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CROMODINÁMICA CUÁNTICA RELATIVISTA

RELATIVISTIC QUANTUM CHROMODYNAMICS

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Investigador Independiente

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Cromodinámica Cuántica Relativista

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RESUMEN

Se conoce hasta la saciedad, que la cromodinámica cuántica estudia la interacción fuerte, esto es, la dinámica de relación existente entre bariones y mesones respectivamente, a propósito de la existencia de los hadrones. Para la teoría cuántica de campos relativistas o curvos formulada por este autor, existen premisas propias de la cromodinámica cuántica que engranan con sus lineamientos generales, más concretamente la existencia de una partícula supermasiva llamada quark – top, cuya masa alcanza la medida de 151 y 197 GeV/ $^{c^2}$ lo que la vuelve la partícula más pesada dentro del modelo estándar, idónea ciertamente para la validación de los cálculos matemáticos desarrollados a lo largo de artículos anteriores, en relación con la existencia de una partícula supermasiva. Ciertamente, el quark top, califica como una partícula supermasiva, a razón de su masa extremadamente densa. Lo propio aplicaría para una antipartícula supermasiva, que es el caso del antihiperhidrógeno-4. En definitiva, la cromodinámica cuántica ofrece un escenario experimentalmente apropiado para aplicar los postulados teorizados y subyacentes al espacio – tiempo cuántico relativista o curvo.

Palabras clave: cromodinámica cuántica, quark top, antihiperhidrógeno-4, campo cuántico relativista o curvo, partícula supermasiva

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Relativistic Quantum Chromodynamics

ABSTRACT

It is known ad nauseam that quantum chromodynamics studies the strong interaction, that is, the dynamics of the relationship between baryons and mesons respectively, regarding the existence of hadrons. For the quantum theory of relativistic or curved fields formulated by this author, there are premises of quantum chromodynamics that mesh with its general guidelines, more specifically the existence of a supermassive particle called quark-top, whose mass reaches the measurement of 151 and 197 GeV/$^{c^2}$ which makes it the heaviest particle within the standard model. It is certainly suitable for the validation of the mathematical calculations developed throughout previous articles, in relation to the existence of a supermassive particle. Certainly, the top quark qualifies as a supermassive particle because of its extremely dense mass. The same would apply to a supermassive antiparticle, which is the case of antihyperhydrogen-4. In short, quantum chromodynamics offers an experimentally appropriate scenario to apply the theorized and underlying postulates to relativistic or curved quantum space-time.

Keywords: quantum chromodynamics, top quark, antihyperhydrogen-4, relativistic or curved quantum field, supermassive particle

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INTRODUCCIÓN

Como me he referido ya en trabajos anteriores, el espacio – tiempo cuántico, es susceptible de deformación, lo que se conocen como campos cuánticos relativistas o curvos. En este contexto, se ha teorizado la existencia de partículas supermasivas, las mismas, cuya masa es tan excesivamente densa, que curva el espacio – tiempo cuántico en el que interactúa, sin embargo, la gravedad de la que está dotada, obedece a dos escenarios posibles, siendo éstos, por gravedad endógena, es decir, cuando la partícula, a propósito de su masa, por sí misma, deforma el espacio – tiempo cuántico, en tanto que, por gravedad exógena, se entiende que una partícula, no es, sino se vuelve supermasiva, cuando interactúa con el gravitón, el dilatón y el inflatón, simultáneamente, o con el gravitino, el dilatino o el inflatino, en tratándose de las antipartículas, es decir, cuando el campo – cuántico local, es permeado por el campo cuántico gravitónico. El propósito de este trabajo, es proporcionar un modelo matemático ajustado, es decir, sostenido en las premisas antes referidas, pero en relación a una partícula específica, esto es, el quark top, el mismo que, como se ha dicho, califica como una partícula supermasiva capaz de deformar el espacio – tiempo cuántico a propósito de la superdensidad de su masa, repercutiendo así, en las trayectorias de propagación (libertad asintónica) y por ende, en los propagadores de las partículas repercutidas, lo que, en sentido estricto, comporta una antisimetría susceptible de reparación, a través de transformaciones de gauge en invariancia y covariancia, según sea el caso. Asimismo, se vuelve necesario en este trabajo, referir a la antipartícula supermasiva antihiperhidrógeno-4, la misma que, considero capaz de generar el mismo efecto de deformación gravitacional de un espacio – cuántico específico. Para estos efectos, trabajaremos en un espacio de Hilbert – Einstein en 4 dimensiones, sobre una superficie de Riemann – Minkowski, con la finalidad, de definir la curvatura de Dirac resultante de las interacciones de acoplamiento y entrelazamiento y colisión entre partículas y antipartículas supermasivas y partículas repercutidas, y en la medida de lo posible, definir el cuanto de gravedad y las cargas de color y sabor, esto, a la luz de la QCD. Además, se examinarán distintos escenarios de aniquilación entre una partícula supermasiva y una partícula repercutida. Finalmente, es indispensable señalar, que el gravitón, podría ser una partícula de carácter gluónica, en tratándose de la interacción fuerte.

RESULTADOS Y DISCUSIÓN

El comportamiento de los bariones y mesones, en un campo cuántico curvo o relativista y a propósito de su hadronización, queda expresado así:

$$R_{e^+e^-} = \frac{\sigma(e^+ + e^- \rightarrow \text{ hadrons-barions - mesons })}{\sigma(e^+ + e^- \rightarrow \mu^+ + \mu^-)}.$$

$$R_{e^+e^-} = R_0\left(1 + \frac{g_s^2}{4\pi^2}\right)$$

Cuyo centro de masa – energía es, a escala renormalizable:

$$R_{e^+e^-} = R_{e^+e^-}(E, \mu, g_s(\mu))$$

$$\mu\frac{d}{d\mu}R_{e^+e^-}(E, \mu, g_s(\mu)) = 0 \longrightarrow \left(\mu\frac{\partial}{\partial\mu} + \beta(g_s)\frac{\partial}{\partial g_s}\right)R_{e^+e^-} = 0,$$

$$\beta(g_s) = \mu\frac{dg_s(\mu)}{d\mu}$$

Cuyo análisis dimensional, va expresado así:

$$R_{e^+e^-}(E, \mu, g_s(\mu)) = f\left(\frac{E}{\mu}, g_s(\mu)\right).$$

$$\left(-\frac{\partial}{\partial\log\, z} + \beta(g_s)\frac{\partial}{\partial g_s}\right)f(z, g_s(\mu)) = 0$$

$$f(z, g_s(\mu)) = \hat{f}\left(\overline{g_s}(z, g_s)\right)$$

$$\frac{\partial\overline{g_s}}{\partial\log\, z} = \beta(\overline{g_s})$$

$$\beta(x) = -\frac{\beta_0}{(4\pi)^2}x^3$$

$$\bar{g}_s^2(E/\mu, g_s(\mu)) = \frac{g_s^2(\mu)}{1 + \frac{\beta_0}{(4\pi)^2}g_s^2(\mu)\log\, E^2/\mu^2}$$

En el que, la renormalización de masa – energía, queda expresada así:

$$\beta_0 < 0: \quad \lim_{E\to 0}\bar{g}(E) = 0$$
$$\beta_0 > 0: \quad \lim_{E\to\infty}\bar{g}(E) = 0$$

$$R_{e^+e^-} = R_0\left(1 + \frac{g_s^2(E)}{4\pi^2} + O(g_s^4(E))\right)$$

$$\varepsilon\mu = 1$$

$$\beta_0^{\text{QCD}} = \frac{1}{3}(11N_c - 2N_F)$$

Ahora bien, la condición de Fermi queda expresada así (simetría spin – sabor):

$$|\Delta^{++}(S_z = 3/2)\rangle \sim |u \uparrow u \uparrow u \uparrow\rangle.$$

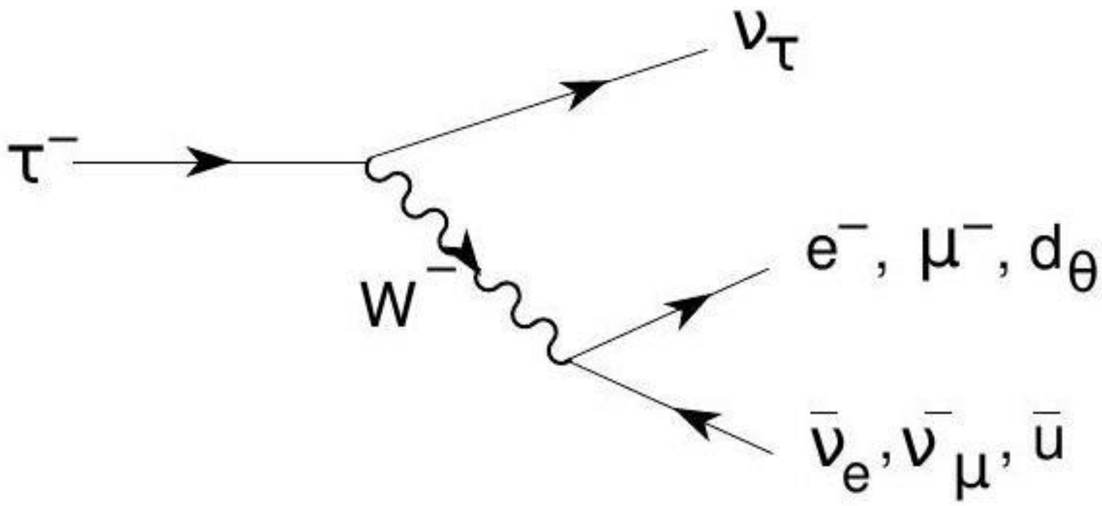


Figura 1. Aniquilación entre una partícula supermasiva y una partícula repercutida.

$$R_\tau = \frac{\Gamma(\tau^- \to \nu_\tau + \text{ hadrons-barions - mesons })}{\Gamma(\tau^- \to \nu_\tau e^- \overline{\nu_e})}$$
$$= N_c(|V_{ud}|^2 + |V_{us}|^2)\big(1 + O(\alpha_s)\big)$$

Más, la invariancia de gauge $SU(3)$ en lagrangiano, en un espacio – tiempo cuántico relativista o curvo, y sus transformaciones de covariancia y propagadores, quedan expresadas así:

$$\mathcal{L}_0 = \bar{\psi}(x) i\, \partial\!\!\!/ \psi(x) - m\bar{\psi}(x)\psi(x), \qquad a\!\!\!/ := \gamma^\mu a_\mu$$

$$\psi(x) \longrightarrow \psi'(x) = e^{-iQ\varepsilon}\psi(x)$$

$$\partial_\mu\psi(x) \longrightarrow e^{-iQ\varepsilon(x)}\big(\partial_\mu - iQ\partial_\mu\varepsilon(x)\big)\psi(x).$$

$$D_\mu\psi(x) = \big(\partial_\mu + iQA_\mu(x)\big)\psi(x)$$

$$A_\mu(x) \longrightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu\varepsilon(x)$$

$$D_\mu\psi(x) \longrightarrow \big(D_\mu\psi\big)'(x) = e^{-iQ\varepsilon(x)}D_\mu\psi(x)$$

$$\mathcal{L} = \bar{\psi}(x)(i D\!\!\!/ - m)\psi(x) = \mathcal{L}_0 - QA_\mu(x)\bar{\psi}(x)\gamma^\mu\psi(x)$$

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\mathcal{L}_0 = \sum_{i=1}^{3} \overline{q_i}(i\not{\partial} - m_q)q_i$$

$$q_i \longrightarrow q_i' = U_{ij}q_j, UU^\dagger = U^\dagger U = \mathbb{1}$$

$$\underline{3}^* \otimes \underline{3} = \underline{1} \oplus \underline{8}, \underline{3} \otimes \underline{3} \otimes \underline{3} = \underline{1} \oplus \underline{8} \oplus \underline{8} \oplus \underline{10},$$

$$U(\varepsilon_a) = \exp\left\{-i\sum_{a=1}^{8} \varepsilon_a \frac{\lambda_a}{2}\right\}$$

$$[\lambda_a, \lambda_b] = 2if_{abc}\lambda_c$$

$$(D^\mu q)_i = \left(\partial^\mu \delta_{ij} + ig_s \sum_{a=1}^{8} G_a^\mu \frac{\lambda_{a,ij}}{2}\right)q_j =: \{(\partial^\mu + ig_s G^\mu)q\}_i$$

$$G_{ij}^\mu := G_a^\mu \frac{\lambda_{a,ij}}{2},$$

$$G_\mu \longrightarrow G_\mu' = U(\varepsilon)G_\mu U^\dagger(\varepsilon) + \frac{i}{g_s}\big(\partial_\mu U(\varepsilon)\big)U^\dagger(\varepsilon)$$

$$G_a^\mu \longrightarrow G_a^{\mu\prime} = G_a^\mu + \frac{1}{g_s}\partial^\mu \varepsilon_a + f_{abc}\varepsilon_b G_c^\mu + O(\varepsilon^2)$$

$$\big[D_\mu, D_\nu\big] = \big[\partial_\mu + ig_s G_\mu, \partial_\nu + ig_s G_\nu\big] =: ig_s G_{\mu\nu}$$

$$\begin{aligned} G^{\mu\nu} &= \partial^\mu G^\nu - \partial^\nu G^\mu + ig_s[G^\mu, G^\nu] \\ G_a^{\mu\nu} &= \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f_{abc} G_b^\mu G_c^\nu \end{aligned}$$

$$G_{\mu\nu} \longrightarrow G_{\mu\nu}' = U(\varepsilon)G_{\mu\nu}U^\dagger(\varepsilon)$$

$$\mathrm{tr}\big(G_{\mu\nu}G^{\mu\nu}\big) = \frac{1}{2}G_a^{\mu\nu}G_{\mu\nu}^a$$

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2}\mathrm{tr}\big(G_{\mu\nu}G^{\mu\nu}\big) + \sum_{f=1}^{N_F} \bar{q}_f\big(i\not{D} - m_f \mathbb{1}_c\big)q_f$$

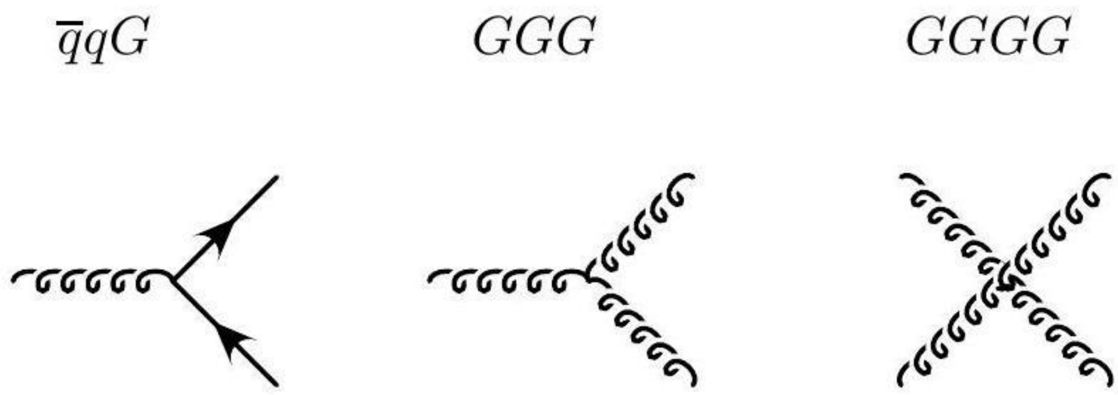


Figura 2. Formas de aniquilación entre partículas supermasivas.

$$(t_a^F)_{ij} = \frac{1}{2}(\lambda_a)_{ij}, (t_a^A)_{bc} = -if_{abc}$$

$$[t_a, t_b] = if_{abc}t_c$$

$$\mathrm{tr}\left(t_a^R t_b^R\right) = T_R\delta_{ab}, \sum_a (t_a^R)_{ij}(t_a^R)_{jk} = C_R\delta_{ik}\ (R = F, A),$$

$$d_R C_R = n_G T_R$$

$$d_A = n_G \longrightarrow C_A = T_A = n \text{ for } SU(n);$$

$$d_F = n, n_G = n^2 - 1, T_F = 1/2 \longrightarrow C_F = \frac{n^2 - 1}{2n}$$

$$C_F = 1.30 \pm 0.01(\mathrm{stat}) \pm 0.09(\mathrm{sys}), C_A = 2.89 \pm 0.03(\mathrm{stat}) \pm 0.21(\mathrm{sys})$$

$$\mathcal{L}_{\mathrm{QCD}} \longrightarrow \mathcal{L}_{\mathrm{QCD}} - \frac{\xi}{2}\left(\partial_\mu G_a^\mu\right)^2 + \mathcal{L}_{\mathrm{ghost}}$$

$$\Delta_{ab}^{\mu\nu}(k) = \delta_{ab}\frac{-i}{k^2 + i\epsilon}\left(g^{\mu\nu} + (\xi^{-1} - 1)\frac{k^\mu k^\nu}{k^2}\right) \stackrel{\xi=1}{=} \delta_{ab}\frac{-ig^{\mu\nu}}{k^2 + i\epsilon}$$

En este punto, el modelo QCD perturbativo, en un campo cuántico curvo o relativista, a propósito de la interacción de hadrones y leptones supermasivos, queda configurado así:

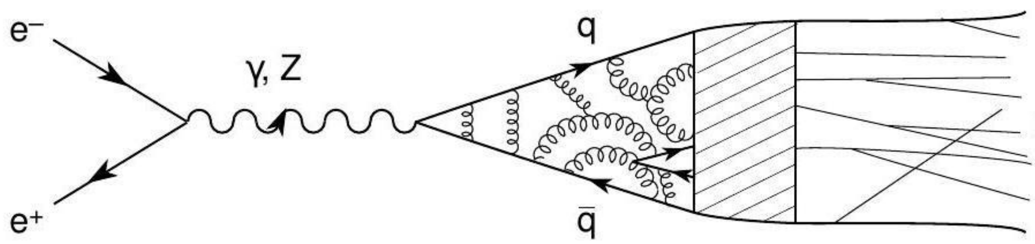


Figura 3. Explosión de energía y formación de un agujero negro cuántico, por aniquilación.

$$\Pi_{\text{em}}^{\mu\nu}(q) = i\int\, d^4x e^{iq\cdot x}\langle 0|TJ_{\text{em}}^{\mu}(x)J_{\text{em}}^{\nu}(0)|0\rangle = (-g^{\mu\nu}q^2 + q^{\mu}q^{\nu})\Pi_{\text{em}}(q^2)$$

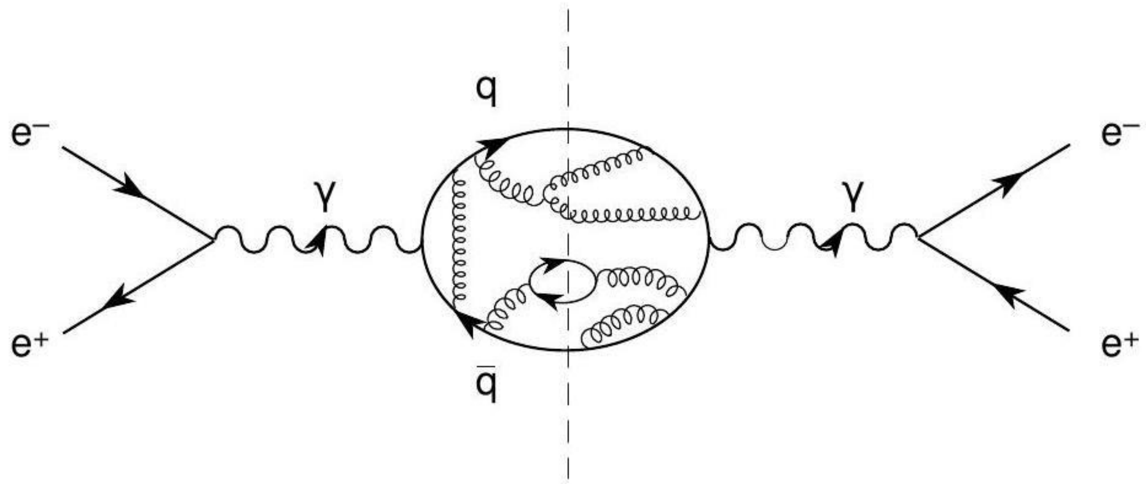


Figura 4. Singularidad de un agujero negro cuántico provocado por la aniquilación de dos partículas supermasivas.

$$e^+e^- \to \gamma^*(Z^*) \to \bar{q}q$$

$$A\left(e^+e^- \to \bar{q}_f^i q_f^i\right) = \frac{Q_f}{e} A(e^+e^- \to \mu^+\mu^-)$$

$$R_{e^+e^-} = \frac{\sigma(e^+e^- \to \text{ hadrons-barions - mesons })}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_{i,f} Q_f^2/e^2 = N_c \sum_f Q_f^2/e^2.$$

$$R_Z = \Gamma(Z \to \text{hadrons-barions - mesons })/\Gamma(Z \to e^+e^-) = N_c(1+\delta_{\text{EW}}) \sum_f \left(v_f^2 + a_f^2\right)/(v_e^2 + a_e^2),$$

$$e^+e^- \to \text{ jets}$$

$$e^+(q_1)e^-(q_2) \to q(p_1)\bar{q}(p_2)G(p_3).$$

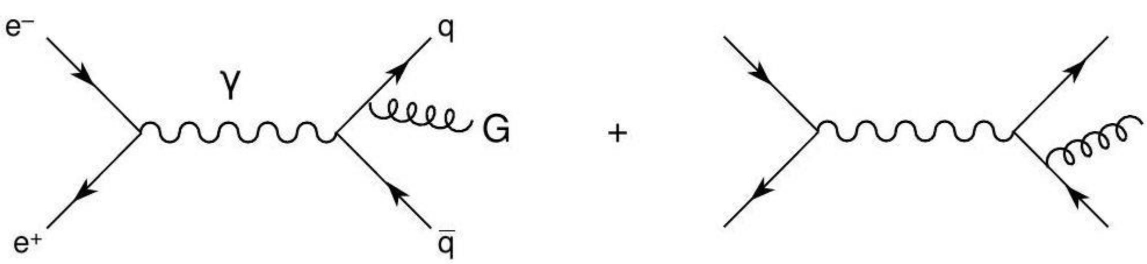


Figura 5. Interacciones entre una partícula supermasiva y el gravitón.

$$\sum_a \mathrm{tr}(t_a^F t_a^F) = T_F \sum_a \delta_{aa} = T_F n_{SU(3)} = d_F C_F = 3C_F = 4.$$

$$s = (q_1 + q_2)^2, \left(p_i + p_j\right)^2 = (q_1 + q_2 - p_k)^2 =: s(1 - x_k)$$

$$x_1 + x_2 + x_3 = 2, \qquad \text{CMS:}\, x_i = 2E_i/\sqrt{s}$$

$$\frac{d^2\sigma}{dx_1 dx_2} = \frac{2\alpha_s \sigma_0}{3\pi} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)} \quad \text{with} \ \sigma_0 = \frac{4\pi\alpha^2}{s} \sum_f \left(Q_f/e\right)^2$$

$$(p_2 + p_3)^2 - m_q^2 = 2p_2 \cdot p_3 = s(1 - x_1)$$

En el que, las correcciones escalares, renormalizaciones de gauge, divergencias de calibre e invariancias masa – energía, en un espacio – tiempo cuántico curvo o relativista, se expresan así:

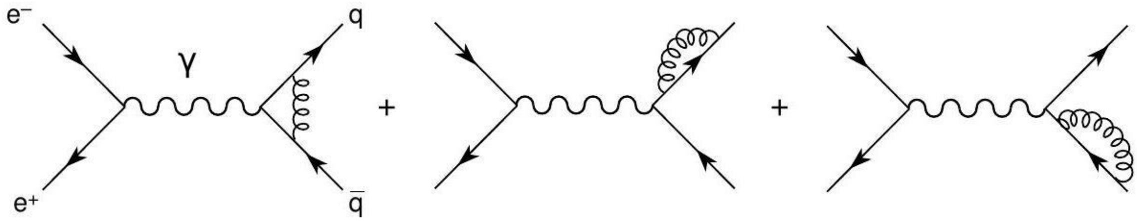
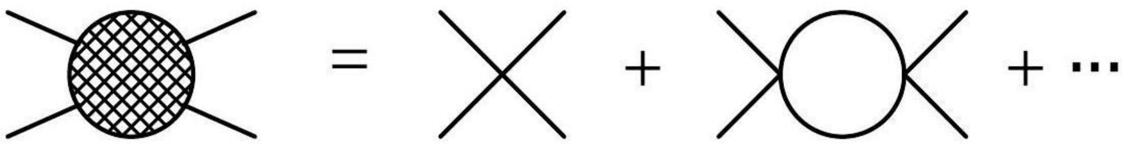


Figura 6. Deformaciones espaciales cruzada, a propósito de la deformación del espacio – tiempo cuántico.

$$\phi\phi \to \phi\phi$$

$$\lambda_r(\mu) := A(s = -t = \mu^2)$$



$$\lambda_r(\mu) = A(s = -t = \mu^2) \ = \lambda + \beta_0 \lambda^2 \log \Lambda/\mu$$
$$+\mu\text{-términos independientes de } O(\lambda^2) + O(\lambda^3)$$

$$\lambda_r(\mu + \delta\mu) - \lambda_r(\mu) = \beta_0 \lambda^2 \log\left(\frac{\Lambda}{\mu + \delta\mu} \frac{\mu}{\Lambda}\right) + O(\lambda^3)$$
$$= \beta_0 \lambda_r^2 \log \frac{\mu}{\mu + \delta\mu} + O(\lambda_r^3) = -\beta_0 \lambda_r^2 \frac{\delta\mu}{\mu} + O[(\delta\mu)^2] + O(\lambda_r^3).$$

$$\mu \frac{d\lambda_r(\mu)}{d\mu} = -\beta_0 \lambda_r^2(\mu) + O(\lambda_r^3) = \beta(\lambda_r(\mu))$$

$$\Pi^{\mu\nu}(q) = (-g^{\mu\nu}q^2 + q^\mu q^\nu)\Pi(q^2)$$

$$\Pi(q^2) = \frac{8e^2\Gamma(\varepsilon)}{(4\pi)^{2-\varepsilon}}\int_0^1 \frac{dxx(1-x)}{[-q^2x(1-x)]^\varepsilon}$$

$$\Gamma(x) = 1/x - \gamma + O(x), 2\varepsilon = 4 - d.$$

$$1 = (c\mu)^{-2\varepsilon}(c\mu)^{2\varepsilon} = (c\mu)^{-2\varepsilon}[1 + \varepsilon\log\,\mu^2 + 2\varepsilon\log\,c + O(\varepsilon^2)].$$

$$\begin{array}{ll} \text{MS} & c = 1 \\ \text{MS} & \log\,c = (\gamma - \log\,4\pi)/2 \end{array}$$

$$\Pi(q^2) = \frac{e^2}{12\pi^2}\left\{\frac{(c\mu)^{-2\varepsilon}}{\varepsilon} - \log\,(-q^2/\mu^2) + \frac{5}{3}\right\} + O(\varepsilon) = \Pi_{\text{div}}^{\overline{\text{MS}}}(\varepsilon,\mu) - \frac{e^2}{12\pi^2}\left\{\log\,(-q^2/\mu^2) - \frac{5}{3}\right\}$$

$$\sigma_{\bar{q}q}^{\text{interference}} = \sigma_0 C_F \frac{\alpha_s}{4\pi} H(\varepsilon)\left\{-\frac{4}{\varepsilon^2} - \frac{6}{\varepsilon} - 16 + O(\varepsilon)\right\}, H(0) = 1$$

$$\sigma_{\bar{q}qG} = \sigma_0 C_F \frac{\alpha_s}{4\pi} H(\varepsilon)\left\{\frac{4}{\varepsilon^2} + \frac{6}{\varepsilon} + 19 + O(\varepsilon)\right\}.$$

$$\sigma(e^+e^- \to \text{hadrons-barions - mesons}) = \sigma_0\left(1 + 3C_F\frac{\alpha_s}{4\pi} + O(\alpha_s^2)\right) = \sigma_0\left(1 + \frac{\alpha_s}{\pi} + O(\alpha_s^2)\right)$$

$$R_{e^+e^-}(s) = N_c \sum_f Q_f^2/e^2\left\{1 + \sum_{n\geq 1} C_n\left(\frac{\alpha_s(\sqrt{s})}{\pi}\right)^n\right\}$$
$$= R_{e^+e^-}^{(0)}\left\{1 + C_1\frac{\alpha_s(\mu)}{\pi} + \left[C_2 - C_1\frac{\beta_0}{4}\log\,(s/\mu^2)\right]\left(\frac{\alpha_s(\mu)}{\pi}\right)^2 + \cdots\right\}.$$

$$\alpha_s(E) = \frac{4\pi}{\beta_0\log\left(E^2/\Lambda_{\text{QCD}}^2\right)}$$

$$\mu\frac{d\alpha_s(\mu)}{d\mu} = 2\beta(\alpha_s) = -\frac{\beta_0}{2\pi}\alpha_s^2 - \frac{\beta_1}{4\pi^2}\alpha_s^3 + \cdots$$

$$\beta_0 = 11 - 2N_F/3, \qquad \beta_1 = 51 - 19N_F/3$$

$$\log\,(\mu_2^2/\mu_1^2) = \int_{\alpha_s(\mu_1)}^{\alpha_s(\mu_2)} \frac{dx}{\beta(x)}$$

$$R_{e^+e^-}(M_Z) = R_{e^+e^-}^{(0)}(M_Z)\left(1 + \frac{\alpha_s(M_Z)}{\pi}\right)$$

$$\alpha_s(M_Z) = 0.123 \pm 0.004$$

$$\alpha_s(M_Z) = 0.1182 \pm 0.0027$$

$$\alpha_s(M_Z) = 0.1187 \pm 0.0020$$

$$\Pi_L^{\mu\nu}(q) = i\int d^4x e^{iq\cdot x}\langle 0|TL^\mu(x)L^\nu(0)^\dagger|0\rangle = (-g^{\mu\nu}q^2 + q^\mu q^\nu)\Pi_L^{(1)}(q^2) + q^\mu q^\nu \Pi_L^{(0)}(q^2)$$

$$R_\tau = 12\pi \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left\{\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}_L^{(1)}(s) + \mathrm{Im}_L^{(0)}(s)\right\}$$

$$\mathrm{Im}\Pi_L^{(0,1)}(s) = \frac{1}{2i}\left[\Pi_L^{(0,1)}(s+i\varepsilon) - \Pi_L^{(0,1)}(s-i\varepsilon)\right]$$

$$R_\tau = 3(|V_{ud}|^2 + |V_{us}|^2)S_{\mathrm{EW}}\{1 + \delta'_{\mathrm{EW}} + \delta_{\mathrm{pert}} + \delta_{\mathrm{nonpert}}\}$$

$$\delta_{\mathrm{pert}} = \frac{\alpha_s(m_\tau)}{\pi} + \left(C_2 + \frac{19}{48}\beta_0\right)\left(\frac{\alpha_s(m_\tau)}{\pi}\right)^2 + \cdots$$

$$= \frac{\alpha_s(m_\tau)}{\pi} + 5.2\left(\frac{\alpha_s(m_\tau)}{\pi}\right)^2 + 26.4\left(\frac{\alpha_s(m_\tau)}{\pi}\right)^3 + O(\alpha_s(m_\tau)^4)$$

$$\delta_{\mathrm{nonpert}} = -0.014 \pm 0.005$$

$$\alpha_s(M_Z) = 0.121 \pm 0.0007(\mathrm{exp}) \pm 0.003(\mathrm{th})$$

$$W^2 = m^2, Q^2 = 2m\nu, x = 1$$

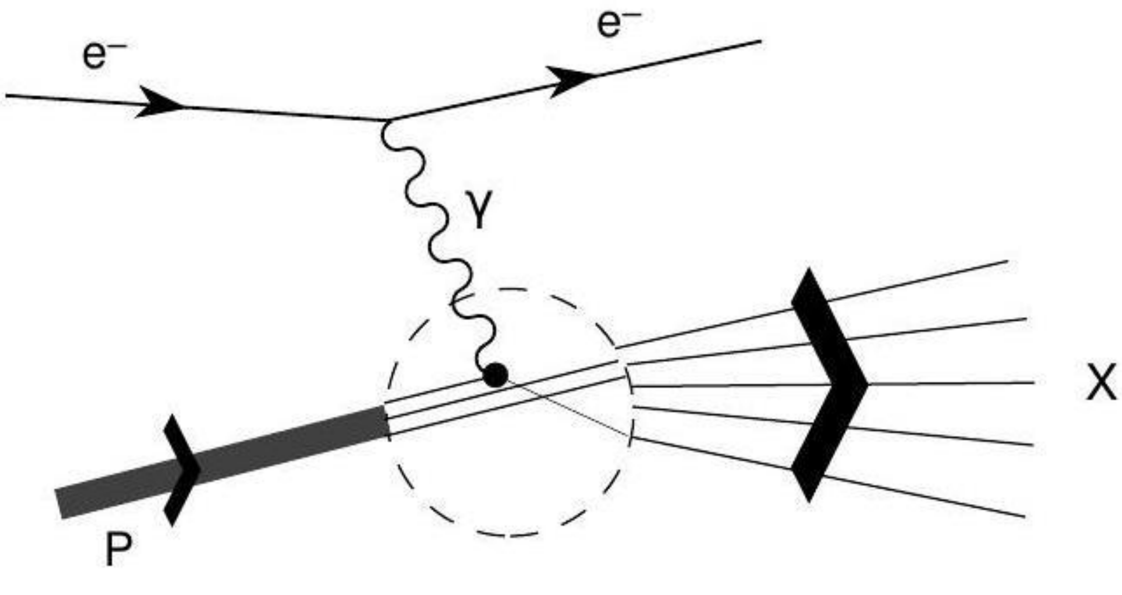


Figura 7. Formación de un agujero negro cuántico por implosión de una partícula supermasiva, esto es, cuando el centro de masa y energía es extremo, causando su colapso.

$$
\begin{aligned}
&q = k - k', Q^2 = -q^2 > 0, p^2 = m^2 \\
&\nu = p \cdot q / m = E - E' \text{ (target rest frame)} \\
&x = \frac{Q^2}{2m\nu}, y = \frac{p \cdot q}{p \cdot k} = 1 - E'/E \\
&W^2 = p_X^2 = (p+q)^2 = m^2 + 2m\nu - Q^2 \geq m^2
\end{aligned}
$$

$$
\frac{d^2\sigma}{dxdy} = x(s - m^2)\frac{d^2\sigma}{dxdQ^2} = \frac{2\pi y \alpha^2}{Q^4} L_{\mu\nu} H^{\mu\nu}
$$

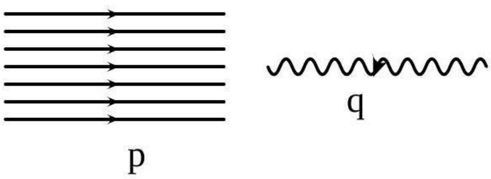
$$
L_{\mu\nu} = 2\left(k_\mu k'_\nu + k'_\mu k_\nu - k \cdot k' g_{\mu\nu}\right)
$$

$$
H^{\mu\nu}(p,q) = \frac{1}{4\pi}\int d^4 z e^{iq\cdot z}\langle p,s|\left[J_{\text{elm}}^{\mu}(z), J_{\text{elm}}^{\nu}(0)\right]|p,s\rangle.
$$

$$
\frac{d^2\sigma}{dxdy} = \frac{Q^2}{y}\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{xyQ^2}\left\{\left(1 - y - \frac{x^2y^2m^2}{Q^2}\right)F_2(x,Q^2) + y^2 x F_1(x,Q^2)\right\}
$$

$$
Q^2 \gg m^2, \nu \gg m \text{ with } x = \frac{Q^2}{2m\nu} \text{ fixed}
$$

$$
F_i(x,Q^2) \longrightarrow F_i(x)
$$



$$
\begin{aligned}
p &\simeq (P,P,0,0) \text{ with } P \gg m \\
q &= (0, -\sqrt{Q^2}, 0, 0)
\end{aligned}
$$

$$
(q + \xi p)^2 \simeq -Q^2 + 2\xi p \cdot q = 0
$$

$$
\xi = x, P = \frac{\sqrt{Q^2}}{2x}, q + xp = \left(xP, -\sqrt{Q^2}/2, 0, 0\right)
$$

$$
e^-(k) + q(\xi p) \rightarrow e^-(k') + q(\xi p + q)
$$

$$
\frac{d^2\sigma_{(q)}}{dxdy} = \frac{4\pi\alpha^2}{yQ^2}\left[1 + (1-y)^2\right]\frac{Q_q^2}{2}\delta(x - \xi)
$$

$$
F_{2(q)} = xQ_q^2\delta(x - \xi) = 2xF_{1(q)}
$$

$$
F_2(x) = \sum_{q,\bar{q}}\int_0^1 d\xi q(\xi) x Q_q^2 \delta(x - \xi) = \sum_{q,\bar{q}} Q_q^2 x q(x)
$$

$$
F_2(x) = 2xF_1(x)
$$

$$F_{2(q)}(x,Q^2)=Q_q^2x\left[\delta(1-x)+\frac{\alpha_s}{2\pi}\left(P_{qq}(x)\log\frac{Q^2}{\kappa^2}+C_q(x)\right)\right].$$

$$P_{qq}(x)=\frac{4}{3}\left(\frac{1+x^2}{[1-x]_+}\right)+2\delta(1-x)$$

$$\int_0^1 dxf(x)[F(x)]_+=\int_0^1 dx(f(x)-f(1))F(x)$$

$$P_{qq}(x)=\frac{4}{3}\left[\frac{1+x^2}{(1-x)}\right]_+$$

$$F_2(x,Q^2)=x\sum_{q,\bar{q}}Q_q^2\left[q_0(x)+\frac{\alpha_s}{2\pi}\int_x^1\frac{dy}{y}q_0(y)\left\{P_{qq}(x/y)\log\frac{Q^2}{\kappa^2}+C_q(x/y)\right\}\right]$$

$$q(x,\mu^2)=q_0(x)+\frac{\alpha_s}{2\pi}\int_x^1\frac{dy}{y}q_0(y)\left\{P_{qq}(x/y)\log\frac{\mu^2}{\kappa^2}+C_q'(x/y)\right\}$$

$$F_2(x,Q^2)=x\sum_{q,\bar{q}}Q_q^2\int_x^1\frac{dy}{y}q(y,\mu^2)\left[\delta(1-x/y)+\frac{\alpha_s}{2\pi}\left\{P_{qq}(x/y)\log\frac{Q^2}{\mu^2}+C_q^{\overline{\text{MS}}}(x/y)\right\}\right]=x\sum_{q,\bar{q}}Q_q^2\int_x^1\frac{dy}{y}q(y,Q^2)\left[\delta(1-x/y)+\frac{\alpha_s}{2\pi}C_q^{\overline{\text{MS}}}(x/y)\right]$$

$$\mu^2\frac{dF_2(x,Q^2)}{d\mu^2}=0\longrightarrow\mu^2\frac{dq(x,\mu^2)}{d\mu^2}=\frac{\alpha_s(\mu)}{2\pi}\int_x^1\frac{dy}{y}P_{qq}(x/y,\alpha_s(\mu))q(y,\mu^2)$$

$$P_{qq}(x,\alpha_s(\mu))=P_{qq}^{(0)}(x)+\frac{\alpha_s(\mu)}{2\pi}P_{qq}^{(1)}(x)+\cdots$$

$$\mu^2\frac{dq(x,\mu^2)}{d\mu^2}=\frac{2\alpha_s(\mu)}{3\pi}\int_x^1\frac{dz}{z}q(x/z,\mu^2)\frac{1+z^2}{1-z}-\frac{2\alpha_s(\mu)}{3\pi}q(x,\mu^2)\int_0^1 dz\frac{1+z^2}{1-z}.$$

$$\mathcal{L}_{\text{eff}}=\mathcal{L}_{d\leq 4}+\sum_{d>4}\frac{1}{\Lambda^{d-4}}\sum_{i_d}g_{i_d}O_{i_d}$$

$$m_t=174.3\pm 5.1\text{GeV}$$

$$\mu\frac{dm_q(\mu)}{d\mu}=-\gamma(\alpha_s(\mu))m_q(\mu)$$

$$\gamma(\alpha_s)=\sum_{n=1}^{4}\gamma_n\left(\frac{\alpha_s}{\pi}\right)^n$$

$$m_q(\mu_2)=m_q(\mu_1)\exp\left\{-\int_{\alpha_s(\mu_1)}^{\alpha_s(\mu_2)}dx\frac{\gamma(x)}{2\beta(x)}\right\}$$

$$\frac{m_q(1\text{GeV})}{m_q(M_Z)}=2.30\pm 0.05$$

$$p^\mu=m_Qv^\mu+k^\mu$$

$$\mathcal{L}_Q = \bar{Q}(i\not{D} - m_Q)Q,$$

$$Q(x) = e^{-im_Q v\cdot x}(h_v(x) + H_v(x))$$
$$h_v(x) = e^{im_Q v\cdot x}P_v^+ Q(x), H_v(x) = e^{im_Q v\cdot x}P_v^- Q(x).$$

$$\mathcal{L}_Q = \bar{Q}(i\not{D} - m_Q)Q = \overline{h_v} iv\cdot Dh_v - \overline{H_v}(iv\cdot D + 2m_Q)H_v$$

$$\mathcal{L}_Q = \overline{h_v} iv\cdot Dh_v + \overline{h_v} i\not{D}_\perp \frac{1}{iv\cdot D + 2m_Q - i\epsilon} i\not{D}_\perp h_v \text{ with } D_\perp^\mu = (g^{\mu\nu} - v^\mu v^\nu)D_\nu$$

$$\mathcal{L}_{b,c} = \overline{b_v} iv\cdot Db_v + \overline{c_v} iv\cdot Dc_v$$

$$\langle\mathcal{M}(v')|\overline{h_{v'}}\Gamma h_v|\mathcal{M}(v)\rangle \sim \xi(v\cdot v')$$

$$\xi(v\cdot v' = 1) = 1$$

$$\Pi(q^2) = \Pi_{\text{pert}}(q^2) + \sum_d C_d(q^2)\langle 0|O_d|0\rangle$$

$$a_\mu^{\text{had,LO}} = a_\mu^{\text{vac.pol.}} = \int_{4M_\pi^2}^{\infty} dtK(t)\sigma_0(e^+e^- \to \text{ hadrons-leptons })(t)$$

$$\sigma_0(e^+e^- \to \pi^+\pi^-)(t) = h(t)\frac{d\Gamma(\tau^- \to \pi^0\pi^-\nu_\tau)}{dt}$$

$$\rho_{\text{em}}(s) = \text{Im}\Pi_{\text{elm}}(s) \text{ and } \rho_V^{I=1}(s) = \text{Im}\Pi_{L,ud}(s)$$

$$\int_0^{s_0} w(s)\rho(s)ds = -\frac{1}{2\pi}\oint_{|s|=s_0} w(s)\Pi(s)ds$$

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (7.6 \pm 8.9)\cdot 10^{-10}$$

La simetría quiral en lagrangiano, en relación a un campo cuántico relativista o curvo, queda expresada así:

Modalidad no perturbativa

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2}\text{tr}(G_{\mu\nu}G^{\mu\nu}) + \sum_{f=1}^{N_F} \bar{q}_f(i\not{D} - m_f \mathbb{1}_c)q_f$$

$$\mathcal{L}_{\text{kin}} = i\sum_{f=1}^{6} \bar{q}_f \not{\partial} q_f = i\sum_{f=1}^{6} \{\bar{q}_{fL} \not{\partial} q_{fL} + \bar{q}_{fR}\not{D}q_{fR}\}$$

$$q_L = \frac{1}{2}(1-\gamma_5)q, q_R = \frac{1}{2}(1+\gamma_5)q$$

$$\mathcal{L}_q = \sum_{f=1}^{6} \left\{\bar{q}_{fL} i \not{D} q_{fL} + \bar{q}_{fR} i \not{D} q_{fR} - m_f\left(\bar{q}_{fR} q_{fL} + \bar{q}_{fL} q_{fR}\right)\right\}$$

$$\mathcal{M}_q = \mathrm{diag}(m_u, m_d, m_s, m_c, m_b, m_t)$$

$$B = (N_u + N_d + N_s + N_c + N_b + N_t)/3$$

$$U(n_F) \times U(1)^{6-n_F} \simeq SU(n_F) \times U(1) \times U(1)^{6-n_F}.$$

$$\begin{array}{lll} n_F = 2: & m_u = m_d & \longrightarrow \quad \text{isoespin } SU(2) \\ n_F = 3: & m_u = m_d = m_s & \longrightarrow \quad \text{sabor } SU(3) \end{array}$$

$$SU(n_F)_L \times SU(n_F)_R \times U(1)_V \times U(1)_A [\times U(1)^{6-n_F}].$$

$$\lim_{k\to 0} \omega(k) = \lim_{\lambda\to\infty} \omega(k) = 0$$

$$\lim_{p\to 0} E = \lim_{p\to 0} \sqrt{p^2 + m^2} = 0 \iff m = 0$$

$$Q|0\rangle = 0$$

$$\|Q|0\rangle\| = \infty$$

$$\langle 0|[Q, A]|0\rangle$$

$$\langle 0|J^0(0)|\mathrm{G}\rangle\langle \mathrm{G}|A|0\rangle \neq 0.$$

$$\langle 0|J^0(0)|\mathrm{G}\rangle \neq 0$$

$$\mathcal{L}_{\text{Goldstone}} = \partial_\mu \phi \partial^\mu \phi^\dagger - \lambda\left(\phi\phi^\dagger - \frac{v^2}{2}\right)^2 \quad (\lambda, v)$$

$$\phi(x) = (R(x) + iG(x))/\sqrt{2}$$
$$\langle 0|R(x)|0\rangle = v, \langle 0|G(x)|0\rangle = 0$$

$$\begin{array}{ll} \text{Goldstone field } G(x) & M_G = 0 \\ \text{massive field } H(x) = R(x) - v & M_H = \sqrt{2\lambda} v \end{array}$$

$$A(GG \to GG) = O(p_G^4), A(GH \to GH) = O(p_G^2)$$

$$\phi(x) = \frac{1}{\sqrt{2}}[h(x) + v]e^{ig(x)/v}$$

$$\begin{aligned} \mathcal{L}_{\text{Goldstone}} &= \frac{1}{2}\left(\partial_\mu g\right)^2 + \frac{1}{2v^2}(h^2 + 2vh)\left(\partial_\mu g\right)^2 \\ &+ \frac{1}{2}\left(\partial_\mu h\right)^2 - \lambda v^2 h^2 - \frac{\lambda}{4}(h^4 + 4vh^3) \end{aligned}$$

$$\begin{array}{ll} \text{Goldstone field } g(x) & M_g = 0 \\ \text{massive field } h(x) & M_h = \sqrt{2\lambda}v \end{array}$$

$$\lim_{p_G \to 0} A(p_G) = 0.$$

Modalidad perturbativa

$$\mathcal{L}_{\text{QCD}}^0 = \overline{q_L} i \not{D} q_L + \overline{q_R} i \not{D} q_R + \mathcal{L}_{\text{heavy quarks}} + \mathcal{L}_{\text{gauge}}$$

$$q^\top = (ud[s]).$$

$$SU(n_F)_L \times SU(n_F)_R \times U(1)_V \times U(1)_A [\times U(1)^{6-n_F}].$$

$$U(\phi) = \exp\left(i\sqrt{2}\Phi/F\right), \Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}$$

$$\mathcal{L}_2^{(0)} = \frac{F^2}{4} \operatorname{tr}_{n_F}\left(\partial_\mu U \partial^\mu U^\dagger\right) =: \frac{F^2}{4} \langle \partial_\mu U \partial^\mu U^\dagger \rangle = \partial_\mu \pi^+ \partial^\mu \pi^- + \frac{1}{2} \partial_\mu \pi^0 \partial^\mu \pi^0 + O(\pi^4)$$

$$\begin{array}{lll} m_u, m_d & \ll M_\rho & n_F = 2 \\ m_s & < M_\rho & n_F = 3 \end{array}$$

$$M_M^2 \sim m_q + O\left(m_q^2\right)$$

$$m_q = O(M_M^2) = O(p^2)$$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots$$

$$\mathcal{L}_2 = \frac{F^2}{4} \langle \partial_\mu U \partial^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U \rangle$$

$$F_\pi = F\left[1 + O\left(m_q\right)\right], \langle 0|\bar{u}u|0\rangle = -F^2 B\left[1 + O\left(m_q\right)\right]$$

$$A_2(s,t,u) = \frac{s - M_\pi^2}{F_\pi^2}$$

$$n_F = 2: \frac{p^2}{(4\pi F)^2} = 0.014 \frac{p^2}{M_\pi^2}, \qquad n_F = 3: \frac{p^2}{(4\pi F)^2} = 0.18 \frac{p^2}{M_K^2}.$$

$$\begin{aligned} M_{\pi^+}^2 &= 2\hat{m}B, M_{\pi^0}^2 = 2\hat{m}B + O[(m_u - m_d)^2/(m_s - \hat{m})] \\ M_{K^+}^2 &= (m_u + m_s)B, M_{\eta_8}^2 = \frac{2}{3}(\hat{m} + 2m_s)B + O[(m_u - m_d)^2/(m_s - \hat{m})] \\ M_{K^0}^2 &= (m_d + m_s)B, \hat{m} := \frac{1}{2}(m_u + m_d). \end{aligned}$$

$$F_\pi^2 M_\pi^2 = -2\hat{m}\langle 0|\bar{u}u|0\rangle$$

$$B=\frac{M_\pi^2}{2\hat{m}}=\frac{M_{K^+}^2}{m_s+m_u}=\frac{M_{K^0}^2}{m_s+m_d}$$

$$3M_{\eta_8}^2=4M_K^2-M_\pi^2 \text{ (isospin limit)}$$

$$\frac{m_u}{m_d}=0.55,\frac{m_s}{m_d}=20.1,\frac{m_s}{\hat{m}}=25.9.$$

	m_u/m_d	m_s/m_d	$m_s/\hat{m}$
$O(p^2)$	0.55	20.1	25.9
$O(p^4)$	0.55 ± 0.04	18.9 ± 0.8	24.4 ± 1.5

$$A_2(s,t,u)=\frac{s-M_\pi^2}{F_\pi^2}$$

a_0^0	r	$B(\nu=1\text{GeV})$
0.16	26	1.4 GeV
0.26	10	F_π

$$a_0^0=0.220\pm0.005, a_0^2=-0.0444\pm0.0010$$

$$M_\pi^2=M^2-\frac{\bar{l}_3}{32\pi^2F^2}M^4+O(M^6)$$
$$M^2=(m_u+m_d)|\langle0|\bar{u}u|0\rangle|/F^2$$

$$M_\sigma=441_{-8}^{+16}\text{MeV},\Gamma_\sigma=544_{-18}^{+25}\text{MeV}.$$

$$|V_{ud}|=0.9738(5),|V_{us}|=0.2200(26),$$

$$\sum_{j=d,s,b}|V_{uj}|^2-1=-0.0033(15).$$

$$\langle\pi^-(p_\pi)|\bar{s}\gamma_\mu u|K^0(p_K)\rangle=f_+^{K^0\pi^-}(t)(p_K+p_\pi)_\mu+f_-^{K^0\pi^-}(t)(p_K-p_\pi)_\mu$$

$$f_+^{K^0\pi^-}(0)=1+f_{p^4}+f_{e^2p^2}+f_{p^6}+O[(m_u-m_d)p^4,e^2p^4]$$

$$r_{+0} = \left(\frac{2\Gamma\left(K^{+}_{e3(\gamma)}\right) M^{5}_{K^{0}} I_{K^{0}}}{\Gamma\left(K^{0}_{e3(\gamma)}\right) M^{5}_{K^{+}} I_{K^{+}}} \right)^{1/2} = \frac{\left| f_{+}^{K^{+}\pi^{0}}(0) \right|}{\left| f_{+}^{K^{0}\pi^{-}}(0) \right|}$$

$$r_{+0}^{\text{th}} = 1.023 \pm 0.003$$

$$r_{+0}^{\text{exp}} = 1.036 \pm 0.008$$

$$\begin{aligned} f_{p^6}^{L=1,2}(M_\rho) &= 0.0093 \pm 0.0005 \\ f_{p^6}^{\text{tree}}(M_\rho) &= 8\frac{(M_K^2 - M_\pi^2)^2}{F_\pi^2} \left[\frac{\left(L_5^r(M_\rho)\right)^2}{F_\pi^2} - C_{12}^r(M_\rho) - C_{34}^r(M_\rho) \right] \\ &= -\frac{(M_K^2 - M_\pi^2)^2}{2M_S^4} \left(1 - \frac{M_S^2}{M_P^2}\right)^2 \end{aligned}$$

$$f_{p^6}^{\text{tree}}(M_\rho) = -0.002 \pm 0.008_{1/N_c} \pm 0.002_{M_S}\,{}^{+0.000}_{-0.002}{}_{P'}$$

$$f_{p^6} = 0.007 \pm 0.012$$

$$f_{+}^{K^0\pi^-}(0) = 0.984 \pm 0.012$$

$$|V_{us}| = 0.2208 \pm 0.0027_{f_+(0)} \pm 0.0008_{\text{exp}}$$

$$\lambda_0 = (13 \pm 3) \cdot 10^{-3}$$
$$\lambda_0 = (13.72 \pm 1.31) \cdot 10^{-3}$$

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2}\text{tr}\left(G_{\mu\nu}G^{\mu\nu}\right) + \sum_{f=1}^{N_F} \bar{q}_f \left(i\not{D} - m_f \mathbb{1}_c\right) q_f$$

Por otro lado, la dimensión cromodinámica en un campo de gauge axial y curvo, viene dada por el hamiltoniano del centro de masa y energía inherente a la partícula supermasiva de orden hadrónico (bariones y mesones), según sea el caso:

$$H_{\text{KS}} = \sum_{f=u,d} \left[\frac{1}{2a} \sum_{n=0}^{2L-2} \left(\phi_n^{(f)\dagger} U_n \phi_{n+1}^{(f)} + \text{ h.c. } \right) + m_f \sum_{n=0}^{2L-1} (-1)^n \phi_n^{(f)\dagger} \phi_n^{(f)} + \frac{a g^2}{2} \sum_{n=0}^{2L-2} \sum_{a=1}^{8} \left| \mathbf{E}_n^{(a)} \right|^2 \right]$$
$$- \frac{\mu_B}{3} \sum_{f=u,d} \sum_{n=0}^{2L-1} \phi_n^{(f)\dagger} \phi_n^{(f)} - \frac{\mu_I}{2} \sum_{n=0}^{2L-1} \left(\phi_n^{(u)\dagger} \phi_n^{(u)} - \phi_n^{(d)\dagger} \phi_n^{(d)} \right)$$

$$H = \sum_{f=u,d}\left[\frac{1}{2}\sum_{n=0}^{2L-2}\left(\phi_n^{(f)\dagger}\phi_{n+1}^{(f)} + \text{ h.c. }\right) + m_f\sum_{n=0}^{2L-1}(-1)^n\phi_n^{(f)\dagger}\phi_n^{(f)}\right] + \frac{g^2}{2}\sum_{n=0}^{2L-2}\sum_{a=1}^{8}\left(\sum_{m\le n} Q_m^{(a)}\right)^2$$

$$-\frac{\mu_B}{3}\sum_{f=u,d}\sum_{n=0}^{2L-1}\phi_n^{(f)\dagger}\phi_n^{(f)} - \frac{\mu_I}{2}\sum_{n=0}^{2L-1}\left(\phi_n^{(u)\dagger}\phi_n^{(u)} - \phi_n^{(d)\dagger}\phi_n^{(d)}\right)$$

$$Q_m^{(a)} = \phi_m^{(u)\dagger}T^a\phi_m^{(u)} + \phi_m^{(d)\dagger}T^a\phi_m^{(d)}$$

$$\mathbf{E}_n^{(a)} = \sum_{m\le n} Q_m^{(a)}$$

$$H = H_{kin} + H_m + H_{el} + H_{\mu_B} + H_{\mu_I}$$

$$H_{kin} = -\frac{1}{2}\sum_{n=0}^{2L-2}\sum_{f=0}^{1}\sum_{c=0}^{2}\left[\sigma_{6n+3f+c}^{+}\left(\bigotimes_{i=1}^{5}\sigma_{6n+3f+c+i}^{z}\right)\sigma_{6(n+1)+3f+c}^{-} + \text{h.c.}\right]$$

$$H_m = \frac{1}{2}\sum_{n=0}^{2L-1}\sum_{f=0}^{1}\sum_{c=0}^{2} m_f\left[(-1)^n\sigma_{6n+3f+c}^{z} + 1\right]$$

$$H_{el} = \frac{g^2}{2}\sum_{n=0}^{2L-2}(2L-1-n)\left(\sum_{f=0}^{1} Q_{n,f}^{(a)}Q_{n,f}^{(a)} + 2Q_{n,0}^{(a)}Q_{n,1}^{(a)}\right)$$

$$+g^2\sum_{n=0}^{2L-3}\sum_{m=n+1}^{2L-2}(2L-1-m)\sum_{f=0}^{1}\sum_{f'=0}^{1} Q_{n,f}^{(a)}Q_{m,f'}^{(a)}$$

$$H_{\mu_B} = -\frac{\mu_B}{6}\sum_{n=0}^{2L-1}\sum_{f=0}^{1}\sum_{c=0}^{2}\sigma_{6n+3f+c}^{z}$$

$$H_{\mu_I} = -\frac{\mu_I}{4}\sum_{n=0}^{2L-1}\sum_{f=0}^{1}\sum_{c=0}^{2}(-1)^f\sigma_{6n+3f+c}^{z}$$

$$Q_{n,f}^{(a)}Q_{n,f}^{(a)} = \frac{1}{3}\left(3 - \sigma_{6n+3f}^{z}\sigma_{6n+3f+1}^{z} - \sigma_{6n+3f}^{z}\sigma_{6n+3f+2}^{z} - \sigma_{6n+3f+1}^{z}\sigma_{6n+3f+2}^{z}\right)$$

$$Q_{n,f}^{(a)}Q_{m,f'}^{(a)}$$

$$= \frac{1}{4}\left[2\left(\sigma_{6n+3f}^{+}\sigma_{6n+3f+1}^{-}\sigma_{6m+3f'}^{-}\sigma_{6m+3f'+1}^{+}\right.\right.$$

$$+ \sigma_{6n+3f}^{+}\sigma_{6n+3f+1}^{z}\sigma_{6n+3f+2}^{-}\sigma_{6m+3f'}^{-}\sigma_{6m+3f'+1}^{z}\sigma_{6m+3f'+2}^{+} + \sigma_{6n+3f+1}^{+}\sigma_{6n+3f+2}^{-}\sigma_{6m+3f'+1}^{-}\sigma_{6m+3f'+2}^{+}$$

$$\left.\left. + \text{ h.c. }\right) + \frac{1}{6}\sum_{c=0}^{2}\sum_{c'=0}^{2}(3\delta_{cc'} - 1)\sigma_{6n+3f+c}^{z}\sigma_{6m+3f'+c'}^{z}\right]$$

$$H_{\mathbf{1}} = \frac{h^2}{2}\sum_{n=0}^{2L-1}\left(\sum_{f=0}^{1} Q_{n,f}^{(a)}Q_{n,f}^{(a)} + 2Q_{n,0}^{(a)}Q_{n,1}^{(a)}\right) + h^2\sum_{n=0}^{2L-2}\sum_{m=n+1}^{2L-1}\sum_{f=0}^{1}\sum_{f'=0}^{1} Q_{n,f}^{(a)}Q_{m,f'}^{(a)},$$

$$\langle H_{el}\rangle = 2(\langle\Delta|H_{el}|\Delta\rangle - \langle\Omega|H_{el}|\Omega\rangle) - (\langle\Delta\Delta|H_{el}|\Delta\Delta\rangle - \langle\Omega|H_{el}|\Omega\rangle).$$

$$S_L = 1 - \mathrm{Tr}[\rho_q^2]$$

$$H_{kin} \sim \sigma^+ ZZZZZ\sigma^- + \text{h.c.}$$

$$\sigma^+\sigma^-\sigma^-\sigma^+ + \text{h.c.} = \frac{1}{8}(XXXX + YYXX + YXYX - YXXY - XYYX + XYXY + XXYY + YYYY)$$

$$G^\dagger(\sigma^+\sigma^-\sigma^-\sigma^+ + \text{h.c.})G = \frac{1}{8}(IIZI - ZIZZ - ZZZZ + ZIZI + IZZI - IIZZ - IZZZ + ZZZI)$$

$$\tilde{G}^\dagger(\sigma^+\sigma^-\sigma^-\sigma^+ + \text{h.c.})\tilde{G} = \frac{1}{8}(IIIZ - IZZZ - IIZZ + ZIIZ + IZIZ - ZZZZ - ZIZZ + ZZIZ)$$

$$G^\dagger(IZZI + IZIZ + ZIIZ)G = IZII + IIIZ + ZIII,$$
$$\tilde{G}^\dagger(ZIZI + IZZI + ZIIZ)\tilde{G} = IIZI + IZII + ZIII.$$

$$C = \left[\sum_{n=0}^{2L-1} Q_n^{(a)}, Q_m^{(\tilde{b})}\cdot Q_l^{(\tilde{b})}\right],$$

$$(T^a)^i_j(T^a)^k_l = \left(\hat{O}^a_{27}\right)^{ik}_{jl} - \frac{2}{5}\left[\delta^i_j\left(\hat{O}^a_8\right)^k_l + \delta^k_l\left(\hat{O}^a_8\right)^i_j\right] + \frac{3}{5}\left[\delta^i_l\left(\hat{O}^a_8\right)^k_j + \delta^k_j\left(\hat{O}^a_8\right)^i_l\right] + \frac{1}{8}\left(\delta^i_l\delta^k_j - \frac{1}{3}\delta^i_j\delta^k_l\right)\hat{O}^a_1$$

$$\left(\hat{O}^a_{27}\right)^{ik}_{jl} = \frac{1}{2}\left[(T^a)^i_j(T^a)^k_l + (T^a)^i_l(T^a)^k_j\right] - \frac{1}{10}\left[\delta^i_j\left(\hat{O}^a_8\right)^k_l + \delta^i_l\left(\hat{O}^a_8\right)^k_j + \delta^k_j\left(\hat{O}^a_8\right)^i_l + \delta^k_l\left(\hat{O}^a_8\right)^i_j\right]$$

$$-\frac{1}{24}\left(\delta^i_j\delta^k_l + \delta^i_l\delta^k_j\right)\hat{O}^a_1\left(\hat{O}^a_8\right)^i_j = (T^a)^i_\beta(T^a)^\beta_j - \frac{1}{3}\delta^i_j\hat{O}^a_1, \hat{O}^a_1 = (T^a)^\alpha_\beta(T^a)^\beta_\alpha = \frac{1}{2}$$

$$\mathcal{O}_i = \begin{cases}(\sigma^+ I\sigma^-\sigma^- Z\sigma^+ - \sigma^+ Z\sigma^-\sigma^- I\sigma^+) - \text{h.c.}\\(I\sigma^-\sigma^+ Z\sigma^+\sigma^- - Z\sigma^-\sigma^+ I\sigma^+\sigma^-) - \text{h.c.}\\(\sigma^+\sigma^- Z\sigma^-\sigma^+ I - \sigma^+\sigma^- I\sigma^-\sigma^+ Z) - \text{h.c.}\end{cases}$$

$$U_m: 2N_cN_fL \mid R_Z$$

$$U_{\mu_B}: 2N_cN_fL \mid R_Z$$

$$U_{\mu_I}: 2N_cN_fL \mid R_Z$$

$$U_{kin}: 2N_cN_f(2L-1) \mid R_Z 2N_cN_f(2L-1) \mid 2N_cN_f(8L-3) - 4 \mid \text{CNOT}$$

$$U_{el}: \frac{1}{2}(2L-1)N_cN_f[3-4N_c+N_f(2L-1)(5N_c-4)] \mid R_Z\frac{1}{2}(2L-1)(N_c-1)N_cN_f[N_f(2L-1)-1] \mid \frac{1}{6}(2L-1)(N_c-1)N_cN_f[(2L-1)(2N_c+17)N_f-2N_c-11] \Big| \text{ CNOT}$$

$$e^{-i\alpha Q_n^{(a)}Q_m^{(a)}} = \exp\Big\{-i\frac{\alpha}{2}[\sigma_n^+\sigma_{n+1}^-\sigma_m^-\sigma_{m+1}^+ + \sigma_n^-\sigma_{n+1}^+\sigma_m^+\sigma_{m+1}^- + \sigma_{n+1}^+\sigma_{n+2}^-\sigma_{m+1}^-\sigma_{m+2}^+ + \sigma_{n+1}^-\sigma_{n+2}^+\sigma_{m+1}^+\sigma_{m+2}^- + \sigma_n^+\sigma_{n+1}^z\sigma_{n+2}^-\sigma_m^-\sigma_{m+1}^z\sigma_{m+2}^+ + \sigma_n^-\sigma_{n+1}^z\sigma_{n+2}^+\sigma_m^+\sigma_{m+1}^z\sigma_{m+2}^- + \frac{1}{6}(\sigma_n^z\sigma_m^z + \sigma_{n+1}^z\sigma_{m+1}^z + \sigma_{n+2}^z\sigma_{m+2}^z) - \frac{1}{12}(\sigma_n^z\sigma_{m+1}^z + \sigma_n^z\sigma_{m+2}^z + \sigma_{n+1}^z\sigma_m^z + \sigma_{n+1}^z\sigma_{m+2}^z + \sigma_{n+2}^z\sigma_m^z + \sigma_{n+2}^z\sigma_{m+1}^z)]\Big\}$$

$$\left\|e^{-iHt} - \left[U_1\left(\frac{t}{N_{\text{Trott}}}\right)\right]^{N_{\text{Trott}}}\right\| \leq \frac{1}{2}\sum_i \sum_{j>i} \|[H_i, H_j]\| \frac{t^2}{N_{\text{Trott}}}$$

$$N_{\text{Trott}} = 0.0393(5)t^2 + 4.13(10)t - 22(5)$$

$$|\Omega_0\rangle, \frac{1}{\sqrt{3}}(|q_r\bar{q}_r\rangle - |q_g\bar{q}_g\rangle + |q_b\bar{q}_b\rangle)$$

$$|q_r\bar{q}_rq_g\bar{q}_gq_b\bar{q}_b\rangle, \frac{1}{\sqrt{3}}(|q_r\bar{q}_rq_g\bar{q}_g\rangle - |q_r\bar{q}_rq_b\bar{q}_b\rangle + |q_g\bar{q}_gq_b\bar{q}_b\rangle)$$

$$\theta_{10} = \theta_{01}, \qquad \theta_{00} = -2\sin^{-1}[\tan(\theta_0/2)\cos(\theta_{01}/2)]$$

$$\theta_{01} = -2\sin^{-1}[\cos(\theta_{11}/2)\tan(\theta_1/2)], \qquad \theta_0 = -2\sin^{-1}[\tan(\theta/2)\cos(\theta_1/2)]$$

$$\langle\Omega_0|U_{var}^\dagger(\theta)\tilde{H}U_{var}(\theta)|\Omega_0\rangle,$$

$$\left(P_{\text{pred}}^{(\text{phys})} - \frac{1}{8}\right) = \left(P_{\text{meas}}^{(\text{phys})} - \frac{1}{8}\right) \times \left(\frac{1-\frac{1}{8}}{P_{\text{meas}}^{(\text{mit})} - \frac{1}{8}}\right),$$

$$H_{el} = \frac{g^2}{2}\sum_{n=0}^{2L-2}\left(\sum_{m\leq n} Q_m^{(a)}\right)^2, Q_m^{(a)} = \phi_m^\dagger T^a \phi_m$$

$$H = H_{kin} + H_m + H_{el} + H_{\mu_B}$$

$$H_{kin} = \frac{1}{2}\sum_{n=0}^{2L-2}\sum_{f=0}^{N_f-1}\sum_{c=0}^{N_c-1}\left[\sigma^{+}_{i(n,f,c)}\left(\bigotimes_{j=1}^{N_cN_f-1}\left(-\sigma^{z}_{i(n,f,c)+j}\right)\right)\sigma^{-}_{i(n,f,c)+N_cN_f} + \text{h.c.}\right]$$

$$H_m = \frac{1}{2}\sum_{n=0}^{2L-1}\sum_{f=0}^{N_f-1}\sum_{c=0}^{N_c-1} m_f\left[(-1)^n\sigma^{z}_{i(n,f,c)} + 1\right]$$

$$H_{el} = \frac{g^2}{2}\sum_{n=0}^{2L-2}(2L-1-n)\left(\sum_{f=0}^{N_f-1} Q^{(a)}_{n,f}Q^{(a)}_{n,f} + 2\sum_{f=0}^{N_f-2}\sum_{f'=f+1}^{N_f-1} Q^{(a)}_{n,f}Q^{(a)}_{n,f'}\right)$$

$$+g^2\sum_{n=0}^{2L-3}\sum_{m=n+1}^{2L-2}(2L-1-m)\sum_{f=0}^{N_f-1}\sum_{f'=0}^{N_f-1} Q^{(a)}_{n,f}Q^{(a)}_{m,f'}$$

$$H_{\mu_B} = -\frac{\mu_B}{2N_c}\sum_{n=0}^{2L-1}\sum_{f=0}^{N_f-1}\sum_{c=0}^{N_c-1}\sigma^{z}_{i(n,f,c)}$$

$$4Q^{(a)}_{n,f}Q^{(a)}_{n,f} = \frac{N_c^2-1}{2} - \left(1+\frac{1}{N_c}\right)\sum_{c=0}^{N_c-2}\sum_{c'=c+1}^{N_c-1}\sigma^{z}_{i(n,f,c)}\sigma^{z}_{i(n,f,c')}$$

$$8Q^{(a)}_{n,f}Q^{(a)}_{m,f'} = 4\sum_{c=0}^{N_c-2}\sum_{c'=c+1}^{N_c-1}\left[\sigma^{+}_{i(n,f,c)}(\otimes Z)_{(n,f,c,c')}\sigma^{-}_{i(n,f,c')}\sigma^{-}_{i(m,f',c)}(\otimes Z)_{(m,f',c,c')}\sigma^{+}_{i(m,f',c')}\right.$$

$$\left.+\text{h.c.}\right] + \sum_{c=0}^{N_c-1}\sum_{c'=0}^{N_c-1}\left(\delta_{cc'} - \frac{1}{N_c}\right)\sigma^{z}_{i(n,f,c)}\sigma^{z}_{i(m,f',c')}(\otimes Z)_{(n,f,c,c')}$$

$$\equiv \bigotimes_{k=1}^{c'-c-1}\sigma^{z}_{i(n,f,c)+k}$$

$$4Q^{(a)}_{n,f}Q^{(a)}_{n,f} = \sum_{c=0}^{N_c-2}\sum_{c'=c+1}^{N_c-1}\left(1-\sigma^{z}_{i(n,f,c)}\sigma^{z}_{i(n,f,c')}\right)$$

$$8Q^{(a)}_{n,f}Q^{(a)}_{m,f'} = 4\sum_{c=0}^{N_c-2}\sum_{c'=c+1}^{N_c-1}\left[\sigma^{+}_{i(n,f,c)}(\otimes Z)_{(n,f,c,c')}\sigma^{-}_{i(n,f,c')}\sigma^{-}_{i(m,f',c)}(\otimes Z)_{(m,f',c,c')}\sigma^{+}_{i(m,f',c')}\right.$$

$$\left.+\text{h.c.}\right]$$

En el que, el mapeo de qubits viene dado por:

$$\mathbf{E}^{(a)}_{n+1} - \mathbf{E}^{(a)}_{n} = Q^{(a)}_{n}$$

$$\mathbf{E}_n^{(a)} = \mathbf{F}^{(a)} + \sum_{i\le n} Q_i^{(a)} + \frac{1}{2}\sum_{n=0}^{2L-2}\sum_{f=0}^{1}\sum_{c=0}^{2}\left(\psi_{6n+3f+c}^{\dagger}\psi_{6(n+1)+3f+c} + \text{ h.c.}\right) + \frac{g^2}{2}\sum_{n=0}^{2L-2}\left(\sum_{m\le n}\sum_{f=0}^{1} Q_{m,f}^{(a)}\right)^2$$

$$Q_{m,f}^{(a)} = \sum_{c=0}^{2}\sum_{c'=0}^{2} \psi_{6m+3f+c}^{\dagger} T_{cc'}^{a} \psi_{6m+3f+c'}$$

$$\psi_n = \bigotimes_{l<n} (-\sigma_l^z)\sigma_n^-, \psi_n^{\dagger} = \bigotimes_{l<n} (-\sigma_l^z)\sigma_n^+$$

$$Q_{m,f}^{(8)} = \frac{1}{4\sqrt{3}}\left(\sigma_{6m+3f}^z + \sigma_{6m+3f+1}^z - 2\sigma_{6m+3f+2}^z\right)$$

$$Q_{m,f}^{(1)} = \frac{1}{2}\sigma_{6m+3f}^{+}\sigma_{6m+3f+1}^{-} + \text{h.c.}$$

$$Q_{m,f}^{(2)} = -\frac{i}{2}\sigma_{6m+3f}^{+}\sigma_{6m+3f+1}^{-} + \text{h.c.}$$

$$Q_{m,f}^{(3)} = \frac{1}{4}\left(\sigma_{6m+3f}^z - \sigma_{6m+3f+1}^z\right)$$

$$Q_{m,f}^{(4)} = -\frac{1}{2}\sigma_{6m+3f}^{+}\sigma_{6m+3f+1}^{z}\sigma_{6m+3f+2}^{-} + \text{h.c.}$$

$$Q_{m,f}^{(5)} = \frac{i}{2}\sigma_{6m+3f}^{+}\sigma_{6m+3f+1}^{z}\sigma_{6m+3f+2}^{-} + \text{h.c.}$$

$$Q_{m,f}^{(6)} = \frac{1}{2}\sigma_{6m+3f+1}^{+}\sigma_{6m+3f+2}^{-} + \text{h.c.}$$

$$Q_{m,f}^{(7)} = -\frac{i}{2}\sigma_{6m+3f+1}^{+}\sigma_{6m+3f+2}^{-} + \text{h.c.}$$

$$H = H_{kin} + H_m + H_{el} + H_{\mu_B} + H_{\mu_I}$$

$$\begin{aligned}H_{kin} = -\tfrac{1}{2}(&\sigma_6^+\sigma_5^z\sigma_4^z\sigma_3^z\sigma_2^z\sigma_1^z\sigma_0^- + \sigma_6^-\sigma_5^z\sigma_4^z\sigma_3^z\sigma_2^z\sigma_1^z\sigma_0^+ + \sigma_7^+\sigma_6^z\sigma_5^z\sigma_4^z\sigma_3^z\sigma_2^z\sigma_1^- + \sigma_7^-\sigma_6^z\sigma_5^z\sigma_4^z\sigma_3^z\sigma_2^z\sigma_1^+\\ &+\sigma_8^+\sigma_7^z\sigma_6^z\sigma_5^z\sigma_4^z\sigma_3^z\sigma_2^- + \sigma_8^-\sigma_7^z\sigma_6^z\sigma_5^z\sigma_4^z\sigma_3^z\sigma_2^+ + \sigma_9^+\sigma_8^z\sigma_7^z\sigma_6^z\sigma_5^z\sigma_4^z\sigma_3^- + \sigma_9^-\sigma_8^z\sigma_7^z\sigma_6^z\sigma_5^z\sigma_4^z\sigma_3^+\\ &+\sigma_{10}^+\sigma_9^z\sigma_8^z\sigma_7^z\sigma_6^z\sigma_5^z\sigma_4^- + \sigma_{10}^-\sigma_9^z\sigma_8^z\sigma_7^z\sigma_6^z\sigma_5^z\sigma_4^+ + \sigma_{11}^+\sigma_{10}^z\sigma_9^z\sigma_8^z\sigma_7^z\sigma_6^z\sigma_5^- + \sigma_{11}^-\sigma_{10}^z\sigma_9^z\sigma_8^z\sigma_7^z\sigma_6^z\sigma_5^+)\end{aligned}$$

$$H_m = \tfrac{1}{2}[m_u(\sigma_0^z + \sigma_1^z + \sigma_2^z - \sigma_6^z - \sigma_7^z - \sigma_8^z + 6) + m_d(\sigma_3^z + \sigma_4^z + \sigma_5^z - \sigma_9^z - \sigma_{10}^z - \sigma_{11}^z + 6)]$$

$$\begin{aligned}H_{el} = \tfrac{g^2}{2}\Big[&\tfrac{1}{3}(3 - \sigma_1^z\sigma_0^z - \sigma_2^z\sigma_0^z - \sigma_2^z\sigma_1^z) + \sigma_4^+\sigma_3^-\sigma_1^-\sigma_0^+ + \sigma_4^-\sigma_3^+\sigma_1^+\sigma_0^- + \sigma_5^+\sigma_4^z\sigma_3^-\sigma_2^-\sigma_1^z\sigma_0^+ + \sigma_5^-\sigma_4^z\sigma_3^+\sigma_2^+\sigma_1^z\sigma_0^-\\ &+\sigma_5^+\sigma_4^-\sigma_2^-\sigma_1^+ + \sigma_5^-\sigma_4^+\sigma_2^+\sigma_1^-\\ &+\tfrac{1}{12}(2\sigma_3^z\sigma_0^z + 2\sigma_4^z\sigma_1^z + 2\sigma_5^z\sigma_2^z - \sigma_5^z\sigma_0^z - \sigma_5^z\sigma_1^z - \sigma_4^z\sigma_2^z - \sigma_4^z\sigma_0^z - \sigma_3^z\sigma_1^z - \sigma_3^z\sigma_2^z)\Big]\end{aligned}$$

$$H_{\mu_B} = -\tfrac{\mu_B}{6}(\sigma_0^z + \sigma_1^z + \sigma_2^z + \sigma_3^z + \sigma_4^z + \sigma_5^z - \sigma_6^z + \sigma_7^z + \sigma_8^z + \sigma_9^z + \sigma_{10}^z + \sigma_{11}^z)$$

$$H_{\mu_I} = -\tfrac{\mu_I}{4}(\sigma_0^z + \sigma_1^z + \sigma_2^z - \sigma_3^z - \sigma_4^z - \sigma_5^z + \sigma_6^z + \sigma_7^z + \sigma_8^z - \sigma_9^z - \sigma_{10}^z - \sigma_{11}^z)$$

$$H=\sum_{f=0}^{1}\sum_{c=0}^{2}\left[m\sum_{n=0}^{2L-1}(-1)^{n}\psi_{6n+3f+c}^{\dagger}\psi_{6n+3f+c}+\frac{1}{2}\sum_{n=0}^{2L-2}\left(\psi_{6n+3f+c}^{\dagger}\psi_{6(n+1)+3f+c}+\text{ h.c. }\right)\right]$$

$$H=\Psi_{i}^{\dagger}M_{ij}\Psi_{j}$$

$$M=\begin{bmatrix} m & 1/2\\ 1/2 & -m\end{bmatrix},$$

$$\tilde{M}=\begin{bmatrix}\lambda & 0\\ 0 & -\lambda\end{bmatrix},\lambda=\frac{1}{2}\sqrt{1+4m^{2}}$$

$$\tilde{\psi}_{i}=\frac{1}{\sqrt{2}}\left(\sqrt{1+\frac{\lambda}{m}}\psi_{i}+\sqrt{1-\frac{\lambda}{m}}\psi_{6+i}\right),\tilde{\psi}_{6+i}=\frac{1}{\sqrt{2}}\left(-\sqrt{1-\frac{\lambda}{m}}\psi_{i}+\sqrt{1+\frac{\lambda}{m}}\psi_{6+i}\right)$$

$$H=\sum_{i=0}^{5}\lambda\left(\tilde{\psi}_{i}^{\dagger}\tilde{\psi}_{i}-\tilde{\psi}_{6+i}^{\dagger}\tilde{\psi}_{6+i}\right)$$

$$|\Omega_{0}\rangle=\prod_{i=0}^{i=5}\tilde{\psi}_{6+i}^{\dagger}|\omega_{0}\rangle$$

$$H=\sum_{i=0}^{5}\lambda\left(\tilde{\psi}_{i}^{\dagger}\tilde{\psi}_{i}-\tilde{\psi}_{6+i}^{\dagger}\tilde{\psi}_{6+i}\right)=\lambda\left(\tilde{\Psi}^{\dagger}\tilde{\Psi}-6\right)$$

$$F=\langle\Psi|\tilde{H}|\Psi\rangle-\eta\langle\Psi\mid\Psi\rangle=\sum_{\alpha\beta}^{n_{s}}a_{\alpha}a_{\beta}\left[\langle\psi_{\alpha}|\tilde{H}|\psi_{\beta}\rangle-\eta\langle\psi_{\alpha}\mid\psi_{\beta}\rangle\right]=\sum_{\alpha\beta}^{n_{s}}a_{\alpha}a_{\beta}\left(\tilde{H}_{\alpha\beta}-\eta\delta_{\alpha\beta}\right)=\sum_{\alpha\beta}^{n_{s}}a_{\alpha}a_{\beta}h_{\alpha\beta},$$

$$a_{\alpha}^{(z+1)}=a_{\alpha}^{(z)}+\sum_{i=1}^{K}2^{i-K-z}(-1)^{\delta_{iK}}q_{i}^{\alpha}$$

$$F=\sum_{\alpha,\beta}^{n_{s}}\sum_{i,j}^{K}Q_{\alpha,i;\beta,j}q_{i}^{\alpha}q_{j}^{\beta},Q_{\alpha,i;\beta,j}=2^{i+j-2K-2z}(-1)^{\delta_{iK}+\delta_{jK}}h_{\alpha\beta}+2\delta_{\alpha\beta}\delta_{ij}2^{i-K-z}(-1)^{\delta_{iK}}\sum_{\gamma}^{n_{s}}a_{\gamma}^{(z)}h_{\gamma\beta}$$

		$\lvert\Omega\rangle$		$\lvert\sigma\rangle$		$\lvert\pi\rangle$
		1		1		1
Step	δE_{Ω}	$-\lvert\langle\Psi_{\Omega}^{\text{exact}}$	δM_{σ}	$-\lvert\langle\Psi_{\sigma}^{\text{exact}}$	δM_{π}	$-\lvert\langle\Psi_{\pi}^{\text{exact}}$
		$\mid\Psi_{\Omega}^{\text{Adv}}\rangle\rvert^{2}$		$\mid\Psi_{\sigma}^{\text{Adv}}\rangle\rvert^{2}$		$\mid\Psi_{\pi}^{\text{Adv}}\rangle\rvert^{2}$

0	$4^{+2}_{-2} \times 10^{-1}$	$10^{+3}_{-5} \times 10^{-2}$	$4^{+2}_{-2} \times 10^{-1}$	$11^{+7}_{-5} \times 10^{-2}$	$3^{+1}_{-1} \times 10^{-1}$	$11^{+71}_{-4} \times 10^{-2}$
1	$9^{+4}_{-3} \times 10^{-3}$	$2^{+6}_{-5} \times 10^{-3}$	$3^{+1}_{-1} \times 10^{-2}$	$7^{+2}_{-2} \times 10^{-3}$	$9^{+4}_{-3} \times 10^{-3}$	$3^{+3}_{-1} \times 10^{-3}$
2	$6^{+2}_{-2} \times 10^{-4}$	$12^{+3}_{-5} \times 10^{-5}$	$4^{+1}_{-1} \times 10^{-3}$	$12^{+3}_{-4} \times 10^{-4}$	$7^{+2}_{-3} \times 10^{-4}$	$3^{+2}_{-2} \times 10^{-4}$
3	$4^{+1}_{-2} \times 10^{-5}$	$9^{+3}_{-4} \times 10^{-6}$	$2^{+1}_{-1} \times 10^{-4}$	$6^{+1}_{-2} \times 10^{-5}$	$4^{+2}_{-2} \times 10^{-5}$	$12^{+6}_{-3} \times 10^{-6}$
4	$16^{+6}_{-6} \times 10^{-7}$	$3^{+2}_{-1} \times 10^{-7}$	$10^{+6}_{-3} \times 10^{-6}$	$9^{+1}_{-1} \times 10^{-6}$	$7^{+9}_{-5} \times 10^{-7}$	$8^{+2}_{-2} \times 10^{-6}$

$$\sigma^+\sigma^-\sigma^-\sigma^+ + \text{h.c.} = \frac{1}{8}(XXXX + XXYY + XYXY - XYYX + YXYX - YXXY + YYXX + YYYY)$$

$$G^\dagger(XXXX + YYXX + YXYX - YXXY - XYYX + XYXY + XXYY + YYYY)G$$
$$= IIZI - ZIZZ - ZZZZ + ZIZI + IZZI - IIZZ - IZZZ + ZZZI$$

$$G^\dagger(IZZI + IZIZ + ZIIZ)G = IZII + IIIZ + ZIII$$

$$\left|\langle\Omega_0|e^{-iH_{kin}t}|\Omega_0\rangle\square\right|^2 = \cos^6(t/2), \left|\langle q_r\bar{q}_r|e^{-iH_{kin}t}|\Omega_0\rangle\square\right|^2 = \cos^4(t/2)\sin^2(t/2), \left|\langle B\bar{B}|e^{-iH_{kin}t}|\Omega_0\rangle\square\right|^2 = \sin^6(t/2)$$

$$\sum_{b=1}^{8}\left|\mathbf{E}_n^{(a)}\right|^2|\mathbf{R},\alpha,\beta\rangle_n = \frac{1}{3}(p^2 + q^2 + pq + 3p + 3q)|\mathbf{R},\alpha,\beta\rangle_n$$

$$U_{el}: (2L-1)N_f\left[9(2L-1)N_f - 7\right] \mid \text{CNOT}\,.$$

$$U_{kin}: 2(2L-1)N_c(N_c+1) \mid \text{CNOT}\,.$$

$$\left(T^{(a)}\right)^\alpha_\beta\left(T^{(a)}\right)^\gamma_\delta = \frac{1}{2}\left(\delta^\alpha_\delta\delta^\gamma_\beta - \frac{1}{N_c}\delta^\alpha_\beta\delta^\gamma_\delta\right)$$

