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LAS PARTÍCULAS SUPERMASIVAS: NATURALEZA FENOMENOLÓGICA DE LA PARTÍCULA OSCURA

SUPERMASSIVE PARTICLES: PHENOMENOLOGICAL NATURE OF THE DARK PARTICLE

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Las partículas supermasivas: naturaleza fenomenológica de la partícula oscura

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RESUMEN

Las partículas supermasivas, teorizadas por este autor a lo largo de los trabajos anteriores, comporta la existencia de partículas conexas al Modelo Estándar de Física de Partículas, las mismas que pueden ser de naturaleza fermiónica o bosónica, según sea el caso, que interactúan en un campo de gauge y en supersimetría, cuya masa, es extremadamente densa, capaz de deformar el espacio – tiempo cuántico en el que interactúa (a lo que llamamos gravedad endógena), afectando la configuración morfológica de las partículas circundantes (masa y energía) así como sus coordenadas en espacio y tiempo, y desdoblando multidimensiones o supermembranas, sin que sea necesario que la partícula supermasiva o masiva, deba interactuar con un campo gravitónico y en consecuencia, con un gravitón, como se ha propuesto en relación al modelo de gravedad exógena. Esta partícula, a lo largo de anteriores trabajos, ha sido denominada también como "partícula cosmológica", "partícula supermasiva" o "superpartícula", más, llámesela también "partícula oscura", para propósitos de este artículo.

Palabras Clave: partícula supermasiva, partícula oscura, teoría cuántica de campos relativistas, agujeros negros cuánticos, supermembranas

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Supermassive particles: phenomenological nature of the dark particle

ABSTRACT

The supermassive particles, theorized by this author throughout the previous works, involve the existence of particles related to the Standard Model of Particle Physics, which can be of fermionic or bosonic nature, as the case may be, interacting in a gauge field and in supersymmetry, whose mass is extremely dense, capable of deforming the quantum space-time in which it interacts (what we call endogenous quantum gravity), affecting the morphological configuration of the surrounding particles (mass and energy) as well as their coordinates in space and time, and unfolding multidimensions or supermembranes, without it being necessary for the supermassive or massive particle to interact with a gravitonic field and consequently with a graviton, as has been proposed in relation to the exogenous gravity model. This particle, throughout previous works, has also been called "cosmological particle", "supermassive particle" or "superparticle", but, for the purposes of this article, let us also call it "dark particle".

Keywords: supermassive particle, dark particle, relativistic quantum field theory, quantum black holes, supermembranes

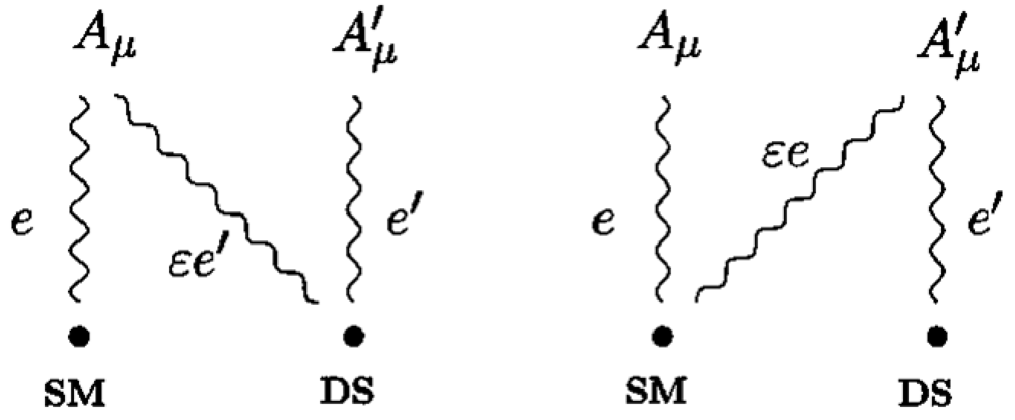
INTRODUCCIÓN

El propósito de mis trabajos anteriores, ha sido, establecer un marco teórico paralelo, cuyo objetivo es unificar la relatividad general y especial con la mecánica cuántica, de ahí nace lo que he denominado "Teoría Cuántica de Campos Relativistas" o "Teoría Cuántica de Campos Curvos", cuyos postulados esenciales, son: 1) La existencia de supergravedad cuántica exógena, esto es, que una partícula deforma el espacio – tiempo en el que interactúa, por la permeabilidad de gravedad que le es inherente, al interactuar con un campo gravitónico y por ende, con un gravitón; 2) La existencia de gravedad cuántica endógena, esto es, cuando una partícula, a propósito de su masa extremadamente densa, curva el espacio – tiempo cuántico en el que interactúa, engendrando supermembranas y multidimensiones, así como deformaciones a los sistemas de referencia de las partículas interactuantes, entendiendo que todos los campos cuánticos, sean de naturaleza fermiónica o bosónica, según sea el caso, están interrelacionados, es decir, conectados por partículas comunes, verbigracia, el bosón de Higgs; y, 3) La existencia de agujeros negros cuánticos, a propósito de la creación de supermembranas y multidimensiones causados por la deformación masiva del espacio – tiempo cuántico, esto es, la singularidad cuántica de la gravitación por densidad.

Los postulados anteriormente mencionados, han sido desarrollados cuidadosamente en trabajos anteriores, sin embargo, las partículas supermasivas, como concepto esencial de la Teoría Cuántica de Campos Relativistas, requieren de un modelo matemático adicional que describa su comportamiento, fenomenología y características particulares, en relación a lo mencionado anteriormente, es decir, en relación a la gravedad cuántica que le es implícita, sus capacidades de deformación debido a sus interacciones de gauge y supersimetrías y el modelamiento de superespacios por la curvatura del tejido del espacio – tiempo cuántico.

RESULTADOS Y DUSCUSIÓN

El campo de una partícula supermasiva, se presenta así:



$\sin\theta = \varepsilon\,,\ \cos\theta = \sqrt{1-\varepsilon^2}$ $\quad\quad$ $\sin\theta = 0\,,\ \cos\theta = 1$

Cuyos grupos de gauge en lagrangiano, son:

$$\mathcal{L}_0 = -\frac{1}{4}F_{a\mu\nu}F_a^{\mu\nu} - \frac{1}{4}F_{b\mu\nu}F_b^{\mu\nu} - \frac{\varepsilon}{2}F_{a\mu\nu}F_b^{\mu\nu}.$$

$$\mathcal{L} = eJ_\mu A_b^\mu + e'J'_\mu A_a^\mu,$$

$$\mathcal{L}' = \left[\frac{e'\cos\,\theta}{\sqrt{1-\varepsilon^2}}J'_\mu + e\left(\sin\,\theta - \frac{\varepsilon\cos\,\theta}{\sqrt{1-\varepsilon^2}}\right)J_\mu\right]A'^\mu + \left[-\frac{e'\sin\,\theta}{\sqrt{1-\varepsilon^2}}J'_\mu + e\left(\cos\,\theta + \frac{\varepsilon\sin\,\theta}{\sqrt{1-\varepsilon^2}}\right)J_\mu\right]A^\mu$$

$$\mathcal{L}' = \left[\frac{e'}{\sqrt{1-\varepsilon^2}}J'_\mu - \frac{e\varepsilon}{\sqrt{1-\varepsilon^2}}J_\mu\right]A'^\mu + eJ_\mu A^\mu$$

$$\mathcal{L}' = e'J'_\mu A'^\mu + \left[-\frac{e'\varepsilon}{\sqrt{1-\varepsilon^2}}J'_\mu + \frac{e}{\sqrt{1-\varepsilon^2}}J_\mu\right]A^\mu$$

Más en términos kinéticos, tenemos:

$$\begin{pmatrix} A_a^\mu \\ A_b^\mu \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\varepsilon^2}} & 0 \\ -\frac{\varepsilon^2}{\sqrt{1-\varepsilon^2}} & 1 \end{pmatrix} \begin{pmatrix} \cos\,\theta & -\sin\,\theta \\ \sin\,\theta & \cos\,\theta \end{pmatrix} \begin{pmatrix} A'^\mu \\ A^\mu \end{pmatrix}$$

$$\sin\,\theta = \frac{\delta\sqrt{1-\varepsilon^2}}{\sqrt{1-2\delta\varepsilon+\delta^2}}\ \cos\,\theta = \frac{1-\delta\varepsilon}{\sqrt{1-2\delta\varepsilon+\delta^2}}$$

Sin embargo, si se rompe la simetría de gauge, tenemos en lagrangiano:

$$\mathcal{L}_{Stu} = -\frac{1}{2}M_a^2 A_{a\mu}A_a^\mu - \frac{1}{2}M_b^2 A_{b\mu}A_b^\mu - M_a M_b A_{a\mu}A_b^\mu$$

$$\mathcal{L}'' = \frac{1}{\sqrt{1-2\delta\varepsilon+\delta^2}}\left[\frac{e'(1-\delta\varepsilon)}{\sqrt{1-\varepsilon^2}}J'_\mu + \frac{e(\delta-\varepsilon)}{\sqrt{1-\varepsilon^2}}J_\mu\right]A'^\mu + \frac{1}{\sqrt{1-2\delta\varepsilon+\delta^2}}[eJ_\mu - \delta e' J'_\mu]A^\mu$$

Cuyo acoplamiento viene dado por:

$$\mathcal{L} \supset -\frac{e\varepsilon}{\sqrt{1-\varepsilon^2}}J_\mu A'^\mu \simeq -e\varepsilon J_\mu A'^\mu,$$

Por otro lado, la hiper – carga de la partícula supermasiva viene dada por:

$$\tilde{\mathcal{L}} = -\frac{\varepsilon}{2\cos\,\theta_W}\tilde{F}'_{\mu\nu}B^{\mu\nu}.$$

Cuya simetría de gauge viene dada por:

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \\ \tilde{A}'_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W & -s_W\varepsilon \\ -s_W & c_W & -c_W\varepsilon \\ t_W\varepsilon & 0 & 1 \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \\ A'_\mu \end{pmatrix},$$

Y en lagrangiano:

$$\mathcal{L} \supset -e\varepsilon J^\mu A'_\mu + e'\varepsilon t_W J'^\mu Z_\mu + e' J^{\mu\prime} A'_\mu,$$

Más sus operadores y osciladores vienen dados por:

$$\mathcal{L} = \frac{e_D}{2\Lambda_5}\bar{\psi}^i\sigma_{\mu\nu}(\mathbb{D}_M^{ij} + i\gamma_5\mathbb{D}_E^{ij})\psi^j F'^{\mu\nu}$$

$$\mathcal{L} = \frac{e_D}{2\Lambda^2}\bar{\psi}_L^i\sigma_{\mu\nu}(\mathbb{D}_M^{ij} + i\gamma_5\mathbb{D}_E^{ij})H\psi_R^j F'^{\mu\nu} + \text{ H.c.}$$

$$\mathcal{L}' = \frac{e_D}{2\Lambda^2}\bar{\psi}^i\gamma_\mu(\mathbb{R}_r^{ij} + i\gamma_5\mathbb{R}_a^{ij})D_\nu\psi^j F'^{\mu\nu}$$

En los que la densidad de la partícula supermasiva, por isotropización, se mide así:

$$m_\chi\left(\frac{0.01}{\alpha_D}\right)^{2/3} \gtrsim 300\text{GeV}$$

$$\frac{G_N m_\chi^4 N}{8\alpha_D^2} \gtrsim 50\log\frac{G_N m_\chi^2 N}{2\alpha_D}$$

$$\chi\chi \to A'A'$$

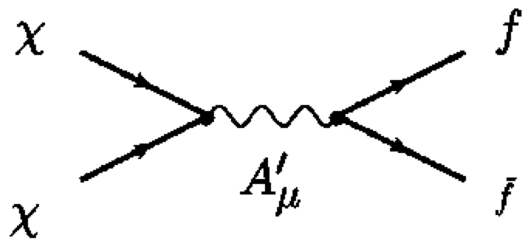
$$\langle\sigma_{\chi\chi\to A'A'}v\rangle = \frac{2\pi\alpha_D^2}{m_\chi^2}$$

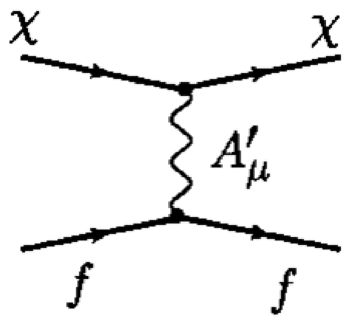
$$\Omega_\chi h^2 \approx \frac{2.5\times 10^{-10}\text{GeV}^{-2}}{\langle\sigma_{\chi\chi\to A'A'}v\rangle}$$

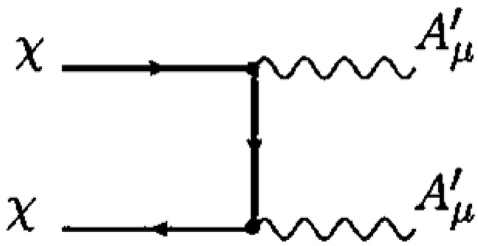
$$\langle \sigma_{\chi\chi\to f\bar{f}} v \rangle \simeq \frac{2\pi\alpha_L^2}{m_S^2}$$

$$2\pi\alpha_L^2 \left(\frac{10\text{TeV}}{m_S}\right)^2 \simeq 0.1$$

En este punto y usando los diagramas de Feynman, pasa a explicarse las interacciones de las partículas supermasivas en distintos grupos de gauge, cuya aniquilación provoca un agujero negro cuántico supermasivo:







$$\chi\bar{\chi} \to A' \to \bar{f}f$$

$$\sigma_{\chi\chi\to ff} = \frac{4\pi}{3}\varepsilon^2\alpha\alpha_D m_\chi^2 \left(1+\frac{2m_e^2}{s}\right)\left(1+\frac{2m_\chi^2}{s}\right) \times \frac{s}{\left(s-m_{A'}^2\right)^2 + m_{A'}^2\Gamma_{A'}^2}\frac{\sqrt{1-\frac{4m_e^2}{m_{A'}^2}}}{\sqrt{1-\frac{4m_\chi^2}{m_{A'}^2}}}$$

$$\langle\sigma_{\chi\chi\to ff}v\rangle \simeq \varepsilon^2\alpha\alpha_D\frac{16\pi m_\chi^2}{\left(4m_\chi^2 - m_{A'}^2\right)^2}$$

$$\Omega_\chi h^2 \approx \frac{2.5\times 10^{-10}\mathrm{GeV}^{-2}}{\langle\sigma_{\chi\chi\to ff}v\rangle}$$

$$y \equiv \varepsilon^2\alpha_D\left(\frac{m_\chi}{m_{A'}}\right)^4$$

$$\langle\sigma_{\chi\chi\to ff}v\rangle \simeq \frac{16\pi\alpha y}{m_\chi^2}$$

Por otro lado, la producción iónica viene dada por:

$$\sigma_e = \frac{16\pi\mu_{\chi e}^2\alpha\alpha_D\varepsilon^2}{\left(m_{A'}^2 + \alpha^2 m_e^2\right)^2}|F(q^2)|^2$$

$$F(q^2) = \frac{m_{A'}^2 + \alpha^2 m_e^2}{m_{A'}^2 + q^2}$$

$$\frac{dR}{d\ln\ E} = N_T\frac{\rho_\chi}{m_\chi}\frac{d\langle\sigma_e v\rangle}{d\ln\ E}$$

Desde una perspectiva relativista, la partícula supermasiva en cuanto a su densidad, queda expresada así:

$$\sigma_{p.e.} = 4\alpha^4\sqrt{2}Z^5\frac{8\pi r_e^2}{3}\left(\frac{m_e}{\omega}\right)^{7/2}$$

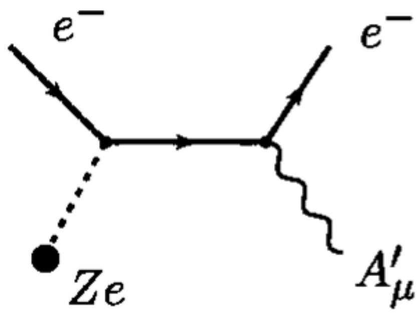
$$\sigma_{A'} = \varepsilon^2\sigma_{p.e.}$$

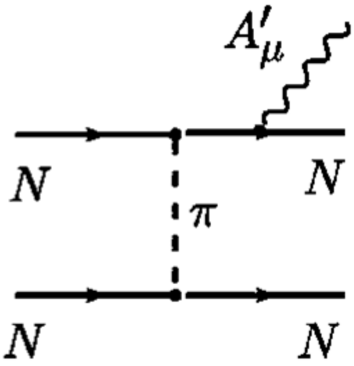
$$\Gamma_{A'} = \frac{\rho_{A'}}{m_{A'}}\sigma_{A'}v_{A'}$$

Ahora bien, la fenomenología relativista de la masa y energía de una partícula supermasiva (nucleosíntesis), se expresa así:

$$d_M^{ij} \equiv \left|\mathbb{D}_M^{ij}\right|$$

$$\mathcal{M}_{A'_\mu} \approx \bar{\psi}(\mathbf{k}\times\boldsymbol{\epsilon})\cdot\boldsymbol{\sigma}\psi$$





$$\mathcal{M}_a \approx \bar{\psi}\mathbf{k}\cdot\boldsymbol{\sigma}\psi$$

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \sum_j Z_j^2 n_j \frac{4\alpha^2\alpha'_{ae}}{\pi}\frac{|\mathbf{p}_1||\mathbf{p}_2|\omega^2}{(\mathbf{q}^2+\kappa_F^2)^2}\left[2\omega^2\frac{p_1\cdot p_2 - m_e^2 + (p_2-p_1)\cdot k}{(p_1\cdot k)(p_2\cdot k)} + 2 - \frac{p_1\cdot k}{p_2\cdot k} - \frac{p_2\cdot k}{p_1\cdot k}\right]$$

$$Q/\rho = \frac{\pi^2\alpha^2\alpha'_{ae}}{15}\frac{T^4}{m_e^2}\sum_j Z_j^2 n_j F(\kappa_F) \simeq \alpha'_{ae} 1.08\times 10^{27}\left(\frac{T}{10^8\ \text{K}}\right)^4 \frac{Z^2}{A}F(\kappa_F)$$

$$F(\kappa_F) \simeq \frac{2+\kappa_F^2}{2}\ln\frac{2+\kappa_F^2}{\kappa_F^2} - 1$$

$$\alpha'_{ae} \leq 3.0\times 10^{-27}$$

$$\alpha'_{ae} = 2\frac{1}{4\pi}\left(2e_D d_M^e \frac{v_h m_e}{\Lambda^2}\right)^2$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^e} \gtrsim 4.5\times 10^6\text{TeV}^2$$

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \frac{16(4\pi)^3\alpha_\pi^2\alpha'_{aN}}{3m_N^2}\left[\left(\frac{\mathbf{k}^2}{\mathbf{k}^2+m_\pi^2}\right)^2 + \left(\frac{\mathbf{l}^2}{\mathbf{l}^2+m_\pi^2}\right)^2 + \frac{\mathbf{k}^2\mathbf{l}^2 - 3(\mathbf{k}\cdot\mathbf{l})^2}{(\mathbf{k}^2+m_\pi^2)(\mathbf{l}^2+m_\pi^2)}\right]$$

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \frac{32(4\pi)^3\alpha_\pi^2\alpha'_{aN}}{m_N^2}$$

$$Q/\rho \simeq \alpha'_{aN} 1.74\times 10^{33}\frac{\rho}{10^{15}}\left(\frac{T}{\text{MeV}}\right)^6$$

$$\alpha'_{aN} \leq 1.3\times 10^{-18}$$

$$\alpha'_{aN} = 2\frac{1}{4\pi}\left(2e_D d_M^q \frac{v_h m_N}{\Lambda^2}\right)^2$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^q} \gtrsim 4.3\times 10^5\text{TeV}^2$$

$$\alpha'_{aN} \geq 0.7 \times 10^{-14}$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^q} \lesssim 5.9 \times 10^3 \mathrm{TeV}^2$$

$$N_{\mathrm{eff}} = 2.878 \pm 0.278$$

$$H(T_d) = \frac{T_d^2}{M_{Pl}} \left(\frac{\pi^2}{90} g_*(T_d) \right)^{1/2}$$

$$\Gamma_{A'} = n_{A'} \langle \sigma v \rangle$$

$$n_{A'} = \frac{2\zeta(3)}{\pi^2} T^3$$

$$\langle \sigma v \rangle \simeq \frac{\alpha_D d_M^2 v_h^2}{\Lambda^4}$$

$$\frac{2\zeta(3)}{\pi^2} T_d^3 \langle \sigma v \rangle < \frac{T_d^2}{M_{Pl}} \left(\frac{2\pi^2}{45} g_*(T_d) \right)^{1/2}$$

$$\left(\frac{T_{BBN}}{T_d} \right)^4 = \left(\frac{g_*(T_{BBN})}{g_*(T_d)} \right)^{4/3} < \frac{7}{4} \Delta N_{\mathrm{eff}}$$

$$g_*(T_d) > (43/7)^{4/3} \Delta N_{\mathrm{eff}}^{-3/4}$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^\ell} \geq 6.6 \times 10^3 \mathrm{TeV}^2$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^q} \geq 4.3 \times 10^3 \mathrm{TeV}^2$$

$$V(r) = -\frac{\alpha_D v^2 d_M^a d_M^b}{4\Lambda^4 r^3} [\boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b - 3(\boldsymbol{\sigma}_a \cdot \hat{\boldsymbol{r}})(\boldsymbol{\sigma}_b \cdot \hat{\boldsymbol{r}})]$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^e} \gtrsim 872 \mathrm{GeV}^2$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^e} \gtrsim 1.61 \mathrm{TeV}^2$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} \sqrt{d_M^e d_M^q}} \gtrsim 1.94 \mathrm{TeV}^2$$

$$a_\ell^{A'} = -\frac{3}{2} \frac{\alpha_D}{\pi} \left(\frac{m_\ell v_h d_M^\ell}{\Lambda^2} \right)^2 \left[\frac{5}{4} + \log \frac{\mu^2}{m^2} \right]$$

$$\delta_{\Delta a_e} < 8.1 \times 10^{-13}$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^e} \gtrsim 0.075 \text{TeV}^2$$

$$\Delta a_\mu < 2.74 \times 10^{-9}$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^\mu} \gtrsim 0.5 \text{TeV}^2$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^\mu} \simeq 0.27 \text{TeV}^2$$

$$\text{BR}(\mu \to eX^0) < 5.8 \times 10^{-5}$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^{\mu e}} \gtrsim 5.1 \times 10^5 \text{TeV}^2$$

$$\text{BR}(K^+ \to \pi^+ \nu\bar{\nu}) < 1.85 \times 10^{-10}$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^{sd}} \gtrsim 9.5 \times 10^6 \text{TeV}^2$$

$$6.5 \times 10^{-14} \leq \alpha'_{aN} \leq 8.0 \times 10^{-8}$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^q} \gtrsim 1.9 \times 10^3 \text{TeV}^2$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^e} \gtrsim 1.2 \text{TeV}^2$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^q} \gtrsim 4.3 \text{TeV}^2$$

En la que, la interacción y acoplamiento de Yukawa, en relación a la partícula supermasiva, en lagrangiano queda expresada así:

$$\mathcal{L} \supset -g_L\left(\phi_L^\dagger \bar{\chi}_R l_L + S_L^{U\dagger} \bar{Q}_R^U q_L + S_L^{D\dagger} \bar{Q}_R^D q_L\right) - g_R\left(\phi_R^\dagger \bar{\chi}_L e_R + S_R^{U\dagger} \bar{Q}_L^D u_R + S_R^{U\dagger} \bar{Q}_L^D d_R\right) + \text{ H.c}$$

$$\mathcal{L} \supset -\lambda_s S_0\left(H^\dagger \phi_R^\dagger \phi_L + \tilde{H}^\dagger S_R^{U\dagger} S_L^U + H^\dagger S_R^{D\dagger} S_L^D\right) + \text{ h.c.}$$

Cuyos escalares, son:

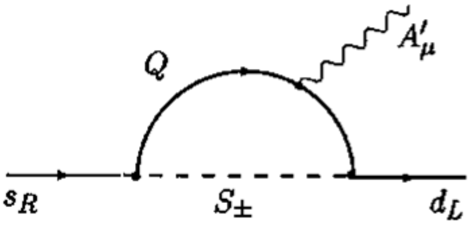
$$m_\pm = m_{\phi,S}\sqrt{1 \pm \eta_s}$$

