

Ciencia Latina Revista Científica Multidisciplinar, Ciudad de México, México.
ISSN 2707-2207 / ISSN 2707-2215 (en línea), septiembre-octubre 2025,
Volumen 9, Número 5.

https://doi.org/10.37811/cl_rcm.v9i5

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RELATIVISTA. SUPERSIMETRÍAS ADS/SCFT
EN SUPERESPACIOS DE WEYL –DE SITTER
Y ANTI – DE SITTER. VOLUMEN I**

RELATIVISTIC QUANTUM SUPERGRAVITY. ADS/SCFT
SUPERSYMMETRIES IN WEYL–DE SITTER AND ANTI–DE
SITTER SUPERSPACES. VOLUME I

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DOI: https://doi.org/10.37811/cl_rcm.v9i5.19763

SUPERGRAVEDAD CUÁNTICA RELATIVISTA. SUPERSIMETRÍAS ADS/SCFT EN SUPERESPACIOS DE WEYL –DE SITTER Y ANTI – DE SITTER. VOLUMEN I

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RESUMEN

En recientes trabajos, este autor ha esbozado planteamientos generales a propósito de la supergravedad cuántica relativista, la misma que, en sentido estricto, se tiene como una teoría del todo. La supergravedad cuántica relativista, es un pilar esencial en la Teoría Cuántica de Campos Relativistas o Curvos, planteada por este investigador, en la medida en que, unifica la relatividad general y la relatividad especial con la mecánica cuántica. Sin embargo, a diferencia de los postulados que son propios de la supergravedad, la supergravedad cuántica relativista, usa la supersimetría en grupos de gauge infinitos (supersimetrías de gauge) con la finalidad de integrar la gravedad a escala subatómica, a partir de la existencia de superpartículas, capaces de deformar el espacio – tiempo cuántico, en dimensiones más altas (supersimetrías de gauge en dimensiones D), lo que ocurre, por interacción de la propia superpartícula a razón de su masa y energía extremadamente densas, o por interacción de la superpartícula con el supergravitón, incluyendo, aquellas interacciones con regiones de antimateria, lo que supone, la permeabilización del campo cuántico repercutido por el campo supergravitónico. En este trabajo, se robustecerá este concepto esencial que sostiene la Teoría Cuántica de Campos Relativistas o Curvos.

Palabras clave: Supersimetría, Supergravedad en D dimensiones, relatividad general, grupos de gauge, superpartículas, supergravitón.

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RELATIVISTIC QUANTUM SUPERGRAVITY. ADS/SCFT SUPERSYMMETRIES IN WEYL-DE SITTER AND ANTI-DE SITTER SUPERSPACES. VOLUME I

ABSTRACT

In recent works, this author has outlined general approaches to relativistic quantum supergravity, which, strictly speaking, is considered a theory of everything. Relativistic quantum supergravity is an essential pillar in the Quantum Theory of Relativistic or Curved Fields, proposed by this researcher, insofar as it unifies general relativity and special relativity with quantum mechanics. However, unlike the postulates that are typical of supergravity, relativistic quantum supergravity uses supersymmetry in infinite gauge groups (gauge supersymmetries) in order to integrate gravity at the subatomic scale, based on the existence of superparticles, capable of deforming quantum space-time, in higher dimensions (gauge supersymmetries in D dimensions). which occurs by interaction of the superparticle itself due to its extremely dense mass and energy, or by interaction of the superparticle with the supergraviton, including those interactions with regions of antimatter, which supposes the permeabilization of the supergravitonic field, which supposes the permeabilization of the quantum field impacted by the supergravitonic field. In this work, this essential concept that explains the Quantum Theory of Relativistic or Curved Fields will be strengthened.

Keywords: Supersymmetry, Supergravity in D dimensions, general relativity, gauge groups, superparticles, supergraviton.

Artículo recibido 11 agosto 2025

Aceptado para publicación: 13 setiembre 2025



INTRODUCCIÓN.

A diferencia de otras líneas de investigación relacionadas a la supergravedad, en este trabajo, abordaré la supergravedad a escala cuántica y en dimensión relativista, en D dimensiones, sin regulación por campos de gauge finitos, sino en contrario, suponiendo que, a escala cuántica, la supersimetría de gauge, está dada en infinitas dimensiones, a propósito de la distorsión del espacio – tiempo cuántico, en dimensiones más altas, a causa de la interacción de la superpartícula, por su propio centro de masa o energía (en el caso de las partículas estrella o blancas), o por interacción de la superpartícula con el supergravitón, incluyendo, aquellas interacciones con regiones de antimateria.

En este trabajo, esbozaré un modelo matemático unitario, a propósito de la supergravedad cuántica relativista, concebida así, en la Teoría Cuántica de Campos Relativistas o Curvos, esto es, la deformación del espacio – tiempo cuántico, y su desdoblamiento en dimensiones infinitas, por efectos de la gravedad causada por la interacción de la superpartícula, con el supergravitón, lo que supone, la permeabilización del campo cuántico repercutido por el campo supergravitónico, o en su propio campo cuántico, en condiciones relativistas, a razón de su masa y energía, extremadamente densas, como ocurre con las partículas blancas.

RESULTADOS Y DISCUSIÓN.

Este trabajo, se encuentra dividido en dos volúmenes. En consecuencia, los cálculos aquí desarrollados se complementan con el volumen II de este manuscrito.

DESARROLLO MATEMÁTICO: En un escenario inflacionario oscilatorio, la supergravedad cuántica relativista en un campo de gauge perturbativo, se calcula así:

Métrica EMDA.

$$\mathcal{L} = \sqrt{-g} \left[R - \frac{1}{2} (\partial\phi)^2 - e^{a\phi} F^2 - \frac{1}{2} e^{c\phi} (\partial\chi)^2 + b\chi \tilde{F}^{\mu\nu} F_{\mu\nu} \right]$$

$$\phi \rightarrow \phi + \lambda, A_\mu \rightarrow e^{-\frac{1}{2}a\lambda} A_\mu$$

$$\chi \rightarrow e^{-\frac{1}{2}c\lambda} \chi$$

$$c = -2a.$$

$$V^\pm = \pm \partial_{r_*} f + f^2 + 2\beta f = \pm \partial_{r_*} W + W^2 - \beta^2$$



$$\begin{aligned}
R_{\mu\nu} &= \frac{1}{2} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} e^{c\phi} \partial_\mu \chi \partial_\nu \chi + 2e^{a\phi} \left(F_{\mu\rho} F_\nu^\rho - \frac{1}{4} F^{\rho\sigma} F_{\rho\sigma} g_{\mu\nu} \right) \\
\nabla_\nu (e^{a\phi} F^{\mu\nu} - b\chi \tilde{F}^{\mu\nu}) &= 0 \\
\Box \phi &= ae^{a\phi} F^2 + \frac{1}{2} ce^{c\phi} (\partial\chi)^2 \\
\nabla_\mu (e^{c\phi} \nabla^\mu \chi) + b\tilde{F}^{\mu\nu} F_{\mu\nu} &= 0
\end{aligned}$$

$$\begin{aligned}
ds^2 &= -\Delta dt^2 + \frac{dr^2}{\Delta} + R^2(d\theta^2 + \sin^2 \theta d\varphi^2) \\
e^{-a\phi} &= f_-^{\frac{2a^2}{1+a^2}}, \chi = 0, A = \frac{Q}{r} dt \\
\Delta &= f_+ f_-^{\frac{1-a^2}{1+a^2}}, R = r f_-^{\frac{a^2}{1+a^2}}, f_\pm = 1 - \frac{r_\pm}{r} \\
Q^2 &= \frac{r_+ r_-}{1+a^2}
\end{aligned}$$

$$\begin{aligned}
ds^2 &= -e^{2\nu} dt^2 + e^{2\mu_1} dr^2 + e^{2\mu_2} d\theta^2 + e^{2\psi} (d\varphi - \mathcal{B}_\alpha dx^\alpha)^2 \\
e^{\underline{0}} &= e^\nu dt, e^{\underline{1}} = e^{\mu_1} dr, e^{\underline{2}} = e^{\mu_2} d\theta, e^{\underline{3}} = e^\psi (d\varphi - \mathcal{B}_\alpha dx^\alpha)
\end{aligned}$$

$$\begin{aligned}
E_{\underline{0}}^\mu \partial_\mu &= e^{-\nu} \left(\frac{\partial}{\partial t} + \mathcal{B}_0 \frac{\partial}{\partial \varphi} \right), E_{\underline{1}}^\mu \partial_\mu = e^{-\mu_1} \left(\frac{\partial}{\partial r} + \mathcal{B}_1 \frac{\partial}{\partial \varphi} \right) \\
E_{\underline{2}}^\mu \partial_\mu &= e^{-\mu_2} \left(\frac{\partial}{\partial \theta} + \mathcal{B}_2 \frac{\partial}{\partial \varphi} \right), E_{\underline{3}}^\mu \partial_\mu = e^{-\psi} \frac{\partial}{\partial \varphi}
\end{aligned}$$

$$\begin{aligned}
\gamma_{axial} : \quad & (\delta \mathcal{B}_0, \delta \mathcal{B}_1, \delta \mathcal{B}_2, \delta F_{\underline{0}\underline{3}}, \delta F_{\underline{1}\underline{3}}, \delta F_{\underline{2}\underline{3}}, \delta \chi) \\
\lambda_{polar} : \quad & (\delta \nu, \delta \mu_1, \delta \mu_2, \delta \psi, \delta F_{\underline{0}\underline{2}}, \delta F_{\underline{1}\underline{2}}, \delta \phi)
\end{aligned}$$

$$\begin{aligned}
\nu &= \bar{\nu} + \epsilon \delta \nu, \mu_1 = \bar{\mu}_1 + \epsilon \delta \mu_1, \mu_2 = \bar{\mu}_2 + \epsilon \delta \mu_2, \psi = \bar{\psi} + \epsilon \delta \psi \\
\mathcal{B}_\alpha &= \epsilon \delta \mathcal{B}_\alpha, \phi = \bar{\phi} + \epsilon \delta \phi, \chi = \epsilon \delta \chi
\end{aligned}$$

$$F_{ab} = \bar{F}_{ab} + \epsilon \delta F_{ab}$$

$$\bar{\nu} = \frac{1}{2} \log \Delta, \bar{\mu}_1 = -\bar{\nu}, \bar{\mu}_2 = \log R, \bar{\psi} = \log R + \log \sin \theta$$

$$\bar{\mathcal{B}}_\alpha = 0, a\bar{\phi} = 2\log r - 2\log R, \bar{F}_{\underline{0}\underline{1}} = \frac{Q}{r^2}$$

$$\begin{aligned}
(\mathfrak{E}_{einstein})_{ab} &\equiv R_{ab} - \frac{1}{2} R \eta_{ab} - \frac{1}{2} \phi_a \phi_b + \frac{1}{4} \phi^c \phi_c \eta_{ab} - \frac{1}{2} e^{c\phi} \chi_a \chi_b + \frac{1}{4} e^{c\phi} \chi^c \chi_c \eta_{ab} \\
&\quad - 2e^{a\phi} \left(F_{ac} F_b^c - \frac{1}{4} \eta_{ab} F^{cd} F_{cd} \right) = 0
\end{aligned}$$

$$(\mathfrak{M}_{maxwell})^a \equiv e_\mu^a \partial_\nu \left(\sqrt{g} E^{b\mu} E^{c\nu} (e^{a\phi} F_{bc} - b\chi \tilde{F}_{bc}) \right) = 0$$

$$E_\phi \equiv \Box \phi - ae^{a\phi} F^{ab} F_{ab} - \frac{1}{2} ce^{c\phi} \chi^a \chi_a = 0$$

$$E_\chi \equiv \partial_\mu (\sqrt{g} e^{c\phi} E^{a\mu} \chi_a) + b\sqrt{g} \tilde{F}^{ab} F_{ab} = 0$$

$$(\mathfrak{I}_{Bianchi})^a \equiv e_\mu^a \varepsilon^{\mu\nu\rho\sigma} \partial_\nu (e_\rho^b e_\sigma^c F_{bc}) = 0$$



$$\begin{aligned}
(\mathfrak{E}_{\text{einstein}})_{1\underline{3}}: \quad & e^{\bar{\nu}-\bar{\mu}_2} \frac{1}{\sin^2 \theta} \partial_\theta (\sin^3 \theta \delta \mathcal{G}_{12}) + e^{-\bar{\nu}+\bar{\mu}_2} \sin \theta \partial_t \delta \mathcal{G}_{01} - \frac{4Q}{r^2} e^{a\bar{\phi}} \delta F_{\underline{0}\underline{3}} = 0 \\
(\mathfrak{E}_{\text{einstein}})_{2\underline{3}}: \quad & e^{-2\bar{\mu}_2} \partial_r (e^{2\bar{\nu}+2\bar{\mu}_2} \delta \mathcal{G}_{12}) - e^{-2\bar{\nu}} \partial_t \delta \mathcal{G}_{02} = 0
\end{aligned}$$

$$\begin{aligned}
(\mathfrak{M}_{\text{maxwell}})^{\underline{3}}: \quad & e^{-\bar{\nu}-\bar{\mu}_2} \partial_t \delta F_{\underline{0}\underline{3}} - e^{-2\bar{\mu}_2-a\bar{\phi}} \partial_r (e^{\bar{\nu}+\bar{\mu}_2+a\bar{\phi}} \delta F_{\underline{1}\underline{3}}) - \\
& e^{-2\bar{\mu}_2} \partial_\theta \left(\delta F_{\underline{2}\underline{3}} + \frac{bQ}{r^2} \delta \chi \right) + \frac{Q \sin \theta}{r^2} \delta \mathcal{G}_{01} = 0
\end{aligned}$$

$$\begin{aligned}
E_\chi: \quad & -e^{-2\bar{\nu}+c\bar{\phi}} \partial_t^2 \delta \chi + e^{-2\bar{\mu}_2} \partial_r (e^{2\bar{\nu}+2\bar{\mu}_2+c\bar{\phi}} \partial_r \delta \chi) \\
& + e^{-2\bar{\mu}_2+c\bar{\phi}} \frac{1}{\sin \theta} \partial_\theta (\sin \theta \partial_\theta \delta \chi) - \frac{4bQ}{r^2} \delta F_{\underline{2}\underline{3}} = 0
\end{aligned}$$

$$\begin{aligned}
(\mathfrak{B}_{\text{Bianchi}})^{\underline{1}}: \quad & \partial_t \delta F_{\underline{2}\underline{3}} = e^{\bar{\nu}-\bar{\mu}_2} \frac{1}{\sin \theta} \partial_\theta (\sin \theta \delta F_{\underline{0}\underline{3}}) \\
(\mathfrak{B}_{\text{Bianchi}})^{\underline{2}}: \quad & \partial_t \delta F_{\underline{1}\underline{3}} = e^{\bar{\nu}-\bar{\mu}_2} \partial_r (e^{\bar{\nu}+\bar{\mu}_2} \delta F_{\underline{0}\underline{3}})
\end{aligned}$$

$$\delta F_{\underline{0}\underline{3}}(t, r, \theta) = \frac{i\omega \ell(\ell+1)e^{-\bar{\nu}}}{r \sin \theta} e^{-i\omega t} B_{\underline{0}\underline{3}}(r) C_{\ell+1}^{-\frac{1}{2}}(\cos \theta) = \frac{-i\omega e^{-\bar{\nu}}}{r} e^{-i\omega t} B_{\underline{0}\underline{3}}(r) \partial_\theta P_\ell(\cos \theta)$$

$$\begin{aligned}
\delta \mathcal{G}_{12}(t, r, \theta) &= \frac{2i\omega \mu \ell(\ell+1)e^{-2\bar{\nu}-\bar{\mu}_2}}{3 \sin^3 \theta} e^{-i\omega t} \mathcal{G}_{12}(r) C_{\ell+2}^{-\frac{3}{2}}(\cos \theta) \\
&= \frac{-2i\omega e^{-2\bar{\nu}-\bar{\mu}_2}}{\mu} e^{-i\omega t} \mathcal{G}_{12}(r) \partial_\theta (\csc \theta \partial_\theta P_\ell(\cos \theta)),
\end{aligned}$$

$$\delta F_{\underline{2}\underline{3}}(t, r, \theta) = \frac{e^{-\bar{\mu}_2}}{r} e^{-i\omega t} B_{\underline{2}\underline{3}}(r) P_\ell(\cos \theta),$$

$$\delta \chi(t, r, \theta) = -2\sqrt{\ell(\ell+1)} e^{-\bar{\mu}_2-\frac{1}{2}c\bar{\phi}} e^{-i\omega t} \chi(r) P_\ell(\cos \theta),$$

$$\mu = \sqrt{\ell(\ell+1)-2}$$

$$\begin{aligned}
(\partial_{r_*}^2 + \omega^2) B_{\underline{0}\underline{\pm}} + T_1 B_{\underline{0}\underline{3}} + S_1 \mathcal{G}_{12} + P_1 \chi &= 0, \\
(\partial_{r_*}^2 + \omega^2) \mathcal{G}_{12} + T_2 \mathcal{G}_{12} + S_1 B_{\underline{0}\underline{3}} &= 0, \\
(\partial_{r_*}^2 + \omega^2) \chi + T_3 \chi + P_1 B_{\underline{0}\underline{3}} &= 0,
\end{aligned}$$

$$\begin{aligned}
T_1 &= -e^{4\bar{\nu}} \left(a\bar{\nu}'\bar{\phi}' + \frac{1}{4}a^2\bar{\phi}'^2 + \frac{1}{2}a\bar{\phi}'' \right) - \frac{1}{r^2} e^{2\bar{\nu}+a\bar{\phi}} \left(\frac{4Q^2}{r^2} + \ell(\ell+1) \right) \\
T_2 &= -e^{4\bar{\nu}} \left(a\bar{\nu}'\bar{\phi}' + \frac{1}{4}a^2\bar{\phi}'^2 + \frac{1}{2}a\bar{\phi}'' + \frac{2}{r^2} - \frac{2}{r}\bar{\nu}' - \frac{a}{r}\bar{\phi}' \right) - \frac{1}{r^2} e^{2\bar{\nu}+a\bar{\phi}} (\ell(\ell+1)-2) \\
T_3 &= -e^{4\bar{\nu}} \left(-(a-c)\bar{\nu}'\bar{\phi}' + \frac{1}{4}(a-c)^2\bar{\phi}'^2 - \frac{1}{2}(a-c)\bar{\phi}'' + \frac{2}{r}\bar{\nu}' - \frac{(a-c)}{r}\bar{\phi}' \right)
\end{aligned}$$

$$S_1 = -\frac{1}{r^2} e^{2\bar{\nu}+a\bar{\phi}} \ell(\ell+1) - \frac{2\mu Q}{r^3} e^{2\bar{\nu}+a\bar{\phi}}$$

$$P_1 = -\frac{2b\sqrt{\ell(\ell+1)}Q}{r^3} e^{2\bar{\nu}-\frac{c}{2}\bar{\phi}}$$

$$\partial_{r_*} = e^{2\bar{\nu}} \partial_r$$



$$e^{\bar{\mu}_2} = r e^{-\frac{1}{2}a\bar{\phi}}$$

$$B_{\underline{2}\underline{3}} = -\ell(\ell+1)B_{\underline{0}\underline{3}}.$$

$$a = 1, c = -2$$

$$\delta \mathcal{G}_{01} = 2e^{-\bar{\mu}_2} e^{-i\omega t} \mathcal{G}_{01}(r) \csc \theta \partial_\theta P_\ell$$

$$\delta \mathcal{G}_{02} = \frac{2e^{-\bar{\mu}_2}}{\mu} e^{-i\omega t} \mathcal{G}_{02}(r) \partial_\theta (\csc \theta \partial_\theta P_\ell)$$

$$\delta \mathcal{G}_{12} = -2i\omega \frac{e^{-2\bar{\nu}-\bar{\mu}_2}}{\mu} e^{-i\omega t} \mathcal{G}_{12}(r) \partial_\theta (\csc \theta \partial_\theta P_\ell)$$

$$\delta F_{\underline{0}\underline{3}} = -i\omega \frac{e^{-\bar{\nu}}}{r} e^{-i\omega t} B_{\underline{0}\underline{3}}(r) \partial_\theta P_\ell$$

$$\delta F_{\underline{1}\underline{3}} = \frac{e^{-\bar{\nu}}}{r} e^{-i\omega t} B_{\underline{1}\underline{3}}(r) \partial_\theta P_\ell$$

$$\delta F_{\underline{2}\underline{3}} = \frac{e^{-\bar{\mu}_2}}{r} e^{-i\omega t} B_{\underline{2}\underline{3}}(r) P_\ell$$

$$\delta \chi = -2\sqrt{\ell(\ell+1)} e^{-\bar{\mu}_2 - \frac{1}{2}c\bar{\phi}} e^{-i\omega t} \chi(r) P_\ell$$

$$(\mathfrak{E}_{\text{einstein}})_{\underline{1}\underline{3}}: \frac{2Q}{r} B_{\underline{0}\underline{3}} - e^{2\bar{\mu}_2} \mathcal{G}_{01} + \sqrt{(\ell+2)(\ell-1)} \mathcal{G}_{12} = 0$$

$$(\mathfrak{E}_{\text{einstein}})_{\underline{2}\underline{3}}: e^{-\bar{\mu}_2} \partial_r (e^{\bar{\mu}_2} \mathcal{G}_{12}) - e^{-2\bar{\nu}} \mathcal{G}_{02} = 0$$

$$(\mathfrak{B}_{\text{Bianchi}})^{\underline{1}}: \ell(\ell+1) B_{\underline{0}\underline{3}} + B_{\underline{2}\underline{3}} = 0$$

$$(\mathfrak{B}_{\text{Bianchi}})^{\underline{2}}: r e^{-\bar{\mu}_2} \partial_r \left(\frac{1}{r} e^{\bar{\mu}_2} B_{\underline{0}\underline{3}} \right) - e^{-2\bar{\nu}} B_{\underline{1}\underline{3}} = 0$$

$$(\mathfrak{M}_{\text{maxwell}})^{\underline{3}}: \frac{1}{r} e^{\bar{\mu}_2} \partial_r (r e^{-\bar{\mu}_2} B_{\underline{1}\underline{3}}) + \omega^2 e^{-2\bar{\nu}} B_{\underline{0}\underline{3}} + e^{-2\bar{\mu}_2} B_{\underline{2}\underline{3}}$$

$$- \frac{2Q}{r} \mathcal{G}_{01} - \frac{2b\sqrt{\ell(\ell+1)}Q}{r^3} e^{-\frac{1}{2}c\bar{\phi}} \chi = 0$$

$$E_\chi: \chi'' + 2\bar{\nu}' \chi' + \left[\omega^2 e^{-4\bar{\nu}} - \bar{\mu}_2' (\bar{\mu}_2' + 2\bar{\nu}') - c\bar{\phi}' \left(\bar{\mu}_2' + \bar{\nu}' + \frac{1}{4} c\bar{\phi}' \right) - \bar{\mu}_2'' \right.$$

$$\left. - \frac{1}{2} c\bar{\phi}'' - \ell(\ell+1) e^{-2\bar{\mu}_2-2\bar{\nu}} \right] \chi + \frac{2bQ}{\sqrt{\ell(\ell+1)}} \frac{e^{-2\bar{\nu}-\frac{1}{2}c\bar{\phi}}}{r^3} B_{\underline{2}} = 0$$

$$d\mathcal{G}: e^{\bar{\mu}_2} \partial_r (e^{-\bar{\mu}_2} \mathcal{G}_{02}) + \omega^2 e^{-2\bar{\nu}} \mathcal{G}_{12} - \sqrt{(\ell+2)(\ell-1)} \mathcal{G}_{01} = 0$$

$$\{\mathcal{G}_{02}, \mathcal{G}_{12}, B_{\underline{0}\underline{3}}, B_{\underline{1}\underline{3}}, \chi, \xi\}$$

$$n = \frac{1}{2} \ell(\ell+1) - 1 = \frac{1}{2} \mu^2$$



$$\begin{aligned}
\mathcal{G}'_{02} &= \bar{\mu}'_2 \mathcal{G}_{02} + (2ne^{-2\bar{\mu}_2} - \omega^2 e^{-2\bar{v}}) \mathcal{G}_{12} + \frac{2\sqrt{2n}Q}{r} e^{-2\bar{\mu}_2} B_{\underline{0}\underline{3}} \\
\mathcal{G}'_{12} &= e^{-2\bar{v}} \mathcal{G}_{02} - \bar{\mu}'_2 \mathcal{G}_{12} \\
B'_{\underline{0}\underline{3}} &= \left(\frac{1}{r} - \bar{\mu}'_2 \right) B_{\underline{0}\underline{3}} + e^{-2\bar{v}} B_{\underline{1}\underline{3}} \\
B'_{\underline{1}\underline{3}} &= \left(2(n+1)e^{-2\bar{\mu}_2} - \omega^2 e^{-2\bar{v}} + \frac{4Q^2}{r^2} e^{-2\bar{\mu}_2} \right) B_{\underline{0}\underline{3}} - \left(\frac{1}{r} - \bar{\mu}'_2 \right) B_{\underline{1}\underline{3}} + \frac{2\sqrt{2n}Q}{r} e^{-2\bar{\mu}_2} \mathcal{G}_{12} + \frac{2b\sqrt{2(n+1)}Q}{r^3} e^{-\frac{1}{2}c\bar{\phi}} \chi \\
\chi' &= \xi \\
\xi' &= \left[2(n+1)e^{-2\bar{\mu}_2-2\bar{v}} + \bar{\mu}'_2(\bar{\mu}'_2 + 2\bar{v}') + c\bar{\phi}' \left(\bar{\mu}'_2 + \bar{v}' + \frac{1}{4}c\bar{\phi}' \right) + \bar{\mu}''_2 + \frac{1}{2}c\bar{\phi}'' - \omega^2 e^{-4\bar{v}} \right] \chi + \frac{2b\sqrt{2(n+1)}Q}{r^3} e^{-2\bar{v}-\frac{1}{2}c\bar{\phi}} B_{\underline{0}\underline{3}} - 2\bar{v}' \xi
\end{aligned}$$

$$(\partial_{r_*}^2 + \omega^2)Z^- - V^- Z^- = 0$$

$$Z^- = f_1 \mathcal{G}_{12} + f_2 B_{\underline{0}\underline{3}} + f_3 \chi$$

$$e_1 \mathcal{G}_{02} + e_2 \mathcal{G}_{12} + e_3 B_{\underline{0}\underline{3}} + e_4 B_{\underline{1}\underline{3}} + e_5 \chi + e_6 \xi = 0$$

$$e_i = 0, 1 \leq i \leq 6$$

$$e_1 = 2f_+ f_-^{\frac{1-a^2}{1+a^2}} f'_1 = 0 \Rightarrow f_1 = c_1$$

$$e_4 = 2f_+ f_-^{\frac{1-a^2}{1+a^2}} f'_2 = 0 \Rightarrow f_2 = c_2$$

$$e_6 = 2(f_+ f_-)^2 f_-^{\frac{-4a^2}{1+a^2}} f'_3 = 0 \Rightarrow f_3 = c_3$$

$$V^- = \frac{1}{(1+a^2)^2 c_1 r^5} f_+ f_-^{\frac{-4a^2}{1+a^2}} [2(n+1)(1+a^2)^2 c_1 r^3$$

$$- (1+a^2) [(2n(1+a^2) + (5-a^2))r_- + 3(1+a^2)r_+] c_1$$

$$- 2\sqrt{2n}Q(1+a^2)c_2] r^2 + [(3r_- + 7(1+a^2)r_+)r_- c_1 - 2\sqrt{2n}Q(1+a^2)^2 c_2 r_-] r$$

$$- (4+a^2)c_1 r_-^2 r_+]$$

$$\left(\frac{3-a^2}{1+a^2} r_- + 3r_+ \right) c_1 c_2 + 2\sqrt{2n}Q(c_1^2 - c_2^2) + 2b\sqrt{2(n+1)}Q f_-^{\frac{a(2a+c)}{1+a^2}} c_1 c_3 = 0$$

$$\sqrt{2n}Qr(r-r_-)c_2 c_3 - b\sqrt{2(n+1)}Qr(r-r_-) f_-^{\frac{a(2a+c)}{1+a^2}} c_1 c_2 - B(r)c_1 c_3 = 0$$

$$B(r) = \frac{1}{2(1+a^2)^2} [4(1+a^2)[(1+a^2)r_+ + (1-a^2)r_-]r^2 + r_- [(a^2 c^2 - 4)r_- - (1+a^2)(ac+10)r_+]r + (ac+2)(3+a^2-ac)r_-^2 r_+]$$

$$B(r) = \frac{(r-r_-)}{1+a^2} [2[(1+a^2)r_+ + (1-a^2)r_-]r - 3(1-a^2)r_- r_+]$$

$$a = 1, c = -2$$



$$(r_- + 3r_+)c_1c_2 + 2\sqrt{2n}Q(c_1^2 - c_2^2) + 2b\sqrt{2(n+1)}Qc_1c_3 = 0$$

$$(\sqrt{2n}c_3 - b\sqrt{2(n+1)}c_1)Qc_2 - 2r_+c_1c_3 = 0$$

$$c_1 = -\frac{pc_2r_-}{4\sqrt{2n}Q},$$

$$c_3 = -\frac{b\sqrt{2(n+1)}Qc_2p}{2(p+2n)r_+}$$

$$\tilde{\Lambda}(p) \equiv p^3r_- + p^2[2n(1+b^2)r_- - 2(1-b^2)r_- - 6r_+] - 4np(r_- + 7r_+) - 32n^2r_+ = 0.$$

$$Z_{(p)}^- = c_1\mathcal{G}_{12} + c_2B_{03} + c_3\chi$$

$$(\partial_{r_*}^2 + \omega^2)Z_{(p)}^- - V_{(p)}^- Z_{(p)}^- = 0$$

$$V_{(p)}^- = \frac{(r-r_+)[8(n+1)r^3 - 4(2(n+1)r_- + 3r_+)r^2 + (3r_- + 14r_+)r - 5r_-^2r_+]}{4(r-r_-)^2r^4}$$

$$- \frac{8n(r-r_+)r_+}{p(r-r_-)r^3}$$

$$D_p = 64n^2[16(n+1)^3r_-^3r_+b^6 + (n+1)^2(r_-^2 + (48n-34)r_-r_+ - 95r_+^2)r_-^2b^4$$

$$+ 2(n+1)[(n-1)r_-^3 + (24n^2 + 2n + 7)r_-^2r_+ + (157n + 53)r_-r_+^2 + 69r_+^3]r_-b^2$$

$$+ [(n+1)r_- - r_+]^2[r_-^2 + 2(8n+3)r_-r_+ + 9r_+^2]]$$

$$D_p = 64n^2[16(n+1)b^2r_-r_+[3r_+ - (n+1)b^2r_-]^2 + r_+^2[3r_+ - (n+1)(3+b^2)r_-]^2$$

$$+ 4(n+1)(77n+25)b^2r_-^2r_+^2 + r_-[r_- + 2(8n+3)r_+][r_+ - (n+1)r_-]^2$$

$$+ 2(n+1)b^2r_-^3[(n-1)r_- + (24n^2 + 2n + 7)r_+]$$

$$+ (n+1)^2b^4r_-^3[r_- + (48n-34)r_+]]$$

$$p_1p_2p_3 = \frac{32n^2r_+}{r_-} > 0 \text{ and } p_1p_2 + p_2p_3 + p_1p_3 = -\frac{4n(r_- + 7r_+)}{r_-} < 0$$

$$p_1 \leq p_2 < 0, p_3 > 0$$

$$b^2=1$$

$$\tilde{\Lambda}_1(p) \equiv p^3r_- + p^2[4nr_- - 6r_+] - 4np(r_- + 7r_+) - 32n^2r_+ = 0$$

$$V^\pm = \pm \partial_{r_*} f + \alpha f^2 + 2\beta f$$

$$V^+ - V^- = 2\partial_{r_*} f$$

$$r_+ = \frac{p[p^2 + 4np - 4n]r_-}{2(2n+p)(8n+3p)}$$

$$f_{(p)} = \frac{r_-(r-r_+)[(4n+2p)r - pr_-]}{2r^2(r-r_-)(4nr + pr_-)}$$



$$\alpha_{(p)} = 1, \beta_{(p)} = \frac{4n(n+1)}{(2n+p)r_-}$$

$$V_{(p)}^{\pm} = \pm \partial_{r_*} f_{(p)} + f_{(p)}^2 + \frac{8n(n+1)}{(2n+p)r_-} f_{(p)}.$$

$$[\partial_{r_*}^2 + \omega^2 - V^{\pm}]Z^{\pm} = 0$$

$$D_- D_+ Z^- + (\omega^2 + \beta^2) Z^- = 0$$

$$D_+ D_- Z^+ + (\omega^2 + \beta^2) Z^+ = 0$$

$$D_{\pm} = \partial_{r_*} \pm (f + \beta),$$

$$D_- Z^+ = (i\omega - \beta) Z^-, D_+ Z^- = (i\omega + \beta) Z^+$$

$$\omega = \pm i\beta$$

$$D_+ Z^- = 0, D_- Z^+ = 0$$

$$Z^{\pm} = \exp \left[\pm \int^r (f(r') + \beta) (f(r') + \beta) e^{-2\bar{v}(r')} dr' \right]$$

$$Z_{(p)}^+ = e^{\beta_{(p)} r_*} \left(1 - \frac{r_-}{r} \right)^{\frac{1}{2}} \left(4n + \frac{pr_-}{r} \right)^{-1}$$

$$Z_{(p)}^- = e^{-\beta_{(p)} r_*} \left(1 - \frac{r_-}{r} \right)^{-\frac{1}{2}} \left(4n + \frac{pr_-}{r} \right)$$

$$e^{-i\omega t} Z_{(p)}^+ = e^{\beta_{(p)}(r_* \pm t)} \left(1 - \frac{r_-}{r} \right)^{\frac{1}{2}} \left(4n + \frac{pr_-}{r} \right)^{-1}$$

$$e^{-i\omega t} Z_{(p)}^- = e^{-\beta_{(p)}(r_* \mp t)} \left(1 - \frac{r_-}{r} \right)^{-\frac{1}{2}} \left(4n + \frac{pr_-}{r} \right)$$

$$\tilde{Z}_{(p)}^+(r) = Z_{(p)}^+(r) \int^r \frac{e^{-2\bar{v}(r')}}{(Z_{(p)}^+(r'))^2} \frac{e^{-2\bar{v}(r')}}{(Z_{(p)}^+(r'))^2} dr', \tilde{Z}_{(p)}^-(r) = Z_{(p)}^-(r) \int^r \frac{e^{-2\bar{v}(r')}}{(Z_{(p)}^-(r'))^2} \frac{e^{-2\bar{v}(r')}}{(Z_{(p)}^-(r'))^2} dr'$$

$$r_+ - r_- = \frac{[p + 2(\ell - 1)][p - 2(\ell + 2)][p + 2(\ell + 2)(\ell - 1)]r_-}{2[p + (\ell + 2)(\ell - 1)][3p + 4(\ell + 2)(\ell - 1)]}$$

$$(1): p > 2(\ell + 2)$$

$$(2): -(\ell + 2)(\ell - 1) < p < -2(\ell - 1)$$

$$(3): -2(\ell + 2)(\ell - 1) < p < -\frac{4}{3}(\ell + 2)(\ell - 1)$$

$$\ell = 2 + y, r = r_+(1 + z)$$

$$p = 2(\ell + 2)(1 + x)$$



$$p = -(\ell + 2)(\ell - 1) + \frac{\ell(\ell - 1)}{1 + x}$$

$$p = -2(\ell + 2)(\ell - 1) + \frac{2(\ell + 2)(\ell - 1)}{3(1 + x)}$$

$$V_i^- = R_i P_i, i = 1, 2, 3$$

$$\mathcal{L} = \sqrt{-g} \left[R - \frac{1}{2} (\partial \phi)^2 - e^\phi F^2 - \frac{1}{2} e^{-2\phi} (\partial \chi)^2 + \chi \tilde{F}^{\mu\nu} F_{\mu\nu} \right. \\ \left. - \frac{e^{-2\phi}}{1 + \chi^2 e^{2\phi}} (e^\phi H^2 + \chi \tilde{H}^{\mu\nu} H_{\mu\nu}) \right]$$

$$\delta H_{\underline{03}}(t,r,\theta)=-\frac{\mathrm{i}\omega e^{-\bar{\nu}+\bar{\phi}}}{r}e^{-\mathrm{i}\omega t}Z_H^-(r)\partial_\theta P_\ell(\cos\theta),$$

$$\delta H_{\underline{12}}(t,r,\theta)=-\frac{\mathrm{i}\omega e^{-\bar{\nu}+\bar{\phi}}}{r^2}e^{-\mathrm{i}\omega t}Z_H^+(r)\partial_\theta P_\ell(\cos\theta)$$

$$\delta H_{\underline{13}}=e^{\bar{\nu}-\bar{\mu}_2}\partial_r(re^{-\bar{\mu}_2}Z_H^-(r))e^{-\mathrm{i}\omega t}\partial_\theta P_\ell(\cos\theta)$$

$$\delta H_{\underline{23}}=-\frac{\ell(\ell+1)e^{-\bar{\mu}_2+\bar{\phi}}}{r}Z_H^-(r)e^{-\mathrm{i}\omega t}P_\ell(\cos\theta)$$

$$\delta H_{\underline{12}}=-\frac{\mathrm{i}\omega e^{-\bar{\nu}+\bar{\phi}}}{r^2}Z_H^+(r)e^{-\mathrm{i}\omega t}\partial_\theta P_\ell(\cos\theta)$$

$$\delta H_{\underline{01}}=\frac{\ell(\ell+1)e^{-\bar{\mu}_2+\bar{\phi}}}{r^2}Z_H^+(r)e^{-\mathrm{i}\omega t}P_\ell(\cos\theta)$$

$$(\partial_{r_*}^2+\omega^2-V_H^\pm)Z_H^\pm=0$$

$$V_H^\pm=\frac{2(n+1)e^{2\bar{\nu}+\bar{\phi}}}{r^2}+e^{4\bar{\nu}}\left[\frac{1}{4}\bar{\phi}'^2\pm\left(\bar{\nu}'\bar{\phi}'+\frac{1}{2}\bar{\phi}''\right)\right]$$

$$V_H^\pm=\pm\partial_{r_*}f_H+f_H^2+2\beta_Hf_H$$

$$f_H=\frac{1}{2}e^{2\bar{\nu}}\bar{\phi}',\beta_H=\frac{2(n+1)e^{\bar{\phi}}}{r^2\bar{\phi}'}=-\frac{2(n+1)}{r_-}$$

$$V_H^- = \frac{(r-r_+)[8(n+1)r^3-4(2n+3)r_-r^2+3(r_-+2r_+)r_-r-5r_-^2r_+]}{4r^4(r-r_-)^2}$$

$$V_H^+ = \frac{(r-r_+)[8(n+1)r^3-4(2n+1)r_-r^2-(r_-+6r_+)r_-r+3r_-^2r_+]}{4r^4(r-r_-)^2}$$

$$f_H=-\frac{r_-(r-r_+)}{2r^2(r-r_-)},\beta_H=-\frac{2(n+1)}{r_-}$$

$$\omega=\pm\mathrm{i}\beta_H=\mp\frac{2\mathrm{i}(n+1)}{r_-}$$

$$Z_H^+=e^{\beta_H r_*}\left(1-\frac{r_-}{r}\right)^{-\frac{1}{2}}, Z_H^-=e^{-\beta_H r_*}\left(1-\frac{r_-}{r}\right)^{\frac{1}{2}}$$



$$e^{-i\omega t} Z_H^+ = e^{\beta_H(r_* \pm t)} \left(1 - \frac{r_-}{r}\right)^{-\frac{1}{2}}$$

$$e^{-i\omega t} Z_H^- = e^{-\beta_H(r_* \mp t)} \left(1 - \frac{r_-}{r}\right)^{\frac{1}{2}}$$

$$\tilde{Z}_H^+ = \frac{(2\beta_H r_- e^{\beta_H(r-2r_*)} \text{Ei}[1+2\beta_H r_+, 2\beta_H(r-r_*)] - e^{-\beta_H r}) \sqrt{r}}{2\beta_H \sqrt{r-r_-} (r-r_*)^{\beta_H r_+}}$$

$$r_+ - r_- = \frac{r_-(p^3 + [2n(1+b^2) - 2(1-b^2) - 6]p^2 - 32np - 32n^2)}{2(p+2n)(3p+8n)}$$

$$b^2 = 0: \tilde{\Lambda}(p) = (p+2n)[r_-p^2 - 2(r_-+3r_+)p - 16nr_+]$$

$$b^2 = \frac{4n+3}{12(n+1)}: \tilde{\Lambda}(p) = \frac{1}{6}(3p+8n)[2r_-p^2 - 3(r_-+4r_+)p - 24nr_+]$$

$$\text{Fixed : } p = -2n$$

$$\text{Range (i) : } -\frac{8n}{3} < p < 4(1 - \sqrt{n+1}) < 0$$

$$\text{Range (ii) : } 0 < 4(1 + \sqrt{n+1}) < p < \infty$$

$$\text{Fixed : } p = -\frac{8n}{3}$$

$$\text{Range (i) : } -2n < p < \frac{15}{4} \left(1 - \sqrt{1 + \frac{64n}{75}}\right) < 0$$

$$\text{Range (ii) : } 0 < \frac{15}{4} \left(1 + \sqrt{1 + \frac{64n}{75}}\right) < p < \infty.$$

$$b^2 = \frac{(p_z + 2n)[2(\ell + 2) - p_z][p_z + 2(\ell - 1)]}{2p_z^2(n + 1)} > 0 \quad \forall p_z < -2n$$

$$b^2 = \frac{4(4n+3)x^3 + 12(2n+1)x^2 + 9nx}{3(n+1)(3+4x)^2(1+x)}, \text{ with } 0 < x < \infty$$

$$\text{Range (i) : } -\frac{8n}{3} < p < p_a < -2n \text{ where } p_a = -\frac{2n(3+4x)}{3(1+x)},$$

$$\text{Range (ii) : } -2n < p < p_{-,a} < 0 \text{ where } p_{-,a} = \frac{4(3(1+x)(3+5x) - \sqrt{P_a})}{(3+4x)^2},$$

$$\text{Range (iii) : } 0 < p_{+,a} < p < \infty \text{ where } p_{+,a} = \frac{4(3(1+x)(3+5x) + \sqrt{P_a})}{(3+4x)^2},$$

$$\text{where } P_a = 3n(1+x)(3+4x)^3 + [3(1+x)(3+5x)]^2.$$

$$b^2 = \frac{(1+4x)(4nx^2 + 8nx + 4n + 12x + 3)}{12(1+x)^2(n+1)} < \infty, \text{ with } 0 < x < \infty$$



$$\text{Range (i): } -\infty < p_b < p < -\frac{8n}{3} \text{ where } p_b = -\frac{8n}{3}(1+x)$$

$$\text{Range (ii): } -2n < p < p_{-,b} < 0 \text{ where } p_{-,b} = \frac{3(5+8x) - \sqrt{P_b}}{4(1+x)^2}$$

$$\text{Range (iii): } 0 < p_{+,b} < p < \infty \text{ where } p_{+,b} = \frac{3(5+8x) + \sqrt{P_b}}{4(1+x)^2}$$

$$\text{where } P_b = 3n[4(1+x)]^3 + [3(5+8x)]^2$$

$$V_{(p)}^- = -\frac{A}{p} + B$$

$$A = \frac{8n(r-r_+)r_+}{(r-r_-)r^3}$$

$$B = \frac{(r-r_+)[8(n+1)r^3 - 4(2(n+1)r_- + 3r_+)r^2 + (3r_- + 14r_+)r_-r - 5r_-^2r_+]}{4(r-r_-)^2r^4}$$

$$r = r_+(1+z), r_+ = r_-(1+w)$$

$$B = \frac{(r-r_+)r_-^2r_+}{4r^4(r-r_-)^2} [8(n+1)(1+w)^2z^3 + 4(1+w)[4n+1+3(2n+1)w]z^2 + [8n+1+2(16n-1)w+24nw^2]z + 2w[4n-1+2(2n-1)w]]$$

$$r_+ = 2r_-, \ell = 2, b = 5$$

$$V_{(p)}^- = \frac{(r-r_+)}{4r^4(r-r_-)^2} U(r)$$

$$U(r) = 8(n+1)r^3 - 4[(2(n+1)r_- + 3r_+)r^2 + (3r_- + 14r_+)r_-r - 5r_-^2r_+] - \frac{32n(r-r_-)r_+r}{p}$$

$$U(r) \rightarrow -\infty \text{ when } r \rightarrow -\infty$$

$$U(r) \rightarrow +\infty \text{ when } r \rightarrow +\infty$$

$$U(0) = -5r_+r_-^2 < 0, U(r_-) = -3(r_+ - r_-)r_-^2 < 0$$

$$r_1r_2r_3 = \frac{5}{8(n+1)}r_+r_-^2 > 0$$

$$\frac{r_1}{r_+} \frac{r_2}{r_+} \frac{r_3}{r_+} = \frac{5}{8(n+1)} \frac{r_-^2}{r_+^2}$$

$$\frac{r_1}{r_-} = \frac{5}{8(n+1)} \frac{r_+r_-}{r_3r_2}$$

$$U(r) = \frac{(r-r_3)}{r_3(r_3-r_-)} P_2(r)$$

$$P_2(r) = 8(n+1)r_3(r_3-r_-)r^2 - r_-[8(n+1)r_3^2 - (8n+5)r_3r_- + 2r_3r_+ - 5r_-r_+]r + 5(r_3-r_-)r_-^2r_+$$



$$U(r_+) = \frac{2(r_+ - r_-)r_+}{p} [pr_- + (4np - 2p - 16n)r_+],$$

$$p = \frac{ur_- + 16nr_+}{2(2n-1)r_+ + r_-},$$

$$U'(r_+) = \frac{r_-^3}{[2(2n-1)r_+ + r_-]} [48n + (8n+1)u + 2[64n^2 + 72n + (16n-1)u]w + 8n(32n + 28 + 3u)w^2 + 128n(n+1)w^3]$$

$$U(r_+) = 2(r_+ - r_-)r_+ W(r_+), W(r_+) = \frac{pr_- - 2(p + 8n - 2np)r_+}{p}$$

$$U(r_+) = 0 \Rightarrow p = \frac{16nr_+}{2(2n-1)r_+ + r_-}$$

$$b^2 = \frac{[r_- + 2(2n+3)r_+][3r_-^2 - 2r_-r_+ + (n+1)(2n-1)r_+^2]}{16(n+1)r_-r_+[r_- + 2(2n-1)r_+]}$$

$$r_+ = r_-(1+w), n = 2+y$$

$$\frac{\partial b^2}{\partial w} > 0, \forall w \geq 0$$

$$r_+ \longrightarrow r_-$$

$$p = \frac{16n}{4n-1}, b^2 = \frac{(4n+7)(16n^2+8n-7)}{16(n+1)(4n-1)}$$

$$b_{\min} = \{1.80525, 2.53171, 3.23273, 3.93299, \cdots\}$$

$$\mathcal{L} = \sqrt{-g} \left[R - \frac{1}{2} (\partial \phi)^2 - e^{\phi} F^2 - \frac{1}{2} e^{-2\phi} (\partial \chi)^2 + b \chi \tilde{F}^{\mu\nu} F_{\mu\nu} \right]$$

$$V^{\pm} = \pm \partial_{r_*} W + W^2 - \beta^2$$

$$\mathcal{Q} = (\mathbf{p} + \mathrm{i}W)\mathbf{b}$$

$$\{\mathbf{b}^\dagger,\mathbf{b}\}=1,\{\mathbf{b},\mathbf{b}\}=0,\{\mathbf{b}^\dagger,\mathbf{b}^\dagger\}=0, [\mathbf{p},\mathbf{b}]=0, [\mathbf{p},\mathbf{b}^\dagger]=0$$

$$H = \frac{1}{2}(-\partial_{r_*}^2 + V^+ + \beta^2)\mathbf{b}\mathbf{b}^\dagger + \frac{1}{2}(-\partial_{r_*}^2 + V^- + \beta^2)\mathbf{b}^\dagger\mathbf{b}$$

$$H = \frac{1}{2} \begin{pmatrix} -\partial_{r_*}^2 + V^+ + \beta^2 & 0 \\ 0 & -\partial_{r_*}^2 + V^- + \beta^2 \end{pmatrix}$$

$$\mathbf{z} = \begin{pmatrix} Z^+ \\ Z^- \end{pmatrix}$$

$$H\mathbf{Z} = \frac{1}{2}(\omega^2 + \beta^2)\mathbf{Z}$$



$$a = 1, c = -2$$

$$(\partial_{r_*}^2 + \omega^2 + T_1)\Psi + \frac{e^{2\bar{v}+\bar{\phi}}}{r^3}W\Psi = 0, \Psi = \begin{pmatrix} G_{12} \\ B_{03} \\ \chi \end{pmatrix}$$

$$W = \begin{pmatrix} r_- + 3r_+ & -2\sqrt{2n}Q & 0 \\ -2\sqrt{2n}Q & 0 & -2\sqrt{2(n+1)}bQ \\ 0 & -2\sqrt{2(n+1)}bQ & r_- - r_+ \end{pmatrix}$$

$$\frac{S_{i1}}{S_{i2}} = \frac{2\sqrt{n}\sqrt{r_-r_+}}{(r_- + 3r_+ - s_i)}, \frac{S_{i3}}{S_{i2}} = -\frac{4nr_-r_+ + (r_- + 3r_+ - s_i)s_i}{2b\sqrt{n+1}\sqrt{r_-r_+}(r_- + 3r_+ - s_i)},$$

$$s^3-2(r_-+r_+)s^2+(r_-^2-3r_+^2+2r_-r_+[1-2n-2(n+1)b^2])s+4r_-r_+[n(r_- - r_+) + (n+1)b^2(r_- + 3r_+)] = 0$$

$$\frac{S_{i1}}{S_{i2}} = \frac{c_1}{c_2}, \frac{S_{i3}}{S_{i2}} = \frac{c_3}{c_2},$$

$$s=r_-+\left(3+\frac{8n}{p}\right)r_+$$

Cuyos superespacios de Sitter y anti – de Sitter, se calculan así:

$$ds^2_{I,st}=f(\vec{v})\text{AdS}_{d+1}+g_{ij,(9-d)}(\vec{v})dv^i dv^j, \Phi(\vec{v})$$

$$\text{AdS}_{d+1}=\frac{u^2}{l^2}(\lambda dt^2+d\vec{x}_{d-2}^2+dy^2)+\frac{l^2 du^2}{u^2}$$

$$ds^2_{II,st}=f(\vec{v},u)[s_1(u)(\lambda dt^2+d\vec{x}_{d-3}^2+dy^2)+s_2(u)du^2+s_3(u)d\phi^2]+$$

$$+g_{ij,(9-d)}(\vec{v},u)(dv^i-A^i)(dv^j-A^j), \Phi(\vec{v},u)$$

$$A^i=s_{4,i}(u)d\phi$$

$$ds^2_{III,st}=\hat{h}(u)^{-\frac{1}{2}}(\lambda dt^2+dy^2+d\vec{x}_{d-2}^2)+\hat{h}(u)^{\frac{1}{2}}e^{2k(u)}du^2+g_{ij,(9-d)}(u,\vec{v})dv^i dv^j, \Phi(u)$$

$$\Sigma_{8,I}=\Sigma_{8,III}=[u,\vec{x}_{d-2},\vec{v}_{9-d}], \Sigma_{8,II}=[u,\vec{x}_{d-3},\phi,\vec{v}_{9-d}]$$

$$t(u), y(u)$$

$$ds^2_{\Sigma_{8,I}}=f(\vec{v})\left(du^2\left[\frac{l^2}{u^2}+\frac{u^2}{l^2}(\lambda t'^2+y'^2)\right]+\frac{u^2}{l^2}d\vec{x}_{d-2}^2\right)+g_{ij,(9-d)}(\vec{v})dv^i dv^j$$

$$ds^2_{\Sigma_{8,II}}=f(\vec{v},u)(du^2[s_2(u)+s_1(u)(\lambda t'^2+y'^2)]+s_1(u)d\vec{x}_{d-3}^2+s_3(u)d\phi^2)+g_{ij,(9-d)}(\vec{v},u)(dv^i-A^i)(dv^j-A^j)$$

$$ds^2_{\Sigma_{8,III}}=du^2\left[\hat{h}(u)^{\frac{1}{2}}e^{2k}+\hat{h}(u)^{-\frac{1}{2}}(\lambda t'^2+y'^2)\right]+\hat{h}(u)^{-\frac{1}{2}}d\vec{x}_{d-2}^2+g_{ij,(9-d)}(\vec{v},u)dv^i dv^j$$

$$4G_{10}S_{EE}=\int_{\Sigma_8}\sqrt{e^{-4\Phi}\det[g_{ind,\Sigma_8}]}$$



$$e^{-4\Phi}\det[g_{\Sigma_{8,I}}]=e^{-4\Phi(\vec{v})}f(\vec{v})^{d-1}\det[g_{9-d}]\left[\left(\frac{u}{l}\right)^{2(d-3)}+\left(\frac{u}{l}\right)^{2(d-1)}(y'^2+\lambda t'^2)\right]$$

$$4G_{10}S_{EE,I}=\mathcal{N}_I\int du\sqrt{F^2(u)(y'^2+\lambda t'^2)+G^2(u)},$$

$$F(u)=\left(\frac{u}{l}\right)^{(d-1)},G(u)=\left(\frac{u}{l}\right)^{(d-3)},\mathcal{N}_I=\int d^{9-d}vd^{d-2}x\sqrt{e^{-4\Phi(\vec{v})}f(\vec{v})^{d-1}\det[g_{9-d}]}$$

$$e^{-4\Phi}\det[g_{\Sigma_{8,II}}]=e^{-4\Phi(\vec{v},u)}f(\vec{v},u)^{d-1}\det[g_{9-d}(\vec{v},u)]s_3(u)s_1(u)^{d-3}[s_1(u)(\lambda t'^2+y'^2)+s_2(u)]$$

$$e^{-4\Phi}\det[g_{\Sigma_{8,III}}]=e^{-4\Phi(u)}\det[g_{9-d}(\vec{v},u)]\hat{h}(u)^{\frac{(1-d)}{2}}[\lambda t'^2+y'^2+e^{2k}\hat{h}(u)]$$

$$4G_{10}S_{EE}=\mathcal{N}\int_{u_0}^{\infty}du\sqrt{F^2(u)(y'^2+\lambda t'^2)+G^2(u)}$$

$$\frac{F^2t'}{\sqrt{G^2+F^2(\lambda t'^2+y'^2)}}=\lambda c_t,\frac{F^2y'}{\sqrt{G^2+F^2(\lambda t'^2+y'^2)}}=c_y$$

$$t'^2(u)=\frac{c_t^2G^2(u)}{F^2(u)(F^2(u)-F^2(u_0))},y'^2(u)=\frac{c_y^2G^2(u)}{F^2(u)(F^2(u)-F^2(u_0))}$$

$$F^2(u_0)=c_y^2+\lambda c_t^2$$

$$T=2c_t\int_{u_0}^{\infty}du\frac{G(u)}{F(u)\sqrt{F^2(u)-F^2(u_0)}},Y=2c_y\int_{u_0}^{\infty}du\frac{G(u)}{F(u)\sqrt{F^2(u)-F^2(u_0)}}$$

$$\frac{4G_{10}}{\mathcal{N}}S_{tEE}=2\int_{u_0}^{\infty}du\frac{G(u)F(u)}{\sqrt{F^2(u)-F^2(u_0)}}-2\int_{u_*}^{\infty}G(u)du$$

$$T_{app}(u_0)=\pi\frac{G(u_0)}{F'(u_0)}$$

$$\frac{dS(u_0)}{dY(u_0)}=F(u_0).$$

$$\frac{dS(u_0)}{dT(u_0)}\approx\frac{dS_{\rm approx}(u_0)}{dT_{\rm app}(u_0)}\approx F(u_0),$$

$$S_{\rm approx}(u_0)=\int F(u_0)dT_{\rm app}(u_0)=\pi\int F(u_0)\frac{d}{du_0}\left(\frac{G(u_0)}{F'(u_0)}\right)du_0$$

$$ds_{I,st}^2=f(\vec{v})\mathrm{AdS}_{d+1}+g_{ij,(9-d)}(\vec{v})dv^i dv^j,\Phi(\vec{v})$$

$$\mathrm{AdS}_{d+1}=\frac{u^2}{l^2}(\lambda dt^2+d\vec{x}_{d-2}^2+dy^2)+\frac{l^2du^2}{u^2}$$

or,

$$AdS_{d+1}=u^2\left(\lambda dt^2+t^2d\Omega_{d-2}^{(\lambda)}+dy^2\right)+\frac{du^2}{u^2}$$



$$T = 2c_t l^{d+1} \int_{u_0}^{\infty} \frac{du}{u^2} \frac{1}{\sqrt{u^{2(d-1)} - u_0^{2(d-1)}}}, Y = 2c_y l^{d+1} \int_{u_0}^{\infty} \frac{du}{u^2} \frac{1}{\sqrt{u^{2(d-1)} - u_0^{2(d-1)}}}$$

$$\frac{4G_{10}}{\mathcal{N}} S_{EE} = \frac{2}{l^{d-3}} \left[\int_{u_0}^{\infty} du \frac{u^{2d-4}}{\sqrt{u^{2d-2} - u_0^{2d-2}}} - \int_0^{\infty} u^{d-3} du \right]$$

$$u_0 = (c_y^2 - c_t^2)^{\frac{1}{2(d-1)}}$$

$$u_0 = e^{\frac{i\pi}{2(d-1)}} \left[\sqrt{|c_y^2 - c_t^2|} \right]^{\frac{1}{d-1}} = (i\tilde{u}_0)^{\frac{1}{d-1}}$$

$$T = \frac{2c_t l^{d+1}}{u_0^d} I_1, Y = \frac{2c_y l^{d+1}}{u_0^d} I_1$$

$$I_1 = \int_0^1 dr \frac{r^{d-1}}{\sqrt{1-r^{2(d-1)}}} = \frac{\sqrt{\pi} \Gamma\left(\frac{d}{2d-2}\right)}{\Gamma\left(\frac{1}{2(d-1)}\right)}$$

$$\frac{2G_{10}l^{d-3}}{\mathcal{N}u_0^{d-2}} S_{EE} = I_2 - I_3,$$

$$I_2 = \int_{\epsilon}^1 \frac{dr}{r^{d-1} \sqrt{1-r^{2d-2}}} = -\frac{1}{(d-2)r^{d-2}} {}_2F_1\left(\frac{1}{2}, \frac{2-d}{2d-2}; \frac{d}{2(d-1)}; r^{2d-2}\right) \Big|_{\epsilon}^1$$

$$I_3 = \int_{\epsilon}^{\infty} \frac{dr}{r^{d-1}} = \frac{1}{(d-2)\epsilon^{d-2}}$$

$$\frac{2G_{N,10}l^{d-3}}{\mathcal{N}u_0^{d-2}} S_{EE} = \frac{1}{(2-d)} {}_2F_1\left(\frac{1}{2}, \frac{2-d}{2d-2}; \frac{d}{2d-2}; 1\right)$$

$$c_t = \frac{T}{2l^{d+1}I_1} u_0^d, c_y = \frac{Y}{2l^{d+1}I_1} u_0^d, u_0 = \frac{2l^{d+1}I_1}{\sqrt{Y^2 - T^2}}$$

$$S_{EE} = \frac{\mathcal{N}(I_2 - I_3)2^{d-3}l^{(d-1)^2}I_1^{d-2}}{G_{10}} \times \frac{1}{(Y^2 - T^2)^{\frac{(d-2)}{2}}}$$

$$S_{EE} = \frac{\mathcal{N}(I_2 - I_3)2^{d-3}l^{(d-1)^2}I_1^{d-2}}{G_{10}} \times \frac{e^{-i\pi\frac{(d-2)}{2}}}{|Y^2 - T^2|^{\frac{(d-2)}{2}}}$$

$$T_{app} = \pi \frac{G(u_0)}{F'(u_0)} = \frac{\pi}{(d-1)\sqrt{\lambda}u_0}$$

$$S_{EE, \text{ approx }} = \int^{u_0} d dz F(z) T'_{app}(z) = -\frac{\pi}{(d^2 - 3d + 2)} u_0^{d-2}$$



$$S_{EE, \text{ approx}} = - \frac{\pi^{d-1}}{\sqrt{\lambda^{d-2}}(d-1)^{d-2}(d^2-3d+2)} \frac{1}{|T_{app}|^{d-2}}$$

$$ds_8^2|_{\Sigma_8} = f(\vec{v})[1+\lambda u^4 t'^2(u)]\frac{du^2}{u^2} + f(\vec{v})u^2 t^2 d\Omega_{d-2}^{(\lambda)} + g_{ij}(\vec{v})dv^i dv^j$$

$$S_{EE}^{(\lambda)}[\Sigma_8] = \frac{1}{4G_{10}} \int d^8x \sqrt{e^{-4\Phi} \det g_8} = \frac{\hat{\mathcal{N}}}{4G_{10}} \int du u^{d-3} t^{d-2} \sqrt{1+\lambda u^4 t'^2(u)}$$

$$\hat{\mathcal{N}} = \text{Vol}\left(\Omega_{d-2}^{(\lambda)}\right) \int d^{9-d}v \sqrt{e^{-4\Phi} \det g_{ij}} f^{\frac{d-1}{2}}(\vec{v})$$

$$\lambda u^3 t(u)((d-1)\lambda u^4 t'(u)^3 + (d+1)t'(u) + ut''(u)) - (d-2)(\lambda u^4 t'(u)^2 + 1) = 0$$

$$t(u) = \frac{\sqrt{R^2 u^2 - \lambda}}{u}, \lambda = \pm 1$$

$$\frac{4G_{10}}{\hat{\mathcal{N}}\sqrt{\lambda^{d-2}}}S_{EE}^{(\lambda)}[\Sigma_8] = \int_1^{\frac{R}{\sqrt{\lambda}\epsilon}} dx (x^2-1)^{\frac{d-3}{2}}$$

$$\frac{4G_{10}}{\hat{\mathcal{N}}\sqrt{\lambda^{d-2}}}S_{EE}^{(\lambda)}[\Sigma_8] = \sum_{j=0}^{\left[\frac{d-3}{2}\right]} \frac{\left(\frac{3-d}{2}\right)_j}{j!(d-2j-2)} \left(\frac{R}{\sqrt{\lambda}\epsilon}\right)^{d-2j-2} - (-1)^{\frac{d+1}{2}} \frac{\sqrt{\pi}\Gamma\left(\frac{(d-1)}{2}\right)}{2\Gamma\left(\frac{d}{2}\right)}$$

$$\frac{4G_{10}}{\hat{\mathcal{N}}\sqrt{\lambda^{d-2}}}S_{EE}^{(\lambda)}[\Sigma_8] = \sum_{j=0}^{\left[\frac{d-3}{2}\right]} \frac{\left(\frac{3-d}{2}\right)_j}{j!(d-2j-2)} \left(\frac{R}{\sqrt{\lambda}\epsilon}\right)^{d-2j-2}$$

$$- \frac{\Gamma\left(\frac{d-1}{2}\right)}{\Gamma\left(\frac{d}{2}\right)} \frac{(-1)^{d/2}}{\sqrt{\pi}} \left(\log\left(\frac{2R}{\epsilon\sqrt{\lambda}}\right) + \frac{1}{2}\mathcal{H}_{\frac{d-2}{2}} \right)$$

$$(d-2)!!\,c_{LM,odd}=(R\partial_R-1)\cdots(R\partial_R-d+2)S_{EE}^{(\lambda)}[\Sigma_8].$$

$$(d-2)!!\,c_{LM,\text{ even}}=R\partial_R\cdots(R\partial_R-d+2)S_{EE}^{(\lambda)}[\Sigma_8]$$

$$c_{slab}=\kappa\frac{T^{d-2}}{L^{d-2}}T\partial_TS_{EE}^{(\lambda)}[\Sigma_8]$$

$$c_{slab} = \frac{\mathcal{N}}{4G_{10}} \frac{\kappa}{L^{d-2}} \left(\frac{2\sqrt{\pi}\Gamma\left(\frac{d}{2d-2}\right)}{\Gamma\left(\frac{1}{2(d-1)}\right)} \right)^{d-1} \frac{1}{\sqrt{\lambda^{d-2}}}$$

$$F_i=2N_i-N_{i-1}-N_{i+1},$$

$$G_{\text{global}}=SU(F_1)\times...\times SU(F_p), G_{\text{local}}=SU(N_1)\times...\times SU(N_p).$$



$$\mathcal{R}(\eta) = \begin{cases} N_1\eta & 0 \leq \eta \leq 1 \\ N_l + (N_{l+1} - N_l)(\eta - l) & l \leq \eta \leq l+1, l = 1, \dots, P-2 \\ N_{P-1}(P - \eta) & (P-1) \leq \eta \leq P. \end{cases}$$

$$\mathcal{R}''(\eta) = \sum_{i=1}^P F_i \delta(\eta - i).$$

$$R_k = \frac{2}{P} \int_0^P \mathcal{R}(\eta) \sin\left(\frac{k\pi\eta}{P}\right) d\eta = \frac{2P}{\pi^2 k^2} \sum_{j=1}^{P-1} F_j \sin\left(\frac{k\pi j}{P}\right).$$

$$ds_{10}^2 = f_1[ds_{AdS_4}^2 + f_2 d\Omega_2(\theta, \phi) + f_3 d\tilde{\Omega}_2(\tilde{\theta}, \tilde{\phi}) + f_4(d\sigma^2 + d\eta^2)]$$

$$ds_{AdS_4}^2 = u^2(\lambda dt^2 + dx^2 + dy^2) + \frac{du^2}{u^2}, e^{-4\Phi} = f_5^2$$

$$f_1 = \frac{\pi}{2} \sqrt{\frac{\sigma^3 \partial_{\sigma\eta}^2 V}{\partial_{\sigma}(\sigma \partial_{\eta} V)}}, f_2 = -\frac{\partial_{\eta} V \partial_{\sigma}(\sigma \partial_{\eta} V)}{\sigma \Lambda}, f_3 = \frac{\partial_{\sigma}(\sigma \partial_{\eta} V)}{\sigma \partial_{\sigma\eta}^2 V}$$

$$f_4 = -\frac{\partial_{\sigma}(\sigma \partial_{\eta} V)}{\sigma^2 \partial_{\eta} V}, f_5 = -16\Lambda \frac{\partial_{\eta} V}{\partial_{\sigma\eta}^2 V}, \Lambda = \partial_{\eta} V \partial_{\sigma\eta}^2 V + \sigma(\partial_{\sigma\eta}^2 V)^2 + \sigma(\partial_{\eta}^2 V)^2$$

$$V(\sigma,\eta)=\frac{1}{\sigma}\sum_{k=1}^{\infty}R_k\cos\left(\frac{k\pi\eta}{P}\right)e^{-\frac{k\pi|\sigma|}{P}}$$

$$ds_8^2 = \frac{f_1}{u^2} (1 - \lambda u^4 t'^2(u)) du^2 + f_1 u^2 dx_1^2 + f_1 f_2 d\Omega_2(\theta, \phi) + f_1 f_3 d\tilde{\Omega}_2(\tilde{\theta}, \tilde{\phi}) + f_1 f_4 (d\sigma^2 + d\eta^2)$$

$$\frac{4G_{10}}{\mathcal{N}}S_{EE}[\Sigma_8]=\begin{cases} -\frac{4\pi\Gamma\left(\frac{3}{4}\right)^2}{\Gamma\left(\frac{1}{4}\right)^2}\frac{1}{|T|} & (\lambda=+1) \\ \frac{4\pi i\Gamma\left(\frac{3}{4}\right)^2}{\Gamma\left(\frac{1}{4}\right)^2}\frac{1}{|T|} & (\lambda=-1) \end{cases}$$

$$\mathcal{N}=-16\pi^6L_{x_1}\int_0^\infty d\sigma\int_0^P d\eta\sigma^2\partial_\eta V\partial_\sigma(\sigma\partial_\eta V)$$

$$T_{app} = \begin{cases} \frac{\pi}{2u_0} & (\lambda = +1) \\ \frac{-i\pi}{2u_0} & (\lambda = -1) \end{cases}$$

$$S_{EE,\text{approx}} = \begin{cases} -\frac{\pi^2}{4}\frac{1}{|T_{app}|} & (\lambda = +1) \\ \frac{i\pi^2}{4}\frac{1}{|T_{app}|} & (\lambda = -1) \end{cases}$$

$$\frac{4G_{10}}{\hat{\mathcal{N}}}S_{EE}^{(\lambda)}[\Sigma_8]=\begin{cases}\frac{R}{\epsilon}-1 & (\lambda=+1) \\ \frac{R}{\epsilon}-i & (\lambda=-1)\end{cases}$$

$$\hat{\mathcal{N}}=-16\pi^6\mathrm{Vol}\left(\Omega_1^{(\lambda)}\right)\int_0^\infty d\sigma\int_0^P d\eta\sigma^2\partial_\eta V\partial_\sigma(\sigma\partial_\eta V)$$

$$c_{LM}=\frac{\hat{\mathcal{N}}}{4G_{10}},c_{\mathrm{slab}}=\frac{\mathcal{N}}{4G_{10}}\frac{\kappa}{L}\frac{4\pi\Gamma\left(\frac{3}{4}\right)^2}{\Gamma\left(\frac{1}{4}\right)^2}$$

$$c_{LM}\propto c_{slab}\propto c_{hol}\sim\sum_{k=1}^{\infty}kR_k^2$$

$$ds_{10}^2=\sqrt{f_1^3f_5}[4ds_{AdS_5}^2+f_2d\Omega_2(\theta,\phi)+f_3d\chi^2+f_4(d\sigma^2+d\eta^2)]\\ ds_{AdS_5}^2=u^2(\lambda dt^2+dx_1^2+dx_2^2+dy^2)+\frac{du^2}{u^2}\\ e^{-4\Phi}=(f_1f_5)^{-3}$$

$$f_1^3=\frac{\dot{V}\Delta}{2V''},f_2=\frac{2V''\dot{V}}{\Delta},f_3=\frac{4\sigma^2V''}{2\dot{V}-\ddot{V}},f_4=\frac{2V''}{\dot{V}},f_5=\frac{2(2\dot{V}-\ddot{V})}{\dot{V}\Delta}\\ \Delta=(2\dot{V}-\ddot{V})V''+(\dot{V}')^2,\dot{V}=\sigma\partial_\sigma V,V''=\partial_\eta^2V$$

$$V(\sigma,\eta)=-\sum_{k=1}^{\infty}R_k\sin\left(\frac{k\pi\eta}{P}\right)K_0\left(\frac{k\pi\sigma}{P}\right)$$

$$\frac{4G_{10}}{\mathcal{N}}S_{EE}\left[\Sigma_8^{(\lambda)}\right]=\begin{cases}-\frac{4\pi^{3/2}\Gamma\left(\frac{2}{3}\right)^3}{\Gamma\left(\frac{1}{6}\right)^3}\frac{1}{|T|^2} & (\lambda=+1) \\ \frac{4\pi^{3/2}\Gamma\left(\frac{2}{3}\right)^3}{\Gamma\left(\frac{1}{6}\right)^3}\frac{1}{|T|^2} & (\lambda=-1)\end{cases}$$