En cuanto al teorema cromodinámico de factorización cuántica, para un campo de gauge curvo o relativista, causado por la interacción de una partícula hadrónica, mesónica o bariónica supermasiva, según sea el caso, se calcula así:

$$\mathcal{L}_{QCD} = \bar{\psi}(i\not{D}_a T_a - m)\psi - \frac{1}{4}F_a^{\mu\nu}F_{\mu\nu a}$$

$$D_a^\mu = \partial^\mu + igA_a^\mu$$
$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - gf_{abc}A_b^\mu A_c^\nu$$

$$\left[T_a^{(F)}, T_b^{(F)}\right] = if_{abc}T_c^{(F)}, \left(T_a^{(A)}\right)_{bc} = -if_{abc}$$

$$\mathcal{L}_{QCD} = \bar{\psi}(i\not{D}_a T_a - m)\psi - \frac{1}{4}F_a^{\mu\nu}F_{\mu\nu a} - \frac{1}{2}\lambda\left(\partial_\mu A_a^\mu\right)^2 + \partial_\mu\eta_a^\dagger\left(\partial^\mu + gf_{abc}A_c^\mu\right)\eta_b,$$

$$\Gamma_{3g} = -gf_{a_1a_2a_3}[g^{\nu_1\nu_2}(p_1 - p_2)^{\nu_3} + g^{\nu_2\nu_3}(p_2 - p_3)^{\nu_1} + g^{\nu_3\nu_1}(p_3 - p_1)^{\nu_2}$$
$$\Gamma_{4g} = -ig^2\left[f_{ea_1a_2}f_{ea_3a_4}(g^{\nu_1\nu_3}g^{\nu_2\nu_4} - g^{\nu_1\nu_4}g^{\nu_2\nu_3}) + f_{ea_1a_3}f_{ea_4a_2}(g^{\nu_1\nu_4}g^{\nu_3\nu_2} - g^{\nu_1\nu_2}g^{\nu_3\nu_4})\right.$$
$$\left.+ f_{ea_1a_4}f_{ea_2a_3}(g^{\nu_1\nu_2}g^{\nu_4\nu_3} - g^{\nu_1\nu_3}g^{\nu_4\nu_2})\right]$$

$$\int \frac{d^4l}{(2\pi)^4}(-ig\gamma^\nu T_a)\frac{i\left(\not{p}_1 - \not{l}\right)}{(p_1 - l)^2 + i\epsilon}(-ie\gamma_\mu)\frac{-i\left(\not{p}_2 - \not{l}\right)}{(p_2 - l)^2 + i\epsilon}(-ig\gamma_\nu T_a)\frac{-i}{l^2 + i\epsilon},$$

$$l^\pm = \frac{l^0 \pm l^z}{\sqrt{2}}, \mathbf{l}_T = (l^x, l^y)$$

$$\int \frac{dl^+dl^-d^2l_T}{(2\pi)^4}\frac{1}{2(l^+ - p_1^+)l^- - l_T^2 + i\epsilon}\frac{1}{2l^+(l^- - p_2^-) - l_T^2 + i\epsilon}\frac{1}{2l^+l^- - l_T^2 + i\epsilon}$$

$$l^- = \frac{l_T^2}{2(l^+ - p_1^+)} + i\epsilon, l^- = p_2^- + \frac{l_T^2}{2l^+} - i\epsilon, l^- = \frac{l_T^2}{2l^+} - i\epsilon$$

$$\frac{-i}{2p_1^+}\int \frac{dl^+d^2l_T}{(2\pi)^3}\frac{p_1^+ - l^+}{2p_2^-l^+(p_1^+ - l^+) + p_1^+l_T^2}\frac{1}{l_T^2} \approx \frac{-i}{4p_1 \cdot p_2}\frac{1}{(2\pi)^3}\int \frac{dl^+}{l^+}\int \frac{d^2l_T}{l_T^2}$$

$$\sigma^{(0)} = N_c\frac{4\pi\alpha^2}{3Q^2}\sum_f Q_f^2$$

$$\sigma^{(1)V} = -2N_cC_F\sum_f Q_f^2\frac{\alpha\alpha_s}{\pi}Q^2\left(\frac{4\pi\mu^2}{Q^2}\right)^{2\epsilon}\frac{1 - \epsilon}{\Gamma(2 - 2\epsilon)}\left[\frac{1}{\epsilon^2} + \frac{3}{2}\frac{1}{\epsilon} - \frac{\pi^2}{2} + 4 + O(\epsilon)\right],$$

$$\sigma^{(1)R} = 2N_cC_F\sum_f Q_f^2\frac{\alpha\alpha_s}{\pi}Q^2\left(\frac{4\pi\mu^2}{Q^2}\right)^{2\epsilon}\frac{1 - \epsilon}{\Gamma(2 - 2\epsilon)}\left[\frac{1}{\epsilon^2} + \frac{3}{2}\frac{1}{\epsilon} - \frac{\pi^2}{2} + \frac{19}{4} + O(\epsilon)\right].$$

$$\sigma = N_c\frac{4\pi\alpha^2}{3Q^2}\sum_f Q_f^2\left[1 + \frac{3}{4}\frac{\alpha_s(Q)}{\pi}C_F\right]$$

$$F_2^q(x, Q^2) = x\left\{\delta(1 - x) + \frac{\alpha_s}{2\pi}C_F\left[\frac{1 + x^2}{1 - x}\left(\ln\frac{1 - x}{x} - \frac{3}{4}\right) + \frac{1}{4}(9 + 5x)\right]_+ + \frac{\alpha_s}{2\pi}C_F\left(\frac{1 + x^2}{1 - x}\right)_+(4\pi\mu e^{-\gamma_E})^\epsilon\int_0^{Q^2}\frac{dk_T^2}{k_T^{2+2\epsilon}} + \cdots\right\}$$

$$\int_0^1 dx \frac{f(x)}{(1-x)_+} \equiv \int_0^1 dx \frac{f(x)-f(1)}{1-x}$$

$$\int_0^{Q^2} \frac{dk_T^2}{k_T^{2+2\epsilon}} = \frac{1}{-\epsilon}(Q^2)^{-\epsilon}$$

$$F_2^q(x,Q^2) = H^{(0)} \otimes \phi_{f/N}^{(0)} + \frac{\alpha_s}{2\pi} H^{(1)} \otimes \phi_{q/N}^{(0)} + \frac{\alpha_s}{2\pi} H^{(0)} \otimes \phi_{q/N}^{(1)} + \cdots,$$

$$H \otimes \phi_{q/N} \equiv \int_x^1 \frac{d\xi}{\xi} H(x/\xi, Q, \mu)\phi_{q/N}(\xi,\mu)$$

$$H^{(0)}(x/\xi, Q, \mu) = \delta(1 - x/\xi), \phi_{q/N}^{(0)}(\xi,\mu) = \delta(1-\xi)$$

$$H^{(1)}(x,Q,\mu) = P_{qq}^{(1)}(x)\ln\frac{Q^2}{\mu^2} + \cdots$$

$$\phi_{q/N}^{(1)}(\xi,\mu) = (4\pi\mu e^{-\gamma})^{\epsilon} P_{qq}^{(1)}(\xi) \int_0^{\mu^2} \frac{dk_T^2}{k_T^{2+2\epsilon}}$$

$$P_{qq}^{(1)}(x) = C_F\left(\frac{1+x^2}{1-x}\right)_+$$

$$\phi_{q/N}(\xi,\mu) = \int \frac{dy^-}{2\pi} \exp\left(-i\xi p^+ y^-\right)$$
$$\times \frac{1}{2}\sum_\sigma \langle N(p,\sigma)|\bar{q}(0,y^-,0_T)\frac{1}{2}\gamma^+ W(y^-,0) q(0,0,0_T)|N(p,\sigma)\rangle$$

$$W(y^-) = \mathcal{P}\exp\left[-ig\int_0^\infty dzn_- \cdot A(y+zn_-)\right]$$

$$\not{p}'\gamma^\nu \frac{\not{p}'+\not{l}}{(p'+l)^2}\gamma^\mu \frac{\not{p}+\not{l}}{(p+l)^2}\gamma_\nu \approx \not{p}'\gamma^- \frac{\not{p}'+\not{l}}{(p'+l)^2}\gamma^\mu \frac{\not{p}+\not{l}}{(p+l)^2}\gamma^+ \approx \not{p}'\gamma^- \frac{\not{p}'}{2p'\cdot l}\gamma^\mu \frac{\not{p}+\not{l}}{(p+l)^2}\gamma^+$$

$$\not{p}'\gamma^\mu \frac{\not{p}+\not{l}}{(p+l)^2}\gamma^+ \frac{p'^-}{p'\cdot l} \approx \not{p}'\gamma^\mu \frac{\not{p}+\not{l}}{(p+l)^2}\gamma^+ \frac{n_-^-}{n_-\cdot l} \approx \not{p}'\gamma^\mu \frac{\not{p}+\not{l}}{(p+l)^2}\gamma_\nu \frac{n_-^\nu}{n_-\cdot l},$$

$$-ig\int_0^\infty dzn_- \cdot \int d^4l \exp\left[iz(n_-\cdot l + i\epsilon)\right]\tilde{A}(l)$$
$$= -ig\int d^4l \frac{\exp\left[iz(n_-\cdot l + i\epsilon)\right]}{i(n_-\cdot l + i\epsilon)}\Bigg|_{z=0}^{z=\infty} n_-\cdot\tilde{A}(l) = \int d^4l \frac{gn_-^\nu}{n_-\cdot l + i\epsilon}\tilde{A}_\nu(l)$$

$$I_{ij}I_{lk} = \frac{1}{4}I_{ik}I_{lj} + \frac{1}{4}(\gamma_\alpha)_{ik}(\gamma^\alpha)_{lj} + \frac{1}{4}(\gamma^5\gamma_\alpha)_{ik}(\gamma^\alpha\gamma^5)_{lj}$$
$$+\frac{1}{4}(\gamma^5)_{ik}(\gamma^5)_{lj} + \frac{1}{8}\left(\gamma^5\sigma_{\alpha\beta}\right)_{ik}\left(\sigma^{\alpha\beta}\gamma^5\right)_{lj}$$

$$I_{ij}I_{lk} = \frac{1}{N_c}I_{ik}I_{lj} + 2(T^c)_{ik}(T^c)_{lj}$$

$$F_2(x,Q^2)=\sum_f\int_x^1\frac{d\xi}{\xi}H_f(x/\xi,Q,\mu)\phi_{f/N}(\xi,\mu)$$

Cuyo escalar y factores de gauge van como sigue:

$$\left|\sum_i \mathcal{M}_{i/N}\right|^2\approx\sum_i \left|\mathcal{M}_f\right|^2\phi_{f/N}$$

$$\mu\frac{d}{d\mu}\phi_{f/N}(\xi,\mu)=\gamma_f\phi_{f/N}(\xi,\mu)$$
$$\mu\frac{d}{d\mu}H_f(x/\xi,Q,\mu)=-\gamma_fH_f(x/\xi,Q,\mu)$$

$$\phi_{f/N}(\xi,Q)=\phi_{f/N}(\xi,Q_0)\exp\left[\int_{Q_0}^{Q}\frac{d\mu}{\mu}\gamma_f(\alpha_s(\mu))\right]$$

$$F_2(x,Q^2)=\sum_f\int_x^1\frac{d\xi}{\xi}\int d^2k_TH_f(x/\xi,k_T,Q,\mu)\Phi_{f/N}(\xi,k_T,\mu)$$

$$\Phi_{q/N}(\xi,k_T,\mu)=\int\frac{dy^-}{2\pi}\int\frac{d^2y_T}{(2\pi)^2}e^{-i\xi p^+y^-+i\mathbf{k}_T\cdot\mathbf{y}_T}$$
$$\times\frac{1}{2}\langle N(p,\sigma)|\bar q(0,y^-,y_T)\frac{1}{2}\gamma^+W(y^-,y_T,0,0_T)q(0,0,0_T)|N(p,\sigma)\rangle$$

$$J(p,n)u(p)=\langle 0|\mathcal{P}\exp\left[-ig\int_0^\infty dzn\cdot A(nz)\right]q(0)|p\rangle$$

$$p^+\frac{d}{dp^+}J=-\frac{n^2}{v\cdot n}v_\alpha\frac{d}{dn_\alpha}J,$$

$$-\frac{n^2}{v\cdot n}v_\alpha\frac{d}{dn_\alpha}\frac{n_\mu}{n\cdot l}=\frac{n^2}{v\cdot n}\left(\frac{v\cdot l}{n\cdot l}n_\mu-v_\mu\right)\frac{1}{n\cdot l}\equiv\frac{\hat n_\mu}{n\cdot l},$$

$$p^+\frac{d}{dp^+}J=[K(m/\mu,\alpha_s(\mu))+G(p^+v/\mu,\alpha_s(\mu))]J,$$

$$K=-ig^2C_F\mu^\epsilon\int\frac{d^{4-\epsilon}l}{(2\pi)^{4-\epsilon}}\frac{\hat n_\mu}{n\cdot l}\frac{g^{\mu\nu}}{l^2-m^2}\frac{v_\nu}{v\cdot l}-\delta K$$

$$G=-ig^2C_F\mu^\epsilon\int\frac{d^{4-\epsilon}l}{(2\pi)^{4-\epsilon}}\frac{\hat n_\mu}{n\cdot l}\frac{g^{\mu\nu}}{l^2}\left(\frac{\not p+\not l}{(p+l)^2}\gamma_\nu-\frac{v_\nu}{v\cdot l}\right)-\delta G$$

$$p^+\frac{d}{dp^+}\Phi(x,k_T)=2\bar\Phi(x,k_T)$$

$$\bar\Phi(x,k_T)=\bar\Phi_s(x,k_T)+\bar\Phi_h(x,k_T)$$

$$\bar{\Phi}_s = \left[-ig^2 C_F \mu^\epsilon \int \frac{d^{4-\epsilon}l}{(2\pi)^{4-\epsilon}} \frac{\hat{n}\cdot v}{n\cdot l l^2 v\cdot l} - \delta K\right]\Phi(x,k_T)$$

$$-ig^2 C_F \mu^\epsilon \int \frac{d^{4-\epsilon}l}{(2\pi)^{4-\epsilon}} \frac{\hat{n}\cdot v}{n\cdot l v\cdot l} 2\pi i\delta(l^2)\Phi(x + l^+/p^+, |\mathbf{k}_T + \mathbf{l}_T|)$$

$$\Phi(x + l^+/p^+, |\mathbf{k_T} + \mathbf{l_T}|) \approx \Phi(x, |\mathbf{k_T} + \mathbf{l_T}|)$$

$$p^+ \frac{d}{dp^+}\Phi(x,b) = 2[K(1/(b\mu), \alpha_s(\mu)) + G(xp^+\nu/\mu, \alpha_s(\mu))]\Phi(x,b)$$

$$\Phi(x,b) = \Delta_k(x,b)\Phi_i(x)$$

$$\Delta_k(x,b) = \exp\left[-2\int_{1/b}^{xp^+} \frac{dp}{p}\int_{1/b}^{p} \frac{d\mu}{\mu}\gamma_K(\alpha_s(\mu))\right]$$

$$\gamma_K = \frac{\alpha_s}{\pi}C_F + \left(\frac{\alpha_s}{\pi}\right)^2 C_F\left[C_A\left(\frac{67}{36} - \frac{\pi^2}{12}\right) - \frac{5}{18}n_f\right]$$

$$\bar{\Phi}(x,k_T) = -ig^2 N_c \int \frac{d^4 l}{(2\pi)^4} \frac{\hat{n}\cdot v}{n\cdot l v\cdot l}\left[\frac{\theta(k_T^2 - l_T^2)}{l^2}\Phi(x,k_T)\right.$$

$$\left. + 2\pi i\delta(l^2)\phi(x, |\mathbf{k}_T + \mathbf{l}_T|)\right],$$

$$\frac{d\phi(x,k_T)}{d\ln(1/x)} = \bar{\alpha}_s \int \frac{d^2 l_T}{\pi l_T^2}[\phi(x, |\mathbf{k}_T + \mathbf{l}_T|) - \theta(k_T^2 - l_T^2)\phi(x,k_T)]$$

$$\sigma \approx \frac{1}{t}\left(\frac{s}{t}\right)^{\omega_P - 1}$$

$$\Phi(x + l^+/p^+, |\mathbf{k_T} + \mathbf{l_T}|) \approx \Phi(x + l^+/p^+, k_T)$$

$$\bar{\phi}_s(N) = \int_0^1 dx x^{N-1}\bar{\phi}_s(x)$$

$$p^+ \frac{d\phi}{dp^+} = \frac{p^+}{N}\frac{d\phi}{d(p^+/N)}$$

$$\phi(N) = \Delta_t(N)\phi_i$$

$$\Delta_t(N) = \exp\left[-2\int_{p^+/N}^{p^+} \frac{dp}{p}\int_{p^+}^{p} \frac{d\mu}{\mu}\gamma_K(\alpha_s(\mu))\right]$$

$$\Delta_t(N) = \exp\left[\int_0^1 dz \frac{1 - z^{N-1}}{1 - z}\int_{(1-z)^2}^{1} \frac{d\lambda}{\lambda}\gamma_K\left(\alpha_s(\sqrt{\lambda}p^+)\right)\right]$$

$$p^+ \frac{d}{dp^+}\phi(x) = \int_x^1 \frac{d\xi}{\xi}P(x/\xi)\phi(\xi)$$

$$P(z) = \frac{\alpha_s(p^+)}{\pi}C_F \frac{2}{(1-z)_+}$$

$$\frac{\partial}{\partial \ln Q^2}\begin{pmatrix}\phi_q\\ \phi_g\end{pmatrix} = \begin{pmatrix}P_{qq} & P_{qg}\\ P_{gq} & P_{gg}\end{pmatrix} \otimes \begin{pmatrix}\phi_q\\ \phi_g\end{pmatrix}$$

$$\bar{\Phi}_s(N,b) = K(p^+/(N\mu), 1/(b\mu), \alpha_s(\mu))\Phi(N,b)$$

$$K = -ig^2 C_F \mu^\epsilon \int_0^1 dz \int \frac{d^{4-\epsilon}l}{(2\pi)^{4-\epsilon}} \frac{\hat{n}\cdot v}{n\cdot lv\cdot l}\left[\frac{\delta(1-z)}{l^2} + 2\pi i\delta(l^2)\delta\left(1-z-\frac{l^+}{p^+}\right) z^{N-1} e^{i\mathbf{l}_T\cdot\mathbf{b}}\right] - \delta K = \frac{\alpha_s(\mu)}{\pi} C_F\left[\ln\frac{1}{b\mu} - K_0\left(\frac{2\nu p^+ b}{N}\right)\right]$$

$$\Phi(N,b) = \Delta_u(N,b)\Phi_i$$

$$\Delta_u(N,b) = \exp\left[-2\int_{p^+\chi^{-1}(N,b)}^{p^+} \frac{dp}{p}\int_{p^+\chi^{-1}(1,b)}^{p} \frac{d\mu}{\mu}\gamma_K(\alpha_s(\mu))\right]$$

$$\chi(N,b) = \left(N + \frac{p^+ b}{2}\right)e^{\gamma_E}$$

$$\Phi(x + l^+/p^+, b) = \theta((1-x)p^+ - l^+)\Phi(x,b) + [\Phi(x + l^+/p^+, b) - \theta((1-x)p^+ - l^+)\Phi(x,b)]$$

$$-iN_c g^2 \int \frac{d^4 l}{(2\pi)^4}\frac{\hat{n}\cdot v}{n\cdot lv\cdot l} 2\pi i\delta(l^2) e^{i\mathbf{l}_T\cdot\mathbf{b}}[\Phi(x + l^+/p^+, b) - \theta((1-x)p^+ - l^+)\Phi(x,b)]$$

$$p^+\frac{d}{dp^+}\Phi(x,b) = -2\left[\int_{1/b}^{xp^+}\frac{d\mu}{\mu}\gamma_K(\alpha_s(\mu)) - \bar{\alpha}_s(xp^+)\ln(p^+ b)\right]\Phi(x,b)$$

$$+2\bar{\alpha}_s(xp^+)\int_x^1 dz P_{gg}(z)\Phi(x/z,b)$$

$$P_{gg} = \left[\frac{1}{(1-z)_+} + \frac{1}{z} - 2 + z(1-z)\right]$$

$$\Delta(x,b,Q_0) = \exp\left(-2\int_{xQ_0}^{xp^+}\frac{dp}{p}\left[\int_{1/b}^{p}\frac{d\mu}{\mu}\gamma_K(\alpha_s(\mu)) - \bar{\alpha}_s(p)\ln\frac{pb}{x}\right]\right)$$

$$\Phi(x,b) = \Delta(x,b,Q_0)\Phi_i + 2\int_x^1 dz\int_{Q_0}^{p^+}\frac{d\mu}{\mu}\bar{\alpha}_s(x\mu)\Delta_k(x,b)P_{gg}(z)\Phi(x/z,b)$$

$$3\ln\delta + 4\ln\delta\ln(2\epsilon) + \frac{\pi^2}{3} - \frac{5}{2}$$

$$\Delta R_{ij}^2 \equiv (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2 < R^2$$

$$d_{ij} = \min(k_{Ti}^2, k_{Tj}^2)\frac{\Delta R_{ij}^2}{R^2}, d_{iB} = k_{Ti}^2, d_{jB} = k_{Tj}^2$$

$$d_{ij} = \frac{\Delta R_{ij}^2}{R^2}, d_{iB} = 1, d_{jB} = 1$$

$$d_{ij} = \min\left(k_{Ti}^{-2}, k_{Tj}^{-2}\right)\frac{\Delta R_{ij}^2}{R^2}, d_{iB} = k_{Ti}^{-2}, d_{jB} = k_{Tj}^{-2}$$

$$W = \mathcal{P}\exp\left[-ig\int_0^\infty dzn \cdot A(zn)\right]$$

$$J_q\left(M_J^2, P_T, v^2, R, \mu^2\right) = \frac{(2\pi)^3}{2\sqrt{2}\left(P_J^0\right)^2 N_c}\sum_{N_J} \mathrm{Tr}\{\not\Leftarrow \langle 0|q(0)W^{(\bar{q})\dagger}|N_J\rangle\langle N_J|W^{(\bar{q})}\bar{q}(0)|0\rangle\}$$

$$J_g\left(M_J^2, P_T, v^2, R, \mu^2\right) = \frac{(2\pi)^3}{2\left(P_J^0\right)^3 N_c}\sum_{N_J} \langle 0|\xi_\sigma F^{\sigma v}(0)W^{(g)\dagger}|N_J\rangle\langle N_J|W^{(g)}F_v^\rho(0)\xi_\rho|0\rangle$$

$$\times\,\delta\left(M_J^2 - \hat{M}_J^2\left(N_J, R\right)\right)\delta^{(2)}\left(\hat{e} - \hat{e}\left(N_J\right)\right)\delta\left(P_J^0 - \omega\left(N_J\right)\right)$$

$$\times\,\delta\left(M_J^2 - \hat{M}_J^2\left(N_J, R\right)\right)\delta^{(2)}\left(\hat{e} - \hat{e}\left(N_J\right)\right)\delta\left(P_J^0 - \omega\left(N_J\right)\right)$$

$$v^\mu = (1, \beta, 0,0), \beta = \sqrt{1 - \left(M_J/P_J^0\right)^2}. \xi^\mu = (1, -1,0,0)$$

$$\Psi(r) = \frac{1}{N_J}\sum_J \frac{\sum_{r_i<r, i\in J} P_{Ti}}{\sum_{r_i<R, i\in J} P_{Ti}},$$

$$\begin{aligned} J_q^{E(1)}\left(M_J^2, P_T, v^2, R, r, \mu^2\right) = &\frac{(2\pi)^3}{2\sqrt{2}\left(P_J^0\right)^2 N_c}\sum_{\sigma,\lambda}\int \frac{d^3p}{(2\pi)^3 2p^0}\frac{d^3k}{(2\pi)^3 2k^0} \\ &\times\left[p^0\Theta\left(r-\theta_p\right) + k^0\Theta\left(r-\theta_k\right)\right] \\ &\times \mathrm{Tr}\left\{z\langle 0|q(0)W^{(\bar{q})\dagger}|p,\sigma;k,\lambda\rangle\langle k,\lambda;p,\sigma|W^{(\bar{q})}\bar{q}(0)|0\rangle\right\} \end{aligned}$$

$$\begin{aligned} J_g^{E(1)}\left(M_J^2, P_T, v^2, R, r, \mu^2\right) = &\frac{(2\pi)^3}{2\left(P_J^0\right)^3 N_c}\sum_{\sigma,\lambda}\int \frac{d^3p}{(2\pi)^3 2p^0}\frac{d^3k}{(2\pi)^3 2k^0} \\ &\times\left[p^0\Theta\left(r-\theta_p\right) + k^0\Theta\left(r-\theta_k\right)\right] \\ &\times\langle 0|\xi_\sigma F^{\sigma v}(0)W^{(g)\dagger}|p,\sigma;k,\lambda\rangle\langle k,\lambda;p,\sigma|W^{(g)}F_v^\rho(0)\xi_\rho|0\rangle \\ &\times\delta\left(M_J^2 - (p+k)^2\right)\delta^{(2)}\left(\hat{e} - \hat{e}_{\mathbf{p+k}}\right)\delta\left(P_J^0 - p^0 - k^0\right) \end{aligned}$$

$$-\frac{n^2}{v\cdot n}v_\alpha\frac{d}{dn_\alpha}\bar{J}_q^E(N=1, P_T, v^2, R, r) = 2(\bar{K}+G)\bar{J}_q^E(N=1, P_T, v^2, R, r),$$

$$\mathcal{H}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*\left[C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)\right]$$

$$O_1 = (\bar{d}b)_{V-A}(\bar{c}u)_{V-A}, O_2 = (\bar{c}b)_{V-A}(\bar{d}u)_{V-A}$$

$$a_1^{\text{eff}} = C_2(\mu) + C_1(\mu)\left[\frac{1}{N_c} + \chi_1(\mu)\right]$$
$$a_2^{\text{eff}} = C_1(\mu) + C_2(\mu)\left[\frac{1}{N_c} + \chi_2(\mu)\right]$$

$$\int duu(1-u)\theta(q^2u(1-u) - m_c^2)$$

$$\ln\frac{m_B}{\Lambda_h}(1+\rho_A e^{i\delta_A}), \ln\frac{m_B}{\Lambda_h}(1+\rho_H e^{i\delta_H})$$

$$A(B \to M_1M_2) = \phi_B \otimes H \otimes J \otimes S \otimes \phi_{M_1} \otimes \phi_{M_2}.$$

$$\frac{1}{xm_B^2 - k_T^2 + i\epsilon} = \frac{P}{xm_B^2 - k_T^2} - i\pi\delta(xm_B^2 - k_T^2).$$

$$B(B^0 \to \pi^{\mp}\pi^{\pm}) = (5.10 \pm 0.19) \times 10^{-6}$$
$$B(B^0 \to \pi^0\pi^0) = (1.91^{+0.22}_{-0.23}) \times 10^{-6}$$

$$A_{CP}(B^0 \to K^{\pm}\pi^{\mp}) = -0.086 \pm 0.007$$
$$A_{CP}(B^{\pm} \to K^{\pm}\pi^0) = 0.040 \pm 0.021$$

Más para el grupo invariante de gauge $SU(N_c)$, tenemos que:

$$q^i \to U^i{}_j q^j \text{ or } q \to Uq,$$

$$U^+U = 1, \det U = 1$$

$$\bar{q}_i \to U_i^j \bar{q}_j \text{ or } \bar{q} \to \bar{q}U^+, \text{ where } U_i^j = (U_j^i)^*$$

$$\bar{q}q' \to \bar{q}U^+Uq' = \bar{q}q'$$

$$\delta_j^i \to \delta_{j'}^{i'} U^i{}_{i'} U_j{}^{j'} = U^i{}_k U_j{}^k = \delta_j^i.$$

$$\varepsilon_{ijk} q_1^i q_2^j q_3^k \to \varepsilon_{ijk} U^i{}_{i'} U^j{}_{j'} U^k{}_{k'} q_1^{i'} q_2^{j'} q_3^{k'} = \det U \cdot \varepsilon_{i'j'k'} q_1^{i'} q_2^{j'} q_3^{k'} = \varepsilon_{ijk} q_1^i q_2^j q_3^k$$

$$\varepsilon_{ijk} \to \varepsilon_{i'j'k'} U_i^{i'} U_j^{j'} U_k^{k'} = \det U^+ \cdot \varepsilon_{ijk} = \varepsilon_{ijk}$$

$$\varepsilon^{ijk} \bar{q}_{1i} \bar{q}_{2j} \bar{q}_{3k} \to \varepsilon^{ijk} \bar{q}_{1i} \bar{q}_{2j} \bar{q}_{3k}$$

$$U = 1 + i\alpha^a t^a,$$

$$U^+U = 1 + i\alpha^a(t^a - (t^a)^+) = 1 \quad \Rightarrow \quad (t^a)^+ = t^a$$
$$\det U = 1 + i\alpha^a \mathrm{Tr} t^a = 1 \quad \Rightarrow \quad \mathrm{Tr} t^a = 0$$

$$\mathrm{Tr} t^a t^b = T_F \delta^{ab}$$

$$[t^a, t^b] = if^{abc} t^c$$

$$f^{abc} = \frac{1}{iT_F} \mathrm{Tr}[t^a, t^b] t^c$$

$$A^a \to \bar{q}U^+ t^a U q' = U^{ab} A^b$$

$$U^+ t^a U = U^{ab} t^b$$

$$U^{ab} = \frac{1}{T_F} \mathrm{Tr} U^+ t^a U t^b$$

$$(t^a)^i{}_j \to U^{ab} U^i_{i'} U_j{}^{j'} (t^b)^{i'}{}_{j'} = (t^a)^i{}_j,$$

$$A^a \to U^{ab} A^b = \bar{q}(1 - i\alpha^c t^c) t^a (1 + i\alpha^c t^c) q' = \bar{q}\left(t^a + i\alpha^c i f^{acb} t^b\right) q'$$

$$U^{ab} = \delta^{ab} + i\alpha^c (t^c)^{ab}$$

$$(t^c)^{ab} = i f^{acb}$$

$$(t^a)^{dc} \left(t^b\right)^{ce} - \left(t^b\right)^{dc} (t^a)^{ce} = i f^{abc} (t^c)^{de}$$

$$\left[t^a, [t^b, t^d]\right] + \left[t^b, [t^d, t^a]\right] + \left[t^d, [t^a, t^b]\right] = 0$$

$$\left(i f^{bdc} i f^{ace} + i f^{dac} i f^{bce} + i f^{abc} i f^{dce}\right) t^e = 0$$

Respecto de lo cual, la simetría lagrangiana y el módulo tensorial se calculan así:

$$L = \bar{q}\left(i\gamma^\mu \partial_\mu - m\right) q$$

$$D_\mu q = \left(\partial_\mu - i g A_\mu\right) q, A_\mu = A_\mu^a t^a.$$

$$\left(\partial_\mu - i g A'_\mu\right) U q = U\left(\partial_\mu - i g A_\mu\right) q,$$

$$A'_\mu = U A_\mu U^{-1} - \frac{i}{g}\left(\partial_\mu U\right) U^{-1}.$$

$$q(x) \to q'(x) = (1 + i\alpha^a(x) t^a) q(x)$$

$$A_\mu^a(x) \to A'^a_\mu(x) = A_\mu^a(x) + \frac{1}{g} D_\mu^{ab} \alpha^b(x)$$

$$D_\mu^{ab} = \delta^{ab} \partial_\mu - i g (t^c)^{ab} A_\mu^c.$$

$$\begin{aligned}\left[D_\mu, D_\nu\right] q = \partial_\mu \partial_\nu q - i g\left(\partial_\mu A_\nu\right) q - i g A_\nu \partial_\mu q - i g A_\mu \partial_\nu q - g^2 A_\mu A_\nu q \\ -\partial_\nu \partial_\mu q + i g\left(\partial_\nu A_\mu\right) q + i g A_\mu \partial_\nu q + i g A_\nu \partial_\mu q + g^2 A_\nu A_\mu q\end{aligned}$$

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i g\left[A_\mu, A_\nu\right] = G_{\mu\nu}^a t^a$$
$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

$$G_{\mu\nu} \to U G_{\mu\nu} U^{-1}, G_{\mu\nu}^a \to U^{ab} G_{\mu\nu}^b;$$

$$L = L_q + L_A$$

$$L_q = \sum_f \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f$$

$$L_A = -\frac{1}{4T_F} \mathrm{Tr} G_{\mu\nu} G^{\mu\nu} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$$q_f \to e^{i\alpha} q_f \approx (1 + i\alpha) q_f$$

$$q_f \to U_{ff'} q_{f'}$$

$$U = 1 + i\alpha + i\alpha^a \tau^a,$$

$$q_f = q_{Lf} + q_{Rf}, q_{L,R} = \frac{1 \pm \gamma_5}{2} q, \gamma_5 q_{L,R} = \pm q_{L,R}$$

$$L_q = \sum_f \bar{q}_{Lf} i\gamma^\mu D_\mu q_{Lf} + \sum_f \bar{q}_{Rf} i\gamma^\mu D_\mu q_{Rf}$$

$$q_L \to (1 + i\alpha_L + i\alpha_L^a \tau^a) q_L, q_R \to (1 + i\alpha_R + i\alpha_R^a \tau^a) q_L$$

$$q \to (1 + i\alpha_V + i\alpha_V^a \tau^a + i\alpha_A \gamma_5 + i\alpha_A^a \tau^a \gamma_5) q$$

$$x^\mu \to \lambda x^\mu, A_\mu \to \lambda^{-1} A_\mu, q \to \lambda^{-3/2} q$$

$$x^\mu \to \frac{x^\mu}{x^2}$$

$$x^\mu \to \frac{x^\mu + a^\mu x^2}{1 + 2a \cdot x + a^2 x^2}$$

Más, la cuantización cromodinámica, se calcula así:

$$< T\{O(x), O(y)\} >= \frac{\int \prod_{x,a,\mu} dA^a_\mu(x) e^{i\int Ld^4x} O(x) O(y)}{\int \prod_{x,a,\mu} dA^a_\mu(x) e^{i\int Ld^4x}} = \frac{1}{i^2} \frac{1}{Z[j]} \frac{\delta^2 Z[j]}{\delta j(x) \delta j(y)}\bigg|_{j=0}$$

$$Z[j] = \int \prod_{x,a,\mu} dA^a_\mu(x) e^{i\int (L + jO) d^4x}$$

$$G^a(A(x)) = \partial^\mu A^a_\mu(x)$$

$$\Delta^{-1}[A] = \int \prod_x dU(x) \prod_{x,a} \delta\big(G^a(A^U(x))\big)$$

$$\delta G(A(x)) = \hat{M} \alpha(x),$$

$$\Delta^{-1}[A] = \int \prod_x d\alpha(x) \delta(\hat{M}\alpha(x)) = 1/\det\hat{M}$$

$$\delta G^a(x) = \frac{1}{g}\partial^\mu D_\mu^{ab}\alpha^b(x) \Rightarrow \hat{M} = \frac{1}{g}\partial^\mu D_\mu^{ab}$$

$$\hat{M} = \frac{1}{g}n^\mu D_\mu^{ab} = \frac{\delta^{ab}}{g}n^\mu \partial_\mu$$

$$\Delta^{-1}[A^{U_0}] = \int \prod_x dU \prod_{x,a} \delta\big(G^a(A^{U_0 U}(x))\big)$$
$$= \int \prod_x D(U_0 U) \prod_{x,a} \delta\big(G^a(A^{U_0 U}(x))\big) = \Delta^{-1}[A]$$

$$Z[J] = \int \prod_x dA(x) e^{iS[A]} = \int \prod_x dU(x) \prod_x dA(x)\Delta[A] \prod_x \delta\big(G(A^U(x))\big)e^{iS[A]}$$

$$= \left(\prod_x \int dU\right) \times \int \prod_x dA(x)\Delta[A] \prod_x \delta(G(A(x)))e^{iS[A]}$$

$$\int dz^* dz e^{-az^* z} \sim \frac{1}{a}$$

$$\int \prod_i dz_i^* dz_i e^{-M_{ij} z_i^* z_j} \sim \frac{1}{\det M}$$

$$\int dc = 0, \int cdc = 1$$

$$e^{-ac^* c} = 1 - ac^* c$$

$$\int dc^* dc e^{-ac^* c} = a$$

$$\int \prod_i dc_i^* dc_i e^{-M_{ij} c_i^* c_j} \sim \det M$$

$$\Delta[A] = \det\hat{M} = \int \prod_{x,a} d\bar{c}^a(x) dc^a(x) e^{i\int L_c d^4x}, L_c = -\bar{c}^a M^{ab} c^b$$

$$L_c = -\bar{c}^a \partial^\mu D_\mu^{ab} c^b \Rightarrow (\partial^\mu \bar{c}^a) D_\mu^{ab} c^b$$

$$Z[J] = \int \prod_{x,a} dA^a(x) d\bar{c}^a(x) dc^a(x) \prod_{x,a} \delta\big(\partial^\mu A_\mu^a(x) - \omega^a(x)\big) e^{i\int (L_A + L_c + JO)d^4x}$$

$$Z[J] = \int \prod_{x,a} dA^a(x) d\bar{c}^a(x) dc^a(x) e^{i\int (L+JO)d^4x}$$

$$L_A = -\frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu}, L_a = -\frac{1}{2a}\left(\partial^\mu A_\mu^a\right)^2, L_c = (\partial^\mu \bar{c}^a) D_\mu^{ab} c^b$$

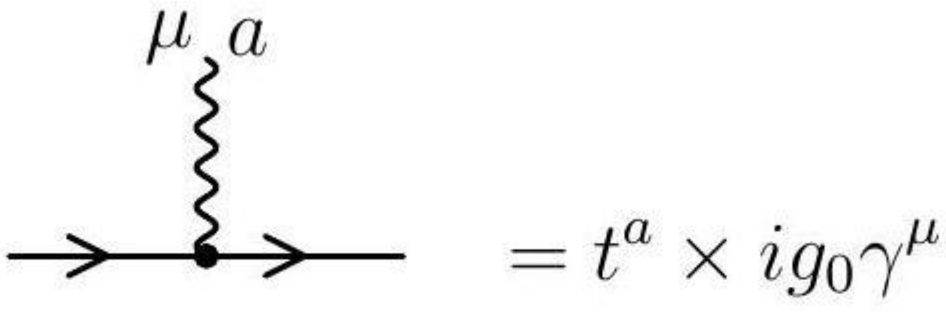
$$\delta A_\mu^a = \lambda^+ D_\mu^{ab} c^b, \delta \bar{c}^a = -\frac{1}{a}\lambda^+ \partial^\mu A_\mu^a, \delta c^a = -\frac{g}{2} f^{abc} \lambda^+ c^b c^c$$

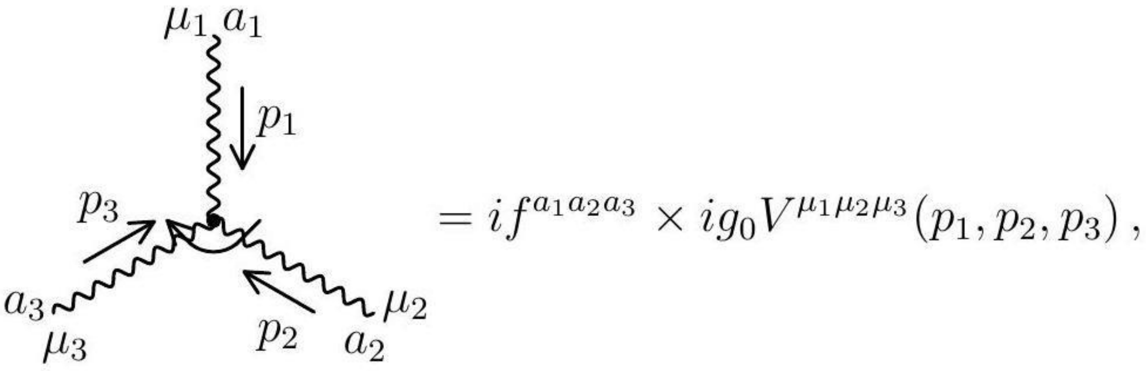
Por otro lado, los propagadores se describen así:

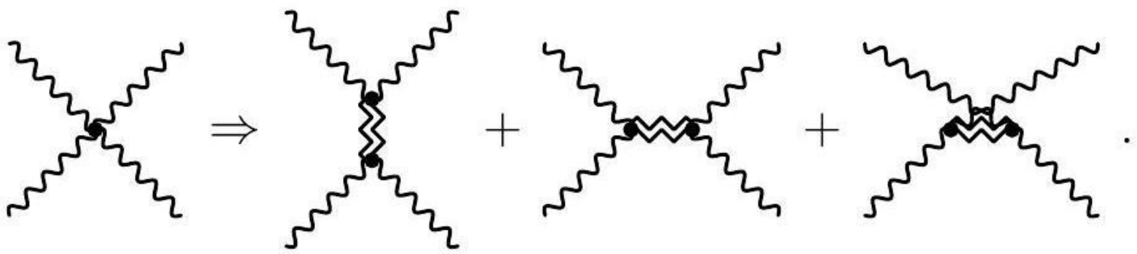
$$\longmapsto p = iS_0(p), S_0(p) = \frac{1}{\not{p} - m} = \frac{\not{p} + m}{p^2 - m^2}$$

$$a \nsim m_p {}^b_\nu = -i\delta^{ab} D^0_{\mu\nu}(p), D^0_{\mu\nu}(p) = \frac{1}{p^2}\left[g_{\mu\nu} - (1 - a_0)\frac{p_\mu p_\nu}{p^2}\right].$$

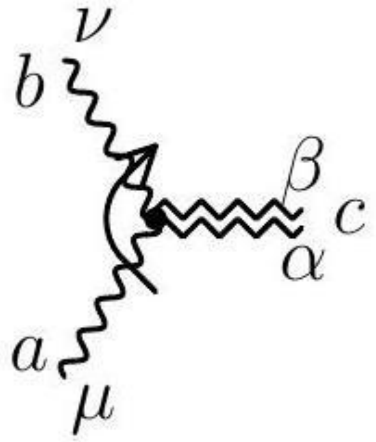
$$a \cdot -> p - -b^b = i\delta^{ab} G_0(p), G_0(p) = \frac{1}{p^2}$$







$$a\overset{\nu}{\underset{\mu}{\approx}}\overset{\beta}{\underset{\alpha}{\approx}}b = \frac{i}{2}\delta^{ab}\left(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha}\right).$$



$$= if^{abc} \times \sqrt{2} g_0 g^{\mu\alpha} g^{\nu\beta} .$$

$$c^{\mu\_a}_{--\to \frac{1}{\beta\_-}} \Big\}^{b} = if^{abc} \times ig_0 p^{\mu}$$

$$L = \sum_i \bar{q}_{0i} i\gamma^{\mu} D_{\mu} q_{0i} - \frac{1}{4} G^{a}_{0\mu\nu} G^{a\mu\nu}_{0} - \frac{1}{2a_0} \left(\partial_{\mu} A^{a\mu}_{0}\right)^2 + (\partial^{\mu} \bar{c}^{a}_{0})(D_{\mu} c^{a}_{0}),$$

$$D_{\mu} q_0 = \left(\partial_{\mu} - ig_0 A_{0\mu}\right) q_0, A_{0\mu} = A^{a}_{0\mu} t^{a}$$
$$\left[D_{\mu}, D_{\nu}\right] q_0 = -ig_0 G_{0\mu\nu} q_0, G_{0\mu\nu} = G^{a}_{0\mu\nu} t^{a}$$
$$G^{a}_{0\mu\nu} = \partial_{\mu} A^{a}_{0\nu} - \partial_{\nu} A^{a}_{0\mu} + g_0 f^{abc} A^{b}_{0\mu} A^{c}_{0\nu}$$
$$D_{\mu} c^{a}_{0} = \left(\partial_{\mu} \delta^{ab} - ig_0 A^{ab}_{0\mu}\right) c^{b}_{0}, A^{ab}_{0\mu} = A^{c}_{0\mu} (t^{c})^{ab}$$

$$q_0 = Z_q^{1/2} q, A_0 = Z_A^{1/2} A, a_0 = Z_A a, g_0 = Z_{\alpha}^{1/2} g$$

$$Z_i(\alpha_s) = 1 + \frac{z_1}{\varepsilon} \frac{\alpha_s}{4\pi} + \left(\frac{z_{22}}{\varepsilon^2} + \frac{z_{21}}{\varepsilon}\right) \left(\frac{\alpha_s}{4\pi}\right)^2 + \cdots$$

$$\frac{\alpha_s(\mu)}{4\pi} = \mu^{-2\varepsilon} \frac{g^2}{(4\pi)^{d/2}} e^{-\gamma\varepsilon}$$

$$\frac{g_0^2}{(4\pi)^{d/2}} = \mu^{2\varepsilon} \frac{\alpha_s(\mu)}{4\pi} Z_{\alpha}(\alpha_s(\mu)) e^{\gamma\varepsilon},$$

$$-iD_{\mu\nu}(p) = -iD^{0}_{\mu\nu}(p) + (-i) D^{0}_{\mu\alpha}(p) i\Pi^{\alpha\beta}(p) (-i) D^{0}_{\beta\nu}(p)$$
$$+ (-i) D^{0}_{\mu\alpha}(p) i\Pi^{\alpha\beta}(p) (-i) D^{0}_{\beta\gamma}(p) i\Pi^{\gamma\delta}(p) (-i) D^{0}_{\gamma\nu}(p) + \cdots$$

$$D_{\mu\nu}(p) = D^{0}_{\mu\nu}(p) + D^{0}_{\mu\alpha}(p) \Pi^{\alpha\beta}(p) D_{\beta\nu}(p)$$

$$A_{\mu\nu} = A_{\perp} \left[g_{\mu\nu} - \frac{p_{\mu} p_{\nu}}{p^2}\right] + A_{\parallel} \frac{p_{\mu} p_{\nu}}{p^2}$$

$$A^{-1}_{\mu\nu} = A^{-1}_{\perp} \left[g_{\mu\nu} - \frac{p_{\mu} p_{\nu}}{p^2}\right] + A^{-1}_{\parallel} \frac{p_{\mu} p_{\nu}}{p^2}$$

$$A^{-1}_{\mu\lambda} A^{\lambda\nu} = \delta^{\nu}_{\mu}$$

$$D_{\mu\nu}^{-1}(p) = (D^0)_{\mu\nu}^{-1}(p) - \Pi_{\mu\nu}(p)$$

$$\Pi_{\mu\nu}(p)p^\nu = 0$$

$$\Pi_{\mu\nu}(p) = \left(p^2 g_{\mu\nu} - p_\mu p_\nu\right)\Pi(p^2).$$

$$D_{\mu\nu}(p) = \frac{1}{p^2\left(1-\Pi(p^2)\right)}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right] + a_0\frac{p_\mu p_\nu}{(p^2)^2}$$

$$D_{\mu\nu}^r(p;\mu) = D_\perp^r(p^2;\mu)\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right] + a(\mu)\frac{p_\mu p_\nu}{(p^2)^2}$$

$$D_\perp^r(p^2;\mu) = Z_A^{-1}(\alpha(\mu))\frac{1}{p^2\left(1-\Pi(p^2)\right)}$$

$$< T\left\{\partial^\mu A_\mu^a(x), \bar{c}^b(y)\right\} >= 0$$

$$< T\left\{\partial^\mu A_\mu^a(x), \partial^\nu A_\nu^b(y)\right\} > -a < T\left\{\partial^\mu D_\mu^{ac} c^c(x), \bar{c}^b(y)\right\} >= 0$$

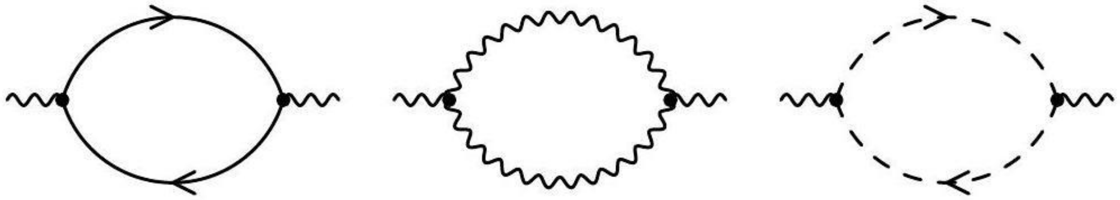
$$< T\left\{\partial^\mu A_\mu^a(x), \partial^\nu A_\nu^b(y)\right\} >= 0$$

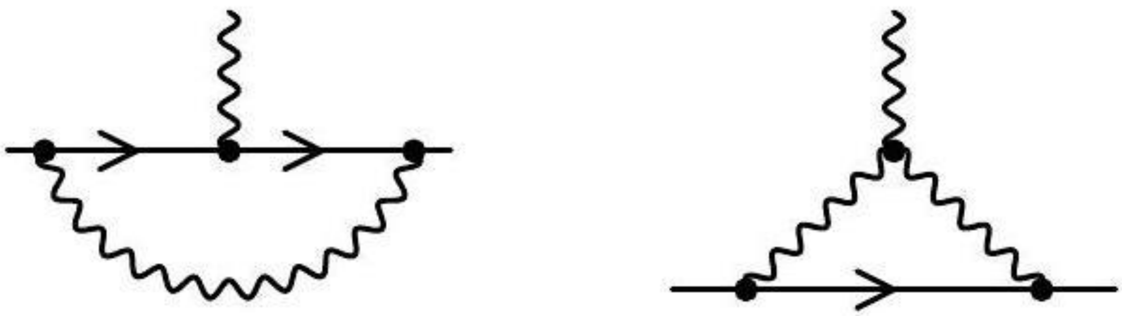
$$\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial y_\nu} < T\left\{A_\mu^a(x), A_\nu^b(y)\right\} >$$

$$p^\mu p^\nu D_{\mu\nu}(p) = p^\mu p^\nu D_{\mu\nu}^0(p)$$

$$\Pi(p^2) = \frac{g_0^2(-p^2)^{-\varepsilon}}{(4\pi)^{d/2}}\frac{G_1}{2(d-1)}\Big\{-4T_F n_f(d-2) + C_A\left[3d-2+(d-1)(2d-7)\xi - \frac{1}{4}(d-1)(d-4)\xi^2\right]\Big\}$$

$$G_1 = -\frac{2g_1}{(d-3)(d-4)}, g_1 = \frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}$$



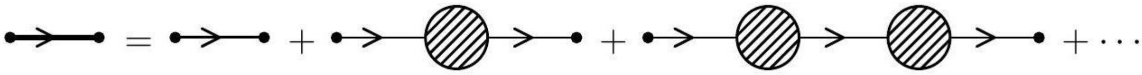


Figuras 8 y 9. Propagadores de una partícula supermasiva, deformando el espacio – tiempo cuántico.

$$p^2 D_\perp(p^2) = 1 - \frac{\alpha_s(\mu)}{4\pi\varepsilon} e^{-L\varepsilon} e^{\gamma\varepsilon} \frac{g_1}{4(1-2\varepsilon)(3-2\varepsilon)} [16(1-\varepsilon)T_F n_f - (\varepsilon(3-2\varepsilon)a^2(\mu) - 2(3-2\varepsilon)(1-3\varepsilon)a(\mu) + 26 - 37\varepsilon + 7\varepsilon^2)C_A]$$

$$p^2 D_\perp(p^2) = 1 + \frac{\alpha_s(\mu)}{4\pi\varepsilon} e^{-L\varepsilon} \left[-\frac{1}{2}\left(a - \frac{13}{3}\right) C_A - \frac{4}{3} T_F n_f + \left(\frac{9a^2 + 18a + 97}{36} C_A - \frac{20}{9} T_F n_f \right) \varepsilon \right].$$

$$Z_A(\alpha_s, a) = 1 - \frac{\alpha_s}{4\pi\varepsilon} \left[\frac{1}{2}\left(a - \frac{13}{3}\right) C_A + \frac{4}{3} T_F n_f \right]$$



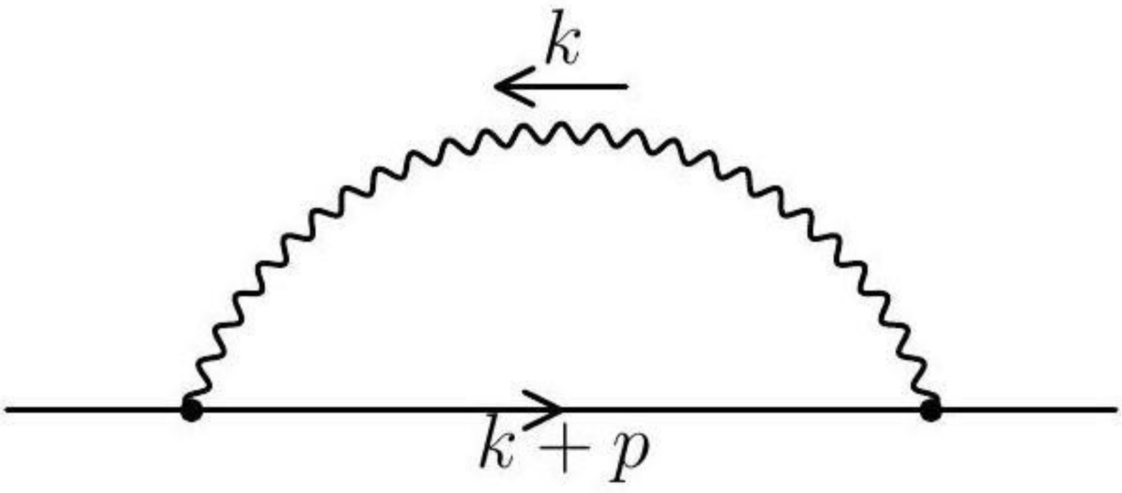
$$iS(p) = iS_0(p) + iS_0(p)(-i)\Sigma(p) iS_0(p) + iS_0(p)(-i)\Sigma(p) iS_0(p)(-i)\Sigma(p) iS_0(p) + \cdots$$

$$S(p) = S_0(p) + S_0(p)\Sigma(p)S(p);$$

$$S(p) = \frac{1}{S_0^{-1}(p) - \Sigma(p)}$$

$$S(p) = \frac{1}{1 - \Sigma_V(p^2)} \frac{1}{\not p}$$

$$\Sigma_V(p^2) = -C_F \frac{g_0^2(-p^2)^{-\varepsilon}}{(4\pi)^{d/2}} \frac{d-2}{2} a_0 G_1$$



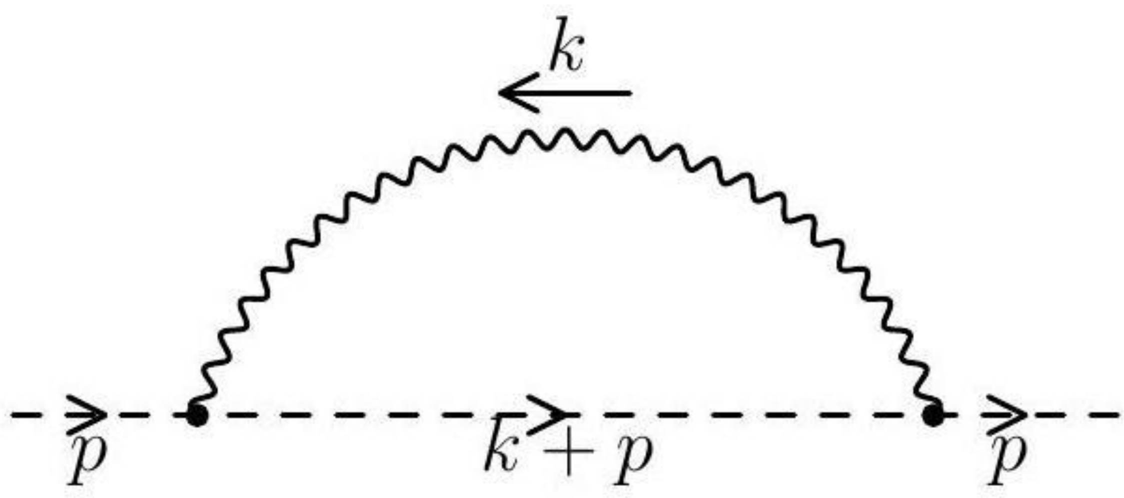


Figura 10 y 11. Propagador de una partícula repercutida por deformación del espacio – tiempo cuántico.

$$\not{p}S(p) = 1 + C_F \frac{\alpha_s(\mu)}{4\pi} e^{-L\varepsilon} e^{\gamma\varepsilon} g_1 a(\mu) \frac{d-2}{(d-3)(d-4)}$$
$$= 1 - C_F \frac{\alpha_s(\mu)}{4\pi\varepsilon} a(\mu) e^{-L\varepsilon}(1 + \varepsilon + \cdots)$$

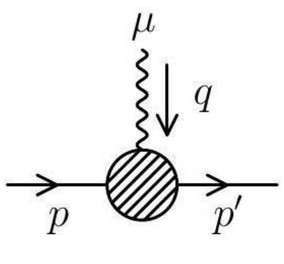
$$Z_q(\alpha, a) = 1 - C_F a \frac{\alpha_s}{4\pi\varepsilon}$$

$$G(p) = \frac{1}{p^2 - \Sigma(p^2)}$$

$$\Sigma(p^2) = -\frac{1}{4} C_A \frac{g_0^2(-p^2)^{1-\varepsilon}}{(4\pi)^{d/2}} G_1[d - 1 - (d-3)a_0]$$

$$p^2 G(p) = 1 + C_A \frac{\alpha_s(\mu)}{4\pi\varepsilon} e^{-L\varepsilon} \frac{3 - a + 4\varepsilon}{4},$$

$$Z_c(\alpha_s, a) = 1 + C_A \frac{3-a}{4} \frac{\alpha_s}{4\pi\varepsilon}$$



$$= i g_0 t^a \Gamma^\mu(p, p'), \qquad \Gamma^\mu(p, p') = \gamma^\mu + \Lambda^\mu(p, p')$$

$$Z_\alpha = \left(Z_\Gamma Z_q\right)^{-2} Z_A^{-1}$$

$$\Lambda_1^\alpha = a\left(C_F - \frac{C_A}{2}\right)\frac{\alpha_s}{4\pi\varepsilon}\gamma^\alpha, \Lambda_2^\alpha = \frac{3}{4}(1+a) C_A \frac{\alpha_s}{4\pi\varepsilon}\gamma^\alpha$$

$$Z_\Gamma = 1 + \left(C_F a + C_A \frac{a+3}{4}\right)\frac{\alpha_s}{4\pi\varepsilon}.$$

$$Z_\Gamma Z_q = 1 + C_A \frac{a+3}{4} \frac{\alpha_s}{4\pi\varepsilon}$$

$$Z_\alpha = 1 - \left(\frac{11}{3} C_A - \frac{4}{3} T_F n_f\right) \frac{\alpha_s}{4\pi\varepsilon}.$$

$$\frac{d\log\ \alpha_s(\mu)}{d\log\ \mu} = -2\varepsilon - 2\beta(\alpha_s(\mu))$$

$$\beta(\alpha_s(\mu)) = \frac{1}{2} \frac{d\log\ Z_\alpha(\alpha_s(\mu))}{d\log\ \mu}$$

$$Z_\alpha(\alpha_s) = 1 + z_1 \frac{\alpha_s}{4\pi\varepsilon} + \cdots$$

$$\beta(\alpha_s) = \beta_0 \frac{\alpha_s}{4\pi} + \cdots = -z_1 \frac{\alpha_s}{4\pi} + \cdots$$

$$Z_\alpha(\alpha_s) = 1 - \beta_0 \frac{\alpha_s}{4\pi\varepsilon} + \cdots$$

$$\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F n_f$$

$$\frac{d\log\ \alpha_s(\mu)}{d\log\ \mu} = -2\beta(\alpha_s(\mu))$$

$$\frac{d}{d\log\ \mu} \frac{\alpha_s(\mu)}{4\pi} = -2\beta_0 \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2$$

$$\frac{d}{d\log\ \mu} \frac{4\pi}{\alpha_s(\mu)} = 2\beta_0$$

$$\frac{4\pi}{\alpha_s(\mu')} - \frac{4\pi}{\alpha_s(\mu)} = 2\beta_0 \log \frac{\mu'}{\mu}$$

$$\alpha_s(\mu') = \frac{\alpha_s(\mu)}{1 + 2\beta_0 \frac{\alpha_s(\mu)}{4\pi} \log \frac{\mu'}{\mu}}$$

$$\alpha_s(\mu) = \frac{2\pi}{\beta_0 \log \frac{\mu}{\Lambda_{\mathrm{MS}}}},$$

$$m_0 = Z_m(\alpha(\mu)) m(\mu),$$

$$\Sigma(p) = p\Sigma_V(p^2) + m_0 \Sigma_S(p^2)$$

$$S(p) = \frac{1}{\not{p} - m_0 - \not{p}\Sigma_V(p^2) - m_0\Sigma_S(p^2)} = \frac{1}{1 - \Sigma_V(p^2)} \frac{1}{\not{p} - \frac{1 + \Sigma_S(p^2)}{1 - \Sigma_V(p^2)} m_0}$$

$$(1 - \Sigma_V) Z_q = \text{ finite }, \frac{1 + \Sigma_S}{1 - \Sigma_V} Z_m = \text{ finite}$$

$$(1+\Sigma_S)Z_q Z_m = \text{ finite}$$

$$\Sigma_S = C_F(3+a(\mu))\frac{\alpha_s(\mu)}{4\pi\varepsilon}.$$

$$Z_m(\alpha_s) = 1 - 3C_F\frac{\alpha}{4\pi\varepsilon} + \cdots$$

$$\frac{dm(\mu)}{d\log\,\mu} + \gamma_m(\alpha_s(\mu))m(\mu) = 0$$

$$\gamma_m(\alpha_s(\mu)) = \frac{d\log\,Z_m(\alpha_s(\mu))}{d\log\,\mu}$$

$$\gamma_m(\alpha_s) = \gamma_{m0}\frac{\alpha_s}{4\pi} + \cdots = -2z_1\frac{\alpha_s}{4\pi} + \cdots$$

$$Z_m(\alpha_s) = 1 - \frac{\gamma_{m0}}{2}\frac{\alpha_s}{4\pi\varepsilon} + \cdots$$

$$\gamma_m(\alpha_s) = 6C_F\frac{\alpha_s}{4\pi} + \cdots$$

$$\frac{d\log\,m}{d\log\,\alpha_s} = \frac{\gamma_m(\alpha_s)}{2\beta(\alpha_s)}$$

$$m(\mu') = m(\mu)\exp\int_{\alpha_s(\mu)}^{\alpha_s(\mu')}\frac{\gamma_m(\alpha_s)}{2\beta(\alpha_s)}\frac{d\alpha_s}{\alpha_s}$$

$$m(\mu') = m(\mu)\left(\frac{\alpha_s(\mu')}{\alpha_s(\mu)}\right)^{\gamma_{m0}/(2\beta_0)}$$

$$m_b(\bar{m}_b) = \bar{m}_b$$

Apéndice A.

Modelo de Cromodinámica Cuántica para una partícula supermasiva de naturaleza hadrónica, bariónica o mesónica.

1. Cálculos Preliminares.

$$\mathrm{d}n_\gamma = e_e^2\frac{\alpha}{\pi}\cdot\frac{\mathrm{d}\omega}{\omega}\cdot\frac{\mathrm{d}b_\perp^2}{b_\perp^2}.$$

$$\mathrm{d}n_\gamma = \frac{\alpha}{\pi}\cdot\frac{\mathrm{d}\omega}{\omega}\cdot\frac{\mathrm{d}k_\perp^2}{k_\perp^2}.$$

$$|e\rangle_{\text{phys}} = |e\rangle + |e\gamma\rangle + |e\gamma\gamma\rangle + \cdots,$$

$$P = p + k \;\longrightarrow 0 = 2p\cdot k + k^2.$$

$$k^2 \approx -k_\perp^2 = \vec{k}_\perp^2$$

$$\tau_\gamma \sim \frac{1}{\delta\omega} \approx \frac{2\omega}{k_\perp^2} = \frac{2xE}{k_\perp^2}$$

2. Ecuaciones de Dokshitser-Gribov-Lipatov-Altarelli-Parisi.

$$\ell(x, k_\perp^2 = 0) = \delta(1-x) \ \text{and} \ \gamma(x, k_\perp^2 = 0) = 0$$

$$\frac{\mathrm{d}\ell(x,k_\perp^2)}{\mathrm{dlog}\ k_\perp^2} = \frac{\alpha(k_\perp^2)}{2\pi}\int_x^1 \frac{\mathrm{d}\xi}{\xi}\mathcal{P}_{\ell\ell}\left(\frac{x}{\xi}, \alpha(k_\perp^2)\right)\ell(\xi, k_\perp^2)$$

$$\frac{\mathrm{d}\gamma(x,k_\perp^2)}{\mathrm{dlog}\ k_\perp^2} = \frac{\alpha(k_\perp^2)}{2\pi}\int_x^1 \frac{\mathrm{d}\xi}{\xi}\mathcal{P}_{\gamma\ell}\left(\frac{x}{\xi}, \alpha(k_\perp^2)\right)\ell(\xi, k_\perp^2)$$

$$\mathcal{P}_{\ell\ell}(z) = e_q^2\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

$$\mathcal{P}_{\gamma\ell}(z) = e_q^2\left[\frac{1+(1-z)^2}{z}\right]$$

$$\int_0^1 \mathrm{d}z[f(z)]_+ g(z) = \int_0^1 \mathrm{d}z f(z)[g(z) - g(1)]$$

3. Radiación y centro de masa.

$$\frac{\mathrm{d}^2 I}{\mathrm{d}\omega\ \mathrm{d}\Omega} = \frac{e^2}{4\pi^2}\left|\vec{\epsilon}^* \cdot \left(\frac{\vec{v}}{1 - \vec{v}\cdot\vec{n}} - \frac{\vec{v}'}{1 - \vec{v}'\cdot\vec{n}}\right)\right|^2$$

$$\mathrm{W}_{\beta',\beta} = \frac{1 - \cos\ \theta_{vv'}}{(1 - \cos\ \theta_{nv})(1 - \cos\ \theta_{nv'})}$$

$$\mathrm{d}N = \frac{\alpha}{\pi}\left|\epsilon_\mu^*\left(\frac{p^\mu}{p\cdot k} - \frac{p'^\mu}{p'\cdot k}\right)\right|^2 \frac{\mathrm{d}^3 k}{(2\pi)^3 2k_0}$$

$$\mathcal{W}(p, p'; k, \epsilon) = \epsilon_\mu^*\left(\frac{p^\mu}{p\cdot k} - \frac{p'^\mu}{p'\cdot k}\right)$$

$$\mathcal{M}_{X\to\mu^+\mu^-\gamma} = e\bar{u}_{\mu^-}(p)\left[\gamma^\mu\frac{\not{p} + \not{k}}{(p+k)^2}\Gamma - \Gamma\frac{\not{p}' - \not{k}}{(p'-k)^2}\gamma^\mu\right]u_{\mu^+}(p')\epsilon_\mu^*(k)$$

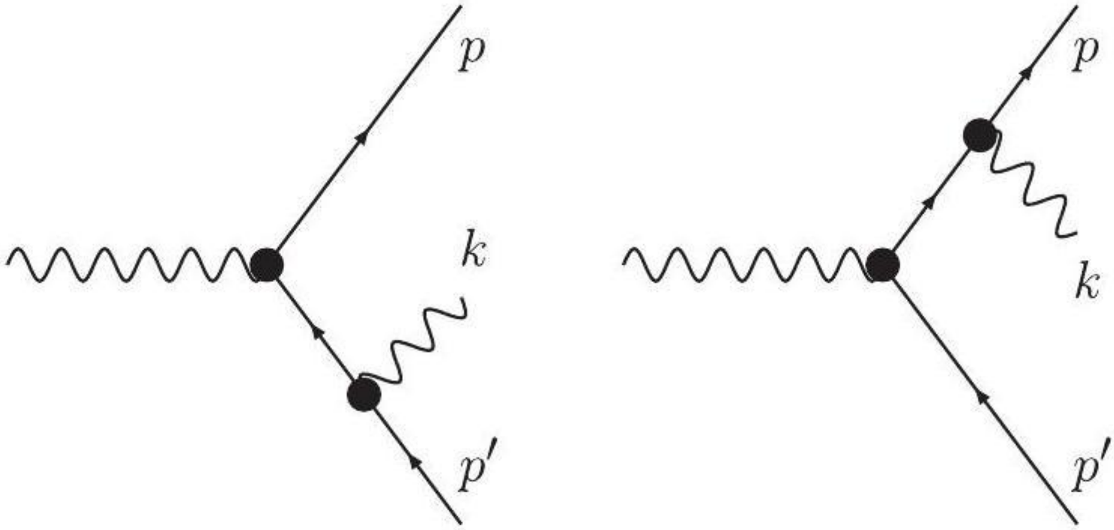


Figura 1. Comportamiento del gravitón, como partícula gluónica.

$$= e\bar{u}_{\mu^-}(p)\left[\frac{2p^\mu + k^\mu - \frac{1}{2}[\gamma^\mu, \not{k}]}{2p \cdot k}\Gamma - \Gamma\frac{2p'^\mu - k^\mu + \frac{1}{2}[\gamma^\mu, \not{k}']}{2p' \cdot k}\right]u_{\mu^+}(p')\epsilon_\mu^*(k),$$

$$\not{p}'u(p') = \bar{u}(p)\not{p} = 0 \ \text{ and } \ p^2 = p'^2 = k^2 = 0$$

$$\begin{aligned}\mathcal{M}_{X\to\mu^+\mu^-\gamma} &= e\epsilon_\mu^*(k)\left[\frac{p^\mu}{p \cdot k} - \frac{p'^\mu}{p' \cdot k}\right]\bar{u}_{\mu^-}(p')\Gamma u_{\mu^+}(p) \\ &= e\mathcal{W}(p, p'; k, \epsilon)\mathcal{M}_{X\to\mu^+\mu^-}\end{aligned}$$

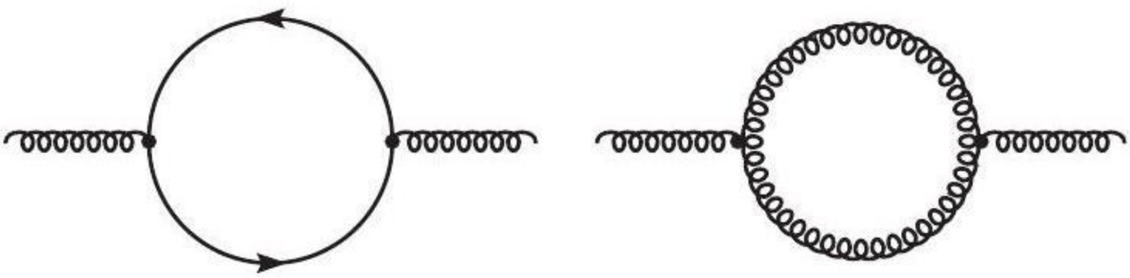


Figura 2. Propagadores de una partícula supermasiva hadronizada.

$$\mu_R^2\frac{\partial\alpha(\mu_R^2)}{\partial\mu_R^2} = \beta(\alpha),$$

$$-\beta(\alpha) = \sum_{n=0}^{\infty} b_n\alpha^{2+n} = \frac{\beta_0}{4\pi}\alpha_s^2 + \frac{\beta_1}{(4\pi)^2}\alpha_s^3 + \cdots.$$

$$\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_R n_f$$

$$\beta_1 = \frac{34}{3}C_A^2 - \frac{20}{3}C_A T_R n_f - 4C_F T_R n_f$$

$$\beta_2 = \frac{2857}{54}C_A^3 + 2C_F^2 T_R n_f - \frac{205}{9}C_F C_A T_R n_f - \frac{1415}{27}C_A^2 T_R n_f + \frac{44}{9}C_F T_R^2 n_f^2 + \frac{158}{27}C_A T_R^2 n_f^2$$

$$C_F \equiv C_q = \frac{N_c^2 - 1}{2N_c} \text{ and } C_A \equiv C_g = N_c$$

$$T_R = \frac{1}{2}$$

$$\alpha(\mu_R^2) \equiv \frac{g^2(\mu_R^2)}{4\pi} = \frac{\alpha(Q^2)}{1 + \alpha(Q^2)\frac{\beta_0}{4\pi}\log\frac{\mu_R^2}{Q^2}}$$

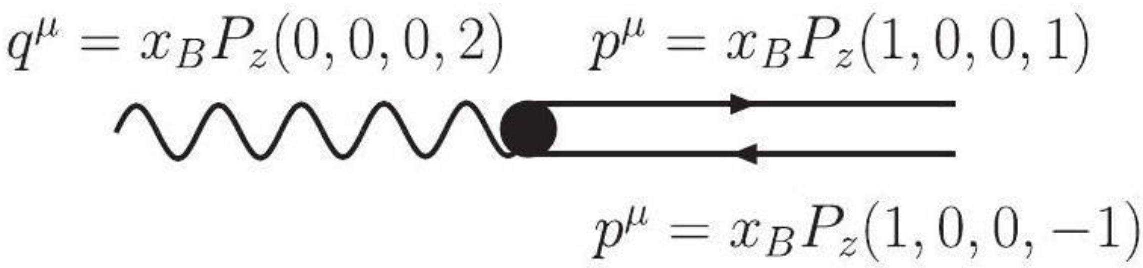
$$\alpha_s(\mu_R^2) \equiv \frac{g_s^2(\mu_R^2)}{4\pi} = \frac{1}{\frac{\beta_0}{4\pi}\log\frac{\mu_R^2}{\Lambda_{\text{QCD}}^2}}$$

$$\beta_0 = -\frac{2n_f}{3}$$

4. Cuantización equivalente de Landau.

$$\mathrm{d}n_g^{q,g} = C_{q,g} \cdot \frac{\alpha_s(k_\perp^2)}{\pi} \cdot \frac{\mathrm{d}\omega}{\omega} \cdot \frac{\mathrm{d}k_\perp^2}{k_\perp^2}$$

$$P^\mu = \left(P_0, \vec{0}, P_z\right) \text{ with } P_z = \sqrt{P_0^2 - m_P^2} \approx P_0$$
$$q^\mu = \left(0, \vec{0}, -q_z\right) \text{ with } Q^2 = -q^2 = q_z^2.$$



$$x_B = -\frac{q^2}{2P \cdot q} = \frac{q_z^2}{2P_z q_z}$$

$$\tau_{\text{int}} \sim \lambda_z \sim \frac{1}{q_z}$$

$$\sigma_{ep} \sim \sum_q e_q^2 f_{q/p}(x, Q^2)$$

$$\frac{\partial}{\partial \log Q^2}\begin{pmatrix} f_{q/h}(x,Q^2) \\ f_{g/h}(x,Q^2)\end{pmatrix}$$
$$= \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{\mathrm{d}z}{z}\begin{pmatrix} \mathcal{P}_{qq}\left(\frac{x}{z}\right) \mathcal{P}_{qg}\left(\frac{x}{z}\right) \\ \mathcal{P}_{gq}\left(\frac{x}{z}\right) \\ \mathcal{P}_{gg}\left(\frac{x}{z}\right)\end{pmatrix}\begin{pmatrix} f_{q/h}(z,Q^2) \\ f_{g/h}(z,Q^2)\end{pmatrix},$$

$$\frac{\partial}{\partial \log Q^2}\begin{pmatrix} f_{q/h}(Q^2) \\ f_{g/h}(Q^2)\end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi}\begin{pmatrix} \mathcal{P}_{qq} & \mathcal{P}_{qg} \\ \mathcal{P}_{gq} & \mathcal{P}_{gg}\end{pmatrix} \otimes \begin{pmatrix} f_{q/h}(Q^2) \\ f_{g/h}(Q^2)\end{pmatrix},$$

$$\mathcal{P}_{qq}^{(1)}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right] = \left[P_{qq}^{(1)}(x)\right]_+ + \gamma_q^{(1)}\delta(1-x)$$

$$\mathcal{P}_{qg}^{(1)}(x) = T_R[x^2+(1-x)^2] = P_{qg}^{(1)}(x)$$

$$\mathcal{P}_{gq}^{(1)}(x) = C_F\left[\frac{1+(1-x)^2}{x}\right] = P_{gq}^{(1)}(x)$$
$$\mathcal{P}_{gg}^{(1)}(x) = 2C_A\left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right]$$
$$+ \frac{11C_A - 4n_f T_R}{6}\delta(1-x) = \left[P_{gg}^{(1)}(x)\right]_+ + \gamma_g^{(1)}\delta(1-x).$$

$$\sum_i \int_0^1 \mathrm{d}z \mathcal{P}_{ij}(z) = 0$$
$$\int_0^1 \mathrm{d}z \mathcal{P}_{qq}(z) = 0$$
$$\int_0^1 \mathrm{d}z z \mathcal{P}_{gg}(z) = 0$$

5. Emisiones Cromodinámicas – Hadronización - Jets.

$$\mathrm{d}w^{q\to qg} = \frac{\alpha_S(k_\perp^2)}{2\pi} C_F \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\frac{\mathrm{d}\omega}{\omega}\left[1+\left(1-\frac{\omega}{E}\right)^2\right]$$
$$= \frac{\alpha_S(k_\perp^2)}{2\pi} C_F \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\mathrm{d}z\frac{1+z^2}{1-z} = \frac{\alpha_S(k_\perp^2)}{2\pi} C_F \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\mathrm{d}z P_{qg}^{(1)}(z).$$

$$t^{(\mathrm{had})} \approx \begin{cases} ER^2 & \text{for light quarks} \\ \dfrac{ER}{m_Q} & \text{for heavy quarks.}\end{cases}$$

$$k \approx k_\parallel \approx k_\perp \approx R^{-1}$$

$$t \approx R \approx \frac{1}{k_\perp},$$

$$t^{(\mathrm{had})} \approx \frac{k_\parallel}{k_\perp^2} \approx \frac{ER}{m},$$

$$t^{(\mathrm{form})} \approx \frac{1}{m_{qg}} \frac{E}{m_{qg}} \approx \frac{E}{kE\theta_{qg}^2} \approx \frac{k}{k_\perp^2} \approx \frac{k_\parallel}{k_\perp^2}.$$

$$\frac{k_\parallel}{k_\perp^2} \approx t^{(\mathrm{form})} \leq t^{(\mathrm{had})} \approx k_\parallel R^2 \longrightarrow k_\perp \geq \frac{1}{R} = \mathcal{O}(\text{ few } \Lambda_{\mathrm{QCD}})$$

$$\tau_Q \sim \left(\frac{m_W}{m_Q}\right)^3 \frac{E}{m_q} \ll \frac{E}{m_q} \sim t^{(\mathrm{had})}$$

$$\Delta R_{ij} = \sqrt{\Delta\eta_{ij}^2 + \Delta\phi_{ij}^2} = \sqrt{(\eta_j - \eta_i)^2 + (\phi_j - \phi_i)^2}$$

$$\sum_{i \in \text{ hadrons}} \left[E_{\perp,i}\Theta(\delta - \Delta R_{i\gamma})\right] \leq \varepsilon_\gamma E_{\perp,\gamma} \left[\frac{1 - \cos\ \delta}{1 - \cos\ \delta_0}\right]^n \ \forall\delta < \delta_0$$

$$\lim_{\delta \to 0} \left[\frac{1 - \cos\ \delta}{1 - \cos\ \delta_0}\right]^n = 0.$$

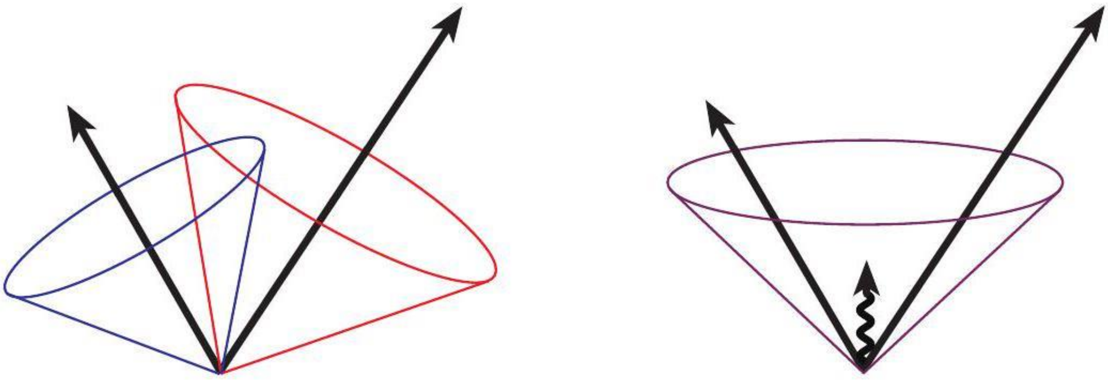


Figura 3. Curvatura del espacio – tiempo provocada por una partícula supermasiva, sea por su masa extremadamente densa o por colapso y encapsulamiento de energía.

$$\begin{aligned} d_{iB} &= (p_{\perp i})^{2p} \\ d_{ij} &= \min\left\{(p_{\perp i})^{2p}, (p_{\perp j})^{2p}\right\} \frac{R_{ij}}{R_0} \end{aligned} \quad .$$

$$p_\perp^2 = \min\{p_{\perp i}, p_{\perp j}\}^2 R_{ij}$$

$$\begin{aligned} p_i &= p_\perp^i(\cosh\ \eta, \cos\ \phi, \sin\ \phi, \sinh\ \eta) \\ p_j &= p_\perp^j(1,1,0,0) \end{aligned}$$

$$\Delta R_{ij} = \sqrt{\eta^2 + \phi^2}, \left(p_i + p_j\right)^2 = 2p_\perp^i p_\perp^j (\cosh\,\eta - \cos\,\phi).$$

$$\left(p_i + p_j\right)^2 = p_\perp^i p_\perp^j \left(\eta^2 + \phi^2 + \mathcal{O}(\eta^4, \phi^4)\right) = p_\perp^i p_\perp^j \left(\Delta R_{ij}\right)^2 + \mathcal{O}(\eta^4, \phi^4)$$

$$m_H^2 \approx z(1-z) p_\perp^2 \left(\Delta R_{ij}\right)^2$$

$$\Delta R_{ij} \approx \frac{m_H}{\sqrt{z(1-z)} p_\perp}$$

6. Teorema de Factorización Cromodinámica, simetrías, supersimetrías, antisimetrías, masa excesiva y aniquilaciones.

$$\sigma_{2\to n} = \sum_{a,b} \int_0^1 \mathrm{d}x_a\, \mathrm{d}x_b f_{a/h_1}(x_a, \mu_F) f_{b/h_2}(x_b, \mu_F) \hat{\sigma}_{ab\to n}(\mu_F, \mu_R)$$

$$= \sum_{a,b} \int_0^1 \mathrm{d}x_a\, \mathrm{d}x_b f_{a/h_1}(x_a, \mu_F) f_{b/h_2}(x_b, \mu_F) \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_n |\mathcal{M}_{ab\to n}|^2 (\Phi_n; \mu_F, \mu_R)$$

$$= \frac{1}{2s} \sum_{a,b} \int_0^1 \frac{\mathrm{d}x_a}{x_a} \frac{\mathrm{d}x_b}{x_b} f_{a/h_1}(x_a, \mu_F) f_{b/h_2}(x_b, \mu_F) \int \mathrm{d}\Phi_n |\mathcal{M}_{ab\to n}|^2 (\Phi_n; \mu_F, \mu_R)$$

$$\hat{s} = x_a x_b s$$

$$\frac{1}{4\sqrt{(p_a \cdot p_b)^2 - p_a^2 p_b^2}} \overset{m_{a,b}\to 0}{\rightarrow} \frac{1}{2\hat{s}} = \frac{1}{2x_a x_b s}$$

$$\mathrm{d}\Phi_n = \prod_{i=1}^{n} \left[\frac{\mathrm{d}p_i}{(2\pi)^4} (2\pi) \delta(p_i^2 - m_i^2) \Theta\left(p_i^{(0)}\right)\right] (2\pi)^4 \delta^4 \left(p_a + p_b - \sum_{i=1}^{n} p_i\right)$$

7. Distribuciones partónicas.

$$f_{u/p}(x, \mu^2) = 2\delta\left(x - \frac{1}{3}\right)$$
$$f_{d/p}(x, \mu^2) = \delta\left(x - \frac{1}{3}\right)$$

$$\int_0^1 \mathrm{d}x \left[f_{u/p}(x, \mu^2) - f_{\bar{u}/p}(x, \mu^2)\right] = 2$$

$$\int_0^1 \mathrm{d}x \left[f_{d/p}(x, \mu^2) - f_{\bar{d}/p}(x, \mu^2)\right] = 1$$

$$\int_0^1 \mathrm{d}x \left[f_{q/p}(x, \mu^2) - f_{\bar{q}/p}(x, \mu^2)\right] = 0 \text{ for } q \in \{s, c, b\}$$

$$\int_0^1 \mathrm{d}x x \sum_i f_{i/h}(x,\mu^2) = 1 \; \forall \mu^2$$

$$\left\langle x|_{f_{u,d/p}(x,\mu^2)}\right\rangle = \frac{1}{3}.$$

8. Efectos QCD.

$$f_{\mathrm{sea}/p}(x,\mu^2) \propto x^{-\lambda}$$

$$f_{e/e}(x,0) = \delta(1-x) \;\text{ and }\; f_{\gamma/e}(x,0) = 0$$

9. Métrica partónica de Fermi.

$$g_W = \left[4\sqrt{2}G_F m_W^2\right]^{\frac{1}{2}} = \frac{e}{\sin\,\theta_W}$$

$$\alpha(\mu) = \frac{e^2(\mu)}{4\pi} \approx \begin{cases} \dfrac{1}{137} & \text{for } \mu \to 0 \\ \dfrac{1}{128} & \text{for } \mu = m_Z \end{cases}$$

$$\sin\,\theta_W \approx 0.23$$

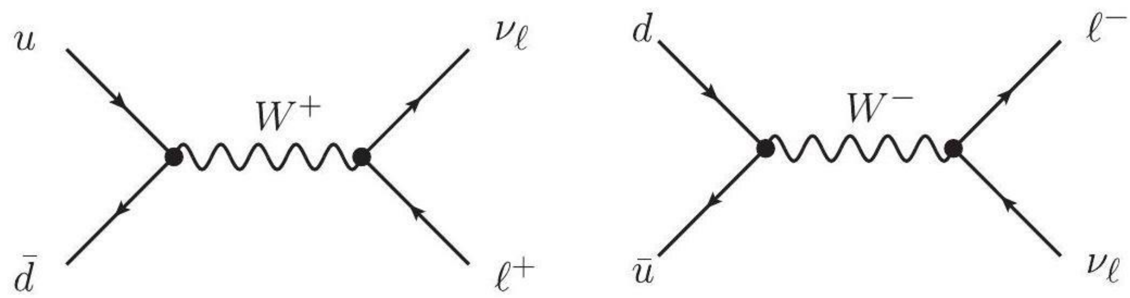
$$G_F \approx 1.166 \cdot 10^{-5}\mathrm{GeV}^{-2}$$

$$\mathcal{M}_{u\bar{d}\to W^+} = -\frac{iV_{ud}g_W\delta_{ij}}{\sqrt{2}}\bar{d}_i(p_2)\gamma^\mu \frac{1-\gamma_5}{2}u_j(p_1)\epsilon_\mu^{(W)},$$

10. Métrica de Cabibbo-Kobayashi-Maskawa.

$$\bar{\sum} |\mathcal{M}_{u\bar{d}\to W^+}|^2 = \frac{3}{9\cdot 4}\frac{|V_{ud}|^2 g_W^2}{2}\mathrm{Tr}\left[\not{p}_2\gamma^\mu \not{p}_1\gamma^\nu \frac{1-\gamma_5}{2}\right]\left[-g_{\mu\nu} + \frac{Q_\mu Q_\nu}{m_W^2}\right]$$
$$= \frac{|V_{ud}|^2 g_W^2}{12}Q^2 = \frac{|V_{ud}|^2 g_W^2}{12}m_W^2$$

$\hat{s} = (p_1+p_2)^2 = 2(p_1p_2) = Q^2 = m_W^2$



$$\mathcal{M}_{u\bar{d}\to\nu_\ell\bar{\ell}} = \left[\bar{v}_{\bar{d}}\left(\frac{-ig_W V_{ud}}{\sqrt{2}}\gamma_{\mu L}\right)u_u\right]\left[\bar{u}_\nu\left(\frac{-ig_W}{\sqrt{2}}\gamma_{\nu L}\right)v_{\bar{\ell}}\right]$$

$$\times\frac{-i}{(p_u+p_{\bar{d}})^2 - m_W^2 + im_W\Gamma_W}\left[g^{\mu\nu} - \frac{(p_u+p_{\bar{d}})^\mu(p_u+p_{\bar{d}})^\nu}{m_W^2}\right]$$

$$\gamma_{\mu L} = \gamma_\mu\frac{1-\gamma_5}{2}$$

$$\bar{\sum}\left|\mathcal{M}_{u\bar{d}\to\ell^+\nu_\ell}\right|^2$$

$$= \frac{3}{9\cdot 4}\frac{|V_{ud}|^2 g_W^4}{4}\mathrm{Tr}\left[\not{p}_{\bar{d}}\gamma^\mu\not{p}_u\gamma^\rho\frac{1-\gamma_5}{2}\right]\mathrm{Tr}\left[\not{p}_{\nu_\ell}\gamma^\nu\not{p}_{\bar{\ell}}\gamma^\sigma\frac{1-\gamma_5}{2}\right]$$

$$\times\frac{\left(g_{\mu\nu} - \frac{Q_\mu Q_\nu}{m_W^2}\right)\left(g_{\rho\sigma} - \frac{Q_\rho Q_\sigma}{m_W^2}\right)}{(Q^2-m_W^2)^2 + m_W^2\Gamma_W^2} = \frac{|V_{ud}|^2 g_W^4}{12}\frac{\hat{t}^2}{(Q^2-m_W^2)^2+m_W^2\Gamma_W^2}$$

$$\hat{s} = Q^2 = (p_u+p_{\bar{d}})^2 \text{ and } \hat{t} = (p_u - p_{\bar{\ell}})^2$$

$$\int \mathrm{d}\Phi_n = \int\frac{\mathrm{d}^4 p_\ell}{(2\pi)^4}(2\pi)\delta(p_\ell^2)\frac{\mathrm{d}^4 p_\nu}{(2\pi)^4}(2\pi)\delta(p_\nu^2)(2\pi)^4\delta^4(p_u+p_d-p_\ell-p_\nu)$$
$$= \frac{1}{32\pi^2}\int \mathrm{d}^2\Omega_\ell^*$$

$$\hat{t} = -2p_u\cdot p_\ell \overset{\text{cms}}{\to} -\frac{\hat{s}}{2}(1-\cos\,\theta^*),$$

$$\hat{\sigma}^{(LO)} = \frac{1}{2\hat{s}}\int\frac{\mathrm{d}^2\Omega_\ell^*}{32\pi^2}|\mathcal{M}|^2_{u\bar{d}\to\nu_\ell\bar{\ell}} = \frac{g_W^4|V_{ud}|^2}{12\cdot 2\hat{s}}\int_{-1}^{1}\frac{2\pi\,\mathrm{d}\cos\,\theta^*}{4\cdot 32\pi^2}\frac{\hat{s}^2(1-\cos\,\theta^*)^2}{[(\hat{s}-m_W^2)^2+m_W^2\Gamma_W^2]}$$

$$= \frac{g_W^4|V_{ud}|^2}{576\pi}\frac{\hat{s}}{(\hat{s}-m_W^2)^2+m_W^2\Gamma_W^2}$$

$$\sigma^{(\mathrm{LO})}_{h_1h_2\to\nu_\ell\bar{\ell}} = \frac{g_W^4|V_{ud}|^2}{576\pi}\int \mathrm{d}y_W \quad \mathrm{d}\hat{s}\left[\frac{1}{[\hat{s}-m_W^2]^2+m_W^2\Gamma_W^2}\right.$$
$$\left.\times\sum_{u,\bar{d}} x_u f_{u/h_1}(x_u,\mu_F)x_{\bar{d}}f_{\bar{d}/h_2}(x_{\bar{d}},\mu_F)\right]$$

$$\mathrm{d}x_u\,\mathrm{d}x_{\bar{d}} = \frac{\mathrm{d}\hat{s}}{s}\,\mathrm{d}y_W.$$

$$\hat{s} \qquad = x_u x_{\bar{d}} s$$
$$y_{\mathrm{c.m.}} = \frac{1}{2}\log\frac{x_u}{x_{\bar{d}}}.(2.77)$$

$$y_{\bar{\ell}} = \hat{y}_{\bar{\ell}} + y_W$$

$$\hat{y}_{\bar{\ell}} = \log \cot \frac{\theta^*}{2} = \frac{1}{2}\log \frac{1+\cos \theta^*}{1-\cos \theta^*}$$

$$\sin \theta^* = \frac{1}{\cosh \hat{y}_{\bar{\ell}}}$$

$$\mathrm{d}\cos \theta^* = \sin^2 \theta^* \,\mathrm{d}\hat{y}_{\bar{\ell}} = \sin^2 \theta^* \,\mathrm{d}y_{\bar{\ell}}.$$

$$\frac{\mathrm{d}\sigma_{h_1h_2\to\bar{\ell}\nu_\ell}}{\mathrm{d}y_{\bar{\ell}}} = \int \mathrm{d}x_u \,\mathrm{d}x_{\bar{d}} f_{u/h_1}(x_u,\mu_F) f_{\bar{d}/h_2}(x_{\bar{d}},\mu_F) \frac{\sin^2 \theta^* \,\mathrm{d}\hat{\sigma}_{u\bar{d}\to\bar{\ell}\nu_\ell}}{\mathrm{d}\cos \theta^*}.$$

