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LAS PARTÍCULAS SUPERMASIVAS: NATURALEZA FENOMENOLÓGICA DE LA PARTÍCULA OSCURA

SUPERMASSIVE PARTICLES: PHENOMENOLOGICAL NATURE OF THE DARK PARTICLE

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Las partículas supermasivas: naturaleza fenomenológica de la partícula oscura

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RESUMEN

Las partículas supermasivas, teorizadas por este autor a lo largo de los trabajos anteriores, comporta la existencia de partículas conexas al Modelo Estándar de Física de Partículas, las mismas que pueden ser de naturaleza fermiónica o bosónica, según sea el caso, que interactúan en un campo de gauge y en supersimetría, cuya masa, es extremadamente densa, capaz de deformar el espacio – tiempo cuántico en el que interactúa (a lo que llamamos gravedad endógena), afectando la configuración morfológica de las partículas circundantes (masa y energía) así como sus coordenadas en espacio y tiempo, y desdoblando multidimensiones o supermembranas, sin que sea necesario que la partícula supermasiva o masiva, deba interactuar con un campo gravitónico y en consecuencia, con un gravitón, como se ha propuesto en relación al modelo de gravedad exógena. Esta partícula, a lo largo de anteriores trabajos, ha sido denominada también como "partícula cosmológica", "partícula supermasiva" o "superpartícula", más, llámesela también "partícula oscura", para propósitos de este artículo.

Palabras Clave: partícula supermasiva, partícula oscura, teoría cuántica de campos relativistas, agujeros negros cuánticos, supermembranas

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Supermassive particles: phenomenological nature of the dark particle

ABSTRACT

The supermassive particles, theorized by this author throughout the previous works, involve the existence of particles related to the Standard Model of Particle Physics, which can be of fermionic or bosonic nature, as the case may be, interacting in a gauge field and in supersymmetry, whose mass is extremely dense, capable of deforming the quantum space-time in which it interacts (what we call endogenous quantum gravity), affecting the morphological configuration of the surrounding particles (mass and energy) as well as their coordinates in space and time, and unfolding multidimensions or supermembranes, without it being necessary for the supermassive or massive particle to interact with a gravitonic field and consequently with a graviton, as has been proposed in relation to the exogenous gravity model. This particle, throughout previous works, has also been called "cosmological particle", "supermassive particle" or "superparticle", but, for the purposes of this article, let us also call it "dark particle".

Keywords: supermassive particle, dark particle, relativistic quantum field theory, quantum black holes, supermembranes

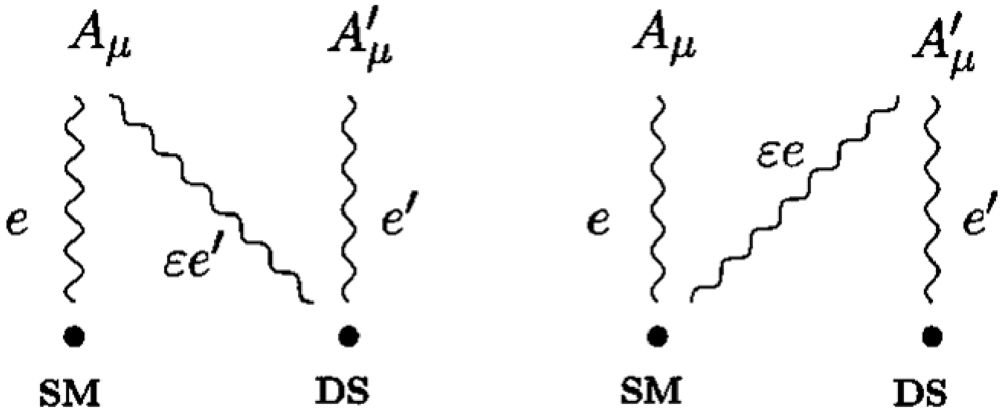
INTRODUCCIÓN

El propósito de mis trabajos anteriores, ha sido, establecer un marco teórico paralelo, cuyo objetivo es unificar la relatividad general y especial con la mecánica cuántica, de ahí nace lo que he denominado "Teoría Cuántica de Campos Relativistas" o "Teoría Cuántica de Campos Curvos", cuyos postulados esenciales, son: 1) La existencia de supergravedad cuántica exógena, esto es, que una partícula deforma el espacio – tiempo en el que interactúa, por la permeabilidad de gravedad que le es inherente, al interactuar con un campo gravitónico y por ende, con un gravitón; 2) La existencia de gravedad cuántica endógena, esto es, cuando una partícula, a propósito de su masa extremadamente densa, curva el espacio – tiempo cuántico en el que interactúa, engendrando supermembranas y multidimensiones, así como deformaciones a los sistemas de referencia de las partículas interactuantes, entendiendo que todos los campos cuánticos, sean de naturaleza fermiónica o bosónica, según sea el caso, están interrelacionados, es decir, conectados por partículas comunes, verbigracia, el bosón de Higgs; y, 3) La existencia de agujeros negros cuánticos, a propósito de la creación de supermembranas y multidimensiones causados por la deformación masiva del espacio – tiempo cuántico, esto es, la singularidad cuántica de la gravitación por densidad.

Los postulados anteriormente mencionados, han sido desarrollados cuidadosamente en trabajos anteriores, sin embargo, las partículas supermasivas, como concepto esencial de la Teoría Cuántica de Campos Relativistas, requieren de un modelo matemático adicional que describa su comportamiento, fenomenología y características particulares, en relación a lo mencionado anteriormente, es decir, en relación a la gravedad cuántica que le es implícita, sus capacidades de deformación debido a sus interacciones de gauge y supersimetrías y el modelamiento de superespacios por la curvatura del tejido del espacio – tiempo cuántico.

RESULTADOS Y DUSCUSIÓN

El campo de una partícula supermasiva, se presenta así:



$\sin\theta = \varepsilon$, $\cos\theta = \sqrt{1-\varepsilon^2}$ $\qquad$ $\sin\theta = 0$, $\cos\theta = 1$

Cuyos grupos de gauge en lagrangiano, son:

$$\mathcal{L}_0 = -\frac{1}{4}F_{a\mu\nu}F_a^{\mu\nu} - \frac{1}{4}F_{b\mu\nu}F_b^{\mu\nu} - \frac{\varepsilon}{2}F_{a\mu\nu}F_b^{\mu\nu}.$$

$$\mathcal{L} = eJ_\mu A_b^\mu + e'J'_\mu A_a^\mu,$$

$$\mathcal{L}' = \left[\frac{e'\cos\ \theta}{\sqrt{1-\varepsilon^2}}J'_\mu + e\left(\sin\ \theta - \frac{\varepsilon\cos\ \theta}{\sqrt{1-\varepsilon^2}}\right)J_\mu\right]A'^\mu + \left[-\frac{e'\sin\ \theta}{\sqrt{1-\varepsilon^2}}J'_\mu + e\left(\cos\ \theta + \frac{\varepsilon\sin\ \theta}{\sqrt{1-\varepsilon^2}}\right)J_\mu\right]A^\mu$$

$$\mathcal{L}' = \left[\frac{e'}{\sqrt{1-\varepsilon^2}}J'_\mu - \frac{e\varepsilon}{\sqrt{1-\varepsilon^2}}J_\mu\right]A'^\mu + eJ_\mu A^\mu$$

$$\mathcal{L}' = e'J'_\mu A'^\mu + \left[-\frac{e'\varepsilon}{\sqrt{1-\varepsilon^2}}J'_\mu + \frac{e}{\sqrt{1-\varepsilon^2}}J_\mu\right]A^\mu$$

Más en términos kinéticos, tenemos:

$$\begin{pmatrix} A_a^\mu \\ A_b^\mu \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\varepsilon^2}} & 0 \\ -\frac{\varepsilon^2}{\sqrt{1-\varepsilon^2}} & 1 \end{pmatrix} \begin{pmatrix} \cos\ \theta & -\sin\ \theta \\ \sin\ \theta & \cos\ \theta \end{pmatrix} \begin{pmatrix} A'^\mu \\ A^\mu \end{pmatrix}$$

$$\sin\ \theta = \frac{\delta\sqrt{1-\varepsilon^2}}{\sqrt{1-2\delta\varepsilon+\delta^2}} \quad \cos\ \theta = \frac{1-\delta\varepsilon}{\sqrt{1-2\delta\varepsilon+\delta^2}}$$

Sin embargo, si se rompe la simetría de gauge, tenemos en lagrangiano:

$$\mathcal{L}_{Stu} = -\frac{1}{2}M_a^2 A_{a\mu}A_a^\mu - \frac{1}{2}M_b^2 A_{b\mu}A_b^\mu - M_a M_b A_{a\mu}A_b^\mu$$

$$\mathcal{L}'' = \frac{1}{\sqrt{1-2\delta\varepsilon+\delta^2}}\left[\frac{e'(1-\delta\varepsilon)}{\sqrt{1-\varepsilon^2}}J'_\mu + \frac{e(\delta-\varepsilon)}{\sqrt{1-\varepsilon^2}}J_\mu\right]A'^\mu + \frac{1}{\sqrt{1-2\delta\varepsilon+\delta^2}}\left[eJ_\mu - \delta e' J'_\mu\right]A^\mu$$

Cuyo acoplamiento viene dado por:

$$\mathcal{L} \supset -\frac{e\varepsilon}{\sqrt{1-\varepsilon^2}}J_\mu A'^\mu \simeq -e\varepsilon J_\mu A'^\mu,$$

Por otro lado, la hiper – carga de la partícula supermasiva viene dada por:

$$\tilde{\mathcal{L}} = -\frac{\varepsilon}{2\cos\,\theta_W}\tilde{F}'_{\mu\nu}B^{\mu\nu}.$$

Cuya simetría de gauge viene dada por:

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \\ \tilde{A}'_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W & -s_W\varepsilon \\ -s_W & c_W & -c_W\varepsilon \\ t_W\varepsilon & 0 & 1 \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \\ A'_\mu \end{pmatrix},$$

Y en lagrangiano:

$$\mathcal{L} \supset -e\varepsilon J^\mu A'_\mu + e'\varepsilon t_W J'^\mu Z_\mu + e' J^{\mu\prime} A'_\mu,$$

Más sus operadores y osciladores vienen dados por:

$$\mathcal{L} = \frac{e_D}{2\Lambda_5}\bar{\psi}^i\sigma_{\mu\nu}(\mathbb{D}_M^{ij} + i\gamma_5\mathbb{D}_E^{ij})\psi^j F'^{\mu\nu}$$

$$\mathcal{L} = \frac{e_D}{2\Lambda^2}\bar{\psi}_L^i\sigma_{\mu\nu}(\mathbb{D}_M^{ij} + i\gamma_5\mathbb{D}_E^{ij})H\psi_R^j F'^{\mu\nu} + \text{ H.c.}$$

$$\mathcal{L}' = \frac{e_D}{2\Lambda^2}\bar{\psi}^i\gamma_\mu(\mathbb{R}_r^{ij} + i\gamma_5\mathbb{R}_a^{ij})D_\nu\psi^j F'^{\mu\nu}$$

En los que la densidad de la partícula supermasiva, por isotropización, se mide así:

$$m_\chi\left(\frac{0.01}{\alpha_D}\right)^{2/3} \gtrsim 300\text{GeV}$$

$$\frac{G_N m_\chi^4 N}{8\alpha_D^2} \gtrsim 50\log\frac{G_N m_\chi^2 N}{2\alpha_D}$$

$$\chi\chi \to A'A'$$

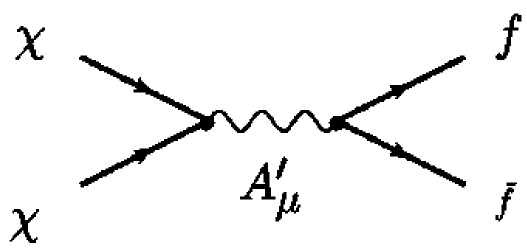
$$\langle\sigma_{\chi\chi\to A'A'}v\rangle = \frac{2\pi\alpha_D^2}{m_\chi^2}$$

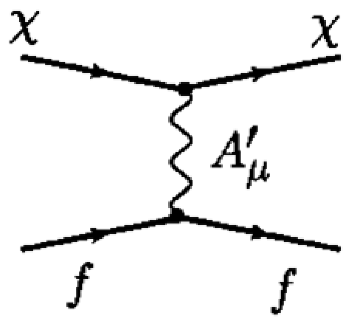
$$\Omega_\chi h^2 \approx \frac{2.5\times 10^{-10}\text{GeV}^{-2}}{\langle\sigma_{\chi\chi\to A'A'}v\rangle}$$

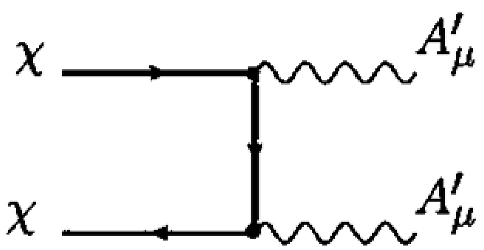
$$\langle \sigma_{\chi\chi \to f\bar{f}} v \rangle \simeq \frac{2\pi\alpha_L^2}{m_S^2}$$

$$2\pi\alpha_L^2 \left(\frac{10\text{TeV}}{m_S}\right)^2 \simeq 0.1$$

En este punto y usando los diagramas de Feynman, pasa a explicarse las interacciones de las partículas supermasivas en distintos grupos de gauge, cuya aniquilación provoca un agujero negro cuántico supermasivo:







$$\chi\bar{\chi} \to A' \to \bar{f}f$$

$$\sigma_{\chi\chi \to ff} = \frac{4\pi}{3}\varepsilon^2 \alpha\alpha_D m_\chi^2 \left(1 + \frac{2m_e^2}{s}\right)\left(1 + \frac{2m_\chi^2}{s}\right) \times \frac{s}{\left(s - m_{A'}^2\right)^2 + m_{A'}^2\Gamma_{A'}^2} \frac{\sqrt{1 - \frac{4m_e^2}{m_{A'}^2}}}{\sqrt{1 - \frac{4m_\chi^2}{m_{A'}^2}}}$$

$$\langle \sigma_{\chi\chi\to ff} v\rangle \simeq \varepsilon^2 \alpha\alpha_D \frac{16\pi m_\chi^2}{\left(4m_\chi^2 - m_{A'}^2\right)^2}$$

$$\Omega_\chi h^2 \approx \frac{2.5\times 10^{-10}\mathrm{GeV}^{-2}}{\langle \sigma_{\chi\chi\to ff} v\rangle}$$

$$y \equiv \varepsilon^2 \alpha_D \left(\frac{m_\chi}{m_{A'}}\right)^4$$

$$\langle \sigma_{\chi\chi\to ff} v\rangle \simeq \frac{16\pi\alpha y}{m_\chi^2}$$

Por otro lado, la producción iónica viene dada por:

$$\sigma_e = \frac{16\pi\mu_{\chi e}^2 \alpha\alpha_D \varepsilon^2}{\left(m_{A'}^2 + \alpha^2 m_e^2\right)^2} |F(q^2)|^2$$

$$F(q^2) = \frac{m_{A'}^2 + \alpha^2 m_e^2}{m_{A'}^2 + q^2}$$

$$\frac{dR}{d\ln\ E} = N_T \frac{\rho_\chi}{m_\chi} \frac{d\langle \sigma_e v\rangle}{d\ln\ E}$$

Desde una perspectiva relativista, la partícula supermasiva en cuanto a su densidad, queda expresada así:

$$\sigma_{p.e.} = 4\alpha^4 \sqrt{2} Z^5 \frac{8\pi r_e^2}{3} \left(\frac{m_e}{\omega}\right)^{7/2}$$

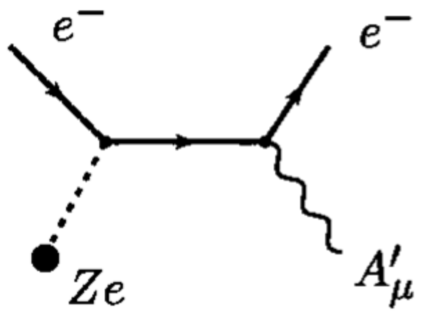
$$\sigma_{A'} = \varepsilon^2 \sigma_{p.e.}$$

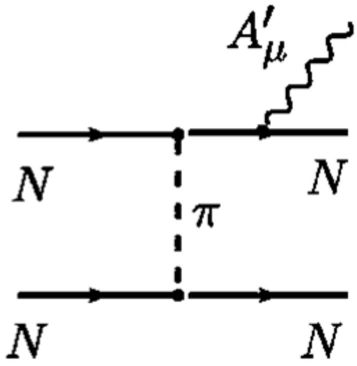
$$\Gamma_{A'} = \frac{\rho_{A'}}{m_{A'}} \sigma_{A'} v_{A'}$$

Ahora bien, la fenomenología relativista de la masa y energía de una partícula supermasiva (nucleosíntesis), se expresa así:

$$d_M^{ij} \equiv \left|\mathbb{D}_M^{ij}\right|$$

$$\mathcal{M}_{A'_\mu} \approx \bar{\psi}(\mathbf{k}\times\boldsymbol{\epsilon})\cdot\boldsymbol{\sigma}\psi$$





$$\mathcal{M}_a \approx \bar{\psi}\mathbf{k}\cdot\boldsymbol{\sigma}\psi$$

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \sum_j Z_j^2 n_j \frac{4\alpha^2\alpha'_{ae}}{\pi}\frac{|\mathbf{p}_1||\mathbf{p}_2|\omega^2}{(\mathbf{q}^2+\kappa_F^2)^2}\left[2\omega^2\frac{p_1\cdot p_2 - m_e^2 + (p_2-p_1)\cdot k}{(p_1\cdot k)(p_2\cdot k)} + 2 - \frac{p_1\cdot k}{p_2\cdot k} - \frac{p_2\cdot k}{p_1\cdot k}\right]$$

$$Q/\rho = \frac{\pi^2\alpha^2\alpha'_{ae}}{15}\frac{T^4}{m_e^2}\sum_j Z_j^2 n_j F(\kappa_F) \simeq \alpha'_{ae} 1.08\times 10^{27}\left(\frac{T}{10^8\ \mathrm{K}}\right)^4 \frac{Z^2}{A} F(\kappa_F)$$

$$F(\kappa_F) \simeq \frac{2+\kappa_F^2}{2}\ln\frac{2+\kappa_F^2}{\kappa_F^2} - 1$$

$$\alpha'_{ae} \leq 3.0\times 10^{-27}$$

$$\alpha'_{ae} = 2\frac{1}{4\pi}\left(2e_D d_M^e \frac{v_h m_e}{\Lambda^2}\right)^2$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D d_M^e}} \gtrsim 4.5\times 10^6\,\mathrm{TeV}^2$$

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \frac{16(4\pi)^3\alpha_\pi^2\alpha'_{aN}}{3m_N^2}\left[\left(\frac{\mathbf{k}^2}{\mathbf{k}^2+m_\pi^2}\right)^2 + \left(\frac{\mathbf{l}^2}{\mathbf{l}^2+m_\pi^2}\right)^2 + \frac{\mathbf{k}^2\mathbf{l}^2 - 3(\mathbf{k}\cdot\mathbf{l})^2}{(\mathbf{k}^2+m_\pi^2)(\mathbf{l}^2+m_\pi^2)}\right]$$

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \frac{32(4\pi)^3\alpha_\pi^2\alpha'_{aN}}{m_N^2}$$

$$Q/\rho \simeq \alpha'_{aN} 1.74\times 10^{33}\frac{\rho}{10^{15}}\left(\frac{T}{\mathrm{MeV}}\right)^6$$

$$\alpha'_{aN} \leq 1.3\times 10^{-18}$$

$$\alpha'_{aN} = 2\frac{1}{4\pi}\left(2e_D d_M^q \frac{v_h m_N}{\Lambda^2}\right)^2$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D d_M^q}} \gtrsim 4.3\times 10^5\,\mathrm{TeV}^2$$

$$\alpha'_{aN} \geq 0.7 \times 10^{-14}$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^q} \lesssim 5.9 \times 10^3 \text{TeV}^2$$

$$N_{\text{eff}} = 2.878 \pm 0.278$$

$$H(T_d) = \frac{T_d^2}{M_{Pl}} \left(\frac{\pi^2}{90} g_*(T_d) \right)^{1/2}$$

$$\Gamma_{A'} = n_{A'} \langle \sigma v \rangle$$

$$n_{A'} = \frac{2\zeta(3)}{\pi^2} T^3$$

$$\langle \sigma v \rangle \simeq \frac{\alpha_D d_M^2 v_h^2}{\Lambda^4}$$

$$\frac{2\zeta(3)}{\pi^2} T_d^3 \langle \sigma v \rangle < \frac{T_d^2}{M_{Pl}} \left(\frac{2\pi^2}{45} g_*(T_d) \right)^{1/2}$$

$$\left(\frac{T_{BBN}}{T_d} \right)^4 = \left(\frac{g_*(T_{BBN})}{g_*(T_d)} \right)^{4/3} < \frac{7}{4} \Delta N_{\text{eff}}$$

$$g_*(T_d) > (43/7)^{4/3} \Delta N_{\text{eff}}^{-3/4}$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^\ell} \geq 6.6 \times 10^3 \text{TeV}^2$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^q} \geq 4.3 \times 10^3 \text{TeV}^2$$

$$V(r) = -\frac{\alpha_D v^2 d_M^a d_M^b}{4\Lambda^4 r^3} [\boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b - 3(\boldsymbol{\sigma}_a \cdot \hat{\boldsymbol{r}})(\boldsymbol{\sigma}_b \cdot \hat{\boldsymbol{r}})]$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^e} \gtrsim 872 \text{GeV}^2$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^e} \gtrsim 1.61 \text{TeV}^2$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} \sqrt{d_M^e d_M^q}} \gtrsim 1.94 \text{TeV}^2$$

$$a_\ell^{A'} = -\frac{3}{2} \frac{\alpha_D}{\pi} \left(\frac{m_\ell v_h d_M^\ell}{\Lambda^2} \right)^2 \left[\frac{5}{4} + \log \frac{\mu^2}{m^2} \right]$$

$$\delta_{\Delta a_e} < 8.1 \times 10^{-13}$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^e} \gtrsim 0.075\text{TeV}^2$$

$$\Delta a_\mu < 2.74 \times 10^{-9}$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^\mu} \gtrsim 0.5\text{TeV}^2$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^\mu} \simeq 0.27\text{TeV}^2$$

$$\text{BR}(\mu \to eX^0) < 5.8 \times 10^{-5}$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^{\mu e}} \gtrsim 5.1 \times 10^5\text{TeV}^2$$

$$\text{BR}(K^+ \to \pi^+ \nu\bar{\nu}) < 1.85 \times 10^{-10}$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^{sd}} \gtrsim 9.5 \times 10^6\text{TeV}^2$$

$$6.5 \times 10^{-14} \leq \alpha'_{aN} \leq 8.0 \times 10^{-8}$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^q} \gtrsim 1.9 \times 10^3\text{TeV}^2$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^e} \gtrsim 1.2\text{TeV}^2$$

$$\frac{\Lambda^2}{\sqrt{\alpha_D} d_M^q} \gtrsim 4.3\text{TeV}^2$$

En la que, la interacción y acoplamiento de Yukawa, en relación a la partícula supermasiva, en lagrangiano queda expresada así:

$$\mathcal{L} \supset -g_L\left(\phi_L^\dagger \bar{\chi}_R l_L + S_L^{U\dagger} \bar{Q}_R^U q_L + S_L^{D\dagger} \bar{Q}_R^D q_L\right) - g_R\left(\phi_R^\dagger \bar{\chi}_L e_R + S_R^{U\dagger} \bar{Q}_L^D u_R + S_R^{U\dagger} \bar{Q}_L^D d_R\right) + \text{ H.c}$$

$$\mathcal{L} \supset -\lambda_s S_0\left(H^\dagger \phi_R^\dagger \phi_L + \tilde{H}^\dagger S_R^{U\dagger} S_L^U + H^\dagger S_R^{D\dagger} S_L^D\right) + \text{ h.c.}$$

Cuyos escalares, son:

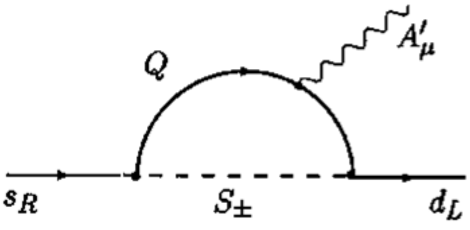
$$m_\pm = m_{\phi,S}\sqrt{1 \pm \eta_s}$$

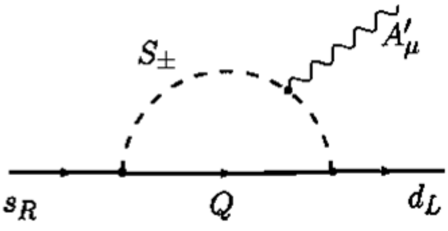
$$\eta_{\phi,S} \equiv \frac{\lambda_s \mu_{\phi,S} v_h}{m_S^2}.$$

$$\mathcal{L}^{(lep)} \supset -g_L\phi_{L\nu}^{\dagger}(\bar{\chi}_R\nu_L) - \frac{g_L}{\sqrt{2}}(\phi_+^{\dagger} + \phi_-^{\dagger})(\bar{\chi}_R e_L) - \frac{g_R}{\sqrt{2}}(\phi_+^{\dagger} - \phi_-^{\dagger})(\bar{\chi}_L e_R) + \text{ h.c.}$$

$$\eta_{\phi,S} < 1 - \left(\frac{M}{m_{\phi,S}}\right)^2.$$

Y cuyos diagramas de vórtice, son:





$$S_L^{U_i^\dagger}\bar{Q}_R^{U_i}q_L^i \to S_L^{U_i\dagger}\bar{Q}_R^{U_i}(\rho_L^U)_{ij}q_L^j S_R^{U_i^\dagger}\bar{Q}_L^{U_i}q_R^i \to S_R^{U_i^\dagger\dagger}\bar{Q}_L^{U_i}(\rho_L^U)_{ij}q_R^j$$

$$\frac{v_h}{\Lambda^2} \simeq \frac{m_{Q^i}}{m_S^2}$$

$$\mathbb{D}_M^{ij} = \rho_{jj}\rho_{ij}^*\mathrm{Re}\left[\frac{g_L g_R}{(4\pi)^2}\right]F_M(x,\eta_s)$$

$$F_M(x,y) = \frac{1}{2}[f(x,y) - f(x,-y)]$$

$$f(x,y) = \frac{1 - x + y + (1+y)\log\left(\frac{x}{1+y}\right)}{(1-x+y)^2}$$

$$\rho_{nm}\rho_{mm}^* - \rho_{nm}^*\rho_{mm} = 2i\sin\,\delta_{\mathrm{CP}}$$

$$m_S^i \gtrsim 940\mathrm{GeV}$$

$$m_\phi \gtrsim 290\mathrm{GeV}$$

$$\frac{m_\phi^2/m_{\chi^e}}{\sqrt{\alpha_D\alpha_L\alpha_R}|\rho_{ee}|^2 \mid F_M(x_e,\eta_\phi)} \gtrsim 2.1\times10^6\mathrm{TeV},$$

$$\frac{m_S^2/m_{Q^u}}{\sqrt{\alpha_D\alpha_L\alpha_R}|\rho_{uu}|^2 F_M(x_u,\eta_S)} \gtrsim 2.0\times10^5\mathrm{TeV},$$

$$\frac{m_\phi^2/m_{\chi^e}}{\sqrt{\alpha_L\alpha_R}|\rho_{ee}|^2 G_M(x_e,\eta_\phi)} \gtrsim 9.8\times10^4\mathrm{TeV}$$

$$G_M(x,y) = \frac{1}{2}[g(x,y) - g(x,-y)]$$

$$g(x,y)=\frac{(1+y)^2-x^2+2x(1+y)\log\left(\frac{x}{1+y}\right)}{2(x-1-y)^3}$$

$$\frac{m_\phi^2/m_{\chi^\mu}}{\sqrt{\alpha_L\alpha_R}\left|\rho_{\mu\mu}\right|^2 G_M\left(x_\mu,\eta_\phi\right)}\gtrsim 6.3\times 10^3\text{TeV}$$

$$\frac{m_\phi^2/m_{\chi^\mu}}{\sqrt{\alpha_L\alpha_R}\left|\rho_{\mu\mu}\rho_{\mu e}^\star\right|G_M\left(x_\mu,\eta_\phi\right)}\gtrsim 4.9\times 10^8\text{TeV}.$$

$$\frac{m_S^2/m_{Q^b}}{\sqrt{\alpha_L\alpha_R}\left|\rho_{bb}\rho_{bs}^\star\right|G_M\left(x_b,\eta_S\right)}\gtrsim 1.3\times 10^4\text{TeV},$$

$$\frac{m_S^2}{(\alpha_L^2+\alpha_R^2)\left|\rho_{ss}\rho_{sd}^*\right|^2}\gtrsim 3\times 10^5\text{TeV}^2$$

La estructura morfológica de una partícula supermasiva, viene dada por:

1. **Modelo Drell-Yan.**

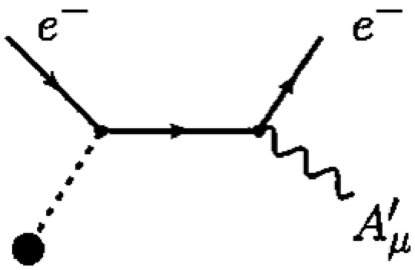
$$\mathcal{L}=-\varepsilon eJ^\mu A'_\mu$$

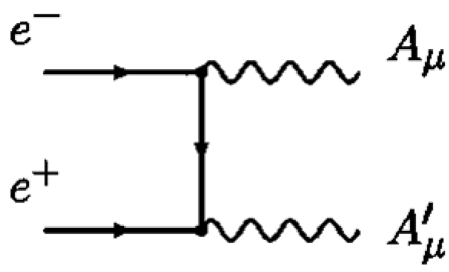
$$\Gamma(A'\to\ell^+\ell^-)=\frac{1}{3}\alpha\varepsilon^2 m_{A'}\sqrt{1-\frac{4m_\ell^2}{m_{A'}^2}}\left(1+\frac{2m_\ell^2}{m_{A'}^2}\right),$$

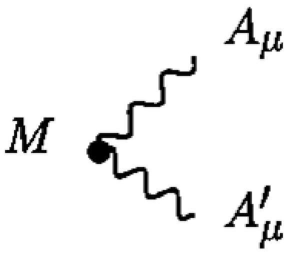
$$\Gamma(A'\to\text{ partícula-supermasiva })=\frac{1}{3}\alpha\varepsilon^2 m_{A'}\sqrt{1-\frac{4m_\mu^2}{m_{A'}^2}}\left(1+\frac{2m_\mu^2}{m_{A'}^2}\right)R$$

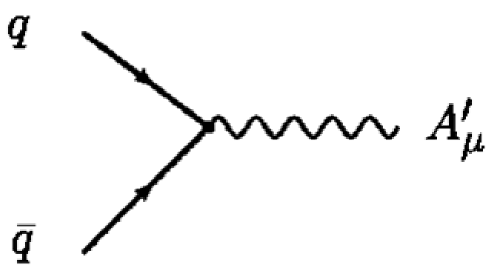
$$\Gamma(A'\to\chi\bar{\chi})=\frac{1}{3}\alpha_D m_{A'}\sqrt{1-\frac{4m_\chi^2}{m_{A'}^2}}\left(1+\frac{2m_\chi^2}{m_{A'}^2}\right)$$

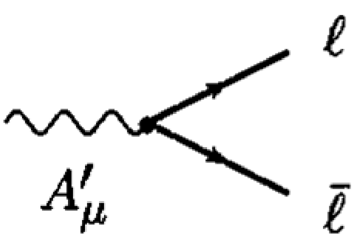
2. **Diagramas de caída de masa.**

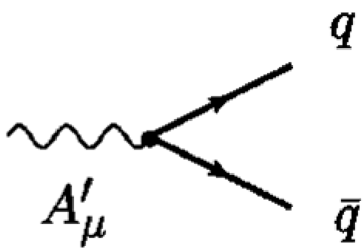


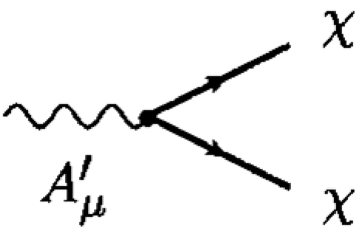












3. Singularidades cuánticas.

$$\frac{\varepsilon}{2} F_{\mu\nu} F^{\mu\nu\prime},$$

$$\frac{a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{f_a} \partial_\mu a \bar{\psi} \gamma^\mu \gamma_5 \psi,$$

$$(\mu S + \lambda S^2) H^\dagger H,$$

$$y_N \bar{L} H N,$$

4. Superdensidad de masa.

$$\Gamma = n\sigma v,$$

$$H(T) = \frac{\pi \sqrt{g_*(T)}}{\sqrt{90}} \frac{T^2}{m_{Pl}}$$

$$g_*(T) = \sum_{\text{bosons}} g_b \left(\frac{T_b}{T}\right)^4 + \frac{7}{8} \sum_{\text{fermions}} g_f \left(\frac{T_f}{T}\right)^4,$$

$$n_{eq}(T) = g_* \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{E/T} \pm 1} = \begin{cases} g_* \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} & \text{no relativista } (T \ll m) \\ \frac{\zeta(3)}{\pi^2} g_* T^3 & \text{bosones relativistas } (T \gg m) \\ \frac{3}{4} \frac{\zeta(3)}{\pi^2} g_* T^3 & \text{fermiones relativistas } (T \gg m), \end{cases}$$

$$\dot{n}(t) + 3H(t)n(t) = -\langle \sigma_{\chi\chi \to ff} v \rangle \left(n^2(t) - n_{eq}^2(t)\right),$$

$$\langle \sigma_{\chi\chi \to ff} v \rangle = \frac{\int_\infty^{4m_\chi^2} ds \sqrt{s}(s - 4m_\chi^2) K_1\left(\frac{\sqrt{s}}{T}\right) \sigma_{\chi\chi \to ff}}{8 m_\chi^4 T \left[K_2\left(\frac{m_\chi}{T}\right)\right]^2}$$

$$\langle \sigma_{\chi\chi \to ff} v \rangle = \langle s_0 + s_1 v^2 + O(v^4) \rangle$$

$$x = m_\chi / T = \sqrt{2tH(T = m_\chi)}$$

$$\frac{dY}{dx} = -\frac{\lambda(x)}{x^2} \left[Y^2(x) - Y_{eq}^2\right]$$

$$\lambda(x) = \frac{m_\chi^3 \langle \sigma_{\chi\chi \to ff} v \rangle}{H(T = m_\chi)},$$

$$\langle \sigma_{\chi\chi \to ff} v \rangle = \sigma_{\chi\chi \to ff} v + O(v^2)$$

$$\lambda(x) = \frac{\sqrt{180} m_{Pl} m_\chi}{\pi \sqrt{g_* x}} \sigma_{\chi\chi \to ff}$$

$$H(T = m_\chi) = \frac{\pi\sqrt{g_*(T = m_\chi)}}{90}\frac{m_\chi^2}{m_{Pl}}.$$

$$Y(x') = \frac{x_d}{\lambda}$$

$$\rho_\chi = m_\chi n(x') = m_\chi^4 \frac{Y(x')}{28x_d}$$

$$\Omega_\chi h^2 \simeq 0.12\frac{x_d}{23}\frac{\sqrt{g_*}}{10}\frac{1.7\times 10^{-9}\text{GeV}^{-2}}{\langle\sigma_{\chi\chi\to ff}v\rangle} \simeq \frac{2.5\times 10^{-10}\text{GeV}^{-2}}{\langle\sigma_{\chi\chi\to ff}v\rangle}$$

$$Q = \prod_{i=1}\int \frac{d^3\mathbf{p}_i}{2E_i(2\pi)^3}f_i(E_i)\prod_{f=1}\int \frac{d^3\mathbf{p}_f}{2E_f(2\pi)^3}[1\pm f_f(E_f)]\int \frac{d^3\mathbf{p}_a}{2\omega_a(2\pi)^3}\omega_a \times\frac{1}{S}\sum_{\text{spin and pol.}}|\mathcal{M}|^2(2\pi)^4\delta^4\left(\sum p_i - \sum p_f - p_a\right)$$

$$n_j(E_j) = g_j\int \frac{d^2\mathbf{p}_j}{(2\pi)^3}f(E_j)$$

$$L = \int dVQe^{-\tau}$$

$$\Pi^{\mu\nu}(k) = 16\pi\alpha\int \frac{d^3\mathbf{p}_i}{(2\pi)^3}\frac{1}{2E}[n_e(E) + n_{\bar{e}}(E)]\times\frac{p\cdot k(p^\mu k^\nu + k^\mu p^\nu) - k^2p^\mu p^\nu - (p\cdot k)^2 g^{\mu\nu}}{(p\cdot k)^2 - (k^2)^2/4}$$

$$\Pi_T(\omega,\mathbf{k}) = \frac{1}{2}(\delta^{ij} - k^ik^j)\Pi^{ij}(\omega,\mathbf{k})$$

$$\Pi_L(\omega,\mathbf{k}) = \Pi^{00}(\omega,\mathbf{k})$$

$$D^{00}(\omega,k) = \frac{1}{k^2 - \Pi_L(\omega,k)}$$

$$D^{ij}(\omega,k) = \frac{1}{k^2 - \Pi_T(\omega,k)}(\delta^{ij} - k^ik^j)$$

$$\omega_T^2 = k^2 + \Pi_T(\omega_T,k) \text{ and } \omega_L^2 = \frac{\omega_L^2}{k^2} + \Pi_T(\omega_L,k)$$

5. Momentum de Fermi.

$$\Pi_T(\omega,\mathbf{k}) = \omega_P^2\frac{3\omega^2}{2v_F^2k^2}\left(1 - \frac{\omega^2 - v_F^2k^2}{2v_F\omega k}\log\frac{\omega + v_Fk}{\omega - v_Fk}\right)$$

$$\Pi_L(\omega,\mathbf{k}) = \omega_P^2\frac{3\omega}{2v_F^3k}\left(\frac{\omega}{2v_Fk}\log\frac{\omega + v_Fk}{\omega - v_Fk} - 1\right)$$

$$Q = -\int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{\mathrm{Im}\Pi_L(\omega,\mathbf{k}) + \mathrm{Im}\Pi_T(\omega,\mathbf{k})}{\omega(e^{\omega/T}-1)}$$

6. Modelo inflacionario - relativista de una partícula supermasiva a propósito de la superdensidad de su masa y la superdensidad de su energía.

$$\frac{\Omega_A}{\Omega_{\mathrm{DM}}} \simeq \sqrt{\frac{m}{6\cdot 10^{-6}\mathrm{eV}}}\left(\frac{H_I}{10^{14}\mathrm{GeV}}\right)^2$$

$$S = \int dt d^3x\sqrt{-g}\left[-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{1}{2}m^2 g^{\mu\nu}A_\mu A_\nu\right]$$

$$S_T = \int \frac{a^3 d^3k dt}{(2\pi)^3}\frac{1}{2a^2}\left[\left|\partial_t\vec{A}_T\right|^2 - \left(\frac{k^2}{a^2}+m^2\right)\left|\vec{A}_T\right|^2\right]$$
$$S_L = \int \frac{a^3 d^3k dt}{(2\pi)^3}\frac{1}{2a^2}\left[\frac{a^2m^2}{k^2+a^2m^2}|\partial_t A_L|^2 - m^2|A_L|^2\right]$$

$$\left\langle X^*(t,\vec{k})X(t,\vec{k}')\right\rangle \equiv (2\pi)^3\delta^3(\vec{k}-\vec{k}')\frac{2\pi^2}{k^3}\mathcal{P}_X(t,k)$$

$$\langle\rho\rangle = \int \mathrm{dlog}\ k\frac{1}{2a^2}\left[\frac{a^2m^2}{k^2+a^2m^2}\mathcal{P}_{\partial_t A_L} + m^2\mathcal{P}_{A_L}\right]$$

$$\left[\partial_t^2 + \frac{3k^2+a^2m^2}{k^2+a^2m^2}H\partial_t + \frac{k^2}{a^2}+m^2\right]A_L = 0$$

$$\mathcal{P}_{A_L}(t,k) = \left(\frac{kH_I}{2\pi m}\right)^2\left(\frac{A_L(t,k)}{A_{L,0}}\right)^2$$

$$\mathcal{P}_{A_L}(t,k) \simeq \left(\frac{k_\star H_I}{2\pi m}\right)^2\left(\frac{a_\star}{a}\right)\frac{(k/k_\star)^2}{1+(k/k_\star)^3}$$

$$a_0\lambda_\star = \frac{2\pi a_0}{m a_\star} \simeq 10^{11}\ \mathrm{km}\left(\frac{10^{-5}\mathrm{eV}}{m}\right)^{1/2}$$

$$\delta(x) \equiv \frac{\rho(x)-\bar{\rho}}{\bar{\rho}}$$

$$\mathcal{P}_\delta(t,k) \simeq \frac{\sqrt{3}(k/k_\star)^3}{\pi((k/k_\star)^{3/2}+1)^{8/3}}$$

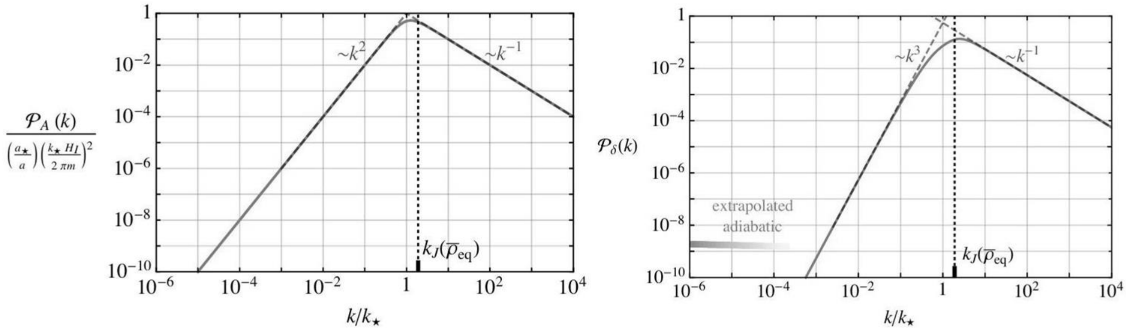


Figura 1. Fluctuaciones inflacionarias de la partícula supermasiva en un campo de gauge específico.

7. Modelo Schrödinger-Poisson para una partícula supermasiva.

$$A_i \equiv \frac{1}{\sqrt{2m^2a^3}}(\psi_i e^{-imt} + \text{ c.c. })\left(i\partial_t + \frac{\nabla^2}{2m} - m\Phi)\psi_i = \nabla^2\Phi = \frac{4\pi G}{a}\sum_i (|\psi_i|^2 - \langle|\psi_i|^2\rangle\right)$$

$$\partial_t\rho_i + 3H\rho_i + a^{-1}\nabla\cdot(\rho_i\vec{v}_i) = 0$$
$$\partial_t\vec{v}_i + H\vec{v}_i + a^{-1}(\vec{v}_i\cdot\nabla)\vec{v}_i = -a^{-1}(\nabla\Phi + \nabla\Phi_{Qi})$$
$$\nabla^2\Phi = 4\pi G a^2(\rho - \bar{\rho})$$

$$\Phi_{Qi} \equiv -\frac{\hbar^2}{2a^2m^2}\frac{\nabla^2\sqrt{\rho_i}}{\sqrt{\rho_i}}$$

$$k_J(\rho) \equiv a(16\pi G\rho m^2)^{1/4}$$

$$\left.\frac{k_J(\bar{\rho})}{k_\star}\right|_{a=a_{\text{eq}}} = \frac{\left(16\pi G\bar{\rho}_{\text{eq}}m^2\right)^{1/4}}{m(a_\star/a_{\text{eq}})} = g_R\left(12\frac{\bar{\rho}_{\text{eq}}}{\rho_{\text{eq}}^{\text{tot}}}\right)^{1/4} = g_R\left(6\frac{\Omega_A}{\Omega_{\text{M}}}\right)^{1/4}$$

$$\left.\frac{k_J(\bar{\rho})}{k_\star}\right|_{a=a_{\text{eq}}} \simeq 1.9$$

$$H^2 = \frac{8\pi G}{3}\rho_{\text{tot}} = \frac{8\pi G}{3}\frac{\bar{\rho}_{\text{eq}}}{2}\left[\left(\frac{a_{\text{eq}}}{a}\right)^3 + \left(\frac{a_{\text{eq}}}{a}\right)^4\right] \equiv \frac{H_{\text{eq}}^2}{2}\left[\left(\frac{a_{\text{eq}}}{a}\right)^3 + \left(\frac{a_{\text{eq}}}{a}\right)^4\right]$$

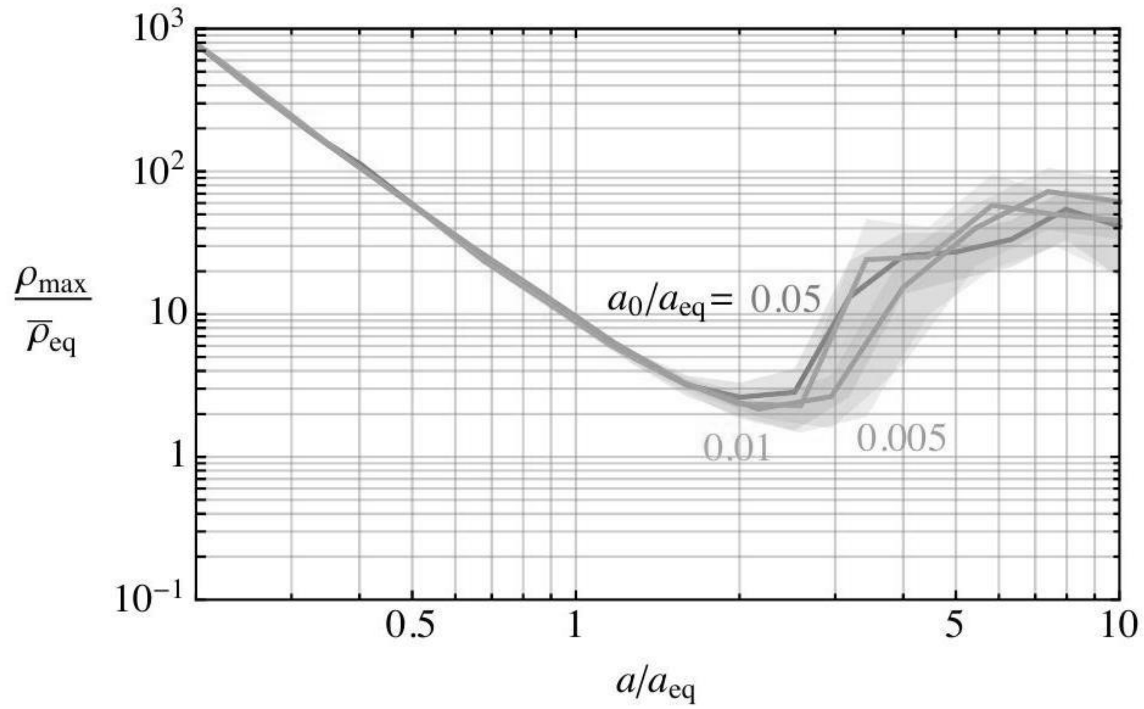
$$\frac{da}{dt} = a\frac{H_{\text{eq}}}{\sqrt{2}}\sqrt{\left(\frac{a_{\text{eq}}}{a}\right)^3 + \left(\frac{a_{\text{eq}}}{a}\right)^4}$$

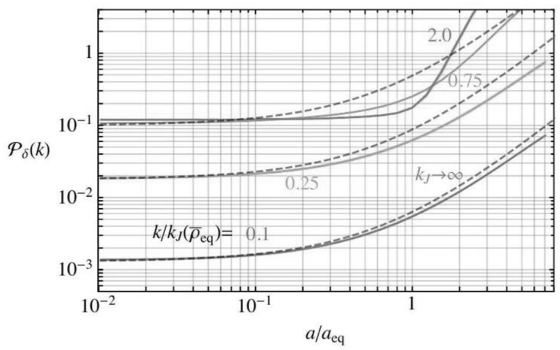
$$\left(i\partial_{\tilde{t}} + \frac{\nabla'^2}{2} - \Phi'\right)\psi_i' = 0$$

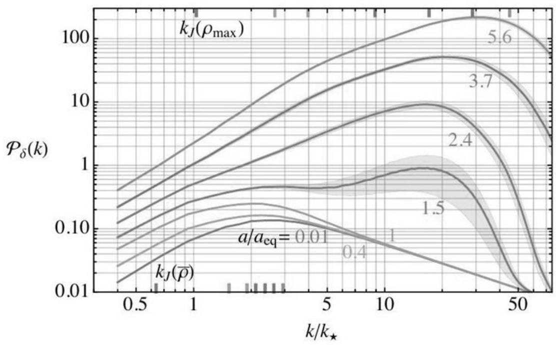
$$\nabla'^2\Phi' = a\sum_i (|\psi_i'|^2 - \langle|\psi_i'|^2\rangle)$$

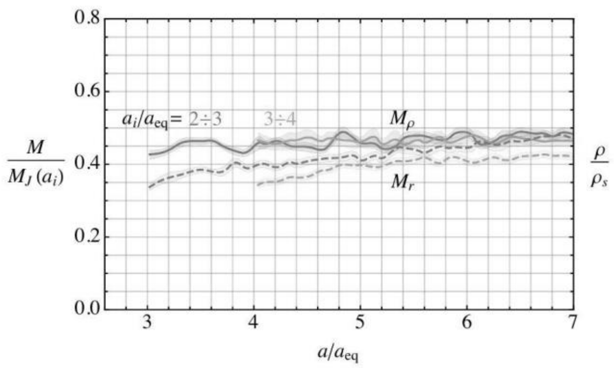
$$\frac{da}{d\tilde{t}} = \beta a^3\sqrt{\left(\frac{a_{\text{eq}}}{a}\right)^3 + \left(\frac{a_{\text{eq}}}{a}\right)^4}$$

$$\langle|\psi'|^2\rangle = \frac{\langle|\psi|^2\rangle}{(\sqrt{4\pi G}T)^{-2}} = \frac{f\bar{\rho}_{\text{eq}}/2}{\left(\sqrt{4\pi G}\sqrt{2}\beta H_{\text{eq}}^{-1}\right)^{-2}} = \frac{3\Omega_{\text{A}}/\Omega_{\text{M}}}{2}\beta^2$$









Figuras 1, 2 y 3. Fluctuaciones del Modelo Schrödinger-Poisson para una partícula supermasiva.

8. Modelo Gravitacional de una partícula supermasiva a propósito de la superdensidad de su masa y energía respectivamente.

$$M(a) = c_M M_J(a), \text{ con } M_J(a) \equiv \frac{4\pi}{3}\bar{\rho}a^3\lambda_J^3(a) \propto a^{-3/4}$$

$$\mathcal{N} \equiv \frac{1}{m}\int d^3x \sum_i |\psi_i|^2$$

$$E = M + \int d^3x \sum_i \left(\frac{1}{2m^2}|\nabla\psi_i|^2 + \frac{1}{2}\Phi|\psi_i|^2\right)$$

$$\psi_i = \frac{m}{\sqrt{4\pi G}}e^{-i\gamma mt}\chi_1(mr)u_i, \Phi = \Phi_1(mr)$$

$$M \simeq \frac{2\alpha}{Gm}, R \simeq \frac{1.9}{\alpha m}$$

$$MR \simeq \frac{3.9}{Gm^2}$$

$$\rho_s = \frac{\alpha^4 m^2}{4\pi G} \simeq \frac{1}{Gm^2R^4} \simeq \frac{G^3m^6M^4}{64\pi}$$

$$L_p = \frac{i}{2m}\epsilon_{pqr}\int d^3x\left(\psi_m^*\partial_q\psi_m x^r - \text{ c.c. }\right)$$
$$S_p = \frac{i}{m}\epsilon_{pqr}\int d^3x\psi_q\psi_r^*$$

$$\epsilon_{\hat{z}}^{(\pm 1)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm i\\ 0\end{pmatrix}, \epsilon_{\hat{z}}^{(0)} = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$\psi_i = \frac{m}{\sqrt{4\pi G}}e^{-i\gamma mt}\sum_\lambda \chi^{(\lambda)}(\vec{x})\left(\epsilon_{\hat{z}}^{(\lambda)}\right)_i$$

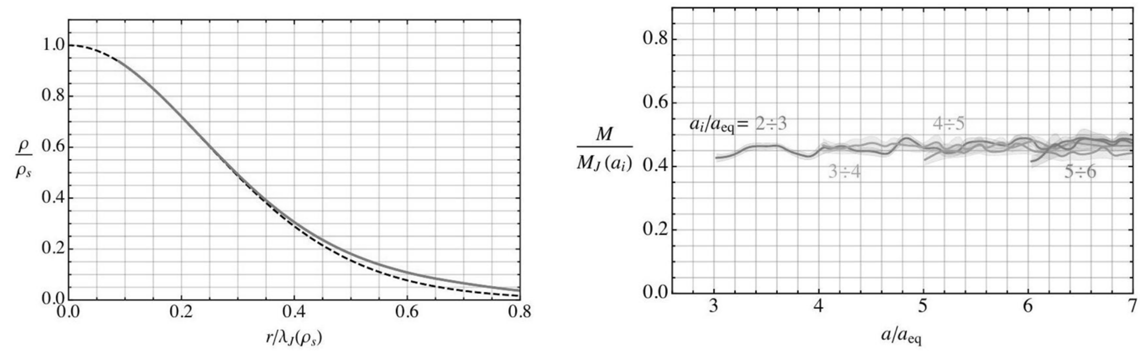


Figura 1. Fluctuaciones de la superdensidad de masa y energía de una partícula supermasiva en un campo de gauge específico.

9. Modelo de distribución de la masa en relación a una partícula supermasiva.

$$M_J^{\rm eq} \equiv M_J(a_{\rm eq}) = 5.2 \cdot 10^{-23} M_\odot \left(\frac{\rm eV}{m}\right)^{3/2}$$

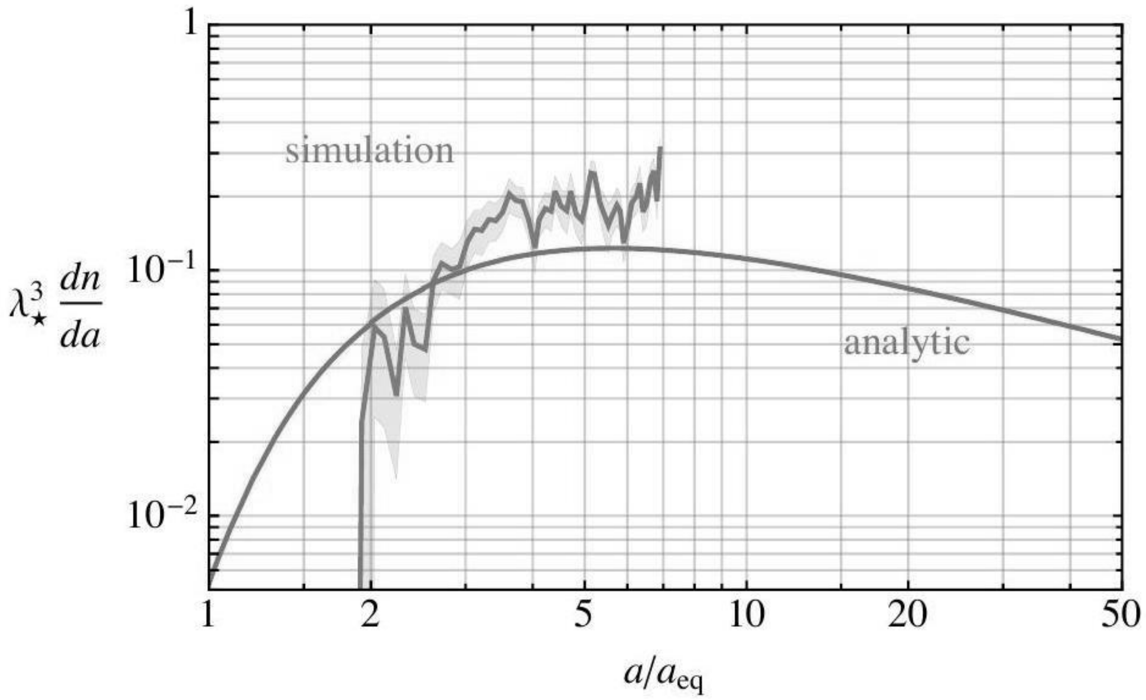
$$M(a) = 2.3 \cdot 10^{-23} M_\odot \left(\frac{c_M}{0.45}\right)\left(\frac{a_{\rm eq}}{a}\right)^{3/4}\left(\frac{\rm eV}{m}\right)^{3/2}$$

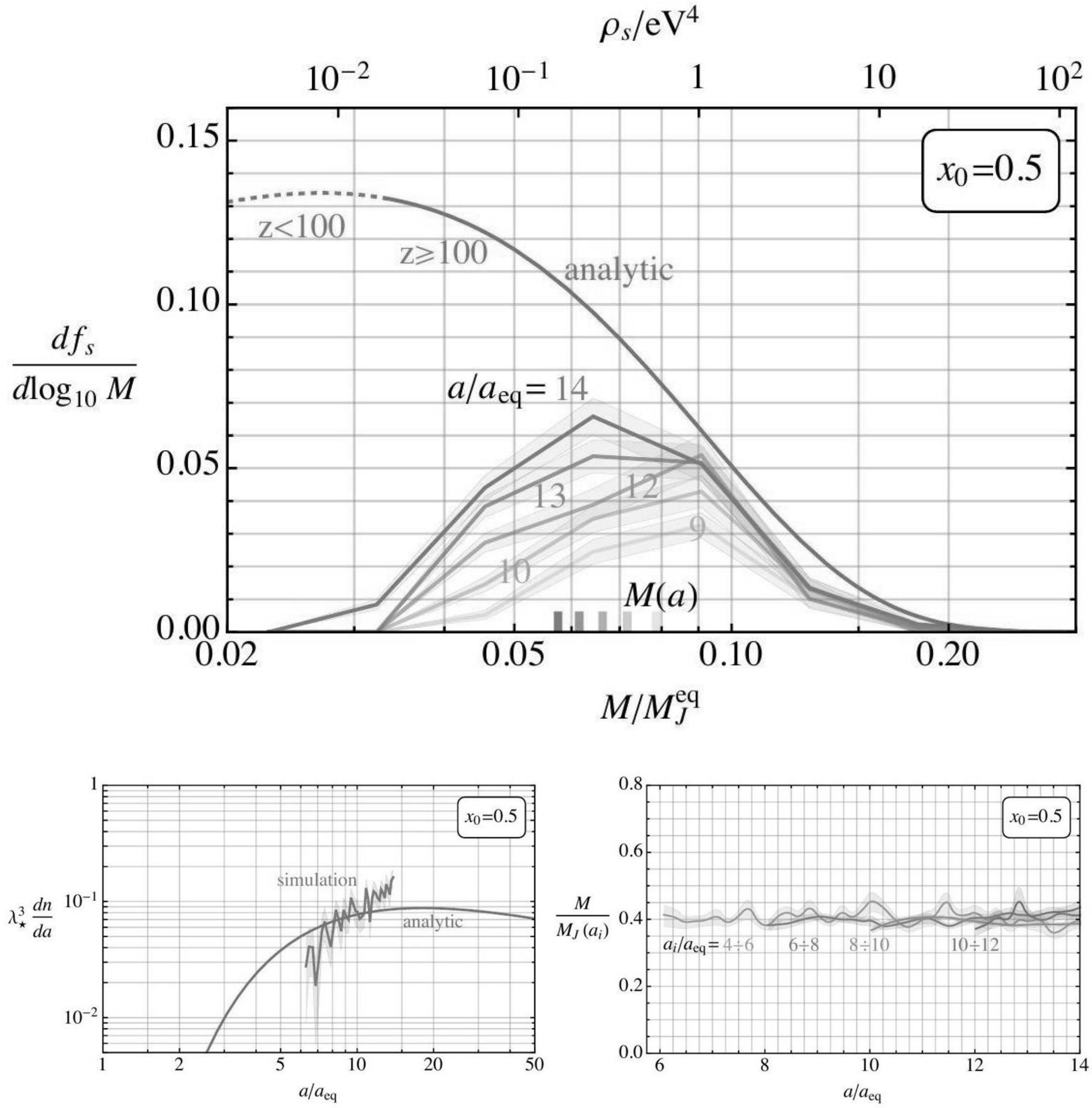
$$\frac{df_s(a,M)}{d\log\, M} = \frac{M}{\bar\rho}\frac{dn(a,M)}{d\log\, M}$$

$$\frac{\rho(r)}{\rho_s} = \frac{\rho_0}{\frac{r/\lambda_J(\rho_s)}{r_0}\left(1+\frac{r/\lambda_J(\rho_s)}{r_0}\right)^2}$$

$$\frac{df_s(a_2,M)}{d\log\, M} = \frac{M^2}{\bar\rho(a_2)}\int_{a_1}^{a_2} da\,\frac{d\Pi_{\delta>\delta_c}(a_1,a)}{da}\frac{\bar\rho(a)}{M(a)}\left(\frac{a}{a_2}\right)^3 \delta[M-M(a)] = \frac{d\Pi_{\delta>\delta_c}(a_1,a_M)}{d\log\, M}$$

$$\frac{df_s}{d\log\, M} \simeq \frac{\delta_c}{\sqrt{2\pi}\sigma(M)} e^{-\frac{\delta_c^2}{2\sigma^2(M)}}\left|\frac{d\log\,\sigma(M)}{d\log\, M}\right|,$$



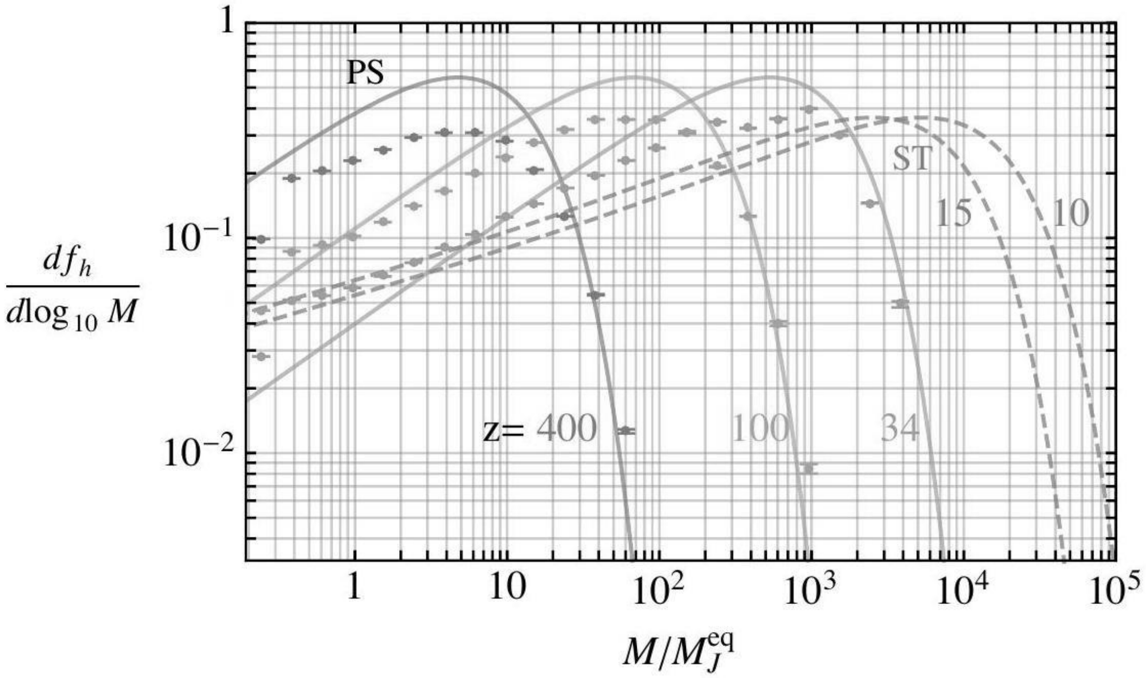


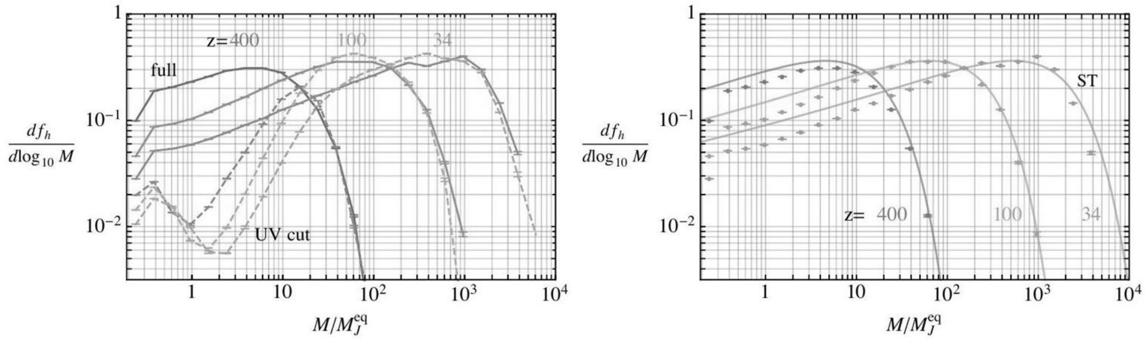
Figuras 1, 2 y 3. Fluctuaciones relativas a la distribución de masa y energía en relación a una partícula supermasiva.

10. Modelo de interacciones gravitacionales Press-Schechter de una partícula supermasiva.

$$\partial_t \rho + 3H\rho + a^{-1}\nabla \cdot (\rho\vec{v}) = 0$$
$$\partial_t \vec{v} + H\vec{v} + a^{-1}(\vec{v}\cdot\nabla)\vec{v} = -a^{-1}\nabla\Phi$$
$$\nabla^2\Phi = 4\pi G a^2(\rho - \bar{\rho})$$

$$\frac{df_{\mathrm{h}}(a,M)}{d\log\, M} = \sqrt{\frac{2}{\pi}}\nu e^{-\nu^2/2}\left|\frac{d\log\,\nu}{d\log\,M}\right| \simeq \sqrt{\frac{8M}{M_\star}}\frac{\pi\delta_c}{3^{3/4}D^2[a]}\exp\left[-\frac{\pi^3\delta_c^2}{3\sqrt{3}D^2[a]}\frac{8M}{M_\star}\right]$$





Figuras 1 y 2. Fluctuationes gravitacionales de una partícula supermasiva.

$$\frac{df_{\mathrm{h}}}{d\log\ M} = A(p)(1 + (q\nu^2)^{-p})(q\nu^2)^{1/2}e^{-q\nu^2/2}\left|\frac{d\log\ \nu}{d\log\ M}\right|$$

$$\rho_0 \ = \frac{4.4\cdot 10^{-3}c_\Delta^3\nu_c^3\Delta}{(c_\Delta + 1)^{-1} + \log\left(c_\Delta + 1\right) - 1}\left[\frac{M_J^{\mathrm{eq}}}{M}\right]^{3/2}\bar{\rho}^{\mathrm{eq}}$$

$$r_0 \ = \frac{4.2}{c_\Delta\nu_c\Delta^{1/3}}\left[\frac{M}{M_J^{\mathrm{eq}}}\right]^{5/6}\lambda_J^{\mathrm{eq}}$$

$$M \simeq 9.5M_\star\left[\frac{1.7}{\delta_c}\right]^2\left[\frac{100}{z+1}\right]^2 \simeq 65M_J^{\mathrm{eq}}\left[\frac{1.7}{\delta_c}\right]^2\left[\frac{100}{z+1}\right]^2$$

$$M_{\mathrm{h}} = (3 \div 5)\cdot 10^3 M_J^{\mathrm{eq}} \simeq (1.5 \div 3)\cdot 10^{-19}M_\odot(\mathrm{eV/m})^{3/2}$$

$$\rho(r) = \frac{\rho_0}{r/r_0(1 + r/r_0)^2}$$

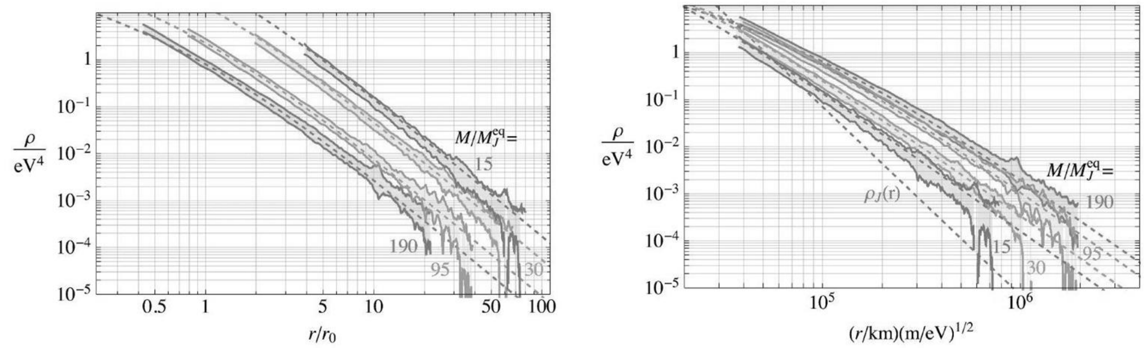


Figura 3: Fluctuaciones de masa a propósito de la gravedad cuántica endógena de la partícula supermasiva.

$$\rho_0 \simeq 0.7 \left[\frac{10^3 M_J^{\mathrm{eq}}}{M}\right]^{3/2} \bar{\rho}^{\mathrm{eq}} \simeq 0.3\mathrm{eV}^4 \left[\frac{10^3 M_J^{\mathrm{eq}}}{M}\right]^{3/2}$$

$$r_0 \simeq 5.4 \left[\frac{M}{10^3 M_J^{\mathrm{eq}}}\right]^{5/6} \lambda_J^{\mathrm{eq}} \simeq 7 \cdot 10^5 \text{ km} \left[\frac{M}{10^3 M_J^{\mathrm{eq}}}\right]^{5/6} \left[\frac{1\mathrm{eV}}{m}\right]^{1/2}$$

$$\langle \rho \rangle \simeq \frac{3\rho_0}{c_\Delta^3} (\log\ c_\Delta - 1)$$

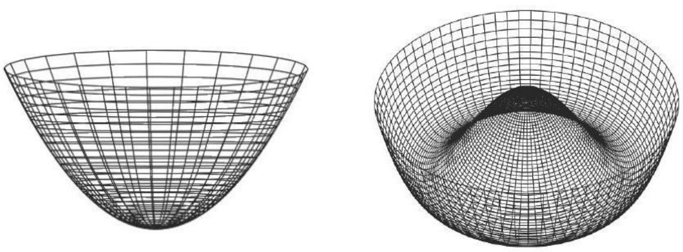


Figura. 4. Simulación de la curvatura del espacio – tiempo cuántico provocado por la interacción de una partícula supermasiva.

11. Modelo de aniquilación de una partícula supermasiva.

$$\rho_s \simeq 1.51 \cdot 10^4 \mathrm{eV}^4 \left(\frac{M}{M_J^{\mathrm{eq}}}\right)^4$$

$$\lambda_J(\rho_s) = 4.6 \cdot 10^3 \text{ km} \left(\frac{\mathrm{eV}}{m}\right)^{1/2} \left(\frac{M_J^{\mathrm{eq}}}{M}\right)$$

$$n = f_s \bar{\rho}(t_0)/M \simeq 10^{20} \mathrm{pc}^{-3} \left(\frac{f_s}{0.05}\right) \left(\frac{\rho_{\mathrm{local}}}{0.5\mathrm{GeV/cm}^3}\right) \left(\frac{0.1 M_J^{\mathrm{eq}}}{M}\right) \left(\frac{m}{\mathrm{eV}}\right)^{3/2}$$

$$\Gamma \simeq n\pi R^2 v_{\rm rel} \simeq \frac{0.1}{\rm yr}\left(\frac{m}{\rm eV}\right)^{1/2}\left(\frac{0.1M_J^{\rm eq}}{M}\right)^3\left(\frac{v_{\rm rel}}{10^{-3}}\right)\left(\frac{f_s}{0.05}\right)\left(\frac{\rho_{\rm local}}{0.5{\rm GeV/cm^3}}\right)$$

$$\aleph_{aniquilación} \simeq 10^2\ {\rm s}\left(\frac{0.1M_J^{\rm eq}}{M}\right)\left(\frac{\rm eV}{m}\right)^{1/2}$$

$$\mathcal{N} \simeq \left(\frac{2\pi}{k_\star/a_{\rm eq}}\right)^3\frac{\rho_{\rm eq}}{m} \simeq 90\frac{\rho_{\rm eq}^{1/4}}{G^{3/4}m^{5/2}} \simeq 30\left(\frac{10^{17}{\rm eV}}{m}\right)^{5/2} \gg 1$$

$$\mathcal{P}_{A_L}(t,k) = \left(\frac{k_\star H_I}{2\pi m}\right)^2\left(\frac{a_\star}{a}\right)F_{A_L}^2[k/k_\star] \simeq \left(\frac{k_\star H_I}{2\pi m}\right)^2\left(\frac{a_\star}{a}\right)\frac{(k/k_\star)^2}{1+(k/k_\star)^3}$$

$$\mathcal{P}_\delta(t,k) = \frac{k^2}{8\langle A_L^2\rangle^2}\int_0^\infty dq\int_{|q-k|}^{q+k} dp\,\frac{(k^2-q^2-p^2)^2}{q^4p^4}\mathcal{P}_{A_L}(t,p)\mathcal{P}_{A_L}(t,q) \simeq \frac{\sqrt{3}(k/k_\star)^3}{\pi((k/k_\star)^{3/2}+1)^{8/3}}$$

$$\Delta E = \frac{4}{3}\frac{G^2M_s^2r^2}{v_{\rm rel}^2b^4}dm$$

$$b_{\rm crit}^2 = \frac{M_s}{v_{\rm rel}}\left(\frac{G}{\pi\bar{\rho}(r)}\right)^{1/2}$$

$$p_{\rm dest} = \pi n\frac{S}{v_{\rm rel}}\left(\frac{G}{\pi\bar{\rho}(r)}\right)^{1/2}$$

$$p_{\rm dest} = 0.4\left(\frac{n}{100}\right)\left(\frac{0.05{\rm eV}^4}{\bar{\rho}(r)}\right)^{1/2}\left(\frac{S}{140M_\odot{\rm pc}^{-2}}\right)\left(\frac{10^{-3}}{v_{\rm rel}}\right)$$

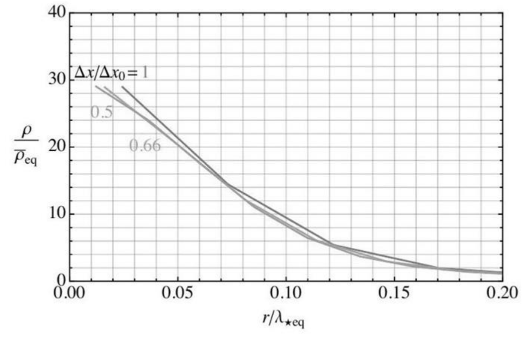
12. Modelo de superdensidades de masa y energía de una partícula supermasiva.

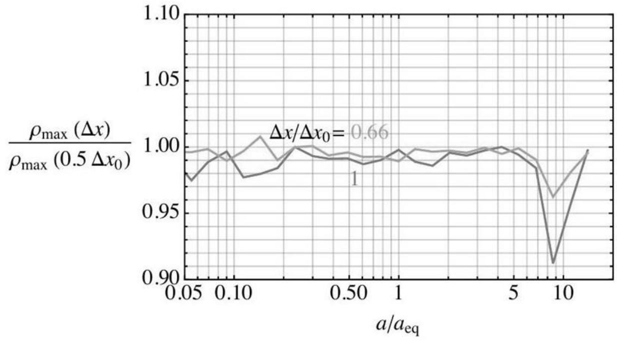
$$\ddot{\delta} + \frac{2}{a}\dot{a}\dot{\delta} - 4\pi G f\rho_{\rm nr}\delta = 0$$

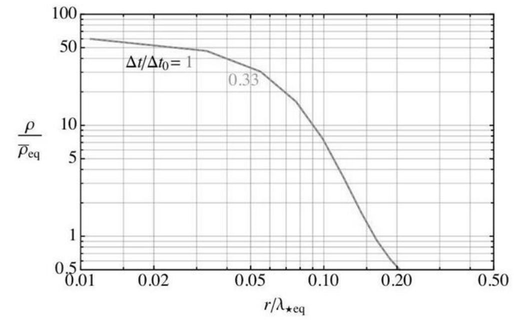
$$\frac{\partial^2\delta}{\partial y^2} + \frac{2+3y}{2y(1+y)}\frac{\partial\delta}{\partial y} - \frac{3f}{2y(1+y)}\delta = 0$$

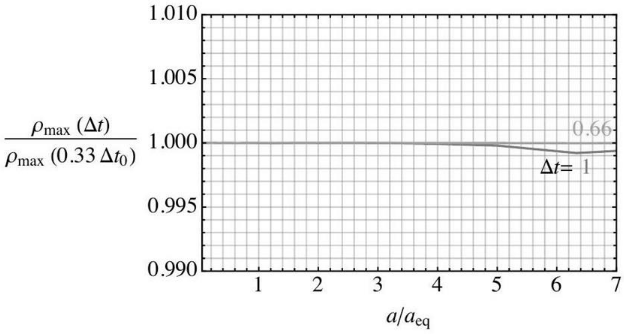
$$\frac{\partial^2\delta}{\partial y^2} + \frac{2+3y}{2y(1+y)}\frac{\partial\delta}{\partial y} - \frac{3f}{2y(1+y)}\left(-1+\left(\frac{k}{k_J}\right)^4\right)\delta$$

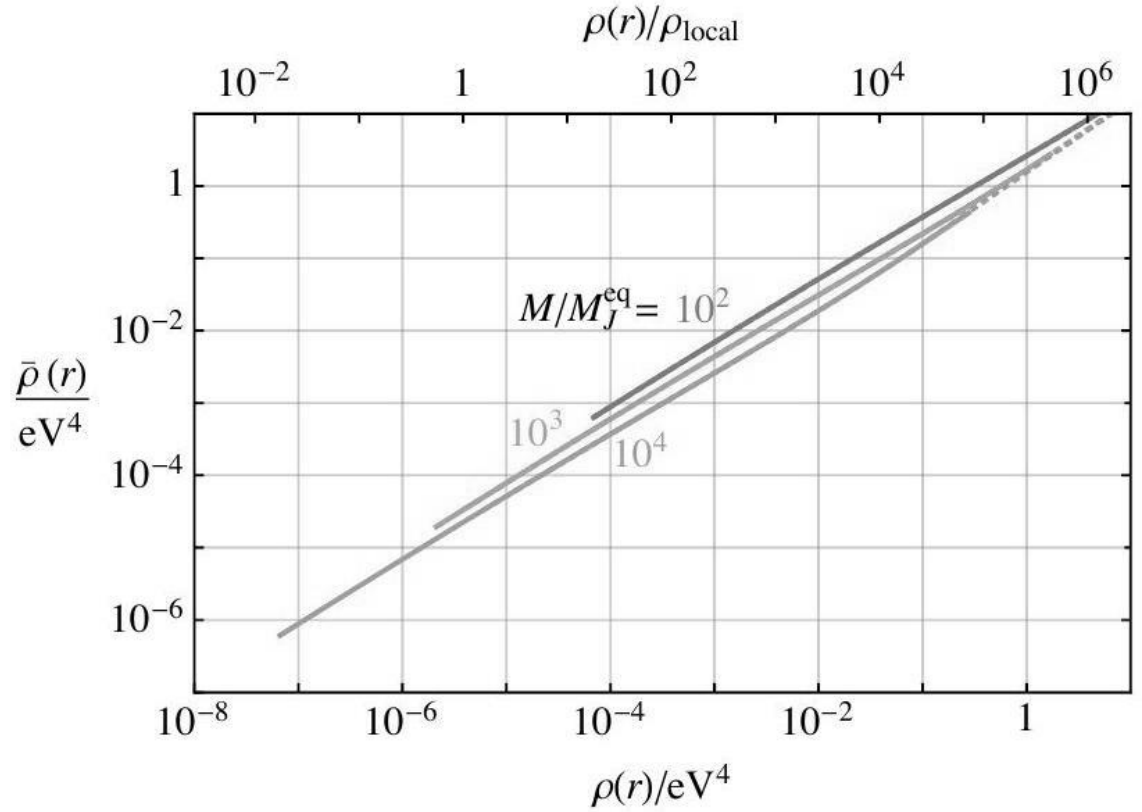
$$\psi_i(t+\Delta t) = \left(\prod_{\alpha=1}^{8} e^{-id_\alpha\Delta\tilde{t}\Phi(\mathbf{x})}e^{-ic_\alpha\Delta\tilde{t}k^2/2}\right)\psi_i(t)$$











Figuras 1, 2 y 3: Fluctuaciones de superdensidad de masa y energía de una partícula supermasiva.

$$\Delta E = 2(2\pi G\sigma_s(r))^2 \frac{(\Delta z)^2}{v_z^2} dm(1 + a^2)^{-3/2}$$

$$\bar{\rho}(r) \lesssim 10^{-6}\mathrm{eV}^4 \left(\frac{\sigma_s}{10^8 M_{\odot}/\mathrm{kpc}^2}\right)^2 \left(\frac{10^{-3}}{v_z}\right)^2$$

13. Modelo de coordenadas de una partícula repercutida por una partícula supermasiva.

$$r_t = r_{\text{orbit}} \left(\frac{M_{\text{clump}}(r_t)}{M_{\text{host}}(r_{\text{orbit}})}\right)^{1/3} \left(3 - \frac{d\log M_{\text{host}}(r)}{d\log r}\bigg|_{r=r_{\text{orbit}}}\right)^{-1/3}$$

$$t_{\text{decay}} = t_{\text{orbit}} \frac{M_{\text{host}}}{M_{\text{clump}}} \left(\log\left(\frac{M_{\text{host}}}{M_{\text{clump}}}\right)\right)^{-1} \simeq \frac{1}{\sqrt{G\bar{\rho}_{\text{host}}}} \frac{M_{\text{host}}}{M_{\text{clump}}} \left(\log\left(\frac{M_{\text{host}}}{M_{\text{clump}}}\right)\right)^{-1}$$

$$\Delta E \simeq \frac{4\pi}{3}\gamma_1 G\bar{\rho}_{\text{host}} r^2 dm$$

$$t_{\text{shock}} \simeq \frac{\bar{\rho}(r)}{\sqrt{G}\gamma_1\gamma_2\bar{\rho}_{\text{host}}^{3/2}} \simeq 10^9 \left(\frac{\bar{\rho}}{0.01\mathrm{eV}^4}\right)\left(\frac{1}{\gamma_1\gamma_2}\right)\left(\frac{10^{-4}\mathrm{eV}^4}{\bar{\rho}_{\text{host}}}\right)^{3/2}$$

$$t_{\text{dyn}} \simeq 3\cdot 10^{11} \left(\frac{10^{-4}\mathrm{eV}}{\bar{\rho}(r)_{\text{host}}}\right)^{1/2} \frac{M_{\text{host}}/M_s}{10^5} \frac{\log(10^5)}{\log(M_{\text{host}}/M_s)}$$

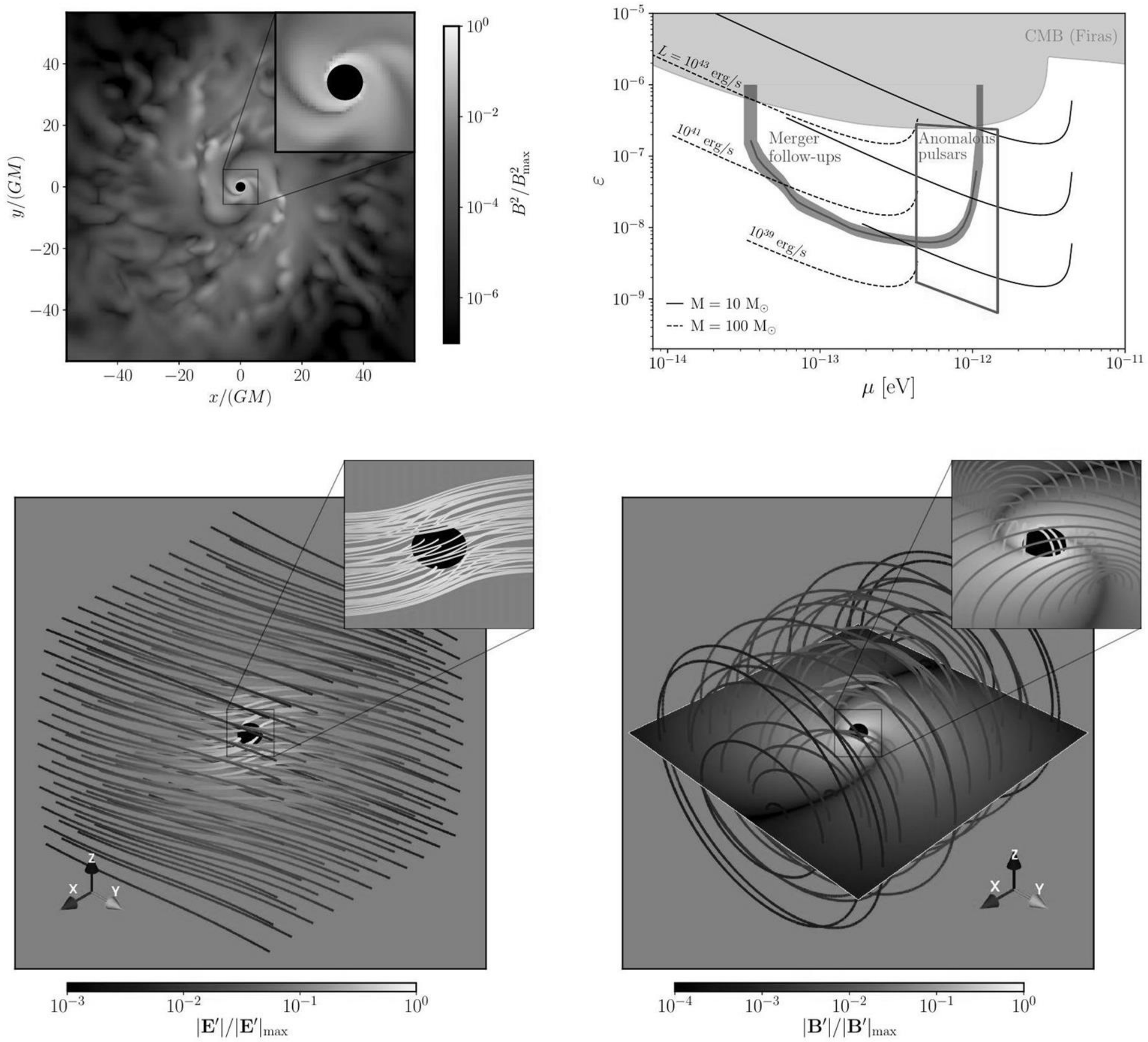
$$\rho_s/\bar{\rho}_{\text{host}}(r_{\text{orbit}}) \gtrsim 100$$

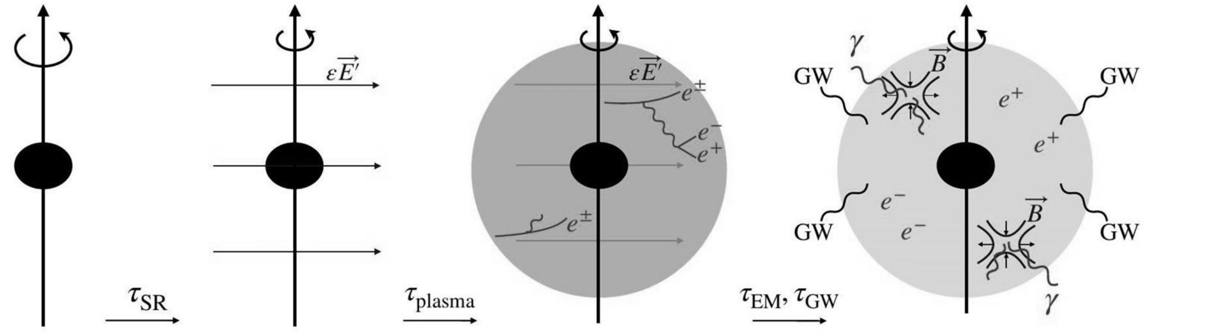
CONCLUSIONES.

La partícula oscura o partícula supermasiva, es un componente esencial pero no principal de la Teoría Cuántica de Campos Relativistas o Teoría Cuántica de Campos Curvos, formulada por este investigador, sin embargo, se destacan ideas nucleares, (i) en primer término, que la partícula oscura o supermasiva, se caracteriza por tener una masa excesivamente densa, lo que la vuelve superlativamente pesada, (ii) por otro lado, que la partícula supermasiva u oscura, aunque se trate de una partícula subatómica extremadamente densa, produce energía cuya densidad es superior a cero, más sin embargo, este fenómeno no sucede en todos los casos, (iii) esta propiedad de la partícula supermasiva u oscura, le permite deformar el espacio – tiempo cuántico en el que ineteractúa, curvándolo, repercutiendo así en el sistema de coordenadas de las partículas repercutidas, lo que se identifica como gravedad cujántica endógena, o en su defecto, provocando agujeros negros cuánticos, a propósito de su aniquilación, que puede ser causada por colisiones o en su defecto, por implosiones, esto último cuando se está ante una partícula supermasiva cuya masa y energía son superdensas. Ahora bien, en este punto es importante reflexionar que si una partícula supermasiva, provoca un agujero negro cuántico, la singularidad cuántica de éste, es la transformación de la materia y la energía, a propósito de la supergravedad cuántica y por ende, la existencia de multidimensiones o supermembranas de realidad microscópica. En esta singularidad, el tiempo abandona su condición de dimensión y pasa a convertirse en un objeto tridimensional, en tanto que el espacio, se funde con el tiempo, volviéndose multidimensional, infinitas veces, es decir, desplegando distintas capas de realidad, sí y sólo sí, se está ante un escenario de aniquilación y supergravedad cuántica.

Apéndice A.

Agujeros negros cuánticos provocados por una partícula supermasiva. Características funcionales y morfológicas y configuraciones de campo.