$$\mathcal{N}=256\pi^2L_{x_1}L_{x_2}\int_0^\infty d\sigma\int_0^P d\eta\sigma\dot{V}V''$$

$$T_{app}=\begin{cases}\frac{\pi}{3u_0} & (\lambda=+1) \\ \frac{-i\pi}{3u_0} & (\lambda=-1)\end{cases}$$



$$S_{EE, \text{ approx}} = \begin{cases} -\frac{\pi^3}{54} \frac{1}{|T_{app}|^2} & (\lambda = +1) \\ \frac{\pi^3}{54} \frac{1}{|T_{app}|^2} & (\lambda = -1) \end{cases}$$

$$\frac{4G_{10}}{\hat{\mathcal{N}}} S_{EE} \left[\hat{\Sigma}_8^{(\lambda)} \right] = \begin{cases} \frac{R^2}{2\epsilon^2} - \frac{1}{2} \log \left(\frac{2R}{\epsilon} \right) - \frac{1}{4} & (\lambda = +1) \\ \frac{R^2}{2\epsilon^2} + \frac{1}{2} \log \left(\frac{2R}{\epsilon} \right) + \frac{1}{4} (1 - i\pi) & (\lambda = -1) \end{cases}$$

$$\hat{\mathcal{N}} = 256\pi^2 \text{Vol} \left(\Omega_2^{(\lambda)} \right) \int_0^\infty d\sigma \int_0^P d\eta \sigma \dot{V} V''$$

$$c_{LM} = \frac{\hat{\mathcal{N}}}{8G_{10}}, c_{slab} = \frac{\mathcal{N}}{4G_{10}} \frac{\kappa}{L_{x_1} L_{x_2}} \frac{8\pi^{3/2} \Gamma\left(\frac{2}{3}\right)^3}{\Gamma\left(\frac{1}{6}\right)^3}$$

$$c_{LM} \propto c_{slab} \propto c_{hol} \sim P \sum_{k=1}^{\infty} R_k^2$$

$$\begin{aligned} ds_{10}^2 &= f_1 [ds_{AdS_6}^2 + f_2 d\Omega_2(\theta, \phi) + f_3 (d\sigma^2 + d\eta^2)] \\ ds_{AdS_6}^2 &= u^2 (\lambda dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dy^2) + \frac{du^2}{u^2} \\ e^{-4\Phi} &= f_6^2 \end{aligned}$$

$$\begin{aligned} f_1 &= \frac{2}{3} \left(\frac{\sigma(\sigma \partial_\eta^2 V + 3 \partial_\sigma V)}{\partial_\eta^2 V} \right)^{1/2}, f_2 = \frac{\partial_\sigma V \partial_\eta^2 V}{3 \tilde{\Lambda}}, f_3 = \frac{\partial_\eta^2 V}{3 \sigma \partial_\sigma V} \\ f_6 &= 18^2 \frac{3 \sigma^2 \partial_\sigma V \partial_\eta^2 V \tilde{\Lambda}}{(3 \partial_\sigma V + \sigma \partial_\eta^2 V)^2}, \tilde{\Lambda} = \sigma (\partial_{\sigma \eta}^2 V)^2 + \partial_\eta^2 V (\partial_\sigma V - \sigma \partial_\sigma^2 V) \end{aligned}$$

$$V(\sigma,\eta)=-\frac{1}{\sigma}\sum_{k=1}^{\infty}\frac{P}{2\pi k}R_k\sin\left(\frac{k\pi\eta}{P}\right)e^{-\frac{k\pi|\sigma|}{P}}$$

$$\frac{4G_{10}}{\mathcal{N}} S_{EE} \left[\Sigma_8^{(\lambda)} \right] = \begin{cases} -\frac{16\pi^2 \Gamma\left(\frac{5}{8}\right)^4}{3\Gamma\left(\frac{1}{8}\right)^4} \frac{1}{|T|^3} & (\lambda = +1) \\ -\frac{16i\pi^2 \Gamma\left(\frac{5}{8}\right)^4}{3\Gamma\left(\frac{1}{8}\right)^4} \frac{1}{|T|^3} & (\lambda = -1) \end{cases}$$

$$\mathcal{N} = \frac{2^8\pi}{3} L_{x_1} L_{x_2} L_{x_3} \int_0^\infty d\sigma \int_0^P d\eta \sigma^3 \partial_\sigma V \partial_\eta^2 V$$



$$T_{app} = \begin{cases} \frac{\pi}{4u_0} & (\lambda = +1) \\ -i\pi & (\lambda = -1) \end{cases}$$

$$S_{EE, \text{approx}} = \begin{cases} -\frac{\pi^4}{768} \frac{1}{|T_{app}|^3} & (\lambda = +1) \\ -\frac{i\pi^4}{768} \frac{1}{|T_{app}|^3} & (\lambda = -1) \end{cases}$$

$$\frac{4G_{10}}{\hat{\mathcal{N}}} S_{EE} \left[\hat{\Sigma}_8^{(\lambda)} \right] = \begin{cases} \left(\frac{R^3}{3\epsilon^3} - \frac{R}{\epsilon} + \frac{2}{3} \right) & (\lambda = +1) \\ \left(\frac{R^3}{3\epsilon^3} + \frac{R}{\epsilon} - i\frac{2}{3} \right) & (\lambda = -1) \end{cases}$$

$$\hat{\mathcal{N}} = \frac{2^8\pi}{3} \text{Vol}\left(\Omega_3^{(\lambda)}\right) \int_0^\infty d\sigma \int_0^P d\eta \sigma^3 \partial_\sigma V \partial_\eta^2 V$$

$$c_{LM} = \frac{\hat{\mathcal{N}}}{4G_{10}} \frac{2}{3}, c_{\text{slab}} = \frac{\mathcal{N}}{4G_{10}} \frac{\kappa}{L_{x_1} L_{x_2} L_{x_3}} \frac{16\pi^2 \Gamma\left(\frac{5}{8}\right)^4}{\Gamma\left(\frac{1}{8}\right)^4}$$

$$c_{LM} \propto c_{slab} \propto c_{hol} \sim P^2 \sum_{k=1}^{\infty} \frac{R_k^2}{k}$$

$$ds_{10}^2 = f_1 ds_{AdS_7}^2 + f_2 d\eta^2 + f_3 d\Omega_2(\theta,\phi)$$

$$ds_{AdS_6}^2 = u^2 (\lambda dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 + dy^2) + \frac{du^2}{u^2}$$

$$e^{-4\Phi} = f_6^{-4}$$

$$f_1=8\sqrt{2}\pi\sqrt{-\frac{V}{V''}}, f_2=\sqrt{2}\pi\sqrt{-\frac{V''}{V}}$$

$$f_3=f_2\frac{V^2}{(V'^2-2VV'')}, f_6^4=\frac{2^5\pi^{10}3^{16}\left(-\frac{V}{V''}\right)^3}{(V'^2-2VV'')^2}$$

$$V''' = -162\pi^3 F_0$$

$$\frac{4G_{10}}{\mathcal{N}} S_{EE} \left[\Sigma_8^{(\lambda)} \right] = \begin{cases} -\frac{8\pi^{5/2}\Gamma\left(\frac{3}{5}\right)^5}{\Gamma\left(\frac{1}{10}\right)^5} \frac{1}{|T|^4} & (\lambda = +1) \\ -\frac{8\pi^{5/2}\Gamma\left(\frac{3}{5}\right)^5}{\Gamma\left(\frac{1}{10}\right)^5} \frac{1}{|T|^4} & (\lambda = -1) \end{cases}$$

$$\mathcal{N} = \frac{4}{3} \left(\frac{2}{3}\right)^7 L_{x_1} \ldots L_{x_4} \int_0^P (-V''V) d\eta$$



$$T_{app} = \begin{cases} \frac{\pi}{5u_0} & (\lambda = +1) \\ -i\pi & (\lambda = -1) \\ \frac{\pi}{5u_0} & (\lambda = -1) \end{cases}$$

$$S_{EE, \text{ approx}} = \begin{cases} -\frac{\pi^5}{12500} \frac{1}{|T_{app}|^4} & (\lambda = +1) \\ -\frac{\pi^5}{12500} \frac{1}{|T_{app}|^4} & (\lambda = -1) \end{cases}$$

$$\frac{4G_{10}}{\hat{\mathcal{N}}} S_{EE}^{(\lambda)} \left[\hat{\Sigma}_8^{(\lambda)} \right] = \begin{cases} \frac{R^4}{4\epsilon^4} - \frac{3}{4} \frac{R^2}{\epsilon^2} + \frac{3}{8} \left(\log \left(\frac{2R}{\epsilon} \right) + \frac{3}{4} \right) & (\lambda = +1) \\ \frac{R^4}{4\epsilon^4} + \frac{3}{4} \frac{R^2}{\epsilon^2} + \frac{3}{8} \left(\log \left(\frac{2R}{\epsilon} \right) - \frac{i\pi}{2} + \frac{3}{4} \right) & (\lambda = -1) \end{cases}$$

$$\hat{\mathcal{N}} = \frac{4}{3} \left(\frac{2}{3} \right)^7 \text{Vol} \left(\Omega_4^{(\lambda)} \right) \int_0^P (-V''V) d\eta$$

$$c_{LM} = \frac{\hat{\mathcal{N}}}{4G_{10}} \frac{3}{8}, c_{slab} = \frac{\mathcal{N}}{4G_{10}} \frac{\kappa}{L_{x_1} \dots L_{x_4}} \frac{32\pi^{5/2} \Gamma\left(\frac{3}{5}\right)^5}{\Gamma\left(\frac{1}{10}\right)^5}$$

$$c_{LM} \propto c_{slab} \propto c_{hol} \sim P^3 \sum_{k=1}^{\infty} \frac{R_k^2}{k^2}$$

$$ds^2 = \frac{\hat{G}}{\sqrt{\hat{h}_4 h_8}} \left(ds^2(\text{AdS}_3) + \frac{h_8 \hat{h}_4}{4 h_8 \hat{h}_4 + (\hat{G}')^2} ds^2(S^2) \right) + \sqrt{\frac{\hat{h}_4}{h_8}} ds^2(\text{CY}_2) + \frac{\sqrt{\hat{h}_4 h_8}}{\hat{G}} d\rho^2$$

$$e^{-\Phi} = \frac{h_8^{\frac{3}{4}}}{2\hat{h}_4^{\frac{1}{4}}\sqrt{\hat{G}}} \sqrt{4h_8\hat{h}_4 + (\hat{G}')^2} \cdot ds^2(\text{AdS}_3) = \frac{u^2}{l^2} (-dt^2 + dy^2) + \frac{l^2 du^2}{u^2}$$

$$ds_{\Sigma_8}^2 = \frac{\hat{G}}{\sqrt{\hat{h}_4 h_8}} \left(du^2 \left[\frac{l^2}{u^2} + \frac{u^2}{l^2} (y'^2 - t'^2) \right] + \frac{h_8 \hat{h}_4}{4 h_8 \hat{h}_4 + (\hat{G}')^2} ds^2(S^2) \right) + \sqrt{\frac{\hat{h}_4}{h_8}} ds^2(\text{CY}_2) + \frac{\sqrt{\hat{h}_4 h_8}}{\hat{G}} d\rho^2$$

$$e^{-4\Phi} \text{det}[g_{\Sigma_8}] = \frac{\hat{h}_4^2 h_8^2}{16} \text{Vol}_{S^2} \text{Vol}_{\text{CY}_2} \left[\frac{l^2}{u^2} + \frac{u^2}{l^2} (y'^2 - t'^2) \right]$$

$$\frac{4G_{10}}{\mathcal{N}} S_{EE} = \int_{u_0}^{\infty} du \sqrt{\frac{l^2}{u^2} + \frac{u^2}{l^2} (y'^2 - t'^2)}$$

$$\mathcal{N} = \pi \text{Vol}_{\text{CY}_2} \int_0^P \hat{h}_4 h_8 d\rho$$



$$\frac{T}{2c_t l^3} = \frac{Y}{2c_y l^3} = \int_{u_0}^{\infty} \frac{du}{u^2 \sqrt{u^2 - u_0^2}} = \frac{1}{u_0^2}$$

$$\frac{4G_{10}}{\mathcal{N}} S_{EE} = \lim_{\Lambda \rightarrow \infty} \int_{u_0}^{\Lambda} \frac{du}{\sqrt{u^2 - u_0^2}} - \int_{\frac{1}{\Lambda}}^{\Lambda} \frac{du}{u} = \log \left[\frac{\sqrt{Y^2 - T^2}}{\Lambda l^3} \right]$$

$$ds_{10}^2 = \frac{u^2}{l^2} [\lambda dt^2 + dy^2 + dx^2 + f(u) d\phi^2] + \frac{l^2 du^2}{f(u) u^2} + l^2 d\tilde{\Omega}_5^2$$

$$d\tilde{\Omega}_5^2 = d\theta^2 + \sin^2 \theta d\psi^2 + \sin^2 \theta \sin^2 \psi (d\varphi_1 - A_1)^2 + \sin^2 \theta \cos^2 \psi (d\varphi_2 - A_1)^2 + \cos^2 \theta (d\varphi_3 - A_1)^2$$

$$A_1 = Q \left(1 - \frac{l^2 Q^2}{u^2} \right) d\phi, f(u) = 1 - \left(\frac{Ql}{u} \right)^6$$

$$ds_8^2 = \left[\frac{u^2}{l^2} (\lambda t'^2 + y'^2) + \frac{l^2}{u^2 f(u)} \right] du^2 + \frac{u^2}{l^2} dx^2 + \frac{u^2}{l^2} f(u) d\phi^2 + l^2 d\tilde{\Omega}_5^2$$

$$S_{EE}^{(\lambda)} = \frac{\hat{\mathcal{N}}}{4G_{10}} \int_{u_0}^{\infty} \sqrt{G^2(u) + F^2(u) (\lambda t'^2(u) + y'^2(u))}$$

$$G(u) = \frac{u}{l}, F(u) = \frac{u^3}{l^3} \sqrt{f(u)}, \hat{\mathcal{N}} = l^5 L_x L_\phi \text{Vol}(\tilde{S}^5), L_\phi = \frac{1}{3Q}$$

$$\frac{T}{2l^5 c_t} = \frac{Y}{2l^5 c_y} = \int_{u_0}^{\infty} \frac{u du}{\sqrt{u^6 - u_\Lambda^6} \sqrt{u^6 - u_0^6}}$$

$$\frac{2lG_{10}}{\hat{\mathcal{N}}} S_{EE} = \int_{u_0}^{\infty} du \frac{u \sqrt{u^6 - u_\Lambda^6}}{\sqrt{u^6 - u_0^6}} - \int_{u_\Lambda}^{\infty} u du$$

$$u_0 = l(Q^6 + c_y^2 - c_t^2)^{1/6}$$

$$u_0 = l e^{i\pi/6} |Q^6 + c_y^2 - c_t^2|^{1/6}$$

$$r = \frac{u_0}{u}, \gamma = \frac{u_\Lambda}{u_0}$$

$$J_1 = \int_0^1 dr \frac{r^3}{\sqrt{(1-r^6)(1-\gamma^6 r^6)}} = \frac{\sqrt{\pi} \Gamma(\frac{5}{3})}{4\Gamma(\frac{7}{6})} {}_2F_1\left[\frac{1}{2}, \frac{2}{3}, \frac{7}{6}, \gamma^6\right]$$

$$J_2 = \int_{\epsilon}^1 \frac{dr}{r^3} \sqrt{\frac{1-\gamma^6 r^6}{1-r^6}} = \mathcal{J}_2 + \frac{1}{2\epsilon^2} + O(\epsilon^4)$$

$$\mathcal{J}_2 = -\frac{\sqrt{\pi} \Gamma(\frac{2}{3})}{2\Gamma(\frac{1}{6})} {}_2F_1\left[-\frac{1}{2}, -\frac{1}{3}, \frac{1}{6}, \gamma^6\right], J_3 = \int_{\epsilon}^{1/\gamma} \frac{dr}{r^3} = \frac{1}{2\epsilon^2} - \frac{\gamma^2}{2}$$

$$c_t = \frac{Tu_0^4}{2J_1 l^5}, c_y = \frac{Yu_0^4}{2J_1 l^5}, \frac{2lG_{10}}{\hat{\mathcal{N}}u_0^2} S_{EE} = J_2 - J_3$$



$$\Delta^2 = (1 - \gamma^6) \frac{4l^4 J_1^2}{u_0^2}, S_{EE} = \frac{\hat{\mathcal{N}}}{2lG_N} (J_2 - J_3) u_0^2$$

$$ds^2 = \frac{\zeta(u, \theta)}{L^2} \left[u^2 (-dt^2 + dy^2 + dx^2 + L^2 \tilde{f}(u) d\phi^2) + \frac{L^2 du^2}{\tilde{f}(u) u^2 \lambda^6(u)} + L^4 d\theta^2 \right] \\ + \frac{L^2}{\zeta(u, \theta)} [\cos^2 \theta d\psi^2 + \cos^2 \theta \sin^2 \psi D\phi_1^2 + \cos^2 \theta \cos^2 \psi D\phi_2^2 + \lambda^6(u) \sin^2 \theta D\phi_3^2]$$

$$A^1 = A^2 = q_1 [\lambda^6(u) - \lambda^6(u_*)] L \, d\phi, A^3 = q_2 \left[\frac{1}{\lambda^6(u)} - \frac{1}{\lambda^6(u_*)} \right] L \, d\phi.$$

$$\zeta(u,\theta)=\sqrt{1+\varepsilon\frac{\ell^2}{u^2}\cos^2\theta}, \lambda^6(u)=\frac{u^2+\varepsilon\ell^2}{u^2} \\ \tilde{f}(u)=\frac{1}{L^2}-\frac{\varepsilon\ell^2L^2}{u^4}\left(q_1^2-\frac{q_2^2}{\lambda^6(u)}\right)$$

$$ds_{\Sigma_8}^2 = \frac{\zeta(u, \theta)}{L^2} \left[u^2 (dx^2 + L^2 \tilde{f}(u) d\phi^2) + \frac{L^2 du^2}{\tilde{f}(u) u^2 \lambda^6(u)} \left[1 + \frac{\tilde{f}(u) \lambda^6(u) u^4}{L^2} (-t'^2 + y'^2) \right] + L^4 d\theta^2 \right] \\ + \frac{L^2}{\zeta(u, \theta)} [\cos^2 \theta d\psi^2 + \cos^2 \theta \sin^2 \psi D\phi_1^2 + \cos^2 \theta \cos^2 \psi D\phi_2^2 + \lambda^6(u) \sin^2 \theta D\phi_3^2] \\ e^{-4\Phi} \det[g_{\Sigma_8}] = L^8 \cos^6 \theta \sin^2 \theta \cos^2 \psi \sin^2 \psi \left[u^2 + \frac{\tilde{f}(u) \lambda^6(u) u^6}{L^2} (y'^2 - t'^2) \right]$$

$$\frac{4G_{10}}{\mathcal{N}_{II}}S_{EE}=\int_{u_0}^{\infty}du\sqrt{G^2(u)+F^2(u)(y'^2-t'^2)}$$

$$G(u)=u, F(u)=\frac{u^3\lambda^3(u)\sqrt{\tilde{f}(u)}}{L}, \text{ and}$$

$$\mathcal{N}_{II}=L^4L_xL_\phi\int d\theta d\psi d\phi_1d\phi_2d\phi_3\cos^3\theta\sin\theta\cos\psi\sin\psi$$

$$G(u)=u, \tilde{f}(u)=\frac{[u^6+\epsilon \ell^2 u^4-L^6 q^6]}{L^2 u^4 (u^2+\epsilon \ell^2)}$$

$$F(u)=\frac{1}{L^2}[u^6+\epsilon \ell^2 u^4-L^6 q^6]^{\frac{1}{2}}$$

$$\frac{T}{2c_tL^4}=\frac{Y}{2c_yL^4}=\int_{u_0}^{\infty}du\frac{u}{\sqrt{(u^6+\epsilon\ell^2u^4-L^6q^6)[(u^6-u_0^6)+\epsilon\ell^2(u^4-u_0^4)]}}$$

$$\frac{2G_{10}}{\mathcal{N}_{II}}S_{EE}=\int_{u_0}^{\infty}duu\sqrt{\frac{(u^6+\epsilon\ell^2u^4-L^6q^6)}{[(u^6-u_0^6)+\epsilon\ell^2(u^4-u_0^4)]}}-\int_{u_*}^{\infty}udu$$

$$u=\frac{u_0}{r},\quad \mu=\frac{\epsilon\ell^2}{u_0^2}, v=\frac{L^6q^6}{u_0^6}$$



$$\frac{Tu_0^4}{2c_tL^4} = \frac{Yu_0^4}{2c_yL^4} = \int_0^1 dr \frac{r^3}{\sqrt{(1+\mu r^2-vr^6)[(1-r^6)+\mu r^2(1-r^4)]}}$$

$$\frac{2G_{10}}{\mathcal{N}_Iu_0^2}S_{EE}=\int_0^1\frac{dr}{r^3}\sqrt{\frac{(1+\mu r^2-vr^6)}{[(1-r^6)+\mu r^2(1-r^4)]}}-\int_0^{\frac{u_0}{u_*}}\frac{dr}{r^3}$$

$$\Delta^2=Y^2-T^2=\frac{4L^2}{u_0^2}(1+\mu-v)J_1^2.$$

$$F(u_0)^2=c_y^2-c_t^2,\rightarrow u_0^6+\epsilon\ell^2u_0^4=L^6(q^6+c_y^2-c_t^2).$$

$$27u_0^6=\left[-\epsilon\ell^2+\frac{\ell^4}{\mathcal{Z}}+\mathcal{Z}\right]^3,$$

$$\mathcal{Z}^3=\frac{27m}{2}\left(1-\frac{2\epsilon\ell^6}{27m}+\sqrt{1-\frac{4\epsilon\ell^6}{27m}}\right),\,m=L^6(q^6+c_y^2-c_t^2).$$

$$\mathrm{d}s^2=f_1^{\frac{3}{2}}f_5^{\frac{1}{2}}[4\,\mathrm{d}s_5^2+f_2D\mu_iD\mu_i+f_4(\,\mathrm{d}\sigma^2+\mathrm{d}\eta^2)+f_3(\,\mathrm{d}\chi+\mathcal{A})^2],e^{\frac{4}{3}\Phi}=f_1f_5$$

$$\mathrm{d}s_5^2=\frac{u^2}{l^2}(\lambda\,\mathrm{d}t^2+\mathrm{d}y^2+\mathrm{d}x^2+f(u)\mathrm{d}\phi^2)+\frac{l^2\,\mathrm{d}u^2}{u^2f(u)}$$

$$f(u)=1-\frac{Q^2l^2}{u^6},\mathcal{A}=Q\left(\frac{1}{u^2}-\frac{1}{u_*^2}\right)\mathrm{d}\phi,u_*(Ql)^{1/3}$$

$$\mu_1=\sin\theta\sin\varphi,\mu_2=\sin\theta\cos\varphi,\mu_3=\cos\theta$$

$$D\mu_1=\mathrm{d}\mu_1+2\mu_2\mathcal{A},D\mu_2=\mathrm{d}\mu_2-2\mu_1\mathcal{A},D\mu_3=\mathrm{d}\mu_3$$

$$f_1=\left(\frac{\dot{V}\tilde{\Delta}}{2V''}\right)^{\frac{1}{3}},f_2=\frac{2V''\dot{V}}{\tilde{\Delta}},f_3=\frac{4\sigma^2}{\Lambda},f_4=\frac{2V''}{\dot{V}}$$

$$f_5=\frac{2\Lambda V''}{\dot{V}\tilde{\Delta}},\tilde{\Delta}=\Lambda(V'')^2+(\dot{V}')^2,\Lambda=\frac{2\dot{V}-\ddot{V}}{V''}$$

$$ds_8^2=(f_1^3f_5)^{\frac{1}{2}}\bigg(4\bigg[\frac{u^2}{l^2}(\lambda t'^2+y'^2)+\frac{l^2}{u^2f(u)}\bigg]du^2+\frac{4u^2}{l^2}dx^2+\frac{4u^2}{l^2}f(u)d\phi^2+f_2D\mu_iD\mu_i+f_4(\,\mathrm{d}\sigma^2+\mathrm{d}\eta^2)+f_3(\,\mathrm{d}\chi+\mathcal{A})^2\bigg)$$

$$e^{-4\Phi}\mathrm{det}[g_{\Sigma_8}]=f_1^9f_2^2f_4^2f_3f_5\mathrm{Vol}_{S^2(\theta,\varphi)}\mathrm{Vol}_{S^1(\chi)}\left[\frac{u^2}{l^2}+\frac{u^6}{l^6}f(u)(y'^2+\lambda t'^2)\right]$$

$$S_{EE}^{(\lambda)}=\frac{\tilde{\mathcal{N}}}{4G_{10}}\int_{u_0}^{\infty}\sqrt{G^2(u)+F^2(u)(\lambda t'^2(u)+y'^2(u))}$$

$$G(u)=\frac{u}{l},F(u)=\frac{u^3}{l^3}\sqrt{f(u)},\tilde{\mathcal{N}}=256\pi^2L_xL_\phi\int_0^\infty d\sigma\int_0^P d\eta\sigma\dot{V}V'',L_\phi=\frac{1}{3Q}$$

$$\mathrm{d}s_{10}^2=\tilde{f}_1^{\frac{3}{2}}\tilde{f}_5^{\frac{1}{2}}[4\tilde{\gamma}\,\mathrm{d}s_5^2+\tilde{f}_2D\mu_iD\mu^i+\tilde{f}_3(\,\mathrm{d}\chi+B)^2+\tilde{f}_4(\,\mathrm{d}\sigma^2+\mathrm{d}\eta^2)]$$

$$e^{\frac{4}{3}\Phi}=\tilde{f}_1\tilde{f}_5$$



$$\begin{aligned}\tilde{f}_1 &= \left(\frac{\dot{V}\tilde{\Delta}}{2V''}\right)^{1/3}, \tilde{V} = \frac{Z}{X(u)} = \frac{Z}{\lambda^2(u)}, \tilde{f}_2 = \frac{2\dot{V}V''}{Z^2\tilde{\Delta}}, \tilde{f}_3 = \frac{4X^3\sigma^2V''}{2X^3\dot{V}-\ddot{V}}Z \\ \tilde{f}_4 &= \frac{2V''}{\dot{V}}Z, \tilde{f}_5 = \frac{2(2X^3\dot{V}-\ddot{V})}{Z^2\dot{V}\tilde{\Delta}} \\ \tilde{\Delta} &= (\dot{V}')^2 + V''(2\dot{V}-\ddot{V}), Z = \left[\frac{(\dot{V}')^2 + V''(2X^3\dot{V}-\ddot{V})}{(\dot{V}')^2 + V''(2\dot{V}-\ddot{V})}\right]^{1/3}\end{aligned}$$

$$ds_5^2 = \frac{u^2\lambda(u)^2}{L^2}(-dt^2 + dy^2 + dx^2 + L^2\tilde{f}(u)d\phi^2) + \frac{du^2}{u^2\lambda(u)^4\tilde{f}(u)}$$

$$\begin{aligned}\mu_1 &= \sin\theta\cos\varphi, \mu_2 = \sin\theta\sin\varphi, \mu_3 = \cos\theta \\ D\mu_i &= (d\mu_1 + 2\mu_2A_{1\phi}d\phi)\delta_{i,1} + (d\mu_2 - 2\mu_1A_{1\phi}d\phi)\delta_{i,2} + d\mu_3\delta_{i,3}\end{aligned}$$

$$e^{-4\Phi}\det[g_{\Sigma_8}]=4^3\tilde{f}_1^9\tilde{f}_2^2\tilde{f}_4^2\tilde{f}_3\tilde{f}_5\tilde{\gamma}^3\left[\frac{u^2}{L^2}+\frac{u^6\lambda^6(u)\tilde{f}(u)}{L^4}(y'^2-t'^2)\right]\mathrm{Vol}_{S^2(\theta,\varphi)}\mathrm{Vol}_{S^1(\chi)}$$

$$\tilde{\mathcal{N}}_{II}=256\pi^2L_xL_\phi\int_0^\infty d\sigma\int_0^Pd\eta\sigma\dot{V}V''$$

$$\begin{aligned}ds_{st}^2 &= e^{\Phi}\hat{h}^{-\frac{1}{2}}(u)[\lambda dt^2 + dy^2 + dx_1^2 + dx_2^2 + \hat{h}(u)e^{2k(u)}du^2 + \hat{h}(u)e^{2h(u)}(d\theta^2 + \sin^2\theta d\varphi^2) \\ &\quad + \frac{\hat{h}(u)e^{2g(u)}}{4}\big[(\omega_1 - A^{(1)})^2 + (\omega_2 - A^{(2)})^2\big] + \frac{\hat{h}(u)e^{2k(u)}}{4}(\omega_3 - A^{(3)})^2] \\ \Phi &= \Phi(u)\end{aligned}$$

$$\omega_1=\cos\tilde{\psi}d\tilde{\theta}+\sin\tilde{\psi}\sin\tilde{\theta}d\tilde{\varphi},\omega_2=-\sin\tilde{\psi}d\tilde{\theta}+\cos\tilde{\psi}\sin\tilde{\theta}d\tilde{\varphi},\omega_3=d\tilde{\psi}+\cos\tilde{\theta}d\tilde{\varphi}\\ A^{(1)}=-a(u)d\theta,A^{(2)}=a(u)\sin\theta d\varphi,A^{(3)}=-\sin\theta d\varphi$$

$$\begin{aligned}e^{2h}&=\frac{(P^2-Q^2)}{4(P\coth(2u)-Q)},e^{2g}=(P\coth(2u)-Q),e^{2k}=\frac{P'}{2},e^{4\Phi}=2e^{4\Phi_0}\frac{\sinh^2(2u)}{(P^2-Q^2)P'}\\ a&=\frac{P}{P\cosh(2u)-Q\sinh(2u)},\hat{h}=1-\kappa^2e^{2\Phi},Q=N_c(2u\coth(2u)-1)\end{aligned}$$

$$P''+P'\left(\frac{P'-Q'}{P+Q}+\frac{P'+Q'}{P-Q}-4\coth(2u)\right)=0$$

$$P=2N_cu$$

$$\begin{aligned}P &= e^{\frac{4u}{3}}[c_+ + \frac{e^{-\frac{8u}{3}}N_c^2}{c_+}\Big(4u^2 - 4u + \frac{13}{4}\Big) + e^{-4u}\Big(c_- - \frac{8c_+}{3}u\Big) \\ &\quad + \frac{N_c^4e^{-\frac{16u}{3}}}{c_+^3}\Big(\frac{18567}{512} + \frac{2781}{32}u + \frac{27}{4}u^2 + 36u^3\Big) + O\Big(e^{-\frac{20u}{3}}\Big)]\end{aligned}$$

$$P=h_1u+\frac{4h_1}{15}\left(1-\frac{4N_c^2}{h_1^2}\right)u^3+\frac{16h_1}{525}\left(1-\frac{4N_c^2}{3h_1^2}-\frac{32N_c^4}{3h_1^4}\right)u^5+O(u^7).$$



$$\begin{aligned}
ds_{\Sigma_8}^2 &= e^{\Phi} \hat{h}^{-\frac{1}{2}}(u) [dx_1^2 + dx_2^2 + (\hat{h}(u)e^{2k(u)} + \lambda t'^2 + y'^2) du^2 + \hat{h}(u)e^{2h(u)} (d\theta^2 + \sin^2 \theta d\varphi^2) \\
&\quad + \frac{\hat{h}(u)e^{2g(u)}}{4} [(\omega_1 - A^{(1)})^2 + (\omega_2 - A^{(2)})^2] + \frac{\hat{h}(u)e^{2k(u)}}{4} (\omega_3 - A^{(3)})^2] \\
e^{-4\Phi} \det[g_{\Sigma_8}] &= \frac{\hat{h}^2}{64} e^{4\Phi+4h+4g+4k} \left(\frac{e^{-2k}}{\hat{h}} (\lambda t'^2 + y'^2) + 1 \right) \text{Vol}_{S^2} \text{Vol}_{S^3} \\
4G_{10} S_{EE} &= \mathcal{N}_{III} \int du \sqrt{G^2(u) + F^2(u)(y'^2 + \lambda t'^2)} \\
G^2(u) &= e^{4\Phi+4h+4g+4k} \hat{h}^2 = \frac{e^{4\Phi_0}}{32} \sinh^2(2u) (P^2 - Q^2) P' \hat{h}^2 \\
F^2(u) &= \frac{e^{-2k} G^2(u)}{\hat{h}} = \frac{e^{4\Phi_0}}{16} \sinh^2(2u) (P^2 - Q^2) \hat{h}
\end{aligned}$$

En tanto que la supersimetría de gauge respecto de un campo cuántico relativista en supergravedad, queda expresada así:

$$\begin{aligned}
S_{\text{SYM}} &= \int d^{10}x \text{Tr} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \chi \gamma^\mu \mathcal{D}_\mu \chi \right] \\
(\bar{\gamma}^\mu)^{\alpha\lambda} (\gamma^\nu)_{\lambda\beta} + (\bar{\gamma}^\nu)^{\alpha\lambda} (\gamma^\mu)_{\lambda\beta} &= 2\eta^{\mu\nu} \delta_\beta^\alpha \\
\delta_\Lambda A_\mu &= \mathcal{D}_\mu \Lambda, \delta_\Lambda \chi = [\chi, \Lambda] \\
\delta_\xi A_\mu &= \xi^\rho \partial_\rho A_\mu + \partial_\mu \xi^\rho A_\rho, \delta_\xi \chi = \xi^\mu \partial_\mu \chi + \frac{1}{4} \omega^{\mu\nu} \gamma_{\mu\nu} \chi \\
\delta_\epsilon A_\mu &= i\epsilon \gamma_\mu \chi, \delta_\epsilon \chi = \frac{1}{2} F_{\mu\nu} \gamma^{\mu\nu} \epsilon \\
(\gamma^{\mu\nu})_\beta^\alpha &:= \frac{1}{2} [(\bar{\gamma}^\mu)^{\alpha\lambda} (\gamma^\nu)_{\lambda\beta} - (\bar{\gamma}^\nu)^{\alpha\lambda} (\gamma^\mu)_{\lambda\beta}]. \\
\mathcal{D}_\nu F^{\nu\mu} - \frac{i}{2} [\chi, \gamma^\mu \chi] &= 0, i\gamma^\mu \mathcal{D}_\mu \chi = 0 \\
\varphi &= \partial \cdot A \\
S_{\text{SYM}} &= \int d^{10}x \text{Tr} \left[\frac{1}{2} A_\mu \square A^\mu + (\partial \cdot A) \varphi - \frac{1}{2} \varphi^2 + \frac{i}{2} \chi \gamma^\mu \partial_\mu \chi \right. \\
&\quad \left. - \partial_\mu A_\nu [A^\mu, A^\nu] + \frac{i}{2} \chi \gamma^\mu [A_\mu, \chi] \right. \\
&\quad \left. - \frac{1}{4} [A_\mu, A_\nu] [A^\mu, A^\nu] \right]. \\
\mathcal{X} &= \bigoplus_{i \in \mathbb{Z}} X_i \\
B_n: \mathcal{X}^{\otimes n} &\longrightarrow \mathcal{X}, |B_n| = 1, n \geq 1,
\end{aligned}$$



$$S = \sum_{n=1}^{\infty} \frac{1}{(n+1)!} \langle \mathcal{A}, B_n(\underbrace{\mathcal{A}, \dots, \mathcal{A}}_{n \text{ times}}) \rangle$$

$$\frac{\delta S}{\delta \mathcal{A}} = \sum_{n=1}^{\infty} \frac{1}{n!} B_n(\underbrace{\mathcal{A}, \dots, \mathcal{A}}_{n \text{ times}})$$

$$\begin{array}{ccccccc} \xrightarrow{B_1} & X_{-1} & \xrightarrow{B_1} & X_0 & \xrightarrow{B_1} & X_1 & \xrightarrow{B_1} & X_2 & \xrightarrow{B_1} & \\ & \Lambda & & \mathcal{A} & & \mathcal{E} & & \mathcal{N} & & \end{array}$$

$$\mathcal{X}_{\text{SYM}} = \mathcal{X}^{\text{B}} \oplus \mathcal{X}^{\text{F}}$$

$$\begin{array}{ccccccc} X_{-1}^{\text{B}} & \xrightarrow{B_1} & X_0^{\text{B}} & \xrightarrow{B_1} & X_1^{\text{B}} & \xrightarrow{B_1} & X_2^{\text{B}} \\ \Lambda & & A_\mu & & E & & \\ & & \varphi & & E_\mu & & N \end{array}$$

$$B_1(\Lambda) = \begin{pmatrix} \partial_\mu \Lambda \\ \square \Lambda \end{pmatrix} \in X_0^{\text{B}}$$

$$B_1 \begin{pmatrix} A_\mu \\ \varphi \end{pmatrix} = \begin{pmatrix} \partial \cdot A - \varphi \\ \square A_\mu - \partial_\mu \varphi \end{pmatrix} \in X_1^{\text{B}}$$

$$B_1 \begin{pmatrix} E \\ E^\mu \end{pmatrix} = \begin{pmatrix} \emptyset \\ \square E - \partial_\mu E^\mu \end{pmatrix} \in X_2^{\text{B}}$$

$$b^2 = 0, bB_1 + B_1b = \square.$$

$$\begin{array}{ccccccc} X_{-1}^{\text{B}} & \xrightarrow{B_1} & X_0^{\text{B}} & \xrightarrow{B_1} & X_1^{\text{B}} & \xrightarrow{B_1} & X_2^{\text{B}} \\ \Lambda & & A_\mu & & E & & \\ & \nwarrow b & \nwarrow b & \nwarrow b & \nwarrow b & & \\ & & \varphi & & E_\mu & & N \end{array} .$$

$$b \begin{pmatrix} \emptyset \\ N \end{pmatrix} = \begin{pmatrix} N \\ 0 \end{pmatrix} \in X_1^{\text{B}},$$

$$b \begin{pmatrix} E \\ E_\mu \end{pmatrix} = \begin{pmatrix} E_\mu \\ 0 \end{pmatrix} \in X_0^{\text{B}},$$

$$b \begin{pmatrix} A_\mu \\ \varphi \end{pmatrix} = \begin{pmatrix} \varphi \\ \emptyset \end{pmatrix} \in X_{-1}^{\text{B}}.$$



$$0 \longrightarrow X_0^{\text{F}} \xrightarrow{B_1} X_1^{\text{F}} \longrightarrow 0$$

$$\chi \qquad \qquad \mathcal{E}_\chi$$

$$b(\mathcal{E}_\chi) = -i\gamma^\mu \partial_\mu \mathcal{E}_\chi \in X_0^{\text{F}}, b(\chi) = 0,$$

$$B_1(\chi) = \begin{pmatrix} i\gamma^\mu \partial_\mu \chi \\ \square \chi \end{pmatrix} \in X_1^{\text{F}}$$

$$b \begin{pmatrix} \mathcal{E}_\chi \\ E_\chi \end{pmatrix} = E_\chi \in X_0^{\text{F}}, b(\chi) = 0.$$

$$0 \longrightarrow X_0^{\text{F}} \xrightarrow{B_1} X_1^{\text{F}} \xrightarrow{B_1} X_2^{\text{F}} \xrightarrow{B_1} \dots$$

$$\begin{array}{ccccccc} \chi & \xrightarrow{i\gamma^\mu \partial_\mu} & \mathcal{E}_\chi & \xrightarrow{i\gamma^\nu \partial_\nu} & \mathcal{N}_\chi & \xrightarrow{i\gamma^\rho \partial_\rho} & \dots \\ & \searrow \square & & \nearrow 1 & & \searrow \square & \\ & & E_\chi & \xrightarrow{-i\gamma^\mu \partial_\mu} & N_\chi & \xrightarrow{-i\gamma^\nu \partial_\nu} & \dots \end{array}$$

$$\begin{array}{ccccccc} X_{-1} & \xrightarrow{B_1} & X_0 & \xrightarrow{B_1} & X_1 & \xrightarrow{B_1} & X_2 \xrightarrow{B_1} \dots \\ \Lambda & \swarrow b & A_\mu, \chi & \swarrow b & E, \mathcal{E}_\chi & \swarrow b & \mathcal{N}_\chi \dots \\ & & \varphi & & E_\mu, E_\chi & & N, N_\chi \dots \end{array}$$

$$\mathcal{K}_{\text{SYM}} = \bigoplus_{i=0}^{\infty} K_i$$

$$m_n\colon \mathcal{K}^{\otimes n} \rightarrow \mathcal{K}, |m_n| = 2-n, n \geq 1$$

$$\mathcal{X}_{\text{SYM}} = \mathcal{K}_{\text{SYM}} \otimes \mathfrak{g}, \mathfrak{g}$$

$$B_1(x) = m_1(u^a) \otimes t_a$$

$$B_2(x_1,x_2) = (-1)^{x_1} f^a{}_{bc} m_2(u_1^b,u_2^c) \otimes t_a$$

$$B_3(x_1,x_2,x_3) = f^a{}_{bef} f^e{}_{cd} [(-1)^{x_2} m_3(u_1^b,u_2^c,u_3^d) + (-1)^{x_1(x_2+1)} m_3(u_2^b,u_1^c,u_3^d)] \otimes t_a$$

$$\begin{array}{ccccccc} K_0 & \xrightarrow{m_1} & K_1 & \xrightarrow{m_1} & K_2 & \xrightarrow{m_1} & K_3 \xrightarrow{m_1} \dots \\ \Lambda & & A_\mu, \chi & & E, \mathcal{E}_\chi & & \mathcal{N}_\chi \dots \\ & & \varphi & & E_\mu, E_\chi & & N, N_\chi \dots \end{array}$$



$$b^2 = 0, bm_1 + m_1b = \square, |b| = -1.$$

$$H_{m,n}(\Xi_1, \dots, \Xi_m): \mathcal{X}_{\text{SYM}}^{\otimes n} \rightarrow \mathcal{X}_{\text{SYM}},$$

$$\Sigma_n(\Xi): \mathcal{X}_{\text{SYM}}^{\otimes n} \rightarrow \mathcal{X}_{\text{SYM}}, |\Sigma_n(\Xi)| = 0, n \geq 1$$

$$\delta_{\Xi} \mathcal{A} =: \Sigma_1(\Xi \mid \mathcal{A}) + \frac{1}{2} \Sigma_2(\Xi \mid \mathcal{A}, \mathcal{A}) + \cdots$$

$$\begin{aligned}\Sigma_1(\xi \mid \Lambda) &= \begin{pmatrix} \xi \cdot \partial \Lambda \\ \emptyset \end{pmatrix} \in X_{-1} \\ \Sigma_1(\xi \mid \mathcal{A}) &= \begin{pmatrix} \xi \cdot \partial A_\mu + \partial_\mu \xi \cdot A, \xi \cdot \partial \chi + \frac{1}{4} \omega_{\mu\nu} \gamma^{\mu\nu} \chi \\ \xi \cdot \partial \varphi \end{pmatrix} \in X_0 \\ \Sigma_1(\xi \mid \mathcal{E}) &= \begin{pmatrix} \xi \cdot \partial E, \xi \cdot \partial \mathcal{E}_\chi + \frac{1}{4} \omega_{\mu\nu} \gamma^{\mu\nu} \mathcal{E}_\chi \\ \xi \cdot \partial E_\mu + \partial_\mu \xi^\nu E_\nu, \xi \cdot \partial E_\chi + \frac{1}{4} \omega_{\mu\nu} \gamma^{\mu\nu} E_\chi \end{pmatrix} \in X_1 \\ \Sigma_1(\xi \mid \mathcal{N}) &= \begin{pmatrix} \xi \cdot \partial \mathcal{N}_\chi + \frac{1}{4} \omega_{\mu\nu} \gamma^{\mu\nu} \mathcal{N}_\chi \\ \xi \cdot \partial N, \xi \cdot \partial N_\chi + \frac{1}{4} \omega_{\mu\nu} \gamma^{\mu\nu} N_\chi \end{pmatrix} \in X_2\end{aligned}$$

$$\begin{aligned}\Sigma_1(\epsilon \mid \Lambda) &= 0 \\ \Sigma_1(\epsilon \mid \mathcal{A}_B) &= \begin{pmatrix} \partial_\mu A_\nu \gamma^{\mu\nu} \epsilon \\ \emptyset \end{pmatrix} \in X_0^{\text{F}}, \Sigma_1(\epsilon \mid \mathcal{A}_F) = \begin{pmatrix} i \epsilon \gamma_\mu \chi \\ 0 \end{pmatrix} \in X_0^{\text{B}} \\ \Sigma_1(\epsilon \mid \mathcal{E}_B) &= \begin{pmatrix} i(E_\mu - \partial_\mu E) \gamma^\mu \epsilon \\ \partial_\mu E_\nu \gamma^{\mu\nu} \epsilon \end{pmatrix} \in X_1^{\text{F}}, \Sigma_1(\epsilon \mid \mathcal{E}_F) = \begin{pmatrix} \epsilon \mathcal{E}_\chi \\ i \epsilon \gamma_\mu E_\chi \end{pmatrix} \in X_1^{\text{B}} \\ \Sigma_1(\epsilon \mid \mathcal{N}_B) &= \begin{pmatrix} N \epsilon \\ -i \partial N \epsilon \end{pmatrix} \in X_2^{\text{F}}, \Sigma_1(\epsilon \mid \mathcal{N}_F) = \begin{pmatrix} \emptyset \\ \epsilon N_\chi \end{pmatrix} \in X_2^{\text{B}}\end{aligned}$$

$$\begin{aligned}B_1 \Sigma_1(\Xi \mid x) &= \Sigma_1(\Xi \mid B_1(x)) \\ B_1 \Sigma_2(\Xi \mid x_1, x_2) &+ B_2(\Sigma_1(\Xi \mid x_1), x_2) + B_2(x_1, \Sigma_1(\Xi \mid x_2)) \\ &= \Sigma_1(\Xi \mid B_2(x_1, x_2)) + \Sigma_2(\Xi \mid B_1(x_1), x_2) + (-1)^{x_1} \Sigma_2(\Xi \mid x_1, B_1(x_2)) \\ &\quad B_1 \Sigma_3(\Xi \mid x_1, x_2, x_3) + B_2(\Sigma_2(\Xi \mid x_1, x_2), x_3) \\ &\quad + B_3(\Sigma_1(\Xi \mid x_1), x_2, x_3) \\ &= \Sigma_1(\Xi \mid B_3(x_1, x_2, x_3)) + \Sigma_2(\Xi \mid B_2(x_1, x_2), x_3) \\ &\quad \vdots\end{aligned}$$

$$\sum_{i+j=n} [B_i, \Sigma_j(\Xi)] = 0, \forall n \geq 2$$

$$[\Sigma(\Xi_1), \Sigma(\Xi_2)] = \Sigma([\Xi_1, \Xi_2]_g)$$

$$\begin{aligned}[\Sigma_1(\Xi_1), \Sigma_1(\Xi_2)] &= \Sigma_1(\Xi_{12}) + [B_1, H_{2,1}(\Xi_1, \Xi_2)], \\ 2[\Sigma_1(\Xi_{[1]}, \Sigma_2(\Xi_{[2]}))] &= \Sigma_2(\Xi_{12}) + [B_1, H_{2,2}(\Xi_1, \Xi_2)] + [H_{2,1}(\Xi_1, \Xi_2), B_2],\end{aligned}$$

$$H_{m,n}(\Xi_1, \dots, \Xi_m): \mathcal{X}_{\text{SYM}}^{\otimes n} \rightarrow \mathcal{X}_{\text{SYM}}, m, n \geq 1$$



$$[\Sigma_1(\epsilon_1), \Sigma_1(\epsilon_2)] = \Sigma_1(a_{12}) + [B_1, H_{2,1}(\epsilon_1, \epsilon_2)]$$

$$[\Sigma_1(\epsilon_1), \Sigma_2(\epsilon_2)] - [\Sigma_1(\epsilon_2), \Sigma_2(\epsilon_1)] = [B_1, H_{2,2}(\epsilon_1, \epsilon_2)] + [H_{2,1}(\epsilon_1, \epsilon_2), B_2]$$

$$\epsilon_{12}^{\mu[n]} = \epsilon_1 \gamma^{\mu[n]} \epsilon_2 \equiv \epsilon_1 \gamma^{\mu_1 \dots \mu_n} \epsilon_2$$

$$\chi_{12} := \frac{7}{8} \epsilon_{12}^\mu \gamma_\mu - \frac{1}{16} \frac{1}{5!} \epsilon_{12}^{\mu[5]} \gamma_{\mu[5]}$$

$$[\Sigma_1(\epsilon_1), \Sigma_1(\epsilon_2)]\Lambda = \Sigma_1(a_{12} \mid \Lambda) + H_{2,1}(\epsilon_1, \epsilon_2 \mid B_1\Lambda)$$

$$[\Sigma_1(\epsilon_1), \Sigma_1(\epsilon_2)]\mathcal{A}_B = \Sigma_1(a_{12} \mid \mathcal{A}_B) + B_1 H_{2,1}(\epsilon_1, \epsilon_2 \mid \mathcal{A}_B) + H_{2,1}(\epsilon_1, \epsilon_2 \mid B_1\mathcal{A}_B)$$

$$[\Sigma_1(\epsilon_1), \Sigma_1(\epsilon_2)]\mathcal{A}_F = \Sigma_1(a_{12} \mid \mathcal{A}_F) + H_{2,1}(\epsilon_1, \epsilon_2 \mid B_1\mathcal{A}_F)$$

$$[\Sigma_1(\epsilon_1), \Sigma_1(\epsilon_2)]\mathcal{E}_B = \Sigma_1(a_{12} \mid \mathcal{E}_B) + B_1 H_{2,1}(\epsilon_1, \epsilon_2 \mid \mathcal{E}_B)$$

$$[\Sigma_1(\epsilon_1), \Sigma_1(\epsilon_2)]\mathcal{E}_F = \Sigma_1(a_{12} \mid \mathcal{E}_F) + B_1 H_{2,1}(\epsilon_1, \epsilon_2 \mid \mathcal{E}_F) + H_{2,1}(\epsilon_1, \epsilon_2 \mid B_1\mathcal{E}_F)$$

$$[\Sigma_1(\epsilon_1), \Sigma_1(\epsilon_2)]\mathcal{N}_B = \Sigma_1(a_{12} \mid \mathcal{N}_B)$$

$$[\Sigma_1(\epsilon_1), \Sigma_1(\epsilon_2)]\mathcal{N}_F = \Sigma_1(a_{12} \mid \mathcal{N}_F) + B_1 H_{2,1}(\epsilon_1, \epsilon_2 \mid \mathcal{N}_F) + H_{2,1}(\epsilon_1, \epsilon_2 \mid B_1\mathcal{N}_F)$$

$$\Sigma_1(a_{12} \mid -) := a_{12}^\mu \partial_\mu (-), \quad \Lambda_{12} := 2i\epsilon_{12}^\mu A_\mu$$

$$H_{2,1}(\epsilon_1, \epsilon_2 \mid \mathcal{A}_B) := \begin{pmatrix} \Lambda_{12} \\ \emptyset \end{pmatrix} \in X_{-1}^B, \quad \varphi_{12} := -2i\epsilon_{12}^\mu E_\mu$$

$$H_{2,1}(\epsilon_1, \epsilon_2 \mid \mathcal{E}_B) := \begin{pmatrix} 0 \\ \varphi_{12} \end{pmatrix} \in X_0^B, \quad \chi_{12} := \chi_{12} \mathcal{E}_\chi$$

$$H_{2,1}(\epsilon_1, \epsilon_2 \mid \mathcal{E}_F) := \begin{pmatrix} \chi_{12} \\ \emptyset \end{pmatrix} \in X_0^F, \quad \mathcal{E}_\chi^{12} := \epsilon_{12}^\mu \gamma_\mu \mathcal{N}_\chi$$

$$H_{2,1}(\epsilon_1, \epsilon_2 \mid \mathcal{N}_F) := \begin{pmatrix} \mathcal{E}_\chi^{12} \\ E_\chi^{12} \end{pmatrix} \in X_1^F, \quad E_\chi^{12} := -\chi_{12} N_\chi$$

$$\mathcal{N}_\chi^{12} := \epsilon_{12}^\mu \gamma_\mu \mathcal{R}_\chi$$

$$H_{2,1}(\epsilon_1, \epsilon_2 \mid \mathcal{R}_F) := \begin{pmatrix} \mathcal{N}_\chi^{12} \\ N_\chi^{12} \end{pmatrix} \in X_2^F, \quad N_\chi^{12} := -\epsilon_{12}^\mu \gamma_\mu R_\chi$$

$$2[\Sigma_1(\epsilon_{[1]}, \Sigma_2(\epsilon_{[2]}))](\Lambda, \Lambda) = 0$$

$$2[\Sigma_1(\epsilon_{[1]}, \Sigma_2(\epsilon_{[2]}))](\mathcal{A}_B, \mathcal{A}_B) = H_{2,1}(\epsilon_1, \epsilon_2 \mid B_2(A, A)) + 2B_2(A, H_{2,1}(\epsilon_1, \epsilon_2 \mid A))$$

$$2[\Sigma_1(\epsilon_{[1]}, \Sigma_2(\epsilon_{[2]}))](\mathcal{A}_B, \mathcal{A}_F) = H_{2,1}(\epsilon_1, \epsilon_2 \mid B_2(A, \chi)) + B_2(\chi, H_{2,1}(\epsilon_1, \epsilon_2 \mid A))$$

$$2[\Sigma_1(\epsilon_{[1]}, \Sigma_2(\epsilon_{[2]}))](\mathcal{A}_F, \mathcal{A}_F) = H_{2,1}(\epsilon_1, \epsilon_2 \mid B_2(\chi, \chi))$$

$$[\Sigma_1(\xi_1), \Sigma_1(\xi_2)] = \Sigma_1(\xi_{12}), \xi_{12} = \mathcal{L}_{\xi_1} \xi_2^\mu$$

$$[\Sigma_1(\xi), \Sigma_1(\epsilon)] = \Sigma_1(\epsilon_{\text{eff}})$$

$$\epsilon_{\text{eff}} := \frac{1}{4} \omega_{\mu\nu} \gamma^{\mu\nu} \epsilon$$

$$\rho_n(\Xi): \mathcal{K}_{\text{SYM}}^{\otimes n} \rightarrow \mathcal{K}_{\text{SYM}}, |\rho_n(\Xi)| = 1 - n$$

$$h_{m,n}(\Xi_1, \dots, \Xi_m): \mathcal{K}_{\text{SYM}}^{\otimes n} \rightarrow \mathcal{K}_{\text{SYM}}, |h_{m,n}(\Xi_1, \dots, \Xi_m)| = 2 - m - n$$

$$\Sigma_1(\Xi \mid x) = \rho_1(\Xi \mid u^a) \otimes t_a$$

$$\Sigma_2(\Xi \mid x_1, x_2) = (-1)^{x_1} f_{bc}^a \rho_2(\Xi \mid u_1^b, u_2^c) \otimes t_a$$



$$\begin{aligned} H_{2,1}(\Xi_1,\Xi_2\mid x) &= h_{2,1}(\Xi_1,\Xi_2\mid u^a)\otimes t_a \\ H_{2,2}(\Xi_1,\Xi_2\mid x_1,x_2) &= (-1)^{x_1}f^a{}_{bc}h_{2,2}(\Xi_1,\Xi_2\mid u_1^b,u_2^c)\otimes t_a. \end{aligned}$$

$$[b,\rho_1(\Xi)]=[b,h_{2,1}(\Xi_1,\Xi_2)]=0$$

$$\square\equiv\widetilde{\square},$$

$$\left. \begin{array}{l} \mathcal{K}_{\text{SYM}} = \mathcal{K}_{\text{SYM}} \otimes \mathfrak{g} \xrightarrow{\text{color-strip}} \mathcal{K}_{\text{SYM}} \\ \tilde{\mathcal{K}}_{\text{SYM}} = \tilde{\mathcal{K}}_{\text{SYM}} \otimes \tilde{\mathfrak{g}} \xrightarrow{\text{color-strip}} \tilde{\mathcal{K}}_{\text{SYM}} \end{array} \right\} \xrightarrow[\text{product}]{\text{tensor}} \mathcal{X}_{\text{SDFT}} \coloneqq (\mathcal{K}_{\text{SYM}} \otimes \tilde{\mathcal{K}}_{\text{SYM}})|_{\text{section}}$$

$$b^- := \frac{1}{2}(b \otimes \tilde{1} - 1 \otimes \tilde{b}), (b^-)^2 = 0.$$

$$\begin{array}{ccccccccccc} X_{-2} & \xrightarrow{\mathbb{B}_1} & X_{-1} & \xrightarrow{\mathbb{B}_1} & X_0 & \xrightarrow{\mathbb{B}_1} & X_1 & \xrightarrow{\mathbb{B}_1} & X_2 & \xrightarrow{\mathbb{B}_1} & X_3 & \xrightarrow{\mathbb{B}_1} & \cdots \\ \mathbb{C} & & \mathbb{A} & & \mathbb{H} & & \mathbb{E} & & \mathbb{N} & & \mathbb{R} & & \cdots \end{array},$$

$$\mathbb{B}_1\coloneqq m_1\otimes\tilde{1}+1\otimes\tilde{m}_1,$$

$$\begin{array}{c} \mathbb{B}_2\coloneqq -\frac{1}{2}b^-(m_2\otimes\tilde{m}_2) \\ \vdots \end{array}$$

$$\mathbb{B}_1(\mathbb{H})+\frac{1}{2}\mathbb{B}_2(\mathbb{H},\mathbb{H})+\frac{1}{6}\mathbb{B}_3(\mathbb{H},\mathbb{H},\mathbb{H})+\cdots=0$$

$$\delta_{\wedge}\mathbb{H}=\mathbb{B}_1(\wedge)+\mathbb{B}_2(\wedge,\mathbb{H})+\cdots.$$

$$\Lambda=\left(\begin{smallmatrix}\Lambda_\mu,\tilde{\Lambda}_{\tilde{\mu}}\\ \eta\end{smallmatrix}\right)\oplus\left(\begin{smallmatrix}\varepsilon^{\tilde{\alpha}},\tilde{\varepsilon}^{\alpha}\\ \emptyset\end{smallmatrix}\right)$$

$$\Lambda_\mu^{\rm triv}=\partial_\mu\tau,\tilde{\Lambda}_{\tilde{\mu}}^{\rm triv}=\tilde{\partial}_{\tilde{\mu}}\tau,\eta^{\rm triv}=\Box\,\tau.$$

$$\mathbb{H}=\mathbb{H}_{\text{NS-NS}}\oplus\mathbb{H}_{\text{NS-R}}\oplus\mathbb{H}_{\text{R-R}}$$

$$\mathbb{H}_{\text{NS-NS}}=\begin{pmatrix} e_{\mu\tilde{\nu}},e,\tilde{e}\\ f_{\mu},\tilde{f}_{\tilde{\mu}}\end{pmatrix}$$

$$\begin{array}{lll} \Box\, e_{\mu\tilde{\nu}} + \tilde{\partial}_{\tilde{\nu}} f_{\mu} - \partial_{\mu} \tilde{f}_{\tilde{\nu}} & = & 0, \\ \partial_{\mu} \tilde{e} - \tilde{\partial}^{\tilde{\nu}} e_{\mu\tilde{\nu}} - f_{\mu} & = & 0, \quad \tilde{\partial}_{\tilde{\nu}} e + \partial^{\mu} e_{\mu\tilde{\nu}} - \tilde{f}_{\tilde{\nu}} = 0 \\ \Box\, e - \partial^{\mu} f_{\mu} & = & 0, \quad \Box\, \tilde{e} - \tilde{\partial}^{\tilde{\mu}} \tilde{f}_{\tilde{\mu}} = 0 \end{array}$$

$$\mathbb{B}_1(\Lambda)=\delta^{\rm lin}_{\Lambda}\mathbb{H}_{\text{NS-NS}}$$

$$\begin{array}{lll} \delta_{\wedge}e_{\mu\tilde{\nu}} & = & \partial_{\mu}\tilde{\Lambda}_{\tilde{\nu}}-\tilde{\partial}_{\tilde{\nu}}\Lambda_{\mu}, \\ \delta_{\wedge}e & = & \partial^{\mu}\Lambda_{\mu}-\eta, \qquad \delta_{\wedge}\tilde{e} \quad = \quad \tilde{\partial}^{\tilde{\mu}}\tilde{\Lambda}_{\tilde{\mu}}-\eta \\ \delta_{\wedge}f_{\mu} & = & \Box\,\Lambda_{\mu}-\partial_{\mu}\eta, \qquad \delta_{\wedge}\tilde{f}_{\tilde{\mu}} = \Box\,\tilde{\Lambda}_{\tilde{\mu}}-\tilde{\partial}_{\tilde{\mu}}\eta \end{array}$$



$$\mathbb{H}_{\text{NS-R}} = \begin{pmatrix} \psi_{\mu}^{\tilde{\alpha}}, \rho_{\tilde{\alpha}}; \tilde{\psi}_{\tilde{\mu}}^{\alpha}, \tilde{\rho}_{\alpha} \\ \varphi^{\tilde{\alpha}}; \tilde{\varphi}^{\alpha} \end{pmatrix}$$

$$\begin{aligned} \gamma^{\tilde{\nu}} \tilde{\partial}_{\tilde{\nu}} \psi_{\mu} - \partial_{\mu} \rho &= 0, \gamma^{\nu} \partial_{\nu} \tilde{\psi}_{\tilde{\mu}} - \tilde{\partial}_{\tilde{\mu}} \tilde{\rho} = 0, \\ \partial^{\mu} \psi_{\mu} - \varphi &= 0, \tilde{\partial}^{\tilde{\mu}} \tilde{\psi}_{\tilde{\mu}} - \tilde{\varphi} = 0, \\ \gamma^{\tilde{\mu}} \tilde{\partial}_{\tilde{\mu}} \rho - \varphi &= 0, \gamma^{\mu} \partial_{\mu} \tilde{\rho} - \tilde{\varphi} = 0, \end{aligned}$$

$$\begin{aligned} \delta_{\wedge} \psi_{\mu} &= \partial_{\mu} \varepsilon, \delta_{\wedge} \tilde{\psi}_{\tilde{\mu}} = \tilde{\partial}_{\tilde{\mu}} \tilde{\varepsilon} \\ \delta_{\wedge} \rho &= \gamma^{\tilde{\mu}} \tilde{\partial}_{\tilde{\mu}} \varepsilon, \delta_{\wedge} \tilde{\rho} = \gamma^{\mu} \partial_{\mu} \tilde{\varepsilon} \\ \delta_{\wedge} \varphi &= \square \varepsilon, \delta_{\wedge} \tilde{\varphi} = \square \tilde{\varepsilon} \end{aligned}$$

$$\mathbb{H}_{\text{R-R}} = \begin{pmatrix} F^{\alpha\tilde{\beta}} \\ \emptyset \end{pmatrix}$$

$$\gamma^{\tilde{\mu}} \tilde{\partial}_{\tilde{\mu}} F = 0, \gamma^{\mu} \partial_{\mu} F = 0.$$

$$\Xi := (\Xi, \tilde{\Xi}) \equiv (\xi, \epsilon),$$

$$\begin{aligned} \mathbb{Z}_1(\Xi) &:= \rho_1(\Xi) \otimes \tilde{1} + 1 \otimes \tilde{\rho}_1(\tilde{\Xi}) \\ \Sigma_2(\Xi) &:= \frac{1}{2} b^- (\rho_2(\Xi) \otimes \tilde{m}_2 + m_2 \otimes \tilde{\rho}_2(\tilde{\Xi})) \end{aligned}$$

$$[\mathbb{B}_1, \Sigma_1(\Xi)] = 0, [\mathbb{B}_1, \Sigma_2(\Xi)] = [\mathbb{Z}_1(\Xi), \mathbb{B}_2]$$

$$\begin{aligned} [\mathbb{Z}_1(\Xi_1), \Sigma_1(\Xi_2)] &= \mathbb{Z}_1(\Xi_{12}) + [\mathbb{B}_1, \mathbb{H}_{2,1}(\Xi_1, \Xi_2)] \\ 2[\mathbb{Z}_1(\Xi_{[1]}, \Sigma_2(\Xi_{[2]})) &= \Sigma_2(\Xi_{12}) + [\mathbb{B}_1, \mathbb{H}_{2,2}(\Xi_1, \Xi_2)] + [\mathbb{H}_{2,1}(\Xi_1, \Xi_2), \mathbb{B}_2] \end{aligned}$$

$$\begin{aligned} \mathbb{H}_{2,1}(\Xi_1, \Xi_2) &:= h_{2,1}(\Xi_1, \Xi_2) \otimes \tilde{1} + 1 \otimes \tilde{h}_{2,1}(\tilde{\Xi}_1, \tilde{\Xi}_2) \\ \mathbb{H}_{2,2}(\Xi_1, \Xi_2) &:= -\frac{1}{2} b^- (h_{2,2}(\Xi_1, \Xi_2) \otimes \tilde{m}_2 + m_2 \otimes \tilde{h}_{2,2}(\tilde{\Xi}_1, \tilde{\Xi}_2) \\ &\quad + \rho_2(\Xi_1) \otimes \tilde{\rho}_2(\tilde{\Xi}_2) - \rho_2(\Xi_2) \otimes \tilde{\rho}_2(\tilde{\Xi}_1)) \end{aligned}$$

$$\begin{aligned} [\Sigma_1(\Xi_1), \Sigma_1(\Xi_2)] &= [\rho_1(\Xi_1) \otimes \tilde{1} + 1 \otimes \tilde{\rho}_1(\tilde{\Xi}_1), \rho_1(\Xi_2) \otimes \tilde{1} + 1 \otimes \tilde{\rho}_1(\tilde{\Xi}_2)] \\ &= \{\rho_1(\Xi_{12}) + [m_1, h_{2,1}(\Xi_1, \Xi_2)]\} \otimes \tilde{1} \\ &\quad + 1 \otimes \{\tilde{\rho}_1(\tilde{\Xi}_{12}) + [\tilde{m}_1, \tilde{h}_{2,1}(\tilde{\Xi}_1, \tilde{\Xi}_2)]\} \\ &= \{\rho_1(\Xi_{12}) \otimes \tilde{1} + [m_1 \otimes \tilde{1} + 1 \otimes \tilde{m}_1, h_{2,1}(\Xi_1, \Xi_2) \otimes \tilde{1}]\} \\ &\quad + \{1 \otimes \tilde{\rho}_1(\tilde{\Xi}_{12}) + [m_1 \otimes \tilde{1} + 1 \otimes \tilde{m}_1, 1 \otimes \tilde{h}_{2,1}(\tilde{\Xi}_1, \tilde{\Xi}_2)]\} \\ &= \Sigma_1(\Xi_{12}) + [\mathbb{B}_1, \mathbb{H}_{2,1}(\Xi_1, \Xi_2)], \end{aligned}$$

$$\begin{aligned} &[\mathbb{Z}_1(\Xi_1), \mathbb{Z}_2(\Xi_2)] - (1 \leftrightarrow 2) \\ &= \left[\rho_1(\Xi_1) \otimes \tilde{1} + 1 \otimes \tilde{\rho}_1(\tilde{\Xi}_1), \frac{1}{2} b^- (\rho_2(\Xi_2) \otimes \tilde{m}_2 + m_2 \otimes \tilde{\rho}_2(\tilde{\Xi}_2)) \right] - (1 \leftrightarrow 2) \\ &= \frac{1}{2} b^- \{ (\rho_2(\Xi_{12}) + [m_1, h_{2,2}(\Xi_1, \Xi_2)] + [h_{2,1}(\Xi_1, \Xi_2), m_2]) \otimes \tilde{m}_2 \\ &\quad - \rho_2(\Xi_1) \otimes [\tilde{m}_1, \tilde{\rho}_2(\tilde{\Xi}_2)] - (1 \leftrightarrow 2) + [m_1, \rho_2(\Xi_1)] \otimes \tilde{\rho}_2(\tilde{\Xi}_2) - (1 \leftrightarrow 2) \\ &\quad + m_2 \otimes (\tilde{\rho}_2(\tilde{\Xi}_{12}) + [\tilde{m}_1, \tilde{h}_{2,2}(\tilde{\Xi}_1, \tilde{\Xi}_2)] + [\tilde{h}_{2,1}(\tilde{\Xi}_1, \tilde{\Xi}_2), \tilde{m}_2]) \} \\ &= \Sigma_2(\Xi_{12}) + [\mathbb{B}_1, \mathbb{H}_{2,2}(\Xi_1, \Xi_2)] + [\mathbb{H}_{2,1}(\Xi_1, \Xi_2), \mathbb{B}_2] \end{aligned}$$



$$[b, \rho_1(\Xi)] = [b, h_{2,1}(\Xi_1, \Xi_2)] = 0$$

$$\delta_{\xi}\mathbb{H}=\mathcal{L}_{\xi}\mathbb{H}$$

$$\begin{aligned}\delta_{\epsilon}e_{\mu\tilde{\nu}} &= i\tilde{\epsilon}^{\tilde{\alpha}}\gamma_{\tilde{\nu}}\tilde{\alpha}^{\tilde{\beta}}\psi_{\mu}^{\tilde{\beta}}-i\epsilon^{\alpha}\gamma_{\mu\alpha\beta}\tilde{\psi}_{\tilde{\nu}}^{\beta},\\ \delta_{\epsilon}e &= i\epsilon^{\alpha}\tilde{\rho}_{\alpha}, & \delta_{\epsilon}\tilde{e} &= i\tilde{\epsilon}^{\tilde{\alpha}}\rho_{\tilde{\alpha}}\\ \delta_{\epsilon}f_{\mu} &= i\epsilon^{\alpha}\gamma_{\mu\alpha\beta}\tilde{\varphi}^{\beta}, & \delta_{\epsilon}\tilde{f}_{\tilde{\mu}} &= i\tilde{\epsilon}^{\tilde{\alpha}}\gamma_{\tilde{\mu}}\tilde{\alpha}^{\tilde{\beta}}\tilde{\varphi}^{\tilde{\beta}};\\ \delta_{\epsilon}\psi_{\mu}^{\tilde{\alpha}} &= \tilde{\partial}_{\tilde{\nu}}e_{\mu\tilde{\rho}}(\gamma^{\tilde{\nu}\tilde{\rho}}\tilde{\epsilon})^{\tilde{\alpha}}-\epsilon^{\beta}\gamma_{\mu\beta\lambda}F^{\lambda\tilde{\alpha}}, & \delta_{\epsilon}\tilde{\psi}_{\tilde{\mu}}^{\alpha} &= -\partial_{\nu}e_{\rho\tilde{\mu}}(\gamma^{\nu\rho}\epsilon)^{\alpha}-\tilde{\epsilon}^{\tilde{\beta}}\gamma_{\tilde{\mu}\tilde{\beta}\tilde{\lambda}}F^{\alpha\tilde{\lambda}},\\ \delta_{\epsilon}\rho_{\tilde{\alpha}} &= \tilde{f}_{\tilde{\mu}}\gamma^{\tilde{\mu}}{}_{\tilde{\alpha}\tilde{\beta}}\tilde{\epsilon}^{\tilde{\beta}}-\tilde{\partial}_{\tilde{\mu}}\tilde{e}^{\tilde{\mu}}{}_{\tilde{\alpha}\tilde{\beta}}\tilde{\epsilon}^{\tilde{\beta}}, & \delta_{\epsilon}\tilde{\rho}_{\alpha} &= f_{\mu}\gamma^{\mu}{}_{\alpha\beta}\epsilon^{\beta}-\partial_{\mu}e\gamma^{\mu}{}_{\alpha\beta}\epsilon^{\beta},\\ \delta_{\epsilon}\varphi^{\tilde{\alpha}} &= \tilde{\partial}_{\tilde{\mu}}\tilde{f}_{\tilde{\nu}}\gamma^{\tilde{\mu}\tilde{\nu}}\tilde{\alpha}^{\tilde{\beta}}\tilde{\epsilon}^{\tilde{\beta}}, & \delta_{\epsilon}\tilde{\varphi}^{\alpha} &= \partial_{\mu}f_{\nu}\gamma^{\mu\nu\alpha}{}_{\beta}\epsilon^{\beta};\\ \delta_{\epsilon}F^{\alpha\tilde{\beta}} &= i\partial_{\mu}\psi_{\nu}{}^{\tilde{\beta}}\gamma^{\mu\nu\alpha}{}_{\lambda}\epsilon^{\lambda}+i\tilde{\partial}_{\tilde{\mu}}\tilde{\psi}_{\tilde{\nu}}{}^{\alpha}\gamma^{\tilde{\mu}\tilde{\nu}}\tilde{\beta}^{\tilde{\lambda}}\tilde{\epsilon}^{\tilde{\lambda}}\end{aligned}$$