11. Aproximación Narrow-Width.

$$\frac{\mathrm{d}\hat{s}}{(\hat{s}-M_X^2)^2 + M_X^2\Gamma_X^2} \longrightarrow \frac{\pi}{M_X\Gamma_X} \mathrm{d}\hat{s}\delta(\hat{s}-M_X^2),$$

$$\sigma^{(\mathrm{LO})}_{h_1h_2\to\nu_\ell\bar{\ell}} = \frac{g_W^4|V_{ud}|^2}{576s}\frac{m_W}{\Gamma_W}\int_{-y_{\max}}^{y_{\max}} \mathrm{d}y_W \sum_{u,\bar{d}} f_{u/h_1}\left(\frac{m_W e^{y_W}}{\sqrt{s}},\mu_F\right) f_{\bar{d}/h_2}\left(\frac{m_W e^{-y_W}}{\sqrt{s}},\mu_F\right),$$

$$|y_W| \le y_{\max} = \frac{1}{2}\log \frac{s}{m_W^2}$$

$$|\mathcal{M}|^2_{ab\to X\to cd} \propto \mathcal{BR}_{X\to ab}\mathcal{BR}_{X\to cd}$$

$$|\mathcal{M}|^2_{u\bar{d}\to W^+} = \frac{g_W^2|V_{ud}|^2 m_W^2}{12}$$

$$\hat{\sigma}^{(\mathrm{LO})}_{u\bar{d}\to W^+} = \frac{1}{2\hat{s}}\int \frac{\mathrm{d}^4p_W}{(2\pi)^4}(2\pi)^4\delta^4(p_u+p_{\bar{d}}-p_W)(2\pi)\delta(p_W^2-m_W^2)|\mathcal{M}|^2_{u\bar{d}\to W^+}$$

$$= \frac{\pi\delta(\hat{s}-m_W^2)}{\hat{s}}|\mathcal{M}|^2_{u\bar{d}\to W^+} = \frac{\pi\delta(\hat{s}-m_W^2)}{\hat{s}}\frac{g_W^2|V_{ud}|^2 m_W^2}{12} \longrightarrow \frac{4\pi^2\alpha|V_{ud}|^2}{12\sin^2 \theta_W m_W^2}$$

$$\sigma^{(\mathrm{LO})}_{h_1h_2\to W^+} = \int_0^1 \mathrm{d}x_u \,\mathrm{d}x_{\bar{d}} \sum_{u,\bar{d}} f_{u/h_1}(x_u,\mu_F) f_{\bar{d}/h_2}(x_{\bar{d}},\mu_F)\hat{\sigma}^{(\mathrm{LO})}_{u\bar{d}\to W^+} \frac{\pi g_W^2|V_{ud}|^2}{12}\int_{y_{\max}} \frac{m_W^2\,\mathrm{d}\hat{s}}{\hat{s}^2}\delta(\hat{s}$$

$$- m_W^2) \times \int_{-y_{\max}}^{y_{\max}} \mathrm{d}y_W \sum_{u,\bar{d}} x_u f_{u/h_1}(x_u,\mu_F) x_{\bar{d}} f_{\bar{d}/h_2}(x_{\bar{d}},\mu_F)\Bigg|_{x_u x_{\bar{d}} s = m_W^2}$$

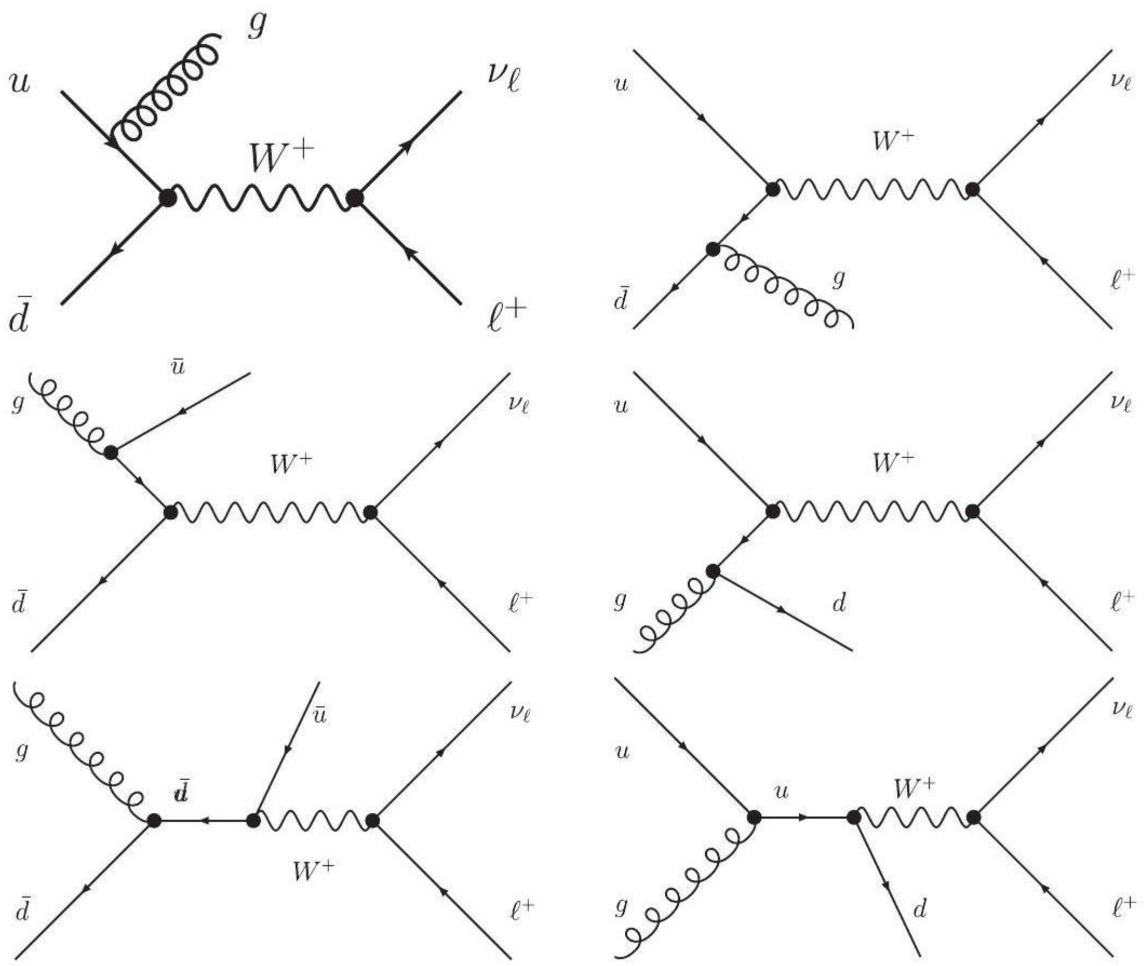
$$= \frac{\pi g_W^2|V_{ud}|^2}{12s}\int_{-y_{\max}}^{y_{\max}} \mathrm{d}y_W \sum_{u,\bar{d}} f_{u/h_1}(x_u,\mu_F) f_{\bar{d}/h_2}(x_{\bar{d}},\mu_F)$$

$$\sigma^{(LO)}_{u\bar{d}\to W^+} = \frac{\pi g_W^2|V_{ud}|^2}{12s}$$

$$\sigma^{(\mathrm{LO})}_{h_1h_2\to W^+} = \sigma^{(LO)}_{u\bar{d}\to W^+}(s)\int_{-y_{\max}}^{y_{\max}} \mathrm{d}y_W \sum_{u,\bar{d}} f_{u/h_1}(x_u,\mu_F) f_{\bar{d}/h_2}(x_{\bar{d}},\mu_F)$$

$$\mathcal{A}_\ell = \frac{\left(\frac{\mathrm{d}\sigma_{\ell^+}}{\mathrm{d}y_{\ell^+}}\right) - \left(\frac{\mathrm{d}\sigma_{\ell^-}}{\mathrm{d}y_{\ell^-}}\right)}{\left(\frac{\mathrm{d}\sigma_{\ell^+}}{\mathrm{d}y_{\ell^+}}\right) + \left(\frac{\mathrm{d}\sigma_{\ell^-}}{\mathrm{d}y_{\ell^-}}\right)},$$

$$\mathcal{M}_{u\bar{d}\to gW^+} = \frac{ig_s g_W V_{ud}}{\sqrt{2}} \bar{v}_{d,i} \left[\gamma_\nu T^a_{ij} \frac{\not{p}_{\bar{d}} - \not{p}_g}{\left(p_{\bar{d}} - p_g\right)^2} \gamma_{\mu L} \right.$$
$$\left. + \gamma_{\mu L} \frac{\not{p}_u - \not{p}_g}{\left(p_u - p_g\right)^2} \gamma_\nu T^a_{ij} \right] u_{u,j} \epsilon^\mu_W \epsilon^{\nu,a}_g$$



$$\mathcal{M}_{ug\to dW^+} = \frac{ig_s g_W V_{ud}}{\sqrt{2}} \bar{u}_{d,i} \left[\gamma_\nu T^a_{ij} \frac{\not{p}_g - \not{p}_d}{\left(p_g - p_d\right)^2} \gamma_{\mu L} \right.$$
$$\left. + \gamma_{\mu L} \frac{\not{p}_u + \not{p}_g}{\left(p_u + p_g\right)^2} \gamma_\nu T^a_{ij} \right] u_{u,j} \epsilon^\mu_W \epsilon^{*\nu,a}_g$$

$$\mathcal{M}_{\bar{d}g\to \bar{u}W^+} = \frac{ig_s g_W V_{ud}}{\sqrt{2}} \bar{v}_{d,i} \left[\gamma_\nu T^a_{ij} \frac{\not{p}_g + \not{p}_{\bar{d}}}{\left(p_g + p_{\bar{d}}\right)^2} \gamma_{\mu L} + \gamma_{\mu L} \frac{\not{p}_g - \not{p}_{\bar{u}}}{\left(p_g - p_{\bar{u}}\right)^2} \gamma_\nu T^a_{ij} \right] v_{u,j} \epsilon^\mu_W \epsilon^{*\nu,a}_g$$

$$\frac{1}{\left(p_q - p_g\right)^2} = -\frac{1}{2E_q E_g (1 - \cos\ \theta)}$$

$$|\mathcal{M}|^2_{u\bar{d}\to gW^+} = \frac{4\pi\alpha_s C_F g_W^2 |V_{ud}|^2}{12} \frac{\hat{t}^2 + \hat{u}^2 + 2m_W^2 \hat{s}}{\hat{t}\hat{u}}$$

$$|\mathcal{M}|^2_{ug\to dW^+} = |\mathcal{M}|^2_{\bar{d}g\to \bar{u}W^+} = \frac{4\pi\alpha_s T_R g_W^2 |V_{ud}|^2}{12} \frac{\hat{s}^2 + \hat{u}^2 + 2m_W^2 \hat{t}}{-\hat{s}\hat{u}}.$$

$$\hat{s} = (p_a + p_b)^2 = (p_1 + p_2)^2 ,$$
$$\hat{t} = (p_a - p_1)^2 = (p_b - p_2)^2,$$
$$\hat{u} = (p_a - p_2)^2 = (p_b - p_1)^2,$$

$$\hat{s} + \hat{t} + \hat{u} = m_a^2 + m_b^2 + m_1^2 + m_2^2,$$

$$|\mathcal{M}|^2_{u\bar{d}\to gW^+} = \frac{\left|\mathcal{M}^{(\mathrm{LO})}\right|^2_{u\bar{d}\to W^+}}{m_W^2} \cdot (4\pi\alpha_s C_F) \frac{\hat{t}^2 + \hat{u}^2 + 2m_W^2 \hat{s}}{\hat{t}\hat{u}}$$
$$|\mathcal{M}|^2_{ug\to dW^+} = |\mathcal{M}|^2_{\bar{d}g\to \bar{u}W^+} = \frac{\left|\mathcal{M}^{(\mathrm{LO})}\right|^2_{u\bar{d}\to W^+}}{m_W^2} \cdot (4\pi\alpha_s T_R) \frac{\hat{s}^2 + \hat{u}^2 + 2m_W^2 \hat{t}}{-\hat{s}\hat{u}}.$$

12. Densidad excesiva de masa.

$$p_W^\mu \ = (m_{\perp W}\cosh\ y_W, p_\perp \cos\ \phi, p_\perp \sin\ \phi, m_{\perp W}\sinh\ y_W)$$
$$p_{q,g}^\mu \ = \left(p_\perp \cosh\ y_{q,g}, -p_\perp \cos\ \phi, -p_\perp \sin\ \phi, p_\perp \sinh\ y_{q,g}\right)$$

$$m_{\perp W} = \sqrt{m_W^2 + p_{\perp W}^2} = \sqrt{m_W^2 + p_\perp^2}.$$

$$p_{1,2}^\mu = x_{1,2} \frac{\sqrt{s}}{2} (1,0,0,\pm 1)$$

$$s = \hat{x}_1 x_2 s$$
$$\hat{t} = -2p_1 p_{q,g} = -x_1 x_\perp s e^{-y_{q,g}}$$
$$\hat{u} = -2p_2 p_{q,g} = -x_2 x_\perp s e^{+y_{q,g}}$$

$$\frac{\hat{t}^2 + \hat{u}^2 + 2m_W^2 \hat{s}}{\hat{t}\hat{u}} = \frac{x_\perp^2\left(x_1^2 e^{-2y_g} + x_2^2 e^{2y_g}\right) + 2x_1 x_2 x_M^2}{x_1 x_2 x_\perp^2}$$

$$\frac{\mathrm{d}^4 p_W}{(2\pi)^4} \frac{\mathrm{d}^4 q}{(2\pi)^4} (2\pi)^4 \delta(p_u + p_{\bar{d}} - p_W - q)(2\pi)\delta(p_W^2 - m_W^2)(2\pi)\delta(q^2)$$

$$= \frac{m_{\perp W}\ \mathrm{d}m_{\perp W}\ \mathrm{d}y_W\ \mathrm{d}^2 Q_\perp}{(2\pi)^2} \delta(m_{\perp W}^2 - Q_\perp^2 - m_W^2)\delta(\hat{s} + \hat{t} + \hat{u} - m_W^2)$$

$$= \frac{\mathrm{d}y_W\ \mathrm{d}Q_\perp^2}{4\pi} \delta(\hat{s} + \hat{t} + \hat{u} - m_W^2)$$

$$\delta(q^2) = \delta((p_u + p_{\bar{d}} - p_W)^2) = \delta(\hat{s} + m_W^2 - 2(p_u + p_{\bar{d}}) \cdot p_W) = \delta(\hat{s} + \hat{t} + \hat{u} - m_W^2)$$

$$d\sigma_{AB\to Wg} = \int_0^1 dx_A\, dx_B \mathcal{L}_{u\bar{d}}(x_A, x_B, \mu_F) \int \frac{dy_W\, dQ_\perp^2}{4\pi} |\mathcal{M}|^2_{u\bar{d}\to gW^+} \delta(\hat{s} + \hat{t} + \hat{u} - m_W^2)$$

$$\mathcal{L}_{u\bar{d}}(x_A, x_B, \mu_F) = \frac{1}{2\hat{s}} [f_{u/A}(x_A, \mu_F) f_{\bar{d}/B}(x_B, \mu_F) + \{u \leftrightarrow \bar{d}\}]$$
$$= \frac{1}{2x_A x_B s} [f_{u/A}(x_A, \mu_F) f_{\bar{d}/B}(x_B, \mu_F) + \{u \leftrightarrow \bar{d}\}]$$

$$\frac{d\sigma_{AB\to Wg}}{dQ_\perp^2\, dy_W} = \frac{1}{4\pi} \int_0^1 dx_A\, dx_B \mathcal{L}_{u\bar{d}}(x_A, x_B, \mu_F) |\mathcal{M}|^2_{u\bar{d}\to gW^+} \delta(\hat{s} + \hat{t} + \hat{u} - m_W^2)$$

$$= \int_{\tilde{x}_A}^1 \frac{dx_A}{x_A} \int_{\tilde{x}_B}^1 \frac{dx_B}{x_B} \Big[f_{u/A}(x_A, \mu_F) f_{\bar{d}/B}(x_B, \mu_F) \delta(\hat{s} + \hat{t} + \hat{u}$$

$$- m_W^2) \times \sigma^{(LO)}_{u\bar{d}\to W^+}(s) \frac{1}{Q_\perp^2} \frac{\alpha_s C_F}{2\pi} \frac{\hat{t}^2 + \hat{u}^2 + 2m_W^2 \hat{s}}{\hat{s}} \Big]$$

$$Q_\perp^2 = \frac{\hat{t}\hat{u}}{\hat{s}} \text{ and } \sigma^{(LO)}_{u\bar{d}\to W^+}(s) = \frac{1}{s} \frac{\pi g_W^2 |V_{ud}|^2}{12}$$

$$|\mathcal{M}|^2_{u\bar{d}\to gW^+} = \sigma^{(LO)}_{u\bar{d}\to W^+} \frac{8\pi s}{Q_\perp^2} \frac{\alpha_s C_F}{2\pi} \frac{\hat{t}^2 + \hat{u}^2 + 2m_W^2 \hat{s}}{\hat{s}}$$

$$m_W^2 = \tilde{x}_A \tilde{x}_B s \text{ and } \tilde{x}_{A,B} = \frac{m_W}{\sqrt{s}} e^{\pm y}$$

$$\frac{d^4 p}{(2\pi)^4} (2\pi) \delta(p^2 - m^2) \Theta(E) \longrightarrow \frac{d^D p}{(2\pi)^D} (2\pi) \delta(p^2 - m^2) \Theta(E)$$

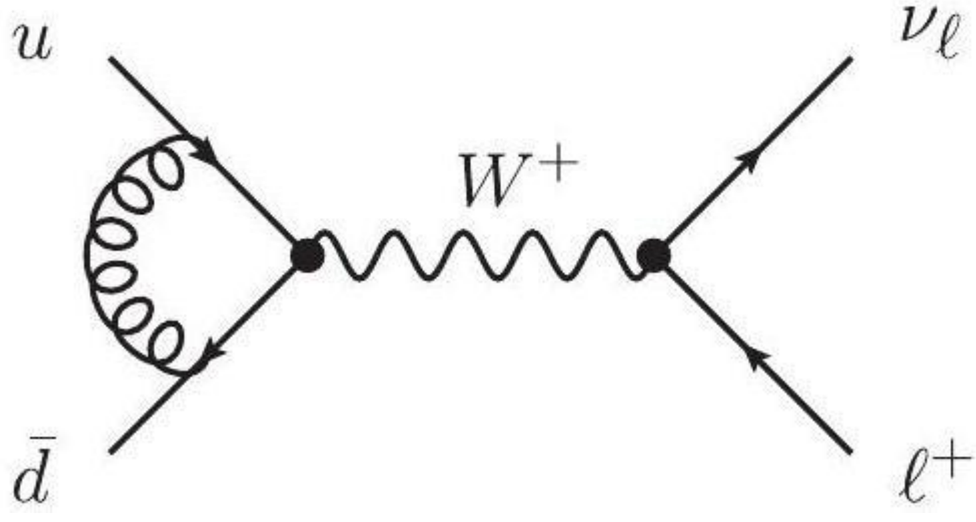


Figura 4. Vórtices de curvatura.

$$\mathcal{M}^{(1)}_{u\bar{d}\to W^+} = \frac{g_W}{\sqrt{2}} g_s^2 \mu^{4-D} \bar{v}_i(\bar{d}) \left\{ \int \frac{d^D k}{(2\pi)^D} \frac{g^{\nu\rho} \delta^{ab}}{k^2} \left[\gamma_\nu T^a_{ik} \frac{\not{p}_d + \not{k}}{(p_d + k)^2} \gamma^{\mu L} \frac{\not{p}_u - \not{k}}{(p_u - k)^2} \gamma_\rho T^b_{kj} \right] \right\} u_j(u) \epsilon_\mu(W^+)$$

$$2\left|\mathcal{M}^{(1*)}_{u\bar{d}\to W^+}\mathcal{M}^{(0)}_{u\bar{d}\to W^+}\right| = \left|\mathcal{M}^{(0)}_{u\bar{d}\to W^+}\right|^2 \frac{\alpha_s}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\varepsilon} c_\Gamma \left(-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 + \pi^2\right)$$

13. Correlaciones.

$$\mu^{4-D}\int \frac{\mathrm{d}^D k}{(2\pi)^D}\left|\mathcal{M}^{(0)}_{u\bar{d}\to W^+ g}\right|^2$$

$$= \left|\mathcal{M}^{(0)}_{u\bar{d}\to W^+}\right|^2 \frac{\alpha_s}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\varepsilon} c_\Gamma$$

$$\times\left[\left(\frac{2}{\varepsilon^2}+\frac{3}{\varepsilon}+\frac{\pi^2}{3}\right)\delta(1-z)+\left(\frac{4}{1-x}\log\frac{(1-z)^2}{z}\right)_+ - 2(1$$

$$+ z)\log\frac{(1-z)^2}{z} - \frac{2}{\varepsilon}\frac{\mathcal{P}^{(1)}_{qq}(z)}{C_F}\right]$$

$$\hat{\sigma}^{(\mathrm{NLO})}_{u\bar{d}\to W^+} = \hat{\sigma}^{(\mathrm{LO})}_{u\bar{d}\to W^+}\left\{1+\frac{\alpha_s(\mu_R)}{2\pi}C_F\left[\left(\frac{4\pi^2}{3}-8\right)\delta(1-z)\right.\right.$$

$$\left.\left.+\left(\frac{4}{1-x}\log\frac{(1-z)^2}{z}\right)_+ - 2(1+z)\log\frac{(1-z)^2}{z} - 2\frac{\mathcal{P}^{(1)}_{qq}(z)}{C_F}\log\frac{\mu_F^2}{Q^2}\right]\right\}$$

$$\hat{\sigma}^{(\mathrm{NLO})}_{ug\to dW^+} = \hat{\sigma}^{(\mathrm{LO})}_{u\bar{d}\to W^+}\cdot\frac{\alpha_S(\mu_R)}{2\pi}T_R\left[\mathcal{P}^{(1)}_{qg}(z)\left(\log\frac{(1-z)^2}{z} - \log\frac{\mu_F^2}{m_W^2}\right)+\frac{1}{2}(1-z)(1+7z)\right]$$

14. Escalares.

$$H_T = \sum_{j\in\mathrm{jets}} p_{\perp,j} + \sum_{l\in\ell} p_{\perp,l} + E_\perp$$

$$\frac{\mathrm{d}\sigma^{(\mathrm{LO})}}{\mathrm{d}p_\perp} = f_{q/p}(\mu_F) f_{\bar{q}/\bar{p}}(\mu_F)\otimes\alpha_S^2(\mu_R)\hat{\sigma}^{(0)}$$

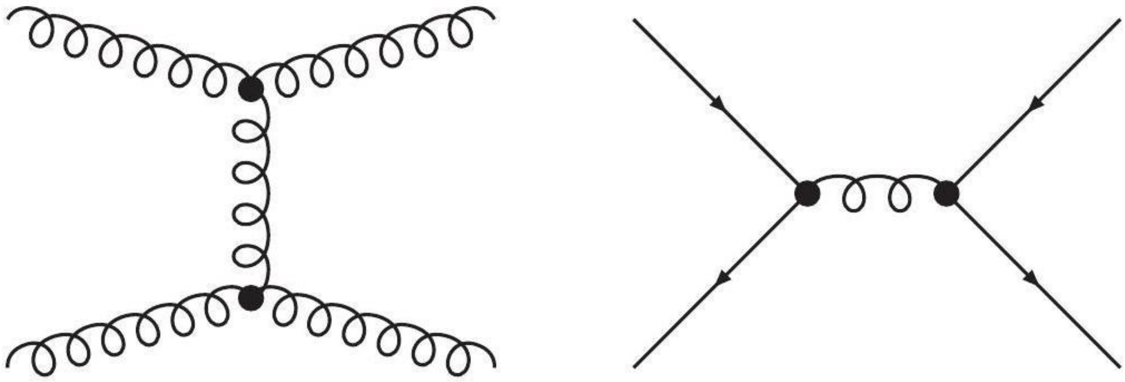


Figura 5. Interacciones de una partícula supermasiva de orden hadrónico e interacción de una partícula supermasiva no hidrónica.

$$\frac{\mathrm{d}\sigma^{(\mathrm{NLO})}}{\mathrm{d}p_{\perp}} = f_{q/p}(\mu_F) f_{\bar{p}/\bar{q}}(\mu_F) \cdot \left[\alpha_s^2(\mu_R)\hat{\sigma}^{(0)} + \alpha_s^3(\mu_R)\left(\hat{\sigma}^{(1)} + 2b_0 \log\frac{\mu_R}{p_{\perp}}\hat{\sigma}^{(0)} - 2\mathcal{P}_{qq}\log\frac{\mu_F}{p_{\perp}}\hat{\sigma}^{(0)}\right)\right]$$

$$\frac{\partial\alpha_s(\mu_R)}{\partial(\log\ \mu_R)} = -b_0\alpha_s^2(\mu_R) - b_1\alpha_s^3(\mu_R) + \mathcal{O}(\alpha_s^4)$$

$$\frac{\partial f_{j/h}(\mu_F)}{\partial(\log\ \mu_F)} = \frac{\alpha_s(\mu_F)}{2\pi}\mathcal{P}_{ji} \otimes f_{i/h}(\mu_F),$$

15. Órdenes Perturbativos.

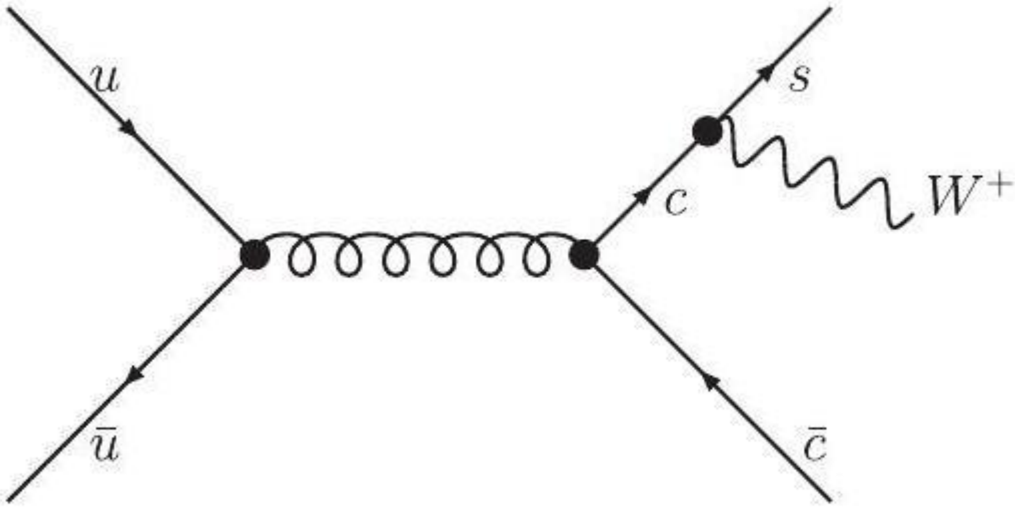
$$\mu_F = \mu_R = p_{\perp}^{\mathrm{jet}}/2$$

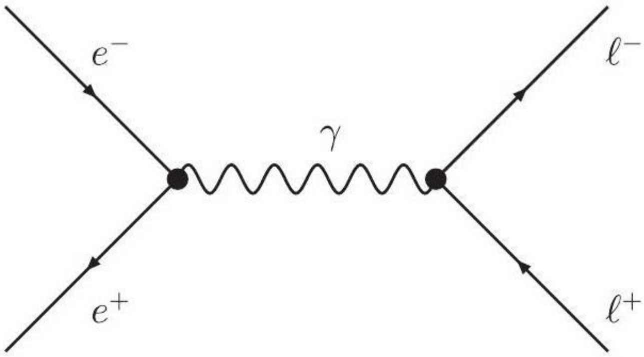
$$K_X^{(\mathrm{N})\mathrm{NLO}} = \frac{\sigma_{\mathrm{tot}}^{(\mathrm{N})\mathrm{NLO}}(X)}{\sigma_{\mathrm{tot}}^{\mathrm{LO}}(X)}$$

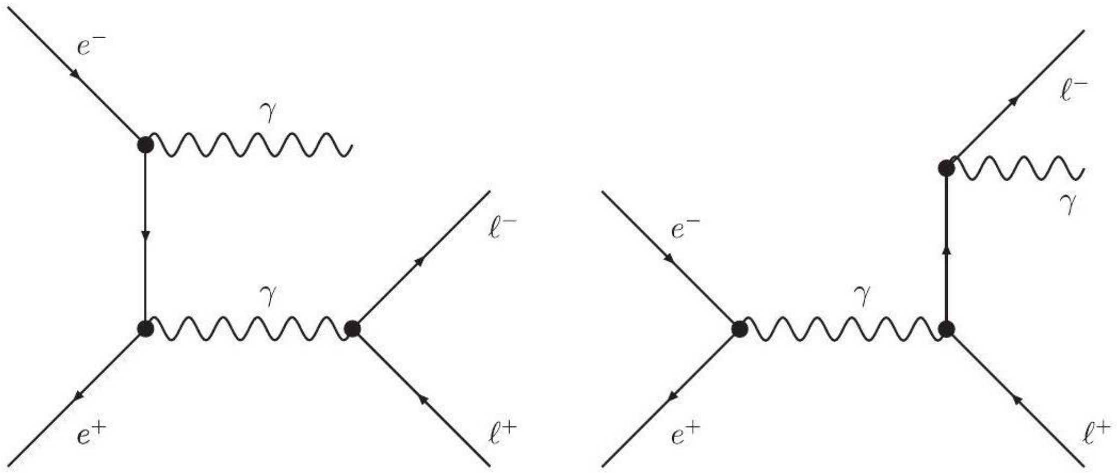
$$K_{e^+e^-\to\ \text{hadrons-leptons}}^{\mathrm{N}NLO} = \frac{\sigma_{\mathrm{tot}}^{\mathrm{NNLO}}(e^+e^- \to\ \text{hadrons-leptons})}{\sigma_{\mathrm{tot}}^{\mathrm{LO}}(e^+e^- \to\ \text{hadrons-leptons})}$$
$$= 1 + \frac{\alpha_s}{2\pi}(C_F) + \left(\frac{\alpha_s}{2\pi}\right)^2(\ldots)$$

$$\mathcal{M}_{gg\to H}^{(1)} = \mathcal{M}_{gg\to H}^{(0)} \times \frac{\alpha_s}{4\pi}C_A\left(\frac{\mu^2}{Q^2}\right)^{\varepsilon} c_{\Gamma}\left[-\frac{2}{\varepsilon^2} + \frac{11}{3} + \pi^2\right]$$

$$\mathcal{M}_{u\bar{d}\to W^+}^{(1)} = \mathcal{M}_{u\bar{d}\to W^+}^{(0)} \times \frac{\alpha_s}{4\pi}C_F\left(\frac{\mu^2}{Q^2}\right)^{\varepsilon} c_{\Gamma}\left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 7 + \pi^2\right].$$







Figuras 6 y 7. Interacción de dos partículas supermasivas.

$$C_{i_1} + C_{i_2} - C_{f,\max}$$

$$|\mathcal{M}|^2_{e^- e^+ \to \gamma\gamma^*} = 32\pi^2\alpha^2 \frac{\hat{t}^2 + \hat{u}^2 + 2Q^2\hat{s}}{\hat{t}\hat{u}}$$

$$\hat{s} + \hat{t} + \hat{u} = M^2 = Q^2$$

$$Q^2 - \hat{t} - \hat{u} = \hat{s}$$

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}} = \frac{|\mathcal{M}|^2}{16\pi\hat{s}^2}$$

$$\frac{\mathrm{d}\hat{\sigma}_{e^-e^+\to\gamma^*\gamma}}{\mathrm{d}\hat{t}} = \frac{2\pi\alpha^2}{\hat{s}^2}\frac{\hat{t}^2+\hat{u}^2+2Q^2\hat{s}}{\hat{t}\hat{u}}$$

$$\hat{\sigma}^{(\mathrm{LO})}_{e^-e^+\to\gamma^*} = \frac{4\pi^2\alpha}{Q^2} \approx \frac{4\pi^2\alpha}{\hat{s}}$$

$$\frac{\mathrm{d}\hat{\sigma}_{e^-e^+\to\gamma^*\gamma}}{\mathrm{d}\hat{t}} = \hat{\sigma}^{(\mathrm{LO})}_{e^-e^+\to\gamma^*}\cdot\frac{\alpha}{2\pi\hat{s}}\frac{\hat{t}^2+\hat{u}^2+2Q^2\hat{s}}{\hat{t}\hat{u}}$$

$$\hat{\sigma}^{(\mathrm{LO})}_{e^-e^+\to\ell^-\ell^+} = \frac{4\pi\alpha^2}{3Q^2}$$

$$\frac{\mathrm{d}\hat{\sigma}_{e^-e^+\to\ell^-\ell^+\gamma}}{\mathrm{d}\hat{t}\;\mathrm{d}Q^2} = \hat{\sigma}^{(\mathrm{LO})}_{e^-e^+\to\ell^-\ell^+}\cdot\frac{\alpha}{2\pi\hat{s}Q^2}\frac{\hat{t}^2+\hat{u}^2+2Q^2\hat{s}}{\hat{t}\hat{u}}$$

$$\hat{t}\to -Q_\perp^2 \to 0 \;\text{ and }\; \hat{u} = Q^2-\hat{s}-\hat{t}\to Q^2-\hat{s}$$

$$Q^2 \le \hat{s} - Q_\perp^2.$$

$$\frac{\mathrm{d}\hat{\sigma}_{e^-e^+\to\ell^-\ell^+\gamma}}{\mathrm{d}Q_\perp^2} = \hat{\sigma}^{(\mathrm{LO})}_{e^-e^+\to\ell^-\ell^+}\frac{\alpha}{2\pi\hat{s}}\cdot\int^{\hat{s}-Q_\perp^2}\frac{\mathrm{d}Q^2}{Q^2}\frac{\mathrm{d}Q^2}{Q^2}\frac{\hat{s}^2+Q^4}{\hat{s}-Q^2}$$
$$= \hat{\sigma}_0\frac{\alpha}{\pi}\frac{1}{Q_\perp^2}\left[\log\frac{\hat{s}}{Q_\perp^2}+\mathcal{O}(1)\right]\longrightarrow \mathrm{d}\hat{\sigma}_R \approx \hat{\sigma}_0\frac{\alpha}{\pi}\frac{\mathrm{d}Q_\perp^2}{Q_\perp^2}\log\frac{\hat{s}}{Q_\perp^2}$$

$$\frac{\alpha}{\pi}\int_0^{p_\perp^2}\frac{\mathrm{d}Q_\perp^2}{Q_\perp^2}\log\frac{\hat{s}}{Q_\perp^2} = \frac{1}{\hat{\sigma}_0}\int_0^{p_\perp^2}\mathrm{d}Q_\perp^2\frac{\mathrm{d}\hat{\sigma}_R}{\mathrm{d}Q_\perp^2},$$

$$\Sigma^{(1)}(\hat{s}) = \frac{1}{\hat{\sigma}_0}\int_0^{\hat{s}}\mathrm{d}Q_\perp^2\frac{\mathrm{d}(\hat{\sigma}_R+\hat{\sigma}_V)}{\mathrm{d}Q_\perp^2} = 1+\mathcal{O}(\alpha),$$

$$\frac{1}{\hat{\sigma}_0}\int_0^{\hat{s}}\mathrm{d}Q_\perp^2\frac{\mathrm{d}(\hat{\sigma}_R+\hat{\sigma}_V)}{\mathrm{d}Q_\perp^2} = \frac{1}{\hat{\sigma}_0}\left[\int_0^{p_\perp^2}\mathrm{d}Q_\perp^2\frac{\mathrm{d}(\hat{\sigma}_R+\hat{\sigma}_V)}{\mathrm{d}Q_\perp^2}+\int_{p_\perp^2}^{\hat{s}}\mathrm{d}Q_\perp^2\frac{\mathrm{d}\hat{\sigma}_R}{\mathrm{d}Q_\perp^2}\right]$$

$$\Sigma^{(1)}(p_\perp^2) = \frac{1}{\hat{\sigma}_0}\int_0^{p_\perp^2}\mathrm{d}Q_\perp^2\frac{\mathrm{d}(\hat{\sigma}_R+\hat{\sigma}_V)}{\mathrm{d}Q_\perp^2} \approx 1-\frac{1}{\hat{\sigma}_0}\int_{p_\perp^2}^{\hat{s}}\mathrm{d}Q_\perp^2\frac{\mathrm{d}\hat{\sigma}_R}{\mathrm{d}Q_\perp^2}$$
$$\approx 1-\frac{\alpha}{2\pi}\log^2\frac{\hat{s}}{p_\perp^2}$$

$$e\epsilon_\mu(k)\bar{u}(p)\gamma^\mu\frac{\not{p}+\not{k}}{(p+k)^2}\ldots \approx e\bar{u}(p)\frac{p\cdot\epsilon}{p\cdot k}\ldots,$$

$$\frac{e^2}{2!}\frac{p\cdot\epsilon_1}{p\cdot k_1}\frac{p\cdot\epsilon_2}{p\cdot k_2}\ldots,$$

$$\frac{e^n}{n!}\frac{p\cdot\epsilon_1}{p\cdot k_1}\frac{p\cdot\epsilon_2}{p\cdot k_2}\cdots\frac{p\cdot\epsilon_n}{p\cdot k_n}\cdots$$

$$\Sigma^{(n)}(p_\perp^2)=\frac{1}{n!}\left[\frac{\alpha}{\pi}\int_0^{p_\perp^2}\mathrm{dlog}\ Q_\perp^2\log\frac{\hat{s}}{Q_\perp^2}\right]^n=\frac{1}{n!}\left[-\frac{\alpha}{2\pi}\log^2\frac{\hat{s}}{p_\perp^2}\right]^n$$

$$\Sigma(Q_\perp^2)=\sum_{n=0}^{\infty}\frac{1}{n!}\left[-\frac{\alpha}{2\pi}\log^2\frac{\hat{s}}{Q_\perp^2}\right]^n=\exp\left[-\frac{\alpha}{2\pi}\log^2\frac{\hat{s}}{Q_\perp^2}\right]$$

$$\frac{1}{\hat{\sigma}_0}\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}Q_\perp^2}=-\frac{\mathrm{d}}{\mathrm{d}Q_\perp^2}\Sigma(Q_\perp^2)=\frac{\alpha}{\pi}\frac{1}{Q_\perp^2}\log\frac{\hat{s}}{Q_\perp^2}\exp\left[-\frac{\alpha}{2\pi}\log^2\frac{\hat{s}}{Q_\perp^2}\right].$$

$$\vec{Q}_\perp=-\sum_{i=0}^{n}\vec{k}_{\perp,i}$$

$$\delta^2\left(\vec{Q}_\perp+\sum_{i=0}^{n}\vec{k}_{\perp,i}\right)=\frac{1}{2\pi}\int\ \mathrm{d}^2b_\perp\exp\left[i\vec{b}_\perp\cdot\left(\vec{Q}_\perp+\sum_{i=0}^{n}\vec{k}_{\perp,i}\right)\right]$$

$$\nu(k_{\perp,i})=\frac{\alpha}{\pi}\frac{1}{k_{\perp,i}}\log\frac{\hat{s}}{k_{\perp,i}^2}$$

$$\nu(b_{\perp,i})=\frac{1}{2\pi}\int\ \mathrm{d}^2k_{\perp,i}\exp\left[-i\vec{b}_{\perp,i}\cdot\vec{k}_{\perp,i}\right]\nu(k_{\perp,i})$$

$$\frac{1}{\hat{\sigma}_0}\frac{\mathrm{d}\hat{\sigma}^{(n)}}{\mathrm{d}Q_\perp^2}=\frac{1}{2\pi n!}\int\ \mathrm{d}^2b_\perp\exp\left[i\vec{b}_\perp\cdot\vec{Q}_\perp\right][\nu(b_\perp)]^n$$

$$\begin{aligned}\frac{1}{\hat{\sigma}_0}\frac{\mathrm{d}\hat{\sigma}^{(\mathrm{all})}}{\mathrm{d}Q_\perp^2}&=\frac{1}{2\pi}\int\ \mathrm{d}^2b_\perp\exp\left[i\vec{b}_\perp\cdot\vec{Q}_\perp\right]\exp\left[\nu(b_\perp)\right]\\&=\frac{1}{2\pi}\int\ b_\perp\mathrm{d}b_\perp\mathrm{J}_0(b_\perp Q_\perp)\exp\left[\nu(b_\perp)\right]\end{aligned}$$

$$\nu^{(QCD)}(k_\perp)=\frac{\alpha_S(k_\perp^2)}{\pi}C_F\frac{1}{k_\perp^2}\log\frac{\hat{s}}{k_\perp^2}.$$

$$\frac{\mathrm{d}\sigma_{AB\to X}}{\mathrm{d}y\ \mathrm{d}Q_\perp^2}=\sum_{ij}\pi\hat{\sigma}_{ij\to X}^{(LO)}\left\{\int\ \frac{\mathrm{d}^2b_\perp}{(2\pi)^2}\left[\exp\left(i\vec{b}_\perp\cdot\vec{Q}_\perp\right)\tilde{W}_{ij}(b;Q,x_A,x_B)\right]\right\},$$

$$\hat{\sigma}_{ij\to X}^{(LO)}\equiv\hat{\sigma}_{ij\to W^+}^{(LO)}=\frac{4\pi^2\alpha\left|V_{ij}\right|^2}{12\sin^2\ \theta_W m_W^2}.$$

$$\tilde{W}_{ij}(b;Q,x_A,x_B)=f_{i/A}\left(x_A,\frac{1}{b_\perp}\right)f_{j/B}\left(x_B,\frac{1}{b_\perp}\right)\exp\left[-\int_{1/b_\perp^2}^{Q^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\left(A(k_\perp^2)\log\frac{Q^2}{k_\perp^2}\right)\right]$$

$$x_{A,B}=\frac{M_X}{\sqrt{s}}e^{\pm y}$$

$$A(k_\perp^2) = C_F \frac{\alpha_s(k_\perp^2)}{\pi}$$

$$\frac{d\sigma_{AB\to X}}{dy\, dQ_\perp^2} = \sum_{ij} \pi \hat{\sigma}_{ij\to X}^{(LO)} \left\{ \int \frac{d^2 b_\perp}{(2\pi)^2} \left[\exp\left(i\vec{b}_\perp \cdot \vec{Q}_\perp\right) \tilde{W}_{ij}(b; Q, x_A, x_B) \right] + Y_{ij\to X}(Q_\perp; Q, x_A, x_B) \right\}$$

$$\tilde{W}_{ij}(b; Q, x_A, x_B)$$

$$= \sum_{ab} \left\{ \left(\int_{x_A}^1 \frac{d\xi_A}{\xi_A} \int_{x_B}^1 \frac{d\xi_B}{\xi_B} \left[f_{a/A}\left(\xi_A, \frac{1}{b_\perp}\right) f_{b/B}\left(\xi_B, \frac{1}{b_\perp}\right) \right. \right. \right.$$

$$\left. \left. \times C_{ia}\left(\frac{x_A}{\xi_A}, b_\perp; \mu\right) C_{jb}\left(\frac{x_B}{\xi_B}, b_\perp; \mu\right) H_{ab}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}; \mu\right) \times \exp\left[-\int_{1/b_\perp^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \left(A(k_\perp^2) \log \frac{Q^2}{k_\perp^2} + B(k_\perp^2) \right) \right] \right] \right\}$$

$$A(\mu) = \sum_N \left(\frac{\alpha_S(\mu)}{2\pi} \right)^N A^{(N)}$$

$$C_{ia}^{(0)}(z) = \delta_{ia} \delta(1-z)$$

$$H_{ab}^{(0)}(z_A, z_B; \mu) = \delta_{ia} \delta_{jb} \delta(1-z_A) \delta(1-z_B),$$

$$Y_{ij\to X}(Q_\perp; Q, x_A, x_B) = \int_{x_A}^1 \frac{d\xi_A}{\xi_A} \int_{x_A}^1 \frac{d\xi_B}{\xi_B} \left\{ f_{i/A}(\xi_A, \mu) f_{j/B}(\xi_B, \mu) \sum_N \left[\left(\frac{\alpha_s}{2\pi} \right)^N R_{ij\to X}^{(N)}\left(Q, \frac{x_A}{\xi_A}, \frac{x_B}{\xi_B} \right) \right] \right\}$$

$$\hat{s} = \frac{1}{\xi_A \xi_B} Q^2 \text{ and } \hat{t}, \hat{u} = \left(1 - \frac{\sqrt{1 + \frac{Q_\perp^2}{Q^2}}}{\xi_{B,A}} \right) Q^2$$

$$\rho(b_\perp) = \exp\left(-\frac{b_\perp^2}{4A} \right)$$

$$A = \left\langle p_{\perp,\text{ intrinsic}}^2 \right\rangle^{-1}$$

$$\Sigma^{(QCD)}(b_\perp) \longrightarrow \Sigma^{(QCD)}(b_\perp) \rho(b_\perp)$$

$$\nu^{(\mathrm{QCD})}(k_\perp) = \frac{\alpha_s}{\pi} C_F \frac{1}{k_\perp} \log \frac{Q^2}{k_\perp^2} \longrightarrow \nu_{q\to qg}^{(\mathrm{QCD})}(k_\perp; z) = \frac{\alpha_s}{\pi} C_F \frac{1}{k_\perp} P_{qq}(z)$$

$$P_{qq}(z) = \frac{1+z^2}{1-z}$$

$$S(Q, Q_0) = \exp\left[-\int_{Q_0^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \left(\frac{\alpha_s(k_\perp^2)}{2\pi} \int_0^{1-\varepsilon} dz P_{qq}(z) \right) \right]$$

$$\int_0^{1-\varepsilon} \mathrm{d}z P_{qq}(z) = C_F \int_0^{1-\varepsilon} \mathrm{d}z \frac{1+z^2}{1-z} \approx C_F \left[\int_0^{1-\varepsilon} \mathrm{d}z \frac{2}{1-z} - \int_0^1 \mathrm{d}z(1+z)\right]$$
$$= 2C_F \left[\log \frac{Q^2}{k_\perp^2} - \frac{3}{4}\right] \equiv \Gamma_q(Q^2, k_\perp^2)$$

$$S(Q^2, Q_0^2) \equiv \Delta(Q^2, Q_0^2) = \exp\left[-\int_{Q_0^2}^{Q^2} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \frac{\alpha_s(k_\perp^2)}{\pi} \Gamma(Q^2, k_\perp^2)\right]$$

16. Métrica de Sudakov.

$$S(Q^2, Q_0^2) \in [0,1]$$

$$\Gamma_{q,g}(Q^2, q^2) = A_{q,g}^{(1)} \log \frac{Q^2}{q^2} + B_{q,g}^{(1)}$$

$$\Delta_{q,g}(Q^2, q^2) = \exp\left[-\frac{\alpha_s}{2\pi}\left(A_{q,g}^{(1)} \log^2 \frac{Q^2}{q^2} + B_{q,g}^{(1)} \log \frac{Q^2}{q^2}\right)\right]$$

$$\Delta_{q,g}(Q^2, q^2) = \exp\left[-\frac{\alpha_s}{2\pi}\left(A_{q,g}^{(1)} \log^2 \frac{\alpha_s(Q^2)}{\alpha_s(q^2)} + B_{q,g}^{(1)} \log \frac{\alpha_s(Q^2)}{\alpha_s(q^2)}\right)\right]$$

$$\Re_2(Q_{\mathrm{cut}}) = \left[\Delta_q(Q^2, Q_{\mathrm{cut}}^2)\right]^2.$$

$$\frac{\mathrm{d}\mathcal{P}_{\mathrm{rad}}(q_\perp^2)}{\mathrm{d}q_\perp^2} = -\frac{\mathrm{d}\Delta_q(Q^2, Q_{\mathrm{cut}}^2)}{\mathrm{d}q_\perp^2} = \frac{\alpha_s(q_\perp^2)}{\pi} C_F \frac{1}{q_\perp^2} \Gamma_q(Q^2, q_\perp^2) \Delta_q(Q^2, Q_{\mathrm{cut}}^2) \Theta(Q^2 - q_\perp^2) \Theta(q_\perp^2 - Q_{\mathrm{cut}}^2)$$

$$\Re_3(Q_{\mathrm{cut}}) = 2\Delta_q(Q^2, Q_{\mathrm{cut}}^2) \int_{Q_{\mathrm{cut}}^2}^{Q^2} \frac{\mathrm{d}q_\perp^2}{q_\perp^2} \left[\left(\frac{\alpha_s(q_\perp^2)}{\pi} C_F \Gamma_q(Q^2, q_\perp^2) \frac{\Delta_q(Q^2, Q_{\mathrm{cut}}^2)}{\Delta_q(Q^2, q_\perp^2)}\right) \times \Delta_q(q_\perp^2, Q_{\mathrm{cut}}^2) \Delta_g(q_\perp^2, Q_{\mathrm{cut}}^2)\right]$$

17. Patrones escalares y producción de jets.

$$R_{(n+1)/n} = \frac{\sigma_{n+1}}{\sigma_n} \equiv R \text{ or } R_{(n+1)/n} = \frac{\Re_{n+1}}{\Re_n} = R \text{ with } \Re_n = \frac{\sigma_n}{\sigma_{\mathrm{tot}}}$$

$$\Re_n = \frac{\bar{n}^n e^{-\bar{n}}}{n!} \text{ or } R_{(n+1)/n} = \frac{\bar{n}}{n+1}$$

$$\sigma_{q\to qgg}^{(1)} \propto \frac{1}{2}\left[\int_{Q_0^2}^{Q^2} \mathrm{d}t \Gamma_{q\to qg}(Q^2, t) \Delta_g(Q^2, t)\right]\left[\int_{Q_0^2}^{Q^2} \mathrm{d}t' \Gamma_{q\to qg}(Q^2, t') \Delta_g(Q^2, t')\right]$$

$$\sigma_{q\to qgg}^{(2)} \propto \left[\int_{Q_0^2}^{Q^2} \mathrm{d}t \Gamma_{q\to qg}(Q^2, t) \Delta_g(Q^2, t)\right]\left[\int_{Q_0^2}^{t} \mathrm{d}t' \Gamma_{g\to gg}(t, t') \Delta_g(t, t')\right]$$

$$\sigma^{(2)}_{q\to qgg} \propto \frac{1}{4}\left[\frac{\alpha_s}{C_A}\log^2\frac{Q}{Q_0} - \sqrt{4\alpha_s}C_A^3\log\frac{Q}{Q_0} + \mathcal{O}\left(\frac{Q_0^2}{Q^2}\right)\right]$$

$$\sigma^{(2)}_{q\to qgg} \propto \frac{1}{4}\left[(\sqrt{2}-1)\sqrt{\alpha_s}C_A^3\log\frac{Q}{Q_0} + \mathcal{O}\left(\frac{Q_0^2}{Q^2}\right)\right]$$

$$\sigma^{(1)}_{q\to qgg} \propto \frac{\alpha_s^2}{4(2\pi)^2}\log^4\frac{Q}{Q_0} + \mathcal{O}\left(\alpha_s^3\log^6\frac{Q}{Q_0}\right) \propto \sigma^{(2)}_{q\to qgg}$$

18. Teoría perturbativa - isotrópica de campo cuántico curvo o relativista.

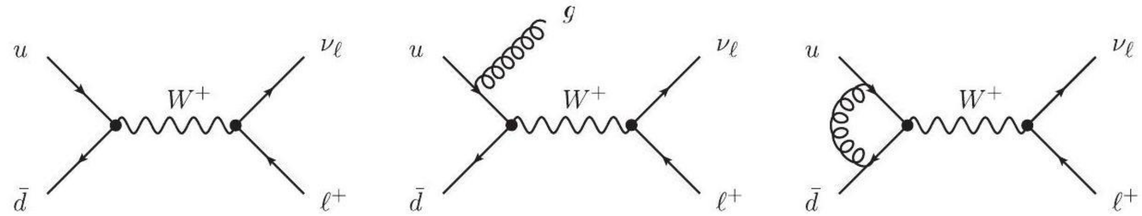


Figura 8. Configuraciones de Born de una partícula supermasiva.

$$\sigma_n^{(\mathrm{LO})} \equiv \sigma_n^{(\mathrm{Born})} = \int \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}(\Phi_{\mathcal{B}})$$

$$= \sum_{a,b}\int_0^1 \mathrm{d}x_a\,\mathrm{d}x_b f_{a/h_1}(x_a,\mu_F)f_{b/h_2}(x_b,\mu_F)\int \mathrm{d}\hat{\sigma}^{(\mathrm{LO})}_{ab\to n}(\mu_F,\mu_R)$$

$$= \sum_{a,b}\int_0^1 \mathrm{d}x_a\,\mathrm{d}x_b \int \mathrm{d}\Phi_n f_{a/h_1}(x_a,\mu_F)f_{b/h_2}(x_b,\mu_F)\frac{1}{2\hat{s}}|\mathcal{M}_{ab\to n}|^2(\Phi_n;\mu_F,\mu_R)$$

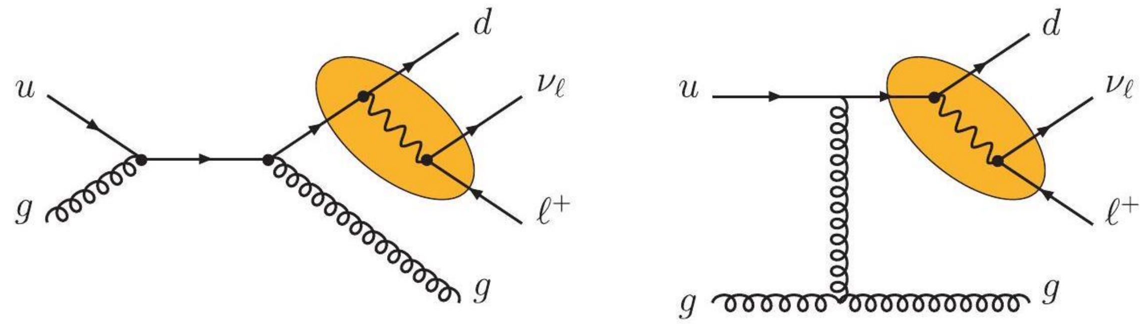


Figura 9. Interacciones de partículas supermasivas en entornos yuxtapuestos o supradimensionales.

$$\not{p} + m = \frac{1}{2}\sum_{\lambda}\left[\left(1+\frac{m}{\sqrt{p^2}}\right)u(p,\lambda)\bar{u}(p,\lambda) + \left(1-\frac{m}{\sqrt{p^2}}\right)v(p,\lambda)\bar{v}(p,\lambda)\right].$$

$$-g_{\mu\nu} + \frac{q_\mu p_\nu + q_\nu p_\mu}{pq} = \sum_{\lambda=\pm}\epsilon_\mu(p,\lambda)\epsilon^*_\nu(p,\lambda).$$

$$\zeta_a(k) = \begin{pmatrix}\sqrt{k^+}\\ \sqrt{k^-}e^{i\phi_k}\end{pmatrix}$$

$$\eta_a(k)\zeta^a(q) \ = \eta_a(k)\epsilon^{ab}\zeta_b(q) = \langle kq\rangle$$
$$\eta_{\dot{a}}(k)\zeta^{\dot{a}}(q) \ = [kq] = \langle kq\rangle^*$$

$$\epsilon_{ab} = \epsilon^{ab} = \epsilon_{\dot{a}\dot{b}} = \epsilon^{\dot{a}\dot{b}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$k^\mu = \sigma^\mu_{\dot{a}b}\zeta^{\dot{a}}(k)\zeta^b(k),$$

$$\epsilon^\mu_\pm(p,q) = \pm\frac{1}{\sqrt{2}}\frac{\langle q^\mp|\sigma^\mu|p^\mp\rangle}{\langle q^\mp p^\pm\rangle}.$$

$$2k\cdot q = \langle kq\rangle[kq].$$

$$T^a_{i\bar{j}}T^b_{j\bar{i}} = \delta^{ab}$$

$$if^{abc}T^c_{i\bar{j}} = [T^a, T^b]_{i\bar{j}}.$$

$$\mathcal{A}(1,2,\dots,n) = \sum_{\sigma\in S_{n-1}} \mathrm{Tr}[T^{a_1}T^{a_2}\dots T^{a_n}]\mathrm{A}(1,\sigma_2,\dots,\sigma_n),$$

$$T^a_{i\bar{j}}T^a_{k\bar{l}} = \delta_{i\bar{l}}\delta_{k\bar{j}} - \frac{1}{N_c}\delta_{i\bar{j}}\delta_{k\bar{l}} \leftrightarrow \bar{j} \longrightarrow \bar{l}l - \frac{1}{N_c}{}^{i}_{\bar{j}}\Big)^{i} \dots\dots\dots (.^{\bar{l}}\ {}^{\bar{l}}\ {}_{k})$$

$$+\sum_{\mathcal{P}_3(\pi)}\sum_{\mathcal{V}_\alpha^{\alpha_1\alpha_2\alpha_3}}\left[S(\pi_1,\pi_2,\pi_3)\mathcal{V}_\alpha^{\alpha_1\alpha_2\alpha_3}\mathcal{J}_{\alpha_1}(\pi_1)\mathcal{J}_{\alpha_2}(\pi_2)\mathcal{J}_{\alpha_3}(\pi_3)\right]\Bigg\}$$

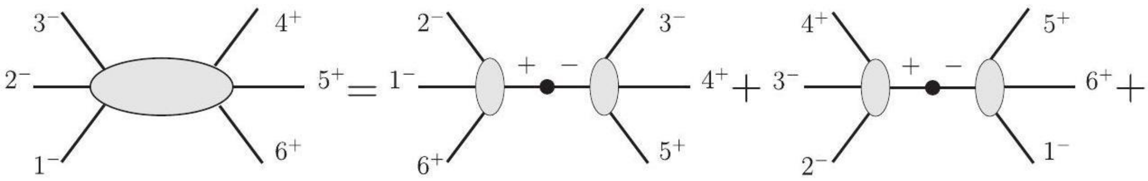
$$\mathcal{A}(\pi) = \mathcal{J}_{\alpha_\rho}(\rho)\frac{1}{P_{\bar{\alpha}_\rho}(\pi\mid\rho)}\bar{\alpha}_\backslash(\pi\mid\rho),$$

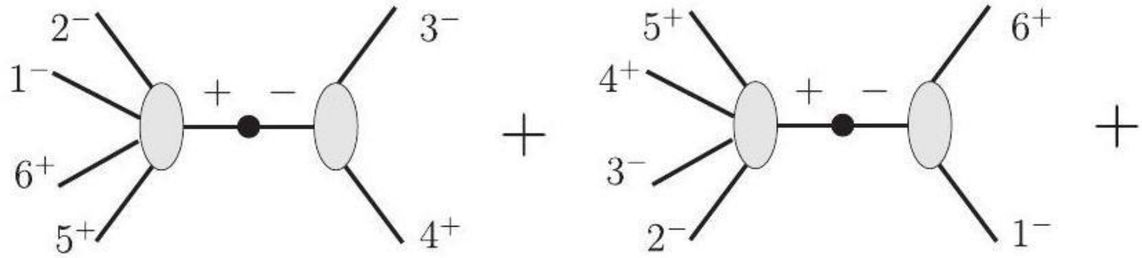
$$J^\mu(1^+,2^+,\dots,n^+) = g_s^{n-2}\frac{\langle q^-|\gamma^\mu P_{1,n}|q^+\rangle}{\sqrt{2}\langle q1\rangle\langle 12\rangle\langle 23\rangle\dots\langle(n-1)n\rangle\langle nq\rangle},$$

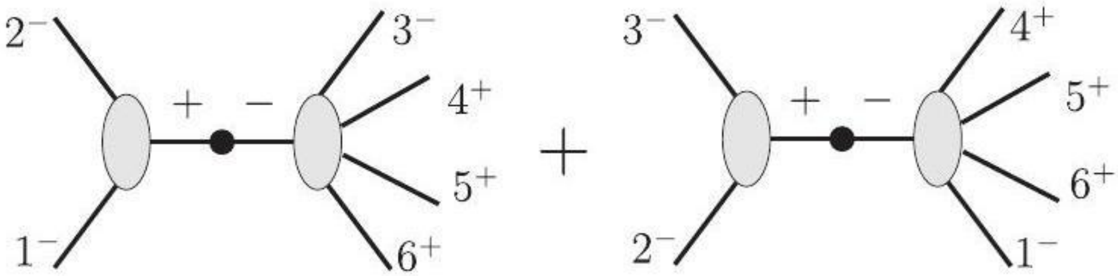
$$P^\mu_{i,j} = \sum_{l=1}^{j-1} p^\mu_l$$

$$\mathcal{A}(1^+,2^+,\dots,i^-,\dots,j^-,\dots\dots,n^+) = ig_s^{n-2}\frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\dots\langle(n-1)n\rangle\langle n1\rangle}$$
$$\mathcal{A}(1^-,2^-,\dots,i^+,\dots,j^+,\dots\dots,n^-) = ig_s^{n-2}\frac{[ij]^4}{[12][23]\dots[(n-1)n][n1]}$$

$$\mathcal{A}_n(1,2,\dots,n) = \sum_{k=2}^{n-2}\mathcal{A}_{k+1}\left(\hat{1},2,\dots,k,-\hat{p}^{-h}_{1,k}\right)\frac{1}{p^2_{1,k}}\mathcal{A}_{n-k+1}\left(\hat{p}^h_{1,k},k+1,\dots,\hat{n}\right)$$







$$p_{1,k}^{\mu} = \sum_{i=1}^{k} p_i^{\mu},$$

$$\hat{p}_{1,k} = p_{1,k} + z\lambda_n\tilde{\lambda}_1$$
$$\hat{p}_1 = p_1 + z\lambda_n\tilde{\lambda}_1$$
$$\hat{p}_n = p_n + z\lambda_n\tilde{\lambda}_1.$$

$$p_i^{a\dot{b}} = \lambda_i^a\tilde{\lambda}_i^{\dot{b}},$$

$$z = \frac{p_{1,k}^2}{\langle n|p_{i,k} \mid 1]}$$

$$I = \int_V \mathrm{d}^D x f(\vec{x}) \Longrightarrow \langle I(f)\rangle_x = \frac{V}{N}\sum_{i=1}^{N} f(\vec{x}_i) = \langle f\rangle_x$$

$$\langle E(f)\rangle_x = \sqrt{\frac{1}{N}\sum_{i=1}^{N} \left(f^2(\vec{x}_i)\right) - \left(\frac{1}{N}\sum_{i=1}^{N} f(\vec{x}_i)\right)^2} = \sqrt{\langle f^2\rangle_x - \langle f\rangle_x^2}$$

$$\langle I(f)\rangle_x = \int \mathrm{d}^D x f(\vec{x}) = \int \mathrm{d}^D x g(\vec{x}) \frac{f(\vec{x})}{g(\vec{x})} = \int \mathrm{d}^D \rho \frac{f(\vec{\rho})}{g(\vec{\rho})} = \langle I(f/g)\rangle_\rho = \left\langle \frac{f}{g} \right\rangle_g$$

$$\int_V \mathrm{d}^D x g(\vec{x}) = 1,$$

$$\langle I(f)\rangle_x = \sum_b \langle I(f)\rangle_{x\in b}$$

$$\langle E(f)\rangle_x^2 = \sum_b \langle E(f)\rangle_{x\in b}^2$$

$$0 = P^\mu - \sum_{i=1}^{n} p_i^\mu = (E, \vec{0}) - \sum_{i=1}^{n} p_i^\mu$$

$$0 = p_i^2 - m_i^2 = s_i - m_i^2 \ \forall i \in \{1, \dots, N\}.$$

$$\mathrm{d}\Phi_N = \prod_{i=1}^{N} \left[\frac{\mathrm{d}^4 p_i}{(2\pi)^4} (2\pi)\delta\left(p_i^2 - m_i^2\right)\Theta(E_i) \right] (2\pi)^4 \delta^4 \left(P^\mu - \sum_{i=1}^{N} p_i^\mu \right)$$

$$\int \mathrm{d}\tilde{\Phi}_N = \int \prod_{i=1}^{N} \left[\frac{\mathrm{d}^4 q_i}{(2\pi)^4} (2\pi)\delta\left(q_i^2\right)\Theta\left(q_i^0\right) f\left(q_i^0\right) \right] = \left[\frac{1}{(2\pi)^2} \int_0^\infty \mathrm{d}x x f(x) \right]^N$$

$$\overset{f(x)\to e^{-x}}{\to} \frac{1}{(2\pi)^{2N}}$$

$$p_i^\mu = x H_{\vec{b}}^\mu(q_i) = x \begin{pmatrix} \gamma q_i^0 + \vec{b}\cdot\vec{q}_i \\ \vec{q}_i + \vec{b} q_i^0 + \frac{(\vec{b}\cdot\vec{q}_i)\vec{b}}{1+\gamma} \end{pmatrix}.$$

$$q_i^\mu = \frac{1}{x} H_{-\vec{b}}^\mu(p_i)$$

$$\vec{b} = -\frac{\vec{Q}}{M}, \gamma = \frac{Q^0}{M}, \text{ and } x = \frac{E}{M}$$

$$\int \mathrm{d}\tilde{\Phi}_N = \int \frac{\mathrm{d}x \, \mathrm{d}^3\vec{b}}{(2\pi)^4} \frac{E^4}{x^{2N+1}\gamma} \Bigg\{ (2\pi)^4 \delta\left(E - \sum_{i=1}^{N} E_i \right) \delta^3 \left(\sum_{i=1}^{N} \vec{p}_i \right)$$

$$\prod_{i=1}^{N} \left[\frac{\mathrm{d}^4 p_i (2\pi)\delta\left(p_i^2\right)\Theta(E_i)}{(2\pi)^4} f\left(\frac{1}{x} H_{-\vec{b}}^0(p_i) \right) \right] \Bigg\}$$

$$= \int \frac{\mathrm{d}x \, \mathrm{d}^3\vec{b}}{(2\pi)^4} \left\{ \frac{E^4}{x^{2N+1}\gamma} \prod_{i=1}^{N} \left[f\left(\frac{1}{x} H_{-\vec{b}}^0(p_i) \right) \right] \mathrm{d}\Phi_N \right\}$$

$$\prod_{i=1}^{N} f\left(\frac{1}{x} H_{-\vec{b}}^0(p_i) \right) = \exp\left[-\frac{\gamma E}{x} \right],$$

$$S_N = \int \frac{\mathrm{d}^3\vec{b}}{(2\pi)^3}\int_0^\infty \frac{\mathrm{d}x}{(2\pi)} \frac{E^4 \exp\left(-\frac{\gamma E}{x}\right)}{\gamma x^{2N+1}} = \frac{E^{4-2N}}{(2\pi)^3} \frac{\Gamma\left(\frac{3}{2}\right)\Gamma(n-1)\Gamma(2n)}{\Gamma\left(n+\frac{1}{2}\right)}.$$

$$\int \mathrm{d}\Phi_N = \frac{E^{2N-4}}{2(4\pi)^{2N-3}\Gamma(N)\Gamma(N-1)}$$

$$\mathcal{A}_n^{\mathrm{MHV},\overline{\mathrm{MHV}}} \propto \prod_{i=1}^{n} \frac{1}{\hat{s}_{i(i+1)}} \quad \text{with} \quad \hat{s}_{n(n+1)} = \hat{s}_{n1}$$

$$\mathrm{d}\mathcal{A}_{ij}^k = \frac{1}{\pi}\,\mathrm{d}^4 p_k \delta p_k^2 \Theta(p_k^0) \frac{(p_i p_j)}{(p_i p_k)(p_j p_k)} g(\xi_{ij}^{ik}) g(\xi_{ij}^{jk}),$$

$$\xi_{ij}^{jk} = \frac{(p_j p_k)}{(p_i p_j)}$$

$$g(\xi) = \frac{1}{2\log\,\xi_m}\Theta(\xi - \xi_m^{-1})\Theta(\xi_m - \xi)$$

$$\xi_m = \frac{\hat{s}}{s_0} - \frac{(n+1)(n+2)}{2}$$

$$g(\vec{x}) = \frac{1}{\sum_j a_j}\sum_j a_j g_j(\vec{x})$$

$$\langle I \rangle_x = \langle f \rangle_x = \langle f/g \rangle_g$$

$$\langle E(a) \rangle = \sqrt{\left\langle \left(\frac{f}{g}\right)^2 \right\rangle_g - \left(\left\langle \frac{f}{g} \right\rangle_g\right)^2}.$$

$$I = \int_V \mathrm{d}^D x g(\vec{x}) \frac{f(\vec{x})}{g(\vec{x})} = \int_V \mathrm{d}^D x f(\vec{x})$$

$$E = \int_V \mathrm{d}^D x g(\vec{x}) \left(\frac{f(\vec{x})}{g(\vec{x})}\right)^2 - I^2 \int_V \mathrm{d}^D x \frac{f^2(\vec{x})}{g(\vec{x})} - I^2.$$

$$\sigma^{(\mathrm{NLO})} = \sum_{a,b} \int_0^1 \mathrm{d}x_a\, \mathrm{d}x_b f_{a/h_1}(x_a, \mu_F) f_{b/h_2}(x_b, \mu_F) \int \mathrm{d}\hat{\sigma}_{ab\to n}^{(\mathrm{NLO})}(\mu_F, \mu_R)$$

$$= \int \mathrm{d}\Phi_{\mathcal{B}}[\mathcal{B}_n(\Phi_{\mathcal{B}}; \mu_F, \mu_R) + \mathcal{V}_n(\Phi_{\mathcal{B}}; \mu_F, \mu_R)] + \int \mathrm{d}\Phi_{\mathcal{R}} \mathcal{R}_n(\Phi_{\mathcal{R}}; \mu_F, \mu_R)$$

$$\mathcal{B}_n(\Phi_{\mathcal{B}};\mu_F,\mu_R)=\overline{\sum_h}\left|\mathcal{M}_n^{(b)}(\Phi_{\mathcal{B}},h;\mu_F,\mu_R)\right|^2$$

$$\mathcal{V}_n(\Phi_{\mathcal{B}};\mu_F,\mu_R)=2\overline{\sum_h}\,\Re\mathrm{e}\left[\mathcal{M}_n^{(b)}(\Phi_{\mathcal{B}},h;\mu_F,\mu_R)\mathcal{M}_n^{*(b+1)}(\Phi_{\mathcal{B}},h;\mu_F,\mu_R)\right]$$

$$\mathcal{R}_n(\Phi_{\mathcal{R}};\mu_F,\mu_R)=2\overline{\sum_h}\left|\mathcal{M}_{n+1}^{(b+1)}(\Phi_{\mathcal{R}},h;\mu_F,\mu_R)\right|^2$$

$$\mathrm{d}\Phi_{\mathcal{B}}=\mathrm{d}x_a\,\mathrm{d}x_b f_{a/h_1}(x_a,\mu_F)f_{b/h_2}(x_b,\mu_F)\frac{1}{2\hat{s}_{ab}}\,\mathrm{d}\Phi_n$$

$$\mathrm{d}\Phi_{\mathcal{R}}=\mathrm{d}x_{a'}\mathrm{d}x_{b'}f_{a'/h_1}(x_{a'},\mu_F)f_{b'/h_2}(x_{b'},\mu_F)\frac{1}{2\hat{s}_{a'b'}}\mathrm{d}\Phi_{n+1}$$

$$\mathrm{d}\Phi_n=\prod_{i=1}^{n}\frac{\mathrm{d}^4p_i}{(2\pi)^4}(2\pi)\delta\left(p_i^2-m_i^2\right)(2\pi)^4\delta^4\left(p_a+p_b-\sum_i p_i\right)\Theta(E_i),$$

19. Proceso de Drell-Yan.

$$\mathcal{M}_{u\bar{d}\to W^+}^{(1)}=g_Wg_s^2C_F\mu^{2\varepsilon}\int\frac{\mathrm{d}^Dk}{(2\pi)^D}\left\{\frac{g^{\nu\rho}}{k^2}\bar{v}(p_d)\left[\gamma_\nu\frac{\not p_d+\not k}{(p_d+k)^2}\gamma^{\mu L}\frac{\not p_u-\not k}{(p_u-k)^2}\gamma_\rho\right.\right.$$

$$\left.\left.+\gamma_\nu\frac{\not p_d+\not k}{(p_d+k)^2}\gamma_\rho\frac{\not p_d}{p_d^2}\gamma^{\mu L}+\gamma^{\mu L}\frac{\not p_u}{p_u^2}\gamma_\nu\frac{\not p_u-\not k}{(p_u-k)^2}\gamma_\rho\right]u(p_u)\epsilon_\mu(W^+)\right\}.$$

$$\mathcal{M}_{u\bar{d}\to W^+}^{(\text{vertex})}=g_Wg_s^2C_F\mu^{4-D}\int\frac{\mathrm{d}^Dk}{(2\pi)^D}\frac{V^\mu\epsilon_\mu(W^+)}{k^2(p_d+k)^2(p_u-k)^2}$$

$$V^\mu=\bar{v}(p_d)\gamma_\nu\left(\not p_d+///\right)\gamma^{\mu L}\left(\not p_u-///\right)\gamma^\nu u(p_u).$$

$$V^\mu=\bar{v}(p_d)\left[-2\left(\not p_u-///\right)\gamma^{\mu R}\left(\not p_d+///\right)+2\varepsilon a^{\mathrm{CDR}}\left(\not p_d+/\not p\right)\gamma^{\mu R}\left(\not p_u-\not k\right)\right]u(p_u).$$

$$\frac{1}{(p_d+k)^2(p_u-k)^2k^2}=\int_0^1\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\frac{2\delta(1-x-y-z)}{[x(p_d+k)^2+y(p_u-k)^2+zk^2]^3}$$
$$=\int_0^1\mathrm{d}x\int_0^{1-x}\mathrm{d}y\frac{2}{[k^2+2k\cdot(xp_d-yp_u)]^3}$$

$$\ell^\alpha\ell^\beta\to g^{\alpha\beta}\ell^2/D$$

$$\int\frac{\mathrm{d}^Dk}{(2\pi)^D}\frac{V^\mu}{k^2(p_d+k)^2(p_u-k)^2}=\int_0^1\mathrm{d}x\int_0^{1-x}\mathrm{d}y\int\frac{\mathrm{d}^D\ell}{(2\pi)^D}\frac{4N\bar{v}(p_d)\gamma^{\mu L}u(p_u)}{(\ell^2+Q^2xy)^3}$$

$$N=Q^2\left[(1-x)(1-y)-\varepsilon a^{\mathrm{CDR}}xy\right]-\ell^2\left(1-a^{\mathrm{CDR}}\varepsilon\right)(1-2/D)$$

$$\int \frac{\mathrm{d}^D\ell}{(2\pi)^D}\frac{\ell^2}{(\ell^2+Q^2xy)^3} = \frac{i}{(4\pi)^{D/2}}\left(\frac{D}{4}\right)\Gamma(\varepsilon)[-Q^2xy]^{-\varepsilon}$$
$$\int \frac{\mathrm{d}^D\ell}{(2\pi)^D}\frac{1}{(\ell^2+Q^2xy)^3} = -\frac{i}{(4\pi)^{D/2}}\left(\frac{1}{2}\right)\Gamma(1+\varepsilon)[-Q^2xy]^{-1-\varepsilon}.$$

$$\int \frac{\mathrm{d}^D\ell}{(2\pi)^D}\frac{4N}{(\ell^2+Q^2xy)^3} = \frac{i\Gamma(1+\varepsilon)}{(4\pi)^{D/2}}(-Q^2)^{-\varepsilon}$$
$$\times\left\{2\left[(1-x)(1-y)-\varepsilon a^{\mathrm{CDR}}xy\right]x^{-1-\varepsilon}y^{-1-\varepsilon}-2\left(1-a^{\mathrm{CDR}}\varepsilon\right)(1-\varepsilon)\frac{1}{\varepsilon}x^{-\varepsilon}y^{-\varepsilon}\right\}$$

$$\int \frac{\mathrm{d}^Dk}{(2\pi)^D}\frac{V^\mu}{k^2(p_d+k)^2(p_u-k)^2}$$
$$=\bar{v}(p_d)\gamma^{\mu L}u(p_u)\frac{i(-Q^2)^{-\varepsilon}}{(4\pi)^{2-\varepsilon}}\frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[\frac{2}{\varepsilon^2}+\frac{3}{\varepsilon}+7+a^{\mathrm{CDR}}\right]$$
$$=-i\bar{v}(p_d)\gamma^{\mu L}u(p_u)\left(-\frac{4\pi}{Q^2}\right)^{\varepsilon}\frac{1}{16\pi^2}\frac{1}{\Gamma(1-\varepsilon)}\left[-\frac{2}{\varepsilon^2}-\frac{3}{\varepsilon}-7-a^{\mathrm{CDR}}\right].$$

$$\mathcal{M}^{(\text{vertex})}_{u\bar{d}\to W^+} = \mathcal{M}^{(0)}_{u\bar{d}\to W^+}\frac{\alpha_s}{4\pi}C_F\left(-\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\frac{1}{\Gamma(1-\varepsilon)}\left[-\frac{2}{\varepsilon^2}-\frac{3}{\varepsilon}-7-a^{\mathrm{CDR}}\right]$$

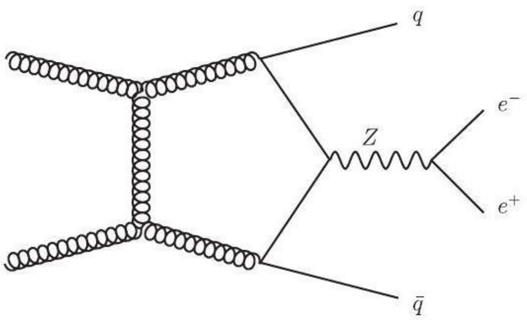
$$c_\Gamma = (4\pi)^\varepsilon/\Gamma(1-\varepsilon) = 1+\varepsilon(1+\log(4\pi)-\gamma_E)+\mathcal{O}(\varepsilon^2),$$

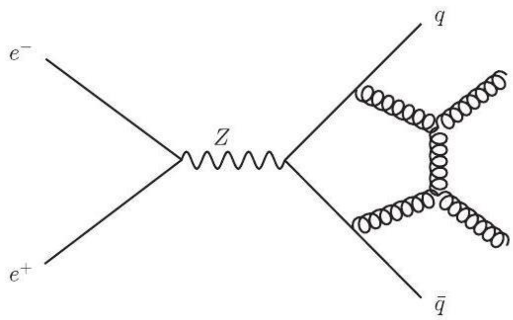
$$(-1)^\varepsilon = 1+i\pi\varepsilon-\pi^2\varepsilon^2/2+\mathcal{O}(\varepsilon^3)$$

$$\mathcal{M}^{(\text{vertex})}_{u\bar{d}\to W^+} = \mathcal{M}^{(0)}_{u\bar{d}\to W^+}\frac{\alpha_s}{4\pi}C_F\left(\frac{\mu^2}{Q^2}\right)^{\varepsilon}c_\Gamma\times\left[-\frac{2}{\varepsilon^2}-\frac{3}{\varepsilon}-7-a^{\mathrm{CDR}}+\pi^2-i\pi\left(\frac{2}{\varepsilon}+3\right)\right]$$

$$\mathcal{V} = 2\Re e\left[\mathcal{M}^{(1)}_{u\bar{d}\to W^+}\mathcal{M}^{*(0)}_{u\bar{d}\to W^+}\right]$$

$$\int \frac{\mathrm{d}^D\ell}{(2\pi)^D}\frac{1}{(\ell^2)^2}$$





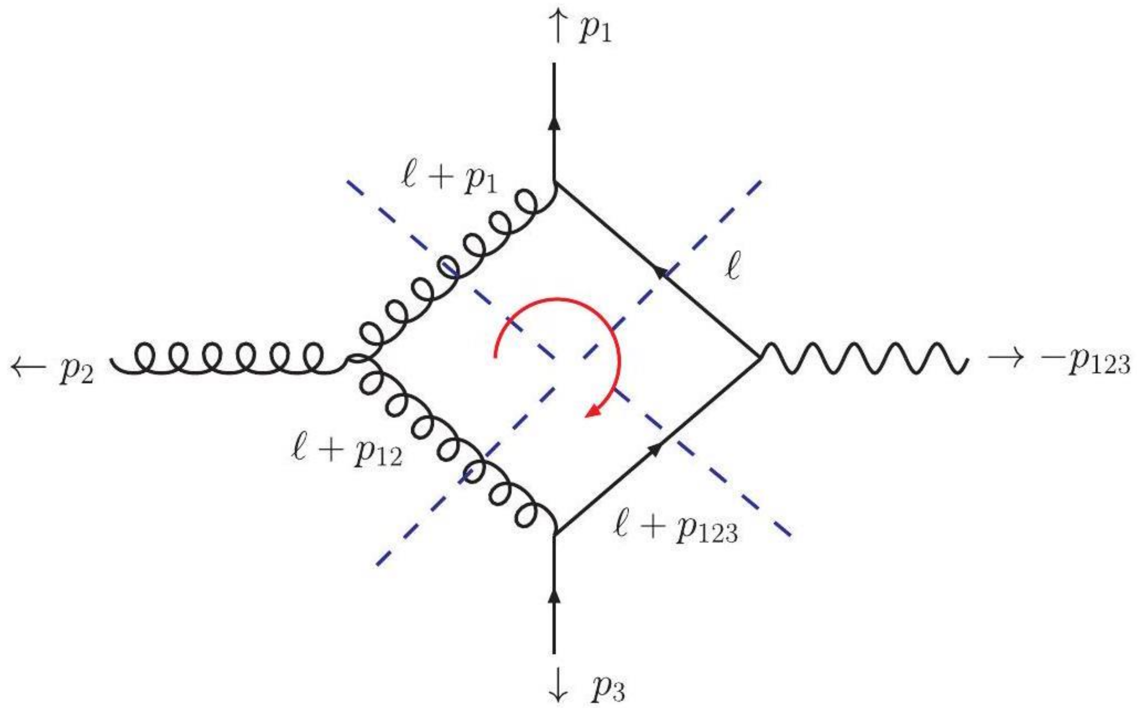


Figura 10. Emisiones de radiación de una partícula supermasiva colapsada.

