho)-k'(\rho)f'(\rho)\right)\right)$$

$$\mathcal{M}_4=[t,x,r(x),\theta=\tilde{\theta},\varphi=2\pi-\tilde{\varphi},\psi=\pi]$$

$$ds_{ind}^2=\alpha' Ne^{\phi}\left[-dt^2+(1+\dot{r}^2)dx^2+\left(e^{2h}+\frac{1}{4}(1-a)^2\right)(d\theta^2+\sin^2\,\theta d\varphi^2)\right]$$

$$V_{S^2}(r) \equiv \frac{1}{4}(1-a(r))^2 + e^{2h(r)} = r\tanh\, r$$

$$S_{DBI} = -T_{D3}\int d^4\sigma e^{-\phi}\sqrt{g_{ind}}$$

$$S_{eff} = 4\pi T_{D3}\mathcal{T}(\alpha' N)^2\int e^{\phi} r\tanh\, r\sqrt{1+r^2}dx$$

$$S_B = -T_{Dp}\int d^{p+1}\sigma e^{-\varphi}\sqrt{-\det(g+2\pi\alpha' F)} + T_{Dp}\int P[C_{p+1}]$$

$$S_{Dp}^{(F)} = \frac{T_{Dp}}{2}\int d^{p+1}\sigma\sqrt{-\mathrm{Det}(M)}\bar{\Theta}(1-\Gamma_{Dp})(\tilde{M}^{-1})^{\alpha\beta}\Gamma_\beta D_\alpha \Theta$$

$$D_\alpha = (\partial_\alpha x^m)\left(\nabla_m + \frac{1}{16\cdot 5!}F_{npqrt}\Gamma^{npqrt}(i\sigma_2)\Gamma_m\right), \nabla_m = \partial_m + \frac{1}{4}\omega_m^{\frac{np}{m}}\Gamma_{\underline{np}}$$

$$\Gamma_{Dp} = \frac{\sqrt{-\mathrm{Det}(g)}}{\sqrt{-\mathrm{Det}(g+F)}}\Gamma_{Dp}^{(0)} \otimes (\sigma_3)^{\frac{p+1}{2}}(-i\sigma_2)\sum_q \Gamma^{\alpha_1\ldots\alpha_{2q}}F_{\alpha_1\alpha_2}\ldots F_{\alpha_{2q-1}\alpha_{2q}} \otimes \frac{\sigma_3^q}{2^q q!}$$

$$\Gamma_{Dp}^{(0)} = \frac{\epsilon^{\alpha_1\ldots\alpha_{p+1}}}{(p+1)!\sqrt{-\mathrm{Det}(g)}}\Gamma_{\alpha_1\ldots\alpha_{p+1}}$$

$$ds^2 = L^2\left(\frac{\cosh^2(u)}{r^2}(-dt^2+dr^2) + du^2 + \sinh^2(u)(d\theta^2 + \sin^2\theta d\phi^2)\right) + L^2(d\alpha_1^2 + \sin^2\alpha_1 d\Omega_4^2)$$

$$F_5 = 4L^4\frac{\sinh^2(u)\cosh^2(u)}{r^2}dt\wedge dr\wedge du\wedge d\theta\wedge d\phi \equiv dC_4$$

$$C_4 = 4L^4\frac{f(u)}{r^2}dt\wedge dr\wedge d\theta\wedge d\phi$$

$$F = \frac{q}{r^2}dt\wedge dr + k\sin\,\theta d\theta\wedge d\phi$$

$$S_B = -T_{D3}\int d^4\sigma\sqrt{-\det(g+F)} + T_{D3}\int P[C_4]$$

$$q = \coth(u_{eq})\sqrt{L^4\sinh^2(u_{eq}) - k^2}$$

$$u = u_{eq} + \delta u, \theta^{\hat{\imath}} = \theta_0^{\hat{\imath}} + \delta\theta^{\hat{\imath}}, F = \frac{q}{r^2}dt\wedge dr + k\sin\,\theta d\theta\wedge d\phi + f$$

$$\begin{aligned} S_B^{(2)} =& \frac{T_{D3}}{4}\int d^4\sigma\Bigg[\frac{1}{r^2}\Bigg(\frac{1}{L^4\sinh^4(u_{eq})+k^2}\Big[2kr^2\sqrt{\sinh^2(u_{eq})L^4 - k^2}(f_{t\phi}f_{r\theta} - f_{t\theta}f_{r\phi} \\ &+ f_{tr}f_{\theta\phi}) + L^4\cosh(u_{eq})\csc(\theta)\sinh^3(u_{eq})\Big(\big(-f_{\theta\phi}^2 + r^2(f_{t\phi}^2 - f_{r\phi}^2)\big) \\ &+ r^2\sin\,\theta^2(f_{t\theta}^2 - f_{r\theta}^2 + r^2 f_{tr}^2)\Big)\Big]\Bigg) \\ &- \frac{L^4\cosh(u_{eq})\sinh(u_{eq})}{r^2}\Big(\csc(\theta)\partial_\phi\delta u^2 + \sin(\theta)\big((\partial_r\delta u^2 - \partial_t\delta u^2)r^2 + \partial_\theta\delta u^2\big)\Big)\Bigg] \end{aligned}$$

$$\hat{g}_{\alpha\beta} = g_{\alpha\beta} - F_{\alpha\gamma}g^{\gamma\delta}F_{\delta\beta} = \frac{\sinh^4(u_{eq})L^4 + k^2}{L^2\sinh^2(u_{eq})}\begin{pmatrix} -\frac{1}{r^2} & 0 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sin^2\theta \end{pmatrix}$$

$$S_B^{(2)} = \frac{T_{D3}}{2}\left(\frac{L^4\cosh\left(u_{eq}\right)\sinh^3\left(u_{eq}\right)}{L^4\sinh^4\left(u_{eq}\right)+k^2}\right)\int d^4\sigma\left[\sqrt{-\hat{g}}\left(L^2\hat{g}^{\alpha\beta}\partial_\alpha\delta u\partial_\beta\delta u+\frac{1}{2}\hat{g}^{\alpha\beta}\hat{g}^{\gamma\delta}f_{\alpha\gamma}f_{\beta\delta}\right)\right.$$
$$\left.+\frac{k\sqrt{\sinh^2\left(u_{eq}\right)L^4-k^2}}{L^4\cosh\left(u_{eq}\right)\sinh^3\left(u_{eq}\right)}\left(f_{t\phi}f_{r\theta}-f_{t\theta}f_{r\phi}+f_{tr}f_{\theta\phi}\right)\right]$$
$$=\frac{T_{D3}}{2}\left(\frac{L^4\cosh\left(u_{eq}\right)\sinh^3\left(u_{eq}\right)}{L^4\sinh^4\left(u_{eq}\right)+k^2}\right)\int d^4\sigma\sqrt{-\hat{g}}\left(L^2\hat{g}^{\alpha\beta}\partial_\alpha\delta u\partial_\beta\delta u+\frac{1}{2}\hat{g}^{\alpha\beta}\hat{g}^{\gamma\delta}f_{\alpha\gamma}f_{\beta\delta}\right.$$
$$\left.+\frac{kq}{2L^4\sinh^2\left(2u_{eq}\right)}\hat{\epsilon}^{\alpha\beta\gamma\delta}f_{\alpha\beta}f_{\gamma\delta}\right),$$
$$S_F^{(2)} = \frac{T_{D3}}{2}\left(\frac{L^4\cosh\left(u_{eq}\right)\sinh^3\left(u_{eq}\right)}{L^4\sinh^4\left(u_{eq}\right)+k^2}\right)\int d^4\sigma\sqrt{\hat{g}}\bar{\Theta}\hat{\Gamma}^\alpha\hat{\nabla}_\alpha\Theta$$
$$\langle W\rangle \simeq e^{-A}$$
$$\mathcal{S}_\mathcal{A} = -\mathrm{Tr}_\mathcal{A}(\rho_\mathcal{A}\ln\,\rho_\mathcal{A})$$
$$\mathcal{S}_\mathcal{A} = \frac{2\pi\mathrm{Area}(\gamma_\mathcal{A})}{\kappa^2}$$
$$\kappa_{(4)}^2\mathcal{L} = R-2\Lambda-\frac{1}{4}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$
$$F_{\mu\nu}^a = \partial_\mu A_\nu^a-\partial_\nu A_\mu^a+g_{YM}\epsilon^{abc}A_\mu^b A_\nu^c$$
$$G_{\mu\nu} = R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R = \frac{3}{R^2}g_{\mu\nu}+\frac{1}{2}\mathrm{Tr}\left[F_{\mu\gamma}F_\nu^\gamma\right]-\frac{g_{\mu\nu}}{8}\mathrm{Tr}\left[F_{\gamma\rho}F^{\gamma\rho}\right]$$
$$D_\mu F^{\mu\nu} = 0$$
$$ds^2 = -M(r)\sigma(r)^2dt^2+\frac{1}{M(r)}dr^2+r^2h(r)^2dx^2+r^2h(r)^{-2}dy^2$$
$$A = \phi(r)\tau^3dt+\omega(r)\tau^1dx$$
$$\omega(r) = 0$$
$$h(r) = 1$$
$$\sigma(r) = 1$$
$$\phi(r) = \mu\left(1-\frac{r_h}{r}\right)$$
$$M(r) = r^2+\frac{\mu^2r_h^2}{r^2}-\left(\frac{\mu^2}{8}+r_h^2\right)\frac{r_h}{r}$$
$$M = M_1(r-r_h)+M_2(r-r_h)^2+\cdots$$
$$h = h_0+h_2(r-r_h)^2+\cdots$$
$$\sigma = \sigma_0+\sigma_1(r-r_h)+\sigma_2(r-r_h)^2+\cdots$$
$$\omega = \omega_0+\omega_2(r-r_h)^2+\omega_3(r-r_h)^3+\cdots$$
$$\phi = \phi_1(r-r_h)+\phi_2(r-r_h)^2+\cdots$$
$$M' = \frac{3r}{\hat{R}^2}-\frac{1}{8\sigma^2}\left(\frac{g_{YM}^2\phi^2\omega^2}{rh^2M}+r\phi'^2\right)-M\left(\frac{1}{r}+\frac{rh'^2}{h^2}+\frac{\omega'^2}{8rh^2}\right)$$
$$\sigma' = \frac{\sigma}{h^2}\left(rh'^2+\frac{\omega'^2}{8r}\right)+\frac{g_{YM}^2\phi^2\omega^2}{8rM^2h^2\sigma};$$
$$h'' = \frac{1}{8r^2h}\left(-\omega'^2+\frac{g_{YM}^2\phi^2\omega^2}{M^2\sigma^2}\right)-h'\left(\frac{2}{r}-\frac{h'}{h}+\frac{M'}{M}+\frac{\sigma'}{\sigma}\right);$$
$$\omega'' = -\frac{g_{YM}^2\phi^2\omega}{M^2\sigma^2}+\omega'\left(\frac{2h'}{h}-\frac{M'}{M}-\frac{\sigma'}{\sigma}\right);$$
$$\phi'' = \frac{g_{YM}^2\phi\omega^2}{r^2h^2M}-\phi'\left(\frac{2}{r}-\frac{\sigma'}{\sigma}\right)$$

$$\sigma \to \lambda\sigma, \phi \to \lambda\phi$$
$$\omega \to \lambda\omega, h \to \lambda h$$
$$M \to \lambda^{-2}M, \sigma \to \lambda\sigma, g_{YM} \to \lambda^{-1}g_{YM}, \hat{R} \to \lambda\hat{R}$$
$$M \to \lambda^2 M, r \to \lambda r, \phi \to \lambda\phi, \omega \to \lambda\omega$$
$$T = \frac{M_1\sigma_0}{2\pi} = \frac{1}{16\pi}(24\sigma_0^2 - \phi_1^2)r_h$$
$$S = \frac{2\pi}{\kappa_{(4)}^2}A_h = \frac{2\pi^2 VT^2}{\kappa_{(4)}^2}\frac{12^2}{(24\sigma_0^2 - \phi_1^2)^2}$$
$$\tilde{S}_{bulk} = -\int dxdydr\sqrt{-g}\mathcal{L}$$
$$G_{yy} = \frac{r^2}{2h^2}\left(\kappa_{(4)}^2\mathcal{L} - R\right)$$
$$G_\mu^\mu = -R = G_r^r + G_t^t + G_x^x + \frac{1}{2}\left(\kappa_{(4)}^2\mathcal{L} - R\right)$$
$$\mathcal{L} = \frac{2}{r^2\sigma\kappa_{(4)}^2}\left[\frac{r^3 M\sigma}{h}\left(\frac{h}{r}\right)'\right]'$$
$$\tilde{S}_{bulk} = -\int dxdydr\sqrt{-g}\mathcal{L} = -\frac{2V}{\kappa_{(4)}^2}\left[\frac{r^3 M\sigma}{h}\left(\frac{h}{r}\right)'\right]_{r=r_\infty}$$
$$\tilde{S}_{GH} = -\frac{1}{\kappa_{(4)}^2}\int dxdy\sqrt{-g_\infty}\nabla_\mu n^\mu = -\frac{V}{\kappa_{(4)}^2}r^2\sigma\left[\frac{M'}{2} + M\left(\frac{\sigma'}{\sigma} + \frac{2}{r}\right)\right]_{r=r_\infty}$$
$$\tilde{S}_{ct} = \frac{1}{\kappa_{(4)}^2}\int dxdy\sqrt{-g_\infty} = \frac{V}{\kappa_{(4)}^2}\left[r^2\sqrt{M}\sigma\right]_{r=r_\infty}$$

$$\Omega = \lim_{r_\infty \to \infty} \tilde{S}_{\text{on-shell}}$$
$$= \lim_{r_\infty \to \infty}\left(\tilde{S}_{bulk} + \tilde{S}_{GH} + \tilde{S}_{ct}\right)$$
$$ds^2 = -M(r)dt^2 + r^2 h(r)^2(dx^2 + dy^2) + \frac{dr^2}{M(r)}$$
$$A = \phi(r)\tau^3 dt + \omega(r)(\tau^1 dx + \tau^2 dy)$$
$$h'' = -\frac{h}{2}\left[\frac{1}{r^2} - \frac{3}{\hat{R}^2 M} + \frac{M'}{rM} + \frac{\phi'^2}{8M} + \frac{\omega'^2}{4r^2h^2}\right] - \frac{h'}{2}\left[\frac{6}{r} + \frac{h'}{h} + \frac{M'}{M}\right] - \frac{g_{YM}^2\omega^2}{8r^2hM}\left[\frac{\phi^2}{M} + \frac{\omega^2}{2r^2h^2}\right]$$
$$M'' = \frac{3}{\hat{R}^2} + \frac{M}{r}\left[-\frac{M'}{M} + \frac{1}{r} + \frac{\omega'^2}{4rh^2}\right] - \frac{h'}{h}\left[M' - \frac{h'}{h} - \frac{2}{r}\right] + \frac{3}{8}\phi'^2 + \frac{g_{YM}^2\omega^2}{4r^2h^2}\left[\frac{\phi^2}{M} + \frac{3\omega^2}{2r^2h^2}\right]$$
$$\omega'' = \frac{g_{YM}^2\omega}{M}\left[\frac{\omega^2}{r^2h^2} - \frac{\phi^2}{M}\right] - \frac{M'\omega'}{M}$$
$$\phi'' = \frac{2g_{YM}^2\phi\omega^2}{r^2h^2M} - 2\phi'\left[\frac{1}{r} + \frac{h'}{h}\right]$$
$$0 = -\frac{3}{\hat{R}^2} + \frac{M}{r^2}\left[1 - \frac{\omega'^2}{4h^2} + \frac{M'}{M}r\right] + \frac{h'}{h}\left[M\left(\frac{2}{r} + \frac{h'}{h}\right) + M'\right] + \frac{1}{8}\phi'^2$$
$$\omega \to \lambda\omega, h \to \lambda h$$
$$M \to \lambda^{-2}M, \phi \to \frac{\phi}{\lambda}, \hat{R} \to \lambda\hat{R}, g_{YM} \to \frac{g_{YM}}{\lambda}$$
$$M \to \lambda^2 M, h \to \frac{h}{\lambda}, \phi \to \lambda\phi, r \to \lambda r$$
$$M = M_1(r - r_h) + M_2(r - r_h)^2 + \cdots$$
$$h = h_0 + h_1(r - r_h) + h_2(r - r_h)^2 + \cdots$$
$$\omega = \omega_0 + \omega_1(r - r_h) + \omega_2(r - r_h)^2 + \cdots$$
$$\phi = \phi_1(r - r_h) + \phi_2(r - r_h)^2 + \cdots$$

$$M = r^2 + 2h_1^b r + \left(h_1^b\right)^2 + \frac{M_1^b}{r} + \frac{-8h_1^b M_1^b + \rho^2 + 2\left(\omega_1^b\right)/3}{8r^2} + \cdots$$

$$h = 1 + \frac{h_1^b}{r} - \frac{\left(\omega_1^b\right)^2}{48r^4} + \cdots$$

$$\omega = \frac{\omega_1^b}{r} - \frac{h_1^b \omega_1^b}{r^2} + \cdots$$

$$\phi = \mu + \frac{\rho}{r} - \frac{\rho h_1^b}{r^2} + \cdots$$

$$T = \frac{M_1}{2\pi}$$

$$S = \frac{2\pi}{\kappa_{(4)}^2} A_h = \frac{2\pi}{\kappa_{(4)}^2} r_h^2 h_0^2$$

$$\Omega = \frac{V M_1^b}{\kappa_{(4)}^2}$$

$$\mathcal{S}_{\mathcal{A}} = \frac{2\pi}{\kappa_{(d+1)}^2} \int_{\gamma_{\mathcal{A}}} d^{(d-1)}\sigma \sqrt{g_{\text{ind}}^{(d-1)}}$$

$$ds_{d+1}^2 = -g_{tt}(r)dt^2 + g_{x_i x_i}(r)dx_i^2 + g_{rr}(r)dr^2, i = 1 \ldots d-1$$

$$\mathcal{S}_{\mathcal{A}} = \frac{2\pi\Lambda}{\kappa_{(d+1)}^2} \int d\zeta \sqrt{g_{x_2 x_2}(r) \ldots g_{x_{d-1} x_{d-1}}(r)} \sqrt{g_{rr}(r) r'^2 + g_{x_1 x_1}(r) x_1'^2}$$

$$f^2(r) = g_{\chi\chi}(r) g_{x_1 x_1}(r), \eta^2(r) = g_{\chi\chi}(r) g_{rr}(r)$$

$$\mathcal{S}_{\mathcal{A}} = \frac{2\pi\Lambda}{\kappa_{(d+1)}^2} \int d\zeta \sqrt{\eta^2(r) r'^2 + f^2(r) x_1'^2}$$

$$x_1'(\zeta) = \pm \frac{f(r_0)\eta(r)}{f(r)} \frac{r'(\zeta)}{\sqrt{f^2(r) - f^2(r_0)}}$$

$$L = 2\int_{r_0}^{\infty} dr \frac{dx_1}{dr} = 2\int_{r_0}^{\infty} dr \frac{\eta(r)}{f(r)} \frac{f(r_0)}{\sqrt{f^2(r) - f^2(r_0)}}$$

$$\mathcal{S}_{\mathcal{A}}(r_0) = 2\frac{2\pi\Lambda}{\kappa_{(d+1)}^2} \int_{r_0}^{\infty} dr \frac{f(r)\eta(r)}{\sqrt{f(r)^2 - f(r_0)^2}}$$

$$\mathcal{S}_{\mathcal{A}_{disc}} = 2\frac{2\pi\Lambda}{\kappa_{(d+1)}^2} \int_{r_{min}}^{\infty} dr \eta(r)$$

$$\Delta\mathcal{S}_{\mathcal{A}} = \frac{4\pi\Lambda}{\kappa_{(d+1)}^2} \left(\int_{r_0}^{\infty} dr \frac{f(r)\eta(r)}{\sqrt{f(r)^2 - f(r_0)^2}} - \int_{r_{\min}}^{\infty} dr \eta(r) \right)$$

$$f_p^2(r) = g_{yy} g_{xx} = r^4, \eta_p^2(r) = g_{yy} g_{rr} = \frac{r^2}{h^2 N}$$

$$\Delta\mathcal{S}_{\mathcal{A}} = \frac{4\pi\Lambda}{\kappa_{(4)}^2} \left(\int_{r_0}^{\infty} dr \frac{r^3}{h\sqrt{N}\sqrt{r^4 - r_0^4}} - \int_{r_{\min}}^{\infty} dr \frac{r}{h\sqrt{N}} \right)$$

$$\mathcal{S}_{\mathcal{A}}(r_0) = \frac{4\pi\Lambda}{\kappa_{(4)}^2} \int_{r_0}^{\mathcal{R}} dr \frac{r^3}{h\sqrt{N}\sqrt{r^4 - r_0^4}} = \mathcal{S}_{\mathcal{A}} + \frac{4\pi\Lambda}{\kappa_{(d+1)}^2} \mathcal{R}$$

$$f_{p+ip}^2(r) = g_{yy} g_{xx} = r^4 h^4, \eta_{p+ip}^2(r) = g_{yy} g_{rr} = \frac{r^2 h^2}{M}$$

$$\Delta\mathcal{S}_{\mathcal{A}} = \frac{4\pi\Lambda}{\kappa_{(4)}^2} \left(\int_{r_0}^{\infty} dr \frac{r^3 h^3}{\sqrt{M}\sqrt{r^4 h^4 - r_0^4 h(r_0)^4}} - \int_{r_{\min}}^{\infty} dr \frac{rh}{\sqrt{M}} \right)$$

$$\mathcal{S}_{\mathcal{A}}(r_0) = \frac{4\pi\Lambda}{\kappa_{(4)}^2} \int_{r_0}^{\mathcal{R}} dr \frac{r^3 h^3}{\sqrt{M}\sqrt{r^4 h^4 - r_0^4 h(r_0)^4}} = \mathcal{S}_{\mathcal{A}} + \frac{4\pi\Lambda}{\kappa_{(4)}^2} \mathcal{R}$$

$$\left[-\frac{d}{dr}\left(P(r, r_0) \frac{d}{dr} \right) + U(r, r_0) \right] \Phi(r) = \omega^2 Q(r, r_0) \Phi(r), r_0 \le r < \infty$$

$$y=\int_{r_0}^{r}\sqrt{\frac{Q}{P}}dr, \Phi(r)=(PQ)^{-\frac{1}{4}}\Psi(y)$$
$$\left[-\frac{d^2}{dy^2}+V\right]\Psi=\omega^2\Psi, 0\le y\le y_0$$
$$V=\frac{U}{Q}+\left[(PQ)^{-\frac{1}{4}}\frac{d^2}{dy^2}\right](PQ)^{\frac{1}{4}}$$
$$=\frac{U}{Q}+\left[\frac{P^{\frac{1}{4}}}{Q^{\frac{3}{4}}}\frac{d}{dr}\left(\sqrt{\frac{P}{Q}}\frac{d}{dr}\right)\right](PQ)^{\frac{1}{4}}$$
$$\delta x|_{r=\infty}=0\Rightarrow \Psi|_{y=y_0}=0$$
$$\text{Even solutions}: \left.\frac{d\Psi}{dy}\right|_{y=0}=0$$
$$\text{Odd solutions}: \Psi|_{y=0}=0$$
$$ds^2=\frac{R^2}{z^2}(-dt^2+dx_idx_i+dz^2)+R^2d\Omega_5^2$$
$$\left(\frac{dz_{\text{cl}}}{dx}\right)^2=\frac{z_0^4-(z_{\text{cl}})^4}{(z_{\text{cl}})^4}$$
$$x_{\text{cl}}(z)=\pm z_0\left[\frac{(2\pi)^{\frac{3}{2}}}{2\Gamma\left[\frac{1}{4}\right]^2}-\frac{1}{4}\,\mathrm{B}\left(\frac{z^4}{z_0^4};\frac{3}{4},\frac{1}{2}\right)\right]$$
$$x\text{-gauge}: \left[\partial_t^2-\frac{z_{\text{cl}}^4(x)}{z_0^4}\partial_x^2\right]\delta x_m(t,x)=0$$
$$r\text{-gauge}: \left[\partial_t^2-\left(1-\frac{z^4}{z_0^4}\right)\partial_z^2+\frac{2}{z}\partial_z\right]\delta x_m(t,z)=0\ m=2,3$$
$$\left[(1-\tilde{z}^4)\partial_{\tilde{z}}^2-\frac{2}{\tilde{z}}+\xi^2\right]f(\tilde{z})=0, 0\le\tilde{z}\le 1$$
$$f(\tilde{z})=\sqrt{1+\xi^2\tilde{z}^2}F(q)$$
$$q(\tilde{z})=\pm 2\int_{\tilde{z}}^{1}\frac{t^2}{(1+(\xi t)^2)\sqrt{1-t^4}}dt$$
$$\frac{d^2F}{dq}+\frac{1}{4}\xi^2(\xi^4-1)F=0, q\in[-q_*,q_*]$$
$$\omega_n z_0\sqrt{\omega_n^4z_0^4-1}\int_0^1\frac{t^2dt}{(1+w_n^2z_0^2)\sqrt{1-t^4}}=\frac{n\pi}{2}, n=1,2,\ldots$$
$$\tilde{M}^{\alpha\beta}=\frac{L^2\sinh^2(u_{eq})}{k^2+L^4\sinh^4(u_{eq})}\begin{pmatrix}-r^2 & \frac{2r^2\sqrt{L^4\sinh^2(u_{eq})-k^2}}{L^2\sinh(2u_{eq})}\tilde{\Gamma} & 0 & 0\\ -\frac{2r^2\sqrt{L^4\sinh^2(u_{eq})-k^2}}{L^2\sinh(2u_{eq})}\tilde{\Gamma} & r^2 & 0 & 0\\ 0 & 0 & 1 & -\frac{k}{L^2\sin\theta\sinh^2(u_{eq})}\tilde{\Gamma}\\ 0 & 0 & \frac{k}{L^2\sin\theta\sinh^2(u_{eq})}\tilde{\Gamma} & \frac{1}{\sin^2\theta}\end{pmatrix}$$
$$\Gamma_t=\frac{L\cosh(u_k)}{r}\Gamma_{\underline{0}}, \Gamma_r=\frac{L\cosh(u_k)}{r}\Gamma_{\underline{1}}$$
$$\Gamma_\theta=L\sinh(u_k)\Gamma_{\underline{2}}, \Gamma_\phi=L\sinh(u_k)\sin\theta\Gamma_{\underline{3}}$$

$$\tilde{M}^{\alpha\beta}\Gamma_\beta D_\alpha = \frac{L\sinh(u_{eq})}{\sqrt{k^2+L^4\sinh^4(u_{eq})}}\left[-re^{2R_e\tilde{\Gamma}}\Gamma_{\underline{0}}D_t + re^{2R_e\tilde{\Gamma}}\Gamma_{\underline{1}}D_r + e^{2R_m\tilde{\Gamma}}\Gamma_{\underline{2}}D_\theta + \frac{1}{\sin\,\theta}e^{2R_m\tilde{\Gamma}}\Gamma_{\underline{3}}D_\phi\right]$$

$$R_e = -\frac{1}{2}\sinh^{-1}\left(\sqrt{\frac{L^4\sinh^2(u_{eq})-k^2}{L^4\sinh^4(u_{eq})+k^2}}\right)\Gamma_{01}, R_m = \frac{1}{2}\arcsin\left(\frac{k}{\sqrt{L^4\sinh^4(u_{eq})+k^2}}\right)\Gamma_{23}$$

$$\hat{e}^{\underline{0}} = \frac{\sqrt{\sinh^4(u_{eq})L^4+k^2}}{Lr\sinh(u_{eq})}dt, \hat{e}^{\underline{1}} = \frac{\sqrt{\sinh^4(u_{eq})L^4+k^2}}{Lr\sinh(u_{eq})}dr$$

$$\hat{e}^{\underline{2}} = \frac{\sqrt{\sinh^4(u_{eq})L^4+k^2}}{L\sinh(u_{eq})}d\theta, \hat{e}^{\underline{3}} = \frac{\sqrt{\sinh^4(u_{eq})L^4+k^2}}{L\sinh(u_{eq})}\sin\,\theta d\phi$$

$$\tilde{M}^{\alpha\beta}\Gamma_\beta D_\alpha = e^{\mathcal{R}\tilde{\Gamma}}[\hat{\Gamma}^\alpha e^{\mathcal{R}\tilde{\Gamma}}D_\alpha e^{-\mathcal{R}\tilde{\Gamma}}]e^{\mathcal{R}\tilde{\Gamma}}$$

$$\nabla_\alpha d\sigma^\alpha = d + \frac{1}{4}\omega\underline{\mu\nu}\Gamma_{\underline{\mu\nu}} + \frac{1}{4}\omega^{\underline{ij}}\Gamma_{\underline{ij}} + \frac{1}{2}\sinh(u_k)e^{\underline{\mu}}\Gamma_{\underline{\mu 4}} + \frac{1}{2}\cosh(u_k)e^{-}\Gamma_{\underline{i}}$$

$$e^{\mathcal{R}\tilde{\Gamma}}\nabla_\alpha d\sigma^\alpha e^{-\mathcal{R}\tilde{\Gamma}} = \hat{\nabla}_\alpha d\sigma^\alpha + \frac{1}{2}\sinh(u_k)e^{\underline{\mu}}\Gamma_{\underline{\mu 4}}e^{-2R_e\tilde{\Gamma}} + \frac{1}{2}\cosh(u_k)e^{\underline{i}-\Gamma_{\underline{i4}}e^{-2R_m\tilde{\Gamma}}},$$

$$\tilde{M}^{\alpha\beta}\Gamma_\beta\nabla_\alpha = e^{\mathcal{R}\tilde{\Gamma}}\left[\hat{\Gamma}^\alpha\hat{\nabla}_\alpha + \frac{L\sinh(u_{eq})}{\sqrt{L^4\sinh^4(u_{eq})+k^2}}\Gamma_4\left(\sinh(u_{eq})e^{-2R_e\tilde{\Gamma}} + \cosh(u_{eq})e^{-2R_m\tilde{\Gamma}}\right)\right]e^{\mathcal{R}\tilde{\Gamma}}$$

$$F_5\Gamma_\alpha d\sigma^\alpha \otimes (i\sigma_2) = -4\Gamma_{\underline{01234}} \otimes (i\sigma_2)\left(\cosh(u_{eq})e^{\underline{\mu}}\Gamma_{\underline{\mu}} + \sinh(u_{eq})e^{\underline{i}}\Gamma_{\underline{i}}\right)(1+\Gamma^{11})$$

$$\tilde{M}^{\alpha\beta}\Gamma_\beta F_5\Gamma_\alpha \otimes (i\sigma_2)$$
$$= -16e^{\mathcal{R}\tilde{\Gamma}}\Gamma_{\underline{01234}}$$
$$\otimes (i\sigma_2)\frac{L\sinh(u_{eq})}{\sqrt{L^4\sinh^4(u_{eq})+k^2}}(\cosh(u_{eq})e^{-2R_m\tilde{\Gamma}} + \sinh(u_{eq})e^{-2R_e\tilde{\Gamma}})e^{\mathcal{R}\tilde{\Gamma}}$$

$$\tilde{M}^{\alpha\beta}\Gamma_\beta D_\alpha = e^{\mathcal{R}\tilde{\Gamma}}\left[\hat{\Gamma}^\alpha\hat{\nabla}_\alpha\right.$$
$$\left. + \left(1-\Gamma_{D3}^{(0)}\right)\frac{L\sinh(u_{eq})}{\sqrt{L^4\sinh^4(u_{eq})+k^2}}\Gamma_{\underline{4}}\left(\cosh(u_{eq})e^{-2R_m\tilde{\Gamma}} + \sinh(u_{eq})e^{-2R_e\tilde{\Gamma}}\right)\right]e^{\mathcal{R}\tilde{\Gamma}}$$

$$\Gamma_{D3}^{(0)} = \Gamma_{\underline{0123}} \otimes (i\sigma_2)$$

$$\Gamma_{D3} = -\frac{\epsilon^{\alpha_1\alpha_2\alpha_3\alpha_4}\Gamma_{\alpha_1\alpha_2\alpha_3\alpha_4}}{(p+1)!\sqrt{\det(g+F)}} \otimes (i\sigma_2) \times \sum_q \Gamma^{\alpha_1\cdots\alpha_{2q}}F_{\alpha_1\alpha_2}\cdots F_{\alpha_{2q-1}\alpha_{2q}} \otimes \frac{(\sigma_3)^q}{q!\,2^q}$$

$$\Gamma_{D3} = -\Gamma_{D3}^{(0)}\frac{L^4\cosh(u_{eq})\sinh^3(u_{eq})}{L^4\sinh^4(u_{eq})+k^2}\left[1 + \left(\frac{k}{L^2\sinh^2(u_{eq})}\Gamma_{\underline{23}} - \frac{\sqrt{L^4\sinh^2(u_{eq})-k^2}}{L^2\cosh(u_{eq})\sinh(u_{eq})}\Gamma_{\underline{01}}\right) \otimes \sigma_3\right].$$

$$\bar{\Theta}\Gamma_{D3} = -\bar{\Theta}e^{\mathcal{R}\tilde{\Gamma}}\Gamma_{D3}^{(0)}e^{-\mathcal{R}\tilde{\Gamma}}$$

$$\mathcal{L}_F = \bar{\Theta} e^{\mathcal{R}\tilde{\Gamma}} \left(1 + \Gamma_{D3}^{(0)}\right) \left[\hat{\Gamma}^{\alpha} \hat{\nabla}_{\alpha} + \left(1 - \Gamma_{D3}^{(0)}\right) \frac{L \sinh\left(u_{eq}\right)}{\sqrt{L^4 \sinh^4\left(u_{eq}\right) + k^2}} \Gamma_{\underline{4}} \left(\cosh\left(u_{eq}\right) e^{-2R_m \tilde{\Gamma}} + \sinh\left(u_{eq}\right) e^{-2R_e \tilde{\Gamma}}\right) \right] e^{\mathcal{R}\tilde{\Gamma}} \Theta.$$

$$\mathcal{L}_F = \bar{\Theta}' \left(1 + \Gamma_{D3}^{(0)}\right) \hat{\Gamma}^{\alpha} \hat{\nabla}_{\alpha} \Theta'.$$

$$\tilde{\Gamma}\Theta' = \Theta'$$

$$\bar{\Theta}(1 - \Gamma_{D3}) \tilde{M}^{\alpha\beta} \Gamma_{\beta} D_{\alpha} \Theta = \bar{\Theta} \hat{\Gamma}^{\alpha} \hat{\nabla}_{\alpha} \Theta$$

$$S_F^{(2)} = \frac{T_{D3}}{2} \left(\frac{L^4 \cosh\left(u_{eq}\right) \sinh^3\left(u_{eq}\right)}{L^4 \sinh^4\left(u_{eq}\right) + k^2} \right) \int d^4\sigma \sqrt{\hat{g}}\, \bar{\Theta} \hat{\Gamma}^{\alpha} \hat{\nabla}_{\alpha} \Theta$$

$$ds^2 = \frac{g_{\mu\nu}(x^{\rho}, z) dx^{\mu} dx^{\nu} + dz^2}{z^2} \equiv g_{\alpha\beta}^{5D} dx^{\alpha} dx^{\beta}$$

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta}^{5D} R - 6 g_{\alpha\beta}^{5D} = 0$$

$$g_{\mu\nu}(x^{\rho}, z) = \eta_{\mu\nu} + g_{\mu\nu}^{(4)}(x^{\rho}) z^4 + \mathcal{O}(z^6) \left\langle T_{\mu\nu}(x^{\rho}) \right\rangle = \frac{N_c^2}{2\pi^2} g_{\mu\nu}^{(4)}(x^{\rho})$$

$$\varepsilon(\tau) \equiv N_c^2 \cdot \frac{3}{8} \pi^2 \cdot T_{eff}^4$$

$$\Delta p_L \equiv 1 - \frac{p_L}{\varepsilon/3} \sim 0.7$$

$\langle T_{--}(x^-) \rangle \propto \mu \delta(x^-)$ o más generalmente $\langle T_{--}(x^-) \rangle \propto f(x^-)$

$$ds^2 = \frac{-dx^- dx^+ + f(x^-) z^4 dx^{-2} + dx_{\perp}^2}{z^2} + \frac{dz^2}{z^2}$$

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Apéndice I.

1. Supergravedad cuántica, supermembranas, agujeros negros cuánticos y supersimetrías en campos cuánticos relativistas. Formalización matemática.

$$\Gamma^{012}\varepsilon=-\varepsilon, \Gamma^{013456}\varepsilon=\varepsilon$$
$$\delta\psi_\mu \equiv \nabla_\mu\epsilon+\frac{1}{288}\left(\Gamma_\mu^{\nu\rho\lambda\sigma}-8\delta_\mu^\nu\Gamma^{\rho\lambda\sigma}\right)F_{\nu\rho\lambda\sigma}\epsilon$$
$$\Gamma^{012345678910}=\mathbb{1}$$
$$\Gamma^{0178910}\varepsilon=-\varepsilon$$
$$ds_{11}^2=e^{2A_0}[-dt^2\ +dy^2+e^{-3A_0}(-\partial_z w)^{-\frac{1}{2}}d\vec{u}\cdot d\vec{u}+e^{-3A_0}(-\partial_z w)^{\frac{1}{2}}d\vec{v}\cdot d\vec{v}$$
$$+(-\partial_z w)\left(dz+(\partial_z w)^{-1}\left(\vec{\nabla}_{\vec{u}}w\right)\cdot d\vec{u}\right)^2\Big]$$
$$e^0=e^{A_0}dt, e^1=e^{A_0}dy, e^2=e^{A_0}(-\partial_z w)^{\frac{1}{2}}\left(dz+(\partial_z w)^{-1}\left(\vec{\nabla}_{\vec{u}}w\right)\cdot d\vec{u}\right)$$
$$e^{i+2}=e^{-\frac{1}{2}A_0}(-\partial_z w)^{-\frac{1}{4}}du_i, e^{i+6}=e^{-\frac{1}{2}A_0}(-\partial_z w)^{\frac{1}{4}}dv_i, i=1,2,3,4$$
$$C^{(3)}=-e^0\wedge e^1\wedge e^2+\frac{1}{3!}\epsilon_{ijk\ell}\left((\partial_z w)^{-1}\left(\partial_{u_\ell}w\right)du^i\wedge du^j\wedge du^k-\left(\partial_{v_\ell}w\right)dv^i\wedge dv^j\wedge dv^k\right)$$
$$\mathcal{L}_u\equiv\nabla_{\vec{u}}\cdot\nabla_{\vec{u}}, \mathcal{L}_v\equiv\nabla_{\vec{v}}\cdot\nabla_{\vec{v}}$$
$$\mathcal{L}_v G_0=(\mathcal{L}_u G_0)(\partial_z\partial_z G_0)-(\nabla_{\vec{u}}\partial_z G_0)\cdot(\nabla_{\vec{u}}\partial_z G_0)$$
$$w=\partial_z G_0, e^{-3A_0}(-\partial_z w)^{\frac{1}{2}}=\mathcal{L}_v G_0$$
$$e^{-3A_0}(\partial_z w)^{-\frac{1}{2}}-(\partial_z w)^{-1}(\nabla_{\vec{u}}w)\cdot(\nabla_{\vec{u}}w)=-\mathcal{L}_u G_0$$
$$ds_{11}^2=e^{2A_0}\Big[-dt^2+dy^2+(-\partial_z w)(dz+(\partial_z w)^{-1}(\partial_u w)du)^2$$
$$+e^{-3A_0}(-\partial_z w)^{-\frac{1}{2}}(du^2+u^2d\Omega_3^2)+e^{-3A_0}(-\partial_z w)^{\frac{1}{2}}(dv^2+v^2d\Omega_3'^2)\Big]$$
$$e^0=e^{A_0}dt, e^1=e^{A_0}dy, e^2=e^{A_0}(-\partial_z w)^{\frac{1}{2}}(dz+(\partial_z w)^{-1}(\partial_u w)du)$$
$$e^3=e^{-\frac{1}{2}A_0}(-\partial_z w)^{-\frac{1}{4}}du, e^4=e^{-\frac{1}{2}A_0}(-\partial_z w)^{\frac{1}{4}}dv$$
$$e^{i+4}=e^{-\frac{1}{2}A_0}(-\partial_z w)^{-\frac{1}{4}}\sigma_i, e^{i+7}=e^{-\frac{1}{2}A_0}(-\partial_z w)^{\frac{1}{4}}\tilde{\sigma}_i, i=1,2,3$$
$$C^{(3)}=-e^0\wedge e^1\wedge e^2+(\partial_z w)^{-1}(u^3\partial_u w)\mathrm{Vol}(S^3)+(v^3\partial_v w)\mathrm{Vol}(S'^3)$$
$$G_0=-\frac{1}{2}z^2\hat{g}_2(u,v)+z\hat{g}_1(u,v)+\hat{g}_0(u,v)$$
$$\mathcal{L}_{\vec{v}}\hat{g}_2+\hat{g}_2\mathcal{L}_{\vec{u}}\hat{g}_2-2\left(\vec{\nabla}_{\vec{u}}\hat{g}_2\right)^2=0,$$
$$\mathcal{L}_{\vec{v}}\hat{g}_1+\hat{g}_2\mathcal{L}_{\vec{u}}\hat{g}_1-2\left(\vec{\nabla}_{\vec{u}}\hat{g}_1\right)\cdot\left(\vec{\nabla}_{\vec{u}}\hat{g}_2\right)=0,$$
$$\mathcal{L}_{\vec{v}}\hat{g}_0+\hat{g}_2\mathcal{L}_{\vec{v}}\hat{g}_0-\left(\vec{\nabla}_{\vec{u}}\hat{g}_1\right)^2=0.$$
$$\mathcal{L}_{\vec{v}}\hat{g}_2-\hat{g}_2^3\mathcal{L}_{\vec{u}}(\hat{g}_2^{-1})=0$$
$$\hat{g}_2=\frac{h_2(\vec{v})}{h_1(\vec{u})}$$
$$G_0=-\frac{1}{2}z^2\frac{h_2(\vec{v})}{h_1(\vec{u})}+\hat{g}_0(u,v)$$
$$\frac{1}{h_1(\vec{u})}\mathcal{L}_{\vec{u}}\hat{g}_0+\frac{1}{h_2(\vec{v})}\mathcal{L}_{\vec{v}}\hat{g}_0=0$$
$$w=\partial_z G_0=-z\frac{h_2(\vec{v})}{h_1(\vec{u})}$$
$$e^{-A_0}e^2=(-\partial_z w)^{\frac{1}{2}}\left(dz+(\partial_z w)^{-1}\left(\vec{\nabla}_{\vec{u}}w\right)\cdot d\vec{u}\right)=\left(\frac{h_2(\vec{v})}{h_1(\vec{u})}\right)^{\frac{1}{2}}\left[dz-\frac{z}{h_1(\vec{u})}\left(\vec{\nabla}_{\vec{u}}h_1(\vec{u})\right)\cdot d\vec{u}\right]$$
$$=(h_1(\vec{u})h_2(\vec{v}))^{\frac{1}{2}}\left[\frac{dz}{h_1(\vec{u})}-\frac{z}{(h_1(\vec{u}))^2}\left(\vec{\nabla}_{\vec{u}}h_1(\vec{u})\right)\cdot d\vec{u}\right]=(h_1(\vec{u})h_2(\vec{v}))^{\frac{1}{2}}d\hat{z}$$

$$\hat{z} \equiv \frac{z}{h_1(\vec{u})}$$
$$\mathcal{L}_{\vec{u}}\hat{g}_0 = -h_0(\vec{u},\vec{v})h_1(\vec{u}), \mathcal{L}_{\vec{v}}\hat{g}_0 = h_0(\vec{u},\vec{v})h_2(\vec{v})$$
$$\hat{g}_0 = f_2(\vec{v})h_1(\vec{u}) - f_1(\vec{u})h_2(\vec{v}), \text{ donde } \mathcal{L}_{\vec{u}}f_1 = h_1^2, \mathcal{L}_{\vec{v}}f_2 = h_2^2$$
$$G_0 = -\frac{1}{2}z^2\frac{h_2(\vec{v})}{h_1(\vec{u})} + f_2(\vec{v})h_1(\vec{u}) - f_1(\vec{u})h_2(\vec{v})$$
$$w = -z\frac{h_2(\vec{v})}{h_1(\vec{u})}, e^{-3A_0}\left(\frac{h_2(\vec{v})}{h_1(\vec{u})}\right)^{\frac{1}{2}} = h_1(\vec{u})h_2^2(\vec{v}) \Rightarrow e^{-2A_0} = h_1(\vec{u})h_2(\vec{v})$$
$$ds_{11}^2 = (h_1(\vec{u})h_2(\vec{v}))^{-1}(-dt^2 + dy^2) + d\hat{z}^2 + h_1(\vec{u})d\vec{u}\cdot d\vec{u} + h_2(\vec{v})d\vec{v}\cdot d\vec{v}$$
$$ds_{11}^2 = e^{2A}\left(\hat{f}_1^2 ds_{AdS_3}^2 + \hat{f}_2^2 ds_{S^3}^2 + \hat{f}_3^2 ds_{S'^3}^2 + h_{ij}d\sigma^i d\sigma^j\right)$$
$$C^{(3)} = b_1\hat{e}^{012} + b_2\hat{e}^{345} + b_3\hat{e}^{678}$$
$$h_{ij}d\sigma^i d\sigma^j = \frac{\partial_w h \partial_{\bar{w}} h}{h^2}|dw|^2$$
$$\partial_w\partial_{\bar{w}}h = 0$$
$$w = \xi + i\rho \Rightarrow \partial_w = \frac{1}{2}\left(\partial_\xi - i\partial_\rho\right), \partial_{\bar{w}} = \frac{1}{2}\left(\partial_\xi + i\partial_\rho\right)$$
$$\partial_{\bar{w}}(-\tilde{h} + ih) = 0$$
$$-\tilde{h} + ih = \beta w = \beta(\xi + i\rho)$$
$$h_{ij}d\sigma^i d\sigma^j = \frac{d\xi^2 + d\rho^2}{4\rho^2}$$
$$\partial_w G = \frac{1}{2}(G + \bar{G})\partial_w \log(h)$$
$$\partial_\xi g_1 + \partial_\rho g_2 = 0, \partial_\xi g_2 - \partial_\rho g_1 = -\frac{1}{\rho}g_1$$
$$\partial_w \Phi = \bar{G}\partial_w h \Leftrightarrow \partial_\xi \Phi = -\beta g_2, \partial_\rho \Phi = \beta g_1$$
$$\left(\partial_\xi^2 + \partial_\rho^2 - \frac{1}{\rho}\partial_\rho\right)\Phi$$
$$\partial_\xi\tilde{\Phi} = -\frac{\beta}{\rho}g_1 = -\frac{1}{\rho}\partial_\rho\Phi, \partial_\rho\tilde{\Phi} = -\frac{\beta}{\rho}g_2 = \frac{1}{\rho}\partial_\xi\Phi$$
$$\partial_\xi^2\tilde{\Phi} + \frac{1}{\rho}\partial_\rho\left(\rho\partial_\rho\tilde{\Phi}\right)$$
$$W_\pm \equiv |G \pm i|^2 + \gamma^{\pm 1}(G\bar{G} - 1)$$
$$\gamma(G\bar{G} - 1) \geq 0$$
$$\gamma > 0, |G| \geq 1$$
$$c_1 = \gamma^{1/2} + \gamma^{-1/2} > 0, c_2 = -\gamma^{1/2} < 0, c_3 = -\gamma^{-1/2} < 0, \sigma = +1$$
$$\hat{f}_1^{-2} = \gamma^{-1}(\gamma + 1)^2(G\bar{G} - 1), \hat{f}_2^{-2} = W_+, \hat{f}_3^{-2} = W_-$$
$$e^{6A} = h^2(G\bar{G} - 1)W_+W_- = \gamma(\gamma + 1)^{-2}h^2\hat{f}_1^{-2}\hat{f}_2^{-2}\hat{f}_3^{-2}$$
$$b_1 = \frac{\nu_1}{c_1^3}\left[\frac{h(G + \bar{G})}{(G\bar{G} - 1)} + \gamma^{-1}(\gamma + 1)^2\Phi - (\gamma - \gamma^{-1})\tilde{h}\right]$$
$$b_2 = \frac{\nu_2}{c_2^3}\left[-\frac{h(G + \bar{G})}{W_+} + (\Phi - \tilde{h})\right], b_3 = \frac{\nu_3}{c_3^3}\left[\frac{h(G + \bar{G})}{W_-} - (\Phi + \tilde{h})\right]$$
$$ds_{AdS_3}^2 = \frac{d\mu^2}{\mu^2} + \mu^2(-dt^2 + dy^2)$$
$$\mu \to \lambda\mu, (t,y) \to \lambda^{-1}(t,y)$$
$$(u,v) \to \sqrt{\lambda}(u,v), z \to \lambda^{-1}z$$
$$e^{A_0} \to \lambda e^{A_0}, w \to \lambda^{-1}w$$
$$u = \sqrt{\mu}m_1(\rho,\xi), \qquad v = \sqrt{\mu}m_2(\rho,\xi), z = \mu^{-1}m_3(\rho,\xi),$$
$$w = \mu^{-1}m_4(\rho,\xi), \quad e^{A_0} = \mu m_5(\rho,\xi),$$
$$e^{2A}\hat{f}_1^2\mu^2 = e^{2A_0}, e^{2A}\hat{f}_2^2 = e^{-A_0}(-\partial_z w)^{-\frac{1}{2}}u^2, e^{2A}\hat{f}_3^2 = e^{-A_0}(-\partial_z w)^{\frac{1}{2}}v^2$$

$$e^{2A}\left(\hat{f}_1^2\frac{d\mu^2}{\mu^2}+\frac{d\xi^2+d\rho^2}{4\rho^2}\right)=e^{-A_0}\left((-\partial_z w)^{-\frac{1}{2}}du^2+(-\partial_z w)^{\frac{1}{2}}dv^2\right)$$
$$+e^{2A_0}(-\partial_z w)(dz+(-\partial_z w)^{-1}(\partial_u w)du)^2$$
$$\frac{\gamma}{(1+\gamma)^2}\frac{1}{(G\bar{G}-1)}\frac{d\mu^2}{\mu^2}+\frac{d\xi^2+d\rho^2}{4\rho^2}$$
$$=\frac{1}{W_+}\frac{du^2}{u^2}+\frac{1}{W_-}\frac{dv^2}{v^2}+\frac{1}{\beta^2\rho^2(G\bar{G}-1)}\frac{W_+}{W_-}\left(u^2dz+(\partial_z w)^{-1}(u^3\partial_u w)\frac{du}{u}\right)^2$$
$$u^2v^2=\frac{\beta^2\gamma}{(\gamma+1)^2}\mu^2\rho^2,(-\partial_z w)\frac{v^2}{u^2}=\frac{W_+}{W_-},e^{A_0}=\frac{\beta\sqrt{\gamma}\mu\rho}{(\gamma+1)}e^{-2A}(W_+W_-)^{\frac{1}{2}}$$
$$u=\sqrt{a\mu\rho}e^{\alpha(\rho,\xi)},v=\sqrt{a\mu\rho}e^{-\alpha(\rho,\xi)},z=\mu^{-1}e^{-2\alpha(\rho,\xi)}p(\rho,\xi)$$
$$a\equiv\frac{\beta\sqrt{\gamma}}{(\gamma+1)}$$
$$(\partial_z w)^{-1}(u^3\partial_u w)=b_2,(v^3\partial_v w)=b_3$$
$$\frac{\gamma}{(1+\gamma)^2}\frac{1}{(G\bar{G}-1)}\frac{d\mu^2}{\mu^2}+\frac{d\xi^2+d\rho^2}{4\rho^2}$$
$$=\frac{1}{W_+}\frac{du^2}{u^2}+\frac{1}{W_-}\frac{dv^2}{v^2}+\frac{1}{\beta^2\rho^2(G\bar{G}-1)}\frac{W_+}{W_-}\left(u^2dz+b_2\frac{du}{u}\right)^2$$
$$\partial_\xi\alpha=-\frac{\varepsilon_1}{2\rho}g_1,\partial_\rho\alpha=\frac{1}{2\rho}g_2,b_2=2a\rho p+\frac{\varepsilon_2\beta\rho g_1}{g_1^2+g_2^2+g_2}$$
$$\partial_\xi p=-\frac{\varepsilon_1\varepsilon_2\beta}{2a\rho}(g_2-1),\partial_\rho p=-\frac{1}{\rho}\left(p+\frac{\varepsilon_2\beta}{2a}g_1\right)$$
$$\alpha=-\frac{1}{2\beta}\tilde{\Phi}$$
$$p=-\frac{\varepsilon_2}{2a\rho}(\Phi+\beta\xi)$$
$$b_2=\varepsilon_2\left(\frac{\beta\rho g_1}{g_1^2+g_2^2+g_2}-(\Phi+\beta\xi)\right)=\varepsilon_2\left(\frac{h(G+\bar{G})}{W_+}-(\Phi-\tilde{h})\right)$$
$$\gamma=1,u=\left(\frac{1}{2}\beta\mu\rho\right)^{\frac{1}{2}}e^{-\frac{1}{2\beta}\tilde{\Phi}},v=\left(\frac{1}{2}\beta\mu\rho\right)^{\frac{1}{2}}e^{+\frac{1}{2\beta}\tilde{\Phi}},z=-\frac{\varepsilon_2}{\beta\rho\mu}e^{\frac{1}{\beta}\tilde{\Phi}}(\Phi+\beta\xi)$$
$$\partial_z w=-\frac{g_1^2+g_2^2+g_2}{g_1^2+g_2^2-g_2}e^{-\frac{2}{\beta}\tilde{\Phi}},(\partial_z w)^{-1}(u^3\partial_u w)=b_2,(v^3\partial_v w)=b_3$$
$$dw=(\partial_z w)dz+(\partial_u w)du+(\partial_v w)dv=d\left[\frac{\varepsilon_2}{\beta\rho\mu}e^{-\frac{1}{\beta}\tilde{\Phi}}(\Phi-\beta\xi)\right]$$
$$w=\frac{\varepsilon_2}{\beta\rho\mu}e^{-\frac{1}{\beta}\tilde{\Phi}}(\Phi-\beta\xi)$$
$$b_3=\varepsilon_2\left(\frac{\beta\rho g_1}{g_1^2+g_2^2-g_2}-(\Phi-\beta\xi)\right)=\varepsilon_2\left(\frac{h(G+\bar{G})}{W_-}-(\Phi+\tilde{h})\right)$$
$$u^2z=-\frac{1}{2}\varepsilon_2(\Phi+\beta\xi),v^2w=\frac{1}{2}\varepsilon_2(\Phi-\beta\xi)$$
$$\Phi\to-\Phi,\tilde{\Phi}\to-\tilde{\Phi}\ \Rightarrow\ u\leftrightarrow v,z\leftrightarrow w$$
$$\omega\equiv e^{3A_0}(-\partial_z w)^{\frac{1}{2}}(dz+(\partial_z w)^{-1}(\partial_u w)du)$$
$$\omega=\frac{W_+\mu^2}{4(G\bar{G}-1)}\left(u^2dz+(\partial_z w)^{-1}(u^3\partial_u w)\frac{du}{u}\right)$$
$$=\frac{\varepsilon_2}{4}\left[\left(\frac{\beta\rho g_1}{g_1^2+g_2^2-1}+2\Phi\right)\mu d\mu-d(\mu^2\Phi)\right]$$
$$=\frac{\varepsilon_2}{8}\left[\left(\frac{h(G+\bar{G})}{(G\bar{G}-1)}+4\Phi\right)\mu d\mu-d(2\mu^2\Phi)\right]=\frac{\varepsilon_2}{v_1}b_1\mu d\mu-\frac{\varepsilon_2}{4}d(\mu^2\Phi)$$

$$b_1 = \frac{\nu_1}{4}\left(\frac{\beta\rho g_1}{g_1^2 + g_2^2 - 1} + 2\Phi\right)$$

$$C^{(3)}_{tyz} = -e^0 \wedge e^1 \wedge e^2 = -dt \wedge dy \wedge \omega = -\frac{\varepsilon_2}{\nu_1} b_1 \mu dt \wedge dy \wedge d\mu + \frac{\varepsilon_2}{4} d(\mu^2 \Phi dt \wedge dy)$$

$$C^{(3)}_{tyz} = b_1 \mu dt \wedge dy \wedge d\mu$$

$$e^0 = \frac{\mu e^A}{2\sqrt{G\bar{G} - 1}} dt, e^1 = \frac{\mu e^A}{2\sqrt{G\bar{G} - 1}} dy$$

$$e^2 = \frac{\varepsilon_2 e^A}{\rho\sqrt{(G\bar{G} - 1)W_+ W_-}}\left(\rho g_1 \frac{d\mu}{\mu} + (G\bar{G} - 1)(g_2 d\xi - g_1 d\rho)\right),$$

$$e^3 = \frac{e^A}{2\sqrt{W_+}}\left(\frac{d\mu}{\mu} + \frac{d\rho}{\rho} + \frac{1}{\rho}(g_1 d\xi + g_2 d\rho)\right),$$

$$e^4 = \frac{e^A}{2\sqrt{W_-}}\left(\frac{d\mu}{\mu} + \frac{d\rho}{\rho} - \frac{1}{\rho}(g_1 d\xi + g_2 d\rho)\right),$$

$$e^{i+4} = \frac{e^A}{2\sqrt{W_+}} \sigma_i, e^{i+7} = \frac{e^A}{2\sqrt{W_-}} \tilde{\sigma}_i, i = 1,2,3$$

$$\partial_\xi b_1 = \varepsilon_2 \partial_\xi \left[-\frac{\beta\rho g_1}{4(G\bar{G} - 1)}\right] + \frac{1}{2}\varepsilon_2 \beta g_2, \quad \partial_\rho b_1 = \varepsilon_2 \partial_\rho \left[-\frac{\beta\rho g_1}{4(G\bar{G} - 1)}\right] - \frac{1}{2}\varepsilon_2 \beta g_1,$$

$$\partial_\xi b_2 = \varepsilon_2 \partial_\xi \left[-\frac{2\beta\rho g_1}{W_+} + \beta\xi\right] - \varepsilon_2 \beta g_2, \quad \partial_\rho b_2 = \varepsilon_2 \partial_\rho \left[-\frac{2\beta\rho g_1}{W_+} + \varepsilon_2 \beta\xi\right] + \varepsilon_2 \beta g_1,$$

$$\partial_\xi b_3 = \varepsilon_2 \partial_\xi \left[-\frac{2\beta\rho g_1}{W_-} - \beta\xi\right] - \varepsilon_2 \beta g_2, \quad \partial_\rho b_3 = \varepsilon_2 \partial_\rho \left[-\frac{2\beta\rho g_1}{W_-} - \beta\xi\right] + \varepsilon_2 \beta g_1.$$

$$b_1 = -\varepsilon_2\left(\frac{\beta\rho g_1}{4(G\bar{G} - 1)} + \frac{1}{2}\Phi\right), b_2 = -\varepsilon_2\left(\frac{2\beta\rho g_1}{W_+} - (\Phi + \beta\xi)\right), b_3 = -\varepsilon_2\left(\frac{2\beta\rho g_1}{W_-} - (\Phi - \beta\xi)\right)$$

$$\Gamma^{012}\varepsilon = \eta_1 \varepsilon, \Gamma^{013567}\varepsilon = \eta_2 \varepsilon, \Gamma^{0148910}\varepsilon = -\eta_1 \eta_2 \varepsilon$$

$$b_1 = \varepsilon_2 \eta_1 \left(\frac{\beta\rho g_1}{4(G\bar{G} - 1)} + \frac{1}{2}\Phi\right), b_2 = \varepsilon_2 \eta_1 \eta_2 \left(\frac{2\beta\rho g_1}{W_+} - (\Phi + \beta\xi)\right)$$

$$b_3 = -\varepsilon_2 \eta_2 \left(\frac{2\beta\rho g_1}{W_-} - (\Phi - \beta\xi)\right)$$

$$\eta_1 = -1, \eta_2 = +1, \varepsilon_2 = +1$$

$$ds^2_{IIB} = \sqrt{h_{11}}\left[-e^{3A} dt^2 + e^{3A} h_{ab} dr^a dr^b + \frac{e^{-3A}}{\det h} dw_2^2 + d\mathbf{y}_6^2\right]$$

$$e^{2\phi} = \frac{h_{11}^2}{\det h}, C_0 = -\frac{h_{12}}{h_{11}}, B_2 = e^{3A} h_{1a} dt \wedge dr^a, C_2 = e^{3A} h_{2a} dt \wedge dr^a$$

$$h_{ab} = \frac{1}{2}\partial_a \partial_b K(r^1, r^2, \mathbf{y})$$

$$\Delta_y K + 2e^{-3A} = 0$$

$$r^1 \to z, r^2 \to u_1, w_2 \to u_2, y_1 \to u_3, y_2 \to y, y_{3,4,5,6} \to v_{3,4,5,6}$$

$$ds^2 = \frac{1}{\sqrt{\det h}}(-dt^2 + dy^2) + \frac{\sqrt{\det h}}{h_{11}}(du_2^2 + du_3^2) + \sqrt{\det h}\left(e^{3A} h_{ab} dr^a dr^b + ds^2_{\mathbb{R}^4}\right)$$

$$e^{2\phi} = \frac{\sqrt{\det h}}{h_{11}}, B_2 = \frac{h_{12}}{h_{11}} du_2 \wedge du_3$$

$$C_3 = e^{3A} h_{1a} dt \wedge dr^a \wedge dy - \frac{v^3}{2}\partial_v \partial_z K d\Omega_3'$$

$$C_5 = \frac{1}{h_{11}} dt \wedge du_1 \wedge du_2 \wedge du_3 \wedge dy + \frac{v^3}{2}\left(\frac{h_{12}}{h_{11}}\partial_v \partial_z K - \partial_v \partial_{u_1} K\right) du_2 \wedge du_3 \wedge d\Omega_3'$$

$$ds_{11}^2 = e^{\frac{-2\phi}{3}} ds_{10}^2 + e^{\frac{4\phi}{3}} (dx + C_1)^2$$
$$C_3' = C_3 + B_2 \wedge dx$$
$$ds_{11}^2 = \frac{h_{11}^{1/3}}{(\det h)^{2/3}} (-dt^2 + dy^2) + \frac{(\det h)^{1/3}}{h_{11}^{2/3}} (du_2^2 + du_3^2 + du_4^2)$$
$$+ (\det h)^{1/3} h_{11}^{1/3} \left(e^{3A} h_{ab} dr^a dr^b + ds_{\mathbb{R}^4}^2\right)$$
$$C_3 = \frac{h_{11}}{\det h} dt \wedge dz \wedge dy + \frac{h_{12}}{\det h} dt \wedge du_1 \wedge dy - \frac{h_{12}}{h_{11}} du_2 \wedge du_3 \wedge du_4 - \frac{v^3}{2} \partial_v \partial_z K d\Omega_3'$$
$$ds_{11}^2 = \frac{h_{11}^{1/3}}{(\det h)^{2/3}} (-dt^2 + dy^2) + \frac{(\det h)^{1/3}}{h_{11}^{2/3}} (du_1^2 + du_2^2 + du_3^2 + du_4^2)$$
$$+ \frac{h_{11}^{4/3}}{(\det h)^{2/3}} \left(dz + \frac{h_{12}}{h_{11}} du_1\right)^2 + (\det h)^{1/3} h_{11}^{1/3} (dv^2 + v^2 d\Omega_3'^2)$$
$$ds_{11}^2 = e^{2A_0} (-dt^2 + dy^2) + e^{-A_0} (-\partial_z w)^{-\frac{1}{2}} (du_1^2 + du_2^2 + du_3^2 + du_4^2)$$
$$+ e^{2A_0} (-\partial_z w) \left(dz + (\partial_z w)^{-1} \left(\partial_{u_1} w\right) du_1\right)^2 + e^{-A_0} (-\partial_z w)^{\frac{1}{2}} (dv^2 + v^2 d\Omega_3'^2)$$
$$C^{(3)} = -e^{3A_0} (-\partial_z w)^{\frac{1}{2}} dt \wedge dy \wedge dz + e^{3A_0} (-\partial_z w)^{-\frac{1}{2}} \left(\partial_{x_1} w\right) dt \wedge dy \wedge dx_1$$
$$+ (-\partial_z w)^{-1} \left(\partial_{u_1} w\right) du_2 \wedge du_3 \wedge du_4 + (v^3 \partial_v w) d\Omega_3'$$
$$e^{2A_0} = \frac{h_{11}^{1/3}}{(\det h)^{2/3}}, h_{11} = -\partial_z w, h_{12} = -\partial_{u_1} w$$
$$t = \eta_0, y = \eta_1, z = \eta_2, \vec{u}, \vec{v} \text{ constante}$$
$$C_{tyz}^{(3)} = -e^0 \wedge e^1 \wedge e^2$$
$$\rho = k_1 \mu^{-1}, \tilde{\Phi}(\xi, \rho) = k_2$$
$$t = \eta_0, y = \eta_1, \mu = e^{\eta_2}, \xi = \sigma_1(\eta_2), \rho = \sigma_2(\eta_2)$$
$$d\hat{s}_3^2 = e^{2A} \left[\hat{f}_1^2 \left(d\eta_2^2 + e^{2\eta_2} (-d\eta_0^2 + d\eta_1^2)\right) + \frac{(\sigma_1')^2 + (\sigma_2')^2}{4\sigma_2^2} d\eta_2^2\right]$$
$$\mathcal{L}_{DBI} = e^{3A} \hat{f}_1^2 e^{2\eta_2} \left(\hat{f}_1^2 + \frac{(\sigma_1')^2 + (\sigma_2')^2}{4\sigma_2^2}\right)^{\frac{1}{2}}$$
$$= h \hat{f}_1^2 e^{2\eta_2} \left[(G\bar{G} - 1) W_+ W_- \left(\hat{f}_1^2 + \frac{(\sigma_1')^2 + (\sigma_2')^2}{4\sigma_2^2}\right)\right]^{\frac{1}{2}}$$
$$\frac{(\sigma_1')^2 + (\sigma_2')^2}{\sigma_2^2} = \frac{g_1^2 + g_2^2}{g_1^2}$$
$$\left[(G\bar{G} - 1) W_+ W_- \left(\hat{f}_1^2 + \frac{(\sigma_1')^2 + (\sigma_2')^2}{4\sigma_2^2}\right)\right] = \left(\frac{W_+ W_-}{4 g_1}\right)^2$$
$$\mathcal{L}_{DBI} = e^{2\eta_2} \frac{\beta \sigma_2 ((g_1^2 + g_2^2)^2 - g_2^2)}{4 g_1 (g_1^2 + g_2^2 - 1)}$$
$$\hat{C}^{(3)} = b_1 e^{2\eta_2} d\eta_0 \wedge d\eta_1 \wedge d\eta_2$$
$$\tilde{C}^{(3)} = e^{2\eta_2} \left(b_1 + 2\Lambda + (\partial_\xi \Lambda) \sigma_1' + (\partial_\rho \Lambda) \sigma_2'\right) d\eta_0 \wedge d\eta_1 \wedge d\eta_2$$
$$\tilde{C}^{(3)} = e^{2\eta_2} \frac{v_1}{c_1^3} \left[\frac{h(G + \bar{G})}{(G\bar{G} - 1)} - 2\left(\partial_\xi \Phi\right) \sigma_1' - 2\left(\partial_\rho \Phi\right) \sigma_2'\right] d\eta_0 \wedge d\eta_1 \wedge d\eta_2$$
$$= v_1 e^{2\eta_2} \frac{\beta \sigma_2}{4} \left[\frac{g_1}{(g_1^2 + g_2^2 - 1)} + g_2 \frac{\sigma_1'}{\sigma_2} - g_1 \frac{\sigma_2'}{\sigma_2}\right] d\eta_0 \wedge d\eta_1 \wedge d\eta_2$$
$$\sigma_2 = k_1 e^{-\eta_2}, \frac{\sigma_1'}{\sigma_2} = \frac{g_2}{g_1}$$
$$g_1 \sigma_1' + g_2 \sigma_2' = 0 \Leftrightarrow \partial_\xi \tilde{\Phi} \sigma_1' + \partial_\rho \tilde{\Phi} \sigma_2' = 0$$
$$\eta_0 = t, \eta_1 = -u_1, \eta_2 = u_2, \eta_3 = u_3, \eta_4 = y$$

$$d\tilde{s}_5^2 = \frac{1}{\sqrt{\det h}}(-dt^2 + dy^2) + \frac{h_{22}}{\sqrt{\det h}} du_1^2 + \frac{\sqrt{\det h}}{h_{11}}(du_2^2 + du_3^2)$$

$$\tilde{B}_2 = \frac{h_{12}}{h_{11}} du_2 \wedge du_3$$

$$\tilde{C}_3 = -\frac{h_{12}}{\det h} dt \wedge du_1 \wedge dy$$

$$\tilde{C}_5 = -\frac{1}{h_{11}} dt \wedge du_1 \wedge du_2 \wedge du_3 \wedge dy$$

$$S_{DBI} = -T_4 \int d^5\sigma e^{-\phi} \sqrt{-\det(\tilde{G}_{\alpha\beta} + F_{\alpha\beta} + \tilde{B}_{\alpha\beta})} = -T_4 \int d^5\sigma \frac{h_{22}}{\det h}$$

$$S_{WZ} = -T_4 \int e^{\tilde{B}_2 + \tilde{F}_2} \wedge \bigoplus_n \tilde{C}_n = T_4 \int d^5\sigma \frac{h_{22}}{\det h}$$

$$h = -iw + i\bar{w}, G = \pm\left[i + \sum_{a=1}^{n+1} \frac{\zeta_a \mathrm{Im}(w)}{(\bar{w} - \xi_a)|w - \xi_a|}\right]$$

$$g_1 = \pm \sum_{a=1}^{n+1} \frac{\zeta_a \rho(\xi - \xi_a)}{((\xi - \xi_a)^2 + \rho^2)^{\frac{3}{2}}}, g_2 = \pm\left[1 + \sum_{a=1}^{n+1} \frac{\zeta_a \rho^2}{((\xi - \xi_a)^2 + \rho^2)^{\frac{3}{2}}}\right]$$

$$\tilde{\Phi} = \pm 2\left[-\log \rho + \sum_{a=1}^{n+1} \frac{\zeta_a}{\sqrt{(\xi - \xi_a)^2 + \rho^2}}\right], \Phi = \mp 2\left[\xi + \sum_{a=1}^{n+1} \frac{\zeta_a(\xi - \xi_a)}{\sqrt{(\xi - \xi_a)^2 + \rho^2}}\right]$$

$$ds_{11} = -e^{2A_0}dt^2 + e^{2A_1}(dy - Pdt)^2 + e^{2A_2}du^2 + e^{2A_3}dv^2 + u^2 e^{2A_4} d\Omega_3^2 + v^2 e^{2A_5} d\Omega_3'^2 + e^{2A_6}(dz + B_1 du)^2$$

$$e^0 = e^{A_0}dt, e^1 = e^{A_1}(dy - Pdt), e^2 = e^{A_6}(dz + B_1 du),$$

$$e^3 = e^{A_2}du, e^4 = e^{A_3}dv, e^{i+4} = ue^{A_4}\sigma_i, e^{i+7} = ve^{A_5}\tilde{\sigma}_i, i = 1,2,3$$

$$\Gamma^{01}\varepsilon = -\varepsilon, \Gamma^{012}\varepsilon = -\varepsilon, \Gamma^{013456}\varepsilon = \varepsilon$$

$$\Gamma^{0178910}\varepsilon = -\varepsilon$$

$$\delta\psi_\mu \equiv \nabla_\mu \epsilon + \frac{1}{288}\left(\Gamma_\mu^{\nu\rho\lambda\sigma} - 8\delta_\mu^\nu \Gamma^{\rho\lambda\sigma}\right) F_{\nu\rho\lambda\sigma}\epsilon$$

$$P \equiv 1 - e^{A_0 - A_1}$$

$$\hat{A}_0 \equiv \frac{1}{2}(A_0 + A_1), \hat{A}_1 \equiv \frac{1}{2}(A_0 - A_1)$$

$$ds_{11} = e^{2\hat{A}_0}\Big[-e^{2\hat{A}_1}dt^2 + e^{-2\hat{A}_1}\left(dy + \left(e^{2\hat{A}_1} - 1\right)dt\right)^2 + (-\partial_z w)(dz + (\partial_z w)^{-1}(\partial_u w)du)^2$$
$$+ e^{-3\hat{A}_0}(-\partial_z w)^{-\frac{1}{2}}(du^2 + u^2 d\Omega_3^2) + e^{-3\hat{A}_0}(-\partial_z w)^{\frac{1}{2}}(dv^2 + v^2 d\Omega'^2)\Big]$$

$$e^0 = e^{\hat{A}_0 + \hat{A}_1}dt, e^1 = e^{\hat{A}_0 - \hat{A}_1}\left(dy + \left(e^{2\hat{A}_1} - 1\right)dt\right)$$

$$e^2 = e^{\hat{A}_0}(-\partial_z w)^{\frac{1}{2}}(dz + (\partial_z w)^{-1}(\partial_u w)du)$$

$$e^3 = e^{-\frac{1}{2}\hat{A}_0}(-\partial_z w)^{-\frac{1}{4}}du, e^4 = e^{-\frac{1}{2}\hat{A}_0}(-\partial_z w)^{\frac{1}{4}}dv,$$

$$e^{i+4} = \frac{1}{2}ue^{-\frac{1}{2}\hat{A}_0}(-\partial_z w)^{-\frac{1}{4}}\sigma_i, e^{i+7} = \frac{1}{2}ve^{-\frac{1}{2}\hat{A}_0}(-\partial_z w)^{\frac{1}{4}}\tilde{\sigma}_i, i = 1,2,3$$

$$C^{(3)} = -e^0 \wedge e^1 \wedge e^2 + (\partial_z w)^{-1}(u^3 \partial_u w)\mathrm{Vol}(S^3) + (v^3 \partial_v w)\mathrm{Vol}(S'^3)$$

$$F_1 \equiv (-\partial_z w)^{\frac{1}{2}} e^{-3\hat{A}_0}, F_2 \equiv (-\partial_z w)^{-\frac{1}{2}} e^{-3\hat{A}_0} + (-\partial_z w)^{-1}(\partial_u w)^2$$

$$\mathcal{L}(H) = e^{2\hat{A}_0}(-\partial_z w)^{-\frac{1}{2}}\Big[(-\partial_z w)\frac{1}{u^3}\partial_u(u^3 \partial_u H) + \frac{1}{v^3}\partial_v(v^3 \partial_v H) + 2(\partial_u w)\partial_u \partial_z H$$
$$+ \left((-\partial_z w)^{-\frac{1}{2}} e^{-3\hat{A}_0} + (-\partial_z w)^{-1}(\partial_u w)^2\right)\partial_z^2 H\Big]$$

$$\mathcal{L}\left(e^{-2\hat{A}_1}\right) = 0$$

$$ds_{11}^2 = e^{2A}[\hat{f}_1^2\left(\frac{d\mu^2}{\mu^2} + \mu^2\left(-e^{2\hat{A}_1}dt^2 + e^{-2\hat{A}_1}\left(dy + \left(e^{2\hat{A}_1} - 1\right)dt\right)^2\right)\right)$$

$$+\hat{f}_2^2 ds_{S^3}^2 + \hat{f}_3^2 ds_{S'^3}^2 + \frac{d\xi^2 + d\rho^2}{4\rho^2}\Bigg]$$

$$\mathcal{L}(H) = 4e^{-A}\left[(G\bar{G} - 1)\frac{1}{\mu}\partial_\mu\left(\mu^3\partial_\mu H\right) + \frac{1}{\rho}\partial_\rho\left(\rho^3\partial_\rho H\right) + \rho^2\partial_\xi^2 H\right]$$

$$\frac{1}{\rho}\partial_\rho\left(\rho^3\partial_\rho K\right) + \rho^2\partial_\xi^2 K + p(p+2)(G\bar{G} - 1)K$$

$$K = \frac{c_1}{u^2} + \frac{c_2}{v^2} = \frac{2}{\beta\rho}\left(c_1 e^{\frac{1}{\beta}\tilde{\Phi}} + c_2 e^{-\frac{1}{\beta}\tilde{\Phi}}\right)$$

$$(u^2 + v^2)^{-3} \sim \mu^{-3}\rho^{-3}$$

$$\frac{1}{\rho^3}\partial_\rho\left(\rho^3\partial_\rho K\right) + \partial_\xi^2 K + \frac{p(p+2)}{\rho^2}(G\bar{G} - 1)K$$

$$\mathcal{L}_4(K) \equiv \frac{1}{\rho^3}\partial_\rho\left(\rho^3\partial_\rho K\right) + \partial_\xi^2 K$$

$$ds_5^2 \equiv d\rho^2 + \rho^2 d\Omega_3^2 + d\xi^2$$

$$\mathcal{L}_4\left(\frac{1}{(\rho^2 + \xi^2)^{\frac{3}{2}}}\right)$$

$$\frac{p(p+2)}{\rho^2}(G\bar{G} - 1) \sim \frac{c_0}{(\rho^2 + \xi^2)^{\frac{3}{2}}}$$

$$K = \frac{Q}{(\rho^2 + \xi^2)^{\frac{3}{2}}}\left(1 - \frac{c_0}{4}\frac{1}{\sqrt{\rho^2 + \xi^2}} + \cdots\right)$$

$$e^{-2\hat{A}_1} = V_0 + V_1\mu^{-2}$$

$$V_0 = 1 + \sum_{a=1}^{m}\frac{k_a}{\left(\left(\xi - \tilde{\xi}_a\right)^2 + \rho^2\right)^{\frac{3}{2}}}, V_1 = q_0 + \sum_{a=1}^{m'}\frac{q_a}{\left(\left(\xi - \hat{\xi}_a\right)^2 + \rho^2\right)^{\frac{3}{2}}}$$

$$e^{-2\hat{A}_1} = 1 + \alpha + Q\mu^{-2}$$

$$\frac{d\mu^2}{\mu^2} + \mu^2\left(-e^{2\hat{A}_1}dt^2 + e^{-2\hat{A}_1}\left(dy + \left(e^{2\hat{A}_1} - 1\right)dt\right)^2\right)$$

$$= \frac{d\mu^2}{\mu^2} - \frac{\mu^4}{Q + \mu^2}dt^2 + (Q + \mu^2)\left(dy - \frac{Q}{Q + \mu^2}dt\right)^2$$

$$= \frac{d\mu^2}{\mu^2} + \mu^2(-dt^2 + dy^2) + Q(dy - dt)^2$$

$$ds_{11}^2 = e^{2\alpha_0}(-dt^2 + dy^2) + e^{2\alpha_1}d\Omega_3^2 + e^{2\alpha_2}d\Omega_3'^2 + g_{ij}dz^i dz^j$$

$$ds_3^2 = g_{ij}dz^i dz^j = e^{2\alpha_3}dz^2 + e^{2\alpha_4}du^2 + e^{2\alpha_5}dv^2$$

$$e^0 = e^{\alpha_0}dt, \quad e^1 = e^{\alpha_0}dy, \quad e^2 = e^{\alpha_1}dz\; e^3 = e^{\alpha_2}du, e^4 = e^{\alpha_3}dv,$$

$$e^{i+4} = e^{\alpha_4}\sigma_i, \quad e^{i+7} = e^{\alpha_5}\tilde{\sigma}_i, \quad i = 1,2,3$$

$$\Gamma^{012}\varepsilon = -\varepsilon, \Gamma^{013567}\varepsilon = \varepsilon, \Gamma^{0148910}\varepsilon = -\varepsilon$$

$$e^0 = e^{A_0}dt, \quad e^1 = e^{A_0}dy, e^2 = e^{A_1}(dz + B_1 du + B_2 dv)$$

$$e^3 = e^{A_2}du, e^4 = e^{A_3}dv, e^{i+4} = ue^{A_4}\sigma_i, e^{i+7} = ve^{A_5}\tilde{\sigma}_i, i = 1,2,3$$

$$B_2 \equiv 0$$

$$e^0 = e^{A_0}dt, \quad e^1 = e^{A_0}dy, \quad e^2 = e^{A_1}(dz + B_1 du),$$

$$e^3 = e^{A_2}du, \quad e^4 = e^{A_3}dv, \quad e^{i+4} = ue^{A_4}\sigma_i, e^{i+7} = ve^{A_5}\tilde{\sigma}_i, i = 1,2,3$$

$$ds_{11}^2 = e^{2A_0}(-dt^2 + dy^2) + e^{2A_2}du^2 + e^{2A_3}dv^2 + u^2 e^{2A_4}d\Omega_3^2 + v^2 e^{2A_5}d\Omega_3'^2$$

$$+e^{2A_1}(dz + B_1 du)^2$$

$$F^{(4)} = e^0 \wedge e^1 \wedge (b_1 e^2 \wedge e^3 + b_2 e^2 \wedge e^4 + b_3 e^3 \wedge e^4)$$
$$+ (b_4 e^2 + b_5 e^3 + b_6 e^4) \wedge e^5 \wedge e^6 \wedge e^7 + (b_7 e^2 + b_8 e^3 + b_9 e^4) \wedge e^8 \wedge e^9 \wedge e^{10}$$
$$\epsilon = e^{\frac{1}{2}A_0} \epsilon_0$$
$$\partial_u (A_5 - A_3) = \partial_z (A_5 - A_3) = 0, \partial_v (A_4 - A_2) = \partial_z (A_4 - A_2)$$
$$A_4 = A_2, A_5 = A_3$$
$$ds_{11}^2 = e^{2A_0}(-dt^2 + dy^2) + e^{2A_2}(du^2 + u^2 d\Omega_3^2) + e^{2A_3}(dv^2 + v^2 d\Omega'^2)$$
$$+ e^{2A_1}(dz + B_1 du)^2$$
$$A_3 = -(A_0 + A_2)$$
$$A_1 = -2A_2$$
$$ds_{11}^2 = e^{2A_0}[(-dt^2 + dy^2) + e^{2(A_2 - A_0)}(du^2 + u^2 d\Omega_3^2) + e^{-2(A_2 + 2A_0)}\left(dv^2 + v^2 d\Omega'^{\,2}_{3\,3}\right)$$
$$+ e^{-2(A_0 + 2A_2)}(dz + B_1 du)^2]$$
$$\partial_z\left(B_1 e^{-2(A_0 + 2A_2)}\right) = \partial_u\left(e^{-2(A_0 + 2A_2)}\right)$$
$$B_1 e^{-2(A_0 + 2A_2)} = -\partial_u w, e^{-2(A_0 + 2A_2)} = -\partial_z w$$
$$B_1 = (\partial_z w)^{-1} \partial_u w, e^{-2(A_0 + 2A_2)} = -\partial_z w$$
$$F_1 \equiv (-\partial_z w)^{\frac{1}{2}} e^{-3A_0}, F_2 \equiv (-\partial_z w)^{-\frac{1}{2}} e^{-3A_0} + (-\partial_z w)^{-1} (\partial_u w)^2$$
$$H_1 \equiv \mathcal{L}_v w - \partial_z F_1, H_2 \equiv \mathcal{L}_u w + \partial_z F_2$$
$$\partial_z H_1 = \partial_u H_1 = \partial_z H_2 = \partial_v H_2 = 0$$
$$w = \partial_z G_0, F_1 = \mathcal{L}_v G_0, F_2 = -\mathcal{L}_u G_0$$
$$\mathcal{L}_v G_0 = (\partial_z^2 G_0)(\mathcal{L}_u G_0) - (\partial_u \partial_z G_0)^2$$
$$\left(\frac{\partial F}{\partial \eta}\right)_{\zeta, \xi}$$
$$dw = \left(\frac{\partial w}{\partial z}\right)_{\vec{u}, \vec{v}} dz + \left(\frac{\partial w}{\partial u_i}\right)_{z, \vec{v}} du_i + \left(\frac{\partial w}{\partial v_i}\right)_{z, \vec{u}} dv_i$$
$$\left(\frac{\partial z}{\partial u_i}\right)_{w, \vec{v}} = -\left(\left(\frac{\partial w}{\partial z}\right)_{\vec{u}, \vec{v}}\right)^{-1} \left(\frac{\partial w}{\partial u_i}\right)_{z, \vec{v}}, \left(\frac{\partial z}{\partial v_i}\right)_{w, \vec{u}} = -\left(\left(\frac{\partial w}{\partial z}\right)_{\vec{u}, \vec{v}}\right)^{-1} \left(\frac{\partial w}{\partial v_i}\right)_{z, \vec{u}}$$

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$$\left(\frac{\partial z}{\partial w}\right)_{\vec{u}, \vec{v}} = \left(\left(\frac{\partial w}{\partial z}\right)_{\vec{u}, \vec{v}}\right)^{-1}$$
$$e^2 = (-\partial_z w)^{\frac{1}{2}}\left(dz + (\partial_z w)^{-1}(\vec{\nabla}_{\vec{u}} w) \cdot d\vec{u}\right)$$
$$= -(-\partial_z w)^{-\frac{1}{2}}\left((\partial_z w) dz + (\vec{\nabla}_{\vec{u}} w) \cdot d\vec{u}\right) = -(-\partial_z w)^{-\frac{1}{2}}\left(dw - (\vec{\nabla}_{\vec{v}} w) \cdot d\vec{v}\right)$$
$$= -\left((-\partial_w z)_{\vec{u}, \vec{v}}\right)^{\frac{1}{2}}\left(dw + \left(\left(\frac{\partial z}{\partial w}\right)_{\vec{v}, \vec{u}}\right)^{-1}\left(\frac{\partial z}{\partial v_i}\right)_{w, \vec{v}} dv_i\right)$$
$$= -(-\partial_w z)^{\frac{1}{2}}\left(dw + (\partial_w z)^{-1}(\vec{\nabla}_{\vec{v}} z) \cdot d\vec{v}\right)$$
$$C^{(3)} = -e^0 \wedge e^1 \wedge e^2 + \frac{1}{3!}\epsilon_{ijk\ell}\left(-(\partial_{u_\ell} z) du^i \wedge du^j \wedge du^k + (\partial_w z)^{-1}(\partial_{v_\ell} z) dv^i \wedge dv^j \wedge dv^k\right)$$
$$ds^2 = G_{xx}\left(dx + A_\mu dx^\mu\right)^2 + \hat{g}_{\mu\nu} dx^\mu dx^\nu,$$
$$B_2 = B_{\mu x} dx^\mu \wedge \left(dx + A_\mu dx^\mu\right) + \hat{B}_2,$$
$$C_p = C_{(p-1)x} \wedge \left(dx + A_\mu dx^\mu\right) + \hat{C}_p,$$
$$d\tilde{s}^2 = G_{xx}^{-1}\left(dx + B_{\mu x} dx^\mu\right)^2 + \hat{g}_{\mu\nu} dx^\mu dx^\nu,$$
$$e^{2\tilde{\phi}} = G_{xx}^{-1} e^{2\phi}$$
$$\tilde{B}_2 = A_\mu dx^\mu \wedge dx + \hat{B}_2$$
$$\tilde{C}_p = \hat{C}_{p-1} \wedge \left(dx + B_{\mu x} dx^\mu\right) + C_{(p)x}.$$

$$\tilde{g}_{\mu\nu} = \sqrt{C_0^2 + e^{-2\phi}}g_{\mu\nu}, e^{-\tilde{\phi}} = \frac{e^{-\phi}}{C_0^2 + e^{-2\phi}}, \tilde{C}_0 = -\frac{C_0}{C_0^2 + e^{-2\phi}},$$

$$\tilde{B}_2 = -C_2, \tilde{C}_2 = B_2, \tilde{C}_4 = C_4 + B_2 \wedge C_2$$

$$ds^2 = \frac{\sqrt{\det h}}{h_{11}}(du_2^2 + du_3^2) + \frac{1}{\sqrt{\det h}}(-dt^2 + dy^2) + \sqrt{\det h}\left(e^{3A}h_{ab}dr^a dr^b + ds_{\mathbb{R}^4}^2\right)$$

$$e^{2\phi} = \frac{\sqrt{\det h}}{h_{11}}, B_2 = \frac{h_{12}}{h_{11}}du_2 \wedge du_3$$

$$C_3 = e^{3A}h_{1a}dt \wedge dr^a \wedge dy, C_5 = \frac{1}{h_{11}}dt \wedge du_1 \wedge du_2 \wedge du_3 \wedge dy$$

$$F_p = dC_{p-1} \quad \text{para } p < 3,$$

$$F_p = dC_{p-1} + H_3 \wedge C_{p-3} \quad \text{para } p \geq 3,$$

$$F_6 = \star F_4, F_8 = \star F_2.$$

$$v_3 = v\cos\phi_1$$

$$v_4 = v\sin\phi_1\cos\phi_2$$

$$v_5 = v\sin\phi_1\sin\phi_2\cos\phi_3$$

$$v_6 = v\sin\phi_1\sin\phi_2\sin\phi_3$$

$$ds_{\mathbb{R}^4}^2 = dv^2 + v^2\left(d\phi_1^2 + \sin^2\phi_1(d\phi_2^2 + \sin^2\phi_2 d\phi_3^2)\right)$$

$$dC_5^e = -\frac{1}{h_{11}^2}(\partial_z h_{11}dz + \partial_v h_{11}dv) \wedge dt \wedge du_1 \wedge du_2 \wedge du_3 \wedge dy$$

$$H_3 \wedge C_3^e = \left[\partial_z\left(\frac{h_{12}}{h_{11}}\right)e^{3A}h_{12} - \partial_{u_1}\left(\frac{h_{12}}{h_{11}}\right)e^{3A}h_{11}\right]dt \wedge du_1 \wedge du_2 \wedge du_3 \wedge dz \wedge dy$$

$$-\partial_v\left(\frac{h_{12}}{h_{11}}\right)e^{3A}h_{11}dz \wedge dt \wedge dv \wedge du_2 \wedge du_3 \wedge dy$$

$$-\partial_v\left(\frac{h_{12}}{h_{11}}\right)e^{3A}h_{12}du_1 \wedge dt \wedge dv \wedge du_2 \wedge du_3 \wedge dy$$

$$F_6^e = f_1 dt \wedge dz \wedge du_1 \wedge du_2 \wedge du_3 \wedge dy$$

$$+f_2 dt \wedge du_1 \wedge du_2 \wedge du_3 \wedge dy \wedge dv$$

$$-f_3 dt \wedge dz \wedge du_2 \wedge du_3 \wedge dy \wedge dv$$

$$f_1 = \frac{1}{h_{11}^2}\partial_z h_{11} - \partial_z\left(\frac{h_{12}}{h_{11}}\right)e^{3A}h_{12} + \partial_{u_1}\left(\frac{h_{12}}{h_{11}}\right)e^{3A}h_{11},$$

$$f_2 = \frac{1}{h_{11}^2}\partial_v h_{11} - \partial_v\left(\frac{h_{12}}{h_{11}}\right)e^{3A}h_{12},$$

$$f_3 = \partial_v\left(\frac{h_{12}}{h_{11}}\right)e^{3A}h_{11}.$$

$$F_4^m = -v^3 h_{11}(f_2 h_{11} + f_3 h_{12})dz \wedge d\Omega_3' - v^3 h_{11}(f_2 h_{12} + f_3 h_{22})du_1 \wedge d\Omega_3'$$

$$+r^3 f_1 h_{11}\det h dr \wedge d\Omega_3'$$

$$F_4^m = -(v^3\partial_v h_{11}dz + v^3\partial_v h_{12}du_1) \wedge d\Omega_3'$$

$$+v^3\left(h_{22}\partial_z h_{11} - h_{12}\partial_z h_{12} - h_{12}\partial_{u_1}h_{11} + h_{11}\partial_{u_1}h_{12}\right)dv \wedge d\Omega_3'$$

$$-\frac{1}{2}v^3\partial_v\partial_z^2 K dz - \frac{1}{2}v^3\partial_v\partial_z\partial_{u_1}K du_1 = -\frac{1}{2}d(v^3\partial_v\partial_z K) + \frac{1}{2}\partial_v(v^3\partial_v\partial_z K)dv$$

$$= -\frac{1}{2}d(v^3\partial_v\partial_z K) + \frac{v^3}{2}\partial_z\left(\frac{1}{v^3}\partial_v(v^3\partial_v K)\right)dv = -\frac{1}{2}d(v^3\partial_v\partial_z K) + \frac{v^3}{2}\partial_z\Delta_y K dv$$

$$\frac{v^3}{4}\left(\partial_{u_1}^2 K\partial_z^3 K - 2\partial_z\partial_{u_1}K\partial_z^2\partial_{u_1}K + \partial_z^2 K\partial_z\partial_{u_1}^2 K\right)dv = v^3\partial_z(\det h)$$

$$F_4^m = -\frac{1}{2}d(v^3\partial_v\partial_z K) \wedge d\Omega_3' + \frac{v^3}{2}\partial_z\left(\Delta_y K + 2\det h\right)dv \wedge d\Omega_3'$$

$$C_3^m = -\frac{1}{2}v^3\partial_v\partial_z K d\Omega_3'$$

$$dC_3^e = -\left[\partial_{u_1}(e^{3A}h_{11}) - \partial_z(e^{3A}h_{12})\right]dt\wedge du_1\wedge dz\wedge dy$$

$$dC_3^e = -\left[\partial_{u_1}(e^{3A}h_{11}) - \partial_z(e^{3A}h_{12})\right]dt\wedge du_1\wedge dz\wedge dy$$
$$-[\partial_v(e^{3A}h_{11})dz + \partial_v(e^{3A}h_{12})du_1]\wedge dt\wedge dv\wedge dy$$

$$F_6^m = \frac{v^3\det h}{h_{11}}[h_{12}\partial_v(e^{3A}h_{11}) - h_{11}\partial_v(e^{3A}h_{12})]dz\wedge du_2\wedge du_3\wedge d\Omega_3'$$
$$+\frac{v^3\det h}{h_{11}}[h_{22}\partial_v(e^{3A}h_{11}) - h_{12}\partial_v(e^{3A}h_{12})]du_1\wedge du_2\wedge du_3\wedge d\Omega_3'$$
$$-\frac{v^3(\det h)^2}{h_{11}}\left[\partial_{u_1}(e^{3A}h_{11}) - \partial_z(e^{3A}h_{12})\right]du_2\wedge du_3\wedge dv\wedge d\Omega_3',$$

$$F_6^m = v^3\left(\frac{h_{12}}{h_{11}}\partial_v h_{11} - \partial_v h_{12}\right)dz\wedge du_2\wedge du_3\wedge d\Omega_3'$$
$$+v^3\left(\frac{h_{12}}{h_{11}}\partial_v h_{12} - \partial_v h_{22}\right)du_1\wedge du_2\wedge du_3\wedge d\Omega_3'$$
$$+v^3\left(\partial_{u_1}(\det h) - \frac{h_{12}}{h_{11}}\partial_z(\det h)\right)dv\wedge dx_2\wedge du_3\wedge d\Omega_3'$$

$$\frac{1}{2}\frac{h_{12}}{h_{11}}d_\Sigma(v^3\partial_v\partial_z K)\wedge du_2\wedge du_3\wedge d\Omega_3' - \frac{1}{2}d_\Sigma(v^3\partial_v\partial_1 K)\wedge du_2\wedge du_3\wedge d\Omega_3'$$

$$F_6^m - H\wedge C_3^m = \left[\frac{h_{12}}{h_{11}}d_\Sigma\left(\frac{v^3}{2}\partial_v\partial_z K\right) + d\left(\frac{h_{12}}{h_{11}}\right)\frac{v^3}{2}\partial_v\partial_z K\right]\wedge du_2\wedge du_3\wedge d\Omega_3'$$
$$-d_\Sigma\left(\frac{v^3}{2}\partial_v\partial_{u_1}K\right)\wedge du_2\wedge du_3\wedge d\Omega_3'$$
$$+v^3\left(\partial_{u_1}(\det h) - \frac{h_{12}}{h_{11}}\partial_z(\det h)\right)dv\wedge dx_2\wedge dx_3\wedge d\Omega_3'$$

$$d\left(\frac{v^3}{2}\frac{h_{12}}{h_{11}}\partial_v\partial_z K\right)\wedge du_2\wedge du_3\wedge d\Omega_3' - \frac{h_{12}}{h_{11}}\partial_v\left(\frac{v^3}{2}\partial_v\partial_z K\right)dv\wedge du_2\wedge du_3\wedge d\Omega_3'$$
$$-d\left(\frac{v^3}{2}\partial_v\partial_{u_1}K\right)\wedge du_2\wedge du_3\wedge d\Omega_3' + \partial_v\left(\frac{v^3}{2}\partial_v\partial_{u_1}K\right)dv\wedge du_2\wedge du_3\wedge d\Omega_3'$$

$$\frac{v^3}{2}\partial_{u_1}\left[2\det h + \frac{1}{v^3}\partial_v(v^3\partial_v K)\right] - \frac{v^3}{2}\frac{h_{12}}{h_{11}}\partial_z\left[2\det h + \frac{1}{v^3}\partial_v(v^3\partial_v K)\right]$$

$$F_6^m - H\wedge C_3^m = d\left(\frac{v^3}{2}\frac{h_{12}}{h_{11}}\partial_v\partial_z K - \frac{v^3}{2}\partial_v\partial_{u_1}K\right)\wedge du_2\wedge du_3\wedge d\Omega_3'$$

$$C_5^m = \frac{v^3}{2}\left(\frac{h_{12}}{h_{11}}\partial_v\partial_z K - \partial_v\partial_{u_1}K\right)du_2\wedge du_3\wedge d\Omega_3'$$

$$ds^2 = \frac{1}{\sqrt{\det h}}(-dt^2 + dy^2) + \frac{\sqrt{\det h}}{h_{11}}(du_2^2 + du_3^2) + \sqrt{\det h}\left(e^{3A}h_{ab}dr^a dr^b + ds^2_{\mathbb{R}^4}\right)$$

$$e^{2\phi} = \frac{\sqrt{\det h}}{h_{11}}, B_2 = \frac{h_{12}}{h_{11}}du_2\wedge du_3$$

$$C_3 = e^{3A}h_{1a}dt\wedge dr^a\wedge dy - \frac{v^3}{2}\partial_v\partial_z K d\Omega_3'$$

$$C_5 = \frac{1}{h_{11}}dt\wedge du_1\wedge du_2\wedge du_3\wedge dy + \frac{v^3}{2}\left(\frac{h_{12}}{h_{11}}\partial_v\partial_z K - \partial_v\partial_{u_1}K\right)du_2\wedge du_3\wedge d\Omega_3'$$

$$ds^2 = Z^{-1/2}(-dt^2 + dx_1^2 + dx_2^2) + Z^{1/2}(dx_3^2 + \cdots + dx_9^2)$$
$$e^\Phi = Z^{1/4}$$
$$C_3 = Z^{-1}dt\wedge dx_1\wedge dx_2$$

$$Z = 1 + \frac{Q}{r^2}, r^2 \equiv x_6^2 + \cdots + x_9^2$$
$$x_2 = x_2' c + x_3' s$$
$$x_3 = -x_2' s + x_3' c$$
$$W \equiv c^2 Z + s^2$$
$$\begin{aligned} ds^2 =& Z^{-1/2}(-dt^2 + dx_1^2) + Z^{1/2}(dx_4^2 + \cdots + dx_9^2) \\ &+ Z^{-1/2} W (dx_3 - cs(Z-1)W^{-1}dx_2)^2 + Z^{1/2}W^{-1}dx_2^2, \\ e^{2\Phi} =& Z^{1/2}, \\ C_3 =& Z^{-1} s dt \wedge dx_1 \wedge (dx_3 - cs(Z-1)W^{-1}dx_2) \\ &+ W^{-1} c dt \wedge dx_1 \wedge dx_2. \end{aligned}$$
$$\begin{aligned} ds^2 &= Z^{-1/2}(-dt^2 + dx_1^2) + Z^{1/2}(dx_4^2 + \cdots + dx_9^2) + Z^{1/2}W^{-1}(dx_2^2 + dx_3^2) \\ e^{2\Phi} &= W^{-1}Z, B_2 = -cs(Z-1)W^{-1}dx_2 \wedge dx_3 \\ C_2 &= Z^{-1} s dt \wedge dx_1, C_4 = W^{-1} c dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \end{aligned}$$
$$ds^2 = Z^{-1/2}(-dt^2 + dx_1^2 + dx_4^2) + Z^{1/2}\left(dx_5^2 + \cdots + dx_9^2\right) + Z^{1/2}W^{-1}(dx_2^2 + dx_3^2)$$
$$e^{2\Phi} = Z^{1/2}W^{-1}$$
$$B_2 = -cs(Z-1)W^{-1}dx_2 \wedge dx_3$$
$$C_3 = Z^{-1} s dt \wedge dx_1 \wedge dx_4$$
$$C_5 = W^{-1} c dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4$$
$$h_{11} = c^2 Z + s^2, h_{22} = s^2 Z + c^2$$
$$h_{12} = cs(Z-1), \det h = Z$$
$$\begin{aligned} dB_2 \wedge C_3 &= [-cs\partial_l((Z-1)W^{-1})dx_l \wedge dx_2 \wedge dx_3] \wedge [Z^{-1} s dt \wedge dx_1 \wedge dx_4] \\ &= cs^2 W^{-2} Z^{-1}(\partial_l Z) dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 \wedge dx_l \\ dC_5 &= c\partial_l(W^{-1})dx_l \wedge dt \wedge dx_1 \wedge \cdots \wedge dx_4 \\ &= c^3 W^{-2}(\partial_l Z) dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 \wedge dx_l \\ F_6 &= cW^{-1}Z^{-1}(\partial_l Z) dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 \wedge dx_l \\ &= cZ^{-1}(\partial_l Z) e^0 \wedge e^1 \wedge e^2 \wedge e^3 \wedge e^4 \wedge e^l \\ F_4^{(m)} &= c\frac{\partial_l Z}{Z}\frac{\epsilon_{(l-4),abcd}}{4!} e^{4+a} \wedge e^{4+b} \wedge e^{4+c} \wedge e^{4+d} \\ &= c(\partial_l Z)\frac{\epsilon_{(l-4),abcd}}{4!} dx^{4+a} \wedge dx^{4+b} \wedge dx^{4+c} \wedge dx^{4+d} \\ F_4^{(m)} &= -c\frac{x_l}{r}(\partial_r Z)\frac{\epsilon_{(l-5),abc}}{3!} dx^5 \wedge dx^{5+a} \wedge dx^{5+b} \wedge dx^{5+c} \\ C_3^{(m)} &= -c\frac{x_5 x_l}{r}(\partial_r Z)\frac{\epsilon_{(l-5),abc}}{3!} dx^{5+a} \wedge dx^{5+b} \wedge dx^{5+c} \\ &= -c x_5 r^3 (\partial_r Z) d\Omega_3' \end{aligned}$$
$$\begin{aligned} ds^2 =& Z^{-1/6}W^{1/3}\left[Z^{-1/2}(-dt^2 + dx_1^2 + dx_4^2) + Z^{1/2}\left(dx_5^2 + \cdots + dx_9^2\right)\right] \\ &+ Z^{1/3}W^{-2/3}(dx_2^2 + dx_3^2 + dx_{11}^2) \\ C_3 =& Z^{-1} s dt \wedge dx_1 \wedge dx_4 - cs(Z-1)W^{-1}dx_2 \wedge dx_3 \wedge dx_{11} - c x_5 r^3 (\partial_r Z) d\Omega_3' \end{aligned}$$

$$x_4 \to cu_1 + sz, x_5 \to -su_1 + cz$$
$$x_1 \to y, x_2 \to u_2, x_3 \to u_3, x_{11} \to -u_4, x_{6,7,8,9} \to v_{1,2,3,4}$$
$$\begin{aligned} ds^2 =& W^{1/3}Z^{-2/3}(-dt^2 + dy^2) + W^{4/3}Z^{-2/3}(dz - cs(Z-1)W^{-1}du_1)^2 \\ &+ W^{1/3}Z^{1/3}(dv_1^2 + \cdots + dv_4^2) + W^{-2/3}Z^{1/3}(du_1^2 + du_2^2 + du_3^2 + du_4^2) \end{aligned}$$
$$C_3 = Z^{-1} s dt \wedge dy \wedge (s dz + c du_1) + cs(Z-1)W^{-1}du_2 \wedge du_3 \wedge du_4 + c(su_1 - cz)v^3(\partial_v Z) d\Omega_3'$$
$$e^{A_0} = W^{1/6}Z^{-1/3}, (-\partial_z w) = W$$
$$\left(\partial_{u_1} w\right) = \mathrm{cs}(Z-1)$$
$$\left(\partial_{v_l} w\right) = c(su_1 - cz)v_l \frac{\partial_v Z}{v}$$
$$\delta C_3 = -c^2 dt \wedge dy \wedge dz + cs dt \wedge dy \wedge du_1$$
$$w \equiv -zW + cs(Z-1)u_1$$

$$G_0 = -\frac{1}{2}Z(cz - su_1)^2 - \frac{1}{2}(sz + cu_1)^2 + f(v)$$

Entiéndase que la supergravedad cuántica, para efectos de este trabajo, comporta la simetría entre dos partículas o antipartículas, según sea el caso, de las cuales, una de ellas, es una superpartícula (Véase la definición proporcionada por este autor respecto de las superpartículas en sentido lato), a propósito de la deformación o perforación del espacio cuántico de que se trate, combinando en consecuencia, relatividad general y supersimetría. Entiéndase por supersimetría, para efectos de este trabajo, comporta la interacción de dos partículas o antipartículas, según sea el caso, de las cuales, una de ellas, es una superpartícula (Véase la definición proporcionada por este autor), a propósito de la deformación o perforación del espacio cuántico de que se trate, por acción de las superpartículas. Entiéndase que las supermembranas, para efectos de este trabajo, comporta la existencia de infinitas dimensiones a propósito de la deformación o perforación del espacio cuántico de que se trate, por acción de las superpartículas. Finalmente, entiéndase por superespacio, para efectos de este trabajo, como la existencia de un espacio cuántico relativista, el mismo que posee dimensiones ordinarias y anticonmutativas, a propósito de la deformación o perforación del espacio cuántico de que se trate, por acción de las superpartículas.

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Iosif Bena, Anthony Houppe, Dimitrios Toulikas y Nicholas P. Warner, Maze Topiary in Supergravity, arXiv:2312.02286v2 [hep-th] 21 Jan 2025.

Apéndice J.