Figuras 1, 2 y 3. Simulación de un agujero negro cuántico provocado por la deformación del espacio – tiempo cuántico, a propósito de las interacciones de una partícula supermasiva.

$$\mathcal{L}' = -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{1}{2}\mu^2 A'^{\mu}A'_{\mu}.$$

$$\omega \simeq \mu\left(1 - \frac{\alpha^2}{2}\right)$$

$$\Gamma_{\rm SR} \equiv \tau_{\rm SR}^{-1} \simeq 4\alpha^7(\Omega_{\rm BH} - \omega) \simeq 4a_*\alpha^6\mu,$$

$$\Omega_{\rm BH} = \frac{1}{2}\left(\frac{a_*}{1 + \sqrt{1 - a_*^2}}\right)r_g^{-1}.$$

$$\Omega_{\rm BH} \leq \omega,$$

$$\alpha \lesssim 1/2$$

$$M_c \simeq 10^{-2}\left(\frac{\Delta a_*}{0.1}\right)\left(\frac{\alpha}{0.1}\right)M$$

$$A'_0 = \frac{\sqrt{M_c}}{\sqrt{\pi}\mu^2 r_c^{5/2}}e^{-r/r_c}\sin\,\theta\sin\,(\omega t - \phi)$$

$$\mathbf{A}' = -\frac{\sqrt{M_c}}{\sqrt{\pi}\mu r_c^{3/2}}e^{-r/r_c}\{\cos\,\omega t, \sin\,\omega t, 0\}$$

$$P_{\rm GW} \simeq 17\frac{\alpha^{10}}{G}\left(\frac{M_c(t)}{M}\right)^2$$

$$M_c(t) = \frac{M_c(t_0)}{1 + (t - t_0)/\tau_{\rm GW}},$$

$$\tau_{\rm GW} \simeq \frac{GM}{17\alpha^{11}\Delta a_*} \sim \zeta\left(\frac{0.1}{\Delta a_*}\right)\left(\frac{0.1}{\alpha}\right)^{11}\left(\frac{M}{10M_\odot}\right)$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{\mu^2}{2}A'_{\mu}A'^{\mu} - \varepsilon\mu^2 A'_{\mu}A^{\mu} + I_{\mu}A^{\mu}$$

$$\begin{aligned}\nabla_\alpha F^{\alpha\beta} &= -I^\beta + \varepsilon\mu^2 A'^\beta\\ \nabla_\alpha F'^{\alpha\beta} &= \mu^2 A'^\beta + \varepsilon\mu^2 A^\beta\end{aligned}$$

$$\begin{aligned}\nabla_\alpha T^{\alpha\beta} &= -F^{\beta\gamma}\left(I_\gamma - \varepsilon\mu^2 A'_\gamma\right),\\ \nabla_\alpha T'^{\alpha\beta} &= \varepsilon\mu^2 F'^{\beta\gamma} A_\gamma.\end{aligned}$$

$$|\varepsilon\mathbf{E}'| \simeq \frac{\varepsilon\sqrt{\Delta a_*}\alpha^{5/2}\mu}{\sqrt{G}} \simeq 2\cdot 10^{13}\ \mathrm{V/m}\sqrt{\Delta a_*}\left(\frac{\varepsilon}{10^{-7}}\right)\left(\frac{\alpha}{0.1}\right)^{5/2}\left(\frac{\mu}{10^{-12}\mathrm{eV}}\right)$$

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

$$\begin{aligned}\rho &\simeq \varepsilon\nabla\cdot\mathbf{E}' \simeq \pm\frac{\varepsilon\sqrt{\Delta a_*}\alpha^{7/2}\mu^2}{\sqrt{G}}\\ &\simeq \pm 5\cdot 10^7\ \mathrm{cm}^{-3}\sqrt{\Delta a_*}\left(\frac{\varepsilon}{10^{-7}}\right)\left(\frac{\alpha}{0.1}\right)^{7/2}\left(\frac{\mu}{10^{-12}\mathrm{eV}}\right)^2\end{aligned}$$

$$\partial_t\mathbf{B} = \frac{\varepsilon\mu^2\mathbf{B}'}{\sigma} + \frac{1}{\sigma}\nabla^2\mathbf{B} + \nabla\times(\mathbf{v}\times\mathbf{B}),$$

$$\gamma_e \simeq \frac{e\varepsilon|\mathbf{E}'|}{m_e\mu} \simeq e\varepsilon\alpha^{5/2}\sqrt{\Delta a_*}\frac{M_{\mathrm{pl}}}{m_e} \simeq 10^{12}\left(\frac{\varepsilon}{10^{-7}}\right)\left(\frac{\alpha}{0.1}\right)^{5/2}\left(\frac{\Delta a_*}{0.1}\right)^{1/2}$$

$$\Gamma_{\mathrm{syn}} \simeq \frac{2}{3}\frac{e^2\gamma_e}{r_c} \simeq \frac{2}{3}\frac{e^3\varepsilon|\mathbf{E}'|}{m_e} \simeq \frac{2}{3}e^3\varepsilon\alpha^{5/2}\sqrt{\Delta a_*}\mu\frac{M_{\mathrm{Pl}}}{m_e},$$

$$\varepsilon > \frac{1}{e^3\alpha^{5/2}\sqrt{\Delta a_*}}\frac{m_e}{M_{\mathrm{Pl}}} \simeq 10^{-18}\left(\frac{0.1}{\alpha}\right)^{5/2}\left(\frac{0.1}{\Delta a_*}\right)^{1/2}$$

$$\frac{\Gamma_{e^\pm}}{V} = \frac{(e|\mathbf{E}|)^2}{4\pi^3}\sum_n^\infty \frac{1}{n^2}\exp\left(-\frac{\pi m_e^2}{e|\mathbf{E}|}n\right)$$

$$S_{\mathrm{B}} = -\gamma_\theta + \left(\frac{2m_e^2}{e\varepsilon|\mathbf{E}'|} + \frac{e\varepsilon|\mathbf{E}'|}{2m_e^2}\gamma_\theta^2\right)\arctan\frac{2m_e^2}{e\varepsilon|\mathbf{E}'|\gamma_\theta},$$

$$\Gamma_{e^\pm}^\gamma = \frac{\alpha_{\mathrm{EM}}}{2\pi}\frac{e\varepsilon|\mathbf{E}'|}{m_e}\exp\left(-\frac{2m_e^2}{e\varepsilon|\mathbf{E}'|}\frac{2m_e}{\omega_{\mathrm{syn}}}\right) = \frac{\alpha_{\mathrm{EM}}}{2\pi}\frac{e\varepsilon|\mathbf{E}'|}{m_e}\exp\left[-\frac{4m_e^6\mu^2}{(e\varepsilon|\mathbf{E}'|)^4}\right]$$

$$\varepsilon \gg \left(\ln\frac{\alpha_{\mathrm{EM}}\gamma_e}{2\pi\alpha}\right)^{-1/4}\frac{m_e^{3/2}\sqrt{2\mu}}{e|\mathbf{E}'|} \approx \left(\ln\frac{\alpha_{\mathrm{EM}}\gamma_e}{2\pi\alpha}\right)^{-1/4}\frac{1}{e\alpha^{5/2}\sqrt{\Delta a_*}}\frac{m_e}{M_{\mathrm{pl}}}\sqrt{\frac{2m_e}{\mu}} \simeq 10^{-10}\left(\frac{0.1}{\alpha}\right)^{5/2}\left(\frac{0.1}{\Delta a_*}\right)^{1/2}\left(\frac{10^{-12}\mathrm{eV}}{\mu}\right)^{1/2}\left(\frac{\log\frac{\alpha_{\mathrm{EM}}\gamma_e}{2\pi\alpha}}{20}\right)^{-1/4}$$

$$\tau_{\mathrm{plasma}} \simeq \frac{1}{2\Gamma_{e^\pm}^\gamma}\ln\left(\frac{n_e^f}{n_e^0}\right) \simeq \frac{1}{2\alpha\mu}\ln\frac{m_e^{3/2}}{e^2\alpha^2\mu^{3/2}}$$

$$\frac{\tau_{\mathrm{plasma}}}{\tau_{\mathrm{SR}}} \simeq 2a_*\alpha^5\ln\frac{m_e^{3/2}}{e^2\alpha^2\mu^{3/2}}$$

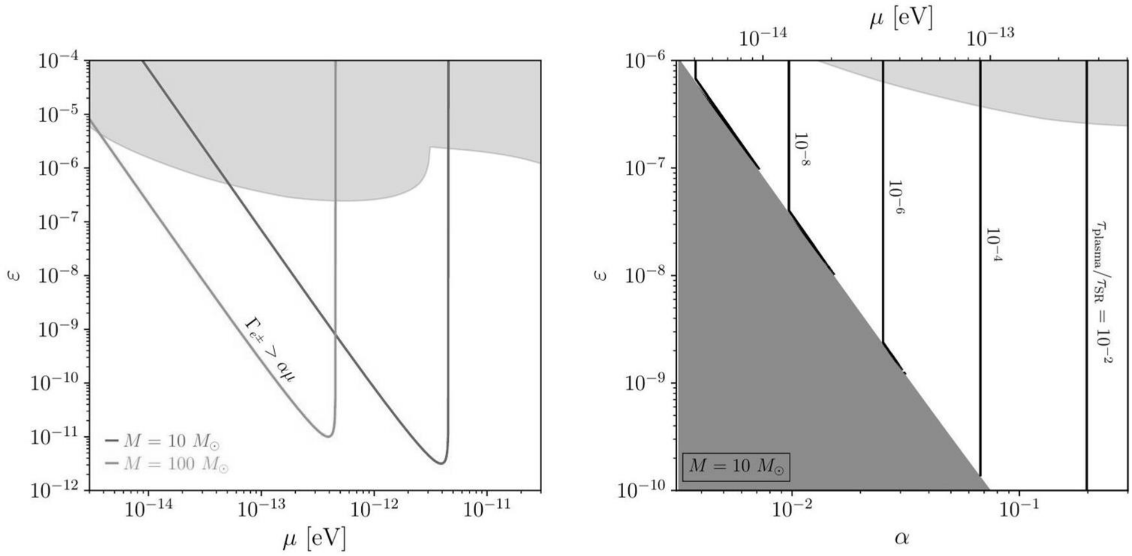


Figura 4. Espectro de radiación de un agujero negro cuántico.

$$M_{\text{plasma}} \approx \frac{m_e \rho}{e(\alpha\mu)^3} \simeq \varepsilon \alpha^{1/2} (\Delta a_*)^{1/2} \frac{m_e M_{\text{pl}}}{\mu} \simeq 10^{-29} M_\odot \left(\frac{\epsilon}{10^{-7}}\right) \left(\frac{\alpha}{0.1}\right)^{1/2} \left(\frac{\Delta a_*}{0.1}\right)^{1/2} \left(\frac{10^{-12}\text{eV}}{\mu}\right)$$

$$\tau_{\text{acc}} = \frac{M_{\text{plasma}}}{\dot{M}_{\text{Bondi}}} \approx \varepsilon \alpha^{-3/2} (\Delta a_*)^{1/2} \frac{c_s^3 M_{\text{pl}} \mu}{\pi n_M} \simeq 10 \text{ years} \left(\frac{\varepsilon}{10^{-7}}\right) \left(\frac{\alpha}{0.1}\right)^{-3/2} \left(\frac{\Delta a_*}{0.1}\right)^{1/2} \left(\frac{\mu}{10^{-12}\text{eV}}\right) \left(\frac{1/\text{cm}^3}{n_M}\right) \left(\frac{c_s}{1}\right)^3$$

$$E^i \equiv n_\nu F^{i\nu}, B^i \equiv n_\nu (*F)^{i\nu} = \frac{1}{2} n_\nu \varepsilon^{i\nu\alpha\beta} F_{\alpha\beta}$$

$$J^i = I^i - \rho_q n^i, \qquad \rho_q = -n_\mu I^\mu$$

$$\rho_q = D_i E^i - \varepsilon \mu^2 n_\mu A'^\mu$$

$$j^\mu = \sigma e^\mu$$

$$e^\mu \equiv u_\nu F^{\mu\nu}, j^\mu \equiv I^\mu + (u^\nu I_\nu) u^\mu$$

$$v^i_{d,\text{ ideal}} = \frac{\varepsilon^{ijk} E_j B_k}{B^2},$$

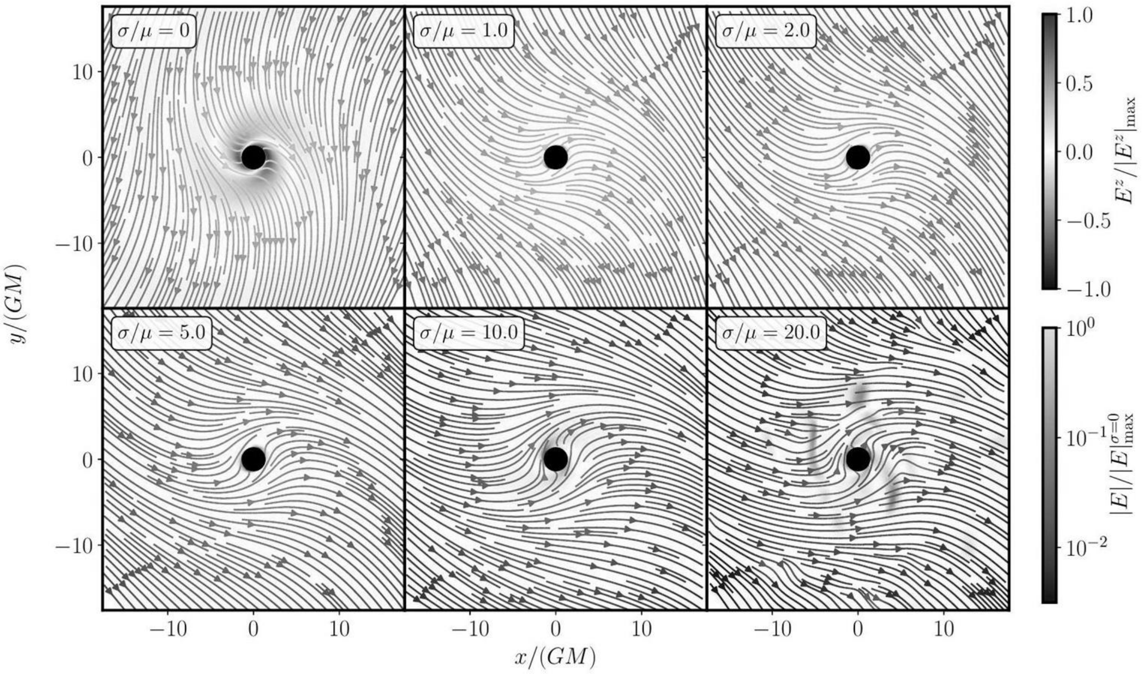
$$v^i_d = \frac{\varepsilon^{ijk} E_j B_k}{B^2 + E_0^2}, E_0^2 = B_0^2 + E^2 - B^2$$

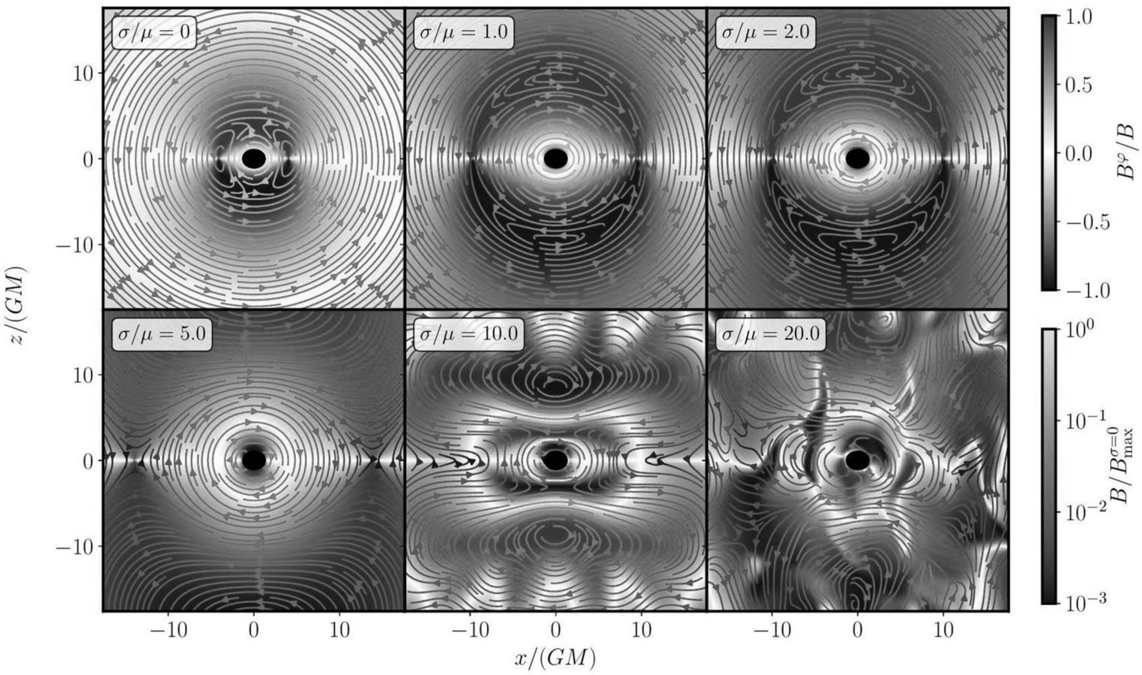
$$B_0^2 = \frac{1}{2}\left[B^2 - E^2 + \sqrt{(B^2 - E^2)^2 + 4(E_i B^i)^2}\right]$$

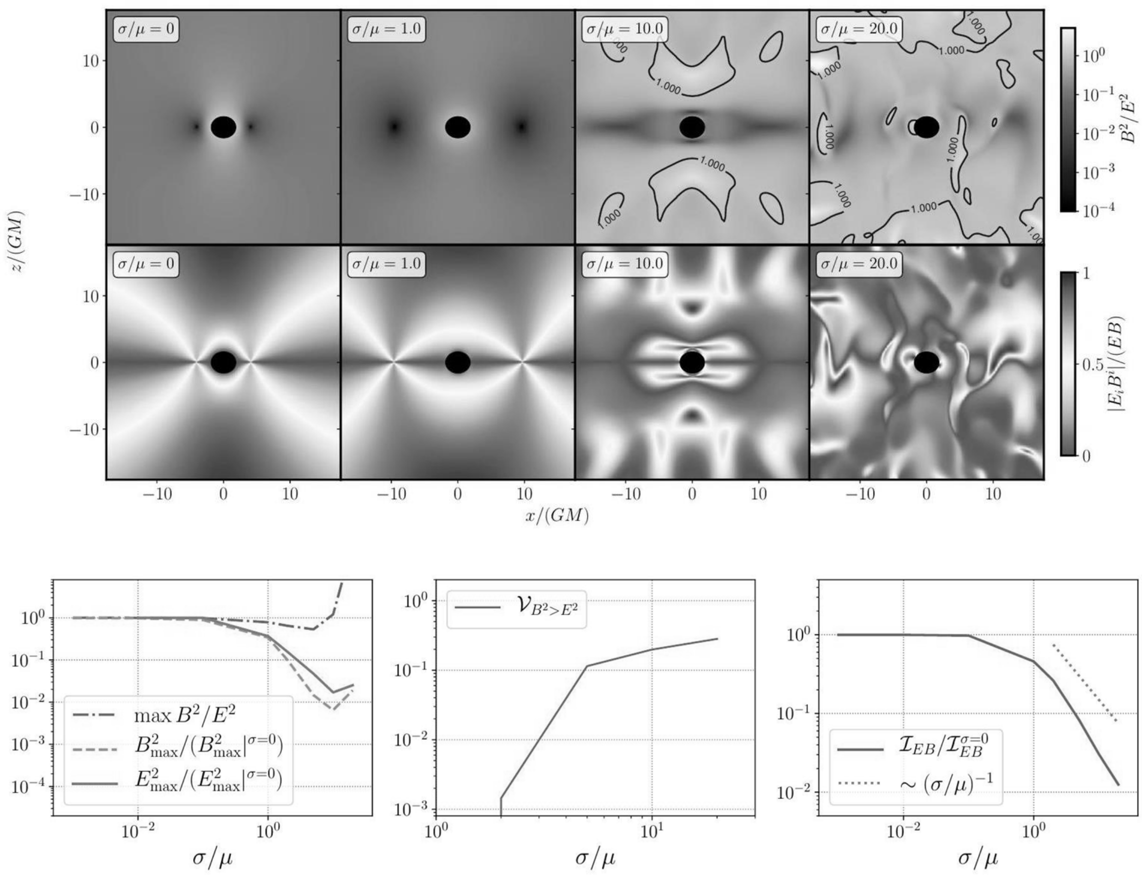
$$J^i = \rho_q v^i_d + \sigma E_0 \sqrt{\frac{B^2 + E_0^2}{B_0^2 + E_0^2}} \left(\frac{E_0 E^i + B_0 B^i}{B^2 + E_0^2}\right)$$

$$\partial_t E^i \approx -\sigma E^i + \varepsilon \mu^2 A'^i$$

$$\mathbf{E} \propto \begin{pmatrix} \sigma + i\mu \\ \mu - i\sigma \\ 0 \end{pmatrix} e^{-i\omega t} + c.c.$$



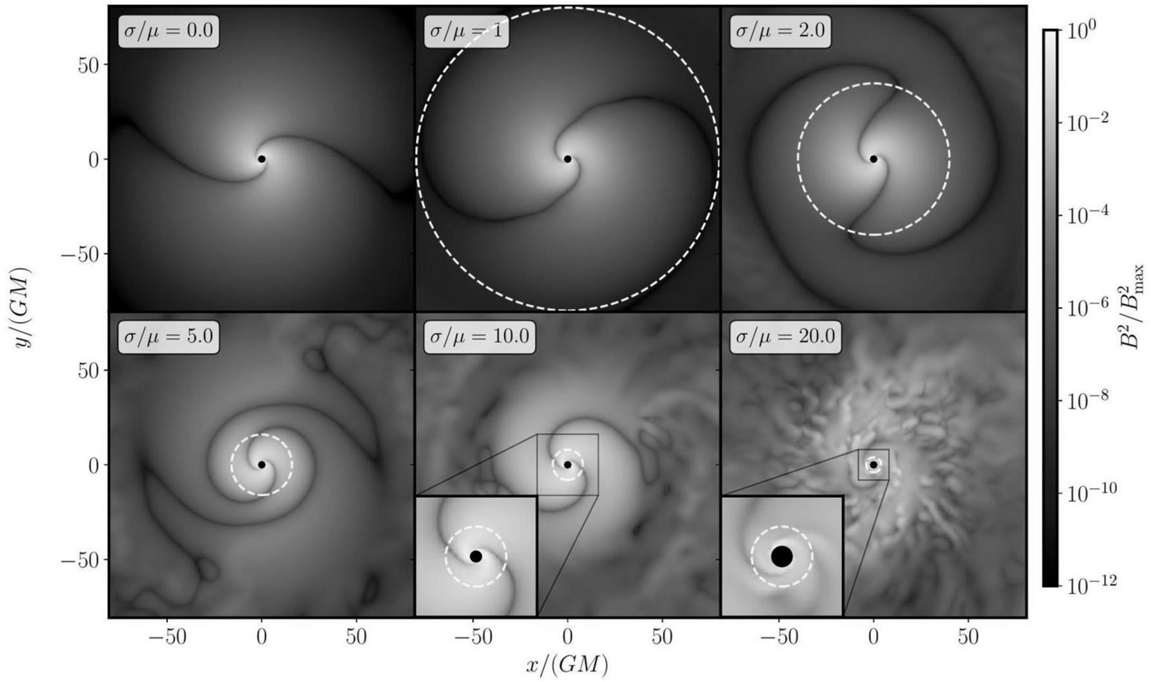


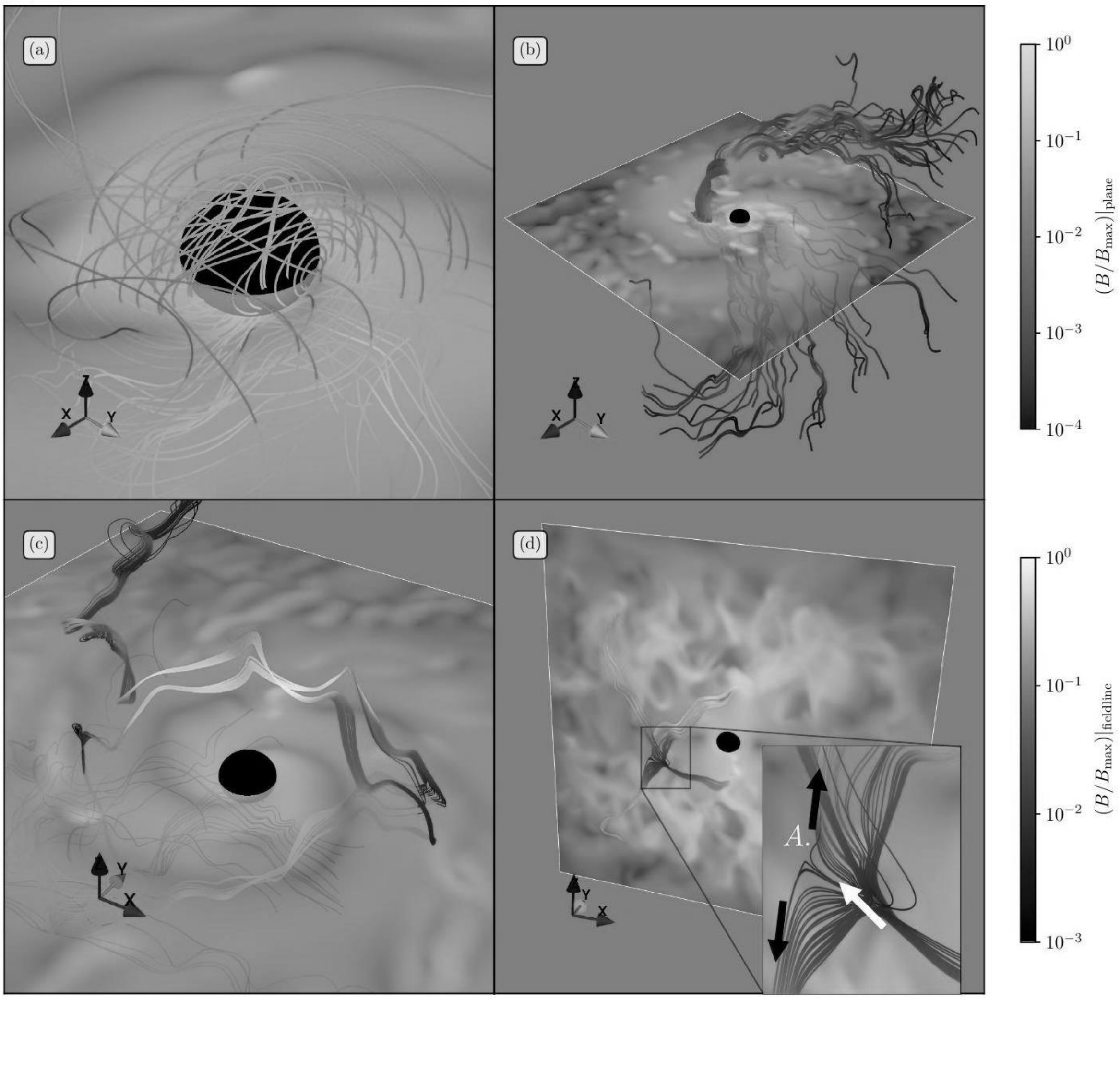


Figuras 5, 6, 7 y 8. Campos de energía desplegados por un agujero negro cuántico provocado por una partícula supermasiva.

$$\partial_t B^i = \frac{\varepsilon \mu^2 B'^i}{\sigma} + \frac{1}{\sigma}\partial_j \partial^j B^i + \varepsilon^{ijk}\varepsilon_{klm}\partial_j v^l B^m$$

$$r_* \approx 80\mu GM/\sigma$$


$\sigma/\mu = 0.0$
$\sigma/\mu = 1$
$\sigma/\mu = 2.0$
$\sigma/\mu = 5.0$
$\sigma/\mu = 10.0$
$\sigma/\mu = 20.0$
$y/(GM)$
$x/(GM)$
$B^2/B^2_{\rm max}$
50
0
−50
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}


(a)
(b)
(c)
(d)
A.
X
Y
Z
$(B/B_{\rm max})|_{\rm plane}$
$(B/B_{\rm max})|_{\rm fieldline}$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}

Figuras 9 y 10. Configuraciones de campo del agujero negro cuántico – deformación del espacio – tiempo cuántico provocado por una partícula supermasiva.

$$\mathcal{E} = \int_D d^3x\sqrt{\gamma}T^{\alpha}{}_{\mu}n_{\alpha}\xi^{\mu}$$

$$T_{\mu\nu} = F_{\mu}{}^{\lambda}F_{\lambda\nu} - \frac{1}{4}g_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$$

$$\partial_t\mathcal{E} = -P_{\mathrm{EM}} - \dot{\mathcal{E}}_{\mathrm{BH}} - L_{\mathrm{diss}} + \dot{\mathcal{E}}_{\mathrm{A}'}$$

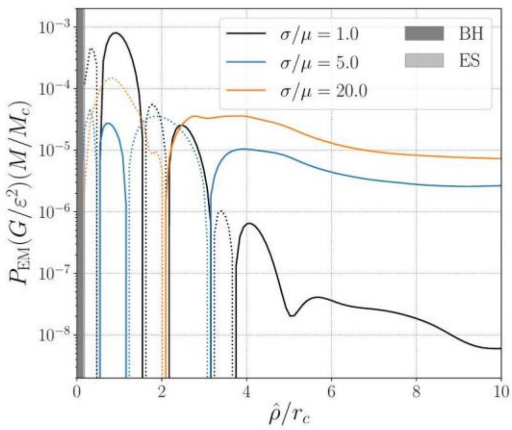
$$P_{\mathrm{EM}} = -\oint_{S^2_{\hat{\rho}}} d\Omega_{\mu}T^{\mu}_{\nu}\xi^{\nu} \overset{\text{curved}}{=} \oint_{S^2_{\hat{\rho}}} d\Omega\hat{\boldsymbol{\rho}}\cdot(\boldsymbol{E}\times\boldsymbol{B})$$

$$\dot{\mathcal{E}}_{\mathrm{BH}} = -\oint_{S^2_{\mathrm{BH}}} d\Omega_{\mu}T^{\mu}{}_{\nu}\xi^{\nu}$$

$$L_{\mathrm{diss}} = -\int_D d^3x\sqrt{-g}F^{\alpha\beta}\xi_{\alpha}I_{\beta} \overset{\text{curved}}{=} \int_D d^3x\boldsymbol{E}\cdot\boldsymbol{J}$$

$$\rho_{\mathrm{diss}} = NF^{\alpha\beta}\xi_{\alpha}I_{\beta}$$

$$\dot{\mathcal{E}}_{\mathrm{A}'} = -\varepsilon\mu^2\int_D d^3x\sqrt{-g}F^{\alpha\beta}\xi_{\alpha}A'_{\beta} \overset{\text{flat}}{=} \varepsilon\mu^2\int_D d^3x\boldsymbol{E}\cdot\boldsymbol{A}'$$



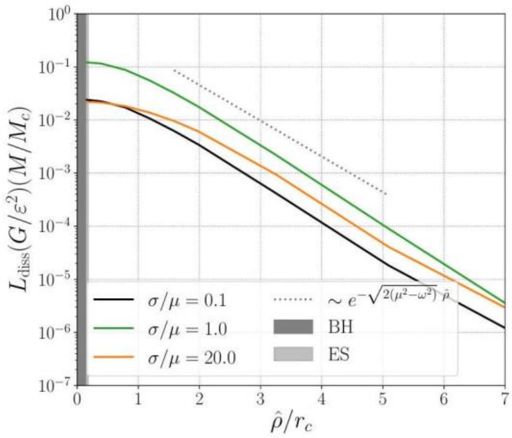


Figura 11. Coordenadas de un agujero negro cuántico.

$$L_{\mathrm{diss}} = L^{\mathrm{bulk}}_{\mathrm{diss}} + L^{\mathrm{turb}}_{\mathrm{diss}}$$

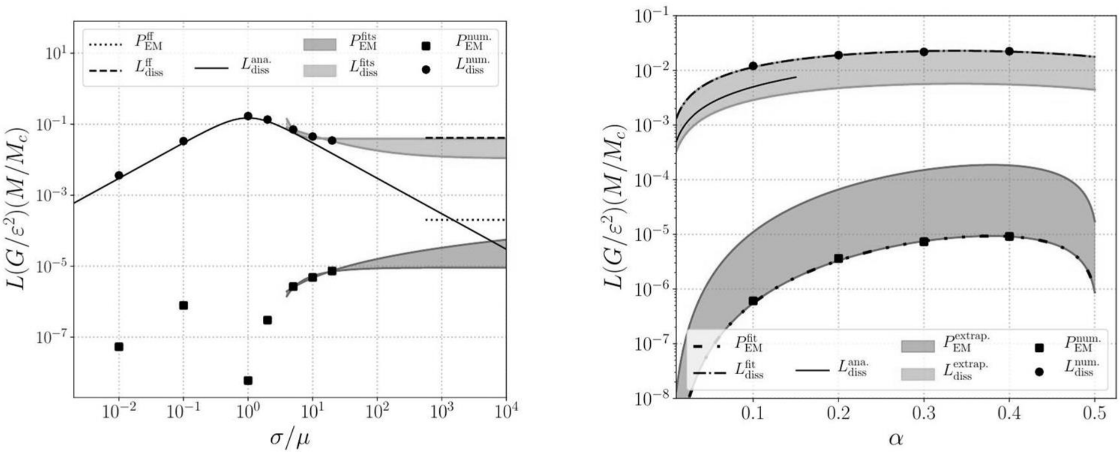


Figura 12. Disipación de un agujero negro cuántico.

$$\mathbf{J} = \sigma\mathbf{E} = -\frac{e^{-r/r_c}\sqrt{M_c\mu}\,\alpha^{3/2}\varepsilon\sigma\omega^2}{2\sqrt{\pi}(\sigma^2+\omega^2)}\begin{pmatrix}\sigma + i\omega\\ -\omega + i\sigma\\ 0\end{pmatrix}e^{-i\omega t} + c.c.$$

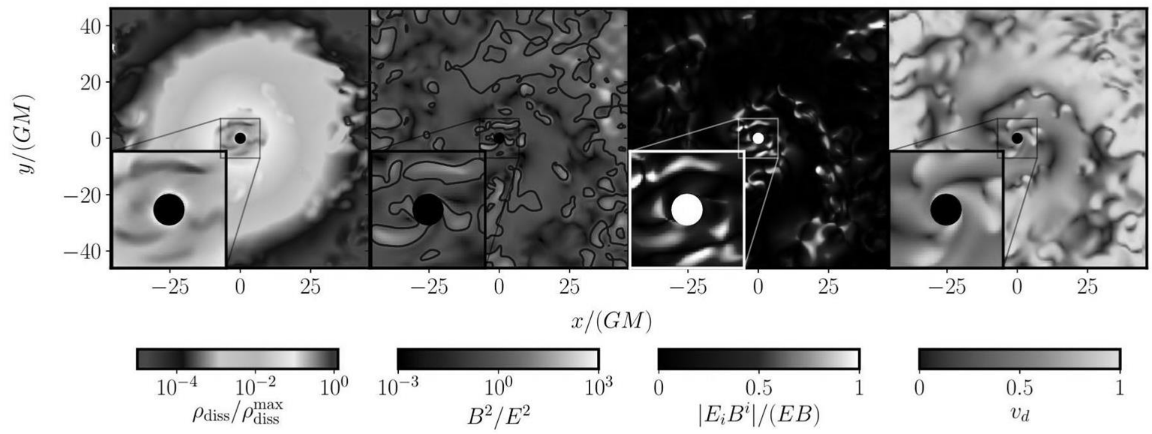
$$L_{\rm diss}^{\rm bulk} = \frac{\sigma\alpha\varepsilon^2}{\mu(1+(\sigma/\mu)^2)}\frac{M_c}{GM}$$

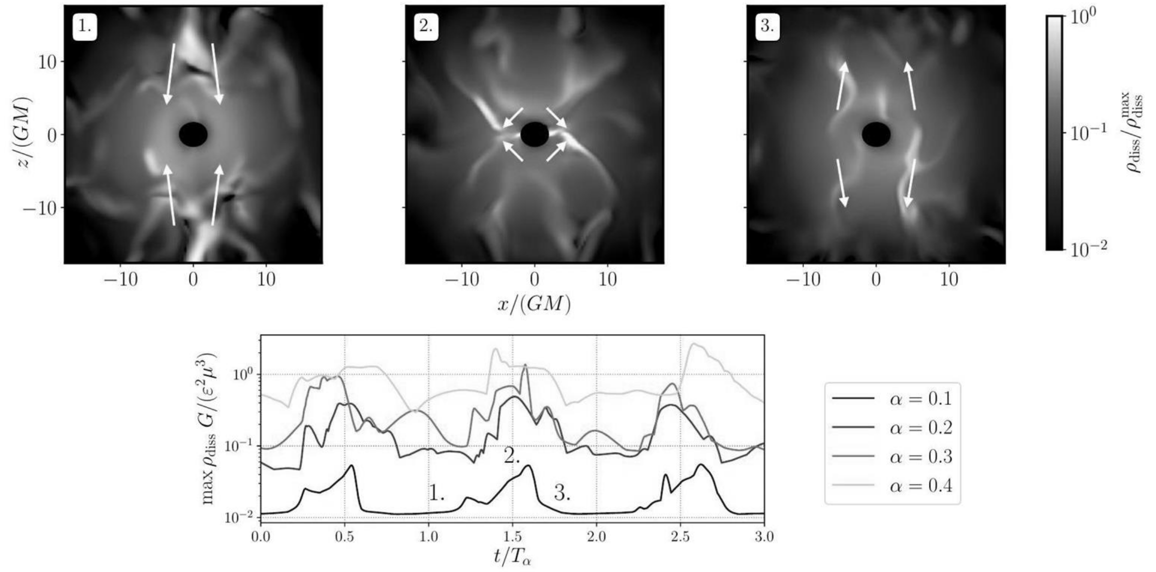
$$L_{\rm diss}^{\rm fit} = \varepsilon^2 F(\alpha)\frac{M_c}{GM},$$

$$P_{\rm EM}^{\rm fit} = \varepsilon^2 G(\alpha)\frac{M_c}{GM},$$

$$F(\alpha) = 1.31\times 10^{-1}\alpha - 1.88\times 10^{-1}\alpha^2$$

$$G(\alpha) = 6.86\times 10^{-4}\alpha^3 - 1.36\times 10^{-3}\alpha^4$$





Figuras 13, 14 y 15. Densidad de un agujero negro cuántico.

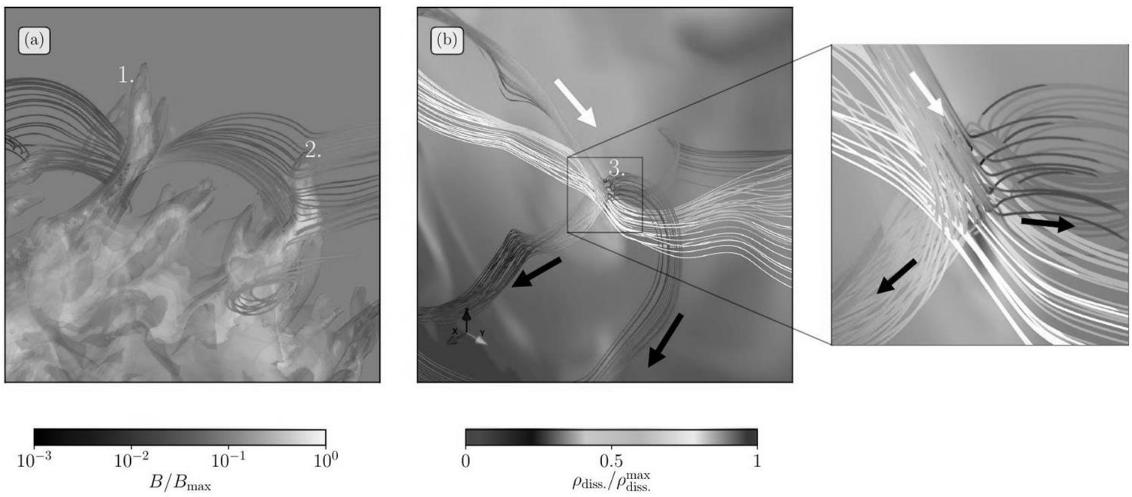


Figura 16. Comportamiento de la materia y la energía en un agujero negro cuántico.

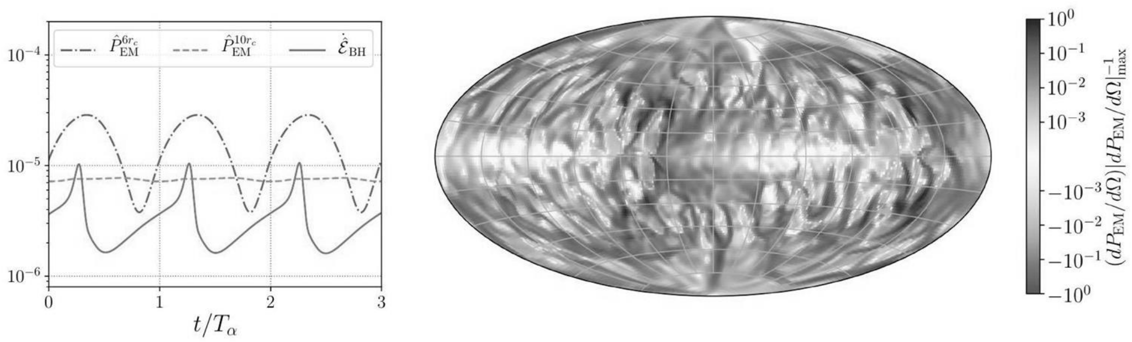


Figura 17. Radiación de un gujero negro cuántico

$$\langle B^2\rangle^{1/2} = 2.5\times 10^8 \text{ Gauss } \left(\frac{\varepsilon}{10^{-7}}\right)\left(\frac{M_\odot}{M}\right)\left(\frac{\alpha}{0.1}\right)^{5/2}$$

$$\gamma_c \approx 2.2\times 10^7 \left(\frac{\varepsilon}{10^{-7}}\right)\left(\frac{\alpha}{0.1}\right)^{3/2}$$

$$\gamma_r = 3\times 10^3 \left(\frac{10^{-7}}{\varepsilon}\right)^{1/2}\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{0.1}{\alpha}\right)^{5/4}$$

$$\nu_{\text{peak}} = 12\text{keV}\left(\frac{\gamma}{10^2}\right)^2\left(\frac{\varepsilon}{10^{-7}}\right)\left(\frac{M_\odot}{M}\right)\left(\frac{\alpha}{0.1}\right)^{5/2},$$

$$t_{\text{growth}} \sim \ln\left(M_c/\mu\right)\tau_{\text{SR}} \approx 10^4 \text{ s}\left(\frac{M}{10M_\odot}\right)\left(\frac{0.7}{a_*}\right)\left(\frac{0.1}{\alpha}\right)^7$$

$$L_{\text{EM}} = \varepsilon^2 F(\alpha)\frac{M_c}{GM} \simeq \varepsilon^2\frac{\alpha^2\Delta a_*}{G} \simeq 4\times 10^{41}\text{erg/s}\left(\frac{\varepsilon}{10^{-7}}\right)^2\left(\frac{\alpha}{0.1}\right)^2\left(\frac{\Delta a_*}{0.1}\right)$$

$$\tau_{\text{GW}} \approx \frac{GM}{17\alpha^{11}\Delta a_*} \approx 10^6 \text{ s}\left(\frac{M}{10M_\odot}\right)\left(\frac{0.1}{\alpha}\right)^{11}\left(\frac{0.1}{\Delta a_*}\right)$$

$$\tau_{\text{EM}} \approx \frac{GM\ln 2}{\varepsilon^2 F(\alpha)} \approx 10^{11} \text{ s}\left(\frac{M}{10M_\odot}\right)\left(\frac{10^{-7}}{\varepsilon}\right)^2\left(\frac{10^{-2}}{F(0.1)}\right)$$

$$\frac{M_c(t)}{M_c(t_0)} = \begin{cases}\left[1+(t-t_0)/\tau_{\text{GW}}\right]^{-1} & \tau_{\text{GW}} \ll \tau_{\text{EM}}\\ e^{-(t-t_0)/\tau_{\text{EM}}\ln 2} & \tau_{\text{GW}} \gg \tau_{\text{EM}}\end{cases}$$

$$\tau_{\text{SR}} \simeq 0.98\tau_{\text{GW}}^{7/11} r_g^{4/11}\left[a_*\left(\frac{\Delta a_*}{0.5}\right)^{7/11}\right]$$

$$\dot{f}_{\text{int}} \simeq \frac{5}{8\pi}\alpha\mu^2 G P_{\text{GW}}$$

$$g^{\alpha\beta}\nabla_\alpha\nabla_\beta A'^\gamma = \mu^2 A'^\gamma$$

$$A'^\mu = B^{\mu\nu}\nabla_\nu Z, Z = R(r)S(\theta)e^{-i(\omega T - m\varphi)}$$

$$R_{\text{near}}(r) = \hat{r}^{i\kappa}(1+\hat{a}_1\hat{r}+\hat{a}_2\hat{r}^2+\cdots),$$

$$\tau = T + \frac{M^2 \log \frac{r - r_+}{r - r_-}}{\sqrt{M^2 - a^2}} + M \log \Delta$$
$$x = \sin \theta (r\cos \bar{\phi} - a\sin \bar{\phi})$$
$$y = \sin \theta (a\cos \bar{\phi} + r\sin \bar{\phi})$$
$$z = r\cos \theta$$

$$T = t - \frac{M^2 \log \frac{r - r_+}{r - r_-}}{\sqrt{M^2 - a^2}} - M \log \Delta$$
$$r = 2^{-1/2} \left[-a^2 + x^2 + y^2 + z^2 + \sqrt{4a^2 z^2 + (a^2 - x^2 - y^2 - z^2)^2} \right]^{1/2}$$
$$\phi = \arctan \left[\frac{rx + ay}{-ax + ry} \right] - \frac{a \log \frac{r - r_+}{r - r_-}}{2\sqrt{M^2 - a^2}}, x > 0$$
$$\theta = \arccos \frac{z}{r}$$

$$L_D(f) \equiv \int_D d^3 x \sqrt{\gamma} |f|$$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + \frac{2Mr^3}{r^4 + a^2 M^2 z^2} \left[dt + \frac{z}{r} dz + \frac{r(xdx + ydy)}{r^2 + a^2 M^2} - \frac{aM(xdy - ydx)}{r^2 + a^2 M^2} \right]^2$$

$$D_t E^i = NKE^i + \varepsilon^{ijk} D_j (NB_k) - NJ^i + N\varepsilon\mu^2 \gamma^i{}_\mu A'^\mu$$
$$D_t B^i = NKB^i - \varepsilon^{ijk} D_j (NE_k)$$
$$D_i E^i = \rho_q + \varepsilon\mu^2 n_\mu A'^\mu$$
$$D_i B^i = 0$$

$$D_t \Psi = -N\left(D_i E^i - \varepsilon\mu^2 n_\mu A'^\mu - \rho_q\right) - N\kappa\Psi$$
$$D_t \Phi = -N D_i B^i - N\kappa\Phi$$

$$E^i \rightarrow E^j \left(\delta_j^i - \frac{B_j B^i}{B^2} \right)$$
$$E^i \rightarrow E^i \left\{ 1 - \hat{\theta}(\lambda) + \hat{\theta}(\lambda) \frac{B}{E} \right\}$$

$$J^a = \rho_q v_d^a + J_\perp^a$$

$$v_d^a = \frac{\varepsilon^{ajk} E_j B_k}{B^2}$$

$$J_{\perp,\mathrm{FF}}^a = \frac{B^a}{B^2} \left[2KB_i E^i - 2K_{ij} E^i B^j + B_i \epsilon^{ijk} D_j B_k - E_i \epsilon^{ijk} D_j E_k + \varepsilon\mu^2 B_i \gamma_\mu^i A'^\mu \right]$$

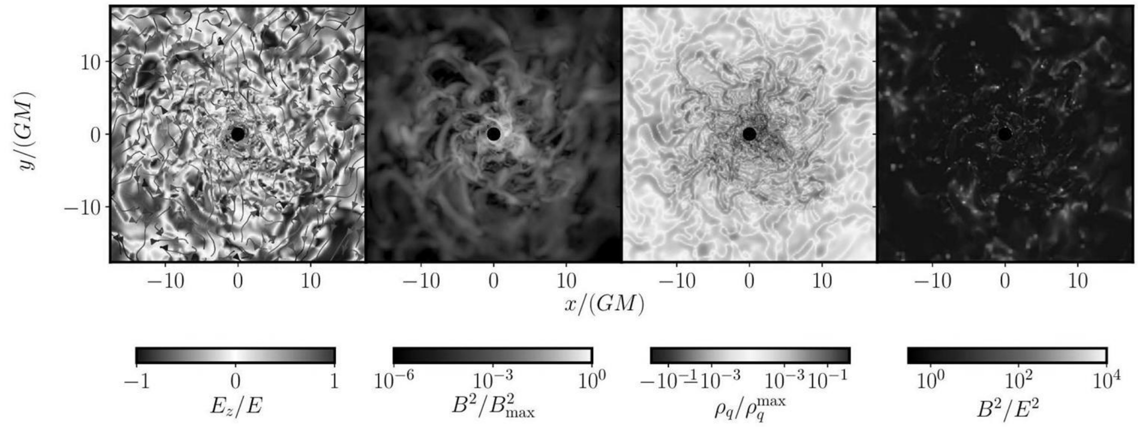


Figura 18. Radio y horizonte de eventos de un agujero negro cuántico.

$$v_d^i = \frac{\varepsilon^{ijk} E_j B_k}{B^2 + E_0^2}, E_0^2 = B_0^2 + E^2 - B^2,$$

$$B_0^2 = \frac{1}{2}\left[B^2 - E^2 + \sqrt{(B^2 - E^2)^2 + 4(E_i B^i)^2}\right].$$

$$J^a_{\perp,(B)} = \sigma E_0 \sqrt{\frac{B^2 + E_0^2}{B_0^2 + E_0^2}} \frac{E_0 E^a + B_0 B^a}{B^2 + E_0^2}$$

$$J^a_{\perp,(C)} = \frac{\sigma}{(\sigma + \kappa)}\left(J^a_{\perp,\mathrm{FF}} + \kappa E^i B_i \frac{B^a}{B^2}\right)$$

$$D_t(E_i B^i) = -\kappa N\left(E^i - \frac{1}{\sigma} J^i\right) B_i,$$

$$D_t \rho_q = N \rho_q K - D_i(N J^i) = N \rho_q K - D_i(N \rho_q v_d^i) = 0$$

$$\mathcal{I}_\rho = \int_D d^3x \sqrt{\gamma} |\rho_q|$$

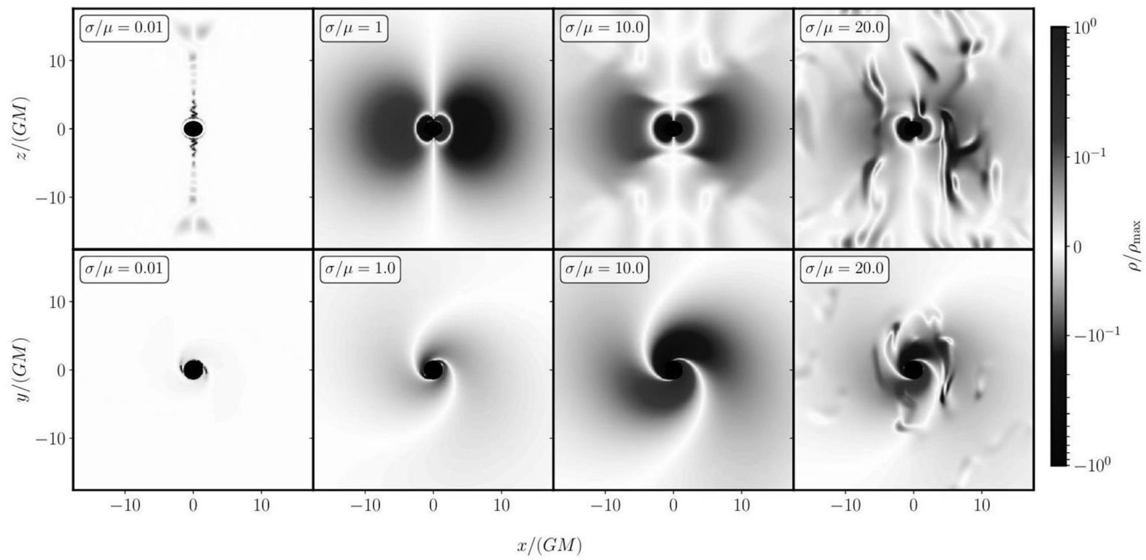
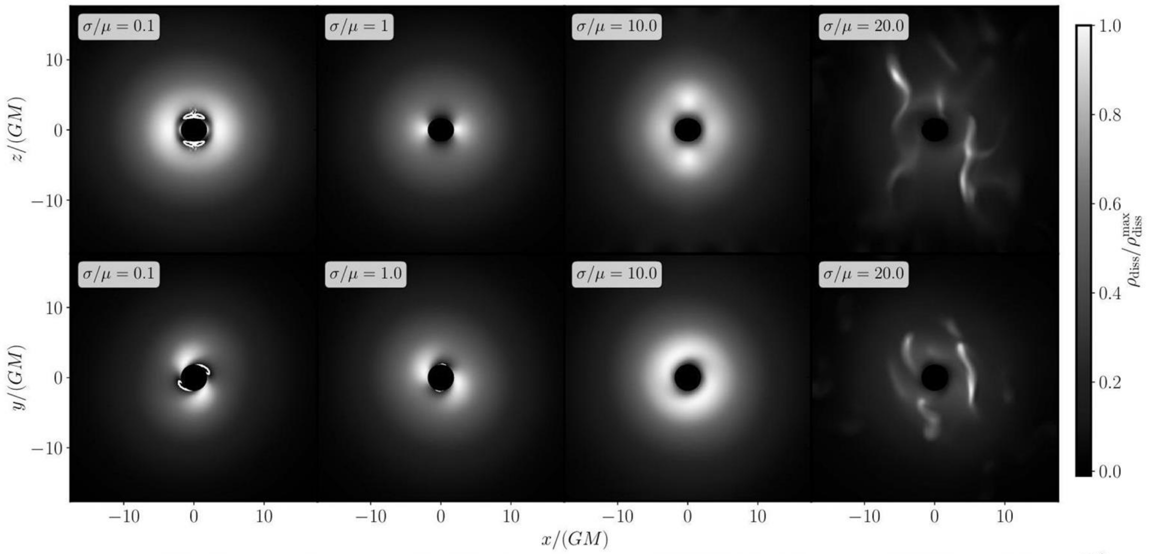


Figura 19. Fluctuaciones gravitacionales de un agujero negro cuántico.

$$\mathcal{I}_{E'} = \int_D d^3x\sqrt{\gamma}\varepsilon\left|D_i E'^i\right|$$



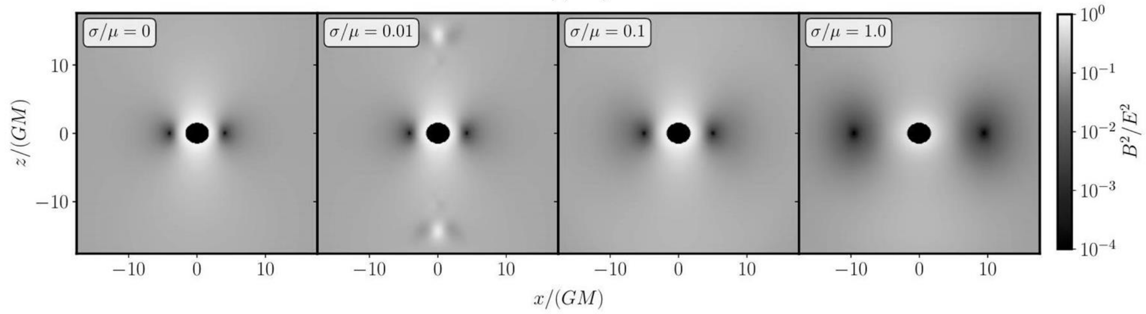


Figura 20. Simulación de un agujero negro cuántico.

$$\mathcal{L}_{\rm mass} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{4}\mathcal{F}'_{\mu\nu}\mathcal{F}'^{\mu\nu} - \frac{\mu^2}{2}\mathcal{A}'_\mu\mathcal{A}'^\mu + I_\mu(\mathcal{A}^\mu + \varepsilon\mathcal{A}'^\mu)$$

$$\mathcal{L}_{\rm inter} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu}$$

$$-\frac{\mu^2}{2}A'_\mu A'^\mu - \varepsilon\mu^2 A'_\mu A^\mu + I_\mu A^\mu$$

$$\nabla_\alpha\mathcal{F}^{\alpha\beta} = -I^\beta$$

$$\nabla_\alpha\mathcal{F}'^{\alpha\beta} = \mu^2\mathcal{A}'^\beta - \varepsilon I^\beta$$

$$\nabla_\alpha F^{\alpha\beta} = -I^\beta + \varepsilon\mu^2 A'^\beta$$

$$\nabla_\alpha F'^{\alpha\beta} = \mu^2 A'^\beta + \varepsilon\mu^2 A^\beta$$

$$\nabla_\alpha\mathcal{T}^{\alpha\beta} = -\mathcal{F}^{\beta\gamma}I_\gamma$$

$$\nabla_\alpha\mathcal{T}'^{\alpha\beta} = -\varepsilon\mathcal{F}'^{\beta\gamma}I_\gamma$$

$$\nabla_\alpha T^{\alpha\beta} = -F^{\beta\gamma}\left(I_\gamma - \varepsilon\mu^2 A'_\gamma\right)$$

$$\nabla_\alpha T'^{\alpha\beta} = \varepsilon\mu^2 F'^{\beta\gamma} A_\gamma$$

$$\nabla^\mu A'_\mu = 0, \qquad \nabla_\mu I^\mu = 0$$

Apéndice B.

Electrodinámica cuántica de una partícula supermasiva. Modelo matemático.

$$\frac{\mathrm{d}S_\alpha}{\mathrm{d}\tau} = \Gamma^\lambda_{\alpha\nu} S_\lambda \frac{\mathrm{d}x^\nu}{\mathrm{d}\tau}$$

$$\frac{\mathrm{d}\mathbf{S}}{\mathrm{d}t} = -\mathbf{S}\frac{\partial\phi}{\partial t} - 2\mathbf{v}\cdot\mathbf{S}\nabla\phi - \mathbf{S}(\mathbf{v}\cdot\nabla\phi) + \mathbf{v}(\mathbf{S}\cdot\nabla\phi) + \frac{1}{2}\mathbf{S}\times(\nabla\times\mathbf{S})$$

$$\mathbf{S}_1 = (1+\phi)\mathbf{S} - \frac{1}{2}\mathbf{v}(\mathbf{v}\cdot\mathbf{S}),$$

$$\frac{\mathrm{d}\mathbf{S}_1}{\mathrm{d}t} = \mathbf{\Omega}\times\mathbf{S}_1$$

$$\frac{\delta S}{S} \sim Tv\frac{\phi}{Rc^2}$$

$$\frac{\delta S}{S} \sim 4\times 10^{-3}\left(\frac{v}{2\times 10^{-3}}\right)^3 \frac{T}{13\times 10^9\mathrm{yr}}\frac{8\mathrm{kpc}}{R}$$

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{\sin\ \alpha}{2}F^{\mu\nu}X_{\mu\nu} + eJ^\mu_{\mathrm{EM}}A_\mu + \frac{m_X^2\cos^2\ \alpha}{2}X^\mu X_\mu$$

$$\mathcal{L} \supset -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{4}\tilde{X}_{\mu\nu}\tilde{X}^{\mu\nu} + \frac{e}{\cos\ \alpha}J^\mu_{\mathrm{EM}}\tilde{A}_\mu + \frac{m_X^2\cos^2\ \alpha}{2}\left(\tilde{X}^\mu\tilde{X}_\mu + 2\chi\tilde{X}_\mu\tilde{A}^\mu + \chi^2\tilde{A}^\mu\tilde{A}_\mu\right)$$

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + eJ^\mu_{\mathrm{EM}}A_\mu + \frac{m_X^2}{2}\left(X^\mu X_\mu + 2\chi X_\mu A^\mu\right)$$

$$-K^2 A^\nu = \chi m_X^2 X^\nu.$$

$$X^\mu_c(t,\mathbf{x}) = \sqrt{V}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}X^\mu(\mathbf{k})e^{-i(\omega t - \mathbf{kx} + \delta(\mathbf{k}))}.$$

$$\rho = \frac{1}{V}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\omega(\mathbf{k})^2}{2}|\mathbf{X}(\mathbf{k})|^2$$

$$\langle X^\mu_c(t)\rangle = \frac{1}{V}\int \mathrm{d}^3\mathbf{x}X^\mu_c(t,\mathbf{x}) = \frac{X^\mu(\mathbf{k}=0)}{\sqrt{V}}e^{-im_X t} \equiv X^\mu_0 e^{-im_X t}$$

$$\rho = \rho \int \mathrm{d}^3\mathbf{v} f_{\mathrm{lab}}(\mathbf{v})$$

$$f_{\mathrm{lab}}(\mathbf{v}) = \frac{m_X^3 \omega^2}{2(2\pi)^3 \rho} |\mathbf{X}(\mathbf{k})|^2$$

$$\rho = \frac{m_X^2}{2} \left\langle |\mathbf{X}_c(t)\right|^2 \right\rangle$$

$$\rho \simeq \frac{m_X^2}{2} |\langle \mathbf{X}_c(t)\ |\rangle|^2 = \frac{m_X^2}{2} |\mathbf{X}_0|^2$$

$$|\mathbf{E}_0| = \left| \frac{\chi m_X}{\epsilon} \mathbf{x}_0 \right|.$$

$$|\mathbf{E}_0| = \left| \frac{\chi m_X}{\epsilon} \mathbf{X}_0 \cos\ \theta \right|,$$

$$\nabla \times \mathbf{B}_X = \mathbf{J}_X$$

$$\nabla \times \mathbf{E}_X = -\frac{\partial \mathbf{B}_X}{\partial t}$$

$$P(t) = P_X \cos^2\ \theta(t)$$

$$\frac{S}{N} = \frac{P}{T_{\mathrm{sys}}} \sqrt{\frac{T}{\Delta v_{\mathrm{DP}}}}$$

$$\frac{S}{N} = 2\left(\sqrt{n_s + n_d} - \sqrt{n_d}\right)$$

$$n_s = \eta \int \mathrm{d}t \frac{P(t)}{\omega}$$

$$\frac{1}{T} \int \mathrm{d}t P(t) \equiv P_X \langle \cos^2\ \theta \rangle_T$$

$$T \gg \tau = \frac{2\pi}{m_X v^2} \simeq 400\mu\,\mathrm{s} \left(\frac{10\mu\mathrm{eV}}{m_X}\right)$$

$$\frac{S}{N} \simeq \frac{S_1 + S_2}{\sqrt{2N_1}}$$

$$\propto \frac{P_X}{T} \int_0^T \mathrm{d}t \cos^2\ \theta(t) + \frac{P_X}{T} \int_{T_{\mathrm{wait}}}^{T_{\mathrm{wait}}+T} \mathrm{d}t \cos^2\ \theta(t)$$

$$\int_0^T \mathrm{d}t \cos^2\ \theta(t) \gg \int_{T_{\mathrm{wait}}}^{T_{\mathrm{wait}}+T} \mathrm{d}t \cos^2\ \theta(t)$$

$$P = P_a + N$$

$$\frac{P_a}{\sigma_N} > \Phi^{-1}[0.95] = 1.64$$

$$P = P_X + N \equiv P_X^0 \langle \cos^2 \theta \rangle_T + N$$

$$\Phi(Z) = \mathbb{P}(z \le Z) = \int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{Z-x} \mathrm{d}y f(x) f(y)$$

$$\Phi[0] \equiv \int_{-\infty}^{+\infty} \mathrm{d}P_X \int_{-\infty}^{0-P_X} \mathrm{d}N f(P_X) f(N) = 1 - 0.95$$

$$\int_0^1 \mathrm{d}\langle \cos^2 \theta \rangle_T \frac{f(\langle \cos^2 \theta \rangle_T)}{2} \left[1 + \mathrm{erf}\left(\frac{-P_X^0 \langle \cos^2 \theta \rangle_T}{\sqrt{2}\sigma_N}\right)\right] = 1 - 0.95$$

$$\langle \cos^2 \theta \rangle_T^{\text{excl.}} = \frac{1.64\sigma_N}{P_X^0}$$

$$\int_0^1 \frac{f(\langle \cos^2 \theta \rangle_T)}{2} \left[1 + \mathrm{erf}\left(\frac{5\sigma_N - P_X^0 \langle \cos^2 \theta \rangle_T}{\sqrt{2}\sigma_N}\right)\right] \mathrm{d}\langle \cos^2 \theta \rangle_T = 1 - 0.95$$

$$\langle \cos^2 \theta \rangle_T^{\text{disc.}} = \frac{(5 + 1.64)\sigma_N}{P_X^0}$$

$$\hat{\mathcal{Z}}(t) = \begin{pmatrix} \cos \lambda_{\text{lab}} \cos \omega_\oplus t \\ \cos \lambda_{\text{lab}} \sin \omega_\oplus t \\ \sin \lambda_{\text{lab}} \end{pmatrix}$$

$$\hat{\mathbf{X}} = \begin{pmatrix} \sin \theta_X \cos \phi_X \\ \sin \theta_X \sin \phi_X \\ \cos \theta_X \end{pmatrix}$$

$$\cos^2 \theta(t) = (\hat{\mathbf{X}} \cdot \hat{\mathcal{Z}}(t))^2$$

$$\langle \cos^2 \theta(t) \rangle_T \equiv \frac{1}{T} \int_0^T \cos^2 \theta(t) \mathrm{d}t$$

$$\frac{1}{4\pi} \int \langle \cos^2 \theta(t) \rangle_T \, \mathrm{d}\cos \theta_X \, \mathrm{d}\phi = \frac{1}{3}$$

$$\langle \cos^2 \theta(t) \rangle_{T=\eta} = \frac{1}{8}(3 + \cos 2\theta_X - (1 + 3\cos 2\theta_X)\cos 2\lambda_{\text{lab}})$$

$$\hat{\mathcal{W}}(t) = \begin{pmatrix} \sin \omega_\oplus t \\ -\cos \omega_\oplus t \\ 0 \end{pmatrix}$$

$$\hat{\mathcal{N}}(t) = \begin{pmatrix} \sin\lambda_{\text{lab}}\cos\omega_{\oplus}t \\ -\sin\lambda_{\text{lab}}\sin\omega_{\oplus}t \\ \cos\lambda_{\text{lab}} \end{pmatrix}$$

$$\langle\cos^2\theta(t)\rangle_\eta = \begin{cases} \frac{1}{8}(3+\cos 2\lambda_{\text{lab}} + (1+3\cos 2\lambda_{\text{lab}})\cos 2\theta_X) \\ \frac{\sin^2\theta_X}{2} \\ \frac{1}{8}(3+\cos 2\theta_X - (1+3\cos 2\theta_X)\cos 2\lambda_{\text{lab}}) \end{cases}$$

$$\lambda_{\text{lab}} = \frac{1}{2}\cos^{-1}\left(-\frac{1}{3}\right) \approx \pm 54.74^\circ \qquad \lambda_{\text{lab}} = \frac{1}{2}\cos^{-1}\left(\frac{1}{3}\right) \approx \pm 35.26^\circ$$

$$\cos\theta(t) = \sqrt{1-(\hat{\mathbf{X}}\cdot\hat{\mathcal{N}}(t))^2},$$

$$\langle\cos^2\theta(t)\rangle_\eta = \begin{cases} \frac{1}{8}(5-\cos 2\lambda_{\text{lab}} - (1+3\cos 2\lambda_{\text{lab}})\cos 2\theta_X) \\ \frac{1}{4}(3+\cos 2\theta_X) \\ \frac{1}{8}(5+\cos 2\theta_X + (3\cos 2\lambda_{\text{lab}} - 1)\cos 2\theta_X) \end{cases}$$

$$\lambda_{\text{lab}} = \frac{1}{2}\cos^{-1}\left(-\frac{1}{3}\right) \approx \pm 54.74^\circ$$

$$\lambda_{\text{lab}} = \frac{1}{2}\cos^{-1}\left(\frac{1}{3}\right) \approx \pm 35.26^\circ$$

$$P_{\text{cav}} = \kappa\mathcal{G}_X V Q \rho_{\text{DM}}\chi^2 m_X$$
$$P_{\text{cav}} = \kappa\mathcal{G}_a V \frac{Q}{m_a}\rho_{\text{DM}} g_{a\gamma}^2 B^2$$

$$\mathcal{G}_X = \frac{\left(\int dV \mathbf{E}_\alpha\cdot\hat{\mathbf{X}}\right)^2}{V\frac{1}{2}\int dV\epsilon(\mathbf{x})\mathbf{E}_\alpha^2 + \mathbf{B}_\alpha^2}$$
$$\mathcal{G}_a = \frac{\left(\int dV \mathbf{E}_\alpha\cdot\mathbf{B}\right)^2}{VB^2\frac{1}{2}\int dV\epsilon(\mathbf{x})\mathbf{E}_\alpha^2 + \mathbf{B}_\alpha^2}$$

$$\chi = g_{a\gamma}\frac{B}{m_X|\cos\theta|}$$

$$P_X\langle\cos^2\theta\rangle_T^{\text{disc.}} > (3+\Phi^{-1}[0.5])\sigma_N \approx 3\sigma_N,$$

$$\xi = \int_0^1 \frac{f(\langle\cos^2\theta\rangle_T)}{2}\left[1+\text{erf}\left(\frac{3\sigma_N - P_X\langle\cos^2\theta\rangle_T}{\sqrt{2}\sigma_N}\right)\right]\mathrm{d}\langle\cos^2\theta\rangle_T$$

$$\Lambda(\theta_X,\phi_X) = 2\left[\ln\ \mathcal{L}\big(d \mid \mathcal{M}_t, \{\hat{P}_X, \theta_X, \phi_X, \hat{P}_N\}\big) - \ln\ \mathcal{L}\left(d \mid \mathcal{M}_0, \left\{\hat{\hat{P}}_X, \hat{\hat{P}}_N\right\}\right)\right]$$

$$\ln\ \mathcal{L}\big(P^{\mathrm{obs}} \mid \mathcal{M}_t, \{P_X^0, \theta_X, \phi_X, P_N\}\big) = -\frac{1}{2\sigma_N^2}\sum_{j=1}^{N_t}\left[P_j^{\mathrm{obs}} - P_X\langle c(\theta_X,\phi_X)\rangle_j - \frac{\Delta t}{T}P_N\right]^2$$

$$\langle c(\theta_X,\phi_X)\rangle_j \equiv \langle \cos^2\ \theta\rangle_j = \frac{1}{\Delta t}\int_{t_j-\Delta t/2}^{t_j+\Delta t/2} \cos^2\ \theta\ \mathrm{d}t$$

$$\ln\ \mathcal{L}\big(P^{\mathrm{obs}} \mid \mathcal{M}_t, \{P_X, P_N\}\big) = -\frac{1}{2\sigma_N^2}\big(P^{\mathrm{obs}} - P_X - P_N\big)^2$$

$$P_j^{\mathrm{obs}} = P_j^{\mathrm{Asi}} \equiv P_X\langle c(\theta_X,\phi_X)\rangle_j + \frac{\Delta t}{T}P_N$$

$$\mathrm{TS}_{\mathrm{mod}}(\theta_X,\phi_X) = \frac{1}{\sigma_N^2}\left(\sum_j P_X\langle c(\theta_X,\phi_X)\rangle_j - \hat{\hat{P}}_X\right)^2$$

$$\mathrm{TS}_{\mathrm{mod}}(\theta_X,\phi_X) = \left(\frac{P_X}{\sigma_N}\right)^2\left(\sum_j \langle c\rangle_j - \langle c\rangle_T\right)^2 = \left(\frac{P_X}{\sigma_N}\right)^2\left(\frac{1}{\Delta t}\int_0^T c\ \mathrm{d}t - \langle c\rangle_T\right)^2$$

$$= \left(\frac{T\langle c\rangle_T P_X}{\sigma_N}\right)^2\left(\frac{1}{\Delta t} - \frac{1}{T}\right)^2 \approx \left(\frac{T\langle c\rangle_T P_X}{\Delta t\sigma_N}\right)^2$$

$$\mathrm{TS}_{\mathrm{mod}} \approx \left(\frac{3T\langle c\rangle_T}{\Delta t\langle \cos^2\ \theta\rangle_{\Delta t}^{\mathrm{disc.}}}\right)^2$$

$$\Lambda(\theta_X,\phi_X) = \left(\frac{3}{\langle \cos^2\ \theta\rangle_{\Delta t}^{\mathrm{disc.}}}\right)^2\left[\left(T\langle c_{\mathrm{true}}\rangle_T\left(\frac{1}{\Delta t} - \frac{1}{T}\right)\right)^2 - \frac{1}{\Delta t}\int_0^T (c_{\mathrm{true}} - c)^2\ \mathrm{d}t\right]$$

$$\begin{pmatrix} \sin\ \theta_X \cos\ \phi_X \\ \sin\ \theta_X \sin\ \phi_X \\ \cos\ \theta_X \end{pmatrix} = R_{\mathrm{gal}}\begin{pmatrix} \cos\ l \cos\ b \\ \sin\ l \cos\ b \\ \sin\ b \end{pmatrix}$$

$$R_{\mathrm{gal}} = \begin{pmatrix} -0.05487556 & +0.49410943 & -0.86766615 \\ -0.87343709 & -0.44482963 & -0.19807637 \\ -0.48383502 & +0.74698225 & +0.45598378 \end{pmatrix}$$

Apéndice C.

Modelo de Multidimensiones y Supermembranas a propósito de la deformación dfel espacio – tiempo cuántico por interacción de una partícula supermasiva. Modelo Sigma.

$$S_{\text{sigma-model}} = -T \int d^2\sigma \frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}$$

$$T_{ab} = \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu\right) g_{\mu\nu}$$

$$\frac{1}{\sqrt{-\gamma}}\partial_a\left(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda}$$

$$\Phi(\varphi) = \frac{1}{2}\epsilon_{ij}\epsilon^{ab}\partial_a\varphi^i\partial_b\varphi^j$$

$$S = -\int d^2\sigma\Phi(\varphi)\left(\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{\epsilon^{ab}}{2\sqrt{-\gamma}}F_{ab}(A)\right)$$

$$\gamma_{ab} \to J\gamma_{ab}$$

$$\varphi^i \to \varphi'^i = \varphi'^i\left(\varphi^i\right)$$

$$\Phi \to \Phi' = J\Phi$$

$$\epsilon^{ab}\partial_b\varphi^i\partial_a\left(\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd}\right) = 0$$

$$\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = M = \text{ const.}$$

$$T_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{1}{2}\gamma_{ab}\frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = 0$$

$$\epsilon^{ab}\partial_b\left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\right) = 0$$

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = T$$

$$S_{\text{sigma-model}} = -T \int d^d\sigma \frac{1}{2}\sqrt{-\gamma}\left(\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} + 2\Lambda\right)$$

$$T_{ab} = \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu\right) g_{\mu\nu} - \gamma_{ab}\Lambda$$

$$\frac{1}{\sqrt{-\gamma}}\partial_a\left(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda}$$

$$\Phi(\varphi) = \epsilon_{ijk\ldots m}\epsilon^{abc\ldots.d}\partial_a\varphi^i\partial_b\varphi^j \ldots\ldots \partial_d\varphi^m$$

$$S = -\int d^d\sigma\Phi(\varphi)\left(\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{\epsilon^{abcd...}}{2\sqrt{-\gamma}}F_{abcd...}(A)\right)$$

$$\gamma_{ab} \to J\gamma_{ab}$$
$$\varphi^i \to \varphi'^i = \lambda^{ij}(\varphi^j)$$

$$\Phi \to \Phi' = J\Phi$$

$$A_{bcd......} \to J^{\frac{d-2}{2}}A_{bcd......}$$

$$K_b^a\partial_a\left(\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cdef}...}{\sqrt{-\gamma}}F_{cdef}...\right)$$

$$\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cdef}...}{\sqrt{-\gamma}}F_{cdef...} = M = \text{const.}$$

$$T_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{1}{2}\gamma_{ab}\frac{\epsilon^{cdef}...}{\sqrt{-\gamma}}F_{cdef...} = 0$$

$$\gamma_{ab} = \frac{1-p}{M}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}$$

$$\epsilon^{abc..d}\partial_d\left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\right) = 0$$

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = T$$

$$S_{\text{current}} = \int d^{p+1}\sigma A_{a_2...a_{p+1}}j^{a_2...a_{p+1}}$$

$$\epsilon^{a_1...a_{p+1}}\partial_{a_1}\left(\frac{\Phi}{\sqrt{-\gamma}}\right) = j^{a_2...a_{p+1}}$$

$$j^{a_1...a_{p+1}} = e\partial_\mu\phi\frac{\partial X^\mu}{\partial\sigma^a}\epsilon^{aa_2...a_{p+1}} \equiv e\partial_a\phi\epsilon^{aa_2...a_{p+1}}$$

$$T = \frac{\Phi}{\sqrt{-\gamma}} = e\phi + T_i$$

$$g_{\mu\nu} \to \omega g_{\mu\nu}$$

$$A_a \to \omega A_a$$
$$\Phi(\varphi) \to \omega^{-1}\Phi(\varphi)$$

$$\phi \to \omega^{-1}\phi$$

$$\Phi = \sqrt{-\gamma}(e\phi + T_i)$$

$$S_i = -\int d^2\sigma(e\phi + T_i)\frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} + \int d^2\sigma A_{\mu\nu}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu + \int d^2\sigma\sqrt{-\gamma}\varphi R$$

$$g_{\mu\nu}^{i} = (e\phi + T_i) g_{\mu\nu}$$

$$R_{\mu\nu}\left(g_{\alpha\beta}^{1}\right) = 0$$

$$R_{\mu\nu}\left(g_{\alpha\beta}^{2}\right) = 0$$

$$ds_2^2 = -(1 - 2GM/r)dt^2 + \frac{dr^2}{1 - 2GM/r} + r^2 d\Omega^2 + dy_4^2 + dy_5^2 + \cdots \ldots + dy_{25}^2$$

$$ds_2^2 = -dt^2 + t^{2p_1} dx_1^2 + t^{2p_2} dx_2^2 + \cdots \ldots + t^{2p_{25}} dx_{25}^2$$

$$e\phi + T_1 = \Omega^2 (e\phi + T_2)$$

$$e\phi = \frac{\Omega^2 T_2 - T_1}{1 - \Omega^2}$$

$$e\phi + T_1 = \frac{\Omega^2 (T_2 - T_1)}{1 - \Omega^2}$$

$$e\phi + T_2 = \frac{(T_2 - T_1)}{1 - \Omega^2}$$

$$ds^2 = -dt^2 + t^2 (d\chi^2 + \sinh^2 \chi d\Omega_2^2)$$

$$ds_2^2 = -dt^2 + t^2 (d\chi^2 + \sinh^2 \chi d\Omega_{D-2}^2)$$

$$ds_1^2 = \frac{\sigma}{t^4}\left(-dt^2 + t^2 (d\chi^2 + \sinh^2 \chi d\Omega_{D-2}^2)\right)$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \left(\frac{1 - \Omega^2}{T_2 - T_1}\right)\left(-dt^2 + t^2 (d\chi^2 + \sinh^2 \chi d\Omega_{D-2}^2)\right)$$

$$dT = \sqrt{\frac{1 + \frac{K}{t^4}}{T_1 - T_2}}\, dt$$

$$ds^2 = l^2 dt^2 - l^2 \cosh^2 t \left(\frac{dr^2}{1 - r^2} + r^2 d\Omega_2^2\right)\Bigg) - dl^2$$

$$ds^2 = \frac{\Lambda l^2}{3}\left(dt^2\left(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}\right) - \frac{dr^2}{1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}} - r^2 d\Omega_2^2\right) - dl^2$$

$$ds_2^2 = l^2 \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu - dl^2$$

$$ds_1^2 = \sigma l^{-2} \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu - \sigma \frac{dl^2}{l^4} = \sigma l^{-4} ds_2^2$$

$$ds^2 = \left(\frac{1 - \Omega^2}{T_2 - T_1}\right)\left(l^2 \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu - dl^2\right)$$

$$dL = \sqrt{\frac{1+\frac{K}{l^4}}{T_1 - T_2}}\,dl$$

$$ds_1^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta$$

$$ds_2^2 = \Omega(x)^2 \eta_{\alpha\beta} dx^\alpha dx^\beta$$

$$x''^{\mu} = \frac{(x^\mu + a^\mu x^2)}{(1 + 2a_\nu x^\nu + a^2 x^2)}$$

$$g_{\alpha\beta}^2 = \Omega^2 \eta_{\alpha\beta} = \frac{1}{\left(1 + 2a_\mu x^\mu + a^2 x^2\right)^2} \eta_{\alpha\beta}$$

$$e\phi + T_1 = \frac{(T_2 - T_1)\left(1 + 2a_\mu x^\mu + a^2x^2\right)^2}{\left(1 + 2a_\mu x^\mu + a^2x^2\right)^2 - 1} = \frac{(T_2 - T_1)\left(1 + 2a_\mu x^\mu + a^2x^2\right)^2}{\left(2a_\mu x^\mu + a^2x^2\right)\left(2 + 2a_\mu x^\mu + a^2x^2\right)}$$

$$e\phi + T_2 = \frac{(T_2 - T_1)}{\left(1 + 2a_\mu x^\mu + a^2x^2\right)^2 - 1} = \frac{(T_2 - T_1)}{\left(2a_\mu x^\mu + a^2x^2\right)\left(2 + 2a_\mu x^\mu + a^2x^2\right)}$$

$$e\phi + T_1 = e\phi + T_2 = \frac{K}{4t}$$

$$g_{\mu\nu} = \frac{1}{(e\phi + T_1)} g_{\mu\nu}^1 = \frac{4t}{K} \eta_{\mu\nu}$$

$$(T_2 - T_1)\Delta t \approx \text{ constant}$$

$$2a_\mu x^\mu + a^2 x^2 = 0$$

$$2 + 2a_\mu x^\mu + a^2 x^2 = 0$$

$$\left(2a_\mu x^\mu + a^2x^2\right)\left(2 + 2a_\mu x^\mu + a^2x^2\right) = \left(2At + A^2(t^2 - x^2)\right)\left(2 + 2At + A^2(t^2 - x^2)\right)$$

$$x_1^2 + x_2^2 + x_3^2 \ldots\ldots + x_{D-1}^2 - \left(t + \frac{1}{A}\right)^2 = -\frac{1}{A^2}$$

$$A(x_1^2 + x_2^2 + x_3^2 \ldots\ldots + x_{D-1}^2) - At^2 - 2t = 0$$

$$x_1^2 + x_2^2 + x_3^2 \ldots\ldots + x_{D-1}^2 - \left(t + \frac{1}{A}\right)^2 = \frac{1}{A^2}$$

$$\sqrt{\frac{1}{A^2} + \left(t + \frac{1}{A}\right)^2} - \sqrt{-\frac{1}{A^2} + \left(t + \frac{1}{A}\right)^2} \to \frac{1}{tA^2} \to 0$$

$$\pi T = (\kappa^2)^{-\frac{1}{d-2}}$$

$$\kappa^2 = 8\pi G = \frac{1}{M_P^2}$$

$$S = \int d^4x \left(\sqrt{-g}\left(-\frac{\epsilon}{2}\phi^2 R + X - V(\phi)\right)\right.$$

$$V(\phi) = \frac{1}{8}\lambda(\phi^2 - v^2)^2$$

$$\Phi(\varphi) \to \omega^{-1}\Phi(\varphi)$$

$$\Delta\phi/M_P < O(1)$$

$$M_P \frac{dV/d\phi}{V} > O(1)$$

$$-M_P^2 \frac{d^2V/d\phi^2}{V} > O(1)$$

$$e\phi + T_1 = \Omega_{12}^2(e\phi + T_2)$$

$$e\phi = \frac{\Omega_{12}^2 T_2 - T_1}{1 - \Omega_{12}^2}$$

$$e\phi + T_1 = \frac{\Omega_{12}^2(T_2 - T_1)}{1 - \Omega_{12}^2}$$

$$e\phi + T_2 = \frac{(T_2 - T_1)}{1 - \Omega_{12}^2}$$

$$e\phi + T_2 = \Omega_{23}^2(e\phi + T_3)$$

$$e\phi = \frac{\Omega_{23}^2 T_3 - T_2}{1 - \Omega_{23}^2}$$

$$\Omega_{23} = \frac{T_1 + \Omega_{12}^2(T_3 - T_2)}{\Omega_{12}^2(T_3 - T_2) - T_3}$$

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Anexo A.