$$0 \to q(p_1) + g(p_2) + \bar{q}(p_3) + V(-p_{123}),$$

$$V(p_{123}) \to q(p_1) + g(p_2) + \bar{q}(p_3),$$
$$\bar{q}(-p_1) + q(-p_3) \to g(p_2) + V(-p_{123}),$$

$$\bar{q}(-p_1) + g(-p_2) \to q(p_3) + V(-p_{123}).$$

$$\int \frac{\mathrm{d}^D \ell}{(2\pi)^D} \bar{u}(p_1)\gamma_\mu \frac{\ell}{\ell^2} \gamma_\sigma \frac{\ell + \not{p}_{123}}{(\ell + p_{123})^2} \gamma_\nu u(p_3) \frac{1}{(\ell + p_1)^2 (\ell + p_{12})^2} \times V^{\mu\nu\rho}(\ell + p_1, -\ell - p_{12}, p_2)\epsilon_\rho(p_2) J^\sigma$$

$$V^{\mu\nu\rho}(\ell + p_1, -\ell - p_{12}, p_2) = \left(2\ell^\rho + 2p_1^\rho + p_2^\rho\right) g^{\mu\nu} - \left(\ell^\nu + 2p_2^\mu + p_1^\mu\right) g^{\nu\rho} + (p_2^\gamma - p_1^\gamma - \ell^\nu) g^{\mu\rho}$$

$$\int \frac{d^D \ell}{(2\pi)^D} \frac{\{1, \ell^\alpha, \ell^\alpha \ell^\beta, \ell^\alpha \ell^\beta \ell^\gamma\}}{\ell^2 (\ell + p_1)^2 (\ell + p_{12})^2 (\ell + p_{123})^2}$$

$$I^\mu = \int \frac{d^D \ell}{(2\pi)^D} \frac{\ell^\mu}{\ell^2 (\ell + p_1)^2 (\ell + p_{12})^2 (\ell + p_{123})^2}$$

$$I^\mu = p_1^\mu D_1 + p_2^\mu D_2 + p_3^\mu D_3$$

$$\begin{pmatrix} I \cdot p_1 \\ I \cdot p_2 \\ I \cdot p_3 \end{pmatrix} = \begin{pmatrix} 0 & p_1 \cdot p_2 & p_1 \cdot p_3 \\ p_1 \cdot p_2 & 0 & p_2 \cdot p_3 \\ p_1 \cdot p_3 & p_2 \cdot p_3 & 0 \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix}.$$

$$I \cdot p_3 = \frac{1}{2}\int \frac{d^D\ell}{(2\pi)^D}\frac{[(\ell+p_{123})^2-\ell^2-p_{123}^2]-[(\ell+p_{12})^2-\ell^2-p_{12}^2]}{\ell^2(\ell+p_1)^2(\ell+p_{12})^2(\ell+p_{123})^2}$$

$$= \frac{1}{2}\int \frac{d^D\ell}{(2\pi)^D}\left\{\frac{1}{\ell^2(\ell+p_1)^2(\ell+p_{12})^2}\right.$$

$$\left. - \frac{1}{\ell^2(\ell+p_1)^2(\ell+p_{123})^2} - \frac{2p_{12}\cdot p_3}{\ell^2(\ell+p_1)^2(\ell+p_{12})^2(\ell+p_{123})^2}\right\}$$

$$I^{\mu\nu} = \sum_i p_i^\mu p_i^\nu D_{ii} + \sum_{i\neq j}\left(p_i^\mu p_j^\nu + p_j^\mu p_i^\nu\right)D_{ij} + g^{\mu\nu}D_{00}.$$

$$C_0(p_1,p_2) = \int \frac{d^D\ell}{(2\pi)^D}\frac{\ell^\mu}{\ell^2(\ell+p_1)^2(\ell+p_{12})^2}$$

$$C^\mu = \int \frac{d^D\ell}{(2\pi)^D}\frac{\ell^\mu}{\ell^2(\ell+p_1)^2(\ell+p_{12})^2} = C_1 p_1^\mu + C_2 p_2^\mu.$$

$$\begin{pmatrix} C\cdot p_1 \\ C\cdot p_2 \end{pmatrix} = \begin{pmatrix} p_1^2 & p_1\cdot p_2 \\ p_1\cdot p_2 & p_2^2 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix},$$

$$C\cdot p_1 = \frac{1}{2}[B_0(p_{12}) - B_0(p_2) - p_1^2 C_0(p_1,p_2)]$$
$$C\cdot p_2 = \frac{1}{2}[B_0(p_1) - B_0(p_{12}) + (p_1^2 - p_{12}^2)C_0(p_1,p_2)]$$

$$B_0(q) = \int \frac{d^D\ell}{(2\pi)^D}\frac{1}{\ell^2(\ell+q)^2}$$

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \frac{1}{\Delta}\begin{pmatrix} p_2^2 & -p_1\cdot p_2 \\ -p_1\cdot p_2 & p_1^2 \end{pmatrix}\begin{pmatrix} C\cdot p_1 \\ C\cdot p_2 \end{pmatrix}$$

$$(C\cdot p_1)p_2^2 - (C\cdot p_2)p_1\cdot p_2 = [p_1^2 p_2^2 - (p_1\cdot p_2)^2]C_1 = \Delta C_1$$

$$\Delta C_1 = -\frac{1}{2}[p_1^2 p_2^2 + p_1\cdot p_2(p_1^2 - p_{12}^2)]C_0(p_1,p_2) + \{\,\zeta_{\text{scalar bubbles}}\,\}$$

$$C_0(p_1,p_2) = \frac{2}{p_1\cdot p_2(p_{12}^2 - p_1^2) - p_1^2 p_2^2}[\Delta C_1 + \{\,\zeta_{\text{scalar bubbles}}\,\}]$$

$$C^\mu = C_1^\star p_1^\mu$$

$$C_1 = \frac{2}{p_1\cdot p_2(p_{12}^2 - p_1^2) - p_1^2 p_2^2}\left[\Delta\sum_{i,j}\alpha_{ij}C_{ij} + \{\,\zeta_{\text{scalar bubbles}}, C_0\}\right],$$

$$v_1^\mu = \frac{\delta^{\mu k_2 k_3}_{k_1 k_2 k_3}}{\Delta}, v_2^\mu = \frac{\delta^{k_1 \mu k_3}_{k_1 k_2 k_3}}{\Delta}, v_3^\mu = \frac{\delta^{k_1 k_2 \mu}_{k_1 k_2 k_3}}{\Delta}$$

$$\delta^{\mu\nu\rho}_{k_1k_2k_3} = \begin{vmatrix} k_1^\mu & k_2^\mu & k_3^\mu \\ k_1^\nu & k_2^\nu & k_3^\nu \\ k_1^\rho & k_2^\rho & k_3^\rho \end{vmatrix},$$

$$\Delta = \delta^{\mu\nu\rho}_{k_1k_2k_3} k_{1\mu} k_{2\nu} k_{3\rho} \equiv \delta^{k_1k_2k_3}_{k_1k_2k_3}$$

$$v_i \cdot k_j = \delta_{ij}$$

$$n^\mu = \frac{\epsilon^{\mu k_1 k_2 k_3}}{\sqrt{\Delta}},$$

$$n_i \cdot k_j = n_i \cdot v_j = 0$$

$$\ell^\mu = \sum_{i=1}^{3} (\ell \cdot k_i) v_i^\mu + (\ell \cdot n) n^\mu.$$

$$d_0 = \ell^2, d_i = (\ell + k_i)^2, \text{ for } i = 1,2,3.$$

$$\ell \cdot k_i = \frac{1}{2}(d_i - d_0) - \frac{1}{2} k_i^2.$$

$$\begin{aligned} \ell^\mu \ell^\nu \ell^\rho &= \frac{1}{2} \ell^\mu \ell^\nu \left(\sum_{i=1}^{3} (d_i - d_0 - k_i^2) v_i^\rho + (\ell \cdot n) n^\rho \right) \\ &= \frac{1}{4} \ell^\mu \left(\sum_{j=1}^{3} (d_j - d_0 - k_j^2) v_j^\nu + (\ell \cdot n) n^\nu \right) \\ &\quad \times \left(-\sum_{i=1}^{3} k_i^2 v_i^\rho + (\ell \cdot n) n^\rho \right) + \ \xi_{\text{triangles}} \\ &= \frac{1}{8} \left(-\sum_{m=1}^{3} k_m^2 v_m^\mu + (\ell \cdot n) n^\mu \right) \left(-\sum_{j=1}^{3} k_j^2 v_j^\nu + (\ell \cdot n) n^\nu \right) \\ &\quad \times \left(-\sum_{i=1}^{3} k_i^2 v_i^\rho + (\ell \cdot n) n^\rho \right) + \ \xi_{\text{triangles}}, \aleph_{\text{bubbles}} \\ &\equiv \delta_0^{\mu\nu\rho} + \delta_1^{\mu\nu\rho}(\ell \cdot n) + \delta_2^{\mu\nu\rho}(\ell \cdot n)^2 + \delta_3^{\mu\nu\rho}(\ell \cdot n)^3 + \ \daleth_{\text{lower points}} \end{aligned}$$

$$\ell^2 = \sum_{i,j=1}^{3} (\ell \cdot k_i)(\ell \cdot k_j) v_i \cdot v_j + (\ell \cdot n)^2 \Rightarrow (\ell \cdot n)^2 = d_0 - \frac{1}{4} \sum_{i,j=1}^{3} (d_i - d_0 - k_i^2)(d_j - d_0 - k_j^2) v_i \cdot v_j$$

$$\ell^\mu \ell^\nu \ell^\rho \longrightarrow \delta_0^{\mu\nu\rho} + \delta_1^{\mu\nu\rho}(\ell \cdot n) + \ \daleth_{\text{lower points}}$$

$$\mathcal{D} = \frac{\{\ell^\mu \ell^\nu \ell^\rho A_{\mu\nu\rho} + \cdots\}}{d_0 d_1 d_2 d_3} = c_4 I_4 + \sum_i c_3^{(i)} I_3^{(i)} + \sum_{i,j} c_2^{(i,j)} I_2^{(i,j)}$$

$$\mathcal{D} = \frac{1}{d_0 d_1 d_2 d_3}\left[c_4 + \sum_i c_3^{(i)} d_i + \sum_{i,j} c_2^{(i,j)} d_i d_j\right]$$

$$\ell^\mu = -\frac{1}{2}\sum_{i=1}^{3} k_i^2 v_i^\mu + \frac{1}{2}\left(\sum_{i,j=1}^{3} k_i^2 k_j^2 v_i \cdot v_j\right)^{\frac{1}{2}} n^\mu$$

$$\mathcal{D} - \frac{c_4}{d_0 d_1 d_2 d_3} = \frac{1}{d_0 d_1 d_2 d_3}\left[\sum_i c_3^{(i)} d_i + \sum_{i,j} c_2^{(i,j)} d_i d_j\right]$$

$$\ell^\mu = \sum_{i=1}^{2} (\ell \cdot k_i) v_i^\mu + (\ell \cdot n_1) n_1^\mu + (\ell \cdot n_2) n_2^\mu$$

$$\ell^\mu = \sum_{i=1}^{2} (\ell \cdot k_i) v_i^\mu + (\ell \cdot n_1) n_1^\mu + (\ell \cdot n_2) n_2^\mu + (\ell \cdot n_\varepsilon) n_\varepsilon^\mu.$$

$$(\ell \cdot n_1)^2 + (\ell \cdot n_2)^2 + (\ell \cdot n_\varepsilon)^2 = d_0 - \frac{1}{4}\sum_{i,j=1}^{3} \left(d_i - d_0 - k_i^2\right)\left(d_j - d_0 - k_j^2\right) v_i \cdot v_j$$

$$\int d^D \ell \frac{(\ell \cdot n_\varepsilon)^2}{d_0 d_1 d_2} = n_\varepsilon^\mu n_\varepsilon^\nu \left[g_{\mu\nu} C_{00}\right] = (-2\varepsilon) C_{00}$$

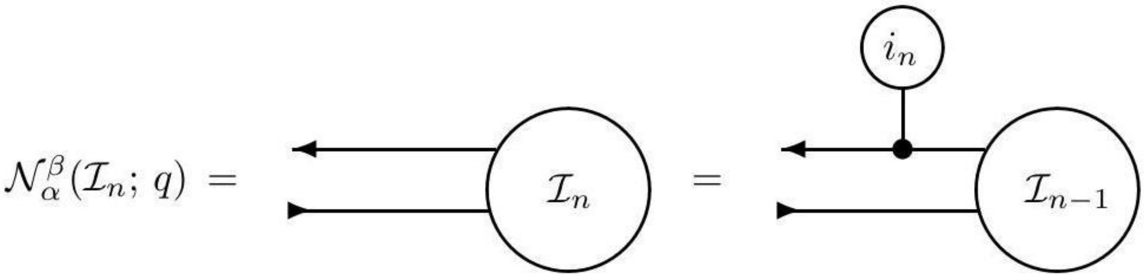
$$\mathcal{A}^{(D)} = \int \frac{\mathrm{d}^D q \mathcal{N}(\mathcal{I}_n; q)}{D_0 D_1 \dots D_{n-1}},$$

$$\mathcal{I}_n = \{i_1, i_2, \dots, i_n\}$$

$$D_i = (q + p_i)^2 - m^2 + i\epsilon$$

$$\mathcal{N}(\mathcal{I}_n; q) = \sum_{r=0}^{R} \mathcal{N}_{\mu_1 \mu_2 \dots \mu_r}^{(r)}(\mathcal{I}_n) q^{\mu_1} q^{\mu_2} \dots q^{\mu_n}$$

$$\mathcal{T}_{n,r}^{\mu_1 \mu_2 \dots \mu_r} = \int \frac{\mathrm{d}^D q q^{\mu_1} q^{\mu_2} \dots q^{\mu_n}}{D_0 D_1 \dots D_{n-1}}$$



$$\mathcal{N}_{\alpha}^{\beta}(\mathcal{J}_n;q)=\mathcal{N}_{\alpha}^{\gamma}(\mathcal{J}_{n-1};q)\mathcal{X}_{\gamma\delta}^{\beta}(\mathcal{J}_n,i_n,\mathcal{J}_{n-1})w^{\delta}(i_n).$$

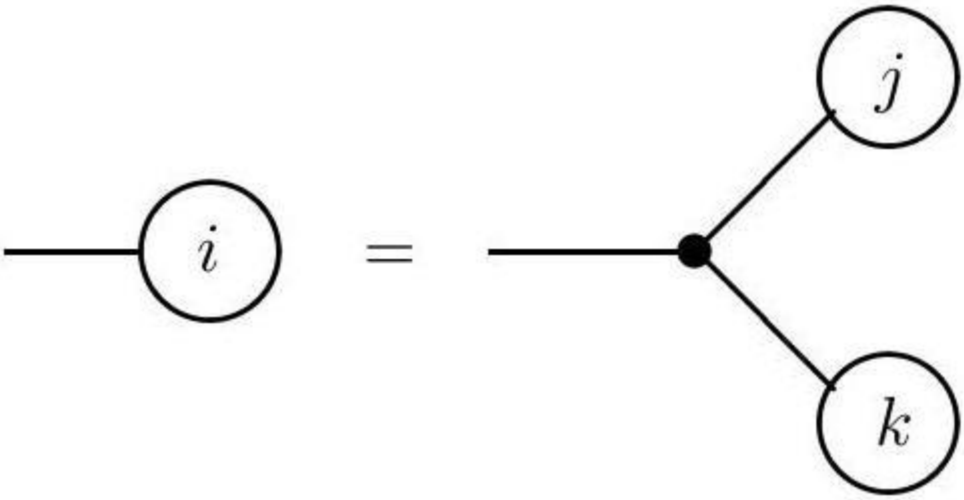


Figura 11. Escisión de una partícula supermasiva por brecha de masa.

$$w^{\beta}(i)=\frac{\mathcal{X}_{\gamma\delta}^{\beta}(i,j,k)w^{\gamma}(j)w^{\delta}(k)}{p_i^2-m_i^2+i\epsilon},$$

$$\mathcal{X}_{\gamma\delta}^{\beta}=\mathcal{Y}_{\gamma\delta}^{\beta}+q^{\nu}\mathcal{Z}_{\nu;\gamma\delta}^{\beta},$$

$$N_{\mu_1\mu_2\ldots\mu_r;\alpha}^{\beta}(\mathcal{J}_n)=\left[\mathcal{Y}_{\gamma\delta}^{\beta}N_{\mu_1\mu_2\ldots\mu_r;\alpha}^{\gamma}(\mathcal{J}_{n-1})+\mathcal{Z}_{\nu;\gamma\delta}^{\beta}N_{\mu_2\ldots\mu_r;\alpha}^{\gamma}(\mathcal{J}_{n-1})+\right]w^{\delta}(i_n).$$

$$\sigma^{(\mathrm{NLO})}=\sum_{a,b}\int_0^1 \mathrm{d}x_a\,\mathrm{d}x_b f_{a/h_1}(x_a,\mu_F)f_{b/h_2}(x_b,\mu_F)\int \mathrm{d}\hat{\sigma}_{ab\to n}^{(\mathrm{NLO})}(\mu_F,\mu_R)$$

$$=\int \mathrm{d}\Phi_{\mathcal{B}}[\mathcal{B}_n(\Phi_{\mathcal{B}};\mu_F,\mu_R)+\mathcal{V}_n(\Phi_{\mathcal{B}};\mu_F,\mu_R)]+\int \mathrm{d}\Phi_{\mathcal{R}}\mathcal{R}_n(\Phi_{\mathcal{R}};\mu_F,\mu_R)$$

$$=\int \mathrm{d}\Phi_{\mathcal{B}}\left[\mathcal{B}_n(\Phi_{\mathcal{B}};\mu_F,\mu_R)+\mathcal{V}_n(\Phi_{\mathcal{B}};\mu_F,\mu_R)+\mathcal{I}_n^{(\mathcal{S})}(\Phi_{\mathcal{B}};\mu_F,\mu_R)\right]$$

$$+\int \mathrm{d}\Phi_{\mathcal{R}}[\mathcal{R}_n(\Phi_{\mathcal{R}};\mu_F,\mu_R)-\mathcal{S}_n(\Phi_{\mathcal{R}};\mu_F,\mu_R)]$$

$$0\equiv\int \mathrm{d}\Phi_{\mathcal{B}}\mathcal{I}_n^{(\mathcal{S})}(\Phi_{\mathcal{B}};\mu_F,\mu_R)-\int \mathrm{d}\Phi_{\mathcal{R}}\mathcal{S}_n(\Phi_{\mathcal{R}};\mu_F,\mu_R)$$

$$\mathcal{B}_n=\sum\left|\mathcal{M}_n^{(\mathcal{B})}\right|^2$$

$$\mathcal{V}_n=\frac{V_n}{\varepsilon}=\sum\left|\mathcal{M}_n^{(\mathcal{B})}\mathcal{M}_n^{*(\mathcal{V})}\right|,$$

$$\mathcal{R}_n(x)=\frac{R_n)(x)}{x}=\sum\left|\mathcal{M}_{n+1}^{(\mathcal{R})}(x)\right|^2,$$

$$\sigma^{(\mathrm{NLO})} = [\mathcal{B}_n + \mathcal{V}_n]F_n^J + \int_0^1 \mathrm{d}x\mathcal{R}_n(x)F_{n+1}^J(x)$$

$$= \left[\mathcal{B}_n + \frac{V_n}{\varepsilon}\right]F_n^J + \int_0^1 \frac{\mathrm{d}x}{x}R_n(x)F_{n+1}^J(x)$$

$$\lim_{x\to 0} F_{n+1}^J(x) = F_{n+1}^J(0) = F_n^J$$

$$\lim_{x\to 0} R_n(x) = R_n(0) = V$$

$$\sigma^{(1)} = \frac{V_n}{\varepsilon}F_n^J + \int_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}}R_n(x)F_{n+1}^J(x)$$
$$= \frac{V_n}{\varepsilon}F_n^J + \int_0^\delta \frac{\mathrm{d}x}{x^{1+\epsilon}}R_n(x)F_{n+1}^J(x) + \int_\delta^1 \frac{\mathrm{d}x}{x^{1+\epsilon}}R_n(x)F_{n+1}^J(x)$$

$$\sigma^{(1)} = \frac{V_n}{\varepsilon}F_n^J + R_n(0)F_n^J\int_0^\delta \frac{\mathrm{d}x}{x^{1+\epsilon}} + \int_\delta^1 \frac{\mathrm{d}x}{x^{1+\epsilon}}R_n(x)F_{n+1}^J(x) + \mathcal{O}(\varepsilon)$$
$$= [1-\delta^{-\epsilon}]\frac{V_n}{\varepsilon}F_n^J + \int_\delta^1 \frac{\mathrm{d}x}{x^{1+\epsilon}}R_n(x)F_{n+1}^J(x) + \mathcal{O}(\varepsilon)$$
$$= \log\,\delta\cdot V_nF_n^J + \int_\delta^1 \frac{\mathrm{d}x}{x^{1+\epsilon}}R_n(x)F_{n+1}^J(x) + \mathcal{O}(\varepsilon),$$

$$R_n(0)F_n^J\int_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}}$$

20. Métrica NLO.

$$\sigma^{(1)} = \frac{V_n}{\varepsilon}F_n^J + \int_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}}R_n(x)F_{n+1}^J(x)$$

$$= \frac{V_n}{\varepsilon}F_n^J + R_n(0)F_n^J\int_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}} - R_n(0)F_n^J\int_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}} + \int_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}}R_n(x)F_{n+1}^J(x)$$

$$= \frac{V_n}{\varepsilon}F_n^J[1-1] + \int_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}}\left[R_n(x)F_n^J(x) - V_nF_n^J\right]$$

21. Modelo Toy.

$$\sigma^{(\mathrm{LO})} = \int \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}_n(\Phi_{\mathcal{B}};\mu_F,\mu_R)$$

$$\sigma^{(\mathrm{NLO})} = \int \mathrm{d}\Phi_{\mathcal{B}}\left[\mathcal{B}_n(\Phi_{\mathcal{B}};\mu_F,\mu_R) + \mathcal{V}_n(\Phi_{\mathcal{B}};\mu_F,\mu_R) + \mathcal{J}_n^{(\mathcal{S})}(\Phi_{\mathcal{B}};\mu_F,\mu_R)\right]$$

$$+ \int \mathrm{d}\Phi_{\mathcal{R}}[\mathcal{R}_n(\Phi_{\mathcal{R}};\mu_F,\mu_R) - \mathcal{S}_n(\Phi_{\mathcal{R}};\mu_F,\mu_R)]$$

$$\pm R_m(0)F_m^J\int_0^1 \frac{\mathrm{d}x}{x^{1+\varepsilon}},$$

$$\frac{\hat{t}^2 + \hat{u}^2 + 2m_W^2\hat{s}}{\hat{t}\hat{u}} = \frac{(\hat{t} + \hat{u})^2 + 2m_W^2\hat{s}}{\hat{t}\hat{u}} - 2$$

$$= \left(\frac{1}{\hat{t}} + \frac{1}{\hat{u}}\right)\frac{(\hat{t} + \hat{u})^2 + 2m_W^2\hat{s}}{\hat{t} + \hat{u}} - 2 = \left(\frac{1}{\hat{t}} + \frac{1}{\hat{u}}\right)\left[(m_W^2 - \hat{s}) + \frac{2m_W^2\hat{s}}{m_W^2 - \hat{s}}\right] - 2$$

$$x = m_W^2/\hat{s}$$

$$\left|\mathcal{M}_{u\bar{d}\to gW^+}^{(\mathrm{LO})}\right|^2 = \frac{2\pi C_F\alpha_s(\mu_R)}{m_W^2}\left|\mathcal{M}_{u\bar{d}\to W^+}^{(\mathrm{LO})}\right|^2 \cdot \left[\frac{\hat{t}^2 + \hat{u}^2 + 2m_W^2\hat{s}}{\hat{t}\hat{u}}\right]$$
$$= \frac{2\pi C_F\alpha_s(\mu_R)}{x}\left|\mathcal{M}_{u\bar{d}\to W^+}^{(\mathrm{LO})}\right|^2 \cdot \left[\left(\frac{1}{\hat{t}} + \frac{1}{\hat{u}}\right)\left(-\frac{2}{1-x} + x + 1\right) - \frac{2x}{m_W^2}\right]$$

$$\left|\mathcal{M}_{u\bar{d}\to gW^+}^{(\mathrm{LO})}\right|^2 = \frac{1}{x}\left|\mathcal{M}_{u\bar{d}\to W^+}^{(\mathrm{LO})}\right|^2 \cdot [\mathcal{D}(\hat{t}, x) + \mathcal{D}(\hat{u}, x) + \mathcal{R}(x)]$$

$$\mathcal{D}(\hat{t}, x) = 8\pi\alpha_s C_F\left[-\frac{1}{\hat{t}}\left(\frac{2}{1-x} - 1 - x\right)\right]$$

$$\mathcal{R}(x) = 8\pi\alpha_s C_F\left[-\frac{2x}{m_W^2}\right]$$

$$\mathcal{S}(\Phi_{\mathcal{R}}) = \frac{1}{x}\left|\mathcal{M}_{u\bar{d}\to W^+}^{(\mathrm{LO})}\right|^2 [\mathcal{D}(\hat{t}, x) + \mathcal{D}(\hat{u}, x)]$$

$$\mathrm{d}\Phi_{Wg} = \frac{\mathrm{d}^D p_W}{(2\pi)^D}(2\pi)\delta(p_W^2 - m_W^2)\frac{\mathrm{d}^D p_g}{(2\pi)^D}(2\pi)\delta(p_g^2)(2\pi)^D\delta^D(p_a + p_b - p_W - p_g) = (2\pi)^{2-D}\frac{\mathrm{d}^{D-1}p_g}{2E}\delta\left((p_a + p_b - p_g)^2 - m_W^2\right)$$

$$\mathrm{d}\Phi_{Wg} = \frac{(2\pi)^{2\varepsilon-2}}{2\sqrt{\hat{s}}}\left(\frac{\hat{t}\hat{u}}{\hat{s}}\right)^{-\varepsilon}\mathrm{d}\left(-\frac{\hat{t} + \hat{u}}{2\sqrt{\hat{s}}}\right)\mathrm{d}\hat{t}\ \mathrm{d}\Omega^{1-2\varepsilon}\delta(\hat{s} + \hat{t} + \hat{u} - m_W^2),$$

$$\int \mathrm{d}\Omega^{1-2\varepsilon} = \frac{2\pi}{\pi^\varepsilon\Gamma(1-\varepsilon)}$$

$$x = \frac{\hat{s} + \hat{t} + \hat{u}}{\hat{s}} = \frac{m_W^2}{\hat{s}}, v = -\frac{\hat{t}}{\hat{s}}$$

$$\mathrm{d}\Phi_{Wg} = \frac{\hat{s}^{1-\varepsilon}}{16\pi^2}\frac{\mathrm{d}\Omega^{1-2\varepsilon}}{(2\pi)^{1-2\varepsilon}}\ \mathrm{d}x\ \mathrm{d}v v^{-\varepsilon}(1 - x - v)^{-\varepsilon}[2\pi\delta(x\hat{s} - m_W^2)]$$

$$\mathrm{d}\phi(x, v, \hat{s}) = \frac{\hat{s}^{1-\varepsilon}}{16\pi^2}\frac{\mathrm{d}\Omega^{1-2\varepsilon}}{(2\pi)^{1-2\varepsilon}}\ \mathrm{d}v v^{-\varepsilon}(1 - x - v)^{-\varepsilon}$$

$$\mu^{2\varepsilon}\int \mathcal{D}(\hat{t}, x)\mathrm{d}\phi(x, v, \hat{s})$$

$$= \frac{\alpha_s C_F}{2\pi}\left(\frac{\mu^2}{m_W^2}\right)^\varepsilon c_\Gamma x^\varepsilon \int_0^{1-x} \mathrm{d}v v^{-\varepsilon}(1 - x - v)^{-\varepsilon}\frac{1}{v}\left[\frac{2}{1-x} - 1 - x\right].$$

$$\int_0^{1-x} \mathrm{d}v v^{-\varepsilon}(1-x-v)^{-\varepsilon}\frac{1}{v} = -\frac{1}{\varepsilon}\frac{\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}(1-x)^{-2\varepsilon}$$

$$\frac{x^\varepsilon(1-x)^{-2\varepsilon}}{1-x} \to \left[\frac{x^\varepsilon(1-x)^{-2\varepsilon}}{1-x}\right]_+ - \frac{1}{2\varepsilon}\left(1+\frac{\varepsilon^2\pi^2}{3}\right)\delta(1-x)$$
$$= \left[\frac{1+\varepsilon\log\, x - 2\varepsilon\log\,(1-x)}{1-x}\right]_+ - \frac{1}{2\varepsilon}\left(1+\frac{\varepsilon^2\pi^2}{3}\right)\delta(1-x)$$

$$-\frac{1}{\varepsilon}x^\varepsilon(1-x)^{-2\varepsilon}\left(\frac{2}{1-x}-1-x\right)$$

$$= \frac{1}{\varepsilon}\left\{\frac{\mathcal{P}_{qq}^{(1)}x}{C_F} - \frac{3}{2\varepsilon}\delta(1-x) - \varepsilon\left[\frac{2}{1-x}\log\frac{(1-x)^2}{x}\right]_+ - \frac{1}{\varepsilon}\left(1+\frac{\varepsilon^2\pi^2}{3}\right)\delta(1-x)\right.$$

$$\left. + \varepsilon(1+x)\log\frac{(1-x)^2}{x}\right\}$$

$$= \left(\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}+\frac{\pi^2}{3}\right)\delta(1-x) - \frac{1}{\varepsilon}\frac{\mathcal{P}_{qq}^{(1)}(x)}{C_F} + \left[\frac{2}{1-x}\log\,\frac{(1-x)^2}{x}\right]_+ - (1$$

$$+ x)\log\,\frac{(1-x)^2}{x}$$

$$\mu^{2\varepsilon}\int\, \mathcal{D}(\hat{t},x)\mathrm{d}\phi(x,v,\hat{s})$$

$$= \frac{\alpha_s C_F}{2\pi}\left(\frac{\mu^2}{m_W^2}\right)^\varepsilon c_\Gamma\left[\left(\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}+\frac{\pi^2}{6}\right)\delta(1\right.$$

$$\left. -x) + \left[\frac{2}{1-x}\log\,\frac{(1-x)^2}{x}\right]_+ - (1+x)\log\,\frac{(1-x)^2}{x}\right] - \frac{\alpha_s}{2\pi}\left(\frac{\mu^2}{m_W^2}\right)^\varepsilon c_\Gamma\frac{1}{\varepsilon}\mathcal{P}_{qq}^{(1)}(x)$$

$$\frac{\alpha_s}{2\pi}\left(\frac{\mu_F^2}{m_W^2}\right)^\varepsilon c_\Gamma\left\{-\frac{1}{\varepsilon}\mathcal{P}_{qq}^{(1)}(x)\right\},$$

$$f_{q/h}(y) = f_{q/h}^{(B)}(y) + \frac{\alpha_s}{2\pi}\left(-\frac{c_\Gamma}{\varepsilon}\right)\int_y^1 \frac{dx}{x}\mathcal{P}_{qq}^{(1)}(x)f_{q/h}^{(B)}\left(\frac{y}{x}\right)$$

$$\frac{1}{\varepsilon}\mathcal{P}_{qq}^{(1)}(x) \to \frac{1}{\varepsilon}\mathcal{P}_{qq}^{(1)}(x) + C_F(1-x) - \frac{C_F}{2}\delta(1-x)$$

$$\mu^{2\varepsilon}\int \mathcal{D}(\hat{t},x)\mathrm{d}\phi(x,v,\hat{s})$$

$$= \frac{\alpha_s C_F}{2\pi}\left(\frac{\mu^2}{m_W^2}\right)^{\varepsilon} c_\Gamma\left[\left(\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}+\frac{\pi^2}{6}-\frac{1-a^{\mathrm{CDR}}}{2}\right)\delta(1\right.$$

$$\left.-x)+1-x+\left[\frac{2}{1-x}\log\frac{(1-x)^2}{x}\right]_+-(1+x)\log\frac{(1-x)^2}{x}\right]$$

$$-\frac{\alpha_s}{2\pi}\log\left(\frac{\mu^2}{m_W^2}\right)\mathcal{P}_{qq}^{(1)}(x)$$

$$\mu^{2\varepsilon}\int \mathcal{R}(x)\mathrm{d}\phi(x,v,\hat{s}) = \frac{\alpha_S C_F}{2\pi}\left(\frac{\mu^2}{m_W^2}\right)^{\varepsilon} c_\Gamma[-2(1-x)]$$

22. Métrica Catani-Seymour.

$$\mathrm{W}(p_1,p_2;k) = -\frac{\mathbf{T}_1\cdot\mathbf{T}_2}{2}\left(\frac{p_1^\mu}{p_1k}-\frac{p_2^\mu}{p_2k}\right)^2 = \mathbf{T}_1\cdot\mathbf{T}_2\frac{p_1p_2}{(p_1k)(p_2k)}$$

$$= \mathbf{T}_1\cdot\mathbf{T}_2\left[\frac{p_1p_2}{(p_1k)(p_1k+p_2k)}+\frac{p_1p_2}{(p_2k)(p_1k+p_2k)}\right] = \tilde{\mathcal{D}}_{1k;2}+\tilde{\mathcal{D}}_{2k;1}$$

$$\Phi_{\mathcal{R}} = \Phi_{\mathcal{B}}\otimes\Phi_1.$$

$$\mathcal{S}(\Phi_{\mathcal{R}}) = \sum_{\text{dipoles}}\mathcal{D}(p_a,p_b;p_1,p_2,\dots,p_{n+1}) = \sum_{ij,k}\mathcal{B}_{ij;k}(\Phi_{\mathcal{B}})\otimes\tilde{\mathcal{D}}_{ij;k}(\Phi_1)\longrightarrow\mathcal{B}(\Phi_{\mathcal{B}})\otimes\mathbf{D}(\Phi_1)$$

$$\mathcal{I}^{(\mathcal{S})}(\Phi_{\mathcal{B}},\varepsilon)= \sum_{\text{dipoles}}\mathcal{I}^{(\mathcal{D})}(p_a,p_b;p_1,p_2,\dots,p_{i-1},p_{i+1},\dots,p_{n+1})$$

$$= \sum_{ij,k}\mathcal{B}_{ij;k}(\Phi_{\mathcal{B}})\otimes\mathcal{I}_{ij;k}^{(\mathcal{D})}(\Phi_{\mathcal{B}})\longrightarrow\mathcal{B}(\Phi_{\mathcal{B}})\otimes\mathbf{D}(\Phi_{\mathcal{B}})$$

$$\mathcal{D}_{ij;k}(p_a,p_b;p_1,p_2,\dots,p_n) = \mathcal{B}\big(p_a,p_b;p_1,p_2,\dots,\tilde{p}_{ij},\dots,\tilde{p}_k,\dots,p_n\big)\otimes\tilde{\mathcal{D}}_{ij;k}\big(p_i,p_j,p_k\big)$$

$$\tilde{p}_{ij} = p_i+p_j-\frac{y_{ij,k}}{1-y_{ij,k}}p_k$$

$$\tilde{p}_k = \frac{1}{1-y_{ij,k}}p_k$$

$$y_{ij,k} = \frac{p_ip_j}{p_ip_j+p_jp_k+p_kp_i}$$

$$\tilde{z}_i = \frac{p_ip_k}{(p_i+p_j)p_k} = \frac{p_i\tilde{p}_k}{\tilde{p}_{ij}\tilde{p}_k}\ \text{ and }\ \tilde{z}_j = 1-\tilde{z}_i$$

$$\frac{p_i p_j + p_j p_k + p_k p_i}{(p_i + p_j) p_k} = \frac{1}{1 - \tilde{z}_i(1 - y_{ij,k})}$$

$$p_i^\mu = z p^\mu + k_\perp^\mu - \frac{k_\perp^2}{z} \frac{n^\mu}{2pn} \text{ and } p_j^\mu = (1 - z) p^\mu - k_\perp^\mu - \frac{k_\perp^2}{1 - z} \frac{n^\mu}{2pn}$$

$$2 p_i p_j = -\frac{k_\perp^2}{z(1 - z)} \text{ with } k_\perp \to 0$$

$$y_{ij,k} \to -\frac{k_\perp^2}{2\tilde{z}_i(1 - \tilde{z}_i)\tilde{p}_{ij}\tilde{p}_k}$$

$$\tilde{\mathcal{D}}_{ij,k} = -\frac{1}{2\tilde{p}_{ij}\tilde{p}_k} \frac{\mathbf{T}_{ij} \cdot \mathbf{T}_k}{\mathbf{T}_{ij}^2} \langle s | V_{ij;k} | s' \rangle.$$

$$\langle s | V_{q_i g_j;k} | s' \rangle = 8\pi \mu_R^{2\varepsilon} C_F \alpha_s(\mu_R) \left[\frac{2}{1 - \tilde{z}_i(1 - y_{ij,k})} - (1 + \tilde{z}_i) - \varepsilon(1 - \tilde{z}_i) \right] \delta_{ss'}$$

$$\mathcal{J}_{ij;k}^{(\mathcal{D})}(\varepsilon) = -\frac{\alpha_s(\mu_R^2)}{2\pi\Gamma(1 - \varepsilon)} \left(\frac{4\pi\mu_R^2}{2\tilde{p}_{ij}\tilde{p}_k} \right)^\varepsilon \frac{\mathbf{T}_{ij} \cdot \mathbf{T}_k}{\mathbf{T}_{ij}^2} \mathcal{V}_{ij}(\varepsilon),$$

$$\mathcal{V}_{ij}(\varepsilon) = \mathbf{T}_i^2 \left(\frac{1}{\varepsilon^2} - \frac{\pi^2}{3} \right) + \gamma_i \left(\frac{1}{\varepsilon} + 1 \right) + K_i$$

$$K_{q,g} = \begin{cases} C_F \left(\frac{7}{2} - \frac{\pi^2}{6} \right) & \text{for } i = q \\ C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{10}{9} T_R n_f & \text{for } i = g \end{cases}$$

$$\gamma_q = \begin{cases} \frac{3}{2} C_F & \text{for } i = q \\ \frac{11}{6} C_A - \frac{2}{3} n_f T_R & \text{for } i = g \end{cases}$$

$$\mathcal{J}^{(\mathcal{S})}(\Phi_{\mathcal{B}}) = \sum_{ij;k} \mathcal{B}_{ij;k}(\Phi_{\mathcal{B}}) \otimes \mathcal{J}_{ij;k}^{(\mathcal{D})}(\varepsilon) \longrightarrow \mathcal{B}(\Phi_{\mathcal{B}}) \otimes \mathbf{I}(\Phi_{\mathcal{B}}; \varepsilon).$$

$$y_{ij,k} = \frac{2 p_i p_j}{Q^2},$$

$$\sigma^{(\mathrm{LO})} = \int \mathrm{d}\Phi_{\mathcal{B}} \mathcal{B}(\Phi_{\mathcal{B}}) = \frac{4\pi\alpha^2 e_q^2}{3Q^2}$$

$$F_J^{(n+1)} \text{ soft, collinear } F_J^{(n)}.$$

$$\mathcal{R}(p_1, p_2, p_3) = \frac{8\pi C_F \alpha_s(\mu_R)}{Q^2} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)} \mathcal{B}(\Phi_{\mathcal{B}})$$

$$\mathrm{d}\Phi_{\mathcal{R}} = \mathrm{d}\Phi_{\mathcal{B}} \frac{Q^2}{16\pi^2} \, \mathrm{d}x_1 \, \mathrm{d}x_2 \Theta(1 - x_1)\Theta(1 - x_x)\Theta(x_1 + x_2 - 2)$$

$$-\frac{1}{2p_i p_j}\otimes\frac{\mathbf{T}_{ij}\cdot\mathbf{T}_k}{T_{ij}^2}V_{ij,k}$$

$$-\frac{1}{2p_i p_j}\otimes\frac{\mathbf{T}_{ij}\cdot\mathbf{T}_k}{T_{ij}^2}=\frac{1}{2p_i p_j}$$

$$\mathcal{D}_{13;2}(p_1,p_2,p_3)^{(\varepsilon=0)}=\mathcal{B}(\tilde{p}_{13},\tilde{p}_2)\otimes\left[-\frac{1}{2p_1p_3}V^{(\varepsilon=0)}_{q_1g_3,\bar{q}_2}\frac{\mathbf{T}_{13}\cdot\mathbf{T}_2}{T_{13}^2}\right]$$

$$=\mathcal{B}(\tilde{p}_{13},\tilde{p}_2)\otimes\frac{8\pi C_F\alpha_s(\mu_R)}{2p_1p_3}\left[\frac{2}{1-\tilde{z}_1\left(1-y_{13,2}\right)}-(1+\tilde{z}_1)\right]$$

$$\tilde{p}_2^\mu=\frac{1}{x_2}p_2^\mu \text{ and } \tilde{p}_{13}^\mu=Q^\mu-\tilde{p}_2^\mu=Q^\mu-\frac{1}{x_2}p_2^\mu,$$

$$y_{13,2}=1-x_2 \text{ and } \tilde{z}_1=\frac{1-x_3}{x_2}$$

$$\mathcal{D}_{13;2}(p_1,p_2,p_3)^{(\varepsilon=0)}=\mathcal{B}(\tilde{p}_{13},\tilde{p}_2)\cdot\frac{8\pi C_F\alpha_s(\mu_R)}{(1-x_2)Q^2}\left[\frac{2}{2-x_1-x_2}-1+\frac{1-x_1-x_2}{x_2}\right]$$

$$=\mathcal{B}(\tilde{p}_{13},\tilde{p}_2)\cdot\frac{8\pi C_F\alpha_s(\mu_R)}{Q^2}\left[\frac{1}{1-x_2}\left(\frac{2}{2-x_1-x_2}-1-x_1\right)+\frac{1-x_1}{x_2}\right]$$

$$\mathcal{S}(\Phi_{\mathcal{R}})=\mathcal{S}(p_1,p_2,p_3)=\mathcal{D}_{13;2}(p_1,p_2,p_3)^{(\varepsilon=0)}+\mathcal{D}_{23;1}(p_1,p_2,p_3)^{(\varepsilon=0)}$$

$$\mathrm{d}\sigma^{(R-S)}=\mathrm{d}\Phi_{\mathcal{R}}[\mathcal{R}(\Phi_{\mathcal{R}})-\mathcal{S}(\Phi_{\mathcal{R}})]$$

$$=\frac{C_F\alpha_s(\mu_R)}{2\pi}\int_0^1\mathrm{d}x_1\,\mathrm{d}x_2\left\{\mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}-\mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}(\tilde{p}_{13},\tilde{p}_2)\right.$$

$$\cdot\left[\frac{1}{1-x_2}\left(\frac{2}{2-x_1-x_2}-1-x_1\right)+\frac{1-x_1}{x_2}\right]-\mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}(\tilde{p}_{23},\tilde{p}_1)$$

$$\left.\cdot\left[\frac{1}{1-x_1}\left(\frac{2}{2-x_1-x_2}-1-x_2\right)+\frac{1-x_2}{x_1}\right]\right\}$$

$$\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}=\frac{1}{1-x_2}\left(\frac{2}{2-x_1-x_2}-1-x_1\right)+\{x_1\leftrightarrow x_2\}$$

$$\mathrm{d}\sigma^{(R-S)}=-\frac{C_F\alpha_s(\mu_R)}{4\pi}\,\mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}.$$

$$\mathrm{d}\sigma^{(V+I)}=\mathrm{d}\Phi_{\mathcal{B}}\left[\mathcal{V}(\Phi_{\mathcal{B}})+\mathcal{I}^{(S)}(\Phi_{\mathcal{B}};\varepsilon)\right]=\mathrm{d}\Phi_{\mathcal{B}}[\mathcal{V}(\Phi_{\mathcal{B}})+\mathcal{B}(\Phi_{\mathcal{B}})\otimes\mathbf{I}(\Phi_{\mathcal{B}};\varepsilon)]$$

$$\mathcal{I}^{(\mathcal{D})}_{qg;\bar{q}}(\Phi_{\mathcal{B}},\varepsilon)=-\frac{\mathbf{T}_q\cdot\mathbf{T}_{\bar{q}}}{\mathbf{T}_q^2}\frac{\alpha_s(\mu_R)}{2\pi\Gamma(1-\varepsilon)}\left(\frac{4\pi\mu_R^2}{\hat{s}}\right)^\varepsilon\mathcal{V}_{qg}(\varepsilon)$$

$$=\frac{C_F\alpha_s(\mu_R)}{2\pi\Gamma(1-\varepsilon)}\left(\frac{4\pi\mu_R^2}{\hat{s}}\right)^\varepsilon\left[\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}+5-\frac{\pi^2}{2}\right].$$

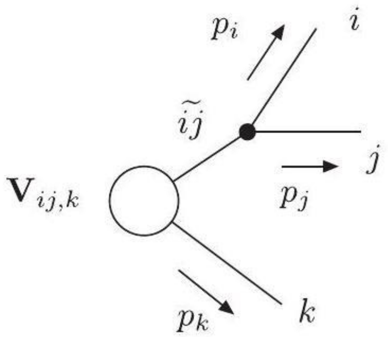
$$d\sigma^{(V+I)} = d\Phi_{\mathcal{B}}\mathcal{B}(\Phi_{\mathcal{B}})\left[\frac{C_F\alpha_s(\mu_R)}{2\pi\Gamma(1-\varepsilon)}\left(\frac{4\pi\mu_R^2}{Q^2}\right)^{\varepsilon}\left(-\frac{2}{\varepsilon^2}-\frac{3}{\varepsilon}-8+\pi^2 + \mathcal{O}(\varepsilon)\right)+\frac{C_F\alpha_s(\mu_R)}{2\pi\Gamma(1-\varepsilon)}\left(\frac{4\pi\mu_R^2}{Q^2}\right)^{\varepsilon}\left(\frac{2}{\varepsilon^2}+\frac{3}{\varepsilon}+10-\pi^2+\mathcal{O}(\varepsilon)\right)\right]$$

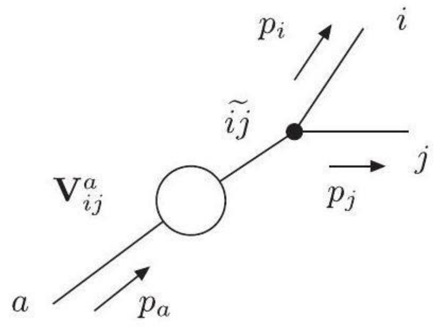
$$= d\Phi_{\mathcal{B}}\mathcal{B}(\Phi_{\mathcal{B}})\frac{C_F\alpha_s(\mu_R)}{\pi}$$

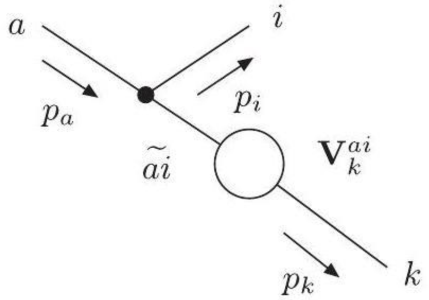
$$\sigma^{(\mathrm{NLO})}_{e^+e^-\to q\bar{q}} = \sigma^{(\mathrm{LO})}_{e^+e^-\to q\bar{q}}\left(1+\frac{3}{4}\frac{C_F\alpha_s(\mu_R)}{\pi}\right).$$

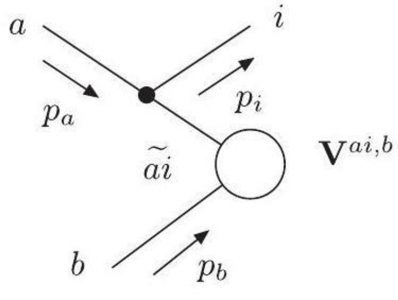
$$d\sigma^{(S)}(p_a, p_b; p_1, p_2, \ldots, p_{n+1})$$

$$= d\Phi_{\mathcal{R}}\left[\sum_{\substack{y\{ij\}\\k\neq i,j}} \mathcal{D}_{ij;k}(p_a, p_b; p_1, p_2, \ldots, p_{n+1}) + \sum_{\substack{\{ij\}\\a}} \mathcal{D}_{ij}^{a}(p_a, p_b; p_1, p_2, \ldots, p_{n+1}) + \sum_{\substack{\{aj\}\\k\neq j}} \mathcal{D}_{k}^{aj}(p_a, p_b; p_1, p_2, \ldots, p_{n+1}) + \sum_{\substack{\{aj\}\\b\neq a}} \mathcal{D}^{aj;b}(p_a, p_b; p_1, p_2, \ldots, p_{n+1})\right]$$









$$\mathcal{D}_{ij;k}(p_a,p_b;\{p_i\}) = \mathcal{B}\big(p_a,p_b;\{\dots,\tilde{p}_{ij},\dots,\tilde{p}_k,\dots\}\big) \otimes \left[-\frac{1}{2p_ip_j}V_{ij,k}\frac{\mathbf{T}_{ij}\cdot\mathbf{T}_k}{T_{ij}^2}\right]$$

$$\mathcal{D}_{ij}^{a}(p_a,p_b;\{p_i\}) = \mathcal{B}\big(\tilde{p}_a,p_b;\{\dots,\tilde{p}_{ij},\dots\}\big) \otimes \left[-\frac{1}{2p_ip_j}\frac{1}{x_{ij,a}}V_{ij}^{a}\frac{\mathbf{T}_{ij}\cdot\mathbf{T}_a}{T_{ij}^2}\right]$$

$$\mathcal{D}_{k}^{aj}(p_a,p_b;\{p_i\}) = \mathcal{B}\big(\tilde{p}_{aj},p_b;\{\dots,\tilde{p}_k,\dots\}\big) \otimes \left[-\frac{1}{2p_ap_j}\frac{1}{x_{jk,a}}V_{k}^{aj}\frac{\mathbf{T}_{aj}\cdot\mathbf{T}_k}{T_{aj}^2}\right]$$

$$\mathcal{D}^{aj;b}(p_a,p_b;\{p_i\}) = \mathcal{B}\big(\tilde{p}_{aj},\tilde{p}_b;\{\dots,\tilde{p}_k,\dots\}\big) \otimes \left[-\frac{1}{2p_ap_j}\frac{1}{x_{j,ab}}V^{aj,b}\frac{\mathbf{T}_{aj}\cdot\mathbf{T}_b}{T_{aj}^2}\right],$$

$$\mathrm{d}\sigma^{(I)} = \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}(\Phi_{\mathcal{B}}) \otimes \mathbf{I}(\Phi_{\mathcal{B}};\varepsilon),$$

$$\mathbf{I}(\Phi_{\mathcal{B}};\varepsilon) = \mathbf{I}_{FF}(\Phi_{\mathcal{B}};\varepsilon) + \mathbf{I}_{FI}(\Phi_{\mathcal{B}};\varepsilon) + \mathbf{I}_{IF}(\Phi_{\mathcal{B}};\varepsilon) + \mathbf{I}_{II}(\Phi_{\mathcal{B}};\varepsilon).$$

$$\mathbf{I}_{FF}(\Phi_{\mathcal{B}};\varepsilon) = -\frac{\alpha_s(\mu_R)}{2\pi\Gamma(1-\varepsilon)}\sum_{\{ij\}}\sum_{k\neq\{ij\}}\left[\left(\frac{4\pi\mu_R^2}{2p_{\{ij\}}p_k}\right)^{\varepsilon}\frac{\mathbf{T}_{\{ij\}}\cdot\mathbf{T}_k}{\mathbf{T}_{\{ij\}}^2}\mathcal{V}_{\{ij\}}(\varepsilon)\right]$$

$$\mathrm{d}\Phi_{\mathcal{B}} \xrightarrow{\text{FF}\to\text{FI,IF,II}} \mathrm{d}\xi\ \mathrm{d}\Phi_{\mathcal{B}}(\xi)$$
$$\mathcal{B}(\Phi_{\mathcal{B}}) \xrightarrow{\text{FF}\to\text{FI,IF,II}} \mathcal{B}(\Phi_{\mathcal{B}},\xi).$$

$$\mathrm{d}\sigma_a^{(C)} = \sum_{a'}\int_0^1 \mathrm{d}\xi\ \mathrm{d}\Phi_{\mathcal{B}}(\xi)\mathcal{B}(\Phi_{\mathcal{B}},\xi)\left[-\frac{1}{\varepsilon}\left(\frac{4\pi\mu_R^2}{\mu_F^2}\right)^{\varepsilon}\mathcal{P}_{a'a}^{(1)}(\xi) + K_{(\text{F.S.})}^{aa'}(\xi)\right]$$

$$K_{(\text{F.S.})}^{aa'}(\xi) \overset{\overline{\text{MS}}}{=} 0$$

$$\mathrm{d}\sigma^{(I+C)} = \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}_{ab}(p_a,p_b)\otimes\mathbf{I}(\varepsilon) + \sum_{a'}\int_0^1 \mathrm{d}\xi_a\ \mathrm{d}\Phi_{\mathcal{B}}(\xi_a)\mathcal{B}_{a'b}(\xi_ap_a,p_b)$$
$$\otimes\left[\mathbf{K}^{aa'}(\xi_a) + \mathbf{P}^{aa'}(\xi_ap_a,\xi_a;\mu_F^2)\right] + \sum_{b'}\int_0^1 \mathrm{d}\xi_b\ \mathrm{d}\Phi_{\mathcal{B}}(\xi_b)\mathcal{B}_{ab'}(p_a,\xi_bp_b)$$
$$\otimes\left[\mathbf{K}^{bb'}(\xi_b) + \mathbf{P}^{bb'}(\xi_bp_b,\xi_b;\mu_F^2)\right]$$

$$\mathbf{I}(\varepsilon) \equiv \mathbf{I}(p_a,p_b;p_1,\dots,p_m;\varepsilon)$$
$$= -\frac{\alpha_s(\mu_R)}{2\pi\Gamma(1-\varepsilon}\left\{\sum_i\frac{\mathcal{V}_i(\varepsilon)}{\mathbf{T}_i^2}\left[\sum_{k\neq i}\mathbf{T}_i\cdot\mathbf{T}_k\left(\frac{4\pi\mu_R^2}{2p_ip_k}\right)^{\varepsilon} + \sum_{c\in\{a,b\}}\mathbf{T}_i\right.\right.$$
$$\left.\left.\cdot\,\mathbf{T}_c\left(\frac{4\pi\mu_R^2}{2p_ip_c}\right)^{\varepsilon}\right] + \sum_{c\in\{a,b\}}\frac{\mathcal{V}_c(\varepsilon)}{\mathbf{T}_c^2}\left[\sum_i\mathbf{T}_c\cdot\mathbf{T}_i\left(\frac{4\pi\mu_R^2}{2p_cp_i}\right)^{\varepsilon} + \mathbf{T}_c\cdot\mathbf{T}_d\left(\frac{4\pi\mu_R^2}{2p_cp_d}\right)^{\varepsilon}\bigg|_{d\neq c}\right]\right\}$$

$$\mathbf{K}^{aa'}(\xi_a) = \frac{\alpha_s(\mu_R)}{2\pi}\Bigg\{\bar{K}^{aa'}(\xi_a)$$

$$+\delta^{aa'}\sum_{\{ij\}}\gamma_{\{ij\}}^{(1)}\frac{\mathbf{T}_{\{ij\}}\cdot\mathbf{T}_{a'}}{\mathbf{T}_{\{ij\}}^2}\left[\left(\frac{1}{1-\xi_a}\right)_+ + \delta(1-\xi_a)\right] - \frac{\mathbf{T}_b\cdot\mathbf{T}_{a'}}{\mathbf{T}_{a'}^2}\tilde{K}^{aa'}(\xi_a)$$

$$-K_{F.S.}^{aa'}(\xi_a)\Bigg\}$$

$$\mathbf{P}^{aa'}(\xi_a p_a, \xi_a;, \mu_F^2) = \frac{\alpha_s(\mu_R)}{2\pi}P_{aa'}^{(1)}(\xi_a)$$

$$\left[\sum_{\{ij\}}\frac{\mathbf{T}_{\{ij\}}\cdot\mathbf{T}_{a'}}{\mathbf{T}_{a'}^2}\log\frac{\mu_F^2}{2\xi_a p_a p_{\{ij\}}} + \frac{\mathbf{T}_b\cdot\mathbf{T}_{a'}}{\mathbf{T}_{a'}^2}\log\frac{\mu_F^2}{2\xi_a p_a p_b}\right],$$

$$\mathrm{d}\sigma^{(R-S)} = \mathrm{d}\Phi_{\mathcal{R}}[\mathcal{R}(\Phi_{\mathcal{R}}) - \mathcal{S}(\Phi_{\mathcal{R}})^{\varepsilon=0}]$$

$$\mathrm{d}\sigma^{(R)} = \mathrm{d}\Phi_{\mathcal{R}}\frac{2\pi C_F\alpha_s(\mu_R)}{x}\left|\mathcal{M}_{u\bar{d}\to W^+}^{(\mathrm{LO})}\right|^2\left[\left(\frac{1}{\hat{t}}+\frac{1}{\hat{u}}\right)\left(-\frac{2}{1-x}+x+1\right)-\frac{2x}{m_W^2}\right],$$

$$\mathrm{d}\Phi_{\mathcal{R}} = \frac{1}{2\hat{s}}\,\mathrm{d}x_u\,\mathrm{d}x_{\bar{d}} f_{u/h_1}(x_u,\mu_F) f_{\bar{d}/h_2}(x_{\bar{d}},\mu_F)\mathrm{d}\Phi_{Wg}$$

$$\mathrm{d}\sigma^{(S)} = \mathrm{d}\Phi_{\mathcal{R}}\frac{1}{4}\left|\mathcal{M}_{u\bar{d}\to W^+}^{(\mathrm{LO})}\right|^2$$

$$\times\left[-\frac{1}{2p_u p_g}\frac{1}{x_{g,u\bar{d}}}V^{ug,\bar{d}}\frac{\mathbf{T}_{ug}\cdot\mathbf{T}_{\bar{d}}}{\mathbf{T}_{ug}^2} - \frac{1}{2p_{\bar{d}}p_g}\frac{1}{x_{g,\bar{d}u}}V^{\bar{d}g,u}\frac{\mathbf{T}_{\bar{d}g}\cdot\mathbf{T}_u}{\mathbf{T}_{\bar{d}g}^2}\right].$$

$$x_{g,u\bar{d}} = \frac{p_u p_{\bar{d}} - p_g(p_u+p_{\bar{d}})}{p_u p_{\bar{d}}} = \frac{\hat{s}+\hat{t}+\hat{u}}{\hat{s}} = \frac{m_W^2}{\hat{s}} = x_{g,\bar{d}u} \stackrel{!}{=} x.$$

$$V^{ug,\bar{d}} = 8\pi C_F\alpha_s(\mu_R)\left[\frac{2}{1-x}-(1+x)\right].$$

$$\mathrm{d}\sigma^{(S)} = \mathrm{d}\Phi_{\mathcal{R}}\left[\frac{8\pi C_F\alpha_s(\mu_R)}{x}\frac{1}{4}\left|\mathcal{M}_{u\bar{d}\to W^+}^{(\mathrm{LO})}\right|^2\left(\frac{1}{\hat{t}}+\frac{1}{\hat{u}}\right)\left(-\frac{2}{1-x}+(1+x)\right)\right]$$

$$\mathrm{d}\sigma^{(R-S)} = \mathrm{d}\Phi_{\mathcal{R}}\left|\mathcal{M}_{u\bar{d}\to W^+}^{(\mathrm{LO})}\right|^2\frac{2\pi C_F\alpha_s(\mu_R)}{x}\Bigg\{\left[\frac{1}{\hat{t}}+\frac{1}{\hat{u}}\right]\left[-\frac{2}{1-x}+(1+x)\right]$$

$$-\frac{2x}{m_W^2}-\left[\frac{1}{\hat{t}}+\frac{1}{\hat{u}}\right]\left[-\frac{2}{1-x}+(1+x)\right]\Bigg\} = -\mathrm{d}\Phi_{\mathcal{R}}\frac{4\pi C_F\alpha_s(\mu_R)}{m_W^2}\left|\mathcal{M}_{u\bar{d}\to W^+}^{(\mathrm{LO})}\right|^2$$

$$\mathbf{I}_{u\bar{d}\to W}(\varepsilon) = \mathbf{I}_{u\bar{d}\to W}(p_u,p_{\bar{d}};\varepsilon) = -\frac{\alpha_s(\mu_R)}{2\pi\Gamma(1-\varepsilon)}\left(\frac{4\pi\mu_R^2}{2p_u p_{\bar{d}}}\right)^{\varepsilon}\left\{\frac{\mathbf{T}_u\cdot\mathbf{T}_{\bar{d}}}{\mathbf{T}_u^2}\mathcal{V}_u(\varepsilon)+\frac{\mathbf{T}_{\bar{d}}\cdot\mathbf{T}_u}{\mathbf{T}_{\bar{d}}^2}\mathcal{V}_{\bar{d}}(\varepsilon)\right\}$$

$$= \frac{C_F\alpha_s(\mu_R)}{\pi}c_\Gamma\left(\frac{\mu_R^2}{m_W^2}\right)^{\varepsilon}\left[\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}+5-\frac{1-a^{\mathrm{CDR}}}{2}-\frac{\pi^2}{2}\right]$$

$$\mathrm{d}\sigma^{(C)} = \int_0^1 \mathrm{d}\xi_u\, \mathrm{d}\Phi_{\mathcal{B}}(\xi_u)\mathcal{B}_{u\bar{d}\to W}(\xi_u p_u, p_{\bar{d}}) \otimes [\mathbf{K}^{qq}(\xi_u) + \mathbf{P}^{qq}(\xi_u p_u, \xi_u; \mu_F^2)]$$
$$+ \int_0^1 \mathrm{d}\xi_{\bar{d}}\, \mathrm{d}\Phi_{\mathcal{B}}(\xi_{\bar{d}})\mathcal{B}_{u\bar{d}\to W}(p_u, \xi_{\bar{d}} p_{\bar{d}}) \otimes [\mathbf{K}^{qq}(\xi_{\bar{d}}) + \mathbf{P}^{qq}(\xi_{\bar{d}} p_{\bar{d}}, \xi_{\bar{d}};, \mu_F^2)].$$

$$\bar{K}^{qq}(\xi) = C_F\left[\left(\frac{2}{1-\xi}\log\frac{1-\xi}{\xi}\right)_+ - (1+\xi)\log\frac{1-\xi}{\xi} + (1-\xi) - \delta(1-\xi)(5-\pi^2)\right]$$

$$\tilde{K}^{qq}(\xi) = C_F\left[\left(\frac{2}{1-\xi}\log(1-\xi)\right)_+ - (1+\xi)\log(1-\xi) - \frac{\pi^2}{3}\delta(1-\xi)\right].$$

$$\mathrm{d}\sigma_u^{(C)} = \int_0^1 \mathrm{d}x_u\, \mathrm{d}x_{\bar{d}} \int_{x_u}^1 \mathrm{d}\xi_u f_{u/h_1}\left(\frac{x_u}{\xi_u}, \mu_F^2\right) f_{\bar{d}/h_1}(x_{\bar{d}}, \mu_F^2)\frac{\pi\delta(x_u x_{\bar{d}} s - m_W^2)}{m_W^2}$$
$$\cdot \left|\mathcal{M}^{(\mathrm{LO})}_{u\bar{d}\to W^+}\right|^2 \frac{\alpha_s(\mu_R)}{2\pi}\left\{\bar{K}^{qq}(\xi_u) + \left[\tilde{K}^{qq}(\xi_u) - P_{qq}^{(1)}(\xi_u)\log\frac{\mu_F^2}{m_W^2}\right]\right\}$$
$$= \int_0^1 \mathrm{d}x_u\, \mathrm{d}x_{\bar{d}} \int_{x_u}^1 \mathrm{d}\xi_u f_{u/h_1}\left(\frac{x_u}{\xi_u}, \mu_F^2\right) f_{\bar{d}/h_1}(x_{\bar{d}}, \mu_F^2)\frac{\pi\delta(x_u x_{\bar{d}} s - m_W^2)}{m_W^2}$$
$$\cdot \left|\mathcal{M}^{(\mathrm{LO})}_{u\bar{d}\to W^+}\right|^2 \frac{\alpha_s(\mu_R)C_F}{2\pi}\left\{\left[\frac{2}{1-\xi_u}\left(\log\frac{1-\xi_u}{\xi_u} + \log(1-\xi_u) - \log\frac{\mu_F^2}{m_W^2}\right)\right]_+\right.$$
$$- (1+\xi_u)\left(\log\frac{1-\xi_u}{\xi_u} + \log(1-\xi_u) - \log\frac{\mu_F^2}{m_W^2}\right) + (1-\xi_u)$$
$$\left. - \delta(1-\xi_u)\left(5 - \frac{2\pi^2}{3} + \frac{3}{2}\log\frac{\mu_F^2}{m_W^2}\right)\right\}$$

$$\mathrm{d}\sigma^{(V+I+C)}_{u\bar{d}\to W^+}(p_u, p_{\bar{d}}) = \frac{C_F\alpha_s(\mu_R)}{2\pi}\left|\mathcal{M}^{(\mathrm{LO})}_{u\bar{d}\to W^+}\right|^2 \int_0^1 \mathrm{d}x_u\, \mathrm{d}x_{\bar{d}} \frac{\delta(x_u x_{\bar{d}} s - m_W^2)}{m_W^2}$$
$$\int_{x_u}^1 \mathrm{d}\xi_u \int_{x_d}^1 \mathrm{d}\xi_d f_{\frac{u}{h_1}}\left(\frac{x_u}{\xi_u}, \mu_F^2\right) f_{\frac{\bar{d}}{h_1}}\left(\frac{x_{\bar{d}}}{\xi_{\bar{d}}}, \mu_F^2\right) \times \left\{\left[c_\Gamma\left(\frac{\mu_R^2}{m_W^2}\right)^\varepsilon\left(-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 7 - a^{\mathrm{CDR}} + \pi^2\right)\right]\delta(1-\xi_u)\delta(1-\xi_{\bar{d}})\right.$$
$$+ \left[c_\Gamma\left(\frac{\mu_R^2}{m_W^2}\right)^\varepsilon\left(+\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + 9 + a^{\mathrm{CDR}} - \pi^2\right) + 2\left(\frac{2\pi^2}{3} - 5 - \frac{3}{2}\log\frac{\mu_F^2}{m_W^2}\right)\right]\delta(1-\xi_u)\delta(1-\xi_{\bar{d}})$$
$$+ \left[\left(\frac{2}{1-\xi_u}\left(\log\frac{(1-\xi_u)^2}{\xi_u} - \log\frac{\mu_F^2}{m_W^2}\right)\right)_+ - (1+\xi_u)\left(\log\frac{(1-\xi_u)^2}{\xi_u} - \log\frac{\mu_F^2}{m_W^2}\right) + (1-\xi_u)\right]\delta(1-\xi_{\bar{d}})$$
$$\left. + \left[\left(\frac{2}{1-\xi_{\bar{d}}}\left(\log\frac{(1-\xi_{\bar{d}})^2}{\xi_{\bar{d}}} - \log\frac{\mu_F^2}{m_W^2}\right)\right) - (1+\xi_{\bar{d}})\left(\log\frac{(1-\xi_{\bar{d}})^2}{\xi_{\bar{d}}} - \log\frac{\mu_F^2}{m_W^2}\right) + (1-\xi_{\bar{d}})\right]\delta(1-\xi_u)\right\}$$

$$c_\Gamma\left(\frac{\mu_R^2}{m_W^2}\right)^\varepsilon \longrightarrow 1$$

$$d\sigma^{(V+I+C)}_{u\bar{d}\to W^+}(p_u, p_{\bar{d}}) = \frac{C_F \alpha_s(\mu_R)}{2\pi} \left|\mathcal{M}^{(\mathrm{LO})}_{u\bar{d}\to W^+}\right|^2 \int_0^1 dx_u \, dx_{\bar{d}} \frac{\delta(x_u x_{\bar{d}} s - m_W^2)}{m_W^2}$$

$$\int_{x_u}^1 d\xi_u \int_{x_{\bar{d}}}^1 d\xi_{\bar{d}} f_{\frac{u}{h_1}}\left(\frac{x_u}{\xi_u}, \mu_F^2\right) f_{\frac{\bar{d}}{h_1}}\left(\frac{x_{\bar{d}}}{\xi_{\bar{d}}}, \mu_F^2\right)$$

$$\times \left\{ \left[\frac{4\pi^2}{3} - 8 + 3\log\frac{m_W^2}{\mu_F^2}\right] \delta(1-\xi_u)\delta(1-\xi_{\bar{d}}) + \left[\left(\frac{2}{1-\xi_u}\log\frac{(1-\xi_u)^2 m_W^2}{\xi_u \mu_F^2}\right)_+ \right.\right.$$

$$\left. -(1+\xi_u)\left(\log\frac{(1-\xi_u)^2 m_W^2}{\xi_u \mu_F^2}\right) + (1-\xi_u)\right] \delta(1-\xi_{\bar{d}})$$

$$+ \left[\left(\frac{2}{1-\xi_{\bar{d}}}\log\frac{(1-\xi_{\bar{d}})^2 m_W^2}{\xi_{\bar{d}} \mu_F^2}\right)_+ -(1+\xi_{\bar{d}})\left(\log\frac{(1-\xi_{\bar{d}})^2 m_W^2}{\xi_{\bar{d}} \mu_F^2}\right) + (1-\xi_{\bar{d}})\right] \delta(1$$

$$\left. - \xi_u) \right\}$$

$$\sigma^{(\mathrm{NLO})} = \int d\Phi_{\mathcal{B}} \left[\mathcal{B}_n(\Phi_{\mathcal{B}}; \mu_F, \mu_R) + \mathcal{V}_n(\Phi_{\mathcal{B}}; \mu_F, \mu_R) + \mathcal{I}_n^{(\mathcal{S})}(\Phi_{\mathcal{B}}; \mu_F, \mu_R)\right]$$

$$+ \int d\Phi_{\mathcal{R}} [\mathcal{R}_n(\Phi_{\mathcal{R}}; \mu_F, \mu_R) - \mathcal{S}_n(\Phi_{\mathcal{R}}; \mu_F, \mu_R)]$$

$$\int_0^1 dz [f(z)]_+ g(z) = \int_0^1 dz f(z)[g(z) - g(1)]$$

$$\mathcal{D}'_{ij;k} = \mathcal{D}_{ij;k}\Theta(\alpha - y_{ij;k})$$

23. Loops y predicciones NNLO.

$$\Re\mathrm{e}[\mathcal{A}^{2-\mathrm{loop}}(Zq\bar{q}g) \times \mathcal{A}^{\mathrm{tree}}(Zq\bar{q}g)^*].$$

$$|\mathcal{A}^{1-\mathrm{loop}}(Zq\bar{q}g)|^2.$$

$$\Re\mathrm{e}[\mathcal{A}^{1-\mathrm{loop}}(Zq\bar{q}gg) \times \mathcal{A}^{\mathrm{tree}}(Zq\bar{q}gg)^*].$$

$$|\mathcal{A}^{\mathrm{tree}}(Zq\bar{q}ggg)|^2.$$

$$d\hat{\sigma}_{ij}^{\mathrm{NNLO}} = \int \Phi_1 \left[d\hat{\sigma}_{ij}^{(a)} + \hat{\sigma}_{ij}^{(b)} - \hat{\sigma}_{ij}^{C_1}\right] + \int \Phi_2 \left[d\hat{\sigma}_{ij}^{(c)} - \hat{\sigma}_{ij}^{C_2}\right] + \int \Phi_3 \left[d\hat{\sigma}_{ij}^{(d)} - \hat{\sigma}_{ij}^{C_3}\right]$$

$$\frac{d\sigma^{NNLO}_{\mathrm{LoopSim}}}{dA} = \frac{d\sigma^{NNLO}}{dA}\left[1 + \mathcal{O}\left(\frac{\alpha_s^2}{K^{NNLO}(A)}\right)\right]$$

$$\hat{\sigma}_{ij}(m_H^2, \hat{s}) \propto \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^k \eta_{ij}^{(k)}(z)$$

$$\hat{\sigma}_{\text{EW real}} \sim \frac{\alpha_{\text{w}}}{4\pi}\log^2\left(\frac{s}{m_W^2}\right)\hat{\sigma}_0$$

$$\hat{\sigma}_{\text{EW virtual}} \sim -\frac{\alpha_{\text{w}}}{4\pi}\log^2\left(\frac{s}{m_W^2}\right)\hat{\sigma}_0$$

$$\delta^{\text{EW}} = \frac{\hat{\sigma}^{\text{EW virtual}}}{\hat{\sigma}_0} = -(\text{ constant })\frac{\alpha_{\text{w}}}{4\pi}\log^2\left(\frac{s}{m_W^2}\right).$$

24. Producción Dijet.

$$\sigma_{2-\text{jet}} = \frac{1}{2s}\sum_{a,b,c,d}\int_0^1 \frac{\mathrm{d}x_a}{x_a}\frac{\mathrm{d}x_b}{x_b} f_{a/h_1}(x_a,\mu_F) f_{b/h_2}(x_b,\mu_F)\int \mathrm{d}\Phi_n |\mathcal{M}_{ab\to cd}|^2$$

$$\left|\mathcal{M}_{qq'\to qq'}\right|^2 = \frac{1}{4N^2} m^{(0)}_{q\bar{q}'\to q\bar{q}'} = \frac{V}{2N^2}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right)$$

$$\left|\mathcal{M}_{qg\to qg}\right|^2 = \frac{1}{4NV} m^{(0)}_{qg\to qg} = \frac{-1}{2N^2}\left(\frac{V}{\hat{u}\hat{s}} - \frac{2N^2}{\hat{t}^2}\right)(\hat{s}^2+\hat{u}^2)$$

$$\left|\mathcal{M}_{q\bar{q}\to gg}\right|^2 = \frac{1}{4N^2} m^{(0)}_{q\bar{q}\to gg} = \frac{V}{2N^3}\left(\frac{V}{\hat{u}\hat{t}} - \frac{2N^2}{\hat{s}^2}\right)(\hat{t}^2+\hat{u}^2)$$

$$\left|\mathcal{M}_{gg\to gg}\right|^2 = \frac{1}{4V^2} m^{(0)}_{gg\to gg} = \frac{2N^2}{V}\left(3 - \frac{\hat{u}\hat{t}}{\hat{s}^2} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2}\right)$$

$$\mathrm{d}\Phi_2 = \frac{p_\perp \mathrm{d}p_\perp \mathrm{d}\eta\,\mathrm{d}\phi}{2(2\pi)^3}(2\pi)\delta((p_1+p_2-p_3)^2)$$

$$p_3 = p_\perp(\cosh\,\eta, \sin\,\phi, \cos\,\phi, \sinh\,\eta)$$

$$\mathrm{d}\Phi_2 = \frac{1}{4\pi}\frac{p_\perp^2}{\hat{s}}\,\mathrm{d}\eta$$

$$p_4 = p_\perp(\cosh\,\eta', -\sin\,\phi, -\cos\,\phi, \sinh\,\eta')$$

$$\sqrt{\hat{s}} = 2p_\perp\cosh\left(\frac{\eta_3-\eta_4}{2}\right) = p_\perp\left(\frac{\chi+1}{\sqrt{\chi}}\right)$$

$$\hat{t} = -\frac{\hat{s}}{2}(1-\cos\,\theta) = -\frac{\hat{s}}{\chi+1}$$

$$\hat{u} = -\frac{\hat{s}}{2}(1+\cos\,\theta) = -\frac{\hat{s}\chi}{\chi+1}$$

$$\chi = \frac{1+\cos\,\theta}{1-\cos\,\theta}$$

$$\mathrm{d}\Phi_2 = \frac{1}{4\pi}\frac{\mathrm{d}\chi}{(\chi+1)^2}$$

$$\mathrm{d}\Phi_2\left|\mathcal{M}_{q\bar{q}'\to q\bar{q}'}\right|^2 = \frac{1}{4\pi}\frac{V}{2N^2}\,\mathrm{d}\chi\left[1+\left(\frac{\chi}{\chi+1}\right)^2\right]$$

$$\mathrm{d}\Phi_2\left|\mathcal{M}_{qg\to qg}\right|^2 = \frac{1}{4\pi}\frac{1}{2N^2}\,\mathrm{d}\chi$$

$$\times\left(V\left[\frac{1}{\chi(\chi+1)}+\frac{\chi}{(\chi+1)^3}\right]+2N^2\left[1+\left(\frac{\chi}{\chi+1}\right)^2\right]\right)$$

$$\mathrm{d}\Phi_2\left|\mathcal{M}_{q\bar{q}\to gg}\right|^2 = \frac{1}{4\pi}\frac{V}{2N^3}\frac{\mathrm{d}\chi}{(\chi+1)^2}\left[V\left(\chi+\frac{1}{\chi}\right)-2N^2\frac{1+\chi^2}{(\chi+1)^2}\right]$$

$$\mathrm{d}\Phi_2\left|\mathcal{M}_{gg\to gg}\right|^2 = \frac{1}{4\pi}\frac{2N^2}{V}\,\mathrm{d}\frac{(1+\chi+\chi^2)^3}{\chi^2(\chi+1)^4}$$

$$\mathcal{L}_{\text{contact}} = \frac{2\pi}{\Lambda^2}(\bar{\psi}_L\gamma^\mu\psi_L)(\bar{\psi}_L\gamma_\mu\psi_L)$$

$$\mathrm{d}\Phi_2\left|\mathcal{M}_{q\bar{q}'\to q\bar{q}'}\right|^2 = \frac{\mathrm{d}\chi}{4\pi}\left\{\frac{V}{2N^2}\left[1+\left(\frac{\chi}{\chi+1}\right)^2\right]+\left(\frac{\hat{s}^2}{\alpha_s^2\Lambda^4}\right)\frac{\chi^2}{(\chi+1)^4}\right\}$$

$$\frac{\sqrt{s}}{2}(x_1+x_2) = p_\perp(\cosh\,\eta+\cosh\,\eta')$$
$$\frac{\sqrt{s}}{2}(x_1-x_2) = p_\perp(\sinh\,\eta+\sinh\,\eta')$$

$$x_1 = \frac{p_\perp}{\sqrt{s}}\left(e^{\eta}+e^{\eta'}\right), x_2 = \frac{p_\perp}{\sqrt{s}}\left(e^{-\eta}+e^{-\eta'}\right)$$

$$m_{ab\to cd} = m_{ab\to cd}^{(0)}+\left(\frac{\alpha_S}{2\pi}\right)m_{ab\to cd}^{(v)}+\cdots$$

$$m_{qq'\to qq'}^{(v)} = c_\Gamma\left\{\left[C_F\left(-\frac{4}{\varepsilon^2}-\frac{1}{\varepsilon}(6+8l(s)-8l(u)-4l(t))\right)\right.\right.$$
$$+\frac{N_c}{\varepsilon}(4l(s)-2l(u)-2l(t))+n_fT_R\left(\frac{4}{3}(l(t)-l(-\mu^2))-\frac{20}{9}\right)$$
$$+\left(C_F\left(-16-2l^2(t)+l(t)(6+8l(s)-8l(u)\right)\right)$$
$$\left.+N_c\left(\frac{85}{9}+\pi^2+2l(t)(l(t)+l(u)-2l(s))+\frac{11}{3}(l(-\mu^2)-l(t))\right)\right]m_{qq'\to qq'}^{(0)}$$
$$-4VC_F\frac{s^2-u^2}{t^2}(2\pi^2+2l^2(t)+l^2(s)+l^2(u)-2l(s)l(t)-2l(t)l(u))$$
$$+N_cV\left(\frac{s^2-u^2}{t^2}(3\pi^2+3l^2(t)+2l^2(s)+l^2(u)-4l(s)l(t)-2l(t)l(u))\right)$$
$$+4V\left(C_F\left((l(u)-l(s))-\frac{u-s}{t}(2l(t)-l(s)\right.\right.$$
$$\left.\left.\left.-l(u))\right)+N_c\left(-\frac{s}{2t}(l(t)-l(u))+\frac{u}{t}(l(t)-l(s))\right)\right)\right\}$$

$$l(x) = \log\left(-\frac{x}{Q^2}\right)$$

$$l^2(t) = \log^2\left(-\frac{t}{Q^2}\right) \to \log^2\left(\frac{t}{Q^2}\right) - \pi^2 \;\; \text{if } t > 0.$$

$$m^{(v)}_{q\bar{q}\to gg} = c_\Gamma \Bigg\{ \Bigg[C_F\left(-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 7\right)$$

$$+ N_c\left(-\frac{2}{\varepsilon^2} - \frac{11}{3\varepsilon} + \frac{11}{3} l(-\mu^2)\right) + n_f T_R\left(\frac{4}{3\varepsilon} - \frac{4}{3} l(-\mu^2)\right)\Bigg] m^{(0)}_{q\bar{q}\to gg}$$

$$+ \frac{l(s)}{\varepsilon}\left[\left(2N_c^2 V + \frac{2V}{N_c^2}\right)\frac{t^2+u^2}{ut} - 4V^2\frac{t^2+u^2}{s^2}\right]$$

$$+ \frac{4N_c^2 V}{\varepsilon}\left[l(t)\left(\frac{u}{t} - \frac{2u^2}{s^2}\right) + l(u)\left(\frac{t}{u} - \frac{2t^2}{s^2}\right)\right] - \frac{4V}{\varepsilon}\left(\frac{u}{t} + \frac{t}{u}\right)\left(l(t) + l(u)\right)\Bigg\}$$

$$+ f_1(s,t,u) + f_1(s,u,t)$$

$$f_1(s,t,u) = 4N_c V\{\frac{l(t)l(u)}{N_c}\frac{t^2+u^2}{2tu} + l^2(s)\left[\frac{1}{4N^3}\frac{s^2}{tu} + \frac{1}{4N_c}\left(\frac{1}{2} + \frac{t^2+u^2}{tu} - \frac{t^2+u^2}{s^2}\right) - \frac{N_c}{4}\frac{t^2+u^2}{s^2}\right]$$

$$+ l(s)\left[\left(\frac{5}{8}\frac{V}{N_c} - \frac{1}{2N_c} - \frac{1}{N_c^3}\right) - \left(N_c + \frac{1}{N_c^3}\right)\frac{t^2+u^2}{2tu} - \frac{V}{4N_c}\frac{t^2+u^2}{s^2}\right]$$

$$+ \pi^2\left[\frac{1}{8N_c} + \frac{1}{N_c^3}\left(\frac{3(t^2+u^2)}{8tu} + \frac{1}{2}\right) + N_c\left(\frac{t^2+u^2}{8tu} - \frac{t^2+u^2}{2s^2}\right)\right]$$

$$+ \left(N_c + \frac{1}{N_c}\right)\left(\frac{1}{8} - \frac{t^2+u^2}{4s^2}\right)$$

$$+ l^2(t)\left[N_c\left(\frac{s}{4t} - \frac{u}{s} - \frac{1}{4}\right) + \frac{1}{N_c}\left(\frac{t}{2u} - \frac{u}{4s}\right) + \frac{1}{N_c^3}\left(\frac{u}{4t} - \frac{s}{2u}\right)\right]$$

$$+ l(t)\left[N_c\left(\frac{t^2+u^2}{s^2} + \frac{3t}{4s} - \frac{5u}{4t} - \frac{1}{4}\right) - \frac{1}{N_c}\left(\frac{u}{4s} + \frac{2s}{u} + \frac{s}{2t}\right)\right.$$

$$\left. - \frac{1}{N_c^3}\left(\frac{3s}{4t} + \frac{1}{4}\right)\right] + l(s)l(t)\left[N_c\left(\frac{t^2+u^2}{s^2} - \frac{u}{2t}\right) + \frac{1}{N_c}\left(\frac{u}{2s} - \frac{t}{u}\right) + \frac{1}{N_c^3}\left(\frac{s}{u} - \frac{u}{2t}\right)\right]\}$$

$$m^{(v)}_{gg\to gg} = c_\Gamma\left\{\left[-\frac{4N_c}{\varepsilon^2} - \frac{22N_c}{3\varepsilon} + \frac{8n_fT_R}{3\varepsilon} - \frac{67N_c}{9}\right.\right.$$

$$\left. + \frac{20n_fT_R}{9} + N_c\pi^2 + \frac{11N_c}{3}l(-\mu^2) - \frac{4n_fT_R}{3}l(-\mu^2)\right]m^{(0)}_{gg\to gg}$$

$$+ \frac{16VN_c^3}{\varepsilon}\left[l(s)\left(3 - \frac{2tu}{s^2} + \frac{t^4+u^4}{t^2u^2}\right)\right.$$

$$\left.\left. + l(t)\left(3 - \frac{2us}{t^2} + \frac{u^4+s^4}{u^2s^2}\right) + l(u)\left(3 - \frac{2st}{u^2} + \frac{s^4+t^4}{s^2t^2}\right)\right]\right\}$$

$$+ 4VN_c^2[f_2(s,t,u) + f_2(t,u,s) + f_2(u,s,t)]$$

$$f_2(s,t,u) = N_c\left\{\left(\frac{2(t^2+u^2)}{tu}\right)l^2(s)\right.$$

$$+ \left(\frac{4s(t^3+u^3)}{t^2u^2} - 6\right)l(t)l(u) + \left[\frac{4}{3}\frac{tu}{s^2} - \frac{14}{3}\frac{t^2+u^2}{tu} - 14 - 8\left(\frac{t^2}{u^2} + \frac{u^2}{t^2}\right)\right]l(s) - 1$$

$$\left. - \pi^2\right\}$$

$$+ n_fT_R\left\{\left(\frac{10}{3}\frac{t^2+u^2}{tu} + \frac{16}{3}\frac{tu}{s^2} - 2\right)l(s) - \frac{s^2+tu}{tu}l^2(s) - \frac{2(t^2+u^2)}{tu}l(t)l(u) + 2\right.$$

$$\left. - \pi^2\right\}$$

$$\left(\frac{11N_c}{3} - \frac{4}{3}n_fT_R\right)l(-\mu^2) = \beta_0 l(-\mu^2).$$

$$R_n = \frac{\sigma(n + 1\text{jet})}{\sigma(n\text{jet})}$$

$$0 \to \bar{q}^+(p_1) + q^-(p_2) + g^-(p_3) + g^+(p_4).$$

$$\mathcal{M}(\bar{q}_1^+, q_2^-, g_3^-, g_4^+) = ig^2\left[(T^{a_3}T^{a_4})_{i_1i_2}M(\bar{q}_1^+, q_2^-, g_3^-, g_4^+) + (T^{a_4}T^{a_3})_{i_1i_2}M(\bar{q}_1^+, q_2^-, g_4^+, g_3^-)\right]$$

$$[T^{a_3}, T^{a_4}]_{i_1i_2} = if^{a_3a_4b}T^b_{i_1i_2}$$

$$M(\bar{q}_1^+, q_2^-, g_3^-, g_4^+) = \frac{\langle 13\rangle\langle 23\rangle^3}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

$$M(\bar{q}_1^+, q_2^-, g_4^+, g_3^-) = -\frac{\langle 13\rangle\langle 23\rangle^3}{\langle 12\rangle\langle 24\rangle\langle 34\rangle\langle 31\rangle}$$

$$0 \to \bar{q}^+(p_1) + q^-(p_2) + \gamma^-(p_3) + g^+(p_4).$$

$$\mathcal{M}(\bar{q}_1^+, q_2^-, \gamma_3^-, g_4^+) = ieQ_qg(T^{a_4})_{i_1i_2}[M(\bar{q}_1^+, q_2^-, g_3^-, g_4^+) + M(\bar{q}_1^+, q_2^-, g_4^+, g_3^-)] \equiv ieQ_qg(T^{a_4})_{i_1i_2}M(\bar{q}_1^+, q_2^-, \gamma_3^-, g_4^+)$$

$$M(\bar{q}_1^+, q_2^-, \gamma_3^-, g_4^+) = \frac{\langle 13\rangle\langle 23\rangle^3}{\langle 12\rangle\langle 23\rangle\langle 24\rangle\langle 34\rangle\langle 31\rangle\langle 41\rangle}(\langle 24\rangle\langle 31\rangle - \langle 23\rangle\langle 41\rangle) = \frac{\langle 13\rangle\langle 23\rangle^3}{\langle 23\rangle\langle 24\rangle\langle 31\rangle\langle 41\rangle}$$

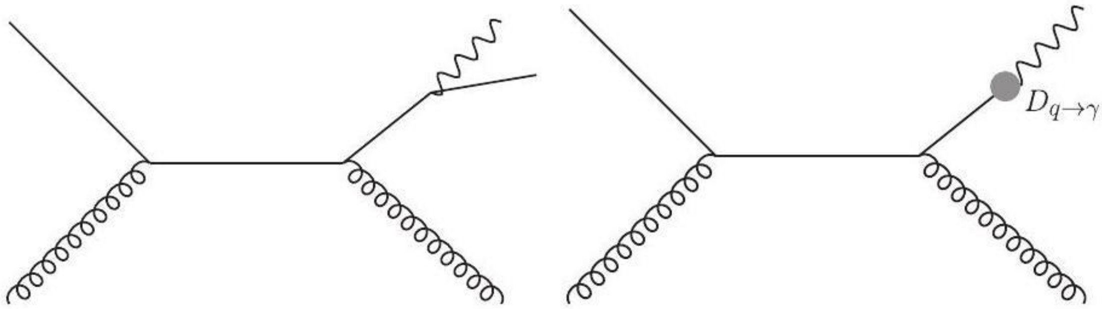


Figura 12. Trayectoria de colisión de una partícula repercutida por efecto gravitacional cuántico de una partícula supermasiva.