1. Agujeros negros cuánticos, supermembranas, superespacios, dimensión temporal y supergravedad cuántica para campos cuánticos relativistas o curvos.

$$S_{EH}[g]=\frac{c^3}{16\pi G_N^{(d)}}\int_{\mathcal{M}} d^d x\sqrt{|g|}R(g)+(-1)^d\frac{c^3}{8\pi G_N^{(d)}}\int_{\partial\mathcal{M}} d^{d-1}\Sigma\mathcal{K}+S_{\text{materia}}$$

$$d^{d-1}\Sigma\equiv n^2 d^{d-1}\Sigma_\rho n^\rho$$

$$d^{d-1}\Sigma_\rho=\frac{1}{(d-1)!\sqrt{|g|}}\epsilon_{\rho\mu_1\cdots\mu_{d-1}}dx^{\mu_1}\wedge\ldots dx^{\mu_{d-1}}$$

$$\vec{F}=-\frac{8(d-3)\pi G_N^{(d)}mM}{(d-2)\omega_{d-2}}\frac{\vec{x}_{d-1}}{|\vec{x}_{d-1}|^{d-1}}$$

$$\mathcal{Z}=\int Dge^{+iS_{EH}/\hbar}$$

$$\frac{S_{EH}}{\hbar}=\frac{2\pi}{\ell_{\text{Planck}}^{d-2}}\int d^d x\ldots$$

$$\frac{\ell_{\text{Planck}}^{d-2}}{2\pi}=\frac{16\pi G_N^{(d)}\hbar}{c^3}$$

$$\ell_{\text{Planck}}=\frac{\ell_{\text{Planck}}}{2\pi}.$$

$$\lambda_{\text{Compton}}=\frac{\hbar}{Mc}$$

$$R_s=\left(\frac{16\pi MG_N^{(d)}c^{-2}}{(d-2)\omega_{(d-2)}}\right)^{\frac{1}{d-3}}$$

$$M_{\text{Planck}}=\left(\frac{\hbar^{d-3}}{G_N^{(d)}c^{d-5}}\right)^{\frac{1}{d-2}}$$

$$\frac{c^3}{G_N^{(d)}\hbar}=\left(\frac{M_{\text{Planck}}\,c}{\hbar}\right)^{d-2}$$

$$M\sim M_{\text{Planck}}\Rightarrow\lambda_{\text{Compton}}\sim R_s\sim\ell_{\text{Planck}}$$

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=0.\Rightarrow R_{\mu\nu}$$

$$ds^2=W(r)(dct)^2-W^{-1}(r)dr^2-R^2(r)d\Omega_{(2)}^2$$

$$d\Omega_{(2)}^2=d\theta^2+\sin^2\theta d\varphi^2$$

$$ds^2=W(dct)^2-W^{-1}dr^2-r^2d\Omega_{(2)}^2, W=1+\frac{\omega}{r}$$

$$M=-\frac{\omega c^2}{2G_N^{(4)}},\Rightarrow\omega=-R_S$$

$$M=\frac{1}{8\pi G_N^{(4)}}\int_{S_\infty^2} d^2S_i\left(\partial_j g_{ij}-\partial_i g_{jj}\right)$$

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\frac{8\pi G_{(N)}^{(4)}}{c^4}T_{\text{materia }\mu\nu}$$

$$ds^2=\left(1-\frac{R_s}{r}\right)dv^2-2dvdr-r^2d\Omega_{(2)}^2$$

$$v = ct + r + R_S \log \left|1 - \frac{R_s}{r}\right|$$
$$ds^2 = \frac{4R_S^3 e^{-r/R_S}}{r}[(dcT)^2 - dX^2] - r^2 d\Omega_{(2)}^2$$

$$\left(\frac{r}{R_S} - 1\right) e^{r/R_S} = X^2 - c^2T^2$$
$$\frac{ct}{R_S} = \ln\left(\frac{X + cT}{X - cT}\right) = 2\text{arcth}(cT/X)$$
$$ds^2 = W^{\frac{2M}{\omega}-1} W dt^2 - W^{1-\frac{2M}{\omega}}\left[W^{-1}dr^2 + r^2 d\Omega_{(2)}^2\right]$$
$$\varphi = \varphi_0 + \frac{\Sigma}{\omega}\ln\ W$$
$$W = 1 + \frac{\omega}{r}, \omega = \pm 2\sqrt{M^2 + \Sigma^2}$$
$$A = \int_{r=R_S} d\theta d\varphi r^2 = 4\pi R_S^2$$
$$\kappa^2 = -\frac{1}{2}(\nabla^\mu k^\nu)(\nabla_\mu k_\nu)\Big|_{\text{horizonte}}$$
$$ds^2 = g_{tt}(r)dt^2 + g_{rr}(r)dr^2 - r^2 d\Omega_{(2)}^2$$
$$\kappa = \frac{1}{2}\frac{\partial_r g_{tt}}{\sqrt{-g_{tt}g_{rr}}},$$
$$\kappa = \frac{c^4}{4G_N^{(4)}M}$$
$$dE = TdS$$
$$dM \sim \frac{1}{G_N^{(4)}}\kappa dA$$
$$dM = \frac{1}{8\pi G_N^{(4)}}\kappa dA$$
$$M = \frac{1}{4\pi G_N^{(4)}}\kappa A$$
$$T = \frac{\hbar\kappa}{2\pi c}$$
$$S = \frac{Ac^3}{4\hbar G_N^{(4)}}$$
$$S = \frac{1}{32\pi^2}\frac{A}{\ell_{\text{Planck}}^2}$$
$$T = \frac{\hbar c^3}{8\pi G_N^{(4)}M}, S = \frac{4\pi G_N^{(4)}M^2}{\hbar c}$$
$$dMc^2 = TdS, Mc^2 = 2TS$$

$$C^{-1} = \frac{\partial T}{\partial M} = \frac{-\hbar c^3}{8\pi G_N^{(4)}M^2} < 0$$
$$S(E) = \log\ \rho(E)$$
$$\rho(M) \sim e^{M^2}$$

$$\mathcal{Z} = \mathrm{Tr} e^{-(H-\mu_i C_i)/T}$$
$$W = E - TS - \mu_i C_i$$
$$e^{-\beta W} = \mathcal{Z}$$
$$S = \frac{1}{T}(E - \mu_i C_i) + \log \mathcal{Z}$$
$$\mathcal{Z} = \int Dg e^{-\tilde{S}_{EH}/\hbar}$$
$$S_{EH}[g] = \frac{c^3}{16\pi G_N^{(4)}} \int_{\mathcal{M}} d^4x\sqrt{|g|}R + \frac{c^3}{8\pi G_N^{(4)}} \int_{\partial\mathcal{M}} (\mathcal{K} - \mathcal{K}_0)$$
$$\mathcal{Z} = e^{-\tilde{S}_{EH}(\text{ on-shell })}$$
$$\left(\frac{r}{R_S} - 1\right) e^{r/R_S} = X^2 - T^2$$
$$\left(\frac{r}{R_S} - 1\right) e^{r/R_S} = X^2 + \mathcal{T}^2 > 0$$
$$\frac{X+T}{X-T} = e^{t/R_S}$$
$$\frac{X - i\mathcal{T}}{X + \mathcal{T}} = e^{-2i\mathrm{Arg}(X+i\mathcal{T})} = e^{-i\tau/R_S}$$
$$n_\mu = -\frac{\delta_{\mu r}}{\sqrt{-n^2}} = -\sqrt{-g_{rr}}\delta_{\mu r}$$
$$ds_{(3)}^2 = h_{\mu\nu}dx^\mu dx^\nu = g_{tt}dt^2 - r^2 d\Omega_{(2)}^2\Big|_{r=r_0}$$
$$\nabla_\mu n_\nu = -\sqrt{-g_{rr}}\{\delta_{\mu r}\delta_{\nu r}\partial_r \log \sqrt{-g_{rr}} - \Gamma_{\mu\nu}^r\}$$
$$\mathcal{K} = h^{\mu\nu}\nabla_\mu n_\nu = \frac{1}{\sqrt{-g_{rr}}}\left\{\frac{1}{2}\partial_r \log g_{tt} + \frac{2}{r}\right\}\Bigg|_{r=r_0}$$
$$\mathcal{K}_0 = \frac{2}{r}\Big|_{r=r_0}$$
$$g_{tt} \sim 1 - \frac{2M}{r}, g_{rr} \sim -\left(1 + \frac{2M}{r}\right)$$
$$(\mathcal{K} - \mathcal{K}_0)|_{r=r_0} \sim -\frac{M}{r_0^2}$$
$$\frac{i}{8\pi}\int_{r_0\to\infty} d^3x\sqrt{|h|}(\mathcal{K} - \mathcal{K}_0) = \lim_{r_0\to\infty}\frac{i}{8\pi}\int_0^{-i\beta} dt \int_{S^2} d\Omega^2 r_0^2\sqrt{g_{tt}(r_0)}(\mathcal{K} - \mathcal{K}_0)$$
$$= \lim_{r_0\to\infty}\frac{\beta}{2}r_0^2(\mathcal{K} - \mathcal{K}_0) = -\frac{\beta M}{2}$$
$$S = \beta M + \log \mathcal{Z} = \frac{\beta M}{2} = 4\pi M^2$$
$$S_{EM}[g, A] = S_{EH}[g] + \frac{1}{c}\int d^dx\sqrt{|g|}\left[-\frac{1}{4}F^2\right]$$
$$F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$$
$$A'_\mu = A_\mu + \partial_\mu \Lambda$$
$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{8\pi G_N^{(4)}}{c^3}T_{\mu\nu} = 0$$
$$\nabla_\mu F^{\mu\nu} = 0$$
$$T_{\mu\nu} = \frac{-2c}{\sqrt{|g|}}\frac{\delta S_M[A]}{\delta g^{\mu\nu}} = F_{\mu\rho}F_\nu^\rho - \frac{1}{4}g_{\mu\nu}F^2$$
$$\nabla_\mu {}^\star F^{\mu\nu}$$

$$A \equiv A_\mu dx^\mu, F = \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu \equiv dA$$
$$d^\star F = 0, dF = 0. \left(\partial_{[\alpha} F_{\beta\gamma]} = 0\right)$$
$$\frac{1}{c^2} \int d^d x \sqrt{|g|} [-A_\mu j^\mu]$$
$$\nabla_\mu j^\mu = 0, (d^\star j = 0)$$
$$\partial_\mu \mathrm{j}^\mu = 0$$
$$\nabla_\mu F^{\mu\nu} = \frac{1}{c} j^\nu, \left(d^\star F = \frac{1}{c} \star j\right)$$
$$j^\mu(y) = qc \int_\gamma dX^\mu \frac{1}{\sqrt{|g|}} \delta^{(4)}(y - X(\xi))$$
$$j^\mu(y^0, \vec{y}) = qc \int dX^0 \frac{dX^\mu}{dX^0} \frac{1}{\sqrt{|g|}} \delta^{(3)}(\vec{y} - \vec{X}) \delta(y^0 - X^0) = qV^\mu \frac{\delta^{(3)}\left(\vec{y} - \vec{X}(y^0)\right)}{\sqrt{|g|}}$$
$$j^\mu(y^0, \vec{y}) = qc\delta^{\mu 0} \frac{\delta^{(3)}(\vec{y})}{\sqrt{|g|}}$$
$$-\frac{q}{c} \int_{\gamma(\xi)} A_\mu \dot{x}^\mu d\xi = -\frac{q}{c} \int_\gamma A$$
$$S_{M,q}[X^\mu(\xi)] = -Mc \int d\xi \sqrt{g_{\mu\nu}(X) \dot{X}^\mu \dot{X}^\nu} - \frac{q}{c} \int A_\mu \dot{X}^\mu$$
$$0 = \int_V d^\star j$$
$$\int_V d^\star j = \int_{x^0 = x_2^0} {}^\star j - \int_{x^0 = x_1^0}^\star j = 0$$
$$q = \frac{1}{c} \int_{x^0 = \text{ constant}} {}^\star j,$$
$$q = \int_{S^2_\infty} {}^\star F$$
$$S_{EM}[g, A] = \frac{1}{16\pi G_N^{(d)}} \int d^d x \sqrt{|g|} \left[R - \frac{1}{4} F^2\right]$$
$$q = \frac{1}{16\pi G_N^{(d)}} \int_{S_\infty^{d-2}} {}^\star F$$
$$E_r = F_{0r} \sim \frac{4G_N^{(4)} q}{r^2}$$
$$F_{tr} \sim \pm \frac{1}{R^2(r)}$$
$$ds^2 = f(r)dt^2 - f^{-1}(r)dr^2 - r^2 d\Omega^2_{(2)}$$
$$F_{tr} = -\frac{4G_N^{(4)} q}{r^2}$$
$$f(r) = r^{-2}(r - r_+)(r - r_-)$$
$$r_\pm = G_N^{(4)} M \pm r_0, r_0 = G_N^{(4)} (M^2 - 4q^2)^{1/2}$$
$$A_\mu = \delta_{\mu t} \frac{-4G_N^{(4)} q}{r}$$
$$A = 4\pi r_+^2$$

$$A_{\text{extreme}} = 4\pi r_+^2 = 4\pi\left(G_N^{(4)}M\right)^2$$

$$F_{12} = -G_N^{(4)}\frac{M_1M_2}{r_{12}^2} + 4G_N^{(4)}\frac{q_1q_2}{r_{12}^2}$$

$$ds^2 = H^{-2}dt^2 - H^2 d\vec{x}_3^2$$

$$A_\mu = -2\text{sign}(q)(H^{-1}-1)\delta_{\mu t},$$

$$H = 1 + \frac{G_N^{(4)}M}{|\vec{x}_3|}$$

$$\partial_{\underline{i}}\partial_{\underline{i}}H = 0$$

$$ds^2 = H^{-2}dt^2 - H^2 d\vec{x}_3^2,$$

$$A_\mu = \delta_{\mu t}\alpha(H^{-1}-1), \alpha = \pm 2$$

$$\partial_{\underline{i}}\partial_{\underline{i}}H = 0.$$

$$H(\vec{x}_3) = 1 + \sum_{i=1}^{N}\frac{2G_N^{(4)}|q_i|}{|\vec{x}_3 - \vec{x}_{3,i}|}$$

$$ds^2 = \frac{\rho^2}{R_{AdS}^2}dt^2 - R_{AdS}^2\frac{d\rho^2}{\rho^2} - R_{AdS}^2 d\Omega_{(2)}^2$$

$$A_t = -\frac{2\rho}{R_{AdS}}, F_{\rho t} = -\frac{2}{R_{AdS}}$$

$$ds^2 = H^{-2}Wdt^2 - H^2\left[W^{-1}d\rho^2 + \rho^2 d\Omega_{(2)}^2\right]$$

$$A_\mu = \delta_{\mu t}\alpha(H^{-1}-1)$$

$$H = 1 + \frac{h}{\rho}, W = 1 + \frac{\omega}{\rho}, \omega = h\left[1 - \left(\frac{\alpha}{2}\right)^2\right]$$

$$\alpha = -\frac{4G_N^{(4)}q}{r_\pm}, h = r_\pm, \omega = \pm 2r_0$$

$$S[g, A^I] = \frac{1}{16\pi G_N^{(4)}}\int d^4x\sqrt{|g|}\left[R - \frac{1}{4}\sum_{I=1}^{I=N}(F^I)^2\right]$$

$$ds^2 = H^{-2}Wdt^2 - H^2\left[W^{-1}d\rho^2 + \rho^2 d\Omega_{(2)}^2\right]$$

$$A_\mu^i = \delta_{\mu t}\alpha^i(H^{-1}-1)$$

$$H = 1 + \frac{h}{\rho}, W = 1 + \frac{\omega}{\rho}, \omega = h\left[1 - \sum_{i=1}^{i=N}\left(\frac{\alpha^i}{2}\right)^2\right]$$

$$\alpha^i = -\frac{4G_N^{(4)}q^i}{r_\pm}, h = r_\pm, \omega = \pm 2r_0$$

$$r_\pm = G_N^{(4)}M \pm r_0, r_0 = G_N^{(4)}\sqrt{M^2 - 4\sum_{i=1}^{i=N}q_i^2}$$

$$dM = \frac{1}{8\pi G_N^{(4)}}\kappa dA + \Phi dq$$

$$\kappa = \frac{1}{G_N^{(4)}}\frac{\sqrt{M^2-4q^2}}{\left(M+\sqrt{M^2-4q^2}\right)^2}$$

$$T=\frac{1}{2\pi G_N^{(4)}}\frac{\sqrt{M^2-4q^2}}{\left(M+\sqrt{M^2-4q^2}\right)^2}, S=\pi G_N^{(4)}\left(M+\sqrt{M^2-4q^2}\right)^2$$

$$\tilde{\vec{E}}=\vec{B},\tilde{\vec{B}}=-\vec{E}$$

$$\tilde{F}=aF+b^{\star}F,\Rightarrow^{\star}\tilde{F}=-bF+a^{\star}F,a^2+b^2\neq 0$$

$$\vec{F}\equiv\begin{pmatrix}F\\ ^{\star}F\end{pmatrix},\star\vec{F}=\begin{pmatrix}0&1\\-1&0\end{pmatrix}\vec{F}$$

$$\nabla_\mu\vec{F}^{\mu\nu}=0$$

$$\tilde{\vec{F}}=M\vec{F},M=\begin{pmatrix}a&b\\-b&a\end{pmatrix}$$

$$\int_{S^2_\infty}{}^{\star}\vec{F}=\begin{pmatrix}16\pi G_N^{(4)}q\\ p\end{pmatrix}\equiv 16\pi G_N^{(4)}\vec{q},\vec{q}=\begin{pmatrix}q\\ p/16\pi G_N^{(4)}\end{pmatrix}$$

$$\tilde{A}_\mu(x)=-\int_0^1 d\lambda\lambda x^\nu\frac{\epsilon_{\mu\nu}^{\rho\sigma}}{\sqrt{|g|}}\partial_\rho A_\sigma(\lambda x)$$

$$G_{\mu\nu}-\vec{F}_\mu^{T\rho}\vec{F}_{\nu\rho}$$

$$\vec{F}\equiv\begin{pmatrix}e^{-2}F\\ ^{\star}F\end{pmatrix},G_{\mu\nu}+\left(\vec{F}_\mu^{\rho}\right)^T\begin{pmatrix}0&1\\-1&0\end{pmatrix}\vec{F}_{\nu\rho}$$

$$M=\begin{pmatrix}a&0\\0&1/a\end{pmatrix},\quad e'=a^{-1}e$$

$$M=\begin{pmatrix}0&1\\-1&0\end{pmatrix},\quad e'=\frac{1}{e}$$

$$\begin{cases}\tilde{F}&=\cos\ \xi F+\sin\ \xi^{\star}F\\ \star\tilde{F}&=-\sin\ \xi F+\cos\ \xi^{\star}F\end{cases}$$

$$\begin{cases}\tilde{F}_{tr}=\dfrac{-4G_N^{(4)}\cos\ \xi q}{r^2}\\ \tilde{F}_{\theta\varphi}=4G_N^{(4)}\sin\ \xi q\sin\ \theta\end{cases}$$

$$\tilde{q}=\cos\ \xi q,\tilde{p}=-16G_N^{(4)}\sin\ \xi q,\Rightarrow\tilde{q}^2+\left(\frac{\tilde{p}}{16\pi G_N^{(4)}}\right)=q^2$$

$$ds^2=f(r)dt^2-f^{-1}(r)dr^2-r^2d\Omega^2_{(2)}$$

$$F_{tr}=-\frac{4G_N^{(4)}q}{r^2},F_{\theta\varphi}=-\frac{p}{4\pi}\sin\ \theta,$$

$$f(r)=r^{-2}(r-r_+)(r-r_-),$$

$$r_\pm=G_N^{(4)}M\pm r_0,r_0=G_N^{(4)}\left\{M^2-4\left[q^2+\left(\frac{p}{16\pi G_N^{(4)}}\right)^2\right]\right\}^{1/2}$$

$$[Q^\alpha,M_{ab}]=\Gamma_s(M_{ab})^\alpha{}_\beta Q^\beta$$

$$\{Q^\alpha,Q^\beta\}=i(\gamma^a C^{-1})^{\alpha\beta}P_a$$

$$[M_{ab},M_{cd}]=-M_{eb}\Gamma_v(M_{cd})^e{}_a-M_{ae}\Gamma_v(M_{cd})^e{}_b$$

$$[P_a,M_{bc}]=-P_e\Gamma_v(M_{bc})^e{}_a,$$

$$[Q^\alpha,M_{ab}]=\Gamma_s(M_{ab})^\alpha{}_\beta Q^\beta,$$

$$\{Q^\alpha,Q^\beta\}=i(\gamma^a C^{-1})^{\alpha\beta}P_a.$$

$$A_\mu=e^a{}_\mu P_a+\frac{1}{2}\omega_\mu{}^{ab}M_{ab}+\bar{\psi}_{\mu\alpha}Q^\alpha$$

$$\Lambda = \sigma^a P_a + \frac{1}{2}\sigma^{ab} M_{ab} + \bar{\epsilon}_\alpha Q^\alpha$$
$$\delta A_\mu = \partial_\mu \Lambda + \Lambda, A_\mu \equiv \mathbb{D}_\mu \Lambda$$
$$S[e^a{}_\mu, \omega_\mu{}^{ab}, \psi_\mu] = \int d^4 x e\left[R(e,\omega) + 2e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu \gamma_5 \gamma_\nu \nabla_\rho \psi_\sigma\right]$$
$$R(e,\omega) = e_a{}^\mu e_b{}^\nu R_{\mu\nu}{}^{ab}(\omega)$$
$$\omega_{abc} = -\Omega_{abc} + \Omega_{bca} - \Omega_{cab},$$
$$\Omega_{\mu\nu}{}^a = \Omega_{\mu\nu}{}^a(e) + \frac{1}{2}T_{\mu\nu}{}^a,$$
$$\Omega_{\mu\nu}{}^a(e) = \partial_{[\mu} e^a{}_{\nu]}, T_{\mu\nu}{}^a = i\bar{\psi}_\mu \gamma^a \psi_\nu.$$
$$\begin{cases} \delta_\xi x^\mu &= \xi^\mu \\ \delta_\xi e^a{}_\mu &= -\xi^\nu \partial_\nu e^a{}_\mu - \partial_\mu \xi^\nu e^a{}_\nu \\ \delta_\xi \psi_\mu &= -\xi^\nu \partial_\nu \psi_\mu - \partial_\mu \xi^\nu \psi_\nu \end{cases}$$
$$\begin{cases} \delta_\sigma e^a{}_\mu &= \sigma^a{}_b e^b{}_\mu \\ \delta_\sigma \psi_\mu &= \frac{1}{2}\sigma^{ab}\gamma_{ab}\psi_\mu \end{cases}$$
$$\begin{cases} \delta_\epsilon e^a{}_\mu &= -i\bar{\epsilon}\gamma\psi_\mu \\ \delta_\epsilon \psi_\mu &= \nabla_\mu \epsilon \end{cases}$$
$$[M_{ab}, M_{cd}] = -M_{eb}\Gamma_v(M_{cd})^e{}_a - M_{ae}\Gamma_v(M_{cd})^e_b,$$
$$[P_a, M_{bc}] = -P_e \Gamma_v(M_{bc})^e{}_a,$$
$$[Q^{\alpha i}, M_{ab}] = \Gamma_s(M_{ab})^\alpha{}_\beta Q^{\beta i},$$
$$\{Q^{\alpha i}, Q^{\beta j}\} = i\delta^{ij}(\gamma^a \mathcal{C}^{-1})^{\alpha\beta} P_a - i(\mathcal{C}^{-1})^{\alpha\beta} Q^{ij} - \gamma_5 (\mathcal{C}^{-1})^{\alpha\beta} P^{ij}$$
$$A_\mu = e^a{}_\mu P_a + \frac{1}{2}\omega_\mu{}^{ab} M_{ab} + \frac{1}{2}A^{ij}{}_\mu Q^{ij} + \bar{\psi}^i{}_{\mu\alpha} Q^{i\alpha}$$
$$\frac{1}{n!}(\gamma^{a_1\cdots a_n}\mathcal{C}^{-1})^{\alpha\beta} Z^{ij}_{a_1\cdots a_n}$$
$$\left[Z^{kl}_{c_1\cdots c_n}, M_{ab}\right] = -n\Gamma_v(M_{ab})^e{}_{[c_1} Z^{kl}_{|e|c_2\cdots c_n]}$$
$$\mathcal{C}^{-1}, \gamma_5 \mathcal{C}^{-1}, \gamma_5 \gamma_a \mathcal{C}^{-1}, \gamma_{abc}\mathcal{C}^{-1}, \gamma_{abcd}\mathcal{C}^{-1}$$
$$\gamma_a \mathcal{C}^{-1}, \gamma_{ab}\mathcal{C}^{-1}, \gamma_5 \gamma_{ab}\mathcal{C}^{-1}, \gamma_5 \gamma_{abc}\mathcal{C}^{-1}$$
$$\begin{aligned} \{Q^{\alpha i}, Q^{\beta j}\} =& i\delta^{ij}(\gamma^a \mathcal{C}^{-1})^{\alpha\beta} P_a + i(\mathcal{C}^{-1})^{\alpha\beta} Z^{[ij]} + \gamma_5 (\mathcal{C}^{-1})^{\alpha\beta}\tilde{Z}^{[ij]} \\ &+ (\gamma^a \mathcal{C}^{-1})^{\alpha\beta} Z_a^{(ij)} + i(\gamma_5 \gamma^a \mathcal{C}^{-1})^{\alpha\beta} Z_a^{[ij]} \\ &+ i(\gamma^{ab}\mathcal{C}^{-1})^{\alpha\beta} Z_{ab}^{(ij)} + (\gamma_5 \gamma^{ab}\mathcal{C}^{-1})^{\alpha\beta}\tilde{Z}_{ab}^{(ij)} \end{aligned}$$
$$\delta g_{\mu\nu} = -2\nabla_{(\mu} k_{\nu)}$$
$$\delta_\epsilon B \sim \epsilon F = 0,$$
$$\delta_\epsilon F \sim \begin{Bmatrix} \partial\epsilon + B\epsilon \\ \partial B\epsilon + B\epsilon \end{Bmatrix} = 0$$
$$\delta_\epsilon \psi_\mu = \nabla_\mu \epsilon = 0$$
$$(1 - \gamma^0 \gamma^1)\epsilon = 0$$
$$k^\mu \sim \bar{\epsilon}\gamma^\mu \epsilon$$
$$\delta_\epsilon |s\rangle \sim \bar{\epsilon}^i_\alpha Q^{i\alpha}|s\rangle = 0$$
$$\bar{\epsilon}\mathfrak{M}_\epsilon = 0,$$
$$\mathfrak{M} \equiv i\delta^{ij}\gamma^a P_a + iZ^{[ij]} + \gamma_5 \tilde{Z}^{[ij]} + \gamma^a Z_a^{(ij)} + i\gamma_5 \gamma^a Z_a^{[ij]} + i\gamma^{ab} Z_{ab}^{(ij)} + \gamma_5 \gamma^{ab}\tilde{Z}_{ab}^{(ij)}$$
$$\mathfrak{M} = ip\gamma^0(1 \pm \gamma^0 \gamma^1)$$
$$\mathfrak{M} = i\gamma^0 M\left(\delta^{ij} + \frac{Q}{M}\gamma^0 \epsilon^{ij}\right)$$
$$(\delta^{ij} \pm \gamma^0 \epsilon^{ij})\epsilon^j = 0$$

$$M = |Z|$$
$$M = |Z_1| = |Z_2|$$
$$M = |Z_1| \neq |Z_2|.$$
$$M = |Z_1| = |Z_2| = |Z_3| = |Z_4|,$$
$$M = |Z_1| = |Z_2| \neq |Z_{3,4}|,$$
$$M = |Z_1| \neq |Z_{2,3,4}|.$$
$$(\delta^{ij} + \gamma^0 \alpha^{ij})$$
$$\mathfrak{M} = i\gamma^0 M \left(\delta^{ij} + \frac{Z^{(p)}}{M} \gamma^0 \gamma^1 \cdots \gamma^p \alpha^{ij} \right)$$
$$M \geq |Z_i|, i = 1, \dots, [N/2]$$
$$\left\{ e^a{}_\mu, \psi_\mu = \begin{pmatrix} \psi_\mu^1 \\ \psi_\mu^2 \end{pmatrix}, A_\mu \right\}$$
$$S = \int d^4x e \{ R(e,\omega) + 2e^{-1} \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu \nabla_\rho \psi_\sigma - \mathcal{F}^2 + \mathcal{J}_{(m)}{}^{\mu\nu} (\mathcal{J}_{(e)\mu\nu} + \mathcal{J}_{(m)\mu\nu}) \}$$
$$\begin{cases} \mathcal{F}_{\mu\nu} = \tilde{F}_{\mu\nu} + \mathcal{J}_{(m)\mu\nu} \\ \tilde{F}_{\mu\nu} = F_{\mu\nu} + \mathcal{J}_{(e)\mu\nu} \\ F_{\mu\nu} = 2\partial_{[\mu} A_{\nu]} \end{cases}$$
$$\begin{cases} \mathcal{J}_{(e)\mu\nu} = i\bar{\psi}_\mu \sigma^2 \psi_\nu \\ \mathcal{J}_{(m)\mu\nu} = -\frac{1}{2e} \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\rho \gamma_5 \sigma^2 \psi_\sigma \end{cases}$$
$$T_{\mu\nu}{}^a = i\bar{\psi}_\mu \gamma^a \psi_\nu (\equiv i\bar{\psi}_{j\mu} \gamma^a \psi_\nu^j)$$
$$\begin{cases} \delta_\epsilon e^a{}_\mu = -i\bar{\epsilon}\gamma^a \psi_\mu \\ \delta_\epsilon A_\mu = -i\bar{\epsilon}\sigma^2 \psi_\mu \\ \delta_\epsilon \psi_\mu = \tilde{\nabla}_\mu \epsilon \end{cases}$$
$$\tilde{\nabla}_\mu = \nabla_\mu + \frac{1}{4} \tilde{F} \gamma_\mu \sigma^2$$
$$\begin{cases} \tilde{F}'_{\mu\nu} = \cos\theta \tilde{F}_{\mu\nu} + \sin\theta^\star \tilde{F}_{\mu\nu} \\ \psi'_\mu = e^{\frac{i}{2}\theta\gamma_5} \psi_\mu \end{cases}$$
$$\{ e^a{}_\mu, A^{(n)}{}_\mu, \phi, a, \psi_\mu^i, \lambda^i \}$$
$$S = \int d^4x \sqrt{|g|} \left\{ R + 2(\partial\phi)^2 + \frac{1}{2} e^{4\phi} (\partial a)^2 - e^{-2\phi} \sum_{n=1}^{6} F^{(n)} F^{(n)} + a \sum_{n=1}^{6} F^{(n)\star} F^{(n)} \right\}$$
$$\tau = a + ie^{-2\phi}$$
$$\tilde{F}_{\mu\nu}^{(n)} = \partial_\mu \tilde{A}_\nu^{(n)} - \partial_\nu \tilde{A}_\mu^{(n)}$$
$$\Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, ad - bc = 1$$
$$\begin{pmatrix} \tilde{F}^{(n)}{}_{\mu\nu} \\ F^{(n)}{}_{\mu\nu} \end{pmatrix} \longrightarrow \Lambda \begin{pmatrix} \tilde{F}^{(n)}{}_{\mu\nu} \\ F^{(n)}{}_{\mu\nu} \end{pmatrix}$$
$$\tau \longrightarrow \frac{a\tau + b}{c\tau + d}$$
$$M^2 + \frac{|Z_1 Z_2|^2}{M^2} - |Z_1|^2 - |Z_2|^2 \geq 0$$
$$\partial_{\underline{i}} \partial_{\underline{i}} \mathcal{H}_1 = \partial_{\underline{i}} \partial_{\underline{i}} \mathcal{H}_2 = 0$$

$$\sum_{n=1}^{N}\left(k^{(n)}\right)^2=0,\sum_{n=1}^{N}\left|k^{(n)}\right|^2=\frac{1}{2}$$

$$e^{-2U}=2\Im\,\mathrm{m}\left(\mathcal{H}_1\overline{\mathcal{H}}_2\right)$$

$$\partial_{[\underline{i}}\omega_{\underline{j}]}=\epsilon_{ijk}\Re\mathrm{e}\left(\mathcal{H}_1\partial_{\underline{k}}\overline{\mathcal{H}}_2-\overline{\mathcal{H}}_2\partial_{\underline{k}}\mathcal{H}_1\right)$$

$$ds^2=e^{2U}\left(dt^2+\omega_{\underline{i}}dx^{\underline{i}}\right)^2-e^{-2U}d\vec{x}^2$$

$$\lambda=\frac{\mathcal{H}_1}{\mathcal{H}_2}$$

$$A_t^{(n)}=2e^{2U}\Re\mathrm{e}\left(k^{(n)}\mathcal{H}_2\right)$$

$$\tilde{A}_t^{(n)}=-2e^{2U}\Re\mathrm{e}\left(k^{(n)}\mathcal{H}_1\right)$$

$$\mathcal{H}_1=i\mathcal{H}_2=\frac{1}{\sqrt{2}}V^{-1}$$

$$\Upsilon=-2\frac{\sum_n\overline{\Gamma^{(n)}}^2}{M}$$

$$M^2+|\Upsilon|^2-4\sum_n\left|\Gamma^{(n)}\right|^2=0$$

$$\frac{1}{2}\left|Z_{1,2}\right|^2=\sum_n\left|\Gamma^{(n)}\right|^2\pm\left[\left(\sum_n\left|\Gamma^{(n)}\right|^2\right)^2-\left|\sum_n\Gamma^{(n)2}\right|^2\right]^{\frac{1}{2}}$$

$$A=4\pi(|M|^2-|\Upsilon|^2)=4\pi\left||Z_1|^2-|Z_2|^2\right|$$

$$A=8\pi\sqrt{\left|\det\left[\begin{pmatrix}\vec{p}^t\\\vec{q}^t\end{pmatrix}(\vec{p}\vec{q})\right]\right|}$$

$$\begin{pmatrix}\vec{p}\\\vec{q}\end{pmatrix}'=R\otimes S\begin{pmatrix}\vec{p}\\\vec{q}\end{pmatrix}$$

$$\left\{\hat{\hat{Q}}^{\hat{\hat{\alpha}}},\hat{\hat{Q}}^{\hat{\hat{\beta}}}\right\}=i\left(\hat{\hat{\Gamma}}^{\hat{\hat{a}}}\hat{\hat{\mathcal{C}}}^{-1}\right)^{\hat{\hat{\alpha}}\hat{\hat{\beta}}}\hat{\hat{P}}_{\hat{\hat{a}}}+\frac{1}{2}\left(\hat{\hat{\Gamma}}^{\hat{\hat{a}}_1\hat{\hat{a}}_2}\hat{\hat{\mathcal{C}}}^{-1}\right)^{\hat{\hat{\alpha}}\hat{\hat{\beta}}}\hat{\hat{\mathcal{Z}}}^{(2)}_{\hat{\hat{a}}_1\hat{\hat{a}}_2}+\frac{i}{5!}\left(\hat{\hat{\Gamma}}^{\hat{\hat{a}}_1\cdots\hat{\hat{a}}_5}\hat{\hat{\mathcal{C}}}^{-1}\right)^{\hat{\hat{\alpha}}\hat{\hat{\beta}}}\hat{\hat{\mathcal{Z}}}^{(5)}_{\hat{\hat{a}}_1\cdots\hat{\hat{a}}_5}$$

$$+\frac{1}{6!}\left(\hat{\hat{\Gamma}}^{\hat{\hat{a}}_1\cdots\hat{\hat{a}}_6}\hat{\hat{\mathcal{C}}}^{-1}\right)\Bigg)^{\hat{\hat{\alpha}}\hat{\hat{\beta}}}\hat{\hat{\mathcal{Z}}}^{(6)}_{\hat{\hat{a}}_1\cdots\hat{\hat{a}}_6}+\frac{i}{9!}\left(\hat{\hat{\Gamma}}^{\hat{\hat{a}}_1\cdots\hat{\hat{a}}_9}\hat{\hat{\mathcal{C}}}^{-1}\right)\Bigg)^{\hat{\hat{\alpha}}\hat{\hat{\beta}}}\mathcal{Z}^{(9)}_{a_1\cdots a_9}$$

$$+\frac{1}{10!}\left(\hat{\hat{\Gamma}}^{\hat{\hat{a}}_1\cdots\hat{\hat{a}}_{10}}\hat{\hat{\mathcal{C}}}^{-1}\right)\hat{\hat{\alpha}}^{\hat{\hat{\alpha}}\hat{\hat{\beta}}}\hat{\hat{\mathcal{Z}}}^{(10)}_{\hat{\hat{a}}_1\cdots\cdots\hat{\hat{a}}_{10}}.$$

$$\left\{\hat{\hat{e}}^{\hat{\hat{a}}}_{\hat{\hat{\mu}}},\hat{\hat{C}}_{\hat{\hat{\mu}}\hat{\hat{\rho}}\hat{\hat{L}}},\hat{\hat{\psi}}_{\hat{\hat{\mu}}}\right\}$$

$$\hat{\hat{S}}=\frac{1}{16\pi G_N^{(11)}}\int d^{11}\hat{\hat{x}}\sqrt{|\hat{\hat{g}}|}\left[\hat{\hat{R}}-\frac{1}{2\cdot4!}\hat{\hat{G}}^2-\frac{1}{(144)^2}\frac{1}{\sqrt{|\hat{\hat{g}}}}\hat{\hat{\epsilon}}\hat{\hat{G}}\hat{\hat{G}}\hat{\hat{C}}\hat{\hat{C}}\right]$$

$$\hat{\hat{G}}=4\partial\hat{\hat{C}}$$

$$\delta_{\hat{\hat{\chi}}}\hat{\hat{C}}=3\partial\hat{\hat{\chi}}$$

$$\begin{cases} \delta_{\hat{\hat{\epsilon}}}\hat{\hat{e}}_{\hat{\hat{\mu}}}{}^{\hat{\hat{a}}} = -\frac{i}{2}\bar{\hat{\hat{\epsilon}}}\hat{\hat{\Gamma}}^{\hat{\hat{a}}}\hat{\hat{\psi}}_{\hat{\hat{\mu}}}, \\ \delta_{\hat{\hat{\epsilon}}}\hat{\hat{\psi}}_{\hat{\hat{\mu}}} = 2\nabla_{\hat{\hat{\mu}}}\hat{\hat{\epsilon}} + \frac{i}{144}\left(\hat{\hat{\Gamma}}^{\hat{\hat{\alpha}}\hat{\hat{\beta}}\hat{\hat{\gamma}}\hat{\hat{\delta}}}{}_{\hat{\hat{\mu}}} - 8\hat{\hat{\Gamma}}^{\hat{\hat{\beta}}\hat{\hat{\gamma}}\hat{\hat{\delta}}}\hat{\hat{\eta}}_{\hat{\hat{\mu}}}{}^{\hat{\hat{\alpha}}}\right)\hat{\hat{\epsilon}}\hat{\hat{G}}_{\hat{\hat{\alpha}}\hat{\hat{\beta}}\hat{\hat{\gamma}}\hat{\hat{\delta}}}, \\ \delta_{\hat{\hat{\epsilon}}}\hat{\hat{C}}_{\hat{\hat{\mu}}\hat{\hat{\nu}}\hat{\hat{\rho}}} = \frac{3}{2}\bar{\hat{\hat{\epsilon}}}\hat{\hat{\Gamma}}_{[\hat{\hat{\mu}}\hat{\hat{\nu}}}\hat{\hat{\psi}}_{\hat{\hat{\rho}}]}. \end{cases}$$

$$\partial\left({}^{\star}\hat{\hat{G}} + \frac{35}{2}\hat{\hat{C}}\hat{\hat{G}}\right)$$

$$*\,\hat{\hat{G}} = 7(\partial\hat{\hat{\hat{C}}} - 10\hat{\hat{C}}\partial\hat{\hat{C}}) \equiv \hat{\hat{\hat{G}}}$$

$$\delta_{\hat{\hat{\chi}}}\hat{\hat{\hat{C}}} = 6\partial\hat{\hat{\chi}}$$

$$\delta_{\hat{\hat{\chi}}}\hat{\hat{\hat{C}}} = -30\partial\hat{\hat{\chi}}\hat{\hat{C}}$$

$$\hat{\hat{P}}_{\hat{\hat{a}}} = (\hat{P}_{\hat{a}}, \hat{Z}^{(0)}), \hat{\hat{Z}}^{(2)}_{\hat{\hat{a}}\hat{\hat{b}}} = \left(\hat{Z}^{(2)}_{\hat{a}\hat{b}}, \hat{Z}^{(1)}_{\hat{a}}\right), \hat{\hat{Z}}^{(5)}_{\hat{\hat{a}}_1\cdots\hat{\hat{a}}_5} = \left(\hat{Z}^{(5)}_{\hat{a}_1\cdots\hat{a}_5}, \hat{Z}^{(4)}_{\hat{a}_1\cdots\hat{a}_4}\right)$$

$$\hat{\hat{Z}}^{(6)}_{\hat{\hat{a}}_1\cdots\hat{\hat{a}}_6} = \left(\hat{Z}^{(6)}_{\hat{a}_1\cdots\hat{a}_6}, \cdot\right), \hat{\hat{Z}}^{(9)}_{\hat{\hat{a}}_1\cdots\hat{\hat{a}}_9} = \left(\cdot, \hat{Z}^{(8)}_{\hat{a}_1\cdots\hat{a}_8}\right)$$

$$\left\{\hat{Q}^{\hat{\alpha}}, \hat{Q}^{\hat{\beta}}\right\} = i\left(\hat{\Gamma}^{\hat{a}}\hat{C}^{-1}\right)^{\hat{\alpha}\hat{\beta}}\hat{P}_{\hat{a}} + \sum_{n=0,1,4,8}\frac{c_n}{n!}\left(\hat{\Gamma}^{\hat{a}_1\cdots\hat{a}_n}\hat{\Gamma}_{11}\hat{C}^{-1}\right)\hat{\alpha}\hat{\beta}\hat{Z}^{(n)}_{\hat{a}_1\cdots\hat{a}_n}$$

$$+\sum_{n=2,5,6}\frac{c_n}{n!}\left(\hat{\Gamma}^{\hat{a}_1\cdots\hat{a}_n}\hat{\Gamma}_{11}\hat{C}^{-1}\right)\hat{\alpha}\hat{\beta}\hat{Z}^{(n)}_{\hat{a}_1\cdots\hat{a}_n}$$

$$\{\hat{g}_{\hat{\mu}\hat{\nu}}, \hat{B}_{\hat{\mu}\hat{\nu}}, \hat{\phi}, \hat{C}^{(3)}{}_{\hat{\mu}\hat{\nu}\hat{\rho}}, \hat{C}^{(1)}{}_{\hat{\mu}}, \}.$$

$$\frac{c_5}{5!}\left(\hat{\Gamma}^{\hat{a}_1\cdots\hat{a}_5}\hat{\Gamma}_{11}\hat{C}^{-1}\right)^{\hat{\alpha}\hat{\beta}}\hat{Z}^{(5)}_{\hat{a}_1\cdots\hat{a}_5} + \frac{c_9}{9!}\left(\hat{\Gamma}^{\hat{a}_1\cdots\hat{a}_9}\hat{C}^{-1}\right)\hat{\alpha}^{\hat{\beta}\hat{\beta}}\hat{Z}^{(9)}_{\hat{a}_1\cdots\hat{a}_9}$$

$$\begin{aligned} \hat{\hat{g}}_{\hat{\mu}\hat{\nu}} &= e^{-\frac{2}{3}\hat{\phi}}\hat{g}_{\hat{\mu}\hat{\nu}} - e^{\frac{4}{3}\hat{\phi}}\hat{C}^{(1)}\hat{\mu}^{(1)}\hat{\nu}, & \hat{\hat{C}}_{\hat{\mu}\hat{\nu}\hat{\rho}} &= \hat{C}^{(3)}\hat{\mu}\hat{\nu}\hat{\rho} \\ \hat{\hat{g}}_{\hat{\mu}\underline{z}} &= -e^{\frac{4}{3}\hat{\phi}}\hat{C}^{(1)}\hat{\mu}, & \hat{\hat{C}}_{\hat{\mu}\hat{\nu}\underline{z}} &= \hat{B}_{\hat{\mu}\hat{\nu}}, \\ \hat{\hat{g}}_{\underline{zz}} &= -e^{\frac{4}{3}\hat{\phi}}. \end{aligned}$$

$$\left(\hat{e}_{\hat{\mu}}\hat{\hat{a}}^{\hat{u}} = \begin{pmatrix} e^{-\frac{1}{3}\hat{\phi}\hat{e}^{\hat{\mu}}_{\hat{e}}}\ \hat{a} & e^{\frac{2}{3}\hat{\phi}}\hat{C}^{(1)}\hat{\mu} \\ 0 & e^{\frac{2}{3}\hat{\phi}} \end{pmatrix}\right),$$

$$(\hat{\hat{e}}_{\hat{\hat{a}}}\hat{\hat{\mu}}) = \begin{pmatrix} e^{\frac{1}{3}}\hat{\phi}^{\hat{e}}\hat{e}^{\hat{\mu}}_{\hat{a}} & -e^{\frac{1}{3}\hat{\phi}}\hat{C}^{(1)}\hat{a} \\ 0 & e^{-\frac{2}{3}\hat{\phi}} \end{pmatrix}.$$

$$\hat{S} = \frac{2\pi t^{(11)}_{\text{lanck}}}{16\pi G^{(1)}_N}\int d^{10}\hat{x}\sqrt{|\hat{g}|}\left\{e^{-2\hat{\phi}}\left[\hat{R} - 4(\partial\hat{\phi})^2 + \frac{1}{2\cdot 3!}\hat{H}^2\right]\right.$$

$$\left.-\left[\frac{1}{4}\left(\hat{G}^{(2)}\right)^2 + \frac{1}{2\cdot 4!}\left(\hat{G}^{(4)}\right)^2\right] - \frac{1}{144}\frac{1}{\sqrt{|\hat{g}|}}\hat{\epsilon}\partial\hat{C}^{(3)}\partial\hat{C}^{(3)}\hat{B}\right\}$$

$$\hat{g}_{\hat{\mu}\hat{\nu}} \rightarrow e^{\frac{2}{3}\hat{\phi}_0}\hat{\eta}_{\hat{\mu}\hat{\nu}}$$

$$\hat{g}_{\hat{\mu}\hat{\nu}} \rightarrow e^{\frac{2}{3}\hat{\phi}_0}\hat{g}_{\hat{\mu}\hat{\nu}}, \quad \hat{C}^{(1)}\hat{\mu} \rightarrow e^{\frac{1}{3}\hat{\phi}_0}\hat{C}^{(1)}\hat{\mu},$$

$$\hat{B}_{\hat{\mu}\hat{\nu}} \rightarrow e^{\frac{2}{3}\hat{\phi}_0}\hat{B}_{\hat{\mu}\hat{\nu}}, \quad \hat{C}^{(3)}\hat{\mu}\hat{\nu}\hat{\rho} \rightarrow e^{\hat{\phi}_0}\hat{C}^{(3)}{}_{\hat{\mu}\hat{\nu}\hat{\rho}}$$

$$\hat{S} = \frac{g_A^2}{16\pi G_{NA}^{100}} \int d^{10}\hat{x}\sqrt{|\hat{g}|}\left\{e^{-2\hat{\phi}}\left[\hat{R} - 4(\partial\hat{\phi})^2 + \frac{1}{2\cdot 3!}\hat{H}^2\right]\right.$$

$$\left. -\left[\frac{1}{4}\left(\hat{G}^{(2)}\right)^2 + \frac{1}{2\cdot 4!}\left(\hat{G}^{(4)}\right)^2\right] - \frac{1}{144}\frac{1}{\sqrt{|\hat{g}|}}\hat{\epsilon}\partial\hat{C}^{(3)}\partial\hat{C}^{(3)}\hat{B}\right\}$$

$$g_A = e^{\hat{\phi}_0}$$

$$\frac{2\pi\ell_{\text{Planck}}^{(11)}\, e^{\frac{8}{3}\hat{\phi}_0}}{16\pi G_N^{(11)}} = \frac{g_A^2}{16\pi G_{NA}^{(10)}}$$

$$G_N^{(10)} = \frac{G_N^{(11)}}{2\pi\ell_{\text{Planck}}^{(11)}\, g_A^{2/3}}$$

$$R_{11} = \frac{1}{2\pi}\lim_{r\to\infty}\int \sqrt{|\hat{\hat{g}}_{\underline{zz}}|}dz = \ell_{\text{Planck}}^{(11)}\, e^{\frac{2}{3}\hat{\phi}_0} = \ell_{\text{Planck}}^{(11)}\, g_A^{2/3}$$

$$G_{NA}^{(10)} = \frac{G_N^{(11)}}{2\pi R_{11}} = \frac{G_N^{(11)}}{V_{11}}$$

$$G_{NA}^{(10)} = \frac{\left(\ell_{\text{Planck}}^{(11)}\right)^8}{32\pi^2 g_A^{2/3}}$$

$$G_{NA}^{(10)} = 8\pi^6 g_A^2 \ell_s^8$$

$$\ell_{\text{Planck}}^{(11)} = 2\pi\ell_s g_A^{1/3}$$

$$R_{11} = \ell_s g_A$$

$$\hat{G}^{(10-k)} = (-1)^{[k/2]\star}\hat{G}^{(k)}$$

$$\hat{G} = d\hat{C} - \hat{H}\wedge\hat{C}$$

$$d\hat{G} - \hat{H}\wedge\hat{G} = 0, d^\star\hat{G} + \hat{H}\wedge^\star\hat{G}$$

$$\hat{H}^{(7)} = e^{-2\hat{\phi}\star}\hat{H}$$

$$dH = 0, d\left(e^{2\phi\star}\hat{H}^{(7)}\right)$$

$$d\left(e^{-2\phi\star}\hat{H}\right) + \frac{1}{2}\star\hat{G}\wedge\hat{G} = 0, d\hat{H}^{(7)} + \frac{1}{2}\star\hat{G}\wedge\hat{G}$$

$$\hat{H}^{(7)} = d\hat{B}^{(6)} - \frac{1}{2}\sum_{n=1}^{n=4}\star\hat{G}^{(2n+2)}\wedge\hat{C}^{(2n-1)}$$

$$\begin{cases} \hat{\hat{\epsilon}} = e^{-\frac{1}{6}(\hat{\phi}^{-}-\hat{\phi}_0)}\hat{\epsilon} \\ \hat{\hat{\psi}}_{\hat{a}} = e^{\frac{1}{6}(\hat{\phi}-\hat{\phi}_0)}\left(2\hat{\psi}_{\hat{a}} - \frac{1}{3}\hat{\Gamma}_{\hat{a}}\hat{\lambda}\right) \\ \hat{\hat{\psi}}_z = \frac{2i}{3}e^{\frac{1}{6}(\hat{\phi}-\hat{\phi}_0)}\hat{\Gamma}_{11}\hat{\lambda} \end{cases}$$

$$\delta_{\hat{\epsilon}}\hat{e}_{\hat{\mu}}^{\hat{a}} = -i\bar{\hat{\epsilon}}\hat{\Gamma}^{\hat{a}}\hat{\psi}_{\hat{\mu}},$$

$$\delta_{\hat{\epsilon}}\hat{\psi}_{\hat{\mu}} = \left\{\partial_{\hat{\mu}} - \frac{1}{4}\left(\psi_{\hat{\mu}} + \frac{1}{2}\Gamma_{11}\hat{H}_{\mu}\right)\right\}\hat{\epsilon} + \frac{i}{8}e^{\hat{e}}\sum_{n=1,2}\frac{1}{(2n)!}\hat{\epsilon}^{(2n)}\hat{\Gamma}_{\hat{\mu}}\left(-\hat{\Gamma}_{11}\right)^{n}\hat{\epsilon},$$

$$\delta_{\hat{\epsilon}}\hat{\beta}_{\hat{\mu}\hat{\nu}} = -2i\bar{\hat{\epsilon}}\hat{\Gamma}_{[\hat{\mu}}\hat{\Gamma}_{11}\hat{\psi}_{\hat{\nu}]},$$

$$\delta_{\hat{\epsilon}}\hat{C}^{(1)}{}_{\hat{\mu}} = -e^{\hat{\phi}\hat{\epsilon}}\hat{\Gamma}_{11}\left(\hat{\psi}_{\hat{\mu}} - \frac{1}{2}\hat{\Gamma}_{\hat{\mu}}\hat{\lambda}\right),$$

$$\delta_{\hat{\epsilon}}\hat{C}^{(3)}{}_{\hat{\mu}\hat{\nu}\hat{\rho}} = 3e^{\hat{\phi}}\bar{\epsilon}\hat{\Gamma}_{\hat{\mu}\hat{\nu}}\left(\hat{\psi}_{\hat{\rho}]} - \frac{1}{3!}\hat{\Gamma}_{\hat{\rho}]}\hat{\lambda}\right) + 3\hat{C}^{(1)}{}_{[\hat{\mu}}\delta_{\hat{\epsilon}}\hat{B}_{\hat{\mu}\hat{\nu}]},$$

$$\delta_{\hat{\epsilon}}\hat{\lambda} = \left(\partial\partial\phi\hat{\phi} + \frac{1}{12}\hat{\Gamma}_{11}\hat{H}\right)\hat{\epsilon} + \frac{i}{4}e^{\hat{\phi}}\sum_{n=1,2}\frac{5-2n}{(2n)!}\hat{\zeta}^{(2n)}\left(-\hat{\Gamma}_{11}\right)^{n}\hat{\epsilon},$$

$$\delta_{\hat{\epsilon}}\hat{\phi} = -\frac{i}{2}\hat{\epsilon}\hat{\epsilon}$$

$$\left\{\hat{Q}^{i\hat{\alpha}},\hat{Q}^{j\hat{\beta}}\right\} = i\delta^{ij}\left(\hat{\Gamma}^{\hat{a}}\hat{C}^{-1}\right)^{\hat{\alpha}\hat{\beta}}\hat{P}_{\hat{a}} + \left(\hat{\Gamma}^{\hat{a}}\hat{C}^{-1}\right)\hat{\alpha}\hat{\beta}\hat{Z}_{\hat{a}}^{(1)(ij)} + \frac{i}{3!}\left(\hat{\Gamma}^{\hat{a}_1\hat{a}_2\hat{a}_3}\hat{C}^{-1}\right)\hat{\alpha}\hat{\beta}\hat{Z}_{\hat{a}_1\hat{a}_2\hat{a}_3}^{(3)[ij]}$$
$$+\frac{i}{5!}\left(\hat{\Gamma}^{\hat{a}_1\cdots\hat{a}_5}\hat{C}^{-1}\right)\hat{\alpha}\hat{\beta}\hat{Z}_{\hat{a}_1\cdots\hat{a}_5}^{(5)(ij)} + \frac{i}{7!}\left(\hat{\Gamma}^{\hat{a}_1\cdots\hat{a}_7}\hat{C}^{-1}\right)\hat{\alpha}\hat{\alpha}\hat{Z}_{\hat{a}_1\cdots\hat{a}_7}^{(\hat{7})[\iota\jmath]}$$
$$+\frac{i}{9!}\left(\hat{\Gamma}^{\hat{a}_1\cdots\hat{a}_9}\hat{C}^{-1}\right)\hat{\alpha}\hat{\beta}\hat{Z}_{\hat{a}_1\cdots\hat{a}_9}^{(9)(ij)}.$$

$$\hat{Z}_{\hat{a}}^{(1)(ij)} = \hat{Z}_{\hat{a}}^{(1)0}\delta^{ij}\hat{Z}_{\hat{a}}^{(1)1}\sigma^1 + \hat{Z}_{\hat{a}}^{(1)3}\sigma^3$$

$$\hat{Z}_{\hat{a}_1\hat{a}_2\hat{a}_3}^{(3)} = \hat{Z}_{\hat{a}_1\hat{a}_2\hat{a}_3}^{(i)}i\sigma^2$$

$$\{\hat{\jmath}_{\hat{\mu}\hat{\nu}},\hat{\mathcal{B}}_{\hat{\mu}\hat{\nu}},\hat{\varphi}\}$$

$$\left\{C^{\hat{(0)}},C^{\hat{(2)}}\hat{\mu}\hat{\nu},C^{\hat{(4)}}\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\right\}$$

$$\begin{cases}\hat{\mathcal{H}} = 3\partial\mathcal{B}\\ \hat{G}^{(1)} = \partial\hat{C}^{(0)}\\ \hat{G}^{(3)} = 3\left(\partial\hat{C}^{(2)} - \partial\hat{\mathcal{B}}\hat{C}^{(0)}\right)\\ \hat{G}^{(5)} = 5\left(\partial\hat{C}^{(4)} - 6\partial\hat{\mathcal{B}}\hat{C}^{(2)}\right)\end{cases}$$

$$\hat{G}^{(5)} = +{}^{\star}\hat{G}^{(5)}$$

$$\left(\hat{G}^{(5)}\right)^2 = \left({}^{\star}\hat{G}^{(5)}\right)^2 = -\left(\hat{G}^{(5)}\right)^2 \Rightarrow = 0$$

$$S_{\mathrm{NSD}} = \frac{g_B^2}{16\pi G_{NB}^{(10)}}\int d^{10}\hat{x}\sqrt{|\hat{\jmath}|}\left\{e^{-2\hat{\varphi}}\left[\hat{R}(\hat{\jmath}) - 4(\partial\hat{\varphi})^2 + \frac{1}{2\cdot 5!}\hat{\mathcal{H}}^2\right]\right.$$
$$+\frac{1}{2}\left(\hat{G}^{(0)}\right)^2 + \frac{1}{2\cdot 3!}\left(\hat{G}^{(3)}\right)^2 + \frac{1}{4\cdot 3!}\left(\hat{G}^{(5)}\right)^2$$
$$\left.-\frac{1}{192}\frac{1}{\sqrt{|\hat{\jmath}|}}\in\partial\hat{C}^{(4)}\partial\hat{C}^{(2)}\hat{\mathcal{B}}\right\}$$

$$g_B = e^{\hat{\varphi}_0}$$

$$\delta_{\hat{\varepsilon}} \hat{e}_{\hat{\mu}}^{\hat{a}} = -i\bar{\hat{\varepsilon}}\hat{\Gamma}^{\hat{a}}\hat{\zeta}_{\hat{\mu}},$$

$$\delta_{\hat{\varepsilon}}\hat{\zeta}_{\hat{\mu}} = \nabla_{\hat{\mu}}\hat{\varepsilon} - \frac{1}{8}\hat{\mathcal{H}}_{\hat{\mu}}\sigma_3\hat{\varepsilon} + \frac{1}{8}e^{\hat{\varphi}}\sum_{n=1,2,3}\frac{1}{(2n-1)!}\hat{l}^{(2n-1)}\hat{\Gamma}_{\hat{\mu}}\mathcal{P}_n\hat{\varepsilon},$$

$$\delta_{\hat{\varepsilon}}\hat{\mathcal{B}}_{\hat{\mu}\hat{\nu}} = -2i\bar{\hat{\varepsilon}}\sigma^3\hat{\Gamma}_{[\hat{\mu}}\hat{\zeta}_{\hat{\nu}]},$$

$$\delta_{\hat{\varepsilon}}\hat{C}^{(2n-2)}{}_{\hat{\mu}_1\cdots\hat{\mu}_{2n-2}} = i(2n-2)e^{-\hat{\varphi}}\bar{\hat{\varepsilon}}\mathcal{P}_n\hat{\Gamma}_{[\hat{\mu}_1\cdots\hat{\mu}_{2n-3}}\left(\hat{\zeta}_{\hat{\mu}_{2n-2}]} - \frac{1}{2(2n-2)}\hat{\Gamma}_{\hat{\mu}_{2n-2}]\hat{\chi})}\right)$$

$$+\frac{1}{2}(2n-2)(2n-3)\hat{C}^{(2n-4)}{}_{[\hat{\mu}_1\cdots\hat{\mu}_{2n-4}}\delta_{\hat{\varepsilon}}\hat{\mathcal{B}}_{\hat{\mu}_{2n-3}\hat{\mu}_{2n-4}},$$

$$\delta_{\hat{\varepsilon}}\hat{\chi} = \left(\partial\hat{\varphi} - \frac{1}{12}\hat{\mathcal{H}}\sigma^3\right)\hat{\varepsilon} + \frac{1}{2}e^{\hat{\varphi}}\sum_{n=1,2,3}\frac{(n-3)}{(2n-1)!}\hat{\zeta}^{(2n-1)}\mathcal{P}_n\varepsilon,$$

$$\delta_{\hat{\varepsilon}}\hat{\varphi} = -\frac{i}{2}\bar{\hat{\varepsilon}}\hat{\chi}$$

$$\mathcal{P}_n = \begin{cases}\sigma^1, & n\\ i\sigma^2, & n\end{cases}$$

$$\hat{J}_{E\mu\nu} = e^{-\varphi/2}J_{\mu\nu}$$

$$\begin{cases}\hat{\vec{\mathcal{B}}} = \begin{pmatrix}\hat{C}^{(2)}\\ \hat{\mathcal{B}}\end{pmatrix}\\ \hat{D} = \hat{C}^{(4)} - 3\hat{\mathcal{B}}\hat{C}^{(2)}\end{cases}$$

$$\begin{cases}\hat{\vec{\mathcal{H}}} = 3\partial\hat{\vec{\mathcal{B}}}\\ \hat{F} = \hat{G}^{(5)} = +{}^{\star}\hat{F}\\ = 5\left(\partial\hat{D} - \hat{\vec{\mathcal{B}}}^T\eta\hat{\vec{\mathcal{H}}}\right)\end{cases}$$

$$\eta = i\sigma^2 = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix} = -\eta^{-1} = -\eta^T$$

$$\Lambda\eta\Lambda^T = \eta, \Rightarrow \eta\Lambda\eta^T = (\Lambda^{-1})^T, \Lambda \in SL(2,\mathbb{R})$$

$$\hat{\mathcal{M}} = e^{\hat{\varphi}}\begin{pmatrix}|\hat{\tau}|^2 & \hat{C}^{(0)}\\ \hat{C}^{(0)} & 1\end{pmatrix}, \hat{\mathcal{M}}^{-1} = e^{\hat{\varphi}}\begin{pmatrix}1 & -\hat{C}^{(0)}\\ -\hat{C}^{(0)} & |\hat{\tau}|^2\end{pmatrix}$$

$$\hat{\tau} = \hat{C}^{(0)} + ie^{-\hat{\varphi}}$$

$$\hat{\mathcal{M}}' = \Lambda\hat{\mathcal{M}}\Lambda^T,$$

$$\hat{\mathcal{B}}' = \Lambda\hat{\vec{\mathcal{B}}}$$

$$\hat{\tau}' = \frac{a\hat{\tau} + b}{c\hat{\tau} + d}$$

$$\hat{S}_{\mathrm{NSD}} = \frac{g_B^2}{16\pi G_N^{100}}\int d^{10}\hat{x}\sqrt{|\hat{\jmath}_E|}\left\{\hat{R}(\hat{\jmath}_E) + \frac{1}{4}\mathrm{Tr}\left(\partial\hat{\mathcal{M}}\hat{\mathcal{M}}^{-1}\right)^2\right.$$

$$\left.+\frac{1}{2\cdot 3!}\hat{\vec{\mathcal{H}}}^T\hat{\mathcal{M}}^{-1}\hat{\vec{\mathcal{H}}} + \frac{1}{4\cdot 5!}\hat{F}^2 - \frac{1}{2^7\cdot 3^3}\frac{1}{\sqrt{|\hat{\jmath}_E|}} \in \hat{D}\hat{\vec{\mathcal{H}}}^T\eta\hat{\vec{\mathcal{H}}}\right\}$$

$$g_B' = 1/g_B$$

$$\hat{\jmath}' = |c\hat{\lambda} + d|\hat{\jmath}$$

$$\hat{\jmath}' = e^{-\hat{\varphi}}\hat{\jmath}$$

$$R' = R/g_B$$

$$\{Q^{i\alpha}, Q^{j\beta}\} = i(\Gamma^a \mathcal{C}^{-1})^{\alpha\beta}\left(\delta^{ij} P_a + \sigma^{1ij} Z_a^{(1)1} + \sigma^{3ij} Z_a^{(1)3}\right)$$
$$+(\mathcal{C}^{-1})^{\alpha\beta}\left(\delta^{ij} Z^{(0)0} + \sigma^{1ij} Z^{(0)1} + \sigma^{3ij} Z^{(0)3}\right)$$
$$+\frac{i}{2!}(\Gamma^{a_1a_2}\mathcal{C}^{-1})^{\alpha\beta}\sigma^{2ij} Z^{(2)}_{a_1a_2} + \frac{1}{3!}\left(\Gamma^{a_1a_2a_3}\mathcal{C}^{-1}\right)^{\alpha\beta}\sigma^{2ij} Z^{(3)}_{a_1a_2a_3}$$
$$+\frac{1}{4!}(\Gamma^{a_1\cdot a_4}\mathcal{C}^{-1})^{\alpha\beta}\left(\sigma^{1ij} Z^{(4)1}_{a_1\cdot a_4} + \sigma^{3ij} Z^{(4)3}_{a_1\cdot a_4}\right)$$
$$+\frac{i}{5!}(\Gamma^{a_1\cdot a_5}\mathcal{C}^{-1})^{\alpha\beta}\left(\delta^{ij} Z^{(5)0}_{a_1\cdot a_5} + \sigma^{1ij} Z^{(5)1}_{a_1\cdot a_5} + \sigma^{3ij} Z^{(5)3}_{a_1\cdot a_5}\right)$$
$$+\frac{i}{6!}(\Gamma^{a_1\cdots a_6}\mathcal{C}^{-1})^{\alpha\beta}\sigma^{2ij}\left(Z^{(6)}_{a_1\cdots a_6} + Z^{(6)!}_{a_1\cdots a_6}\right)$$
$$+\frac{1}{7!}(\Gamma^{a_1\cdots a_7}\mathcal{C}^{-1})^{\alpha\beta}\sigma^{2ij}\left(Z^{(7)}_{a_1\cdots a_7} + Z^{(7)!}_{a_1\cdots a_7}\right)$$
$$+\frac{1}{8!}(\Gamma^{a_1\cdot a_8}\mathcal{C}^{-1})^{\alpha\beta}\left(\sigma^{1ij} Z^{(8)1}_{a_1\cdot a_8} + \sigma^{3ij} Z^{(8)3}_{a_1\cdot a_8}\right)$$

$$\hat{g}_{\mu\nu} = g_{\mu\nu} - k^2 A^{(1)}{}_{\mu} A^{(1)}{}_{\nu}, g_{\mu\nu} = \hat{g}_{\mu\nu} - \hat{g}_{\mu\underline{x}}\hat{g}_{\nu\underline{x}}/\hat{g}_{\underline{xx}},$$
$$\hat{B}_{\mu\nu} = B_{\mu\nu} + A^{(1)}{}_{[\mu} A^{(2)}{}_{\nu]}, B_{\mu\nu} = \hat{B}_{\mu\nu} + \hat{g}_{[\mu|\underline{x}|}\hat{B}_{\nu]\underline{x}}/\hat{g}_{\underline{xx}},$$
$$\hat{\phi} = \phi + \frac{1}{2}\log\, k, \phi \ = \hat{\phi} - \frac{1}{4}\log\left|\hat{g}_{\underline{xx}}\right|,$$
$$\hat{g}_{\mu\underline{x}} = -k^2 A^{(1)}{}_{\mu}, A^{(1)}{}_{\mu} = \hat{g}_{\mu\underline{x}}/\hat{g}_{\underline{xx}},$$
$$\hat{B}_{\mu\underline{x}} = -A^{(2)}{}_{\mu}, A^{(2)}{}_{\mu} = -\hat{B}_{\mu\underline{x}},$$
$$\hat{g}_{\underline{xx}} = -k^2, k = \left|\hat{g}_{\underline{xx}}\right|^{1/2}$$
$$\hat{C}^{(2n-1)}{}_{\mu_1\cdots\mu_{2n-1}} = C^{(2n-1)}{}_{\mu_1\cdots\mu_{2n-1}} + (2n-1)A^{(1)}{}_{[\mu_1} C^{(2n-2)}{}_{\mu_2\cdots\mu_{2n-1}]},$$
$$\hat{C}^{(2n+1)}{}_{\mu_1\cdots\mu_{2n}\underline{x}} = C^{(2n)}{}_{\mu_1\cdots\mu_{2n}},$$
$$C^{(2n-1)}{}_{\mu_1\cdots\mu_{2n-1}} = \hat{C}^{(2n-1)}{}_{\mu_1\cdots\mu_{2n-1}} - (2n-1)\hat{g}_{[\mu_1|\underline{x}|}\hat{C}^{(2n-1)}{}_{\mu_2\cdots\mu_{2n-1}]\underline{x}}/\hat{g}_{\underline{xx}},$$
$$C^{(2n)}{}_{\mu_1\cdots\mu_{2n}} = \hat{C}^{(2n+1)}{}_{\mu_1\cdots\mu_{2n}\underline{x}}$$
$$\hat{\jmath}_{\mu\nu} = g_{\mu\nu} - k^{-2} A^{(2)}{}_{\mu} A^{(2)}{}_{\nu}, g_{\mu\nu} = \hat{\jmath}_{\mu\nu} - \hat{\jmath}_{\mu\underline{y}}\hat{\jmath}_{\nu\underline{y}}/\hat{\jmath}_{\underline{yy}},$$
$$\hat{\mathcal{B}}_{\mu\nu} = B_{\mu\nu} + A^{(1)}{}_{[\mu} A^{(2)}{}_{\nu]}, B_{\mu\nu} = \hat{\mathcal{B}}_{\mu\nu} + \hat{\jmath}_{[\mu|\underline{y}|}\hat{\mathcal{B}}_{\nu]\underline{y}}/\hat{\jmath}_{\underline{yy}},$$
$$\hat{\varphi} = \phi - \frac{1}{2}\log\, k, \phi \ = \hat{\varphi} - \frac{1}{4}\log\left|\hat{\jmath}_{\underline{yy}}\right|,$$
$$\hat{\jmath}_{\mu\underline{y}} = -k^{-2} A^{(2)}{}_{\mu}, A^{(1)}{}_{\mu} = \hat{\mathcal{B}}_{\mu\underline{y}},$$
$$\hat{\mathcal{B}}_{\mu\underline{y}} = A^{(1)}{}_{\mu}, A^{(2)}{}_{\mu} = \hat{\jmath}_{\mu\underline{y}}/\jmath_{\underline{yy}},$$
$$\hat{\jmath}_{\underline{yy}} = -k^{-2}, k = \left|\hat{\jmath}_{\underline{yy}}\right|^{-1/2}$$
$$\hat{C}^{(2n)}{}_{\mu_1\cdots\mu_{2n}} = C^{(2n)}{}_{\mu_1\cdots\mu_{2n}} - (2n)A^{(2)}{}_{[\mu_1} C^{(2n-1)}{}_{\mu_2\cdots\mu_{2n}]},$$
$$\hat{C}^{(2n)}{}_{\mu_1\cdots\mu_{2n-1}\underline{y}} = -C^{(2n-1)}{}_{\mu_1\cdots\mu_{2n-1}},$$
$$C^{(2n)}{}_{\mu_1\cdots\mu_{2n}} = \hat{C}^{(2n)}{}_{\mu_1\cdots\mu_{2n}} + (2n)\hat{\jmath}_{[\mu_1|\underline{y}}\hat{C}^{(2n)}{}_{\mu_2\cdots\mu_{2n}]\underline{y}}/\hat{\jmath}_{\underline{y}},$$
$$C^{(2n-1)}{}_{\mu_1\cdots\mu_{2n-1}} = -\hat{C}^{(2n)}{}_{\mu_1\cdots\mu_{2n-1}\underline{y}}$$

$$\hat{\jmath}_{\mu\nu} = \hat{g}_{\mu\nu} - \left(\hat{g}_{\mu\underline{x}}\hat{g}_{\nu\underline{x}} - \hat{B}_{\mu\underline{x}}\hat{B}_{\nu\underline{x}}\right)/\hat{g}_{\underline{xx}}, \hat{\jmath}_{\mu\underline{y}} = \hat{B}_{\mu\underline{x}}/\hat{g}_{\underline{\underline{xx}}},$$

$$\hat{\mathcal{B}}_{\mu\nu} = \hat{B}_{\mu\nu} + 2\hat{g}_{[\mu|\underline{x}}\hat{B}_{\nu]\underline{x}}/\hat{g}_{\underline{xx}}, \hat{\mathcal{B}}_{\mu\underline{y}} = \hat{g}_{\mu\underline{x}}/\hat{g}_{\underline{xx}},$$

$$\hat{\varphi} = \hat{\phi} - \frac{1}{2}\log\left|\hat{g}_{\underline{xx}}\right|, \hat{\jmath}_{\underline{yy}} = 1/\hat{g}_{\underline{xx}},$$

$$\hat{C}^{(2n)}{}_{\mu_1\dots\mu_{2n}} = \hat{C}^{(2n+1)}{}_{\mu_1\dots\mu_{2n}\underline{x}} + 2n\hat{B}_{[\mu_1|\underline{x}|}\hat{C}^{(2n-1)}{}_{\mu_2\dots\mu_{2n}]}$$

$$-2n(2n-1)\hat{B}_{[\mu_1|\underline{x}|}\hat{g}_{\mu_2|\underline{x}|}\hat{C}^{(2n-1)}{}_{\mu_3\dots\mu_{2n}]\underline{x}}/\hat{g}_{\underline{xx}},$$

$$\hat{C}^{(2n)}{}_{\mu_1\dots\mu_{2n-1}\underline{y}} = -\hat{C}^{(2n-1)}{}_{\mu_1\dots\mu_{2n-1}}$$

$$+(2n-1)\hat{g}_{[\mu_1|\underline{x}|}\hat{C}^{(2n-1)}{}_{\mu_2\dots\mu_{2n-1}]\underline{x}}/\hat{g}_{\underline{xx}}$$

$$\hat{g}_{\mu\nu} = \hat{\jmath}_{\mu\nu} - \left(\hat{\jmath}_{\mu\underline{y}}\hat{\jmath}_{\underline{y}} - \hat{\mathcal{B}}_{\mu\underline{y}}\hat{\mathcal{B}}_{\nu\underline{y}}\right)/\hat{\jmath}_{\underline{yy}}, \hat{g}_{\mu\underline{x}} = \hat{\mathcal{B}}_{\mu\underline{\underline{y}}}/\hat{\jmath}_{\underline{yy}},$$

$$\hat{B}_{\mu\nu} = \hat{\mathcal{B}}_{\mu\nu} + 2\hat{\jmath}_{[\mu|\underline{\underline{1}}}\hat{\mathcal{B}}_{\nu]\underline{y}}/J_{\underline{\underline{y}}}, \hat{B}_{\mu\underline{x}} = \hat{\jmath}_{\mu\underline{y}}/\hat{\jmath}_{\underline{yy}},$$

$$\hat{\phi} = \hat{\varphi} - \frac{1}{2}\log\left|\underline{\hat{y}_{\underline{yy}}}\right|, \hat{g}_{\underline{xx}} = 1/\hat{\jmath}_{\underline{yy}},$$

$$\hat{C}^{(2n+1)}{}_{\mu_1\dots\mu_{2n+1}} = -\hat{C}^{(2n+2)}{}_{\mu_1\dots\mu_{2n+1}\underline{y}} + (2n+1)\hat{\mathcal{B}}_{[\mu_1|\underline{y}|}\hat{C}^{(2n)}{}_{\mu_2\dots\mu_{2n+1}]}$$

$$-2n(2n+1)\hat{\mathcal{B}}_{[\mu_1|\underline{y}|}\hat{\jmath}_{\mu_2|\underline{y}}\hat{C}^{(2n)}{}_{\mu_3\dots\mu_{2n+1}]\underline{\underline{1}}}/\hat{\jmath}_{\underline{yy}},$$

$$\hat{C}^{(2n+1)}{}_{\mu_1\dots\mu_{2n}\underline{x}} = \hat{C}^{(2n)}{}_{\mu_1\dots\mu_{2n}}$$

$$+2n\hat{\jmath}_{[\mu_1|\underline{y}|}\hat{C}^{(2n)}{}_{\mu_2\dots\mu_{2n}]\underline{y}}/\hat{\jmath}_{\underline{yy}}$$

$$\hat{\jmath}_{\underline{yy}} = 1/\hat{g}_{\underline{xx}}, \hat{g}_{\underline{xx}} = 1/\hat{\jmath}_{\underline{yy}}$$

$$\hat{g}_{\underline{xx}} \to (R_A/\ell_s)^2, \hat{\jmath}_{\underline{yy}} \to (R_B/\ell_s)^2$$

$$R_{A,B} = \ell_s^2/R_{B,A}$$

$$g_{A,B} = g_{B,A}/R_{B,A}$$

$$g \ll 1, \ell_s \ll 1$$

$$\ell_s/R_c \ll 1$$

$$S_{NG}^{(p)}[X^\mu(\xi)] = -T_{(p)}\int d^{p+1}\xi\sqrt{|g_{ij}|}$$

$$g_{ij} = g_{\mu\nu}(X)\partial_i X^\mu \partial_j X^\nu$$

$$S_P^{(p)}[X^\mu, \gamma_{ij}] = -\frac{T_{(p)}}{2}\int d^{p+1}\xi\sqrt{|\gamma|}\left[\gamma^{ij}\partial_i X^\mu \partial_j X^\nu g_{\mu\nu} + (1-p)\right]$$

$$\gamma_{ij} = g_{ij}$$

$$\gamma_{ij} = \Omega(\xi)g_{ij}$$

$$S_{NG}^{(p)}[X^\mu(\xi)] = -\frac{T_{(p)}}{K_0^\alpha}\int d^{p+1}\xi K(X)^\alpha\sqrt{|g_{ij}|} - \frac{\mu}{K_0^\alpha(p+1)!}\int d^{p+1}\xi\epsilon^{i_1\cdots i_{p+1}}A_{(p+1)i_1\cdots i_{p+1}}$$

$$A_{(p+1)i_1\cdots i_{p+1}} = A_{(p+1)\mu_1\cdots\mu_{p+1}}(X)\partial_{i_1}X^{\mu_1}\dots\partial_{i_{p+1}}X^{\mu_{p+1}}$$

$$\delta A_{(p+1)} = (p+1)\partial\Lambda_{(p)}$$

$$S_{NG}^{(p)}[X^\mu(\xi)] = -\frac{T_{(p)}}{K_0^\alpha}\int d^{p+1}\xi K^\alpha(X)\sqrt{|g_{ij}+F_{ij}|} + \cdots, F_{ij} = 2\partial_{[i}V_{j]}$$

$$S_{NG}^{(p)}[X^\mu(\xi)] = -\frac{T_{(p)}}{K_0^\alpha}\int d^{p+1}\xi K^\alpha(X)\sqrt{|g_{ij}|}\left\{1 - \frac{1}{2}\mathcal{F}^2 + \cdots\right\}$$

$$S = \frac{1}{16\pi G_N^{(d)}}\int d^d x\sqrt{|g|}\left[R + 2(\partial\log K)^2 + \frac{(-1)^{p+1}}{2\cdot(p+2)!}K^\beta F_{(p+2)}^2\right]$$

$$F_{(p+2)} = (p+2)\partial A_{(p+1)}$$

$$S=-T\int d^2\xi\sqrt{|\hat{g}_{ij}|}-\frac{T}{2}\int d^2\xi\epsilon^{ij}\hat{B}_{ij}$$
$$S^{(p)}=-T_{(p)}e^{2\phi_0}\int d^{p+1}\xi e^{-2\phi}\sqrt{|g_{ij}|}+\cdots$$
$$S=-T_{S5}e^{2\hat{\phi}_0}\int d^6\xi e^{-2\hat{\phi}}\sqrt{|\hat{g}_{ij}|}-\frac{T_{S5}e^{2\hat{\phi}_0}}{6!}\int d^6\xi\epsilon^{i_1\cdots i_6}\hat{B}^{(6)}_{i_1\cdots i_6}$$
$$S=-T_{Dp}e^{\hat{\phi}_0}\int d^{p+1}\xi e^{-\hat{\phi}}\sqrt{|\hat{g}_{ij}+2\pi\alpha'\mathcal{F}_{ij}|}-\frac{T_{SDp}e^{\hat{\phi}_0}}{6!}\int d^6\xi\epsilon^{i_1\cdots i_{p+1}}\hat{C}^{(p+1)}_{i_1\cdots i_{p+1}}$$
$$\mathcal{F}_{ij}=F_{ij}+\frac{1}{2\pi\alpha'}\hat{B}_{ij},F_{ij}=2\partial_{[i}V_{j]}$$
$$R_{A,B}=\ell_s^2/R_{B,A}$$
$$g_{A,B}=g_B\ell_s/R_{B,A}$$
$$g'=1/g$$
$$R_i'=R_i/\sqrt{g}$$
$$M'=g^{1/2}M$$
$$M_{F1}=\frac{R_9}{\ell_s^2}$$
$$M_{D1}=M_{F1}'=g^{1/2}M_{F1}=g^{1/2}\frac{R_9}{\ell_s^2}=\frac{R_9'}{g'\ell_s^2}$$
$$M_{D0}=M_{D1}'=\frac{R_9}{g\ell_s^2}=\frac{\ell_s^2/R_9'}{g'\ell_s/R_9'\ell_s^2}=\frac{1}{g'\ell_s}$$
$$M_{D0}=M_{D1}'=\frac{R_9}{g\ell_s^2}=\frac{R_9}{g'\ell_s/R_8'\ell_s^2}=\frac{R_8R_9}{g'\ell_s^3}$$
$$M_{Dp}=\frac{R_{10-p}\ldots R_9}{g\ell_s^{p+1}}$$
$$M_{S5}=g^{1/2}M_{D5}'=g^{1/2}\frac{R_5\ldots R_9}{g\ell_s^6}=g'^{-1/2}\frac{R_5'/g'^{1/2}\ldots R_9'/g^{1/2}}{g'^{-1}\ell_s^6}=\frac{R_5\ldots R_9}{g^2\ell_s^6}$$

Objeto Supermasivo	Masa	Masa	Objeto Masivo
F1m	R_9^{-1}		
D0	$g_A^{-1}\ell_s^{-1}$	R_{10}^{-1}	$WM(+,-^{10})$
F1w	$R_9\ell_s^{-2}$	$R_{10}R_9\left(\ell^{(11)}_{\text{Planck}}\right)^{-3}$	$M2(+,-^{8},+^{2})$
D2	$R_9R_8g_A^{-1}\ell_s^{-3}$	$R_9R_8\left(\ell^{(11)}_{\text{Planck}}\right)^{-3}$	$M2(+,-^{7},+^{2},-)$
D4	$R_9\ldots R_6g_A^{-1}\ell_s^{-5}$	$R_{10}R_9\ldots R_5\left(\ell^{(11)}_{\text{Planck}}\right)^{-6}$	$M5(+,-^{5},+^{5})$
S5A	$R_9\ldots R_5g_A^{-2}\ell_s^{-6}$	$R_9\ldots R_5\left(\ell^{(11)}_{\text{Planck}}\right)^{-6}$	$M5(+,-^{4},+^{5},-)$
D6	$R_9\ldots R_4g_A^{-1}\ell_s^{-7}$	$R_{10}^2R_9\ldots R_4\left(\ell^{(11)}_{\text{Planck}}\right)^{-9}$	$KK7M(+,-^{3},+^{6},-^{*})$

KK6A	$R_9^2 R_8 \dots R_4 g_A^{-2} \ell_s^{-8}$	$R_{10} R_9^2 \dots R_4 \left(\ell_{\text{Planck}}^{(11)} \right)^{-9}$	$\text{KK7M}(+,\ ^3, +^5, +^\star, +)$
D8	$R_9 \dots R_2 g_A^{-1} \ell_s^{-9}$	$R_{10}^3 R_9 \dots R_4 \left(\ell_{\text{Planck}}^{(11)} \right)^{-12}$	$\text{KK9M}(+, -, +^8, +^\star)$
KK8A	$R_9^3 R_8 \dots R_2 g_A^{-3} \ell_s^{-11}$	$R_{10} R_9^3 R_8 \dots R_2 \left(\ell_{\text{Planck}}^{(11)} \right)^{-12}$	$\text{KK9M}(+, -, +^7, +^\star, +)$
KK9A	$R_9^3 R_8 \dots R_1 g_A^{-4} \ell_s^{-12}$	$R_{10} R_9^3 R_8 \dots R_1 \left(\ell_{\text{Planck}}^{(11)} \right)^{-12}$	$\text{KK9M}(+, +^8, +^\star, -)$

Objeto Supermasivo	Masa	Objeto Masivo	Masa
F1m	R_9^{-1}	KK6A	$R_9^2 R_8 \dots R_4 g_B^{-2} \ell_s^{-8}$
F1w	$R_9 \ell_s^{-2}$	D7	$R_9 \dots R_3 g_B^{-1} \ell_s^{-8}$
D1	$R_9 g_B^{-1} \ell_s^{-2}$	Q7	$R_9 \dots R_3 g_B^{-3} \ell_s^{-8}$
D3	$R_9 \dots R_7 g_B^{-1} \ell_s^{-4}$	D9	$R_9 \dots R_1 g_B^{-1} \ell_s^{-10}$
D5	$R_9 \dots R_5 g_B^{-1} \ell_s^{-6}$	Q9	$R_9 \dots R_1 g_B^{-4} \ell_s^{-10}$
S5B	$R_9 \dots R_5 g_B^{-2} \ell_s^{-6}$		

$$S = -T_{M2} \int d^3\xi \sqrt{|\hat{\hat{g}}_{ij}|} - \frac{T_{M2}}{3!} \int d^3\xi \epsilon^{i_1 \cdots i_3} \hat{\hat{C}}_{i_1 \cdots i_3}$$

$$\ell_s = \ell_{\text{Planck}}^{(11)} / R_{10}^{1/2}$$

$$g_A = R_{10}^{3/2} / \ell_{\text{Planck}}^{(11)}$$

$$M_{M2} = M_{F1A} = \frac{R_9}{\ell_s^2} = \frac{R_9 R_{10}}{\left(\ell_{\text{Planck}}^{(11)} \right)^3}$$

$$M_{M2} = \frac{R_8 R_9}{\left(\ell_{\text{Planck}}^{(11)} \right)^3} = \frac{R_8 R_9}{g_A \ell_s^3}$$

$$M_{M5} = M_{D4} = \frac{R_6 \dots R_9}{g_A \ell_s^5} = \frac{R_6 \dots R_{10}}{\left(\ell_{\text{Planck}}^{(11)} \right)^6}$$

$$M_{M5} = \frac{R_5 \dots R_9}{\left(\ell_{\text{Planck}}^{(11)} \right)^6} = \frac{R_5 \dots R_9}{g_A^2 \ell_s^6}$$

$$M_{D0} = \frac{1}{g_A \ell_s} = \frac{1}{R_{10}}$$

Objeto Súper	Masa
WM	0
M2	$R_{10} R_9 \left(\ell_{\text{Planck}}^{(11)} \right)^{-3}$

M5	$R_{10} \ldots R_6 \left(\ell_{\text{Planck}}^{(11)}\right)^{-6}$
KK7M	$R_{10}^2 R_9 \ldots R_4 \left(\ell_{\text{Planck}}^{(11)}\right)^{-9}$
KK9M	$R_{10}^3 R_9 \ldots R_4 \left(\ell_{\text{Planck}}^{(11)}\right)^{-12}$

$$S = \frac{1}{16\pi G_N^{(d)}} \int d^d x \sqrt{|g|} \left[R + 2(\partial\varphi)^2 + \frac{(-1)^{p+1}}{2 \cdot (p+2)!} e^{-2a\varphi} F_{(p+2)}^2 \right]$$

$$F_{(p+2)} = dA_{(p+1)}, F_{(p+2)\mu_1 \ldots \mu_{p+2}} = (p+2)\partial_{[\mu_1} A_{(p+1)\mu_2 \ldots \mu_{p+2}]}$$

$$ds^2 = f\left[W dt^2 - d\vec{y}_p^2\right] - g^{-1}\left[W^{-1} d\rho^2 - \rho^2 d\Omega_{(\tilde{p}+2)}^2\right]$$

$$A_{\underline{t}\underline{t}\underline{y}^1 \ldots \underline{y}^p} = \alpha(H^{-1} - 1)$$

$$\tilde{p} \equiv d - p - 4$$

$$H = 1 + \frac{h}{\rho^{\tilde{p}+1}}, W = 1 + \frac{\omega}{\rho^{\tilde{p}+1}}$$

$$ds^2 = H^{\frac{2x-2}{p+1}}\left[W dt^2 - d\vec{y}_p^2\right] - H^{\frac{-(2x-2)}{\tilde{p}+1}}\left[W^{-1} d\rho^2 + \rho^2 d\Omega_{(\tilde{p}+2)}^2\right]$$

$$e^{-2a\varphi} = e^{-2a\varphi_0} H^{2x}, A_{\underline{t}\underline{y}^1 \ldots \underline{y}^p} = e^{a\varphi_0}\alpha(H^{-1} - 1),$$

$$H = 1 + \frac{h}{\rho^{\tilde{p}+1}}, W = 1 + \frac{\omega}{\rho^{\tilde{p}+1}},$$

$$\omega = h\left[1 - \frac{a^2}{4x}\alpha^2\right],$$

$$x = \frac{\frac{a^2}{2}c}{1 + \frac{a^2}{2}c}, c = \frac{(p+1) + (\tilde{p}+1)}{(p+1)(\tilde{p}+1)}$$

$$ds^2 = H^{\frac{2x-2}{p+1}}\left(dt^2 - d\vec{y}_p^2\right) - H^{\frac{-(2x-2)}{\tilde{p}+1}} d\vec{x}_{(\tilde{p}+3)}^2,$$

$$e^{-2a\varphi} = e^{-2a\varphi_0} H^{2x}, A_{\underline{t}\underline{t}\underline{g}^1 \ldots \underline{y}^p} = e^{a\varphi_0}\alpha(H^{-1} - 1)$$

$$\partial_{\underline{m}}\partial_{\underline{m}} H = 0,$$

$$x = \frac{\frac{a^2}{2}}{1 + \frac{a^2}{2}}, c = \frac{(p+1) + (\tilde{p}+1)}{(p+1)(\tilde{p}+1)}, \alpha^2 = \frac{4x}{a^2}$$

$$H = 1 + \frac{h}{\left|\vec{x}_{(\tilde{p}+3)}\right|^{\tilde{p}+1}}$$

$$H = 1 + \sum_{I=1}^{N} \frac{h_I}{\left|\vec{x}_{(\tilde{p}+3)} - \vec{x}_{(\tilde{p}+3)I}\right|^{\tilde{p}+1}}$$

$$ds^2 = W dt^2 - d\vec{y}_p^2 - W^{-1} d\rho^2 + \rho^2 d\Omega_{(\tilde{p}+2)}^2$$

$$W = 1 + \frac{\omega}{\rho^{\tilde{p}+1}}$$

$$S = S_a + S_p$$

$$S_p[X^\mu,\gamma_{ij}] = -\frac{T}{2}\int\ d^{p+1}\xi\sqrt{|\gamma|}[e^{-2b\varphi}\gamma^{ij}\partial_i X^\mu \partial_j X^\nu g_{\mu\nu} - (p-1)]$$

$$-\frac{\mu}{(p+1)!}\int\ d^{p+1}\xi A_{(p+1)\mu_1\cdots\mu_{p+1}}\partial_{i_1}X^{\mu_1}\dots\partial_{i_{p+1}}X^{\mu_{p+1}}$$

$$Y^i(\xi) = \xi^i$$

$$X^m(\xi) = 0$$

$$a = -(p+1)b$$

$$\mu = T/\alpha$$

$$H = \epsilon + \frac{h}{|\vec{x}_{(\tilde{p}+3)}|^{\tilde{p}+1}}$$

$$h = \frac{16\pi G_N^{(d)}T}{(\tilde{p}+1)\alpha^2\omega_{(\tilde{p}+2)}}$$

$$d\hat{\hat{s}}_E^2 = H_{M2}^{-2/3}[Wdt^2 - d\vec{y}_2^2] - H_{M2}^{1/3}[W^{-1}d\rho^2 + \rho^2 d\Omega_{(7)}^2]$$

$$\hat{\hat{C}}_{t\underline{y}^1\underline{y}^2} = \alpha(H_{M2}^{-1} - 1),$$

$$H_{M2} = 1 + \frac{h_{M2}}{\rho^6}, W = 1 + \frac{\omega}{\rho^6},$$

$$\omega = h_{M2}[1-\alpha^2]$$

$$d\hat{\hat{s}}^2 = H_{M2}^{-2/3}[dt^2 - d\vec{y}_2^2] - H_{M2}^{1/3}d\vec{x}_8^2,$$

$$\hat{\hat{C}}_{ty_1y_2} = \pm(H_{M2}^{-1} - 1),$$

$$H_{M2} = 1 + \frac{h_{M2}}{|\vec{x}_8|^6}$$

$$h_{M2} = \frac{16\pi G_N^{(11)}T_{M2}}{6\omega_{(7)}}$$

$$T_{M2} = \frac{M_{M2}}{(2\pi)^2 R_9 R_{10}} = \frac{1}{(2\pi)^2\left(\ell_{\text{Planck}}^{(11)}\right)^3} = \frac{2\pi}{\left(\ell_{\text{Planck}}^{(11)}\right)^3}$$

$$h_{M2} = \frac{\left(\ell_{\text{Planck}}^{(11)}\right)^6}{6\omega_{(7)}}$$

$$d\hat{\hat{s}}^2 = H_{M5}^{-1/3}[Wdt^2 - d\vec{y}_5^2] - H_{M5}^{2/3}[W^{-1}d\rho^2 + \rho^2 d\Omega_{(4)}^2]$$

$$\tilde{\hat{\hat{C}}}_{t\underline{ty}^1\cdots\underline{y}^5} = \alpha(H_{M5s}^{-1} - 1),$$

$$H_{M5} = 1 + \frac{h_{M5}}{\rho^3}, W = 1 + \frac{\omega}{\rho^3},$$

$$\omega = h_{M5}[1-\alpha^2]$$

$$d\hat{\hat{s}}^2 = H_{M5}^{-1/3}[dt^2 - d\vec{y}_5^2] - H_{M5}^{2/3}d\vec{x}_5^2,$$

$$\tilde{\hat{\hat{C}}}_{t\underline{t}^1\cdots\underline{y}^5} = \pm(H_{M5}^{-1} - 1),$$

$$H_{M5} = 1 + \frac{h_{M5}}{|\vec{x}_5|^3}$$

$$h_{M5} = \frac{\left(\ell_{\text{Planck}}^{(11)}\right)^3}{3\omega_{(4)}}$$

$$S = \frac{1}{16\pi G_N^{(d)}} \int d^d x \sqrt{|g_s|} \left\{ e^{-2\phi} \left[R_s - 4(\partial\phi)^2 + \frac{(-1)^{p_1+1}}{2 \cdot (p_1+2)!} F^2_{(p_1+2)} \right] + \frac{(-1)^{p_2+1}}{2 \cdot (p_2+2)!} F^2_{(p_2+2)} \right\}$$

$$g_{s\mu\nu} = e^{\frac{4}{(d-2)}\phi} g_{\mu\nu}$$

$$S = \frac{1}{16\pi G_N^{(d)}} \int d^d x \sqrt{|g|} \left[R + \frac{4}{(d-2)} (\partial\phi)^2 \right.$$

$$\left. + \frac{(-1)^{p_1+1}}{2 \cdot (p_1+2)!} e^{-4\frac{(p_1+1)}{(d-2)}\phi} F^2_{(p_1+2)} + \frac{(-1)^{p_2+1}}{2 \cdot (p_2+2)!} e^{2\frac{(\tilde{p}_2 - p_1)}{(d-2)}\phi} F^2_{(p_2+2)} \right\}$$

$$\phi = \sqrt{\frac{(d-2)}{2}} \varphi$$

$$a_1 = \frac{2(p_1+1)}{\sqrt{2(d-2)}}, \quad (\mathrm{NS-NS})$$

$$a_2 = \frac{-(\tilde{p}_2 - p_2)}{\sqrt{2(d-2)}}. \quad (\mathrm{RR})$$

$$a_3 = -\frac{2(\tilde{p}_1 + 1)}{\sqrt{2(d-2)}}$$

$$d\tilde{\hat{s}}_E^2 = H_{F1}^{-3/4}[W dt^2 - dy^2] - H_{F1}^{1/4}[W^{-1} d\rho^2 + \rho^2 d\Omega^2_{(7)}],$$

$$d\hat{s}_S^2 = H_{F1}^{-1}[W dt^2 - dy^2] - [W^{-1} d\rho^2 + \rho^2 d\Omega^2_{(7)}],$$

$$e^{-2(\hat{\phi} - \hat{\phi}_0)} = H_{F1},$$

$$\hat{B}_{t\underline{y}} = \alpha(H_{F1}^{-1} - 1),$$

$$H_{F1} = 1 + \frac{h_{F1}}{\rho^6}, W = 1 + \frac{\omega}{\rho^6},$$

$$\omega = h_{F1}[1 - \alpha^2]$$

$$d\tilde{\hat{s}}_E^2 = H_{F1}^{-3/4}[dt^2 - dy^2] - H_{F1}^{1/4} d\vec{x}_8^2$$

$$d\hat{s}_S^2 = H_{F1}^{-1}[dt^2 - dy^2] - d\vec{x}_8^2,$$

$$e^{-2(\hat{\phi} - \hat{\phi}_0)} = H_{F1},$$

$$\hat{B}_{t\underline{y}} = \pm(H_{F1}^{-1} - 1),$$

$$H_{F1} = 1 + \frac{h_{F1}}{|\vec{x}_8|^6}$$

$$h_{F1} = \frac{2^5 \pi^6 \ell_s^6 g^2}{3\omega_{(7)}}$$

$$d\tilde{\hat{s}}_E^2 = H_{S5}^{-1/4}[W dt^2 - d\vec{y}_5^2] - H_{S5}^{3/4}[W^{-1} d\rho^2 + \rho^2 d\Omega^2_{(3)}]$$

$$d\hat{s}_S^2 = [W dt^2 - d\vec{y}_5^2] - H_{S5}[W^{-1} d\rho^2 + \rho^2 d\Omega^2_{(3)}]$$

$$e^{-2(\hat{\phi} - \hat{\phi}_0)} = H_{S5}^{-1},$$

$$\hat{B}^{(6)}{}_{t\underline{y}^1 \ldots \underline{y}^5} = \alpha e^{-2\hat{\phi}_0}(H_{S5}^{-1} - 1)$$

$$H_{S5} = 1 + \frac{h_{ps}}{\rho^2}, W = 1 + \frac{\omega}{\rho^2},$$

$$\omega = h_{S5}[1 - \alpha^2]$$

$$d\tilde{\hat{s}}_E^2 = H_{S5}^{-1/4}[dt^2 - d\vec{y}_5^2] - H_{S5}^{3/4} d\vec{x}_4^2,$$
$$d\hat{s}_S^2 = [dt^2 - d\vec{y}_5^2] - H_{S5} d\vec{x}_4^2,$$
$$e^{-2(\hat{\phi}-\hat{\phi}_0)} = H_{S5}^{-1},$$
$$\tilde{\hat{B}}^{(6)}{}_{\underline{t}\underline{y}^1\cdots\underline{y}^5} = \pm e^{-2\hat{\phi}_0}(H_{S5}^{-1} - 1),$$
$$H_{S5} = 1 + \frac{h_{S5}}{|\vec{x}_4|^2}$$
$$h_{S5} = \ell_s^2$$
$$d\tilde{\hat{s}}_E^2 = H_{D_p}^{-\frac{(7-p)}{8}}\left[W dt^2 - d\vec{y}_p^2\right] - H_{D_p}^{\frac{(p+1)}{8}}\left[W^{-1} d\rho^2 + \rho^2 d\Omega_{(8-p)}^2\right]$$
$$d\hat{s}_S^2 = H_{D_p}^{-1/2}\left[W dt^2 - d\vec{y}_p^2\right] - H_{Dp}^{1/2}\left[W^{-1} d\rho^2 + \rho^2 d\Omega_{(8-p)}^2\right],$$
$$e^{-2(\hat{\phi}-\hat{\phi}_0)} = H_{D_p}^{\frac{(p-3)}{2}},$$
$$\hat{C}^{(p+1)}{}_{\underline{t}\underline{y}^1\ldots\underline{y}^p} = \alpha e^{-\hat{\phi}_0}\left(H_{Dp}^{-1} - 1\right),$$
$$H_{Dp} = 1 + \frac{h_{Dp}}{\rho^{7-p}}, W = 1 + \frac{\omega}{\rho^{7-p}},$$
$$\omega = h_{D_p}[1 - \alpha^2]$$
$$H \sim h\log\rho, W \sim \omega\log\rho$$
$$H \sim h\rho, W \sim \omega\rho$$
$$d\tilde{\hat{s}}_E^2 = H_{Dp}^{\frac{p-7}{8}}\left[dt^2 - d\vec{y}_p^2\right] - H_{Dp}^{\frac{p+1}{8}} d\vec{x}_{9-p}^2,$$
$$d\hat{s}_S^2 = H_{Dp}^{-1/2}\left[dt^2 - d\vec{y}_p^2\right] - H_{Dp}^{1/2} d\vec{x}_{9-p}^2,$$
$$e^{-2(\hat{\phi}-\hat{\phi}_0)} = H_{Dp}^{\frac{(p-3)}{2}},$$
$$\hat{C}^{(p+1)}\underline{t}\underline{y}^1\cdots\underline{y}^p = \pm e^{-\hat{\phi}_0}\left(H_{Dp}^{-1} - 1\right),$$
$$H_{Dp} = 1 + \frac{h_{Dp}}{\left|\vec{x}_{9-p}\right|^{7-p}}$$
$$H_{D7} = 1 + h_{D7}\log|\vec{x}_2|$$
$$H_{D8} = 1 + h_{D8}|x|$$
$$h_{Dp} = \frac{(2\pi\ell_s)^{(7-p)} g}{(7-p)\omega_{(8-p)}}$$
$$h_{D7} =, h_{D8} = \frac{g}{4\pi\ell_s}$$
$$\begin{cases} d\hat{\hat{s}}^2 = H_{M2}^{-2/3}\left[dt^2 - dy^2 - e^{\frac{4}{3}\hat{\phi}_0} dz^2\right] - H_{M2}^{1/3} d\vec{x}_8^2 \\ \hat{\hat{C}}_{\underline{t}\underline{y}\underline{z}} = \pm e^{\frac{2}{3}\hat{\phi}_0}(H_{M2}^{-1} - 1) \\ H_{M2} = 1 + \frac{h_{M2}}{|\vec{x}_8|^6} \end{cases}$$
$$h_{M2} = \frac{\left(\ell_{\text{Planck}}^{(11)}\right)^6}{6\omega_{(7)}} = \frac{\left(2\pi\ell_s g^{1/3}\right)^6}{6\omega_{(7)}} = \frac{(2\pi\ell_s)^6 g^2}{6\omega_{(7)}} = h_{D2}$$

$$H_{M2} = 1 + h_{M2} \sum_{n=-\infty}^{n=+\infty} \frac{1}{(|\vec{x}_7|^2 + (z + 2\pi n R_{11})^2\ |)^3}$$

$$u_n = \frac{(z - 2\pi n R_{11})}{|\vec{x}_7|}, u_n \in \left[\frac{2\pi n R_{11}}{|\vec{x}_7|}, \frac{2\pi(n+1)R_{11}}{|\vec{x}_7|}\right]$$

$$H_{M2} = 1 + \frac{h_{M2}}{|\vec{x}_7|^6} \sum_{n=-\infty}^{n=+\infty} \frac{1}{(1+u_n^2)^3} \sim 1 + \frac{h_{M2}}{|\vec{x}_7|^6} \frac{1}{2\pi R_{11}/|\vec{x}_7|} \int_{-\infty}^{+\infty} \frac{du}{(1+u^2)^3}$$

$$= 1 + \frac{h_{M2}\omega_{(5)}}{2\pi R_{11}\omega_{(4)}} \frac{1}{|\vec{x}_7|^5}$$

$$\frac{h_{M2}\omega_{(5)}}{2\pi R_{11}\omega_{(4)}} = h_{D2}$$

$$H_p = 1 + h_p \sum_{n=-\infty}^{n=+\infty} \frac{1}{(|\vec{x}_{n+1}|^2 + (z + 2\pi n R)^2\ |)^{n/2}} \sim 1 + \frac{h_p \omega_{(n-1)}}{2\pi R \omega_{(n-2)}} \frac{1}{|\vec{x}_{n+1}|^{n-1}}$$

$$h'_p = \frac{h_p \omega_{(n-1)}}{2\pi R \omega_{(n-2)}}$$

$$h'_p = \frac{h_p \omega_{(n-1)}}{V^m \omega_{(n-m-1)}}, V^m = (2\pi)^m R_1 \dots R_m$$

$$\frac{h_{Dp}\omega_{(6-p)}}{2\pi R \omega_{(5-p)}} = \frac{(2\pi \ell_s)^{7-p} g}{2\pi R} \frac{\omega_{(6-p)}}{(7-p)\omega_{(8-p)}\omega_{(6-p)}}$$

$$\frac{\omega_{(n-1)}}{n\omega_{(n+1)}\omega_{(n-2)}} = \frac{1}{(n-1)\omega_{(n)}}$$

$$\begin{cases} \hat{\hat{e}}_{\underline{i^2}}{}^{j} = H_{M2}^{-1/3}\delta_i^j \\ \hat{\hat{e}}_{\underline{m}}^{n} = H_{M2}^{1/6}\delta_m^n \end{cases}$$

$$\begin{cases} \hat{\hat{\omega}}_{\underline{\underline{m}}}{}^{nl} = -\frac{1}{3} H_{M2}^{-1} \partial_{\underline{q}} H_{M2} \eta_m^{[n} \eta^{p]q} \\ \hat{\hat{\omega}}_{\underline{\underline{2}}}{}^{mj} = \frac{2}{3} H_{M2}^{-3/2} \partial_{\underline{q}} H_{M2} \eta_i^{[m} \eta^{j]q} \\ \hat{\hat{G}}_{\underline{m}ty^1y^2} = \mp H_{M2}^{-2} \partial_{\underline{m}} H_{M2} \end{cases}$$

$$\begin{cases} \delta_{\hat{\hat{\epsilon}}} \hat{\hat{\psi}}_{\underline{i}} = \frac{1}{3} H_{M2}^{-3/2} \partial_{\underline{n}} H_{M2} \hat{\hat{\Gamma}}_{(i)}^{n} \left(1 \mp \frac{i}{2} \epsilon(i)jk \hat{\hat{\Gamma}}^{(i)jk}\right) \hat{\hat{\epsilon}} = 0, \\ \delta_{\hat{\hat{\epsilon}}}^{\hat{\hat{\psi}}} \underline{\underline{m}} \end{cases}$$

$$= 2\partial_{\underline{m}} \hat{\hat{\epsilon}} - \frac{1}{6} H_{M2}^{-1} \partial_{\underline{n}} H_{M2} \left[\hat{\hat{\Gamma}}^{mn} \mp i \left(\hat{\hat{\Gamma}}^{mn} + 2\delta^{mn}\right) \hat{\hat{\Gamma}}012\right] \hat{\hat{\epsilon}}$$

$$\left(1 \mp i\hat{\hat{\Gamma}}^{012}\right) \hat{\hat{\epsilon}} = 0$$

$$\delta_{\hat{\hat{\epsilon}}} \hat{\hat{\psi}}_{\underline{m}} = 2\left(\partial_{\underline{m}} + \frac{1}{6} H_{M2}^{-1} \partial_{\underline{m}} H_{M2}\right) \hat{\hat{\epsilon}}$$

$$\hat{\hat{\epsilon}} = H_{M2}^{-1/6} \hat{\hat{\epsilon}}_0, \left(1 \mp i\hat{\hat{\Gamma}}^{012}\right) \hat{\hat{\epsilon}}_0$$

$$\begin{cases} \hat{\hat{e}}_{\underline{i}}^{j} = H_{M5}^{-1/6} \delta_i{}^{j}, \\ \hat{\hat{e}}_{\underline{m}}{}^{n} = H_{M5}^{1/3} \delta_m{}^{n} \end{cases}$$

$$\begin{cases} \hat{\hat{\omega}}_{\underline{m}}{}^{nl} = -\frac{2}{3} H_{M5}^{-1} \partial_{\underline{q}} H_{M2} \eta_m^{[n} \eta^{p]q} \\ \hat{\hat{\omega}}_{\underline{\underline{m}}}{}^{mj} = \frac{1}{3} H_{M5}^{-3/2} \partial_{\underline{q}} H_{M2} \eta_i^{[m} \eta^{j]q} \\ \hat{\hat{G}}_{\underline{m}_1 \cdots \underline{m}_4} = \pm \epsilon_{m_1 \cdots m_5} \partial_{\underline{m}_5} H_{M5} \end{cases}$$

$$\hat{\hat{\epsilon}} = H_{M5}^{-1/12} \hat{\hat{\epsilon}}_0, \left(1 \mp \hat{\hat{\Gamma}}^{012345}\right) \hat{\hat{\epsilon}}_0$$

$$\begin{cases} \delta_{\hat{\epsilon}} \hat{\psi}_{\hat{\mu}} = \left\{ \partial_{\hat{\mu}} - \frac{1}{4} \ddot{\psi}_{\hat{\mu}} + \frac{i}{8} e^{\hat{\phi}} \frac{1}{(p+2)!} \hat{\epsilon}^{(p+2)} \hat{\Gamma}_{\hat{\mu}} \left(-\hat{\Gamma}_{11}\right)^{\frac{p+2}{2}} \right\} \hat{\epsilon} \\ \delta_{\hat{\epsilon}} \hat{\lambda} = \left\{ \partial \hat{\phi} - \frac{i}{4} e^{\hat{\phi}} \frac{(p-3)}{(p+2)!} \hat{\zeta}^{(p+2)} \left(-\hat{\Gamma}_{11}\right)^{\frac{p+2}{2}} \right\} \hat{\epsilon} \end{cases}$$

$$\begin{cases} \delta_{\hat{\varepsilon}} \hat{\zeta}_{\hat{\mu}} = \left\{ \partial_{\hat{\mu}} - \frac{1}{4} \psi_{\hat{\mu}} + \frac{1}{8} e^{\hat{\varphi}} \frac{1}{(p+2)!} \hat{\epsilon}^{(p+2)} \hat{\Gamma}_{\hat{\mu}} \mathcal{P}_{\underline{\frac{p+3}{2}}} \right\} \hat{\varepsilon} \\ \delta_{\hat{\varepsilon}} \hat{\chi} = \left\{ \partial \hat{\varphi} + \frac{1}{4} e^{\hat{\varphi}} \left(\frac{(p-3)}{(p+2)!} \hat{\epsilon}^{(p+2)} \mathcal{P}_{\underline{\frac{p+3}{2}}} \right\} \hat{\varepsilon} \end{cases}$$

$$\mathcal{P}_n = \begin{cases} \sigma^1, n \text{ par} \\ i\sigma^2, n \text{ impar} \end{cases}$$

$$\begin{cases} \ddot{\psi}_{\underline{i}} = -\frac{1}{2} H_{Dp}^{-3/2} \partial_{\underline{n}} H_{Dp} \eta_{ij} \Gamma^{nj} \\ \psi_{\underline{m}} = \frac{1}{2} H_{Dp}^{-1} \partial_{\underline{n}} H_{Dp} \eta_{mq} \hat{\Gamma}^{nq} \\ \hat{G}^{(p+2)} = \mp e^{-\hat{\phi}_0} H_{Dp}^{\frac{p}{4}-2} \partial_{\underline{m}} H_{Dp} \hat{\Gamma}^m \hat{\Gamma}^{01\cdots p} \end{cases}$$

$$\text{IIA: } \hat{\epsilon} = H_{Dp}^{-1/8} \hat{\epsilon}_0, \quad \left[1 \mp i\hat{\Gamma}^{01\cdots p} \left(-\hat{\Gamma}_{11}\right)^{\frac{p+2}{2}}\right] \hat{\epsilon}_0 = 0$$

$$\text{IIB: } \hat{\varepsilon} = H_{Dp}^{-1/8} \hat{\varepsilon}_0, \quad \left(1 \pm i\hat{\Gamma}^{01\cdots p} \mathcal{P}_{\underline{\frac{p+3}{2}}}\right) \hat{\varepsilon}_0 = 0$$

$$\begin{cases} \delta_{\hat{\epsilon}} \hat{\psi}_{\hat{\mu}} = \left\{ \partial_{\hat{\mu}} - \frac{1}{4} \left(\psi_{\hat{\mu}} + \frac{1}{2} \hat{H}_{\hat{\mu}} \mathcal{O} \right) \right\} \hat{\epsilon} \\ \delta_{\hat{\epsilon}} \hat{\lambda} = \left\{ \partial \partial \hat{\phi} - \frac{1}{12} \hat{H} \mathcal{O} \right\} \epsilon \end{cases}$$

$$\psi_{\underline{i}} = -H_{F1}^{-3/2} \partial_{\underline{m}} H_{F1} \Gamma_i^m$$

$$H_{\underline{i}} = \mp 2 H_{F1}^{-1} \partial_{\underline{m}} H_{F1} \Gamma^{01}$$

$$H_{\underline{m}} = \pm \epsilon_{ij} H_{F1}^{-3/2} \partial_{\underline{m}} H_{F1} \Gamma^{mj}$$

$$\hat{\epsilon} = H_{F1}^{1/4} \hat{\epsilon}_0, \left(1 \pm \hat{\Gamma}^{01} \mathcal{O}\right) \hat{\epsilon}_0$$

$$\begin{cases} \delta_{\hat{\epsilon}} \hat{\psi}_{\hat{\mu}} = \left\{ \partial_{\hat{\mu}} - \frac{1}{4} \left(\widehat{\omega}_{\hat{\mu}} + \frac{1}{7!} e^{2\hat{\phi}} \hat{H}^{(7)} \hat{a}_1 \cdots \hat{a}_7 \right. \right. \\ \left. \left. \hat{\Gamma}_{\hat{\mu} \hat{a}_1 \cdots \hat{a}_7} \mathcal{O} \right) \right\} \hat{\epsilon} \\ \delta_{\hat{\epsilon}} \hat{\lambda} = \left\{ \partial \partial \hat{\phi} + \frac{1}{2} \hat{H}^{(7)} \mathcal{O} \right\} \hat{\epsilon} \end{cases}$$

$$\hat{\epsilon} = \hat{\epsilon}_0, \left(1 \pm \hat{\Gamma}^{0\cdots 5} \mathcal{O}\right) \hat{\epsilon}_0$$

$$\begin{cases} d\tilde{s}_E^2 &= H_{F1}^{-6/7}dt^2 - H_{F1}^{1/7}d\vec{x}_8^2 \\ ds_s^2 &= H_{F1}^{-1}dt^2 - d\vec{x}_8^2 \\ A_t &= \pm(H_{F1}^{-1} - 1) \\ e^{-2(\phi-\phi_0)} &= H_{F1}^{1/2} \\ K/K_0 &= H_{F1}^{-1/2} \end{cases}$$

$$\begin{cases} d\tilde{s}_E^2 &= H_{S5}^{-2/3}dt^2 - H_{S5}^{1/3}d\vec{x}_4^2 \\ ds_s^2 &= dt^2 - H_{S5}d\vec{x}_4^2, \\ A_t &= \pm(H_{S5}^{-1} - 1), \\ e^{-2(\phi-\phi_0)} &= H_{S5}^{-1}, \\ K/K_0 &= 1 \end{cases}$$

$$\begin{cases} d\tilde{s}_E^2 &= H_{Dp}^{-\frac{7-p}{8-p}}dt^2 - H_{Dp}^{\frac{1}{8-p}}d\vec{x}_{9-p}^2 \\ ds_s^2 &= H_{Dp}^{-1/2}dt^2 - H_{Dp}^{1/2}d\vec{x}_4^2 \\ A_t &= \pm\left(H_{Dp}^{-1} - 1\right) \\ e^{-2(\phi-\phi_0)} &= H_{Dp}^{\frac{p-6}{4}} \\ K/K_0 &= H_{Dp}^{-p/4} \end{cases}$$

$5 -$ pluridimensión $\|+| + + + + + - - -$

$$P_p\epsilon = \left(1 \pm \Gamma^{01\cdots p}\mathcal{O}_p\right)\epsilon$$

$$\left[P_p, P_{p'}\right] = 0$$

$$\mathrm{D}_p \perp \mathrm{D}_{(p+4)}(p)$$

$$q_{F1} \sim \int_{S^7} e^{-2\varphi\star}\hat{\mathcal{H}}$$

$$de^{-2\varphi\star}\hat{\mathcal{H}} = 0$$

$$q_{F1} \sim \int_{S^7} \left(e^{-2\varphi\star}\hat{\mathcal{H}} - {}^{\star}\hat{G}^{(3)}\hat{C}^{(0)} - \hat{G}^{(5)}\hat{C}^{(2)}\right)$$

$$\int_{S^5} \hat{G}^{(5)} \int_{S^2} \hat{C}^{(2)}$$

$$q_{F1} \sim \int_{S^2} dV^{(1)}$$

$$\mathrm{F1} \perp \mathrm{D}_p(0)$$

$$\mathrm{D}_p \perp \mathrm{D}_{p+2}(p-1), p \geq 1$$

$$\mathrm{D}_p \perp \mathrm{D}_{p+4}(p)$$

$$\mathrm{F1} \perp \ \mathrm{S5\ B}(0)$$

$$\mathrm{D}_p \perp \mathrm{D}_p(p-2), p \geq 2$$

$$\mathrm{D}_p \perp \ \mathrm{S5}(p-1), p \geq 1$$

$\mathrm{F1}\|\mathrm{S5},\ \mathrm{F1} \perp \mathrm{D}_p(0),$

$\mathrm{S5} \perp \ \mathrm{S5}(1), \mathrm{S5} \perp \ \mathrm{S5}(1), \mathrm{S5} \perp \mathrm{D}_p(p-1)(p > 1),$

$\mathrm{D}_p \perp \mathrm{D}_{p'}(m)p + p' = 4 + 2m,$

$\mathrm{W}\|\ \mathrm{F1},\ \mathrm{W}\|\mathrm{S5},\ \mathrm{W}\|\mathrm{D}_p,$