Modelo matemático para calcular agujeros negros cuánticos en campos cuánticos relativistas o curvos con aproximaciones en supergravedad cuántica.

$$S_{\mathrm{EH}}[g_{\mu\nu}] = \frac{1}{16\pi G_N} \int_x \sqrt{g}(R - 2\Lambda),$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{Z_h G} h_{\mu\nu}$$

$$Z_h = Z_h(\Delta), G = G(\Delta)$$

$$\Delta = -g^{\mu\nu}\nabla_\mu\nabla_\nu$$

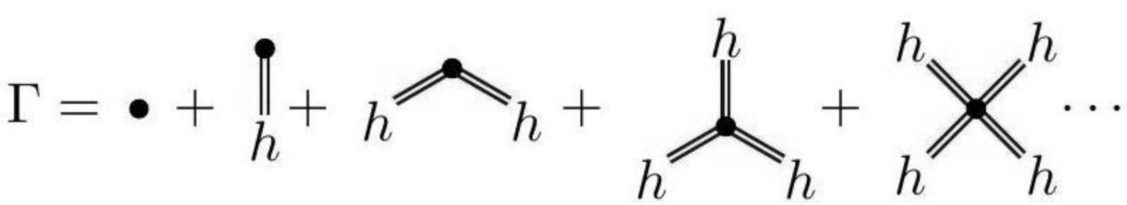
$$g_{\mu\nu} = \eta_{\mu\nu} = \delta_{\mu\nu}$$

$$\Gamma[g] = \Gamma[g, h = 0],$$

$$\Gamma[g_{\mu\nu}] = \frac{1}{16\pi} \int_x \sqrt{g}\left[\mathcal{R}(\Delta, R) + R f_{R^2}(\Delta) R + R_{\mu\nu} f_{R^2_{\mu\nu}}(\Delta) R^{\mu\nu}\right]$$

$$\int_x \sqrt{g} \frac{1}{G(\Delta)} R \simeq \int_x \sqrt{g} \frac{1}{G(0)} R$$

$$\mathrm{IR}: \mathcal{R}(\Delta, R) \overset{G_N\Delta \to 0}{\to} \frac{1}{G_N} R,$$
$$\mathrm{UV}: \mathcal{R}(\Delta, R) \overset{G_N\Delta \to \infty}{\to} 0,$$



$$\Gamma^{(n)}[g] = \Gamma^{(n,0)}[g, 0] \approx (Z_h G)^{-n/2} \Gamma^{(0,n)}[g, 0]$$

$$\Gamma^{(n,m)}[\bar{g},h] = \frac{\delta^{n+m}\Gamma[\bar{g},h]}{\delta\bar{g}^n \delta h^m}$$

$$\Gamma_{\mathrm{tt}}^{(2,0)}(p) = \frac{1}{16\pi G_N} Z_g(p) p^2 \Pi_{\mathrm{tt}}$$
$$\Gamma_{\mathrm{tt}}^{(0,2)}(p) = \frac{1}{16\pi} Z_h(p) p^2 \Pi_{\mathrm{tt}}$$

$$G(p) = \frac{G_N}{Z_g(p)}$$

$$\Gamma_{\mathrm{tt}}^{(n)}(\boldsymbol{p}) = \gamma_g^{(n)}(p)\mathcal{T}_{R,\mathrm{tt}}^{(n)}(\boldsymbol{p})$$

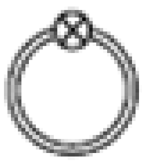
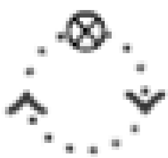
$$\Gamma_{\mathrm{tt}}^{(0,n)}(\boldsymbol{p}) = \prod_{i=1}^{n} \sqrt{Z_h(p_i)G(p_i)}\gamma_g^{(n)}(p)\mathcal{T}_{R,\mathrm{tt}}^{(n)}(\boldsymbol{p})$$

$$\gamma_h^{(n)}(\boldsymbol{p}) = \prod_{i=1}^{n} \sqrt{G(p_i)}\gamma_g^{(n)}(\boldsymbol{p})$$

$$\bar{\Gamma}^{(0,n)}(\boldsymbol{p}) = \frac{\Gamma^{(0,n)}(\boldsymbol{p})}{\prod_{i=1}^{n} \sqrt{Z_h(p_i)}}$$

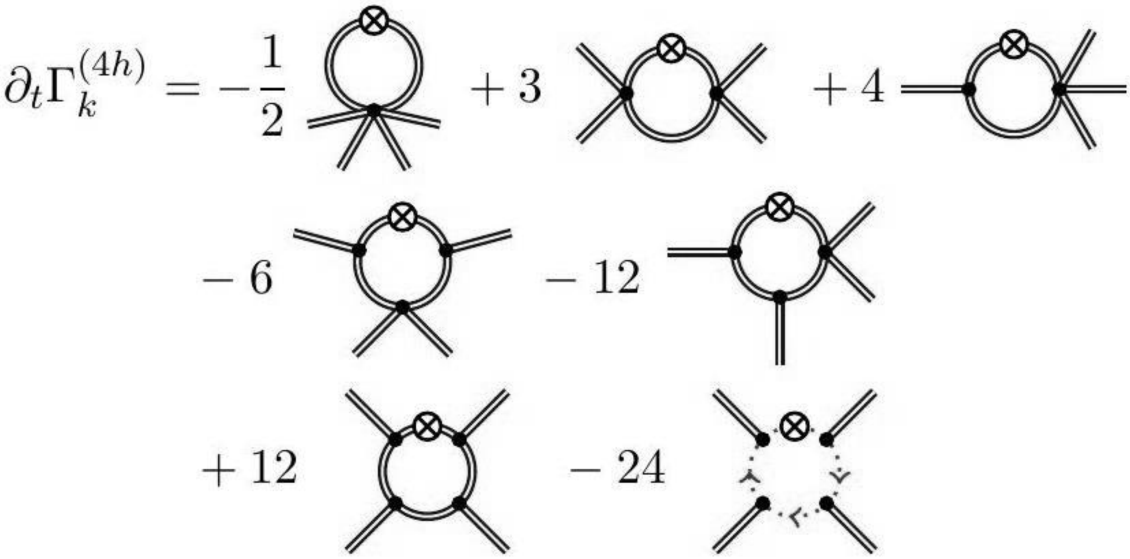
$$G(p) = \left[\gamma_h^{(3)}(p)\right]^2 = \left[\gamma_g^{(3)}(p)\right]^{-1}$$

$$\bar{g}_{\mu\nu} = \delta_{\mu\nu},$$

$$\partial_t \Gamma_k[\bar{g},h] = \frac{1}{2}$$  $-$ 

$$\partial_t \Gamma_k[\bar{g},h] = \frac{1}{2}\mathrm{Tr}\mathcal{G}_k[\bar{g},h]\partial_t \mathcal{R}_k$$

$$\mathcal{G}_k[\bar{g}, h] = \frac{1}{\Gamma_k^{(0,2)}[\bar{g}, h] + \mathcal{R}_k},$$



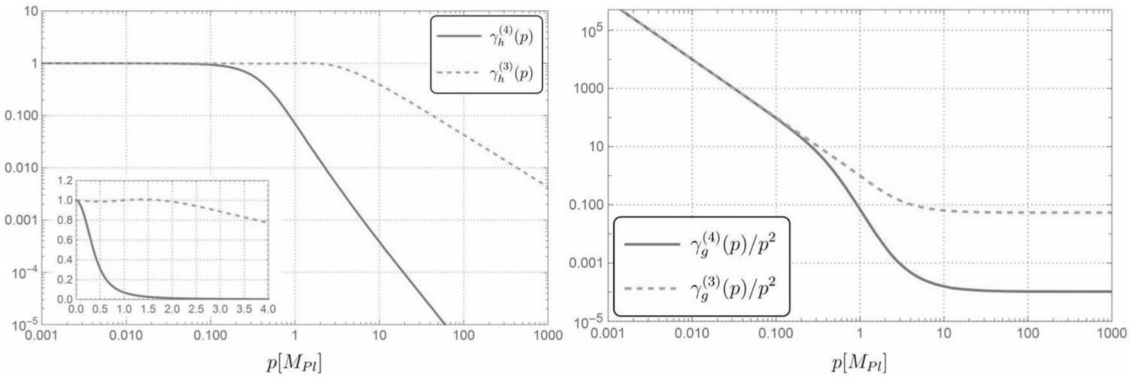


Figura 1. Fluctuaciones de materia y energía de un agujero negro cuántico causado por una partícula supermasiva.

$$\Gamma_{\text{tt}}^{(n)} \circ \mathcal{T}_{R,\text{tt}}^{(n)}/p^2 = C_R^{(n)} \gamma_g^{(n)} p^2$$

$$C_R^{(n)}\gamma_g^{(n)}p^2 = C_{\mathcal{R}}^{(n)} g_{\mathcal{R}} p^2 + C_{R^2}^{(n)} f_{R^2} p^4 + C_{R_{\mu\nu}^2}^{(n)} f_{R_{\mu\nu}^2} p^4$$

$$f_{R_{\mu\nu}^2}(p^2) = \frac{1}{C_{R_{\mu\nu}^2}^{(3)}} \frac{C_R^{(3)}\gamma_g^{(3)} - C_{\mathcal{R}}^{(3)} g_{\mathcal{R}}}{p^2}$$

$$f_{R^2}(p^2) = \frac{1}{C_{R^2}^{(4)}} \frac{C_R^{(4)}\gamma_g^{(4)} - C_{\mathcal{R}}^{(4)} g_{\mathcal{R}} - C_{R_{\mu\nu}^2}^{(4)} f_{R_{\mu\nu}^2} p^2}{p^2}$$

$$\mathcal{R}(\Delta, R) = R\frac{\gamma_g^{(3)}(\Delta) - \bar{\gamma}_3\Delta}{\Delta + R}R$$

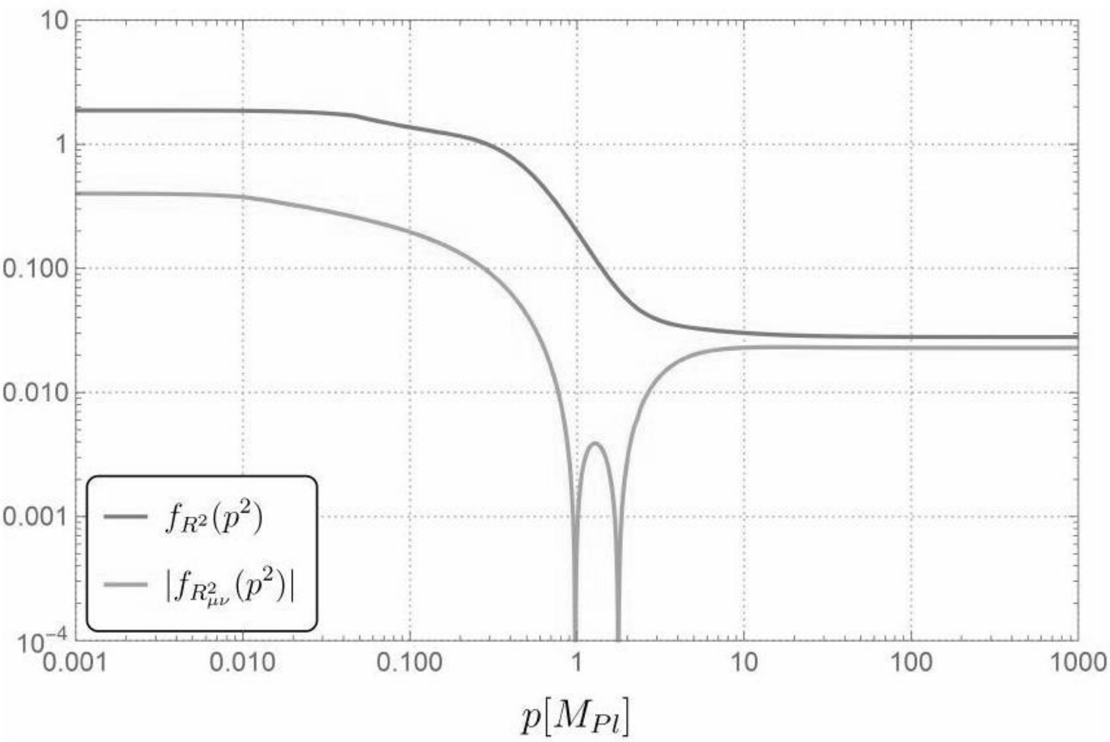


Figura 2. Rangos de momentum del agujero negro cuántico.

$$f_{R_{\mu\nu}^2}(\Delta) \simeq \tilde{g}_{R_{\mu\nu}^2} + \frac{g_{R_{\mu\nu}^2} - \tilde{g}_{R_{\mu\nu}^2}}{1 + p_0^{-2}\Delta}$$

$$f_{R^2}(\Delta) \simeq \tilde{g}_{R^2} + \frac{g_{R^2} - \tilde{g}_{R^2}}{1 + p_1^{-2}\Delta}$$

$$f_{R_{\mu\nu}^2}(\Delta) \simeq g_{R_{\mu\nu}^2} + c_1\Delta$$

$$f_{R^2}(\Delta) \simeq g_{R^2} + c_2\Delta$$

$$g_{R_{\mu\nu}^2} = c_3\left[\gamma_g^{(3)}\right]'(0)$$

$$g_{R^2} = c_4\left[\gamma_g^{(4)}\right]'(0) - c_5\left[\gamma_g^{(3)}\right]'(0)$$

$$2\left(R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu} + g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right)f_{R^2}(\Box)R = 0$$

$$\left(2R_{\mu\sigma}g_{\nu\rho} - \frac{1}{2}g_{\mu\nu}R_{\rho\sigma} + g_{\rho\mu}g_{\sigma\nu}\Box\right)f_{R_{\mu\nu}^2}(\Box)R^{\rho\sigma}$$
$$+(g_{\mu\nu}\nabla_\rho\nabla_\sigma - 2g_{\mu\sigma}\nabla_\rho\nabla_\nu)f_{R_{\mu\nu}^2}(\Box)R^{\rho\sigma} = 0$$

$$(\Box + R)\mathcal{G}(x,x') = \frac{1}{\sqrt{-g}}\delta(x - x')$$

$$\Gamma_{\mathrm{IR}}[g_{\mu\nu}] = \frac{1}{16\pi}\int_x \sqrt{-g}\left(G_N^{-1}R + g_{R_{\mu\nu}^2}R_{\mu\nu}R^{\mu\nu} + g_{R^2}R^2 + c_1R_{\mu\nu}\Box R^{\mu\nu} + c_2R\Box R\right)$$

$$ds^2 = -f(r)\mathrm{d}t^2 + \frac{1}{g(r)}\mathrm{d}r^2 + r^2 d\Omega^2$$

$$d\Omega^2 = \mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\varphi^2$$

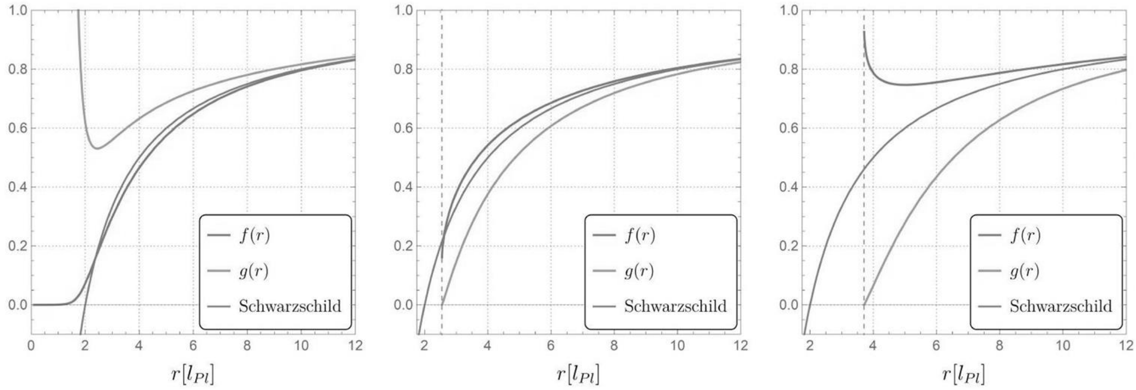


Figura 3. Fluctuaciones de simetría para agujeros negros cuánticos.

$$f(r \gg 1) = 1 - \frac{2M}{r} + S_0 \frac{e^{-m_0 r}}{r} + S_2 \frac{e^{-m_2 r}}{r}$$
$$g(r \gg 1) = 1 - \frac{2M}{r} - S_0 \frac{e^{-m_0 r}}{r}(1 + m_0 r) + \frac{1}{2} S_2 \frac{e^{-m_2 r}}{r}(1 + m_2 r)$$

$$T = \frac{1}{4\pi}\sqrt{|f'(r_h)g'(r_h)|}$$

$$S_{gf}[\bar{g}, h] = \frac{1}{2\alpha}\int_x \sqrt{\bar{g}}\,\bar{g}^{\mu\nu} F_\mu F_\nu$$

$$F_\mu[\bar{g}, h] = \bar{\nabla}^\nu h_{\mu\nu} - \frac{1+\beta}{4}\bar{\nabla}_\mu h_\nu^\nu$$

$$S_{gh}[\bar{g}, h, c, \bar{c}] = \int_x \sqrt{\bar{g}}\,\bar{c}^\mu \mathcal{M}_{\mu\nu} c^\nu$$

$$\mathcal{M}_{\mu\nu} = \bar{\nabla}^\rho \left(g_{\mu\nu}\nabla_\rho + g_{\rho\nu}\nabla_\mu\right) - \frac{1+\beta}{2}\bar{g}^{\sigma\rho}\bar{\nabla}_\mu g_{\nu\sigma}\nabla_\rho$$

$$\Gamma_k[\bar{g}, \phi] = \sum_{n=0}^{\infty} \frac{1}{n!}\Gamma_k^{(0,\phi_{i_1}\ldots\phi_{i_n})}[\bar{g}, 0]\prod_{l=1}^{n}\phi_{i_l}$$

$$\phi = (h, \bar{c}, c)$$

$$\Gamma_k^{(0,\phi_1\ldots\phi_n)}(\boldsymbol{p}) = \left(\prod_{i=1}^{n} Z_{\phi_i}^{\frac{1}{2}}(p_i^2)\right)\gamma_h^{(n)}(\boldsymbol{p})\mathcal{T}^{(\phi_1\ldots\phi_n)}(\boldsymbol{p}; \Lambda_n)$$

$$\mathcal{T}_{\text{EH}}^{(\phi_1\ldots\phi_n)}(\boldsymbol{p};\Lambda_n) = G_N S_{\text{EH}}^{(\phi_1\ldots\phi_n)}(\boldsymbol{p};\Lambda \to \Lambda_n).$$

$$\mathfrak{F}^{(n)}(\hat{p}) := \frac{\partial_t \Gamma^{(n)}(\hat{p})}{Z_h^{n/2} k^{4-n}},$$

$$g_n := \left(\gamma_h^{(n)}\right)^{\frac{2}{n-2}} k^2, \lambda_n := \Lambda_n k^{-2}, \mu_h := -2\lambda_2$$

$$\Pi_{\text{tt}}^{\mu\nu\alpha\beta} := \frac{1}{2}\left(\Pi_\perp^{\mu\alpha}\Pi_\perp^{\nu\beta} + \Pi_\perp^{\mu\beta}\Pi_\perp^{\nu\alpha}\right) - \frac{1}{3}\Pi_\perp^{\mu\nu}\Pi_\perp^{\alpha\beta}$$

$$\Pi_\perp^{\mu\nu} := \delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}$$

$$\mathcal{T}_{R,\text{tt}}^{(n)}(\boldsymbol{p}) = \prod_{i=1}^{n} \Pi_{\text{tt}}(p_i)\mathcal{T}_{\text{EH}}^{(n)}(\boldsymbol{p},0)$$

$$\Gamma^{(0,n)}(\boldsymbol{p}) = \prod_{i=1}^{n} \sqrt{Z_h(p_i)G(p_i)}\gamma_g^{(n)}(\boldsymbol{p})\mathcal{T}_{R,\text{tt}}^{(n)}(\boldsymbol{p}) + \cdots$$

$$\Gamma^{(n)}(\boldsymbol{p}) = \gamma_g^{(n)}(\boldsymbol{p})\mathcal{T}_{R,\text{tt}}^{(n)}(\boldsymbol{p}) + \cdots$$

$$\gamma_g^{(n)}(\boldsymbol{p}) = \prod_{i=1}^{n} \frac{1}{\sqrt{Z_h(p_i)G(p_i)}} \times \frac{\Gamma^{(0,n)}_{\mu_1\nu_1\cdots\mu_n\nu_n}\mathcal{T}_{R,\text{tt}}^{(n)\mu_1\nu_1\cdots\mu_n\nu_n}}{\mathcal{T}^{(n)}_{R,\text{tt}\ \mu_1\nu_1\cdots\mu_n\nu_n}\mathcal{T}_{R,\text{tt}}^{(n)\mu_1\nu_1\cdots\mu_n\nu_n}}$$

$$\gamma_g^{(n)}(\boldsymbol{p}) = \frac{\Gamma^{(n)}_{\mu_1\nu_1\cdots\mu_n\nu_n}\mathcal{T}_{R,\text{tt}}^{(n)\mu_1\nu_1\cdots\mu_n\nu_n}}{\mathcal{T}^{(n)}_{R,\text{tt}\ \mu_1\nu_1\cdots\mu_n\nu_n}\mathcal{T}_{R,\text{tt}}^{(n)\mu_1\nu_1\cdots\mu_n\nu_n}}$$

$$\Gamma_G^{(0,n)}(p^2) := \mathcal{T}_{R,\text{tt}}^{(n)}/p^2 \circ \Gamma_k^{(0,n)} = Z_h^{\frac{n}{2}} g_n^{\frac{n}{2}-1} k^{2-n}\left(C_{\Lambda_n}^{(n)}\lambda_n k^2 + C_R^{(n)}p^2\right)$$

$$\mathcal{T}_{R,\text{tt}}^{(n)}/p^2 \circ S^{(n)}_{R^2+R^2_{\mu\nu}} = C_{R^2}^{(n)} g_{R^2} p^4 + C_{R^2_{\mu\nu}}^{(n)} g_{R^2_{\mu\nu}} p^4$$

$$C_{\Lambda_3}^{(3)} = -\frac{9}{4096\pi^2}, \qquad C_{\Lambda_4}^{(4)} = \frac{222485}{60466176\pi^2}$$

$$C_R^{(3)} = \frac{171}{32768\pi^2}, \qquad C_R^{(4)} = \frac{6815761}{544195584\pi^2}$$

$$C_{R_{\mu\nu}^2}^{(3)} = -\frac{405}{32768\pi^2}, \qquad C_{R_{\mu\nu}^2}^{(4)} = -\frac{3676621}{51018336\pi^2}$$

$$C_{R^2}^{(3)} = 0 \qquad , \quad C_{R^2}^{(4)} = -\frac{96203921}{1632586752\pi^2}$$

$$\mathcal{R}_k(p^2) = \Gamma_k^{(0,hh)}(p^2)\Big|_{\mu_h=0} r(\hat{p}^2) = Z_h(p^2)\mathcal{T}^{(2)}(p^2;0)r(\hat{p}^2)$$

$$r(\hat{p}^2) = \left(\frac{1}{\hat{p}^2} - 1\right)\Theta(1-\hat{p}^2)$$

$$\frac{\dot{g}_4(p^2)}{g_4(p^2)} = 2\big(1+\eta_h(p^2)\big) + 2C_4\frac{k^2}{p^2}(\eta_h(p^2)-\eta_h(0))\lambda_4 + \frac{1}{C_R^{(4)}}\frac{k^2}{p^2}\left(\frac{\mathfrak{F}_G^{(4)}(p^2)}{g_4(p^2)} - \frac{\mathfrak{F}_G^{(4)}(0)}{g_4(0)}\right)$$

$$\eta_{\phi_i}(p^2) := -\partial_t \ln\left(Z_{\phi_i}(p^2)\right)$$

$$g_6 = g_5 = g_4 \ \text{ and } \ \lambda_6 = \lambda_5 = \lambda_3$$

$$\mu_h = -0.26, \lambda_3 = 0.145 \ \text{ and } \ \lambda_4 = 0.025$$

$$\mathfrak{F}_G^{(4)}(\hat{p}) = \frac{g_3^2}{g_4}(c_1 + c_2\eta_h) + \sqrt{g_3 g_4}(c_3 + c_4\eta_h) + g_3(c_5 + c_6\eta_h) + g_4(c_7 + c_8\eta_h)$$

$$(g_3^*, g_4^*) = (2.15, 1.48)$$

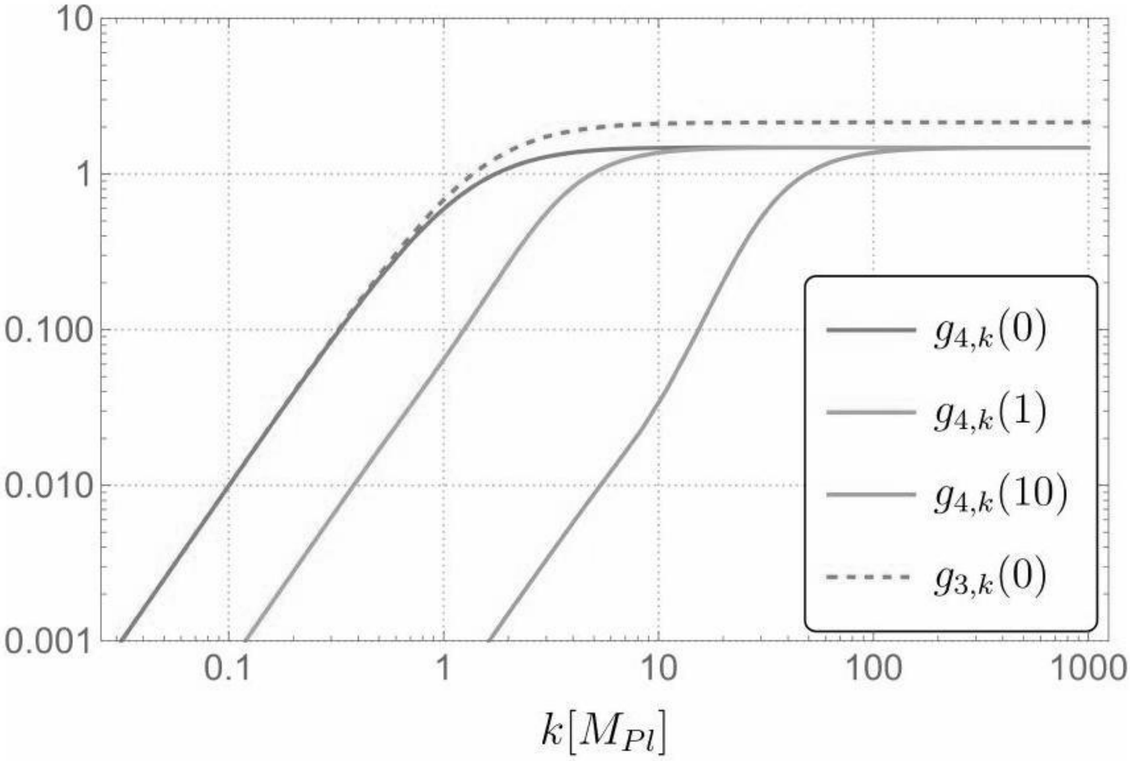


Figura 4. Coordenadas de un agujero negro cuántico en curvatura.

$$\bar{\gamma}_3 = \lim_{p^2\to\infty}\left(\frac{\gamma_g^{(3)}(p^2)}{p^2}\right)$$

$$\bar{\gamma}_3 \approx 0.054$$

$$R^{(n)}\left(\frac{\gamma_g^{(3)} - \bar{\gamma}_3\Delta}{\Delta + R}R\right)\Bigg|_{\Delta\to 0, R\to 0}$$

$$g_{R_{\mu\nu}^2} \approx -0.40, g_{R^2} \approx 1.9$$

$$\tilde{g}_{R_{\mu\nu}^2} \approx -0.023, \tilde{g}_{R^2} \approx 0.028$$

$$c_1 = 344.09, c_2 = -136.75$$

$$c_3 = \frac{C_R^{(3)}}{C_{R_{\mu\nu}^2}^{(3)}}, c_4 = \frac{C_R^{(4)}}{C_{R^2}^{(4)}}, c_5 = \frac{C_{R_{\mu\nu}^2}^{(4)}}{C_{R^2}^{(4)}}c_3$$

$$m_2^2 = \frac{-G_N^{-1}}{g_{R_{\mu\nu}^2}} \approx 2.5 M_{\text{pl}}^2$$
$$m_0^2 = \frac{G_N^{-1}}{6g_{R^2} + 2g_{R_{\mu\nu}^2}} \approx 0.095 M_{\text{pl}}^2$$

$$\gamma_{h,\,\text{simple}}^{(4)}(p^2) = \frac{G_4^*}{G_4^* + p^2}$$

$$\gamma_{h,\text{fit}}^{(4)}(p^2) = \frac{a_1 + a_2 p^2 + a_3 p^4 + a_4 p^6 + a_5 p^8}{b_1 + b_2 p^2 + b_3 p^4 + b_4 p^6 + b_5 p^8 + b_6 p^{10}}$$

$$a_1 = 0.0075542, \quad a_2 = 7.3967, \quad a_3 = 109.42,$$
$$a_4 = 122.40, \quad a_5 = 8.4140,$$
$$b_1 = 0.0075403, \quad b_2 = 7.4773, \quad b_3 = 153.00,$$
$$b_4 = 876.09, \quad b_5 = 2261.9, \quad b_6 = 228.87.$$

$$p_0 \approx 0.091916, p_1 \approx 0.27551$$

$$f_{R_{\mu\nu}^2,\,\text{fit}}(p^2) = a_0 + \sum_{i=1}^{4} \frac{a_i}{(p/p_i)^2 + 1}$$

$$a_0 = -0.023601, \quad a_1 = -0.13727, \quad a_2 = 0.13138$$
$$a_3 = -0.22100, \quad a_4 = -0.15080, \quad p_1 = 0.12436$$
$$p_2 = 1.2476, \quad p_3 = 0.56405, \quad p_4 = 0.021230$$

$$f_{R^2,\text{fit}}(p^2) = a_0 + \sum_{i=1}^{3} \frac{a_i}{((p/p_i)^2 + 1)^2}$$

$$a_0 = 0.028373, \quad a_1 = 0.012637, \quad a_2 = 1.2661$$
$$a_3 = 0.57040, \quad p_1 = 5.7131, \quad p_2 = 0.73200$$
$$p_3 = 0.092956$$

$$\Gamma_{\text{QG}} = \Gamma_{\text{L}} + \Gamma_{\text{NL}} + \Gamma_{\text{matter}}$$

$$\Gamma_{\text{L}} = \int d^4x \sqrt{|g|} \left[\frac{M_P^2}{2}(R - 2\Lambda) + c_1(\mu) R^2 + c_2(\mu) R_{\mu\nu} R^{\mu\nu} \right.$$
$$\left. + c_3(\mu) R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + c_4(\mu) \Box R + O(M_P^{-2}) \right]$$

$$\Gamma_{\mathrm{NL}} = -\int d^4x\sqrt{|g|}[\alpha R\ln\left(\frac{\Box}{\mu^2}\right)R + \beta R_{\mu\nu}\ln\left(\frac{\Box}{\mu^2}\right)R^{\mu\nu} + \gamma R_{\mu\nu\rho\sigma}\ln\left(\frac{\Box}{\mu^2}\right)R^{\mu\nu\rho\sigma} + O(M_P^{-2})\Big]$$

$$M_P = \sqrt{\hbar c/(8\pi G_N)} = 2.4\times 10^{18}\mathrm{GeV}$$

	α	β	γ
Scalar	$5(6\xi-1)^2$	-2	2
Fermion	-5	8	7
Vector	-50	176	-26
Boson	250	-244	424

$$c_1(\mu) = c_1(\mu_*) - \alpha\ln\left(\frac{\mu^2}{\mu_*^2}\right),$$
$$c_2(\mu) = c_2(\mu_*) - \beta\ln\left(\frac{\mu^2}{\mu_*^2}\right),$$
$$c_3(\mu) = c_3(\mu_*) - \gamma\ln\left(\frac{\mu^2}{\mu_*^2}\right).$$

$$\Gamma_{\mathrm{L}} = \int d^4x\sqrt{|g|}\left[\frac{R}{16\pi G_{\mathrm{N}}} + \bar{c}_1(\mu)R^2 + \bar{c}_2(\mu)R_{\mu\nu}R^{\mu\nu}\right]$$
$$\Gamma_{\mathrm{NL}} = -\int d^4x\sqrt{|g|}\left[\bar{\alpha}R\ln\left(\frac{\Box}{\mu^2}\right)R + \bar{\beta}R_{\mu\nu}\ln\left(\frac{\Box}{\mu^2}\right)R^{\mu\nu}\right]$$

$$E_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + 16\pi G_{\mathrm{N}}\left(H_{\mu\nu}^{\mathrm{L}} + H_{\mu\nu}^{\mathrm{NL}}\right) = 8\pi G_{\mathrm{N}}T_{\mu\nu},$$

$$H_{\mu\nu}^{\mathrm{L}} = \bar{c}_1(\mu)\left(2R_{\mu\nu}R - \frac{1}{2}g_{\mu\nu}R^2 + 2g_{\mu\nu}\nabla^2R - \nabla_\mu\nabla_\nu R - \nabla_\nu\nabla_\mu R\right)$$
$$+\bar{c}_2(\mu)\left(-\frac{1}{2}g_{\mu\nu}R_{\rho\sigma}R^{\rho\sigma} + 2R_\mu^\rho R_{\nu\rho} + \nabla^2R_{\mu\nu} - \nabla_\rho\nabla_\mu R_\nu^\rho - \nabla_\rho\nabla_\nu R_\mu^\rho + g_{\mu\nu}\nabla_\sigma\nabla_\rho R^{\rho\sigma}\right)$$

$$H_{\mu\nu}^{\mathrm{NL}} = -\bar{\alpha}\left(2R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + 2g_{\mu\nu}\nabla^2 - \nabla_\mu\nabla_\nu - \nabla_\nu\nabla_\mu\right)\ln\left(\frac{\Box}{\mu^2}\right)R$$
$$-\bar{\beta}\left(-\frac{1}{2}g_{\mu\nu}R^{\rho\sigma} + 2\delta_\nu^\sigma R_\mu^\rho + \delta_\mu^\rho\delta_\nu^\sigma\nabla^2 - \delta_\nu^\sigma\nabla^\rho\nabla_\mu - \delta_\mu^\sigma\nabla^\rho\nabla_\nu + g_{\mu\nu}\nabla^\sigma\nabla^\rho\right)\ln\left(\frac{\Box}{\mu^2}\right)R_{\rho\sigma}$$

$$ds^2 = h(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2 d\theta^2 - r^2 \sin^2\,\theta d\phi^2$$

$$S = \int\, d^4x\sqrt{|g|}\left(\frac{M_P^2}{2}R + \tilde{c}_1(\mu)R^2 + \tilde{c}_2(\mu)C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} + \tilde{\alpha}R\ln\left(\frac{\Box}{\mu^2}\right)R + \tilde{\beta}C_{\mu\nu\alpha\beta}\ln\left(\frac{\Box}{\mu^2}\right)C^{\mu\nu\alpha\beta}\right)$$

$$h(r) = k\big((r - r_0) + h_2(r - r_0)^2 + h_3(r - r_0)^3\big) + \mathcal{O}((r - r_0)^4)$$
$$f(r) = f_1(r - r_0) + f_2(r - r_0)^2 + f_3(r - r_0)^3 + \mathcal{O}((r - r_0)^4)$$

$$\bar{c}_1(\mu) \to \bar{c}_1(\mu) + 2\bar{\alpha}\ln\,\mu,$$
$$\bar{c}_2(\mu) \to \bar{c}_2(\mu) + 2\bar{\beta}\ln\,\mu,$$
$$\bar{c}_3(\mu) \to \bar{c}_3(\mu) + 2\bar{\gamma}\ln\,\mu,$$

$$h_2 = \frac{1 - 2f_1r_0}{f_1r_0^2} + \frac{1 - f_1r_0}{8\big(\tilde{c}_2(\mu) + 2\tilde{\beta}\ln\,\mu\big)f_1^2r_0}$$
$$f_2 = \frac{1 - 2f_1r_0}{r_0^2} - \frac{3(1 - f_1r_0)}{8\big(\tilde{c}_2(\mu) + 2\tilde{\beta}\ln\,\mu\big)f_1r_0}$$

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ANEXO B.

Materia y energía oscuras inherentes a un espacio – tiempo cuántico relativista por interacciones de una partícula oscura. Métrica de Einstein - Friedmann-Lemaître-Robertson-Walker. Modelo Inflacionario.

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\ \theta d\phi^2)\right]$$

$$G^{\mu v} = R^{\mu v} - \frac{1}{2}g^{\mu v}R = \frac{8\pi G}{c^4}T^{\mu v}$$

$$R_0^0 = \frac{3\ddot{a}}{a}$$
$$R_j^i = \left(\frac{\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2} + \frac{2K}{a^2}\right)\delta_j^i$$
$$R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{K}{a^2}\right)$$

$$T_\xi^\alpha = \mathrm{diag}(\rho, -p, -p, -p)$$

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{K}{a^2}$$
$$\dot{H} = -4\pi G(p+\rho) + \frac{K}{a^2}$$

$$\dot{\rho} + 3H(\rho + p) = 0$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p).$$

$$\omega = p/\rho$$

$$a \propto (t-t_0)^{\frac{2}{3(1+w)}}$$
$$\rho \propto a^{-3(1+w)}$$

$$a \propto e^{H_0 t}$$
$$\rho = \text{ Const}$$

Radiation: $a \propto (t-t_0)^{1/2}, \rho \propto a^{-4}$
Dust: $a \propto (t-t_0)^{2/3}, \rho \propto a^{-3}$

$$S = \int\ \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}g^{\mu v}\partial_\mu\phi\partial_v\phi - V(\phi)\right]$$

$$\square\ \phi - V'(\phi) = \frac{1}{\sqrt{-g}}\partial_\mu\left(g^{\mu v}\sqrt{-g}\partial_v\phi\right) - V'(\phi) = 0$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0.$$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right]$$

$$\rho = T_0^0 = \frac{1}{2}\dot{\phi}^2 + V(\phi), p = -T_i^i = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$
$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left[\dot{\phi}^2 - V(\phi)\right].$$

$$\frac{1}{\kappa^2}G_{\mu\nu} = \Lambda(\phi)g_{\mu\nu} - \frac{1}{2}\nabla_\mu\phi\nabla_\nu\phi.$$

$$\rho_{\text{sca}} = \frac{1}{2}\dot{\phi}^2 + \Lambda(\phi)$$
$$p_{\text{sca}} = -\Lambda(\phi)$$

$$\ddot{\phi} + \frac{3}{2}H\dot{\phi} + \Lambda'(\phi) = 0.$$

$$\rho_{\text{sca}} = \frac{C}{a^3} + \Lambda,$$
$$p_{\text{sca}} = -\Lambda.$$

$$\frac{1}{2}mv^2 + V(x) = E = \text{ Const },$$

$$\mathcal{S} = \int dt\mathcal{L} = \int dt\frac{dB}{dt}\left(\frac{1}{2}mv^2 + V(x)\right),$$

$$m\dot{v} = -V'(x),$$

$$m\frac{d}{dt}\left(\frac{dB}{dt}\frac{dx}{dt}\right) = \frac{dV(x)}{dx}\frac{dB}{dt}.$$

$$\mathcal{H} = \frac{dB}{dt}p_a + \frac{dx}{dt}p_x - \mathcal{L} = \frac{dx}{dt}p_x = mv^2\frac{dB}{dt} = \text{ Const },$$

$$\frac{dB}{dt} = C/v^2.$$

$$m\frac{d}{dt}\left(\frac{C}{v}\right) = V'(x)\frac{C}{v^2},$$

$$\mathcal{S}_{(\chi)} = \int d^4x\sqrt{-g}\chi_{\mu;\nu}T_{(\chi)}^{\mu\nu}$$

$$\Gamma_{\mu\nu}^\rho = \left\{\begin{matrix}\rho \\ \mu\nu\end{matrix}\right\} = \frac{1}{2}g^{\rho\lambda}(g_{\lambda\mu,\nu} + g_{\lambda\nu,\mu} - g_{\mu\nu,\lambda}).$$

$$\pi_{\chi_0} = \frac{\partial \mathcal{L}}{\partial \dot{\chi}^0} = T_0^0(\chi)$$

$$\nabla_\mu T^{\mu\nu}_{(\chi)} = 0$$

$$G^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} [\mathcal{L}_\chi + \mathcal{L}_m], \nabla_\mu G^{\mu\nu} = 0$$

$$\nabla_\mu T^{\mu\nu} = 3\sigma j^\nu$$

$$S_{(\chi,A)} = \int d^4x\sqrt{-g}\chi_{\mu;\nu}T^{\mu\nu}_{(\chi)} + \frac{\sigma}{2}\int d^4x\sqrt{-g}\left(\chi_\mu + \partial_\mu A\right)^2$$

$$\nabla_\nu T^{\mu\nu}_{(\chi)} = \sigma(\chi^\mu + \partial^\mu A) = f^\mu$$

$$\nabla_\mu f^\mu = \nabla_\mu(\chi^\mu + \partial^\mu A) = 0$$

$$f_\mu = \sigma\left(\chi_\mu + \partial_\mu A\right) = 0 \;\Rightarrow\; \chi_\mu = -\partial_\mu A$$

$$\mathcal{S} = -\int d^4x\sqrt{-g}T^{\mu\nu}_{(\chi)}\nabla_\mu\nabla_\nu A$$

$$T^{\mu\nu} = -\frac{1}{2}\phi^\mu\phi^\nu + U(\phi)g^{\mu\nu},$$

$$-\frac{1}{2}\nabla_\mu(\phi^\mu\phi^\nu) = 0 \Rightarrow \dot{\phi}^2 \sim \frac{1}{a^3}.$$

$$\mathcal{S}_{(\chi)} = \int d^4x\sqrt{-g}\chi_{\mu;\nu}T^{\mu\nu}_{(\chi)}$$

$$\pi_{\chi_0} = \frac{\partial \mathcal{L}}{\partial \dot{\chi}^0} = T_0^0(\chi).$$

$$\nabla_\mu T^{\mu\nu}_{(\chi)} = 0.$$

$$G^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} [\mathcal{L}_\chi + \mathcal{L}_m], \nabla_\mu G^{\mu\nu} = 0$$

$$\chi_\mu \to \chi_\mu + k_\mu, T^{\mu\nu}_{(\chi)} \to T^{\mu\nu}_{(\chi)} + \Lambda g^{\mu\nu},$$

$$\sqrt{-g}\chi^\mu_{;\mu}\mathcal{L}_1 = \partial_\mu\left(\sqrt{-g}\chi^\mu\right)\mathcal{L}_1 = \Phi\mathcal{L}_1,$$

$$\partial_\alpha\mathcal{L}_1 = 0 \Rightarrow \mathcal{L}_1 = M = \text{ const.}$$

$$\Phi = \frac{1}{4!}\varepsilon^{\alpha\beta\gamma\delta}\varepsilon_{abcd}\partial_\alpha\varphi^{(a)}\partial_\beta\varphi^{(b)}\partial_\gamma\varphi^{(c)}\partial_\delta\varphi^{(d)},$$

$$S = \int d^4x\Phi\mathcal{L}_1 + \int d^4x\sqrt{-g}\mathcal{L}_2$$

$$\mathcal{L} = -\frac{1}{2}R + \chi_{\mu;\nu}T^{\mu\nu}_{(\chi)} - \frac{1}{2}g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} - V(\phi).$$

$$T^{\mu\nu}_{(\chi)} = -\frac{1}{2}\phi^{\mu}\phi^{\nu} + U(\phi)g^{\mu\nu}.$$

$$ds^2 = -dt^2 + a(t)^2\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right)$$

$$\ddot{\phi} + \frac{3}{2}\mathcal{H}\dot{\phi} + U'(\phi) = 0.$$

$$\chi^{\lambda}_{;\lambda}U'(\phi) - V'(\phi) = \nabla_{\mu}j^{\mu},$$
$$j^{\mu} = \frac{1}{2}\phi_{,\nu}(\chi^{\mu;\nu} + \chi^{\nu;\mu}) + \phi^{,\mu},$$

$$\ddot{\phi}(\dot{\chi}_0 - 1) + \dot{\phi}[\ddot{\chi}_0 + 3\mathcal{H}(\dot{\chi}_0 - 1)]$$
$$= U'(\phi)(\dot{\chi}_0 + 3\mathcal{H}\chi_0) - V'(\phi).$$

$$[1-2\dot{\chi}_0 - 3\mathcal{H}\chi_0]\ddot{\phi} - \left[\ddot{\chi}_0 - 3\mathcal{H} + \frac{9}{2}\mathcal{H}(\dot{\chi}_0 + \chi_0\mathcal{H})\right]\dot{\phi}$$
$$+V'(\phi) = 0.$$

$$G^{\mu\nu} = g^{\mu\nu}\left(\frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} + V(\phi) + \frac{1}{2}\chi^{\alpha;\beta}\phi_{,\alpha}\phi_{,\beta} + \chi^{\lambda}\phi_{,\lambda}U'(\phi)\right)$$
$$-\frac{1}{2}\phi^{\mu}\left((\chi^{\lambda}_{;\lambda} + 2)\phi^{\nu} + \chi^{\lambda;\nu}\phi_{,\lambda} + \chi^{\lambda}\phi^{\nu}_{;\lambda}\right)$$
$$-\frac{1}{2}(\chi^{\lambda}\phi^{\mu}_{;\lambda}\phi^{\nu} + \chi^{\lambda;\mu}\phi_{,\lambda}\phi^{\nu})$$

$$\rho = \dot{\phi}^2\left(\dot{\chi}_0\left(1 - \frac{3}{2}\mathcal{H}\right) - \frac{1}{2}\right) + V(\phi) - \dot{\phi}\dot{\chi}_0\left(U'(\phi) + \ddot{\phi}\right)$$

$$p = \frac{1}{2}\dot{\phi}^2(\dot{\chi}_0 - 1) - V(\phi) - \chi_0\dot{\phi}U'(\phi)$$

$$\rho = \left(\dot{\chi}_0 - \frac{1}{2}\right)\dot{\phi}^2 + V(\phi),$$

$$U(\phi) = C, V(\phi) = \Omega_{\Lambda}.$$

$$\dot{\phi}^2 = \frac{2\Omega_m}{a^3},$$

$$\dot{\chi}_0 = 1 - \kappa a^{-1.5},$$

$$\rho = \Omega_{\Lambda} - \frac{\Omega_{\kappa}}{a^{4.5}} + \frac{\Omega_m}{a^3},$$
$$p = -\Omega_{\Lambda} - \frac{1}{2}\frac{\Omega_{\kappa}}{a^{4.5}},$$

$$U(\phi) = C, V(\phi) = \Omega_{\Lambda}e^{-\beta\phi},$$

$$\ddot{\phi} = -\frac{3}{2}\mathcal{H}\dot{\phi}$$

$$\delta = \dot{\chi}_0 - 1,$$

$$\dot{\phi}\left(\dot{\delta} + \frac{3}{2}\mathcal{H}\delta\right) = \beta V(\phi)$$

$$x = \frac{\dot{\phi}}{\sqrt{6}H}, y = \frac{\sqrt{V(\phi)}}{\sqrt{3}H}$$

$$(1 + 2\delta)x^2 + y^2 = 1$$

$$\frac{dx}{d\tau} = -\frac{3x}{4}(x^2 - 1 + 3y^2)$$
$$\frac{dy}{d\tau} = -\frac{y}{4}\left(-9 + 3x^2 + 9y^2 + 2\sqrt{6}x\beta\right)$$

$$\omega_\chi = \frac{1}{2}(1 - x^2 - 3y^2)$$

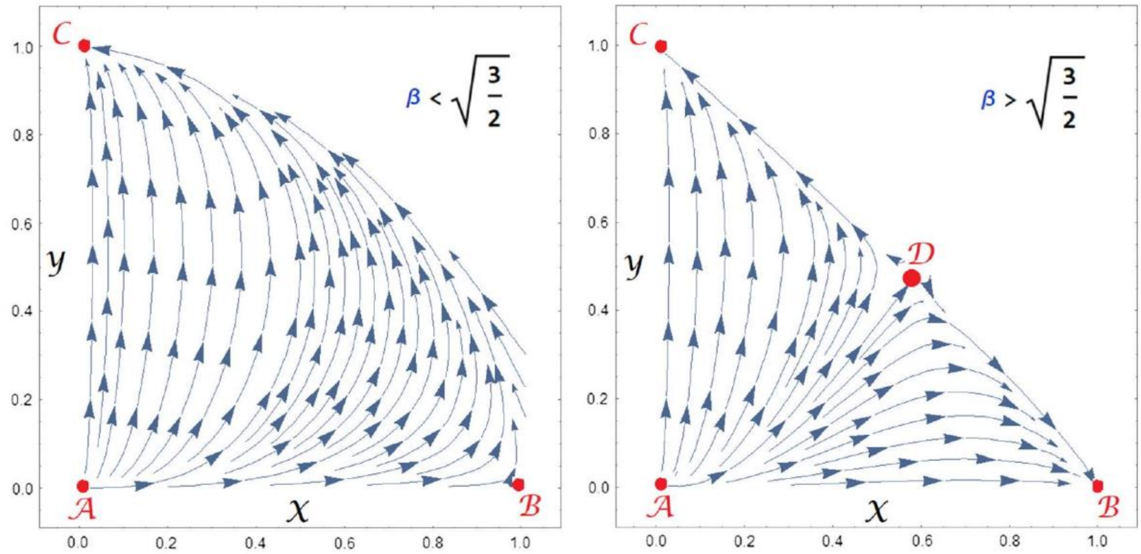


Figura 1. Deformaciones del espacio – tiempo cuántico por interacción de la partícula oscura.

$$\frac{d}{dt} = -\mathcal{H}(z)(z+1)\frac{d}{dz}$$

$$\left(x = \sqrt{\frac{3}{2}}\frac{1}{\beta}, y = \frac{\sqrt{2\beta^2 - 3}}{\sqrt{6}\beta}\right)$$

$$\mathcal{S} = \int \sqrt{-g}\left[\frac{1}{2\kappa^2}R + \chi_{\mu;\nu}T^{\mu\nu}_{(\phi)} - \frac{1}{2}g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} - V(\phi)\right]d^4x$$

$$T^{\mu\nu}_{(\phi)} = -\frac{1}{2}\phi^{,\mu}\phi^{,\nu} + U(\phi)g^{\mu\nu}.$$

$$\nabla_\mu T^{\mu\nu}_{(\phi)} = 0$$

$$\chi^{\lambda}_{;\lambda} U'(\phi) - V'(\phi) \ = \nabla_\mu j^\mu$$
$$j^\mu \ = \frac{1}{2}\phi_{,\nu}(\chi^{\mu;\nu} + \chi^{\nu;\mu}) + \phi^{,\mu}$$

$$\frac{1}{\kappa^2} G^{\mu\nu} = g^{\mu\nu}\left(\frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} + V(\phi) + \frac{1}{2}\chi^{\alpha;\beta}\phi_{,\alpha}\phi_{,\beta} + \chi^\lambda \phi_{,\lambda} U'(\phi)\right)$$
$$-\frac{1}{2}\phi^{,\mu}\left[(\chi^{\lambda}_{;\lambda} + 2)\phi^{,\nu} + \chi^{\lambda;\nu}\phi_{,\lambda} + \chi^\lambda \phi^{;\nu}_{;\lambda}\right] - \frac{1}{2}\left(\chi^\lambda \phi^{;\mu}_{;\lambda}\phi^{,\nu} + \chi^{\lambda;\mu}\phi_{,\lambda}\phi^{,\nu}\right).$$

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left[\frac{\mathrm{d}r^2}{1 - Kr^2} + r^2(\ \mathrm{d}\theta^2 + \sin^2\ \theta\ \mathrm{d}\phi^2)\right]$$

$$\ddot{\phi} + \frac{3}{2}H\dot{\phi} + U'(\phi) = 0,$$

$$j^\mu = \left(\dot{\phi}(1 - \dot{\chi}_0), 0{,}0{,}0\right)$$

$$j^\mu_{;\mu} = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g} j^\mu\right) = \frac{1}{a^3}\partial_\mu(a^3 j^0) = -\ddot{\phi}(\dot{\chi}_0 - 1) + \dot{\phi}[3H(\dot{\chi}_0 - 1) - \ddot{\chi}_0]$$

$$\chi^{\lambda}_{;\lambda} U'(\phi) - V'(\phi) \ = U'(\phi)\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\chi^\mu\right) - V'(\phi)$$
$$= -U'(\phi)\frac{1}{a^3}\partial_\mu(a^3\chi^0) - V'(\phi) = U'(\phi)[\dot{\chi}_0 + 3H\chi_0] + V'(\phi).$$

$$\ddot{\phi}(\dot{\chi}_0 - 1) + \dot{\phi}[3H(\dot{\chi}_0 - 1) + \ddot{\chi}_0] = U'(\phi)[\dot{\chi}_0 + 3H\chi_0] - V'(\phi).$$

$$\rho \ = \dot{\phi}^2\left(\dot{\chi}_0\left(1 - \frac{3}{2}\mathcal{H}\right) - \frac{1}{2}\right) + V(\phi) - \dot{\phi}_0\left(U'(\phi) + \ddot{\phi}\right)$$
$$p \ = \frac{1}{2}\dot{\phi}^2(\dot{\chi}_0 - 1) - V(\phi) - \chi_0\dot{\phi}U'(\phi)$$

$$\rho \ = \left(\dot{\chi}_0 - \frac{1}{2}\right)\dot{\phi}^2 + V(\phi)$$
$$p \ = \frac{1}{2}\dot{\phi}^2(\dot{\chi}_0 - 1) - V(\phi) - \chi_0\dot{\phi}U'(\phi)$$

$$U(\phi) = \ \text{Const.}$$

$$\dot{\phi}^2 = \frac{2C_{m0}}{a^3}$$

$$\delta = \dot{\chi}_0 - 1$$

$$\dot{\phi}\left(\dot{\delta} + \frac{3}{2}H\delta\right) = -V'(\phi)$$

$$\rho = \left(\delta + \frac{1}{2}\right)\dot{\phi}^2 + V(\phi)$$
$$p = \frac{\delta}{2}\dot{\phi}^2 - V(\phi)$$

$$\delta = \frac{1}{2}\xi a^{-3/2}$$

$$\rho = \Lambda + \frac{\xi C_{m0}}{a^{9/2}} + \frac{C_{m0}}{a^3}$$
$$p = -\Lambda + \frac{\xi C_{m0}}{2a^{9/2}}$$

$$H(z) = H_0\left[\Omega_\Lambda + \Omega_{m0}(1+z)^3 + \Omega_{\xi 0}(1+z)^{9/2}\right]^{1/2}$$

$$\delta = \xi = 0$$

$$\rho = \Lambda + \frac{C_{m0}}{a^3}$$
$$p = -\Lambda$$

$$\frac{d\delta}{dz} = \frac{V'(\phi)}{(z+1)^{5/2}\sqrt{C_{m0}}H(\phi,\delta)} + \frac{3\delta}{2(z+1)}$$
$$\frac{d\phi}{dz} = -\frac{\sqrt{2C_{m0}(z+1)}}{H(\phi,\delta)}$$

$$H(\phi,\delta) = H_0[(2\delta+1)\Omega_{m0}(z+1)^3 + \Omega_{DE}(\phi)]^{1/2}$$

$$x = \frac{\dot{\phi}}{\sqrt{6}H}, y = \frac{\sqrt{V(\phi)}}{\sqrt{3}H}, z = -\frac{V'(\phi)}{V(\phi)}.$$

$$x' = -\frac{3}{4}x(x^2 + 3y^2 - 1)$$
$$y' = -\frac{1}{4}y\left(3x^2 + 9y^2 - 9 + 2\sqrt{6}xz\right)$$
$$z' = -\sqrt{6}z^2x(\Gamma - 1)$$

$$\Gamma = \frac{V(\phi)V''(\phi)}{V'(\phi)^2}$$

$$(1+2\delta)x^2 + y^2 = 1$$

$$\beta > \sqrt{\frac{3}{2}}\left(\beta > \sqrt{\frac{3}{2}}\right)\left(\sqrt{\frac{3}{2}}\frac{1}{\beta}, \sqrt{\frac{2\beta^2-3}{\sqrt{6}\beta}}\right)$$

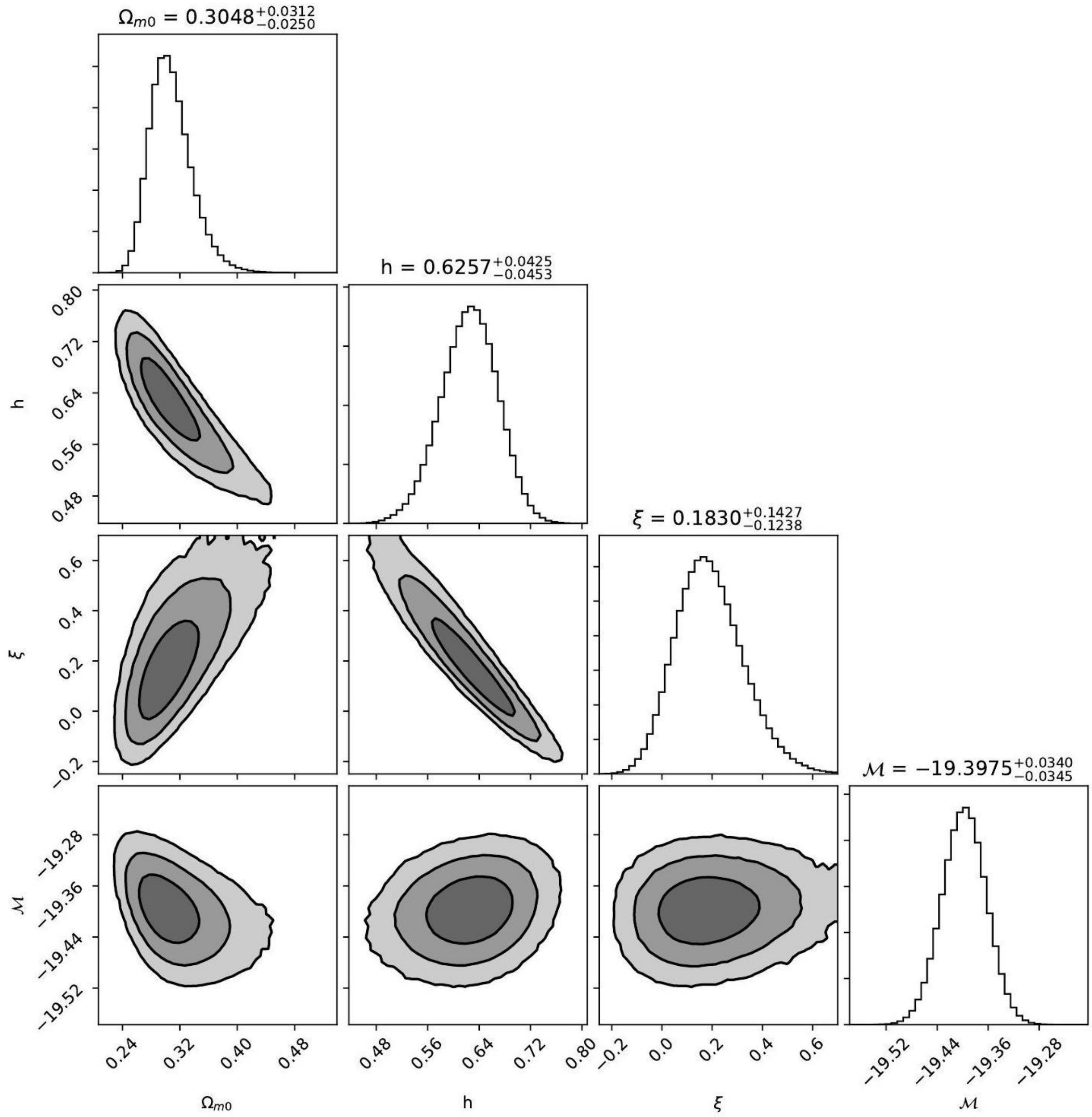
$$z' = \sqrt{6}x(1+z^2)$$

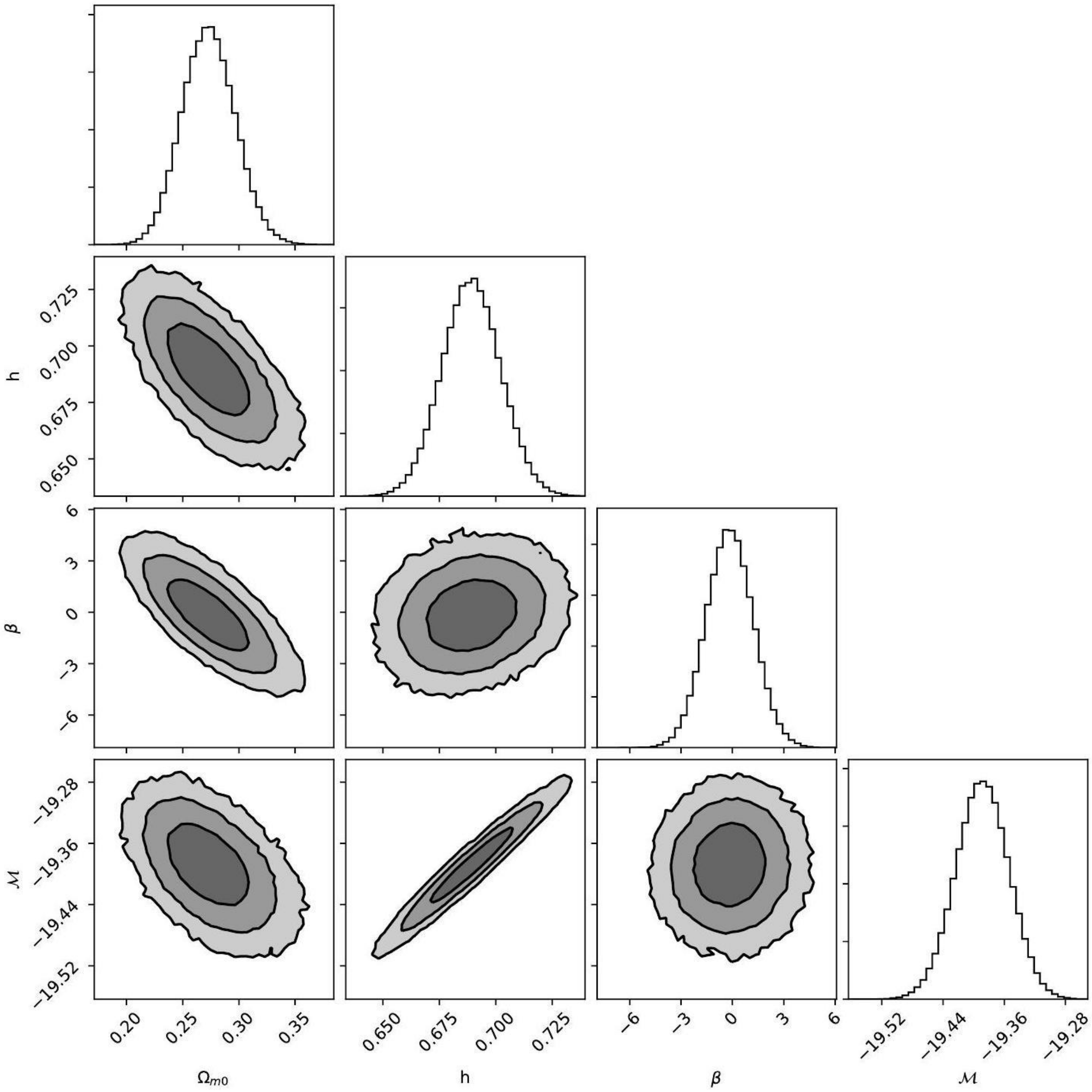
$$z' = \sqrt{6}x(z-\alpha)^2$$

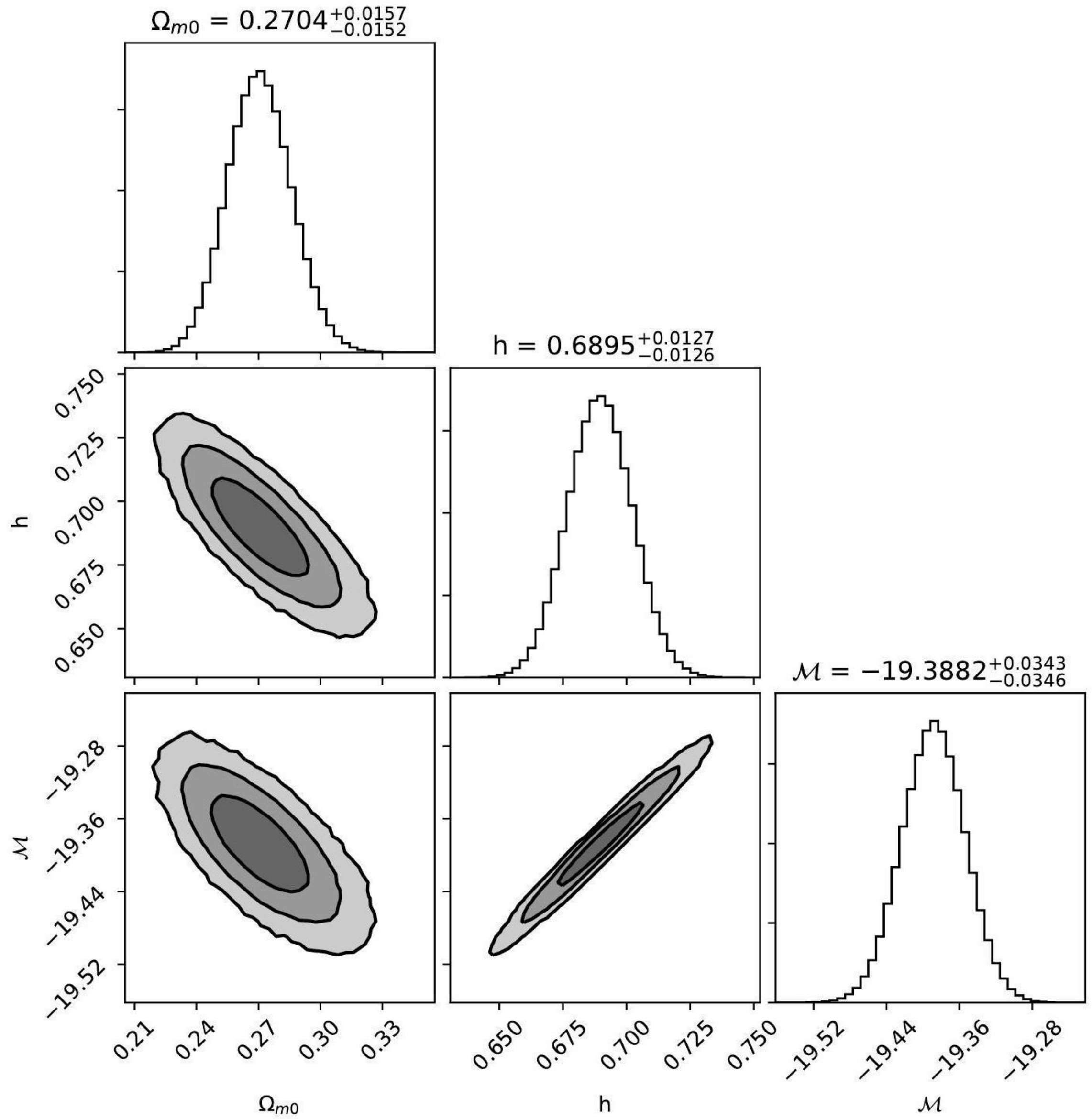
$$\left(\sqrt{\frac{3}{2\alpha}}, \sqrt{\frac{2\alpha-3}{6\alpha}}\right)\alpha < \sqrt{\frac{5}{6}}\alpha > \sqrt{\frac{5}{6}}$$

$$\chi_H^2(\phi^\nu) = \mathcal{H}\mathbf{C}_{H,\mathrm{cov}}^{-1}\mathcal{H}^T$$

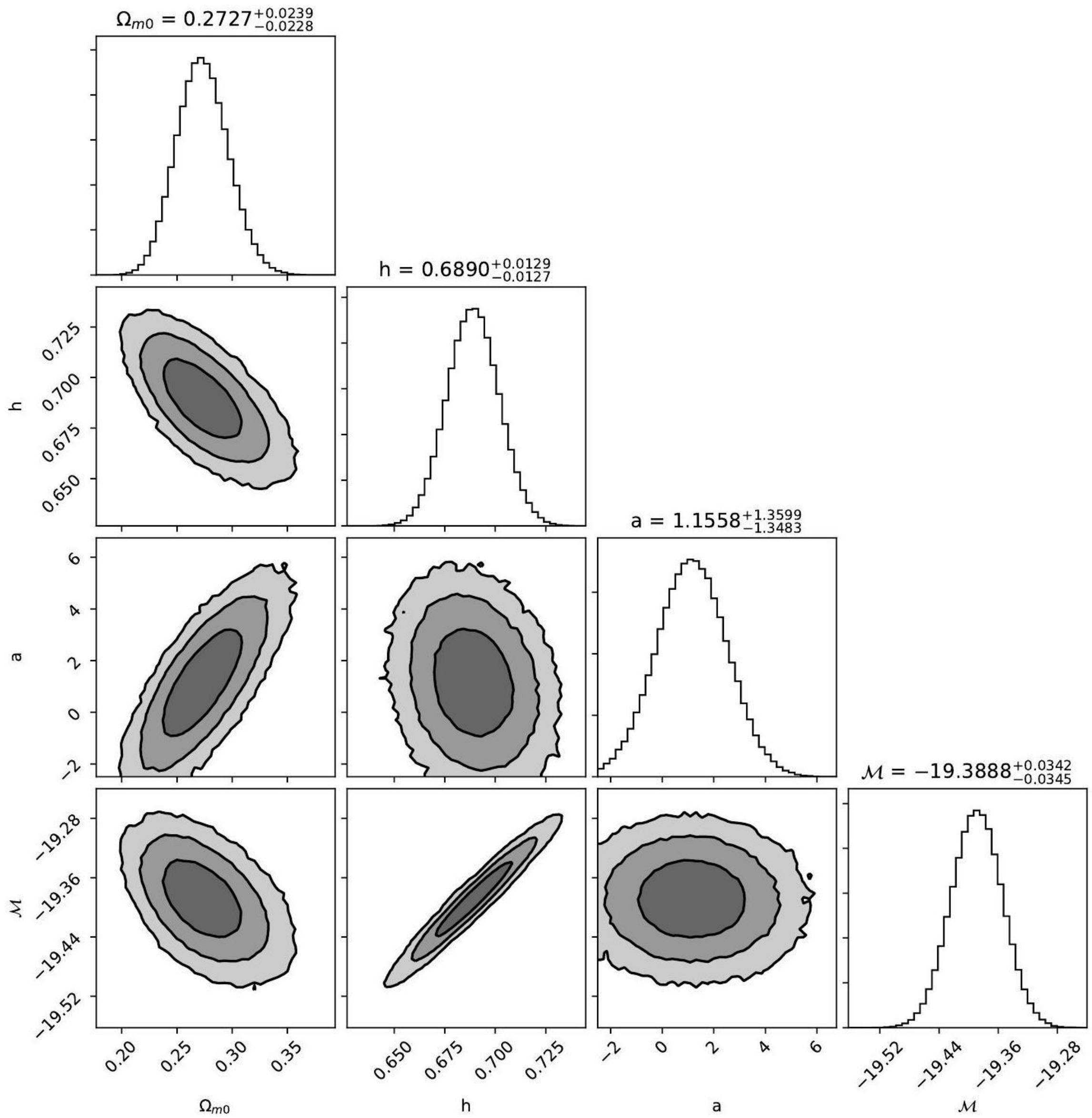
$$\chi_\mathrm{s}^2(\phi_\mathrm{s}^\nu) = \mu_\mathrm{s}\mathbf{C}_{\mathrm{s,cov}}^{-1}\mu_\mathrm{s}^T,$$




h
β
$\mathcal{M}$
Ω_{m0}
h
β
$\mathcal{M}$



$\Omega_{m0} = 0.2704^{+0.0157}_{-0.0152}$
$h = 0.6895^{+0.0127}_{-0.0126}$
$\mathcal{M} = -19.3882^{+0.0343}_{-0.0346}$
h
0.650
0.675
0.700
0.725
0.750
$\mathcal{M}$
−19.28
−19.36
−19.44
−19.52
0.21
0.24
0.27
0.30
0.33
Ω_{m0}
0.650
0.675
0.700
0.725
0.750
h
−19.52
−19.44
−19.36
−19.28
$\mathcal{M}$



Figuras 2, 3, 4 y 5. Fluctuaciones de materia y energía de la partícula oscura.

$$\mu_{\text{th}} = 5\log\left(\frac{D_L(z)}{\text{Mpc}}\right) + 25$$

$$D_L(z) = c(1+z)\int_0^z \frac{dx}{H(x,\theta^\nu)}$$

$$\mathcal{L}_{\text{tot}}(\phi^\psi) = \prod_{p=1}^{P} \exp\left(-\chi_p^2\right)$$

$$\chi_{\text{tot}}^2 = \sum_{p=1}^{P} \chi_P^2,$$

Model	Ω_{m0}	h	α or ξ	β	$\mathcal{M}$	$\chi^2_{\min}$
$V, U const.$	$0.305^{+0.031}_{-0.025}$	$0.6257^{+0.0428}_{-0.0455}$	$0.183^{+0.143}_{-0.125}$	—	$-19.397^{+0.034}_{-0.035}$	84.114
V_1	$0.277^{+0.024}_{-0.023}$	$0.6885^{+0.0130}_{-0.0128}$	—	$-0.593^{+1.367}_{-1.355}$	$-19.390^{+0.034}_{-0.035}$	88.100
$V_2(cosine)$	0.270 ± 0.015	$0.6895^{+0.0128}_{-0.0127}$	− − −	1	-19.388 ± 0.035	87.954
V_3	$0.273^{+0.024}_{-0.023}$	$0.6890^{+0.0130}_{-0.0127}$	$1.152^{+1.370}_{-1.352}$	1	-19.389 ± 0.034	87.942
ΛCDM	$0.281^{+0.016}_{-0.015}$	0.686 ± 0.013	—	—	-19.403 ± 0.035	85.700

$$\mathrm{AIC} = -2\ln(\mathcal{L}_{\max}) + 2\psi + \frac{2\psi(\psi+1)}{N_{\mathrm{tot}} - \psi - 1}$$

$$\mathrm{BIC} = -2\ln(\mathcal{L}_{\max}) + \psi\log(N_{\mathrm{tot}}).$$

$$\mathrm{DIC} = D\left(\overline{\phi^{\psi}}\right) + 2C_B$$

$$100\% \times \frac{\left(H_{\Lambda CDM}(z_{\mathrm{BBN}}, \Omega_{m0}, h) - H_i(z_{\mathrm{BBN}}, \theta_i^{\nu})\right)^2}{H_{\Lambda CDM}(z_{\mathrm{BBN}}, \Omega_{m0}, h)^2} < 10\%$$

$$ds^2 = -dt^2 + R(t)^2 \frac{\Sigma dx^j dx^j}{f_D(x)^2} + R(t)^2 \frac{\Sigma dx^p dx^p}{f_d(x)^2}$$

$$f_D = 1 + \frac{k_D}{4}\Sigma\left(x^i\right)^2, f_d = 1 + \frac{k_d}{4}\Sigma(y^p)^2$$

$$\mathcal{H}_R = \frac{\dot{R}}{R}\ \mathcal{H}_r = \frac{\dot{r}}{r}.$$

$$V = R^D r^d,$$

$$\mathcal{H} = \frac{\dot{V}}{V}.$$

$$\mathcal{H} = D\mathcal{H}_R + d\mathcal{H}_d.$$

$$T_\nu^\mu = \mathrm{diag}(\rho, -p, -p, \dots, -p', -p', \dots).$$

$$\frac{1}{2}D(D-1)\left[\frac{\dot{R}^2}{R^2} + \frac{k_D}{R^2}\right] + \frac{1}{2}d(d-1)\left[\frac{\dot{R}^2}{R^2} + \frac{k_d}{R^2}\right]$$
$$+Dd\frac{\dot{R}}{R}\frac{\dot{r}}{r} = 8\pi\rho$$
$$(D-1)\frac{\ddot{R}}{R} + d\frac{\ddot{r}}{r} - d\frac{\dot{R}}{R}\frac{\dot{r}}{r} - (D-1)\left[\frac{\dot{R}^2}{R^2} + \frac{k_D}{R^2}\right]$$
$$= -8\pi(\rho + p)$$
$$D\frac{\ddot{R}}{R} + (d-1)\frac{\ddot{r}}{r} - D\frac{\dot{R}}{R}\frac{\dot{r}}{r} - (d-1)\left[\frac{\dot{r}^2}{r^2} + \frac{k_d}{r^2}\right]$$
$$= -8\pi(\rho + p').$$

$$D\frac{\ddot{R}}{R} + d\frac{\ddot{r}}{r} = \frac{8\pi\rho}{D+d-1}[1 - (D+d)\gamma].$$

$$\frac{\ddot{V}}{V}=\frac{D+d}{D+d-1}8\pi(\rho-p).$$

$$E=\frac{1}{2}\dot{V}^2-\frac{D+d}{D+d-1}\Omega V^2,$$

$$\frac{\dot{R}}{R}=\frac{1}{(D+d)V}\left[\dot{V}+\sqrt{\frac{2Ed}{D}(D+d-1)}\right]$$
$$\frac{\dot{r}}{r}=\frac{1}{(D+d)V}\left[\dot{V}-\sqrt{\frac{2ED}{d}(D+d-1)}\right].$$

$$R(t)=V^{\frac{1}{D+d}}\exp\left[+\frac{1}{D+d}\sqrt{\frac{2Ed(D+d-1)}{D}}\int\frac{dt}{V}\right]$$

$$r(t)=V^{\frac{1}{D+d}}\exp\left[-\frac{1}{D+d}\sqrt{\frac{2ED(D+d-1)}{d}}\int\frac{dt}{V}\right].$$

$$p=\frac{1}{D+d}\left(1+\sqrt{\frac{d(D+d+1)}{D}}\right)$$
$$q=\frac{1}{D+d}\left(1-\sqrt{\frac{D(D+d+1)}{d}}\right),$$

$$Dp+dq=Dp^2+dq^2=1.$$

$$\mathcal{L}=-\frac{1}{2}R+\chi_{\mu;\nu}T^{\mu\nu}_{(\phi)}-\frac{1}{2}g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}-V(\phi),$$

$$\nabla_\mu T^{\mu\nu}_{(\phi)}=0.$$

$$T^{\mu\nu}_{(\phi)}=-\frac{1}{2}\phi^\mu\phi^\nu+U(\phi)g^{\mu\nu},$$

$$\ddot{\phi}+\frac{1}{2}\mathcal{H}\dot{\phi}+U'(\phi)=0.$$

$$\chi^{;\lambda}_{;\lambda}U'(\phi)-V'(\phi)=\nabla_\mu j^\mu$$
$$j^\mu=\frac{1}{2}\phi_{,\nu}(\chi^{\mu;\nu}+\chi^{\nu;\mu})+\phi^\mu,$$

$$\ddot{\phi}(\dot{\chi}_0-1)+\dot{\phi}(\mathcal{H}(\dot{\chi}_0-1)+\ddot{\chi}_0)$$
$$=U'(\phi)(\dot{\chi}_0+\mathcal{H}\chi_0)-V'(\phi).$$

$$G^{\mu\nu} = g^{\mu\nu}\left(\frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} + V(\phi) + \frac{1}{2}\chi^{\alpha;\beta}\phi_{,\alpha}\phi_{,\beta} + \chi^{\lambda}\phi_{,\lambda}U'(\phi)\right)$$
$$-\frac{1}{2}\phi^{\mu}\left((\chi^{\lambda}_{;\lambda} + 2)\phi^{\nu} + \chi^{\lambda;\nu}\phi_{,\lambda} + \chi^{\lambda}\phi^{\nu}_{;\lambda}\right)$$
$$-\frac{1}{2}\left(\chi^{\lambda}\phi^{\mu}_{;\lambda}\phi^{\nu} + \chi^{\lambda;\mu}\phi_{,\lambda}\phi^{\nu}\right)$$

$$\rho = \left(\dot{\chi}_0 - \frac{1}{2}\right)\dot{\phi}^2 + V(\phi)$$

$$U(\phi) = C, V(\phi) = \Omega_\Lambda.$$

$$\dot{\phi}^2 = \frac{2\Omega_m}{V},$$

$$\dot{\chi}_0 = 1 + \frac{\kappa}{V^2},$$

$$\Omega = \Omega_\Lambda + \frac{\Omega_m}{V} + \frac{\Omega_\kappa}{V^{3/2}},$$

$$E = \frac{1}{2}\dot{V}^2 + U_{\text{eff}}(V)$$

$$U_{\text{eff}}(V) = -\frac{D+d}{D+d-1}\left(\Omega_\Lambda V^2 + \Omega_m V + \Omega_\kappa\sqrt{V}\right)$$

$$R(t) = V_C^{\frac{1}{D+d}}\exp\left[+\frac{1}{D+d}\sqrt{\frac{2Ed(D+d-1)}{D}}\frac{t}{V_C}\right]$$

$$r(t) = V_C^{\frac{1}{D+d}}\exp\left[-\frac{1}{D+d}\sqrt{\frac{2ED(D+d-1)}{d}}\frac{t}{V_C}\right]$$

$$V(\phi) = \frac{\Lambda_{+\infty} - \Lambda_{-\infty}}{2}\tanh(\beta\phi) + \frac{\Lambda_{+\infty} + \Lambda_{-\infty}}{2},$$

$$V(\phi) = \frac{\Lambda_{+\infty} - \Lambda_{-\infty}}{2}\text{Sign}(\phi) + \frac{\Lambda_{+\infty} + \Lambda_{-\infty}}{2}.$$

$$\xi = \dot{\chi}_0 - 1,$$

$$\dot{\phi}\left(\dot{\xi} + \frac{1}{2}\mathcal{H}\xi\right) = -V'(\phi).$$

$$\frac{2\Omega_m}{V^{\frac{3}{2}}}d\left(\xi V^{\frac{1}{2}}\right) = -dV(\phi)$$

$$\frac{2\Omega_m}{V}\left(\frac{d}{d\phi}\xi+\frac{1}{2V}\frac{d}{d\phi}V\right)=-(\Lambda_{+\infty}-\Lambda_{-\infty})\delta(\phi)$$

$$\xi(\phi<0)=\frac{\kappa_-}{V_{(\phi=0)}^{\frac{1}{2}}},\xi(\phi>0)=\frac{\kappa_+}{V_{(\phi=0)}^{\frac{1}{2}}}.$$

$$\Delta\xi=-\frac{V_{(\phi=0)}}{2\Omega_m}(\Lambda_{+\infty}-\Lambda_{-\infty}).$$

$$\kappa_+-\kappa_-=-\frac{V_{(\phi=0)}^{\frac{3}{2}}}{2\Omega_m}(\Lambda_{+\infty}-\Lambda_{-\infty}).$$

$$E=\frac{1}{2}\dot{V}^2-\frac{D+d}{D+d-1}V^2\left[\frac{2\Omega_m}{V}\left(\xi+\frac{1}{2}\right)+\frac{\Lambda_{+\infty}-\Lambda_{-\infty}}{2}\mathrm{Sign}(\phi)+\frac{\Lambda_{+\infty}+\Lambda_{-\infty}}{2}\right].$$

$$\frac{1}{2}\Delta\dot{V}^2=\frac{D+d}{D+d-1}\left(\frac{2\Omega_m}{V_{(\phi=0)}}\Delta\xi+\Lambda_{+\infty}-\Lambda_{-\infty}\right)V_{(\phi=0)}^2=0,$$

$$V(t)=V_0\exp(\chi t),$$

$$\chi^2=2\frac{D+d}{D+d-1}\Omega$$

$$R(t)\sim e^{\frac{\chi t}{D+d}}\exp\left[-\sqrt{\frac{2Ed(D+d-1)}{D}}\frac{V_0^{\frac{1}{D+d}}}{\chi}e^{-\chi t}\right]$$

$$r(t)\sim e^{\frac{\chi t}{D+d}}\exp\left[+\sqrt{\frac{2ED(D+d-1)}{d}}\frac{V_0^{\frac{1}{D+d}}}{\chi}e^{-\chi t}\right].$$

$$R(t)\to R_0\exp\left(\frac{\chi}{D+d}t\right)$$
$$r(t)\to r_0\exp\left(\frac{\chi}{D+d}t\right)$$

$$\dot{\rho}+(p+\rho)D\frac{\dot{R}}{R}+(p'+\rho)\frac{\dot{r}}{r}=0.$$

$$R_{00} = -\left(D\frac{\ddot{R}}{R} + d\frac{\ddot{r}}{r}\right)$$

$$R_{DD} = \dot{H}_D + (DH_D + dH_d)H_D + (D-1)\frac{k_D}{R^2}$$

$$R_{dd} = \dot{H}_d + (DH_D + dH_d)H_d + (d-1)\frac{k_d}{r^2}.$$

$$R = 2D\frac{\ddot{R}}{R} + 2d\frac{\ddot{r}}{r} + 2DdH_DH_d + D(D-1)\left(H_D^2 + \frac{k_D}{R^2}\right) + d(d-1)\left(H_d^2 + \frac{k_d}{R^2}\right).$$

$$\nabla_\mu T^{\mu\nu} = 3\sigma j^\nu$$

$$-\nabla_\mu T^{\mu\nu}_{(\Lambda)} = \nabla_\mu T^{\mu\nu}_{(\mathrm{Dust})} = J^\nu, \nabla_\mu J^\mu = 0$$

$$S = \int \mathrm{d}^4x\Phi\mathcal{L}_1 + \int \mathrm{d}^4x\sqrt{-g}\mathcal{L}_2$$

$$A^\alpha_a\partial_\alpha\mathcal{L}_1 = 0$$

$$S = \mathcal{S}_{(\chi)} + \mathcal{S}_{(R)} = \int \mathrm{d}^4x\sqrt{-g}\chi_{\mu;\nu}T^{\mu\nu}_{(\chi)} + \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}R$$

$$S_{(\chi)} = \int \mathrm{d}^4x\sqrt{-g}\chi,\mu;\nu T^{\mu\nu}_{(\chi)}$$

$$\nabla_\mu T^{\mu\nu}_{(\chi)} = f^\nu; \nabla_\nu f^\nu = 0$$

$$T^{\mu\nu}_{(G)} = \frac{-2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_M\right)}{\delta g^{\mu\nu}}; \nabla_\mu T^{\mu\nu}_{(G)} = 0$$

$$\chi \to \chi + C_\chi; T^{\mu\nu}_{(\chi)} \to T^{\mu\nu}_{(\chi)} + g^{\mu\nu}C_T$$

$$S_{(\chi,A)} = \int \mathrm{d}^4x\sqrt{-g}\chi_{\mu;\nu}T^{\mu\nu}_{(\chi)}\mathrm{d}^4x + \frac{\sigma}{2}\int \mathrm{d}^4x\sqrt{-g}\left(\chi_\mu + \partial_\mu A\right)^2 \mathrm{d}^4x$$

$$\nabla_\mu T^{\mu\nu}_{(\chi)} = \sigma(\chi^\mu + \partial^\mu A)$$

$$f^\mu = \sigma(\chi^\mu + \partial^\mu A)$$

$$S = \int \ddot{B}\left[\frac{1}{2}m\dot{x}^2 + V(x)\right]\mathrm{d}t$$

$$S = \int \ddot{B}\left[\frac{1}{2}m\dot{x}^2 + V(x)\right]\mathrm{d}t$$

$$\frac{1}{2}m\dot{x}^2 + V(x) = E(t) = Pt + E_0$$

$$m\ddot{x}\frac{\mathrm{d}^2B}{\mathrm{d}t^2} + m\dot{x}\frac{\mathrm{d}^3B}{\mathrm{d}t^3} = V'(x)\frac{\mathrm{d}^2B}{\mathrm{d}t^2}$$

$$\frac{\dot{\ddot{B}}}{\ddot{B}} = \frac{2V'(x)}{\sqrt{2m(E(t) - V(x))}} - \frac{P}{2(E(t) - V(x))}$$

$$\pi_x = \frac{\partial\mathcal{L}}{\partial\dot{x}} = m\dot{x}\ddot{B}$$

$$\pi_B = \frac{\partial\mathcal{L}}{\partial\dot{B}} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial\mathcal{L}}{\partial\ddot{B}} = -\frac{\mathrm{d}}{\mathrm{d}t}E(t)$$

$$\Pi_B = \frac{\partial\mathcal{L}}{\partial\ddot{B}} = E(t)$$

$$\begin{aligned}\mathcal{H} &= \dot{x}\pi_x + \dot{B}\pi_B + \ddot{B}\Pi_B - \mathcal{L}\\ &= \pi_x\sqrt{\frac{2}{m}(\Pi_B - V(x))} + \dot{B}\pi_B = \text{Const.}\end{aligned}$$

$$\begin{aligned}\mathcal{S} = &\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}R + \int \mathrm{d}^4x\sqrt{-g}\mathcal{L}(\phi, X)\\ &+ \int \mathrm{d}^4x\sqrt{-g}\chi,\mu;\nu T^{\mu\nu}_{(\chi)}\end{aligned}$$

$$\mathcal{L}(\phi, X) = \sum_{N=1}^{\infty} A_n(\phi)X^n - V(\phi)$$

$$T^{\mu\nu}_{(\chi)} = g^{\mu\nu}\Lambda(\phi, X) \Rightarrow \mathcal{S}_{(\chi)} = \int \mathrm{d}^4x\Lambda\,\Box\,\chi$$

$$j_\alpha = 2(\Box\,\chi + 1)\phi_{,\alpha}$$

$$T^{\mu\nu}_{(G)} = g^{\mu\nu}\left(-\Lambda + \chi^{,\sigma}\Lambda_{,\sigma}\right) + j^\mu\phi^{,\nu} - \chi^{,\mu}\Lambda^{,\nu} - \chi^{,\nu}\Lambda^{,\mu}$$