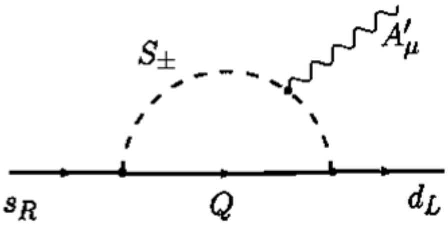
$$\eta_{\phi,S} \equiv \frac{\lambda_s \mu_{\phi,S} v_h}{m_S^2}.$$

$$\mathcal{L}^{(lep)} \supset -g_L \phi_{L\nu}^{\dagger}(\bar{\chi}_R \nu_L) - \frac{g_L}{\sqrt{2}}(\phi_+^{\dagger} + \phi_-^{\dagger})(\bar{\chi}_R e_L) - \frac{g_R}{\sqrt{2}}(\phi_+^{\dagger} - \phi_-^{\dagger})(\bar{\chi}_L e_R) + \text{ h.c.}$$

$$\eta_{\phi,S} < 1 - \left(\frac{M}{m_{\phi,S}}\right)^2.$$

Y cuyos diagramas de vórtice, son:





$$S_L^{U_i^{\dagger}} \bar{Q}_R^{U_i} q_L^i \to S_L^{U_i \dagger} \bar{Q}_R^{U_i} (\rho_L^U)_{ij} q_L^j S_R^{U_i^{\dagger}} \bar{Q}_L^{U_i} q_R^i \to S_R^{U_i^{\dagger}\dagger} \bar{Q}_L^{U_i} (\rho_L^U)_{ij} q_R^j$$

$$\frac{v_h}{\Lambda^2} \simeq \frac{m_{Q^i}}{m_S^2}$$

$$\mathbb{D}_M^{ij} = \rho_{jj}\rho_{ij}^{*} \mathrm{Re}\left[\frac{g_L g_R}{(4\pi)^2}\right] F_M(x, \eta_s)$$

$$F_M(x,y) = \frac{1}{2}[f(x,y) - f(x,-y)]$$

$$f(x,y) = \frac{1 - x + y + (1+y)\log\left(\frac{x}{1+y}\right)}{(1-x+y)^2}$$

$$\rho_{nm}\rho_{mm}^{*} - \rho_{nm}^{*}\rho_{mm} = 2i\sin\,\delta_{\mathrm{CP}}$$

$$m_S^i \gtrsim 940\mathrm{GeV}$$

$$m_\phi \gtrsim 290\mathrm{GeV}$$

$$\frac{m_\phi^2/m_{\chi^e}}{\sqrt{\alpha_D \alpha_L \alpha_R}|\rho_{ee}|^2 \mid F_M(x_e, \eta_\phi)} \gtrsim 2.1 \times 10^6\mathrm{TeV},$$

$$\frac{m_S^2/m_{Q^u}}{\sqrt{\alpha_D \alpha_L \alpha_R}|\rho_{uu}|^2 F_M(x_u, \eta_S)} \gtrsim 2.0 \times 10^5\mathrm{TeV},$$

$$\frac{m_\phi^2/m_{\chi^e}}{\sqrt{\alpha_L \alpha_R}|\rho_{ee}|^2 G_M(x_e, \eta_\phi)} \gtrsim 9.8 \times 10^4\mathrm{TeV}$$

$$G_M(x,y) = \frac{1}{2}[g(x,y) - g(x,-y)]$$

$$g(x,y) = \frac{(1+y)^2 - x^2 + 2x(1+y)\log\left(\frac{x}{1+y}\right)}{2(x-1-y)^3}$$

$$\frac{m_\phi^2/m_{\chi^\mu}}{\sqrt{\alpha_L\alpha_R}\left|\rho_{\mu\mu}\right|^2 G_M(x_\mu, \eta_\phi)} \gtrsim 6.3 \times 10^3 \text{TeV}$$

$$\frac{m_\phi^2/m_{\chi^\mu}}{\sqrt{\alpha_L\alpha_R}\left|\rho_{\mu\mu}\rho_{\mu e}^\star\right| G_M(x_\mu, \eta_\phi)} \gtrsim 4.9 \times 10^8 \text{TeV}.$$

$$\frac{m_S^2/m_{Q^b}}{\sqrt{\alpha_L\alpha_R}\left|\rho_{bb}\rho_{bs}^\star\right| G_M(x_b, \eta_S)} \gtrsim 1.3 \times 10^4 \text{TeV},$$

$$\frac{m_S^2}{(\alpha_L^2 + \alpha_R^2)\left|\rho_{ss}\rho_{sd}^*\right|^2} \gtrsim 3 \times 10^5 \text{TeV}^2$$

La estructura morfológica de una partícula supermasiva, viene dada por:

1. **Modelo Drell-Yan.**

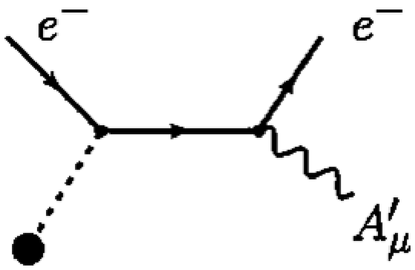
$$\mathcal{L} = -\varepsilon e J^\mu A'_\mu$$

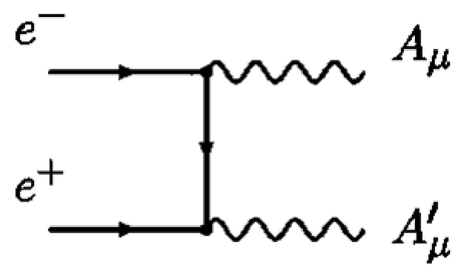
$$\Gamma(A' \to \ell^+\ell^-) = \frac{1}{3}\alpha\varepsilon^2 m_{A'}\sqrt{1 - \frac{4m_\ell^2}{m_{A'}^2}}\left(1 + \frac{2m_\ell^2}{m_{A'}^2}\right),$$

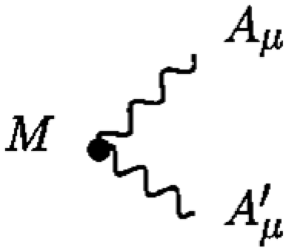
$$\Gamma(A' \to \text{ partícula-supermasiva }) = \frac{1}{3}\alpha\varepsilon^2 m_{A'}\sqrt{1 - \frac{4m_\mu^2}{m_{A'}^2}}\left(1 + \frac{2m_\mu^2}{m_{A'}^2}\right)R$$

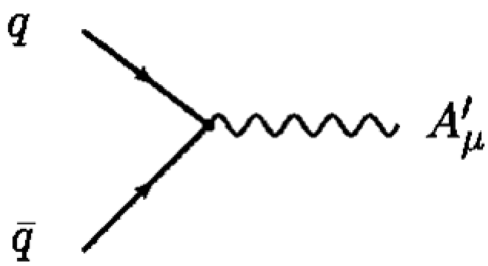
$$\Gamma(A' \to \chi\bar{\chi}) = \frac{1}{3}\alpha_D m_{A'}\sqrt{1 - \frac{4m_\chi^2}{m_{A'}^2}}\left(1 + \frac{2m_\chi^2}{m_{A'}^2}\right)$$

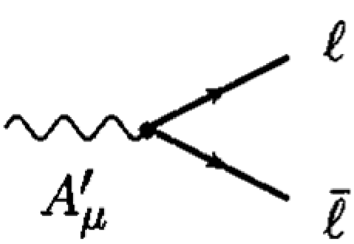
2. **Diagramas de caída de masa.**

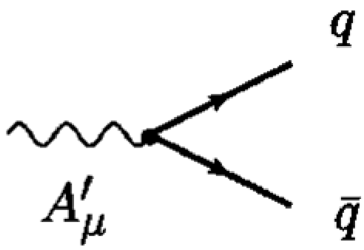


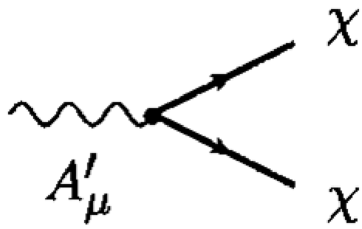












3. Singularidades cuánticas.

$$\frac{\varepsilon}{2} F_{\mu\nu} F^{\mu\nu\prime},$$

$$\frac{a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{f_a} \partial_\mu a \bar{\psi} \gamma^\mu \gamma_5 \psi,$$

$$(\mu S + \lambda S^2) H^\dagger H,$$

$$y_N \bar{L} H N,$$

4. Superdensidad de masa.

$$\Gamma = n\sigma v,$$

$$H(T) = \frac{\pi \sqrt{g_*(T)}}{\sqrt{90}} \frac{T^2}{m_{Pl}}$$

$$g_*(T) = \sum_{\text{bosons}} g_b \left(\frac{T_b}{T}\right)^4 + \frac{7}{8} \sum_{\text{fermions}} g_f \left(\frac{T_f}{T}\right)^4,$$

$$n_{eq}(T) = g_* \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{E/T} \pm 1} = \begin{cases} g_* \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} & \text{no relativista } (T \ll m) \\ \frac{\zeta(3)}{\pi^2} g_* T^3 & \text{bosones relativistas } (T \gg m) \\ \frac{3}{4} \frac{\zeta(3)}{\pi^2} g_* T^3 & \text{fermiones relativistas } (T \gg m), \end{cases}$$

$$\dot{n}(t) + 3H(t)n(t) = -\langle \sigma_{\chi\chi \to ff} v \rangle \left(n^2(t) - n_{eq}^2(t)\right),$$

$$\langle \sigma_{\chi\chi \to ff} v \rangle = \frac{\int_{\infty}^{4m_\chi^2} ds \sqrt{s} \left(s - 4m_\chi^2\right) K_1 \left(\frac{\sqrt{s}}{T}\right) \sigma_{\chi\chi \to ff}}{8 m_\chi^4 T \left[K_2 \left(\frac{m_\chi}{T}\right)\right]^2}$$

$$\langle \sigma_{\chi\chi \to ff} v \rangle = \langle s_0 + s_1 v^2 + O(v^4) \rangle$$

$$x = m_\chi / T = \sqrt{2tH\left(T = m_\chi\right)}$$

$$\frac{dY}{dx} = -\frac{\lambda(x)}{x^2} \left[Y^2(x) - Y_{eq}^2\right]$$

$$\lambda(x) = \frac{m_\chi^3 \langle \sigma_{\chi\chi \to ff} v \rangle}{H\left(T = m_\chi\right)},$$

$$\langle \sigma_{\chi\chi \to ff} v \rangle = \sigma_{\chi\chi \to ff} v + O(v^2)$$

$$\lambda(x) = \frac{\sqrt{180} m_{Pl} m_\chi}{\pi \sqrt{g_* x}} \sigma_{\chi\chi \to ff}$$

$$H(T=m_\chi) = \frac{\pi\sqrt{g_*(T=m_\chi)}}{90}\frac{m_\chi^2}{m_{Pl}}.$$

$$Y(x') = \frac{x_d}{\lambda}$$

$$\rho_\chi = m_\chi n(x') = m_\chi^4 \frac{Y(x')}{28x_d}$$

$$\Omega_\chi h^2 \simeq 0.12\frac{x_d}{23}\frac{\sqrt{g_*}}{10}\frac{1.7\times10^{-9}\text{GeV}^{-2}}{\langle\sigma_{\chi\chi\to ff}v\rangle} \simeq \frac{2.5\times10^{-10}\text{GeV}^{-2}}{\langle\sigma_{\chi\chi\to ff}v\rangle}$$

$$Q = \prod_{i=1}\int \frac{d^3\mathbf{p}_i}{2E_i(2\pi)^3}f_i(E_i)\prod_{f=1}\int \frac{d^3\mathbf{p}_f}{2E_f(2\pi)^3}[1\pm f_f(E_f)]\int \frac{d^3\mathbf{p}_a}{2\omega_a(2\pi)^3}\omega_a \times\frac{1}{S}\sum_{\text{spin and pol.}}|\mathcal{M}|^2(2\pi)^4\delta^4\left(\sum p_i - \sum p_f - p_a\right)$$

$$n_j(E_j) = g_j\int \frac{d^2\mathbf{p}_j}{(2\pi)^3}f(E_j)$$

$$L = \int dVQe^{-\tau}$$

$$\Pi^{\mu\nu}(k) = 16\pi\alpha\int \frac{d^3\mathbf{p}_i}{(2\pi)^3}\frac{1}{2E}[n_e(E)+n_{\bar{e}}(E)]\times\frac{p\cdot k(p^\mu k^\nu + k^\mu p^\nu) - k^2p^\mu p^\nu - (p\cdot k)^2g^{\mu\nu}}{(p\cdot k)^2-(k^2)^2/4}$$

$$\Pi_T(\omega,\mathbf{k}) = \frac{1}{2}(\delta^{ij}-k^ik^j)\Pi^{ij}(\omega,\mathbf{k})$$

$$\Pi_L(\omega,\mathbf{k}) = \Pi^{00}(\omega,\mathbf{k})$$

$$D^{00}(\omega,k) = \frac{1}{k^2-\Pi_L(\omega,k)}$$

$$D^{ij}(\omega,k) = \frac{1}{k^2-\Pi_T(\omega,k)}(\delta^{ij}-k^ik^j)$$

$$\omega_T^2 = k^2+\Pi_T(\omega_T,k) \text{ and } \omega_L^2 = \frac{\omega_L^2}{k^2}+\Pi_T(\omega_L,k)$$

5. Momentum de Fermi.

$$\Pi_T(\omega,\mathbf{k}) = \omega_P^2\frac{3\omega^2}{2v_F^2k^2}\left(1-\frac{\omega^2-v_F^2k^2}{2v_F\omega k}\log\frac{\omega+v_Fk}{\omega-v_Fk}\right)$$

$$\Pi_L(\omega,\mathbf{k}) = \omega_P^2\frac{3\omega}{2v_F^3k}\left(\frac{\omega}{2v_Fk}\log\frac{\omega+v_Fk}{\omega-v_Fk}-1\right)$$

$$Q = -\int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{\mathrm{Im}\Pi_L(\omega,\mathbf{k}) + \mathrm{Im}\Pi_T(\omega,\mathbf{k})}{\omega(e^{\omega/T}-1)}$$

6. Modelo inflacionario - relativista de una partícula supermasiva a propósito de la superdensidad de su masa y la superdensidad de su energía.

$$\frac{\Omega_A}{\Omega_{\mathrm{DM}}} \simeq \sqrt{\frac{m}{6\cdot 10^{-6}\mathrm{eV}}}\left(\frac{H_I}{10^{14}\mathrm{GeV}}\right)^2$$

$$S = \int dtd^3x\sqrt{-g}\left[-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{1}{2}m^2 g^{\mu\nu}A_\mu A_\nu\right]$$

$$S_T = \int \frac{a^3 d^3kdt}{(2\pi)^3}\frac{1}{2a^2}\left[\left|\partial_t\vec{A}_T\right|^2 - \left(\frac{k^2}{a^2}+m^2\right)\left|\vec{A}_T\right|^2\right]$$
$$S_L = \int \frac{a^3 d^3kdt}{(2\pi)^3}\frac{1}{2a^2}\left[\frac{a^2m^2}{k^2+a^2m^2}|\partial_t A_L|^2 - m^2|A_L|^2\right]$$

$$\langle X^*(t,\vec{k})X(t,\vec{k}')\rangle \equiv (2\pi)^3\delta^3(\vec{k}-\vec{k}')\frac{2\pi^2}{k^3}\mathcal{P}_X(t,k)$$

$$\langle\rho\rangle = \int \mathrm{dlog}\ k\frac{1}{2a^2}\left[\frac{a^2m^2}{k^2+a^2m^2}\mathcal{P}_{\partial_t A_L} + m^2\mathcal{P}_{A_L}\right]$$

$$\left[\partial_t^2 + \frac{3k^2+a^2m^2}{k^2+a^2m^2}H\partial_t + \frac{k^2}{a^2} + m^2\right]A_L = 0$$

$$\mathcal{P}_{A_L}(t,k) = \left(\frac{kH_I}{2\pi m}\right)^2\left(\frac{A_L(t,k)}{A_{L,0}}\right)^2$$

$$\mathcal{P}_{A_L}(t,k) \simeq \left(\frac{k_\star H_I}{2\pi m}\right)^2\left(\frac{a_\star}{a}\right)\frac{(k/k_\star)^2}{1+(k/k_\star)^3}$$

$$a_0\lambda_\star = \frac{2\pi a_0}{m a_\star} \simeq 10^{11}\ \mathrm{km}\left(\frac{10^{-5}\mathrm{eV}}{m}\right)^{1/2}$$

$$\delta(x) \equiv \frac{\rho(x)-\bar{\rho}}{\bar{\rho}}$$

$$\mathcal{P}_\delta(t,k) \simeq \frac{\sqrt{3}(k/k_\star)^3}{\pi((k/k_\star)^{3/2}+1)^{8/3}}$$

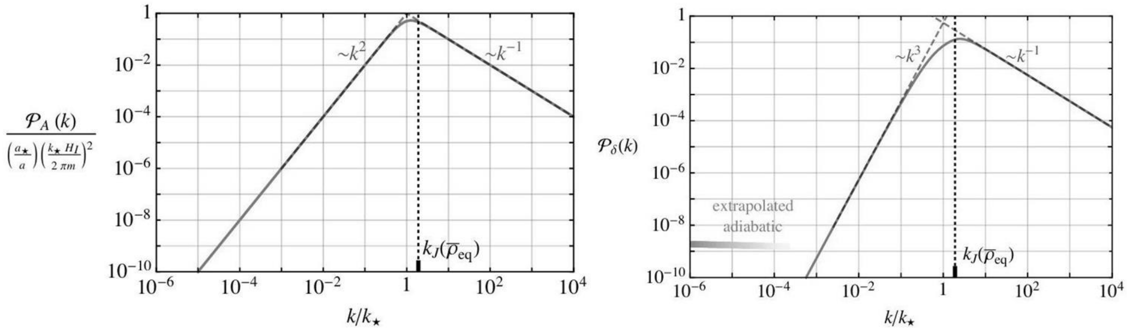


Figura 1. Fluctuaciones inflacionarias de la partícula supermasiva en un campo de gauge específico.

7. Modelo Schrödinger-Poisson para una partícula supermasiva.

$$A_i \equiv \frac{1}{\sqrt{2m^2a^3}}(\psi_i e^{-imt} + \text{ c.c. })\left(i\partial_t + \frac{\nabla^2}{2m} - m\Phi)\psi_i = \nabla^2\Phi = \frac{4\pi G}{a}\sum_i (|\psi_i|^2 - \langle|\psi_i|^2\rangle\right)$$

$$\partial_t\rho_i + 3H\rho_i + a^{-1}\nabla\cdot(\rho_i\vec{v}_i) = 0$$
$$\partial_t\vec{v}_i + H\vec{v}_i + a^{-1}(\vec{v}_i\cdot\nabla)\vec{v}_i = -a^{-1}\left(\nabla\Phi + \nabla\Phi_{Qi}\right)$$
$$\nabla^2\Phi = 4\pi G a^2(\rho - \bar{\rho})$$

$$\Phi_{Qi} \equiv -\frac{\hbar^2}{2a^2m^2}\frac{\nabla^2\sqrt{\rho_i}}{\sqrt{\rho_i}}$$

$$k_J(\rho) \equiv a(16\pi G\rho m^2)^{1/4}$$

$$\left.\frac{k_J(\bar{\rho})}{k_\star}\right|_{a=a_{\text{eq}}} = \frac{\left(16\pi G\bar{\rho}_{\text{eq}}m^2\right)^{1/4}}{m\left(a_\star/a_{\text{eq}}\right)} = g_R\left(12\frac{\bar{\rho}_{\text{eq}}}{\rho_{\text{eq}}^{\text{tot}}}\right)^{1/4} = g_R\left(6\frac{\Omega_A}{\Omega_{\text{M}}}\right)^{1/4}$$

$$\left.\frac{k_J(\bar{\rho})}{k_\star}\right|_{a=a_{\text{eq}}} \simeq 1.9$$

$$H^2 = \frac{8\pi G}{3}\rho_{\text{tot}} = \frac{8\pi G}{3}\frac{\bar{\rho}_{\text{eq}}}{2}\left[\left(\frac{a_{\text{eq}}}{a}\right)^3 + \left(\frac{a_{\text{eq}}}{a}\right)^4\right] \equiv \frac{H_{\text{eq}}^2}{2}\left[\left(\frac{a_{\text{eq}}}{a}\right)^3 + \left(\frac{a_{\text{eq}}}{a}\right)^4\right]$$

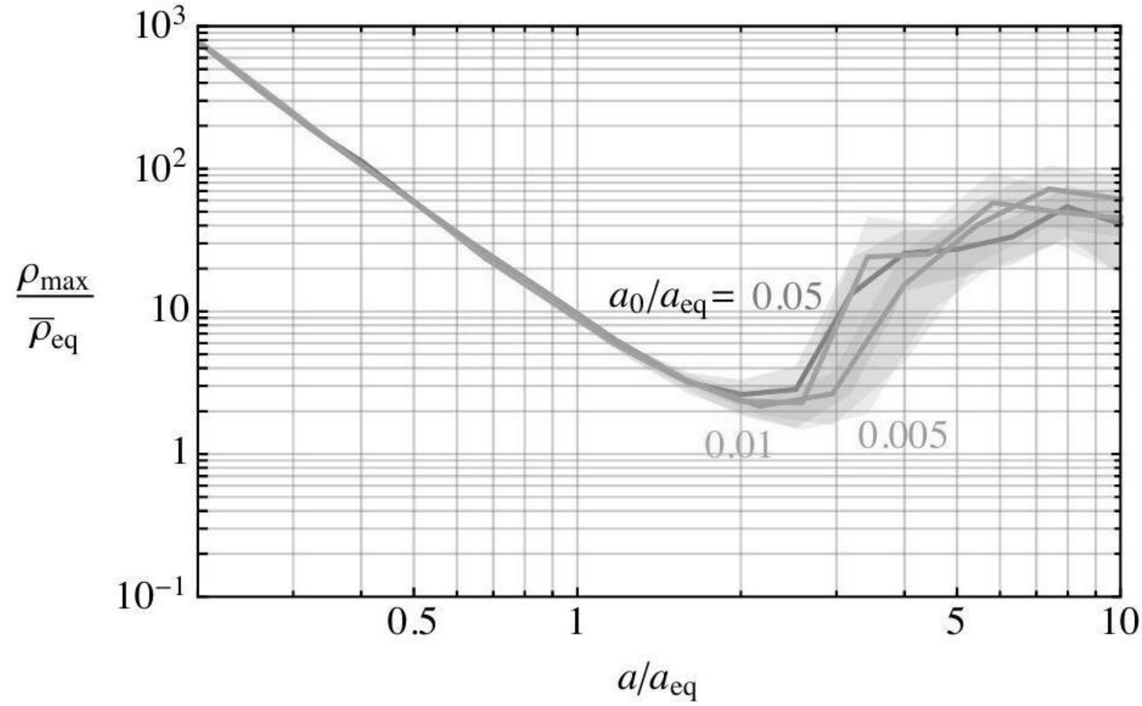
$$\frac{da}{dt} = a\frac{H_{\text{eq}}}{\sqrt{2}}\sqrt{\left(\frac{a_{\text{eq}}}{a}\right)^3 + \left(\frac{a_{\text{eq}}}{a}\right)^4}$$

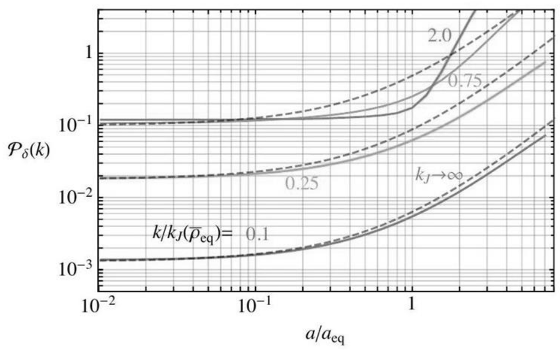
$$\left(i\partial_{\tilde{t}} + \frac{\nabla'^2}{2} - \Phi'\right)\psi_i' = 0$$

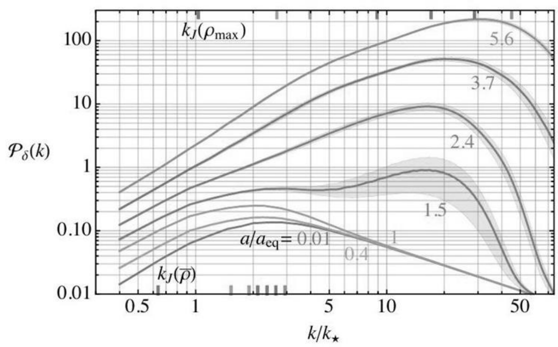
$$\nabla'^2\Phi' = a\sum_i (|\psi_i'|^2 - \langle|\psi_i'|^2\rangle)$$

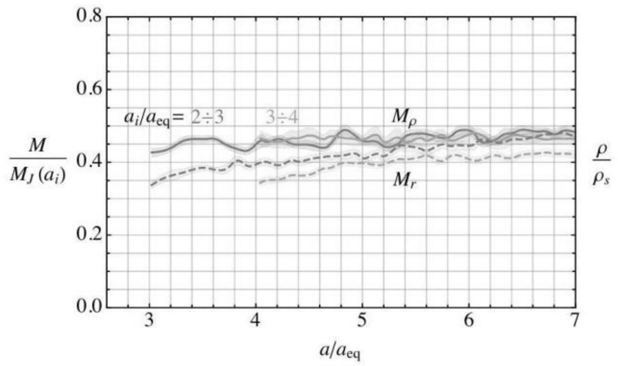
$$\frac{da}{d\tilde{t}} = \beta a^3\sqrt{\left(\frac{a_{\text{eq}}}{a}\right)^3 + \left(\frac{a_{\text{eq}}}{a}\right)^4}$$

$$\langle|\psi'|^2\rangle = \frac{\langle|\psi|^2\rangle}{(\sqrt{4\pi G}T)^{-2}} = \frac{f\bar{\rho}_{\text{eq}}/2}{\left(\sqrt{4\pi G}\sqrt{2}\beta H_{\text{eq}}^{-1}\right)^{-2}} = \frac{3\Omega_{\text{A}}/\Omega_{\text{M}}}{2}\beta^2$$









Figuras 1, 2 y 3. Fluctuaciones del Modelo Schrödinger-Poisson para una partícula supermasiva.

8. Modelo Gravitacional de una partícula supermasiva a propósito de la superdensidad de su masa y energía respectivamente.

$$M(a) = c_M M_J(a),\ \text{con}\ M_J(a) \equiv \frac{4\pi}{3}\bar{\rho} a^3 \lambda_J^3(a) \propto a^{-3/4}$$

$$\mathcal{N} \equiv \frac{1}{m}\int d^3x \sum_i |\psi_i|^2$$

$$E = M + \int d^3x \sum_i \left(\frac{1}{2m^2}|\nabla\psi_i|^2 + \frac{1}{2}\Phi|\psi_i|^2\right)$$

$$\psi_i = \frac{m}{\sqrt{4\pi G}} e^{-i\gamma m t}\chi_1(mr)u_i, \Phi = \Phi_1(mr)$$

$$M \simeq \frac{2\alpha}{Gm}, R \simeq \frac{1.9}{\alpha m}$$

$$MR \simeq \frac{3.9}{Gm^2}$$

$$\rho_s = \frac{\alpha^4 m^2}{4\pi G} \simeq \frac{1}{Gm^2R^4} \simeq \frac{G^3m^6M^4}{64\pi}$$

$$L_p = \frac{i}{2m}\epsilon_{pqr}\int d^3x(\psi_m^*\partial_q\psi_m x^r - \text{ c.c. })$$
$$S_p = \frac{i}{m}\epsilon_{pqr}\int d^3x\psi_q\psi_r^*$$

$$\epsilon_{\hat{z}}^{(\pm 1)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm i\\ 0\end{pmatrix}, \epsilon_{\hat{z}}^{(0)} = \begin{pmatrix}0\\ 0\\ 1\end{pmatrix}$$

$$\psi_i = \frac{m}{\sqrt{4\pi G}} e^{-i\gamma m t}\sum_\lambda \chi^{(\lambda)}(\vec{x})\left(\epsilon_{\hat{z}}^{(\lambda)}\right)_i$$

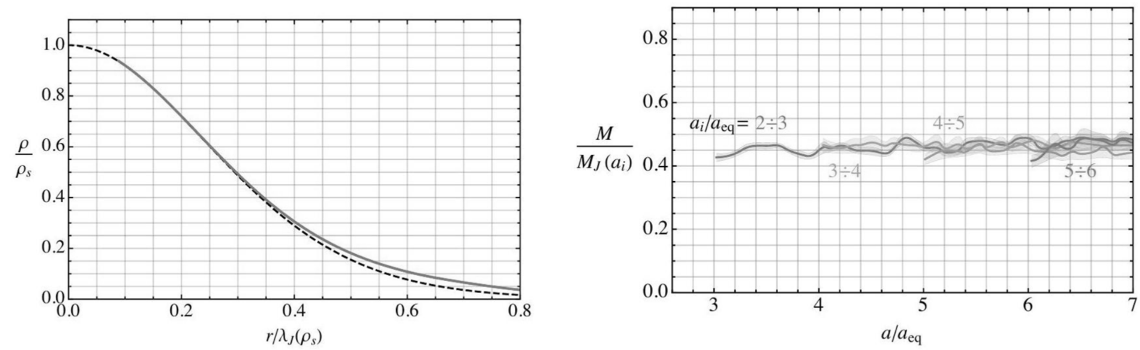


Figura 1. Fluctuaciones de la superdensidad de masa y energía de una partícula supermasiva en un campo de gauge específico.