$\mathrm{KK} \perp D_p(p-2).$

$$\mathrm{M2} \perp \mathrm{M2}(0), \mathrm{M2} \perp \mathrm{M5}(1), \mathrm{M5} \perp \mathrm{M5}(1), \mathrm{M5} \perp \mathrm{M5}(3),$$
$$\mathrm{W}\|\mathrm{M2}, \ \mathrm{W}\|\mathrm{M5},$$
$$\mathrm{KK}\|\mathrm{M2}, \mathrm{KK} \perp \mathrm{M2}(0), \mathrm{KK}\|\mathrm{M5}, \mathrm{KK} \perp \mathrm{M5}(1), \mathrm{KK} \perp \mathrm{M5}(3),$$
$$\mathrm{W}\|\mathrm{KK}, \ \mathrm{W} \perp \mathrm{KK}(2), \mathrm{W} \perp \mathrm{KK}(4).$$

$$d\hat{s}_s^2 = H_{Dp}^{-1/2} H_{F1}^{-1} dt^2 - H_{Dp}^{+1/2} H_{F1}^{-1} dy^2 - H_{Dp}^{-1/2} d\vec{z}_p^2 - H_{Dp}^{+1/2} d\vec{x}_{8-p}^2,$$

$$e^{-2\left(\hat{\phi}^{\hat{\phi}}\hat{\phi}_0\right)} = H_{D_p^2}^{\frac{(p-3)}{2}} H_{F1},$$

$$\hat{C}^{(p+1)} t_{t\underline{1}} \cdots\cdots \underline{\underline{z}}^p = \pm e^{-\hat{\phi}_0}\left(H_{Dp}^{-1} - 1\right),$$

$$\hat{B}_{t\underline{t}} = \pm(H_{F1}^{-1} - 1),$$

$$H_{Dp,F1} = 1 + \frac{h_{Dp,F1}}{\left|\vec{x}_{8-p}\right|^{6-p}}$$

$$\begin{cases} ds^2 \ = H^\alpha\left[W dt^2 - d\vec{y}_{p-1}^2 - dz^2\right] - H^\beta[W^{-1} d\rho^2 + \rho^2 d\Omega^2] \\ e^{-2(\phi-\phi_0)} \ = H^\gamma \\ A_{(p+1)\underline{t y^1 \cdots y^{p-1} z}} \ = \alpha(H^{-1} - 1) \\ W \ = 1 + \dfrac{\omega}{\rho^n}, H = 1 + \dfrac{h}{\rho^n} \end{cases}$$

$$\begin{pmatrix} t \\ z \end{pmatrix} \to \begin{pmatrix} \cosh\ \gamma & \sinh\ \gamma \\ \sinh\ \gamma & \cosh\ \gamma \end{pmatrix} \begin{pmatrix} t \\ z \end{pmatrix}$$

$$W dt^2 - dz^2 \to dt^2 - dz^2 + \cosh^2\ \gamma(W - 1)(dt + \tanh^2\ \gamma dz)^2$$

$$H_W^{-1} dt^2 - H_W[dz - (H_W^{-1} - 1)dt]^2, H_W \ = 1 + \frac{h_W}{\rho^n}$$

$$ds^2 \ = H^\alpha\left\{H_W^{-1} dt^2 - H_W[dz - (H_W^{-1} - 1)dt]^2 - d\vec{y}_{p-1}^2\right\} - H^\beta d\vec{x}^2$$

$$ds^2 = H_W^{-1} dt^2 - H_W[dz + \alpha(H_W^{-1} - 1)dt]^2 - d\vec{x}_{d-2}^2,$$

$$H_W \ = 1 + \frac{h_W}{|\vec{x}_{d-2}|^{d-4}}, \alpha \ = \pm 1$$

$$H_W \ = 1 + \frac{h_W}{|\vec{x}_{d-2}|^{d-4}} \delta(u_\alpha). u_\alpha \ = \frac{1}{\sqrt{2}}(t - \alpha z)$$

$$h_W \ = -\alpha \frac{\sqrt{2}|p^z| 8\pi G_N^{(d)}}{(d - 4)\omega_{(d-3)}}$$

$$\delta(u_\alpha) \sim -\alpha \frac{\sqrt{2}}{2\pi R_z}$$

$$h_W \ = \frac{|N| 8 G_N^{(d)}}{R_z^2 (d - 4)\omega_{(d-3)}}$$

$$\begin{cases} d\hat{s}_s^2 \ = H_{D1}^{-1/2} H_{D5}^{-1/2}\{H_W^{-1} dt^2 - H_W[dy^1 + \alpha_W(H_W^{-1} - 1)dt]^2\} \\ -H_{D1}^{1/2} H_{D5}^{-1/2} d\vec{y}_4^2 - H_{D1}^{1/2} H_{D5}^{1/2} d\vec{x}_4^2, \\ e^{-2(\hat{\phi}-\hat{\phi}_0)} \\ \hat{C}_{D5}/H_{D1} \\ \underline{t\hat{t}y}^{(2)} = \alpha_{D1}(H_{D1}^{-1} - 1) \\ \hat{C}^{(6)}{}_{t\underline{t}^1 \cdots \underline{y}^5} = \alpha_{D5}(H_{D5}^{-1} - 1) \end{cases}$$

$$H_i = 1 + \frac{r_i^2}{|\vec{x}_4|^2}, i = D1, D5, W$$
$$r_{D5}^2 = N_{D5} h_{D5} = N_{D5} \ell_s^2 g$$
$$r_{D1}^2 = N_{D1} h_{D1} \frac{\omega_{(5)}}{V^4 \omega_{(1)}} = \frac{N_{D1} \ell_s^6 g}{V}, V \equiv R_2 \dots R_5$$
$$r_W^2 = h_W \frac{\omega_{(5)}}{V^4 \omega_{(1)}} = \frac{N_W \ell_s^8 g^2}{R^2 V}$$
$$d\tilde{s}_E^2 = (H_{D1} H_{D5} H_W)^{-2/3} dt^2 - (H_{D1} H_{D5} H_W)^{1/3} d\vec{x}_4^2$$
$$ds_S^2 = (H_{D1} H_{D5})^{-1/2} H_W^{-1} dt^2 - (H_{D1} H_{D5})^{1/2} d\vec{x}_4^2$$
$$A^{(D1,D5,W)t} = \alpha_{D1,D5,W} \left(H_{D1,D5,W}^{-1} - 1 \right)$$
$$K_V / K_{V0} = H_{D1} / H_{D5}, e^{-2(\phi - \phi_0)} = K_R / K_{R0} = (H_{D1} H_{D5})^{-1/4} H_W^{1/2}$$
$$A = \omega_{(3)} \left(\lim_{|\vec{x}_4| \to 0} |\vec{x}_4|^6 H_{D1} H_{D5} H_W \right)^{1/2} = 2\pi^2 (r_{D1} r_{D5} r_W)^{1/2}$$
$$= 2\pi^2 \sqrt{N_{D1} N_{D5} N_W} \frac{\ell_s^8 g^2}{RV}$$
$$S = \frac{A}{4 G_N^{(5)}}, G_N^{(5)} = \frac{G_N^{(10)}}{(2\pi)^5 RV} = \frac{\pi}{4} \frac{\ell_s^8 g^2}{RV}$$
$$S = 2\pi \sqrt{N_{D1} N_{D5} N_W}$$
$$M = \frac{N_{D1} R}{g \ell_s} + \frac{N_{D5} RV}{g \ell_s^6} + \frac{N_W}{R}$$
$$ds^2 = H^{-2} dt^2 - H d\vec{x}_4^2, H = H_{D1} = H_{D5} = H_W$$
$$\rho(E) \sim e^{\sqrt{\pi (c - 24 E_0) EL/3}}$$
$$\rho(E) = e^{2\pi \sqrt{N_{D1} N_{D5} N_W}}$$
$$e_a{}^\mu e_b{}^\nu g_{\mu\nu} = \eta_{ab}, e_\mu{}^a e_\nu^b \eta_{ab} = g_{\mu\nu}$$
$$\nabla_\mu \xi^\nu = \partial_\mu \xi^\nu + \Gamma_{\mu\rho}{}^\nu \xi^\rho,$$
$$\mathcal{D}_\mu \xi^a = \partial_\mu \xi^a + \omega_{\mu b}{}^a \xi^b,$$
$$\nabla_\mu \psi = \partial_\mu \psi - \frac{1}{4} \omega_\mu{}^{ab} \Gamma_{ab} \psi$$
$$[\nabla_\mu, \nabla_\nu] \xi^\rho = R_{\mu\nu\sigma}{}^\rho (\Gamma) \xi^\sigma + T_{\mu\nu}{}^\sigma \nabla_\sigma \xi^\rho$$
$$[\mathcal{D}_\mu, \mathcal{D}_\nu] \xi^a = R_{\mu\nu b}{}^a (\omega) \xi^b$$
$$R_{\mu\nu\rho}^\sigma (\Gamma) = 2 \partial_{[\mu} \Gamma_{\nu]\rho}^\sigma + 2 \Gamma_{[\mu|\lambda}{}^\sigma \Gamma_{\nu]\rho}$$
$$R_{\mu\nu a}{}^b (\omega) = 2 \partial_{[\mu} \omega_{\nu]a}^b - 2 \omega_{[\mu|a}{}^c \omega_{|\nu]c}^b$$
$$\nabla_\mu e_a^\mu = 0$$
$$\omega_{\mu a}{}^b = \Gamma_{\mu a}{}^b + e_a{}^\nu \partial_\mu e_\nu^b$$
$$R_{\mu\nu\rho}{}^\sigma (\Gamma) = e_\rho{}^a e^\sigma{}_b R_{\mu\nu a}{}^b (\omega)$$
$$\nabla_\mu g_{\rho\sigma} = 0$$
$$\Gamma_{\mu\nu}^\rho = \left\{ \begin{matrix} \rho \\ \mu\nu \end{matrix} \right\} + K_{\mu\nu}^\rho = \Gamma_{\mu\nu}^\rho (g) + K_{\mu\nu}^\rho$$
$$\left\{ \begin{matrix} \rho \\ \mu\nu \end{matrix} \right\} = \frac{1}{2} g^{\rho\sigma} \{ \partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \}$$
$$K_{\mu\nu}^\rho = \frac{1}{2} g^{\rho\sigma} \{ T_{\mu\sigma\nu} + T_{\nu\sigma\mu} - T_{\mu\nu\sigma} \}$$
$$\omega_{abc} = \omega_{abc}(e) + K_{abc}, \omega_{abc}(e) = -\Omega_{abc} + \Omega_{bca} - \Omega_{cab}, \Omega_{ab}^c = e_a{}^\mu e_b{}^\nu \partial_{[\mu} e^c{}_{\nu]}$$

$$\left\{\hat{\hat{\Gamma}}\hat{\hat{a}}, \hat{\hat{\Gamma}}^{\hat{\hat{b}}}\right\} = +2\hat{\hat{\eta}}^{\hat{\hat{a}}\hat{\hat{b}}}$$

$$\hat{\hat{\Gamma}}_{\hat{1}\hat{0}} = i\hat{\hat{\Gamma}}^{\hat{\hat{0}}} \ldots \hat{\hat{\Gamma}}^{\hat{\hat{\rho}}} \equiv -i\hat{\Gamma}_{1\hat{1}}$$

$$\hat{\hat{\Gamma}}^{\hat{\hat{a}}\star} = -\hat{\hat{\Gamma}}^{\hat{\hat{a}}}$$

$$\hat{\hat{\Gamma}}^{\hat{\hat{0}}\dagger} = +\hat{\hat{\Gamma}}^{\hat{\hat{0}}}.$$

$$\hat{\hat{\Gamma}}^{\hat{\hat{\imath}}\dagger} = -\hat{\hat{\Gamma}}^{\hat{\imath}}, \hat{\hat{\imath}} = 1, \ldots, 10.$$

$$\hat{\hat{\Gamma}}^{\hat{\hat{0}}T} = -\hat{\hat{\Gamma}}^{\hat{\hat{0}}}.$$

$$\hat{\hat{\Gamma}}\hat{\hat{\imath}}T = +\hat{\hat{\Gamma}}^{\hat{\hat{\imath}}}, \hat{\hat{\imath}} = 1, \ldots, 10.$$

$$\hat{\hat{\Gamma}}_{\hat{\hat{0}}}^{\hat{\hat{}}}\hat{\Gamma}^{\hat{a}\hat{}\hat{\hat{\hat{0}}}} = \hat{\hat{\Gamma}}^{\hat{\hat{a}}\dagger}$$

$$\hat{\hat{\mathcal{D}}} = i\hat{\hat{\Gamma}}^{0}$$

$$\hat{\hat{\mathcal{D}}}\hat{\hat{\Gamma}}^{\hat{a}_1 \ldots \hat{\hat{a}}_n}\hat{\hat{\mathcal{D}}}^{-1} = (-1)^{[n/2]}\left(\hat{\hat{\Gamma}}^{\hat{\hat{a}}_1 \ldots \hat{a}_n}\right)^{\dagger}$$

$$\hat{\hat{\mathcal{C}}} = \hat{\hat{\mathcal{D}}} = i\hat{\hat{\Gamma}}^{0}$$

$$\hat{\hat{\mathcal{C}}}^{T} = \hat{\hat{\mathcal{C}}}^{\dagger} = \hat{\hat{\mathcal{C}}}^{-1} = -\hat{\hat{\mathcal{C}}}$$

$$\hat{\hat{\mathcal{C}}}\hat{\hat{\Gamma}}\hat{\hat{a}}\hat{\hat{\mathcal{C}}}^{-1} = -\hat{\hat{\Gamma}}\overline{\hat{\hat{a}}}^{\hat{2}}$$

$$\hat{\hat{\mathcal{C}}}\hat{\hat{\Gamma}}^{\hat{\hat{a}}_1 \ldots \hat{\hat{a}}_n}\hat{\hat{\mathcal{C}}}^{-1} = (-1)^{n+[n/2]}\left(\hat{\hat{\Gamma}}^{\hat{\hat{a}}_1 \ldots \hat{\hat{a}}_n}\right)^{T}$$

$$\overline{\hat{\lambda}} = \hat{\hat{\lambda}}^{\dagger}\hat{\mathcal{D}}$$

$$\hat{\hat{\lambda}}^{c} = \hat{\hat{\lambda}}^{T}\hat{\hat{\mathcal{C}}}$$

$$\overline{\hat{\hat{\lambda}}} = \hat{\hat{\lambda}}^{c}$$

$$\overline{\hat{\hat{\epsilon}}}\hat{\hat{a}}^{\hat{a}_1 \cdots \hat{\hat{a}}_n}\hat{\hat{\psi}} = (-1)^{n+[n/2]\hat{}}\overline{\hat{\hat{N}}}^{\hat{a}_1 \cdots \hat{\hat{a}}_n}\hat{\hat{\epsilon}}$$

$$\left(\overline{\hat{\hat{\epsilon}}}\hat{\hat{\Gamma}}^{\hat{\hat{a}}_1 \cdots \hat{\hat{a}}_n}\hat{\hat{\psi}}\right)^{\dagger} = (-1)^{[n/2]\bar{\hat{}}}\hat{\hat{\Gamma}}^{\hat{\hat{a}}_1 \ldots \hat{\hat{a}}_n}\hat{\hat{\epsilon}}$$

$$\hat{\hat{\Gamma}}^{\hat{\hat{a}}_1 \ldots \hat{\hat{a}}_n} = i\frac{(-1)^{[n/2]+1}}{(11-n)!}\hat{\hat{\epsilon}}^{\hat{\hat{a}}_1 \ldots \hat{\hat{a}}_n\hat{\hat{b}}_1 \ldots \hat{\hat{b}}_{11-n}}\hat{\hat{\Gamma}}_{\hat{\hat{b}}_1 \ldots \hat{\hat{b}}_{11-n}}$$

$$\begin{cases}\hat{\hat{\Gamma}}^{\hat{a}} = \hat{\Gamma}^{\hat{a}}, \hat{a} = 0, \ldots, 9 \\ \hat{\hat{\Gamma}}^{10} = +i\hat{\Gamma}^{0} \ldots \hat{\Gamma}^{9}\end{cases}$$

$$\hat{\Gamma}_{11} = -\hat{\Gamma}^{0} \ldots \hat{\Gamma}^{9} = i\hat{\hat{\Gamma}}^{10}$$

$$\hat{\Gamma}_{11}\hat{\psi}^{(\pm)} = \pm\hat{\psi}^{(\pm)}$$

$$\hat{\Gamma}_{11} = \mathbb{I}_{16\times16} \otimes \sigma^{3} = \begin{pmatrix}\mathbb{I}_{16\times16} & 0 \\ 0 & -\mathbb{I}_{16\times16}\end{pmatrix}$$

$$\hat{\psi} = \begin{pmatrix}\hat{\psi}^{(+)} \\ \hat{\psi}^{(-)}\end{pmatrix}$$

$$\Gamma_{11}\hat{\Gamma}^{\hat{a}_1 \cdots \hat{a}_n} = \frac{(-1)^{[(10-n)/2]+1}}{(10-n)!}\hat{\epsilon}^{\hat{a}_1 \cdots \hat{a}_n\hat{b}_1 \cdots \hat{b}_{10-n}}\hat{\Gamma}_{\hat{b}_1 \cdots \hat{b}_{10-n}}$$

$$\begin{cases}\hat{\Gamma}^{a} = \Gamma^{a} \otimes \sigma^{2}, a = 0, \ldots, 8 \\ \hat{\Gamma}^{9} = \mathbb{I}_{16\times16} \otimes i\sigma^{1}\end{cases}$$

$$\Gamma^{8} = \Gamma^{0} \cdots \Gamma^{7}$$

$$\Gamma_{(8)9} = i\Gamma^{8} = i\Gamma^{0} \cdots \Gamma^{7}$$

$$\gamma_5 = -i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3} = \frac{i}{4!}\epsilon_{abcd}\gamma^{abcd}$$

$$\gamma^{a_1 \cdots a_n} = \frac{(-1)^{[n/2]}i}{(4-n)!}\epsilon^{a_1 \cdots a_n b_1 \cdots b_{4-n}}\gamma_{b_1 \cdots b_{4-n}}\gamma_5$$

$$n^{\mu}n_{\mu} = \varepsilon, \begin{cases} \varepsilon = +1, & \Sigma \text{ espacio} \\ \varepsilon = -1, & \Sigma \text{ tiempo} \end{cases}$$

$$h_{\mu\nu} = g_{\mu\nu} - \varepsilon n_{\mu} n_{\nu}$$

$$\mathcal{K}_{\mu\nu} \equiv h_{\mu}{}^{\alpha} h_{\nu}{}^{\beta} \nabla_{(\alpha} n_{\beta)}$$

$$\mathcal{K}_{\mu\nu} = \frac{1}{2} \pounds_{n} h_{\mu\nu}$$

$$\mathcal{K} = h^{\mu\nu}\mathcal{K}_{\mu\nu} = h^{\mu\nu}\nabla_{\mu} n_{\nu}$$

$$\begin{cases} x^1 = \rho_{n-1} \sin\ \varphi \\ x^2 = \rho_{n-1} \cos\ \varphi \\ x^3 = \rho_{n-2} \cos\ \theta_1 \\ \vdots\ \vdots \\ x^k = \rho_{n-k+1} \cos\ \theta_{k-2}, 3 \leq k \leq n+1 \end{cases}$$

$$\begin{cases} \rho_l = [(x^1)^2 + \cdots + (x^{n+1-l})^2]^{1/2} = r \prod_{m=1}^{l} \sin\ \theta_{n-m}, \\ \rho_0 = r = [(x^1)^2 + \cdots + (x^{n+1})^2]^{1/2} \end{cases}$$

$$d\Omega^n \equiv d\varphi \prod_{i=1}^{n-1} \sin^i\ \theta_i d\theta_i$$

$$d\Omega^n = \frac{1}{n!\, r^{n+1}} \epsilon_{\mu_1 \cdots \mu_{n+1}} x^{\mu_{n+1}} dx^{\mu_1} \ldots dx^{\mu_n}$$

$$\begin{cases} d^{n+1}x = r^n dr d\Omega^n, \\ r^n d\Omega^n = d^n y \sqrt{|g|} \end{cases}$$

$$\omega_{(n)} = \int_{S^n} d\Omega^n = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma\left(\frac{n+1}{2}\right)}$$

$$\Gamma(x+1) = x\Gamma(x), \Gamma(0) = 1, \Gamma(1/2) = \pi^{1/2}$$

$$\begin{aligned} d\vec{x}^2 &= d\rho_0^2 + \rho_0^2 d\theta_{n-1}^2 + \cdots + \rho_{n-2}^2 d\theta_1^2 + \rho_{n-1}^2 d\varphi^2 \\ &= dr^2 + r^2\{d\theta_{n-1}^2 + \sin^2\ \theta_{n-1}[d\theta_{n-2}^2 + \sin^2\ \theta_{n-2}(d\theta_{n-3}^2 + \sin^2\ \theta_{n-3}(\cdots \\ &\quad \cdots \sin^2\ \theta_2(d\theta_1^2 + \sin^2\ \theta_1 d\varphi^2) \cdots]\} \\ &= dr^2 + r^2 d\Omega_{(n)}^2 \end{aligned}$$

$$\boldsymbol{P} = P^A \boldsymbol{G}_A$$

$$[\boldsymbol{P}, \boldsymbol{Q}] = \boldsymbol{P}\boldsymbol{Q} - (-1)^{pq} \boldsymbol{Q}$$

$$[\boldsymbol{P}, \boldsymbol{Q}] = P^A Q^B [\boldsymbol{G}_A, \boldsymbol{G}_B],\ \text{si } P^A \text{ o } Q^B$$

$$[\boldsymbol{P}, \boldsymbol{Q}] = P^A Q^B \{\boldsymbol{G}_A, \boldsymbol{G}_B\},\ \text{si } P^A \text{ y } Q^B$$

$$[\boldsymbol{G}_A, \boldsymbol{G}_B] = \boldsymbol{G}_A \boldsymbol{G}_B - \boldsymbol{G}_B \boldsymbol{G}_A$$

$$\{\boldsymbol{G}_A, \boldsymbol{G}_B\} = \boldsymbol{G}_A \boldsymbol{G}_B + \boldsymbol{G}_B \boldsymbol{G}_A$$

$$\mathrm{D}\boldsymbol{Z} = \mathrm{d}\boldsymbol{Z} + [\boldsymbol{A}, \boldsymbol{Z}]$$

$$\boldsymbol{A} \to \boldsymbol{A}' = g(\boldsymbol{A} - g^{-1}\, \mathrm{d}g) g^{-1}$$

$$\delta \boldsymbol{A} = -\mathrm{D}\boldsymbol{\lambda}$$

$$\langle \cdots \rangle_r : \underbrace{\mathfrak{g} \times \cdots \times \mathfrak{g}}_{r} \to \mathbb{C}$$

$$\langle \cdots \boldsymbol{P}\boldsymbol{Q} \cdots \rangle_r = (-1)^{pq} \langle \cdots \boldsymbol{Q}\boldsymbol{P} \cdots \rangle_r$$

$$\langle (g\boldsymbol{Z}_1 g^{-1}) \cdots (g\boldsymbol{Z}_r g^{-1}) \rangle_r = \langle \boldsymbol{Z}_1 \cdots \boldsymbol{Z}_r \rangle_r,$$

$$\langle [\boldsymbol{\lambda}, \boldsymbol{Z}_1] \boldsymbol{Z}_2 \cdots \boldsymbol{Z}_r \rangle_r + \cdots + \langle \boldsymbol{Z}_1 \cdots \boldsymbol{Z}_{r-1} [\boldsymbol{\lambda}, \boldsymbol{Z}_r] \rangle_r$$

$$\begin{aligned} &\langle [\boldsymbol{A}, \boldsymbol{Z}_1] \boldsymbol{Z}_2 \cdots \boldsymbol{Z}_r \rangle_r + (-1)^{p_1} \langle \boldsymbol{Z}_1 [\boldsymbol{A}, \boldsymbol{Z}_2] \boldsymbol{Z}_3 \cdots \boldsymbol{Z}_r \rangle_r + \\ &\qquad + \cdots + (-1)^{p_1 + \cdots + p_{r-1}} \langle \boldsymbol{Z}_1 \cdots \boldsymbol{Z}_{r-1} [\boldsymbol{A}, \boldsymbol{Z}_r] \rangle_r \end{aligned}$$

$$\langle \mathrm{D}(\boldsymbol{Z}_1 \cdots \boldsymbol{Z}_r) \rangle_r = \mathrm{d} \langle \boldsymbol{Z}_1 \cdots \boldsymbol{Z}_r \rangle_r$$

$$\langle \mathrm{D}(\boldsymbol{Z}_1 \cdots \boldsymbol{Z}_r)\rangle_r = \langle (\mathrm{D}\boldsymbol{Z}_1)\boldsymbol{Z}_2 \cdots \boldsymbol{Z}_r\rangle_r + (-1)^{p_1}\langle \boldsymbol{Z}_1(\mathrm{D}\boldsymbol{Z}_2)\boldsymbol{Z}_3 \cdots \boldsymbol{Z}_r\rangle_r + + \cdots + (-1)^{p_1+\cdots+p_{r-1}}\langle \boldsymbol{Z}_1 \cdots \boldsymbol{Z}_{r-1}(\mathrm{D}\boldsymbol{Z}_r)\rangle_r$$

$$[\boldsymbol{P}_a, \boldsymbol{P}_b] = 0$$

$$[\boldsymbol{J}_{ab}, \boldsymbol{P}_c] = \eta_{cb}\boldsymbol{P}_a - \eta_{ca}\boldsymbol{P}_b$$

$$[\boldsymbol{J}_{ab}, \boldsymbol{J}_{cd}] = \eta_{cb}\boldsymbol{J}_{ad} - \eta_{ca}\boldsymbol{J}_{bd} + \eta_{db}\boldsymbol{J}_{ca} - \eta_{da}\boldsymbol{J}_{cb}$$

$$\langle \boldsymbol{J}_{ab}\boldsymbol{P}_c\rangle = \varepsilon_{abc}$$

$$\langle [\boldsymbol{A}, \boldsymbol{J}_{ab}]\boldsymbol{P}_c\rangle + \langle \boldsymbol{J}_{ab}[\boldsymbol{A}, \boldsymbol{P}_c]\rangle$$

$$\boldsymbol{A} = e^a\boldsymbol{P}_a + \frac{1}{2}\omega^{ab}\boldsymbol{J}_{ab}$$

$$\omega_a^e\varepsilon_{ebc} + \omega_b^e\varepsilon_{aec} + \omega_c^e\varepsilon_{abe} = 0$$

$$\mathrm{D}_\omega\varepsilon_{abc} = 0$$

$$\delta_{abcd}^{efgh} = 0$$

$$\begin{aligned} 0 &= \omega^d{}_e\delta_{abcd}^{efgh} \\ &= \omega^d{}_e\left(\delta_a^e\delta_{bcd}^{fgh} - \delta_b^e\delta_{acd}^{fgh} + \delta_c^e\delta_{abd}^{fgh} - \delta_d^e\delta_{abc}^{fgh}\right) \\ &= \omega^d{}_a\delta_{bbd}^{fgh} - \omega^d{}_b\delta_{acd}^{fgh} + \omega^d{}_c\delta_{abd}^{fgh} \\ &= \omega^e{}_a\delta_{ebc}^{fgh} + \omega^e{}_b\delta_{aec}^{fgh} + \omega^e{}_c\delta_{abe}^{fgh}. \end{aligned}$$

$$\omega_a^e\varepsilon_{ebc} + \omega_b^e\varepsilon_{aec} + \omega^e{}_c\varepsilon_{abe} = 0$$

$$\mathcal{Q}_{\mathrm{CS}}^{(2n+1)} \equiv (n+1)\int_0^1 dt\langle \boldsymbol{A}(t\,\mathrm{d}\boldsymbol{A} + t^2\boldsymbol{A}^2)^n\rangle$$

$$\mathcal{Q}_{\mathrm{CS}}^{(3)} = \left\langle \boldsymbol{A}\mathrm{d}\boldsymbol{A} + \frac{2}{3}\boldsymbol{A}^3\right\rangle$$

$$\mathcal{Q}_{\mathrm{CS}}^{(5)} = \left\langle \boldsymbol{A}(\mathrm{d}\boldsymbol{A})^2 + \frac{3}{2}\boldsymbol{A}^3\,\mathrm{d}\boldsymbol{A} + \frac{3}{5}\boldsymbol{A}^5\right\rangle$$

$$\mathrm{d}\mathcal{Q}_{\mathrm{CS}}^{(2n+1)} = \langle \boldsymbol{F}^{n+1}\rangle$$

$$\mathrm{d}\delta_{\text{gauge}}\,\mathcal{Q}_{\mathrm{CS}}^{(2n+1)} = 0$$

$$\mathcal{Q}_{\mathrm{CS}}^{(2n+1)}(\boldsymbol{A}') = \mathcal{Q}_{\mathrm{CS}}^{(2n+1)}(\boldsymbol{A}) + (-1)^{n+1}\frac{n!\,(n+1)!}{(2n+1)!}\langle (g^{-1}\,\mathrm{d}g)^{2n+1}\rangle + \mathrm{d}\Omega_{\text{fin}}^{(2n)}$$

$$\delta_{\text{gauge}}\,\mathcal{Q}_{\mathrm{CS}}^{(2n+1)} = \mathrm{d}\Omega^{(2n)}$$

$$L_{\mathrm{YM}} = -\frac{1}{4}\langle \boldsymbol{F} \wedge\star \boldsymbol{F}\rangle$$

$$L_{\mathrm{CS}}^{(2n+1)} = (n+1)k\int_0^1 dt\langle \boldsymbol{A}(t\,\mathrm{d}\boldsymbol{A} + t^2\boldsymbol{A}^2)^n\rangle$$

$$[\boldsymbol{P}_a, \boldsymbol{P}_b] = \boldsymbol{J}_{ab}$$

$$[\boldsymbol{J}_{ab}, \boldsymbol{P}_c] = \eta_{cb}\boldsymbol{P}_a - \eta_{ca}\boldsymbol{P}_b$$

$$[\boldsymbol{J}_{ab}, \boldsymbol{J}_{cd}] = \eta_{cb}\boldsymbol{J}_{ad} - \eta_{ca}\boldsymbol{J}_{bd} + \eta_{db}\boldsymbol{J}_{ca} - \eta_{da}\boldsymbol{J}_{cb}$$

$$\boldsymbol{A} = \frac{1}{\ell}e^a\boldsymbol{P}_a + \frac{1}{2}\omega^{ab}\boldsymbol{J}_{ab}$$

$$\boldsymbol{\lambda} = \frac{1}{\ell}\lambda^a\boldsymbol{P}_a + \frac{1}{2}\lambda^{ab}\boldsymbol{J}_{ab}$$

$$\delta e^a = \lambda^a{}_b e^b - \mathrm{D}_\omega\lambda^a$$

$$\delta\omega^{ab} = -\mathrm{D}_\omega\lambda^{ab} + \frac{1}{\ell^2}(\lambda^a e^b - \lambda^b e^a)$$

$$\boldsymbol{F} = \frac{1}{\ell}T^a\boldsymbol{P}_a + \frac{1}{2}\left(R^{ab} + \frac{1}{\ell^2}e^a e^b\right)\boldsymbol{J}_{ab}$$

$$T^a = \mathrm{D}_\omega e^a$$

$$R^{ab} = \mathrm{d}\omega^{ab} + \omega^a{}_c\omega^{cb}$$

$$\langle \boldsymbol{J}_{a_1a_2}\cdots\boldsymbol{J}_{a_{2n-1}a_{2n}}\boldsymbol{P}_{a_{2n+1}}\rangle = \frac{2^n}{n+1}\varepsilon_{a_1\cdots a_{2n+1}}$$

$$\boldsymbol{e} = \frac{1}{\ell}e^a\boldsymbol{P}_a$$

$$\boldsymbol{\omega} = \frac{1}{2}\omega^{ab}\boldsymbol{J}_{ab}$$

$$\boldsymbol{T} = \mathrm{d}\boldsymbol{e} + [\boldsymbol{\omega},\boldsymbol{e}]$$

$$\boldsymbol{R} = \mathrm{d}\boldsymbol{\omega} + \boldsymbol{\omega}^2$$

$$\boldsymbol{T} = \frac{1}{\ell}T^a\boldsymbol{P}_a$$

$$\boldsymbol{R} = \frac{1}{2}R^{ab}\boldsymbol{J}_{ab}$$

$$L_{\mathrm{CS}}^{(2n+1)} = (n+1)k\int_0^1 dt\langle(\boldsymbol{R}+t^2\boldsymbol{e}^2)^n\boldsymbol{e}\rangle + \mathrm{d}B_{\mathrm{CS}}^{(2n)}$$

$$L_{\mathrm{CS}}^{(2n+1)} = \frac{k}{\ell}\varepsilon_{a_1\cdots a_{2n+1}}\int_0^1 dt\left(R^{a_1a_2}+\frac{t^2}{\ell^2}e^{a_1}e^{a_2}\right)\times\cdots\times$$

$$\times\left(R^{a_{2n-1}a_{2n}}+\frac{t^2}{\ell^2}e^{a_{2n-1}}e^{a_{2n}}\right)e^{a_{2n+1}} + \mathrm{d}B_{\mathrm{CS}}^{(2n)}$$

$$Q_{\boldsymbol{A}\leftarrow\overline{\boldsymbol{A}}}^{(2n+1)} \equiv (n+1)\int_0^1 dt\langle\boldsymbol{\Theta}\boldsymbol{F}_t^n\rangle$$

$$\boldsymbol{\Theta} \equiv \boldsymbol{A} - \overline{\boldsymbol{A}}$$

$$\boldsymbol{A}_t \equiv \overline{\boldsymbol{A}} + t\boldsymbol{\Theta}$$

$$\boldsymbol{F}_t \equiv \mathrm{d}\boldsymbol{A}_t + \boldsymbol{A}_t^2$$

$$\langle\boldsymbol{F}^{n+1}\rangle - \left\langle\overline{\boldsymbol{F}}^{n+1}\right\rangle = \int_0^1 dt\frac{d}{dt}\langle\boldsymbol{F}_t^{n+1}\rangle$$

$$\langle\boldsymbol{F}^{n+1}\rangle - \left\langle\overline{\boldsymbol{F}}^{n+1}\right\rangle = (n+1)\int_0^1 dt\left\langle\boldsymbol{F}_t^n\frac{d}{dt}\boldsymbol{F}_t\right\rangle$$

$$\frac{d}{dt}\boldsymbol{F}_t = \mathrm{D}_t\boldsymbol{\Theta}$$

$$\langle\boldsymbol{F}^{n+1}\rangle - \left\langle\overline{\boldsymbol{F}}^{n+1}\right\rangle = (n+1)\int_0^1 dt\langle\boldsymbol{F}_t^n\mathrm{D}_t\boldsymbol{\Theta}\rangle$$

$$\langle\boldsymbol{F}^{n+1}\rangle - \left\langle\overline{\boldsymbol{F}}^{n+1}\right\rangle = (n+1)\int_0^1 dt\langle\mathrm{D}_t(\boldsymbol{F}_t^n\boldsymbol{\Theta})\rangle$$

$$\langle\boldsymbol{F}^{n+1}\rangle - \left\langle\overline{\boldsymbol{F}}^{n+1}\right\rangle = (n+1)\mathrm{d}\int_0^1 dt\langle\boldsymbol{F}_t^n\boldsymbol{\Theta}\rangle$$

$$\langle\boldsymbol{F}^{n+1}\rangle - \left\langle\overline{\boldsymbol{F}}^{n+1}\right\rangle = \mathrm{d}Q_{\boldsymbol{A}\leftarrow\overline{\boldsymbol{A}}}^{(2n+1)}$$

$$\langle\boldsymbol{F}^{n+1}\rangle = \mathrm{d}Q_{\boldsymbol{A}\leftarrow 0}^{(2n+1)}$$

$$S_{\mathrm{T}}[\boldsymbol{A},\overline{\boldsymbol{A}}] = k\int_M Q_{\boldsymbol{A}\leftarrow\overline{\boldsymbol{A}}}^{(2n+1)}$$

$$\delta_{\mathrm{dif}}\boldsymbol{A} = -\pounds_\xi\boldsymbol{A}$$

$$\delta_{\mathrm{dif}}\overline{\boldsymbol{A}} = -\pounds_\xi\overline{\boldsymbol{A}}$$

$$\delta_{\mathrm{gauge}}\boldsymbol{A} = -\mathrm{D}\boldsymbol{\lambda}$$

$$\delta_{\mathrm{gauge}}\overline{\boldsymbol{A}} = -\overline{\mathrm{D}}\boldsymbol{\lambda}$$

$$S_{\mathrm{T}}^{(2n+1)}[\overline{\boldsymbol{A}},\boldsymbol{A}] = -S_{\mathrm{T}}^{(2n+1)}[\boldsymbol{A},\overline{\boldsymbol{A}}]$$

$$\begin{aligned}\boldsymbol{\Theta} &\rightarrow -\boldsymbol{\Theta} \\ \boldsymbol{A}_t &\rightarrow \boldsymbol{A}_{1-t} \\ \boldsymbol{F}_t &\rightarrow \boldsymbol{F}_{1-t}\end{aligned}$$

$$\int_0^1 f(t)dt = \int_0^1 f(1-t)dt$$

$$\delta S_{\mathrm{T}}^{(2n+1)} = (n+1)k\int_M \left(\langle\delta\boldsymbol{A}\boldsymbol{F}^n\rangle - \left\langle\delta\overline{\boldsymbol{A}}\overline{\boldsymbol{F}}^n\right\rangle\right) + \int_{\partial M} \Xi$$

$$\Xi \equiv n(n+1)k\int_0^1 dt\langle\delta\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle$$

$$Q_{\boldsymbol{A}\leftarrow\overline{\boldsymbol{A}}}^{(2n+1)} = (n+1)\int_0^1 dt\langle\boldsymbol{\Theta}\boldsymbol{F}_t^n\rangle$$

$$\begin{aligned}\delta\boldsymbol{\Theta} &= \delta\boldsymbol{A} - \delta\overline{\boldsymbol{A}} \\ \delta\boldsymbol{A}_t &= \delta\overline{\boldsymbol{A}} + t\delta\boldsymbol{\Theta} \\ \delta\boldsymbol{F}_t &= \mathrm{D}_t\delta\boldsymbol{A}_t\end{aligned}$$

$$\delta Q_{\boldsymbol{A}\leftarrow\overline{\boldsymbol{A}}}^{(2n+1)} = (n+1)\int_0^1 dt\langle\delta\boldsymbol{\Theta}\boldsymbol{F}_t^n\rangle + n(n+1)\int_0^1 dt\langle\boldsymbol{\Theta}\mathrm{D}_t\delta\boldsymbol{A}_t\boldsymbol{F}_t^{n-1}\rangle$$

$$\langle\boldsymbol{\Theta}\mathrm{D}_t\delta\boldsymbol{A}_t\boldsymbol{F}_t^{n-1}\rangle = \langle\mathrm{D}_t\boldsymbol{\Theta}\delta\boldsymbol{A}_t\boldsymbol{F}_t^{n-1}\rangle + \mathrm{d}\langle\delta\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle$$

$$\begin{aligned}\frac{d}{dt}\boldsymbol{F}_t &= \mathrm{D}_t\boldsymbol{\Theta} \\ \frac{d}{dt}\delta\boldsymbol{A}_t &= \delta\boldsymbol{\Theta}\end{aligned}$$

$$n\langle\mathrm{D}_t\boldsymbol{\Theta}\delta\boldsymbol{A}_t\boldsymbol{F}_t^{n-1}\rangle = \frac{d}{dt}\langle\delta\boldsymbol{A}_t\boldsymbol{F}_t^n\rangle - \langle\delta\boldsymbol{\Theta}\boldsymbol{F}_t^n\rangle$$

$$n\langle\boldsymbol{\Theta}\mathrm{D}_t\delta\boldsymbol{A}_t\boldsymbol{F}_t^{n-1}\rangle = \frac{d}{dt}\langle\delta\boldsymbol{A}_t\boldsymbol{F}_t^n\rangle - \langle\delta\boldsymbol{\Theta}\boldsymbol{F}_t^n\rangle + n\,\mathrm{d}\langle\delta\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle$$

$$\delta Q_{\boldsymbol{A}\leftarrow\overline{\boldsymbol{A}}}^{(2n+1)} = (n+1)\int_0^1 dt\frac{d}{dt}\langle\delta\boldsymbol{A}_t\boldsymbol{F}_t^n\rangle + n(n+1)\mathrm{d}\int_0^1 dt\langle\delta\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle$$

$$\delta Q_{\boldsymbol{A}\leftarrow\overline{\boldsymbol{A}}}^{(2n+1)} = (n+1)\left(\langle\delta\boldsymbol{A}\boldsymbol{F}^n\rangle - \left\langle\delta\overline{\boldsymbol{A}}\overline{\boldsymbol{F}}^n\right\rangle\right) + n(n+1)\mathrm{d}\int_0^1 dt\langle\delta\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle$$

$$\begin{aligned}\langle\boldsymbol{F}^n\boldsymbol{G}_A\rangle &= 0 \\ \left\langle\overline{\boldsymbol{F}}^n\boldsymbol{G}_A\right\rangle &= 0\end{aligned}$$

$$\left.\int_0^1 dt\langle\delta\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle\right|_{\partial M}$$

$$\mathrm{d}\star J = 0$$

$$\star J_{\mathrm{gauge}} = n(n+1)k\,\mathrm{d}\int_0^1 dt\langle\boldsymbol{\lambda}\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle$$

$$\star J_{\mathrm{dif}} = n(n+1)k\,\mathrm{d}\int_0^1 dt\left\langle\mathrm{I}_\xi\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\right\rangle$$

$$Q_{\mathrm{gauge}}(\boldsymbol{\lambda}) = n(n+1)k\int_{\partial\Sigma}\int_0^1 dt\langle\boldsymbol{\lambda}\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle$$

$$Q_{\mathrm{dif}}(\xi) = n(n+1)k\int_{\partial\Sigma}\int_0^1 dt\left\langle\mathrm{I}_\xi\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\right\rangle$$

$$\begin{aligned}\delta_{\boldsymbol{\lambda}}Q_{\mathrm{gauge}}(\boldsymbol{\eta}) &= -Q_{\mathrm{gauge}}([\boldsymbol{\lambda},\boldsymbol{\eta}]) \\ \delta_{\boldsymbol{\lambda}}Q_{\mathrm{dif}}(\xi) &= -Q_{\mathrm{gauge}}(\pounds_\xi\boldsymbol{\lambda})\end{aligned}$$

$$\{Q_{\boldsymbol{\eta}}, Q_{\boldsymbol{\lambda}}\} = Q_{[\boldsymbol{\eta},\lambda]}$$

$$\mathrm{C}\left(S_{\mathrm{T}}^{(2n+1)}\right) = -S_{\mathrm{T}}^{(2n+1)}$$

$$\mathrm{PT}\left(S_{\mathrm{T}}^{(2n+1)}\right) = -S_{\mathrm{T}}^{(2n+1)}$$

$$\mathrm{CPT}\left(S_{\mathrm{T}}^{(2n+1)}\right) = S_{\mathrm{T}}^{(2n+1)}$$

$$\delta L_{\mathrm{T}}^{(2n+1)} = (n+1)k\left(\langle \delta \boldsymbol{A}\boldsymbol{F}^n\rangle - \left\langle \delta \overline{\boldsymbol{A}}\overline{\boldsymbol{F}}^n\right\rangle\right) + \mathrm{d}\Xi$$

$$\Xi = n(n+1)k\int_0^1 dt\langle \delta \boldsymbol{A}_t \boldsymbol{\Theta} \boldsymbol{F}_t^{n-1}\rangle$$

$$\delta_{\mathrm{gauge}}\, \boldsymbol{A} = -\mathrm{D}\boldsymbol{\lambda}$$

$$\delta_{\mathrm{gauge}}\, \overline{\boldsymbol{A}} = -\overline{\mathrm{D}}\boldsymbol{\lambda}$$

$$\delta_{\mathrm{gauge}}\, L_{\mathrm{T}}^{(2n+1)} = -(n+1)k\,\mathrm{d}\left(\langle \boldsymbol{\lambda}\boldsymbol{F}^n\rangle - \left\langle \boldsymbol{\lambda}\overline{\boldsymbol{F}}^n\right\rangle\right) + \mathrm{d}\Xi_{\mathrm{gauge}}$$

$$\star J'_{\mathrm{gauge}} \equiv (n+1)k\left(\langle \boldsymbol{\lambda}\boldsymbol{F}^n\rangle - \left\langle \boldsymbol{\lambda}\overline{\boldsymbol{F}}^n\right\rangle\right) - \Xi_{\mathrm{gauge}}$$

$$\Xi_{\mathrm{gauge}} = -n(n+1)k\,\mathrm{d}\int_0^1 dt\langle \boldsymbol{\lambda}\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle + (n+1)k\left(\langle \boldsymbol{\lambda}\boldsymbol{F}^n\rangle - \left\langle \boldsymbol{\lambda}\overline{\boldsymbol{F}}^n\right\rangle\right)$$

$$\star J'_{\mathrm{gauge}} = n(n+1)k\,\mathrm{d}\int_0^1 dt\langle \boldsymbol{\lambda}\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle$$

$$\delta_{\mathrm{dif}}\boldsymbol{A} = -\pounds_{\xi}\boldsymbol{A}$$

$$\delta_{\mathrm{dif}}\overline{\boldsymbol{A}} = -\pounds_{\xi}\overline{\boldsymbol{A}}$$

$$\delta_{\mathrm{dif}} L_{\mathrm{T}}^{(2n+1)} = -(n+1)k\left(\langle \pounds_{\xi}\boldsymbol{A}\boldsymbol{F}^n\rangle - \left\langle \pounds_{\xi}\overline{\boldsymbol{A}}\overline{\boldsymbol{F}}^n\right\rangle\right) + \mathrm{d}\Xi_{\mathrm{dif}}$$

$$\pounds_{\xi}\boldsymbol{A} = \mathrm{I}_{\xi}\boldsymbol{F} + \mathrm{D}\mathrm{I}_{\xi}\boldsymbol{A}$$

$$\langle \pounds_{\xi}\boldsymbol{A}\boldsymbol{F}^n\rangle = \langle \mathrm{I}_{\xi}\boldsymbol{F}\boldsymbol{F}^n\rangle + \langle \mathrm{D}\mathrm{I}_{\xi}\boldsymbol{A}\boldsymbol{F}^n\rangle$$

$$\langle \pounds_{\xi}\boldsymbol{A}\boldsymbol{F}^n\rangle = \mathrm{d}\langle \mathrm{I}_{\xi}\boldsymbol{A}\boldsymbol{F}^n\rangle$$

$$\delta_{\mathrm{dif}} L_{\mathrm{T}}^{(2n+1)} = -(n+1)k\,\mathrm{d}\left(\langle \mathrm{I}_{\xi}\boldsymbol{A}\boldsymbol{F}^n\rangle - \left\langle \mathrm{I}_{\xi}\overline{\boldsymbol{A}}\overline{\boldsymbol{F}}^n\right\rangle\right) + \mathrm{d}\Xi_{\mathrm{dif}}$$

$$\begin{aligned}\delta_{\mathrm{dif}}\, L_{\mathrm{T}}^{(2n+1)} &= -\pounds_{\xi} L_{\mathrm{T}}^{(2n+1)} \\ &= -\mathrm{d}\mathrm{I}_{\xi} L_{\mathrm{T}}^{(2n+1)},\end{aligned}$$

$$\star J'_{\mathrm{dif}} \equiv (n+1)k\left(\langle \mathrm{I}_{\xi}\boldsymbol{A}\boldsymbol{F}^n\rangle - \left\langle \mathrm{I}_{\xi}\overline{\boldsymbol{A}}\overline{\boldsymbol{F}}^n\right\rangle\right) - \Xi_{\mathrm{dif}} - \mathrm{I}_{\xi} L_{\mathrm{T}}^{(2n+1)}$$

$$\begin{aligned}\Xi_{\mathrm{dif}} + \mathrm{I}_{\xi} L_{\mathrm{T}}^{(2n+1)} &= -n(n+1)k\,\mathrm{d}\int_0^1 dt\langle \mathrm{I}_{\xi}\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle + \\ &+(n+1)k\left(\langle \mathrm{I}_{\xi}\boldsymbol{A}\boldsymbol{F}^n\rangle - \left\langle \mathrm{I}_{\xi}\overline{\boldsymbol{A}}\overline{\boldsymbol{F}}^n\right\rangle\right)\end{aligned}$$

$$\star J'_{\mathrm{dif}} = n(n+1)k\,\mathrm{d}\int_0^1 dt\langle \mathrm{I}_{\xi}\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle$$

$$S_{\mathrm{T}}^{(2n+1)}[\boldsymbol{A},\overline{\boldsymbol{A}}] = S_{\mathrm{CS}}^{(2n+1)}[\boldsymbol{A}] - S_{\mathrm{CS}}^{(2n+1)}[\overline{\boldsymbol{A}}] + \int_{\partial M}\mathcal{B}^{(2n)}$$

$$-S_{\mathrm{CS}}^{(2n+1)}[\overline{\boldsymbol{A}}] = -\int_M L_{\mathrm{CS}}^{(2n+1)}(\overline{\boldsymbol{A}}) = \int_{-M} L_{\mathrm{CS}}^{(2n+1)}(\overline{\boldsymbol{A}})$$

$$L_{\mathrm{T}}^{(2n+1)}(\boldsymbol{A},\overline{\boldsymbol{A}}) = (n+1)k\int_0^1 dt\langle \boldsymbol{\Theta}\boldsymbol{F}_t^n\rangle$$

$$\mathrm{d}\mathcal{Q}^{(2n+1)}_{\boldsymbol{A}\leftarrow\bar{\boldsymbol{A}}} + \mathrm{d}\mathcal{Q}^{(2n+1)}_{\tilde{\boldsymbol{A}}\leftarrow\tilde{\boldsymbol{A}}} + \mathrm{d}\mathcal{Q}^{(2n+1)}_{\tilde{\boldsymbol{A}}\leftarrow\boldsymbol{A}} = 0$$

$$\begin{aligned}\mathrm{d}\mathcal{Q}^{(2n+1)}_{\boldsymbol{A}\leftarrow\bar{\boldsymbol{A}}} + \mathrm{d}\mathcal{Q}^{(2n+1)}_{\bar{\boldsymbol{A}}\leftarrow\tilde{\boldsymbol{A}}} + \mathrm{d}\mathcal{Q}^{(2n+1)}_{\tilde{\boldsymbol{A}}\leftarrow\boldsymbol{A}} =& \langle \boldsymbol{F}^{n+1}\rangle - \left\langle \overline{\boldsymbol{F}}^{n+1}\right\rangle + \left\langle \overline{\boldsymbol{F}}^{n+1}\right\rangle + \\ & -\left\langle \tilde{\boldsymbol{F}}^{n+1}\right\rangle + \left\langle \tilde{\boldsymbol{F}}^{n+1}\right\rangle - \langle \boldsymbol{F}^{n+1}\rangle \\ =& 0.\end{aligned}$$

$$\mathcal{Q}^{(2n+1)}_{\boldsymbol{A}\leftarrow\bar{\boldsymbol{A}}} + \mathcal{Q}^{(2n+1)}_{\bar{\boldsymbol{A}}\leftarrow\tilde{\boldsymbol{A}}} + \mathcal{Q}^{(2n+1)}_{\tilde{\boldsymbol{A}}\leftarrow\boldsymbol{A}} = \mathrm{d}\mathcal{Q}^{(2n)}_{\boldsymbol{A}\leftarrow\tilde{\boldsymbol{A}}\leftarrow\bar{\boldsymbol{A}}}$$

$$\mathcal{Q}^{(2n+1)}_{\boldsymbol{A}\leftarrow\bar{\boldsymbol{A}}} = \mathcal{Q}^{(2n+1)}_{\boldsymbol{A}\leftarrow\tilde{\boldsymbol{A}}} + \mathcal{Q}^{(2n+1)}_{\tilde{\boldsymbol{A}}\leftarrow\bar{\boldsymbol{A}}} + \mathrm{d}\mathcal{Q}^{(2n)}_{\boldsymbol{A}\leftarrow\tilde{\boldsymbol{A}}\leftarrow\bar{\boldsymbol{A}}}$$

$$t^i \geq 0, i = 0, \dots, r+1$$

$$\sum_{i=0}^{r+1} t^i = 1$$

$$\boldsymbol{A}_t = \sum_{i=0}^{r+1} t^i \boldsymbol{A}_i$$

$$\boldsymbol{F}_t = \mathrm{d}\boldsymbol{A}_t + \boldsymbol{A}_t^2$$

$$T_{r+1} = (\boldsymbol{A}_0 \boldsymbol{A}_1 \cdots \boldsymbol{A}_{r+1})$$

$$\mathrm{d}: \Omega^a(M) \times \Omega^b(T_{r+1}) \to \Omega^{a+1}(M) \times \Omega^b(T_{r+1})$$

$$\mathrm{d}_t: \Omega^a(M) \times \Omega^b(T_{r+1}) \to \Omega^a(M) \times \Omega^{b+1}(T_{r+1})$$

$$l_t: \Omega^a(M) \times \Omega^b(T_{r+1}) \to \Omega^{a-1}(M) \times \Omega^{b+1}(T_{r+1})$$

$$l_t \boldsymbol{A}_t = 0$$

$$l_t \boldsymbol{F}_t = \mathrm{d}_t \boldsymbol{A}_t$$

$$\mathrm{d}^2 = 0$$

$$\mathrm{d}_t^2 = 0$$

$$[l_t, \mathrm{d}] = \mathrm{d}_t$$

$$[l_t, \mathrm{d}_t] = 0$$

$$\{\mathrm{d}, \mathrm{d}_t\} = 0$$

$$\int_{\partial T_{r+1}} \frac{l_t^p}{p!}\pi = \int_{T_{r+1}} \frac{l_t^{p+1}}{(p+1)!}\mathrm{d}\pi + (-1)^{p+q}\,\mathrm{d}\int_{T_{r+1}} \frac{l_t^{p+1}}{(p+1)!}\pi$$

$$\pi = \sum_p \alpha_p \left\langle \boldsymbol{A}_t^{a_p} \boldsymbol{F}_t^{b_p} (\mathrm{d}_t \boldsymbol{A}_t)^{c_p} (\mathrm{d}_t \boldsymbol{F}_t)^{d_p} \right\rangle$$

$$a_p + 2b_p + c_p + 2d_p = m$$

$$c_p + d_p = q$$

$$(p+1)\mathrm{d}_t t_t^p \pi = l_t^{p+1}\,\mathrm{d}\pi - \mathrm{d}l_t^{p+1}\pi$$

$$\left[l_t^{p+1}, \mathrm{d}\right] = (p+1)\mathrm{d}_t l_t^p$$

$$\begin{aligned}[l_t^2, \mathrm{d}] &= l_t[l_t, \mathrm{d}] + [l_t, \mathrm{d}]l_t \\ &= l_t\,\mathrm{d}_t + \mathrm{d}_t l_t \\ &= 2\,\mathrm{d}_t l_t.\end{aligned}$$

$$\begin{aligned}[l_t^{k+2}, \mathrm{d}] &= l_t[l_t^{k+1}, \mathrm{d}] + [l_t, \mathrm{d}]l_t^{k+1} \\ &= (k+1)l_t\,\mathrm{d}_t l_t^k + \mathrm{d}_t l_t^{k+1} \\ &= (k+2)\mathrm{d}_t l_t^{k+1}\end{aligned}$$

$$(p+1)\int_{\partial T_{r+1}} l_t^p \pi = \int_{T_{r+1}} l_t^{p+1}\,\mathrm{d}\pi - \int_{T_{r+1}} \mathrm{d}l_t^{p+1}$$

$$\mathrm{d}\int_{T_s} \alpha = (-1)^s \int_{T_s} \mathrm{d}\alpha$$

$$(p+1)\int_{\partial T_{r+1}} l_t^p \pi = \int_{T_{r+1}} l_t^{p+1}\, \mathrm{d}\pi + (-1)^{p+q}\, \mathrm{d}\int_{T_{r+1}} l_t^{p+1}\pi$$
$$\pi = \langle \boldsymbol{F}_t^{n+1} \rangle$$
$$\int_{\partial T_{p+1}} \frac{l_t^p}{p!}\langle \boldsymbol{F}_t^{n+1} \rangle = (-1)^p\, \mathrm{d}\int_{T_{p+1}} \frac{l_t^{p+1}}{(p+1)!}\langle \boldsymbol{F}_t^{n+1} \rangle$$
$$\int_{\partial T_1} \langle \boldsymbol{F}_t^{n+1} \rangle = \mathrm{d}\int_{T_1} l_t \langle \boldsymbol{F}_t^{n+1} \rangle$$
$$\boldsymbol{A}_t = t^0 \boldsymbol{A}_0 + t^1 \boldsymbol{A}_1$$
$$\partial T_1 = (\boldsymbol{A}_1) - (\boldsymbol{A}_0)$$
$$\int_{\partial T_1} \langle \boldsymbol{F}_t^{n+1} \rangle = \langle \boldsymbol{F}_1^{n+1} \rangle - \langle \boldsymbol{F}_0^{n+1} \rangle$$
$$l_t \langle \boldsymbol{F}_t^{n+1} \rangle = (n+1)\langle (l_t \boldsymbol{F}_t)\boldsymbol{F}_t^n \rangle$$
$$\begin{aligned} l_t \boldsymbol{F}_t &= \mathrm{d}_t \boldsymbol{A}_t \\ &= \mathrm{d}t^0 \boldsymbol{A}_0 + \mathrm{d}t^1 \boldsymbol{A}_1 \\ &= \mathrm{d}t^1 (\boldsymbol{A}_1 - \boldsymbol{A}_0). \end{aligned}$$
$$\langle \boldsymbol{F}_1^{n+1} \rangle - \langle \boldsymbol{F}_0^{n+1} \rangle = (n+1)\mathrm{d}\int_{T_1} \mathrm{d}t^1 \langle (\boldsymbol{A}_1 - \boldsymbol{A}_0)\boldsymbol{F}_t^n \rangle$$
$$\langle \boldsymbol{F}_1^{n+1} \rangle - \langle \boldsymbol{F}_0^{n+1} \rangle = \mathrm{d}Q^{(2n+1)}_{\boldsymbol{A}_1 \leftarrow \boldsymbol{A}_0}$$
$$\begin{aligned} Q^{(2n+1)}_{\boldsymbol{A}_1 \leftarrow \boldsymbol{A}_0} &= \int_{(\boldsymbol{A}_0 \boldsymbol{A}_1)} l_t \langle \boldsymbol{F}_t^{n+1} \rangle \\ &= (n+1)\int_0^1 \mathrm{d}t^1 \langle (\boldsymbol{A}_1 - \boldsymbol{A}_0)\boldsymbol{F}_t^n \rangle. \end{aligned}$$
$$\int_{\partial T_2} l_t \langle \boldsymbol{F}_t^{n+1} \rangle = -\mathrm{d}\int_{T_2} \frac{l_t^2}{2}\langle \boldsymbol{F}_t^{n+1} \rangle$$
$$\boldsymbol{A}_t = t^0 \boldsymbol{A}_0 + t^1 \boldsymbol{A}_1 + t^2 \boldsymbol{A}_2$$
$$\partial T_2 = (\boldsymbol{A}_1 \boldsymbol{A}_2) - (\boldsymbol{A}_0 \boldsymbol{A}_2) + (\boldsymbol{A}_0 \boldsymbol{A}_1)$$
$$\int_{\partial T_2} l_t \langle \boldsymbol{F}_t^{n+1} \rangle = \int_{(\boldsymbol{A}_1 \boldsymbol{A}_2)} l_t \langle \boldsymbol{F}_t^{n+1} \rangle - \int_{(\boldsymbol{A}_0 \boldsymbol{A}_2)} l_t \langle \boldsymbol{F}_t^{n+1} \rangle + \int_{(\boldsymbol{A}_0 \boldsymbol{A}_1)} l_t \langle \boldsymbol{F}_t^{n+1} \rangle$$
$$\int_{\partial T_2} l_t \langle \boldsymbol{F}_t^{n+1} \rangle = Q^{(2n+1)}_{\boldsymbol{A}_2 \leftarrow \boldsymbol{A}_1} - Q^{(2n+1)}_{\boldsymbol{A}_2 \leftarrow \boldsymbol{A}_0} + Q^{(2n+1)}_{\boldsymbol{A}_1 \leftarrow \boldsymbol{A}_0}$$
$$l_t^2 \langle \boldsymbol{F}_t^{n+1} \rangle = n(n+1)\langle (\mathrm{d}_t \boldsymbol{A}_t)^2 \boldsymbol{F}_t^{n-1} \rangle$$
$$\int_{T_2} \frac{l_t^2}{2}\langle \boldsymbol{F}_t^{n+1} \rangle = Q^{(2n)}_{\boldsymbol{A}_2 \leftarrow \boldsymbol{A}_1 \leftarrow \boldsymbol{A}_0}$$
$$Q^{(2n)}_{\boldsymbol{A}_2 \leftarrow \boldsymbol{A}_1 \leftarrow \boldsymbol{A}_0} \equiv n(n+1)\int_0^1 dt \int_0^t ds \langle (\boldsymbol{A}_2 - \boldsymbol{A}_1)(\boldsymbol{A}_1 - \boldsymbol{A}_0)\boldsymbol{F}_t^{n-1} \rangle$$
$$\boldsymbol{A}_t = \boldsymbol{A}_0 + s(\boldsymbol{A}_2 - \boldsymbol{A}_1) + t(\boldsymbol{A}_1 - \boldsymbol{A}_0)$$
$$Q^{(2n+1)}_{\boldsymbol{A}_2 \leftarrow \boldsymbol{A}_1} - Q^{(2n+1)}_{\boldsymbol{A}_2 \leftarrow \boldsymbol{A}_0} + Q^{(2n+1)}_{\boldsymbol{A}_1 \leftarrow \boldsymbol{A}_0} = -\mathrm{d}Q^{(2n)}_{\boldsymbol{A}_2 \leftarrow \boldsymbol{A}_1 \leftarrow \boldsymbol{A}_0}$$
$$Q^{(2n+1)}_{\boldsymbol{A}_2 \leftarrow \boldsymbol{A}_0} = Q^{(2n+1)}_{\boldsymbol{A}_2 \leftarrow \boldsymbol{A}_1} + Q^{(2n+1)}_{\boldsymbol{A}_1 \leftarrow \boldsymbol{A}_0} + \mathrm{d}Q^{(2n)}_{\boldsymbol{A}_2 \leftarrow \boldsymbol{A}_1 \leftarrow \boldsymbol{A}_0}$$
$$L_{\mathrm{T}}^{(2n+1)}(\boldsymbol{A}, \overline{\boldsymbol{A}}) = kQ^{(2n+1)}_{\boldsymbol{A} \leftarrow \overline{\boldsymbol{A}}}$$
$$Q^{(2n+1)}_{\boldsymbol{A} \leftarrow \overline{\boldsymbol{A}}} = Q^{(2n+1)}_{\boldsymbol{A} \leftarrow \tilde{\boldsymbol{A}}} + Q^{(2n+1)}_{\tilde{\boldsymbol{A}} \leftarrow \overline{\boldsymbol{A}}} + \mathrm{d}Q^{(2n)}_{\boldsymbol{A} \leftarrow \tilde{\boldsymbol{A}} \leftarrow \overline{\boldsymbol{A}}}$$
$$Q^{(2n)}_{\boldsymbol{A} \leftarrow \tilde{\boldsymbol{A}} \leftarrow \overline{\boldsymbol{A}}} \equiv n(n+1)\int_0^1 dt \int_0^t ds \big\langle (\boldsymbol{A} - \tilde{\boldsymbol{A}})(\tilde{\boldsymbol{A}} - \overline{\boldsymbol{A}})\boldsymbol{F}_{st}^{n-1} \big\rangle$$
$$\boldsymbol{A}_{st} = \overline{\boldsymbol{A}} + s(\boldsymbol{A} - \tilde{\boldsymbol{A}}) + t(\tilde{\boldsymbol{A}} - \overline{\boldsymbol{A}})$$

$$\boldsymbol{A} = \boldsymbol{a}_0 + \boldsymbol{a}_1,$$
$$\overline{\boldsymbol{A}} = \overline{\boldsymbol{a}}_0 + \overline{\boldsymbol{a}}_1$$
$$\mathcal{Q}^{(2+1}_{\boldsymbol{a}_0+\boldsymbol{a}_1\leftarrow\overline{\boldsymbol{a}}_0+\overline{\boldsymbol{a}}_1} = \mathcal{Q}^{(2+1)}_{\boldsymbol{a}_0+\boldsymbol{a}_1\leftarrow\boldsymbol{a}_0} + \mathcal{Q}^{(2n+1)}_{\boldsymbol{a}_0\leftarrow\overline{\boldsymbol{a}}_0+\overline{\boldsymbol{a}}_1} + \mathrm{d}\mathcal{Q}^{(2n)}_{\boldsymbol{a}_0+\boldsymbol{a}_1\leftarrow\boldsymbol{a}_0\leftarrow\overline{\boldsymbol{a}}_0+\overline{\boldsymbol{a}}_1}$$
$$\mathcal{Q}^{(2n+1)}_{a_0\leftarrow\overline{\boldsymbol{a}}_0+\bar{a}_1} = \mathcal{Q}^{(2n+1)}_{a_0\leftarrow\overline{\boldsymbol{a}}_0} + \mathcal{Q}^{(2n+1)}_{\overline{\boldsymbol{a}}_0\leftarrow\overline{\boldsymbol{a}}_0+\overline{\boldsymbol{a}}_1} + \mathrm{d}\mathcal{Q}^{(2n)}_{a_0\leftarrow\overline{\boldsymbol{a}}_0\leftarrow\bar{a}_0+\overline{\boldsymbol{a}}_1}$$
$$\mathcal{Q}^{(2n+1)}_{\boldsymbol{a}_0+\boldsymbol{a}_1\leftarrow\overline{\boldsymbol{a}}_0+\overline{\boldsymbol{a}}_1} = \mathcal{Q}^{(2n+1)}_{\boldsymbol{a}_0+\boldsymbol{a}_1\leftarrow\boldsymbol{a}_0} + \mathcal{Q}^{(2n+1)}_{\overline{\boldsymbol{a}}_0\leftarrow\overline{\boldsymbol{a}}_0+\overline{\boldsymbol{a}}_1} + \mathcal{Q}^{(2n+1)}_{\boldsymbol{a}_0\leftarrow\overline{\boldsymbol{a}}_0} +$$
$$+\mathrm{d}\mathcal{Q}^{(2n)}_{\boldsymbol{a}_0\leftarrow\overline{\boldsymbol{a}}_0\leftarrow\overline{\boldsymbol{a}}_0+\overline{\boldsymbol{a}}_1} + \mathrm{d}\mathcal{Q}^{(2n)}_{a_0+\boldsymbol{a}_1\leftarrow a_0\leftarrow\overline{\boldsymbol{a}}_0+\overline{\boldsymbol{a}}_1}$$
$$\mathcal{Q}^{(2n+1)}_{\boldsymbol{a}_0+\boldsymbol{a}_1\leftarrow\overline{\boldsymbol{a}}_0+\overline{\boldsymbol{a}}_1} = \mathcal{Q}^{(2n+1)}_{\boldsymbol{a}_0+\boldsymbol{a}_1\leftarrow\boldsymbol{a}_0} - \mathcal{Q}^{(2n+1)}_{\overline{\boldsymbol{a}}_0+\overline{\boldsymbol{a}}_1\leftarrow\overline{\boldsymbol{a}}_0} + \mathcal{Q}^{(2n+1)}_{\boldsymbol{a}_0\leftarrow\overline{\boldsymbol{a}}_0} +$$
$$+\mathrm{d}\mathcal{Q}^{(2n)}_{\boldsymbol{a}_0\leftarrow\overline{\boldsymbol{a}}_0\leftarrow\overline{\boldsymbol{a}}_0+\overline{\boldsymbol{a}}_1} + \mathrm{d}\mathcal{Q}^{(2n}_{\boldsymbol{a}_0+\boldsymbol{a}_1\leftarrow a_0\leftarrow\overline{\boldsymbol{a}}_0+\overline{\boldsymbol{a}}_1}.$$
$$\mathcal{Q}^{(2n+1)}_{\boldsymbol{A}\leftarrow\overline{\boldsymbol{A}}} = \mathcal{Q}^{(2n+1)}_{\boldsymbol{A}\leftarrow\tilde{\boldsymbol{A}}} + \mathcal{Q}^{(2n+1)}_{\tilde{\boldsymbol{A}}\leftarrow\overline{\boldsymbol{A}}} + \mathrm{d}\mathcal{Q}^{(2n)}_{\boldsymbol{A}\leftarrow\tilde{\boldsymbol{A}}\leftarrow\overline{\boldsymbol{A}}}$$
$$\mathcal{Q}^{(2n+1)}_{\mathrm{CS}}(\boldsymbol{A}) = \mathcal{Q}^{(2n+1)}_{\boldsymbol{A}\leftarrow 0}$$
$$\mathcal{Q}^{(2n+1)}_{\boldsymbol{A}\leftarrow\overline{\boldsymbol{A}}} = \mathcal{Q}^{(2n+1)}_{\mathrm{CS}}(\boldsymbol{A}) - \mathcal{Q}^{(2n+1)}_{\mathrm{CS}}(\overline{\boldsymbol{A}}) + \mathrm{d}\mathcal{B}^{(2n)}$$
$$\mathcal{B}^{(2n)} = -n(n+1)\int_0^1 dt \int_0^t ds\left\langle \boldsymbol{A}\overline{\boldsymbol{A}}\boldsymbol{F}_{st}^{n-1}\right\rangle$$
$$\boldsymbol{A}_{st} = s\boldsymbol{A} + (1-t)\overline{\boldsymbol{A}}$$
$$\boldsymbol{F}_{st} = \overline{\boldsymbol{F}} + \overline{\mathrm{D}}(s\boldsymbol{A} - t\overline{\boldsymbol{A}}) + (s\boldsymbol{A} - t\overline{\boldsymbol{A}})^2$$
$$[\boldsymbol{P}_a, \boldsymbol{P}_b] = \boldsymbol{J}_{ab}$$
$$[\boldsymbol{J}_{ab}, \boldsymbol{P}_c] = \eta_{cb}\boldsymbol{P}_a - \eta_{ca}\boldsymbol{P}_b$$
$$[\boldsymbol{J}_{ab}, \boldsymbol{J}_{cd}] = \eta_{cb}\boldsymbol{J}_{ad} - \eta_{ca}\boldsymbol{J}_{bd} + \eta_{db}\boldsymbol{J}_{ca} - \eta_{da}\boldsymbol{J}_{cb}$$
$$\left\langle \boldsymbol{J}_{a_1a_2}\cdots\boldsymbol{J}_{a_{2n-1}a_{2n}}\boldsymbol{P}_{a_{2n+1}}\right\rangle = \frac{2^n}{n+1}\varepsilon_{a_1\cdots a_{2n+1}}$$
$$L^{(2n+1)}_{\mathrm{G}} = k\mathcal{Q}^{(2n+1)}_{\boldsymbol{A}\leftarrow\overline{\boldsymbol{A}}}$$
$$\bar{A} = \bar{\omega}$$
$$A = e + \omega$$
$$\overline{\boldsymbol{F}} = \overline{\boldsymbol{R}}$$
$$\boldsymbol{F} = \boldsymbol{R} + e^2 + \boldsymbol{T}$$
$$\boldsymbol{R} = \mathrm{d}\boldsymbol{\omega} + \boldsymbol{\omega}^2$$
$$\boldsymbol{T} = \mathrm{d}\boldsymbol{e} + [\boldsymbol{\omega}, \boldsymbol{e}]$$
$$\tilde{\boldsymbol{A}} = \omega$$
$$\mathcal{Q}^{(2n+1)}_{e+\omega\leftarrow\bar{\omega}} = \mathcal{Q}^{(2n+1)}_{e+\omega\leftarrow\omega} + \mathcal{Q}^{(2n+1)}_{\omega\leftarrow\bar{\omega}} + \mathrm{d}\mathcal{Q}^{(2n)}_{e+\omega\leftarrow\omega\leftarrow\bar{\omega}}$$
$$\mathcal{Q}^{(2n+1)}_{e+\omega\leftarrow\boldsymbol{\omega}} = (n+1)\int_0^1 dt\langle \boldsymbol{e}\boldsymbol{F}_t^n\rangle$$
$$\boldsymbol{A}_t = \boldsymbol{\omega} + t\boldsymbol{e}$$
$$\boldsymbol{F}_t = \boldsymbol{R} + t^2\boldsymbol{e}^2 + t\boldsymbol{T}$$
$$\mathcal{Q}^{(2n+1)}_{\boldsymbol{e}+\boldsymbol{\omega}\leftarrow\boldsymbol{\omega}} = (n+1)\int_0^1 dt\langle \boldsymbol{e}(\boldsymbol{R} + t^2\boldsymbol{e}^2)^n\rangle$$
$$\mathcal{Q}^{(2n+1)}_{\omega\leftarrow\bar{\omega}} = 0$$
$$\mathcal{Q}^{(2n)}_{e+\omega\leftarrow\omega\leftarrow\bar{\omega}} = n(n+1)\int_0^1 dt\int_0^t ds\langle \boldsymbol{e}\boldsymbol{\theta}\boldsymbol{F}_{st}^{n-1}\rangle$$
$$\theta \equiv \omega - \bar{\omega}$$
$$\boldsymbol{A}_{st} = \overline{\boldsymbol{\omega}} + s\boldsymbol{e} + t\boldsymbol{\theta}$$
$$\boldsymbol{F}_{st} = \overline{\boldsymbol{R}} + \mathrm{D}_{\overline{\boldsymbol{\omega}}}(s\boldsymbol{e} + t\boldsymbol{\theta}) + s^2\boldsymbol{e}^2 + st[\boldsymbol{e}, \boldsymbol{\theta}] + t^2\boldsymbol{\theta}^2$$

$$\mathcal{Q}^{(2n)}_{\boldsymbol{e}+\boldsymbol{\omega}\leftarrow\boldsymbol{\omega}\leftarrow\overline{\boldsymbol{\omega}}} = n(n+1)\int_0^1 dt\int_0^t ds\left\langle \boldsymbol{e}\boldsymbol{\theta}\left(\overline{\boldsymbol{R}}+t\mathrm{D}_{\bar{\omega}}\boldsymbol{\theta}+s^2\boldsymbol{e}^2+t^2\boldsymbol{\theta}^2\right)^{n-1}\right\rangle$$

$$L_{\mathrm{G}}^{(2n+1)} = (n+1)k\int_0^1 dt\langle \boldsymbol{e}(\boldsymbol{R}+t^2\boldsymbol{e}^2)^n\rangle +$$

$$+n(n+1)k\,\mathrm{d}\int_0^1 dt\int_0^t ds\left\langle \boldsymbol{e}\boldsymbol{\theta}\left(\overline{\boldsymbol{R}}+t\mathrm{D}_{\overline{\boldsymbol{\omega}}}\boldsymbol{\theta}+s^2\boldsymbol{e}^2+t^2\boldsymbol{\theta}^2\right)^{n-1}\right\rangle$$

$$\langle \boldsymbol{J}_{ab}(\boldsymbol{R}+\boldsymbol{e}^2)^{n-1}\boldsymbol{T}\rangle = 0$$

$$\langle \boldsymbol{P}_a(\boldsymbol{R}+\boldsymbol{e}^2)^n\rangle = 0$$

$$\mathcal{R}_{abc} \equiv \langle \boldsymbol{F}^{n-1}\boldsymbol{J}_{ab}\boldsymbol{P}_c\rangle$$

$$\mathcal{R}_{abc}T^c = 0$$

$$\mathcal{R}_{abc}\left(R^{ab}+\frac{1}{\ell^2}e^a e^b\right) = 0$$

$$\mathcal{R}_{abc} = \frac{2}{n+1}\varepsilon_{abca_1\cdots a_{2n-2}}\left(R^{a_1a_2}+\frac{1}{\ell^2}e^{a_1a_2}\right)\cdots$$

$$\cdots\left(R^{a_{2n-3}a_{2n-2}}+\frac{1}{\ell^2}e^{a_{2n-3}a_{2n-2}}\right)$$

$$\int_0^1 dt\langle(\delta\overline{\boldsymbol{\omega}}+t\delta\boldsymbol{\theta}+t\delta\boldsymbol{e})(\boldsymbol{\theta}+\boldsymbol{e})\boldsymbol{F}_t^{n-1}\rangle\Big|_{\partial M}$$

$$\boldsymbol{A}_t = \overline{\boldsymbol{\omega}}+t(\boldsymbol{e}+\boldsymbol{\theta})$$

$$\boldsymbol{F}_t = \overline{\boldsymbol{R}}+t\mathrm{D}_{\overline{\boldsymbol{\omega}}}(\boldsymbol{e}+\boldsymbol{\theta})+t^2(\boldsymbol{e}^2+[\boldsymbol{e},\boldsymbol{\theta}]+\boldsymbol{\theta}^2)$$

$$\delta\overline{\boldsymbol{\omega}}|_{\partial M} = 0$$

$$\int_0^1 dt\left\langle t(\delta\boldsymbol{\theta}\boldsymbol{e}-\boldsymbol{\theta}\delta\boldsymbol{e})\left(\overline{\boldsymbol{R}}+t^2\boldsymbol{e}^2+t^2\boldsymbol{\theta}^2\right)^{n-1}\right\rangle\Big|_{\partial M}$$

$$\delta\theta^{[ab}e^{c]} = \theta^{[ab}\delta e^{c]}$$

$$\overline{\boldsymbol{\omega}} \to \overline{\boldsymbol{\omega}}+\overline{\boldsymbol{e}}_{\mathrm{g}}$$

$$\mathcal{Q}^{(2n+1)}_{\omega\leftarrow\bar{\omega}} = 0 \to \mathcal{Q}^{(2n+1)}_{\omega+e_g\leftarrow\bar{\omega}+\bar{e}_g} \neq 0$$

$$\lambda_{\alpha_1}\cdots\lambda_{\alpha_n} = \lambda_{\gamma(\alpha_1,\dots,\alpha_n)}$$

$$K^{\rho}_{\alpha_1\cdots\alpha_n} = \begin{cases}1, & \text{cuando } \rho=\gamma(\alpha_1,\dots,\alpha_n)\\ 0, & \infty\end{cases}$$

$$K_{\alpha_1\cdots\alpha_n}{}^{\rho} = K_{\alpha_1\cdots\alpha_{n-1}}{}^{\sigma}K_{\sigma\alpha_n}{}^{\rho} = K_{\alpha_1\sigma}{}^{\rho}K_{\alpha_2\cdots\alpha_n}{}^{\sigma}$$

$$(\lambda_\alpha)_\mu{}^\nu = K_{\mu\alpha}{}^\nu$$

$$(\lambda_\alpha)_\mu{}^\sigma(\lambda_\beta)_\sigma{}^\nu = K_{\alpha\beta}{}^\sigma(\lambda_\sigma)_\mu{}^\nu = \left(\lambda_{\gamma(\alpha,\beta)}\right)_\mu{}^\nu$$

$$S_p\times S_q = \left\{\lambda_\gamma \mid \lambda_\gamma = \lambda_{\alpha_p}\lambda_{\alpha_q},\ \text{con } \lambda_{\alpha_p}\in S_p, \lambda_{\alpha_q}\in S_q\right\}$$

$$0_S\lambda_\alpha = \lambda_\alpha 0_S = 0_S$$

$$\left[\boldsymbol{T}_{a_0},\boldsymbol{T}_{b_0}\right] = C_{a_0b_0}{}^{c_0}\boldsymbol{T}_{c_0}+C_{a_0b_0}{}^{c_1}\boldsymbol{T}_{c_1}$$

$$\left[\boldsymbol{T}_{a_0},\boldsymbol{T}_{b_1}\right] = C_{a_0b_1}{}^{c}\boldsymbol{T}_{c_1}$$

$$\left[\boldsymbol{T}_{a_1},\boldsymbol{T}_{b_1}\right] = C_{a_1b_1}{}^{c}\boldsymbol{T}_{c_0}+C_{a_1b_1}{}^{c_1}\boldsymbol{T}_{c_1}$$

$$C_{(A,\alpha)(B,\beta)}{}^{(C,\gamma)} = K_{\alpha\beta}{}^{\gamma}C_{AB}{}^{C}$$

$$\begin{aligned}\left[\boldsymbol{T}_{(A,\alpha)},\boldsymbol{T}_{(B,\beta)}\right] &\equiv \lambda_\alpha\lambda_\beta[\boldsymbol{T}_A,\boldsymbol{T}_B]\\ &= \lambda_{\gamma(\alpha,\beta)}C_{AB}{}^{C}\boldsymbol{T}_C\\ &= C_{AB}^{C}\boldsymbol{T}_{(C,\gamma(\alpha,\beta))}\end{aligned}$$

$$K^{\rho}_{\alpha\beta} = \begin{cases}1, & \text{cuando } \rho=\gamma(\alpha,\beta)\\ 0, & \infty\end{cases}$$

$$[\boldsymbol{T}_{(A,\alpha)}, \boldsymbol{T}_{(B,\beta)}] = K_{\alpha\beta}{}^{\rho} C_{AB}{}^{C} \boldsymbol{T}_{(C,\rho)}$$
$$C_{(A,\alpha)(B,\beta)}{}^{(C,\gamma)} = K_{\alpha\beta}{}^{\gamma} C_{AB}{}^{C}$$
$$C_{(A,\alpha)(B,\beta)}{}^{(C,\gamma)} = -(-1)^{\mathfrak{q}(A)\mathfrak{q}(B)} C_{(B,\beta)(A,\alpha)}{}^{(C,\gamma)}$$
$$K_{i,N+1}^{j} = K_{N+1,i}^{j} = 0$$
$$K_{i,N+1}{}^{N+1} = K_{N+1,i}^{N+1} = 1$$
$$K_{N+1,N+1}^{j} = 0$$
$$K_{N+1,N+1}{}^{N+1} = 1$$
$$[\boldsymbol{T}_{(A,i)}, \boldsymbol{T}_{(B,j)}] = K_{ij}{}^{k} C_{AB}{}^{C} \boldsymbol{T}_{(C,k)} + K_{ij}{}^{N+1} C_{AB}{}^{C} \boldsymbol{T}_{(C,N+1)}$$
$$[\boldsymbol{T}_{(A,N+1)}, \boldsymbol{T}_{(B,j)}] = C_{AB}{}^{C} \boldsymbol{T}_{(C,N+1)}$$
$$[\boldsymbol{T}_{(A,N+1)}, \boldsymbol{T}_{(B,N+1)}] = C_{AB}{}^{C} \boldsymbol{T}_{(C,N+1)}$$
$$[\boldsymbol{T}_{(A,i)}, \boldsymbol{T}_{(B,j)}] = C_{(A,i)(B,j)}{}^{(C,k)} \boldsymbol{T}_{(C,k)}$$
$$C_{(A,i)(B,j)}{}^{(C,k)} = K_{ij}{}^{k} C_{AB}{}^{C}$$
$$\boldsymbol{T}_{(A,N+1)} = 0_S \boldsymbol{T}_A = \boldsymbol{0}$$
$$C_{(A,i)(B,j)}{}^{(C,k)} = \begin{cases} 0, & \text{cuando } i+j \neq k \\ C_{AB}^{C}, & \text{cuando } i+j = k \end{cases}$$
$$S_{\mathrm{E}}^{(N)} = \{\lambda_\alpha, \alpha = 0, \dots, N, N+1\}$$
$$\lambda_\alpha \lambda_\beta = \begin{cases} \lambda_{\alpha+\beta}, & \text{cuando } \alpha+\beta \leq N \\ \lambda_{N+1}, & \text{cuando } \alpha+\beta \geq N+1 \end{cases}$$
$$K_{\alpha\beta}^{\gamma} = \begin{cases} \delta_{\alpha+\beta}^{\gamma}, & \text{cuando } \alpha+\beta \leq N \\ \delta_{N+1}^{\gamma}, & \text{cuando } \alpha+\beta \geq N+1 \end{cases}$$
$$C_{(A,\alpha)(B,\beta)}{}^{(C,\gamma)} = \begin{cases} \delta_{\alpha+\beta}^{\gamma} C_{AB}{}^{C}, & \text{cuando } \alpha+\beta \leq N \\ \delta_{N+1}^{\gamma} C_{AB}^{C}, & \text{cuando } \alpha+\beta \geq N+1 \end{cases}$$
$$C_{(A,i)(B,j)}{}^{(C,k)} = \delta_{i+j}^{k} C_{AB}{}^{C}$$
$$\lambda^\alpha \lambda^\beta = \lambda^{\alpha+\beta}$$
$$\lambda^\alpha = 0 \text{ cuando } \alpha > N$$
$$[V_p, V_q] \subset \bigoplus_{r \in i_{(p,q)}} V_r$$
$$S_p \times S_q \subset \bigcap_{r \in i_{(p,q)}} S_r$$
$$W_p \equiv S_p \otimes V_p$$
$$\mathfrak{G}_{\mathrm{R}} \equiv \bigoplus_{p \in I} W_p$$
$$[W_p, W_q] \subset (S_p \times S_q) \otimes [V_p, V_q]$$
$$\subset \bigcap_{s \in i_{(p,q)}} S_s \otimes \bigoplus_{r \in i_{(p,q)}} V_r$$
$$\subset \bigoplus_{r \in i_{(p,q)}} \left(\bigcap_{s \in i_{(p,q)}} S_s \right) \otimes V_r.$$
$$\bigcap_{s \in i_{(p,q)}} S_s \subset S_r$$

$$[W_p, W_q] \subset \bigoplus_{r \in i_{(p,q)}} S_r \otimes V_r$$

$$[W_p, W_q] \subset \bigoplus_{r \in i_{(p,q)}} W_r$$

$$C_{(a_p,\alpha_p)(b_q,\beta_q)}{}^{(c_r,\gamma_r)} = K_{\alpha_p \beta_q}{}^{\gamma_r} C_{a_p b_q}{}^{c_r}$$

$$[V_0, V_0] \subset V_0,$$
$$[V_0, V_1] \subset V_1,$$
$$[V_1, V_1] \subset V_0.$$

$$S_0 = \left\{\lambda_{2m}, \text{con } m = 0, \dots, \left[\frac{N}{2}\right]\right\} \cup \{\lambda_{N+1}\},$$

$$S_1 = \left\{\lambda_{2m+1}, \text{con } m = 0, \dots, \left[\frac{N-1}{2}\right]\right\} \cup \{\lambda_{N+1}\}$$

$$S_0 \times S_0 \subset S_0,$$
$$S_0 \times S_1 \subset S_1,$$
$$S_1 \times S_1 \subset S_0$$

$$\mathfrak{G}_{\mathrm{R}} = W_0 \oplus W_1$$
$$W_0 = S_0 \otimes V_0$$
$$W_1 = S_1 \otimes V_1$$

$$C_{(a_p,\alpha_p)(b_q,\beta_q)}{}^{(c_r,\gamma_r)} = \begin{cases} \delta^{\gamma_r}_{\alpha_p+\beta_q} C_{a_p b_q}{}^{c_r}, & \text{cuando } \alpha_p + \beta_q \leq N \\ \delta^{\gamma_r}_{N+1} C_{a_p b_q} c_r, & \text{cuando } \alpha_p + \beta_q \geq N+1 \end{cases}$$

$$C_{(a_p,\alpha_p)(b_q,\beta_q)}{}^{(c_r,\gamma_r)} = \begin{cases} \delta^{\gamma_r}_{\alpha_p+\beta_q} C^{c_r}_{a_p b_q b_q}, & \text{cuando } \alpha_p + \beta_q \leq N \\ \delta^{\gamma_r}_{N+1} C_{a_p b_q}, & \text{cuando } \alpha_p + \beta_q \geq N+1 \end{cases}$$

$$N_0 = 2\left[\frac{N}{2}\right]$$

$$N_1 = 2\left[\frac{N-1}{2}\right] + 1$$

$$S_0 = \{\lambda_0, \lambda_2, \lambda_4\},$$
$$S_1 = \{\lambda_1, \lambda_3, \lambda_4\}$$

$$[V_0, V_0] \subset V_0,$$
$$[V_0, V_1] \subset V_1,$$
$$[V_0, V_2] \subset V_2,$$
$$[V_1, V_1] \subset V_0 \oplus V_2,$$
$$[V_1, V_2] \subset V_1,$$
$$[V_2, V_2] \subset V_0 \oplus V_2$$

$$S_p = \left\{\lambda_{2m+p}, \text{con } m = 0, \dots, \left[\frac{N-p}{2}\right]\right\} \cup \{\lambda_{N+1}\}, p = 0,1,2$$

$$S_0 \times S_0 \subset S_0,$$
$$S_0 \times S_1 \subset S_1,$$
$$S_0 \times S_2 \subset S_2,$$
$$S_1 \times S_1 \subset S_0 \cap S_2,$$
$$S_1 \times S_2 \subset S_1,$$
$$S_2 \times S_2 \subset S_0 \cap S_2$$

$$C_{(a_p,\alpha_p)(b_q,\beta_q)}{}^{(c_r,\gamma_r)} = \begin{cases} \delta^{\gamma_q}_{\alpha_p+\beta_q} C_{a_p b_c}{}^{c_r}, & \text{cuando } \alpha_p + \beta_q \leq N \\ \delta^{\gamma_r}_{N+1} C_{a_p b_q}, & \text{cuando } \alpha_p + \beta_q \geq N+1 \end{cases}$$

$$C_{(a_p,\alpha_p)(b_q,\beta_q)}{}^{(c_r,\gamma_r)} = \begin{cases} \delta^{\gamma_r}_{\alpha_p+\beta_q} C^{c_r}_{a_p b_q b_r}, & \text{cuando } \alpha_p + \beta_q \leq N \\ \delta^{\gamma_r}_{N+1} C_{a_p b_q}, & \text{cuando } \alpha_p + \beta_q \geq N+1 \end{cases}$$

$$N_p = 2\left[\frac{N-p}{2}\right] + p, p = 0,1,2$$

$$[\boldsymbol{S}_{\kappa\lambda}, \boldsymbol{S}_{\mu\nu}] = -i(\delta_{\mu\lambda}\boldsymbol{S}_{\kappa\nu} - \delta_{\mu\kappa}\boldsymbol{S}_{\lambda\nu} + \delta_{\nu\lambda}\boldsymbol{S}_{\mu\kappa} - \delta_{\nu\kappa}\boldsymbol{S}_{\mu\lambda})$$

	a	b
a	a	b
b	b	a

$$[\boldsymbol{G}_i, \boldsymbol{G}_j] = i\varepsilon_{ijk}\boldsymbol{G}_k,$$
$$\boldsymbol{J}_i = a\boldsymbol{G}_i,$$
$$\boldsymbol{K}_i = b\boldsymbol{G}_i,$$
$$[\boldsymbol{J}_i, \boldsymbol{J}_j] = i\varepsilon_{ijk}\boldsymbol{J}_k,$$
$$[\boldsymbol{J}_i, \boldsymbol{K}_j] = i\varepsilon_{ijk}\boldsymbol{K}_k,$$
$$[\boldsymbol{K}_i, \boldsymbol{K}_j] = i\varepsilon_{ijk}\boldsymbol{J}_k$$
$$\boldsymbol{M}_{ij} = \varepsilon_{ijk}\boldsymbol{J}_k,$$
$$\boldsymbol{M}_{i4} = \boldsymbol{K}_i,$$
$$[\boldsymbol{M}_{\kappa\lambda}, \boldsymbol{M}_{\mu\nu}] = -i(\delta_{\mu\lambda}\boldsymbol{M}_{\kappa\nu} - \delta_{\mu\kappa}\boldsymbol{M}_{\lambda\nu} + \delta_{\nu\lambda}\boldsymbol{M}_{\mu\kappa} - \delta_{\nu\kappa}\boldsymbol{M}_{\mu\lambda})$$
$$[\boldsymbol{P}_a, \boldsymbol{P}_b] = \boldsymbol{J}_{ab},$$
$$[\boldsymbol{J}^{ab}, \boldsymbol{P}_c] = \delta^{ab}_{ec}\boldsymbol{P}^e,$$
$$[\boldsymbol{P}_a, \boldsymbol{Z}_{b_1\cdots b_5}] = -\frac{1}{5!}\varepsilon_{ab_1\cdots b_5 c_1\cdots c_5}\boldsymbol{Z}^{c_1\cdots c_5},$$
$$[\boldsymbol{J}^{ab}, \boldsymbol{J}_{cd}] = \delta^{abf}_{\mathrm{ecd}}\boldsymbol{J}^e{}_f,$$
$$[\boldsymbol{J}^{ab}, \boldsymbol{Z}_{c_1\cdots c_5}] = \frac{1}{4!}\delta^{abe_1\cdots e_4}_{dc_1\cdots c_5}\boldsymbol{Z}^d_{e_1\cdots e_4},$$
$$[\boldsymbol{Z}^{a_1\cdots a_5}, \boldsymbol{Z}_{b_1\cdots b_5}] = \eta^{[a_1\cdots a_5][c_1\cdots c_5]}\varepsilon_{c_1\cdots c_5 b_1\cdots b_5 e}\boldsymbol{P}^e + \delta^{a_1\cdots a_5 e}_{db_1\cdots b_5}\boldsymbol{J}^d{}_e$$
$$-\frac{1}{3!\,3!\,5!}\varepsilon_{c_1\cdots c_{11}}\delta^{a_1\cdots a_5 c_4 c_5 c_6}_{d_1 d_2 d_3 b_1\cdots b_5}{}^{[c_1c_2c_3][d_1d_2d_3]}\boldsymbol{Z}^{c_7\cdots c_{11}},$$
$$[\boldsymbol{P}_a, \boldsymbol{Q}] = -\frac{1}{2}\Gamma_a\boldsymbol{Q},$$
$$[\boldsymbol{J}_{ab}, \boldsymbol{Q}] = -\frac{1}{2}\Gamma_{ab}\boldsymbol{Q},$$
$$[\boldsymbol{Z}_{abcde}, \boldsymbol{Q}] = -\frac{1}{2}\Gamma_{abcde}\boldsymbol{Q},$$
$$\{\boldsymbol{Q}, \overline{\boldsymbol{Q}}\} = \frac{1}{8}\left(\Gamma^a\boldsymbol{P}_a - \frac{1}{2}\Gamma^{ab}\boldsymbol{J}_{ab} + \frac{1}{5!}\Gamma^{abcde}\boldsymbol{Z}_{abcde}\right)$$

Superespacios de $\mathfrak{G}_{\mathrm{R}}$	Generadores
	$\boldsymbol{J}_{ab} = \lambda_0\boldsymbol{J}^{((\mathrm{o.5})}_{ab}$
$S_0 \otimes V_0$	$\boldsymbol{Z}_{ab} = \lambda_2\boldsymbol{J}^{(\mathrm{o.sp})}_{ab}$
	$\boldsymbol{0} = \lambda_3\boldsymbol{J}^{(\mathrm{osp})}_{ab}$

$S_1 \otimes V_1$	$\boldsymbol{Q} = \lambda_1 \boldsymbol{Q}^{(\mathrm{osp})}$ $\boldsymbol{0} = \lambda_3 \boldsymbol{Q}^{(\mathrm{osp})}$
$S_2 \otimes V_2$	$\boldsymbol{P}_a = \lambda_2 \boldsymbol{P}_a^{(\mathrm{osp})}$ $\boldsymbol{Z}_{\mathrm{abcde}} = \lambda_2 \boldsymbol{Z}_{\mathrm{abcde}}^{(\mathrm{osp})}$ $\boldsymbol{0} = \lambda_3 \boldsymbol{P}_a^{(\mathrm{osp})}$ $0 = \lambda_3 \boldsymbol{Z}_{\mathrm{abcde}}^{(\mathrm{osp})}$

$$
\begin{aligned}
[\boldsymbol{J}^{ab}, \boldsymbol{J}_{cd}] &= \delta^{abf}_{ecd} \boldsymbol{J}^{e}_{f}, \\
[\boldsymbol{J}^{ab}, \boldsymbol{P}_{c}] &= \delta^{ab}_{ec} \boldsymbol{P}^{e}, \\
[\boldsymbol{J}^{ab}, \boldsymbol{Z}_{cd}] &= \delta^{aff}_{ecd} \boldsymbol{Z}^{e}_{f}, \\
\left[\boldsymbol{J}^{ab}, \boldsymbol{Z}_{c_1 \cdots c_5}\right] &= \frac{1}{4!} a^{abe_4}_{dc_1 \cdots c_5} \boldsymbol{Z}^{d}_{e_1 \cdots e_4}, \\
[\boldsymbol{J}_{ab}, \boldsymbol{Q}] &= -\frac{1}{2} \Gamma_{ab} \boldsymbol{Q}, \\
[\boldsymbol{P}_a, \boldsymbol{P}_b] &= \boldsymbol{0}, \\
[\boldsymbol{P}_a, \boldsymbol{Z}_{bc}] &= \boldsymbol{0}, \\
\left[\boldsymbol{P}_a, \boldsymbol{Z}_{b_1 \cdots b_5}\right] &= \boldsymbol{0}, \\
[\boldsymbol{Z}_{ab}, \boldsymbol{Z}_{cd}] &= \boldsymbol{0}, \\
\left[\boldsymbol{Z}_{ab}, \boldsymbol{Z}_{c_1 \cdots c_5}\right] &= \boldsymbol{0}, \\
\left[\boldsymbol{Z}_{a_1 \cdots a_5}, \boldsymbol{Z}_{b_1 \cdots b_5}\right] &= \boldsymbol{0}, \\
[\boldsymbol{P}_a, \boldsymbol{Q}] &= \boldsymbol{0}, \\
[\boldsymbol{Z}_{ab}, \boldsymbol{Q}] &= \boldsymbol{0}, \\
[\boldsymbol{Z}_{abcde}, \boldsymbol{Q}] &= \boldsymbol{0}, \\
\{\boldsymbol{Q}, \overline{\boldsymbol{Q}}\} &= \frac{1}{8}\left(\Gamma^{a} \boldsymbol{P}_a - \frac{1}{2} \Gamma^{ab} \boldsymbol{Z}_{ab} + \frac{1}{5!} \Gamma^{abcde} \boldsymbol{Z}_{abcde}\right) \\
[\boldsymbol{P}_a, \boldsymbol{Q}] &= -\frac{1}{2} \Gamma_a \boldsymbol{Q}', \\
[\boldsymbol{Z}_{ab}, \boldsymbol{Q}] &= -\frac{1}{2} \Gamma_{ab} \boldsymbol{Q}', \\
[\boldsymbol{Z}_{abcde}, \boldsymbol{Q}] &= -\frac{1}{2} \Gamma_{abcde} \boldsymbol{Q}'
\end{aligned}
$$

Superespacios de $\mathfrak{G}_{\mathrm{R}}$	Generadores
$S_0 \otimes V_0$	$\boldsymbol{J}_{ab} = \lambda_0 \boldsymbol{J}_{ab}^{(\mathrm{osp})}$ $\boldsymbol{Z}_{ab} = \lambda_2 \boldsymbol{J}_{ab}^{(\mathrm{osp})}$ $\boldsymbol{0} = \lambda_4 \boldsymbol{J}_{ab}^{(\mathrm{osp})}$
$S_1 \otimes V_1$	$\boldsymbol{Q} = \lambda_1 \boldsymbol{Q}^{(\mathrm{osp}\ )}$ $\boldsymbol{Q}' = \lambda_3 \boldsymbol{Q}^{((\mathrm{osp})}$

$S_2 \otimes V_2$

$$\mathbf{0} = \lambda_4 \boldsymbol{Q}^{((\mathrm{s.p})}$$
$$\boldsymbol{P}_a = \lambda_2 \boldsymbol{P}_a^{(\mathrm{osp})}$$
$$\boldsymbol{Z}_{\mathrm{abcde}} = \lambda_2 \boldsymbol{Z}_{\mathrm{abcde}}^{(\mathrm{osp})}$$
$$\mathbf{0} = \lambda_4 \boldsymbol{P}_a^{(\mathrm{osp})}$$
$$\mathbf{0} = \lambda_4 \boldsymbol{Z}_{\mathrm{abcde}}^{(\mathrm{osp})}$$

$$\lambda_\alpha \lambda_\beta = \lambda_{(\alpha+\beta)\mathrm{mod}4}$$
$$S_0 = \{\lambda_0, \lambda_2\},$$
$$S_1 = \{\lambda_1, \lambda_3\},$$
$$S_2 = \{\lambda_0, \lambda_2\}.$$

$S_E^{(2)}$				$\mathbb{Z}_4$			
0	1	2	3	0	1	2	3
1	2	3	3	1	2	3	0
2	3	3	3	2	3	0	1
3	3	3	3	3	0	1	2

Subespacios de $\mathfrak{G}_{\mathrm{R}}$	Generadores
$S_0 \otimes V_0$	$\boldsymbol{J}_{ab} = \lambda_0 \boldsymbol{J}_{ab}^{((\mathrm{osp}))}$ $\boldsymbol{Z}_{ab} = \lambda_2 \boldsymbol{J}_{ab}^{(\mathrm{osp}))}$
$S_1 \otimes V_1$	$\boldsymbol{Q} = \lambda_1 \boldsymbol{Q}^{((\mathrm{o.5p}))}$ $\boldsymbol{Q}' = \lambda_3 \boldsymbol{Q}^{((\mathrm{asp}\,)}$
$S_2 \otimes V_2$	$\boldsymbol{P}'_a = \lambda_0 \boldsymbol{P}_a^{(\mathrm{osp}\,)}$ $abccde = \lambda_0 \boldsymbol{Z}_{abcde}^{(\mathrm{osp}\,)}$ $\boldsymbol{P}_a = \lambda_2 \boldsymbol{P}_a^{(\mathrm{osp}))}$ $a_{bccee} = \lambda_2 \boldsymbol{Z}_{abcde}^{(\mathrm{osp}\,)}$

$$\{\boldsymbol{Q}, \boldsymbol{Q}\} \sim \boldsymbol{P} + \boldsymbol{Z}_2 + \boldsymbol{Z}_5,$$
$$\{\boldsymbol{Q}', \boldsymbol{Q}'\} \sim \boldsymbol{P} + \boldsymbol{Z}_2 + \boldsymbol{Z}_5,$$
$$\{\boldsymbol{Q}, \boldsymbol{Q}'\} \sim \boldsymbol{P}' + \boldsymbol{J} + \boldsymbol{Z}'_5$$
$$[\boldsymbol{P}, \boldsymbol{P}] \sim \boldsymbol{J}$$
$$[\boldsymbol{P}', \boldsymbol{P}'] \sim \boldsymbol{J}$$
$$[\boldsymbol{P}, \boldsymbol{P}'] \sim \boldsymbol{Z}_2$$

$$[\boldsymbol{Z}_2, \boldsymbol{Z}_2] \sim \boldsymbol{J},$$
$$[\boldsymbol{Z}_2, \boldsymbol{Z}_5] \sim \boldsymbol{Z}'_5,$$
$$[\boldsymbol{Z}_2, \boldsymbol{Z}'_5] \sim \boldsymbol{Z}_5,$$
$$[\boldsymbol{Z}_5, \boldsymbol{Z}_5] \sim \boldsymbol{P}' + \boldsymbol{J} + \boldsymbol{Z}'_5,$$
$$[\boldsymbol{Z}_5, \boldsymbol{Z}'_5] \sim \boldsymbol{P} + \boldsymbol{Z}_2 + \boldsymbol{Z}_5,$$
$$[\boldsymbol{Z}'_5, \boldsymbol{Z}'_5] \sim \boldsymbol{P}' + \boldsymbol{J} + \boldsymbol{Z}'_5$$
$$[\boldsymbol{P}, \boldsymbol{Q}] \sim \boldsymbol{Q}',$$
$$[\boldsymbol{Z}_2, \boldsymbol{Q}] \sim \boldsymbol{Q}',$$
$$[\boldsymbol{Z}_5, \boldsymbol{Q}] \sim \boldsymbol{Q}'$$
$$[\boldsymbol{P}, \boldsymbol{Q}'] \sim \boldsymbol{Q},$$
$$[\boldsymbol{Z}_2, \boldsymbol{Q}'] \sim \boldsymbol{Q},$$
$$[\boldsymbol{Z}_5, \boldsymbol{Q}'] \sim \boldsymbol{Q},$$
$$[\boldsymbol{P}', \boldsymbol{Q}] \sim \boldsymbol{Q},$$
$$[\boldsymbol{J}, \boldsymbol{Q}] \sim \boldsymbol{Q},$$
$$[\boldsymbol{Z}'_5, \boldsymbol{Q}] \sim \boldsymbol{Q},$$
$$[\boldsymbol{P}', \boldsymbol{Q}'] \sim \boldsymbol{Q}'$$
$$[\boldsymbol{J}, \boldsymbol{Q}'] \sim \boldsymbol{Q}'$$
$$[\boldsymbol{Z}'_5, \boldsymbol{Q}'] \sim \boldsymbol{Q}'$$
$$\left|\boldsymbol{T}_{(A_1,\alpha_1)} \cdots \boldsymbol{T}_{(A_n,\alpha_n)}\right| \equiv \alpha_\gamma K_{\alpha_1\cdots\alpha_n}{}^{\gamma} \left|\boldsymbol{T}_{A_1} \cdots \boldsymbol{T}_{A_n}\right|$$
$$\sum_{p=1}^{n} X^{(p)}_{A_0\cdots A_n} = 0$$
$$X^{(p)}_{A_0\cdots A_n} = (-1)^{\mathrm{q}(A_0)\left(\mathrm{q}(A_1)+\cdots+\mathrm{q}(A_{p-1})\right)} C^{B}_{A_0 A_p} \times$$
$$\times \left|\boldsymbol{T}_{A_1} \cdots \boldsymbol{T}_{A_{p-1}} \boldsymbol{T}_B \boldsymbol{T}_{A_{p+1}} \cdots \boldsymbol{T}_{A_n}\right|,$$
$$X^{(p)}_{(A_0,\alpha_0)\cdots(A_n,\alpha_n)} = (-1)^{\mathrm{q}(A_0,\alpha_0)\left(\mathrm{q}(A_1,\alpha_1)+\cdots+\mathrm{q}(A_{p-1},\alpha_{p-1})\right)} \times$$
$$\times\, C_{(A_0,\alpha_0)(A_p,\alpha_p)}{}^{(B,\beta)} \mid \boldsymbol{T}_{(A_1,\alpha_1)} \cdots \boldsymbol{T}_{(A_{p-1},\alpha_{p-1})}$$
$$\times\, \boldsymbol{T}_{(B,\beta)} \boldsymbol{T}_{(A_{p+1},\alpha_{p+1})} \cdots \boldsymbol{T}_{(A_n,\alpha_n)} \mid,$$
$$X^{(p)}_{(A_0,\alpha_0)\cdots(A_n,\alpha_n)} = \alpha_\gamma K_{\alpha_0\cdots\alpha_n}{}^{\gamma} X^{(p)}_{A_0\cdots A_n}$$
$$\sum_{p=1}^{n} X^{(p)}_{(A_0,\alpha_0)\cdots(A_n,\alpha_n)} = 0$$
$$\left|\boldsymbol{T}_{(A_1,\alpha_1)} \cdots \boldsymbol{T}_{(A_n,\alpha_n)}\right| = \sum_{m=0}^{M} \alpha_\gamma^{\beta_1\cdots\beta_m} K_{\beta_1\cdots\beta_m\alpha_1\cdots\alpha_n}{}^{\gamma} \left|\boldsymbol{T}_{A_1} \cdots \boldsymbol{T}_{A_n}\right|$$
$$\mathrm{STr}\left(\boldsymbol{T}_{(A_1,\alpha_1)} \cdots \boldsymbol{T}_{(A_n,\alpha_n)}\right) = K_{\gamma\alpha_1\cdots\alpha_n}{}^{\gamma} \mathrm{Str}\left(\boldsymbol{T}_{A_1} \cdots \boldsymbol{T}_{A_n}\right)$$
$$\left|\boldsymbol{T}_{(a_{p_1},\alpha_{p_1})} \cdots \boldsymbol{T}_{(a_{p_n},\alpha_{p_n})}\right| = \alpha_\gamma K^{\gamma}_{\alpha_{p_1}\cdots\alpha_{p_n}} \left|\boldsymbol{T}_{a_{p_1}} \cdots \boldsymbol{T}_{a_{p_n}}\right|$$
$$\left|\boldsymbol{T}_{(A_1,i_1)} \cdots \boldsymbol{T}_{(A_n,i_n)}\right| \equiv \alpha_j K_{i_1\cdots i_n}{}^{j} \left|\boldsymbol{T}_{A_1} \cdots \boldsymbol{T}_{A_n}\right|$$
$$Y^{(p)}_{(A_0,i_0)\cdots(A_n,i_n)} = (-1)^{\mathrm{q}(A_0,i_0)\left(\mathrm{q}(A_1,i_1)+\cdots+\mathrm{q}(A_{p-1},i_{p-1})\right)} \times$$
$$\times\, C_{(A_0,i_0)(A_p,i_p)}{}^{(B,j)} \mid \boldsymbol{T}_{(A_1,i_1)} \cdots \boldsymbol{T}_{(A_{p-1},i_{p-1})}$$
$$\times\, \boldsymbol{T}_{(B,j)} \boldsymbol{T}_{(A_{p+1},i_{p+1})} \cdots \boldsymbol{T}_{(A_n,i_n)} \mid.$$
$$Y^{(p)}_{(A_0,i_0)\cdots(A_n,i_n)} = \alpha_k K_{i_0 i_p}{}^{j} K_{i_1\cdots i_{p-1} j i_{p+1}\cdots i_n}{}^{k} X^{(p)}_{A_0\cdots A_n}$$

$$K_{i_0 i_p}{}^{j} K_{i_1\cdots i_{p-1} j i_{p+1}\cdots i_n}{}^{k} = K_{i_0 i_p}{}^{\gamma} K_{i_1\cdots i_{p-1}\gamma i_{p+1}\cdots i_n}{}^{k} + \\ - K_{i_0 i_p}{}^{N+1} K_{i_1\cdots i_{p-1}(N+1) i_{p+1}\cdots i_n}{}^{k} \\ = K_{i_0\cdots i_n}{}^{k} - K_{i_0 i_p}{}^{N+1} K_{i_1\cdots i_{p-1}(N+1) i_{p+1}\cdots i_n}{}^{k}.$$

$$K_{i_0 i_p}{}^{j} K_{i_1\cdots i_{p-1} j i_{p+1}\cdots i_n}{}^{k} = K_{i_0\cdots i_n}{}^{k}$$

$$Y^{(p}_{(A_0,i_0)\cdots(A_n,i_n)} = \alpha_k K_{i_0\cdots i_n}{}^{k} X^{(p)}_{A_0\cdots A_n}$$

$$\sum_{p=1}^{n} Y^{(p)}_{(A_0,i_0)\cdots(A_n,i_n)} = 0$$

$$\left|\boldsymbol{T}_{(a_{p_1},i_{p_1})}\cdots \boldsymbol{T}_{(a_{p_n},i_{p_n})}\right| = \alpha_j K_{i_{p_1}\cdots i_{p_n}}{}^{j} \left|\boldsymbol{T}_{a_{p_1}}\cdots \boldsymbol{T}_{a_{p_n}}\right|$$

$$\mathrm{STr}\left(\boldsymbol{T}_{(A_1,i_1)}\cdots \boldsymbol{T}_{(A_n,i_n)}\right) = K_{j_1 i_1}{}^{2} K_{j_2 i_2}{}^{j_3} \cdots K_{j_{n-1} i_{n-1}}{}^{j_n} \times \\ \times K_{j_n i_n}{}^{1} \mathrm{Str}\left(\boldsymbol{T}_{A_1}\cdots \boldsymbol{T}_{A_n}\right).$$

$$K_{j_1 i_1\cdots i_n}{}^{j_1} = K_{j_1 i_1}{}^{2} K_{\gamma_2 i_2}{}^{3} \cdots K_{\gamma_{n-1} i_{n-1}}{}^{\gamma_n} K_{\gamma_n i_n}{}^{j_1} \\ = K_{j_1 i_1}{}^{2} K_{j_2 i_2}{}^{2} \cdots K_{j_{n-1} i_{n-1}}{}_{n} K_{j_n i_n}{}_{1},$$

$$\mathrm{STr}\left(\boldsymbol{T}_{(A_1,i_1)}\cdots \boldsymbol{T}_{(A_n,i_n)}\right) = K_{j_1 i_1\cdots i_n}{}^{j_1} \mathrm{Str}\left(\boldsymbol{T}_{A_1}\cdots \boldsymbol{T}_{A_n}\right)$$

$$\mathrm{STr}\left(\boldsymbol{T}_{(A_1,i_1)}\cdots \boldsymbol{T}_{(A_n,i_n)}\right) = K_{i_1\cdots i_n}{}^{0} \mathrm{Str}\left(\boldsymbol{T}_{A_1}\cdots \boldsymbol{T}_{A_n}\right)$$

$$\mathrm{STr}\left(\boldsymbol{T}_{(A_1,0)}\cdots \boldsymbol{T}_{(A_n,0)}\right) = \mathrm{Str}\left(\boldsymbol{T}_{A_1}\cdots \boldsymbol{T}_{A_n}\right)$$

$$\{\boldsymbol{Q},\overline{\boldsymbol{Q}}\} = 2\gamma^{a}\boldsymbol{P}_a$$

$$\{\boldsymbol{Q},\overline{\boldsymbol{Q}}\} = \frac{1}{8}\left(\Gamma^{a}\boldsymbol{P}_a - \frac{1}{2}\Gamma^{ab}\boldsymbol{Z}_{ab} + \frac{1}{5!}\Gamma^{abcde}\boldsymbol{Z}_{abcde}\right)$$

$$[\boldsymbol{J}^{ab},\boldsymbol{Z}_{cd}] = \delta^{abf}_{ecd}\boldsymbol{Z}^{e}_{f},$$

$$[\boldsymbol{J}^{ab},\boldsymbol{Z}_{c_1\cdots c_5}] = \frac{1}{4!}\delta^{abe_1\cdots e_4}_{dc_1\cdots c_5}\boldsymbol{Z}^{d}_{e_1\cdots e_4}$$

$$\lambda_\alpha\lambda_\beta = \begin{cases}\lambda_{\alpha+\beta}, & \text{cuando } \alpha+\beta\le 2\\ \lambda_3, & \text{cuando } \alpha+\beta\ge 3\end{cases}$$

$$\left\langle \boldsymbol{T}_{(A_1,i_1)}\cdots \boldsymbol{T}_{(A_n,i_n)}\right\rangle = \alpha_j K_{i_1\cdots i_n}{}^{j} \left\langle \boldsymbol{T}_{A_1}\cdots \boldsymbol{T}_{A_n}\right\rangle$$

$$K_{i_1\cdots i_n}{}^{j} = \delta^{j}_{i_1+\cdots+i_n}$$

$$\left\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_6b_6}\right\rangle_{\mathrm{M}} = \alpha_0\left\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_6b_6}\right\rangle_{osp}$$

$$\left\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_5b_5}\boldsymbol{P}_c\right\rangle_{\mathrm{M}} = \alpha_2\left\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_5b_5}\boldsymbol{P}_c\right\rangle_{\mathrm{osp}},$$

$$\left\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_5b_5}\boldsymbol{Z}_{cd}\right\rangle_{\mathrm{M}} = \alpha_2\left\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_5b_5}\boldsymbol{J}_{cd}\right\rangle_{\mathrm{osp}},$$

$$\left\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_5b_5}\boldsymbol{Z}_{c_1\cdots c_5}\right\rangle_{\mathrm{M}} = \alpha_2\left\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_5b_5}\boldsymbol{Z}_{c_1\cdots c_5}\right\rangle_{osp},$$

$$\left\langle \boldsymbol{Q}\boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_4b_4}\overline{\boldsymbol{Q}}\right\rangle_{\mathrm{M}} = \alpha_2\left\langle \boldsymbol{Q}\boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_4b_4}\overline{\boldsymbol{Q}}\right\rangle_{\mathrm{osp}}$$

$$G = \begin{bmatrix} C_{\alpha\beta} & 0\\ 0 & 1\end{bmatrix}$$

$$\boldsymbol{P}_a = \begin{bmatrix} \frac{1}{2}(\Gamma_a)^\alpha & 0 \\ 0 & 0 \end{bmatrix},$$

$$\boldsymbol{J}_{ab} = \begin{bmatrix} \frac{1}{2}(\Gamma_{ab})^\alpha & 0 \\ 0 & 0 \end{bmatrix},$$

$$\boldsymbol{Z}_{abcde} = \begin{bmatrix} \frac{1}{2}(\Gamma_{abcde})^\alpha & 0 \\ 0 & 0 \end{bmatrix}$$

$$\boldsymbol{Q}^\gamma = \begin{bmatrix} 0 & C^{\gamma\alpha} \\ \delta_\beta^\gamma & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} A^\alpha{}_\beta & A^\alpha \\ A_\beta & 0 \end{bmatrix}$$

$$\{A_1\} = A_1$$

$$\{A_1 \cdots A_n\} = \frac{1}{n}\sum_{p=1}^{n} A_p\{A_1 \cdots \hat{A}_p \cdots A_n\}$$

$$\langle A_1 \cdots A_6\rangle = \mathrm{STr}\{A_1 \cdots A_6\}$$

$$\langle A_1 \cdots A_6\rangle = \mathrm{Tr}\{A_1 \cdots A_6\}$$

$$\langle A_1 \cdots A_6\rangle = \mathrm{Tr}(\{A_1 \cdots A_5\}A_6)$$

$$\langle \chi\zeta A_1 \cdots A_4\rangle = -\frac{2}{5}\bar{\chi}\{A_1 \cdots A_4\}\zeta$$

$$\{A_1A_2\} = 2\mathrm{Tr}(A_1A_2)\mathbb{1} + A_1A_2\Gamma_{[4]},$$

$$\{A_1A_2A_3\} = \sum_{\langle ijk\rangle}\left(\mathrm{Tr}(A_iA_j)A_k - \frac{4}{3}[A_iA_jA_k]\right)\Gamma_{[2]} + A_1A_2A_3\Gamma_{[6]},$$

$$\{A_1 \cdots A_4\} = \sum_{\langle ijkl\rangle}\left(\frac{1}{2}\mathrm{Tr}(A_iA_j)\mathrm{Tr}(A_kA_l) - \frac{2}{3}\mathrm{Tr}(A_iA_jA_kA_l)\right)\mathbb{1} +$$

$$+\sum_{\langle ijkl\rangle}\left(\frac{1}{2}\mathrm{Tr}(A_iA_j)A_kA_l - \frac{4}{3}A_i[A_jA_kA_l]\right)\Gamma_{[4]} +$$

$$+(A_1 \cdots A_4)\Gamma_{[8]},$$

$$\{A_1 \cdots A_5\} = \sum_{\langle ijklm\rangle}\left(\frac{1}{2}\mathrm{Tr}(A_iA_j)\mathrm{Tr}(A_kA_l)A_m - \frac{2}{3}\mathrm{Tr}(A_iA_jA_kA_l)A_m +\right.$$

$$\left.-\frac{4}{3}\mathrm{Tr}(A_iA_j)[A_kA_lA_m] + \frac{32}{15}[A_iA_jA_kA_lA_m]\right)\Gamma_{[2]} +$$

$$+\sum_{\langle ijklm\rangle}\left(\frac{1}{6}\mathrm{Tr}(A_iA_j)A_kA_lA_m - \frac{2}{3}A_iA_j[A_kA_lA_m]\right)\Gamma_{[6]} +$$

$$+(A_1 \cdots A_5)\Gamma_{[10]}$$

$$\mathrm{Tr}(A_{i_1} \cdots A_{i_n}) = (A_{i_1})^{c_1}{}_{c_1}(A_{i_1})^{c_2}{}_{c_3} \cdots (A_{i_n})^{c_n}$$

$$[A_{i_1} \cdots A_{i_n}]^{ab} = (A_{i_1})^a{}_{c_1}(A_{i_2})^{c_1}_1{}_{c_2} \cdots (A_{i_n})^{c_{n-1}b}$$

$$\langle \boldsymbol{J}^5\boldsymbol{P}\rangle_{\text{osp}} = \frac{1}{2}\varepsilon_{a_1\cdots a_{11}} L_1^{a_1a_2}\cdots L_5^{a_9a_{10}} B_1^{a_{11}},$$

$$\langle \boldsymbol{J}^6\rangle_{\text{osp}} = \frac{1}{3}\sum_{\langle i_1\cdots i_6\rangle} \left[\frac{1}{4}\mathrm{Tr}(L_{i_1}L_{i_2})\mathrm{Tr}(L_{i_3}L_{i_4})\mathrm{Tr}(L_{i_5}L_{i_6}) + \right.$$

$$\left. - \mathrm{Tr}(L_{i_1}L_{i_2}L_{i_3}L_{i_4})\mathrm{Tr}(L_{i_5}L_{i_6}) + \frac{16}{15}\mathrm{Tr}(L_{i_1}L_{i_2}L_{i_3}L_{i_4}L_{i_5}L_{i_6})\right],$$

$$\langle \boldsymbol{J}^5\boldsymbol{Z}\rangle_{\text{osp}} = \frac{1}{3}\varepsilon_{a_1\cdots a_{11}}\sum_{\langle i_1\cdots i_5\rangle}\left[-\frac{5}{4}L_{i_1}^{a_1a_2}\cdots L_{i_4}^{a_7a_8}(L_{i_5})_{bc}B_5^{bca_9a_{10}a_{11}} + \right.$$

$$+10L_{i_1}^{a_1a_2}L_{i_2}^{a_3a_4}L_{i_3}^{a_5a_6}(L_{i_4})^{a_7}{}_{}(L_{i_5})^{a_8}{}_{c}B_5^{ba_9a_{10}a_{11}} +$$

$$+\frac{1}{4}L_{i_1}^{a_1a_2}L_{i_2}^{a_3a_4}L_{i_3}^{a_5a_6}B_5^{a_7\cdots a_{11}}\mathrm{Tr}(L_{i_4}L_{i_5}) +$$

$$\left. -L_{i_1}^{a_1a_2}L_{i_2}^{a_3a_4}[L_{i_3}L_{i_4}L_{i_5}]_5^{a_5a_6}B_5^{a_7\cdots a_{11}}\right],$$

$$\langle \boldsymbol{Q}\boldsymbol{J}^4\overline{\boldsymbol{Q}}\rangle_{\text{osp}} = -\frac{1}{240}\varepsilon_{a_1\cdots a_8abc}L_1^{a_1a_2}\cdots L_4^{a_7a_8}(\bar{\chi}\Gamma^{abc}\zeta) +$$

$$+\frac{1}{60}\sum_{\langle i_1\cdots i_4\rangle}\left[\frac{3}{4}\mathrm{Tr}(L_{i_1}L_{i_2})L_{i_3}^{a_1a_2}L_{i_4}^{a_3a_4}(\bar{\chi}\Gamma_{a_1\cdots a_4}\zeta) + \right.$$

$$-2L_{i_1a_2}^{a_1}[L_{i_2}L_{i_3}L_{i_4}]^{a_3a_4}(\bar{\chi}\Gamma_{a_1\cdots a_4}\zeta) +$$

$$\left.+\frac{3}{4}\mathrm{Tr}(L_{i_1}L_{i_2})\mathrm{Tr}(L_{i_3}L_{i_4})\bar{\chi}\zeta - \mathrm{Tr}(L_{i_1}L_{i_2}L_{i_3}L_{i_4})\bar{\chi}\zeta\right]$$

$$\langle \boldsymbol{J}^5\boldsymbol{P}\rangle_{\text{osp}} = L_1^{a_1b_1}\cdots L_5^{a_5b_5}B_1^c\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_5b_5}\boldsymbol{P}_c\rangle_{\text{osp}}$$

$$\langle \boldsymbol{J}^6\rangle_{\text{osp}} = L_1^{a_1b_1}\cdots L_6^{a_6b_6}\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_6b_6}\rangle_{\text{osp}}$$