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\,\mathrm{d}\Omega^2\right]$$

$$2\dot{\phi}\ddot{\phi} = \frac{C_2}{a^3}$$

$$\dot{\phi}^2 = C_1 + C_2\int \frac{\mathrm{d}t}{a^3}$$

$$2\dot{\phi}(\Box\,\chi + 1) = \frac{C_3}{a^3}$$

$$\dot{\chi} = \frac{1}{a^3}\int a^3\,\mathrm{d}t + \frac{C_4}{a^3} - \frac{C_3}{2a^3}\int \frac{\mathrm{d}t}{\dot{\phi}}$$

$$\rho_{de} = \dot{\phi}^2 + 2\dot{\chi}\dot{\phi}\ddot{\phi}$$

$$\rho_{dm} = \frac{C_3}{a^3}\dot{\phi} - 4\dot{\chi}\dot{\phi}\ddot{\phi}$$

$$\lim_{t\to\infty}\dot{x} = \frac{1}{3H_0}$$

$$\rho_{de} = C_1 + O\left(\frac{1}{a^6}\right);$$

$$\rho_{dm} = \left(C_3\sqrt{C_1} - \frac{2C_2}{3H_0}\right)\frac{1}{a^3} + O\left(\frac{1}{a^6}\right)$$

$$\mathcal{S} = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}R + \int \mathrm{d}^4x(\Phi + \sqrt{-g})\mathcal{L}(X,\phi)$$

$$\rho_{DE} = \dot{\phi}^2 = C_1$$

$$\rho_{\mathrm{Dust}} = \frac{\sqrt{C_1}C_3}{a^3}$$

$$a_0(t) = \left(\frac{C_3}{\sqrt{C_1}}\right)^{\frac{1}{3}}\sinh^{2/3}(\alpha t)$$

$$\Omega_\Lambda = \frac{C_1}{H};\ \Omega_m = \frac{C_1\sqrt{C_3}}{H}$$

$$\lambda_1(t,t_0) = \frac{C_2}{C_1}\int_{t_0}^{t}\frac{\mathrm{d}t}{a^3}$$

$$\lambda_2(t,t_0) = \frac{C_2}{\sqrt{C_1C_3}}\dot{\chi}(t,t_0)$$

$$\rho_{de} = C_1\left(1+\lambda_1+\frac{C_3}{\sqrt{C_1}}\lambda_2\right)+O_2(\lambda_1,\lambda_2)$$

$$\rho_{dm} = \frac{\sqrt{C_1}C_3}{a^3}\left(1-\frac{1}{2}(\lambda_1+\lambda_2)\right)+O_2(\lambda_1,\lambda_2)$$

$$\mathcal{S} = \frac{1}{16\pi G}\int\sqrt{-g}R+\int\sqrt{-g}\Lambda+\int\sqrt{-g}\chi_{\mu;\nu}T^{\mu\nu}_{(\chi)} + \frac{\sigma}{2}\int\sqrt{-g}\left(\chi_\mu+\partial_\mu A\right)^2$$

$$\nabla_\mu\Lambda = f_\mu = \sigma\left(\chi_\mu+\partial_\mu A\right)$$

$$f^{\mu}_{;\mu} = 0$$

$$j_\alpha = 2\left(\chi^{\lambda}_{;\lambda}+1\right)\phi_{,\alpha};\, j^{\alpha}_{;\alpha} = 0$$

$$T^{\mu\nu}_{(G)} = g^{\mu\nu}\left(-\Lambda+\chi^{,\lambda}\Lambda_{,\lambda}-\frac{1}{2\sigma}\Lambda^{,\lambda}\Lambda_{,\lambda}\right) + j^{\mu}\phi^{,\nu}-\chi^{,\mu}\Lambda^{,\nu}-\chi^{,\nu}\Lambda^{,\mu}+\frac{1}{\sigma}\Lambda^{,\mu}\Lambda^{,\mu}$$

$$\mathcal{S} = \frac{1}{16\pi G}\int\sqrt{-g}R+\int\sqrt{-g}g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} - \int\sqrt{-g}\chi_\mu\nabla_\nu T^{\mu\nu}_{(\chi)}+\frac{\sigma}{2}\int\sqrt{-g}\left(\chi_\mu+\partial_\mu A\right)^2.$$

$$0 = -\nabla_\nu T^{\mu\nu}_{(\chi)}+\sigma\left(\chi_\mu+\partial_\mu A\right)$$

$$\mathcal{S} = \frac{1}{16\pi G}\int\sqrt{-g}R+\int\sqrt{-g}g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} - \frac{1}{2\sigma}\int\sqrt{-g}\left(\nabla_\nu T^{\mu\nu}_{(\chi)}\right)^2+\int\sqrt{-g}\partial_\nu A\nabla_\alpha T^{\nu\alpha}_{(\chi)}$$

$$\mathcal{Z} = \int\mathcal{D}\phi\,\delta(\nabla_\nu f^\nu)\ \exp\left[\frac{1}{2\sigma}\int \mathrm{d}^4x\sqrt{g}f_\mu f^\mu-\int \mathrm{d}^4x\sqrt{g}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right]$$

$$\rho_{de} = C_1+C_2\int\frac{\mathrm{d}t}{a^3}$$

$$\rho_{dm} = \frac{C_3}{a^3} - \frac{C_2 t}{a^3}$$

$$\rho_{de} = C_1 + C_2 \int \frac{\mathrm{d}t}{a^3}$$

$$\rho_{dm} = \frac{C_3}{a^3} \dot{\phi}$$

$$\frac{\partial g^{\alpha\beta}}{\partial g_{\mu\nu}} = -\frac{1}{2}\left(g^{\alpha\mu} g^{\beta\nu} + g^{\alpha\nu} g^{\beta\mu}\right)$$

$$\frac{\partial \Gamma^{\tau}_{\lambda\sigma}}{\partial g_{\mu\nu}} = -\frac{1}{2}\left(g^{\mu\tau} \Gamma^{\nu}_{\lambda\sigma} + g^{\nu\tau} \Gamma^{\mu}_{\lambda\sigma}\right)$$

$$\frac{\partial \Gamma^{\tau}_{\lambda\alpha}}{\partial g_{\mu\nu,\sigma}} = \frac{1}{4}\left[g^{\mu\tau}\left(\delta^{\nu}_{\alpha}\delta^{\sigma}_{\lambda} + \delta^{\nu}_{\lambda}\delta^{\sigma}_{\alpha}\right) + g^{\tau\nu}\left(\delta^{\mu}_{\alpha}\delta^{\sigma}_{\lambda} + \delta^{\mu}_{\lambda}\delta^{\sigma}_{\alpha}\right) - g^{\tau\sigma}\left(\delta^{\mu}_{\alpha}\delta^{\nu}_{\lambda} + \delta^{\mu}_{\lambda}\delta^{\nu}_{\alpha}\right)\right]$$

$$T^{\alpha\beta}_{(G)} = \frac{-2}{\sqrt{-g}} \frac{\partial\left(\sqrt{-g}\,\chi_{\mu;\nu} T^{\mu\nu}_{(\chi)}\right)}{\partial g_{\alpha\beta}}$$

$$+ \frac{2}{\sqrt{-g}} \frac{\partial}{\partial x^{\sigma}} \frac{\partial\left(\sqrt{-g}\,\chi_{\mu;\nu} T^{\mu\nu}_{(\chi)}\right)}{\partial g_{\alpha\beta,\sigma}}$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = \frac{C_2}{a^3}$$

$$\nabla_{\mu} T^{\mu\nu}_{(\mathrm{Dust})} = \gamma^2 j^{\nu}$$

$$\nabla_{\mu} T^{\mu\nu}_{(\mathrm{Dust})} = -\nabla_{\mu} T^{\mu\nu}_{(\Lambda)} = \gamma^2 j^{\nu}$$

$$\nabla_{\mu}\left(T^{\mu\nu}_{(\mathrm{Dust})} + T^{\mu\nu}_{(\Lambda)}\right) = 0$$

$$T^{\mu}_{\nu} = \mathbf{Diag}(\rho, -p, -p, -p)$$

$$\dot{\rho}_{\mathrm{dust}} + 3H(1 + \tilde{\omega})\rho_{\mathrm{dust}} = \frac{\gamma^2}{a^3}$$

$$\dot{\rho}_{\Lambda} + 3H(1 + \omega)\rho_{\Lambda} = -\frac{\gamma^2}{a^3}.$$

$$x = \frac{\rho_{\mathrm{dust}}}{3H^2}, y = \frac{\rho_{\Lambda}}{3H^2}, \delta = \frac{\gamma^2}{a^3 H \rho_{\mathrm{dust}}}$$

$$x' = 6x^2(\tilde{\omega} - \omega) + x(\gamma\delta + 3 + 6\omega + 3\tilde{\omega})$$

$$\delta' = \delta(\gamma\delta + 3(x-1)(\omega - \tilde{\omega})).$$

Name	The point	Eigenvalues	Densities fraction
A	$(0, \frac{3}{\gamma}(\omega - \tilde{\omega}))$	$3(3\omega + 1),\ 3(\omega - \tilde{\omega})$	0
B	$\left(\frac{\omega + 1/3}{\omega - \tilde{\omega}}, -\frac{3\tilde{\omega}+1}{\gamma}\right)$	$\frac{1}{2}\left(\pm\sqrt{36\omega^2 - 72\omega\tilde{\omega} + 9(\tilde{\omega}-2)\tilde{\omega} - 3} - 6\omega - 3\tilde{\omega} - 3\right)$	$-\frac{3\omega+1}{3\tilde{\omega}+1}$
C	(0,0)	$3(\tilde{\omega} - \omega),\ 3(2\omega + \tilde{\omega} + 1)$	0
D	$\left(\frac{2\omega+\tilde{\omega}+1}{2(\omega-\tilde{\omega})}, 0\right)$	$-3(2\omega + \tilde{\omega} + 1),\ \frac{3}{2}(1 + 3\tilde{\omega})$	$-\frac{2\omega+\tilde{\omega}+1}{3\tilde{\omega}+1}$

$$\Phi = \varepsilon^{\alpha\beta\gamma\delta}\varepsilon_{abcd}\partial_\alpha\varphi_a\partial_\beta\varphi_b\partial_\gamma\varphi_c\partial_\delta\varphi_d.$$

$$S = \int d^4x\Phi\mathcal{L}_1 + \int d^4x\sqrt{-g}\mathcal{L}_2$$

$$A_a^\alpha\partial_\alpha\mathcal{L}_1 = 0,$$

$$\mathcal{S} = \int d^4x\sqrt{-g}\chi_{\mu;\nu}T^{\mu\nu}_{(\chi)}$$

$$\delta\chi_\mu : \nabla_\mu T^{\mu\nu}_{(\chi)} = 0.$$

$$\int d^4x\sqrt{-g}\chi_{\mu;\nu}T^{\mu\nu}_{(\chi)} = \int d^4x\sqrt{-g}\chi^\lambda_{;\lambda}\mathcal{L}_m$$
$$= \int d^4x\partial_\mu\left(\sqrt{-g}\chi^\mu\right)\mathcal{L}_m = \int d^4x\Phi\mathcal{L}_m$$

$$\pi_{\chi^0} = \frac{\partial\mathcal{L}}{\partial\dot{\chi}^0} = T^0_0(\chi) := \rho_{(\chi)}$$

$$S_{(\chi,A)} = \int d^4x\sqrt{-g}\chi_{\mu;\nu}T^{\mu\nu}_{(\chi)} + \frac{\kappa}{2}\int d^4x\sqrt{-g}\left(\chi_\mu + \partial_\mu A\right)^2$$

$$\nabla_\nu T^{\mu\nu}_{(\chi)} = \kappa(\chi^\mu + \partial^\mu A) = f^\mu,$$

$$\nabla_\mu f^\mu = \kappa\nabla_\mu(\chi^\mu + \partial^\mu A) = 0.$$

$$T^{\mu\nu}_{(G)} = \frac{-2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_M\right)}{\delta g^{\mu\nu}}, \nabla_\mu T^{\mu\nu}_{(G)} = 0.$$

$$f_\mu = \kappa\left(\chi_\mu + \partial_\mu A\right) = 0 \Rightarrow \chi_\mu = -\partial_\mu A$$

$$\mathcal{S} = -\int d^4x\sqrt{-g}\nabla_\mu\nabla_\nu\chi T^{\mu\nu}_{(\chi)}$$

$$\mathcal{L} = \frac{1}{2}\mathcal{R} + \chi_{,\mu;L}T^{\mu\nu}_{(\chi)} - \frac{1}{2}\phi^{,\mu}\phi_{,\mu} - V(\phi)$$

$$T^{\mu\nu}_{(\chi)} = -\frac{\lambda_1}{2}\phi^{\mu}\phi^{\nu} - \frac{\lambda_2}{2}g^{\mu\nu}\left(\phi_{,\alpha}\phi^{,\alpha}\right) + g^{\mu\nu}U(\phi),$$

$$\rho_{(\chi)} = (\lambda_1+\lambda_2)\frac{\dot{\phi}^2}{2} + U(\phi),$$

$$p_{(\chi)} = -\lambda_2\frac{\dot{\phi}^2}{2} - U(\phi).$$

$$\nabla_\mu\nabla_\nu T^{\mu\nu}_{(\chi)} = 0$$

$$j^{\mu} = \frac{\lambda_1}{2}(\chi^{\mu;\nu} + \chi^{\nu;\mu})\phi_{,\nu} + (1 + \lambda_2 \,\square\, \chi)\phi^{,\mu},$$

$$\nabla_\mu j^{\mu} = V'(\phi) - \square\, \chi U'(\phi)$$

$$\begin{aligned} G^{\mu\nu} =& g^{\mu\nu}\left(-\chi_{,\alpha;\beta}T^{\alpha\beta}_{(\chi)} + \frac{1}{2}\phi^{,\alpha}\phi_{,\alpha} + V(\phi)\right) \\ & -\phi^{,\mu}\phi^{\nu} + \chi_{,\alpha;\beta}\frac{\partial T^{\alpha\beta}_{(\chi)}}{g_{\mu\nu}} \\ & +\nabla_\lambda\left(\chi^{\mu}T^{\nu\lambda}_{(\chi)} + \chi^{\nu}T^{\mu\lambda}_{(\chi)} - \chi^{\lambda}T^{\mu\nu}_{(\chi)}\right) \end{aligned}$$

$$\begin{aligned} \chi_{,\alpha;\beta}\frac{\partial T^{\alpha\beta}}{\partial g_{\mu\nu}} =& -\frac{\lambda_1}{2}\chi^{(,\mu}\phi^{,\nu)} \,\square\, \phi + \left(\frac{\lambda_1}{2} + \lambda_2\right)\phi^{,\mu}\phi^{,\nu} \,\square\, \chi \\ & + \frac{\lambda_1}{2}\chi^{\gamma;\mu}\phi^{,\nu}\phi_{,\gamma} - \lambda_2\phi^{,\mu;\lambda}\chi^{,\nu}\phi_{,\lambda} - \lambda_2\chi^{\mu}\phi^{,\gamma;\nu}\phi_{,\nu} \\ & + \frac{\lambda_1}{2}\phi^{,\mu}\chi^{\gamma;\nu}\phi_{,\gamma} - \frac{\lambda_1}{2}\chi^{(,\nu}\phi^{,\mu);\gamma}\phi_{,\gamma} \\ & + \frac{\lambda_1}{2}\pi^{(,\nu}\phi^{,\mu);\gamma}\chi_{,\gamma} + \chi^{(,\mu}\phi^{,\nu)}U'(\phi). \end{aligned}$$

$$ds^2 = -dt^2 + a(t)^2\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right)$$

$$(\lambda_1+\lambda_2)\dot{\phi}\ddot{\phi} + U'(\phi)\dot{\phi} + 3H\lambda_1\dot{\phi}^2 = \frac{\sigma}{a^3}$$

$$\dot{\phi}^2 = \dot{\phi}^2_{(0)}a^{-\frac{3\lambda_1}{\lambda_1+\lambda_2}} + \frac{\sigma}{\lambda_1+\lambda_2}a^{-\frac{3\lambda_1}{\lambda_1+\lambda_2}}\int_0^t ds\, a^{-\frac{3\lambda_2}{\lambda_1+\lambda_2}}$$

$$\left(\frac{\lambda_1}{2} - \lambda_2\right)\ddot{\chi} + (1 - 3H\dot{\chi})\lambda_2 = \frac{\tilde{\sigma}}{\dot{\phi}a^3}$$

$$\begin{aligned} \rho =& \frac{3}{2}H(\lambda_1 - 2\lambda_2)\dot{\chi}\dot{\phi}^2 + \frac{1}{2}\dot{\phi}^2(1 - 2(\lambda_1+\lambda_2)\ddot{\chi}) \\ & + \dot{\chi}\dot{\phi}\left((\lambda_1+\lambda_2)\ddot{\phi}\right) + V, \end{aligned}$$

$$p = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\lambda_1\ddot{\chi}\dot{\phi}^2 + \lambda_2\dot{\chi}\dot{\phi}\ddot{\phi} - V$$

$$a \sim t^{\alpha}.$$

$$\dot{\phi} = \sqrt{\frac{2\sigma}{3\alpha(\lambda_1 - \lambda_2) + \lambda_1 + \lambda_2}} t^{\frac{1}{2} - \frac{3\alpha}{2}},$$

$$\dot{\chi} = Ct$$

$$C = \frac{2\lambda_2}{-6\alpha\lambda_2 + \lambda_1 - 2\lambda_2}.$$

$$\rho = \frac{\alpha_1}{a^3} + \frac{\alpha_2 t}{a^3} + V$$

$$\alpha_1 = \frac{18\alpha^2\lambda_2(2\lambda_2 - \lambda_1)}{2(\lambda_1 - 2\lambda_2(3\alpha + 1))}$$

$$\alpha_2 = \frac{(6\alpha + 2)\lambda_1\lambda_2 + 2(3\alpha + 1)(\lambda_2 - 1)\lambda_2 + \lambda_1}{2(\lambda_1 - 2\lambda_2(3\alpha + 1))}$$

$$a \sim t.$$

$$\dot{\phi}^2 = \dot{\phi}_0^2 a^{-\frac{3\lambda_1}{\lambda_1 + \lambda_2}} - \sigma_0 H_0 \frac{\lambda_1 + \lambda_2}{3\lambda_2} \frac{1}{a^3}$$

$$\dot{\chi} = \frac{1}{3H_0} + \mathcal{O}\left(\frac{1}{a^3}\right).$$

$$\rho = H_0(3\lambda_2 - 1)\sigma \frac{\lambda_1 + \lambda_2}{6\lambda_2} \frac{1}{a^3} + V + \frac{1}{2}\dot{\phi}_0^2(1 - 2\lambda_2) a^{-\frac{3\lambda_1}{\lambda_1 + \lambda_2}}.$$

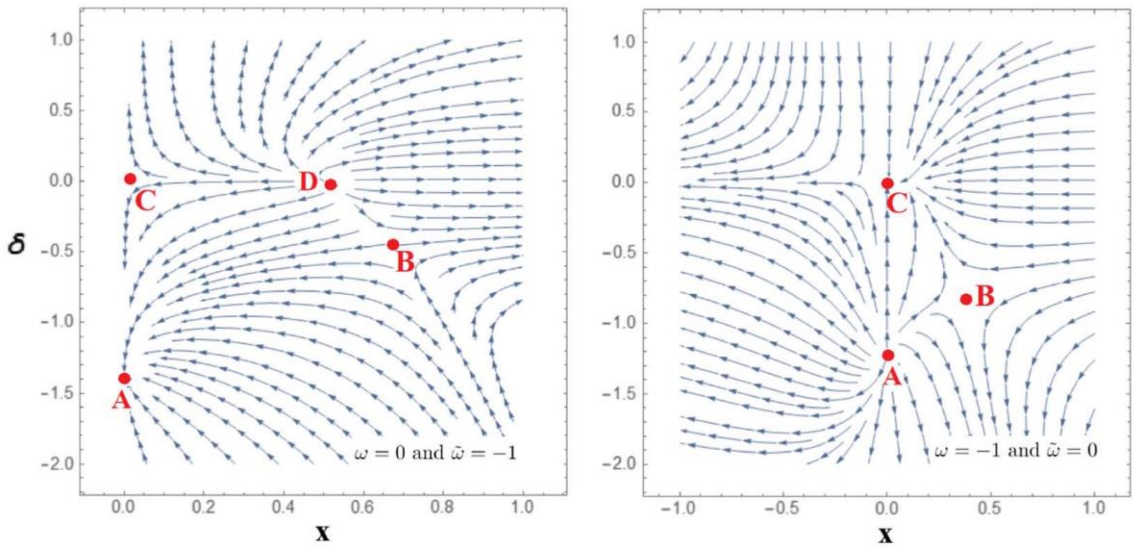


Figura 6. Comportamiento vectorial de la partícula oscura en un espacio – tiempo cuántico relativista.

$$T^{\mu\nu}_{(\chi)} = -\frac{\lambda_1}{2}\phi^{,\mu}\phi^{\nu} + g^{\mu\nu}U(\phi)$$

$$\dot{\phi}^2 = \frac{\dot{\phi}^2_{(0)}}{a^3} + \frac{\sigma}{\lambda_1}\frac{t}{a^3}$$

$$\dot{\chi}(t) = \dot{\chi}(0) - \frac{2}{\lambda_1}t + \int dt \frac{2\tilde{\sigma}}{\lambda_1 \dot{\phi} a^3}$$

$$\dot{\chi}(t \to \infty) \to -\frac{2t}{\lambda_1}$$

$$\rho = V + \frac{\alpha_1}{a^3} + \frac{\alpha_2}{a^{4.5}}$$

$$\alpha_1 = \frac{5\dot{\phi}_0^2\lambda_1 + \lambda_1\sigma\chi_0 + 3\sigma t}{2\lambda_1}$$

$$\alpha_2 = -\frac{2\tilde{\sigma}}{3\dot{\phi}_0 H_0 \lambda_1}\left(3\dot{\phi}_0^2 H_0 \lambda_1 + 3H_0\sigma t + \sigma\right)$$

$$\chi \to \chi + ct$$

$$\chi^0 \to \chi^0 + c$$

$$\partial_t \rho_{\text{dust}} + 3H\rho_{\text{dust}} = \frac{\gamma^2}{a^3},$$

$$\rho_{\text{dust}} = \frac{C_1}{a^3} + \frac{\gamma^2 t}{a^3},$$

$$\partial_t \rho_\Lambda = -\frac{\gamma^2}{a^3},$$

$$\rho_\Lambda = C_2 - \gamma^2 \int \frac{dt}{a^3}$$

$$\rho = \frac{C_1 + \gamma^2 t}{a^3} + C_2 + \mathcal{O}\left(\frac{1}{a^6}\right),$$

$$C_1 = \frac{5\dot{\phi}_0^2 + \sigma\chi_0}{2}$$

$$\gamma^2 = \frac{3\sigma}{2\lambda_1}, C_2 = V.$$

$$\frac{d}{dt} = -H(z)(z+1)\frac{d}{dz}$$

$$\left(\frac{\lambda_1}{2} - \lambda_2\right)\ddot{\chi} + (1 - 3H\dot{\chi})\lambda_2 = \frac{\tilde{\sigma}}{\dot{\phi} a^3}$$

$$\lambda_1\ddot{\chi} - \lambda_2(\ddot{\chi} - 3H\dot{\chi}) + 1 = \tilde{\sigma}/\left(\dot{\phi} a^3\right).$$

$$\dot{\chi} = \frac{1}{3H_0} + \mathcal{O}\left(\frac{1}{a^3}\right),$$

$$\dot{\chi} = \frac{1}{3\lambda_2 H_0} + \mathcal{O}\left(\frac{1}{a^3}\right).$$

$$\rho = H_0(3\lambda_2 - 1)\sigma\frac{\lambda_1 + \lambda_2}{6\lambda_2}\frac{1}{a^3} + V + \frac{1}{2}\dot{\phi}_0^2(1 - 2\lambda_2)a^{-\frac{3\lambda_1}{\lambda_1+\lambda_2}}$$

$$\rho = \frac{H_0\sigma_0(\lambda_1 + \lambda_2)}{3\lambda_2}\frac{1}{a^3} + V - \frac{1}{2}\dot{\phi}_0^2 a^{-\frac{3\lambda_1}{\lambda_1+\lambda_2}}.$$

$$\dot{\chi}(t) = \dot{\chi}(0) - \frac{1}{\lambda_1}t + \int dt\frac{\tilde{\sigma}}{\lambda_1\dot{\phi}a^3}$$

$$\dot{\chi}(t) = \dot{\chi}(0) - \frac{2}{\lambda_1}t + \int dt\frac{2\tilde{\sigma}}{\lambda_1\dot{\phi}a^3}.$$

$$\dot{\chi}(t \to \infty) \to -\frac{t}{\lambda_1}$$

$$\dot{\chi}(t \to \infty) \to -\frac{2t}{\lambda_1}.$$

$$\alpha_1 = \sigma\dot{\chi}_0 + \frac{\sigma t}{\lambda_1} + 3\dot{\phi}_0^2$$
$$\alpha_2 = \frac{\tilde{\sigma}}{\sqrt{\lambda_1(\lambda_1\dot{\phi}_0^2 + \sigma t)}}\left(3H_0(\lambda_1\dot{\phi}_0^2 + \sigma t) + 2\sigma\right)$$

$$\alpha_1 = \frac{5\dot{\phi}_0^2\lambda_1 + \lambda_1\sigma\chi_0 + 3\sigma t}{2\lambda_1}, \alpha_2 = -\frac{2\tilde{\sigma}}{3\dot{\phi}_0 H_0\lambda_1}\left(3\dot{\phi}_0^2 H_0\lambda_1 + 3H_0\sigma t + \sigma\right)$$

$$C_1 = \frac{3\dot{\phi}_0^2 + \sigma\dot{\chi}_0}{2},$$
$$\gamma^2 = \frac{\sigma}{\lambda_1}, C_2 = V$$

$$C_1 = \frac{5\dot{\phi}_0^2 + \sigma\chi_0}{2}, \gamma^2 = \frac{3\sigma}{2\lambda_1}, C_2 = V$$

SECCIÓN II.

$$p_{dm} = 0, \rho_{dm} = nm$$

$$m = \lambda\varphi$$

$$\rho_{dm} = \lambda n\varphi.$$

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \rho_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi).$$

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV_{eff}}{d\varphi} = 0$$
$$V_{eff} = V(\varphi) + \lambda n\varphi$$

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV_{eff}}{d\varphi} = -\lambda n_0 a^{-3}.$$

$$m_\psi = \lambda\langle\varphi\rangle.$$

$$V(\varphi) = u_0\varphi^{-p}\ (p > 0).$$

$$V_{eff}(\varphi) = V(\varphi) + \lambda n_\psi \varphi$$

$$\langle\varphi\rangle = \left(\frac{pu_0}{\lambda n_\psi}\right)^{1/1+p}$$

$$\nabla_\mu\left(T^{\mu}_{(dm)^{\mu}_{\nu}} + T^{\mu}_{(\varphi)^{\mu}_{\nu}}\right) = \nabla_\mu T^{\mu}_{(dm)^{\mu}_{\nu}} + \nabla_\mu T^{\mu}_{(\varphi)^{\mu}_{\nu}} = 0,$$

$$\nabla_\mu T^{\mu}_{(dm)^{\mu}_{\nu}} = -\nabla_\mu T^{\mu}_{(\varphi)^{\mu}_{\nu}}$$

$$T^{\mu}_{(\varphi)^{\mu}_{\nu}} = \partial^\mu\varphi\partial_\nu\varphi - \delta^{\mu}_{\nu}\left[\frac{1}{2}\partial^\alpha\varphi\partial_\alpha\varphi - V(\varphi)\right]$$

$$\nabla_\mu T^{\mu}_{(\varphi)^{\mu}_{\nu}} = \partial_\mu T^{\mu}_{(\varphi)^{\mu}_{\nu}} + \Gamma^{\mu}_{\mu\beta} T^{\beta}_{(\varphi)^{\beta}_{\nu}} - \Gamma^{\beta}_{\mu\nu} T^{\mu}_{(\varphi)^{\mu}_{\beta}} = -\left(\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi}\right)\partial_\nu\varphi.$$

$$\nabla_\mu T^{\mu}_{(\varphi)^{\mu}_{\nu}} = \lambda n \partial_\nu\varphi$$

$$\nabla_\mu T^{\mu}_{(dm)^{\mu}_{0}} = \dot{\rho}_{dm} + 3H\rho_{dm} = \lambda n\dot{\varphi}$$

$$T^{\mu}_{\nu} = V g^{\mu}_{\nu}$$

$$T^{\mu}_{\nu} = (\rho + p)u^\mu u_\nu - p g^{\mu}_{\nu}$$

$$\nabla_\mu \hat{T}^{\mu}_{\nu} = F_\nu, F_\mu \equiv \nabla_\mu V$$

$$\nabla_\mu\left(T^{\mu}_{(de)^{\mu}_{\nu}} + T^{\mu}_{(dm)^{\mu}_{\nu}}\right) = 0.$$

$$\nabla_\mu T^{\mu}_{(de)^{\mu}_{\nu}} = -T^{\mu}_{(dm)^{\mu}_{\nu}} = F_\nu,$$

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = 8\pi G\left(T_{(de)_{\mu\nu}} + T_{(dm)_{\mu\nu}}\right)$$

$$u^\mu\nabla^\nu T_{(dm)\mu\nu} = -u^\mu F_\mu$$
$$u^\mu\nabla^\nu T_{(de)\mu\nu} = u^\mu F_\mu$$

$$h^{\mu\beta}\nabla^\nu T_{(dm)\mu\nu} = -h^{\mu\beta}F_\mu$$
$$h^{\mu\beta}\nabla^\nu\nabla^\nu T_{(de)\mu\nu} = h^{\mu\beta}F_\mu$$

$$h^{\mu\beta}\nabla_\mu p_{dm} + (\rho_{dm} + p_{dm})u^\mu\nabla_\mu u^\beta = -h^{\mu\beta}F_\mu$$
$$h^{\mu\beta}\nabla_\mu p_{de} + (\rho_{de} + p_{de})u^\mu\nabla_\mu u^\beta = h^{\mu\beta}F_\mu$$

$$\nabla_\mu u^\mu = 3H$$
$$u^\mu \nabla_\mu u^\nu = 0$$

$$\dot{\rho}_{dm} + 3H\rho_{dm} = Q$$
$$\dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = -Q$$

$$L = i\bar{\psi}\gamma_\mu \nabla^\mu \psi - m_\psi \bar{\psi}\psi - \frac{1}{2}\nabla_\mu \varphi \nabla^\mu \varphi - \frac{1}{2}m_\varphi \varphi^2 + g\varphi\bar{\psi}\psi$$

$$V(r) = -\frac{Gm_\psi^2}{r}\left[1 + \alpha \exp\left(-\frac{r}{r_s}\right)\right]$$

$$\mathcal{L} = \frac{1}{2}\left[\frac{R}{\kappa} - \nabla^a \varphi \nabla_a \varphi\right] + V(\varphi) - C(\varphi)\mathcal{L}_{\mathrm{DM}} + \mathcal{L}_{\mathrm{S}}$$

$$T_{ab} = \nabla_a \varphi \nabla_b \varphi - g_{ab}\left[\frac{1}{2}\nabla^c \nabla_c \varphi - V(\varphi)\right] + C(\varphi)T_{ab}^{\mathrm{DM}} + T_{ab}^{\mathrm{S}}$$

$$\frac{d^2\mathbf{r}}{dt^2} = -\vec{\nabla}\Phi - \frac{C_\varphi(\varphi)}{C(\varphi)}\vec{\nabla}\varphi$$

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2}\partial^\mu \phi \partial_\mu \phi + U(\phi) + m(\phi)\bar{\psi}\psi - \mathcal{L}_{kin}[\psi]\right],$$

$$\nabla_\nu T^\nu_{(\alpha)\mu} = Q_{(\alpha)\mu},$$

$$\sum_\alpha Q_{(\alpha)\mu} = 0$$

$$\frac{d\rho_\phi}{d\eta} = -3\mathcal{H}(p_\phi + \rho_\phi) + \beta(\phi)\frac{d\phi}{d\eta}(1 - 3w_\alpha)\rho_\alpha$$
$$\frac{d\rho_\alpha}{d\eta} = -3\mathcal{H}(p_\phi + \rho_\phi) - \beta(\phi)\frac{d\phi}{d\eta}(1 - 3w_\alpha)\rho_\alpha$$

$$Q_{(\phi)\mu} = \frac{\partial \ln\, m(\phi)}{\partial \phi}T_\alpha \partial_\mu \phi\,, m_\alpha = \bar{m}_\alpha e^{-\beta(\phi)\phi}.$$

$$\frac{d\mathbf{v}_\alpha}{d\eta} + \left(\mathcal{H} - \beta(\phi)\frac{d\phi}{d\eta}\right)\mathbf{v}_\alpha - \nabla[\Phi_\alpha + \beta\phi] = 0.$$

$$\dot{\mathbf{v}}_\alpha = -\tilde{H}\mathbf{v}_\alpha - \nabla\frac{\tilde{G}_\alpha m_\alpha}{r}.$$

$$\nabla[\Phi_\alpha + \beta\phi]\tilde{G}_\alpha = G_N[1 + 2\beta^2(\phi)]$$

$$\tilde{H}\mathbf{v}_\alpha \equiv H\left(1 - \beta(\phi)\frac{\dot{\phi}}{H}\right)\mathbf{v}_\alpha$$

$$\dot{\rho}_{dm} + 3H\rho_{dm} = Q$$
$$\dot{\rho}_{de} + 3H(1 + w_{de})\rho_{de} = -Q$$

$$Q\begin{cases}>0\\<0\end{cases}\rightarrow \text{ energy transfer is } \begin{cases}\text{dark energy } \rightarrow \text{ dark matter}\\ \text{dark matter } \rightarrow \text{ dark energy}\end{cases}$$

$$w_{de}=w_{\varphi}=\frac{p_{\varphi}}{\rho_{\varphi}}=\frac{\frac{1}{2}\dot{\varphi}^2-V(\varphi)}{\frac{1}{2}\dot{\varphi}^2+V(\varphi)}$$

$$\ddot{\varphi}^2+3H\dot{\varphi}+\frac{dV}{d\varphi}=-\frac{Q}{\dot{\varphi}}.$$

$$\dot{\rho}_i+3H\left(1+w_{eff,i}\right)\rho_i=0, i=de,dm$$

$$w_{eff,dm}=-\frac{Q}{3H\rho_{dm}}, w_{eff,de}=w_{de}+\frac{Q}{3H\rho_{de}},$$

$$Q>0\rightarrow\begin{cases}w_{\text{eff,dm}}<0 & \text{dark matter redshifts slower than } a^{-3}\\ w_{\text{eff,de}}>w_{\text{de}} & \text{dark energy has les accelerating power}\end{cases}$$
$$Q<0\rightarrow\begin{cases}w_{\text{eff,dm}}>0 & \text{dark matter redshifts faster than } a^{-3}\\ w_{\text{eff,de}}<w_{\text{de}} & \text{dark energy has more accelerating power}\end{cases}$$

$$w_{\text{eff},dm}=w_{dm}=0, w_{\text{eff,de}}=w_{de}$$

$$Q\equiv-3H\Pi_{dm}=+3H\Pi_{de}$$

$$\dot{\rho}_{dm}+3H(\rho_{dm}+\Pi_{dm})=0$$
$$\dot{\rho}_{de}+3H(\rho_{de}+p_{de}+\Pi_{de})=0$$

$$\dot{r}=\frac{\rho_{dm}}{\rho_{de}}\left(\frac{\dot{\rho}_m}{\rho_m}-\frac{\dot{\rho}_{de}}{\rho_{de}}\right)=3Hr\left(w_{de}+\frac{1+r}{\rho_{dm}}\frac{Q}{3H}\right)$$

$$\frac{Q}{3H\rho_{dm}}=-\frac{w_{de}+\frac{\xi}{3}}{1+r}$$

$$\frac{\rho'_{dm}}{\rho_{dm}}=-1-\frac{\Pi}{\rho_{dm}}$$
$$\frac{\rho'^{de}_{de}}{\rho_{de}}=-(1+w_{de})+\frac{\Pi}{\rho_{de}}$$

$$\rho'=-\left(1+\frac{w_{de}}{1+r}\right)\rho$$
$$r'=r\left[w_{de}-\frac{(1+r)^2}{r\rho}\Pi\right]$$

$$\Omega'_{dm}=w_{de}\Omega_{dm}\Omega_{de}$$
$$\Omega'_{de}=-w_{de}\Omega_{dm}\Omega_{de}$$

$$r_c=-(1+w_{de})$$

$$\rho_c=-\frac{w_{de}}{1+w_{de}}\Pi_c$$

$$\frac{d}{dt} = \frac{d}{dz}\frac{dz}{da}\frac{da}{dt} = -(1+z)H(z)\frac{d}{dz}$$

$$\frac{d\rho_{dm}}{dz} - \frac{3}{1+z}\rho_{dm} = -\frac{Q(z)}{(1+z)H(z)}$$
$$\frac{d\rho_{de}}{dz} - \frac{3}{1+z}(1+w_{de})\rho_{de} = \frac{Q(z)}{(1+z)H(z)}$$

$$I_Q(z) \equiv \frac{1}{\rho_{crit}^0(1+z)^3 H(z)}Q(z)$$

$$\frac{d\Omega_{dm}}{dz} - \frac{3}{1+z}\Omega_{dm} = -(1+z)^2 I_Q(z)$$
$$\frac{d\Omega_{de}}{dz} - \frac{3}{1+z}(1+w_{de})\Omega_{de} = (1+z)^2 I_Q(z)$$

$$q = q_0^* + q_{dm}^*(\rho_{dm} - \rho_{dm,0}) + q_{de}^*(\rho_{de} - \rho_{de,0})$$

$$q = q_0 + q_{dm}\rho_{dm} + q_{de}\rho_{de}$$

$$q \propto \rho_{dm}, q_0 = q_{de} = 0$$
$$q \propto \rho_{de}, q_0 = q_{dm} = 0$$
$$q \propto \rho_{total}, q_0 = 0, q_{dm} = q_{de}$$

$$\delta(a) = \frac{d\ln\ m_\psi(a)}{d\ln\ a}$$

$$\dot{\rho}_{dm} + 3H\rho_{dm} - \delta(a)H\rho_{dm} = 0$$

$$\rho_{dm}(a) = \rho_{dm}^{(0)} a^{-3} \exp\left(-\int_a^1 \delta(a')d\ln\ a'\right)$$

$$\rho_{dm}(a) = \rho_{dm,0}a^{-3+\delta}$$

$$\dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) + \delta(a)H\rho_{dm} = 0$$

$$\rho_{de}(a) = \rho_{de,0}a^{-3(1+w_{de})} + \frac{\delta\rho_{dm,0}}{\delta + 3w_{de}}\left(a^{-3(1+w_{de})} - a^{-3+\delta}\right)$$

$$\rho_\Lambda(a) = \rho_{\Lambda,0} - \frac{\delta\rho_{dm,0}}{3-\delta}a^{-3+\delta}$$

$$\rho_{de}(a) = \rho_{de0}a^{-[3(1+w_{de})+\delta]}$$
$$\rho_{dm}(a) = \frac{-\delta\rho_{de0}}{3w_{de}+\delta}a^{-[3(1+w_{de})+\delta]} + \left(\rho_{dm0} + \frac{\delta\rho_{de0}}{3w_{de}+\delta}\right)a^{-3}$$

$$p_1 = (\gamma_1 - 1)\rho_1$$
$$p_2 = (\gamma_2 - 1)\rho_2$$

$$\dot{\rho}_1 + 3H\gamma_1\rho_1 = -\beta H\rho_1 + \alpha H\rho_2$$
$$\dot{\rho}_2 + 3H\gamma_2\rho_2 = \beta H\rho_1 - \alpha H\rho_2$$

$$\ddot{H} + H\dot{H}(\alpha + \beta + 3\gamma_1 + 3\gamma_2) + \frac{3}{2}H^3(\alpha\gamma_1 + \beta\gamma_2 + 3\gamma_1\gamma_2)$$
$$= \ddot{H} + AH\dot{H} + BH^3 = 0$$
$$A \equiv \alpha + \beta + 3\gamma_1 + 3\gamma_2, B \equiv \frac{3}{2}(\alpha\gamma_1 + \beta\gamma_2 + 3\gamma_1\gamma_2)$$

$$H = \frac{h}{t}, h \neq 0$$

$$Bh^2 - Ah + 2 = 0$$

$$h_\pm = \frac{A \pm \sqrt{A^2 - 8B}}{2B}$$

$$H^2 = a^{-A/2}\left(c_1 a^{\sqrt{A^2-8B}/2} + c_2 a^{-\sqrt{A^2-8B}/2}\right)$$

$$H^2 \to a^{-\left(A-\sqrt{A^2-8B}\right)/2}$$

$$H^2 \to a^{-\left(A+\sqrt{A^2-8B}\right)/2}$$

$$a_\pm \propto t^{\left(A\pm\sqrt{A^2-8B}\right)/2B}$$

$$\frac{\rho_2''}{\rho_2} + A\frac{\rho_2'}{\rho_2} + 2B = 0$$

$$\rho_2 = \rho_{20}a^M$$

$$\rho_1 = \rho_{10}a^M$$

$$\frac{\rho_2}{\rho_1} = \frac{\beta}{N + 3\gamma_1 + \alpha}$$

$$A = \beta + 4$$
$$B = 2\beta$$
$$\delta = \frac{B}{A^2} = \frac{2\beta}{(\beta + 4)^2}$$
$$h_+ = \frac{1}{2}, h_- = \frac{2}{\beta}$$

$$\rho_{dm} = \frac{r}{1+r}\rho, \rho_{de} = \frac{1}{1+r}\rho, r \equiv \frac{\rho_{dm}}{\rho_{de}}, \rho = \rho_{dm} + \rho_{de}$$

$$\Pi = -\gamma\rho^m r^n (1+r)^s$$

$$Q = 3\gamma H \rho_{de}^{m-n}\rho_{dm}^n = 3\gamma H \rho_{de}^m r^n$$

$$\rho' = -\left(1 + \frac{w_{de}}{1+r}\right)\rho,$$
$$r' = r(w_{de} + \gamma).$$

$$r = r_0 a^{3(w_{de}+\gamma)},$$

$$\rho = \rho_0 a^{-3(1+w_{de})}\left[\frac{1+r_0 a^{3(w_{de}+\gamma)}}{1+r_0}\right]^{\frac{w_{de}}{w_{de}+\gamma}},$$

$$\rho_{dm} = \rho_{dm0} a^{-3(1-\gamma)}\left[\frac{1+r_0 a^{3(w_{de}+\gamma)}}{1+r_0}\right]^{-\frac{\gamma}{w_{de}+\gamma}},$$

$$\rho_{de} = \rho_{de0} a^{-3(1+w)}\left[\frac{1+r_0 a^{3(w_{de}+\gamma)}}{1+r_0}\right]^{-\frac{\gamma}{w_{de}+\gamma}}.$$

$$r = r_0 \frac{w_{de}}{(w_{de}+\gamma r_0)^{-3w_{de}-\gamma r_0}},$$

$$\rho = \rho_0 a^{-3\left(1-\frac{\gamma w_{de}}{w_{de}-\gamma}\right)}\left[\frac{(w_{de}+\gamma r_0)a^{-3w_{de}}+r_0(w_{de}-\gamma)}{w_{de}(1+r_0)}\right]$$

$$r = \left(r_0 - \frac{\gamma}{|w_{de}|}\right)a^{-3|w_{de}|} + \frac{\gamma}{|w_{de}|}$$

$$\rho = \rho_0 a^{-3\left(1-\frac{w_{de}^2}{|w_{de}|+\gamma}\right)}\left[\frac{|w_{de}|+\gamma+(|w_{de}|r_0-\gamma)a^{-3|w_{de}|}}{|w_{de}|(1+r_0)}\right]^{|w_{de}|}.$$

$$r = \frac{\rho_{dm}}{\rho_{de}} = f(a).$$

$$\dot{\rho}_{dm} = \dot{\rho}_{de} f + \rho_{de} f'\dot{a}$$

$$\dot{\rho}_{de} = \frac{\dot{\rho}_{dm}}{f} - \frac{\rho_{dm} f'\dot{a}}{f^2}, f' = \frac{df}{da}$$

$$Q = \frac{f}{1+f}\left(\frac{f'}{f}a - 3w_{de}\right)H\rho_{de}$$

$$Q = \left(\frac{f'}{f}a - 3w_{de}\right)H\rho_{de}\Omega_{dm}$$

$$Q = (\xi - 3w_{de})H\rho_{de}\Omega_{dm}$$

$$Q = q(\alpha\dot{\rho} + 3\beta H\rho)$$

$$Q = q(\alpha\dot{\rho}_m + 3\beta H\rho_m),$$
$$Q = q(\alpha\dot{\rho}_{tot} + 3\beta H\rho_{tot}),$$
$$Q = q(\alpha\dot{\rho}_{DE} + 3\beta H\rho_{DE}).$$

$$\dot{\rho}_\Lambda = -Q$$

$$H^2 = \frac{\kappa^2}{3}\rho_{tot} = \frac{\kappa^2}{3}(\rho_\Lambda + \rho_m)$$
$$\dot{H} = -\frac{\kappa^2}{2}(\rho_{tot} + p_{tot}) = -\frac{\kappa^2}{2}\rho_m$$

$$\dot{\rho}_m = \frac{\beta q - 1}{1-\alpha q}\cdot 3H\rho_m$$

$$Q=\frac{\beta-\alpha}{1-\alpha q}\cdot 3qH\rho_m$$

$$\rho_m=-\frac{2}{\kappa^2}\dot{H}$$

$$\ddot{H}=\frac{\beta q-1}{1-\alpha q}\cdot 3H\dot{H}$$

$$aH''+\frac{a}{H}H'^2+H'=\frac{\beta q-1}{1-\alpha q}\cdot 3H'$$

$$q=-1-\frac{\dot{H}}{H^2}=-1-\frac{a}{H}H'$$

$$Q=3\beta qH\rho_m$$

$$H(a)=C_{12}\left[3C_{11}(1+\beta)-(2+3\beta)a^{-3(1+\beta)}\right]^{1/(2+3\beta)}$$

$$\Omega_m\equiv\frac{\kappa^2\rho_m}{3H^2}=-\frac{2\dot{H}}{3H^2}=-\frac{2aH'}{3H}.$$

$$\Omega_m=\frac{2(1+\beta)}{2+3\beta-3C_{11}(1+\beta)a^{3(1+\beta)}}$$

$$C_{11}=\frac{\Omega_{m0}(2+3\beta)-2(1+\beta)}{3\Omega_{m0}(1+\beta)}$$
$$C_{12}=H_0[3C_{11}(1+\beta)-(2+3\beta)]^{-1/(2+3\beta)}$$

$$E\equiv\frac{H}{H_0}=\left\{1-\frac{2+3\beta}{2(1+\beta)}\Omega_{m0}\left[1-(1+z)^{3(1+\beta)}\right]\right\}^{1/(2+3\beta)}$$

$$q(z)=-\frac{(1+z)}{E(z)}\frac{d}{dz}\left(\frac{1}{E(z)}\right)-1$$

$$q(z)=-1+\frac{3}{2}\Omega_{m0}\frac{(1+z)^{3(1+\beta)}}{E^{(2+3\beta)}}$$

$$w_{\text{eff}}\equiv\frac{p_{tot}}{\rho_{tot}}=\frac{(2q-1)}{3}$$

$$Q=\frac{3qH^3}{\kappa^2}\left(2\alpha\frac{\dot{H}}{H^2}+3\beta\right)$$

$$aH''+\frac{a}{H}H'^2+(4+3\alpha q)H'+\frac{9\beta qH}{2a}=0$$

$$H(a)=C_{22}\cdot a^{-3(2-3\beta+r_1)/8}\cdot\left(a^{3r_1/2}+C_{21}\right)^{1/2}$$

$$\Omega_m=\frac{1}{4}\left[2-3\beta+\left(\frac{2C_{21}}{a^{3r_1/2}+C_{21}}-1\right)r_1\right]$$

$$C_{21} = -1 + \frac{2r_1}{2 - 3\beta - 4\Omega_{m0} + r_1}, C_{22} = H_0(1 + C_{21})^{-1/2}$$

$$E \equiv \frac{H}{H_0} = (1+z)^{3(2-3\beta+r_1)/8} \cdot \left[\frac{(1+z)^{-3r_1/2} + C_{21}}{1 + C_{21}}\right]^{1/2}$$

$$Q = \frac{3\beta q H \rho_\Lambda}{1 + \alpha q}$$

$$\rho_\Lambda = \frac{3}{\kappa^2} H^2 - \rho_m = \frac{1}{\kappa^2}(3H^2 + 2\dot{H})$$

$$aH'' + \frac{a}{H} H'^2 + \left(4 + \frac{3\beta q}{1 + \alpha q}\right) H' + \frac{9\beta q H}{2a(1 + \alpha q)} = 0$$

$$H(a) = C_{32} \cdot a^{-3(2-5\beta+r_2)/[4(2-3\beta)]} \cdot \left(a^{3r_2/2} + C_{31}\right)^{1/(2-3\beta)},$$

$$\Omega_m = \frac{1}{2(2-3\beta)}\left[2 - 5\beta + \left(\frac{2C_{31}}{a^{3r_2/2} + C_{31}} - 1\right) r_2\right].$$

$$C_{31} = -1 + \frac{2r_2}{2 - 5\beta + r_2 + 2\Omega_{m0}(3\beta - 2)}, C_{32} = H_0(1 + C_{31})^{1/(3\beta-2)}$$

$$E \equiv \frac{H}{H_0} = (1+z)^{3(2-5\beta+r_2)/[4(2-3\beta)]} \cdot \left[\frac{(1+z)^{-3r_2/2} + C_{31}}{1 + C_{31}}\right]^{1/(2-3\beta)}$$

$$w(z) = \frac{H(z)^2 - \frac{2}{3} H(z) H'(z)(1+z)}{H_0^2 \Omega_m (1+z)^3 - H^2(z)}, H'(z) = \frac{dH}{dz}$$

$$\dot{\rho}_{dm} + 3H\rho_{dm} = Q(t)$$
$$\dot{\rho}_{de} + 3H(1+w)\rho_{de} = -Q(t),$$
$$Q(t) = \gamma H \rho_{dm}, \gamma = \text{const}$$

$$\rho_{dm} = \rho_{dm0}(1+z)^{3-\gamma}$$
$$\rho_{de} = \left(\rho_{de0} + \rho_{dm0}\frac{\gamma}{\gamma + 3w}\right)(1+z)^{3(1+w)}$$
$$H^2 = H_0^2\left[\Omega_{dm0}\left(1 - \frac{\gamma}{\gamma + 3w}\right)(1+z)^{3-\gamma} + \left(1 - \frac{3\Omega_{dm0} w}{\gamma + 3w}\right)(1+z)^{3(1+w)}\right]$$

$$H \to \bar{H} = -H, \rho \to \bar{\rho} = \rho, p \to \bar{p} = -2\rho - p.$$

$$\gamma = \frac{\rho + p}{\rho} \to \bar{\gamma} = \frac{\bar{\rho} + \bar{p}}{\bar{\rho}} = -\bar{\gamma}.$$

$$3H^2 = \rho_1 + \rho_2$$
$$\dot{\rho}_1 + \dot{\rho}_2 + 3H(\rho_1 + \rho_2 + p_1 + p_2) = 0$$

$$\bar{\rho}_1 = \alpha\rho_1 + (1-\beta)\rho_2$$
$$\bar{\rho}_2 = (1-\alpha)\rho_1 + \beta\rho_2$$
$$\bar{H} = -H$$

$$\alpha = \frac{\bar{\gamma}_2 + \gamma_1}{\bar{\gamma}_1 + \bar{\gamma}_2}, \beta = -\frac{\gamma_2 + \bar{\gamma}_1}{\bar{\gamma}_1 + \bar{\gamma}_2}.$$

$$3H^2 = \rho_1 + \rho_2$$
$$\dot{\rho}_1 + 3H\gamma_1\rho_1 = -3H\Pi,$$
$$\dot{\rho}_2 + 3H\gamma_2\rho_2 = 3H\Pi$$

$$w = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}{\frac{1}{2}\dot{\varphi}^2 + V(\varphi)}$$

$$\rho = \rho_s + \rho_{dm}, p = p_s + p_{dm}$$

$$\dot{\rho}_{dm} + 3H(\rho_{dm} + p_{dm}) = Q$$
$$\dot{\rho}_s + 3H(\rho_s + p_s) = -Q$$

$$Q \equiv -3H\Pi_{dm} = 3H\Pi_s.$$

$$\dot{\rho}_{dm} + 3H(\rho_{dm} + p_{dm} + \Pi_{dm}) = 0$$
$$\dot{\rho}_s + 3H(\rho_s + p_s + \Pi_s) = 0$$

$$\dot{r} = r\left(\frac{\dot{\rho}_{dm}}{\rho_{dm}} - \frac{\dot{\rho}_s}{\rho_s}\right)$$

$$\dot{r} = -3Hr\left[\gamma_{dm} - \gamma_s + \frac{1+r}{r}\Pi_{dm}\right]$$

$$\Pi_{dm} = (\gamma_s - \gamma_{dm})\frac{r}{1+r}$$

$$Q = -3H(\gamma_s - 1)\frac{r}{1+r}\rho_s$$

$$Q = -\sqrt{3}(\gamma_s - 1)\frac{\rho_s\rho_{dm}}{\sqrt{\rho_s + \rho_{dm}}}$$

$$w_\varphi = \frac{p_\varphi}{\rho_\varphi} = \frac{\dot{\varphi}^2 + 2V(\varphi)}{\dot{\varphi}^2 - 2V(\varphi)}$$

$$a(t) = a(t_m)\left[-w_\varphi + (1 + w_\varphi)\left(\frac{t}{t_m}\right)\right]^{\frac{2}{3(1+w_\varphi)}}$$

$$\dot{\rho}_{dm} + 3H\rho_{dm} = \delta H\rho_{dm}$$
$$\dot{\rho}_{de} + 3H(1 + w_{de})\rho_{de} = -\delta H\rho_{dm}$$

$$\rho_{dm} = \rho_{dm0}a^{-3}e^{\int \delta(a)d\log a}$$

$$r \equiv \frac{\rho_{dm}}{\rho_{de}} = \frac{\rho_{dm0}}{\rho_{de0}} a^{-\xi} = A^{-1} a^{-\xi}, A \equiv \frac{\rho_{de0}}{\rho_{dm0}} = \frac{\Omega_{de0}}{\Omega_{dm0}}$$

$$\rho_{de} = \frac{A a^{\xi}}{1 + A a^{\xi}} \rho_{tot}, \rho_{dm} = \frac{1}{1 + A a^{\xi}} \rho_{tot}$$

$$\frac{d\rho_{tot}}{da} + \frac{3}{a} \frac{1 + (1 + w_{de}) A a^{\xi}}{1 + A a^{\xi}} \rho_{tot} = 0$$

$$\rho_{tot} = \rho_{tot0} a^{-3} \left[1 - \Omega_{de0}\left(1 - a^{\xi}\right)\right]^{-3w_{de}/\xi}, \rho_{tot0} = \rho_{de0} + \rho_{dm0}$$

$$H^2 = H_0^2 a^{-3} \left[1 - \Omega_{de0}\left(1 - a^{\xi}\right)\right]^{-3w_{de}/\xi}$$

$$\delta = 3 + \frac{1}{H} \frac{\dot{\rho}_{dm}}{\rho_{dm}} = -\frac{(\xi + 3w_{de}) A a^{\xi}}{1 + A a^{\xi}} = -(\xi + 3w_{de}) \frac{\rho_{de}}{\rho_{tot}}$$

$$\delta = \frac{\delta_0}{\Omega_{de0} + (1 - \Omega_{de0}) a^{-\xi}}, \delta_0 \equiv -\Omega_{de0}(\xi + 3w_{de})$$

$$q = -\frac{\ddot{a}}{aH^2} = -1 + \frac{\dot{H}}{H^2} = -1 + \frac{3}{2} \frac{1 - \Omega_{de0} + (1 + w_{de}) \Omega_{de0} a^{\xi}}{1 - \Omega_{de0}(1 - a^{\xi})}.$$

$$L = -V(T) \sqrt{1 - g_{00} \dot{T}^2}$$

$$\rho_T = \frac{V(T)}{\sqrt{1 - \dot{T}^2}}$$

$$p_T = -V(T) \sqrt{1 - \dot{T}^2}$$

$$\frac{\ddot{T}}{1 - \dot{T}^2} + 3H\dot{T} + \frac{1}{V(T)} \frac{dV}{dT} = 0.$$

$$\dot{\rho}_T = -3H\dot{T}^2 \rho_T$$

$$Q = 3H \frac{r}{(r+1)^2} (1 - \dot{T}^2)(\rho_T + \rho_{dm})$$

$$p = -\frac{A}{\rho},$$

$$\rho = \sqrt{A + \frac{B}{a^6}},$$

$$L = \frac{1}{2} \dot{\varphi}^2 - V(\varphi),$$

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) = \rho = \sqrt{A + \frac{B}{a^6}}$$

$$p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) = -\frac{A}{\rho} = -\frac{A}{\sqrt{A + \frac{B}{a^6}}}$$

$$\dot{\varphi}^2 = \frac{B}{a^6\sqrt{A + \frac{B}{a^6}}}, V(\varphi) = \frac{1}{2}\sqrt{A}\left(\cosh\ 3\varphi + \frac{1}{\cosh\ 3\varphi}\right),$$

$$p = -\frac{A}{\rho^\alpha}$$

$$\rho = \left[A + \frac{B}{a^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}}$$

$$p = \frac{A(a)}{\rho^\alpha}$$

$$\rho = \left[Aa^{-3(1+w_{de})(1+\alpha)} + Ba^{-3(1+\alpha)}\right]^{\frac{1}{1+\alpha}}$$

$$A(a) = -w_{de}Aa^{-3(1+w_{de})(1+\alpha)}$$

$$\rho = \rho_{de} + \rho_{dm}$$

$$\rho_{de} = \frac{p}{w_{de}} = \frac{Aa^{-3(1+w_{de})(1+\alpha)}}{[Aa^{-3(1+w_{de})(1+\alpha)} + Ba^{-3(1+\alpha)}]^{\frac{\alpha}{1+\alpha}}}$$

$$\rho_{dm} = \frac{Ba^{-3(1+\alpha)}}{[Aa^{-3(1+w_{de})}(1+\alpha) + Ba^{-3(1+\alpha)}]^{\frac{\alpha}{1+\alpha}}}$$

$$\frac{\rho_{dm}}{\rho_{de}} = \frac{B}{A}a^{3w_{de}(1+\alpha)}$$

$$w_{eff,de} = w_{de} - \frac{\alpha w_{de}(1-\Omega_{0de})a^{3w}d_{de}(1+\alpha)}{\Omega_{0de}(1-\Omega_{0de})a^{3w}de^{(1+\alpha)}}$$

$$w_{eff,dm} = \frac{\alpha w_{de}\Omega_{0de}}{\Omega_{0de} + (1-\Omega_{0de})a^{3w}de^{(1+\alpha)}}$$

$$\rho_{de}(z) = \rho_{de,0}(1+z)^{3(1+w_{de})} + \frac{\delta\rho_{dm,0}}{\delta + 3w_{de}}\left[(1+z)^{3(1+w_{de})} - (1+z)^{3-\delta}\right]$$

$$w_{eff,de} = \frac{p_{eff,de}}{\rho_{de}} = \frac{p_{de} + Q/3H}{\rho_{de}} = -1 + \Delta, \Delta \equiv \frac{1}{3}\frac{d\ln\ \rho_{de}}{d\ln\ (1+z)}.$$

$$\frac{d\rho_{de}(z)}{dz} = 3(1+w_{de})\rho_{de,0}(1+z)^{2+3w_{de}} + \frac{\delta\rho_{dm,0}}{\delta+3w_{de}}\left[3(1+w_{de})(1+z)^{2+3w_{de}} - (3-\delta)(1+z)^{2-\delta}\right]$$

$$\dot{x} = f(x,y,t)$$
$$\dot{y} = g(x,y,t)$$

$$f(x_c,y_c) = g(x_c,y_c) = 0$$

$$(x(t),y(t)) \to (x_c,y_c) \ \text{ for } t \to \infty.$$

$$x = x_c + \delta x, y = y_c + \delta y.$$

$$\frac{d}{dN}\begin{pmatrix}\delta x\\ \delta y\end{pmatrix} = \hat{M}\begin{pmatrix}\delta x\\ \delta y\end{pmatrix}$$

$$\hat{M} = \begin{pmatrix}\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y}\\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y}\end{pmatrix}$$

$$\delta x = C_1 e^{\lambda_1 N} + C_2 e^{\lambda_2 N}$$
$$\delta y = C_3 e^{\lambda_1 N} + C_4 e^{\lambda_2 N}$$

$$\rho'_{de} + 3(1+w_{de})\rho_{de} = -Q$$
$$\rho'_{dm} + 3(1+w_{dm})\rho_{dm} = Q$$

$$Q_1 = 3\gamma_m\rho_{dm}, Q_2 = 3\gamma_d\rho_{de}, Q_3 = 3\gamma_{tot}\rho_{tot}$$

$$Q = Q_1, w_{eff,de} = w_{de}(\Omega_{de}) + \gamma_m\frac{1-\Omega_{de}}{\Omega_{de}}, w_{eff,dm} = w_{dm} - \gamma_m$$
$$Q = Q_2, w_{eff,de} = w_{de}(\Omega_{de}) + \gamma_d, w_{eff,dm} = w_{dm} - \gamma_d\frac{\Omega_{de}}{1-\Omega_{de}}$$

$$Q = Q_3, w_{eff,de} = w_{de}(\Omega_{de}) + \gamma_m\frac{1}{\Omega_{de}}, w_{eff,dm} = w_{dm} - \frac{\gamma_{tot}}{1-\Omega_{de}}.$$

$$\Omega'_{dm} = 3f_j\Omega_{dm}\Omega_{de}$$
$$\Omega'_{de} = -3f_j\Omega_{dm}\Omega_{de}$$

$$f_j = w_{eff,dej} - w_{eff,dmj}$$
$$f_1 = f_0 + \frac{\gamma_m}{\Omega_{de}}$$
$$f_2 = f_0 + \frac{\gamma_d}{1-\Omega_{\Omega_d}}$$
$$f_3 = f_0 + \frac{\gamma_{tot}}{\Omega_{de}(1-\Omega_{de})}$$

$$\hat{M} = \begin{pmatrix} 3f(\bar{\Omega}_{de})\bar{\Omega}_{de} & 3(f(\bar{\Omega}_{de})\bar{\Omega}_{dm} + f'\bar{\Omega}_{dm}\bar{\Omega}_{de}) \\ -3f(\bar{\Omega}_{de})\bar{\Omega}_{de} & -3(f(\bar{\Omega}_{de})\bar{\Omega}_{dm} + f'\bar{\Omega}_{dm}\bar{\Omega}_{de}) \end{pmatrix}$$

$$\lambda_1 = 0$$
$$\lambda_2 = 3f(2\Omega_{de} - 1) - 3f'\Omega_{de}(1 - \Omega_{de})$$

$$\lambda^2 + \left[2 + w_{de} - w_{de}(1 + w_{de})\frac{\partial_r \Pi}{\Pi}\right]\lambda + \left(1 + w_{de} + w_{de}\partial_\rho \Pi\right) = 0$$
$$\partial_r \Pi \equiv \frac{\partial \Pi}{\partial r}, \partial_\rho \Pi \equiv \frac{\partial \Pi}{\partial \rho}$$

$$\lambda_\pm = \frac{1}{2}\left\{\begin{matrix} \left[w_{de}(1 + w_{de})\frac{\partial_r \Pi}{\Pi} - (2 + w_{de})\right] \\ \pm\sqrt{\left(2 + w_{de} - w_{de}(1 + w_{de})\frac{\partial_r \Pi}{\Pi}\right)^2 - 4\left(1 + w_{de} + w_{de}\partial_\rho \Pi\right)} \end{matrix}\right\}$$

$$G^{\mu\nu} = 8\pi G T^{\mu\nu} \to G^{\mu\nu} = 8\pi G T^{\mu\nu} + \Lambda g^{\mu\nu}$$

$$u_\mu T^{\mu\nu}_{dm;\nu} = -u_\mu \left(\frac{\Lambda}{8\pi G} g^{\mu\nu}\right)_{;\nu}$$

$$\dot{\rho}_{dm} + 3H\rho_{dm} = -\dot{\rho}_\Lambda$$

$$T\frac{dS}{dt} = -\frac{\dot{\Lambda}a^3}{8\pi G}$$

$$\dot{\rho}_{dm} + 3H\rho_{dm} = Q$$
$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = -Q$$

$$Q = 3H\rho_{dm}\frac{\Lambda' + 2\Lambda''\rho_{dm}}{1 + 3\Lambda' + 2\Lambda''\rho_{dm}}$$

$$H^2 = \frac{1}{3}\rho + f(t)$$
$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p) + g(t)$$

$$\dot{\rho} + 3H(\rho + p) = 6H\left(-f(t) + \frac{\dot{f}(t)}{2H} + g(t)\right)$$

$$\dot{\rho} + 3H(\rho + p) = 6H\left(-f(t) + \frac{\dot{f}(t)}{2H} + g(t)\right) \to \dot{\rho} + 3H(\rho + p) = 0$$

$$\dot{\rho} + 3H(\rho + p) = 6H\left(-f(t) + \frac{\dot{f}(t)}{2H} + g(t)\right) \to \dot{\rho} + 3H(\rho + p) = \dot{\Lambda}(t)$$

$$S_{\mathrm{EH}} = \int d^4x\sqrt{-g}\frac{1}{16\pi G}R = \int d^4x\sqrt{-g}\frac{M_{\mathrm{pl}}^2}{2}R$$

$$S_\phi = -\int d^4x\sqrt{-g}\left\{\frac{1}{2}(\partial\phi)^2 + V(\phi)\right\}$$

$$S_m = -\int d^4x L_m\left(\psi_m^{(i)}, g_{\mu\nu}^{(i)}\right)$$

$$g_{\mu\nu}^{(i)} = e^{\frac{2\beta_i\varphi}{M_{Pl}}} g_{\mu\nu}$$

$$S = \int d^4x\sqrt{-g}\left\{\frac{M_{Pl}^2}{2}R + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi) - \frac{1}{\sqrt{-g}}L_m\left(\psi_m^{(i)}, g_{\mu\nu}^{(i)}\right)\right\}$$

$$\nabla^2\varphi = \frac{dV(\varphi)}{d\varphi} + \sum_i \frac{1}{\sqrt{-g}}\frac{\partial L_m\left(\psi_m^{(i)}, g_{\mu\nu}^{(i)}\right)}{\partial g_{\mu\nu}^{(i)}}\frac{2\beta_i}{M_{Pl}} g_{\mu\nu}^{(i)}.$$

$$\frac{1}{\sqrt{-g}}\frac{\partial L_m\left(\psi_m^{(i)}, g_{\mu\nu}^{(i)}\right)}{\partial g_{\mu\nu}^{(i)}} g_{\mu\nu}^{(i)} = \frac{1}{2}\rho_i(1-3w_i)e^{(1-3w_i)\beta_i\varphi/M_{Pl}}.$$

$$\nabla^2\varphi = \frac{dV(\varphi)}{d\varphi} + \sum_i (1-3w_i)\frac{\beta_i}{M_{Pl}}\rho_i e^{(1-3w_i)\beta_i\varphi/M_{Pl}}.$$

$$\nabla^2\varphi = \frac{dV_{eff}(\varphi)}{d\varphi};\ V_{eff} = V(\varphi) + \sum_i \rho_i e^{(1-3w_i)\beta_i\varphi/M_{Pl}}$$

$$V_{eff} = V(\varphi) + \sum_i \rho_i e^{\beta_i\varphi/M_{Pl}}$$

$$V_{eff} = V(\varphi) + U(\beta\varphi/M_{Pl})\rho.$$

$$V(\phi) = \frac{M^{4+n}}{\phi^n}$$

$$V(\phi) = M^4 \exp\left(\frac{M^n}{\phi^n}\right)$$

$$V(\phi) = M^4 \exp\left(\frac{M^n}{\phi^n}\right)$$

$$g_{\mu\nu} = \mathrm{diag}(-1, a^2, a^2, a^2)$$

$$\begin{aligned}\nabla^2\phi &= g^{\mu\nu}\nabla_\mu\nabla_\nu\phi \\ &= g^{\mu\nu}\partial_\mu\partial_\nu\phi - g^{\mu\nu}\Gamma^\rho_{\nu\mu}\phi_{,\rho} \\ &= g^{00}\partial_0\partial_0\phi - (a^{-2}\Gamma^0_{11} + a^{-2}\Gamma^0_{22} + a^{-2}\Gamma^0_{33})\phi_{,0} \\ &= -\ddot{\phi} - a^{-2}(3a\dot{a})\dot{\phi} \\ &= -(\ddot{\phi} + 3H\dot{\phi})\end{aligned}$$

$$\ddot{\phi} + 3H\dot{\phi} = -V_{\mathrm{eff},\phi}(\phi),$$

$$3H^2M_{\mathrm{pl}}^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_{\mathrm{m}} e^{\beta\phi/M_{\mathrm{pl}}} + \rho_{\mathrm{r}}$$

$$\rho_{\mathrm{critical}} \equiv \frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_{\mathrm{m}} e^{\beta\phi/M_{\mathrm{pl}}} + \rho_{\mathrm{r}}$$

$$\Omega_{\mathrm{m}} \equiv \frac{\rho_{\mathrm{m}} e^{\beta\phi_{\min}/M_{\mathrm{pl}}}}{\rho_{\mathrm{critical}}}$$

$$\ddot{x}^\rho + \tilde{\Gamma}^\rho_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = 0$$

$$\tilde{g}_{\mu\nu,\sigma} = \left(\frac{2\beta_i}{M_{\mathrm{pl}}}\phi_{,\sigma} g_{\mu\nu} + g_{\mu\nu,\sigma}\right) e^{2\beta_i\phi/M_{\mathrm{pl}}}$$

$$\begin{aligned}
\tilde{\Gamma}^\rho_{\mu\nu} &= \frac{1}{2}\tilde{g}^{\sigma\rho}\left(\tilde{g}_{\sigma\nu,\mu} + \tilde{g}_{\sigma\mu,\nu} - \tilde{g}_{\mu\nu,\sigma}\right) \\
&= \frac{1}{2} e^{-2\beta_i\phi/M_{\mathrm{pl}}} g^{\sigma\rho} \begin{pmatrix} \frac{2\beta_i}{M_{\mathrm{pl}}}\phi_{,\mu} g_{\sigma\nu} + g_{\sigma\nu,\mu} + \frac{2\beta_i}{M_{\mathrm{pl}}}\phi_{,\nu} g_{\sigma\mu} \\ + g_{\sigma\mu,\nu} - \frac{2\beta_i}{M_{\mathrm{pl}}}\phi_{,\sigma} g_{\mu\nu} - g_{\mu\nu,\sigma} \end{pmatrix} e^{2\beta_i\phi/M_{\mathrm{pl}}} \\
&= \frac{1}{2} g^{\sigma\rho}\left(g_{\sigma\nu,\mu} + g_{\sigma\mu,\nu} - g_{\mu\nu,\sigma}\right) + \frac{\beta_i}{M_{\mathrm{pl}}} g^{\sigma\rho}\left(\phi_{,\mu} g_{\sigma\nu} + \phi_{,\nu} g_{\sigma\mu} - \phi_{,\sigma} g_{\mu\nu}\right) \\
&= \Gamma^\rho_{\mu\nu} + \frac{\beta_i}{M_{\mathrm{pl}}}\left(\phi_{,\mu}\delta^\rho_\nu + \phi_{,\nu}\delta^\rho_\mu - g^{\sigma\rho}\phi_{,\sigma} g_{\mu\nu}\right)
\end{aligned}$$

$$\begin{aligned}
0 &= \ddot{x}^\rho + \Gamma^\rho_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + \frac{\beta_i}{M_{\mathrm{pl}}}\left(\phi_{,\mu}\delta^\rho_\nu + \phi_{,\nu}\delta^\rho_\mu - g^{\sigma\rho}\phi_{,\sigma} g_{\mu\nu}\right)\dot{x}^\mu\dot{x}^\nu \\
&= \ddot{x}^\rho + \Gamma^\rho_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + \frac{\beta_i}{M_{\mathrm{pl}}}\left(\phi_{,\mu}\dot{x}^\mu\dot{x}^\rho + \phi_{,\nu}\dot{x}^\rho\dot{x}^\nu - g^{\sigma\rho}\phi_{,\sigma} g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu\right) \\
&= \ddot{x}^\rho + \Gamma^\rho_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + \frac{\beta_i}{M_{\mathrm{pl}}}\left(2\phi_{,\mu}\dot{x}^\mu\dot{x}^\rho + g^{\sigma\rho}\phi_{,\sigma}\right)
\end{aligned}$$

$$\frac{\vec{F}_\phi}{m} = -\frac{\beta_i}{M_{\mathrm{pl}}}\vec{\nabla}\phi$$

$$L_{\text{scalar + matter}} = w\rho f(\phi),$$

$$p = w\rho$$

$$H^2 = \varepsilon$$

$$\varepsilon = \frac{\dot{\phi}^2}{2} + V(\phi) + \rho f(\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) + w\rho f'(\phi) = 0,$$

$$\dot{\varepsilon} + 3H(\varepsilon + P) = 0$$

$$P = w\rho + \frac{\dot{\phi}^2}{2} - V(\phi)$$

$$H^2 = \frac{\dot{\phi}^2}{2} + V + \frac{\rho_0}{f^{1-w}a^{3(1+w)}}$$
$$\ddot{\phi} + 3H\dot{\phi} + V' + \frac{w\rho_0 f'}{f^{1-w}a^{3(1+w)}} = 0$$

$$V(\phi) = \frac{8\cosh^4 \frac{\phi}{2\phi_0}}{3(1+w)}\left(6\alpha^2(1+w) + 3\phi_0^2(1-w) + 4\alpha\tanh\frac{\phi}{2\phi_0}\right),$$
$$f(\phi) = \left(-\frac{16\cosh^4\frac{\phi}{2\phi_0}\exp\left(3\alpha(1+w)\frac{\phi}{\phi_0}\right)}{3Mt_R^2(1+w)}\left(3\phi_0^2 + 2\alpha\tanh\frac{\phi}{2\phi_0}\right)\right)^{\frac{1}{w}},$$

$$\phi_0 \geq \sqrt{\frac{2\alpha}{3}}$$
$$w < -1$$

$$\phi_0 > \sqrt{\frac{4\alpha - 6\alpha^2(1+w)}{3(1-w)}}$$

$$\sqrt{\frac{2\alpha}{3}} < \phi_0 < \sqrt{\frac{4\alpha - 6\alpha^2(1+w)}{3(1-w)}}$$

$$\phi = 2\phi_0\text{arctanh}\frac{5\alpha^2(1+w) + 3\phi_0^2(1-w)}{4\alpha}.$$

$$w = \frac{2m+1}{n},$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3\phi}\epsilon_m + \frac{\omega}{6}\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi}{3\phi}\left[\rho_m\left(\frac{3+\omega}{3+2\omega}\right) + 3p_m\left(\frac{\omega}{3+2\omega}\right)\right] - \frac{\omega}{3}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{4\pi}{3+2\omega}S$$
$$\frac{\ddot{\phi}}{\phi} + 3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} = \frac{8\pi}{(3+2\omega)\phi}(\rho_m - 3p_m) - \frac{8\pi}{3+2\omega}S$$

$$\dot{\rho}_m + 3\gamma\frac{\dot{a}}{a}\rho_m = \frac{1}{2}S\dot{\phi}$$
$$\dot{\rho}_\phi + 6\frac{\dot{a}}{a}\rho_\phi = -\frac{1}{16\pi}R\dot{\phi} - \frac{1}{2}S\dot{\phi}$$

$$\rho_\phi \equiv \frac{\omega\dot{\phi}^2}{16\pi\phi}$$

$$X = \frac{\dot{a}}{a}, Y = \frac{\dot{\phi}}{\phi} \text{ and } Z = \frac{\rho}{\phi}$$

$$\dot{X} = \left(Q_3 - \frac{2Q_1Q_4}{Q_2}\right)^{-1}\left[\left(\frac{\omega A}{6} - \frac{\omega}{3} + \frac{Q_4}{Q_2} - \frac{\omega B}{6\alpha^2} - \frac{\omega B Q_4}{6\alpha^2 Q_2} - \frac{\gamma\omega}{2\alpha}\frac{X}{Y}\right)Y^2 + \left(1 - A + \frac{B}{\alpha^2} + \frac{3\gamma}{2\alpha}\frac{X}{Y}\right)X^2 + \left(1 - A - \frac{3Q_4}{Q_2} + \frac{B}{\alpha^2} + \frac{BQ_4}{\alpha^2 Q_1} + \frac{3\gamma}{\alpha}\frac{X}{Y}\right)XY\right]$$

$$\dot{Y} = \left(1 - 2\frac{Q_1}{Q_2Q_3}\right)^{-1}\left\{\left[\frac{1}{Q_2}\left(1 - \frac{BC}{3} - \frac{\omega B}{6\alpha^2} - \frac{3\omega B}{6\alpha^2}\frac{X}{Y}\right) + \frac{\omega Q_1}{Q_2Q_3}\left(\frac{A}{3} - \frac{2}{3} - \frac{B}{3\alpha^2} - \frac{\gamma B}{2\alpha}\frac{X}{Y}\right)\right]Y^2 + \left[\frac{1}{Q_2}\left(\frac{2BC}{\alpha} + \frac{B}{\alpha^2} + \frac{3B}{2\alpha^2}\frac{X}{Y}\right) + \frac{Q_1}{Q_2Q_3}\left(2 - 2A + \frac{2B}{\alpha^2} + \frac{3(\gamma+1)B}{\alpha^2}\frac{X}{Y}\right)\right]X^2 + \left[\frac{1}{Q_2}\left(-3 + \frac{2BC}{\alpha} + \frac{B}{\alpha^2} + \frac{3B}{2\alpha^2}\frac{X}{Y}\right) + \frac{Q_1}{Q_2Q_3}\left(-2A + \frac{2B}{\alpha^2} + \frac{3\gamma B}{\alpha^2}\frac{X}{Y}\right)\right]XY\right\}$$

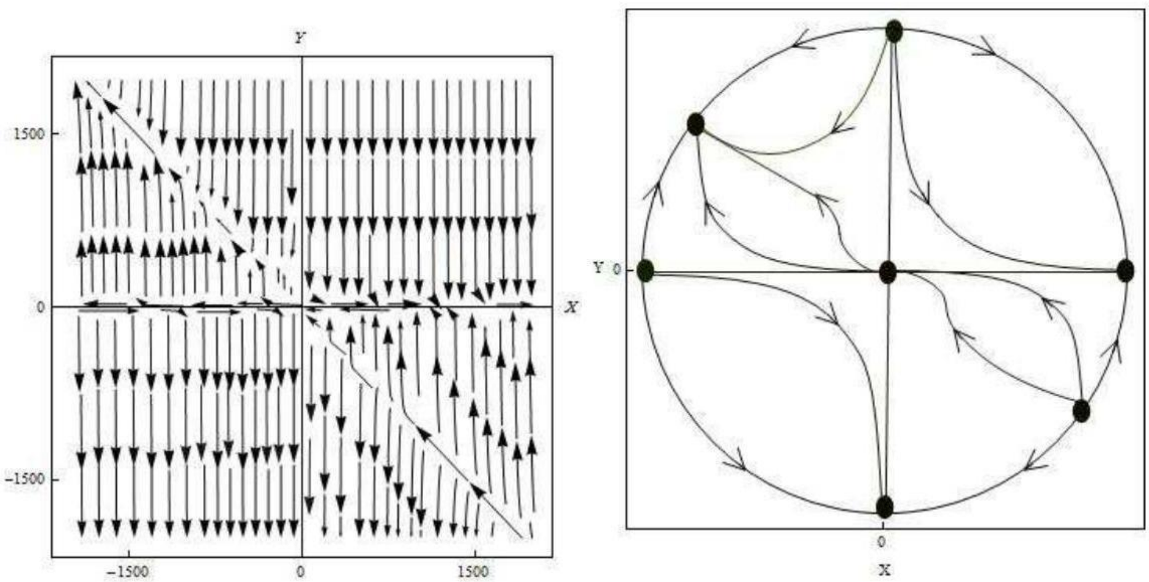
$$\alpha = \frac{8\pi}{3}, A = \frac{3 - 2\omega + 3\gamma}{3 + 2\omega}, B = \frac{4\pi}{3 + 2\omega} \text{ and } C = 2 - 3\gamma$$

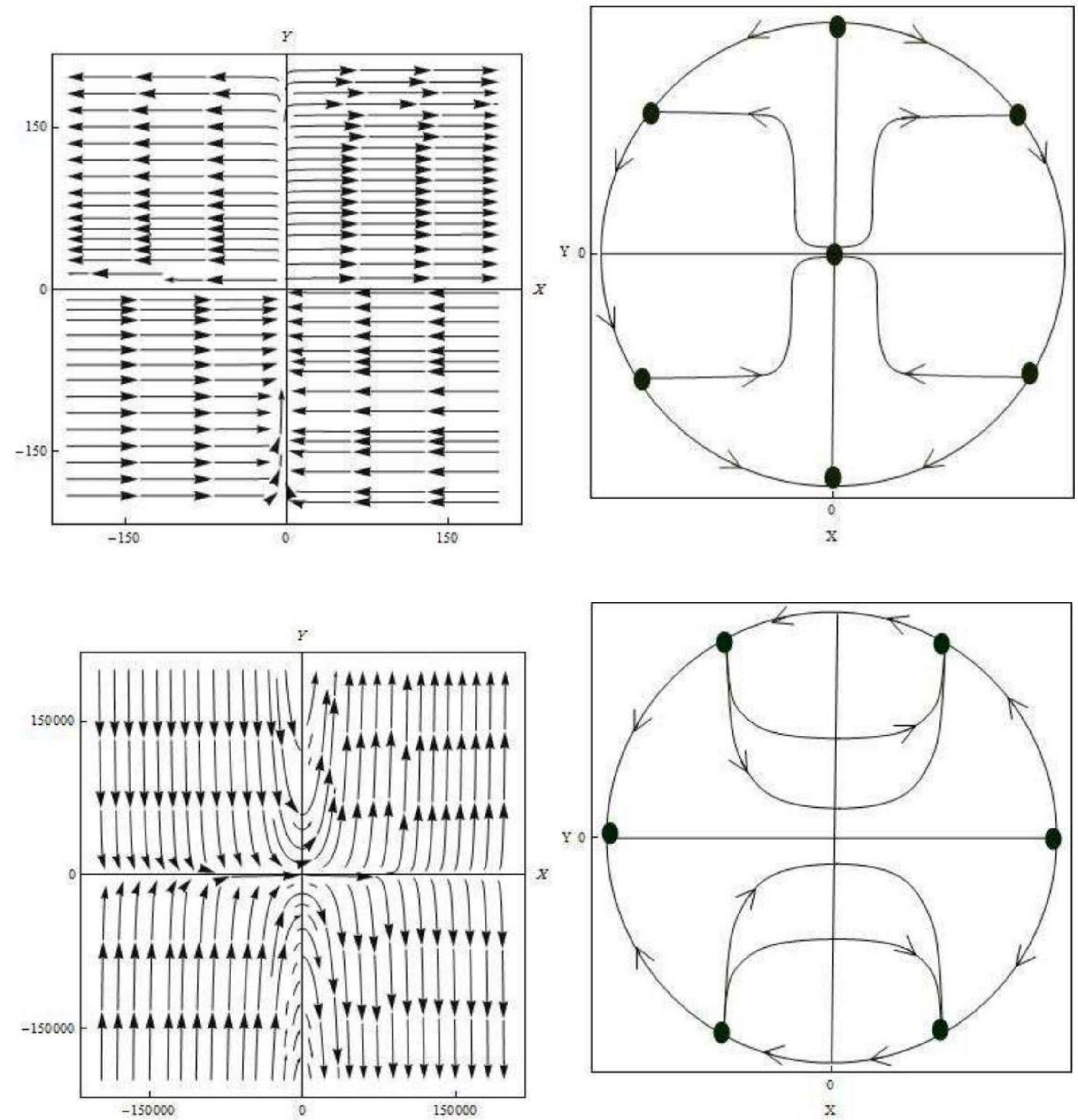
$$Q_1 = \frac{B}{\alpha}\left(1 - 2\frac{X}{Y}\right)$$

$$Q_2 = 1 + \frac{2B\omega}{3\alpha} - \frac{2B}{\alpha}\frac{X}{Y}$$

$$Q_3 = 1 - \frac{B}{\alpha} - \frac{2B}{\alpha}\frac{X}{Y}$$

$$Q_4 = \frac{B}{\alpha}\left(\frac{X}{Y} - \frac{\omega}{3}\right).$$

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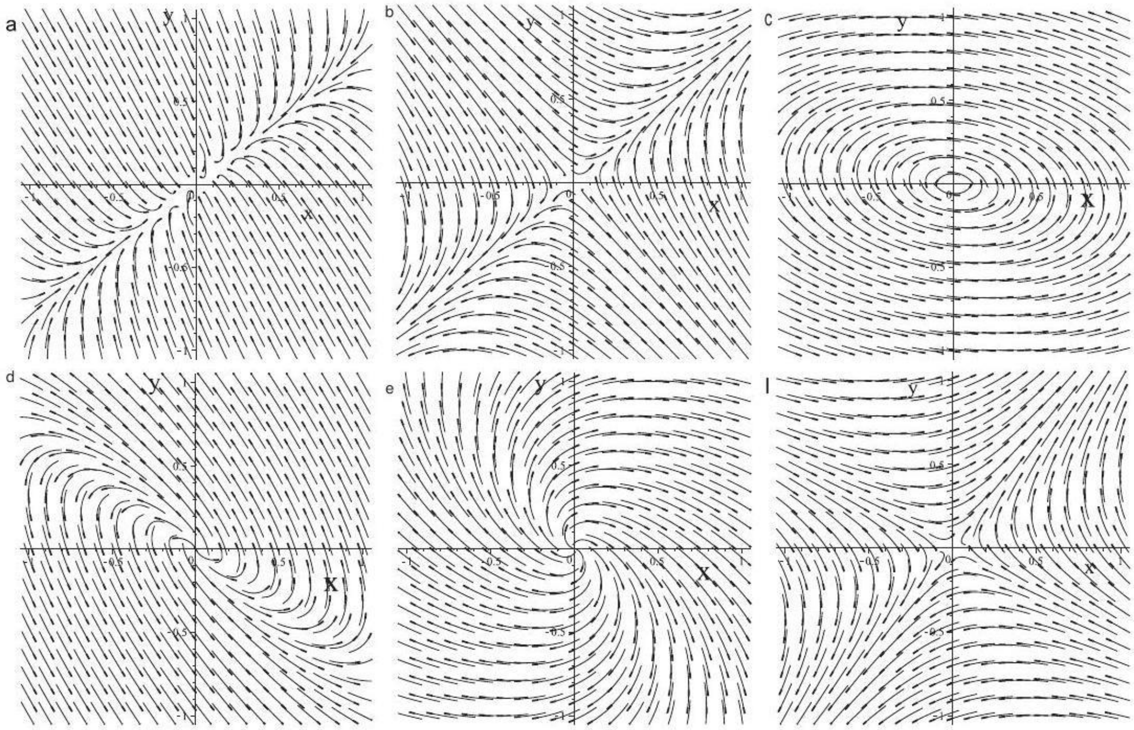
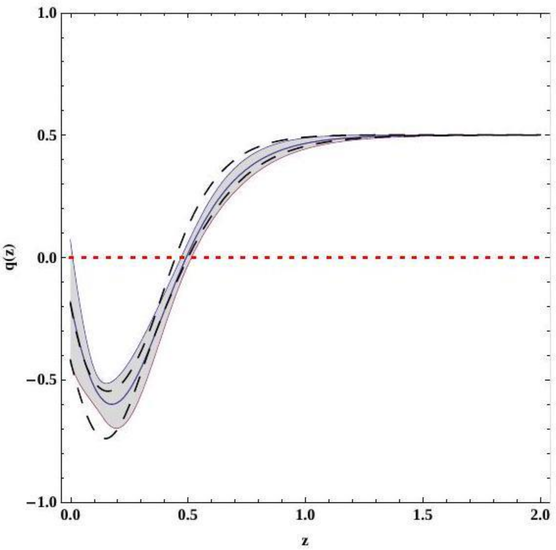
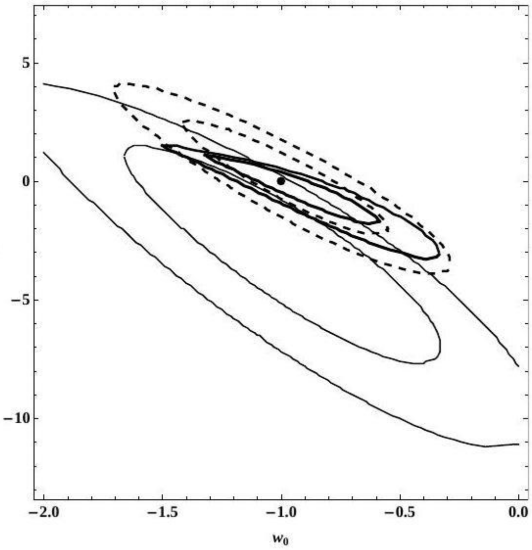






Figuras 6, 7, 8, 9 y 10. Deformación del espacio – tiempo cuántico, por interacción de la partícula oscura.

$$S = \frac{1}{2\kappa}\int \; d^4x\sqrt{-g}f(R) + S_m(g^{\mu\iota},\psi), S_m = \int \; d^4x\sqrt{-g}L_m(g^{\mu\iota},\psi)$$

$$f'R_{\mu\nu} - \frac{1}{2}fg_{\mu\nu} - \nabla_\mu\nabla_\nu f' + g_{\mu\nu}\,\square\, f'' = kT_{\mu\nu}^{(m)}, f' \equiv \frac{df}{dR}$$

$$T_{\mu\nu}^{(m)} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}L_m\right)}{\delta(g^{\mu\nu})}$$

$$\phi \equiv f'(R)$$
$$U(\phi) \equiv R(\phi)f' - f[R(\phi)]$$

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}(\phi R - U(\phi) + L_m)$$

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = f' g_{\mu\nu}$$

$$d\tilde{\phi} = \sqrt{\frac{3}{2k}\frac{d\phi}{\phi}}.$$

$$S = \int d^4x\sqrt{-g}\left[\frac{\tilde{R}}{2\kappa} - \frac{1}{2}\partial^\mu\tilde{\phi}\partial_\mu\tilde{\phi} - V(\tilde{\phi})\right] + S_m\left(e^{-2\beta\tilde{\phi}}\tilde{g}_{\mu\nu}, \psi\right)$$

$$V(\tilde{\phi}) = \frac{Rf' - f}{2\kappa f'^2}$$

$$\tilde{G}_{\mu\nu} = \kappa\left(\tilde{T}^{\tilde{\phi}}_{\mu\nu} + \tilde{T}^m_{\mu\nu}\right).$$
$$\Box\,\tilde{\phi} - \frac{dV(\tilde{\phi})}{d\tilde{\phi}} = -\beta\sqrt{\kappa}\tilde{T}^m,$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = -\beta\sqrt{\kappa}\rho_m$$

$$\dot{\rho}_\phi + 3H\left(1 + w_\phi\right)\rho_\phi = -Q, Q = \beta\sqrt{\kappa}\dot{\phi}\rho_m.$$

$$d\tilde{t} = \sqrt{F}dt, \tilde{a} = \sqrt{F}a, \tilde{H} = \frac{1}{\tilde{a}}\frac{d\tilde{a}}{d\tilde{t}} = \frac{1}{\sqrt{F}}\left(H + \frac{\dot{F}}{2F}\right), F = e^{-2\beta\sqrt{\kappa}\phi}$$

$$\dot{\rho}_m + 3H\rho_m = Q$$

$$(2q - 1)H = -\beta\sqrt{k}\dot{\phi}$$

$$\frac{f''}{f'}\dot{R} = -2(2q - 1)H$$

$$I = \frac{1}{16\pi G}\int d^4x\sqrt{-g}[f(T) + L_m]$$

$$H^2 + \frac{k}{a^2} = \frac{1}{3}(\rho_m + \rho_T)$$

$$\dot{H} - \frac{k}{a^2} = -\frac{1}{2}(\rho_m + \rho_T + p_T),$$

$$\rho_T = \frac{1}{2}(2Tf' - f - T),$$
$$p_T = -\frac{1}{2}[-8\dot{H}Tf'' + (2T - 4\dot{H})f' - f + 4\dot{H} - T],$$

$$T = -6\left(H^2 + \frac{k}{a^2}\right)$$

$$\rho_m = \frac{1}{2}[f - 2Tf'].$$

$$\dot{\rho_m} + 3H\rho_m = Q,$$

$$\dot{\rho_T} + 3H(\rho_T + p_T) = -Q,$$

$$\dot{\rho_T} + 3H\rho_T(1 + w_{eff}) = 0.$$

$$w_{eff} = w_T + \frac{Q}{3H\rho_T}.$$

$$w_{eff} = -1 + \left(\frac{4k}{a^2} - \frac{\dot{T}}{3H}\right)\left(\frac{2Tf'' + f' - 1}{2Tf' - f - T}\right) + \frac{Q}{3H\rho_T}.$$

$$\dot{\rho_T} = \frac{\dot{T}}{2}[f' + 2Tf'' - 1],$$

$$w_T = -\left[1 + \frac{Q}{3H\rho_T} + \frac{\dot{T}}{3H}\frac{(2Tf'' + f' - 1)}{(2Tf' - f - T)}\right].$$

$$\dot{T} = \frac{12H}{(f' + 2Tf'')}\left[\frac{(f - 2Tf')}{4} + \frac{k}{a^2}(f' + 2Tf'' - 1)\right].$$

$$w_T = -\left[1 + \frac{Q}{3H\rho_T} + \frac{4}{(f' + 2Tf'')}\frac{(2Tf'' + f' - 1)}{(2Tf' - f - T)}\left(\frac{(f - 2Tf')}{4} + \frac{k}{a^2}(f' + 2Tf'' - 1)\right)\right]$$

$$q = \frac{1}{2} - \frac{k}{2a^2}\left[\frac{T}{6} + \frac{k}{a^2}\right]^{-1} + \left[\frac{T}{6} + \frac{k}{a^2}\right]^{-1}\left[\frac{(2Tf' - f - T)}{4} + \frac{Q}{6H} + \frac{(2Tf'' + f' - 1)}{(f' + 2Tf'')} \times \left(\frac{(f - 2Tf')}{4} + \frac{k}{a^2}(f' + 2Tf'' - 1)\right)\right]$$

$$S = S_g + S_{\mathrm{m}}$$

$$S_g = \frac{M_p^2}{2}\int \mathrm{d}\varrho(x)\sqrt{-g}\left(R - 2\lambda - \omega\partial_\mu v\partial^\mu v\right)$$

$$S_{\mathrm{m}} = \int \mathrm{d}\varrho\sqrt{-g}\mathcal{L}_{\mathrm{m}}$$

$$\left(\frac{D}{2} - 1\right)H^2 + H\frac{\dot{v}}{v} - \frac{1}{2}\frac{\omega}{D - 1}\dot{v}^2 = \frac{1}{M_p^2(D - 1)}\rho + \frac{\lambda}{D - 1} - \frac{k}{a^2}$$

$$\frac{\square\, v}{v} - (D - 2)\left(H^2 + \dot{H} - H\frac{\dot{v}}{v} + \frac{\omega}{D - 1}\dot{v}^2\right) + \frac{2\lambda}{D - 1} = \frac{1}{M_p^2(D - 1)}[(D - 3)\rho + (D - 1)p]$$

$$\dot{H} + (D - 1)H^2 + \frac{2k}{a^2} + \frac{\square\, v}{v} + H\frac{\dot{v}}{v} + \omega(v\,\square\, v - \dot{v}^2) = 0$$

$$\dot{\rho}+\left[(D-1)H+\frac{\dot{v}}{v}\right](\rho+p)=0$$

$$H^2=\frac{1}{3M_p^2}\rho+\frac{\lambda}{3}-\frac{k}{a^2}$$
$$H^2+\dot{H}=-\frac{1}{6M_p^2}(3p+\rho)+\frac{\lambda}{3}$$

$$v=t^{-\beta}$$

$$\dot{\rho}_m+(3H-\beta t^{-1})\rho_m=Q$$
$$\dot{\rho}_x+(1+w)(3H-\beta t^{-1})\rho_x=-Q$$

$$\Omega_m=\frac{\rho_m}{3M_p^2H^2},\Omega_x=\frac{\rho_x}{3M_p^2H^2}$$

$$\dot{\Omega}_m+(3H-\beta t^{-1})\Omega_m+2\Omega_m\frac{\dot{H}}{H}=\frac{Q}{3M_p^2H^2},$$
$$\dot{\Omega}_x+(1+w_x)(3H-\beta t^{-1})\Omega_x+2\Omega_x\frac{\dot{H}}{H}=-\frac{Q}{3M_p^2H^2},$$

$$\dot{H}+H^2-\frac{\beta H}{2t}+\frac{\beta(\beta+1)}{2t^2}+\frac{\omega\beta^2}{3t^{2(\beta+1)}}=-\frac{1}{2}((1+3w)\Omega_x+\Omega_m)H^2.$$

$$\Omega_f=\frac{\omega\dot{v}^2}{6H^2}-\frac{\dot{v}}{Hv}.$$

$$\Omega_f=\frac{\omega\beta^2}{6H^2t^{2(\beta+1)}}+\frac{\beta}{Ht}$$

$$\sum_{\alpha=k,f,x,m}\Omega_\alpha\equiv 1.$$

$$Q\equiv H(\delta\rho_x+\gamma\rho_m)$$

$$\dot{\Omega}_m+(3H-\beta t^{-1})\Omega_m+2\Omega_m\frac{\dot{H}}{H}=H(\delta\Omega_x+\gamma\Omega_m),$$
$$\dot{\Omega}_x+(1+w_x)(3H-\beta t^{-1})\Omega_x+2\Omega_x\frac{\dot{H}}{H}=-H(\delta\Omega_x+\gamma\Omega_m),$$
$$\dot{\Omega}_f+\left(\frac{\dot{H}}{H}+2(1+\beta)t^{-1}\right)\Omega_f-\frac{(1+2\beta)\beta}{Ht}=0.$$

$$\Omega_m'=\theta\Omega_m+\sigma\delta\Omega_x$$
$$\Omega_x'=-\delta\gamma\Omega_m+v\Omega_x$$

$$\tau^2-(\theta+v)\tau+\delta^2\sigma\gamma+\theta v=0$$

$$\tau_\pm=\frac{\theta+v}{2}\left[1\pm\sqrt{1-4\frac{(\delta^2\sigma\gamma+\theta v)}{(\theta+v)^2}}\right]$$

$$\tau_{\pm} \in \Re, \tau_{\pm} < 0, \tau_{+} > \tau_{-} > 0, \theta + v < 0{,}4(\delta^2\sigma\gamma + \theta v) < (\theta + v)^2, \delta^2\sigma\gamma + \theta v > 0.$$

$$\tau_{\pm} \in \Re, \tau_{\pm} > 0, \tau_{+} > \tau_{-} > 0, \theta + v > 0{,}4(\delta^2\sigma\gamma + \theta v) < (\theta + v)^2, \delta^2\sigma\gamma + \theta v > 0.$$

$$\tau_{\pm} \in \Re, \tau_{+}\tau_{-} < 0, \delta^2\sigma\gamma + \theta v < 0.$$

$$\tau_{\pm} \in \mathbb{C}, \tau\pm = \tau_1 \pm i\tau_2, \tau_1, \tau_2 \in \Re\tau_1, \tau_2 > 0, \theta + v < 0, (\theta + v)^2 < 4(\delta^2\sigma\gamma + \theta v).$$

$$\tau_{\pm} \in \mathbb{C}, \tau\pm = \tau_1 \pm i\tau_2, \tau_1, \tau_2 \in \Re\tau_1, \tau_2\langle 0, \theta + v\rangle\, 0, (\theta + v)^2 < 4(\delta^2\sigma\gamma + \theta v).$$

$$\tau_{\pm} \in \Im, \tau_{\pm} = \pm i\tau, \tau \in \Re, \theta = v, \delta^2\sigma\gamma + \theta v > 0.$$