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\gamma+X}(M_F) + \sum_i \mathrm{d}\sigma_{i+X} \otimes D_{i\to\gamma}(M_F)$$

$$q + \bar{q} \to \gamma\gamma$$

$$\mathcal{M}_n = g^n \left[M_n^{(0)} + g^2 M_n^{(1)} + g^4 M_n^{(2)} + \cdots\right]$$

$$p_T^{\gamma_1} > 40\mathrm{GeV}, p_T^{\gamma_2} > 40 + \delta\mathrm{GeV}$$

$$0 \to q^+(p_1) + g^+(p_2) + \bar{q}^-(p_3) + \bar{\ell}^-(p_4) + \ell^+(p_5),$$

$$A^{\mathrm{LO}} = 2e^2 g T_{i_1 i_3}^{a_2} A^{\mathrm{tree}}$$

$$A^{\mathrm{tree}} = -i\frac{\langle 34\rangle^2}{\langle 12\rangle\langle 23\rangle\langle 45\rangle}.$$

$$A^{1-\mathrm{loop}} = 2e^2 g\left(\frac{\alpha_S N_c}{4\pi}\right) T_{i_1 i_3}^{a_2}\left(A^{\mathrm{lc}} + \frac{1}{N_c^2} A^{\mathrm{slc}}\right).$$

$$A^{\mathrm{lc}} = c_\Gamma A^{\mathrm{tree}} \left\{ -\frac{1}{\varepsilon^2}\left(\frac{\mu^2}{-s_{12}}\right)^{\varepsilon} - \frac{1}{\varepsilon^2}\left(\frac{\mu^2}{-s_{23}}\right)^{\varepsilon} - \frac{3}{2\varepsilon}\left(\frac{\mu^2}{-s_{23}}\right)^{\varepsilon} - 3 \right\}$$

$$+ i\left\{ \frac{\langle 34\rangle^2}{\langle 12\rangle\langle 23\rangle\langle 45\rangle} \mathrm{Ls}_{-1}\left(\frac{-s_{12}}{-s_{45}}, \frac{-s_{23}}{-s_{45}}\right) - \frac{\langle 34\rangle\langle 13\rangle[15]}{\langle 12\rangle\langle 23\rangle} \frac{\mathrm{L}_0\left(\frac{-s_{23}}{-s_{45}}\right)}{s_{45}} \right.$$

$$\left. + \frac{1}{2}\frac{\langle 13\rangle^2[15]^2\langle 45\rangle}{\langle 12\rangle\langle 23\rangle} \frac{\mathrm{L}_1\left(\frac{-s_{23}}{-s_{45}}\right)}{s_{45}^2} \right\}$$

$$\mathrm{L}_0(x) = \frac{\ln(x)}{1-x}, \ \mathrm{L}_1(x) = \frac{\mathrm{L}_0(x)+1}{1-x}$$

$$\mathrm{Ls}_{-1}(x,y) = \mathrm{Li}_2(1-x) + \mathrm{Li}_2(1-y) + \ln x \ln y - \frac{\pi^2}{6},$$

$$A^{\mathrm{slc}} = -c_\Gamma A^{\mathrm{tree}} \left\{ -\frac{1}{\varepsilon^2}\left(\frac{\mu^2}{-s_{13}}\right)^{\varepsilon} - \frac{3}{2\varepsilon}\left(\frac{\mu^2}{-s_{45}}\right)^{\varepsilon} - \frac{7}{2} \right\} + i\left\{ -\frac{\langle 34\rangle^2}{\langle 32\rangle\langle 21\rangle\langle 45\rangle} \mathrm{Ls}_{-1}\left(\frac{-s_{13}}{-s_{45}}, \frac{-s_{12}}{-s_{45}}\right) \right.$$

$$+ \frac{\langle 34\rangle(\langle 13\rangle\langle 24\rangle - \langle 14\rangle\langle 32\rangle)}{\langle 32\rangle\langle 12\rangle^2\langle 45\rangle} \mathrm{Ls}_{-1}\left(\frac{-s_{13}}{-s_{45}}, \frac{-s_{23}}{-s_{45}}\right) + 2\frac{[12]\langle 14\rangle\langle 34\rangle}{\langle 12\rangle\langle 45\rangle} \frac{\mathrm{L}_0\left(\frac{-s_{23}}{-s_{45}}\right)}{s_{45}}$$

$$+ \frac{\langle 14\rangle^2\langle 32\rangle}{\langle 12\rangle^3\langle 45\rangle} \mathrm{Ls}_{-1}\left(\frac{-s_{13}}{-s_{45}}, \frac{-s_{23}}{-s_{45}}\right) - \frac{1}{2}\frac{\langle 14\rangle^2[12]^2\langle 32\rangle}{\langle 12\rangle\langle 45\rangle} \frac{\mathrm{L}_1\left(\frac{-s_{45}}{-s_{23}}\right)}{s_{23}^2}$$

$$+ \frac{\langle 14\rangle^2\langle 23\rangle[12]}{\langle 12\rangle^2\langle 45\rangle} \frac{\mathrm{L}_0\left(\frac{-s_{45}}{-s_{23}}\right)}{s_{23}}$$

$$- \frac{\langle 31\rangle[12]\langle 42\rangle[25]}{\langle 12\rangle} \frac{\mathrm{L}_1\left(\frac{-s_{45}}{-s_{13}}\right)}{s_{13}^2} - \frac{\langle 31\rangle[12]\langle 24\rangle\langle 14\rangle}{\langle 12\rangle^2\langle 45\rangle} \frac{\mathrm{L}_0\left(\frac{-s_{45}}{-s_{13}}\right)}{s_{13}}$$

$$\left. - \frac{1}{2}\frac{[25]([12][35] + [32][15])}{[13][32]\langle 12\rangle[45]} \right\}$$

$$R_n = \frac{\sigma(W + n \text{ jets})}{\sigma(W + (n-1) \text{ jets})}$$

$$\sigma^{\mathrm{NLO}}_{\mathrm{anti}-k_T} = 48.7^{+3.8}_{-7.9}\mathrm{fb}, \sigma^{\mathrm{NLO}}_{\mathrm{SISCone}} = 40.3^{+8.6}_{-8.5}\mathrm{fb}.$$

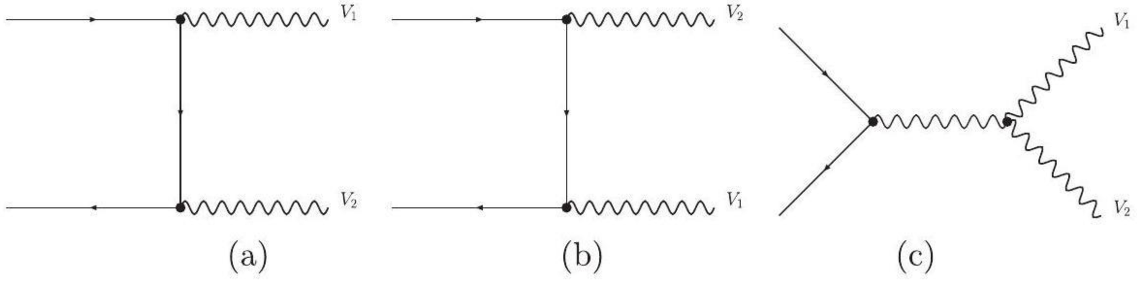


Figura 13. Emisión de energía por entrelazamiento de dos partículas supermasivas.

$$0 \to u^-(p_1) + \bar{u}^+(p_2) + \ell^-(p_3) + \bar{\nu}^+(p_4) + \bar{\ell}'^+(p_5) + \nu'^-(p_6)$$

$$A^{\text{tree}} = \left(\frac{e^2}{\sin^2 \theta_W}\right)^2 \delta_{i_1 i_2} P_W(s_{34}) P_W(s_{56}) \left[A^{\text{tree},a} + C_{L,u} A^{\text{tree},b}\right]$$

$$P_W(s) = \frac{s}{s - m_W^2 + i\Gamma_W m_W}$$

$$C_{L,u} = 2Q_u \sin^2 \theta_W + \frac{s_{12}(1 - 2Q_u \sin^2 \theta_W)}{s_{12} - m_Z^2}$$

$$A^{\text{tree},a} = i\frac{\langle 13\rangle[25]\langle 6|(2+5)|4\rangle}{s_{34}s_{56}t_{134}}$$

$$A^{\text{tree},b} = \frac{i}{s_{12}s_{34}s_{56}}\left[\langle 13\rangle[25]\langle 6|(2+5)|4\rangle + [24]\langle 16\rangle\langle 3|(1+6)|5\rangle\right]$$

$$A^{1-\text{loop}} = \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{N_c^2-1}{N_c}\right)\left(\frac{e^2}{\sin^2 \theta_W}\right)^2 \delta_{i_1 i_2} P_W(s_{34}) P_W(s_{56}) \left[A^a + C_{L,u} A^b\right]$$

$$A^a = c_\Gamma\left[A^{\text{tree},a} V + iF^a\right]$$

$$V = -\frac{1}{\varepsilon^2}\left(\frac{\mu^2}{-s_{12}}\right)^\varepsilon - \frac{3}{2\varepsilon}\left(\frac{\mu^2}{-s_{12}}\right)^\varepsilon - \frac{7}{2},$$

$$F^b = 0$$

$$\begin{aligned} F^a = &\left[\frac{\langle 13\rangle^2[25]^2}{\langle 34\rangle[56]t_{134}\langle 1|(5+6)|2\rangle} - \frac{\langle 2|(5+6)|4\rangle^2\langle 6|(2+5)|1\rangle^2}{[34]\langle 56\rangle t_{134}\langle 2|(5+6)|1\rangle^3}\right]\widetilde{\mathrm{Ls}}_{-1}^{2\,\mathrm{mh}}(s_{12}, t_{134}; s_{34}, s_{56}) \\ &+\left[\frac{1}{2}\frac{\langle 6|1|4\rangle^2 t_{134}}{[34]\langle 56\rangle\langle 2|(5+6)|1\rangle}\frac{\mathrm{L}_1\left(\frac{-s_{34}}{-t_{134}}\right)}{t_{134}^2} + 2\frac{\langle 6|1|4\rangle\langle 6|(2+5)|4\rangle}{[34]\langle 56\rangle\langle 2|(5+6)|1\rangle}\frac{\mathrm{L}_0\left(\frac{-t_{134}}{-s_{34}}\right)}{s_{34}}\right. \\ &-\frac{\langle 16\rangle\langle 26\rangle[14]^2 t_{134}}{[34]\langle 56\rangle\langle 2|(5+6)|1\rangle^2}\frac{\mathrm{L}_0\left(\frac{-t_{134}}{-s_{34}}\right)}{s_{34}} - \frac{1}{2}\frac{\langle 26\rangle[14]\langle 6|(2+5)|4\rangle}{[34]\langle 56\rangle\langle 2|(5+6)|1\rangle^2}\log\left(\frac{(-t_{134})(-s_{12})}{(-s_{34})^2}\right) \\ &\left.-\frac{3}{4}\frac{\langle 6|(2+5)|4\rangle^2}{[34]\langle 56\rangle t_{134}\langle 2|(5+6)|1\rangle}\log\left(\frac{(-t_{134})(-s_{12})}{(-s_{34})^2}\right) + L_{34/12}\log\left(\frac{-s_{34}}{-s_{12}}\right) - \text{flip}\right] \end{aligned}$$

$$+TI_3^{3\,\mathrm{m}}(s_{12},s_{34},s_{56})+\frac{1}{2}\frac{(t_{234}\delta_{12}+2s_{34}s_{56})}{\langle 2|(5+6)|1\rangle\Delta_3}\left(\frac{[45]^2}{[34][56]}+\frac{\langle 36\rangle^2}{\langle 34\rangle\langle 56\rangle}\right)+\frac{\langle 36\rangle[45](t_{134}-t_{234})}{\langle 2|(5+6)|1\rangle\Delta_3}$$

$$-\frac{1}{2}\frac{\langle 6|(2+5)|4\rangle^2}{[34]\langle 56\rangle t_{134}\langle 2|(5+6)|1\rangle}$$

$$\text{flip}:\ 1\leftrightarrow 2, 3\leftrightarrow 5, 4\leftrightarrow 6, \langle ab\rangle\leftrightarrow [ab].$$

$$\delta_{12}\equiv s_{12}-s_{34}-s_{56}, \delta_{34}\equiv s_{34}-s_{12}-s_{56}, \delta_{56}\equiv s_{56}-s_{12}-s_{34}$$

$$\Delta_3\equiv -4\begin{vmatrix} s_{12} & p_{12}\cdot p_{34}\\ p_{12}\cdot p_{34} & s_{34}\end{vmatrix}=s_{12}^2+s_{34}^2+s_{56}^2-2s_{12}s_{34}-2s_{12}s_{56}-2s_{34}s_{56}$$

$$\widetilde{\mathrm{Ls}}_{-1}^{2\,\mathrm{m}h}(s,t,m_1^2,m_2^2)\equiv -\mathrm{Li}_2\left(1-\frac{m_1^2}{t}\right)-\mathrm{Li}_2\left(1-\frac{m_2^2}{t}\right)-\frac{1}{2}\log^2\left(\frac{-s}{-t}\right)+\frac{1}{2}\log\left(\frac{-s}{-m_1^2}\right)\log\left(\frac{-s}{-m_2^2}\right)$$

$$I_3^{3\,\mathrm{m}}(s_{12},s_{34},s_{56})=-\frac{1}{\sqrt{\Delta_3}}\mathrm{Re}[2(\mathrm{Li}_2(-\rho x)+\mathrm{Li}_2(-\rho y))+\log(\rho x)\log(\rho y)$$
$$+\log\left(\frac{y}{x}\right)\log\left(\frac{1+\rho y}{1+\rho x}\right)+\frac{\pi^2}{3}\right]$$

$$x=\frac{s_{12}}{s_{56}}, y=\frac{s_{34}}{s_{56}}, \rho=\frac{2s_{56}}{\delta_{56}+\sqrt{\Delta_3}}$$

$$L_{\frac{34}{12}}=\frac{3}{2}\frac{\delta_{56}(t_{134}-t_{234})\langle 3|1+2|4\rangle\langle 6|1+2|5\rangle}{\langle 2|5+6|1\rangle\Delta_3^2}+\frac{3}{2}\frac{\langle 36\rangle[4|(1+2)(3+4)|5]}{\langle 2|5+6|1\rangle\Delta_3}$$
$$+\frac{1}{2}\frac{\langle 3|4|5\rangle[4|(5+6)(1+2)|5]}{[56]\langle 2|5+6|1\rangle\Delta_3}+\frac{[14]\langle 26\rangle t_{134}(\langle 36\rangle\delta_{12}-2\langle 3|45|6\rangle)}{\langle 56\rangle\langle 2|(5+6)|1\rangle^2\Delta_3}$$

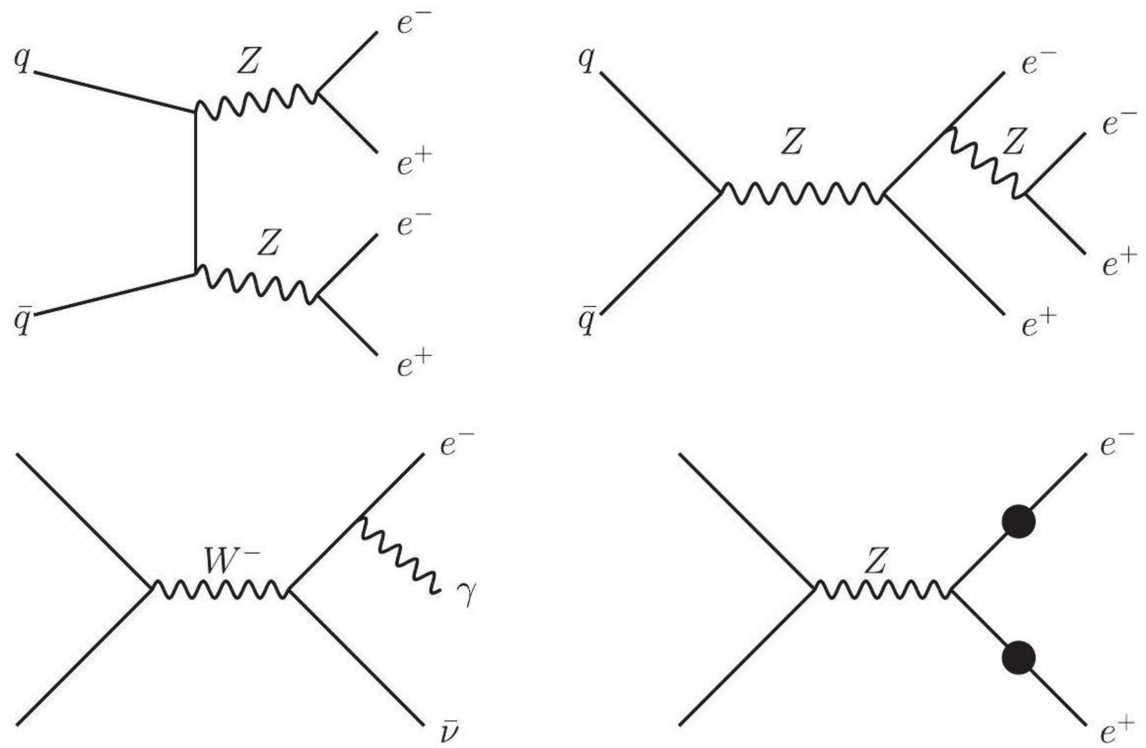
$$+\frac{1}{2}\frac{t_{134}}{\langle 2|(5+6)|1\rangle\Delta_3}\left(\frac{\langle 34\rangle[45]^2}{[56]}+\frac{[34]\langle 36\rangle^2}{\langle 56\rangle}-2\langle 36\rangle[45]\right)$$

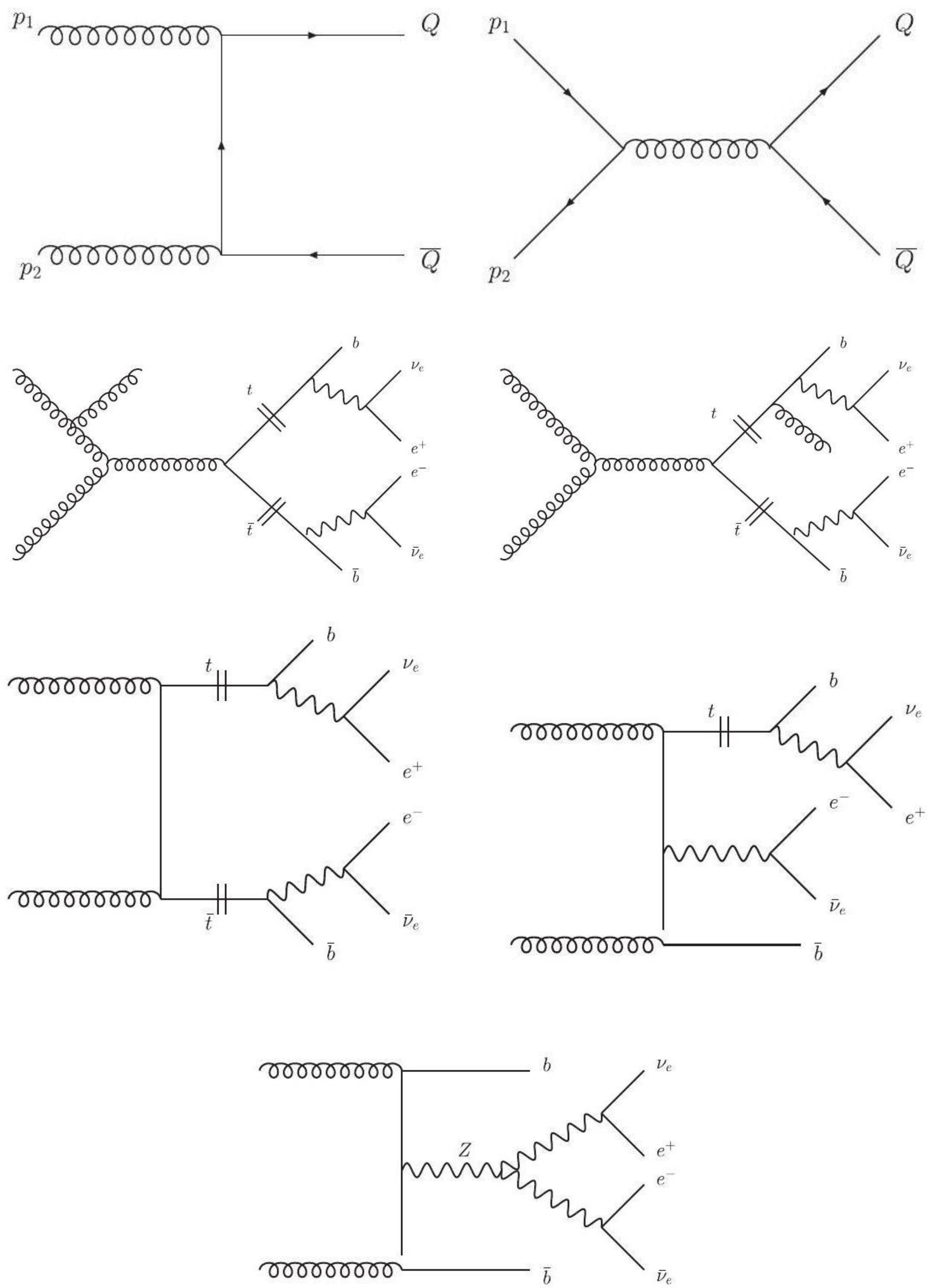
$$+\left(\frac{\langle 3|(1+4)|5\rangle}{[56]}-\frac{\langle 34\rangle[14]\langle 26\rangle}{\langle 2|(5+6)|1\rangle}\right)\frac{[45]\delta_{12}-2[4|36|5]}{\langle 2|(5+6)|1\rangle\Delta_3}$$

$$+4\frac{\langle 3|4|5\rangle\langle 6|(1+3)|4\rangle+\langle 6|3|4\rangle\langle 3|(2+4)|5\rangle}{\langle 2|(5+6)|1\rangle\Delta_3}$$

$$+2\frac{\delta_{12}}{\langle 2|(5+6)|1\rangle\Delta_3}\left(\frac{[45]\langle 3|(2+4)|5\rangle}{[56]}-\frac{\langle 36\rangle\langle 6|(1+3)|4\rangle}{\langle 56\rangle}\right)$$

$$
\begin{aligned}
T = & \frac{3}{2}\frac{s_{12}\delta_{12}(t_{134}-t_{234})\langle 6|(1+2)|5\rangle\langle 3|(1+2)|4\rangle}{\langle 2|(5+6)|1\rangle\Delta_3^2} \\
& -\frac{1}{2}\frac{(3s_{12}+2t_{134})\langle 6|(1+2)|5\rangle\langle 3|(1+2)|4\rangle}{\langle 2|(5+6)|1\rangle\Delta_3} \\
& +\frac{t_{134}}{\langle 2|(5+6)|1\rangle^2\Delta_3}[[14]\langle 26\rangle(\langle 3|6|5\rangle\delta_{56}-\langle 3|4|5\rangle\delta_{34}) \\
& -[15]\langle 23\rangle(\langle 6|5|4\rangle\delta_{56}-\langle 6|3|4\rangle\delta_{34})] \\
& +\frac{\langle 36\rangle[45]s_{12}t_{134}}{\langle 2|(5+6)|1\rangle\Delta_3}-\frac{\langle 34\rangle[56]\langle 6|(1+2)|4\rangle^2}{\langle 2|(5+6)|1\rangle\Delta_3}+2\frac{\langle 16\rangle[24](\langle 6|5|4\rangle\delta_{56}-\langle 6|3|4\rangle\delta_{34})}{[34]\langle 56\rangle\Delta_3} \\
& +2\frac{\langle 6|(2+5)|4\rangle}{\langle 2|(5+6)|1\rangle\Delta_3}\Big[\frac{\langle 6|5|2\rangle\langle 2|1|4\rangle\delta_{56}-\langle 6|2|1\rangle\langle 1|3|4\rangle\delta_{34}+\langle 6|(2+5)|4\rangle s_{12}\delta_{12}}{[34]\langle 56\rangle} \\
& +2\langle 3|(2+6)|5\rangle s_{12}] \\
& -\frac{[14]\langle 26\rangle\langle 3|(2+6)|5\rangle}{\langle 2|(5+6)|1\rangle^2}+2\frac{[15]\langle 23\rangle\langle 6|(2+5)|4\rangle}{\langle 2|(5+6)|1\rangle^2}-\frac{[14]\langle 26\rangle\langle 6|(2+5)|4\rangle\delta_{12}}{[34]\langle 56\rangle\langle 2|(5+6)|1\rangle^2} \\
& +\frac{1}{2}\frac{1}{\langle 2|(5+6)|1\rangle}\Big[3\frac{\langle 6|2|4\rangle\langle 6|1|4\rangle}{[34]\langle 56\rangle}+\frac{\langle 3|2|5\rangle\langle 3|1|5\rangle}{\langle 34\rangle[56]}+\frac{[14]\langle 16\rangle[45]}{[34]}-\frac{[24]\langle 26\rangle\langle 36\rangle}{\langle 56\rangle} \\
& +\frac{\langle 23\rangle[25]\langle 36\rangle}{\langle 34\rangle}-\frac{\langle 13\rangle[15][45]}{[56]}+4\langle 36\rangle[45]\Big] \\
& +\frac{1}{2}\frac{1}{\langle 1|(5+6)|2\rangle}\Big[\frac{\langle 16\rangle^2[24]^2}{[34]\langle 56\rangle}-\frac{\langle 13\rangle^2[25]^2]}{\langle 34\rangle[56]}-\frac{1}{2}\frac{[14]^2\langle 26\rangle^2(t_{134}\delta_{12}+2s_{34}s_{56})}{[34]\langle 56\rangle\langle 2|(5+6)|1\rangle^3}
\end{aligned}
$$





Figuras 14, 15, 16 y 17. Escenarios de colisión por interacción de dos o más partículas supermasivas en distintas dimensiones.

25. Radiación cero.

$$0 \to u^{-}(p_1) + \bar{d}^{+}(p_2) + \ell^{-}(p_3) + \bar{\nu}^{+}(p_4) + \gamma^{+}(p_5)$$

$$A^{\text{tree}} = i\sqrt{2}\left(\frac{e^3}{\sin^2\ \theta_W}\right) V_{ud}\delta_{i_1 i_2}\frac{P_W(s_{34})}{(s_{12}-s_{34})}\frac{\langle 13\rangle^2}{\langle 34\rangle\langle 15\rangle\langle 25\rangle}(Q_u s_{25} + Q_d s_{15})$$

$$Q_u p_2 \cdot p_5 + Q_d p_1 \cdot p_5$$

$$Q_u(1+\cos\ \theta^\star) + Q_d(1-\cos\ \theta^\star).$$

$$y_\gamma^\star = \frac{1}{2}\log\left(\frac{1+\cos\ \theta^\star}{1-\cos\ \theta^\star}\right) \approx -0.35.$$

$$y_W^\star \approx \frac{1}{2}\log\left(\frac{m_W - p_T^\gamma \cos\ \theta^\star}{m_W + p_T^\gamma \cos\ \theta^\star}\right),$$

$$y_W^\star \approx \frac{p_T^{\gamma,\min}}{3m_W}$$

$$\Gamma_t = \frac{G_F m_t^3}{8\sqrt{2}\pi}[(1-\beta^2)^2 + \omega^2(1+\beta^2) - 2\omega^4]\sqrt{1+\omega^4+\beta^4-2(\omega^2+\beta^2+\omega^2\beta^2)},$$

$$q(p_1) + \bar{q}(p_2) \to t(p_3) + \bar{t}(p_4)$$
$$g(p_1) + g(p_2) \to t(p_3) + \bar{t}(p_4)$$

$$\left|\mathcal{M}_{q\bar{q}\to t\bar{t}}\right|^2 = \frac{V}{2N^2}\left(\frac{\hat{t}^2 + \hat{u}^2 + 2m_t^2\hat{s}}{\hat{s}^2}\right)$$
$$\left|\mathcal{M}_{gg\to t\bar{t}}\right|^2 = \frac{1}{2VN}\left(\frac{V}{\hat{t}\hat{u}} - \frac{2N^2}{\hat{s}^2}\right)\left(\hat{t}^2 + \hat{u}^2 + 4m_t^2\hat{s} - \frac{4m_t^4\hat{s}^2}{\hat{t}\hat{u}}\right)$$

$$p_t^\mu = (m_T\cosh\ y_3, \vec{p}_T, m_T\sinh\ y_3)$$

$$(p_3 - p_1)^2 - m_t^2 = \hat{t} = -\sqrt{s}x_1 m_T(\cosh\ y_3 - \sinh\ y_3),$$

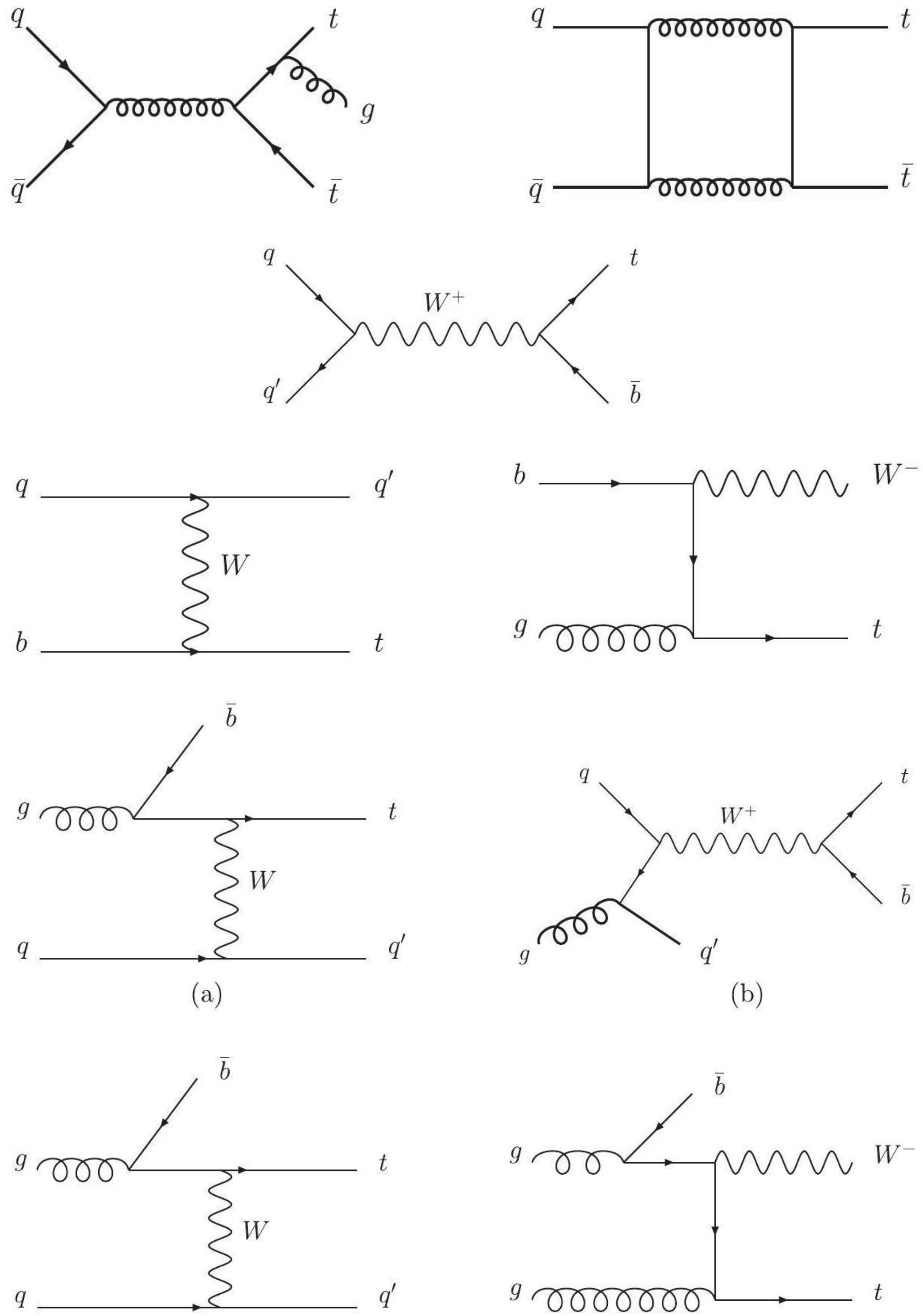
$$x_1 = \frac{m_T}{\sqrt{s}}(e^{y_3} + e^{y_4})$$

$$(p_3 - p_1)^2 - m_t^2 = -m_T^2(1 + e^{y_4 - y_3})$$

26. Partículas supermasivas - TEVATRON.

$$m_t = m_t^{\overline{\text{MS}}}(\mu_R)\left[1 + c_1\frac{\alpha_S}{\pi} + c_2\left(\frac{\alpha_S}{\pi}\right)^2 + \cdots\right],$$

$$m_t = m_t^{\overline{\text{MS}},NLO}(m_t)\left(1 + \frac{4\alpha_S}{3\pi}\right)$$



Figuras 18, 19, 20 y 21. Ciclos de origen y colapso del tevatrón en un espacio cuántico relativista o curvo.

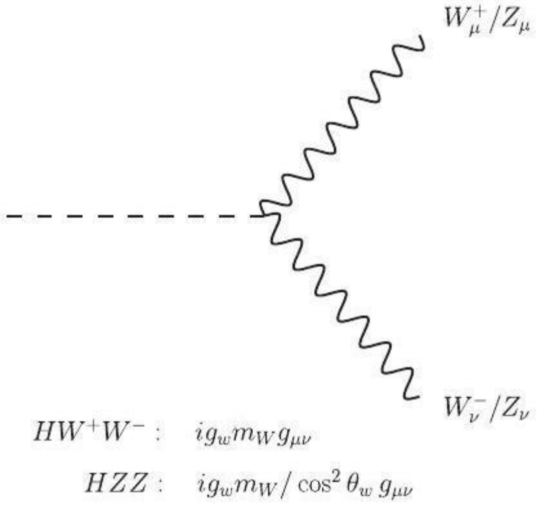
$$A_{\text{lab}}^{t\bar{t}} = \frac{\sigma(y_t > 0) - \sigma(y_t < 0)}{\sigma(y_t > 0) + \sigma(y_t < 0)}.$$

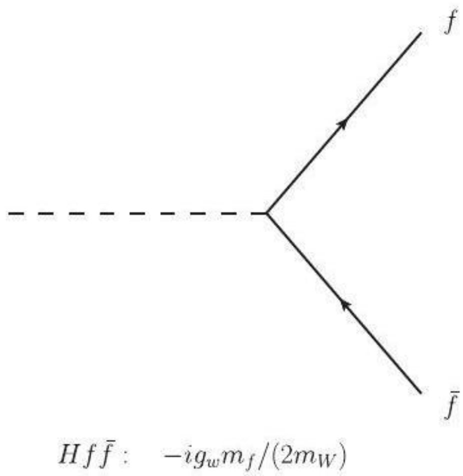
$$g(p_1) + q(p_2) \rightarrow t(p_3) + \bar{b}(p_4) + q'(p_5)$$

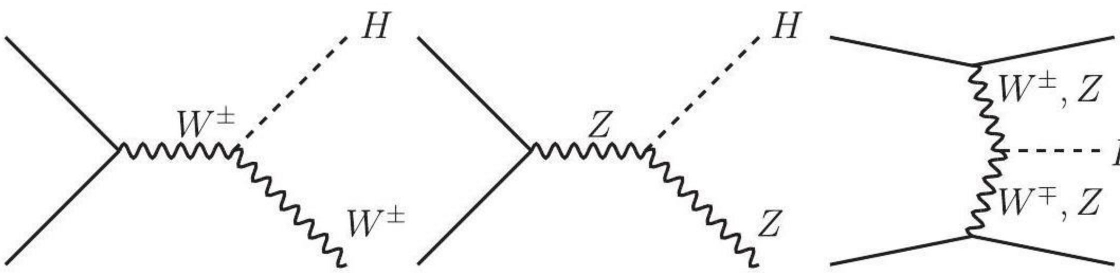
$$D^{(a)} = T^{a_1}_{i_3 i_4} \delta_{i_5 i_2} K^{(a)}, D^{(b)} = T^{a_1}_{i_5 i_2} \delta_{i_3 i_4} K^{(b)},$$

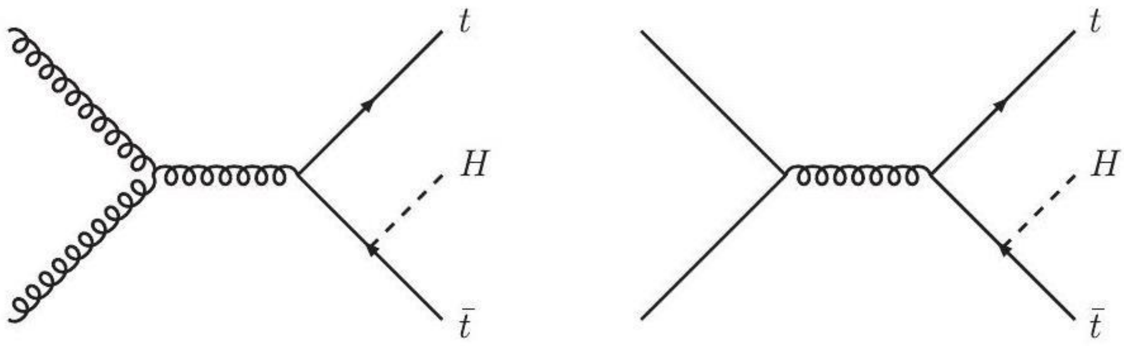
$$\left|D^{(a)} + D^{(b)}\right|^2 = \left|D^{(a)}\right|^2 + \left|D^{(b)}\right|^2 = N_c^2 C_F \left(\left|K^{(a)}\right|^2 + \left|K^{(b)}\right|^2\right)$$

$$f_b(x, \mu^2) = \frac{\alpha_s}{2\pi} \log\left(\frac{\mu^2}{m_b^2}\right) \int_x^1 \frac{\mathrm{d}z}{z} \mathcal{P}_{qg}(z) f_g\left(\frac{x}{z}, \mu^2\right) + \mathcal{O}(\alpha_s^2)$$









$$\overline{|\mathcal{M}|^2} = \frac{1}{8V}\left(\frac{\alpha_s}{3\pi v}\right)^2 p_H^4 \left|\frac{3}{4} I_q(m_t^2/m_H^2)\right|^2,$$

$$I_q(x) = 4x[2 + (4x-1)F(x)],$$

$$F(x) = \begin{cases} \frac{1}{2}[\log((1+\sqrt{1-4x})/(1-\sqrt{1-4x})) - i\pi]^2 & x < \frac{1}{4} \\ -2\left[\sin^{-1}(1/2\sqrt{x})\right]^2 & x \geq \frac{1}{4} \end{cases}$$

$$\Gamma(H \to VV^\star) = \frac{1}{S_V}\frac{g_W^2 m_H^3}{64\pi m_W^2}\left(\frac{m_V \Gamma_V}{\pi}\right)\int_0^{(m_H-m_V)^2} dp^2 \frac{\sqrt{\lambda(p^2)}\left(\lambda(p^2) + \frac{12 m_V^2 p^2}{m_H^4}\right)}{(p^2 - m_V^2)^2 + m_V^2\Gamma_V^2}$$

$$\lambda(p^2) = \left(1 - \frac{p^2}{m_H^2} - \frac{m_V^2}{m_H^2}\right)^2 - \frac{4 m_V^2 p^2}{m_H^4}$$

$$\Gamma(H \to VV) = \frac{1}{S_V}\frac{g_W^2 m_H^3}{128\pi m_W^2}\sqrt{1 - \frac{4m_V^2}{m_H^2}}\left(1 - \frac{4m_V^2}{m_H^2} + \frac{12 m_V^4}{m_H^4}\right),$$

$$\Gamma(H \to f\bar{f}) = d_f \frac{g_W^2 m_H m_f^2}{32\pi m_W^2}\left(1 - \frac{4m_f^2}{m_H^2}\right)^{3/2},$$

$$|\mathcal{M}|^2 = \frac{e^4}{16\pi^2}\frac{G_F m_H^4}{8\sqrt{2}\pi^2}\left|N_c\left(Q_t^2 I_q(m_t^2/m_H^2) + Q_b^2 I_q(m_b^2/m_H^2)\right) + I_W(m_W^2/m_H^2)\right|^2$$

$$I_W(x) = -2[(6x+1) + 6x(2x-1)F(x)].$$

$$|\mathcal{M}|^2 = \frac{e^4}{16\pi^2}\frac{G_F m_H^4}{8\sqrt{2}\pi^2}|(1.838) + (-0.016 + 0.019i) + (-8.323)|^2.$$

$$\mathcal{L}^{\text{eff}} = \frac{\alpha_s}{12\pi v}\left(1 + \frac{11}{4}\frac{\alpha_s}{\pi}\right) H \operatorname{tr} G_{\mu\nu} G^{\mu\nu} + \mathcal{O}(\alpha_s^3),$$

$$\left|\mathcal{M}_{gg\to H}^{(\text{LO})}\right|^2 = \frac{1}{8V}\left(\frac{\alpha_s}{3\pi v}\right)^2 m_H^4.$$

$$R \equiv \frac{\sigma_{\text{full}}}{\sigma_{\text{effective}}} = \left|\frac{3}{4} I_q(m_t^2/m_H^2)\right|^2.$$

$$2\mathcal{R}e\left[\mathcal{M}_{gg\to H}^{(1-\text{loop})} \times \mathcal{M}_{gg\to H}^{*(\text{LO})}\right] = \frac{\alpha_s N_c}{2\pi}\left(\frac{\mu^2}{m_H^2}\right)^{\varepsilon} c_\Gamma \left(-\frac{2}{\varepsilon^2} + \pi^2\right)\left|\mathcal{M}_{gg\to H}^{(\text{LO})}\right|^2$$

$$-2\frac{b_0}{\varepsilon} c_\Gamma \frac{\alpha_s}{2\pi}\left|\mathcal{M}_{gg\to H}^{(\text{LO})}\right|^2$$

$$+2\frac{N_c}{6}\frac{\alpha_s}{2\pi}\left|\mathcal{M}_{gg\to H}^{(\text{LO})}\right|^2$$

$$\frac{\alpha_s N_c}{2\pi} c_\Gamma \left[-\frac{2}{\varepsilon^2} \left(\frac{\mu^2}{m_H^2} \right)^\varepsilon - \frac{2}{\varepsilon} \frac{b_0}{N_c} + 4 + \pi^2 \right] \left| \mathcal{M}_{gg\to H}^{(\mathrm{LO})} \right|^2 .$$

$$\mathrm{d}\sigma_{gg\to H}^{NLO}(p_{g_1}, p_{g_2})$$

$$= \frac{\alpha_s N_c}{2\pi} \left| \mathcal{M}_{gg\to H}^{(\mathrm{LO})} \right|^2 \int_0^1 \mathrm{d}x_1\, \mathrm{d}x_2 \frac{\delta(x_1 x_2 s - m_H^2)}{m_H^2} \int_{x_1}^1 \mathrm{d}\xi_1 \int_{x_2}^1 \mathrm{d}\xi_2 f_{g/h_1}\left(\frac{x_1}{\xi_1}, \mu_F^2\right) f_{g/h_1}\left(\frac{x_2}{\xi_2}, \mu_F^2\right) \Bigg\{ \Bigg[\frac{11}{3}$$

$$+ \frac{4\pi^2}{3} \Bigg] \delta(1-\xi_1)\delta(1-\xi_2)$$

$$+ \Bigg[\left(\frac{2}{1-\xi_1} \log \frac{(1-\xi_1)^2 m_H^2}{\xi_1 \mu_F^2} \right)_+$$

$$+ 2\left(\frac{1}{\xi_1} - 2 + \xi_1 - \xi_1^2 \right) \left(\log \frac{(1-\xi_1)^2 m_H^2}{\xi_1 \mu_F^2} \right) + \frac{11}{6} \frac{(1-\xi_1)^3}{\xi_1} \Bigg] \delta(1-\xi_2)$$

$$+ \Bigg[\left(\frac{2}{1-\xi_2} \log \frac{(1-\xi_2)^2 m_H^2}{\xi_2 \mu_F^2} \right)_+$$

$$+ 2\left(\frac{1}{\xi_2} - 2 + \xi_2 - \xi_2^2 \right) \left(\log \frac{(1-\xi_2)^2 m_H^2}{\xi_2 \mu_F^2} \right) + \frac{11}{6} \frac{(1-\xi_2)^3}{\xi_2} \Bigg] \delta(1-\xi_1) \Bigg\}$$

$$(a): \quad 0 \to g(p_1) + \bar{q}(p_2) + q(p_3) + H$$
$$(b): \quad 0 \to g(p_1) + g(p_2) + g(p_3) + H$$

$$M_{gq\to qH} = -i \frac{g_s^2}{16\pi^2} \frac{g_W}{4m_W} \frac{1}{2} (t^A)_{i_3 i_2} g_s$$

$$\frac{1}{s_{23}} \bar{u}(p_3) \gamma_\mu u(p_2) \left(g^{\alpha\mu} - \frac{p_1^\mu (p_2^\alpha + p_3^\alpha)}{p_1 \cdot (p_2 + p_3)} \right) \epsilon_\alpha(p_1) F(s_{23}, s_H)$$

$$F(s_{23}, s_H) = -8m_q^2 \big[2 - (s_H - s_{23} - 4m_q^2) C_0(p_1, p_{23}; m_q, m_q, m_q)$$

$$+ \frac{2 s_{23}}{s_H - s_{23}} \big(B_0(p_{123}; m_q, m_q) - B_0(p_{23}; m_q, m_q) \big) \big]$$

$$B_0(p_1; m_1, m_2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}}} \Gamma(1$$

$$- \varepsilon) \int \mathrm{d}^D l \frac{1}{(l^2 - m_1^2 + i\epsilon)((l+p_1)^2 - m_2^2 + i\epsilon)} C_0(p_1, p_2; m_1, m_2, m_3)$$

$$= \frac{1}{i\pi^2} \times \int \mathrm{d}^4 l \frac{1}{(l^2 - m_1^2 + i\epsilon)((l+p_1)^2 - m_2^2 + i\epsilon)((l+p_1+p_2)^2 - m_3^2 + i\epsilon)}$$

$$\left| M_{gq\to qH} \right|^2 = \left(\frac{g_s^3 g_W}{64\pi^2 m_W} \right)^2 N_c C_F \frac{s_{12}^2 + s_{13}^2}{s_{23}(s_H - s_{23})^2} |F(s_{23}, s_H)|^2$$

$$M_{gg\to gH} = -\frac{g_W}{m_W}\frac{g_s^3}{32\pi^2}s_H^2 f_{ABC}\epsilon_\alpha(p_1)\epsilon_\beta(p_2)\epsilon_\gamma(p_3)$$
$$\Big[F_2^{\alpha\beta\gamma}(p_1,p_2,p_3)A_3(p_1,p_2,p_3) + F_1^{\alpha\beta\gamma}(p_1,p_2,p_3)A_2(p_1,p_2,p_3)$$
$$+F_1^{\beta\gamma\alpha}(p_2,p_3,p_1)A_2(p_2,p_3,p_1) + F_1^{\gamma\alpha\beta}(p_3,p_1,p_2)A_2(p_3,p_1,p_2)\Big],$$

$$F_1^{\alpha\beta\gamma}(p_1,p_2,p_3) = \left(\frac{g^{\alpha\beta}}{p_1\cdot p_2} - \frac{p_1^\beta p_2^\alpha}{p_1\cdot p_2^2}\right)\left(\frac{p_2^\gamma}{p_2\cdot p_3} - \frac{p_1^\gamma}{p_1\cdot p_3}\right)$$

$$F_2^{\alpha\beta\gamma}(p_1,p_2,p_3) = \frac{p_3^\alpha p_1^\beta p_2^\gamma - p_2^\alpha p_3^\beta p_1^\gamma}{p_1\cdot p_2 p_1\cdot p_3 p_2\cdot p_3} + \frac{g^{\alpha\beta}}{p_1\cdot p_2}\left(\frac{p_1^\gamma}{p_3\cdot p_1} - \frac{p_2^\gamma}{p_3\cdot p_2}\right)$$
$$+\frac{g^{\beta\gamma}}{p_2\cdot p_3}\left(\frac{p_2^\alpha}{p_1\cdot p_2} - \frac{p_3^\alpha}{p_1\cdot p_3}\right) + \frac{g^{\alpha\gamma}}{p_1\cdot p_3}\left(\frac{p_3^\beta}{p_2\cdot p_3} - \frac{p_1^\beta}{p_2\cdot p_1}\right).$$

$$A_3(p_1,p_2,p_3) = \frac{1}{2}[A_2(p_1,p_2,p_3) + A_2(p_2,p_3,p_1) + A_2(p_3,p_1,p_2) - A_4(p_1,p_2,p_3)].$$

$$A_2(p_1,p_2,p_3) = b_2(s_{12},s_{13},s_{23}) + b_2(s_{12},s_{23},s_{13})$$

$$A_4(p_1,p_2,p_3) = b_4(s_{12},s_{13},s_{23}) + b_4(s_{13},s_{23},s_{12}) + b_2(s_{23},s_{12},s_{13})$$

$$b_4(s,t,u) = \frac{m_q^2}{s_H}\left[-\frac{2}{3} + \left(\frac{m_q^2}{s_H} - \frac{1}{4}\right)\left(W_2(s) - W_2(s_H) + W_3(s,t,u,s_H)\right)\right]$$

$$b_2(s,t,u) = \frac{m_q^2}{s_H^2}\left[\frac{s(u-s)}{s+u} + \frac{2ut(u+2s)}{(s+u)^2}\left(W_1(t) - W_1(s_H)\right)\right.$$

$$+\left(m_q^2 - \frac{s}{4}\right)\left(\frac{1}{2}W_2(s) + \frac{1}{2}W_2(s_H) - W_2(t) + W_3(s,t,u,s_H)\right)$$

$$+s^2\left(\frac{2m_q^2}{(s+u)^2} - \frac{1}{2(s+u)}\right)\left(W_2(t) - W_2(s_H)\right)$$

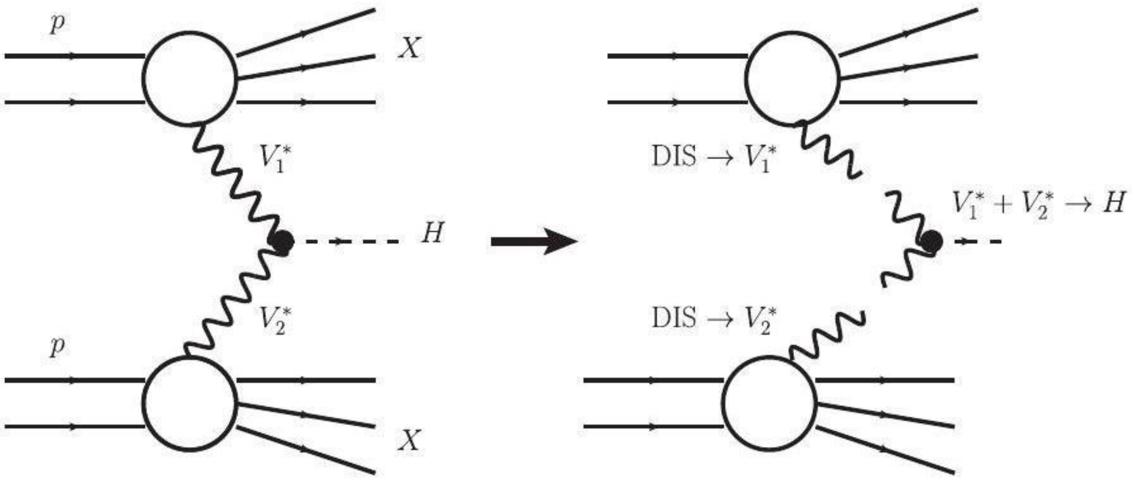
$$\left.+\frac{ut}{2s}\left(W_2(s_H) - 2W_2(t)\right) + \frac{1}{8}\left(s - 12m_q^2 - \frac{4ut}{s}\right)W_3(t,s,u,s_H)\right]$$

$$W_1(s) = 2 + \int_0^1 \mathrm{d}x\log\left(1 - \frac{s}{m_q^2}x(1-x) - i\epsilon\right)$$

$$W_2(s) = 2\int_0^1 \frac{\mathrm{d}x}{x}\log\left(1 - \frac{s}{m_q^2}x(1-x) - i\epsilon\right)$$
$$W_3(s,t,u,v) = I_3(s,t,u,v) - I_3(s,t,u,s) - I_3(s,t,u,u)$$
$$I_3(s,t,u,v) = \int_0^1 \mathrm{d}x\left(\frac{m_q^2 t}{us} + x(1-x)\right)^{-1}\log\left(1 - \frac{v}{m_q^2}x(1-x) - i\epsilon\right)$$

$$\left|M_{gg\to gH}\right|^2 = \frac{g_W^2 g_s^6}{256\pi^4}\frac{8N_c^2 C_F}{s_{12}s_{13}s_{23}}\left[|A_2(p_1,p_2,p_3)|^2 + |A_2(p_2,p_3,p_1)|^2 + |A_2(p_3,p_1,p_2)|^2 + |A_4(p_1,p_2,p_3)|^2\right]$$

$$C_{HVV}^{\mu\nu}(p_1,p_2) = a_1(p_1,p_2)g^{\mu\nu} + a_2(p_1,p_2)\left(p_1\cdot p_2 g^{\mu\nu} - p_1^{\nu}p_2^{\mu}\right) + a_3(p_1,p_2)\epsilon^{\mu\nu\rho\sigma}p_{1,\rho}p_{2,\sigma}$$

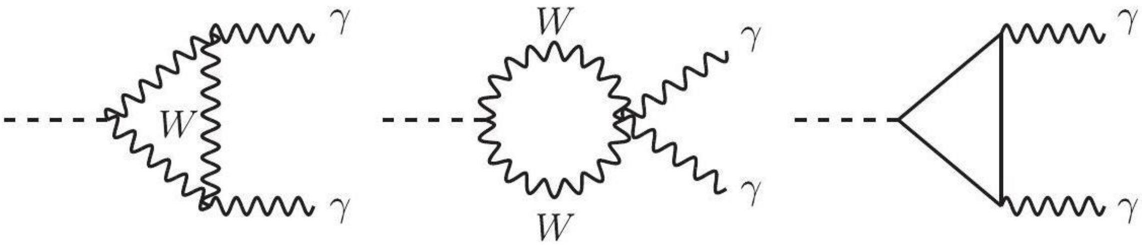


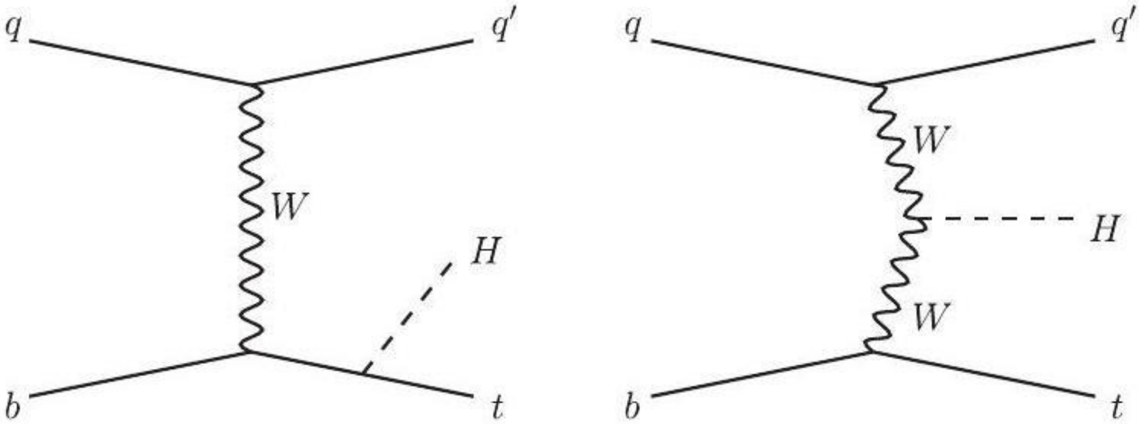
$$V_1 + V_2 \to V_3 + V_4$$

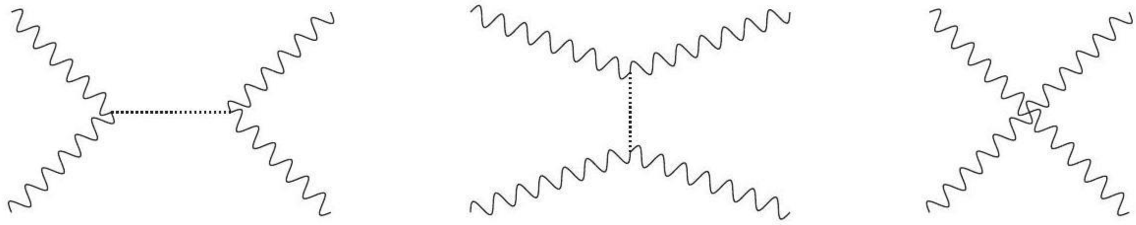
$$V_1 = V_3 = W^+, V_2 = V_4 = W^-$$

$$m_H < \left(\frac{8\sqrt{2}\pi}{3G_F}\right)^{\frac{1}{2}} \approx 1\text{TeV}$$

27. Radiación QCD.







Figuras 22, 23 y 24. Radiación de una partícula supermasiva.

$$\mathrm{d}w^{q\to qg} = \frac{\alpha_s(k_\perp^2)}{2\pi} C_F \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\frac{\mathrm{d}\omega}{\omega}\left[1+\left(1-\frac{\omega}{E}\right)\right]$$

$$\frac{1}{R} \ll k_\perp \sim \omega \sim Q \longrightarrow w^{q\to qg} \sim \alpha_s(k_\perp^2) \ll 1,$$

$$\frac{1}{R} \leq k_\perp \leq \omega \ll Q \longrightarrow w^{q\to qg} \sim \alpha_s(k_\perp^2)\log\, k_\perp^2 \sim 1$$

$$\frac{1}{R} \leq k_\perp \ll \omega \ll Q$$

$$\frac{1}{R} \leq k_\perp \ll \omega \sim Q$$

$$\frac{1}{R} \leq k_\perp \sim \omega \ll Q$$

$$t^{\text{form}} = \frac{k_\parallel}{k_\perp^2} \text{ and } t^{\text{had}} = k_\parallel R^2.$$

$$\mathrm{d}N_{(\text{hadrons})} \sim \int_{k_\perp > 1/R}^{Q} \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\frac{C_F\alpha_s(k_\perp^2)}{2\pi}\left[1+\left(1-\frac{\omega}{E}\right)\right]\frac{\mathrm{d}\omega}{\omega}$$

$$\sim \frac{C_F\alpha_s(1/R^2)}{\pi}\log\,(Q^2R^2)\frac{\mathrm{d}\omega}{\omega} = \frac{C_F\alpha_s(1/R^2)}{\pi}\log\,(Q^2R^2)\mathrm{d}\log\,\omega$$

$$\mathrm{d}N_{(\text{hadrons})}/\mathrm{d}\log\,\epsilon = \text{const.}$$

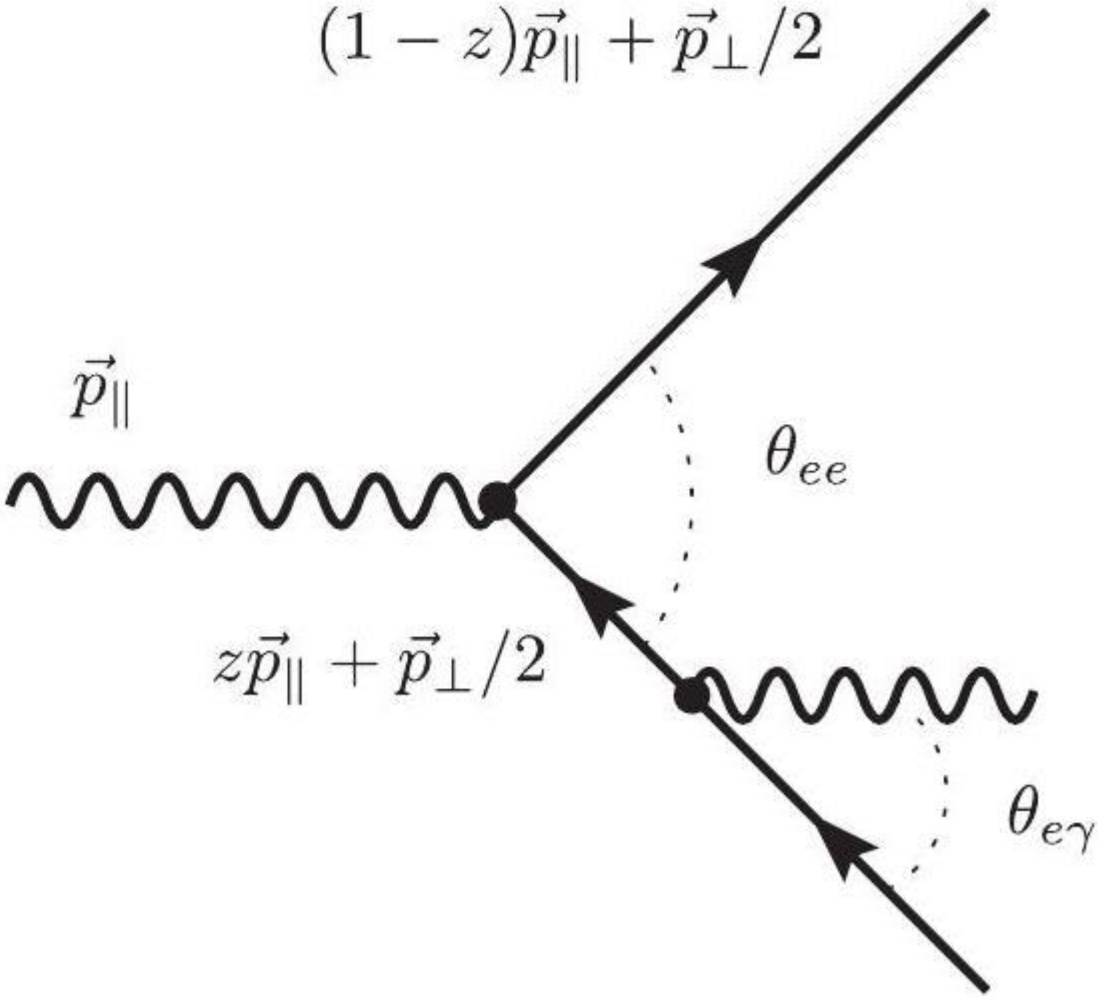
$$t^{\text{form}} \sim \frac{k_\parallel}{k_\perp^2}$$

$$t^{\text{sep}} \sim R\theta \sim t^{\text{form}}(Rk_\perp)$$

$$t^{\text{had}} \sim k_\parallel R^2 \sim t^{\text{form}}(Rk_\perp)^2$$

$$1/R \lesssim \omega_{(\text{hadron})} \lesssim 1/(R\theta)$$

$$\sin\,\theta_{ee} \approx \theta_{ee} \approx \frac{p_\perp}{p_\parallel}$$



$$k_{\perp}^{2} \approx \left(p^{(+)}\right)^{2} \sin^{2}\ \theta_{e\gamma} \approx \left(zp_{||}\right)^{2} \theta_{e\gamma}^{2}$$

$$\Delta E \approx \frac{k_{\perp}^{2}}{zp_{||}} \approx zp_{||}\theta_{e\gamma}^{2}$$

$$\Delta t \approx \frac{1}{\Delta E} \approx \frac{1}{zp_{||}\theta_{e\gamma}^{2}}$$

$$\Delta b = \theta_{ee}\Delta t = \frac{p_{\perp}}{p_{||}}\Delta t$$

$$\Delta b = \frac{\theta_{ee}}{zp_{||}\theta_{e\gamma}^{2}}$$

$$\lambda_{\perp}^{\gamma} \approx \frac{1}{k_{\perp}} \approx \frac{1}{zp_{||}\theta_{e\gamma}}$$

$$\Delta b \approx \frac{\theta_{ee}}{zp_{||}\theta_{e\gamma}^{2}} > \frac{1}{zp_{||}\theta_{e\gamma}} \approx \lambda_{\perp}^{\gamma}$$

$$\theta_{ee} > \theta_{\gamma e}$$

$$\mathcal{W}(p,p';k,\epsilon)=\epsilon_\mu^*\left(\frac{p'^\mu}{p'\cdot k}-\frac{p^\mu}{p\cdot k}\right)$$

$$\mathrm{W}_{e^+e^-}=\frac{2(1-\vec{n}_+\vec{n}_-)}{(1-\vec{n}\vec{n}_+)(1-\vec{n}\vec{n}_-)}=\frac{2(1-\cos\,\theta_{e^+e^-})}{(1-\cos\,\theta_{\gamma e^+})(\cos\,\theta_{\gamma e^-})}$$

$$\mathrm{W}_{e^+e^-}=\mathrm{W}_{e^+e^-}^{(+)}+\mathrm{W}_{e^+e^-}^{(-)}$$

$$\mathrm{W}_{e^+e^-}^{(\pm)}=\mathrm{W}_{e^+e^-}+\frac{1}{1-\cos\,\theta_{\gamma e^\pm}}-\frac{1}{1-\cos\,\theta_{\gamma e^\mp}}$$

$$\mathrm{d}^2\Omega_\gamma=\mathrm{dcos}\,\theta_{\gamma e^+}\mathrm{d}\phi_{\gamma e^+}$$

$$1-\cos\,\theta_{\gamma e^-}=(1-\cos\,\theta_{e^+e^-}\cos\,\theta_{\gamma e^+})-(\sin\,\theta_{e^+e^-}\sin\,\theta_{\gamma e^+})\cos\,\phi_{\gamma e^+}$$
$$=a-b\cos\,\phi_{\gamma e^+}$$

$$\mathrm{d}\phi=i\frac{\mathrm{dz}}{z}$$

$$\cos\,\phi=\frac{z+z^*}{2}$$

$$I=\int_0^{2\pi}\frac{\mathrm{d}\phi_{\gamma e^+}}{2\pi}\frac{1}{1-\cos\,\theta_{\gamma e^-}}=\frac{i}{2\pi}\oint_{|z|=1}\frac{\mathrm{dz}}{z\left(a-b\frac{z+z^*}{2}\right)}$$
$$=\frac{1}{i\pi}\oint_{|z|=1}\frac{\mathrm{dz}}{bz^2-2az+b}=\frac{1}{i\pi b}\oint_{|z|=1}\frac{\mathrm{dz}}{(z-z_+)(z-z_-)}$$

$$z_\pm=\frac{a}{b}\pm\sqrt{\frac{a^2}{b^2}-1}$$

$$I=\frac{1}{\sqrt{a^2-b^2}}=\frac{1}{|\cos\,\theta_{\gamma e^+}-\cos\,\theta_{e^+e^-}|}$$

$$\int_0^{2\pi}\frac{\mathrm{d}\phi_{\gamma e^+}}{2\pi}\mathrm{W}_{e^+e^-}^{(+)}=\frac{1}{1-\cos\,\theta_{\gamma e^+}}\left[1+\frac{\cos\,\theta_{\gamma e^+}-\cos\,\theta_{e^+e^-}}{|\cos\,\theta_{\gamma e^+}-\cos\,\theta_{e^+e^-}|}\right]=\begin{cases}0 & \text{if } \theta_{\gamma e^+}>\theta_{e^+e^-}\\ \frac{2}{1-\cos\,\theta_{\gamma e^+}} & \text{else.}\end{cases}$$

28. Colisiones hadrónicas.

$$t^{(\mathrm{form})}\sim\lambda_\perp/\theta$$

$$\rho_\perp=t^{(\mathrm{form})}\theta_{ac}$$

$$\theta_{ak},\theta_{ck}\leq\theta_{ac}$$

$$p_\perp\approx\sqrt{-\hat{t}}$$

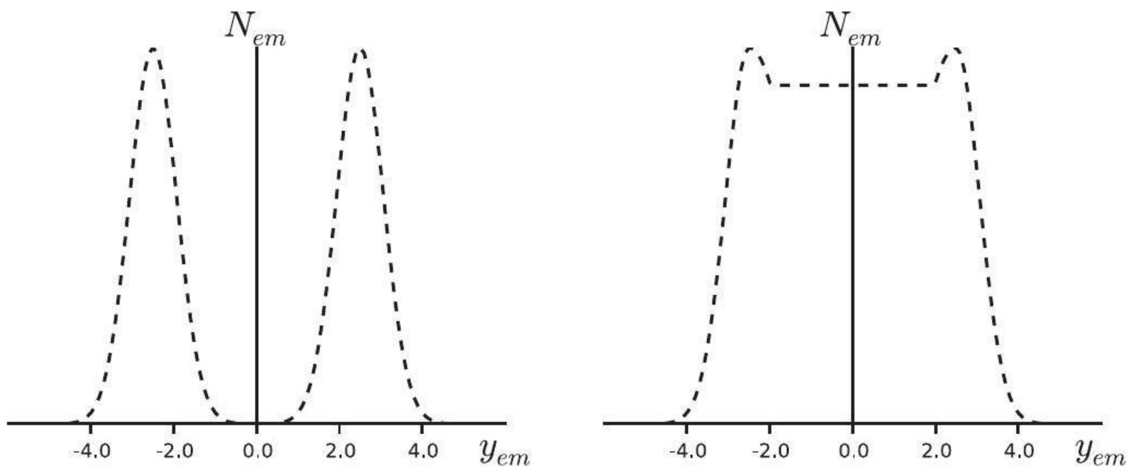


Figura 25. Fluctaciones de energía por colisiones hadrónicas.

Hump-backed plateau:

$$\theta \sim k_\perp/k_\parallel \sim k_\perp/\omega > 1/(\omega R).$$

$$p_\perp \sim \epsilon\theta \sim 1/R$$

$$N \propto \int^E \frac{\mathrm{d}\epsilon}{\epsilon}\frac{\mathrm{d}\epsilon}{\epsilon}\int^1 \frac{\mathrm{d}\theta}{\theta}\frac{\mathrm{d}\theta}{\theta}\delta(\epsilon\theta - 1/R)$$

$$1/R \le \epsilon \le \omega$$

$$p_\perp \sim \epsilon\theta > 1/R$$

$$N \propto \int^E \frac{\mathrm{d}\epsilon}{\epsilon}\frac{\mathrm{d}\epsilon}{\epsilon}\int^1 \frac{\mathrm{d}\theta}{\theta}\frac{\mathrm{d}\theta}{\theta}\delta(\epsilon\theta - 1/R)$$

$$+\alpha_s \int^E \frac{\mathrm{d}\omega}{\omega}\frac{\mathrm{d}\omega}{\omega}\int^1 \frac{\mathrm{d}\theta_0}{\theta_0}\frac{\mathrm{d}\theta_0}{\theta_0}\Theta(\omega\theta_0 - 1/R)\int^\omega \frac{\mathrm{d}\epsilon}{\epsilon}\frac{\mathrm{d}\epsilon}{\epsilon}\int^{\theta_{\max}} \frac{\mathrm{d}\theta}{\theta}\frac{\mathrm{d}\theta}{\theta}\delta(\epsilon\theta - 1/R)$$

$$\int^1 \frac{\mathrm{d}\theta_0}{\theta_0}\frac{\mathrm{d}\theta_0}{\theta_0}\int^{\theta_0} \frac{\mathrm{d}\theta}{\theta}\frac{\mathrm{d}\theta}{\theta} \approx \frac{1}{2}\int^1 \frac{\mathrm{d}\theta_0}{\theta_0}\frac{\mathrm{d}\theta_0}{\theta_0}\int^1 \frac{\mathrm{d}\theta}{\theta}\frac{\mathrm{d}\theta}{\theta}$$

$$\frac{\mathrm{d}N}{\mathrm{dlog}\ \epsilon} = \begin{cases} 1 + \frac{\alpha_s}{2}[\log^2(ER) - \log^2(\epsilon R)] & \text{for incoherent sum, } \theta_{\max} = 1 \\ 1 + \alpha_s \log\frac{E}{\epsilon}\log \epsilon R & \text{for coherent sum, } \theta_{\max} = \theta_0 \end{cases}$$

$$\langle\epsilon\rangle = \langle E_{\text{had}}\rangle = \frac{1}{R} \sim m_{\text{had}}$$

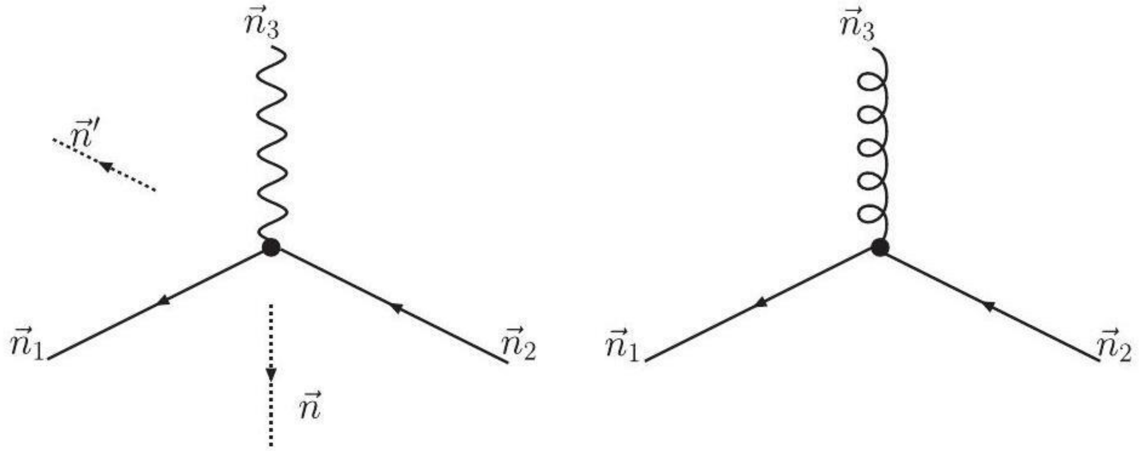
El efecto drag:

$$\mathrm{d}w^{q\bar{q}} = C_F\frac{\alpha_s}{2\pi}\frac{\mathrm{d}\omega}{\omega}\frac{\mathrm{d}^2\Omega_{\vec{n}}}{4\pi}\frac{2p_ip_j}{(p_ik)(p_jk)} = C_F\frac{\alpha_s}{2\pi}\frac{\mathrm{d}\omega}{\omega}\frac{\mathrm{d}^2\Omega_{\vec{n}}}{4\pi}W_{q\bar{q}}(\vec{n})$$

$$W_{q\bar{q}}(\vec{n}) = \frac{2(1-\vec{n}_q\vec{n}_{\bar{q}})}{(1-\vec{n}\vec{n}_q)(1-\vec{n}\vec{n}_{\bar{q}})}$$

$$= \left[\frac{1}{1-\vec{n}\vec{n}_q} + \frac{\vec{n}_q(\vec{n}_{\bar{q}}-\vec{n})}{(1-\vec{n}\vec{n}_q)(1-\vec{n}\vec{n}_{\bar{q}})}\right] + \left[\frac{1}{1-\vec{n}\vec{n}_{\bar{q}}} + \frac{\vec{n}_{\bar{q}}(\vec{n}_q-\vec{n})}{(1-\vec{n}\vec{n}_q)(1-\vec{n}\vec{n}_{\bar{q}})}\right]$$

$$= W_q(\vec{n};\vec{n}_{\bar{q}}) + W_{\bar{q}}(\vec{n};\vec{n}_q)$$



$$\langle W_q(\vec{n};\vec{n}_{\bar{q}})\rangle = \int \frac{\mathrm{d}\phi}{2\pi} W_q(\vec{n};\vec{n}_{\bar{q}}) \approx \frac{2}{1-\cos\,\theta qg}\Theta(\theta_{q\bar{q}}-\theta qg).$$

$$E_q \approx E_{\bar{q}} \approx E_{\gamma,g} \approx \frac{E}{3}$$
$$\theta_{qg} \approx \theta_{\bar{q}g} \approx \theta_{q\bar{q}} \approx \frac{2\pi}{3},$$

$$\mathrm{d}w^{q\bar{q}\gamma} = C_F \frac{\alpha_s}{2\pi}\frac{\mathrm{d}\omega}{\omega}\frac{\mathrm{d}^2\Omega_{\vec{n}}}{4\pi} W_{q\bar{q}}(\vec{n}),$$

$$C_F = \frac{N_c^2-1}{2N_c} \overset{N_c\to\infty}{\rightarrow} \frac{N_c}{2}.$$

$$\mathrm{d}w''^{q\bar{q}g''} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}^3k}{(2\pi^3)2\omega}\left(\frac{p_1^\mu}{p_1k}+\frac{p_2^\mu}{p_2k}-2\frac{p_3^\mu}{p_3k}\right)^2$$
$$= \frac{\alpha_s}{2\pi}\frac{\mathrm{d}\omega}{\omega}\frac{\mathrm{d}^2\Omega_{\vec{n}}}{4\pi}\left[W_{qg}(\vec{n}) + W_{g\bar{q}}(\vec{n}) - \frac{1}{2}W_{q\bar{q}}(\vec{n})\right]$$

$$\left[W_{qg}(\vec{n}) + W_{g\bar{q}}(\vec{n}) - \frac{1}{2}W_{q\bar{q}}(\vec{n})\right] = \left[2\cdot\frac{2\left(1-\cos\frac{4\pi}{3}\right)}{\left(1-\cos\frac{2\pi}{3}\right)(1-\cos\,\pi)} - \frac{1}{2}\cdot\frac{2\left(1-\cos\frac{4\pi}{3}\right)}{\left(1-\cos\frac{2\pi}{3}\right)^2}\right]$$

$$= \left(1-\cos\frac{4\pi}{3}\right)\left[2\cdot 2 - \frac{1}{2}\cdot 8\right] = 0$$

$$\left[W_{qg}(\vec{n}') + W_{g\bar{q}}(\vec{n}') - \frac{1}{2}W_{q\bar{q}}(\vec{n}')\right] = \frac{3}{2}\cdot 9.$$

$$W_{q\bar{q}}(\vec{n}) = \frac{2\left(1-\cos\frac{4\pi}{3}\right)}{\left(1-\cos\frac{2\pi}{3}\right)^2} = \frac{3}{2}\cdot 8$$

$$W_{q\bar{q}}(\vec{n}') = \frac{2\left(1-\cos\frac{4\pi}{3}\right)}{\left(1-\cos\frac{2\pi}{3}\right)(1-\cos\pi)} = \frac{3}{2}\cdot 2$$

$$\frac{\mathrm{d}w^{q\bar{q}\gamma}(\vec{n}')}{\mathrm{d}w^{q\bar{q}\gamma}(\vec{n})} = \frac{1}{4}, \frac{\mathrm{d}w^{q\bar{q}g}(\vec{n}')}{\mathrm{d}w^{q\bar{q}g}(\vec{n})} = \frac{5N_c^2-1}{2N_c^2-4} = \frac{22}{7}$$

$$\frac{\mathrm{d}w^{q\bar{q}\gamma}(\vec{n}_\perp)}{\mathrm{d}w^{q\bar{q}\gamma}(\vec{n})} = \frac{1}{4}, \frac{\mathrm{d}w^{q\bar{q}g}(\vec{n}_\perp)}{\mathrm{d}w^{q\bar{q}g}(\vec{n})} = \frac{N_c+2C_F}{2(4C_F-N_c)} = \frac{17}{14}$$

$$\frac{\mathrm{d}w^{q\bar{q}g}(\vec{n})}{\mathrm{d}w^{q\bar{q}\gamma}(\vec{n})} = \frac{N_c^2-2}{2(N_c^2-1)} = \frac{7}{16}$$

$$\frac{1}{Q_{\perp,X}^2}\alpha_s^n \log^m \frac{Q_{\perp,X}^2}{Q_X^2} \quad \text{donde } m \leq 2n-1$$

$$\frac{\mathrm{d}\sigma_{AB\to X}}{\mathrm{d}y\,\mathrm{d}Q_\perp^2} = \sum_{ij} \pi\hat{\sigma}_{ij\to X}^{(LO)} \left\{\int \frac{\mathrm{d}^2 b_\perp}{(2\pi)^2}\left[\exp\left(i\vec{b}_\perp\cdot\vec{Q}_\perp\right)\tilde{W}_{ij}(b_\perp;Q,x_A,x_B)\right] + Y_{ij\to X}(Q_\perp;Q,x_A,x_B)\right\}$$

$$\tilde{W}_{ij}(b_\perp;Q,x_A,x_B)$$

$$= \sum_{ab}\int_{x_A}^1 \frac{\mathrm{d}\xi_A}{\xi_A}\int_{x_B}^1 \frac{\mathrm{d}\xi_B}{\xi_B}\left\{f_{a/A}(\xi_A,\mu_F^2)f_{b/B}(\xi_B,\mu_F^2)\right.$$

$$\times C_{ia}\left(\frac{x_A}{\xi_A},\mu_R^2,\frac{1}{b_\perp},\mu_F^2\right)C_{jb}\left(\frac{x_B}{\xi_B},\mu_R^2,\frac{1}{b_\perp},\mu_F^2\right)H_{ab}(\mu_R^2)\times\exp\left[-\int_{b_0^2/b_\perp^2}^{Q^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\left(A(k_\perp^2)\log\frac{Q^2}{k_\perp^2}\right.\right.$$

$$\left.\left.\left. + B(k_\perp^2)\right)\right]\right\}$$

$$Y_{ij}(Q_\perp;Q,x_A,x_B) = \int_{x_A}^1 \frac{\mathrm{d}\xi_A}{\xi_A}\int_{x_A}^1 \frac{\mathrm{d}\xi_B}{\xi_B}\left[f_{i/A}(\xi_A,\mu_F^2)f_{j/B}(\xi_B,\mu_F^2) \times R_{ij\to X}\left(Q_\perp;Q,\frac{x_A}{\xi_A},\frac{x_B}{\xi_B}\right)\right]$$

$$b_0 = 2e^{-\gamma_E}$$

$$A(\mu_R^2) = \sum_{N=1}\left(\frac{\alpha_s(\mu_R^2)}{2\pi}\right)^N A^{(N)}$$

$$B(\mu_R^2) = \sum_{N=1} \left(\frac{\alpha_s(\mu_R^2)}{2\pi}\right)^N B^{(N)}$$

$$C_{ia}\left(\frac{x_A}{\xi_A}, \mu_R^2, \frac{1}{b_\perp}, \mu_F^2\right) = \delta_{ia}\delta\left(1 - \frac{x_A}{\xi_A}\right) + \sum_{N=1} \left(\frac{\alpha_s(\mu_R^2)}{2\pi}\right)^N C_{ia}^{(N)}\left(\frac{x_A}{\xi_A}, \frac{1}{b_\perp}, \mu_F^2\right)$$

$$H_{ab\to X}(\mu_R^2) = 1 + \sum_{N=1} \left(\frac{\alpha_s(\mu_R^2)}{2\pi}\right)^N H_{ab\to X}^{(N)}$$

$$R_{ij\to X}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q, \mu_R^2\right) = \sum_{N=1} \left(\frac{\alpha_s(\mu_R^2)}{2\pi}\right)^N R_{ij\to X}^{(N)}\left(Q, \frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}\right)$$

$$C_q = C_F \text{ and } C_g = C_A$$

$$A_{q,g}^{(1)} = 2C_{q,g}$$

$$A_{q,g}^{(2)} = 2C_{q,g}K = 2C_{q,g}\left[C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{10}{9}T_R n_f\right]$$

$$A_{q,g}^{(3)} = 2C_{q,g}K' = 2C_{q,g}\left\{C_A^2\left[\frac{245}{24} - \frac{67}{9}\frac{\pi^2}{6} + \frac{11}{6}\zeta(3) + \frac{11}{5}\left(\frac{\pi^2}{6}\right)^2\right]\right.$$

$$\left. + C_F n_f\left[-\frac{55}{24} + 2\zeta(3)\right] + C_A n_f\left[-\frac{209}{108} + \frac{10}{9}\frac{\pi^2}{6} - \frac{7}{3}\zeta(3)\right] + n_f^2\left[-\frac{1}{27}\right]\right\}$$

$$B_a^{(1)} = -2\gamma_a^{(1)}$$

$$B_q^{(1)} = -3C_F$$

$$B_g^{(1)} = -2\beta_0 = -\left(\frac{11}{3}C_A - \frac{4}{3}T_R n_f\right)$$

$$C_{ia}^{(0)}\left(z, \frac{1}{b_\perp}, \mu_F^2\right) = \delta_{ia}\delta(1 - z)$$

$$C_{ia}^{(1)}\left(z, \frac{1}{b_\perp}, \mu_F^2\right) = P_{ia}^{(1)}\log\frac{b_0^2}{b_\perp^2\mu_F^2} - P_{ia}^{\epsilon}(z) + \delta_{ia}\delta(1 - z)C_a\frac{\pi^2}{6}$$

$$P_{qq}^{\epsilon}(z) = -C_F(1 - z)$$

$$P_{gq}^{\epsilon}(z) = -C_F z$$

$$P_{qg}^{\epsilon}(z) = -2T_R z(1 - z)$$

$$P_{gg}^{\epsilon}(z) = 0$$

$$B_a^{(2)} = -2\gamma_a^{(2)} + \beta_0\left(\frac{2\pi^2}{3}C_a + \mathcal{A}_a^{(\text{loop})}\right)$$

$$\gamma_q^{(2)} = C_F^2\left[\frac{3}{8}-\frac{\pi^2}{2}+6\zeta(3)\right]+C_FC_A\left[\frac{17}{24}+\frac{11\pi^2}{18}-3\zeta(3)\right]-C_FT_Rn_f\left[\frac{1}{6}-\frac{2\pi^2}{9}\right]$$

$$\gamma_g^{(2)} = C_A^2\left[\frac{8}{3}+3\zeta(3)\right]-C_FT_Rn_f-\frac{4}{3}C_AT_Rn_f$$

$$B_{q\bar{q}\to Z}^{(2)} = C_F^2\left[\pi^2-\frac{3}{4}-12\zeta(3)\right]+C_FC_A\left[\frac{11\pi^2}{9}-\frac{193}{12}+6\zeta(3)\right]+C_FT_Rn_f\left[\frac{17}{3}-\frac{4\pi^2}{9}\right]$$

$$B_{gg\to H}^{(2)H} = C_A^2\left[\frac{23}{6}+\frac{22\pi^2}{9}-6\zeta(3)\right]+4C_FT_Rn_f-C_AT_Rn_f\left[\frac{2}{3}+\frac{8\pi^2}{9}\right]-\frac{11}{2}C_FC_A.$$

$$H_{ab\to X}^{(1)} = \mathcal{A}_{ab\to X}^{(1-\text{ loop })}$$

$$\mathcal{A}_{q\bar{q}\to Z,q\bar{q}'\to W\pm}^{(1-\text{ loop })} = C_F\left(-8+\frac{2\pi^2}{3}\right)$$

$$\mathcal{A}_{gg\to H}^{(1-\text{ loop })} = C_A\left(5+\frac{2\pi^2}{3}\right)-3C_F.$$

$$\frac{\mathrm{d}\sigma_{AB\to Wg}}{\mathrm{d}Q_\perp^2\,\mathrm{d}y_W} = \int_{\tilde{x}_A}^{1}\frac{\mathrm{d}x_A}{x_A}\int_{\tilde{x}_B}^{1}\frac{\mathrm{d}x_B}{x_B}\Bigg[f_{u/A}(\xi_A,\mu_F)f_{\bar{d}/B}(\xi_B,\mu_F)\delta(\hat{s}+\hat{t}+\hat{u}$$

$$-m_W^2)\times\sigma_{u\bar{d}\to W^+}^{(LO)}(s)\frac{1}{Q_\perp^2}\frac{\alpha_sC_F}{2\pi}\frac{\hat{t}^2+\hat{u}^2+2m_W^2\hat{s}}{\hat{s}}\Bigg]$$

$$\frac{1}{Q_\perp^2}\delta(1-z_A)P(z_B)$$

$$\frac{\delta(1-z_A)\delta(1-z_B)}{Q_\perp^2}\left[A^{(1)}\log\frac{Q_\perp^2}{Q^2}+B^{(1)}\right]$$

$$R_{q\bar{q}'\to W}^{(1)} = \frac{C_F}{\pi Q_\perp^2}\Bigg\{\frac{\hat{t}^2+\hat{u}^2+2m_W^2\hat{s}}{\hat{s}}\delta(\hat{s}+\hat{t}+\hat{u}-Q^2)$$

$$-\delta(1-z_A)\delta(1-z_B)\left(2\log\frac{Q^2}{Q_\perp^2}-3\right)$$

$$-\delta(1-z_A)\left(\frac{1+z_B^2}{1-z_B}\right)_+-\delta(1-z_B)\left(\frac{1+z_A^2}{1-z_A}\right)_+\Bigg\}$$

$$\hat{t}^2+\hat{u}^2+2m_W^2\hat{s} = (Q^2-\hat{t})^2+(Q^2-\hat{u})^2$$

$$m_W^2 \equiv Q^2 = \hat{s}+\hat{t}+\hat{u}$$

$$R_{qg\to W}^{(1)} = R_{\bar{q}g\to W}^{(1)} = \frac{1}{4\pi}\Bigg\{-\frac{(\hat{s}+\hat{t})^2+(\hat{t}+\hat{u})^2}{\hat{s}\hat{u}}\delta(\hat{s}+\hat{t}+\hat{u}-Q^2)$$

$$-\delta(1-z_A)\frac{z_B^2+(1-z_B)^2}{Q_\perp^2}\Bigg\}$$

$$R_{gq\to W}^{(1)}=R_{g\bar{q}\to W}^{(1)}=\frac{1}{4\pi}\Bigg\{-\frac{(\hat{s}+\hat{u})^2+(\hat{t}+\hat{u})^2}{\hat{s}\hat{t}}\delta(\hat{s}+\hat{t}+\hat{u}-Q^2)$$

$$-\delta(1-z_B)\frac{z_A^2+(1-z_A)^2}{Q_\perp^2}\Bigg\}$$

$$\hat{s}=\frac{1}{\xi_A\xi_B}Q^2,\hat{t},\hat{u}=\left(1-\frac{\sqrt{1+\frac{Q_\perp^2}{Q^2}}}{\xi_{B,A}}\right)Q^2,Q_\perp^2=\frac{\hat{t}\hat{u}}{\hat{s}}.$$

$$\tilde{W}_{ij}(b_\perp,\dots)\longrightarrow\tilde{W}^{(\mathrm{NP})}(b_\perp,\dots)\tilde{W}_{ij}(b_*,\dots)$$

$$b_*=\frac{b_\perp}{\sqrt{1+(b_\perp/b_{\max})^2}}$$

$$\tilde{W}_{ij}^{(\mathrm{CSS})}(b_\perp^2)=\exp\left[-F_1(b_\perp)\log\left(\frac{Q^2}{Q_0^2}\right)-F_{i/h_1}(x_1,b_\perp)-F_{j/h_1}(x_2,b_\perp)\right]$$

$$\tilde{W}_{ij}^{(\mathrm{DWS})}(b_\perp^2)=\exp\left[-g_1b_\perp^2-g_2b_\perp^2\log\left(\frac{Q}{2Q_0}\right)\right]$$

$$\tilde{W}_{ij}^{(\mathrm{LY})}(b_\perp^2)=\exp\left[-g_1b_\perp^2-g_2b_\perp^2\log\left(\frac{Q}{2Q_0}\right)-g_1g_3b_\perp\log\left(100x_1x_2\right)\right]$$

$$\tilde{W}_{ij}^{(\mathrm{BLNY})}(b_\perp^2)=\exp\left[-g_1b_\perp^2-g_2b_\perp^2\log\left(\frac{Q}{2Q_0}\right)-g_1g_3b_\perp^2\log\left(100x_1x_2\right)\right]$$

$$Q_0=1.6\mathrm{GeV}\text{ and }b_{\max}=0.5\mathrm{GeV}^{-1}$$

$$\int\frac{\mathrm{d}^2b_\perp}{(2\pi)^2}\exp\left(-i\vec{b}_\perp\cdot\vec{Q}_\perp\right)f(b_\perp)=\frac{1}{2\pi}\int_0^\infty\mathrm{d}b_\perp b_\perp J_0(Q_\perp b_\perp)f(b_\perp)$$

$$=\frac{1}{4\pi}\int_0^\infty\mathrm{d}b_\perp b_\perp[h_1(Q_\perp b_\perp,v)+h_2(Q_\perp b_\perp,v)]f(b_\perp)$$

$$h_1(z,v)=-\frac{1}{\pi}\int_{-iv\pi}^{-\pi+iv\pi}\mathrm{d}\theta e^{-iz\sin\,\theta}\text{ and }h_2(z,v)=-\frac{1}{\pi}\int_{\pi+iv\pi}^{-iv\pi}\mathrm{d}\theta e^{-iz\sin\,\theta}$$

$$h_1(z,v)+h_2(z,v)=2J_0(z)$$

$$g_W^2\left|V_{ij}\right|^2=\frac{e^2\left|V_{ij}\right|^2}{\sin^2\,\theta_W}\overset{W\to Z}{\rightarrow}\frac{e^2[(1-4|e_i|\sin^2\,\theta_W)^2+1]\delta_{ij}}{4\sin^2\,\theta_W\cos^2\,\theta_W}$$