$$\langle \boldsymbol{J}^5\boldsymbol{Z}\rangle_{\text{osp}} = L_1^{a_1b_1}\cdots L_5^{a_5b_5}B_5^{c_1\cdots c_5}\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_5b_5}\boldsymbol{Z}_{c_1\cdots c_5}\rangle_{\text{osp}}$$

$$\langle \boldsymbol{Q}\boldsymbol{J}^4\overline{\boldsymbol{Q}}\rangle_{\text{osp}} = L_1^{a_1b_1}\cdots L_4^{a_4b_4}\bar{\chi}_\alpha\zeta^\beta\left\langle \boldsymbol{Q}^\alpha \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_4b_4}\overline{\boldsymbol{Q}}_\beta\right\rangle_{\text{osp}}$$

$$\mathrm{Tr}(A_{i_1}\cdots A_{i_n}) = (A_{i_1})^{c_1}{}_{c_2}(A_{i_1})^{c_2}\cdots(A_{i_n})^{c_n}{}_{c_1},$$

$$[A_{i_1}\cdots A_{i_n}]^{ab} = (A_{i_1})^{a}{}_{c_1}(A_{i_2})^{c_1}{}_{c_2}\cdots(A_{i_n})^{c_{n-1}b}.$$

$$\langle \boldsymbol{J}^5\boldsymbol{P}\rangle_{\text{osp}} = \frac{1}{2}\varepsilon_{a_1\cdots a_{11}} L_1^{a_1a_2}\cdots L_5^{a_9a_{10}} B_1^{a_{11}},$$

$$\langle \boldsymbol{J}^6\rangle_{\text{osp}} = \frac{1}{3}\sum_{\langle i_1\cdots i_6\rangle} \left[\frac{1}{4}\mathrm{Tr}(L_{i_1}L_{i_2})\mathrm{Tr}(L_{i_3}L_{i_4})\mathrm{Tr}(L_{i_5}L_{i_6}) + \right.$$

$$\left. -\mathrm{Tr}(L_{i_1}L_{i_2}L_{i_3}L_{i_4})\mathrm{Tr}(L_{i_5}L_{i_6}) + \frac{16}{15}\mathrm{Tr}(L_{i_1}L_{i_2}L_{i_3}L_{i_4}L_{i_5}L_{i_6})\right]$$

$$\langle \boldsymbol{J}^5\boldsymbol{Z}\rangle_{\text{osp}} = \frac{1}{3}\varepsilon_{a_1\cdots a_{11}} \sum_{\langle i_1\cdots i_5\rangle} \Big[-\frac{5}{4}L_{i_1}^{a_1a_2}\cdots L_{i_4}^{a_7a_8}\left(L_{i_5}\right)_{bc}B_5^{bca_9a_{10}a_{11}} + $$
$$+10L_{i_1}^{a_1a_2}L_{i_2}^{a_3a_4}L_{i_3}^{a_5a_6}\left(L_{i_4}\right)^{a_7}{}_b\left(L_{i_5}\right)^{a_8}{}_cB_5^{bca_9a_{10}a_{11}} + $$
$$+\frac{1}{4}L_{i_1}^{a_1a_2}L_{i_2}^{a_3a_4}L_{i_3}^{a_5a_6}B_5^{a_7\cdots a_{11}}\mathrm{Tr}\left(L_{i_4}L_{i_5}\right) + $$
$$-L_{i_1}^{a_1a_2}L_{i_2}^{a_3a_4}\left[L_{i_3}L_{i_4}L_{i_5}\right]^{a_5a_6}B_5^{a_7\cdots a_{11}}\Big],$$
$$\langle \boldsymbol{Q}\boldsymbol{J}^4\overline{\boldsymbol{Q}}\rangle_{\text{osp}} = -\frac{1}{240}\varepsilon_{a_1\cdots a_8abc}L_1^{a_1a_2}\cdots L_4^{a_7a_8}\left(\bar{\chi}\Gamma^{abc}\zeta\right) + $$
$$+\frac{1}{60}\sum_{\langle i_1\cdots i_4\rangle}\Big[\frac{3}{4}\mathrm{Tr}\left(L_{i_1}L_{i_2}\right)L_{i_3}^{a_1a_2}L_{i_4}^{a_3a_4}\left(\bar{\chi}\Gamma_{a_1\cdots a_4}\zeta\right) + $$
$$-2L_{i_1}^{a_1a_2}\left[L_{i_2}L_{i_3}L_{i_4}\right]^{a_3a_4}\left(\bar{\chi}\Gamma_{a_1\cdots a_4}\zeta\right) + $$
$$+\frac{3}{4}\mathrm{Tr}\left(L_{i_1}L_{i_2}\right)\mathrm{Tr}\left(L_{i_3}L_{i_4}\right)\bar{\chi}\zeta - \mathrm{Tr}\left(L_{i_1}L_{i_2}L_{i_3}L_{i_4}\right)\bar{\chi}\zeta\Big],$$
$$\langle \boldsymbol{J}^5\boldsymbol{P}\rangle_{\text{osp}} = L_1^{a_1b_1}\cdots L_5^{a_5b_5}B_1^c\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_5b_5}\boldsymbol{P}_c\rangle_{\text{osp}},$$
$$\langle \boldsymbol{J}^6\rangle_{\text{osp}} = L_1^{a_1b_1}\cdots L_6^{a_6b_6}\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_6b_6}\rangle_{\text{osp}},$$
$$\langle \boldsymbol{J}^5\boldsymbol{Z}\rangle_{\text{osp}} = L_1^{a_1b_1}\cdots L_5^{a_5b_5}B_5^{c_1\cdots c_5}\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_5b_5}\boldsymbol{Z}_{c_1\cdots c_5}\rangle_{\text{osp}},$$
$$\langle \boldsymbol{Q}\boldsymbol{J}^4\overline{\boldsymbol{Q}}\rangle_{\text{osp}} = L_1^{a_1b_1}\cdots L_4^{a_4b_4}\bar{\chi}_\alpha\zeta^\beta\left\langle \boldsymbol{Q}^\alpha\boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_4b_4}\overline{\boldsymbol{Q}}_\beta\right\rangle_{\text{osp}}.$$

$$\langle \boldsymbol{J}^6\rangle_{\text{M}} = \frac{1}{3}\alpha_0\sum_{\langle i_1\cdots i_6\rangle}\Big[\frac{1}{4}\mathrm{Tr}\left(L_{i_1}L_{i_2}\right)\mathrm{Tr}\left(L_{i_3}L_{i_4}\right)\mathrm{Tr}\left(L_{i_5}L_{i_6}\right) + $$
$$-\mathrm{Tr}\left(L_{i_1}L_{i_2}L_{i_3}L_{i_4}\right)\mathrm{Tr}\left(L_{i_5}L_{i_6}\right) + \frac{16}{15}\mathrm{Tr}\left(L_{i_1}L_{i_2}L_{i_3}L_{i_4}L_{i_5}L_{i_6}\right)\Big]$$
$$\langle \boldsymbol{J}^5\boldsymbol{P}\rangle_{\text{M}} = \frac{1}{2}\alpha_2\varepsilon_{a_1\cdots a_{11}}L_1^{a_1a_2}\cdots L_5^{a_9a_{10}}B_1^{a_{11}}$$
$$\langle \boldsymbol{J}^5\boldsymbol{Z}_2\rangle_{\text{M}} =$$

$$\alpha_2\sum_{\langle i_1\cdots i_5\rangle}\Big[\frac{1}{2}\mathrm{Tr}\left(L_{i_1}L_{i_2}\right)\mathrm{Tr}\left(L_{i_3}L_{i_4}\right)\mathrm{Tr}\left(L_{i_5}B_2\right) + $$
$$-\frac{4}{3}\mathrm{Tr}\left(L_{i_1}L_{i_2}\right)\mathrm{Tr}\left(L_{i_3}L_{i_4}L_{i_5}B_2\right) + $$
$$-\frac{2}{3}\mathrm{Tr}\left(L_{i_1}L_{i_2}L_{i_3}L_{i_4}\right)\mathrm{Tr}\left(L_{i_5}B_2\right) + $$

$$+\frac{32}{15}\mathrm{Tr}\left(L_{i_1}L_{i_2}L_{i_3}L_{i_4}L_{i_5}B_2\right)\Big]$$

$$\langle \boldsymbol{J}^5\boldsymbol{Z}_5\rangle_{\mathrm{M}} = \frac{1}{3}\alpha_2\varepsilon_{a_1\cdots a_{11}}\sum_{\langle i_1\cdots i_5\rangle}\Big[-\frac{5}{4}L_{i_1}^{a_1a_2}\cdots L_{i_4}^{a_7a_8}\left(L_{i_5}\right)_{bc}B_5^{bca_9a_{10}a_{11}} +$$

$$+10L_{i_1}^{a_1a_2}L_{i_2}^{a_3a_4}L_{i_3}^{a_5a_6}\left(L_{i_4}\right)^{a_7}{}_b\left(L_{i_5}\right)^{a_8}{}_cB_5^{bca_9a_{10}a_{11}} +$$

$$+\frac{1}{4}\mathrm{Tr}\left(L_{i_1}L_{i_2}\right)L_{i_3}^{a_1a_2}L_{i_4}^{a_3a_4}L_{i_5}^{a_5a_6}B_5^{a_7\cdots a_{11}} +$$

$$-L_{i_1}^{a_1a_2}L_{i_2}^{a_3a_4}\left[L_{i_3}L_{i_4}L_{i_5}\right]^{a_5a_6}B_5^{a_7\cdots a_{11}}\Big],$$

$$\langle \boldsymbol{Q}\boldsymbol{J}^4\overline{\boldsymbol{Q}}\rangle_{\mathrm{M}} = -\frac{\alpha_2}{240}\varepsilon_{a_1\cdots a_8abc}L_1^{a_1a_2}\cdots L_4^{a_7a_8}(\bar{\chi}\Gamma^{abc}\zeta) +$$

$$+\frac{\alpha_2}{60}\sum_{\langle i_1\cdots i_4\rangle}\Big[\frac{3}{4}\mathrm{Tr}\left(L_{i_1}L_{i_2}\right)L_{i_3}^{a_1a_2}L_{i_4}^{a_3a_4}(\bar{\chi}\Gamma_{a_1\cdots a_4}\zeta) +$$

$$-2L_{i_1}^{a_1a_2}\left[L_{i_2}L_{i_3}L_{i_4}\right]^{a_3}{}_{3a_4}(\bar{\chi}\Gamma_{a_1\cdots a_4}\zeta) +$$

$$+\frac{3}{4}\mathrm{Tr}\left(L_{i_1}L_{i_2}\right)\mathrm{Tr}\left(L_{i_3}L_{i_4}\right)\bar{\chi}\zeta - \mathrm{Tr}\left(L_{i_1}L_{i_2}L_{i_3}L_{i_4}\right)\bar{\chi}\zeta\Big],$$

$$\langle \boldsymbol{J}^6\rangle_{\mathrm{M}} = L_1^{a_1b_1}\cdots L_6^{a_6b_6}\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_6b_6}\rangle_{\mathrm{M}},$$

$$\langle \boldsymbol{J}^5\boldsymbol{P}\rangle_{\mathrm{M}} = L_1^{a_1b_1}\cdots L_5^{a_5b_5}B_1^c\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_5b_5}\boldsymbol{P}_c\rangle_{\mathrm{M}},$$

$$\langle \boldsymbol{J}^5\boldsymbol{Z}_2\rangle_{\mathrm{M}} = L_1^{a_1b_1}\cdots L_5^{a_5b_5}B_2^{cd}\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_{55}b_5}\boldsymbol{Z}_{cd}\rangle_{\mathrm{M}}$$

$$\langle \boldsymbol{J}^5\boldsymbol{Z}_5\rangle_{\mathrm{M}} = L_1^{a_1b_1}\cdots L_5^{a_5b_5}B_5^{c_1\cdots c_5}\left\langle \boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_5}\,\boldsymbol{c}_{c_1c_5}\right\rangle_{\mathrm{M}},$$

$$\langle \boldsymbol{Q}\boldsymbol{J}^4\overline{\boldsymbol{Q}}\rangle_{\mathrm{M}} = L_1^{a_1b_1}\cdots L_4^{a_4b_4}\bar{\chi}_\alpha\zeta^\beta\langle \boldsymbol{Q}^\alpha\boldsymbol{J}_{a_1b_1}\cdots \boldsymbol{J}_{a_4b_4}\boldsymbol{Q}_\beta\rangle_{\mathrm{M}}.$$

$$\langle\cdots\rangle_{\mathrm{M}}' = \langle\cdots\rangle_{6=6} + \beta_{4+2}\langle\cdots\rangle_{6=4+2} + \beta_{2+2+2}\langle\cdots\rangle_{6=2+2+2}$$

$$\langle \boldsymbol{J}^6\rangle_{\mathrm{M}}' = \frac{1}{3}\alpha_0\sum_{\langle i_1\cdots i_6\rangle}\Big[\frac{1}{4}\gamma_5\mathrm{Tr}\left(L_{i_1}L_{i_2}\right)\mathrm{Tr}\left(L_{i_3}L_{i_4}\right)\mathrm{Tr}\left(L_{i_5}L_{i_6}\right) +$$

$$-\kappa_{15}\mathrm{Tr}\left(L_{i_1}L_{i_2}L_{i_3}L_{i_4}\right)\mathrm{Tr}\left(L_{i_5}L_{i_6}\right) + \frac{16}{15}\mathrm{Tr}\left(L_{i_1}L_{i_2}L_{i_3}L_{i_4}L_{i_5}L_{i_6}\right)\Big]$$

$$\langle \boldsymbol{J}^5\boldsymbol{P}\rangle_{\mathrm{M}}' = \frac{1}{2}\alpha_2\varepsilon_{a_1\cdots a_{11}}L_1^{a_1a_2}\cdots L_5^{a_9a_{10}}B_1^{a_{11}}$$

$$\langle \boldsymbol{J}^5\boldsymbol{Z}_2\rangle_{\mathrm{M}}' = \alpha_2\sum_{\langle i_1\cdots i_5\rangle}\Big[\frac{1}{2}\gamma_5\mathrm{Tr}\left(L_{i_1}L_{i_2}\right)\mathrm{Tr}\left(L_{i_3}L_{i_4}\right)\mathrm{Tr}\left(L_{i_5}B_2\right) +$$

$$-\frac{4}{3}\kappa_{15}\mathrm{Tr}\left(L_{i_1}L_{i_2}\right)\mathrm{Tr}\left(L_{i_3}L_{i_4}L_{i_5}B_2\right) +$$

$$-\frac{2}{3}\kappa_{15}\mathrm{Tr}\left(L_{i_1}L_{i_2}L_{i_3}L_{i_4}\right)\mathrm{Tr}\left(L_{i_5}B_2\right) +$$

$$+\frac{32}{15}\mathrm{Tr}\left(L_{i_1}L_{i_2}L_{i_3}L_{i_4}L_{i_5}B_2\right)\Big],$$

$$
\begin{aligned}
\langle J^5 \boldsymbol{Z}_5\rangle'_{\mathrm{M}} = & \frac{1}{3}\alpha_2 \varepsilon_{a_1\cdots a_{11}} \sum_{\langle i_1\cdots i_5\rangle} \Big[-\frac{5}{4} L_{i_1}^{a_1a_2}\cdots L_{i_4}^{a_4a_8}\left(L_{i_5}\right)_{bc} b_5^{bca_9a_{10}a_{11}} + \\
& +10 L_{i_1}^{a_1a_2} L_{i_2a_4}^{a_3} L_{i_3}^{a_3a_6}\left(L_{i_4}\right)^{a_7}{}_b \left(L_{i_5}\right)^{a_8}{}_c B_5^{bca_9a_{10}a_{11}} + \\
& +\frac{1}{4}\kappa_{15}\mathrm{Tr}\left(L_{i_1}L_{i_2}\right) L_{i_3}^{a_1a_2} L_{i_4a_4a_4}^{a_{55}L_5a_6} B_5^{a_7\cdots a_{11}} + \\
& -L_{i_1a_2a_2}^{L_{i_2}^{a_3a_4}[L_{i_3}L_{i_4}L_{i_5}a_5a_6} B_5^{a_7\cdots a_{11}}\Big],
\end{aligned}
$$

$$
\begin{aligned}
\left\langle \boldsymbol{Q}\boldsymbol{J}^4\overline{\boldsymbol{Q}}\right\rangle'_{\mathrm{M}} = & -\frac{\alpha_2}{240}\varepsilon_{a_1\cdots a_8abc} L_1^{a_1a_2}\cdots L_4^{a_7a_8}(\bar{\chi}\Gamma^{abc}\zeta) + \\
& +\frac{\alpha_2}{60}\sum_{\langle i_1\cdots i_4\rangle}\Big\{\frac{3}{4}\kappa_9\mathrm{Tr}\left(L_{i_1}L_{i_2}\right) L_{i_3}^{a_1a_2} L_{i_4}^{a_3a_4}\left(\bar{\chi}\Gamma_{a_1\cdots a_4}\zeta\right) + \\
& -2L_{i_1}^{a_2a_2}\left[L_{i_2}L_{i_3}L_{i_4}\right]_3^{a_3a_4}\left(\bar{\chi}\Gamma_{a_1\cdots a_4}\zeta\right) + \\
& +\frac{3}{4}(5\gamma_9-4)\mathrm{Tr}\left(L_{i_1}L_{i_2}\right)\mathrm{Tr}\left(L_{i_3}L_{i_4}\right)\bar{\chi}\zeta + \\
& -\kappa_3\mathrm{Tr}\left(L_{i_1}L_{i_2}L_{i_3}L_{i_4}\right)\bar{\chi}\zeta\Big\},
\end{aligned}
$$

$$\kappa_n = 1 + \frac{1}{n}\beta_{4+2}\mathrm{Tr}(\mathbb{1}),$$

$$\gamma_n = \kappa_n + \frac{1}{15}\beta_{2+2+2}[\mathrm{Tr}(\mathbb{1})]^2$$

$$\beta_{4+2} = \frac{1}{\mathrm{Tr}(\mathbb{1})} n(\kappa_n - 1),$$

$$\beta_{2+2+2} = \frac{15}{[\mathrm{Tr}(\mathbb{1})]^2}(\gamma_n - \kappa_n)$$

$$\kappa_m = 1 + \frac{n}{m}(\kappa_n - 1)$$

$$\gamma_m = \gamma_n + \left(\frac{n}{m} - 1\right)(\kappa_n - 1)$$

$$\beta_{4+2} = 0 \quad \Leftrightarrow \quad \kappa_n = 1$$

$$\beta_{2+2+2} = 0 \quad \Leftrightarrow \quad \gamma_n = \kappa_n.$$

$$\boldsymbol{A} = \boldsymbol{\omega} + \boldsymbol{e} + \boldsymbol{b}_2 + \boldsymbol{b}_5 + \overline{\boldsymbol{\psi}}$$

$$\overline{\boldsymbol{A}} = \overline{\boldsymbol{\omega}} + \overline{\boldsymbol{e}} + \overline{\boldsymbol{b}}_2 + \overline{\boldsymbol{b}}_5 + \bar{\chi}$$

$$\boldsymbol{\omega} = \frac{1}{2}\omega^{ab}\boldsymbol{J}_{ab},$$

$$\boldsymbol{e} = \frac{1}{\ell}e^a\boldsymbol{P}_a,$$

$$\boldsymbol{b}_2 = \frac{1}{2}b_2^{ab}\boldsymbol{Z}_{ab},$$

$$\boldsymbol{b}_5 = \frac{1}{5!}b_5^{abcde}\boldsymbol{Z}_{abcde},$$

$$\overline{\boldsymbol{\psi}} = \bar{\psi}_\alpha \boldsymbol{Q}^\alpha$$

$$\boldsymbol{F} = \boldsymbol{R} + \boldsymbol{F}_P + \boldsymbol{F}_2 + \boldsymbol{F}_5 + \mathrm{D}_\omega\overline{\boldsymbol{\psi}}$$

$$\boldsymbol{R} = \frac{1}{2}R^{ab}\boldsymbol{J}_{ab},$$
$$\boldsymbol{F}_P = \left(\frac{1}{\ell}T^a + \frac{1}{16}\bar{\psi}\Gamma^a\psi\right)\boldsymbol{P}_a,$$
$$\boldsymbol{F}_2 = \frac{1}{2}\left(\mathrm{D}_\omega b^{ab} - \frac{1}{16}\bar{\psi}\Gamma^{ab}\psi\right)\boldsymbol{Z}_{ab},$$
$$\boldsymbol{F}_5 = \frac{1}{5!}\left(\mathrm{D}_\omega b^{a_1\cdots a_5} + \frac{1}{16}\bar{\psi}\Gamma^{a_1\cdots a_5}\psi\right)\boldsymbol{Z}_{a_1\cdots a_5},$$
$$\mathrm{D}_\omega\overline{\boldsymbol{\psi}} = \mathrm{D}_\omega\bar{\psi}\boldsymbol{Q}$$
$$\mathrm{D}_\omega\psi = \mathrm{d}\psi + \frac{1}{4}\omega^{ab}\Gamma_{ab}\psi,$$
$$\mathrm{D}_\omega\bar{\psi} = \mathrm{d}\bar{\psi} - \frac{1}{4}\omega^{ab}\bar{\psi}\Gamma_{ab}$$
$$\delta_{\text{gauge}}\boldsymbol{A} = -\mathrm{D}\boldsymbol{\lambda},$$
$$\delta_{\text{gauge}}\overline{\boldsymbol{A}} = -\overline{\mathrm{D}}\boldsymbol{\lambda}$$
$$\boldsymbol{\lambda} = \frac{1}{2}\lambda^{ab}\boldsymbol{J}_{ab} + \frac{1}{\ell}\kappa^a\boldsymbol{P}_a + \frac{1}{2}\kappa^{ab}\boldsymbol{Z}_{ab} + \frac{1}{5!}\kappa^{abcde}\boldsymbol{Z}_{abcde} + \bar{\varepsilon}\boldsymbol{Q}$$

- $\boldsymbol{\lambda} = (1/\ell)\kappa^a\boldsymbol{P}_a,$

$$\delta e^a = -\mathrm{D}_\omega\kappa^a,$$
$$\delta b_2^{ab} = 0,$$
$$\delta b_5^{abcde} = 0,$$
$$\delta\omega^{ab} = 0,$$
$$\delta\psi = 0$$

- $\boldsymbol{Z}_2: \boldsymbol{\lambda} = (1/2)\kappa^{ab}\boldsymbol{Z}_{ab},$

$$\delta e^a = 0,$$
$$\delta b_2^{ab} = -\mathrm{D}_\omega\kappa^{ab},$$
$$\delta b_5^{abcde} = 0,$$
$$\delta\omega^{ab} = 0,$$
$$\delta\psi = 0.$$

- $\boldsymbol{Z}_5: \boldsymbol{\lambda} = (1/5!)\kappa^{abcde}\boldsymbol{Z}_{abcde},$

$$\delta e^a = 0,$$
$$\delta b_2^{ab} = 0,$$
$$\delta b_5^{abcde} = -\mathrm{D}_\omega\kappa^{abcde},$$
$$\delta\omega^{ab} = 0,$$
$$\delta\psi = 0.$$

- $\boldsymbol{\lambda} = (1/2)\lambda^{ab}\boldsymbol{J}_{ab},$

$$\delta e^a = \lambda^a{}_b e^b,$$
$$\delta b_2^{ab} = -2\lambda^{[a}{}_c b_2^{b]c},$$
$$\delta b_5^{abcde} = 5\lambda^{[a}{}_f b_5^{bced]},$$
$$\delta\omega^{ab} = -\mathrm{D}_\omega\lambda^{ab},$$
$$\delta\psi = \frac{1}{4}\lambda^{ab}\Gamma_{ab}\psi.$$

- $\boldsymbol{\lambda} = \bar{\varepsilon}\boldsymbol{Q},$

$$\delta e^{a} = \frac{\ell}{8}\bar{\varepsilon}\Gamma^{a}\psi,$$

$$\delta b_2^{ab} = -\frac{1}{8}\bar{\varepsilon}\Gamma^{ab}\psi,$$

$$\delta b_5^{abcde} = \frac{1}{8}\bar{c}\Gamma^{abcde}\psi,$$

$$\delta\omega^{ab} = 0,$$

$$\delta\psi = -\mathrm{D}_{\omega}\varepsilon$$

$$L_{\mathrm{M}}^{(11)}(\boldsymbol{A},\overline{\boldsymbol{A}}) = \mathcal{Q}^{(11)}_{\boldsymbol{A}\leftarrow\overline{\boldsymbol{A}}}$$

$$L_{\mathrm{M}}^{(11)}(\boldsymbol{A},\overline{\boldsymbol{A}}) = \mathcal{Q}^{(11)}_{\boldsymbol{A}\leftarrow\overline{\boldsymbol{\omega}}} + \mathcal{Q}^{(11)}_{\overline{\boldsymbol{\omega}}\leftarrow\overline{\boldsymbol{A}}} + \mathrm{d}\mathcal{Q}^{(10)}_{\boldsymbol{A}\leftarrow\overline{\boldsymbol{\omega}}\leftarrow\overline{\boldsymbol{A}}}$$

$$\mathcal{Q}^{(11)}_{A\leftarrow\bar{\omega}} = \mathcal{Q}^{(11)}_{\boldsymbol{A}\leftarrow\omega} + \mathcal{Q}^{(11)}_{\omega\leftarrow\bar{\omega}} + \mathrm{d}\mathcal{Q}^{(10)}_{\boldsymbol{A}\leftarrow\omega\leftarrow\bar{\omega}}$$

$$L_{\mathrm{M}}^{(11)}(\boldsymbol{A},\overline{\boldsymbol{A}}) = \mathcal{Q}^{(11)}_{\boldsymbol{A}\leftarrow\boldsymbol{\omega}} - \mathcal{Q}^{(11)}_{\overline{\boldsymbol{A}}\leftarrow\bar{\omega}} + \mathcal{Q}^{(11)}_{\boldsymbol{\omega}\leftarrow\overline{\boldsymbol{\omega}}} + \mathrm{d}\mathcal{B}_{\mathrm{M}}^{(10)}$$

$$\mathcal{B}_{\mathrm{M}}^{(10)} = \mathcal{Q}^{(10)}_{\boldsymbol{A}\leftarrow\boldsymbol{\omega}\leftarrow\bar{\omega}} + \mathcal{Q}^{(10)}_{\boldsymbol{A}\leftarrow\bar{\omega}\leftarrow\overline{\boldsymbol{A}}}$$

$$\boldsymbol{A}_0 = \boldsymbol{\omega}$$

$$\boldsymbol{A}_1 = \boldsymbol{\omega} + \boldsymbol{e}$$

$$\boldsymbol{A}_2 = \boldsymbol{\omega} + \boldsymbol{e} + \boldsymbol{b}_2$$

$$\boldsymbol{A}_3 = \boldsymbol{\omega} + \boldsymbol{e} + \boldsymbol{b}_2 + \boldsymbol{b}_5$$

$$\boldsymbol{A}_4 = \boldsymbol{\omega} + \boldsymbol{e} + \boldsymbol{b}_2 + \boldsymbol{b}_5 + \overline{\boldsymbol{\psi}}$$

$$\mathcal{Q}^{(11}_{\boldsymbol{A_4}\leftarrow\boldsymbol{A}_0} = \mathcal{Q}^{(11)}_{\boldsymbol{A}_4\leftarrow\boldsymbol{A}_3} + \mathcal{Q}^{(11)}_{\boldsymbol{A_3}\leftarrow\boldsymbol{A}_0} + \mathrm{d}\mathcal{Q}^{(10)}_{\boldsymbol{A}_4\leftarrow\boldsymbol{A}_3\leftarrow\boldsymbol{A}_0}$$

$$\mathcal{Q}^{(11)}_{\boldsymbol{A_3}\leftarrow\boldsymbol{A}_0} = \mathcal{Q}^{(11)}_{\boldsymbol{A}_3\leftarrow\boldsymbol{A_2}} + \mathcal{Q}_{\boldsymbol{A}_2\leftarrow\boldsymbol{A}_0} + \mathrm{d}\mathcal{Q}_{\boldsymbol{A}_3\leftarrow\boldsymbol{A}_2\leftarrow\boldsymbol{A}_0}$$

$$\mathcal{Q}_{\boldsymbol{A}_2\leftarrow\boldsymbol{A}_0} = \mathcal{Q}_{\boldsymbol{A_2}\leftarrow\boldsymbol{A}_1} + \mathcal{Q}_{\boldsymbol{A}_1\leftarrow\boldsymbol{A}_0} + \mathrm{d}\mathcal{Q}_{\boldsymbol{A_2}\leftarrow\boldsymbol{A}_1\leftarrow\boldsymbol{A}_0}$$

$$\mathcal{Q}^{(11)}_{\boldsymbol{A}_4\leftarrow\boldsymbol{A}_0} = 6\left[H_a e^a + \frac{1}{2}H_{ab}b_2^{ab} + \frac{1}{5!}H_{abcde}b_5^{abcde} - \frac{5}{2}\bar{\psi}\mathcal{R}\mathrm{D}_{\omega}\psi\right]$$

$$H_a \equiv \langle \boldsymbol{R}^5\boldsymbol{P}_a\rangle_{\mathrm{M}},$$

$$H_{ab} \equiv \langle \boldsymbol{R}^5\boldsymbol{Z}_{ab}\rangle_{\mathrm{M}},$$

$$H_{abcce} \equiv \langle \boldsymbol{R}^5\boldsymbol{Z}_{abcde}\rangle_{\mathrm{M}},$$

$$\mathcal{R}^{\alpha}{}_{\beta} \equiv \left\langle \boldsymbol{Q}^{\alpha}\boldsymbol{R}^4\overline{\boldsymbol{Q}}_{\beta}\right\rangle_{\mathrm{M}},$$

$$H_a = \frac{\alpha_2}{64}R_a^{(5)},$$

$$H_{ab} = \alpha_2\left[\frac{5}{2}\left(R^4 - \frac{3}{4}R^2R^2\right)R_{ab} + 5R^2R^3_{ab} - 8R^5_{ab}\right]$$

$$H_{abcde} = -\frac{5}{16}\alpha_2\left[5R_{[ab}R^{(4)}_{cde]} - 40R^{f}_{[a}R^{g}{}_{b}R^{(3)}_{cde]fg} + \right.$$

$$\left. -R^2R^{(3)}_{abcde} + 4R^{(2)}_{abcdefg}(R^3)^{fg}\right],$$

$$\mathcal{R} = -\frac{\alpha_2}{40}\left\{\left(R^4 - \frac{3}{4}R^2R^2\right)\mathbb{1} + \frac{1}{96}R^{(4)}_{abc}\Gamma^{abc} + \right.$$

$$\left. -\frac{3}{4}\left[R^2R^{ab} - \frac{8}{3}(R^3)^{ab}\right]R^{cd}\Gamma_{abcd}\right\}$$

$$R^n = R^{a_1}_{a_{a_2}\cdots}\cdots R^{a_n}_{a_1}$$

$$R^n_{ab} = R^{c_1}_{ac_1}{}^{c}{}_{c_2}\cdots R^{c_{n-1}}{}_{b}$$

$$R^{(n)}_{a_1\cdots a_{d-2n}} = \varepsilon_{a_1\cdots a_{d-2n}b_1\cdots b_{2n}}R^{b_1b_2}\ldots R^{b_{2n-1}b_{2n}}$$

$$\mathcal{Q}^{(11)}_{\omega\leftarrow\bar{\omega}} = 3\int_0^1 dt\theta^{ab}L_{ab}(t)$$
$$L_{ab}(t) = \langle \boldsymbol{R}_t^5 \boldsymbol{J}_{ab}\rangle_{\mathrm{M}}$$
$$\boldsymbol{R}_t = \frac{1}{2}R_t^{ab}\boldsymbol{J}_{ab},$$
$$R_t^{ab} = \bar{R}^{ab} + t\mathrm{D}_{\bar{\omega}}\theta^{ab} + t^2\theta^a{}_c\theta^{cb}$$
$$L_{ab}(t) = \alpha_0\left[\frac{5}{2}\left(R_t^4 - \frac{3}{4}R_t^2R_t^2\right)(R_t)_{ab} + 5R_t^2(R_t^3)_{ab} - 8(R_t^5)_{ab}\right]$$
$$\mathcal{Q}^{(11)}_{\omega\leftarrow\bar{\omega}} = \mathcal{Q}^{(11)}_{\omega\leftarrow 0} - \mathcal{Q}^{(11)}_{\bar{\omega}\leftarrow 0} + \mathrm{d}\mathcal{Q}^{(10)}_{\omega\leftarrow \boldsymbol{A}_3\leftarrow\bar{\omega}}$$
$$\langle \boldsymbol{F}^5\boldsymbol{G}_A\rangle_{\mathrm{M}} = 0$$
$$H_a = 0,$$
$$H_{ab} = 0,$$
$$H_{abcde} = 0,$$
$$\mathcal{R}\mathrm{D}_\omega\psi = 0$$
$$L_{ab} - 10(\mathrm{D}_\omega\bar{\psi})\mathcal{Z}_{ab}(\mathrm{D}_\omega\psi) + 5H_{abc}\left(T^c + \frac{1}{16}\bar{\psi}\Gamma^c\psi\right) +$$
$$+\frac{5}{2}H_{abcd}\left(\mathrm{D}_\omega b^{cd} - \frac{1}{16}\bar{\psi}\Gamma^{cd}\psi\right) + \frac{1}{24}H_{abc_1\cdots c_5}\left(\mathrm{D}_\omega b^{c_1\cdots c_5} + \frac{1}{16}\bar{\psi}\Gamma^{c_1\cdots c_5}\psi\right) = 0$$
$$L_{ab} \equiv \langle \boldsymbol{R}^5\boldsymbol{J}_{ab}\rangle_{\mathrm{M}},$$
$$(\mathcal{Z}_{ab})^\alpha \equiv \left\langle \boldsymbol{Q}^\alpha \boldsymbol{R}^3\boldsymbol{J}_{ab}\overline{\boldsymbol{Q}}_\beta\right\rangle_{\mathrm{M}},$$
$$H_{abc} \equiv \langle \boldsymbol{R}^4\boldsymbol{J}_{ab}\boldsymbol{P}_c\rangle_{\mathrm{M}},$$
$$H_{abcd} \equiv \langle \boldsymbol{R}^4\boldsymbol{J}_{ab}\boldsymbol{Z}_{cd}\rangle_{\mathrm{M}},$$
$$H_{abcdefg} \equiv \left\langle \boldsymbol{R}^4\boldsymbol{J}_{ab}\boldsymbol{Z}_{cdefg}\right\rangle_{\mathrm{M}}.$$
$$H_c = \frac{1}{2}R^{ab}H_{abc},$$
$$H_{cd} = \frac{1}{2}R^{ab}H_{abcd},$$
$$H_{cdefg} = \frac{1}{2}R^{ab}H_{abcdefg},$$
$$\mathcal{R}^\alpha_\beta = \frac{1}{2}R^{ab}(\mathcal{Z}_{ab})^\alpha{}_\beta$$
$$L_{ab} = \alpha_0\left[\frac{5}{2}\left(R^4 - \frac{3}{4}R^2R^2\right)R_{ab} + 5R^2R^3_{ab} - 8R^5_{ab}\right]$$
$$\mathcal{Z}_{ab} = \frac{\alpha_2}{40}\left\{2\left(R^3_{ab} - \frac{3}{4}R^2R_{ab}\right)\mathbb{1} - \frac{1}{48}R^{(3)}_{abcde}\Gamma^{cde} + \right.$$
$$-\frac{3}{4}\left(R_{ab}R^{cd} - \frac{1}{2}R^2\delta^{cd}_{ab}\right)R^{ef}\Gamma_{cdef} +$$
$$\left.-\left[\delta^{cg}_{ab}R_{gh}R^{hd}R^{ef} - R^c_aR^d_bR^{ef} + \frac{1}{2}\delta^{ef}_{ab}(R^3)^{cd}\right]\Gamma_{cdef}\right)$$
$$H_{abc} = \frac{\alpha_2}{32}R^{(4)}_{abc}$$

$$H_{abcd} =\alpha_2\delta^{ef}_{ab}g^{gh}_{cd}\Big[\frac{3}{4}R^2R_{ef}R_{gh} - R^3_{ef}R_{gh} - R_{ef}R^3_{gh} +$$
$$-\frac{4}{5}\left(R_{eh}R^3_{fg} + R^3_{eh}R_{fg} - R^2_{eh}R^2_{fg}\right) + \frac{1}{2}R^2R_{eh}R_{fg} +$$
$$+\frac{1}{8}\eta_{[ef][gh]}\left(R^4 - \frac{3}{4}R^2R^2\right) - \eta_{fg}\left(R^2R^2_{eh} - \frac{8}{5}R^4_{eh}\right)\Big],$$
$$H_{abc_1\cdots c_5} =\frac{\alpha_2}{80}\delta^{\mathrm{clefg}}_{c_1\ldots c_5}\Big(-\frac{5}{3}R^{(3)}_{abccde}R_{fg} + 10R^{(2)}_{\mathrm{abcdepq}}R^p{}_fR^q{}_g +$$
$$-\frac{1}{6}R_{ab}R^{(3)}_{\mathrm{cdefg}} + \frac{1}{4}R^2R^{(2)}_{\mathrm{abcdefg}} - \frac{2}{3}R^{(1)}_{\mathrm{abcdefgpq}}(R^3)^{pq} +$$
$$+\frac{1}{3}R^p{}_aR^q{}_bR^{(2)}_{\mathrm{clefgpq}} - \frac{1}{3}R^q{}_aR^{(2)}_{bcdefgp}R^p{}_q + \frac{1}{3}R^q{}_bR^{(2)}_{\mathrm{acclefgp}}R^p{}_q +$$
$$-\frac{10}{3}\eta_{ga}R^{(3)}_{bcdep}R^p{}_f + \frac{10}{3}\eta_{gb}R^{(3)}_{\mathrm{acdep}}R^p{}_f - \frac{5}{24}\eta_{[ab][cd]}R^{(4)}_{\mathrm{efg}}\Big).$$
$$\langle \boldsymbol{F}^n\boldsymbol{G}_A\rangle = 0$$
$$\mathrm{d}s^2 = e^{-2\xi|z|}\left(\mathrm{d}z^2 + \tilde{g}^{(d)}_{\alpha\beta}\mathrm{d}x^\alpha\mathrm{d}x^\beta\right) + \gamma^{(10-d)}_{\kappa\lambda}\mathrm{d}y^\kappa\mathrm{d}y^\lambda$$
$$R^{ab} = \tilde{R}^{ab} - \xi^2\tilde{e}^a\tilde{e}^b + 2\xi\theta(z)(\tilde{e}^a\kappa^b - \tilde{e}^b\kappa^a) - \kappa^a\kappa^b,$$
$$R^{aZ} = -2e^{\xi|z|}\xi\delta(z)E^Z\tilde{e}^a - 2\xi\theta(z)\tilde{T}^a + \mathrm{D}_{\tilde{\omega}}\kappa^a,$$
$$T^a = \kappa^aE^Z + e^{-\xi|z|}\tilde{T}^a,$$
$$T^Z = -e^{-\xi|z|}\kappa^a\tilde{e}_a$$
$$\xi e^{\xi|z|}\delta(z)E^Z\varepsilon_{abcd}\left(\tilde{R}^{ab} - \xi^2\tilde{e}^a\tilde{e}^b\right)\tilde{e}^c = \mathcal{T}_d$$
$$\varepsilon_{abcd}\left(\tilde{R}^{ab} - \xi^2\tilde{e}^a\tilde{e}^b\right)\left(\tilde{R}^{cd} - \xi^2\tilde{e}^c\tilde{e}^d\right) = \mathcal{T}$$
$$\xi e^{\xi|z|}\delta(z)E^Z\varepsilon_{abcd}\left(\tilde{R}^{ab} - \xi^2\tilde{e}^a\tilde{e}^b\right)\tilde{e}^c = \mathcal{T}_d,$$
$$\varepsilon_{abcd}\left(\tilde{R}^{ab} - \xi^2\tilde{e}^a\tilde{e}^b\right)\left(\tilde{R}^{cd} - \xi^2\tilde{e}^c\tilde{e}^d\right) = \mathcal{T},$$
$$\mathcal{T}_d =2E^Ze^{\xi|z|}\xi\delta(z)\varepsilon_{abcd}\left(\frac{1}{2}\kappa^a\kappa^b - \xi\theta(z)(\tilde{e}^a\kappa^b - \tilde{e}^b\kappa^a)\right)\tilde{e}^c +$$
$$+\varepsilon_{abcd}\left[\tilde{R}^{ab} - \xi^2\tilde{e}^a\tilde{e}^b + 2\xi\theta(z)(\tilde{e}^a\kappa^b - \tilde{e}^b\kappa^a) - \kappa^a\kappa^b\right] \times$$
$$\times\left(\frac{1}{2}\mathrm{D}_{\tilde{\omega}}\kappa^c - \xi\theta(z)\tilde{T}^c\right),$$
$$\mathcal{T} = -4\varepsilon_{abcd}\left(\tilde{R}^{ab} - \xi^2\tilde{e}^a\tilde{e}^b + \xi\theta(z)(\tilde{e}^a\kappa^b - \tilde{e}^b\kappa^a) - \frac{1}{2}\kappa^a\kappa^b\right) \times$$
$$\times\left(\xi\theta(z)(\tilde{e}^c\kappa^d - \tilde{e}^d\kappa^c) - \frac{1}{2}\kappa^c\kappa^d\right)$$
$$\delta L = E(\phi)\delta\phi + \mathrm{d}\Xi(\phi,\delta\phi)$$
$$\delta_{\mathrm{gauge}}L = \mathrm{d}\Omega$$
$$\star J_{\mathrm{gauge}} = \Omega - \Xi_{\mathrm{gauge}}$$
$$\delta_{\mathrm{dif}}L = -\pounds_\xi L$$
$$= -\left(\mathrm{dI}_\xi + \mathrm{I}_\xi\mathrm{d}\right)L$$
$$= -\mathrm{dI}_\xi L,$$
$$\star J_{\mathrm{dif}} = -\Xi_{\mathrm{dif}} - \mathrm{I}_\xi L$$
$$\Xi = n(n+1)k\int_0^1 dt\langle\delta\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}^{n-1}_t\rangle$$
$$\star J_{\mathrm{gauge}} = \Omega - \Xi_{\mathrm{gauge}}$$
$$\Omega = 0$$

$$\delta_{\text{gauge}}\,\boldsymbol{A} = -\mathrm{D}\boldsymbol{\lambda},$$
$$\delta_{\text{gauge}}\,\overline{\boldsymbol{A}} = -\overline{\mathrm{D}}\boldsymbol{\lambda}$$
$$\delta_{\text{gauge}}\,\boldsymbol{A}_t = -\mathrm{D}_t\boldsymbol{\lambda}$$
$$\Xi_{\text{gauge}} = -n(n+1)k\int_0^1 dt\langle \mathrm{D}_t\boldsymbol{\lambda}\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle$$
$$\Xi_{\text{gauge}} = -n(n+1)k\,\mathrm{d}\int_0^1 dt\langle \boldsymbol{\lambda}\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle + n(n+1)k\int_0^1 dt\langle \boldsymbol{\lambda}\mathrm{D}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle$$
$$\Xi_{\text{gauge}} = -n(n+1)k\,\mathrm{d}\int_0^1 dt\langle \boldsymbol{\lambda}\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle + (n+1)k\int_0^1 dt\frac{d}{dt}\langle \boldsymbol{\lambda}\boldsymbol{F}_t^{n}\rangle$$
$$\Xi_{\text{gauge}} = -n(n+1)k\,\mathrm{d}\int_0^1 dt\langle \boldsymbol{\lambda}\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle + (n+1)k\left(\langle \boldsymbol{\lambda}\boldsymbol{F}^{n}\rangle - \left\langle \boldsymbol{\lambda}\overline{\boldsymbol{F}}^{n}\right\rangle\right)$$
$$\star J_{\text{gauge}} = n(n+1)k\,\mathrm{d}\int_0^1 dt\langle \boldsymbol{\lambda}\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\rangle$$
$$\star J_{\text{dif}} = -\Xi_{\text{dif}} - \mathrm{I}_\xi L_{\mathrm{T}}^{(2n+1)}$$
$$\delta_{\text{dif}}\boldsymbol{A} = -\pounds_\xi\boldsymbol{A},$$
$$\delta_{\text{dif}}\overline{\boldsymbol{A}} = -\pounds_\xi\overline{\boldsymbol{A}}$$
$$\delta_{\text{dif}}\boldsymbol{A}_t = -\pounds_\xi\boldsymbol{A}_t$$
$$\Xi_{\text{dif}} = -n(n+1)k\int_0^1 dt\left\langle \pounds_\xi\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\right\rangle$$
$$\pounds_\xi\boldsymbol{A}_t = \mathrm{I}_\xi\boldsymbol{F}_t + \mathrm{D}_t\mathrm{I}_\xi\boldsymbol{A}_t$$
$$\Xi_{\text{dif}} = -n(n+1)k\int_0^1 dt\left\langle \mathrm{I}_\xi\boldsymbol{F}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\right\rangle - n(n+1)k\int_0^1 dt\left\langle \mathrm{D}_t\mathrm{I}_\xi\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\right\rangle$$
$$\Xi_{\text{dif}} = -\mathrm{I}_\xi L_{\mathrm{T}}^{(2n+1)} + (n+1)k\int_0^1 dt\left\langle \mathrm{I}_\xi\boldsymbol{\Theta}\boldsymbol{F}_t^{n}\right\rangle +$$
$$-n(n+1)k\,\mathrm{d}\int_0^1 dt\left\langle \mathrm{I}_\xi\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\right\rangle +$$
$$+n(n+1)k\int_0^1 dt\left\langle \mathrm{I}_\xi\boldsymbol{A}_t\mathrm{D}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\right\rangle.$$
$$\frac{d}{dt}\boldsymbol{F}_t = \mathrm{D}_t\boldsymbol{\Theta}$$
$$\frac{d}{dt}\mathrm{I}_\xi\boldsymbol{A}_t = \mathrm{I}_\xi\boldsymbol{\Theta}$$
$$\Xi_{\text{dif}} = -\mathrm{I}_\xi L_{\mathrm{T}}^{(2n+1)} + (n+1)k\int_0^1 dt\frac{d}{dt}\left\langle \mathrm{I}_\xi\boldsymbol{A}_t\boldsymbol{F}_t^{n}\right\rangle +$$
$$-n(n+1)k\,\mathrm{d}\int_0^1 dt\left\langle \mathrm{I}_\xi\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\right\rangle,$$
$$\Xi_{\text{dif}} + \mathrm{I}_\xi L_{\mathrm{T}}^{(2n+1)} = -n(n+1)k\,\mathrm{d}\int_0^1 dt\left\langle \mathrm{I}_\xi\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\right\rangle +$$
$$+(n+1)k\left(\left\langle \mathrm{I}_\xi\boldsymbol{A}\boldsymbol{F}^{n}\right\rangle - \left\langle \mathrm{I}_\xi\overline{\boldsymbol{A}\boldsymbol{F}}^{n}\right\rangle\right).$$
$$\star J_{\text{dif}} = n(n+1)k\,\mathrm{d}\int_0^1 dt\left\langle \mathrm{I}_\xi\boldsymbol{A}_t\boldsymbol{\Theta}\boldsymbol{F}_t^{n-1}\right\rangle$$

$$\delta^{a_1\cdots a_n}_{b_1\cdots b_n} \equiv \det\begin{bmatrix} \delta^{a_1}_{b_1} & \delta^{a_1}_{b_2} & \cdots & \delta^{a_1}_{b_n} \\ \delta^{a_2}_{b_1} & \delta^{a_2}_{b_2} & \cdots & \delta^{a_2}_{b_n} \\ \vdots & \vdots & \ddots & \vdots \\ \delta^{a_n}_{b_1} & \delta^{a_n}_{b_2} & \cdots & \delta^{a_n}_{b_n} \end{bmatrix}$$

$$\delta^{a_1\cdots a_r a_{r+1}\cdots a_n}_{b_1\cdots b_r a_{r+1}\cdots a_n} = \frac{(d-r)!}{(d-n)!}\delta^{a_1\cdots a_r}_{b_1\cdots b_r}$$

$$\delta^{a_1\cdots a_n}_{a_1\cdots a_n} = \frac{d!}{(d-n)!}$$

$$\delta^{a_1\cdots a_n}_{b_1\cdots b_n} A^{b_1\cdots b_n} = n!\, A^{a_1\cdots a_n},$$

$$\delta^{a_n}_{b_1\cdots b_n} A_{a_1\cdots a_n} = n!\, A_{b_1\cdots b_n}$$

$$\varepsilon_{a_1\cdots a_d} = \delta^{1\cdots\cdots}_{a_1\cdots a_d},$$

$$\varepsilon^{a_1\cdots a_d} = \delta^{a_1\cdots a_d}_{1\cdots d}$$

$$\varepsilon^{a_1\cdots a_d}\varepsilon_{b_1\cdots b_d} = \delta^{a_1\cdots a_d}_{b_1\cdots b_d}$$

$$\delta^{a_1\cdots a_n}_{b_1\cdots b_n} = \sum_{p=1}^{n} (-1)^{p+1}\delta^{a_1}_{b_p}\mathcal{V}^{a_2\cdots a_n}_{b_1\cdots b_p\cdots b_n}$$

$$\delta^{a_1\cdots a_n}_{b_1\cdots b_n} = \sum_{p=1}^{n-1}\sum_{q=p+1}^{n} (-1)^{p+q+1}\delta^{a_1 a_2}_{b_p b_q}\delta^{a_2\cdots\cdots}_{b_1\cdots\hat{b}_p\cdots\hat{b}_q\cdots b_n}$$

$$\delta^{a_1\cdots a_n}_{b_1\cdots b_n} = (-1)^{r(r+1)/2}\sum_{p_1=1}^{n-r+1}\sum_{p_2=p_1+1}^{n-r+2}\cdots\sum_{p_{r-1}=p_{r-2}+1}^{n-1}$$

$$\sum_{p_r=p_{r-1}+1}^{n} (-1)^{p_1+\cdots+p_r}\delta^{\delta^{a_{r+1}\cdots a_n}_{b_1\cdots b_{p_1}\cdots b_{p_r}\cdots b_n}\cdot}_{b^{a_{11}\cdots a_r}_{p_1\cdots b_{p_r}}}$$

$$\delta^{a_1\cdots a_n}_{b_1\cdots b_n} = (-1)^{r(r+1)/2}\sum_{p_1=1}^{n-r+1}\sum_{p_2=p_1+1}^{n-r+2}\cdots\sum_{p_{r-1}=p_{r-2}+1}^{n-1}$$

$$\sum_{p_r=p_{r-1}+1}^{n} (-1)^{p_1+\cdots+p_r}\delta^{a_{p_1}\cdots a_{p_r}}_{b_1\cdots b_r}\delta^{a_1\cdots\hat{a}_{p_1}\cdots\hat{a}_{p_r}\cdots a_n}_{b_{r+1}\cdots b_n}.$$

$$\delta^{a_1\cdots a_n}_{b_1\cdots b_n} = \binom{n}{p}\delta^{[a_1\cdots a_p}_{b_1\cdots b_p}\delta^{a_{p+1}\cdots a_n]}_{b_{p+1}\cdots b_n} = \binom{n}{p}\delta^{a_1\cdots a_p}_{[b_1\cdots b_p}\delta^{a_{p+1}\cdots a_n}_{b_{p+1}\cdots b_n]}$$

$$\eta_{[a_1\cdots a_p][b_1\cdots b_p]} \equiv \delta^{c_1\cdots c_p}_{a_1\cdots a_p}\left(\eta_{b_1 c_1}\cdots\eta_{b_p c_p}\right),$$

$$\eta^{[a_1\cdots a_p][b_1\cdots b_p]} \equiv \delta^{a_1\cdots a_p}_{c_1\cdots c_p}\left(\eta^{b_1 c_1}\cdots\eta_p b_p c_p\right)$$

$$\eta_{[b_1\cdots b_p][a_1\cdots a_p]} = \eta_{[a_1\cdots a_p][b_1\cdots b_p]},$$

$$\eta^{b_1\cdots b_p][a_1\cdots a_p]} = \eta^{a_1\cdots a_p][b_1\cdots b_p]}.$$

$$\eta_{[a_1\cdots a_p][b_1\cdots b_p]}A^{b_1\cdots b_p} = p!\, A_{a_1\cdots a_p},$$

$$\eta^{[a_1\cdots a_p][b_1\cdots b_p]}A_{b_1\cdots b_p} = p!\, A^{a_1\cdots a_p}.$$

$$\eta^{[a_1\cdots a_p][c_1\cdots c_p]}\eta_{[c_1\cdots c_p][b_1\cdots b_p]} = p!\,\delta^{a_1}_{b_1\cdots b_p\cdots a_p}$$

$$\{\Gamma_a,\Gamma_b\} = 2\eta_{ab}$$

$$\Gamma_{a_1\cdots a_p} \equiv \Gamma_{[a_1}\cdots\Gamma_{a_p]}$$

$$= \frac{1}{p!}\delta^{b_1\cdots b_p}_{a_1\cdots a_p}\Gamma_{b_1}\cdots\Gamma_{b_p}$$

$$
\begin{aligned}
\Gamma_* &\equiv \Gamma_0 \cdots \Gamma_{d-1} \\
&= \frac{1}{d!}\varepsilon^{a_1\cdots a_d}\Gamma_{a_1}\cdots\Gamma_{a_d} \\
&= \frac{1}{d!}\varepsilon^{a_1\cdots a_d}\Gamma_{a_1\cdots a_d} \\
\Gamma_*^2 &= (-1)^{(d-2)(d+1)/2} \\
\Gamma_*\Gamma_a &= (-1)^{d+1}\Gamma_a\Gamma_*, \\
\Gamma_*\Gamma_{a_1\cdots a_p} &= (-1)^{p(d+1)}\Gamma_{a_1\cdots a_p}\Gamma_*. \\
\Gamma_{a_1\cdots a_{d-k}} &= \frac{1}{k!}(-1)^{k(k-1)/2}\varepsilon_{a_1\cdots a_d}\Gamma^{a_{d-k+1}\cdots a_d}\Gamma_* \\
\Gamma_a^T &= \xi C\Gamma_a C^{-1} \\
C^T &= \lambda C
\end{aligned}
$$

$d$mod8	λ	ξ	S	A
0	+1	+1	1,4	2,3
0	+1	-1	3,4	1,2
1	+1	+1	1,4	2,3
2	+1	+1	1,4	2,3
2	-1	-1	1,2	3,4
3	-1	-1	1,2	3,4
4	-1	+1	2,3	1,4
4	-1	-1	1,2	3,4
5	-1	+1	2,3	1,4
6	-1	+1	2,3	1,4
6	+1	-1	3,4	1,2
7	+1	-1	3,4	1,2

$$
\left(C\Gamma_{a_1\cdots a_p}\right)^T = (-1)^{p(p-1)/2}\lambda\xi^p\left(C\Gamma_{a_1\cdots a_p}\right)
$$

$$
\mathrm{Tr}\left(\Gamma^{a_1\cdots a_p}\Gamma_{b_1\cdots b_q}\right) = (-1)^{p(p-1)/2}m\delta^{a_1\cdots a_p}_{b_1\cdots b_q}
$$

$$
M = \sum_{p\geq 0}\frac{1}{p!}k_{a_1\cdots a_p}\Gamma^{a_1\cdots a_p}
$$

$$
k_{a_1\cdots a_p} = \frac{1}{m}(-1)^{p(p-1)/2}\mathrm{Tr}\left(M\Gamma_{a_1\cdots a_p}\right)
$$

$$
\Gamma_{a_1\cdots a_i}\Gamma^{b_1\cdots b_j} = \sum_{k=|i-j|}^{i+j}\frac{i!\,j!}{s!\,t!\,u!}\delta^{[b_1}_{[a_i}\cdots\delta^{b_s}_{a_{t+1}}\Gamma_{a_1\cdots a_t]}{}^{b_{s+1}\cdots b_j]}
$$

$$s=\frac{1}{2}(i+j-k)$$
$$t=\frac{1}{2}(i-j+k)$$
$$u=\frac{1}{2}(-i+j+k)$$
$$\Gamma^{a_1\cdots a_i}\Gamma_{b_1\cdots b_j}=\sum_{s=0}^{\min(i,j)}\frac{1}{t!\,u!}(-1)^{s(s-1)/2}\delta^{a_1\cdots a_i e_1\cdots e_u}_{d_1\cdots d_t b_1\cdots b_j}\Gamma^{d_1\cdots d_t}{}_{e_1\cdots e_u}$$
$$t=i-s,$$
$$u=j-s$$
$$\left[\Gamma^{a_1\cdots a_i},\Gamma_{b_1\cdots b_j}\right]=\sum_{s=0}^{\min(i,j)}\frac{1}{t!\,u!}(-1)^{s(s-1)/2}\times$$
$$\times\left[1-(-1)^{ij-s^2}\right]\delta^{a_1\cdots a_i e_1\cdots e_u}_{d_1\cdots d_t b_1\cdots b_j}\Gamma^{d_1\cdots d_t}_{e_1\cdots e_u},$$
$$\left\{\Gamma^{a_1\cdots a_i},\Gamma_{b_1\cdots b_j}\right\}=\sum_{s=0}^{\min(i,j)}\frac{1}{t!\,u!}(-1)^{s(s-1)/2}\times$$
$$\times\left[1+(-1)^{ij-s^2}\right]\delta^{a_1\cdots a_i e_1\cdots e_u}_{d_1\cdots d_t b_1\cdots b_j}\Gamma^{d_1\cdots d_t}{}_{e_1\cdots e_u}.$$
$$\Gamma_{a_1\cdots a_i}\Gamma_{b_1\cdots b_j}=\sum_{s=0}^{\min(i,j)}D_{a_1\cdots a_i b_1\cdots b_j}(s)$$
$$D_{a_1\cdots a_i b_1\cdots b_j}(s)=(-1)^{s(i-s)+s(s-1)/2}\sum_{p_1=1}^{1+j-s}\cdots\sum_{p_s=p_{s-1}+1}^{j}$$
$$\sum_{q_1=1}^{1+i-s}\cdots\sum_{q_s=q_{s-1}+1}^{i}(-1)^{p_1+\cdots+p_s+q_1+\cdots+q_s}$$
$$\eta_{[a_{q_1}\cdots a_{q_s}][b_{p_1}\cdots b_{p_s}]}\Gamma_{a_1\cdots\hat{a}_{q_1}\cdots\hat{a}_{q_s}\cdots a_i b_1\cdots\hat{b}_{p_1}\cdots\hat{b}_{p_s}\cdots b_j}.$$
$$A_iB_j=\sum_{s=0}^{\min(i,j)}\binom{i}{s}\binom{j}{s}(-1)^{s(s-1)/2}\eta_{[b_1\cdots b_s][c_1\cdots c_s]}$$
$$A^{a_1\cdots a_{i-s}b_1\cdots b_s}B^{c_1\cdots c_s a_{i-s+1}\cdots a_{i+j-2s}}\Gamma_{a_1\cdots a_{i+j-2s}},$$
$$A_iB_j=\sum_{s=0}^{\min(i,j)}\binom{i}{s}\binom{j}{s}(-1)^{s(s-1)/2}\eta_{[b_1\cdots b_s][c_1\cdots c_s]}$$
$$A^{a_1\cdots a_{i-s}b_1\cdots b_s}B^{c_1\cdots c_s a_{i-s+1}\cdots a_{i+j-2s}}\Gamma_{a_1\cdots a_{i+j-2s}},$$
$$A_i=A^{a_1\cdots a_i}\Gamma_{a_1\cdots a_i},$$
$$B_j=B^{b_1\cdots b_j}\Gamma_{b_1\cdots b_j}$$
$$\psi'=\exp\left(\frac{1}{4}\lambda^{ab}\Gamma_{ab}\right)\psi$$
$$\delta\psi=\frac{1}{4}\lambda^{ab}\Gamma_{ab}\psi$$
$$\bar{\psi}_\alpha=\psi^\beta C_{\beta\alpha}$$
$$\psi^\alpha=\bar{\psi}_\beta C^{\beta\alpha}$$
$$C^{\alpha\gamma}C_{\gamma\beta}=C_{\beta\gamma}C^{\gamma\alpha}=\delta^\alpha_\beta$$

$$\bar{\chi}\zeta = \bar{\zeta}\chi$$
$$\bar{\chi}S\zeta = -\bar{\zeta}S\chi$$
$$\bar{\chi}A\zeta = \bar{\zeta}A\chi$$

$$AdS_2 \times S^2 \times S^4 \times \Sigma$$

$$(a;q)_0 := 1, (a;q)_n := \prod_{k=0}^{n-1} (1-aq^k), (q)_n := \prod_{k=1}^{n} (1-q^k),$$

$$(a;q)_\infty := \prod_{k=0}^{\infty} (1-aq^k), (q)_\infty := \prod_{k=1}^{\infty} (1-q^k)$$

$$\langle W_{\mathcal{R}_1} \cdots W_{R_k} \rangle^G (t;q)$$
$$= \frac{1}{|\mathrm{Weyl}(G)|} \frac{(q)_\infty^{2\mathrm{rank}(G)}}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^{\mathrm{rank}(G)}} \oiiint \prod_{\alpha \in \mathrm{root}(G)} ds \frac{(s^\alpha;q)_\infty (qs^\alpha;q)_\infty}{\left(q^{\frac{1}{2}}t^2 s^\alpha;q\right)_\infty \left(q^{\frac{1}{2}}t^{-2}s^\alpha;q\right)_\infty} \prod_{i=1}^{k} \chi_{\mathcal{R}_i}$$

$$\mathcal{I}^G(t;q) := \mathrm{Tr}(-1)^F q^{J+\frac{H+C}{4}} t^{H-C}$$

$$\langle \mathcal{W}_{\mathcal{R}_1} \cdots \mathcal{W}_{\mathcal{R}_k} \rangle^G (t;q) = \frac{\langle W_{\mathcal{R}_1} \cdots W_{\mathcal{R}_k} \rangle^G (t;q)}{\mathcal{I}^G(t;q)}$$

$$\langle W_{\mathcal{R}_1} \cdots W_{\mathcal{R}_k} \rangle^G_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q})$$
$$= \frac{1}{|\mathrm{Weyl}(G)|} \frac{1}{(1-\mathfrak{q}^2)^{\mathrm{rank}(G)}} \oiiint \prod_{\alpha \in \mathrm{root}(G)} ds \frac{(1-s^\alpha)}{(1-\mathfrak{q}^2 s^\alpha)} \prod_{i=1}^{k} \chi^{\mathfrak{g}}_{\mathcal{R}_i}$$

$$\frac{1}{N!} \oiiint \prod_{i=1}^{N} \frac{ds_i}{2\pi i s_i} \frac{\prod_{i\neq j} 1 - \frac{s_i}{s_j}}{\prod_{i,j} 1 - \mathrm{t}\frac{s_i}{s_j}} P_\mu(s;\mathrm{t}) P_\lambda(s^{-1};\mathrm{t}) = \frac{\delta_{\mu\lambda}}{(\mathrm{t};\mathrm{t})_{N-l(\mu)} \prod_{j\geq 1} (\mathrm{t};\mathrm{t})_{m_j(\mu)}}$$

$$\frac{1}{|\mathrm{Weyl}(G)|} \oiiint \prod_{\alpha \in \mathrm{root}(G)} ds(1-s^\alpha) \prod_{i=1}^{k} \chi^{\mathfrak{g}}_{\mathcal{R}_i}$$

$$\langle T_B T_B \rangle^G (t;q) = \sum_{v \in \mathrm{Rep}(B)} \frac{1}{|\mathrm{Weyl}(B)|} \frac{(q)_\infty^{2\mathrm{rank}(G)}}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^{\mathrm{rank}(G)}} \oint \prod_{\alpha \in \mathrm{root}(G)} ds$$
$$\times \frac{\left(q^{\frac{|\alpha(B)|}{2}} s^\alpha;q\right)_\infty \left(q^{1+\frac{|\alpha(B)|}{2}} s^\alpha;q\right)_\infty}{\left(q^{\frac{1+|\alpha(B)|}{2}} t^2 s^\alpha;q\right)_\infty \left(q^{\frac{1+|\alpha(B)|}{2}} t^{-2} s^\alpha;q\right)_\infty} Z^{(B,v)}_{\mathrm{bubb}}(t,s;q).$$

$$(z_e, z_m) \in \mathbb{Z}_2 \times \mathbb{Z}_2$$
$$(z_e, z_m) = (0,0), (z_e, z_m) = (1,0)$$
$$(z_e, z_m) = (0,0), (z_e, z_m) = (0,1)$$
$$(z_e, z_m) = (0,0), (z_e, z_m) = (1,1)$$

$$\chi^{\mathrm{sp}(2N+1)}_{\mathrm{sp}} = \prod_{i=1}^{N} \left(s_i^{\frac{1}{2}} + s_i^{-\frac{1}{2}}\right)$$

$$\chi_{\square}^{\mathrm{so}\,(2N+1)} = 1 + \sum_{i=1}^{N} (s_i + s_i^{-1}).$$

$$\chi_{\lambda}^{\mathrm{so}\,(2N+1)} = \frac{\det\left(s_j^{\lambda_i+N-i+1/2} - s_j^{-(\lambda_i+N-i+1/2)}\right)}{\det\left(s_j^{N-i+1/2} - s_j^{-(N-i+1/2)}\right)}$$

$$\langle W_{\mathcal{R}_1} \cdots W_{\mathcal{R}_k} \rangle^{SO(2N+1)}$$

$$= \int d\mu^{SO(2N+1)} \exp\left[\sum_{n=1}^{\infty} \frac{1}{n} f_n(q,t) \frac{\bar{P}_n(s)^2 - \bar{P}_{2n}(s)}{2}\right] \prod_{i=1}^{k} \chi_{\mathcal{R}_i}^{\mathfrak{so}(2\mathfrak{N}+1)}(s)$$

$$d\mu^{SO(2N+1)} = \frac{1}{2^N N!} \prod_{i=1}^{N} \frac{ds_i}{2\pi i s_i} (1-s_i)(1-s_i^{-1})$$

$$\times \prod_{1 \le i < j \le N} (1 - s_i s_j)(1 - s_i^{-1} s_j^{-1})(1 - s_i s_j^{-1})(1 - s_i^{-1} s_j)$$

$$f_n(q,t) = \frac{q^{n/2}(t^{2n} + t^{-2n}) - 2q^n}{1 - q^n}$$

$$P_m(s) := \sum_{i=1}^{N} (s_i^m + s_i^{-m})$$

$$\bar{P}_m(s) := 1 + P_m(s) = 1 + \sum_{i=1}^{N} (s_i^m + s_i^{-m})$$

$$\bar{M}_n(s) = \frac{\bar{P}_n(s)^2 - \bar{P}_{2n}(s)}{2} = P_n(s) + \frac{P_n(s)^2 - P_{2n}(s)}{2}$$

$$\exp\left(\sum_{n=1}^{\infty} \frac{1}{n} f_n(q,t) \bar{M}_n(s)\right) = \sum_{\lambda} \frac{1}{z_\lambda} f_\lambda(q,t) \bar{M}_\lambda(s)$$

$$z_\lambda = \prod_{i=1}^{\infty} i^{m_i} m_i!, \; f_\lambda(q,t) = \prod_{i=1}^{\ell(\lambda)} f_{\lambda_i}(q,t), \; \bar{M}_\lambda(s) = \prod_{i=1}^{\ell(\lambda)} \bar{M}_{\lambda_i}(s)$$

$$\langle W_{\mathcal{R}_1} \cdots W_{\mathcal{R}_k} \rangle^{SO(2N+1)} = \sum_{\lambda} \frac{1}{z_\lambda} f_\lambda(q,t) \int d\mu^{SO(2N+1)} \bar{M}_\lambda(s) \prod_{i=1}^{k} \chi_{\mathcal{R}_i}^{\mathrm{so}(2\mathrm{N}+1)}(s)$$

$$\int d\mu^{SO(2N+1)} \bar{M}_\lambda(s) \prod_{i=1}^{k} \chi_{\mathcal{R}_i}^{\mathfrak{so}(2N+1)}(s)$$

$$\int d\mu^{SO(2N+1)} \bar{P}_\mu(s) = \sum_{\nu \in R_{2N+1}(|\mu|)} \chi_\nu^S(\mu) + \sum_{\nu \in W_{2N+1}(|\mu|)} \chi_\nu^S(\mu)$$

$$R_n(p) = \{\lambda \vdash p \mid \ell(\lambda) \le n \text{ and } \forall \lambda_i \text{ is even}\}$$

$$W_n(p) = \{\lambda \vdash p \mid \ell(\lambda) = n \text{ and } \forall \lambda_i \text{ is odd}\}$$

$$s_\lambda = \sum_{\mu \vdash \lambda} \frac{\chi_\lambda^S(\mu)}{z_\mu} p_\mu$$

$$\bar{M}_\lambda(s) \prod_{i=1}^{k} \chi_{\mathcal{R}_i}^{\mathrm{so}\,(2N+1)}(s) = \sum_{\mu} a_{\lambda,\mathcal{R}}^{\mu} \bar{P}_\mu(s)$$

$$\int d\mu^{SO(2N+1)}\bar{M}_\lambda(s)\prod_{i=1}^{k}\chi_{\mathcal{R}_i}^{\text{soo }(2N+1)}(s)=\sum_\mu a_{\lambda,\mathcal{R}}^{\mu}\int d\mu^{SO(2N+1)}\bar{P}_\mu(s)$$

$$=\sum_\mu a_{\lambda,\mathcal{R}}^{\mu}\left(\sum_{\nu\in R_{2N+1}(|\mu|)}\chi_\nu^S(\mu)+\sum_{\nu\in W_{2N+1}(|\mu|)}\chi_\nu^S(\mu)\right)$$

$$\left(\chi_{\text{sp}}^{\text{so}(2N+1)}\right)^2=\prod_{i=1}^{N}(1+s_i)(1+s_i^{-1})$$

$$d\mu^{SO(2N+1)}\left(\chi_{\text{sp}}^{\text{so}(2N+1)}\right)^2=d\mu^{USp(2N)}$$

$$d\mu^{USp(2N)}=\frac{1}{2^N N!}\prod_{i=1}^{N}\frac{ds_i}{2\pi i s_i}(1-s_i^2)(1-s_i^{-2})$$
$$\times\prod_{1\le i<j\le N}(1-s_is_j)(1-s_i^{-1}s_j^{-1})(1-s_is_j^{-1})(1-s_i^{-1}s_j)$$

$$\langle W_{\text{sp}}W_{\text{sp}}\rangle^{\text{Spin}(2N+1)}=\int d\mu^{USp(2N)}\exp\left[\sum_{n=1}^{\infty}\frac{1}{n}f_n(q,t)\bar{M}_n(s)\right]$$

$$=\int d\mu^{USp(2N)}\exp\left[\sum_{n=1}^{\infty}\frac{1}{n}f_n(q,t)\left(P_n(s)+\frac{P_n(s)^2-P_{2n}(s)}{2}\right)\right]$$

$$\mathcal{J}^{SO(3)}(t;q)=\mathcal{J}^{USp(2)}(t;q)$$
$$=-\frac{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty}{(q;q)_\infty^2}\sum_{\substack{p_1,p_2\in\mathbb{Z}\\p_1<p_2}}\frac{\left(q^{\frac{1}{2}}t^{-2}\right)^{p_1+p_2-2}}{\left(1-q^{p_1-\frac{1}{2}}t^2\right)\left(1-q^{p_2-\frac{1}{2}}t^2\right)}.$$

$$\langle W_{\text{sp}}W_{\text{sp}}\rangle^{\text{Spin}(3)}(t;q)$$
$$=\frac{1}{2}\frac{(q)_\infty^2}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty}\oint\frac{ds}{2\pi i s}\frac{(s^\pm;q)_\infty(qs^\pm;q)_\infty}{\left(q^{\frac{1}{2}}t^2s^\pm;q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s^\pm;q\right)_\infty}\left(s^{\frac{1}{2}}+s^{-\frac{1}{2}}\right)^2$$

$$\langle T_{\left(\frac{1}{2}\right)}T_{\left(\frac{1}{2}\right)}\rangle^{USp(2)/\mathbb{Z}_2}(t;q)$$
$$=\frac{(q)_\infty^2}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty}\oint\frac{ds}{2\pi i s}\frac{\left(q^{\frac{1}{2}}s^{\pm 2};q\right)_\infty\left(q^{\frac{3}{2}}s^{\pm 2};q\right)_\infty}{(qt^2s^{\pm 2};q)_\infty(qt^{-2}s^{\pm 2};q)_\infty}$$

$$\langle W_{\text{sp}}W_{\text{sp}}\rangle^{\text{Spin}(3)}(t;q)=\left\langle T_{\left(\frac{1}{2}\right)}T_{\left(\frac{1}{2}\right)}\right\rangle^{USp(2)/\mathbb{Z}_2}(t;q)$$

$$\langle W_{\text{sp}}W_{\text{sp}}\rangle^{Spin(3)}(t;q)=\langle T_{\left(\frac{1}{2}\right)}T_{\left(\frac{1}{2}\right)}\rangle^{USp(2)/\mathbb{Z}_2}(t;q)$$
$$=\langle W_\square W_\square\rangle^{SU(2)}(t;q)=\frac{\left(q^{\frac{1}{2}}t^{22};q\right)_\infty}{(qt^{\pm 4};q)_\infty}\sum_{m\in\mathbb{Z}\setminus\{0,n\}}\frac{t^{2m}-t^{-2m}}{t^2-t^{-2}}\frac{q^{\frac{m-1}{2}}}{1-q^m}.$$

$$\left\langle W_{\text{sp}} W_{\text{sp}} \right\rangle^{Spin(3)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \left\langle T_{\left(\frac{1}{2}\right)} T_{\left(\frac{1}{2}\right)} \right\rangle^{USS(2)/\mathbb{Z}_2}_{\frac{1}{2}\text{BPS}}(\mathfrak{q})$$

$$= \langle W_{\square} W_{\square} \rangle^{SU(2)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{1+\mathfrak{q}^2}{1-\mathfrak{q}^4} = \frac{1}{1-\mathfrak{q}^2}.$$

$$\langle \underbrace{W_{\text{sp}} W_{\text{sp}} \cdots W_{\text{sp}}}_{2k} \rangle^{Spin(3)}(t;q) = \langle \underbrace{W_{\square} W_{\square} \cdots W_{\square}}_{2k} \rangle^{SU(2)}(t;q)$$

$$\left\langle W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} \right\rangle^{\text{Sin}(3)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{2+3\mathfrak{q}^2+\mathfrak{q}^4}{1-\mathfrak{q}^4}$$

$$\left\langle W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} \right\rangle^{\text{Sin}(3)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{5+9\mathfrak{q}^2+5\mathfrak{q}^4+\mathfrak{q}^6}{1-\mathfrak{q}^4}$$

$$\left\langle W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} \right\rangle^{Spin(3)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{14+28\mathfrak{q}^2+20\mathfrak{q}^4+7\mathfrak{q}^6+\mathfrak{q}^8}{1-\mathfrak{q}^4}$$

$$\left\langle W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} W_{\text{sp}} \right\rangle^{\text{Sin}(3)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{42+90\mathfrak{q}^2+75\mathfrak{q}^4+35\mathfrak{q}^6+9\mathfrak{q}^8+\mathfrak{q}^{10}}{1-\mathfrak{q}^4}$$

$$\langle \underbrace{W_{\text{sp}} \cdots W_{\text{sp}}}_{2k} \rangle^{\text{Spin}(3)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \mathcal{I}^{SO(3)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) \sum_{i=0}^{k} a^{\mathfrak{so}(3)}_{k\text{sp}}(i)\mathfrak{q}^{2i}$$

$$= \frac{1}{1-\mathfrak{q}^4} \sum_{i=0}^{k} a^{\mathfrak{so}(3)}_{k\text{sp}}(i)\mathfrak{q}^{2i}$$

$$a^{\text{so}(3)}_{k\text{sp}}(i) = (2i+1)\frac{(2k)!}{(k-i)!\,(k+i+1)!}$$

$$= C_{k+i+1,2i+1}$$

$$C_{n,m} = \frac{m}{n}\binom{2n-m-1}{n-1}$$

$$C_k = \frac{1}{k+1}\binom{2k}{k}$$

$$= \prod_{1\le i\le j\le k-1} \frac{i+j+2}{i+j}$$

$$\frac{1-\sqrt{1-4x}}{2x} = \sum_{k=0}^{\infty} a^{\text{so}(3)}_{k\text{sp}}(0)x^k$$

$$\frac{1}{x^{i+1}}\left(\frac{1-\sqrt{1-4x}}{2}\right)^{2i+1} = \sum_{k=0}^{\infty} a^{\text{so}(3)}_{k\text{sp}}(i)x^k$$

$$\sum_{k=0}^{\infty} x^k \langle \underbrace{W_{\text{sp}} \cdots W_{\text{sp}}}_{2k} \rangle^{\text{Sin}(3)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{1}{1-\mathfrak{q}^4} \cdot \frac{2(1-\sqrt{1-4x})}{4x-(1-\sqrt{1-4x})^2\mathfrak{q}^2}$$

$$\langle W_{\square} W_{\square} \rangle^{SO(3)}(t;q)$$

$$= \frac{1}{2}\frac{(q)^2_\infty}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty} \oint \frac{ds}{2\pi i s} \frac{(s^{\pm};q)_\infty (qs^{\pm};q)_\infty}{\left(q^{\frac{1}{2}}t^2 s^{\pm};q\right)_\infty \left(q^{\frac{1}{2}}t^{-2}s^{\pm};q\right)_\infty}(1+s+s^{-1})^2$$

$$\langle W_\square W_\square\rangle^{SO(3)}(t;q) = \langle W_{\square\square} W_{\square\square}\rangle^{SU(2)}(t;q)$$

$$= \frac{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty}{(qt^{\pm 4};q)_\infty}\left[\frac{3}{2}\sum_{m\in\mathbb{Z}\setminus\{0\}}\left(\frac{t^{2m}-t^{-2m}}{t-t^{-2}}\frac{q^{\frac{m-1}{2}}}{1-q^m}\right)-\frac{2}{1-q}-\frac{q^{\frac{1}{2}}(t^2+t^{-2})}{1-q^2}\right]$$

$$\langle W_\square W_\square\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \langle W_\square W_{\square\square}\rangle^{SU(2)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$= \frac{1+\mathfrak{q}^2+\mathfrak{q}^4}{1-\mathfrak{q}^4}$$

$$= \frac{1-\mathfrak{q}^6}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)}$$

$$\langle \underbrace{W_\square\cdots W_\square}_{k}\rangle^{SO(3)}(t;q) = \sum_{i=0}^{k}\binom{k}{i}(-1)^i\langle\underbrace{W_{\mathrm{sp}}\cdots W_{\mathrm{sp}}\rangle}_{2(k-i)}^{SO(3)}(t;q)$$

$$\langle W_\square\rangle^{SO(3)}(t;q) = \langle W_{\mathrm{sp}}W_{\mathrm{sp}}\rangle^{SO(3)}(t;q) - \mathcal{I}^{SO(3)}(t;q)$$

$$= -\frac{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty}{(q;q)^2_\infty}\sum_{\substack{p_1,p_2\in\mathbb{Z}\\ p_1+1<p_2}}\frac{\left(q^{\frac{1}{2}}t^{-2}\right)^{p_1+p_2-2}}{\left(1-q^{p_1-\frac{1}{2}}t^2\right)\left(1-q^{p_2-\frac{1}{2}}t^2\right)}$$

$$\langle W_\square\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \langle W_{\mathrm{sp}}W_{\mathrm{sp}}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) - \mathcal{I}^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$= \frac{\mathfrak{q}^2}{1-\mathfrak{q}^4}.$$

$$\langle W_\square W_\square W_\square\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \langle W_{\mathrm{sp}}W_{\mathrm{sp}}W_{\mathrm{sp}}W_{\mathrm{sp}}W_{\mathrm{sp}}W_{\mathrm{sp}}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) - 3\langle W_{\mathrm{sp}}W_{\mathrm{sp}}W_{\mathrm{sp}}W_{\mathrm{sp}}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$+3\langle W_{\mathrm{sp}}W_{\mathrm{sp}}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) - \mathcal{I}^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$= \frac{1+3\mathfrak{q}^2+2\mathfrak{q}^4+\mathfrak{q}^6}{1-\mathfrak{q}^4}.$$

$$\langle W_\square W_\square W_\square W_\square\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{3+6\mathfrak{q}^2+6\mathfrak{q}^4+3\mathfrak{q}^6+\mathfrak{q}^8}{1-\mathfrak{q}^4}$$

$$\langle W_\square W_\square W_\square W_\square W_\square\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{6+15\mathfrak{q}^2+15\mathfrak{q}^4+10\mathfrak{q}^6+4\mathfrak{q}^8+\mathfrak{q}^{10}}{1-\mathfrak{q}^4}$$

$$\langle W_\square W_\square W_\square W_\square W_\square W_\square\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{15+36\mathfrak{q}^2+40\mathfrak{q}^4+29\mathfrak{q}^6+15\mathfrak{q}^8+5\mathfrak{q}^{10}+\mathfrak{q}^{12}}{1-\mathfrak{q}^4}$$

$$\langle \underbrace{W_\square\cdots W_\square}_{k}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\sum_{i=0}^{k} a_k^{\mathfrak{so}(3)}(i)\mathfrak{q}^{2i}}{1-\mathfrak{q}^4}$$

$$a_k^{\mathrm{so}\,(3)}(i) = c_k^{(i)} - c_k^{(i+1)}$$

$$(1+x+x^2)^n = \sum_{i=-k}^{k} c_k^{(i)} x^{k+i}$$

$$R_n = \sum_{i=0}^{n} (-1)^{n-i} \binom{n}{i} C_i$$

$$\sum_{n=1}^{\infty} R_n x^n = \frac{1}{2x}\left(1 - \frac{\sqrt{1-3x}}{\sqrt{1+x}}\right)$$

$$\left\langle W_{(k)} W_{(k)} \right\rangle^{SO(3)}(t;q) = \frac{\left(q^{\frac{1}{2}t^{\pm 2}};q\right)_\infty}{(qt^{\pm 4};q)_\infty}\left[\frac{2k+1}{2}\sum_{m\in\mathbb{Z}\setminus\{0\}}\left(\frac{t^{2m}-t^{-2m}}{t-t^{-2}}\frac{q^{\frac{m-1}{2}}}{1-q^m}\right)\right.$$

$$\left.-\sum_{m=1}^{2k}(2k-m+1)\left(\frac{t^{2m}-t^{-2m}}{t-t^{-2}}\frac{q^{\frac{m-1}{2}}}{1-q^m}\right)\right]$$

$$\langle W_{(k)} W_{(k)} \rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1+\mathfrak{q}^2+\cdots+\mathfrak{q}^{4k}}{1-\mathfrak{q}^4}$$

$$= \frac{1-\mathfrak{q}^{4k+2}}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)}$$

$$\langle W_{(\infty)} W_{(\infty)} \rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)}$$

$$\langle \underbrace{W_{\square\square}\cdots W_{\square}}_{k} \rangle^{SO(3)}(t;q) = \sum_{k_1+k_2+k_3=k}\binom{k}{k_1,k_2,k_3}(-3)^{k_2}\langle \underbrace{W_{\mathrm{sp}}\cdots W_{\mathrm{sp}}}_{4k_1+2k_2}\rangle^{SO(3)}(t;q)$$

$$\langle W_{\square\square}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \langle W_{\mathrm{sp}} W_{\mathrm{sp}} W_{\mathrm{sp}} W_{\mathrm{sp}}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) - 3\langle W_{\mathrm{sp}} W_{\mathrm{sp}}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) + \mathcal{I}^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$= \frac{\mathfrak{q}^4}{1-\mathfrak{q}^4},$$

$$\langle W_{\square} W_{\square} W_{\square\square}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \langle \underbrace{W_{\mathrm{sp}}\cdots W_{\mathrm{sp}}}_{12}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) - 9\langle \underbrace{W_{\mathrm{sp}}\cdots W_{\mathrm{sp}}}_{10}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$+30\langle \underbrace{W_{\mathrm{sp}}\cdots W_{\mathrm{sp}}}_{8}\rangle\frac{1}{2}\frac{\mathrm{BPS}}{SO(3)}(\mathfrak{q}) - 45\langle \underbrace{W_{\mathrm{sp}}\cdots W_{\mathrm{sp}}}_{6}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$+30\langle \underbrace{W_{\mathrm{sp}}\cdots W_{\mathrm{sp}}}_{4}\rangle^{SO(\mathfrak{q})}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) - 9\langle W_{\mathrm{sp}} W_{\mathrm{sp}}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) + \mathcal{I}^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$= \frac{1+3\mathfrak{q}^2+5\mathfrak{q}^4+4\mathfrak{q}^6+3\mathfrak{q}^8+2\mathfrak{q}^{10}+\mathfrak{q}^{12}}{1-\mathfrak{q}^4}.$$

$$\langle \underbrace{W_{\square\square\square}\cdots W_{\square\square}}_{k}\rangle^{SO(3)}(t;q)$$

$$= \sum_{k_1+k_2+k_3+k_4=k}\binom{k}{k_1,k_2,k_3,k_4}(-1)^{k_2+k_4}5^{k_2}6^{k_3}\langle \underbrace{W_{\mathrm{sp}}\cdots W_{\mathrm{sp}}}_{6k_2+4k_1+2k_2}\rangle^{SO(3)}(t;q).$$

$$\langle W^{\square}_{\square\square}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$= \langle \underbrace{W_{\mathrm{sp}}\cdots W_{\mathrm{sp}}}_{6}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) - 5\langle \underbrace{W_{\mathrm{sp}}\cdots W_{\mathrm{sp}}}_{4}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) + 6\langle W_{\mathrm{sp}} W_{\mathrm{sp}}\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) - \mathcal{I}^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$= \frac{\mathfrak{q}^6}{1-\mathfrak{q}^4},$$

$$\left\langle W_{\square}^{W_{\square\square} W_{\square\square}} \right\rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(3)}(\mathfrak{q})$$

$$= \langle \underbrace{W_{\mathrm{sp}} \cdots W_{\mathrm{sp}}}_{18} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(3)}(\mathfrak{q}) - 15 \langle \underbrace{W_{\mathrm{sp}} \cdots W_{\mathrm{sp}}}_{16} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(3)}(\mathfrak{q}) + 93 \langle \underbrace{W_{\mathrm{sp}} \cdots W_{\mathrm{sp}}}_{14} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(3)}(\mathfrak{q})$$

$$-308 \langle \underbrace{W_{\mathrm{sp}} \cdots W_{\mathrm{sp}}}_{12} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(3)}(\mathfrak{q}) + 588 \langle \underbrace{W_{\mathrm{sp}} \cdots W_{\mathrm{sp}}}_{10} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(3)}(\mathfrak{q}) - 651 \langle \underbrace{W_{\mathrm{sp}} \cdots W_{\mathrm{sp}}}_{8} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(3)}(\mathfrak{q})$$

$$+399 \langle \underbrace{W_{\mathrm{sp}} \cdots W_{\mathrm{sp}}}_{6} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(3)}(\mathfrak{q}) - 123 \langle \underbrace{W_{\mathrm{sp}} \cdots W_{\mathrm{sp}}}_{4} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(3)}(\mathfrak{q}) + 18 \left\langle W_{\mathrm{sp}} W_{\mathrm{sp}} \right\rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(3)}(\mathfrak{q}) - \mathcal{I}_{\frac{1}{2}\mathrm{BPS}}^{SO(3)}(\mathfrak{q})$$

$$= \frac{1 + 3\mathfrak{q}^2 + 5\mathfrak{q}^4 + 7\mathfrak{q}^6 + 6\mathfrak{q}^8 + 5\mathfrak{q}^{10} + 4\mathfrak{q}^{12} + 3\mathfrak{q}^{14} + 2\mathfrak{q}^{16} + \mathfrak{q}^{18}}{1 - \mathfrak{q}^4}.$$

$$\chi_{(k)}^{\mathrm{so}(3)} = \sum_{n=0}^{k} (-1)^n \binom{2k-n}{n} \chi_{\mathrm{sp}}^{\mathrm{so}(3)^{2k-2n}}$$

$$\langle W_{(k)} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(3)}(\mathfrak{q}) = \frac{\mathfrak{q}^{2k}}{1 - \mathfrak{q}^4}$$

$$\langle W_{(k)} W_{(l)} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(3)}(\mathfrak{q}) = \frac{\mathfrak{q}^{2(l-k)}(1 - \mathfrak{q}^{4k+2})}{(1 - \mathfrak{q}^2)(1 - \mathfrak{q}^4)}$$

$$\langle W_{(k)} W_{(k)} W_{(k)} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(3)}(\mathfrak{q}) = \frac{1 + \mathfrak{q}^2 - 3\mathfrak{q}^{2k+2} + \mathfrak{q}^{6k+4}}{(1 - \mathfrak{q}^2)^2(1 - \mathfrak{q}^4)}$$

$$\langle W_{(\infty)} W_{(\infty)} W_{(\infty)} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(3)}(\mathfrak{q}) = \frac{1}{(1 - \mathfrak{q}^2)^3}$$

$$\langle W_{(k)} W_{(k)} W_{(k)} W_{(k)} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(3)}(\mathfrak{q}) = \frac{2k + 1 - 3\mathfrak{q}^2 - (2k+1)\mathfrak{q}^4 + 4\mathfrak{q}^{4k+4} - \mathfrak{q}^{8k+6}}{(1 - \mathfrak{q}^2)^3(1 - \mathfrak{q}^4)}$$

$$\left\langle W_{\mathrm{sp}} W_{\mathrm{sp}} \right\rangle^{\mathrm{Spin}(5)}(t;q)$$

$$= \frac{1}{8} \frac{(q)_\infty^4}{\left(q^{\frac{1}{2}} t^{\pm}; q\right)_\infty^2} \oint \prod_{i=1}^{2} \frac{ds_i}{2\pi i s_i} \frac{(s_i^{\pm}; q)_\infty (q s_i^{\pm}; q)_\infty}{\left(q^{\frac{1}{2}} t^2 s_i^{\pm}; q\right)_\infty \left(q^{\frac{1}{2}} t^{-2} s_i^{\pm}; q\right)_\infty}$$

$$\times \frac{(s_1^{\pm} s_2^{\mp}; q)_\infty (s_1^{\pm} s_2^{\pm}; q)_\infty (q s_1^{\pm} s_2^{\mp}; q)_\infty (q s_1^{\pm} s_2^{\pm}; q)_\infty}{\left(q^{\frac{1}{2}} t^2 s_1^{\pm} s_2^{\mp}; q\right)_\infty \left(q^{\frac{1}{2}} t^2 s_1^{\pm} s_2^{\pm}; q\right)_\infty \left(q^{\frac{1}{2}} t^{-2} s_1^{\pm} s_2^{\mp}; q\right)_\infty \left(q^{\frac{1}{2}} t^{-2} s_1^{\pm} s_2^{\pm}; q\right)_\infty} \prod_{i=1}^{2} \left(s_i^{\frac{1}{2}} + s_i^{-\frac{1}{2}}\right)^2$$

$$\left\langle T_{\left(\frac{1}{2},\frac{1}{2}\right)} T_{\left(\frac{1}{2},\frac{1}{2}\right)} \right\rangle^{USp(4)/\mathbb{Z}_2}(t;q)$$

$$= \frac{1}{2} \frac{(q)_\infty^4}{\left(q^{\frac{1}{2}} t^{\pm 2}; q\right)_\infty^2} \oint \prod_{i=1}^{2} \frac{ds_i}{2\pi i s_i} \frac{\left(q^{\frac{1}{2}} s_i^{\pm 2}; q\right)_\infty \left(q^{\frac{3}{2}} s_i^{\pm 2}; q\right)_\infty}{(q t^2 s_i^{\pm 2}; q)_\infty (q t^{-2} s_i^{\pm 2}; q)_\infty}$$

$$\times \frac{(s_1^{\pm} s_2^{\mp}; q)_\infty \left(q^{\frac{1}{2}} s_1^{\pm} s_2^{\pm}; q\right)_\infty (q s_1^{\pm} s_2^{\mp}; q)_\infty \left(q^{\frac{3}{2}} s_1^{\pm} s_2^{\pm}; q\right)_\infty}{\left(q^{\frac{1}{2}} t^2 s_1^{\pm} s_2^{\mp}; q\right)_\infty (q t^2 s_1^{\pm} s_2^{\pm}; q)_\infty \left(q^{\frac{1}{2}} t^{-2} s_1^{\pm} s_2^{\mp}; q\right)_\infty (q t^{-2} s_1^{\pm} s_2^{\pm}; q)_\infty}$$

$$\left\langle W_{\mathrm{sp}} W_{\mathrm{sp}} \right\rangle^{Spin(5)}(t;q) = \left\langle T_{\left(\frac{1}{2},\frac{1}{2}\right)} T_{\left(\frac{1}{2},\frac{1}{2}\right)} \right\rangle^{USp(4)/\mathbb{Z}_2}(t;q)$$

$$\left\langle W_{\text{sp}} W_{\text{sp}} \right\rangle_{\frac{1}{2}\text{BPS}}^{\text{Spin}(5)}(\mathfrak{q}) = \left\langle T_{\left(\frac{1}{2},\frac{1}{2}\right)} T_{\left(\frac{1}{2},\frac{1}{2}\right)} \right\rangle_{\frac{1}{2}\text{BPS}}^{USp(4)/\mathbb{Z}_2}(\mathfrak{q})$$

$$= \frac{1+\mathfrak{q}^2+\mathfrak{q}^4+\mathfrak{q}^6}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}$$

$$= \frac{1}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)}$$

$$\langle \underbrace{V_{\text{sp}} \cdots W_{\text{sp}}}_{4} \rangle_{\frac{1}{2}\text{BPS}}^{S\text{Sin}(5)}(\mathfrak{q}) = \frac{3+6\mathfrak{q}^2+8\mathfrak{q}^4+9\mathfrak{q}^6+6\mathfrak{q}^8+3\mathfrak{q}^{10}+\mathfrak{q}^{12}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}$$

$$\langle \underbrace{\text{S}_{\text{sp}} \cdots W_{\text{sp}}}_{6} \rangle_{\frac{1}{2}\text{BPS}}^{S\text{Sin}(5)}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}(14+40\mathfrak{q}^2+66\mathfrak{q}^4+85\mathfrak{q}^6$$

$$+81\mathfrak{q}^8+59\mathfrak{q}^{10}+34\mathfrak{q}^{12}+15\mathfrak{q}^{14}+5\mathfrak{q}^{16}+\mathfrak{q}^{18})$$

$$\langle \underbrace{\text{S}_{\text{sp}} \cdots W_{\text{sp}}}_{8} \rangle_{\frac{1}{2}\text{BPS}}^{\text{Sin}(5)}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}(84+300\mathfrak{q}^2+581\mathfrak{q}^4+840\mathfrak{q}^6+945\mathfrak{q}^8+842\mathfrak{q}^{10}$$

$$+616\mathfrak{q}^{12}+378\mathfrak{q}^{14}+195\mathfrak{q}^{16}+83\mathfrak{q}^{18}+28\mathfrak{q}^{20}+7\mathfrak{q}^{22}+\mathfrak{q}^{24})$$

$$\langle \underbrace{W_{\text{sp}} \cdots W_{\text{sp}}}_{2k} \rangle_{\frac{1}{2}\text{BPS}}^{Sp(5)}(\mathfrak{q}) = \frac{\sum_{i=0}^{3k} a_{k\,sp}^{\text{so}\,(5)}(i)\mathfrak{q}^{2i}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}$$

$$a_{k\,\text{sp}}^{a_{\text{sp}}\,(0)} = C_k C_{k+2} - C_{k+1}^2$$

$$= \frac{24(2k+1)!\,(2k-1)!}{(k-1)!\,k!\,(k+2)!\,(k+3)!}$$

$$= \prod_{1\le i\le j\le k-1} \frac{i+j+4}{i+j},$$

$${}_3F_2\left(1,\frac{1}{2},\frac{3}{2};3,4;16x\right) = \sum_{k=0}^{\infty} a_{ksp}^{sso(5)}(0)x^k$$

$${}_pF_q(a_1,\cdots,a_p;b_1,\cdots,b_q;z) = \sum_{k=0}^{\infty} \frac{(a_1)_k(a_2)_k\cdots(a_p)_k}{(b_1)_k(b_2)_k\cdots(b_q)_k}\frac{z^k}{k!}$$

$$a_{ksp}^{\text{so}\,(5)}(1) = \frac{60(2k)!\,(2k+2)!}{(k-1)!\,k!\,(k+3)!\,(k+4)!}$$

$$\langle \underbrace{W_{\Box} \cdots W_{\Box}}_{k} \rangle_{\frac{1}{2}\text{BPS}}^{SO(5)}(\mathfrak{q}) = \frac{\sum_{i=0}^{2k} a_k^{\text{son}(5)}(i)\mathfrak{q}^{2i}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}$$

$$\langle W_{\square}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})=\frac{\mathfrak{q}^4}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)},$$

$$\langle W_{\square}W_{\square}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})=\frac{1+\mathfrak{q}^2+\mathfrak{q}^4+\mathfrak{q}^6+\mathfrak{q}^8}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}$$

$$=\frac{1-\mathfrak{q}^{10}}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)},$$

$$\langle W_{\square}W_{\square}W_{\square}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})=\frac{\mathfrak{q}^2+3\mathfrak{q}^4+3\mathfrak{q}^6+3\mathfrak{q}^8+2\mathfrak{q}^{10}+\mathfrak{q}^{12}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}$$

$$\langle W_{\square}W_{\square}W_{\square}W_{\square}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})=\frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}(3+3\mathfrak{q}^2+9\mathfrak{q}^4+15\mathfrak{q}^6+12\mathfrak{q}^8$$
$$+12\mathfrak{q}^{10}+6\mathfrak{q}^{12}+\mathfrak{q}^{16})$$

$$\langle W_{\square}W_{\square}W_{\square}W_{\square}W_{\square}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})=\frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}(1+10\mathfrak{q}^2+24\mathfrak{q}^4+36\mathfrak{q}^6+44\mathfrak{q}^8$$
$$+41\mathfrak{q}^{10}+31\mathfrak{q}^{12}+19\mathfrak{q}^{14}+10\mathfrak{q}^{16}+4\mathfrak{q}^{18}+\mathfrak{q}^{20})$$

$$a^{\mathrm{so}(5)}_{k\square}(0)=\sum_{i=0}^{\lfloor\frac{k}{2}\rfloor}C_iC_{i+1}\binom{k}{2i}-\sum_{i=0}^{\lfloor\frac{k+1}{2}\rfloor}C_i^2\binom{k}{2i-1}$$

$$=-k\,{}_3F_2\left(\frac{3}{2},\frac{1}{2}-\frac{k}{2};3,3;16\right)+\ {}_3F_2\left(\frac{3}{2},\frac{1}{2}-\frac{k}{2};2,3;16\right)$$

$$\langle\langle\underbrace{W_{\square}\cdots W_{\square}}_{k}\ {}^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})=\frac{\sum_{i=0}^{3k}\ a_k^{\mathfrak{fMs}(5)}(i)\mathfrak{q}^{2i}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}$$

$$\langle W_{\square}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})=\frac{\mathfrak{q}^2+\mathfrak{q}^6}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}$$

$$=\frac{\mathfrak{q}^2}{(1-\mathfrak{q}^4)^2},$$

$$\langle W_{\square}W_{\square}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})=\frac{1+\mathfrak{q}^2+3\mathfrak{q}^4+2\mathfrak{q}^6+3\mathfrak{q}^8+\mathfrak{q}^{10}+\mathfrak{q}^{12}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}$$

$$=\frac{(1-\mathfrak{q}^6)(1-\mathfrak{q}^8)}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)^3},$$

$$\langle W_{\square}W_{\square}W_{\square}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})=\frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}(1+6\mathfrak{q}^2+9\mathfrak{q}^4+16\mathfrak{q}^6+15\mathfrak{q}^8$$
$$+15\mathfrak{q}^{10}+9\mathfrak{q}^{12}+6\mathfrak{q}^{14}+2\mathfrak{q}^{16}+\mathfrak{q}^{18})$$

$$\langle W_{\square}W_{\square}W_{\square}W_{\square}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})=\frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}(6+22\mathfrak{q}^2+54\mathfrak{q}^4+82\mathfrak{q}^6+15\mathfrak{q}^8$$
$$+15\mathfrak{q}^{10}+9\mathfrak{q}^{12}+6\mathfrak{q}^{14}+2\mathfrak{q}^{16}+\mathfrak{q}^{18})$$

$$\langle\underbrace{W_{(2)}\cdots W_{(2)}}_{k}\rangle\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})=\frac{\sum_{i=0}^{4k}\ a_k^{\mathfrak{son}(5)}(i)\mathfrak{q}^{2i}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}$$

$$\langle W_{\square\square}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(q) = \frac{q^4+q^8}{(1-q^4)(1-q^8)} = \frac{q^4}{(1-q^4)^2},$$

$$\langle W_{\square\square}W_{\square\square}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(q) = \frac{1+q^2+2q^4+2q^6+3q^8+2q^{10}+3q^{12}+q^{14}+q^{16}}{(1-q^4)(1-q^8)}$$

$$\langle W_{\square\square}W_{\square\square}W_{\square\square}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(q) = \frac{1}{(1-q^4)(1-q^8)} \times (1+3q^2+9q^4+13q^6+20q^8+21q^{10} +22q^{12}+18q^{14}+15q^{16}+9q^{18}+6q^{20}+2q^{22}+q^{24})$$

$$\langle W_{(k)}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(q) = \frac{q^{4k-4}+q^{4k}}{(1-q^4)(1-q^8)} = \frac{q^{4k-4}}{(1-q^4)^2}.$$

$$\langle W_{(k)}W_{(k)}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(q) = \frac{\sum_{i=0}^{8k} a_2^{\mathfrak{so}(5)}(i)q^{2i}}{(1-q^4)(1-q^8)}$$

$$\langle W_{\square\square\square}W_{\square\square}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(q) = \frac{1}{(1-q^4)(1-q^8)}(1+q^2+2q^4+3q^6+4q^8+4q^{10} +6q^{12}+5q^{14}+5q^{16}+4q^{18}+3q^{20}+q^{22}+q^{24})$$

$$\langle W_{\square\square\square}W_{\square\square\square}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(q) = \frac{1}{(1-q^4)(1-q^8)}(1+q^2+2q^4+3q^6+5q^8+5q^{10} +8q^{12}+8q^{14}+10q^{16}+9q^{18}+10q^{20}+7q^{22}+7q^{24} +4q^{26}+3q^{28}+q^{30}+q^{32})$$

$$\langle W_{(\infty)}W_{(\infty)}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(q) = 1+q^2+3q^4+4q^6+9q^8+11q^{10}+21q^{12}+26q^{14}+44q^{16}+54q^{18}+84q^{20}+\cdots$$

$$\langle W_{(\infty)}W_{(\infty)}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(q) = \frac{1-q^{24}}{(1-q^2)(1-q^4)^2(1-q^6)(1-q^8)^2(1-q^{12})}$$

$$\left\langle W_{\mathrm{sp}}W_{\mathrm{sp}}\right\rangle^{\mathrm{Spin}(7)}(t;q) = \frac{1}{48}\frac{(q)_\infty^6}{\left(q^{\frac{1}{2}}tt^{\pm};q\right)_\infty^3}\int\prod_{i=1}^{3}\frac{ds_i}{2\pi i s_i}\frac{\left(s_i^{\pm};q\right)_\infty\left(qs_i^{\pm};q\right)_\infty}{\left(q^{\frac{1}{2}}t^2s_i^{\pm};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm};q\right)_\infty}$$

$$\times\prod_{i<j}\frac{\left(s_i^{\pm}s_j^{\mp};q\right)_\infty\left(s_i^{\pm}s_j^{\pm};q\right)_\infty\left(qs_i^{\pm}s_j^{\mp};q\right)_\infty\left(qs_i^{\pm}s_j^{\pm};q\right)_\infty}{\left(q^{\frac{1}{2}}t^2s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^2s_i^{\pm}s_j^{\pm};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm}s_j^{\pm};q\right)_\infty}\prod_{i=1}^{3}\left(s_i^{\frac{1}{2}}+s_i^{-\frac{1}{2}}\right)^2.$$

$$\left\langle T_{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)} T_{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)} \right\rangle^{USp(6)/\mathbb{Z}_2} (t;q)$$

$$= \frac{1}{6} \frac{(q)_\infty^6}{\left(q^{\frac{1}{2}} \pm^{\pm 2}; q\right)_\infty^3} \oint \prod_{i=1}^{3} \frac{ds_i}{2\pi i s_i} \frac{\left(q^{\frac{1}{2}} s_i^{\pm 2}; q\right)_\infty \left(q^{\frac{3}{2}} s_i^{\pm 2}; q\right)_\infty}{(qt^2 s_i^{\pm 2}; q)_\infty (qt^{-2} s_i^{\pm 2}; q)_\infty}$$

$$\times \prod_{i<j} \frac{(s_i^{\pm} s_j^{\mp}; q)_\infty \left(q^{\frac{1}{2}} s_i^{\pm} s_j^{\pm}; q\right)_\infty (q s_i^{\pm} s_j^{\mp}; q)_\infty \left(q^{\frac{3}{2}} s_i^{\pm} s_j^{\pm}; q\right)_\infty}{\left(q^{\frac{1}{2}} t^2 s_i^{\pm} s_j^{\mp}; q\right)_\infty (qt^2 s_i^{\pm} s_j^{\pm}; q)_\infty \left(q^{\frac{1}{2}} t^{-2} s_i^{\pm} s_j^{\mp}; q\right)_\infty (qt^{-2} s_i^{\pm} s_j^{\pm}; q)_\infty}.$$

$$\langle W_{\mathrm{sp}} W_{\mathrm{sp}} \rangle_{\frac{1}{2}\mathrm{BPS}}^{\mathrm{Spin}(7)}(\mathfrak{q}) = \langle T_{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)} T_{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)} \rangle_{\frac{1}{2}\mathrm{BPS}}^{USp(6)/\mathbb{Z}_2}(\mathfrak{q})$$

$$= \frac{1 + \mathfrak{q}^2 + \mathfrak{q}^4 + 2\mathfrak{q}^6 + \mathfrak{q}^8 + \mathfrak{q}^{10} + \mathfrak{q}^{12}}{(1 - \mathfrak{q}^4)(1 - \mathfrak{q}^8)(1 - \mathfrak{q}^{12})}$$

$$= \frac{1}{(1 - \mathfrak{q}^2)(1 - \mathfrak{q}^4)(1 - \mathfrak{q}^6)}.$$

$$\langle \underbrace{W_{\mathrm{sp}} \cdots W_{\mathrm{sp}}}_{2k} \rangle_{\frac{1}{2}\mathrm{BPin}(7)}^{\mathrm{SPS}}(\mathfrak{q}) = \frac{\sum_{i=0}^{6k} a_{k\,\mathrm{sp}}^{\mathfrak{so}(7)}(i)\mathfrak{q}^{2i}}{(1 - \mathfrak{q}^4)(1 - \mathfrak{q}^8)(1 - \mathfrak{q}^{12})}$$

$$\langle W_{\mathrm{sp}} W_{\mathrm{sp}} W_{\mathrm{sp}} W_{\mathrm{sp}} \rangle_{\frac{1}{2}\mathrm{BPS}}^{\mathrm{Spin}(7)}(\mathfrak{q}) = \frac{1}{(1 - \mathfrak{q}^4)(1 - \mathfrak{q}^8)(1 - \mathfrak{q}^{12})}(4 + 9\mathfrak{q}^2 + 15\mathfrak{q}^4$$
$$+25\mathfrak{q}^6 + 29\mathfrak{q}^8 + 32\mathfrak{q}^{10} + 33\mathfrak{q}^{12} + 26\mathfrak{q}^{14} + 20\mathfrak{q}^{16}$$
$$+13\mathfrak{q}^{18} + 6\mathfrak{q}^{20} + 3\mathfrak{q}^{22} + \mathfrak{q}^{24}),$$

$$\langle W_{\mathrm{sp}} W_{\mathrm{sp}} W_{\mathrm{sp}} W_{\mathrm{sp}} W_{\mathrm{sp}} W_{\mathrm{sp}} \rangle_{\frac{1}{2}\mathrm{BPS}}^{\mathrm{Spin}(7)}(\mathfrak{q}) = \frac{1}{(1 - \mathfrak{q}^4)(1 - \mathfrak{q}^8)(1 - \mathfrak{q}^{12})}(30 + 105\mathfrak{q}^2 + 235\mathfrak{q}^4$$
$$+435\mathfrak{q}^6 + 650\mathfrak{q}^8 + 855\mathfrak{q}^{10} + 1010\mathfrak{q}^{12} + 1055\mathfrak{q}^{14}$$
$$+1006\mathfrak{q}^{16} + 865\mathfrak{q}^{18} + 665\mathfrak{q}^{20} + 470\mathfrak{q}^{22} + 299\mathfrak{q}^{24}$$
$$+170\mathfrak{q}^{26} + 89\mathfrak{q}^{28} + 40\mathfrak{q}^{30} + 15\mathfrak{q}^{32} + 5\mathfrak{q}^{34} + \mathfrak{q}^{36}).$$

$$a_{k\mathrm{sp}}^{\mathrm{so}\,(7)}(0) = \prod_{1 \le i \le j \le k-1} \frac{i + j + 6}{i + j}$$

$$_4F_3\left(1, \frac{1}{2}, \frac{5}{2}, \frac{3}{2}; 4,5,6; 64x\right) = \sum_{k=0}^{\infty} a_{k\mathrm{sp}}^{\mathrm{so}\,(7)}(0)x^k$$

$$\langle \underbrace{W_{\square} \cdots W_{\square}}_{k} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(7)}(\mathfrak{q}) = \frac{\sum_{i=0}^{3k} a_k^{\mathfrak{son}(7)}(i)\mathfrak{q}^{2i}}{(1 - \mathfrak{q}^4)(1 - \mathfrak{q}^8)(1 - \mathfrak{q}^{12})}$$

$$\langle W_{\square} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(7)}(\mathfrak{q}) = \frac{\mathfrak{q}^6}{(1 - \mathfrak{q}^4)(1 - \mathfrak{q}^8)(1 - \mathfrak{q}^{12})},$$

$$\langle W_{\square} W_{\square} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(7)}(\mathfrak{q}) = \frac{1 + \mathfrak{q}^2 + \mathfrak{q}^4 + \mathfrak{q}^6 + \mathfrak{q}^8 + \mathfrak{q}^{10} + \mathfrak{q}^{12}}{(1 - \mathfrak{q}^4)(1 - \mathfrak{q}^8)(1 - \mathfrak{q}^{12})}$$

$$= \frac{1 - \mathfrak{q}^{14}}{(1 - \mathfrak{q}^2)(1 - \mathfrak{q}^4)(1 - \mathfrak{q}^8)(1 - \mathfrak{q}^{12})},$$

$$\langle W_{\square} W_{\square} W_{\square} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(7)}(\mathfrak{q}) = \frac{\mathfrak{q}^4 + 3\mathfrak{q}^6 + 3\mathfrak{q}^8 + 3\mathfrak{q}^{10} + 3\mathfrak{q}^{12} + 3\mathfrak{q}^{14} + 2\mathfrak{q}^{16} + \mathfrak{q}^{18}}{(1 - \mathfrak{q}^4)(1 - \mathfrak{q}^8)(1 - \mathfrak{q}^{12})}$$

$$\Big\langle \underbrace{W_{\substack{\square\\\square}} \cdots W_{\substack{\square\\\square}}}_{k} \Big\rangle^{SO(7)}_{\frac{1}{2}\mathrm{SPS}}(\mathfrak{q}) = \frac{\sum_{i=0}^{5k} \, a^{\mathrm{so}(7)}_{k\substack{\square\\\square}}(i)\mathfrak{q}^{2i}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}$$

$$\Big\langle \underbrace{W_{\substack{\square\\\square\\\square}} \cdots W_{\substack{\square\\\square\\\square}}}_{k} \Big\rangle^{SO(7)}_{\frac{1}{2}\mathrm{SPS}}(\mathfrak{q}) = \frac{\sum_{i=0}^{6k} \, a^{\mathrm{so}(7)}_{k\substack{\square\\\square\\\square}}(i)\mathfrak{q}^{2i}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}$$

$$\left\langle W_{\substack{\square\\\square}} \right\rangle^{SO(7)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\mathfrak{q}^2+\mathfrak{q}^6+\mathfrak{q}^{10}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})},$$

$$\left\langle W_{\substack{\square\\\square}} W_{\substack{\square\\\square}} \right\rangle^{SO(7)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}$$
$$\times (1+\mathfrak{q}^2+3\mathfrak{q}^4+2\mathfrak{q}^6+5\mathfrak{q}^8+3\mathfrak{q}^{10}$$
$$+5\mathfrak{q}^{12}+2\mathfrak{q}^{14}+3\mathfrak{q}^{16}+\mathfrak{q}^{18}+\mathfrak{q}^{20}).$$

$$\left\langle W_{\substack{\square\\\square\\\square}} \right\rangle^{SO(7)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\mathfrak{q}^4+\mathfrak{q}^8+\mathfrak{q}^{12}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})},$$

$$\left\langle W_{\substack{\square\\\square\\\square}} \right\rangle^{SO(7)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}$$
$$\times (1+\mathfrak{q}^2+3\mathfrak{q}^4+4\mathfrak{q}^6+7\mathfrak{q}^8+6\mathfrak{q}^{10}$$
$$+9\mathfrak{q}^{12}+6\mathfrak{q}^{14}+7\mathfrak{q}^{16}+4\mathfrak{q}^{18}+3\mathfrak{q}^{20}+\mathfrak{q}^{22}+\mathfrak{q}^{24}).$$

$$\langle \underbrace{W_{(l)} \cdots W_{(l)}}_{k} \rangle^{SO(7)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\sum_{i=0}^{3lk} \, a^{50}_{k(l)}(7)}{}(i)\mathfrak{q}^{2i}$$

$$\langle W_{\square\square} \rangle^{SO(7)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\mathfrak{q}^4+\mathfrak{q}^8+\mathfrak{q}^{12}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}$$

$$\langle W_{\square\square} W_{\square\square} \rangle^{SO(7)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}$$
$$\times (1+\mathfrak{q}^2+2\mathfrak{q}^4+2\mathfrak{q}^6+4\mathfrak{q}^8+3\mathfrak{q}^{10}+5\mathfrak{q}^{12}$$
$$+3\mathfrak{q}^{14}+5\mathfrak{q}^{16}+2\mathfrak{q}^{18}+3\mathfrak{q}^{20}+\mathfrak{q}^{22}+\mathfrak{q}^{24})$$

$$\left\langle W_{\mathrm{sp}} W_{\mathrm{sp}} \right\rangle^{\mathrm{Spin}(2N+1)}(t;q)$$
$$= \frac{1}{2^N N!} \frac{(q)_\infty^{2N}}{\left(q^{\frac{1}{2}} t^{\pm}; q\right)_\infty^N} \oint \prod_{i=1}^{N} \frac{ds_i}{2\pi i s_i} \frac{\left(s_i^{\pm}; q\right)_\infty \left(q s_i^{\pm}; q\right)_\infty}{\left(q^{\frac{1}{2}} t^2 s_i^{\pm}; q\right)_\infty \left(q^{\frac{1}{2}} t^{-2} s_i^{\pm}; q\right)_\infty}$$
$$\times \prod_{i<j} \frac{\left(s_i^{\pm} s_j^{\mp}; q\right)_\infty \left(s_i^{\pm} s_j^{\pm}; q\right)_\infty \left(q s_i^{\pm} s_j^{\mp}; q\right)_\infty \left(q s_i^{\pm} s_j^{\pm}; q\right)_\infty}{\left(q^{\frac{1}{2}} t^2 s_i^{\pm} s_j^{\mp}; q\right)_\infty \left(q^{\frac{1}{2}} t^2 s_i^{\pm} s_j^{\pm}; q\right)_\infty \left(q^{\frac{1}{2}} t^{-2} s_i^{\pm} s_j^{\mp}; q\right)_\infty \left(q^{\frac{1}{2}} t^{-2} s_i^{\pm} s_j^{\pm}; q\right)_\infty} \prod_{i=1}^{N} \left(s_i^{\frac{1}{2}} + s_i^{-\frac{1}{2}}\right)^2.$$