$\gamma = -2, \sigma = 3, \beta = -1, \delta = 3, \gamma = -3, \sigma = 1, \beta = 2, \delta = 3, \gamma = 3, \sigma = 3, \beta = -1, \delta = 1, \gamma = 3, \sigma = 1, \beta = 2, \delta = 3, \gamma = 3, \sigma = -3, \beta = 2\,, \delta = 3, \gamma = 3, \sigma = -3, \beta = 1, \delta = -2.$

$$L^3\Lambda^3 \leq S_{BH} \equiv \frac{1}{4}\frac{A_{BH}}{l_p^2} = \pi L^2 M_p^2$$

$$L \sim \Lambda^{-3} M_p^2$$

$$\rho_{vac} \approx \frac{\Lambda^4}{16\pi^2}$$

$$L^3\rho_\Lambda \leq L M_{Pl}^2 \equiv 2M_{BH}$$

$$L^3\rho_\Lambda \leq M_{BH} \sim L M_{Pl}^2$$

$$\rho_\Lambda \sim L^{-2} M_{Pl}^2 \sim H^2 M_{Pl}^2$$

$$M_{Pl} \simeq 1.2 \times 10^{19}\text{GeV};\; H_0 \simeq 1.6 \times 10^{-42}\text{GeV}$$

$$\rho_L = 3c^2 M_p^2 L^{-2}$$

$$\begin{aligned} \dot{\rho}_{dm} + 3H\rho_{dm} &= Q \\ \dot{\rho}_L + 3H(\rho_L + p_L) &= -Q \end{aligned}$$

$$\Omega_L = \frac{8\pi\rho_L}{3M_{Pl}^2 H^2}, \Omega_m = \frac{8\pi\rho_m}{3M_{Pl}^2 H^2}, \Omega_k = \frac{k}{H^2 a^2}$$

$$H^2 = \frac{8\pi G}{3}(\rho_L + \rho_m) - \frac{k}{a^2},$$

$$\Omega_L + \Omega_m = 1 + \Omega_k$$

$$r = \frac{1 - \Omega_L + \Omega_k}{\Omega_L}$$

$$\dot{r} = 3Hr\left[w_L - w_m + \frac{1 + r}{r}\frac{\Gamma}{3H}\right] = 3Hr\left[w_L^{\text{eff}} - w_m^{\text{eff}}\right].$$

$$\frac{1}{H}\frac{dH}{dx} = -\frac{3}{2} - \frac{1}{2}\Omega_k - \frac{3}{2}w_L\Omega_L$$

$$\frac{d\Omega_L}{dx} = 3\Omega_L\left[\frac{1}{3}\Omega_k + w_L(\Omega_L - 1) - \frac{\Gamma}{3H}\right]$$
$$\frac{d\Omega_k}{dx} = \Omega_k(1 + \Omega_k + 3w_L\Omega_L)$$

$$\Gamma = 3H(-1 - w_L) + 2\frac{\dot{L}}{L}$$

$$\frac{d\Omega_L}{dx} = -3\Omega_L(1 - \Omega_L)\left(w_L^{\text{eff}} - w_m^{\text{eff}}\right) + \Omega_k\Omega_L\left(1 + 3w_m^{\text{eff}}\right)$$
$$\frac{d\Omega_k}{dx} = 3\Omega_k\Omega_L\left(w_L^{\text{eff}} - w_m^{\text{eff}}\right) + \Omega_k(1 + \Omega_k)\left(1 + 3w_m^{\text{eff}}\right)$$

$$Q = 3\alpha H\rho$$

$$Q = 3\alpha H\rho_L$$

$$\dot{\rho}_L = -2\rho_L\frac{\dot{L}}{L},$$

$$w_L \equiv \frac{p_L}{\rho_L} = \frac{2}{3}\frac{\dot{L}}{LH} - \alpha - 1$$

$$\rho_H = 3c^2M_p^2H^2$$

$$w_H = -\frac{2}{3}\frac{\dot{H}}{H^2} - \alpha - 1$$

$$H = \frac{2(1 - \alpha c^2)}{3 - 2\alpha c^2}\frac{A}{t}$$

$$a(t) = a_0 t^{\frac{2(1-\alpha c^2)}{3-2\alpha c^2}}$$

$$q(t) = \frac{1}{2(1 - 2\alpha c^2)}$$

$$\frac{Q}{3H\rho_m} = \mu\left(\frac{H}{H_0}\right)^{-n}$$

$$\frac{H}{H_0} = \left(\frac{1}{3}\right)^{1/n}\left[1 - 2q_0 + 2(1 + q_0)a^{-3n/2}\right]^{1/n},$$

$$\mu = \frac{1}{3}(1 - 2q_0)$$

$$L_f = a(t)\int_t^{\infty}\frac{1}{a(t')}dt'$$

$$\dot{L_f} = HL_f - 1$$

$$w = -\frac{1}{3} - \frac{2}{3c}\sqrt{\Omega_f} - \alpha$$

$$q = -\frac{\ddot{a}}{aH^2} = \frac{1}{2} + \frac{3}{2}w\Omega_f = \frac{1}{2} - (1+3\alpha)\Omega_f - \frac{1}{c}\Omega_f^{\frac{3}{2}}.$$

$$\rho_{\mathcal{R}} = 3\alpha M_p^2\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right)$$

$$\mathcal{R} = -6\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right)$$

$$H^2 = \frac{1}{3M_p^2}(\rho_{\mathcal{R}} + \rho_m + \rho_\gamma + \rho_k)$$

$$Q = \gamma H \rho_{\mathcal{R}}$$

$$\frac{\alpha}{2}\frac{d^2H^2}{dx^2} - \left(1 - \frac{7\alpha}{2} - \frac{\alpha\gamma}{2}\right)\frac{dH^2}{dx} - (3 - 6\alpha - 2\alpha\gamma)H^2 - \frac{\rho_{\gamma 0}}{3M_p^2}e^{-4x} - \{1 - \alpha(1+\gamma)\}ke^{-2x} = 0$$

$$\frac{H^2}{H_0^2} = A_+e^{\sigma_+ x} + A_-e^{\sigma_- x} + A_\gamma e^{-4x} + A_k e^{-2x}$$

$$\sigma_\pm = \frac{2 - 7\alpha - \alpha\gamma \pm \sqrt{(2-\alpha)^2 - 2\alpha(\alpha+2)\gamma + \alpha^2\gamma^2}}{2\alpha},$$

$$A_\gamma = \Omega_{\gamma 0},$$
$$A_k = \Omega_{k0},$$
$$A_\pm = \pm\frac{\alpha(\sigma_\mp + 3)\Omega_{k0} + 2\Omega_{\Lambda 0} - \alpha(1 - \Omega_{\gamma 0})(\sigma_\mp + 4)}{\alpha(\sigma_+ - \sigma_-)}.$$

$$\rho_{\mathcal{R}} = \rho_{c0}\sum_{i=+,-}\alpha\left(\frac{\sigma_i}{2} + 2\right)A_i e^{\sigma_i x}.$$

$$\rho_m = \rho_{c0}\sum_{i=+,-}\left\{1 - \alpha\left(\frac{\sigma_i}{2} + 2\right)\right\}A_i e^{\sigma_i x}$$

$$w_{\mathcal{R}} = -1 - \frac{1}{3}\left(\gamma + \frac{1}{\rho_{\mathcal{R}}}\frac{d\rho_{\mathcal{R}}}{dx}\right)$$

$$3H^2 = \rho = \rho_c + \rho_x$$

$$\rho' = d\rho/d\eta = -\rho_c - (1+\omega_x)\rho_x$$

$$\rho' = -\alpha\rho_c - \beta\rho_x$$

$$\omega_x = (\alpha - 1)r + \beta - 1$$

$$\rho_c = -(\beta\rho + \rho')/\Delta, \rho_x = (\alpha\rho + \rho')/\Delta$$

$$\rho'' + (\alpha + \beta)\rho' + \alpha\beta\rho = Q\Delta.$$

$$Q = c_1 \frac{(\gamma_s - \alpha)(\gamma_s - \beta)}{\Delta}\rho + c_2(\gamma_s - \alpha)\rho_c$$
$$-c_3(\gamma_s - \beta)\rho_x - c_4 \frac{(\gamma_s - \alpha)(\gamma_s - \beta)}{\gamma_s\Delta}\rho'$$

$$Q = \frac{u\rho + \gamma_s^{-1}[u - (\gamma_s - \alpha)(\gamma_s - \beta)]\rho'}{\Delta}$$

$$\rho'' + (\gamma_s + \gamma^+)\rho' + \gamma_s\gamma^+\rho = 0$$

$$\rho_c = \frac{(\gamma_s - \beta)b_1 a^{-3\gamma_s} + (1 - \beta)b_2 a^{-3}}{\Delta}$$
$$\rho_x = \frac{(\alpha - \gamma_s)b_1 a^{-3\gamma_s} + (\alpha - 1)b_2 a^{-3}}{\Delta}$$

$$Q = \frac{(\alpha\beta - 1)}{\Delta\gamma}\rho + \frac{(\alpha + \beta - \nu - 2)}{\Delta\gamma}\rho' - \frac{\nu\rho'^2}{\rho\Delta\gamma},$$

$$\rho = \left[\rho_{10}a^{-3} + \rho_{20}a^{-3(1+\nu)}\right]^{1/(1+\nu)}$$

$$\rho_c = \frac{-\rho}{\alpha - \beta}\left[\beta - 1 + \frac{\nu}{(1 + \nu)(1 + \rho_{20}a^{-3\nu}/\rho_{10})}\right]$$
$$\rho_x = \frac{\rho}{\alpha - \beta}\left[\alpha - 1 + \frac{\nu}{(1 + \nu)(1 + \rho_{20}a^{-3\nu}/\rho_{10})}\right]$$
$$p = -\frac{\nu\rho_{10}}{1 + \nu}\frac{a^{-3}}{\rho^{\nu}}$$

$$\omega = -\frac{\nu\rho_{10}}{(1 + \nu)(\rho_{10} + \rho_{20}a^{-3\nu})}$$

$$\delta t = \beta t_p^{2/3} t^{1/3}$$

$$E_{\delta t^3} \sim t^{-1}$$

$$\rho_q \sim \frac{E_{\delta t^3}}{\delta t^3} \sim \frac{1}{t_p^2 t^2}$$

$$\rho_q = \frac{3n^2 M_p^2}{T^2}$$

$$T = \int_0^a \frac{da'}{Ha'}$$

$$\Omega_q = \frac{n^2}{H^2 T^2}$$

$$\Omega_q' = \Omega_q\left(-2\frac{\dot{H}}{H^2} - \frac{2}{n}\sqrt{\Omega_q}\right).$$

$$-\frac{\dot{H}}{H^2} = \frac{3}{2}\left(1-\Omega_q\right) + \frac{\Omega_q^{3/2}}{n} - \frac{Q}{6M_p^2H^3}.$$

$$\Omega_q' = \Omega_q\left[\left(1-\Omega_q\right)\left(3 - \frac{2}{n}\sqrt{\Omega_q}\right) - \frac{Q}{3M_p^2H^3}\right],$$

$$\frac{Q}{3M_p^2H^3} = \begin{cases} 3\alpha\Omega_q & \text{for } Q = 3\alpha H\rho_q \\ 3\beta\left(1-\Omega_q\right) & \text{for } Q = 3\beta H\rho_m \\ 3\gamma & \text{for } Q = 3\gamma H\rho_{tot} \end{cases}$$

$$w_q = -1 + \frac{2}{3n}\sqrt{\Omega_q} - \frac{Q}{3H\rho_q},$$

$$\frac{Q}{3H\rho_q} = \begin{cases} \alpha & \text{for } \ Q = 3\alpha H\rho_q \\ \beta\left(\Omega_q^{-1} - 1\right) & \text{for } \ Q = 3\beta H\rho_m \\ \gamma\Omega_q^{-1} & \text{for } \ Q = 3\gamma H\rho_{tot} \end{cases}$$

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2} = \frac{1}{2} - \frac{3}{2}\Omega_q + \frac{\Omega_q^{3/2}}{n} - \frac{Q}{6M_p^2H^3}.$$

$$\dot{\rho}_{dm} + 3H\rho_{dm} = \frac{\dot{f}}{f}\rho_{dm}$$

$$\dot{\rho}_{de} + 3H(1+w)\rho_{de} = -\frac{\dot{f}}{f}\rho_{dm}$$

$$3H^2 = 8\pi G\rho,$$

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}\left(1 + \frac{p}{\rho}\right),$$

$$\rho_{dm} = \rho_{dm,0}\left(\frac{a_0}{a}\right)^3\frac{f}{f_0}$$

$$\dot{\rho}_{de} + 3H\left(1 + w_{eff}\right)\rho_{de} = 0, w_{eff} \equiv w + \frac{\dot{f}}{3Hf}r$$

$$w_{eff} = -\frac{\dot{f}}{3Hf}, w = (1+r)w_{eff}$$

$$\frac{p}{\rho} = \frac{p_{de}}{\rho_{de} + \rho_{dm}} = \frac{w}{1+r} = w_{eff}$$

$$q = \frac{1}{2}\left(\frac{3p}{\rho} + 1\right)$$

$$q = \frac{1}{2}\left(1 - \frac{\dot{f}}{Hf}\right)$$

$$\dot{r} = 3Hr\left[w_{de} + \frac{Q}{9H^3}\frac{(r+1)^2}{r}\right].$$

$$\dot{r} = \dot{H}\frac{dr}{dH}, \dot{H} = -\frac{1}{2}(\rho_{dm} + \rho_{de} + p_{de}) = -\frac{3}{2}\frac{1 + w_{de} + r}{1 + r}H^2$$

$$\frac{dr}{dH} = \frac{I}{H}, I \equiv -2r\frac{1 + r}{1 + w_{de} + r}\left[w_{de} + \frac{Q}{9H^3}\frac{(r+1)^2}{r}\right]$$

$$Q = 3\alpha H(\rho_{dm} + \rho_{de}), Q = 3\beta H\rho_{dm}, Q = 3\gamma H\rho_{de},$$

$$r_s^{\pm} = -1 + 2b \pm 2\sqrt{b(b-1)}, b = -\frac{w}{4\alpha} > 1$$

$$r(x) = \frac{r_s^{-} + xr_s^{+}}{1 + x}$$

$$w(z) = w_0 + \frac{w_a z}{1 + z}$$

$$w(z) = -\frac{1 + \tanh\left[(z - z_t)\Delta\right]}{2}$$

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}\rho_m = -\dot{\rho}_\Lambda$$

$$\rho_m = \rho_{m,0}a^{-3+\epsilon(a)}$$

$$\rho_\Lambda = \rho_{m0}\int_a^1 \frac{\epsilon(\tilde{a}) + \tilde{a}\epsilon'\ln(\tilde{a})}{\tilde{a}^{4-\epsilon(a)}}d\tilde{a} + \mathrm{X}$$

$$H = H_0\left[\Omega_{b,0}a^{-3} + \Omega_{m0}\varphi(a) + \Omega_{\mathrm{X},0}\right]^{1/2}$$

$$\varphi(a) = a^{-3+\epsilon(a)} + \int_a^1 \frac{\epsilon(\tilde{a}) + \tilde{a}\epsilon'\ln(\tilde{a})}{\tilde{a}^{4-\epsilon(a)}}d\tilde{a}$$

$$\epsilon(a) = \epsilon_0 a^{\xi} = \epsilon_0(1 + z)^{-\xi}$$

$$\rho_\Lambda = \rho_{m0}\epsilon_0\int_a^1 \frac{\left[1 + \ln\left(\tilde{a}^{\xi}\right)\right]}{\tilde{a}^{4-\xi-\epsilon_0\tilde{a}^{\xi}}}d\tilde{a} + \mathrm{X}$$

$$\Omega_b(a) = \frac{a^{-3}}{\mathrm{A} + a^{-3} + \mathrm{B}^{-1}\varphi(a)}$$

$$\Omega_m(a) = \frac{a^{-3+\epsilon(a)}}{\mathrm{D} + \mathrm{B}a^{-3} + \varphi(a)}$$

$$\Omega_\Lambda(a) = \frac{\mathrm{D} + \varphi(a) - a^{-3+\epsilon(a)}}{\mathrm{D} + \mathrm{B}a^{-3} + \varphi(a)}$$

$$q(a) = \frac{3}{2}\frac{\Omega_{b,0}a^{-3} + \Omega_{m0}a^{\epsilon(a)-3}}{\Omega_{b,0}a^{-3} + \Omega_{m0}\varphi(a) + \Omega_{\mathrm{X},0}} - 1$$

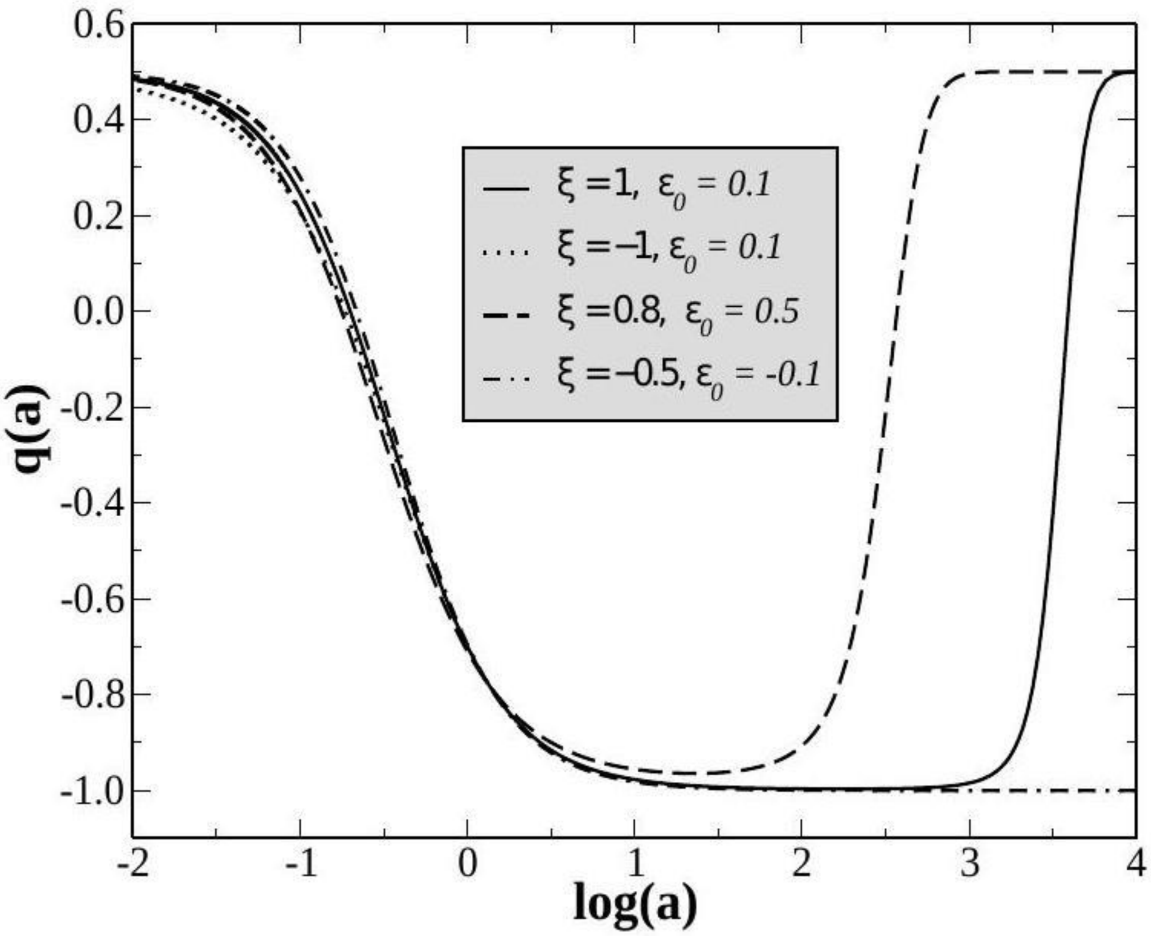


Figura 10. Trayectorias orbitales de la partícula oscura.

$$3M_{Pl}^2H^2 = \rho_{DE} + \rho_m + \rho_b$$

$$\dot{\rho}_b + 3H\rho_b = 0 \Rightarrow \rho_b = \rho_{b0}\left(\frac{a_0}{a}\right)^3$$

$$\rho_m = \tilde{\rho}_{m0}\left(\frac{a_0}{a}\right)^3 f(a)$$

$$Q = \rho_m\frac{\dot{f}}{f} = \tilde{\rho}_{m0}\left(\frac{a_0}{a}\right)^3 \dot{f}$$

$$f(a) = 1 + g(a)$$

$$\dot{f} = \dot{g} = \frac{dg}{da}\dot{a}$$

$$Q = \tilde{\rho}_{m0}\frac{dg}{da}\dot{a}\left(\frac{a_0}{a}\right)^3$$

$$\rho_m = \tilde{\rho}_{m0}(1+g)\left(\frac{a_0}{a}\right)^3$$

$$\rho_{m0} = \tilde{\rho}_{m0}(1+g_0)$$

$$\dot{\rho}_{DE} + 3H(1+w)\rho_{DE} = -\tilde{\rho}_{m0}\frac{dg}{da}\dot{a}\left(\frac{a_0}{a}\right)^3$$

$$\rho_{DE} = (\rho_{m0} + \tilde{\rho}_{m0}g_0)\left(\frac{a_0}{a}\right)^{3(1+w)} - \tilde{\rho}_{m0}\left(\frac{a_0}{a}\right)^3 g + 3w\tilde{\rho}_{m0}a_0^3 a^{-3(1+w)}\int_{a_0}^{a} daga^{3w-1}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}\left\{\tilde{\rho}_{m0}(1+g)\left(\frac{a_0}{a}\right)^3 + \rho_{b0}\left(\frac{a_0}{a}\right)^3 + (1+3w)\times\right.$$
$$\left.\times\left[(\rho_{m0} + \tilde{\rho}_{m0}g_0)\left(\frac{a_0}{a}\right)^{3(1+w)} - \tilde{\rho}_{m0}\left(\frac{a_0}{a}\right)^3 g + 3w\tilde{\rho}_{m0}a_0^3 a^{-3(1+w)}\int_{a_0}^{a} daga^{3w-1}\right]\right\}$$

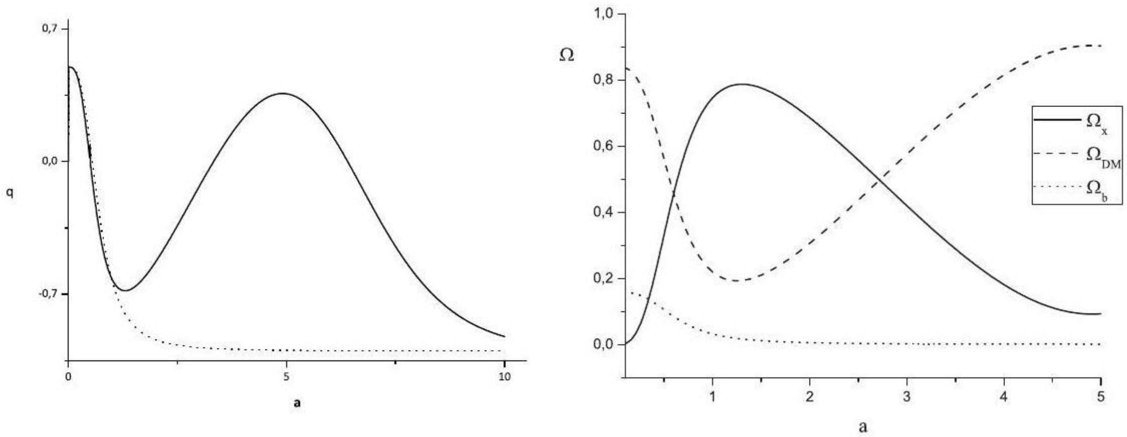


Figura 11. Trayectorias orbitales de la partícula oscura.

$$V(\phi) = \rho_{\phi 0}\left[1 - \frac{\lambda}{6}(1 + \alpha\sqrt{\sigma}\phi)^2\right]\exp\left[-\lambda\sqrt{\sigma}\left(\phi + \frac{\alpha\sqrt{\sigma}}{2}\phi^2\right)\right],$$

$$h^2 = \frac{U(y) + x}{1 - \frac{1}{2}\left(\frac{dy}{dN}\right)^2}$$

$$\frac{d^2y}{dN^2} - \frac{3}{2}\left(\frac{dy}{dN}\right)^3 + 3\frac{dy}{dN} = \left(\frac{\Gamma}{\frac{dy}{dN}} + 1.5\frac{dy}{dN}x - U'(y)\right)h^{-2}$$

$$\frac{dx}{dN} = -\Gamma - 3x$$

$$Q = 3\beta(a)H\rho_{de}$$

$$\beta(a) = \beta_0 a^{\xi}$$

$$\dot{\rho}_{dm} + 3H\rho_{dm} = Q$$
$$\dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = -Q$$

$$\rho_{de} = \rho_{de0}a^{-3(1+w_0)}\cdot\exp\left[\frac{3\beta_0(1-a^{\xi})}{\xi}\right],$$

$$\rho_{dm} = f(a)\rho_{dm0}$$

$$f(a) \equiv \frac{1}{a^3}\left\{1 - \frac{\Omega_{de0}}{\Omega_{dm0}}\frac{3\beta_0 a^{-3w_0} e^{\frac{3\beta_0}{\xi}}}{\xi} \cdot \left[a^{\xi} E_{\frac{3w_0}{\xi}}\left(\frac{3\beta_0 a^{\xi}}{\xi}\right) - a^{3w_0} E_{\frac{3w_0}{\xi}}\left(\frac{3\beta_0}{\xi}\right)\right]\right\},$$

$$\rho_{de} = \rho_{de0} a^{-3(1+w_0+\beta_0)}$$

$$\rho_{dm} = \rho_{dm0} a^{-3}\left[1 + \frac{\Omega_{de0}}{\Omega_{dm0}}\frac{\beta_0}{w_0 + \beta_0}\left(1 - a^{-3(w_0+\beta_0)}\right)\right].$$

$$E^2(z) \equiv \frac{H^2}{H_0^2} = \Omega_{b0} a^{-3} + \Omega_{dm0} f(a) + \Omega_{de0} a^{-3(1+w_0)} e^{\frac{3\beta_0(1-a}{\xi}\xi}$$

$$\Omega_b(a) = \frac{a^{-3}}{a^{-3} + Af(a) + Ba^{-3(1+w_0)} e^{\frac{3\beta_0(1-a^{\xi})}{\xi}}}$$

$$\Omega_{dm}(a) = \frac{f(a)}{A^{-1}a^{-3} + f(a) + A^{-1}Ba^{-3(1+w_0)} e^{\frac{3\beta_0(1-a^{\xi})}{\xi}}}$$

$$\Omega_{de}(a) = \frac{a^{-3(1+w_0)} e^{\frac{3\beta_0(1-a^{\xi})}{\xi}}}{B^{-1}a^{-3} + AB^{-1}f(a) + a^{-3(1+w_0)} e^{\frac{3\beta_0(1-a^{\xi})}{\xi}}},$$

$$q \equiv -\frac{\ddot{a}}{aH^2} = -1 + \frac{3}{2}\left[\frac{\Omega_b + \Omega_m + (1 + w_0)\Omega_{de}}{\Omega_b + \Omega_m + \Omega_{de}}\right]$$

$$q = -1 + \frac{3}{2}\left[\frac{a^{-3} + Af(a) + B(1 + w_0)a^{-3(1+w_0)} e^{\frac{3\beta_0(1-a}{\xi}\xi)}}{a^{-3} + Af(a) + Ba^{-3(1+w_0)} e^{\frac{3\beta_0(1-a\xi)}{\xi}}}\right].$$

$$q = \frac{1}{2} + \frac{w_0\Omega_{de0}}{w_0\Omega_{de0}/(w_0 + \beta_0) + \left(1 - w_0\Omega_{de0}/(w_0 + \beta_0)\right)a^{3(w_0+\beta_0)}}.$$

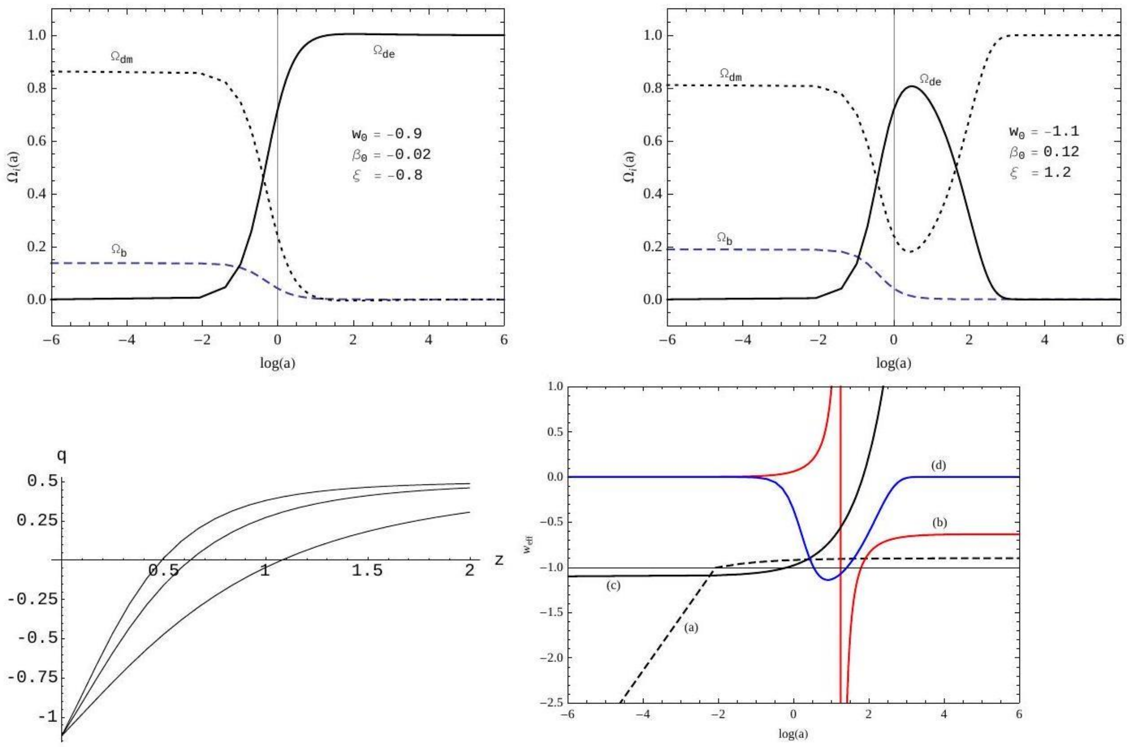


Figura 12. Trayectorias orbitales de la partícula oscura.

$$H^2 = \frac{8\pi G}{3}\left(\rho_\varphi + \rho_{dm} + \rho_b\right)$$

$$\dot{H} + H^2 = -\frac{4\pi G}{3}\left(\rho_\varphi + \rho_{dm} + \rho_b + 3\rho_\varphi\right)$$

$$\delta(a) = \frac{d\ln\ m_\psi(a)}{d\ln\ a}$$

$$\dot{\rho}_{dm} + 3H\rho_{dm} - \delta(a)H\rho_{dm} = 0$$

$$\dot{\rho}_\varphi + 3H\left(\rho_\varphi + p_\varphi\right) + \delta(a)H\rho_{dm} = 0$$

$$W(\varphi(a)) = \exp\left(-\int_a^1 \delta(a')d\ln\ a'\right)$$

$$\ddot{\varphi} + 3H\dot{\varphi} + \left(\frac{dV}{d\varphi} + \frac{\rho_{dm}^{(0)}}{a^3}\frac{dW}{d\varphi}\right) = 0$$

$$\rho_{dm}(a) = \rho_{dm}^{(0)} a^{-3} \exp\left(-\int_a^1 \delta(a')d\ln\ a'\right)$$

$$\rho_{dm}(u) = \rho_{dm}^{(0)} e^{-3u} \exp\left(-\int_a^1 \delta(u')du'\right)$$

$$\rho'_\varphi(u) + 3\left(1 + w_\varphi(u)\right)\rho_\varphi(u) + \delta(u)\rho_{dm}(u) = 0$$

$$\frac{H^2(u)}{H_0^2} = \Omega_b e^{-3u} + \Omega_{dm} e^{-3u} \exp\left(-\int_u^0 \delta(u')du'\right) + \Omega_\varphi f(u)$$

$$\left(\frac{d\tilde{\varphi}}{du}\right)^2 = -\frac{1}{4\pi}\left(\frac{d\ln\ H(u)}{du} + \frac{3}{2}(\Omega_{dm}(u) + \Omega_b(u))\right),$$

$$\Omega_{dm,b}(u) = \frac{\rho_{dm,b}(u)}{\rho_\varphi(u) + \rho_{dm}(u) + \rho_b(u)}$$

$$\tilde{V}(\tilde{\varphi}) \equiv \frac{V(u(\tilde{\varphi}))}{\rho_c^{(0)}} = \frac{1}{3}\frac{H(u)}{H_0}\frac{d(H/H_0)}{du} + \frac{H^2(u)}{H_0^2} - \frac{1}{2}\Omega_b e^{-3u} - \frac{1}{2}\Omega_{dm} e^{-3u} \exp\left(-\int_u^0 \delta(u')du'\right)$$

$$W(u(\tilde{\varphi})) = \exp\left(-\int_u^0 \delta(u')du'\right)$$

$$\rho_{dm} = \rho_{dm}^{(0)} a^{-3+\delta}$$

$$\rho_\varphi(a) = \rho_\varphi^{(0)} a^{-3(1+w_\varphi)} + \frac{\delta}{\delta + 3w_\varphi}\rho_{dm}^{(0)}()\left(a^{-3(1+w_\varphi)} - a^{-3+\delta}\right)$$

$$W(\tilde{\varphi}(u)) = e^{\delta u}$$

$$H(t) = \frac{1}{a}\frac{da}{dt}$$

$$q(t) = -\frac{1}{a}\frac{d^2a}{dt^2}\left(\frac{1}{a}\frac{da}{dt}\right)^{-2}$$

$$j(t) = \frac{1}{a}\frac{d^3a}{dt^3}\left(\frac{1}{a}\frac{da}{dt}\right)^{-3}$$

$$s(t) = \frac{1}{a}\frac{d^4a}{dt^4}\left(\frac{1}{a}\frac{da}{dt}\right)^{-4}$$

$$l(t) = \frac{1}{a}\frac{d^5a}{dt^5}\left(\frac{1}{a}\frac{da}{dt}\right)^{-5}$$

$$p = p_{de}, \rho = \rho_{de} + \rho_{dm}$$

$$\frac{1}{a^3}\frac{d}{dt}(\rho_{dm}a^3) + \frac{1}{a^{3(1+w_{de})}}\frac{d}{dt}\left(\rho_{de}a^{3(1+w_{de})}\right) = 0$$

$$\frac{1}{a^3}\frac{d}{dt}(\rho_{dm}a^3) = \gamma(t), \frac{1}{a^{3(1+w_{de})}}\frac{d}{dt}\left(\rho_{de}a^{3(1+w_{de})}\right) = -\gamma(t).$$

$$\rho_{dm}a^3 = \rho_{dm0}a_0^3 + \int_{t_0}^t \gamma(t)a^3 dt$$

$$\rho_{de}a^{3(1+w_{de})} = \rho_{de0}a_0^{3(1+w_{de})} + \int_{t_0}^t \gamma(t)a^{3(1+w_{de})} dt$$

$$\ddot{a} = \frac{1}{2}\left[-\frac{A(a)}{a^2} - \frac{(1+3w_{de})B(a)}{a^{2+3w_{de}}}\right]$$

$$A(a) \equiv \frac{1}{3}\rho_{dm}a^3, B(a) \equiv \frac{1}{3}a^{3(1+w_{de})}$$

$$qH^2 = \frac{1}{2}\left[\frac{A(a)}{a^3} + \frac{(1+3w_{de})B(a)}{a^{3(1+w_{de})}}\right]$$

$$\nu(t) \equiv \frac{\gamma(t)}{3H^3}$$

$$j - \frac{3}{2}w_{de}\nu = \Omega_{dm} + \frac{1}{2}(1+3w_{de})(2+3w_{de})\Omega_{de}$$

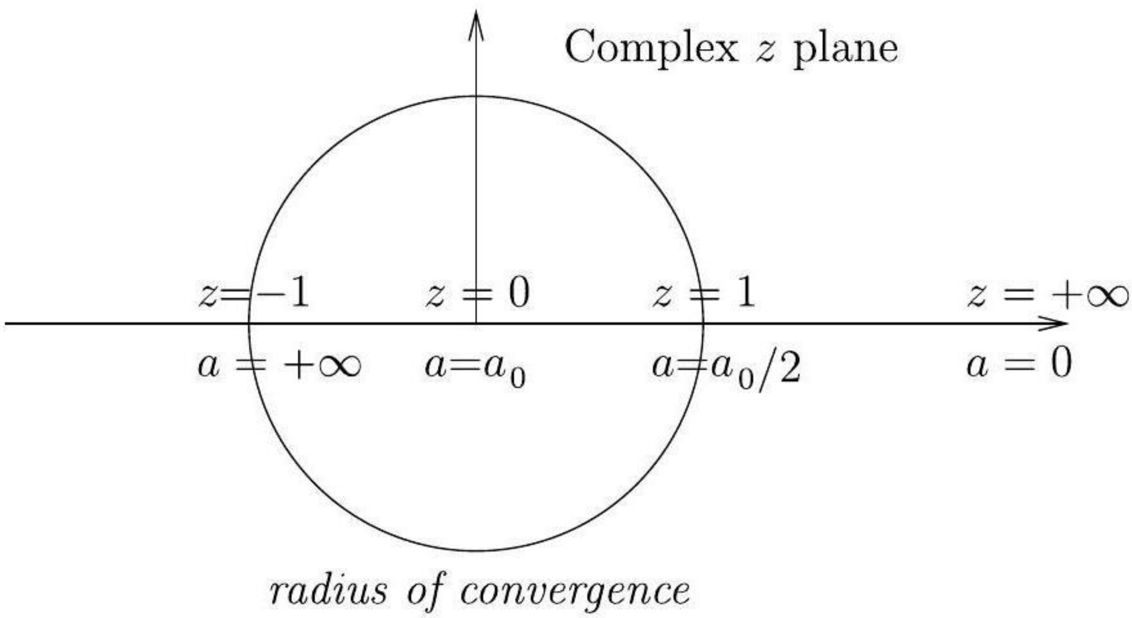
$$j - \frac{3}{2}w_{de}\nu - 1 = \frac{9}{2}w_{de}(1+w_{de})\Omega_{de} - \Omega_c, \Omega_c \equiv -\frac{k}{a^2H^2}$$

$$q = \frac{1}{2}\Omega_{dm} + \frac{1+3w_{de}}{2}\Omega_{de}$$

$$j - \frac{3}{2}w_{de}\nu + q = \frac{3}{2}\Omega_{dm} + \frac{1}{2}(1+3w_{de})(1+w_{de})\Omega_{de}$$

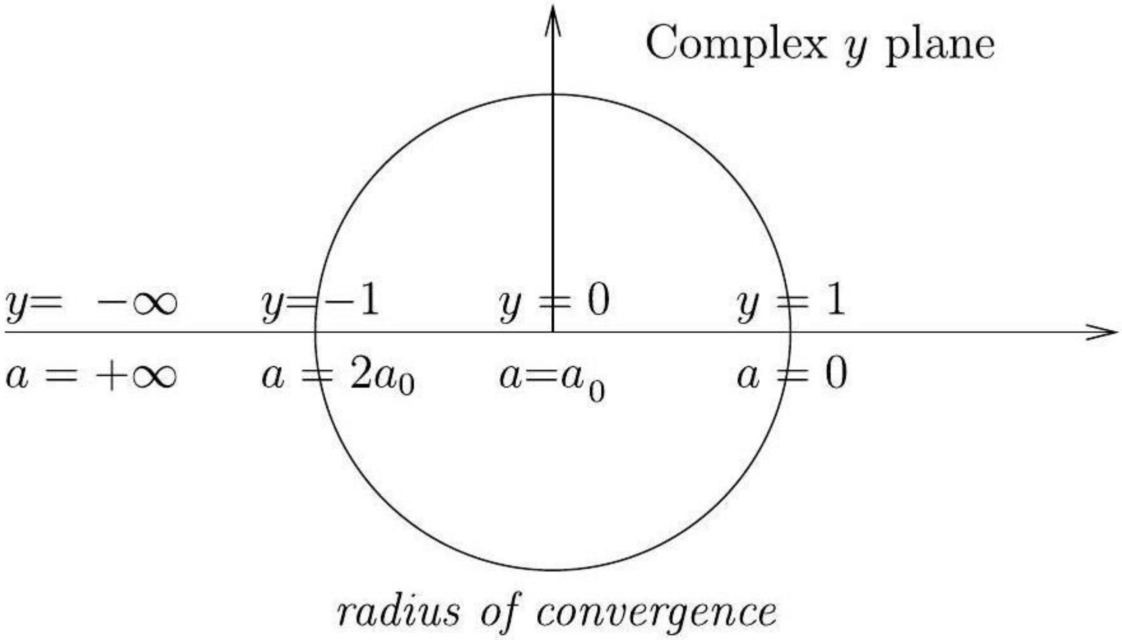
$$j - \frac{3}{2}w_{de}\nu + q = -\frac{3}{2}\Omega_{dm}(4+3w_{de})w_{de} + \frac{3}{2}(1+3w_{de})(1+w_{de})$$

$$\frac{1}{1+z} = \frac{a(t)}{a_0} = 1 + H_0(t-t_0) - \frac{q_0H_0^2}{2!}(t-t_0)^2 + \frac{j_0H_0^3}{3!}(t-t_0)^3 + O([t-t_0]^4)$$



$$y = \frac{z}{1+z};\ z = \frac{y}{1-y}$$

$$y = \frac{\lambda_0 - \lambda_e}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0}$$



$$Om(x) \equiv \frac{h^2(x)-1}{x^3-1}, x = 1+z, h(x) = \frac{H(x)}{H_0}$$

$$h^2(x) = \Omega_{m0}x^3 + (1-\Omega_{m0})x^{\alpha}, \alpha = 3(1+w)$$

$$Om(x) = \Omega_{m0} + (1-\Omega_{m0})\frac{x^{\alpha}-1}{x^3-1}$$

$$q = -1 - (\dot{H}/H)^2 = 1/2(1+3w_{de}\Omega_{de})$$

$$\frac{\ddot{H}}{H^3} = \frac{9}{2}\left(1+\frac{p_{de}}{\rho}\right) + \frac{9}{2}\left[w_{de}(1+w_{de})\frac{\rho_{de}}{\rho} - w_{de}\frac{\Pi}{\rho} - \frac{\dot{w}_{de}}{3H}\frac{\rho_{de}}{\rho}\right],$$

$$r \equiv \frac{\ddot{a}}{aH^3}, s \equiv \frac{r-1}{3(q-1/2)},$$

$$r = 1 + \frac{9}{2}\Omega_{de}w_{de}(1+w_{de}) - \frac{3}{2}\Omega_{de}\frac{\dot{w}_{de}}{H};$$

$$s = 1 + w_{de} - \frac{1}{3}\frac{\dot{w}_{de}}{w_{de}H}; \; w_{de} \equiv \frac{p_{de}}{\rho_{de}}$$

$$\{r,s\} = \{1{,}0\}$$

$$\{r,s\} = \left\{1 + \frac{9}{2}\Omega_{DE}(1+w_{de}), 1 + w_{de}\right\}$$

$$\{r,s\} = \left\{1 + \frac{12\pi G\dot{\varphi}^2}{H^2} + \frac{8\pi G\dot{V}}{H^3}, \frac{2\left(\dot{\varphi}^2 + \frac{2\dot{V}}{H}\right)}{\dot{\varphi}^2 - 2V}\right\}'$$

$$r = 1 + \frac{9}{2}\frac{w_{de}}{1+R}\left[1 + w_{de} - \frac{\Pi}{\rho_{de}} - \frac{\dot{w}_{de}}{3w_{de}H}\right], R \equiv \frac{\rho_{dm}}{\rho_{de}}$$

$$s = 1 + w - \frac{\Pi}{\rho_{de}} - \frac{\dot{w}_{de}}{3Hw_{de}},$$

$$\Pi = \rho_{de}\left(w_{de} + \frac{\xi}{3}\right)\frac{R_0(1+z)^{\xi}}{1+R_0(1+z)^{\xi}}$$

$$r = 1 + \frac{9}{2}\frac{w_{de}}{1+R_0(1+z)^{\xi}}\left[1 + w_{de} - \left(w_{de} + \frac{\xi}{3}\right)\frac{R_0(1+z)^{\xi}}{1+R_0(1+z)^{\xi}}\right]$$

$$s = 1 + w_{de} - \left(w_{de} + \frac{\xi}{3}\right)\frac{R_0(1+z)^{\xi}}{1+R_0(1+z)^{\xi}}$$

$$\rho_{de} = 3M_p^2(\alpha H^2 + \beta\dot{H})$$

$$\dot{r}_\rho = 3H\left[wr_\rho + b(1+r_\rho)^2\right].$$

$$3M_p^2H^2 = \rho_{de} + \rho_m$$

$$\dot{H} = -\frac{3}{2}H^2\left(1 + \frac{w}{1+r_\rho}\right)$$

$$\Omega_{de} = \frac{1}{1+r_\rho}$$

$$w = \left(\frac{2\alpha}{3\beta} - 1\right)(1+r_\rho) - \frac{2}{3\beta}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p),$$

$$r = 1 + \frac{9(\rho+p)}{2\rho}\frac{\dot{p}}{\dot{\rho}}, s = \frac{(\rho+p)}{p}\frac{\dot{p}}{\dot{\rho}},$$

$$q = -\frac{\ddot{a}}{aH^2} = \frac{1}{2} + \frac{3p}{2\rho}$$

$$w^{\text{eff}} = w + b(1+r_\rho)$$

$$\dot{\rho}_{de} + 3H(1+w^{\text{eff}})\rho_{de} = 0$$

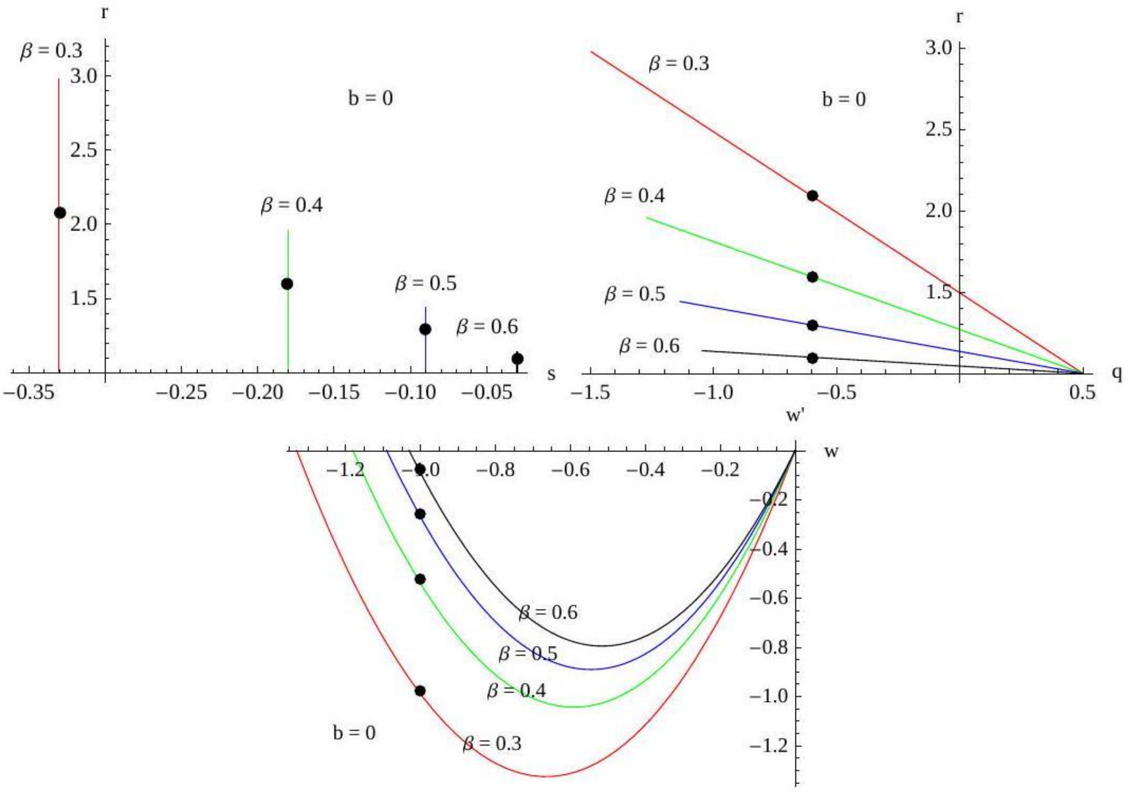
$$r = 1 - \frac{3}{2}\Omega_{de}[w' - 3w(1+w^{\text{eff}})]$$

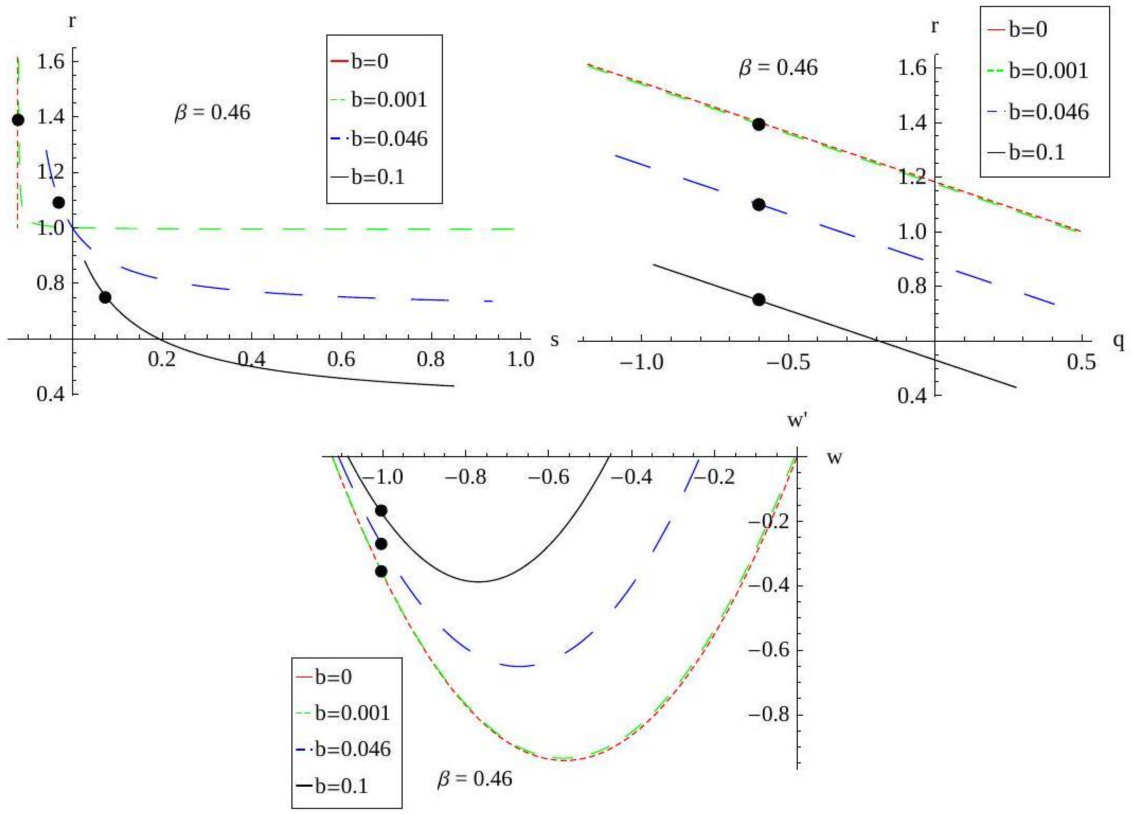
$$s = 1 + w^{\text{eff}} - \frac{w'}{3w}$$

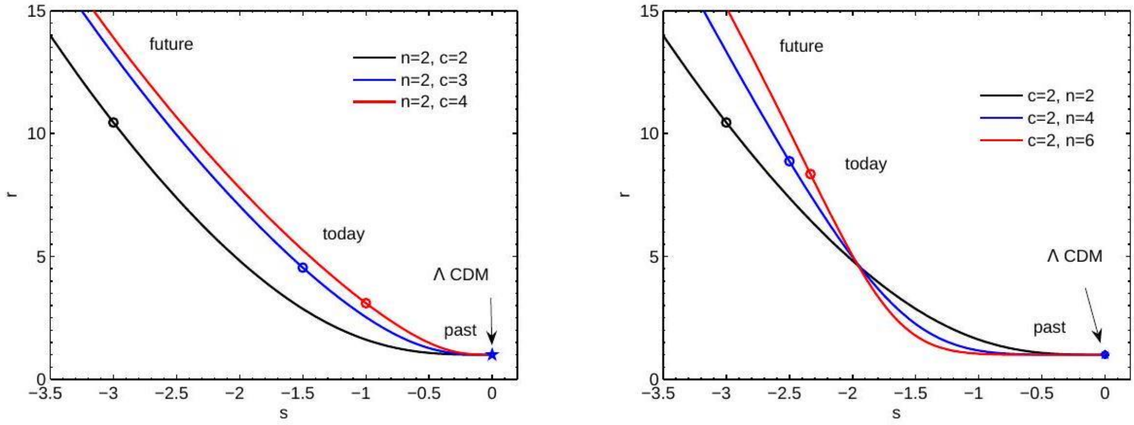
$$q = \frac{1}{2} + \frac{3}{2}w\Omega_{de}$$

$$\{r,s\}|_{\text{LCDM}} = \{1,0\}$$

$$H^2 = \frac{1}{3M_p^2}(\rho_m + \rho_\Lambda)$$







Figuras 13, 14 y 15. Trayectorias orbitales de la partícula oscura.

$$\rho_\Lambda = \alpha H$$

$$\Omega_m = \frac{\rho_m}{\rho_c} = \frac{\rho_m}{3M_p^2H^2}, \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{\rho_\Lambda}{3M_p^2H^2}$$

$$r = \frac{\ddot{H}}{H^3} - 3q - 2.$$

$$r = 1 + \frac{9}{4}w_\Lambda\Omega_\Lambda(w_\Lambda\Omega_\Lambda + 1) - \frac{3}{2}\Omega_\Lambda w'_\Lambda$$

$$s = \frac{1}{2}(1 + w_\Lambda\Omega_\Lambda) - \frac{w'_\Lambda}{3w_\Lambda}$$

$$p_\Lambda = K\rho_\Lambda^{1+\frac{1}{n}},$$

$$\rho_\Lambda = \left(\frac{1}{Ba^{\frac{3(1+\alpha)}{n}} - \widetilde{K}}\right)^n,$$

$$\dot{\rho_\Lambda} + 3H(1 + \alpha + w_\Lambda)\rho_\Lambda = 0$$

$$\dot{\rho_\Lambda} = -3BH(1+\alpha)a^{\frac{3(1+\alpha)}{n}}\rho_\Lambda^{1+\frac{1}{n}}$$

$$\frac{\ddot{H}}{H^3} = -\frac{9}{2}\Omega_\Lambda(1+\alpha)(\alpha + w_\Lambda)[(1+\alpha)(-w_\Lambda + \Omega_\Lambda\alpha + \Omega_\Lambda w_\Lambda) - \alpha(\alpha+2)]$$
$$-\frac{3}{2}\Omega_\Lambda(1+\alpha)w'_\Lambda + \frac{9}{2}[\Omega_\Lambda(1+\alpha)(\alpha + w_\Lambda) + 1]^2$$

$$r = \frac{\ddot{a}}{aH^3} = \frac{\ddot{H}}{H^3} - 3q - 2$$

$$r = 1 + \frac{3}{2}\Omega_\Lambda(1+\alpha)[3(1+\alpha)(\alpha + w_\Lambda)(1 + \alpha + w_\Lambda) - w'_\Lambda]$$

$$s=\frac{2}{3}\frac{3\alpha(\alpha+1)^2+3\alpha w_\Lambda(2\alpha+w_\Lambda+3)+3w_\Lambda(1+w_\Lambda)-w'_\Lambda}{\alpha+w_\Lambda}$$

$$H(z)=-\frac{1}{1+z}\frac{dz}{dt}$$

$$\chi_H^2=\sum_{i=1}^{13}\frac{[H(z_i)-H_{obs}(z_i)]^2}{\sigma_{hi}^2},$$

$$A=\frac{\sqrt{\Omega_m}}{E(z_{\mathrm{BAO}})^{1/3}}\left[\frac{1}{z_{BAO}}\int_0^{z_{BAO}}\frac{dz'}{E(z')}\right]^{2/3}$$

$$\chi_{BAO}^2=\frac{(A-A_{\mathrm{obs}})^2}{\sigma_A^2}$$

$$R=\sqrt{\Omega_{m0}}\int_0^{z_{rec}}\frac{dz'}{E(z')}$$

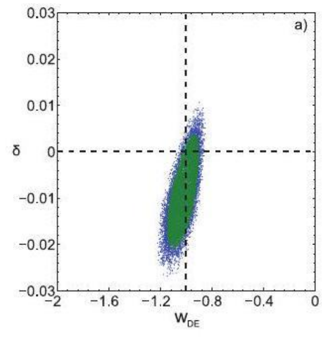
$$\chi_{\mathrm{CMB}}^2=\frac{(R-R_{\mathrm{obs}})^2}{\sigma_R^2}$$

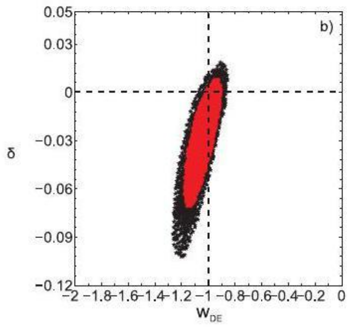
$$\mu=5\log\frac{d_L}{Mpc}+25=5\log_{10}H_0d_L-\mu_0$$

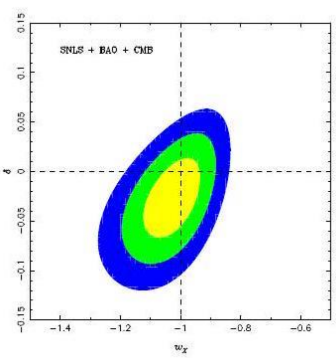
$$\chi_{SN}^2=A-\frac{B^2}{C}+\ln\left(\frac{C}{2\pi}\right)$$

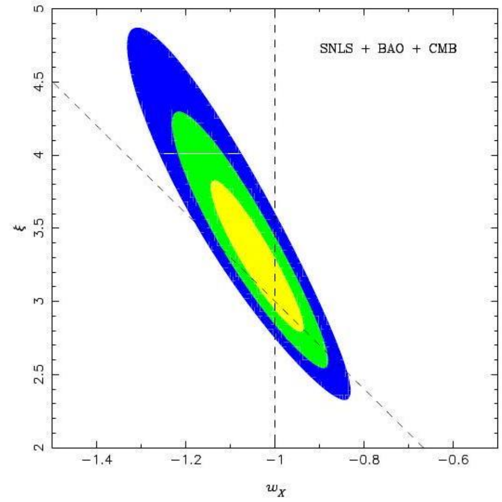
$A=\sum_i^{557}\left(\mu^{\mathrm{data}}-\mu^{\mathrm{th}}\right)^2/\sigma_i^2, B=\sum_i^{557}\mu^{\mathrm{data}}-\mu^{\mathrm{th}}/\sigma_i^2, C=\sum_i^{557}1/\sigma_i^2, \mu^{\mathrm{data}}$

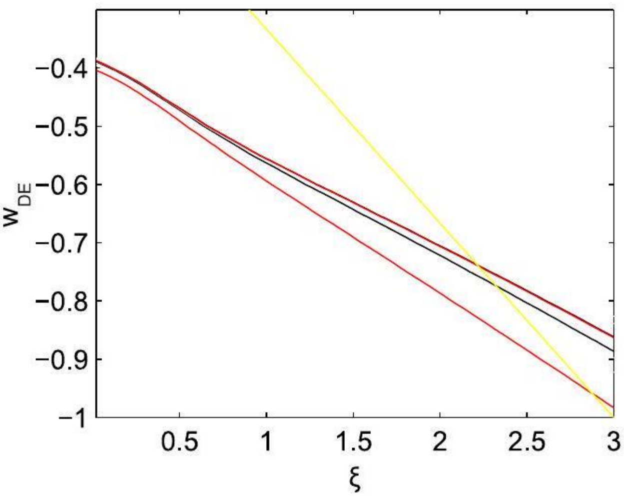
Model	$\Omega_{m,0}$	w_{DE}	δ
$Q=3\delta H\rho_m$	$0.274^{+0.029}_{-0.029}$	$-1.02^{+0.12}_{-0.13}$	$-0.009^{+0.013}_{-0.012}$
$Q=3\delta H\rho_{DE}$	$0.272^{+0.030}_{-0.030}$	$-1.02^{+0.09}_{-0.09}$	$-0.023^{+0.039}_{-0.040}$
$\rho_m=\rho_{m0}a^{-3+\delta}$	$0.270^{+0.040}_{-0.050}$	$-1.03^{+0.12}_{-0.15}$	$-0.03^{+0.06}_{-0.05}$
ΛCDM	$0.270^{+0.019}_{-0.019}$	$-1.0710^{+0.0775}_{-0.0775}$	0

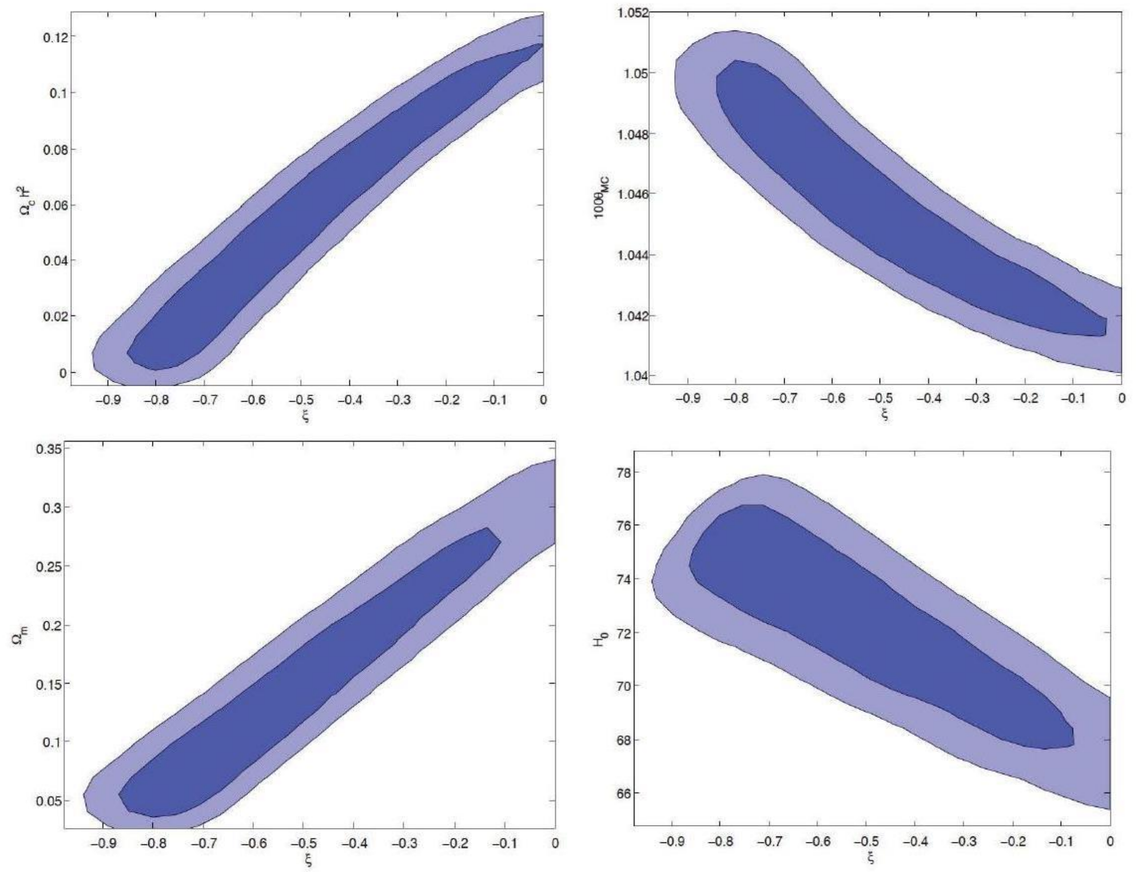

a)
δ
w_{DE}


b)
δ
w_{DE}


SNLS + BAO + CMB
δ
w_x


SNLS + BAO + CMB
ξ
w_x


w_{DE}
ξ


$\Omega_c h^2$
$100\theta_{MC}$
Ω_m
H_0
ξ

Figuras 16, 17 y 18. Agujeros negros cuánticos provocados por una partícula oscura.

$$\frac{\rho_{DE}}{\rho_m} = \frac{\rho_{DE0}}{\rho_{m0}} a^{\xi}$$

$$\nabla_\mu T^{\mu}_{(dm)\nu} = Q u_\nu^{(dm)}/a$$
$$T^{\mu}_{(de)\nu} = -Q u_\nu^{(dm)}/a$$
$$Q = \xi H \rho_{de}$$

$$\dot{\rho}_{dm} + 3\mathcal{H}\rho_{dm} = \xi\mathcal{H}\rho_{de}$$
$$\dot{\rho}_{de} + 3\mathcal{H}(1+w)\rho_{de} = -\xi\mathcal{H}\rho_{de}$$

$$\dot{\delta}_{dm} = -\left(kv_{dm} + \frac{1}{2}\dot{h}\right) + \xi\mathcal{H}\frac{\rho_{de}}{\rho_{dm}}(\delta_{de} - \delta_{dm})$$
$$+\xi\frac{\rho_{de}}{\rho_{dm}}\left(\frac{kv_T}{3} + \frac{\dot{h}}{6}\right)$$
$$\dot{\delta}_{de} = -(1+w)\left(kv_{de} + \frac{1}{2}\dot{h}\right) - 3\mathcal{H}(1-w)$$
$$\left[\delta_{de} + \mathcal{H}(3(1+w)+\xi)\frac{v_{de}}{k}\right] - \xi\left(\frac{kv_T}{3} + \frac{\dot{h}}{6}\right)$$
$$\dot{v}_{dm} = -\mathcal{H}v_{dm}$$
$$\dot{v}_{de} = 2\mathcal{H}\left(1 + \frac{\xi}{1+w}\right)v_{de} + \frac{k}{1+w}\delta_{de} - \xi\mathcal{H}\frac{v_{dm}}{1+w}$$

$$\dot{\rho}_{dm} + 3H\rho_{dm} = -\beta(\phi)\dot{\phi}\rho_{dm}$$
$$\dot{\rho}_{\phi} + 3H\rho_{\phi} = +\beta(\phi)\dot{\phi}\rho_{dm}$$

$$m_c(a) = m_c(a_0)e^{-\int \beta(\phi)d\phi}$$

$$\ddot{\delta}_{dm} + (2H - \beta\dot{\phi})\dot{\delta}_{dm} - \frac{3}{2}H^2[(1+2\beta^2)\Omega_{dm}\delta_{dm} + \Omega_b\delta_b] = 0$$

$$-\beta\dot{\phi}\dot{\delta}_{dm}$$

$$\dot{\vec{v}}_i = \beta\dot{\phi}\vec{v}_i + \sum_{j\neq i}\frac{G(1+2\beta^2)m_j\vec{r}_{ij}}{|\vec{r}_{ij}|^3},$$

$$\vec{a}_v = \beta\dot{\phi}\vec{v}$$

$$
\begin{aligned}
\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} &= \sqrt{\frac{2}{3}}\beta_c(\phi)\frac{\rho_c}{M_{\rm Pl}} \\
\dot{\rho}_c + 3H\rho_c &= -\sqrt{\frac{2}{3}}\beta_c(\phi)\frac{\rho_c\dot{\phi}}{M_{\rm Pl}} \\
\dot{\rho}_b + 3H\rho_b &= 0 \\
\dot{\rho}_r + 4H\rho_r &= 0 \\
3H^2 &= \frac{1}{M_{\rm Pl}^2}(\rho_r + \rho_c + \rho_b + \rho_\phi)
\end{aligned}
$$

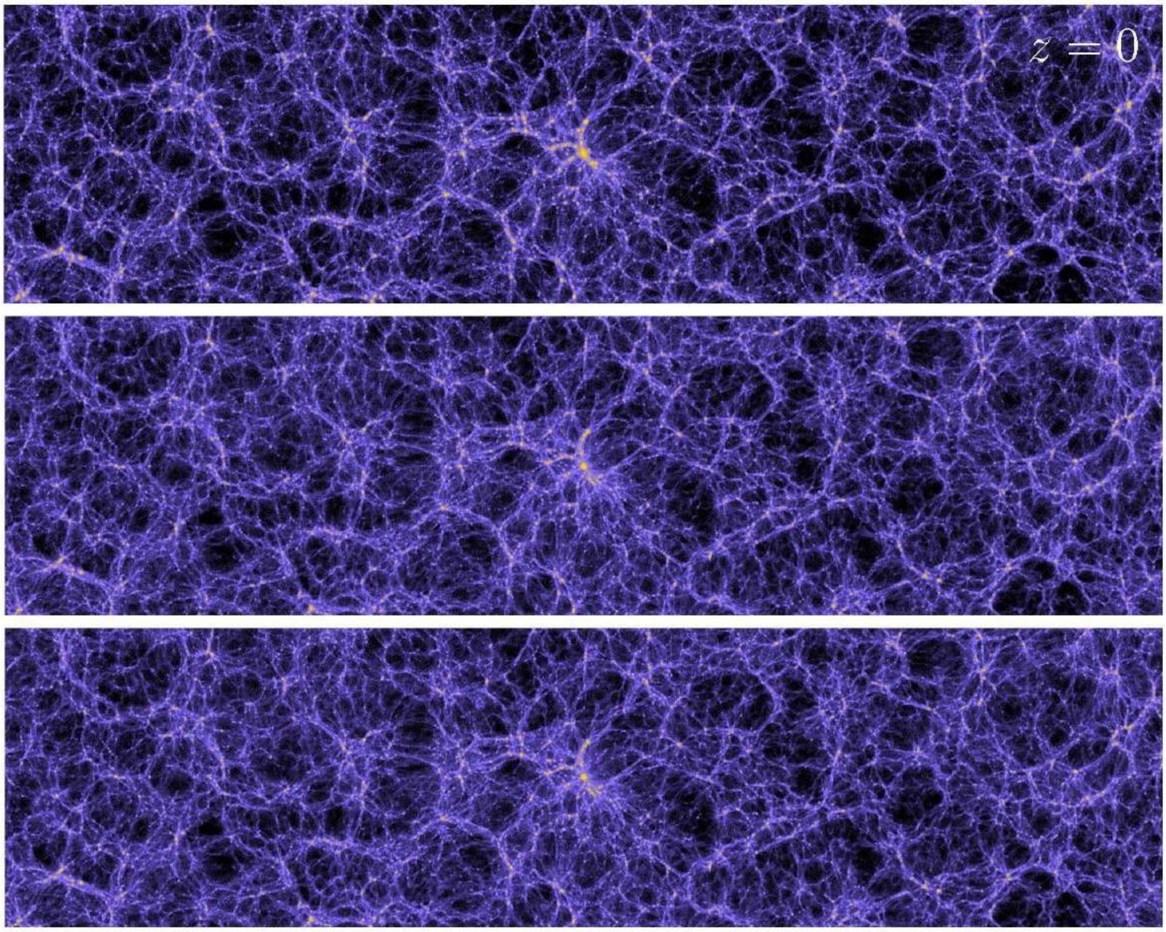
$$V(\phi) = Ae^{-\alpha\phi}$$

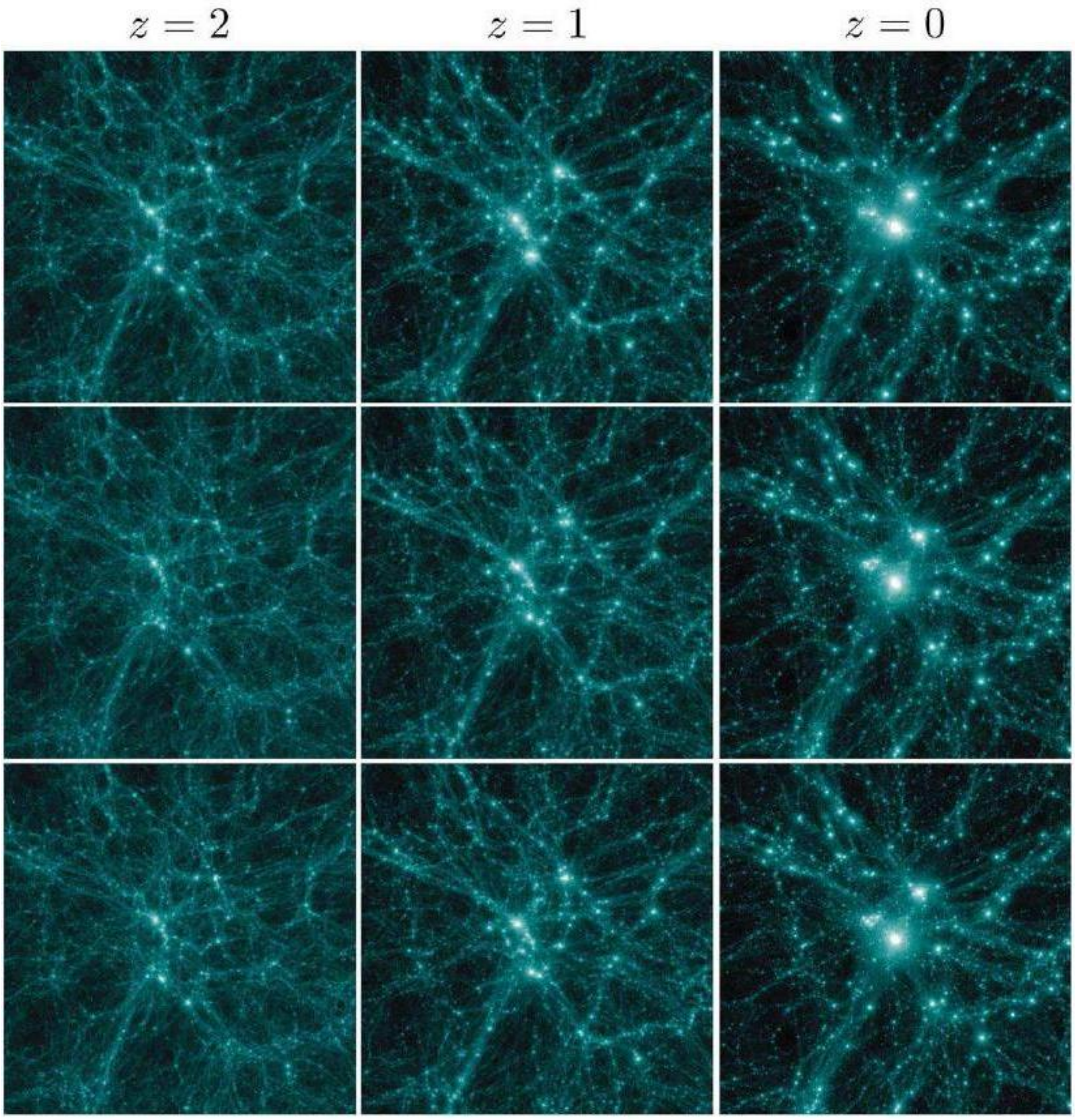
$$V(\phi) = A\phi^{-\alpha}e^{\phi^2/2}$$

$$\beta_c(\phi) \equiv \beta_0 e^{\beta_1\phi}$$

Parameter	Value
H_0	70.3 km s^{-1} Mpc^{-1}
$\Omega_{\rm CDM}$	0.226
$\Omega_{\rm DE}$	0.729
$\mathcal{A}_s$	2.42×10^{-9}
Ω_b	0.0451
n_s	0.966

Model	Potential	α	β_0	β_1	Scalar field normalization	Potential normalization	$w_\phi(z=0)$	$\mathcal{A}_s(z_{\rm CMB})$	$\sigma_8(z=0)$
ΛCDM	$V(\phi) = A$	–	–	–	–	$A = 0.0219$	-1.0	2.42×10^{-9}	0.809
EXP001	$V(\phi) = Ae^{-\alpha\phi}$	0.08	0.05	0	$\phi(z=0) = 0$	$A = 0.0218$	-0.997	2.42×10^{-9}	0.825
EXP002	$V(\phi) = Ae^{-\alpha\phi}$	0.08	0.1	0	$\phi(z=0) = 0$	$A = 0.0218$	-0.995	2.42×10^{-9}	0.875
EXP003	$V(\phi) = Ae^{-\alpha\phi}$	0.08	0.15	0	$\phi(z=0) = 0$	$A = 0.0218$	-0.992	2.42×10^{-9}	0.967
EXP008e3	$V(\phi) = Ae^{-\alpha\phi}$	0.08	0.4	3	$\phi(z=0) = 0$	$A = 0.0217$	-0.982	2.42×10^{-9}	0.895
SUGRA003	$V(\phi) = A\phi^{-\alpha}e^{\phi^2/2}$	2.15	-0.15	0	$\phi(z \to \infty) = \sqrt{\alpha}$	$A = 0.0202$	-0.901	2.42×10^{-9}	0.806


$z = 0$



Figuras 19 y 20. Materia y energía oscuras en un espacio – tiempo cuántico relativista.

SECCIÓN III.