9. Modelo de distribución de la masa en relación a una partícula supermasiva.

$$M_J^{\mathrm{eq}} \equiv M_J(a_{\mathrm{eq}}) = 5.2 \cdot 10^{-23} M_\odot \left(\frac{\mathrm{eV}}{m}\right)^{3/2}$$

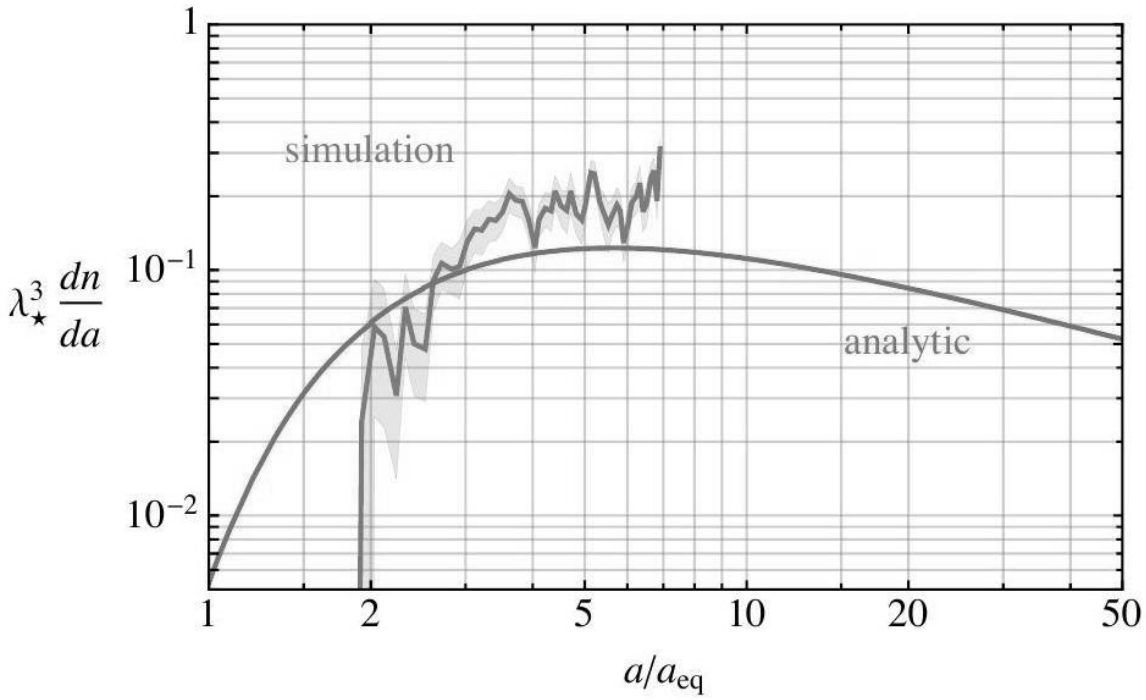
$$M(a) = 2.3 \cdot 10^{-23} M_\odot \left(\frac{c_M}{0.45}\right)\left(\frac{a_{\mathrm{eq}}}{a}\right)^{3/4}\left(\frac{\mathrm{eV}}{m}\right)^{3/2}$$

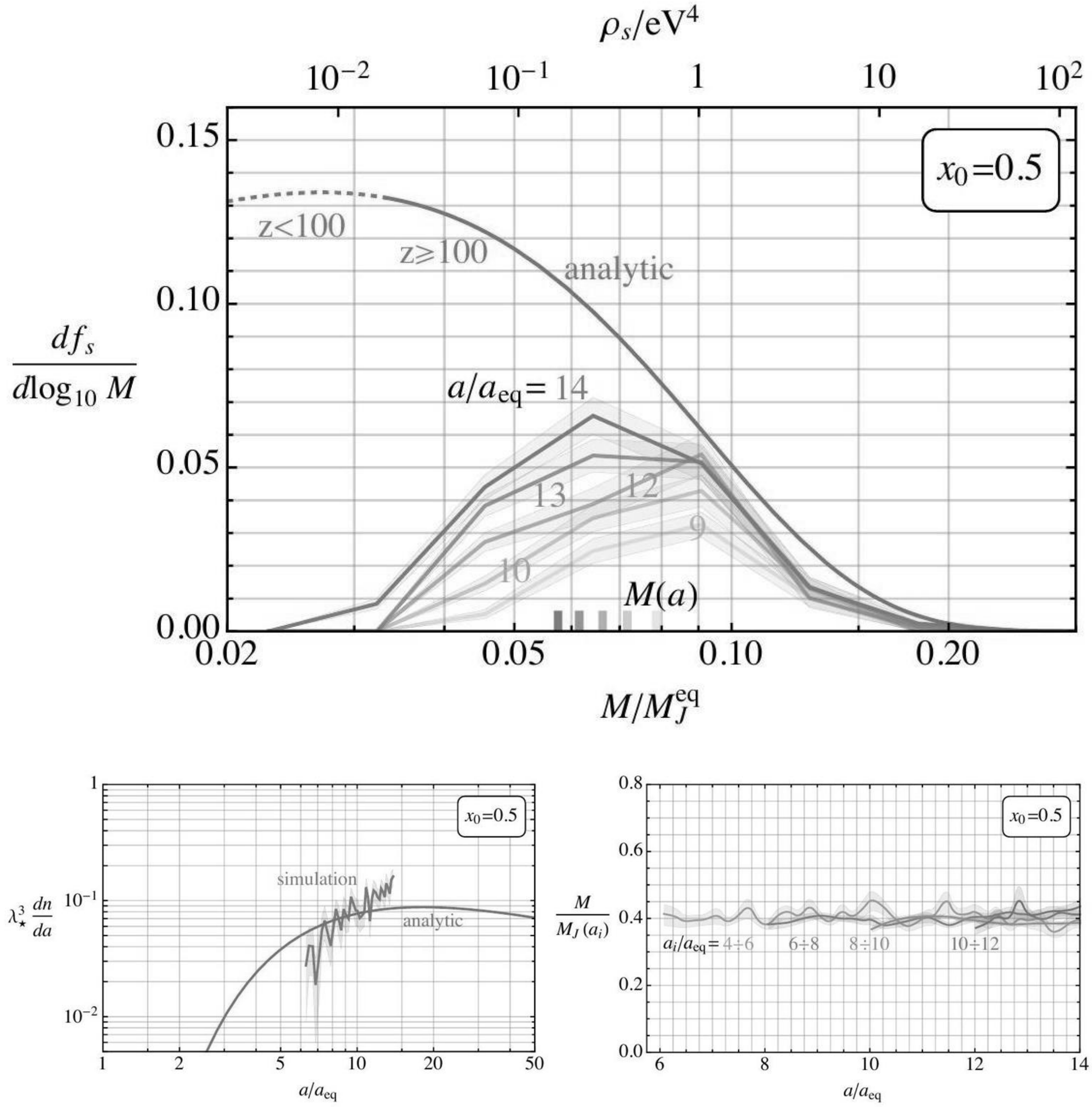
$$\frac{df_s(a, M)}{d\log\, M} = \frac{M}{\bar{\rho}}\frac{dn(a, M)}{d\log\, M}$$

$$\frac{\rho(r)}{\rho_s} = \frac{\rho_0}{\frac{r/\lambda_J(\rho_s)}{r_0}\left(1 + \frac{r/\lambda_J(\rho_s)}{r_0}\right)^2}$$

$$\frac{df_s(a_2, M)}{d\log\, M} = \frac{M^2}{\bar{\rho}(a_2)}\int_{a_1}^{a_2} da \frac{d\Pi_{\delta>\delta_c}(a_1, a)}{da}\frac{\bar{\rho}(a)}{M(a)}\left(\frac{a}{a_2}\right)^3 \delta[M - M(a)] = \frac{d\Pi_{\delta>\delta_c}(a_1, a_M)}{d\log\, M}$$

$$\frac{df_s}{d\log\, M} \simeq \frac{\delta_c}{\sqrt{2\pi}\sigma(M)} e^{-\frac{\delta_c^2}{2\sigma^2(M)}}\left|\frac{d\log\, \sigma(M)}{d\log\, M}\right|,$$



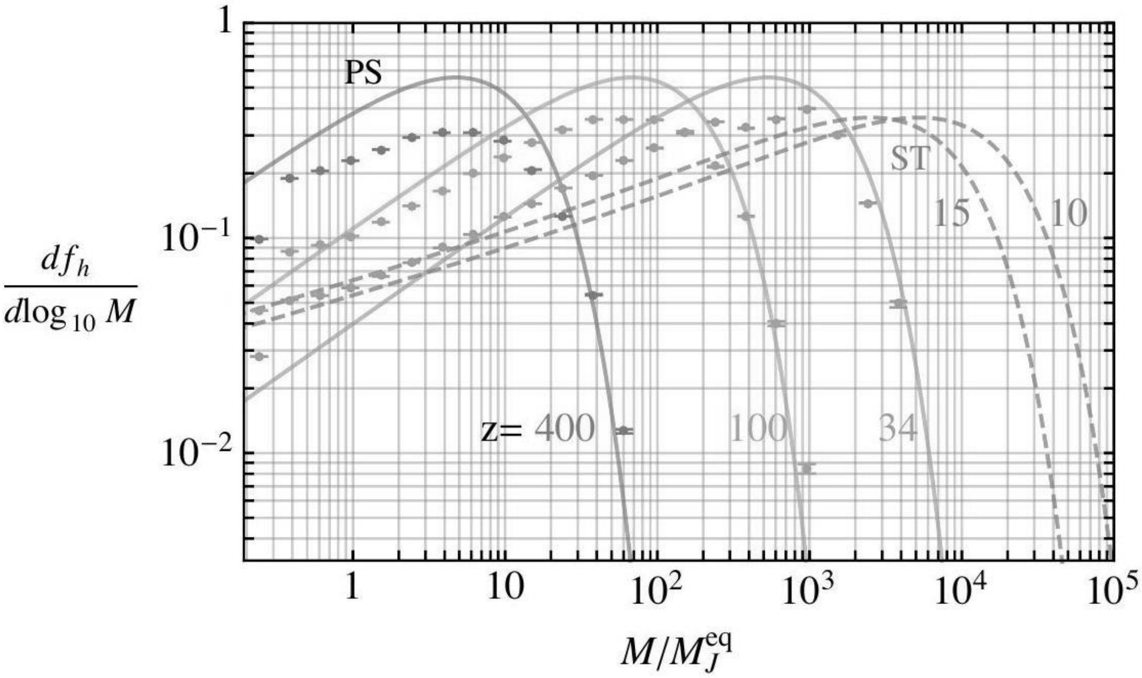


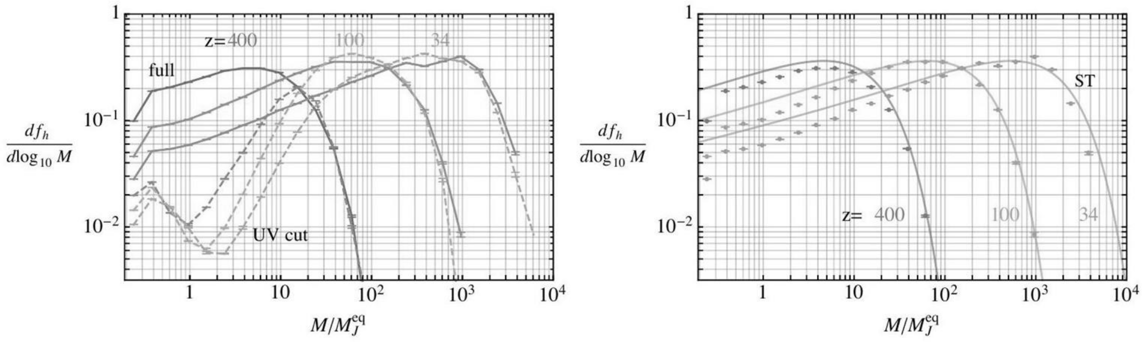
Figuras 1, 2 y 3. Fluctuaciones relativas a la distribución de masa y energía en relación a una partícula supermasiva.

10. Modelo de interacciones gravitacionales Press-Schechter de una partícula supermasiva.

$$\partial_t \rho + 3H\rho + a^{-1}\nabla \cdot (\rho \vec{v}) \ = 0$$
$$\partial_t \vec{v} + H\vec{v} + a^{-1}(\vec{v} \cdot \nabla)\vec{v} \ = -a^{-1}\nabla\Phi$$
$$\nabla^2 \Phi \ = 4\pi G a^2 (\rho - \bar{\rho})$$

$$\frac{df_{\mathrm{h}}(a,M)}{d\log\, M} = \sqrt{\frac{2}{\pi}}\nu e^{-\nu^2/2}\left|\frac{d\log\, \nu}{d\log\, M}\right| \simeq \sqrt{\frac{8M}{M_\star}}\frac{\pi\delta_c}{3^{3/4}D^2[a]}\exp\left[-\frac{\pi^3\delta_c^2}{3\sqrt{3}D^2[a]}\frac{8M}{M_\star}\right]$$





Figuras 1 y 2. Fluctuationes gravitacionales de una partícula supermasiva.

$$\frac{df_{\mathrm{h}}}{d\log\ M} = A(p)(1 + (q\nu^2)^{-p})(q\nu^2)^{1/2} e^{-q\nu^2/2} \left|\frac{d\log\ \nu}{d\log\ M}\right|$$

$$\rho_0 \ = \frac{4.4 \cdot 10^{-3} c_\Delta^3 \nu_c^3 \Delta}{(c_\Delta + 1)^{-1} + \log{(c_\Delta + 1)} - 1} \left[\frac{M_J^{\mathrm{eq}}}{M}\right]^{3/2} \bar{\rho}^{\mathrm{eq}}$$

$$r_0 \ = \frac{4.2}{c_\Delta \nu_c \Delta^{1/3}} \left[\frac{M}{M_J^{\mathrm{eq}}}\right]^{5/6} \lambda_J^{\mathrm{eq}}$$

$$M \simeq 9.5 M_\star \left[\frac{1.7}{\delta_c}\right]^2 \left[\frac{100}{z+1}\right]^2 \simeq 65 M_J^{\mathrm{eq}} \left[\frac{1.7}{\delta_c}\right]^2 \left[\frac{100}{z+1}\right]^2$$

$$M_{\mathrm{h}} = (3 \div 5) \cdot 10^3 M_J^{\mathrm{eq}} \simeq (1.5 \div 3) \cdot 10^{-19} M_\odot (\mathrm{eV/m})^{3/2}$$

$$\rho(r) = \frac{\rho_0}{r/r_0 (1 + r/r_0)^2}$$

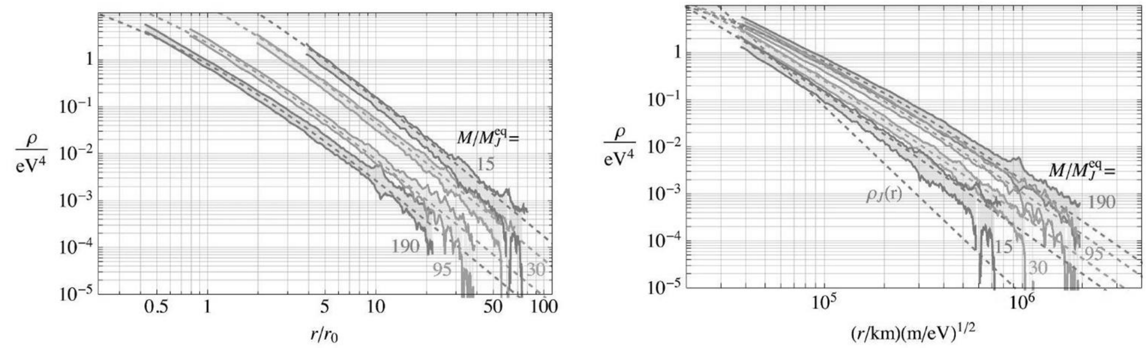


Figura 3: Fluctuaciones de masa a propósito de la gravedad cuántica endógena de la partícula supermasiva.

$$\rho_0 \simeq 0.7\left[\frac{10^3 M_J^{\mathrm{eq}}}{M}\right]^{3/2} \bar{\rho}^{\mathrm{eq}} \simeq 0.3\mathrm{eV}^4\left[\frac{10^3 M_J^{\mathrm{eq}}}{M}\right]^{3/2}$$

$$r_0 \simeq 5.4\left[\frac{M}{10^3 M_J^{\mathrm{eq}}}\right]^{5/6} \lambda_J^{\mathrm{eq}} \simeq 7\cdot 10^5\ \mathrm{km}\left[\frac{M}{10^3 M_J^{\mathrm{eq}}}\right]^{5/6}\left[\frac{1\mathrm{eV}}{m}\right]^{1/2}$$

$$\langle\rho\rangle \simeq \frac{3\rho_0}{c_\Delta^3}(\log\ c_\Delta - 1)$$

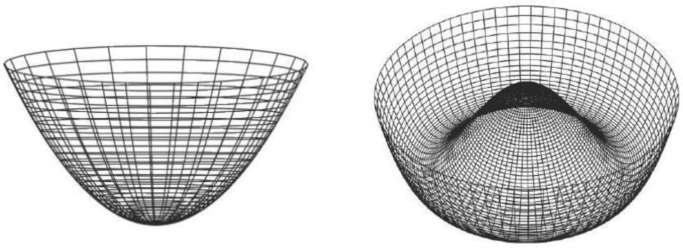


Figura. 4. Simulación de la curvatura del espacio – tiempo cuántico provocado por la interacción de una partícula supermasiva.

11. Modelo de aniquilación de una partícula supermasiva.

$$\rho_s \simeq 1.51\cdot 10^4\mathrm{eV}^4\left(\frac{M}{M_J^{\mathrm{eq}}}\right)^4$$

$$\lambda_J(\rho_s) = 4.6\cdot 10^3\ \mathrm{km}\left(\frac{\mathrm{eV}}{m}\right)^{1/2}\left(\frac{M_J^{\mathrm{eq}}}{M}\right)$$

$$n = f_s\bar{\rho}(t_0)/M \simeq 10^{20}\mathrm{pc}^{-3}\left(\frac{f_s}{0.05}\right)\left(\frac{\rho_{\mathrm{local}}}{0.5\mathrm{GeV/cm}^3}\right)\left(\frac{0.1M_J^{\mathrm{eq}}}{M}\right)\left(\frac{m}{\mathrm{eV}}\right)^{3/2}$$

$$\Gamma \simeq n\pi R^2 v_{\rm rel} \simeq \frac{0.1}{\rm yr}\left(\frac{m}{\rm eV}\right)^{1/2}\left(\frac{0.1M_J^{\rm eq}}{M}\right)^3\left(\frac{v_{\rm rel}}{10^{-3}}\right)\left(\frac{f_s}{0.05}\right)\left(\frac{\rho_{\rm local}}{0.5{\rm GeV/cm^3}}\right)$$

$$\aleph_{aniquilación} \simeq 10^2\ {\rm s}\left(\frac{0.1M_J^{\rm eq}}{M}\right)\left(\frac{\rm eV}{m}\right)^{1/2}$$

$$\mathcal{N} \simeq \left(\frac{2\pi}{k_\star/a_{\rm eq}}\right)^3\frac{\rho_{\rm eq}}{m} \simeq 90\frac{\rho_{\rm eq}^{1/4}}{G^{3/4}m^{5/2}} \simeq 30\left(\frac{10^{17}{\rm eV}}{m}\right)^{5/2} \gg 1$$

$$\mathcal{P}_{A_L}(t,k) = \left(\frac{k_\star H_I}{2\pi m}\right)^2\left(\frac{a_\star}{a}\right)F_{A_L}^2[k/k_\star] \simeq \left(\frac{k_\star H_I}{2\pi m}\right)^2\left(\frac{a_\star}{a}\right)\frac{(k/k_\star)^2}{1+(k/k_\star)^3}$$

$$\mathcal{P}_\delta(t,k) = \frac{k^2}{8\langle A_L^2\rangle^2}\int_0^\infty dq\int_{|q-k|}^{q+k}dp\,\frac{(k^2-q^2-p^2)^2}{q^4p^4}\mathcal{P}_{A_L}(t,p)\mathcal{P}_{A_L}(t,q) \simeq \frac{\sqrt{3}(k/k_\star)^3}{\pi((k/k_\star)^{3/2}+1)^{8/3}}$$

$$\Delta E = \frac{4}{3}\frac{G^2M_s^2r^2}{v_{\rm rel}^2b^4}dm$$

$$b_{\rm crit}^2 = \frac{M_s}{v_{\rm rel}}\left(\frac{G}{\pi\bar{\rho}(r)}\right)^{1/2}$$

$$p_{\rm dest} = \pi n\frac{S}{v_{\rm rel}}\left(\frac{G}{\pi\bar{\rho}(r)}\right)^{1/2}$$

$$p_{\rm dest} = 0.4\left(\frac{n}{100}\right)\left(\frac{0.05{\rm eV}^4}{\bar{\rho}(r)}\right)^{1/2}\left(\frac{S}{140M_\odot{\rm pc}^{-2}}\right)\left(\frac{10^{-3}}{v_{\rm rel}}\right)$$

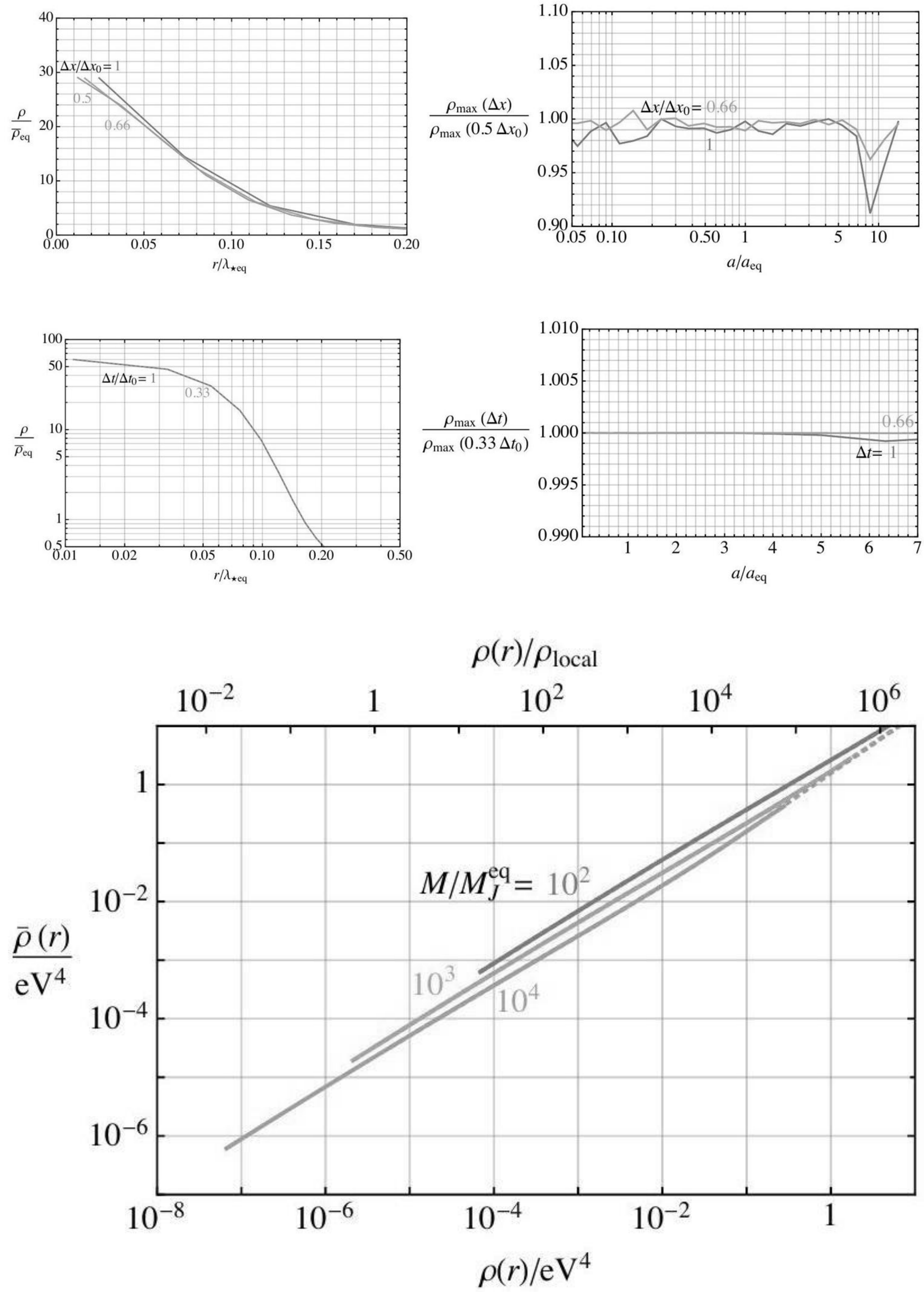
12. Modelo de superdensidades de masa y energía de una partícula supermasiva.

$$\ddot{\delta} + \frac{2}{a}\dot{a}\dot{\delta} - 4\pi G f\rho_{\rm nr}\delta = 0$$

$$\frac{\partial^2\delta}{\partial y^2} + \frac{2+3y}{2y(1+y)}\frac{\partial\delta}{\partial y} - \frac{3f}{2y(1+y)}\delta = 0$$

$$\frac{\partial^2\delta}{\partial y^2} + \frac{2+3y}{2y(1+y)}\frac{\partial\delta}{\partial y} - \frac{3f}{2y(1+y)}\left(-1+\left(\frac{k}{k_J}\right)^4\right)\delta$$

$$\psi_i(t+\Delta t) = \left(\prod_{\alpha=1}^{8} e^{-id_\alpha\Delta\tilde{t}\Phi(\mathbf{x})}e^{-ic_\alpha\Delta\tilde{t}k^2/2}\right)\psi_i(t)$$