$$\langle T_{\left(\frac{1}{2}\right)^N} T_{\left(\frac{1}{2}\right)^N} \rangle^{USp(2N)/\mathbb{Z}_2}(t;q)$$

$$= \frac{1}{N!} \frac{(q)_\infty^{2N}}{\left(q^{\frac{1}{2}} t^{\pm 2}; q\right)_\infty^N} \oint \prod_{i=1}^{N} \frac{ds_i}{2\pi i s_i} \frac{\left(q^{\frac{1}{2}} s_i^{\pm 2}; q\right)_\infty \left(\left(q^{\frac{3}{2}} s_i^{\pm 2}; q\right)_\infty^{\pm 2}; q\right)_\infty}{(q t^{-2} s_i^{\pm 2}; q)_\infty}$$

$$\times \prod_{i<j} \frac{(s_i^{\pm} s_j^{\mp}; q)_\infty \left(q^{\frac{1}{2}} s_i^{\pm} s_j^{\pm}; q\right)_\infty (q s_i^{\pm} s_j^{\mp}; q)_\infty \left(q^{\frac{3}{2}} s_i^{\pm} s_j^{\pm}; q\right)_\infty}{(t^2 s_i^{\pm} s_j^{\mp}; q)_\infty (q t^2 s_i^{\pm} s_j^{\pm}; q)_\infty \left(q^{\frac{1}{2}} t^{-2} s_i^{\pm} s_j^{\mp}; q\right)_\infty (q t^{-2} s_i^{\pm} s_j^{\pm}; q)_\infty}.$$

$$\langle W_{\mathrm{sp}} W_{\mathrm{sp}} \rangle_{\frac{1}{2}\mathrm{BPS}}^{Spin(2N+1)}(\mathfrak{q}) = \langle T_{\left(\frac{1}{2}^N\right)} T_{\left(\frac{1}{2}^N\right)} \rangle_{\frac{1}{2}\mathrm{BPS}}^{USp(2N)/\mathbb{Z}_2}(\mathfrak{q})$$

$$= \prod_{n=1}^{N} \frac{1}{(1-\mathfrak{q}^{2n})}.$$

$$\mathcal{I}_{\frac{1}{2}\mathrm{BPS}}^{Spin(2N+1)}(\mathfrak{q}) = \prod_{n=1}^{N} \frac{1}{1-\mathfrak{q}^{4n}}$$

$$\langle \mathcal{W}_{\mathrm{sp}} \mathcal{W}_{\mathrm{sp}} \rangle_{\frac{1}{2}\mathrm{BPS}}^{\mathrm{Spin}(2N+1)}(\mathfrak{q}) = \prod_{n=1}^{N} (1+\mathfrak{q}^{2n})$$

$$\langle \underbrace{W_{\mathrm{sp}} \cdots W_{\mathrm{sp}}}_{2k} \rangle_{\frac{1}{2}\mathrm{BPS}}^{\mathrm{Spin}(2N+1)}(\mathfrak{q}) = \frac{\sum_{i=0}^{\frac{N(N+1)k}{2}} a_{k\,\mathrm{sp}}^{\mathrm{so}\,(2N+1)}(i)\mathfrak{q}^{2i}}{\prod_{n=1}^{N} (1-\mathfrak{q}^{4n})}$$

$$a_{k\mathrm{sp}}^{\mathrm{so}\,(2N+1)}(0) = \det(C_{2N-i-j+k})$$

$$= \prod_{1\le i\le j\le k-1} \frac{i+j+2N}{i+j},$$

$$\langle \underbrace{W_\square \cdots W_\square}_{k} \rangle_{\frac{1}{2}\mathrm{BPS}}^{S(2N+1)}(\mathfrak{q}) = \frac{\sum_{i=0}^{Nk} a_{k\square}^{so(2N+1)}(i)\mathfrak{q}^{2i}}{\prod_{n=1}^{N} (1-\mathfrak{q}^{4n})}$$

$$\langle W_\square \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(2N+1)}(\mathfrak{q}) = \frac{\mathfrak{q}^{2N}}{\prod_{n=1}^{N} (1-\mathfrak{q}^{4n})},$$

$$\langle W_\square W_\square \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(2N+1)}(\mathfrak{q}) = \frac{1+\mathfrak{q}^2+\mathfrak{q}^4+\cdots+\mathfrak{q}^{4N}}{\prod_{n=1}^{N} (1-\mathfrak{q}^{4n})}$$

$$= \frac{1-\mathfrak{q}^{4N+2}}{(1-\mathfrak{q}^2)\prod_{n=1}^{N} (1-\mathfrak{q}^{4n})}$$

$$\langle \mathcal{W}_\square \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(2N+1)}(\mathfrak{q}) = \mathfrak{q}^{2N},$$

$$\langle \mathcal{W}_\square \mathcal{W}_\square \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(2N+1)}(\mathfrak{q}) = \frac{1-\mathfrak{q}^{4N+2}}{1-\mathfrak{q}^2}$$

$$\langle \mathcal{W}_\square \mathcal{W}_\square \rangle_{\frac{1}{2}\mathrm{BPS},c}^{SO(2N+1)}(\mathfrak{q}) = \langle \mathcal{W}_\square \mathcal{W}_\square \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(2N+1)}(\mathfrak{q}) - \langle \mathcal{W}_\square \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(2N+1)}(\mathfrak{q})^2$$

$$= \frac{1-\mathfrak{q}^{4N}}{1-\mathfrak{q}^2}.$$

$$\langle \underbrace{W_{\square} \cdots W_{\square}}_{k} \rangle^{\frac{1}{2}\mathrm{BPS}\ SO(2N+1)}(\mathfrak{q}) = \frac{\sum_{i=0}^{(2N-1)k} a_{k\square\square}^{\mathfrak{so}(2N+1)}(i)\mathfrak{q}^{2i}}{\prod_{n=1}^{N} (1-\mathfrak{q}^{4n})}$$

$$\langle W_{\square} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(2N+1)}(\mathfrak{q}) = \frac{\mathfrak{q}^2 + \mathfrak{q}^6 + \cdots + \mathfrak{q}^{4N-2}}{\prod_{n=1}^{N} (1-\mathfrak{q}^{4n})}$$

$$= \frac{\mathfrak{q}^2(1-\mathfrak{q}^{NN})}{(1-\mathfrak{q}^4)\prod_{n=1}^{N} (1-\mathfrak{q}^{4n})}.$$

$$\left\langle \mathcal{W}_{\substack{\square\\ \square}} \right\rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(2N+1)}(\mathfrak{q}) = \frac{\mathfrak{q}^2(1-\mathfrak{q}^{4N})}{1-\mathfrak{q}^4}$$

$$\langle \underbrace{W_{(l)} \cdots W_{(l)}}_{k} \rangle_{\frac{1}{2}\mathrm{BPS}}^{SO(2N+1)}(\mathfrak{q}) = \frac{\sum_{i=0}^{Nlk} a_{k(l)}^{\mathrm{so}(2N+1)}(i)\mathfrak{q}^{2i}}{\prod_{n=1}^{N} (1-\mathfrak{q}^{4n})}$$

$$(z_e, z_m) \in \mathbb{Z}_2 \times \mathbb{Z}_2$$

$$(z_e, z_m) = (0{,}0), (z_e, z_m) = (1{,}0)$$

$$(z_e, z_m) = (0{,}0), (z_e, z_m) = (0{,}1)$$

$$(z_e, z_m) = (0{,}0), (z_e, z_m) = (1{,}1)$$

$$\chi_{\square}^{\mathrm{usp}(2N)} = \sum_{i=1}^{N} (s_i + s_i^{-1})$$

$$\chi_{\square}^{\mu\mathrm{sp}(2N)} = \frac{\det\left(s_j^{\lambda_i+N-i+1} - s_j^{-\lambda_i-N+i-1}\right)}{\det\left(s_j^{N-i+1} - s_j^{-N+i-1}\right)}$$

$$\left\langle W_{\mathcal{R}_1} \cdots W_{\mathcal{R}_k} \right\rangle^{USp(2N)}$$

$$= \int d\mu^{USp(2N)} \exp\left(\sum_{n=1}^{\infty} \frac{1}{n} f_n(q,t) L_n(s)\right) \prod_{i=1}^{k} \chi_{\mathcal{R}_i}^{\boldsymbol{usp}(2N)}(s),$$

$$L_n(s) = \frac{P_n(s)^2 + P_{2n}(s)}{2}$$

$$\exp\left(\sum_{n=1}^{\infty} \frac{1}{n} f_n(q,t) L_n(s)\right) = \sum_{\lambda} \frac{1}{z_\lambda} f_\lambda(q,t) L_\lambda(s)$$

$$\left\langle W_{\mathcal{R}_1} \cdots W_{\mathcal{R}_k} \right\rangle^{USp(2N)} = \sum_{\lambda} \frac{1}{z_\lambda} f_\lambda(q,t) \int d\mu^{USp(2N)} L_\lambda(s) \prod_{i=1}^{k} \chi_{\mathcal{R}_i}^{\mathrm{usp}(2N)}(s)$$

$$L_\lambda(s) \prod_{i=1}^{k} \chi_{\mathcal{R}_i}^{\mathbf{usp}(2N)}(s) = \sum_{\mu} b_{\lambda,\mathcal{R}}^{\mu} P_\mu(s)$$

$$\int d\mu^{USp(2N)} P_\mu(s) = \sum_{\nu \in R_{2N}^c(|\mu|)} \chi_\nu^S(\mu),$$

$$R_n^c(p) = \{\lambda \vdash p \mid \ell(\lambda) \leq n \text{ and } \forall \lambda_i' \text{ is even }\}$$

$$\int d\mu^{USp(2N)} L_\lambda(s) \prod_{i=1}^{k} \chi_{\mathcal{R}_i}^{\mu\mathrm{sp}(2N)}(s) = \sum_{\mu} b_{\lambda,\mathcal{R}}^{\mu} \int d\mu^{USp(2N)} P_\mu(s)$$

$$= \sum_{\mu} b_{\lambda,\mathcal{R}}^{\mu} \sum_{\nu \in R_{2N}^c(|\mu|)} \chi_\nu^S(\mu).$$

$$\left\langle W_{\mathcal{R}_1} \cdots W_{\mathcal{R}_k} \right\rangle^{USp(2N)} = \sum_{\lambda} \frac{1}{z_\lambda} f_\lambda(q,t) \sum_{\mu} b_{\lambda,\mathcal{R}}^{\mu} \sum_{\nu \in R_{2N}^c(|\mu|)} \chi_\nu^S(\mu)$$

$$\langle W_\square W_\square \rangle^{USp(2)}(t;q) = \frac{1}{2}\frac{(q)_\infty^2}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty}\oint \frac{ds}{2\pi i s}\frac{(s^{\pm 2};q)_\infty (qs^{\pm 2};q)_\infty}{\left(q^{\frac{1}{2}}t^2 s^{\pm 2};q\right)_\infty \left(q^{\frac{1}{2}}t^{-2}s^{\pm 2};q\right)_\infty}(s+s^{-1})^2$$

$$\left\langle T_{(1)}T_{(1)}\right\rangle^{SO(3)}(t;q) = \frac{(q)_\infty^2}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty}\oint \frac{ds}{2\pi i s}\frac{\left(q^{\frac{1}{2}}s^{\pm};q\right)_\infty \left(q^{\frac{3}{2}}s^{\pm};q\right)_\infty}{(qt^2 s^{\pm};q)_\infty (qt^{-2}s^{\pm};q)_\infty}$$

$$\langle W_\square W_\square \rangle^{USp(2)}(t;q) = \left\langle W_{\mathrm{sp}} W_{\mathrm{sp}}\right\rangle^{\mathrm{Spin}(3)}(t;q) = \left\langle T_{(1)}T_{(1)}\right\rangle^{SO(3)}(t;q) = \left\langle T_{\left(\frac{1}{2}\right)}T_{\left(\frac{1}{2}\right)}\right\rangle^{USp(2)/\mathbb{Z}_2}(t;q)$$

$$\langle W_\square W_\square \rangle^{USp(2)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \left\langle T_{(1)}T_{(1)}\right\rangle^{SO(3)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1+\mathfrak{q}^2}{1-\mathfrak{q}^4} = \frac{1}{1-\mathfrak{q}^2}$$

$$\langle W_{\square\square} W_{\square\square} \rangle^{USp(2)}(t;q) = \frac{1}{2}\frac{(q)_\infty^2}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty}\oint \frac{ds}{2\pi i s}\frac{(s^{\pm 2};q)_\infty (qs^{\pm 2};q)_\infty}{\left(q^{\frac{1}{2}}t^2 s^{\pm 2};q\right)_\infty \left(q^{\frac{1}{2}}t^{-2}s^{\pm 2};q\right)_\infty}(1+s^2+s^{-2})^2$$

$$\left\langle W_{(2k)}\right\rangle^{USp(2)}(t;q) = \left\langle W_{(k)}\right\rangle^{SO(3)}(t;q).$$

$$\left\langle W_{(2k)}\right\rangle^{USp(2)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\mathfrak{q}^{2k}}{(1-\mathfrak{q}^4)}$$

$$\left\langle W_{(k)}W_{(k)}\right\rangle^{USp(2)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1-\mathfrak{q}^{2k+2}}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)}$$

$$\left\langle W_{(\infty)}W_{(\infty)}\right\rangle^{USS(2)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)}$$

$$\langle W_\square W_\square \rangle^{USp(4)}(t;q) = \frac{1}{8}\frac{(q)_\infty^4}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^2}\oint \prod_{i=1}^{2}\frac{ds_i}{2\pi i s_i}\frac{\left(s_i^{\pm 2};q\right)_\infty \left(qs_i^{\pm 2};q\right)_\infty}{\left(q^{\frac{1}{2}}t^2 s_i^{\pm 2};q\right)_\infty \left(q^{\frac{1}{2}}t^{-2}s_i^{\pm 2};q\right)_\infty}$$

$$\times \frac{\left(s_1^{\pm}s_2^{\mp};q\right)_\infty \left(s_1^{\pm}s_2^{\pm};q\right)_\infty \left(qs_1^{\pm}s_2^{\mp};q\right)_\infty \left(qs_1^{\pm}s_2^{\pm};q\right)_\infty}{\left(q^{\frac{1}{2}}t^2 s_1^{\pm}s_2^{\mp};q\right)_\infty \left(q^{\frac{1}{2}}t^2 s_1^{\pm}s_2^{\pm};q\right)_\infty \left(q^{\frac{1}{2}}t^{-2} s_1^{\pm}s_2^{\mp};q\right)_\infty \left(q^{\frac{1}{2}}t^{-2} s_1^{\pm}s_2^{\pm};q\right)_\infty}\left[\sum_{i=1}^{2}(s_i+s_i^{-1})\right]^2,$$

$$\left\langle T_{(1,0)}T_{(1,0)}\right\rangle^{SO(5)}(t;q)$$

$$=\frac{1}{2}\frac{(q)_\infty^4}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^2}\oint\prod_{i=1}^{2}\frac{ds_i}{2\pi i s_i}\frac{\left(q^{\frac{1}{2}}s_1^{\pm};q\right)_\infty\left(s_2^{\pm};q\right)_\infty\left(q^{\frac{3}{2}}s_1^{\pm};q\right)_\infty;q\Big)_\infty\left(q^{\frac{1}{2}}t^2s_2^{\pm};qs_\infty^{\pm};q\right)_\infty\left(qt^{-2}s_1^{\pm};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_2^{\pm};q\right)_\infty}{}$$

$$\times\frac{\left(q^{\frac{1}{2}}s_1^{\pm}s_2^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}s_1^{\pm}s_2^{\pm};q\right)_\infty\left(q^{\frac{3}{2}}s_1^{\pm}s_2^{\mp};q\right)_\infty\left(q^{\frac{3}{2}}s_1^{\pm}s_2^{\pm};q\right)_\infty}{\left(qt^2s_1^{\pm}s_2^{\mp};q\right)_\infty\left(qt^2s_1^{\pm}s_2^{\pm};q\right)_\infty\left(qt^{-2}s_1^{\pm}s_2^{\mp};q\right)_\infty\left(qt^{-2}s_1^{\pm}s_2^{\pm};q\right)_\infty}.$$

$$\langle W_{\square}W_{\square}\rangle^{USp(4)}(t;q)=\left\langle W_{\mathrm{sp}}W_{\mathrm{sp}}\right\rangle^{\mathrm{Spin}(5)}(t;q)$$

$$=\left\langle T_{(1,0)}T_{(1,0)}\right\rangle^{SO(5)}(t;q)=\left\langle T_{\left(\frac{1}{2},\frac{1}{2}\right)}T_{\left(\frac{1}{2},\frac{1}{2}\right)}\right\rangle^{USp(4)/\mathbb{Z}_2}(t;q).$$

$$\langle W_{\square}W_{\square}\rangle^{USp(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})=\langle T_{(1,0)}T_{(1,0)}\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$=\frac{1}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)},$$

$$\left\langle W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}\right\rangle^{USp(4)}(t;q)=\left\langle W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}\right\rangle^{SO(5)}(t;q)$$

$$\langle\underbrace{W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}\cdots W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}}_{k}\rangle^{USp(4)}(t;q)=\langle\underbrace{W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}\cdots W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}}_{k}\rangle^{SO(5)}(t;q)$$

$$\langle\underbrace{W_{\square\square}\cdots W_{\square\square}}_{k}\rangle^{USp(4)}(t;q)=\langle\underbrace{W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}\cdots W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}}_{k}\rangle^{SO(5)}(t;q)$$

$$\langle\underbrace{W_{(2l)}\cdots W_{(2l)}}_{k}\rangle^{USp(4)}(t;q)=\langle\underbrace{W_{(l^2)}\cdots W_{(l^2)}}_{k}\rangle^{SO(5)}(t;q)$$

$$\langle\underbrace{W_{(2l)}\cdots W_{(2l)}}_{k}\rangle^{USp(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})=\frac{\sum_{i=0}^{3lk}a_k^{\mathrm{usp}\,(4)}(i)\mathfrak{q}^{2i}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}$$

$$\left\langle W_{(\infty)}W_{(\infty)}\right\rangle^{USp(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})=\left\langle W_{(\infty^2)}W_{(\infty^2)}\right\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$=1+\mathfrak{q}^2+4\mathfrak{q}^4+5\mathfrak{q}^6+13\mathfrak{q}^8+16\mathfrak{q}^{10}+33\mathfrak{q}^{12}+41\mathfrak{q}^{14}+73\mathfrak{q}^{16}+90\mathfrak{q}^{18}+145\mathfrak{q}^{20}+\cdots.$$

$$\left\langle W_{(\infty)}W_{(\infty)}\right\rangle^{USp(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})=\left\langle W_{(\infty^2)}W_{(\infty^2)}\right\rangle^{SO(5)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$=\frac{1-\mathfrak{q}^{16}}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)^3(1-\mathfrak{q}^6)(1-\mathfrak{q}^8)^2}.$$

$$\langle\underbrace{W_{(l^2)}\cdots W_{(l^2)}}_{k}\rangle^{USp(4)}(t;q)=\langle\underbrace{W_{(l)}\cdots W_{(l)}}_{k}\rangle^{SO(5)}(t;q)$$

$$\langle W_{\square}W_{\square}\rangle^{USp(6)}(t;q)$$

$$=\frac{1}{48}\frac{(q)_\infty^6}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^3}\oint\prod_{i=1}^{3}\frac{ds_i}{2\pi i s_i}\frac{\left(s_i^{\pm 2};q\right)_\infty\left(qs_i^{\pm 2};q\right)_\infty}{\left(q^{\frac{1}{2}}t^2s_i^{\pm 2};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm 2};q\right)_\infty}$$

$$\times\prod_{i<j}\frac{\left(s_i^{\pm}s_j^{\mp};q\right)_\infty\left(s_i^{\pm}s_j^{\pm};q\right)_\infty\left(qs_i^{\pm}s_j^{\mp};q\right)_\infty\left(qs_i^{\pm}s_j^{\pm};q\right)_\infty}{\left(q^{\frac{1}{2}}t^2s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^2s_i^{\pm}s_j^{\pm};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm}s_j^{\pm};q\right)_\infty}\left[\sum_{i=1}^{3}\left(s_i+s_i^{-1}\right)\right]^2.$$

$$\langle T_{(1,0,0)}T_{(1,0,0)}\rangle^{SO(7)}(t;q)$$

$$= \frac{1}{8}\frac{(q)_\infty^6}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^3}\oint\prod_{i=1}^{3}\frac{ds_i}{2\pi i s_i}\frac{\left(q^{\frac{1}{2}\delta_{i,1}}s_i^{\pm};q\right)_\infty\left(q^{1+\frac{1}{2}\delta_{i,1}}s_i^{\pm};q\right)_\infty}{\left(q^{\frac{1+\delta_{i,1}}{2}}t^2 s_i^{\pm};q\right)_\infty\left(q^{\frac{1+\delta_{i,1}}{2}}t^{-2}s_i^{\pm};q\right)_\infty}$$

$$\times\prod_{i<j}\frac{\left(q^{\frac{1}{2}\delta_{i+j,1}}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}\delta_{i+j,1}}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{1+\frac{1}{2}\delta_{i+j,1}}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{1+\frac{1}{2}\delta_{i+j,1}}s_i^{\pm}s_j^{\pm};q\right)_\infty}{\left(q^{\frac{1+\delta_{i+j,1}}{2}}t^2 s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1+\delta_{i+j,1}}{2}}t^2 s_i^{\pm}s_j^{\pm};q\right)_\infty\left(q^{\frac{1+\delta_{i+j,1}}{2}}t^{-2} s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1+\delta_{i+j,1}}{2}}t^{-2} s_i^{\pm}s_j^{\pm};q\right)_\infty}.$$

$$\langle W_{\square}W_{\square}\rangle^{USp(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \langle T_{(1,0,0)}T_{(1,0,0)}\rangle^{SO(7)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$= \frac{1+\mathfrak{q}^2+\mathfrak{q}^4+\mathfrak{q}^6+\mathfrak{q}^8+\mathfrak{q}^{10}}{(1-\mathfrak{q}^4)\big(1-\mathfrak{q}^8(1-\mathfrak{q}^{12})\big)}$$

$$= \frac{1}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}.$$

$$\langle\underbrace{W_{\square}\cdots W_{\square}}_{2k}\rangle^{USp(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\sum_{i=0}^{5k} a^{\mathrm{usp}(6)}_{k\square}(i)\mathfrak{q}^{2i}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}$$

$$\langle W_{\square}W_{\square}W_{\square}W_{\square}\rangle^{USp(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}(3+6\mathfrak{q}^2+9\mathfrak{q}^4+12\mathfrak{q}^6+14\mathfrak{q}^8$$

$$+15\mathfrak{q}^{10}+12\mathfrak{q}^{12}+9\mathfrak{q}^{14}+6\mathfrak{q}^{16}+3\mathfrak{q}^{18}+\mathfrak{q}^{20})$$

$$\det\begin{pmatrix} F_0 & F_1 & F_2 \\ F_1 & F_2 & F_3 \\ F_2 & F_3 & F_4 \end{pmatrix} = \sum_{k=0}^{\infty}\frac{a^{\mathrm{usp}\,(6)}_{k\square}(0)}{(2k)!}x^{2k}$$

$$F_m(x) := \sum_{j=0}^{m}\binom{m}{j}\big(I_{2j-m}(2x) - I_{2j-m+2}(2x)\big)$$

$$I_k(2x) := \sum_{n=0}^{\infty}\frac{x^{2n+k}}{n!\,(n+k)!}$$

$$\left\langle W_{\substack{\square\\\square}}\right\rangle^{USp(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\mathfrak{q}^4+\mathfrak{q}^8}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})},$$

$$\left\langle W_{\substack{\square\\\square}}W_{\substack{\square\\\square}}\right\rangle^{USp(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1+\mathfrak{q}^2+2\mathfrak{q}^4+2\mathfrak{q}^6+3\mathfrak{q}^8+2\mathfrak{q}^{10}+3\mathfrak{q}^{12}+\mathfrak{q}^{14}+\mathfrak{q}^{16}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}.$$

$$\left\langle W_{\substack{\square\\\square\\\square}}W_{\substack{\square\\\square\\\square}}\right\rangle^{USS(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1+\mathfrak{q}^2+\mathfrak{q}^4+2\mathfrak{q}^6+2\mathfrak{q}^8+2\mathfrak{q}^{10}+2\mathfrak{q}^{12}+\mathfrak{q}^{14}+\mathfrak{q}^{16}+\mathfrak{q}^{18}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}$$

$$\langle\underbrace{W_{\square\square}\cdots W_{\square\square}}_{k}\rangle^{USp(6)}(t;q) = \langle\underbrace{W_{\substack{\square\\\square}}\cdots W_{\substack{\square\\\square}}}_{k}\rangle^{SO(7)}(t;q)$$

$$\langle W_{\square\square}\rangle^{USp(6)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) \;=\frac{\mathfrak{q}^2+\mathfrak{q}^6+\mathfrak{q}^{10}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})},$$

$$\langle W_{\square\square}W_{\square\square}\rangle^{USp(6)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) \;=\frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}$$

$$\times(1+\mathfrak{q}^2+3\mathfrak{q}^4+2\mathfrak{q}^6+5\mathfrak{q}^8+3\mathfrak{q}^{10}$$

$$+5\mathfrak{q}^{12}+2\mathfrak{q}^{14}+3\mathfrak{q}^{16}+\mathfrak{q}^{18}+\mathfrak{q}^{20})$$

$$\langle \underbrace{W_{(2l)}\cdots W_{(2l)}}_{k}\rangle^{USp(6)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q})=\frac{\sum_{i=0}^{5lk}\ a^{uspp(6)}_{k(2l)}(i)\mathfrak{q}^{2i}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}$$

$$\langle W_{\square\square\square}\rangle^{USp(6)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q})=\frac{\mathfrak{q}^4+\mathfrak{q}^8+2\mathfrak{q}^{12}+\mathfrak{q}^{16}+\mathfrak{q}^{20}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}$$

$$\langle W_{\square}W_{\square}\rangle^{USp(2N)}(t;q)$$

$$=\frac{1}{2^N N!}\frac{(q)_\infty^{2N}}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^N}\oint\prod_{i=1}^{N}\frac{ds_i}{2\pi i s_i}\frac{(s_i^{\pm 2};q)_\infty(qs_i^{\pm 2};q)_\infty}{\left(q^{\frac{1}{2}}t^2s_i^{\pm 2};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm 2};q\right)_\infty}$$

$$\times\prod_{i<j}\frac{(s_i^{\pm}s_j^{\mp};q)_\infty(s_i^{\pm}s_j^{\pm};q)_\infty(qs_i^{\pm}s_j^{\mp};q)_\infty(qs_i^{\pm}s_j^{\pm};q)_\infty}{\left(q^{\frac{1}{2}}t^2s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^2s_i^{\pm}s_j^{\pm};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm}s_j^{\pm};q\right)_\infty}\left[\sum_{i=1}^{N}(s_i+s_i^{-1})\right]^2.$$

$$\langle T_{(1,0^{N-1})}T_{(1,0^{N-1})}\rangle^{SO(2N+1)}(t;q)$$

$$=\frac{1}{2^{N-1}(N-1)!}\frac{(q)_\infty^{2N}}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^N}\oint\prod_{i=1}^{N}\frac{ds_i}{2\pi i s_i}\frac{\left(q^{\frac{1}{2}\delta_{i,1}}s_i^{\pm};q\right)_\infty\left(q^{1+\frac{1}{2}\delta_{i,1}}s_i^{\pm};q\right)_\infty}{\left(\frac{1+\delta_{i,1}}{2}t^2s_i^{\pm};q\right)_\infty\left(q^{\frac{1+\delta_{i,1}}{2}}t^{-2}s_i^{\pm};q\right)_\infty}$$

$$\times\prod_{i<j}\frac{\left(q^{\frac{1}{2}\delta_{i+j,1}}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}\delta_{i+j,1}}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{1+\frac{1}{2}\delta_{i+j,1}}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{1+\frac{1}{2}\delta^{\pm}_{i+j,1,1}s_i^{\pm}s_j^{\pm};q}\right)_\infty}{\left(q^{\frac{1+\delta_{i+j,1}}{2}}t^2s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1+\delta_{i+j,1}}{2}}t^2s_i^{\pm}s_j^{\pm};q\right)_\infty\left(q^{\frac{1+\delta^2_{i+j,1}}{2}}t^{-2}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1+\delta_{i+j,1}}{2}}t^{-2}s_i^{\pm}s_j^{\pm};q\right)_\infty}.$$

$$\langle W_{\square}W_{\square}\rangle^{USp(2N)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q})=\left\langle T_{(1,0^{N-1})}T_{(1,0^{N-1})}\right\rangle^{SO(2N+1)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q})$$

$$=\frac{1}{(1-\mathfrak{q}^2)\prod_{n=1}^{N-1}(1-\mathfrak{q}^{4n})}.$$

$$\mathcal{I}^{USp(2N)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q})=\prod_{n=1}^{N}\frac{1}{1-\mathfrak{q}^{4n}}$$

$$\langle \mathcal{W}_{\square}\mathcal{W}_{\square}\rangle^{USp(2N)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q})=\frac{1-\mathfrak{q}^{4N}}{1-\mathfrak{q}^2}$$

$$\left\langle\underbrace{W_{\square}\cdots W_{\square}}_{2k}\right\rangle^{USp(2N)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q})=\frac{\sum_{i=0}^{(2N-1)k}\ a^{\text{usp}\,(2N)}_{k\square}(i)\mathfrak{q}^{2i}}{\prod_{n=1}^{N}(1-\mathfrak{q}^{4n})}$$

$$\det\left(F_{i+j-2}(x)\right)=\sum_{k=0}^{\infty}\frac{a^{\text{usp}\,(2N)}_{k\square}(0)}{(2k)!}x^{2k}$$

$$\langle \underbrace{W_{\substack{\square\\\square}} \cdots W_{\substack{\square\\\square}}}_{k} \rangle^{USp(2N)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\sum_{i=0}^{2(N-1)k} a^{\mathrm{usp}(2N)}_{k\substack{\square\\\square}}(i)\mathfrak{q}^{2i}}{\prod_{n=1}^{N}(1-\mathfrak{q}^{4n})}$$

$$\langle \underbrace{W_{\square\square} \cdots W_{\square\square}}_{k} \rangle^{USp(2N)}(t;q) = \langle \underbrace{W_{\substack{\square\\\square}} \cdots W_{\substack{\square\\\square}}}_{k} \rangle^{SO(2N+1)}(t;q)$$

$$\langle W_{\square\square} \rangle^{USp(2N)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\mathfrak{q}^2+\mathfrak{q}^6+\cdots+\mathfrak{q}^{4N-2}}{\prod_{n=1}^{N}(1-\mathfrak{q}^{4n})}$$

$$= \frac{\mathfrak{q}^2(1-\mathfrak{q}^{4N})}{(1-\mathfrak{q}^4)\prod_{n=1}^{N}(1-\mathfrak{q}^{4n})}.$$

$$\langle \underbrace{W_{(2l)} \cdots W_{(2l)}}_{k} \rangle^{USp(2N)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\sum_{i=0}^{(2N-1)lk} a^{\mathrm{usp}(2N)}_{k(2l)}(i)\mathfrak{q}^{2i}}{\prod_{n=1}^{N}(1-\mathfrak{q}^{4n})}$$

$$(z_{e,S}, z_{e,C}; z_{m,S}, z_{m,C}) \in (\mathbb{Z}_2\times\mathbb{Z}_2)\times(\mathbb{Z}_2\times\mathbb{Z}_2)$$

$$\begin{aligned}
&\mathrm{Spin}(2N),\\
&SO(2N) = \mathrm{Spin}(2N)/\mathbb{Z}_2^V,\\
&Ss(2N) = \mathrm{Spin}(2N)/\mathbb{Z}_2^S,\\
&Sc(2N) = \mathrm{Spin}(2N)/\mathbb{Z}_2^C,\\
&SO(2N)/\mathbb{Z}_2 = \mathrm{Spin}(2N)/(\mathbb{Z}_2^S\times\mathbb{Z}_2^C).
\end{aligned}$$

$$\begin{aligned}
Ss(2N)_+ :\ &(z_{e,S}, z_{e,C}; z_{m,S}, z_{m,C}) = (0,0;0,0),\\
&(z_{e,S}, z_{e,C}; z_{m,S}, z_{m,C}) = (1,0;0,0),\\
&(z_{e,S}, z_{e,C}; z_{m,S}, z_{m,C}) = (0,0;0,1),\\
Ss(2N)_- :\ &(z_{e,S}, z_{e,C}; z_{m,S}, z_{m,C}) = (0,0;0,0),\\
&(z_{e,S}, z_{e,C}; z_{m,S}, z_{m,C}) = (1,0;0,0),\\
&(z_{e,S}, z_{e,C}; z_{m,S}, z_{m,C}) = (0,1;0,1).\\
Ss(2N)_+ :\ &(z_{e,S}, z_{e,C}; z_{m,S}, z_{m,C}) = (0,0;0,0),\\
&(z_{e,S}, z_{e,C}; z_{m,S}, z_{m,C}) = (1,0;0,0),\\
&(z_{e,S}, z_{e,C}; z_{m,S}, z_{m,C}) = (0,0;1,0),\\
Ss(2N)_- :\ &(z_{e,S}, z_{e,C}; z_{m,S}, z_{m,C}) = (0,0;0,0),\\
&(z_{e,S}, z_{e,C}; z_{m,S}, z_{m,C}) = (1,0;0,0),\\
&(z_{e,S}, z_{e,C}; z_{m,S}, z_{m,C}) = (0,1;1,0).
\end{aligned}$$

$$\begin{aligned}
&(z_{e,S}, z_{e,C}; z_{m,S}, z_{m,C}) = (0,0;0,0)\\
&(z_{e,S}, z_{e,C}; z_{m,S}, z_{m,C}) = (n_{SS}, n_{SC};1,0),\\
&(z_{e,S}, z_{e,C}; z_{m,S}, z_{m,C}) = (n_{CS}, n_{CC};0,1),
\end{aligned}$$

$$(z_e, z_m) \in \mathbb{Z}_4\times\mathbb{Z}_4$$

$$\begin{aligned}
&\mathrm{Spin}(2N),\\
&SO(2N) = \mathrm{Spin}(2N)/\mathbb{Z}_2,\\
&SO(2N)/\mathbb{Z}_2 = \mathrm{Spin}(2N)/\mathbb{Z}_4.
\end{aligned}$$

$$(z_e, z_m) = (0,0), (z_e, z_m) = (n, 1)$$

$$\chi^{\mathfrak{so}(2N)}_{\mathrm{sp}} = \frac{1}{2}\left[\prod_{i=1}^{N}\left(s_i^{\frac{1}{2}}+s_i^{-\frac{1}{2}}\right)+\prod_{i=1}^{N}\left(s_i^{\frac{1}{2}}-s_i^{-\frac{1}{2}}\right)\right]$$

$$\chi_{\mathrm{sp}}^{\mathfrak{so}(2N)} = \frac{1}{2}\left[\prod_{i=1}^{N}\left(s_i^{\frac{1}{2}} + s_i^{-\frac{1}{2}}\right) - \prod_{i=1}^{N}\left(s_i^{\frac{1}{2}} - s_i^{-\frac{1}{2}}\right)\right]$$

$$\chi_{\square}^{\mathfrak{so}\,(2N)} = \sum_{i=1}^{N}\left(s_i + s_i^{-1}\right)$$

$$\chi_{\lambda}^{\mathfrak{so}(2N)} = \frac{\det\left(s_j^{\lambda_i+N-i} + s_j^{-\lambda_i-N+i}\right) + \det\left(s_j^{\lambda_i+N-i} - s_j^{-\lambda_i-N+i}\right)}{\det\left(s_j^{N-i} + s_j^{-N+i}\right)}$$

$$\left\langle W_{\mathcal{R}_1}\cdots W_{\mathcal{R}_k}\right\rangle^{SO(2N)} = \int d\mu^{SO(2N)}\exp\left(\sum_{n=1}^{\infty}\frac{1}{n}f_n(q,t)M_n(s)\right)\prod_{i=1}^{k}\chi_{\mathcal{R}_i}^{\mathfrak{so}(2N)}(s),$$

$$d\mu^{SO(2N)} = \frac{1}{2^{N-1}N!}\prod_{i=1}^{N}\frac{ds_i}{2\pi i s_i}\prod_{1\le i<j\le N}\left(1-s_is_j\right)\left(1-s_i^{-1}s_j^{-1}\right)\left(1-s_is_j^{-1}\right)\left(1-s_i^{-1}s_j\right)$$

$$M_n(s) = \frac{P_n(s)^2 + P_{2n}(s)}{2}$$

$$\mathcal{I}^{SO(4)}(t;q) = \mathcal{I}^{SU(2)}(t;q)\times\mathcal{I}^{SU(2)}(t;q) = \frac{\left(q^{\frac{1}{2}}t^{\pm2};q\right)_\infty^2}{(q;q)_\infty^4}\left(\sum_{\substack{p_1,p_2\in\mathbb{Z}\\ p_1<p_2}}\frac{\left(q^{\frac{1}{2}}t^{-2}\right)^{p_1+p_2-2}}{\left(1-q^{p_1-\frac{1}{2}}t^2\right)\left(1-q^{p_2-\frac{1}{2}}t^2\right)}\right)^2.$$

$$\left\langle W_{\mathrm{sp}}W_{\mathrm{sp}}\right\rangle^{\mathrm{Spin}(4)}(t;q) = \frac{1}{4}\frac{(q)_\infty^4}{\left(q^{\frac{1}{2}}t^{\pm2};q\right)_\infty^2}\oint\prod_{i=1}^{2}\frac{ds_i}{2\pi i s_i}$$
$$\times\frac{\left(s_1^{\pm}s_2^{\mp};q\right)_\infty\left(s_1^{\pm}s_2^{\pm};q\right)_\infty\left(qs_1^{\pm}s_2^{\mp};q\right)_\infty\left(qs_1^{\pm}s_2^{\pm};q\right)_\infty}{\left(q^{\frac{1}{2}}t^2s_1^{\pm}s_2^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^2s_1^{\pm}s_2^{\pm};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_1^{\pm}s_2^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_1^{\pm}s_2^{\pm};q\right)_\infty}$$
$$\times\left(s_1^{\frac{1}{2}}s_2^{\frac{1}{2}} + s_1^{-\frac{1}{2}}s_2^{-\frac{1}{2}}\right)^2.$$

$$\left\langle T_{\left(\frac{1}{2},\frac{1}{2}\right)}T_{\left(\frac{1}{2},\frac{1}{2}\right)}\right\rangle^{SO(4)/\mathbb{Z}_2}(t;q) = \frac{1}{2}\frac{(q)_\infty^4}{\left(q^{\frac{1}{2}}t^{\pm2};q\right)_\infty^2}\oint\prod_{i=1}^{2}\frac{ds_i}{2\pi i s_i}$$
$$\times\frac{\left(q^{\frac{1}{2}}s_1^{\pm}s_2^{\mp};q\right)_\infty\left(s_1^{\pm}s_2^{\pm};q\right)_\infty\left(q^{\frac{3}{2}}s_1^{\pm}s_2^{\mp};q\right)_\infty\left(qs_1^{\pm}s_2^{\pm};q\right)_\infty}{\left(qt^2s_1^{\pm}s_2^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^2s_1^{\pm}s_2^{\pm};q\right)_\infty\left(qt^{-2}s_1^{\pm}s_2^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_1^{\pm}s_2^{\pm};q\right)_\infty}.$$

$$\left\langle W_{\mathrm{sp}}W_{\mathrm{sp}}\right\rangle^{Spin(4)}(t;q) = \left\langle T_{\left(\frac{1}{2},\frac{1}{2}\right)}T_{\left(\frac{1}{2},\frac{1}{2}\right)}\right\rangle^{SO(4)/\mathbb{Z}_2}(t;q) = \mathcal{I}^{SU(2)}(t;q)\left\langle W_\square W_\square\right\rangle^{SU(2)}(t;q),$$

$$\langle W_{sp}W_{sp}\rangle^{\mathrm{Spin}(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \left\langle T_{\left(\frac{1}{2},\frac{1}{2}\right)}T_{\left(\frac{1}{2},\frac{1}{2}\right)}\right\rangle^{SO(4)/\mathbb{Z}_2}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$= \frac{1+\mathfrak{q}^2}{(1-\mathfrak{q}^4)^2}$$

$$= \frac{1}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)}.$$

$$\langle \underbrace{W_{sp}\cdots W_{sp}}_{2k}\rangle^{Spin(4)}(t;q) = \mathcal{I}^{SU(2)}(t;q)\langle \underbrace{W_{\square}\cdots W_{\square}}_{2k}\rangle^{SU(2)}(t;q)$$

$$\langle \underbrace{W_{sp}\cdots W_{sp}}_{2k}\rangle^{\mathrm{Spin}(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \mathcal{I}^{SO(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})\sum_{i=0}^{k} a^{\mathrm{so}\,(4)}_{ksp}(i)\mathfrak{q}^{2i}$$

$$= \frac{1}{(1-\mathfrak{q}^4)^2}\sum_{i=0}^{k} a^{\mathrm{so}\,(4)}_{ksp}(i)\mathfrak{q}^{2i}$$

$$a^{\mathrm{so}\,(4)}_{ksp}(i) = \frac{(2i+1)(2k)!}{(k-i)!\,(k+i+1)!}$$

$$\langle W_{sp}W_{sp}W_{sp}W_{sp}\rangle^{\mathrm{Spin}(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{2+3\mathfrak{q}^2+\mathfrak{q}^4}{(1-\mathfrak{q}^4)^2},$$

$$\langle W_{sp}W_{sp}W_{sp}W_{sp}W_{sp}W_{sp}\rangle^{Spin(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{5+9\mathfrak{q}^2+5\mathfrak{q}^4+\mathfrak{q}^6}{(1-\mathfrak{q}^4)^2},$$

$$\langle W_{sp}W_{sp}W_{sp}W_{sp}W_{sp}W_{sp}W_{sp}W_{sp}\rangle^{Spin(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{14+28\mathfrak{q}^2+20\mathfrak{q}^4+7\mathfrak{q}^6+\mathfrak{q}^8}{(1-\mathfrak{q}^4)^2}.$$

$$\langle W_{sp}^{2k}W_{sp}^{2m}\rangle^{\mathrm{Spin}(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \left(\mathcal{I}^{SO(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})\right)^{-1}\langle W_{sp}^{2k}\rangle^{\mathrm{Spin}(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})\langle W_{sp}^{2m}\rangle^{\mathrm{Spin}(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$= (1-\mathfrak{q}^4)^2\langle W_{sp}^{2k}\rangle^{\mathrm{Sin}(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})\langle W_{sp}^{2m}\rangle^{\mathrm{Sin}(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}).$$

$$\langle W_{sp}^{2k}W_{sp}^{2m}\rangle^{\mathrm{Spin}(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^4)^2}\left(\sum_{i=0}^{k} a^{\mathrm{so}(4)}_{ksp}(i)\mathfrak{q}^{2i}\right)\left(\sum_{j=0}^{m} a^{\mathrm{so}(4)}_{msp}(j)\mathfrak{q}^{2j}\right)$$

$$\langle W_{\square}W_{\square}\rangle^{SO(4)}(t;q) = \frac{1}{4}\frac{(q)_\infty^4}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^2}\oint\prod_{i=1}^{2}\frac{ds_i}{2\pi i s_i}$$

$$\times\frac{(s_1^{\pm}s_2^{\mp};q)_\infty(s_1^{\pm}s_2^{\pm};q)_\infty(qs_1^{\pm}s_2^{\mp};q)_\infty(qs_1^{\pm}s_2^{\pm};q)_\infty}{\left(q^{\frac{1}{2}}t^2s_1^{\pm}s_2^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^2s_1^{\pm}s_2^{\pm};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_1^{\pm}s_2^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_1^{\pm}s_2^{\pm};q\right)_\infty}$$

$$\times(s_1+s_2+s_1^{-1}+s_2^{-1})^2.$$

$$\left\langle T_{(1,0)}T_{(1,0)}\right\rangle^{SO(4)}(t;q) = \frac{(q)_\infty^4}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^2}\oint\prod_{i=1}^{2}\frac{ds_i}{2\pi i s_i}$$

$$\times\frac{\left(q^{\frac{1}{2}}s_1^{\pm}s_2^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}s_1^{\pm}s_2^{\pm};q\right)_\infty\left(q^{\frac{3}{2}}s_1^{\pm}s_2^{\mp};q\right)_\infty\left(q^{\frac{3}{2}}s_1^{\pm}s_2^{\pm};q\right)_\infty}{(qt^2s_1^{\pm}s_2^{\mp};q)_\infty(qt^2s_1^{\pm}s_2^{\pm};q)_\infty(qt^{-2}s_1^{\pm}s_2^{\mp};q)_\infty(qt^{-2}s_1^{\pm}s_2^{\pm};q)_\infty}.$$

$$\langle W_\square W_\square\rangle^{SO(4)}(t;q) = \langle W_\square W_\square\rangle^{SU(2)}(t;q)^2$$

$$\left\langle W_\square W_\square\right\rangle^{SO(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^2)^2}$$

$$\langle \underbrace{W_\square \cdots W_\square}_{2k}\rangle^{SO(4)}(t;q) = \langle \underbrace{W_\square \cdots W_\square}_{k}\rangle^{SU(2)}(t;q)^2$$

$$\langle \underbrace{W_\square \cdots W_\square}_{2k}\rangle^{SO(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\sum_{i=0}^{2k} a_k^{\mathfrak{so}(4)}(i)\mathfrak{q}^{2i}}{(1-\mathfrak{q}^4)^2}$$

$$\langle W_\square W_\square W_\square W_\square\rangle^{SO(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{4+12\mathfrak{q}^2+13\mathfrak{q}^4+6\mathfrak{q}^6+\mathfrak{q}^8}{(1-\mathfrak{q}^4)^2}$$

$$\left\langle W_\square W_\square W_\square W_\square W_\square W_\square\right\rangle^{SO(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{25+90\mathfrak{q}^2+131\mathfrak{q}^4+100\mathfrak{q}^6+43\mathfrak{q}^8+10\mathfrak{q}^{10}+\mathfrak{q}^{12}}{(1-\mathfrak{q}^4)^2}$$

$$\langle W_\square W_\square W_\square W_\square W_\square W_\square W_\square W_\square\rangle^{SO(4)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^4)^2}(196+784\mathfrak{q}^2+1344\mathfrak{q}^4+1316\mathfrak{q}^6$$
$$+820\mathfrak{q}^8+336\mathfrak{q}^{10}+89\mathfrak{q}^{12}+14\mathfrak{q}^{14}+\mathfrak{q}^{16}).$$

$$a_{k\square}^{\mathfrak{so}(4)}(0) = C_k^2$$

$$\langle W_\square W_\square\rangle^{SO(4)^-}(t;q) = \frac{1}{2}\frac{(q)_\infty^2(-q;q)_\infty^2}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty\left(-q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty}\oint\frac{ds}{2\pi i s}$$
$$\times\frac{(s^\pm;q)_\infty(-s;q)_\infty(qs^\pm;q)_\infty(-qs^\pm;q)_\infty}{\left(q^{\frac{1}{2}}t^2s^\pm;q\right)_\infty\left(-q^{\frac{1}{2}}t^2s^\pm;q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s^\pm;q\right)_\infty\left(-q^{\frac{1}{2}}t^{-2}s^\pm;q\right)_\infty}(s+s^{-1})^2.$$

$$\left\langle T_{(1)}T_{(1)}\right\rangle^{SO(4)^-}(t;q) = \frac{(q)_\infty^2(-q;q)_\infty^2}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty\left(-q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty}\oint\frac{ds}{2\pi i s}$$
$$\times\frac{\left(q^{\frac{1}{2}}s^\pm;q\right)_\infty\left(-q^{\frac{1}{2}}s;q\right)_\infty\left(q^{\frac{3}{2}}s^\pm;q\right)_\infty\left(-q^{\frac{3}{2}}s^\pm;q\right)_\infty}{(qt^2s^\pm;q)_\infty(-qt^2s^\pm;q)_\infty(qt^{-2}s^\pm;q)_\infty(-qt^{-2}s^\pm;q)_\infty}.$$

$$\langle W_\square W_\square\rangle^{SO(4)^-}(t;q) = \left\langle T_{(1)}T_{(1)}\right\rangle^{SO(4)^-}(t;q)$$
$$= \langle W_\square W_\square\rangle^{SU(2)}(t^2;q^2).$$

$$\langle W_\square W_\square\rangle^{SO(4)^-}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \left\langle T_{(1)}T_{(1)}\right\rangle^{SO(4)^-}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$
$$= \frac{1}{1-\mathfrak{q}^4}.$$

$$\langle \underbrace{W_\square \cdots W_\square}_{2k}\rangle^{SO(4)^-}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\sum_{i=0}^{k} a_{k\square}^{\mathfrak{so}(4)^-}(i)\mathfrak{q}^{4i}}{1-\mathfrak{q}^8}$$

$$a_{k\square}^{\mathfrak{so}(4)^-}(i) = (2i+1)\frac{(2k)!}{(k-i)!\,(k+i+1)!},$$

$$\left\langle W_{\lambda_1}\cdots W_{\lambda_k}\right\rangle^{O(4)^+}(t;q) = \frac{1}{2}\left[\left\langle W_{\lambda_1}\cdots W_{\lambda_k}\right\rangle^{SO(4)}(t;q) + \left\langle W_{\lambda_1}\cdots W_{\lambda_k}\right\rangle^{SO(4)^-}(t;q)\right].$$

$$\langle W_\square W_\square\rangle^{O(4)^+}(t;q) = \left\langle T_{(1)}T_{(1)}\right\rangle^{O(4)^+}(t;q)$$

$$\langle \underbrace{W_{\square} \cdots W_{\square}}_{2k} \rangle^{O(4)^+}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{\sum_{i=0}^{2k+1} a^{O(4)}_{k\square}(i)\mathfrak{q}^{2i}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}$$

$$\langle W_{\square} W_{\square} \rangle^{O(4)^+}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{1+\mathfrak{q}^2+\mathfrak{q}^4+\mathfrak{q}^6}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)} = \frac{1}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)},$$

$$\langle W_{\square} W_{\square} W_{\square} W_{\square} \rangle^{O(4)^+}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{3+6\mathfrak{q}^2+9\mathfrak{q}^4+9\mathfrak{q}^6+6\mathfrak{q}^8+3\mathfrak{q}^{10}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)},$$

$$\langle W_{\square} W_{\square} W_{\square} W_{\square} W_{\square} W_{\square} \rangle^{O(4)^+}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}(15+45\mathfrak{q}^2+80\mathfrak{q}^4+95\mathfrak{q}^6+85\mathfrak{q}^8 +55\mathfrak{q}^{10}+20\mathfrak{q}^{12}+5\mathfrak{q}^{14}).$$

$$a^{\tiny{o}(4)}_{k\square}(0) = \frac{1}{2}(C_k^2 + C_k)$$

$$\substack{\square\\\square} = (1,1),\quad \substack{\square\\\square} = (1,-1)$$

$$\langle \underbrace{W_{\substack{\square\\\square}} \cdots W_{\substack{\square\\\square}}}_{k} \rangle^{SO(4)}(t;q) = \langle \underbrace{W_{\substack{\square\\\square}} \cdots W_{\substack{\square\\\square}}}_{k} \rangle^{SO(4)}(t;q) = \mathcal{I}^{SU(2)}(t;q)\langle \underbrace{W_{\square\square} \cdots W_{\square\square}}_{k} \rangle^{SU(2)}(t;q).$$

$$\Big\langle \underbrace{W_{\square} \cdots W_{\square}}_{k} \Big\rangle^{SO(4)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{\sum_{i=0}^{k} a^{SO(4)}_{k\substack{\square\\\square}}(i)\mathfrak{q}^{2i}}{(1-\mathfrak{q}^4)^2}$$

$$\Big\langle W_{\substack{\square\\\square}} \Big\rangle^{SO(4)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{\mathfrak{q}^2}{(1-\mathfrak{q}^4)^2},$$

$$\Big\langle W_{\substack{\square\\\square}} W_{\substack{\square\\\square}} \Big\rangle^{SO(4)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{1+\mathfrak{q}^2+\mathfrak{q}^4}{(1-\mathfrak{q}^4)^2},$$

$$\Big\langle W_{\substack{\square\\\square}} W_{\substack{\square\\\square}} W_{\substack{\square\\\square}} \Big\rangle^{SO(4)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{1+3\mathfrak{q}^2+2\mathfrak{q}^4+\mathfrak{q}^6}{(1-\mathfrak{q}^4)^2}$$

$$\Big\langle W_{\substack{\square\\\square}} W_{\substack{\square\\\square}} W_{\substack{\square\\\square}} W_{\substack{\square\\\square}} \Big\rangle^{SO(4)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{3+6\mathfrak{q}^2+6\mathfrak{q}^4+3\mathfrak{q}^6+\mathfrak{q}^8}{(1-\mathfrak{q}^4)^2}$$

$$\Big\langle W_{\substack{\square\\\square}} W_{\substack{\square\\\square}} W_{\substack{\square\\\square}} W_{\substack{\square\\\square}} W_{\substack{\square\\\square}} \Big\rangle^{SO(4)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{6+15\mathfrak{q}^2+15\mathfrak{q}^4+10\mathfrak{q}^6+4\mathfrak{q}^8+\mathfrak{q}^{10}}{(1-\mathfrak{q}^4)^2},$$

$$\Big\langle W_{\substack{\square\\\square}} W_{\substack{\square\\\square}} W_{\substack{\square\\\square}} W_{\substack{\square\\\square}} W_{\substack{\square\\\square}} W_{\substack{\square\\\square}} \Big\rangle^{SO(4)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{15+36\mathfrak{q}^2+40\mathfrak{q}^4+29\mathfrak{q}^6+15\mathfrak{q}^8+5\mathfrak{q}^{10}+\mathfrak{q}^{12}}{(1-\mathfrak{q}^4)^2}$$

$$\Big\langle \big(W_{\substack{\square\\\square}}\big)^k \big(W_{\substack{\square\\\square}}\big)^m \Big\rangle^{SO(4)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = (1-\mathfrak{q}^4)^2 \Big\langle \big(W_{\substack{\square\\\square}}\big)^k \Big\rangle^{SO(4)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) \Big\langle \big(W_{\substack{\square\\\square}}\big)^m \Big\rangle^{SO(4)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q})$$

$$\langle \underbrace{W_{(l)} \cdots W_{(l)}}_{k} \rangle^{SO(4)} (t;q) = \langle \underbrace{W_{(l)} \cdots W_{(l)}}_{k} \rangle^{SU(2)} (t;q)^2$$

$$\langle \underbrace{W_{\square\square} \cdots W_{\square\square}}_{k} \rangle^{SO(4)}_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q}) = \frac{\sum_{i=0}^{2k} a^{\mathrm{so}(4)^-}_{k\square\square}(i)\mathfrak{q}^{2i}}{(1-\mathfrak{q}^4)^2}$$

$$\langle W_{\square\square} \rangle^{SO(4)}_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q}) = \frac{\mathfrak{q}^4}{(1-\mathfrak{q}^4)^2}$$

$$\langle W_{\square\square} W_{\square\square} \rangle^{SO(4)}_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q}) = \frac{(1+\mathfrak{q}^2+\mathfrak{q}^4)^2}{(1-\mathfrak{q}^4)^2}$$

$$\langle W_{\square\square} W_{\square\square} W_{\square\square} \rangle^{SO(4)}_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q}) = \frac{(1+3\mathfrak{q}^2+2\mathfrak{q}^4+\mathfrak{q}^6)^2}{(1-\mathfrak{q}^4)^2}$$

$$a^{\mathrm{so}(4)^-}_{k\square\square}(0) = R_k^2$$

$$a^{\mathrm{so}(4)^-}_{k\square\square}(1) = 2R_k R_{k+1}$$

$$\langle (W_{\square\square})^k \rangle^{SO(4)} = \left\langle \left(W_{\substack{\square\\\square}}\right)^k \left(W_{\substack{\square\\\square}}\right)^k \right\rangle^{SO(4)}$$

$$\langle W_{(2k)} \rangle^{SO(4)}_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q}) = \frac{\mathfrak{q}^{4k}}{(1-\mathfrak{q}^4)^2}$$

$$\langle W_{(k)} W_{(k)} \rangle^{SO(4)}_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q}) = \frac{(1-\mathfrak{q}^{2k+2})^2}{(1-\mathfrak{q}^2)^2(1-\mathfrak{q}^4)^2}$$

$$\langle W_{(\infty)} W_{(\infty)} \rangle^{SO(4)}_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^2)^2(1-\mathfrak{q}^4)^2}$$

$$\langle \underbrace{W_{(l)} \cdots W_{(l)}}_{k} \rangle^{SO(4)^-} (t;q) = \langle \underbrace{W_{(l)} \cdots W_{(l)}}_{k} \rangle^{SU(2)} (t^2;q^2)$$

$$\langle W_{(2l)} \rangle^{SO(4)^-}_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q}) = \frac{\mathfrak{q}^{4l}}{1-\mathfrak{q}^8},$$

$$\langle W_{(l)} W_{(l)} \rangle^{SO(4)^-}_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q}) = \frac{1-\mathfrak{q}^{4l+4}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}.$$

$$\langle W_{(2l)} \rangle^{O(4)^+}_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q}) = \frac{\mathfrak{q}^{4l}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)},$$

$$\langle W_{(l)} W_{(l)} \rangle^{O(4)^+}_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q}) = \frac{1-\mathfrak{q}^2+\mathfrak{q}^4-\mathfrak{q}^{2l+2}-\mathfrak{q}^{2l+6}+\mathfrak{q}^{4l+6}}{(1-\mathfrak{q}^2)^2(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}$$

$$\langle W_{(\infty)} W_{(\infty)} \rangle^{O(4)^+}_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q}) = \frac{1-\mathfrak{q}^2+\mathfrak{q}^4}{(1-\mathfrak{q}^2)^2(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}$$

$$\begin{aligned} \langle \underbrace{W_{(l,l)} \cdots W_{(l,l)}}_{k} \rangle^{SO(4)} (t;q) &= \langle \underbrace{W_{(l,-l)} \cdots W_{(l,-l)}}_{k} \rangle^{SO(4)} (t;q) \\ &= \mathcal{I}^{SU(2)}(t;q) \langle \underbrace{W_{(2l)} \cdots W_{(2l)}}_{k} \rangle^{SU(2)} (t;q). \end{aligned}$$

$$\langle W_{(l,l)} \rangle^{SO(4)}_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q}) = \langle W_{(l,-l)} \rangle^{SO(4)}_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q}) = \frac{\mathfrak{q}^{2l}}{(1-\mathfrak{q}^4)^2},$$

$$\langle W_{(l,l)} W_{(l,l)} \rangle^{SO(4)}_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q}) = \langle W_{(l,-l)} W_{(l,-l)} \rangle^{SO(4)}_{\frac{1}{2}\mathrm{BPS}} (\mathfrak{q}) = \frac{1-\mathfrak{q}^{4l+2}}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)^2}.$$

$$\langle W^k_{(l,l)} W^k_{(l,-l)} \rangle^{SO(4)} = \langle W^k_{(2l)} \rangle^{SO(4)}$$

$$\mathcal{I}^{SO(6)}(t;q) = \mathcal{I}^{SU(4)}(t;q)$$
$$= -\frac{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty}{(q;q)_\infty^2} \sum_{\substack{p_1,p_2,p_3,p_4\in\mathbb{Z} \\ p_1<p_2<p_3<p_4}} \frac{\left(q^{\frac{1}{2}}t^{-2}\right)^{p_1+p_2+p_3+p_4-8}}{(1-q^{p_1-1}t^4)(1-q^{p_2-1}t^4)(1-q^{p_3-1}t^4)(1-q^{p_4-1}t^4)}$$

$$\langle W_{\mathrm{sp}} W_{\overline{\mathrm{sp}}} \rangle^{\mathrm{Spin}(6)}(t;q) = \frac{1}{24} \frac{(q)_\infty^6}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^3} \oint \prod_{i=1}^{3} \frac{ds_i}{2\pi i s_i}$$
$$\times \prod_{i\neq j} \frac{(s_i^\pm s_j^\mp;q)_\infty (s_i^\pm s_j^\pm;q)_\infty (q s_i^\pm s_j^\mp;q)_\infty (q s_i^\pm s_j^\pm;q)_\infty}{\left(q^{\frac{1}{2}}t^2 s_i^\pm s_j^\mp;q\right)_\infty \left(q^{\frac{1}{2}}t^2 s_i^\pm s_j^\pm;q\right)_\infty \left(q^{\frac{1}{2}}t^{-2} s_i^\pm s_j^\mp;q\right)_\infty \left(q^{\frac{1}{2}}t^{-2} s_i^\pm s_j^\pm;q\right)_\infty}$$
$$\times (1 + s_1 s_2 + s_1 s_3 + s_2 s_3)(1 + s_1^{-1}s_2^{-1} + s_1^{-1}s_3^{-1} + s_2^{-1}s_3^{-1})$$
$$\left\langle T_{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)} T_{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)} \right\rangle^{SO(6)/\mathbb{Z}_2}(t;q) = \frac{1}{6} \frac{(q)_\infty^6}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^3} \oint \prod_{i=1}^{3} \frac{ds_i}{2\pi i s_i}$$
$$\times \prod_{i<j} \frac{(s_i^\pm s_j^\mp;q)_\infty \left(q^{\frac{1}{2}} s_i^\pm s_j^\pm;q\right)_\infty (q s_i^\pm s_j^\mp;q)_\infty \left(q^{\frac{3}{2}} s_i^\pm s_j^\pm;q\right)_\infty}{\left(q^{\frac{1}{2}}t^2 s_i^\pm s_j^\mp;q\right)_\infty (q t^2 s_i^\pm s_j^\pm;q)_\infty \left(q^{\frac{1}{2}}t^{-2} s_i^\pm s_j^\mp;q\right)_\infty (q t^{-2} s_i^\pm s_j^\pm;q)_\infty}.$$
$$\langle W_{\mathrm{sp}} W_{\overline{\mathrm{sp}}} \rangle^{Spin(6)}(t;q) = \langle T_{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)} T_{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)} \rangle^{SO(6)/\mathbb{Z}_2}(t;q)$$
$$= \langle W_{\square} W_{\overline{\square}} \rangle^{SU(4)}(t;q).$$
$$\langle W_{\mathrm{sp}} W_{\overline{\mathrm{sp}}} \rangle^{Spin(6)}(t;q) = \left\langle T_{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)} T_{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)} \right\rangle^{SO(6)/\mathbb{Z}_2}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)(1-\mathfrak{q}^6)}$$
$$\langle \underbrace{W_{\mathrm{sp}} W_{\mathrm{sp}} \cdots W_{\mathrm{sp}} W_{\mathrm{sp}}}_{2k} \rangle^{Spin(6)}(t;q) = \langle \underbrace{W_{\square} W_{\overline{\square}} \cdots W_{\square} W_{\overline{\square}}}_{2k} \rangle^{SU(4)}(t;q)$$
$$\langle \underbrace{W_{\mathrm{sp}} W_{\overline{\mathrm{sp}}} \cdots W_{\mathrm{sp}} W_{\overline{\mathrm{sp}}}}_{2k} \rangle^{Spin(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\sum_{i=0}^{3k} a_{k\ \mathrm{sp}}^{\mathrm{so}\,(6)}(i)\mathfrak{q}^{2i}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^6)(1-\mathfrak{q}^8)}$$
$$\left\langle W_{\mathrm{sp}} W_{\overline{\mathrm{sp}}} W_{\mathrm{sp}} W_{\overline{\mathrm{sp}}} \right\rangle^{Spin(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{2 + 4\mathfrak{q}^2 + 6\mathfrak{q}^4 + 7\mathfrak{q}^6 + 5\mathfrak{q}^8 + 3\mathfrak{q}^{10} + \mathfrak{q}^{12}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^6)(1-\mathfrak{q}^8)}$$
$$\langle W_{\mathrm{sp}} W_{\overline{\mathrm{sp}}} W_{\mathrm{sp}} W_{\overline{\mathrm{sp}}} W_{\mathrm{sp}} W_{\overline{\mathrm{sp}}} \rangle^{Spin(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^6)(1-\mathfrak{q}^8)}(6 + 18\mathfrak{q}^2 + 35\mathfrak{q}^4 + 50\mathfrak{q}^6$$
$$+53\mathfrak{q}^8 + 45\mathfrak{q}^{10} + 29\mathfrak{q}^{12} + 14\mathfrak{q}^{14} + 5\mathfrak{q}^{16} + \mathfrak{q}^{18}).$$

$$\det \begin{pmatrix} I_0(2x) & I_1(2x) & I_2(2x) & I_3(2x) \\ I_1(2x) & I_0(2x) & I_1(2x) & I_2(2x) \\ I_2(2x) & I_1(2x) & I_0(2x) & I_1(2x) \\ I_3(2x) & I_2(2x) & I_1(2x) & I_0(2x) \end{pmatrix} = \sum_{k=0}^{\infty} \frac{a_{k\ \mathrm{sp}}^{\mathrm{so}(6)}(0)}{(k!)^2} x^{2k}$$

$$
\langle W_{\square} W_{\square} \rangle^{SO(6)}(t;q) = \frac{1}{24} \frac{(q)_\infty^6}{\left(q^{\frac{1}{2}} t^{\pm 2}; q\right)_\infty^3} \oint \prod_{i=1}^{3} \frac{ds_i}{2\pi i s_i}
$$

$$
\times \prod_{i \neq j} \frac{(s_i^{\pm} s_j^{\mp}; q)_\infty (s_i^{\pm} s_j^{\pm}; q)_\infty (q s_i^{\pm} s_j^{\mp}; q)_\infty (q s_i^{\pm} s_j^{\pm}; q)_\infty}{\left(q^{\frac{1}{2}} t^2 s_i^{\pm} s_j^{\mp}; q\right)_\infty \left(q^{\frac{1}{2}} t^2 s_i^{\pm} s_j^{\pm}; q\right)_\infty \left(q^{\frac{1}{2}} t^{-2} s_i^{\pm} s_j^{\mp}; q\right)_\infty \left(q^{\frac{1}{2}} t^{-2} s_i^{\pm} s_j^{\pm}; q\right)_\infty}
$$

$$
\times (s_1 + s_2 + s_3 + s_1^{-1} + s_2^{-1} + s_3^{-1})^2.
$$

$$
\left\langle T_{(1,0,0)} T_{(1,0,0)} \right\rangle^{SO(6)}(t;q) = \frac{1}{4} \frac{(q)_\infty^6}{\left(q^{\frac{1}{2}} t2; q\right)_\infty^3} \oint \prod_{i=1}^{3} \frac{ds_i}{2\pi i s_i}
$$

$$
\times \prod_{i<j} \frac{\left(q^{\frac{1}{2}\delta_{i+j,1}} s_i^{\pm} s_j^{\mp}; q\right)_\infty \left(q^{\frac{1}{2}\delta_{i+j,1}} s_i^{\pm} s_j^{\pm}; q\right)_\infty}{\left(q^{\frac{1}{2}(1+\delta_{i+j,1})} t^2 s_i^{\pm} s_j^{\mp}; q\right)_\infty \left(q^{\frac{1}{2}(1+\delta_{i+j,1})} t^2 s_i^{\pm} s_j^{\pm}; q\right)_\infty}
$$

$$
\times \frac{\left(q^{1+\frac{1}{2}\delta_{i+j,1'}} s_i^{\pm} s_j^{\mp}; q\right)_\infty \left(q^{1+\frac{1}{2}\delta_{i+j,1}} s_i^{\pm} s_j^{\pm}; q\right)_\infty}{\left(q^{\frac{1}{2}(1+\delta_{i+j,1,1}} t^{-2} s_i^{\pm} s_j^{\mp}; q\right)_\infty \left(q^{\frac{1}{2}(1+\delta_{i+j,1})} t^{-2} s_i^{\pm} s_j^{\pm}; q\right)_\infty}.
$$

$$
\begin{aligned}
\langle W_{\square} W_{\square} \rangle^{SO(6)}(t;q) &= \left\langle T_{(1,0,0)} T_{(1,0,0)} \right\rangle^{SO(6)}(t;q) \\
&= \left\langle W_{\substack{\square\\\square}} \cdots W_{\substack{\square\\\square}} \right\rangle^{SU(4)}(t;q).
\end{aligned}
$$

$$
\begin{aligned}
\langle W_{\square} W_{\square} \rangle^{SO(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) &= \left\langle T_{(1,0,0)} T_{(1,0,0)} \right\rangle^{SO(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) \\
&= \frac{1 + \mathfrak{q}^2 + 2\mathfrak{q}^4 + \mathfrak{q}^6 + \mathfrak{q}^8}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^6)(1-\mathfrak{q}^8)} \\
&= \frac{1}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)^2}.
\end{aligned}
$$

$$
\langle \underbrace{W_{\square} \cdots W_{\square}}_{2k} \rangle^{SO(6)}(t;q) = \langle \underbrace{W_{\substack{\square\\\square}} \cdots W_{\substack{\square\\\square}}}_{2k} \rangle^{SU(4)}(t;q)
$$

$$
\langle \langle \underbrace{W_{\square} \cdots W_{\square}}_{2k} \rangle^{SO(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\sum_{i=0}^{4k} \; a_{k\square}^{\mathfrak{son}(6)}(i) \mathfrak{q}^{2i}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^6)(1-\mathfrak{q}^8)}
$$

$$
\begin{aligned}
\langle W_{\square} W_{\square} W_{\square} W_{\square} \rangle^{SO(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^6)(1-\mathfrak{q}^8)} &(3 + 7\mathfrak{q}^2 + 15\mathfrak{q}^4 + 18\mathfrak{q}^6 + 20\mathfrak{q}^8 \\
&+14\mathfrak{q}^{10} + 9\mathfrak{q}^{12} + 3\mathfrak{q}^{14} + \mathfrak{q}^{16}),
\end{aligned}
$$

$$
\begin{aligned}
\langle W_{\square} W_{\square} W_{\square} W_{\square} W_{\square} W_{\square} \rangle^{SO(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^6)(1-\mathfrak{q}^8)} &(16 + 60\mathfrak{q}^2 + 149\mathfrak{q}^4 + 249\mathfrak{q}^6 \\
&+334\mathfrak{q}^8 + 347\mathfrak{q}^{10} + 301\mathfrak{q}^{12} + 206\mathfrak{q}^{14} + 119\mathfrak{q}^{16} \\
&+53\mathfrak{q}^{18} + 20\mathfrak{q}^{20} + 5\mathfrak{q}^{22} + \mathfrak{q}^{24}).
\end{aligned}
$$

$$\langle W_{\square} W_{\square}\rangle^{SO(6)^-}(t;q) = \frac{1}{8}\frac{(q)_\infty^4(-q;q)_\infty^2}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^2\left(-q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty}$$

$$\times \oint \prod_{i=1}^{2}\frac{ds_i}{2\pi i s_i}\frac{(s_i^{\pm};q)_\infty(-s_i^{\pm};q)_\infty(qs_i^{\pm};q)_\infty(-qs_i^{\pm};q)_\infty}{\left(q^{\frac{1}{2}}t^2 s_i^{\pm};q\right)_\infty\left(-q^{\frac{1}{2}}t^2 s_i^{\pm};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2} s_i^{\pm};q\right)_\infty\left(-q^{\frac{1}{2}}t^{-2} s_i^{\pm};q\right)_\infty}$$

$$\times \frac{(s_1^{\pm}s_2^{\mp};q)_\infty(s_1^{\pm}s_2^{\pm};q)_\infty(qs_1^{\pm}s_2^{\mp};q)_\infty(qs_1^{\pm}s_2^{\pm};q)_\infty}{\left(q^{\frac{1}{2}}t^2 s_1^{\pm}s_2^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^2 s_1^{\pm}s_2^{\pm};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2} s_1^{\pm}s_2^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2} s_1^{\pm}s_2^{\pm};q\right)_\infty}$$

$$\times (s_1 + s_2 + s_1^{-1} + s_2^{-1})^2.$$

$$\langle T_{(1,0)} T_{(1,0)}\rangle^{SO(6)^-}(t;q) = \frac{1}{2}\frac{(q)_\infty^4(-q;q)_\infty^2}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^2\left(-q^{\frac{1}{2}}t^{\pm};q\right)_\infty}\oint \prod_{i=1}^{2}\frac{ds_i}{2\pi i s_i}$$

$$\times \frac{\left(q^{\frac{1}{2}}s_1^{\pm};q\right)_\infty(s_2^{\pm};q)_\infty\left(-q^{\frac{1}{2}}s_1^{\pm};q\right)_\infty(-s_2^{\pm};q)_\infty}{(qt^2 s_1^{\pm};q)_\infty\left(q^{\frac{1}{2}}t^2 s_2^{\pm};q\right)_\infty(-qt^2 s_1^{\pm};q)_\infty\left(-q^{\frac{1}{2}}t^2 s_2^{\pm};q\right)_\infty}$$

$$\times \frac{\left(q^{\frac{3}{2}}s_1^{\pm};q\right)_\infty(qs_2^{\pm};q)_\infty\left(-q^{\frac{3}{2}}s_1^{\pm};q\right)_\infty(-qs_2^{\pm};q)_\infty}{(qt^{-2} s_1^{\pm};q)_\infty\left(q^{\frac{1}{2}}t^{-2} s_2^{\pm};q\right)_\infty(-qt^{-2} s_1^{\pm};q)_\infty\left(-q^{\frac{1}{2}}t^{-2} s_2^{\pm};q\right)_\infty}$$

$$\times \frac{\left(q^{\frac{1}{2}}s_1^{\pm}s_2^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}s_1^{\pm}s_2^{\pm};q\right)_\infty\left(q^{\frac{3}{2}}s_1^{\pm}s_2^{\mp};q\right)_\infty\left(q^{\frac{3}{2}}s_1^{\pm}s_2^{\pm};q\right)_\infty}{(qt^2 s_1^{\pm}s_2^{\mp};q)_\infty(qt^2 s_1^{\pm}s_2^{\pm};q)_\infty(qt^{-2} s_1^{\pm}s_2^{\mp};q)_\infty(qt^{-2} s_1^{\pm}s_2^{\pm};q)_\infty},$$

$$\langle W_{\square} W_{\square}\rangle^{SO(6)^-}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \langle T_{(1,0)} T_{(1,0)}\rangle^{SO(6)^-}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$
$$= \frac{1+\mathfrak{q}^2+\mathfrak{q}^6+\mathfrak{q}^8}{(1+\mathfrak{q}^6)(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}$$
$$= \frac{1}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^8)}.$$

$$\langle \underbrace{W_{\square}\cdots W_{\square}}_{2k}\rangle^{SO(6)^-}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\sum_{i=0}^{4k} a_k^{so(6)^-}(i)\mathfrak{q}^{2i}}{(1+\mathfrak{q}^6)(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)^8},$$

$$\langle W_{\square}W_{\square}W_{\square}W_{\square}\rangle^{SO(6)^-}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{3+5\mathfrak{q}^2+3\mathfrak{q}^4+6\mathfrak{q}^6+8\mathfrak{q}^8+4\mathfrak{q}^{10}+3\mathfrak{q}^{12}+3\mathfrak{q}^{14}+\mathfrak{q}^{16}}{(1+\mathfrak{q}^6)(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}$$

$$\langle W_{\square}W_{\square}W_{\square}W_{\square}W_{\square}W_{\square}\rangle^{SO(6)^-}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1+\mathfrak{q}^6)(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}(14+30\mathfrak{q}^2+31\mathfrak{q}^4+49\mathfrak{q}^6$$
$$+66\mathfrak{q}^8+55\mathfrak{q}^{10}+49\mathfrak{q}^{12}+46\mathfrak{q}^{14}+29\mathfrak{q}^{16}+15\mathfrak{q}^{18}$$
$$+10\mathfrak{q}^{20}+5\mathfrak{q}^{22}+\mathfrak{q}^{24})$$

$$a_k^{so(6)^-}(0) = C_k C_{k+2} - C_{k+1}^2$$

$$\langle W_{\lambda_1}\cdots W_{\lambda_k}\rangle^{O(6)^+}(t;q) = \frac{1}{2}\left[\langle W_{\lambda_1}\cdots W_{\lambda_k}\rangle^{SO(6)^+}(t;q) + \langle W_{\lambda_1}\cdots W_{\lambda_k}\rangle^{SO(6)^-}(t;q)\right]$$

$$\langle W_{\square} W_{\square} \rangle^{O(6)^+}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1+\mathfrak{q}^2+\mathfrak{q}^4+\mathfrak{q}^6+\mathfrak{q}^8+\mathfrak{q}^{10}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}$$
$$= \frac{1}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)},$$
$$\langle W_{\square} W_{\square} W_{\square} W_{\square} \rangle^{O(6)^+}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}(3+6\mathfrak{q}^2+9\mathfrak{q}^4+12\mathfrak{q}^6$$
$$+15\mathfrak{q}^8+15\mathfrak{q}^{10}+12\mathfrak{q}^{12}+9\mathfrak{q}^{14}+6\mathfrak{q}^{16}+3\mathfrak{q}^{18}).$$

$$\langle \underbrace{W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \cdots W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}}_{k} \rangle^{SO(6)}(t;q) = \langle \underbrace{W_{\begin{smallmatrix}\square\square\\\square\\\square\end{smallmatrix}} W_{\begin{smallmatrix}\square\square\\\square\\\square\end{smallmatrix}}}_{k} \rangle^{SU(4)}(t;q),$$

$$\left\langle \underbrace{W_{\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}} \cdots W_{\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}}}_{2k} \right\rangle^{SO(6)}(t;q) = \langle \underbrace{W_{\square\square} \cdots W_{\square\square}}_{2k} \rangle^{SU(4)}(t;q)$$

$$\left\langle W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \right\rangle^{SO(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\mathfrak{q}^2}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)},$$
$$\left\langle W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}} W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \right\rangle^{SO(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}$$
$$\times(1+2\mathfrak{q}^2+4\mathfrak{q}^4+6\mathfrak{q}^6+7\mathfrak{q}^8+7\mathfrak{q}^{10}$$
$$+6\mathfrak{q}^{12}+5\mathfrak{q}^{14}+3\mathfrak{q}^{16}+\mathfrak{q}^{18}),$$
$$\left\langle W_{\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}} W_{\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}} \right\rangle^{SO(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}$$
$$\times(1+\mathfrak{q}^2+2\mathfrak{q}^4+3\mathfrak{q}^6+3\mathfrak{q}^8+3\mathfrak{q}^{10}$$
$$+3\mathfrak{q}^{12}+2\mathfrak{q}^{14}+\mathfrak{q}^{16}+\mathfrak{q}^{18}).$$
$$\left\langle W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \right\rangle^{SO(6)^-}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\mathfrak{q}^2}{(1+\mathfrak{q}^2)(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)},$$
$$\left\langle W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}} W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \right\rangle^{SO(6)^-}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1+\mathfrak{q}^2+2\mathfrak{q}^4+\mathfrak{q}^8}{(1+\mathfrak{q}^2)(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}.$$
$$\left\langle W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \right\rangle^{O(6)^+}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\mathfrak{q}^2}{(1-\mathfrak{q}^4)^2(1-\mathfrak{q}^8)},$$
$$\left\langle W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}} W_{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \right\rangle^{O(6)^+}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1+\mathfrak{q}^2+2\mathfrak{q}^4+\mathfrak{q}^6+2\mathfrak{q}^8}{(1-\mathfrak{q}^4)^2(1-\mathfrak{q}^8)}.$$
$$\langle \underbrace{W_{(l)} \cdots W_{(l)}}_{2k} \rangle^{SO(6)}(t;q) = \langle \underbrace{W_{(l^2)} \cdots W_{\overline{(l^2)}}}_{k} \rangle^{SU(4)}(t;q)$$

$$\langle W_{\square\square}\rangle^{SO(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\mathfrak{q}^4+\mathfrak{q}^8}{(1+\mathfrak{q}^4)(1-\mathfrak{q}^6)(1-\mathfrak{q}^8)},$$

$$\langle W_{\square\square}W_{\square\square}\rangle^{SO(6)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}$$
$$\times (1+\mathfrak{q}^2+3\mathfrak{q}^4+4\mathfrak{q}^6+6\mathfrak{q}^8+6\mathfrak{q}^{10}+7\mathfrak{q}^{12}$$
$$+6\mathfrak{q}^{14}+4\mathfrak{q}^{16}+4\mathfrak{q}^{18}+\mathfrak{q}^{20}+\mathfrak{q}^{22})$$

$$\langle W_{\square\square}\rangle^{SO(6)^-}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\mathfrak{q}^4+\mathfrak{q}^8}{(1+\mathfrak{q}^6)(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)},$$

$$\langle W_{\square\square}W_{\square\square}\rangle^{SO(6)^-}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1+2\mathfrak{q}^8-2\mathfrak{q}^{10}+\mathfrak{q}^{12}-\mathfrak{q}^{14}-\mathfrak{q}^{18}}{(1+\mathfrak{q}^6)(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)}.$$

$$\langle W_{\square\square}\rangle^{O(6)^+}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{\mathfrak{q}^4+\mathfrak{q}^8}{(1+\mathfrak{q}^6)(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)},$$

$$\langle W_{\square\square}W_{\square\square}\rangle^{O(6)^+}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1+\mathfrak{q}^2+2\mathfrak{q}^4+2\mathfrak{q}^6+4\mathfrak{q}^8+3\mathfrak{q}^{10}+4\mathfrak{q}^{12}+2\mathfrak{q}^{14}+2\mathfrak{q}^{16}+\mathfrak{q}^{18}}{(1-\mathfrak{q}^4)(1-\mathfrak{q}^8)(1-\mathfrak{q}^{12})}.$$

$$\left\langle W_{\mathrm{sp}}W_{\mathrm{sp}}\right\rangle^{\mathrm{Spin}(2N)}(t;q) = \frac{1}{2^{N-1}N!}\frac{(q)_\infty^{2N}}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^N}\oint\prod_{i=1}^N\frac{ds_i}{2\pi i s_i}$$
$$\times\prod_{i\neq j}\frac{(s_i^{\pm}s_j^{\mp};q)_\infty(s_i^{\pm}s_j^{\pm};q)_\infty(qs_i^{\pm}s_j^{\mp};q)_\infty(qs_i^{\pm}s_j^{\pm};q)_\infty}{\left(q^{\frac{1}{2}}t^2s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^2s_i^{\pm}s_j^{\pm};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm}s_j^{\pm};q\right)_\infty}$$
$$\times\frac{1}{4}\left[\prod_{i=1}^N\left(s_i^{\frac{1}{2}}+s_i^{-\frac{1}{2}}\right)+\prod_{i=1}^N\left(s_i^{\frac{1}{2}}-s_i^{-\frac{1}{2}}\right)\right]^2.$$

$$\left\langle W_{\mathrm{sp}}W_{\overline{\mathrm{sp}}}\right\rangle^{\mathrm{Spin}(2N)}(t;q) = \frac{1}{2^{N-1}N!}\frac{(q)_\infty^{2N}}{\left(q^{\frac{1}{2}}tt^{\pm 2};q\right)_\infty^N}\oint\prod_{i=1}^N\frac{ds_i}{2\pi i s_i}$$
$$\times\prod_{i\neq j}\frac{(s_i^{\pm}s_j^{\mp};q)_\infty(s_i^{\pm}s_j^{\pm};q)_\infty(qs_i^{\pm}s_j^{\mp};q)_\infty(qs_i^{\pm}s_j^{\pm};q)_\infty}{\left(q^{\frac{1}{2}}t^2s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^2s_i^{\pm}s_j^{\pm};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm}s_j^{\pm};q\right)_\infty}$$
$$\times\frac{1}{4}\left[\prod_{i=1}^N\left(s_i^{\frac{1}{2}}+s_i^{-\frac{1}{2}}\right)+\prod_{i=1}^N\left(s_i^{\frac{1}{2}}-s_i^{-\frac{1}{2}}\right)\right]\left[\prod_{i=1}^N\left(s_i^{\frac{1}{2}}+s_i^{-\frac{1}{2}}\right)-\prod_{i=1}^N\left(s_i^{\frac{1}{2}}-s_i^{-\frac{1}{2}}\right)\right]$$

$$\langle T_{\left(\frac{1}{2}^N\right)}T_{\left(\frac{1}{2}^N\right)}\rangle^{SO(2N)/\mathbb{Z}_2}(t;q) = \frac{1}{N!}\frac{(q)_\infty^{2N}}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^N}\oint\prod_{i=1}^N\frac{ds_i}{2\pi i s_i}$$
$$\times\prod_{i<j}\frac{(s_i^{\pm}s_j^{\mp};q)_\infty\left(q^{\frac{1}{2}}s_i^{\pm}s_j^{\pm};q\right)_\infty(qs_i^{\pm}s_j^{\mp};q)_\infty\left(q^{\frac{3}{2}}s_i^{\pm}s_j^{\pm};q\right)_\infty}{\left(q^{\frac{1}{2}}t^2s_i^{\pm}s_j^{\mp};q\right)_\infty(qt^2s_i^{\pm}s_j^{\pm};q)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm}s_j^{\mp};q\right)_\infty(qt^{-2}s_i^{\pm}s_j^{\pm};q)_\infty}.$$

$$\langle W_{\rm sp}W_{\rm sp}\rangle^{Spin(2N)}_{\frac{1}{2}\rm BPS}(\mathfrak{q}) = \langle T_{\left(\frac{1}{2}^{N}\right)}T_{\left(\frac{1}{2}^{N}\right)}\rangle^{SO(2N)/\mathbb{Z}_2}_{\frac{1}{2}\rm BPS}(\mathfrak{q})$$
$$= \prod_{n=1}^{N}\frac{1}{1-\mathfrak{q}^{2n}}$$

$$\langle W_{\rm sp}W_{\overline{\rm sp}}\rangle^{Spin(2N)}_{\frac{1}{2}\rm BPS}(\mathfrak{q}) = \left\langle T_{\left(\frac{1}{2}^{N}\right)}T_{\left(\frac{1}{2}^{N}\right)}\right\rangle^{SO(2N)/\mathbb{Z}_2}_{\frac{1}{2}\rm BPS}(\mathfrak{q})$$
$$= \prod_{n=1}^{N}\frac{1}{1-\mathfrak{q}^{2n}}$$
$$\mathcal{I}^{Spin(2N)}_{\frac{1}{2}\rm BPS}(\mathfrak{q}) = \frac{1}{1-\mathfrak{q}^{2N}}\prod_{n=1}^{N-1}\frac{1}{1-\mathfrak{q}^{4n}}$$
$$\langle \mathcal{W}_{\rm sp}\mathcal{W}_{\rm sp}\rangle^{\rm Spin(2N)}_{\frac{1}{2}\rm BPS}(\mathfrak{q}) = \langle \mathcal{W}_{\rm sp}\mathcal{W}_{\overline{\rm sp}}\rangle^{\rm Spin(2N)}_{\frac{1}{2}\rm BPS}(\mathfrak{q})$$
$$= \prod_{n=1}^{N-1}(1+\mathfrak{q}^{2n}).$$
$$\langle \underbrace{W_{\rm sp}\cdots W_{\rm sp}}_{2k}\rangle^{\rm Spin(2N=4n)}_{\frac{1}{2}\rm BPS}(\mathfrak{q}) = \langle \underbrace{W_{\rm sp}\cdots W_{\overline{\rm sp}}}_{2k}\rangle^{\rm Spin(2N=4n+2)}_{\frac{1}{2}\rm BPS}(\mathfrak{q})$$
$$= \frac{\sum_{i=0}^{\frac{N(N+1)k}{2}} a^{\rm sp(2N)}_{k\rm sp}(i)\mathfrak{q}^{2i}}{1-\mathfrak{q}^{2N}\prod_{n=1}^{N-1} 1-\mathfrak{q}^{4n}},$$
$$\langle W_{\Box}W_{\Box}\rangle^{SO(2N)}(t;q) = \frac{1}{2^{N-1}N!}\frac{(q)_\infty^{2N}}{\left(q^{\frac{1}{2}t}t^{\pm 2};q\right)_\infty^{N}}\oint\prod_{i=1}^{N}\frac{ds_i}{2\pi i s_i}$$
$$\times\prod_{i\neq j}\frac{(s_i^{\pm}s_j^{\mp};q)_\infty(s_i^{\pm}s_j^{\pm};q)_\infty(qs_i^{\pm}s_j^{\mp};q)_\infty(qs_i^{\pm}s_j^{\pm};q)_\infty}{\left(q^{\frac{1}{2}}t^2s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^2s_i^{\pm}s_j^{\pm};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm}s_j^{\pm};q\right)_\infty}\left[\sum_{i=1}^{N}(s_i+s_i^{-1})\right]^2.$$
$$\left\langle T_{(1,0^{N-1})}T_{(1,0^{N-1})}\right\rangle^{SO(2N)}(t;q) = \frac{1}{2^{N-2}(N-1)!}\frac{(q)_\infty^{2N}}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^{N}}$$
$$\times\oint\prod_{i=1}^{N}\frac{ds_i}{2\pi i s_i}\prod_{i<j}\frac{\left(q^{\frac{1}{2}\delta_{i+j,1}}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}\delta_{i+j,1}}s_i^{\pm}s_j^{\pm};q\right)_\infty}{\left(q^{\frac{1}{2}(1+\delta_{i+j,1})}t^2s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}(1+\delta_{i+j,1})}t^2s_i^{\pm}s_j^{\pm};q\right)_\infty}$$
$$\times\frac{\left(q^{1+\frac{1}{2}\delta_{i+j,1}}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{1+\frac{1}{2}\delta_{i+j,1}}s_i^{\pm}s_j^{\pm};q\right)_\infty}{\left(q^{\frac{1}{2}(1+\delta_{i+j,1})}t^{-2}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}(1+\delta_{i+j,1})}t^{-2}s_i^{\pm}s_j^{\pm};q\right)_\infty}.$$

$$\langle W_\square W_\square\rangle^{SO(2N)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{1+\mathfrak{q}^2+\cdots \mathfrak{q}^{2N-4}+2\mathfrak{q}^{2N-2}+\mathfrak{q}^{2N}+\cdots \mathfrak{q}^{4N-4}}{(1-\mathfrak{q}^{2N})\prod_{n=1}^{N-1}(1-\mathfrak{q}^{4n})}$$

$$= \frac{1}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^{2(N-1)})\prod_{n=1}^{N-2}(1-\mathfrak{q}^{4n})}.$$

$$\langle \underbrace{W_\square \cdots W_\square}_{2k} \rangle^{SO(2N)}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \frac{\sum_{i=0}^{(2N-2)k} a_k^{\mathfrak{so}(2N)}(i)\mathfrak{q}^{2i}}{(1-\mathfrak{q}^{2N})\prod_{n=1}^{N-1}(1-\mathfrak{q}^{4n})}.$$

$$\langle W_\square W_\square\rangle^{SO(2N)^-}(t;q) = \frac{1}{2^{N-1}(N-1)!}\frac{(q)_\infty^{2N-2}(-q;q)_\infty^2}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^{N-1}\left(-q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty}$$

$$\times \oint \prod_{i=1}^{N-1}\frac{ds_i}{2\pi i s_i}\frac{(s_i^{\pm};q)_\infty(-s_i^{\pm};q)_\infty(qs_i^{\pm};q)_\infty(-qs_i^{\pm};q)_\infty}{t^2s_i^{\pm};q)_\infty\left(-q^{\frac{1}{2}}t^2s_i^{\pm};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm};q\right)_\infty\left(-q^{\frac{1}{2}}t^{-2}s_i^{\pm};q\right)_\infty}$$

$$\times \prod_{i<j}\frac{(s_i^{\pm}s_j^{\mp};q)_\infty(s_i^{\pm}s_j^{\pm};q)_\infty(qs_i^{\pm}s_j^{\mp};q)_\infty(qs_i^{\pm}s_j^{\pm};q)_\infty}{\left(q^{\frac{1}{2}}t^2s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^2s_i^{\pm}s_j^{\pm};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}}t^{-2}s_i^{\pm}s_j^{\pm};q\right)_\infty}\left[\sum_{i=1}^{N-1}(s_i+s_i^{-1})\right]^2.$$

$$\langle T_{(1,0^{N-2})}T_{(1,0^{N-2})}\rangle^{SO(2N)^-}(t;q)$$

$$= \frac{1}{2^{N-2}(N-2)!}\frac{(q)_\infty^{2N-2}(-q;q)_\infty^2}{\left(q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty^{N-1}\left(-q^{\frac{1}{2}}t^{\pm 2};q\right)_\infty}\oint\prod_{i=1}^{N-1}\frac{ds_i}{2\pi i s_i}$$

$$\times\frac{\left(q^{\frac{1}{2}\delta_{i,1}}s_i^{\pm};q\right)_\infty\left(-q^{\frac{1}{2}\delta_{i,1}}s_i^{\pm};q\right)_\infty\left(q^{1+\frac{1}{2}\delta_{i,1}}s_i^{\pm};q\right)_\infty\left(-q^{1+\frac{1}{2}\delta_{i,1}}s_i^{\pm};q\right)_\infty}{\left(q^{\frac{1}{2}(1+\delta_{i,1})}t^2s_i^{\pm};q\right)_\infty\left(-q^{\frac{1}{2}(1+\delta_{i,1})}t^2s_i^{\pm};q\right)_\infty\left(q^{\frac{1}{2}(1+\delta_{i,1})}t^{-2}s_i^{\pm};q\right)_\infty\left(-q^{\frac{1}{2}(1+\delta_{i,1})}t^{-2}s_i^{\pm};q\right)_\infty}$$

$$\times\prod_{i<j}\frac{\left(q^{\frac{1}{2}\delta_{i+j,1}}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}\delta_{i+j,1}}s_i^{\pm}s_j^{\pm};q\right)_\infty}{\left(q^{\frac{1}{2}(1+\delta_{i+j,1})}t^2s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}(1+\delta_{i+j,1})}t^2s_i^{\pm}s_j^{\pm};q\right)_\infty}$$

$$\times\frac{\left(q^{1+\frac{1}{2}\delta_{i+j,1}}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{1+\frac{1}{2}\delta_{i+j,1}}s_i^{\pm}s_j^{\pm};q\right)_\infty}{\left(q^{\frac{1}{2}(1+\delta_{i+j,1})}t^{-2}s_i^{\pm}s_j^{\mp};q\right)_\infty\left(q^{\frac{1}{2}(1+\delta_{i+j,1})}t^{-2}s_i^{\pm}s_j^{\pm};q\right)_\infty}.$$

$$\langle W_\square W_\square\rangle^{SO(2N)^-}_{\frac{1}{2}\text{BPS}}(\mathfrak{q}) = \left\langle T_{(1,0^{N-2})}T_{(1,0^{N-2})}\right\rangle^{SO(2N)^-}_{\frac{1}{2}\text{BPS}}(\mathfrak{q})$$

$$= \frac{1+\mathfrak{q}^2+\cdots+\mathfrak{q}^{2N-4}}{\prod_{n=1}^{N-1}(1-\mathfrak{q}^{4n})}$$

$$= \frac{1-\mathfrak{q}^{2(N-1)}}{1-\mathfrak{q}^2}\prod_{n=1}^{N-1}\frac{1}{1-\mathfrak{q}^{4n}}$$

$$\langle W_{\square} W_{\square}\rangle^{O(2N)^+}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1+\mathfrak{q}^2+\cdots+\mathfrak{q}^{4N-2}}{\prod_{n=1}^{N}(1-\mathfrak{q}^{4n})}$$

$$= \frac{1}{1-\mathfrak{q}^2}\prod_{n=1}^{N-1}\frac{1}{1-\mathfrak{q}^{4n}}.$$

$$\mathcal{J}^{O(2N)^+}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \prod_{n=1}^{N}\frac{1}{1-\mathfrak{q}^{4n}}$$

$$\langle \mathcal{W}_{\square} \mathcal{W}_{\square}\rangle^{O(2N)^+}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1-\mathfrak{q}^{4N}}{1-\mathfrak{q}^2}$$

$$\mathcal{J}^{SO(2\infty+1)}(t;q) = \mathcal{J}^{USp(2\infty)}(t;q) = \mathcal{J}^{SO(2\infty)}(t;q) = \mathcal{J}^{O(2\infty)^+}(t;q)$$

$$= \prod_{n,m,l=0}^{\infty} \frac{\left(1-q^{n+m+l+\frac{3}{2}}t^{-4m+4l\pm 2}\right)^2}{(1-q^{n+m+l+1}t^{-4m+4l\pm 4})(1-q^{n+m++1}t^{-4m+4l})(1-q^{n+m+l+3}t^{-4m+4l})}.$$

$$i^{AdS_5\times\mathbb{RP}^5}(t;q) = \frac{q^{\frac{1}{2}}(t^2+t^{-2})-q-q^2}{(1-qt^4)(1-qt^{-4})} - \frac{q^{\frac{1}{2}}(t^2+t^{-2})}{\left(1+q^{\frac{1}{2}}t^2\right)\left(1+q^{\frac{1}{2}}t^{-2}\right)(1-q)}$$

$$i^X(t;q) := \mathrm{Tr}(-1)^F q^{\frac{h+j}{2}} t^{2(q_2-q_3)}$$

$$i^X_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) := \mathrm{Tr}(-1)^F \mathfrak{q}^{2(q_2-q_3)}$$

$$\langle \mathcal{W}_{\square} \mathcal{W}_{\square}\rangle^{SO(2\infty+1)}_{\frac{1}{2}\mathrm{BPS},c}(\mathfrak{q}) = \langle \mathcal{W}_{\square} \mathcal{W}_{\square}\rangle^{USp(2\infty)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$= \langle \mathcal{W}_{\square} \mathcal{W}_{\square}\rangle^{SO(2\infty}_{\frac{1}{2}\mathrm{BPS},c}(\mathfrak{q}) = \langle \mathcal{W}_{\square} \mathcal{W}_{\square}\rangle^{O(2\infty)^+}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \frac{1}{1-\mathfrak{q}^2}.$$

$$\langle \mathcal{W}_{\square} \mathcal{W}_{\square}\rangle^{SO(2\infty+1)}(t;q) = \langle \mathcal{W}_{\square} \mathcal{W}_{\square}\rangle^{USp(2\infty)}(t;q)$$

$$= \langle \mathcal{W}_{\square} \mathcal{W}_{\square}\rangle^{SO(2\infty)}(t;q) = \langle \mathcal{W}_{\square} \mathcal{W}_{\square}\rangle^{O(2\infty)^+}(t;q) = \frac{1-q}{\left(1-q^{\frac{1}{2}t^2}\right)\left(1-q^{\frac{1}{2}}t^{-2}\right)}.$$

$$i^{\mathrm{string}}(t;q) = -q + q^{\frac{1}{2}}t^2 + q^{\frac{1}{2}}t^{-2}$$

$$\left\langle \mathcal{W}_{\substack{\square\\\square}}\right\rangle^{SO(2\infty+1)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \langle \mathcal{W}_{\square\square}\rangle^{USp(2\infty)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \langle \mathcal{W}_{\square\square}\rangle^{O(2\infty)^+}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$= \frac{\mathfrak{q}^2}{(1-\mathfrak{q}^4)}$$

$$= \mathfrak{q}^2 + \mathfrak{q}^6 + \mathfrak{q}^{10} + \mathfrak{q}^{14} + \mathfrak{q}^{18} + \cdots.$$

$$\left\langle \mathcal{W}_{\substack{\square\\\square}} \mathcal{W}_{\substack{\square\\\square}}\right\rangle^{SO(2\infty+1)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \langle \mathcal{W}_{\square\square} \mathcal{W}_{\square\square}\rangle^{USp(2\infty)}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q}) = \left\langle \mathcal{W}_{\substack{\square\\\square}} \mathcal{W}_{\substack{\square\\\square}}\right\rangle^{O(2\infty)^+}_{\frac{1}{2}\mathrm{BPS}}(\mathfrak{q})$$

$$= \frac{1+\mathfrak{q}^2+\mathfrak{q}^4}{(1-\mathfrak{q}^4)^2}$$

$$= 1 + \mathfrak{q}^2 + 3\mathfrak{q}^4 + 2\mathfrak{q}^6 + 5\mathfrak{q}^8 + 3\mathfrak{q}^{10} + 7\mathfrak{q}^{12} + 4\mathfrak{q}^{14} + 9\mathfrak{q}^{16} + \cdots.$$

$$\left\langle \mathcal{W}_{\square}\mathcal{W}_{\square}\right\rangle_{\frac{1}{2}\text{BPS},c}^{SO(2\infty+1)}(\mathfrak{q}) = \langle \mathcal{W}_{\square\square}\mathcal{W}_{\square\square}\rangle_{\frac{1}{2}\text{BPS},c}^{USp(2\infty)}(\mathfrak{q})\langle \mathcal{W}_{\square}\mathcal{W}_{\square}\rangle_{\frac{1}{2}\text{BPS},c}^{O(2\infty)^{+}}$$

$$= \frac{1}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)} = 1+\mathfrak{q}^2 2\mathfrak{q}^4 + 2\mathfrak{q}^6 + 3\mathfrak{q}^8 + 3\mathfrak{q}^{10} + 4\mathfrak{q}^{12} + 4\mathfrak{q}^{14} + 5\mathfrak{q}^{16} + 5\mathfrak{q}^{18} + \cdots$$

$$\langle \mathcal{W}_{\lambda}\mathcal{W}_{\lambda}\rangle_{\frac{1}{2}\text{BPS},c}^{G}(\mathfrak{q}) := \langle \mathcal{W}_{\lambda}\mathcal{W}_{\lambda}\rangle_{\frac{1}{2}\text{BPS}}^{G}(\mathfrak{q}) - \langle \mathcal{W}_{\lambda}\rangle_{\frac{1}{2}\text{BPS}}^{G}(\mathfrak{q})^2.$$

$$\left\langle \mathcal{W}_{\square}\right\rangle^{SO(2\infty+1)}(t;q) = \langle \mathcal{W}_{\square\square}\rangle^{USp(2\infty)}(t;q) = \left\langle \mathcal{W}_{\square}\right\rangle^{O(2\infty)^{+}}(t;q)$$

$$= \frac{q^{\frac{1}{2}}(t^2+t^{-2}) - q - q^2}{(1-qt^4)(1-qt^4)}$$

$$\left\langle \mathcal{W}_{\square}\mathcal{W}_{\square}\right\rangle^{SO(2\infty+1)}(t;q) = \langle \mathcal{W}_{\square\square}\mathcal{W}_{\square\square}\rangle^{USp(2\infty)}(t;q) = \left\langle \mathcal{W}_{\square}\mathcal{W}_{\square}\right\rangle^{O(2\infty)^{+}}(t;q)$$

$$= \frac{1}{(1-qt^4)(1-qt^{-4})}\Big(1+(t^2+t^{-2})q^{\frac{1}{2}} + (3+t^4+t^{-4})q - 3(t^2+t^{-2})q^{\frac{3}{2}}$$

$$-(t^2+t^{-2})q^2 - 3(t^2+t^{-2})q^{\frac{5}{2}} + (3+t^4+t^{-4})q^3 + (t^2+t^{-2})q^{\frac{7}{2}} + q^4\Big).$$

$$\left\langle \mathcal{W}_{\square}\mathcal{W}_{\square}\right\rangle_{c}^{SO(2\infty+1)}(t;q) = \langle \mathcal{W}_{\square\square}\mathcal{W}_{\square\square}\rangle_{c}^{USp(2\infty)}(t;q) = \left\langle \mathcal{W}_{\square}\mathcal{W}_{\square}\right\rangle_{c}^{O(2\infty)^{+}}(t;q)$$

$$= \frac{(1-q)\left(1+q-q^{\frac{3}{2}}(t^2+t^{-2})\right)}{\left(1-q^{\frac{1}{2}}t^2\right)\left(1-q^{\frac{1}{2}}t^{-2}\right)(1-qt^4)(1-qt^{-4})}.$$

$$\langle \mathcal{W}_{\text{sp}}\mathcal{W}_{\text{sp}}\rangle_{\frac{1}{2}\text{BPS}}^{\text{Spin}(2\infty+1)}(\mathfrak{q}) = \langle \mathcal{W}_{\text{sp}}\mathcal{W}_{\text{sp}}\rangle_{\frac{1}{2}\text{BPS}}^{\text{Sin}(4\infty)}(\mathfrak{q}) = \left\langle \mathcal{W}_{\text{sp}}\mathcal{W}_{\text{sp}}\right\rangle_{\frac{1}{2}\text{BPS}}^{S^{\text{Sin}(4\infty+2)}}(\mathfrak{q})$$

$$= \prod_{n=1}^{\infty}\frac{1}{1-\mathfrak{q}^{4n-2}}.$$

$$\langle \mathcal{W}_{\text{sp}}\mathcal{W}_{\text{sp}}\rangle_{\frac{1}{2}\text{BPS}}^{\text{Spin}(\infty)}(\mathfrak{q}) = \sum_{n\geq 0} d_{\{\text{sp},\text{sp}\}}^{(H)}(n)\mathfrak{q}^{2n}$$

$$i_{\frac{1}{2}\text{ BPS}}^{\text{fat string}}(\mathfrak{q}) = \frac{\mathfrak{q}^2}{1-\mathfrak{q}^4} = \mathfrak{q}^2 + \mathfrak{q}^6 + \mathfrak{q}^{10} + \cdots$$

$$S_{\text{D5}} = T_5\int d^6\sigma\sqrt{\det(g+2\pi\alpha' F)} - iT_5\int 2\pi\alpha' F\wedge C_{(4)}$$

$$S_{\text{D5}AdS_2\times\mathbb{RP}^4} = T_5\int d^6\sigma\sqrt{\det g} = T_5\text{vol}(AdS_2)\text{vol}(\mathbb{RP}^4)$$

$$ds_{AdS_2}^2 = \frac{1}{r^2}(-dt^2+dr^2), ds_4^2 = g_{ij}d\sigma^i d\sigma^j, i,j = 1,2,3,4$$

$$S = T_5\int d^6\sigma\sqrt{g^{(4)}}\frac{1}{2}\frac{1}{r^2}\left[r^2(\partial_t\phi)^2 - r^2(\partial_r\phi)^2 + \left(\nabla_i\phi\nabla^i\phi - 4\phi^2\right)\right].$$

$$\phi(t,r,\Theta) = \sum_{w}\phi_w(t,r)Y^w(\Theta).$$

$$S = T_5 \sum_w \frac{2}{3}\pi^2 \int d^2\sigma \frac{1}{r^2}\left(r^2(\partial_t\phi_w)^2 - r^2(\partial_r\phi_w)^2 - w(w+1)\phi_w^2\right)$$

$$h = \frac{1}{2} + \sqrt{\frac{1}{4} + m^2}$$

$$\langle \mathcal{W}_{\text{sp}}\mathcal{W}_{\text{sp}}\rangle^{\text{Spin}(2\infty+1)}(t;q) = \langle \mathcal{W}_{\text{sp}}\mathcal{W}_{\text{sp}}\rangle^{\text{Spin}(4\infty)}(t;q) = \langle \mathcal{W}_{\text{sp}}\mathcal{W}_{\text{sp}}\rangle^{\text{Spin}(4\infty+2)}(t;q)$$
$$= \prod_{n=0}^{\infty}\prod_{m=0}^{\infty} \frac{(1-q^{1+n+m}t^{4n-4m})(1-q^{2+n+m}t^{4n-4m})}{\left(1-q^{\frac{1}{2}+n+m}t^{2+4n-4m}\right)\left(1-q^{\frac{1}{2}+n+m}t^{-2+4n+4m}\right)}$$

$$\langle \mathcal{W}_{\text{sp}}\mathcal{W}_{\text{sp}}\rangle^{\text{Spin}(2\infty+1)}(q) = \langle \mathcal{W}_{\text{sp}}\mathcal{W}_{\text{sp}}\rangle^{\text{Spin}(4\infty)}(q) = \langle \mathcal{W}_{\text{sp}}\mathcal{W}_{\text{sp}}\rangle^{\text{Spin}(4\infty+2)}(q)$$
$$= \prod_{n=1}^{\infty} \frac{(1-q^n)^{2n-1}}{\left(1-q^{n-\frac{1}{2}}\right)^{2n}}$$
$$= 1 + 2q^{1/2} + 2q^2 + 6q^{3/2} + 7q^2 + 10q^{5/2} + 21q^3 + 22q^{7/2} + \cdots.$$

$$i^{\text{fat string}}(t;q) = \frac{q^{\frac{1}{2}}(t^2+t^{-2}) - q - q^2}{(1-qt^4)(1-qt^{-4})}.$$

$$\langle \mathcal{W}_{\square\square}\rangle_{\frac{1}{2}\text{BPS}}^{SO(2\infty+1)}(\mathfrak{q}) = \left\langle \mathcal{W}_{\substack{\square\\\square}}\right\rangle_{\frac{1}{2}\text{BPS}}^{USp(2\infty)}(\mathfrak{q}) = \langle \mathcal{W}_{\square\square}\rangle_{\frac{1}{2}\text{BPS}}^{O(2\infty)^+}(\mathfrak{q})$$
$$= \frac{\mathfrak{q}^4}{(1-\mathfrak{q}^4)}$$

$$\langle \mathcal{W}_{\square\square}\mathcal{W}_{\square\square}\rangle_{\frac{1}{2}\text{BPS}}^{SO(2\infty+1)}(\mathfrak{q}) = \left\langle \mathcal{W}_{\substack{\square\\\square}}\mathcal{W}_{\substack{\square\\\square}}\right\rangle_{\frac{1}{2}\text{BPS}}^{USp(2\infty)}(\mathfrak{q}) = \langle \mathcal{W}_{\square\square}\mathcal{W}_{\square\square}\rangle_{\frac{1}{2}\text{BPS}}^{O(2\infty)^+}(\mathfrak{q})$$
$$= \frac{1+\mathfrak{q}^2+\mathfrak{q}^8}{(1-\mathfrak{q}^4)^2}.$$

$$\langle \mathcal{W}_{\square\square}\mathcal{W}_{\square\square}\rangle_{\frac{1}{2}\text{BPS},c}^{SO(2\infty+1)}(\mathfrak{q}) == \left\langle \mathcal{W}_{\substack{\square\\\square}}\mathcal{W}_{\substack{\square\\\square}}\right\rangle_{\frac{1}{2}\text{BPS}}^{USp(2\infty)}(\mathfrak{q}) = \langle \mathcal{W}_{\square\square}\mathcal{W}_{\square\square}\rangle_{\frac{1}{2}\text{BPS}}^{O(2\infty)^+}(\mathfrak{q})$$
$$= \frac{1}{(1-\mathfrak{q}^2)(1-\mathfrak{q}^4)},$$

$$\langle \mathcal{W}_{\square\square}\rangle^{SO(2\infty+1)}(t;q) = \left\langle \mathcal{W}_{\substack{\square\\\square}}\right\rangle^{USp(2\infty)}(t;q) = \langle \mathcal{W}_{\square\square}\rangle^{O(2\infty)^+}(t;q)$$
$$= \frac{q(1+t^4+t^{-4}) - q^{\frac{3}{2}}(t^2+t^{-2}) - q^2}{(1-qt^4)(1-qt^{-4})}$$

$$\langle \mathcal{W}_{\square\square}\mathcal{W}_{\square\square}\rangle^{SO(2\infty+1)}(t;q) = \left\langle \mathcal{W}_{\substack{\square\\\square}}\mathcal{W}_{\substack{\square\\\square}}\right\rangle^{USp(2\infty)}(t;q) = \langle \mathcal{W}_{\square\square}\mathcal{W}_{\square\square}\rangle^{O(2\infty)^+}(t;q)$$
$$= \frac{1}{(1-qt^4)(1-qt^{-4})}\Big(1 + (t^2+t^{-2})q^{\frac{1}{2}} + q - (t^2+t^{-2})q^{\frac{3}{2}} + (t^8+t^4+t^{-4}+t^{-8})q^2$$
$$-(2t^6+5t^2+5t^{-2}+2t^{-6})q^{\frac{5}{2}} + q^3 + 3(t^2+t^{-2})q^{\frac{7}{2}} + q^4\Big).$$

$$\langle \mathcal{W}_{\square\square}\mathcal{W}_{\square\square}\rangle_c^{SO(2\infty+1)}(t;q) = \left\langle \mathcal{W}_{\substack{\square\\\square}}\mathcal{W}_{\substack{\square\\\square}}\right\rangle_c^{USp(2\infty)}(t;q) = \langle \mathcal{W}_{\square\square}\mathcal{W}_{\square\square}\rangle_c^{O(2\infty)^+}(t;q)$$

$$= \frac{(1-q)\left(1+q-q^{\frac{3}{2}}(t^2+t^{-2})\right)}{\left(1-q^{\frac{1}{2}}t^2\right)\left(1-q^{\frac{1}{2}}t^{-2}\right)(1-qt^4)(1-qt^{-4})}.$$

$$\chi_\lambda^{\mathrm{usp}(2N)} = \det\left(E_{\lambda_i'-i+j} - E_{\lambda_i'-i-j}\right)_{1\le i,j\le l(\lambda')}$$

$$\chi_\lambda^{\mathrm{so}(2N+1)} = \det\left(\bar{H}_{\lambda_i-i+j} - \bar{H}_{\lambda_i-i-j}\right)_{1\le i,j\le l(\lambda)}$$

$$E_k = e_k(s_1,\dots,s_N,s_1^{-1},\dots,s_N^{-1})$$

$$\bar{H}_k = h_k(s_1,\dots,s_N,s_1^{-1},\dots,s_N^{-1},1)$$

$$P_k = p_k(s_1,\dots,s_N,s_1^{-1},\dots,s_N^{-1})$$

$$\bar{P}_k = p_k(s_1,\dots,s_N,s_1^{-1},\dots,s_N^{-1},1)$$

$$p_k(x_1,\dots,x_n) = \sum_{i=1}^{n} x_i^k$$

$$\chi_{\square}^{\mathrm{usp}\,(2N)} = P_1, \qquad \chi_{\square}^{\mathrm{so}\,(2N+1)} = \bar{P}_1,$$

$$\chi_{\square\square}^{\mathrm{usp}\,(2N)} = \frac{P_2}{2}+\frac{P_1^2}{2}, \qquad \chi_{\square\square}^{\mathrm{so}\,(2N+1)} = \frac{\bar{P}_2}{2}+\frac{\bar{P}_1^2}{2}-1,$$

$$\chi_{\substack{\square\\\square}}^{\mathrm{usp}\,(2N)} = -\frac{P_2}{2}+\frac{P_1^2}{2}-1, \qquad \chi_{\substack{\square\\\square}}^{\mathrm{so}\,(2N+1)} = -\frac{\bar{P}_2}{2}+\frac{\bar{P}_1^2}{2},$$

$$\chi_{\square\square\square}^{\mathrm{usp}\,(2N)} = \frac{P_3}{3}+\frac{P_2P_1}{2}+\frac{P_1^3}{6}, \qquad \chi_{\square\square\square}^{\mathrm{so}\,(2N+1)} = \frac{\bar{P}_3}{3}+\frac{\bar{P}_2\bar{P}_1}{2}+\frac{\bar{P}_1^3}{6}-\bar{P}_1,$$

$$\chi_{\substack{\square\square\\\square}}^{\mathrm{usp}\,(2N)} = -\frac{P_3}{3}+\frac{P_1^3}{3}-P_1, \qquad \chi_{\substack{\square\square\\\square}}^{\mathrm{so}\,(2N+1)} = -\frac{\bar{P}_3}{3}+\frac{\bar{P}_1^3}{3}-\bar{P}_1,$$

$$\chi_{\substack{\square\\\square\\\square}}^{\mathrm{usp}\,(2N)} = \frac{P_3}{3}-\frac{P_2P_1}{2}+\frac{P_1^3}{6}-P_1, \qquad \chi_{\substack{\square\\\square\\\square}}^{\mathrm{so}\,(2N+1)} = \frac{\bar{P}_3}{3}-\frac{\bar{P}_2\bar{P}_1}{2}+\frac{\bar{P}_1^3}{6}.$$