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2 d\theta^2 + r^2 \sin^2\ \theta d^2\phi\right]$$

$H_0/[\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}]$	67.74 ± 0.46
$\Omega_{b,0}h^2$	0.02230 ± 0.00014
$\Omega_{c,0}h^2$	0.1188 ± 0.0010
$\Omega_{\Lambda,0}$	0.6911 ± 0.0062
$\Omega_{K,0}$	$0.0008^{+0.0040}_{-0.0039}$
ω_d	$-1.019^{+0.075}_{-0.080}$

$$ds^2 = a^2(\tau)\left[-d\tau^2 + \frac{dr^2}{1-Kr^2} + r^2 d\theta^2 + r^2 \sin^2\ \theta d^2\phi\right]$$

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\sum\ \rho_i + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum\ (\rho_i + 3p_i) + \frac{\Lambda}{3}$$

$$1 + z = \frac{a(t_0)}{a(t_e)}$$

$$E(z) = \frac{H(z)}{H_0} = \left(\sum_i\ \Omega_{i,0} f_i(z)\right)^{1/2}, f_i(z) = \exp\left[3\int_0^z \frac{1+\omega_i(z')}{1+z'} dz'\right]$$

$$D_L = (1+z)cH_0^{-1}\int_0^z \frac{dz'}{E(z')}$$

$$D_A = \frac{cH_0^{-1}}{(1+z)}\int_0^z \frac{dz'}{E(z')}$$

$$t_L(z) = H_0^{-1}\int_0^z \frac{dz'}{(1+z')E(z')} \equiv t_0 - t_{age}(z) - df$$

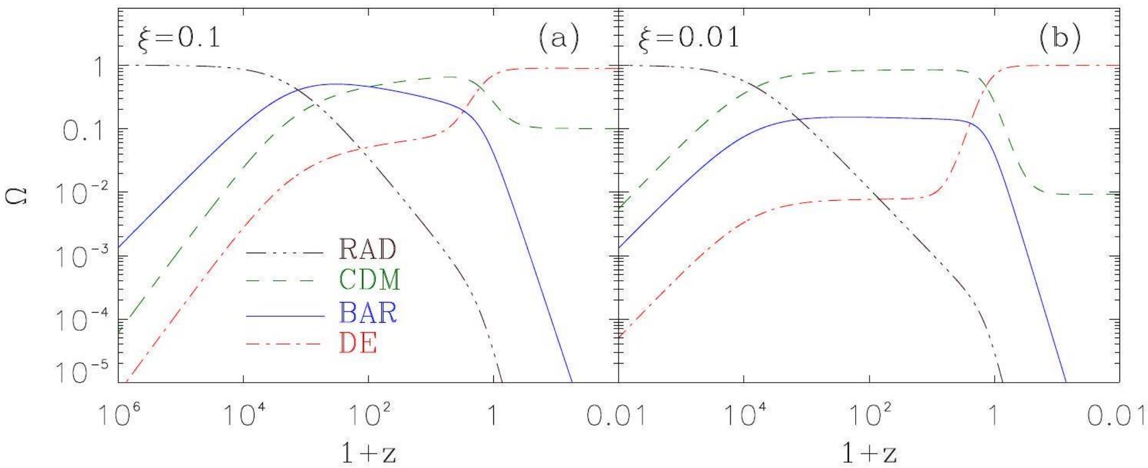
$$\frac{\rho_\Lambda}{\rho_m(t_{\text{Planck}})} = \frac{\rho_\Lambda}{\rho_{m,0}}\left(\frac{T_{\text{Planck}}}{T_0}\right)^{-3} \simeq 10^{-95}$$

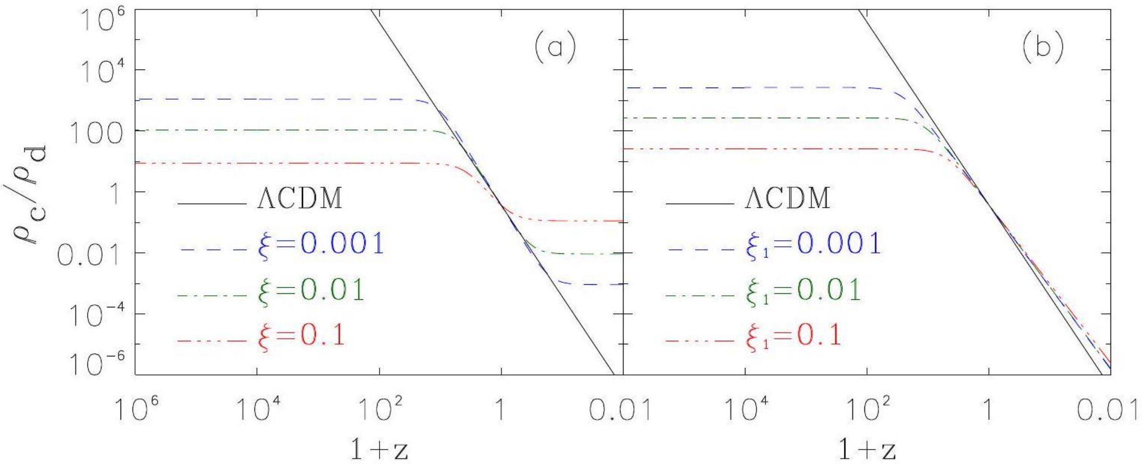
$$\frac{d\rho_i}{dt} + 3H(1+\omega_i)\rho_i = 0$$

Model	Q	DE EoS
I	$\xi_2 H\rho_d$	$-1 < \omega_d < 0$
II	$\xi_2 H\rho_d$	$\omega_d < -1$
III	$\xi_1 H\rho_c$	$\omega_d < -1$
IV	$\xi H(\rho_c + \rho_d)$	$\omega_d < -1$

$$\dot{\rho}_c + 3H\rho_c = Q$$

$$\dot{\rho}_d + 3H(1+\omega_d)\rho_d = -Q$$





Figuras 21 y 22. Densidades de masa y energía de una partícula oscura.

$$Q = H\xi_1\rho_c;\ Q = H\xi_2\rho_d;\ Q = H\xi(\rho_d + \rho_c).$$

$$\frac{dr}{dt} = -3\Gamma Hr, \Gamma = -\omega_d - \xi^2\frac{(\rho_c + \rho_d)^2}{3\rho_c\rho_d},$$

$$r_s^{\pm} = -1 + 2b \pm 2\sqrt{b(b-1)}, b = -\frac{3\omega_d}{4\xi} > 1$$

$$\dot{r} = H[\xi_1(1 + r) + 3\omega_d];$$
$$r = -(3\omega_d + \xi_1)r_0/\{\xi_1 r_0 - (1 + z)^{-(3\omega_d+\xi_1)}[\xi_1(1 + r_0) + 3\omega_d]\},$$

$$\chi = \frac{1}{aH^3}\frac{d^3a}{dt^3}, s = \frac{\chi - 1}{3\left(q - \frac{1}{2}\right)},$$

$$\chi = 1 + \frac{9}{2}\frac{\omega_d}{1 + r_0(1 + z)^\zeta}\left[1 + \omega_d - \left(\omega_d + \frac{\zeta}{3}\right)\frac{r_0(1 + z)^\zeta}{1 + r_0(1 + z)^\zeta}\right]$$
$$s = 1 + \omega_d - \left(\omega_d + \frac{\zeta}{3}\right)\frac{r_0(1 + z)^\zeta}{1 + r_0(1 + z)^\zeta}$$

$$\mathcal{L} = \sqrt{-g}\bar{\Upsilon}(i \not{\mathcal{D}} - m)\Upsilon + \mathfrak{N}_{\text{non derivative interactions}}$$

$$T_{\mu\nu} = \frac{i}{4}\left(\bar{\Upsilon}\gamma_\mu\nabla_\nu\Upsilon + \bar{\Upsilon}\gamma_\nu\nabla_\mu\Upsilon - \nabla_\mu\bar{\Upsilon}\gamma_\nu\Upsilon - \nabla_\nu\bar{\Upsilon}\gamma_\mu\Upsilon\right)$$

$$\mathcal{L} = \bar{\Upsilon}(i\not{\partial})\Upsilon + \mathcal{L}_s(\varphi) + F(\varphi)\bar{\Upsilon}\Upsilon$$

$$\mathcal{L}_s(\varphi) = \ell\frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - V(\varphi)$$

$$H^2 = \frac{8\pi G}{3}\left(\rho_r + \rho_b + \rho_c + \frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right)$$

$$\dot{\rho}_c + 3H\rho_c = -\rho_c\dot{\varphi}\varrho/(1-\varrho\varphi)$$
$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = \rho_c\varrho/(1-\varrho\varphi)$$

$$\mathcal{L} = p(\varphi, X), X = \frac{1}{2}\left(D_\mu\varphi D^\mu\varphi\right)$$

$$\mathcal{L}_{\text{tach}} = -V(\varphi)\sqrt{\left(1-\alpha\partial^\mu\varphi\partial_\mu\varphi\right)}$$

$$\mathcal{L} = \mathcal{L}_{\text{tach}} + \frac{i}{2}\left[\bar{\Upsilon}\gamma^\mu\nabla_\mu\Upsilon - \bar{\Upsilon}\overleftarrow{\nabla}_\mu\gamma^\mu\Upsilon\right] - F(\varphi)\bar{\Upsilon}\Upsilon$$

$$i\gamma^\mu\nabla_\mu\Upsilon - (M-\beta\varphi)\Upsilon = 0$$
$$\alpha\nabla_\mu\partial^\mu\varphi + \alpha^2\frac{\partial^\mu\varphi\left(\nabla_\mu\partial_\sigma\varphi\right)\partial^\sigma\varphi}{1-\alpha\partial_\mu\varphi\partial^\mu\varphi} + \frac{d\ln V(\varphi)}{d\varphi} = \frac{\beta\bar{\Upsilon}\Upsilon}{V(\varphi)}\sqrt{1-\alpha\partial^\mu\varphi\partial_\mu\varphi}$$

$$\ddot{\varphi} = -(1-\alpha\dot{\varphi}^2)\left[\frac{1}{\alpha}\frac{d\ln V(\varphi)}{d\varphi} + 3H\dot{\varphi} - \frac{\beta\bar{\Upsilon}\Upsilon}{\alpha V(\varphi)}\sqrt{1-\alpha\dot{\varphi}^2}\right]$$

$$\frac{d(a^3\bar{\Upsilon}\Upsilon)}{dt} = 0, \Rightarrow \bar{\Upsilon}\Upsilon = \bar{\Upsilon}_0\Upsilon_0 a^{-3}$$

$$\rho_\varphi = \frac{V(\varphi)}{\sqrt{1-\alpha\dot{\varphi}^2}}, \qquad p_\varphi = -V(\varphi)\sqrt{1-\alpha\dot{\varphi}^2}$$
$$\rho_\Upsilon = (M-\beta\varphi)\bar{\Upsilon}\Upsilon, \quad p_\Upsilon = 0$$

$$\dot{\rho}_\varphi + 3H\rho_\varphi\left(\omega_\varphi + 1\right) = \beta\dot{\varphi}\bar{\Upsilon}_0\Upsilon_0 a^{-3}$$
$$\dot{\rho_\Upsilon} + 3H\rho_\Upsilon = -\beta\dot{\varphi}\bar{\Upsilon}_0\Upsilon_0 a^{-3}$$

$$H^2 = \frac{8\pi G}{3}\left[\rho_r + \rho_b + (M-\beta\varphi)\bar{\Upsilon}_0\Upsilon_0 a^{-3} + \frac{V(\varphi)}{\sqrt{1-\alpha\dot{\varphi}^2}}\right]$$

$$\rho_d = \frac{3\wp}{8\pi G L^2}$$

$$\dot{r} = 3Hr\left[\omega_d + \frac{1+r}{r}\frac{\xi_1}{3H}\right]$$

$$\omega_d = -\left(1+\frac{1}{r}\right)\left[\frac{\xi_1}{3H}+\frac{\dot{\wp}}{3H_{\wp}}\right]$$

$$S=\int d^4x\sqrt{-g}\left(-\frac{R}{4}+\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right.$$
$$\left.+\frac{i}{2}\left[\bar{\Upsilon}\gamma^\mu\nabla_\mu\Upsilon-\bar{\Upsilon}\overleftarrow{\nabla}_\mu\gamma^\mu\Upsilon\right]-F(\varphi)\bar{\Upsilon}\Upsilon\right)$$

$$\ddot{\varphi}+3H\dot{\varphi}+V'=-F'\bar{\Upsilon}\Upsilon$$
$$H^2=\frac{1}{3M_p^2}\left\{\frac{\dot{\varphi}^2}{2}+V(\varphi)+F(\varphi)\bar{\Upsilon}\Upsilon\right\}$$

$$\dot{H}=-\frac{1}{2M_p^2}\{\dot{\varphi}^2+F(\varphi)\bar{\Upsilon}\Upsilon\}$$

$$-W_\varphi\dot{\varphi}2M_p^2=\dot{\varphi}^2+F(\varphi)\frac{\bar{\Upsilon}_0\Upsilon_0}{a^3}$$

$$\dot{\varphi}^{n-1}+\frac{\left[\dot{\varphi}+2M_p^2W_\varphi\right]}{F(\varphi)\bar{\Upsilon}_0\Upsilon_0\sigma J(\varphi)}=0$$

$$\ddot{\varphi}+3H_0\dot{\varphi}+V'=\frac{\beta\bar{\Upsilon}_0\Upsilon_0}{a^3}$$

$$\varphi(t)=K_1+K_2e^{-3H_0t}+K_3e^{-\frac{3}{2}H_0t}$$

$$\varphi(t)=Y_1+Y_2\left[\frac{(\ln\ t)^2}{2}+Y_3\ln\ t\right]$$

$$F(\varphi)=-C_1\frac{\mathrm{e}^{\left(3\varphi^2/8M_p^2\right)}}{\varphi^4},V_3(\varphi)=\frac{3\mu^8}{4M_p^2\varphi^2}$$
$$\varphi(t)=(6\mu^4t)^{1/3},a(t)=\left(\frac{\bar{\Upsilon}_0\Upsilon_0C_1}{2\mu^8}\right)^{1/3}e^{\left(6\mu^4t\right)^{2/3}/8M_p^2}$$

$$\rho_d(a)=\frac{\mu^8}{\left(32M_p^4\right)}\frac{1}{\ln^2(\gamma a)}(1+3\ln(\gamma a))$$
$$p_d(a)=\frac{\mu^8}{\left(32M_p^4\right)}\frac{1}{(\ln(\gamma a))^2}(1-3\ln(\gamma a))$$
$$w_d(a)=\frac{1-3\ln(\gamma a)}{1+3\ln(\gamma a)}$$

$$H^2=\frac{8\pi G}{3}[X(f+2f_1)+\rho_c+\rho_r]$$
$$\dot{H}=-4\pi G\left[2X(f+f_1)+\rho_c+\frac{4}{3}\rho_r\right]$$
$$\ddot{\varphi}=-3AH(f+f_1)\dot{\varphi}-\lambda X\big(1-A(f+2f_1)\big)-AQ\rho_c$$

$$Q = -\frac{1}{\rho_c\sqrt{-g}}\frac{\partial\mathcal{L}_m}{\partial\varphi}$$

$$x = \frac{\sqrt{4\pi G}\dot{\varphi}}{3H}, y = \frac{\sqrt{8\pi G e^{-\lambda\varphi/2}}}{3H}, z = \frac{\sqrt{8\pi G\rho_r}}{3H},$$
$$\Omega_c = \frac{8\pi G\rho_c}{9H^2} = 1 - \Omega_\varphi - z^2, \Omega_\varphi = x^2(f + 2f_1).$$

$$ds^2 = a^2(\tau)\left[-(1+2\psi)d\tau^2 + 2\partial_i B d\tau dx^i + (1+2\phi)\delta_{ij}dx^i dx^j + D_{ij}E dx^i dx^j\right]$$

$$\delta\nabla_\mu T^{\mu 0}_{(\lambda)} = \frac{1}{a^2}\{-2[\rho'_\lambda + 3\mathcal{H}(p_\lambda + \rho_\lambda)]\psi + \delta\rho'_\lambda + (p_\lambda + \rho_\lambda)\theta_\lambda$$
$$+3\mathcal{H}(\delta p_\lambda + \delta\rho_\lambda) + 3(p_\lambda + \rho_\lambda)\phi'\} = \delta Q^0_\lambda$$
$$\partial_i\delta\nabla_\mu T^{\mu i}_{(\lambda)} = \frac{1}{a^2}\{[p'_\lambda + \mathcal{H}(p_\lambda + \rho_\lambda)]\nabla^2 B + [(p'_\lambda + \rho'_\lambda) + 4\mathcal{H}(p_\lambda + \rho_\lambda)]\theta_\lambda$$
$$+(p_\lambda + \rho_\lambda)\nabla^2 B' + \nabla^2\delta p_\lambda + (p_\lambda + \rho_\lambda)\theta'_\lambda + (p_\lambda + \rho_\lambda)\nabla^2\psi\}$$
$$= \partial_i\delta Q^i_{(\lambda)}$$

$$-4\pi G a^2\delta\rho = \nabla^2\phi + 3\mathcal{H}(\mathcal{H}\psi - \phi') + \mathcal{H}\nabla^2 B - \frac{1}{6}[\nabla^2]^2 E$$
$$-4\pi G a^2(\rho + p)\theta = \mathcal{H}\nabla^2\psi - \nabla^2\phi' + 2\mathcal{H}^2\nabla^2 B - \frac{a''}{a}\nabla^2 B + \frac{1}{6}[\nabla^2]^2 E'$$
$$8\pi G a^2\Pi^i_j = -\partial^i\partial_j\psi - \partial^i\partial_j\phi + \frac{1}{2}\partial^i\partial_j E'' + \mathcal{H}\partial^i\partial_j E' + \frac{1}{6}\partial^i\partial_j\nabla^2 E$$
$$-2\mathcal{H}\partial^i\partial_j B - \partial^i\partial_j B$$

$$\tilde{x}^\mu = x^\mu + \delta x^\mu, \delta x^0 = \xi^0(x^\mu), \delta x^i = \partial^i\beta(x^\mu) + v^i_*(x^\mu)$$

$$\tilde{\psi} = \psi - \xi^{0\prime} - \frac{a'}{a}\xi^0, \qquad \tilde{B} = B + \xi^0 - \beta',$$
$$\tilde{\phi} = \phi - \frac{1}{3}\nabla^2\beta - \frac{a'}{a}\xi^0, \qquad \tilde{E} = E - 2\beta$$
$$\tilde{v} = v + \beta', \qquad \tilde{\theta} = \theta + \nabla^2\beta'$$

$$\tilde{\delta Q^0} = \delta Q^0 - Q^{0\prime}\xi^0 + Q^0\xi^{0\prime}, \tilde{\delta Q_p} = \delta Q_p + Q^0\beta'$$

$$\delta Q^i = \partial^i\delta Q_p + \delta Q^i_*$$

$$\mathcal{L}_{\delta x}Q^\nu = \delta x^\sigma Q^\nu_{,\sigma} - Q^\sigma\delta x^\nu_{,\sigma}, \delta\tilde{Q}^\nu = \delta Q^\nu - \mathcal{L}_{\delta x}Q^\nu$$

$$\tilde{\psi}Y^{(s)} = \left(\psi - \xi^{0\prime} - \frac{a'}{a}\xi^0\right)Y^{(s)} \qquad \tilde{B}Y^{(s)}_i = (B - k\xi^0 - \beta')Y^{(s)}_i$$
$$\tilde{\phi}Y^{(s)} = \left(\phi - \frac{1}{3}k\beta - \frac{a'}{a}\xi^0\right)Y^{(s)} \qquad \tilde{E}Y^{(s)}_{ij} = (E + 2k\beta)Y^{(s)}_{ij}$$
$$\tilde{\theta}Y^{(s)} = (\theta + k\beta')Y^{(s)}$$

$$\delta'_\lambda + 3\mathcal{H}\left(\frac{\delta p_\lambda}{\delta\rho_\lambda} - \omega_\lambda\right)\delta_\lambda = -(1+\omega_\lambda)kv_\lambda - 3(1+\omega_\lambda)\phi'$$

$$+(2\psi - \delta_\lambda)\frac{a^2 Q^0_\lambda}{\rho_\lambda} + \frac{a^2\delta Q^0_\lambda}{\rho_\lambda}$$

$$(v_\lambda + B)' + \mathcal{H}(1-3\omega_\lambda)(v_\lambda + B) = \frac{k}{1+\omega_\lambda}\frac{\delta p_\lambda}{\delta\rho_\lambda}\delta_\lambda - \frac{\omega'_\lambda}{1+\omega_\lambda}(v_\lambda + B)$$

$$+k\psi - \frac{a^2 Q^0_\lambda}{\rho_\lambda}v_\lambda - \frac{\omega_\lambda a^2 Q^0_\lambda}{(1+\omega_\lambda)\rho_\lambda}B + \frac{a^2\delta Q_{p\lambda}}{(1+\omega_\lambda)\rho_\lambda}$$

$$\Psi = \psi - \frac{1}{k}\mathcal{H}\left(B + \frac{E'}{2k}\right) - \frac{1}{k}\left(B' + \frac{E''}{2k}\right), \Phi = \phi + \frac{1}{6}E - \frac{1}{k}\mathcal{H}\left(B + \frac{E'}{2k}\right),$$

$$\delta Q^{0I}_\lambda = \delta Q^0_\lambda - \frac{Q^{0'}_\lambda}{\mathcal{H}}\left(\phi + \frac{E}{6}\right) + Q^0_\lambda\left[\frac{1}{\mathcal{H}}\left(\phi + \frac{E}{6}\right)\right]', V_\lambda = v_\lambda - \frac{E'}{2k},$$

$$\delta Q^I_{p\lambda} = \delta Q_{p\lambda} - Q^0_\lambda\frac{E'}{2k}, D_\lambda = \delta_\lambda - \frac{\rho'_\lambda}{\rho_\lambda\mathcal{H}}\left(\phi + \frac{E}{6}\right),$$

$$D'_c + \left\{\left(\frac{a^2 Q^0_c}{\rho_c\mathcal{H}}\right)' + \frac{\rho'_c}{\rho_c\mathcal{H}}\frac{a^2 Q^0_c}{\rho_c}\right\}\Phi + \frac{a^2 Q^0_c}{\rho_c}D_c + \frac{a^2 Q^0_c}{\rho_c\mathcal{H}}\Phi' = -kV_c$$

$$+2\Psi\frac{a^2 Q^0_c}{\rho_c} + \frac{a^2\delta Q^{0I}_c}{\rho_c} + \frac{a^2 Q^{0'}_c}{\rho_c\mathcal{H}}\Phi - \frac{a^2 Q^0_c}{\rho_c}\left(\frac{\Phi}{\mathcal{H}}\right)',$$

$$V'_c + \mathcal{H}V_c = k\Psi - \frac{a^2 Q^0_c}{\rho_c}V_c + \frac{a^2\delta Q^I_{pc}}{\rho_c},$$

$$D'_d + \left\{\left(\frac{a^2 Q^0_d}{\rho_d\mathcal{H}}\right)' - 3\omega'_d + 3(C_e^2 - \omega_d)\frac{\rho'_d}{\rho_d} + \frac{\rho'_d}{\rho_d\mathcal{H}}\frac{a^2 Q^0_d}{\rho_d}\right\}\Phi$$

$$+\left\{3\mathcal{H}(C_e^2 - \omega_d) + \frac{a^2 Q^0_d}{\rho_d}\right\}D_d + \frac{a^2 Q^0_d}{\rho_d\mathcal{H}}\Phi'$$

$$= -(1+\omega_d)kV_d + 3\mathcal{H}(C_e^2 - C_a^2)\frac{\rho'_d}{\rho_d}\frac{V_d}{k} + 2\Psi\frac{a^2 Q^0_d}{\rho_d} + \frac{a^2\delta Q^{0I}_d}{\rho_d}$$

$$+\frac{a^2 Q^{0'}_d}{\rho_d\mathcal{H}}\Phi - \frac{a^2 Q^0_d}{\rho_d}\left(\frac{\Phi}{\mathcal{H}}\right)'$$

$$V'_d + \mathcal{H}(1-3\omega_d)V_d = \frac{kC_e^2}{1+\omega_d}D_d + \frac{kC_e^2}{1+\omega_d}\frac{\rho'_d}{\rho_d\mathcal{H}}\Phi - \frac{a^2 Q^0_d}{\rho_d}V_d$$

$$-(C_e^2 - C_a^2)\frac{V_d}{1+\omega_d}\frac{\rho'_d}{\rho_d} - \frac{\omega'_d}{1+\omega_d}V_d + k\Psi + \frac{a^2\delta Q^I_{pd}}{(1+\omega_d)\rho_d}$$

$$\frac{\delta p_d}{\rho_d} = C_e^2\delta_d - (C_e^2 - C_a^2)\frac{\rho'_d}{\rho_d}\frac{v_d + B}{k}$$

$$U_\lambda = (1+\omega_d)V_\lambda$$

$$D_c' + \left\{\left(\frac{a^2 Q_c^0}{\rho_c \mathcal{H}}\right)' + \frac{\rho_c'}{\rho_c \mathcal{H}}\frac{a^2 Q_c^0}{\rho_c}\right\}\Phi + \frac{a^2 Q_c^0}{\rho_c}D_c + \frac{a^2 Q_c^0}{\rho_c \mathcal{H}}\Phi' =$$
$$-kU_c + 2\Psi\frac{a^2 Q_c^0}{\rho_c} + \frac{a^2 \delta Q_c^{0I}}{\rho_c} + \frac{a^2 Q_c^{0'}}{\rho_c \mathcal{H}}\Phi - \frac{a^2 Q_c^0}{\rho_c}\left(\frac{\Phi}{\mathcal{H}}\right)'$$
$$U_c' + \mathcal{H}U_c = k\Psi - \frac{a^2 Q_c^0}{\rho_c}U_c + \frac{a^2 \delta Q_{pc}^I}{\rho_c}$$

$$D_d' + \left\{\left(\frac{a^2 Q_d^0}{\rho_d \mathcal{H}}\right)' - 3\omega_d' + 3(C_e^2 - \omega_d)\frac{\rho_d'}{\rho_d} + \frac{\rho_d'}{\rho_d \mathcal{H}}\frac{a^2 Q_d^0}{\rho_d}\right\}\Phi$$
$$+\left\{3\mathcal{H}(C_e^2 - \omega_d) + \frac{a^2 Q_d^0}{\rho_d}\right\}D_d + \frac{a^2 Q_d^0}{\rho_d \mathcal{H}}\Phi' = 3\mathcal{H}(C_e^2 - C_a^2)\frac{\rho_d'}{\rho_d}\frac{U_d}{(1+\omega_d)k}$$
$$-kU_d + 2\Psi\frac{a^2 Q_d^0}{\rho_d} + \frac{a^2 \delta Q_d^{0I}}{\rho_d} + \frac{a^2 Q_d'}{\rho_d \mathcal{H}}\Phi - \frac{a^2 Q_d^0}{\rho_d}\left(\frac{\Phi}{\mathcal{H}}\right)'$$
$$U_d' + \mathcal{H}(1 - 3\omega_d)U_d = kC_e^2 D_d + kC_e^2\frac{\rho_d'}{\rho_d \mathcal{H}}\Phi - (C_e^2 - C_a^2)\frac{U_d}{1+\omega_d}\frac{\rho_d'}{\rho_d}$$
$$+(1+\omega_d)k\Psi - \frac{a^2 Q_d^0}{\rho_d}U_d + \frac{a^2 \delta Q_{pd}^I}{\rho_d}$$

$$\Phi = \frac{4\pi G a^2 \sum \rho_i\{D^i + 3\mathcal{H}U^i/k\}}{k^2 - 4\pi G a^2 \sum \rho_i'/\mathcal{H}}.$$

$$Q_{(\lambda)}^{\mu} = Q_{(\lambda\eta)}U_{(\eta)}^{\mu} + F_{(\lambda\eta)}^{\mu}$$

$$Q_{(\lambda)}^{\nu} = \left[\frac{Q_{(\lambda)}}{a}, 0{,}0{,}0\right]^T$$

$$Q_{(\lambda)}{}^{\mu} Q_{(\lambda)\mu} = g_{00}\left(Q_{(\lambda)}{}^0\right)^2 = -Q_{(\lambda)}{}^2$$

$$\delta Q_{(\lambda)}^0 = -\frac{\psi}{a}Q_{(\lambda)} + \frac{1}{a}\delta Q_{(\lambda)}$$

$$\delta Q_{p\lambda} = \delta Q_{p\lambda}^I\big|_t + Q_{(\lambda)}^0 v_t$$

$$Q_c = -Q_d = 3H(\xi_1\rho_c + \xi_2\rho_d)$$

$$\delta Q_c = -\delta Q_d = 3H(\xi_1\delta\rho_c + \xi_2\delta\rho_d)$$
$$\delta Q_c^0 = -\delta Q_d^0 = -3H(\xi_1\rho_c + \xi_2\rho_d)\frac{\psi}{a} + 3H(\xi_1\delta\rho_c + \xi_2\delta\rho_d)\frac{1}{a}$$

$$\frac{a^2 \delta Q_c^{0I}}{\rho_c} = -3\mathcal{H}(\xi_1 + \xi_2/r)\Psi + 3\mathcal{H}\{\xi_1 D_c + \xi_2 D_d/r\} + 3\left(\xi_1\frac{\rho_c'}{\rho_c} + \frac{\xi_2}{r}\frac{\rho_d'}{\rho_d}\right)\Phi$$
$$-\frac{a^2}{\rho_c}\frac{Q_c^{0'}}{\mathcal{H}}\Phi + \frac{a^2 Q_c^0}{\rho_c}\left[\frac{\Phi}{\mathcal{H}}\right]',$$
$$\frac{a^2 \delta Q_d^{0I}}{\rho_d} = 3\mathcal{H}(\xi_1 r + \xi_2)\Psi - 3\mathcal{H}\{\xi_1 D_c r + \xi_2 D_d\} - 3\left(\xi_1 r\frac{\rho_c'}{\rho_c} + \frac{\rho_d'}{\rho_d}\xi_2\right)\Phi$$

$$-\frac{a^2 Q_d^{0'}}{\rho_d}\Phi+\frac{a^2 Q_d^0}{\rho_d}\left[\frac{\Phi}{\mathcal{H}}\right]'$$

$$\begin{aligned}
D_c' &= -kU_c + 3\mathcal{H}\Psi(\xi_1+\xi_2/r) - 3(\xi_1+\xi_2/r)\Phi' + 3\mathcal{H}\xi_2(D_d-D_c)/r \\
U_c' &= -\mathcal{H}U_c + k\Psi - 3\mathcal{H}(\xi_1+\xi_2/r)U_c \\
D_d' &= -3\mathcal{H}(C_e^2-\omega_d)D_d + \{3\omega_d' - 9\mathcal{H}(\omega_d - C_e^2)(\xi_1 r+\xi_2+1+\omega_d)\}\Phi \\
&\quad -9\mathcal{H}^2(C_e^2-C_a^2)\frac{U_d}{k} + 3(\xi_1 r+\xi_2)\Phi' - 3\Psi\mathcal{H}(\xi_1 r+\xi_2) \\
&\quad -9\mathcal{H}^2(C_e^2-C_a^2)(\xi_1 r+\xi_2)\frac{U_d}{(1+\omega_d)k} - kU_d + 3\mathcal{H}\xi_1 r(D_d-D_c) \\
U_d' &= -\mathcal{H}(1-3\omega_d)U_d - 3kC_e^2(\xi_1 r+\xi_2+1+\omega_d)\Phi \\
&\quad +3\mathcal{H}(C_e^2-C_a^2)(\xi_1 r+\xi_2)\frac{U_d}{(1+\omega_d)} + 3(C_e^2-C_a^2)\mathcal{H}U_d \\
&\quad +kC_e^2 D_d + (1+\omega_d)k\Psi + 3\mathcal{H}(\xi_1 r+\xi_2)U_d
\end{aligned}$$

$$\frac{D_c}{1-\xi_1-\xi_2/r} = \frac{D_d}{1+\omega_d+\xi_1 r+\xi_2}$$

$$D_d' \approx (-1+\omega_d+\xi_1 r)3\mathcal{H}D_d - 9\mathcal{H}^2(1-\omega_d)\left(1+\frac{\xi_1 r+\xi_2}{1+\omega_d}\right)\frac{U_d}{k}$$

$$U_d' \approx 2\left[1+\frac{3}{1+\omega_d}(\xi_1 r+\xi_2)\right]\mathcal{H}U_d + kD_d$$

$$D_d' \approx -3\mathcal{H}D_d - 9\mathcal{H}^2\frac{1-\omega_d}{1+\omega_d}\frac{U_d}{k}, U_d' \approx 2\frac{1-2\omega_d}{1+\omega_d}\mathcal{H}U_d + kD_d$$

$$D_d'' \approx \left(2\frac{\mathcal{H}'}{\mathcal{H}} - \frac{1+7\omega_d}{1+\omega_d}\mathcal{H}\right)D_d' + 3(\mathcal{H}'-\mathcal{H}^2)D_d$$

$\mathcal{H} \sim \tau^{-1}, \mathcal{H}' \sim -\tau^{-2}, (\mathcal{H}'/\mathcal{H}) \sim -\tau^{-1}$ $\quad D_d'' \approx -3\frac{1+3\omega_d}{1+\omega_d}\frac{D_d'}{\tau} - \frac{6}{\tau^2}D_d$, $\quad D_d \approx C_1\tau^{r_1} + C_2\tau^{r_2}$, $\quad r_1 = -\frac{1+4\omega_d-\sqrt{-5-4\omega_d+10\omega_d^2}}{1+\omega_d}, r_2 = -\frac{1+4\omega_d+\sqrt{-5-4\omega_d+10\omega_d^2}}{1+\omega_d}$

$$D_d' \approx (-1+\omega_d)3\mathcal{H}D_d - 9\mathcal{H}^2(1-\omega_d)\left(1+\frac{\xi_2}{1+\omega_d}\right)\frac{U_d}{k},$$

$$U_d' \approx 2\left(1+\frac{3\xi_2}{1+\omega_d}\right)\mathcal{H}U_d + kD_d.$$

$$\begin{aligned}
D_d'' &= \left[\left(-1+3\omega_d+\frac{6\xi_2}{1+\omega_d}\right)\mathcal{H} + 2\frac{\mathcal{H}'}{\mathcal{H}}\right]D_d' \\
&\quad +3(1-\omega_d)\left[\mathcal{H}' + \mathcal{H}^2\left(-1+\frac{3\xi_2}{1+\omega_d}\right)\right]D_d
\end{aligned}$$

$$D_d'' = \left(-3+3\omega_d+\frac{6\xi_2}{1+\omega_d}\right)\frac{D_d'}{\tau} + 3(1-\omega_d)\left(-2+\frac{3\xi_2}{1+\omega_d}\right)\frac{D_d}{\tau^2}$$

$$D_d \sim C_1 \tau^{r_1} + C_2 \tau^{r_2}$$

$$r_1 = \frac{1}{2}\frac{\Gamma}{1+\omega_d} + \frac{1}{2}\frac{\sqrt{\Delta}}{1+\omega_d}, r_2 = \frac{1}{2}\frac{\Gamma}{1+\omega_d} - \frac{1}{2}\frac{\sqrt{\Delta}}{1+\omega_d}$$

$$D_d \sim C_1 \tau^{\frac{1}{2}\frac{\Gamma}{1+\omega_d}} \cos \frac{1}{2}\frac{\sqrt{|\Delta|}}{1+\omega_d} \ln \tau + C_2 \tau^{\frac{1}{2}\frac{\Gamma}{1+\omega_d}} \sin \frac{1}{2}\frac{\sqrt{|\Delta|}}{1+\omega_d} \ln \tau$$

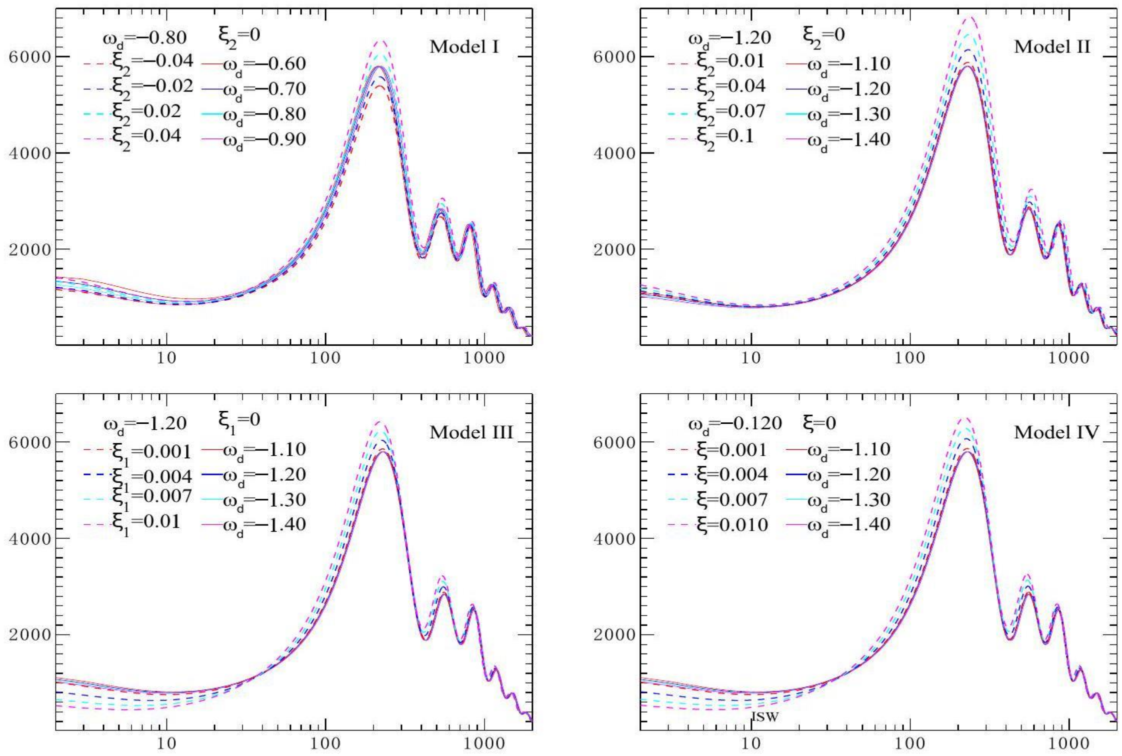
$$C_\ell = 4\pi \int \frac{dk}{k} \mathcal{P}_\chi(k) |\Delta_\ell(k, \tau_0)|^2$$

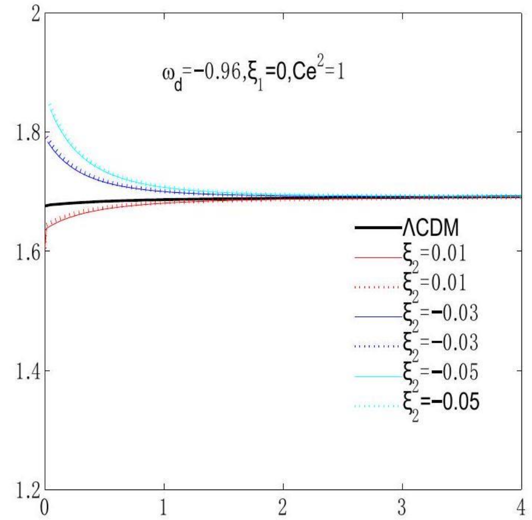
$$\Delta_\ell(k, \tau_0) = \Delta_\ell^{SW}(k) + \Delta_\ell^{ISW}(k),$$

$$\Delta_\ell^{ISW} = \int_{\tau_i}^{\tau_0} d\tau j_\ell(k[\tau_0 - \tau]) e^{\kappa(\tau_0) - \kappa(\tau)} [\Psi' - \Phi']$$

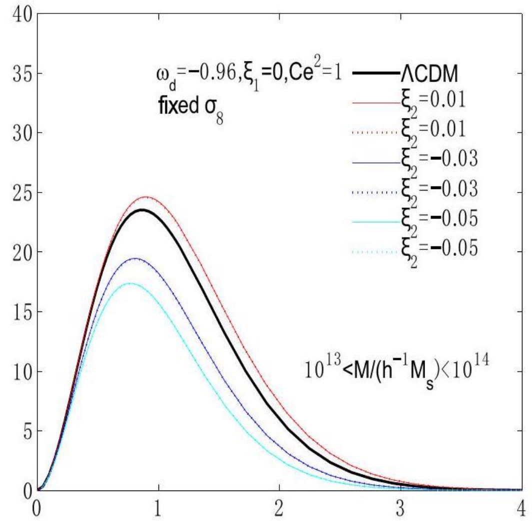
$$\Psi' - \Phi' = 2\mathcal{H}\left[\Phi + 4\pi G a^2 \sum_i U^i \rho^i / (\mathcal{H}k) + \mathcal{T}\right] - \mathcal{T}'$$

$$\Phi' = -\mathcal{H}\Phi - \mathcal{H}\mathcal{T} - 4\pi G a^2 \sum_i U^i \rho^i / k, \mathcal{T} = \frac{8\pi G a^2}{k^2}\{p^\gamma \Pi^\gamma + p^\nu \Pi^\nu\}$$

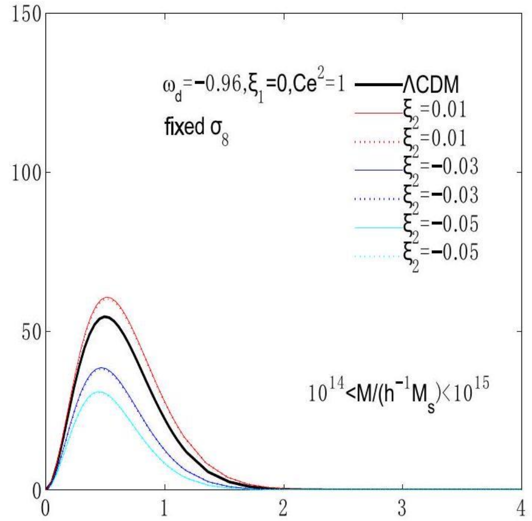



ωd=−0.96,ξ1=0,Ce2=1
ΛCDM
ξ2=0.01
ξ2=0.01
ξ2=−0.03
ξ2=−0.03
ξ2=−0.05
ξ2=−0.05

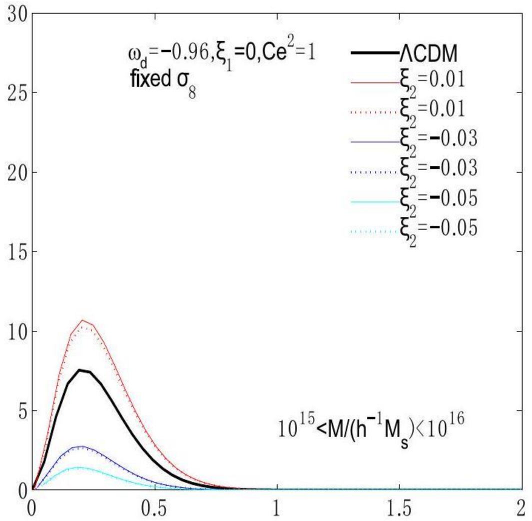
(a)


ωd=−0.96,ξ1=0,Ce2=1
fixed σ8
ΛCDM
ξ2=0.01
ξ2=0.01
ξ2=−0.03
ξ2=−0.03
ξ2=−0.05
ξ2=−0.05
10^13<M/(h^−1Ms)<10^14

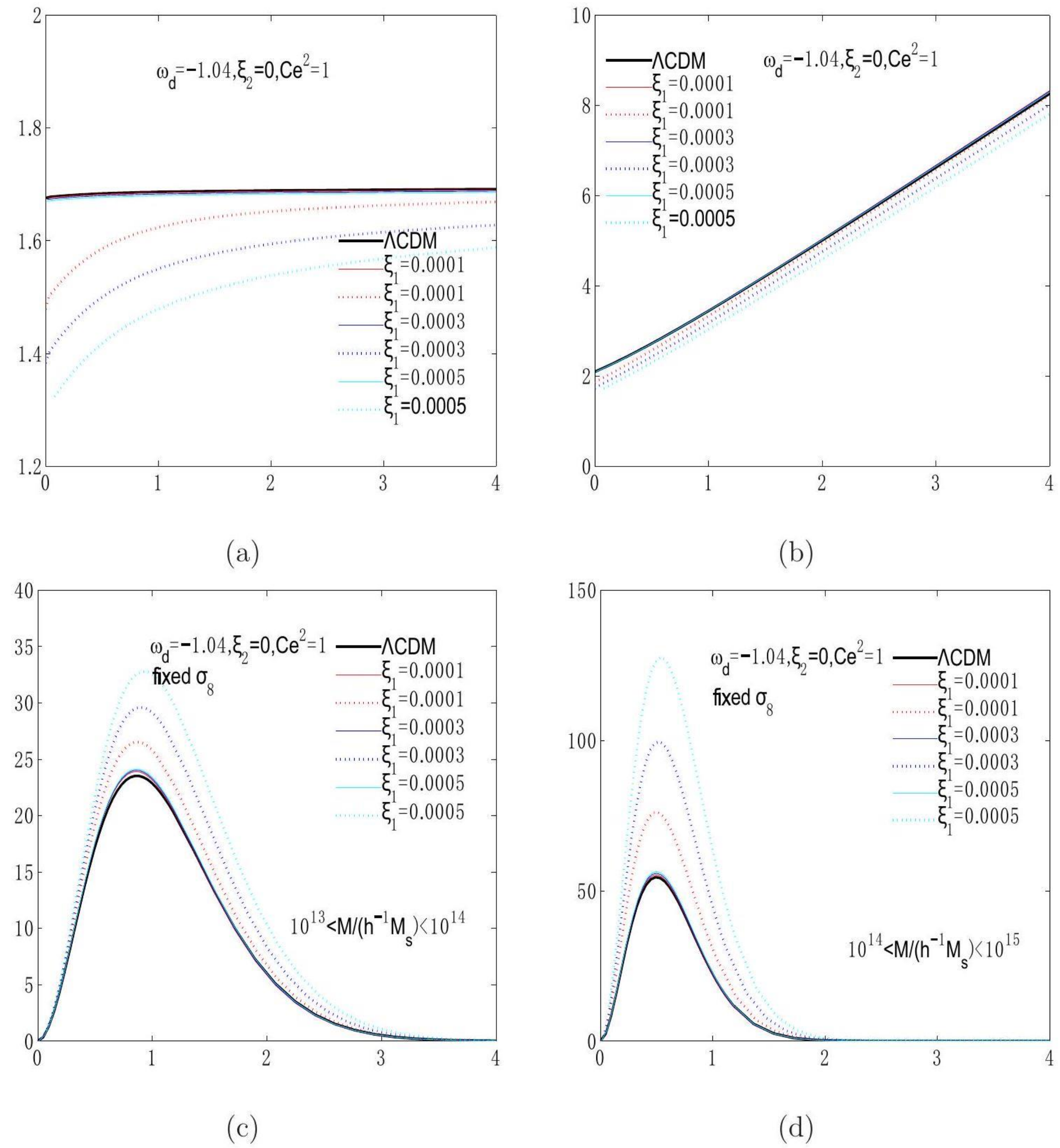
(b)


ωd=−0.96,ξ1=0,Ce2=1
fixed σ8
ΛCDM
ξ2=0.01
ξ2=0.01
ξ2=−0.03
ξ2=−0.03
ξ2=−0.05
ξ2=−0.05
10^14<M/(h^−1Ms)<10^15

(c)


ωd=−0.96,ξ1=0,Ce2=1
fixed σ8
ΛCDM
ξ2=0.01
ξ2=0.01
ξ2=−0.03
ξ2=−0.03
ξ2=−0.05
ξ2=−0.05
10^15<M/(h^−1Ms)<10^16

(d)



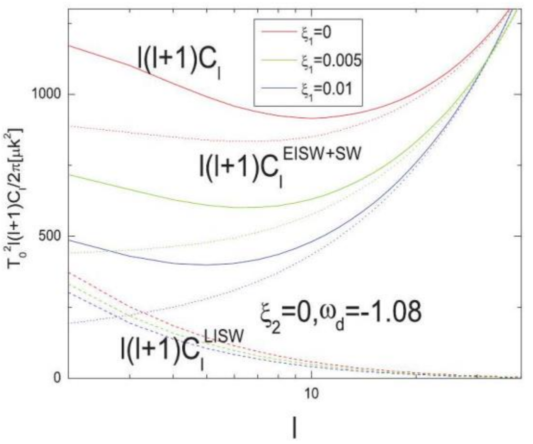
Figuras 23, 23A y 23B. Fluctuaciones espectrales de una partícula oscura en relación a sectores oscuros.

$$C_l^{gg} = 4\pi \int \frac{dk}{k} \mathcal{P}_\chi(k) I_l^g(k) I_l^g(k)$$

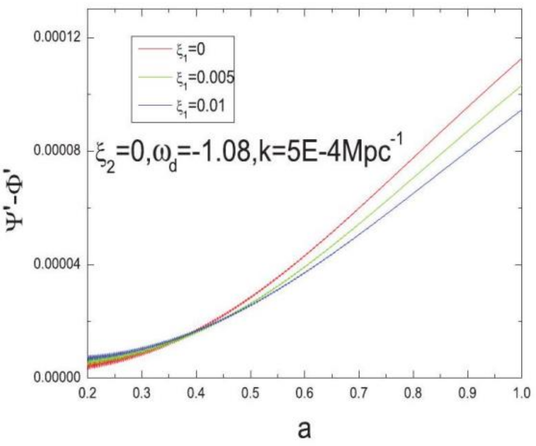
$$C_l^{gI} = 4\pi \int \frac{dk}{k} \mathcal{P}_\chi(k) I_l^g(k) \Delta_l^{ISW}(k),$$

$I_l^g(k) = \int dz b_g(z) n(z)(D_c + D_b) j_l[k\chi(z)]$, $z = 0, \chi(z) = \int_0^z \frac{dz'}{H(z)} = \int_{\tau_i(z)}^{\tau_0} d\tau = \tau_0 - \tau_i(z)$ $n(z) =$

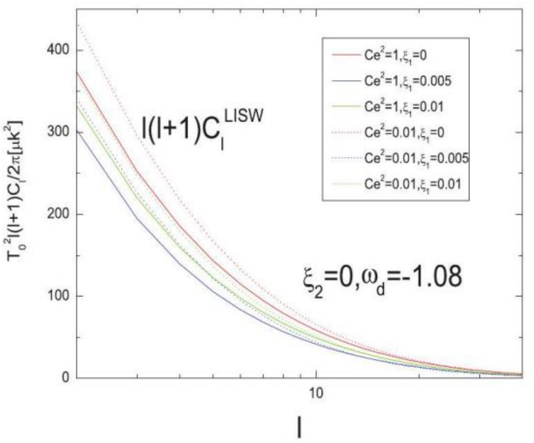
$\frac{3}{2} \frac{z^2}{z_0^3} \exp\left[-\left(\frac{z}{z_0}\right)^{3/2}\right] \int n(z) dz = 1$


ξ_1=0
ξ_1=0.005
ξ_1=0.01
l(l+1)C$_l$
l(l+1)C$_l^{EISW+SW}$
l(l+1)C$_l^{LISW}$
ξ_2=0,ω_d=-1.08
T$_0^2$l(l+1)C$_l$/2π[μk^2]
1000
500
0
10
l

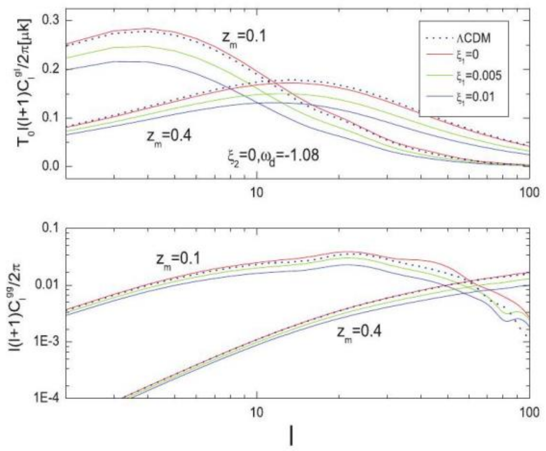
(a)


ξ_1=0
ξ_1=0.005
ξ_1=0.01
ξ_2=0,ω_d=-1.08,k=5E-4Mpc^{-1}
Ψ'-Φ'
0.00012
0.00008
0.00004
0.00000
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
a

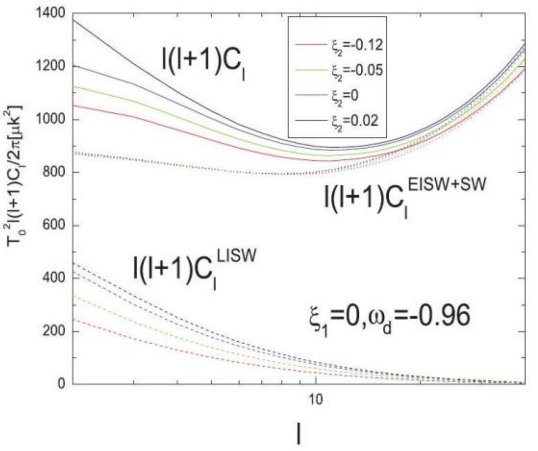
(b)


Ce^2=1,ξ_1=0
Ce^2=1,ξ_1=0.005
Ce^2=1,ξ_1=0.01
Ce^2=0.01,ξ_1=0
Ce^2=0.01,ξ_1=0.005
Ce^2=0.01,ξ_1=0.01
l(l+1)C$_l^{LISW}$
ξ_2=0,ω_d=-1.08
T$_0^2$l(l+1)C$_l$/2π[μk^2]
400
300
200
100
0
10
l

(c)

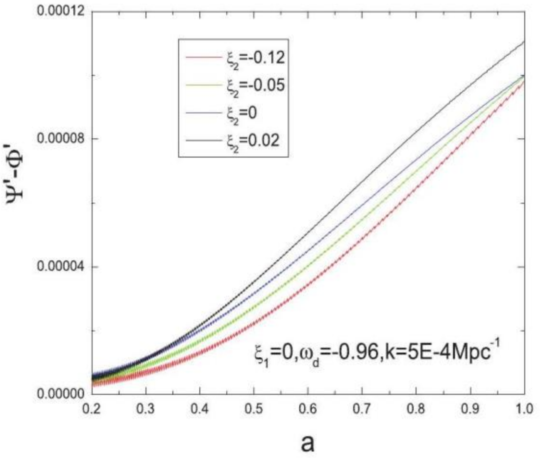

ΛCDM
ξ_1=0
ξ_1=0.005
ξ_1=0.01
z$_m$=0.1
z$_m$=0.4
ξ_2=0,ω_d=-1.08
T$_0$l(l+1)C$_l^{gI}$/2π[μk]
0.3
0.2
0.1
0.0
10
100
z$_m$=0.1
z$_m$=0.4
l(l+1)C$_l^{gg}$/2π
0.1
0.01
1E-3
1E-4
10
100
l

(d)



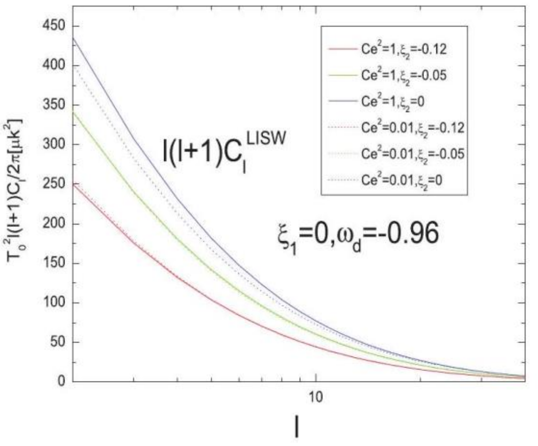
l(l+1)C_l
l(l+1)C_l^EISW+SW
l(l+1)C_l^LISW
ξ_1=0,ω_d=-0.96
ξ_2=-0.12
ξ_2=-0.05
ξ_2=0
ξ_2=0.02
T_0^2 l(l+1)C_l/2π[μk^2]
l

(a)



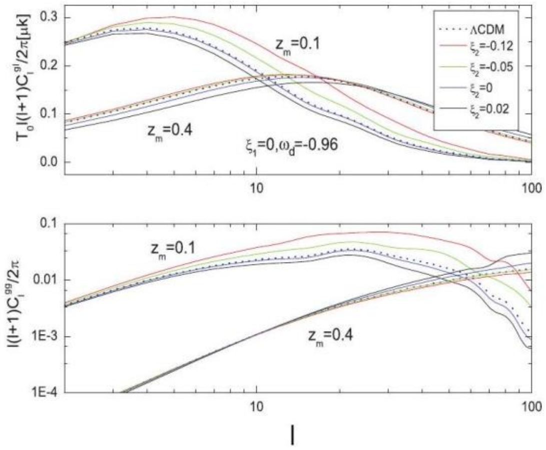
ξ_2=-0.12
ξ_2=-0.05
ξ_2=0
ξ_2=0.02
Ψ'-Φ'
ξ_1=0,ω_d=-0.96,k=5E-4Mpc^-1
a

(b)



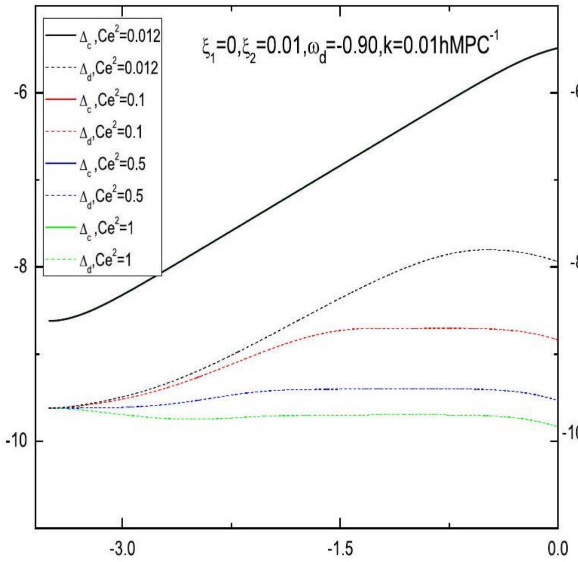
Ce^2=1,ξ_2=-0.12
Ce^2=1,ξ_2=-0.05
Ce^2=1,ξ_2=0
Ce^2=0.01,ξ_2=-0.12
Ce^2=0.01,ξ_2=-0.05
Ce^2=0.01,ξ_2=0
l(l+1)C_l^LISW
ξ_1=0,ω_d=-0.96
T_0^2 l(l+1)C_l/2π[μk^2]
l

(c)

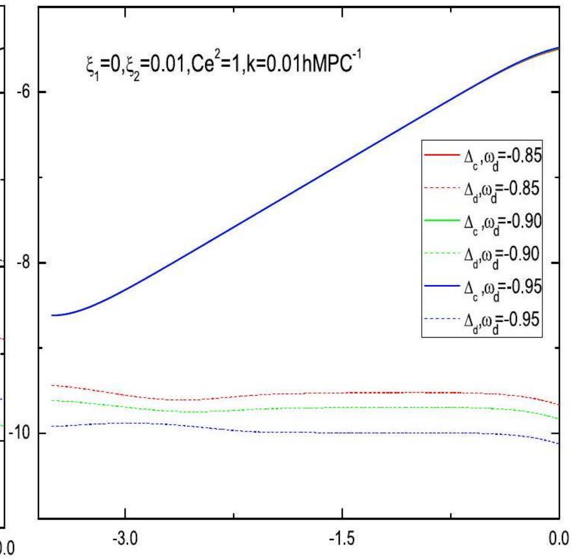


ΛCDM
ξ_2=-0.12
ξ_2=-0.05
ξ_2=0
ξ_2=0.02
z_m=0.1
z_m=0.4
ξ_1=0,ω_d=-0.96
T_0 l(l+1)C_l^gI/2π[μk]
l(l+1)C_l^gg/2π
l

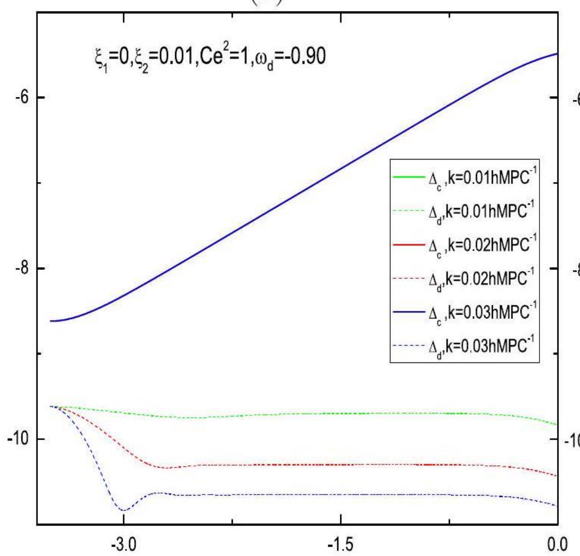
(d)


$\xi_1=0,\xi_2=0.01,\omega_d=-0.90,k=0.01hMPC^{-1}$
$\Delta_c,Ce^2=0.012$
$\Delta_d,Ce^2=0.012$
$\Delta_c,Ce^2=0.1$
$\Delta_d,Ce^2=0.1$
$\Delta_c,Ce^2=0.5$
$\Delta_d,Ce^2=0.5$
$\Delta_c,Ce^2=1$
$\Delta_d,Ce^2=1$
-6
-8
-10
-3.0
-1.5
0.0

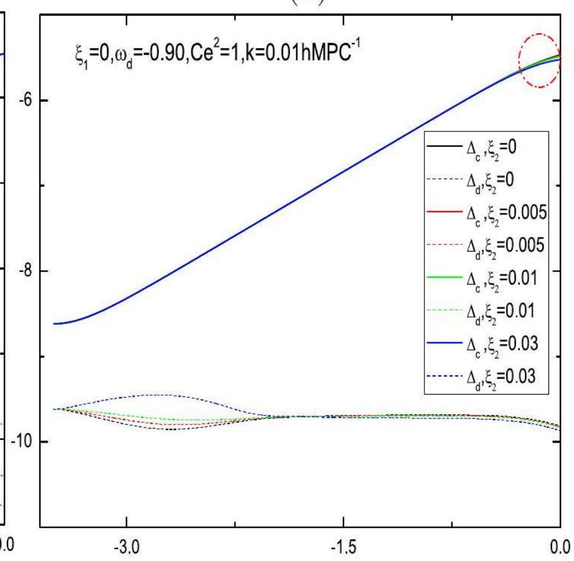
(a)


$\xi_1=0,\xi_2=0.01,Ce^2=1,k=0.01hMPC^{-1}$
$\Delta_c,\omega_d=-0.85$
$\Delta_d,\omega_d=-0.85$
$\Delta_c,\omega_d=-0.90$
$\Delta_d,\omega_d=-0.90$
$\Delta_c,\omega_d=-0.95$
$\Delta_d,\omega_d=-0.95$
-6
-8
-10
-3.0
-1.5
0.0

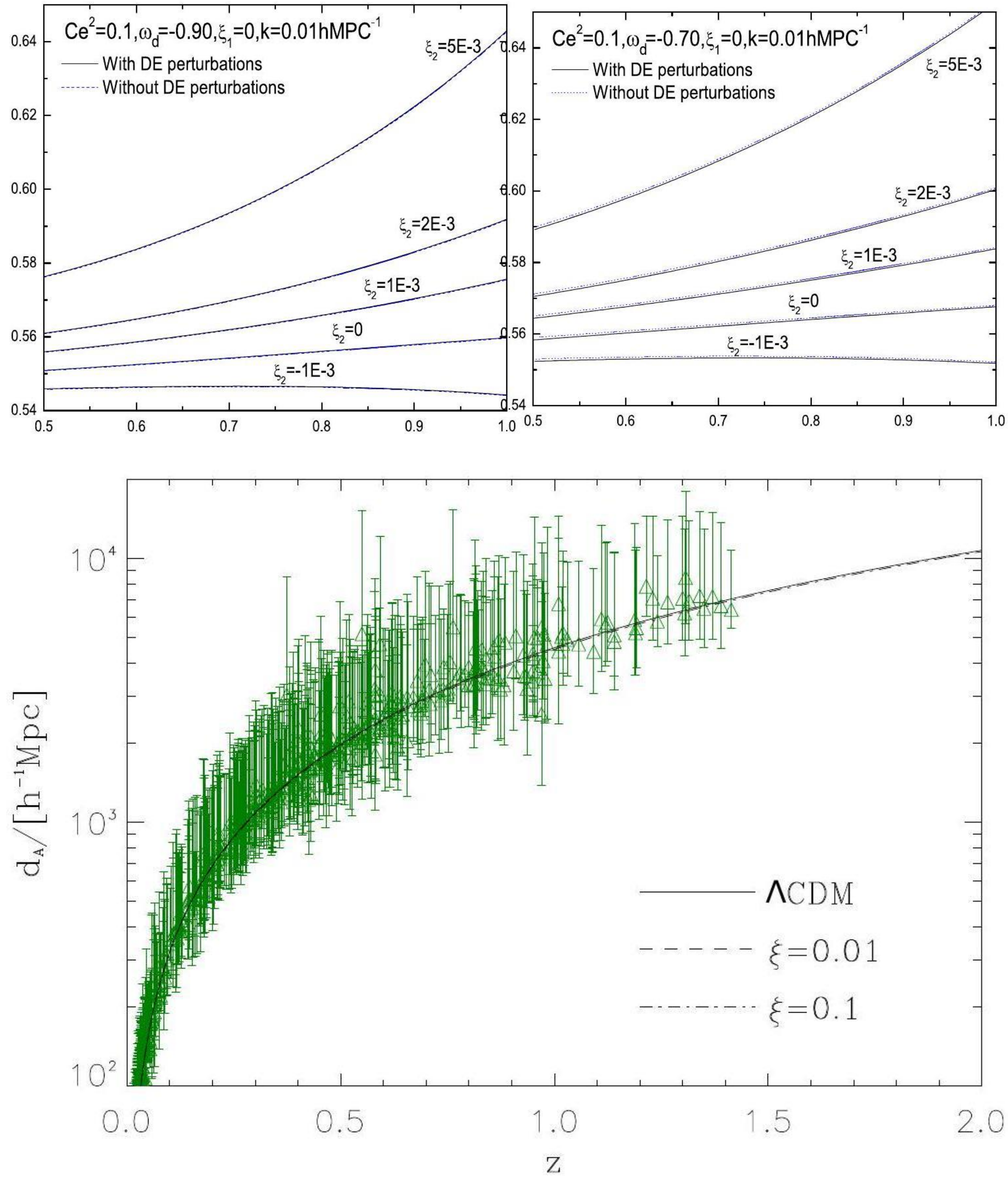
(b)


$\xi_1=0,\xi_2=0.01,Ce^2=1,\omega_d=-0.90$
$\Delta_c,k=0.01hMPC^{-1}$
$\Delta_d,k=0.01hMPC^{-1}$
$\Delta_c,k=0.02hMPC^{-1}$
$\Delta_d,k=0.02hMPC^{-1}$
$\Delta_c,k=0.03hMPC^{-1}$
$\Delta_d,k=0.03hMPC^{-1}$
-6
-8
-10
-3.0
-1.5
0.0

(c)


$\xi_1=0,\omega_d=-0.90,Ce^2=1,k=0.01hMPC^{-1}$
$\Delta_c,\xi_2=0$
$\Delta_d,\xi_2=0$
$\Delta_c,\xi_2=0.005$
$\Delta_d,\xi_2=0.005$
$\Delta_c,\xi_2=0.01$
$\Delta_d,\xi_2=0.01$
$\Delta_c,\xi_2=0.03$
$\Delta_d,\xi_2=0.03$
-6
-8
-10
-3.0
-1.5
0.0

(d)



Figuras 24, 25, 26, 27 y 28. Campos oscuros perturbativos por interacción de una partícula oscura.

$$\delta\rho_\lambda^I = \delta\rho_\lambda - \rho_\lambda' \frac{v_\lambda + B}{k}, \delta p_\lambda^I = \delta p_\lambda - p_\lambda' \frac{v_\lambda + B}{k}$$

$$\Delta_\lambda = \delta_\lambda - \frac{\rho_\lambda'}{\rho_\lambda} \frac{v_\lambda + B}{k}, V_\lambda = v_\lambda - \frac{E'}{2k}$$

$$\Phi = 4\pi G \frac{a^2}{k^2} \sum_\lambda \left(\Delta_\lambda + \frac{a^2 Q_\lambda^0}{\rho_\lambda} \frac{V_\lambda}{k} \right) \rho_\lambda,$$

$$k(\mathcal{H}\Psi - \Phi') = 4\pi G a^2 \sum_\lambda (\rho_\lambda + p_\lambda) V_\lambda, \Psi = -\Phi,$$

$$\Delta_c' + \left[\frac{\rho_c'}{\rho_c}\frac{V_c}{k}\right]' = -kV_c - 3\Phi' + 2\Psi\frac{a^2Q_c^0}{\rho_c} - \Delta_c\frac{a^2Q_c^0}{\rho_c} - \frac{\rho_c'}{\rho_c}\frac{V_c}{k}\frac{a^2Q_c^0}{\rho_c}$$
$$+\frac{a^2\delta Q_c^{0I}}{\rho_c} + \frac{a^2Q_c^{0'}}{\rho_c\mathcal{H}}\Phi - \frac{a^2Q_c^0}{\rho_c}\left[\frac{\Phi}{\mathcal{H}}\right]'$$
$$V_c' = -\mathcal{H}V_c + k\Psi - \frac{a^2Q_c^0}{\rho_c}V_c + \frac{a^2\delta Q_{pc}^I}{\rho_c}.$$

$$\Delta_d' + \left[\frac{\rho_d'}{\rho_d}\frac{V_d}{k}\right]' + 3\mathcal{H}C_e^2\left(\Delta_d + \frac{\rho_d'}{\rho_d}\frac{V_d}{k}\right) - 3\mathcal{H}(C_e^2 - C_a^2)\frac{\rho_d'}{\rho_d}\frac{V_d}{k}$$
$$-3\omega_d\mathcal{H}\left(\Delta_d + \frac{\rho_d'}{\rho_d}\frac{V_d}{k}\right) = -k(1+\omega_d)V_d - 3(1+\omega_d)\Phi' + 2\Psi\frac{a^2Q_d^0}{\rho_d}$$
$$-\left(\Delta_d + \frac{\rho_d'}{\rho_d}\frac{V_d}{k}\right)\frac{a^2Q_d^0}{\rho_d} + \frac{a^2\delta Q_d^{0I}}{\rho_d} + \frac{a^2Q_d^{0'}}{\rho_d\mathcal{H}}\Phi - \frac{a^2Q_d^0}{\rho_d}\left[\frac{\Phi}{\mathcal{H}}\right]'$$
$$V_d' + \mathcal{H}(1-3\omega_d)V_d = \frac{k}{1+\omega_d}\left[C_e^2\left(\Delta_d + \frac{\rho_d'}{\rho_d}\frac{V_d}{k}\right) - (C_e^2 - C_a^2)\frac{\rho_d'}{\rho_d}\frac{V_d}{k}\right]$$
$$-\frac{\omega_d'}{1+\omega_d}V_d + k\Psi - \frac{a^2Q_d^0}{\rho_d}V_d + \frac{a^2\delta Q_{pd}^I}{(1+\omega_d)\rho_d}$$

$$\Delta_c' = -kV_c - \Delta_c\frac{a^2Q_c^0}{\rho_c} + \frac{a^2\delta Q_c^{0I}}{\rho_c}$$
$$V_c' = -\mathcal{H}V_c + k\Psi - \frac{a^2Q_c^0}{\rho_c}V_c + \frac{a^2\delta Q_{pc}^I}{\rho_c};$$
$$\Delta_d' = 3\mathcal{H}(\omega_d - C_e^2)\Delta_d - k(1+\omega_d)V_d - \Delta_d\frac{a^2Q_d^0}{\rho_d} + \frac{a^2\delta Q_d^{0I}}{\rho_d}$$
$$V_d' + \mathcal{H}(1-3\omega_d)V_d = \frac{kC_e^2}{1+\omega_d}\Delta_d + \frac{C_a^2}{1+\omega_d}\frac{\rho_d'}{\rho_d}V_d + k\Psi - \frac{\omega_d'}{1+\omega_d}V_d$$
$$-\frac{a^2Q_d^0}{\rho_d}V_d + \frac{a^2\delta Q_{pd}^I}{(1+\omega_d)\rho_d}$$

$$\Delta_c'' = -\left(\mathcal{H} + \frac{2a^2Q_c^0}{\rho_c}\right)\Delta_c' + \left(-\Delta_c\frac{a^2Q_c^0}{\rho_c} + \frac{a^2\delta Q_c^{0I}}{\rho_c}\right)\left(\mathcal{H} + \frac{a^2Q_c^0}{\rho_c}\right)$$
$$-\Delta_c\left(\frac{a^2Q_c^0}{\rho_c}\right)' + \left(\frac{a^2\delta Q_c^{0I}}{\rho_c}\right)' - \frac{a^2k\delta Q_{pc}^I}{\rho_c} - k^2\Psi.$$

$$\Delta_d'' = -3\mathcal{H}'C_e^2\Delta_d - \left(\frac{a^2Q_d^0}{\rho_d}\right)'\Delta_d + \left\{\mathcal{H}(1-3\omega_d) - \frac{\omega_d}{1+\omega_d}\frac{\rho_d'}{\rho_d} + \frac{a^2Q_d^0}{\rho_d}\right\}$$
$$\times\left\{-3\mathcal{H}C_e^2 + 3\omega_d\mathcal{H} - \frac{a^2Q_d^0}{\rho_d}\right\}\Delta_d$$
$$-\left[\mathcal{H} + 3\mathcal{H}C_e^2 - 6\omega_d\mathcal{H} + \frac{2a^2Q_d^0}{\rho_d} - \frac{\omega_d}{1+\omega_d}\frac{\rho_d'}{\rho_d}\right]\Delta_d'$$
$$-k\left(\frac{a^2\delta Q_{pd}^I}{\rho_d}\right) - k^2C_e^2\Delta_d - k^2(1+\omega_d)\Psi + 3(\omega_d'\mathcal{H} + \omega_d\mathcal{H}')\Delta_d$$
$$+\frac{a^2\delta Q_d^{0I}}{\rho_d}\left[\mathcal{H}(1-3\omega_d) - \frac{\omega_d}{1+\omega_d}\frac{\rho_d'}{\rho_d} + \frac{a^2Q_d^0}{\rho_d}\right] + \left(\frac{a^2\delta Q_d^{0I}}{\rho_d}\right)'$$

$$-\frac{k^2}{a^2}\Psi = \frac{3}{2}H^2\{\Omega_c\Delta_c + (1-\Omega_c)\Delta_d\}$$

$$\frac{a^2\delta Q_c^{0I}}{\rho_c} \approx 3\mathcal{H}(\xi_1\Delta_c + \xi_2\Delta_d/r), \frac{a^2\delta Q_d^{0I}}{\rho_d} \approx -3\mathcal{H}(\xi_1\Delta_c r + \xi_2\Delta_d),$$

$$\Delta_c' + \nabla_{\bar{x}}\cdot V_c = 3\mathcal{H}\xi_2(\Delta_d - \Delta_c)/r$$
$$V_c' + \mathcal{H}V_c = -\nabla_{\bar{x}}\Psi - 3\mathcal{H}(\xi_1 + \xi_2/r)V_c$$
$$\Delta_d' + (1+\omega_d)\nabla_{\bar{x}}\cdot V_d = 3\mathcal{H}(\omega_d - C_e^2)\Delta_d + 3\mathcal{H}\xi_1 r(\Delta_d - \Delta_c)$$
$$V_d' + \mathcal{H}V_d = -\nabla_{\bar{x}}\Psi - \frac{C_e^2}{1+\omega_d}\nabla_{\bar{x}}\Delta_d - \frac{\omega_d'}{1+\omega_d}V_d$$
$$+3\mathcal{H}\left\{(\omega_d - C_a^2) + \frac{1+\omega_d - C_a^2}{1+\omega_d}(\xi_1 r + \xi_2)\right\}V_d$$

$$\dot{\sigma}_c + 3H\sigma_c + \nabla_x(\rho_c V_c) = 3H(\xi_1\sigma_c + \xi_2\sigma_d)$$
$$\frac{\partial}{\partial t}(aV_c) = -\nabla_x(a\Psi) - 3H(\xi_1 + \xi_2/r)(aV_c)$$
$$\dot{\sigma}_d + 3H(1+C_e^2)\sigma_d + (1+\omega_d)\nabla_x(\rho_d V_d) = -3H(\xi_1\sigma_c + \xi_2\sigma_d)$$
$$\frac{\partial}{\partial t}(aV_d) = -\nabla_x(a\Psi) - \frac{C_e^2}{1+\omega_d}\nabla_x\cdot(a\Delta_d)$$
$$+3H\left[(\omega_d - c_a^2) + \frac{1+\omega_d - c_a^2}{1+\omega_d}(\xi_1 r + \xi_2)\right](aV_d)$$

$$\nabla^2\psi_\lambda = 4\pi G(1+3\omega_\lambda)\sigma_\lambda$$

$$\psi_c = -4\pi G\int dV'\frac{\sigma_c}{|x-x'|}, \psi_d = -4\pi G\int dV'\frac{(1+3\omega_d)\sigma_d}{|x-x'|}$$

$$\frac{\partial}{\partial t}(aV_c) = -\nabla_x(a\psi_c + a\psi_d) - 3H(\xi_1 + \xi_2/r)(aV_c)$$

$$\frac{\partial}{\partial t}(a^2T_c) - a^2 3H(\xi_1 + \xi_2/r)T_c$$

$$-\int aV_c\nabla_x(a\psi_c + a\psi_d)\rho_c\hat{\varepsilon} = a^2\int \nabla_x(\rho_c V_c)\psi_c\hat{\varepsilon} + a^2\int \nabla_x(\rho_c V_c)\psi_d\hat{\varepsilon}$$

$$-\int aV_c\nabla_x(a\psi_c + a\psi_d)\rho_c\hat{\varepsilon} = -a^2(\dot{U}_{cc} + HU_{cc}) - a^2\int \psi_d\frac{\partial}{\partial t}(\sigma_c\hat{\varepsilon})$$
$$+3a^2H\{\xi_1U_{cd} + \xi_2U_{dc} + 2\xi_1U_{cc} + 2\xi_2U_{dd}\}$$

$$U_{cc} = \tfrac{1}{2}\int \sigma_c\psi_c\hat{\varepsilon}, U_{dc} = \int \sigma_d\psi_c\hat{\varepsilon}, U_{cd} = \int \sigma_c\psi_d\hat{\varepsilon}, U_{dd} = \tfrac{1}{2}\int \sigma_d\psi_d\hat{\varepsilon}$$
$$-\int (aV_c)^2 3H(\xi_1 + \xi_2/r)\rho_c\hat{\varepsilon} = -a^2 6H(\xi_1 + \xi_2/r)T_c$$

$$-\int (aV_c)^2 3H(\xi_1 + \xi_2/r)\rho_c\hat{\varepsilon} = -a^2 6H(\xi_1 + \xi_2/r)T_c$$

$$\dot{T}_c + \dot{U}_{cc} + H(2T_c + U_{cc}) = -\int \psi_d\frac{\partial}{\partial t}(\sigma_c\hat{\varepsilon}) - 3H(\xi_1 + \xi_2/r)T_c$$
$$+3H\{\xi_1U_{cd} + \xi_2U_{dc} + 2\xi_1U_{cc} + 2\xi_2U_{dd}\}$$

$$\dot{T}_c + \dot{U}_{cc} + H(2T_c + U_{cc}) = -3H(\xi_1 + \xi_2/r)T_c + 6H\xi_1 U_{cc}$$

$$\frac{\partial}{\partial t}(a^2 T_d) + 3a^2 H(\omega_d + \xi_1 r + \xi_2)T_d$$

$$-\int aV_d \nabla_x (a\psi_c + a\psi_d)\rho_d \hat{\varepsilon} = -\frac{a^2}{1+\omega_d}(\dot{U}_{dd} + HU_{dd})$$
$$-\frac{a^2}{1+\omega_d}3H\{2(C_e^2 + \xi_2)U_{dd} + 2\xi_1 U_{cc} + \xi_1 U_{cd} + (C_e^2 + \xi_2)U_{dc}\}$$
$$-\frac{a^2}{1+\omega_d}\int \psi_c \frac{\partial}{\partial t}(\sigma_d \hat{\varepsilon})$$

$$3H\left[(w - c_a^2) + \frac{1 + w - C_a^2}{1+\omega_d}(\xi_1 r + \xi_2)\right]\int (aV_d)^2 \rho_d \hat{\varepsilon}$$

$$-\frac{c_e^2}{1+\omega_d}\int aV_d \nabla_x (a\Delta_d)\rho_d \hat{\varepsilon}$$
$$= 6a^2 H\left[(\omega_d - C_a^2) + \frac{1+\omega_d - c_a^2}{1+\omega_d}(\xi_1 r + \xi_2)\right]T_d$$
$$-\frac{c_e^2}{1+\omega_d}a^2 \int V_d \nabla_x (\sigma_d)\hat{\varepsilon}$$

$$(1 + \omega_d)\dot{T}_d + \dot{U}_{dd} + H[2(1+\omega_d)T_d + U_{dd}] = -3H\{2(C_e^2 + \xi_2)U_{dd} + 2\xi_1 U_{cc}$$
$$+\xi_1 U_{cd} + (C_e^2 + \xi_2)U_{dc}\} - \int \psi_c \frac{\partial}{\partial t}(\sigma_d \hat{\varepsilon}) - c_e^2 \int V_d \nabla_x (\sigma_d)\hat{\varepsilon}$$
$$+3H[(1+\omega_d)(\omega_d - 2C_a^2) + (1 + \omega_d - 2C_a^2)(\xi_1 r + \xi_2)]T_d$$

$$\dot{\theta} = -\frac{1}{3}\theta^2 - 4\pi G \sum_\lambda (\rho_\lambda + 3p_\lambda)$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\sum_\lambda (\rho_\lambda + 3p_\lambda)$$

$$\dot{\rho}_c^{cl} + 3\frac{\dot{R}}{R}\rho_c^{cl} = 3H(\xi_1 \rho_c^{cl} + \xi_2 \rho_d),$$

$$\ddot{R} = -\frac{4\pi G}{3}[\rho_c^{cl} + (1 + 3\omega_d)\rho_d]R,$$

$$2a^2\left(1 + \frac{1}{r}\right)\frac{d^2 R}{da^2} - \frac{dR}{da}[1 + (3\omega_d + 1)/r]a = -R[(3\omega_d + 1)/r + \zeta]$$

$$\frac{d\zeta}{da} = \frac{3}{a}(1 - \xi_2/r)\zeta - 3\frac{1}{R}\frac{dR}{da}\zeta + \frac{3}{a}\xi_2/r$$

$$u_{(d)}^a = \gamma\left(u_{(c)}^a + v_d^a\right)$$

$$T_{(c)}^{ab} = \rho_c u_{(c)}^a u_{(c)}^b$$
$$T_{(d)}^{ab} = \rho_d u_{(d)}^a u_{(d)}^b + p_d h_{(d)}^{ab}$$

$$T^{ab}_{(d)} = \rho_d u^a u^b + p_d h^{ab} + 2u^a q^b_{(d)}$$

$$\nabla_a T^{ab}_{(\lambda)} = Q^b_{(\lambda)}$$

$$\dot{\rho}^{cl}_c + 3h\rho^{cl}_c = 3H\left(\xi_1 \rho^{cl}_c + \xi_2 \rho^{cl}_d\right)$$
$$\dot{\rho}^{cl}_d + 3h(1+\omega_d)\rho^{cl}_d = -\vartheta(1+\omega_d)\rho^{cl}_d - 3H\left(\xi_1 \rho^{cl}_c + \xi_2 \rho^{cl}_d\right)$$

$$\dot{q}^a_{(d)} + 4hq^a_{(d)} = 0$$

$$\dot{\vartheta} + h(1-3\omega_d)\vartheta = 3H(\xi_1 \Gamma + \xi_2)\vartheta.$$

$$\vartheta' + \frac{R'}{R}(1-3\omega_d)\vartheta = \frac{3}{a}(\xi_1 \Gamma + \xi_2)\vartheta$$
$$\zeta'_c = \frac{3}{a}\{1 - \xi_2/r\}\zeta_c - 3\frac{R'}{R}\zeta_c + \frac{3}{a}\xi_2\zeta_d/r$$

$$\zeta'_d = \frac{3}{a}(1+\omega_d+\xi_1 r)\zeta_d - 3(1+\omega_d)\frac{R'}{R}\zeta_d - \frac{3}{a}\xi_1\zeta_c r - \vartheta(1+\omega_d)\zeta_d$$

$$2a^2\left(1+\frac{1}{r}\right)R'' - R'[1+(3\omega_d+1)/r]a = -R[(3\omega_d+1)\zeta_d/r + \zeta_c]$$

$$\frac{dn(M,z)}{dM} = \sqrt{\frac{2}{\pi}}\frac{\rho_m}{3M^2}\frac{\delta_c}{\sigma}e^{-\delta_c^2/2\sigma^2}\left[-\frac{R}{\sigma}\frac{d\sigma}{dR}\right]$$

$$\sigma(R,z) = \sigma_8\left(\frac{R}{8h^{-1}\text{Mpc}}\right)^{-\gamma(R)} D(z)$$

$$\gamma(R) = (0.3\Gamma + 0.2)\left[2.92 + \log_{10}\left(\frac{R}{8h^{-1}\text{Mpc}}\right)\right]$$

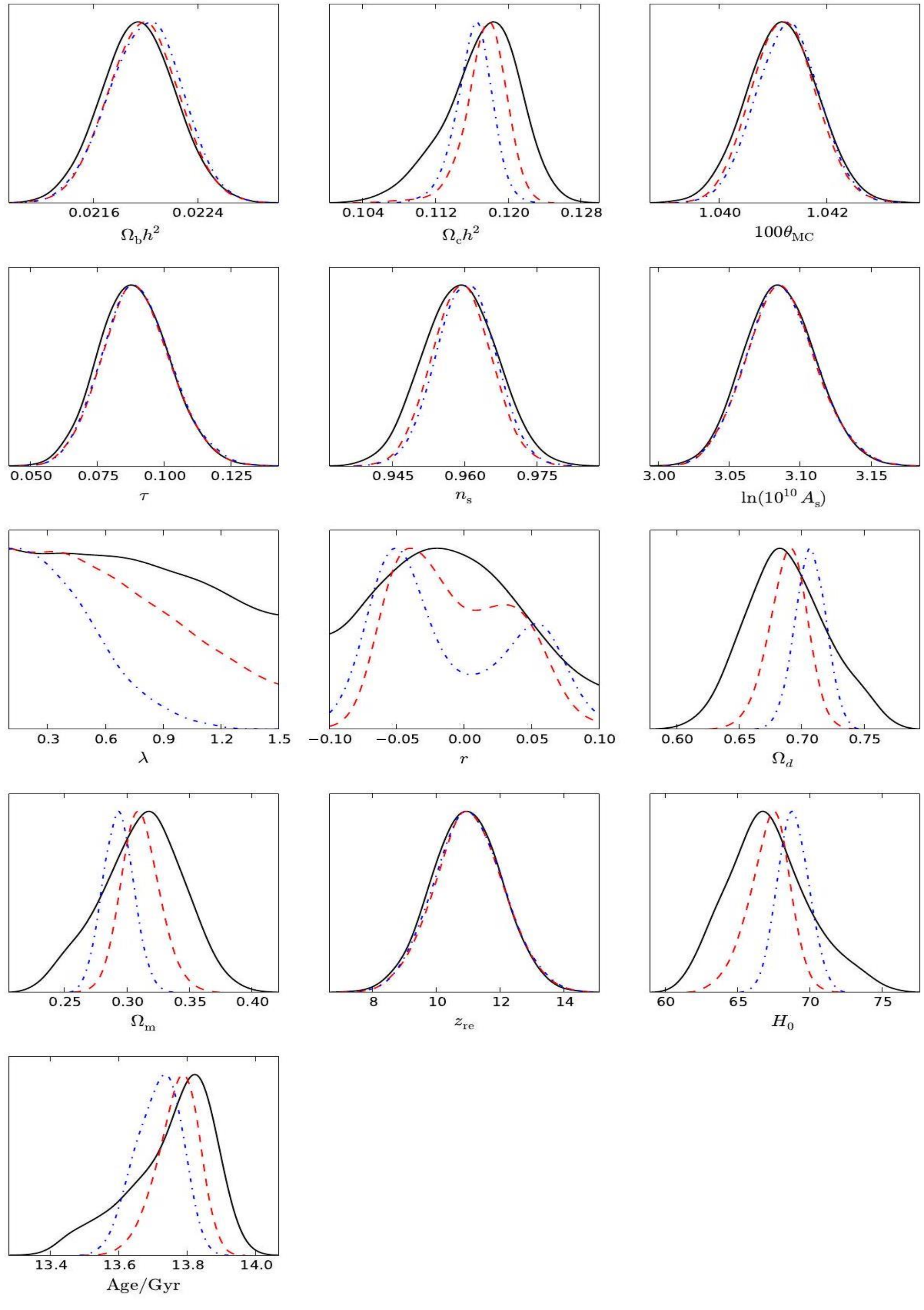
$$R = 0.951h^{-1}\text{Mpc}\left(\frac{M}{\Omega_m 10^{12}h^{-1}M_{\odot}}\right)^{1/3}$$

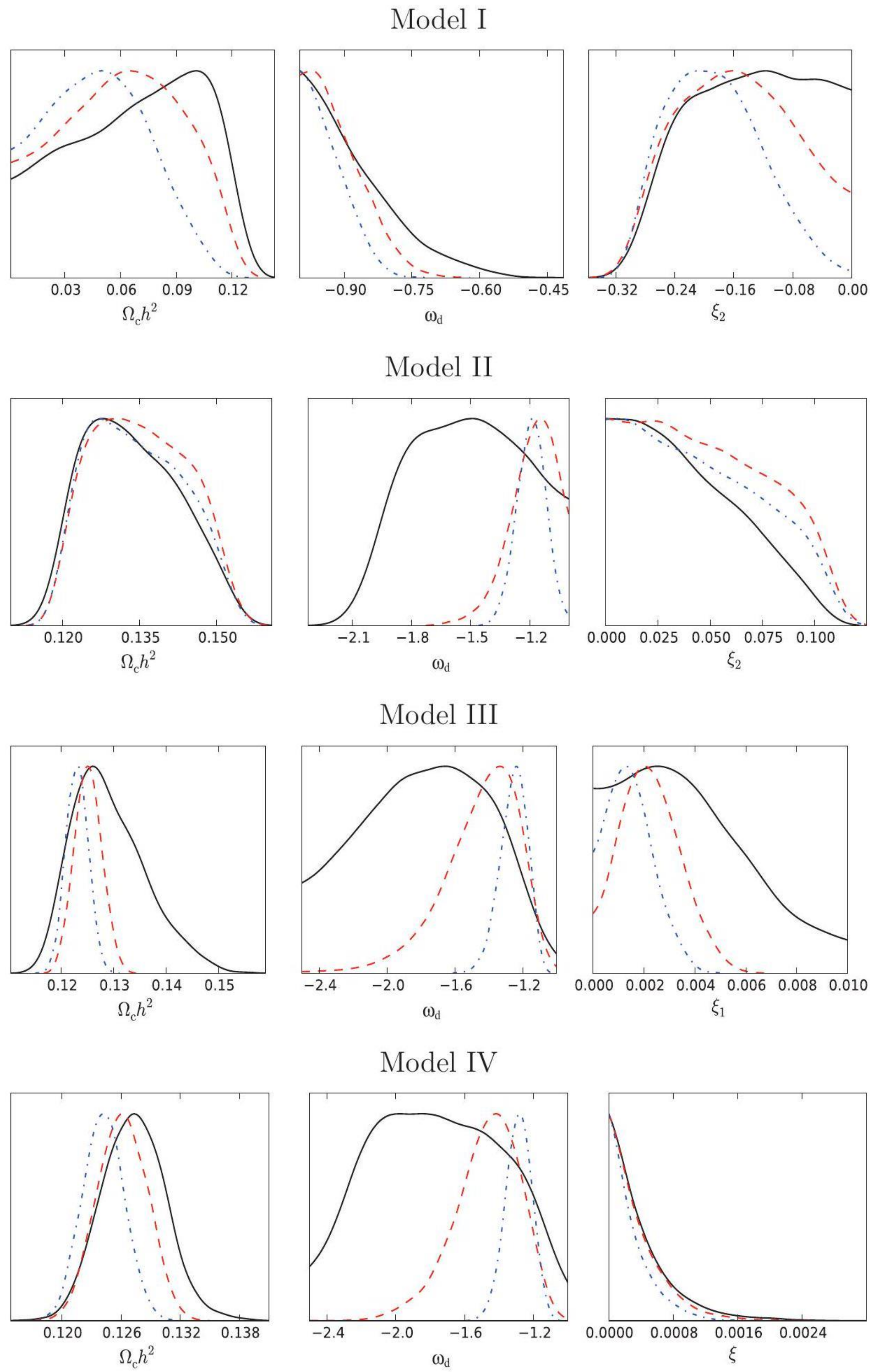
$$\frac{dN}{dz} = \int d\Omega \frac{dV}{dzd\Omega}\int n(M)dM$$

$dV/dzd\Omega = r^2(z)/H(z)\ r(z) = \int_0^z \frac{dz'}{H(z)}$

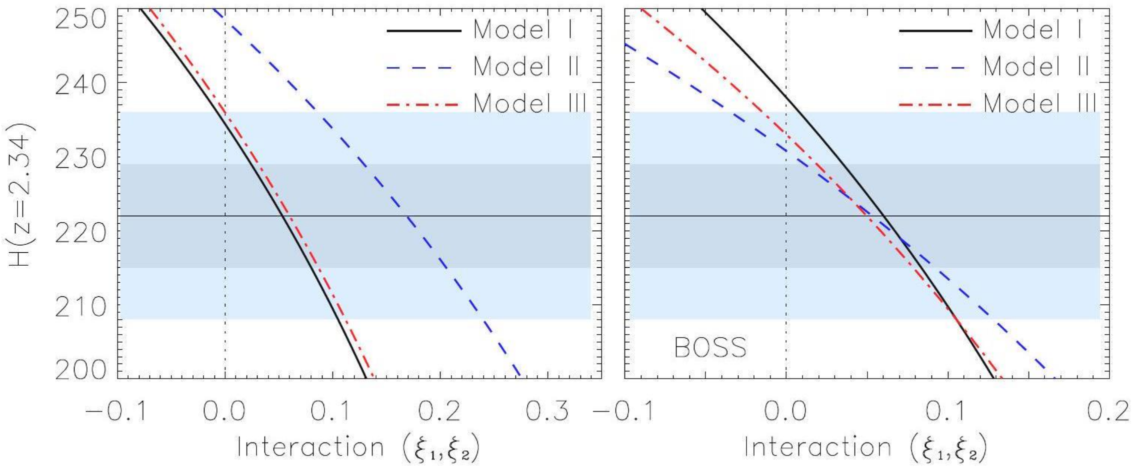
$$\varphi(\hat{n}) = -2\int_0^{\varsigma_{rec}} d\varsigma \frac{\varsigma_{rec} - \varsigma}{\varsigma_{rec}\varsigma}\Psi(\varsigma\hat{n}; \tau_0 - \tau)$$

$$AIC = 2k - 2\ln\hat{\mathcal{L}}(\theta);\ BIC = -2\ln\hat{\mathcal{L}}(\theta) + k\ln N.$$


0.0216
0.0224
$\Omega_b h^2$
0.104
0.112
0.120
0.128
$\Omega_c h^2$
1.040
1.042
$100\theta_{MC}$
0.050
0.075
0.100
0.125
τ
0.945
0.960
0.975
n_s
3.00
3.05
3.10
3.15
$\ln(10^{10} A_s)$
0.3
0.6
0.9
1.2
1.5
λ
−0.10
−0.05
0.00
0.05
0.10
r
0.60
0.65
0.70
0.75
Ω_d
0.25
0.30
0.35
0.40
Ω_m
8
10
12
14
z_{re}
60
65
70
75
H_0
13.4
13.6
13.8
14.0
Age/Gyr



Model I
0.03 0.06 0.09 0.12
Ωch2
−0.90 −0.75 −0.60 −0.45
ωd
−0.32 −0.24 −0.16 −0.08 0.00
ξ2
Model II
0.120 0.135 0.150
Ωch2
−2.1 −1.8 −1.5 −1.2
ωd
0.000 0.025 0.050 0.075 0.100
ξ2
Model III
0.12 0.13 0.14 0.15
Ωch2
−2.4 −2.0 −1.6 −1.2
ωd
0.000 0.002 0.004 0.006 0.008 0.010
ξ1
Model IV
0.120 0.126 0.132 0.138
Ωch2
−2.4 −2.0 −1.6 −1.2
ωd
0.0000 0.0008 0.0016 0.0024
ξ



Figuras 29, 30 y 31. Fluctuaciones de gauge en un espacio – tiempo cuántico relativista o curvo, en el que interactúa una partícula oscura.

$$\rho_d = (1+z)^{3(-1+\omega_d+\xi_2)}\rho_d^0$$

$$\rho_c = (1+z)^3\left\{\frac{\xi_2\left[1-(1+z)^{3(\xi_2+\omega_d)}\right]\rho_d^0}{\xi_2+\omega_d}+\rho_c^0\right\}$$

$$\rho_d = (1+z)^{3(1+\omega_d)}\left(\rho_d^0+\frac{\xi_1\rho_c^0}{\xi_1+\omega_d}\right)-\frac{\xi_1}{\xi_1+\omega_d}(1+z)^{3(1-\xi_1)}\rho_c^0$$

$$\rho_c = \rho_c^0(1+z)^{3(1-\xi_1)}$$

$$r_s^+ \sim \frac{1}{\xi} \text{ and } r_s^- \sim \xi$$

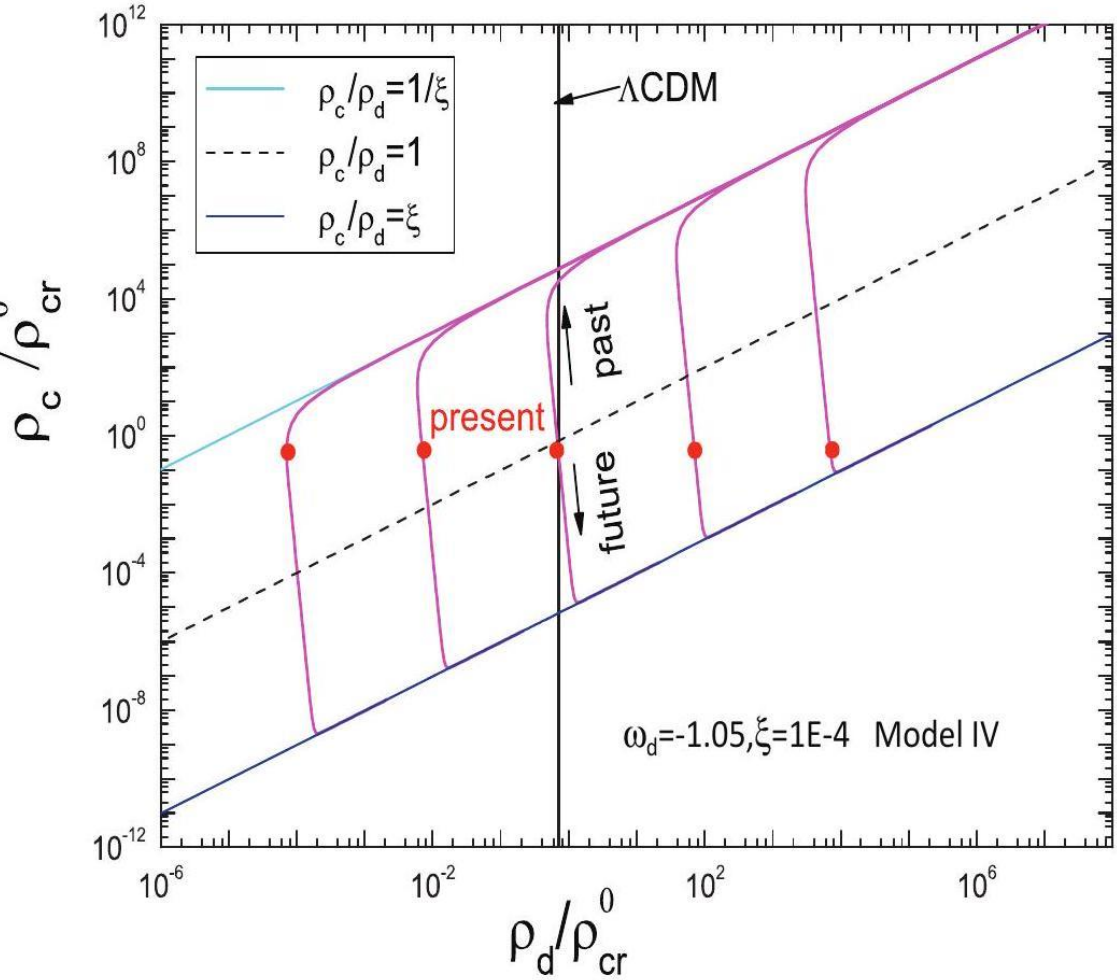


Figura 32. Comportamiento de un agujero negro provocado por una partícula oscura. Crisis inflacionaria y perturbativa de campo.