$$e_{\mu\nu}=h_{\mu\nu}+B_{\mu\nu},\;h_{\mu\nu}:=e_{(\mu\nu)},B_{\mu\nu}:=e_{[\mu\nu]}$$

$$\delta_{\wedge}h_{\mu\nu}=\partial_{\mu}\lambda_{\nu}+\partial_{\nu}\lambda_{\mu},\delta_{\wedge}B_{\mu\nu}=\partial_{\mu}\zeta_{\nu}-\partial_{\nu}\zeta_{\mu}$$

$$\lambda_{\mu}:=\frac{1}{2}(\tilde{\Lambda}_{\mu}-\Lambda_{\mu}),\zeta_{\mu}:=\frac{1}{2}(\tilde{\Lambda}_{\mu}+\Lambda_{\mu})$$

$$\phi\!:=\!\frac{1}{2}(\tilde{e}-e-h).$$

$$\lambda_{\tilde{\alpha}}\!:=\!\rho_{\tilde{\alpha}}-(\gamma^{\mu})_{\tilde{\alpha}\tilde{\beta}}\psi_{\mu}^{\tilde{\beta}},\tilde{\lambda}_{\alpha}\!:=\!\tilde{\rho}_{\alpha}-(\gamma^{\mu})_{\alpha\beta}\tilde{\psi}_{\mu}^{\beta},$$

$$\psi_{\mu}^{\mathrm{RS}}\!:=\!\psi_{\mu}+\frac{1}{8}\gamma_{\mu}\lambda,\tilde{\psi}_{\mu}^{\mathrm{RS}}\!:=\!\tilde{\psi}_{\mu}+\frac{1}{8}\gamma_{\mu}\tilde{\lambda}$$

$$h_{\mu\nu}^{\mathrm{E}}:=h_{\mu\nu}+\frac{1}{4}\eta_{\mu\nu}\phi.$$

$$\mathrm{Spin}(1,9)\equiv\widetilde{\mathrm{Spin}}(1,9)$$

$$F^{\alpha\beta}=(\gamma^{\mu})^{\alpha\beta}F_{\mu}+\frac{1}{3!}(\gamma^{\mu\nu\rho})^{\alpha\beta}F_{\mu\nu\rho}+\frac{1}{2\cdot5!}(\gamma^{\mu\nu\rho\sigma\tau})^{\alpha\beta}F_{\mu\nu\rho\sigma\tau},$$

$$\begin{aligned}\partial_{[\mu}F_{\nu]}&=0,\partial^{\mu}F_{\mu}=0\\ \partial_{[\mu}F_{\nu\rho\sigma]}&=0,\partial^{\mu}F_{\mu\nu\rho}=0\\ \partial_{[\mu}F_{\nu\rho\sigma\tau\omega]}&=0,\partial^{\mu}F_{\mu\nu\rho\sigma\tau}=0\end{aligned}$$

$$\psi_{\mu}:=\psi_{\mu}^{\mathrm{RS}}+i\tilde{\psi}_{\mu}^{\mathrm{RS}},\mathrm{R}=1/2$$

$$\lambda:=\lambda-i\tilde{\lambda},\mathrm{R}=3/2$$

$$A:=\phi-4iC,\mathrm{R}=2$$

$$A_{\mu\nu}:=B_{\mu\nu}+2iC_{\mu\nu},\mathrm{R}=1$$



$$\begin{aligned}\delta_\epsilon h^{\rm E}_{\mu\nu} &= \frac{1}{2}(\epsilon\gamma_{(\mu}\psi_{\nu)}^* - \epsilon^*\gamma_{(\mu}\psi_{\nu)}) \\ \delta_\epsilon A &= \frac{1}{2}\epsilon\lambda \\ \delta_\epsilon A_{\mu\nu} &= -\epsilon\gamma_{[\mu}\psi_{\nu]} - \frac{1}{8}\epsilon^*\gamma_{\mu\nu}\lambda \\ \delta_\epsilon C_{\mu\nu\rho\sigma} &= \frac{i}{2}(\epsilon^*\gamma_{[\mu\nu\rho}\psi_{\sigma]} + \epsilon\gamma_{[\mu\nu\rho}\psi_{\sigma]}^*) \\ \delta_\epsilon\lambda &= -2i\gamma^\mu\partial_\mu A\epsilon^* - \frac{i}{3}\gamma^{\mu\nu\rho}\mathbf{F}_{\mu\nu\rho}\epsilon \\ \delta_\epsilon\psi_\mu &= -i\partial_\nu h^{\rm E}_{\mu\rho}\gamma^{\nu\rho}\epsilon + \frac{i}{24}(\gamma_\mu^{\nu\rho\sigma}\mathbf{F}_{\nu\rho\sigma} - 9\gamma^{\nu\rho}\mathbf{F}_{\mu\nu\rho})\epsilon^* - \frac{1}{2\cdot 5!}\gamma^{\nu[5]}\gamma_\mu F_{\nu[5]}\epsilon\end{aligned}$$

$$\tilde{X}^{\tilde{\alpha}}\equiv\tilde{X}_{\alpha}$$

$$\Psi_{\mu} \coloneqq (\psi_{\mu}^{\mathrm{RS}\alpha}, \tilde{\psi}_{\mu\alpha}^{\mathrm{RS}}), \lambda \coloneqq (\lambda_{\alpha}, \tilde{\lambda}^{\alpha})$$

$$F^{\alpha}{}_{\beta}=\delta^{\alpha}_{\beta}F+\frac{1}{2!}(\gamma^{\mu\nu})^{\alpha}{}_{\beta}F_{\mu\nu}+\frac{1}{4!}(\gamma^{\mu\nu\rho\sigma})^{\alpha}{}_{\beta}F_{\mu\nu\rho\sigma},$$

$$\partial_\mu F=0,$$

$$\begin{aligned}\partial_{[\mu}F_{\nu\rho]}&=0,\partial^\mu F_{\mu\nu}=0\\ \partial_{[\mu}F_{\nu\rho\sigma\tau]}&=0,\partial^\mu F_{\mu\nu\rho\sigma}=0\end{aligned}$$

$$\eta_{\mu\nu}\coloneqq(-1,1,\ldots,1)$$

$$\epsilon^{0123456789}=-\epsilon_{0123456789}=1,\epsilon^{\mu_1,\ldots,\mu_{10}}\epsilon_{\mu_1,\ldots,\mu_{10}}=-10!$$

$$\{\Gamma^\mu,\Gamma^\nu\}=2\eta^{\mu\nu}$$

$$\Psi^C\coloneqq \Psi^T C = \Psi^\dagger \Gamma_0$$

$$C\coloneqq\begin{pmatrix}0&\delta_{\alpha}^{\dot{\beta}}\\-\delta_{\gamma}^{\dot{\beta}}&0\end{pmatrix}$$

$$\Psi =: \begin{pmatrix} \chi^\alpha \\ \bar{\chi}_{\dot{\beta}} \end{pmatrix}$$

$$C\Gamma^\mu =: \begin{pmatrix} (\gamma^\mu)_{\alpha\beta} & 0 \\ 0 & (\bar{\gamma}^\mu)^{\dot{\beta}\dot{\alpha}} \end{pmatrix}$$

$$(\gamma^\mu)_{\alpha\lambda}(\bar{\gamma}^\nu)^{\lambda\beta}+(\gamma^\nu)_{\alpha\lambda}(\bar{\gamma}^\mu)^{\lambda\beta}=2\eta^{\mu\nu}\delta_\alpha^\beta$$

$$\gamma^{\mu[n]}=\gamma^{\mu_1\ldots\mu_n}\coloneqq\gamma^{[\mu_1}\ldots\gamma^{\mu_n]}, n\leq D,$$

$$(\gamma^\mu)_{\alpha(\beta}(\gamma_\mu)_{\lambda\delta)}=0$$



$$\begin{aligned}
(\gamma^{[10]})_{\alpha}{}^{\beta} &= -\epsilon^{[10]} \delta_{\alpha}{}^{\beta}, & (\gamma^{[10]})^{\alpha}{}_{\beta} &= +\epsilon^{\mu[10]} \delta^{\alpha}{}_{\beta} \\
(\gamma^{\mu[9]})_{\alpha\beta} &= -\epsilon^{\mu[9]\nu[1]} (\gamma_{\nu[1]})_{\alpha\beta}, & (\gamma^{\mu[9]})^{\alpha\beta} &= +\epsilon^{\mu[9]\nu[1]} (\gamma_{\nu[1]})^{\alpha\beta} \\
(\gamma^{\mu[8]})_{\alpha}{}^{\beta} &= +\frac{1}{2} \epsilon^{\mu[8]\nu[2]} (\gamma_{\nu[2]})_{\alpha}{}^{\beta}, & (\gamma^{\mu[8]})^{\alpha}{}_{\beta} &= -\frac{1}{2} \epsilon^{\mu[8]\nu[2]} (\gamma_{\nu[2]})^{\alpha}{}_{\beta},
\end{aligned}$$

$$\begin{aligned}
(\gamma^{\mu[7]})_{\alpha\beta} &= +\frac{1}{3!} \epsilon^{\mu[7]\nu[3]} (\gamma_{\nu[3]})_{\alpha\beta}, & (\gamma^{\mu[7]})^{\alpha\beta} &= -\frac{1}{3!} \epsilon^{\mu[7]\nu[3]} (\gamma_{\nu[3]})^{\alpha\beta} \\
(\gamma^{\mu[6]})_{\alpha}{}^{\beta} &= -\frac{1}{4!} \epsilon^{\mu[6]\nu[4]} (\gamma_{\nu[4]})_{\alpha}{}^{\beta}, & (\gamma^{\mu[6]})^{\alpha}{}_{\beta} &= +\frac{1}{4!} \epsilon^{\mu[6]\nu[4]} (\gamma_{\nu[4]})^{\alpha}{}_{\beta} \\
(\gamma^{\mu[5]})_{\alpha\beta} &= -\frac{1}{5!} \epsilon^{\mu[5]\nu[5]} (\gamma_{\nu[5]})_{\alpha\beta}, & (\gamma^{\mu[5]})^{\alpha\beta} &= +\frac{1}{5!} \epsilon^{\mu[5]\nu[5]} (\gamma_{\nu[5]})^{\alpha\beta}
\end{aligned}$$

$$\begin{aligned}
\xi_{\alpha} \chi^{\beta} &= \frac{1}{16} \delta_{\alpha}^{\beta} (\xi \chi) + \frac{1}{2!} \frac{1}{16} (\gamma^{\mu\nu})_{\alpha}^{\beta} (\xi \gamma_{\mu\nu} \chi) + \frac{1}{4!} \frac{1}{16} (\gamma^{\mu\nu\rho\sigma})_{\alpha}^{\beta} (\xi \gamma_{\mu\nu\rho\sigma} \chi) \\
\xi^{\alpha} \chi^{\beta} &= \frac{1}{16} (\gamma^{\mu})^{\alpha\beta} (\xi \gamma_{\mu} \chi) + \frac{1}{3!} \frac{1}{16} (\gamma^{\mu\nu\rho})^{\alpha\beta} (\xi \gamma_{\mu\nu\rho} \chi) + \frac{1}{5!} \frac{1}{32} (\gamma^{\mu\nu\rho\sigma\tau})^{\alpha\beta} (\xi \gamma_{\mu\nu\rho\sigma\tau} \chi)
\end{aligned}$$

$$\begin{aligned}
\gamma^{\mu} \gamma^{\nu} &= \gamma^{\mu\nu} + \eta^{\mu\nu}, & \gamma^{\mu} \gamma_{\mu} &= D \\
\gamma^{\mu\nu} \gamma^{\rho} &= \gamma^{\mu\nu\rho} + 2\gamma^{[\mu} \eta^{\nu]\rho}, & \gamma^{\mu\nu} \gamma_{\nu} &= (D-1) \gamma^{\mu} \\
\gamma^{\mu\nu\rho} \gamma^{\sigma} &= \gamma^{\mu\nu\rho\sigma} + 3\gamma^{[\mu\nu} \eta^{\rho]\sigma}, & \gamma^{\mu\nu\rho} \gamma_{\rho} &= (D-2) \gamma^{\mu\nu} \\
\gamma^{\mu\nu\rho\sigma} \gamma^{\tau} &= \gamma^{\mu\nu\rho\sigma\tau} + 4\gamma^{[\mu\nu\rho} \eta^{\sigma]\tau}, & \gamma^{\mu\nu\rho\sigma} \gamma_{\sigma} &= (D-3) \gamma^{\mu\nu\rho} \\
\gamma^{\mu\nu\rho\sigma\tau} \gamma^{\lambda} &= \gamma^{\mu\nu\rho\sigma\tau\lambda} + 5\gamma^{[\mu\nu\rho\sigma} \eta^{\tau]\lambda}, & \gamma^{\mu\nu\rho\sigma\tau} \gamma_{\tau} &= (D-4) \gamma^{\mu\nu\rho\sigma} \\
&\vdots & &\vdots \\
\gamma^{\mu_1 \dots \mu_p} \gamma^{\nu} &= \gamma^{\mu_1 \dots \mu_p \nu} + p \gamma^{[\mu_1 \dots \mu_{p-1}} \eta^{\mu_p] \nu}, & \gamma^{\mu_1 \dots \mu_p \nu} \gamma_{\nu} &= (D-p) \gamma^{\mu_1 \dots \mu_p}
\end{aligned}$$

$$\begin{aligned}
\gamma_{\mu\nu} \gamma^{\rho\sigma} &= \gamma_{\mu\nu}^{\rho\sigma} - 4\gamma_{[\mu}^{\rho} \delta_{\nu]}^{\sigma]} - 2\delta_{\mu\nu}^{\rho\sigma} \\
\gamma_{\mu\nu\rho} \gamma^{\sigma\tau} &= \gamma_{\mu\nu\rho}^{\sigma\tau} - 6\gamma_{[\mu\nu}^{\sigma} \delta_{\rho]}^{\tau]} - 6\gamma_{[\mu}^{\sigma} \delta_{\nu\rho]}^{\tau} \\
\gamma_{\mu\nu\rho\sigma} \gamma^{\tau\omega} &= \gamma_{\mu\nu\rho\sigma}^{\tau\omega} - 8\gamma_{[\mu\nu\rho}^{\tau} \delta_{\sigma]}^{\omega]} - 12\gamma_{[\mu\nu}^{\tau} \delta_{\rho\sigma]}^{\omega} \\
\gamma_{\mu\nu\rho\sigma\tau} \gamma^{\omega\lambda} &= \gamma_{\mu\nu\rho\sigma\tau}^{\omega\lambda} - 10\gamma_{[\mu\nu\rho\sigma}^{\omega} \delta_{\tau]}^{\lambda]} - 20\gamma_{[\mu\nu\rho}^{\omega} \delta_{\sigma\tau]}^{\lambda}
\end{aligned}$$

$$\gamma_{\mu_1 \dots \mu_p} \gamma^{\nu\rho} = \gamma_{\mu_1 \dots \mu_p}^{\nu\rho} - 2p \gamma_{[\mu_1 \dots \mu_{p-1}}^{\nu} \delta_{\mu_p]}^{\rho]} - p(p-1) \gamma_{[\mu_1 \dots \mu_{p-2}}^{\nu\rho} \delta_{\mu_{p-1} \mu_p]}$$

$$\begin{aligned}
\gamma_{\mu\rho} \gamma^{\rho\nu} &= (D-1) \delta_{\mu}^{\nu} + (D-2) \gamma_{\mu}^{\nu} \\
\gamma_{\mu\nu} \gamma^{\mu\nu} &= (1-D) D \\
\gamma^{\mu} \gamma^{\nu} \gamma_{\mu} &= (2-D) \gamma^{\nu} \\
\gamma^{\mu} \gamma^{\nu} \gamma^{\rho} \gamma_{\mu} &= (D-4) \gamma^{\nu\rho} + D \eta^{\nu\rho} \\
\gamma_{\mu\nu} \gamma_{\rho} \gamma^{\mu\nu} &= -(D-1)(D-4) \gamma_{\rho} \\
\gamma_{\rho\sigma} \gamma^{\mu\nu} \gamma^{\rho\sigma} &= -[(D-7)(D-2) + 2] \gamma^{\mu\nu} \\
\gamma^{\nu} \gamma_{\mu[p} \gamma_{\nu} &= -(D + (-1)^p 2p) \gamma_{\mu[p} \\
\gamma^{\nu} \gamma_{\mu[5} \gamma_{\nu} &= 0 \text{ for } D = 10, p = 5
\end{aligned}$$

$$\begin{aligned}
[P_{\mu}, P_{\nu}] &= 0 \\
[P_{\mu}, M_{\rho\sigma}] &= -i(\eta_{\mu\rho} P_{\sigma} - \eta_{\mu\sigma} P_{\rho}) \\
[M_{\mu\nu}, M_{\rho\sigma}] &= i(\eta_{\mu\rho} M_{\nu\sigma} - \eta_{\mu\sigma} M_{\nu\rho} - \eta_{\nu\rho} M_{\mu\sigma} + \eta_{\nu\sigma} M_{\mu\rho})
\end{aligned}$$

$$L = \bigoplus_{i=0}^{i=n} L_i,$$



$$[\cdot, \cdot]: L \times L \rightarrow L,$$

$$\begin{aligned} [L_i, L_j] &\in L_{i+j} \bmod n + 1 \\ [L_i, L_j] &= -(-1)^{\eta_i \eta_j} [L_j, L_i] \\ [L_i, \{L_j, L_k\}] &(-1)^{\eta_i \eta_k} + [L_j, [L_k, L_i]] (-1)^{\eta_i \eta_j} + [L_k, [L_i, L_j]] (-1)^{\eta_j \eta_k} \end{aligned}$$

$$L_{\rm SP} = L_0 \oplus L_1,$$

$$\begin{aligned} [P_\mu, Q_\alpha^I] &= [P_\mu, \bar{Q}_\alpha^I] = 0, & \{Q_\alpha^I, \bar{Q}_\alpha^J\} &= 2\delta^{IJ}\sigma_{\alpha\dot{\alpha}}^\mu P_\mu \\ [M_{\mu\nu}, Q_\alpha^I] &= -i(\sigma_{\mu\nu})_\alpha{}^\beta Q_\beta^I, & \{Q_\alpha^I, Q_\beta^J\} &= \epsilon_{\alpha\beta} Z^{IJ} \\ [M_{\mu\nu}, \bar{Q}_\alpha^I] &= -i(\bar{\sigma}_{\mu\nu})_{\dot{\alpha}\dot{\beta}} \bar{Q}^{I\dot{\beta}}, & \{\bar{Q}_\alpha^I, \bar{Q}_\beta^J\} &= \epsilon_{\dot{\alpha}\dot{\beta}} (Z^{IJ})^* \end{aligned}$$

$$J^2|j,m\rangle=j(j+1)|j,m\rangle,J_3|j,m\rangle=m|j,m\rangle$$

$$J_3(Q_1^I|j,m))=\left(m-\frac{1}{2}\right)Q_1^I|j,m),$$

$$\begin{aligned} \langle \psi | \{ Q_\alpha^I, \bar{Q}_\alpha^I \} | \psi \rangle &= 2 \mathcal{N} \sigma_{\alpha\dot{\alpha}}^\mu \langle \psi | P_\mu | \psi \rangle \\ &\stackrel{|}{=} \langle \psi | [Q_\alpha^I (Q_\alpha^I)^\dagger + (Q_\alpha^I)^\dagger Q_\alpha^I] | \psi \rangle = \| Q_\alpha^I | \psi \rangle \|^2 + \| (Q_\alpha^I)^\dagger | \psi \rangle \|^2 \end{aligned}$$

$$\langle \psi | P_0 | \psi \rangle \geq 0$$

$$\mathrm{Tr}(-1)^{N_{\mathrm{F}}} = n_{\mathrm{B}} - n_{\mathrm{F}}.$$

$$\{(-1)^{N_{\mathrm{F}}},Q_{\alpha}^I\}=0$$

$$\begin{aligned} 0 &= \mathrm{Tr}[(-1)^{N_{\mathrm{F}}}Q_{\alpha}^I\bar{Q}_{\dot{\alpha}}^I - \bar{Q}_{\dot{\alpha}}^I(-1)^{N_{\mathrm{F}}}Q_{\alpha}^I] = \mathrm{Tr}[(-1)^{N_{\mathrm{F}}}\{Q_{\alpha}^I,\bar{Q}_{\dot{\alpha}}^I\}] = \\ &= 2\mathcal{N}[\mathrm{Tr}(-1)^{N_{\mathrm{F}}}] \sigma_{\alpha\dot{\alpha}}^\mu P_\mu = 2\mathcal{N}(n_{\mathrm{B}} - n_{\mathrm{F}}) \sigma_{\alpha\dot{\alpha}}^\mu P_\mu \end{aligned}$$

$$\{Q_{\alpha}^I,\bar{Q}_{\dot{\alpha}}^J\}=2\delta^{IJ}\sigma_{\alpha\dot{\alpha}}^\mu P_\mu=\begin{pmatrix}4E&0\\0&0\end{pmatrix}\delta^{IJ}$$

$$a_I=\frac{1}{2\sqrt{E}}Q_{1I},a_I^\dagger=\frac{1}{2\sqrt{E}}\bar{Q}_{\dot{1}I}=(a_I)^\dagger$$

$$\{a_I,a_j^\dagger\}=\delta_{IJ},\{a_I,a_j\}=\{a_I^\dagger,a_j^\dagger\}=0$$

$$a_I|\lambda_0\rangle=0\,\forall I$$

$$|\lambda_0\rangle,a_I^\dagger|\lambda_0\rangle=\left|\lambda_0+\frac{1}{2}\right\rangle_I,\ldots a_{\mathcal{N}+}^\dagger\ldots a_1^\dagger|\lambda_0\rangle=\left|\lambda_0+\frac{\mathcal{N}}{2}\right\rangle,$$

$$\left\{\lambda_0,\lambda_0+\frac{1}{2}\right\}\oplus\left\{-\left(\lambda_0+\frac{1}{2}\right),-\lambda_0\right\},$$

$$\left\{0,\frac{1}{2}\right\}\oplus\left\{-\frac{1}{2},0\right\}$$



$$\left\{\frac{1}{2},1\right\}\oplus\left\{-1,-\frac{1}{2}\right\}$$

$$\left\{-\frac{1}{2},0,0,\frac{1}{2}\right\}\oplus\left\{-\frac{1}{2},0,0,\frac{1}{2}\right\}$$

$$\{Q_\alpha,\bar{Q}_{\dot{\alpha}}\}=2M\delta_{\alpha\dot{\alpha}}$$

$$b_\alpha=\frac{1}{\sqrt{2M}}Q_\alpha,b_\alpha^\dagger=\frac{1}{\sqrt{2M}}\bar{Q}_{\dot{\alpha}}$$

$$\{b_\alpha,b_\alpha^\dagger\}=\delta_{\alpha\dot{\alpha}},\{b_\alpha,b_\beta\}=\{b_\alpha^\dagger,b_\beta^\dagger\}=0$$

$$\begin{array}{l} |j_0=0\rangle\rightarrow\left\{-\frac{1}{2},0,0,\frac{1}{2}\right\} \\ |j_0=\frac{1}{2}\rangle\rightarrow\left\{0,2\times\frac{1}{2},1\right\}\oplus\left\{-1,2\times-\frac{1}{2},0\right\} \end{array}$$

$$\left\{c^r_{\alpha},(c^s_{\beta})^{\dagger}\right\}=(2m+Z_r)\delta^{rs}\delta_{\alpha\beta},\left\{d^r_{\alpha},(d^s_{\beta})^{\dagger}\right\}=(2m-Z_r)\delta^{rs}\delta_{\alpha\beta}$$

$$|Z_r|\leq 2m\,\forall r,$$

$$[\bar{Q}_{\dot{\alpha}},\phi]=0$$

$$[Q_\alpha,\phi]\equiv\psi_\alpha$$

$$\{Q_\alpha,\psi_\beta\}=F_{\alpha\beta},\{\bar{Q}_{\dot{\alpha}},\psi_\beta\}=H_{\dot{\alpha}\beta}$$

$$F_{\alpha\beta}+F_{\beta\alpha}=0\;\leftrightarrow\;F_{\alpha\beta}=F\epsilon_{\alpha\beta},H_{\dot{\alpha}\beta}=2\sigma^\mu_{\beta\dot{\alpha}}[P_\mu,\phi]\sim\partial_\mu\phi$$

$$[Q_\alpha,F]=0, [\bar{Q}_{\dot{\alpha}},F]=-2(\partial_\mu\psi\sigma^\mu)_{\dot{\alpha}}$$

$$\{\phi,\psi,F\}.$$

$$\delta_\epsilon=i(\epsilon Q+\bar{\epsilon}\bar{Q})$$

$$\delta\phi=\sqrt{2}\epsilon\psi$$

$$\delta\psi=i\sqrt{2}(\sigma^\mu\bar{\epsilon})\partial_\mu\phi+\sqrt{2}F\epsilon$$

$$\delta F=i\sqrt{2}\bar{\epsilon}\bar{\sigma}^\mu\partial_\mu\psi$$

$$\left[\delta_{\epsilon_1},\delta_{\epsilon_2}\right]=-2i(\epsilon_1\sigma^\mu\bar{\epsilon}_2-\epsilon_2\sigma^\mu\bar{\epsilon}_1)\partial_\mu$$

$$\theta\rightarrow\theta+\epsilon,\bar{\theta}\rightarrow\bar{\theta}+\bar{\epsilon},\delta_\epsilon x^\mu=i(\epsilon\sigma^\mu\bar{\theta}-\theta\sigma\bar{\epsilon})$$

$$G(x,\theta,\bar{\theta})=e^{i(-x^\mu P_\mu+\epsilon Q+\bar{\epsilon}\bar{Q})}$$

$$\begin{array}{l} F(x^\mu,\theta_\alpha,\bar{\theta}_{\dot{\alpha}})=f(x)+\theta\psi(x)+\bar{\theta}\bar{\chi}(x)+\theta^2M(x)+\bar{\theta}^2\bar{N}(x)+\theta\sigma^\mu\bar{\theta}A_\mu(x)\\ +\theta^2\bar{\theta}\bar{\lambda}(x)+\bar{\theta}^2\theta\zeta(x)+\theta^2\bar{\theta}^2D(x) \end{array}$$

$$e^{i(\epsilon Q+\bar{\epsilon}\bar{Q})}F(x,\theta,\bar{\theta})e^{-i(\epsilon Q+\bar{\epsilon}\bar{Q})}$$



$$Q_\alpha = -i\partial_\alpha - (\sigma^\mu \bar{\theta})_\alpha \partial_\mu, \bar{Q}_{\dot{\alpha}} = i\bar{\partial}_{\dot{\alpha}} + (\theta \sigma^\mu)_{\dot{\alpha}} \partial_\mu$$

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = -2i\sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu$$

$$D_\alpha = \partial_\alpha + i(\sigma^\mu \bar{\theta})_\alpha \partial_\mu, \bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} - i(\theta \sigma^\mu)_{\dot{\alpha}} \partial_\mu$$

$$\{D, D\} = \{\bar{D}, \bar{D}\} = \{D, Q\} = \{D, \bar{Q}\} = \{\bar{D}, Q\} = \{\bar{D}, \bar{Q}\} = 0$$

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i\sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu$$

$$\int d^4x d^2\theta d^2\bar{\theta} F(x, \theta, \bar{\theta})$$

$$\begin{aligned} \delta_\epsilon \int d^4x d^2\theta d^2\bar{\theta} F(x, \theta, \bar{\theta}) &= \int d^4x d^2\theta d^2\bar{\theta} \delta_\epsilon F(x, \theta, \bar{\theta}) \\ &= \int d^4x d^2\theta d^2\bar{\theta} (i\epsilon Q + i\bar{\epsilon} \bar{Q}) F(x, \theta, \bar{\theta}) \\ &= \int d^4x d^2\theta d^2\bar{\theta} [\epsilon^\alpha \partial_\alpha + \bar{\epsilon}_{\dot{\alpha}} \partial^{\dot{\alpha}} + i(\epsilon \sigma^\mu \bar{\theta} - \theta \sigma^\mu \bar{\epsilon}) \partial_\mu] F(x, \theta, \bar{\theta}) \end{aligned}$$

$$\bar{D}_{\dot{\alpha}}\Phi=0$$

$$x^\mu \longrightarrow y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$$

$$D_\alpha = \partial_\alpha + 2i(\sigma^\mu \bar{\theta})_\alpha \partial_\mu, \bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}}$$

$$\Phi(y,\theta)=\phi(y)+\sqrt{2}\theta\psi(y)+\theta^2F(y)$$

$$\begin{aligned} \Phi(x,\theta,\bar{\theta}) = & \phi(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) \\ & - \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \frac{1}{4}\theta^2\bar{\theta}^2\Box\phi(x) \end{aligned}$$

$$S=\int d^4x d^2\theta d^2\bar{\theta} \bar{\Phi} \Phi$$

$$\mathcal{L} = -\partial_\mu \bar{\phi} \partial^\mu \phi - i \psi \sigma^\mu \partial_\mu \bar{\psi} + \bar{F} F,$$

$$\int d^4x d^2\theta d^2\bar{\theta} K(\bar{\Phi},\Phi)$$

$$K(\bar{\Phi},\Phi)\longrightarrow K(\bar{\Phi},\Phi)+\Lambda(\Phi)+\bar{\Lambda}(\bar{\Phi}),$$

$$K_i\equiv\partial_iK\equiv\frac{\partial K}{\partial\phi^{\bar{i}}},$$

$$\begin{aligned} \int d^2\theta d^2\bar{\theta} K(\Phi,\bar{\Phi}) = & K_{i\bar{j}} \left[-\partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{j}} - \left(\frac{i}{2} \bar{\psi}^{\bar{j}} \bar{\sigma}^\mu \partial_\mu \psi^i + h.c. \right) + F^i \bar{F}^{\bar{j}} \right] \\ & - \left[\frac{i}{2} K_{i\bar{j}\bar{k}} (\psi^i \sigma^\mu \bar{\psi}^{\bar{k}} \partial_\mu \phi^j - i \psi^i \psi^j \bar{F}^{\bar{k}}) + h.c. \right] \\ & + \frac{1}{4} K_{i\bar{j}\bar{k}\bar{l}} \psi^i \psi^j \bar{\psi}^{\bar{k}} \bar{\psi}^{\bar{l}} \end{aligned}$$

$$K_{i\bar{j}}=\partial_i\partial_{\bar{j}}K$$



$$\Gamma_{jk}^i = K^{i\bar{l}} K_{j\bar{k}\bar{l}}, \Gamma_{j\bar{k}}^{\bar{l}} = K^{l\bar{i}} K_{j\bar{k}l}, R_{i\bar{j}k\bar{l}} = K_{m\bar{l}} \partial_j \Gamma_{ik}^m$$

$$D_\mu \psi^i = \partial_\mu \psi^i + \Gamma_{jk}^i (\sigma^\mu \psi^j) \partial_\mu \phi^k, D_i V_j = \partial_i V_j - \Gamma_{ij}^l V_l$$

$$\int d^2\theta W(\Phi) + h.c.$$

$$\int d^2\theta W(\Phi) = W_i F^i - \frac{1}{2} W_{ij} \psi^i \psi^j$$

$$S_{\rm chiral} = \int d^4x \left\{ \int d^2\theta d^2\bar\theta K(\Phi, \bar\Phi) + \left(\int d^2\theta W(\Phi) + \text{h.c.} \right) \right\}$$

$$\begin{aligned} \mathcal{L}_{\rm chiral} = & K_{i\bar{j}} \left[-\partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{j}} - \left(\frac{i}{2} \bar{\psi}^{\bar{j}} \bar{\sigma}^\mu \partial_\mu \psi^i + \text{h.c.} \right) + F^i \bar{F}^{\bar{j}} \right] \\ & - \left[\frac{i}{2} K_{ij\bar{k}} (\psi^i \sigma^\mu \bar{\psi}^{\bar{k}} \partial_\mu \phi^j - i \psi^i \psi^j \bar{F}^{\bar{k}}) + \text{h.c.} \right] \\ & + \frac{1}{4} K_{ij\bar{k}\bar{l}} \psi^i \psi^j \bar{\psi}^{\bar{k}} \bar{\psi}^{\bar{l}} + \left(W_i F^i - \frac{1}{2} W_{ij} \psi^i \psi^j + \text{h.c.} \right) \end{aligned}$$

$$F^i = -K^{i\bar{j}} \bar{W}_{\bar{j}} + \Gamma_{jk}^i \psi^j \psi^k,$$

$$\begin{aligned} \mathcal{L}_{\rm chiral} = & -K_{i\bar{j}} \left[\partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{j}} + \left(\frac{i}{2} \bar{\psi}^{\bar{j}} \bar{\sigma}^\mu D_\mu \psi^i + \text{h.c.} \right) \right] - K^{i\bar{j}} D_i W \bar{D}_{\bar{j}} \bar{W} \\ & - \frac{1}{2} (D_i D_j W \psi^i \psi^j + \text{h.c.}) + \frac{1}{4} R_{i\bar{j}k\bar{l}} \psi^i \psi^k \bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{l}} \end{aligned}$$

$$V(\phi,\bar{\phi})=K^{i\bar{j}}D_iW\bar{D}_{\bar{j}}\bar{W},$$

$$K_{\rm WZ}(\Phi)=\bar{\Phi}\Phi, W_{\rm WZ}(\Phi)=\frac{m}{2}\Phi^2+\frac{\lambda}{3}\Phi^3$$

$$\mathcal{L}_{\rm WZ} = -\partial_\mu \phi \partial^\mu \bar{\phi} - \left(\frac{i}{2} \bar{\psi} \bar{\sigma}^\mu \partial_\mu \psi + \text{h.c.} \right) - \frac{m}{2} (\psi \psi + \bar{\psi} \bar{\psi}) - |m\phi + \lambda\phi^2|^2 - \lambda(\phi\psi\psi + \text{h.c.}),$$

$$\delta\phi=\sqrt{2}\epsilon\psi,\delta\psi=i\sqrt{2}(\sigma^\mu\bar{\epsilon})\partial_\mu\phi-\sqrt{2}(m\bar{\phi}+\lambda\bar{\phi}^2)\epsilon,$$

$$V=\bar{V}.$$

$$\begin{aligned} V(x,\theta,\bar{\theta}) = & C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) - \theta\sigma^\mu\bar{\theta}A_\mu(x) \\ & + \frac{i}{2}\theta^2[M(x) + iN(x)] - \frac{i}{2}\bar{\theta}^2[M(x) - iN(x)] \\ & + i\theta^2\bar{\theta}\left[\bar{\lambda}(x) + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)\right] - i\bar{\theta}^2\theta\left[\lambda(x) + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)\right] \\ & + \frac{1}{2}\theta^2\bar{\theta}^2[D(x) + \square C(x)]. \end{aligned}$$

$$V\rightarrow V+i\Lambda-i\bar{\Lambda},$$

$$A_\mu\rightarrow A_\mu+\partial_\mu(\alpha+\bar{\alpha})$$



$$V(x, \theta, \bar{\theta}) = -\theta \sigma^\mu \bar{\theta} A_\mu(x) + i\theta^2 \bar{\theta} \bar{\lambda}(x) - i\bar{\theta}^2 \theta \lambda(x) + \frac{1}{2} \theta^2 \bar{\theta}^2 D(x)$$

$$\begin{aligned}\delta A_\mu &= i(\bar{\epsilon} \bar{\sigma}_\mu \lambda - \bar{\lambda} \bar{\sigma}_\mu \epsilon) \\ \delta \lambda &= iD\epsilon + F_{\mu\nu} \sigma^{\mu\nu} \epsilon \\ \delta D &= -\epsilon \sigma^\mu \partial_\mu \bar{\lambda} - \partial_\mu \lambda \sigma^\mu \bar{\epsilon}\end{aligned}$$

$$W_\alpha = -\frac{1}{4} \bar{D}^2 D_\alpha V$$

$$W_\alpha(y,\theta)=-i\lambda_\alpha(y)+\Big[D(y)\delta^\beta_\alpha-iF_{\mu\nu}(y)(\sigma^{\mu\nu})^\beta_\alpha\Big]+\theta^2(\sigma^\mu\partial_\mu\bar\lambda)_\alpha$$

$$\mathcal{L}=\frac{1}{4}\int d^2\theta W^\alpha W_\alpha+\frac{1}{4}\int d^2\bar\theta\bar W_{\dot\alpha}\bar W^{\dot\alpha}=-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-\left(\frac{i}{2}\lambda\sigma^\mu\partial_\mu\bar\lambda+h.c.\right)+\frac{1}{2}D^2,$$

$$\mathcal{L}_{\rm FI}=2\xi\int d^2\theta d^2\bar\theta V=\xi D$$

$$V=V_aT^a,\Lambda=\Lambda_aT^a,[T^a,T^b]=f^a{}_{bc}T^c,\mathrm{Tr}(T^aT^b)=\delta^{ab}$$

$$W_\alpha=-\frac{1}{8g}\bar{D}^2(e^{-2gV}D_\alpha e^{2gV})$$

$$W_\alpha=-\frac{1}{4}\bar{D}^2D_\alpha V+\frac{g}{4}\bar{D}^2[V,D_\alpha V]$$

$$W^a_\alpha(y,\theta)=-i\lambda^a_\alpha(y)+\Big[D^a(y)\delta^\beta_\alpha-iF^a_{\mu\nu}(y)(\sigma^{\mu\nu})^\beta_\alpha\Big]\theta_\beta+\theta^2(\sigma^\mu\mathcal{D}_\mu\bar\lambda^a)_\alpha$$

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu-gf^a{}_{bc}A^b_\mu A^c_\nu,\mathcal{D}_\mu\lambda^a=\partial_\mu\lambda^a-gf^a{}_{bc}A^b_\mu\lambda^c$$

$$\Phi\,\longrightarrow\,e^{-i\Lambda}\Phi$$

$$\bar{\Phi}e^{2gV}\Phi$$

$$\mathcal{L}_{\rm SYM}=\int d^2\theta d^2\bar{\theta}\bar{\Phi}e^{2gV}\Phi+\left(\int d^2\theta\left[\frac{1}{4}\mathrm{Tr}(W^\alpha W_\alpha)+W(\Phi)\right]+h.c.\right)$$

$$\begin{aligned}\delta\phi^i&=\sqrt{2}\bar{\epsilon}\psi^i, & \delta A^a_\mu&=i(\bar{\epsilon}\bar{\sigma}_\mu\lambda^a-\bar{\lambda}^a\bar{\sigma}_\mu\epsilon) \\ \delta\psi^i&=i\sqrt{2}(\sigma^\mu\bar{\epsilon})\mathcal{D}_\mu\phi^i+\sqrt{2}F^i\epsilon, & \delta\lambda^a&=iD^a\epsilon+F^a_{\mu\nu}\sigma^{\mu\nu}\epsilon \\ \delta F^i&=\sqrt{2}\bar{\epsilon}\bar{\sigma}^\mu\mathcal{D}_\mu\psi+2ig(T_a)^i{}_j\phi^j\bar{\epsilon}\bar{\lambda}^a & \delta D^a&=-\epsilon\sigma^\mu\mathcal{D}_\mu\bar{\lambda}^a-\mathcal{D}_\mu\lambda^a\sigma^\mu\bar{\epsilon}\end{aligned}$$

$$\mathcal{D}_\mu\phi^i=\partial_\mu\phi^i+igA^a_\mu(T_a)^i{}_j\phi^j,\mathcal{D}_\mu\psi^i=\partial_\mu\psi^i+igA^a_\mu(T_a)^i{}_j\psi^j$$

$$V(\phi,\bar{\phi})=\sum_i|F^i|^2+\frac{1}{2}\sum_aD^aD_a,$$

$$F^i=-\bar{W}_{\bar{i}},D^a=-g(T^a)_{ij}\phi^i\bar{\phi}^{\bar{j}}$$

$$F^i=-K^{i\bar{j}}\bar{W}_{\bar{j}}=0,D^a=-g(T^a)_{ij}\phi^i\bar{\phi}^{\bar{j}}-\xi^a=0$$



$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} (\bar{\Phi}_1 e^{2gV} \Phi_1 + \bar{\Phi}_2 e^{-2gV} \Phi_2 + 2\xi V) + \left[\int d^2\theta \left(\frac{1}{4} W^\alpha W_\alpha + m \Phi_1 \Phi_2 \right) + h.c. \right]$$

$$V=m^2(|\phi_1|^2+|\phi_2|^2)+\frac{1}{2}[g(|\phi_1|^2-|\phi_2|^2)+2\xi],$$

$$F_1=-m\bar{\phi}_2=0, F_2=-m\bar{\phi}_1=0, D=-\xi-g(|\phi_1|^2-|\phi_2|^2)=0.$$

$$W(\Phi)=a_i\Phi_i+\frac{1}{2}b_{ij}\Phi^i\Phi^j+\frac{1}{3}c_{ijk}\Phi^i\Phi^j\Phi^k+\cdots$$

$$a_i+b_{ij}\phi^j+c_{ijk}\phi^i\phi^j+\cdots=0.$$

$$K(\Phi,\bar{\Phi})=\bar{\Phi}\Phi, W(\Phi)=f\Phi$$

$$W_{\rm O,R}=\frac{h}{2}X\Phi_1^2+m\Phi_1\Phi_2+fX$$

$$\bar{F}_1=-hX\phi_1-m\phi_2=0$$

$$\bar{F}_2=-m\phi_1=0$$

$$\bar{F}_X=-\frac{h}{2}\phi_1^2-f=0$$

$$y\equiv\left|\frac{hf}{m^2}\right|$$

$$\phi_2=-\frac{h}{m}X\phi, \phi_1=\pm i\sqrt{\frac{2f}{h}\left(\frac{y-1}{y}\right)},$$

$$\langle \delta \psi^i \rangle \sim \langle F^i \rangle \epsilon, \langle \delta \lambda^a \rangle \sim \langle D^a \rangle \epsilon$$

$$G=\frac{1}{f}(\langle F_i\rangle\psi^i+\langle D_a\rangle\lambda^a)$$

$$-\frac{1}{2}(\psi^i\quad\lambda^a)\begin{pmatrix}W_{ij}&-i\sqrt{2}g\bar{\phi}^j(T_b)_{ij}\\-i\sqrt{2}g\bar{\phi}^j(T_a)_{ij}&0\end{pmatrix}\begin{pmatrix}\psi^j\\ \lambda^b\end{pmatrix}$$

$$\chi'(x')=\chi(x)+f\epsilon$$

$$\delta_\epsilon\chi(x)=\chi'(x)-\chi(x)=f\epsilon+\frac{i}{f}[\chi(x)\sigma^\mu\bar{\epsilon}-\epsilon\sigma^\mu\bar{\chi}(x)]\partial_\mu\chi(x)$$

$$[\delta_1,\delta_2]\chi=-2i(\epsilon_1\sigma^\mu\bar{\epsilon}_2-\epsilon_2\sigma^\mu\bar{\epsilon}_1)\partial_\mu\chi$$

$$a^\alpha_\mu=\delta^\alpha_\mu+\frac{i}{f^2}(\chi\sigma^\alpha\partial_\mu\bar{\chi}-\partial_\mu\chi\sigma^\alpha\bar{\chi})$$

$$\mathcal{L}_{\rm VA}=-\frac{f^2}{2}{\rm deta}=-\frac{f^2}{2}-\frac{i}{2}(\chi\sigma^\mu\partial_\mu\bar{\chi}-\partial_\mu\chi\sigma^\mu\bar{\chi})+\cdots$$

$$S^2=0$$



$$S = \frac{G^2}{2F_S} + \sqrt{2}\theta G + \theta^2 F_S$$

$$K(S, \bar{S}) = \bar{S}S, W(S) = fS$$

$$F_S = -f \left(1 + \frac{\bar{G}^2}{4f^4} \square G^2 - \frac{3}{16f^8} G^2 \bar{G}^2 \square G^2 \square \bar{G}^2 \right),$$

$$\mathcal{L}_S = -f^2 - iG\sigma^\mu\partial_\mu\bar{G} + \frac{1}{4f^2}\bar{G}^2\square G^2 - \frac{1}{16f^6}G^2\bar{G}^2\square G^2\square\bar{G}^2,$$

$$S(\Phi - \bar{\Phi}) = 0$$

$$\Phi = \phi + \sqrt{2}\theta\chi_\phi + \theta^2 F_\phi, \phi = A + iB$$

$$B = -\frac{G\sigma^\mu\bar{G}}{2|F_S|^2}\partial_\mu A$$

$$\chi_\phi = -i\sigma^\mu\frac{\bar{G}}{\bar{F}_S}\partial_\mu\phi$$

$$F_\phi = -\partial_\nu\left(\frac{\bar{G}}{\bar{F}_S}\right)\bar{\sigma}^\mu\sigma^\nu\frac{\bar{G}}{\bar{F}_S}\partial_\mu A + \frac{1}{2}\left(\frac{\bar{G}}{\bar{F}_S}\right)^2\square A$$

$$K = h(\mathcal{A})\mathcal{B}^2 + \bar{S}S, W = f(\Phi)S + g(\Phi)$$

$$\Phi = \mathcal{A} + i\mathcal{B} \leftrightarrow \mathcal{A} = \frac{1}{2}(\Phi + \bar{\Phi}), \mathcal{B} = \frac{1}{2i}(\Phi - \bar{\Phi})$$

$$\begin{aligned} \mathcal{L} = & |F_S|^2 + \frac{h}{2}\partial_\mu A\partial^\mu A - \left(\frac{i}{2}\partial_\mu G\sigma^\mu\bar{G} + \text{h.c.}\right) + \frac{1}{4}\frac{\bar{G}^2}{\bar{F}_S}\square\left(\frac{G^2}{F_S}\right) \\ & + \frac{h}{2}\left[i\partial^\mu\left(\frac{G}{F_S}\right)\sigma^\nu\left(\frac{\bar{G}}{\bar{F}_S}\right)\partial_\mu A\partial_\nu A - \frac{i}{2}\partial_\mu\left(\frac{G}{F_S}\right)\sigma^\mu\left(\frac{\bar{G}}{\bar{F}_S}\right)(\partial A)^2 + \text{h.c.}\right] \\ & + \left[fF_S + \frac{i}{2}f'\frac{G\sigma^\mu\bar{G}}{\bar{F}_S}\partial_\mu A + \frac{g''}{2}\left(\frac{\bar{G}}{\bar{F}_S}\right)^2(\partial A)^2 + \frac{g'}{2}\left(\frac{\bar{G}}{\bar{F}_S}\right)^2\square A \right. \\ & \left. - g'\partial_\nu\left(\frac{\bar{G}}{\bar{F}_S}\right)\bar{\sigma}^\mu\sigma^\nu\frac{\bar{G}}{\bar{F}_S}\partial_\mu A + \text{h.c.}\right]. \end{aligned}$$

$$\begin{aligned} \mathcal{L} = & -|f|^2 - \frac{h}{2}\partial_\mu A\partial^\mu A + \left(\frac{i}{2}\partial_\mu G\sigma^\mu\bar{G} + \text{h.c.}\right) - \left(i\frac{f'}{2f} + \text{h.c.}\right)G\sigma^\mu\bar{G}\partial_\mu A \\ & + \frac{h}{2|f|^2}\left[i\partial^\mu G\sigma^\nu\bar{G}\partial_\mu A\partial_\nu A - \frac{i}{2}\partial_\mu G\sigma^\mu\bar{G}\partial_\nu A\partial^\nu A + \text{h.c.}\right] \\ & - \left[\frac{\bar{g}'}{\bar{f}^2}\left(\frac{1}{2}G^2\square A - \partial_\mu G\sigma^\mu\bar{\sigma}^\nu G\partial_\nu A\right) + \xi G^2\partial_\mu A\partial^\mu A + \text{h.c.}\right] \\ & + \frac{1}{4|f|^2}\bar{G}^2\square G^2, \end{aligned}$$

$$\xi \equiv \frac{\bar{g}''}{2\bar{f}^2} - \frac{\bar{g}'\bar{f}'}{\bar{f}^3}$$



$$\delta G^\alpha = \frac{h}{4|f|^2} (\partial A)^2 G^\alpha, \delta G^\alpha = -i \frac{g'}{f^2} (\bar{G} \bar{\sigma}^\mu)^\alpha \partial_\mu A, \delta A = \frac{\bar{g}'}{2h\bar{f}^2} G^2 + h.c.,$$

$$\begin{aligned} \mathcal{L} = & -|f|^2 - \frac{h}{2} \partial_\mu A \partial^\mu A + \left(\frac{i}{2} \partial_\mu G \sigma^\mu \bar{G} + \text{h.c.} \right) + (m G^2 + \text{h.c.}) + \lambda G^2 \bar{G}^2 \\ & - \left(i \frac{f'}{2f} G \sigma^\mu \bar{G} \partial_\mu A + \bar{\xi} G^2 \partial_\mu A \partial^\mu A + \text{h.c.} \right) - \frac{1}{8h} \left(\frac{\bar{g}'^2}{\bar{f}^4} G^2 \square G^2 + \text{h.c.} \right) \\ & + \frac{1}{2|f|^2} \left(h - \frac{2|g'|^2}{|f|^2} \right) (i \partial^\mu G \sigma^\nu \bar{G} \partial_\mu A \partial_\nu A + \text{h.c.}) \\ & + \frac{1}{4|f|^2} \left(1 - \frac{|g'|^2}{h|f|^2} \right) \bar{G}^2 \square G^2, \end{aligned}$$

$$m = \frac{g'f'}{hf}, \lambda = \frac{(f'^2 + ff'')g'^2}{2h^2f^4}, \bar{\xi} = \xi - \frac{g'h'}{4hf^2}$$

$$\mathcal{A}(G, \bar{G} \rightarrow G, \bar{G}) = \frac{h|f|^2 - |g'|^2}{h|f|^4} s^2$$

$$\mathcal{A}(G, G \rightarrow \bar{G}, \bar{G}) = \frac{2\bar{g}'^2}{h\bar{f}^4} s^2$$

$$\mathcal{A}(G, A \rightarrow G, A) = \frac{h|f|^2 - 2|g'|^2}{2|f|^4} s^2$$

$$\left. \frac{d^2}{ds^2} \mathcal{A}(G, \bar{G} \rightarrow G, \bar{G}) \right|_{s=t=0} \geq 0 \Leftrightarrow h|f|^2 \geq |g'|^2$$

$$\left. \frac{d^2}{ds^2} \mathcal{A}(G, A \rightarrow G, A) \right|_{s=t=0} \geq 0 \Leftrightarrow h|f|^2 \geq 2|g'|^2$$

$$\left. \frac{d^2}{ds^2} \left[\mathcal{A}(G, \bar{G} \rightarrow G, \bar{G}) - \frac{1}{2} |\mathcal{A}(G, G \rightarrow \bar{G}, \bar{G})| \right] \right|_{\blacksquare \mathfrak{N}|s=t=0} \geq 0 \Leftrightarrow h|f|^2 \geq 2|g'|^2$$

$$h(0)|f(0)|^2 \geq 2|g'(0)|^2$$

$$\left\{ \left(1 + \frac{h\dot{A}^2}{2|f|^2} \right)^2 k_0^2 - \left[\left(1 - \frac{h\dot{A}^2}{2|f|^2} \right)^2 + 4\dot{A}^2 \frac{|g'|^2}{|f|^4} \right] k_3^3 \right\} G_\alpha = 0$$

$$c_s^2 = 1 - \frac{4\dot{A}^2}{\left(|f(A)|^2 + \frac{h(A)\dot{A}^2}{2}\right)^2} \left(\frac{h(A)}{2} |f(A)|^2 - |g'(A)|^2 \right)$$

$$h(A)|f(A)|^2 \geq 2|g'(A)|^2,$$

$$\mathcal{H} = H + \sqrt{2}\theta\chi_H + \theta^2 F_H,$$

$$S\overline{\mathcal{H}} = \text{chiral} \, .$$



$$\begin{aligned}\chi_H &= -i\sigma^\mu \frac{\bar{G}}{\bar{F}_S} \partial_\mu H \\ F_H &= -\partial_\nu \left(\frac{\bar{G}}{\bar{F}_S} \right) \bar{\sigma}^\mu \sigma^\nu \frac{\bar{G}}{\bar{F}_S} \partial_\mu H + \frac{1}{2} \left(\frac{\bar{G}}{\bar{F}_S} \right)^2 \square H, \\ K &= \kappa(\mathcal{H}, \bar{\mathcal{H}}) + \bar{S}S, W(\mathcal{H}) = f(\mathcal{H})S + g(\mathcal{H}),\end{aligned}$$

$$\begin{aligned}\mathcal{L} &= -|f|^2 + \left(\frac{i}{2} \partial_\mu G \sigma^\mu \bar{G} + \text{h.c.} \right) - \kappa_{H\bar{H}} \partial_\mu H \partial^\mu \bar{H} \\ &+ \frac{1}{4|f|^2} \bar{G}^2 \square G^2 + \frac{\kappa_H \bar{H}}{2|f|^2} (i\partial^\mu G \sigma^\nu \bar{G} + \text{h.c.}) (\partial_\mu H \partial_\nu \bar{H} + \partial_\nu H \partial_\mu \bar{H}) \\ &- \frac{\kappa_{H\bar{H}}}{2|f|^2} (i\partial_\mu G \sigma^\mu \bar{G} + \text{h.c.}) \partial_\nu H \partial^\nu \bar{H} - \left[i \frac{f_H}{f} G \sigma^\mu \bar{G} \partial_\mu H + \text{h.c.} \right] \\ &+ \frac{\kappa_{H\bar{H}}}{|f|^2} \left[\frac{1}{2} G \sigma^\mu \bar{G} (i\partial_\mu H \square \bar{H} + \text{h.c.}) - \epsilon^{\mu\nu\alpha\beta} \partial_\mu G \sigma_\nu \bar{G} \partial_\alpha H \partial_\beta \bar{H} \right] \\ &+ \left[\frac{g'}{f^2} \bar{G} \bar{\sigma}^\mu \sigma^\nu \partial_\nu \bar{G} \partial_\mu H - \frac{g''}{2f^2} \bar{G}^2 (\partial H)^2 - \frac{g'}{2f^2} \bar{G}^2 \square H + \text{h.c.} \right].\end{aligned}$$

$$\begin{aligned}\delta G^\alpha &= -\frac{\kappa_H \bar{H}}{2|f|^2} \partial_\mu H \partial^\mu \bar{H} G^\alpha, & \delta H &= -\frac{i}{2|f|^2} G \sigma^\mu \bar{G} \partial_\mu H, \\ \delta G^\alpha &= i \frac{g'}{f^2} (\bar{G} \bar{\sigma}^\mu)^\alpha \partial_\mu H, & \delta H &= \frac{\bar{g}'}{2\kappa_{H\bar{H}} f^2} G^2,\end{aligned}$$

$$\begin{aligned}\mathcal{L} &= -|f|^2 - \kappa_{H\bar{H}} \partial_\mu H \partial^\mu \bar{H} + \left(\frac{i}{2} \partial_\mu G \sigma^\mu \bar{G} + \text{h.c.} \right) - (m_1 G^2 + \text{h.c.}) \\ &+ 2[\bar{m}_1(\bar{k}\bar{H} - kH)\bar{G}^2 + \text{h.c.}] + \frac{1}{4|f|^2} \left(1 - \frac{|g'|^2}{|f|^2 \kappa_{H\bar{H}}} \right) \bar{G}^2 \square G^2 \\ &+ \frac{1}{2} (k^2 H^2 + \bar{k}^2 \bar{H}^2 - 2|k|^2 |H|^2) (m_1 G^2 + \bar{m}_1 \bar{G}^2) - m_2 G^2 \bar{G}^2 \\ &+ \frac{1}{2|f|^2} \left(\kappa_{H\bar{H}} + \frac{|g'|^2}{|f|^2} \right) (i\partial^\mu G \sigma^\nu \bar{G} \partial_\mu H \partial_\nu \bar{H} + \text{h.c.}) \\ &+ \frac{1}{2|f|^2} \left(\kappa_{H\bar{H}} - 3 \frac{|g'|^2}{|f|^2} \right) (i\partial^\mu G \sigma^\nu \bar{G} \partial_\mu \bar{H} \partial_\nu H + \text{h.c.}) \\ &- [\lambda_1 G^2 \partial_\mu H \partial^\mu \bar{H} + \lambda_2 \bar{G}^2 (\partial H)^2 + \text{h.c.}] \\ &- \left[\frac{i(2f_H \bar{f} - \partial_H |f|^2)}{2|f|^2} G \sigma^\mu \bar{G} \partial_\mu H + \text{h.c.} \right] \\ &- \frac{1}{|f|^2} \left(\kappa_{H\bar{H}} + \frac{|g'|^2}{|f|^2} \right) \epsilon^{\mu\nu\alpha\beta} \partial_\mu G \sigma_\nu \bar{G} \partial_\alpha H \partial_\beta \bar{H},\end{aligned}$$

$$\kappa_{H,\bar{H}}(0)|f(0)|^2 \geq |g'(0)|$$

$$c_s^2 = 1 - \frac{4|\dot{H}|^2}{(|f|^2 + \kappa_{H\bar{H}}|\dot{H}|^2)^2} [\kappa_{H\bar{H}}|f|^2 - |g'|^2],$$



$$\kappa_{H,\bar{H}}(H,\bar{H})|f(H)|^2 \geq |g'(H)|,$$

$$SQ=0,$$

$$Q=Q+\sqrt{2}\theta\chi_Q+\theta^2F_Q.$$

$$Q=\frac{\chi_Q G}{F_S}-\frac{F_Q}{2F_S^2}G^2.$$

$$K=S\bar{S}+Q\bar{Q}+\frac{1}{2}Q\bar{Q}(h_1Q+\bar{h}_1\bar{Q})+\frac{h_2}{4}(Q\bar{Q})^2, W=f(Q)S+g(Q),$$

$$\begin{aligned} \mathcal{L} = & \left(\frac{i}{2} \partial_\mu G \sigma^\mu \bar{G} + \text{h.c.} \right) + \left(\frac{i}{2} \partial_\mu \chi_q \sigma^\mu \bar{\chi}_q + \text{h.c.} \right) + \frac{1}{2|f|^2} \left[\frac{g'}{f} \partial_\mu (\chi_q G) \partial^\mu \bar{G}^2 + \text{h.c.} \right] \\ & + \frac{1}{|f|^2} (\bar{\chi}_q \bar{G}) \square (\chi_q G) + \frac{1}{4} |f|^2 \left(1 + \frac{|g'|^2}{|f|^2} \right) \bar{G}^2 \square G^2 + \cdots, \end{aligned}$$

$$\bar{S}SQ_L=0,$$

$$Q_{\rm L}=-2\frac{\bar{D}_{\dot{\alpha}}\bar{S}\bar{D}^{\dot{\alpha}}Q_{\rm L}}{\bar{D}^2\bar{S}}-\frac{\bar{S}\bar{D}^2Q_{\rm L}}{\bar{D}^2\bar{S}}-2\frac{D^{\alpha}SD_{\alpha}\bar{D}^2(\bar{S}Q_{\rm L})}{D^2S\bar{D}^2\bar{S}}-S\frac{D^2\bar{D}^2(\bar{S}Q_{\rm L})}{D^2S\bar{D}^2\bar{S}},$$

$$\bar{S}S(\Phi-\bar{\Phi})=0$$

$$\bar{S}SD_{\alpha}\Phi=0$$

$$\bar{S}SD^2\Phi=0$$

$$\bar{S}S(\Phi-\bar{\Phi})=0,\bar{S}SD_{\alpha}\Phi=0$$

$$\begin{aligned} B &= \frac{i}{4} \left[\left(\frac{\bar{G}}{\bar{F}_S} \right)^2 \bar{F}_\phi - \left(\frac{G}{F_S} \right)^2 F_\phi \right] - \frac{G \sigma^\mu \bar{G}}{2|F_S|^2} \partial_\mu A + \cdots \\ \psi_\alpha &= \frac{F_\phi}{F_S} G_\alpha - i \frac{(\sigma^\mu \bar{G})_\alpha}{\bar{F}_S} \partial_\mu A + \frac{i}{4} \frac{(\sigma^\mu \bar{G})_\alpha}{\bar{F}_S} \partial_\mu \left[\left(\frac{G}{F_S} \right)^2 F_\phi \right] + \cdots \end{aligned}$$

$$K=\bar{S}S+\kappa(\Phi,\bar{\Phi}), W(\Phi)=f(\Phi)S+g(\Phi)$$

$$\begin{aligned} \mathcal{L} = & - \left[|f|^2 + \frac{|g'|^2}{\kappa_{\phi\bar{\phi}}} \right] - \kappa_{\phi\bar{\phi}} \partial_\mu A \partial^\mu A + Z \left(\frac{i}{2} \partial_\mu G \sigma^\mu \bar{G} + \text{h.c.} \right) + (mG^2 + \text{h.c.}) \\ & + cG^2 \bar{G}^2 - Z \left(i \frac{f'}{2f} + \text{h.c.} \right) G \sigma^\mu \bar{G} \partial_\mu A + (dG^2 + \text{h.c.}) (\partial A)^2 \\ & + \left[\frac{\bar{g}'}{\bar{f}^2} \partial_\mu G \sigma^\mu \bar{\sigma}^\nu G \partial_\nu A + \frac{\kappa_{\phi\bar{\phi}}}{|f|^2} i \partial^\mu G \sigma^\nu \bar{G} \partial_\mu A \partial_\nu A - \frac{\kappa_{\phi\bar{\phi}}}{2|f|^2} i \partial_\mu G \sigma^\mu \bar{G} (\partial A)^2 + \text{h.c.} \right] \\ & + \frac{1}{4|f|^2} \left(1 + \frac{|g'|^2}{2\kappa_{\phi\bar{\phi}}|f|^2} \right) \bar{G}^2 \square G^2 - \frac{1}{16\kappa_{\phi\bar{\phi}}} \left(\frac{\bar{g}'^2}{\bar{f}^4} G^2 \square G^2 + \text{h.c.} \right), \end{aligned}$$

$$Z\equiv 1+\frac{|g'|^2}{\kappa_{\phi\bar{\phi}}|f|^2}$$



$$\delta G^\alpha = \frac{\kappa_{\phi\bar{\phi}}}{2Z|f|^2} G^\alpha (\partial A)^2, \delta G^\alpha = -\frac{i}{Z} \frac{g'}{f^2} (\bar{G}\bar{\sigma}^\mu)^\alpha \partial_\mu A$$

$$\begin{aligned} \mathcal{L} = & -\left[|f|^2 + \frac{|g'|^2}{\kappa_{\phi\bar{\phi}}}\right] - \kappa_{\phi\bar{\phi}} \partial_\mu A \partial^\mu A + Z \left(\frac{i}{2} \partial_\mu G \sigma^\mu \bar{G} + \text{h.c.} \right) \\ & + (mG^2 + \text{h.c.}) + \lambda G \sigma^\mu \bar{G} \partial_\mu A + cG^2 \bar{G}^2 + (\tilde{d}G^2 + \text{h.c.}) \partial_\mu A \partial^\mu A \\ & + \left(\frac{\kappa_{\phi\bar{\phi}}}{|f|^2 + \frac{|g'|^2}{\kappa_{\phi\bar{\phi}}}} \right) (i\partial^\mu G \sigma^\nu \bar{G} \partial_\mu A \partial_\nu A + \text{h.c.}) + \frac{1}{4|f|^2} \left(1 + \frac{|g'|^2}{2\kappa_{\phi\bar{\phi}}|f|^2} \right) \bar{G}^2 \square G^2 \\ & - \frac{1}{16\kappa_{\phi\bar{\phi}}} \left(\frac{\bar{g}'^2}{\bar{f}^4} G^2 \square G^2 + \text{h.c.} \right), \end{aligned}$$

$$c_s^2=1-\frac{4\dot{A}^2|f|^2\kappa_{\phi\bar{\phi}}}{\left(|f|^2+\frac{|g'|^2}{\kappa_{\phi\bar{\phi}}}+\kappa_{\phi\bar{\phi}}\dot{A}^2\right)^2},$$

$$\epsilon \longrightarrow \epsilon(x).$$

$$\delta\psi_\mu\sim\partial_\mu\epsilon,$$

$$\frac{1}{M_{\rm P}}\bar{\psi}_\mu \Xi^\mu,$$

$$\delta \Xi^\mu \sim T^{\mu\nu} \gamma_\nu \epsilon.$$

$$g_{\mu\nu}T^{\mu\nu}.$$

$$\delta\psi_\mu\sim M_{\rm P}\partial_\mu\epsilon,$$

$$\delta g_{\mu\nu}\sim \frac{1}{M_{\rm P}}\bar{\epsilon}\gamma_{(\mu}\psi_{\nu)}.$$

$$T_A=\{P_a,M_{ab},Q_\alpha\},$$

$$A_\mu^AT_A=e_\mu{}^aP_a+\frac{1}{2}\omega_\mu^{ab}M_{ab}+\bar{\psi}_\mu Q.$$

$$\delta A_\mu^A=\partial_\mu\epsilon^A+f^A{}_{BC}A_\mu^B\epsilon^C,$$

$$\delta e_\mu^a=\frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu,\delta\psi_\mu=D_\mu\epsilon,$$

$$D_\mu \epsilon = \partial_\mu \epsilon + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} \epsilon.$$

$$T_{\mu\nu}^a=2\big(\partial_{[\mu}e_{\nu]}^a+\omega_{[\mu b}^ae_{\nu]}^b\big)-\frac{1}{2}\bar{\psi}_{[\mu}\gamma^a\psi_{\nu]},$$



$$\hat{\omega}_\mu^{ab} \equiv \omega_\mu^{ab}(e, \psi) = \omega_\mu^{ab}(e) - \frac{1}{4}(\bar{\psi}^{[a}\gamma_\mu\psi^{b]} - \bar{\psi}_\mu\gamma^{[a}\psi^{b]} - \bar{\psi}^{[a}\gamma^{b]}\psi_\mu).$$

$$e^{-1}\mathcal{L}_{\text{SUGRA}}=\frac{1}{2}R(\hat{\omega})-\frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu(\hat{\omega})\psi_\rho,$$

$$\begin{aligned}\delta e_\mu{}^a &= \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu, \\ \delta\psi_\mu &= D_\mu(\hat{\omega})\epsilon.\end{aligned}$$

$$e^{-1}\mathcal{L}_{\text{SUGRA}}=\frac{1}{2}R-\frac{1}{2}\psi_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho+\mathcal{L}_{\text{torsion}},$$

$$e^{-1}\mathcal{L}_{\text{torsion}}=-\frac{1}{32}\big[(\bar{\psi}^\mu\gamma^\nu\psi^\rho)(\bar{\psi}_\mu\gamma_\nu\psi_\rho+\bar{\psi}_\mu\gamma_\rho\psi_\nu)-4(\bar{\psi}_\mu\gamma^\rho\psi_\rho)(\bar{\psi}^\mu\gamma^\rho\psi_\rho)\big].$$

$$[\delta_1,\delta_2]=\delta_\xi-\delta_{\hat\Lambda}-\delta_{\hat\epsilon},$$

$$\xi^a=-\frac{1}{2}\bar{\epsilon}_1\gamma^a\epsilon_2,\hat{\Lambda}^{ab}=\xi^\rho\omega_\rho^{ab},\hat{\epsilon}=\xi^\rho\psi_\rho$$

$$K(\phi,\bar{\phi})\,\rightarrow\,K(\phi,\bar{\phi})+\Lambda(\phi)+\bar{\Lambda}(\bar{\phi}),$$

$$\psi_\mu\rightarrow e^{-\frac{i}{2}\Im\mathrm{m}(\Lambda)\gamma_5}\psi_\mu$$

$$\mathcal{D}_\mu\psi_\nu=D_\mu\psi_\nu+\frac{i}{2}Q_\mu\gamma_5\psi_\nu,$$

$$Q_\mu=\frac{i}{2}\big[K_{\bar i}\partial_\mu\bar\phi^{\bar i}-K_i\partial_\mu\phi^i\big].$$

$$\chi_{\mu L}^i\rightarrow e^{\frac{i}{2}\Im\mathrm{m}(\Lambda)\gamma_5}\chi_{\mu L}^i,\mathcal{D}_\mu\chi_{\nu L}^i=D_\mu\chi_{\nu}^i+\Gamma_{jk}^i\partial_\mu\phi^j\chi_L^k-\frac{i}{2}Q_\mu\gamma_5\chi_L^i,$$

$$e^{\frac{K}{2}W}$$

$$W(\phi)\rightarrow e^{-\Lambda(\phi)}W(\phi)$$

$$e^{\frac{K}{2}}\mathcal{D}_iW=e^{\frac{K}{2}}(\partial_i+K_i)W.$$

$$\mathcal{G}\equiv K+\log\,|W|^2,$$

$$\begin{aligned}e^{-1}\mathcal{L}=&\frac{1}{2}R-\frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}\mathcal{D}_\mu\psi_\nu-K_{i\bar{j}}\Big(\partial_\mu\phi^i\partial^\mu\bar{\phi}^{\bar{j}}+\bar{\chi}_L^i\Big/\mathcal{D}_{R\bar{j}}+\bar{\chi}_R^{\bar{j}}\Big/\mathcal{D}_L^i\Big)\\&+\Big[\frac{1}{2}e^{\frac{K}{2}}W\bar{\psi}_{\mu R}\gamma^{\mu\nu}\psi_{\nu R}-e^{\frac{K}{2}}\mathcal{D}_i\mathcal{D}_{\bar{j}}W\bar{\chi}_L^i\chi_L^{\bar{j}}+\text{h.c.}\Big]\\&+\Big[K_{i\bar{j}}\bar{\psi}_{\mu L}\gamma^\nu\gamma^\mu\chi_L^i\partial_\nu\bar{\phi}^{\bar{j}}+\bar{\psi}_{\mu R}\gamma^\mu\chi_L^ie^{\frac{K}{2}}\mathcal{D}_{\bar{i}}W+\text{h.c.}\Big]-V(\phi,\bar{\phi})\\&+\mathcal{L}_{4\text{ F}}.\end{aligned}$$