Figuras 1, 2 y 3: Fluctuaciones de superdensidad de masa y energía de una partícula supermasiva.

$$\Delta E = 2(2\pi G\sigma_s(r))^2 \frac{(\Delta z)^2}{v_z^2} dm(1 + a^2)^{-3/2}$$

$$\bar{\rho}(r) \lesssim 10^{-6}\mathrm{eV}^4 \left(\frac{\sigma_s}{10^8 M_\odot/\mathrm{kpc}^2}\right)^2 \left(\frac{10^{-3}}{v_z}\right)^2$$

13. Modelo de coordenadas de una partícula repercutida por una partícula supermasiva.

$$r_t = r_{\mathrm{orbit}} \left(\frac{M_{\mathrm{clump}}(r_t)}{M_{\mathrm{host}}(r_{\mathrm{orbit}})}\right)^{1/3} \left(3 - \frac{d\log M_{\mathrm{host}}(r)}{d\log r}\bigg|_{r=r_{\mathrm{orbit}}}\right)^{-1/3}$$

$$t_{\mathrm{decay}} = t_{\mathrm{orbit}} \frac{M_{\mathrm{host}}}{M_{\mathrm{clump}}} \left(\log\left(\frac{M_{\mathrm{host}}}{M_{\mathrm{clump}}}\right)\right)^{-1} \simeq \frac{1}{\sqrt{G\bar{\rho}_{\mathrm{host}}}} \frac{M_{\mathrm{host}}}{M_{\mathrm{clump}}} \left(\log\left(\frac{M_{\mathrm{host}}}{M_{\mathrm{clump}}}\right)\right)^{-1}$$

$$\Delta E \simeq \frac{4\pi}{3}\gamma_1 G\bar{\rho}_{\mathrm{host}} r^2 dm$$

$$t_{\mathrm{shock}} \simeq \frac{\bar{\rho}(r)}{\sqrt{G}\gamma_1\gamma_2\bar{\rho}_{\mathrm{host}}^{3/2}} \simeq 10^9 \left(\frac{\bar{\rho}}{0.01\mathrm{eV}^4}\right)\left(\frac{1}{\gamma_1\gamma_2}\right)\left(\frac{10^{-4}\mathrm{eV}^4}{\bar{\rho}_{\mathrm{host}}}\right)^{3/2}$$

$$t_{\mathrm{dyn}} \simeq 3\cdot 10^{11}\left(\frac{10^{-4}\mathrm{eV}}{\bar{\rho}(r)_{\mathrm{host}}}\right)^{1/2} \frac{M_{\mathrm{host}}/M_s}{10^5} \frac{\log(10^5)}{\log(M_{\mathrm{host}}/M_s)}$$

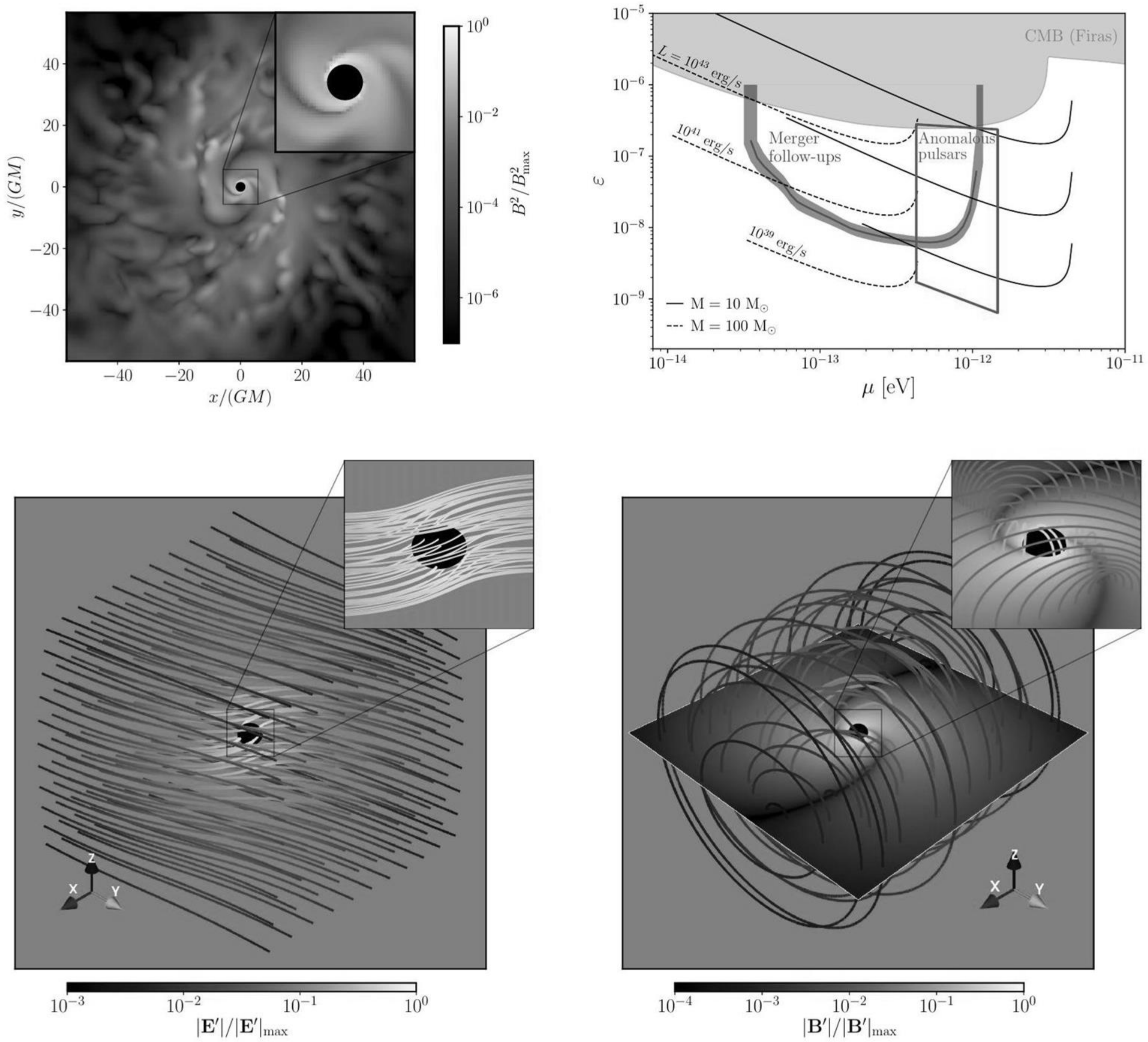
$$\rho_s/\bar{\rho}_{\mathrm{host}}(r_{\mathrm{orbit}}) \gtrsim 100$$

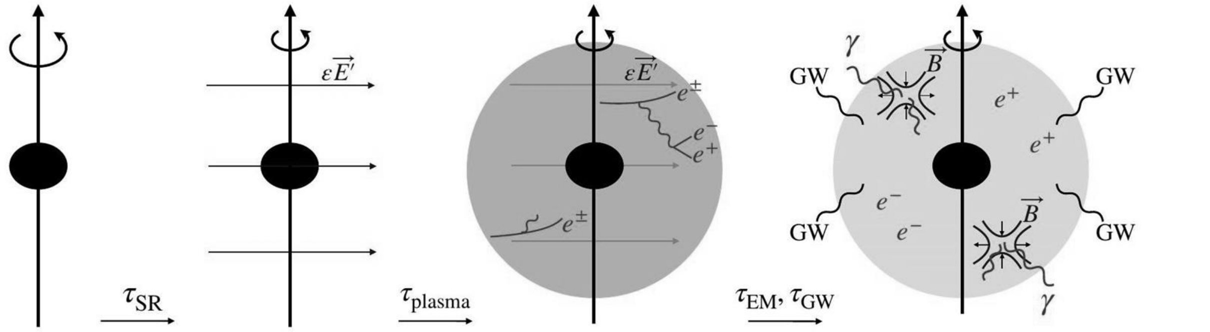
CONCLUSIONES.

La partícula oscura o partícula supermasiva, es un componente esencial pero no principal de la Teoría Cuántica de Campos Relativistas o Teoría Cuántica de Campos Curvos, formulada por este investigador, sin embargo, se destacan ideas nucleares, (i) en primer término, que la partícula oscura o supermasiva, se caracteriza por tener una masa excesivamente densa, lo que la vuelve superlativamente pesada, (ii) por otro lado, que la partícula supermasiva u oscura, aunque se trate de una partícula subatómica extremadamente densa, produce energía cuya densidad es superior a cero, más sin embargo, este fenómeno no sucede en todos los casos, (iii) esta propiedad de la partícula supermasiva u oscura, le permite deformar el espacio – tiempo cuántico en el que ineteractúa, curvándolo, repercutiendo así en el sistema de coordenadas de las partículas repercutidas, lo que se identifica como gravedad cujántica endógena, o en su defecto, provocando agujeros negros cuánticos, a propósito de su aniquilación, que puede ser causada por colisiones o en su defecto, por implosiones, esto último cuando se está ante una partícula supermasiva cuya masa y energía son superdensas. Ahora bien, en este punto es importante reflexionar que si una partícula supermasiva, provoca un agujero negro cuántico, la singularidad cuántica de éste, es la transformación de la materia y la energía, a propósito de la supergravedad cuántica y por ende, la existencia de multidimensiones o supermembranas de realidad microscópica. En esta singularidad, el tiempo abandona su condición de dimensión y pasa a convertirse en un objeto tridimensional, en tanto que el espacio, se funde con el tiempo, volviéndose multidimensional, infinitas veces, es decir, desplegando distintas capas de realidad, sí y sólo sí, se está ante un escenario de aniquilación y supergravedad cuántica.

Apéndice A.

Agujeros negros cuánticos provocados por una partícula supermasiva. Características funcionales y morfológicas y configuraciones de campo.





Figuras 1, 2 y 3. Simulación de un agujero negro cuántico provocado por la deformación del espacio – tiempo cuántico, a propósito de las interacciones de una partícula supermasiva.

$$\mathcal{L}' = -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{1}{2}\mu^2 A'^{\mu}A'_{\mu}.$$

$$\omega \simeq \mu\left(1 - \frac{\alpha^2}{2}\right)$$

$$\Gamma_{\rm SR} \equiv \tau_{\rm SR}^{-1} \simeq 4\alpha^7(\Omega_{\rm BH} - \omega) \simeq 4a_*\alpha^6\mu,$$

$$\Omega_{\rm BH} = \frac{1}{2}\left(\frac{a_*}{1+\sqrt{1-a_*^2}}\right)r_g^{-1}.$$

$$\Omega_{\rm BH} \leq \omega,$$

$$\alpha \lesssim 1/2$$

$$M_c \simeq 10^{-2}\left(\frac{\Delta a_*}{0.1}\right)\left(\frac{\alpha}{0.1}\right)M$$

$$A'_0 = \frac{\sqrt{M_c}}{\sqrt{\pi}\mu^2 r_c^{5/2}}e^{-r/r_c}\sin\,\theta\sin\,(\omega t - \phi)$$

$$\mathbf{A}' = -\frac{\sqrt{M_c}}{\sqrt{\pi}\mu r_c^{3/2}}e^{-r/r_c}\{\cos\,\omega t, \sin\,\omega t, 0\}$$

$$P_{\rm GW} \simeq 17\frac{\alpha^{10}}{G}\left(\frac{M_c(t)}{M}\right)^2$$

$$M_c(t) = \frac{M_c(t_0)}{1 + (t - t_0)/\tau_{\rm GW}},$$

$$\tau_{\rm GW} \simeq \frac{GM}{17\alpha^{11}\Delta a_*} \sim \zeta\left(\frac{0.1}{\Delta a_*}\right)\left(\frac{0.1}{\alpha}\right)^{11}\left(\frac{M}{10M_\odot}\right)$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{\mu^2}{2}A'_{\mu}A'^{\mu} - \varepsilon\mu^2 A'_{\mu}A^{\mu} + I_{\mu}A^{\mu}$$

$$\nabla_\alpha F^{\alpha\beta} = -I^\beta + \varepsilon\mu^2 A'^\beta$$
$$\nabla_\alpha F'^{\alpha\beta} = \mu^2 A'^\beta + \varepsilon\mu^2 A^\beta$$

$$\nabla_\alpha T^{\alpha\beta} = -F^{\beta\gamma}\left(I_\gamma - \varepsilon\mu^2 A'_\gamma\right),$$
$$\nabla_\alpha T'^{\alpha\beta} = \varepsilon\mu^2 F'^{\beta\gamma} A_\gamma.$$

$$|\varepsilon\mathbf{E}'| \simeq \frac{\varepsilon\sqrt{\Delta a_*}\alpha^{5/2}\mu}{\sqrt{G}} \simeq 2\cdot 10^{13}\ \mathrm{V/m}\sqrt{\Delta a_*}\left(\frac{\varepsilon}{10^{-7}}\right)\left(\frac{\alpha}{0.1}\right)^{5/2}\left(\frac{\mu}{10^{-12}\mathrm{eV}}\right)$$

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

$$\rho \simeq \varepsilon\nabla\cdot\mathbf{E}' \simeq \pm\frac{\varepsilon\sqrt{\Delta a_*}\alpha^{7/2}\mu^2}{\sqrt{G}}$$
$$\simeq \pm 5\cdot 10^7\ \mathrm{cm}^{-3}\sqrt{\Delta a_*}\left(\frac{\varepsilon}{10^{-7}}\right)\left(\frac{\alpha}{0.1}\right)^{7/2}\left(\frac{\mu}{10^{-12}\mathrm{eV}}\right)^2$$

$$\partial_t\mathbf{B} = \frac{\varepsilon\mu^2\mathbf{B}'}{\sigma} + \frac{1}{\sigma}\nabla^2\mathbf{B} + \nabla\times(\mathbf{v}\times\mathbf{B}),$$

$$\gamma_e \simeq \frac{e\varepsilon|\mathbf{E}'|}{m_e\mu} \simeq e\varepsilon\alpha^{5/2}\sqrt{\Delta a_*}\frac{M_{\mathrm{pl}}}{m_e} \simeq 10^{12}\left(\frac{\varepsilon}{10^{-7}}\right)\left(\frac{\alpha}{0.1}\right)^{5/2}\left(\frac{\Delta a_*}{0.1}\right)^{1/2}$$

$$\Gamma_{\mathrm{syn}} \simeq \frac{2}{3}\frac{e^2\gamma_e}{r_c} \simeq \frac{2}{3}\frac{e^3\varepsilon|\mathbf{E}'|}{m_e} \simeq \frac{2}{3}e^3\varepsilon\alpha^{5/2}\sqrt{\Delta a_*}\mu\frac{M_{\mathrm{Pl}}}{m_e},$$

$$\varepsilon > \frac{1}{e^3\alpha^{5/2}\sqrt{\Delta a_*}}\frac{m_e}{M_{\mathrm{Pl}}} \simeq 10^{-18}\left(\frac{0.1}{\alpha}\right)^{5/2}\left(\frac{0.1}{\Delta a_*}\right)^{1/2}$$

$$\frac{\Gamma_{e^\pm}}{V} = \frac{(e|\mathbf{E}|)^2}{4\pi^3}\sum_n^\infty \frac{1}{n^2}\exp\left(-\frac{\pi m_e^2}{e|\mathbf{E}|}n\right)$$

$$S_{\mathrm{B}} = -\gamma_\theta + \left(\frac{2m_e^2}{e\varepsilon|\mathbf{E}'|} + \frac{e\varepsilon|\mathbf{E}'|}{2m_e^2}\gamma_\theta^2\right)\arctan\frac{2m_e^2}{e\varepsilon|\mathbf{E}'|\gamma_\theta},$$

$$\Gamma_{e^\pm}^\gamma = \frac{\alpha_{\mathrm{EM}}}{2\pi}\frac{e\varepsilon|\mathbf{E}'|}{m_e}\exp\left(-\frac{2m_e^2}{e\varepsilon|\mathbf{E}'|}\frac{2m_e}{\omega_{\mathrm{syn}}}\right) = \frac{\alpha_{\mathrm{EM}}}{2\pi}\frac{e\varepsilon|\mathbf{E}'|}{m_e}\exp\left[-\frac{4m_e^6\mu^2}{(e\varepsilon|\mathbf{E}'|)^4}\right]$$

$$\varepsilon \gg \left(\ln\frac{\alpha_{\mathrm{EM}}\gamma_e}{2\pi\alpha}\right)^{-1/4}\frac{m_e^{3/2}\sqrt{2\mu}}{e|\mathbf{E}'|} \approx \left(\ln\frac{\alpha_{\mathrm{EM}}\gamma_e}{2\pi\alpha}\right)^{-1/4}\frac{1}{e\alpha^{5/2}\sqrt{\Delta a_*}}\frac{m_e}{M_{\mathrm{pl}}}\sqrt{\frac{2m_e}{\mu}} \simeq 10^{-10}\left(\frac{0.1}{\alpha}\right)^{5/2}\left(\frac{0.1}{\Delta a_*}\right)^{1/2}\left(\frac{10^{-12}\mathrm{eV}}{\mu}\right)^{1/2}\left(\frac{\log\frac{\alpha_{\mathrm{EM}}\gamma_e}{2\pi\alpha}}{20}\right)^{-1/4}$$

$$\tau_{\mathrm{plasma}} \simeq \frac{1}{2\Gamma_{e^\pm}^\gamma}\ln\left(\frac{n_e^f}{n_e^0}\right) \simeq \frac{1}{2\alpha\mu}\ln\frac{m_e^{3/2}}{e^2\alpha^2\mu^{3/2}}$$

$$\frac{\tau_{\mathrm{plasma}}}{\tau_{\mathrm{SR}}} \simeq 2a_*\alpha^5\ln\frac{m_e^{3/2}}{e^2\alpha^2\mu^{3/2}}$$

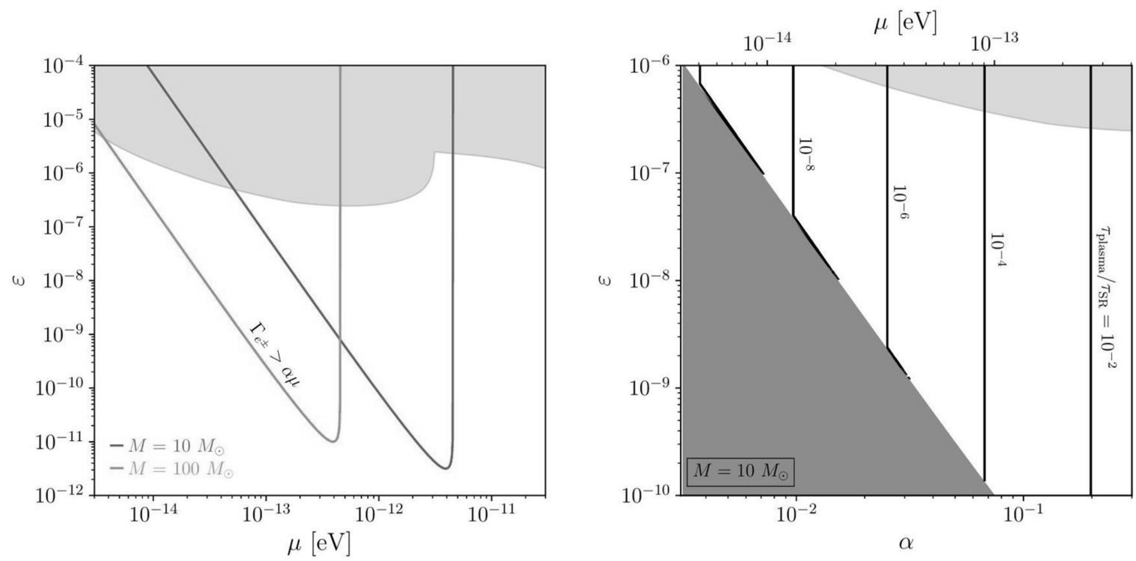


Figura 4. Espectro de radiación de un agujero negro cuántico.

$$M_{\text{plasma}} \approx \frac{m_e \rho}{e(\alpha\mu)^3} \simeq \varepsilon \alpha^{1/2} (\Delta a_*)^{1/2} \frac{m_e M_{\text{pl}}}{\mu} \simeq 10^{-29} M_\odot \left(\frac{\epsilon}{10^{-7}}\right) \left(\frac{\alpha}{0.1}\right)^{1/2} \left(\frac{\Delta a_*}{0.1}\right)^{1/2} \left(\frac{10^{-12}\text{eV}}{\mu}\right)$$

$$\tau_{\text{acc}} = \frac{M_{\text{plasma}}}{\dot{M}_{\text{Bondi}}} \approx \varepsilon \alpha^{-3/2} (\Delta a_*)^{1/2} \frac{c_s^3 M_{\text{pl}} \mu}{\pi n_M} \simeq 10 \text{ years} \left(\frac{\varepsilon}{10^{-7}}\right) \left(\frac{\alpha}{0.1}\right)^{-3/2} \left(\frac{\Delta a_*}{0.1}\right)^{1/2} \left(\frac{\mu}{10^{-12}\text{eV}}\right) \left(\frac{1/\text{cm}^3}{n_M}\right) \left(\frac{c_s}{1}\right)^3$$

$$E^i \equiv n_\nu F^{i\nu}, B^i \equiv n_\nu (* F)^{i\nu} = \frac{1}{2} n_\nu \varepsilon^{i\nu\alpha\beta} F_{\alpha\beta}$$

$$J^i = I^i - \rho_q n^i, \qquad \rho_q = -n_\mu I^\mu$$

$$\rho_q = D_i E^i - \varepsilon \mu^2 n_\mu A'^\mu$$

$$j^\mu = \sigma e^\mu$$

$$e^\mu \equiv u_\nu F^{\mu\nu}, j^\mu \equiv I^\mu + (u^\nu I_\nu) u^\mu$$

$$v^i_{d,\text{ ideal}} = \frac{\varepsilon^{ijk} E_j B_k}{B^2},$$

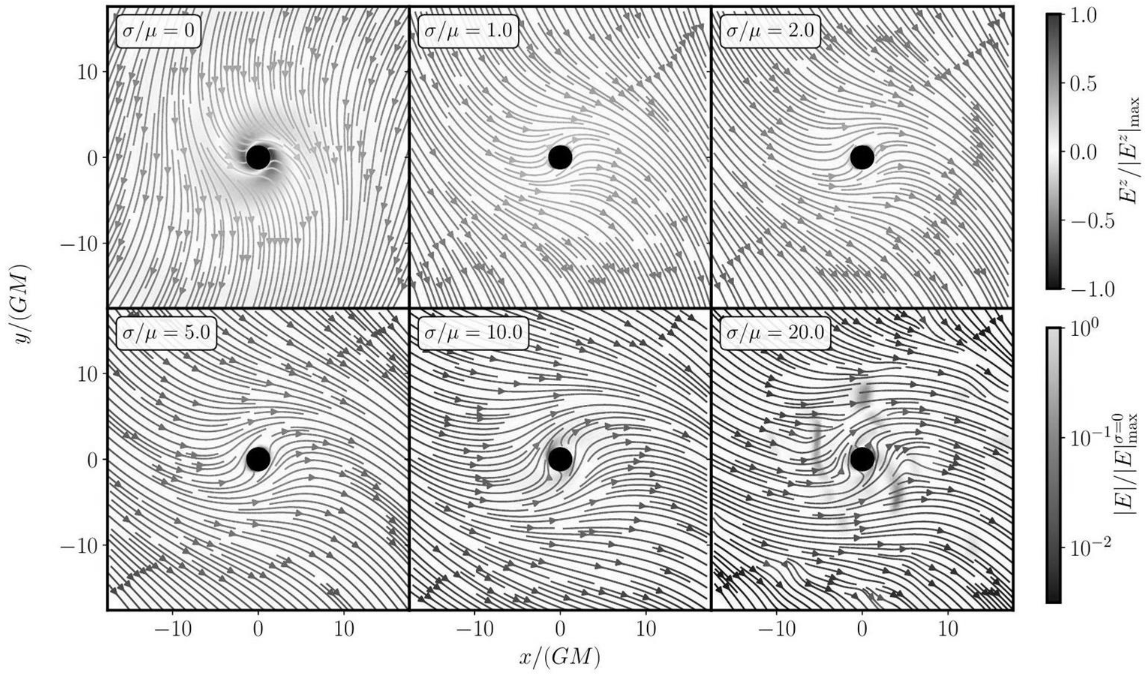
$$v_d^i = \frac{\varepsilon^{ijk} E_j B_k}{B^2 + E_0^2}, E_0^2 = B_0^2 + E^2 - B^2$$

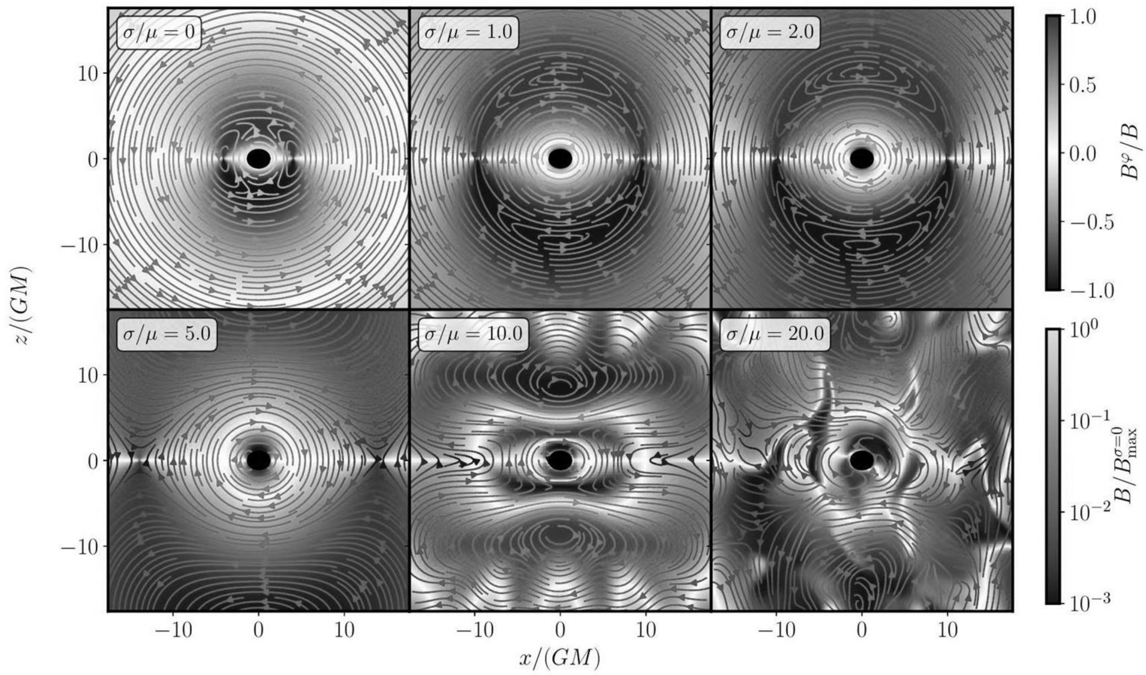
$$B_0^2 = \frac{1}{2}\left[B^2 - E^2 + \sqrt{(B^2 - E^2)^2 + 4(E_i B^i)^2}\right]$$