SECCIÓN IV.

$$\dot{\rho}_c + 3\mathcal{H}\rho_c = Q, \dot{\rho}_d + 3\mathcal{H}(1+\omega_d)\rho_d = -Q,$$

$$Q_1 = \mathcal{H}\xi\rho_c, \quad Q_2 = \mathcal{H}\xi\rho_d, \quad Q_3 = \mathcal{H}\xi(\rho_c + \rho_d)$$
$$Q_4 = \mathcal{H}\xi(\rho_c + (1+\omega_d)\rho_d), \quad Q_5 = \mathcal{H}\xi\sqrt{\rho_d\rho_c}, \quad Q_6 = \mathcal{H}\xi\frac{\rho_c\rho_d}{\rho_c + \rho_d}$$

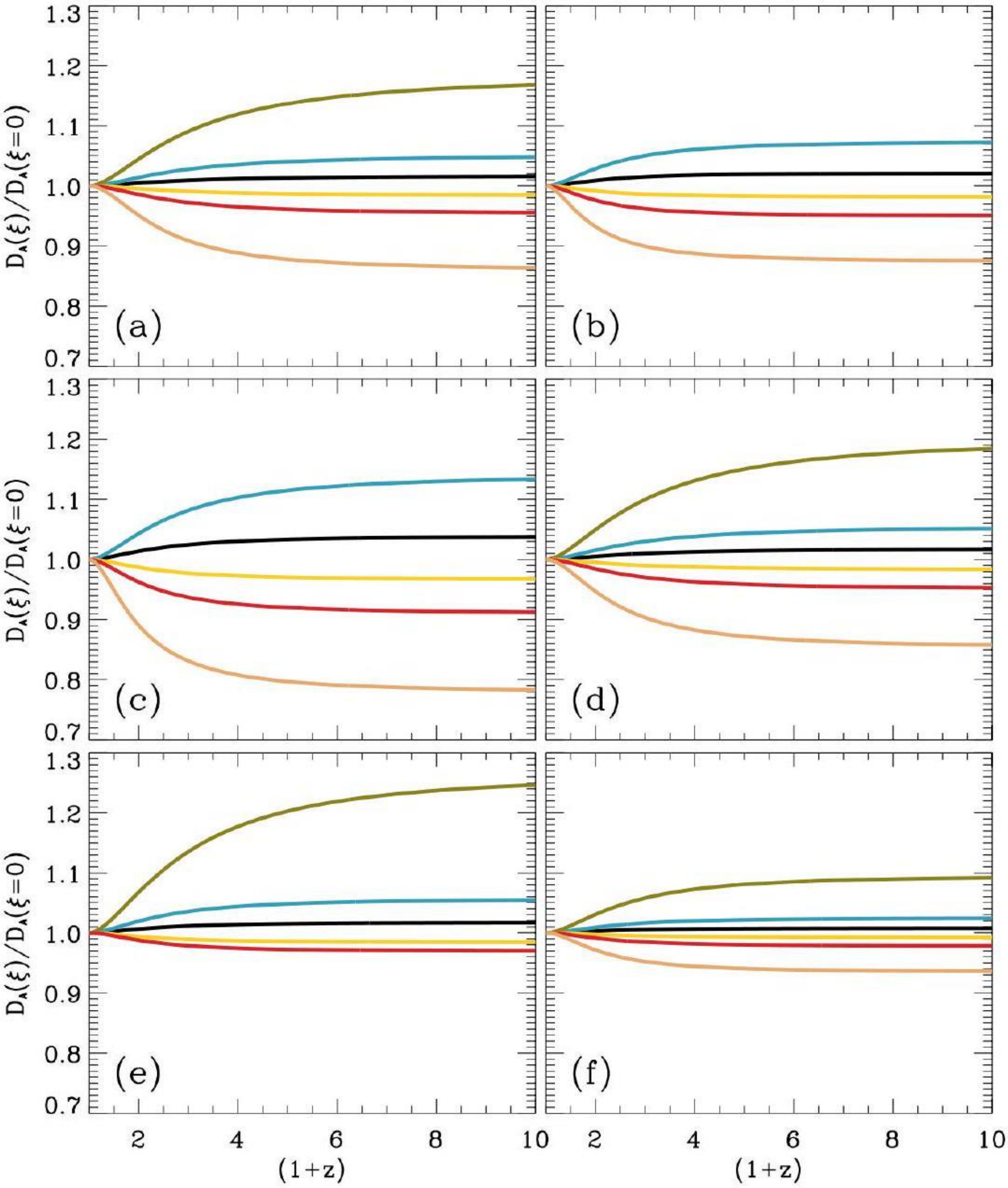


Figura 33. Kernels de campo a propósito de la interacción de una partícula oscura.

$$S = S_g(g_{\mu\nu}) + S_\phi(\phi_1, \phi_2) + S_m$$

$$S_\phi(\phi_1, \phi_2) = -\int d^4x\sqrt{-g}\left(\frac{1}{2}(\partial\phi_1)^2 + \frac{1}{2}(\partial\phi_2)^2 + V(\phi_1, \phi_2)\right)$$

$$V(\phi_1, \phi_2) = V_1(\phi_1) + V_2(\phi_2) + V_{\text{int}}(\phi_1, \phi_2)$$

$$V_1(\phi_1) = \mu_1^4\left[1 - \cos\left(\frac{\phi_1}{f_1}\right)\right], V_2(\phi_2) = \mu_2^4\left[1 - \cos\left(\frac{\phi_2}{f_2}\right)\right],$$

$$V_{\text{int}}(\phi_1,\phi_2)=\mu_3^4\left[1-\cos\left(\frac{\phi_1}{f_1}-n\frac{\phi_2}{f_2}\right)\right].$$

$$T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta(S_m+S_\phi)}{\delta g^{\mu\nu}}$$

$$\begin{aligned}3H^2&=\rho+\frac{1}{2}\dot{\phi_1}^2+\frac{1}{2}\dot{\phi_2}^2\\&+2\mu_1^4\sin^2\left(\frac{\phi_1}{2f_1}\right)+2\mu_2^4\sin^2\left(\frac{\phi_2}{2f_2}\right)\\&+2\mu_3^4\sin^2\left(\frac{\phi_1}{2f_1}-n\frac{\phi_2}{2f_2}\right)\end{aligned}$$

$$2\dot{H}=-\rho-\dot{\phi_1}^2-\dot{\phi_2}^2.$$

$$\ddot{\phi}_1+3H\dot{\phi}_1+\frac{\mu_1^4}{f_1}\sin\left(\frac{\phi_1}{f_1}\right)+\frac{\mu_3^4}{f_1}\sin\left(\frac{\phi_1}{f_1}-n\frac{\phi_2}{f_2}\right)=0,$$
$$\ddot{\phi}_2+3H\dot{\phi}_2+\frac{\mu_2^4}{f_2}\sin\left(\frac{\phi_2}{f_2}\right)-n\frac{\mu_3^4}{f_2}\sin\left(\frac{\phi_1}{f_1}-n\frac{\phi_2}{f_2}\right)=0.$$

$$\dot{\rho}+3H\rho=0.$$

$$\nabla_\mu T^{\mu\nu}_{(\lambda)}=Q^\nu_{(\lambda)}$$

$$\sum_\lambda \nabla_\mu T^{\mu\nu}_{(\lambda)}=0$$

$$T^{\mu\nu}=\rho U^\mu U^\nu+p(g^{\mu\nu}+U^\mu U^\nu)$$

$$\begin{aligned}ds^2=&a^2(\tau)\big[-(1+2\psi)d\tau^2+2\partial_i B d\tau dx^i\\&+(1+2\phi)\delta_{ij}dx^i dx^j+D_{ij}E dx^i dx^j\big]\end{aligned}$$

$$D_{ij}=(\,\partial_i\partial_j-\tfrac{1}{3}\delta_{ij}\nabla^2\,)$$

$$\begin{aligned}\Psi&=\psi-\frac{1}{k}\mathcal{H}\left(B+\frac{E'}{2k}\right)-\frac{1}{k}\left(B'+\frac{E''}{2k}\right)\\\Phi&=\phi+\frac{1}{6}E-\frac{1}{k}\mathcal{H}\left(B+\frac{E'}{2k}\right)\\D_\lambda&=\delta_\lambda-\frac{\rho'_\lambda}{\rho_\lambda\mathcal{H}}\left(\phi+\frac{E}{6}\right)\\V_\lambda&=v_\lambda-\frac{E'}{2k}\end{aligned}$$

$$\begin{aligned}\Psi&=\psi\\\Phi&=\phi\\D_\lambda&=\delta_\lambda-\frac{\rho'_\lambda}{\rho_\lambda\mathcal{H}}\Phi\\V_\lambda&=v_\lambda\end{aligned}$$

$$D_c' = -kU_c + 6\mathcal{H}\Psi(\xi_1 + \xi_2/r) - 3(\xi_1 + \xi_2/r)\Phi' + 3\mathcal{H}\xi_2(D_d - D_c)/r$$

$$U_c' = -\mathcal{H}U_c + k\Psi - 3\mathcal{H}(\xi_1 + \xi_2/r)U_c$$

$$\begin{aligned} D_d' =& -3(C_e^2 - w_d)D_d - 9\mathcal{H}^2(C_e^2 - C_a^2)\frac{U_d}{k} \\ &+ [3w_d' - 9\mathcal{H}(w_d - C_e^2)(\xi_1 r + \xi_2 + 1 + w_d)]\Phi \\ &+ 3(\xi_1 r + \xi_2)\Phi' - 3\Psi\mathcal{H}(\xi_1 r + \xi_2) \\ &- 9\mathcal{H}^2(C_e^2 - C_a^2)(\xi_i r + \xi_2)\frac{U_d}{(1 + w_d)k} \\ &- kU_d + 3\mathcal{H}\xi_1 r(D_d - D_c), \\ U_d' =& -\mathcal{H}(1 - 3w_d)U_d + 3(C_e^2 - C_a^2)\mathcal{H}U_d \\ &- 3kC_e^2(\xi_1 r + \xi_2 + 1 + w_d)\Phi + kC_e^2 D_d \\ &+ 3\mathcal{H}(C_e^2 - C_a^2)(\xi_1 r + \xi_2)\frac{U_d}{1 + w_d} \\ &+ (1 + w_d)k\Psi + 3\mathcal{H}(\xi_1 r + \xi_2)U_d, \end{aligned}$$

$$-k^2\Psi = \frac{3}{2}\mathcal{H}^2[\Omega_c \triangle_c + (1 - \Omega_c)\triangle_d]$$

$$\triangle_\lambda = \delta_\lambda - \frac{\rho_\lambda'}{\rho_\lambda}\frac{V_\lambda}{k}, \Omega_\lambda = \frac{\rho_\lambda}{\rho_{\mathrm{crit}}}$$

$$\nabla^2\Psi = -\tfrac{3}{2}\mathcal{H}^2[\Omega_c \triangle_c + (1 - \Omega_c)\triangle_d]$$

$$V_c' + [\mathcal{H} + 3\mathcal{H}(\xi_1 + \xi_2/r)]V_c - k\Psi = 0.$$

$$\nabla V_c' + [\mathcal{H} + 3\mathcal{H}(\xi_1 + \xi_2/r)]\nabla V_c + \frac{3}{2}\mathcal{H}^2[\Omega_c \triangle_c + (1 - \Omega_c)\triangle_d] = 0$$

$$\alpha(a) = -3\mathcal{H}(\xi_1 + \xi_2/r)a.$$

$$\mathbf{x}(\mathbf{q}) = \mathbf{q} + \nabla_q\Psi^{(1)} + \nabla_q\Psi^{(2)}$$

$$\mathbf{v}(\mathbf{q}) = f_1 H\nabla_q\Psi^{(1)} + f_2 H\nabla_q\Psi^{(2)}$$

$$f_{1,2} = \frac{d\ln(D_{1,2})}{d\ln a}$$

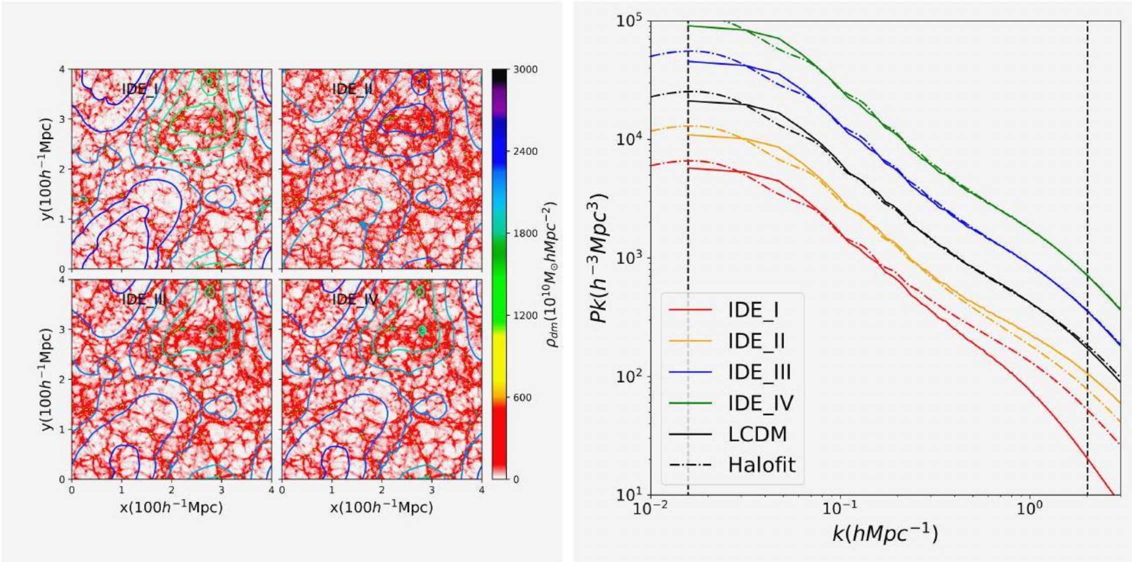


Figura 34. Energía y materia oscuras inherentes a un sector de campo cuántico – relativista oscuro.

$$\bar{T}_{\mathrm{b}}(z) = \frac{3}{32\pi} \frac{\left(h_{\mathrm{p}}c\right)^3 \bar{n}_{\mathrm{HI}} A_{10}}{k_{\mathrm{B}} E_{21}^2 (1+z) H(z)} = 0.188 h \Omega_{\mathrm{HI}}(z) \frac{(1+z)^2}{E(z)} \mathrm{K}$$

$$\mathrm{d}s^2 = a^2(\eta)\left[(1+2\Psi)\mathrm{d}\eta^2 - (1-2\Phi)\delta_{ij}\, \mathrm{d}x^i\, \mathrm{d}x^j\right]$$

$$\dot{\mathbf{v}} + \mathcal{H}\mathbf{v} + \nabla\Psi = -\mathbf{v}\frac{aQ}{\rho_{\mathrm{m}}},$$

$$\Delta_{T_{\mathrm{b}}}(z, \hat{\mathbf{n}}) = \delta_n - \frac{1}{\mathcal{H}}\hat{\mathbf{n}} \cdot (\hat{\mathbf{n}} \cdot \nabla \mathbf{v}) + \left(\frac{\mathrm{dln}\,(\mathrm{a}^3 \bar{\mathrm{n}}_{\mathrm{HI}})}{\mathrm{d}\eta} - \frac{\dot{\mathcal{H}}}{\mathcal{H}} - 2\mathcal{H}\right)\delta\eta + \frac{1}{\mathcal{H}}\dot{\Phi} + \Psi - \frac{1}{\mathcal{H}}\hat{\mathbf{n}} \cdot \mathbf{v}\frac{aQ}{\rho_{\mathrm{m}}}$$

$$\delta_n = b_{\mathrm{HI}}\delta_{\mathrm{m}}^{\mathrm{syn}} + \left(\frac{\mathrm{dln}\,(\mathrm{a}^3 \bar{\mathrm{n}}_{\mathrm{HI}})}{\mathrm{d}\eta} - 3\mathcal{H}\right)\frac{v_{\mathrm{m}}}{k},$$

$$\Delta_{T_{\mathrm{b}}}(z, \hat{\mathbf{n}}) = \sum_{\ell m} \Delta_{T_{\mathrm{b}},\ell m}(z) Y_{\ell m}(\hat{\mathbf{n}})$$

$$\Delta_{T_{\mathrm{b}},\ell m}(z) = 4\pi i^l \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}} \Delta_{T_{\mathrm{b}},\ell}(\mathbf{k}, z) Y_{\ell m}^*(\hat{\mathbf{k}})$$

$$\Delta_{T_{\mathrm{b}},\ell}(\mathbf{k},z)= \delta_n j_\ell(k\chi)+\frac{kv}{\mathcal{H}}j_\ell''(k\chi)+\left(\frac{1}{\mathcal{H}}\dot{\Phi}+\Psi\right)j_\ell(k\chi)$$
$$-\left(\frac{1}{\mathcal{H}}\frac{\mathrm{dln}\,(\mathrm{a}^3\overline{\mathrm{n}}_{\mathrm{HI}})}{\mathrm{d}\eta}-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-2\right)\Big[\Psi j_\ell(k\chi)$$
$$+vj_\ell'(k\chi)+\int_0^\chi(\dot{\Psi}+\dot{\Phi})j_\ell(k\chi')d\chi'\Big]$$
$$+\frac{1}{\mathcal{H}}vj_\ell'(k\chi)\frac{aQ}{\rho_{\mathrm{m}}}$$

$$\Delta^W_{T_{\mathrm{b}},\ell}(\mathbf{k})=\int_0^\infty \mathrm{d}zW(z)\Delta_{T_{\mathrm{b}},\ell}(\mathbf{k},z)$$

$$W(z)=\begin{cases}\frac{1}{\Delta z}, & z-\frac{\Delta z}{2}\leq z\leq z+\frac{\Delta z}{2}\\ 0, & \text{otherwise}\end{cases}$$

$$C_\ell^{WW'}=4\pi\int \mathrm{dln}\;k\mathcal{P}_{\mathcal{R}}(k)\Delta^W_{T_{\mathrm{b}},\ell}(k)\Delta^{W'}_{T_{\mathrm{b}},\ell}(k)$$

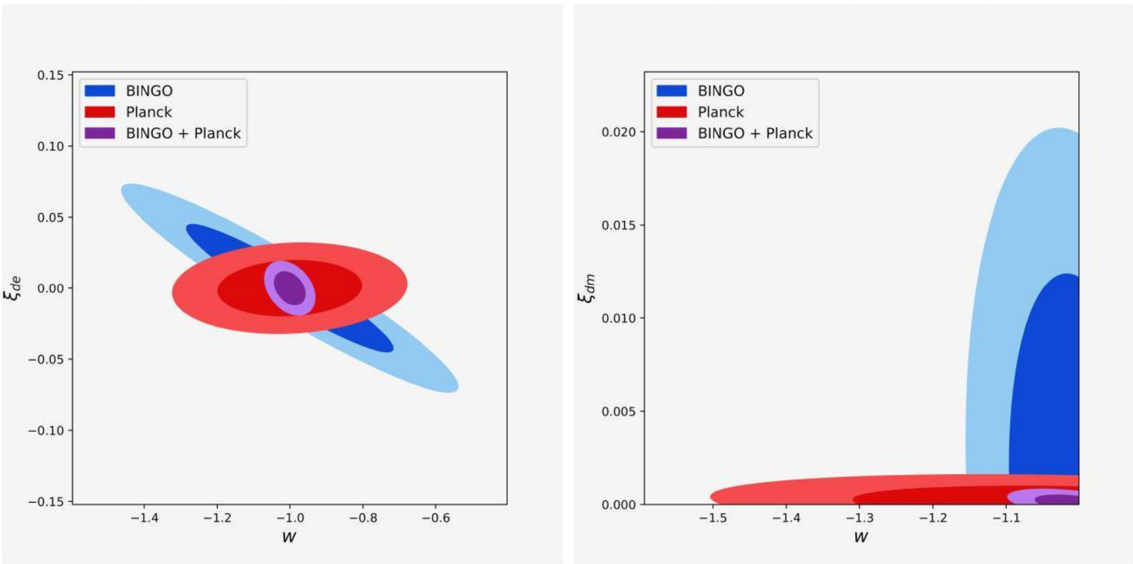


Figura 35. Deformación del espacio – tiempo causada por una partícula supermasiva.

$$\widehat{\mathcal{DM}}=\widehat{\mathcal{DM}}_{MW}+\widehat{\mathcal{DM}}_{\mathrm{halo}}+\widehat{\mathcal{DM}}_{IGM}+\frac{\widehat{\mathcal{DM}}_{\mathrm{host}}+\widehat{\mathcal{DM}}_{\mathrm{src}}}{1+z}$$

$$\langle\widehat{\mathcal{D}}_{IGM}(z)\rangle=\frac{3cH_0\Omega_b f_{\mathrm{IGM}}}{8\pi Gm_p}\int_0^z\frac{\chi(z')(1+z')dz'}{E(z')}$$

$$\chi(z)\simeq\frac{3}{4}\chi_{\mathrm{e,H}}(z)+\frac{1}{8}\chi_{\mathrm{e,He}}(z)$$

$$h_+(t)=\left(\frac{G\mathcal{M}_z}{c^2}\right)^{5/3}\left(\frac{\pi f(t)}{c}\right)^{2/3}\frac{2(1+\cos^2\iota)}{D_L}\cos\left(\Psi(t,\mathcal{M}_z,\eta)\right)$$
$$h_\times(t)=\left(\frac{G\mathcal{M}_z}{c^2}\right)^{5/3}\left(\frac{\pi f(t)}{c}\right)^{2/3}\frac{4\cos\iota}{D_L}\sin\left(\Psi(t,\mathcal{M}_z,\eta)\right)$$

SECCIÓN V.

$$S_{\mathrm{EsGB}} = \int \ dx^4\sqrt{-g}\left[\frac{1}{2\kappa}(R-2\Lambda)+f(\phi)\mathcal{G}+\mathcal{L}_\phi(\phi)+\mathcal{L}_m\big(g_{\mu\nu},\phi;\psi\big)\right]$$

$$\mathcal{L}_\phi = -\frac{1}{2}(D^\mu\phi)\big(D_\mu\phi\big)-V(\phi)$$
$$\mathcal{G} = R^2-4R_{\mu\nu}R^{\mu\nu}+R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu} = T_{\mu\nu}^{GB}+\kappa T_{\mu\nu}^{\phi}+\kappa T_{\mu\nu}^{m}$$
$$D^2\phi = \frac{dV}{d\phi}-\alpha\frac{df}{d\phi}G-Q$$

$$T_{\mu\nu}^{GB} \equiv 2\big(D_\mu D_\nu f\big)R-2g_{\mu\nu}\big(D_\rho D^\rho f\big)R-4(D^\rho D_\nu f)R_{\mu\rho}-4\big(D^\rho D_\mu f\big)R_{\nu\rho}$$
$$+4\big(D^\rho D_\rho f\big)R_{\mu\nu}+4g_{\mu\nu}(D^\rho D^\sigma f)R_{\rho\sigma}-4(D^\rho D^\sigma f)R_{\mu\rho\nu\sigma}$$
$$T_{\mu\nu}^{\phi} \equiv \frac{1}{2}\big(D_\mu\phi\big)D_\nu\phi-\frac{1}{4}g_{\mu\nu}[(D^\alpha\phi)D_\alpha\phi+2V(\phi)]$$
$$T_{\mu\nu}^{m} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta\big(\sqrt{-g}\mathcal{L}_m\big)}{\delta g^{\mu\nu}}$$
$$Q \equiv 2\kappa\frac{\delta\mathcal{L}_m}{\delta\phi}$$

$$3H^2 = \rho_m+\rho_\phi+24f'(\phi)\dot{\phi}H^3$$

$$\dot{\rho}_m+3H\rho_m \ = Q\rho_m\dot{\phi}$$
$$\ddot{\phi}+V_{,\phi}+3H\dot{\phi} \ = -Q\rho_m-24f'H^2(H^2+\dot{H})$$

$$V(\phi) = V_0e^{-\lambda\phi}, f'(\phi) = \alpha e^{\mu\phi}$$

$$x = \frac{\dot{\phi}}{\sqrt{6}H}, y = \frac{\sqrt{V}}{\sqrt{3}H}, z = f'H^2, \Omega_M = \frac{\rho_m}{3H^2}, \Omega_\phi = \frac{\rho_\phi}{3H^2}.$$

$$\Omega_M = 1-\Omega_\phi-\Omega_{\mathrm{GB}}.$$

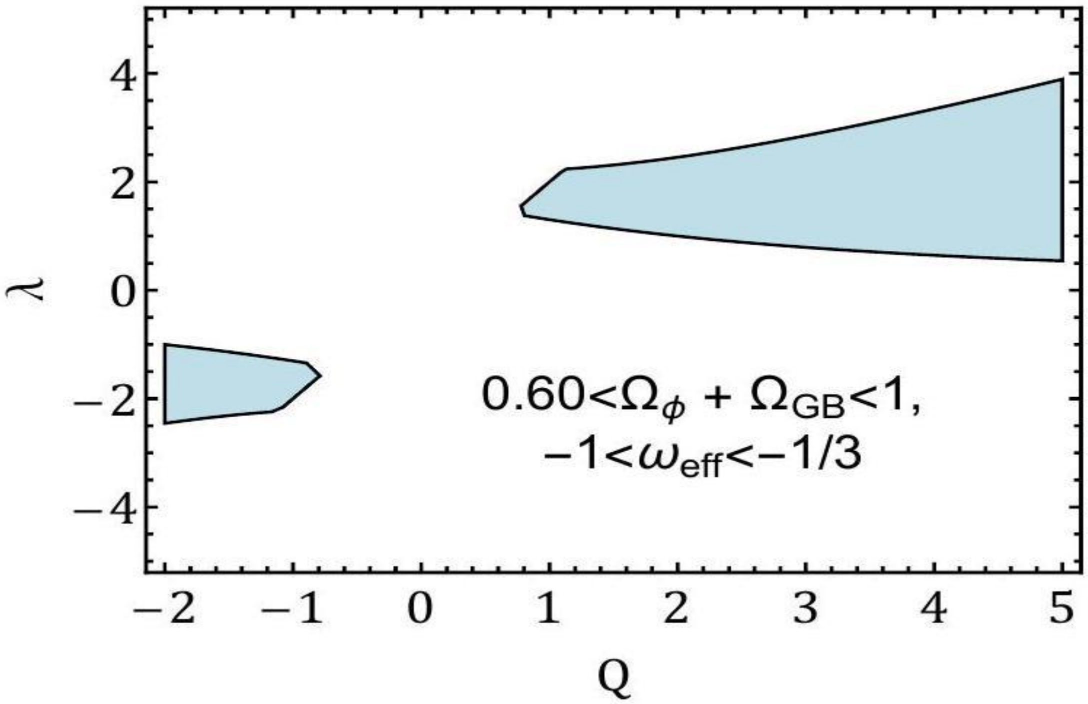
$$0\le\Omega_M\le 1, \Omega_\phi>0, 0<\Omega_\phi+\Omega_{\mathrm{GB}}<1$$

$$\frac{\dot{H}}{H^2} = \frac{12Q\Omega_M z+24\mu x^2 z-3x^2-12\sqrt{6}xz-4\sqrt{6}xz+12\lambda y^2 z-\frac{3\Omega_M}{2}-96z^2}{-8\sqrt{6}xz+96z^2+1}.$$

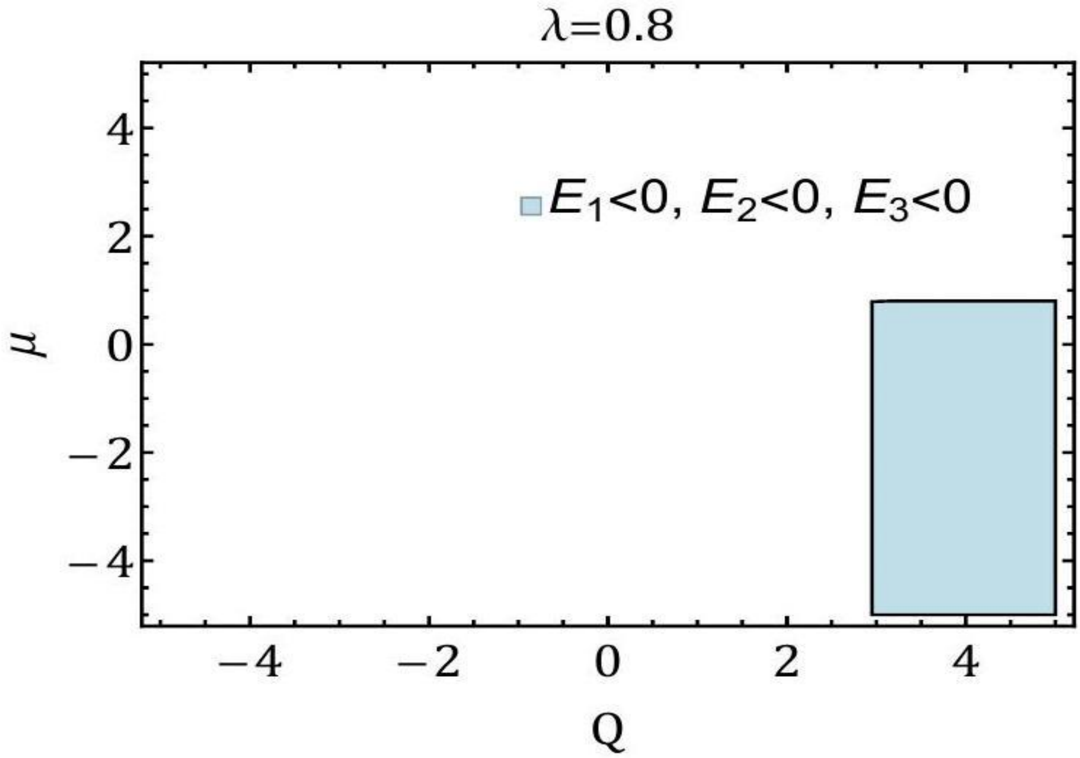
$$x' = \frac{\dot{x}}{H} = -\frac{\dot{H}}{H^2}\left(x+\frac{24z}{\sqrt{6}}\right)-\frac{3Q\Omega_M}{\sqrt{6}}-3x+\frac{3\lambda y^2}{\sqrt{6}}-\frac{24z}{\sqrt{6}}$$
$$y' = \frac{\dot{y}}{H} = \frac{-1}{2}\sqrt{6}\lambda xy-y\frac{\dot{H}}{H^2}$$
$$z' = \frac{\dot{z}}{H} = \sqrt{6}\mu xz+2z\frac{\dot{H}}{H^2}$$

Points	(x_*, y_*, z_*)	Ω_ϕ	$\Omega_{\rm GB}$	$\omega_{\rm eff}$	Eigenvalues
$P_{1,2}$	$(\mp 1, 0, 0)$	1	0	1	$\left(3 \mp \sqrt{6}Q, 3 \pm \sqrt{\frac{3}{2}}\lambda, \sqrt{6}(\mp\mu) - 6\right)$
P_3	$\left(-\sqrt{\frac{2}{3}}Q, 0, 0\right)$	$\frac{2Q^2}{3}$	0	$\frac{2Q^2}{3}$	$\left(Q^2 - \frac{3}{2}, Q(\lambda + Q) + \frac{3}{2}, -2Q(\mu + Q) - 3\right)$
P_4	$\left(\frac{\lambda}{\sqrt{6}}, \left\lvert\frac{\sqrt{-\left(\left(\lambda^2-6\right)(\lambda+Q)\right)}}{\sqrt{6}\sqrt{\lambda+Q}}\right\rvert, 0\right)$	1	0	$\frac{1}{3}\left(\lambda^2 - 3\right)$	$\left(\frac{1}{2}\left(\lambda^2 - 6\right), \lambda(\lambda + Q) - 3, \lambda(\mu - \lambda)\right)$
P_5	$\left(\frac{\sqrt{\frac{3}{2}}}{\lambda+Q}, \left\lvert\frac{\sqrt{Q^2+\lambda Q+\frac{3}{2}}}{\lambda+Q}\right\rvert, 0\right)$	$\frac{Q^2+\lambda Q+3}{(\lambda+Q)^2}$	0	$-\frac{Q}{\lambda+Q}$	$(\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3)$ –
P_6	$\left(0, \frac{\sqrt{2Q^2+1}}{\sqrt{2}\sqrt{Q}\sqrt{\lambda+Q}}, \frac{1}{16Q}\right)$	$\frac{2Q^2+1}{2Q^2+2\lambda Q}$	0	-1	$(\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3)$ –
P_7	$(0, 1, \lambda/8)$	1	0	-1	$6\lambda Q - 3, -\frac{3}{2} \mp \frac{\sqrt{3}\sqrt{\left(3\lambda^2+2\right)\left(17\lambda^2-8\lambda\mu+6\right)}}{6\lambda^2+4}$
$P_{8,9}$	–	–	–	–	–

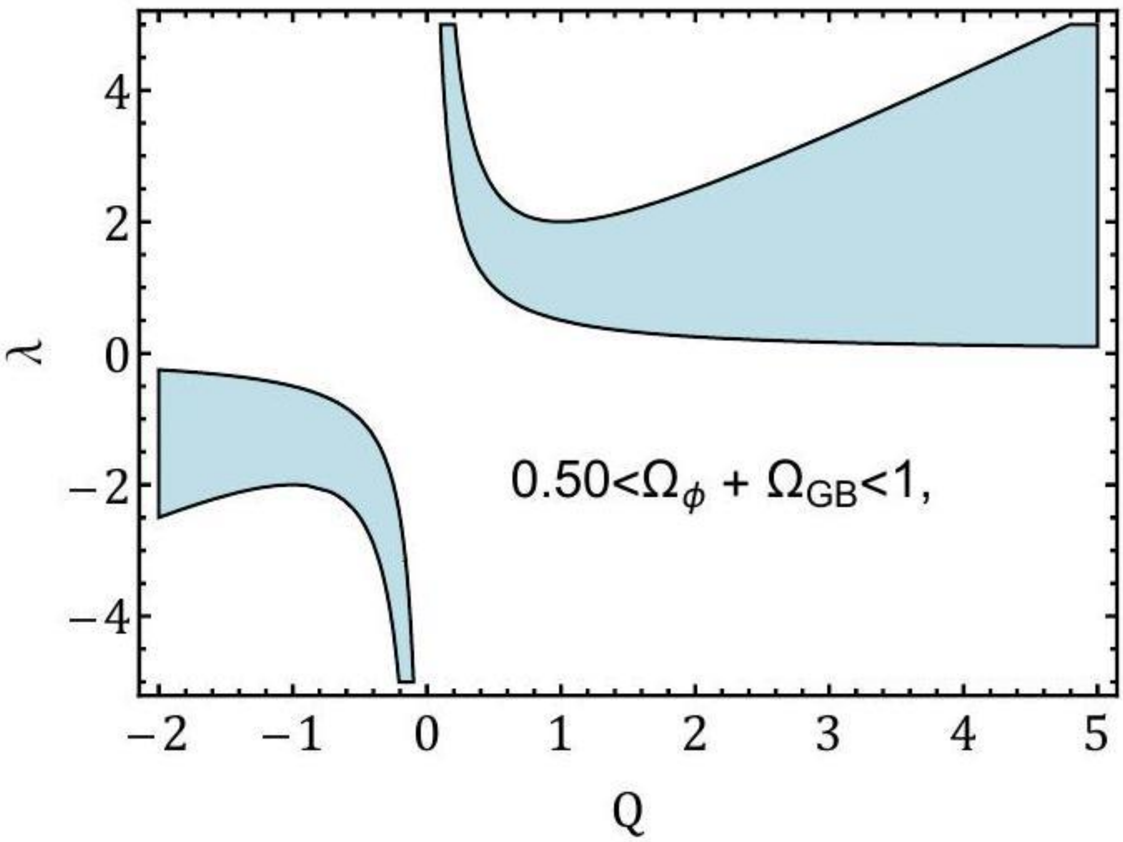
$$\omega_{\rm eff} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2}.$$

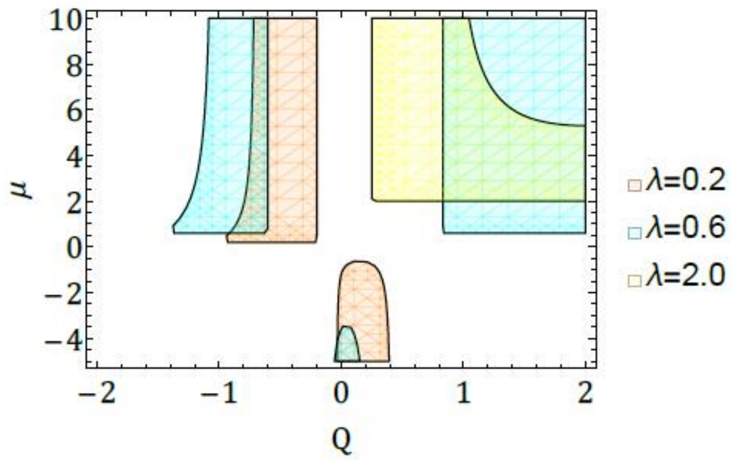


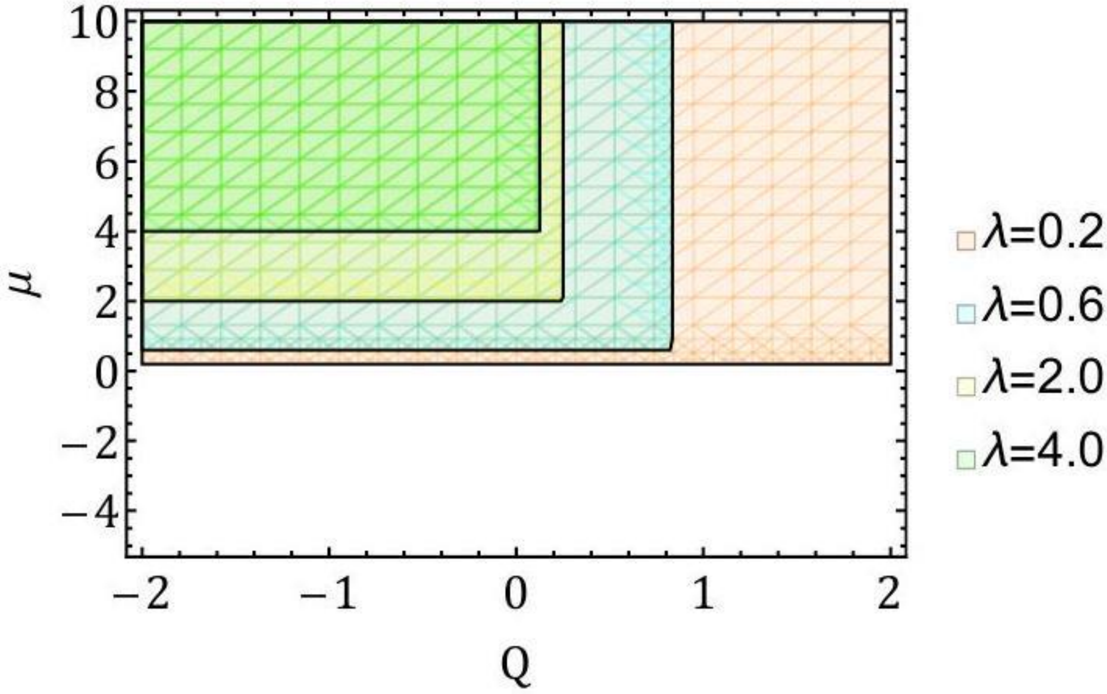
(a)



(b)

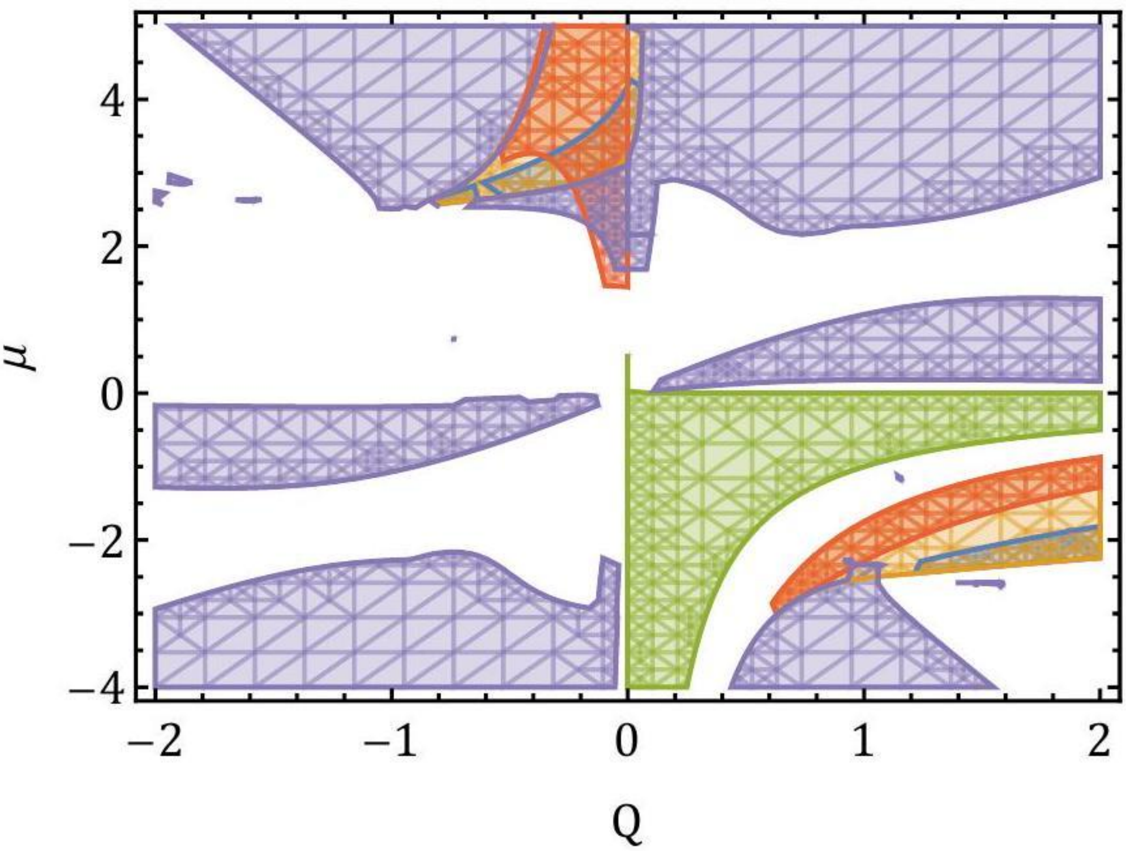

0.50<Ωϕ + ΩGB<1,
λ
Q
4
2
0
−2
−4
−2
−1
0
1
2
3
4
5

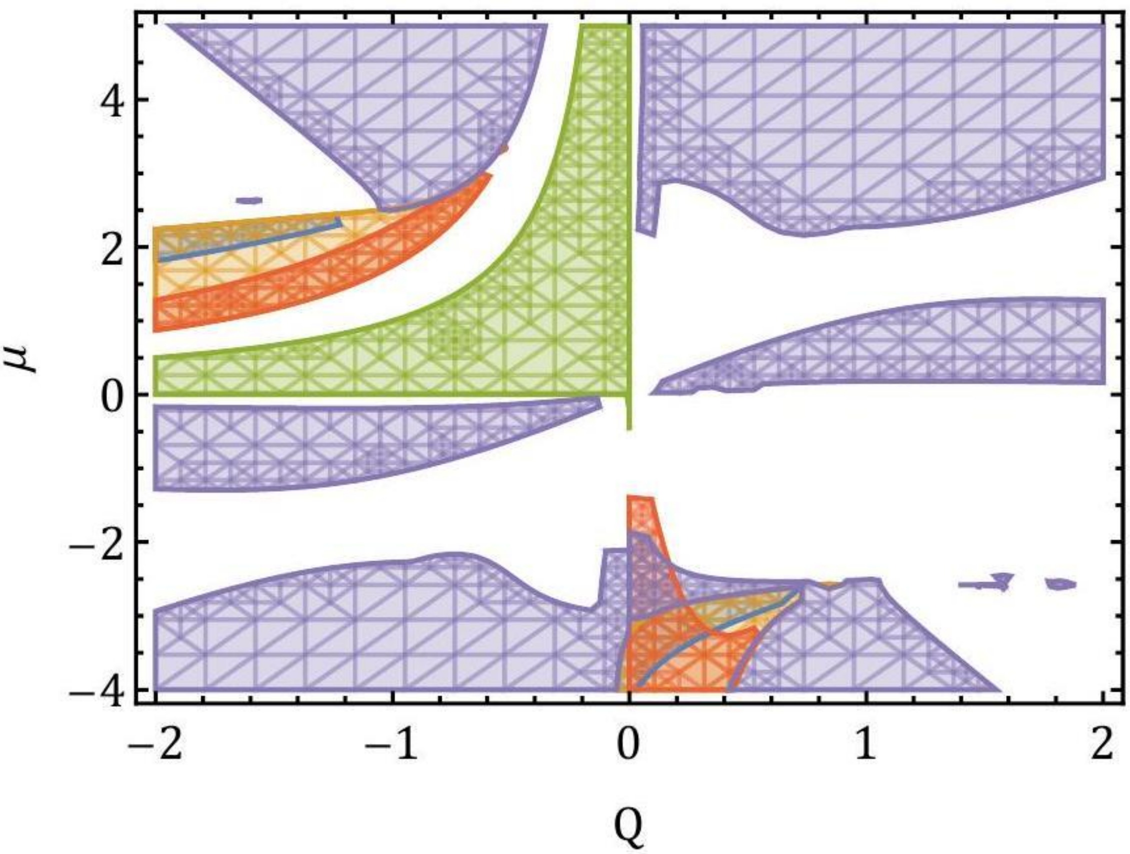

μ
Q
λ=0.2
λ=0.6
λ=2.0



Figuras 36, 37, 38 y 39. Densidades de energía oscura.

$$(x_*, y_*, z_*) = \left(\frac{5\mu + \sqrt{\mu^2 - 48\mu Q^3 + (16 - 48\mu^2)Q^2 - 64\mu Q} - 4Q}{2\sqrt{6}\mu(\mu + Q)} \right.$$
$$\left. \frac{\mu - \sqrt{\mu^2 - 48\mu Q^3 + (16 - 48\mu^2)Q^2 - 64\mu Q} + 4Q}{-9 - 96\mu Q} \right).$$


μ
Q
4
2
0
−2
−4
−2
−1
0
1
2

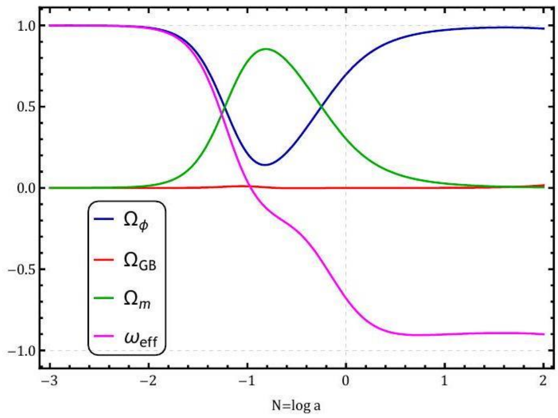

μ
Q
4
2
0
−2
−4
−2
−1
0
1
2

Figuras 39 y 40. Puntos y efectos gravitacionales críticos.

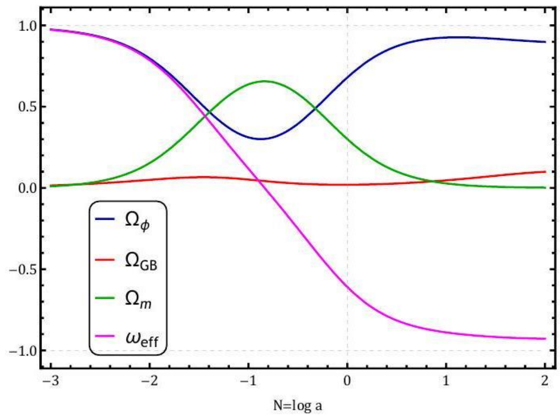
$$\Omega_{\phi} = \frac{\left(5\mu + \sqrt{\mu^2 - 48\mu Q^3 + (16 - 48\mu^2)Q^2 - 64\mu Q} - 4Q\right)^2}{24\mu^2(\mu + Q)^2}$$

$$(x_*, y_*, z_*) = \left(\frac{5\mu - \sqrt{\mu^2 - 48\mu Q^3 + (16 - 48\mu^2)Q^2 - 64\mu Q} - 4Q}{2\sqrt{6}\mu(\mu + Q)}\right.$$
$$\left.\frac{\mu + \sqrt{\mu^2 - 48\mu Q^3 + (16 - 48\mu^2)Q^2 - 64\mu Q} + 4Q}{96\mu Q}\right)$$

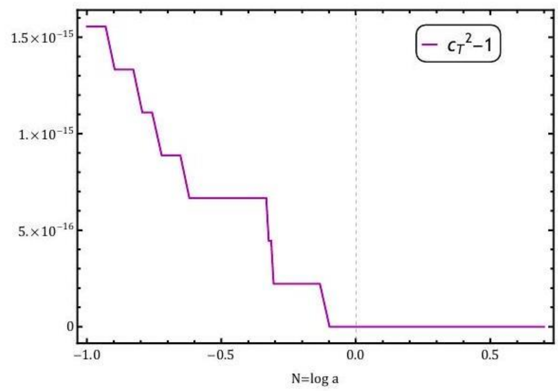
$$\Omega_{\phi} = \frac{\left(-5\mu + \sqrt{\mu^2 - 48\mu Q^3 + (16 - 48\mu^2)Q^2 - 64\mu Q} + 4Q\right)^2}{24\mu^2(\mu + Q)^2}$$



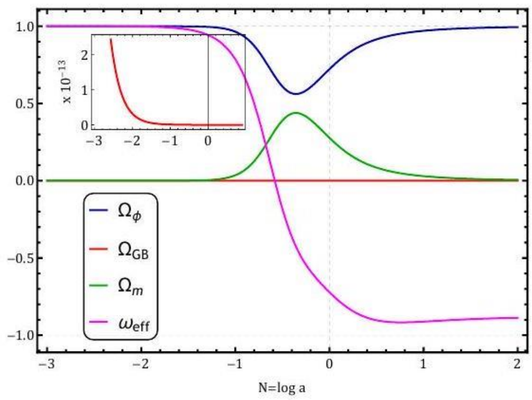
(a)



(b)



(a)



(b)

Figuras 41 y 42. Parámetro cosmológico – cuántico en un campo escalar oscuro de materia.

$$c_T^2 = \frac{1 - 8\ddot{f}}{1 - 8H\dot{f}}.$$

$$\left|c_T^2 - 1\right| \leq 5 \times 10^{-16},$$

$$c_T^2 = \frac{1 - 8\left(6\mu z x^2 + \sqrt{6} z (x' + x\dot{H}/H^2)\right)}{1 - 8\sqrt{6} z x}.$$

$$\mu = 5\log_{10}(D_L) + 25$$

$$D_L = c(1+z)\int_0^z \frac{1}{H} dz$$

$$c_T^2 = \frac{1 - 8\ddot{f}}{1 - 8H\dot{f}} = 1 \Longrightarrow \dot{f} = a/f_0$$

$$f_{,\phi} = \frac{a}{f_0}\frac{1}{\dot{\phi}}, f_{,\phi\phi} = \left(\frac{\dot{a}}{f_0} - \frac{a}{f_0}\frac{\ddot{\phi}}{\dot{\phi}}\right)\frac{1}{\dot{\phi}^2}.$$

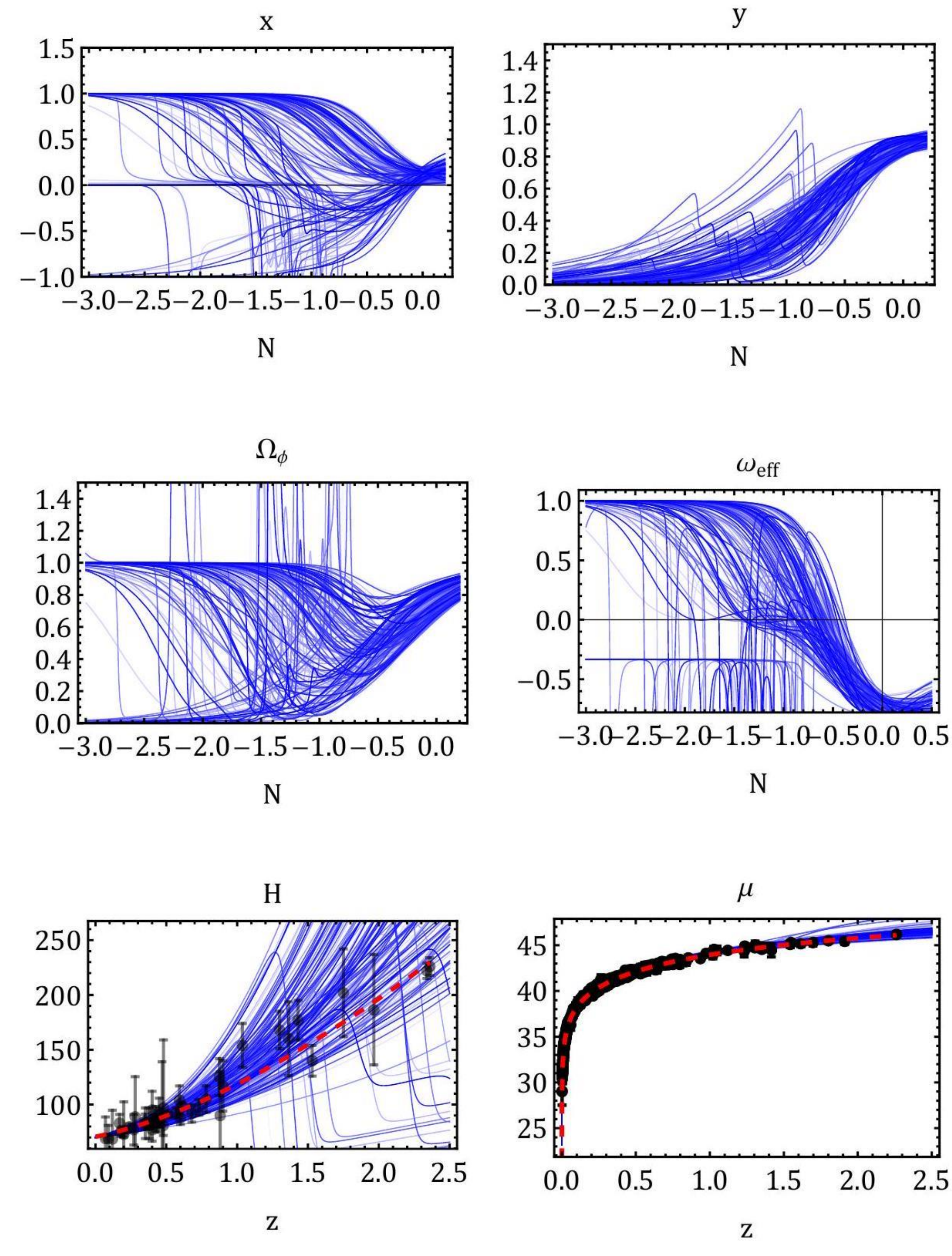


Figura 43. Fluctuaciones gravitacionales de campo causadas por una partícula supermasiva – sector oscuro.

$$\xi = \frac{aH}{a_0 H_0 + aH}$$

$$\xi = \begin{cases} 0, & aH \to 0 \\ \frac{1}{2}, & aH = a_0 H_0 \\ 1, & aH \gg a_0 H_0 \end{cases}$$

$$\dot{\alpha} = \frac{-\xi \mathcal{M}_0 f_0}{(1-\xi)} \left(\frac{\dot{H}}{H^2} \right) + \frac{\mathcal{M}_0 f_0 \xi'}{(1-\xi)^2}$$

$$\xi' = \xi(1-\xi)\left(1 + \frac{\dot{H}}{H^2}\right)$$

$$\frac{\dot{H}}{H^2}\left(1 - \frac{4\mathcal{M}_0 \xi}{1-\xi}\right) = \frac{-3}{2}\Omega_M - \frac{4\mathcal{M}_0 \xi}{1-\xi} - 3x^2 + \frac{4\mathcal{M}_0 \xi'}{(1-\xi)^2}$$

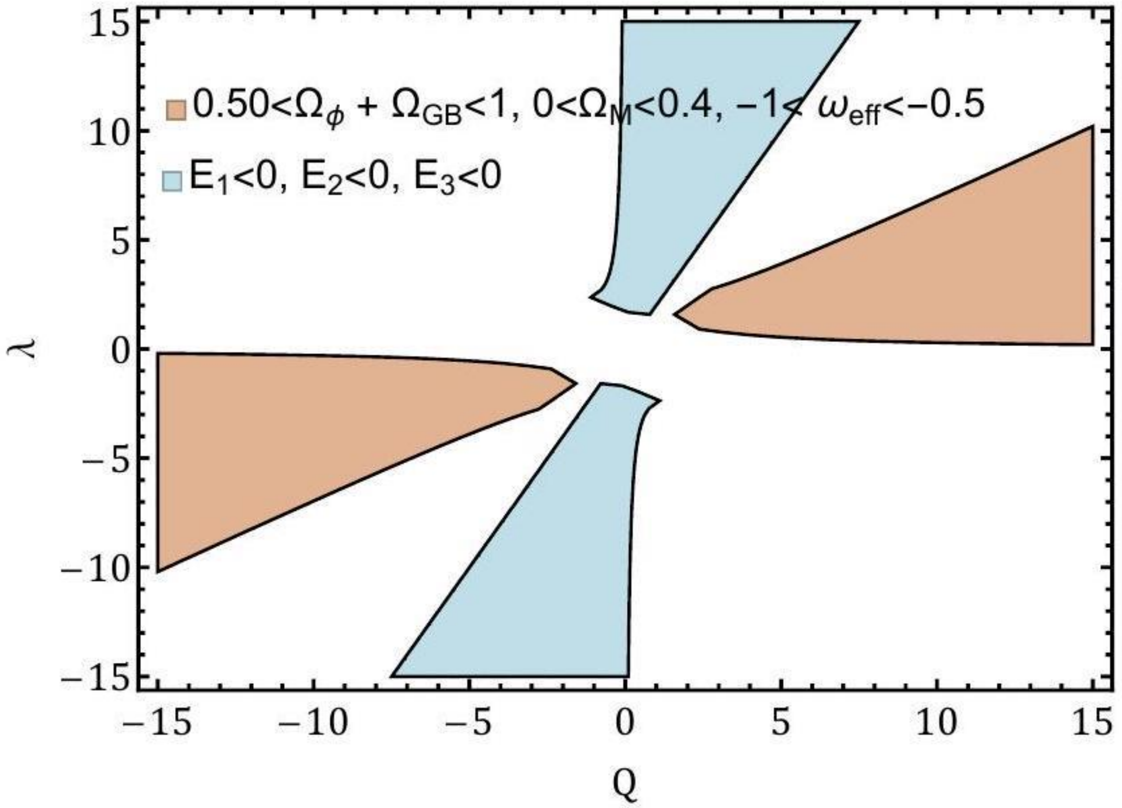
$$\frac{\dot{H}}{H^2} = \frac{1}{1 - \frac{8\mathcal{M}_0 \xi}{1-\xi}} \left(\frac{-3}{2}\Omega_M - 3x^2 \right)$$

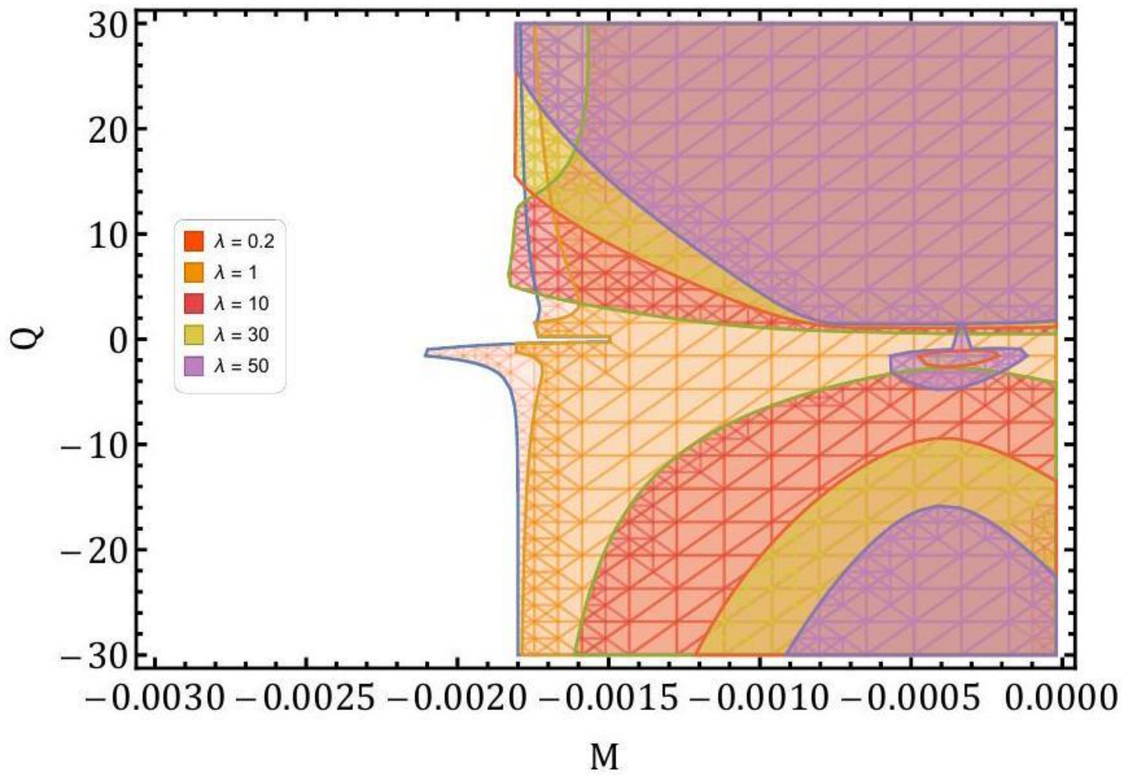
$$\begin{aligned} x' &= \frac{-3Q\Omega_M}{\sqrt{6}} + \frac{3\lambda y^2}{\sqrt{6}} - 3x - \frac{4\mathcal{M}_0 \xi}{(1-\xi)x} - \frac{\dot{H}}{H^2}\left(x + \frac{4\mathcal{M}_0 \xi}{(1-\xi)x}\right) \\ y' &= \frac{-1}{2}\sqrt{6}\lambda x y - y\frac{\dot{H}}{H^2} \\ \xi' &= \xi(1-\xi)\left(1 + \frac{\dot{H}}{H^2}\right) \end{aligned}$$

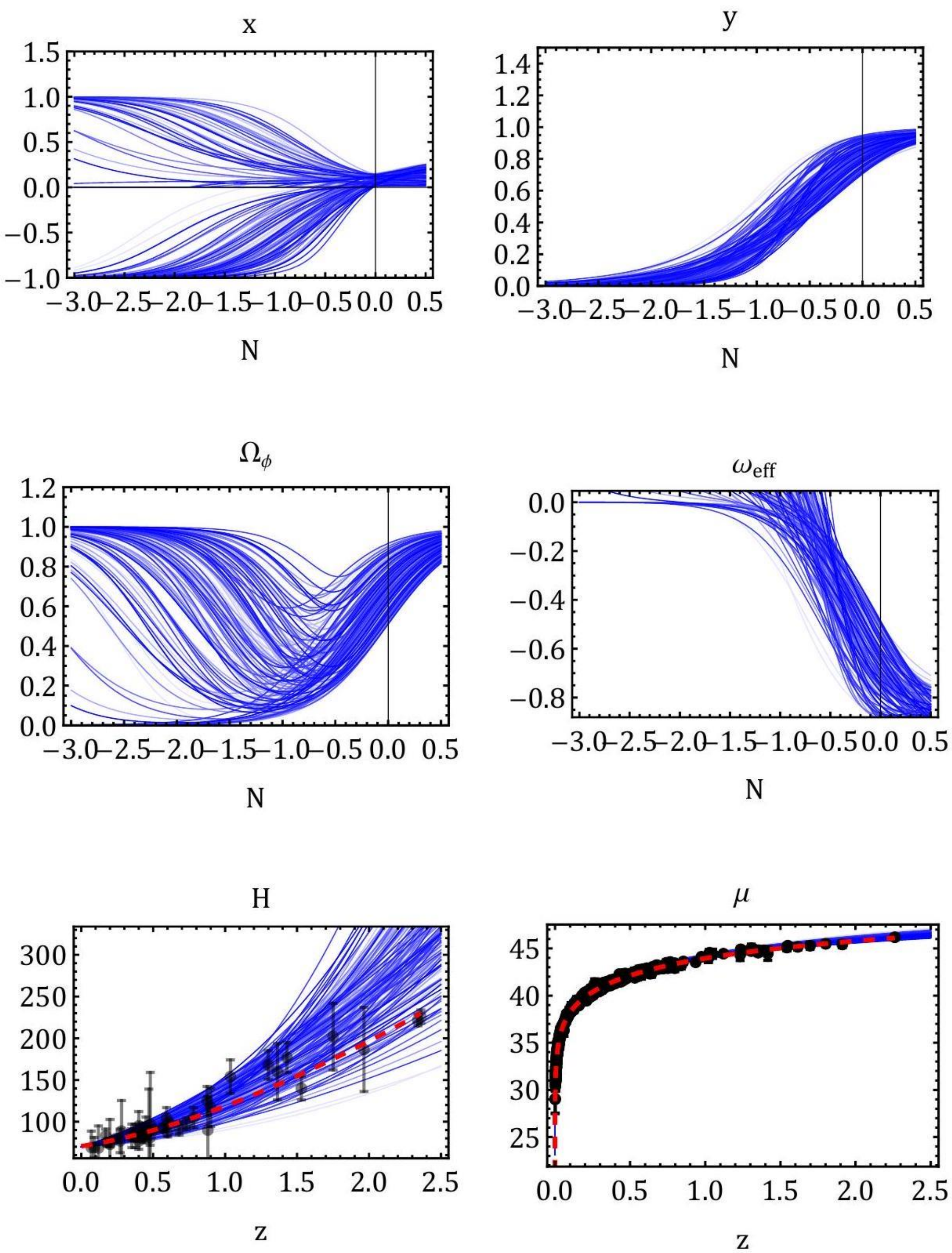
Points	(x_*, y_*, ξ_*)	Ω_ϕ	Ω_{GB}	ω_{eff}
$P_{1,2}$	$(\mp 1, 0, 0)$	1	0	1
P_3	$\left(-\sqrt{\frac{2}{3}}Q, 0, 0\right)$	$\frac{2Q^2}{3}$	0	$\frac{2Q^2}{3}$
P_4	$\left(\frac{\lambda}{\sqrt{6}}, \frac{\sqrt{-\left((\lambda^2-6)(\lambda+Q)\right)}}{\sqrt{6}\sqrt{\lambda+Q}}, 0\right)$	1	0	$\frac{1}{3}\left(\lambda^2 - 3\right)$
P_5	$\left(\frac{\sqrt{\frac{3}{2}}}{\lambda+Q}, \frac{\sqrt{Q^2+\lambda Q+\frac{3}{2}}}{\lambda+Q}, 0\right)$	$\frac{Q^2+\lambda Q+3}{(\lambda+Q)^2}$	0	$-\frac{Q}{\lambda+Q}$
P_6	$\left(\frac{\sqrt{\frac{2}{3}}}{\lambda}, \frac{2}{\sqrt{3}\lambda}, \frac{\lambda^2-2}{\lambda^2(8\mathcal{M}_0+1)-2}\right)$	$\frac{2}{\lambda^2}$	$1 - \frac{2}{\lambda^2}$	$-\frac{1}{3}$
P_7	$\left(-\frac{1}{2\sqrt{6}Q}, \text{Any}, 1\right)$	$\frac{1}{24Q^2} + y_*^2$	$-\frac{8\mathcal{M}\xi_*}{\xi_*-1}$	$-\frac{(\xi_*-1)\left(24Q^2 y_*^2-1\right)}{24Q^2\left(8\mathcal{M}_0\xi_*+\xi_*-1\right)}$

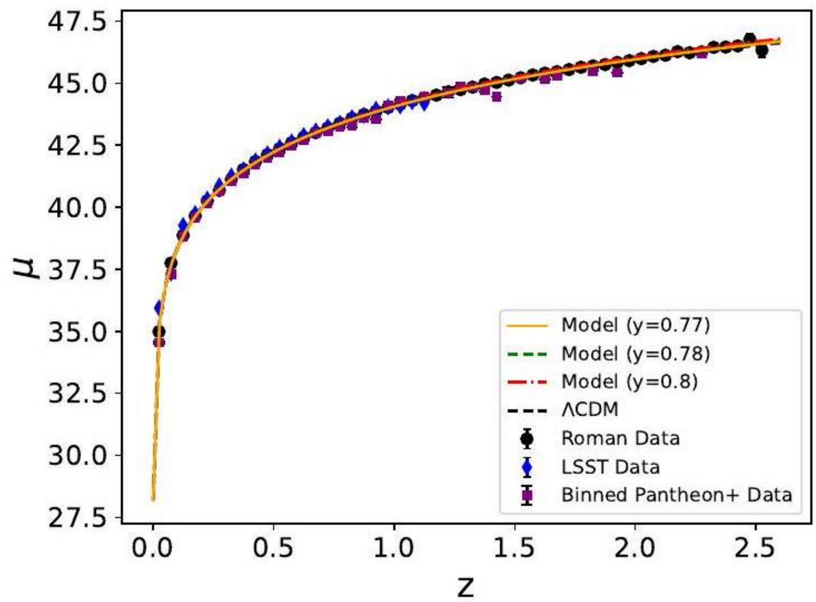
$$E_1 < 0 \rightarrow \left(\lambda > 0, Q < \frac{\lambda}{2}\right)$$

$$\Omega_M = 1 - x^2 - y^2 - \frac{8\mathcal{M}_0\xi}{(1-\xi)}$$

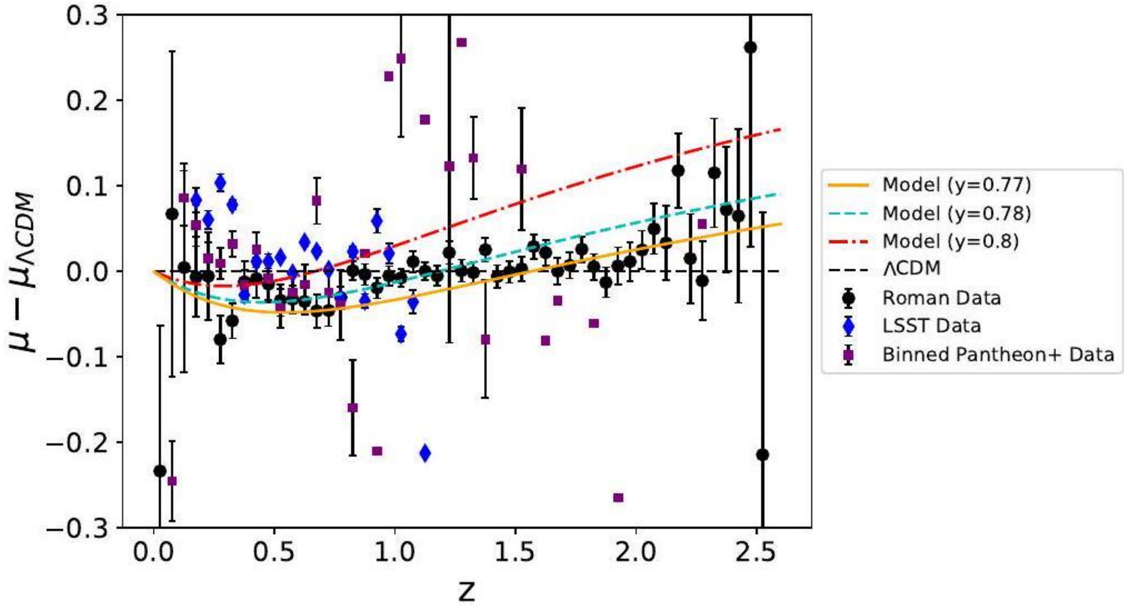





x
N
y
N
Ωϕ
N
ωeff
N
H
z
μ
z

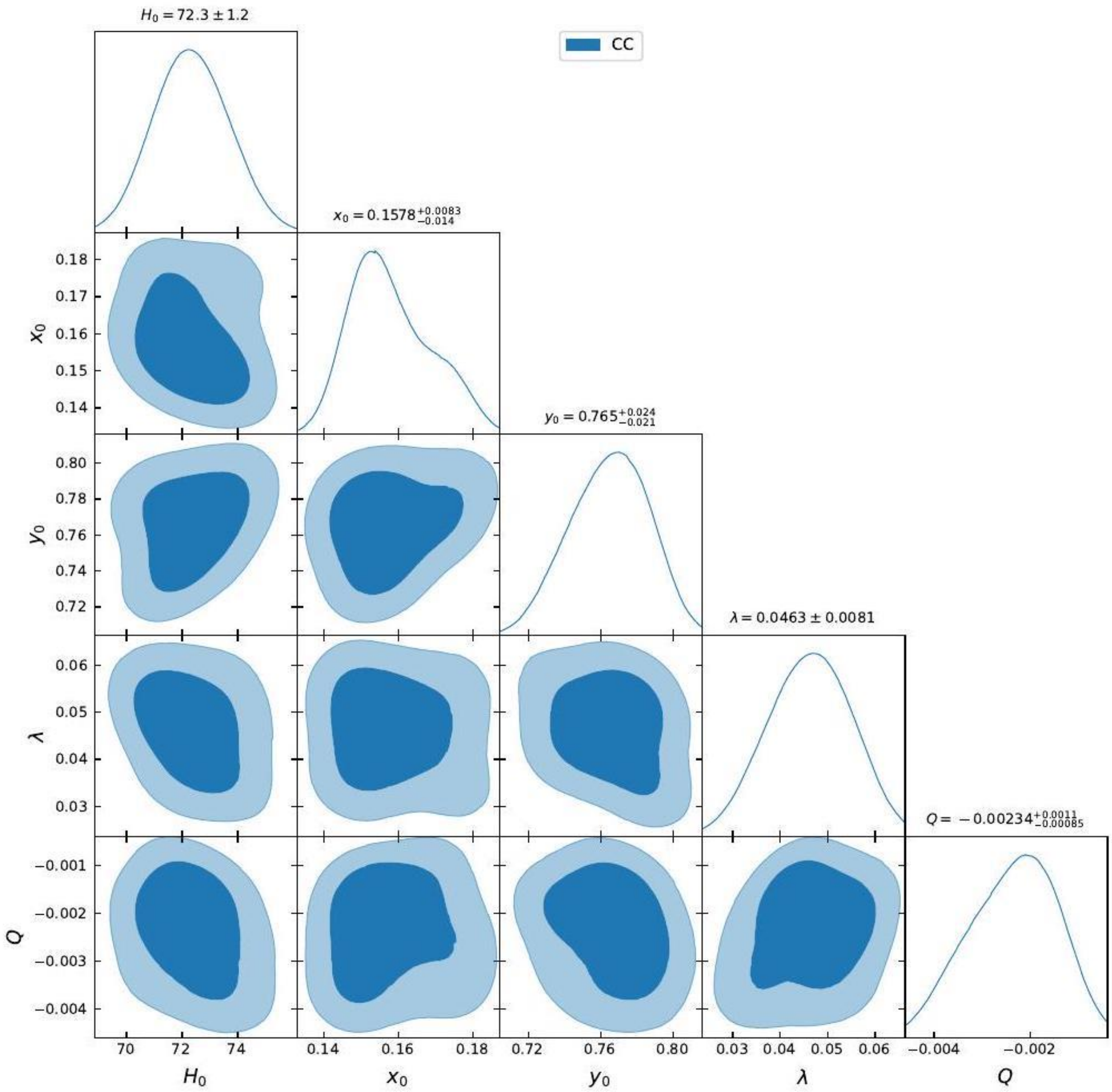


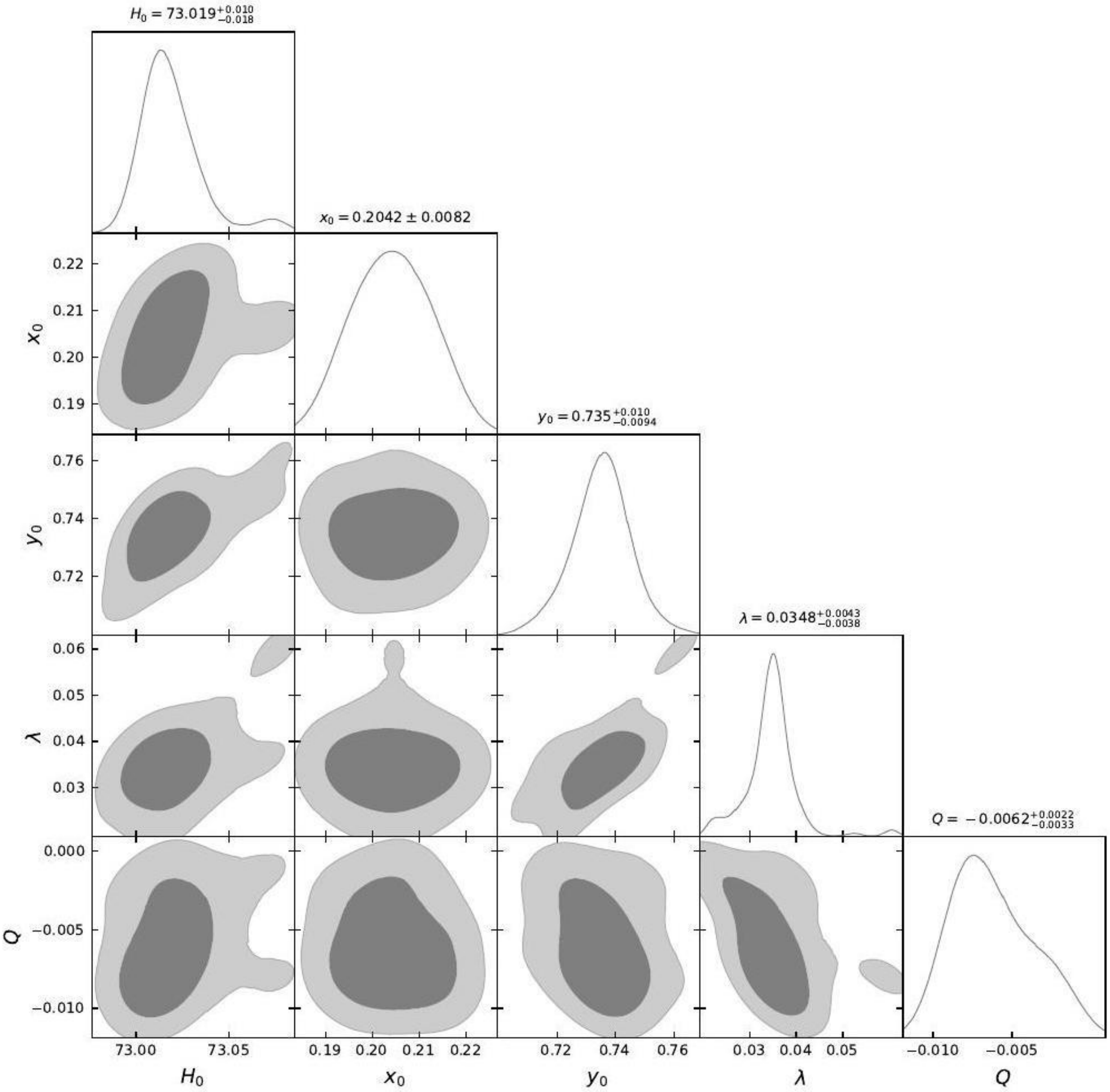
Model (y=0.77)
Model (y=0.78)
Model (y=0.8)
ΛCDM
Roman Data
LSST Data
Binned Pantheon+ Data
μ
z



Model (y=0.77)
Model (y=0.78)
Model (y=0.8)
ΛCDM
Roman Data
LSST Data
Binned Pantheon+ Data
μ − μ_ΛCDM
z

Figuras 44, 45, 46 y 47. Curvatura perturbativa y fluctuaciones gravitacionales extraordinarias.





Figuras 48 y 49. Deformaciones gravitacionales extraordinarias. Parametrización.

SECCIÓN VI.

$$\dot{\rho}_{\mathrm{dm}} + 3H\rho_{\mathrm{dm}} = Q \quad ; \quad \dot{\rho}_{\mathrm{de}} + 3H\rho_{\mathrm{de}}(1+\omega) = -Q$$
$$\dot{\rho}_{\mathrm{bm}} + 3H\rho_{\mathrm{bm}} = 0 \quad ; \quad \dot{\rho}_{\mathrm{r}} + 3H\rho_{\mathrm{r}}(1+1/3) = 0$$

$$Q = \begin{cases} > 0 & \text{Dark Energy} \rightarrow \text{Dark Matter (iDEDM regime)} \\ < 0 & \text{Dark Matter} \rightarrow \text{Dark Energy (iDMDE regime)} \\ = 0 & \text{No interaction.} \end{cases}$$

$$H^2(a) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_{\mathrm{r}} + \rho_{\mathrm{bm}} + \rho_{\mathrm{dm}} + \rho_{\mathrm{de}}) - \frac{kc^2}{a^2}$$
$$q = \Omega_{\mathrm{r}} + \frac{1}{2}(\Omega_{\mathrm{bm}} + \Omega_{\mathrm{dm}}) + \frac{1}{2}\Omega_{\mathrm{de}}(1+3\omega)$$
$$\omega^{\mathrm{eff}} = \frac{P_{\mathrm{tot}}}{\rho_{\mathrm{tot}}} = \frac{\frac{1}{3}\Omega_{\mathrm{r}} + \omega_{\mathrm{de}}\Omega_{\mathrm{de}}}{\Omega_{\mathrm{r}} + \Omega_{\mathrm{bm}} + \Omega_{\mathrm{dm}} + \Omega_{\mathrm{de}}}$$

$$\omega_{\mathrm{dm}}^{\mathrm{eff}} = -\frac{Q}{3H\rho_{\mathrm{dm}}}\ ;\ \omega_{\mathrm{de}}^{\mathrm{eff}} = \omega_{\mathrm{de}} + \frac{Q}{3H\rho_{\mathrm{de}}}.$$

$$Q > 0(\mathrm{iDEDM})\begin{cases}\omega_{\mathrm{dm}}^{\mathrm{eff}} < 0 & \text{Dark matter redshifts slower than } a^{-3}\text{(less DM in past),}\\ \omega_{\mathrm{de}}^{\mathrm{eff}} > \omega_{\mathrm{de}} & \text{Dark energy has less accelerating pressure,}\end{cases}$$
$$Q < 0(\mathrm{iDMDE})\begin{cases}\omega_{\mathrm{dm}}^{\mathrm{eff}} > 0 & \text{Dark matter redshifts faster than } a^{-3}\text{(more DM in past),}\\ \omega_{\mathrm{de}}^{\mathrm{eff}} < \omega_{\mathrm{de}} & \text{Dark energy has more accelerating pressure.}\end{cases}$$

$\omega_{\mathrm{dm}}^{\mathrm{eff}} = \omega_{\mathrm{dm}} = 0$ and $\omega_{\mathrm{de}}^{\mathrm{eff}} = \omega_{\mathrm{de}}$

$$r_{\mathrm{IDE}} = \frac{\rho_{\mathrm{dm}}}{\rho_{\mathrm{de}}} = \frac{\rho_{(\mathrm{dm},0)}a^{-3\left(1+\omega_{\mathrm{dm}}^{\mathrm{eff}}\right)}}{\rho_{(\mathrm{de},0)}a^{-3\left(1+\omega_{\mathrm{de}}^{\mathrm{eff}}\right)}} = r_0 a^{-\zeta_{\mathrm{IDE}}}\ ;\ \text{ with } \zeta_{\mathrm{IDE}} = 3\left(\omega_{\mathrm{dm}}^{\mathrm{eff}} - \omega_{\mathrm{de}}^{\mathrm{eff}}\right)$$

$$\zeta_{\mathrm{IDE}} = 3\left(\omega_{\mathrm{dm}}^{\mathrm{eff}} - \omega_{\mathrm{de}}^{\mathrm{eff}}\right)\begin{cases}Q > 0(\mathrm{iDEDM}): & \zeta_{\mathrm{IDE}} < \zeta_{\Lambda\mathrm{CDM}} & \text{alleviates coincidence problem,}\\ Q < 0(\mathrm{iDMDE}): & \zeta_{\mathrm{IDE}} > \zeta_{\Lambda\mathrm{CDM}} & \text{worsens coincidence problem.}\end{cases}$$

$$\mathbf{d} = \frac{Q}{3H\rho_{\mathrm{de}}(1+\omega)}$$

$$\dot{\Omega}_{\mathrm{x}} = \left[\frac{8\pi\dot{G}}{3H^2}\rho_{\mathrm{x}}\right] = \frac{8\pi G}{3}\left[\frac{\dot{\rho}_{\mathrm{x}}}{H^2} - \rho\frac{2\dot{H}}{H^3}\right] = \frac{8\pi G}{3H^2}\left[\dot{\rho}_{\mathrm{x}} - \rho\frac{2\dot{H}}{H}\right]$$

$$\dot{\rho}_{\mathrm{x}} = -3H\rho_{\mathrm{x}}(1+\omega_{\mathrm{x}}) \pm Q$$

$$\dot{\Omega}_x = \frac{8\pi G}{3H^2}\left[-3H\rho_{\mathrm{x}}(1+\omega_{\mathrm{x}}) \pm Q - \rho_x\frac{2\dot{H}}{H}\right] = \frac{8\pi G}{3H^2}\rho_{\mathrm{x}}H\left[-3\left(1+\omega_{\mathrm{x}} - \frac{2\dot{H}}{H^2}\right) \pm \frac{8\pi G}{3H^2}Q\right.$$

$$\dot{H} = \frac{d}{dt}\dot{a}a^{-1} = \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 \ \rightarrow\ \frac{\dot{H}}{H^2} = \frac{\ddot{a}a}{a^2} - 1 = -q - 1.$$

$$\dot{\Omega}_{\mathrm{x}} = \Omega_{\mathrm{x}}H[-3(1+\omega_{\mathrm{x}}) + 2q + 1] \pm \frac{8\pi G}{3H^2}Q = \Omega_{\mathrm{x}}H[2q - 1 - 3\omega_{\mathrm{x}}] \pm \frac{8\pi G}{3H^2}Q.$$

$$\begin{aligned}\dot{\Omega}_{\mathrm{x}} &= \Omega_{\mathrm{x}}H\left[2\left(\Omega_{\mathrm{r}} + \frac{1}{2}\Omega_{\mathrm{bm}} + \frac{1}{2}\Omega_{\mathrm{dm}} + \frac{1}{2}\Omega_{\mathrm{de}}(1+3\omega_{\mathrm{de}})\right) - 1 - 3\omega_{\mathrm{x}}\right] \pm \frac{8\pi G}{3H^2}Q\\ &= \Omega_{\mathrm{x}}H[2\Omega_{\mathrm{r}} + \Omega_{\mathrm{bm}} + \Omega_{\mathrm{dm}} + \Omega_{\mathrm{de}}(1+3\omega_{\mathrm{de}}) - 1 - 3\omega_{\mathrm{x}}] \pm \frac{8\pi G}{3H^2}Q\end{aligned}$$

$$\begin{aligned}\dot{\Omega}_{\mathrm{de}} &= \Omega_{\mathrm{de}}H[2\Omega_{\mathrm{r}} + \Omega_{\mathrm{bm}} + \Omega_{\mathrm{dm}} + \Omega_{\mathrm{de}}(1+3\omega_{\mathrm{de}}) - 1 - 3\omega_{\mathrm{de}}] - \frac{8\pi G}{3H^2}Q\\ \dot{\Omega}_{\mathrm{dm}} &= \Omega_{\mathrm{dm}}H[2\Omega_{\mathrm{r}} + \Omega_{\mathrm{bm}} + \Omega_{\mathrm{dm}} + \Omega_{\mathrm{de}}(1+3\omega_{\mathrm{de}}) - 1] + \frac{8\pi G}{3H^2}Q\\ \dot{\Omega}_{\mathrm{bm}} &= \Omega_{\mathrm{bm}}H[2\Omega_{\mathrm{r}} + \Omega_{\mathrm{bm}} + \Omega_{\mathrm{dm}} + \Omega_{\mathrm{de}}(1+3\omega_{\mathrm{de}}) - 1]\\ \dot{\Omega}_{\mathrm{r}} &= \Omega_{\mathrm{r}}H[2\Omega_{\mathrm{r}} + \Omega_{\mathrm{bm}} + \Omega_{\mathrm{dm}} + \Omega_{\mathrm{de}}(1+3\omega_{\mathrm{de}}) - 2]\end{aligned}$$

$$\frac{d\Omega_{\mathrm{de}}}{d\Omega_{\mathrm{dm}}} = \frac{\Omega_{\mathrm{de}}H[2\Omega_{\mathrm{r}} + \Omega_{\mathrm{bm}} + \Omega_{\mathrm{dm}} + \Omega_{\mathrm{de}}(1+3\omega_{\mathrm{de}}) - 1 - 3\omega_{\mathrm{de}}] - \frac{8\pi G}{3H^2}Q}{\Omega_{\mathrm{dm}}H[2\Omega_{\mathrm{r}} + \Omega_{\mathrm{bm}} + \Omega_{\mathrm{dm}} + \Omega_{\mathrm{de}}(1+3\omega_{\mathrm{de}}) - 1] + \frac{8\pi G}{3H^2}Q}$$

$$Q = \delta H \rho_{\mathrm{de}}\ ;\ Q = \delta H \rho_{\mathrm{dm}}$$

$$\frac{d\Omega_{\mathrm{de}}}{d\Omega_{\mathrm{dm}}} = \frac{\Omega_{\mathrm{de}}[2\Omega_{\mathrm{r}} + \Omega_{\mathrm{bm}} + \Omega_{\mathrm{dm}} + \Omega_{\mathrm{de}}(1 + 3\omega_{\mathrm{de}}) - 1 - 3\omega_{\mathrm{de}} - \delta]}{\Omega_{\mathrm{dm}}[2\Omega_{\mathrm{r}} + \Omega_{\mathrm{bm}} + \Omega_{\mathrm{dm}} + \Omega_{\mathrm{de}}(1 + 3\omega_{\mathrm{de}}) - 1] + \delta\Omega_{\mathrm{de}}}$$

$$\frac{d\Omega_{\mathrm{de}}}{d\Omega_{\mathrm{m}}} = \frac{\Omega_{\mathrm{de}}[\Omega_{\mathrm{m}} + \Omega_{\mathrm{de}}(1 + 3\omega_{\mathrm{de}}) - 1 - 3\omega_{\mathrm{de}} - \delta]}{\Omega_{\mathrm{m}}[\Omega_{\mathrm{m}} + \Omega_{\mathrm{de}}(1 + 3\omega_{\mathrm{de}}) - 1] + \delta\Omega_{\mathrm{de}}}$$

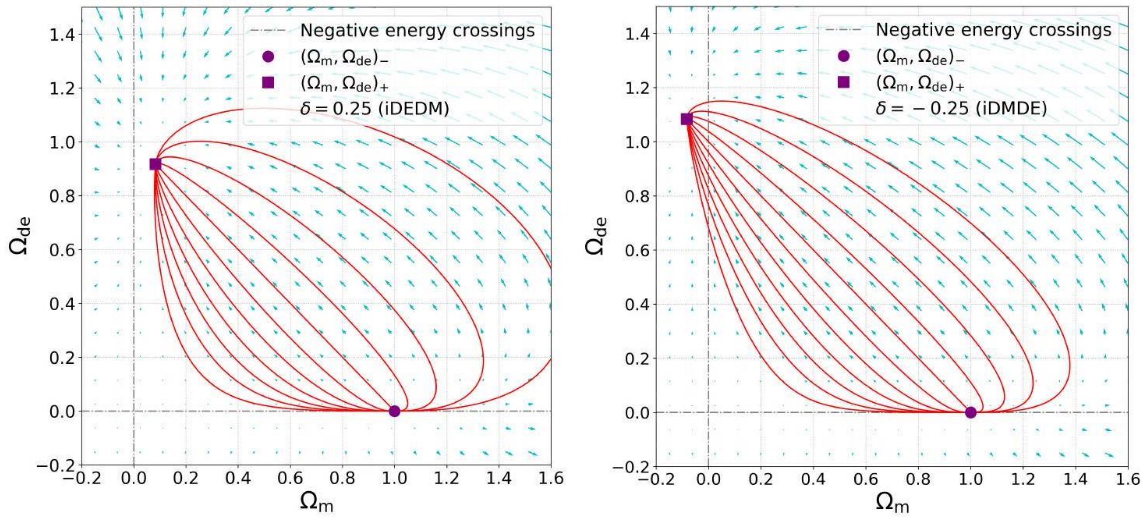


Figura 50. Deformaciones gravitacionales en pluridimensionalidad.