$$T_p\mathcal{M}^{\mathbb{C}} = T_p\mathcal{M}^+ \oplus T_p\mathcal{M}^-,$$

$$T_p\mathcal{M}^{\pm} = \{Z \in T_p\mathcal{M}^{\mathbb{C}} / \mathcal{J}_p Z = \pm iZ\}$$

$$\mathcal{J}_p(\mathcal{P}^{\pm}Z) = \mathcal{J}_p Z^{\pm} = \pm i(\mathcal{P}^{\pm}Z) = \pm iZ^{\pm}$$

$$\mathcal{N}(u,v) \equiv [\mathcal{J}u,\mathcal{J}v] - \mathcal{J}[u,\mathcal{J}v] - \mathcal{J}[\mathcal{J}u,v] - [u,v]$$

$$\phi(p) \equiv z^\mu \equiv x^\mu + iy^\mu, \psi(p) \equiv w^\mu \equiv u^\mu + iv^\mu, 1 \le \mu,\nu \le n$$

$$\frac{\partial u^\nu}{\partial x^\mu} = \frac{\partial v^\nu}{\partial y^\mu}, \frac{\partial u^\nu}{\partial y^\mu} = -\frac{\partial v^\nu}{\partial x^\mu}.$$

$$\mathcal{J}_p\frac{\partial}{\partial x^\mu} = \frac{\partial}{\partial y^\mu}, \mathcal{J}_p\frac{\partial}{\partial y^\mu} = -\frac{\partial}{\partial x^\mu}, \text{ entonces}$$

$$\mathcal{J}_p\frac{\partial}{\partial u^\mu} = \frac{\partial}{\partial v^\mu}, \mathcal{J}_p\frac{\partial}{\partial v^\mu} = -\frac{\partial}{\partial u^\mu}$$

$$(\mathcal{J}_p) = \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix}, \forall p \in \mathcal{M}$$

$$(\mathcal{J}_p) = \begin{pmatrix} i\mathbb{I} & 0 \\ 0 & -i\mathbb{I} \end{pmatrix}, \forall p \in \mathcal{M}$$

$$\alpha = \sum_{p+q=k} \alpha^{(p,q)}, \alpha^{(p,q)} = \frac{1}{p!\,q!}\alpha_{\mu_1\dots\mu_p\bar{\nu}_1\dots\bar{\nu}_q} dz^{\mu_1}\wedge\dots\wedge dz^{\mu_p}\wedge d\bar{z}^{\bar{\nu}_1}\dots\wedge d\bar{z}^{\bar{\nu}_q}$$

$$d = \partial + \bar{\partial},$$
$$\partial\alpha^{(p,q)} = \alpha^{(p+1,q)}, \bar{\partial}\alpha^{(p,q)} = \alpha^{(p,q+1)},$$
$$\alpha^{(p+1,q)} \ = \frac{1}{p!\,q!}\partial_\mu \alpha^{(p,q)}_{\mu_1\ldots\mu_p\bar{\nu}_1\ldots\bar{\nu}_q} dz^\mu \wedge dz^{\mu_1} \wedge \ldots \wedge dz^{\mu_p} \wedge d\bar{z}^{\bar{\nu}_1} \ldots \wedge d\bar{z}^{\bar{\nu}_q}.$$
$$\alpha^{(p,q+1)} \ = \frac{1}{p!\,q!}\partial_{\bar{\nu}} \alpha^{(p,qz^{\mu_1}}_{\mu_1\ldots\mu_p\bar{\nu}_1\ldots\bar{\nu}_q} \wedge \ldots \wedge dz^{\mu_p} \wedge d\bar{z}^{\bar{\nu}} \wedge d\bar{z}^{\bar{\nu}_1} \ldots \wedge d\bar{z}^{\bar{\nu}_q}.$$
$$\partial^2 = \partial\bar{\partial} + \bar{\partial}\partial = \bar{\partial}^2$$
$$\mathcal{G}_p(\mathcal{J}_p u, \mathcal{J}_p v) = \mathcal{G}_p(u, v)\ \forall p \in \mathcal{M}, u, v \in T_p\mathcal{M}$$
$$\mathcal{G}_p(u, v) \equiv g_p(u, v) + g_p(\mathcal{J}_p u, \mathcal{J}_p v)\ \forall p \in \mathcal{M}, u, v \in T_p\mathcal{M}$$
$$\mathcal{G}_p(\mathcal{J}_p u, u) = \mathcal{G}_p(\mathcal{J}_p^2 u, \mathcal{J}_p u) = -\mathcal{G}_p(u, \mathcal{J}_p u) = -\mathcal{G}_p(\mathcal{J}_p u, u)$$
$$\mathcal{G}_{\mu\nu} \equiv \mathcal{G}\left(\frac{\partial}{\partial z^\mu}, \frac{\partial}{\partial z^\nu}\right) = \mathcal{G}\left(\mathcal{J}_p\frac{\partial}{\partial z^\mu}, \mathcal{J}_p\frac{\partial}{\partial z^\nu}\right) = -\mathcal{G}\left(\frac{\partial}{\partial z^\mu}, \frac{\partial}{\partial z^\nu}\right)$$
$$\mathcal{G}_{\overline{\mu}\overline{\nu}} \equiv \mathcal{G}\left(\frac{\partial}{\partial \bar{z}^{\bar{\mu}}}, \frac{\partial}{\partial \bar{z}^{\bar{\nu}}}\right) = \mathcal{G}\left(\mathcal{J}_p\frac{\partial}{\partial \bar{z}^{\bar{\mu}}}, \mathcal{J}_p\frac{\partial}{\partial \bar{z}^{\bar{\nu}}}\right) = -\mathcal{G}\left(\frac{\partial}{\partial \bar{z}^{\bar{\mu}}}, \frac{\partial}{\partial \bar{z}^{\bar{\nu}}}\right)$$
$$\mathcal{G}_{\mu\bar{\nu}} \equiv \mathcal{G}\left(\frac{\partial}{\partial z^\mu}, \frac{\partial}{\partial \bar{z}^{\bar{\nu}}}\right) = \mathcal{G}\left(\mathcal{J}_p\frac{\partial}{\partial z^\mu}, \mathcal{J}_p\frac{\partial}{\partial \bar{z}^{\bar{\nu}}}\right) = \mathcal{G}\left(\frac{\partial}{\partial \bar{z}^{\bar{\nu}}}, \frac{\partial}{\partial z^{\bar{\mu}}}\right) = \mathcal{G}_{\bar{\nu}\mu}$$
$$\mathcal{G} = \mathcal{G}_{\mu\bar{\nu}} dz^\mu \otimes d\bar{z}^{\bar{\nu}} + \mathcal{G}_{\bar{\nu}\mu} d\bar{z}^{\bar{\nu}} \otimes dz^\mu$$
$$\omega_p(u, v) \equiv \mathcal{G}_p(\mathcal{J}_p u, v)\ \forall u, v \in T_p\mathcal{M}.$$
$$\omega_p(u, v) = \mathcal{G}_p(\mathcal{J}_p^2 u, \mathcal{J}_p v) = \mathcal{G}_p(-u, \mathcal{J}_p v) = -\mathcal{G}_p(\mathcal{J}_p v, u) = -\omega(u, v).$$
$$\omega_p(\mathcal{J}_p u, \mathcal{J}_p v) = \mathcal{G}_p(\mathcal{J}_p^2 u, \mathcal{J}_p v) = \mathcal{G}_p(-u, \mathcal{J}_p v) = \omega(u, v)$$
$$\omega = i\mathcal{G}_{\mu\bar{\nu}} dz^\mu \wedge d\bar{z}^{\bar{\nu}}$$
$$[\omega] \in H^{(1,1)}_{\bar{\partial}}(\mathcal{M}; \mathbb{C})$$
$$\omega = i\partial_\mu\partial_{\bar{\nu}}\mathcal{K}(z, \bar{z}) dz^\mu \wedge d\bar{z}^{\bar{\nu}}$$
$$\mathcal{K}' = \mathcal{K} + f + f',$$
$$\omega(\mathcal{K}') = i\partial\bar{\partial}(\mathcal{K} + f + f') = i\partial\bar{\partial}\mathcal{K} = \omega(\mathcal{K}).$$
$$\Gamma^\rho_{\mu\nu} = \mathcal{G}^{\rho\bar{\rho}}\partial_\mu\mathcal{G}_{\bar{\rho}\nu}, \Gamma^{\bar{\rho}}_{\bar{\mu}\bar{\nu}} = \mathcal{G}^{\bar{\rho}\rho}\partial_{\bar{\mu}}\mathcal{G}_{\rho\bar{\nu}}$$
$$R_{\mu\bar{\nu}} = \frac{1}{2}\partial_\mu\partial_{\bar{\nu}}(\log\ \det\mathcal{G}).$$
$$\Re \equiv iR_{\mu\bar{\nu}} dz^\mu \wedge d\bar{z}^{\bar{\nu}}$$
$$\Psi_{(i)} = e^{-\left(qf_{(i,j)} + \bar{q}\bar{f}_{(i,j)}\right)}\Psi_{(j)},$$
$$\mathcal{K}_{(i)} = \mathcal{K}_{(j)} + f_{(i,j)} + \bar{f}_{(i,j)}.$$
$$\mathcal{Q} \equiv (2i)^{-1}\left(dz^\mu\partial_\mu\mathcal{K} - d\bar{z}^{\bar{\nu}}\partial_{\bar{\nu}}\mathcal{K}\right)$$
$$\mathcal{Q}_{(i)} = \mathcal{Q}_{(j)} - \frac{i}{2}\partial f_{(i,j)},$$
$$\mathfrak{D}_\mu \equiv \nabla_\mu + iq\mathcal{Q}_\mu, \mathfrak{D}_{\bar{\nu}} \equiv \nabla_{\bar{\nu}} - i\bar{q}\mathcal{Q}_{\bar{\nu}}$$
$$D_\mu \equiv \partial_\mu + iq\mathcal{Q}_\mu, D_{\bar{\nu}} \equiv \partial_{\bar{\nu}} - i\bar{q}\mathcal{Q}_{\bar{\nu}}$$
$$\Omega \equiv \begin{pmatrix}\mathcal{X}^\Lambda \\ \mathcal{F}_\Sigma\end{pmatrix} \to \begin{cases}\langle\Omega \mid \bar{\Omega}\rangle & \equiv \overline{\mathcal{X}}^\Lambda\mathcal{F}_\Lambda - \mathcal{X}^\Lambda\overline{\mathcal{F}}_\Lambda = -ie^{-\mathcal{K}} \\ \partial_{\bar{\nu}}\Omega & = 0 \\ \langle\partial_\mu\Omega \mid \Omega\rangle & = 0\end{cases}$$
$$\mathcal{V} \equiv \begin{pmatrix}\mathcal{L}^\Lambda \\ \mathcal{M}_\Sigma\end{pmatrix} \to \begin{cases}\langle\mathcal{V} \mid \overline{\mathcal{V}}\rangle & \equiv \overline{\mathcal{L}}^\Lambda\mathcal{M}_\Lambda - \mathcal{L}^\Lambda\overline{\mathcal{M}}_\Lambda = -i \\ \mathfrak{D}_{\bar{\nu}}\mathcal{V} & = \left(\partial_{\bar{\nu}} + \frac{1}{2}\partial_{\bar{\mathcal{L}}}\mathcal{K}\right)\mathcal{V} = 0, \\ \langle\mathfrak{D}_\mu\mathcal{V} \mid \mathcal{V}\rangle & = 0\end{cases}$$

$$\mathcal{U}_\mu \equiv \mathfrak{D}_\mu \mathcal{V} = \begin{pmatrix} f^\Lambda{}_\mu \\ h_{\Sigma\mu} \end{pmatrix}, \overline{\mathcal{U}}_{\bar{\nu}} = \overline{\mathcal{U}_\nu},$$
$$\mathfrak{D}_{\bar{\nu}} \mathcal{U}_\mu = \mathcal{G}_{\mu\bar{\nu}} \mathcal{V} \langle \mathcal{U}_\mu \mid \overline{\mathcal{U}}_{\bar{\nu}} \rangle = i\mathcal{G}_{\mu\bar{\nu}}$$
$$\langle \mathcal{U}_\mu \mid \overline{\mathcal{V}} \rangle = 0, \langle \mathcal{U}_\mu \mid \mathcal{V} \rangle = 0$$
$$\langle \mathfrak{D}_\mu \mathcal{U}_\nu \mid \mathcal{V} \rangle = \langle \mathcal{U}_\nu \mid \mathcal{U}_\mu \rangle = 0.$$
$$\mathcal{A} = i\langle \mathcal{A} \mid \overline{\mathcal{V}} \rangle \mathcal{V} - i\langle \mathcal{A} \mid \mathcal{V} \rangle \overline{\mathcal{V}} + i\langle \mathcal{A} \mid \mathcal{U}_\mu \rangle \mathcal{G}^{\mu\bar{\nu}} \overline{\mathcal{U}}_{\bar{\nu}} - i\langle \mathcal{A} \mid \overline{\mathcal{U}}_{\bar{\nu}} \rangle \mathcal{G}^{\mu\bar{\nu}} \mathcal{U}_\mu$$
$$\mathcal{C}_{\mu\nu\rho} \equiv \langle \mathfrak{D}_\mu \mathcal{U}_\nu \mid \mathcal{U}_\rho \rangle \ \rightarrow \ \mathfrak{D}_\mu \mathcal{U}_\nu = i\mathcal{C}_{\mu\nu\rho} \mathcal{G}^{\rho\bar{\epsilon}} \overline{\mathcal{U}}_{\bar{\epsilon}}$$
$$\mathfrak{D}_{\bar{\mu}} \mathcal{C}_{\nu\rho\epsilon} = 0, \mathfrak{D}_{[\mu} \mathcal{C}_{\nu]\rho\epsilon} = 0$$
$$\mathcal{M}_\Lambda = \mathcal{N}_{\Lambda\Sigma} \mathcal{L}^\Sigma, h_{\Lambda\mu} = \overline{\mathcal{N}}_{\Lambda\Sigma} f^\Sigma{}_\mu.$$
$$\mathcal{L}^\Lambda \Im m \mathcal{N}_{\Lambda\Sigma} \overline{\mathcal{L}}^\Sigma \ = -\frac{1}{2},$$
$$\mathcal{L}^\Lambda \Im m \mathcal{N}_{\Lambda\Sigma} f^\Sigma{}_\mu \ = \mathcal{L}^\Lambda \Im m \mathcal{N}_{\Lambda\Sigma} \bar{f}^\Sigma{}_{\bar{\nu}} = 0$$
$$f^\Lambda{}_\mu \Im m \mathcal{N}_{\Lambda\Sigma} \bar{f}^\Sigma{}_{\bar{\nu}} \ = -\frac{1}{2} \mathcal{G}_{\mu\bar{\nu}}.$$
$$(\partial_\mu \mathcal{N}_{\Lambda\Sigma}) \mathcal{L}^\Sigma \ = -2i \Im m(\mathcal{N})_{\Lambda\Sigma} f^\Sigma{}_\mu$$
$$\partial_\mu \overline{\mathcal{N}}_{\Lambda\Sigma} f^\Sigma{}_\nu \ = -2\mathcal{C}_{\mu\nu\rho} \mathcal{G}^{\rho\bar{\rho}} \Im m \mathcal{N}_{\Lambda\Sigma} \bar{f}^\Sigma{}_{\bar{\rho}}$$
$$\mathcal{C}_{\mu\nu\rho} \ = f^\Lambda{}_\mu f^\Sigma{}_\nu \partial_\rho \overline{\mathcal{N}}_{\Lambda\Sigma}$$
$$\mathcal{L}^\Sigma \partial_{\bar{\nu}} \mathcal{N}_{\Lambda\Sigma} \ = 0,$$
$$\partial_{\bar{\nu}} \overline{\mathcal{N}}_{\Lambda\Sigma} f^\Sigma{}_\mu \ = 2i\mathcal{G}_{\mu\bar{\nu}} \Im m \mathcal{N}_{\Lambda\Sigma} \mathcal{L}^\Sigma.$$
$$U^{\Lambda\Sigma} \equiv f^\Lambda{}_\mu \mathcal{G}^{\mu\bar{\nu}} \bar{f}^\Sigma{}_{\bar{\nu}} \ = -\frac{1}{2} \Im m(\mathcal{N})^{-1|\Lambda\Sigma} - \overline{\mathcal{L}}^\Lambda \mathcal{L}^\Sigma$$
$$\mathcal{T}_\Lambda \ \equiv 2i\mathcal{L}_\Lambda = 2i\mathcal{L}^\Sigma \Im_{\mathrm{m}} \mathcal{N}_{\Sigma\Lambda},$$
$$\mathcal{T}^\mu{}_\Lambda \ \equiv -\bar{f}_\Lambda{}^\mu = -\mathcal{G}^{\mu\bar{\nu}} \bar{f}^\Sigma{}_{\bar{\nu}} \Im \mathrm{m} \mathcal{N}_{\Sigma\Lambda}.$$
$$\partial_\mu \mathcal{N}_{\Lambda\Sigma} = 4\mathcal{T}_{\mu(\Lambda} \mathcal{T}_{\Sigma)},$$
$$\partial_{\bar{\nu}} \mathcal{N}_{\Lambda\Sigma} = 4\overline{\mathcal{C}}_{\bar{\nu}\bar{\rho}\bar{\epsilon}} \mathcal{T}^{\bar{\nu}}{}_{(\Lambda} \mathcal{T}^{\bar{\rho}}{}_{\Sigma)}.$$
$$e^{-\mathcal{K}} \ = -2\Im m \mathcal{N}_{\Lambda\Sigma} \mathcal{X}^\Lambda \overline{\mathcal{X}}^\Sigma$$
$$\partial_\mu \mathcal{X}^\Lambda [2\mathcal{F}_\Lambda - \partial_\Lambda(\mathcal{X}^\Sigma \mathcal{F}_\Sigma)] = 0.$$
$$\mathcal{F}_\Lambda = \partial_\Lambda \mathcal{F}(\mathcal{X})$$
$$\mathcal{N}_{\Lambda\Sigma} = \overline{\mathcal{F}}_{\Lambda\Sigma} + 2i \frac{\Im\ \mathrm{m} \mathcal{F}_{\Lambda\Lambda'} \mathcal{X}^{\Lambda'} \Im \mathrm{sm} \mathcal{F}_{\Sigma\Sigma'} \mathcal{X}^{\Sigma'}}{\mathcal{X}^\Omega \Im \mathrm{m} \mathcal{F}_{\Omega\Omega'} \mathcal{X}^{\Omega'}}.$$
$$\mathcal{C}_{\mu\nu\rho} = e^{\mathcal{K}} \partial_\mu \mathcal{X}^\Lambda \partial_\nu \mathcal{X}^\Sigma \partial_\rho \mathcal{X}^\Omega \mathcal{F}_{\Lambda\Sigma\Omega},$$
$$[P_\mu, P_\nu] \ = 0$$
$$[P_\mu, J_{\nu\rho}] \ = (\eta_{\mu\nu} P_\rho - \eta_{\mu\rho} P_\nu),$$
$$[J_{\mu\nu}, J_{\rho\gamma}] \ = -(\eta_{\mu\rho} J_{\nu\gamma} + \eta_{\nu\gamma} J_{\mu\rho} - \eta_{\mu\gamma}, J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\gamma}),$$
$$[T_r, T_s] = f_{rs}^t T_t,$$
$$[P_\mu, T_s] = [J_{\mu\nu}, T_s] = 0$$
$$[Q_\alpha^L, J_{\mu\nu}] \ = (\sigma_{\mu\nu})_\alpha^\beta Q_\beta^L,$$
$$[Q_\alpha^L, P_\mu] \ = [\bar{Q}_{\dot{\alpha}}^L, P_\mu] = 0$$
$$\{Q_\alpha^L, \bar{Q}_{\dot{\beta}M}\} \ = 2(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu \delta_M^L,$$
$$\{Q_\alpha^L, Q_\beta^M\} \ = \epsilon_{\alpha\beta} Z^{LM},$$
$$[Q_\alpha^L, T_r] = S_r{}^L{}_M Q_\alpha^M \neq 0,$$

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = (\bar{\epsilon}_1 \gamma^\mu \epsilon_2)\partial_\mu + \cdots$$
$$\delta_\epsilon B \sim \bar{\epsilon} F,$$
$$\delta_\epsilon F \sim B\epsilon.$$
$$\delta_\epsilon B \sim \bar{\epsilon} F,$$
$$\delta_\epsilon F \sim \partial\epsilon + B\epsilon.$$
$$S_{EH}[\mathbf{e}, \omega] = \int \star \mathbf{R}(\omega) \wedge \mathbf{e} \wedge \mathbf{e}.$$
$$S = \int d^4x\sqrt{|g|}\{R + \mathcal{G}_{ij}(\phi)\partial_\mu \phi^i \partial^\mu \phi^j + 2\Im m\mathcal{N}_{\Lambda\Sigma}(\phi)F^\Lambda{}_{\mu\nu}F^{\Sigma\mu\nu}$$
$$-2\Re e\mathcal{N}_{\Lambda\Sigma}(\phi)F^\Lambda{}_{\mu\nu} \star F^{\Sigma\mu\nu}\},$$
$$\mathcal{E}_{\mu\nu} = G_{\mu\nu} + \mathcal{G}_{ij}\left[\partial_\mu \phi^i \partial_\nu \phi^j - \frac{1}{2}\partial_\rho \phi^i \partial^\rho \phi^j\right] + 8\Im m\mathcal{N}_{\Lambda\Sigma}F_\mu^{\Lambda+\rho}F_{\nu\rho}^{\Sigma-} = 0,$$
$$\mathcal{E}_i = \nabla_\mu(\mathcal{G}_{ij}\partial^\mu \phi^j) - \frac{1}{2}\partial_i \mathcal{G}_{jk}\partial_\rho \phi^j \partial^\rho \phi^k + \partial_i[\tilde{F}_\Lambda^{\nu\mu *}F_{\mu\nu}^\Lambda] = 0,$$
$$\mathcal{E}_\Lambda^\mu = \nabla_\nu \star \tilde{F}_\Lambda^{\nu\mu} = 0,$$
$$\tilde{F}_{\Lambda\mu\nu} \equiv -\frac{1}{4\sqrt{|g|}}\frac{\delta S}{\delta \star F^{\Lambda\mu\nu}} = \Re e\mathcal{N}_{\Lambda\Sigma}F_{\mu\nu}^\Sigma + \Im m\mathcal{N}_{\Lambda\Sigma}^* F_{\mu\nu}^\Sigma.$$
$$\mathcal{B}^{\Lambda\mu} \equiv \nabla_\nu \star F^{\Lambda\nu\mu} = 0.$$
$$\mathcal{E}_\mu^M \equiv \begin{pmatrix} \mathcal{B}_\mu^\Lambda \\ \mathcal{E}_{\Lambda\mu} \end{pmatrix}$$
$$\mathcal{E}_\mu^M = 0 \to m^M{}_N \mathcal{E}_\mu^N = 0, m^M{}_N \in \mathrm{GL}(2n_v + 2, \mathbb{R}).$$
$$F_\mu^M \equiv \begin{pmatrix} F^\Lambda \\ \tilde{F}_\Lambda \end{pmatrix}, F'^M = m^M{}_N F^N.$$
$$\tilde{F}'_{\Lambda\mu\nu} \equiv -\frac{1}{4\sqrt{|g|}}\frac{\delta S'}{\delta \star F'^{\Lambda\mu\nu}}.$$
$$\mathrm{i}: \mathrm{Diff}(\mathcal{M}_{\text{escalar}}) \to \mathrm{GL}(2n_v + 2, \mathbb{R})$$
$$\{\phi, F^M, \mathcal{N}_{\Sigma\Lambda}(\phi)\} \xrightarrow{\xi} \{\xi(\phi), (\mathrm{i}(\xi))^M{}_N F^N, \mathcal{N}'_{\Sigma\Lambda}(\xi(\phi))\}.$$
$$\mathrm{i}: \mathrm{Diff}(\mathcal{M}_{\text{escalar}}) \to \mathrm{Sp}(2n_v + 2, \mathbb{R})$$
$$\mathcal{N}'(\xi(\phi)) = (A\mathcal{N}(\phi) + B)(C\mathcal{N}(\phi) + D)^{-1}$$
$$m \equiv \begin{pmatrix} D & C \\ B & A \end{pmatrix} \in \mathrm{Sp}(2n_v + 2, \mathbb{R})$$
$$\mathrm{i}: \text{Isometrías}(\mathcal{M}_{\text{escalar}}, \mathcal{G}_{ij}) \to \mathrm{Sp}(2n_v + 2, \mathbb{R}).$$
$$S = \int d^4x\sqrt{|g|}\{R + h_{uv}(q)\partial_\mu q^u \partial^\mu q^v + \mathcal{G}_{i\bar{j}}(z, \bar{z})\partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}}$$
$$+2\Im_{\Lambda\Sigma}(z, \bar{z})F^\Lambda{}_{\mu\nu}F^{\Sigma\mu\nu} - 2\Re_{\Lambda\Sigma}(z, \bar{z})F^\Lambda{}_{\mu\nu} \star F^{\Sigma\mu\nu}\}$$
$$\mathcal{F} = -\frac{1}{3!}\kappa_{ijk}^0 z^i z^j z^k + \frac{ic}{2} + \frac{i}{(2\pi)^3}\sum_{\{d_i\}} n_{\{d_i\}} Li_3\left(e^{2\pi i d_i z^i}\right)$$
$$\mathcal{F}_{\mathrm{P}} = -\frac{1}{3!}\kappa_{ijk}^0 z^i z^j z^k + \frac{ic}{2},$$
$$\mathcal{F}_{\mathrm{NP}} = \frac{i}{(2\pi)^3}\sum_{\{d_i\}} n_{\{d_i\}} Li_3\left(e^{2\pi i d_i z^i}\right).$$
$$F(\mathcal{X}) = -\frac{1}{3!}\kappa_{ijk}^0 \frac{\mathcal{X}^i \mathcal{X}^j \mathcal{X}^k}{\mathcal{X}^0} + \frac{ic(\mathcal{X}^0)^2}{2} + \frac{i(\mathcal{X}^0)^2}{(2\pi)^3}\sum_{\{d_i\}} n_{\{d_i\}} Li_3\left(e^{2\pi i d_i \frac{\mathcal{X}^i}{\mathcal{X}^0}}\right)$$

$$z^i = \frac{\mathcal{X}^i}{\mathcal{X}^0}$$
$$\mathcal{B} \equiv \mathcal{M} - I^-(\mathcal{I}^+),$$
$$\nabla^\mu \xi^2 = -2\kappa \xi^\mu.$$
$$\delta M = \frac{1}{8\pi}\kappa \delta A + \Omega \delta J + \Phi \delta Q$$
$$\delta A \geq 0.$$
$$T = \frac{\kappa}{2\pi}.$$
$$S_{\text{bh}} = \frac{A}{4}.$$
$$\delta_\epsilon B \sim \bar{\epsilon} F = 0,$$
$$\delta_\epsilon F \sim \partial \epsilon + B\epsilon = 0,$$
$$\mathbf{g} = \left(1 - \frac{2M}{r}\right) dt \otimes dt - \left(1 - \frac{2M}{r}\right)^{-1} dr \otimes dr - r^2(d\theta \otimes d\theta + \sin^2 \theta d\phi \otimes d\phi)$$
$$\delta_\epsilon \Psi_\mu \big|_{\text{Schw.}} = 0$$
$$\mathbf{g} = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right) dt \otimes dt - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr \otimes dr - r^2(d\theta \otimes d\theta + \sin^2 \theta d\phi \otimes d\phi)$$
$$T = \frac{\kappa}{2\pi} = \frac{r_+ - r_-}{4\pi r_+^2} = \frac{\sqrt{M^2 - q^2}}{2\pi^2 r_+^2} = 0.$$
$$\delta_\epsilon \Psi_\mu \big|_{\text{RN}} \text{ extr.} = 0$$
$$\mathbf{g} = e^{2U(\tau)} dt \otimes dt - e^{-2U(\tau)} \gamma_{\underline{mn}} dx^{\underline{m}} \otimes dx^{\underline{n}},$$
$$\gamma_{\underline{mn}} dx^{\underline{m}} \otimes dx^{\underline{n}} = \frac{r_0^2}{\sinh^2 r_0 \tau}\left[\frac{r_0^2}{\sinh^2 r_0 \tau} d\tau \otimes d\tau + h_{S^2}\right]$$
$$h_{S^2} = d\theta \otimes d\theta + \sin^2 \theta d\phi \otimes d\phi,$$
$$\mathbf{g} = e^{2U(\tau)} dt \otimes dt - e^{-2U(\tau)} [\delta_{ab} dx^a \otimes dx^b],$$
$$\lim_{\tau \to -\infty} e^{-2U} = \frac{A}{4\pi} \lim_{\tau \to -\infty} \tau^2, \lim_{\tau \to -\infty} \tau \frac{d\phi^i}{d\tau} = 0, i = 1, \dots, n_v$$
$$\lim_{\tau \to -\infty} \phi^i = \phi_h^i, \mathcal{G}^{ij}(\phi_h) \partial_j V_{\text{bh}}(\phi_h) = 0$$
$$\partial_j V_{\text{bh}}(\phi_h) = 0,$$
$$S = \pi V_{\text{bh}}(\phi_h(\mathcal{Q})) = 0,$$
$$I_{\text{FGK}}[U, z^i] = \int d\tau \{(\dot{U})^2 + \mathcal{G}_{ij} \dot{z}^i \dot{\bar{z}}^{\bar{j}} - e^{2U} V_{\text{bh}}(z, \bar{z}, \mathcal{Q})\},$$
$$(\dot{U})^2 + \mathcal{G}_{ij} \dot{z}^i \dot{\bar{z}}^j + e^{2U} V_{\text{bh}}(z, \bar{z}, \mathcal{Q}) = r_0^2.$$
$$V_{\text{bh}}(z, \bar{z}, \mathcal{Q}) \equiv \frac{1}{2} \mathcal{M}_{MN}(\mathcal{N}) \mathcal{Q}^M \mathcal{Q}^N,$$
$$(\mathcal{Q}^M) = \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix},$$
$$(\mathcal{M}_{MN}(\mathcal{N})) \equiv \begin{pmatrix} I + RI^{-1}R & -RI^{-1} \\ -I^{-1}R & I^{-1} \end{pmatrix}$$
$$X \equiv \frac{1}{\sqrt{2}} e^{U + i\alpha}$$
$$\mathcal{F}_\Lambda \equiv \frac{\partial \mathcal{F}}{\partial \mathcal{X}^\Lambda} \text{ y } \mathcal{F}_{\Lambda\Sigma} \equiv \frac{\partial^2 \mathcal{F}}{\partial \mathcal{X}^\Lambda \partial \mathcal{X}^\Sigma}, \text{ se tiene } \mathcal{F}_\Lambda = \mathcal{F}_{\Lambda\Sigma} \mathcal{X}^\Sigma$$
$$(\mathcal{V}^M) = \begin{pmatrix} \mathcal{L}^\Lambda \\ \mathcal{M}_\Lambda \end{pmatrix} = e^{\mathcal{K}/2} \begin{pmatrix} \mathcal{X}^\Lambda \\ \mathcal{F}_\Lambda \end{pmatrix}$$

$$\frac{\mathcal{M}^{\Lambda}}{X}=\mathcal{F}_{\Lambda\Sigma}\frac{\mathcal{L}^{\Lambda}}{X}.$$
$$\mathcal{R}^{M}\equiv\Re\mathfrak{e}(\mathcal{V}^{M}/X),\mathcal{J}^{M}\equiv\Im\mathfrak{m}(\mathcal{V}^{M}/X),$$
$$\mathcal{R}^{M}=-\mathcal{M}_{MN}(\mathcal{F})\mathcal{J}^{M}$$
$$d\mathcal{R}^{M}=-\mathcal{M}_{MN}(\mathcal{F})d\mathcal{J}^{M}$$
$$\frac{\partial\mathcal{J}^{M}}{\partial\mathcal{R}_{N}}=\frac{\partial\mathcal{J}^{N}}{\partial\mathcal{R}_{M}}=-\frac{\partial\mathcal{R}^{M}}{\partial\mathcal{J}_{N}}=-\frac{\partial\mathcal{R}^{N}}{\partial\mathcal{J}_{M}}=-\mathcal{M}^{MN}(\mathcal{F}).$$
$$H^{M}\equiv\mathcal{J}^{M}(X,z,\bar{X},\bar{z})$$
$$z^{i}=\frac{\mathcal{V}^{i}/X}{\mathcal{V}^{0}/X}=\frac{\tilde{H}^{i}(H)+iH^{i}}{\tilde{H}^{0}(H)+iH^{0}},e^{-2U}=\frac{1}{2|X|^{2}}=\tilde{H}_{M}(H)H^{M}.$$
$$\dot{\alpha}=2|X|^{2}\dot{H}^{M}H_{M}-\left[\frac{1}{2i}\dot{z}^{i}\partial_{i}\mathcal{K}+c.c.\right].$$
$$\mathbf{W}(H)\equiv\tilde{H}_{M}(H)H^{M}=e^{-2U},$$
$$\tilde{H}_{M}=\frac{1}{2}\frac{\partial\,\mathrm{W}}{\partial H^{M}}\equiv\frac{1}{2}\partial_{M}\,\mathrm{W},H^{M}=\frac{1}{2}\frac{\partial\,\mathrm{W}}{\partial\tilde{H}^{M}}.$$
$$-I_{\mathrm{H-FGK}}[H]=\int d\tau\left\{\frac{1}{2}g_{MN}\dot{H}^{M}\dot{H}^{N}-V\right\}$$
$$r_{0}^{2}=\frac{1}{2}g_{MN}\dot{H}^{M}\dot{H}^{N}+V$$
$$g_{MN}\equiv\partial_{M}\partial_{N}\log\ \mathrm{W}-2\frac{H_{M}H_{N}}{\mathrm{W}}$$
$$V(H)\equiv\left\{-\frac{1}{4}g_{MN}+\frac{H_{M}H_{N}}{2\mathbf{W}^{2}}\right\}Q^{M}Q^{N}$$
$$V_{\mathrm{bh}}=-\mathrm{W}V.$$
$$g_{MN}\ddot{H}^{N}+[PQ,M]\dot{H}^{P}\dot{H}^{Q}+\partial_{M}V=0$$
$$[PQ,M]\equiv\partial_{(P}g_{Q)M}-\frac{1}{2}\partial_{M}g_{PQ}$$
$$\tilde{H}_{M}(\ddot{H}^{M}-r_{0}^{2}H^{M})+\frac{(\dot{H}^{M}H_{M})^{2}}{\mathrm{W}}=0,$$
$$\dot{H}^{M}H_{M}=0,$$
$$\tilde{H}_{M}(\ddot{H}^{M}-r_{0}^{2}H^{M})=0,$$
$$H^{M}=A^{M}-\frac{B^{M}}{\sqrt{2}}\tau,$$
$$H^{M}=A^{M}\cosh(r_{0}\tau)+B^{M}\sinh(r_{0}\tau)$$
$$H^{M}=A^{M}-\frac{Q^{M}}{\sqrt{2}}\tau,$$
$$H^{0}=H_{0}=H_{i}=0,p^{0}=p_{0}=q_{i}=0,$$
$$H^{i}=a^{i}-\frac{p^{i}}{\sqrt{2}}\tau,r_{0}=0,$$
$$H^{M}=H^{M}(a,b),$$
$$\{H^{P}=0,Q^{P}=0\}\Rightarrow\mathcal{E}_{P}=0,$$
$$\begin{pmatrix}iH^{i}\\ \tilde{H}_{i}\end{pmatrix}=\frac{e^{\mathcal{K}/2}}{X}\begin{pmatrix}\mathcal{X}^{i}\\ \frac{\partial F(\mathcal{X})}{\partial\mathcal{X}^{i}}\end{pmatrix},\begin{pmatrix}\tilde{H}^{0}\\ 0\end{pmatrix}=\frac{e^{\mathcal{K}/2}}{X}\begin{pmatrix}\mathcal{X}^{0}\\ \frac{\partial F(\mathcal{X})}{\partial\mathcal{X}^{0}}\end{pmatrix},$$
$$e^{-2U}=\tilde{H}_{i}H^{i},z^{i}=i\frac{H^{i}}{\tilde{H}^{0}},$$
$$\frac{\partial F(H)}{\partial\tilde{H}^{0}}=0$$

$$F(H)=\frac{i}{3!}\kappa^0_{ijk}\frac{H^iH^jH^k}{\tilde{H}^0}+\frac{ic\left(\tilde{H}^0\right)^2}{2}+\frac{i\left(\tilde{H}^0\right)^2}{(2\pi)^3}\sum_{\{d_i\}} n_{\{d_i\}}Li_3\left(e^{-2\pi d_i\frac{H^i}{\tilde{H}^0}}\right)$$

$$\tilde{H}_i=-i\frac{\partial F(H)}{\partial H^i},$$

$$-\frac{1}{3!}\kappa^0_{ijk}\frac{H^iH^jH^k}{\left(\tilde{H}^0\right)^3}+c+\frac{1}{4\pi^3}\sum_{\{d_i\}} n_{\{d_i\}}\left[Li_3\left(e^{-2\pi d_i\frac{H^i}{\tilde{H}^0}}\right)+Li_2\left(e^{-2\pi d_i\frac{H^i}{\tilde{H}^0}}\right)\left[\frac{\pi d_iH^i}{\tilde{H}^0}\right]\right]$$

$$\lim_{|w|\to 0} Li_s(w)=w, \forall s\in\mathbb{N}$$

$$-\frac{1}{3!}\kappa^0_{ijk}\frac{H^iH^jH^k}{\left(\tilde{H}^0\right)^3}+c+\frac{1}{4\pi^3}\sum_{\{d_i\}} n_{\{d_i\}}\left[e^{-2\pi d_i\frac{H^i}{\tilde{H}^0}}+e^{-2\pi d_i\frac{H^i}{\tilde{H}^0}}\left[\frac{\pi d_iH^i}{\tilde{H}^0}\right]\right]=0, \Im\,\mathrm{m}z^i\gg 1$$

$$-\frac{1}{3!}\kappa^0_{ijk}\frac{H^iH^jH^k}{\left(\tilde{H}^0\right)^3}+c+\frac{1}{4\pi^3}\sum_{\{d_i\}} n_{\{d_i\}}e^{-2\pi d_i\frac{H^i}{\tilde{H}^0}}\left[\frac{\pi d_iH^i}{\tilde{H}^0}\right]$$

$$-\frac{1}{3!}\kappa^0_{ijk}\frac{H^iH^jH^k}{\left(\tilde{H}^0\right)^3}+c+\frac{\hat{n}}{4\pi^3}e^{-2\pi\hat{d}_i\frac{H^i}{\tilde{H}^0}}\left[\frac{\pi\hat{d}_iH^i}{\tilde{H}^0}\right]$$

$$e^{-2U}=\mathbf{W}(H)=\alpha\left|\kappa^0_{ijk}H^iH^jH^k\right|^{2/3}$$

$$V_{\mathrm{bh}}=\frac{\mathrm{W}(H)}{4}\partial_{ij}\log\ \mathrm{W}(H)Q^iQ^j,$$

$$z^i=i(3!\,c)^{1/3}\frac{H^i}{\left(\kappa^0_{ijk}H^iH^jH^k\right)^{1/3}},$$

$$\kappa^0_{ijk}\Im\,\mathrm{m}z^i\Im\,\mathrm{m}z^j\Im\,\mathrm{m}z^k>\frac{3c}{2}.$$

$$c>\frac{c}{4}$$

$$e^{-\mathcal{K}}=6c$$

$$c>0\Rightarrow h^{11}>h^{21}$$

$$X^{3,1}\Rightarrow\kappa^0_{111}=48,\kappa^0_{222}=\kappa^0_{333}=8$$

$$Y^{3,1}\Rightarrow\kappa^0_{122}=6,\kappa^0_{222}=18,\kappa^0_{333}=8$$

$$\mathbf{W}(H)=\alpha|48(H^1)^3+8[(H^2)^3+(H^3)^3]|^{2/3},$$

$$\mathbf{W}(H)=\alpha|18(H^2)^2[H^1+H^2]+8(H^3)^3|^{2/3}.$$

$$H=a\cosh(r_0\tau)+\frac{b}{r_0}\sinh(r_0\tau), b=s_b\sqrt{r_0^2a^2+\frac{p^2}{2}},$$

$$z^1=i(3!\,c)^{1/3}\lambda^{-1/3}=s_{2,3}z^{2,3}$$

$$\lambda=[48+8(s_2+s_3)]\ \text{para}\ X^{3,1}$$

$$\lambda=[18+18s_2+8s_3]\ \text{para}\ Y^{3,1}.$$

$$ds^2=\left[\frac{1}{2}(3!\,c)^{1/3}\left[a\cosh(r_0\tau)+\frac{b}{r_0}\sinh(r_0\tau)\right]^2\right]^{-1}dt^2$$

$$-\frac{1}{2}(3!\,c)^{1/3}\left[a\cosh(r_0\tau)+\frac{b}{r_0}\sinh(r_0\tau)\right]^2\left[\frac{r_0^4}{\sinh^4\ r_0\tau}d\tau^2+\frac{r_0^2}{\sinh^2\ r_0\tau}d\Omega^2_{(2)}\right]$$

$$2\pi id_iz^i\sim-\frac{1}{3}\sum_{i=1}^{3}d_i, d_i\geq 1$$

$$s_2 = s_3 = -1,\ \text{para } X^{3,1},$$
$$s_2 = -s_3 = 1,\ \text{para } Y^{3,1}.$$
$$\mathcal{G}_{ij^*} = \partial_i \partial_{j^*} \mathcal{K}$$
$$a = -s_b \frac{\Im m z^1}{\sqrt{3c}}.$$
$$M = r_0 \sqrt{1 + \frac{3cp^2}{2r_0^2 (\Im\, \mathrm{m} z^1)^2}},$$
$$S_\pm = r_0^2 \pi \left(\sqrt{1 + \frac{3cp^2}{2r_0^2 (\Im\, \mathrm{m} z^1)^2}} \pm 1 \right)^2.$$
$$S_+ S_- = \frac{\pi^2 \alpha^2}{4} p^4 \lambda^{4/3}.$$
$$e^{-2U} = \mathbf{W}(H) = \alpha |H^1 H^2 H^3|^{2/3},$$
$$z^i = i c^{1/3} \frac{H^i}{(H^1 H^2 H^3)^{1/3}}.$$
$$H^i = a^i \cosh(r_0 \tau) + \frac{b^i}{r_0} \sinh(r_0 \tau), b^i = s_b^i \sqrt{r_0^2 (a^i)^2 + \frac{(p^i)^2}{2}}.$$
$$a^i = -s_b^i \frac{\Im m z_\infty^i}{\sqrt{3c}}.$$
$$M = \frac{r_0}{3} \sum_i \sqrt{1 + \frac{3c(p^i)^2}{2r_0^2 (\Im m z_\infty^i)^2}}$$
$$S_\pm = r_0^2 \pi \prod_i \left(\sqrt{1 + \frac{3c(p^i)^2}{2r_0^2 (\Im\, \mathrm{m} z_\infty^i)^2}} \pm 1 \right)^{2/3}$$
$$S_+ S_- = \frac{\pi^2 \alpha^2}{4} \prod_i (p^i)^{4/3}$$
$$-\frac{1}{3!} \kappa_{ijk}^0 \frac{H^i H^j H^k}{(\tilde{H}^0)^3} + \frac{\hat{n}}{4\pi^3} e^{-2\pi \hat{d}_l \frac{H^l}{H^0}} \left[\frac{\pi \hat{d}_n H^n}{\tilde{H}^0} \right]$$
$$\tilde{H}^0 = \frac{\pi \hat{d}_l H^l}{W_a \left(s_a \sqrt{\frac{3\hat{n} (\hat{d}_n H^n)^3}{2\kappa_{ijk}^0 H^i H^j H^k}} \right)},$$
$$\tilde{H}_i = \frac{1}{2} \kappa_{ijk}^0 \frac{H^j H^k}{\pi \hat{d}_l H^l} W_a \left(s_a \sqrt{\frac{3\hat{n} (\hat{d}_m H^m)^3}{2\kappa_{pqr}^0 H^p H^q H^r}} \right).$$
$$e^{-2U} = \mathrm{W}(H) = \frac{\kappa_{ijk}^0 H^i H^j H^k}{2\pi \hat{d}_m H^m} W_a \left(s_a \sqrt{\frac{3\hat{n} (\hat{d}_l H^l)^3}{2\kappa_{pqr}^0 H^p H^q H^r}} \right)$$
$$z^i = i \frac{H^i}{\pi \hat{d}_m H^m} W_a \left(s_a \sqrt{\frac{3\hat{n} (\hat{d}_l H^l)^3}{2\kappa_{pqr}^0 H^p H^q H^r}} \right).$$

$$s_0 \equiv \mathrm{sign}\left[\kappa^0_{ijk}\frac{H^iH^jH^k}{\hat{d}_mH^m}\right],$$
$$s_{-1} \equiv -1$$
$$H^i = a^i - \frac{p^i}{\sqrt{2}}\tau, r_0 = 0.$$
$$S = \frac{1}{2}\kappa^0_{ijk}\frac{p^ip^jp^k}{\hat{d}_mp^m}W_a(s_a\beta),$$
$$\beta = \sqrt{\frac{3\hat{n}\left(\hat{d}_lp^l\right)^3}{2\kappa^0_{pqr}p^pp^qp^r}},$$
$$M = \dot{U}(0) = \frac{1}{2\sqrt{2}}\left[\frac{3\kappa^0_{ij}p^ia^ja^k}{\kappa^0_{pqr}a^pa^qa^r}\left[1-\frac{1}{1+W_a(s_a\alpha)}\right]-\frac{d_lp^l}{d_na^n}\left[1-\frac{3}{2\left(1+W_a(s_a\alpha)\right)}\right]\right],$$
$$\alpha = \sqrt{\frac{3\hat{n}(d_la^l)^3}{2\kappa^0_{pqr}a^pa^qa^r}}.$$
$$W_a(x)e^{-2W_a(x)} \gg e^{-2W_a(x)}.$$
$$e^{-2U} = \frac{\kappa^0_{ijk}H^iH^jH^k}{2\pi\hat{d}_mH^m}W_0\left(\sqrt{\frac{3\hat{n}\left(\hat{d}_lH^l\right)^3}{2\kappa^0_{pqr}H^pH^qH^r}}\right),$$
$$z^i = i\frac{H^i}{\pi\hat{d}_mH^m}W_0\left(\sqrt{\frac{3\hat{n}\left(\hat{d}_lH^l\right)^3}{2\kappa^0_{pqr}H^pH^qH^r}}\right).$$
$$\frac{\kappa^0_{ijk}H^iH^jH^k}{2\pi\hat{d}_nH^n} > 0\ \forall\tau \in (-\infty, 0],$$
$$\frac{\kappa^0_{ijk}a^ia^ja^k}{2\pi\hat{d}_ma^m}W_0(\alpha) = 1,$$
$$e^{-2U} \overset{\tau\to-\infty}{\to} \frac{\kappa^0_{ijk}p^ip^jp^k}{8\pi\hat{d}_mp^m}W_0(\beta)\tau^2.$$
$$L = \bigoplus_{k=0}^{N} L_k,$$
$$u_j \circ u_k \in L_{(j+k)\mathrm{mod}(N+1)}.$$
$u_0 \circ v_0 \in L_0, \forall u_0, v_0 \in L_0,$
$u_0 \circ u_1 \in L_1, \forall u_0 \in L_0, v_1 \in L_1,$
$u_1 \circ v_1 \in L_0, \forall u_1, v_1 \in L_1,$

- Supersimetrización: $\forall x_i \in L_i, \forall x_j \in L_j i, j = 0,1, x_i \circ x_j = -(-1)^{ij}x_j \circ x_i.$
- Identidades de Jacobi: $\forall x_k \in L_k, \forall x_l \in L_l, \forall x_m \in L_m, k, l, m = 0,1, \quad x_k \circ (x_l \circ x_m)(-1)^{km} + x_l \circ (x_m \circ x_k)(-1)^{lk} + x_m \circ (x_k \circ x_l)(-1)^{ml} = 0.$

$$x_\mu \circ x_\nu \equiv x_\mu x_\nu - (-1)^{g_\mu g_\nu}x_\nu x_\mu = c^\omega_{\mu\nu}x_\omega$$
$$Li_w(z) = \sum_{j=1}^{\infty}\frac{z^j}{j^w}, z, w \in \mathbb{C}.$$
$$Li_{w-1}(z) = z\frac{\partial Li_w(z)}{\partial z}.$$

$$Li_1(z) = -\log(1-z)$$

$$Li_0(z) = \frac{z}{1-z}, Li_{-n}(z) = \left(z\frac{\partial}{\partial z}\right)^n \frac{z}{1-z}.$$

$$Li_w(z) = \int_0^z \frac{Li_{w-1}(s)}{s} ds$$

$$z = W(z)e^{W(z)}, \forall z \in \mathbb{C}.$$

$$\frac{dW(z)}{dz} = \frac{W(z)}{z(1+W(z))}, \forall z \notin \{0, -1/e\}, \left.\frac{dW(z)}{dz}\right|_{z=0}$$

$$\lim_{x\to -1/e} \frac{dW_0(x)}{dx} = \infty, \lim_{x\to -1/e} \frac{dW_{-1}(x)}{dx} = -\infty.$$

$$X \times S$$

$$\mu^{1,1} \in \Omega^{0,1}(X, \mathrm{T}_X) \otimes C^\infty(S)$$

$$\mu^{1,1} = \mu_j^i(z, \bar{z}, t)\mathrm{d}\bar{z}_i \frac{\partial}{\partial z_j}$$

$$\gamma^{1,0} \in \Omega^{1,0}(X) \otimes C^\infty(S), \gamma^{1,2} \in \Omega^{1,2}(X) \otimes C^\infty(S)$$

$$\bar{\partial}\mu^{1,1} + \frac{1}{2}[\mu^{1,1}, \mu^{1,1}] + \Omega^{-1} \vee (\partial\gamma^{1,0} \wedge \partial\gamma^{1,2}) = 0$$

$$\Omega^{0,\cdot}(X, V) = \left(\Omega^{0,j}(X, V)[-j], \bar{\partial}\right)$$

$$\partial_\Omega(\mu) \wedge \Omega = L_\mu(\Omega)$$

$$\partial_\Omega : \Omega^{0,\cdot}(X, \mathrm{T}_X) \to \Omega^{0,\cdot}(X)$$

$$\Omega^{0,\cdot}(X, \mathrm{T}_X) \overset{\partial_\Omega}{\to} \Omega^{0,\cdot}(X)$$

$$[\mu, \mu'] = L_\mu \mu'$$

$$[\mu, \nu] = L_\mu \nu$$

$$(\mathrm{Sym}(\mathcal{L}^\vee[-1]), \delta_{\mathcal{L}})$$

$$\Phi^* : \mathrm{C}^\cdot(\mathcal{L}') \to \mathrm{C}^\cdot(\mathcal{L})$$

$$\Psi_\infty : \nu \mapsto 1 - e^{-\nu}, \mu \mapsto e^{-\nu}\mu$$

$$[\mu]_1 = \bar{\partial}\mu + \partial_\Omega \mu$$

$$[\mu_1, \mu_2]_2 = \partial_\Omega(\mu_1 \wedge \mu_2)$$

$$[\nu, \mu_1, \mu_2]_3 = \partial_\Omega(\nu\mu_1 \wedge \mu_2)$$

$$[\nu_1, \dots, \nu_{k-2}, \mu_1, \mu_2]_k = \partial_\Omega(\nu_1 \cdots \nu_k \mu_1 \wedge \mu_2)$$

$$[\nu_1, \dots, \nu_{k-3}, \mu_1, \mu_2, \gamma]_k = \nu_1 \cdots \nu_{k-3}(\mu \wedge \mu') \vee \partial\gamma.$$

$$[\nu_1, \dots, \nu_{k-2}, \mu, \gamma]_k = \nu_1 \cdots \nu_{k-2}\mu \vee \partial\gamma.$$

$$[x \otimes a]_1 = [x]_1^{\mathcal{L}} \otimes a + (-1)^{|x|} x \otimes \mathrm{d}_{\mathcal{A}} a$$

$$[x_1 \otimes a_1, \dots, x_k \otimes a_k]_k = [x_1, \dots, x_k]_k^{\mathcal{L}} \otimes (a_1 \cdots a_k), k \geqslant 2.$$

$$\mathrm{F}_A = [A]_1 + \frac{1}{2}[A, A]_2 + \frac{1}{3!}[A, A, A]_3 + \cdots$$

$$(A, B) \in \mathcal{L}[1] \oplus \mathcal{L}^![-2]$$

$$-n \qquad -n+1 \quad -1 \qquad 0$$

$$\Omega^0(X; S) \overset{\partial}{\to} \Omega^1(X; S) \qquad \mathrm{PV}^1(X; S) \overset{\partial_\Omega}{\to} \mathrm{PV}^0(X; S).$$

$$\Omega^i(X; S) = \Omega^{i,\cdot;\cdot}(X; S)$$

$$= \bigoplus\nolimits_{j,k} \mathrm{PV}^{i,j}(X) \otimes \Omega^k(S)[-j-k]$$

$$n = \dim_{\mathbb{C}}(X) + \dim_{\mathbb{R}}(S) - 1$$

$$\int_{X\times S}^{\Omega} \mu \vee \gamma + \int_{X\times S}^{\Omega} \nu\beta$$

$$S_{BF,0} = \int^{\Omega} \left[\beta \wedge (\bar{\partial} + \mathrm{d}_S)\nu + \gamma \wedge (\bar{\partial} + \mathrm{d}_S)\mu + \beta \wedge \partial_\Omega \mu + \frac{1}{2}[\mu, \mu] \vee \gamma + [\mu, \nu]\beta\right]\left[\beta \wedge (\bar{\partial} + \mathrm{d}_S)\nu + \gamma \wedge (\bar{\partial} + \mathrm{d}_S)\mu + \beta \wedge \partial_\Omega \mu + \frac{1}{2}[\mu, \mu] \vee \gamma + [\mu, \nu]\beta\right].$$

$$S_{BF,\infty} = \int^{\Omega} \left[\beta \wedge (\bar{\partial} + \mathrm{d}_S)\nu + \gamma \wedge (\bar{\partial} + \mathrm{d}_S)\mu + \beta \wedge \partial_\Omega \mu + \frac{1}{2}\frac{1}{1-\nu}\mu^2 \vee \partial\gamma\right]\left[\beta \wedge (\bar{\partial} + \mathrm{d}_S)\nu + \gamma \wedge (\bar{\partial} + \mathrm{d}_S)\mu + \beta \wedge \partial_\Omega \mu + \frac{1}{2}\frac{1}{1-\nu}\mu^2 \vee \partial\gamma\right].$$

$$\mu \mapsto e^{-\nu}\mu, \nu \mapsto 1 - e^{-\nu}, \beta \mapsto (\beta - \mu \vee \gamma)e^{\nu}, \gamma \mapsto e^{\nu}\gamma$$

$$\Omega^0(X;S)_\beta \xrightarrow{\partial} \Omega^1(X;S)_\gamma \qquad\qquad \mathrm{PV}^1(X;S)_\mu \xrightarrow{\partial_\Omega} \mathrm{PV}^0(X;S)_\nu.$$

$$S_{BF} + gJ$$

$$\{S_{BF} + gJ, S_{BF} + gJ\} = 0$$

$$\{S_{BF}, J\} = \{J, J\} = 0$$

$$J = \frac{1}{6}\gamma \wedge \partial\gamma \wedge \partial\gamma$$

$$\gamma^{1,i;j} \in \Omega^{1,i}(X) \otimes \Omega^j(S)$$

$$\deg(J) = 6$$

$$\{\beta \wedge \partial_\Omega \mu, J\} = \frac{1}{2}\partial\beta \wedge \partial\gamma \wedge \partial\gamma = 0$$

$$\left\{\frac{1}{2}\frac{1}{1-\nu}\partial\gamma \vee \mu^2, \frac{1}{6}\gamma \wedge \partial\gamma \wedge \partial\gamma\right\} = \frac{1}{2}(\mu \vee \partial\gamma) \wedge \partial\gamma \wedge \partial\gamma$$

$$\mathrm{Sym}^3(\boxminus) \cong \boxplus \oplus \nexists$$

$$\wedge^3(\exists) \cong \boxminus \oplus \boxminus$$

$$\gamma \mapsto \sqrt{g}\gamma, \beta \mapsto \sqrt{g}\beta$$

$$\frac{1}{\sqrt{g}}\left(S_{BF,\infty} + J\right)$$

$$\tilde{J} = \frac{1}{6}e^{\nu}\gamma \wedge \partial(e^{\nu}\gamma) \wedge \partial(e^{\nu}\gamma)$$

$$\bar{\partial}\nu + \mathrm{d}_S\nu + \partial_\Omega\mu = 0$$

$$\bar{\partial}\mu + \mathrm{d}_S\mu + \frac{1}{2}\frac{1}{1-\nu}\partial_\Omega(\mu^2) + \frac{1}{2}(\partial\gamma \wedge \partial\gamma) \vee (g\Omega^{-1}) = 0$$

$$(\bar{\partial} + \mathrm{d}_S)\gamma + \partial\beta + \frac{1}{1-\nu}(\mu \vee \partial\gamma) = 0$$

$$(\bar{\partial} + \mathrm{d}_S)\beta + \frac{1}{2}\frac{1}{(1-\nu)^2}\mu^2 \vee \partial\gamma = 0$$

- $\mu = \sum_{i,j} \mu^{i;j}\ \mu^{i;j} \in \mathrm{PV}^{1,i}(X) \otimes \Omega^j(\mathbb{R}), i = 0, \ldots, 5, j = 0,1$
- $\nu = \sum_{i,j} \nu^{i;j}\ \nu^{i;j} \in \mathrm{PV}^{0,i}(X,\ \mathrm{T}_X) \otimes \Omega^j(\mathbb{R}), i = 0, \ldots, 5, j = 0,1$
- $\gamma = \sum_{i,j} \gamma^{i;j}\ \gamma^{i;j} \in \Omega^{1,i}(X) \otimes \Omega^j(\mathbb{R}), i = 0, \ldots, 5, j = 0,1$
- $\beta = \sum_{i,j} \beta^{i;j}\ \beta^{i;j} \in \Omega^{0,i}(X) \otimes \Omega^j(\mathbb{R}), i = 0, \ldots, 5, j = 0,1$

$$\bar{\partial}\mu^{1;0} + \frac{1}{2}[\mu^{1;0}, \mu^{1;0}] + \left(\frac{1}{2}\partial\gamma^{1;0} \wedge \partial\gamma^{1;0} + \partial\gamma^{2;0} \wedge \partial\gamma^{0;0}\right) \vee (g\Omega^{-1}) = 0$$

$$\alpha \stackrel{\text{def}}{=} \partial\gamma^{0;0}$$

$$\Omega_X^{2,\,\mathrm{hol}} \xrightarrow{\wedge\alpha} \Omega_X^{4,\,\mathrm{hol}} \cong {}_{\Omega}\mathcal{T}_X^{\mathrm{hol}}$$

$$\bar{\partial}\xi + \frac{1}{2}[\xi, \xi] = \alpha \vee \rho$$

$$\mu^{1;0} = \mu_i^j(z, \bar{z}, t)\mathrm{d}\bar{z}_j\partial_{z_i}$$

$$\mu^{0;1} = \mu_i^t(z, \bar{z}, t)\mathrm{d}t\partial_{z_i}$$

$$\mu^{0;0} = \mu_i(z, \bar{z}, t)\partial_{z_i}$$

$$\beta^{3;0} = \beta^{ijk}(z, \bar{z}, t)\mathrm{d}\bar{z}_i\ \mathrm{d}\bar{z}_j\ \mathrm{d}\bar{z}_k, \beta^{2;1} = \beta_t^{ij}(z, \bar{z}, t)\mathrm{d}\bar{z}_i\ \mathrm{d}\bar{z}_j\ \mathrm{d}t$$

$$\gamma^{2;0} = \gamma^{ijk}(z, \bar{z}, t)\mathrm{d}z_i\ \mathrm{d}\bar{z}_j\ \mathrm{d}\bar{z}_k, \gamma^{1;1} = \gamma_t^{ij}(z, \bar{z}, t)\mathrm{d}z_i\ \mathrm{d}\bar{z}_j\ dt.$$

$$\mu^i \partial_{z_i} \in \mathrm{Vect}(\mathbb{C}^5) \cong \mathcal{O}(\mathbb{C}^5)\partial_{z_i}, \nu \in \mathcal{O}(\mathbb{C}^5)$$

$$\beta \in \mathcal{O}(\mathbb{C}^5), \gamma^i\, \mathrm{d}z_i \in \Omega^1(\mathbb{C}^5) \cong \mathcal{O}(\mathbb{C}^5)\mathrm{d}z_i$$

$$\boldsymbol{\mu}^i_{(m_j)}: \mu^i \mapsto \partial_{z_1}^{m_1}\partial_{z_2}^{m_2}\partial_{z_3}^{m_3}\partial_{z_4}^{m_4}\partial_{z_5}^{m_5}\mu^i$$

$$\boldsymbol{\nu}_{(m_j)}: \nu \mapsto \partial_{z_1}^{m_1}\partial_{z_2 m_2}^{\partial_{z_3}^{m_3}\partial_{z_4}^{m_4}\partial_{z_5}^{m_5}\nu}$$

$$\boldsymbol{\gamma}^i_{(m_j)}: \gamma^i \mapsto \partial_{z_1}^{m_1}\partial_{z_2 m_2}^{\partial_{z_3}^{m_3}\partial_{z_4}^{m_4}\partial_{z_5}^{m_5}\gamma^i}$$

$$\boldsymbol{\beta}_{(m_j)}: \beta \mapsto \partial_{z_1}^{m_1}\partial_{z_2}^{m_2}\partial_{z_3}^{m_3}\partial_{z_4}^{m_4}\partial_{z_5}^{m_5}\beta$$

$$i(q_1, \ldots, q_5) = \frac{\sum_{i=1}^5 q_i}{\prod_{i=1}^5 (1-q_i)} + \frac{\sum_{i=1}^5 q_i^{-1}}{\prod_{i=1}^5 (1-q_i^{-1})}$$

$$q_1^{m_1+1} \cdots q_i^{m_i} \cdots q_5^{m_5+1}.$$

$$q_1^{m_1} \cdots q_i^{m_i+1} \cdots q_5^{m_5}$$

$$\sum_{i=1}^5 \left(\sum_{(m_i)\in\mathbb{Z}^5_{\geqslant 0}} q_1^{m_1} \cdots q_i^{m_i+1} \cdots q_5^{m_5} - \sum_{(m_i)\in\mathbb{Z}^5_{\geqslant 0}} q_1^{m_1+1} \cdots q_i^{m_i} \cdots q_5^{m_5+1} \right)$$

$$-\frac{\sum_{i=1}^5 q_1 \cdots \hat{q}_i \cdots q_5}{\prod_{i=1}^5 (1-q_i)} + \frac{\sum_{i=1}^5 q_i}{\prod_{i=1}^5 (1-q_i)}$$

$$\prod_{i=1}^5 \prod_{(m_i)\in\mathbb{Z}^5_{\geqslant 0}} \frac{1-q_1^{m_1+1} \cdots q_i^{m_i} \cdots q_5^{m_5+1}}{1-q_1^{m_1} \cdots q_i^{m_i+1} \cdots q_5^{m_5}}$$

$$E(5,10)_+ = \mathrm{Vect}_0(\mathbb{C}^5)$$

$$E(5,10)_- = \Omega^2_{cl}(\mathbb{C}^5)$$

$$[\alpha, \alpha'] = \Omega^{-1} \vee (\alpha \wedge \alpha')$$

$$\delta = \{S_{BF,\infty} + J, -\}$$

$$\delta^{(1)} = \bar{\partial} + \mathrm{d}_{\mathbb{R}} + \partial_\Omega|_{\mu\to\nu} + \partial|_{\beta\to\gamma}$$

$$\mathrm{Vect}(\mathbb{C}^5) \xrightarrow{\partial_\Omega} \mathcal{O}(\mathbb{C}^5)$$

$$\mathcal{O}(\mathbb{C}^5) \xrightarrow{\partial} \Omega^1(\mathbb{C}^5)$$

$$\mu = \mu \otimes 1 \in \Pi\mathrm{Vect}_0(\mathbb{C}^5) \otimes \Omega^0(\mathbb{R})$$

$$[\gamma] = [\gamma] \otimes 1 \in \left(\Omega^1(\mathbb{C}^5)/\mathrm{d}\left(\mathcal{O}(\mathbb{C}^5)\right) \otimes \Omega^0(\mathbb{R})\right)$$

$$(\mu, [\gamma], b) \in \mathrm{Vect}_0(\mathbb{C}^5) \oplus \Pi\Omega^1(\mathbb{C}^5)/\partial\mathcal{O}(\mathbb{C}^5) \oplus \mathbb{C}$$

$$[[\gamma], [\gamma']] = \Omega^{-1} \vee (\partial\gamma \wedge \partial\gamma') \in \mathrm{Vect}_0(\mathbb{C}^5)$$

$$\partial: \Omega^1(\mathbb{C}^5)/\mathrm{d}\left((\mathbb{C}^5) \overset{\cong}{\Rightarrow} \Omega^2_{cl}(\mathbb{C}^5)\right)_q$$

$$_K \leftrightarrow \left(\Pi\mathcal{E}, \delta^{(1)}\right) \rightleftarrows (E(5,10) \oplus \mathbb{C}_b, 0),$$

$$\widetilde{K}\partial\gamma + \partial K\gamma = \gamma$$

$$\gamma - \widetilde{K}\partial\gamma = \partial K\gamma$$

$$[\mu, \mu', [\gamma]]_3 = \varphi(\mu, \mu', [\gamma])$$

$$\varphi: E(5,10) \times E(5,10) \times E(5,10) \to \mathbb{C}_b$$

$$\varphi(\mu, \mu', \alpha) = \langle \mu \wedge \mu', \alpha \rangle|_{z=0}$$

$$\mathrm{C}^{\mathrm{even}}(\mathcal{L}) = \mathrm{C}^{2\cdot,+}(\mathcal{L}) \oplus \mathrm{C}^{2\cdot+1,-}(\mathcal{L})$$

$$\mathrm{C}^{\mathrm{odd}}(\mathcal{L}) = \mathrm{C}^{2\cdot,-}(\mathcal{L}) \oplus \mathrm{C}^{2\cdot+1,+}(\mathcal{L})$$

$$\mathfrak{t}_{11d} = V \oplus \Pi S$$

$$\mathrm{Sym}^2(S) \cong V \oplus \wedge^2 V \oplus \wedge^5 V$$

$$[\psi,\psi'] = \Gamma_{\wedge^1}(\psi,\psi')$$
$$\mathfrak{siso}_{11d} = \mathfrak{so}(11,\mathbb{C}) \ltimes \mathfrak{t}_{11d}$$
$$c_{M2} \in \mathrm{C}^{2,+}(\mathfrak{siso}_{11d}; \Omega^{\cdot}(\mathbb{R}^{11})[2])$$
$$c_{M2}(\psi,\psi') = \Gamma_{\wedge^2}(\psi,\psi') \in \Omega^2(\mathbb{R}^{11})$$
$$V = L \oplus L^{\vee} \oplus \mathbb{C}_t, S = \wedge^{\cdot} L$$
$$\mathfrak{sl}(L) \oplus \wedge^2 L \oplus \wedge^2 L^{\vee} \oplus L \oplus L^{\vee} \oplus \mathbb{C}$$
$$S = \wedge^{\cdot}(L) = \mathbb{C} \oplus L \oplus \wedge^2 L \oplus \wedge^3 L \oplus \wedge^4 L \oplus \wedge^5 L$$
$$\mathrm{Stab}(Q) = \mathfrak{sl}(L) \oplus \wedge^2 L^{\vee} \oplus L^{\vee} \subset \mathfrak{so}(11,\mathbb{C})$$
$$L \oplus \mathrm{Stab}(Q) \oplus \Pi(\wedge^2 L^{\vee}) \oplus \mathbb{C}$$
$$[Q,-]: \mathfrak{so}(11,\mathbb{C}) \to S$$
$$[\psi,\psi']_2 = \psi \wedge \psi' \in \wedge^4 L^{\vee} \cong L_v$$
$$[v,v',\psi]_3 = 4\langle v \wedge v', \psi\rangle \in \mathbb{C}_b$$
$$\left[z_i \wedge z_j, z_k \wedge z_l\right]_2 = \epsilon_{ijklm}\partial_{z_m}$$
$$\left[\partial_{z_i}, \partial_{z_j}, z_k \wedge z_\ell\right]_3 = 4\left(\delta_k^i \delta_\ell^j - \delta_\ell^i \delta_k^j\right)$$
$$H^{\cdot}(\mathfrak{m2brane}^{Q}) \oplus \left(L^{\vee} \overset{\mathbb{B}}{\to} \Pi L^{\vee}\right)$$
$$[v,\lambda] = \langle v,\lambda\rangle \in \mathbb{C}_b$$
$$[v,\psi] = \langle v,\psi\rangle \in \Pi L_{\tilde{\lambda}}$$
$$\mathfrak{g} \rightsquigarrow \mathfrak{m2brane}^{Q}$$
$$H: \Omega^2(\mathbb{R}^{11}) \to \Omega^1(\mathbb{R}^{11})$$
$$\tilde{\psi} = \psi - H\Gamma_{\wedge^2}(Q,\psi) \in \Pi S \oplus \Pi\Omega^1$$
$$[v,\tilde{\psi}] = -L_v(H\Gamma_{\wedge^2}(Q,\psi)) = -\langle v, \Gamma_{\wedge^2}(Q,\psi)\rangle - \mathrm{d}\langle v, H\Gamma_{\wedge^2}(Q,\psi)\rangle$$
$$v \otimes \psi \mapsto \langle v, H\Gamma_{\wedge^2}(Q,\psi)\rangle \in L_\lambda$$
$$K \circlearrowright (\mathfrak{g},\delta) \underset{i}{\overset{q}{\rightleftarrows}} (H^{\cdot}(\mathfrak{m2brane}^{Q}), 0)$$
$$H^{\cdot}(\mathfrak{m2brane}^{Q}) \rightsquigarrow \mathcal{L}(\mathbb{C}^5 \times \mathbb{R})$$
$$L^{\vee} \mapsto 0$$
$$\wedge^2 L_1^{\vee} \mapsto 0$$
$$z_i \wedge z_j \in \wedge^2 L_2^{\vee} \mapsto \frac{1}{2}\left(z_i\, \mathrm{d}z_j - z_j\, \mathrm{d}z_i\right) \in \Omega^{1,0}(\mathbb{C}^5)\, \widehat{\otimes}\, \Omega^0(\mathbb{R})$$
$$A_{ij} \in \mathfrak{sl}(5) \mapsto \sum_{ij} A_{ij} z_i \partial_{z_j} \in \mathrm{PV}^{1,0}(\mathbb{C}^5)\, \widehat{\otimes}\, \Omega^0(\mathbb{R})$$
$$\partial_{z_j} \in L \mapsto \partial_{z_i} \in \mathrm{PV}^{1,0}(\mathbb{C}^5)\, \widehat{\otimes}\, \Omega^0(\mathbb{R}^5)$$
$$1 \in \mathbb{C}_b \mapsto 1 \in \Omega^{0,0}(\mathbb{C}^5)\, \hat{\otimes}\, \Omega^0(\mathbb{R}).$$
$$\left[z_i \wedge z_j, z_k \wedge z_l\right] = \epsilon_{ijklm}\partial_{z_m}$$
$$\left[\partial_{z_i}, z_j\, \mathrm{d}z_k - z_k\, \mathrm{d}z_j\right] = \delta_j^i\, \mathrm{d}z_k - \delta_k^i\, \mathrm{d}z_j$$
$$\Phi^{(2)}\left(\partial_{z_i}, z_j \wedge z_k\right) = \frac{1}{2}\left(\delta_j^i z_k - \delta_k^i z_j\right).$$
$$\left[\Phi^{(1)}(\partial_{z_i}), \Phi^{(1)}(z_j \wedge z_k)\right] = \partial\Phi^{(2)}\left(\partial_{z_i}, z_j \wedge z_k\right)$$
$$\Phi^{(1)}\left[\partial_{z_i}, \partial_{z_j}, z_k \wedge z_l\right]_3 = \left[\Phi^{(1)}(\partial_{z_i}), \Phi^{(1)}(\partial_{z_i}), \Phi^{(1)}(z_k \wedge z_l)\right]_3$$
$$+\left[\partial_{z_i}, \Phi^{(2)}\left(\partial_{z_j}, z_k \wedge z_l\right)\right] + \left[\partial_{z_j}, \Phi^{(2)}\left(\partial_{z_i}, z_k \wedge z_l\right)\right]$$
$$H^{\cdot}(\mathfrak{m2brane}^{Q}) \to E\widehat{(5,10)}$$
$$L \oplus \mathrm{Stab}(Q) \oplus \Pi(\wedge^3 L) \oplus \mathbb{C}_b \to E(5,10) \oplus \mathbb{C}_{b'}$$

$$L_1^\vee \mapsto 0$$
$$\wedge^2 L_1^\vee \mapsto 0$$
$$z_i \wedge z_j \in \wedge^2 L_2 \mapsto \mathrm{d}z_i \wedge \mathrm{d}z_j \in \Omega^2_{cl}(\mathbb{C}^5)$$
$$A_{ij} \in \mathfrak{sl}(5) \mapsto \sum_{ij} A_{ij} z_i \partial_{z_j} \in \mathrm{Vect}_0(\mathbb{C}^5)$$
$$\partial_{z_i} \in L \mapsto \partial_{z_i} \in \mathrm{Vect}_0(\mathbb{C}^5)$$
$$b \in \mathbb{C}_b \mapsto b \in \mathbb{C}_{b'}.$$
$$\mu_2(\psi, \psi', v, v') = \langle v \wedge v', \Gamma(\psi, \psi')\rangle$$
$$A \in \Pi\Omega^{0,\cdot}(\mathbb{C}^2) \mathbin{\widehat{\otimes}} \Omega^{\cdot}(\mathbb{R}^7),$$
$$\bar{\partial}A + \mathrm{d}_{\mathbb{R}^7} A + \partial_{z_1} A \wedge \partial_{z_2} A = 0$$
$$\mathbb{C}^2 \times \mathbb{R}^7$$
$$Q + Q_{nm}$$
$$Q_{nm} \in \wedge^2 (L^\vee)$$
$$\mathbb{C}^5 \times \mathbb{R} = \mathbb{C}^2_{z_i} \times \mathbb{C}^3_{w_a} \times \mathbb{R}$$
$$Q_{nm} = \mathrm{d}z_1 \wedge \mathrm{d}z_2$$
$$\prod_{(n_1,n_2)\in\mathbb{Z}^2_{\geqslant 0}} \frac{1}{1 - q^{-n_1+n_2}}$$
$$\boldsymbol{A}_{(n_1,n_2)}: A \mapsto \partial_{w_1}^{n_1} \partial_{w_2}^{n_2} A(0)$$
$$\sum_{(n_1,n_2)\in\mathbb{Z}^2_{\geqslant 0}} q^{-n_1+n_2}$$
$$\prod_{(n_1,n_2)\in\mathbb{Z}^2_{\geqslant 0}} \frac{1}{1 - q^{-n_1+n_2}}$$
$$q_1 q_2 = 1, q_3 q_4 q_5 = 1$$
$$i(q) = \frac{1}{(1-q)(1-q^{-1})}$$
$$\gamma_{nm} = \frac{1}{2}(z_1\, \mathrm{d}z_2 - z_2\, \mathrm{d}z_1) \in \Omega^{1,0}(\mathbb{C}^5) \otimes \Omega^0(\mathbb{R})$$
$$\mathrm{d}z_1 \wedge \mathrm{d}z_2 \in \Omega^2_{cl}(\mathbb{C}^5)$$
$$[f_i \partial_{z_i}, \mathrm{d}z_1 \wedge \mathrm{d}z_2] = \partial f_i \wedge \mathrm{d}z_j - \partial f_j \wedge \mathrm{d}z_i$$
$$[g_a \partial_{w_a}, \mathrm{d}z_1 \wedge \mathrm{d}z_2] = 0$$
$$[h^{ab}\, \mathrm{d}w_a \wedge \mathrm{d}w_b, \mathrm{d}z_1 \wedge \mathrm{d}z_2] = \epsilon_{abc} h^{ab} \partial_{w_c}.$$
$$\partial_{z_1} f + \partial_{z_2} g = 0$$
$$H^{\cdot}(E(5,10), [\mathrm{d}z_1 \wedge \mathrm{d}z_2, -]) \simeq \mathrm{Vect}_0(\mathbb{C}^2)$$
$$0 \to \mathbb{C} \to \mathcal{O}(\mathbb{C}^2) \to \mathrm{Vect}_0(\mathbb{C}^2) \to 0$$
$$H^{\cdot}(\widehat{E(5,10)}, [\mathrm{d}z_1 \wedge \mathrm{d}z_2, -]) \simeq \mathcal{O}(\mathbb{C}^2)$$
$$\varphi(\mu, \mu', \alpha) = \langle \mu \wedge \mu', \alpha \rangle|_{z=0}$$
$$\left(f_i \partial_{z_i}, g_j \partial_{z_j}\right) \mapsto (f_1 g_2 - f_2 g_1)(z_1 = z_2 = 0)$$
$$Z \times M$$
$$\alpha \in \Pi\Omega^{0,\cdot}(Z) \mathbin{\widehat{\otimes}} \Omega^{\cdot}(M)$$
$$\{\alpha^I(z,\bar{z})\mathrm{d}\bar{z}_I, \alpha^J(z,\bar{z})\mathrm{d}\bar{z}_J\}_{pb} = \left(\partial_{z_1}\alpha^I \partial_{z_2}\alpha^J \pm \partial_{z_2}\alpha^I \partial_{z_1}\alpha^J\right)\mathrm{d}\bar{z}_I \wedge \mathrm{d}\bar{z}_J$$
$$\frac{1}{2}\int_{Z\times M} (\alpha \wedge \mathrm{d}\alpha) \wedge \omega_Z^{2,0} + \frac{1}{6}\int_{Z\times M} \alpha \wedge \{\alpha, \alpha\}_{pb} \wedge \omega_Z^{2,0}$$

$$\mu_z \in \mathrm{PV}^{1,\cdot}(\mathbb{C}_z^2) \otimes \mathrm{PV}^{0,\cdot}(\mathbb{C}_w^3) \otimes \Omega^{\cdot}(\mathbb{R})$$
$$\mu_w \in \mathrm{PV}^{0,\cdot}(\mathbb{C}_z^2) \otimes \mathrm{PV}^{1,\cdot}(\mathbb{C}_w^3) \otimes \Omega^{\cdot}(\mathbb{R})$$
$$\frac{1}{2}\int_{\mathbb{C}^2\times\mathbb{C}^3\times\mathbb{R}} \frac{1}{1-\nu}(\partial\gamma \vee \mu^2) \wedge (\mathrm{d}^2 z \wedge \ \mathrm{d}^3 w)$$
$$\frac{1}{6}\int_{\mathbb{C}^2\times\mathbb{C}^3\times\mathbb{R}} \gamma\partial\gamma\partial\gamma$$
$$\int \frac{1}{1-\nu}\left(\frac{1}{2}\partial^w\gamma_w \vee \mu_w^2 + \partial^z\gamma_w \vee \mu_w\mu_z + \partial^w\gamma_z \vee \mu_w\mu_z + \frac{1}{2}\partial^z\gamma_z \vee \mu_z^2\right) \wedge (\mathrm{d}^2 z \wedge \ \mathrm{d}^3 w)$$
$$+\frac{1}{2}\int \frac{1}{1-\nu}(\mathrm{d}^2 z \vee \mu_z^2) \wedge (\mathrm{d}^2 z \wedge \ \mathrm{d}^3 w)$$
$$\frac{1}{6}\int (\gamma_w\partial^z\gamma_w\partial^z\gamma_w + \gamma_w\partial^w\gamma_w\partial^z\gamma_z + \gamma_w\partial^w\gamma_z\partial^w\gamma_z) + \frac{1}{2}\int (\gamma_w\partial^w\gamma_w) \wedge \mathrm{d}^2 z$$
$$\frac{1}{2}\int (\mathrm{d}^2 z \vee \mu_z^2) \wedge (\mathrm{d}^2 z \wedge \ \mathrm{d}^3 w) + \frac{1}{2}\int (\gamma_w \wedge \partial^w\gamma_w) \wedge \mathrm{d}^2 z$$
$$\Omega_Z^{1,\cdot} \mathbin{\widehat{\otimes}} \Omega_W^{0,\cdot} \overset{\omega_Z^{2,0}\otimes\mathbb{1}}{\rightarrow} \mathrm{PV}_Z^{1,\cdot} \mathbin{\widehat{\otimes}} \mathrm{PV}_W^{0,\cdot}$$
$$\Omega_Z^{0,\cdot} \mathbin{\widehat{\otimes}} \Omega_W^{1,\cdot} \overset{\mathbb{1}\otimes\partial^w}{\rightarrow} \Omega_Z^{0,\cdot} \mathbin{\widehat{\otimes}} \Omega_W^{2,\cdot} \overset{\mathbb{1}\otimes\Omega_W}{\rightarrow} \mathrm{PV}_Z^{0,\cdot} \mathbin{\widehat{\otimes}} \mathrm{PV}_W^{1,\cdot}$$
$$\Omega_Z^{0,\cdot} \mathbin{\widehat{\otimes}} \Omega_W^{\cdot\cdot} \mathbin{\hat{\otimes}} \Omega_L^{\cdot} = \bigoplus_{k=0}^{3} \Omega_Z^{0,\cdot} \mathbin{\hat{\otimes}} \Omega_W^{k,\cdot} \mathbin{\hat{\otimes}} \Omega_L^{\cdot}$$
$$\mu_z = (1-\tilde{\alpha}^3)\left(\partial_{z_1} \wedge \partial_{z_2}\right) \vee \partial^z\alpha^0, \mu_w = \left(\partial_{w_1} \wedge \partial_{w_2} \wedge \partial_{w_3}\right) \vee \alpha^2, \nu = \tilde{\alpha}^3$$
$$\beta = \alpha^0, \gamma_w = \alpha^1, \gamma_z = 0$$
$$\int \sum_{k=0}^{3} \alpha^k\left(\bar{\partial} + \mathrm{d}_{\mathbb{R}}\right)\alpha^{3-k}$$
$$\int \alpha^0\partial^w\alpha^2 - \int \alpha^0\partial^z\alpha^0\partial^z\alpha^3$$
$$\int \frac{1}{2}\alpha^1\partial^w\alpha^1$$
$$\frac{1}{2}\int \frac{1}{1-\tilde{\alpha}^3}\partial^w\alpha^1(\tilde{\alpha}^2)^2\ \mathrm{d}^2 z + \int \alpha^2\partial^z\alpha^0\partial^z\alpha^1 + \frac{1}{2}\int (1-\alpha^3)\partial^z\alpha^0\partial^z\alpha^0$$
$$\frac{1}{6}\int \alpha^1\partial^z\alpha^1\partial^z\alpha^1$$
$$S_{pCS}(\alpha) + \int \frac{1}{2}\frac{1}{1-\tilde{\alpha}^3}\partial^w\alpha^1(\tilde{\alpha}^2)^2\ \mathrm{d}^2 z$$
$$\frac{1}{6}\int \frac{1}{1-\tilde{\alpha}^3}\alpha^2(\tilde{\alpha}^2)^2$$
$$\frac{1}{2}\int \frac{1}{1-\tilde{\alpha}^3}\partial^w\alpha^1(\tilde{\alpha}^2)^2 + \frac{1}{6}\int \frac{1}{1-\tilde{\alpha}^3}\partial^w(\alpha^2)\alpha^2(\tilde{\alpha}^2)^2$$
$$\mathrm{PV}^{i,j}(X) = \Omega^{0,j}\left(X, \wedge^i \ \mathrm{T}_X\right)$$
$$\partial_\Omega : \mathrm{PV}^{i,\cdot}(X) \rightarrow \mathrm{PV}^{i-1,\cdot}(X)$$
$$\left(\mathrm{PV}^{\cdot\cdot}(X)[[u]][2], \bar{\partial} + u\partial_\Omega\right)$$
$$(\partial_\Omega \otimes 1)\delta_{\Delta\subset X\times X} \in [\mathrm{PV}^{\cdot\cdot}(X)]^{\hat{\otimes}2}$$
$$I_{BCOV}(\Sigma) = \mathrm{Tr}_X\langle\exp\ \Sigma\rangle_0 = \sum_{n\geqslant 0} \mathrm{Tr}_X\left\langle\Sigma^{\otimes n}\right\rangle_0$$
$$\langle u^{k_1}\mu_1 \otimes \cdots \otimes u^{k_m}\mu_m\rangle_0 := \left(\int_{\overline{\mathcal{M}}_{0,m}} \psi_1^{k_1} \cdots \psi_m^{k_m}\right)\mu_1 \cdots \mu_m = \binom{m-3}{k_1, \cdots, k_m}\mu_1 \cdots \mu_m$$
$$\Sigma \mapsto [u(\exp(\Sigma/u) - 1)]_+$$

$$\left(\bigoplus_{i+j\leqslant d-1} u^i \mathrm{PV}^{j,\cdot}(X)[2], \bar{\partial} + u\partial_\Omega\right)$$

$$\mathrm{PV}^{1,\cdot} \xrightarrow{u\partial_\Omega} u\mathrm{PV}^{0,\cdot}$$

$$\mathrm{PV}^{2,\cdot} \xrightarrow{u\partial_\Omega} u\mathrm{PV}^{1,\cdot} \xrightarrow{u\partial_\Omega} u^2\mathrm{PV}^{0,\cdot}$$

$$\mathrm{PV}^{3,\cdot} \xrightarrow{u\partial_\Omega} u\mathrm{PV}^{2,\cdot} \xrightarrow{u\partial_\Omega} u^2\mathrm{PV}^{1,\cdot} \xrightarrow{u\partial_\Omega} u^3\mathrm{PV}^{0,\cdot}$$

$$\alpha = \sum_n \alpha_n u^n \in \mathcal{E}_{mKS}(\mathbb{C}^4) \otimes \Omega^\cdot(\mathbb{R}^2)$$

$$I_{IIA} = \int_{\mathbb{C}^4\times\mathbb{R}^2} \alpha_0^3 + \cdots$$

$$\eta \in \mathrm{PV}^{0,\cdot}(\mathbb{C}^4) \otimes \Omega^\cdot(\mathbb{R}^2), \mu + u\nu \in \mathrm{PV}^{1,\cdot}(\mathbb{C}^4) \otimes \Omega^\cdot(\mathbb{R}^2) \bigoplus u\mathrm{PV}^{0,\cdot}(\mathbb{C}^4) \otimes \Omega^\cdot(\mathbb{R}^2)$$

$$\Pi \in \mathrm{PV}^{3,\cdot}(\mathbb{C}^4) \otimes \Omega^\cdot(\mathbb{R}^2), \sigma \in \mathrm{PV}^{3,\cdot}(\mathbb{C}^4) \otimes \Omega^\cdot(\mathbb{R}^2)$$

$$I_{IIA} = \frac{1}{2}\mathrm{Tr}_{\mathbb{C}^4\times\mathbb{R}^2} \frac{1}{1-\nu}\mu^2 \wedge \Pi + \mathrm{Tr}_{\mathbb{C}^4\times\mathbb{R}^2} \frac{1}{1-\nu}\eta \wedge \mu \wedge \sigma + \frac{1}{2}\mathrm{Tr}_{\mathbb{C}^4\times\mathbb{R}^2} \frac{1}{1-\nu}\eta \wedge \Pi^2 + \cdots$$

$$\mathrm{PV}^{0,\cdot}(\mathbb{C}^4) \otimes \Omega^\cdot(\mathbb{R}^2)_\eta$$

$$u^{-1}\Omega^{0,\cdot}(\mathbb{C}^4) \otimes \Omega^\cdot(\mathbb{R}^2)_\beta \xrightarrow{u\partial} \Omega^{1,\cdot}(\mathbb{C}^4) \otimes \Omega^\cdot(\mathbb{R}^2)_\gamma$$

$$\Omega^{0,\cdot}(\mathbb{C}^4) \otimes \Omega^\cdot(\mathbb{R}^2)_\theta$$

$$\int_{\mathbb{C}^4\times\mathbb{R}^2}^{\Omega} \eta\theta + \int_{\mathbb{C}^4\times\mathbb{R}^2}^{\Omega} \mu \vee \gamma + \int_{\mathbb{C}^4\times\mathbb{R}^2}^{\Omega} \nu\beta$$

$$\tilde{I}_{IIA} = \frac{1}{2}\int_{\mathbb{C}^4\times\mathbb{R}^2}^{\Omega} \frac{1}{1-\nu}\mu^2 \vee \partial\gamma + \int_{\mathbb{C}^4\times\mathbb{R}^2}^{\Omega} \frac{1}{1-\nu}(\eta\wedge\mu) \vee \partial\theta + \frac{1}{2}\int_{\mathbb{C}^4\times\mathbb{R}^2} \frac{1}{1-\nu}\eta \wedge \partial\gamma \wedge \partial\gamma$$

$$M \times V \to M \times V_{\mathbb{R}}$$

$$\mathbb{C}^4 \times \mathbb{C} \times \mathbb{R}_t \to \mathbb{C}^4 \times \mathbb{R}_x \times \mathbb{R}_t \cong \mathbb{C}^4 \times \mathbb{R}^2$$

$$\nu_{11d} \in \mathrm{PV}^{0,\cdot}(\mathbb{C}^5) \otimes \Omega^\cdot(\mathbb{R})$$

$$\nu(z_i, x, t) = \nu_{11d}(z_i, x, y=0, t)|_{\mathrm{d}\bar{z}_5=\mathrm{d}x}$$

$$\beta(z_i, x, t) = \beta_{11d}(z_i, x, y=0, t)|_{\mathrm{d}\bar{z}_5=\mathrm{d}x}.$$

$$\mu_{11d} = \mu_{11d}^0 + \theta_{11d}\partial_{z_5}$$

$$\mu_{11d}^0 \in \mathrm{PV}^{1,\cdot}(\mathbb{C}^4) \otimes \Omega^{0,\cdot}(\mathbb{C}_{z_5}) \otimes \Omega^\cdot(\mathbb{R}_t)$$

$$\theta_{11d} \in \Omega^{0,\cdot}(\mathbb{C}^4) \otimes \Omega^{0,\cdot}(\mathbb{C}_{z_5}) \otimes \Omega^\cdot(\mathbb{R}_t).$$

$$\mu(z_i, x, t) = \mu_{11d}^0(z_i, x, y=0, t)\big|_{\mathrm{d}\bar{z}_5=\mathrm{d}x}.$$

$$\theta(z_i, x, t) = \theta_{11d}(z_i, x, y=0, t)|_{\mathrm{d}\bar{z}_5=\mathrm{d}x}$$

$$\gamma_{11d} = \gamma_{11d}^0 + \eta_{11d}\,\mathrm{d}z_5$$

$$\gamma_{11d}^0 \in \Omega^{1,\cdot}(\mathbb{C}^4) \otimes \Omega^{0,\cdot}(\mathbb{C}_{z_5}) \otimes \Omega^\cdot(\mathbb{R}_t)$$

$$\eta_{11d} \in \mathrm{PV}^{0,\cdot}(\mathbb{C}^4) \otimes \Omega^{0,\cdot}(\mathbb{C}_{z_5}) \otimes \Omega^\cdot(\mathbb{R}_t)$$

$$\gamma(z_i, x, t) = \gamma_{11d}^0(z_i, x, y=0, t)\big|_{\mathrm{d}\bar{z}_5=\mathrm{d}x}$$

$$\eta(z_i, x, t) = \eta_{11d}(z_i, x, y=0, t)|_{\mathrm{d}\bar{z}_5=\mathrm{d}x}$$

$$\int_{\mathbb{C}^4\times\mathbb{R}^2}^{\Omega_{\mathbb{C}^4}} \frac{1}{1-\nu}\mu^2 \vee \partial\gamma + \int_{\mathbb{C}^4\times\mathbb{R}^2}^{\Omega_{\mathbb{C}^4}} \frac{1}{1-\nu}(\theta\wedge\mu) \vee \partial\eta$$

$$\int_{\mathbb{C}^4\times\mathbb{R}^2} \eta \wedge \partial\gamma \wedge \partial\gamma$$

$$\tilde{\theta} = \frac{1}{1-\nu}\theta, \tilde{\eta} = (1-\nu)\eta, \tilde{\beta} = \beta + \frac{1}{1-\nu}\eta \wedge \theta$$

$$\int_{\mathbb{C}^4\times\mathbb{R}^2}^{\Omega_{\mathbb{C}^4}} \frac{1}{1-\nu}\mu^2 \vee \partial\gamma + \int_{\mathbb{C}^4\times\mathbb{R}^2}^{\Omega_{\mathbb{C}}{}^{4}} \frac{1}{1-\nu}\tilde{\eta} \wedge \partial\gamma \wedge \partial\gamma + \int_{\mathbb{C}^4\times\mathbb{R}^2}^{\Omega_{\mathbb{C}}^4} (\tilde{\theta} \wedge \mu) \vee \partial\left(\frac{1}{1-\nu}\tilde{\eta}\right)$$
$$+ \int_{\mathbb{C}^4\times\mathbb{R}^2}^{\Omega_{\mathbb{C}^4}} \frac{1}{1-\nu}(\tilde{\eta} \wedge \tilde{\theta})\partial_\Omega \mu$$
$$- \int_{\mathbb{C}^4\times\mathbb{R}^2}^{\Omega_{\mathbb{C}^4}} \left(\frac{1}{1-\nu}\tilde{\eta}\right) \partial_\Omega(\tilde{\theta}\mu) + \int_{\mathbb{C}^4\times\mathbb{R}^2}^{\Omega_{\mathbb{C}^4}} \left(\frac{1}{1-\nu}\tilde{\eta}\right) \tilde{\theta}\partial_\Omega \mu$$

$$\begin{array}{cccc} - & + & - & + \\ \hline \mathrm{PV}^{1,\bullet}(\mathbb{C}^5) & \xrightarrow{u\partial_\Omega} & u\mathrm{PV}^{0,\bullet}(\mathbb{C}^5) & \end{array}.$$

$$\mathrm{PV}^{3,\bullet}(\mathbb{C}^5) \xrightarrow{u\partial_\Omega} u\mathrm{PV}^{2,\bullet}(\mathbb{C}^5) \xrightarrow{u\partial_\Omega} u^2\mathrm{PV}^{1,\bullet}(\mathbb{C}^5) \xrightarrow{u\partial_\Omega} u^3\mathrm{PV}^{0,\bullet}(\mathbb{C}^5)$$

$$I_{\text{typeI}} = \mathrm{Tr}_{\mathbb{C}^5} \frac{1}{1-\nu}\mu^2 \vee \sigma + \cdots$$

$$\begin{array}{ccc} - & + & - \\ \hline \mathrm{PV}^{1,\bullet}(\mathbb{C}^5)_\mu \xrightarrow{\partial_\Omega} \mathrm{PV}^{0,\bullet}(\mathbb{C}^5)_\nu & & \\ & \Omega^{0,\bullet}(\mathbb{C}^5)_{\tilde{\beta}} \xrightarrow{\partial} \Omega^{1,\bullet}(\mathbb{C}^5)_{\tilde{\gamma}}. & \end{array}$$

$$\tilde{I}_{\text{typeI}} = \frac{1}{2}\int_{\mathbb{C}^5}^{\Omega} \frac{1}{1-\nu}\mu^2 \vee \partial\tilde{\gamma}$$

$$\begin{array}{cc} - & + \\ \hline \mathrm{PV}^{1,\bullet}(\mathbb{C}^5)_\mu \xrightarrow{\partial_\Omega} \mathrm{PV}^{0,\bullet}(\mathbb{C}^5)_\nu & \\ \Omega^{0,\bullet}(\mathbb{C}^5)_\beta \xrightarrow{\partial} \Omega^{1,\bullet}(\mathbb{C}^5)_\gamma. & \end{array}$$

$$\mathcal{M}_{t=0}, \mathcal{M}_{t=1} \subset \mathcal{E}_\partial$$
$$\mathcal{M}_{t=0} \overset{\trianglerighteq}{\times} \mathcal{E}_\partial \mathcal{M}_{t=1}.$$
$$\mathcal{M}_{t=0}\colon \gamma|_{t=0} = \beta|_{t=0} = 0$$
$$\mathcal{M}_{t=1}\colon \gamma|_{t=1} = \beta|_{t=1} = 0$$
$$\mathbb{C}^5 \times [0,1] \to \mathbb{C}^5$$

$$\begin{array}{cc} - & + \\ \hline \mathrm{PV}^{1,\bullet}(\mathbb{C}^5)_\mu \xrightarrow{\partial_\Omega} \mathrm{PV}^{0,\bullet}(\mathbb{C}^5)_\nu. & \end{array}$$

$$\begin{array}{ccc} - & + & - \\ \hline \mathrm{PV}^{1,\bullet}(\mathbb{C}^5)_\mu \xrightarrow{\partial_\Omega} \mathrm{PV}^{0,\bullet}(\mathbb{C}^5)_\nu & & \\ \Omega^{0,\bullet}(\mathbb{C}^5)_\beta \xrightarrow{\partial} \Omega^{1,\bullet}(\mathbb{C}^5)_\gamma & & \\ \quad\overset{\mathbb{1}}{\searrow} \qquad\qquad \overset{\mathbb{1}}{\searrow} & & \\ & \Omega^{0,\bullet}(\mathbb{C}^5)_{\tilde{\beta}} \xrightarrow{\partial} \Omega^{1,\bullet}(\mathbb{C}^5)_{\tilde{\gamma}}. & \end{array}$$

$$\mathcal{M}_{t=0} \hookrightarrow \tilde{\mathcal{M}}_{t=0} \to \mathcal{E}_\partial$$

$$\begin{array}{ccc} - & + & - \\ \hline \mathrm{PV}^{1,\bullet}(\mathbb{C}^5)_\mu \xrightarrow{\partial_\Omega} \mathrm{PV}^{0,\bullet}(\mathbb{C}^5)_\nu & & \\ & \Omega^{0,\bullet}(\mathbb{C}^5)_{\tilde{\beta}} \xrightarrow{\partial} \Omega^{1,\bullet}(\mathbb{C}^5)_{\tilde{\gamma}} & \end{array}$$

$$\mathbb{C}^5 \times S^1 \to \mathbb{C}^5$$
$$\mu + \epsilon\mu' \in \Pi\mathrm{PV}^{1,\cdot}(\mathbb{C}^5)[\epsilon]$$

$$\mathrm{PV}^{1,\bullet}(\mathbb{C}^5)_\mu \xrightarrow{\partial} \mathrm{PV}^{0,\bullet}(\mathbb{C}^5)_\nu$$

$$\epsilon\Omega^{0,\bullet}(\mathbb{C}^5)_{\beta'} \xrightarrow{\partial_\Omega} \epsilon\Omega^{1,\bullet}(\mathbb{C}^5)_{\gamma'}.$$

$$\epsilon\mathrm{PV}^{1,\bullet}(\mathbb{C}^5)_{\mu'} \xrightarrow{\partial_\Omega} \epsilon\mathrm{PV}^{0,\bullet}(\mathbb{C}^5)_{\nu'}$$

$$\Omega^{0,\bullet}(\mathbb{C}^5)_\beta \xrightarrow{\partial_\Omega} \Omega^{1,\bullet}(\mathbb{C}^5)_\gamma.$$

$$\int_{\mathbb{C}^5}^{\Omega} \left(\beta' \wedge \bar\partial\nu + \beta \wedge \bar\partial\nu' + \gamma' \wedge \bar\partial\mu + \gamma \wedge \bar\partial\mu' + \beta' \wedge \partial_\Omega\mu + \beta \wedge \partial_\Omega\mu'\right)$$

$$+ \int_{\mathbb{C}^5}^{\Omega} \left(\frac{1}{2}\frac{1}{1-\nu}\mu^2 \vee \partial\gamma' + \frac{1}{1-\nu}(\mu \wedge \mu') \vee \partial\gamma' + \frac{1}{2}\frac{\nu'}{(1-\nu)^2}\mu^2 \vee \partial\gamma\right)$$

$$+ \frac{1}{2}\int_{\mathbb{C}^5} \gamma' \wedge \partial\gamma \wedge \partial\gamma$$

$$X \times \mathbb{C}^2 \times \mathbb{R} \to \mathbb{C}^2 \times \mathbb{R}$$

$$\alpha, \eta \in \Pi\Omega^{0,\cdot}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R})$$

$$A_{\mathrm{grav}}, B_{\mathrm{grav}} \in \Pi\Omega^{0,\cdot}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R})$$

$$\int_{\mathbb{C}^2\times\mathbb{R}}^{\Omega} \left(\eta\bar\partial\alpha + B_{\mathrm{grav}}\bar\partial A_{\mathrm{grav}} + B\bar\partial A + \psi\bar\partial\chi\right)$$

$$+ \int_{\mathbb{C}^2\times\mathbb{R}}^{\Omega} \left(\frac{1}{2}\eta\{\alpha,\alpha\} + B_{\mathrm{grav}}\{\alpha, A_{\mathrm{grav}}\} + B\{\alpha, A\} + \psi\{\alpha,\chi\}\right)$$

$$+ \frac{1}{6}\int_{\mathbb{C}^2\times\mathbb{R}} B_{\mathrm{grav}}\,\partial B_{\mathrm{grav}}\,\partial B_{\mathrm{grav}}$$

$$\mathrm{PV}^{0,\cdot}(X \times \mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R}) \simeq H^\cdot(X, \mathcal{O}) \otimes \mathrm{PV}^{0,\cdot}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R})$$

$$= \mathrm{PV}^{0,\cdot}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R}) \oplus \Pi\bar\Omega_X\mathrm{PV}^{0,\cdot}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R})$$

$$\nu_{11d} = \nu + \bar\Omega_X\tilde\nu$$

$$\Pi\mathrm{PV}^{1,\cdot}(X \times \mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R}) \simeq \Pi H^\cdot(X, \mathcal{O}) \otimes \mathrm{PV}^{1,\cdot}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R})$$

$$\oplus\, \Pi H^\cdot(X,\ \mathrm{T}_X) \otimes \mathrm{PV}^{0,\cdot}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R})$$

$$= \Pi\mathrm{PV}^{1,\cdot}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R}) \oplus \bar\Omega_X\mathrm{PV}^{1,\cdot}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R})$$

$$\oplus\, H^1(X,\ \mathrm{T}_X) \otimes \mathrm{PV}^{0,\cdot}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R}) \oplus \Pi H^2(X,\ \mathrm{T}_X) \otimes \mathrm{PV}^{0,\cdot}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R})$$

$$\mu_{11d} = \mu + \bar\Omega_X\tilde\mu$$

$$+ e^i\chi_i + f^aA_a + (\Omega_X^{-1} \vee \omega^2)A_{\mathrm{grav}}$$

$$H^2(X, \Omega_X^2)_\perp \subset H^2(X, \Omega_X^2) \cong H^2(X,\ \mathrm{T}_X)$$

$$\beta_{11d} = \beta + \bar\Omega_X\tilde\beta$$

$$\gamma_{11d} = \gamma + \bar\Omega_X\tilde\gamma + e_i\psi^i + f_aB^a + \omega \wedge B_{\mathrm{grav}}$$

$$\alpha, \chi \in \Omega^{0,\cdot}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R})$$

$$\eta = (\mathrm{d}^2z)^{-1} \vee \partial\tilde\gamma, \psi = (\mathrm{d}^2z)^{-1} \vee \partial\gamma \in \Omega^{0,\cdot}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R})$$

$\alpha, A_{\mathrm{grav}}$	$\in \Pi\Omega^{0,\cdot}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R}), \eta, B_{\mathrm{grav}}$					$\in \Omega^{0,\cdot}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R})$
χ, χ_i	$\in$	$\Omega^0 \cdot (\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R})$,	ψ, ψ^i	$\in$	$\Omega^{0,}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R})$,	$i = 1, \ldots, h^{2,1}$
A_a	$\in$	$\Pi\Omega^{0,\cdot}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R})$,	B^a	$\in$		$\Pi\Omega^{0,}(\mathbb{C}^2) \otimes \Omega^\cdot(\mathbb{R}), a = 1, \ldots, h^{1,1} - 1.$

$$\int_{\mathbb{C}^2\times\mathbb{R}}^{\Omega} \left(\frac{1}{2}\partial\alpha \wedge \partial\alpha \wedge \eta + \partial A_{\mathrm{grav}} \wedge \partial A_{\mathrm{grav}} \wedge B_{\mathrm{grav}}\right)$$

$$+ \int_{\mathbb{C}^2\times\mathbb{R}}^{\Omega} \left(\partial\alpha \wedge \partial\chi \wedge \psi + \partial\alpha \wedge \partial\chi_i \wedge \psi^i + \partial\alpha \wedge \partial A_a \wedge B^a\right)$$

$$\frac{1}{6}\int_{\mathbb{C}^2\times\mathbb{R}} B_{\mathrm{grav}}\,\partial B_{\mathrm{grav}}\,\partial B_{\mathrm{grav}}$$

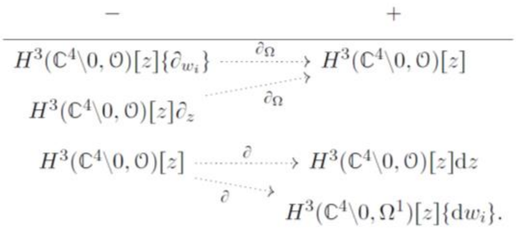
$$H^\cdot(X, \Omega^\cdot) = \oplus_{i,j} H^i\left(X, \Omega_X^j\right)$$

$$H^{\cdot}(X,\Omega^{\cdot})\otimes\mathcal{O}(\mathbb{C}^2)$$
$$[[\omega]\otimes f,[\omega]\otimes g]=[\omega^2]\otimes\{f,g\}\in H^{2,2}(X,\Omega^2)\otimes\mathcal{O}(\mathbb{C}^2)$$
$$\mathbb{C}_w^4\times\mathbb{C}_z\times\mathbb{R}$$
$$I_{M2}(\gamma)=N\int_{\mathbb{C}_z}\gamma+\cdots$$
$$I_{D2}(\gamma)=N\int_{\mathbb{R}\times\mathbb{C}_z}\gamma+\cdots$$
$$\bar\partial\mu+\frac{1}{2}[\mu,\mu]+\partial\gamma\partial\gamma=N\Omega^{-1}\delta_{w=0}$$
$$\partial_\Omega\mu=0$$
$$F_{M2}=\frac{6}{(2\pi i)^4}\frac{\sum_{a=1}^4\ \bar w_a\ \mathrm{d}\bar w_1\cdots\widehat{\mathrm{d}}_a\cdots\mathrm{d}\bar w_4}{\|w\|^8}\partial_z$$
$$\bar\partial(NF_{M2})+\frac{1}{2}[NF_{M2},NF_{M2}]=N\Omega^{-1}\delta_{w=0}$$
$$\partial_\Omega(NF_{M2})=0$$
$$[F_{M2},F_{M2}]=0$$
$$\frac{\partial}{\partial z},z\frac{\partial}{\partial z}-\frac{1}{4}\sum_{a=1}^4 w_a\frac{\partial}{\partial w_a},z\left(z\frac{\partial}{\partial z}-\frac{1}{2}\sum_{a=1}^4 w_a\frac{\partial}{\partial w_a}\right)\in\mathrm{PV}^{1,0}(\mathbb{C}^5)\otimes\Omega^0(\mathbb{R})$$
$$\sum_{a,b=1}^4 B_{ab}w_a\frac{\partial}{\partial w_b}\in\mathrm{PV}^{1,0}(\mathbb{C}^5)\otimes\Omega^0(\mathbb{R}),(B_{ab})\in\mathfrak{sl}(4)$$
$$(\wedge^2\mathbb{C}^4)_{+1}\oplus(\wedge^2\mathbb{C}^4)_{-1}$$
$$\frac{1}{2}(w_a\,\mathrm{d}w_b-w_b\,\mathrm{d}w_a)\in\Omega^{1,0}(\mathbb{C}^5)\otimes\Omega^0(\mathbb{R}),a,b=1,2,3,4$$
$$\frac{1}{2}z(w_a\,\mathrm{d}w_b-w_b\,\mathrm{d}w_a)\in\Omega^{1,0}(\mathbb{C}^5)\otimes\Omega^0(\mathbb{R}),a,b=1,2,3,4$$
$$i_{M2}:\mathfrak{osp}(6\mid 1)\hookrightarrow E(5,10)$$
$$\mathrm{d}w_a\wedge\ \mathrm{d}w_b,a,b=1,2,3,4$$
$$z\,\mathrm{d}w_a\wedge\ \mathrm{d}w_b+\frac{1}{2}\ \mathrm{d}z\wedge(w_a\,\mathrm{d}w_b-w_b\,\mathrm{d}w_a),a,b=1,2,3,4$$
$$(\mathbb{C}^5\times\mathbb{R})\setminus\{w=0\}\cong(\mathbb{C}_w^4\setminus 0)\times\mathbb{C}_z\times\mathbb{R}$$
$$[F,-]:\mathrm{PV}^{i,\cdot}(\mathbb{C}_w^4\setminus 0)\otimes\mathrm{PV}^{j,\cdot}(\mathbb{C}_z)\otimes\Omega^{\cdot}(\mathbb{R})\to\mathrm{PV}^{i,\cdot+3}(\mathbb{C}_w^4\setminus 0)\otimes\mathrm{PV}^{j,\cdot}(\mathbb{C}_z)\otimes\Omega^{\cdot}(\mathbb{R})$$
$$[F,-]:\Omega^{i,\cdot}(\mathbb{C}_w^4\setminus 0)\otimes\Omega^{j,\cdot}(\mathbb{C}_z)\otimes\Omega^{\cdot}(\mathbb{R})\to\Omega^{i,\cdot+3}(\mathbb{C}_w^4\setminus 0)\otimes\Omega^{j,\cdot}(\mathbb{C}_z)\otimes\Omega^{\cdot}(\mathbb{R})$$
$$\delta^{(1)}=\bar\partial+\mathrm{d}_{\mathbb{R}}+\partial_\Omega|_{\mu\to\nu}+\partial|_{\beta\to\gamma}$$