$$V(\phi,\bar{\phi})=e^K(K^{i\bar{j}}\mathcal{D}_iW\overline{\mathcal{D}_{\bar{j}}W}-3|W|^2),$$



$$V = e^{\mathcal{G}}(\mathcal{G}^{i\bar{j}}\mathcal{G}_i\mathcal{G}_{\bar{j}} - 3)$$

$$\mathcal{L}_{4\text{ F}} = \mathcal{L}_{\text{torsion}} - \frac{1}{2}K_{i\bar{j}}\bar{\psi}_{\mu}\chi^i\bar{\psi}^{\mu}\chi^{\bar{j}} + \frac{1}{4}(R_{i\bar{j}k\bar{l}} - K_{i\bar{j}}K_{k\bar{l}})\bar{\chi}^i\chi^k\bar{\chi}^{\bar{j}}\chi^{\bar{l}}$$

$$\delta e_{\mu}^a = \frac{1}{2}\bar{\epsilon}\gamma^a\psi_{\mu}, \delta\psi_{\mu L} = \mathcal{D}_{\mu}\epsilon_L + \frac{1}{2}e^{\frac{K}{2}}W\gamma_{\mu}\epsilon_R$$

$$\delta\phi^i = \bar{\epsilon}_L\chi_L^i, \delta\chi_L^i = \frac{1}{2}\partial\phi^i\epsilon_R - \frac{1}{2}e^{\frac{K}{2}}K^{i\bar{j}}\overline{\mathcal{D}}_{\bar{j}}\bar{W}\epsilon_L$$

$$V(\phi,\bar{\phi})=e^K(K^{i\bar{j}}\mathcal{D}_iW\overline{\mathcal{D}}_{\bar{j}}\bar{W}-3|W|^2)+\frac{1}{2}(\Re f_{IJ})D^ID^J,$$

$$\langle\delta\psi_{\mu L}\rangle=\langle\mathcal{D}_{\mu}\epsilon_L\rangle+\frac{1}{2}\left\langle e^{\frac{K}{2}}W\right\rangle\gamma_{\mu}\epsilon_R=0$$

$$\langle\delta\chi_L^i\rangle=-\frac{1}{2}\left\langle K^{i\bar{j}}e^{\frac{K}{2}}\overline{\mathcal{D}}_{\bar{j}}\bar{W}\right\rangle\epsilon_L=0$$

$$\langle\delta\lambda_L^I\rangle=\frac{i}{2}\langle D^I\rangle\epsilon_L=0$$

$$\langle\mathcal{D}_iW\rangle=0,\langle D^I\rangle=0$$

$$V_0=-3\langle e^K|W|^2\rangle$$

$$\Lambda_{\rm supy}^4=\langle e^K K^{i\bar{j}}\mathcal{D}_iW\overline{\mathcal{D}}_{\bar{j}}\bar{W}\rangle+\frac{1}{2}\langle(\Re f_{IJ})D^ID^J\rangle,$$

$$V_0=\Lambda_{\rm supy}^4-3\langle e^K|W|^2\rangle.$$

$$G_L=\left\langle\frac{\mathcal{D}_iW}{W}\right\rangle\chi_L^i$$

$$\delta G_L=-\sqrt{3}\left\langle e^{\frac{K}{2}}\bar{W}\right\rangle\epsilon_L$$

$$\begin{aligned}\mathcal{L} = & -\frac{1}{2}\bar{\psi}_{\mu}\gamma^{\mu\nu\rho}\partial_{\nu}\psi_{\rho} - (\bar{G}_L\,\partial G_R + \bar{G}_R\,\partial G_L) \\ & + \frac{1}{2}\left[\left\langle e^{\frac{K}{2}}W\right\rangle\left(\bar{G}_L\,G_L + 2\sqrt{3}\bar{\psi}_{\mu R}\gamma^{\mu}G_L + \bar{\psi}_{\mu R}\gamma^{\mu\nu}\psi_{\nu R}\right) + h.c.\right].\end{aligned}$$

$$\psi_{\mu L}\rightarrow\psi_{\mu L}+\frac{2}{\sqrt{3}}\left\langle e^{\frac{K}{2}}\bar{W}\right\rangle^{-1}\partial_{\mu}G_L+\frac{1}{\sqrt{3}}\gamma_{\mu}G_R$$

$$\mathcal{L}=-\frac{1}{2}\bar{\psi}_{\mu}\gamma^{\mu\nu\rho}\partial_{\nu}\psi_{\rho}+\frac{1}{2}(m_{3/2}\bar{\psi}_{\mu R}\gamma^{\mu\nu}\psi_{\nu R}+\text{ h.c. }\,)$$

$$\delta\psi_{\mu L}=\partial_{\mu}\epsilon_L+\left\langle e^{\frac{K}{2}}\Big|W|\right\rangle\gamma_{\mu}\epsilon_R$$

$$\epsilon=\frac{2}{\sqrt{3}}\left\langle e^{\frac{K}{2}}\Big|W|\right\rangle^{-1}G$$



$$|m_{3/2}|=\frac{1}{M_{\rm P}^2}\Big\langle e^{\frac{K}{2}}\Big|W|\Big\rangle\sim\frac{\Lambda_{\rm susy}^2}{M_{\rm P}},$$

$$\psi_\mu\rightarrow\frac{1}{m_{3/2}}\partial_\mu G$$

$$\frac{1}{M_{\rm P}}\,\bar{\Xi}^\mu\psi_\mu\longrightarrow\frac{1}{m_{3/2}M_{\rm P}}\,\bar{\Xi}^\mu\partial_\mu G\sim\frac{1}{\Lambda_{\rm su/y}^2}\partial_\mu\bar{\Xi}^\mu G$$

$$e^{-1}\mathcal{L}=\frac{1}{2}R-\frac{1}{2}\bar{\Psi}_\mu\gamma^{\mu\nu\rho}D_\nu\Psi_\rho+\frac{m(\phi)}{2}\bar{\Psi}_\mu\gamma^{\mu\nu}\Psi_\nu-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)+e^{-1}\mathcal{L}_{\rm torsion}\,,$$

$$m(\phi)=g(\phi)$$

$$V(\phi)=f^2(\phi)-3g^2(\phi).$$

$$g_{\mu\nu}=a(\tau)^2\eta_{\mu\nu}$$

$$G_{\mu\nu}=T_{\mu\nu}^{(\phi)},$$

$$G_{\mu\nu}=a^2\left[\left(4H^2-\frac{R}{3}\right)\delta_\mu^0\delta_\nu^0+\left(H^2-\frac{R}{3}\right)\eta_{\mu\nu}\right], R=6\frac{a''}{a^3},$$

$$H=\frac{a'}{a^2},$$

$$\begin{aligned} T_{\mu\nu}^{(\phi)}&= \partial_\mu\phi\partial_\nu\phi-g_{\mu\nu}\left(\frac{1}{2}\partial_\rho\phi\partial^\rho\phi+V(\phi)\right)=\\ &= a^2\delta_\mu^0\delta_\nu^0\left(\frac{\phi'^2}{2a^2}+V(\phi)\right)+a^2\delta_{ij}\delta_\mu^i\delta_\nu^j\left(\frac{\phi'^2}{2a^2}-V(\phi)\right). \end{aligned}$$

$$3(H^2+m^2)\!=\!\frac{\phi'^2}{2a^2}+f^2,$$

$$\left(H^2-3m^2-\frac{R}{3}\right)\!=\!\frac{\phi'^2}{2a^2}-f^2,$$

$$e^{-1}\mathcal{L}_\Psi=-\frac{1}{2}\bar{\Psi}_\mu\gamma^{\mu\nu\rho}\nabla_\nu\Psi_\rho+\frac{m}{2}\bar{\Psi}_\mu\gamma^{\mu\nu}\Psi_\nu$$

$$\gamma^{\mu\nu\rho}\nabla_\nu\Psi_\rho=m\gamma^{\mu\nu}\Psi_\nu$$

$$\gamma^{\mu\nu}\nabla_\mu\Psi_\nu=\frac{3}{2}m\gamma^\mu\Psi_\mu$$

$$G_{\mu\nu}\gamma^\mu\Psi^\nu\,=\,3m^2\gamma^\mu\Psi_\mu+2(\partial_\mu m)\gamma^{\mu\nu}\Psi_\nu$$

$$\triangle\Psi^\mu-\nabla^\mu(\gamma^\rho\Psi_\rho)=-\frac{m}{2}(\gamma^{\mu\rho}\Psi_\rho+3\Psi^\mu)$$

$$\Psi_\mu=\frac{1}{\sqrt{a}}\psi_\mu$$



$$\mathcal{L}_\psi = -\frac{1}{2}\bar{\psi}_\mu\bar{\gamma}^{\mu\nu\rho}\partial_\nu\psi_\rho + \frac{am}{2}\bar{\psi}_\mu\bar{\gamma}^{\mu\nu}\psi_\nu - aH\bar{\psi}_\rho\bar{\gamma}^\rho\psi_0$$

$$\bar{\gamma}^{\mu\nu\rho}\partial_\nu\psi_\rho - am\bar{\gamma}^{\mu\rho}\psi_\rho + aH(\bar{\gamma}^\mu\psi_0 - \delta_0^\mu\bar{\gamma}^\rho\psi_\rho) = 0$$

$$\gamma^\mu = e^\mu{}_a \gamma^a = a(\tau)^{-1} \delta^\mu{}_a \gamma^a \equiv a(\tau)^{-1} \bar{\gamma}^\mu$$

$$\partial_\mu\psi^\mu - \partial(\bar{\gamma}^\rho\psi_\rho) - 2aH\psi_0 + \frac{1}{2}(3am - aH\bar{\gamma}^0)(\bar{\gamma}^\rho\psi_\rho) = 0$$

$$\bar{\gamma}^0\psi_0 = C(\tau)\bar{\gamma}^i\psi_i$$

$$C(\tau) = C_{\rm R}(\tau) + C_{\rm I}(\tau)\bar{\gamma}_0 = \frac{H^2 - 3m^2 - R/3}{3(H^2 + m^2)} + \frac{2m'}{3a(H^2 + m^2)}\bar{\gamma}_0$$

$$\psi_\mu(\tau,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \psi_{\mu,k}(\tau) e^{i\vec{k}\cdot\vec{x}}$$

$$\psi_{i,k} = \psi_{i,k}^{\rm t} + (P_\gamma)_i \vec{\gamma} \cdot \vec{\psi}_k + (P_k)_i \vec{k} \cdot \vec{\psi}_k$$

$$(P_\gamma)_i = \frac{1}{2} \left[\frac{k_i}{\vec{k}^2} (\vec{k} \cdot \vec{\gamma}) - \bar{\gamma}_i \right], (P_k)_i = \frac{1}{2\vec{k}^2} [\bar{\gamma}_i (\vec{k} \cdot \vec{\gamma}) - 3k_i].$$

$$(P_\gamma)^i \psi_{i,k}^{\rm t} = (P_k)^i \psi_{i,k}^{\rm t} = 0$$

$$\vec{k} \cdot \vec{\psi}_k = [\vec{k} \cdot \vec{\gamma} + ia(m - H\bar{\gamma}^0)]\vec{\gamma} \cdot \vec{\psi}_k, \bar{\gamma}^0\psi_{0,k} = C(\tau)(\vec{\gamma} \cdot \vec{\psi}_k)$$

$$\psi_{i,k} = \psi_{i,k}^{\rm t} + \hat{O}_i \theta_k$$

$$\hat{O}_i = \frac{1}{\vec{k}^2} \left\{ -k_i (\vec{k} \cdot \vec{\gamma}) + \frac{ia}{2} [\bar{\gamma}_i (\vec{k} \cdot \vec{\gamma}) - 3k_i] (m - H\bar{\gamma}^0) \right\}.$$

$$\theta_k = \sqrt{\frac{3(m^2 + H^2)a^2}{2\vec{k}^2}} \bar{\gamma}^0 \vartheta_k$$

$$\mathcal{L}_\psi = \mathcal{L}_{\rm t} + \mathcal{L}_{\vartheta},$$

$$\mathcal{L}_{\rm t} = -\frac{1}{2} \sum_{i=1}^3 \bar{\psi}_{i,k}^{\rm t} [\bar{\gamma}^0 \partial_0 + i(\vec{k} \cdot \vec{\gamma}) + am] \psi_{i,k}^{\rm t}$$

$$\mathcal{L}_{\vartheta} = -\frac{1}{2} \bar{\vartheta}_k [\bar{\gamma}^0 \partial_0 - i(\vec{k} \cdot \vec{\gamma}) C + aM] \vartheta_k$$

$$c_s^2 = |C^2(\tau)| = C_{\rm R}^2(\tau) + C_{\rm I}^2(\tau),$$

$$M \equiv -\left[\frac{m}{2} + \frac{3}{2}(mC_{\rm R} + HC_{\rm I})\right]$$



$$c_s^2 = 1 - \frac{2\dot{\phi}^2}{\left(\frac{\dot{\phi}^2}{2} + 3m^2 + V\right)^2} [V + 3m^2 - 2(m')^2]$$

$$c_s^2 = 1 - \frac{2\dot{\phi}^2}{\left(\frac{\dot{\phi}^2}{2} + f(\phi)^2\right)^2} [f(\phi)^2 - 2g'(\phi)^2]$$

$$c_s \leq 1 \leftrightarrow f(\phi)^2 \leq 2g'(\phi)^2,$$

$$V_{\text{improved}} = f(\phi)^2 + 2g'(\phi)^2 - 3g(\phi)^2,$$

$$c_s^2 = 1 - \frac{2\dot{\phi}^2 f(\phi)^2}{\left(\frac{\dot{\phi}^2}{2} + f(\phi)^2 + g'(\phi)^2\right)^2},$$

$$\begin{pmatrix} a_{\text{out}} \\ a_{\text{out}}^\dagger \end{pmatrix} = \begin{pmatrix} \alpha & \beta^\dagger \\ \beta & \alpha^\dagger \end{pmatrix} \begin{pmatrix} a_{\text{in}} \\ a_{\text{in}}^\dagger \end{pmatrix}.$$

$$|\alpha|^2 - |\beta|^2 = 1,$$

$$\langle N_{\text{out}} \rangle_{\text{in}} = |\beta|^2.$$

$$\partial_\tau^2 \varphi_k + \omega_k^2 \varphi_k = 0$$

$$A_k(\tau) = \frac{\partial_\tau \omega_k}{\omega_k^2}$$

$$\omega_k^2 = c_s^2 k^2 + a^2 M^2,$$

$$A_k(\tau) = \frac{c_s(\partial_\tau c_s) \vec{k}^2 + a^3 H M^2 + a^2 M \partial_\tau M}{(c_s^2 k^2 + a^2 M^2)^{\frac{3}{2}}}$$

$$\begin{aligned} S_\Phi &= \int d^4x \sqrt{-g} \mathcal{L}_\Phi = \\ &= -\frac{1}{2} \int d^4x \sqrt{-g} (\partial_\mu \Phi \partial^\mu \Phi + m^2 \Phi^2) = \int d^4x \frac{a^2}{2} [(\Phi')^2 - (\vec{\nabla} \Phi)^2 - a^2 m^2 \Phi^2] \end{aligned}$$

$$\Phi \equiv \frac{\varphi}{a},$$

$$S_\varphi = \int d^4x \mathcal{L}_\varphi = \frac{1}{2} \int d^4x \left\{ (\varphi')^2 - \varphi \left(-\vec{\nabla}^2 + a^2 m^2 - \frac{a''}{a} \right) \varphi \right\}$$

$$\varphi(\tau, \vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \{ \varphi_{\mathbf{k}}(\tau) e^{i\mathbf{k}\cdot\mathbf{x}} + \bar{\varphi}_{\mathbf{k}}(\tau) e^{-i\mathbf{k}\cdot\mathbf{x}} \}$$

$$\varphi''_{\mathbf{k}} + \omega^2 \varphi_{\mathbf{k}} = 0$$



$$\omega^2 \equiv \vec{k}^2 + a^2 m^2 - \frac{a''}{a} \equiv \omega_0^2 - \frac{a''}{a}$$

$$\Pi_\varphi \equiv \frac{\partial \mathcal{L}_\varphi}{\partial \varphi'} = \varphi'$$

$$\mathcal{H}_\varphi = \frac{1}{2} \left[\Pi_\varphi^2 + \varphi \left(-\vec{\nabla}^2 + a^2 m^2 - \frac{a''}{a} \right) \varphi \right].$$

$$[\varphi(\tau, \vec{x}), \Pi_\varphi(\tau, \vec{y})] = i\delta^{(3)}(\vec{x} - \vec{y})$$

$$\begin{aligned}\varphi(\tau, \vec{x}) &= \int \frac{d^3k}{(2\pi)^{3/2}} \{ \varphi_k(\tau) e^{i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}} + \bar{\varphi}_k(\tau) e^{-i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}}^\dagger \} \\ \Pi_\varphi(\tau, \vec{x}) &= \int \frac{d^3k}{(2\pi)^{3/2}} \{ \Pi_k(\tau) e^{i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}} + \bar{\Pi}_k(\tau) e^{-i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}}^\dagger \}\end{aligned}$$

$$[a_{\mathbf{k}}, a_{\mathbf{k}'}^\dagger] = \delta^{(3)}(\vec{k} - \vec{k}'), [a_{\mathbf{k}}, a_{\mathbf{k}'}] = 0 = [a_{\mathbf{k}}^\dagger, a_{\mathbf{k}'}^\dagger]$$

$$\varphi_k \bar{\Pi}_k - \bar{\varphi}_k \Pi_k = i$$

$$H_\varphi = \int d^3k \mathcal{H}_\varphi = \frac{1}{2} \int d^3k \begin{pmatrix} a_{\mathbf{k}}^\dagger & a_{-\mathbf{k}} \end{pmatrix} \begin{pmatrix} \mathcal{E}_k & \mathcal{F}_k^\dagger \\ \mathcal{F}_k & \mathcal{E}_k \end{pmatrix} \begin{pmatrix} a_{\mathbf{k}} \\ a_{-\mathbf{k}}^\dagger \end{pmatrix}$$

$$\mathcal{E}_k = |\Pi_k|^2 + \omega^2 |\varphi_k|^2, \mathcal{F}_k = \Pi_k^2 + \omega^2 \varphi_k^2$$

$$\varphi_k = \frac{1}{\sqrt{2\omega}}(\alpha_k + \beta_k), \quad \Pi_k = -\frac{i\omega}{\sqrt{2\omega}}(-\alpha_k + \beta_k)$$

$$|\alpha_k|^2 - |\beta_k|^2 = 1,$$

$$\alpha'_k = -i\omega\alpha_k + \frac{\omega'}{2\omega}\beta_k, \quad \beta'_k = i\omega\beta_k + \frac{\omega'}{2\omega}\alpha_k$$

$$\begin{pmatrix} \tilde{a}_{\mathbf{k}} \\ \tilde{a}_{-\mathbf{k}}^\dagger \end{pmatrix} = \begin{pmatrix} \alpha_k & \beta_k^\dagger \\ \beta_k & \alpha_k^\dagger \end{pmatrix} \begin{pmatrix} a_{\mathbf{k}} \\ a_{-\mathbf{k}}^\dagger \end{pmatrix},$$

$$\begin{aligned}H_\varphi &= \frac{1}{2} \int d^3k \begin{pmatrix} a_{\mathbf{k}}^\dagger & a_{-\mathbf{k}} \end{pmatrix} \begin{pmatrix} \alpha_k^\dagger & \beta_k^\dagger \\ \beta_k & \alpha_k \end{pmatrix} \begin{pmatrix} \omega & 0 \\ 0 & \omega \end{pmatrix} \begin{pmatrix} \alpha_k & \beta_k^\dagger \\ \beta_k & \alpha_k^\dagger \end{pmatrix} \begin{pmatrix} a_{\mathbf{k}} \\ a_{-\mathbf{k}}^\dagger \end{pmatrix} = \\ &= \int d^3k \frac{\omega}{2} (\tilde{a}_{\mathbf{k}}^\dagger \tilde{a}_{\mathbf{k}} + \tilde{a}_{\mathbf{k}} \tilde{a}_{\mathbf{k}}^\dagger)\end{aligned}$$

$$\alpha_l(\tau_0)=1, \beta_k(\tau_0)=0$$

$$a_{\mathbf{k}}|0\rangle_{\text{BD}}\equiv\tilde{a}_{\mathbf{k}}(\tau_0)|0\rangle_{\text{BD}}\equiv 0$$

$$\langle N_\varphi(\tau) \rangle_{\text{BD}} = |\beta_k(\tau)|^2$$

$$T_{\mu\nu} = \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{\mu\nu} [(\partial \Phi)^2 + m^2 \Phi^2]$$



$$E_\varphi = \int d^3x \sqrt{-g} T_0^0 = \int d^3x \frac{a^2}{2} \{(\Phi')^2 + (\vec{\nabla}\Phi)^2 + a^2 m^2 \Phi^2\} = \\ = \frac{1}{2} \int d^3x \{(\varphi' - aH\varphi)^2 + \varphi(-\vec{\nabla}^2 + a^2 m^2)\varphi\}$$

$$E_\varphi = \frac{1}{2} \int d^3k (\tilde{a}_k^\dagger \quad \tilde{a}_{-k}) \begin{pmatrix} \mathcal{E}_k & \mathcal{F}_k^\dagger \\ \mathcal{F}_k & \mathcal{E}_k \end{pmatrix} \begin{pmatrix} \tilde{a}_k \\ \tilde{a}_{-k}^\dagger \end{pmatrix}$$

$$\mathcal{E}_k = \omega_0^2 + \omega^2 + a^2 H^2 \\ \mathcal{F}_k = \omega_0^2 - \omega^2 + a^2 H^2 - 2iaH\omega$$

$$\begin{pmatrix} \tilde{a}_k \\ \tilde{a}_{-k}^\dagger \end{pmatrix} \equiv \begin{pmatrix} \hat{\alpha}_k & \hat{\beta}_k^\dagger \\ \hat{\beta}_k & \hat{\alpha}_k^\dagger \end{pmatrix} \begin{pmatrix} \hat{a}_k \\ \hat{a}_{-k}^\dagger \end{pmatrix}$$

$$\hat{\alpha}_k = \frac{\omega + \omega_0 + iaH}{2\sqrt{\omega\omega_0}}, \quad \hat{\beta}_k = \frac{\omega - \omega_0 - iaH}{2\sqrt{\omega\omega_0}}$$

$$E_\varphi = \int d^3k \frac{\omega_0}{2} (\hat{a}_k^\dagger \hat{a}_k + \hat{a}_k \hat{a}_k^\dagger)$$

$$\langle E_\varphi(\tau_0) \rangle_{\text{BD}} = V_3 \left[\omega_0 \left(\frac{1}{2} + |\hat{\beta}_k|^2 \right) \right] (\tau_0) = V_3 \left[\frac{\omega^2 + \omega_0^2 + a^2 H^2}{4\omega} \right] (\tau_0),$$

$$\lim_{k\rightarrow\infty}\hat{\beta}_k=0$$

$$\mathcal{H}_{\rm t} = \frac{1}{2} \sum_{i=1}^3 \bar{\psi}_{i,k}^{\rm t} [i(\vec{k} \cdot \vec{\gamma}) + am] \psi_{i,k}^{\rm t}, \mathcal{H}_{\vartheta} = \frac{1}{2} \bar{\vartheta}_k [-i(\vec{k} \cdot \vec{\gamma}) C + aM] \vartheta_k.$$

$$\Pi_\vartheta \equiv \frac{\partial \mathcal{L}_\vartheta}{\partial \partial_0 \vartheta} = \frac{i}{2} \vartheta^\dagger,$$

$$\{\vartheta(\tau,\vec{x}),\Pi_\vartheta(\tau,\vec{y})\}=\delta^{(3)}(\vec{x}-\vec{y}),\{\vartheta(\tau,\vec{x}),\vartheta(\tau,\vec{y})\}=\{\Pi_\vartheta(\tau,\vec{x}),\Pi_\vartheta(\tau,\vec{y})\}=0$$

$$\vartheta(\tau,\vec{x})=\int\frac{d^3k}{(2\pi)^{3/2}}\sum_re^{i\vec{k}\cdot\vec{x}}[\vartheta_{r,k}a_{r,k}+v_{r,k}a_{r,-k}^\dagger]$$

$$\vartheta_{r,k}^\dagger \vartheta_{s,k} = \delta_{rs} = v_{r,k}^\dagger v_{s,k}$$

$$\bar{v}_{r,k} = \vartheta_{r,-k}^{\rm T} C_4,$$

$$\vartheta_{r,k} = \begin{pmatrix} \vartheta_A(r,\boldsymbol{k}) \\ \vartheta_B(r,\boldsymbol{k}) \end{pmatrix} = \frac{1}{2} \begin{pmatrix} (\vartheta_+ - r\vartheta_-)\psi_r \\ (\vartheta_+ + r\vartheta_-)\psi_r \end{pmatrix},$$

$$\psi_+=\begin{pmatrix}1\\0\end{pmatrix},\psi_-=\begin{pmatrix}0\\1\end{pmatrix},\quad \sigma^3\psi_r=r\psi_r$$

$$\{a_{r,\boldsymbol{k}},a_{s,\boldsymbol{q}}^\dagger\}=\delta_{rs}\delta^{(3)}(\vec{k}-\vec{q}),\{a_{r,\boldsymbol{k}},a_{s,\boldsymbol{q}}\}=0=\{a_{r,\boldsymbol{k}}^\dagger,a_{s,\boldsymbol{q}}^\dagger\}$$

$$|\vartheta_+|^2+|\vartheta_-|^2=2$$



$$\vartheta'_{\pm} = -i(C_R \mp iC_I)k\vartheta_{\mp} \mp iaM\vartheta_{\pm}.$$

$$\mathcal{H}_{\vartheta} = \frac{1}{2} \sum_r \left[(a_{r,k}^{\dagger} \quad a_{-r,-k}) \begin{pmatrix} \mathcal{E}_k & \mathcal{F}_k^{\dagger} \\ \mathcal{F}_k & -\mathcal{E}_k \end{pmatrix} \begin{pmatrix} a_{r,k} \\ a_{-r,-k}^{\dagger} \end{pmatrix} \right],$$

$$\varepsilon_k = -\frac{C_R k}{2}(\vartheta_+^* \vartheta_- + \vartheta_+ \vartheta_-^*) + \frac{i}{2}C_I k(\vartheta_+^* \vartheta_- - \vartheta_+ \vartheta_-^*) + \frac{aM}{2}(|\vartheta_+|^2 - |\vartheta_-|^2)$$

$$\mathcal{F}_k = -\frac{C_R k}{2}(\vartheta_+^2 - \vartheta_-^2) - \frac{i}{2}C_I k(\vartheta_+^2 + \vartheta_-^2) - aM\vartheta_+\vartheta_-$$

$$\begin{aligned}\vartheta_+ &= c_+(k)\alpha_k - c_-(k)\beta_k \\ \vartheta_- &= c_-(k)\alpha_k + c_+(k)\beta_k\end{aligned}$$

$$\alpha_{-k}=\alpha_k,\beta_{-k}=-\beta_k$$

$$|\alpha_k|^2+|\beta_k|^2=1,$$

$$c_+^2(k)+c_-^2(k)=2, c_+(-k)=c_+(k), c_-(-k)=-c_-(k)$$

$$\begin{pmatrix} \tilde{a}_{r,k} \\ \tilde{a}_{-r,-k}^{\dagger} \end{pmatrix} = \begin{pmatrix} \alpha_k & -\beta_k^{\dagger} \\ \beta_k & \alpha_k^{\dagger} \end{pmatrix} \begin{pmatrix} a_{r,k} \\ a_{-r,-k}^{\dagger} \end{pmatrix}$$

$$\mathcal{H}_{\vartheta} = \frac{1}{2} \sum_r \left(\tilde{a}_{r,k}^{\dagger} \quad \tilde{a}_{-r,-k} \right) \begin{pmatrix} A_k & B_k \\ B_k^{\dagger} & -A_k \end{pmatrix} \begin{pmatrix} \tilde{a}_{r,k} \\ \tilde{a}_{-r,-k}^{\dagger} \end{pmatrix}$$

$$A_k = -[c_+(k)c_-(k)]C_Rk + \left[\frac{c_+^2(k)-c_-^2(k)}{2}\right]aM$$

$$B_k = -\left[\frac{c_+^2(k)-c_-^2(k)}{2}\right]C_Rk + iC_Ik - [c_+(k)c_-(k)]aM$$

$$\begin{pmatrix} \hat{b}_{r,k} \\ \hat{b}_{-r,-k}^{\dagger} \end{pmatrix} = \begin{pmatrix} \hat{\alpha}_k & -\hat{\beta}_k^{\dagger} \\ \hat{\beta}_k & \hat{\alpha}_k^{\dagger} \end{pmatrix} \begin{pmatrix} \tilde{a}_{r,k} \\ \tilde{a}_{-r,-k}^{\dagger} \end{pmatrix}$$

$$\hat{\alpha}_k^{\dagger} = -\frac{B_k}{\omega-A_k}\hat{\beta}_k, |\hat{\alpha}_k|^2 = \frac{\omega+A_k}{2\omega}, |\hat{\beta}_k|^2 = \frac{\omega-A_k}{2\omega}$$

$$\omega^2=c_S^2k^2+a^2M^2, c_S^2=C_R^2+C_I^2$$

$$\mathcal{H}_{\vartheta} = \frac{\omega}{2} \sum_r \left(\hat{b}_{r,k}^{\dagger} \hat{b}_{r,k} - \hat{b}_{r,k} \hat{b}_{r,k}^{\dagger} \right).$$

$$\begin{aligned}T^{\mu\nu} = & \frac{1}{2}\bar{\Psi}\rho\gamma^{\rho\sigma(\mu}(\nabla_{\sigma}\Psi^{\nu)}-\nabla^{\nu)}\Psi_{\sigma})-\frac{m}{2}\bar{\Psi}\rho\gamma^{\rho(\mu}\Psi^{\nu)}\\ & -\frac{1}{2}\nabla_{\rho}(\bar{\Psi}\rho\gamma^{(\mu}\Psi^{\nu)})-\frac{1}{2}\nabla^{(\mu}(\bar{\Psi}^{\nu)}\gamma^{\rho}\Psi_{\rho})-\frac{1}{2}g^{\mu\nu}\nabla_{\rho}(\bar{\Psi}_{\sigma}\gamma^{\sigma}\Psi^{\rho}).\end{aligned}$$

$$E_{\psi} = \int \, d^3x \sqrt{-g} T_0^0 = \frac{1}{2} \int \, d^3x \{ \bar{\psi}_i \bar{\gamma}^{ijl} \partial_j \psi_l - am \bar{\psi}_i \bar{\gamma}^{ij} \psi_j \}$$



$$E_\psi = \int d^3k [\mathcal{E}_t + \mathcal{E}_\theta]$$

$$\varepsilon_t = \frac{1}{2} \sum_{j=1}^3 [\bar{\psi}_{j,k}^t (\vec{k} \cdot \vec{\gamma}) \psi_{j,k}^t + am \bar{\psi}_{j,k}^t \psi_{j,k}^t]$$

$$\varepsilon_\vartheta = \frac{1}{2} \bar{\vartheta}_k [(\vec{k} \cdot \vec{\gamma}) D + am] \vartheta_k$$

$$D = d_{\rm R} + id_{\rm I} \bar{\gamma}^0 \equiv \frac{3m^2 + H^2}{3(m^2 + H^2)} + \frac{2mH}{3(m^2 + H^2)} \bar{\gamma}_0$$

$$\mathcal{E}_\vartheta = \frac{\omega_d}{2} \sum_r (\hat{a}_{r,k}^\dagger \hat{a}_{r,k} - \hat{a}_{r,k} \hat{a}_{r,k}^\dagger)$$

$$\omega_d^2 = d_s^2 k^2 + a^2 m^2, d_s^2 = d_{\rm R}^2 + d_{\rm I}^2 = \frac{m^2 + H^2/9}{m^2 + H^2}$$

$$\left(\begin{array}{c} \hat{a}_{r,k} \\ \hat{a}_{-r,-k}^\dagger \end{array}\right) = \left(\begin{array}{cc} x_k & -y_k^\dagger \\ y_k & x_k^\dagger \end{array}\right) \left(\begin{array}{c} \tilde{a}_{r,k} \\ \tilde{a}_{-r,-k}^\dagger \end{array}\right) = \left(\begin{array}{cc} x_k & -y_k^\dagger \\ y_k & x_k^\dagger \end{array}\right) \left(\begin{array}{cc} \alpha_k & -\beta_k^\dagger \\ \beta_k & \alpha_k^\dagger \end{array}\right) \left(\begin{array}{c} a_{r,k} \\ a_{-r,-k}^\dagger \end{array}\right)$$

$$x_k^\dagger = -\frac{B_k^{\rm D}}{\omega_d - A_k^{\rm D}} y_k, |x_k|^2 = \frac{\omega_d + A_k^{\rm D}}{2\omega_d}, |y_k|^2 = \frac{\omega_d - A_k^{\rm D}}{2\omega_d},$$

$$A_k^{\rm D} = [c_+(k)c_-(k)]d_{\rm R}k + \left(\frac{c_+^2(k) - c_-^2(k)}{2}\right)am$$

$$B_k^{\rm D} = \left(\frac{c_+^2(k) - c_-^2(k)}{2}\right)d_{\rm R}k - [c_+(k)c_-(k)]am - id_{\rm I}k$$

$$\left(\begin{array}{c} \hat{a}_{r,k} \\ \hat{a}_{-r,-k}^\dagger \end{array}\right) = \left(\begin{array}{cc} \mathcal{A}_k & -\mathcal{B}_k^\dagger \\ \mathcal{B}_k & \mathcal{A}_k^\dagger \end{array}\right) \left(\begin{array}{c} \hat{b}_{r,k} \\ \hat{b}_{-r,-k}^\dagger \end{array}\right),$$

$$\mathcal{A}_k = x_k \hat{\alpha}_k^\dagger + y_k^\dagger \hat{\beta}_k, \mathcal{B}_k = y_k \hat{\alpha}_k^\dagger - x_k^\dagger \hat{\beta}_k$$

$$|\mathcal{A}_k|^2 = \frac{1}{2} \left[1 - \frac{(C_{\rm R} d_{\rm R} + C_{\rm I} d_{\rm I})}{\omega \omega_d} k^2 + \frac{a^2 m M}{\omega \omega_d} \right]$$

$$|\mathcal{B}_k|^2 = \frac{1}{2} \left[1 + \frac{(C_{\rm R} d_{\rm R} + C_{\rm I} d_{\rm I})}{\omega \omega_d} k^2 - \frac{a^2 m M}{\omega \omega_d} \right]$$

$$\lim_{k \rightarrow \infty} |\mathcal{B}_k|^2 = \frac{1}{2} \left(1 + \frac{C_{\rm R} d_{\rm R} + C_{\rm I} d_{\rm I}}{c_s d_s} \right)$$

$$\theta(t,\vec{k}) = \left(\begin{array}{c} \chi_k(t) \\ \bar{\chi}_{-\vec{k}}(t) \end{array}\right)$$

$$\mathcal{L}_\theta = \frac{a^2 f^2}{2 \vec{k}^2} \left\{ \left(1 + \frac{\dot{\phi}^2}{2 f^2} \right) \frac{i}{2} \partial_0 \chi_k \sigma^0 \bar{\chi}_k + \left(1 - \frac{\dot{\phi}^2}{2 f^2} \right) \frac{1}{2} \chi_k (\vec{k} \cdot \vec{\sigma}) \bar{\chi}_k + \right. \\ \left. + \frac{g' \dot{\phi}}{f^2} \chi_k (\vec{k} \cdot \vec{\sigma}) \bar{\sigma}^0 \chi_{-k} + \frac{1}{2} \left[\left(\frac{\dot{\phi}^2}{f^2} - 1 \right) am + \left(\frac{3 g' \dot{\phi}}{f^2} \right) aH \right] \chi_k \chi_{-k} + \text{h.c.} \right\}$$



$$\mathcal{L}_G = \left(1 + \frac{\dot{\phi}^2}{2f^2}\right) \frac{i}{2} \partial_0 G_{\mathbf{k}} \sigma^0 \bar{G}_{\mathbf{k}} + \left(1 - \frac{\dot{\phi}^2}{2f^2}\right) \frac{1}{2} G_{\mathbf{k}} (\vec{k} \cdot \vec{\sigma}) \bar{G}_{\mathbf{k}} + \\ + i \frac{g' \dot{\phi}}{f^2} G_{\mathbf{k}} (\vec{k} \cdot \vec{\sigma}) \bar{\sigma}^0 G_{-\mathbf{k}} + \text{h.c.}$$

$$\chi_k = \frac{2k}{af} G_k$$

$$\psi_\mu \longrightarrow \frac{M_{\rm P}}{f} \partial_\mu G$$

$$D^2\mathcal{S}^\mu=0,$$

$$(\sigma_\mu)_{\alpha\dot\alpha}\bar D^{\dot\alpha}\mathcal{S}^\mu_{\rm FZ}=D_\alpha X,$$

$$\mathcal{S}^\mu_{\rm FZ} = \{J^\mu, \Xi^\mu, T^{\mu\nu}, x\},$$

$$\delta J^\mu = -i\bar{\epsilon}\gamma_5\Xi^\mu \\ \delta \Xi^\mu = T^{\mu\nu}\gamma_\nu\epsilon + \frac{1}{8}\varepsilon^{\mu\nu\rho\sigma}\partial_\rho J_\sigma\gamma_\nu\epsilon + \frac{i}{4}\partial_\rho J^\mu\gamma_5\gamma^\rho\epsilon + \frac{i}{4}\partial^\mu(A-iB\gamma_5)\gamma_5\epsilon \\ \delta T^{\mu\nu} = -\frac{1}{4}\bar{\epsilon}\gamma_{(\mu}{}^\lambda\partial_\lambda\Xi_{\nu)} \\ \delta x = 2i\bar{\epsilon}_R\gamma^\rho\Xi_{\rho L}$$

$$\delta g_{\mu\nu} = \bar{\epsilon}\gamma_{(\mu}\psi_{\nu)} \\ \delta\psi_\mu = \partial_\mu\epsilon - \frac{1}{4}\partial_\nu g_{\rho\mu}\gamma^{\nu\rho}\epsilon - \frac{1}{6}(M+iN\gamma_5)\gamma_\mu\epsilon + \frac{i}{2}\Big(V_\mu - \frac{1}{3}\gamma_\mu V\Big)\gamma_5\epsilon \\ \delta V_\mu = \frac{i}{4}\bar{\epsilon}\gamma_5\Big(\gamma_\mu^{\rho\sigma} - 4\delta_\mu^{[\rho}\gamma^{\sigma]}$$

$$e^{-1}\mathcal{L}_{\rm OM}=\mathcal{L}_{\rm SUGRA}+\frac{1}{3}(|Z|^2+V_\mu V^\mu)$$

$$(\sigma_\mu)_{\alpha\dot\alpha}\bar D^{\dot\alpha}\mathcal{S}^\mu_{\rm R}=\chi_\alpha$$

$$\bar{D}_{\dot{\alpha}}\chi_{\alpha}=0, D^{\alpha}\chi_{\alpha}=\bar{D}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}$$

$$\mathcal{S}^\mu_{\rm R} = \{J^\mu_{\rm R}, \Xi^\mu, T^{\mu\nu}, X^{\mu\nu}\}$$

$$X^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma}\partial_\rho X_\sigma$$

$$\delta J_\mu = -i\bar{\epsilon}\gamma_5\Xi_\mu \\ \delta \Xi_\mu = T_{\mu\nu}\gamma^\nu\epsilon + X_{\mu\nu}\gamma^\nu\epsilon + \frac{1}{8}\varepsilon_{\mu\nu\rho\sigma}\partial^\rho J^\sigma\gamma^\nu\epsilon + \frac{i}{4}\partial_\rho J_\mu\gamma_5\gamma^\rho\epsilon \\ \delta T_{\mu\nu} = -\frac{1}{4}\bar{\epsilon}\gamma_{(\mu}{}^\lambda\partial_\lambda\Xi_{\nu)} \\ \delta X_{\mu\nu} = -\frac{i}{8}\varepsilon_{\mu\nu\rho\sigma}\bar{\epsilon}\gamma_5\gamma^\rho\gamma^\lambda\partial^\sigma\Xi_\lambda$$



$$\begin{aligned}
\delta g_{\mu\nu} &= \bar{\epsilon} \gamma_{(\mu} \psi_{\nu)} \\
\delta \psi_{\mu} &= \partial_{\mu} \epsilon - \frac{1}{4} \partial_{\nu} g_{\rho\mu} \gamma^{\nu\rho} \epsilon + i \gamma_5 V_{\mu} \epsilon - \frac{i}{4} \gamma_{\mu} G_{\nu} \gamma_5 \gamma^{\nu} \epsilon \\
\delta V_{\mu} &= -\frac{i}{4} \bar{\epsilon} \gamma_5 \gamma_{\mu} \gamma^{\nu\rho} \partial_{\nu} \psi_{\rho} + \frac{3}{8} \varepsilon_{\mu\nu}^{\rho\sigma} \bar{\epsilon} \gamma^{\nu} \partial_{\rho} \psi_{\sigma} \\
\delta C_{\mu\nu} &= \bar{\epsilon} \gamma_{[\mu} \psi_{\nu]}
\end{aligned}$$

$$e^{-1} \mathcal{L}_{\text{NM}} = \mathcal{L}_{\text{SUGRA}} - 3V_{\mu} V^{\mu} + 2V^{\mu} C_{\mu}$$

$$\begin{aligned}
(\sigma_{\mu})_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} S^{\mu} &= D_{\alpha} X + \chi_{\alpha} \\
\bar{D}_{\dot{\alpha}} X &= \bar{D}_{\dot{\alpha}} \chi_{\alpha} = 0 \\
D^{\alpha} \chi_{\alpha} &= \bar{D}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}}
\end{aligned}$$

$$\mathcal{L}_{\text{FP}} = \partial_{\mu} h_{\nu\rho} \partial^{\nu} h^{\mu\rho} - \frac{1}{2} \partial_{\rho} h_{\mu\nu} \partial^{\rho} h^{\mu\nu} - \partial_{\mu} h^{\mu\nu} \partial_{\nu} h + \frac{1}{2} \partial_{\mu} h \partial^{\mu} h - \frac{m^2}{2} (h^{\mu\nu} h_{\mu\nu} - h^2)$$

$$\Lambda_{\lambda} = (m^{\lambda-1} M_{\text{P}})^{\frac{1}{\lambda}}$$

$$\{h_{\mu\nu}, \lambda_{\mu}, \chi_{\mu}, A_{\mu}\}$$

$$\begin{aligned}
\delta A_{\mu} &= \frac{1}{2} \bar{\epsilon} \lambda_{\mu} \\
\delta \lambda_{\mu} &= -\frac{i}{4} (m h_{\mu\rho} + \tilde{F}_{\mu\rho} + 2i \partial_{\rho} A_{\mu} \gamma_5) \gamma_5 \gamma^{\rho} \epsilon \\
\delta h_{\mu\nu} &= \frac{i}{m} \bar{\epsilon} \gamma_5 (\partial_{(\mu} \lambda_{\nu)} - m \gamma_{(\mu} \chi_{\nu)}) \\
\delta \chi_{\mu} &= \frac{i}{4} \partial_{\rho} h_{\mu\sigma} \gamma^{\rho\sigma} \gamma_5 \epsilon + \frac{1}{8m} \partial_{\mu} F_{\rho\sigma} \gamma^{\rho\sigma} \epsilon + \frac{m}{4} A_{\rho} (2\delta_{\mu}^{\rho} + \gamma^{\rho}{}_{\mu}) \epsilon
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\text{SFP}} = & -\frac{3}{8} F^{\mu\nu} F_{\mu\nu} - \frac{3}{4} m^2 A^{\mu} A_{\mu} - \frac{1}{2} \bar{\lambda}_{\mu} \gamma^{\mu\nu\rho} \partial_{\nu} \lambda_{\rho} - \frac{1}{2} \bar{\chi}_{\mu} \gamma^{\mu\nu\rho} \partial_{\nu} \chi_{\rho} - m \bar{\lambda}_{\mu} \gamma^{\mu\nu} \chi_{\nu} \\
& + \frac{1}{4} \partial_{\mu} h_{\nu\rho} \partial^{\nu} h^{\mu\rho} - \frac{1}{8} \partial_{\rho} h_{\mu\nu} \partial^{\rho} h^{\mu\nu} - \frac{1}{4} \partial_{\mu} h^{\mu\nu} \partial_{\nu} h + \frac{1}{8} \partial_{\mu} h \partial^{\mu} h - \frac{m^2}{8} (h^{\mu\nu} h_{\mu\nu} - h^2)
\end{aligned}$$

$$\begin{aligned}
\partial_{\mu} F^{\mu\nu} - m^2 A^{\nu} &= 0, & \partial_{[\mu} F_{\nu\rho]} &= 0, \\
\gamma^{\mu\nu\rho} \partial_{\nu} \lambda_{\rho} + m \gamma^{\mu\nu} \chi_{\nu} &= 0, & \gamma^{\mu\nu\rho} \partial_{\nu} \chi_{\rho} + m \gamma^{\mu\nu} \lambda_{\nu} &= 0,
\end{aligned}$$

$$\square h_{\mu\nu} - 2\partial_{\rho} \partial_{(\mu} h_{\nu)}^{\rho} + \eta_{\mu\nu} \partial_{\rho} \partial_{\sigma} h^{\rho\sigma} + \partial_{\mu} \partial_{\nu} h - \eta_{\mu\nu} \square h - m^2 (h_{\mu\nu} - \eta_{\mu\nu} h) = 0$$

$$\begin{aligned}
\square A_{\mu} &= m^2 A_{\mu}, & \partial^{\mu} A_{\mu} &= 0, \\
\square h_{\mu\nu} &= m^2 h_{\mu\nu}, & \partial^{\rho} h_{\rho\mu} &= 0, h = 0, \\
\partial \lambda_{\mu} &= m \chi_{\mu}, \square \lambda_{\mu} = m^2 \lambda_{\mu}, & \partial^{\mu} \lambda_{\mu} &= 0, \gamma^{\mu} \lambda_{\mu} = 0, \\
\partial \chi_{\mu} &= m \lambda_{\mu}, \square \chi_{\mu} = m^2 \chi_{\mu}, & \partial^{\mu} \chi_{\mu} &= 0, \gamma^{\mu} \chi_{\mu} = 0.
\end{aligned}$$

$$[\delta_1, \delta_2] = -\frac{1}{2} (\bar{\epsilon}_1 \gamma^{\rho} \epsilon_2) \partial_{\rho}$$

$$\lambda_{\mu} \rightarrow e^{i\alpha\gamma_5} \lambda_{\mu}, \chi_{\mu} \rightarrow e^{-i\alpha\gamma_5} \chi_{\mu}, \epsilon \rightarrow e^{-i\alpha\gamma_5} \epsilon$$

$$J_0^{\mu} = \varepsilon^{\mu\nu\rho\sigma} (\bar{\lambda}_{\rho} \gamma_{\nu} \lambda_{\sigma} - \bar{\chi}_{\rho} \gamma_{\nu} \chi_{\sigma})$$



$$\Xi_0^\mu = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} \{ (mh_{\rho\tau} + \tilde{F}_{\rho\tau} - 2i\partial_\tau A_\rho \gamma_5) \gamma^\tau \gamma_\nu \lambda_\sigma + \left[\partial_\lambda h_{\tau\rho} \gamma^{\lambda\tau} \gamma_5 - \frac{i}{2m} \partial_\rho F_{\lambda\tau} \gamma^{\lambda\tau} + imA_\tau (2\delta_\rho^\tau + \gamma_\rho^\tau) \right] \gamma_\nu \gamma_5 \chi_\sigma \}$$

$$X_0^\mu = \frac{1}{2} A_\nu (mh^{\nu\mu} + \tilde{F}^{\nu\mu}) + \frac{i}{4} \bar{\lambda}^\nu \gamma_5 \gamma^\mu \lambda_\nu$$

$$T_0^{\mu\nu} = T_K^{\mu\nu} + \partial_\rho B_0^{\rho\mu\nu} + \partial_\rho \partial_\sigma G_0^{\mu\sigma\nu\rho}$$

$$T_K^{\mu\nu} = \frac{3}{2} T_P^{\mu\nu} + T_{RS}^{\mu\nu} + \frac{1}{4} T_{FP}^{\mu\nu}$$

$$T_P^{\mu\nu} = F^{\mu\rho} F_\rho^\nu - \frac{1}{4} \eta^{\mu\nu} F^2 + m^2 A^\mu A^\nu - \frac{m^2}{2} \eta^{\mu\nu} A^2,$$

$$T_{RS}^{\mu\nu} = \frac{1}{2} (\bar{\lambda}^\sigma \gamma^{(\mu} \partial^{\nu)} \lambda_\sigma + \bar{\chi}^\sigma \gamma^{(\mu} \partial^{\nu)} \chi_\sigma) - (\bar{\lambda}^\sigma \gamma^{(\mu} \partial_\sigma \psi^{\nu)} + \bar{\chi}^\sigma \gamma^{(\mu} \partial_\sigma \chi^{\nu)}),$$

$$T_{FP}^{\mu\nu} = \partial^\mu h^{\rho\sigma} \partial^\nu h_{\rho\sigma} + \partial^\rho h^{\mu\sigma} \partial_\rho h_\sigma^\nu - 2\partial^{(\mu} h^{\rho\sigma} \partial_\rho h^{\nu)}{}_\sigma + m^2 h^{\mu\rho} h_\rho^\nu + \frac{\eta^{\mu\nu}}{2} (-\partial^\lambda h^{\rho\sigma} \partial_\lambda h_{\rho\sigma} + \partial^\lambda h^{\rho\sigma} \partial_\rho h_{\lambda\sigma} - m^2 h^{\rho\sigma} h_{\rho\sigma}).$$

$$B_0^{\rho\mu\nu} = \frac{1}{4} (h^{(\nu}{}_\sigma \partial^{\mu)} h^{\rho\sigma} - \partial^{(\mu} h^{\nu)}{}_\sigma h^{\rho\sigma}) + \frac{5}{4} (A^{(\nu} \partial^{\mu)} A^\rho - A^\rho \partial^{(\mu} A^{\nu)}) + \frac{1}{4m} (\partial^{(\mu} \tilde{F}^{\rho\sigma} h^{\nu)}{}_\sigma - \tilde{F}^{\rho\sigma} \partial^{(\mu} h^{\nu)}{}_\sigma + \tilde{F}^{(\nu}{}_\sigma \partial^{\mu)} h^{\rho\sigma} - \partial^{(\mu} \tilde{F}^{\nu)}{}_\sigma h^{\rho\sigma}) + \frac{1}{2m} (\partial^{(\mu} \bar{\chi}^{\nu)} \lambda^\rho - \bar{\chi}^{(\nu} \partial^{\mu)} \lambda^\rho + \partial^{(\mu} \bar{\lambda}^{\nu)} \chi^\rho - \bar{\lambda}^{(\nu} \partial^{\mu)} \chi^\rho),$$

$$G_0^{\mu\sigma\nu\rho} = \frac{1}{2m^2} \eta^{\nu[\sigma} F^{\mu]\lambda} F_\lambda^{[\rho} + \frac{1}{8m^2} \eta^{\nu[\sigma} \eta^{\mu]\rho} F^2 + 3\eta^{\nu[\sigma} A^{\mu]} A^{[\rho} + \frac{1}{4} \eta^{\nu[\mu} \eta^{\sigma]\rho} A^2 + \frac{1}{2m} \eta^{\rho][\mu} (\tilde{F}^{\sigma]}{}_\lambda h^{\lambda[\nu} - h^{\sigma]}{}_\lambda \tilde{F}^{\lambda[\nu)} - h^{\mu[\rho} h^{\nu]\sigma} + \frac{1}{2} \eta^{\rho][\mu} h^{\sigma]}{}_\lambda h^{\lambda[\nu} + \frac{1}{m} (\bar{\lambda}^{[\sigma} \gamma^{\mu]}{}_{[\rho} \chi^{\nu]} + \bar{\lambda}^{[\rho} \gamma^{\nu]}{}_{[\sigma} \chi^{\mu]})$$

$$\partial_\rho B_0^{\rho\mu\nu} = \partial_\rho B_0^{\rho(\mu\nu)} = \partial_\rho B_0^{[\rho\mu]\nu} = \partial_\rho B_0^{[\rho|\mu|\nu]} \\ \partial_\rho \partial_\sigma G_0^{\mu\sigma\nu\rho} = \partial_\rho \partial_\sigma G_0^{(\mu|\sigma|\nu)\rho} = \partial_\rho \partial_\sigma G_0^{[\mu\sigma]\nu\rho} = \partial_\rho \partial_\sigma G_0^{\mu\sigma[\nu\rho]}$$

$$\mathcal{S}_0^\mu = \{J_0^\mu, \Xi_0^\mu, T_0^{\mu\nu}, X_0^{\mu\nu}\}$$

$$\frac{4}{5} \partial_\rho B_A^{\rho\mu\nu} = \partial_\rho (A^{(\nu} \partial^{\mu)} A^\rho - A^\rho \partial^{(\mu} A^{\nu)}) = \\ = \partial_\rho (A^\nu \partial^{[\mu} A^{\rho]} + A^{[\mu} \partial^{\rho]} A^\nu + A^{[\mu|\nu} \partial^\rho A^{|\rho]}) + \underbrace{\partial_\rho (A^{[\nu|} \partial^\rho A^{|\mu]})}_{=0} = \frac{4}{5} \partial_\rho B_A^{[\rho\mu]\nu}$$

$$\mathcal{L} \sim -\frac{1}{2} (g_{\mu\nu} - \eta_{\mu\nu}) T^{\mu\nu} - \frac{1}{2} \bar{\psi}_\mu \Xi^\mu - \frac{1}{2} V_\mu J^\mu + C_\mu X^\mu,$$

$$g_{\mu\nu} \partial_\rho \partial_\sigma G_0^{\mu\sigma\nu\rho} \approx (\partial_\rho \partial_{[\sigma} g_{\mu][\nu]}) G_0^{[\mu\sigma][\nu\rho]} \sim -\frac{1}{2} R_{\mu\sigma\nu\rho} G_0^{\mu\sigma\nu\rho}$$

$$R_{\mu\sigma\nu\rho} \sim -2\partial_\rho \partial_{[\sigma} \partial_{\mu][\nu]}$$



$$g_{\mu\nu}\partial_\rho B_0^{\rho\mu\nu} \approx \left(\partial_{[\rho}g_{\mu]\nu} + \frac{1}{2}\partial_\nu g_{\mu\rho}\right)B_0^{[\rho\mu]\nu} \sim \Gamma_{\rho\mu\nu}B_0^{\rho\mu\nu},$$

$$g_{\rho\sigma}\Gamma_{\mu\nu}^\sigma \sim \partial_{[\mu}g_{\rho]\nu} + \frac{1}{2}\partial_\nu g_{\mu\rho}$$

$$\mathcal{S}_0^\mu \longrightarrow \mathcal{S}^\mu = \mathcal{S}_0^\mu + \Delta \mathcal{S}^\mu,$$

$$\Delta \mathcal{S}^\mu = \{\Delta J^\mu, \Delta \Xi^\mu, \Delta T^{\mu\nu}, \Delta X^{\mu\nu}\}$$

$$\Delta \mathcal{S}^\mu = \partial_\nu \mathcal{L}^{\mu\nu} + \sigma^{\mu\alpha\dot\beta} [D_\alpha, \bar{D}_{\dot\beta}] \mathcal{U}$$

$$D^2\mathcal{L}^{\mu\nu}=0$$

$$\Delta J^\mu = \frac{1}{m}\partial_\nu L^{\mu\nu},$$

$$L^{\mu\nu}=c_{\rm B1}h_\lambda^{[\mu}\partial^\lambda A^{\nu]}+c_{\rm B2}h_\lambda^{[\mu}\partial^{\nu]}A^\lambda+\frac{c_{\rm B3}}{m}\varepsilon^{\mu\nu\rho\sigma}\partial_\lambda A_\rho\partial_\sigma A^\lambda\\ +c_{\rm F1}\varepsilon^{\mu\nu\rho\sigma}\bar\chi_\rho\lambda_\sigma+ic_{\rm F2}\bar\chi^{[\mu}\gamma_5\lambda^{\nu]}$$

$$\delta_{\rm L}^2 L^{\mu\nu} = 0$$

$$\epsilon_{\rm L}\bar{\epsilon}_{\rm L}=-\frac{1}{2}(\bar{\epsilon}_{\rm L}\epsilon_{\rm L})P_{\rm L}$$

$$\delta_{\rm L}^2(h^{[\mu}{}_{\sigma}\partial^{\sigma}A^{\nu]})=\frac{i}{4}(\bar{\epsilon}_{\rm L}\epsilon_{\rm L})\Big[-\frac{1}{m}\partial_{\sigma}\bar{\lambda}_{\rm L}^{[\mu}\partial^{\nu]}\lambda_{\rm L}^{\sigma}+\bar{\chi}_{\rm R}^{\sigma}\gamma^{[\mu}\partial^{\nu]}\lambda_{\sigma{\rm L}}\Big],$$

$$\delta_{\rm L}^2(h^{[\mu}{}_{\sigma}\partial^{\nu]}A^{\sigma})=\frac{i}{4}(\bar{\epsilon}_{\rm L}\epsilon_{\rm L})\Big[\frac{1}{m}\partial_{\sigma}\bar{\lambda}_{\rm L}^{[\mu}\partial^{\nu]}\lambda_{\rm L}^{\sigma}+\bar{\chi}_{\rm R}^{\sigma}\gamma^{[\mu}\partial^{\nu]}\lambda_{\sigma{\rm L}}\Big],$$

$$\delta_{\rm L}^2\left(\frac{1}{m}\varepsilon^{\mu\nu\rho\sigma}\partial_\lambda A_\rho\partial_\sigma A^\lambda\right)=\frac{i}{4}(\bar{\epsilon}_{\rm L}\epsilon_{\rm L})\left[\frac{2}{m}\partial_\sigma\bar{\lambda}_{\rm L}^{[\mu}\partial^{\nu]}\lambda_{\rm L}^{\sigma}-2\bar{\chi}_{\rm R}^{\sigma}\gamma^{[\mu}\partial^{\nu]}\lambda_{\sigma{\rm L}}\right],$$

$$\delta_{\rm L}^2(\varepsilon^{\mu\nu\rho\sigma}\bar{\chi}_\rho\lambda_\sigma)=0,$$

$$\delta_{\rm L}^2(\bar{\chi}^{[\mu}\gamma_5\lambda^{\nu]})=0.$$

$$c_{\rm B2}=0, c_{\rm B1}=2c_{\rm B3}\equiv 2c_{\rm B}$$

$$L^{\mu\nu}=c_{\rm B}\left(2h_\lambda^{[\mu}\partial^\lambda A^{|\nu]}+\frac{1}{m}\varepsilon^{\mu\nu\rho\sigma}\partial_\lambda A_\rho\partial_\sigma A^\lambda\right)+c_{\rm F1}\varepsilon^{\mu\nu\rho\sigma}\bar\chi_\rho\lambda_\sigma+ic_{\rm F2}\bar\chi^{[\mu}\gamma_5\lambda^{\nu]}$$

$$\begin{aligned}\partial_\rho\Delta B_{\rm B}^{\rho\mu\nu}&=\frac{1}{8}\partial_\rho(A^{(\mu}\partial^{\nu)}A^\rho-A^\rho\partial^{(\mu}A^{\nu)})\\&+\frac{1}{8m^2}\partial_\rho(\partial^\sigma A^{(\mu}\partial_\sigma\partial^{\nu)}A^\rho-\partial_\sigma A^\rho\partial^\sigma\partial^{(\mu}A^{\nu)}),\\ \partial_\rho\Delta B_{\rm F1}^{\rho\mu\nu}&=\frac{1}{2m}\partial_\rho(\partial^{(\mu}\bar{\chi}^{\nu)}\lambda^\rho-\bar{\chi}^{(\nu}\partial^{\mu)}\lambda^\rho+\partial^{(\mu}\bar{\lambda}^{\nu)}\chi^\rho-\bar{\lambda}^{(\nu}\partial^{\mu)}\chi^\rho)\\&+\frac{1}{2m}\partial_\rho(\partial^{(\mu}\bar{\lambda}_\sigma\gamma^\rho\gamma^{|\nu)}\chi^\sigma-\partial^\rho\bar{\lambda}_\sigma\gamma^{(\mu}\gamma^{\nu)}\chi^\sigma),\\ \partial_\rho\Delta B_{\rm F2}^{\rho\mu\nu}&=-\frac{1}{4}\partial_\rho B_0^{\rho\mu\nu}.\end{aligned}$$



$$c_B = c_{F1} = 0, c_{F2} = 4$$

$$\mathcal{S}^\mu = \{J^\mu, \Xi^\mu, T^{\mu\nu}, X^{\mu\nu}\}$$

$$\begin{aligned} J^\mu &= \varepsilon^{\mu\nu\rho\sigma}(\bar{\lambda}_\rho\gamma_\nu\lambda_\sigma - \bar{\chi}_\rho\gamma_\nu\chi_\sigma) + \frac{4i}{m}\partial_\nu(\bar{\chi}^{[\mu}\gamma_5\lambda^{\nu]}) \\ \Xi^\mu &= \Xi_0^\mu - \frac{i}{m}\partial_\nu\left[\left(mh_\lambda^{[\nu} + \tilde{F}_\lambda^{[\nu} - 2i\partial_\lambda A^{[\nu}\gamma_5\right)\gamma^\lambda\gamma_5\chi^{\mu]} \right. \\ &\quad \left. + \left(\partial_\rho h^{[\mu}{}_\sigma\gamma^{\rho\sigma}\gamma_5 - \frac{i}{2m}\partial^{[\mu}F_{\rho\sigma}\gamma^{\rho\sigma} + 2imA^{[\mu} - imA_\rho\gamma^{\rho[\mu}\right)\lambda^{\nu]}\right] \\ T^{\mu\nu} &= T_K^{\mu\nu} + \partial_\rho\partial_\sigma\mathcal{G}^{\mu\sigma\nu\rho} \\ X^\mu &= \frac{i}{4}\bar{\lambda}^\nu\gamma_5\gamma^\mu\lambda_\nu + \frac{3m}{4}A_\nu h^{\nu\mu} + \frac{9}{8}A_\nu\tilde{F}^{\nu\mu} + \frac{1}{4m^2}\partial_\rho\partial_\sigma(\tilde{F}^{\mu\rho}A^\sigma) \\ &\quad + \frac{1}{8m}\partial_\nu(F^{\nu\rho}h_\rho^\mu - F^{\mu\rho}h_\rho^\nu) - \frac{3}{4m}\partial_\nu(\partial^\mu A^\rho h_\rho^\nu + \partial_\rho A^\nu h^{\rho\mu}) \\ &\quad + \frac{1}{4}\varepsilon^{\mu\nu\rho\sigma}\partial_\nu h_\rho^\lambda h_{\sigma\lambda} - \frac{1}{4m}\varepsilon^{\mu\nu\rho\sigma}(\bar{\chi}_\nu\partial_\rho\lambda_\sigma + \bar{\lambda}_\nu\partial_\rho\chi_\sigma) \\ \mathcal{G}^{\mu\sigma\nu\rho} &= -\frac{1}{2}h^{\rho[\mu}h^{\sigma]\nu} + \frac{1}{2}\eta^{\rho[\mu}h^{\sigma]}\lambda h^{\lambda[\nu} + 2\eta^{\rho[\mu}A^{\sigma]}A^{[\nu} - \frac{3}{4m^2}F^{\mu\sigma}F^{\nu\rho} + \frac{1}{2m^2}\eta^{\rho[\mu}F^{\sigma]}\lambda F^{\lambda[\nu} \\ &\quad + \frac{1}{m^2}(\eta^{\sigma][\nu}F^{\rho]}\partial^\lambda A^{[\mu} + \eta^{\rho][\mu}F^{\sigma]}\lambda\partial^\lambda A^{[\nu]} + \frac{1}{2m}\eta^{\rho][\mu}(\tilde{F}^{\sigma]}\lambda h^{\lambda[\nu} - h^{\sigma]}\lambda\tilde{F}^{\lambda[\nu]} \\ &\quad + \frac{1}{2m}(\varepsilon^{\mu\sigma\lambda\tau}A_\lambda\partial^{[\nu}h^{\rho]}\tau + \varepsilon^{\nu\rho\lambda\tau}A_\lambda\partial^{[\mu}h^{\sigma]}\tau) + \frac{1}{m}(\bar{\lambda}^{[\sigma}\gamma^\mu]^{[\rho}\chi^{\nu]} + \bar{\lambda}^{[\rho}\gamma^\nu]^{[\sigma}\chi^{\mu]}) \end{aligned}$$

$$\mathcal{L} = \mathcal{L}_{\text{SUGRA}} + \mathcal{L}_{\text{SFP}} + \mathcal{L}_\psi + \mathcal{L}_{\text{non-min}} + \mathcal{L}_{\text{quartic}} + \cdots,$$

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{SFP}} &= -\frac{3}{8}F^{\mu\nu}F_{\mu\nu} - \frac{3}{4}m^2A^\mu A_\mu - \frac{1}{2}\bar{\lambda}_\mu\gamma^{\mu\nu\rho}D_\nu\lambda_\rho - \frac{1}{2}\bar{\chi}_\mu\gamma^{\mu\nu\rho}D_\nu\chi_\rho - m\bar{\lambda}_\mu\gamma^{\mu\nu}\chi_\nu \\ &\quad + \frac{1}{4}\nabla_\mu h_{\nu\rho}\nabla^\nu h^{\mu\rho} - \frac{1}{8}\nabla_\rho h_{\mu\nu}\nabla^\rho h^{\mu\nu} - \frac{1}{4}\nabla_\mu h^{\mu\nu}\nabla_\nu h + \frac{1}{8}\nabla_\mu h\nabla^\mu h - \frac{m^2}{8}(h^{\mu\nu}h_{\mu\nu} - h^2) \end{aligned}$$

$$\begin{aligned} e^{-1}\mathcal{L}_\psi &= -\frac{e^{-1}}{4}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\left[(mh_{\rho\tau} + \tilde{F}_{\rho\tau} - 2i\nabla_\tau A_\rho\gamma_5)\gamma^\tau\gamma_\nu\lambda_\sigma \right. \\ &\quad \left. + \left(\nabla_\lambda h_{\tau\rho}\gamma^{\lambda\tau}\gamma_5 - \frac{i}{2m}\nabla_\rho F_{\lambda\tau}\gamma^{\lambda\tau} + imA_\tau(2\delta_\rho^\tau + \gamma_\rho^\tau)\right)\gamma_\nu\gamma_5\chi_\sigma\right] \\ &\quad + \frac{i}{2m}\bar{\rho}_{\mu\nu}\left[(mh_\lambda^\nu + \tilde{F}_\lambda^\nu - 2i\nabla_\lambda A^\nu\gamma_5)\gamma^\lambda\gamma_5\chi^\mu \right. \\ &\quad \left. + \left(\nabla_\rho h^\mu{}_\sigma\gamma^{\rho\sigma}\gamma_5 - \frac{i}{2m}\nabla^\mu F_{\rho\sigma}\gamma^{\rho\sigma} + 2imA^\mu - imA_\rho\gamma^{\rho\mu}\right)\lambda^\nu\right] \end{aligned}$$

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{non-min}} &= \frac{1}{4}R_{\mu\sigma\nu\rho}\left[-\frac{1}{2}h^{\rho\mu}h^{\sigma\nu} + \frac{1}{2}g^{\rho\mu}h^\sigma{}_\lambda h^{\lambda\nu} + 2g^{\rho\mu}A^\sigma A^\nu - \frac{3}{4m^2}F^{\mu\sigma}F^{\nu\rho} + \frac{1}{2m^2}g^{\rho\mu}F^\sigma{}_\lambda F^{\lambda\nu} \right. \\ &\quad \left. + \frac{2}{m^2}g^{\sigma\nu}F^{\rho}{}_\lambda \nabla^\lambda A^\mu + \frac{e^{-1}}{m}\varepsilon^{\mu\sigma\lambda\tau}A_\lambda\nabla^\nu h^{\rho}{}_\tau + \frac{1}{m}g^{\rho\mu}\tilde{F}^\sigma{}_\lambda h^{\lambda\nu} + \frac{2}{m}\bar{\lambda}^\sigma\gamma^{\mu\rho}\chi^\nu\right] \end{aligned}$$



$$e^{-1}\mathcal{L}_{\text{quartic}}=-\frac{1}{4}\left(e^{-1}\varepsilon_{\mu}{}^{\nu\rho\sigma}(\bar{\lambda}_{\rho}\gamma_{\nu}\lambda_{\sigma}-\bar{\chi}_{\rho}\gamma_{\nu}\chi_{\sigma})+\frac{4i}{m}\nabla^{\nu}(\bar{\chi}_{[\mu}\gamma_5\lambda_{\nu]})\right)\left[\frac{i}{4}\bar{\lambda}^{\tau}\gamma_5\gamma^{\mu}\lambda_{\tau}+\frac{3m}{4}A_{\tau}h^{\tau\mu}\right.\\ \left.+\frac{9}{8}A_{\tau}\tilde{F}^{\tau\mu}+\frac{1}{8m}\nabla_{\tau}(F^{\tau\lambda}h^{\mu}{}_{\lambda}-F^{\mu\lambda}h^{\tau}{}_{\lambda})-\frac{3}{4m}\nabla_{\tau}(\nabla^{\mu}A^{\lambda}h^{\tau}{}_{\lambda}+\nabla_{\lambda}A^{\tau}h^{\lambda\mu})\right.\\ \left.+\frac{1}{4m^2}\nabla_{\lambda}\nabla_{\pi}(\tilde{F}^{\mu\lambda}A^{\pi})+\frac{e^{-1}}{4}\varepsilon^{\mu\tau\lambda\pi}\nabla_{\tau}h^{\alpha}{}_{\lambda}h_{\pi\alpha}-\frac{e^{-1}}{4m}\varepsilon^{\mu\tau\lambda\pi}(\bar{\chi}_{\tau}D_{\lambda}\lambda_{\pi}+\bar{\lambda}_{\tau}D_{\lambda}\chi_{\pi})\right]$$

$$T^{\mu\nu}\rightarrow T^{\mu\nu}+(\partial^{\mu}\partial^{\nu}-\eta^{\mu\nu}\Box)U,\Xi^{\mu}\rightarrow\Xi^{\mu}-2\gamma^{\mu\nu}\partial_{\nu}Y$$

$$U=\mathcal{U}|_{\vartheta=0},\delta U=\bar{\epsilon}Y$$

$$e^{-1}\mathcal{L}_u=-\frac{1}{2}RU+\bar{\rho}_{\mu\nu}\gamma^{\mu\nu}Y$$

$$E^a=\phi^{-\frac{1}{2}}e^a,\qquad \mathcal{A}=a(dy+B)+A\\ E^4=\phi(dy+B),\quad \Psi=\phi^{\frac{5}{4}}\zeta(dy+B)+\phi^{-\frac{1}{4}}\Big(\psi-\frac{1}{2}e^a\gamma_a\gamma_5\zeta\Big)$$

$$\psi_{\mu}(-y)=\sigma_3\gamma_5\psi_{\mu}(y),\zeta(-y)=-\sigma_3\gamma_5\zeta(y)$$

$$B_{\mu}(x,y)=0,\phi(x,y)=\phi(x),a(x,y)=a(x),\zeta(x,y)=\frac{1}{2}(1-\sigma_3\gamma_5)\zeta(x)$$

$$\delta B_{\mu}(x,y)=\frac{\phi(x)^{-\frac{3}{2}}}{2}\bar{\epsilon}(x)\gamma_5\psi_{\mu}(x,y)+\phi(x)^{-3}g_{\mu\nu}(x,y)\partial_y\xi^{\nu}(x,y)+\partial_{\mu}\xi^5(x,y)\\ \delta\phi(x,y)=\frac{\phi(x)}{2}\bar{\epsilon}(x)\gamma_5\zeta(x)+\xi^{\mu}(x,y)\partial_{\mu}\phi(x)+\partial_y\xi^5(x,y)\phi(x)$$

$$\partial_y\xi^5(x,y)=-\xi^{\mu}(x,y)\partial_{\mu}\log\phi(x)$$

$$\partial_y\xi^{\mu}(x,y)+\phi(x)^3g^{\mu\nu}(x,y)\partial_{\nu}\xi^5(x,y)=-\frac{\phi(x)^{\frac{3}{2}}}{2}g^{\mu\nu}(x,y)\bar{\epsilon}(x)\gamma_5\psi_{\nu}(x,y)$$

$$\delta e_{\mu}{}^a(x,y)=\frac{1}{2}\bar{\epsilon}(x)\gamma^a\psi_{\mu}(x,y)+D_{\mu}(\xi^{\nu}(x,y)e_{\nu}{}^a(x,y))+\xi^5(x,y)\partial_y e_{\mu}{}^a(x,y),$$