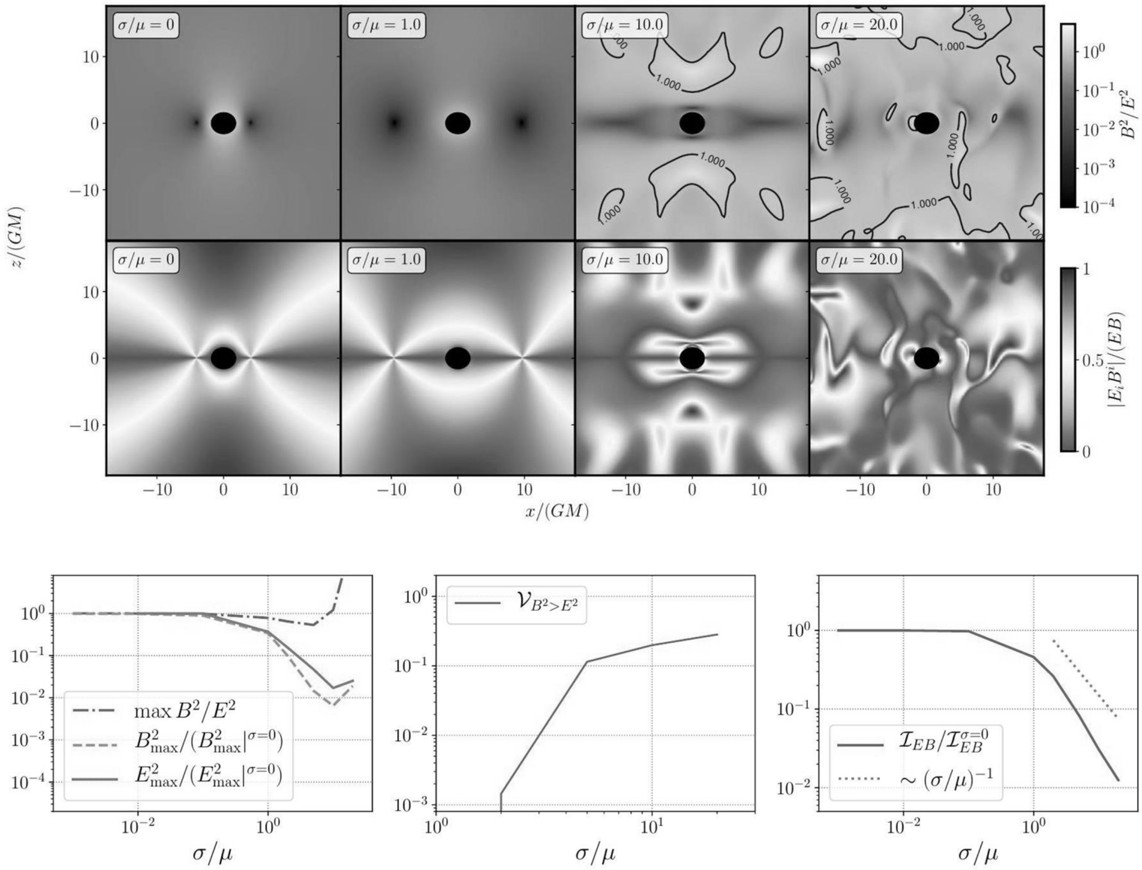
$$J^i = \rho_q v_d^i + \sigma E_0 \sqrt{\frac{B^2 + E_0^2}{B_0^2 + E_0^2}} \left(\frac{E_0 E^i + B_0 B^i}{B^2 + E_0^2}\right)$$

$$\partial_t E^i \approx -\sigma E^i + \varepsilon \mu^2 A'^i$$

$$\mathbf{E} \propto \begin{pmatrix} \sigma + i\mu \\ \mu - i\sigma \\ 0 \end{pmatrix} e^{-i\omega t} + c.c.$$



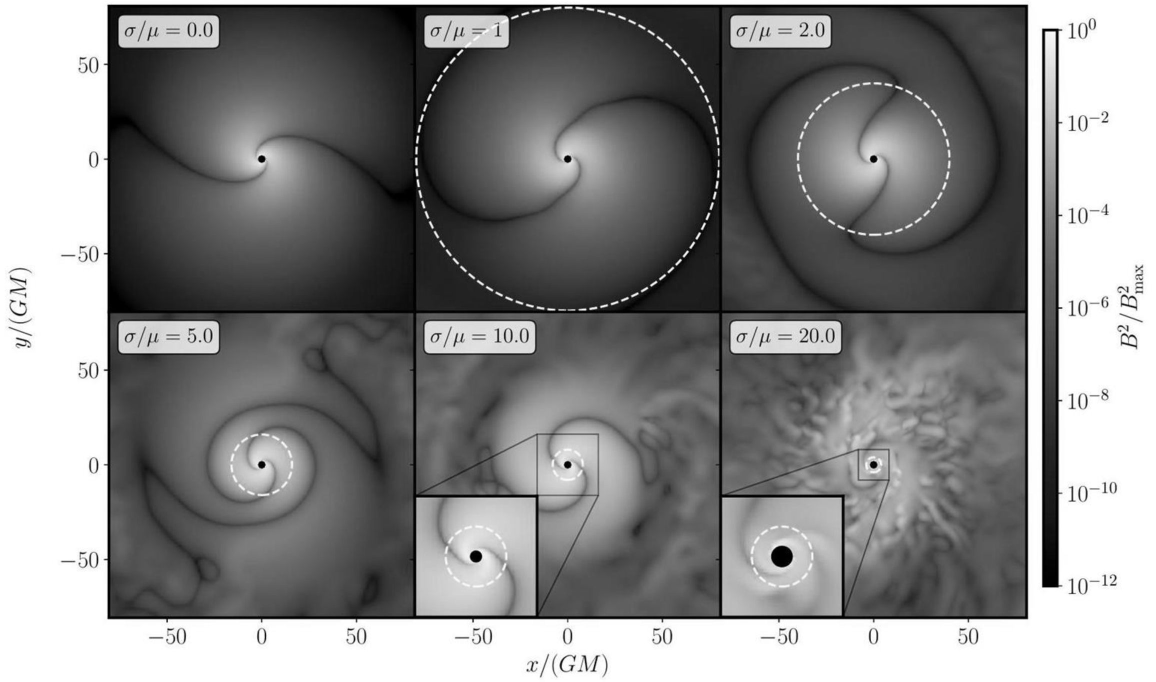




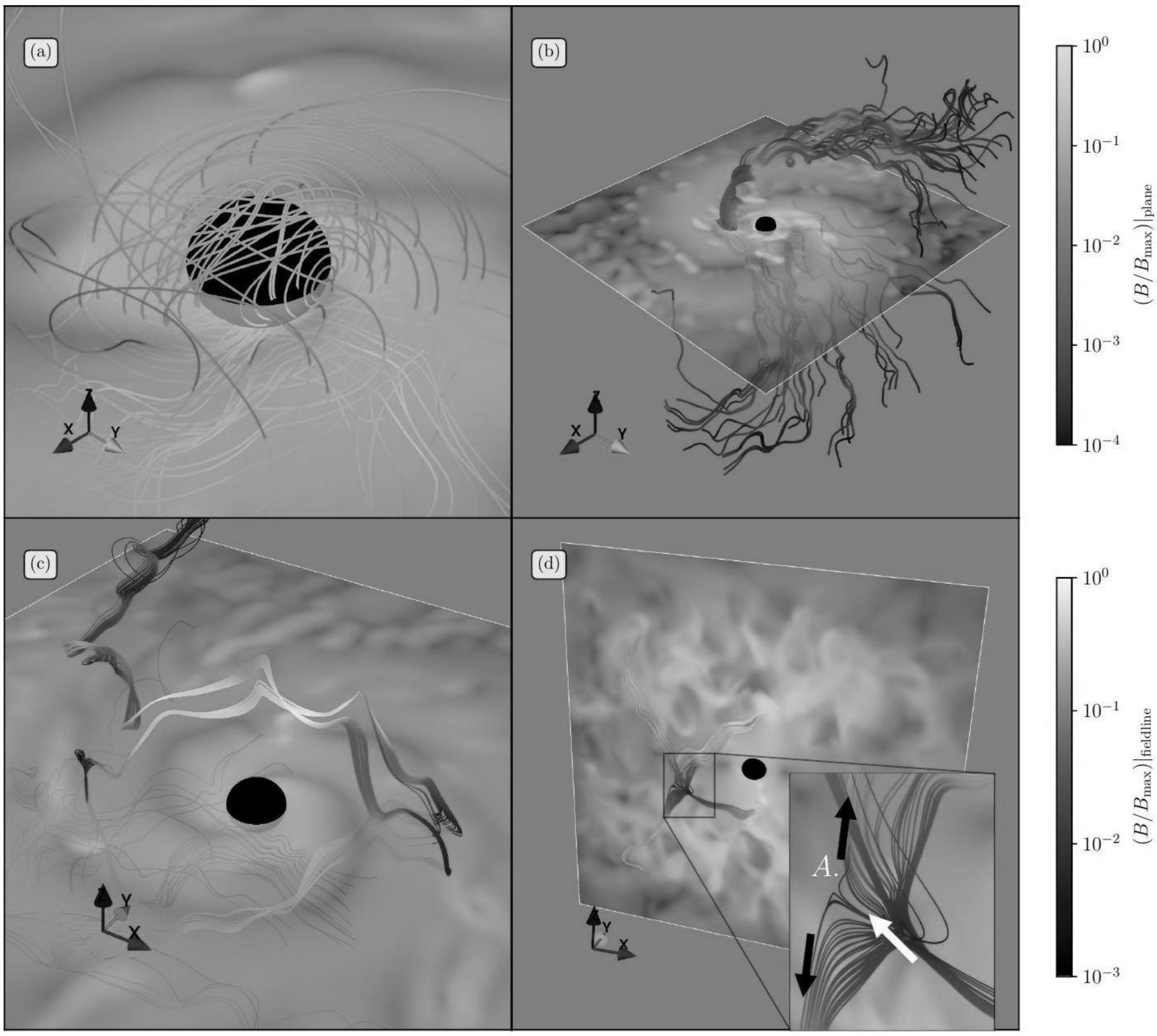
Figuras 5, 6, 7 y 8. Campos de energía desplegados por un agujero negro cuántico provocado por una partícula supermasiva.

$$\partial_t B^i = \frac{\varepsilon \mu^2 B'^i}{\sigma} + \frac{1}{\sigma} \partial_j \partial^j B^i + \varepsilon^{ijk} \varepsilon_{klm} \partial_j v^l B^m$$

$$r_* \approx 80 \mu GM/\sigma$$



$\sigma/\mu = 0.0$
$\sigma/\mu = 1$
$\sigma/\mu = 2.0$
$\sigma/\mu = 5.0$
$\sigma/\mu = 10.0$
$\sigma/\mu = 20.0$
$y/(GM)$
$x/(GM)$
$B^2/B^2_{\max}$



(a)
(b)
(c)
(d)
A.
$(B/B_{\max})|_{\mathrm{plane}}$
$(B/B_{\max})|_{\mathrm{fieldline}}$

Figuras 9 y 10. Configuraciones de campo del agujero negro cuántico – deformación del espacio – tiempo cuántico provocado por una partícula supermasiva.

$$\mathcal{E} = \int_D d^3x\sqrt{\gamma}T^{\alpha}{}_{\mu}n_{\alpha}\xi^{\mu}$$

$$T_{\mu\nu} = F_{\mu}{}^{\lambda}F_{\lambda\nu} - \frac{1}{4}g_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$$

$$\partial_t\mathcal{E} = -P_{\text{EM}} - \dot{\mathcal{E}}_{\text{BH}} - L_{\text{diss}} + \dot{\mathcal{E}}_{\text{A}'}$$

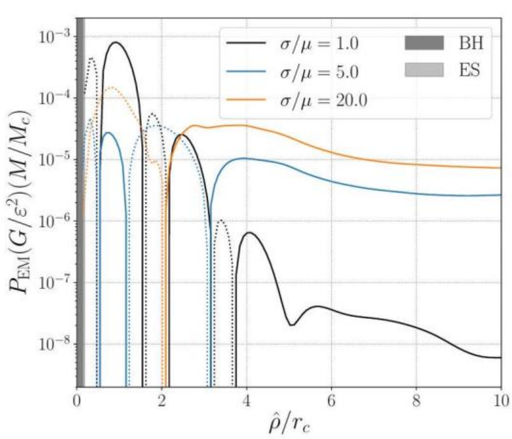
$$P_{\text{EM}} = -\oint_{S^2_{\hat{\rho}}} d\Omega_{\mu}T^{\mu}_{\nu}\xi^{\nu} \overset{\text{curved}}{=} \oint_{S^2_{\hat{\rho}}} d\Omega\hat{\boldsymbol{\rho}}\cdot(\boldsymbol{E}\times\boldsymbol{B})$$

$$\dot{\mathcal{E}}_{\text{BH}} = -\oint_{S^2_{\text{BH}}} d\Omega_{\mu}T^{\mu}{}_{\nu}\xi^{\nu}$$

$$L_{\text{diss}} = -\int_D d^3x\sqrt{-g}F^{\alpha\beta}\xi_{\alpha}I_{\beta} \overset{\text{curved}}{=} \int_D d^3x\boldsymbol{E}\cdot\boldsymbol{J}$$

$$\rho_{\text{diss}} = NF^{\alpha\beta}\xi_{\alpha}I_{\beta}$$

$$\dot{\mathcal{E}}_{\text{A}'} = -\varepsilon\mu^2\int_D d^3x\sqrt{-g}F^{\alpha\beta}\xi_{\alpha}A'_{\beta} \overset{\text{flat}}{=} \varepsilon\mu^2\int_D d^3x\boldsymbol{E}\cdot\boldsymbol{A}'$$



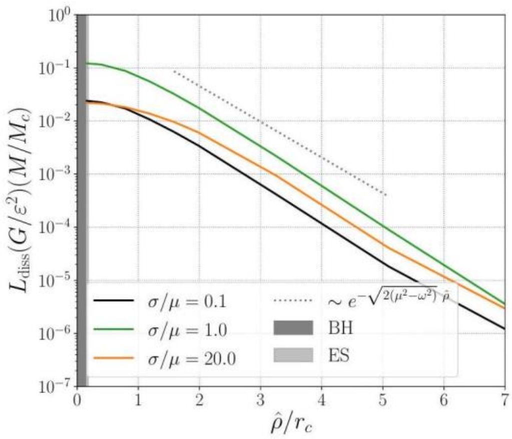


Figura 11. Coordenadas de un agujero negro cuántico.

$$L_{\text{diss}} = L_{\text{diss}}^{\text{bulk}} + L_{\text{diss}}^{\text{turb}}$$

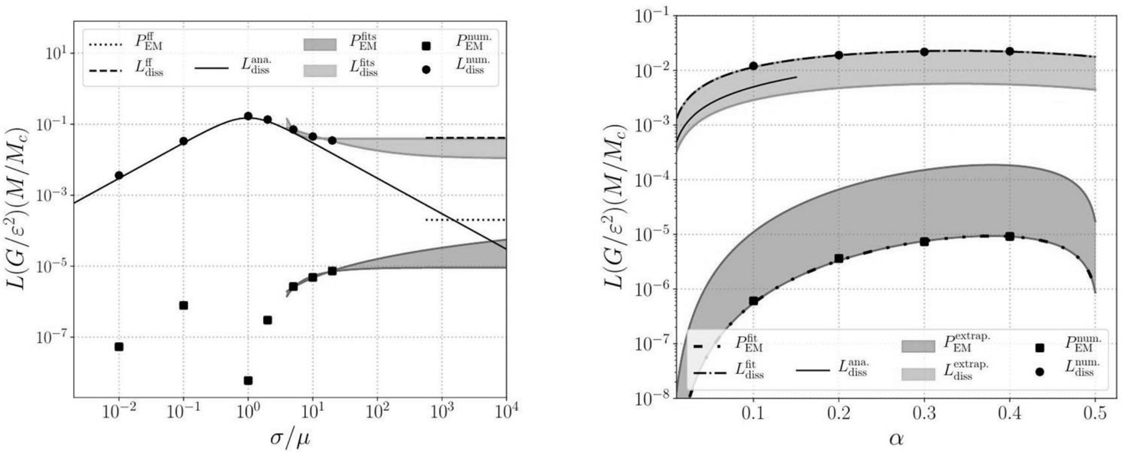


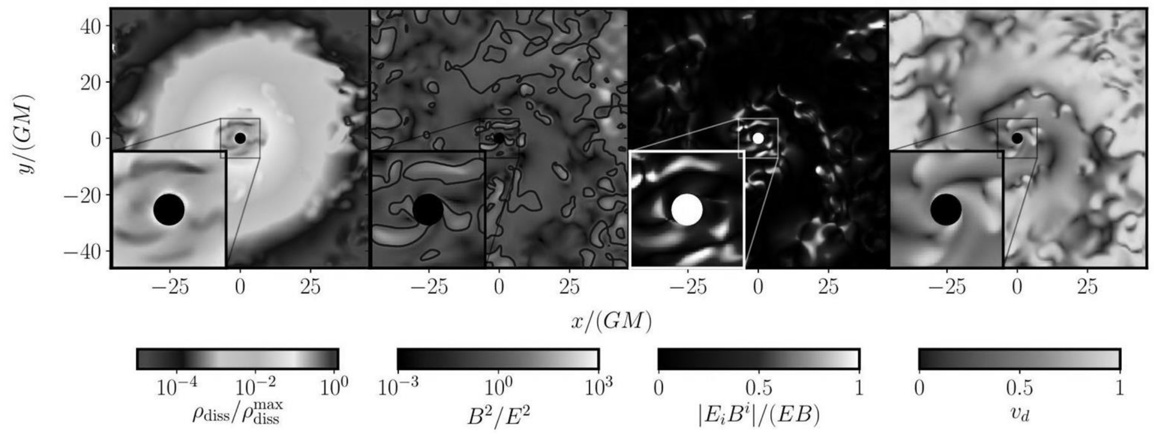
Figura 12. Disipación de un agujero negro cuántico.

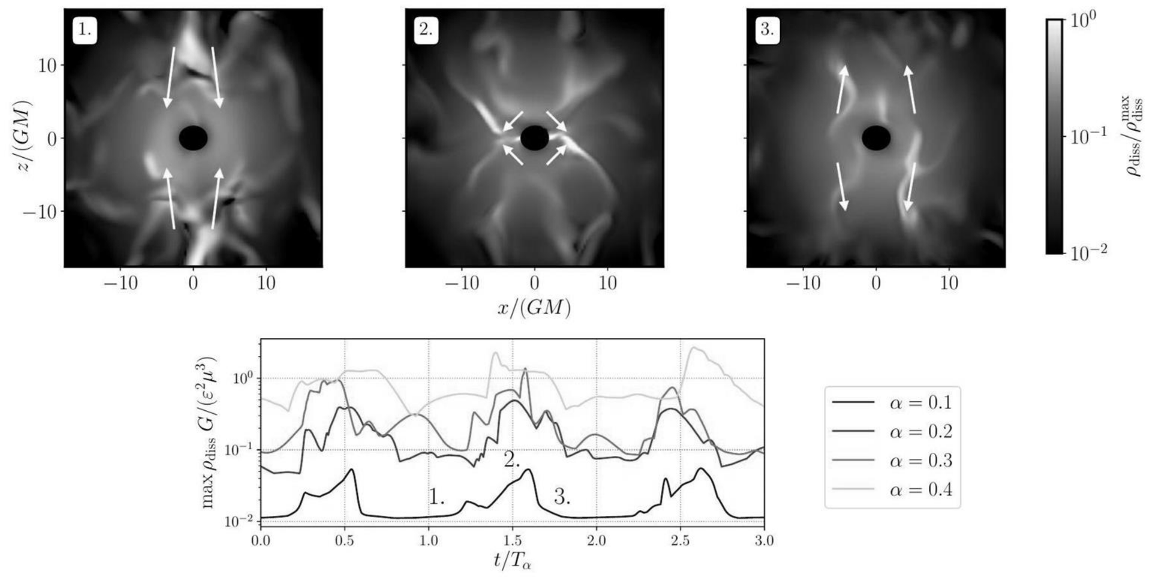
$$\mathbf{J} = \sigma\mathbf{E} = -\frac{e^{-r/r_c}\sqrt{M_c\mu}\,\alpha^{3/2}\varepsilon\sigma\omega^2}{2\sqrt{\pi}(\sigma^2+\omega^2)}\begin{pmatrix}\sigma + i\omega\\ -\omega + i\sigma\\ 0\end{pmatrix}e^{-i\omega t} + c.c.$$

$$L_{\rm diss}^{\rm bulk} = \frac{\sigma\alpha\varepsilon^2}{\mu(1+(\sigma/\mu)^2)}\frac{M_c}{GM}$$

$$L_{\rm diss}^{\rm fit} = \varepsilon^2 F(\alpha)\frac{M_c}{GM},$$
$$P_{\rm EM}^{\rm fit} = \varepsilon^2 G(\alpha)\frac{M_c}{GM},$$

$$F(\alpha) = 1.31\times10^{-1}\alpha - 1.88\times10^{-1}\alpha^2$$
$$G(\alpha) = 6.86\times10^{-4}\alpha^3 - 1.36\times10^{-3}\alpha^4$$





Figuras 13, 14 y 15. Densidad de un agujero negro cuántico.

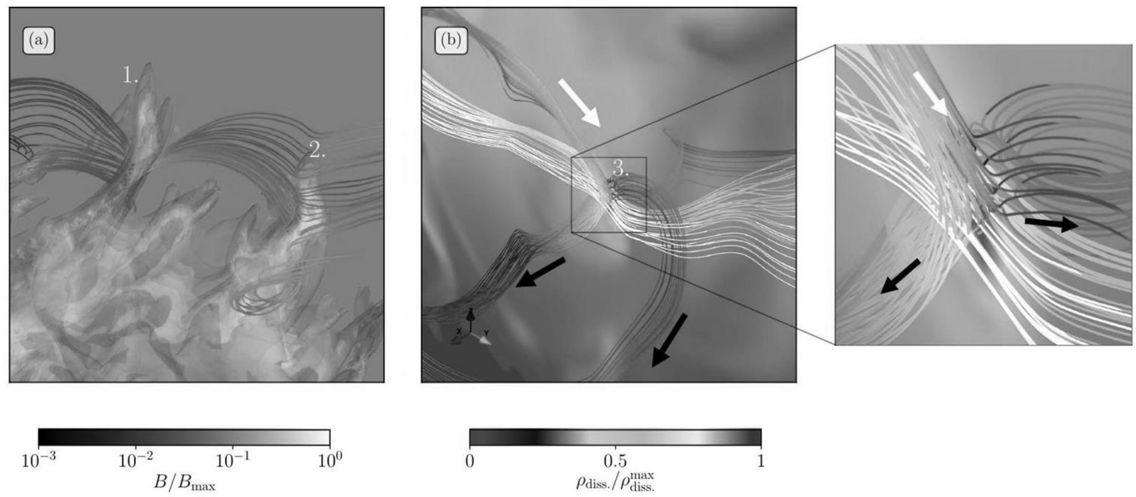


Figura 16. Comportamiento de la materia y la energía en un agujero negro cuántico.