+	−
$H^\bullet(\mathbb{C}^4\backslash 0,\mathrm{T})\otimes H^\bullet(\mathbb{C},\mathcal{O})$	$H^\bullet(\mathbb{C}^4\backslash 0,\mathcal{O})\otimes H^\bullet(\mathbb{C},\mathcal{O})$
$H^\bullet(\mathbb{C}^4\backslash 0,\mathcal{O})\otimes H^\bullet(\mathbb{C},\mathrm{T})$	
$H^\bullet(\mathbb{C}^4\backslash 0,\mathcal{O})\otimes H^\bullet(\mathbb{C},\mathcal{O})$	$H^\bullet(\mathbb{C}^4\backslash 0,\mathcal{O})\otimes H^\bullet(\mathbb{C},\Omega^1)$
	$H^\bullet(\mathbb{C}^4\backslash 0,\Omega^1)\otimes H^\bullet(\mathbb{C},\mathcal{O})$

$$\mathbb{C}[z]\hookrightarrow H^{\cdot}(\mathbb{C},\mathcal{F})$$
$$\mathbb{C}[w_1,\dots,w_4]\hookrightarrow H^0(\mathbb{C}^4\setminus 0,\mathcal{O})$$
$$\mathbb{C}[w_1,\dots,w_4]\{\partial_{w_i}\}\hookrightarrow H^0(\mathbb{C}^4\setminus 0,\mathrm{T})$$
$$\mathbb{C}[w_1,\dots,w_4]\{\mathrm{d}w_i\}\hookrightarrow H^0(\mathbb{C}^4\setminus 0,\Omega^1)$$

$$(w_1 \cdots w_4)^{-1}\mathbb{C}[w_1^{-1}, \dots, w_4^{-1}] \hookrightarrow H^3(\mathbb{C}^4 \setminus 0, \mathcal{O})$$
$$(w_1 \cdots w_4)^{-1}\mathbb{C}[w_1^{-1}, \dots, w_4^{-1}]\{\partial_{w_i}\} \hookrightarrow H^3(\mathbb{C}^4 \setminus 0, \mathrm{T})$$
$$(w_1 \cdots w_4)^{-1}\mathbb{C}[w_1^{-1}, \dots, w_4^{-1}]\{\mathrm{d}w_i\} \hookrightarrow H^3(\mathbb{C}^4 \setminus 0, \Omega^1)$$
$$H^{\cdot}\left(\mathcal{L}(\mathbb{C}^5 \times \mathbb{R} \setminus \{w = 0\}), \bar{\partial}\right)$$



$$[F] = (w_1 \cdots w_4)^{-1}\partial_z \in H^3(\mathbb{C}^4 \setminus 0, \mathcal{O})[z]\partial_z$$
$$[[F], z(w_a\, \mathrm{d}w_b - w_b\, \mathrm{d}w_a)] = (w_1 \cdots w_4)^{-1}(w_a\, \mathrm{d}w_b - w_b\, \mathrm{d}w_a) = 0$$
$$\{w_1 = w_2 = t = 0\} \subset \mathbb{C}_z^3 \times \mathbb{C}_w^2 \times \mathbb{R}$$
$$I_{M5} = N \int_{\mathbb{C}_z^3} \partial_\Omega^{-1}\mu \vee \Omega + \cdots$$
$$N \int_{\mathbb{C}^2 \times \mathbb{R}} \partial_\Omega^{-1}\mu \vee \Omega_{\mathbb{C}^4} + \cdots$$
$$\bar{\partial}\partial\gamma + \partial_\Omega\left(\frac{1}{1-\nu}\mu\right) \wedge \partial\gamma = N\delta_{w_1=w_2=t=0}$$
$$\left(\bar{\partial} + \mathrm{d}_{\mathbb{R}}\right)\mu + \partial\gamma\partial\gamma = 0$$
$$F_{M5} = \frac{1}{(2\pi i)^3}\frac{\bar{w}_1\, \mathrm{d}\bar{w}_2 \wedge\, \mathrm{d}t - \bar{w}_2\, \mathrm{d}\bar{w}_1 \wedge\, \mathrm{d}t + t\, \mathrm{d}\bar{w}_1 \wedge\, \mathrm{d}\bar{w}_2}{(\|w\|^2 + t^2)^{5/2}} \wedge\, \mathrm{d}w_1 \wedge\, \mathrm{d}w_2$$
$$\bar{\partial}(NF_{M5}) + \mathrm{d}_{\mathbb{R}}(NF_{M5}) = N\delta_{w_1=w_2=t=0}$$
$$(NF_{M5}) \wedge (NF_{M5}) = 0.$$
$$\bar{\partial}F + \mathrm{d}_{\mathbb{R}}F = N\delta_{w_1=w_2=t=0}$$
$$(\mathbb{C}^2 \times \mathbb{R}) \setminus 0 \simeq S^4 \times \mathbb{R}$$
$$\oint_{S^4} NF = N$$
$$\mathbb{C}^5 \times \mathbb{R} = \mathbb{C}_z^3 \times \mathbb{C}_w^2 \times \mathbb{R}_t$$
$$\mathfrak{sl}(4) \oplus \mathfrak{sl}(2)$$
$$\frac{\partial}{\partial z_i} \in \mathrm{PV}^{1,0}(\mathbb{C}^5) \otimes \Omega^0(\mathbb{R}), i = 1,2,3$$
$$A_{ij} z_i \frac{\partial}{\partial z_j} \in \mathrm{PV}^{1,0}(\mathbb{C}^5) \otimes \Omega^0(\mathbb{R}), \left(A_{ij}\right) \in \mathfrak{sl}(3)$$
$$\sum_{i=1}^{3} z_i \frac{\partial}{\partial z_i} - \frac{3}{2}\sum_{a=1}^{2} w_a \frac{\partial}{\partial w_a} \in \mathrm{PV}^{1,0}(\mathbb{C}^5) \otimes \Omega^0(\mathbb{R})$$
$$z_j\left(\sum_{i=1}^{3} z_i \frac{\partial}{\partial z_i} - 2\sum_{a=1}^{2} w_a \frac{\partial}{\partial w_a}\right) \in \mathrm{PV}^{1,0}(\mathbb{C}^5) \otimes \Omega^0(\mathbb{R})$$
$$w_1 \frac{\partial}{\partial w_2}, w_2 \frac{\partial}{\partial w_1}, \frac{1}{2}\left(w_1 \frac{\partial}{\partial w_1} - w_2 \frac{\partial}{\partial w_2}\right) \in \mathrm{PV}^{1,0}(\mathbb{C}^5) \otimes \Omega^0(\mathbb{R})$$
$$L \otimes R \oplus \wedge^2 L \otimes R \cong \mathbb{C}^3 \otimes \mathbb{C}^2 \oplus \wedge^2 \mathbb{C}^3 \otimes \mathbb{C}$$
$$z_i\, \mathrm{d}w_a \in \Omega^{1,0}(\mathbb{C}^5) \otimes \Omega^0(\mathbb{R}), a = 1,2, i = 1,2,3$$

$$\frac{1}{2} w_a \left(z_i \, \mathrm{d}z_j - z_j \, \mathrm{d}z_i\right) \in \Omega^{1,0}(\mathbb{C}^5) \otimes \Omega^0(\mathbb{R}), a = 1,2, k = 1,2,3$$

$$i_{M5}: \mathfrak{osp}(6 \mid 1) \hookrightarrow E.$$

$$\mathrm{d}z_i \wedge \, \mathrm{d}w_a, i = 1,2,3, a = 1,2.$$

$$w_a \, \mathrm{d}z_i \wedge \, \mathrm{d}z_j + \frac{1}{2} \, \mathrm{d}w_a \wedge \left(z_i \, \mathrm{d}z_j - z_j \, \mathrm{d}z_i\right), i, j = 1,2,3, a = 1,2$$

$$\Omega^{0,\cdot}(\mathbb{C}^5) \otimes \Omega^{\cdot}(\mathbb{R})$$

$$\Omega^{0,\cdot}(\mathbb{C}^5) \otimes \Omega^{\cdot}(\mathbb{R}) \cong \Omega^{\cdot}(\mathbb{C}^5 \times \mathbb{R})/(\mathrm{d}z_1, \dots, \mathrm{d}z_5)$$

$$\mathbb{C}^5 \times \mathbb{R} \setminus \mathbb{C}^3 \cong \mathbb{C}_z^3 \times (\mathbb{C}_w^2 \times \mathbb{R} \setminus 0)$$

$$\Omega^{\cdot}(\mathbb{C}^5 \times \mathbb{R} \setminus \mathbb{C}^3)/(\mathrm{d}z_1, \mathrm{d}z_2, \mathrm{d}z_3, \mathrm{d}w_1, \mathrm{d}w_2)$$

$$\Omega^{0,\cdot}(\mathbb{C}_z^3) \otimes \left(\Omega^{\cdot}(\mathbb{C}_w^2 \times \mathbb{R} \setminus 0)/(\mathrm{d}w_1, \mathrm{d}w_2)\right)$$

$$\mathbb{C}[w_1, w_2] \hookrightarrow H^0\left(\Omega^{\cdot}(\mathbb{C}_w^2 \times \mathbb{R} \setminus 0)/(\mathrm{d}w_1, \mathrm{d}w_2)\right)$$

$$w_1^{-1} w_2^{-1} \mathbb{C}[w_1, w_2] \hookrightarrow H^2\left(\Omega^{\cdot}(\mathbb{C}_w^2 \times \mathbb{R} \setminus 0)/(\mathrm{d}w_1, \mathrm{d}w_2)\right)$$

$$\frac{\bar{w}_1 \, \mathrm{d}\bar{w}_2 \wedge \, \mathrm{d}t - \bar{w}_2 \, \mathrm{d}\bar{w}_1 \wedge \, \mathrm{d}t + t \, \mathrm{d}\bar{w}_1 \wedge \, \mathrm{d}\bar{w}_2}{(\|w\|^2 + t^2)^{5/2}}$$

$$\delta^{(1)} = \bar{\partial} + \mathrm{d}_{\mathbb{R}} + \partial_\Omega|_{\mu \to \nu} + \partial|_{\beta \to \gamma}.$$

$$\mathbb{C}^5 \times \mathbb{R} \setminus \{w_1 = w_2 = t = 0\} \cong \mathbb{C}_z^3 \times (\mathbb{C}_w^2 \times \mathbb{R} \setminus 0)$$

$$\begin{array}{ccc}
+ & & - \\
\hline
w_1^{-1} w_2^{-1} \mathbb{C}[w_1^{-1}, w_2^{-1}][z_1, z_2, z_3]\{\partial_{w_i}\} & \xrightarrow{\partial_\Omega} & w_1^{-1} w_2^{-1} \mathbb{C}[w_1^{-1}, w_2^{-1}][z_1, z_2, z_3] \\
w_1^{-1} w_2^{-1} \mathbb{C}[w_1^{-1}, w_2^{-1}][z_1, z_2, z_3]\{\partial_{z_i}\} & \nearrow_{\partial_\Omega} & \\
w_1^{-1} w_2^{-1} \mathbb{C}[w_1^{-1}, w_2^{-1}][z_1, z_2, z_3] & \xrightarrow{\partial} & w_1^{-1} w_2^{-1} \mathbb{C}[w_1^{-1}, w_2^{-1}][z_1, z_2, z_3]\{\mathrm{d}z_i\} \\
 & \searrow^{\partial} & w_1^{-1} w_2^{-1} \mathbb{C}[w_1^{-1}, w_2^{-1}][z_1, z_2, z_3]\{\mathrm{d}w_i\}.
\end{array}$$

$$\left[[F], f^i(z, w)\mathrm{d}z_i\right] = \epsilon_{ijk} w_1^{-1} w_2^{-1} \partial_{z_j} f^i(z, w) \partial_{z_k}$$

$$\mathbb{C}[w_1^{-1}, w_2^{-1}][z_1, z_2, z_3]\left\{\partial_{z_i}\right\} \subset H^0(\mathbb{C}^3, \mathrm{T}) \otimes H^2\left(\Omega^{\cdot}(\mathbb{C}^2 \times \mathbb{R} \setminus 0)/(\mathrm{d}w_1, \mathrm{d}w_2)\right)$$

$$\left[[F], w_a\left(z_i \, \mathrm{d}z_j - z_j \, \mathrm{d}z_i\right)\right] = 2\epsilon_{ijk}(w_1^{-1} w_2^{-1}) \cdot w_a \partial_{z_k} = 0$$

$$\partial_t A_i = \Delta A_i + \left[A_j, 2\partial_j A_i - \partial_i A_j + \left[A_j, A_i\right]\right] + (C_{\mathrm{A}}^\varepsilon A)_i + \xi_i^\varepsilon$$

$$\lim_{\varepsilon \downarrow 0} |C_{\mathrm{A}}^\varepsilon - \bar{C}_{\mathrm{A}}^\varepsilon| = 0$$

$$|\mathbf{E} W_\ell[\mathcal{F}_s(A^a)] - \mathbf{E} W_\ell[\mathcal{F}_s(A^b)]| \gtrsim t^{\frac{10}{9}}$$

$$\chi^\varepsilon(t, x) = \varepsilon^{-5} \chi(\varepsilon^{-2} t, \varepsilon^{-1} x)$$

$$E = \mathfrak{g}^3 \oplus \mathbf{V}$$

$$\mathfrak{g} \ni v \mapsto \mathrm{Ad}_g v \in \mathfrak{g} \ \text{ and } \ \mathfrak{g}^3 \ni (v_1, v_2, v_3) \mapsto \left(\mathrm{Ad}_g v_1, \mathrm{Ad}_g v_2, \mathrm{Ad}_g v_3\right) \in \mathfrak{g}^3$$

$$C_{\mathrm{YM}}^\varepsilon \in L_G(\mathfrak{g}), C_{\mathrm{Higgs}}^\varepsilon \in L_G(\mathbf{V})$$

$$C_{\mathrm{A}}^\varepsilon = C_{\mathrm{YM}}^\varepsilon + \dot{C}_{\mathrm{A}} \in L(\mathfrak{g}^3) \ \text{ and } \ C_\Phi^\varepsilon = C_{\mathrm{Higgs}}^\varepsilon + \dot{C}_\Phi \in L(\mathbf{V}).$$

$$\begin{aligned}
\partial_t A_i = \Delta A_i &+ \left[A_j, 2\partial_j A_i - \partial_i A_j + \left[A_j, A_i\right]\right] \\
&- \mathbf{B}\left((\partial_i \Phi + A_i \Phi) \otimes \Phi\right) + (C_{\mathrm{A}}^\varepsilon A)_i + \xi_i^\varepsilon
\end{aligned}$$

$$\partial_t \Phi = \Delta \Phi + 2A_j \partial_j \Phi + A_j^2 \Phi - |\Phi|^2 \Phi + C_\Phi^\varepsilon \Phi + \xi_{\mathrm{H}}^\varepsilon$$

$$(A(0), \Phi(0)) = (a, \varphi) \in \mathcal{C}^\infty,$$

$$\langle \mathbf{B}(u \otimes v), h \rangle_{\mathfrak{g}} = \langle u, hv \rangle_{\mathbf{V}}.$$

$$\partial_t X = \Delta X + X\partial X + X^3 + C^\varepsilon X + \chi^\varepsilon * \xi$$

$$X = (A, \Phi): [0, T] \times \mathbf{T}^3 \to E$$

$$C = \{C^\varepsilon\}_{\varepsilon \in (0,1)} = \{C_{\mathrm{A}}^\varepsilon, C_\Phi^\varepsilon\}_{\varepsilon \in (0,1)}$$

$$\mathfrak{G}^\varrho \stackrel{\mathrm{def}}{=} \mathcal{C}^\varrho(\mathbf{T}^3, G)$$

$$g \cdot A \stackrel{\text{def}}{=} \mathrm{Ad}_g(A) - (\mathrm{d}g)g^{-1}, \text{ and } g \cdot \Phi \stackrel{\text{def}}{=} g\Phi$$
$$g \cdot \mathrm{SYMH}(C,(a,\varphi)) \stackrel{\text{law}}{=} \mathrm{SYMH}(C, g(0)\cdot(a,\varphi))$$
$$g^{-1}(\partial_t g) = \partial_j\left(g^{-1}\partial_j g\right) + \left[A_j, g^{-1}\partial_j g\right]$$
$$\check{C} \in L_G(\mathfrak{g})$$
$$[\mathrm{SYMH}(C,(a,\varphi))] \stackrel{\text{law}}{=} [\mathrm{SYMH}(C, g(0)\cdot(a,\varphi))]$$
$$\partial_s A_i = \Delta A_i + \left[A_j, 2\partial_j A_i - \partial_i A_j + [A_j, A_i]\right]$$
$$A(0) = a$$
$$\mathrm{d}y_t = y_t \,\mathrm{d}\ell_A, y_0 = 1$$
$$\ell_A(t) = \int_0^t \langle A(\ell_s), \dot{\ell}_s\rangle \mathrm{d}s$$
$$W_\ell(A) = \mathrm{Trhol}(A,\ell)$$
$$\mathring{C}_{\mathrm{A}} = \check{C} + c \in L_G(\mathfrak{g}^3), C^\varepsilon = \left(C^\varepsilon_{\mathrm{YM}} + \dot{C}_{\mathrm{A}}, C^\varepsilon_\Phi\right) \in L_G(\mathfrak{g}^3) \oplus L_G(\mathbf{V})$$
$$\mathrm{SYMH}(C,x) = \left(A^{(1)}, \Phi^{(1)}\right), \mathrm{SYMH}(C, g\cdot x) = \left(A^{(2)}, \Phi^{(2)}\right)$$
$$\left|\mathbf{E}W_\ell\left[\mathcal{F}_s\left(A_t^{(1)}\right)\right] - \mathbf{E}W_\ell\left[\mathcal{F}_s\left(A_t^{(2)}\right)\right]\right| \geq \sigma t^{1+r}$$
$$\left|\mathbf{E}W_\ell[\mathcal{F}_s(A_t)] - \mathbf{E}W_\ell[\mathcal{F}_s(\tilde{A}_t)]\right| \geq \sigma t^{1+r}$$
$$\partial_t \tilde{X} = \Delta\tilde{X} + \tilde{X}\partial\tilde{X} + \tilde{X}^3 + \chi^\varepsilon * \xi + C^\varepsilon\tilde{X} + (c\,\mathrm{d}\tilde{g}\tilde{g}^{-1}, 0), \tilde{X}(0) = \tilde{x}$$
$$\partial_t\tilde{g} = \Delta\tilde{g} - (\partial_j\tilde{g})\tilde{g}^{-1}(\partial_j\tilde{g}) + [\tilde{A}_j, (\partial_j\tilde{g})\tilde{g}^{-1}]\tilde{g}$$
$$(\partial_t g)g^{-1} = \partial_j\left((\partial_j g)g^{-1}\right) + [B_j, (\partial_j g)g^{-1}]$$
$$\partial_t Y = \Delta Y + Y\partial Y + Y^3 + C^\varepsilon Y + (C^\varepsilon \mathrm{d}g g^{-1}, 0) + \mathrm{Ad}_g(\chi^\varepsilon * \xi)$$
$$\mathbf{E}W_\ell\left[\mathcal{F}_s\left(A_t^{(1)}\right)\right] = \mathbf{E}W_\ell[\mathcal{F}_s(\tilde{A}_t)] + O(t^M)$$
$$\left|\mathbf{E}W_\ell[\mathcal{F}_s(\tilde{A}_t)] - \mathbf{E}W_\ell\left[\mathcal{F}_s\left(A_t^{(2)}\right)\right]\right| \geq \sigma t^{1+r}$$
$$\mathcal{P}_t \star f = \int_0^t P_{t-s} f_s \,\mathrm{d}s$$
$$|f|_{\mathcal{C}^\beta} = \sup_{s\in(0,1)} s^{-\beta/2}|P_s f|_\infty$$
$$|f|_{\mathcal{C}^\beta(\mathbf{R}^3)} \stackrel{\text{def}}{=} \max_{|k|<\lfloor\beta\rfloor} |\partial^k f|_\infty + \max_{|k|=\lfloor\beta\rfloor} |\partial^k f|_{\mathcal{C}^{\beta-\lfloor\beta\rfloor}} < \infty,$$
$$|f|_{\mathcal{C}^\eta} = \sup_{x\neq y} |x-y|^{-\eta}|f(x) - f(y)|$$
$$O = [-1,2] \times \mathbf{T}^3$$
$$|\xi|_{\mathcal{C}^\beta(O)} = \sup_{z\in O}\sup_{\varphi\in\mathcal{B}^r}\sup_{\lambda\in(0,1]} \lambda^{-\beta}\left|\langle\xi, \varphi_z^\lambda\rangle\right|$$
$$\varphi^\lambda_{(s,y)}(t,x) = \lambda^{-5}\varphi((t-s)\lambda^{-2}, (x-y)\lambda^{-1}).$$
$$\partial_t X = \Delta X + X\partial X + X^3 + \chi^\varepsilon * \xi + C^\varepsilon X + (ch, 0),$$
$$\partial_t h_i = \Delta h_i - [h_j, \partial_j h_i] + \left[[A_j, h_j], h_i\right] + \partial_i[A_j, h_j],$$
$$\omega \in (-1/2, 0), \kappa \stackrel{\text{def}}{=} \frac{1}{100}(\omega + 1/2) \wedge \frac{1}{100}(-\omega) \in \left(0, \frac{1}{200}\right)$$
$$|f_1 f_2|^{\gamma_n}_{\mathcal{D}^{\gamma,\eta}_\alpha} \lesssim |f_1|_{\mathcal{D}^{\gamma_1,\eta_1}_1} |f_2|_{\mathcal{D}^{\gamma_2,\eta_2}_2}$$
$$|\partial f|_{\mathcal{D}^{\gamma-1,\eta-1}_{\alpha-1}} \lesssim |f|_{\mathcal{D}^{\gamma,\eta}_\alpha}$$
$$\mathcal{X} = PX(0) + \mathbf{\Psi} + \mathcal{P}\mathbf{1}_+\{\mathcal{X}\partial\mathcal{X} + \mathcal{X}^3 + \dot{C}\mathcal{X} + c\mathcal{H}\}$$
$$\stackrel{\text{def}}{=} PX(0) + \mathbf{\Psi} + \mathcal{P}\mathbf{1}_+\{Q^{\mathrm{YMH}}(\mathcal{X}) + c\mathcal{H}\}$$
$$\mathcal{H} = Ph(0) + \mathcal{P}\mathbf{1}_+(\mathcal{H}\partial\mathcal{H} + \mathcal{X}\mathcal{H}^2) + \mathcal{P}'\mathbf{1}_+(\mathcal{X}\mathcal{H})$$

$$\boldsymbol{\Psi} = \mathcal{P}^{\mathbf{1}_+\xi}\mathbf{1}_+\Xi \in \mathcal{D}^{\frac{3}{2}+2\kappa,-\frac{1}{2}-\kappa}_{-\frac{1}{2}-\kappa}$$

$$\mathcal{X} = \mathcal{Y} + \boldsymbol{\Psi},$$

$$\mathcal{Y} = PX(0) + \mathcal{P}\mathbf{1}_+\{\mathcal{X}^3 + c\mathcal{H} + \check{C}\mathcal{X}\} + \mathcal{P}^{\Psi\partial\Psi}(\Psi\partial\Psi)$$

$$+\mathcal{P}\mathbf{1}_+(\mathcal{Y}\partial\Psi + \boldsymbol{\Psi}\partial\mathcal{Y} + \mathcal{Y}\partial\mathcal{Y})$$

$$= PX(0) + \mathcal{P}\mathbf{1}_+\{\tilde{Q}^{\mathrm{YMH}}(\mathcal{Y}) + c\mathcal{H}\} + \mathcal{P}^{\Psi\partial\Psi}(\Psi\partial\Psi),$$

$$\tilde{Q}^{\mathrm{YMH}}(\mathcal{Y}) = \mathcal{X}^3 + \dot{C}\mathcal{X} + \mathcal{Y}\partial\Psi + \boldsymbol{\Psi}\partial\mathcal{Y} + \mathcal{Y}\partial\mathcal{Y}$$

$$\tilde{Q}^{\mathrm{YMH}}: \mathcal{D}^{\frac{3}{2}+2\kappa,\omega}_{-\kappa} \to \mathcal{D}^{\kappa,\omega-\frac{3}{2}-\kappa}_{-\frac{3}{2}-2\kappa}$$

$$\mathcal{D}^{\frac{3}{2}+2\kappa,\omega}_{-\kappa} \times \mathcal{D}^{\frac{1}{2}+2\kappa,-\frac{3}{2}-\kappa}_{-\frac{3}{2}-\kappa} \to \mathcal{D}^{\kappa,\omega-\frac{3}{2}-\kappa}_{-\frac{3}{2}-2\kappa}.$$

$$(\mathcal{Y},\mathcal{H}) \in \mathcal{D}^{\frac{3}{2}+2\kappa,\omega}_{-\kappa} \times \mathcal{D}^{1+2\kappa,0}_{0}$$

$$|\mathcal{P}\mathbf{1}_+f|_{\mathcal{D}^{\gamma+2,\bar{\eta}}} \lesssim t^{\theta/2}|f|_{\mathcal{D}^{\gamma,\eta}}, |\mathcal{P}'\mathbf{1}_+f|_{\mathcal{D}^{\gamma+1,\bar{\eta}-1}} \lesssim t^{\theta/2}|f|_{\mathcal{D}^{\gamma,\eta}}$$

$$|\mathcal{P}^w\mathbf{1}_+f|_{\mathcal{D}^{\gamma+2,\bar{\eta}}} \lesssim t^{\theta/2}\left(|f|_{\mathcal{D}^{\gamma,\eta}} + |w|_{\mathcal{C}^{\eta\wedge\alpha}(O)}\right)$$

$$\tau^{-1/q} \lesssim 2 + \|Z\|_{\frac{3}{2}+2\kappa;O} + |X(0)|_{\mathcal{C}^3} + |h(0)|_{\mathcal{C}^3}$$

$$+|\mathcal{P}\star\mathbf{1}_+\xi|_{\mathcal{C}([-1,3],\mathcal{C}^{-1/2-\kappa})} + |\mathcal{P}\star(\Psi\partial\Psi)|_{\mathcal{C}([-1,3],\mathcal{C}^{-2\kappa})}$$

$$|\mathcal{P}'\mathbf{1}_+(\mathcal{X}\mathcal{H})|_{\mathcal{D}^{1+2\kappa,-\kappa}_0} \lesssim t^{1/4}|\mathcal{X}\mathcal{H}|_{\mathcal{D}^{\frac{1}{2}+\kappa,-\frac{1}{2}-\kappa}_{-\frac{1}{2}-\kappa}}$$

$$\lesssim t^{1/4}|\mathcal{X}|_{\mathcal{D}^{\frac{3}{2}+2\kappa,-\frac{1}{2}-\kappa}_{-\frac{1}{2}-\kappa}}|\mathcal{H}|_{\mathcal{D}^{1+2\kappa,0}_0} \lesssim t^{1/4}$$

$$|\mathcal{P}\mathbf{1}_+(\mathcal{H}\partial\mathcal{H})|_{\mathcal{D}^{1+2\kappa,0}_0} \lesssim t, |\mathcal{P}\mathbf{1}_+(\mathcal{X}\mathcal{H}^2)|_{\mathcal{D}^{\frac{3}{2}+\kappa,-\kappa}_0} \lesssim t^{3/4}$$

$$\left|\mathcal{P}\mathbf{1}_+\left(\tilde{Q}^{\mathrm{YMH}}(\mathcal{Y})\right)\right|_{\mathcal{D}^{\frac{3}{2}+2\kappa,\omega}_0} \lesssim t^{1/4-\kappa/2}\left|\tilde{Q}^{\mathrm{YMH}}(\mathcal{Y})\right|_{\mathcal{D}^{\kappa,\omega-\frac{3}{2}-2\kappa}_{-\frac{3}{2}-\kappa}} \lesssim t^{1/4-\kappa/2}$$

$$\left|\mathcal{P}^{\Psi\partial\Psi}(\boldsymbol{\Psi}\partial\boldsymbol{\Psi})\right|_{\mathcal{D}^{\frac{3}{2}+2\kappa,\omega}_{-\kappa}} \lesssim t^{-(\kappa+\omega)/2}$$

$$|\mathcal{P}\mathcal{H}|^{\frac{3}{2}}_{\mathcal{D}^{3}_{-\kappa}+2\kappa,\omega} \lesssim |\mathcal{P}Ph(0)|_{\mathcal{D}^{\frac{3}{2}+2\kappa,\omega}_{-\kappa}} + \left|\mathcal{P}O_{\mathcal{D}^{1+2\kappa,-\kappa}_0}\left(t^{1/4}\right)\right|_{\mathcal{D}^{\frac{3}{2}+2\kappa,\omega}_{-\kappa}}$$

$$\lesssim t^{1-\frac{\omega}{2}}|\mathrm{Ph}(0)|_{\mathcal{D}^{0+,0}_0} + t^{1/4+1-\frac{\kappa}{2}-\frac{\omega}{2}} \lesssim t^{1-\omega/2}$$

$$B_0 = PX(0) + \boldsymbol{\Psi}, h_0 = Ph(0)$$

$$B_n = \mathcal{P}\left(\dot{C}B_{n-4}\mathbf{1}_{n\geq 4}\right) + \sum_{k_1+k_2=n-1}\mathcal{P}\left(B_{k_1}\partial B_{k_2}\right) + \sum_{k_1+k_2+k_3=n-2}\mathcal{P}\left(B_{k_1}B_{k_2}B_{k_3}\right)$$

$$\mathcal{X} = \sum_{i=0}^{n} B_i + c\mathbf{1}_{n=5}\mathcal{P}h_0 + r_n, \mathcal{H} = h_0 + q_0$$

$$r_0 = O_{\mathcal{D}^{\frac{3}{2}+2\kappa,\omega}_{-\kappa}}\left(t^{-\omega/2-\kappa/2}\right), q_0 = O_{\mathcal{D}^{1+2\kappa,-\kappa}_0}\left(t^{1/4}\right)$$

$$\eta(0) = -1/2 - \kappa, \qquad \eta(n) = -1/2 + 2\kappa\ (1 \leq n \leq 5)$$

$$b(0) = 0, \qquad b(n) = (1/4 - \kappa/2)n - 3\kappa/2\ (1 \leq n \leq 5)$$

$$|B_n|_{\mathcal{D}^{\frac{3}{2}+2\kappa,\eta(n)}_{\alpha(n)}} \lesssim t^{b(n)}\ \forall 0 \leq n \leq 5$$

$$r_0 = O_{\mathcal{D}^{\frac{3}{2}+2\kappa,\omega}_{-\kappa}}\left(t^{-\omega/2-\kappa/2}\right), r_n = O_{\mathcal{D}^{\frac{3}{2}+2\kappa,\omega}_{0}}\left(t^{(n+1)/4-\kappa_n}\right) \forall 1 \le n \le 5,$$

$$\frac{1}{2}\eta(n) + b(n) = -\frac{1}{4} - \frac{\kappa}{2} + \left(\frac{1}{4} - \frac{\kappa}{2}\right) n \ \forall n \ge 0$$

$$\begin{aligned} &\left|\mathcal{P}\left(B_{k_1}\partial B_{k_2}\right)\right|^{\frac{3}{2}+2\kappa,-\frac{1}{2}+2\kappa}_{\mathcal{D}^2_{\alpha(n)}} \\ &\lesssim t^{\frac{1}{2}\left(\eta(k_1)+\eta(k_2)+\frac{3}{2}-2\kappa\right)}\left|B_{k_1}\partial B_{k_2}\right|_{\mathcal{D}^{\kappa,\eta(k_1)+\eta(k_2)-1}_{\alpha(k_1)+\alpha(k_2)-1}} \\ &\lesssim t^{\frac{1}{2}\left(\eta(k_1)+\eta(k_2)+\frac{3}{2}-2\kappa\right)}\left|B_{k_1}\right|_{\mathcal{D}^{\frac{3}{2}+2\kappa,\eta(k_1)}_{\alpha(k_1)}}\left|\partial B_{k_2}\right|_{\mathcal{D}^{\frac{1}{2}+2\kappa(k_2)-1}_{\alpha(k_2)-1}} \\ &\lesssim t^{\frac{1}{2}\left(\eta(k_1)+\eta(k_2)+\frac{3}{2}-2\kappa\right)} \cdot t^{b(k_1)} \cdot t^{b(k_2)} = t^{b(n)}. \end{aligned}$$

$$\begin{aligned} \left|\mathcal{P}\left(B_{k_1}B_{k_2}B_{k_3}\right)\right|_{\mathcal{D}^{\frac{3}{2}+2\kappa,-\frac{1}{2}+2\kappa}_{\alpha(n)}} &\lesssim t^{\frac{1}{2}\left(\sum_i \eta(k_i)+\frac{5}{2}-2\kappa\right)}\prod_{i=1}^{3}\left|B_{k_i}\right|_{\mathcal{D}^{\frac{3}{2}+2\kappa,\eta(k_i)}_{\alpha(k_i)}} \\ &\lesssim t^{\frac{1}{2}\left(\sum_i \eta(k_i)+\frac{5}{2}-2\kappa\right)} \cdot t^{\sum_i b(k_i)} = t^{b(n)}. \end{aligned}$$

$$\begin{aligned} \left|\mathcal{P}\left(B_{n-4}\right)\right|_{\mathcal{D}^{\frac{3}{2}+2\kappa,-\frac{1}{2}+2\kappa}_{\alpha(n)}} &\lesssim t^{\frac{1}{2}\left(\eta(n-4)+\frac{5}{2}-2\kappa\right)}\left|B_{n-4}\right|_{\mathcal{D}^{\frac{3}{2}+2\kappa,\eta(n-4)}_{\alpha(n-4)}} \\ &\lesssim t^{\left(\frac{1}{4}-\frac{\kappa}{2}\right)n+\frac{\kappa}{2}} \le t^{b(n)} \end{aligned}$$

$$\sum_{i=0}^{n} B_i + c\mathbf{1}_{n=5}\mathcal{P}h_0 + r_n = B_0 + \mathcal{P}\mathbf{1}_+\{Q^{\mathrm{YMH}}(\mathcal{X}) + c\mathcal{H}\}$$

$$r_n = \mathcal{P}\mathbf{1}_+\left\{Q^{\mathrm{YMH}}\left(\sum_{i=0}^{n-1} B_i + r_{n-1}\right) + c(h_0 + q_0 - \mathbf{1}_{n=5}h_0)\right\} - \sum_{i=1}^{n} B_i$$

$$\begin{aligned} r_n =&\mathcal{P}\mathbf{1}_+\Bigg(\sum_{k_1+k_2\ge n} B_{k_1}\partial B_{k_2} + \sum_{k_1+k_2+k_3\ge n-1} B_{k_1}B_{k_2}B_{k_3} \\ &+r_{n-1}\partial r_{n-1} + \left(\sum_{i=0}^{n-1} B_i\right)\partial r_{n-1} + r_{n-1}\partial\left(\sum_{i=0}^{n-1} B_i\right) \\ &+r_{n-1}^3 + 3r_{n-1}^2\left(\sum_{i=0}^{n-1} B_i\right) + 3r_{n-1}\left(\sum_{i=0}^{n-1} B_i\right)^2 + c(h_0 + q_0 - \mathbf{1}_{n=5}h_0) \\ &+\mathring{C}\left(\sum_{i=(n-3)\vee 0}^{n-1} B_i + r_{n-1}\right)\Bigg), \end{aligned}$$

$$\begin{aligned} \left|\mathcal{P}\left(B_{k_1}\partial B_{k_2}\right)\right|_{\mathcal{D}^{\frac{3}{2}+2\kappa,\omega}_{0}} &\lesssim t^{\frac{1}{2}(\eta(k_1)+\eta(k_2)+1-\omega)} \cdot t^{b(k_1)} \cdot t^{b(k_2)} \\ &\le t^{-\omega/2-\kappa+(1/4-\kappa/2)n} = t^{(n+1)/4-\kappa_n} \end{aligned}$$

$$\begin{aligned} \left|\mathcal{P}\left(B_{k_1}B_{k_2}B_{k_3}\right)\right|_{\mathcal{D}^{\frac{3}{2}+2\kappa,\omega}_{0}} &\lesssim t^{\frac{1}{2}(\sum_i \eta(k_i)+2-\omega)} \cdot t^{\sum_i b(k_i)} \\ &\le t^{\frac{1}{4}-\frac{\omega}{2}-\frac{3}{2}\kappa+(1/4-\kappa/2)(n-1)} = t^{(n+1)/4-\kappa_n} \end{aligned}$$

$$|\mathcal{P}B_i|_{\mathcal{D}_0^{\frac{3}{2}+2\kappa,\omega}} \lesssim t^{\frac{1}{2}(\eta(i)+2-\omega)}|B_i|_{\mathcal{D}_{\alpha(i)}^{\frac{3}{2}+2\kappa,\eta(i)}}$$

$$\lesssim t^{\frac{1}{2}(\eta(i)+2-\omega)} \cdot t^{b(i)} = t^{\frac{3}{4}-\frac{\omega}{2}-\frac{\kappa}{2}+\left(\frac{1}{4}-\frac{\kappa}{2}\right)i} \le t^{(n+1)/4-\kappa_n}$$

$$|\mathcal{P}(B_0\partial r_{n-1})|_{\mathcal{D}_0^{\frac{3}{2}+2\kappa,\omega}} \lesssim t^{1/4-\kappa/2}|B_0\partial r_{n-1}|_{\mathcal{D}_{-\frac{3}{2}-2\kappa}^{\kappa,\omega-\frac{3}{2}-\kappa}}$$

$$\lesssim t^{1/4-\kappa/2}|B_0|_{\mathcal{D}_{-\frac{1}{2}-\kappa}^{\frac{3}{2}+2\kappa,-\frac{1}{2}-\kappa}}|\partial r_{n-1}|_{\mathcal{D}_{-1-\kappa}^{\frac{1}{2}+2\kappa,\omega-1}}$$

$$\lesssim t^{1/4-\kappa/2}t^{n/4-\kappa_{n-1}} = t^{(n+1)/4-\kappa_n}$$

$$|\mathcal{P}(r_{n-1}\partial B_0)|_{\mathcal{D}_0^{\frac{3}{2}+2\kappa,\omega}} \lesssim t^{(n+1)/4-\kappa_n}$$

$$|\mathcal{P}(B_0^2 r_{n-1})|_{\mathcal{D}_0^{\frac{3}{2}+2\kappa,\omega}} \lesssim t^{1/2-\kappa}|B_0^2 r_{n-1}|_{\mathcal{D}_{-1-3\kappa}^{\frac{1}{2},\omega-1-2\kappa}}$$

$$\lesssim t^{1/2-\kappa}|B_0|^2_{\mathcal{D}_{-\frac{1}{2}-\kappa}^{\frac{3}{2}+2\kappa,-\frac{1}{2}-\kappa}}|r_{n-1}|_{\mathcal{D}_{-\kappa}^{\frac{3}{2}+2\kappa,\omega}}$$

$$\lesssim t^{1/2-\kappa}t^{n/4-\kappa_{n-1}} = t^{\frac{1}{4}-\frac{\kappa}{2}}t^{(n+1)/4-\kappa_n} \le t^{(n+1)/4-\kappa_n}$$

$$|\mathcal{P}(B_0 r_{n-1}^2)|_{\mathcal{D}_0^{\frac{3}{2}+2\kappa,\omega}} \lesssim t^{\frac{\omega}{2}+\frac{3}{4}-\frac{\kappa}{2}}|B_0 r_{n-1}^2|_{\mathcal{D}}^{1,2\omega-\frac{1}{2}-3\kappa}$$

$$\lesssim t^{\frac{\omega}{2}+\frac{3}{4}-\frac{\kappa}{2}}|B_0|_{\mathcal{D}_{-\frac{1}{2}-\kappa}^{\frac{3}{2}+2\kappa,-\frac{1}{2}-\kappa}}|r_{n-1}|^2_{\mathcal{D}_{-\kappa}^{\frac{3}{2}}+2\kappa,\omega}$$

$$\lesssim t^{\frac{\omega}{2}+\frac{3}{4}-\frac{\kappa}{2}}\left(t^{n/4-\kappa_{n-1}}\right)^2 \le t^{(n+1)/4-\kappa_n}$$

$$|\mathcal{P}(r_{n-1}\partial r_{n-1})|_{\mathcal{D}_0^{\frac{3}{2}+2\kappa,\omega}} \lesssim t^{(1+\omega)/2}|r_{n-1}\partial r_{n-1}|_{\mathcal{D}_{-1-2\kappa}^{\frac{1}{2}+\kappa,2\omega-1}}$$

$$\lesssim t^{(1+\omega)/2}|r_{n-1}|_{\mathcal{D}_{-\kappa}^{\frac{3}{2}+2\kappa,\omega}}\Big|\,|\partial r_{n-1}|_{\mathcal{D}_{-1-\kappa}^{1+2\kappa-1}}^{\frac{1}{2}+2}$$

$$\lesssim t^{(1+\omega)/2}\left(t^{n/4-\kappa_{n-1}}\right)^2 \le t^{(n+1)/4-\kappa_n}$$

$$|\mathcal{P}(r_{n-1}^3)|_{\mathcal{D}_0^{\frac{3}{2}+2\kappa,\omega}} \lesssim t^{1+\omega}|r_{n-1}^3|_{\mathcal{D}_{-3\kappa}^{\frac{3}{2},3\omega}} \lesssim t^{1+\omega}|r_{n-1}|^3_{\mathcal{D}_{-\kappa}^{\frac{3}{2}}} + 2\kappa,\omega$$

$$\lesssim t^{1+\omega}\left(t^{n/4-\kappa_{n-1}}\right)^3 \le t^{(n+1)/4-\kappa_n}$$

$$|\mathcal{P}q_0|_{\mathcal{D}_0^{\frac{3}{2}+2\kappa,\omega}} \lesssim t^{\frac{1}{2}(2-\kappa-\omega)}|q_0|_{\mathcal{D}_0^{\frac{1}{2}+2\kappa,-\kappa}} \lesssim t^{\frac{1}{2}(2-\kappa-\omega)} \cdot t^{\frac{1}{4}} \le t^{(n+1)/4-\kappa_n}$$

$$|\mathcal{P}h_0|_{\mathcal{D}_0^{\frac{3}{2}+2\kappa,\omega}} \lesssim t^{\frac{1}{2}(2-\omega)}|h_0|_{\mathcal{D}_0^{\frac{1}{2}+2\kappa,0}} \lesssim t^{(n+1)/4-\kappa_n}$$

$$\Psi = \mathcal{R}\Psi = \mathcal{P} \star \mathbf{1}_+\xi$$

$$(\mathcal{R}\mathcal{X})(t) = X(0) + \Psi_t + \mathcal{P}_t \star (\Psi\partial\Psi) + O_{L^\infty}\left(t^{\frac{1}{4}-3\kappa/2}\right).$$

$$\mathcal{R}B_1 = \mathcal{P} \star (PX(0)P'X(0) + PX(0)\partial\Psi + P'X(0)\Psi + \Psi\partial\Psi)$$

$$|\Psi|_{L^\infty} \lesssim t^{-\frac{1}{4}-\kappa/2}, |\partial\Psi|_{L^\infty} \lesssim t^{-\frac{3}{4}-\kappa/2}$$

$$|r_1|_{\mathcal{D}_0^{\frac{3}{2}+2\kappa,\omega}} \lesssim t^{\frac{1}{2}-\kappa_1}, |(\mathcal{R}r_1)(t)|_{L^\infty} \lesssim t^{\frac{1}{2}-\kappa_1+\omega/2} = t^{\frac{1}{4}-3\kappa/2}$$

$$\mathcal{N}(A)\colon (0,\infty) \to \mathcal{C}^\infty(\mathbf{T}^3, \mathfrak{g}^3 \otimes (\mathfrak{g}^3)^3), \mathcal{N}_s(A) \stackrel{\text{def}}{=} P_sA \otimes \nabla P_sA$$

$$\mathcal{L} = \mathbf{T}^3 \times \{v \in \mathbf{R}^3\colon |v| \le 1/4\}$$

$$|f|_{\gamma-\mathrm{gr}} = \sup_{\ell\in\mathcal{L}} \frac{|\int_\ell f|}{|\ell|^\gamma},$$

$$\int_\ell f = \int_0^1 |v| f(y+tv)\mathrm{d}t \in F$$

$$[\,[A;B]\,]_{\gamma,\delta} \stackrel{\text{def}}{=} \sup_{s\in(0,1)} s^\delta |\mathcal{N}_s A - \mathcal{N}_s B|_{\gamma-\mathrm{gr}}, [\,[A]\,]_{\gamma,\delta} = [\,[A;0]\,]_{\gamma,\delta}$$

$$[\,[A;B]\,]_{\beta,\delta} \stackrel{\text{def}}{=} \sup_{s\in(0,1)} s^\delta |\mathcal{N}_s A - \mathcal{N}_s B|_{\mathcal{C}^\beta}$$

$$\|A\|_{\alpha,\theta} \stackrel{\text{def}}{=} \sup_{s\in(0,1)} |P_s A|_{\alpha-\mathrm{gr};<s^\theta}$$

$$|A|_{\alpha-\mathrm{gr};<r} \stackrel{\text{def}}{=} \sup_{\ell\in\mathcal{L},|\ell|<r} \frac{|A(\ell)|}{|\ell|^\alpha}$$

$$|A|_{\mathcal{C}^\eta} \lesssim \|A\|_{\alpha,\theta}$$

$$\Sigma(A) \stackrel{\text{def}}{=} \|A\|_{\alpha,\theta} + [\,[A]\,]_{\gamma,\delta} < \infty$$

$$\Sigma(A,B) \stackrel{\text{def}}{=} \|A-B\|_{\alpha,\theta} + [\,[A;B]\,]_{\gamma,\delta} < \infty$$

$$\alpha \in (0,1/2), \theta > 0, \gamma \in (1/2,1], \delta \in (0,1)$$

$$\mu \stackrel{\text{def}}{=} \gamma - 1 + 2(1-\delta) \in (-1/2,0), \text{ and } \eta + \mu > -1$$

$$|\mathcal{F}_s A|_\infty \lesssim s^{\frac{\eta}{2}}\Sigma(A), |\partial \mathcal{F}_s A|_\infty \lesssim s^{\frac{\eta}{2}-\frac{1}{2}}\Sigma(A)$$

$$|P_s A|_\infty \lesssim s^{\frac{\eta}{2}}|A|_{\mathcal{C}^\eta}, |\partial P_s A|_\infty \lesssim s^{\frac{\eta}{2}-\frac{1}{2}}|A|_{\mathcal{C}^\eta}$$

$$\lambda \stackrel{\text{def}}{=} (1-\zeta)\eta/2 - \theta(1-\alpha)\zeta < 0$$

$$|P_s A|_{\gamma-\mathrm{gr}} \lesssim s^\lambda \|A\|_{\alpha,\theta}$$

$$0 < \nu \le \min\left\{\frac{\eta}{2} + \frac{\mu}{2} + \frac{1}{2}, 1 + 3\eta/2, \mu + \frac{1}{2}\right\}$$

$$|\mathcal{P}_s \star \mathcal{N}A|_{\gamma-\mathrm{gr}} \lesssim \int_0^s |\mathcal{N}_r A|_{\gamma-\mathrm{gr}}\,\mathrm{d}r \lesssim \int_0^s r^{-\delta}[\,[A]\,]_{\gamma,\delta}\mathrm{d}r \lesssim s^{1-\delta}[\,[A]\,]_{\gamma,\delta}$$

$$\mathcal{F}_s A = P_s A + \mathcal{P}_s \star \mathcal{N}A + R_s A$$

$$|R_s A|_\infty \lesssim s^\nu(\Sigma(A) + \Sigma(A)^3)$$

$$R_s = \int_0^s P_{s-r}\{(P_r A + \mathcal{P}_r \star \mathcal{N}A + R_r)\partial(P_r A + \mathcal{P}_r \star \mathcal{N}A + R_r)$$

$$+(\mathcal{F}_r A)^3\}\mathrm{d}r - \mathcal{P}_s \star \mathcal{N}A$$

$$|\mathcal{P}_s \star \mathcal{N}A|_\infty \lesssim \int_0^s |P_{s-r}(\mathcal{N}_r A)|_\infty \mathrm{d}r \lesssim \int_0^s (s-r)^{\frac{\gamma-1}{2}}|\mathcal{N}_r A|_{\mathcal{C}^{\gamma-1}}\,\mathrm{d}s$$

$$\lesssim \int_0^s (s-r)^{\frac{\gamma-1}{2}}|\mathcal{N}_r A|_{\gamma-\mathrm{gr}}\,\mathrm{d}s$$

$$\lesssim \int_0^s (s-r)^{\frac{\gamma-1}{2}} r^{-\delta}[\,[A]\,]_{\gamma,\delta}\mathrm{d}s \asymp s^{\mu/2}[\,[A]\,]_{\gamma,\delta},$$

$$|\partial \mathcal{P}_s \star \mathcal{N}A|_\infty \lesssim s^{\mu/2-\frac{1}{2}}[\,[A]\,]_{\gamma,\delta}$$

$$\int_0^s \left\{r^{\frac{\eta}{2}+\frac{\mu}{2}-\frac{1}{2}} + r^{\mu-\frac{1}{2}} + r^{3\eta/2}\right\}\mathrm{d}r \asymp s^{\frac{\eta}{2}+\frac{\mu}{2}+\frac{1}{2}} + s^{\mu+\frac{1}{2}} + s^{1+3\eta/2}$$

$$|R|_{\mathcal{B}} \stackrel{\text{def}}{=} \sup_{s\in(0,T)} \left\{s^{-\nu}|R_s|_\infty + s^{-\nu+\frac{1}{2}}|\partial R_s|_\infty\right\}$$

$$s^{-\nu}|R|_\infty \lesssim \Sigma(A) + \Sigma(A)^3 + s^{\kappa'}|R|_{\mathcal{B}}(\Sigma(A) + \Sigma(A)^2)$$

$$+s^{\kappa''}|R|_{\mathcal{B}}^2(1+\Sigma(A)) + s^{1+2\nu}|R|_{\mathcal{B}}^3$$

$$\kappa' = \min\left\{\frac{\eta}{2}+\frac{1}{2}, 1+\eta\right\} = \frac{\eta}{2}+\frac{1}{2}$$

$$\kappa'' = \min\left\{\frac{1}{2}+\nu, 1+\frac{\eta}{2}+\nu\right\} = \frac{1}{2}+\nu$$

$$|R|_{\mathcal{B}} \lesssim \Sigma(A)+\Sigma(A)^3+T^{\kappa}(|R|_{\mathcal{B}}+|R|_{\mathcal{B}}^3)(1+\Sigma(A)^2)$$

$$\mathcal{F}_s A = P_s A + O_{\Omega_{\gamma-\mathrm{gr}[\mathfrak{g},\mathfrak{g}]}\left(s^{\nu}(\Sigma(A)+\Sigma(A)^3)\right), P_s A = O_{\Omega_{\gamma-\mathrm{gr}}}(s^{\lambda})\ldots\ldots}$$

$$\mathcal{F}_s \tilde{A} = \mathcal{F}_s A + P_s r + O_{L^\infty[\mathfrak{g},\mathfrak{g}]}\left(s^{\eta/2+\frac{1}{2}}|r|_\infty\right)$$

$$\mathcal{F}_s A + Q_s = P_s(A+r) + \int_0^s P_{s-u}\{(\mathcal{F}_u A+Q_u)\partial(\mathcal{F}_u A+Q_u)+(\mathcal{F}_u A+Q_u)^3\}\mathrm{d}u$$

$$Q_s = P_s r + \int_0^s P_{s-u}(Q_u\partial\mathcal{F}_u A + (\mathcal{F}_u A)\partial Q_u + Q_u\partial Q_u$$

$$+(\mathcal{F}_u A)^2 Q_u + (\mathcal{F}_u A)Q_u^2 + Q_u^3)\mathrm{d}u$$

$$\int_0^s \left\{|Q_u|_\infty u^{\eta/2-\frac{1}{2}} + u^{\eta/2}|Q_u|_{\mathcal{C}^1} + |Q_u|_\infty |Q_u|_{\mathcal{C}^1}\right.$$

$$\left.+u^{\eta}|Q_u|_\infty + u^{\eta/2}|Q_u|_\infty^2 + |Q_u|_\infty^3\right\}\mathrm{d}u$$

$$\lesssim |Q|_\infty s^{\eta/2+\frac{1}{2}} + s^{\eta/2+\frac{1}{2}}|Q|_{L^\infty_{1/2}\mathcal{C}^1} + s^{1/2}|Q|_\infty |Q|_{L^\infty_{1/2}\mathcal{C}^1}$$

$$+s^{\eta+1}|Q|_\infty + s^{\eta/2+1}|Q|_\infty^2 + s|Q|_\infty^3,$$

$$|Q|_\infty + |Q|_{L^\infty_{1/2}\mathcal{C}^1} \lesssim |r|_\infty$$

$$Q_s = P_s r + O_{L^\infty}\left(s^{\eta/2+\frac{1}{2}}|r|_\infty\right)$$

$$\tilde{\mathcal{X}} = B + c\bar{h} + O_{\mathcal{D}_0^{\frac{3}{2}+2\kappa,\omega}}\left(t^{3/2-\kappa_5}\right)$$

$$\tilde{\mathcal{X}} = B + c\mathcal{P}\mathrm{Ph}(0) + O_{\mathcal{D}_0^{\frac{3}{2}+2\kappa,\omega}}\left(t^{3/2-\kappa_5}\right),$$

$$\tilde{A} = A + ctP_t h(0) + O_{L^\infty}\left(t^{5/4-7\kappa/2}\right).$$

$$\mathcal{F}_s\tilde{A} = \mathcal{F}_s A + P_s\left(ctP_t h(0) + O_{L^\infty}\left(t^{5/4-7\kappa/2}\right)\right) + O_{L^\infty[\mathfrak{g},\mathfrak{g}]}\left(s^{\eta/2+\frac{1}{2}}t\right)$$

$$|f|_{p-\mathrm{var}} \overset{\mathrm{def}}{=} \sup_{P\subset[0,1]}\left(\sum_{[s,t]\in P}|f(t)-f(s)|^p\right)^{1/p}$$

$$\mathrm{d}J^{\gamma}(x) = J^{\gamma}(x)\mathrm{d}\gamma(x), J^{\gamma}(0) = \mathrm{id}$$

$$J^{\gamma+\zeta}(1) = J^{\gamma}(1) + \int_0^1 \mathrm{d}\boldsymbol{\zeta}(x) + \int_0^1\int_0^x \{\mathrm{d}\boldsymbol{\zeta}(x)\mathrm{d}\boldsymbol{\gamma}(y) + \mathrm{d}\boldsymbol{\gamma}(x)\mathrm{d}\boldsymbol{\zeta}(y)\}$$

$$+O\{v(w^2+w^{L-1}) + w^L + w^{L+1} + v^{L+1} + v^2(1+w+v+w^{L-3})\}$$

$$J^{\gamma}(1) = \mathrm{id} + I^{\gamma} + \int_0^1 \cdots \int_0^{x_{L-1}} J^{\gamma}(x_L)\mathrm{d}\gamma(x_L)\mathrm{d}\gamma(x_{L-1})\ldots\mathrm{d}\gamma(x_1)$$

$$I^{\gamma+\zeta} = I^{\gamma} + \int_0^1 \mathrm{d}\boldsymbol{\zeta}(x) + \int_0^1\int_0^x \{\mathrm{d}\boldsymbol{\zeta}(x)\mathrm{d}\boldsymbol{\gamma}(y) + \mathrm{d}\boldsymbol{\gamma}(x)\mathrm{d}\boldsymbol{\zeta}(y)\}$$

$$+O\{v^2 + v(w^2+w^{L-1})\} + O\{v^2(w+v+w^{L-3}+v^{L-3})\}$$

$$|\ell_a|_{\frac{1}{\gamma}\text{-var}} \leq |\ell_a|_{\gamma-\text{Höl}} \leq |a|_{\gamma-\mathrm{gr}}$$

$$W_\ell(\mathcal{F}_s\tilde{A}) = W_\ell(\mathcal{F}_sA) + t\mathrm{Tr}\int_\ell \mathrm{ch}(0) + t\mathrm{Tr}\int_{[0,1]^2} \mathrm{d}\ell_{A(0)}(x_1)\mathrm{d}\ell_{\mathrm{ch}(0)}(x_2)$$

$$+O\left(t^{5/4-7\kappa/2} + ts + ts^\lambda\|\Psi_t^{\mathrm{YM}}\|_{\alpha,\theta} + ts^\lambda\|\mathcal{P}_t \star (\Psi\partial\Psi)^{\mathrm{YM}}\|_{\alpha,\theta} + |A(0)|_{L^\infty}s^{\eta/2+\frac{1}{2}}t\right.$$

$$\left.+s^\nu t(\Sigma(A) + \Sigma(A)^3) + t(u^2 + u^{L-1}) + s^{\nu L} + u^L + u^{L+1} + t^2u\right)$$

$$\gamma(x) = \int_0^x (\mathcal{F}_sA)_1(y,0,0)\mathrm{d}y = \ell_{\mathcal{F}_sA}(x)$$

$$W_\ell(\mathcal{F}_sA) = \mathrm{Tr}J^\gamma(1), W_\ell(\mathcal{F}_s\tilde{A}) = \mathrm{Tr}J^{\gamma+\zeta}(1)$$

$$D_h \overset{\mathrm{def}}{=} ctP_{t+s}h(0) = cth(0) + O_{L^\infty}(t(t+s)),$$

$$D_{\mathrm{err}} \overset{\mathrm{def}}{=} O_{L^\infty[\mathfrak{g},\mathfrak{g}]}\left(s^{\eta/2+\frac{1}{2}}t\right) + O_{L^\infty}\left(t^{5/4-7\kappa/2}\right).$$

$$W_\ell(\mathcal{F}_s\tilde{A}) = W_\ell(\mathcal{F}_sA) + \mathrm{Tr}\left(\int_0^1 \mathrm{d}\zeta(x)\right)$$

$$+\mathrm{Tr}\int_0^1\int_0^x \{\mathrm{d}\zeta(x)\mathrm{d}\gamma(y) + \mathrm{d}\gamma(x)\mathrm{d}\zeta(y)\}$$

$$+O\{v(w^2 + w^{L-1}) + w^L + w^{L+1} + v^{L+1} + v^2(1 + w + v + w^{L-3})\}$$

$$\mathrm{Tr}\left(\int_0^1 \mathrm{d}\zeta(x)\right) = t\mathrm{Tr}\int_\ell \mathrm{ch}(0) + O\left(t^{5/4-7\kappa/2} + ts\right)$$

$$\mathrm{Tr}\int_0^1\int_0^x \{\mathrm{d}\boldsymbol{\zeta}(x)\mathrm{d}\boldsymbol{\gamma}(y) + \mathrm{d}\boldsymbol{\gamma}(x)\mathrm{d}\boldsymbol{\zeta}(y)\} = \mathrm{Tr}\int_{[0,1]^2}\{\mathrm{d}\boldsymbol{\zeta}(x)\mathrm{d}\boldsymbol{\gamma}(y)\}$$

$$\mathcal{F}_sA = P_sA + O_{\gamma-\mathrm{gr}}\left(s^\nu(\Sigma(A) + \Sigma(A)^3)\right)$$

$$A = A(0) + \Psi_t^{\mathrm{YM}} + \mathcal{P}_t \star (\Psi\partial\Psi)^{\mathrm{YM}} + O_{L^\infty}\left(t^{1/4-3\kappa/2}\right)$$

$$\int_0^1 \mathrm{d}\gamma(x) = \int_0^1 \mathrm{d}\ell_{A(0)}(x) + O\left(s^\lambda\|\Psi_t^{\mathrm{YM}}\|_{\alpha,\theta} + s^\lambda\|\mathcal{P}_t \star (\Psi\partial\Psi)^{\mathrm{YM}}\|_{\alpha,\theta}\right)$$

$$+O\left(t^{1/4-3\kappa/2} + s^\nu(\Sigma(A) + \Sigma(A)^3)\right)$$

$$\int_0^1 \mathrm{d}\boldsymbol{\zeta}(y) = t\int_0^1 \mathrm{d}\ell_{\mathrm{ch}(0)}(y) + O\left(t^{5/4-7\kappa/2} + s^{\eta/2+\frac{1}{2}}t\right)$$

$$\mathrm{Tr}\int_{[0,1]^2}\{\mathrm{d}\boldsymbol{\zeta}(x)\mathrm{d}\gamma(y)\} = t\mathrm{Tr}\left(\int_{[0,1]^2} \mathrm{d}\ell_{A(0)}(x_1)\mathrm{d}\ell_{ch(0)}(x_2)\right)$$

$$+O\left(ts^\lambda\|\Psi_t^{\mathrm{YM}}\|_{\alpha,\theta} + ts^\lambda\|\mathcal{P}_t \star (\Psi\partial\Psi)^{\mathrm{YM}}\|_{\alpha,\theta} + t^{5/4-7\kappa/2}\right)$$

$$+O\left(ts^\nu(\Sigma(A) + \Sigma(A)^3) + s^{\eta/2+\frac{1}{2}}t|A(0)|_{L^\infty}\right)$$

$$w = |\gamma|_{\frac{1}{\gamma}-\mathrm{var}} \le |\mathcal{F}_sA|_{\gamma-\mathrm{gr}} = |\mathcal{P}_sA|_{\gamma-\mathrm{gr}} + O(s^\nu) = O(s^\lambda)\|A\|_{\alpha,\theta} + O(s^\nu)$$

$$v = |\boldsymbol{\zeta}|_{\frac{1}{\gamma}-\mathrm{var}} \le |D_h + D_{\mathrm{err}}|_{\gamma-\mathrm{gr}} \lesssim s^{\eta/2+\frac{1}{2}}t + t = O(t)$$

$$[\,[A+B]\,]_{\gamma,\delta} \lesssim [\,[A]\,]_{\gamma,\delta} + |B|_{\mathcal{C}^{\bar{\eta}}}\left(|A|_{\mathcal{C}^\eta} + |B|_{\mathcal{C}^{\bar{\eta}}}\right)$$

$$\mathcal{N}_s(A+B) = \mathcal{N}_sA + \mathcal{N}_sB + P_sA \otimes \nabla P_sB + P_sB \otimes \nabla P_sA$$

$$Z_{s,t} = s^\delta\mathcal{N}_s\Psi_t$$

$$\mathbf{E}\left[\left|\sup_{(s,t)\neq(\bar{s},\bar{t})}\frac{|Z_{s,t}-Z_{\bar{s},\bar{t}}|_{\gamma-\mathrm{gr}}}{(|t-\bar{t}|+|s-\bar{s}|)^{\bar{\kappa}}}\right|^{p}\right]^{1/p}<\infty$$

$$\left(\mathbf{E}\mid Z_{s,t}-Z_{\bar{s},\bar{t}}\bar{\tau}^{p}_{\gamma-\mathrm{gr}}\right)^{1/p}\lesssim(|t-\bar{t}|+|s-\bar{s}|)^{\kappa/2}$$

$$\mathbf{E}\left|\sup_{t\in[0,1]}t^{-\bar{\kappa}}[\,[\Psi_t]\,]_{\gamma,\delta}\right|^{p}<\infty$$

$$|C_{s,\bar{s};,t,\bar{t}}(x)|\lesssim(|s-\bar{s}|+|t-\bar{t}|)^{\kappa}|x|^{4\delta-4-2\kappa}$$

$$|\nabla C_{s,\bar{s};,t\bar{t}}|\lesssim(|s-\bar{s}|+|t-\bar{t}|)^{\kappa}|x-y|^{4\delta-5-2\kappa}$$

$$|\nabla C_{r,s}(x)|\lesssim|x|^{-2},\left|\nabla\left(C_{r,r}-C_{r,s}\right)(x)\right|\lesssim|r-s|^{\kappa}|x|^{-2-2\kappa}$$

$$d((x,v),(\bar{x},\bar{v}))\stackrel{\mathrm{def}}{=}|x-\bar{x}|\vee|x+v-(\bar{x}+\bar{v})|$$

$$\left(\mathbf{E}\left|\int_{\ell}(Z_{s,t}-Z_{\bar{s},\bar{t}}|^{p}\right)^{1/p}\lesssim(|t-\bar{t}|+|s-\bar{s}|)^{\kappa}\left|\ell\right|^{2\delta-1-\kappa}$$

$$\left(\mathbf{E}\left|\left(\int_{\ell}-\int_{\bar{\ell}}\right)(Z_{s,t}-Z_{\bar{s},\bar{t}})\right|^{p}\right)^{1/p}\lesssim(|t-\bar{t}|+|s-\bar{s}|)^{\kappa/2}d(\ell,\bar{\ell})^{2\delta-3/2-\kappa}$$

$$\begin{aligned}\mathbf{E}\left|\int_{\ell}(Z_{s,t}-Z_{\bar{s},\bar{t}})\right|^{2}&=|\ell|^{2}\int_{[0,1]^2}C((r-\bar{r})v)\mathrm{d}r\,\mathrm{d}\bar{r}\\&\lesssim(|t-\bar{t}|+|s-\bar{s}|)^{\kappa}|\ell|^{4\delta-2-2\kappa}\int_{[0,1]^2}|r-\bar{r}|^{4\delta-4-2\kappa}\,\mathrm{d}r\,\mathrm{d}\bar{r}\\&\lesssim(|t-\bar{t}|+|s-\bar{s}|)^{\kappa}|\ell|^{4\delta-2-2\kappa}\end{aligned}$$

$$\begin{aligned}\mathbf{E}\mid\left(\int_{\ell}-\int_{\bar{\ell}}\right)\left(Z_{s,t}-Z_{\bar{s},\bar{t}}\right|^{2}&=|\ell|^{2}\int_{[0,1]^2}\{C((r-\bar{r})v)-C(rv-\bar{r}\bar{v})\\&\quad-C(r\bar{v}-\bar{r}v)+C((r-\bar{r})\bar{v})\}\mathrm{d}r\,\mathrm{d}\bar{r}\end{aligned}$$

$$=|\ell|^{2}\int_{[0,1]^2}\{2C((r-\bar{r})v)-2C(rv-\bar{r}\bar{v})\}\mathrm{d}r\,\mathrm{d}\bar{r},$$

$$(|t-\bar{t}|+|s-\bar{s}|)^{\kappa}|\ell|^{4\delta-2-2\kappa}\int_{0}^{h}r^{4\delta-4-2\kappa}\,\mathrm{d}r$$

$$\begin{aligned}&\asymp(|t-\bar{t}|+|s-\bar{s}|)^{\kappa}|\ell|^{4\delta-2-2\kappa}h^{4\delta-3-2\kappa}\\&=(|t-\bar{t}|+|s-\bar{s}|)^{\kappa}|\ell||v-\bar{v}|^{4\delta-3-2\kappa}\end{aligned}$$

$$(|t-\bar{t}|+|s-\bar{s}|)^{\kappa}|\ell|^{4\delta-3-2\kappa}|v-\bar{v}|\int_{h}^{1}r^{4\delta-5-2\kappa}\,\mathrm{d}r$$

$$\begin{aligned}&\asymp(|t-\bar{t}|+|s-\bar{s}|)^{\kappa}|\ell|^{4\delta-3-2\kappa}|v-\bar{v}|h^{4\delta-4-2\kappa}\\&=(|t-\bar{t}|+|s-\bar{s}|)^{\kappa}|\ell||v-\bar{v}|^{4\delta-3-2\kappa}\end{aligned}$$

$$\mathbf{E}\mid\left(\int_{\ell}-\int_{\bar{\ell}}\right)\left(Z_{s,t}-Z_{\bar{s},\bar{t}}\right|^{2}\lesssim(|t-\bar{t}|+|s-\bar{s}|)^{\kappa}|\ell||v-\bar{v}|^{4\delta-3-2\kappa}$$

$$\left(\mathbf{E}\left|\int_{\ell}A\right|^{p}\right)^{1/p}\leq M_p|\ell|^{\alpha}$$

$$\left(\mathbf{E}\left|\left(\int_{\ell}-\int_{\bar{\ell}}\right)A\right|^{p}\right)^{1/p}\leq M_p d(\ell,\bar{\ell})^{\beta}$$

$$\left(\mathbf{E}|A|^{p}_{\gamma-\mathrm{gr}}\right)^{1/p}\leq\lambda M_p$$

$$\sup_{\ell \in D} \frac{\left|\int_\ell A\right|}{|\ell|^\gamma} \lesssim \sup_{N\ge 1} \sup_{\substack{a\in D_N \\ |a|\le K2^{-N/\omega}}} \frac{\left|\int_a A\right|}{2^{-\gamma N/\omega}} + \sup_{N\ge 1} \sup_{\substack{a,b\in D_N \\ d(a,b)\le K2^{-N}}} \frac{\left|\left(\int_a - \int_b\right)A\right|}{2^{-\gamma N/\omega}}.$$

$$\mathbf{E}\left|\sup_{\ell\in D} \frac{\left|\int_\ell A\right|}{|\ell|^\gamma}\right|^p \lesssim M_p^p \sum_{N\ge 1} \left\{2^{N(6-p(\alpha-\gamma)/\omega)} + 2^{N(12-p(\beta-\gamma/\omega))}\right\}$$

$$\alpha = \frac{1}{2} - \varepsilon, \theta = \varepsilon, \gamma = \frac{1}{2} + \varepsilon, \delta = 1 - \varepsilon, \nu = -\bar{\eta} = \varepsilon/2$$

$$\|f\|_{\alpha,\theta} \lesssim |f|_{\mathcal{C}^{\bar{\eta}}}$$

$$\mathcal{P} \star (\Psi\partial\Psi) = \lim_{\varepsilon\downarrow 0} \mathcal{P} \star (\Psi_\varepsilon \partial \Psi_\varepsilon),$$

$$\mathbf{E}\|\Psi_t\|_{\alpha,\theta}^p + \mathbf{E}|\mathcal{P}_t \star (\Psi\partial\Psi)|_{\mathcal{C}^{\bar{\eta}}}^p + \mathbf{E}[\,[\Psi]\,]_{\gamma,\delta}^p = O(t^{p\varepsilon}).$$

$$Q_t = \Big\{\|Z\|_{\frac{3}{2}+2\kappa;O} + |\mathcal{P} \star \mathbf{1}_+\xi|_{\mathcal{C}([-1,3],\mathcal{C}^{-1/2-\kappa})} + |\mathcal{P} \star (\Psi\partial\Psi)|_{\mathcal{C}([-1,3],\mathcal{C}-2\kappa)}$$

$$+t^{-\varepsilon}\|\Psi_t\|_{\alpha,\theta} + t^{-\varepsilon}|\mathcal{P}_t \star (\Psi\partial\Psi)|_{\mathcal{C}^{\bar{\eta}}} + t^{-\varepsilon}[\,[\Psi_t]\,]_{\gamma,\delta} < M\Big\}$$

$$\{\|Z\|_{\frac{3}{2}+2\kappa;O} + |\mathcal{P}\star\mathbf{1}_+\xi|_{\mathcal{C}([-1,3],\mathcal{C}^{-1/2-\kappa})} + |\mathcal{P}\star(\Psi\partial\Psi)|_{\mathcal{C}([-1,3],\mathcal{C}^{-2\kappa})} < M\} \supset Q_t\,,$$

$$\mathbf{E}W_\ell(\mathcal{F}_s\tilde{A})\mathbf{1}_{Q_t} - \mathbf{E}W_\ell(\mathcal{F}_sA)\mathbf{1}_{Q_t} = \mathbf{P}[Q_t]\left\{t\mathrm{Tr}\left(\int_\ell \mathrm{ch}(0)\right)\right.$$

$$\left.+t\mathrm{Tr}\left(\int_{[0,1]^2} \mathrm{d}\ell_{A(0)}(x_1)\mathrm{d}\ell_{ch(0)}(x_2)\right) + O\left(t^{1+r+\beta/6} + t^{1+3r/2} + t^{1+r+\nu\beta}\right)\right\}$$

$$\|A\|_{\alpha,\theta} \le |A(0)|_\infty + \|\Psi_t\|_{\alpha,\theta} + \|\mathcal{P}_t \star (\Psi\partial\Psi)\|_{\alpha,\theta} + O\left(t^{1/4-3\kappa/2}\right) \lesssim t^r$$

$$\beta = -\frac{r}{4\lambda} > 0$$

$$u = s^\lambda\|A\|_{\alpha,\theta} \lesssim s^\lambda t^r = t^{3r/4}$$

$$[\,[A]\,]_{\gamma,\delta} \lesssim [\,[\Psi_t]\,]_{\gamma,\delta} + |\Psi|_{\mathcal{C}^\eta}\left\{|A(0)|_{L^\infty} + |\mathcal{P}_t(\Psi\partial\Psi)|_{\mathcal{C}^{\bar{\eta}}} + O\left(t^{1/4-3\kappa/2}\right)\right\} \lesssim t^r$$

$$t^{5/4-7\kappa/2} + ts \lesssim t^{1+3r/2},$$

$$ts^\lambda\left(\|\Psi_t\|_{\alpha,\theta} + \|\mathcal{P}_t \star (\Psi\partial\Psi)\|_{\alpha,\theta}\right) \lesssim s^\lambda t^{1+\varepsilon} \lesssim t^{1+3r/2},$$

$$|A(0)|_{L^\infty} s^{\eta/2+\frac{1}{2}} t \lesssim t^{r+1} t^{\left(\frac{\eta}{2}+\frac{1}{2}\right)\beta} \lesssim t^{1+r+\beta/6},$$

$$s^\nu t(\Sigma(A) + \Sigma(A)^3) \lesssim t^{1+r+\nu\beta},$$

$$t(u^2 + u^{L-1}) \lesssim t^{1+3r/2},$$

$$\left|\mathbf{E}\{W_\ell(\mathcal{F}_s\tilde{A}) - W_\ell(\mathcal{F}_sA)\}\mathbf{1}_{Q_t}\right| \gtrsim t^{1+r}$$

$$A(0) = t^r\mathrm{ch}(0).$$

$$\int_\ell \mathrm{ch}(0) = \int_\ell c_1^{(1)} h_1(0) = c_1^{(1)}\zeta(1).$$

$$\left|\mathbf{E}W_\ell(\mathcal{F}_s\tilde{A})\mathbf{1}_{Q_t} - \mathbf{E}W_\ell(\mathcal{F}_sA)\mathbf{1}_{Q_t}\right| \gtrsim t$$

$$\left|\mathbf{E}W_\ell(\mathcal{F}_s\tilde{A})\mathbf{1}_{Q_t} - \mathbf{E}W_\ell(\mathcal{F}_sA)\mathbf{1}_{Q_t}\right| \gtrsim t^{1+r}\left|\mathrm{Tr}\left(\left\{c_1^{(1)}\zeta(1)\right\}^2\right)\right| - o(t^{1+r}) \gtrsim t^{1+r}$$

$$u(x,y,z) = \begin{cases} e^{\psi(y)X} & \text{if } y \in \left[0,\frac{1}{4}\right] \\ 1 & \text{if } y \in \left[\frac{1}{4},\frac{3}{4}\right] \\ e^{\psi(y-1)X} & \text{if } y \in \left[\frac{3}{4},1\right] \end{cases}$$

$$h_2(x,0,0) \stackrel{\text{def}}{=} (\partial_2 u)u^{-1}(x,0,0) = X, x \in [0,1]$$

$$\int_\ell ch = \int_0^1 c_1^{(2)} h_2(x,0,0)\mathrm{d}x = c_1^{(2)} X$$

$$\delta\psi_{\bar{a}}^+ = D_{\bar{a}}\epsilon^+ + \frac{1}{16}F_\# \gamma_{\bar{a}}\epsilon^-, \quad \delta\rho^+ = \gamma^a D_a \epsilon^+,$$

$$\delta\psi_a^- = D_a\epsilon^- + \frac{1}{16}F_\#^T \gamma_a \epsilon^+, \quad \delta\rho^- = \gamma^{\bar{a}} D_{\bar{a}}\epsilon^-.$$

$$R_{a\bar{b}} + \frac{1}{16}\Phi^{-1}\langle F, \Gamma_{a\bar{b}}F\rangle = 0, S = 0, \Gamma^A D_A F = 0,$$

$$\gamma^b D_b \psi_{\bar{a}}^+ - D_{\bar{a}}\rho^+ = \frac{1}{16}\gamma^b F_\# \gamma_{\bar{a}} \psi_b^-, \qquad \gamma^a D_a \rho^+ - D^{\bar{a}}\psi_{\bar{a}}^+ = -\frac{1}{16}F_\# \rho^-,$$

$$\gamma^{\bar{b}} D_{\bar{b}} \psi_a^- - D_a \rho^- = \frac{1}{16}\gamma^{\bar{b}} F_\#^T \gamma_a \psi_{\bar{b}}^+, \qquad \gamma^{\bar{a}} D_{\bar{a}}\rho^- - D^a \psi_a^- = -\frac{1}{16}F_\#^T \rho^+.$$

$$\left\{g_{\mu\nu}, B_{\mu\nu}, \phi, A^{(n)}_{\mu_1\ldots\mu_n}, \psi_\mu^\pm, \lambda^\pm\right\}$$

$$\psi_\mu = \psi_\mu^+ + \psi_\mu^- \ \gamma^{(10)}\psi_\mu^\pm = \mp\psi_\mu^\pm$$

$$\lambda = \lambda^+ + \lambda^- \ \ \gamma^{(10)}\lambda^\pm = \pm\lambda^\pm$$

$$\psi_\mu = \begin{pmatrix}\psi_\mu^+ \\ \psi_\mu^-\end{pmatrix} \quad \gamma^{(10)}\psi_\mu^\pm = \psi_\mu^\pm$$

$$\lambda = \begin{pmatrix}\lambda^+ \\ \lambda^-\end{pmatrix} \quad \gamma^{(10)}\lambda^\pm = -\lambda^\pm.$$

$$\rho^\pm := \gamma^\mu \psi_\mu^\pm - \lambda^\pm,$$

$$S_{\mathrm{B}} = \frac{1}{2\kappa^2}\int \sqrt{-g}\left[\mathrm{e}^{-2\phi}\left(\mathcal{R} + 4(\partial\phi)^2 - \frac{1}{12}H^2\right) - \frac{1}{4}\sum_n \frac{1}{n!}\left(F_{(n)}^{(B)}\right)^2\right],$$

$$F^{(B)} = \sum_n F_{(n)}^{(B)} = \sum_n \mathrm{e}^B \wedge \mathrm{d}A_{(n-1)},$$

$$F_{(n)}^{(B)} = (-)^{[n/2]} * F_{(10-n)}^{(B)},$$

$$\begin{aligned} S_{\mathrm{F}} = -\frac{1}{2\kappa^2}\int \sqrt{-g}\big[&\mathrm{e}^{-2\phi}\big(2\bar{\psi}^{+\mu}\gamma^\nu\nabla_\nu\psi_\mu^+ - 4\bar{\psi}^{+\mu}\nabla_\mu\rho^+ - 2\bar{\rho}^+\nabla\rho^+ \\ &-\frac{1}{2}\bar{\psi}^{+\mu}H\psi_\mu^+ - \bar{\psi}_\mu^+ H^{\mu\nu\lambda}\gamma_\nu\psi_\lambda^+ - \frac{1}{2}\rho^+ H^{\mu\nu\lambda}\gamma_{\mu\nu}\psi_\lambda^+ + \frac{1}{2}\rho^+ H/\rho^+\big) \\ &+\mathrm{e}^{-2\phi}\big(2\bar{\psi}^{-\mu}\gamma^\nu\nabla_\nu\psi_\mu^- - 4\bar{\psi}^{-\mu}\nabla_\mu\rho^- - 2\bar{\rho}^-\nabla\rho^- \\ &+\frac{1}{2}\bar{\psi}^{-\mu}HH\psi_\mu^- + \bar{\psi}_\mu^- H^{\mu\nu\lambda}\gamma_\nu\psi_\lambda^- + \frac{1}{2}\rho^- H^{\mu\nu\lambda}\gamma_{\mu\nu}\psi_\lambda^- - \frac{1}{2}\rho^- H/\rho^-\big) \\ &-\frac{1}{4}\mathrm{e}^{-\phi}\big(\bar{\psi}_\mu^+\gamma^\nu F^{(B)}\gamma^\mu\psi_\nu^- + \rho^+ F^{(B)}\rho^-\big)\big]. \end{aligned}$$

$$\mathcal{R}_{\mu\nu} - \frac{1}{4}H_{\mu\lambda\rho}H_\nu{}^{\lambda\rho} + 2\nabla_\mu\nabla_\nu\phi - \frac{1}{4}\mathrm{e}^{2\phi}\sum_n \frac{1}{(n-1)!}F^{(B)}_{\mu\lambda_1\ldots\lambda_{n-1}}F_\nu^{(B)\lambda_1\ldots\lambda_{n-1}} = 0$$

$$\nabla^\mu\left(\mathrm{e}^{-2\phi}H_{\mu\nu\lambda}\right) - \frac{1}{2}\sum_n \frac{1}{(n-2)!}F^{(B)}_{\mu\nu\lambda_1\ldots\lambda_{n-2}}F^{(B)\lambda_1\ldots\lambda_{n-2}} = 0$$

$$\nabla^2\phi - (\nabla\phi)^2 + \frac{1}{4}\mathcal{R} - \frac{1}{48}H^2 = 0$$

$$\mathrm{d}F^{(B)} - H \wedge F^{(B)} = 0$$

$$\gamma^{\nu}\left[\left(\nabla_{\nu} \mp \frac{1}{24} H_{\nu\lambda\rho}\gamma^{\lambda\rho} - \partial_{\nu}\phi\right)\psi_{\mu}^{\pm} \pm \frac{1}{2} H_{\nu\mu}{}^{\lambda}\psi_{\lambda}^{\pm}\right] - \left(\nabla_{\mu} \mp \frac{1}{8} H_{\mu\nu\lambda}\gamma^{\nu\lambda}\right)\rho^{\pm}$$
$$= \frac{1}{16} \mathrm{e}^{\phi} \sum_{n} (\pm)^{[(n+1)/2]} \gamma^{\nu} \psi_{(n)}^{(B)} \gamma_{\mu} \psi_{\nu}^{\mp},$$
$$\left(\nabla_{\mu} \mp \frac{1}{8} H_{\mu\nu\lambda}\gamma^{\nu\lambda} - 2\partial_{\mu}\phi\right)\psi^{\mu\pm} - \gamma^{\mu}\left(\nabla_{\mu} \mp \frac{1}{24} H_{\mu\nu\lambda}\gamma^{\nu\lambda} - \partial_{\mu}\phi\right)\rho^{\pm}$$
$$= \frac{1}{16} \mathrm{e}^{\phi} \sum_{n} (\pm)^{[(n+1)/2]} F_{(n)}^{(B)} \rho^{\mp},$$
$$\epsilon = \epsilon^{+} + \epsilon^{-} \quad \gamma^{(10)}\epsilon^{\pm} = \mp\epsilon^{\pm}$$
$$\epsilon = \begin{pmatrix} \epsilon^{+} \\ \epsilon^{-} \end{pmatrix} \quad \gamma^{(10)}\epsilon^{\pm} = \epsilon^{\pm}$$
$$\delta e_{\mu}^{a} = \bar{\epsilon}^{+}\gamma^{a}\psi_{\mu}^{+} + \bar{\epsilon}^{-}\gamma^{a}\psi_{\mu}^{-}$$
$$\delta B_{\mu\nu} = 2\bar{\epsilon}^{+}\gamma_{[\mu}\psi_{\nu]}^{+} - 2\bar{\epsilon}^{-}\gamma_{[\mu}\psi_{\nu]}^{-},$$
$$\delta\phi - \frac{1}{4}\delta\log(-g) = -\frac{1}{2}\bar{\epsilon}^{+}\rho^{+} - \frac{1}{2}\bar{\epsilon}^{-}\rho^{-},$$
$$(\mathrm{e}^{B} \wedge \delta A)_{\mu_1\ldots\mu_n}^{(n)} = \frac{1}{2}\left(\mathrm{e}^{-\phi}\bar{\psi}_{\nu}^{+}\gamma_{\mu_1\ldots\mu_n}\gamma^{\nu}\epsilon^{-} - \mathrm{e}^{-\phi}\bar{\epsilon}^{+}\gamma_{\mu_1\ldots\mu_n}\rho^{-}\right)$$
$$\mp \frac{1}{2}\left(\mathrm{e}^{-\phi}\bar{\epsilon}^{+}\gamma^{\nu}\gamma_{\mu_1\ldots\mu_n}\psi_{\nu}^{-} + \mathrm{e}^{-\phi}\bar{\rho}^{+}\gamma_{\mu_1\ldots\mu_n}\epsilon^{-}\right),$$
$$\delta\psi_{\mu}^{\pm} = \left(\nabla_{\mu} \mp \frac{1}{8} H_{\mu\nu\lambda}\gamma^{\nu\lambda}\right)\epsilon^{\pm} + \frac{1}{16}\mathrm{e}^{\phi}\sum_{n}(\pm)^{[(n+1)/2]} F_{(n)}^{(B)}\gamma_{\mu}\epsilon^{\mp},$$
$$\delta\rho^{\pm} = \gamma^{\mu}\left(\nabla_{\mu} \mp \frac{1}{24} H_{\mu\nu\lambda}\gamma^{\nu\lambda} - \partial_{\mu}\phi\right)\epsilon^{\pm}.$$
$$B_{(i)} = B_{(j)} - \mathrm{d}\Lambda_{(ij)}$$
$$\Lambda_{(ij)} + \Lambda_{(jk)} + \Lambda_{(ki)} = \mathrm{d}\Lambda_{(ijk)},$$
$$A_{(i)} = \mathrm{e}^{\mathrm{d}\Lambda_{(ij)}} \wedge A_{(j)} + \mathrm{d}\hat{\Lambda}_{(ij)},$$
$$B'_{(i)} = B_{(i)} - \mathrm{d}\lambda_{(i)}, A'_{(i)} = \mathrm{e}^{\mathrm{d}\lambda_{(i)}} A_{(i)},$$
$$\Omega_{\mathrm{cl}}^{2}(M) \longrightarrow G_{\mathrm{NS}} \longrightarrow \mathrm{Diff}(M),$$
$$\delta_{v+\lambda} g = \mathcal{L}_{v} g, \delta_{v+\lambda}\phi = \mathcal{L}_{v}\phi, \delta_{v+\lambda} B_{(i)} = \mathcal{L}_{v} B_{(i)} - \mathrm{d}\lambda_{(i)},$$
$$\mathrm{d}\lambda_{(i)} = \mathrm{d}\lambda_{(j)} - \mathcal{L}_{v}\,\mathrm{d}\Lambda_{(ij)}$$
$$\lambda_{(i)} = \lambda_{(j)} - i_{v}\,\mathrm{d}\Lambda_{(ij)},$$
$$0 \longrightarrow T^{*}M \longrightarrow E \longrightarrow TM \longrightarrow 0$$
$$v_{(i)} + \lambda_{(i)} = v_{(j)} + \left(\lambda_{(j)} - i_{v_{(j)}}\mathrm{d}\Lambda_{(ij)}\right),$$
$$\langle V, V\rangle = i_{v}\lambda,$$
$$\tilde{E} = \det T^{*}M \otimes E$$
$$\left\langle \hat{E}_{A}, \hat{E}_{B}\right\rangle = \Phi^{2}\eta_{AB}, \ \eta = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$
$$\tilde{F} = \left\{\left(x, \{\hat{E}_{A}\}\right): x \in M, \text{and } \{\hat{E}_{A}\}\ \tilde{E}_{x}\right\}.$$
$$V^{A} \mapsto V'^{A} = M_{B}^{A} V^{B}, \hat{E}_{A} \mapsto \hat{E}'_{A} = \hat{E}_{B}(M^{-1})^{B}{}_{A}.$$
$$V^{M} = \begin{cases} v^{\mu} & \text{for } M = \mu \\ \lambda_{\mu} & \text{for } M = \mu + d \end{cases}$$
$$E_{(p)}^{\otimes n} = (\det T^{*}M)^{p} \otimes E \otimes \cdots \otimes E.$$
$$\{\Gamma_{A}, \Gamma_{B}\} = 2\eta_{AB}$$
$$V^{A}\Gamma_{A}\Psi_{(i)} = i_{v}\Psi_{(i)} + \lambda_{(i)} \wedge \Psi_{(i)}$$
$$\Psi_{(i)} = \mathrm{e}^{\mathrm{d}\Lambda_{(ij)}} \wedge \Psi_{(j)}$$

$$S^{\pm}_{(p)} = (\det T^*M)^p \otimes S^{\pm}(E)$$
$$\langle \Psi, \Psi' \rangle = \sum_n (-)^{[(n+1)/2]} \Psi^{(d-n)} \wedge \Psi'^{(n)} \in \Gamma((\det T^*M)^{2p})$$
$$\hat{E}_A = \begin{cases} \hat{E}_a = (\det e)\left(\hat{e}_a + i_{\hat{e}_a} B\right) & \text{para } A = a \\ E^a = (\det e) e^a & \text{para } A = a + d \end{cases}$$
$$\left\langle \hat{E}_A, \hat{E}_B \right\rangle = (\det e)^2 \eta_{AB}$$
$$\begin{aligned} V^{(B)} &= v^a (\det e) \hat{e}_a + \lambda_a (\det e) e^a \\ &= v_{(i)} + \lambda_{(i)} - i_{v_{(i)}} B_{(i)}, \end{aligned}$$
$$M = (\det A)^{-1} \begin{pmatrix} 1 & 0 \\ \omega & 1 \end{pmatrix} \begin{pmatrix} A & 0 \\ 0 & (A^{-1})^T \end{pmatrix},$$
$$\hat{E}_A = \begin{cases} \hat{E}_a = \mathrm{e}^{-2\phi} (\det e)\left(\hat{e}_a + i_{\hat{e}_a} B\right) & \text{para } A = a \\ E^a = \mathrm{e}^{-2\phi} (\det e) e^a & \text{para } A = a + d \end{cases}$$
$$V^{(B,\phi)} = \mathrm{e}^{2\phi} \left(v_{(i)} + \lambda_{(i)} - i_{v_{(i)}} B_{(i)} \right).$$
$$\Psi^{(B)} = \sum_n \frac{1}{n!} \Psi^{(B)}_{a_1 \dots a_n} e^{a_1} \wedge \dots \wedge e^{a_n} = \mathrm{e}^{B_{(i)}} \wedge \Psi_{(i)},$$
$$\Psi^{(B,\phi)} = \mathrm{e}^{p\phi} \mathrm{e}^{B_{(i)}} \wedge \Psi_{(i)}.$$
$$L_V W = \mathcal{L}_v w + \mathcal{L}_v \zeta - i_w \, \mathrm{d}\lambda$$
$$\mathcal{L}_v w^\mu = v^\nu \partial_\nu w^\mu - w^\nu \partial_\nu v^\mu + p(\partial_\nu v^\nu) w^\mu,$$
$$\mathcal{L}_v \zeta_\mu = v^\nu \partial_\nu \zeta_\mu + \left(\partial_\mu v^\nu\right) \zeta_\nu + p(\partial_\nu v^\nu) \zeta^\mu.$$
$$\partial_M = \begin{cases} \partial_\mu & \text{para } M = \mu \\ 0 & \text{para } M = \mu + d \end{cases}.$$
$$L_V W^M = V^N \partial_N W^M + (\partial^M V^N - \partial^N V^M) W_N + p(\partial_N V^N) W^M$$
$$m \cdot W = \begin{pmatrix} a & 0 \\ -\omega & -a^T \end{pmatrix} \begin{pmatrix} w \\ \zeta \end{pmatrix} - p\mathrm{tr} a \begin{pmatrix} w \\ \zeta \end{pmatrix}$$
$$\begin{aligned} L_V \alpha^{M_1 \dots M_n} =& V^N \partial_N \alpha^{M_1 \dots M_n} + (\partial^{M_1} V^N - \partial^N V^{M_1}) \alpha_N{}^{M_2 \dots M_n} \\ &+ \dots + (\partial^{M_n} V^N - \partial^N V^{M_n}) \alpha^{M_1 \dots M_{n-1}}{}_N + p(\partial_N V^N) W^M \end{aligned}$$
$$L_V \Psi = V^N \partial_N \Psi + \frac{1}{4} (\partial_M V_N - \partial_N V_M) \Gamma^{MN} \Psi + p(\partial_M V^M) \Psi$$
$$\begin{aligned} [\,[V, W]\,] &= \frac{1}{2} (L_V W - L_W V) \\ &= [v, w] + \mathcal{L}_v \zeta - \mathcal{L}_w \lambda - \frac{1}{2} \, \mathrm{d}(i_v \zeta - i_w \lambda), \end{aligned}$$
$$[\,[U, V]\,]^M = U^N \partial_N V^M - V^N \partial_N U^M - \frac{1}{2} (U_N \partial^M V^N - V_N \partial^M U^N).$$
$$(\mathrm{d}\Psi)_{(i)} = \frac{1}{2} \Gamma^M \partial_M \Psi_{(i)} = \mathrm{d}\Psi_{(i)},$$
$$D_M W^A = \partial_M W^A + \tilde{\Omega}_M{}^A{}_B W^B$$
$$\tilde{\Omega}_M{}^A{}_B = \Omega_M{}^A{}_B - \Lambda_M \delta^A{}_B,$$
$$\Omega_M^{AB} = -\Omega_M{}^{BA}.$$
$$\begin{aligned} D_M \alpha^{A_1 \dots A_n} &= \partial_M \alpha^{A_1 \dots A_n} + \Omega_M{}^{A_1}{}_B \alpha^{B A_2 \dots A_n} \\ &+ \dots + \Omega_M{}^{A_n}{}_B \alpha^{A_1 \dots A_{n-1} B} - p \Lambda_M \alpha^{A_1 \dots A_n}. \end{aligned}$$
$$D_M \Psi = \left(\partial_M + \frac{1}{4} \Omega_M^{AB} \Gamma_{AB} - p \Lambda_M \right) \Psi$$
$$W = W^A \hat{E}_A = w^a \hat{E}_a + \zeta_a E^a$$
$$(D_M^\nabla W^A) \hat{E}_A = \begin{cases} (\nabla_\mu w^a) \hat{E}_a + (\nabla_\mu \zeta_a) E^a & \text{para } M = \mu \\ 0 & \text{para } M = \mu + d \end{cases}.$$

$$T(V)\cdot\alpha = L_V^D\alpha - L_V\alpha$$
$$T_{ABC} = -3\tilde{\Omega}_{[ABC]} + \tilde{\Omega}_D{}^D{}_B\eta_{AC} - \Phi^{-2}\langle \hat{E}_A, L_{\Phi^{-1}\hat{E}_B}\hat{E}_C\rangle$$
$$T \in \Gamma(\Lambda^3 E \oplus E)$$
$$T^M{}_{PN} = (T_1)^M{}_{PN} - (T_2)_P\delta^M{}_N,$$
$$(T_1)_{MNP} = -3\tilde{\Omega}_{[MNP]} = -3\Omega_{[MNP]}$$
$$(T_2)_M = -\tilde{\Omega}_{Q\ M} = \Lambda_M - \Omega_{Q\ M}$$
$$\Gamma^M D_M\Psi = \Gamma^M\left(\partial_M\Psi + \frac{1}{4}\Omega_{MNP}\Gamma^{NP}\Psi - \frac{1}{2}\Lambda_M\Psi\right)$$
$$= \Gamma^M\partial_M\Psi + \frac{1}{4}\Omega_{[MNP]}\Gamma^{MNP}\Psi - \frac{1}{2}(\Lambda_M - \Omega_{N\ M})\Gamma^M\Psi$$
$$= 2\,\mathrm{d}\Psi - \frac{1}{12}(T_1)_{[MNP]}\Gamma^{MNP}\Psi - \frac{1}{2}(T_2)_M\Gamma^M\Psi.$$
$$\Gamma^M D_M\Psi = 2\,\mathrm{d}\Psi$$
$$L^{\hat{E}_A}_{\Phi-1}\hat{E}_B = \left(L_{\Phi^{-1}\hat{E}_A}\Phi\right)\Phi^{-1}\hat{E}_B + \Phi\left(L_{\Phi^{-1}\hat{E}_A}\left(\Phi^{-1}\hat{E}_B\right)\right)$$
$$L_{\Phi^{-1}\hat{E}_A}\Phi = \begin{cases} -\mathrm{e}^{-2\phi}(\det e)\left(i_{\hat{e}_a}i_{\hat{e}_b}\,\mathrm{d}e^b + 2i_{\hat{e}_a}\,\mathrm{d}\phi\right) & \text{para } A = a \\ 0 & \text{para } A = a + d \end{cases}$$
$$L_{\Phi^{-1}\hat{E}_A}\Phi^{-1}\hat{E}_B = \begin{pmatrix} [\hat{e}_a,\hat{e}_b] + i_{\hat{e}_a,\hat{e}_b}B - i_{\hat{e}_a}i_{\hat{e}_b}H & \mathcal{L}_{\hat{e}_a}e^b \\ -\hat{e}_{\hat{e}_b}e^a & 0 \end{pmatrix}_{AB}$$
$$T_1 = -4H, T_2 = -4\,\mathrm{d}\phi,$$
$$R(U,V,W) = [D_U, D_V]W - D_{[\,[U,V]\,]}W$$
$$[D_{fU}, D_{gV}]hW - D_{[\,[fU,gV]\,]}hW$$
$$= \mathrm{fgh}\left([D_U, D_V]W - D_{[\,[U,V]\,]}W\right) - \frac{1}{2}h\langle U,V\rangle D_{(f\,\mathrm{d}g - g\,\mathrm{d}f)}W,$$
$$E = C_+ \oplus C_-,$$
$$\langle \hat{E}_a^+, \hat{E}_b^+\rangle = \Phi^2\eta_{ab},$$
$$\langle \hat{E}_{\bar{a}}^-, \hat{E}_{\bar{b}}^-\rangle = -\Phi^2\eta_{\bar{a}\bar{b}},$$
$$\langle \hat{E}_a^+, \hat{E}_{\bar{a}}^-\rangle = 0.$$
$$\hat{E}_A = \begin{cases} \hat{E}_a^+ & \text{para } A = a \\ \hat{E}_{\bar{a}}^- & \text{para } A = \bar{a} + d \end{cases}$$
$$\langle \hat{E}_A, \hat{E}_B\rangle = \Phi^2\eta_{AB}, \text{ where } \eta_{AB} = \begin{pmatrix} \eta_{ab} & 0 \\ 0 & -\eta_{\bar{a}\bar{b}} \end{pmatrix}$$
$$\hat{E}^A = \begin{cases} \hat{E}^{+a} & \text{para } A = a \\ -\hat{E}^{-\bar{a}} & \text{para } A = \bar{a} + d' \end{cases}$$
$$\hat{E}_a^+ = \mathrm{e}^{-2\phi}\sqrt{-g}\left(\hat{e}_a^+ + e_a^+ + i_{\hat{e}_a^+}B\right),$$
$$\hat{E}_{\bar{a}}^- = \mathrm{e}^{-2\phi}\sqrt{-g}\left(\hat{e}_{\bar{a}}^- - e_{\bar{a}}^- + i_{\hat{e}_{\bar{a}}}B\right),$$
$$\Phi = \mathrm{e}^{-2\phi}\sqrt{-g}$$
$$g = \eta_{ab}e^{+a}\otimes e^{+b} = \eta_{\bar{a}\bar{b}}e^{-\bar{a}}\otimes e^{-\bar{b}},$$
$$g(\hat{e}_a^+, \hat{e}_b^+) = \eta_{ab}, g\left(\hat{e}_{\bar{a}}^-, \hat{e}_{\bar{b}}^-\right) = \eta_{\bar{b}\bar{b}}.$$
$$G = \Phi^{-2}\left(\eta^{ab}\hat{E}_a^+\otimes\hat{E}_b^+ + \eta^{\bar{a}\bar{b}}\hat{E}_{\bar{a}}^-\otimes\hat{E}_{\bar{b}}^-\right)$$
$$G_{MN} = \frac{1}{2}\begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}_{MN}$$
$$\Gamma^{(+)} = \frac{1}{d!}\epsilon^{a_1\dots a_d}\Gamma_{a_1}\dots\Gamma_{a_d}, \qquad \Gamma^{(-)} = \frac{1}{d!}\epsilon^{\bar{a}_1\dots\bar{a}_d}\Gamma_{\bar{a}_1}\dots\Gamma_{\bar{a}_d}.$$
$$\Gamma_a\cdot\Psi^{(B)} = i_{\hat{e}_a^+}\Psi^{(B)} + e_a^+\wedge\Psi^{(B)}, \Gamma_{\bar{a}}\cdot\Psi^{(B)} = i_{\hat{e}_{\bar{a}}^-}\Psi^{(B)} - e_a^-\wedge\Psi^{(B)}$$

$$\Gamma^{(+)}\Psi_{(n)}^{(B)} = (-)^{[n/2]} * \Psi_{(n)}^{(B)}, \qquad \Gamma^{(-)}\Psi_{(n)}^{(B)} = (-)^{d}(-)^{[n+1/2]} * \Psi_{(n)}^{(B)}$$

$$\Gamma^{(-)}\Gamma^{A}\Gamma^{(-)-1} = G_{B}^{A}\Gamma^{B}$$

$$DG = 0, D\Phi = 0$$

$$W = w_{+}^{a}\hat{E}_{a}^{+} + w_{-}^{\bar{a}}\hat{E}_{\bar{a}}^{-},$$

$$D_{M}W^{A} = \begin{cases} \partial_{M}w_{+}^{a} + \Omega_{M}{}^{a}{}_{b}w_{+}^{b} & \text{para } A = a \\ \partial_{M}w_{-}^{\bar{a}} + \Omega_{M}{}^{\bar{a}}{}_{\bar{b}}w_{-}^{\bar{b}} & \text{para } A = \bar{a} \end{cases},$$

$$\Omega_{Mab} = -\Omega_{Mba}, \Omega_{M\bar{a}\bar{b}} = -\Omega_{M\bar{b}\bar{a}}.$$

$$\nabla_{\mu}v^{\nu} = \left(\partial_{\mu}v^{a} + \omega_{\mu}^{+a}{}_{b}v^{b}\right)(\hat{e}_{a}^{+})^{\nu} = \left(\partial_{\mu}v^{\bar{a}} + \omega_{\mu}^{-\bar{a}}\bar{b}v^{\bar{b}}\right)(\hat{e}_{\bar{a}}^{-})^{\nu}.$$

$$D_{M}^{\nabla}W^{a} = \begin{cases} \nabla_{\mu}w_{+}^{a} & \text{para } M = \mu \\ 0 & \text{para } M = \mu + d \end{cases}, D_{M}^{\nabla}W^{\bar{a}} = \begin{cases} \nabla_{\mu}w_{-}^{\bar{a}} & \text{para } M = \mu \\ 0 & \text{para } M = \mu + d \end{cases}.$$

$$W = w_{+}^{a}\hat{E}_{a}^{+} + w_{-}^{\bar{a}}\hat{E}_{\bar{a}}^{-} = (w_{+}^{a} + w_{-}^{a})\hat{E}_{a} + (w_{+a} - w_{-a})E^{a},$$

$$T_{1} = -4H, T_{2} = -4\,\mathrm{d}\phi$$

$$D_{M}W^{A} = D_{M}^{\nabla}W^{A} + \Sigma_{M}{}^{A}{}_{B}W^{B}$$

$$\Sigma_{Mab} = -\Sigma_{Mba}, \Sigma_{M\bar{a}\bar{b}} = -\Sigma_{M\bar{b}\bar{a}}.$$

$$(T_{1})_{ABC} = -4H_{ABC} - 3\Sigma_{[ABC]}, (T_{2})_{A} = -4\,\mathrm{d}\phi_{A} - \Sigma_{C}{}^{C}{}_{A}.$$

$$\mathrm{d}x^{\mu} = \frac{1}{2}\Phi^{-1}\left(\hat{e}_{a}^{+\mu}\hat{E}^{+a} - \hat{e}_{\bar{a}}^{-\mu}\hat{E}^{-\bar{a}}\right),$$

$$\mathrm{d}\phi = \frac{1}{2}\partial_{a}\phi\left(\Phi^{-1}\hat{E}^{+a}\right) - \frac{1}{2}\partial_{\bar{a}}\phi\left(\Phi^{-1}\hat{E}^{-\bar{a}}\right).$$

$$\Lambda^{3}T^{*}M \hookrightarrow \Lambda^{3}E \simeq \Lambda^{3}C_{+} \oplus (\Lambda^{2}C_{+} \otimes C_{-}) \oplus (C_{+} \otimes \Lambda^{2}C_{-}) \oplus \Lambda^{3}C_{-},$$

$$\mathrm{d}\phi_{A} = \begin{cases} \frac{1}{2}\partial_{a}\phi & A = a \\ \frac{1}{2}\partial_{\bar{a}}\phi & A = \bar{a} + d \end{cases}, H_{ABC} = \begin{cases} \frac{1}{8}H_{abc} & (A,B,C) = (a,b,c) \\ \frac{1}{8}H_{ab\bar{c}} & (A,B,C) = (a,b,\bar{c}+d) \\ \frac{1}{8}H_{a\bar{b}\bar{c}} & (A,B,C) = (a,\bar{b}+d,\bar{c}+d) \\ \frac{1}{8}H_{\bar{a}\bar{b}\bar{c}} & (A,B,C) = (\bar{a}+d,\bar{b}+d,\bar{c}+d) \end{cases}$$

$$\Sigma_{[abc]} = -\frac{1}{6}H_{abc}, \quad \Sigma_{\bar{a}bc} = -\frac{1}{2}H_{\bar{a}bc}, \quad \Sigma_{a}{}^{a}{}_{b} = -2\partial_{b}\phi,$$

$$\Sigma_{[\bar{a}\bar{b}\bar{c}]} = +\frac{1}{6}H_{\bar{a}\bar{b}\bar{c}}, \quad \Sigma_{a\bar{b}\bar{c}} = +\frac{1}{2}H_{a\bar{b}\bar{c}}, \quad \Sigma_{\bar{a}}{}_{\bar{b}} = -2\partial_{\bar{b}}\phi.$$

$$D_{a}w_{+}^{b} = \nabla_{a}w_{+}^{b} - \frac{1}{6}H_{a}{}^{b}{}_{c}w_{+}^{c} - \frac{2}{9}(\delta_{a}{}^{b}\partial_{c}\phi - \eta_{ac}\partial^{b}\phi)w_{+}^{c} + A_{a}^{+b}{}_{c}w_{+}^{c},$$

$$D_{\bar{a}}w_{+}^{b} = \nabla_{\bar{a}}w_{+}^{b} - \frac{1}{2}H_{\bar{a}}{}^{b}{}_{c}w_{+}^{c},$$

$$D_{a}w_{-}^{\bar{b}} = \nabla_{a}w_{-}^{\bar{b}} + \frac{1}{2}H_{a}{}^{\bar{b}}{}_{\bar{c}}w_{-}^{\bar{c}},$$

$$D_{\bar{a}}w_{-}^{\bar{b}} = \nabla_{\bar{a}}w_{-}^{\bar{b}} + \frac{1}{6}H_{\bar{a}}^{\bar{b}}{}_{\bar{c}}w_{-}^{\bar{c}} - \frac{2}{9}\left(\delta_{\bar{a}}{}^{\bar{b}}\partial_{\bar{c}}\phi - \eta_{\bar{a}\bar{c}}\partial^{\bar{b}}\phi\right)w_{-}^{\bar{c}} + A_{\bar{a}}^{-\bar{b}}\bar{c}_{\bar{c}}w_{-}^{\bar{c}},$$

$$A_{abc}^{+} = -A_{acb}^{+}, \quad A_{[abc]}^{+} = 0, \quad A_{a}^{+a}{}_{b} = 0,$$

$$A_{\bar{a}\bar{b}\bar{c}}^{-} = -A_{\bar{a}\bar{c}\bar{b}}^{-}, \quad A_{[\bar{a}\bar{b}\bar{c}]}^{-} = 0, \quad A_{\bar{a}}^{-\bar{a}}\bar{b} = 0$$

$$D_{\bar{a}}w_{+}^{b} = \nabla_{\bar{a}}w_{+}^{b} - \frac{1}{2}H_{\bar{a}}{}^{b}{}_{c}w_{+}^{c},$$

$$D_{a}w_{-}^{\bar{b}} = \nabla_{a}w_{-}^{\bar{b}} + \frac{1}{2}H_{a}{}^{\bar{b}}{}_{\bar{c}}w_{-}^{\bar{c}},$$

$$D_a w_+^a = \nabla_a w_+^a - 2(\partial_a \phi) w_+^a,$$
$$D_{\bar{a}} w_-^{\bar{a}} = \nabla_{\bar{a}} w_-^{\bar{a}} - 2(\partial_{\bar{a}} \phi) w_-^{\bar{a}}.$$
$$D_M \epsilon^+ = \partial_M \epsilon^+ + \frac{1}{4} \Omega_M{}^{ab} \gamma_{ab} \epsilon^+,$$
$$D_M \epsilon^- = \partial_M \epsilon^- + \frac{1}{4} \Omega_M{}^{\bar{a}\bar{b}} \gamma_{\bar{a}\bar{b}} \epsilon^-.$$

$$D_{\bar{a}} \epsilon^+ = \left(\nabla_{\bar{a}} - \frac{1}{8} H_{\bar{a}bc} \gamma^{bc}\right) \epsilon^+,$$
$$D_a \epsilon^- = \left(\nabla_a + \frac{1}{8} H_{a\bar{b}} \gamma^{\bar{b}\bar{c}}\right) \epsilon^-,$$
$$\gamma^a D_a \epsilon^+ = \left(\gamma^a \nabla_a - \frac{1}{24} H_{abc} \gamma^{abc} - \gamma^a \partial_a \phi\right) \epsilon^+,$$
$$\gamma^{\bar{a}} D_{\bar{a}} \epsilon^- = \left(\gamma^{\bar{a}} \nabla_{\bar{a}} + \frac{1}{24} H_{\bar{a}\bar{b}\bar{c}} \gamma^{\bar{a}\bar{c}} - \gamma^{\bar{a}} \partial_{\bar{a}} \phi\right) \epsilon^-.$$
$$R_{a\bar{b}} w_+^a = [D_a, D_{\bar{b}}] w_+^a$$
$$R_{\bar{a}b} w_-^{\bar{a}} = [D_{\bar{a}}, D_b] w_-^{\bar{a}}$$
$$\frac{1}{2} R_{a\bar{b}} \gamma^a \epsilon^+ = [\gamma^a D_a, D_{\bar{b}}] \epsilon^+,$$
$$\frac{1}{2} R_{\bar{a}} \gamma \gamma^{\bar{a}} \epsilon^- = [\gamma^{\bar{a}} D_{\bar{a}}, D_b] \epsilon^-.$$
$$-\frac{1}{4} S \epsilon^+ = (\gamma^a D_a \gamma^b D_b - D^{\bar{a}} D_{\bar{a}}) \epsilon^+$$
$$-\frac{1}{4} S \epsilon^- = \left(\gamma^{\bar{a}} D_{\bar{a}} \gamma^{\bar{b}} D_{\bar{b}} - D^a D_a\right) \epsilon^-$$
$$R_{ab} = \mathcal{R}_{ab} - \frac{1}{4} H_{acd} H_b^{cd} + 2\nabla_a \nabla_b \phi + \frac{1}{2} \mathrm{e}^{2\phi} \nabla^c \left(\mathrm{e}^{-2\phi} H_{cab}\right),$$
$$S = \mathcal{R} + 4\nabla^2 \phi - 4(\partial \phi)^2 - \frac{1}{12} H^2$$
$$\mathrm{d}s_{10}^2 = \mathrm{d}s^2(\mathbb{R}^{9-d,1}) + \mathrm{d}s_d^2$$
$$\{g, B, \phi\} \in \frac{O(10,10)}{O(9,1) \times O(1,9)} \times \mathbb{R}^+$$
$$\delta_V G = L_V G, \delta_V \Phi = L_V \Phi$$
$$\psi_{\bar{a}}^+ \in \Gamma\left(C_- \otimes S^{\mp}(C_+)\right), \psi_a^- \in \Gamma\left(C_+ \otimes S^+(C_-)\right),$$
$$\rho^+ \in \Gamma\left(S^{\pm}(C_+)\right), \rho^- \in \Gamma\left(S^+(C_-)\right)$$
$$\epsilon^+ \in \Gamma\left(S^{\mp}(C_+)\right), \epsilon^- \in \Gamma\left(S^+(C_-)\right)$$
$$F \in \Gamma\left(S_{(1/2)}^{\pm}\right),$$
$$F^{(B)} = \mathrm{e}^{B_{(i)}} \wedge F_{(i)} = \mathrm{e}^{B_{(i)}} \wedge \sum_n \mathrm{d}A_{(i)}^{(n-1)}.$$
$$\Gamma^A = \begin{cases} \gamma^a \otimes 1 & \text{para } A = a \\ \gamma^{(10)} \otimes \gamma^{\bar{a}} \gamma^{(10)} & \text{para } A = \bar{a} + d \end{cases}$$
$$S_{(1/2)} \simeq S(C_+) \otimes S(C_-)$$
$$F_\#: S(C_-) \to S(C_+).$$
$$F_\#^T = (C F_\# C^{-1})^T,$$
$$\not{F}^{(B,\phi)} = \sum_n \frac{1}{n!} F_{a_1 \dots a_n}^{(B,\phi)} \gamma^{a_1 \dots a_n}.$$

$$F_{\#} = \Lambda^{+} F^{(B,\phi)} (\Lambda^{-})^{-1}$$

$$F_{(i)} = \mathrm{e}^{-B_{(i)}} \wedge F^{(B)} = \mathrm{e}^{-\phi} \mathrm{e}^{-B_{(i)}} \wedge F^{(B,\phi)}$$

$$= \mathrm{e}^{-\phi} \mathrm{e}^{-B_{(i)}} \wedge \sum_{n} \left[\frac{1}{32(n!)} (-)^{[n/2]} \mathrm{tr}\big(\gamma_{(n)} (\Lambda^{+})^{-1} F_{\#} \Lambda^{-} \big) \right].$$

$$\Gamma^{(-)} F = -F,$$

$$\delta \psi_{\bar{a}}^{+} = D_{\bar{a}} \epsilon^{+} + \frac{1}{16} F_{\#} \gamma_{\bar{a}} \epsilon^{-},$$

$$\delta \psi_{a}^{-} = D_{a} \epsilon^{-} + \frac{1}{16} F_{\#}^{T} \gamma_{a} \epsilon^{+}$$

$$\delta \rho^{+} = \gamma^{a} D_{a} \epsilon^{+},$$

$$\delta \rho^{-} = \gamma^{\bar{a}} D_{\bar{a}} \epsilon^{-},$$

$$\tilde{\delta} \hat{E}_{a}^{+} = (\delta \log \Phi) \hat{E}_{a}^{+} - \big(\delta \Lambda_{a\bar{b}}^{+} \big) \hat{E}^{-\bar{b}},$$

$$\tilde{\delta} \hat{E}_{\bar{a}}^{-} = (\delta \log \Phi) \hat{E}_{\bar{a}}^{-} - (\delta \Lambda_{\bar{a}b}^{-}) \hat{E}^{+b},$$

$$\delta \Lambda_{a\bar{a}}^{+} = \bar{\epsilon}^{+} \gamma_{a} \psi_{\bar{a}}^{+} + \bar{\epsilon}^{-} \gamma_{\bar{a}} \psi_{a}^{-},$$

$$\delta \Lambda_{a\bar{a}}^{-} = \bar{\epsilon}^{+} \gamma_{a} \psi_{\bar{a}}^{+} + \bar{\epsilon}^{-} \gamma_{\bar{a}} \psi_{a}^{-},$$

$$\delta \log \Phi = -2 \delta \phi + \frac{1}{2} \delta \log (-g) = \bar{\epsilon}^{+} \rho^{+} + \bar{\epsilon}^{-} \rho^{-}$$

$$\tilde{\delta} e_{\mu}^{+a} = \bar{\epsilon}^{+} \gamma_{\mu} \psi^{+a} + \bar{\epsilon}^{-} \gamma^{a} \psi_{\mu}^{-},$$

$$\tilde{\delta} e_{\mu}^{-\bar{a}} = \bar{\epsilon}^{+} \gamma^{\bar{a}} \psi_{\mu}^{+} + \bar{\epsilon} \gamma_{\mu} \psi^{-\bar{a}},$$

$$\tilde{\delta} g_{\mu\nu} = 2 \bar{\epsilon}^{+} \gamma_{(\mu} \psi_{\nu)}^{+} + 2 \bar{\epsilon} \gamma_{(\mu} \psi_{\nu)}^{-}$$

$$\tilde{\delta} e_{\mu}^{+a} = \delta e_{\mu}^{+a} - (\bar{\epsilon}^{+} \gamma^{a} \psi^{+b} - \bar{\epsilon}^{+} \gamma^{b} \psi^{+a}) e_{\mu b}^{+},$$

$$\tilde{\delta} e_{\mu}^{-\bar{a}} = \delta e_{\mu}^{+\bar{a}} - \big(\bar{\epsilon}^{-} \gamma^{\bar{a}} \psi^{-\bar{b}} - \bar{\epsilon} \gamma^{\bar{b}} \psi^{-\bar{a}} \big) e_{\mu \bar{b}}^{-}.$$

$$\delta G_{a\bar{a}} = \delta G_{\bar{a}a} = 2 (\bar{\epsilon}^{+} \gamma_{a} \psi_{\bar{a}}^{+} + \bar{\epsilon} \gamma_{\bar{a}} \psi_{a}^{-})$$

$$\frac{1}{16} (\delta A_{\#}) = (\gamma^{a} \epsilon^{+} \bar{\psi}_{a}^{-} - \rho^{+} \bar{\epsilon}^{-}) \mp (\psi_{\bar{a}}^{+} \bar{\epsilon}^{-} \gamma^{\bar{a}} + \epsilon^{+} \bar{\rho}^{-})$$

$$R_{a\bar{b}} + \frac{1}{16} \Phi^{-1} \langle F, \Gamma_{a\bar{b}} F \rangle = 0$$

$$S = 0$$

$$\frac{1}{2} \Gamma^{A} D_{A} F = \mathrm{d} F = 0$$

$$S_{B} = \frac{1}{2\kappa^{2}} \int \left(\Phi S + \frac{1}{4} \big\langle F, \Gamma^{(-)} F \big\rangle \right)$$

$$\begin{aligned} S_{F} = -\frac{1}{2\kappa^{2}} \int \; 2\Phi [& \bar{\psi}^{+\bar{a}} \gamma^{b} D_{b} \psi_{\bar{a}}^{+} + \bar{\psi}^{-a} \gamma^{\bar{b}} D_{\bar{b}} \psi_{a}^{-} \\ & + 2 \bar{\rho}^{+} D_{\bar{a}} \psi^{+\bar{a}} + 2 \bar{\rho}^{-} D_{a} \psi^{-a} \\ & - \bar{\rho}^{+} \gamma^{a} D_{a} \rho^{+} - \bar{\rho}^{-} \gamma^{\bar{a}} D_{\bar{a}} \rho^{-} \\ & - \frac{1}{8} (\bar{\rho}^{+} F_{\#} \rho^{-} + \bar{\psi}_{\bar{a}}^{+} \gamma^{a} F_{\#} \gamma^{\bar{a}} \psi_{a}^{-}) \Big]. \end{aligned}$$

$$\gamma^{b} D_{b} \psi_{\bar{a}}^{+} - D_{\bar{a}} \rho^{+} = + \frac{1}{16} \gamma^{b} F_{\#} \gamma_{\bar{a}} \psi_{b}^{-},$$

$$\gamma^{\bar{b}} D_{\bar{b}} \psi_{a}^{-} - D_{a} \rho^{-} = + \frac{1}{16} \gamma^{\bar{b}} F_{\#}^{T} \gamma_{a} \psi_{\bar{b}}^{+},$$

$$\gamma^{a} D_{a} \rho^{+} - D^{\bar{a}} \psi_{\bar{a}}^{+} = - \frac{1}{16} F_{\#} \rho^{-},$$

$$\gamma^{\bar{a}} D_{\bar{a}} \rho^{-} - D^{a} \psi_{a}^{-} = - \frac{1}{16} F_{\#}^{T} \rho^{+},$$

$$\omega_{(k)} = \frac{1}{k!}\omega_{\mu_1\dots\mu_k}\, \mathrm{d}x^{\mu_1}\wedge\cdots\wedge\, \mathrm{d}x^{\mu_k}$$
$$\omega_{(k)}\wedge\eta_{(l)} = \frac{1}{(k+l)!}\left(\frac{(k+l)!}{k!\,l!}\omega_{[\mu_1\dots\mu_k}\eta_{\mu_{k+1}\dots\mu_{k+l}]}\right)\mathrm{d}x^{\mu_1}\wedge\cdots\wedge\, \mathrm{d}x^{\mu_{k+l}}$$
$$*\,\omega_{(k)} = \frac{1}{(10-k)!}\left(\frac{1}{k!}\sqrt{-g}\epsilon_{\mu_1\dots\mu_{10-k}\nu_1\dots\nu_k}\omega^{\nu_1\dots\nu_k}\right)\mathrm{d}x^{\mu_1}\wedge\cdots\wedge\, \mathrm{d}x^{\mu_{10-k}}$$
$$\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu}, \gamma^{\mu_1\dots\mu_k} = \gamma^{[\mu_1}\dots\gamma^{\mu_k]}$$
$$C\gamma^\mu C^{-1} = -(\gamma^\mu)^T, C^T = -C$$
$$C\gamma^{\mu_1\dots\mu_k}C^{-1} = (-)^{[(k+1)/2]}(\gamma^{\mu_1\dots\mu_k})^T,$$
$$\bar{\epsilon}\gamma^{\mu_1\dots\mu_k}\chi = (-)^{[(k+1)/2]}\bar{\chi}\gamma^{\mu_1\dots\mu_k}\epsilon,$$
$$\gamma^{(10)} = \gamma^0\gamma^1\dots\gamma^9 = \frac{1}{10!}\epsilon_{\mu_1\dots\mu_{10}}\gamma^{\mu_1\dots\mu_{10}}$$
$$\gamma_{\mu_1\dots\mu_k}\gamma^{(10)} = (-)^{[k/2]}\frac{1}{(10-k)!}\sqrt{-g}\epsilon_{\mu_1\dots\mu_k\nu_1\dots\nu_{10-k}}\gamma^{\nu_1\dots\nu_{10-k}},$$
$$\gamma^{(k)}\gamma^{(10)} = (-)^{[k/2]} * \gamma^{(10-k)}$$
$$\Psi = \sum_k \frac{1}{k!}\Psi_{\mu_1\dots\mu_k}\gamma^{\mu_1\dots\mu_k}$$
$$F = \{(x,\{\hat{e}_a\}): x\in M \text{ and } \{\hat{e}_a\}\ T_xM\}.$$
$$v^a \mapsto v'^a = A^a{}_b v^b, \hat{e}_a \mapsto \hat{e}'_a = \hat{e}_b(A^{-1})^b{}_a.$$
$$\mathcal{L}_v w = -\mathcal{L}_w v = [v,w],$$
$$\mathcal{L}_v\alpha^{\mu_1\dots\mu_p}_{\nu_1\dots\nu_q} = v^\mu\partial_\mu\alpha^{\mu_1\dots\mu_p}_{\nu_1\dots\nu_q}$$
$$+\left(\partial_\mu v^{\mu_1}\right)\alpha^{\mu\mu_2\dots\mu_p}_{\nu_1\dots\nu_q} + \cdots + \left(\partial_\mu v^{\mu_p}\right)\alpha^{\mu_1\dots\mu_{p-1}\mu}_{\nu_1\dots\nu_q}$$
$$-\left(\partial_{\nu_1}v^\mu\right)\alpha^{\mu_1\dots\mu_p}_{\mu\nu_2\dots\nu_q} - \cdots - \left(\partial_{\nu_q}v^\mu\right)\alpha^{\mu_1}_{\nu_1\dots\nu_{q-1}\mu}.$$
$$T(v,w) = \nabla_v w - \nabla_w v - [v,w].$$
$$T^\mu{}_{\nu\lambda} = \omega_\nu{}^\mu{}_\lambda - \omega_\lambda{}^\mu{}_\nu,$$
$$T^a{}_{bc} = \omega_b{}^a{}_c - \omega_c{}^a{}_b + [\hat{e}_b,\hat{e}_c]^a.$$
$$(i_v T)\alpha = \mathcal{L}_v^\nabla\alpha - \mathcal{L}_v\alpha,$$
$$\mathcal{R}(u,v)w = [\nabla_u,\nabla_v]w - \nabla_{[u,v]}w,$$
$$\mathcal{R}_{\mu\nu}{}^\lambda{}_\rho v^\rho = \left[\nabla_\mu,\nabla_\nu\right]v^\lambda - T^\rho{}_{\mu\nu}\nabla_\rho v^\lambda.$$
$$\mathcal{R}_{\mu\nu} = \mathcal{R}_{\lambda\mu\nu}{}^\lambda.$$
$$\mathcal{R} = g^{\mu\nu}\mathcal{R}_{\mu\nu}$$
$$P = \{(x,\{\hat{e}_a\})\in F: g(\hat{e}_a,\hat{e}_b) = \delta_{ab}\},$$
$$g|_x \in GL(d,\mathbb{R})/O(d)$$
$$\nabla_{\partial/\partial x^\mu}\hat{e}_a = \omega_\mu{}^b{}_a\hat{e}_b.$$