$$\sigma_{gg\to H}^{(LO)} = \frac{\sqrt{2}G_F\alpha_s(m_H^2)}{576\pi}$$

$$\vec{Q}_{\perp,X} = \sum_i \vec{q}_{\perp,i}$$

$$\frac{Q^2\,d\sigma_{AB\to X}^{(\mathrm{res})}}{dQ_\perp^2\,dQ^2} = \sum_{ij}\pi\hat{\sigma}_{ij\to X}^{(LO)}\int_0^1 dx_A\,dx_B\int\frac{d^2b_\perp}{(2\pi)^2}J_0(Q_\perp b_\perp)\tilde{W}_{ij}(b_\perp;Q,x_A,x_B)$$

$$\tau = \frac{Q^2}{S}$$

$$\Sigma_{AB\to X}(N) = \int_0^{1-2Q_\perp/Q} d\tau\tau^N\frac{Q^2Q_\perp^2}{\pi\sigma_{AB\to X}^{(LO)}}\frac{d\sigma_{AB\to X}^{(\mathrm{res})}}{dQ_\perp^2\,dQ^2}$$

$$\Sigma_{AB\to X}(N) = \sum_{ij}\left[f_{i/A}(N,\mu_F)f_{j/B}(N,\mu_F)\hat{\Sigma}_{ij}(N)\right]$$

$$\hat{\Sigma}_{ij}(N) = Q_\perp^2\sum_{a,b}\int_0^\infty\frac{b_\perp db_\perp}{(2\pi)}\Big\{J_0(b_\perp Q_\perp)C_{ia}\left(N,\alpha_s(b_0^2/b_\perp^2)\right)C_{jb}\left(N,\alpha_s(b_0^2/b_\perp^2)\right)$$

$$\times\exp\left[-\int_{b_0^2/b_\perp^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\left(A\left(\alpha_s(k_\perp^2)\right)\log\frac{Q^2}{k_\perp^2}\right.\right.$$

$$\left.\left.+B\left(\alpha_s(k_\perp^2)\right)\right)-\int_{b_0^2/b_\perp^2}^{\mu_F^2}\frac{dk_\perp^2}{k_\perp^2}\left(\gamma_{ia}\left(N,\alpha_s(k_\perp^2)\right)+\gamma_{jb}\left(N,\alpha_s(k_\perp^2)\right)\right)\right]\Big\}$$

$$C_{ia}\left(N,\mu,\alpha_s(b_0^2/b_\perp^2)\right) = \mathbf{M}_N\left[C\left(z,\frac{1}{b_\perp},\mu_F\right)\right]$$

$$= \int_0^1 dzz^N\left[\delta_{ia}\delta(1-z)+\frac{\alpha_s(\mu)}{2\pi}\left(-P_{ia}^{\epsilon}(z)+\delta_{ia}\delta(1-z)C_a\frac{\pi^2}{6}\right)\right.$$

$$\left.+\mathcal{O}(\alpha_s^2)\right]\overset{i=a=q}{\to}1+\frac{C_F\alpha_s(\mu)}{2\pi}\left(\frac{1}{(N+1)(N+2)}+\frac{\pi^2}{6}\right)+\mathcal{O}(\alpha_s^2)$$

$$\hat{\Sigma}_{ij}(N) = \frac{Q_\perp^2}{2\pi}\sum_{a,b}\int_0^\infty db_\perp b_\perp J_0(b_\perp Q_\perp)\times\exp\left\{\sum_{n=1}^\infty\sum_{m=0}^{n+1}\left[{}_nD_m(N,L;a,b)\left(\frac{\alpha_s(\mu)}{2\pi}\right)^n\left(\log\frac{Q^2b_\perp^2}{b_0^2}\right)^m\right]\right\}$$

$${}_1D_2(N,L) = -\frac{1}{2}A^{(1)}\;{}_1D_1(N,L) = -B^{(1)}-2\gamma^{(1)}(N)\;{}_1D_0(N,L) = 2\gamma^{(1)}(N)L+2C^{(1)}(N)\;{}_2D_3(N,L)$$

$$= -\frac{1}{3}\beta_0\;{}_2D_2(N,L) = -\frac{1}{2}A^{(2)}+\beta_0\left[\frac{1}{2}A^{(1)}L-\frac{1}{2}B^{(1)}-\gamma^{(1)}(N)\right]\;{}_2D_1(N,L)$$

$$= -B^{(2)}-2\gamma^{(2)}(N)+\beta_0\left[B^{(1)}L+2\gamma^{(1)}(N)L+2C^{(1)}(N)\right]$$

$$\hat{\Sigma}_{ij}(N) = \frac{1}{\pi}\left\{\sum_{n=1}^{\infty}\sum_{m=0}^{n+1}\left[{}_{n}C_{m}(N,L;a,b)\left(\frac{\alpha_S(\mu)}{2\pi}\right)^{n}\left(\log\frac{Q^2}{Q_\perp^2}\right)^{m}\right]\right\}.$$

$$\frac{\mathrm{d}[xJ_1(x)]}{\mathrm{d}x} = xJ_0(x)$$

$$\hat{\Sigma}_{ij}(N) = -\frac{Q_\perp^2}{2\pi}\sum_{a,b}\int_0^{\infty}\mathrm{d}xJ_1(x)\cdot\frac{\mathrm{d}^2}{\mathrm{d}Q_\perp^2}\exp\left\{\sum_{n=1}^{\infty}\sum_{m=0}^{n+1}\left[{}_{n}D_{m}(N,L)\left(\frac{\alpha_s(\mu)}{2\pi}\right)^{n}\left(\log\frac{Q^2x^2}{Q_\perp^2b_0^2}\right)^{m}\right]\right\}$$

$$\int_0^{\infty}\mathrm{d}xJ_1(x)\log^m(x/b_0)$$

$$\int_0^{\infty}\mathrm{d}b_\perp b_\perp J_0(b_\perp Q_\perp)\exp\left[-\int_{b_0^2/b_*^2}^{Q^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\left(A\log\frac{Q^2}{k_\perp^2}+B\right)\right]\tilde{W}_{ij}^{(\text{non-pert.})}(b_\perp)$$

$$\frac{\mathrm{d}}{\mathrm{d}Q_\perp^2}\exp\left[-\int_{Q_\perp^2}^{Q^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\left(\tilde{A}\log\frac{Q^2}{k_\perp^2}+\tilde{B}\right)\right]$$

$$\alpha_s^n\left[\frac{\log^k(1-z)}{1-z}\right]_+.$$

$$\frac{\mathrm{d}\sigma_{AB\to X}}{\mathrm{d}Q^2} = \int_0^1\mathrm{d}\tau\int_0^1\mathrm{d}x_i\,\mathrm{d}x_j f_{i/A}(x_i,\mu_F)f_{j/B}(x_j,\mu_F)\delta\left(\tau-\frac{Q^2}{Sx_ix_j}\right)\cdot W_{ij}^{(\text{thres})}(\tau,Q,\mu_R,\mu_F)$$

$$\mathbf{M}_N\left[W_{ij}^{(\text{thres})}(\tau,Q,\mu_R,\mu_F)\right] \equiv \mathbf{M}_N\left[W_{ij}\right] = \int_0^1\mathrm{d}\tau\tau^N W_{ij}^{(\text{thres})}(\tau,Q,\mu_R,\mu_F)$$

$$\tau \longrightarrow \frac{Q^2}{Sx_ix_j} \approx 1$$

$$\mathrm{d}w(k) = -(4\pi)\alpha_s C_F\frac{\mathrm{d}^3k}{(2\pi^3)(2\epsilon)}\left(\frac{p_a^\mu}{p_ak}-\frac{p_b^\mu}{p_bk}\right)^2$$

$$w^0 + \int\mathrm{d}w(k) = 0$$

$$\mathcal{W}_{\text{eik}} = (1+w^0)\delta(1-\tau) + \int\mathrm{d}w(k)\delta\left(1-\tau-\frac{\epsilon}{E}\right)$$

$$\mathbf{M}_N\left[W_{ij}^{(\text{thres})}\right]^{(1)} = \frac{4C_F}{2\pi}\int_0^1\mathrm{d}z\frac{z^N-1}{1-z}\int_{(1-z)Q^2}^{(1-z)^2Q^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\alpha_s(k_\perp^2)$$

$$q^2 = |(p_a - k)^2| \approx \frac{k_\perp^2}{1-z}$$

$$\mathbf{M}_N\left[W_{ij}^{(\text{thres})}\right]^{(1,LL)} = \frac{4C_F}{2\pi}\int_0^1 \,\mathrm{d}z\frac{z^N-1}{1-z}\int_{(1-z)Q^2}^{(1-z)^2Q^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\alpha_s = \frac{4C_F\alpha_s}{2\pi}\int_0^1 \,\mathrm{d}z\frac{z^N-1}{1-z}\log(1-z)$$

$$= \frac{4C_F\alpha_s}{2\pi}\int_0^1 \,\mathrm{d}z\left[\frac{\log(1-z)}{1-z}\right]_+ = \frac{4C_F\alpha_s}{4\pi}\{\psi'(N)+\zeta(2)+[\psi(N)+\gamma_E]^2\}$$

$$\mathbf{M}_N\left[W_{ij}^{(\text{thres})}\right]^{(1,LL)} \overset{N\to\infty}{\to} \lim_{N\to\infty}\frac{C_F\alpha_s}{\pi}\{\psi'(N)+\zeta(2)+[\psi(N)+\gamma_E]^2\}$$
$$\frac{C_F\alpha_s}{\pi}\left[\frac{\pi^2}{6}+(\log N+\gamma_E)^2\right].$$

$$z^N - 1 \approx \Theta\left(1-\frac{1}{N}-z\right)$$

$$\mathbf{M}_N\left[W_{ij}^{(\text{thres})}\right]^{(1,alt)} \approx \frac{4C_F\alpha_s}{2\pi}\int_0^1 \,\mathrm{d}z\frac{\log(1-z)}{1-z}\Theta\left(1-\frac{1}{N}-z\right) = \frac{4C_F\alpha_s}{2\pi}\int_0^{1-\frac{1}{N}} \,\mathrm{d}z\frac{\log(1-z)}{1-z}$$

$$= \frac{4C_F\alpha_s}{4\pi}\log^2 N$$

$$\mathbf{M}_N\left[W_{ij}^{(\text{thres})}\right]^{(LL)} = \exp\left\{\frac{C_F\alpha_s}{\pi}\left[\frac{\pi^2}{6}+(\log N+\gamma_E)^2\right]\right\}.$$

$$\mathbf{M}_{N\to\infty}\left[W_{ij}^{(\text{thres})}\right] = \exp\left\{\int_0^1 \,\mathrm{d}z\frac{z^N-1}{1-z}\int_{(1-z)Q^2}^{(1-z)^2Q^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}[A(k_\perp^2)+D(k_\perp^2)]\right\}$$

$$A(\mu^2) = \sum_{n=1}^{\infty} A^{(n)}\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^n \text{ and } D(\mu^2) = \sum_{n=2}^{\infty} D^{(n)}\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^n$$

$$A^{(1)} = 2C_F, A^{(2)} = 2C_F K, \text{ and } D^{(1)} = 0$$

$$K = C_A\left(\frac{67}{18}-\frac{\pi^2}{6}\right)-\frac{10}{9}T_R n_f$$

29. Ecuaciones BFKL.

$$|f\rangle = \hat{S}|i\rangle$$

$$\hat{S} = \hat{\mathbf{1}} + i\hat{T}$$

$$1 \overset{!}{=} \hat{S}^\dagger\hat{S} = 1 + i(\hat{T}-\hat{T}^\dagger) + \hat{T}^\dagger\hat{T}$$

$$i\hat{T}^\dagger\hat{T} = (\hat{T}-\hat{T}^\dagger) = \mathfrak{Im}(\hat{T})$$

$$\langle f|\hat{T}|i\rangle = T_{fi} = (2\pi)^4\delta^4\left(\sum p_f^\mu - \sum p_i^\mu\right)\mathcal{T}_{fi}$$

$$\left(\mathcal{T}_{fi}-\mathcal{T}_{if}^{*}\right)=i\sum_{n}\left[(2\pi)^{4}\delta^{4}\left(\sum p_{n}^{\mu}-\sum p_{i}^{\mu}\right)\mathcal{T}_{fn}^{*}\mathcal{T}_{ni}\right]$$

$$\sigma_{\text{tot}}=\frac{1}{2s}\sum_{n}\left[(2\pi)^{4}\delta^{4}\left(\sum p_{n}^{\mu}-\sum p_{i}^{\mu}\right)\mathcal{T}_{in}^{*}\mathcal{T}_{ni}\right]\propto\frac{1}{2s}\mathfrak{Im}(\mathcal{T}_{ii})$$

$$\mathfrak{Im}[\mathcal{A}]=\frac{\mathcal{A}(\hat{s},\hat{t})-\mathcal{A}^{*}(\hat{s},\hat{t})}{2i}$$

$$\mathcal{A}^{*}(\hat{s},\hat{t})=\mathcal{A}(\hat{s}^{*},\hat{t})$$

$$\mathfrak{Im}[\mathcal{A}]=\lim_{\epsilon\to0}\frac{\mathcal{A}(\hat{s}+i\epsilon,\hat{t})-\mathcal{A}(\hat{s}-i\epsilon,\hat{t})}{2i}=\mathfrak{Disc}[\mathcal{A}(\hat{s},\hat{t})]$$

$$\mathcal{A}(\hat{s},\hat{t})=\int_{-\infty}^{0}\frac{\mathrm{d}s'}{2\pi i}\frac{\mathfrak{Disc}[\mathcal{A}(s',\hat{t})]}{s'-\hat{s}}+\int_{-\hat{t}}^{\infty}\frac{\mathrm{d}s'}{2\pi i}\frac{\mathfrak{Disc}[\mathcal{A}(s',\hat{t})]}{s'-\hat{s}}$$

$$z_{t}=-\cos\,\theta_{t}=-\left(1+\frac{2\hat{s}}{\hat{t}}\right)$$

$$\mathcal{A}(\hat{s},\hat{t})=\int_{-\infty}^{-1}\frac{\mathrm{d}z_{t}'}{2\pi i}\frac{\mathfrak{Disc}[\mathcal{A}(z_{t}',\hat{t})]}{z_{t}'-z_{t}}+\int_{1}^{\infty}\frac{\mathrm{d}s'}{2\pi i}\frac{\mathfrak{Disc}[\mathcal{A}(z_{t}',\hat{t})]}{z_{t}'-z_{t}}$$

$$\mathcal{A}(\hat{s},\hat{t})=\sum_{l}(2l+1)\mathcal{A}_{l}(\hat{s},\hat{t})P_{l}(z_{t})$$

$$Q_{l}(z')=\frac{1}{2}\int_{-1}^{1}\frac{\mathrm{d}z'}{z'-z}P_{l}(z)$$

$$\mathcal{A}_{l}(\hat{s},\hat{t})=[1+(-1)^{l+L}]\int_{1}^{\infty}\frac{\mathrm{d}z'}{2\pi i}Q_{l}(z')\mathfrak{Disc}[\mathcal{A}(z',\hat{t})]$$

$$z_{t}\longrightarrow-\frac{2\hat{s}}{\hat{t}}\longrightarrow\infty$$

$$\mathcal{A}(\hat{s},\hat{t})=\int_{\delta-i\infty}^{\delta+i\infty}\frac{\mathrm{d}l}{2i}(2l+1)[1+(-1)^{l+L}]\int_{1}^{\infty}\frac{\mathrm{d}z_{t}'}{2\pi i}\frac{P_{l}(-z_{t})Q_{l}(z_{t}')}{\sin\,(\pi l)}\mathfrak{Disc}[\mathcal{A}(z_{t}',\hat{t})]$$

$$\longrightarrow-\frac{1}{4\pi}\int_{\delta-i\infty}^{\delta+i\infty}\mathrm{d}l\,\frac{(-1)^{l}+(-1)^{L}}{\sin\,(\pi l)}e^{ly}\mathcal{F}_{l}(\hat{t})$$

$$P_{l}(z)\longrightarrow\frac{1}{\sqrt{\pi}}(2z)^{l}\frac{\Gamma\left(l+\frac{1}{2}\right)}{\Gamma(l+1)}$$
$$Q_{l}(z)\longrightarrow\frac{1}{\sqrt{\pi}}(2z)^{-(l+1)}\frac{\Gamma(l+1)}{\Gamma\left(l+\frac{3}{2}\right)}$$

$$\mathcal{F}_{l}(\hat{t})=\int_{0}^{\infty}\mathrm{d}y\,e^{-ly}\mathfrak{Disc}[\mathcal{A}(z_{t}',\hat{t})]$$

$$p^{\mu} = \alpha P_{+}^{\mu} + \beta P_{-}^{\mu} + p_{\perp}^{\mu}$$

$$p_a = \sqrt{s}(x_A, 0; \vec{0}) \text{ and } p_b = \sqrt{s}(0, x_B; \vec{0})$$

$$k_i^{\mu} = (k_{0,i} + k_{3,i}, k_{0,i} - k_{3,i}; \vec{k}_{\perp}) \overset{m=0}{\rightarrow} (k_{\perp,i}e^{y_i}, k_{\perp,i}e^{-y_i}; \vec{k}_{\perp,i}),$$

$$g^{\mu\nu} = 2\frac{p_a^{\mu}p_b^{\nu} + p_a^{\nu}p_b^{\mu}}{\hat{s}} - \delta_{\perp}^{\mu\nu}$$

$$\frac{\mathrm{d}^3 k}{(2\pi)^2(2E)} = \frac{\mathrm{d}^2 k_{\perp}}{(2\pi)^2}\frac{\mathrm{d}y}{4\pi}$$

$$\bar{y} = \frac{y_0 + y_1}{2} = \frac{1}{2}\log(x_A/x_B)$$

$$y^* = \frac{y_0 - y_1}{2}$$

$$x_A = \frac{k_{\perp}}{\sqrt{s}}(e^{y_0} + e^{y_1}) = \frac{2k_{\perp}}{\sqrt{s}}e^{\bar{y}}\cosh\ y^*$$
$$x_B = \frac{k_{\perp}}{\sqrt{s}}(e^{-y_0} + e^{-y_1}) = \frac{2k_{\perp}}{\sqrt{s}}e^{-\bar{y}}\cosh\ y^*$$

$$\hat{s} = 4k_{\perp}^2\cosh^2\ y^*, \hat{t} = -2k_{\perp}^2\cosh\ y^* e^{-y^*}, \text{ and } \hat{u} = -2k_{\perp}^2\cosh\ y^* e^{y^*}$$

$$\frac{1}{(4\pi\alpha_s)^2}\left|\mathcal{M}_{qq'\to qq'}\right|^2 = \frac{C_F^2}{4}\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} = \frac{C_F^2}{4}\frac{4\cosh^2\ y^* + e^{2y^*}}{e^{-2y^*}} = \frac{C_F^2}{4}\left(e^{y^*}\cosh\ y^*\right)^2$$

$$\frac{\mathrm{d}\hat{\sigma}_{qq'\to qq'}}{\mathrm{d}\hat{t}} = \frac{(4\pi\alpha_s)^2\left|\mathcal{M}_{qq'\to qq'}\right|^2}{16\pi\hat{s}^2} = \frac{\pi C_F^2\alpha_s^2}{64k_{\perp}^2}e^{2y^*}$$

$$\sigma = \int_{\zeta_A}^{1}\mathrm{d}x_A\int_{\zeta_B}^{1}\mathrm{d}x_B f_{q/A}(x_A,\mu_F^2)f_{\bar{q}/B}(x_B,\mu_F^2)\hat{\sigma}_{qq'\to qq'}(\mu_F^2;\mu_R^2)$$

$$\frac{\mathrm{d}\sigma_{qq'\to qq'}}{\mathrm{d}k_{\perp}^2\,\mathrm{d}y_0\,\mathrm{d}y_1} = x_A f_{a/A}(x_A,\mu_F^2)x_B f_{b/B}(x_B,\mu_F^2) = x_A f_{a/A}(x_A,\mu_F^2)x_B f_{b/B}(x_B,\mu_F^2)\frac{\pi\alpha_s^2}{36k_{\perp}^2}e^{2y^*}$$

$$y^* \to \frac{1}{2}\log\left(-\frac{\hat{s}}{\hat{t}}\right)$$

$$\left|\mathcal{M}_{qq'\to qq'}\right|^2 = \left|\mathcal{M}_{qq\to qq}\right|^2 = \left|\mathcal{M}_{q\bar{q}\to q\bar{q}}\right|^2 = \frac{C_F^2 g_s^4}{2}\frac{\hat{s}^2}{\hat{t}^2}$$

$$\left|\mathcal{M}_{qg\to qg}\right|^2 = \frac{C_F C_A}{2}g_s^4\frac{\hat{s}^2}{\hat{t}^2}$$

$$\left|\mathcal{M}_{gg\to gg}\right|^2 = \frac{C_A^2 g_s^4}{2}\frac{\hat{s}^2}{\hat{t}^2}$$

$$\mathcal{M}_{qq'\to qq'} = [\bar{u}_i(k_0)(-ig_sT^a_{ij})\gamma_\mu\bar{u}_j(p_a)]\frac{-ig^{\mu\nu}}{\hat{t}}[\bar{u}_k(k_1)(-ig_sT^a_{kl})\gamma_\nu\bar{u}_l(p_b)]$$

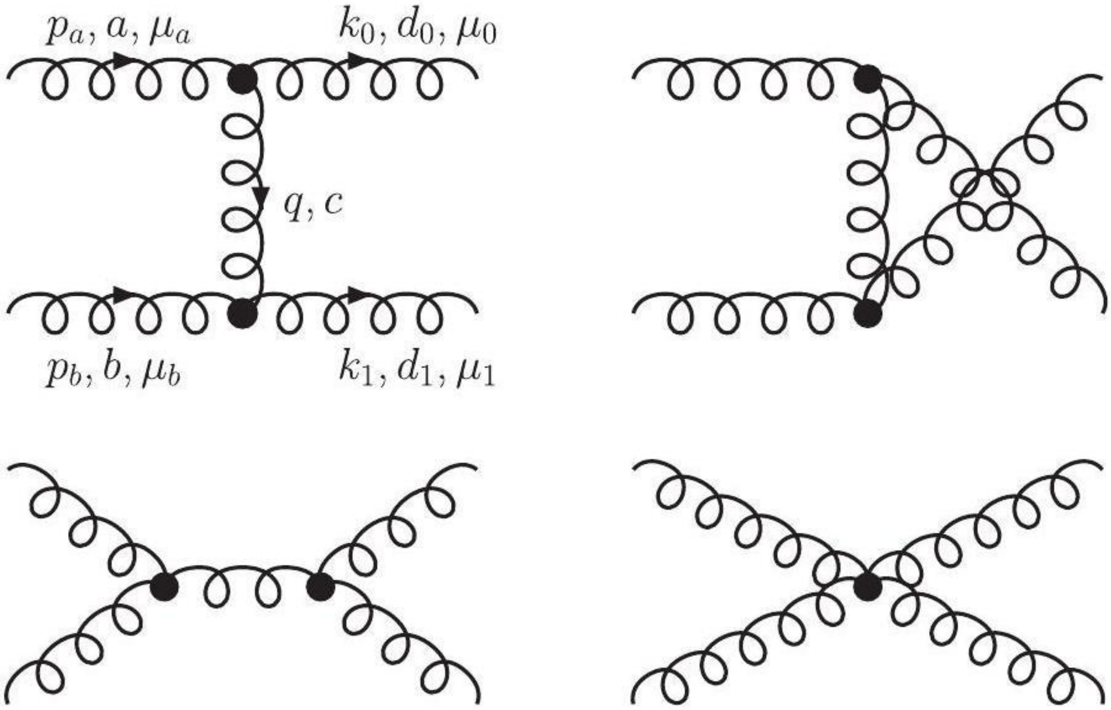


Figura 26. Vórtices y vértices de entrelazamiento de una partícula supermasiva.

$$\mathcal{M}_{qq'\to qq'} = [\bar{u}_i(k_0)(-ig_sT^a_{ij})\gamma_\mu\bar{u}_j(p_a)]\frac{-2ip_b^\mu p_a^\nu}{\hat{s}\hat{t}}[\bar{u}_k(k_1)(-ig_sT^a_{kl})\gamma_\nu\bar{u}_l(p_b)].$$

$$\left|\overline{\mathcal{M}}_{qq'\to qq'}\right|^2 = \frac{4g_s^4\left(\mathrm{Tr}[T^aT^b]\right)^2}{4\cdot 9}\frac{16[2(k_0p_b)(p_ap_b)][2(k_1p_a)(p_ap_b)]}{\hat{s}^2\hat{t}^2} = \frac{g_s^4\left(T_R\delta^{ab}\right)^2}{9}\frac{4\hat{u}^2}{\hat{t}^2} = \frac{8g_s^4}{9}\frac{\hat{u}^2}{\hat{t}^2}$$

$$\approx \frac{8g_s^4}{9}\frac{\hat{s}^2}{\hat{t}^2}$$

$$\begin{aligned}\mathcal{M}_{gg\to gg} =& ig_sf^{ad_0c}\left[g_{\mu_a\mu_0}(p_a+k_0)_\xi + g_{\mu_0\xi}(-k_0+q)_{\mu_a} + g_{\xi\mu_a}(-q-p_a)_{\mu_0}\right]\\ &\cdot ig_sf^{bd_1c}\left[g_{\mu_b\zeta}(p_b-q)_{\mu_1} + g_{\zeta\mu_1}(q+k_1)_{\mu_b} + g_{\mu_1\mu_b}(-k_1-p_b)_\zeta\right]\\ &\cdot\frac{-ig^{\xi\zeta}}{q^2}\cdot\epsilon_{\lambda_a}^{\mu_a*}(p_a)\epsilon_{\lambda_b}^{\mu_b*}(p_b)\epsilon_{\lambda_0}^{\mu_0}(k_0)\epsilon_{\lambda_1}^{\mu_1}(k_1)\end{aligned}$$

$$\approx -i\left(2g_sf^{ad_0c}g_{\mu_a\mu_0}p_{\xi,a}\right)\left(2g_sf^{bd_1c}g_{\mu_1\mu_b}p_{\zeta,b}\right)\frac{2p_a^\zeta p_b^\xi}{\hat{s}\hat{t}}\cdot\epsilon_{\lambda_a}^{\mu_a*}\epsilon_{\lambda_b}^{\mu_b*}\epsilon_{\lambda_0}^{\mu_0}\epsilon_{\lambda_1}^{\mu_1}$$

$$\approx -ig_s^2f^{ad_0c}f^{bd_1c}g_{\mu_a\mu_0}g_{\mu_1\mu_b}\frac{2\hat{s}}{\hat{t}}\cdot\epsilon_{\lambda_a}^{\mu_a*}\epsilon_{\lambda_b}^{\mu_b*}\epsilon_{\lambda_0}^{\mu_0}\epsilon_{\lambda_1}^{\mu_1}$$

$$f^{ad_0c}f^{bd_1c}f^{d_0ac'}f^{d_1bc'} = C_A^2(N_c^2-1)$$

$$\sum_\lambda \epsilon_\lambda^\mu(p)\epsilon_\lambda^{*\mu'}(p) = -\left(g^{\mu\nu} - \frac{n^\mu p^{\mu'} + n^{\mu'}p^\mu}{(n\cdot p)} + \frac{n^2p^\mu p^{\mu'}}{(n\cdot p)^2}\right)$$

$$\sum_{\lambda_a} \epsilon^{\mu}_{\lambda_a}(p_a)\epsilon^{*\mu'}_{\lambda_a}(p_a) = -\left(g^{\mu\mu'} - 2\frac{p_b^{\mu}p_a^{\mu'} + p_b^{\mu'}p_a^{\mu}}{\hat{s}}\right) \equiv \delta_{\perp}^{\mu\mu'}$$

$$g_{\mu_a\mu_0}g_{\mu'_a\mu'_0}\left[\sum_{\lambda_a} \epsilon^{\mu_a}_{\lambda_a}(p_a)\epsilon^{*\mu'_a}_{\lambda_a}(p_a)\right]\left[\sum_{\lambda_0} \epsilon^{\mu_0}_{\lambda_0}(k_0)\epsilon^{*\mu'_0}_{\lambda_0}(k_0)\right] = 2\left[1 + \mathcal{O}\left(\frac{\hat{t}}{\hat{s}}\right)\right].$$

$$\left|\overline{\mathcal{M}}_{gg\to gg}\right|^2 = \frac{4C_A^2 g_s^4}{N_c^2 - 1}\frac{\hat{s}^2}{\hat{t}^2} = \frac{9g_s^4}{2}\frac{\hat{s}^2}{\hat{t}^2}$$

$$\mathrm{d}\Phi_2 = \frac{\mathrm{d}y_0\,\mathrm{d}k_{\perp,0}^2}{4\pi(2\pi)^2}\frac{\mathrm{d}y_1\,\mathrm{d}k_{\perp,1}^2}{4\pi(2\pi)^2}\cdot(2\pi)^4\delta^4(p_a + p_b - k_0 - k_1)$$

$$= \frac{1}{2\hat{s}}\frac{\cosh(y_0 - y_1) + 1}{\sinh(y_0 - y_1)}\frac{\mathrm{d}k_{\perp,0}^2}{(2\pi)^2}\frac{\mathrm{d}k_{\perp,1}^2}{(2\pi)^2}(2\pi)^2\delta^2\left(\vec{k}_{\perp,0} + \vec{k}_{\perp,1}\right) \approx \frac{1}{2\hat{s}}\frac{\mathrm{d}k_{\perp}^2}{(2\pi)^2}$$

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}k_{\perp}^2} = \frac{|\overline{\mathcal{M}}|^2}{16\pi\hat{s}^2}$$

30. Cinemática Multi-Regge.

$$\vec{0} = \sum_{i=0}^{n+1} \vec{k}_{\perp,i}, x_a = \sum_{i=0}^{n+1} \frac{k_{\perp,i}e^{y_i}}{\sqrt{s}}, \text{ and } x_b = \sum_{i=0}^{n+1} \frac{k_{\perp,i}e^{-y_i}}{\sqrt{s}}$$

$$\hat{s} = x_a x_b s = \sum_{i,j=0}^{n+1} k_{\perp,i}k_{\perp,j}e^{-(y_i - y_j)} \approx k_{\perp,0}k_{\perp,n+1}e^{y_0 - y_{n+1}}$$

$$\hat{s}_{ij} = 2k_ik_j = 2k_{\perp,i}k_{\perp,j}\left[\cosh\left(y_i - y_j\right) - \cos\left(\phi_i - \phi_j\right)\right] \approx k_{\perp,i}k_{\perp,j}e^{|y_i - y_j|}$$

$$\hat{t}_{ai} = -2p_ak_i = -\sum_{j=0}^{n+1} k_{\perp,i}k_{\perp,j}e^{-(y_i - y_j)} \approx -k_{\perp,0}k_{\perp,i}e^{y_0 - y_i}$$

$$\hat{t}_{bi} = -2p_bk_i = -\sum_{j=0}^{n+1} k_{\perp,i}k_{\perp,j}e^{y_i - y_j} \approx -k_{\perp,i}k_{\perp,n+1}e^{y_i - y_{n+1}}$$

$$y_0 \gg y_1 \gg y_2 \gg \cdots \gg y_n \gg y_{n+1} \text{ and } k_{\perp,i} \approx k_{\perp}\forall i$$

$$\hat{s} \gg \hat{s}_{ij} \gg k_{\perp}^2$$

$$q_1 = p_a - k_0 \approx \sqrt{s}\left(\frac{k_{\perp,0}e^{y_0} + k_{\perp,1}e^{y_1}}{\sqrt{s}}, 0; \vec{0}\right) - \left(k_{\perp,0}e^{y_0}, k_{\perp,0}e^{-y_0}; \vec{k}_{\perp,i}\right)$$

$$= \left(k_{\perp,1}e^{y_1}, k_{\perp,0}e^{-y_0}; -\vec{k}_{\perp,0}\right)(5.173)$$

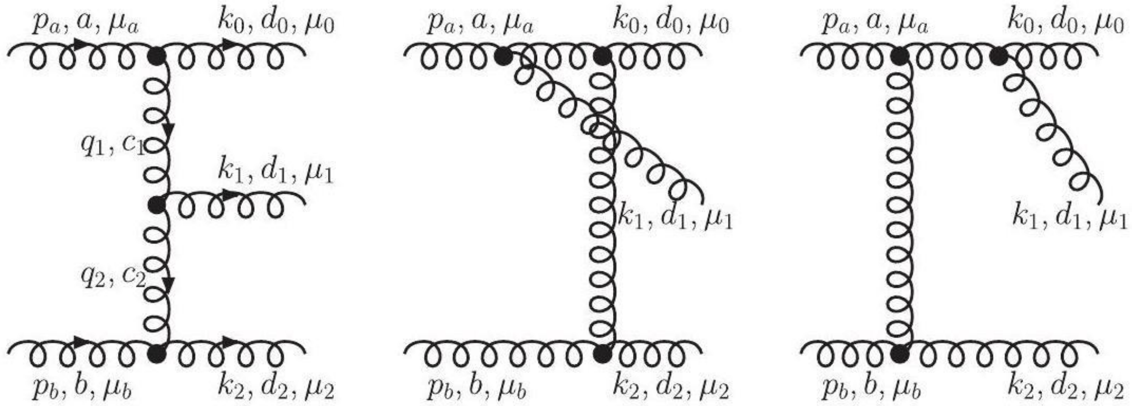
$$q_1^2 = \hat{t}_1 = -k_{\perp,1}k_{\perp,0}e^{y_1 - y_0} - k_{\perp,0}^2 \approx -k_{\perp,0}^2 = q_{\perp,1}^2$$

$$\hat{t}_i = q_i^2 \approx -q_{\perp,i}^2$$

$$\mathcal{M}_{gg\to ggg} = \left(2ig_s f^{ad_0c_1} g_{\mu_a\mu_0} p_a^{\xi_1}\right)\frac{1}{\hat{t}_1}$$

$$\cdot\left(-ig_s f^{c_1c_2d_1}\right)\left[g_{\xi_1\xi_2}(q_1+q_2)_{\mu_1} + g_{\xi_2\mu_1}(-q_2+k_1)_{\xi_1} + g_{\mu_1\xi_1}(-k_1-q_1)_{\xi_2}\right]$$

$$\cdot\left(2ig_s f^{ad_2c_2} g_{\mu_a\mu_0} p_b^{\xi_2}\right)\frac{1}{\hat{t_2}}$$



$$\tilde{C}_{\mu_1} = \frac{2}{\hat{s}} p_a^{\xi_1} p_b^{\xi_2}\left[g_{\xi_1\xi_2}(q_1+q_2)_{\mu_1} + g_{\xi_2\mu_1}(-q_2+k_1)_{\xi_1} + g_{\mu_1\xi_1}(-k_1-q_1)_{\xi_2}\right]$$

$$\approx (q_1+q_2)_{\perp,\mu_1} - \frac{\hat{t}_{a1}}{\hat{s}} p_{\mu_1,b} + \frac{\hat{t}_{b1}}{\hat{s}} p_{\mu_1,a}$$

$$C^{\mu_1}(q_1,q_2) = (q_1+q_2)_\perp^{\mu_1} - \left(\frac{\hat{t}_{a1}}{\hat{s}} + \frac{2\hat{t}_2}{\hat{t}_{b1}}\right)p_b^{\mu_1} + \left(\frac{\hat{t}_{b1}}{\hat{s}} + \frac{2\hat{t}_1}{\hat{t}_{a1}}\right)p_a^{\mu_1}.$$

$$\mathcal{M}_{gg\to ggg} = 2i\hat{s}\epsilon^{\mu_a *}(p_a)\epsilon^{\mu_b *}(p_b)\epsilon^{\mu_0}(k_0)\epsilon^{\mu_1}(k_1)\epsilon^{\mu_2}(k_2)$$

$$\times\left[ig_s f^{ad_0c_1} g_{\mu_a\mu_0}\right]\frac{1}{\hat{\hat{t}_1}}\left[ig_s f^{c_1d_1c_2} C_{\mu_1}(q_1,q_2)\right]\frac{1}{\hat{\hat{t}_2}}\left[ig_s f^{ad_0c_1} g_{\mu_a\mu_0}\right]$$

$$C^{\mu_1}C_{\mu_1} = (q_1+q_2)_\perp^2 - \left(\frac{\hat{t}_{a1}\hat{t}_{b1}}{2\hat{s}} + \hat{t}_2 + \hat{t}_1 + \frac{2\hat{t}_1\hat{t}_2\hat{s}}{\hat{t}_{a1}\hat{t}_{b1}}\right)$$

$$\approx 2\vec{q}_{\perp,1}\vec{q}_{\perp,2} - \frac{k_{\perp,1}^2}{2} - 2\frac{q_{\perp,1}^2 q_{\perp,2}^2}{k_{\perp,1}^2} = \frac{4q_{\perp,1}^2 q_{\perp,2}^2}{k_{\perp,1}^2}$$

$$\hat{t}_{a1}\hat{t}_{b1} \approx k_{\perp,0}k_{\perp,n+1}k_{\perp,1}^2 e^{y_0-y_{n+1}} \approx k_{\perp,1}^2\hat{s}$$

$$\left|\overline{\mathcal{M}}_{gg\to ggg}\right|^2 = \frac{16C_A^3 g_s^6}{N_c^2-1}\frac{\hat{s}^2}{k_{\perp,0}^2 k_{\perp,1}^2 k_{\perp,2}^2}$$

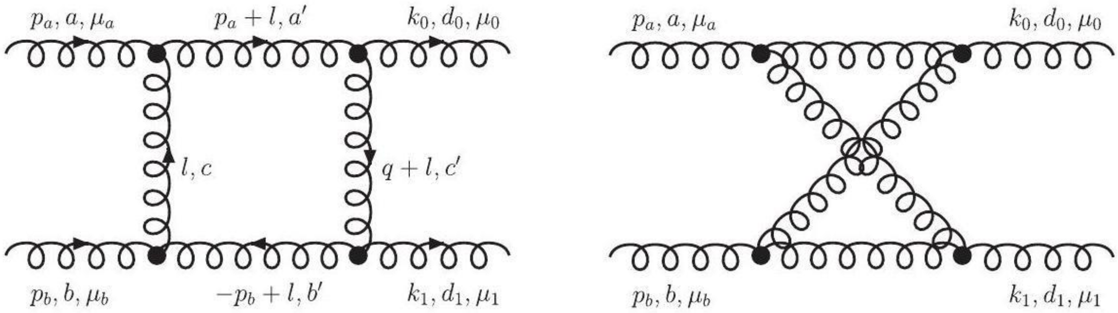
$$\left|\overline{\mathcal{M}}_{gg\to ggg}\right|_{\text{exact}}^2 = 4(\pi\alpha_s C_A)^3 \sum_{i>j}\hat{s}_{ij}^4 \sum_{\text{non-cycl}} \frac{1}{\hat{s}_{a0}\hat{s}_{01}\hat{s}_{12}\hat{s}_{2b}\hat{s}_{ba}},$$

$$\left|\overline{\mathcal{M}}_{gg\to gg}\right|^2 = \frac{4C_A^2 g_s^4}{N_c^2-1}\frac{\hat{s}^2}{k_{\perp,0}^2 k_{\perp,1}^2}$$

$$\mathrm{d}\Phi_3 = \left[\prod_{i=0}^{3}\frac{\mathrm{d}y_i\ \mathrm{d}^2k_{\perp,i}}{4\pi(2\pi)^2}\right]\cdot(2\pi)^4\delta^4\left(p_a+p_b-\sum_{i=0}^{2}k_i\right)$$

$$= \frac{1}{2\hat{s}}\frac{\mathrm{d}^2k_{\perp,0}}{(2\pi)^2}\frac{\mathrm{d}y_1\ \mathrm{d}^2k_{\perp,1}}{4\pi(2\pi)^2}\frac{\mathrm{d}^2k_{\perp,2}}{(2\pi)^2}\cdot(2\pi)^2\delta^2\left(\sum_{i=0}^{2}\vec{k}_{\perp,i}\right)$$

$$\frac{\mathrm{d}\hat{\sigma}_{gg\to ggg}}{\mathrm{d}k_{\perp,0}^2\ \mathrm{d}k_{\perp,2}^2\ \mathrm{d}\phi} = \frac{C_A^3\alpha_s^3}{4\pi}\frac{|y_0-y_2|}{k_{\perp,0}^2k_{\perp,2}^2\left(k_{\perp,0}^2+k_{\perp,0}^2+2k_{\perp,0}k_{\perp,2}\cos\ \phi_{02}\right)}$$



$$\mathcal{M} = \epsilon_{\mu_{a*}}(p_a)\epsilon_{\mu_{b*}}(p_b)\epsilon_{\mu_0}(k_0)\epsilon_{\mu_1}(k_1)$$
$$\times\int\frac{\mathrm{d}^4l}{(2\pi)^4}\left\{\frac{g_s^4 f^{aa'c}f^{a'd_0c'}f^{cb'b}f^{c'd_1b'}}{(p_a+l)^2l^2(p_b-l)^2(q-l)^2}\cdot g_{\mu_a'\mu_0'}g_{\mu_a''\mu_b''}g_{\mu_b'\mu_1'}g_{\mu_0''\mu_1''}\right.$$
$$\times\left[\left(2g^{\mu_a\mu_{a'}}p_a^{\mu_{a''}} - 2g^{\mu_{a'}\mu_{a''}}l^{\mu_a} + g^{\mu_{a''}\mu_a}(l-p_a)^{\mu_{a'}}\right)\right]$$
$$\times\left[\left(g^{\mu_{0'}\mu_0}(p_a+k_0+l)^{\mu_{0''}} + g^{\mu_0\mu_{0''}}(p_a-2k_0+l)^{\mu_{0'}} - 2g^{\mu_{0''}\mu_{0'}}p_a^{\mu_0}\right)\right]$$

$$\times\left[\left(g^{\mu_{b'}\mu_b}(l-2p_b)^{\mu_{b''}} + g^{\mu_b\mu_{b''}}(p_b+l)^{\mu_{b'}}\right.\right.$$

$$\left.\left. - 2g^{\mu_{b''}\mu_{b'}}l^{\mu_b}\right)\right]\times\left[\left(g^{\mu_{1''}\mu_1}(p_a-k_0+l+k_1)^{\mu_{1'}} + g^{\mu_1\mu_{1'}}(l-k_1-p_b)^{\mu_{1''}}\right)\right]\Big\}$$

$$\approx \epsilon_{\mu_{a*}}(p_a)\epsilon_{\mu_{b*}}(p_b)\epsilon_{\mu_0}(k_0)\epsilon_{\mu_1}(k_1)$$

$$\times\int\frac{\mathrm{d}^4l}{(2\pi)^4}\left\{\frac{g_s^4 f^{aa'}c}{(p_a+l)^2l^2(p_bc'-l)^2(q-l)^2}f^{c'd_1b'}\right.$$

$$\times\left[\left(2g^{\mu_a\mu_{a'}}p_a^{\mu_{a''}} - g^{\mu_{a'}\mu_a}p_a^{\mu_{a'}}\right)\right]\left[\left(2g^{\mu_{0'}\mu_0}_{\mu_a''\mu_b''}g_{\mu_b'\mu_1'}g_{\mu_0''\mu_1''}\right.\right.$$

$$\times\left[\left(g^{\mu_b\mu_{b''}}p_b^{\mu_{b'}} - 2g^{\mu_0\mu_{0''}}p_a^{\mu_{0'}}\right)\right] = g_s^4 f^{aa'c}f_b^{a'd_0c'}f^{cb'b}f^{c'd_1b'}\cdot\mathcal{J}$$

$$g_{\mu\nu} = 2\frac{p_{\mu,a}p_{\nu,b}+p_{\mu,b}p_{\nu,a}}{\hat{s}} - \delta_{\mu\nu,\perp}$$

$$\mathcal{I} = 4\hat{s}^2 \int \frac{d^4 l}{(2\pi^4)} \left[\frac{1}{(p_a + l)^2} \cdot \frac{1}{l^2} \cdot \frac{1}{(p_b - l)^2} \cdot \frac{1}{(q - l)^2} \right]$$

$$= 2\hat{s}^3 \int \frac{d\alpha\, d\beta\, d^2 k_\perp}{(2\pi)^4} \left[\frac{1}{(1+\alpha)\beta\hat{s} - k_\perp^2 + i\varepsilon} \cdot \frac{1}{\alpha\beta\hat{s} - k_\perp^2 + i\varepsilon} \cdot \frac{1}{\alpha(\beta - 1)\hat{s} - k_\perp^2 + i\varepsilon} \right.$$

$$\left. \cdot \frac{1}{\alpha\beta\hat{s} - (q_\perp - k_\perp)^2 + i\varepsilon} \right]$$

$$l^\mu = \alpha p_a^\mu + \beta p_b^\mu + l_\perp^\mu \text{ and } d^4 l = \frac{\hat{s}}{2}\, d\alpha\, d\beta\, d^2 l_\perp$$

$$\mathcal{I} \approx -2i\hat{s}^2 \int \frac{d\beta\, d^2 k_\perp}{(2\pi)^4} \left[\frac{1}{\beta\hat{s} - k_\perp^2 + i\varepsilon} \cdot \frac{1}{k_\perp^2 + i\varepsilon} \cdot \frac{1}{(q_\perp - k_\perp)^2 + i\varepsilon} \right]$$

$$\mathcal{M} \approx g^{\mu_a\mu_0} g^{\mu_b\mu_1} \epsilon_{\mu_a *}(p_a) \epsilon_{\mu_b *}(p_b) \epsilon_{\mu_0}(k_0) \epsilon_{\mu_1}(k_1) \times \frac{16\pi\alpha_s}{C_A} \cdot f^{aa'c} f^{a'd_0c'} f^{cb'b} f^{c'd_1b'}$$

$$\cdot \frac{\hat{s}}{-\hat{t}} \log \frac{\hat{s}}{-\hat{t}} \alpha(\hat{t})$$

$$\alpha(\hat{t}) = \alpha_s C_A \hat{t} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{1}{k_\perp^2 (q - k)_\perp^2}$$

$$\alpha(\hat{t}) \approx -\frac{\alpha_S C_A}{4\pi} \log \frac{q_\perp^2}{\mu^2}$$

$$\mathcal{M} \approx -\frac{16\pi\alpha_s}{C_A} \frac{\hat{s}}{-\hat{t}} \alpha(\hat{t}) g^{\mu_a\mu_0} g^{\mu_b\mu_1} \epsilon_{\mu_a *} \epsilon_{\mu_b *} \epsilon_{\mu_0} \epsilon_{\mu_1} f^{aa'c} f^{a'd_0c'}$$

$$\times \left[\log \frac{\hat{s}}{-\hat{t}} f^{cb'b} f^{c'd_1b'} - \left(\log \frac{\hat{s}}{-\hat{t}} + i\pi \right) f^{c'b'b} f^{cd_1b'} \right]$$

$$8 \otimes 8 = [8 \otimes 8]_\mathrm{S} + [8 \otimes 8]_\mathrm{A}$$

$$[8 \otimes 8]_S = \mathbf{1} \oplus \mathbf{8_S} \oplus \mathbf{27}$$
$$[\mathbf{8} \otimes \mathbf{8}]_\mathbf{S} = \mathbf{8_A} \oplus \mathbf{10} \oplus \overline{\mathbf{10}}$$

$$\hat{P}_{bd_1}^{ad_0}(\mathbf{1}) = \frac{1}{N_c^2 - 1} \delta^{ad_0} \delta_{bd_1}$$
$$\hat{P}_{bd_1}^{ad_0}(\mathbf{8_A}) = \frac{1}{N_c} f^{acd_0} f_{bcd_1}$$
$$\hat{P}_{bd_1}^{ad_0}(\mathbf{10} \oplus \overline{\mathbf{10}}) = \frac{1}{2}\left(\delta^a{}_b \delta_{d_1}^{d_0}\right) - \frac{1}{N_c} f^{acd_0} f_{bcd_1}$$

$$\mathcal{M} \approx -8\pi\alpha_s \alpha(\hat{t}) \frac{\hat{s}}{-\hat{t}} \log \frac{\hat{s}}{-\hat{t}} g^{\mu_a\mu_0} g^{\mu_b\mu_1} f^{acd_0} f^{bcd_1}$$

$$\frac{1}{\hat{t}_i} \longrightarrow \frac{1}{\hat{t}_i} \cdot \left(-\frac{\hat{s}_{i-1,i}}{\hat{t}_i} \right)^{\alpha(\hat{t}_i)} \approx \frac{1}{\hat{t}_i} \cdot \exp\left[\alpha(\hat{t}_i)(y_{i-1} - y_i)\right]$$

$$\mathcal{M}_{gg\to(n+2)g} = 2i\hat{s}\left[ig_s f^{ad_0c_1} g_{\mu_a\mu_0}\right]\epsilon^{\mu_a *}(p_a)\epsilon^{\mu_0}(k_0) \times \frac{1}{\hat{t}_1}\exp\left[\alpha(\hat{t}_1)(y_0-y_1)\right]$$

$$\times\left[ig_s f^{c_1d_1c_2} C_{\mu_1}(q_1,q_2)\right]\epsilon^{\mu_1}(k_1) \times \frac{1}{\hat{t}_2}\exp\left[\alpha(\hat{t}_2)(y_1-y_2)\right]$$

$$\times\left[ig_s f^{c_2d_2c_3} C_{\mu_2}(q_2,q_3)\right]\epsilon^{\mu_2}(k_2) \times \frac{1}{\hat{t}_{n+1}}\exp\left[\alpha(\hat{t}_{n+1})(y_n-y_{n+1})\right]$$

$$\times\left[ig_s f^{c_nbd_{n+1}} g_{\mu_b\mu_{n+1}}\right]\epsilon^{\mu_b *}(p_b)\epsilon^{\mu_{n+1}}(k_{n+1})$$

$$\frac{i}{k^2} \longrightarrow 2\pi\delta(k^2)$$

$$\Phi_{n+2} = \prod_{i=0}^{n}\int \frac{\mathrm{d}^4k_i}{(2\pi)^4}\prod_{j=0}^{n+1}(2\pi)\delta(k_j)^2 = \prod_{i=0}^{n+1}\int \frac{\mathrm{d}y_i}{4\pi}\frac{\mathrm{d}^2k_{i,\perp}}{(2\pi)^2}(2\pi)^4\delta^4\left(p_+p_b - \sum_{i=0}^{n+1}k_i\right)$$

$$\Phi_{n+2} = \int \frac{1}{2\hat{s}}\frac{\mathrm{d}^2k_{0,\perp}}{(2\pi)^2}\left[\prod_{i=1}^{n}\int \frac{\mathrm{d}y_i}{4\pi}\frac{\mathrm{d}^2k_{i,\perp}}{(2\pi)^2}\right]\frac{\mathrm{d}^2k_{n+1,\perp}}{(2\pi)^2}(2\pi)^2\delta^2\left(\sum_{i=0}^{n+1}k_{i,p\perp}\right)$$

$$\mathfrak{Disc}\left[i\mathcal{M}^{aba'}_{\mu_a\mu_b\mu_{a'}\mu_{b'}}(\hat{s},\hat{t})\right]$$

$$= \sum_{n=0}^{\infty}\int \frac{1}{2\hat{s}}\frac{\mathrm{d}^2k_{0,\perp}}{(2\pi)^2}\left[\prod_{i=1}^{n}\int \frac{\mathrm{d}y_i}{4\pi}\frac{\mathrm{d}^2k_{i,\perp}}{(2\pi)^2}\right]\frac{\mathrm{d}^2k_{n+1,\perp}}{(2\pi)^2}(2\pi)^2\delta^2\left(\sum_{i=0}^{n+1}k_{i,p\perp}\right)$$

$$\times\left[-2\hat{s}\delta_{\mu_a\mu_{a'}}g_s f^{ad_0c_1}\right]\left[-2\hat{s}\delta_{\mu_b\mu_{b'}}g_s f^{c_1'd_0a'}\right]$$

$$\times\frac{1}{\hat{t}_1}\exp\left[\alpha(\hat{t}_1)(y_0-y_1)\right]\frac{1}{\hat{t}_1'}\exp\left[\alpha(\hat{t}_1')(y_0-y_1)\right]$$

$$\times\left[ig_s f^{c_1d_1c_2}C_{\mu_1}(q_1,q_2)\right]\left[\left(ig_s f^{c_1'd_1c_2'}\right)C^{\mu_1}(q-q_1,q-q_2)\right]$$

$$\times\left[ig_s f^{c_nd_nc_{n+1}}C_{\mu_n}(q_n,q_{n+1})\right]\left[\left(ig_s f^{c_n'd_nc_{n+1}'}\right)C^{\mu_n}(q-q_n,q-q_{n+1})\right]$$

$$\times\frac{1}{\hat{t}_{n+1}}\exp\left[\alpha(\hat{t}_{n+1})(y_n-y_{n+1})\right]\frac{1}{\hat{t}_{n+1}'}\exp\left[\alpha(\hat{t}_{n+1}')(y_n-y_{n+1})\right]$$

$$\times\left(ig_s f^{bd_{n+1}c_{n+1}}\right)\left(ig_s f^{c_{n+1}'d_{n+1}b'}\right)$$

$$C^{\mu_1}(q_i,q_{i+1})C_{\mu_1}(q-q_i,q-q_{i+1}) = -2\left[q_\perp^2 - \frac{(q-q_i)_\perp^2 q_{i+1,\perp}^2 + (q-q_{i+1})_\perp^2 q_{i,\perp}^2}{(q_i-q_{i+1})_\perp^2}\right] = -2\mathcal{K}(q_i,q_{i+1})$$

$$C = \begin{cases} N_c & \text{for singlet} \\ N_c/2 & \text{for octet}\end{cases}$$

$$\mathfrak{Disc}\left[i\mathcal{M}^{aba''}_{\mu_a\mu_b\mu_{a'}\mu_{b'}}(\hat{s},\hat{t})\right] = 2i\hat{s}\sum_{n=0}^{\infty}(-g_s^2C)^{n+2}\int\left[\prod_{i=1}^{n}\frac{\mathrm{d}y_i}{4\pi}\right]\int\left[\prod_{j=1}^{n+1}\frac{\mathrm{d}^2q_{j,\perp}}{(2\pi)^2}\right]$$

$$\times\left[\prod_{k=1}^{n+1}\frac{1}{\hat{t}_k\hat{t}'_k}e^{[y_{k-1}-y_k][\alpha(\hat{t}_k)+\alpha(\hat{t}'_k)]}\right]\left[\prod_{m=1}^{n}2\mathcal{K}(q_m,q_{m+1})\right].$$

$$\mathcal{F}_l(\hat{t}) = -2i(4\pi\alpha_sC)^2\hat{t}\sum_{n=0}^{\infty}\left[\int\prod_{i=1}^{n+1}\frac{\mathrm{d}^2q_{i,\perp}}{(2\pi)^2}\right]$$

$$\times\left[\prod_{j=1}^{n}\frac{1}{\hat{t}_j\hat{t}'_j}\frac{1}{l-1-\alpha(\hat{t}_j)-\alpha(\hat{t}'_j)}(-2\alpha_sC)\mathcal{K}(q_j,q_{j+1})\right]$$

$$\times\left[\frac{1}{\hat{t}_{n+1}\hat{t}'_{n+1}}\frac{1}{l-1-\alpha(\hat{t}_{n+1})-\alpha(\hat{t}'_{n+1})}\right]$$

$$\mathcal{F}_l(\hat{t}) = -2i(4\pi\alpha_sC)^2\hat{t}\int\frac{\mathrm{d}^2q_{1,\perp}}{(2\pi)^2}\frac{1}{q_{1,\perp}^2(q-q_1)_\perp^2}f_l(q_1,t)$$

$$f_l(q_1,t) = \frac{1}{l-1-\alpha(\hat{t}_1)-\alpha(\hat{t}'_1)}\left[1-2\alpha_sC\int\frac{\mathrm{d}^2q_{2,\perp}}{(2\pi)^2}\frac{\mathcal{K}(q_1,q_2)}{q_{2,\perp}^2(q-q_2)_\perp^2}f_l(q_2,t)\right]$$

$$(l-1)f_l^{\mathrm{oct}}(q_1,\hat{t}) = 1-\alpha_sN_cq_\perp^2\int\frac{\mathrm{d}^2k_\perp}{(2\pi)^2}\frac{1}{k_\perp^2(q-k)_\perp^2}f_l^{\mathrm{oct}}(k,\hat{t})$$

$$f_l^{\mathrm{oct}}(q_1,\hat{t}) = f_l^{\mathrm{oct}}(k,\hat{t}) = \frac{1}{l-1-\alpha(\hat{t})}$$

$$\mathcal{M}^{\mathrm{oct}}_{gg\to gg}(\hat{s},\hat{t}) = 4\pi\alpha_sN_c\frac{\pi\alpha(\hat{t})}{\sin[\pi\alpha(\hat{t})]}\left(1+e^{i\pi\alpha(\hat{t})}\right)\left(\frac{\hat{s}}{-\hat{t}}\right)^{1+\alpha(\hat{t})}$$

$$\mathcal{M}^{\mathrm{oct}}_{gg\to gg}(\hat{s},\hat{t}) = -8\pi\alpha_sN_c\frac{\hat{s}}{\hat{t}}e^{\alpha(\hat{t})(y_a-y_b)}$$

$$(l-1)\tilde{f}_l^{\mathrm{sing}}(q_1,k)$$

$$=\frac{1}{2}\delta^2(\vec{q}_{1,\perp}-\vec{k}_\perp)$$

$$+4\alpha_sN_c\int\frac{\mathrm{d}^2q_{2\perp}}{(2\pi)^2}\frac{1}{(q_1-q_2)_\perp^2}\left[\tilde{f}_l^{\mathrm{sing}}(q_2,k)-\frac{q_{1,\perp}^2}{q_{2,\perp}^2+(q_1-q_2)_\perp^2}\tilde{f}_l^{\mathrm{sing}}(q_1,k)\right]$$

$$f_l^{\mathrm{sing}}(q_1,\hat{t}=0) = \int\frac{\mathrm{d}^2k_\perp}{(2\pi)^2}\tilde{f}_l^{\mathrm{sing}}(q_1,k,\hat{t}=0)$$

$$\tilde{f}_l^{\mathrm{sing}}(q_1,k) = \sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}\nu a(\nu,n)\exp\left[i\nu\left(\log\frac{q_1^2}{\mu^2}-\log\frac{k^2}{\mu^2}\right)+in(\phi_1-\phi)\right]$$

$$\delta^2(\vec{q}_{1,\perp}-\vec{k}_\perp)=\frac{1}{k_\perp q_{1,\perp}}\frac{1}{(2\pi)^2}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}\nu\exp\left[i\nu\left(\log\frac{q_1^2}{\mu^2}-\log\frac{k^2}{\mu^2}\right)+in(\phi_1-\phi)\right]$$

$$(l-1)a(\nu,n)=\frac{1}{4\pi^2k_\perp q_{1,\perp}}+\omega(\nu,n)a(\nu,n)$$

$$a(\nu,n)=\frac{1}{4\pi^2k_\perp q_{1,\perp}}\frac{1}{l-1-\omega(\nu,n)}$$

$$\omega(\nu,n)=-\frac{2\alpha_sN_c}{\pi}\Re\left[\psi\left(\frac{|n|+1}{2}+i\nu\right)-\psi(1)\right]$$

$$\psi(x)=\frac{\mathrm{d}\log\Gamma(x)}{\mathrm{d}x}$$

$$\omega(\nu,n=0)=\frac{2\alpha_sN_c}{\pi}(2\log 2-7\zeta(3)\nu^2+\cdots)\approx\frac{4N_c\log 2}{\pi}\alpha_s\approx 2.65\alpha_s$$

$$\tilde{f}_l(k_a,k_b)\approx\frac{1}{4\pi^2k_{a,\perp}k_{b,\perp}}\frac{\pi}{\sqrt{B(l-1-A)}}\exp\left[-\sqrt{\frac{l-1-A}{B}}\log\frac{k_{a,\perp}^2}{k_{b,\perp}^2}\right]$$

$$A=\frac{4N_c\log 2}{\pi}\alpha_s\ \text{ and }\ B=\frac{14\zeta(3)N_c}{\pi}\alpha_s$$

$$\hat{\sigma}_{gg\to gg}^{(\mathrm{tot})}=\frac{8N_c^2}{N_c^2-1}\alpha_s^2\int\frac{\mathrm{d}^2k_{a,\perp}}{k_{a,\perp}^2}\frac{\mathrm{d}^2k_{b,\perp}}{k_{b,\perp}^2}f^{\mathrm{sing}}(k_a,k_b,|y_a-y_b|)$$

$$f^{\mathrm{sing}}(k_a,k_b,y)=\frac{1}{4\pi^2k_{a,\perp}k_{b,\perp}}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}\nu\exp\left[\omega(\nu,n)y+i\nu\log\frac{k_{a,\perp}^2}{k_{b,\perp}^2}+in(\phi_a-\phi_b)\right]$$

$$\frac{\mathrm{d}\hat{\sigma}_{gg\to gg}^{(\mathrm{tot})}}{\mathrm{d}k_{a,\perp}^2\,\mathrm{d}k_{b,\perp}^2}=\frac{N_c^2\alpha_s^2}{4k_{a,\perp}^3k_{b,\perp}^3}\int_{-\infty}^{\infty}\mathrm{d}\nu\exp\left[\omega(\nu,n=0)|y_a-y_b|+i\nu\log\frac{k_{a,\perp}^2}{k_{b,\perp}^2}\right]$$

$$\approx\frac{N_c^2\alpha_s^2\pi}{4k_{a,\perp}^3k_{b,\perp}^3}\frac{1}{\sqrt{\pi B|y_a-y_b|}}\exp\left[A|y_a-y_b|-\frac{1}{4B|y_a-y_b|}\log^2\frac{k_{a,\perp}^2}{k_{b,\perp}^2}\right]$$

$$\hat{\sigma}_{gg\to gg}^{(k_\perp>p_\perp)}=\frac{N_c^2\alpha_s^2(p_\perp^2)}{4p_\perp^2}\int_{-\infty}^{\infty}\mathrm{d}\nu\frac{e^{\omega(\nu,n=0)|y_a-y_b|}}{\nu^2+\frac{1}{4}}\approx\frac{N_c^2\alpha_s^2(p_\perp^2)\pi}{2p_\perp^2}\frac{\exp\left(\frac{4N_c\alpha_s(p_\perp^2)|y_a-y_b|\log 2}{\pi}\right)}{\sqrt{\frac{7\zeta(3)N_c\alpha_s(p_\perp^2)|y_a-y_b|}{2}}}$$

31. Factor de formación de Sudakov.

$$\mathcal{P}^{\mathrm{nodec.}}(t,0)=\exp\left[-\frac{t}{\tau}\right]=\exp\left[-\Gamma t\right]$$

$$\mathcal{P}^{\mathrm{dec.}}(t,0)=1-\mathcal{P}^{\mathrm{nodec.}}(t,0)=1-\exp\left[-\Gamma t\right].$$

$$\frac{\mathrm{d}\mathcal{P}^{\text{dec.}}(t,0)}{\mathrm{d}t} = -\frac{\mathrm{d}\mathcal{P}^{\text{nodec.}}(t,0)}{\mathrm{d}t} = \Gamma\exp\left[-\Gamma t\right] = \Gamma\cdot\mathcal{P}^{\text{nodec.}}(t,0).$$

$$\mathcal{P}^{\text{nodec.}}(t,0) = \exp\left[-\int_0^t \mathrm{d}t'\Gamma(t')\right]$$

$$\frac{\mathrm{d}\mathcal{P}^{\text{dec}\cdot}(t,0)}{\mathrm{d}t} = -\frac{\mathrm{d}\mathcal{P}^{\text{nodec.}}(t,0)}{\mathrm{d}t} = \Gamma(t)\cdot\mathcal{P}^{\text{nodec.}}(t,0)$$

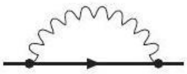
$$\Gamma(t) \longrightarrow \frac{\alpha_{\text{s}}}{\pi}\frac{\Gamma_a(T,t)}{t}$$

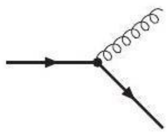
$$\Delta_a(T,t) = \exp\left[-\int_t^T \frac{\mathrm{d}t'}{t'}\frac{\alpha_{\text{s}}}{\pi}\Gamma_a(T,t')\right]$$

$$t' \geq t_{\text{c}} > 0$$

$$\begin{aligned} 1-\Delta_{q,g}(T,t_{\text{c}}) &= 1-\exp\left[-\frac{C_{F,A}\alpha_{\text{s}}}{\pi}\left(\log^2\frac{T}{t_{\text{c}}}-\tilde{\gamma}_{q,g}^{(1)}\log\frac{T}{t_{\text{c}}}\right)\right] \\ &= \frac{C_{F,A}\alpha_{\text{s}}}{\pi}\left(\log^2\frac{T}{t_{\text{c}}}-\tilde{\gamma}_{q,g}^{(1)}\log\frac{T}{t_{\text{c}}}\right)+\mathcal{O}(\alpha_s^2) \end{aligned}$$

$$\begin{aligned} \Delta_{q,g}(T,t_{\text{c}}) &= \exp\left[-\frac{C_{F,A}\alpha_{\text{s}}}{\pi}\left(\log^2\frac{T}{t_{\text{c}}}-\tilde{\gamma}_{q,g}^{(1)}\log\frac{T}{t_{\text{c}}}\right)\right] \\ &= 1-\frac{C_{F,A}\alpha_{\text{s}}}{\pi}\left(\log^2\frac{T}{t_{\text{c}}}-\tilde{\gamma}_{q,g}^{(1)}\log\frac{T}{t_{\text{c}}}\right)+\mathcal{O}(\alpha_s^2) \end{aligned}$$

 +  $\approx 1-\frac{C_F\alpha_S}{\pi}\left(\log^2\frac{T}{t_0}-\frac{3}{2}\log\frac{T}{t_0}\right)$

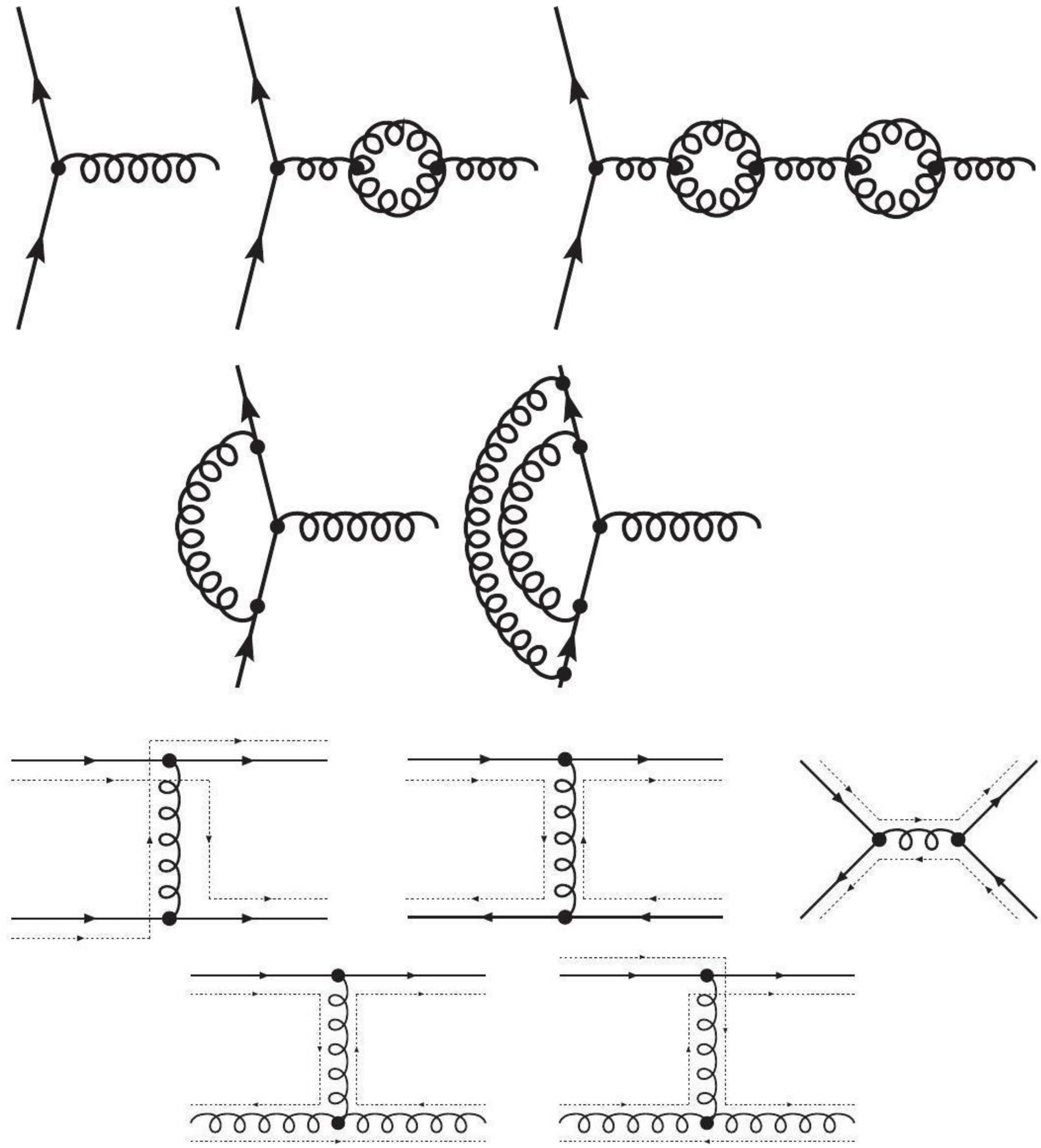
\+  $\approx \frac{C_F\alpha_S}{\pi}\left(\log^2\frac{T}{t_0}-\frac{3}{2}\log\frac{T}{t_0}\right)$

$$\Delta_{a\to bc}(T,t) = \exp\left[-\int_t^T \frac{\mathrm{d}t'}{t'}\int_{z_-}^{z_+}\mathrm{d}z\int_0^{2\pi}\frac{\mathrm{d}\phi}{2\pi}\frac{\alpha_{\text{S}}(p_\perp(t',z))}{\pi}P_{a\to bc}^{(1)}(z)\right].$$

$$\frac{\mathrm{d}f_{b/h}(x,t)}{\mathrm{dlog}\ t} = \frac{\alpha_s(t)}{2\pi}\sum_a\int\frac{\mathrm{d}x'}{x'}\mathcal{P}a\to bc\left(\frac{x}{x'}\right)f_{a/h}(x',t)$$

$$\mathrm{d}\mathcal{P}_b = \frac{\mathrm{d}f_{b/h}(x,t)}{f_{b/h}(x,t)} = -\frac{\mathrm{d}t}{t}\frac{\alpha_s(t)}{2\pi}\sum_a\int\frac{\mathrm{d}x'}{x'}\mathcal{P}_{ba}^{(1)}\left(\frac{x}{x'}\right)\frac{f_{a/h}(x',t)}{f_{b/h}(x,t)}$$

$$\Delta_{a\to bc}(T,t) = \exp\left\{-\int_t^T\frac{\mathrm{d}t'}{t'}\int_{z_-}^{z_+}\frac{\mathrm{d}z}{z}\int_0^{2\pi}\frac{\mathrm{d}\phi}{2\pi}\frac{\alpha_s(p_\perp(t',z))}{2\pi}P_{a\to bc}^{(1)}(z)\frac{f_{a/h}\left(\frac{x}{z},t'\right)}{f_{b/h}(x,t')}\right\}$$



Figuras 27 y 28. Propagadores fantasmas de una partícula supermasiva.

$$\frac{\alpha_s(k_\perp^2)}{2\pi} \longrightarrow \frac{\alpha_s(k_\perp^2)}{2\pi} + \left[\frac{\alpha_s(k_\perp^2)}{2\pi}\right]^2 \cdot \left[C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5n_f}{9}\right]$$

$$k_\perp^{\min} \approx 1\mathrm{GeV} > \Lambda_{\mathrm{QCD}}$$

$$\mathcal{K}_{ij;k}(\Phi_1) \longrightarrow \begin{cases} \dfrac{1}{p_i p_j} \mathcal{P}_{(ij)i}\big(z(\Phi_1)\big) & \text{for} \quad z \nrightarrow 1 \\ \dfrac{1}{p_i p_j} \cdot \dfrac{p_i p_k}{p_j p_k} & \text{for} \quad z \rightarrow 1 \end{cases} \quad \text{(collinear)}$$

$$\mathcal{K}_{ij;k}(\Phi_1) \overset{z\to 1}{\to} \frac{1}{p_i p_j} \cdot \frac{p_i p_k}{(p_i + p_k) p_j}.$$

$$\frac{\mathrm{d}k_{\perp ij}^2}{k_{\perp ij}^2} = \frac{\mathrm{d}m_{ij}^2}{m_{ij}^2} = \frac{\mathrm{d}\theta_{ij}^2}{\theta_{ij}^2},$$

$$\begin{aligned}
&\mathrm{FF}: \tilde{p}_{ij} + \tilde{p}_k \longrightarrow p_i + p_j + p_k \\
&\mathrm{IF}: \tilde{p}_{aj} + \tilde{p}_k \longrightarrow p_a + p_j + p_k \\
&\mathrm{FI}: \tilde{p}_{ij} + \tilde{p}_a \longrightarrow p_i + p_j + p_a \\
&\mathrm{II}: \tilde{p}_{aj} + \tilde{p}_b \longrightarrow p_a + p_j + p_b
\end{aligned}$$

$$\Delta_{ij;k}^{(\mathcal{K})}(T,t) = \exp\left[-\int_t^T \frac{\mathrm{d}t'}{t'} \int \mathrm{d}z \int \frac{\mathrm{d}\phi}{2\pi} J(t',z,\phi) \mathcal{K}_{ij;k}(t',z,\phi)\right] = \exp\left[-\int_t^T \mathrm{d}\Phi_1 \mathcal{K}_{ij;k}(\Phi_1)\right]$$

$$t^{(\mathrm{FF})} = t_{ij} = \tilde{p}_{ij}^2 = \left(p_i + p_j\right)^2 \text{ and } z^{(\mathrm{FF})} = z_{ij} = \frac{E_i}{\tilde{E}_{(ij)}}$$

$$t^{(\mathrm{II})} = t_{aj} = \left|\tilde{p}_{aj}^2\right| = \left|\left(p_a - p_j\right)^2\right| \text{ and } z^{(\mathrm{II})} = z_{aj} = \frac{x_a}{\tilde{x}_{aj}} = \frac{E_a}{\tilde{E}_{aj}}.$$

$$z_\pm = \frac{1}{2}\left(1 \pm \sqrt{1 - \frac{t_{ij}}{E_{(ij)}^2}}\right) \Theta\left(t_{ij} - m_{(ij)}^2\right),$$

$$t \geq t_{\mathrm{c}}^{(ij)} = \sqrt{\tilde{m}_{(ij)}^2 + \frac{Q_0^2}{4}}.$$

$$\alpha_{\mathrm{s}}(k_\perp^2) \text{ with } k_\perp^2 = \begin{cases} z_{ij}(1 - z_{ij}) t_{ij} & (\mathrm{FF}) \\ (1 - z_{aj}) t_{aj} & (\mathrm{II}) \end{cases}$$

$$t = \left(p_i^{(0)} + p_j^{(0)}\right)^2 = 2E_i^{(0)} E_j^{(0)} (1 - \cos\,\theta_{ij})$$

$$E = z_{ij} E + (1 - z_{ij}) E = E_i^{(0)} + E_j^{(0)}$$

$$p_{i,j} = (1 - r_{i,j}) p_{i,j}^{(0)} + r_{j,i} p_{j,i}^{(0)}$$

$$r_{i,j} = \frac{t_{ij} + t_{i,j} - t_{j,i} - \sqrt{\left(t_{ij} - t_i - t_j\right)^2 - 4 t_i t_j}}{2 t_{ij}}$$

$$\int_{x_{ij}}^1 \mathrm{d}z_{ij} \frac{\alpha_{\mathrm{s}}(t_{ij})}{2\pi} \frac{x_i f_{i/h}(x_i, t_{ij})}{x_{ij} f_{ij/h}(x_{ij}, t_{ij})}$$

$$\hat{s}_{a\tilde{b}} = (p_a + p_{\tilde{b}})^2 = \frac{\hat{s}_{\tilde{a}\tilde{b}}}{z_{aj}},$$

$$E_{(aj),(bl)} = \frac{\hat{s}_{\tilde{a}\tilde{b}} \mp \left(Q_{(aj)}^2 - Q_{(bl)}^2\right)}{2\sqrt{\hat{s}_{\tilde{a}\tilde{b}}}}$$

$$p_{(aj),(bl)}^{\parallel} = \pm\sqrt{\frac{\left(\hat{s}_{\tilde{a}\tilde{b}} + Q_{(aj)}^2 + Q_{(bl)}^2\right)^2 - 4Q_{(aj)}^2 Q_{(bl)}^2}{4\hat{s}_{\tilde{a}\tilde{b}}}}$$

$$t_j = p_j^2 \leq \frac{q_{aj}q_{ak} - r_{aj}r_{ak}}{2Q_{(bl)}^2} - Q_{(aj)}^2 - Q_{(ak)}^2$$

$$q_{aj} = \hat{s}_{\tilde{a}\tilde{b}} + Q_{(aj)}^2 - Q_{(bl)}^2$$
$$q_{ak} = \hat{s}_{a\tilde{b}} + Q_{(ak)}^2 - Q_{(bl)}^2 = \frac{\hat{s}_{\tilde{a}\tilde{b}}}{z_{aj}} + Q_{(ak)}^2 - Q_{(bl)}^2$$
$$r_{aj} = q_{aj}^2 - 4Q_{(aj)}^2 Q_{(bl)}^2$$
$$r_{ak} = q_{ak}^2 - 4Q_{(ak)}^2 Q_{(bl)}^2.$$

$$\theta_{ij} \approx \frac{p_{\perp,i}}{E_i} + \frac{p_{\perp,j}}{E_j} \approx \frac{1}{\sqrt{z_{ij}\left(1 - z_{ij}\right)}} \frac{\sqrt{t_{ij}}}{E_{(ij)}}.$$

$$\theta_i < \theta_{ij} \longrightarrow \frac{z_i(1 - z_i)}{t_i} > \frac{1 - z_{ij}}{t_{ij}}$$

$$p_{\perp,ij}^2 = z_{ij}\left(1 - z_{ij}\right)\left(t_{ij} - m_{(ij)}^2\right),$$

$$z_{ij} = \frac{1}{1 - k_1 - k_3}\left[\frac{x_1}{2 - x_2} - k_3\right]$$

$$k_1 = \frac{t_{ij} - \lambda\left(t_{ij}, m_i^2, m_j^2\right) + m_j^2 - m_i^2}{2t_{ij}}$$

$$k_3 = \frac{t_{ij} - \lambda\left(t_{ij}, m_i^2, m_j^2\right) - m_j^2 + m_i^2}{2t_{ij}}$$
$$x_{1/2} = \frac{2\left(p_i + p_j + p_k'\right)p_{i/j}}{\left(p_i + p_j + p_k'\right)^2}$$
$$z_{ij} = \frac{1}{1 - k_1 - k_3}\left[\frac{x_1}{2 - x_2} - k_3\right]$$

$$p_k' = p_k$$

$$p_k' = pk\left[1 - \frac{t_{ij} - m_{(ij)}^2}{\left(p_i + p_j + p_k\right)^2} - 2t_{ij} + 2m_{(ij)}^2\right]$$
$$\times\left[1 + \frac{t_{ij} - m_{(ij)}^2}{\left(p_i + p_j + p_k\right)^2} - 2t_{ij} + 2m_{(ij)}^2\right]^{-1}$$

$$p_\perp^2 = (1 - z)Q^2 - zQ^4/\hat{s}$$

$$t = (1-z)Q^2 \equiv p_{\perp,\mathrm{evol}}^2$$

$$p_{\perp,evol}^2 = (1-z)\left(t_{ij} + m_{(ij)}^2\right)$$

$$z = \frac{2p_{(aj)}p_b}{2p_a p_b}$$

$$\hat{\mathcal{P}}_{qq}(z, p_\perp^2) = C_F\left[\frac{1+z^2}{1-z} - \frac{2z(1-z)m^2}{p_\perp^2 + (1-z)^2 m^2}\right]$$

$$\hat{\mathcal{P}}_{qg}(z, p_\perp^2) = T_R\left[1 - 2z(1-z)\frac{p_\perp^2}{p_\perp^2 + m^2}\right]$$

$$t = \frac{\vec{k}_\perp^2}{z^2(1-z)^2} - \frac{\mu_{ij}^2}{z(1-z)} + \frac{\mu_i^2}{z^2(1-z)} + \frac{\mu_j^2}{z(1-z)^2}$$

$$\to \begin{cases} \dfrac{\vec{k}_\perp^2}{z^2(1-z)^2} + \dfrac{\mu^2}{z^2} + \dfrac{Q_{g,\min}^2}{z(1-z)^2} & \text{for } q \to qg \\ \dfrac{\vec{k}_\perp^2 + \mu^2}{z^2(1-z)^2} & \text{for } g \to q\bar{q} \\ \dfrac{\vec{k}_\perp^2 + Q_{g,\min}^2}{z^2(1-z)^2} & \text{for } g \to gg, \end{cases}$$

$$\mu_R^2 = z^2(1-z)^2 t$$

$$t_{i+1} < z_i^2 t_i \ \text{ and } \ \bar{t}_{i+1} < (1-z_i)^2 t_i$$

$$\mu = \min\{m, Q_{g,\min}\}$$

$$t = \frac{\vec{k}_\perp^2 + zQ_{g,\min}^2}{(1-z)^2}$$

$$\mu_R^2 = (1-z)^2 t.$$

$$q_i^\mu = \alpha_i p^\mu + \beta_i n^\nu + q_{\perp,i}^\mu,$$

$$\vec{p}_{\perp,i} = \vec{q}_{\perp,i} - z_i \vec{q}_{\perp,i-1}.$$

$$q_{i-1}^2 = \frac{q_i^2}{z_i} + \frac{k_i^2}{1-z_i} + \frac{p_{\perp,i}^2}{z_i(1-z_i)},$$

$$\mathcal{K}_{qg,k}^{(FF)} = C_F\left[\frac{2}{1 - z_i\left(1 - y_{ij;k}\right)} - (1 + z_i)\right]$$

$$\mathcal{K}_{gg,k}^{(FF)} = 2C_A\left[\frac{1}{1 - z_i\left(1 - y_{ij;k}\right)} + \frac{1}{1 - (1 - z_i)\left(1 + y_{ij;k}\right)} - 2 + z_i(1 - z_i)\right]$$

$$\mathcal{K}_{q\bar{q},k}^{(FF)} = T_R[1 - 2z_i(1 - z_i)]$$

$$y_{ij;k} = \frac{p_i p_j}{p_i p_j + p_i p_k + p_j p_k}$$

$$z_i = \frac{p_i p_k}{p_i p_k + p_j p_k} = 1 - z_j$$

case	recoil parameter	splitting parameter
FF	$y_{ij;k} = \frac{p_i p_j}{p_i p_j + p_i p_k + p_j p_k}$	$z_i = \frac{p_i p_k}{(p_i + p_j) p_k} = 1 - z_j$
FI	$x_{ij,a} = \frac{p_i p_a + p_j p_a - p_i p_j}{(p_i + p_j) p_a}$	$z_i = \frac{p_i p_a}{(p_i + p_j) p_a} = 1 - z_j$
IF	$u_i = \frac{p_i p_a}{(p_i + p_k) p_a} = 1 - u_k$	$x_{ik,a} = \frac{p_i p_a + p_k p_a - p_i p_k}{(p_i + p_k) p_a}$ $x_{ik,a} = \frac{p_i p_a + p_k p_a - p_i p_k}{(p_i + p_k) p_a}$
II		$x_{i,ab} = \frac{p_a p_b - p_i p_a - p_i p_b}{p_a p_b}$

$$t = k_\perp^2 = \begin{cases} 2p_i p_j z_i(1 - z_i) & \text{for final-state emissions} \\ 2p_a p_j(1 - x_a) & \text{for initial-state emissions} \end{cases}$$

$$t = 2p_i p_j \cdot \begin{cases} z_i(1 - z_i) & \text{if } i, j = g \\ (1 - z_i) & \text{if } i \neq g \text{ and } j = g \\ z_i & \text{if } i = g \text{ and } j \neq g \\ z_i(1 - z_i) & \text{if } i, j \neq g \end{cases}$$

$$t = 2p_a p_j \cdot \begin{cases} 1 - x_{aj,k} & \text{if } j = g \\ 1 & \text{if } j \neq g \end{cases}$$

$$\begin{aligned} p_i &= \quad z_i \tilde{p}_{ij} + (1 - z_i) y_{ij;k} \tilde{p}_k + \vec{k}_\perp \\ p_j &= \quad (1 - z_i) \tilde{p}_{ij} + \\ p_k &= \quad z_i y_{ij;k} \tilde{p}_k - \vec{k}_\perp \\ &\quad (1 - y_{ij;k}) \tilde{p}_k \end{aligned}$$

$$\tilde{p}_i + \tilde{p}_k \to p_i + p_j + p_k$$

$$x_l = \frac{2p_l Q}{Q^2}.$$

$$k_\perp^2 = \frac{s_{ij}s_{jk}}{s_{ijk}} \quad \text{and} \quad y = \frac{1}{2}\log\frac{s_{ij}}{s_{jk}}.$$

$$\mathrm{d}\mathcal{P}_{\tilde{\imath}\tilde{k}\to ijk} = \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\,\mathrm{d}y\frac{\mathrm{d}\phi}{2\pi}\mathcal{K}_{\tilde{\imath}\tilde{k}\to ijk}$$

$$\mathrm{d}\mathcal{P}_{\tilde{\imath}\tilde{k}\to ijk} = \mathrm{d}k_\perp^2\,\mathrm{d}y\frac{\mathrm{d}\phi}{2\pi}\frac{\left|\mathcal{M}_{X\to ijk}\right|^2}{\left|\mathcal{M}_{X\to i\tilde{k}}\right|^2},$$

$$s_{lm} = 2p_l p_m = Q^2(1-x_n)$$

$$\mathrm{d}\mathcal{P}_{\tilde{\imath}\tilde{k}\to ijk} = \mathrm{d}x_q\,\mathrm{d}_{\bar{q}}\frac{\mathrm{d}\phi}{2\pi}\frac{C_F\alpha_s}{2\pi}\frac{x_q^2+x_{\bar{q}}^2}{(1-x_q)(1-x_{\bar{q}})} = \frac{\mathrm{d}s_{\bar{q}g}}{Q^2}\frac{\mathrm{d}s_{qg}}{Q^2}\frac{\mathrm{d}\phi}{2\pi}\frac{C_F\alpha_s}{2\pi}\frac{\left(Q^2-s_{\bar{q}g}\right)^2+\left(Q^2-s_{qg}\right)^2}{s_{\bar{q}g}s_{qg}}$$

$$= \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\,\mathrm{d}y\frac{\mathrm{d}\phi}{2\pi}\frac{2C_F\alpha_s}{2\pi}\frac{(1-x_\perp e^{y})^2+(1-x_\perp e^{-y})^2}{2}$$

$$s_{\bar{q}g,qg} = \sqrt{k_\perp^2 Q^2 e^{\pm y}} = Q^2 x_\perp e^{\pm y}$$

$$\tilde{p}_{i,k} = \frac{Q}{2}.$$

32. Configuración de niveles de Born:

$$\mathrm{d}\sigma_N^{(\mathrm{Born})} = \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}_N(\Phi_{\mathcal{B}})\left\{\Delta_N^{(\mathcal{K})}\left(\mu_Q^2,t_c\right)+\int_{t_c}^{\mu_Q^2}\mathrm{d}\Phi_1\left[\mathcal{K}_N(\Phi_1)\Delta_N^{(\mathcal{K})}\left(\mu_Q^2,t(\Phi_1)\right)\right]\right\}$$

$$\mathcal{K}_N(\Phi_1) = \sum_{\{ij;k\}\in N}\mathcal{K}_{ij;k}(\Phi_1).$$

$$\Delta_N^{(\mathcal{K})}(T,t) = \prod_{\{ij;k\}\in N}\Delta_{ij;k}^{(\mathcal{K})}(T,t).$$

$$\begin{aligned}\mathrm{d}\sigma_N^{(\mathrm{Born})} = \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}_N(\Phi_{\mathcal{B}}) \quad &\left\{\Delta_N^{(\mathcal{K})}\left(\mu_Q^2,t_c\right)+\int_{t_c}^{\mu_Q^2}\mathrm{d}\Phi_1\left[\mathcal{K}_N(\Phi_1)\Delta_N^{(\mathcal{K})}\left(\mu_Q^2,t(\Phi_1)\right)\right.\right.\\ &\times\left\{\Delta_{N+1}^{(\mathcal{K})}(t,t_c)+\int_{t_c}^{t}\mathrm{d}\Phi_1'\left[\mathcal{K}_{N+1}(\Phi_1')\Delta_{N+1}^{(\mathcal{K})}\left(t(\Phi_1),t'(\Phi_1')\right)\right.\right.\\ &\left.\left.\left.\left.\times\left\{\Delta_{N+2}^{(\mathcal{K})}(t',t_c)+\cdots\right.\right]\right\}\right]\right\}.\end{aligned}$$

$$\mathrm{d}\Phi_{N+1}\mathcal{B}_{N+1}(\Phi_{N+1}) = \mathrm{d}\Phi_N\mathcal{B}_N(\Phi_N)\mathcal{K}_N\,\mathrm{d}\Phi_1\Theta\left(\mu_Q^2(\Phi_N)-t(\Phi_1)\right)$$

$$\mathrm{d}\Phi_{N+m}\mathcal{B}_{N+m}(\Phi_{N+m}) = \mathrm{d}\Phi_N\mathcal{B}_N(\Phi_N)\prod_{i=1}^{m}\left[\mathcal{K}_{N+i-1}\,\mathrm{d}\Phi_1^{(i)}\Theta\left(t^{(i-1)}-t^{(i)}\right)\right]$$

$$\mathcal{E}_n^{(\mathcal{K})}(\mu_Q^2, t_c) = \Delta_n^{(\mathcal{K})}(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1 \left[\mathcal{K}_n(\Phi_1)\Delta_n^{(\mathcal{K})}\left(\mu_Q^2, t(\Phi_1)\right) \otimes \mathcal{E}_{n+1}^{(\mathcal{K})}(t(\Phi_1), t_c)\right]$$

$$\mathrm{d}\sigma_N^{(\mathrm{Born})} = \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}_N(\Phi_{\mathcal{B}})\mathcal{E}_N^{(\mathcal{K})}(\mu_Q^2, t_c)$$

$$\mathcal{R}_N(\Phi_{\mathcal{B}} \times \Phi_1) \leq \mathcal{B}_N(\Phi_{\mathcal{B}}) \otimes \mathcal{K}_N(\Phi_1)$$

$$\tilde{\mathcal{K}}_N(\Phi_1) = \mathcal{R}_N(\Phi_{\mathcal{B}} \times \Phi_1)/\mathcal{B}_N(\Phi_{\mathcal{B}})$$

$$\mathrm{d}\sigma_N^{(\mathrm{Born})} = \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}_N(\Phi_{\mathcal{B}})\left\{\Delta_N^{(\tilde{\mathcal{K}})}(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1 \left[\tilde{\mathcal{K}}(\Phi_1)\Delta_N^{(\tilde{\mathcal{K}})}\left(\mu_Q^2, t(\Phi_1)\right)\right]\right\}$$

$$= \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}_N(\Phi_{\mathcal{B}})\left\{\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q} \mathrm{d}\Phi_1 \left[\frac{\mathcal{R}_N(\Phi_{\mathcal{B}} \times \Phi_1)}{\mathcal{B}_N(\Phi_{\mathcal{B}})}\Delta_N^{(\mathcal{R}/\mathcal{B})}\left(\mu_Q^2, t(\Phi_1)\right)\right]\right\}$$

$$\mathcal{P}_{\mathrm{MEC}} = \frac{\tilde{\mathcal{K}}_N(\Phi_1)}{\mathcal{K}_N(\Phi_1)} = \frac{\mathcal{R}_N(\Phi_{\mathcal{B}} \times \Phi_1)}{\mathcal{B}_N(\Phi_{\mathcal{B}}) \times \mathcal{K}_N(\Phi_1)}$$

$$\mathrm{d}\Phi_g\mathcal{R}_{q\bar{q}}(\Phi_{q\bar{q}} \times \Phi_g) = \mathcal{B}_{q\bar{q}}(\Phi_{q\bar{q}}) \times \frac{C_F\alpha_s}{2\pi}\frac{x_1^2 + x_3^2}{(1-x_1)(1-x_3)}\mathrm{d}x_1\,\mathrm{d}x_3$$

$$x_{1,3} = 2E_{q,\bar{q}}/E_{\mathrm{c.m.}} \in [0,1]$$

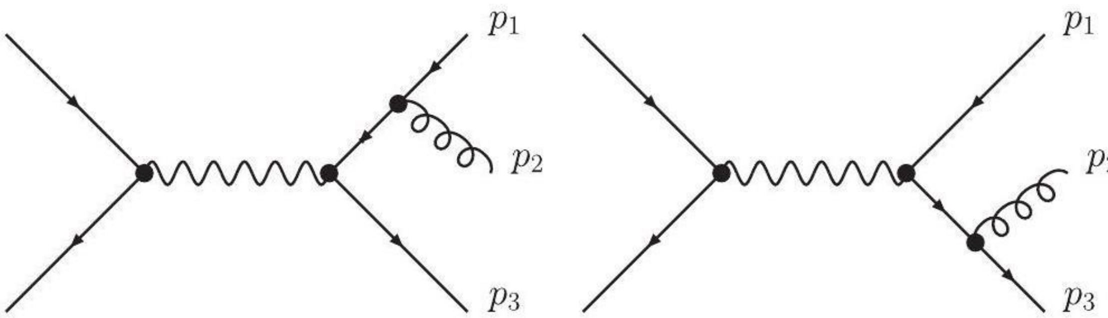


Figura 29. Aniquilaciones en D – dimensiones.