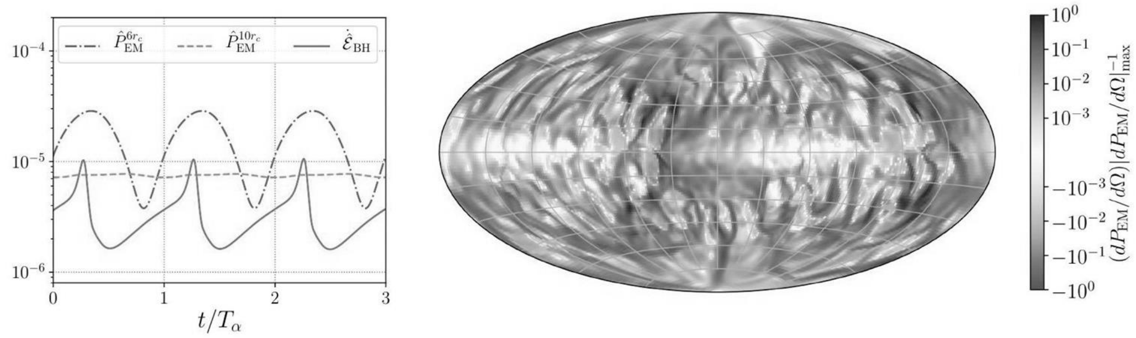


Figura 17. Radiación de un gujero negro cuántico.

$$\langle B^2\rangle^{1/2} = 2.5\times 10^8 \text{ Gauss } \left(\frac{\varepsilon}{10^{-7}}\right)\left(\frac{M_\odot}{M}\right)\left(\frac{\alpha}{0.1}\right)^{5/2}$$

$$\gamma_c \approx 2.2\times 10^7 \left(\frac{\varepsilon}{10^{-7}}\right)\left(\frac{\alpha}{0.1}\right)^{3/2}$$

$$\gamma_r = 3\times 10^3 \left(\frac{10^{-7}}{\varepsilon}\right)^{1/2}\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{0.1}{\alpha}\right)^{5/4}$$

$$\nu_{\text{peak}} = 12\text{keV}\left(\frac{\gamma}{10^2}\right)^2\left(\frac{\varepsilon}{10^{-7}}\right)\left(\frac{M_\odot}{M}\right)\left(\frac{\alpha}{0.1}\right)^{5/2},$$

$$t_{\text{growth}} \sim \ln\left(M_c/\mu\right)\tau_{\text{SR}} \approx 10^4 \text{ s}\left(\frac{M}{10M_\odot}\right)\left(\frac{0.7}{a_*}\right)\left(\frac{0.1}{\alpha}\right)^7$$

$$L_{\text{EM}} = \varepsilon^2 F(\alpha)\frac{M_c}{GM} \simeq \varepsilon^2\frac{\alpha^2\Delta a_*}{G} \simeq 4\times 10^{41}\text{erg/s}\left(\frac{\varepsilon}{10^{-7}}\right)^2\left(\frac{\alpha}{0.1}\right)^2\left(\frac{\Delta a_*}{0.1}\right)$$

$$\tau_{\text{GW}} \approx \frac{GM}{17\alpha^{11}\Delta a_*} \approx 10^6 \text{ s}\left(\frac{M}{10M_\odot}\right)\left(\frac{0.1}{\alpha}\right)^{11}\left(\frac{0.1}{\Delta a_*}\right)$$

$$\tau_{\text{EM}} \approx \frac{GM\ln 2}{\varepsilon^2 F(\alpha)} \approx 10^{11} \text{ s}\left(\frac{M}{10M_\odot}\right)\left(\frac{10^{-7}}{\varepsilon}\right)^2\left(\frac{10^{-2}}{F(0.1)}\right)$$

$$\frac{M_c(t)}{M_c(t_0)} = \begin{cases} \left[1+(t-t_0)/\tau_{\text{GW}}\right]^{-1} & \tau_{\text{GW}} \ll \tau_{\text{EM}} \\ e^{-(t-t_0)/\tau_{\text{EM}}\ln 2} & \tau_{\text{GW}} \gg \tau_{\text{EM}} \end{cases}$$

$$\tau_{\text{SR}} \simeq 0.98\tau_{\text{GW}}^{7/11} r_g^{4/11}\left[a_*\left(\frac{\Delta a_*}{0.5}\right)^{7/11}\right]$$

$$\dot{f}_{\text{int}} \simeq \frac{5}{8\pi}\alpha\mu^2 G P_{\text{GW}}$$

$$g^{\alpha\beta}\nabla_\alpha\nabla_\beta A'^\gamma = \mu^2 A'^\gamma$$

$$A'^\mu = B^{\mu\nu}\nabla_\nu Z, Z = R(r)S(\theta)e^{-i(\omega T - m\varphi)}$$

$$R_{\text{near}}(r) = \hat{r}^{i\kappa}\left(1+\hat{a}_1\hat{r}+\hat{a}_2\hat{r}^2+\cdots\right),$$

$$\tau = T + \frac{M^2\log\frac{r-r_+}{r-r_-}}{\sqrt{M^2-a^2}} + M\log\Delta$$
$$x = \sin\theta\left(r\cos\bar{\phi} - a\sin\bar{\phi}\right)$$
$$y = \sin\theta\left(a\cos\bar{\phi} + r\sin\bar{\phi}\right)$$
$$z = r\cos\theta$$

$$T = t - \frac{M^2 \log \frac{r - r_+}{r - r_-}}{\sqrt{M^2 - a^2}} - M \log \Delta$$

$$r = 2^{-1/2}\Big[-a^2 + x^2 + y^2 + z^2 + \sqrt{4a^2z^2 + (a^2 - x^2 - y^2 - z^2)^2}\Big]^{1/2}$$

$$\phi = \arctan\left[\frac{rx + ay}{-ax + ry}\right] - \frac{a \log \frac{r - r_+}{r - r_-}}{2\sqrt{M^2 - a^2}}, x > 0$$

$$\theta = \arccos \frac{z}{r}$$

$$L_D(f) \equiv \int_D d^3x \sqrt{\gamma} |f|$$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + \frac{2Mr^3}{r^4 + a^2M^2z^2}\left[dt + \frac{z}{r}dz + \frac{r(xdx + ydy)}{r^2 + a^2M^2} - \frac{aM(xdy - ydx)}{r^2 + a^2M^2}\right]^2$$

$$D_tE^i = NKE^i + \varepsilon^{ijk}D_j(NB_k) - NJ^i + N\varepsilon\mu^2\gamma^i{}_{\mu}A'^{\mu}$$

$$D_tB^i = NKB^i - \varepsilon^{ijk}D_j(NE_k)$$

$$D_iE^i = \rho_q + \varepsilon\mu^2 n_\mu A'^{\mu}$$

$$D_iB^i = 0$$

$$D_t\Psi = -N(D_iE^i - \varepsilon\mu^2 n_\mu A'^{\mu} - \rho_q) - N\kappa\Psi$$

$$D_t\Phi = -ND_iB^i - N\kappa\Phi$$

$$E^i \rightarrow E^j\left(\delta^i_j - \frac{B_jB^i}{B^2}\right)$$

$$E^i \rightarrow E^i\left\{1 - \hat{\theta}(\lambda) + \hat{\theta}(\lambda)\frac{B}{E}\right\}$$

$$J^a = \rho_q v^a_d + J^a_\perp$$

$$v^a_d = \frac{\varepsilon^{ajk}E_jB_k}{B^2}$$

$$J^a_{\perp,\mathrm{FF}} = \frac{B^a}{B^2}\left[2KB_iE^i - 2K_{ij}E^iB^j + B_i\epsilon^{ijk}D_jB_k - E_i\epsilon^{ijk}D_jE_k + \varepsilon\mu^2 B_i\gamma^i_\mu A'^{\mu}\right]$$

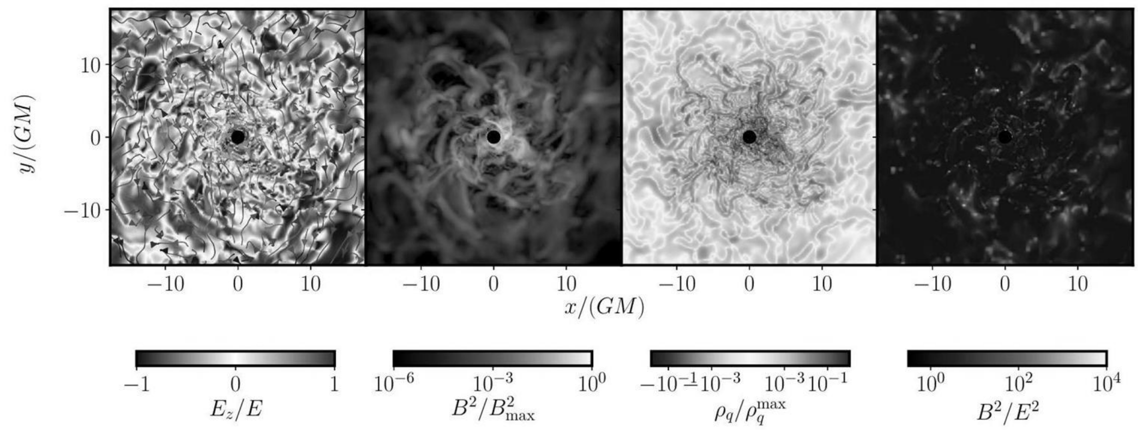


Figura 18. Radio y horizonte de eventos de un agujero negro cuántico.

$$v_d^i = \frac{\varepsilon^{ijk} E_j B_k}{B^2 + E_0^2}, E_0^2 = B_0^2 + E^2 - B^2,$$

$$B_0^2 = \frac{1}{2}\left[B^2 - E^2 + \sqrt{(B^2 - E^2)^2 + 4(E_i B^i)^2}\right].$$

$$J_{\perp,(B)}^a = \sigma E_0 \sqrt{\frac{B^2 + E_0^2}{B_0^2 + E_0^2}} \frac{E_0 E^a + B_0 B^a}{B^2 + E_0^2}$$

$$J_{\perp,(C)}^a = \frac{\sigma}{(\sigma + \kappa)}\left(J_{\perp,\mathrm{FF}}^a + \kappa E^i B_i \frac{B^a}{B^2}\right)$$

$$D_t(E_i B^i) = -\kappa N\left(E^i - \frac{1}{\sigma} J^i\right) B_i,$$

$$D_t \rho_q = N \rho_q K - D_i(N J^i) = N \rho_q K - D_i(N \rho_q v_d^i) = 0$$

$$\mathcal{I}_\rho = \int_D d^3 x \sqrt{\gamma} |\rho_q|$$

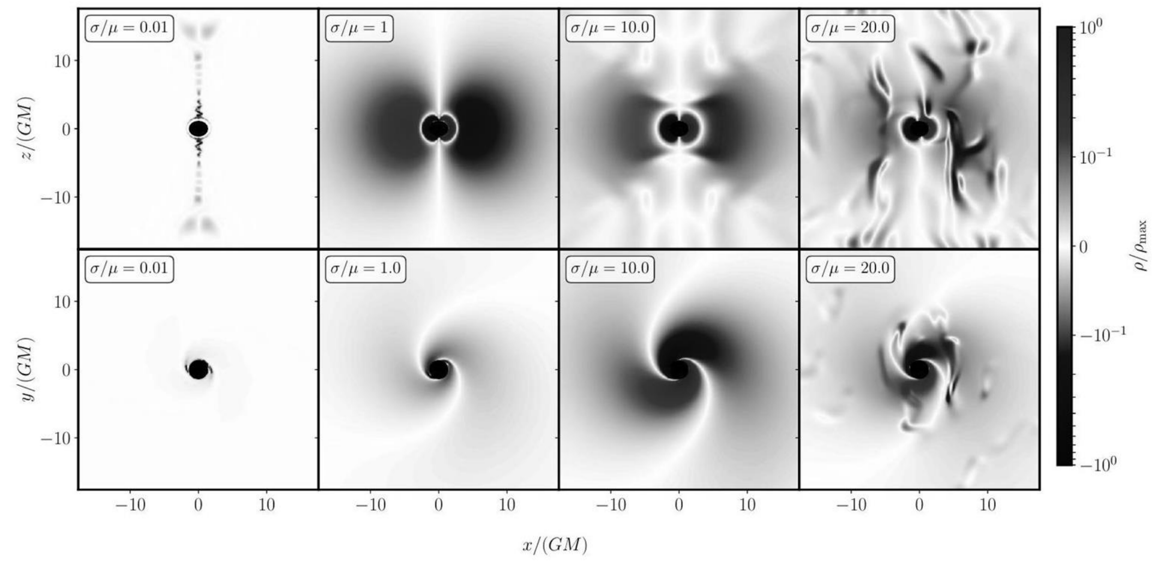
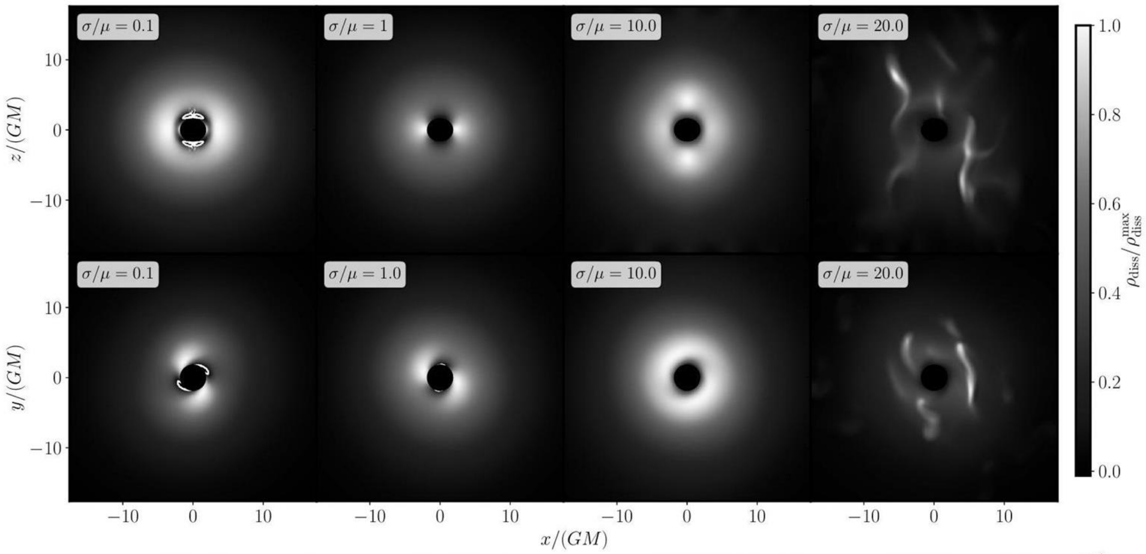


Figura 19. Fluctuaciones gravitacionales de un agujero negro cuántico.

$$\mathcal{I}_{E'} = \int_D d^3x\sqrt{\gamma}\varepsilon\left|D_i E'^i\right|$$



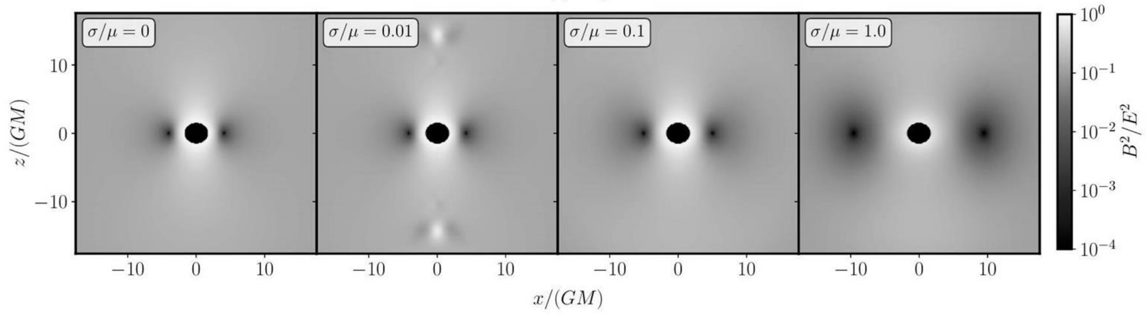


Figura 20. Simulación de un agujero negro cuántico.

$$\mathcal{L}_{\text{mass}} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{4}\mathcal{F}'_{\mu\nu}\mathcal{F}'^{\mu\nu} - \frac{\mu^2}{2}\mathcal{A}'_{\mu}\mathcal{A}'^{\mu} + I_{\mu}(\mathcal{A}^{\mu} + \varepsilon\mathcal{A}'^{\mu})$$

$$\mathcal{L}_{\text{inter}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu}$$

$$-\frac{\mu^2}{2}A'_{\mu}A'^{\mu} - \varepsilon\mu^2 A'_{\mu}A^{\mu} + I_{\mu}A^{\mu}$$

$$\nabla_{\alpha}\mathcal{F}^{\alpha\beta} = -I^{\beta}$$

$$\nabla_{\alpha}\mathcal{F}'^{\alpha\beta} = \mu^2\mathcal{A}'^{\beta} - \varepsilon I^{\beta}$$

$$\nabla_{\alpha}F^{\alpha\beta} = -I^{\beta} + \varepsilon\mu^2 A'^{\beta}$$

$$\nabla_{\alpha}F'^{\alpha\beta} = \mu^2 A'^{\beta} + \varepsilon\mu^2 A^{\beta}$$

$$\nabla_{\alpha}\mathcal{T}^{\alpha\beta} = -\mathcal{F}^{\beta\gamma}I_{\gamma}$$

$$\nabla_{\alpha}\mathcal{T}'^{\alpha\beta} = -\varepsilon\mathcal{F}'^{\beta\gamma}I_{\gamma}$$

$$\nabla_\alpha T^{\alpha\beta} = -F^{\beta\gamma}\left(I_\gamma - \varepsilon\mu^2 A'_\gamma\right)$$

$$\nabla_\alpha T'^{\alpha\beta} = \varepsilon\mu^2 F'^{\beta\gamma} A_\gamma$$

$$\nabla^\mu A'_\mu = 0, \qquad \nabla_\mu I^\mu = 0$$

Apéndice B.

Electrodinámica cuántica de una partícula supermasiva. Modelo matemático.

$$\frac{\mathrm{d}S_\alpha}{\mathrm{d}\tau} = \Gamma^\lambda_{\alpha\nu} S_\lambda \frac{\mathrm{d}x^\nu}{\mathrm{d}\tau}$$

$$\frac{\mathrm{d}\mathbf{S}}{\mathrm{d}t} = -\mathbf{S}\frac{\partial\phi}{\partial t} - 2\mathbf{v}\cdot\mathbf{S}\nabla\phi - \mathbf{S}(\mathbf{v}\cdot\nabla\phi) + \mathbf{v}(\mathbf{S}\cdot\nabla\phi) + \frac{1}{2}\mathbf{S}\times(\nabla\times\mathbf{S})$$

$$\mathbf{S}_1 = (1+\phi)\mathbf{S} - \frac{1}{2}\mathbf{v}(\mathbf{v}\cdot\mathbf{S}),$$

$$\frac{\mathrm{d}\mathbf{S}_1}{\mathrm{d}t} = \mathbf{\Omega}\times\mathbf{S}_1$$

$$\frac{\delta S}{S} \sim T v \frac{\phi}{R c^2}$$

$$\frac{\delta S}{S} \sim 4\times 10^{-3}\left(\frac{v}{2\times 10^{-3}}\right)^3 \frac{T}{13\times 10^9\mathrm{yr}}\frac{8\mathrm{kpc}}{R}$$

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{\sin\ \alpha}{2}F^{\mu\nu}X_{\mu\nu} + eJ^\mu_{\mathrm{EM}}A_\mu + \frac{m_X^2\cos^2\ \alpha}{2}X^\mu X_\mu$$

$$\mathcal{L} \supset -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{4}\tilde{X}_{\mu\nu}\tilde{X}^{\mu\nu} + \frac{e}{\cos\ \alpha}J^\mu_{\mathrm{EM}}\tilde{A}_\mu + \frac{m_X^2\cos^2\ \alpha}{2}\left(\tilde{X}^\mu\tilde{X}_\mu + 2\chi\tilde{X}_\mu\tilde{A}^\mu + \chi^2\tilde{A}^\mu\tilde{A}_\mu\right)$$

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + eJ^\mu_{\mathrm{EM}}A_\mu + \frac{m_X^2}{2}\left(X^\mu X_\mu + 2\chi X_\mu A^\mu\right)$$

$$-K^2 A^\nu = \chi m_X^2 X^\nu.$$

$$X_c^\mu(t,\mathbf{x}) = \sqrt{V}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}X^\mu(\mathbf{k})e^{-i(\omega t - \mathbf{kx} + \delta(\mathbf{k}))}.$$

$$\rho = \frac{1}{V}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\omega(\mathbf{k})^2}{2}|\mathbf{X}(\mathbf{k})|^2$$

$$\langle X_c^\mu(t)\rangle = \frac{1}{V}\int \mathrm{d}^3\mathbf{x}X_c^\mu(t,\mathbf{x}) = \frac{X^\mu(\mathbf{k}=0)}{\sqrt{V}}e^{-im_X t} \equiv X_0^\mu e^{-im_X t}$$

$$\rho = \rho \int d^3\mathbf{v} f_{\mathrm{lab}}(\mathbf{v})$$

$$f_{\mathrm{lab}}(\mathbf{v}) = \frac{m_X^3 \omega^2}{2(2\pi)^3 \rho} |\mathbf{X}(\mathbf{k})|^2$$

$$\rho = \frac{m_X^2}{2} \left\langle |\mathbf{X}_c(t)\right|^2 \Big\rangle$$

$$\rho \simeq \frac{m_X^2}{2} |\langle \mathbf{X}_c(t)\ |\rangle|^2 = \frac{m_X^2}{2} |\mathbf{X}_0|^2$$

$$|\mathbf{E}_0| = \left|\frac{\chi m_X}{\epsilon} \mathbf{x}_0\right|.$$

$$|\mathbf{E}_0| = \left|\frac{\chi m_X}{\epsilon} \mathbf{X}_0 \cos\ \theta\right|,$$

$$\nabla \times \mathbf{B}_X = \mathbf{J}_X$$

$$\nabla \times \mathbf{E}_X = -\frac{\partial \mathbf{B}_X}{\partial t}$$

$$P(t) = P_X \cos^2\ \theta(t)$$

$$\frac{S}{N} = \frac{P}{T_{\mathrm{sys}}} \sqrt{\frac{T}{\Delta \nu_{\mathrm{DP}}}}$$

$$\frac{S}{N} = 2\left(\sqrt{n_s + n_d} - \sqrt{n_d}\right)$$

$$n_s = \eta \int dt \frac{P(t)}{\omega}$$

$$\frac{1}{T} \int dt P(t) \equiv P_X \langle \cos^2\ \theta \rangle_T$$

$$T \gg \tau = \frac{2\pi}{m_X v^2} \simeq 400\mu\,\mathrm{s} \left(\frac{10\mu\mathrm{eV}}{m_X}\right)$$

$$\frac{S}{N} \simeq \frac{S_1 + S_2}{\sqrt{2N_1}}$$

$$\propto \frac{P_X}{T} \int_0^T dt \cos^2\ \theta(t) + \frac{P_X}{T} \int_{T_{\mathrm{wait}}}^{T_{\mathrm{wait}}+T} dt \cos^2\ \theta(t)$$

$$\int_0^T dt \cos^2\ \theta(t) \gg \int_{T_{\mathrm{wait}}}^{T_{\mathrm{wait}}+T} dt \cos^2\ \theta(t)$$

$$P = P_a + N$$

$$\frac{P_a}{\sigma_N} > \Phi^{-1}[0.95] = 1.64$$

$$P = P_X + N \equiv P_X^0 \langle \cos^2 \theta \rangle_T + N$$

$$\Phi(Z) = \mathbb{P}(z \leq Z) = \int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{Z-x} \mathrm{d}y f(x) f(y)$$

$$\Phi[0] \equiv \int_{-\infty}^{+\infty} \mathrm{d}P_X \int_{-\infty}^{0-P_X} \mathrm{d}N f(P_X) f(N) = 1 - 0.95$$

$$\int_0^1 \mathrm{d}\langle \cos^2 \theta \rangle_T \frac{f(\langle \cos^2 \theta \rangle_T)}{2} \left[1 + \mathrm{erf}\left(\frac{-P_X^0 \langle \cos^2 \theta \rangle_T}{\sqrt{2}\sigma_N}\right)\right] = 1 - 0.95$$

$$\langle \cos^2 \theta \rangle_T^{\mathrm{excl.}} = \frac{1.64\sigma_N}{P_X^0}$$

$$\int_0^1 \frac{f(\langle \cos^2 \theta \rangle_T)}{2} \left[1 + \mathrm{erf}\left(\frac{5\sigma_N - P_X^0 \langle \cos^2 \theta \rangle_T}{\sqrt{2}\sigma_N}\right)\right] \mathrm{d}\langle \cos^2 \theta \rangle_T = 1 - 0.95$$

$$\langle \cos^2 \theta \rangle_T^{\mathrm{disc.}} = \frac{(5 + 1.64)\sigma_N}{P_X^0}$$

$$\hat{\mathcal{Z}}(t) = \begin{pmatrix} \cos \lambda_{\mathrm{lab}} \cos \omega_\oplus t \\ \cos \lambda_{\mathrm{lab}} \sin \omega_\oplus t \\ \sin \lambda_{\mathrm{lab}} \end{pmatrix}$$

$$\hat{\mathbf{X}} = \begin{pmatrix} \sin \theta_X \cos \phi_X \\ \sin \theta_X \sin \phi_X \\ \cos \theta_X \end{pmatrix}$$

$$\cos^2 \theta(t) = (\hat{\mathbf{X}} \cdot \hat{\mathcal{Z}}(t))^2$$

$$\langle \cos^2 \theta(t) \rangle_T \equiv \frac{1}{T} \int_0^T \cos^2 \theta(t) \mathrm{d}t$$

$$\frac{1}{4\pi} \int \langle \cos^2 \theta(t) \rangle_T \,\mathrm{d}\cos \theta_X \,\mathrm{d}\phi = \frac{1}{3}$$

$$\langle \cos^2 \theta(t) \rangle_{T=\eta} = \frac{1}{8}(3 + \cos 2\theta_X - (1 + 3\cos 2\theta_X)\cos 2\lambda_{\mathrm{lab}})$$

$$\hat{\mathcal{W}}(t) = \begin{pmatrix} \sin \omega_\oplus t \\ -\cos \omega_\oplus t \\ 0 \end{pmatrix}$$

$$\hat{\mathcal{N}}(t) = \begin{pmatrix} \sin \lambda_{\text{lab}} \cos \omega_{\oplus} t \\ -\sin \lambda_{\text{lab}} \sin \omega_{\oplus} t \\ \cos \lambda_{\text{lab}} \end{pmatrix}$$

$$\langle \cos^2 \theta(t) \rangle_\eta = \begin{cases} \frac{1}{8}(3 + \cos 2\lambda_{\text{lab}} + (1 + 3\cos 2\lambda_{\text{lab}})\cos 2\theta_X) \\ \frac{\sin^2 \theta_X}{2} \\ \frac{1}{8}(3 + \cos 2\theta_X - (1 + 3\cos 2\theta_X)\cos 2\lambda_{\text{lab}}) \end{cases}$$

$$\lambda_{\text{lab}} = \frac{1}{2}\cos^{-1}\left(-\frac{1}{3}\right) \approx \pm 54.74^\circ \qquad \lambda_{\text{lab}} = \frac{1}{2}\cos^{-1}\left(\frac{1}{3}\right) \approx \pm 35.26^\circ$$

$$\cos \theta(t) = \sqrt{1 - (\hat{\mathbf{X}} \cdot \hat{\mathcal{N}}(t))^2},$$

$$\langle \cos^2 \theta(t) \rangle_\eta = \begin{cases} \frac{1}{8}(5 - \cos 2\lambda_{\text{lab}} - (1 + 3\cos 2\lambda_{\text{lab}})\cos 2\theta_X) \\ \frac{1}{4}(3 + \cos 2\theta_X) \\ \frac{1}{8}(5 + \cos 2\theta_X + (3\cos 2\lambda_{\text{lab}} - 1)\cos 2\theta_X) \end{cases}$$

$$\lambda_{\text{lab}} = \frac{1}{2}\cos^{-1}\left(-\frac{1}{3}\right) \approx \pm 54.74^\circ$$

$$\lambda_{\text{lab}} = \frac{1}{2}\cos^{-1}\left(\frac{1}{3}\right) \approx \pm 35.26^\circ$$

$$P_{\text{cav}} = \kappa \mathcal{G}_X V Q \rho_{\text{DM}} \chi^2 m_X$$
$$P_{\text{cav}} = \kappa \mathcal{G}_a V \frac{Q}{m_a} \rho_{\text{DM}} g_{a\gamma}^2 B^2$$

$$\mathcal{G}_X = \frac{\left(\int dV \mathbf{E}_\alpha \cdot \hat{\mathbf{X}}\right)^2}{V \frac{1}{2} \int dV \epsilon(\mathbf{x}) \mathbf{E}_\alpha^2 + \mathbf{B}_\alpha^2}$$
$$\mathcal{G}_a = \frac{\left(\int dV \mathbf{E}_\alpha \cdot \mathbf{B}\right)^2}{V B^2 \frac{1}{2} \int dV \epsilon(\mathbf{x}) \mathbf{E}_\alpha^2 + \mathbf{B}_\alpha^2}$$