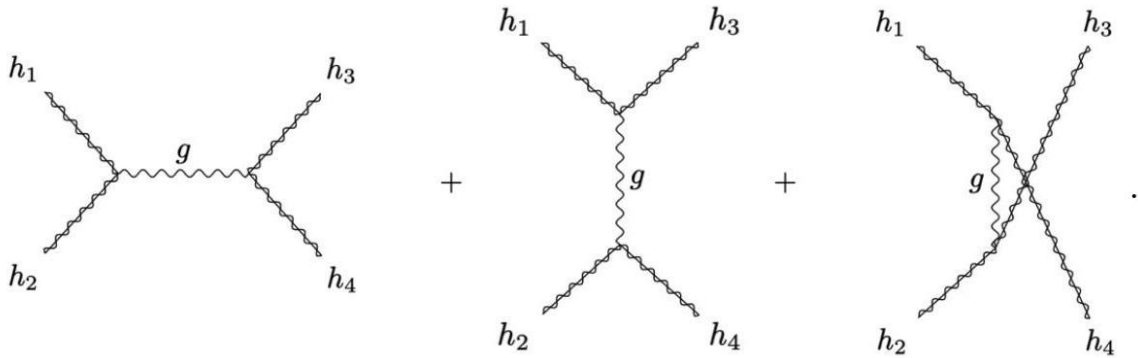
$$\xi^{\mu}(\epsilon_1,\epsilon_2)\sim -\frac{1}{2}\bar{\epsilon}_1\gamma^{\mu}\epsilon_2+\frac{1}{4m^2}\bar{\epsilon}_1h^{\nu\rho}\psi_{\rho}\partial_{\nu}(\bar{\epsilon}_2h^{\mu\sigma}\psi_{\sigma})-\frac{1}{4m^2}\bar{\epsilon}_2h^{\nu\rho}\psi_{\rho}\partial_{\nu}(\bar{\epsilon}_1h^{\mu\sigma}\psi_{\sigma})$$

$$\begin{array}{c} h \\ \diagdown \\ \text{---} g \text{---} \\ \diagup \\ h \end{array} = -i \left[\frac{1}{4} T_{\text{FP}}^{\mu\nu} + \partial_{\rho} \partial_{\sigma} \left(-\frac{1}{2} h^{\rho[\mu} h^{\sigma]\nu} + \frac{1}{2} \eta^{\rho[\mu} h^{\sigma]}{}_{\lambda} h^{\lambda[\nu} \right) \right],$$

$$i(2\pi)^4\delta^{(4)}(p_1+p_2-p_1'-p_2')\mathcal{A}=-\frac{\kappa^2}{2}\langle\text{out}\,|\,\int\,d^4x d^4y:T_h^{\mu\nu}(x)D_{\mu\nu\rho\sigma}(x-y)T_h^{\rho\sigma}(y):\,|\,\text{in}\,\rangle$$



$$D_{\mu\nu\rho\sigma}(p) = -\frac{i}{2p^2}(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma})$$



$$E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}, \mathbf{p} = |\mathbf{p}|(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \equiv |\mathbf{p}| \hat{\mathbf{p}},$$

$$h_{\mu\nu}(x) = 2 \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_{\mathbf{p}}}} \sum_{\sigma} \left(\epsilon_{\mu\nu}(\sigma, \mathbf{p}) e^{ip \cdot x} a_{\sigma, \mathbf{p}} + \bar{\epsilon}_{\mu\nu}(\sigma, \mathbf{p}) e^{-ip \cdot x} a_{\sigma, \mathbf{p}}^{\dagger} \right)$$

$$\epsilon^{\mu\nu}(\sigma,\mathbf{p})\bar{\epsilon}_{\mu\nu}(\sigma',\mathbf{p})=\delta_{\sigma\sigma'}\\ \sum_{\sigma}\epsilon^{\mu\nu}(\sigma,\mathbf{p})\bar{\epsilon}^{\rho\sigma}(\sigma,\mathbf{p})=\frac{1}{2}(P^{\mu\rho}P^{\nu\sigma}+P^{\nu\rho}P^{\nu\sigma})-\frac{1}{3}P^{\mu\nu}P^{\rho\sigma}$$

$$p^\mu \epsilon_{\mu\nu}(\sigma,\mathbf{p})=0, \epsilon_\mu^\mu(\sigma,\mathbf{p})=0$$

$$\begin{aligned}\epsilon^{\mu\nu}(\pm 2,\mathbf{p})&= \epsilon_{\pm 1}^\mu(\mathbf{p})\epsilon_{\pm 1}^\nu(\mathbf{p})\\ \epsilon^{\mu\nu}(\pm 1,\mathbf{p})&= \frac{1}{\sqrt{2}}\big(\epsilon_{\pm 1}^\mu(\mathbf{p})\epsilon_0^\nu(\mathbf{p})+\epsilon_{\pm 1}^\nu(\mathbf{p})\epsilon_0^\mu(\mathbf{p})\big)\\ \epsilon^{\mu\nu}(0,\mathbf{p})&= \frac{1}{\sqrt{6}}\big(\epsilon_1^\mu(\mathbf{p})\epsilon_{-1}^\nu(\mathbf{p})+\epsilon_1^\nu(\mathbf{p})\epsilon_{-1}^\mu(\mathbf{p})+2\epsilon_0^\mu(\mathbf{p})\epsilon_0^\nu(\mathbf{p})\big)\end{aligned}$$

$$\epsilon_{\pm 1}^\mu(\mathbf{p})=\pm \frac{e^{\pm i\phi}}{\sqrt{2}}\left(0,-\cos\theta\cos\phi\pm i\sin\phi,-\cos\theta\sin\phi\mp i\cos\phi,\sin\theta\right)$$

$$\epsilon_0^\mu(\mathbf{p})=\frac{E_{\mathbf{p}}}{m}\left(\sqrt{1-\frac{m^2}{E_{\mathbf{p}}^2}},\hat{\mathbf{p}}\right)$$



$$\mathcal{A}(0,0,0,0) = \frac{\kappa^2}{16m^4s(s-4m^2)} \left[s^5 \sin^2 \theta (3 + \cos^2 \theta) - \frac{4m^2s^4}{3} (21 - 29\cos^2 \theta) \right. \\ \left. + 2m^4s^3 \left(\frac{32}{\sin^2 \theta} + \frac{37 - 231\cos^2 \theta + 18\cos^4 \theta}{3} \right) \right. \\ \left. - 16m^6s^2 \left(\frac{16}{\sin^2 \theta} - 7 + 2\cos^2 \theta - \cos^4 \theta \right) \right. \\ \left. + 32m^8s \left(\frac{4}{\sin^2 \theta} + \frac{11 + 26\cos^2 \theta}{3} \right) + 128m^{10}\cos(2\theta) \right]$$

$$\mathcal{A}(1,1,0,0) = \frac{\kappa^2}{384m^6s(s-4m^2)} \times \\ \times [s^6(5 - \cos^2 \theta) - m^2s^5(49 - 9\cos^4 \theta) + 2m^4s^4(217 - 113\cos^2 \theta + 26\cos^4 \theta) \\ - 4m^6s^3(480 - 265\cos^2 \theta + 69\cos^4 \theta) + 32m^8s^2(103 + 96\cos^2 \theta - 9\cos^4 \theta) \\ - 64m^{10}s(56 + 119\cos^2 \theta + \cos^4 \theta) - 3072m^{12}\cos(2\theta)],$$

$$T_U^{\mu\nu} = (\partial^\mu \partial^\nu - \eta^{\mu\nu} \square) \left(c_1 h^{\rho\sigma} h_{\rho\sigma} + c_2 \partial_\lambda h^{\rho\sigma} \partial^\lambda h_{\rho\sigma} + c_3 \partial_\rho \partial_\sigma (h^{\rho\lambda} h_\lambda^\sigma) \right)$$

$$-\frac{3}{16m^2}R_{\mu\sigma\nu\rho}F^{\mu\sigma}F^{\nu\rho}$$

$$\frac{\kappa}{2}\Big[g_{\mu\nu}^{(1)}\left(\partial^\mu h^{\rho\sigma}\partial^\nu h_{\rho\sigma}-4\partial^\mu h^{\rho\sigma}\partial_\rho h_\sigma^\nu\right)+h^{\mu\nu}\left(\partial_\mu g_{\rho\sigma}^{(1)}\partial_\nu h^{\rho\sigma}-\partial_\rho g_{\mu\sigma}^{(1)}\partial_\nu h^{\rho\sigma}\right) \\ +\partial_\mu\partial^\rho g_{\nu\rho}^{(1)}\left(2h_\sigma^\mu h^{\nu\sigma}-\frac{1}{4}\eta^{\mu\nu}h_{\sigma\lambda}h^{\sigma\lambda}\right)\Big]$$

$$S=-T\int\,dA=-T\int\,d^2\xi\sqrt{-\gamma}$$

$$\gamma_{ab}=\frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b}G_{\mu\nu}=\begin{pmatrix}\dot{X}^\mu\dot{X}_\mu&\dot{X}^\mu X'^\mu_\mu\\ X'^\mu\dot{X}_\mu&X'^\mu X'^\mu_\mu\end{pmatrix}$$

$$S_{\rm NG}=-T\int\,d^2\xi\sqrt{(\dot{X}\cdot X')^2-(\dot{X}\cdot\dot{X})(X'\cdot X')}$$

$$S_{\rm P}=-\frac{T}{2}\int\,d^2\xi\sqrt{-g}g^{ab}\partial_aX^\mu\partial_bX^\nu G_{\mu\nu}$$

$$T_{ab}=\partial_aX^\mu\partial_bX_\mu-\frac{1}{2}g_{ab}(g^{cd}\partial_cX^\mu\partial_dX_\mu)=0$$

$$\xi^\pm=\tau\pm\sigma, ds^2=-d\xi^+d\xi^-,\eta=\begin{pmatrix}0&-\frac{1}{2}\\-\frac{1}{2}&0\end{pmatrix}$$

$$S_{\rm P}=2T\int\,d\xi^+d\xi^-\partial_+X^\mu\partial_-X_\mu$$



$$\begin{aligned}
\delta S_P &= 2T \int d\xi^+ d\xi^- (\partial_+ \delta X^\mu \partial_- X_\mu + \partial_+ X^\mu \partial_- \delta X_\mu) \\
&= 2T \int d\xi^+ d\xi^- [\partial_+ (\delta X^\mu \partial_- X_\mu) + \partial_- (\partial_+ X^\mu \delta X_\mu) - 2(\partial_+ \partial_- X^\mu) \delta X_\mu] \\
&= -4T \int d\xi^+ d\xi^- (\partial_+ \partial_- X^\mu) \delta X_\mu - T \int d^2 \xi \partial_\sigma (\delta X^\mu \partial_\sigma X_\mu) \\
&\quad \int_0^\pi d\sigma \partial_\sigma (\delta X^\mu \partial_\sigma X_\mu) = (\delta X^\mu \partial_\sigma X_\mu)|_{\sigma=\pi} - (\delta X^\mu \partial_\sigma X_\mu)|_{\sigma=0}
\end{aligned}$$

$$\text{NN: } \partial_\sigma X^\mu|_{\sigma=0} = \partial_\sigma X^\mu|_{\sigma=\pi} = 0, \text{DD: } \delta X^\mu|_{\sigma=0} = \delta X^\mu|_{\sigma=\pi} = 0$$

$$\partial_+ \partial_- X^\mu = 0$$

$$X^\mu(\tau, \sigma) = X_L^\mu(\tau + \sigma) + X_R^\mu(\tau - \sigma)$$

$$\begin{aligned}
X_L^\mu(\tau + \sigma) &= x_L^\mu + \frac{\alpha'}{2} p_L^\mu(\tau + \sigma) + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{\alpha_n^\mu}{n} e^{-in(\tau + \sigma)} \\
X_R^\mu(\tau - \sigma) &= x_R^\mu + \frac{\alpha'}{2} p_R^\mu(\tau - \sigma) + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{\bar{\alpha}_n^\mu}{n} e^{-in(\tau - \sigma)}
\end{aligned}$$

$$T_{++} = \partial_+ X^\mu \partial_+ X_\mu, T_{--} = \partial_- X^\mu \partial_- X_\mu$$

$$X^\mu(\tau, \sigma) = x^\mu + \alpha' p^\mu \tau + i \sqrt{\frac{\alpha'}{2}} \sum_{n \in \mathbb{Z} \setminus \{0\}} \frac{1}{n} [\alpha_n^\mu e^{-in(\tau + \sigma)} + \bar{\alpha}_n^\mu e^{-in(\tau - \sigma)}]$$

$$\alpha_0^\mu \equiv \sqrt{\frac{\alpha'}{2}} p^\mu$$

$$T_{++} = \alpha' \sum_n L_n e^{-in(\tau + \sigma)}, T_{--} = \alpha' \sum_n \bar{L}_n e^{-in(\tau - \sigma)}$$

$$L_n = \frac{1}{2} \sum_m \alpha_{n-m}^\mu \alpha_{m\mu}, \bar{L}_n = \frac{1}{2} \sum_m \bar{\alpha}_{n-m}^\mu \bar{\alpha}_{m\mu}$$

$$\{L_n, L_m\} = -i(n-m)L_{n+m}, \{\bar{L}_n, \bar{L}_m\} = -i(n-m)\bar{L}_{n+m}$$

$$L_n = \bar{L}_n = 0 \quad \forall n \in \mathbb{Z}.$$

$$M^2 = \frac{2}{\alpha'} \sum_{m \neq 0} \alpha_{-m}^\mu \alpha_{m\mu}$$

$$\sum_{m \neq 0} \alpha_{-m}^\mu \alpha_{m\mu} = \sum_{m \neq 0} \bar{\alpha}_{-m}^\mu \bar{\alpha}_{m\mu}$$

$$\alpha_n^\mu = \bar{\alpha}_n^\mu$$



$$X^\mu(\tau, \sigma) = x^\mu + 2\alpha' p^\mu \tau + i\sqrt{2\alpha'} \sum_{n \in \mathbb{Z} \setminus \{0\}} \frac{\alpha_n^\mu}{n} \cos(n\sigma) e^{-in\tau}$$

$$M^2 = \frac{1}{2\alpha'} \sum_{m \neq 0} \alpha_{-m}^\mu \alpha_{m\mu}$$

$$X^\mu(\tau, \sigma) = x^\mu + d^\mu \sigma + \sqrt{2\alpha'} \sum_{n \in \mathbb{Z} \setminus \{0\}} \frac{\alpha_n^\mu}{n} \sin(n\sigma) e^{-in\tau}$$

$$[X^\mu(\tau, \sigma), \Pi^\nu(\tau, \sigma')] = i\eta^{\mu\nu} \delta(\sigma - \sigma')$$

$$X^\pm \equiv \frac{1}{\sqrt{2}} (X^0 \pm X^{D-1}), ds^2 = -2dX^+ dX^- + \sum_{i=1}^{D-2} dX^i dX_i$$

$$X^+(\tau, \sigma) = x^+ + 2\alpha' p^+ \tau$$

$$X^-(\tau, \sigma) = x^- + 2\alpha' p^- \tau + i\sqrt{2\alpha'} \sum_{n \neq 0} \frac{\alpha_n^-}{n} \cos(n\sigma) e^{-in\tau}$$

$$X^i(\tau, \sigma) = x^i + 2\alpha' p^i \tau + i\sqrt{2\alpha'} \sum_{n \neq 0} \frac{\alpha_n^i}{n} \cos(n\sigma) e^{-in\tau}$$

$$T_{++} = T_{--} = 0 \Rightarrow \partial_\pm X^- = \frac{1}{2\alpha' p^+} \partial_\pm X^i \partial_\pm X_i$$

$$\alpha_n^- = \sqrt{\frac{2}{\alpha'}} \frac{1}{4p^+} \sum_m \alpha_{n-m}^i \alpha_{mi}.$$

$$M^2 = \frac{1}{2\alpha'} \sum_{m \neq 0} \alpha_{-m}^i \alpha_{mi},$$

$$[\alpha_n^i, \alpha_m^j] = n\delta_{n+m,0}\delta^{ij}$$

$$M^2 = \frac{1}{2\alpha'} \sum_{m \neq 0} : \alpha_{-m}^i \alpha_{mi} : = \frac{1}{\alpha'} \sum_{m > 0} \alpha_{-m}^i \alpha_{mi} - \frac{D-2}{24\alpha'} \equiv \frac{1}{\alpha'} \left(N - \frac{D-2}{24} \right).$$

$$M^2|0\rangle = -\frac{D-2}{24\alpha'}|0\rangle,$$

$$M^2(\alpha_{-1}^i|0\rangle) = -\frac{D-26}{24\alpha'}(\alpha_{-1}^i|0\rangle)$$

$$\alpha_{-2}^i|0\rangle, \alpha_{-1}^i\alpha_{-1}^j|0\rangle$$

$$\begin{aligned} (D-2) + \frac{(D-2)(D-1)}{2} &= \frac{D(D-1)}{2} - 1 \\ &= \left[\frac{(D-2)(D-1)}{2} - 1 \right] + (D-2) + 1. \end{aligned}$$



$$M^2=\frac{2}{\alpha'}\left[\sum_{n>0}(\alpha_{-m}^i\alpha_{mi}+\bar{\alpha}_{-m}^i\bar{\alpha}_{mi})-\frac{D-2}{12}\right]\equiv\frac{2}{\alpha'}\Big(N+\bar{N}-\frac{D-2}{12}\Big),$$

$$\alpha_{-1}^i\bar{\alpha}_{-1}^j|0,\overline{0}\rangle.$$

$$(D-2)(D-2)=\left[\frac{(D-2)(D-1)}{2}-1\right]+\frac{(D-2)(D-3)}{2}+1$$

$$S_{\rm P}=-\frac{1}{4\pi\alpha'}\int\,d^2\xi\{g^{ab}\partial_aX^\mu\partial_bX^\nu G_{\mu\nu}(X)+\varepsilon^{ab}\partial_aX^\mu\partial_bX^\nu B_{\mu\nu}(X)-\alpha'R(g)\phi(X)\}$$

$$\beta_{\mu\nu}^{(G)}\!=\!R_{\mu\nu}+\frac{1}{4}H_{\mu}^{\lambda\rho}H_{\nu\lambda\rho}-2\nabla_{\mu}\nabla_{\nu}\phi+\mathcal{O}(\alpha')=0$$

$$\beta_{\mu\nu}^{(B)}=\nabla_{\lambda}H^{\lambda}{}_{\mu\nu}-2\nabla_{\lambda}\phi H^{\lambda}{}_{\mu\nu}+\mathcal{O}(\alpha')=0$$

$$\beta^{(\phi)}=4\nabla_{\lambda}\phi\nabla^{\lambda}\phi-4\nabla_{\lambda}\nabla^{\lambda}\phi+R+\frac{1}{12}H_{\lambda\rho\sigma}H^{\lambda\rho\sigma}+\mathcal{O}(\alpha')=0$$

$$S=\frac{1}{2}\int\,d^{26}x\sqrt{-G}e^{-2\phi}\left(R-\frac{1}{12}H^{\mu\nu\rho}H_{\mu\nu\rho}+4\partial_{\mu}\phi\partial^{\mu}\phi\right)$$

$$z=\int\,DX(\tau,\sigma)e^{-S_{\rm P}}$$

$$\langle q_{\rm F},t_{\rm F}\mid q_{\rm I},t_{\rm I}\rangle=\int_{q_{\rm I}}^{q_{\rm F}}Dq(t)e^{-\frac{1}{\hbar}S(q,\dot{q})}$$

$$\frac{\phi}{4\pi}\int\,d^2\xi\sqrt{-g}R(g)=\phi\chi$$

$$\chi=2-2h-b-c$$

$$z\sim \frac{1}{g_s^2}[\chi=2]+\frac{1}{g_s}[\chi=1]+[\chi=0]+\sum_{n>0}g_s^n[\chi=n]$$

$$\Gamma=-\log\,z=-\frac{V_D}{2(4\pi)^{\frac{D}{2}}}\int_{\epsilon}^{\infty}\frac{dt}{t^{\frac{D}{2}+1}}\mathrm{Str}(e^{-tM^2})$$

$$M^2=\frac{2}{\alpha'}(N+\bar{N}-2)$$

$$\delta(N-\bar{N})=\int_{-\frac{1}{2}}^{\frac{1}{2}}ds e^{2\pi i(N-\bar{N})s}$$

$$\Gamma=-\frac{V_{26}}{2(4\pi)^{13}}\int_{-\frac{1}{2}}^{\frac{1}{2}}ds\int_{\epsilon}^{\infty}\frac{dt}{t^{14}}\mathrm{Tr}\Big[e^{\frac{-2t}{\alpha'}(N+\bar{N}-2)}e^{2\pi i(N-\bar{N})s}\Big]$$

$$\tau=\tau_1+i\tau_2\equiv s+\frac{i}{\pi\alpha'}t, q=e^{2\pi i\tau}$$



$$\Gamma=-\frac{V_{26}}{2(4\pi^2\alpha')^{13}}\int_{\mathcal{D}}\frac{d^2\tau}{\tau_2^{14}}\mathrm{Tr}(q^{N-1}\bar{q}^{\bar{N}-1})$$

$$\begin{aligned}\mathrm{Tr}(q^{N-1})&=\frac{1}{q}\mathrm{Tr}\Big(q^{\sum_{n>0}\alpha_{-n}^i\alpha_{ni}}\Big)=\frac{1}{q}\prod_{i=1}^{24}\prod_{n=1}^{\infty}\mathrm{Tr}\Big(q^{\alpha_{-n}^i\alpha_n^i}\Big)\\&=\frac{1}{q}\prod_{i=1}^{24}\prod_{n=1}^{\infty}\sum_{k_n}\langle k_n|q^{\alpha_{-n}^i\alpha_n^i}|k_n\rangle=\frac{1}{q}\prod_{i=1}^{24}\prod_{n=1}^{\infty}\sum_{k_n}q^{k_nn}\\&=\frac{1}{q}\prod_{i=1}^{24}\prod_{n=1}^{\infty}\frac{1}{1-q^n}=\left[\frac{1}{q^{\frac{1}{24}}\prod_{n=1}^{\infty}(1-q^n)}\right]^{24}=\frac{1}{\eta(\tau)^{24}}\end{aligned}$$

$$\Gamma=-\frac{V_{26}}{2(4\pi^2\alpha')^{13}}\int_{\mathcal{D}}\frac{d^2\tau}{\tau_2^{14}}\frac{1}{|\eta(\tau)|^{48}}$$

$$\tau\rightarrow\frac{a\tau+b}{c\tau+d}$$

$$T=\begin{pmatrix}1&1\\0&1\end{pmatrix},S=\begin{pmatrix}0&-1\\1&0\end{pmatrix},(S)^2=(ST)^3=-\mathbb{1}$$

$$\mathcal{F}=\mathbb{H}^+\setminus\mathrm{PSL}(2,\mathbb{Z})=\left\{-\frac{1}{2}<\tau_1\leq\frac{1}{2},|\tau|\geq1,\tau_2>0\right\}$$

$$\mathcal{T}=\frac{1}{(4\pi^2\alpha')^{13}}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^{14}}\frac{1}{|\eta(\tau)|^{48}}$$

$$T\colon \eta(\tau+1)=e^{\frac{i\pi}{12}}\eta(\tau), S\colon \eta\left(-\frac{1}{\tau}\right)=\sqrt{-i\tau}\eta(\tau)$$

$$\frac{1}{\eta(q)^{24}\bar{\eta}(\bar{q})^{24}}=\sum_{n,m}c(n,m)q^n\bar{q}^m$$

$$\left.\frac{1}{\eta(q)^{24}\bar{\eta}(\bar{q})^{24}}\right|_{\mathrm{physical}}=\sum_n c(n,n)(q\bar{q})^n=\frac{1}{q\bar{q}}+576+104976q\bar{q}+\cdots$$

$$\mathcal{A}=\frac{1}{2(4\pi^2\alpha')^{24}}\int_0^\infty\frac{d\tau_2}{\tau_2^{14}}\frac{1}{\eta\left(\frac{i\tau_2}{2}\right)^{24}}$$

$$\frac{1}{\eta(q)^{24}}=\frac{1}{q}+24+324q\ldots$$

$$\tilde{\mathcal{A}}=\frac{2^{-13}}{2(4\pi^2\alpha')^{24}}\int_0^\infty d\ell\frac{1}{\eta(i\ell)^{24}}$$

$$\rho^0=\begin{pmatrix}0&-1\\1&0\end{pmatrix},\quad \rho^1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\{\rho^a,\rho^b\}=2\eta^{ab}$$



$$\psi^\mu = \begin{pmatrix} \psi_-^\mu \\ \psi_+^\mu \end{pmatrix}, \psi_\pm^{\mu*} = \psi_\pm^\mu, \bar{\psi}^\mu = i\psi^{\mu\dagger}\rho^0$$

$$S_P = -\frac{1}{4\pi\alpha'} \int d^2\xi (\partial_a X^\mu \partial^a X_\mu + \bar{\psi}^\mu \rho^a \partial_a \psi_\mu)$$

$$\delta X^\mu = \bar{\epsilon} \psi^\mu, \delta \psi^\mu = \rho^a \partial_a X^\mu \epsilon$$

$$T_{\pm\pm} = \partial_\pm X^\mu \partial_\pm X_\mu + \frac{i}{2} \psi_\pm^\mu \partial_\pm \psi_{\pm\mu}, \Xi_\pm = \partial_\pm X^\mu \psi_{\pm\mu}$$

$$\int d\tau (\psi_-^\mu \delta \psi_{-\mu} - \psi_+^\mu \delta \psi_{+\mu}) \Big|_{\sigma=0}^{\sigma=\{2\pi\}} = 0$$

Bosonic	$X^\mu(\tau, \sigma + 2\pi) = X^\mu(\tau, \sigma)$	NN: $\partial_\sigma X^\mu(\tau, 0) = \partial_\sigma X^\mu(\tau, \pi) = 0$ DD: $\delta X^\mu(\tau, 0) = \delta X^\mu(\tau, \pi) = 0$
Fermionic	R-R: $\psi_\pm^\mu(\tau, \sigma + 2\pi) = +\psi_\pm^\mu(\tau, \sigma)$	$\psi_+^\mu(\tau, 0) = \psi_-^\mu(\tau, 0)$
	NS-NS: $\psi_\pm^\mu(\tau, \sigma + 2\pi) = -\psi_\pm^\mu(\tau, \sigma)$	
	R-NS: $\psi_+^\mu(\tau, \sigma + 2\pi) = +\psi_+^\mu(\tau, \sigma)$ $\psi_-^\mu(\tau, \sigma + 2\pi) = -\psi_-^\mu(\tau, \sigma)$	R: $\psi_+^\mu(\tau, \pi) = +\psi_-^\mu(\tau, \pi)$
	NS-R: $\psi_+^\mu(\tau, \sigma + 2\pi) = -\psi_+^\mu(\tau, \sigma)$ $\psi_-^\mu(\tau, \sigma + 2\pi) = +\psi_-^\mu(\tau, \sigma)$	NS: $\psi_+^\mu(\tau, \pi) = -\psi_-^\mu(\tau, \pi)$

$$\begin{aligned} \text{R: } \psi_+^\mu &= \sqrt{2\pi\alpha'} \sum_{n \in \mathbb{Z}} d_n^\mu e^{-2i\pi n(\tau+\sigma)}, \psi_-^\mu = \sqrt{2\pi\alpha'} \sum_{n \in \mathbb{Z}} \tilde{d}_n^\mu e^{-2i\pi n(\tau-\sigma)} \\ \text{NS: } \psi_+^\mu &= \sqrt{2\pi\alpha'} \sum_{r \in \mathbb{Z} + \frac{1}{2}} b_r^\mu e^{-2i\pi r(\tau+\sigma)}, \psi_-^\mu = \sqrt{2\pi\alpha'} \sum_{r \in \mathbb{Z} + \frac{1}{2}} \tilde{b}_r^\mu e^{-2i\pi r(\tau-\sigma)} \end{aligned}$$

$$\text{R: } \psi_\pm^\mu = \sqrt{\pi\alpha'} \sum_{n \in \mathbb{Z}} d_n^\mu e^{-i\pi n(\tau \pm \sigma)}, \text{NS: } \psi_\pm^\mu = \sqrt{\pi\alpha'} \sum_{r \in \mathbb{Z} + \frac{1}{2}} \tilde{b}_r^\mu e^{-i\pi r(\tau \pm \sigma)}$$

$$X^+ = x^+ + 2\pi\alpha' p^+ \tau, \psi_\pm^+ = 0$$

$$\partial_\pm X^- = \frac{1}{2\pi\alpha' p^+} \left(\partial_\pm X^i \partial_\pm X_i + \frac{i}{2} \psi_\pm^i \partial_\pm \psi_{\pm i} \right), \psi_\pm^- = \frac{1}{\pi\alpha' p^+} \psi_\pm^i \partial_\pm X_i$$

$$\text{R: } N_\psi = \sum_{n>0} n d_{-n}^i d_{ni}, \quad \text{NS: } N_\psi = \sum_{r \geq \frac{1}{2}} r b_{-r}^i b_{ri}$$

$$\text{R: } M^2 = \frac{1}{\alpha'} (N_X + N_\psi), \text{NS: } M^2 = \frac{1}{\alpha'} \left(N_X + N_\psi - \frac{D-2}{16} \right).$$

$$\text{R: } M_L^2 = \frac{4}{\alpha'} (N_X + N_\psi), \text{NS: } M_L^2 = \frac{4}{\alpha'} \left(N_X + N_\psi - \frac{D-2}{16} \right),$$

$$M^2|0\rangle_{\text{NS}} = -\frac{(D-2)}{16\alpha'}|0\rangle_{\text{NS}}$$



$$M^2 \left(b_{-\frac{1}{2}}^i |0\rangle_{\text{NS}} \right) = \frac{10-D}{2\alpha'} \left(b_{-\frac{1}{2}}^i |0\rangle_{\text{NS}} \right).$$

$$M^2 |0, \bar{0}\rangle_{\text{NSNS}} = -\frac{(D-2)}{4\alpha'} |0, \bar{0}\rangle_{\text{NSNS}}$$

$$b_{-\frac{1}{2}}^i \bar{b}_{-\frac{1}{2}}^j |0, \bar{0}\rangle_{\text{NSNS}}, M^2 \left(b_{-\frac{1}{2}}^i \bar{b}_{-\frac{1}{2}}^j |0, \bar{0}\rangle_{\text{NSNS}} \right) = \frac{10-D}{2\alpha'} \left(b_{-\frac{1}{2}}^i \bar{b}_{-\frac{1}{2}}^j |0, \bar{0}\rangle_{\text{NSNS}} \right).$$

$$\begin{aligned} |0, \bar{0}\rangle_{\text{RR}} &\simeq (\mathbf{8}_s \oplus \mathbf{8}_c) \otimes (\mathbf{8}_s \oplus \mathbf{8}_c) = \\ &= (\mathbf{8}_s \otimes \mathbf{8}_s) \oplus (\mathbf{8}_c \otimes \mathbf{8}_c) \oplus [(\mathbf{8}_s \otimes \mathbf{8}_c) \oplus (\mathbf{8}_c \otimes \mathbf{8}_s)] = \\ &= (\mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35}_+) \oplus (\mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35}_-) \oplus [(\mathbf{8}'_v \oplus \mathbf{56}) \oplus (\mathbf{8}'_v \oplus \mathbf{56}')]. \end{aligned}$$

$$\begin{aligned} |0, \bar{0}\rangle_{\text{NSR}} &\simeq \mathbf{8}_v \otimes (\mathbf{8}_s \oplus \mathbf{8}_c) = (\mathbf{8}_v \otimes \mathbf{8}_s) \oplus (\mathbf{8}_v \otimes \mathbf{8}_c) \\ &= (\mathbf{56}_s \oplus \mathbf{8}_s) \oplus (\mathbf{56}_c \oplus \mathbf{8}_c) \end{aligned}$$

$$\text{NS: } (-1)^{G_{\text{NS}}} = (-1)^{\sum_{r \geq \frac{1}{2}} b_{-r}^i b_{ri}}, \text{R: } (-1)^{G_{\text{R}}} = \gamma_9 (-1)^{\sum_{n \geq 1} d_{-n}^i d_{ni}},$$

$$(-1)^{G_{\text{NS}}} |0\rangle_{\text{NS}} = |0\rangle_{\text{NS}}$$

$$\mathcal{P}_{\text{GSO}}^{\text{NS}} = \frac{1 - (-1)^{G_{\text{NS}}}}{2}$$

$$\mathcal{P}_{\text{GSO}}^{\text{R}} = \frac{1 \pm (-1)^{G_{\text{R}}}}{2}$$

$$b_{-\frac{1}{2}}^i \bar{b}_{-\frac{1}{2}}^j |0, \bar{0}\rangle_{\text{NSNS}} = b_{-\frac{1}{2}}^i |0\rangle_{\text{NS}} \otimes \bar{b}_{-\frac{1}{2}}^j |\bar{0}\rangle_{\text{NS}}$$

$$\text{type IIA: } |+\rangle_{\text{R}} \otimes |-\rangle_{\text{R}}, \text{ type IIB: } |+\rangle_{\text{R}} \otimes |+\rangle_{\text{R}},$$

		Type IIA	Type IIB
Bosons	graviton	$G_{\mu\nu}$	$G_{\mu\nu}$
	dilaton	ϕ	ϕ
	Kalb–Ramond	$B_{\mu\nu}$	$B_{\mu\nu}$
	R-R forms	$C^{(1)}, C^{(3)}$ $C^{(5)}, C^{(7)}$	$C^{(0)}, C^{(2)}, C_+^{(4)}$ $C^{(6)}, C^{(8)}$
Fermions	gravitini	ψ_{1R}, ψ_{2L}	ψ_{1L}, ψ_{2L}
	dilatini	$\lambda_{1R}, \lambda_{2L}$	$\lambda_{1L}, \lambda_{2L}$



$$\text{type IIA: } b_{-\frac{1}{2}}^i |0\rangle_{\text{NS}} \otimes |-\rangle_{\text{R}} \in \mathbf{56}_s \oplus \mathbf{8}_s,$$

$$\text{type IIB: } b_{-\frac{1}{2}}^i |0\rangle_{\text{NS}} \otimes |+\rangle_{\text{R}} \in \mathbf{56}_s \oplus \mathbf{8}_s,$$

$$\text{type IIA: } |+\rangle_{\text{R}} \otimes \bar{b}_{-\frac{1}{2}}^i |0\rangle_{\text{NS}} \in \mathbf{56}_c \oplus \mathbf{8}_c$$

$$\text{type IIB: } |+\rangle_{\text{R}} \otimes \bar{b}_{-\frac{1}{2}}^i |0\rangle_{\text{NS}} \in \mathbf{56}_s \oplus \mathbf{8}_s$$

$$\begin{aligned} \vartheta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] (z | \tau) &= \sum_n q^{\frac{1}{2}(n+\alpha)^2} e^{2i\pi(n+\alpha)(z+\beta)} \\ &= e^{2\pi i \alpha(z+\beta)} q^{\frac{\alpha^2}{2}} \prod_{n=1}^{\infty} (1-q^n) \left(1 + q^{n+\alpha-\frac{1}{2}} e^{2i\pi(z+\beta)} \right) \left(1 + q^{n+\alpha-\frac{1}{2}} e^{-2i\pi(z+\beta)} \right) \end{aligned}$$

$$\vartheta_1(z | \tau) = \vartheta \left[\begin{smallmatrix} 1 \\ 2 \end{smallmatrix} \right] (z | \tau) = 2 \sin(\pi z) q^{\frac{1}{8}} \prod_{n=1}^{\infty} (1-q^n) (1 - q^n e^{2\pi i z}) (1 - q^n e^{-2\pi i z})$$

$$\vartheta_2(z | \tau) = \vartheta \left[\begin{smallmatrix} 1 \\ 2 \\ 0 \end{smallmatrix} \right] (z | \tau) = 2 \cos(\pi z) q^{\frac{1}{8}} \prod_{n=1}^{\infty} (1-q^n) (1 + q^n e^{2\pi i z}) (1 + q^n e^{-2\pi i z})$$

$$\vartheta_3(z | \tau) = \vartheta \left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix} \right] (z | \tau) = \prod_{n=1}^{\infty} (1-q^n) \left(1 + q^{n-\frac{1}{2}} e^{2\pi i z} \right) \left(1 + q^{n-\frac{1}{2}} e^{-2\pi i z} \right)$$

$$\vartheta_4(z | \tau) = \vartheta \left[\begin{smallmatrix} 0 \\ 1 \\ 2 \end{smallmatrix} \right] (z | \tau) = \prod_{n=1}^{\infty} (1-q^n) \left(1 - q^{n-\frac{1}{2}} e^{2\pi i z} \right) \left(1 - q^{n-\frac{1}{2}} e^{-2\pi i z} \right)$$

$$T: \vartheta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] (z | \tau + 1) = e^{-i\pi\alpha(\alpha-1)} \vartheta \left[\begin{smallmatrix} \alpha \\ \beta + \alpha - \frac{1}{2} \end{smallmatrix} \right] (z | \tau)$$

$$S: \vartheta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] \left(\frac{z}{\tau} \middle| -\frac{1}{\tau} \right) = (-i\tau)^{\frac{1}{2}} e^{2i\pi\alpha\beta + i\pi z^2/\tau} \vartheta \left[\begin{smallmatrix} \beta \\ \alpha \end{smallmatrix} \right] (z | \tau)$$

$$T: \vartheta_{1,2} \rightarrow e^{\frac{i\pi}{4}} \vartheta_{1,2}, \quad \vartheta_{3,4} \rightarrow \vartheta_{4,3}$$

$$S: \vartheta_{1,3} \rightarrow \sqrt{-i\tau} \vartheta_{1,3}, \quad \vartheta_{2,4} \rightarrow \sqrt{-i\tau} \vartheta_{4,2}$$

$$O_{2n}(q) = \frac{\vartheta_3^n + \vartheta_4^n}{2\eta(q)^n}, V_{2n}(q) = \frac{\vartheta_3^n - \vartheta_4^n}{2\eta(q)^n}$$

$$S_{2n}(q) = \frac{\vartheta_2^n + i^{-n}\vartheta_1^n}{2\eta(q)^n}, C_{2n}(q) = \frac{\vartheta_2^n - i^{-n}\vartheta_1^n}{2\eta(q)^n}$$

$$T = e^{-\frac{i n \pi}{12}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & e^{\frac{i n \pi}{4}} & 0 \\ 0 & 0 & 0 & e^{\frac{i n \pi}{4}} \end{pmatrix}, S = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & i^{-n} & -i^{-n} \\ 1 & -1 & -i^{-n} & i^{-n} \end{pmatrix}$$



$$\begin{aligned} O_{2n} &\sim q^{-\frac{n}{24}}, & V_{2n} & \\ S_{2n} &\sim 2^{n-1} q^{\frac{n}{12}}, & C_{2n} q^{\frac{1}{2}-\frac{n}{24}}, & \\ & & & \sim 2^{n-1} q^{\frac{n}{12}} \end{aligned}$$

$$\frac{O_8}{\eta^8}, \frac{V_8}{\eta^8}, \frac{S_8}{\eta^8}, \frac{C_8}{\eta^8},$$

$$\begin{aligned} O_{2n} &= O_{p-1} O_{2n+1-p} + V_{p-1} V_{2n+1-p}, & V_{2n} &= O_{p-1} V_{2n+1-p} + V_{p-1} O_{2n+1-p}, \\ S_{2n} &= S_{p-1} S_{2n+1-p} + C_{p-1} C_{2n+1-p}, & C_{2n} &= S_{p-1} C_{2n+1-p} + C_{p-1} S_{2n+1-p}, \end{aligned}$$

$$\begin{aligned} \text{Str}_{\text{NS}} \left[\frac{1 + (-1)^{G_{\text{NS}}}}{2} q^{N_X + N_\psi - \frac{1}{2}} \right] &= O_8(q), & \text{Str}_{\text{R}} \left[\frac{1 + (-1)^{G_{\text{R}}}}{2} q^{N_X + N_\psi} \right] &= S_8(q), \\ \text{Str}_{\text{NS}} \left[\frac{1 - (-1)^{G_{\text{NS}}}}{2} q^{N_X + N_\psi - \frac{1}{2}} \right] &= V_8(q), & \text{Str}_{\text{R}} \left[\frac{1 - (-1)^{G_{\text{R}}}}{2} q^{N_X + N_\psi} \right] &= C_8(q). \end{aligned}$$

$$\begin{aligned} \mathcal{T}_{\text{IIA}} &= \frac{1}{(4\pi\alpha')^5} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^6} \frac{1}{\eta(q)^8 \bar{\eta}(\bar{q})^8} (V_8 - S_8)(\bar{V}_8 - \bar{C}_8), \\ \mathcal{T}_{\text{IIB}} &= \frac{1}{(4\pi\alpha')^5} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^6} \frac{1}{\eta(q)^8 \bar{\eta}(\bar{q})^8} (V_8 - S_8)(\bar{V}_8 - \bar{S}_8). \end{aligned}$$

$$\vartheta_3^4 - \vartheta_4^4 - \vartheta_2^4 = 0$$

$$\mathcal{T}=\bar{\chi}^i\mathcal{N}_{ij}\chi^j,$$

$$\begin{aligned} \mathcal{T}_{0\text{ A}} &= |V_8|^2 + |O_8|^2 + S_8 \bar{C}_8 + \bar{S}_8 C_8, \\ \mathcal{T}_{0\text{ B}} &= |V_8|^2 + |O_8|^2 + |S_8|^2 + |C_8|^2. \end{aligned}$$

$$\mathcal{A}_{pp} = \frac{1}{(8\pi\alpha')^{\frac{p+1}{2}}} \int_0^\infty \frac{d\tau_2}{\tau_2^{\frac{p+3}{2}}} \frac{V_8 - S_8}{\eta^8} \left(\frac{i\tau_2}{2} \right)$$

$$\tilde{\mathcal{A}}_{pp} = \frac{2^{-\frac{p+1}{2}}}{(8\pi\alpha')^{\frac{p+1}{2}}} \int_0^\infty \frac{d\ell}{\ell^{\frac{9-p}{2}}} \frac{V_8 - S_8}{\eta^8} (i\ell),$$

$$\mathcal{A}_{pp} = O_{p-1} V_{9-p} + V_{p-1} O_{9-p} - S_{p-1} S_{9-p} - C_{p-1} C_{9-p}.$$

$$\mathcal{A}_{pp} = \frac{N\bar{N}}{(8\pi\alpha')^{\frac{p+1}{2}}} \int_0^\infty \frac{d\tau_2}{\tau_2^{\frac{p+3}{2}}} \frac{V_8 - S_8}{\eta^8} \left(\frac{i\tau_2}{2} \right)$$

$$\Omega_{\text{closed}}: \sigma \rightarrow -\sigma$$

$$\alpha_m^\mu \leftrightarrow \bar{\alpha}_m^\mu, b_r^\mu \leftrightarrow \bar{b}_r^\mu, d_n^\mu \leftrightarrow \bar{d}_m^\mu$$

$$\Omega_{\text{open}}: \sigma \rightarrow \pi - \sigma$$

$$\alpha_m^\mu \rightarrow (-1)^m \alpha_m^\mu, b_r^\mu \rightarrow (-1)^{r-\frac{1}{2}} b_r^\mu, d_n^\mu \rightarrow (-1)^n d_n^\mu$$



$$\mathrm{Str}\left[\frac{1+\Omega}{2}\mathcal{P}_{\mathrm{GSO}}q^{N_X+N_\psi-\Delta}\bar{q}^{\bar{N}_X+\bar{N}_\psi-\Delta}\right]$$

$$\mathcal{K}=\frac{1}{(4\pi\alpha')^5}\int_0^\infty\frac{d\tau_2}{\tau_2^6}\frac{V_8-S_8}{\eta^8}(2i\tau_2)$$

$$\begin{aligned} \frac{1}{2}\mathcal{T}+\mathcal{K} &\sim \frac{1}{2}\left|\frac{V_8-S_8}{\eta^8}\right|^2+\frac{V_8-S_8}{2\eta^8}=\\ &= \frac{|V_8|^2+|S_8|^2+V_8-S_8}{2\eta^8}-\frac{V_8\bar{S}_8+\bar{V}_8S_8}{2\eta^8}\sim (35+1+28)-(56+8), \end{aligned}$$

$$\tilde{\mathcal{K}}=\frac{2^5}{(4\pi\alpha')^5}\int_0^\infty d\ell\frac{V_8-S_8}{\eta^8}(i\ell)$$

$$\mathrm{Str}\left[\frac{1+\Omega}{2}\mathcal{P}_{\mathrm{GSO}}q^{N_X+N_\psi+\Delta}\right]$$

$$\begin{aligned}\mathcal{A}&=\frac{N^2}{(8\pi\alpha')^5}\int_0^\infty\frac{d\tau_2}{\tau_2^6}\frac{V_8-S_8}{\eta^8}\Big(\frac{i\tau_2}{2}\Big)\\ \tilde{\mathcal{A}}&=\frac{2^{-5}N^2}{(8\pi\alpha')^5}\int_0^\infty d\ell\frac{V_8-S_8}{\eta^8}(i\ell)\end{aligned}$$

$$\hat{\chi}_i(q)=e^{-i\pi(h_i-\frac{c}{24})}\chi_i(q)$$

$$\mathcal{M}=\frac{\epsilon N}{(8\pi\alpha')^5}\int_0^\infty\frac{d\tau_2}{\tau_2^6}\frac{\hat{V}_8-\hat{S}_8}{\hat{\eta}^8}\Big(\frac{1}{2}+\frac{i\tau_2}{2}\Big)$$

$$\mathcal{A}+\mathcal{M}\sim\frac{N(N+\epsilon)}{2}(8-8)$$

$$P=TST^2S\colon \frac{1}{2}+\frac{i\tau_2}{2}\stackrel{P}{\rightarrow}\frac{1}{2}+\frac{i}{2\tau_2}\equiv\frac{1}{2}+i\ell$$

$$\tilde{\mathcal{M}}=\frac{2\epsilon N}{(8\pi\alpha')^5}\int_0^\infty d\ell\frac{\hat{V}_8-\hat{S}_8}{\hat{\eta}^8}\Big(\frac{1}{2}+i\ell\Big)$$

$$\tilde{\mathcal{K}}+\tilde{\mathcal{A}}+\tilde{\mathcal{M}}\sim 2^5+2^{-5}N^2+2\epsilon N=2^{-5}(N+2^5\epsilon)^2,$$

$$g=(-1)^F\delta$$

$$\delta\colon X^9\rightarrow X^9+\pi R,$$

$$X^9=x^9+2\alpha'\frac{m}{R}\tau+2nR\sigma+[\,\mathfrak{D}_{\mathrm{oscillators}}\,]$$

$$X_{\mathrm{L,R}}^9=x_{\mathrm{L,R}}^9+\alpha'p_{\mathrm{L,R}}^9(\tau\mp\sigma)+[\,\mathfrak{D}_{\mathrm{oscillators}}\,],p_{\mathrm{L,R}}^9=\frac{m}{R}\pm\frac{nR}{\alpha'}$$

$$M^2 = \frac{m^2}{R^2} + \frac{n^2 R^2}{\alpha'^2} + \frac{2}{\alpha'} (N_X + N_\psi + \bar{N}_X + \bar{N}_\psi - 1)$$

$$P_m \equiv q^{\frac{\alpha' m^2}{R^2}}, W_n \equiv q^{\frac{n^2 R^2}{4 \alpha'}}, \Lambda_{m,n} = q^{\frac{\alpha'}{4} \left(\frac{m}{R} + \frac{nR}{\alpha'}\right)^2} \bar{q}^{\frac{\alpha'}{4} \left(\frac{m}{R} - \frac{nR}{\alpha'}\right)^2}$$

$$\mathcal{T} = \frac{1}{2|\eta|^{16}} \sum_{m,n} (|V_8 - S_8|^2 \Lambda_{m,n} + |V_8 + S_8|^2 (-1)^m \Lambda_{m,n} \\ + |O_8 - C_8|^2 \Lambda_{m,n+1/2} + |O_8 + C_8|^2 (-1)^m \Lambda_{m,n+1/2})$$

$$g = (-1)^F \delta^2, \quad \delta^2 \colon X^9 \rightarrow X^9 + 2\pi R$$

$$\mathcal{T} = \frac{1}{|\eta|^{16}} \sum_{m,n} [(|V_8|^2 + |S_8|^2) \Lambda_{m,2n} - (V_8 \bar{S}_8 + \bar{V}_8 S_8) \Lambda_{m+1/2,2n} \\ + (|O_8|^2 + |C_8|^2) \Lambda_{m,2n+1} - (O_8 \bar{C}_8 + \bar{O}_8 C_8) \Lambda_{m+1/2,2n+1}]$$

$$\mathcal{K}_1 = \frac{1}{2} \frac{V_8 - S_8}{\eta^8} \sum_m P_m, \tilde{\mathcal{K}}_1 = \frac{2^5 v}{2} \frac{V_8 - S_8}{\eta^8} \sum_n W_{2n},$$

$$\Omega_2 = \Omega \Pi, \Pi \colon X^9 \rightarrow 2\pi - X^9$$

$$\mathcal{K}_2 = \frac{1}{2\eta^8} \sum_n [(V_8 - S_8)W_{2n} + (O_8 - C_8)W_{2n+1}].$$

$$\tilde{\mathcal{K}}_2 = \frac{2^5}{2v} \frac{1}{\eta^8} \sum_m (V_8 P_{2m} - S_8 P_{2m+1})$$

$$\Omega_3 = \Omega \Pi (-1)^{G_L}$$

$$\mathcal{K}_3 = \frac{1}{2\eta^8} \sum_n [(V_8 - S_8)W_{2n} - (O_8 - C_8)W_{2n+1}]$$

$$\tilde{\mathcal{K}}_3 = \frac{2^5}{2v} \frac{1}{\eta^8} \sum_m (S_8 P_{2m} - V_8 P_{2m+1})$$

$$\Omega_{\rm DP} = \Omega \delta$$

$$\mathcal{T}_{\rm DP} = \left| \frac{V_8 - S_8}{\eta^8} \right|^2 \sum_{m,n} \Lambda_{m,n}$$

$$\mathcal{K}_{\rm DP} = \frac{1}{2} \frac{V_8 - S_8}{\eta^8} \sum_m (-1)^m P_m, \tilde{\mathcal{K}}_{\rm DP} = \frac{2^5 v}{2} \frac{V_8 - S_8}{\eta^8} \sum_n W_{2n+1}.$$

$$\Omega_{\rm DP} \big| G_{\mu\nu}, \phi, C^{(2)}, C^{(6)}, C^{(10)} \big\rangle^{(m)} = (-1)^m \big| G_{\mu\nu}, \phi, C^{(2)}, C^{(6)}, C^{(10)} \big\rangle^{(m)}$$

$$\Omega_{\rm DP} \big| B_{\mu\nu}, \phi, C^{(0)}, C_+^{(4)}, C^{(8)} \big\rangle^{(m)} = -(-1)^m \big| B_{\mu\nu}, \phi, C^{(0)}, C_+^{(4)}, C^{(8)} \big\rangle^{(m)}$$

$$\Omega_{\rm DP} \big| \psi_{1\mu}, \lambda_1 \big\rangle^{(m)} = (-1)^m \big| \psi_{2\mu}, \lambda_2 \big\rangle^{(m)}, \Omega_{\rm DP} \big| \psi_{2\mu}, \lambda_2 \big\rangle^{(m)} = (-1)^m \big| \psi_{1\mu}, \lambda_1 \big\rangle^{(m)}$$



$$\psi_{\mu}^{\prime(m)} \equiv \frac{\psi_{1\mu}^{(m)} + (-1)^m \psi_{2\mu}^{(m)}}{\sqrt{2}}, \lambda^{\prime(m)} \equiv \frac{\lambda_1^{(m)} + (-1)^m \lambda_2^{(m)}}{\sqrt{2}}$$

$$\{G_{\mu\nu}, \phi, C^{(2)}, C^{(6)}, C^{(10)}, \psi_1^{\prime\mu}, \lambda_1^{\prime}\}^{(2m)}, \quad M^2 = \frac{(2m)^2}{R^2},$$

$$\{B_{\mu\nu}, C^{(0)}, C_+^{(4)}, C^{(8)}, \psi_2^{\prime\mu}, \lambda_2^{\prime}\}^{(2m+1)}, \quad M^2 = \frac{(2m+1)^2}{R^2}.$$

$$\tilde{\mathcal{A}}_{55} = \frac{2^{-3}vN^2}{2} \frac{V_4O_4 + O_4V_4 - S_4S_4 - C_4C_4}{\eta^8} \sum_n W_n,$$

$$\tilde{\mathcal{M}}_5 = -N \left(\frac{4\epsilon v}{\hat{\eta}^2 \hat{\vartheta}_2^2} \right) (\hat{V}_4 \hat{O}_4 - \hat{O}_4 \hat{V}_4 + \hat{S}_4 \hat{S}_4 - \hat{C}_4 \hat{C}_4) \sum_n W_{2n+1},$$

$$\mathcal{A}_{55} = \frac{N^2}{2} \frac{V_4O_4 + O_4V_4 - S_4S_4 - C_4C_4}{\eta^8} \sum_m P_m$$

$$\mathcal{M}_5 = \frac{N}{2} \left(\frac{4\epsilon}{\hat{\eta}^2 \hat{\vartheta}_2^2} \right) (\hat{V}_4 \hat{O}_4 - \hat{O}_4 \hat{V}_4 + \hat{S}_4 \hat{S}_4 - \hat{C}_4 \hat{C}_4) \sum_m (-1)^m P_m$$

$$\mathcal{A}_{55} + \mathcal{M}_5 \sim q^{-\frac{1}{3}} \left[\frac{N(N+\epsilon)}{2} (V_4O_4 - C_4C_4) + \frac{N(N-\epsilon)}{2} (O_4V_4 - S_4S_4) \right]$$

$$\begin{aligned} \tilde{\mathcal{A}}_{55} &= \frac{2^{-3}}{2v} \frac{V_8 - S_8}{\eta^8} \sum_m \left[e^{\frac{i\pi m}{2}} N + e^{-\frac{i\pi m}{2}} \bar{N} \right]^2 P_m \\ &= \frac{2^{-3}}{2v} \frac{V_8 - S_8}{\eta^8} \sum_m [(N + \bar{N})^2 P_{2m} - (N - \bar{N})^2 P_{2m+1}] \end{aligned}$$

$$\mathcal{A}_{55} = \frac{V_8 - S_8}{\eta^8} \sum_n \left(N \bar{N} W_n + \frac{N^2 + \bar{N}^2}{2} W_{n+1/2} \right)$$

$$\begin{aligned} \tilde{\mathcal{A}}_{77} &= \frac{2^{-4}}{2v} \frac{1}{\eta^8} \sum_m [(N + \bar{N})^2 (V_8 P_{2m} - S_8 P_{2m+1}) - (N - \bar{N})^2 (V_8 P_{2m+1} - S_8 P_{2m})] \\ &= \frac{2^{-4}}{v} \frac{1}{\eta^8} \sum_m \left[N \bar{N} (V_8 - S_8) + \frac{N^2 + \bar{N}^2}{2} (-1)^m (V_8 + S_8) \right] P_m \end{aligned}$$

$$\mathcal{A}_{77} = \frac{1}{\eta^8} \left[N \bar{N} (V_8 - S_8) \sum_n W_n + \frac{N^2 + \bar{N}^2}{2} (O_8 - C_8) \sum_n W_{n+1/2} \right].$$

$$\tilde{\mathcal{A}}_{77}^{\text{IIB}} = \frac{2^{-4}}{v} \frac{1}{\eta^8} \sum_m [(M \bar{M} + N \bar{N}) (V_8 - S_8) + (M \bar{N} + N \bar{M}) (V_8 + S_8)] P_m$$

$$M = N_1 + e^{2\pi iam} N_2, N = (-1)^m (\bar{N}_1 + e^{2\pi iam} \bar{N}_2)$$



$$\begin{aligned}\tilde{\mathcal{A}}_{77} &= \frac{2^{-4}}{2v} \frac{1}{\eta^8} \sum_m \left[|N_1 + \bar{N}_1 + e^{2\pi iam}(N_2 + \bar{N}_2)|^2 (V_8 P_m^{\text{even}} - S_8 P_m^{\text{odd}}) \right. \\ &\quad \left. + |N_1 - \bar{N}_1 + e^{2\pi iam}(N_2 - \bar{N}_2)|^2 (V_8 P_m^{\text{odd}} - S_8 P_m^{\text{even}}) \right] \\ &= \frac{2^{-3}}{v} \frac{1}{\eta^8} \sum_m P_m \left[(N_1 \bar{N}_1 + N_2 \bar{N}_2 + \cos(2\pi am)(N_1 \bar{N}_2 + N_2 \bar{N}_1))(V_8 - S_8) \right. \\ &\quad \left. + \left(\frac{N_1^2 + \bar{N}_1^2 + N_2^2 + \bar{N}_2^2}{2} + \cos(2\pi am)(N_1 N_2 + \bar{N}_1 \bar{N}_2) \right) (-1)^m (V_8 + S_8) \right]\end{aligned}$$

$$\langle \text{D}p, X^9 = x \mid \text{D}p, X^9 = y \rangle = \sum_m e^{i \frac{m}{R} (y-x)} \left\langle \text{D}p, p^9 = \frac{m}{R} \mid \text{D}p, p^9 = \frac{m}{R} \right\rangle$$

$$\begin{aligned}\mathcal{A}_{77} &= \sum_n \left[N_1 \bar{N}_1 W_n + N_2 \bar{N}_2 W_n + \frac{N_1 \bar{N}_2 + N_2 \bar{N}_1}{2} (W_{n-a} + W_{n+a}) \right] \frac{V_8 - S_8}{\eta^8} \\ &\quad + \sum_n \left[\frac{N_1^2 + \bar{N}_1^2 + N_2^2 + \bar{N}_2^2}{2} W_{n+\frac{1}{2}} + \frac{N_1 N_2 + \bar{N}_1 \bar{N}_2}{2} (W_{n+\frac{1}{2}+a} + W_{n+\frac{1}{2}-a}) \right] \frac{O_8 - C_8}{\eta^8},\end{aligned}$$

$$\begin{aligned}\tilde{\mathcal{A}}_{88} &= \frac{2^{-\frac{9}{2}} v N^2}{2} \frac{1}{\eta^8} (V_7 O_1 + O_7 V_1) \sum_n W_n, \\ \tilde{\mathcal{M}}_8 &= -\frac{2v N \epsilon}{\hat{\eta}^{\frac{13}{2}} \hat{\vartheta}_2^{\frac{1}{2}}} (\hat{V}_7 \hat{O}_1 - \hat{O}_7 \hat{V}_1) \sum_n W_{2n+1},\end{aligned}$$

$$\mathcal{A}_{88} = \frac{N^2}{2} \frac{1}{\eta^8} [(O_7 + V_7)(O_1 + V_1) - 2S'_7 S'_1] \sum_m P_m$$

$$\mathcal{M}_8 = -\epsilon \frac{N}{2} \frac{1}{\hat{\eta}^7} \left(\frac{2\hat{\eta}}{\hat{\vartheta}_2} \right)^{\frac{1}{2}} [\hat{O}_7 \hat{O}_1 + \hat{V}_7 \hat{V}_1 - \hat{O}_7 \hat{V}_1 + \hat{V}_7 \hat{O}_1] \sum_m (-1)^m P_m$$

$$\begin{aligned}\tilde{\mathcal{A}}_{33} &= \frac{2^{-2}}{2v} \frac{1}{\eta^8} \sum_m [(N + \bar{N})^2 (V_8 P_{2m} - S_8 P_{2m+1}) - (N - \bar{N})^2 (V_8 P_{2m+1} - S_8 P_{2m})] \\ &= \frac{2^{-4}}{v} \frac{1}{\eta^8} \sum_m \left[N \bar{N} (V_8 - S_8) + \frac{N^2 + \bar{N}^2}{2} (-1)^m (V_8 + S_8) \right] P_m\end{aligned}$$

$$\mathcal{A}_{33} = N \bar{N} \left(\frac{V_8 - S_8}{\eta^8} \right) \sum_n W_n + \frac{N^2 + \bar{N}^2}{2} \left(\frac{O_8 - C_8}{\eta^8} \right) \sum_n W_{n+1/2}$$

$$\mathcal{A}_{44} = \frac{N^2}{2} \frac{1}{\eta^8} [(O_3 + V_3)(O_5 + V_5) - 2S'_5 S'_5] \sum_m P_m$$

$$\mathcal{M}_4 = \epsilon \frac{N}{2} \frac{1}{\hat{\eta}^3} \left(\frac{2\hat{\eta}}{\hat{\vartheta}_2} \right)^{\frac{5}{2}} [\hat{O}_3 \hat{O}_5 + \hat{V}_3 \hat{V}_5 - \hat{O}_3 \hat{V}_5 + \hat{V}_3 \hat{O}_5] \sum_m (-1)^m P_m$$

$$\begin{aligned}Q_o &= V_4 O_4 - C_4 C_4, & Q_v &= O_4 V_4 - S_4 S_4 \\ Q_s &= O_4 C_4 - S_4 O_4, & Q_c &= V_4 S_4 - C_4 V_4\end{aligned}$$



	tensor/ gravity	hyper/gravitino		tensor/ gravity	hyper
$\mathcal{Q}_o$	$g_{\mu\nu}, C_{\mu\nu+}, \psi_{\mu}^i$ $C_+, B_{\mu\nu-}, \lambda_+^i$	$B_{\mu\nu+}, C_{\mu\nu+I}, \psi_{\mu}^{\hat{i}}$ $e^{-\phi}, C_I, \lambda^{\hat{i}}$	$\mathcal{Q}_o$	$g_{\mu\nu}, C_{\mu\nu+}, \psi_{\mu}^i$	$e^{-\phi}, C_I, \lambda^{\hat{i}}$
$\mathcal{Q}_v$	$v_4, C_{\mu\nu-}, \lambda^i$ $B_{\hat{I}}, C_{\mu\nu-\hat{I}}, \lambda_{\hat{I}}^i$	$C_-, B_I, \lambda^{\hat{i}}$ $C_{\hat{I}}, g_{\hat{I}I}, \lambda_{\hat{I}}^{\hat{i}}$	$\mathcal{Q}_v$	$v_4, C_{\mu\nu-}, \lambda^i$	$C_{\hat{I}}, g_{\hat{I}I}, \lambda_{\hat{I}}^{\hat{i}}$
$\mathcal{Q}_s$	$B_A, B_{\mu\nu-A}, \lambda_A^i$	$C_A, g_{AI}, \lambda_A^{\hat{i}}$	$\mathcal{Q}_s$		$C_A, g_{AI}, \lambda_A^{\hat{i}}$

$$\mathcal{T} = \frac{1}{2} \left[\left| \frac{Q_o + Q_v}{\eta^8} \right|^2 \sum_{m,n} q^{\frac{\alpha'}{4} p_L^{\top} g^{-1} p_L q^{\frac{\alpha'}{4} p_R^{\top} g^{-1} p_R} \right. \\ \left. + \frac{16}{|\eta|^4} \left(\left| \frac{Q_o - Q_v}{\vartheta_2^2} \right|^2 + \left| \frac{Q_s - Q_c}{\vartheta_3^2} \right|^2 + \left| \frac{Q_s + Q_c}{\vartheta_4^2} \right|^2 \right) \right],$$

$$\mathcal{K} = \frac{1}{4} \left[\frac{Q_o + Q_v}{\eta^8} \left(\sum_m q^{\frac{\alpha'}{2} m^{\top} g^{-1} m} + \sum_n q^{\frac{1}{2\alpha'} n^{\top} g n} \right) + 2 \times 16 \frac{Q_s + Q_c}{\eta^2 \vartheta_4^2} \right],$$

$$\tilde{\mathcal{K}} = \frac{2^5}{4} \left[\frac{Q_o + Q_v}{\eta^8} \left(v_4 \sum_n q^{\frac{1}{\alpha'} n^{\top} g n} + \frac{1}{v_4} \sum_m q^{\alpha' m^{\top} g^{-1} m} \right) + 8 \frac{Q_o - Q_v}{\eta^2 \vartheta_2^2} \right] \\ \stackrel{q=0}{\sim} 8 \left[\frac{Q_o}{\eta^8} \Big|_{q=0} \left(\sqrt{v_4} + \frac{1}{\sqrt{v_4}} \right)^2 + \frac{Q_v}{\eta^8} \Big|_{q=0} \left(\sqrt{v_4} - \frac{1}{\sqrt{v_4}} \right)^2 \right].$$

$$\omega^0 = \frac{i}{2} (dz_1 \wedge d\bar{z}_1 + dz_2 \wedge d\bar{z}_2), \quad \omega^+ = dz_1 \wedge dz_2, \omega^- = d\bar{z}_1 \wedge d\bar{z}_2, \omega^{\dagger} = dz_1 \wedge d\bar{z}_2, \omega^{\hat{+}} = d\bar{z}_1 \wedge dz_2$$

$$C^{(0)}=C_++C_-$$

$$C^{(2)}=C_I\omega^I+C_{\hat{I}}\omega^{\hat{I}}+C_A\omega^A+\frac{1}{2}C_{\mu\nu}dx^\mu\wedge dx^\nu$$

$$C^{(4)}=\frac{1}{4}(C_+-C_-)\omega^+\wedge\bar{\omega}^-+\frac{1}{2}(C_{\mu\nu+I}\omega^I+C_{\mu\nu-\hat{I}}\omega^{\hat{I}}+C_{\mu\nu-A}\omega^A)dx^\mu\wedge dx^\nu$$

$$H_2(\text{K3},\mathbb{Z})=\left\langle \frac{1}{2}e_{ij}+\frac{1}{2}\sum_{\substack{A\in(\mathbb{Z}_2)^4\\a_i=a_j=0}}e_A\right\rangle_{i<j},\frac{1}{2}\sum_{\substack{A\in(\mathbb{Z}_2)^4\\a_i=0}}e_A\Bigg\rangle_{i=1},\frac{1}{2}\sum_{A\in(\mathbb{Z}_2)^4}e_A,e_A|_{\sum_ia_i\leq 2}\Bigg\rangle,$$

$$(e_{ij},e_{kl})=2\varepsilon_{ijkl},(e_A,e_B)=-2\delta_{AB}=-2\prod_{i=1}^4\delta_{a_ib_i}$$

$$\delta X_i = X_i + \pi R_i.$$

$$\mathcal{K} = \frac{1}{4} \left[\frac{Q_o + Q_v}{\eta^8} \left(\sum_m (-1)^m q^{\frac{\alpha'}{2} m^{\top} g^{-1} m} + \sum_n q^{\frac{1}{\alpha'} n^{\top} g n} \right) + 2 \times (8-8) \frac{Q_s + Q_c}{\eta^2 \vartheta_4^2} \right].$$



$$\tilde{\mathcal{K}}=\frac{2^5}{4}\Bigg[\frac{Q_o+Q_v}{\eta^8}\Bigg(v_4\sum_{n\text{ odd}}q^{\frac{1}{\alpha}n^{\text{T}}gn}+\frac{1}{v_4}\sum_mq^{\alpha'm^{\text{T}}g^{-1}m}\Bigg)+(1-1)\frac{Q_o-Q_v}{\eta^2\vartheta_2^2}\Bigg].$$

	tensor/ gravity	hyper
$\mathcal{Q}_o$	$g_{\mu\nu},\, C_{\mu\nu+},\, \psi_{\mu}^i$	$e^{-\phi},\, C_I,\, \lambda^{\hat{i}}$
$\mathcal{Q}_v$	$v_4,C_{\mu\nu-},\, \lambda^i$ $B_{\hat{\pm}},\, C_{\mu\nu-\hat{\pm}},\, \lambda_{\hat{\pm}}^i$	$C_{\hat{0}},\, g_{\hat{0}I},\, \lambda_{\hat{0}}^{\hat{i}}$
$\mathcal{Q}_s$	$B_{\hat{a}},\, C_{\mu\nu-\hat{a}},\, \lambda_{\hat{a}}^i$	$C_a,\, g_{aI},\, \lambda_a^{\hat{i}}$

$$\Omega'=\Omega Z_4, (\Omega')^2=Z_2$$

$$Z_4(z_1,z_2)=(iz_1,-iz_2)\leftrightarrow Z_4(X_1,X_2,X_3,X_4)=(-X_2,X_1,X_4,-X_3)$$

$$\mathcal{K}=\frac{1}{4\eta^2}\Bigg[8\frac{Q_o-Q_v}{\vartheta_2^2}+2\times(4+6-6)\frac{Q_s-Q_c}{\vartheta_3^2}\Bigg].$$

$$\tilde{\mathcal{K}}=\frac{16}{\eta^2}\Bigg[\Bigg(\frac{1}{\vartheta_4^2}-\frac{1}{\vartheta_3^2}\Bigg)Q_s+\Bigg(\frac{1}{\vartheta_4^2}+\frac{1}{\vartheta_3^2}\Bigg)Q_c\Bigg].$$