$\nabla_\mu A^\alpha{}_\beta := \partial_\mu A^\alpha{}_\beta - \Gamma^\rho_{\beta\mu}A^\alpha_\rho + \Gamma^\alpha_{\rho\mu}A^\rho{}_\beta.$

$\Gamma^\rho_{\mu\nu} := \left\{{\rho \atop \mu\nu}\right\} + K^\rho_{\mu\nu} + L^\rho{}_{\mu\nu},$

$\left\{{\rho \atop \mu\nu}\right\} := \frac{1}{2}g^{\rho\lambda}\left(\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu}\right),$

$K^\rho{}_{\mu\nu} := \frac{1}{2}\left(T_\mu{}^\rho{}_\nu + T_\nu{}^\rho{}_\mu - T^\rho{}_{\mu\nu}\right),$

$L^\rho{}_{\mu\nu} := \frac{1}{2}\left(Q^\rho{}_{\mu\nu} - Q^\rho_\mu{}_\nu - Q_\nu{}^\rho{}_\mu\right).$

$R^\mu_{\nu\rho\sigma} := \partial_\rho\Gamma^\mu_{\nu\sigma} - \partial_\sigma\Gamma^\mu_{\nu\rho} + \Gamma^\mu_{\tau\rho}\Gamma^\tau_{\nu\sigma} - \Gamma^\mu_{\tau\sigma}\Gamma^\tau_{\nu\rho},$

$T^\mu_{\nu\rho} := 2\Gamma^\mu_{[\rho\nu]} \equiv \Gamma^\mu_{\rho\nu} - \Gamma^\mu_{\nu\rho},$

$Q_{\mu\nu\rho} := \nabla_\mu g_{\nu\rho} \equiv \partial_\mu g_{\nu\rho} - 2\Gamma^\lambda_{(\nu|\mu}g_{\rho)\lambda} \neq 0.$

$$R^{\mu}_{\nu\rho\sigma} = -R^{\mu}_{\nu\sigma\rho},$$
$$T^{\mu}_{\nu\rho} = -T^{\mu}_{\rho\nu},$$
$$Q_{\mu\nu\rho} = Q_{\mu\rho\nu}.$$

$$R^{\mu}_{[\nu\rho\sigma]} = \nabla_{[\nu}T^{\mu}_{\rho\sigma]} + T^{\mu}_{\alpha[\nu}T^{\alpha}_{\rho\sigma]},$$
$$\nabla_{[\alpha}R^{\mu}_{|\nu|\rho\sigma]} = -R^{\mu}_{\nu\tau[\alpha}T^{\tau}_{\rho\sigma]}$$

$$\partial_\mu := \left(\frac{\partial}{\partial x^\mu}\right)_p,$$
$$\mathrm{d}x^\mu \partial_\nu = \delta^\mu_\nu.$$
$$e_A := e^\mu_A \partial_\mu, e^A := e^A_\mu \,\mathrm{d}x^\mu,$$
$$g_{\mu\nu} = \eta_{AB} e^A e^B{}_\nu, \eta_{AB} = g_{\mu\nu} e^\mu_A e^\nu_B.$$

$$\begin{aligned}[e_A, e_B] &:= e_A e_B - e_B e_A \\ &= (e^\mu_A \partial_\mu)(e^\nu_B \partial_\nu) - (e^\nu_B \partial_\nu)(e^\mu_A \partial_\mu) \\ &= [e^\mu_A e^C_\nu(\partial_\mu e^\nu_B) - e^\nu_B e^C_\mu(\partial_\nu e^\mu_B)]e_C \\ &= e^\mu_A e^\nu_B[\partial_\nu e^C_\mu - \partial_\mu e^C_\nu]e_C \\ &= f^C_{AB} e_C,\end{aligned}$$
$$f^C_{AB} := e^\mu_A e^\nu_B[\partial_\nu e^C_\mu - \partial_\mu e^C_\nu]$$
$$\mathrm{d}\omega = \partial_\mu \omega_\nu \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu$$
$$\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu = \mathrm{d}x^\mu \otimes \mathrm{d}x^\nu - \mathrm{d}x^\nu \otimes \mathrm{d}x^\mu$$
$$\mathrm{d}\omega(u,v) = u\omega(v) - v\omega(u) - \omega([u,v]_{\mathcal{L}})$$
$$\mathrm{d}\omega(u,v) := \partial_\mu \omega_\nu (u^\mu v^\nu - u^\nu v^\mu),$$
$$u\omega(v) := u^\mu v^\nu \partial_\mu \omega_\nu + u^\mu \omega_\nu \partial_\mu v^\nu,$$
$$\omega([u,v]_{\mathcal{L}}) := \omega_\nu (u^\mu \partial_\mu v^\nu - v^\mu \partial_\mu u^\nu),$$
$$\begin{aligned}\{\mathrm{d}e^C(e_A, e_B)\}e_C &= \{e_A[e^C(e_B)] - e_B[e^C(e_A)] \\ &\quad -e^C([e_A, e_B]_{\mathcal{L}})\}e_C \\ &= -e^C([e_A, e_B]^L_{\mathcal{L}} e_L)e_C \\ &= -[e_A, e_B]_{\mathcal{L}}.\end{aligned}$$
$$\nabla_{e_A} e_B = \gamma^C_{AB} e_C,$$
$$\begin{aligned}\gamma_{\lambda\nu\mu} &:= e^A_\mu e^B_\lambda \nabla_A (e_\nu)_B \\ &= -e^A_\mu (e_\nu)_B \nabla_A e^B_\lambda \\ &= -e^A_\mu e^B_\nu \nabla_A (e_\lambda)_B = -\gamma_{\nu\lambda\mu},\end{aligned}$$
$$\gamma^C_{AB} = \omega^C_B(e_A) \iff \omega^C_B = \gamma^C_{AB} e^A$$
$$\begin{aligned}[\nabla_\mu, \partial_\nu] &= \nabla_\mu \partial_\nu - \nabla_\nu \partial_\mu \\ &= (\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu})\partial_\lambda \\ &= T^\lambda_{\mu\nu}\partial_\lambda,\end{aligned}$$
$$T(v,u) := \nabla_v u - \nabla_u v - [v,u]_{\mathcal{L}}.$$
$$\begin{aligned}T(e_A, e_B) &= \nabla_{e_A} e_B - \nabla_{e_B} e_A - [e_A, e_B]_{\mathcal{L}} \\ &= [\omega^C_B(e_A) - \omega^C_A(e_B) + \mathrm{d}e^C(e_A, e_B)]e_C \\ &= [(\omega^C_D \wedge e^D + \mathrm{d}e^C)(e_A, e_B)]e_C.\end{aligned}$$
$$T = \Omega^C \otimes e_C$$

$$\begin{aligned} de^C &:= -\omega_A^C \wedge e^A \\ &= -\frac{1}{2}(\gamma_{AB}^C - \gamma_{BA}^C)e^A \wedge e^B \\ &= -\frac{1}{2}e_A^\mu e_B^\nu\left(\partial_\nu e_\mu^C - \partial_\mu e_\nu^C\right)e^A \wedge e^B \\ &= -\frac{1}{2}f_{AB}^C e^A \wedge e^B, \end{aligned}$$

$\eta_{AB} = \eta_{\mu\nu}e_A^\mu e_B^\nu.$
$\Lambda_\nu^\mu: x^\mu \longrightarrow x'^\mu = \Lambda_\nu^\mu(x)x^\nu,$
$\eta_{\mu\nu}x^\mu x^\nu = -t^2 + x^2 + y^2 + z^2.$

$$\Lambda^\alpha{}_\beta = \mathcal{G}\cdot\begin{bmatrix} \gamma & -\gamma\mathcal{R}^i{}_j\frac{v^j}{c} \\ -\gamma\mathcal{R}^i{}_j{}^j\frac{v^j}{c} & \mathcal{R}^i{}_j\left(\delta_j^i + (\gamma-1)\frac{v^iv^j}{v^2}\right) \end{bmatrix},$$

$$\begin{aligned} \mathbb{1} &:= \text{diag}(1,1,1,1) \\ \mathbb{P} &:= \text{diag}(1,-1,-1,-1) \\ \mathbb{T} &:= \text{diag}(-1,1,1,1) \end{aligned}$$

$\bar{e}^A{}_\mu = \Lambda_B^A e^B{}_\mu,$
$g_{\mu\nu} = \eta_{AB}\bar{e}_\mu^A\bar{e}^B{}_\nu \; \eta_{AB} = g_{\mu\nu}\bar{e}_A^\mu\bar{e}_B^\nu.$
$\Lambda^\alpha{}_\beta = \delta_\beta^\alpha + \omega^\alpha{}_\beta + \mathcal{O}\left[\left(\omega_\beta^\alpha\right)^2\right].$
$(J_{AB})_D^C := 2i\eta_{[B|D}\delta_{A]}^C = i(\eta_{BD}\delta_A^C - \eta_{AD}\delta_B^C).$
$\Lambda = e^{\frac{i}{2}\omega_{AB}J^{AB}}.$
$\omega_\mu: J_{AB} \in \mathfrak{L} \longrightarrow \omega_\mu := \frac{1}{2}\omega^{AB}{}_\mu J_{AB},$
$\mathcal{D}_\mu := \partial_\mu - \omega_\mu = \partial_\mu - \frac{i}{2}\omega^{AB}{}_\mu J_{AB},$

$$\begin{aligned} \mathcal{D}_\mu e^C &= \partial_\mu e^C - \frac{i}{2}\omega^{AB}{}_\mu[i(\eta_{BD}\delta_A^C - \eta_{AD}\delta_B^C)]e^D \\ &= \partial_\mu e^C + \frac{1}{2}\left[\omega^A{}_{D\mu}\delta_A^C + \omega^B{}_{D\mu}\delta_B^C\right]e^D \\ &= \partial_\mu e^C + \omega^C{}_{D\mu}e^D. \end{aligned}$$

$$\begin{aligned} \mathcal{D}_\mu\left(e_\lambda^C dx^\lambda\right) &= \mathcal{D}_\mu\left(e_\lambda^C\right)\mathrm{d}x^\lambda + e_\lambda^C\mathcal{D}_\mu\left(\mathrm{d}x^\lambda\right) \\ &= \mathcal{D}_\mu\left(e_\lambda^C\right)\mathrm{d}x^\lambda + e_\lambda^C\left(\delta_\mu^\lambda + e_E^\lambda e_\mu^D\omega^E{}_{D\rho}\,\mathrm{d}x^\rho\right) \\ &= \mathcal{D}_\mu\left(e_\lambda^C\right)\mathrm{d}x^\lambda + e_\mu^C, \end{aligned}$$

$$\begin{aligned} \mathcal{D}_\mu\left(e_\lambda^C dx^\lambda\right) &= \partial_\mu\left(e_\lambda^C dx^\lambda\right) + \omega^C{}_{D\mu}e_\lambda^D\,\mathrm{d}x^\lambda \\ &= \partial_\mu\left(e_\lambda^C\right)dx^\lambda + e_\mu^C + \omega^C{}_{D\mu}e_\lambda^D\,\mathrm{d}x^\lambda. \end{aligned}$$

$\mathcal{D}_\mu\left(e_\lambda^C\right) = \partial_\mu\left(e_\lambda^C\right) + \omega^C{}_{D\mu}e_\lambda^D.$
$\tilde{\nabla}_\mu X_B^A := \partial_\mu + \omega^A{}_{C\mu}X^C{}_B - \omega^C{}_{B\mu}X_C^A.$

$$\begin{aligned} \nabla V &= \left(\nabla_\mu V^\nu\right)\mathrm{d}x^\mu \otimes \partial_\nu \\ &= \left(\partial_\mu V + \Gamma_{\mu\lambda}^\nu V^\lambda\right)\mathrm{d}x^\mu \otimes \partial_\nu. \end{aligned}$$

$$
\begin{aligned}
\tilde{\nabla} V &= \left(\tilde{\nabla}_\mu V^A\right)\mathrm{d}x^\mu \otimes e_A \\
&= \left(\partial_\mu V^A + \omega^A{}_{B\mu} V^B\right)\mathrm{d}x^\mu \otimes e_A \\
&= \left[\partial_\mu\left(e^A_\lambda V^\lambda\right) + \omega^A{}_{B\mu} e^B_\lambda V^\lambda\right]\mathrm{d}x^\mu \otimes \left(e^\nu_A \partial_\nu\right) \\
&= \left[\partial_\mu V^\nu + \left(e^\nu_A \partial_\mu e^A_\lambda + \omega^A{}_{B\mu} e^\nu_A e^B_\lambda\right) V^\lambda\right]\mathrm{d}x^\mu \otimes \partial_\nu \\
&= \left[\partial_\mu V^\nu + \left(e^\nu_A \mathcal{D}_\mu e^A_\lambda\right) V^\lambda\right]\mathrm{d}x^\mu \otimes \partial_\nu .
\end{aligned}
$$

$$\Gamma^\lambda_{\mu\nu} \equiv e^\lambda_A \mathcal{D}_\mu e^A{}_\nu$$

$$\omega^A_{B\mu} = e^A_\lambda e^\nu_B \Gamma^\lambda_{\mu\nu} + e^A_\sigma \partial_\mu e^\sigma_B \equiv e^A_\nu \nabla_\mu e^\nu_B$$

$\omega^{AB} = \omega^{AB}{}_\mu \mathrm{d}x^\mu,$

$$\nabla_\mu e^A_\nu = \partial_\mu e^A_\nu - \Gamma^\lambda_{\mu\nu} e^A_\lambda + \omega^A_{B\mu} e^B_\nu = 0;$$

$$
\begin{aligned}
0 = \nabla_\lambda g_{\mu\nu} &= \partial_\lambda g_{\mu\nu} - \Gamma^\sigma_{\lambda\mu} g_{\sigma\nu} - \Gamma^\sigma_{\lambda\nu} g_{\mu\sigma} \\
&= \partial_\lambda\left(e^A_\mu e^B_\nu \eta_{AB}\right) - e^\sigma_A g_{\sigma\nu} \mathcal{D}_\lambda e^A_\mu - e^\sigma_A g_{\mu\sigma} \mathcal{D}_\lambda e^A_\nu \\
&= -e^A_\nu e^D_\mu \left(\omega_{AD\lambda} - \omega_{DA\lambda}\right),
\end{aligned}
$$

$$
\begin{aligned}
\partial_\mu x'^A &= \partial_\mu\left(\Lambda^A_B(x) x^B\right) \\
&= \left(\partial_\mu x^B\right)\Lambda^A_B(x) + x^B\left(\partial_\mu \Lambda^A_B(x)\right),
\end{aligned}
$$

$$\partial_\mu x'^A = e'{}_\mu \partial'_C x'^A = e'^A{}_\mu = e^C_\mu \Lambda^A_C(x).$$

$$e^A_\mu = \partial_\mu x^A + \dot{\omega}^A{}_{B\mu} x^B \equiv \mathcal{D}_\mu x^A,$$

$$\dot{\omega}^A{}_{B\mu} := \Lambda^A_C(x)\partial_\mu \Lambda^C_B(x)$$

$\omega^A{}_{B\mu} = \underbrace{\Lambda^A_C(x)\omega'^C{}_{D\mu}\Lambda^D_C}_{\text{non inertial}} + \underbrace{\Lambda^A_C \partial_\mu \Lambda^C_B(x)}_{\text{inertial}}.$

$f^C_{AB} = \dot{\omega}^C_{BA} - \dot{\omega}^C_{AB}.$

as

$\dot{\omega}^A_{BC} = \frac{1}{2}\left(f_B{}^A{}_C + f_C{}^A{}_B - f^A_{BC}\right).$

$$R^A_{B\mu\nu} = \partial_\nu \dot{\omega}^A_{B\mu} - \partial_\mu \dot{\omega}^A_{B\nu} + \dot{\omega}^A_{E\nu}\dot{\omega}^E_{B\mu} -$$

$\dot{\omega}^A_{E\mu}\dot{\omega}^E_{B\nu} \equiv 0,$

$T^A_{\nu\mu} = \partial_\nu e^A_\mu - \partial_\mu e^A_\nu + \dot{\omega}^A_{E\nu} e^E_\mu - \dot{\omega}^A_{E\mu} e^E_\nu .$

$g_{\mu\nu}(\varphi(p)) = \eta_{\mu\nu}, \partial_\lambda g_{\mu\nu}(\varphi(p)) = 0.$

$\frac{\mathrm{d}^2\xi^\alpha}{\mathrm{d}s^2} = 0,$

$\frac{\mathrm{d}^2 x^\lambda}{\mathrm{d}s^2} + \mathring{\Gamma}^\lambda_{\mu\nu}\frac{\mathrm{d}x^\mu}{\mathrm{d}s}\frac{\mathrm{d}x^\nu}{\mathrm{d}s} = 0,$

$\mathring{\Gamma}^\lambda_{\mu\nu} := \frac{\partial x^\lambda}{\partial \xi^\sigma}\frac{\partial^2 \xi^\sigma}{\partial x^\mu \partial x^\nu},$

$\left[\mathring{\nabla}_\mu, \mathring{\nabla}_\nu\right] v^\alpha = \mathring{R}^\alpha_{\beta\mu\nu} v^\beta .$

$\mathring{R}_{\mu\nu\alpha\beta} = -\mathring{R}_{\nu\mu\alpha\beta},$

$\mathring{R}_{\mu\nu\alpha\beta} = \mathring{R}_{\alpha\beta\mu\nu}.$

$T^\mu = \frac{\partial x^\mu}{\partial t}, S^\mu = \frac{\partial x^\mu}{\partial s}.$

$V^\mu = T^\nu \mathring{\nabla}_\nu T^\mu,$

$A^\mu = T^\nu \mathring{\nabla}_\nu V^\mu.$

$$A^{\mu} = \overset{\circ}{R}{}^{\mu}{}_{\lambda\alpha\beta} T^{\lambda} T^{\alpha} S^{\beta},$$

$$S_{\mathrm{GR}} := \frac{c^4}{16\pi G} \int \mathrm{d}^4 x \sqrt{-g} (\mathcal{L}_{\mathrm{GR}} + \mathcal{L}_{\mathrm{m}}),$$

$$\overset{\circ}{G}_{\mu\nu} := \overset{\circ}{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \overset{\circ}{R} = \frac{8\pi G}{c^4} T_{\mu\nu},$$

$$T^{\mu\nu} = -\frac{1}{2\sqrt{-g}} \frac{\delta \mathcal{L}_m}{\delta g_{\mu\nu}}$$

$$\mathrm{d}e^C + \overset{\circ}{\omega}{}^A_B \wedge e^B = 0,$$
$$\overset{\circ}{\omega}_{AB} + \overset{\circ}{\omega}_{BA} = \mathrm{d}g_{AB},$$
$$\mathrm{d}\overset{\circ}{\omega}{}^A_B + \overset{\circ}{\omega}{}^A_C \wedge \overset{\circ}{\omega}{}^C{}_B = \frac{1}{2} \overset{\circ}{r}_{BCD} e^C \wedge e^D,$$
$$\overset{\circ}{\omega}{}^A_{B\mu} := e^A_\nu \overset{\circ}{\nabla}_\mu e^\nu_B,$$
$$\overset{\circ}{f}{}^A_{BC} := \dot{\gamma}^A_{BC} - \dot{\gamma}^A_{CB},$$
$$\mathrm{d}g_{AB} = \partial_C g_{AB} e^C,$$

$$\dot{\gamma}^A_{BC} = \frac{1}{2}\left(\overset{\circ}{f}{}^A_{BC} - g_{CL} g^{AM} \overset{\circ}{f}{}^L_{BM} - g_{BL} g^{AM} \overset{\circ}{f}{}^L_{CM}\right)$$

$$+\overset{\circ}{\Gamma}{}^A_{BC}.$$

$$\overset{\circ}{r}{}^A_{BCD} = \partial_D \overset{\circ}{\gamma}{}^A_{BC} - \partial_C \overset{\circ}{\gamma}{}^A_{BD} + \overset{\circ}{\gamma}{}^A_{CM} \overset{\circ}{\gamma}{}^M_{DB}$$
$$-\dot{\gamma}^A_{DM} \overset{\circ}{\gamma}{}^M_{CB} - \dot{\gamma}^A_{MB} \dot{\gamma}^M_{CD}.$$

$$x^A \longrightarrow \bar{x}^A = x^A + \varepsilon^A(x^\mu),$$
$$P_A := \partial_A.$$
$$[P_A, P_B] \equiv [\partial_A, \partial_B] = 0.$$
$$\delta \bar{x}^A = \varepsilon(x^\mu)^B \partial_B x^A = \varepsilon(x^\mu)^A.$$
$$\delta_\varepsilon \Psi = \epsilon^A(x^\mu) \partial_A \Psi.$$
$$\partial_\varepsilon(\partial_\mu \Psi) = \varepsilon^A \partial_A(\partial_\mu \Psi).$$
$$\partial_\varepsilon(\partial_\mu \Psi) = \underbrace{\varepsilon^A(x^\mu)\partial_A(\partial_\mu \Psi)}_{\text{correct}} + \underbrace{\left(\partial_\mu \varepsilon^A(x^\mu)\right)\partial_A \Psi}_{\text{spurious}},$$
$$e'_\mu \Psi \equiv \partial_\mu \Psi = \partial_\mu + B^A_\mu \partial_A \Psi,$$
$$\delta_\varepsilon B^A_\mu = -\partial_\mu \varepsilon^A(x^\mu).$$
$$\partial_\varepsilon(e'_\mu \Psi) = \underbrace{\varepsilon^A(x^\mu)\partial_A(\partial_\mu \Psi)}_{\text{correct}},$$
$$e_\mu = \Psi = e^A_\mu \partial_A \Psi, e^A_\mu = \partial_\mu x^A + B^A_\mu,$$
$$B^A_\mu \longrightarrow \Lambda^A_B(x) B^B_\mu.$$
$$e_\mu \Psi = \partial_\mu + \dot{\omega}^A_{B\mu} x^B \partial_A \Psi + B^A_\mu \partial_A \Psi,$$
$$e^A_\mu = \partial_\mu x^A + \dot{\omega}^A_{B\mu} x^B + B^A_\mu = \dot{\mathcal{D}}_\mu x^A + B^A_\mu.$$
$$\delta_\varepsilon B^A_\mu = -\dot{\mathcal{D}}_\mu \varepsilon^A(x^\mu).$$
$$e'^A_\mu \longrightarrow e^A_\mu,$$
$$\eta_{\mu\nu} \longrightarrow g_{\mu\nu}.$$

$$\partial_\mu \Psi \to \mathcal{D}'_\mu \Psi = \partial_\mu \Psi$$
$$+\frac{1}{2} e'^A{}_\mu (f'_B{}^C{}_A + f'_A{}^C{}_B - f'^C_{BA}) S^B_C \Psi,$$

$$\partial_\mu \Psi \to \mathcal{D}_\mu \Psi = \partial_\mu \Psi + \frac{1}{2} e^A{}_\mu (f_B{}^C{}_A^C + f_A C_B^C - f_{BA}^C) S_C^B \Psi,$$

$$\underbrace{\begin{Bmatrix} e'^A \longrightarrow e^A \\ \partial_\mu \longrightarrow \mathcal{D}_\mu \end{Bmatrix}}_{\text{grav. coupling prescription in TG}} \quad \Leftrightarrow \quad \underbrace{\eta_{\mu\nu} \longrightarrow g_{\mu\nu}}_{\text{grav. coupling prescription in GR}}$$

$$\frac{\mathrm{d}u'^A}{\mathrm{d}\sigma} = 0,$$

$$\frac{\mathrm{d}u'^A}{\mathrm{d}\sigma} = \underbrace{\Lambda_B^A(x)\frac{\mathrm{d}u^B}{\mathrm{d}\sigma}}_{\text{correct}} + \underbrace{\frac{\mathrm{d}\Lambda_B^A(x)}{\mathrm{d}\sigma} u^B}_{\text{spurious}}.$$

$$\frac{\mathrm{d}u'^A}{\mathrm{d}\sigma} = 0 \longrightarrow \frac{\mathrm{d}u^B}{\mathrm{d}\sigma} + \dot{\omega}_{B\mu}^A u^B u^\mu = 0.$$

$$\dot{\gamma}(\tau) := \frac{\mathrm{d}\gamma^\mu}{\mathrm{d}\tau}\partial_\mu.$$

$$\frac{\mathrm{d}Y^\mu}{\mathrm{d}\tau} := \nabla_\gamma Y^\mu \equiv \frac{\mathrm{d}Y^\mu}{\mathrm{d}\tau} + \Gamma_{\alpha\beta}^\mu Y^\alpha \frac{\mathrm{d}\gamma^\beta}{\mathrm{d}\tau} = 0,$$

$$\nabla_\gamma \dot{\gamma} \equiv \frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} + \Gamma_{\alpha\beta}^\mu \frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau} = 0,$$

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} + \mathring{\Gamma}_{\alpha\beta}^\mu \frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau} = -K_{\alpha\beta}^\mu \frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau},$$

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} + \dot{\Gamma}_{\alpha\beta}^\mu \frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau} = -L_{\alpha\beta}^\mu \frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau}.$$

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} + \Gamma_{\alpha\beta}^\mu \frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau} = -\left(\frac{\mathrm{d}\lambda}{\mathrm{d}\tau}\right)^2 \frac{\mathrm{d}^2\tau}{\mathrm{d}\lambda^2}\frac{\mathrm{d}\gamma^\mu}{\mathrm{d}\tau}.$$

$$[e_\mu, e_\nu] = \hat{T}_{\nu\mu}^A \partial_A,$$

$$\hat{T}_{\mu\nu}^A = \partial_\nu B_\mu^A - \partial_\mu B_\nu^A + \dot{\omega}_{B\nu}^A B_\mu^B - \dot{\omega}_{B\mu}^A B_\nu^B$$

$$\dot{\mathcal{D}}_\mu(\dot{\mathcal{D}}_\nu x^A) - \dot{\mathcal{D}}_\nu(\dot{\mathcal{D}}_\mu x^A) \equiv 0$$

$$\hat{T}_{\mu\nu}^A = \partial_\nu e_\mu^A - \partial_\mu e^A + \dot{\omega}_{B\nu}^A e^B - \dot{\omega}_{B\mu}^A e^B{}_\nu.$$

$$\hat{T}_{\mu\nu}^\lambda = e_A^\lambda \hat{T}_{\mu\nu}^\lambda := \Gamma_{\nu\mu}^\lambda - \Gamma_{\mu\nu}^\lambda.$$

$$e_{(\mathrm{r})\mu}^A := \lim_{G\to 0} e^A{}_\mu.$$

$$\hat{T}_{BC}^A(e_\mu^A, \dot{\omega}_{B\mu}^A) = \dot{\omega}_{BC}^A - \dot{\omega}_{BC}^A - f_{BC}^A(e_{(\mathrm{r})}) = 0,$$

$$\dot{\omega}_{BC}^A = \tfrac{1}{2} e_{(\mathrm{r})\mu}^C [f_B{}^A{}_C(e_{(\mathrm{r})}) + f_C{}^A_B(e_{(\mathrm{r})}) - f_{BC}^A(e_{(\mathrm{r})})].$$

$$\dot{\omega}_{AB}^C - \dot{\omega}_{BA}^C = f_{AB}^C + T_{AB}^C.$$

$$\tfrac{1}{2}(f_B^C{}_A + f_A C_B^C - f_{BA}^C) = \dot{\omega}_{BA}^C - \hat{K}_{BA}^C,$$

$$\hat{K}_{BA}^C = \tfrac{1}{2}(\hat{T}_B^C{}_A + \hat{T}_A^C{}_B - \hat{T}_{BA}^C),$$

$$\dot{\omega}_{B\mu}^C - \hat{K}_{B\mu}^C = \mathring{\omega}_{B\mu}^C,$$

$$\hat{S}_A^{\mu\nu} := \hat{K}^{\mu\nu}{}_A - e_A{}^\nu \hat{T}^\mu + e_A^\mu \hat{T}^\nu,$$

$$\hat{T} := \frac{1}{2}\hat{S}_A^{\mu\nu}\hat{T}_{\mu\nu}^A = \frac{1}{4}\hat{T}_{\mu\nu}^\rho \hat{T}_\rho^{\mu\nu} + \frac{1}{2}\hat{T}_{\mu\nu}^\rho \hat{T}_\rho^{\nu\mu} - \hat{T}_\mu \hat{T}^\mu,$$

$$\hat{R} = \mathring{R} + \hat{T} + \tfrac{2}{e}\partial_\mu(e\hat{T}^\mu) = 0,$$

$$\mathring{R} = -\hat{T} - \underbrace{\frac{2}{e^2\partial_\mu(eT^\mu)}}_{\text{boundary term}}. \; .$$

$S_{\text{TEGR}} = -\frac{c^4}{16\pi G}\int \mathrm{d}^4x e \underbrace{\mathcal{L}_{\text{TEGR}}}_{-\hat{T}} + \int \mathrm{d}^4x e\mathcal{L}_m,$

$\hat{G}_{\mu\nu} := \frac{1}{e}\partial_\lambda\left(e\hat{S}_{\mu\nu}{}^{\lambda}\right) - \frac{4\pi G}{c^4}\mathrm{t}_{\mu\nu} = \frac{4\pi G}{c^4}T_{\mu\nu},$

$$\mathrm{t}_{\mu\nu} = \frac{c^4}{4\pi G}\hat{S}_{\lambda\nu}{}^{\rho}\Gamma^{\lambda}_{\rho\mu} - g_{\mu\nu}\frac{c^4}{16\pi G}\hat{T}$$

$\hat{S}_A{}^{\mu\nu} = -\frac{8\pi G}{c^4 e}\frac{\partial\mathcal{L}_{\text{TEGR}}}{\partial(\partial_\nu e^A{}_\mu)}.$

$$\hat{G}_{\mu\nu} := \frac{1}{e}e^A{}_\mu g_{\nu\rho}\partial_\sigma\left(e\hat{S}_A^{\rho\sigma}\right) - \hat{S}_B^{\sigma}{}_\nu\hat{T}^B_{\sigma\mu}$$

$\hat{T}_{\text{gen}} := -\frac{c_1}{4}\hat{T}_{\alpha\mu\nu}\hat{T}^{\alpha\mu\nu} - \frac{c_2}{2}\hat{T}_{\alpha\mu\nu}\hat{T}^{\mu\alpha\nu} + c_3\hat{T}_\alpha\hat{T}^\alpha,$

$\hat{T}_{[\mu\nu]} = e^A_{[\mu}g_{\nu]\rho}\hat{T}^\rho_A = 0.$

$\mathcal{L}_{\text{TEGR}}\left(e^A_\mu, 0\right), \mathcal{L}_{\text{TEGR}}\left(e^A_\mu, \dot{\omega}^A_{B\mu}\right),$

$$\mathcal{L}_{\text{TEGR}}\left(e^A_\mu,\dot{\omega}^A_{B\mu}\right) + \partial_\mu\left[\frac{ec^4}{8\pi G}\hat{T}^\mu\left(e^A_\mu,\dot{\omega}^A_{B\mu}\right)\right] = \mathcal{L}_{\text{TEGR}}\left(e^A_\mu,0\right) + \partial_\mu\left[\frac{ec^4}{8\pi G}\hat{T}^\mu\left(e^A_\mu,0\right)\right],$$

$\hat{T}^\mu\left(e^A_\mu,\dot{\omega}^A_{B\mu}\right) = \hat{T}^\mu\left(e^A_\mu,0\right) - \dot{\omega}^\mu.$

$\mathcal{L}_{\text{TEGR}}\left(e^A_\mu,\dot{\omega}^A_{B\mu}\right) = \mathcal{L}_{\text{TEGR}}\left(e^A_\mu,0\right) + \partial_\mu\left[\frac{ec^4}{8\pi G}\dot{\omega}^\mu\right].$

$$g_{\nu\lambda}\overset{\nabla}{\overset{\nabla}{\mu}}{}^{\lambda} = \overset{\vec{\nabla}}{\mu}v_\nu - v^\lambda\overset{Q}{\mu\nu\lambda}$$

$$T^\lambda\vec{\nabla}_\lambda v\cdot w = T^\lambda v^\mu w^\nu\mathring{Q}_{\lambda\mu\nu}$$

$$T^\lambda\vec{\nabla}_\lambda\left(\frac{v\cdot w}{|v||w|}\right) \neq 0$$

$$a^\mu := u^\lambda\mathring{\nabla}_\lambda u^\mu$$

$$\tilde{a}_\mu := u^\lambda\mathring{\nabla}_\lambda u_\mu = a_\mu + \lambda\mathring{v}\mu u^\lambda u^\nu$$

$$u_\mu a^\mu = u_\mu u^\lambda\mathring{\nabla}_\lambda u^\mu$$

$$= u^\lambda\overset{\nabla}{\overset{\nabla}{\lambda}}(u_\mu u^\mu) - u^\mu u^\lambda\mathring{\nabla}_\lambda u_\mu$$

$$= \mathring{Q}_{\lambda\mu\nu}u^\lambda u^\mu u^\nu + 2u_\mu a^\mu - \tilde{a}_\mu u^\mu,$$

$$a^\mu u_\mu = \tilde{a}_\mu u^\mu - \mathring{Q}_{\lambda\mu\nu}u^\lambda u^\mu u^\nu.$$

$$(\tilde{a}_\mu - a_\mu)u^\mu = \mathring{Q}_{\lambda\mu\nu}u^\lambda u^\mu u^\nu$$

$$a^\mu = 0, \tilde{a}_\mu = \mathring{Q}_{\lambda\nu\mu}u^\lambda u^\nu;$$

$$\mathring{Q}_{(\lambda\mu\nu)} = 0, \mathring{Q}_{(\lambda\mu)\nu} = 0$$

$$S_{\text{STEGR}} := \int \mathrm{d}^4x\sqrt{-g}\left[\frac{c^4}{16\pi G}\underbrace{\mathcal{L}_{\text{STEGR}}}_{\mathring{Q}} + \mathcal{L}_m\right],$$

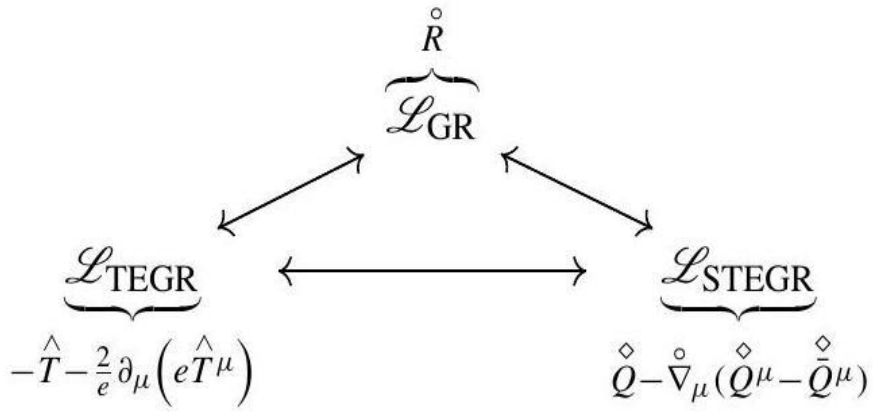
$$\mathring{Q} := g^{\mu\nu}\left(\mathring{L}^{\alpha}{}_{\beta\mu}\mathring{L}^{\beta}{}_{\nu\alpha} - \mathring{L}^{\alpha}{}_{\beta\alpha}\mathring{L}^{\beta}{}_{\mu\nu}\right)$$
$$= \frac{1}{4}\left(\mathring{Q}_{\alpha}\mathring{Q}^{\alpha} - \mathring{Q}_{\alpha\beta\gamma}\mathring{Q}^{\alpha\beta\gamma}\right)$$
$$+ \frac{1}{2}\left(\mathring{Q}_{\alpha\beta\gamma}\mathring{Q}^{\beta\alpha\gamma} - \mathring{Q}_{\alpha}\mathring{Q}^{\alpha}\right),$$
$$\mathring{Q} = \mathring{R} + \mathring{\nabla}_{\mu}\left(\mathring{Q}^{\mu} - \mathring{\bar{Q}}^{\mu}\right),$$
$$\mathring{\nabla}_{\mu}\left(\mathring{Q}^{\mu} - \mathring{\bar{Q}}^{\mu}\right) \equiv \frac{1}{\sqrt{-g}}\partial_{\mu}\left[\sqrt{-g}\left(\mathring{Q}^{\mu} - \mathring{\bar{Q}}^{\mu}\right)\right]$$
$$\mathring{Q}_{\mathrm{gen}} := c_1\mathring{Q}_{\alpha\beta\gamma}\grave{Q}^{\alpha\beta\gamma} + c_2\mathring{Q}_{\alpha\beta\gamma}\mathring{Q}^{\beta\alpha\gamma} + c_3\mathring{Q}_{\alpha}\grave{Q}^{\alpha}$$
$$+ c_4\overset{\varrho}{Q}_{\alpha}\mathring{Q}^{\alpha} + c_5\mathring{Q}_{\alpha}\overset{\varrho}{Q}^{\alpha},$$
$$\mathring{P}^{\alpha}_{\mu\nu} := \frac{1}{2\sqrt{-g}}\frac{\partial(\sqrt{-g}\mathring{Q})}{\partial\grave{Q}_{\alpha}^{\mu\nu}}$$
$$= \frac{1}{4}\mathring{Q}^{\alpha}{}_{\mu\nu} - \frac{1}{4}\mathring{Q}_{(\mu}{}^{\alpha}{}_{\nu)} - \frac{1}{4}g_{\mu\nu}\mathring{Q}^{\alpha\beta}{}_{\beta}$$
$$+ \frac{1}{4}\left[\mathring{Q}^{\beta\alpha}_{\beta}g_{\mu\nu} + \frac{1}{2}\delta^{\alpha}_{(\mu}\mathring{Q}_{\nu)}{}^{\beta}{}_{\beta}\right].$$

$\mathring{Q} := \mathring{Q}_{\alpha\mu\nu}\mathring{P}^{\alpha\mu\nu}$.

$$\frac{1}{\sqrt{-g}}\mathring{q}_{\mu\nu} := \frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\mathring{Q})}{\partial g^{\mu\nu}} - \frac{1}{2}\mathring{Q}g_{\mu\nu}$$
$$= \frac{1}{4}\left(2\mathring{Q}_{\alpha\beta\mu}\mathring{Q}^{\alpha\beta}{}_{\nu} - \mathring{Q}_{\mu\alpha\beta}\mathring{Q}_{\nu}{}^{\alpha\beta}\right)$$
$$- \frac{1}{4}\left(2\mathring{Q}_{\alpha}{}^{\beta}{}_{\beta}\mathring{Q}^{\alpha}{}_{\mu\nu} - \mathring{Q}_{\mu}{}^{\beta}{}_{\beta}\mathring{Q}_{\nu}{}^{\beta}{}_{\beta}\right)$$
$$- \frac{1}{2}\left(\mathring{Q}_{\alpha\beta\mu} - \mathring{Q}_{\nu}{}^{\beta\alpha}{}_{\alpha}\mathring{Q}^{\alpha}{}_{\mu\nu}\right).$$
$$\mathring{G}_{\mu\nu} := -2\nabla_{\alpha}\left(\sqrt{-g}\mathring{P}^{\alpha}_{\mu\nu}\right)$$
$$+ \mathring{q}_{\mu\nu} - \frac{\sqrt{-g}}{2}\mathring{Q}_{g_{\mu\nu}} = \frac{8\pi G}{c^4}T_{\mu\nu},$$

$\nabla_{\mu}\nabla_{\nu}\left(\sqrt{-g}\mathring{P}^{\mu\nu}_{\alpha}\right) = 0,$

$\Gamma^{\alpha}_{\mu\nu} := (e^{-1})^{\alpha}{}_{\beta}\partial_{\mu}e^{\beta}{}_{\nu}$.
$T^{\alpha}_{\mu\nu} := (e^{-1})^{\alpha}_{\beta}\partial_{[\nu}e^{\beta}_{\mu]} = 0,$
$\partial_{\mu}e^{\beta}{}_{\nu} = \partial_{\nu}e^{\beta}{}_{\mu} \Leftrightarrow e^{\alpha}_{\beta} \equiv e'^{\alpha}_{\beta} := \partial_{\beta}\xi^{\alpha},$
$\Gamma^{\alpha}_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial\xi^{\lambda}}\partial_{\mu}\partial_{\nu}\xi^{\lambda}.$
$\xi^{\alpha} := M^{\alpha}_{\beta}x^{\beta} + \xi^{\alpha}_{0},$
$\mathring{\nabla}_{\mu} = \partial_{\mu}, \mathring{L}^{\lambda}_{\mu\nu} = -\mathring{\Gamma}^{\lambda}_{\mu\nu}.$



$$\nabla_\lambda R^\alpha_{\beta\mu\nu}+\nabla_\mu R^\alpha_{\beta\nu\lambda}+\nabla_\nu R^\alpha_{\beta\lambda\mu}$$
$$=T^\rho_{\mu\lambda}R^\alpha_{\beta\nu\rho}+T^\rho_{\nu\lambda}R^\alpha_{\beta\mu\rho}+T^\rho_{\nu\mu}R^\alpha_{\beta\lambda\rho},$$

$$\mathring{\nabla}_\lambda\mathring{R}^\alpha_{\beta\mu\nu}+\mathring{\nabla}_\mu\mathring{R}^\alpha_{\beta\nu\lambda}+\mathring{\nabla}_\nu\mathring{R}^\alpha_{\beta\lambda\mu}=0.$$
$$\partial_\lambda\mathring{R}^\lambda_{\beta\mu\nu}+\partial_\mu\mathring{R}^\lambda_{\beta\nu\lambda}+\partial_\nu\mathring{R}^\lambda_{\beta\lambda\mu}=0.$$
$$\partial_\lambda\mathring{R}^\lambda_{\beta\mu\nu}-\partial_\mu\mathring{R}^\lambda_{\beta\lambda\nu}+\partial_\nu\mathring{R}^\lambda_{\beta\lambda\mu}=0.$$
$$-\partial_\lambda\mathring{R}^\lambda_\nu-\partial_\beta\mathring{R}^{\ \beta}_{\nu}+\partial_\nu\mathring{R}=0,$$
$$\partial_\mu\mathring{R}^\mu_\nu-\frac{1}{2}\partial_\nu\mathring{R}=0.$$
$$\partial_\mu\left(\mathring{R}^{\mu\nu}-\frac{1}{2}g^{\mu\nu}\hat{R}\right)=0\Rightarrow\mathring{\nabla}_\mu\left(\mathring{R}^{\mu\nu}-\frac{1}{2}g^{\mu\nu}\hat{R}\right)=0,$$
$$\mathring{\nabla}_\mu\mathring{G}^{\mu\nu}=0,\Leftrightarrow\ \mathring{\nabla}_\mu T^{\mu\nu}=0.$$
$$\hat{\nabla}_\lambda R^\alpha{}_{\beta\mu\nu}+\hat{\nabla}_\nu R^\alpha_{\beta\lambda\mu}+\hat{\nabla}_\mu R^\alpha_{\beta\nu\lambda}=0,$$

$$\hat{\mathcal{K}}^\alpha_{\beta\mu\nu}:=\mathring{\nabla}_\mu\hat{K}^\alpha_{\beta\nu}-\mathring{\nabla}_\nu\mathring{K}^\alpha_{\beta\mu}$$
$$+\hat{K}^\alpha_{\sigma\mu}\hat{K}^\sigma_{\beta\nu}-\hat{K}^\alpha_{\sigma\nu}\hat{K}^\sigma_{\beta\mu},$$

$$\hat{\mathcal{K}}^\alpha{}_{\beta\mu\nu}=-\hat{\mathcal{K}}_\beta{}^\alpha{}_{\mu\nu},\hat{\mathcal{K}}^\alpha{}_{\beta\mu\nu}=-\hat{\mathcal{K}}^\alpha{}_{\beta\nu\mu}.$$

$$\hat{\nabla}_\lambda\mathring{R}^\lambda_{\beta\mu\nu}+\hat{\nabla}_\mu\mathring{R}^\lambda_{\beta\nu\lambda}+\hat{\nabla}_\nu\mathring{R}^\lambda_{\beta\lambda\mu}$$
$$+\hat{\nabla}_\lambda\hat{\mathcal{K}}^\lambda_{\beta\mu\nu}+\hat{\nabla}_\mu\hat{\mathcal{K}}^\lambda_{\beta\nu\lambda}+\hat{\nabla}_\nu\hat{\mathcal{K}}^\lambda_{\beta\lambda\mu}=0.$$

$$\hat{\nabla}_\mu\left(\mathring{R}^\mu_\nu+\hat{\mathcal{K}}^\mu_\nu\right)-\frac{1}{2}\hat{\nabla}_\nu(\mathring{R}+\hat{\mathcal{K}})=0,$$
$$\mathring{R}_{\mu\nu}-\frac{1}{2}g_{\mu\nu}\mathring{R}=-\hat{\mathcal{K}}_{\mu\nu}+\frac{1}{2}g_{\mu\nu}\hat{\mathcal{K}},$$
$$\hat{\mathcal{K}}_{\mu\nu}-\frac{1}{2}g_{\mu\nu}\hat{\mathcal{K}}=0,$$

$$\hat{\mathcal{K}}_{\mu\nu}=\mathring{\nabla}_\alpha\hat{K}^\alpha{}_{\mu\nu}-\mathring{\nabla}_\nu\hat{K}^\alpha{}_{\mu\alpha}+\hat{K}^\sigma{}_{\mu\nu}\hat{K}^\alpha{}_{\sigma\alpha}-\hat{K}^\sigma{}_{\mu\alpha}\hat{K}^\alpha{}_{\sigma\nu}$$
$$=\mathring{\nabla}_\alpha\hat{K}^\alpha_{\mu\nu}+\mathring{\nabla}_\nu\hat{T}_\mu-\hat{K}_{\sigma\mu\nu}\hat{T}^\sigma-\hat{K}^\sigma_{\mu\alpha}\hat{K}^\alpha_{\sigma\nu}$$
$$=\mathring{\nabla}_\alpha\hat{S}_\nu{}^\alpha{}_\mu+\mathring{\nabla}_\alpha\hat{T}^\alpha g_{\mu\nu}-\hat{K}^\alpha_{\sigma\nu}\hat{S}_\alpha{}^\sigma{}_\mu,$$

$$\hat{K}^\alpha{}_{\mu\alpha}=-\hat{T}_\mu,$$
$$\hat{K}^\alpha{}_{\alpha\mu}=0,$$
$$\hat{K}^\mu_{\nu\lambda}=\hat{S}^{\mu\nu}_\lambda+\delta^\nu_\lambda\hat{T}^\mu-\delta^\mu_\lambda\hat{T}^\nu.$$

$$\hat{\mathcal{K}} = 2\overset{\circ}{\nabla}_\lambda \hat{T}^\lambda + \hat{T} = \frac{2}{e}\partial_\lambda\left(e\hat{T}^\lambda\right) + \hat{T}.$$
$$\overset{\circ}{\nabla}_\alpha \hat{S}_{\nu\mu}{}^{\alpha} + \hat{K}^{\alpha}_{\sigma\nu}\hat{S}_\alpha{}^{\sigma}{}_{\mu} + \frac{1}{2}g_{\mu\nu}\hat{T} = 0,$$

$$\partial_\lambda R^{\alpha}_{\beta\mu\nu} + \partial_\nu R^{\alpha}_{\beta\lambda\mu} + \partial_\mu R^{\alpha}_{\beta\nu\lambda} = 0,$$
$$\overset{\circ}{\mathcal{L}}{}^{\alpha}{}_{\beta\mu\nu} = \overset{\circ}{\nabla}_\mu \overset{\circ}{L}{}^{\alpha}{}_{\beta\nu} - \overset{\circ}{\nabla}_\nu \overset{\circ}{L}{}^{\alpha}{}_{\beta\mu} + \overset{\circ}{L}{}^{\alpha}_{\alpha}\overset{\circ}{L}{}^{\sigma}{}_{\beta\nu} - \overset{\circ}{L}{}^{\alpha}{}_{\sigma\nu}\overset{\circ}{L}{}^{\sigma}{}_{\beta\mu},$$

$$\overset{\diamond}{\mathcal{L}}{}^{\alpha}{}_{\beta\mu\nu} = -\overset{\diamond}{\mathcal{L}}{}^{\beta}{}_{\alpha\mu\nu}, \qquad \overset{\diamond}{\mathcal{L}}{}^{\alpha}{}_{\beta\mu\nu} = -\overset{\diamond}{\mathcal{L}}{}^{\alpha}{}_{\beta\nu\mu},$$

$$\partial_\lambda \overset{\circ}{R}_{\beta\mu\nu} + \partial_\mu \overset{\circ}{R}{}^{\lambda}_{\beta\nu\lambda} + \partial_\nu \overset{\circ}{R}_{\beta\lambda\mu}$$
$$+\partial_\lambda \overset{\circ}{\mathcal{L}}{}^{\lambda}_{\beta\mu\nu} + \partial_\mu \overset{\circ}{\mathcal{L}}{}^{\lambda}_{\beta\nu\lambda} + \partial_\nu \overset{\circ}{\mathcal{L}}{}^{\lambda}_{\beta\lambda\mu} = 0.$$

$$\overset{\circ}{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\overset{\circ}{R} = -\overset{\circ}{\mathcal{L}}_{\mu\nu} + \frac{1}{2}g_{\mu\nu}\overset{\circ}{\mathcal{L}},$$
$$\overset{\circ}{\mathcal{L}}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\overset{\circ}{\mathcal{L}} = 0,$$

$$\overset{\circ}{\mathcal{L}}_{\mu\nu} = \overset{\circ}{\nabla}_\alpha \overset{\circ}{\mathcal{L}}{}^{\alpha}{}_{\mu\nu} - \overset{\circ}{\nabla}_\nu \overset{\circ}{\mathcal{L}}{}^{\alpha}{}_{\mu\alpha} + \overset{\circ}{\mathcal{L}}{}^{\sigma}{}_{\mu\nu}\overset{\circ}{\mathcal{L}}{}^{\alpha}{}_{\sigma\alpha} - \overset{\circ}{L}{}^{\sigma}{}_{\mu\alpha}\overset{\circ}{\mathcal{L}}{}^{\alpha}{}_{\sigma\nu}$$
$$= \overset{\circ}{\nabla}_\alpha \overset{\circ}{\mathcal{L}}{}^{\alpha}{}_{\mu\nu} + \frac{1}{2}{}^{\circ}{}_{\nu}\ {}_{\nu}\overset{\circ}{Q}_\mu - \frac{1}{2}\overset{\circ}{Q}_\alpha \overset{\circ}{\mathcal{L}}{}^{\alpha}{}_{\mu\nu}$$
$$-\frac{1}{4}\left[\overset{\circ}{Q}_\mu{}^{\sigma}{}_{\alpha}\overset{\circ}{Q}_\nu{}^{\alpha}{}_{\sigma} + 2\overset{\circ}{Q}{}^{\alpha}{}_{\sigma\nu}\left(\overset{\circ}{Q}{}^{\sigma}{}_{\alpha\mu} - \overset{\circ}{Q}_\alpha{}^{\sigma}\mu\right)\right],$$
$$\overset{\circ}{L}{}^{\alpha}_{\mu\alpha} = -\frac{1}{2}\overset{\circ}{Q}_\mu$$
$$\overset{\circ}{L}{}^{\alpha}{}_{\mu\nu} = 2\overset{\circ}{P}{}^{\alpha}_{\mu\nu} + \frac{1}{2}g_{\mu\nu}\left(\overset{\circ}{Q}{}^{\alpha} - \overset{\circ}{\tilde{Q}}{}^{\alpha}\right)$$
$$-\frac{1}{4}\left(\delta^{\alpha}_{\mu}\overset{\circ}{Q}_\nu + \delta^{\alpha}_{\nu}\overset{\circ}{Q}_\mu\right)$$
$$\overset{\circ}{\mathcal{L}} = \overset{\circ}{\nabla}_\alpha\left(\overset{\circ}{Q}{}^{\alpha} - \overset{\circ}{Q}{}^{\alpha}\right) + \frac{1}{4}\overset{\circ}{Q}_{\alpha\beta\gamma}\overset{\circ}{Q}{}^{\alpha\beta\gamma} - \frac{1}{2}\overset{\circ}{Q}_{\alpha\beta\gamma}\overset{\circ}{Q}{}^{\gamma\beta\alpha}$$
$$-\frac{1}{4}\overset{\circ}{Q}_\alpha\overset{\circ}{Q}{}^{\alpha} + \frac{1}{2}\overset{\circ}{Q}_\alpha\overset{\circ}{Q}{}^{\alpha}$$
$$= \overset{\circ}{\nabla}_\alpha\left(\overset{\circ}{Q}{}^{\alpha} - \overset{\circ}{Q}{}^{\alpha}\right) - \overset{\circ}{Q}.$$

$$\partial_\alpha \overset{\diamond}{Q}{}^{\alpha} = \overset{\circ}{\nabla}_\alpha \overset{\diamond}{Q}{}^{\alpha} + \overset{\diamond}{L}{}^{\alpha}{}_{\sigma\alpha}\overset{\diamond}{Q}{}^{\sigma} = \overset{\circ}{\nabla}_\alpha \overset{\diamond}{Q}{}^{\alpha} - \frac{1}{2}\overset{\diamond}{Q}_\alpha \overset{\diamond}{Q}{}^{\alpha},$$

$$2\partial_\alpha P^{\alpha}{}_{\mu\nu} + \frac{1}{2}\overset{\circ}{Q}_{\alpha\mu\nu}\left(\overset{\circ}{Q}{}^{\alpha} - \overset{\circ}{\tilde{Q}}{}^{\alpha}\right) + \frac{1}{2}g_{\mu\nu}\partial_\alpha\left(\overset{\circ}{Q}{}^{\alpha} - \overset{\circ}{\tilde{Q}}{}^{\alpha}\right)$$
$$+\frac{1}{2}\overset{\circ}{L}{}^{\sigma}{}_{\mu\nu}\overset{\circ}{Q}_\sigma + \frac{1}{4}\overset{\circ}{Q}_\mu{}^{\alpha}{}_{\sigma}\overset{\circ}{Q}_\nu{}^{\sigma}{}_{\alpha} + \frac{1}{2}\overset{\circ}{Q}{}^{\alpha}{}_{\sigma\mu}\left(\overset{\circ}{Q}{}^{\sigma}{}_{\nu\alpha} - \overset{\circ}{Q}_\alpha{}^{\sigma}{}_{\nu}\right)$$
$$-\frac{1}{2}g_{\mu\nu}\overset{\circ}{\nabla}_\alpha\left(\overset{\circ}{Q}{}^{\alpha} - \overset{\circ}{Q}_\alpha\right) + \frac{1}{2}g_{\mu\nu}\overset{\circ}{Q},$$

$$\frac{2}{\sqrt{-g}}\partial_\alpha\left(\sqrt{-g}\overset{\circ}{P}{}^{\alpha}_{\mu\nu}\right)-\frac{1}{\sqrt{-g}}\overset{\circ}{q}_{\mu\nu}+\frac{1}{2}g_{\mu\nu}\overset{\circ}{Q}=0....$$

$\mathrm{d}s^2=-e^{\nu(t,r)}\mathrm{d}t^2+e^{\lambda(t,r)}\mathrm{d}r^2+r^2\,\mathrm{d}\varphi^2,$

$-e^{\nu(t,r)}\approx -1+\frac{2M}{r},$

$\mathrm{d}s^2=-\left(1-\frac{2M}{r}\right)\mathrm{d}t^2+\left(1-\frac{2M}{r}\right)^{-1}\ \mathrm{d}r^2+r^2\,\mathrm{d}\varphi^2,$

$\dot f(t,r):=\frac{\mathrm{d}f(t,r)}{\mathrm{d}t},f'(t,r):=\frac{\mathrm{d}f(t,r)}{\mathrm{d}r}.$

$\overset{\circ}{G}_{\mu\nu}\equiv\overset{\circ}{R}_{\mu\nu}=0,$

$\overset{\circ}{G}_{tr}\equiv\frac{\dot\lambda(t,r)}{r}=0,\Rightarrow\ \lambda=\lambda(r).$

$\overset{\circ}{G}_{rr}\equiv -e^{\lambda(r)}+r\nu'(t,r)+1=0,$

$\overset{\circ}{G}_{tt}\equiv e^{-\lambda(r)}(r\lambda'(r)-1)+1=0.$

$\left[e^{-\lambda(r)}r\right]'=1\Rightarrow e^{-\lambda(r)}=1-\frac{C_1}{r},$

$\lambda'(r)+\nu'(r)=0,\Rightarrow\ \lambda(r)+\nu(r)=C_2,$

$-e^{\nu(r)}=1-\frac{2M}{r},e^{\lambda(r)}=\frac{1}{1-\frac{2M}{r}}.$

$$e^A_\mu=\begin{pmatrix}\sqrt{-e^{\nu(r)}} & 0 & 0 & 0\\ 0 & \sqrt{e^{\lambda(r)}} & 0 & 0\\ 0 & 0 & r & 0\\ 0 & 0 & 0 & r\sin\,\theta\end{pmatrix}.$$

$\hat T^{t}_{tr}=-\frac{1}{2}\nu'(r)=-\frac{M}{r^2}\left(1-\frac{2M}{r}\right)^{-1},$

$\hat T^{\varphi}_{r\varphi}=\frac{1}{r}.$

$$\hat K_{ttr}=\frac{1}{2}e^{\nu(r)}\nu'(r)=\frac{M}{r^2}$$
$$\hat K_{\varphi r\varphi}=r$$

$\hat S^{tr}_{\hat t}=\frac{2e^{-\lambda(r)}\sqrt{e^{-\nu(r)}}}{r}=\frac{2}{r}\sqrt{1-\frac{2M}{r}},$

$\hat S^{r\varphi}_{\hat\varphi}=-\frac{e^{-\lambda(r)}(r\nu'(r)+2)}{2r^2}=\frac{M-r}{r^3}.$

$\hat T=-\frac{2e^{-\lambda(r)}(r\nu'(r)+1)}{r^2}=-\frac{2}{r^2},$

$$\overset{\circ}{Q}_{r\mu\nu}=\begin{pmatrix}-e^{\nu(r)}\nu'(r) & 0 & 0\\ 0 & e^{\lambda(r)}\lambda'(r) & 0\\ 0 & 0 & 2r\end{pmatrix}$$
$$=\begin{pmatrix}-\frac{2M}{r^2} & 0 & 0\\ 0 & -\frac{2M}{r^2\left(1-\frac{2M}{r}\right)^2} & 0\\ 0 & 0 & 2r\end{pmatrix},$$

$$\overset{\circ}{P}{}^{t}_{tr} = \frac{r\lambda'(r) - r\nu'(r) + 4}{8r} = \frac{1}{8}(\lambda'(r) + \nu'(r)),$$
$$\overset{\circ}{P}{}^{r}_{rr} = \frac{e^{\nu(r)-\lambda(r)}}{r} = \frac{1}{r}\left(1 - \frac{2M}{r}\right)^2,$$
$$\overset{\circ}{P}{}^{r}_{\varphi\varphi} = -\frac{1}{4}re^{-\lambda(r)}(r\nu'(r) + 2) = \frac{M - r}{2},$$
$$\overset{\circ}{P}{}^{\varphi}_{r\varphi} = \frac{1}{8}(\lambda'(r) + \nu'(r)) = 0,$$
$$\frac{\overset{\circ}{q}_{\mu\nu}}{\sqrt{-g}} = \begin{pmatrix} \frac{2e^{\nu(r)-\lambda(r)}\nu'(r)}{r} & 0 & 0 \\ 0 & \frac{2r\nu'(r)+2}{r^2} & 0 \\ 0 & 0 & -\frac{r\nu'(r)+2}{e^{\lambda(r)}} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{4M}{r^3}\left(1 - \frac{2M}{r}\right) & 0 & 0 \\ 0 & \frac{2}{r^2\left(1 - \frac{2M}{r}\right)} & 0 \\ 0 & 0 & \frac{2M}{r} - 2 \end{pmatrix}.$$
$$e^A_\mu g_{\nu\rho}\partial_\sigma \hat{S}_A{}^{\rho\sigma} + e^{-1}e^A_\mu g_{\nu\rho}\partial_\sigma e$$
$$-\hat{S}_B{}^{\sigma}{}_{\nu}T^B{}_{\sigma\mu} + \frac{1}{2}g_{\mu\nu}\hat{T} = 0.$$
$$\partial_\sigma \hat{S}_{\mu\nu}{}^{\sigma} - \hat{S}_{\alpha\nu}{}^{\sigma}\Gamma^{\alpha}{}_{\nu\sigma} - \hat{S}_{\alpha\mu}{}^{\sigma}\Gamma^{\alpha}{}_{\nu\sigma} - \hat{S}_{\mu}{}^{\rho\sigma}\Gamma^{\alpha}{}_{\rho\sigma}g_{\nu\alpha}$$
$$-\hat{S}_{\alpha}{}^{\sigma}{}_{\nu}\hat{T}^{\alpha}_{\sigma\mu} + \Gamma^{\alpha}_{\alpha\sigma}\hat{S}_{\mu\nu}{}^{\sigma} + \frac{1}{2}\hat{T}g_{\mu\nu} = 0.$$
$$\overset{\circ}{\nabla}_\sigma \hat{S}_{\mu\nu}{}^{\sigma} - \hat{K}^{\alpha}{}_{\mu\sigma}\hat{S}_{\alpha\nu}{}^{\sigma} - \hat{K}^{\alpha}{}_{\nu\sigma}\hat{S}_{\mu\alpha}{}^{\sigma}$$
$$+\hat{K}^{\sigma}{}_{\alpha\sigma}\hat{S}_{\mu\nu}{}^{\alpha} - \hat{K}_{\nu\rho\sigma}\hat{S}_{\mu}{}^{\rho\sigma} + \hat{T}_\sigma\hat{S}_{\mu\nu}{}^{\sigma}$$
$$-\hat{S}_{\alpha}{}^{\sigma}{}_{\nu}\hat{T}^{\alpha}{}_{\sigma\mu} + \frac{1}{2}g_{\mu\nu}\hat{T} = 0.$$

$$-\hat{K}^{\alpha}{}_{\nu\sigma}\hat{S}_{\mu\alpha}{}^{\sigma} - \hat{K}_{\nu\rho\sigma}\hat{S}_{\mu}{}^{\rho\sigma} = 0,$$
$$\hat{K}^{\sigma}{}_{\alpha\sigma}\hat{S}_{\mu\nu}{}^{\alpha} + \hat{T}_\sigma\hat{S}_{\mu\nu}{}^{\sigma} = 0.$$
$$\overset{\circ}{\nabla}\hat{S}_{\mu\nu}{}^{\sigma} + \hat{K}^{\alpha}_{\mu\sigma}\hat{S}_{\alpha}{}_{\nu} - \hat{T}^{\alpha}{}_{\sigma\mu}\hat{S}_{\alpha}{}_{\nu} + \frac{1}{2}\hat{T}g_{\mu\nu} = \overset{\circ}{\nabla}_\sigma\hat{S}_{\mu\nu}{}^{\sigma} + \hat{K}^{\alpha}{}_{\sigma\mu}\hat{S}_{\alpha}{}_{\nu}{}_{\nu} + \frac{1}{2}g_{\mu\nu}\hat{T} = 0$$
$$\frac{\partial_\alpha\sqrt{-g}}{\sqrt{-g}} = \overset{\circ}{\Gamma}{}^{\sigma}_{\alpha\sigma} = -\overset{\circ}{L}{}^{\sigma}_{\alpha\sigma} = \frac{1}{2}\overset{\circ}{Q}_\alpha.$$
$$2\partial_\alpha P^{\alpha}{}_{\mu\nu} + \frac{1}{2}\overset{\circ}{Q}_{\alpha\mu\nu}\left(\overset{\circ}{Q}{}^{\alpha} - \overset{\circ}{\tilde{Q}}{}^{\alpha}\right) + \frac{1}{2}g_{\mu\nu}\partial_\alpha\left(\overset{\circ}{Q}{}^{\alpha} - \overset{\circ}{\tilde{Q}}{}^{\alpha}\right)$$
$$+\frac{1}{2}\overset{\circ}{L}{}^{\sigma}{}_{\mu\nu}\overset{\circ}{Q}_\sigma + \frac{1}{4}\overset{\circ}{Q}_{\mu}{}^{\alpha}{}_{\sigma}\overset{\circ}{Q}_{\nu}{}^{\sigma}{}_{\alpha} + \frac{1}{2}\overset{\circ}{Q}{}^{\alpha}{}_{\sigma\mu}\left(\overset{\circ}{Q}{}^{\sigma}{}_{\nu\alpha} - \overset{\circ}{Q}_{\alpha}{}^{\sigma}{}_{\nu}\right)$$
$$-\frac{1}{2}g_{\mu\nu}\overset{\circ}{\nabla}_\alpha\left(\overset{\circ}{Q}{}^{\alpha} - \overset{\circ}{\tilde{Q}}{}^{\alpha}\right) + \frac{1}{2}g_{\mu\nu}{}^{Q} = 0.$$
$$\mathbb{P}(x_n, t_n \mid x_1, t_1; \ldots; x_{n-1}, t_{n-1}) = \mathbb{P}(x_n, t_n \mid x_{n-1}, t_{n-1})$$
$$\hat{\mathcal{E}}(t)(\hat{\rho}(0)) = \hat{\rho}(t)$$
$$\hat{\mathcal{E}}(t_1 + t_2) = \hat{\mathcal{E}}(t_1)\hat{\mathcal{E}}(t_2),$$

$$\frac{d\hat{\rho}}{dt}(t) = \hat{\mathcal{L}}\hat{\rho}(t)$$
$$\hat{H} = \hat{H}_\mathrm{S} + \hat{H}_\mathrm{B} + \hat{H}_\mathrm{SB}$$
$$\hat{H}_\mathrm{S} = \frac{\hat{P}^2}{2M} + V(\hat{Q}),$$
$$\hat{H}_\mathrm{B}^{(k)} = \sum_{k=1}^{N}\left(\frac{\hat{p}_k^2}{2m_k} + \frac{m_k\omega_k^2\hat{q}_k^2}{2}\right),$$
$$\hat{H}_\mathrm{SB}^{(k)} = \sum_{k=1}^{N}\left(g_k\hat{q}_k\hat{Q} + \frac{g_k^2}{2m_k\omega_k^2}\hat{Q}^2\right).$$
$$\frac{d^2\hat{Q}}{dt^2}(t) + V'(\hat{Q}(t)) + \frac{1}{M}\int_0^t k(t-t')\hat{P}(t')dt' + k(t)\hat{Q}(0) = \hat{f}(t)$$
$$k(t) = \sum_{i=1}^{N}\frac{g_k^2}{m_k\omega_k^2}\cos(\omega_k t)$$
$$\hat{f}(t) = \sum_{i=1}^{N}\left(g_k\hat{q}_k(0)\cos(\omega_k t) + \frac{g_k\hat{p}_k(0)}{m_k\omega_k}\sin(\omega_k t)\right)$$
$$\langle\hat{f}(t)\rangle = \mathrm{Tr}\big(\hat{f}(t)\rho_\mathrm{th}\big) = 0$$
$$\langle\{\hat{f}(t),\hat{f}(t')\}\rangle = \sum_{k=1}^{N}\frac{\hbar g_k^2}{m_k\omega_k}\coth\left(\frac{\hbar\omega_k}{2k_BT}\right)\cos\big(\omega_k(t-t')\big),$$

$$\hat{\Theta}\hat{\boldsymbol{q}}\hat{\Theta}^{-1} = \hat{\boldsymbol{q}}, \hat{\Theta}\hat{\boldsymbol{p}}\hat{\Theta}^{-1} = -\hat{\boldsymbol{p}},$$
$$\hat{A}_R(t) = \hat{\Theta}\hat{A}(-t)\hat{\Theta}^{-1}$$
$$\hat{\boldsymbol{q}}_R(t) = \hat{\Theta}\hat{\boldsymbol{q}}(-t)\hat{\Theta}^{-1} = \hat{\boldsymbol{q}}(t), \hat{\boldsymbol{p}}_R(t) = \hat{\Theta}\hat{\boldsymbol{p}}(-t)\hat{\Theta}^{-1} = -\hat{\boldsymbol{p}}(t).$$
$$M\frac{d^2\hat{Q}}{dt^2}(-t) + V'(\hat{Q}(-t)) + \frac{1}{M}\int_0^{-t} k(-t-t')\hat{P}(t')dt' + k(-t)\hat{Q}(0) = \hat{f}(-t)$$
$$\frac{d\hat{\rho}_R}{dt} = -\hat{\Theta}\frac{d\hat{\rho}}{dt}\hat{\Theta}^{-1} = -\hat{\Theta}\hat{\mathcal{L}}\hat{\Theta}^{-1}\hat{\Theta}\hat{\rho}\hat{\Theta}^{-1} = -\hat{\mathcal{L}}_R\hat{\rho}_R.$$
$$\hat{\rho}(t) = \hat{\mathcal{E}}(t)\hat{\rho}(0), \hat{\mathcal{E}}(t) = \exp(-\hat{\mathcal{L}}t).$$
$$\hat{\Theta}\hat{\mathcal{E}}(t)\hat{\Theta}^{-1} = \exp\big(-\hat{\Theta}\hat{\mathcal{L}}\hat{\Theta}^{-1}t\big) = \exp\big(-\hat{\mathcal{L}}_R t\big).$$
$$\hat{\Theta}\hat{\mathcal{E}}(t)\hat{\Theta}^{-1} = \exp(\hat{\mathcal{L}}t) = \hat{\mathcal{E}}(t)^{-1}.$$
$$\hat{\Theta}\hat{U}(t)\hat{\Theta}^{-1} = \hat{U}(-t)$$
$$\frac{d\hat{\rho}}{dt} = \hat{\mathcal{L}}(t)\hat{\rho}(t)$$
$$\frac{d\hat{\rho}_R}{dt}(t) = -\hat{\Theta}\frac{d\hat{\rho}}{dt}(-t)\hat{\Theta}^{-1} = -\hat{\mathcal{L}}_R(-t)\hat{\rho}_R(t)$$
$$\hat{\rho}(t) = \hat{\mathcal{E}}(t,0)\hat{\rho}(0), \hat{\mathcal{E}}(t_2,t_1) = \hat{\mathcal{T}}\exp\left(-\int_{t_1}^{t_2}\hat{\mathcal{L}}(t')dt'\right),$$
$$\hat{\Theta}\hat{\mathcal{E}}(t_1,t_2)\hat{\Theta}^{-1} = \hat{\mathcal{T}}\exp\left(-\int_{t_1}^{t_2}\hat{\Theta}\hat{\mathcal{L}}(t')\hat{\Theta}^{-1}dt'\right) = \hat{\mathcal{T}}\exp\left(-\int_{t_1}^{t_2}\hat{\mathcal{L}}_R(t')dt'\right).$$
$$\hat{\Theta}\hat{\mathcal{E}}(t_1,t_2)\hat{\Theta}^{-1} = \mathcal{T}\exp\left(-\int_{t_2}^{t_1}\hat{\mathcal{L}}(t')dt'\right) = \mathcal{E}(t_2,t_1)^{-1}.$$
$$\int_0^\infty k(t')dt' < \infty$$

$$\int_0^{\tau_B} k(t')dt' \approx \int_0^{\infty} k(t')dt'$$

$$\int_0^{t} k(t-t')\hat{P}(t')dt' = \int_0^{t} k(t')\hat{P}(t-t')dt' \approx \hat{P}(t)\int_0^{\tau_B} k(t')dt' = \hat{P}(t)\int_0^{\infty} k(t')dt'$$

$$\int_0^{t} k(t')dt' = \mathrm{sgn}(t)\int_0^{|t|} k(t')dt',$$

$$\int_0^{t} k(t-t')\hat{P}(t')dt' \approx \mathrm{sgn}(t)\hat{P}(t)\int_0^{\infty} k(t')dt'$$

$$M\frac{d^2\hat{Q}}{dt^2} + V'(\hat{Q}(t)) + \mathrm{sgn}(t)\gamma\hat{P}(t) = \hat{f}(t).$$

$$\int_0^{\infty} k(t')dt' = M\gamma$$

$$\langle\{\hat{f}(t),\hat{f}(t')\}\rangle = \frac{\gamma M\hbar}{\pi}\int_0^{\Lambda} \omega\coth\left(\frac{\hbar\omega}{2k_BT}\right)\cos\left(\omega(t-t')\right)d\omega$$

$$\langle\{\hat{f}(t),\hat{f}(t'\}\rangle = 2\gamma Mk_BT\delta(t-t').$$

$$\mathrm{Tr}_S(\hat{Y}\hat{\mu}(t)) = \mathrm{Tr}_S\left(\hat{\rho}_S\hat{Y}(t)\right)$$

$$\dot{\hat{\mu}}(t) = -\frac{i}{\hbar}\left[\hat{H}_S,\hat{\mu}(t)\right] + \frac{i}{2\hbar}[\{\gamma\mathrm{sgn}(t)\hat{P},\hat{\mu}(t)\},\hat{Q}] + \frac{i}{2\hbar}[\{\hat{f}(t),\hat{\mu}(t)\},\hat{Q}]$$

$$\hat{\rho}(t) = \langle\hat{\mu}(t)\rangle := \mathrm{Tr}_B(\hat{\mu}(t)\hat{\rho}_{th}),$$

$$\dot{\hat{\rho}}(t) = -\frac{i}{\hbar}\left[\hat{H}_S,\hat{\rho}(t)\right] + \frac{i\mathrm{sgn}(t)}{2\hbar}[\{\gamma\hat{P},\hat{\rho}(t)\},\hat{Q}] - \frac{\Gamma(t)}{\hbar^2}[[\hat{\rho}(t),\hat{Q}],\hat{Q}].$$

$$\Gamma(t) = \int_0^{t} \langle\{\hat{f}(t),\hat{f}(t')\}\rangle dt'$$

$$\int_0^{t} \langle\{\hat{f}(t),\hat{f}(t')\}\rangle dt' = \mathrm{sgn}(t)\int_0^{|t|} \langle\{\hat{f}(|t|),\hat{f}(t')\}\rangle dt'$$

$$\lim_{t\to\infty}\int_0^{|t|} \langle\{\hat{f}(|t|),\hat{f}(t')\}\rangle dt' = 2\gamma Mk_BT.$$

$$\dot{\hat{\rho}}(t) = -\frac{i}{\hbar}\left[\hat{H}_S,\hat{\rho}(t)\right] + \frac{i\mathrm{sgn}(t)}{2\hbar}[\{\gamma\hat{P},\hat{\rho}(t)\},\hat{Q}] - \mathrm{sgn}(t)\frac{2\gamma Mk_BT}{\hbar^2}[[\hat{\rho}(t),\hat{Q}],\hat{Q}].$$

$$\frac{d\hat{\rho}}{dt}(t) = \left(i\hat{\mathcal{L}}_H + \mathrm{sgn}(t)\hat{\mathcal{L}}_D\right)\hat{\rho}(t)$$

$$\hat{\mathcal{L}}_H\hat{\rho}(t) = -\frac{1}{\hbar}\left[\hat{H}_S,\hat{\rho}(t)\right]$$

$$\hat{\mathcal{L}}_D\hat{\rho}(t) = \frac{i}{2\hbar}[\{\gamma\hat{P},\hat{\rho}(t)\},\hat{Q}] - \frac{2\gamma Mk_BT}{\hbar^2}[[\hat{\rho}(t),\hat{Q}],\hat{Q}].$$

$$\frac{d\hat{\rho}_R}{dt}(t) = -\hat{\Theta}\frac{d\hat{\rho}}{dt}(t)\hat{\Theta}^{-1} = \left(i\hat{\mathcal{L}}_H + \mathrm{sgn}(t)\hat{\mathcal{L}}_D\right)\hat{\rho}_R(t)$$

$$\hat{\rho}(t) = \hat{\mathcal{E}}(t)\hat{\rho}(0), \hat{\mathcal{E}}(t) = \exp\left(i\hat{\mathcal{L}}_Ht + \hat{\mathcal{L}}_D|t|\right)$$

$$\rho(p,q,t) = \frac{1}{\sqrt{\pi N(t)}}\exp\left(\frac{-(q+\mathrm{sgn}(t)A(t)p)^2}{N(t)} - B(t)p^2\right)$$

$$N(t)=\frac{mk_BT}{\hbar^2}\left(1-e^{-2\gamma|t|}\right)+\frac{e^{-2\gamma|t|}}{\sigma^2}$$

$$A(t)=\frac{i\hbar}{2\sigma^2 m\gamma}e^{-\gamma|t|}\left(1-e^{-\gamma|t|}\right)-\frac{ik_BT}{2\hbar\gamma}\left(1-e^{-\gamma|t|}\right)^2$$

$$B(t)=\frac{\hbar^2}{4\sigma^2m^2\gamma^2}\left(1-e^{-\gamma|t|)}\right)^2+\frac{\sigma^2}{4}+\frac{k_BT}{m\gamma^2}\left(2\gamma|t|-3+4e^{-\gamma|t|}-e^{-2\gamma|t|}\right).$$

$$\psi(x,0)=\frac{1}{(\sigma^2\pi)^{1/4}}\exp\left(-\frac{x^2}{2\sigma^2}\right).$$

$$S_{\mathrm{vN}}(\xi)=\frac{1-\xi}{2\xi}\log\left(\frac{1+\xi}{1-\xi}\right)-\log\left(\frac{2\xi}{1+\xi}\right)$$

$$\hat{H}=\hat{H}_{\mathrm{S}}+\hat{H}_{\mathrm{B}}+\hat{H}_{\mathrm{SB}}$$

$$\hat{H}_{\mathrm{I}}(t)=e^{\frac{i}{\hbar}(\hat{H}_{\mathrm{S}}+\hat{H}_{\mathrm{B}})t}\hat{H}_{\mathrm{SB}}e^{\frac{-i}{\hbar}(\hat{H}_{\mathrm{S}}+\hat{H}_{\mathrm{B}})t}$$

$$\frac{d}{dt}\hat{\rho}_{\mathrm{I}}(t)=-\frac{i}{\hbar}\left[\hat{H}_{\mathrm{I}}(t),\hat{\rho}_{\mathrm{I}}(t)\right]$$

$$\frac{d}{dt}\hat{\rho}_{\mathrm{S}}(t)=-\frac{1}{\hbar^2}\int_0^t \mathrm{Tr}_{\mathrm{B}}\left(\left[\hat{H}_{\mathrm{I}}(t),\left[\hat{H}_{\mathrm{I}}(s),\hat{\rho}_{\mathrm{I}}(s)\right]\right]\right)ds$$

$$\frac{d}{dt}\hat{\rho}_{\mathrm{S}}(t)=-\frac{1}{\hbar^2}\int_0^t \mathrm{Tr}_{\mathrm{B}}\left(\left[\hat{H}_{\mathrm{I}}(t),\left[\hat{H}_{\mathrm{I}}(s),\hat{\rho}_{\mathrm{S}}(t)\otimes\hat{\rho}_{\mathrm{B}}\right]\right]\right)ds$$

$$\hat{H}_{\mathrm{I}}(t)=\sum_{\alpha,\omega} e^{-i\omega t}\hat{A}_\alpha(\omega)\otimes\hat{B}_\alpha(t)$$

$$\frac{d}{dt}\hat{\rho}_{\mathrm{S}}(t)=$$
$$-\frac{1}{\hbar^2}\sum_{\alpha,\beta,\omega}\left[\Gamma_{\alpha\beta}(\omega,t)\hat{A}_\alpha^\dagger(\omega)\hat{A}_\beta(\omega)\hat{\rho}_{\mathrm{S}}(t)+\Gamma_{\beta\alpha}^*(\omega,t)\hat{\rho}_{\mathrm{S}}(t)\hat{A}_\alpha^\dagger(\omega)\hat{A}_\beta(\omega)\left(\Gamma_{\alpha\beta}(\omega,t)+\Gamma_{\beta\alpha}^*(\omega,t)\right)\hat{A}_\beta(\omega)\hat{\rho}_{\mathrm{S}}(\right.$$

$$\Gamma_{\alpha\beta}(\omega,t)=\int_0^t e^{i\omega s}\mathrm{Tr}\left(\hat{B}_\alpha^\dagger(t)\hat{B}_\beta(t-s)\hat{\rho}_{\mathrm{B}}\right)ds$$

$$\Gamma_{\alpha\beta}(\omega,t)+\Gamma_{\beta\alpha}^*(\omega,t)=\mathrm{sgn}(t)\int_{-|t|}^{|t|} e^{i\omega s}\mathrm{Tr}\left(\hat{B}_\alpha^\dagger(s)\hat{B}_\beta(0)\hat{\rho}_B\right)ds.$$

$$\Gamma_{\alpha\beta}(\omega,t)+\Gamma_{\beta\alpha}^*(\omega,t)\approx\mathrm{sgn}(t)\gamma_{\alpha\beta}(\omega)$$

$$\gamma_{\alpha\beta}(\omega)=\int_{-\infty}^{\infty} e^{i\omega s}\mathrm{Tr}\left(\hat{B}_\alpha^\dagger(s)\hat{B}_\beta(0)\hat{\rho}_B\right)ds$$

$$\eta_{\alpha\beta}(\omega)=\frac{1}{2i}\left(\Gamma_{\alpha\beta}(\omega,t)-\Gamma_{\beta\alpha}^*(\omega,t)\right).$$

$$\frac{d}{dt}\hat{\rho}_{\mathrm{S}}(t)=-\frac{i}{\hbar}\left[\hat{H}_{\mathrm{S}},\hat{\rho}_{\mathrm{S}}\right]+\frac{\mathrm{sgn}(t)}{\hbar^2}\sum_{\alpha,\beta,\omega}\hat{\mathcal{D}}_{\alpha\beta}(\omega)\hat{\rho}_{\mathrm{S}}(t)$$

$$\hat{H}_{\mathrm{S}}=\frac{1}{\hbar}\sum_{\alpha,\beta,\omega}\eta_{\alpha\beta}(\omega)\hat{A}_\alpha^\dagger(\omega)\hat{A}_\beta(\omega)$$

$$\hat{\mathcal{D}}_{\alpha\beta}(\omega)\hat{\rho}_{\mathrm{S}}(t)=\gamma_{\alpha\beta}(\omega)\left(\hat{A}_\beta(\omega)\hat{\rho}_{\mathrm{S}}(t)\hat{A}_\alpha^\dagger(\omega)-\frac{1}{2}\left\{\hat{A}_\alpha^\dagger(\omega)\hat{A}_\beta(\omega),\hat{\rho}_{\mathrm{S}}(t)\right\}\right).$$

$$\hat{\Theta}\gamma_{\alpha\beta}(\omega)\hat{\Theta}^{-1}=\int_{-\infty}^{\infty} e^{i\omega s}\mathrm{Tr}\left(\hat{\Theta}\hat{B}_\alpha^\dagger(s)\hat{B}_\beta(0)\hat{\Theta}^{-1}\hat{\rho}_B\right)ds=\gamma_{\alpha\beta}(\omega).$$

$$\frac{d}{dt}\rho_n(t) = \sum_{n'\neq n} \left(W_{n,n'}\rho_{n'}(t) - W_{n',n}\rho_n(t)\right),$$
$$\langle n|\hat{A}^\dagger_\alpha(\omega)\hat{A}_\beta(\omega)|n'\rangle = \delta_{n,n'}\langle n|\hat{A}_\alpha|m\rangle\langle m|\hat{A}_\beta|n\rangle,$$
$$\frac{d}{dt}\rho_n(t) = \frac{\mathrm{sgn}(t)}{\hbar^2}\sum_{n\neq n'} \left(W_{n,n'}\rho_{n'}(t) - W_{n',n}\rho_n(t)\right),$$
$$W_{n',n} = \sum_{\alpha,\beta} \gamma_{\alpha\beta}(\varepsilon_n - \varepsilon_{n'})\langle n|\hat{A}_\alpha|n'\rangle\langle n'|\hat{A}_\beta|n\rangle$$
$$\hat{H} = \hat{H}_0 + \lambda\hat{H}_{\mathrm{SB}}$$
$$\frac{d}{dt}\rho_{\varepsilon,k}(t) = \frac{\lambda^2}{\hbar^2}\sum_{\varepsilon',k'}\int_0^t \Lambda_{\varepsilon,k,\varepsilon',k'}(t-s)\left(\rho_{\varepsilon',k'}(s) - \rho_{\varepsilon,k}(s)\right)ds,$$
$$\Lambda_{\varepsilon,k,\varepsilon',k'}(t) = 2\left|\langle\varepsilon,k|\hat{H}_{\mathrm{SB}}|\varepsilon',k'\rangle\quad\right|^2\cos\left(\frac{\varepsilon-\varepsilon'}{\hbar}t\right)$$
$$\int_0^t \Lambda_{\varepsilon,k,\varepsilon',k'}(t-s)ds = 2\hbar\frac{\left|\langle\varepsilon,k|\hat{H}_{\mathrm{SB}}|\varepsilon',k'\rangle\quad\right|^2}{\varepsilon-\varepsilon'}\sin\left(\frac{\varepsilon-\varepsilon'}{\hbar}t\right).$$
$$\frac{\hbar}{\varepsilon-\varepsilon'}\sin\left(\frac{\varepsilon-\varepsilon'}{\hbar}t\right) \approx \mathrm{sgn}(t)\pi\hbar\delta(\varepsilon-\varepsilon')$$
$$\frac{d}{dt}\rho_{\varepsilon,k}(t) = \mathrm{sgn}(t)\sum_{k'}\left(W^{\varepsilon}_{k,k'}\rho_{\epsilon,k'}(t) - W^{\epsilon}_{k',k}\rho_{\epsilon,k}(t)\right)$$
$$W^{\varepsilon}_{k,k'} = \frac{2\lambda^2}{\hbar}\left|\langle\varepsilon,k|\hat{H}_{\mathrm{SB}}|\varepsilon,k'\rangle\quad\right|^2\eta(\varepsilon)$$
$$\psi_R(x,a+t) = \psi^*(x,a-t)$$
$$\frac{dW}{dt} = -\frac{p}{M}\frac{dW}{dx} + \sum_{n=0}^{\infty}\frac{(-\hbar^2)^n V^{(2n+1)}(x)}{(2n+1)!\,2^{2n}}\frac{d^{2n+1}W}{dp^{2n+1}} + \mathrm{sgn}(t)\gamma\frac{d}{dp}(pW) + \mathrm{sgn}(t)2\gamma Mk_BT\frac{d^2W}{dp^2}$$
$$\frac{dW}{dt} = -\frac{p}{M}\frac{dW}{dx} + \frac{dV}{dx}\frac{dW}{dp} + \mathrm{sgn}(t)\gamma\frac{d}{dp}(pW) + \mathrm{sgn}(t)2\gamma Mk_BT\frac{d^2W}{dp^2}$$
$$\dot{\hat{\mu}}(t) = -\frac{i}{\hbar}\left[\hat{H}_{\mathrm{S}},\hat{\mu}(t)\right] + \frac{i}{2\hbar}[\{\gamma\mathrm{sgn}(t)\hat{P},\hat{\mu}(t)\},\hat{Q}] + \frac{i}{2\hbar}[\{\hat{f}(t),\hat{\mu}(t)\},\hat{Q}],$$
$$\hat{\rho}(t) = \langle\hat{\mu}(t)\rangle := \mathrm{Tr}_{\mathrm{B}}(\hat{\mu}(t)\hat{\rho}_{\mathrm{th}}),$$
$$\hat{A}\hat{\mu}(t) = -\frac{i}{\hbar}\left[\hat{H}_{\mathrm{S}},\hat{\mu}(t)\right] + \frac{i}{2M\hbar}[\{\gamma\mathrm{sgn}(t)\hat{P},\hat{\mu}(t)\},\hat{Q}],$$
$$\hat{B}\hat{\mu}(t) = \frac{i}{\hbar}[\hat{\mu}(t),\hat{Q}],$$
$$\hat{\alpha}(t)\hat{\mu}(t) = \frac{1}{2}\{\hat{f}(t),\hat{\mu}(t)\},$$
$$\hat{\eta}(t) = \exp(-\hat{A}t)\hat{\mu}(t).$$
$$\dot{\hat{\eta}}(t) = \hat{B}(t)\hat{\alpha}(t)\hat{\eta}(t),$$
$$\frac{d}{dt}\langle\hat{\eta}(t)\rangle = \hat{B}(t)\langle\hat{\alpha}(t)\hat{\mu}(0)\rangle + \int_0^t \hat{B}(t)\hat{B}(t')\langle\hat{\alpha}(t)\hat{\alpha}(t')\hat{\mu}(t')\rangle dt'$$
$$\frac{d\hat{\rho}}{dt}(t) = \hat{A}\hat{\rho}(t) + \int_0^t \left\langle\hat{\alpha}(t)\hat{\alpha}(t')\hat{I}\right\rangle\hat{B}\hat{B}(t'-t)\hat{\rho}(t')dt'$$
$$\frac{d\hat{\rho}}{dt}(t) = \hat{A}\hat{\rho}(t) - \int_0^t \left\langle\hat{\alpha}(t)\hat{\alpha}(t')\hat{I}\right\rangle dt'\frac{1}{\hbar}[[\hat{\rho}(t),\hat{Q}],\hat{Q}].$$

$$\dot{\hat{\rho}}(t) = -\frac{i}{\hbar}\left[\hat{H}_{\mathrm{S}}, \hat{\rho}(t)\right] + \frac{i}{2\hbar}[\{\gamma \mathrm{sgn}(t)\hat{P}, \hat{\rho}(t)\}, \hat{Q}] - \frac{\Gamma(t)}{\hbar^2}[[\hat{\rho}(t), \hat{Q}], \hat{Q}],$$

$$\sum_{k=1}^{m_1} \Pr(1 \mid [1,k]) = 1$$

$$\sum_{k=1}^{m_2} \Pr(1 \mid [2,k]) = 1 \frac{\sum_{k=1}^{m_{n-1}} \Pr(1 \mid [n-1,k]) = 1}{\sum_{k=1}^{m_n} \Pr(1 \mid [n,k]) = \begin{cases} & 0, \mathrm{NCHV}, \\ 1, & \mathrm{Q} \end{cases}}$$

$$\cdots$$

$$\sum_{i \in V} \Pr(1 \mid i) - \sum_{(i,j) \in E} \Pr(1{,}1 \mid i,j) \overset{\mathrm{NCHV}}{\leqslant} \alpha(G) \overset{\mathrm{Q}}{\leqslant} \vartheta(G)$$

$$\alpha(G) = n-1, \vartheta(G) = n, \text{and } \chi(\bar{G}) = n$$

$$p_1 := \sum_{k=1}^{19} \Pr(1 \mid k) = 1$$

$$p_2 := \sum_{k=20}^{38} \Pr(1 \mid k) = 1$$

$$p_3 := \sum_{k=39}^{57} \Pr(1 \mid k) = \begin{cases} 0, & \mathrm{NCHV} \\ 1, & \mathrm{Q} \end{cases}$$

$$|\boldsymbol{a}\rangle = \begin{pmatrix} a_1 & a_2 & \cdots \\ & & a_d \end{pmatrix}^{\dagger}$$

$$\leftrightarrow \{|\alpha_1, \Delta t\rangle, |\alpha_2, 2\Delta t\rangle, \cdots, |\alpha_d, d\Delta t\rangle\}$$

$$\langle J \mid \boldsymbol{a}\rangle = \sum_{k=1}^{b} \langle J_k \mid \boldsymbol{a}\rangle$$

$$p_1 = 0.9939(15), p_2 = 0.9980(2), p_3 = 0.9983(2)$$

$$\sum_{i \in V} \Pr(1 \mid i) = 2.9902(4)$$

$$\alpha(G) + \sum_{(i,j) \in E} \Pr(1{,}1 \mid i,j) = 2.651(4)$$

$$\max_{\mathbf{B}} \vartheta = \mathrm{tr}(\mathrm{BJ})$$

$$\mathbf{B} \geqslant 0, \mathrm{tr}(\mathbf{B}) = 1$$

$$B_{ij} = 0, \forall (i,j) \in E(G)$$

$$h_0 = \tilde{\alpha} \mathrm{Re}\left(e^{-i\phi} \sum_{k=1} c_k e^{ik\varphi}\right)$$

$$h_1 = \tilde{\alpha} \mathrm{Re}\left(e^{-i\phi+\pi/2} \sum_{k=1} c_k e^{ik\varphi}\right)$$

$$h_2 = \tilde{\alpha} \mathrm{Re}\left(e^{-i\phi+\pi/2} \sum_{k=4} c_k e^{ik\varphi}\right)$$

$$P(1{,}1 \mid i,j) = P(1 \mid i) P(1 \mid j, i=1)$$

$$\widehat{\mathcal{H}}^{(N)}/\hbar = \omega_0 \hat{a}^{\dagger}\hat{a} + \omega_{\mathrm{a}}\left(\hat{S}_z + \frac{N}{2}\right) + \frac{2g}{\sqrt{N}}(\hat{a}^{\dagger} + \hat{a})\hat{S}_x$$

$$g > \frac{\sqrt{\omega_{\mathrm{a}}\omega_0}}{2}$$

$$\widehat{\mathcal{H}}_{\mathrm{spin}} = \widehat{\mathcal{H}}_{\mathrm{Fe}} + \widehat{\mathcal{H}}_{\mathrm{Er}} + \widehat{\mathcal{H}}_{\mathrm{Fe-Er}}$$

$$\widehat{\mathcal{H}}_{\mathrm{Fe}} = \sum_{s=\mathrm{A,\,B}} \sum_{i=1}^{N_0} \mu_0 \mu_{\mathrm{B}} \mathfrak{g}_{\mathrm{Fe}}^{x} S_{i,x}^{s} H_x^{\mathrm{DC}} + J_{\mathrm{Fe}} \sum_{i,i'} \mathrm{S}_i^{\mathrm{A}} \cdot \mathbf{S}_{i'}^{\mathrm{B}}$$

$$-D_{\mathrm{Fe}}^{y} \sum_{i,j'} \left(S_{i,z}^{\mathrm{A}} S_{i',x}^{\mathrm{B}} - S_{i',z}^{\mathrm{B}} S_{i,x}^{\mathrm{A}}\right)$$

$$- \sum_{s=A,B} \sum_{i=1}^{N_0} \left[A_{\mathrm{Fe}}^{x}\left(S_{i,x}^{s}\right)^2 + A_{\mathrm{Fe}}^{z}\left(S_{i,z}^{s}\right)^2\right],$$

$$\widehat{\mathcal{H}}_{\mathrm{Er}} = \sum_{s=\mathrm{A,\,B}} \sum_{i=1}^{N_0} \mu_0 \mu_{\mathrm{B}} \mathfrak{g}_{\mathrm{Er}}^{x} \mathfrak{a}_{i,x}^{s} H_x^{\mathrm{DC}} + J_{\mathrm{Er}} \sum_{i,i'} \mathfrak{Z}_i^{\mathrm{A}} \cdot \mathfrak{Z}_{i'}^{\mathrm{B}}$$

$$- \sum_{s=\mathrm{A,\,B}} \sum_{i=1}^{N_0} \left[A_{\mathrm{Er}}^{x}\left(3_{i,x}^{s}\right)^2 + A_{\mathrm{Er}}^{z}\left(\Xi_{i,z}^{s}\right)^2\right],$$

$$\widehat{\mathcal{H}}_{\mathrm{Fe-Er}} = \sum_{i=1}^{N_0} \sum_{s,s'=\mathrm{A,\,B}} \left[\,\mathrm{J}_i^{s} \cdot\, \mathrm{S}_i^{s'} + \mathrm{D}^{s,s'} \cdot \left(\mathfrak{Z}_i^{s} \times \mathrm{S}_i^{s'}\right)\right]$$

$$\left\langle \mathfrak{B}_{\parallel}^{s} \right\rangle = -\frac{1}{2}\tanh\left(\frac{\mathfrak{g}_{\mathrm{Er}}\mu_{\mathrm{B}}\left|\overline{\mathbf{B}}_{\mathrm{Er}}^{s}\right|}{2k_{\mathrm{B}}T}\right)$$

$$\left\langle S_{\parallel}^{s} \right\rangle = -B_S\left(\frac{S\mathfrak{g}_{\mathrm{Fe}}\mu_{\mathrm{B}}\left|\overline{\mathbf{B}}_{\mathrm{Fe}}^{s}\right|}{k_{\mathrm{B}}T}\right)$$

$$\hbar\frac{d}{dt}\delta\mathfrak{B}^{s} = -\delta\mathfrak{s}^{s} \times \mathfrak{g}_{\mathrm{Er}}\mu_{\mathrm{B}}\overline{\mathbf{B}}_{\mathrm{Er}}^{s}\left(\mathfrak{g}^{\mathrm{A/B}}, \mathbf{S}^{\mathrm{A/B}}\right) - \bar{\mathfrak{s}}^{s} \times \mathfrak{g}_{\mathrm{Er}}\mu_{\mathrm{B}}\mathbf{B}_{\mathrm{Er}}^{s}\left(\delta\mathfrak{s}^{\mathrm{A/B}}, \delta\mathbf{S}^{\mathrm{A/B}}\right)$$

$$\hbar\frac{d}{dt}\delta\mathbf{S}^{s} = -\delta\mathbf{S}^{s} \times \mathfrak{g}_{\mathrm{Fe}}\mu_{\mathrm{B}}\overline{\mathbf{B}}_{\mathrm{Er}}^{s}\left(\mathfrak{g}^{\mathrm{A/B}}, \mathbf{S}^{\mathrm{A/B}}\right) - \overline{\mathbf{S}}^{s} \times \mathfrak{g}_{\mathrm{Fe}}\mu_{\mathrm{B}}\mathbf{B}_{\mathrm{Er}}^{s}\left(\delta\mathfrak{s}^{\mathrm{A/B}}, \delta\mathbf{S}^{\mathrm{A/B}}\right)$$

$$\frac{\widehat{\mathcal{H}}_{\mathrm{Dicke}}}{\hbar} \sim \omega_0 \hat{a}^{\dagger}\hat{a} + \omega_{\mathrm{a}}\hat{\Sigma}_x^{+} + \frac{ig_z}{\sqrt{N_0}}\left(\hat{a}^{\dagger} - \hat{a}\right)\hat{\Sigma}_z^{-} + \cdots$$

$$\hbar\frac{d\mathfrak{s}^{s}}{dt} = -\mathfrak{B}^{s} \times \mathfrak{g}_{\mathrm{Er}}\mu_{\mathrm{B}}\mathbf{B}_{\mathrm{Er}}^{s}\left(\mathfrak{g}^{\mathrm{A/B}}, \mathbf{S}^{\mathrm{A/B}}\right)$$

$$\hbar\frac{d\mathbf{S}^{s}}{dt} = -\mathbf{S}^{s} \times \mathfrak{g}_{\mathrm{Fe}}\mu_{\mathrm{B}}\mathbf{B}_{\mathrm{Fe}}^{s}\left(\mathfrak{h}^{\mathrm{A/B}}, \mathbf{S}^{\mathrm{A/B}}\right)$$

$$\mathbf{B}_{\mathrm{Er}}^{s} = \mathbf{B}^{\mathrm{DC}} + \frac{2z_{\mathrm{Er}}J_{\mathrm{Er}}}{\mu_{\mathrm{B}}\mathfrak{g}_{\mathrm{Er}}}\mathfrak{s}^{\bar{s}} + \frac{2}{\mu_{\mathrm{B}}\mathfrak{g}_{\mathrm{Er}}} \sum_{s'=\mathrm{A,\,B}} \left[J\mathbf{S}^{s} - \left(\mathbf{D}^{s,s'} \times \mathbf{S}^{s}\right) - \mathbf{A}_{\mathrm{Er}} \cdot \mathfrak{s}^{s}\right]$$

$$\mathbf{B}_{\mathrm{Fe}}^{s} = \mathbf{B}^{\mathrm{DC}} + \frac{2z_{\mathrm{Fe}}\,\mathrm{J}_{\mathrm{Fe}}}{\mu_{\mathrm{B}}\mathfrak{g}_{\mathrm{Fe}}}\boldsymbol{s}^{\bar{s}} - \frac{z_{\mathrm{Fe}}}{\mu_{\mathrm{B}}\mathfrak{g}_{\mathrm{Fe}}}\mathbf{D}_{\mathrm{Fe}} \times \boldsymbol{s}^{\bar{s}} + \frac{2}{\mu_{\mathrm{B}}\mathfrak{g}_{\mathrm{Fe}}} \sum_{s'=\mathrm{A,\,B}} \left[J\boldsymbol{S}^{s} - \left(\mathbf{D}^{s,s'} \times \mathbf{S}^{s}\right) - \mathbf{A}_{\mathrm{Fe}} \cdot \mathfrak{s}^{s}\right]$$

$$\mathcal{H}_{\mathrm{Er}}^{s} = \mathfrak{g}_{\mathrm{Er}}\mu_{\mathrm{B}}\mathfrak{Z}^{s} \cdot \overline{\mathbf{B}}_{\mathrm{Er}}^{s} = \mathfrak{g}_{\mathrm{Er}}\mu_{\mathrm{B}}\mathfrak{Z}_{\parallel}^{s}\left|\overline{\mathbf{B}}_{\mathrm{Er}}^{s}\right|$$

$$\mathcal{H}_{\mathrm{Fe}}^{s} = \mathfrak{g}_{\mathrm{Fe}}\mu_{\mathrm{B}}\mathbf{S}^{s} \cdot \overline{\mathbf{B}}_{\mathrm{Fe}}^{s} = \mathfrak{g}_{\mathrm{Fe}}\mu_{\mathrm{B}}\,\mathrm{S}_{\parallel}^{s}\left|\overline{\mathbf{B}}_{\mathrm{Fe}}^{s}\right|$$

$$\langle \mathfrak{h}_{\parallel}^{s} \rangle = -\frac{1}{2}\tanh\left(\frac{\mathrm{g}_{\mathrm{Er}}\mu_{\mathrm{B}}\left|\overline{\mathbf{B}}_{\mathrm{Er}}^{s}\right|}{2k_{\mathrm{B}}T}\right)$$

$$\langle S_{\parallel}^{s} \rangle = -B_{S}\left(\frac{S\mathrm{g}_{\mathrm{Fe}}\mu_{\mathrm{B}}\left|\overline{\mathbf{B}}_{\mathrm{Fe}}^{s}\right|}{k_{\mathrm{B}}T}\right)$$

$$\hbar\frac{d}{dt}\delta\mathfrak{s}^{s} = -\delta\mathfrak{s}^{s}\times \mathrm{g}_{\mathrm{Er}}\mu_{\mathrm{B}}\overline{\mathbf{B}}_{\mathrm{Er}}^{s}\left(\mathfrak{g}^{\mathrm{A/B}},\mathbf{S}^{\mathrm{A/B}}\right) - \overline{\mathfrak{s}}^{s}\times \mathrm{g}_{\mathrm{Er}}\mu_{\mathrm{B}}\mathbf{B}_{\mathrm{Er}}^{s}\left(\delta\mathfrak{Z}^{\mathrm{A/B}},\delta\mathbf{S}^{\mathrm{A/B}}\right)$$

$$\hbar\frac{d}{dt}\delta\mathbf{S}^{s} = -\delta\mathbf{S}^{s}\times \mathrm{g}_{\mathrm{Fe}}\mu_{\mathrm{B}}\overline{\mathbf{B}}_{\mathrm{Er}}^{s}\left(\mathfrak{s}^{\mathrm{A/B}},\mathbf{S}^{\mathrm{A/B}}\right) - \overline{\mathbf{S}}^{s}\times \mathrm{g}_{\mathrm{Fe}}\mu_{\mathrm{B}}\mathbf{B}_{\mathrm{Er}}^{s}\left(\delta\mathbf{3}^{\mathrm{A/B}},\delta\mathbf{S}^{\mathrm{A/B}}\right)$$

$$C = \sum_{i}\frac{w_i}{N_i}\sum_{j\in i\text{th mode}}\frac{\left(f_{ij}-\tilde{f}_{ij}\right)^2}{\tilde{f}_{ij}^2}$$

$$\overline{\mathbf{S}}_0^{\mathrm{A}} = \begin{pmatrix} S\sin\,\beta_0 \\ 0 \\ -S\cos\,\beta_0 \end{pmatrix}, \overline{\mathbf{S}}_0^{\mathrm{B}} = \begin{pmatrix} S\sin\,\beta_0 \\ 0 \\ S\cos\,\beta_0 \end{pmatrix}$$

$$\beta_0 = -\frac{1}{2}\arctan\left[\frac{z_{\mathrm{Fe}}D_{\mathrm{Fe}}^{y}}{z_{\mathrm{Fe}}J_{\mathrm{Fe}} - A_{\mathrm{Fe}}^{x} + A_{\mathrm{Fe}}^{z}}\right]$$

$$\widehat{\mathcal{H}}_{\mathrm{Fe}} \approx \sum_{k=0,\pi}\hbar\omega_k a_k^{\dagger}a_k + \text{ const}$$

$$\omega_k = \mathrm{g}_{\mathrm{Fe}}\mu_{\mathrm{B}}/\hbar\sqrt{(b\cos\,k-a)(d\cos\,k+c)}$$

$$a = [S/(\mathrm{g}_{\mathrm{Fe}}^{x}\mu_{\mathrm{B}})]\left[-A_{\mathrm{Fe}}^{z} - A_{\mathrm{Fe}}^{x} - (z_{\mathrm{Fe}}J_{\mathrm{Fe}} + A_{\mathrm{Fe}}^{z} - A_{\mathrm{Fe}}^{x})\cos\,(2\beta_0) + z_{\mathrm{Fe}}D_{\mathrm{Fe}}^{y}\sin\,(2\beta_0)\right]$$

$$b = [S/(\mathrm{g}_{\mathrm{Fe}}^{x}\mu_{\mathrm{B}})]z_{\mathrm{Fe}}J_{\mathrm{Fe}}$$

$$c = [S/(\mathrm{g}_{\mathrm{Fe}}^{x}\mu_{\mathrm{B}})]\left[(z_{\mathrm{Fe}}J_{\mathrm{Fe}} + 2A_{\mathrm{Fe}}^{z} - 2A_{\mathrm{Fe}}^{x})\cos\,(2\beta_0) + z_{\mathrm{Fe}}D_{\mathrm{Fe}}^{y}\sin\,(2\beta_0)\right]$$

$$d = [S/(\mathrm{g}_{\mathrm{Fe}}^{x}\mu_{\mathrm{B}})]\left[-z_{\mathrm{Fe}}J_{\mathrm{Fe}}\cos\,(2\beta_0) - z_{\mathrm{Fe}}D_{\mathrm{Fe}}^{y}\sin\,(2\beta_0)\right]$$

$$\delta\mathbf{S}_i^{\mathrm{A}} \approx \sqrt{\frac{S}{2N_0}}\begin{bmatrix} -(T_0-T_\pi)\cos\,\beta_0 \\ (Y_0-Y_\pi) \\ -(T_0-T_\pi)\sin\,\beta_0 \end{bmatrix}$$

$$\delta\mathbf{S}_i^{\mathrm{B}} \approx \sqrt{\frac{S}{2N_0}}\begin{bmatrix} (T_0+T_\pi)\cos\,\beta_0 \\ (Y_0+Y_\pi) \\ -(T_0+T_\pi)\sin\,\beta_0 \end{bmatrix}$$

$$T_k = \left(\frac{b\cos\,k-a}{d\cos\,k-a}\right)^{1/4}\frac{a_{-k}^{\dagger}+a_k}{\sqrt{2}}$$

$$Y_k = \left(\frac{d\cos\,k+c}{b\cos\,k-a}\right)^{1/4i\left(a_{-k}^{\dagger}-a_k\right)}\frac{\sqrt{2}}{}$$

$$\mathfrak{I}_i^{s} = \frac{1}{N_0}\sum_{j=1}^{N_0}\mathfrak{I}_j^{s}$$

$$\equiv \frac{1}{N_0}\Sigma_j^{s}$$

$$\widehat{\mathcal{H}}_{\mathrm{Er}} \approx \mu_0\mu_{\mathrm{B}}\mathfrak{g}_{\mathrm{Er}}^{x}H_x^{\mathrm{DC}}\Sigma_x^{+} + z_{\mathrm{Er}}J_{\mathrm{Er}}\sum_{i=1}^{N_0}\mathfrak{s}_i^{\mathrm{A}}\cdot\sum_{i'=1}^{N_0}\frac{\mathfrak{s}_{i'}^{\mathrm{B}}}{N_0}$$

$$-\sum_{i}\sum_{\xi=x,z}A_{\mathrm{Er}}^{\xi}\sum_{s=\mathrm{A,\,B}}\left(\frac{1}{N_0}\sum_{j}\mathfrak{Z}_{j,\xi}^{s}\right)^2$$

$$= \mu_0\mu_{\mathrm{B}}\mathfrak{g}_{\mathrm{Er}}^{x}H_x^{\mathrm{DC}}\Sigma_x^{+} + \frac{z_{\mathrm{Er}}J_{\mathrm{Er}}}{N_0}\Sigma^{\mathrm{A}}\cdot\Sigma^{\mathrm{B}} - \sum_{s=\mathrm{A,\,B}\xi=x,z}\sum_{\mathrm{Er}}\frac{A_{\mathrm{Er}}^{\xi}}{N_0}\left(\Sigma_{\xi}^{s}\right)^2$$

$$\widehat{\mathcal{H}}_{\mathrm{Fe-Er}} = 4S(J\sin\,\beta_0 + D_y\cos\,\beta_0)\Sigma_x^{+} + (-4SD_x\cos\,\beta_0)\Sigma_y^{-}$$

$$+\sqrt{\frac{S}{N_0}}\left[(J\cos\,\beta_0 - D_y\sin\,\beta_0)T_\pi\Sigma_x^{+} + JY_0\Sigma_y^{+}\right.$$

$$\left.+(D_x\sin\,\beta_0)T_\pi\Sigma_y^{-} + D_xY_\pi\Sigma_z^{-} - (J\sin\,\beta_0 + D_y\cos\,\beta_0)T_0\Sigma_z^{+}\right]$$

$$\widehat{\mathcal{H}}_{\mathrm{Dicke}} \approx \sum_{m=\mathrm{qFM,qAFM}}\hbar\omega_m a_m^{\dagger}a_m + E_x\Sigma_x^{+} + E_y\Sigma_y^{-} + \mu_0\mu_{\mathrm{B}}\mathfrak{g}_{\mathrm{Er}}^{x}H_x^{\mathrm{DC}}\Sigma_x^{+}$$

$$+\frac{z_{\mathrm{Er}}J_{\mathrm{Er}}}{N_0}\Sigma^{\mathrm{A}}\cdot\Sigma^{\mathrm{B}} - \sum_{\xi=x,zs=A,\,\mathrm{B}}\sum_{0}\frac{A_{\mathrm{Er}}^{\xi}}{N_0}\left(\Sigma_{\xi}^{s}\right)^2 + \frac{\hbar g_x}{\sqrt{N_0}}\left(a_{\mathrm{qAFM}}^{\dagger} + a_{\mathrm{qAFM}}\right)\Sigma_x^{+}$$

$$+\frac{i\hbar g_y}{\sqrt{N_0}}\left(a_{\mathrm{qFM}}^{\dagger} - a_{\mathrm{qFM}}\right)\Sigma_y^{+} + \frac{\hbar g_{y'}}{\sqrt{N_0}}\left(a_{\mathrm{qAFM}}^{\dagger} + a_{\mathrm{qAFM}}\right)\Sigma_y^{-}$$

$$+\frac{i\hbar g_z}{\sqrt{N_0}}\left(a_{\mathrm{qAFM}}^{\dagger} - a_{\mathrm{qAFM}}\right)\Sigma_z^{-} + \frac{\hbar g_{z'}}{\sqrt{N_0}}\left(a_{\mathrm{qFM}}^{\dagger} + a_{\mathrm{qFM}}\right)\Sigma_z^{+}$$

$$\hbar g_x = \sqrt{xS}(J\cos\,\beta_0 - D_y\sin\,\beta_0)\left(\frac{b+a}{d-c}\right)^{1/4} = h\sqrt{x}$$

$$\hbar g_y = \sqrt{xS}J\left(\frac{d+c}{b-a}\right)^{1/4} = h\sqrt{x}$$

$$\hbar g_{y'} = \sqrt{xS}D_x\sin\,\beta_0\left(\frac{b+a}{d-c}\right)^{1/4} = h\sqrt{x}$$

$$\hbar g_z = \sqrt{xS}D_x\left(\frac{d-c}{b+a}\right)^{1/4} = h\sqrt{x}$$

$$\hbar g_{z'} = \sqrt{xS}(-J\sin\,\beta_0 - D_y\cos\,\beta_0)\left(\frac{b-a}{d+c}\right)^{1/4} = h\sqrt{x}$$

$$x = \tanh\left[\frac{\left|E_x + \mu_0\mu_{\mathrm{B}}\mathfrak{g}_{\mathrm{Er}}^{x}\mathrm{He}_x^{\mathrm{DC}}\right|}{2k_{\mathrm{B}}T}\right]$$

$$\varepsilon \propto \frac{2\pi}{\hbar}\sum_{\alpha}\left|\sum_{(v_i,c_i,i=1,2,3)}\left\langle e_3h_3\right|\mathbf{e}\cdot\hat{p}\left|e_1h_1e_2h_2\right\rangle_{\alpha}\right|^2\Gamma$$

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