$$t_q = m_{qg}^2 = (p_1 + p_2)^2 = E_{\mathrm{c.m.}}^2(1 - x_3)$$

$$z_q = \frac{E_1}{E_1 + E_2} = \frac{x_1}{2 - x_3}$$

$$\mathrm{d}\Phi_g[\mathcal{B}_{q\bar{q}}(\Phi_{q\bar{q}}) \times \mathcal{K}(\Phi_g)] = \mathcal{B}_{q\bar{q}}(\Phi_{q\bar{q}}) \times \sum_{i\in\{q,\bar{q}\}} \frac{\mathrm{d}t_i}{t_i}\,\mathrm{d}z_i\frac{C_F\alpha_s}{2\pi}\frac{1 + z_i^2}{1 - z_i}$$

$$= \mathcal{B}_{q\bar{q}}(\Phi_{q\bar{q}}) \times \frac{C_F\alpha_s}{2\pi}\frac{\mathrm{d}x_1\,\mathrm{d}x_3}{(1-x_1)(1-x_3)}$$

$$\times \left\{\frac{1-x_1}{x_2}\left[1 + \left(\frac{x_1}{2-x_3}\right)^2\right] + \frac{1-x_3}{x_2}\left[1 + \left(\frac{x_3}{2-x_1}\right)^2\right]\right\}$$

$$\mathrm{d}\Phi_g[\mathcal{B}_{q\bar{q}}(\Phi_{q\bar{q}}) \times \mathcal{K}(\Phi_g)]$$

$$= \mathcal{B}_{q\bar{q}}(\Phi_{q\bar{q}}) \times \frac{C_F\alpha_s}{2\pi} \frac{\mathrm{d}x_1\ \mathrm{d}x_3}{(1-x_1)(1-x_3)} \times \left\{\left[1+\left(\frac{x_1+x_3-\frac{1}{2}}{x_1}\right)^2\right]_{\left[\substack{x_1>1-z(1-z)\\ x_3>1-x_1+zx_1}\right]}\right\}$$

$$+ x_1 \leftrightarrow x_3$$

$$z = \frac{1}{2} + \frac{x_1+2x_3-1}{2x_1}$$

$$\vec{k}_\perp^2 = E^2 y_{ij;k} z_i(1-z_i)$$

$$y_{ij;k} = \frac{p_i p_j}{p_i p_j + p_j p_k + p_k p_i} = 1 - x_k$$

$$z_i = \frac{p_i p_k}{(p_i+p_j)p_k} = \frac{1-x_j}{2-x_j-x_i} = \frac{x_i+x_k-1}{x_k}.$$

$$\frac{\mathrm{d}\vec{k}_\perp^2}{\vec{k}_\perp^2} = \frac{\mathrm{d}y_{ij;k}}{y_{ij;k}}$$

$$\mathrm{d}\Phi_g[\mathcal{B}_{q\bar{q}}(\Phi_{q\bar{q}}) \times \mathcal{K}(\Phi_g)]$$

$$= \mathcal{B}_{q\bar{q}}(\Phi_{q\bar{q}}) \times \frac{C_F\alpha_s}{2\pi} \frac{\mathrm{d}x_1\ \mathrm{d}x_3}{(1-x_1)(1-x_3)}$$

$$\times \left\{[x_1^2+x_3^2] + \left[\frac{(1-x_1)^2(1-x_3)}{x_3} + \frac{(1-x_1)(1-x_3)^2}{x_1}\right]\right\}$$

$$\sigma_N^{(\mathrm{NLO})} = \int \mathrm{d}\Phi_{\mathcal{B}}\left[\mathcal{B}_N(\Phi_{\mathcal{B}}) + \mathcal{V}_N(\Phi_{\mathcal{B}}) + \mathcal{I}_N^{(\mathcal{S})}(\Phi_{\mathcal{B}})\right]$$
$$+ \int \mathrm{d}\Phi_{\mathcal{R}}[\mathcal{R}_N(\Phi_{\mathcal{R}}) - \mathcal{S}_N(\Phi_{\mathcal{R}})]$$

$$\overline{\mathcal{B}}_N(\Phi_{\mathcal{B}}) = \mathcal{B}(\Phi_{\mathcal{B}}) + \tilde{\mathcal{V}}_N(\Phi_{\mathcal{B}}) + \int \mathrm{d}\Phi_1[\mathcal{R}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) - \mathcal{S}_N(\Phi_{\mathcal{B}} \otimes \Phi_1)]$$

$$\tilde{\mathcal{V}}_N(\Phi_{\mathcal{B}}) = \mathcal{V}_N(\Phi_{\mathcal{B}}) + \mathcal{I}_N^{(\mathcal{S})}(\Phi_{\mathcal{B}})$$

$$\Phi_{\mathcal{R}} = \Phi_{\mathcal{B}} \otimes \Phi_1$$

$$\mathrm{d}\sigma_N^{(\mathrm{NLO})} = \mathrm{d}\Phi_{\mathcal{B}}\overline{\mathcal{B}}_N(\Phi_{\mathcal{B}}) \times \left\{\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1\left[\frac{\mathcal{R}_N(\Phi_{\mathcal{B}} \times \Phi_1)}{\mathcal{B}_N(\Phi_{\mathcal{B}})}\Delta_N^{(\mathcal{R}/\mathcal{B})}\left(\mu_Q^2, t(\Phi_1)\right)\right]\right\}$$

$$\mathrm{d}\sigma_N^{(\mathrm{NLO})} \longrightarrow \mathrm{d}\Phi_{\mathcal{B}}\overline{\mathcal{B}}_N\left\{\Delta_N^{(\mathcal{R}/\overline{\mathcal{B}})}(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1\left[\frac{\mathcal{R}_N}{\overline{\mathcal{B}}_N}\Delta_N^{(\mathcal{R}/\overline{\mathcal{B}})}\right]\right\}$$

$$\mathcal{R}_N = \mathcal{R}_N\left(\frac{h^2}{p_\perp^2+h^2} + \frac{p_\perp^2}{p_\perp^2+h^2}\right) = \mathcal{R}_N^{(\mathrm{S})} + \mathcal{R}_N^{(\mathrm{H})}$$

$$d\sigma_N^{(\mathrm{NLO})} = d\Phi_{\mathcal{B}}\tilde{\mathcal{B}}_N \left\{\Delta_N^{(\mathcal{R}^{(s)}/\mathcal{B})}(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} d\Phi_1 \left[\frac{\mathcal{R}_N^{(S)}}{\mathcal{B}_N}\Delta_N^{(\mathcal{R}^{(s)}/\mathcal{B})}(\mu_Q^2, t)\right]\right\} + d\Phi_{\mathcal{R}}\mathcal{R}^{(\mathrm{H})}$$

$$\tilde{\mathcal{B}} = \mathcal{B} + \tilde{\mathcal{V}}_N + \int d\Phi_1[\mathcal{R}^{(S)} - \mathcal{S}]$$

$$\mathcal{R}_N(\Phi_{\mathcal{R}}) = \mathcal{R}_N^{(S)}(\Phi_{\mathcal{R}}) + \mathcal{R}_N^{(H)}(\Phi_{\mathcal{R}}) = \mathcal{S}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) + \mathcal{H}_N(\Phi_{\mathcal{R}}).$$

$$\mathcal{S}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) \equiv \sum_{ijk} \mathcal{B}_N(\Phi_{\mathcal{B}}) \otimes \mathcal{K}_{ij;k}(\Phi_1) = \mathcal{B}_N(\Phi_{\mathcal{B}}) \otimes \mathcal{K}(\Phi_1)$$

$$d\sigma_N^{(\mathrm{NLO})} = d\Phi_{\mathcal{B}}\tilde{\mathcal{B}}_N(\Phi_{\mathcal{B}})\left\{\Delta_N^{(\mathcal{K})}(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} d\Phi_1\mathcal{K}(\Phi_1)\Delta_N^{(\mathcal{K})}\left(\mu_Q^2, t(\Phi_1)\right)\right\} + d\Phi_{\mathcal{R}}\mathcal{H}_N(\Phi_{\mathcal{R}})$$

$$\tilde{\mathcal{B}}_N(\Phi_{\mathcal{B}}) = \mathcal{B}_N(\Phi_{\mathcal{B}}) + \tilde{\mathcal{V}}_N(\Phi_{\mathcal{B}})$$

$$d\sigma_N^{(\mathrm{NLO})} = d\Phi_{\mathcal{B}}\tilde{\mathcal{B}}_N(\Phi_{\mathcal{B}})\mathcal{E}_N^{(\mathcal{K})}(\mu_Q^2, t_c) + d\Phi_{\mathcal{R}}\mathcal{H}(\Phi_{\mathcal{R}})\mathcal{E}_{N+1}^{(\mathcal{K})}(\mu_H^2, t_c),$$

$$\mathfrak{R}_2(Q_{\mathrm{cut}}) = \left[\Delta_q\left(\mu_Q^2, Q_{\mathrm{cut}}^2\right)\right]^2$$

$$\mathfrak{R}_3(Q_{\mathrm{cut}}) = 2\Delta_q\left(\mu_Q^2, Q_{\mathrm{cut}}^2\right)\int_{Q_{\mathrm{cut}}^2}^{\mu_Q^2} \frac{dq_\perp^2}{q_\perp^2}\left[\left(\frac{\alpha_s(q_\perp^2)}{\pi}C_F\Gamma_q\left(\mu_Q^2, q_\perp^2\right)\frac{\Delta_q\left(\mu_Q^2, Q_{\mathrm{cut}}^2\right)}{\Delta_q\left(\mu_Q^2, q_\perp^2\right)}\right)\right.$$

$$\left. \times \Delta_q(q_\perp^2, Q_{\mathrm{cut}}^2)\Delta_g(q_\perp^2, Q_{\mathrm{cut}}^2)\right]$$

$$\mathfrak{R}_3(Q_{\mathrm{cut}}) = 1 - \mathfrak{R}_2(Q_{\mathrm{cut}}) = \int_{Q_{\mathrm{cut}}^2}^{\mu_Q^2} \frac{dq_\perp^2}{q_\perp^2}\left[\frac{2C_F\alpha_s(q_\perp^2)}{\pi}\Gamma_q\left(\mu_Q^2, q_\perp^2\right)\right] + \mathcal{O}(\alpha_s^2).$$

$$\mathfrak{R}_3(Q_{\mathrm{cut}}) = \int_{Q_{\mathrm{cut}}^2}^{\mu_Q^2} d\Phi_1[\mathcal{K}_{qg;\bar{q}}(\Phi_1) + \mathcal{K}_{\bar{q}g;q}(\Phi_1)] + \mathcal{O}(\alpha_s^2) = \frac{d\sigma_3^{(\mathrm{res})}(Q_{\mathrm{cut}})}{\sigma^{(\mathrm{tot})}}.$$

$$Q^{(m)} \leq Q^{(m-1)} \leq \cdots \leq Q^{(2)} \leq Q^{(1)} \leq Q^{(\mathrm{core})} = Q^2$$

$$t^{(m)} \leq t^{(m-1)} \leq \cdots \leq t^{(2)} \leq t^{(1)} \leq t^{(\mathrm{core})} = \mu_Q^2 = \approx Q^2$$

$$\mu_R^{(m)} \leq \mu_R^{(m-1)} \ldots \leq \mu_R^{(2)} \leq \mu_R^{(1)} \leq \mu_R^{(\mathrm{core})}$$

$$\alpha_s^{M+m}(\mu_R^2) = \alpha_s^M\left(\mu_{R,(\mathrm{core})}^2\right) \cdot \prod_{i \in m} \alpha_s\left(\mu_{R,(i)}^2\right),$$

$$\mathfrak{R}_2(Q_{\mathrm{cut}}) = \left[\Delta_q\left(\mu_Q^2, Q_{\mathrm{cut}}^2\right)\right]^2 = \exp\left[-2\int_{Q_{\mathrm{cut}}}^{\mu_Q} \frac{dq_\perp}{q_\perp}\frac{\alpha_s C_F}{\pi}\left(\log\frac{\mu_Q}{q_\perp} - \frac{3}{4}\right)\right]$$

$$\alpha_s \to \text{ const. } \exp\left[-\frac{\alpha_s C_F}{\pi}\left(\log^2\frac{\mu_Q}{Q_{\mathrm{cut}}} - \frac{3}{2}\log\frac{\mu_Q}{Q_{\mathrm{cut}}}\right)\right]$$

$$\mathfrak{R}_2'(Q_J) = \left[\Delta_q(\mu_Q^2, Q_{\text{cut}}^2)\cdot\Delta_q(Q_{\text{cut}}^2, Q_J^2)\right]^2 \overset{\alpha_s\to\text{const.}}{\rightarrow} \exp\left[-\frac{\alpha_s C_F}{\pi}\left(\log^2\frac{\mu_Q}{Q_{\text{cut}}} - \frac{3}{2}\log\frac{\mu_Q}{Q_{\text{cut}}}\right.\right.$$

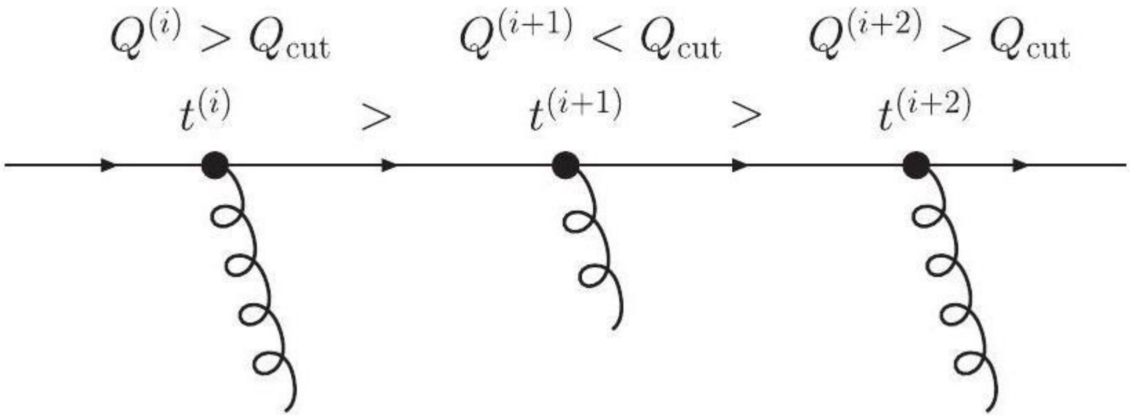
$$\left.\left.+\log^2\frac{Q_{\text{cut}}}{Q_J} - \frac{3}{2}\log\frac{Q_{\text{cut}}}{Q_J}\right)\right]$$

$$= \exp\left[-\frac{\alpha_s C_F}{\pi}\left(\log^2\frac{\mu_Q}{Q_J} - \frac{3}{2}\log\frac{\mu_Q}{Q_J} + 2\log\frac{\mu_Q}{Q_{\text{cut}}}\log\frac{Q_J}{Q_{\text{cut}}}\right)\right] \neq \mathfrak{R}_2(Q_J)\big|_{\alpha_s\to\text{const}}$$

$$1 + \int_{Q_{\text{cut}}}^{\mu_Q}\frac{dq_\perp}{q_\perp}\frac{\alpha_s C_F}{\pi}\Gamma_q(\mu_Q, q_\perp)$$
$$+ \int_{Q_{\text{cut}}}^{\mu_Q}\frac{dq_\perp}{q_\perp}\frac{\alpha_s C_F}{\pi}\Gamma_q(\mu_Q, q_\perp)\int_{Q_{\text{cut}}}^{q_\perp}\frac{dr_\perp}{r_\perp}\frac{\alpha_s C_F}{\pi}\Gamma_q(q_\perp, r_\perp) + \cdots$$

$$= 1 + \int_{Q_{\text{cut}}}^{\mu_Q}\frac{dq_\perp}{q_\perp}\frac{\alpha_s C_F}{\pi}\Gamma_q(\mu_Q, q_\perp) + \frac{1}{2}\left[\int_{Q_{\text{cut}}}^{\mu_Q}\frac{dq_\perp}{q_\perp}\frac{\alpha_s C_F}{\pi}\Gamma_q(\mu_Q, q_\perp)\right]^2 + \cdots$$

$$= \exp\left[\int_{Q_{\text{cut}}}^{\mu_Q}\frac{dq_\perp}{q_\perp}\frac{\alpha_s C_F}{\pi}\Gamma_q(\mu_Q, q_\perp)\right] = \Delta_q^{-1}(\mu_Q, Q_{\text{cut}})$$



$$d\sigma = d\Phi_N \mathcal{B}_N\left[\Delta_N^{(\mathcal{K})}(\mu_N^2, t_c) + \int_{t_c}^{\mu_N^2} d\Phi_1 \mathcal{K}_N \Delta_N^{(\mathcal{K})}(\mu_N^2, t_{N+1})\Theta(Q_{\text{cut}} - Q_{N+1})\right]$$

$$+ d\Phi_{N+1}\mathcal{B}_{N+1}\Delta_N^{(\mathcal{K})}(\mu_{N+1}^2, t_{N+1})\Theta(Q_{N+1} - Q_{\text{cut}})$$

$$d\sigma = \sum_{n=N}^{N_{\max}-1} \{ d\Phi_n \mathcal{B}_n \overbrace{\left[\prod_{j=N}^{n} \Theta(Q_j - Q_{\text{cut}})\right]}^{(n-N)\text{ extra jets}} \overbrace{\left[\prod_{j=N}^{n-1} \Delta_j^{(\mathcal{K})}(t_j, t_{j+1})\right]}^{\text{no emissions off internal lines}}$$

$$\times \left[\underbrace{\Delta_N^{(\mathcal{K})}(t_N, t_c)}_{\text{no emission}} + \underbrace{t_N\ d\Phi_1 \mathcal{K}_N \Delta_N^{(\mathcal{K})}(t_N, t_{N+1}) \Theta(Q_{\text{cut}} - Q_{N+1})}_{\text{next emission no jet \textbackslash\& below last ME emission}}\right]$$

$$+ d\Phi_{N_{\max}} \mathcal{B}_{N_{\max}} \left[\prod_{j=N}^{N_{\max}} \Theta(Q_{j+1} - Q_{\text{cut}})\right] \left[\prod_{j=N}^{N_{\max}-1} \Delta_j^{(\mathcal{K})}(t_j, t_{j+1})\right]$$

$$\times \left[\Delta_{N_{\max}}^{(\mathcal{K})}(t_{N_{\max}}, t_c)\right.$$

$$\left. + \int_{t_c}^{t_{N_{\max}}} d\Phi_1 \mathcal{K}_{N_{\max}} \Delta_{N_{\max}}^{(\mathcal{K})}(t_{N_{\max}}, t_{N_{\max}+1}) \cdot \Theta(Q_{N_{\max}} - Q_{N_{\max}+1})\right]$$

$$\mathcal{K}_N(\Phi_1) \overset{\text{MePS}}{\to} \mathcal{K}_N^{<Q}(\Phi_1) = \mathcal{K}_N(\Phi_1)\Theta(Q - Q_{N+1}).$$

$$\mathcal{E}_N^{(\mathcal{K},<Q)}(\mu_Q^2, t_c) = \Delta_N^{(\mathcal{K})}(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} d\Phi_1 \left[\mathcal{K}_N^{<Q}(\Phi_1)\Delta_N^{(\mathcal{K})}\left(\mu_Q^2, t(\Phi_1)\right) \otimes \mathcal{E}_{N+1}^{(\mathcal{K},<Q)}(t(\Phi_1), t_c)\right]$$

$$d\sigma = \sum_{n=N}^{N_{\max}-1} d\Phi_n \mathcal{B}_n \Theta(Q_n - Q_{\text{cut}}) \mathcal{E}_n^{(\mathcal{K},<Q_{\text{cut}})}(\mu_n^2, t_c)$$

$$+ d\Phi_{N_{\max}} \mathcal{B}_{N_{\max}} \Theta(Q_{N_{\max}} - Q_{\text{cut}}) \mathcal{E}_{N_{\max}}^{(\mathcal{K},<Q_{N_{\max}})}(\mu_{N_{\max}}^2, t_c)$$

$$\int d\sigma^{(LO)} = \int d\Phi_N \mathcal{B}_N \overset{!}{=} \int d\sigma^{(\text{UMEPS})}$$

$$\Delta_N^{(\mathcal{K})}(T, t) = \Delta_N^{(\mathcal{K})}(T, t; Q_{N+1} \geq Q_{\text{cut}}) \times \Delta_N^{(\mathcal{K})}(T, t; Q_{N+1} < Q_{\text{cut}}) = \Delta_N^{\geq Q_{\text{cut}},(\mathcal{K})}(T, t) \times \Delta_N^{<Q_{\text{cut}},(\mathcal{K})}(T, t)$$

$$= \exp\left[-\int_t^T d\Phi_1 \mathcal{K}_N^{\geq Q_{\text{cut}}}(\Phi_1)\right] \times \exp\left[-\int_t^T d\Phi_1 \mathcal{K}_N^{<Q_{\text{cut}}}(\Phi_1)\right]$$

$$d\Phi_N \mathcal{B}_N \left[\Delta_N^{(\mathcal{K})}(\mu_N^2, t_c) + \int_{t_c}^{\mu_N^2} d\Phi_1 \mathcal{K}_N \Delta_N^{(\mathcal{K})}(\mu_N^2, t_{N+1}) \Theta(Q_{\text{cut}} - Q_{N+1})\right]$$

$$= d\Phi_N \mathcal{B}_N \Delta_N^{\geq Q_{\text{cut}},(\mathcal{K})}(\mu_N^2, t_c)$$

$$\times \left[\Delta_N^{<Q_{\text{cut}},(\mathcal{K})}(\mu_N^2, t_c) + \int_{t_c}^{\mu_N^2} d\Phi_1 \mathcal{K}_N^{<Q_{\text{cut}}} \Delta_N^{<Q_{\text{cut}},(\mathcal{K})}(\mu_N^2, t)\right]$$

$$\Delta_N^{\geq Q_{\text{cut}},(\mathcal{K})}(\mu_N^2, t_c) = 1 - \int_{t_c}^{\mu_N^2} d\Phi_1 \mathcal{K}^{\geq Q_{\text{cut}}} \Delta_N^{\geq Q_{\text{cut}},(\mathcal{K})}(\mu_N^2, t)$$

$$\tilde{\Delta}_N^{\geq Q_{\text{cut}},(\mathcal{K})}(\mu_N^2, t_c) = 1 - \int_{t_c}^{\mu_N^2} \mathrm{d}\Phi_1 \frac{\mathcal{B}_{N+1}}{\mathcal{B}_N} \Theta(Q_{N+1} - Q_{\text{cut}}) \Delta_N^{\geq Q_{\text{cut}},(\mathcal{K})}(\mu_N^2, t)$$

$$\mathrm{d}\sigma = \mathrm{d}\Phi_N \mathcal{B}_N \tilde{\Delta}_N^{\geq Q_{\text{cut}},(\mathcal{K})}(\mu_N^2, t_c) \times \left[\Delta_N^{<Q_{\text{cut}},(\mathcal{K})}(\mu_N^2, t_c) + \int_{t_c}^{\mu_N^2} \mathrm{d}\Phi_1 \mathcal{K}_N^{<Q_{\text{cut}}} \Delta_N^{<Q_{\text{cut}},(\mathcal{K})}(\mu_N^2, t)\right]$$

$$+ \mathrm{d}\Phi_{N+1} \mathcal{B}_{N+1} \Delta_N^{\geq Q_{\text{cut}},(\mathcal{K})}(\mu_N^2, t) \Theta(Q_{N+1} - Q_{\text{cut}})$$

$$k_N(\Phi_{N+1}) = \frac{\tilde{\mathcal{B}}_N}{\mathcal{B}_N}\left(1 - \frac{\mathcal{H}_N}{\mathcal{B}_{N+1}}\right) + \frac{\mathcal{H}_N}{\mathcal{B}_{N+1}} \longrightarrow \begin{cases} \tilde{\mathcal{B}}_N / \mathcal{B}_N & \text{for soft emissions} \\ 1 & \text{for hard emissions.} \end{cases}$$

$$\mathcal{H}_N = \mathcal{R}_N - \mathcal{S}_N \overset{\text{soft}}{\rightarrow} 0,$$

$$\mathcal{H}_N = \mathcal{R}_N - \mathcal{S}_N \overset{\text{hard}}{\rightarrow} \mathcal{R}_N = \mathcal{B}_{N+1}.$$

$$\mathrm{d}\sigma = \mathrm{d}\Phi_N \Theta(Q_N - Q_{\text{cut}}) \tilde{\mathcal{B}}_N \times \left[\Delta_N^{(\mathcal{K})}(\mu_N^2, t_{\text{cut}}) + \int_{t_{\text{cut}}}^{\mu_N^2} \mathrm{d}\Phi_1 \mathcal{K}_N \Delta_N^{(\mathcal{K})}(\mu_N^2, t_{N+1}) \Theta(Q_{\text{cut}} - Q_{N+1})\right]$$

$$+ \mathrm{d}\Phi_{N+1} \Theta(Q_N - Q_{\text{cut}}) \Theta(Q_{\text{cut}} - Q_{N+1}) \mathcal{H}_N \Delta_N^{(\mathcal{K})}(\mu_{N+1}^2, t_{N+1})$$

$$+ \mathrm{d}\Phi_{N+1} k_N(\Phi_{N+1}) \Theta(Q_{N+1} - Q_{\text{cut}}) \mathcal{B}_{N+1} \Delta_N^{(\mathcal{K})}(\mu_{N+1}^2, t_{N+1})$$

$$\times \left[\Delta_{N+1}^{(\mathcal{K})}(t_{N+1}, t_{\text{cut}}) + \int_{t_{\text{cut}}}^{t_{N+1}} \mathrm{d}\Phi_1 \mathcal{K}_{N+1} \Delta_{N+1}^{(\mathcal{K})}(t_{N+1}, t_{N+2}) \Theta(Q_{\text{cut}} - Q_{N+2})\right]$$

$$+ \mathrm{d}\Phi_{N+2} k_N(\Phi_{N+2}) \Theta(Q_{N+2} - Q_{\text{cut}}) \mathcal{B}_{N+2} \Delta_{N+1}^{(\mathcal{K})}(\mu_{N+2}^2, t_{N+2}) \Delta_N^{(\mathcal{K})}(t_{N+2}, t_{N+1})$$

$$\times \left[\Delta_{N+1}^{(\mathcal{K})}(t_{N+2}, t_{\text{cut}}) + \cdots\right.$$

$$\mathrm{d}\sigma^{(\text{wrong})} = \mathrm{d}\Phi_N \Theta(Q_N - Q_{\text{cut}}) \tilde{\mathcal{B}}_N \times \left[\Delta_N^{(\mathcal{K})}(\mu_N^2, t_c) + \int_{t_c}^{\mu_N^2} \mathrm{d}\Phi_1 \mathcal{K}_N \Delta_N^{(\mathcal{K})}(\mu_N^2, t_{N+1}) \Theta(Q_{\text{cut}} - Q_{N+1})\right]$$

$$+ \mathrm{d}\Phi_{N+1} \Theta(Q_N - Q_{\text{cut}}) \Theta(Q_{\text{cut}} - Q_{N+1}) \mathcal{H}_N \Delta_N^{(\mathcal{K})}(\mu_N^2, t_{N+1})$$

$$+ \mathrm{d}\Phi_{N+1} \Theta(Q_{N+1} - Q_{\text{cut}}) \tilde{\mathcal{B}}_{N+1}$$

$$\times \left[\Delta_{N+1}^{(\mathcal{K})}(t_{N+1}, t_c) + \int_{t_c}^{t_{N+1}} \mathrm{d}\Phi_1 \mathcal{K}_{N+1} \Delta_{N+1}^{(\mathcal{K})}(t_{N+1}, t_{N+2})\right]$$

$$+ \mathrm{d}\Phi_{N+2} \Theta(Q_{N+1} - Q_{\text{cut}}) \mathcal{H}_{N+1} \Delta_N^{(\mathcal{K})}(\mu_N^2, t_{N+1}) \Delta_{N+1}^{(\mathcal{K})}(t_{N+1}, t_{N+2})$$

$$\left[\mathrm{d}\Phi_N \int_{t_c}^{\mu_N^2} \mathrm{d}\Phi_1 \tilde{\mathcal{B}}_N \mathcal{K}_N + \mathrm{d}\Phi_{N+1}\mathcal{H}_N\right]\Theta(Q_{\mathrm{cut}} - Q_{N+1})\Delta_N^{(\mathcal{K})}(\mu_N^2, t_{N+1})$$

$$= \left[\mathrm{d}\Phi_N \int_{t_c}^{\mu_N^2} \mathrm{d}\Phi_1 (\mathcal{B}_N \otimes \mathcal{K}_N + \mathcal{H}_N) + \mathcal{O}(\alpha_s^2)\right]\Theta(Q_{\mathrm{cut}} - Q_{N+1})\Delta_N^{(\mathcal{K})}(\mu_N^2, t_{N+1})$$

$$= \mathrm{d}\Phi_{N+1}\mathcal{B}_{N+1}\Theta(Q_{\mathrm{cut}} - Q_{N+1})\left[1 - \int_{t_{N+1}}^{\mu_N^2} \mathrm{d}\Phi_1 \mathcal{K}_N + \mathcal{O}(\alpha_s^2)\right]$$

$$\mathrm{d}\sigma = \mathrm{d}\Phi_N \Theta(Q_N - Q_{\mathrm{cut}})\tilde{\mathcal{B}}_N \times \left[\Delta_N^{(\mathcal{K})}(\mu_N^2, t_c) + \int_{t_c}^{\mu_N^2} \mathrm{d}\Phi_1 \mathcal{K}_N \Delta_N^{(\mathcal{K})}(\mu_N^2, t_{N+1})\Theta(Q_{\mathrm{cut}} - Q_{N+1})\right]$$

$$+ \mathrm{d}\Phi_{N+1}\Theta(Q_N - Q_{\mathrm{cut}})\Theta(Q_{\mathrm{cut}} - Q_{N+1})\mathcal{H}_N \Delta_N^{(\mathcal{K})}(\mu_N^2, t_{N+1})$$

$$+ \mathrm{d}\Phi_{N+1}\Theta(Q_{N+1} - Q_{\mathrm{cut}})\tilde{\mathcal{B}}_{N+1}\Delta_N^{(\mathcal{K})}(\mu_{N+1}^2, t_{N+1})\left(1 + \frac{\mathcal{B}_{N+1}}{\tilde{\mathcal{B}}_{N+1}}\int_{t_{N+1}}^{\mu_N^2} \mathrm{d}\Phi_1 \mathcal{K}_N\right)$$

$$\times \left[\Delta_{N+1}^{(\mathcal{K})}(t_{N+1}, t_c) + \int_{t_c}^{t_{N+1}} \mathrm{d}\Phi_1 \mathcal{K}_{N+1}\Delta_{N+1}^{(\mathcal{K})}(t_{N+1}, t_{N+2})\right]$$

$$+ \mathrm{d}\Phi_{N+2}\Theta(Q_{N+1} - Q_{\mathrm{cut}})\Theta(Q_{\mathrm{cut}} - Q_{N+2})\mathcal{H}_{N+1}$$

$$\times \Delta_N^{(\mathcal{K})}(\mu_{N+1}^2, t_{N+1})\Delta_{N+1}^{(\mathcal{K})}(t_{N+1}, t_{N+2})$$

$$\alpha_s^{M+m}(\mu_R^2) = \alpha_s^n(\mu_R^2) = \alpha_s^M(\mu_{R(\,\mathrm{core}\,)}^2) \cdot \prod_{i \in m} \alpha_s(\mu_{R,(i)}^2).$$

$$\alpha_s^n(\mu_R^2) \longrightarrow \alpha_s^n(\tilde{\mu}_R^2)\left(1 - \frac{\alpha_s(\tilde{\mu}_R^2)}{2\pi}\beta_0 \sum_{i=1}^{n} \log \frac{\mu_i^2}{\tilde{\mu}_R^2}\right),$$

$$\mathcal{B}_N \log \frac{\tilde{\mu}_F^2}{\mu_F^2}\left[\sum_{c=q,g} \int_{x_a}^{1} \frac{\mathrm{d}z}{z}\mathcal{P}_{ac}(z) f_{c/h_a}\left(\frac{x_a}{z}, \tilde{\mu}_F^2\right) + \sum_{d=q,g} \int_{x_b}^{1} \frac{\mathrm{d}z}{z}\mathcal{P}_{bd}(z) f_{d/h_b}\left(\frac{x_b}{z}, \tilde{\mu}_F^2\right)\right]$$

$$Q_0 \geq Q_1 \geq Q_2 \ldots \geq Q_{N-1} \geq Q_N = Q_{\mathrm{cut}}$$

$$\mathcal{S} = \prod_{i=1}^{N} \Delta_i(Q_{i-1}^2, Q_i^2) \prod_k \Delta_k(Q_k^2, Q_{\mathrm{cut}}^2).$$

$$1 - \sum_i \Delta^{(1)}(Q_{i-1}^2, Q_i^2) - \sum_k \Delta_k^{(1)}(Q_k^2, Q_{\mathrm{cut}}^2)$$

$$= 1 + \sum_i \int_{Q_i^2}^{Q_{i-1}^2} \frac{\mathrm{d}q_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\Gamma_i(Q_{i-1}^2, q_\perp^2) + \sum_k \int_{Q_{\mathrm{cut}}^2}^{Q_k^2} \frac{\mathrm{d}q_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\Gamma_i(Q_k^2, q_\perp^2)$$

$$\Delta_k^{(1)}(Q^2, Q_0^2) = -\int_{Q_{\text{cut}}^2}^{Q_k^2} \frac{\mathrm{d}q_\perp^2}{q_\perp^2} \frac{\alpha_s(t)}{2\pi} \Gamma_i(Q_k^2, q_\perp^2)$$

$$\alpha_s^{M+N}(\mu_R^2) = \alpha_s^M(\mu_{R,(\text{ core })}) \cdot \prod_{i \in N} \alpha_s(\mu_{R,(i)}),$$

$$\alpha_s^{(M+N+1)} = \frac{1}{M+n}\left[M \alpha_s(\mu_{R,(\text{ core })}^2) + \sum_{i=1}^{N} \alpha_s(\mu_{R,(i)})\right],$$

$$\overline{\mathcal{B}}(\Phi_{\mathcal{B}}) = \alpha_s(m_H^2)\alpha_s(q_\perp)\Delta_g^2(m_H^2, Q_\perp^2) \cdot \{\mathcal{B}(\Phi_{\mathcal{B}})[1 - 2\Delta^{(1)}(m_H^2, Q_\perp^2)]$$

$$+ \alpha_s(Q_\perp^2)\left[\tilde{\mathcal{V}}(\Phi_{\mathcal{B}}) + \int \mathrm{d}\Phi_1 \mathcal{R}(\Phi_{\mathcal{B}} \times \Phi_1)\right]\Big\}$$

$$\Delta B_{2,b_\perp \to q_\perp}^{(q,g)} = 4\zeta(3)\left(A_1^{(q,g)}\right)^2$$

$$B_{2,Q_T}^{(q,g)} = B_{2,b_\perp}^{(q,g)} + \Delta B_2^{(q,g)}$$

$$\Delta_k^{(\mathrm{NNLL})}(Q^2, Q_0^2) = \exp\left\{-\int_{Q_0^2}^{Q^2} \frac{\mathrm{d}q_\perp^2}{q_\perp^2}\left[A(q_\perp^2)\log\frac{Q^2}{q_\perp^2} + B(q_\perp^2)\right]\right\}$$

$$A(q_\perp^2) = \frac{\alpha_s(q_\perp^2)}{2\pi} A_1 + \left(\frac{\alpha_s(q_\perp^2)}{2\pi}\right)^2 A_2$$
$$B(q_\perp^2) = \frac{\alpha_s(q_\perp^2)}{2\pi} B_1 + \left(\frac{\alpha_s(q_\perp^2)}{2\pi}\right)^2 B_{2,Q_T}$$

$$\langle \mathcal{O} \rangle = \left\{\int \mathrm{d}\Phi_{\mathcal{B}}\left[\overline{\mathcal{B}}_N - \int_{t_c} \mathrm{d}\Phi_1 \mathcal{R}_N\right] + \int_{t_c} \mathrm{d}\Phi_{\mathcal{R}}[1 - \Delta_N(t_1, \mu_Q^2)]\mathcal{R}_N(\Phi_{\mathcal{R}})\right\} \mathcal{O}(\Phi_{\mathcal{B}})$$
$$+ \int_{t_c} \mathrm{d}\Phi_{\mathcal{R}} \Delta_N(t_1, \mu_Q^2) \mathcal{R}_N \mathcal{E}_N^{(\mathcal{K})}(t_1, t_c; \mathcal{O})$$

$$\overline{\mathcal{B}}_N(\Phi_{\mathcal{B}}) = \mathcal{B}(\Phi_{\mathcal{B}}) + \tilde{\mathcal{V}}_N(\Phi_{\mathcal{B}}) + \int \mathrm{d}\Phi_1[\mathcal{R}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) - \mathcal{S}_N(\Phi_{\mathcal{B}} \otimes \Phi_1)]$$

$$\mathcal{E}_n^{(\mathcal{K})}(t, t_c; \mathcal{O}) = \Delta_n^{(\mathcal{K})}(t, t_c)\mathcal{O}(\Phi_n)$$
$$+ \int_{t_c}^{t} \mathrm{d}\Phi_1\left[\mathcal{K}_n(\Phi_1)\Delta_n^{(\mathcal{K})}(t, t(\Phi_1)) \otimes \mathcal{E}_{n+1}^{(\mathcal{K})}(t(\Phi_1), t_c; \mathcal{O})\right]$$

33. Ecuaciones PDF (Función de Distribución de Parton) y Ecuaciones DGLAP.

$$\sigma = \sum_{a,b} \int_0^1 \mathrm{d}x_a\, \mathrm{d}x_b \int f_{a/h_1}(x_a, \mu_F) f_{b/h_2}(x_b, \mu_F) \mathrm{d}\hat{\sigma}_{ab \to n}(\mu_F, \mu_R)$$

$$\frac{\partial}{\partial \log\ Q^2}\begin{pmatrix} f_{q/h}(x,Q^2) \\ f_{g/h}(x,Q^2) \end{pmatrix}$$

$$= \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{\mathrm{d}z}{z}\begin{pmatrix} \mathcal{P}_{qq}^{(1)}\left(\frac{x}{z}\right) \mathcal{P}_{qg}^{(1)}\left(\frac{x}{z}\right) \\ \mathcal{P}_{gq}^{(1)}\left(\frac{x}{z}\right) \mathcal{P}_{gg}^{(1)}\left(\frac{x}{z}\right) \end{pmatrix}\begin{pmatrix} f_{q/h}(z,Q^2) \\ f_{g/h}(z,Q^2) \end{pmatrix}$$

$$p^\mu = \frac{\sqrt{\hat{s}}}{2}(1{,}0{,}0{,}1) \text{ and } p'^\mu = \frac{\sqrt{\hat{s}}}{2}(1{,}0{,}0,-1),$$

$$\mathrm{d}\hat{\sigma}^{(0)}_{jj'\to X}(p,p') = \frac{1}{2\hat{s}}\ \mathrm{d}\Phi_X \left|\mathcal{M}^{(0)}_{jj'\to X}(p,p')\right|^2.$$

$$k^\mu = (1-x)p^\mu + \beta p'^\mu + k_\perp^\mu,$$

$$k^\mu = (1-x)p^\mu + \frac{\mathbf{k}_\perp^2}{(1-x)\hat{s}}p'^\mu + k_\perp^\mu.$$

$$q^2 = (p-k)^2 = \frac{\mathbf{k}_\perp^2}{1-x}$$

$$\mathrm{d}\Phi_k = \frac{\mathrm{d}^4k}{(2\pi)^4}(2\pi)\delta(k^2) = \frac{1}{(2\pi)^3}\ \mathrm{d}x\ \mathrm{d}\beta\ \mathrm{d}^2\mathbf{k}_\perp\delta\left((1-x)\beta - \frac{\mathbf{k}_\perp^2}{\hat{s}}\right) = \frac{\mathrm{d}x\ \mathrm{d}\mathbf{k}_\perp^2}{16\pi^2(1-x)}$$

$$\mathrm{d}\hat{\sigma}_{jj'\to X}(p,p') = \frac{1}{2\hat{s}}\ \mathrm{d}\Phi_X\left|\mathcal{M}^{(0)}_{jj'\to X}(p,p')\right|^2$$

$$+\frac{1}{2\hat{s}}\frac{1}{16\pi^2}\ \mathrm{d}\Phi_X\int_0^1\frac{\mathrm{d}x}{1-x}\int\ \ \mathrm{d}^2\mathbf{k}_\perp^2\left|\mathcal{M}^{(0)}_{jj'\to ikj'\to kX}(p,p')\right|^2$$

$$\frac{1-x}{x}\frac{\alpha_s}{2\pi}P_{ji}^{(1)}(x)\left|\mathcal{M}^{(0)}_{ij'\to X}(xp,p')\right|^2 = \frac{1}{16\pi^2}\lim_{\mathbf{k}_\perp\to 0}\left[\mathbf{k}_\perp^2\left|\mathcal{M}^{(0)}_{jj'\to ikj'\to kX}(p,p')\right|^2\right]$$

$$\epsilon_\pm^\mu = \frac{\sqrt{2}\mathbf{k}_\perp}{(1-x)\hat{s}}p'^\mu + \frac{1}{\sqrt{2}\mathbf{k}_\perp}k_\perp^\mu \pm i n_\perp^\mu$$

$$\epsilon\cdot p' = \epsilon\cdot k = 0.$$

$$\epsilon_\pm\cdot k_\perp = \frac{\mathbf{k}_\perp}{\sqrt{2}} \text{ and } \epsilon_\pm\cdot p = \frac{\mathbf{k}_\perp}{\sqrt{2}(1-x)}.$$

$$\left|\mathcal{M}^{(0)}_{ij'\to X}\right|^2 = \bar{u}(xp,\lambda)[\gamma_\mu M^\mu]u(xp,\lambda),$$

$$M^\mu = ap^\mu + bp'^\mu + m_\perp^\mu$$

$$\left|\mathcal{M}^{(0)}_{ij'\to X}\right|^2 = \bar{u}(xp,\lambda)[\gamma_\mu b p'^\mu]u(xp,\lambda).$$

$$|\bar{u}(xp,\lambda)(\gamma\cdot p')u(xp,\lambda)|^2 = (2xpp')^2 = x^2\hat{s}^2$$

$$M^\mu = \frac{p'^\mu}{2x\hat{s}} \left|\mathcal{M}^{(0)}_{ij'\to X}\right|^2 .$$

$$\begin{aligned}
&\left|\mathcal{M}^{(0)}_{jj'\to ikj'\to kX}(p,p')\right|^2 \\
&= \frac{g^2 \mathrm{Tr}[T^a T^a]}{2N_c} \sum_{\lambda,\pm} \bar{u}(p,\lambda) \not{k}_\pm \frac{\not{p} - \not{p}}{(p-k)^2} \left[\frac{\not{p}'}{x\hat{s}} \left|\mathcal{M}^{(0)}_{ij'\to X}\right|^2\right] \frac{\not{p} - \not{h}}{(p-k)^2} \not{k}^*_\pm u(p,\lambda) \\
&= \frac{g^2 \mathrm{Tr}[T^a T^a]}{2N_c x\hat{s}(-2p\cdot k)^2} \sum_{\pm} \mathrm{Tr}\left[\not{p} k_\pm (\not{p} - \not{h}) \not{p}' (\not{p} - \not{h}) \not{k}^*_\pm\right] \left|\mathcal{M}^{(0)}_{ij'\to X}\right|^2
\end{aligned}$$

$$= \frac{2g^2 C_F}{x\mathbf{k}_\perp^2} (1+x^2) \left|\mathcal{M}^{(0)}_{ij'\to X}\right|^2 .$$

$$\begin{aligned}
2kp &= \frac{\mathbf{k}_\perp^2}{1-x} \\
2kp' &= \hat{s}(1-x) \\
\sum_\pm \epsilon^\pm_\mu \epsilon^{*\pm}_\nu &= -g_{\mu\nu} + \frac{k_\mu p'_\nu + k_\nu p'_\mu}{kp'}
\end{aligned}$$

$$\frac{1-x}{x} \frac{\alpha_s}{2\pi} P^{(1)}_{qq}(x) = \frac{1}{16\pi^2} \times \frac{2g^2 C_F}{x} (1+x^2) = \frac{8\pi \alpha_s C_F (1+x^2)}{16\pi^2 x}$$

$$P^{(1)}_{qq}(x) = C_F \frac{1+x^2}{1-x}$$

$$\begin{aligned}
\text{finite } &= \int_0^1 \mathrm{d}x \left[\frac{\alpha_s}{2\pi} P^{(1)}_{qq}(x) \left|\mathcal{M}_{qj'\to X}(xp)\right|^2 + (1+V)\left|\mathcal{M}_{qj'\to X}(xp)\right|^2\right] \\
&= \int_0^1 \mathrm{d}x \left[\frac{\alpha_s}{2\pi} P^{(1)}_{qq}(x) + \delta(1-x)(1+V)\right] \left|\mathcal{M}_{qj'\to X}(xp)\right|^2
\end{aligned}$$

$$\int_0^1 \mathrm{d}x \left[\frac{\alpha_s}{2\pi} P^{(1)}_{qq}(x) + \delta(1-x)(1+V)\right] = 1$$

$$V = -\frac{\alpha_s}{2\pi} \lim_{\delta\to 0} \int_0^{1-\delta} \mathrm{d}x P^{(1)}_{qq}(x) = \frac{\alpha_s C_F}{2\pi} \lim_{\delta\to 0} \int_0^{1-\delta} \mathrm{d}x \left(\frac{2}{1-x} - (1+x)\right) = \frac{\alpha_s C_F}{2\pi} \left(\frac{3}{2} + 2\log\,\delta\right)$$

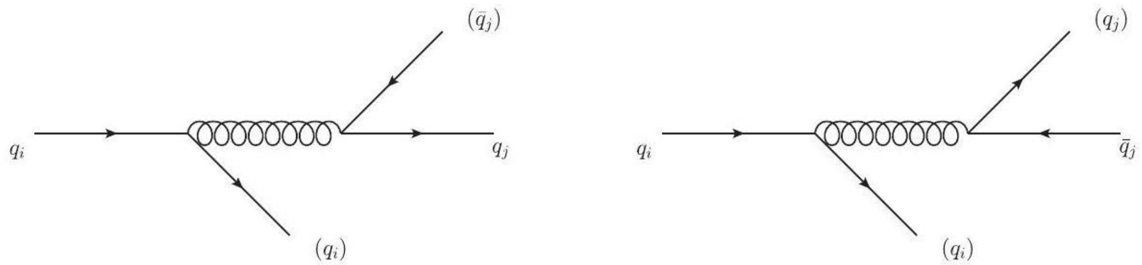
$$\mathcal{P}_{qq}(x) = C_F \lim_{\delta\to 0} \left[\frac{1+x^2}{1-x} \Theta(1-x-\delta) + \delta(1-x)\left(\frac{3}{2} + 2\log\,\delta\right)\right]$$

$$\left(\frac{1}{1-x}\right)_+ f(x) \overset{!}{=} \frac{f(x)-f(1)}{1-x},$$

$$\mathcal{P}_{qq}(x) = C_F \left[\left(\frac{1+x^2}{1-x}\right)_+ + \frac{3}{2}\delta(1-x)\right].$$

$$\int_0^1 \mathrm{d}x \mathcal{P}_{qq}(x) = 0$$

$$\mathcal{P}_{ij}(x) = \mathcal{P}_{ij}^{(1)} + \frac{\alpha_s}{2\pi}\mathcal{P}_{ij}^{(2)} + \mathcal{O}(\alpha_s^2)$$



$$\frac{\partial}{\partial \log Q^2}\begin{pmatrix} f_{q_i/h}(x,Q^2) \\ f_{\bar{q}_i/h}(x,Q^2) \\ f_{g/h}(x,Q^2) \end{pmatrix}$$

$$= \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dz}{z}\begin{pmatrix} \mathcal{P}_{q_iq_k}\left(\frac{x}{z}\right) & \mathcal{P}_{q_i\bar{q}_k}\left(\frac{x}{z}\right) & \mathcal{P}_{qg}\left(\frac{x}{z}\right) \\ \mathcal{P}_{\bar{q}_iq_k}\left(\frac{x}{z}\right) & \mathcal{P}_{\bar{q}_i\bar{q}_k}\left(\frac{x}{z}\right) & \mathcal{P}_{\bar{q}g}\left(\frac{x}{z}\right) \\ \mathcal{P}_{gq_k}\left(\frac{x}{z}\right) & \mathcal{P}_{g\bar{q}_k}\left(\frac{x}{z}\right) & \mathcal{P}_{gg}\left(\frac{x}{z}\right) \end{pmatrix}\begin{pmatrix} f_{q_k/h}(z,Q^2) \\ f_{\bar{q}_k/h}(z,Q^2) \\ f_{g/h}(z,Q^2) \end{pmatrix},$$

$$\mathcal{P}_{q_iq_j} = \delta_{ij}\mathcal{P}_{qq}^{V} + \mathcal{P}_{qq}^{S},$$

$$\mathcal{P}_{q_i\bar{q}_j} = \delta_{ij}\mathcal{P}_{q\bar{q}}^{V} + \mathcal{P}_{q\bar{q}}^{S}. (6.32)$$

$$\mathcal{P}_{(\pm)} = \mathcal{P}_{qq}^{V} \pm \mathcal{P}_{q\bar{q}}^{V},$$

$$\mathcal{P}_{QQ} = \mathcal{P}_{(+)} + 2n_f\mathcal{P}_{qq}^{S}$$

$$\mathcal{P}_{Qg} = 2n_f\mathcal{P}_{qg}$$

$$\mathcal{P}_{gQ} = \mathcal{P}_{gq} (6.33)$$

$$f_{q_i^{(\pm)}/h}(x,Q^2) = f_{q_i/h}(x,Q^2) \pm f_{\bar{q}_i/h}(x,Q^2)$$

$$f_{Q^{(\pm)}/h}(x,Q^2) = \sum_{i=1}^{n_f} f_{q_i^{(\pm)}/h}(x,Q^2)$$

$$\frac{\partial}{\partial \log Q^2} f_{q_i^{(-)}/h}(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dz}{z}\mathcal{P}_{(-)} f_{q_i^{(-)}/h}(z,Q^2)$$

$$\frac{\partial}{\partial \log Q^2} S_i(z,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dz}{z}\mathcal{P}_{(+)} S_i(z,Q^2)$$

$$S_i(x,Q^2) = n_f f_{q_i^{(+)}/h}(x,Q^2) - f_{Q^{(+)}/h}(x,Q^2).$$

$$\frac{\partial}{\partial \log Q^2}\begin{pmatrix} f_{Q^{(+)}/h}(x,Q^2) \\ f_{g/h}(x,Q^2) \end{pmatrix}$$

$$= \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dz}{z}\begin{pmatrix} \mathcal{P}_{QQ}\left(\frac{x}{z}\right) & \mathcal{P}_{Qg}\left(\frac{x}{z}\right) \\ \mathcal{P}_{gQ}\left(\frac{x}{z}\right) & \mathcal{P}_{gg}\left(\frac{x}{z}\right) \end{pmatrix}\begin{pmatrix} f_{Q^{(+)}/h}(z,Q^2) \\ f_{g/h}(z,Q^2) \end{pmatrix}$$

$$\mathcal{P}_{gg}^{(2)}(z) = C_A\mathcal{P}_{ggA}^{(2)}(z) + T_R n_f \mathcal{P}_{ggF}^{(2)}(z)$$

$$\mathcal{P}_{ggA}^{(2)}(z) = \frac{\Gamma_1}{8}\frac{1}{C_A}\left[P_{gg}^{(1)}(z)\right]_+ + \delta(1-z)[C_A(-1+3\zeta_3)+\beta_0]$$

$$+\left[P_{gg}^{(1)}(z)\right]_+\left(-2\ln(1-z)+\frac{1}{2}\ln z\right)\ln z + \left[P_{gg}^{(1)}(-z)\right]_+\left(S_2(z)+\frac{1}{2}\ln^2 z\right)$$

$$+C_A\left(4(1+z)\ln^2 z - \frac{4(9+11z^2)}{3}\ln z - \frac{277}{18z} + 19(1-z) + \frac{277}{18}z^2\right)$$

$$+\beta_0\left(\frac{13}{6z} - \frac{3}{2}(1-z) - \frac{13}{6}z^2 + (1+z)\ln z\right)$$

$$\mathcal{P}_{ggF}^{(2)}(z) = C_F\left[-\delta(1-z) + \frac{4}{3z} - 16 + 8z + \frac{20}{3}z^2 - 2(1+z)\ln^2 z - 2(3+5z)\ln z\right]$$

$$\Gamma_1 = \frac{4}{3}(C_A(4-\pi^2)+5\beta_0)$$

$$S_2(z) = -2\mathrm{Li}_2(-z) - 2\ln(1+z)\ln z - \frac{\pi^2}{6}$$

$$\mathcal{P}_{gq}^{(2)}(z) = C_A\left\{P_{gq}^{(1)}(z)\left[\ln^2(1-z) - 2\ln(1-z)\ln z - \frac{101}{18} - \frac{\pi^2}{6}\right]\right.$$

$$+P_{gq}^{(1)}(-z)S_2(z) + C_F\left(2z\ln(1-z) + (2+z)\ln^2 z - \frac{36+15z+8z^2}{3}\ln z\right.$$

$$\left.\left.+\frac{56-z+88z^2}{18}\right)\right\}$$

$$-C_F\left\{P_{gq}^{(1)}(z)\ln^2(1-z) + \left[3P_{gq}^{(1)}(z) + 2zC_F\right]\ln(1\right.$$

$$\left.-z) + C_F\left(\frac{2-z}{2}\ln^2 z - \frac{4+7z}{2}\ln z + \frac{5+7z}{2}\right)\right\} + \beta_0\left\{\mathcal{P}_{gq}^{(1)}(z)\left[\ln(1-z) + \frac{5}{3}\right] + z\right\}$$

$$\mathcal{P}_{qqV}^{(2)}(z) = \frac{\Gamma_1}{8}C_F\frac{(1+z^2)}{(1-z)_+} + \delta(1-z)C_F\left[C_F\left(\frac{3}{8} - \frac{\pi^2}{2} + 6\zeta_3\right) + C_A\left(\frac{1}{4} - 3\zeta_3\right) + \beta_0\left(\frac{1}{8} + \frac{\pi^2}{6}\right)\right]$$

$$+C_F^2\left\{\frac{1+z^2}{1-z}\left[2\ln(1-z) + \frac{3}{2}\right]\ln z + \frac{1+z}{2}\ln^2 z + \frac{3+7z}{2}\ln z + 5(1-z)\right\}$$

$$+C_AC_F\left[\frac{1}{2}\frac{1+z^2}{1-z}\ln^2 z + (1+z)\ln z + 3(1-z)\right] + \beta_0\left[\frac{1}{2}\frac{1+z^2}{1-z}\ln z + 1 - z\right]$$

$$\mathcal{P}_{q\bar{q}V}^{(2)}(z) = (2C_F - C_A)C_F\left\{\frac{1+z^2}{1+z}\left[S_2(z) + \frac{1}{2}\ln^2 z\right] + (1+z)\ln z + 2(1-z)\right\}$$

$$\mathcal{P}_{qqS}^{(2)}(z) = T_R C_F \left[-(1+z)\ln^2 z + \left(1 + 5z + \frac{8}{3}z^2\right)\ln z + \frac{20}{9z} - 2 + 6z - \frac{56}{9}z^2\right]$$

$$\mathcal{P}_{qg}^{(2)}(z) = C_F T_R \left[(z^2 + (1-z)^2)\left(\ln^2 \frac{1-z}{z} - 2\ln \frac{1-z}{z} - \frac{\pi^2}{3} + 5\right) + 2\ln(1-z) - \frac{1-2z}{2}\ln^2 z\right.$$

$$\left. - \frac{1-4z}{2}\ln z + 2 - \frac{9}{2}z\right]$$

$$+ C_A T_R \left\{(z^2 + (1-z)^2)\left[-\ln^2(1-z) + 2\ln(1-z) + \frac{22}{3}\ln z - \frac{109}{9} + \frac{\pi^2}{6}\right]\right.$$

$$+ (z^2 + (1+z)^2)S_2(z) - 2\ln(1-z) - (1$$

$$\left. + 2z)\ln^2 z + \frac{68z - 19}{3}\ln z + \frac{20}{9z} + \frac{91}{9} + \frac{7}{9}z\right\}$$

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\,\mathrm{d}y} = \frac{2\pi\alpha^2}{xyQ^4}\left[(1 + (1-y)^2)F_2 - (1 - (1-y)^2)xF_3 - y^2 F_L\right]$$

$$2\left(\frac{G_F m_W^2}{4\pi\alpha}\frac{Q^2}{Q^2 + m_W^2}\right)^2$$

$$F_2^{\mathrm{NC}} = x\sum_q C_q\left[f_q(x,Q^2) + f_{\bar{q}}(x,Q^2)\right],$$

$$F_3^{\mathrm{NC}} = \sum_q C'_q\left[f_q(x,Q^2) - f_{\bar{q}}(x,Q^2)\right],$$

$$F_2^{\mathrm{CC}(-)} = 2x[f_u(x,Q^2) + f_{\bar{d}}(x,Q^2) + f_{\bar{s}}(x,Q^2) + f_c(x,Q^2) + \cdots],$$

$$F_3^{\mathrm{CC}(-)} = 2[f_u(x,Q^2) - f_{\bar{d}}(x,Q^2) - f_{\bar{s}}(x,Q^2) + f_c(x,Q^2) + \cdots].$$

$$F(x,Q_0) = x^{A_1}(1-x)^{A_2}P(x; A_3, A_4 \ldots)$$

34. Método Hessiano.

$$\chi^2(\{a\},\{\lambda\}) = \sum_{k=1}^{N}\frac{1}{s_k^2}\left(D_k - T_k(\{a\}) - \sum_{\alpha=1}^{N_\lambda}\lambda_\alpha\beta_{k\alpha}\right)^2 + \sum_{\alpha=1}^{N_\lambda}\lambda_\alpha^2,$$

$$s_k = \sqrt{s_{k,\,\mathrm{stat}}^2 + s_{k,\,\mathrm{uncorr\ sys}}^2}$$

$$\Delta X_{\max}^{+} = \sqrt{\sum_{i=1}^{N}\left[\max\left(X_i^{+} - X_0, X_i^{-} - X_0, 0\right)\right]^2}$$

$$\Delta X_{\max}^{-} = \sqrt{\sum_{i=1}^{N}\left[\max\left(X_0 - X_i^{+}, X_0 - X_i^{-}, 0\right)\right]^2}$$

35. Método Multiplicador de Lagrange, ecuaciones NNLO, distribución y luminosidad partónicas.

$$\langle \mathcal{F}[\{q\}]\rangle = \frac{1}{N_{\text{rep}}}\sum_{k=1}^{N_{\text{rep}}} \mathcal{F}\left[\{q^{(k)}\}\right]$$

$$\sigma_{\mathcal{F}} = \left(\frac{N_{\text{rep}}}{N_{\text{rep}}-1}\left(\langle \mathcal{F}[\{q\}]^2\rangle - \langle \mathcal{F}[\{q\}]\rangle^2\right)\right)^{1/2}$$

$$= \left(\frac{1}{N_{\text{rep}}-1}\sum_{k=1}^{N_{\text{rep}}} \left(\mathcal{F}\left[\{q^{(k)}\}\right] - \langle \mathcal{F}[\{q\}]\rangle\right)^2\right)^{1/2}.$$

$$q^{(0)} \equiv \langle q\rangle = \frac{1}{N_{\text{rep}}}\sum_{k=1}^{N_{\text{rep}}} q^{(k)}$$

$$X = X_0 + \Delta X \cos\,\theta,$$
$$Y = Y_0 + \Delta Y \cos\,(\theta+\varphi),$$

$$\cos\,\varphi = \frac{\vec{\nabla}X\cdot\vec{\nabla}Y}{\Delta X\Delta Y} = \frac{1}{4\Delta X\Delta Y}\sum_{i=1}^{N}\left(X_i^{(+)} - X_i^{(-)}\right)\left(Y_i^{(+)} - Y_i^{(-)}\right).$$

$$\{a_{\text{minor}}, a_{\text{major}}\} = \frac{\sin\,\varphi}{\sqrt{1\pm\cos\,\varphi}}$$

$$\left(\frac{\delta X}{\Delta X}\right)^2 + \left(\frac{\delta Y}{\Delta Y}\right)^2 - 2\left(\frac{\delta X}{\Delta X}\right)\left(\frac{\delta Y}{\Delta Y}\right)\cos\,\varphi = \sin^2\,\varphi.$$

$$\Delta f = |\vec{\nabla} f| = \sqrt{(\Delta X\partial_X f)^2 + 2\Delta X\Delta Y\cos\,\varphi\partial_X f\partial_Y f + (\Delta Y\partial_Y f)^2}.$$

$$\frac{\Delta f}{f_0} = \sqrt{\left(m\frac{\Delta X}{X_0}\right)^2 - 2mn\frac{\Delta X}{X_0}\frac{\Delta Y}{Y_0}\cos\,\varphi + \left(n\frac{\Delta Y}{Y_0}\right)^2}.$$

$$\cos\,\phi[A,B] = \frac{N_{\text{rep}}}{(N_{\text{rep}}-1)}\frac{\langle AB\rangle_{\text{rep}} - \langle A\rangle_{\text{rep}}\langle B\rangle_{\text{rep}}}{\sigma_A\sigma_B}$$

$$\frac{dL_{ij}}{d\hat{s}dy} = \frac{1}{s}\frac{1}{1+\delta_{ij}}\left[f_i(x_1,\mu)f_j(x_2,\mu) + (1\leftrightarrow 2)\right]$$

$$\sigma = \sum_{i,j}\int_0^1 dx_1 dx_2 f_i(x_1,\mu)f_j(x_2,\mu)\hat{\sigma}_{ij}$$

$$\sigma = \sum_{i,j}\int \left(\frac{d\hat{s}}{\hat{s}}dy\right)\left(\frac{dL_{ij}}{d\hat{s}dy}\right)(\hat{s}\hat{\sigma}_{ij})$$

$$W_n^0 = 1, W_n^i = \frac{f(x_1, Q; S_i) f(x_2, Q; S_i)}{f(x_1, Q; S_0) f(x_2, Q; S_0)}$$

$$\sigma_{\text{tot}} > \sigma_{\text{inel}} > \sigma_{\text{el}} > \sigma_{\text{SD}} > \sigma_{\text{DD}} > \sigma_{\text{CXP}}$$

$$\mathcal{A}_{ab\to cd}(s,t) = \sum_{l=0}^{\infty} (2l+1) a_l(s) P_l(\cos\ \theta).$$

$$\cos\ \theta = 1 + \frac{2t}{s}$$

$$\mathcal{A}_{ab\to cd}(s,t) = \frac{1}{2i} \oint_C \mathrm{d}l (2l+1) a(l,t) \frac{P\left(l, 1+\frac{2t}{s}\right)}{\sin\ (\pi l)}.$$

$$a(l,t) < \exp\ (\pi l) \text{ for } |l| \to \infty.$$

$$\mathcal{A}_{ab\to cd}(s,t) = \frac{1}{2i} \oint_C \mathrm{d}l (2l+1) \sum_{\eta=\pm} \left[\frac{\eta + e^{-i\pi l} P\left(l, 1+\frac{2t}{s}\right)}{2\sin\ (\pi l)} a^{(\eta)}(l,t) \right]$$

$$\mathcal{A}_{ab\to cd}(s,t) = \frac{1}{2i} \int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} \mathrm{d}l (2l+1) \sum_{\eta=\pm} \left[\frac{(\eta + e^{-i\pi l}) P\left(l, 1+\frac{2t}{s}\right)}{2\sin\ (\pi l)} a^{(\eta)}(l,t) \right]$$
$$+ \sum_{\eta=\pm} \sum_{j\in n_\eta} \left[\frac{(\eta + e^{-i\pi\alpha_j(t)}) P\left(\alpha_j(t), 1+\frac{2t}{s}\right)}{2\sin\ (\pi\alpha_j(t))} \beta_j(t) \right]$$

$$\mathcal{A}_{ab\to cd}(s,t) \overset{s\to\infty}{\to} \frac{\eta + e^{-i\pi\alpha_j(t)}}{2} \beta(t) s^{\alpha_j(t)}.$$

$$\mathcal{A}_{ab\to cd}(s,t) \overset{s\to\infty}{\to} \frac{\eta + e^{-i\pi\alpha(t)}}{2} \frac{\gamma_{ac}(t)\gamma_{bd}(t)}{\sin\ [\pi\alpha(t)]\Gamma(\alpha(t))} s^{\alpha(t)}$$

$$\alpha(t) = \alpha(0) + \alpha' \cdot t$$

$$\sigma_{\text{tot}} \propto s^{\alpha(0)-1}$$

$$\sigma_{\text{tot}}^{(pp)}(s_{pp}) = \sigma_{\mathbf{P}} \left(\frac{s_{pp}}{\text{GeV}^2}\right)^{\epsilon}$$

$$\sigma_{\mathbf{P}} = 21.7\text{mb} \ \text{ and } \ \epsilon = 0.0808$$

$$\epsilon \lesssim \alpha_{\mathbf{P}} - 1$$

$$A = \frac{4N_c \log\ 2}{\pi} \alpha_s \approx 2.5 \cdot \alpha_s,$$

$$\sigma_{\text{tot}}^{(pp,p\bar{p})} = \sigma_{\mathbf{P}} \left(\frac{s_{pp}}{\text{GeV}^2}\right)^{\epsilon} + \sigma_{\mathbf{R}} \left(\frac{s_{pp}}{\text{GeV}^2}\right)^{-\eta}$$

$$\eta = 0.4525 \text{ and } \sigma_{\mathbf{R}} = \begin{cases} 56.08\text{mb for } pp \\ 98.39\text{mb for } p\bar{p}. \end{cases}$$

36. Función Eikonal.

$$\mathcal{T}(s,t) = 4s \int \mathrm{d}^2 B_\perp e^{i\vec{q}_\perp \cdot \vec{B}_\perp} a(s, \vec{B}_\perp)$$

$$t = \vec{q}^2 = \vec{q}_\perp^2$$

$$a(s, \vec{B}_\perp) = \frac{1}{2i}\left[\exp\left(-\frac{\Omega(s,\vec{B}_\perp)}{2}\right) - 1\right]$$

$$\Omega(s, \vec{B}_\perp) \propto \Omega_P \cdot \left(\frac{s}{\mathrm{GeV}^2}\right)^{\epsilon} + \Omega_R \cdot \left(\frac{s}{\mathrm{GeV}^2}\right)^{\eta} + \cdots$$

$$\sigma_{\text{tot}}(s) = \frac{1}{s}\Im\mathfrak{m}\left(\mathcal{T}(s, t=0) = 2\int \mathrm{d}^2 B_\perp \left[1 - \exp\left(-\frac{\Omega(s,\vec{B}_\perp)}{2}\right)\right]\right.$$

$$\sigma_{\text{el}}(s) = 4\int \mathrm{d}^2 B_\perp \left| a\left(s, \vec{B}_\perp\right|^2 = \int \mathrm{d}^2 B_\perp \left|\exp\left(-\frac{\Omega(s,\vec{B}_\perp)}{2}\right)\right|^2\right.$$

$$\sigma_{\text{inel}}(s) = \sigma_{\text{tot}}(s) - \sigma_{\text{el}}(s) = \int \mathrm{d}^2 B_\perp \left[1 - \exp\left(-\Omega(s,\vec{B}_\perp)\right)\right]$$

$$B(s) = \left[\frac{\mathrm{d}}{\mathrm{d}t}\left(\log \frac{\mathrm{d}\sigma_{\text{el}}(s,t)}{\mathrm{d}t}\right)\right]_{t=0} = \frac{1}{\sigma_{\text{tot}}}\int \mathrm{d}^2 B_\perp^2 B_\perp^2 \left[1 - \exp\left(-\frac{\Omega(s,\vec{B}_\perp)}{2}\right)\right].$$

37. Difracción. Estados de Good – Walker relativos a masa baja.

$$|\psi_j\rangle = \sum_{i=1}^{N} \alpha_{ji} |\phi_i\rangle.$$

$$\langle \phi_i \mid \phi_k \rangle = \delta_{ik} \text{ and } \sum_{i=1}^{N} |\alpha_{ji}|^2 = 1,$$

$$\langle \Psi | \hat{T} | \Psi \rangle = \sum_i |\alpha_{1i}|^2 T_i = \langle \hat{T} \rangle,$$

$$\sigma_{\text{el}} \propto \langle \hat{T} \rangle^2$$

$$\langle \psi_k | \hat{T} | \Psi \rangle = \sum_i \alpha_{1i} \alpha_{ik}^* T_i$$

$$\sum_k \langle \Psi | \hat{T} | \psi_k \rangle \langle \psi_k | \hat{T} | \Psi \rangle = \sum_{ijk} \alpha_{1i} \alpha_{ik}^* \alpha_{j1}^* \alpha_{kj} T_i T_j = \sum_{ij} \alpha_{1i} \alpha_{j1}^* T_i T_j \delta_{ij} = \langle \hat{T}^2 \rangle$$

$$\sigma_{\text{diff. exc.}} \propto \langle \hat{T}^2 \rangle - \langle \hat{T} \rangle^2$$

$$\sigma_{\text{tot}}(Y) = 2\int \mathrm{d}^2 B_\perp \left\{ \sum_{i,k} |\alpha_i|^2 |\alpha_k|^2 \left[1 - \exp\left(-\frac{\Omega_{ik}(Y, B_\perp)}{2} \right) \right] \right\}$$

$$\sigma_{\text{el}}(Y) = \int \mathrm{d}^2 B_\perp \left\{ \sum_{i,k} |\alpha_i|^2 |\alpha_k|^2 \left[1 - \exp\left(-\frac{\Omega_{ik}(Y, B_\perp)}{2} \right) \right] \right\}^2$$

$$\sigma_{\text{inel}}(Y) = \int \mathrm{d}^2 B_\perp \left\{ \sum_{i,k} |\alpha_i|^2 |\alpha_k|^2 \left[1 - \exp\left(-\Omega_{ik}(Y, B_\perp) \right) \right] \right\}$$

$$Y = \log \frac{s}{m_{\text{had}}^2}$$

$$\frac{\mathrm{d}\sigma_{\text{el}}(Y)}{\mathrm{d}t} = \frac{1}{4\pi} \int \mathrm{d}^2 B_\perp \left\{ e^{i\vec{q}_\perp \cdot \vec{B}_\perp} \sum_{i,k} |\alpha_i|^2 |\alpha_k|^2 \left[1 - \exp\left(-\frac{\Omega_{ik}(Y, B_\perp)}{2} \right) \right] \right\}^2$$

$$\frac{\mathrm{d}\sigma_{\text{el+SD}_1}(Y)}{\mathrm{d}t} = \frac{1}{4\pi} \sum_{i,j,k} \left\{ |\alpha_i|^2 |\alpha_j|^2 |\alpha_k|^2 \right.$$

$$\times \int \mathrm{d}^2 B_\perp \exp\left(i\vec{q}_\perp \cdot \vec{B}_\perp \right) \left[1 \right.$$

$$\left. - \exp\left(-\frac{\Omega_{ik}(Y, B_\perp)}{2} \right) \right] \times \int \mathrm{d}^2 B'_\perp \exp\left(-i\vec{q}_\perp \cdot \vec{B}'_\perp \right) \left[1 - \exp\left(-\frac{\Omega_{jk}(Y, B'_\perp)}{2} \right) \right] \Bigg\}$$

38. Grados de sabor y disociación hadrónica.

$$|p\rangle = |uud\rangle = \frac{1}{\sqrt{2}} |u\rangle |(ud)_0\rangle + \frac{1}{\sqrt{6}} |u\rangle |(ud)_1\rangle + \frac{1}{\sqrt{3}} |d\rangle |(uu)_1\rangle.$$

$$\left[\sigma_{2\to 2}(p_{\perp,\min}) \equiv \int_{p_{\perp,\min}^2}^{s} \mathrm{d}p_\perp^2 \frac{\mathrm{d}\hat{\sigma}_{2\to 2}}{\mathrm{d}p_\perp^2} \right]_{p_{\perp,\min} \approx 5\text{GeV}} \geq \sigma_{pp,\text{ tot}}$$

$$\sigma_{2\to 2}(p_{\perp,\min}) \geq \sigma_{pp,\text{ND}} \longrightarrow \langle N_{\text{scatters}}(p_{\perp,\min}) \rangle \equiv \frac{\sigma_{2\to 2}(p_{\perp,\min})}{\sigma_{pp,\text{ND}}} \geq 1$$

$$\Delta^{(\text{UE})}(Q^2, t) = \exp\left[-\frac{1}{\sigma_{pp,\text{ND}}} \int_t^{Q^2} \mathrm{d}p_\perp^2 \frac{\mathrm{d}\hat{\sigma}_{2\to 2}}{\mathrm{d}p_\perp^2} \right]$$

$$\frac{\alpha_s^2(p_\perp^2 + p_{\perp,0}^2)}{\alpha_s(p_\perp^2)} \frac{p_\perp^4}{\left(p_\perp^2 + p_{\perp,0}^2\right)^2}$$

$$p_{\perp,0}(E) = \left(\frac{E}{E_{\text{ref}}} \right)^\eta p_{\perp,0}(E_{\text{ref}})$$

$$f_{i/h_1}\left(x_1,\mu_F;\frac{b}{2}\right)f_{j/h_2}\left(x_2,\mu_F;\frac{b}{2}\right)=f_{i/h_1}(x_1,\mu_F)f_{j/h_2}(x_2,\mu_F)A(b)$$

$$\frac{\mathrm{d}\mathcal{P}(Q^2,t)}{\mathrm{d}p_\perp^2}=\left(\frac{\mathrm{d}\mathcal{P}_{\mathrm{PS}}}{\mathrm{d}p_\perp^2}+\frac{\mathrm{d}\mathcal{P}_{\mathrm{MPI}}}{\mathrm{d}p_\perp^2}\right)\cdot\exp\left[-\int_t^{Q^2}\mathrm{d}p_\perp^2\left(\frac{\mathrm{d}\mathcal{P}_{\mathrm{PS}}}{\mathrm{d}p_\perp^2}+\frac{\mathrm{d}\mathcal{P}_{\mathrm{MPI}}}{\mathrm{d}p_\perp^2}\right)\right]$$

39. Interacciones partónicas y hadronización.

$$\mathrm{d}\hat{\sigma}_{X+Y}=\mathrm{d}\hat{\sigma}_{X+Y}^{\mathrm{dir}}+\frac{m}{2}\frac{\mathrm{d}\hat{\sigma}_X^{\mathrm{dir}}\otimes\ \mathrm{d}\hat{\sigma}_Y^{\mathrm{dir}}}{\sigma_{\mathrm{eff}}},$$

$$f_{p_1p_2/h}(x_1,x_2;\mu_F^2)=(1-x_1-x_2)f_{p_1/h}(x_1;\mu_F^2)f_{p_2/h}(x_2;\mu_F^2)$$

$$\vec{p}_\perp^{\,\ell^+}+\vec{p}_\perp^{\,\ell^-}=\vec{p}_\perp^{\,(\ell\ell)}\approx 0,$$

$$\vec{p}_\perp^{\,(\ell\ell)}>0$$

$$\langle\rho\rangle=\int_0^\infty \mathrm{d}p_\perp p_\perp\rho(p_\perp)=\int_0^\infty \mathrm{d}p_\perp p_\perp \exp\left(-\frac{p_\perp^2}{\sigma^2}\right)\approx\frac{1}{R_{\mathrm{had}}}\approx m_{\mathrm{had}}\approx 1\mathrm{GeV}$$

$$E=\int_0^Y \mathrm{d}y\cosh\,y\int_0^\infty \mathrm{d}p_\perp p_\perp\rho(p_\perp)=\langle\rho\rangle\sinh\,Y$$

$$P=\int_0^Y \mathrm{d}y\sinh\,y\int_0^\infty \mathrm{d}p_\perp p_\perp\rho(p_\perp)=\langle\rho\rangle(\cosh\,Y-1)$$

$$M=E^2-P^2=2\cosh\,Y\langle\rho\rangle^2\approx 2E\langle\rho\rangle,$$

$$\frac{\mathrm{d}\sigma_{e^-e^+\to h+X}(z)}{\mathrm{d}z}=\sigma_{e^-e^+\to q\bar{q}}\left[D_{h/q}(z,\mu_F)+D_{h/\bar{q}}(z,\mu_F)\right]$$

$$\sum_h\int_0^1 \mathrm{d}zD_{q/h}(z,\mu_F)=1$$

$$V(r)=-\frac{\kappa}{r}+\sigma r$$

40. Funciones de fragmentación y parametrizaciones QCD de una partícula supermasiva.

$$x=\frac{2E_h}{\sqrt{s}}\in[0,1]$$

$$\frac{1}{\sigma_{e^+e^-\to q\bar{q}}}\frac{\mathrm{d}^2\sigma^h}{\mathrm{d}x\ \mathrm{d}\cos\,\theta}=\frac{3(1+\cos^2\,\theta)}{8}F_T^h(x)+\frac{3\sin^2\,\theta}{4}F_L^h(x)+\frac{3\cos\,\theta}{4}F_A^h(x)$$

$$\frac{1}{\sigma_{e^+e^-\to q\bar{q}}}\frac{\mathrm{d}\sigma^h}{\mathrm{d}x}=F^h(x,\mu^2)=\sum_i\int_x^1\frac{\mathrm{d}z}{z}C_i\left(z,\alpha_s,\frac{s}{\mu^2}\right)D_{h/i}\left(\frac{x}{z,\mu^2}\right)$$

$$C_i\left(z,\alpha_s,\frac{s}{\mu^2}\right)=g_i(s)\delta(1-z)+\mathcal{O}(\alpha_s)$$

$$\frac{\partial D_{h/i}(x,\mu^2)}{\partial \log\,\mu^2} = \sum_j \int_x^1 \frac{\mathrm{d}z}{z} \mathcal{P}_{ji}(z,\alpha_s) D_{h/j}\left(\frac{x}{z},\mu^2\right)$$

$$D_{h/i}(z,\mu_0^2) = N_i z^{\alpha_i^h}(1-z)^{\beta_i^h}$$

$$D_{h,\bar{h}/i}(z,\mu_0^2) = D_{\bar{h},h/\bar{\imath}}(z,\mu_0^2)$$

$$D_{\pi^+/d}(z,\mu_0^2) = D_{\pi^+/s} < D_{\pi^+/u}(z,\mu_0^2) = D_{\pi^+/\bar{d}}$$

$$D_{K^+/\bar{u}}(z,\mu_0^2) = D_{K^+/d,\bar{d}} < D_{K^+/u}(z,\mu_0^2) < D_{K^+/\bar{s}}$$

$$\beta_d^{\pi^+} = \beta_{s,\bar{s}}^{\pi^+} = \beta_{u,\bar{d}}^{\pi^+} + 1$$
$$\beta_{\bar{u}}^{K^+} = \beta_{d,\bar{d}}^{K^+} = \beta_u^{K^+} + 1 = \beta_{\bar{s}}^{K^+} + 2$$

$$\int_0^1 \mathrm{d}z \left[z \sum_h D_{h/i}(z,\mu^2) \right] = 1$$

$$D_{h/q}(z,\mu_F = 1\mathrm{GeV}) = \frac{N z^{\alpha}(1-z)^{\beta}\left[1+\gamma(1-z)^{\delta}\right]}{B(2+\alpha,1+\beta)+\gamma B(2+\alpha,1+\beta+\delta)},$$

$$D_{H/Q}(z,m_Q^2) \propto \begin{cases} \frac{1}{z}\left(1-\frac{1}{z}-\frac{\epsilon}{1-z}\right)^{-2} \\ z^{\alpha}(1-z) \\ (1-z)^{\alpha} z^{-(1+bm_{h,\perp}^2)} \exp\left(-\frac{bm_{h,\perp}^2}{z}\right) \end{cases}$$

41. Modelo Feynman-Field e interacciones de clústers pesados y fragmentación fuerte.

$$\rho(k_\perp^2) = \mathrm{d}k_\perp^2 \exp\left(-\frac{\pi k_\perp^2}{\sigma}\right)$$

$$\mathcal{P}_{C\to h_1 h_2} \propto n_s^{(h_1)} n_s^{(h_2)} \frac{\sqrt{\left(m_{\mathrm{clus}}^2 - m_1^2 - m_2^2\right)^2 + 4m_1^2 m_2^2}}{8\pi m_{\mathrm{clus}}} \times \left|\langle F_1 \bar{f} \mid \Psi_{h_1}\rangle\right|^2 \left|\langle f \bar{F}_2 \mid \Psi_{h_2}\rangle\right|^2 \mathcal{P}_{\to f\bar{f}}$$

$$M_{1,2} = m_{1,2} + \left(M - m_{1,2} - m_f\right)\#^{1/\eta}$$

$$f(z) = z^{\alpha}(1-z)^{\beta}$$

$$\sigma^2 A = \sigma^2 \frac{8r_0^2}{2} = 4E_0^2 = m^2$$

$$\Gamma = \frac{(eE)^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi m_f^2}{eE}\right)$$

$$\mathcal{P}_{q\bar{q}} \propto \exp\left(-\frac{\pi m_q^2}{\sigma}\right)$$

$$P_{u,d}: P_s: P_{(ud)} \approx 1: 0.3: 0.003$$

$$\mathcal{P}_{q\bar{q}} \longrightarrow \mathcal{P}_{q\bar{q}}(p_\perp) \propto \exp\left(-\frac{\pi m_q^2}{\sigma}\right)\exp\left(-\frac{\pi p_\perp^2}{\sigma}\right) = \exp\left(-\frac{\pi m_\perp^2}{\sigma}\right)$$

$$E_{i,j} = \pm\sigma\left(x_{i,j} - x_{ij}\right) \text{ and } p_{i,j} = \pm\sigma\left(t_{i,j} - t_{ij}\right)$$

$$E_{ij} = E_i + E_j = \sigma\left(x_i - x_j\right) \text{ and } p_{ij} = p_i + p_j = \sigma\left(t_i - t_j\right)$$

$$y_{ij} = \frac{1}{2}\log\frac{\left(x_i - x_j\right) + \left(t_i - t_j\right)}{\left(x_i - x_j\right) - \left(t_i - t_j\right)}$$

$$m_{ij}^2 = E_{ij}^2 - p_{ij}^2 = \sigma^2\left[\left(x_i - x_j\right)^2 - \left(t_i - t_j\right)^2\right] \overset{!}{>} 0,$$

$$\begin{aligned} \text{string}[q_1] &\longrightarrow \text{meson}[q_1\bar{q}_2] + \text{string}[q_2] \\ \text{string}[q_1] &\longrightarrow \text{baryon}[q_1(q_2q_3)] + \text{string}[(\bar{q}_2\bar{q}_3)] \\ \text{string}[(q_1q_2)] &\longrightarrow \text{baryon}[(q_1q_2)q_3] + \text{string}[\bar{q}_3] \end{aligned}$$

$$f(z) = N\frac{(1-z)^a}{z}\exp\left(-\frac{bm_\perp^2}{z}\right)$$

$$\text{string}[(q_1q_2)] \longrightarrow \text{meson}[q_1\bar{q}_3] + \text{string}[(q_2q_3)]$$