$$\frac{1}{36(4\pi)^4}\Big(\frac{n_H+29n_T-273}{5}\mathrm{Tr}R^4+\frac{n_H-7n_T+51}{4}\mathrm{Tr}R^2\mathrm{Tr}R^2\Big)\stackrel{\triangle}{\underset{n_H=12}{\underset{n_T=9}{\mathfrak{S}}}}0.$$

$$\Omega'=\Omega(-1)^{F_L}\delta,$$

$$(\Omega')^2=g,$$

$$\mathcal{K}_4=\frac{1}{2}\frac{V_8+S_8}{\eta^8}\sum_m(-1)^mP_m$$

$$\tilde{\mathcal{K}}_4=\frac{2^5v}{2}\frac{O_8-C_8}{\eta^8}\sum_nW_{2n+1}$$

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}$$

$$\begin{pmatrix} B^{(2)} \\ C^{(2)} \end{pmatrix} = \begin{pmatrix} dB^{(2)} - cC^{(2)} \\ aC^{(2)} - bB^{(2)} \end{pmatrix}$$



$$\{1, (-1)^F\} \rightarrow \text{Mp}(2, \mathbb{Z}) \rightarrow \text{GL}(2, \mathbb{Z})$$

$$\Omega(-1)^{F_L} = S^2$$

$$S(-1)^{F_L} = \Omega S, S^4 = (-1)^F$$

$$\Omega' \left| C^{(0)}, C_+^{(4)}, C^{(8)} \right\rangle^{(m)} = (-1)^m \left| C^{(0)}, C_+^{(4)}, C^{(8)} \right\rangle^{(m)}$$

$$\Omega' \left| B^{(2)}, C^{(2)}, C^{(6)}, C^{(10)} \right\rangle^{(m)} = -(-1)^m \left| B^{(2)}, C^{(2)}, C^{(6)}, C^{(10)} \right\rangle^{(m)}$$

$$\Omega' \left| \psi_1^\mu, \lambda_2 \right\rangle^{(m+\frac{1}{2})} = -i(-1)^m \left| \psi_2^\mu, \lambda_1 \right\rangle^{(m+\frac{1}{2})}$$

$$\Omega' \left| \psi_2^\mu, \lambda_1 \right\rangle^{(m+\frac{1}{2})} = i(-1)^m \left| \psi_1^\mu, \lambda_2 \right\rangle^{(m+\frac{1}{2})}$$

$$|\psi'^\mu, \lambda'\rangle^{(m+\frac{1}{2})} \equiv \frac{|\psi_1^\mu, \lambda_2\rangle^{(m+\frac{1}{2})} - i(-1)^m |\psi_2^\mu, \lambda_1\rangle^{(m+\frac{1}{2})}}{\sqrt{2}}$$

$$\left(G_{\mu\nu}, \phi, C^{(0)}, C_+^{(4)}\right)^{(2m)}, M^2 = \frac{(2m)^2}{R^2}$$

$$\left(B^{(2)}, C^{(2)}\right)^{(2m+1)}, M^2 = \frac{(2m+1)^2}{R^2}$$

$$\left(\psi'_\mu, \lambda'\right)^{(m+\frac{1}{2})}, M^2 = \frac{\left(m+\frac{1}{2}\right)^2}{R^2}$$

$$\left(g_{\mu\nu}, C^{(0)}, C_+^{(4)}\right)(y+\pi R) = \left(g_{\mu\nu}, C^{(0)}, C_+^{(4)}\right)(y)$$

$$\left(B^{(2)}, C^{(2)}\right)(y+\pi R) = -\left(B^{(2)}, C^{(2)}\right)(y)$$

$$\psi_\pm(y+\pi R) = \pm i\psi_\pm(y) \rightarrow \psi_\pm(y+2\pi R) = -\psi_\pm(y)$$

$$\left(g_{\mu\nu}, \tau, C_+^{(4)}\right)(y,x) = \sum_m e^{2mi\frac{y}{R}} \left(g_{\mu\nu}, \tau, C_+^{(4)}\right)^{(m)}(x)$$

$$\left(B^{(2)}, C^{(2)}\right)(y,x) = \sum_m e^{(2m+1)i\frac{y}{R}} \left(B^{(2)}, C^{(2)}\right)^{(m)}(x)$$

$$\psi_+(y,x) = \sum_m e^{\left(2m+\frac{1}{2}\right)i\frac{y}{R}} \psi_+^{(m)}(x)$$

$$\psi_-(y,x) = \sum_m e^{\left(2m-\frac{1}{2}\right)i\frac{y}{R}} \psi_-^{(m)}(x)$$

$$T^2\colon Z\stackrel{\mathbb{Z}_4}{\rightarrow}-Z,\qquad S^1\colon X^9\stackrel{\mathbb{Z}_4}{\rightarrow}X^9+\frac{\pi}{2}R_9$$

$$\left(G_{\mu\nu}, G_{IJ}, C_{\mu IJ}, C_{\mu\nu\rho}\right)^{(2m)}, M^2 = \frac{(2m)^2}{R^2}$$

$$\left(C_{\mu\nu I}, g_{\mu I}\right)^{(2m+1)}, M^2 = \frac{(2m+1)^2}{R^2}$$

$$\left(\Psi_{\mu L}, \Psi_{IR}\right)^{(2m+1/2)}, M^2 = \frac{(2m+1/2)^2}{R^2}$$

$$\left(\Psi_{\mu R}, \Psi_{IL}\right)^{(2m+3/2)}, M^2 = \frac{(2m+3/2)^2}{R^2}$$



$$V = V_{\text{SS}} + V_{\text{K}} = -\left(\frac{1}{2}\mathcal{F} + \mathcal{K}_4\right)$$

$$V_{\text{SS}} = -\frac{1}{2(4\pi^2\alpha')^{\frac{9}{2}}}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^{\frac{11}{2}}}\frac{1}{|\eta|^{16}}\sum_{m,n}\left[|V_8-S_8|^2\Lambda_{m,n}+|V_8+S_8|^2(-1)^m\Lambda_{m,n}\right. \\ \left.+|O_8-C_8|^2\Lambda_{m,n+1/2}+|O_8+C_8|^2(-1)^m\Lambda_{m,n+1/2}\right] \\ V_{\text{K}} = -\frac{1}{2(4\pi^2\alpha')^{\frac{9}{2}}}\int_0^\infty\frac{d\tau_2}{\tau_2^{\frac{11}{2}}}\frac{V_8+S_8}{\eta^8}(2i\tau_2)\sum_m(-1)^mP_m$$

$$V_{\text{SS}}\sim -\frac{1}{2(4\pi^2\alpha')^{\frac{9}{2}}}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^{\frac{11}{2}}}\left|\frac{V_8+S_8}{\eta^8}\right|^2\sum_{m\in\mathbb{Z}}(-1)^m\Lambda_{m,0} \\ \sim -\frac{16^2}{2(4\pi^2\alpha')^{\frac{9}{2}}}\int_0^\infty\frac{d\tau_2}{\tau_2^{\frac{11}{2}}}\sum_{m\in\mathbb{Z}}\left[e^{-\pi\tau_2\frac{\alpha'm^2}{R^2}}-e^{-\pi\tau_2\frac{\alpha'(m+\frac{1}{2})^2}{R^2}}\right] \\ \sim -2\frac{16^2R}{(2\pi)^9\alpha'^5}\int_0^\infty\frac{d\tau_2}{\tau_2^6}\sum_{k=0}^\infty e^{-\frac{4\pi R^2}{\alpha'\tau_2}\left(k+\frac{1}{2}\right)^2}= \\ =-\frac{24}{\pi^{14}R^9}\sum_{k=0}^\infty\frac{1}{(2k+1)^{10}}=-\frac{31}{7560(2\pi)^4R^9}$$

$$V_{\text{K}}=-\frac{1}{2(4\pi^2\alpha')^{\frac{9}{2}}}\int_0^\infty\frac{d\tau_2}{\tau_2^{\frac{11}{2}}}\frac{V_8+S_8}{\eta^8}(2i\tau_2)\sum_m(-1)^me^{-\pi\tau_2\alpha'\frac{m^2}{R^2}} \\ =-\frac{R}{(2\pi)^9\alpha'^5}\int_0^\infty\frac{d\tau_2}{\tau_2^6}\frac{V_8+S_8}{\eta^8}(2i\tau_2)\sum_{k=0}^\infty e^{-\frac{\pi R^2}{\alpha'\tau_2}\left(k+\frac{1}{2}\right)^2} \\ \sim -\frac{24}{\pi^{14}R^9}\sum_{k=0}^\infty\frac{2^5}{(2k+1)^{10}}=-\frac{31}{3780\pi^4R^9}$$

$$V\sim -\frac{1}{(4\pi)^{9/2}}\int_0^\infty\frac{dt}{t^{\frac{11}{2}}}\sum_{m=-\infty}^\infty\left(2\times 36e^{-\left(\frac{2m}{R}\right)^2t}+2\times 28e^{-\left(\frac{2m+1}{R}\right)^2t}-64e^{-\left(\frac{m+\frac{1}{2}}{R}\right)^2t}\right)$$

$$\sum_{n\in\mathbb{Z}}e^{-\pi mAn+2i\pi bn}=\frac{1}{\sqrt{A}}\sum_{m\in\mathbb{Z}}e^{-\pi(m-b)A^{-1}(m-b)}$$

$$\frac{V_8+S_8}{\eta^8}=\frac{\vartheta_2(0)^4}{\eta^{12}}=16\prod_{n\geq 1}\frac{(1+q^n)^8}{(1-q^n)^8}=16\sum_{n=0}^\infty c(n)q^n$$

$$V_{\text{SS}}=\frac{2R}{(2\pi)^9\alpha'^5}\int_0^\infty\frac{d\tau_2}{\tau_2^6}\int_{-\frac{1}{2}}^{\frac{1}{2}}d\tau_1\left|\frac{V_8+S_8}{\eta^8}\right|^2\sum_{k=0}^\infty e^{-\frac{4\pi R^2}{\alpha'\tau_2}\left(k+\frac{1}{2}\right)^2} \\ V_{\text{K}}=-\frac{R}{(2\pi)^9\alpha'^5}\int_0^\infty\frac{d\tau_2}{\tau_2^6}\frac{V_8+S_8}{\eta^8}(2i\tau_2)\sum_{k=0}^\infty e^{-\frac{\pi R^2}{\alpha'\tau_2}\left(k+\frac{1}{2}\right)^2}$$



$$V = -\frac{403}{189(4\pi)^4} \frac{1}{R^9} \\ -\frac{1}{R^4} \sum_{n \geq 1} \sum_{k \geq 0} \frac{\left(\frac{n}{\alpha'}\right)^{\frac{5}{2}}}{\left(k + \frac{1}{2}\right)^5} \left[c(n)^2 K_5 \left(8\pi R \sqrt{\frac{n}{\alpha'}} \left(k + \frac{1}{2} \right) \right) + 2c(n) K_5 \left(4\pi R \sqrt{\frac{n}{\alpha'}} \left(k + \frac{1}{2} \right) \right) \right],$$

$$\hat{V} = -\frac{403}{189(4\pi)^4} \frac{1}{R^9} \\ -\frac{1}{2\ell_{10}^5 R_{(m,n)}^4} \sum_{-\mathbb{Z}^2 \setminus (0,0)} \sum_{k \geq 0} \frac{\left(e^{\frac{\phi}{2}} |n + \tau m|\right)^{\frac{5}{2}}}{\left(k + \frac{1}{2}\right)^5} \left[+2c(\gcd(m,n)) K_5 \left(4\pi \frac{R}{\ell_{10}} \sqrt{e^{\frac{\phi}{2}} |n + \tau m|} \left(k + \frac{1}{2} \right) \right) \right. \\ \left. + c(\gcd(m,n))^2 K_5 \left(8\pi \frac{R}{\ell_{10}} \sqrt{e^{\frac{\phi}{2}} |n + \tau m|} \left(k + \frac{1}{2} \right) \right) \right]$$

$$R \times \min_{m,n} \sqrt{e^{\frac{\phi}{2}} |n + \tau m|} \leq \sqrt{2} \ell_{10}$$

$$M_{\rm F1-D1}^2 = -2 \frac{|n + \tau m|}{\alpha'} + \frac{R^2 |n + \tau m|^2}{\alpha'^2}.$$

$$M_{\rm D1}^2 = -2 \frac{\sqrt{\frac{1}{g_s^2} + C_0^2}}{\alpha'} + \frac{R^2}{\alpha'^2} \left(\frac{1}{g_s^2} + C_0^2 \right) \approx \frac{R^2}{\alpha'^2 g_s^2} - \frac{2}{\alpha' g_s} + \mathcal{O}(g_s^0)$$

$$4e^{-2\phi}\partial_\tau\partial_{\bar\tau}\hat{V}=\frac{1}{16}(\partial_{\log R}^2+22\partial_{\log R}+117)\hat{V}$$

$$\int ds f(s) \frac{\ell_{10}^{4s}}{R^{9+4s}} e^{-\frac{\phi}{2}} K_{s-\frac{1}{2}}(2\pi |N| e^{-\Phi}) e^{2\pi i N C_0}$$

$$S_{\mathrm{D}(-1)}=2\pi |N| e^{-\Phi}+2\pi i N C_0$$

$$V \sim \sum_{h=1}^{\infty} d_h \frac{1}{R^9} \frac{\ell_{10}^{8(h-1)}}{R^{8(h-1)}}$$

$$\sigma(X^8,X^9,X^{10})=(-X^8,X^9+\pi R,-X^{10})$$

$$\mathcal{M}_3 = \mathbb{R}^3/\pi_1(\mathcal{M}_3)$$

$$T_{a_1,a_2,a_3}(X^8,X^9,X^{10})=\left(X^8+2\pi a_1\frac{\alpha'}{R_{\rm B}},X^9+2\pi a_3R,X^{10}+2\pi a_2e^{\frac{2}{3}\Phi_{\rm A}}\sqrt{\alpha'}\right)$$

$$H_1(\mathcal{M}_3,\mathbb{Z})=\pi_1(\mathcal{M}_3)/[\pi_1(\mathcal{M}_3),\pi_1(\mathcal{M}_3)]=\mathbb{Z}\oplus\mathbb{Z}_2\oplus\mathbb{Z}_2,$$



$$\begin{aligned} H_0(\mathcal{M}_3, \mathbb{Z}) &= \mathbb{Z}, & H_1(\mathcal{M}_3, \mathbb{Z}) &= \mathbb{Z} \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2, \\ H_2(\mathcal{M}_3, \mathbb{Z}) &= \mathbb{Z}, & H_3(\mathcal{M}_3, \mathbb{Z}) &= \mathbb{Z}. \end{aligned}$$

$$S^1(X^9) \in H_1(\mathcal{M}_3, \mathbb{Z}), T^2(X^8, X^{10}) \in H_2(\mathcal{M}_3, \mathbb{Z})$$

$$S^1(X^8) \in H_1(\mathcal{M}_3, \mathbb{Z}), S^1(X^{10}) \in H_1(\mathcal{M}_3, \mathbb{Z})$$

$$\begin{aligned} \mathcal{A} &= \frac{1}{8\eta^8} \sum_m [(\alpha^2 + \beta^2 + \gamma^2 + \delta^2)(V_8 P_m - S_8 P_{m+1/2}) \\ &\quad + (\alpha^2 + \beta^2 - \gamma^2 - \delta^2)(V_8 P_{m+1/2} - S_8 P_m) \\ &\quad + (\alpha^2 - \beta^2 + \gamma^2 - \delta^2)(O_8 P_m - C_8 P_{m+1/2}) \\ &\quad + (\alpha^2 - \beta^2 - \gamma^2 + \delta^2)(O_8 P_{m+1/2} - C_8 P_m)] \\ \tilde{\mathcal{A}} &= \frac{2^{-5}v}{2} \frac{1}{\eta^8} \sum_n [(\alpha^2 V_8 - \beta^2 S_8) W_{2n} + (\gamma^2 O_8 - \delta^2 C_8) W_{2n+1}] \end{aligned}$$

$$\begin{aligned} \mathcal{M}_9 &= -\frac{1}{2\eta^8} \sum_m [\epsilon_1 \gamma (-1)^m P_m \hat{O}_8 - i \epsilon_2 \delta (-1)^m P_{m+1/2} \hat{C}_8] \\ \tilde{\mathcal{M}}_9 &= -\frac{v}{\eta^8} \sum_n [\epsilon_1 \gamma \hat{O}_8 + (-1)^n \epsilon_2 \delta \hat{C}_8] W_{2n+1} \end{aligned}$$

$$\begin{aligned} \alpha &= N_1 + \overline{N_1} + N_2 + \overline{N_2}, & \beta &= i(N_1 - \overline{N_1} + N_2 - \overline{N_2}), \\ \gamma &= N_1 + \overline{N_1} - N_2 - \overline{N_2}, & \delta &= i(N_1 - \overline{N_1} - N_2 + \overline{N_2}). \end{aligned}$$

$$\begin{aligned} A_{99} &= \frac{1}{\eta^8} \left[N\bar{N} \sum_m (V_8 P_m - S_8 P_{m+1/2}) + \frac{N^2 + \bar{N}^2}{2} \sum_m (O_8 P_m - C_8 P_{m+1/2}) \right] \\ M_9 &= -\frac{1}{2\hat{\eta}^8} \sum_m (-1)^m [\epsilon_1 (N + \bar{N}) O_8 P_m - \epsilon_2 (N - \bar{N}) C_8 P_{m+1/2}] \end{aligned}$$

$$\tilde{\mathcal{M}}_7 = -\frac{2v}{\hat{\eta}^5 \hat{\vartheta}_2} \sum_n [\epsilon_1 \gamma (\hat{O}_6 \hat{O}_2 + \hat{V}_6 \hat{V}_2) (-1)^n - \epsilon_2 \delta (\hat{S}_6 \hat{C}_2 - \hat{C}_6 \hat{S}_2)] W_{2n+1}$$

$$\begin{aligned} \alpha &= N_1 + \overline{N_1} + N_2 + \overline{N_2}, & \gamma &= i(N_1 - \overline{N_1} + N_2 - \overline{N_2}), \\ \beta &= N_1 + \overline{N_1} - N_2 - \overline{N_2}, & \delta &= i(N_1 - \overline{N_1} - N_2 + \overline{N_2}). \end{aligned}$$

$$\begin{aligned} \mathcal{A}_{77} &= \frac{1}{\eta^8} \left[N\bar{N} \sum_m (V_8 P_m - S_8 P_{m+1/2}) + \frac{N^2 + \bar{N}^2}{2} \sum_m (V_8 P_{m+1/2} - S_8 P_m) \right] \\ \mathcal{M}_7 &= \left(\frac{2}{\hat{\eta}^5 \hat{\vartheta}_2} \right) \frac{N - \bar{N}}{2} \sum_m (-1)^m [\epsilon_1 (\hat{V}_6 \hat{O}_2 - \hat{O}_6 \hat{V}_2) P_{m+1/2} + \epsilon_2 (\hat{S}_6 \hat{S}_2 - \hat{C}_6 \hat{C}_2) P_m] \end{aligned}$$

$$D7_{01234589} \xrightarrow[\text{T-duality}]{} KK6_{0123459}.$$



$$\begin{aligned}
A_{77} &= \frac{V_8 - S_8}{\eta^8} \sum_n \left(N\bar{N}W_n + \frac{N^2 + \bar{N}^2}{2} W_{n+1/2} \right) \\
\tilde{\mathcal{A}}_{77} &= \frac{2^{-4}}{2v} \left(\frac{V_8 - S_8}{\eta^8} \right) \sum_m \left[e^{\frac{i\pi m}{2}} N + e^{-\frac{i\pi m}{2}} \bar{N} \right]^2 P_m \\
&= \frac{2^{-4}}{2v} \left(\frac{V_8 - S_8}{\eta^8} \right) \sum_m [(N + \bar{N})^2 P_{2m} - (N - \bar{N})^2 P_{2m+1}] \\
&= \frac{2^{-4}}{v} \left(\frac{V_8 - S_8}{\eta^8} \right) \sum_m \left[N\bar{N} + (-1)^m \frac{N^2 + \bar{N}^2}{2} \right] P_m
\end{aligned}$$

$$D7_{01234568} \xrightarrow{\text{T-duality}} \text{KK6}_{0123456}$$

$$\begin{aligned}
\tilde{\mathcal{A}}_{55} &= \frac{2^{-3}}{2v} \frac{1}{\eta^8} \sum_m \left[\left(e^{\frac{i\pi m}{2}} N + e^{-\frac{i\pi m}{2}} \bar{N} \right)^2 V_8 + \left(e^{\frac{i\pi m}{2}} N - e^{-\frac{i\pi m}{2}} \bar{N} \right)^2 S_8 \right] P_m \\
&= \frac{2^{-3}}{2v} \frac{1}{\eta^8} \sum_m [(N + \bar{N})^2 (V_8 P_{2m} - S_8 P_{2m+1}) - (N - \bar{N})^2 (V_8 P_{2m+1} - S_8 P_{2m})] \\
&= \frac{2^{-4}}{v} \frac{1}{\eta^8} \sum_m \left[N\bar{N}(V_8 - S_8) + \frac{N^2 + \bar{N}^2}{2} (-1)^m (V_8 + S_8) \right] P_m \\
\mathcal{A}_{55} &= \frac{1}{\eta^8} \left[N\bar{N}(V_8 - S_8) \sum_n W_n + \frac{N^2 + \bar{N}^2}{2} (O_8 - C_8) \sum_n W_{n+1/2} \right]
\end{aligned}$$

KK number	Fields	Representation
$2m$	vectors $A_\mu^{(m)}$ + 6 scalars	$N\bar{N}$
	4 fermions	$\frac{N(N-1)}{2} + \frac{N(N+1)}{2}$
$2m+1$	vectors $A_\mu^{(m)}$ + 6 scalars	$N\bar{N}$
	4 fermions	$\frac{N(N+1)}{2} + \frac{N(N-1)}{2}$
$2m + \frac{1}{2}$	vectors $A_\mu^{(m)}$	$\frac{N(N-1)}{2} + \frac{N(N+1)}{2}$
	4 fermions	$N\bar{N}$
	6 complex scalars	$\frac{N(N+1)}{2} + \frac{N(N-1)}{2}$
$2m + \frac{3}{2}$	vectors $A_\mu^{(m)}$	$\frac{N(N+1)}{2} + \frac{N(N-1)}{2}$
	4 fermions	$N\bar{N}$
	6 complex scalars	$\frac{N(N-1)}{2} + \frac{N(N+1)}{2}$

$$D5_{012348} \xrightarrow{\text{T-duality}} M5_{0123410}$$

$$D5_{012345} \xrightarrow{\text{T-duality}} \text{KK6}_{0123458}$$



$$\mathcal{A}_{33}=\frac{1}{\eta^8}\Bigg[N\bar{N}\sum_m\left(V_8P_m-S_8P_{m+1/2}\right)+\frac{N^2+\bar{N}^2}{2}\sum_m\left(V_8P_{m+1/2}-S_8P_m\right)\Bigg]$$

$$\mathcal{M}_3=\left(\frac{8\hat{\eta}}{\hat{\vartheta}_2^3}\right)\frac{N-\bar{N}}{2}\sum_m(-1)^m\big[\epsilon_1(\hat{O}_2\hat{V}_6-\hat{V}_2\hat{O}_6)P_{m+1/2}+\epsilon_2(\hat{S}_2\hat{S}_6-\hat{C}_2\hat{C}_6)P_m\big]$$

$$\varsigma=\left(\begin{array}{cc}0&i\mathbb{I}_{N\times N}\\ \mathbb{I}_{N\times N}&0\end{array}\right)$$

$$Z_4A_\mu(X)=-\varsigma A_\mu(X+\pi R)^\top\varsigma^\dagger,\quad Z_4\phi_{ij}(X)=\varsigma\phi_{ij}(X+\pi R)^\top\varsigma^\dagger$$

$$Z_4\lambda_{\alpha i}(X)=i\varsigma\lambda_{\alpha i}(X+\pi R)^\top\varsigma^\dagger,\quad Z_4\lambda^i_\alpha(X)=-i\varsigma\lambda^i_\alpha(X+\pi R)^\top\varsigma^\dagger$$

$$\mathrm{D3}_{0189} \stackrel{\mathrm{T-duality}}{\rightarrow} \mathrm{M2}_{019}$$

$$\mathrm{D3}_{0129} \stackrel{\mathrm{T-duality}}{\rightarrow} \mathrm{M5}_{0128910},$$

$$\tilde{\mathcal{A}}_{11}=\frac{2^{-1}v}{2\eta^8}\sum_n\left[(N+\bar{N})^2(V_8W_{2n}+O_8W_{2n+1})+(N-\bar{N})^2(S_8W_{2n}+C_8W_{2n+1})\right]$$

$$\tilde{\mathcal{M}}_1=-v\left(\frac{2\hat{\eta}}{\hat{\vartheta}_2}\right)^4\sum_n\left[\epsilon_1(N+\bar{N})(\hat{O}_0\hat{O}_8-\hat{V}_0\hat{V}_8)+i\epsilon_2(-1)^n(N-\bar{N})(\hat{S}_0\hat{C}_8-\hat{C}_0\hat{S}_8)\right]W_{2n+1}.$$

$$\mathcal{A}_{11}=\frac{1}{\eta^8}\Bigg[N\bar{N}\sum_m\left(V_8P_m-S_8P_{m+1/2}\right)+\frac{N^2+\bar{N}^2}{2}\sum_m\left(O_8P_m-C_8P_{m+1/2}\right)\Bigg]$$

$$\mathcal{M}_1=\left(\frac{2\hat{\eta}}{\hat{\vartheta}_2}\right)^4\sum_m(-1)^m\bigg[\epsilon_1\frac{N+\bar{N}}{2}(\hat{O}_0\hat{O}_8-\hat{V}_0\hat{V}_8)P_m-\epsilon_2\frac{N-\bar{N}}{2}(\hat{S}_0\hat{C}_8-\hat{C}_0\hat{S}_8)P_{m+1/2}\bigg]$$

$$O_8=O_0O_8+V_0V_8, V_8=V_0O_8+O_0V_8$$

$$S_8=S_0S_8+C_0C_8, C_8=S_0C_8+C_0S_8$$

$$\mathrm{D1}_{09} \stackrel{\mathrm{T-duality}}{\rightarrow} \mathrm{M2}_{089},$$

$$Z_2(X^6,X^7,X^8,X^9)=(-X^6,-X^7,-X^8,-X^9)$$

$$\delta_5 X^5 = X^5 + \pi R_5$$

$$\Omega'=\Omega Z_4\delta_5,$$

$$(\Omega')^2=Z_2\delta_5^2$$



$$\mathcal{T} = \frac{1}{2} \left\{ \left| \frac{Q_o + Q_v}{\eta^8} \right|^2 \sum_{m,n} \left(q^{\frac{\alpha'}{4}} p_L^T g^{-1} p_L q^{\frac{\alpha'}{4}} p_R^T g^{-1} p_R \right) \sum_{m_5, n_5} \left(\Lambda_{m_5, 2n_5}^{(1,1)} + \Lambda_{m_5 + \frac{1}{2}, 2n_5}^{(1,1)} \right) \right. \\ \left. + \frac{16}{|\eta|^4} \left| \frac{Q_o - Q_v}{\vartheta_2^2} \right|^2 \sum_{m_5, n_5} \left(\Lambda_{m_5, 2n_5}^{(1,1)} - \Lambda_{m_5 + \frac{1}{2}, 2n_5}^{(1,1)} \right) \right. \\ \left. + \left| \frac{Q_s - Q_c}{\vartheta_3^2} \right|^2 \sum_{m_5, n_5} \left(\Lambda_{m_5, 2n_5 + 1}^{(1,1)} - \Lambda_{m_5 + \frac{1}{2}, 2n_5 + 1}^{(1,1)} \right) \right. \\ \left. + \left| \frac{Q_s + Q_c}{\vartheta_4^2} \right|^2 \sum_{m_5, n_5} \left(\Lambda_{m_5, 2n_5 + 1}^{(1,1)} + \Lambda_{m_5 + \frac{1}{2}, 2n_5 + 1}^{(1,1)} \right) \right\}$$

$$\mathcal{K} = \frac{2^2}{2} \frac{Q_o - Q_v}{\eta^2 \vartheta_2^2} \sum_{m_5} (-1)^{m_5} P_{m_5}$$

$$\tilde{\mathcal{K}} = \frac{2^3}{2} \frac{R_5}{\sqrt{\alpha'}} \frac{Q_s + Q_c}{\eta^2 \vartheta_4^2} \sum_{n_5} W_{2n_5 + 1}$$

$$T_{(\mu\nu)} \equiv \frac{1}{2} (T_{\mu\nu} + T_{\nu\mu}), T_{[\mu\nu]} \equiv \frac{1}{2} (T_{\mu\nu} - T_{\nu\mu})$$

$$T_{\mu]...[\nu} \equiv T_{[\mu|...| \nu]}$$

$$[O_1,O_2]\equiv O_1O_2-O_1O_2=2O_{[1}O_{2]},\{O_1,O_2\}\equiv O_1O_2+O_1O_2=2O_{(1}O_{2)}$$

$$\Gamma_{\nu\rho}^{\mu}=\frac{1}{2}g^{\mu\sigma}(\partial_{\rho}g_{\nu\sigma}+\partial_{\nu}g_{\rho\sigma}-\partial_{\sigma}g_{\nu\rho})$$

$$\omega_{\mu}^{ab}=2e^{v[a}\partial_{[\mu}e_{\nu]}^{b]}-e_c^{\mu}e^{v[a}e^{b]\sigma}\partial_{\nu}e_{\sigma}^c$$

$$R_{\nu\rho\sigma}^{\mu}=2\partial_{[\rho}\Gamma_{\sigma]\nu}^{\mu}+2\Gamma_{\lambda[\rho}^{\mu}\Gamma_{\sigma]\nu}^{\lambda},R_{\mu\nu}=R_{\mu\rho\nu}^{\rho},G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R$$

$$T_{\mu\nu}=-\frac{2}{e}\frac{\delta(e\mathcal{L})}{\delta g^{\mu\nu}}=-2\frac{\delta\mathcal{L}}{\delta g^{\mu\nu}}+\frac{1}{2}g_{\mu\nu}\mathcal{L}$$

$$\varepsilon^{\mu_1\ldots\mu_p\rho_1\ldots\rho_{D-p}}\varepsilon_{\nu_1\ldots\nu_p\rho_1\ldots\rho_{D-p}}=-\frac{1}{p!(D-p)!}\delta^{\mu_1\ldots\mu_p}_{\nu_1\ldots\nu_p}$$

$$\delta^{\mu_1\ldots\mu_p}_{\nu_1\ldots\nu_p}\equiv\delta^{\mu_1}_{[\nu_1}\ldots\delta^{\mu_p}_{\nu_p]}$$

$$\bar{\psi}_{\dot{\alpha}} = (\psi_{\alpha})^{\dagger} = (\psi^{\alpha})^*$$

$$\epsilon_{\alpha\beta}=\epsilon_{\dot{\alpha}\dot{\beta}}=\begin{pmatrix}0&-1\\1&0\end{pmatrix},\epsilon^{\alpha\beta}=\epsilon^{\dot{\alpha}\dot{\beta}}=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$$

$$\psi\chi=\psi^{\alpha}\chi_{\alpha},\bar{\psi}\bar{\chi}=\bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}$$

$$\sigma^1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\sigma^2=\begin{pmatrix}0&-1\\i&0\end{pmatrix},\sigma^3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$



$$\sigma^m = (-1, \sigma^i), \bar{\sigma}^m = (-1, -\sigma^i), \bar{\sigma}^{m\dot{\alpha}\alpha} = \varepsilon^{\dot{\alpha}\beta} \varepsilon^{\alpha\gamma} \sigma_{\beta\gamma}^m$$

$$(\sigma^{mn})_{\alpha}^{\beta} = \frac{1}{4}(\sigma^m \bar{\sigma}^n - \sigma^n \bar{\sigma}^m)_{\alpha}^{\beta}, (\bar{\sigma}^{mn})_{\dot{\beta}}^{\dot{\alpha}} = \frac{1}{4}(\bar{\sigma}^m \sigma^n - \bar{\sigma}^n \sigma^m)_{\dot{\beta}}^{\dot{\alpha}},$$

$$\text{Tr}(\sigma^m \bar{\sigma}^n) = -2\eta^{mn}$$

$$\sigma_{\alpha\dot{\alpha}}^m \bar{\sigma}_m^{\dot{\beta}\beta} = -2\delta_{\alpha}^{\beta} \delta_{\dot{\alpha}}^{\dot{\beta}}$$

$$\sigma^m \bar{\sigma}^n + \sigma^n \bar{\sigma}^m = -2\eta^{mn} \mathbb{1}$$

$$\bar{\sigma}^m \sigma^n + \bar{\sigma}^n \sigma^m = -2\eta^{mn} \mathbb{1}$$

$$\sigma^m \bar{\sigma}^n \sigma^p = (-\eta^{mn} \eta^{rs} + \eta^{mr} \eta^{ns} - \eta^{nr} \eta^{ms} + i\epsilon^{mnrs}) \sigma_s$$

$$\bar{\sigma}^m \sigma^n \bar{\sigma}^p = (-\eta^{mn} \eta^{rs} + \eta^{mr} \eta^{ns} - \eta^{nr} \eta^{ms} - i\epsilon^{mnrs}) \bar{\sigma}_s$$

$$\theta^{\alpha} \theta^{\beta} = -\frac{1}{2} \theta^2 \epsilon^{\alpha\beta},$$

$$\psi \chi = \psi^{\alpha} \chi_{\alpha} = -\chi_{\alpha} \psi^{\alpha} = \chi \psi$$

$$\bar{\theta}^{\dot{\alpha}} \bar{\theta}^{\dot{\beta}} = \frac{1}{2} \bar{\theta}^2 \epsilon^{\dot{\alpha}\dot{\beta}},$$

$$\bar{\psi} \bar{\chi} = \bar{\psi}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}} = -\bar{\chi}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}} = \bar{\chi} \bar{\psi}$$

$$\theta_{\alpha} \theta_{\beta} = \frac{1}{2} \theta^2 \epsilon_{\alpha\beta},$$

$$(\theta \sigma^m \bar{\psi})(\theta \sigma^n \bar{\chi}) = \frac{1}{2} \theta^2 (\bar{\psi} \bar{\sigma}^m \sigma^n \bar{\chi})$$

$$\bar{\theta}_{\dot{\alpha}} \bar{\theta}_{\dot{\beta}} = -\frac{1}{2} \bar{\theta}^2 \epsilon_{\dot{\alpha}\dot{\beta}},$$

$$(\psi \sigma^m \bar{\theta})(\chi \sigma^n \bar{\theta}) = \frac{1}{2} \bar{\theta}^2 (\psi \sigma^m \bar{\sigma}^n \chi)$$

$$\psi \sigma^m \bar{\chi} = -\bar{\chi} \bar{\sigma}^m \psi,$$

$$(\theta \sigma^m \bar{\theta})(\theta \sigma^n \bar{\theta}) = \frac{1}{2} \theta^2 \bar{\theta}^2 \eta^{mn}$$

$$\theta \sigma^m \bar{\sigma}^n \theta = \theta^2 \eta^{mn},$$

$$(\theta \psi)(\theta \chi) = -\frac{1}{2} \theta^2 \psi \chi$$

$$\psi \sigma^m \bar{\sigma}^n \chi = \chi \sigma^n \bar{\sigma}^m \psi,$$

$$(\bar{\theta} \bar{\psi})(\bar{\theta} \bar{\chi}) = -\frac{1}{2} \bar{\theta}^2 \bar{\psi} \bar{\chi}$$

$$\theta_1^2 = \theta_2^2 = \bar{\theta}_1^2 = \bar{\theta}_2^2 = 0 \leftrightarrow \theta^3 = \bar{\theta}^3 = 0.$$

$$F(\vartheta) = F_0 + F_1 \vartheta.$$

$$\int d\vartheta = 0, \int d\vartheta \vartheta = 1$$

$$\int d\vartheta F(\vartheta) = F_1 = \partial_{\vartheta} F(\vartheta), \int d\vartheta \vartheta F(\vartheta) = F_0, \int d\vartheta \partial_{\vartheta} F(\vartheta) = 0$$

$$d^2\theta = -\frac{1}{4}\epsilon_{\alpha\beta}d\theta^{\alpha}d\theta^{\beta} = \frac{1}{2}d\theta^1d\theta^2$$

$$d^2\bar{\theta} = -\frac{1}{4}\epsilon^{\dot{\alpha}\dot{\beta}}d\bar{\theta}_{\dot{\alpha}}d\bar{\theta}_{\dot{\beta}} = -\frac{1}{2}d\bar{\theta}_1d\bar{\theta}_2$$

$$d^4\theta = d^2\theta d^2\bar{\theta}$$

$$\int \theta^2 d^2\theta = \int \bar{\theta}^2 d^2\bar{\theta} = 1$$

$$D_{\alpha} = \partial_{\alpha} + i(\sigma^m \bar{\theta})_{\alpha} \partial_m, \quad \bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} - i(\theta \sigma^m)_{\dot{\alpha}} \partial_m$$



$$\begin{aligned}
D_\alpha D_\beta &= -\frac{1}{2} \varepsilon_{\alpha\beta} D^2, & \bar{D}_{\dot{\alpha}} \bar{D}_{\dot{\beta}} &= \frac{1}{2} \varepsilon_{\dot{\alpha}\dot{\beta}} \bar{D}^2 \\
\{D_\alpha, \bar{D}_{\dot{\alpha}}\} &= -2i\sigma_{\alpha\dot{\alpha}}^m \partial_m, & \{D_\alpha, D_\beta\} &= \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0 \\
D^2 \theta^2 &= -4 + 4i\theta \sigma^m \bar{\theta} \partial_m + \theta^2 \bar{\theta}^2 \square, & [D_\alpha, \bar{D}^2] &= -4i\sigma_{\alpha\dot{\alpha}}^m \bar{D}^{\dot{\alpha}} \partial_m \\
\bar{D}^2 \bar{\theta}^2 &= -4 - 4i\theta \sigma^m \bar{\theta} \partial_m + \theta^2 \bar{\theta}^2 \square, & [D^2, \bar{D}^2] &= -8iD^\alpha \sigma_{\alpha\dot{\alpha}}^m \bar{D}^{\dot{\alpha}} \partial_m + 16 \square
\end{aligned}$$

$$\Psi = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$$

$$\{\gamma^m,\gamma^n\}=2\eta^{mn}$$

$$(\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0 \leftrightarrow (\gamma^0)^\dagger = -\gamma^0, (\gamma^i)^\dagger = \gamma^i$$

$$\bar{\Psi} = i\Psi^\dagger \gamma^0$$

$$\gamma^m = \begin{pmatrix} 0 & -i\sigma^m \\ -i\bar{\sigma}^m & 0 \end{pmatrix}, \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$$

$$P_L=\frac{1-\gamma_5}{2}=\begin{pmatrix}0&0\\0&\mathbb{1}\end{pmatrix}, P_R=\frac{1+\gamma_5}{2}=\begin{pmatrix}\mathbb{1}&0\\0&0\end{pmatrix}$$

$$\Psi_{L,R}\equiv P_{L,R}\Psi, \Psi_L=\begin{pmatrix}0\\ \bar{\chi}^{\dot{\alpha}}\end{pmatrix}, \Psi_R=\begin{pmatrix}\psi_\alpha\\ 0\end{pmatrix}, \bar{\Psi}_{L,R}=\bar{\Psi}P_{L,R}=\overline{\Psi_{R,L}}$$

$$\{\mathbb{1},\gamma^m,\gamma^{mn},\gamma^m\gamma_5,\gamma_5\}$$

$$\gamma^{m_1\dots m_r}=-\frac{i}{(4-r)!}\,\varepsilon^{m_r\dots m_1r_1\dots r_{4-r}}\gamma_{r_1\dots r_{4-r}}\gamma_5$$

$$C_4=\begin{pmatrix}\sigma^2&0\\0&-\sigma^2\end{pmatrix}, C_4^{\rm T}=C_4^\dagger=C_4^{-1}=-C_4$$

$$(C_4\gamma^{m_1\dots m_r})^{\rm T}=-t_rC_4\gamma^{m_1\dots m_r}$$

$$\begin{aligned}
M &= \frac{1}{4} \text{Tr}(M) \mathbb{1} + \frac{1}{4} \text{Tr}(\gamma^m M) \gamma_m - \frac{1}{8} \text{Tr}(\gamma^{mn} M) \gamma_{mn} \\
&\quad - \frac{1}{4} \text{Tr}(\gamma^m \gamma_5 M) \gamma_m \gamma_5 + \frac{1}{4} \text{Tr}(\gamma_5 M) \gamma_5
\end{aligned}$$

$$\lambda=\begin{pmatrix}\psi_\alpha\\ \bar{\psi}^{\dot{\alpha}}\end{pmatrix}$$

$$\bar{\lambda}=\lambda^{\rm T}C_4.$$

$$\{\gamma^\mu,\gamma^\nu\}=2g^{\mu\nu}, \gamma^\mu=e^\mu{}_m\gamma^m$$

$$\gamma^\mu=e^\mu{}_a\gamma^a=a(\tau)^{-1}\gamma^a\equiv a(\tau)^{-1}\bar{\gamma}^\mu$$



$$\vec{k} \cdot \vec{x} = \sum_{i=1}^3 k^i x^i = k^i x_i, \vec{\gamma} \cdot \vec{\psi}_k = \sum_{i=1}^3 \bar{\gamma}^i \psi_{i,k} = \bar{\gamma}^i \psi_{i,k}$$

$$\vec{k} \cdot \vec{\psi}_k = \sum_{i=1}^3 k^i \psi_{i,k} = k^i \psi_{i,k}, \vec{k} \cdot \vec{\gamma} = \sum_{i=1}^3 k^i \bar{\gamma}^i = k_i \bar{\gamma}^i$$

$$(\vec{k} \cdot \vec{\gamma})(\vec{k} \cdot \vec{\gamma}) = \vec{k}^2, (\vec{k} \cdot \vec{\gamma}) \bar{\gamma}^i (\vec{k} \cdot \vec{\gamma}) = 2k^i (\vec{k} \cdot \vec{\gamma}) - \vec{k}^2 \bar{\gamma}^i$$

$$\begin{aligned} \bar{\gamma}^i (P_\gamma)_i &= -1, & \bar{\gamma}^i (P_k)_i &= 0 \\ k^i (P_\gamma)_i &= 0, & k^i (P_k)_i &= -1 \\ \{(P_\gamma)_i, \bar{\gamma}_0\} &= 0, & \{(P_\gamma)_i, \bar{\gamma}_j\} &= -\eta_{ij} + \frac{k_i k_j}{\vec{k}^2}, \\ [(P_\gamma)_k, \bar{\gamma}_0] &= 0, & [(P_k)_i, \bar{\gamma}_j] &= \frac{1}{\vec{k}^2} [\bar{\gamma}_i k_j + \bar{\gamma}_j k_i + \eta_{ij} (\vec{k} \cdot \vec{\gamma})], \\ (P_\gamma)_i^\dagger &= (P_\gamma)_i, & (P_k)_i^\dagger &= \frac{1}{2\vec{k}^2} [(\vec{k} \cdot \vec{\gamma}) \bar{\gamma}_i - 3k_i], \\ (P_\gamma)^i (P_\gamma)_i &= \frac{1}{2}, & (P_\gamma)^i (P_k)_i &= -\frac{1}{2\vec{k}^2} (\vec{k} \cdot \vec{\gamma}), \\ (P_k)^i (P_k)_i &= \frac{1}{4\vec{k}^4}, & (P_k)^i (P_\gamma)_i &= \frac{1}{2\vec{k}^2} (\vec{k} \cdot \vec{\gamma}), \\ (P_k)^{i\dagger} (P_k)_i &= \frac{3}{4\vec{k}^4}, & (P_k)^{i\dagger} (P_\gamma)_i &= -\frac{1}{2\vec{k}^2} (\vec{k} \cdot \vec{\gamma}). \end{aligned}$$

$$\begin{aligned} \bar{\gamma}^0 \hat{\partial}_i^\dagger \bar{\gamma}^0 &= \frac{1}{\vec{k}^2} \left\{ -k_i (\vec{k} \cdot \vec{\gamma}) + \frac{ia}{2} (m + H\bar{\gamma}^0) [(\vec{k} \cdot \vec{\gamma}) \bar{\gamma}_i - 3k_i] \right\} \\ (\vec{k} \cdot \vec{\gamma}) \hat{\partial}_i &= - \left\{ k_i + \frac{ia}{2} \left[\frac{k_i}{\vec{k}^2} (\vec{k} \cdot \vec{\gamma}) + \bar{\gamma}_i \right] (m - H\bar{\gamma}^0) \right\} \end{aligned}$$

$$\begin{aligned} T^{\mu\nu} &= \frac{1}{2} \bar{\Psi}_\rho \gamma^{\rho\sigma\mu} (\nabla_\sigma \Psi^\nu - \nabla^\nu \Psi_\sigma) - \frac{m}{2} \bar{\Psi}_\rho \gamma^{\rho\mu} \Psi^\nu \\ &\quad - \frac{1}{4} \nabla_\rho (\bar{\Psi}^\rho \gamma^\mu \Psi^\nu + \bar{\Psi}^\rho \gamma^\nu \Psi^\mu - \bar{\Psi}^\mu \gamma^\rho \Psi^\nu) \\ &\quad - \frac{1}{2} \nabla^\nu (\bar{\Psi}^\mu \gamma^\rho \Psi_\rho) - \frac{1}{2} g^{\mu\nu} \nabla_\rho (\bar{\Psi}_\sigma \gamma^\sigma \Psi^\rho). \end{aligned}$$



$$\begin{aligned}
T^{[\mu\nu]} &= \frac{1}{2} \bar{\Psi}_\rho \gamma^{\rho\sigma[\mu} (\nabla_\sigma \Psi^{|\nu]} - \nabla^{|\nu]} \Psi_\sigma) - \frac{m}{2} \bar{\Psi}_\rho \gamma^{\rho[\mu} \Psi^{|\nu]} \\
&\quad - \frac{1}{2} \nabla^{[\nu} (\bar{\Psi}^{\mu]} \gamma^\rho \Psi_\rho) + \frac{1}{4} \nabla_\rho (\bar{\Psi}^{[\mu} \gamma^\rho \Psi^{|\nu]}) = \\
&= -\frac{1}{2} (\bar{\Psi}_\rho \gamma^{\rho[\mu} - \bar{\Psi}^{[\mu}) [\nabla^{\nu]} - \nabla^{\nu]} (\gamma^\rho \Psi_\rho)] - \frac{m}{2} \bar{\Psi}_\rho \gamma^{\rho[\mu} \Psi^{|\nu]} \\
&\quad - \frac{1}{2} \bar{\Psi}^\sigma \gamma^{[\mu} (\nabla_\sigma \Psi^{\nu]} - \nabla^{\nu]} \Psi_\sigma) + \frac{1}{2} \bar{\Psi}_\rho \gamma^\rho \nabla^{[\mu} \Psi^{\nu]} = \\
&= \frac{m}{2} \bar{\Psi}_\rho \gamma^{\rho[\mu} \Psi^{\nu]} - \frac{3}{2} \bar{\Psi}_\rho \gamma^{[\rho} \nabla^{\nu]} \Psi^{\mu]} \\
&= \frac{m}{2} \bar{\Psi}_\rho \gamma^{\rho[\mu} \Psi^{\nu]} - \frac{3}{2} \delta_{\alpha\beta\gamma}^{\rho\nu\mu} \bar{\Psi}_\rho \gamma^\alpha \nabla^\beta \Psi^\gamma = \\
&= \frac{m}{2} \bar{\Psi}_\rho \gamma^{\rho[\mu} \Psi^{\nu]} + \frac{i}{4} \varepsilon^{\rho\nu\mu}{}_\sigma \bar{\Psi}_\rho \gamma_5 [\gamma^{\sigma\lambda\tau} \nabla_\lambda \Psi_\tau] = \\
&= \frac{m}{2} \bar{\Psi}_\rho \gamma^{\rho[\mu} \Psi^{\nu]} + \frac{m}{4} \bar{\Psi}_\rho \gamma^{\rho\nu\mu} \gamma^\sigma \Psi_\sigma \\
&= \frac{m}{2} \bar{\Psi}_\rho \gamma^{\rho[\mu} \Psi^{\nu]} - \frac{m}{2} \bar{\Psi}_\rho \gamma^{\rho[\mu} \Psi^{\nu]} = 0
\end{aligned}$$

$$R_{[\nu\rho\sigma]}^\mu = 0$$

$$R_{\mu\nu\rho\sigma} \gamma^{\mu\nu\lambda} \gamma^{\rho\sigma} = 4G_\tau^\lambda \gamma^\tau$$

$$[\nabla_\mu, \nabla_\nu] \Psi^\rho = \frac{1}{4} R_{\mu\nu\lambda\tau} \gamma^{\lambda\tau} \Psi^\rho + R_{\sigma\mu\nu}^\rho \Psi^\sigma$$

$$[\nabla_\mu, \nabla_\nu] V^{\rho_1 \dots \rho_r} = R_{\sigma\mu\nu}^{\rho_1} V^{\sigma\rho_2 \dots \rho_r} + \dots + R_{\sigma\mu\nu}^{\rho_r} V^{\rho_1 \rho_2 \dots \sigma}$$

$$\begin{aligned}
\frac{1}{2} \nabla_\mu [\bar{\Psi}_\rho \gamma^{\rho\sigma\mu} (\nabla_\sigma \Psi^\nu - \nabla^\nu \Psi_\sigma) - m \bar{\Psi}_\rho \gamma^{\rho\mu} \Psi^\nu] &= \\
&= \frac{1}{2} \bar{\Psi}_\rho \bar{\nabla}_\mu \gamma^{\rho\sigma\mu} (\nabla_\sigma \Psi^\nu - \nabla^\nu \Psi_\sigma) + \frac{1}{2} \bar{\Psi}_\rho \gamma^{\rho\sigma\mu} \nabla_\mu \nabla_\sigma \Psi^\nu - \frac{1}{2} \bar{\Psi}_\rho \gamma^{\rho\mu\sigma} \nabla_\mu \nabla^\nu \Psi_\sigma \\
&\quad - \frac{m}{2} \bar{\Psi}_\rho \bar{\nabla}_\mu \gamma^{\rho\mu} \Psi^\nu - \frac{m}{2} \bar{\Psi}_\rho \gamma^{\rho\mu} \nabla_\mu \Psi^\nu - \frac{1}{2} (\partial_\mu m) \bar{\Psi}_\rho \gamma^{\rho\mu} \Psi^\nu = \\
&= -\frac{1}{4} R_{\rho\alpha\beta}^\nu \bar{\Psi}^\alpha \gamma^\rho \Psi^\beta - \frac{1}{2} R_\rho^\nu \bar{\Psi}_\alpha \gamma^\alpha \Psi^\rho
\end{aligned}$$

$$\square_{\boxtimes}^{\prime\prime\odot} - \frac{1}{4} \nabla_\mu \nabla_\rho [\bar{\Psi}^\rho \gamma^\nu \Psi^\mu] = -\frac{i}{8} [\nabla_\mu, \nabla_\rho] V^{\rho\nu\mu} =$$

$$\square_{\boxtimes}^{\prime\prime\odot} = -\frac{1}{8} (R_{\sigma\mu\rho}^\rho V^{\sigma\nu\mu} + R_{\sigma\mu\rho}^\mu V^{\rho\nu\sigma} + R_{\sigma\mu\rho}^\nu V^{\rho\sigma\mu}) = \frac{1}{8} R_{\rho\alpha\beta}^\nu \bar{\Psi}^\alpha \gamma^\rho \Psi^\beta$$

$$\begin{aligned}
\square_{\boxtimes}^{\prime\prime\odot} - \frac{1}{4} \nabla_\mu \nabla_\rho [\bar{\Psi}^\rho \gamma^\mu \Psi^\nu - \bar{\Psi}^\mu \gamma^\rho \Psi^\nu] &= -\frac{1}{4} [\nabla_\mu, \nabla_\rho] V^{\rho\mu\nu} \\
&= -\frac{1}{4} (R_{\sigma\mu\rho}^\rho V^{\sigma\mu\nu} + R_{\sigma\mu\rho}^\mu V^{\rho\sigma\nu} + R_{\sigma\mu\rho}^\nu V^{\rho\mu\sigma}) = -\frac{1}{4} R_{\beta\rho\alpha}^\nu \bar{\Psi}^\alpha \gamma^\rho \Psi^\beta,
\end{aligned}$$

$$\begin{aligned}
&-\frac{1}{2} \nabla_\mu [g^{\mu\nu} \nabla_\rho (\bar{\Psi}_\alpha \gamma^\alpha \Psi^\rho) + \nabla^\nu (\bar{\Psi}^\mu \gamma^\rho \Psi_\rho)] \\
&= -\frac{1}{2} [\nabla^\nu, \nabla_\mu] (\bar{\Psi}_\rho \gamma^\rho \Psi^\mu) V^\mu = \frac{1}{2} R_\rho^\nu \bar{\Psi}_\alpha \gamma^\alpha \Psi^\rho.
\end{aligned}$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\gamma^0 = \mathbb{1} \otimes (i\sigma_1 \otimes \mathbb{1}), \gamma^i = \mathbb{1} \otimes (\sigma_2 \otimes \sigma^i), \gamma^4 = \mathbb{1} \otimes (\sigma_3 \otimes \mathbb{1}),$$



$$\mathcal{C}_5 = \sigma_2 \otimes (\mathbb{1} \otimes \sigma_2) \equiv \sigma_2 \otimes (\gamma_5 \mathcal{C}_4),$$

$$\mathcal{C}_5^{-1} = \mathcal{C}_5^{\rm T} = \mathcal{C}_5^{\dagger} = \mathcal{C}_5.$$

$$(\mathfrak{C}_5\gamma^{M_1\ldots M_r})^{\rm T} = -t_r\mathfrak{C}_5\gamma^{M_1\ldots M_r},$$

$$\bar{\lambda} = \lambda^{\rm T} \mathcal{C}_5.$$

$$\delta E^A = \frac{1}{2} \bar{\varepsilon} \gamma^A \Psi$$

$$\delta \mathcal{A} = -\frac{i}{2} \bar{\varepsilon} \Psi$$

$$\delta \Psi = \mathcal{D} \varepsilon + i E^A \left(\frac{1}{8} \gamma_A^{BC} \mathcal{F}_{BC} - \frac{1}{2} \mathcal{F}_{AB} \gamma^B \right) \varepsilon$$

$$E^a=\phi^{-\frac{1}{2}}e^a$$

$$E^4=\phi(dy+B)$$

$$\mathcal{A}=a(dy+B)+A$$

$$\Psi=\phi^{\frac{5}{4}}\zeta(dy+B)+\phi^{-\frac{1}{4}}\Big(\psi-\frac{1}{2}e^a\gamma_a\gamma_5\zeta\Big)$$

$$\varepsilon = \phi^{-\frac{1}{4}} \epsilon.$$

$$\mathcal{F}_{ab} = \phi e_a{}^\mu e_b{}^\nu F_{\mu\nu}, \mathcal{F}_{a4} = \phi^{-\frac{1}{2}} e_a{}^\mu (\partial_\mu a - \partial_y A_\mu)$$

$$dE^A=\frac{1}{2}\Omega_{BC}{}^AE^B\wedge E^C, (\omega_C)_{AB}=\frac{1}{2}(\Omega_{CAB}-\Omega_{CBA}+\Omega_{BAC}),$$

$$\Omega^a_{bc}=\phi^{\frac{1}{2}}\Omega^{(4)a}_{bc}-\phi^{-\frac{1}{2}}\partial_\mu\phi e^\mu_{[b}\delta^a_{c]},\qquad\Omega^4_{ab}=\phi^2e^\mu_{[a}e^\nu_{b]}G_{\mu\nu}$$

$$\Omega^a_{b4}=\frac{1}{2}\phi^{-2}\partial_y\phi\delta^a_b-\phi^{-1}e^\mu_b\partial_ye^a_\mu,\quad\Omega^4_{a4}=\phi^{-1/2}e^\mu_a(\partial_\mu\phi-\phi\partial_yB_\mu).$$

$$\delta_{\rm LL} E^A = \Lambda^A{}_B E^B$$

$$\Lambda^a_b=\frac{1}{4}\phi^{\frac{1}{4}}(\bar{\varepsilon}\gamma^a{}_b\gamma_5\zeta),\Lambda^a{}_4=\frac{1}{4}\phi^{\frac{1}{4}}(\bar{\varepsilon}\gamma^a\zeta)$$



$$\begin{aligned}
\delta e_\mu^a &= \frac{1}{2} \bar{\epsilon} \gamma^a \psi_\mu \\
\delta \phi &= \frac{\phi}{2} \bar{\epsilon} \gamma_5 \zeta \\
\delta a &= -\frac{i}{2} \phi \bar{\epsilon} \zeta \\
\delta A_\mu &= -\frac{i}{2} \phi^{-\frac{1}{2}} \bar{\epsilon} \left(\psi_\mu - \frac{1}{2} \gamma_\mu \gamma_5 \zeta \right) \\
\delta B_\mu &= \frac{\phi^{-\frac{3}{2}}}{2} \bar{\epsilon} \left(\gamma_5 \psi_\mu + \frac{3}{2} \gamma_\mu \zeta \right) \\
\delta \zeta &= -\frac{\phi^{-\frac{5}{2}}}{4} \partial_y \phi \epsilon + \frac{\phi^{-\frac{3}{2}}}{4} Q_{ab} \gamma^{ab} \epsilon + \frac{i}{8} \phi^{\frac{1}{2}} (F_{\rho\sigma} + i\phi G_{\rho\sigma} \gamma_5) \gamma^{\rho\sigma} \gamma_5 \epsilon \\
&\quad + \frac{i}{2} \phi^{-1} [\partial_\rho a - \partial_y A_\rho + i(\phi \partial_y B_\rho - \partial_\rho \phi) \gamma_5] \gamma^\rho \epsilon \\
\delta \psi_\mu &= D_\mu \epsilon - \frac{3}{8} \phi^{-\frac{5}{2}} \partial_y \phi \gamma_\mu \gamma_5 \epsilon + \frac{\phi^{-\frac{3}{2}}}{2} e_\mu^a \left(P_{ab} \gamma^b + \frac{1}{4} Q_{bc} \gamma_a \gamma^{bc} \right) \gamma_5 \epsilon \\
&\quad - \frac{1}{4} \partial_y B_\rho \gamma_\mu \gamma^\rho \epsilon + \frac{3i}{4} \phi^{-1} (\partial_y A_\mu - \partial_\mu a) \gamma_5 \epsilon - \frac{i}{8} \phi^{\frac{1}{2}} (3F_{\mu\rho}^{+,5} - i\phi G_{\mu\rho}^{+,5} \gamma_5) \gamma^\rho \epsilon
\end{aligned}$$

$$P_{ab} \equiv e_{(a}{}^\lambda \partial_y e_{b)\lambda}, \quad Q_{ab} \equiv e_{[a}{}^\lambda \partial_y e_{b]\lambda}$$

$$F_{\mu\nu}^{\pm,5} \equiv F_{\mu\nu} \pm i\tilde{F}_{\mu\nu} \gamma_5, \quad \tilde{F}^{\mu\rho} \equiv \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

$$\begin{aligned}
e_\mu{}^a(y) &= e_\mu^{(0)a}(x) + \sum_{n=1}^{\infty} e_\mu^{(n)a}(x) \cos\left(\frac{ny}{R}\right), \quad A_\mu(y) = \sum_{n=1}^{\infty} A_\mu^{(n)}(x) \sin\left(\frac{ny}{R}\right) \\
\phi(y) &= \phi^{(0)}(x) \left[1 + \sum_{n=1}^{\infty} \phi^{(n)}(x) \cos\left(\frac{ny}{R}\right) \right] \quad B_\mu(y) = \sum_{n=1}^{\infty} B_\mu^{(n)}(x) \sin\left(\frac{ny}{R}\right) \\
a(y) &= a^{(0)}(x) + \sum_{n=1}^{\infty} a^{(n)}(x) \cos\left(\frac{ny}{R}\right)
\end{aligned}$$

$$\Psi_{i\mu}(-y) = \sigma_{ij}^3 \gamma_5 \Psi_{j\mu}(y), \quad \Psi_{i5}(-y) = -\sigma_{ij}^3 \gamma_5 \Psi_{j5}(y)$$

$$\begin{aligned}
\psi_{1\mu}(-y) &= \gamma_5 \psi_{1\mu}(y), & \zeta_{1\mu}(-y) &= -\gamma_5 \zeta_{1\mu}(y) \\
\psi_{2\mu}(-y) &= -\gamma_5 \psi_{2\mu}(y), & \zeta_{2\mu}(-y) &= \gamma_5 \zeta_{2\mu}(y)
\end{aligned}$$



$$\begin{aligned}\psi_{1\mu} &= \psi_{1\mu R} + \psi_{1\mu L} = \psi_{1\mu}^{(0)} + \sum_{n=1}^{\infty} \left[\psi_{1\mu R}^{(n)} \cos\left(\frac{ny}{R}\right) + i\psi_{1\mu L}^{(n)} \sin\left(\frac{ny}{R}\right) \right] \\ \psi_{2\mu} &= \psi_{1\mu L} + \psi_{1\mu R} = \psi_{2\mu}^{(0)} + \sum_{n=1}^{\infty} \left[\psi_{2\mu L}^{(n)} \cos\left(\frac{ny}{R}\right) + i\psi_{2\mu R}^{(n)} \sin\left(\frac{ny}{R}\right) \right] \\ \zeta_1 &= \zeta_{1L} + \zeta_{1R} = \zeta_{1L}^{(0)} + \sum_{n=1}^{\infty} \left[\zeta_{1L}^{(n)} \cos\left(\frac{ny}{R}\right) + i\zeta_{1R}^{(n)} \sin\left(\frac{ny}{R}\right) \right] \\ \zeta_2 &= \zeta_{2R} + \zeta_{2L} = \zeta_{2R}^{(0)} + \sum_{n=1}^{\infty} \left[\zeta_{2R}^{(n)} \cos\left(\frac{ny}{R}\right) + i\zeta_{2L}^{(n)} \sin\left(\frac{ny}{R}\right) \right] \\ \zeta_{2R}^{(n)} &= -\left(\zeta_{1L}^{(n)}\right)^*, \zeta_{2L}^{(n)} = -\left(\zeta_{1R}^{(n)}\right)^*, \psi_{2\mu R}^{(n)} = \left(\psi_{1\mu L}^{(n)}\right)^*, \psi_{2\mu L}^{(n)} = \left(\psi_{1\mu R}^{(n)}\right)^*\end{aligned}$$

$$\epsilon_i(-y) = \sigma_{ij}^3 \gamma_5 \epsilon_j(y)$$

$$\epsilon_1 = \epsilon_{1R}, \epsilon_2 = \epsilon_{2L}$$

$$\epsilon \equiv \epsilon_{1R} + \epsilon_{2L}$$

$$\begin{aligned}\delta e_{\mu}^{(0)a} &= \frac{1}{2} \left(\bar{\epsilon}_1 \gamma^a \psi_{1\mu}^{(0)} + \bar{\epsilon}_2 \gamma^a \psi_{2\mu}^{(0)} \right) \\ \delta \phi^{(0)} &= \frac{\phi^{(0)}}{2} \left(\bar{\epsilon}_2 \zeta_2^{(0)} - \bar{\epsilon}_1 \zeta_1^{(0)} \right) \\ \delta a^{(0)} &= -\frac{i}{2} \phi^{(0)} \left(\bar{\epsilon}_1 \zeta_1^{(0)} + \bar{\epsilon}_2 \zeta_2^{(0)} \right) \\ \delta \psi_{1\mu R}^{(0)} &= D_{\mu} \epsilon_1 - \frac{3i}{4} \phi^{(0)-1} \partial_{\mu} a^{(0)} \epsilon_1 \\ \delta \psi_{2\mu L}^{(0)} &= D_{\mu} \epsilon_2 + \frac{3i}{4} \phi^{(0)-1} \partial_{\mu} a^{(0)} \epsilon_2 \\ \delta \zeta_{2R}^{(0)} &= \frac{\phi^{(0)-1}}{2} \partial_{\rho} (\phi^{(0)} + i a^{(0)}) \gamma^{\rho} \epsilon_2 \\ \delta \zeta_{1L}^{(0)} &= -\frac{\phi^{(0)-1}}{2} \partial_{\rho} (\phi^{(0)} - i a^{(0)}) \gamma^{\rho} \epsilon_1\end{aligned}$$

$$\psi_{\mu} \equiv \psi_{1\mu R}^{(0)} + \psi_{2\mu L}^{(0)}, \zeta \equiv \phi^{(0)} \left(\zeta_{2R}^{(0)} - \zeta_{1L}^{(0)} \right)$$

$$T \equiv \phi^{(0)} + i a^{(0)}$$

$$\begin{aligned}\delta e_{\mu}^a &= \frac{1}{2} \bar{\epsilon} \gamma^a \psi_{\mu} \\ \delta \psi_{\mu} &= D_{\mu} \epsilon + \frac{i}{2} Q_{\mu} \gamma_5 \epsilon \\ \delta T &= \bar{\epsilon}_R \zeta_R \\ \delta \zeta_R &= \frac{1}{2} \partial T \epsilon_L\end{aligned}$$

$$K=-3\log{(T+\bar{T})}, Q_{\mu}=-\frac{3}{2i(T+\bar{T})}(\partial_{\mu}T-\partial_{\mu}\bar{T})$$



$$m_n = \phi^{(0)-\frac{3}{2}} \frac{n}{R}$$

$$\begin{aligned} \partial_y \phi &= -\phi^{(0)\frac{5}{2}} \sum_n m_n \phi^{(n)} \sin\left(\frac{ny}{R}\right), & P_{ab} &= -\phi^{(0)\frac{3}{2}} \sum_n m_n e_{(a}^{(0)\rho} e_{b)}^{(n)} \rho \sin\left(\frac{ny}{R}\right), \\ \partial_y B_\mu &= \phi^{(0)\frac{3}{2}} \sum_n m_n B_\mu^{(n)} \cos\left(\frac{ny}{R}\right), & Q_{ab} &= -\phi^{(0)\frac{3}{2}} \sum_n m_n e_{[a}^{(0)\rho} e_{b]\rho}^{(n)} \rho \sin\left(\frac{ny}{R}\right), \\ \partial_y A_\mu &= \phi^{(0)\frac{3}{2}} \sum_n m_n A_\mu^{(n)} \cos\left(\frac{ny}{R}\right), & \omega_\mu^{(n)ab} &= 2e^{(0)[a}\partial_{[\mu} e_{\alpha]}^{(n)b]} - e_\alpha^{(0)[a} e_\beta^{(0)b]} e_\mu^{(0)c} \partial^\alpha e_c^{(n)\beta}, \end{aligned}$$

$$\begin{aligned} \delta e_n^a &= \frac{1}{2} \left(\bar{\epsilon}_1 \gamma^a \psi_{1\mu}^{(n)} + \bar{\epsilon}_2 \gamma^a \psi_{2\mu}^{(n)} \right) \\ \delta \phi^{(n)} &= \frac{1}{2} \left(\bar{\epsilon}_2 \zeta_2^{(n)} - \bar{\epsilon}_1 \zeta_1^{(n)} \right) \\ \delta a^{(n)} &= -\frac{i}{2} \phi^{(0)} \left(\bar{\epsilon}_1 \zeta_1^{(n)} + \bar{\epsilon}_2 \zeta_2^{(n)} \right) \\ \delta A_\mu^{(n)} &= \frac{\phi^{(0)-\frac{1}{2}}}{2} \left[\bar{\epsilon}_1 \left(\psi_{1\mu}^{(n)} - \frac{1}{2} \gamma_\mu \zeta_1^{(n)} \right) + \bar{\epsilon}_2 \left(\psi_{2\mu}^{(n)} - \frac{1}{2} \gamma_\mu \zeta_2^{(n)} \right) \right] \\ \delta B_\mu^{(n)} &= \frac{i}{2} \phi^{(0)-\frac{3}{2}} \left[\bar{\epsilon}_2 \left(\psi_{2\mu}^{(n)} + \frac{3}{2} \gamma_\mu \zeta_2^{(n)} \right) - \bar{\epsilon}_1 \left(\psi_{1\mu}^{(n)} - \frac{3}{2} \gamma_\mu \zeta_1^{(n)} \right) \right] \end{aligned}$$

$$\delta \zeta_{1L}^{(n)} = \frac{m_n}{2} \left[-i \left(\phi^{(0)\frac{1}{2}} A_\rho^{(n)} - \frac{\phi^{(0)-1}}{m_n} \partial_\rho a^{(n)} \right) + \left(\phi^{(0)\frac{3}{2}} B_\rho^{(n)} - \frac{1}{m_n} \partial_\rho \phi^{(n)} \right) \right] \gamma^\rho \epsilon_1,$$

$$\delta \zeta_{1R}^{(n)} = \frac{\phi^{(0)\frac{1}{2}}}{8} \left(F_{\rho\sigma}^{(n)} + i \phi^{(0)} G_{\rho\sigma}^{(n)} \right) \gamma^{\rho\sigma} \epsilon_1 + \frac{i}{4} m_n \left(e_a^{(0)\rho} e_b^{(n)} \rho \gamma^{ab} - \phi^{(n)} \right) \epsilon_1,$$

$$\delta \zeta_{2L}^{(n)} = -\frac{\phi^{(0)\frac{1}{2}}}{8} \left(F_{\rho\sigma}^{(n)} - i \phi^{(0)} G_{\rho\sigma}^{(n)} \right) \gamma^{\rho\sigma} \epsilon_2 + \frac{i}{4} m_n \left(e_a^{(0)\rho} e_b^{(n)} \rho \gamma^{ab} - \phi^{(n)} \right) \epsilon_2,$$

$$\delta \zeta_{2R}^{(n)} = \frac{m_n}{2} \left[-i \left(\phi^{(0)\frac{1}{2}} A_\rho^{(n)} - \frac{\phi^{(0)-1}}{m_n} \partial_\rho a^{(n)} \right) - \left(\phi^{(0)\frac{3}{2}} B_\rho^{(n)} - \frac{1}{m_n} \partial_\rho \phi^{(n)} \right) \right] \gamma^\rho \epsilon_2,$$

$$\begin{aligned} \delta \psi_{1\mu R}^{(n)} &= \frac{1}{4} \left(2e^{(0)a\alpha} \partial_{[\mu} e_{\alpha]}^{(n)b} - e_\alpha^{(0)a} e_\beta^{(0)b} e_\mu^{(0)c} \partial^\alpha e_c^{(n)\beta} \right) \gamma_{ab} \epsilon_1 - \frac{m_n}{4} \phi^{(0)\frac{3}{2}} B_\rho^{(n)} \gamma_\mu \gamma^\rho \epsilon_1 \\ &\quad + \frac{3i}{4} m_n \left(\phi^{(0)\frac{1}{2}} A_\mu^{(n)} - \frac{\phi^{(0)-1}}{m_n} \partial_\mu a^{(n)} \right) \epsilon_1 \end{aligned}$$