$$\chi = g_{a\gamma} \frac{B}{m_X |\cos \theta|}$$

$$P_X \langle \cos^2 \theta \rangle_T^{\text{disc.}} > (3 + \Phi^{-1}[0.5])\sigma_N \approx 3\sigma_N,$$

$$\xi = \int_0^1 \frac{f(\langle \cos^2 \theta \rangle_T)}{2} \left[1 + \text{erf}\left(\frac{3\sigma_N - P_X \langle \cos^2 \theta \rangle_T}{\sqrt{2}\sigma_N}\right)\right] d\langle \cos^2 \theta \rangle_T$$

$$\Lambda(\theta_X, \phi_X) = 2\left[\ln\ \mathcal{L}\left(d \mid \mathcal{M}_t, \{\hat{P}_X, \theta_X, \phi_X, \hat{P}_N\}\right) - \ln\ \mathcal{L}\left(d \mid \mathcal{M}_0, \left\{\hat{\hat{P}}_X, \hat{\hat{P}}_N\right\}\right)\right]$$

$$\ln\ \mathcal{L}\left(P^{\text{obs}} \mid \mathcal{M}_t, \{P_X^0, \theta_X, \phi_X, P_N\}\right) = -\frac{1}{2\sigma_N^2}\sum_{j=1}^{N_t}\left[P_j^{\text{obs}} - P_X\langle c(\theta_X, \phi_X)\rangle_j - \frac{\Delta t}{T}P_N\right]^2$$

$$\langle c(\theta_X, \phi_X)\rangle_j \equiv \langle \cos^2\ \theta\rangle_j = \frac{1}{\Delta t}\int_{t_j-\Delta t/2}^{t_j+\Delta t/2} \cos^2\ \theta\ \mathrm{d}t$$

$$\ln\ \mathcal{L}\left(P^{\text{obs}} \mid \mathcal{M}_t, \{P_X, P_N\}\right) = -\frac{1}{2\sigma_N^2}\left(P^{\text{obs}} - P_X - P_N\right)^2$$

$$P_j^{\text{obs}} = P_j^{\text{Asi}} \equiv P_X\langle c(\theta_X, \phi_X)\rangle_j + \frac{\Delta t}{T}P_N$$

$$\mathrm{TS}_{\text{mod}}(\theta_X, \phi_X) = \frac{1}{\sigma_N^2}\left(\sum_j P_X\langle c(\theta_X, \phi_X)\rangle_j - \hat{\hat{P}}_X\right)^2$$

$$\mathrm{TS}_{\text{mod}}(\theta_X, \phi_X) = \left(\frac{P_X}{\sigma_N}\right)^2\left(\sum_j \langle c\rangle_j - \langle c\rangle_T\right)^2 = \left(\frac{P_X}{\sigma_N}\right)^2\left(\frac{1}{\Delta t}\int_0^T c\ \mathrm{d}t - \langle c\rangle_T\right)^2$$

$$= \left(\frac{T\langle c\rangle_T P_X}{\sigma_N}\right)^2\left(\frac{1}{\Delta t} - \frac{1}{T}\right)^2 \approx \left(\frac{T\langle c\rangle_T P_X}{\Delta t\sigma_N}\right)^2$$

$$\mathrm{TS}_{\text{mod}} \approx \left(\frac{3T\langle c\rangle_T}{\Delta t\langle \cos^2\ \theta\rangle_{\Delta t}^{\text{disc.}}}\right)^2$$

$$\Lambda(\theta_X, \phi_X) = \left(\frac{3}{\langle \cos^2\ \theta\rangle_{\Delta t}^{\text{disc.}}}\right)^2\left[\left(T\langle c_{\text{true}}\rangle_T\left(\frac{1}{\Delta t} - \frac{1}{T}\right)\right)^2 - \frac{1}{\Delta t}\int_0^T (c_{\text{true}} - c)^2\ \mathrm{d}t\right]$$

$$\begin{pmatrix}\sin\ \theta_X \cos\ \phi_X \\ \sin\ \theta_X \sin\ \phi_X \\ \cos\ \theta_X\end{pmatrix} = R_{\text{gal}}\begin{pmatrix}\cos\ l \cos\ b \\ \sin\ l \cos\ b \\ \sin\ b\end{pmatrix}$$

$$R_{\text{gal}} = \begin{pmatrix}-0.05487556 & +0.49410943 & -0.86766615 \\ -0.87343709 & -0.44482963 & -0.19807637 \\ -0.48383502 & +0.74698225 & +0.45598378\end{pmatrix}$$

Apéndice C.

Modelo de Multidimensiones y Supermembranas a propósito de la deformación dfel espacio – tiempo cuántico por interacción de una partícula supermasiva. Modelo Sigma.

$$S_{\text{sigma-model}} = -T\int d^2\sigma \frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}$$

$$T_{ab} = \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu\right) g_{\mu\nu}$$

$$\frac{1}{\sqrt{-\gamma}}\partial_a\left(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda}$$

$$\Phi(\varphi) = \frac{1}{2}\epsilon_{ij}\epsilon^{ab}\partial_a\varphi^i\partial_b\varphi^j$$

$$S = -\int d^2\sigma\Phi(\varphi)\left(\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{\epsilon^{ab}}{2\sqrt{-\gamma}}F_{ab}(A)\right)$$

$$\gamma_{ab} \to J\gamma_{ab}$$

$$\varphi^i \to \varphi'^i = \varphi'^i(\varphi^i)$$

$$\Phi \to \Phi' = J\Phi$$

$$\epsilon^{ab}\partial_b\varphi^i\partial_a\left(\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd}\right) = 0$$

$$\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = M = \text{ const.}$$

$$T_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{1}{2}\gamma_{ab}\frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = 0$$

$$\epsilon^{ab}\partial_b\left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\right) = 0$$

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = T$$

$$S_{\text{sigma-model}} = -T\int d^d\sigma \frac{1}{2}\sqrt{-\gamma}\left(\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} + 2\Lambda\right)$$

$$T_{ab} = \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu\right) g_{\mu\nu} - \gamma_{ab}\Lambda$$

$$\frac{1}{\sqrt{-\gamma}}\partial_a\left(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda}$$

$$\Phi(\varphi) = \epsilon_{ijk\ldots m}\epsilon^{abc\ldots.d}\partial_a\varphi^i\partial_b\varphi^j \ldots\ldots \partial_d\varphi^m$$

$$S=-\int d^d\sigma\Phi(\varphi)\left(\frac{1}{2}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu g_{\mu\nu}-\frac{\epsilon^{abcd\ldots}}{2\sqrt{-\gamma}}F_{abcd\ldots}(A)\right)$$

$$\gamma_{ab}\rightarrow J\gamma_{ab}$$
$$\varphi^i\rightarrow\varphi'^i=\lambda^{ij}(\varphi^j)$$

$$\Phi\rightarrow\Phi'=J\Phi$$

$$A_{bcd\ldots\ldots}\rightarrow J^{\frac{d-2}{2}}A_{bcd\ldots\ldots}$$

$$K_b^a\partial_a\left(\gamma^{cd}\partial_c X^\mu\partial_d X^\nu g_{\mu\nu}-\frac{\epsilon^{cdef}\ldots}{\sqrt{-\gamma}}F_{cdef}\ldots\right)$$

$$\gamma^{cd}\partial_c X^\mu\partial_d X^\nu g_{\mu\nu}-\frac{\epsilon^{cdef}\ldots}{\sqrt{-\gamma}}F_{cdef\ldots}=M=\text{ const.}$$

$$T_{ab}=\partial_a X^\mu\partial_b X^\nu g_{\mu\nu}-\frac{1}{2}\gamma_{ab}\frac{\epsilon^{cdef}\ldots}{\sqrt{-\gamma}}F_{cdef\ldots}=0$$

$$\gamma_{ab}=\frac{1-p}{M}\partial_a X^\mu\partial_b X^\nu g_{\mu\nu}$$

$$\epsilon^{abc..d}\partial_d\left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\right)=0$$

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}}=T$$

$$S_{\text{current}}=\int d^{p+1}\sigma A_{a_2\ldots a_{p+1}}j^{a_2\ldots a_{p+1}}$$

$$\epsilon^{a_1\ldots a_{p+1}}\partial_{a_1}\left(\frac{\Phi}{\sqrt{-\gamma}}\right)=j^{a_2\ldots a_{p+1}}$$

$$j^{a_1\ldots a_{p+1}}=e\partial_\mu\phi\frac{\partial X^\mu}{\partial\sigma^a}\epsilon^{aa_2\ldots a_{p+1}}\equiv e\partial_a\phi\epsilon^{aa_2\ldots a_{p+1}}$$

$$T=\frac{\Phi}{\sqrt{-\gamma}}=e\phi+T_i$$

$$g_{\mu\nu}\rightarrow\omega g_{\mu\nu}$$

$$A_a\rightarrow\omega A_a$$
$$\Phi(\varphi)\rightarrow\omega^{-1}\Phi(\varphi)$$

$$\phi\rightarrow\omega^{-1}\phi$$

$$\Phi=\sqrt{-\gamma}(e\phi+T_i)$$

$$S_i=-\int d^2\sigma(e\phi+T_i)\frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu g_{\mu\nu}+\int d^2\sigma A_{\mu\nu}\epsilon^{ab}\partial_a X^\mu\partial_b X^\nu+\int d^2\sigma\sqrt{-\gamma}\varphi R$$

$$g_{\mu\nu}^{i} = (e\phi + T_i)g_{\mu\nu}$$

$$R_{\mu\nu}(g_{\alpha\beta}^{1}) = 0$$

$$R_{\mu\nu}(g_{\alpha\beta}^{2}) = 0$$

$$ds_2^2 = -(1 - 2GM/r)dt^2 + \frac{dr^2}{1 - 2GM/r} + r^2 d\Omega^2 + dy_4^2 + dy_5^2 + \cdots \ldots + dy_{25}^2$$

$$ds_2^2 = -dt^2 + t^{2p_1}dx_1^2 + t^{2p_2}dx_2^2 + \cdots \ldots + t^{2p_{25}}dx_{25}^2$$

$$e\phi + T_1 = \Omega^2(e\phi + T_2)$$

$$e\phi = \frac{\Omega^2 T_2 - T_1}{1 - \Omega^2}$$

$$e\phi + T_1 = \frac{\Omega^2(T_2 - T_1)}{1 - \Omega^2}$$

$$e\phi + T_2 = \frac{(T_2 - T_1)}{1 - \Omega^2}$$

$$ds^2 = -dt^2 + t^2(d\chi^2 + \sinh^2 \chi d\Omega_2^2)$$

$$ds_2^2 = -dt^2 + t^2(d\chi^2 + \sinh^2 \chi d\Omega_{D-2}^2)$$

$$ds_1^2 = \frac{\sigma}{t^4}\left(-dt^2 + t^2(d\chi^2 + \sinh^2 \chi d\Omega_{D-2}^2)\right)$$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \left(\frac{1 - \Omega^2}{T_2 - T_1}\right)\left(-dt^2 + t^2(d\chi^2 + \sinh^2 \chi d\Omega_{D-2}^2)\right)$$

$$dT = \sqrt{\frac{1 + \frac{K}{t^4}}{T_1 - T_2}}dt$$

$$ds^2 = l^2 dt^2 - l^2 \cosh^2 t \left(\frac{dr^2}{1 - r^2} + r^2 d\Omega_2^2\right)\Bigg) - dl^2$$

$$ds^2 = \frac{\Lambda l^2}{3}\left(dt^2\left(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}\right) - \frac{dr^2}{1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}} - r^2 d\Omega_2^2\right) - dl^2$$

$$ds_2^2 = l^2 \bar{g}_{\mu\nu}(x)dx^\mu dx^\nu - dl^2$$

$$ds_1^2 = \sigma l^{-2}\bar{g}_{\mu\nu}(x)dx^\mu dx^\nu - \sigma\frac{dl^2}{l^4} = \sigma l^{-4} ds_2^2$$

$$ds^2 = \left(\frac{1 - \Omega^2}{T_2 - T_1}\right)(l^2 \bar{g}_{\mu\nu}(x)dx^\mu dx^\nu - dl^2)$$

$$dL = \sqrt{\frac{1+\frac{K}{l^4}}{T_1 - T_2}}dl$$

$$ds_1^2 = \eta_{\alpha\beta}dx^\alpha dx^\beta$$

$$ds_2^2 = \Omega(x)^2\eta_{\alpha\beta}dx^\alpha dx^\beta$$

$$x''^\mu = \frac{(x^\mu + a^\mu x^2)}{(1 + 2a_\nu x^\nu + a^2x^2)}$$

$$g_{\alpha\beta}^2 = \Omega^2\eta_{\alpha\beta} = \frac{1}{(1 + 2a_\mu x^\mu + a^2x^2)^2}\eta_{\alpha\beta}$$

$$e\phi + T_1 = \frac{(T_2 - T_1)(1 + 2a_\mu x^\mu + a^2x^2)^2}{(1 + 2a_\mu x^\mu + a^2x^2)^2 - 1} = \frac{(T_2 - T_1)(1 + 2a_\mu x^\mu + a^2x^2)^2}{(2a_\mu x^\mu + a^2x^2)(2 + 2a_\mu x^\mu + a^2x^2)}$$

$$e\phi + T_2 = \frac{(T_2 - T_1)}{(1 + 2a_\mu x^\mu + a^2x^2)^2 - 1} = \frac{(T_2 - T_1)}{(2a_\mu x^\mu + a^2x^2)(2 + 2a_\mu x^\mu + a^2x^2)}$$

$$e\phi + T_1 = e\phi + T_2 = \frac{K}{4t}$$

$$g_{\mu\nu} = \frac{1}{(e\phi + T_1)}g_{\mu\nu}^1 = \frac{4t}{K}\eta_{\mu\nu}$$

$$(T_2 - T_1)\Delta t \approx \text{ constant}$$

$$2a_\mu x^\mu + a^2x^2 = 0$$

$$2 + 2a_\mu x^\mu + a^2x^2 = 0$$

$$(2a_\mu x^\mu + a^2x^2)(2 + 2a_\mu x^\mu + a^2x^2) = (2At + A^2(t^2 - x^2))(2 + 2At + A^2(t^2 - x^2))$$

$$x_1^2 + x_2^2 + x_3^2 \ldots\ldots + x_{D-1}^2 - \left(t + \frac{1}{A}\right)^2 = -\frac{1}{A^2}$$

$$A(x_1^2 + x_2^2 + x_3^2 \ldots\ldots + x_{D-1}^2) - At^2 - 2t = 0$$

$$x_1^2 + x_2^2 + x_3^2 \ldots\ldots + x_{D-1}^2 - \left(t + \frac{1}{A}\right)^2 = \frac{1}{A^2}$$

$$\sqrt{\frac{1}{A^2} + \left(t + \frac{1}{A}\right)^2} - \sqrt{-\frac{1}{A^2} + \left(t + \frac{1}{A}\right)^2} \to \frac{1}{tA^2} \to 0$$

$$\pi T = (\kappa^2)^{-\frac{1}{d-2}}$$

$$\kappa^2 = 8\pi G = \frac{1}{M_P^2}$$

$$S = \int d^4x \left(\sqrt{-g}\left(-\frac{\epsilon}{2}\phi^2 R + X - V(\phi)\right)\right.$$

$$V(\phi) = \frac{1}{8}\lambda(\phi^2 - v^2)^2$$

$$\Phi(\varphi) \to \omega^{-1}\Phi(\varphi)$$

$$\Delta\phi/M_P < O(1)$$

$$M_P \frac{dV/d\phi}{V} > O(1)$$

$$-M_P^2 \frac{d^2V/d\phi^2}{V} > O(1)$$

$$e\phi + T_1 = \Omega_{12}^2(e\phi + T_2)$$

$$e\phi = \frac{\Omega_{12}^2 T_2 - T_1}{1 - \Omega_{12}^2}$$

$$e\phi + T_1 = \frac{\Omega_{12}^2(T_2 - T_1)}{1 - \Omega_{12}^2}$$

$$e\phi + T_2 = \frac{(T_2 - T_1)}{1 - \Omega_{12}^2}$$

$$e\phi + T_2 = \Omega_{23}^2(e\phi + T_3)$$

$$e\phi = \frac{\Omega_{23}^2 T_3 - T_2}{1 - \Omega_{23}^2}$$

$$\Omega_{23} = \frac{T_1 + \Omega_{12}^2(T_3 - T_2)}{\Omega_{12}^2(T_3 - T_2) - T_3}$$

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Anexo A.

Modelo matemático para calcular agujeros negros cuánticos en campos cuánticos relativistas o curvos con aproximaciones en supergravedad cuántica.

$$S_{\mathrm{EH}}[g_{\mu\nu}] = \frac{1}{16\pi G_N}\int_x \sqrt{g}(R - 2\Lambda),$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{Z_h G}h_{\mu\nu}$$

$$Z_h = Z_h(\Delta), G = G(\Delta)$$

$$\Delta = -g^{\mu\nu}\nabla_\mu\nabla_\nu$$

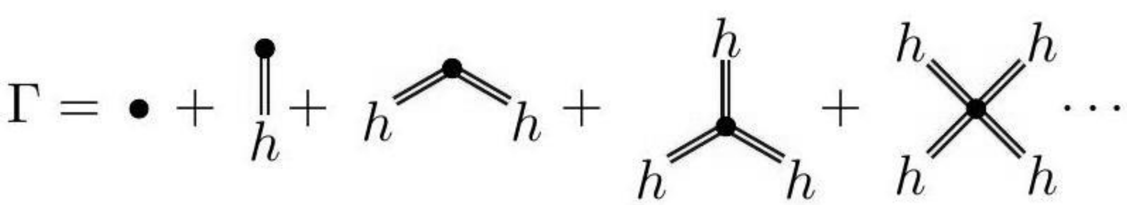
$$g_{\mu\nu} = \eta_{\mu\nu} = \delta_{\mu\nu}$$

$$\Gamma[g] = \Gamma[g, h = 0],$$

$$\Gamma[g_{\mu\nu}] = \frac{1}{16\pi}\int_x \sqrt{g}[\mathcal{R}(\Delta, R) + R f_{R^2}(\Delta) R + R_{\mu\nu} f_{R^2_{\mu\nu}}(\Delta) R^{\mu\nu}]$$

$$\int_x \sqrt{g}\frac{1}{G(\Delta)}R \simeq \int_x \sqrt{g}\frac{1}{G(0)}R$$

$$\mathrm{IR}: \mathcal{R}(\Delta, R) \overset{G_N\Delta\to 0}{\to} \frac{1}{G_N}R,$$
$$\mathrm{UV}: \mathcal{R}(\Delta, R) \overset{G_N\Delta\to\infty}{\to} 0,$$



$$\Gamma^{(n)}[g] = \Gamma^{(n,0)}[g, 0] \approx (Z_h G)^{-n/2}\Gamma^{(0,n)}[g, 0]$$

$$\Gamma^{(n,m)}[\bar{g},h]=\frac{\delta^{n+m}\Gamma[\bar{g},h]}{\delta\bar{g}^{n}\delta h^{m}}$$

$$\Gamma_{\mathrm{tt}}^{(2,0)}(p)=\frac{1}{16\pi G_N}Z_g(p)p^2\Pi_{\mathrm{tt}}$$
$$\Gamma_{\mathrm{tt}}^{(0,2)}(p)=\frac{1}{16\pi}Z_h(p)p^2\Pi_{\mathrm{tt}}$$

$$G(p)=\frac{G_N}{Z_g(p)}$$

$$\Gamma_{\mathrm{tt}}^{(n)}(\boldsymbol{p})=\gamma_g^{(n)}(p)\mathcal{T}_{R,\mathrm{tt}}^{(n)}(\boldsymbol{p})$$

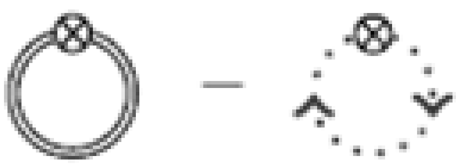
$$\Gamma_{\mathrm{tt}}^{(0,n)}(\boldsymbol{p})=\prod_{i=1}^{n}\sqrt{Z_h(p_i)G(p_i)}\gamma_g^{(n)}(p)\mathcal{T}_{R,\mathrm{tt}}^{(n)}(\boldsymbol{p})$$

$$\gamma_h^{(n)}(\boldsymbol{p})=\prod_{i=1}^{n}\sqrt{G(p_i)}\gamma_g^{(n)}(\boldsymbol{p})$$

$$\bar{\Gamma}^{(0,n)}(\boldsymbol{p})=\frac{\Gamma^{(0,n)}(\boldsymbol{p})}{\prod_{i=1}^{n}\sqrt{Z_h(p_i)}}$$

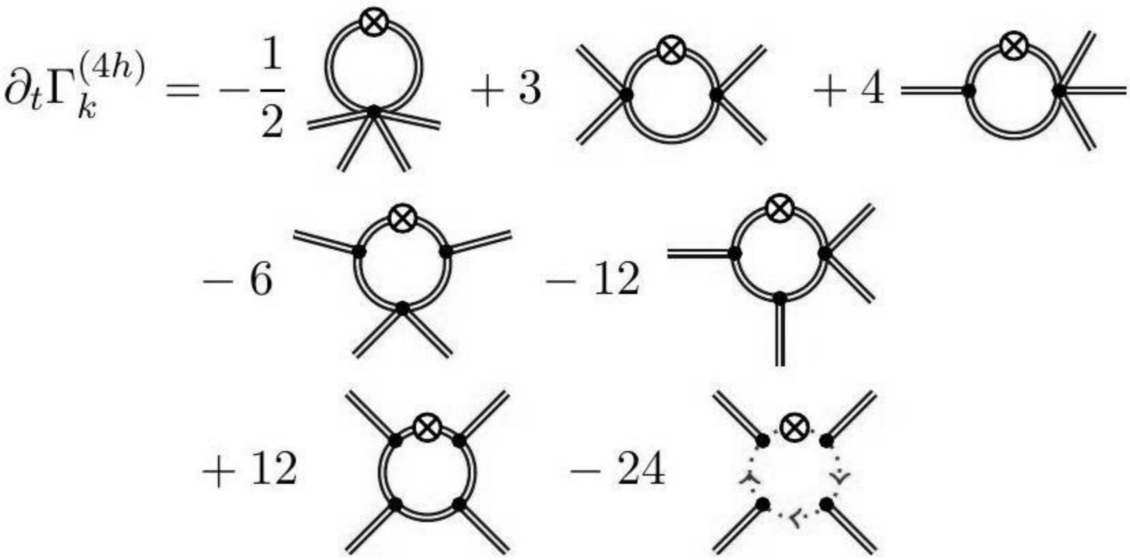
$$G(p)=\left[\gamma_h^{(3)}(p)\right]^2=\left[\gamma_g^{(3)}(p)\right]^{-1}$$

$$\bar{g}_{\mu\nu}=\delta_{\mu\nu},$$

$$\partial_t\Gamma_k[\bar{g},h]=\frac{1}{2}$$


$$\partial_t\Gamma_k[\bar{g},h]=\frac{1}{2}\mathrm{Tr}\mathcal{G}_k[\bar{g},h]\partial_t\mathcal{R}_k$$

$$\mathcal{G}_k[\bar{g},h] = \frac{1}{\Gamma_k^{(0,2)}[\bar{g},h] + \mathcal{R}_k},$$



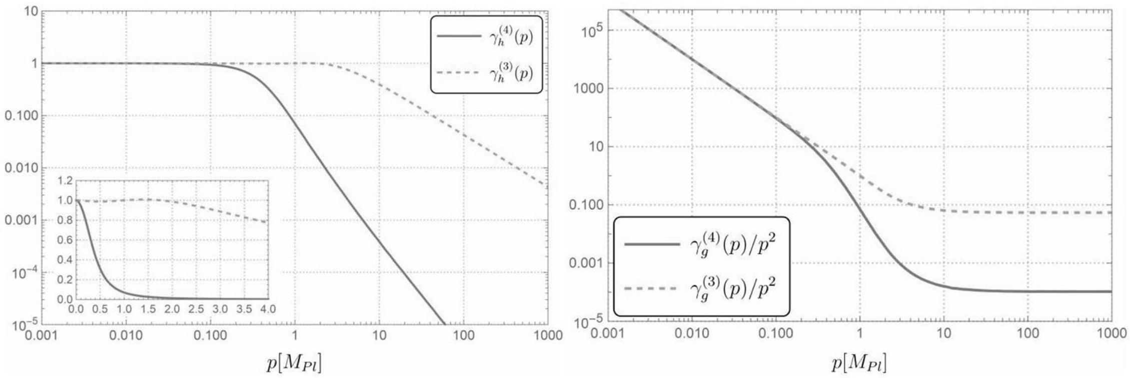


Figura 1. Fluctuaciones de materia y energía de un agujero negro cuántico causado por una partícula supermasiva.

$$\Gamma_{\mathrm{tt}}^{(n)} \circ \mathcal{T}_{R,\mathrm{tt}}^{(n)}/p^2 = C_R^{(n)}\gamma_g^{(n)}p^2$$

$$C_R^{(n)}\gamma_g^{(n)}p^2 = C_{\mathcal{R}}^{(n)}g_{\mathcal{R}}p^2 + C_{R^2}^{(n)}f_{R^2}p^4 + C_{R_{\mu\nu}^2}^{(n)}f_{R_{\mu\nu}^2}p^4$$

$$f_{R_{\mu\nu}^2}(p^2) = \frac{1}{C_{R_{\mu\nu}^2}^{(3)}}\frac{C_R^{(3)}\gamma_g^{(3)} - C_{\mathcal{R}}^{(3)}g_{\mathcal{R}}}{p^2}$$

$$f_{R^2}(p^2) = \frac{1}{C_{R^2}^{(4)}}\frac{C_R^{(4)}\gamma_g^{(4)} - C_{\mathcal{R}}^{(4)}g_{\mathcal{R}} - C_{R_{\mu\nu}^2}^{(4)}f_{R_{\mu\nu}^2}p^2}{p^2}$$

$$\mathcal{R}(\Delta, R) = R\frac{\gamma_g^{(3)}(\Delta) - \bar{\gamma}_3\Delta}{\Delta + R}R$$