42. Caídas hadrónicas y ansimetrías.

$$\mathcal{M}_{\tau\to\nu_\tau\ell\bar{\nu}_\ell} = \frac{G_F}{\sqrt{2}}\left(\bar{u}_\ell\gamma_L^\mu u_{\bar{\nu}_\ell}\right)\left(\bar{u}_{\nu_\tau}\gamma_{\mu L}u_\tau\right) = \frac{G_F}{\sqrt{2}}J_\ell^\mu L_\mu$$

$$\frac{g_W^2}{8}\frac{g^{\mu\nu} - \frac{p^\mu p^\nu}{m_W^2}}{p^2 - m_W^2} = \frac{e^2 g^{\mu\nu}}{8m_W^2\sin^2\,\theta_W} + \mathcal{O}\left(\frac{p^2}{m_W^2}\right) = \frac{G_F}{\sqrt{2}}g^{\mu\nu} + \mathcal{O}\left(\frac{p^2}{m_W^2}\right)$$

$$\Gamma_{\tau\to\nu_\tau\ell\bar{\nu}_\ell} = \frac{G_F^2 m_\tau^5}{192\pi^3}f\left(\frac{m_\ell^2}{m_\tau^2}\right) \text{ with } f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2\log\,x$$

$$J_{h_1h_2\ldots h_N}^\mu = V_{uq}\langle h_1h_2\ldots h_N|\bar{u}_{\bar{u}}\gamma_L^\mu u_q|0\rangle,$$

$$J_{PS}^\mu = V_{uq}\langle PS|\bar{u}_{\bar{u}}\gamma_L^\mu u_q|0\rangle = -iV_{uq}f_{PS}p_{PS}^\mu$$

$$f_\pi = 130.2(1.7)\text{MeV and } f_K = 155.6(0.4)\text{MeV}$$

$$\Gamma_{\tau\to\nu_\tau PS^-} = \frac{G_F^2\left|V_{uq}\right|^2 f_{PS}^2 m_\tau^3}{16\pi}\left(1 - \frac{m_{PS}^2}{m_\tau^2}\right)^2$$

$$J^{\mu}_{h^-h^0} = V_{uq}\langle h^-h^0|\bar{u}_{\bar{u}}\gamma^{\mu}_L u_q|0\rangle = \sqrt{2}V_{uq}\left[\left(g^{\mu\nu} - \frac{q^{\mu}q^{\nu}}{q^2}\right)(p_{h^-,\nu} - p_{h^0,\nu})F_V^{h^-h^0}(q^2) + q^{\mu}F_S^{h^-h^0}(q^2)\right]$$

$$F_V^{\pi^-\pi^0}(s) = \frac{1}{\sum_V \alpha_V}\sum_V \frac{m_V^2}{m_V^2 - s - im_V\Gamma_V(s)}$$

$$\mathcal{M}_{M\to\ell\bar{\nu}_\ell} = \frac{G_F}{\sqrt{2}}V_{qq'}\langle 0|\bar{u}_{\bar{q}}\gamma_{\mu L}u_{q'}|M\rangle(\bar{u}_\ell\gamma^{\mu}_L u_{\bar{\nu}_\ell}),$$

$$f_{D^\pm} \approx 211.9(1.1)\text{MeV}, f_{D_s} \approx 249.0(1.2)\text{MeV},$$

$$f_{B^\pm} \approx 187.1(4.2)\text{MeV}, f_{B^0} \approx 190.9(4.1)\text{MeV}, \text{ and } f_{B_s} \approx 227.2(3.4)\text{MeV}.$$

$$\Gamma_{PS\to\ell\bar{\nu}_\ell} = \frac{G_F^2 f_{PS}^2 m_{PS} m_\ell^2 |V_{qq'}|^2}{8\pi}\left(1 - \frac{m_\ell^2}{m_{PS}^2}\right)^2.$$

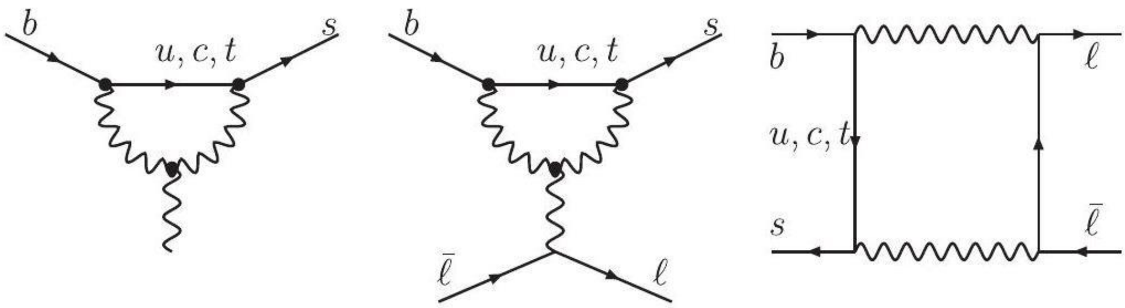
$$r = \left(1 - \tan^2\beta\frac{m_{PS}^2}{m_{H^\pm}^2}\right)^2$$

$$\mathcal{M}_{H\to\ell\bar{\nu}_\ell h} = \frac{G_F}{\sqrt{2}}V_{qq'}\langle h|\bar{u}_{\bar{q}}\gamma_{\mu L}u_{q'}|H\rangle(\bar{u}_\ell\gamma^{\mu}_L u_{\bar{\nu}_\ell}) = \frac{G_F}{\sqrt{2}}J_\mu^{Hh}L^\mu,$$

$$\langle h|\bar{u}_{\bar{q}}\gamma_\mu u_{q'}|H\rangle = \xi(v_h\cdot v_H)(v_h + v_H)_\mu$$

$$\xi(1) = 1 + \mathcal{O}(1/m_Q^2).$$

$$\mathcal{M}_{B\to\pi\pi} = \langle\pi|J_\mu^{b\to q}|B\rangle\langle\pi|J_{\mu,q\bar{q}}|0\rangle\left[1 + \sum r_n\alpha_s^n + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_B}\right)\right]$$



$$\mathcal{BR}_{j/\Psi\to\ell\bar{\ell}} \approx \frac{e_c^2\alpha}{C\alpha_s^3(m_c) + e_c^2\alpha\sum_f e_f^2} \approx \frac{e_c^2\alpha}{5\alpha_s^3(m_c) + 4\alpha e_c^2} \approx 5\%,$$

43. Tevatrón y de las partículas supermasivas fusionadas por aniquilación.

$$\sigma_{\text{tot}}^2 = \frac{16\pi}{1+\rho^2}\left.\frac{d\sigma_{\text{el}}}{d|t|}\right|_{t=0} = \frac{16\pi}{1+\rho^2}\frac{1}{\mathcal{L}}\left.\frac{dN_{\text{el}}}{d|t|}\right|_{t=0}$$

$$\sigma_{\text{inel}} = \sigma_{\text{tot}} - \sigma_{\text{el}}$$

$$\zeta = \frac{M_X^2}{E_{\text{c.m.}}^2},$$

$$\Delta_{jets}^n = \frac{\left|\vec{p}_\perp^{j_1} + \vec{p}_\perp^{j_2}\right|}{\left|\vec{p}_\perp^{j_1}\right| + \left|\vec{p}_\perp^{j_2}\right|}$$

$$z_{ij} = \frac{\min\left(p_{\perp_i}, p_{\perp_j}\right)}{p_{\perp_i} + p_{\perp_j}} < z_{\text{cut}}$$

$$\Delta R_{ij} > D_{\text{cut}} \equiv \alpha \cdot \frac{m_J}{p_\perp}$$

$$\frac{\sigma(Z + (n+1)\text{ jets })}{\sigma(Z + n\text{ jets })} = \frac{\bar{n}}{n},$$

$$\mu_F^2 = \mu_R^2 = m_{\ell\nu}^2 + p_\perp^2(\ell\nu) + \frac{m_b^2 + p_\perp^2(b)}{2} + \frac{m_{\bar{b}}^2 + p_\perp^2(\bar{b})}{2}.$$

$$m_T = \sqrt{\left(E_T^{ll} + p_T^{\nu\nu}\right)^2 - \left|\mathbf{p}_\mathbf{T}^{\text{ll}} + \mathbf{p}_\mathbf{T}^{\nu\nu}\right|^2}$$

$$\Gamma(\alpha) = \int_0^\infty \mathrm{d}x x^{\alpha-1} e^{-x}$$

$$\Gamma(\alpha+1) = \alpha\Gamma(\alpha).$$

$$\Gamma(\alpha) = (\alpha-1)!$$

$$\Gamma(1+\varepsilon) = \exp\left(-\gamma_E\varepsilon + \frac{\pi^2}{12}\varepsilon^2\right) + \mathcal{O}(\varepsilon^3),$$

$$\frac{\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)} = 1 - \frac{\varepsilon^2\pi^2}{6} + \mathcal{O}(\varepsilon^3).$$

$$B(\alpha,\beta) = \int_0^1 \mathrm{d}x x^{\alpha-1}(1-x)^{\beta-1} = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)},$$

$$\int_0^1 \mathrm{d}x f(x) g(x) = \int_0^1 \mathrm{d}x (f(x) - f(1)) g(x) + f(1)\int_0^1 \mathrm{d}x g(x)$$

$$= \int_0^1 \mathrm{d}x f(x)\left([g(x)]_+ + \delta(1-x)\int_0^1 \mathrm{d}y g(y)\right) \text{(A.7)}$$

$$\int_0^1 \mathrm{d}x f(x)[g(x)]_+ = \int_0^1 \mathrm{d}x[f(x) - f(1)]g(x)$$

$$\mathrm{Li}_2(x) = -\int_0^x dy \frac{\ln(1-y)}{y}$$

$$\mathrm{Li}_2(x) = \sum_k^{\infty} \frac{x^k}{k^2}$$

$$\mathrm{Li}_2(0) = 0$$

$$\mathrm{Li}_2(1) = \frac{\pi^2}{6}$$

$$\mathrm{Li}_2(x) + \mathrm{Li}_2(1-x) = \frac{\pi^2}{6} - \log\, x \log(1-x)$$

$$\mathbf{M}_N[f(x)] = \int_0^1 \mathrm{d}x x^N f(x)$$

$$\sigma = \int_0^1 \mathrm{d}x (f \otimes \hat{\sigma})(x) = \int_0^1 \mathrm{d}x \int_x^1 \frac{\mathrm{d}y}{y} f(y) \hat{\sigma}(x/y)$$

$$= \int_0^1 \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z \delta(x-yz) f(y) \hat{\sigma}(z)$$

$$\mathbf{M}_N[(f \otimes \hat{\sigma})(x)] = \int_0^1 \mathrm{d}x x^N (f \otimes \hat{\sigma})(x) = \int_0^1 \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z x^N \delta(x-yz) f(y) \hat{\sigma}(z)$$

$$= \int_0^1 \mathrm{d}y\, \mathrm{d}z (yz)^N f(y) \hat{\sigma}(z) = \mathbf{M}_N[f(x)] \cdot \mathbf{M}_N[\hat{\sigma}(x)]$$

$$\frac{\mathrm{d}}{\mathrm{dlog}\, \mu^2} \mathbf{M}_N\left[f_{i/A}(x,\mu)\right] = \gamma\left(N, \alpha_s(\mu^2)\right) \mathbf{M}_N\left[f_{i/A}(x,\mu)\right]$$

$$\mathbf{M}_N\left[f_{i/A}(x,\mu)\right] = \exp\left[-\int_{\mu^2}^{Q^2} \frac{\mathrm{d}q^2}{q^2} \gamma\left(N, \alpha_s(q^2)\right)\right] \mathbf{M}_N\left[f_{i/A}(x,Q)\right]$$

$$\gamma\left(N, \alpha_s(\mu^2)\right) = \sum_{i=1}^{\infty} \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^i \gamma^{(i)}(N)$$

$$\gamma_{qq}^{(1)}(N) = \mathbf{M}_N\left[P_{qq}^{(1)}(x)\right] = \int_0^1 \mathrm{d}x x^N P_{qq}^{(1)}(x) = C_F \int_0^1 \mathrm{d}x x^N \left(\frac{1+x^2}{1-x}\right)_+$$

$$= C_F \int_0^1 \mathrm{d}x \frac{(1+x^2)(x^N-1)}{1-x} = C_F \xi(N)$$

$$\frac{\mathrm{d}^L}{\mathrm{d}N^L} \mathbf{M}_N[f(x)] = \mathbf{M}_N[\log^L(x) f(x)]$$

$$\mathbf{M}_N[x^k f(x)] = \mathbf{M}_{N+k} f(x)$$

$$\mathbf{M}_N\left[\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right] = x^N f(x)|_0^1 - N \mathbf{M}_{N-1} f(x)$$

$$\mathbf{M}_N[1] = \frac{1}{N+1}$$

$$\mathbf{M}_N\left[\left[\frac{1}{1-x}\right]_+\right] = -\sum_{k=1}^{N}\frac{1}{k}$$

$$\mathbf{M}_N\left[\left[\frac{\log(1-x)}{1-x}\right]_+\right] = \frac{1}{2}\{\psi'(N) + \zeta(2) + [\psi(N) + \gamma_E]^2\}$$

$$\psi(x) = \frac{\mathrm{dlog}\,\Gamma(x)}{\mathrm{d}x} \text{ and } \psi'(x) = \frac{\mathrm{d}\psi(x)}{\mathrm{d}x} = \frac{\mathrm{d}^2\log\,\Gamma(x)}{\mathrm{d}x^2}$$

$$\frac{1}{A_1^{\nu_1}\dots A_n^{\nu_n}} = \frac{\Gamma(\nu)}{\prod_i\,\Gamma(\nu_i)}\int_0^1\,\mathrm{d}^n x_i\delta\left(\sum_i x_i - 1\right)\frac{\prod_i\, x_i^{\nu_i-1}}{(\sum_i\, x_iA_i)^{\sum_i\nu_i}}$$

$$\int\frac{\mathrm{d}^D\ell}{(2\pi)^D}\frac{(\ell^2)^k}{(\ell^2-\Delta+i\varepsilon)^n}$$

$$\int\,\mathrm{d}^D\ell = \int\,\mathrm{d}\ell_E\ell_E^{D-1}\sin^{D-2}\,\theta_{D-1}\sin^{D-3}\,\theta_{D-2}\dots\sin\,\theta_2\,\mathrm{d}\theta_{D-1}\,\mathrm{d}\theta_{D-2}\dots\,\mathrm{d}\theta_1$$

$$\int_0^\pi\,\mathrm{d}\theta\sin^n\,\theta = \sqrt{\pi}\frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right)}$$

$$\int\frac{\mathrm{d}^D\ell}{(2\pi)^D} = \frac{2i\pi^{D/2}}{(2\pi)^D\Gamma(D/2)}\int\,\mathrm{d}\ell_E\ell_E^{D-1}$$

$$\int\frac{\mathrm{d}\ell_E\ell_E^{D-1}(-\ell_E^2)^k}{(-\ell_E^2-\Delta+i\varepsilon)^n} = \frac{(-1)^{n-k}}{2}(\Delta-i\varepsilon)^{D/2-n+k}\int_0^1\,\mathrm{d}x x^{n-k-D/2-1}(1-x)^{D/2+k-1}$$

$$\int\frac{\mathrm{d}^D\ell}{(2\pi)^D}\frac{(\ell^2)^k}{(\ell^2-\Delta)^n} = \frac{i(-1)^{n-k}}{(4\pi)^{D/2}}\frac{\Gamma(D/2+k)}{\Gamma(D/2)}\frac{\Gamma(n-k-D/2)}{\Gamma(n)}(\Delta-i\varepsilon)^{D/2-n+k}.$$

$$(\not{p}-m)u(p,\lambda) = 0, (\not{p}+m)v(p,\lambda) = 0,$$

$$(1\mp\gamma^5\not{\phi})u(p,\pm) = 0, (1\mp\gamma^5\not{\phi})v(p,\pm) = 0$$

$$w(k_0,\lambda)\bar{w}(k_0,\lambda) = \frac{1+\lambda\gamma_5}{2}\not{k}_0,$$

$$w(k_0,\lambda) = \lambda\not{k}_1 w(k_0,-\lambda)$$

$$k_0^2 = 0, k_0\cdot k_1 = 0 \text{ and } k_1^2 = -1.$$

$$u(p,\lambda) = \frac{\not{p}+m}{\sqrt{2p\cdot k_0}}w(k_0,-\lambda)$$
$$v(p,\lambda) = \frac{\not{p}-m}{\sqrt{2p\cdot k_0}}w(k_0,-\lambda)$$

$$\bar{u} = u^\dagger\gamma^0 \text{ and } \bar{v} = v^\dagger\gamma^0,$$

$$\bar{u}(p,\lambda)(\not{p}-m)=0 \text{ and } \bar{v}(p,\lambda)(\not{p}+m)=0,$$

$$\bar{u}(p,\pm)(1\mp\gamma^5\not{\phi})=0 \text{ and } \bar{v}(p,\pm)(1\mp\gamma^5\not{\phi})=0$$

$$\bar{u}(p,\lambda)u(p,\lambda)=2m \text{ and } \bar{v}(p,\lambda)v(p,\lambda)=-2m,$$

$$\bar{u}(p,\lambda)=\bar{w}(k_0,-\lambda)\frac{\not{p}+m}{\sqrt{2p\cdot k_0}}$$
$$\bar{v}(p,\lambda)=\bar{w}(k_0,-\lambda)\frac{\not{p}-m}{\sqrt{2p\cdot k_0}}.$$

$$\bar{u}(p_1,+)u(p_2,p_2) = \frac{(p_1k_0)(p_2k_1)-(p_1k_1)(p_2k_0)+i\epsilon_{\mu\nu\rho\sigma}p_1^\mu p_2^\nu k_0^\rho k_1^\sigma}{\sqrt{(p_1k_0)(p_2k_0)}}$$
$$\bar{u}(p_1,-)u(p_2,+) = [\bar{u}(p_1,+)u(p_2,-)]^*.$$

$$1=\sum_\lambda \frac{\bar{u}(p,\lambda)u(p,\lambda)-\bar{v}(p,\lambda)v(p,\lambda)}{2m}.$$

$$\not{p}+m=\frac{1}{2}\sum_\lambda\left[\left(1+\frac{m}{\sqrt{p^2}}\right)u(p,\lambda)\bar{u}(p,\lambda)+\left(1-\frac{m}{\sqrt{p^2}}\right)v(p,\lambda)\bar{v}(p,\lambda)\right]$$

$$\epsilon_\mu(p,\lambda)p^\mu=0$$
$$\sum_{\lambda=\pm}\epsilon_\mu(p,\lambda)\epsilon_\nu^*(p,\lambda)=-g_{\mu\nu}+\frac{q_\mu p_\nu+q_\nu p_\mu}{pq}$$

$$\epsilon_\mu(p,\lambda)=\frac{1}{2\sqrt{pq}}\bar{u}(q,\lambda)\gamma_\mu u(p,\lambda).$$

$$\sum_{\lambda=\pm,0}\epsilon_\mu(p,\lambda)\epsilon_\nu^*(p,\lambda)=-g_{\mu\nu}+\frac{p_\mu p_\nu}{p^2},$$

$$\epsilon_\mu(p,\lambda)\longrightarrow\sqrt{\frac{3}{8\pi p^2}}\bar{u}(q_1,\lambda)\gamma_\mu u(q_2,\lambda)\bigg|_{p^\mu=q_1^\mu+q_2^\mu}$$

$$\psi_{\dot{a}}=(\psi_a)^* \text{ and } \psi^a=(\psi^{\dot{a}})^*$$

$$\epsilon_{ab}=\epsilon^{ab}=\epsilon_{\dot{a}\dot{b}}=\epsilon^{\dot{a}\dot{b}}=\begin{pmatrix}0&1\\-1&0\end{pmatrix}.$$

$$\langle\zeta\eta\rangle=\zeta_a\eta^a$$
$$[\zeta\eta]=\zeta_{\dot{a}}\eta^{\dot{a}}=\langle\zeta\eta\rangle^*$$

$$\sigma^{\mu\dot{a}b}=(\sigma^0,\vec{\sigma}) \text{ and } \sigma^\mu_{a\dot{b}}=(\sigma^0,-\vec{\sigma})$$

$$k_{\dot{a}b}=\sigma^\mu_{a\dot{b}}k_\mu=\begin{pmatrix}k^+&k_\perp\\k_\perp^*&k^-\end{pmatrix}, \text{ where } \begin{matrix}k^\pm=k^0\pm k^3\\k_\perp=k^1+ik^2\end{matrix}$$

$$k_{\dot{a}b} = \zeta_{\dot{a}}(k)\zeta_b(k), \text{ with } \zeta_a(k) = \begin{pmatrix} \sqrt{k^+} \\ \sqrt{k^-}e^{i\phi_k} \end{pmatrix},$$

$$k^\mu = \sigma^\mu_{\dot{a}b}\zeta^{\dot{a}}(k)\zeta^b(k).$$

$$2k_ik_j = \langle ij\rangle[ij],$$

$$P_{R,L} = P_\pm = \frac{1 \pm \gamma_5}{2}.$$

$$u_+(p,m) = \frac{1}{\sqrt{2|\vec{p}|}}\begin{pmatrix} \sqrt{p_0 - \bar{p}}\chi_+(\hat{p}) \\ \sqrt{p_0 + \bar{p}}\chi_+(\hat{p}) \end{pmatrix}$$
$$u_-(p,m) = \frac{1}{\sqrt{2|\vec{p}|}}\begin{pmatrix} \sqrt{p_0 + \bar{p}}\chi_-(\hat{p}) \\ \sqrt{p_0 - \bar{p}}\chi_-(\hat{p}) \end{pmatrix}$$

$$v_+(p,m) = \frac{1}{\sqrt{2|\vec{p}|}}\begin{pmatrix} \sqrt{p_0 - \bar{p}}\chi_-(\hat{p}) \\ -\sqrt{p_0 + \bar{p}}\chi_-(\hat{p}) \end{pmatrix}$$
$$v_-(p,m) = \frac{1}{\sqrt{2|\vec{p}|}}\begin{pmatrix} -\sqrt{p_0 - \bar{p}}\chi_+(\hat{p}) \\ \sqrt{p_0 + \bar{p}}\chi_+(\hat{p}) \end{pmatrix}$$

$$\chi_+(\hat{p}) = \frac{1}{\sqrt{\hat{p}^+}}\begin{pmatrix} \hat{p}^+ \\ \hat{p}_\perp \end{pmatrix} = \begin{pmatrix} \sqrt{\hat{p}^+} \\ \sqrt{\hat{p}^-}e^{i\phi_p} \end{pmatrix}$$
$$\chi_-(\hat{p}) = \frac{e^{i\pi}}{\sqrt{\hat{p}^+}}\begin{pmatrix} -\hat{p}^*_\perp \\ \hat{p}^+ \end{pmatrix} = \begin{pmatrix} \sqrt{\hat{p}^-}e^{-i\phi_p} \\ -\sqrt{\hat{p}^+} \end{pmatrix}.$$

$$u_\pm(k) = v_\mp(k) = |k^\pm\rangle \text{ and } \bar{u}_\pm(k) = \bar{v}_\mp(k) = \langle k^\pm|$$

$$\epsilon^\mu_\pm(p,q) = \pm\frac{\langle q^\mp|\gamma^\mu|p^\mp\rangle}{\sqrt{2}\langle q^\mp \mid p^\pm\rangle}.$$

$$\epsilon^\mu_\pm(p,q) = \pm\frac{\langle q^\mp|\gamma^\mu|\tilde{p}^\mp\rangle}{\sqrt{2}\langle q^\mp \mid \tilde{p}^\pm\rangle}$$

$$\epsilon^\mu_0(p,q) = \frac{1}{\sqrt{p^2}}\left[\langle\tilde{q}^-|\gamma^\mu|\tilde{q}^-\rangle - \frac{p^2}{2pq}\langle q^-|\gamma^\mu|q-\rangle\right],$$

$$|k^\pm\rangle\langle k^p m| = \frac{1 \pm \gamma_5}{2}\not{k},$$

$$\bar{u}(+,p_1)u(-,p_2) = \frac{\langle p_1p_2\rangle\langle k_0k_1\rangle[p_2k_0][k_1p_1]}{\sqrt{4(p_1k_0)(p_2k_0)}}$$
$$\bar{u}(-,p_1)u(+,p_2) = \frac{\langle p_2k_0\rangle\langle k_1p_1\rangle[p_1p_2][k_0k_1]}{\sqrt{4(p_1k_0)(p_2k_0)}}$$

$$y = \frac{1}{2}\log\frac{E + p_z}{E - p_z}$$

$$E' = E\cosh\,\gamma - p_z\sinh\,\gamma$$
$$p_z' = p_z\cosh\,\gamma - E\sinh\,\gamma$$

$$y' = y - \gamma$$

$$\eta = \log\,\tan\,\frac{\theta}{2}$$

$$p^\mu = p_\perp(\cosh\,\eta, \cos\,\phi, \sin\,\phi, \sinh\,\eta)$$

$$p^\mu = (m_\perp\cosh\,y, p_\perp\cos\,\phi, p_\perp\sin\,\phi, m_\perp\sinh\,y)$$

$$m_\perp^2 = p_\perp^2 + m^2$$

$$\frac{\mathrm{d}^4 p}{(2\pi)^4}(2\pi)\delta(p^2 - m^2)\Theta(p_0) = \frac{\mathrm{d}^3 p}{2E(2\pi)^3} = \frac{p_\perp \mathrm{d}p_\perp \mathrm{d}y\,\mathrm{d}\phi}{2(2\pi)^3}$$

$$P_\pm = (E, 0{,}0, \pm E)$$

$$S = 2P_+P_-$$

$$p^\mu = \alpha P_+^\mu + \beta P_-^\mu + \vec{p}_\perp^\mu$$

$$y = \frac{1}{2}\log\,\frac{E + p_z}{E - p_z} = \frac{1}{2}\log\,\frac{p_+}{p_-} = \frac{1}{2}\log\,\frac{\alpha}{\beta}$$

$$p^2 = \alpha\beta S - p_\perp^2$$

$$\alpha = \frac{m^2 + p_\perp^2}{\beta S} = \frac{m^2 + p_\perp^2}{S}e^{+y} \;\; \text{or} \;\; \beta = \frac{m^2 + p_\perp^2}{\alpha S} = \frac{m^2 + p_\perp^2}{S}e^{-y}$$

$$\alpha_1 + \alpha_2 = \alpha_1 = \sum_{i=3}^{n} \alpha_i$$

$$\beta_1 + \beta_2 = \beta_2 = \sum_{i=3}^{n} \beta_i$$

$$\vec{p}_{\perp,1} + \vec{p}_{\perp,2} = 0 = \sum_{i=3}^{n} \vec{p}_{\perp,i}$$

$$x_1 \equiv \alpha_1 \;\; \text{and} \;\; x_2 \equiv \beta_2$$

44. Invariancia de Gauge $U(1)$ en lagrangiano y transformaciones de gauge en simetría.

$$\mathcal{L} = \bar{\psi}(i\,\partial\!\!\!/ - m)\psi,$$

$$\psi \longrightarrow \psi' = e^{i\theta}\psi \;\; \text{and} \;\; \bar{\psi} \longrightarrow \bar{\psi}' = \bar{\psi}e^{-i\theta}$$

$$\partial\!\!\!/\psi' = e^{i\theta(x)}\,\partial\!\!\!/\psi + e^{i\theta(x)}(i\,\partial\!\!\!/\theta(x))\psi \neq e^{i\theta(x)}\,\partial\!\!\!/\psi$$

$$D_\mu = \partial_\mu - ieA_\mu(x)$$

$$A_\mu(x) \longrightarrow A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu\theta(x)$$

$$(D\psi)' = D\psi\psi$$

$$\mathcal{L} = \bar{\psi}(i D\!\!\!\!/ - m)\psi = \bar{\psi}(i\,\partial\!\!\!/ + e\Big/\mathcal{A} - m)\psi.$$

$$\mathcal{L}_{\text{gauge}} = [D_\mu, D_\nu] = (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

$$\mathcal{L}_{gm} = \frac{m^2}{2}A_\mu A^\mu$$

$$\Psi = (\psi_1, \psi_2, \dots \psi_n)^T$$

$$\Psi \longrightarrow \Psi' = \exp\,(i\theta^a\tau^a)\Psi$$

$$\psi_i \longrightarrow \psi'_i = [\exp\,(i\theta^a\tau^a)]_{ij}\psi_j = U_{ij}\psi_j,$$

$$D_\mu = \partial_\mu - ig\tau^a A^a_\mu(x) = \partial_\mu - igA_\mu.$$

$$A_\mu(x) \longrightarrow A'_\mu(x) = U(x)A_\mu(x)U^\dagger(x) + \frac{i}{g}[\partial_\mu U(x)]U^\dagger(x)$$

$$D_\mu(x) \longrightarrow D'_\mu(x) = U(x)D_\mu(x)U^\dagger(x).$$

$$\Psi_q = (\psi_{q,1}, \psi_{q,2}, \psi_{q,3})^T.$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$[\lambda_a, \lambda_b] = if_{abc}\lambda_c$$

$$C_F = \sum_a \tau_a^2 = \frac{1}{4}\sum_{a=1}^{8} \lambda_a^2 = \frac{4}{3}.$$

$$f_{123} = 1$$

$$f_{147} = f_{165} = f_{246} = f_{257} = f_{345} = f_{376} = \frac{1}{2}$$

$$f_{458} = f_{678} = \frac{\sqrt{3}}{2}$$

$$T^a_{ik} = if_{aik}$$

$$C_A = \sum_a T^a T^a = 3$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[\sigma_a, \sigma_b] = 1\epsilon_{abc}\sigma_c$$

$$\psi = \psi_L + \psi_R = P_L\psi + P_R\psi = \frac{1-\gamma_5}{2}\psi + \frac{1+\gamma_5}{2}\psi$$

Fields	$SU(3)_c$	$SU(2)_L: T_3$	$U(1)_Y: Y_W$	Q
$Q^{(I)}_{L,i} = \begin{pmatrix} u^{(I)}_{L,i} \\ d^{(I)}_{L,i} \end{pmatrix}$	C_F	$+\frac{1}{2} - \frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3} - \frac{1}{3}$
$u^{(I)}_{R,i}$	C_F	0	$+\frac{4}{3}$	$+\frac{2}{3}$
$d^{(I)}_{R,i}$	C_F	0	$-\frac{2}{3}$	$-\frac{1}{3}$
$L^{(I)}_{L,i} = \begin{pmatrix} \nu^{(I)}_{L,i} \\ \ell^{(I)}_{L,i} \end{pmatrix}$	0	$+\frac{1}{2} - \frac{1}{2}$	-1	0
$\ell^{(I)}_{R,i}$	0	0	-2	-1

$$Q = T_3 + \frac{Y_W}{2}.$$

$$D_\mu Q^{(I)}_{L,i,\alpha} = \left(\partial_\mu + ig_3\frac{\lambda^a_{ij}}{2}G^a_\mu\delta_{\alpha\beta} + ig_2\frac{\sigma^a_{\alpha\beta}}{2}W_\mu^a\delta_{ij} + ig_1\frac{Y_W}{2}B_\mu\delta_{ij}\delta_{\alpha\beta}\right)Q^{(I)}_{L,j,\beta}$$

$$D_\mu u^{(I)}_{R,i} = \left(\partial_\mu + ig_3\frac{\lambda^a_{ij}}{2}G^a_\mu + ig_1\frac{Y_W}{2}B_\mu\delta_{ij}\right)u^{(I)}_{R,j}$$

$$D_\mu d^{(I)}_{R,i} = \left(\partial_\mu + ig_3\frac{\lambda^a_{ij}}{2}G^a_\mu + ig_1\frac{Y_W}{2}B_\mu\delta_{ij}\right)u^{(I)}_{R,j}$$

$$D_\mu L_{L,\alpha}^{(I)} = \left(\partial_\mu + ig_2 \frac{\sigma_{\alpha\beta}^a}{2} W_\mu^a + ig_1 \frac{Y_W}{2} B_\mu \delta_{\alpha\beta}\right) L_{L,\beta}^{(I)}$$

$$D_\mu \ell_R^{(I)} = \left(\partial_\mu + ig_1 \frac{Y_W}{2} B_\mu\right) \ell_R^{(I)}$$

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{gauge}}$$

$$\mathcal{L}_{\text{matter}} = \sum_{I=1}^{3} \left[\bar{Q}_L^{(I)} \not{D} Q_L^{(I)} + \bar{u}_R^{(I)} \not{D} u_R^{(I)} + \bar{d}_R^{(I)} \not{D} d_R^{(I)} + \bar{L}_L^{(I)} \not{D} L_L^{(I)} + \bar{\ell}_R^{(I)} \not{D} \ell_R^{(I)}\right]$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + ig_1 f^{abc} G_\mu^b G_\nu^c$$

$$B_\mu B^\mu \longrightarrow B'_\mu B'^\mu = B_\mu B^\mu + \frac{2}{g} B^\mu \partial_\mu \theta + \frac{1}{g^2} (\partial_\mu \theta)(\partial^\mu \theta) \neq B_\mu B^\mu.$$

$$\mathcal{L}_{\text{Dirac,mass}} = m\bar{\psi}\psi = m(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R).$$

45. Mecanismo de Brout-Englert-Higgs.

$$D_\mu \Phi_\beta = \left(\partial_\mu \delta_{\alpha\beta} + ig_2 \frac{\sigma_{\alpha\beta}^a}{2} W_\mu^a + ig_1 \frac{Y_W}{2} B_\mu \delta_{\alpha\beta}\right) \Phi_\beta,$$

$$\mathcal{L}_{\text{H}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$

$$\mathcal{L}_{\text{HF}} = -f_u^{IJ} \bar{Q}_L^{(I)} \tilde{\Phi} u_R^J - f_d^{IJ} \bar{Q}_L^{(I)} \Phi d_R^J - f_e^{IJ} \bar{L}_L^{(I)} \Phi l_R^J$$

$$\tilde{\Phi} = i\sigma^2 \Phi,$$

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{H}} + \mathcal{L}_{\text{HF}},$$

$$-\mu^2 + 2\lambda \Phi^\dagger \Phi = 0$$

$$\langle \Phi^\dagger \Phi \rangle_0 = \frac{\mu^2}{2\lambda} = \frac{v^2}{2}$$

$$\langle \Phi \rangle_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix},$$

$$\Phi' = \Phi - \langle \Phi \rangle_0 = \begin{pmatrix} \phi^+ \\ \phi^0 - \frac{v}{\sqrt{2}} \end{pmatrix}.$$

$$W_\mu^{1,2} \longrightarrow W_\mu^{\pm} = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$$

$$\sigma^{\pm} = \sigma^1 \pm i\sigma^2 = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix},$$

$$\Phi(x) = \exp\left[-\frac{i}{v}\sum_{i=\pm,3} \xi_i(x)\tau^i\right]\begin{pmatrix} 0 \\ \frac{v+\eta(x)}{\sqrt{2}} \end{pmatrix} = U^{-1}(\xi)\frac{v+\eta(x)}{\sqrt{2}}\chi$$

$$\langle v\rangle_0 = \langle \xi_i\rangle_0 = 0.$$

$$\Phi' = \Phi_{\text{unitary}} = U(\xi)\Phi = \frac{v+\eta(x)}{\sqrt{2}}\chi = \begin{pmatrix} 0 \\ \frac{v+\eta(x)}{\sqrt{2}} \end{pmatrix}.$$

$$\left(\sum_{i=\pm,3} W_\mu^i\tau^i\right)' = U(\xi)\left[\sum_{i=\pm,3} W_\mu^i\tau^i\right]U^{-1}(\xi) + \frac{i}{g_2}\left(\partial_\mu U(\xi)\right)U^{-1}(\xi)$$

$$\begin{aligned} B'_\mu &= B_\mu \\ \Psi'_L &= U(\xi)\Psi_L \\ \Psi'_R &= \Psi_R. \end{aligned}$$

$$\left(\Phi^\dagger\Phi\right) \longrightarrow \left(\Phi^\dagger\Phi\right) = \Phi^\dagger U^\dagger(\xi)U(\xi)\Phi = \left(\Phi^\dagger\Phi\right)$$

$$\mathcal{L}_{H,\text{pot}} = -\frac{2\lambda v^2}{2}\eta^2 - \lambda v\eta^3 - \frac{\lambda}{4}\eta^4 + \text{ const.}$$

$$m_H = v\sqrt{2\lambda}$$

$$\left(D_\mu\Phi\right)' = U(\xi)D_\mu\Phi$$

$$\mathcal{L}_{H,\text{ kin}} = \left(D'_\mu\Phi'\right)^\dagger\left(D'^\mu\Phi'\right) = \mathcal{L}_{\eta,\text{ kin}} + \mathcal{L}_M + \mathcal{L}_I$$

$$\mathcal{L}_{\eta,\text{ kin}} = \frac{1}{2}\left(\partial_\mu\eta\right)\left(\partial^\mu\eta\right)$$

$$\begin{aligned} \mathcal{L}_M &= \frac{v^2g_2^2}{4}W_\mu^-W^{+,\mu} + \frac{v^2}{8}\begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}^T\begin{pmatrix} g_1^2 & -g_1g_2 \\ -g_1g_2 & g_2^2 \end{pmatrix}\begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}^T \\ &= m_W^2W_\mu^-W^{+,\mu} + \frac{1}{2}m_Z^2Z_\mu Z^\mu \\ \mathcal{L}_I &= \frac{m_W^2}{v^2}\left(\eta^2 + 2v\eta\right)W_\mu^-W^{+,\mu} + \frac{m_Z^2}{2v^2}\left(\eta^2 + 2v\eta\right)Z_\mu Z^\mu \end{aligned}$$

$$\begin{aligned} A_\mu &= \sin\,\theta_W W_\mu^3 + \cos\,\theta_W B_\mu \\ Z_\mu &= \cos\,\theta_W W_\mu^3 - \sin\,\theta_W B_\mu \end{aligned}$$

$$m_W = \frac{v g_2}{2} \text{ and } m_Z = \frac{v}{2}\sqrt{g_1^2 + g_2^2}$$

$$\tan \theta_W = \frac{g_1}{g_2} \text{ or } \cos \theta_W = \frac{m_W}{m_Z} = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}.$$

$$\mathcal{L}_{\text{HF}} = \frac{v+\eta}{\sqrt{2}}\left[f_u^{IJ}\bar{u}_L^{(I)}u_R^{(J)} + f_d^{IJ}\bar{d}_L^{(I)}d_R^{(J)} + f_\ell^{IJ}\bar{\ell}_L^{(I)}\ell_R^{(J)}\right]$$

$$f_u^{IJ} \longrightarrow \frac{\sqrt{2}m_u^{(I)}}{v}\delta^{IJ}$$

$$M_{\text{diag}} = S^\dagger M T,$$

$$M = HU,$$

$$S^\dagger(M^\dagger M)S = (M^2)_{\text{diag}} \longrightarrow \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}$$

$$S^\dagger F^\dagger(M^\dagger M)FS = (M^2)_{\text{diag}},$$

$$F = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix}.$$

$$H = S M_{\text{diag}} S^\dagger$$

$$U = H^{-1}M \text{ and } U^\dagger = M^\dagger H^{-1}.$$

$$M_{\text{diag}} = S^\dagger H S = S^\dagger M U^\dagger S = S^\dagger M T$$

$$\bar{\psi}_L M \psi_R = (\bar{\psi}_L S)(S^\dagger M T)(T^\dagger \psi_R) = \bar{\psi}'_L M_{\text{diag}} \psi'_R.$$

$$\bar{\psi}_R^I \gamma^\mu \psi_R^I = \bar{\psi}_R'^K \gamma^\mu T_{KI} T_{IL}^\dagger \psi_R'^L = \bar{\psi}_R'^K \gamma^\mu \delta_{KL} \psi_R'^L = \bar{\psi}_R'^K \gamma^\mu \psi_R'^K.$$

$$\bar{u}_L^I \gamma^\mu d_L^I = \bar{u}_L'^K \gamma^\mu S_{u,KI}^\dagger S_{d,IL} d_L^L = \bar{u}_L'^K \gamma^\mu V_{KL}^{(\text{CKM})} d_L'^L$$

$$V_{KL}^{(\text{CKM})} = S_{u,KI}^\dagger S_{d,IL}$$

$$V^{(\text{CKM})} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) - A\lambda^2 & & 1 \end{pmatrix}$$

$$A \approx 0.8, \rho \approx 0.135, \text{ and } \eta \approx 0.35.$$

46. Reglas de Feynman. Interacciones de gauge entre bosones y fermiones.

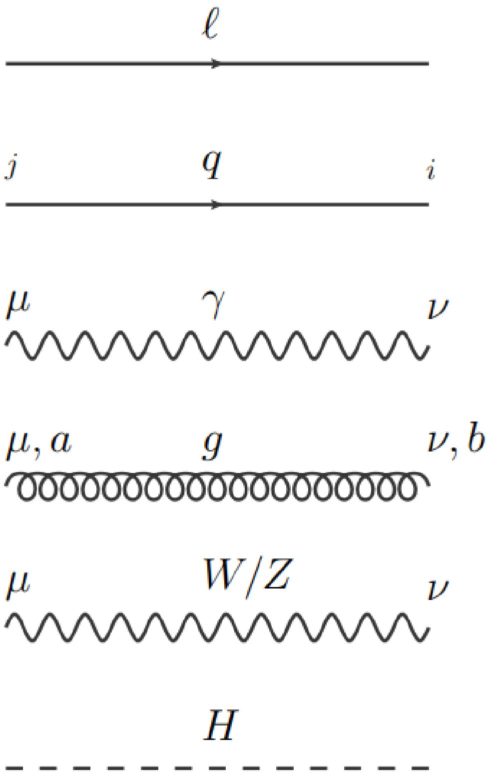
$$\mathcal{S} = i\int \mathrm{d}^4 x \mathcal{L}(x)$$

$$i(\not p - m)^{-1} = \frac{i(\not p + m)}{p^2 - m^2},$$

$$Z_\mu[(p^2 - m_Z^2)g^{\mu\nu} - p^\mu p^\nu]Z_\nu$$

$$\frac{1}{p^2 - m_Z^2}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{m_Z^2}\right)$$

$$\mathcal{L}_{\text{g.f.}} = -\frac{1}{2\xi}\left(\partial^\mu A_\mu\right)^2,$$



$$i\frac{(\not p + m_\ell)}{p^2 - m_\ell^2}$$

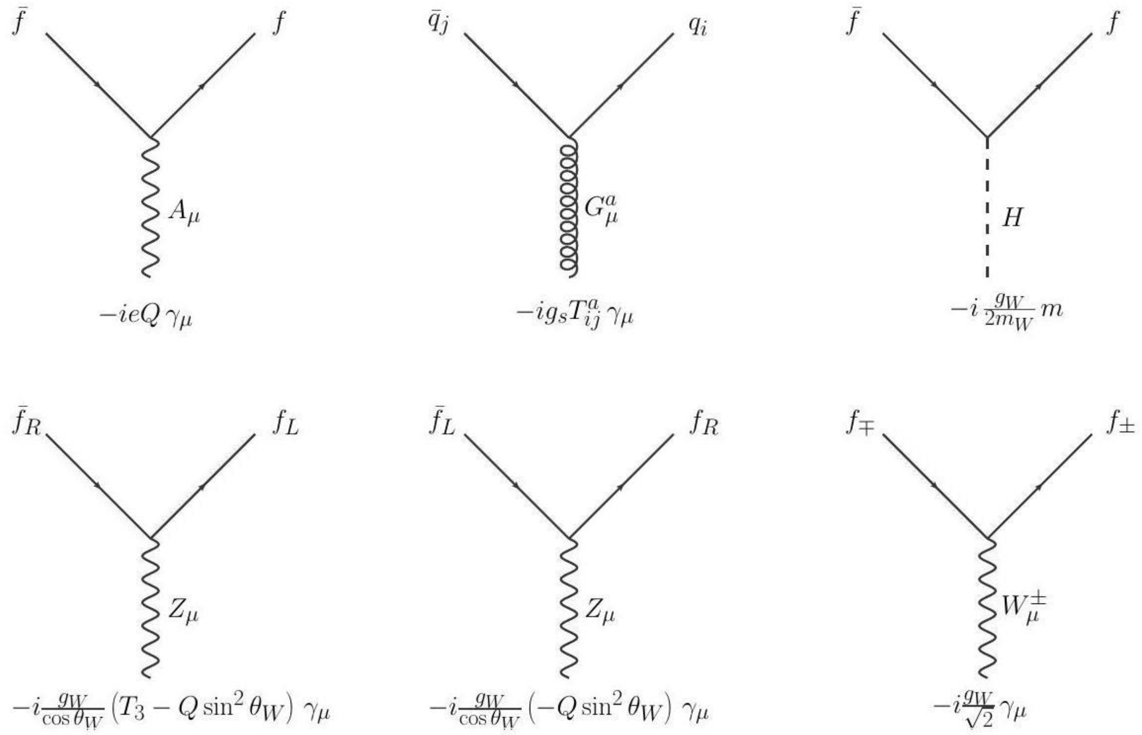
$$i\frac{(\not p + m_q)}{p^2 - m_q^2}\delta_{ij}$$

$$\frac{-i}{p^2}\left[g^{\mu\nu} - (1-\xi)\frac{p^\mu p^\nu}{p^2}\right]$$

$$\frac{-i}{p^2}\left[g^{\mu\nu} - (1-\xi)\frac{p^\mu p^\nu}{p^2}\right]\delta_{ab}$$

$$\frac{-i}{p^2 - m^2}\left[g^{\mu\nu} - (1-\xi)\frac{p^\mu p^\nu}{m^2}\right]$$

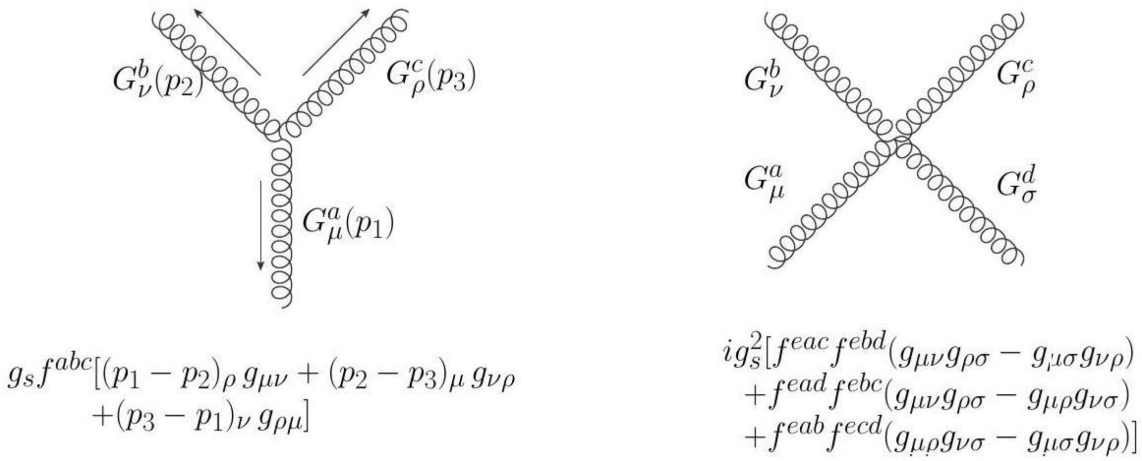
$$\frac{i}{p^2 - m_H^2}$$



$$D_\mu L^{(I)}_{L,\alpha} = \left(\partial_\mu + ig_2\left[\frac{1}{\sqrt{2}}\tau^+_{\alpha\beta}W^+_\mu + \frac{1}{\sqrt{2}}\tau^-_{\alpha\beta}W^-_\mu + \tau^3_{\alpha\beta}W^3_\mu +\right] + ig_1\frac{Y_W}{2}B_\mu\delta_{\alpha\beta}\right)L^{(I)}_{L,\beta},$$

$$D_\mu L^{(I)}_{L,\alpha} = \left(\partial_\mu + \frac{ig_2}{\sqrt{2}}\left[\tau^+_{\alpha\beta}W^+_\mu + \tau^-_{\alpha\beta}W^-_\mu\right] + i[g_2T_3\sin\,\theta_W + g_1(Q-T_3)\cos\,\theta_W]A_\mu\,Z_\mu\delta_{\alpha\beta}\right)L^{(I)}_{L,\beta}$$

$$D_\mu L^{(I)}_{L,\alpha} = \left(\partial_\mu + \frac{ig_W}{\sqrt{2}}\left[\tau^+_{\alpha\beta}W^+_\mu + \tau^-_{\alpha\beta}W^-_\mu\right] + ieQA_\mu + \frac{ig_W}{\cos\,\theta_W}[T_3 - Q\sin^2\,\theta_W]Z_\mu\delta_{\alpha\beta}\right)L^{(I)}_{L,\beta}$$



$$\mathcal{L}_{\text{gauge ,EW}} = \frac{1}{4} W_{\mu\nu}^{a} W^{a,\mu\nu}$$

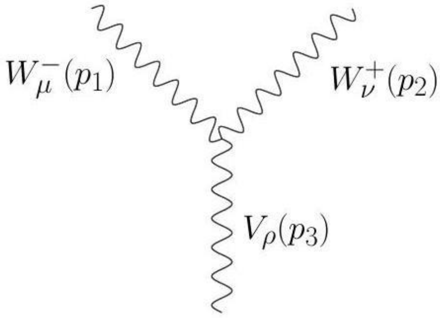
$$= \left[\frac{1}{4} W_{\mu}^{+} W_{\nu}^{+} W_{\rho}^{-} W_{\sigma}^{-} - \frac{1}{2} W_{\mu}^{-} W_{\nu}^{+} \left(Z_{\rho} Z_{\sigma} g_{W}^{2} \cos\ \theta_{W}^{2} + A_{\rho} A_{\sigma} e^{2} + 2 A_{\rho} Z_{\sigma} e g_{W} \cos\ \theta_{W}\right)\right]$$

$$\times \left(2 g_{\mu\nu} g_{\rho\sigma} - g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}\right)$$

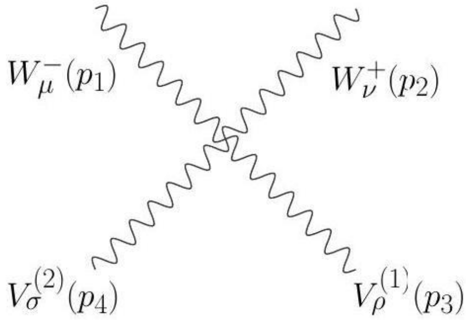
$$+ i W_{\mu}^{-} W_{\nu}^{+} \left(A_{\rho} e + Z_{\rho} g_{W} \cos\ \theta_{W}\right)$$

$$\times \left(\left(p_{\rho}^{W^{-}} - p_{\rho}^{W^{+}}\right) g_{\mu\nu} + \left(p_{\mu}^{W^{+}} - p_{\mu}^{V}\right) g_{\nu\rho} + \left(p_{\nu}^{V} - p_{\nu}^{W^{-}}\right) g_{\mu\rho}\right) + ב_{\text{kinetic terms}}$$

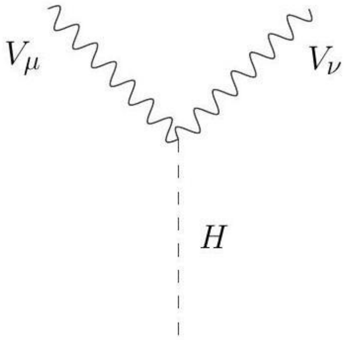
$$c_{\gamma} = e, c_{Z} = g_{W} \cos\ \theta_{W},$$



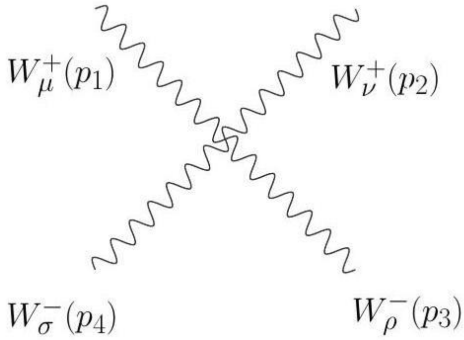
$$-ic_V[(p_1-p_2)_\rho\, g_{\mu\nu} + (p_2-p_3)_\mu\, g_{\nu\rho} + (p_3-p_1)_\nu\, g_{\rho\mu}]$$



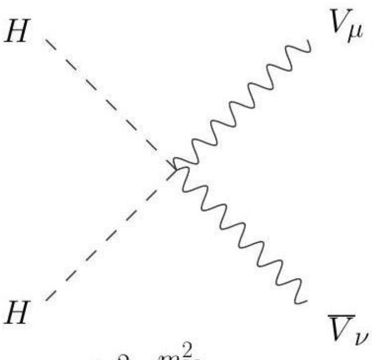
$$-ic_{V^{(1)}}c_{V^{(2)}}(2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})$$



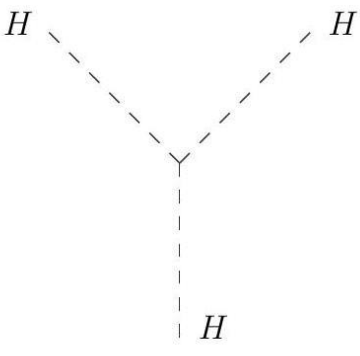
$$ig_W\,\frac{m_V^2}{m_W}\,g_{\mu\nu}$$



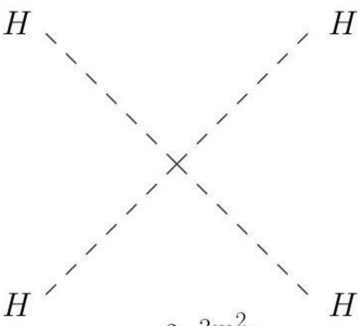
$$ig_W^2\,(2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})$$



$$ig_W^2\frac{m_V^2}{2m_W^2}\,g_{\mu\nu}$$



$$-ig_W\frac{3mh^2}{2m_W}$$



$$-ig_W^2\frac{3m_H^2}{4m_W^2}$$

47. Sustracción Catani-Seymour. Kernels.

$$p_i + p_j + p_k = \tilde{p}_{ij} + \tilde{p}_k$$

$$\tilde{p}_{ij} = p_i + p_j - \frac{y_{ij,k}}{1 - y_{ij,k}} p_k$$

$$\tilde{p}_k = \frac{1}{1 - y_{ij,k}} p_k$$

$$y_{ij,k} = \frac{p_i p_j}{p_i p_j + p_j p_k + p_k p_i}$$

$$\tilde{z}_i = \frac{p_i p_k}{(p_i + p_j) p_k} = \frac{p_i \tilde{p}_k}{\tilde{p}_{ij} \tilde{p}_k} \text{ and } \tilde{z}_j = 1 - \tilde{z}_i$$

$$\langle s | V_{q_i g_j;k} | s' \rangle = 8\pi\mu^{2\varepsilon} C_F \alpha_s \left[\frac{2}{1 - \tilde{z}_i (1 - y_{ij,k})} - (1 + \tilde{z}_i) - \varepsilon(1 - \tilde{z}_i) \right] \delta_{ss'} \langle \mu | V_{q_i \bar{q}_j;k} | \nu \rangle$$

$$= 8\pi\mu^{2\varepsilon} T_R \alpha_s \left[-g^{\mu\nu} - \frac{2}{p_i p_j} (\tilde{z}_i p_i - \tilde{z}_j p_j)^{\mu} (\tilde{z}_i p_i - \tilde{z}_j p_j)^{\nu} \right] \langle \mu | V_{g_i g_j;k} | \nu \rangle$$

$$= 16\pi\mu^{2\varepsilon} C_A \alpha_s \left[-g^{\mu\nu} \left(\frac{1}{1 - \tilde{z}_i (1 - y_{ij,k})} + \frac{1}{1 - \tilde{z}_j (1 - y_{ij,k})} \right. \right.$$

$$\left. \left. - 2 \right) + \frac{1 - \varepsilon}{p_i p_j} (\tilde{z}_i p_i - \tilde{z}_j p_j)^{\mu} (\tilde{z}_i p_i - \tilde{z}_j p_j)^{\nu} \right]$$

$$\frac{\langle V_{q_i g_j,k} \rangle}{8\pi\alpha_s \mu^{2\varepsilon}} = C_F \left[\frac{2}{1 - \tilde{z}_i (1 - y_{ij,k})} - (1 + \tilde{z}_i) - \varepsilon(1 - \tilde{z}_i) \right]$$

$$\frac{\langle V_{q_i} \bar{q}_j; k \rangle}{8\pi\mu^{2\varepsilon}\alpha_s} = T_R \left[1 - \frac{2\tilde{z}_i (1 - \tilde{z}_i)}{1 - \varepsilon} \right]$$

$$\frac{\langle V_{g_i g_j;k} \rangle}{8\pi\mu^{2\varepsilon}\alpha_s} = 2C_A \left[\frac{1}{1 - \tilde{z}_i (1 - y_{ij,k})} + \frac{1}{1 - (1 - \tilde{z}_i)(1 - y_{ij,k})} - 2 + \tilde{z}_i (1 - \tilde{z}_i) \right]$$

$$\mathcal{V}_{ij}(\varepsilon) = \int_0^1 \mathrm{d}z [z(1 - z)]^{-\varepsilon} \int_0^1 \mathrm{d}y (1 - 2y)^{1 - 2\varepsilon} y^{-\varepsilon} \frac{\langle V_{ij,k}(z, y) \rangle}{8\pi\alpha_s \mu^{2\varepsilon}}$$

$$\mathcal{V}_{qg}(\varepsilon) = C_F \left[\frac{1}{\varepsilon^2} + \frac{3}{2\varepsilon} + 5 - \frac{\pi^2}{2} + \mathcal{O}(\varepsilon) \right]$$

$$\mathcal{V}_{q\bar{q}}(\varepsilon) = T_R \left[-\frac{2}{3\varepsilon} - \frac{16}{9} + \mathcal{O}(\varepsilon) \right]$$

$$\mathcal{V}_{gg}(\varepsilon) = 2C_A \left[\frac{1}{\varepsilon^2} + \frac{11}{6\varepsilon} + \frac{50}{9} - \frac{\pi^2}{2} + \mathcal{O}(\varepsilon) \right].$$

$$\mathcal{V}_i(\varepsilon) = T_i^2\left(\frac{1}{\varepsilon^2} - \frac{\pi^2}{3}\right) + \gamma_i^{(1)}\left(\frac{1}{\varepsilon} + 1\right) + K_i + \mathcal{O}(\varepsilon)$$

$$K_q = C_F\left(\frac{7}{2} - \frac{\pi^2}{6}\right)$$
$$K_g = C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - T_R n_f\frac{10}{9}$$

$$p_i + p_j - p_a = \tilde{p}_{ij} - \tilde{p}_a$$

$$\tilde{p}_{ij} = p_i + p_j - (1 - x_{ij,a})p_a$$
$$\tilde{p}_a = (1 - x_{ij,a})p_a,$$

$$x_{ij,a} = \frac{p_i p_a + p_j p_a - p_i p_j}{(p_i + p_j)p_a}$$
$$\tilde{z}_i = \frac{p_i p_a}{(p_i + p_j)p_a} = \frac{p_i\tilde{p}_a}{\tilde{p}_{ij}\tilde{p}_a} \text{ and } \tilde{z}_j = 1 - \tilde{z}_i$$

$$\langle s|V^a_{q_i g_j}|s'\rangle = 8\pi\mu^{2\varepsilon}C_F\alpha_s\left[\frac{2}{1 - \tilde{z}_i(1 - x_{ij,a})} - (1 + \tilde{z}_i) - \varepsilon(1 - \tilde{z}_i)\right]\delta_{ss'}$$

$$\langle\mu|V^a_{q_i\bar{q}_j}|\nu\rangle = 8\pi\mu^{2\varepsilon}T_R\alpha_s\left[-g^{\mu\nu} - \frac{2}{p_i p_j}(\tilde{z}_i p_i - \tilde{z}_j p_j)^\mu(\tilde{z}_i p_i - \tilde{z}_j p_j)^\nu\right]$$

$$\langle\mu|V^a_{g_i g_j}|\nu\rangle = 16\pi\mu^{2\varepsilon}C_A\alpha_s\left[-g^{\mu\nu}\left(\frac{1}{1 - \tilde{z}_i(1 - x_{ij,a})} + \frac{1}{1 - \tilde{z}_j(1 - x_{ij,a})}\right.\right.$$
$$\left.\left.- 2\right) + \frac{1 - \varepsilon}{p_i p_j}(\tilde{z}_i p_i - \tilde{z}_j p_j)^\mu(\tilde{z}_i p_i - \tilde{z}_j p_j)^\nu\right]$$

$$\mathcal{V}_{ij}(x,\varepsilon) = \Theta(x_{ij,a})\Theta(1 - x_{ij,a})\left(\frac{1}{1 - x_{ij,a}}\right)^{1+\varepsilon}\int_0^1 \mathrm{d}z[z(1-z)]^{-\varepsilon}\frac{\langle V_{ij,k}(z, y_{ij,k})\rangle}{8\pi\alpha_s\mu^{2\varepsilon}}$$

$$\mathcal{V}_{ij}(x_{ij,a},\varepsilon) = [\mathcal{V}_{ij}(x_{ij,a},\varepsilon)]_+ + \delta(1 - x_{ij,a})\int_0^1 \mathrm{d}\tilde{x}\mathcal{V}_{ij}(\tilde{x},\varepsilon)$$

$$[\mathcal{V}_{ij}(x_{ij,a},\varepsilon)]_+ = [\mathcal{V}_{ij}(x_{ij,a},0)]_+ + \mathcal{O}(\varepsilon).$$

$$\mathcal{V}_{qg}(x,\varepsilon) = C_F\left[\left(\frac{2}{1-x}\log\frac{1}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ + \frac{2}{1-x}\log(2-x)\right] + \delta(1-x)\left[\mathcal{V}_{qg}(\varepsilon) - \frac{3C_F}{2}\right]$$
$$+ \mathcal{O}(\varepsilon)$$

$$\mathcal{V}_{q\bar{q}}(x,\varepsilon) = \frac{2}{3}T_R\left(\frac{1}{1-x}\right)_+ + \delta(1-x)\left[\mathcal{V}_{q\bar{q}}(\varepsilon) + \frac{2T_R}{3}\right] + \mathcal{O}(\varepsilon)$$

$$\mathcal{V}_{gg}(x,\varepsilon) = 2C_A\left[\left(\frac{2}{1-x}\log\frac{1}{1-x}\right)_+ - \frac{11}{6}\left(\frac{1}{1-x}\right)_+ + \frac{2}{1-x}\log(2-x)\right] + \delta(1 - x)\left[\mathcal{V}_{gg}(\varepsilon) - \frac{11C_A}{3}\right] + \mathcal{O}(\varepsilon)$$

$$p_i + p_j - p_a = \tilde{p}_{ij} - \tilde{p}_a$$

$$\tilde{p}_{ai} = x_{ij,a}p_a$$
$$\tilde{p}_k = p_k + p_i - (1 - x_{ij,a})p_a$$

$$x_{ik,a} = \frac{p_kp_a + p_ip_a - p_ip_k}{(p_i + p_k)p_a}$$

$$u_i = \frac{p_ip_a}{(p_i + p_k)p_a}$$

$$\langle s|V_k^{q_ag_i}|s'\rangle = 8\pi\mu^{2\varepsilon}C_F\alpha_s\left[\frac{2}{1 - x_{ik,a} + u_i} - (1 + x_{ik,a}) - \varepsilon(1 - x_{ik,a})\right]\delta_{ss'}\langle s|V_k^{g_aq_i}|s'\rangle$$
$$= 8\pi\mu^{2\varepsilon}T_R\alpha_s[1 - \varepsilon - 2x_{ik,a}(1 - x_{ik,a})]\delta_{ss'}$$

$$\langle\mu|V_k^{q_a\bar{q}_j}|\nu\rangle = 8\pi\mu^{2\varepsilon}C_F\alpha_s\left[-g^{\mu\nu}x_{ik,a} + \frac{1 - x_{ik,a}}{x_{ik,a}}\frac{2u_i(1 - u_i)}{p_ip_k}q_{ik}^{\mu}q_{ik}^{\nu}\right]$$
$$\langle\mu|V_k^{g_ig_j}|\nu\rangle = 16\pi\mu^{2\varepsilon}C_A\alpha_s\left[-g^{\mu\nu}\left(\frac{1}{1 - x_{ik,a} + u_i} - 1 + x_{ik,a}(1 - x_{ik,a})\right) + (1 - \varepsilon)\frac{1 - x_{ik,a}}{x_{ik,a}}\frac{2u_i(1 - u_i)}{p_ip_k}q_{ik}^{\mu}q_{ik}^{\nu}\right]$$

$$q_{ik}^{\mu} = \frac{p_i^{\mu}}{u_i} - \frac{p_k^{\mu}}{1 - u_i}$$

$$\frac{n_s(\tilde{q})}{n_s(q)}\frac{\langle V_k^{qg}\rangle}{8\pi\alpha_s\mu^{2\varepsilon}} = C_F\left[\frac{2}{1 - x_{ik,a} + u_i} - (1 + x_{ik,a}) - \varepsilon(1 - x_{ik,a})\right]\frac{n_s(\tilde{q})}{n_s(g)}\frac{\langle V_k^{g\bar{q}}\rangle}{8\pi\alpha_s\mu^{2\varepsilon}}$$

$$= T_R\left[1 - \frac{2x_{ik,a}(1 - x_{ik,a})}{1 - \varepsilon}\right]\frac{n_s(\tilde{g})}{n_s(q}\frac{\langle V_k^{qq}\rangle}{8\pi\alpha_s\mu^{2\varepsilon}}$$

$$= C_F\left[(1 - \varepsilon)x_{ik,a} + 2\frac{1 - x_{ik,a}}{x_{ik,a}}\right]\frac{n_s(\tilde{g})}{n_s(g)}\frac{\langle V_k^{gg}\rangle}{8\pi\alpha_s\mu^{2\varepsilon}}$$

$$= 2C_A\left[\left[\frac{1}{1 - x_{ik,a} + u_i} + \frac{1 - x_{ik,a}}{x_{ik,a}} - 1 + x_{ik,a}(1 - x_{ik,a})\right]\right],$$

$$\mathcal{V}^{ai,a}(x,\varepsilon) = \Theta(x)\Theta(1 - x)\left(\frac{1}{1 - x}\right)^{\varepsilon}\int_0^1 \mathrm{d}u_i[u_i(1 - u_i)]^{-\varepsilon}\frac{n_s(\tilde{a\imath})}{n_s(a)}\frac{\langle V_{ij,k}(\tilde{z}_i, y_{ij,k})\rangle}{8\pi\alpha_s\mu^{2\varepsilon}}.$$

$$\mathcal{V}^{ai,a}(x,\varepsilon)=\frac{1}{\varepsilon}\left\{\frac{1}{x}\left[\varepsilon x\mathcal{V}^{ai,a}(x,\varepsilon)\right]_{+}+\varepsilon\delta(1-x)\int_{0}^{1}\mathrm{d}\tilde{x}\tilde{x}\mathcal{V}^{ai,a}(\tilde{x},\varepsilon)\right\}$$

$$\mathcal{V}^{ai,a}(x,\varepsilon)\sim p^{a,\tilde{a\imath}}(x)+\propto\left(\frac{1}{\varepsilon}+\mathcal{O}(1)\right)+\mathcal{O}(\varepsilon)$$

$$\begin{aligned}
p^{qg}(x)&=P_{gq}(x)\\
p^{gq}(x)&=P_{qg}(x)\\
p^{qq}(x)&=\left[P_{qq}(x)\right]_{+}\\
p^{gg}(x)&=\left[P_{gg}(x)\right]_{+}-2C_A+\delta(1-x)\gamma_g^{(1)},
\end{aligned}$$

$$\begin{aligned}
\mathcal{V}^{qg}(x,\varepsilon)&=\left[-\frac{1}{\varepsilon}+\log(1-x)\right]p^{qg}(x)+C_F x+\mathcal{O}(\varepsilon)\\
\mathcal{V}^{gq}(x,\varepsilon)&=\left[-\frac{1}{\varepsilon}+\log(1-x)\right]p^{gq}(x)+2T_R x(1-x)+\mathcal{O}(\varepsilon)\\
\mathcal{V}^{qq}(x,\varepsilon)&=-\frac{1}{\varepsilon}p^{qq}(x)+\delta(1-x)\left[\mathcal{V}_{qg}(\varepsilon)+C_F\left(\frac{2\pi^2}{3}-5\right)\right]\\
&+C_F\left[-\left(\frac{4}{1-x}\log\frac{1}{1-x}\right)_{+}-\frac{2}{1-x}\log(2-x)\right.\\
&\left.+(1-x)-(1+x)\log(1-x)\right]\\
\mathcal{V}^{gg}(x,\varepsilon)&=-\frac{1}{\varepsilon}p^{gg}(x)+\delta(1-x)\left[\frac{1}{2}\mathcal{V}_{gg}(\varepsilon)+n_f\mathcal{V}_{q\bar{q}}(\varepsilon)+C_A\left(\frac{2\pi^2}{3}-\frac{50}{9}\right)+\frac{16}{9}n_f T_R\right]
\end{aligned}$$

$$+C_A\left[-\left(\frac{4}{1-x}\log\frac{1}{1-x}\right)_{+}-\frac{2}{1-x}\log(2-x)\ +2\left(-1+x(1-x)+\frac{1-x}{x}\right)\log(1-x)\right]$$

$$p_a^{\mu}+p_b^{\mu}-p_i^{\mu}-\sum_{j\neq i}k_j^{\mu}=0$$

$$\tilde{p}_{ai}^{\mu}+p_b^{\mu}-\sum_{j\neq i}\tilde{k}_j^{\mu}=0$$

$$\tilde{p}_{ai}=x_{i,ab}p_a$$

$$x_{i,ab}=\frac{p_a p_b-p_i(p_a+p_b)}{p_a p_b}$$

$$\tilde{k}_j^{\mu}=k_j^{\mu}-\frac{2k_j(K+\tilde{K})}{(K+\tilde{K})^2}(K+\tilde{K})^{\mu}-\frac{2k_j K}{K^2}\tilde{K}^{\mu}$$

$$\begin{aligned}
K^{\mu}&=p_a^{\mu}+p_b^{\mu}-p_i^{\mu}\\
\tilde{K}^{\mu}&=\tilde{p}_{ai}^{\mu}+p_b^{\mu}
\end{aligned}$$

$$\langle s|V^{q_a g_i,b}|s'\rangle=8\pi\mu^{2\varepsilon}C_F\alpha_s\left[\frac{2}{1-x_{i,ab}}-\left(1+x_{i,ab}\right)-\varepsilon\left(1-x_{i,ab}\right)\right]\delta_{ss'}$$

$$\langle s|V^{g_a q_i,b}|s'\rangle=8\pi\mu^{2\varepsilon}T_R\alpha_s\left[1-\varepsilon-2x_{i,ab}\left(1-x_{i,ab}\right)\right]\delta_{ss'}$$

$$\langle\mu|V^{q_a\bar{q}_j,b}|\nu\rangle = 8\pi\mu^{2\varepsilon}C_F\alpha_s\left[-g^{\mu\nu}x_{i,ab} + \frac{1-x_{i,ab}}{x_{i,ab}}\frac{2p_ap_b}{p_ip_ap_ip_b}q^\mu q^\nu\right]$$

$$\langle\mu|V^{g_ig_j,b}|\nu\rangle = 16\pi\mu^{2\varepsilon}C_A\alpha_s\left[-g^{\mu\nu}\left(\frac{1}{1-x_{i,ab}} + x_{i,ab}(1-x_{i,ab})\right)\right.$$

$$\left. +(1-\varepsilon)\frac{1-x_{i,ab}}{x_{i,ab}}\frac{2p_ap_b}{p_ip_ap_ip_b}q^\mu q^\nu\right]$$

$$q^\mu = p_i^\mu - \frac{p_ip_a}{p_bp_a}p_b^\mu$$

$$\mathcal{V}^{\tilde{a},ai}(x,\varepsilon) = -\frac{1}{\varepsilon}\frac{\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\Theta(x)\Theta(1-x)(1-x)^{-2\varepsilon}\frac{n_s(\tilde{a\imath})}{n_s(a)}\frac{\langle V^{ai,b}\rangle}{8\pi\alpha_s\mu^{2\varepsilon}}$$

$$\mathcal{V}^{ab}\tilde{(}x,\varepsilon = \mathcal{V}^{ab}(x,\varepsilon) + \delta^{ab}T_a^2\left[\left(\frac{2}{1-x}\log\frac{1}{1-x}\right)_+ + \frac{2}{1-x}\log(2-x)\right] + \tilde{K}^{ab}$$

$$\tilde{K}^{ab}(x) = P_{ab}^{(\text{reg})}(x)\log(1-x) + \delta^{ab}T_a^2\left[\left(\frac{2}{1-x}\log(1-x)\right)_+ - \frac{\pi^2}{3}\delta(1-x)\right]$$

$$P_{ab}^{(\text{reg})}(x) = \mathcal{P}_{ab}(x) - \delta^{ab}\left[2T_a^2\left(\frac{1}{1-x}\right)_+ + \gamma_a^{(1)}\delta(1-x)\right]$$

$$P_{ab}^{(\text{reg})}(x) = P_{ab}(x) \qquad \text{for } a \neq b$$
$$P_{qq}^{(\text{reg})}(x) = -C_F(1+x)$$
$$P_{gg}^{(\text{reg})}(x) = 2C_A\left[\frac{1-x}{x} - 1 + x(1-x)\right].$$

$$\frac{n_s(\tilde{a\imath})}{n_s(a)}\frac{\langle V^{ai,b}(x)\rangle}{8\pi\alpha_s\mu^{2\varepsilon}} = P_{a,\tilde{a\imath}}(x)$$

$$d\sigma^{(I+C)} = d\sigma_{ab}^{(\text{Born})}(p_a,p_b)\otimes I(\varepsilon) + \sum_{a'}\int_0^1 dx\, d\sigma_{a'b}^{(\text{Born})}(xp_a,p_b)$$

$$\otimes\left[K^{aa'}(x) + P^{aa'}(xp_a,x;\mu_F^2)\right] + \sum_{b'}\int_0^1 dx\, d\sigma_{ab'}^{(\text{Born})}(p_a,xp_b)$$

$$\otimes\left[K^{bb'}(x) + P^{bb'}(xp_b,x;,\mu_F^2)\right]$$

$$I(\varepsilon) = -\frac{\alpha_s}{2\pi\Gamma(1-\varepsilon)}$$
$$\left\{\sum_i\frac{\mathcal{V}_i(\varepsilon)}{T_i^2}\left[\sum_{k\neq i}T_i\cdot T_k\left(\frac{4\pi\mu^2}{2p_ip_k}\right)^\varepsilon + T_i\cdot T_a\left(\frac{4\pi\mu^2}{2p_ip_a}\right)^\varepsilon + T_i\cdot T_b\left(\frac{4\pi\mu^2}{2p_ip_b}\right)^\varepsilon\right]\right.$$
$$+\frac{\mathcal{V}_a(\varepsilon)}{T_a^2}\left[\sum_k T_a\cdot T_k\left(\frac{4\pi\mu^2}{2p_ap_k}\right)^\varepsilon + T_a\cdot T_b\left(\frac{4\pi\mu^2}{2p_ap_b}\right)^\varepsilon\right]$$

$$+\frac{\mathcal{V}_b(\varepsilon)}{T_b^2}\left[\sum_k T_b\cdot T_k\left(\frac{4\pi\mu^2}{2p_bp_k}\right)^{\varepsilon}+T_b\cdot T_a\left(\frac{4\pi\mu^2}{2p_bp_a}\right)^{\varepsilon}\right]\Bigg\}$$

$$K^{aa'}(x)=\frac{\alpha_s}{2\pi}\Bigg\{\bar{K}^{aa'}(x)-K^{aa'}_{(\mathrm{F.S.})}(x)$$

$$-\sum_i\frac{T_i\cdot T_a}{T_i^2}\tilde{K}^{aa'}(x)+\delta^{aa'}\sum_i\frac{T_i\cdot T_a}{T_i^2}\gamma_i\left[\left(\frac{1}{1-x}\right)_+ +\delta(1-x)\right]\Bigg\}$$

$$P^{aa'}(xp_a,x;\mu_F^2)=\frac{\alpha_s}{2\pi}\mathcal{P}^{(1)}_{a'a}(x)\left[\sum_i\frac{T_i\cdot T_{a'}}{T_{a'}^2}\log\frac{\mu_F^2}{2xp_ap_i}+\frac{T_b\cdot T_{a'}}{T_{a'}^2}\log\frac{\mu_F^2}{2xp_ap_b}\right],$$

$$\bar{K}^{qq}=C_F\left[\left(\frac{1+x^2}{1-x}\log\frac{1-x}{x}\right)_+ +(1-x)-\delta(1-x)(5-\pi^2)\right]$$

$$\bar{K}^{gg}=2C_A\left[\left(\frac{1}{1-x}\log\frac{1-x}{x}\right)_+ +\left(\frac{1-x}{x}-1+x(1-x)\right)\log\frac{1-x}{x}\right]-\delta(1$$

$$-x)\left[C_A\left(\frac{50}{9}-\pi^2\right)-\frac{16}{9}T_Rn_f\right]$$

$$\bar{K}^{qg}=P^{(1)}_{qg}\log\frac{1-x}{x}+C_Fx$$

$$\bar{K}^{gq}=P^{(1)}_{gq}\log\frac{1-x}{x}+2T_Rx(1-x)$$

$$\tilde{K}^{ab}(x)=P^{(\mathrm{reg})}_{ba}(x)\log(1-x)+\delta^{ab}T_a^2\left[\left(\frac{2\log(1-x)}{1-x}\right)_+ -\frac{\pi^2}{3}\delta(1-x)\right]$$

$$y_{ij;k}=\frac{p_ip_j}{p_ip_j+p_ip_k+p_jp_k}\quad\text{and}\quad z_i=\frac{p_ip_k}{p_ip_k+p_jp_k}=1-z_j$$

$$k_\perp^2=z_i(1-z_i)y_{ij;k}Q^2$$

$$J^{(FF)}=1-y_{ij;k}$$

$$\mathrm{d}\Phi_1=\frac{1}{16\pi^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\frac{\mathrm{d}z_i}{z_i(1-z_i)}\frac{\mathrm{d}\phi}{2\pi}\left(1-\frac{k_\perp^2}{z(1-z)Q^2}\right)$$

$$\mathcal{K}^{(FF)}_{qg,k}=C_F\left[\frac{2}{1-z_i(1-y_{ij;k})}-(1+z_i)\right]$$

$$\mathcal{K}^{(FF)}_{gg,k}=2C_A\left[\frac{1}{1-z_i(1+y_{ij;k})}+\frac{1}{1-(1-z_i)(1+y_{ij;k})}-2+z_i(1-z_i)\right]$$

$$\mathcal{K}^{(FF)}_{q\bar{q},k} = T_R[1 - 2z_i(1 - z_i)]$$

$$\begin{aligned} p_i = &\quad z_i\tilde{p}_{ij} + (1 - z_i)y_{ij;k}\tilde{p}_k + \vec{k}_\perp \\ p_j = &\quad (1 - z_i)\tilde{p}_{ij} + \\ p_k = &\quad z_i y_{ij;k}\tilde{p}_k - \vec{k}_\perp \\ &\quad \left(1 - y_{ij;k}\right)\tilde{p}_k. \end{aligned}$$

$$x_{ij;a} = \frac{p_i p_a + p_j p_a - p_i p_j}{p_i p_a + p_j p_a} \text{ and } z_i = \frac{p_i p_a}{p_i p_a + p_j p_a} = 1 - z_j$$

$$k_\perp^2 = z_i(1 - z_i)\frac{1 - x_{ij;a}}{x_{ij;a}}Q^2$$

$$J^{(FI)} = \frac{f_{a/h}\left(\frac{\eta_a}{x_{ij;a}}, \mu_F^2\right)}{f_{a/h}(\eta_a, \mu_F^2)}$$

$$\mathcal{K}^{(FI)}_{qg,k} = C_F\left[\frac{2}{1 - z_i + \left(1 - x_{ij;a}\right)} - (1 + z_i)\right]$$

$$\mathcal{K}^{(FI)}_{gg,k} = 2C_A\left[\frac{1}{1 - z_i + \left(1 + x_{ij;a}\right)} + \frac{1}{z_i + \left(1 - x_{ij;a}\right)} - 2 + z_i(1 - z_i)\right]$$

$$\mathcal{K}^{(FI)}_{q\bar{q},k} = T_R[1 - 2z_i(1 - z_i)]$$

$$p_i = z_i\tilde{p}_{ij} + \frac{(1 - z_i)\left(1 - x_{ij;a}\right)}{x_{ij;a}}\tilde{p}_k + \vec{k}_\perp$$

$$\begin{aligned} p_j &= (1 - z_i)\tilde{p}_{ij} + \quad \frac{z_i\left(1 - x_{ij;a}\right)}{x_{ij;a}}\tilde{p}_k - \vec{k}_\perp \\ p_k &= \frac{1}{x_{ij;a}}\tilde{p}_k \end{aligned}$$

$$x_{aj;k} = \frac{p_a p_j + p_a p_k - p_j p_k}{p_a p_j + p_a p_k} \text{ and } u_a = \frac{p_a p_j}{p_a p_j + p_a p_k} = 1 - u_k$$

$$k_\perp^2 = u_i(1 - u_i)\frac{1 - x_{aj;k}}{x_{aj;k}}Q^2$$

$$J^{(IF)} = \frac{1}{x_{ij;a}}\frac{1 - u_a}{1 - 2u_a}\frac{f_{A/a}\left(\frac{\eta_a}{x_{ij;a}}, \mu_F^2\right)}{f_{A/a}(\eta_a, \mu_F^2)},$$

$$\mathcal{K}_{qg,k}^{(IF)} = C_F \left[\frac{2}{1 - x_{aj;k} + u_a} - (1 + x_{aj;k}) \right]$$

$$\mathcal{K}_{qq,k}^{(IF)} = C_F \left[2 \frac{1 - x_{aj;k}}{x_{aj;k}} + x_{aj;k} \right]$$

$$\mathcal{K}_{gg,k}^{(IF)} = 2C_A \left[\frac{2}{1 - x_{aj;k} + u_a} + \frac{1 - x_{aj;k}}{x_{aj;k}} - 1 + x_{aj;k}(1 - x_{aj;k}) \right]$$

$$\mathcal{K}_{q\bar{q},k}^{(IF)} = T_R [1 - 2x_{aj;k}(1 - x_{aj;k})];$$

$$p_a = \frac{1}{x_{aj;k}} \tilde{p}_{aj}$$

$$p_j = (1 - u_a) \frac{1 - x_{aj;k}}{x_{aj;k}} \tilde{p}_{aj}$$

$$p_k = u_a \frac{1 - x_{aj;k}}{x_{aj;k}} \tilde{p}_{aj} + (1 - u_a)\tilde{p}_k - \vec{k}_\perp$$

$$p_a = \frac{1 - u_a}{x_{aj;k} - u_a} \tilde{p}_{aj} + \frac{u}{x} \frac{1 - x_{aj;k}}{x_{aj;k} - u_a} \tilde{p}_k + \frac{1}{u_a - x_{aj;k}} \vec{k}_\perp$$

$$p_j = \frac{1 - x_{aj;k}}{x_{aj;k} - u_a} \tilde{p}_{aj} + \frac{u}{x} \frac{1 - u_a}{x_{aj;k} - u_a} \tilde{p}_k + \frac{1}{u_a - x_{aj;k}} \vec{k}_\perp$$

$$p_k = \frac{x_{aj;k} - u_a}{x_{aj;k}} \tilde{p}_k.$$

$$\Lambda_\nu^\mu(K) = g_\nu^\mu + \frac{x_{aj;k}}{(1 - u_a)(1 - x_{aj;k})} \frac{k_\perp^\mu k_{\perp\nu}}{\tilde{p}_{aj}\tilde{p}_k} + \frac{u_a(1 - x_{aj;k})}{x_{aj;k} - u_a} \frac{K^\mu K_\nu}{\tilde{p}_{aj}\tilde{p}_k} + \frac{x_{aj;k}}{x_{aj;k} - u_a} \frac{k_\perp^\mu K_\nu - K^\mu k_{\perp\nu}}{\tilde{p}_{aj}\tilde{p}_k}$$

$$\tilde{p}_{aj} = x_{aj;b} p_a \ \text{ and } \ \tilde{p}_b = p_b$$

$$x_{aj;b} = \frac{p_a p_b - p_a p_j - p_b p_j}{p_a p_b}$$

$$k_\perp^2 = \frac{1 - x_{aj;b} - v_j}{x_{aj;b}} v_j Q^2$$

$$v_j = \frac{p_a p_j}{p_a p_b}$$

$$J^{(II)} = \frac{1}{x_{ij;a}} \frac{1 - x_{ij;a} - v_j}{1 - x_{ij;a} - 2v_j} \frac{f_{A/a}\left(\frac{\eta_a}{x_{ij;a}}, \mu_F^2\right)}{f_{A/a}(\eta_a, \mu_F^2)}$$

$$\mathcal{K}_{qg,k}^{(II)} = C_F \left[\frac{2}{1 - x_{aj;b}} - (1 + x_{aj;b}) \right]$$

$$\mathcal{K}_{qq,k}^{(II)} = C_F\left[\frac{2(1-x_{aj;b})}{x_{aj;b}} - x_{aj;b}\right]$$
$$\mathcal{K}_{gg,k}^{(II)} = 2C_A\left[\frac{1}{1-x_{aj;b}} + \frac{1-x_{aj;b}}{x_{aj;b}} - 1 + x_{aj;b}(1-x_{aj;b})\right]$$
$$\mathcal{K}_{q\bar{q},k}^{(II)} = T_R\left[1 - 2x_{aj;b}(1-x_{aj;b})\right]$$

$$p_a = \frac{1}{x_{aj;k}}\tilde{p}_{aj}$$
$$p_j = \frac{1-x_{aj;k}-v_j}{x_{aj;k}}\tilde{p}_{aj} + v_j\tilde{p}_b + \vec{k}_\perp$$
$$p_b = \tilde{p}_b$$

$$k_j = \Lambda(\tilde{p}_{aj} + p_b, p_a + p_b - p_j)\tilde{k}_j$$

$$\Lambda_\nu^\mu(\tilde{K}, K) = g_\nu^\mu - 2\frac{(\tilde{K}+K)^\mu(\tilde{K}+K)_\nu}{(\tilde{K}+K)^2} + 2\frac{K^\mu\tilde{K}_\nu}{\tilde{K}^2}$$

$$\Sigma_{\mathrm{DDT}}(q_T, Q) = \exp\left[-\int_{q_T^2}^{Q^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\frac{\alpha_s(k_\perp^2)}{\pi}C_F\left(\log\frac{Q^2}{k_\perp^2} - \frac{3}{2}\right)\right]$$

$$\int\frac{\mathrm{d}^3q_i}{(2\pi)^4}(2\pi)\delta(q_i^2) = \int\frac{(q_i^0)^2\,\mathrm{d}^2\Omega}{16\pi^3q_i^0} = \frac{q_i^0}{4\pi^2}$$

$$4\Delta C_{22} + 4(D-1)p_1^2C_{00} + p_1^4C_0(p_1, p_2) = \{\,\aleph_{\text{bubbles}}\,\}$$

$$m_{B^*} - m_B \propto 1/m_B,$$
$$m_{D^*}^2 - m_D^2 \approx m_{D^*}^2 - m_D^2 \approx m_{D_s^*}^2 - m_{D_s}^2 \approx 0.5\mathrm{GeV}^2$$

$$\mathcal{M}_{b\to s\gamma} \propto f\left(\frac{m_t^2}{m_W^2}\right)V_{bt}V_{ts}^* + f\left(\frac{m_c^2}{m_W^2}\right)V_{bc}V_{cs}^* + f\left(\frac{m_u^2}{m_W^2}\right)V_{bu}V_{us}^*$$
$$m_c, m_u \to 0 \quad f\left(\frac{m_t^2}{m_W^2}\right)V_{bt}V_{ts}^* + f(0)(V_{bc}V_{cs}^* + V_{bu}V_{us}^*)$$
$$= \left[f\left(\frac{m_t^2}{m_W^2}\right) - f(0)\right]V_{bt}V_{ts}^*$$

$$0 = \sum_q (V_{bq}V_{qs}^*) = V_{bt}V_{ts}^* + V_{bc}V_{cs}^* + V_{bu}V_{us}^* \longrightarrow V_{bt}V_{ts}^* = -(V_{bc}V_{cs}^* + V_{bu}V_{us}^*)$$

$$f(x) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\mathrm{d}N x^{-N-1}f(N)$$

CONCLUSIONES

Es evidente que la interacción fuerte, es una facción del Modelo Estándar de Física de Partículas, que, a propósito de la existencia del quark – top, el mismo que cumple con los parámetros formulados en la Teoría Cuántica de Campos Relativistas o Curvos, para clasificarse como una partícula supermasiva, dicha partícula, a razón de la densidad de su masa, es capaz, de deformar el espacio – tiempo cuántico en el que interactúa, más, el efecto gravitacional cuántico causado por sus interacciones, se tiene por indeterminado, es decir, que la gravedad local, le es endógena o exógena, según sea el caso. Es criterio de este autor, que la gravedad es endógena, concretamente por la magnitud de masa, sin perjuicio de que, por permeabilidad del campo gravitónico, la partícula supermasiva en referencia haya adquirido gravedad por transferencia o entrelazamiento con un gravitón, que como ha quedado dicho y en este campo en partícula, sería de naturaleza gluónica.

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