$$\begin{aligned} \delta \psi_{1\mu L}^{(n)} &= -\frac{\phi^{(0)\frac{1}{2}}}{8} \left(3F_{\mu\rho}^{(n)+,5} + i \phi^{(0)} G_{\mu\rho}^{(n)+,5} \right) \gamma^\rho \epsilon_1 + \frac{i}{4} m_n g_{\mu\rho}^{(n)} \gamma^\rho \epsilon_1 \\ &\quad + \frac{i}{4} m_n \left(e_a^{(0)\rho} e_b^{(n)} \rho \gamma_\mu \gamma^{ab} - \phi^{(n)} \gamma_\mu \right) \epsilon_1 \end{aligned}$$

$$\begin{aligned} \delta \psi_{2\mu R}^{(n)} &= -\frac{\phi^{(0)\frac{1}{2}}}{8} \left(3F_{\mu\rho}^{(n)+,5} - i \phi^{(0)} G_{\mu\rho}^{(n)+,5} \right) \gamma^\rho \epsilon_2 - \frac{i}{4} m_n g_{\mu\rho}^{(n)} \gamma^\rho \epsilon_2 \\ &\quad - \frac{i}{4} m_n \left(e_a^{(0)\rho} e_b^{(n)} \gamma_\mu \gamma^{ab} - \phi^{(n)} \gamma_\mu \right) \epsilon_2, \end{aligned}$$

$$\begin{aligned} \delta \psi_{2\mu L}^{(n)} &= \frac{1}{4} \left(2e^{(0)a\alpha} \partial_{[\mu} e_{\alpha]}^{(n)b} - e_\alpha^{(0)a} e_\beta^{(0)b} e_\mu^{(0)c} \partial^\alpha e_c^{(n)\beta} \right) \gamma_{ab} \epsilon_2 - \frac{m_n}{4} \phi^{(0)\frac{3}{2}} B_\rho^{(n)} \gamma_\mu \gamma^\rho \epsilon_2 \\ &\quad - \frac{3i}{4} m_n \left(\phi^{(0)\frac{1}{2}} A_\mu^{(n)} - \frac{\phi^{(0)-1}}{m_n} \partial_\mu a^{(n)} \right) \epsilon_2. \end{aligned}$$



$$A_{\mu}^{(n)S} = \phi^{(0)\frac{1}{2}} A_{\mu}^{(n)} - \frac{\phi^{(0)-1}}{m_n} \partial_{\mu} a^{(n)},$$

$$\delta A_{\mu}^{(n)S} = \frac{1}{2} \left[\bar{\epsilon}_1 \left(\psi_{1\mu L}^{(n)} - \frac{1}{2} \gamma_{\mu} \zeta_{1R}^{(n)} + \frac{i}{m_n} \partial_{\mu} \zeta_{1L}^{(n)} \right) + \bar{\epsilon}_2 \left(\psi_{2\mu R}^{(n)} + \frac{1}{2} \gamma_{\mu} \zeta_{2L}^{(n)} + \frac{i}{m_n} \partial_{\mu} \zeta_{2R}^{(n)} \right) \right]$$

$$\psi_{1\mu L}^{(n)} S = \psi_{1\mu L}^{(n)} - \frac{1}{2} \gamma_{\mu} \zeta_{1R}^{(n)} + \frac{i}{m_n} \partial_{\mu} \zeta_{1L}^{(n)}$$

$$\psi_{2\mu R}^{(n)} S = \psi_{2\mu R}^{(n)} + \frac{1}{2} \gamma_{\mu} \zeta_{2L}^{(n)} + \frac{i}{m_n} \partial_{\mu} \zeta_{2R}^{(n)}$$

$$\lambda_{\mu}^{(n)S} \equiv \psi_{1\mu L}^{(n)S} + \psi_{2\mu R}^{(n)S} = \left(\psi_{1\mu L}^{(n)} + \psi_{2\mu R}^{(n)} \right) - \frac{1}{2} \gamma_{\mu} \left(\zeta_{1R}^{(n)} - \zeta_{2L}^{(n)} \right) + \frac{i}{m_n} \partial_{\mu} \left(\zeta_{1L}^{(n)} + \zeta_{2R}^{(n)} \right),$$

$$\chi_{\mu}^{(n)S} = -i \left[\left(\psi_{1\mu R}^{(n)} - \psi_{2\mu L}^{(n)} \right) - \frac{1}{2} \gamma_{\mu} \left(\zeta_{1L}^{(n)} + \zeta_{2R}^{(n)} \right) + \frac{i}{m_n} \partial_{\mu} \left(\zeta_{1R}^{(n)} - \zeta_{2L}^{(n)} \right) \right]$$

$$g_{\mu\nu}^{(n)S} = 2e_{(\mu}^{(0)} e_{\nu)}^{(n)} a - \phi^{(n)} g_{\mu\nu}^{(0)} + \frac{\phi^{(0)\frac{3}{2}}}{m_n} \left(\partial_{\mu} B_{\nu}^{(n)} + \partial_{\nu} B_{\mu}^{(n)} \right) - \frac{2}{m_n^2} \partial_{\mu} \partial_{\nu} \phi^{(n)}$$

$$\tilde{\mathcal{A}}_{77} = \frac{2^{-4}v}{2} \frac{1}{\eta^8} [(N + \bar{N})^2 V_8 + (N - \bar{N})^2 S_8] \sum_n W_n$$

$$\tilde{\mathcal{M}}_7 = \left(\frac{2\epsilon v}{\hat{\eta}^5 \hat{\vartheta}_2} \right) [(N + \bar{N})(\hat{O}_6 \hat{V}_2 - \hat{V}_6 \hat{O}_2) - (N - \bar{N})(\hat{S}_6 \hat{S}_2 - \hat{C}_6 \hat{C}_2)] \sum_n W_{2n+1}$$

$$\mathcal{A}_{77} = \frac{1}{\eta^8} \left[N\bar{N}(V_8 - S_8) + \frac{N^2 + \bar{N}^2}{2} (O_8 - C_8) \right] \sum_m P_m$$

$$\mathcal{M}_7 = - \left(\frac{2\epsilon}{\hat{\eta}^5 \hat{\vartheta}_2} \right) \left[\frac{N + \bar{N}}{2} (\hat{O}_6 \hat{O}_2 + \hat{V}_6 \hat{V}_2) + \frac{N - \bar{N}}{2} (\hat{S}_6 \hat{C}_2 - \hat{C}_6 \hat{S}_2) \right] \sum_m (-1)^m P_m$$

$$\tilde{\mathcal{A}}_{33} = \frac{2^{-2}v}{2} \frac{1}{\eta^8} [(N + \bar{N})^2 V_8 + (N - \bar{N})^2 S_8] \sum_n W_n,$$

$$\tilde{\mathcal{M}}_3 = \left(\frac{8\epsilon v \hat{\eta}}{\hat{\vartheta}_2^3} \right) [(N + \bar{N})(\hat{O}_2 \hat{V}_6 - \hat{V}_2 \hat{O}_6) - (N - \bar{N})(\hat{S}_2 \hat{S}_6 - \hat{C}_2 \hat{C}_6)] \sum_n W_{2n+1},$$

$$\mathcal{A}_{33} = \left[N\bar{N} \left(\frac{V_8 - S_8}{\eta^8} \right) + \frac{N^2 + \bar{N}^2}{2} \left(\frac{O_8 - C_8}{\eta^8} \right) \right] \sum_m P_m$$

$$\mathcal{M}_3 = \left(\frac{8\epsilon \hat{\eta}}{\hat{\vartheta}_2^3} \right) \left[\frac{N + \bar{N}}{2} (\hat{O}_2 \hat{O}_6 + \hat{V}_2 \hat{V}_6) + \frac{N - \bar{N}}{2} (\hat{S}_2 \hat{C}_6 - \hat{C}_2 \hat{S}_6) \right] \sum_m (-1)^m P_m$$

$$\tilde{\mathcal{A}}_{pp} = \frac{2^{-\frac{p+1}{2}} v N^2}{2} \frac{1}{\eta^8} (V_{p-1} O_{9-p} + O_{p-1} V_{9-p}) \sum_n W_n,$$

$$\tilde{\mathcal{M}}_p = - \frac{\sqrt{2} v N \epsilon}{\hat{\eta}^{p-1}} \left(\frac{2\hat{\eta}}{\hat{\vartheta}_2} \right)^{\frac{9-p}{2}} (\hat{V}_{p-1} \hat{O}_{9-p} - \hat{O}_{p-1} \hat{V}_{9-p}) \sum_n W_{2n+1},$$



$$\begin{aligned}
\mathcal{A}_{pp} &= \frac{N^2}{2} \frac{1}{\eta^8} [(O_{p-1} + V_{p-1})(O_{9-p} + V_{9-p}) - 2S'_{p-1}S'_{9-p}] \sum_m P_m \\
\mathcal{M}_p &= -\frac{\epsilon N}{\sqrt{2}} \frac{1}{\hat{\eta}^{p-1}} \left(\frac{2\hat{\eta}}{\hat{\vartheta}_2} \right)^{\frac{9-p}{2}} \left[\sin \frac{(p-5)\pi}{4} (\hat{O}_{p-1}\hat{O}_{9-p} + \hat{V}_{p-1}\hat{V}_{9-p}) \right. \\
&\quad \left. + \cos \frac{(p-5)\pi}{4} (\hat{O}_{p-1}\hat{V}_{9-p} - \hat{V}_{p-1}\hat{O}_{9-p}) \right] \sum_m (-1)^m P_m \\
\tilde{\mathcal{A}}_{pp} &= \frac{2^{-\frac{p+1}{2}}}{2v\eta^8} \left(e^{\frac{i\pi m}{2}} N + e^{-\frac{i\pi m}{2}} \bar{N} \right)^2 (V_{p-1}O_{9-p} + O_{p-1}V_{9-p}) \sum_m P_m \\
\mathcal{A}_{pp} &= \frac{1}{\eta^8} \left(N\bar{N} \sum_n W_n + \frac{N^2 + \bar{N}^2}{2} \sum_n W_{n+\frac{1}{2}} \right) [(O_{p-1} + V_{p-1})(O_{9-p} + V_{9-p}) - 2S'_{p-1}S'_{9-p}] \\
\tilde{\mathcal{A}}_{55} &= \frac{2^{-3}v}{2\eta^8} \sum_n [(N + \bar{N})^2 (V_8 W_{2n} + O_8 W_{2n+1}) + (N - \bar{N})^2 (S_8 W_{2n} + C_8 W_{2n+1})] \\
\tilde{\mathcal{M}}_5 &= -\left(\frac{4v}{\hat{\eta}^2 \hat{\vartheta}_2^2} \right) \sum_n [\epsilon_1 (N + \bar{N}) (\hat{O}_4 \hat{O}_4 - \hat{V}_4 \hat{V}_4) + i(-1)^n \epsilon_2 (N - \bar{N}) (\hat{S}_4 \hat{C}_4 - \hat{C}_4 \hat{S}_4)] W_{2n+1} \\
\mathcal{A}_{55} &= \frac{1}{\eta^8} \left[N\bar{N} \sum_m (V_8 P_m - S_8 P_{m+1/2}) + \frac{N^2 + \bar{N}^2}{2} \sum_m (O_8 P_m - C_8 P_{m+1/2}) \right], \\
\mathcal{M}_5 &= \left(\frac{2}{\hat{\eta}^2 \hat{\vartheta}_2^2} \right) \sum_m (-1)^m [\epsilon_1 (N + \bar{N}) (\hat{O}_4 \hat{O}_4 - \hat{V}_4 \hat{V}_4) P_m + \epsilon_2 (N - \bar{N}) (\hat{S}_4 \hat{C}_4 - \hat{C}_4 \hat{S}_4) P_{m+1/2}].
\end{aligned}$$

$$D5_{012389} \xrightarrow{\text{T-duality}} M5_{0123910}$$

$$D5_{012349} \xrightarrow{\text{T-duality}} KK6_{0123489},$$

$$\begin{aligned}
A_{33} &= \sum_n \left(N\bar{N}W_n + \frac{N^2 + \bar{N}^2}{2} W_{n+1/2} \right) \frac{V_8 - S_8}{\eta^8} \\
\tilde{\mathcal{A}}_{33} &= \frac{2^{-2}}{2v} \left(\frac{V_8 - S_8}{\eta^8} \right) \sum_m \left[e^{\frac{i\pi m}{2}} N + e^{-\frac{i\pi m}{2}} \bar{N} \right]^2 P_m \\
&= \frac{2^{-2}}{v} \left(\frac{V_8 - S_8}{\eta^8} \right) \sum_m \left[N\bar{N} + (-1)^m \frac{N^2 + \bar{N}^2}{2} \right] P_m
\end{aligned}$$

$$D3_{0128} \xrightarrow{\text{T-duality}} M2_{012}, D3_{0123} \xrightarrow{\text{T-duality}} M5_{0123810},$$

$$D1_{01} \xrightarrow{\text{T-duality}} M2_{018}, D1_{08} \xrightarrow{\text{T-duality}} KK(X^{10}),$$



$$\begin{aligned}
\tilde{\mathcal{A}}_{pp} &= \frac{2^{-\frac{p+1}{2}} v}{2\eta^8} \left[(N_1 + N_2)^2 V_8 \sum_n W_{2n} + (N_1 - N_2)^2 O_8 \sum_n W_{2n+1} \right] \\
\tilde{\mathcal{M}}_p &= -\sqrt{2} v (N_1 - N_2) \frac{1}{\hat{\eta}^{p-1}} \left(\frac{2\hat{\eta}}{\hat{\vartheta}_2} \right)^{\frac{9-p}{2}} (\hat{O}_{p-1} \hat{O}_{9-p} + \hat{V}_{p-1} \hat{V}_{9-p}) \sum_n W_{2n+1}, \\
\mathcal{A}_{pp} &= \frac{N_1^2 + N_2^2}{2\eta^8} \left[(O_{p-1} + V_{p-1})(O_{9-p} + V_{9-p}) \sum_m P_m - 2S'_{p-1} S'_{9-p} \sum_m P_{m+\frac{1}{2}} \right] \\
&\quad + \frac{N_1 N_2}{\eta^8} \left[(O_{p-1} + V_{p-1})(O_{9-p} + V_{9-p}) \sum_m P_{m+\frac{1}{2}} - 2S'_{p-1} S'_{9-p} \sum_m P_m \right] \\
\mathcal{M}_p &= -\frac{N_1 - N_2}{\sqrt{2}} \frac{1}{\hat{\eta}^{p-1}} \left(\frac{2\hat{\eta}}{\hat{\vartheta}_2} \right)^{\frac{9-p}{2}} \left[\sin \frac{(p-5)\pi}{4} (\hat{V}_{p-1} \hat{O}_{9-p} - \hat{O}_{p-1} \hat{V}_{9-p}) \right. \\
&\quad \left. + \cos \frac{(p-5)\pi}{4} (\hat{O}_{p-1} \hat{O}_{9-p} + \hat{V}_{p-1} \hat{V}_{9-p}) \right] \sum_m (-1)^m P_m
\end{aligned}$$

En supergravedad cuántica relativista, un agujero negro cuántico, se forma por la interacción de la superpartícula, en la medida en que deforma el espacio – tiempo cuántico, en dimensiones más altas, lo que provoca una suerte de desdoblamiento en D dimensiones del espacio – tiempo cuántico en el que interactúa, esto es, por la gravedad intensa generada por la propia superpartícula, a propósito de su centro de masa y energía extremadamente denso o por interacción con el supergravitón, lo que equivale a que, el campo supergravitónico, permea y por ende, distorsiona el espacio – tiempo cuántico, por lo que, el modelamiento matemático de este fenómeno, se expresa así, utilizando la métrica de Kerr – Hawking:

$$\begin{aligned}
ds^2 &= -\frac{\Delta_r}{W} \left(dt - \frac{a \sin^2 \theta}{\Xi} d\phi \right)^2 + W \left(\frac{dr^2}{\Delta_r} + \frac{d\theta^2}{\Delta_\theta} \right) + \frac{\Delta_\theta \sin^2 \theta}{W} \left[a dt - \frac{r_1 r_2 + a^2}{\Xi} d\phi \right]^2 \\
A_1 &= \frac{2\sqrt{2} m \sinh(\delta_1) \cosh(\delta_1) r_2}{W} \left(dt - \frac{a \sin^2 \theta}{\Xi} d\phi \right) + \alpha_{41} dt \\
A_2 &= \frac{2\sqrt{2} m \sinh(\delta_2) \cosh(\delta_2) r_1}{W} \left(dt - \frac{a \sin^2 \theta}{\Xi} d\phi \right) + \alpha_{42} dt \\
e^{\varphi_1} &= \frac{r_1^2 + a^2 \cos^2 \theta}{W} = 1 + \frac{r_1(r_1 - r_2)}{W} \\
\chi_1 &= \frac{a(r_2 - r_1) \cos \theta}{r_1^2 + a^2 \cos^2 \theta} \\
r_i &\equiv r + 2m \sinh^2(\delta_i), (i = 1, 2) \\
\Delta_r &\equiv r^2 + a^2 - 2mr + g^2 r_1 r_2 (r_1 r_2 + a^2) \\
\Delta_\theta &\equiv 1 - g^2 a^2 \cos^2 \theta \\
W &\equiv r_1 r_2 + a^2 \cos^2 \theta \\
\Xi &\equiv 1 - a^2 g^2
\end{aligned}$$



$$T_{\text{H}} = \frac{\Delta'_r(r_+)}{4\pi(r_1(r_+)r_2(r_+) + a^2)}, \Omega_{\text{H}} = \frac{a(1 + g^2r_1(r_+)r_2(r_+))}{r_1(r_+)r_2(r_+) + a^2}$$

$$\Phi_1 = \Phi_2 = \frac{2m\sinh(\delta_1)\cosh(\delta_1)r_2(r_+)}{r_1(r_+)r_2(r_+) + a^2}, \Phi_3 = \Phi_4 = \frac{2m\sinh(\delta_2)\cosh(\delta_2)r_1(r_+)}{r_1(r_+)r_2(r_+) + a^2}$$

$$S_{\text{BH}} = \frac{\pi \hat{r}_+^2}{\Xi},$$

$$\hat{r}_{\pm}^2 \equiv r_1(r_{\pm})r_2(r_{\pm}) + a^2$$

$$E = \frac{m}{\Xi^2}(1 + \sinh^2(\delta_1) + \sinh^2(\delta_2)), \quad J = \frac{ma}{\Xi^2}(1 + \sinh^2(\delta_1) + \sinh^2(\delta_2))$$

$$Q_1 = Q_2 = \frac{m\sinh(\delta_1)\cosh(\delta_1)}{2\Xi}, \quad Q_3 = Q_4 = \frac{m\sinh(\delta_2)\cosh(\delta_2)}{2\Xi}$$

$$m = \frac{g^2 r_+^3}{2}(1+k_1)^2(1+k_2)^2$$

$$T_{\text{H}} = \frac{g^2 r_+}{4\pi}(3 + k_1 + k_2 - k_1 k_2), \Omega_{\text{H}} = ag^2$$

$$\Phi_1 = \Phi_2 = gr_+ \sqrt{k_1}(1+k_2), \Phi_3 = \Phi_4 = gr_+ \sqrt{k_2}(1+k_1)$$

$$E = \frac{1}{\Xi^2} \frac{g^2 r_+^3}{2} (1+k_1)^2(1+k_2)^2, J = \frac{g^2 r_+^3 a}{2\Xi^2} (1+k_1)^2(1+k_2)^2$$

$$Q_1 = Q_2 = \frac{gr_+^2(1+k_1)(1+k_2)\sqrt{k_1}}{4\Xi}, Q_3 = Q_4 = \frac{gr_+^2(1+k_1)(1+k_2)\sqrt{k_2}}{4\Xi}$$

$$S_{\text{BH}} = \frac{\pi r_+^2(1+k_1)(1+k_2)}{\Xi}$$

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - a\sin^2\theta d\phi)^2 + \frac{\sin^2\theta}{\rho^2}((r^2 + a^2)d\phi - adt)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

$$\Delta \equiv r^2 - 2Mr + a^2, \rho^2 \equiv r^2 + a^2 \cos^2\theta$$

$$r_{\pm} = M \pm \sqrt{M^2 - a^2}.$$

$$T_{\text{H}} = \frac{r_+ - M}{4\pi M r_+}, \Omega_{\text{H}} = \frac{a}{2Mr_+}$$

$$\hat{t} = \frac{\lambda t}{2M}, x = \frac{r - M}{\lambda M}, \hat{\phi} = \phi - \frac{t}{2M},$$

$$ds^2 = 2J\Omega^2 \left(-x^2 d\hat{t}^2 + \frac{dx^2}{x^2} + d\theta^2 + \Lambda^2 (d\hat{\phi} + x d\hat{t})^2 \right)$$

$$\Omega^2 = \frac{1 + \cos^2\theta}{2}, \Lambda = \frac{2\sin\theta}{1 + \cos^2\theta}$$

$$\zeta_\epsilon = \epsilon_n(\phi)\partial_\phi - r\epsilon'_n(\phi)\partial_r, \text{ with } \epsilon_n(\phi) = -e^{-in\phi}.$$

$$T_{\text{L}} = \frac{1}{2\pi}, T_{\text{R}} = 0$$



$$S = \frac{\pi^2}{3} c_L T_L = 2\pi J$$

$$\frac{1}{\sqrt{-g}}\partial_{\mu}(\sqrt{-g}g^{\mu\nu}\partial_{\nu}\Phi(t,r,\theta,\phi))=0$$

$$\Phi(t,r,\theta,\phi)=e^{-i\omega t+im\phi}R(r)S(\theta)$$

$$\left[\frac{1}{\sin\theta}\partial_{\theta}(\sin\theta\partial_{\theta})-\frac{m^2}{\sin^2\theta}+\omega^2a^2\cos^2\theta\right]S(\theta)=-K_{\ell}S(\theta),$$

$$\left[\partial_r\Delta\partial_r+\frac{(2Mr_+\omega-ma)^2}{(r-r_+)(r_+-r_-)}-\frac{(2Mr_-\omega-ma)^2}{(r-r_-)(r_+-r_-)}+(r^2+2M(r+2M))\omega^2\right]R(r)=K_{\ell}R(r)$$

$$\nabla_{S^2}^2S(\theta)=\left[\frac{1}{\sin\theta}\partial_{\theta}(\sin\theta\partial_{\theta})-\frac{m^2}{\sin^2\theta}\right]S(\theta)=-\ell(\ell+1)S(\theta),m=-\ell,\cdots,\ell$$

$$\left[\partial_r\Delta\partial_r+\frac{(2Mr_+\omega-ma)^2}{(r-r_+)(r_+-r_-)}-\frac{(2Mr_-\omega-ma)^2}{(r-r_-)(r_+-r_-)}\right]R(r)=\ell(\ell+1)R(r).$$

$$R(r)=C_1\left(\frac{r-r_+}{r-r_-}\right)^{-\frac{i(\omega-m\Omega_H)}{4\pi T_H}}(r-r_-)^{-1-\ell}$$

$$\times {}_2F_1\left[1+\ell-i\frac{4M}{r_+-r_-}(M\omega-r_+m\Omega_H),1+\ell-i2M\omega;1-\frac{i(\omega-m\Omega_H)}{2\pi T_H};\frac{r-r_+}{r-r_-}\right]$$

$$+C_2\left(\frac{r-r_+}{r-r_-}\right)^{\frac{i(\omega-m\Omega_H)}{4\pi T_H}}(r-r_-)^{-1-\ell}$$

$$\times {}_2F_1\left[1+\ell+i\frac{4M}{r_+-r_-}(M\omega-r_+m\Omega_H),1+\ell+i2M\omega;1+\frac{i(\omega-m\Omega_H)}{2\pi T_H};\frac{r-r_+}{r-r_-}\right]$$

$$\omega^+=\sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_R\phi}$$

$$\omega^-=\sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_L\phi-\frac{t}{2M}}$$

$$y=\sqrt{\frac{r_+-r_-}{r-r_-}}e^{\pi(T_L+T_R)\phi-\frac{t}{4M}}$$

$$T_L\equiv\frac{r_++r_-}{4\pi a},T_R\equiv\frac{r_+-r_-}{4\pi a},$$

$$H_1=i\partial_+$$

$$H_0=i\left(\omega^+\partial_++\frac{1}{2}y\partial_y\right)$$

$$H_{-1}=i((\omega^+)^2\partial_++\omega^+y\partial_y-y^2\partial_-)$$



$$\begin{aligned}\bar{H}_1 &= i\partial_- \\ \bar{H}_0 &= i\left(\omega^-\partial_- + \frac{1}{2}y\partial_y\right) \\ \bar{H}_{-1} &= i((\omega^-)^2\partial_- + \omega^-y\partial_y - y^2\partial_+)\end{aligned}$$

$$\begin{aligned}[H_0, H_{\pm 1}] &= \mp iH_{\pm 1}, \quad [H_{-1}, H_1] = -2iH_0 \\ [\bar{H}_0, \bar{H}_{\pm 1}] &= \mp i\bar{H}_{\pm 1}, \quad [\bar{H}_{-1}, \bar{H}_1] = -2i\bar{H}_0\end{aligned}$$

$$S=\frac{\pi^2}{3}(c_{\rm L}T_{\rm L}+c_{\rm R}T_{\rm R})=2\pi Mr_+,$$

$$\begin{aligned}\omega^+ &= R(r)e^{2\pi T_{\rm R}\phi} \\ \omega^- &= R(r)e^{2\pi T_{\rm L}\phi - \Delta'_r(r_+)t/r_*^2} \\ y &= Q(r)e^{\pi(T_{\rm L}+T_{\rm R})\phi - \Delta'_r(r_+)t/(2r_*^2)}\end{aligned}$$

$$T_{\rm L}\equiv\frac{\Delta'_r(r_+)}{4\pi a\Xi}\frac{\hat{r}_+^2+\hat{r}_-^2}{r_*^2}, T_{\rm R}\equiv\frac{\Delta'_r(r_+)}{4\pi a\Xi}$$

$$r_*^2\equiv\hat{r}_+^2-\hat{r}_-^2$$

$$\begin{aligned}t &= \frac{1}{\Delta'_r(r_+)}\left(\hat{r}_+^2\ln\frac{\omega^+}{\omega^-}+\hat{r}_-^2\ln(\omega^+\omega^-+y^2)\right), \\ r &= r_+ + \frac{\omega^+\omega^-\Delta'_r(r_+)}{\omega^+\omega^-+y^2}, \\ \phi &= \frac{a\Xi}{\Delta'_r(r_+)}\ln\frac{\omega^+(\omega^+\omega^-+y^2)}{\omega^-}.\end{aligned}$$

$$\begin{aligned}ds^2 &= \frac{4W_+}{y^2}d\omega^+d\omega^- + \frac{4\Delta_\theta r_*^4 a^2 \sin^2 \theta}{\Delta'_r(r_+)^2 y^2 W_+} dy^2 + \frac{W_+}{\Delta_\theta} d\theta^2 \\ &\quad - \frac{4\omega^+}{y^3} \left(r_*^2 - \frac{\Delta_\theta a^2 \sin^2 \theta}{W_+ \Delta'_r(r_+)^2} (r_*^4 - \Delta'_r(r_+) r_*^2 \tilde{r}_+) \right) d\omega^- dy \\ &\quad - \frac{4\omega^-}{y^3} \left(W_+ + W_- - \frac{\Delta_\theta a^2 \sin^2 \theta}{W_+ \Delta'_r(r_+)^2} (r_*^4 + \Delta'_r(r_+) r_*^2 \tilde{r}_+) \right) d\omega^+ dy + \cdots\end{aligned}$$

$$\begin{aligned}W_{\pm} &= r_1(r_{\pm})r_2(r_{\pm})+a^2\cos^2\theta \\ \tilde{r}_+ &= r_1(r_+)+r_2(r_+)\end{aligned}$$

$$\epsilon_{\theta+-y}=\frac{4W_+r_*^2a\sin\theta}{\Delta'_r(r_+)y^3}+\cdots$$

$$\Gamma_{+y}^+ + \Gamma_{-y}^- = -\frac{2}{y}$$

$$\zeta(\varepsilon)=\varepsilon\partial_++\frac{1}{2}(\partial_+\varepsilon)y\partial_y, \bar{\zeta}(\bar{\varepsilon})=\bar{\varepsilon}\partial_-+\frac{1}{2}(\partial_-\bar{\varepsilon})y\partial_y$$

$$\varepsilon_n=2\pi T_{\rm R}(\omega^+)^{1+\frac{in}{2\pi T_{\rm R}}}, \bar{\varepsilon}_n=2\pi T_{\rm L}(\omega^-)^{1+\frac{in}{2\pi T_{\rm L}}}$$



$$[\zeta_m, \zeta_n] = i(n-m)\zeta_{n+m}, [\bar{\zeta}_m, \bar{\zeta}_n] = i(n-m)\bar{\zeta}_{n+m}$$

$$\{\mathcal{Q}_n, \mathcal{Q}_m\} = (m-n)\mathcal{Q}_{m+n} + K_{m,n}$$

$$K_{m,n}=\delta\mathcal{Q}(\zeta_n,\mathcal{L}_{\zeta_m}g;g)=\frac{c_{\mathrm{R}}m^3}{12}\delta_{m+n}$$

$$\delta\mathcal{Q}=\delta\mathcal{Q}_{\mathrm{IW}}+\delta\mathcal{Q}_{\mathrm{ct}}$$

$$\delta\mathcal{Q}_{\mathrm{IW}}=\frac{1}{16\pi}\int_{\partial\Sigma}{}^*F_{\mathrm{IW}}$$

$$F_{\mathrm{IW}}^{ab}=\frac{1}{2}\nabla^a\zeta^bh+\nabla^ah^{cb}\zeta_c+\nabla_c\zeta^ah^{cb}+\nabla_ch^{ca}\zeta^b-\nabla^ah\zeta^b-(a\leftrightarrow b)$$

$$F_{\mathrm{IW}}^{-y}=4h_m^{y-}\zeta_n^y\Gamma_{-y}^-$$

$$h_m^{-y}=g^{+-}\partial_+\zeta_m^y=\frac{y^3\varepsilon_m''}{4W_+},\text{ with }\varepsilon_m''\equiv\partial_+^2\varepsilon$$

$$\delta\mathcal{Q}_{\mathrm{ct}}=\frac{1}{16\pi}\int_{\partial\Sigma}F_{\mathrm{ct}}^{ab}d\Sigma_{ab}$$

$$F_{\mathrm{ct}}^{ab}=-2N_d{}^c\nabla_c(\zeta^ah^{db})-(a\leftrightarrow b),$$

$$N_+^+=1, N_-^-= -1$$

$$F_{\mathrm{ct}}^{-y}=2h_m^{-y}\zeta_n^y(\Gamma_{+y}^+-\Gamma_{-y}^-)$$

$$\begin{aligned}\delta Q=&\frac{1}{16\pi}\int d\theta d\omega^+\epsilon_{\theta+-y}(F_{\mathrm{IW}}^{-y}+F_{\mathrm{ct}}^{-y})\\&=i\frac{ar_*^2}{2\Delta_r'(r_+)}m^3\delta_{m+n,0}\end{aligned}$$

$$c_{\mathrm{R}}=\frac{6ar_*^2}{\Delta_r'(r_+)}=\frac{6a}{\Delta_r'(r_+)}\big(r_1(r_+)r_2(r_+)-r_1(r_-)r_2(r_-)\big)$$

$$S_{\mathrm{Cardy}}=\frac{\pi^2}{3}(c_{\mathrm{L}}T_{\mathrm{L}}+c_{\mathrm{R}}T_{\mathrm{R}})$$

$$\begin{aligned}T_{\mathrm{L}}&=\frac{\Delta_r'(r_+)}{4\pi a\Xi}\frac{\hat{r}_+^2+\hat{r}_-^2}{r_*^2}\\&=\frac{\Delta_r'(r_+)\big[(r_++2m\sinh^2(\delta_1))(r_++2\sinh^2(\delta_2))+(r_-+2m\sinh^2(\delta_1))(r_-+2m\sinh^2(\delta_2))+2a^2\big]}{4\pi a(1-a^2g^2)\big[(r_++2m\sinh^2(\delta_1))(r_++2m\sinh^2(\delta_2))-(r_-+2m\sinh^2(\delta_1))(r_-+2m\sinh^2(\delta_2))\big]}\\T_{\mathrm{R}}&=\frac{\Delta_r'(r_+)}{4\pi a\Xi}=\frac{\Delta_r'(r_+)}{4\pi a(1-a^2g^2)}\end{aligned}$$



$$\begin{aligned}
c_L &= c_R \\
&= \frac{6ar_*^2}{\Delta'_r(r_+)} = \frac{6a}{\Delta'_r(r_+)} (r_1(r_+)r_2(r_+) - r_1(r_-)r_2(r_-)) \\
&= \frac{6a}{\Delta'_r(r_+)} \left((r_+ + 2m\sinh^2(\delta_1))(r_+ + 2m\sinh^2(\delta_2)) - (r_- + 2m\sinh^2(\delta_1))(r_- + 2m\sinh^2(\delta_2)) \right)
\end{aligned}$$

$$S_{\text{Cardy}} = \frac{\pi \hat{r}_+^2}{\Xi} = \frac{\pi [(r_1 + 2m\sinh^2(\delta_1))(r_+ + 2m\sinh^2(\delta_2)) + a^2]}{1 - a^2 g^2},$$

$$\begin{aligned}
T_0^{\mu\nu} &= \rho u^\mu u^\nu + \mathcal{P} P^{\mu\nu} \\
J_{i,0}^\mu &= r_i u^\mu \\
J_{S,0}^\mu &= s u^\mu
\end{aligned}$$

$$\begin{aligned}
T_1^{\mu\nu} &= -\zeta \vartheta P^{\mu\nu} - 2\eta \sigma^{\mu\nu} + q^\mu u^\nu + u^\mu q^\nu \\
J_{i,1}^\mu &= q_i^\mu \\
J_{S,1}^\mu &= \frac{1}{\mathcal{T}} (q^\mu - \mu_i q_i^\mu)
\end{aligned}$$

$$\begin{aligned}
\vartheta &= \nabla_\mu u^\mu, \\
\sigma^{\mu\nu} &= \frac{1}{2} (P^{\mu\lambda} \nabla_\lambda u^\nu + P^{\nu\lambda} \nabla_\lambda u^\mu) - \frac{1}{2} \vartheta P^{\mu\nu}, \\
q^\mu &= -\kappa P^{\mu\nu} (\partial_\nu \mathcal{T} + a_\nu \mathcal{T}), \\
q_i^\mu &= -D_{ij} P^{\mu\nu} \partial_\nu \left(\frac{\mu_j}{\mathcal{T}} \right),
\end{aligned}$$

$$\Phi \equiv \mathcal{E} - \mathcal{T} \mathcal{S} - \mu_i \mathcal{R}_i, i = 1, \cdots, 4,$$

$$d\Phi = -Sd\mathcal{T} - \mathcal{P}dV - \mathcal{R}_i d\mu_i.$$

$$\Phi = -\mathrm{vol}(S^2)\mathcal{T}^d h(v_i),$$

$$\begin{aligned}
\rho &= 2\mathcal{P} = 2\mathcal{T}^3 h(v) \\
r_i &= \mathcal{T}^2 h_i(v) \\
s &= \mathcal{T}^2 (3h - v_i h_i)
\end{aligned}$$

$$\begin{aligned}
E &= \frac{2\mathrm{vol}(S^2)\tau^3 h(v)}{(1-\omega_1^2)^2} \\
L_1 &= \frac{2\mathrm{vol}(S^2)\tau^3 h(v)\omega_1}{(1-\omega_1^2)^2} \\
R_i &= \frac{\mathrm{vol}(S^2)\tau^2 h_i(v)}{1-\omega_1^2} \\
S &= \frac{\mathrm{vol}(S^2)\tau^2 [3h(v) - v_i h_i(v)]}{1-\omega_1^2}
\end{aligned}$$



$$T = \left(\frac{\partial E}{\partial S} \right)_{L_1, R_i} = \tau$$

$$\Omega_1 = \left(\frac{\partial E}{\partial L_1} \right)_{S, R_i} = \omega_1$$

$$\zeta_i = \left(\frac{\partial E}{\partial R_i} \right)_{S, L_1, R_j} = \tau v_i$$

$$\mathcal{Z} = \exp \left[-\frac{E - TS - \Omega_1 L_1 - \zeta_i R_i}{T} \right] = \exp \left[\frac{\text{vol}(S^2) T^2 h(\zeta/T)}{1 - \Omega_1^2} \right]$$

$$h = \frac{\mathcal{P}}{\mathcal{T}^3} = \frac{4\pi^2(1+k_1)^2(1+k_2)^2}{\Xi g^4(3+k_1+k_2-k_1k_2)^3}$$

$$h_1 \equiv \frac{\partial h}{\partial v_1} = \frac{r_1}{\mathcal{T}^2} = \frac{\pi(1+k_1)(1+k_2)\sqrt{k_1}}{\Xi g^3(3+k_1+k_2-k_1k_2)^2}, h_2 \equiv \frac{\partial h}{\partial v_2} = \frac{r_2}{\mathcal{T}^2} = h_1$$

$$h_3 \equiv \frac{\partial h}{\partial v_3} = \frac{r_3}{\mathcal{T}^2} = \frac{\pi(1+k_1)(1+k_2)\sqrt{k_2}}{\Xi g^3(3+k_1+k_2-k_1k_2)^2}, h_4 \equiv \frac{\partial h}{\partial v_4} = \frac{r_4}{\mathcal{T}^2} = h_3$$

$$v_1 = v_2 = \frac{4\pi(1+k_2)\sqrt{k_1}}{3+k_1+k_2-k_1k_2}, v_3 = v_4 = \frac{4\pi(1+k_1)\sqrt{k_2}}{3+k_1+k_2-k_1k_2}$$

$$\mathcal{S} = \text{vol}(S^2) \cdot s = \frac{\pi r_+^2(1+k_1)(1+k_2)}{\Xi},$$

$$\log \mathcal{Z} = \frac{16\pi^3(2N)^{3/2}T^2(1+k_1)^2(1+k_2)^2}{3(1-\Omega_1^2)(3+k_1+k_2-k_1k_2)^3}.$$

$$Z(q,q';y_1,y_2) = \text{Tr}_{\mathcal{H}}[e^{-\beta'\{Q,S\}}e^{-\beta(\epsilon+j_3)}e^{-\gamma_1h_1-\gamma_2h_2}]$$

$$q \equiv e^{-\beta}, q' \equiv e^{-\beta'}, y_1 \equiv e^{-\gamma_1}, y_2 \equiv e^{-\gamma_2}$$

$$Z(q,q';y_1,y_2) = \frac{1}{(N!)^2} \sum_{\mathfrak{m}, \tilde{\mathfrak{m}} \in \mathbb{Z}^N} \int \left[\prod_{i=1}^N \frac{d\alpha_i}{2\pi} \frac{d\tilde{\alpha}_i}{2\pi} \right] \prod_{\substack{i < j \\ \mathfrak{m}_i = \mathfrak{m}_j}} \left[2 \sin \left(\frac{\alpha_i - \alpha_j}{2} \right) \right]^2 \prod_{\substack{i < j \\ \tilde{\mathfrak{m}}_i = \tilde{\mathfrak{m}}_j}} \left[2 \sin \left(\frac{\tilde{\alpha}_i - \tilde{\alpha}_j}{2} \right) \right]^2 \times Z_0 Z_{\text{cl}} Z_{\text{ch}} Z_{\text{g}}$$

$$Z_{\text{cl}} = e^{ik \sum_{i=1}^N (\mathfrak{m}_i \alpha_i - \tilde{\mathfrak{m}}_i \tilde{\alpha}_i)}$$

$$Z_{\text{ch}} = \prod_{i,j=1}^N \exp \left[\sum_{n=1}^{\infty} \frac{1}{n} \left(f_{ij}^+(q^n, q'^n; y_1^n, y_2^n) e^{in(\tilde{\alpha}_j - \alpha_i)} + f_{ij}^-(q^n, q'^n; y_1^n, y_2^n) e^{in(\alpha_i - \tilde{\alpha}_j)} \right) \right]$$

$$Z_{\text{g}} = \prod_{i,j=1}^N \exp \left[\sum_{n=1}^{\infty} \frac{1}{n} \left(f_{ij}^{\text{adj}}(q^n, q'^n) e^{-in(\alpha_i - \alpha_j)} + \tilde{f}_{ij}^{\text{adj}}(q^n, q'^n) e^{-in(\tilde{\alpha}_i - \tilde{\alpha}_j)} \right) \right]$$

$$Z_0 = \prod_{i,j=1}^N \exp \left[-\beta \left(|\mathfrak{m}_i - \tilde{\mathfrak{m}}_j| - \sum_{i < j} |\mathfrak{m}_{ij}| - \sum_{i < j} |\tilde{\mathfrak{m}}_{ij}| \right) \right]$$



$$\begin{aligned}
f_{ij}^{\pm B}(q, q'; y_1, y_2) &\equiv \sum_{\substack{4 \text{ scalars} \\ s=\pm 1}} \sum_{j=\frac{|m_i-\tilde{m}_j|}{2}}^{\infty} \sum_{j_3=-j}^j (q^{\epsilon_j+j_3} q'^{\epsilon_j-h_3-j_3} y_1^{h_1} y_2^{h_2}) \\
f_{ij}^{\pm F}(q, q'; y_1, y_2) &\equiv \sum_{\substack{4 \text{ fermions}}} \sum_{j=\frac{|m_i-\tilde{m}_j|\pm 1}{2}}^{\infty} \sum_{j_3=-j}^j (q^{\epsilon_j+j_3} q'^{\epsilon_j-h_3-j_3} y_1^{h_1} y_2^{h_2}) \\
f_{ij}^{+B}(q, q'; y_1, y_2) &= \left(\sqrt{\frac{y_1}{y_2}} + \sqrt{\frac{y_2}{y_1}} \right) \sum_{j=\frac{|m_i-\tilde{m}_j|}{2}}^{\infty} q^{\frac{1}{2}} \sum_{l=0}^{2j} q'^{2j-l} q^l \\
&\quad + \left(\sqrt{y_1 y_2} + \frac{1}{\sqrt{y_1 y_2}} \right) \sum_{j=\frac{|m_i-\tilde{m}_j|}{2}}^{\infty} q' q^{\frac{1}{2}} \sum_{l=0}^{2j} q'^{2j-l} q^l, \\
f_{ij}^{-B}(q, q'; y_1, y_2) &= \left(\sqrt{y_1 y_2} + \frac{1}{\sqrt{y_1 y_2}} \right) \sum_{j=\frac{|\tilde{m}_i-m_j|}{2}}^{\infty} q^{\frac{1}{2}} \sum_{l=0}^{2j} q'^{2j-l} q^l \\
&\quad + \left(\sqrt{\frac{y_1}{y_2}} + \sqrt{\frac{y_2}{y_1}} \right) \sum_{j=\frac{|\tilde{m}_i-m_j|}{2}}^{\infty} q' q^{\frac{1}{2}} \sum_{l=0}^{2j} q'^{2j-l} q^l \\
f_{ij}^{+F}(q, q'; y_1, y_2) &= - \left(\sqrt{y_1 y_2} + \frac{1}{\sqrt{y_1 y_2}} \right) \sum_{j=\frac{|m_i-\tilde{m}_j|+1}{2}}^{\infty} q^{\frac{1}{2}} \sum_{l=0}^{2j} q'^{2j-l} q^l \\
&\quad - \left(\sqrt{\frac{y_1}{y_2}} + \sqrt{\frac{y_2}{y_1}} \right) \sum_{j=\frac{|m_i-\tilde{m}_j|-1}{2}}^{\infty} q' q^{\frac{1}{2}} \sum_{l=0}^{2j} q'^{2j-l} q^l \\
f_{ij}^{-F}(q, q'; y_1, y_2) &= - \left(\sqrt{\frac{y_1}{y_2}} + \sqrt{\frac{y_2}{y_1}} \right) \sum_{j=\frac{|m_i-\tilde{m}_j|+1}{2}}^{\infty} q^{\frac{1}{2}} \sum_{l=0}^{2j} q'^{2j-l} q^l \\
&\quad - \left(\sqrt{y_1 y_2} + \frac{1}{\sqrt{y_1 y_2}} \right) \sum_{j=\frac{|m_i-\tilde{m}_j|-1}{2}}^{\infty} q' q^{\frac{1}{2}} \sum_{l=0}^{2j} q'^{2j-l} q^l.
\end{aligned}$$

$$\begin{aligned}
f_{ij}^{\text{adj}, B}(q, q') &= \sum_{j=\frac{|m_{ij}|}{2}+1}^{\infty} \frac{q^{2j+1} - q'^{2j+1}}{q - q'} + \sum_{j=\frac{|m_{ij}|}{2}}^{\infty} \frac{q q' (q^{2j+1} - q'^{2j+1})}{q - q'} + (1 - \delta_{m_{ij}}) \frac{q' q^{|m_{ij}|} - q q'^{|m_{ij}|}}{q - q'}, \\
f_{ij}^{\text{adj}, F}(q, q') &= \sum_{j=\frac{|m_{ij}|}{2}+1}^{\infty} \frac{q' (q^{2j+1} - q'^{2j+1})}{q - q'} - \sum_{j=\frac{|m_{ij}|}{2}+1}^{\infty} q \frac{q^{2j+1} q'^{2j+1}}{q - q'} - (1 - \delta_{m_{ij}}) \frac{q (q^{m_{ij}} - q'^{m_{ij}})}{q - q'},
\end{aligned}$$

$$t_1 = \frac{1}{\sqrt{y_1 y_2}}, t_2 = \sqrt{y_1 y_2}, t_3 = \sqrt{\frac{y_2}{y_1}}, t_4 = \sqrt{\frac{y_1}{y_2}}$$



$$\begin{aligned}
& f_{ij}^+(q^n, q'^n; t_i^n) \\
&= f_{ij}^{+B}(q^n, q'^n; t_i^n) + (-1)^{n-1} f_{ij}^{+F}(q^n, q'^n; t_i^n) \\
&= \frac{t_{3,4}^n}{q^n - q'^n} \left(\frac{q^{n(1/2+|m_i-\tilde{m}_j|)}(q^n + (-1)^n q'^n)}{1 - q^{2n}} - \frac{q^{n/2} q'^n(1+|m_i-\tilde{m}_j|)(1 + (-1)^n)}{1 - q'^{2n}} \right) \\
&\quad + \frac{t_{1,2}^n}{q^n - q'^n} \left(\frac{q^{n(3/2+|m_i-\tilde{m}_j|)}(q'^n + (-1)^n q^n)}{1 - q^{2n}} - \frac{q^{n/2} q'^n(2+|m_i-\tilde{m}_j|)(1 + (-1)^n)}{1 - q'^{2n}} \right), \\
& f_{ij}^-(q^n, q'^n; t_i^n) \\
&= f_{ij}^{-B}(q^n, q'^n; t_i^n) + (-1)^{n-1} f_{ij}^{-F}(q^n, q'^n; t_i^n) \\
&= \frac{t_{1,2}^n}{q^n - q'^n} \left(\frac{q^{n(1/2+|m_i-\tilde{m}_j|)}(q^n + (-1)^n q'^n)}{1 - q^{2n}} - \frac{q^{n/2} q'^n(1+|m_i-\tilde{m}_j|)(1 + (-1)^n)}{1 - q'^{2n}} \right) \\
&\quad + \frac{t_{3,4}^n}{q^n - q'^n} \left(\frac{q^{n(3/2+|m_i-\tilde{m}_j|)}(q'^n + (-1)^n q^n)}{1 - q^{2n}} - \frac{q^{n/2} q'^n(2+|m_i-\tilde{m}_j|)(1 + (-1)^n)}{1 - q'^{2n}} \right),
\end{aligned}$$

$$\begin{aligned}
& f_{ij}^{\text{adj}}(q^n, q'^n) = f_{ij}^{\text{adj}, B}(q^n, q'^n) + (-1)^{n-1} f_{ij}^{\text{adj}, F}(q^n, q'^n) \\
&= \left(\frac{q^{n(|m_{ij}|+2)}}{1 - q^{2n}} - \frac{q'^n(|m_{ij}|+2)}{1 - q'^{2n}} \right) \frac{q^n + q'^n}{q^n - q'^n} (1 + (-1)^n) \\
&\quad + \frac{1 - \delta_{m_{ij}}}{q^n - q'^n} \left(q^{n|m_{ij}|} (q'^n + (-1)^n q^n) - q^n q'^n |m_{ij}| (1 + (-1)^n) \right) \\
& \tilde{f}_{ij}^{\text{adj}}(q^n, q'^n) = \tilde{f}_{ij}^{\text{adj}, B}(q^n, q'^n) + (-1)^{n-1} \tilde{f}_{ij}^{\text{adj}, F}(q^n, q'^n) \\
&= \left(\frac{q^{n(|\tilde{m}_{ij}|+2)}}{1 - q^{2n}} - \frac{q'^n(|\tilde{m}_{ij}|+2)}{1 - q'^{2n}} \right) \frac{q^n + q'^n}{q^n - q'^n} (1 + (-1)^n) \\
&\quad + \frac{1 - \delta_{\tilde{m}_{ij}}}{q^n - q'^n} \left(q^{n|\tilde{m}_{ij}|} (q'^n + (-1)^n q^n) - q^n q'^n |\tilde{m}_{ij}| (1 + (-1)^n) \right)
\end{aligned}$$

$$\begin{aligned}
m_i &\rightarrow \frac{N^\alpha}{\beta} x_i, \alpha_i \rightarrow y_i \\
\tilde{m}_i &\rightarrow \frac{N^\alpha}{\beta} x_i, \tilde{\alpha}_i \rightarrow \tilde{y}_i,
\end{aligned}$$

$$\sum_{i,j} e^{-nt\beta|m_i-\tilde{m}_j|\pm in(\tilde{\alpha}_j-\alpha_i)} \rightarrow \frac{2N^{2-\alpha}}{nt} \int dx \rho^2(x) e^{\pm in\delta y}$$



$$\begin{aligned}
\log Z_{\text{ch}} = & 2N^{3/2} \int dx \rho^2(x) \sum_{n=1}^{\infty} \left[\left(\frac{t_{3,4}^n q^{3n/2} e^{in\delta y}}{n^2(q^n - q'^n)(1 - q^{2n})} + \frac{(-t_{3,4})^n q^{3n/2} q'^n e^{in\delta y}}{n^2(q^n - q'^n)(1 - q^{2n})} \right. \right. \\
& + \frac{t_{1,2}^n q^{3n/2} q'^n e^{in\delta y}}{n^2(q^n - q'^n)(1 - q^{2n})} + \frac{(-t_{1,2})^n q^{5n/2} e^{in\delta y}}{n^2(q^n - q'^n)(1 - q^{2n})} \\
& + \frac{t_{1,2}^{-n} q^{3n/2} e^{-in\delta y}}{n^2(q^n - q'^n)(1 - q^{2n})} + \frac{(-t_{1,2})^{-n} q^{3n/2} q'^n e^{-in\delta y}}{n^2(q^n - q'^n)(1 - q^{2n})} \\
& \left. \left. + \frac{t_{3,4}^{-n} q^{3n/2} q'^n e^{-in\delta y}}{n^2(q^n - q'^n)(1 - q^{2n})} + \frac{(-t_{3,4})^{-n} q^{5n/2} e^{-in\delta y}}{n^2(q^n - q'^n)(1 - q^{2n})} \right) \right. \\
& \left. - (1 + (-1)^n) \frac{1 + \lambda}{1 - \lambda} \left(\frac{t_{3,4}^n q'^n q^{n/2} e^{in\delta y}}{n^2(q^n - q'^n)(1 - q'^{2n})} + \frac{t_{1,2}^n q'^{2n} q^{n/2} e^{in\delta y}}{n^2(q^n - q'^n)(1 - q^{2n})} \right. \right. \\
& \left. \left. + \frac{t_{1,2}^{-n} q'^n q^{n/2} e^{-in\delta y}}{n^2(q^n - q'^n)(1 - q'^{2n})} + \frac{t_{3,4}^{-n} q'^{2n} q^{n/2} e^{-in\delta y}}{n^2(q^n - q'^n)(1 - q'^{2n})} \right) \right], \\
\log Z_{\text{g}} = & 4N^{3/2} \int dx \rho^2(x) \sum_{n=1}^{\infty} \frac{1 + (-1)^n}{n^2} \left[\frac{q^{3n} + q^{2n} q'^n}{(q^n - q'^n)(1 - q^{2n})} - \frac{1 + \lambda}{1 - \lambda} \left(\frac{q^n q'^{2n} + q'^{3n}}{(q^n - q'^n)(1 - q'^{2n})} \right) \right]
\end{aligned}$$

$$\bar{\omega} = \lambda \bar{\beta} \ll \lambda_i$$

$$\begin{aligned}
\beta &= \frac{1}{2}(\bar{\beta} + \bar{\omega}) = \frac{1}{2}(1 + \lambda)\bar{\beta} \\
\beta' &= \frac{1}{2}(\bar{\beta} - \bar{\omega}) = \frac{1}{2}(1 - \lambda)\bar{\beta}
\end{aligned}$$

$$\begin{aligned}
\frac{q^{l_1 n} q'^{l_2 n}}{(q^n - q'^n)(1 - q^{2n})} &= - \left(\frac{1}{n^2 \lambda (1 + \lambda) \bar{\beta}^2} + \frac{(2 + \lambda) - l_1(1 + \lambda) - l_2(1 - \lambda)}{2n \lambda (1 + \lambda) \bar{\beta}} \right) + \mathcal{O}(\bar{\beta}^0) \\
\frac{q^{l_1 n} q'^{l_2 n}}{(q^n - q'^n)(1 - q'^{2n})} &= - \left(\frac{1}{n^2 \lambda (1 - \lambda) \bar{\beta}^2} + \frac{(2 - \lambda) - l_1(1 + \lambda) - l_2(1 - \lambda)}{2n \lambda (1 - \lambda) \bar{\beta}} \right) + \mathcal{O}(\bar{\beta}^0)
\end{aligned}$$

$$Z = \int \mathcal{D}\rho \mathcal{D}\delta y \exp(-W_{\text{eff}}[x; \rho, \delta y])$$



$$\begin{aligned}
W_{\text{eff}}[x; \rho, \delta y] = & 2N^{3/2} \int dx \left(\frac{ik\rho(x\delta y - \mu)}{\bar{\beta}(1+\lambda)} \right. \\
& + \rho^2(x) \left[\frac{1}{\lambda(1+\lambda)\bar{\beta}^2} (\text{Li}_4[e^{i(\lambda_i+\delta y)}] + \text{Li}_4[e^{-i(\lambda_i+\delta y)}] + \text{Li}_4[e^{i(\pi+\lambda_i+\delta y)}] + \text{Li}_4[e^{-i(\pi+\lambda_i+\delta y)}]) \right. \\
& - \frac{1+\lambda}{8\lambda(1-\lambda)^2\bar{\beta}^2} (\text{Li}_4[e^{2i(\lambda_i+\delta y)}] + \text{Li}_4[e^{-2i(\lambda_i+\delta y)}]) + \frac{4\text{Li}_4[1]}{8\lambda(1+\lambda)\bar{\beta}^2} - \frac{4(1+\lambda)\text{Li}_4[1]}{8\lambda(1-\lambda)^2\bar{\beta}^2} \\
& + \frac{2+\lambda-\frac{3}{2}(1+\lambda)}{2\lambda(1+\lambda)\bar{\beta}} (\text{Li}_3[e^{i(\lambda_{3,4}+\delta y)}] + \text{Li}_3[e^{-i(\lambda_{1,2}+\delta y)}]) \\
& + \frac{2+\lambda-\frac{1}{2}(1+\lambda)-(1-\lambda)}{2\lambda(1+\lambda)\bar{\beta}} (\text{Li}_3[e^{i(\pi+\lambda_{3,4}+\delta y)}] + \text{Li}_3[e^{-i(\pi+\lambda_{1,2}+\delta y)}]) \\
& + \frac{2+\lambda-\frac{3}{2}(1+\lambda)-(1-\lambda)}{2\lambda(1+\lambda)\bar{\beta}} (\text{Li}_3[e^{i(\lambda_{1,2}+\delta y)}] + \text{Li}_3[e^{-i(\lambda_{3,4}+\delta y)}]) \\
& + \frac{2+\lambda-\frac{5}{2}(1+\lambda)}{2\lambda(1+\lambda)\bar{\beta}} (\text{Li}_3[e^{i(\pi+\lambda_{1,2}+\delta y)}] + \text{Li}_3[e^{-i(\pi+\lambda_{3,4}+\delta y)}]) \\
& - \frac{2-\lambda-\frac{1}{2}(1+\lambda)-(1-\lambda)}{4 \cdot 2\lambda(1-\lambda)\bar{\beta}} \frac{1+\lambda}{1-\lambda} (\text{Li}_3[e^{2i(\lambda_{3,4}+\delta y)}] + \text{Li}_3[e^{-2i(\lambda_{1,2}+\delta y)}]) \\
& - \frac{2-\lambda-\frac{1}{2}(1+\lambda)-2(1-\lambda)}{4 \cdot 2\lambda(1-\lambda)\bar{\beta}} \frac{1+\lambda}{1-\lambda} (\text{Li}_3[e^{2i(\lambda_{1,2}+\delta y)}] + \text{Li}_3[e^{-2i(\lambda_{3,4}+\delta y)}]) \\
& + \left(\frac{2(2+\lambda-3(1+\lambda))}{2\lambda(1+\lambda)\bar{\beta}} + \frac{2(2+\lambda-2(1+\lambda)-(1-\lambda))}{2\lambda(1+\lambda)\bar{\beta}} \right) \frac{\text{Li}_3[1]}{4} \\
& \left. - \left(\frac{2(2+\lambda-(1+\lambda)-2(1-\lambda))}{2\lambda(1-\lambda)\bar{\beta}} \frac{1+\lambda}{1-\lambda} + \frac{2(1+\lambda-3(1-\lambda))}{2\lambda(1-\lambda)\bar{\beta}} \frac{1+\lambda}{1-\lambda} \frac{\text{Li}_3[1]}{4} \right) + o(\bar{\beta}^0) \right)
\end{aligned}$$

$$\text{Li}_n[z] = \sum_{k=1}^{\infty} \frac{z^k}{k^n}$$

$$\text{Li}_n[z] + \text{Li}_n[-z] = 2^{1-n} \text{Li}_n[z^2]$$

$$Q_i = \frac{\partial \log Z}{\partial \lambda_i}.$$

$$e^{2\delta_1+2\delta_2}=1+\frac{2}{ag},$$

$$\begin{aligned}
S_{BH} &= \frac{\pi^2}{3} c_L T_L + \frac{\pi^2}{3} c_R T_R \\
&= 2\pi \sqrt{\frac{c_L N_L}{6}} + 2\pi \sqrt{\frac{c_R N_R}{6}}
\end{aligned}$$

$$T_L = \frac{1}{\pi} \sqrt{\frac{6N_L}{c_L}}, T_R = \frac{1}{\pi} \sqrt{\frac{6N_R}{c_R}}.$$



$$Z_R = \sum_{N_R} q^{N_R} d(N_R) = \sum_{N_R} q^{N_R} e^{S_R} = \sum_{N_R} q^{N_R} e^{2\pi\sqrt{c_R N_R/6}}$$

$$N_R = q \frac{\partial}{\partial q} \log Z_R \approx \frac{\pi^2 c_R}{6(\log(q))^2}$$

$$N_L = q \frac{\partial}{\partial q} \log Z_L \approx \frac{\pi^2 c_L}{6(\log(q))^2}$$

$$\rho_L(k_0) = \frac{q^n}{1-q^n} = \frac{e^{-k_0/T_L}}{1-e^{-k_0/T_L}},$$

$$\rho_R(k_0) = \frac{q^n}{1-q^n} = \frac{e^{-k_0/T_R}}{1-e^{-k_0/T_R}},$$

$$\rho_L(k_0) = \frac{e^{-k_0/T_L}}{1-e^{-k_0/T_L}} \approx \frac{T_L}{k_0}, \rho_R(k_0) = \frac{e^{-k_0/T_R}}{1-e^{-k_0/T_R}} \approx \frac{T_R}{k_0}.$$

$$\rho_L(k_0) \approx \rho_R(k_0) \approx \frac{e^{-2k_0/T_H}}{1-e^{-2k_0/T_H}},$$

$$d\Gamma \sim \frac{d^4k}{2k_0} \frac{1}{p_0^L p_0^R} |\mathcal{A}|^2 c_L c_R \rho_L(k_0) \rho_R(k_0)$$

$$d\Gamma \sim (\text{horizon area}) \cdot \frac{e^{-2k_0/T_H}}{1-e^{-2k_0/T_H}} d^4k$$

Un campo cuántico relativista, bajo supergravedad, supone la existencia de supermembranas en dimensiones altas, lo que se calcula así:

$$\frac{\mathcal{G}}{\mathcal{H}} = \frac{SL(2, \mathbb{R}) \times SU(2) \times \mathbb{R}_t \times \mathbb{S}_y^1}{U(1)_L \times U(1)_R} \times \mathcal{M}$$

$$u=t-y, v=t+y$$

$$\begin{aligned} ds^2 = & n_5(d\theta^2 + d\rho^2) + \frac{1}{\Sigma} \left[-f_0 dudv + \frac{s(s+1)}{kR_y} \Delta dv^2 \right. \\ & + (n_5 \sinh^2 \rho + n_5(s+1)^2 + k^2 R_y^2) \sin^2 \theta d\phi^2 + (n_5 \sinh^2 \rho + n_5 s^2 + k^2 R_y^2) \cos^2 \theta d\psi^2 \\ & + 2 \left((kR_y - (s+1)\Delta) dy - (s+1)\Delta dt \right) \sin^2 \theta d\phi + 2 \left((kR_y + s\Delta) dy + s\Delta dt \right) \cos^2 \theta d\psi \Big] \\ & + d\mathbf{z} \cdot d\mathbf{z} \end{aligned}$$

$$\begin{aligned} B = & \frac{1}{\Sigma} \left[\frac{kR_y}{n_5} \Delta dt \wedge dy + (n_5 \sinh^2 \rho + n_5(s+1)^2 + k^2 R_y^2) \cos^2 \theta d\phi \wedge d\psi \right. \\ & + \left((kR_y - (s+1)\Delta) dy - (s+1)\Delta dt \right) \wedge \cos^2 \theta d\psi + \left((kR_y + s\Delta) dy + s\Delta dt \right) \wedge \sin^2 \theta d\phi \Big] \\ e^{2\Phi} = & \frac{kR_y V_4 \Delta}{n_1 \Sigma} \end{aligned}$$

$$\Delta = kR_y + \frac{n_5 s(s+1)}{kR_y}, \Sigma = f_0 + \frac{kR_y}{n_5} \Delta, f_0 = \sinh^2 \rho - s \sin^2 \theta + (s+1) \cos^2 \theta$$



$$\frac{s(s+1)}{k} \in \mathbb{Z}.$$

$$\tilde{t} = \frac{t}{R_y}, \tilde{y} = \frac{y}{R_y},$$

$$\begin{aligned} \frac{1}{n_5} ds^2 = & -\frac{1}{k^2} \cosh^2 \rho d\tilde{t}^2 + d\rho^2 + \frac{1}{k^2} \sinh^2 \rho d\tilde{y}^2 \\ & + d\theta^2 + \cos^2 \theta \left(d\psi + \frac{s}{k} d\tilde{t} + \frac{s+1}{k} d\tilde{y} \right)^2 + \sin^2 \theta \left(d\phi - \frac{s+1}{k} d\tilde{t} - \frac{s}{k} d\tilde{y} \right)^2 \end{aligned}$$

$$\psi_{\text{NS}} = \psi + \frac{s}{k} \tilde{t} + \frac{s+1}{k} \tilde{y}, \phi_{\text{NS}} = \phi - \frac{s+1}{k} \tilde{t} - \frac{s}{k} \tilde{y}, \tilde{t}_{\text{NS}} = t, \tilde{y}_{\text{NS}} = \tilde{y}$$

$$(\tilde{y}, \psi_{\text{NS}}, \phi_{\text{NS}}) \sim (\tilde{y}, \psi_{\text{NS}}, \phi_{\text{NS}}) + 2\pi \left(1, \frac{s+1}{k}, -\frac{s}{k} \right).$$

$$g_{tt} = -\frac{f_0 - \Delta_p}{\Sigma}, \Delta_p \equiv s(s+1) + n_5 \left(\frac{s(s+1)}{kR_y} \right)^2.$$

$$ds^2 = d\theta^2 + \cos^2 \theta d\psi^2 + \sin^2 \theta d\phi^2$$

$$g_{\text{su}} = e^{-i(\phi-\psi)\sigma_3/2} e^{i(\pi/2-\theta)\sigma_2} e^{-i(\phi+\psi)\sigma_3/2} = \begin{pmatrix} e^{-i\phi} \sin \theta & e^{i\psi} \cos \theta \\ -e^{-i\psi} \cos \theta & e^{i\phi} \sin \theta \end{pmatrix}$$

$$g(z,\bar{z})=g_\ell(z)g_r(\bar{z})$$

$$g_\ell(z)=e^{-iv'z\sigma_3/2}, g_r(\bar{z})=e^{-iv'\bar{z}\sigma_3/2}$$

$$g_\ell(z)=e^{-i\alpha'_\ell\sigma_1/2}e^{-iv'z\sigma_3/2}, g_r(\bar{z})=e^{-iv'\bar{z}\sigma_3/2}e^{-i\alpha'_r\sigma_1/2}$$

$$g(\xi_0)=\begin{pmatrix} \cos v'\xi_0\cos\frac{\alpha'_\ell+\alpha'_r}{2}-i\sin v'\xi_0\cos\frac{\alpha'_\ell-\alpha'_r}{2} & -i\cos v'\xi_0\sin\frac{\alpha'_\ell+\alpha'_r}{2}+\sin v'\xi_0\sin\frac{\alpha'_\ell-\alpha'_r}{2} \\ -i\cos v'\xi_0\sin\frac{\alpha'_\ell+\alpha'_r}{2}-\sin v'\xi_0\sin\frac{\alpha'_\ell-\alpha'_r}{2} & \cos v'\xi_0\cos\frac{\alpha'_\ell+\alpha'_r}{2}+i\sin v'\xi_0\cos\frac{\alpha'_\ell-\alpha'_r}{2} \end{pmatrix}$$

$$-\cos 2\theta = \cos (\alpha'_\ell + \alpha'_r) \cos^2 (v'\xi_0) + \cos (\alpha'_\ell - \alpha'_r) \sin^2 (v'\xi_0).$$

$$\theta_{\pm}=\frac{1}{2}(\pi-|\alpha'_\ell\mp\alpha'_r|)$$

$$\mathcal{E}=\frac{n_5}{2}\text{tr}[\partial g\partial g^{-1}]+\frac{n_5}{2}\text{tr}[\bar{\partial} g\bar{\partial} g^{-1}]=\frac{n_5}{2}(v')^2,$$

$$J_{\text{su}}^3=-\frac{in_5}{2}\text{tr}[(\partial g)g^{-1}\sigma_3]=n_5(\cos^2\theta\partial\psi-\sin^2\theta\partial\phi)=-\frac{n_5}{2}v'\cos(\alpha'_\ell),$$

$$\bar{J}_{\text{su}}^3=-\frac{in_5}{2}\text{tr}[g^{-1}(\bar{\partial} g)\sigma_3]=-n_5(\cos^2\theta\bar{\partial}\psi+\sin^2\theta\bar{\partial}\phi)=-\frac{n_5}{2}v'\cos(\alpha'_r);$$

$$j'=\frac{n_5}{2}v',\frac{m'}{j'}=-\cos(\alpha'_\ell),\frac{\bar{m}'}{j'}=-\cos(\alpha'_r)$$

$$ds^2 = -\cosh^2 \rho d\tau^2 + d\rho^2 + \sinh^2 \rho d\sigma^2$$



$$g_{\text{sl}} = e^{i(\tau+\sigma)\sigma_3/2} e^{\rho\sigma_1} e^{i(\tau-\sigma)\sigma_3/2} = \begin{pmatrix} e^{i\tau} \cosh \rho & e^{i\sigma} \sinh \rho \\ e^{-i\sigma} \sinh \rho & e^{-i\tau} \cosh \rho \end{pmatrix}$$

$$g_\ell(z) = e^{ivz\sigma_3/2}, g_r(\bar{z}) = e^{iv\bar{z}\sigma_3/2}$$

$$g_\ell(z) = e^{\alpha_\ell\sigma_1/2} e^{ivz\sigma_3/2}, g_r(\bar{z}) = e^{iv\bar{z}\sigma_3/2} e^{\alpha_r\sigma_1/2}$$

$$g(\xi_0) = \begin{pmatrix} \cos v\xi_0 \cosh \frac{\alpha_\ell + \alpha_r}{2} + i \sin v\xi_0 \cosh \frac{\alpha_\ell - \alpha_r}{2} & \cos v\xi_0 \sinh \frac{\alpha_\ell + \alpha_r}{2} - i \sin v\xi_0 \sinh \frac{\alpha_\ell - \alpha_r}{2} \\ \cos v\xi_0 \sinh \frac{\alpha_\ell + \alpha_r}{2} + i \sin v\xi_0 \sinh \frac{\alpha_\ell - \alpha_r}{2} & \cos v\xi_0 \cosh \frac{\alpha_\ell + \alpha_r}{2} - i \sin v\xi_0 \cosh \frac{\alpha_\ell - \alpha_r}{2} \end{pmatrix}$$

$$\cosh 2\rho = \cos^2 (v\xi_0) \cosh (\alpha_\ell + \alpha_r) + \sin^2 (v\xi_0) \cosh (\alpha_\ell - \alpha_r),$$

$$\rho_{\pm} = \frac{1}{2} |\alpha_{\ell} \pm \alpha_r|.$$

$$\mathcal{E} = -\frac{n_5}{2} \text{tr}[\partial g \partial g^{-1}] - \frac{n_5}{2} \text{tr}[\bar{\partial} g \bar{\partial} g^{-1}] = -\frac{n_5}{2} v^2$$

$$J_{\text{sl}}^3 = -\frac{in_5}{2} \text{tr}[(\partial g)g^{-1}\sigma_3] = n_5(\cosh^2 \rho \partial \tau - \sinh^2 \rho \partial \sigma) = \frac{n_5}{2} v \cosh(\alpha_\ell)$$

$$\bar{J}_{\text{sl}}^3 = -\frac{in_5}{2} \text{tr}[g^{-1}(\bar{\partial} g)\sigma_3] = n_5(\cosh^2 \rho \bar{\partial} \tau + \sinh^2 \rho \bar{\partial} \sigma) = \frac{n_5}{2} v \cosh(\alpha_r);$$

$$j = \frac{n_5}{2} v, \frac{m}{j} = \cosh \alpha_\ell, \frac{\bar{m}}{j} = \cosh \alpha_r.$$

$$m = s + 1, n = s,$$

$$\mathcal{S} = \int d\xi \sqrt{g} (g^{\xi\xi} \partial_\xi X^\mu \partial_\xi X^\nu G_{\mu\nu}(X) + M^2)$$

$$g^{\xi\xi} \partial_\xi X^\mu \partial_\xi X^\nu G_{\mu\nu}(X) = M^2.$$

$$\mathcal{L}_g = \mathcal{L}_{\text{sl}} + \mathcal{L}_{\text{su}} = \frac{n_5}{2} [(-\cosh^2 \rho \dot{\tau}^2 + \dot{\rho}^2 + \sinh^2 \rho \dot{\sigma}^2) + (\dot{\theta}^2 + \cos^2 \theta \dot{\psi}_{\text{NS}}^2 + \sin^2 \theta \dot{\phi}_{\text{NS}}^2)]$$