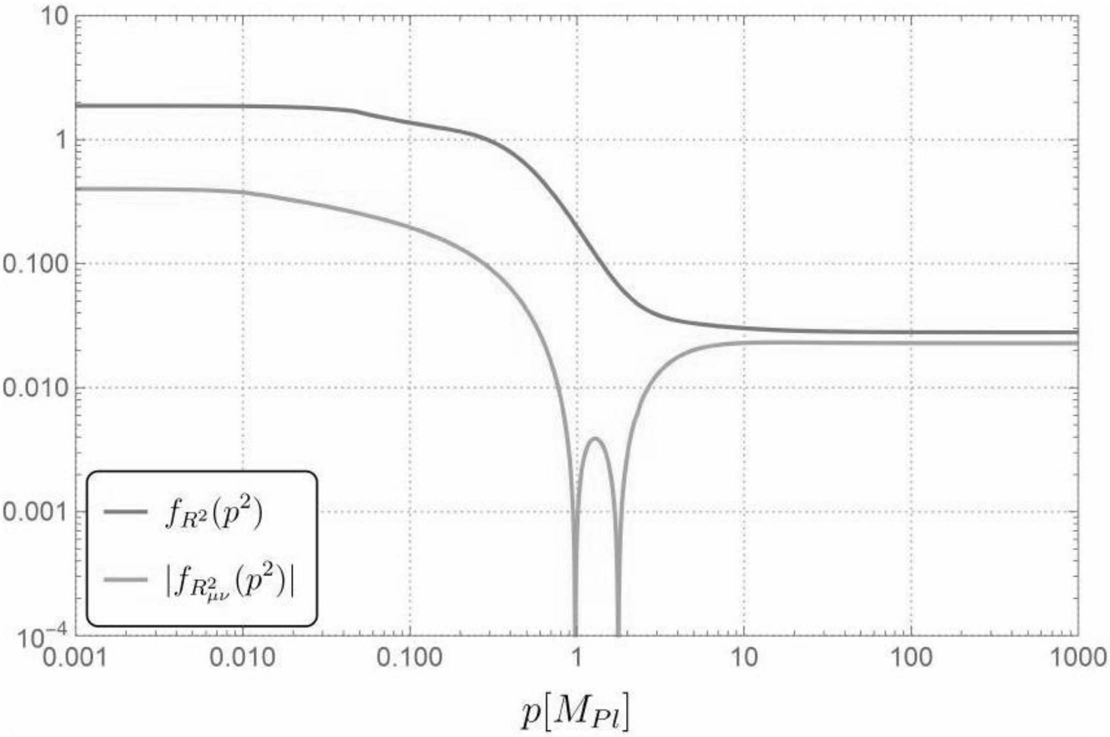


Figura 2. Rangos de momentum del agujero negro cuántico.

$$f_{R_{\mu\nu}^2}(\Delta) \simeq \tilde{g}_{R_{\mu\nu}^2} + \frac{g_{R_{\mu\nu}^2} - \tilde{g}_{R_{\mu\nu}^2}}{1 + p_0^{-2}\Delta}$$

$$f_{R^2}(\Delta) \simeq \tilde{g}_{R^2} + \frac{g_{R^2} - \tilde{g}_{R^2}}{1 + p_1^{-2}\Delta}$$

$$f_{R_{\mu\nu}^2}(\Delta) \simeq g_{R_{\mu\nu}^2} + c_1\Delta$$

$$f_{R^2}(\Delta) \simeq g_{R^2} + c_2\Delta$$

$$g_{R_{\mu\nu}^2} = c_3\left[\gamma_g^{(3)}\right]'(0)$$

$$g_{R^2} = c_4\left[\gamma_g^{(4)}\right]'(0) - c_5\left[\gamma_g^{(3)}\right]'(0)$$

$$2\left(R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu} + g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right)f_{R^2}(\Box)R = 0$$

$$\left(2R_{\mu\sigma}g_{\nu\rho} - \frac{1}{2}g_{\mu\nu}R_{\rho\sigma} + g_{\rho\mu}g_{\sigma\nu}\Box\right)f_{R_{\mu\nu}^2}(\Box)R^{\rho\sigma}$$
$$+(g_{\mu\nu}\nabla_\rho\nabla_\sigma - 2g_{\mu\sigma}\nabla_\rho\nabla_\nu)f_{R_{\mu\nu}^2}(\Box)R^{\rho\sigma} = 0$$

$$(\Box + R)\mathcal{G}(x,x') = \frac{1}{\sqrt{-g}}\delta(x - x')$$

$$\Gamma_{\mathrm{IR}}[g_{\mu\nu}] = \frac{1}{16\pi}\int_x \sqrt{-g}\left(G_N^{-1}R + g_{R_{\mu\nu}^2}R_{\mu\nu}R^{\mu\nu} + g_{R^2}R^2 + c_1R_{\mu\nu}\Box R^{\mu\nu} + c_2R\Box R\right)$$

$$ds^2 = -f(r)\mathrm{d}t^2 + \frac{1}{g(r)}\mathrm{d}r^2 + r^2d\Omega^2$$

$$d\Omega^2 = \mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\varphi^2$$

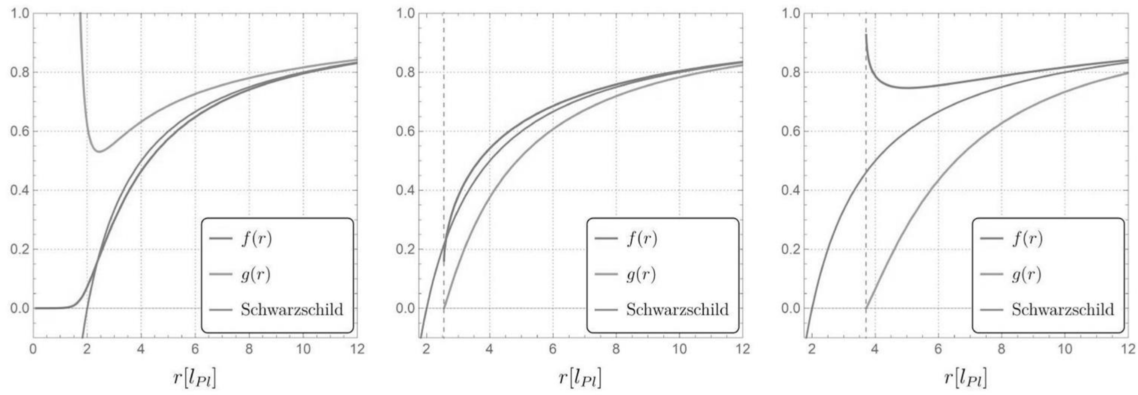


Figura 3. Fluctuaciones de simetría para agujeros negros cuánticos.

$$f(r \gg 1) = 1 - \frac{2M}{r} + S_0 \frac{e^{-m_0 r}}{r} + S_2 \frac{e^{-m_2 r}}{r}$$
$$g(r \gg 1) = 1 - \frac{2M}{r} - S_0 \frac{e^{-m_0 r}}{r}(1 + m_0 r) + \frac{1}{2} S_2 \frac{e^{-m_2 r}}{r}(1 + m_2 r)$$

$$T = \frac{1}{4\pi}\sqrt{|f'(r_h) g'(r_h)|}$$

$$S_{gf}[\bar{g}, h] = \frac{1}{2\alpha}\int_x \sqrt{\bar{g}}\bar{g}^{\mu\nu} F_\mu F_\nu$$

$$F_\mu[\bar{g}, h] = \bar{\nabla}^\nu h_{\mu\nu} - \frac{1+\beta}{4}\bar{\nabla}_\mu h_\nu^\nu$$

$$S_{gh}[\bar{g}, h, c, \bar{c}] = \int_x \sqrt{\bar{g}}\bar{c}^\mu \mathcal{M}_{\mu\nu} c^\nu$$

$$\mathcal{M}_{\mu\nu} = \bar{\nabla}^\rho (g_{\mu\nu}\nabla_\rho + g_{\rho\nu}\nabla_\mu) - \frac{1+\beta}{2}\bar{g}^{\sigma\rho}\bar{\nabla}_\mu g_{\nu\sigma}\nabla_\rho$$

$$\Gamma_k[\bar{g}, \phi] = \sum_{n=0}^{\infty} \frac{1}{n!}\Gamma_k^{(0,\phi_{i_1}\dots\phi_{i_n})}[\bar{g}, 0]\prod_{l=1}^{n} \phi_{i_l}$$

$$\phi = (h, \bar{c}, c)$$

$$\Gamma_k^{(0,\phi_1\dots\phi_n)}(\boldsymbol{p}) = \left(\prod_{i=1}^{n} Z_{\phi_i}^{\frac{1}{2}}(p_i^2)\right)\gamma_h^{(n)}(\boldsymbol{p})\mathcal{T}^{(\phi_1\dots\phi_n)}(\boldsymbol{p}; \Lambda_n)$$

$$\mathcal{T}_{\mathrm{EH}}^{(\phi_1\dots\phi_n)}(\boldsymbol{p};\Lambda_n) = G_N S_{\mathrm{EH}}^{(\phi_1\dots\phi_n)}(\boldsymbol{p};\Lambda\to\Lambda_n).$$

$$\mathfrak{F}^{(n)}(\hat{p}) := \frac{\partial_t \Gamma^{(n)}(\hat{p})}{Z_h^{n/2} k^{4-n}},$$

$$g_n := \left(\gamma_h^{(n)}\right)^{\frac{2}{n-2}} k^2, \lambda_n := \Lambda_n k^{-2}, \mu_h := -2\lambda_2$$

$$\Pi_{\mathrm{tt}}^{\mu\nu\alpha\beta} := \frac{1}{2}\left(\Pi_\perp^{\mu\alpha}\Pi_\perp^{\nu\beta} + \Pi_\perp^{\mu\beta}\Pi_\perp^{\nu\alpha}\right) - \frac{1}{3}\Pi_\perp^{\mu\nu}\Pi_\perp^{\alpha\beta}$$

$$\Pi_\perp^{\mu\nu} := \delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}$$

$$\mathcal{T}_{R,\mathrm{tt}}^{(n)}(\boldsymbol{p}) = \prod_{i=1}^{n} \Pi_{\mathrm{tt}}(p_i)\mathcal{T}_{\mathrm{EH}}^{(n)}(\boldsymbol{p},0)$$

$$\Gamma^{(0,n)}(\boldsymbol{p}) = \prod_{i=1}^{n} \sqrt{Z_h(p_i)G(p_i)}\gamma_g^{(n)}(\boldsymbol{p})\mathcal{T}_{R,\mathrm{tt}}^{(n)}(\boldsymbol{p}) + \cdots$$

$$\Gamma^{(n)}(\boldsymbol{p}) = \gamma_g^{(n)}(\boldsymbol{p})\mathcal{T}_{R,\mathrm{tt}}^{(n)}(\boldsymbol{p}) + \cdots$$

$$\gamma_g^{(n)}(\boldsymbol{p}) = \prod_{i=1}^{n} \frac{1}{\sqrt{Z_h(p_i)G(p_i)}} \times \frac{\Gamma^{(0,n)}_{\mu_1\nu_1\cdots\mu_n\nu_n}\mathcal{T}_{R,\mathrm{tt}}^{(n)\mu_1\nu_1\cdots\mu_n\nu_n}}{\mathcal{T}^{(n)}_{R,\mathrm{tt}\ \mu_1\nu_1\cdots\mu_n\nu_n}\mathcal{T}_{R,\mathrm{tt}}^{(n)\mu_1\nu_1\cdots\mu_n\nu_n}}$$

$$\gamma_g^{(n)}(\boldsymbol{p}) = \frac{\Gamma^{(n)}_{\mu_1\nu_1\cdots\mu_n\nu_n}\mathcal{T}_{R,\mathrm{tt}}^{(n)\mu_1\nu_1\cdots\mu_n\nu_n}}{\mathcal{T}^{(n)}_{R,\mathrm{tt}\ \mu_1\nu_1\cdots\mu_n\nu_n}\mathcal{T}_{R,\mathrm{tt}}^{(n)\mu_1\nu_1\cdots\mu_n\nu_n}}$$

$$\Gamma_G^{(0,n)}(p^2) := \mathcal{T}_{R,\mathrm{tt}}^{(n)}/p^2 \circ \Gamma_k^{(0,n)} = Z_h^{\frac{n}{2}} g_n^{\frac{n}{2}-1} k^{2-n}\left(C_{\Lambda_n}^{(n)}\lambda_n k^2 + C_R^{(n)} p^2\right)$$

$$\mathcal{T}_{R,\mathrm{tt}}^{(n)}/p^2 \circ S_{R^2+R_{\mu\nu}^2}^{(n)} = C_{R^2}^{(n)} g_{R^2} p^4 + C_{R_{\mu\nu}^2}^{(n)} g_{R_{\mu\nu}^2} p^4$$

$$C_{\Lambda_3}^{(3)} = -\frac{9}{4096\pi^2}, \qquad C_{\Lambda_4}^{(4)} = \frac{222485}{60466176\pi^2}$$

$$C_R^{(3)} = \frac{171}{32768\pi^2}, \qquad C_R^{(4)} = \frac{6815761}{544195584\pi^2}$$

$$C_{R_{\mu\nu}^2}^{(3)} = -\frac{405}{32768\pi^2}, \qquad C_{R_{\mu\nu}^2}^{(4)} = -\frac{3676621}{51018336\pi^2}$$

$$C_{R^2}^{(3)} = 0 \qquad , \quad C_{R^2}^{(4)} = -\frac{96203921}{1632586752\pi^2}$$

$$\mathcal{R}_k(p^2) = \Gamma_k^{(0,hh)}(p^2)\Big|_{\mu_h=0} r(\hat{p}^2) = Z_h(p^2)\mathcal{T}^{(2)}(p^2;0)r(\hat{p}^2)$$

$$r(\hat{p}^2) = \left(\frac{1}{\hat{p}^2} - 1\right)\Theta(1-\hat{p}^2)$$

$$\frac{\dot{g}_4(p^2)}{g_4(p^2)} = 2\big(1+\eta_h(p^2)\big) + 2C_4\frac{k^2}{p^2}(\eta_h(p^2)-\eta_h(0))\lambda_4 + \frac{1}{C_R^{(4)}}\frac{k^2}{p^2}\left(\frac{\mathfrak{F}_G^{(4)}(p^2)}{g_4(p^2)} - \frac{\mathfrak{F}_G^{(4)}(0)}{g_4(0)}\right)$$

$$\eta_{\phi_i}(p^2) := -\partial_t \ln\left(Z_{\phi_i}(p^2)\right)$$

$$g_6 = g_5 = g_4 \ \text{ and } \ \lambda_6 = \lambda_5 = \lambda_3$$

$$\mu_h = -0.26, \lambda_3 = 0.145 \ \text{ and } \ \lambda_4 = 0.025$$

$$\mathfrak{F}_G^{(4)}(\hat{p}) = \frac{g_3^2}{g_4}(c_1 + c_2\eta_h) + \sqrt{g_3 g_4}(c_3 + c_4\eta_h) + g_3(c_5 + c_6\eta_h) + g_4(c_7 + c_8\eta_h)$$

$$(g_3^*, g_4^*) = (2.15{,}1.48)$$

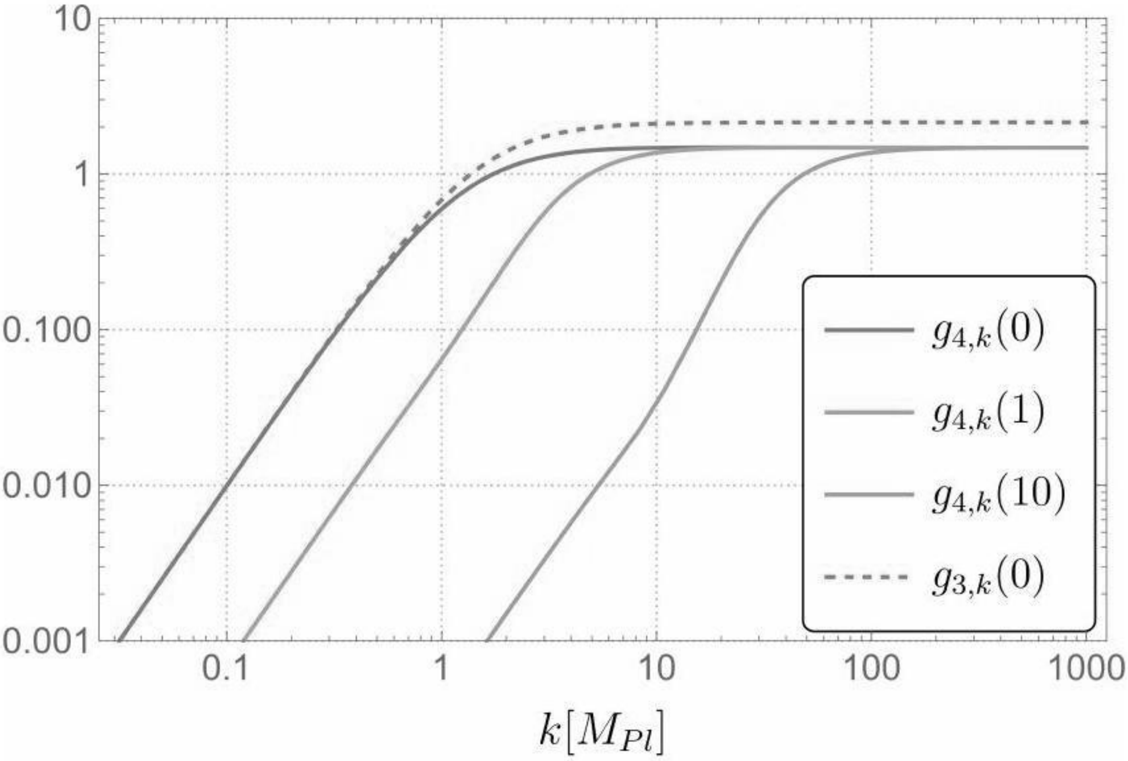


Figura 4. Coordenadas de un agujero negro cuántico en curvatura.

$$\bar{\gamma}_3 = \lim_{p^2\to\infty}\left(\frac{\gamma_g^{(3)}(p^2)}{p^2}\right)$$

$$\bar{\gamma}_3 \approx 0.054$$

$$R^{(n)}\left(\frac{\gamma_g^{(3)} - \bar{\gamma}_3\Delta}{\Delta + R}R\right)\Bigg|_{\Delta\to 0, R\to 0}$$

$$g_{R_{\mu\nu}^2} \approx -0.40, g_{R^2} \approx 1.9$$

$$\tilde{g}_{R_{\mu\nu}^2} \approx -0.023, \tilde{g}_{R^2} \approx 0.028$$

$$c_1 = 344.09, c_2 = -136.75$$

$$c_3 = \frac{C_R^{(3)}}{C_{R_{\mu\nu}^2}^{(3)}}, c_4 = \frac{C_R^{(4)}}{C_{R^2}^{(4)}}, c_5 = \frac{C_{R_{\mu\nu}^2}^{(4)}}{C_{R^2}^{(4)}}c_3$$

$$m_2^2 = \frac{-G_N^{-1}}{g_{R_{\mu\nu}^2}} \approx 2.5 M_{\text{pl}}^2$$

$$m_0^2 = \frac{G_N^{-1}}{6 g_{R^2} + 2 g_{R_{\mu\nu}^2}} \approx 0.095 M_{\text{pl}}^2$$

$$\gamma_{h,\,\text{simple}}^{(4)}(p^2) = \frac{G_4^*}{G_4^* + p^2}$$

$$\gamma_{h,\text{fit}}^{(4)}(p^2) = \frac{a_1 + a_2 p^2 + a_3 p^4 + a_4 p^6 + a_5 p^8}{b_1 + b_2 p^2 + b_3 p^4 + b_4 p^6 + b_5 p^8 + b_6 p^{10}}$$

$$\begin{aligned} &a_1 = 0.0075542, && a_2 = 7.3967, && a_3 = 109.42, \\ &a_4 = 122.40, && a_5 = 8.4140, && \\ &b_1 = 0.0075403, && b_2 = 7.4773, && b_3 = 153.00, \\ &b_4 = 876.09, && b_5 = 2261.9, && b_6 = 228.87. \end{aligned}$$

$$p_0 \approx 0.091916, p_1 \approx 0.27551$$

$$f_{R_{\mu\nu}^2,\,\text{fit}}(p^2) = a_0 + \sum_{i=1}^{4} \frac{a_i}{(p/p_i)^2 + 1}$$

$$\begin{aligned} &a_0 = -0.023601, && a_1 = -0.13727, && a_2 = 0.13138 \\ &a_3 = -0.22100, && a_4 = -0.15080, && p_1 = 0.12436 \\ &p_2 = 1.2476, && p_3 = 0.56405, && p_4 = 0.021230 \end{aligned}$$

$$f_{R^2,\text{fit}}(p^2) = a_0 + \sum_{i=1}^{3} \frac{a_i}{((p/p_i)^2 + 1)^2}$$

$$\begin{aligned} &a_0 = 0.028373, && a_1 = 0.012637, && a_2 = 1.2661 \\ &a_3 = 0.57040, && p_1 = 5.7131, && p_2 = 0.73200 \\ &p_3 = 0.092956 && && \end{aligned}$$

$$\Gamma_{\text{QG}} = \Gamma_{\text{L}} + \Gamma_{\text{NL}} + \Gamma_{\text{matter}}$$

$$\Gamma_{\text{L}} = \int d^4x \sqrt{|g|} \left[\frac{M_P^2}{2}(R - 2\Lambda) + c_1(\mu) R^2 + c_2(\mu) R_{\mu\nu} R^{\mu\nu} \right.$$
$$\left. + c_3(\mu) R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + c_4(\mu) \Box R + O(M_P^{-2}) \right]$$

$$\Gamma_{\mathrm{NL}} = -\int d^4x\sqrt{|g|}[\alpha R \ln\left(\frac{\square}{\mu^2}\right) R + \beta R_{\mu\nu} \ln\left(\frac{\square}{\mu^2}\right) R^{\mu\nu} + \gamma R_{\mu\nu\rho\sigma} \ln\left(\frac{\square}{\mu^2}\right) R^{\mu\nu\rho\sigma} + O(M_P^{-2})\Big]$$

$$M_P = \sqrt{\hbar c/(8\pi G_N)} = 2.4 \times 10^{18}\mathrm{GeV}$$

	α	β	γ
Scalar	$5(6\xi - 1)^2$	-2	2
Fermion	-5	8	7
Vector	-50	176	-26
Boson	250	-244	424

$$c_1(\mu) = c_1(\mu_*) - \alpha \ln\left(\frac{\mu^2}{\mu_*^2}\right),$$
$$c_2(\mu) = c_2(\mu_*) - \beta \ln\left(\frac{\mu^2}{\mu_*^2}\right),$$
$$c_3(\mu) = c_3(\mu_*) - \gamma \ln\left(\frac{\mu^2}{\mu_*^2}\right).$$

$$\Gamma_{\mathrm{L}} = \int d^4x\sqrt{|g|}\left[\frac{R}{16\pi G_{\mathrm{N}}} + \bar{c}_1(\mu)R^2 + \bar{c}_2(\mu)R_{\mu\nu}R^{\mu\nu}\right]$$
$$\Gamma_{\mathrm{NL}} = -\int d^4x\sqrt{|g|}\left[\bar{\alpha} R \ln\left(\frac{\square}{\mu^2}\right) R + \bar{\beta} R_{\mu\nu} \ln\left(\frac{\square}{\mu^2}\right) R^{\mu\nu}\right]$$

$$E_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + 16\pi G_{\mathrm{N}}\left(H_{\mu\nu}^{\mathrm{L}} + H_{\mu\nu}^{\mathrm{NL}}\right) = 8\pi G_{\mathrm{N}} T_{\mu\nu},$$

$$H_{\mu\nu}^{\mathrm{L}} = \bar{c}_1(\mu)\left(2R_{\mu\nu}R - \frac{1}{2} g_{\mu\nu}R^2 + 2g_{\mu\nu}\nabla^2 R - \nabla_\mu\nabla_\nu R - \nabla_\nu\nabla_\mu R\right)$$
$$+\bar{c}_2(\mu)\left(-\frac{1}{2} g_{\mu\nu}R_{\rho\sigma}R^{\rho\sigma} + 2R_\mu^\rho R_{\nu\rho} + \nabla^2 R_{\mu\nu} - \nabla_\rho\nabla_\mu R_\nu^\rho - \nabla_\rho\nabla_\nu R_\mu^\rho + g_{\mu\nu}\nabla_\sigma\nabla_\rho R^{\rho\sigma}\right)$$

$$H_{\mu\nu}^{\mathrm{NL}} = -\bar{\alpha}\left(2R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R + 2g_{\mu\nu}\nabla^2 - \nabla_\mu\nabla_\nu - \nabla_\nu\nabla_\mu\right) \ln\left(\frac{\square}{\mu^2}\right) R$$
$$-\bar{\beta}\left(-\frac{1}{2} g_{\mu\nu}R^{\rho\sigma} + 2\delta_\nu^\sigma R_\mu^\rho + \delta_\mu^\rho \delta_\nu^\sigma \nabla^2 - \delta_\nu^\sigma \nabla^\rho \nabla_\mu - \delta_\mu^\sigma \nabla^\rho \nabla_\nu + g_{\mu\nu}\nabla^\sigma\nabla^\rho\right) \ln\left(\frac{\square}{\mu^2}\right) R_{\rho\sigma}$$

$$ds^2 = h(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2 d\theta^2 - r^2 \sin^2\,\theta d\phi^2$$

$$S = \int d^4x\sqrt{|g|}\left(\frac{M_P^2}{2}R + \tilde{c}_1(\mu)R^2 + \tilde{c}_2(\mu)C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} + \tilde{\alpha}R\ln\left(\frac{\Box}{\mu^2}\right)R + \tilde{\beta}C_{\mu\nu\alpha\beta}\ln\left(\frac{\Box}{\mu^2}\right)C^{\mu\nu\alpha\beta}\right)$$

$$h(r) = k\big((r - r_0) + h_2(r - r_0)^2 + h_3(r - r_0)^3\big) + \mathcal{O}((r - r_0)^4)$$
$$f(r) = f_1(r - r_0) + f_2(r - r_0)^2 + f_3(r - r_0)^3 + \mathcal{O}((r - r_0)^4)$$

$$\bar{c}_1(\mu) \to \bar{c}_1(\mu) + 2\bar{\alpha}\ln\,\mu,$$
$$\bar{c}_2(\mu) \to \bar{c}_2(\mu) + 2\bar{\beta}\ln\,\mu,$$
$$\bar{c}_3(\mu) \to \bar{c}_3(\mu) + 2\bar{\gamma}\ln\,\mu,$$

$$h_2 = \frac{1 - 2f_1r_0}{f_1r_0^2} + \frac{1 - f_1r_0}{8\big(\tilde{c}_2(\mu) + 2\tilde{\beta}\ln\,\mu\big)f_1^2r_0}$$
$$f_2 = \frac{1 - 2f_1r_0}{r_0^2} - \frac{3(1 - f_1r_0)}{8\big(\tilde{c}_2(\mu) + 2\tilde{\beta}\ln\,\mu\big)f_1r_0}$$

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