$$(\Omega_{\mathrm{m}}, \Omega_{\mathrm{de}})_- = (1{,}0)\ ;\ (\Omega_{\mathrm{m}}, \Omega_{\mathrm{de}})_+ = \left(-\frac{\delta}{3\omega}, 1 + \frac{\delta}{3\omega}\right).$$

$$\left(-\frac{\delta}{3\omega}, 1 + \frac{\delta}{3\omega}\right)$$

$$r_- = \frac{\Omega_{(\mathrm{m},-)}}{\Omega_{(\mathrm{de},-)}} = \frac{1}{0} \to \infty\ ;\ r_+ = \frac{\Omega_{(\mathrm{dm},+)}}{\Omega_{(\mathrm{de},-)}} \approx \frac{\Omega_{(\mathrm{m},+)}}{\Omega_{(de,+)}} = \frac{-\frac{\delta}{3\omega}}{1 + \frac{\delta}{3\omega}} \to -\frac{\delta}{\delta + 3\omega}$$

$$\rho_{\mathrm{dm}} = \left(\rho_{(\mathrm{dm},0)} + \rho_{(\mathrm{de},0)} \frac{\delta}{\delta + 3\omega}\left[1 - a^{-(\delta+3\omega)}\right]\right) a^{-3}$$
$$\rho_{\mathrm{de}} = \rho_{(\mathrm{de},0)} a^{-(\delta+3\omega+3)}$$

$$\omega_{\mathrm{dm}}^{\mathrm{eff}} = -\frac{Q}{3H\rho_{\mathrm{de}}} = -\frac{\delta H \rho_{\mathrm{de}}}{3H\rho_{\mathrm{dm}}} = -\frac{\delta}{3}\frac{\rho_{\mathrm{de}}}{\rho_{\mathrm{dm}}} = -\frac{\delta}{3}\frac{1}{r}$$

$$\omega_{\mathrm{de}}^{\mathrm{eff}} = \omega + \frac{Q}{3H\rho_{\mathrm{de}}} = \omega + \frac{\delta H \rho_{\mathrm{de}}}{3H\rho_{\mathrm{de}}} = \omega + \frac{\delta}{3}$$

$$\Omega_{\mathrm{dm}} = \frac{H_0^2}{H^2}\left(\Omega_{(\mathrm{dm},0)} + \Omega_{(\mathrm{de},0)}\frac{\delta}{\delta+3\omega}\left[1-(1+z)^{(\delta+3\omega)}\right]\right)(1+z)^3$$

$$\Omega_{\mathrm{de}} = \frac{H_0^2}{H^2}\Omega_{(\mathrm{de},0)}(1+z)^{(\delta+3\omega+3)}$$

$$\Omega_{\mathrm{bm}} = \frac{H_0^2}{H^2}\Omega_{(\mathrm{bm},0)}(1+z)^3$$

$$\Omega_{\mathrm{r}} = \frac{H_0^2}{H^2}\Omega_{(\mathrm{r},0)}(1+z)^4$$

$$a^{-(\delta+3\omega)} = 1 + r_0\left(\frac{\delta+3\omega}{\delta}\right)$$

$$z_{(\mathrm{dm}=0)} = \left[1 + r_0\left(\frac{\delta+3\omega}{\delta}\right)\right]^{\frac{1}{\delta+3\omega}} - 1$$

$$a^{-(\delta+3\omega)} = 1 + r_0\left(\frac{\delta+3\omega}{\delta}\right) \quad \text{where} \;\; (\delta+3\omega<0)$$

$$\delta<0 \Rightarrow \begin{cases} \text{Past} & (a<1) \;\; \rightarrow \;\; (0<\text{ L.H.S. }<1\,;\;\; \text{R.H.S. }>1) \\ \text{Future} & (a>1) \;\; \rightarrow \;\; (\text{L.H.S. }>1\,;\;\; \text{R.H.S. }>1) \end{cases}$$

$$\delta>0 \Rightarrow \begin{cases} \text{Past} & (a<1) \;\; \rightarrow \;\; (0<\text{ L.H.S. }<1\,;\;\; \text{R.H.S. }<1) \\ \text{Future} & (a>1) \;\; \rightarrow \;\; (\text{L.H.S. }>1\,;\;\; \text{R.H.S. }<1) \end{cases}$$

$$1 + r_0\left(\frac{\delta+3\omega}{\delta}\right) < 0 \;\rightarrow\; \delta < -\frac{3\omega}{\left(1+\frac{1}{r_0}\right)}$$

$$0 < \delta < -\frac{3\omega}{\left(1+\frac{1}{r_0}\right)}$$

$$r(z) = \frac{\rho_{\mathrm{dm}}(z)}{\rho_{\mathrm{de}}(z)} = \left(r_0 + \frac{\delta}{\delta+3\omega}\right)(1+z)^{-(\delta+3\omega)} - \frac{\delta}{\delta+3\omega}$$

$$r \propto a^{(\delta+3\omega)} \;\rightarrow\; \zeta_{\mathrm{Q}_1} = \zeta_{\mathrm{Q}} = -3\omega - \delta$$

$$\zeta_{\mathrm{Q}} = -3\omega-\delta \rightarrow \begin{cases} \text{if } \delta>0(\text{iDEDM}) & \rightarrow \;\; \zeta_{\mathrm{Q}}<\zeta \\ \text{if } \delta<0(\text{iDMDE}) & \rightarrow \;\; \zeta_{\mathrm{Q}}>\zeta \end{cases} \text{worsens coincidence problem.}$$

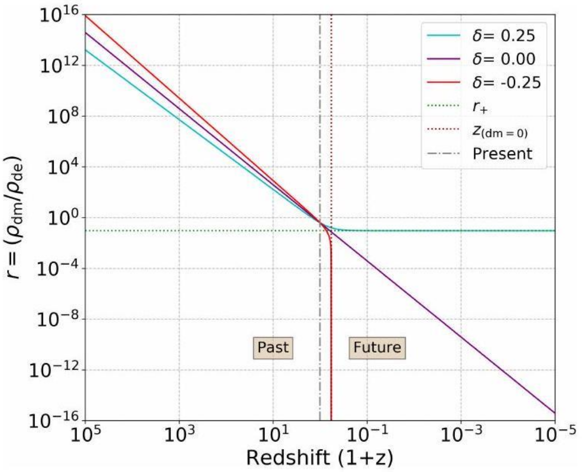
$$\lim_{(1+z)\to\infty} r_- \to \infty,; \;\; \lim_{(1+z)\to 0} r_+ =\to -\frac{\delta}{\delta+3\omega}$$

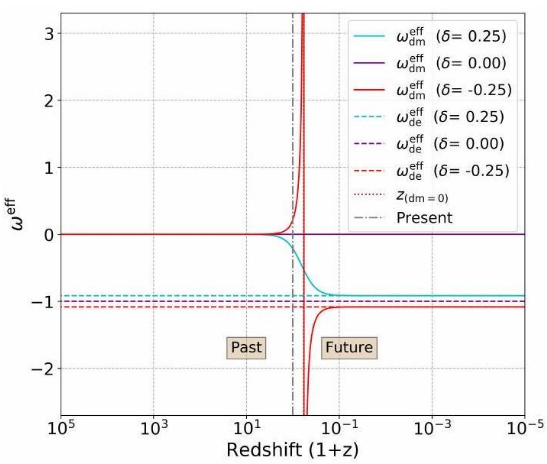
$$\lim_{(1+z)\to 0} r_+ \propto a^0 \;\rightarrow\; \zeta_{(\mathrm{Q},-)} = 0$$

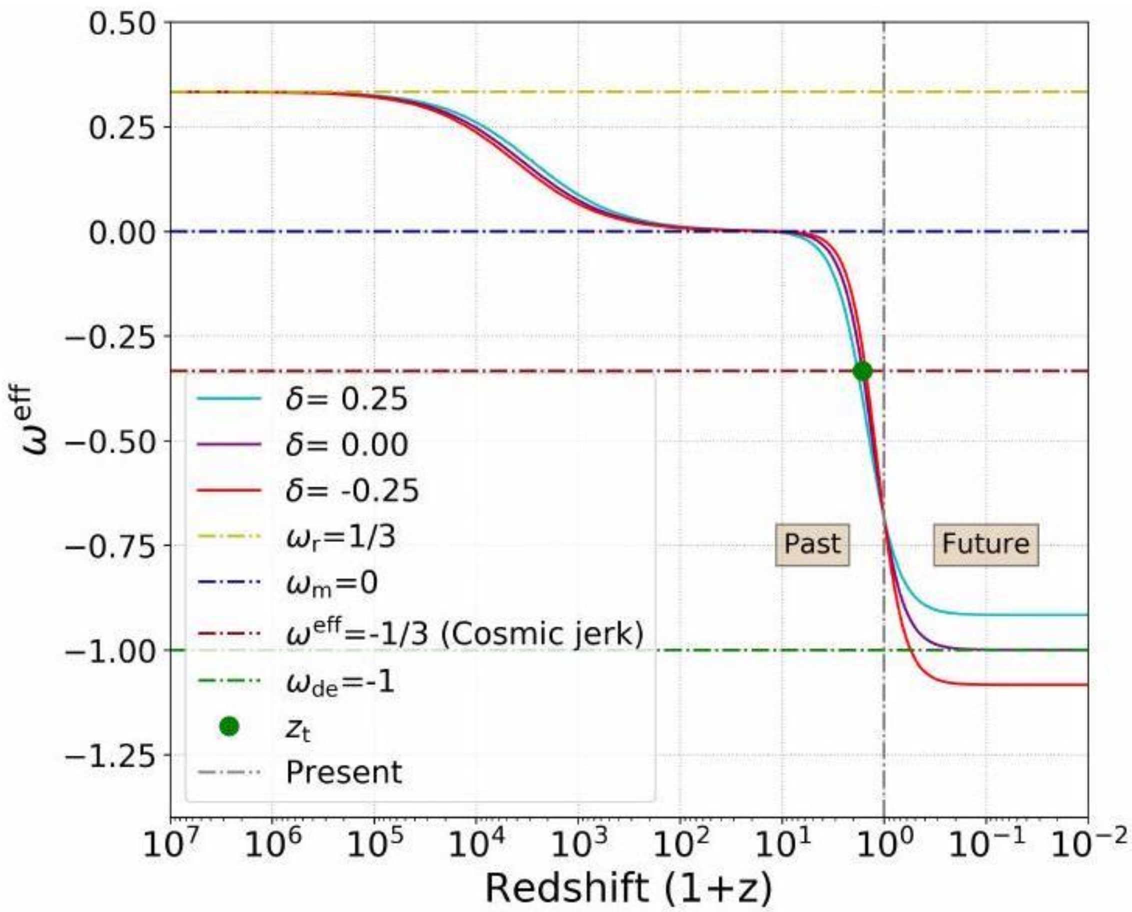
$$\lim_{(1+z)\to 0}\zeta_{\mathrm{Q}} = 0\begin{cases} \text{if } \delta>0 \rightarrow r_- = +\text{constant} & \text{solves coincidence problem} \\ \text{if } \delta<0 \rightarrow r_- = -\text{constant} & \text{negative energy densities (unphysical).} \end{cases}$$

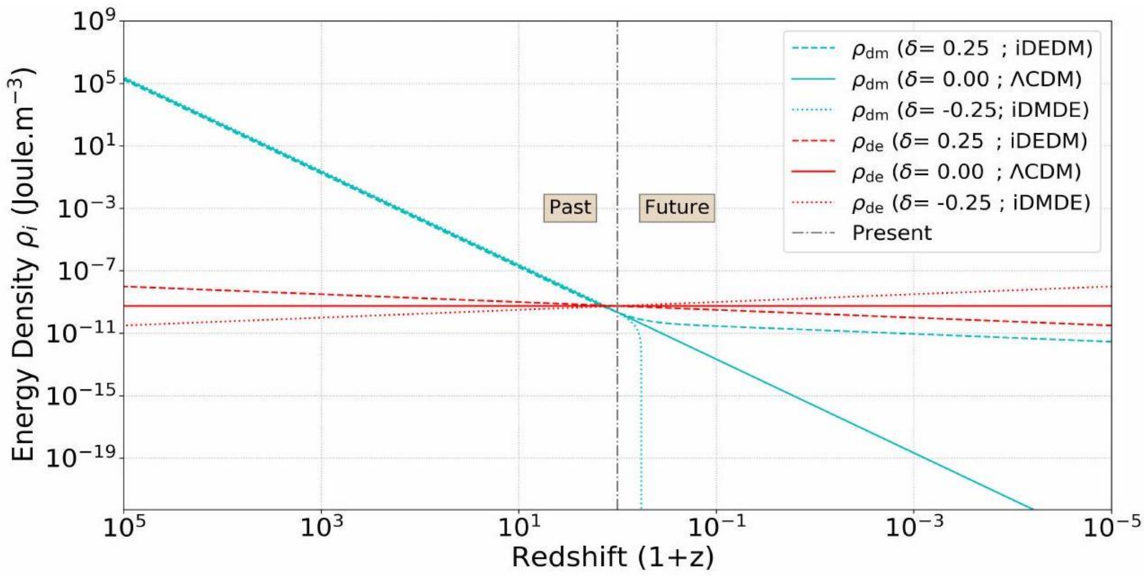
$$\omega_{\mathrm{dm}}^{\mathrm{eff}} = -\frac{\delta}{3}\frac{1}{r} = -\frac{\delta}{3}\frac{1}{\left(r_0+\frac{\delta}{\delta+3\omega}\right)(1+z)^{-(\delta+3\omega)} - \frac{\delta}{\delta+3\omega}}$$

$$\omega_{\mathrm{dm}}^{\mathrm{eff}} = -\frac{\delta}{3}\frac{1}{r}\begin{cases} \text{Distant past } (r=r_{-}): & \omega_{\mathrm{dm}}^{\mathrm{eff}} = -\frac{\delta}{3}\frac{1}{\infty} = 0 = \omega_{\mathrm{dm}} \\ \text{Distant future } (r=r_{+}): & \omega_{\mathrm{dm}}^{\mathrm{eff}} = -\frac{\delta}{3}\left(\frac{\delta}{\delta+3\omega}\right) = \omega + \frac{\delta}{3} = \omega_{\mathrm{de}}^{\mathrm{eff}} \end{cases}$$

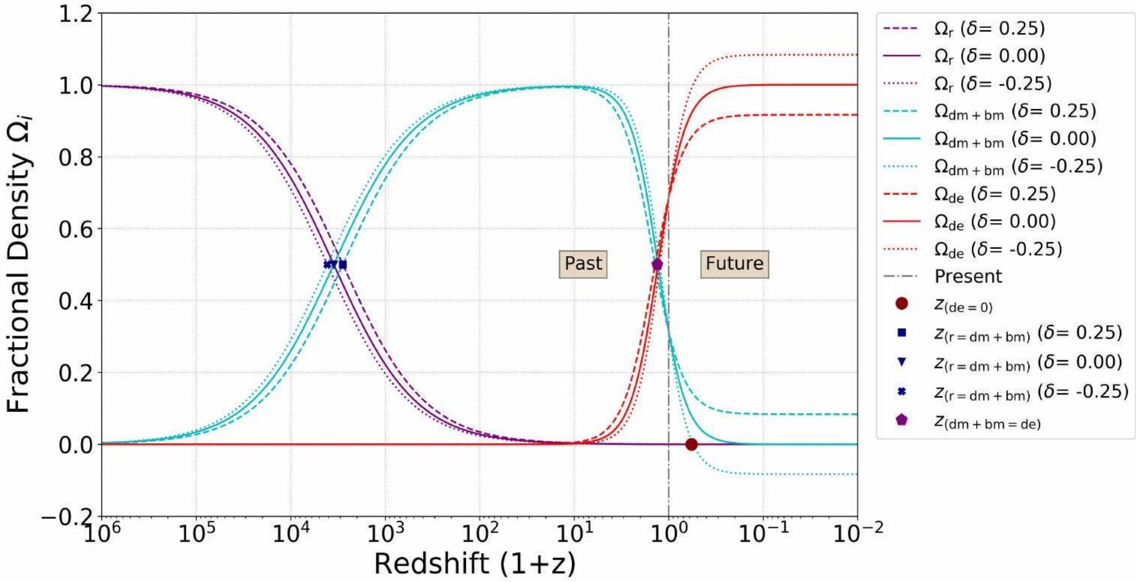




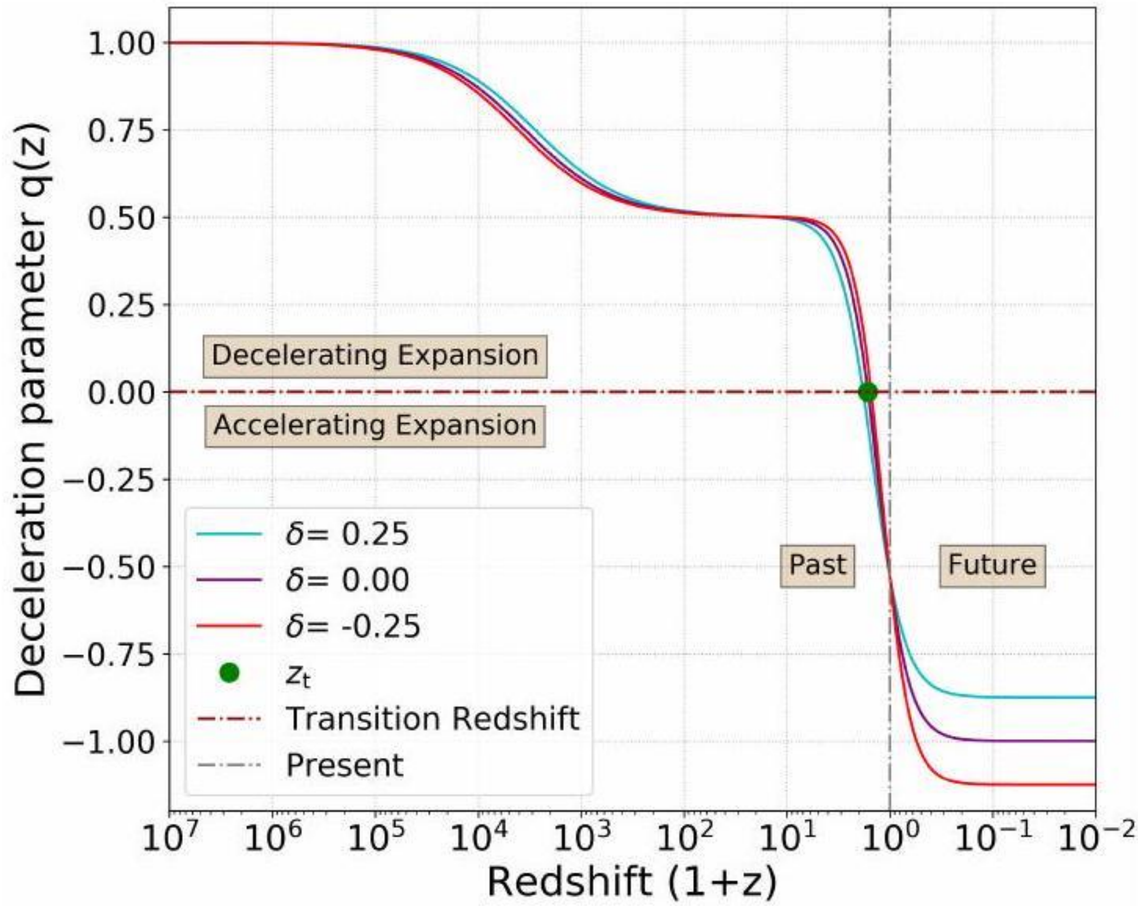




Energy Density ρ_i (Joule.m^{-3})
Redshift (1+z)
ρ_{dm} (δ= 0.25 ; iDEDM)
ρ_{dm} (δ= 0.00 ; ΛCDM)
ρ_{dm} (δ= -0.25; iDMDE)
ρ_{de} (δ= 0.25 ; iDEDM)
ρ_{de} (δ= 0.00 ; ΛCDM)
ρ_{de} (δ= -0.25 ; iDMDE)
Present
Past
Future



Fractional Density Ω_i
Redshift (1+z)
Past
Future
Ω_r (δ= 0.25)
Ω_r (δ= 0.00)
Ω_r (δ= -0.25)
Ω_{dm+bm} (δ= 0.25)
Ω_{dm+bm} (δ= 0.00)
Ω_{dm+bm} (δ= -0.25)
Ω_{de} (δ= 0.25)
Ω_{de} (δ= 0.00)
Ω_{de} (δ= -0.25)
Present
$z_{(de=0)}$
$z_{(r=dm+bm)}$ (δ= 0.25)
$z_{(r=dm+bm)}$ (δ= 0.00)
$z_{(r=dm+bm)}$ (δ= -0.25)
$z_{(dm+bm=de)}$



Figuras 51, 52, 53, 54 y 55. Multidimensiones por aceleración y desaceleración inflacionarias de un campo cuántico – relativista oscuro.

$$\delta > 0(\mathrm{iDEDM})\begin{cases} \text{Past expansion:} & \omega_{\mathrm{de}}^{\mathrm{eff}} > \omega_{\mathrm{de}}(\zeta_Q < \zeta) & \text{alleviates coincidence problem} \\ \text{Future expansion:} & \omega_{\mathrm{dm}}^{\mathrm{eff}} = \omega_{\mathrm{de}}^{\mathrm{eff}}(\zeta_Q = 0) & \text{solves coincidence problem,} \end{cases}$$

$$\delta < 0(\mathrm{iDMDE})\begin{cases} \text{Past expansion:} & \omega_{\mathrm{de}}^{\mathrm{eff}} < \omega_{\mathrm{de}}(\zeta_Q > \zeta) & \text{worsens coincidence problem} \\ \text{Future expansion:} & \omega_{\mathrm{dm}}^{\mathrm{eff}} = \omega_{\mathrm{de}}^{\mathrm{eff}}(\rho_{\mathrm{de}} < 0) & \text{negative energy densities.} \end{cases}$$

$$z_{(\mathrm{r=dm+bm})} \approx \left(\frac{\Omega_{(\mathrm{bm},0)} + \Omega_{(\mathrm{dm},0)} + \Omega_{(\mathrm{de},0)}\dfrac{\delta}{\delta + 3\omega}}{\Omega_{(\mathrm{r},0)}}\right) - 1.$$

$$z_{(\mathrm{dm+bm=de})} = \left(\frac{\dfrac{\Omega_{(\mathrm{bm},0)} + \Omega_{(\mathrm{dm},0)}}{\Omega_{(\mathrm{de},0)}} + \dfrac{\delta}{\delta + 3\omega}}{\left(1 + \dfrac{\delta}{\delta + 3\omega}\right)}\right)^{\left(\frac{1}{\delta+3\omega}\right)} - 1.$$

$$\delta > 0(\mathrm{iDEDM})\begin{cases} \text{Radiation-matter equality:} & z_{\mathrm{IDE}} < z_{\Lambda\mathrm{CDM}} & \text{happens later than } \Lambda\mathrm{CDM} \\ \text{Matter-dark energy equality:} & z_{\mathrm{IDE}} > z_{\Lambda\mathrm{CDM}} & \text{happens earlier than } \Lambda\mathrm{CDM,} \end{cases}$$

$$\delta < 0(\mathrm{iDMDE})\begin{cases} \text{Radiation-matter equality:} & z_{\mathrm{IDE}} > z_{\Lambda\mathrm{CDM}} & \text{happens earlier than } \Lambda\mathrm{CDM} \\ \text{Matter-dark energy equality:} & z_{\mathrm{IDE}} < z_{\Lambda\mathrm{CDM}} & \text{happens later than } \Lambda\mathrm{CDM,} \end{cases}$$

$$(\Omega_{\mathrm{dm}},\Omega_{\mathrm{de}})_{-}=(1,0),\ (\Omega_{\mathrm{dm+bm}},\Omega_{\mathrm{de}})_{+}=\left(-\frac{\delta}{3\omega},1+\frac{\delta}{3\omega}\right)$$

$$\rightarrow z_{\mathrm{t}}=\left(-\frac{\frac{\Omega_{(\mathrm{bm},0)}+\Omega_{(\mathrm{dm},0)}}{\Omega_{(\mathrm{de},0)}}+\frac{\delta}{\delta+3\omega}}{1+3\omega+\frac{\delta}{\delta+3\omega}}\right)^{\left(\frac{1}{\delta+3\omega}\right)}-1$$

$$\text{Cosmic jerk } (z_{\mathrm{t}})\begin{cases}\delta>0(\mathrm{iDEDM}): & z_{\mathrm{IDE}}>z_{\Lambda\mathrm{CDM}} \quad \text{happens earlier than } \Lambda\mathrm{CDM},\\ \delta<0(\mathrm{iDMDE}): & z_{\mathrm{IDE}}<z_{\Lambda\mathrm{CDM}} \quad \text{happens later than } \Lambda\mathrm{CDM}.\end{cases}$$

$$\omega_{+}^{\mathrm{eff}}=\frac{\frac{1}{3}\Omega_{\mathrm{r}}+\omega\Omega_{\mathrm{de}}}{\Omega_{\mathrm{r}}+\Omega_{\mathrm{bm}}+\Omega_{\mathrm{dm}}+\Omega_{\mathrm{de}}}=\frac{\frac{1}{3}(0)+\omega\left(1+\frac{\delta}{3\omega}\right)}{(0)+\left(-\frac{\delta}{3\omega}\right)+\left(1+\frac{\delta}{3\omega}\right)}=\frac{\omega+\frac{\delta}{3}}{1}=\omega+\frac{\delta}{3}=\omega_{\mathrm{de}}^{\mathrm{eff}}$$

$$\mathbf{d}=\frac{Q}{3H\rho_{\mathrm{de}}(1+\omega)}=\frac{\delta H\rho_{\mathrm{de}}}{3H\rho_{\mathrm{de}}(1+\omega)}=\frac{\delta}{3(1+\omega)},$$

$$\mathbf{d}<0\begin{cases}\delta<0 \ ; \ \omega>-1 & \text{(Quintessence regime)}\\ \delta>0 \ ; \ \omega<-1 & \text{(Phantom regime)}\end{cases}\rightarrow \quad \text{No instabilities expected}$$

$$\mathbf{d}>0\begin{cases}\delta>0 \ ; \ \omega>-1\\ \delta<0 \ ; \ \omega<-1\end{cases}\text{(Quintessence regime)} \rightarrow \text{Instabilities can develop if } \mathbf{d}>1$$

$$\rho_{\mathrm{de}}=\rho_{(\mathrm{de},0)}a^{-3\left(1+\omega+\frac{\delta}{3}\right)},-3\left(1+\omega+\frac{\delta}{3}\right)>0 \ \text{ if } \ \omega_{\mathrm{de}}^{\mathrm{eff}}=\omega+\frac{\delta}{3}<-1$$

$$t_{rip}\approx-\frac{2}{3H_0\left(1+\omega+\frac{\delta}{3}\right)\sqrt{\left(1-\frac{\delta}{\delta+3\omega}\right)\left(1-\Omega_{(\mathrm{dm+bm},0)}\right)}}$$

$$3(\omega+1)<\delta<-\frac{3\omega}{\left(1+\frac{1}{r_0}\right)} \quad \text{with} \ \ \omega_{\mathrm{de}}<-1$$

$$\rho_{\mathrm{dm}}=\rho_{(\mathrm{dm},0)}a^{(\delta-3)}$$
$$\rho_{\mathrm{de}}=\left[\rho_{(\mathrm{de},0)}+\rho_{(\mathrm{dm},0)}\frac{\delta}{\delta+3\omega}\left(1-a^{\delta+3\omega}\right)\right]a^{-3(1+\omega)}$$

$$\omega_{\mathrm{dm}}^{\mathrm{eff}}=-\frac{\delta}{3},\omega_{\mathrm{de}}^{\mathrm{eff}}=\omega_{\mathrm{de}}+\frac{\delta}{3}r$$

$$r(z)=\frac{1}{\left(\frac{1}{r_0}+\frac{\delta}{\delta+3\omega}\right)(1+z)^{(\delta+3\omega)}-\frac{\delta}{\delta+3\omega}},$$

$$0<\delta<-\frac{3\omega}{(1+r_0)}$$

$$z_{(\mathrm{de}=0)} = \left[1 + \frac{1}{r_0}\left(\frac{\delta + 3\omega}{\delta}\right)\right]^{-\frac{1}{\delta+3\omega}} - 1$$

$$\delta > 0(\mathrm{iDEDM})\begin{cases} \text{Past expansion:} & \omega_{\mathrm{dm}}^{\mathrm{eff}} = \omega_{\mathrm{de}}^{\mathrm{eff}}(\zeta_{\mathrm{Q}} = 0) & \text{solves coincidence problem} \\ \text{Future expansion:} & \omega_{\mathrm{dm}}^{\mathrm{eff}} < \omega_{\mathrm{dm}}(\zeta_{\mathrm{Q}} < \zeta) & \text{alleviates coincidence problem,} \end{cases}$$

$$\delta < 0(\mathrm{iDMDE})\begin{cases} \text{Past expansion:} & \omega_{\mathrm{dm}}^{\mathrm{eff}} = \omega_{\mathrm{de}}^{\mathrm{eff}}(\rho_{\mathrm{de}} < 0) & \text{negative energy densities} \\ \text{Future expansion:} & \omega_{\mathrm{dm}}^{\mathrm{eff}} > \omega_{\mathrm{dm}}(\zeta_{\mathrm{Q}} > \zeta) & \text{worsens coincidence problem.} \end{cases}$$

$$\mathbf{d} = \frac{Q}{3H\rho_{\mathrm{de}}(1+\omega)} = \frac{\delta H \rho_{\mathrm{dm}}}{3H\rho_{\mathrm{de}}(1+\omega)} = \frac{\delta}{3(1+\omega)}\frac{\rho_{\mathrm{dm}}}{\rho_{\mathrm{de}}},$$

$$t_{\mathrm{rip}} \approx -\frac{2}{3H_0(1+\omega)\sqrt{1 - \Omega_{(\mathrm{bm},0)} - \left(1 - \frac{\delta}{\delta+3\omega}\right)\Omega_{(\mathrm{dm},0)}}},$$

$$\left(\frac{\dot{a}}{a}\right) \approx H_0\sqrt{\left(\Omega_{(\mathrm{dm},0)} + \Omega_{(\mathrm{de},0)}\frac{\delta}{\delta+3\omega}\left[1 - a^{-(\delta+3\omega)}\right]\right)a^{-3} + \Omega_{(\mathrm{de},0)}a^{-3\left(1+\omega+\frac{\delta}{3}\right)}}$$

$$= H_0\sqrt{\left(\Omega_{(\mathrm{dm},0)} + \Omega_{(\mathrm{de},0)}\frac{\delta}{\delta+3\omega}\right)a^{-3} + \left(1 - \frac{\delta}{\delta+3\omega}\right)\Omega_{(\mathrm{de},0)}a^{-3\left(1+\omega+\frac{\delta}{3}\right)}}$$

$$\left(\frac{\dot{a}}{a}\right) \approx H_0\sqrt{\left(1 - \frac{\delta}{\delta+3\omega}\right)\Omega_{(\mathrm{de},0)}a^{-3\left(1+\omega+\frac{\delta}{3}\right)}}$$

$$\left(\frac{\dot{a}}{a}\right) \approx H_0\sqrt{\left(1 - \frac{\delta}{\delta+3\omega}\right)\left(1 - \Omega_{(\mathrm{dm}+\mathrm{bm},0)}\right)}a^{-\frac{3}{2}\left(1+\omega+\frac{\delta}{3}\right)}$$

$$\frac{da}{dt} = H_0\sqrt{\left(1 - \frac{\delta}{\delta+3\omega}\right)\left(1 - \Omega_{(\mathrm{dm}+\mathrm{bm},0)}\right)}a^{-\frac{1}{2}(1+3\omega+\delta)}$$

$$\int_{t_0}^{t_{\mathrm{rip}}} dt = \frac{1}{H_0}\frac{1}{\sqrt{\left(1 - \frac{\delta}{\delta+3\omega}\right)\left(1 - \Omega_{(\mathrm{dm}+\mathrm{bm},0)}\right)}}\int_1^{\infty} a^{-\frac{1}{2}(1+3\omega+\delta)}da$$

$$t_{\mathrm{rip}} - t_0 = \frac{1}{H_0}\frac{1}{\sqrt{\left(1 - \frac{\delta}{\delta+3\omega}\right)\left(1 - \Omega_{(\mathrm{dm}+\mathrm{bm},0)}\right)}}\frac{2}{3\left(1+\omega+\frac{\delta}{3}\right)}a^{\frac{3(1+\omega+\delta/3)}{2}}\Bigg|_1^{\infty}$$

$$t_{\mathrm{rip}} - t_0 = \frac{2}{3H_0\left(1+\omega+\frac{\delta}{3}\right)\sqrt{\left(1 - \frac{\delta}{\delta+3\omega}\right)\left(1 - \Omega_{(\mathrm{dm}+\mathrm{bm},0)}\right)}}\left(0 - 1^{\frac{3(1+\omega+\delta/3)}{2}}\right)$$

$$\to t_{\mathrm{rip}} - t_0 = -\frac{2}{3H_0\left(1+\omega+\frac{\delta}{3}\right)\sqrt{\left(1 - \frac{\delta}{\delta+3\omega}\right)\left(1 - \Omega_{(\mathrm{dm}+\mathrm{bm},0)}\right)}}$$

SECCIÓN VII.

$$\rho_\Lambda(a) = \frac{3H_0^2}{8\pi G_N}\Omega_\Lambda(a) = \frac{\Lambda(a)}{8\pi G_N} = \int_{\tilde{E}_{\rm IR}(a)}^{\tilde{E}_{\rm UV}(a)} d\tilde{E} I(\tilde{E})$$

$$\begin{aligned} S &= -\int_x \int_{\tilde{x}} \sqrt{-g(x)}\sqrt{-\tilde{g}(\tilde{x})}[R(x) + \tilde{R}(\tilde{x}) + \cdots] \\ &= -\int_x \sqrt{-g(x)}\left[R(x)\int_{\tilde{x}} \sqrt{-\tilde{g}(\tilde{x})} + \int_{\tilde{x}} \sqrt{-\tilde{g}(\tilde{x})}\tilde{R}(\tilde{x}) + \cdots\right] \end{aligned}$$

$$1/G_N \sim \int_{\tilde{x}} \sqrt{-\tilde{g}(\tilde{x})}$$

$$\Lambda/G_N \sim \int_{\tilde{x}} \sqrt{-\tilde{g}(\tilde{x})}\tilde{R}(\tilde{x})$$

$$\rho_\Lambda(a) = \frac{\Lambda(a)}{8\pi G_N}$$

$$\rho_\Lambda(a) = \int_{\tilde{E}_{\rm IR}(a)}^{\tilde{E}_{\rm UV}(a)} d\tilde{E} I(\tilde{E})$$

$$w(a) = w_0 + w_a(1-a)$$

$$\rho_\Lambda(a) = \frac{3H_0^2}{8\pi G_N}\Omega_\Lambda(a) = \frac{\Lambda(a)}{8\pi G_N} = \int_{\tilde{E}_{\rm IR}(a)}^{\tilde{E}_{\rm UV}(a)} d\tilde{E} I(\tilde{E})$$

$$\begin{aligned} w(a) &= -1 - \frac{a}{3}\frac{d}{da}\log\,\Omega_\Lambda(a) \\ &= -1 - \frac{a}{3\Lambda(a)}\frac{d\Lambda(a)}{da} \\ &= -1 - \frac{8\pi G_N a}{3\Lambda(a)}\frac{d}{da}\left[\int_{\tilde{E}_{\rm IR}(a)}^{\tilde{E}_{\rm UV}(a)} d\tilde{E} I(\tilde{E})\right] \\ &= -1 - \frac{8\pi G_N a}{3\Lambda(a)}\left[I(\tilde{E}_{\rm UV}(a))\frac{d\tilde{E}_{\rm UV}(a)}{da} - I(\tilde{E}_{\rm IR}(a))\frac{d\tilde{E}_{\rm IR}(a)}{da}\right] \end{aligned}$$

$$I(\tilde{E}_{\rm UV}(a))\frac{d\tilde{E}_{\rm UV}(a)}{da} > I(\tilde{E}_{\rm IR}(a))\frac{d\tilde{E}_{\rm IR}(a)}{da}$$

$$I(\tilde{E}_{\rm UV}(a))\frac{d\tilde{E}_{\rm UV}(a)}{da} < I(\tilde{E}_{\rm IR}(a))\frac{d\tilde{E}_{\rm IR}(a)}{da}$$

$$E_{\rm IR}\tilde{E}_{\rm UV} = E_{\rm UV}\tilde{E}_{\rm IR} = \mu$$

$$E_{\rm IR}(a) = \frac{E_0}{a} \rightarrow \tilde{E}_{\rm UV}(a) = \frac{\mu}{E_{\rm IR}(a)} = \frac{\mu a}{E_0},$$

$$\frac{d\tilde{E}_{\rm UV}(a)}{da} = \frac{\mu}{E_0} > 0$$

$$I_{\rm DE}(\tilde{E}, E_0) = A\tilde{E}^3 e^{-B\tilde{E}/E_0},$$

$$\rho_\Lambda(a)=\frac{\Lambda(a)}{8\pi G_\mathrm{N}}=\int_{\tilde{E}_\mathrm{IR}(a)}^{\tilde{E}_\mathrm{UV}(a)} d\tilde{E} I_\mathrm{DE}\left(\tilde{E},E_0\right)$$

$$\rho_\Lambda(a)=\frac{\Lambda(a)}{8\pi G_\mathrm{N}}=\rho_*[b(\xi_\mathrm{IR}(a))-b(\xi_\mathrm{UV}(a))],$$

$$\xi_\mathrm{UV}(a)=\frac{B\tilde{E}_\mathrm{UV}(a)}{E_0},\xi_\mathrm{IR}(a)=\frac{B\tilde{E}_\mathrm{IR}(a)}{E_0}$$

$$\rho_*=\frac{6A}{B^4}E_0^4,b(\xi)=\left(1+\xi+\frac{1}{2}\xi^2+\frac{1}{6}\xi^3\right)e^{-\xi}$$

$$\begin{aligned}w(a)&=-1-\frac{a}{3\rho_\Lambda(a)}\left[I\left(\tilde{E}_\mathrm{UV}(a)\right)\frac{d\tilde{E}_\mathrm{UV}(a)}{da}-I\left(\tilde{E}_\mathrm{IR}(a)\right)\frac{d\tilde{E}_\mathrm{IR}(a)}{da}\right]\\&=-1-\frac{1}{18[b_\mathrm{IR}(a)-b_\mathrm{UV}(a)]}\left[\xi_\mathrm{UV}^3e^{-\xi_\mathrm{UV}}\left(a\frac{d\xi_\mathrm{UV}}{da}\right)-\xi_\mathrm{IR}^3e^{-\xi_\mathrm{IR}}\left(a\frac{d\xi_\mathrm{IR}}{da}\right)\right]\end{aligned}$$

$$\tilde{E}_\mathrm{UV}(a)=\frac{\mu}{E_0}a,\tilde{E}_\mathrm{IR}(a)=0$$

$$\xi_\mathrm{UV}(a)=\frac{B\mu}{E_0^2}a\equiv\xi_0a,\xi_\mathrm{IR}(a)=0$$

$$a\frac{d\xi_\mathrm{UV}(a)}{da}=\xi_0a=\xi_\mathrm{UV}(a),a\frac{d\xi_\mathrm{IR}(a)}{da}=0$$

$$\rho_\Lambda(a)=\frac{\Lambda(a)}{8\pi G_\mathrm{N}}=\rho_*[1-b(\xi_0a)]$$

$$w(a)=-1-\frac{\xi^4e^{-\xi}}{18\{1-b(\xi)\}},\xi=\xi_0a$$

$$\frac{dw(a)}{da}=\frac{d\xi}{da}\frac{dw(\xi)}{d\xi}=-\frac{\xi_0}{18}\left[\frac{(4-\xi)\xi^3e^{-\xi}}{\{1-b(\xi)\}}-\frac{\xi^7e^{-2\xi}}{6\{1-b(\xi)\}^2}\right].$$

$$\begin{aligned}w_0&=w(1)=-1-\frac{\xi_0^4e^{-\xi_0}}{18\{1-b(\xi_0)\}}\\w_a&=-w'(1)=\frac{\xi_0^4e^{-\xi_0}}{18\{1-b(\xi_0)\}}\left[(4-\xi_0)-\frac{\xi_0^4e^{-\xi_0}}{6\{1-b(\xi_0)\}}\right]\\&=-(4-\xi_0)(w_0+1)-3(w_0+1)^2\end{aligned}$$

$$\begin{aligned}w_0&=-\frac{7}{3}+\frac{4}{15}\xi_0-\frac{2}{225}\xi_0^2-\frac{2}{2625}\xi_0^3+O(\xi_0^4)\\w_a&=-\frac{4}{15}\xi_0+\frac{4}{225}\xi_0^2+\frac{2}{875}\xi_0^3+O(\xi_0^4)\end{aligned}$$

$$\tilde{E}_\mathrm{UV}=\frac{\mu}{E_0}[\alpha+(1-\alpha)a],\tilde{E}_\mathrm{IR}=\frac{\mu}{E_\mathrm{Planck}}\frac{1}{[\beta+(1-\beta)a]}$$

$$\alpha\beta = \frac{E_0}{E_{\text{Planck}}}$$

$$\tilde{E}_{\text{UV}}(0) = \tilde{E}_{\text{IR}}(0)$$

$$\alpha = \beta = \sqrt{\frac{E_0}{E_{\text{Planck}}}} \equiv k, \tilde{E}_{\text{UV}}(0) = \tilde{E}_{\text{IR}}(0) = \frac{\mu}{\sqrt{E_0 E_{\text{Planck}}}}$$

$$\xi_{\text{UV}}(a) = \frac{B\mu}{E_0^2}[k + (1-k)a] = \xi_0[k + (1-k)a]$$

$$\xi_{\text{IR}}(a) = \frac{B\mu}{E_0 E_{\text{Planck}}}[k + (1-k)a]^{-1}$$
$$= \xi_0 \frac{E_0}{E_{\text{Planck}}}[k + (1-k)a]^{-1} = \xi_0 k^2 [k + (1-k)a]^{-1}$$

$$E_{\text{IR}}\tilde{E}_{\text{IR}} = \mu_{\text{IR}}, E_{\text{UV}}\tilde{E}_{\text{UV}} = \mu_{\text{UV}},$$

$$E_{\text{IR}}(a) = \frac{E_0}{a}, E_{\text{UV}}(a) = E_{\text{Planck}}a$$

$$\tilde{E}_{\text{IR}}(a) = \frac{\mu_{\text{IR}}a}{E_0}, \tilde{E}_{\text{UV}}(a) = \frac{\mu_{\text{UV}}}{E_{\text{Planck}}a}$$

$$\tilde{E}_{\text{IR}}(0) < \tilde{E}_{\text{UV}}(0)$$

$$\tilde{E}_{\text{IR}}(a) = \tilde{E}_{\text{UV}}(a)$$

$$\frac{\mu_{\text{IR}}a}{E_0} = \frac{\mu_{\text{UV}}}{E_{\text{Planck}}a}$$
$$\downarrow$$
$$1 < a^2 = \frac{\mu_{\text{UV}}}{\mu_{\text{IR}}}\frac{E_0}{E_{\text{Planck}}},$$

$$\frac{\mu_{\text{UV}}}{\mu_{\text{IR}}} > \frac{E_{\text{Planck}}}{E_0} \gg 1$$

$$\xi_{\text{IR}}(a) = \frac{B\tilde{E}_{\text{IR}}(a)}{E_0} = \frac{B\mu_{\text{IR}}}{E_0^2}a \equiv \xi_{\text{IR}}^{(1)}a$$
$$\xi_{\text{UV}}(a) = \frac{B\tilde{E}_{\text{UV}}(a)}{E_0} = \frac{B\mu_{\text{UV}}}{E_0^2 a} \equiv \frac{\xi_{\text{UV}}^{(1)}}{a}$$

$$\xi_{\text{IR}}(a) \overset{a\to 0}{\to} 0, \xi_{\text{UV}}(a) \overset{a\to 0}{\to} \infty$$

$$b(\xi_{\text{IR}}(a)) \overset{a\to 0}{\to} 1, b(\xi_{\text{UV}}(a)) \overset{a\to 0}{\to} 0$$

$$\rho_\Lambda(0)/\rho_* = 1, w(0) = -1$$

$$\rho_\Lambda(a)/\rho_* \begin{cases} \overset{\xi_{\mathrm{IR}}^{(1)}\to 0}{\to} & 1-b(\xi_{\mathrm{UV}}) \\ \overset{\xi_{\mathrm{UV}}^{(1)}\to\infty}{\to} & b(\xi_{\mathrm{IR}}) \end{cases}$$

$$w(a) \begin{cases} \overset{\xi_{\mathrm{IR}}^{(1)}\to 0}{\to} & -1-\frac{1}{18[1-b(\xi_{\mathrm{UV}})]}\left[\xi_{\mathrm{UV}}^3 e^{-\xi_{\mathrm{UV}}}\left(a\frac{d\xi_{\mathrm{UV}}}{da}\right)\right] \\ \overset{\xi_{\mathrm{UV}}^{(1)}\to\infty}{\to} & -1+\frac{1}{18b(\xi_{\mathrm{IR}})}\left[\xi_{\mathrm{IR}}^3 e^{-\xi_{\mathrm{IR}}}\left(a\frac{d\xi_{\mathrm{IR}}}{da}\right)\right] \end{cases}$$

$$w_0 = w(1) = -1+\frac{\left(\xi_{\mathrm{UV}}^{(1)}\right)^4 e^{-\xi_{\mathrm{UV}}^{(1)}}+\left(\xi_{\mathrm{IR}}^{(1)}\right)^4 e^{-\xi_{\mathrm{IR}}^{(1)}}}{18\left[b\left(\xi_{\mathrm{IR}}^{(1)}\right)-b\left(\xi_{\mathrm{UV}}^{(1)}\right)\right]}$$

$$\overset{\xi_{\mathrm{IR}}^{(1)}\to 0}{\to} -1+\frac{\left(\xi_{\mathrm{UV}}^{(1)}\right)^4 e^{-\xi_{\mathrm{UV}}^{(1)}}}{18\left[1-b\left(\xi_{\mathrm{UV}}^{(1)}\right)\right]}$$

$$\overset{\xi_{\mathrm{UV}}^{(1)}\to\infty}{\to} -1+\frac{\left(\xi_{\mathrm{IR}}^{(1)}\right)^4 e^{-\xi_{\mathrm{IR}}^{(1)}}}{18b\left(\xi_{\mathrm{IR}}^{(1)}\right)} = -1+\frac{\left(\xi_{\mathrm{IR}}^{(1)}\right)^4}{\left[1+\left(\xi_{\mathrm{IR}}^{(1)}\right)+\frac{1}{2}\left(\xi_{\mathrm{IR}}^{(1)}\right)^2+\frac{1}{6}\left(\xi_{\mathrm{IR}}^{(1)}\right)^3\right]}$$

$$w_a = -w'(1) = \frac{\left(4-\xi_{\mathrm{UV}}^{(1)}\right)\left(\xi_{\mathrm{UV}}^{(1)}\right)^4 e^{-\xi_{\mathrm{UV}}^{(1)}}-\left(4-\xi_{\mathrm{IR}}^{(1)}\right)\left(\xi_{\mathrm{IR}}^{(1)}\right)^4 e^{-\xi_{\mathrm{IR}}^{(1)}}}{18\left[b\left(\xi_{\mathrm{IR}}^{(1)}\right)-b\left(\xi_{\mathrm{UV}}^{(1)}\right)\right]}$$

$$-3\left[\frac{\left(\xi_{\mathrm{UV}}^{(1)}\right)^4 e^{-\xi_{\mathrm{UV}}^{(1)}}+\left(\xi_{\mathrm{IR}}^{(1)}\right)^4 e^{-\xi_{\mathrm{IR}}^{(1)}}}{18\left[b\left(\xi_{\mathrm{IR}}^{(1)}\right)-b\left(\xi_{\mathrm{UV}}^{(1)}\right)\right]}\right]^2$$

$$=\left[\frac{\left(4-\xi_{\mathrm{UV}}^{(1)}\right)\left(\xi_{\mathrm{UV}}^{(1)}\right)^4 e^{-\xi_{\mathrm{UV}}^{(1)}}-\left(4-\xi_{\mathrm{IR}}^{(1)}\right)\left(\xi_{\mathrm{IR}}^{(1)}\right)^4 e^{-\xi_{\mathrm{IR}}^{(1)}}}{\left(\xi_{\mathrm{UV}}^{(1)}\right)^4 e^{-\xi_{\mathrm{UV}}^{(1)}}+\left(\xi_{\mathrm{IR}}^{(1)}\right)^4 e^{-\xi_{\mathrm{IR}}^{(1)}}}\right](w_0+1)-3(w_0+1)^2$$

$$\overset{\xi_{\mathrm{IR}}^{(1)}\to 0}{\to} \left(4-\xi_{\mathrm{UV}}^{(1)}\right)(w_0+1)-3(w_0+1)^2$$

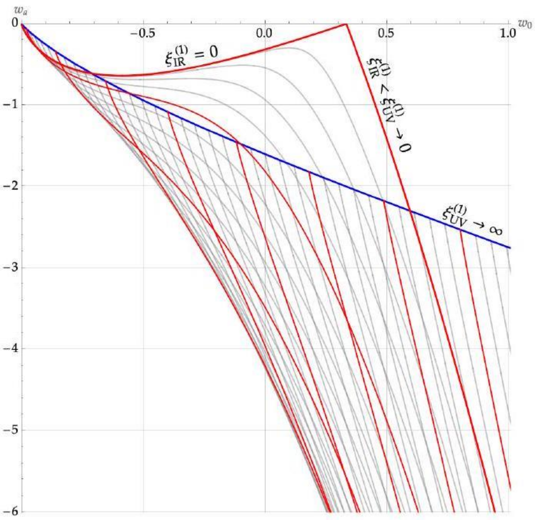
$$\overset{\xi_{\mathrm{UV}}^{(1)}\to\infty}{\to} -\left(4-\xi_{\mathrm{IR}}^{(1)}\right)(w_0+1)-3(w_0+1)^2$$

$$w_0 = -1+\frac{\left(\xi_{\mathrm{IR}}^{(1)}\right)^4\left[\kappa^4 e^{-\kappa\xi_{\mathrm{IR}}^{(1)}}+e^{-\xi_{\mathrm{IR}}^{(1)}}\right]}{18\left[b\left(\xi_{\mathrm{IR}}^{(1)}\right)-b\left(\kappa\xi_{\mathrm{IR}}^{(1)}\right)\right]}$$

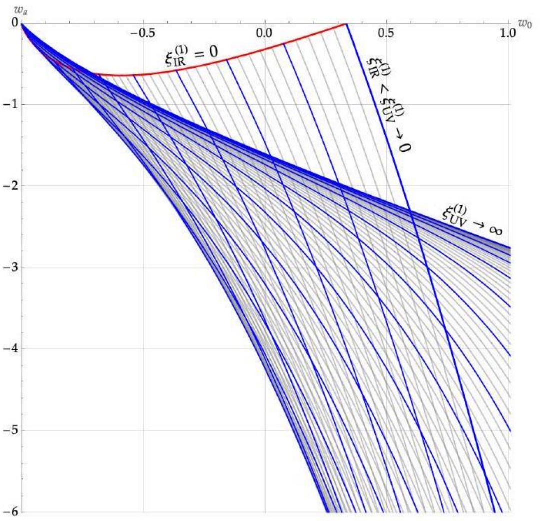
$$\overset{\xi_{\mathrm{IR}}^{(1)}\to 0}{\to} -1+\frac{4(\kappa^4+1)}{3(\kappa^4-1)} = \frac{\kappa^4+7}{3(\kappa^4-1)} \overset{\kappa\to\infty}{\to} \frac{1}{3}$$

$$w_a = \left[\frac{\left(4-\kappa\xi_{\mathrm{IR}}^{(1)}\right)\left(\kappa\xi_{\mathrm{IR}}^{(1)}\right)^4 e^{-\kappa\xi_{\mathrm{IR}}^{(1)}}-\left(4-\xi_{\mathrm{IR}}^{(1)}\right)\left(\xi_{\mathrm{IR}}^{(1)}\right)^4 e^{-\xi_{\mathrm{IR}}^{(1)}}}{\left(\kappa\xi_{\mathrm{IR}}^{(1)}\right)^4 e^{-\kappa\xi_{\mathrm{IR}}^{(1)}}+\left(\xi_{\mathrm{IR}}^{(1)}\right)^4 e^{-\xi_{\mathrm{IR}}^{(1)}}}\right](w_0+1)-3(w_0+1)^2$$

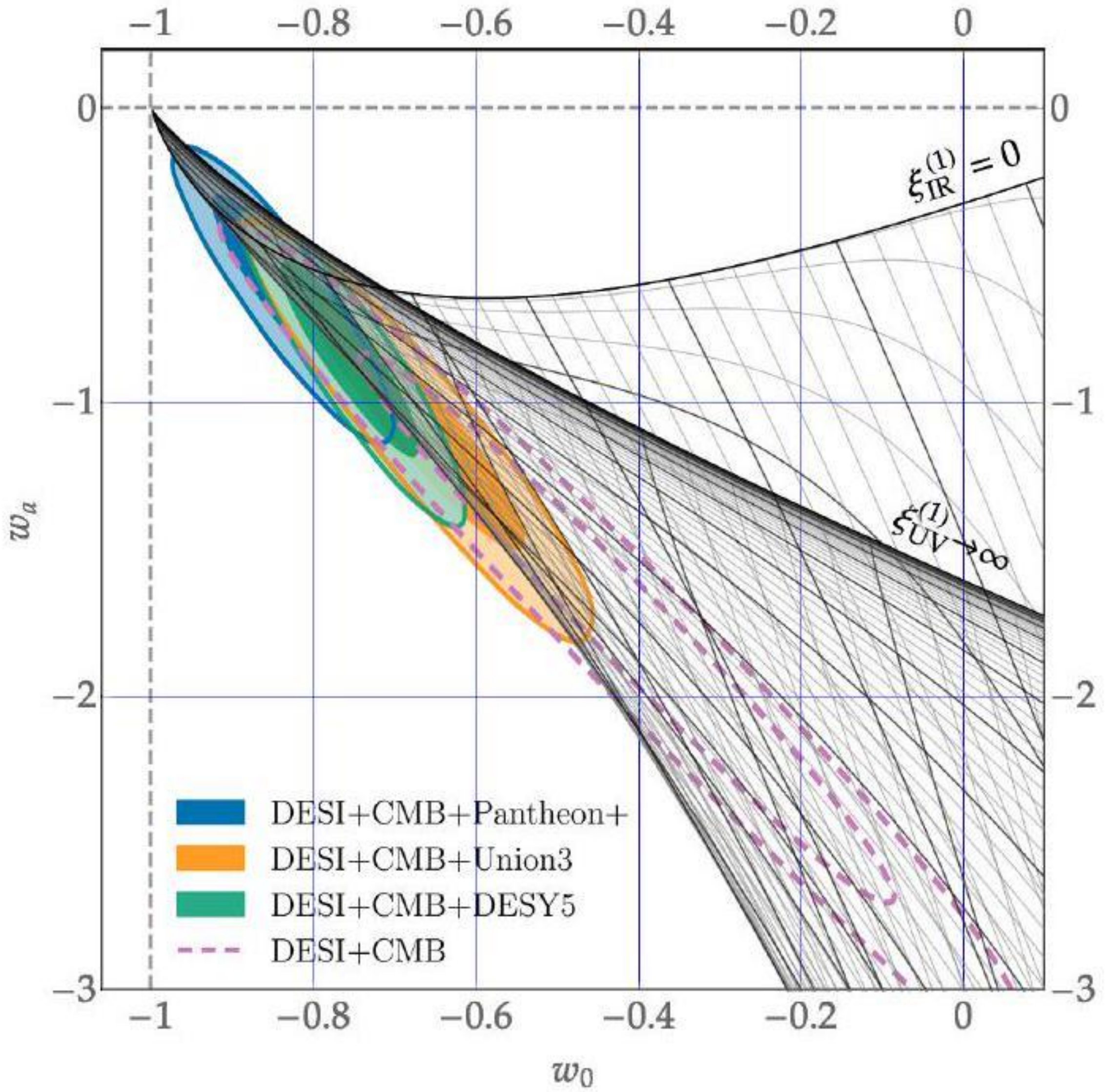
$$\overset{\xi_{\mathrm{IR}}^{(1)}\to 0}{\to} \frac{4(\kappa^4-1)}{\kappa^4+1}(w_0+1)-3(w_0+1)^2 = \frac{16}{3}-3(w_0+1)^2 = -\frac{64\kappa^4}{3(\kappa^4-1)^2} \overset{\kappa\to\infty}{\to} 0$$

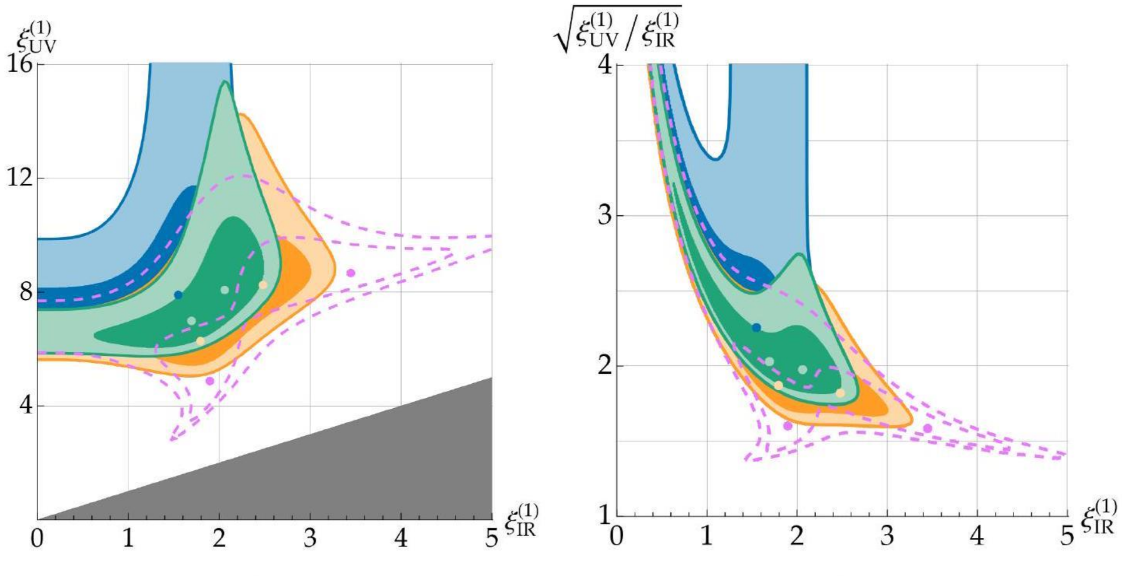

w_a
w_0
$\xi_{IR}^{(1)} = 0$
$\xi_{IR}^{(1)} < \xi_{UV}^{(1)} \to 0$
$\xi_{UV}^{(1)} \to \infty$

(a)


w_a
w_0
$\xi_{IR}^{(1)} = 0$
$\xi_{IR}^{(1)} < \xi_{UV}^{(1)} \to 0$
$\xi_{UV}^{(1)} \to \infty$

(b)


$\xi_{IR}^{(1)} = 0$
$\xi_{UV}^{(1)} \to \infty$
DESI+CMB+Pantheon+
DESI+CMB+Union3
DESI+CMB+DESY5
DESI+CMB
w_a
w_0



(a) (b)

Figuras 56, 57 y 58. Curvatura crítica.

$$(w_0, w_a) = (-0.42, -1.75) \ \rightarrow \ \left(\xi_{\mathrm{IR}}^{(1)}, \xi_{\mathrm{UV}}^{(1)}\right) = (1.90{,}4.90), (3.45.8.71).$$

$$(w_0, w_a) = (-0.838, -0.62) \ \rightarrow \ \left(\xi_{\mathrm{IR}}^{(1)}, \xi_{\mathrm{UV}}^{(1)}\right) = (1.55{,}7.93).$$

$$(w_0, w_a) = (-0.667, -1.09) \ \rightarrow \ \left(\xi_{\mathrm{IR}}^{(1)}, \xi_{\mathrm{UV}}^{(1)}\right) = (1.79{,}6.31), (2.48.8.28).$$

$$(w_0, w_a) = (-0.752, -0.86) \ \rightarrow \ \left(\xi_{\mathrm{IR}}^{(1)}, \xi_{\mathrm{UV}}^{(1)}\right) = (1.69{,}7.02), (2.06.8.10).$$

$$w_a = -\left.\frac{dw}{da}\right|_{a=1}$$

$$H(a)^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\sum_i \rho_i(a) \approx \frac{8\pi G_N}{3}[\rho_m(a) + \rho_\Lambda(a)]$$

$$\rho_\Lambda(a) = \frac{3H(a)^2}{8\pi G_N} - \rho_m(a) = \frac{3H(a)^2}{8\pi G_N}[1 - \Omega_m(a)],$$

$$\Omega_i(a) = \frac{8\pi G_N}{3H(a)^2}\rho_i(a)$$

$$q(a) = -\frac{\ddot{a}}{aH(a)^2} = \frac{4\pi G_N}{3H(a)^2}\sum_i [\rho_i(a) + 3p_i(a)] \approx \frac{1}{2} + \frac{4\pi G_N}{H(a)^2}p_\Lambda(a)$$

$$p_\Lambda(a) = \frac{H(a)^2}{4\pi G_N}\left[q(a) - \frac{1}{2}\right]$$

$$w(a) = \frac{p_\Lambda(a)}{\rho_\Lambda(a)} = \frac{2q(a) - 1}{3[1 - \Omega_m(a)]}$$

$$\Omega_m(a) = \left[\frac{H_0}{H(a)}\right]^2 \frac{\Omega_{m,0}}{a^3}$$

$$H(a) = H_0\sqrt{\frac{\Omega_{m,0}}{a^3} + \Omega_\Lambda(a)}$$

$$3[1 - \Omega_m(a)] = \frac{3H_0^2}{H(a)^2}\Omega_\Lambda(a)$$

$$\frac{dH}{dt} = \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 = -H(a)^2[q(a) + 1]$$

$$q(a) = -\frac{1}{H(a)^2}\frac{dH(a)}{dt} - 1 = -\frac{a}{H(a)}\frac{dH(a)}{da} - 1$$

$$2q(a) - 1 = \frac{3H_0^2}{H(a)^2}\left[-\frac{a}{3}\frac{d\Omega_\Lambda(a)}{da} - \Omega_\Lambda(a)\right]$$

$$w(a) = \frac{-\frac{a}{3}\frac{d\Omega_\Lambda(a)}{da} - \Omega_\Lambda(a)}{\Omega_\Lambda(a)} = -1 - \frac{a}{3}\frac{d}{da}\log\,\Omega_\Lambda(a)$$

$$S_{2d} = \frac{1}{4\pi}\int_\Sigma [\partial_\tau \mathbb{X}^A(\eta_{AB}(\mathbb{X}) + \omega_{AB}(\mathbb{X})) - \partial_\sigma \mathbb{X}^A H_{AB}(\mathbb{X})]\partial_\sigma \mathbb{X}^B$$

$$[\hat{\mathbb{X}}^A, \hat{\mathbb{X}}^B] = i\omega^{AB}$$

$$S \sim \int d\tau d\sigma \mathrm{Tr}[\partial_\tau \hat{\mathbb{X}}^A \partial_\sigma \hat{\mathbb{X}}^B(\omega_{AB}(\hat{\mathbb{X}}) + \eta_{AB}(\hat{\mathbb{X}})) - \partial_\sigma \hat{\mathbb{X}}^A H_{AB}(\hat{\mathbb{X}})\partial_\sigma \hat{\mathbb{X}}^B]$$

$$\partial_\sigma \hat{\mathbb{X}}^A \;\rightarrow\; [\hat{\mathbb{X}}^{26}, \hat{\mathbb{X}}^A], A = 0,1,\cdots,25$$

$$S \sim \int d\tau \mathrm{Tr}(\partial_\tau \hat{\mathbb{X}}^a[\hat{\mathbb{X}}^b, \hat{\mathbb{X}}^c]\eta_{abc}(\hat{\mathbb{X}}) - H_{ac}[\hat{\mathbb{X}}^a, \hat{\mathbb{X}}^b][\hat{\mathbb{X}}^c, \hat{\mathbb{X}}^d]H_{bd}(\hat{\mathbb{X}}))$$

$$\mathbb{S}_{ncM} = \frac{1}{4\pi}\int_\tau (\partial_\tau \hat{\mathbb{X}}^i[\hat{\mathbb{X}}^j, \hat{\mathbb{X}}^k]g_{ijk}(\hat{\mathbb{X}}) - [\hat{\mathbb{X}}^i, \hat{\mathbb{X}}^j][\hat{\mathbb{X}}^k, \hat{\mathbb{X}}^\ell]h_{ijk\ell}(\mathbb{X}))$$

$$S_{\mathrm{eff}}^{nc} = \int_x \int_{\tilde{x}} \mathrm{Tr}\sqrt{g(x,\tilde{x})}[R(x,\tilde{x}) + L_m(x,\tilde{x}) + \cdots]$$

$$[\hat{x}^a, \hat{\tilde{x}}_b] = 2\pi i\lambda^2\delta_b^a, [\hat{x}^a, \hat{x}^b] = [\hat{\tilde{x}}_a, \hat{\tilde{x}}_b] = 0$$

$$\begin{aligned} S_{d=4} &= -\int_x \int_{\tilde{x}} \sqrt{-g(x)}\sqrt{-\tilde{g}(\tilde{x})}[R(x) + \tilde{R}(\tilde{x})] \\ &= -\int_x \sqrt{-g(x)}\left[R(x)\int_{\tilde{x}} \sqrt{-\tilde{g}(\tilde{x})} + \int_{\tilde{x}} \sqrt{-\tilde{g}(\tilde{x})}\tilde{R}(\tilde{x})\right] \end{aligned}$$

$$1/G_N \sim \int_{\tilde{x}} \sqrt{-\tilde{g}(\tilde{x})}$$

$$\Lambda/G_N \sim \int_{\tilde{x}} \sqrt{-\tilde{g}(\tilde{x})}\tilde{R}(\tilde{x})$$

$$\left[\hat{a}_i, \hat{a}_j^\dagger\right]_q = \hat{a}_i\hat{a}_j^\dagger - q\hat{a}_j^\dagger\hat{a}_i = \delta_{ij}$$

$$\hat{a}_i|0\rangle = 0, \hat{a}_i\hat{a}_j^\dagger = \delta_{ij}, \sum_i \hat{a}_i^\dagger\hat{a}_i = \mathbf{1} - |0\rangle\langle 0|$$

$$\hat{N} = \sum_{k=1}^{\infty}\sum_{i_1} \hat{a}_{i_1}^\dagger\left(\sum_{i_2} \hat{a}_{i_2}^\dagger \dots \left(\sum_{i_k} \hat{a}_{i_k}^\dagger\hat{a}_{i_k}\right)\dots\hat{a}_{i_2}\right)\hat{a}_{i_1}$$

$$\hat{N}_1 = \sum_{k=1}^{\infty} \left(\hat{a}^\dagger\right)^k(\hat{a})^k = \frac{\hat{a}^\dagger\hat{a}}{1-\hat{a}^\dagger\hat{a}}$$

Sección VIII.

$$ds^2 = -dt^2 - a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\ \theta d\phi^2)],$$

$$H^2 = \frac{8\pi G}{3}(\rho_b + \rho_{dm} + \rho_r + \rho_{de})$$

$$\begin{aligned} \dot{\rho_{dm}} + 3H(1+\omega_{dm})\rho_{dm} &= Q \\ \dot{\rho_{de}} + 3H(1+\omega_{de})\rho_{de} &= -Q \\ \dot{\rho_r} + 3H(1+\omega_r)\rho_r &= 0 \\ \dot{\rho_b} + 3H(1+\omega_b)\rho_b &= 0 \end{aligned}$$

$$\rho_{dm} = \rho_{dm0}a^{-I}$$

$$\begin{aligned} \frac{d\rho_{dm}}{da} + \frac{3}{a}(1+\omega_{dm})\rho_{dm} &= \kappa\left(\frac{d\rho_{dm}}{da} + \frac{d\rho_{de}}{da}\right) \\ \frac{d\rho_{de}}{da} + \frac{3}{a}(1+\omega_{de})\rho_{de} &= -\kappa\left(\frac{d\rho_{dm}}{da} + \frac{d\rho_{de}}{da}\right) \end{aligned}$$

$$\rho_{de} = \frac{\rho_{dm0}}{\kappa}\left[1 - \frac{3}{I} - \kappa\right]a^{-I} + \tilde{\rho}_{de}$$

$$\omega_{de} = -1 + \left[\frac{\frac{\rho_{dm0}}{\kappa}\left(1 - \frac{3}{I} - \frac{3\kappa}{I}\right)a^{-I}}{\frac{\rho_{dm0}}{\kappa}\left(1 - \frac{3}{I} - \kappa\right)a^{-I} + \tilde{\rho_{de}}}\right]\frac{I}{3}$$

$$\Omega_i = \frac{8\pi G}{3H^2}\rho_i$$

$$\begin{aligned} \Omega_{dm} &= \frac{8\pi G}{3H^2}\rho_{dm0}a^{-I} \\ \Omega_{de} &= \frac{8\pi G}{3H^2}\left[\frac{\rho_{dm0}}{\kappa}\left(1 - \frac{3}{I} - \kappa\right)a^{-I} + \tilde{\rho}_{de}\right] \end{aligned}$$

$$\Omega_{de} = \Omega_{dm}\left(\frac{1}{\kappa} - \frac{3}{I\kappa} - 1\right) + \tilde{\Omega}_{de}$$

$$H^2(a) = H_0^2\left[\Omega_{b0}a^{-3} + \Omega_{r0}a^{-4} + \Omega_{dm0}a^{-I}\left(1-\frac{3}{I}\right)\frac{1}{\kappa} + \tilde{\Omega}_{de0}\right]$$

$$\Omega_b + \Omega_r + \Omega_{dm} + \Omega_{de} = 1$$

$$\tilde{\Omega}_{de0} = 1 - \Omega_{b0} - \Omega_{r0} - \Omega_{dm0}\left(1-\frac{3}{I}\right)\frac{1}{\kappa}$$

$$H^2(z) = H_0^2[\Omega_{b0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_{dm0}(1+z)^I\left(1-\frac{3}{I}\right)\frac{1}{\kappa} + 1 - \Omega_{b0} - \Omega_{r0} - \Omega_{dm0}\left(1-\frac{3}{I}\right)\frac{1}{\kappa}]$$

$$q = -1 - \frac{\dot{H}}{H^2} = -1 - \frac{\dot{a}}{H^2}\frac{dH}{da}$$

$$\dot{a}^2 = H_0^2\left[\Omega_{bo}a^{-1} + \Omega_{ro}a^{-2} + \Omega_{dmo}a^{2-I}\left(1-\frac{3}{I}\right)\frac{1}{\kappa} + \tilde{\Omega}_{deo}a^2\right]$$

$$\begin{aligned}\frac{dH}{da} = \frac{H_0}{2}&\left[\Omega_{b0}a^{-3} + \Omega_{r0}a^{-4} + \Omega_{dm0}a^{-I}\left(1-\frac{3}{I}\right)\frac{1}{\kappa} + \tilde{\Omega}_{de0}\right]^{-1/2}\\ &\times\left[(-3)\Omega_{b0}a^{-4} + (-4)\Omega_{r0}a^{-5} + (-I)\Omega_{dm0}a^{-I-1}\left(1-\frac{3}{I}\right)\frac{1}{\kappa}\right]\end{aligned}$$

$$\begin{aligned}q(a) = -1 - &\left(\frac{H_0\left[\Omega_{bo}a^{-1} + \Omega_{ro}a^{-2} + \Omega_{dmo}a^{2-I}\left(1-\frac{3}{I}\right)\frac{1}{\kappa} + \tilde{\Omega}_{deo}a^2\right]^{1/2}}{H_0^2\left[\Omega_{b0}a^{-3} + \Omega_{r0}a^{-4} + \Omega_{dm0}a^{-I}\left(1-\frac{3}{I}\right)\frac{1}{\kappa} + \tilde{\Omega}_{de0}\right]}\right.\\ &\times\frac{H_0}{2}\left[\Omega_{b0}a^{-3} + \Omega_{r0}a^{-4} + \Omega_{dm0}a^{-I}\left(1-\frac{3}{I}\right)\frac{1}{\kappa} + \tilde{\Omega}_{de0}\right]^{-1/2}\\ &\left.\times\left[(-3)\Omega_{b0}a^{-4} + (-4)\Omega_{r0}a^{-5} + (-I)\Omega_{dm0}a^{-I-1}\left(1-\frac{3}{I}\right)\frac{1}{\kappa}\right]\right)\end{aligned}$$

$$\begin{aligned}q(z) = -1 - &\left(\frac{H_0\left[\Omega_{bo}(1+z)^1 + \Omega_{ro}(1+z)^2 + \Omega_{dmo}(1+z)^{I-2}\left(1-\frac{3}{I}\right)\frac{1}{\kappa} + \tilde{\Omega}_{deo}(1+z)^{-2}\right]^{1/2}}{H_0^2\left[\Omega_{b0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_{dm0}(1+z)^I\left(1-\frac{3}{I}\right)\frac{1}{\kappa} + \tilde{\Omega}_{de0}\right]}\right.\\ &\times\frac{H_0}{2}\left[\Omega_{b0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_{dm0}(1+z)^I\left(1-\frac{3}{I}\right)\frac{1}{\kappa} + \tilde{\Omega}_{de0}\right]^{-1/2}\\ &\left.\times\left[(-3)\Omega_{b0}(1+z)^4 + (-4)\Omega_{r0}(1+z)^5 + (-I)\Omega_{dm0}(1+z)^{I+1}\left(1-\frac{3}{I}\right)\frac{1}{\kappa}\right]\right)\end{aligned}$$

$$\begin{aligned}d_L &= \chi(1+z)\\ d_A &= \frac{\chi}{(1+z)}\end{aligned}$$

$$d_L = \frac{1+z}{H_0}\int_0^z \frac{dz}{\left[\Omega_{b0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_{dm0}(1+z)^I\left(1-\frac{3}{I}\right)\frac{1}{\kappa} + 1 - \Omega_{b0} - \Omega_{r0} - \Omega_{dm0}\left(1-\frac{3}{I}\right)\frac{1}{\kappa}\right]^{1/2}}$$

$$d_A = \frac{(1+z)^{-1}}{H_0}\int_0^z \frac{dz}{\left[\Omega_{b0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_{dm0}(1+z)^I\left(1-\frac{3}{I}\right)\frac{1}{\kappa} + 1 - \Omega_{b0} - \Omega_{r0} - \Omega_{dm0}\left(1-\frac{3}{I}\right)\frac{1}{\kappa}\right]^{1/2}}$$

$$t_0 - t = \int_0^z \frac{dz}{(1+z)H(z)}$$

$$t_0 - t = \int_0^z \frac{dz}{(1+z)H_0\left[\Omega_{b0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_{dm0}(1+z)^I\left(1-\frac{3}{I}\right)\frac{1}{\kappa} + 1 - \Omega_{b0} - \Omega_{r0} - \Omega_{dm0}\left(1-\frac{3}{I}\right)\frac{1}{\kappa}\right]^{1/2}}$$

$$t_0 = \int_0^\infty \frac{dz}{(1+z)H_0\left[\Omega_{b0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_{dm0}(1+z)^I\left(1-\frac{3}{I}\right)\frac{1}{\kappa} + 1 - \Omega_{b0} - \Omega_{r0} - \Omega_{dm0}\left(1-\frac{3}{I}\right)\frac{1}{\kappa}\right]^{1/2}}$$

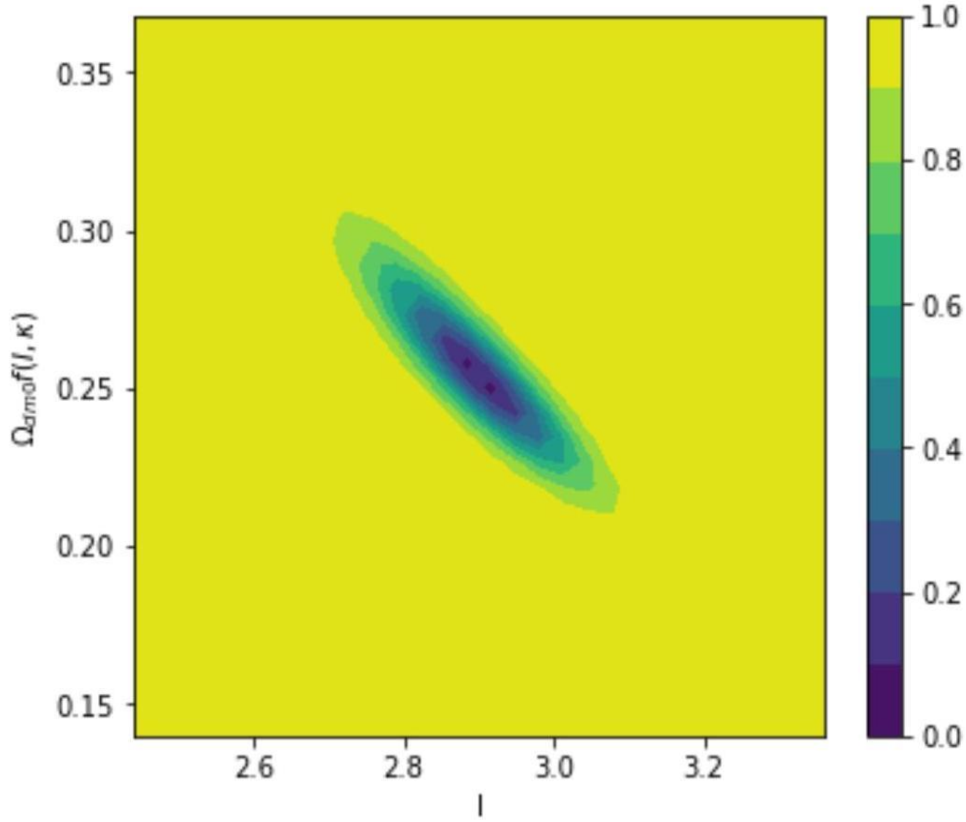
$$m_{\text{model}} = 5\log_{10}\left(\frac{d_L}{1Mpc}\right) + 25 + M$$

$$\chi^2_{SN} = \frac{\left(m_{obs_i} - m_{\text{model}}\right)^2}{\sigma_i^2}$$

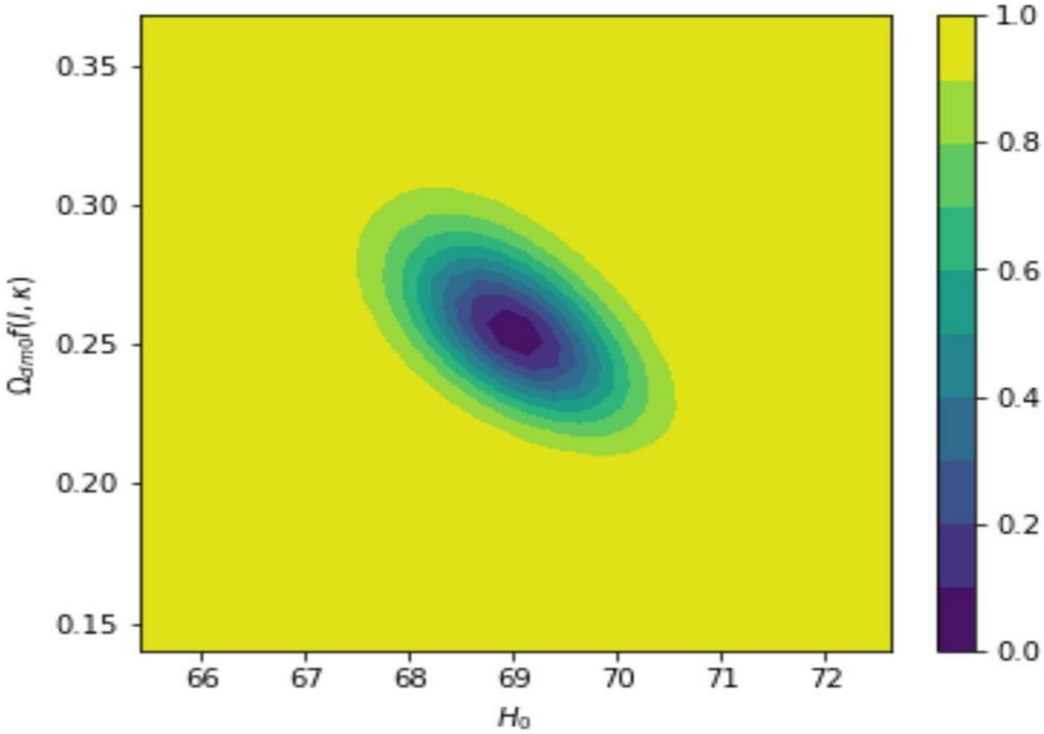
$$\chi^2_{H(z)} = \frac{\left(H(z)_{obs_i} - H(z)_{model}\right)^2}{\sigma_i^2}$$

$$\chi^2 = \chi^2_{SN} + \chi^2_{H(z)}$$

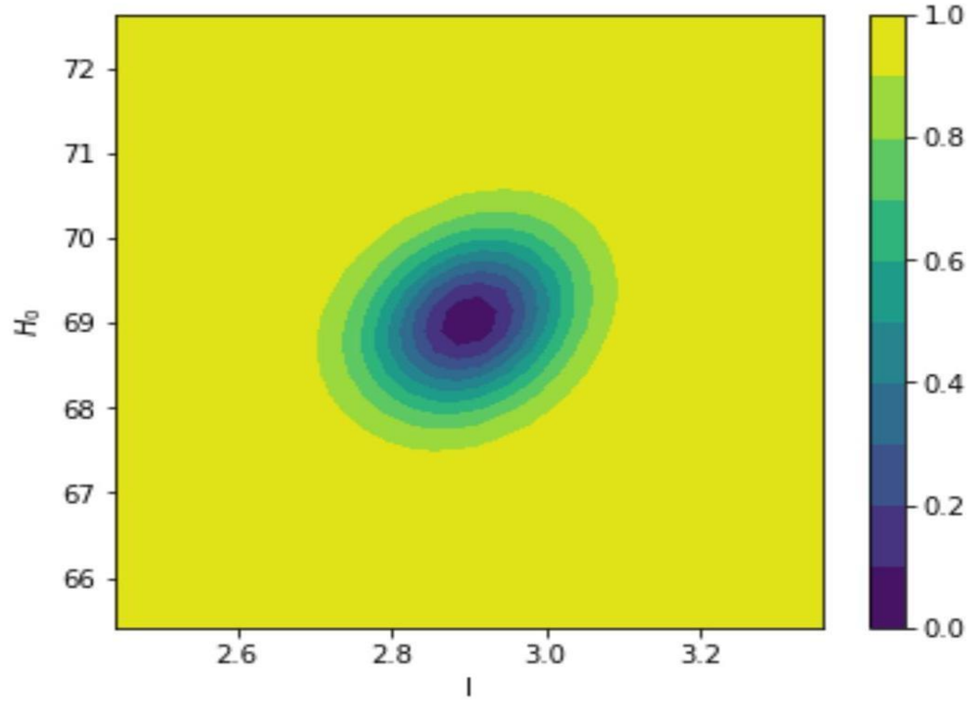
$$\left(\frac{da}{d\tilde{t}}\right)^2 = \left[\Omega_{bo}a^{-1} + \Omega_{dmo}a^{2-I}\left(1-\frac{3}{I}\right)\frac{1}{\kappa} + \tilde{\Omega}_{deo}a^2\right]$$



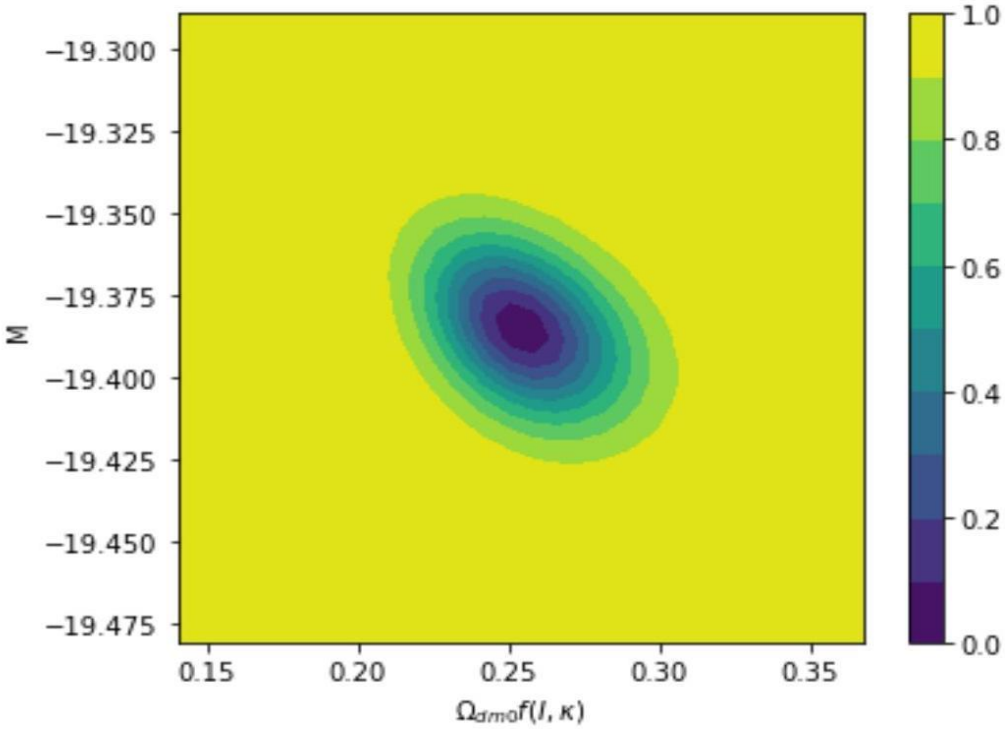
0.35
0.30
0.25
0.20
0.15
2.6
2.8
3.0
3.2
l
$\Omega_{dm0}f(l,\kappa)$
1.0
0.8
0.6
0.4
0.2
0.0



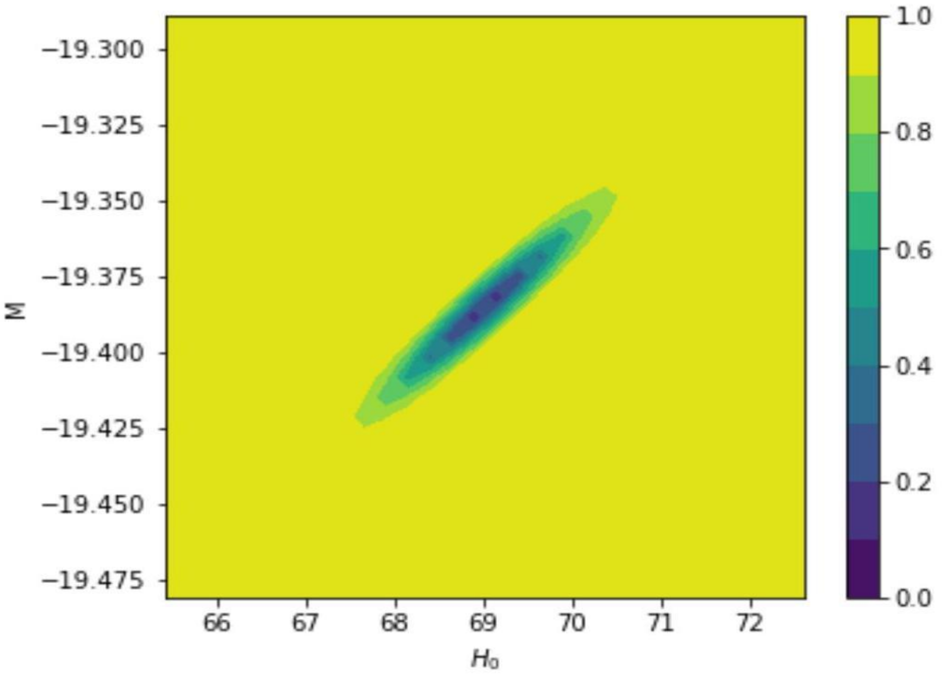
0.35
0.30
0.25
0.20
0.15
66
67
68
69
70
71
72
H_0
$\Omega_{dm0}f(l,\kappa)$
1.0
0.8
0.6
0.4
0.2
0.0

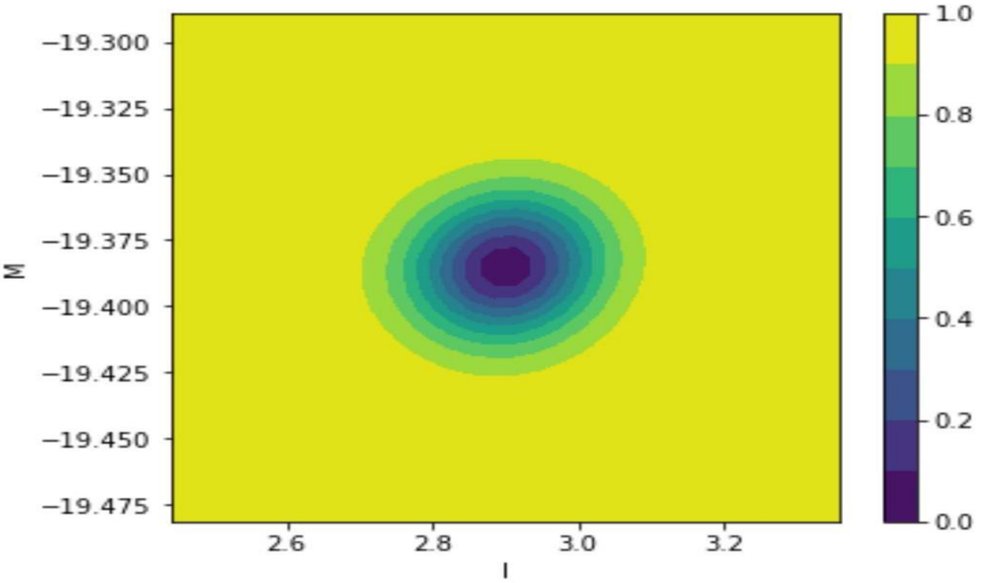


72
71
70
69
68
67
66
H_0
2.6
2.8
3.0
3.2
I
1.0
0.8
0.6
0.4
0.2
0.0



−19.300
−19.325
−19.350
−19.375
−19.400
−19.425
−19.450
−19.475
M
0.15
0.20
0.25
0.30
0.35
$\Omega_{dm0} f(I, \kappa)$
1.0
0.8
0.6
0.4
0.2
0.0





Figuras 59, 60, 61, 62, 63 y 64. Agujero negro cuántico. Mareas gravitacionales.

$$q_0^{IDE} = -0.561; z_t^{IDE} = 0.744$$

$$q_0^{\Lambda CDM} = -0.580; z_t^{\Lambda CDM} = 0.727$$

$$t_0^{IDE} = 14.032\text{Gyr}$$

$$t_0^{\Lambda CDM} = 13.885\text{Gyr}$$

$$\frac{d(m_{dm}n_{dm})}{da} + \frac{3}{a}(1+\omega_{dm})m_{dm}n_{dm} = \kappa\left[\frac{d(m_{dm}n_{dm})}{da} + \frac{d\rho_{de}}{da}\right]$$

$$m_{dm}\frac{dn_{dm}}{da} + \frac{3}{a}m_{dm}n_{dm} = \kappa\left(m_{dm}\frac{dn_{dm}}{da} + \frac{d\rho_{de}}{da}\right)$$

$$\frac{dn_{dm}}{da} + \frac{3}{a(1-\kappa)}n_{dm} = \frac{\kappa}{1-\kappa}\frac{n_{dm}}{\rho_{dm}}\frac{d\rho_{de}}{da}$$

$$n_{dm} = n_{dm0}a^{-I}$$

$$m_{dm}\frac{dn_{dm}}{da} + n_{dm}\frac{dm_{dm}}{da} + \frac{3}{a}m_{dm}n_{dm} = \kappa\left[m_{dm}\frac{dn_{dm}}{da} + n_{dm}\frac{dm_{dm}}{da} + \frac{d\rho_{de}}{da}\right]$$

$$\frac{dn_{dm}}{da} + \frac{3}{a}n_{dm} = 0$$

$$m_{dm} = m_{dm0}a^{3-I}$$

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