$$\psi_{\text{NS}} = \psi + n\dot{\tau} + m\dot{\sigma}, \phi_{\text{NS}} = \phi - m\dot{\tau} - n\dot{\sigma}$$

$$-p_\tau = n_5(\cosh^2 \rho \dot{\tau} - n \cos^2 \theta \dot{\psi}_{\text{NS}} + m \sin^2 \theta \dot{\phi}_{\text{NS}})$$

$$p_\sigma = n_5(\sinh^2 \rho \dot{\sigma} + m \cos^2 \theta \dot{\psi}_{\text{NS}} - n \sin^2 \theta \dot{\phi}_{\text{NS}})$$

$$m_\psi = n_5(\cos^2 \theta \dot{\psi}_{\text{NS}})$$

$$m_\phi = n_5(\sin^2 \theta \dot{\phi}_{\text{NS}})$$

$$k\mathcal{E} = n_5 \cosh^2 \rho \dot{\tau} - nm_\psi + mm_\phi$$

$$kn_y = n_5 \sinh^2 \rho \dot{\sigma} + mm_\psi - nm_\phi$$

$$n_5 \cosh^2 \rho \dot{\tau} = m + \bar{m}, m_\phi = m' + \bar{m}'$$

$$-n_5 \sinh^2 \rho \dot{\sigma} = m - \bar{m}, m_\psi = \bar{m}' - m'$$



$$\begin{aligned}k\varepsilon &= m + \bar{m} + (2s + 1)m' + \bar{m}' \\ -kn_y &= m - \bar{m} + (2s + 1)m' - \bar{m}'\end{aligned}$$

$$\begin{aligned}\mathcal{H}_{\text{sl}} &= n_5^2 \dot{\rho}^2 + \frac{(m - \bar{m})^2}{\sinh^2 \rho} - \frac{(m + \bar{m})^2}{\cosh^2 \rho} = -4j^2 \\ \mathcal{H}_{\text{su}} &= n_5^2 \dot{\theta}^2 + \frac{(m' - \bar{m}')^2}{\cos^2 \theta} + \frac{(m' + \bar{m}')^2}{\sin^2 \theta} = +4(j')^2\end{aligned}$$

$$\mathcal{H}_{\text{tot}} = \mathcal{H}_{\text{sl}} + \mathcal{H}_{\text{su}} = M^2$$

$$m' = -j + \hat{\mathfrak{m}}, m = j + \hat{\mathfrak{m}}$$

$$\sinh^2 \rho = \frac{\hat{\mathfrak{m}}}{2j}, \cos^2 \theta = \frac{\hat{\mathfrak{m}}}{2j}$$

$$\begin{aligned}D_{j'm'\bar{m}'}(\theta, \phi, \psi) &= e^{-im'(\phi+\psi)} e^{-i\bar{m}'(\phi-\psi)} d_{j'm'\bar{m}'}(\theta) \\ d_{j'm'\bar{m}'}(\theta) &= (\cos \theta)^a (\sin \theta)^b P_q^{(a,b)}(\theta) \\ P_q^{(a,b)}(\theta) &= \sum_{p=0}^q \binom{q+a}{q-p} \binom{q+b}{p} (\sin \theta)^{2p} (\cos \theta)^{2q-2p}\end{aligned}$$

$$j' \rightarrow -j, m' \rightarrow m, \bar{m}' \rightarrow \bar{m}, \hat{\mathfrak{m}} \rightarrow \hat{\mathfrak{m}}$$

$$\left(\frac{\pi}{2}-\theta\right)\rightarrow -i\rho, \phi\rightarrow -\tau, \psi\rightarrow \sigma+\frac{\pi}{2}$$

$$e^{-2ij'\phi}(\sin\theta)^{2j'}\longrightarrow e^{-2ij\tau}(\cosh\rho)^{-2j}$$

$$k\varepsilon = -kn_y = \hat{\mathfrak{m}} + (2s + 1)\hat{\mathfrak{m}} - 2sj$$

$$\Delta_{\ell\hat{\mathfrak{m}}\hat{\mathfrak{n}}}e^{iv_{\ell\hat{\mathfrak{m}}\hat{\mathfrak{n}}}}=\left(\frac{r_{\mathfrak{b}}}{\sqrt{r_{\mathfrak{b}}^2+a^2}}\right)^{\hat{\mathfrak{n}}}\left(\frac{a}{\sqrt{r_{\mathfrak{b}}^2+a^2}}\right)^{\ell}\sin^{\ell-\hat{\mathfrak{n}}}\theta\cos^{\hat{\mathfrak{m}}}\theta e^{i[(\hat{\mathfrak{n}}+(2s+1)\hat{\mathfrak{m}}-2sj)v/\mathbf{k}+(\ell-\hat{\mathfrak{n}})\phi-\hat{\mathfrak{m}}\psi]}$$

$$\frac{r_b^2}{a^2} = \sinh^2 \rho$$

$$k\varepsilon = -kn_y = 2j(\sinh^2 \rho + (2s + 1)\cos^2 \theta - s) = n_5 v f_0$$

$$\begin{aligned}k(\varepsilon - n_y) &\approx n_5 v [\cosh (\rho_+ + \rho_-) + (m + n)\cos (\theta_+ + \theta_-)] \\ k(\varepsilon + n_y) &\approx n_5 v [\cosh (\rho_+ - \rho_-) - (m - n)\cos (\theta_+ - \theta_-)]\end{aligned}$$

$$k\varepsilon \approx n_5 v [\cosh \rho_+ \cosh \rho_- - m \sin \theta_+ \sin \theta_- + n \cos \theta_+ \cos \theta_-]$$

$$k(\varepsilon + n_y) \approx \frac{n_5 v}{2} (\alpha_r^2 + (\alpha_r')^2)$$

$$f_0 = \cosh^2 \rho - (s + 1)\sin^2 \theta + s \cos^2 \theta$$



$$\frac{\mathcal{G}}{\mathcal{H}} = \frac{SL(2, \mathbb{R}) \times SU(2) \times \mathbb{R}_t \times \mathbb{S}_y^1}{U(1)_L \times U(1)_R}$$

$$ds^2 = n_5(-\cosh^2 \rho d\tau^2 + d\rho^2 + \sinh^2 \rho d\sigma^2) + n_5(d\theta^2 + \cos^2 \theta d\psi^2 + \sin^2 \theta d\phi^2) - dt^2 + dy^2.$$

$$\begin{array}{l} \mathcal{J} = J_{\text{sl}}^3 + l_2 J_{\text{su}}^3 + l_3 i \partial t + l_4 i \partial y, \quad \lambda = \psi_{\text{sl}}^3 + l_2 \psi_{\text{su}}^3 + l_3 \psi^t + l_4 \psi^y \\ \overline{\mathcal{J}} = \bar{J}_{\text{sl}}^3 + r_2 \bar{J}_{\text{su}}^3 + r_3 i \bar{\partial} t + r_4 i \bar{\partial} y, \quad \bar{\lambda} = \bar{\psi}_{\text{sl}}^3 + r_2 \bar{\psi}_{\text{su}}^3 + r_3 \bar{\psi}^t + r_4 \bar{\psi}^y \end{array}$$

$$n_5(-1+l_2^2)-l_3^2+l_4^2=n_5(-1+r_2^2)-r_3^2+r_4^2=0$$

$$l_2=2s+1, r_2=1, \qquad l_3=r_3=-\Delta=-\left(kR_y+\frac{n_5s(s+1)}{kR_y}\right),$$

$$l_4=kR_y-\frac{n_5s(s+1)}{kR_y}, \quad r_4=-kR_y-\frac{n_5s(s+1)}{kR_y}.$$

$$\Sigma = \sinh^2 \rho + l_2 r_2 \cos^2 \theta + \frac{1-l_2 r_2}{2} + \frac{l_3 r_3 - l_4 r_4}{2 n_5}.$$

$$\begin{array}{ll} \delta\tau = l_1\alpha + r_1\beta = (\alpha + \beta), & \delta\phi = -l_2\alpha - r_2\beta = -(\alpha + \beta)(s+1) - (\alpha - \beta)s, \\ \delta\sigma = l_1\alpha + r_1\beta = (\alpha - \beta), & \delta\psi = l_2\alpha - r_2\beta = (\alpha + \beta)s + (\alpha - \beta)(s+1), \\ \delta t = l_3\alpha + r_3\beta = -(\alpha + \beta)\Delta, & \delta y = -l_4\alpha - r_4\beta = -kR_y(\alpha - \beta) + \frac{n_5s(s+1)}{kR_y}(\alpha + \beta). \end{array}$$

$$\tilde{t}_{\mathrm{gi}} = \tilde{t} + k\tau, \tilde{y}_{\mathrm{gi}} = \tilde{y} + k\sigma,$$

$$\phi_{\mathrm{gi},1} = \phi - \frac{s+1}{k}\tilde{t} - \frac{s}{k}\tilde{y}, \psi_{\mathrm{gi},1} = \psi + \frac{s^2}{k}\tilde{t} + \frac{s+1}{k}\tilde{y},$$

$$\phi_{\mathrm{gi},2} = \phi + (s+1)\tau + s\sigma, \psi_{\mathrm{gi},2} = \psi - s\tau - (s+1)\sigma.$$

$$\delta(\sigma,\psi,\phi)=\frac{2\pi}{\mathsf{k}}\left(1,s+1,-s\right)$$

$$\begin{array}{l} |++\rangle_{\mathsf{k},s}: h=\frac{\mathsf{k}}{4}+\frac{s(s+1)}{\mathsf{k}}, M'=s+\frac{1}{2} \\ \bar{h}=\frac{\mathsf{k}}{4}, \bar{M}'=\frac{1}{2}. \end{array}$$

$$\begin{array}{l} |--\rangle_{\mathsf{k},s} = |++\rangle_{\mathsf{k},s-1}: h=\frac{\mathsf{k}}{4}+\frac{s(s-1)}{\mathsf{k}}, M'=s-\frac{1}{2} \\ \bar{h}=\frac{\mathsf{k}}{4}, \bar{M}'=\frac{1}{2}. \end{array}$$

$$Q_{\mathrm{BRST}} = \oint \, dz [(cT + \gamma G + \, \mathrm{ghosts} \,) + (\tilde{c}\mathcal{J} + \tilde{\gamma}\lambda)]$$

$$\Phi_{j;m,\bar{m}}^{(w)}\Psi_{j';m',\bar{m}'}^{(w',\bar{w}')}e^{-iEt+iP_yy+i\bar{P}_y\bar{y}}$$

$$P_y=\frac{n_y}{R_y}+w_yR_y,\bar{P}_y=\frac{n_y}{R_y}-w_yR_y$$



$$J_{\rm sl}^a = j_{\rm sl}^a - \frac{i}{2} (\epsilon_{\rm sl})_{bc}^a \psi_{\rm sl}^b \psi_{\rm sl}^c, J_{\rm su}^a = j_{\rm su}^a - \frac{i}{2} (\epsilon_{\rm su})_{bc}^a \psi_{\rm su}^b \psi_{\rm su}^c$$

$$L_0-\frac{1}{2}=-\frac{j(j-1)}{n_5}+\frac{j'(j'+1)}{n_5}-Mw-\frac{n_5}{4}w^2+M'w'+\frac{n_5}{4}w'^2-\frac{1}{4}E^2+\frac{1}{4}P_y^2+h_L=0$$

$$\bar{L}_0-\frac{1}{2}=-\frac{j(j-1)}{n_5}+\frac{j'(j'+1)}{n_5}-\bar{M}w-\frac{n_5}{4}w^2+\bar{M}'\bar{w}'+\frac{n_5}{4}\bar{w}'^2-\frac{1}{4}E^2+\frac{1}{4}\bar{P}_y^2+h_R=0$$

$$\Psi_{j',m',\bar{m}'}^{(w',\bar{w}')}=\Lambda_{j',m',\bar{m}'}^{(\alpha_{\rm su},\bar{\alpha}_{\rm su})}\exp\left[i\frac{2}{\sqrt{n_5}}\left(\left(M'+\frac{n_5}{2}w'\right)Y'+\left(\bar{M}'+\frac{n_5}{2}\bar{w}'\right)\bar{Y}'\right)\right]$$

$$J_{\rm su}^3=i\sqrt{n_5}\partial Y',\bar{J}_{\rm su}^3=i\sqrt{n_5}\bar{\partial}\bar{Y}'$$

$$h_{\Lambda}=\frac{j'(j'+1)}{n_5}-\frac{M'^2}{n_5}+\frac{\alpha_{\rm su}^2}{2},$$

$$h_{\Psi}=h_{\Lambda}+\frac{1}{n_5}\Big(M'+\frac{n_5}{2}w'\Big)^2$$

$$(2M+n_5w)+l_2(2M'+n_5w')+l_3E+l_4P_y=0$$

$$(2\bar{M}+n_5w)+r_2(2\bar{M}'+n_5\bar{w}')+r_3E+r_4\bar{P}_y=0$$

$$\mathcal{V}^{\alpha\dot{\alpha}},\mathcal{S}^{AB},$$

$$(|\,+\,+\rangle_{\rm k})^{2j'+1}\longrightarrow |I\rangle_{(2j'+1){\rm k}}$$

$$n_yw_y=0.$$

$$E=\frac{\varepsilon}{R_y}$$

$$j=j'+1$$

$$\mathrm{k}(\varepsilon-n_y)=2M+2(2s+1)M',$$

$$\mathrm{k}(\varepsilon+n_y)=2\bar{M}+2\bar{M}'.$$

$$\mathrm{k}\varepsilon=M+\bar{M}+(2s+1)M'+\bar{M}'$$

$$-\mathrm{k}n_y=M-\bar{M}+(2s+1)M'-\bar{M}'$$

$$\bar{M}=\bar{J}=j+\bar{\epsilon},\bar{M}'=-\bar{J}'=-(j'+\bar{\epsilon}')$$

$$\varepsilon=-n_y.$$

$$M+(2s+1)M'+\mathrm{k}n_y=0$$

$$M=J+\hat{\mathfrak{n}}=(j+\epsilon)+\hat{\mathfrak{n}},M'=-J'+\hat{\mathfrak{m}}=-(j'+\epsilon')+\hat{\mathfrak{m}},$$



$$\hat{n} + (2s + 1)\hat{m} - 2sj' + \epsilon - (2s + 1)\epsilon' + 1 = -kn_y.$$

$$-2sj' + \epsilon - (2s + 1)\epsilon' + 1 = -kn_y$$

$$-2sj' - 2s\epsilon' = -kn_y$$

$$(|++\rangle_{k,s})^{2j'+1} \rightarrow |++\rangle_{(2j'+1)k,(2j'+1)s}$$

$$(|++\rangle_{k,s})^{2j'+1} \rightarrow |--\rangle_{(2j'+1)k,(2j'+1)s} = |++\rangle_{(2j'+1)k,(2j'+1)(s-1)}$$

$$|\hat{m}, \hat{n}; I\rangle_{(2j'+1)k,(2j'+1)s} = \left(J_{-\frac{(2s+1)}{k}}^+\right)^{\hat{m}} \left(L_{-\frac{1}{k}} - J_{-\frac{1}{k}}^3\right)^{\hat{n}} |I\rangle_{(2j'+1)k,(2j'+1)s}$$

$$\hat{n} + (2s + 1)\hat{m} - 2sj' + (2s + 2) = -kn_y$$

$$\left(G_{-\frac{s+1}{k}}^{+1}G_{-\frac{s+1}{k}}^{+2} + \frac{1}{k}J_{-\frac{2s+1}{k}}^+\left(L_{-\frac{1}{k}} - J_{-\frac{1}{k}}^3\right)\right)|\hat{m}, \hat{n}; ++\rangle_{(2j'+1)k,(2j'+1)s}.$$

$$\hat{n} + (2s + 1)\hat{m} - 2sj' + 2 = -kn_y$$

$$\left(G_{-\frac{s+1}{k}}^{+1}G_{-\frac{s+1}{k}}^{+2} + \frac{1}{k}J_{-\frac{2s+1}{k}}^+\left(L_{-\frac{1}{k}} - J_{-\frac{1}{k}}^3\right)\right)|\hat{m}, \hat{n}; --\rangle_{(2j'+1)k,(2j'+1)s}.$$

$$L_{\frac{n}{k}} \rightarrow L_{\frac{n}{k}}, J_{\frac{n}{k}}^{\pm} \rightarrow J_{\frac{n \mp 2s}{k}}^{\pm}, J_{\frac{n}{k}}^3 \rightarrow J_{\frac{n}{k}}^3, G_{\frac{n}{k}}^{\pm, A} \rightarrow G_{\frac{n \mp s}{k}}^{\pm, A},$$

$$E=w_yR_y+\frac{\varepsilon}{R_y}$$

$$k(\varepsilon - n_y) = 2\left(M_{\rm tot} + \frac{n_5}{2}w\right) + (2s+1)2\left(M'_{\rm tot} + \frac{n_5}{2}w'\right) - \frac{2s(s+1)}{k}n_5w_y$$

$$k(\varepsilon + n_y) = 2\left(\bar{M}_{\rm tot} + \frac{n_5}{2}w\right) + 2\left(\bar{M}'_{\rm tot} + \frac{n_5}{2}\bar{w}'\right)$$

$$M_{\rm tot} = M + M_{\rm osc}, M'_{\rm tot} = M' + M'_{\rm osc}$$

$$0=\frac{-j(j-1)+j'(j'+1)}{n_5}-M_{\rm tot}w+M'_{\rm tot}w'-\frac{n_5}{4}(w^2-(w')^2)-\frac{w_y}{2}(\varepsilon-n_y)+h_L,$$

$$0=\frac{-j(j-1)+j'(j'+1)}{n_5}-\bar{M}_{\rm tot}w+\bar{M}'_{\rm tot}\bar{w}'-\frac{n_5}{4}(w^2-(\bar{w}')^2)-\frac{w_y}{2}(\varepsilon+n_y)+h_R,$$

$$h_L=N_L+\frac{\alpha_{\rm sl}^2+\alpha_{\rm su}^2}{2}-\frac{1}{2}, h_R=N_R+\frac{\bar{\alpha}_{\rm sl}^2+\bar{\alpha}_{\rm su}^2}{2}-\frac{1}{2}.$$

$$J_{\rm sl}^3,J_{\rm su}^3,i\partial t\equiv J_t,i\partial y\equiv J_y\;;\;\bar{J}_{\rm sl}^3,\bar{J}_{\rm su}^3,i\bar{\partial}t\equiv\bar{J}_t,i\bar{\partial}y\equiv\bar{J}_y,$$



$$\begin{aligned} M_{\text{tot}} + \frac{n_5}{2}w, \quad M'_{\text{tot}} + \frac{n_5}{2}w', \quad \frac{E}{2}, \quad \frac{1}{2}\left(\frac{n_y}{R_y} + w_y R_y\right) \\ \bar{M}_{\text{tot}} + \frac{n_5}{2}w, \quad \bar{M}'_{\text{tot}} + \frac{n_5}{2}\bar{w}', \quad \frac{E}{2}, \quad \frac{1}{2}\left(\frac{n_y}{R_y} - w_y R_y\right) \end{aligned}$$

$$\begin{aligned} \delta w = q, \delta w' = -l_2 q, \delta \bar{w}' = -r_2 q, \\ \delta E = l_3 q, \delta P_y = -l_4 q, \delta \bar{P}_y = -r_4 q, q \in \mathbb{Z}, \\ \Rightarrow \delta n_y = -n_5 \frac{s(s+1)}{k} q, \delta w_y = -kq. \end{aligned}$$

$$\begin{aligned} \mathcal{M} &= J_{\text{sl}}^3 - \frac{n_5}{l_3} J_t, \quad \overline{\mathcal{M}} = \bar{J}_{\text{sl}}^3 - \frac{n_5}{l_3} \bar{J}_t \\ \mathcal{M}' &= J_{\text{su}}^3 - \frac{n_5 l_2}{l_4} J_y, \quad \overline{\mathcal{M}}' = \bar{J}_{\text{su}}^3 - \frac{n_5 r_2}{r_4} \bar{J}_y \\ \mathcal{L} &= J_t + \frac{l_3}{l_4} J_y, \quad \bar{\mathcal{L}} = \bar{J}_t + \frac{l_3}{r_4} \bar{J}_y \end{aligned}$$

$$\mathcal{M} + l_2 \mathcal{M}' + \frac{(l_3^2 + n_5)}{l_3} \mathcal{L} = \mathcal{J}$$

$$\mathcal{M} - \overline{\mathcal{M}} = M_{\text{tot}} - \bar{M}_{\text{tot}},$$

$$\mathcal{M} + \overline{\mathcal{M}} = M_{\text{tot}} + \bar{M}_{\text{tot}} + n_5 \left(w + \frac{w_y}{k} \right)$$

$$\mathcal{E} = \varepsilon - n_5 \frac{s(s+1)}{k} \frac{w_y}{k}$$

$$\mathcal{N}_y = n_y + n_5 \frac{s(s+1)}{k} \frac{w_y}{k},$$

$$\mathcal{M}' = M'_{\text{tot}} + \frac{n_5}{2} \left(w' - (2s+1) \frac{w_y}{k} \right), \overline{\mathcal{M}}' = \bar{M}'_{\text{tot}} + \frac{n_5}{2} \left(\bar{w}' - \frac{w_y}{k} \right)$$

$$w_y \in \{0, 1, \dots, k-1\}$$

$$\begin{aligned} \mathcal{E} - \mathcal{N}_y = \frac{2}{kw + w_y} \left[\frac{-j(j-1) + j'(j'+1)}{n_5} + h_L \right. \\ \left. + M'_{\text{tot}} \left(w' - (2s+1) \frac{w_y}{k} \right) + (2s+1) M'_{\text{tot}} \left(w + \frac{w_y}{k} \right) \right. \\ \left. + \frac{n_5}{4} \left(\left(w + \frac{w_y}{k} \right)^2 + \left(w' - (2s+1) \frac{w_y}{k} \right)^2 \right) \right. \\ \left. + 2(2s+1) \left(w + \frac{w_y}{k} \right) \left(w' - (2s+1) \frac{w_y}{k} \right) \right] \end{aligned}$$

$$\begin{aligned} \hat{n} = \frac{k}{kw + w_y} \left[\frac{-j(j-1) + j'(j'+1)}{n_5} + h_L \right. \\ \left. - (J + M_{\text{osc}}) \left(w + \frac{w_y}{k} \right) + M'_{\text{tot}} \left(w' - (2s+1) \frac{w_y}{k} \right) \right. \\ \left. + \frac{n_5}{4} \left(- \left(w + \frac{w_y}{k} \right)^2 + \left(w' - (2s+1) \frac{w_y}{k} \right)^2 \right) \right] \end{aligned}$$



$$\varepsilon + \mathcal{N}_y = \frac{2}{kw + w_y} \left[\frac{-j(j-1) + j'(j'+1)}{n_5} + h_R + \bar{M}'_{\text{tot}} \left(\bar{w}' - \frac{w_y}{k} \right) + \bar{M}'_{\text{tot}} \left(w + \frac{w_y}{k} \right) + \frac{n_5}{4} \left(\left(w + \frac{w_y}{k} \right)^2 + \left(\bar{w}' - \frac{w_y}{k} \right)^2 + 2 \left(w + \frac{w_y}{k} \right) \left(\bar{w}' - \frac{w_y}{k} \right) \right) \right]$$

$$\hat{\mathfrak{n}} = \frac{k}{kw + w_y} \left[\frac{-j(j-1) + j'(j'+1)}{n_5} + h_R - (\bar{J} + \bar{M}_{\text{osc}}) \left(w + \frac{w_y}{k} \right) + \bar{M}'_{\text{tot}} \left(\bar{w}' - \frac{w_y}{k} \right) + \frac{n_5}{4} \left(- \left(w + \frac{w_y}{k} \right)^2 + \left(\bar{w}' - \frac{w_y}{k} \right)^2 \right) \right]$$

$$(|++\rangle_{k,s})^{2j'+1+n_5w} \rightarrow |++\rangle_{(2j'+1+n_5w)k,(2j'+1+n_5w)s}$$

$$\Delta J_{\text{su}}^3 = \Delta \bar{J}_{\text{su}}^3 = - \left(j' + \frac{n_5}{2} w \right) \\ \Delta L_0 = -(2j' + n_5 w) \frac{s}{k}, \Delta \bar{L}_0 = 0$$

$$w' = 2s \frac{w_y}{k} - w, \bar{w}' = -w$$

$$\left. \frac{\varepsilon - \mathcal{N}_y}{2} \right|_{\alpha n_5} = -n_5 \frac{s}{k} \left(w + \frac{w_y}{k} \right), \left. \frac{\varepsilon + \mathcal{N}_y}{2} \right|_{\alpha n_5} = 0$$

$$\hat{\mathfrak{n}} = \frac{k}{kw + w_y} \left[\frac{-j(j-1) + j'(j'+1)}{k} + h_L \right] - (J + M_{\text{osc}} + M'_{\text{tot}})$$

$$\hat{\mathfrak{n}} = \frac{k}{kw + w_y} \left[\frac{-j(j-1) + j'(j'+1)}{k} + h_R \right] - (J + M_{\text{osc}} + M'_{\text{tot}}).$$

$$\varepsilon = 2 \left(\frac{-j(j-1) + j'(j'+1)}{n_5(kw + w_y)} \right) + \frac{h_L + h_R}{kw + w_y} + \left(\frac{s}{k} \right) \left[2M'_{\text{tot}} - n_5 \left(w + \frac{w_y}{k} \right) \right] \\ \mathcal{N}_y = -\frac{h_L - h_R}{kw + w_y} - \left(\frac{s}{k} \right) \left[2M'_{\text{tot}} - n_5 \left(w + \frac{w_y}{k} \right) \right]$$

$$s+1=\ell_1m_1, s=\ell_2m_2, k=\ell_1\ell_2,$$

$$j=j'+1, \bar{M}=J=J'=-\bar{M}', w=-\bar{w}', \bar{M}_{\text{osc}}=\bar{M}'_{\text{osc}}=0$$

$$w_y=p\ell_1, w=w'=pm_2, M'=-J', \varepsilon=-n_y=-\frac{2J'm_2}{\ell_1}, p=1,\dots,\ell_2-1$$

$$w_y=-p\ell_2, w=-w'=pm_1, M'=J', \varepsilon=-n_y=\frac{2J'm_1}{\ell_2}, p=1,\dots,\ell_1-1$$

$$(|++\rangle_{k,s})^{2j'+n_5pm_2+\lceil n_5p/\ell_2 \rceil} \rightarrow |\Upsilon_1\rangle|00\rangle_{(2j'+n_5pm_2)k+n_5p\ell_1,2j's+n_5pm_2(s+1)}$$

$$(|++\rangle_{k,s})^{2j'+n_5pm_1-\lfloor n_5p/\ell_1 \rfloor} \rightarrow |\Upsilon_2\rangle|00\rangle_{(2j'+n_5pm_1)k-n_5p\ell_2,2j'(s+1)+n_5pm_1s}$$

$$j=\frac{1}{2}\Big(1+\sqrt{(2j'+1)^2+2n_5(h_L+h_R)}\Big).$$



$$\frac{1}{2} < j < \frac{n_5 + 1}{2}.$$

$$ds^2 = \frac{1}{H_1} (-dt_b^2 + dy_b^2) + H_5 (dr_b^2 + r_b^2 d\Omega_3^2) + d\mathbf{z} \cdot d\mathbf{z}$$

$$e^{2\Phi} = g_s^2 \frac{H_5}{H_1}, H_{1,5} = 1 + \frac{Q_{1,5}}{r_b^2}$$

$$Q_5 = n_5 \alpha', Q_1 = \frac{g_s^2 n_1 (\alpha')^3}{V_4}$$

$$ds^2 = \frac{\tilde{r}^2}{\tilde{r}^2 + n_1/V_4} (-dt_b^2 + dy_b^2) + \left(g_s^2 + \frac{n_5}{\tilde{r}^2}\right) (d\tilde{r}^2 + \tilde{r}^2 d\Omega_3^2) + d\mathbf{z} \cdot d\mathbf{z}$$

$$e^{2\Phi} = \frac{g_s^2 \tilde{r}^2 + n_5}{\tilde{r}^2 + n_1/V_4}$$

$$g_s \ll 1, \frac{n_5}{n_1} V_4 \ll 1,$$

$$r_b = g_s \tilde{r} = ar, a^2 \equiv \frac{g_s^2 n_1 n_5}{V_4 R_y^2}, t = t_b/R_y, y = y_b/R_y$$

$$ds^2 = \frac{r^2 R_y^2}{r^2 + R_y^2/n_5} (-dt^2 + dy^2) + \left(\frac{g_s^2 n_1 n_5}{V_4 R_y^2} + \frac{n_5}{r^2}\right) (dr^2 + r^2 d\Omega_3^2) + d\mathbf{z} \cdot d\mathbf{z}$$

$$e^{2\Phi} = \frac{g_s^2 r^2 + V_4 R_y^2/n_1}{r^2 + R_y^2/n_5}$$

$$\tilde{Q}_1 = \frac{n_1}{V_4}, Q_5 = n_5$$

$$n_1 \gg n_5 V_4 \Rightarrow \tilde{Q}_1 \gg Q_5$$

$$(s+1)^2 + \frac{k^2 R_y^2}{n_5}, s^2 + \frac{k^2 R_y^2}{n_5}.$$

$$e^{2\Phi} = \frac{n_5 V_4}{n_1},$$

Los superestratos y simetrías superconformes, en un campo cuántico relativista bajo supergravedad, se calculan así:

$$ds_{10}^2 = \ell_{\text{AdS}}^2 (d\rho^2 + \cosh^2 \rho dt_{\text{E}}^2 + \sinh^2 \rho d\varphi^2) + R_+^2 d\Omega_+^2 + R_-^2 d\Omega_-^2 + L^2 dy^2$$

$$H = \frac{2}{\ell_{\text{AdS}}} \omega_{\text{AdS}} + \frac{2}{R_+} \omega_+ + \frac{2}{R_-} \omega_-$$

$$\frac{R_{\pm}}{\ell_s} = \sqrt{k_{\pm}}, \frac{L}{\ell_s} = g_s^2 \frac{N_1}{k_+ k_- \sqrt{k_+ + k_-}}$$



$$\alpha = \frac{k_-}{k_+}$$

$$\gamma = \frac{k_-}{k_+ + k_-} = \frac{\alpha}{1 + \alpha}$$

$$\frac{\ell_{\text{AdS}}}{\ell_s} = \sqrt{k}$$

$$c = 6kN_1 \equiv 6k, k \equiv \frac{k_+k_-}{k_+ + k_-},$$

$$h \geq \gamma \ell_+ + (1 - \gamma) \ell_-.$$

$$\begin{aligned} \chi_{\text{L}}^{\text{g}}(h, \ell_+, \ell_-) = & (q^h + q^{h+2}) \chi_{\ell_+}(r_+) \chi_{\ell_-}(r_-) \\ & + \left(q^{h+\frac{1}{2}} + q^{h+\frac{3}{2}} \right) \left(\chi_{\ell_++\frac{1}{2}}(r_+) + \chi_{\ell_--\frac{1}{2}}(r_+) \right) \left(\chi_{\ell_++\frac{1}{2}}(r_-) + \chi_{\ell_--\frac{1}{2}}(r_-) \right) \\ & + q^{h+1} \left(2 \chi_{\ell_+}(r_+) \chi_{\ell_-}(r_-) + \left(\chi_{\ell_++1}(r_+) + \chi_{\ell_+-1}(r_+) \right) \chi_{\ell_-}(r_-) \right. \\ & \left. + \chi_{\ell_+}(r_+) \left(\chi_{\ell_-+1}(r_-) + \chi_{\ell_- -1}(r_-) \right) \right) \end{aligned}$$

$$\chi_{\ell}(r) = \frac{r^{\ell+\frac{1}{2}} - r^{-\ell-\frac{1}{2}}}{r^{\frac{1}{2}} - r^{-\frac{1}{2}}}, \ell \geq 0 \ (\chi_{\ell}(r) = 0, \ell < 0)$$

$$q = e^{2\pi i \tau}, r_{\pm} = e^{2\pi i \rho_{\pm}}$$

$$\begin{aligned} \chi_{\text{S}}^{\text{g}}(\ell_+, \ell_-) = & q^{h_s} \chi_{\ell_+}(r_+) \chi_{\ell_-}(r_-) + q^{\frac{3}{2}} \chi_{\ell_+-\frac{1}{2}}(r_+) \chi_{\ell_--\frac{1}{2}}(r_-) \\ & + q^{h_s+\frac{1}{2}} \left(\chi_{\ell_+-\frac{1}{2}}(r_+) \chi_{\ell_--\frac{1}{2}}(r_-) + \chi_{\ell_++\frac{1}{2}}(r_+) \chi_{\ell_--\frac{1}{2}}(r_-) + \chi_{\ell_+-\frac{1}{2}}(r_+) \chi_{\ell_--\frac{1}{2}}(r_-) \right) \\ & + q^{h_s+1} \left(\chi_{\ell_+}(r_+) \chi_{\ell_- -1}(r_-) + \chi_{\ell_+-1}(r_+) \chi_{\ell_-}(r_-) + \chi_{\ell_+}(r_+) \chi_{\ell_-}(r_-) \right), \end{aligned}$$

$$h_s = h_s(\ell_+, \ell_-) \equiv \gamma \ell_+ + (1 - \gamma) \ell_-.$$

$$\chi_{\text{L}}^{\text{g}}(h, \ell_+, \ell_-) = q^{h-h_s(\ell_+, \ell_-)} \left[\chi_{\text{S}}^{\text{g}}(\ell_+, \ell_-) + \chi_{\text{S}}^{\text{g}}\left(\ell_+ + \frac{1}{2}, \ell_- + \frac{1}{2}\right) \right].$$

$$\chi_{\text{S}}^{\text{g}}\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) = -q^{\frac{1}{2}} \chi_{\frac{1}{2}}(r_+) \chi_{\frac{1}{2}}(r_-) + q(1 + \chi_1(r_+) + \chi_1(r_-)) - q^{\frac{3}{2}} \chi_{\frac{1}{2}}(r_+) \chi_{\frac{1}{2}}(r_-) + q^2.$$

$$\sum_{\ell=0}^{\infty} \left| \chi_{\text{S}}^{\text{g}}(\ell, \ell) + \chi_{\text{S}}^{\text{g}}\left(\ell + \frac{1}{2}, \ell + \frac{1}{2}\right) \right|^2 + \sum_{\ell_+ \neq \ell_-} |\chi_{\text{L}}^{\text{g}}(h, \ell_+, \ell_-)|^2.$$

$$h(\ell_+, \ell_-) = \sqrt{\frac{k}{k_+} \left(\ell_+ + \frac{1}{2} \right)^2 + \frac{k}{k_-} \left(\ell_- + \frac{1}{2} \right)^2} - \frac{1}{2}$$



$$\sum_{\ell_+, \ell_-} |\chi_L^g(h(\ell_+, \ell_-), \ell_+, \ell_-)|^2$$

$$\begin{aligned} h &\geq \frac{1}{k_+ + k_-} [k_+ \ell_- + k_- \ell_+ + (\ell_+ - \ell_-)^2 + u^2] \\ &= \gamma \ell_+ + (1 - \gamma) \ell_- + \frac{(\ell_+ - \ell_-)^2 + u^2}{(k_+ + k_-) N_1} \end{aligned}$$

$$\begin{aligned} h - \frac{c}{24} &\geq \frac{1}{k_+ + k_-} \left[\left(\ell_+ + \ell_- - \frac{1}{2} \right)^2 + u^2 \right] \\ &= \frac{1}{N_1(k_+ + k_-)} \left[\left(\ell_+ + \ell_- - \frac{1}{2} \right)^2 + u^2 \right] \end{aligned}$$

$$h - \frac{c}{24} \geq \frac{6}{c} \frac{\alpha}{(1 + \alpha)^2} \left(\ell_+ + \ell_- - \frac{1}{2} \right)^2$$

$$\chi_{\text{NS}}^\gamma(q,r_+,r_-;\delta) = \text{Tr}_{\text{NS}}\left(e^{i\pi\delta F} q^{L_0 - \frac{c}{24}} r_+^{J_0^{3,+}} r_-^{J_0^{3,-}}\right)$$

$$\chi_{\text{NS}}^\gamma(q,r_+,r_-;\delta) = q^{h-\frac{c}{24}} \frac{\vartheta_{3+\delta}^2\left(\frac{\rho_++\rho_-}{2},\tau\right)\vartheta_{3+\delta}^2\left(\frac{\rho_--\rho_-}{2},\tau\right)}{\eta(\tau)^6\vartheta_1(\rho_+,\tau)\vartheta_1(\rho_-,\tau)}\mu_{\text{NS}}(q,r_+,r_-;\delta)$$

$$\mu_{\text{NS}}(q,r_+,r_-;\delta) = \sum_{n,m \in \mathbb{Z}} q^{n^2 k_+ + m^2 k_- + n(2\ell_+ + 1) + m(2\ell_- + 1)} \sum_{\epsilon_{\pm} = \pm} \epsilon_+ \epsilon_- \hat{\mu}_{\text{NS}}(q, r_+^{\epsilon_+}, r_-^{\epsilon_-})$$

$$\hat{\mu}_{\text{NS}}(q,r_+,r_-) = r_+^{nk_+ + \ell_+ + \frac{1}{2}} r_-^{mk_- + \ell_- + \frac{1}{2}} \times \begin{cases} 1, & \text{long} \\ \frac{1}{1 + e^{i\pi\delta} r_+^{\frac{1}{2}} r_-^{\frac{1}{2}} q^{m+n+\frac{1}{2}}}}, & \text{short} \end{cases}$$

$$\chi_{\text{R}}^\gamma(q,r_+,r_-;\delta) = q^{h-\frac{c}{24}} \frac{\vartheta_{2-\delta}^2\left(\frac{\rho_++\rho_-}{2},\tau\right)\vartheta_{2-\delta}^2\left(\frac{\rho_--\rho_-}{2},\tau\right)}{\eta(\tau)^6\vartheta_1(\rho_+,\tau)\vartheta_1(\rho_-,\tau)}\mu_{\text{R}}(q,r_+,r_-;\delta)$$

$$\mu_{\text{R}}(q,r_+,r_-;\delta) = \sum_{n,m \in \mathbb{Z}} q^{n^2 k_+ + m^2 k_- + 2\ell_+ n + 2\ell_- m} \sum_{\epsilon_{\pm} = \pm} \epsilon_+ \epsilon_- \hat{\mu}_{\text{R}}(q, r_+^{\epsilon_+}, r_-^{\epsilon_-})$$

$$\hat{\mu}_{\text{R}}(q,r_+,r_-) = r_+^{nk_+ + \ell_+} r_-^{mk_- + \ell_-} \times \begin{cases} 1, & \text{long} \\ \frac{1}{\left(r_+^{-1} q^{-n} + e^{i\pi\delta} r_-^{-1} q^{-m}\right)}, & \text{short} \end{cases}$$

$$L_n, \mathcal{G}_r^+ \equiv G_r^3 + i G_r^0, \mathcal{G}_r^- \equiv G_r^3 - i G_r^0, J_n \equiv 2(\gamma J_n^{+,3} - (1-\gamma) J_n^{-,3}),$$

$$\begin{aligned} \{\mathcal{G}_r^+, \mathcal{G}_s^-\} &= 4L_{r+s} + 2(r-s)J_{r+s} + \frac{c}{3}\delta_{r+s,0}\left(r^2 - \frac{1}{4}\right) \\ [J_n, \mathcal{G}_r^\pm] &= \pm \mathcal{G}_{r+n}^\pm \end{aligned}$$

$$\left\{\mathcal{G}_{-\frac{1}{2}}^+, \mathcal{G}_{\frac{1}{2}}^-\right\}=4L_0-2J_0$$

$$\text{Tr}_{\text{NS}}((-1)^{F_s}q^{L_0-J_0/2})$$



$$\left(\frac{d}{d\rho_+}-\frac{d}{d\rho_-}\right)\mathrm{Tr}_{\mathrm{NS}}\left((-1)^{F_{\mathrm{s}}}q^{L_0-\frac{c}{24}}r_+^{J_0^{3,+}}r_-^{J_0^{3,-}}\right)\Big|_{\rho_+=\gamma\tau,\rho_-=- (1-\gamma)\tau}$$

$$\begin{array}{l} \mathcal{S}_+ \colon (h(\ell_+,\ell_-),\ell_+,\ell_-)_{\mathrm{NS}} \mapsto \left(h(\ell_+,\ell_-)-\ell_++\frac{\mathrm{k}_+}{4},\frac{\mathrm{k}_+}{2}+\ell_+,\ell_-+\frac{1}{2}\right)_{\mathrm{R}} \\ \mathcal{S}_- \colon (h(\ell_+,\ell_-),\ell_+,\ell_-)_{\mathrm{NS}} \mapsto \left(h(\ell_+,\ell_-)-\ell_-+\frac{\mathrm{k}_-}{4},\ell_++\frac{1}{2},\frac{\mathrm{k}_-}{2}-\ell_-+\right)_{\mathrm{R}} \end{array}$$

$$\begin{array}{l} (t_{\mathrm{E}},\varphi,\phi_1^+,\phi_2^+,\phi_1^-,\phi_2^-,y)\\ \sim (t_{\mathrm{E}}+\beta,\varphi+i\beta\mu,\phi_1^++i\beta\nu_1^+,\phi_2^++i\beta\nu_2^+,\phi_1^-+i\beta\nu_1^-,\phi_2^-+i\beta\nu_2^-,y) \end{array}$$

$$\tau=\frac{\beta\mu+i\beta}{2\pi},q=e^{2\pi i\tau}$$

$$\rho_{\pm}=\beta(v_2^{\pm}-v_1^{\pm}),\bar{\rho}_{\pm}=\beta(\bar{v}_2^{\pm}+\bar{v}_1^{\pm}),r_{\pm}=e^{2\pi i\rho_{\pm}},$$

$$\mathfrak{t}=\mathfrak{t}_1+i\mathfrak{t}_2,\mathfrak{z}=e^{2\pi i\mathfrak{t}}$$

$$\begin{array}{l} \mathcal{Z}_{\mathfrak{sl}(2)}(\mathfrak{t})\!=\!\frac{\beta\sqrt{k}}{2\pi\sqrt{\mathfrak{t}_2}}\sum_{n,m\in\mathbb{Z}}\mathcal{Z}_{\mathfrak{sl}(2)}^{(n,m)}(\mathfrak{t})\\ \mathcal{Z}_{\mathfrak{sl}(2)}^{(n,m)}(\mathfrak{t})=\frac{\exp\left(-\frac{k\beta^2}{4\pi\mathfrak{t}_2}|m-n\mathfrak{t}|^2+\frac{2\pi}{\mathfrak{t}_2}\big(\mathrm{Im}(\mathfrak{u}_{n,m})\big)^2\right)}{\left|\vartheta_1(\mathfrak{u}_{n,m},\mathfrak{t})\right|^2} \end{array}$$

$$\mathfrak{u}_{n,m}=\tau(n\mathfrak{t}-m)$$

$$\mathcal{Z}_{\mathrm{su}(2)}^{(n,m)}(\mathfrak{t})=e^{\frac{\pi}{\mathfrak{t}_2}\big(\mathrm{Im}(\rho_{n,m})\big)^2[k|\mathfrak{nt}-m|^2-(k-2)]}\sum_{\ell=0}^{\frac{k}{2}-1}\left|\frac{\Theta_{2\ell+1}^{(k)}(\rho_{n,m},\mathfrak{t})-\Theta_{-2\ell-1}^{(k)}(\rho_{n,m},\mathfrak{t})}{\vartheta_1(\rho_{n,m},\mathfrak{t})}\right|^2$$

$$\Theta_l^{(k)}(\rho,\tau)=\sum_{n\in\mathbb{Z}+\frac{l}{2k}}q^{kn^2}r^{nk},q=e^{2\pi i\tau},r=e^{2\pi i\rho}$$

$$\rho_{n,m}=\rho(n\mathfrak{t}-m)$$

$$\mathcal{Z}_{\mathfrak{u}(1)}^{(n,m)}(\mathfrak{t})=\frac{1}{|\eta(\mathfrak{t})|^2}\sum_{p_L,p_R}\mathfrak{z}^{p_L^2\bar{\mathfrak{z}}^2}$$

$$\mathcal{Z}_{bc}(\mathfrak{t})=|\eta(\mathfrak{t})|^4$$

$$\begin{array}{l} (Z_{\mathrm{PF}}^{\pm})^{(n,m)}=\mathrm{Tr}_{\mathrm{NS}}\left(\frac{1-e^{i\pi F}}{2}e^{2\pi i\mathfrak{t}L_0}\right)-\mathrm{Tr}_{\mathrm{R}}\left(\frac{1\pm e^{i\pi F}}{2}e^{2\pi i\mathfrak{t}L_0}\right)\\ =\frac{1}{2}\Big(\mathcal{Z}_3^{(n,m)}-\mathcal{Z}_4^{(n,m)}-\mathcal{Z}_2^{(n,m)}\mp\mathcal{Z}_1^{(n,m)}\Big) \end{array}$$



$$\mathcal{Z}_a^{(n,m)} = \underbrace{e^{-\frac{\pi}{t_2}(\text{Im}(u_{n,m}))^2} \frac{\vartheta_a(u_{n,m}, t) \sqrt{\vartheta_a(0, t)}}{\eta(t)^{\frac{3}{2}}}}_{\text{sl}(2)} \times \underbrace{e^{-\frac{\pi}{t_2}(\text{Im}(\rho_{n,m}^+))^2} \frac{\vartheta_a(\rho_{n,m}^+, t) \sqrt{\vartheta_a(0, t)}}{\eta(t)^{\frac{3}{2}}}}_{\text{su}(2)_+} \\ \times \underbrace{e^{-\frac{\pi}{t_2}(\text{Im}(\rho_{n,m}^-))^2} \frac{\vartheta_a(\rho_{n,m}^-, t) \sqrt{\vartheta_a(0, t)}}{\eta(t)^{\frac{3}{2}}}}_{\text{su}(2)_-} \times \underbrace{\sqrt{\frac{\vartheta_a(0, t)}{\eta(t)}} \times \frac{\eta(t)}{\vartheta_a(0, t)}}_{\text{u}(1)} \frac{\eta(t)}{\beta \gamma}$$

$$\mathcal{Z}_{\text{PF}}^{(n,m)}(t) = \frac{e^{-\frac{\pi}{t_2}[(\text{Im}(u_{n,m}))^2 + (\text{Im}(\rho_{n,m}^+))^2 + (\text{Im}(\rho_{n,m}^-))^2]}}{\eta(t)^4} \\ \times \prod_{\eta_{\pm}=\pm 1} \vartheta_1\left(\frac{-u_{n,m} + \eta_+ \rho_{n,m}^+ + \eta_- \rho_{n,m}^-}{2}, t\right)$$

$$(\mathcal{Z}_{\text{AF}}^{\pm})^{(n,m)} = \text{Tr}_{\text{NS}} \left(\left[\frac{1 - e^{i\pi F}}{2} \frac{1 + e^{i\pi n}}{2} + \frac{1 + e^{i\pi F}}{2} e^{i\pi m} \frac{1 - e^{i\pi n}}{2} \right] e^{2\pi i t L_0} \right) \\ - \text{Tr}_{\text{R}} \left(\left[\frac{1 \pm e^{i\pi F}}{2} \frac{1 - e^{i\pi n}}{2} + \frac{1 + e^{i\pi F}}{2} e^{i\pi m} \frac{1 - e^{i\pi n}}{2} \right] e^{2\pi i t L_0} \right)$$

$$\mathcal{Z}_{\text{IIB}} = \int_{\mathcal{F}} \frac{d^2 t}{t_2} \sum_{n,m \in \mathbb{Z}} \mathcal{Z}_{\text{su}(2)}^{(n,m)}(t) \mathcal{Z}_{\text{su}(2)_+}^{(n,m)}(t) \mathcal{Z}_{\text{su}(2)_-}^{(n,m)}(t) \mathcal{Z}_{bc}(t) \left| \mathcal{Z}_{\text{PF}}^{(n,m)}(t) \right|^2$$

$$\mathcal{Z}_{\text{IIB}}(\tau, \rho_+, \rho_-) = -\beta \sum_{m=1}^{\infty} f_{\text{PF}}(m\tau, \rho_+, \rho_-),$$

$$\mathcal{Z}_{\text{CFT}}(\tau, \rho_+, \rho_-)|_{\text{thermal AdS}_3} = \exp \left(S_{\text{tree}} + \mathcal{Z}_{\text{IIB}} + \cdots \right)$$

$$f_{\text{PF}}(\tau, \rho_+, \rho_-) = f_{\text{PF, disc}}(\tau, \rho_+, \rho_-) + f_{\text{PF, cont}}(\tau, \rho_+, \rho_-)$$

$$f_{\text{PF}} = \frac{\sqrt{k}\beta}{2\pi} \int_0^\infty \frac{dt_2}{t_2^2} \int_{-\frac{1}{2}}^{\frac{1}{2}} dt_1 e^{-\frac{k\beta^2}{4\pi t_2} \mathcal{J}_{\text{PF}}} \\ = \frac{4}{ik} \int_{-\infty}^\infty \zeta d\zeta \int_0^\infty dt_2 \int_{-\frac{1}{2}}^{\frac{1}{2}} dt_1 e^{2i\beta\zeta - \frac{4\pi}{k} t_2 \zeta^2} \mathcal{J}_{\text{PF}}$$

$$\mathcal{J}_{\text{PF}} = \left| \frac{\prod_{\epsilon_{\pm}=\pm} \vartheta_1\left(\frac{\tau + \epsilon_+ \rho_+ + \epsilon_- \rho_-}{2}, t\right)}{\vartheta_1(\tau, t) \vartheta_1(\rho_+, t) \vartheta_1(\rho_-, t) \eta(t)^3} \right|^2 \times \left(\sum_{p_L, p_R} \mathfrak{z}^{p_L^2} \bar{\mathfrak{z}}_R^{-2} \right) \\ \times \prod_{\epsilon \in \{\pm\}} \left[\sum_{\ell_\epsilon=0}^{\frac{1}{2}k_\epsilon-1} \left| \sum_{n_\epsilon \in \mathbb{Z}} \mathfrak{z}^{\frac{(n_\epsilon k_\epsilon + \ell_\epsilon + \frac{1}{2})^2}{k_\epsilon}} \left(r_\epsilon^{n k_\epsilon + \ell_\epsilon + \frac{1}{2}} - r_\epsilon^{-n k_\epsilon - \ell_\epsilon - \frac{1}{2}} \right) \right|^2 \right]$$

$$\mathcal{J}_{\text{PF}} = (q\bar{q})^{-\frac{1}{2}} I_0 \times I_{\text{primaries}} \times I_{\text{oscillators}}$$



$$\begin{aligned}
I_0 &= \left| 1 + \frac{\chi_S^g\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)}{1-q} \right|^2 \\
I_{\text{primaries}} &= \left(\sum_{p_L, p_R} \mathfrak{z}^{p_L^2 p_R^2} \right) \times \prod_{\epsilon \in \{\pm\}} \left(\sum_{\ell_\epsilon=0}^{\frac{k_\epsilon}{2}-1} \left| \sum_{n_\epsilon \in \mathbb{Z}} \mathfrak{z}^{\frac{(n_\epsilon k_\epsilon + \ell_\epsilon + \frac{1}{2})^2}{k_\epsilon}} \chi_{n_\epsilon k_\epsilon + \ell_\epsilon}(r_\epsilon) \right|^2 \right) \\
I_{\text{oscillators}} &= \left| \prod_{n=1}^{\infty} \frac{\prod_{\pm=\pm 1} \left(1 - (qr_+^{\epsilon_+} r_-^{\epsilon_-})^{\frac{1}{2}} \mathfrak{z}^n \right) \left(1 - (qr_+^{\epsilon_+} r_-^{\epsilon_-})^{-\frac{1}{2}} \mathfrak{z}^n \right)}{(1-\mathfrak{z}^n)^2 \prod_{\epsilon=\pm 1} (1-q^\epsilon \mathfrak{z}^n) (1-r_+^{\epsilon} \mathfrak{z}^n) (1-r_-^{\epsilon} \mathfrak{z}^n)} \right|^2 \\
&\prod_{n=1}^{\infty} \frac{1-\mathfrak{z}^n}{(1-q\mathfrak{z}^n)(1-q^{-1}\mathfrak{z}^n)} = \sum_{w=0}^{\infty} q^w \mathfrak{z}^{-\frac{1}{2}w(w+1)} \sum_{N \in \mathbb{Z}} \mathbf{P}_N^w(q) \mathfrak{z}^N \\
I_{\text{oscillators}} &= \left| \sum_{w=0}^{\infty} \sum_{N \in \mathbb{Z}} \mathbf{Q}_N^w(q, r_+, r_-) q^w \mathfrak{z}^{-\frac{1}{2}w(w+1)+N} \right|^2 \\
\mathcal{I}_{\text{PF}} &= (q\bar{q})^{-\frac{1}{2}} I_0 \sum^{\wedge} (q\bar{q})^w \mathbf{Q}_{N, \bar{N}}^w \times (\mathfrak{z}\bar{\mathfrak{z}})^{-\frac{1}{2}w(w+1)} \mathfrak{z}^{p_L^2 + h_{\text{su}(2)} + N} \bar{\mathfrak{z}}_R^{-2} + \bar{h}_{\text{su}(2)} + \bar{N} \\
h_{\text{su}(2)} &= \frac{\left(n_+ k_+ + \ell_+ + \frac{1}{2}\right)^2}{k_+} + \frac{\left(n_- k_- + \ell_- + \frac{1}{2}\right)^2}{k_-} \\
\bar{h}_{\text{su}(2)} &= \frac{\left(\bar{n}_+ k_+ + \ell_+ + \frac{1}{2}\right)^2}{k_+} + \frac{\left(\bar{n}_- k_- + \ell_- + \frac{1}{2}\right)^2}{k_-} \\
\mathbf{Q}_{N, \bar{N}}^w &\equiv \mathbf{Q}_N^w(q, r_+, r_-) \bar{\mathbf{Q}}_{\bar{N}}^w(\bar{q}, \bar{r}_+, \bar{r}_-) \left[\prod_{\epsilon=\pm 1} \chi_{n_\epsilon k_\epsilon + \ell_\epsilon}(r_\epsilon) \chi_{\bar{n}_\epsilon k_\epsilon + \ell_\epsilon}(\bar{r}_\epsilon) \right]; \\
\sum^{\wedge} &\equiv \sum_{w=0}^{\infty} \sum_{N, \bar{N} \in \mathbb{Z}} \sum_{n_{\pm}, \bar{n}_{\pm} \in \mathbb{Z}} \sum_{p_L, p_R} \sum_{\ell_+=0}^{\frac{k_+}{2}-1} \sum_{\ell_-=0}^{\frac{k_-}{2}-1} \\
\delta_{\text{LM}}: p_L^2 + h_{\text{su}(2)} + N &= p_R^2 + \bar{h}_{\text{su}(2)} + \bar{N} \\
f_{\text{PF}} &= \frac{k}{4\pi} I_0 \sum^{\wedge} \delta_{\text{LM}}(q\bar{q})^{w-\frac{1}{2}} \mathbf{Q}_{N, \bar{N}}^w \times \int_{-\infty}^{\infty} \frac{d\zeta}{i\pi} \frac{\zeta e^{2i\beta\zeta}}{\zeta^2 + \Delta_w^2} \left(e^{-\frac{2\beta}{w+1} \frac{\zeta^2 + \Delta_w^2}{k}} - e^{-\frac{2\beta}{w} \frac{\zeta^2 + \Delta_w^2}{k}} \right) \\
\Delta_w^2 &= k \left[p_L^2 + h_{\text{su}(2)} + N - \frac{1}{2} w(w+1) \right] \\
\zeta_* &= i\Delta_w
\end{aligned}$$



$$f_{\text{PF,disc}}(\tau, \rho_{\pm}) = I_0 \sum^{\wedge} \delta_{\text{LM}} Q_{N, \bar{N}}^w (q \bar{q})^{w - \frac{1}{2} + \Delta_w}$$

$$\frac{k}{2}w \leq \Delta_w \leq \frac{k}{2}(w+1)$$

$$f_{\text{PF,cont}}(\tau, \rho_{\pm}) = I_0 \sum_{w=1}^{\infty} \int_{-\infty}^{\infty} \frac{ds}{i\pi} \varrho(s; q, \bar{q}, r_{\pm}, \bar{r}_{\pm}) (q \bar{q})^{\frac{k w}{4} + \frac{1}{w k} (s^2 + \Delta_w^2) + w - \frac{1}{2}}$$

$$\Delta_{w=0} - \frac{1}{2} = \sqrt{k p_L^2 + k \left(\frac{(n_+ k_+ + \ell_+ + \frac{1}{2})^2}{k_+} + \frac{(n_- k_- + \ell_- + \frac{1}{2})^2}{k_-} \right)} - \frac{1}{2}$$

$$h_{\text{CFT}} = \sqrt{\frac{k}{k_+} \left(\ell_+ + \frac{1}{2} \right)^2 + \frac{k}{k_-} \left(\ell_- + \frac{1}{2} \right)^2} - \frac{1}{2}.$$

$$f_{\text{PF,disc}}(\tau, \rho_{\pm})|_{\text{sugra}} = I_0 \sum_{\ell_+=0}^{\frac{k_+-1}{2}} \sum_{\ell_-=0}^{\frac{k_- -1}{2}} (q \bar{q})^{h_{\text{CFT}}} |\chi_{\ell_+}(r_+) \chi_{\ell_-}(r_-)|^2$$

$$\chi^{\rm g}_{\rm S}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)q^h\chi_{\ell_+}(r_+)\chi_{\ell_-}(r_-)=\chi^{\rm g}_{\rm L}(h,\ell_+,\ell_-)-q^h(1-q)\chi_{\ell_+}(r_+)\chi_{\ell_-}(r_-).$$

$$f_{\text{PF,disc}}(\tau, \rho_{\pm})|_{\text{sugra}} = \sum_{\ell_+=0}^{\frac{k_+-1}{2}} \sum_{\ell_-=0}^{\frac{k_- -1}{2}} |\chi^{\rm g}_{\rm L}(h_{\text{CFT}}, \ell_+, \ell_-)|^2$$

$$I_0=1+\frac{\chi^{\rm g}_{\rm S}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)}{1-q}+\frac{\bar{\chi}^{\rm g}_{\rm S}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)}{1-\bar{q}}+\frac{\chi^{\rm g}_{\rm S}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)\bar{\chi}^{\rm g}_{\rm S}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)}{|1-q|^2}$$

$$\exp\left(\sum_{j=1}^{\infty}\frac{1}{j}I_{0,\text{hol}}(j\tau,j\rho_{\pm})\right)=\frac{\vartheta_4^2\left(\frac{\rho_++\rho_-}{2},\tau\right)\vartheta_4^2\left(\frac{\rho_--\rho_-}{2},\tau\right)\left(r_+^{\frac{1}{2}}-r_+^{-\frac{1}{2}}\right)\left(r_-^{\frac{1}{2}}-r_-^{-\frac{1}{2}}\right)(1-q)}{\eta(\tau)^6\vartheta_1(\rho_+,\tau)\vartheta_1(\rho_-,\tau)\prod_{\epsilon_{\pm}=\pm 1}\left(1-r_+^{\frac{1}{2}\epsilon_+}r_-^{\frac{1}{2}\epsilon_-}q^{\frac{1}{2}}\right)}$$

$$\exp\left(\sum_{j=1}^{\infty}\frac{1}{j}I_{0,\text{hol}}(j\tau,j\rho_{\pm})\right)=\chi^{\gamma}_{\text{NS}}(q,r_+,r_-,u)|_{n=m=0}$$

$$\mu_{\text{NS}}(q,r_+,r_-)|_{n=m=0}=\sum_{\epsilon_{\pm}=\pm 1}\frac{\epsilon_+\epsilon_-}{1-r_+^{\frac{1}{2}\epsilon_+}r_-^{\frac{1}{2}\epsilon_-}q^{\frac{1}{2}}}=\frac{\left(r_+^{\frac{1}{2}}-r_+^{-\frac{1}{2}}\right)\left(r_-^{\frac{1}{2}}-r_-^{-\frac{1}{2}}\right)(1-q)}{\prod_{\epsilon_{\pm}=\pm 1}\left(1-r_+^{\frac{1}{2}\epsilon_+}r_-^{\frac{1}{2}\epsilon_-}q^{\frac{1}{2}}\right)}$$



$$f(r_+, r_-) \mapsto \sum_{n, m \in \mathbb{Z}} q^{k_+ n^2 + k_- m^2} r_+^{k_+ n} r_-^{k_- m} f(q^2 r_+, q^{2m} r_-)$$

$$f_{\text{PF}} = -4i \int_{-\infty}^{\infty} \zeta d\zeta \int_0^{\infty} dt_2 \int_{-\frac{1}{2}}^{\frac{1}{2}} dt_1 e^{2i\beta\zeta - 4\pi t_2 \zeta^2} \mathcal{I}_{\text{PF}}$$

$$\mathcal{I}_{\text{PF}} = \left| \frac{\prod_{\pm} \vartheta_1\left(\frac{\tau + \epsilon_+ \rho_+ + \epsilon_- \rho_-}{2}, t\right)}{\eta(t)^6} \times \frac{\eta(t)^3}{\vartheta_1(\tau, t)} \right|^2 \times \left(\sum_{p_L, p_R} 3^{p_L^2} \bar{3}^{p_R^2} \right)$$

$$f_{\text{PF}}(\tau, \rho_+, \rho_-) = I_0 \sum_{N, \bar{N} \in \mathbb{Z}} \sum_{p_L, p_R} \sum_{w=0}^{\infty} \delta_{N+p_L^2-p_R^2, \bar{N}} \tilde{Q}_{N, \bar{N}}^w(q\bar{q})^{w-\frac{1}{2}} \mathfrak{Z}_{k=1}$$

$$\mathfrak{Z}_{k=1} = \int_{-\infty}^{\infty} d\zeta \frac{\zeta e^{2i\beta\zeta}}{\zeta^2 + \Delta_w^2} \left[(q\bar{q})^{\frac{\zeta^2 + \Delta_w^2}{w+1}} - (q\bar{q})^{\frac{\zeta^2 + \Delta_w^2}{w}} \right]$$

$$\Delta_w^2 = p_L^2 + N + \frac{1}{4} - \frac{1}{2} w(w+1)$$

$$\frac{w}{2} < \sqrt{p_L^2 + N + \frac{1}{4} - \frac{1}{2} w(w+1)} < \frac{w+1}{2}$$

$$\frac{(3w-1)(w+1)}{4} < p_L^2 + N < \frac{w(3w+4)}{4}$$

$$f_{\text{PF,cont}} \supset I_0 \sum_{w=1}^{\infty} \sum_{N, p_L, p_R} \delta_{\text{LM}} \int_{-\infty}^{\infty} \frac{ds}{i\pi} (q\bar{q})^{E_w(s)} \left[\frac{\left(s + \frac{iw}{2}\right) \tilde{Q}_N^{w-1}}{\left(s + \frac{iw}{2}\right)^2 + \Delta_{w-1}^2} - \frac{\left(s + \frac{iw}{2}\right) \tilde{Q}_N^w}{\left(s + \frac{iw}{2}\right)^2 + \Delta_w^2} \right]$$

$$E_w(s) = \frac{3}{4}w - 1 + \frac{1}{w} \left(s^2 + \frac{1}{4} + N + p_L^2 \right)$$

$$z_{\text{CFT}}^{\text{NS}}(\tau, \rho_+, \rho_-, \bar{\tau}, \bar{\rho}_+, \bar{\rho}_-) \Big|_{\text{BTZ}} = \text{Tr}_{\text{NS}} \left(q^{L_0} r_+^{J_0^{3,+}} r_-^{J_0^{3,-}} \bar{q}^{\bar{L}_0} \bar{r}_+^{\bar{J}_0^{3,+}} \bar{r}_-^{\bar{J}_0^{3,-}} \right) \Big|_{\text{BTZ}}$$

$$= \text{Tr}_{\text{NS}} \left(q_{\text{AdS}}^{L_0} r_{+, \text{AdS}}^{J_0^{3,+}} r_{-, \text{AdS}}^{J_0^{3,-}} \bar{q}_{\text{AdS}}^{\bar{L}_0} \bar{r}_{+, \text{AdS}}^{\bar{J}_0^{3,+}} \bar{r}_{-, \text{AdS}}^{\bar{J}_0^{3,-}} \right) \Big|_{\text{AdS}}$$

$$z_{\text{CFT}}^{\text{R}}(\tau, \rho_+, \rho_-, \bar{\tau}, \bar{\rho}_+, \bar{\rho}_-) \Big|_{\text{BTZ}} = \text{Tr}_{\text{R}} \left(q^{L_0} r_+^{J_0^{3,+}} r_-^{J_0^{3,-}} \bar{q}^{\bar{L}_0} \bar{r}_+^{\bar{J}_0^{3,+}} \bar{r}_-^{\bar{J}_0^{3,-}} \right) \Big|_{\text{BTZ}}$$

$$= \text{Tr}_{\text{NS}} \left((-1)^{F_S} q_{\text{AdS}}^{L_0} r_{+, \text{AdS}}^{J_0^{3,+}} r_{-, \text{AdS}}^{J_0^{3,-}} \bar{q}_{\text{AdS}}^{\bar{L}_0} \bar{r}_{+, \text{AdS}}^{\bar{J}_0^{3,+}} \bar{r}_{-, \text{AdS}}^{\bar{J}_0^{3,-}} \right) \Big|_{\text{AdS}}$$

$$q_{\text{AdS}} = e^{-\frac{2\pi i}{\tau}}, (r_{\pm})_{\text{AdS}} = e^{-2\pi i \frac{\rho}{\tau}}$$

$$T = \frac{1}{i\pi} \frac{1}{\bar{\tau} - \tau}, \Omega = \pi(\tau + \bar{\tau})$$

$$\tau \rightarrow i\infty, \bar{\tau} \rightarrow -i0, |\tau| < 1.$$

$$\text{Tr}_{\text{NS}} \left(q_{\text{AdS}}^{L_0} r_{+, \text{AdS}}^{J_0^{3,+}} r_{-, \text{AdS}}^{J_0^{3,-}} \bar{q}_{\text{AdS}}^{\bar{L}_0} \bar{r}_{+, \text{AdS}}^{\bar{J}_0^{3,+}} \bar{r}_{-, \text{AdS}}^{\bar{J}_0^{3,-}} \right) \Big|_{\text{AdS}} = e^{S_{\text{tree}}} \times |\chi_{\text{NS}}^\gamma(q, r_+, r_-; 0)|^2 \times \cdots$$



$$z_{\text{CFT}}^{\text{NS}}(\tau, \rho_{\pm}, \bar{\tau}, \bar{\rho}_{\pm})|_{\text{BTZ}} \sim e^{S_{\text{tree}} + \beta + \frac{T}{\gamma T_{\text{gap}}}(\rho_+^2 + \alpha \rho_-^2)} \left(\frac{T}{T_{\text{gap}}} \right)^5 \mathfrak{F}_+(\rho_+) \mathfrak{F}_-(\rho_-),$$

$$\mathfrak{F}_{\pm}(\rho) \equiv \sum_{m \in \mathbb{Z}} \frac{\left(m + \frac{\rho}{2}\right)}{\sin(\pi \rho)} e^{-4 \frac{k_+}{k} \frac{T}{T_{\text{gap}}} \left(m + \frac{\rho}{2}\right)^2}$$

$$\mathfrak{F}_{\pm}(\rho) = \frac{\pi^{\frac{3}{2}}}{4} \left(\frac{k}{k_{\pm}} \frac{T_{\text{gap}}}{T} \right)^{\frac{3}{2}} \sum_{n=1}^{\infty} n \chi_{\frac{n-1}{2}}(r) e^{-\frac{\pi^2}{4} \frac{k}{k_{\pm}} \frac{T_{\text{gap}}}{T} n^2}$$

$$\begin{aligned} \tilde{z}_{\text{CFT}}^{\text{NS}}(T, \rho_+, \rho_-, Q_{\text{U}(1)} = 0)|_{\text{BTZ}} &\sim \frac{\alpha^{\frac{3}{2}}}{(1+\alpha)^3} \left(\frac{T}{T_{\text{gap}}} \right)^{\frac{3}{2}} e^{S_{\text{tree}} + \beta + \frac{\rho_+^2 + \alpha \rho_-^2}{\gamma} \frac{T}{T_{\text{gap}}}} \\ &\times \sum_{m,n=1}^{\infty} m n \chi_{\frac{m-1}{2}}(r_+) \chi_{\frac{n-1}{2}}(r_-) e^{-\beta E_{\text{gap}} \frac{\alpha m^2 + n^2}{1+\alpha}} \end{aligned}$$

$$\tilde{z}_{\text{CFT}}^{\text{NS}}(T, j_+, j_-, Q_{\text{U}(1)} = 0)|_{\text{BTZ}} \sim \left(\frac{T}{T_{\text{gap}}} \right)^{\frac{3}{2}} e^{S_{\text{tree}} + S_1 T - \beta \frac{E_{\text{gap}}}{1+\alpha} (\alpha(2j_+ + 1)^2 + (2j_- + 1)^2 + E_1)}$$

$$\text{Tr}_{\text{NS}} \left((-1)^F q_{\text{AdS}}^{L_0} r_{+, \text{AdS}}^{J_0^{3,+}} r_{-, \text{AdS}}^{J_0^{3,-}} \bar{q}_{\text{AdS}}^{\bar{L}_0} \bar{r}_{+, \text{AdS}}^{\bar{J}_0^{3,+}} \bar{r}_{-, \text{AdS}}^{\bar{J}_0^{3,-}} \right) \Big|_{\text{AdS}} = e^{S_{\text{tree}}} \times \left| \chi_{\text{NS}}^{\gamma}(q, r_+, r_-; 1) \right|^2 \times \dots$$

$$\begin{aligned} z_{\text{CFT}}^{\text{R}}(\tau, \rho_{\pm}, \bar{\tau}, \bar{\rho}_{\pm})|_{\text{BTZ}} &\sim 64 \frac{T}{T_{\text{gap}}} \exp(S_{\text{tree}} + 2y(\rho_+^2 + \alpha \rho_-^2)) \\ &\times \frac{\cos^2\left(\frac{\pi(\rho_+ + \rho_-)}{2}\right) \cos^2\left(\frac{\pi(\rho_+ - \rho_-)}{2}\right)}{\sin(\pi \rho_+) \sin(\pi \rho_-)} \mathfrak{B}_{\alpha}(y, \rho_+, \rho_-) \end{aligned}$$

$$y = \frac{1}{2} \frac{T}{T_{\text{gap}}} \frac{k_+}{k} = \frac{T}{2\gamma T_{\text{gap}}}$$

$$\mathfrak{B}_{\alpha}(y, \rho_+, \rho_-) \equiv \sum_{n_{\pm} \in \mathbb{Z}} \frac{\left(n_+ + \frac{1}{2}\rho_+\right) \left(n_- + \frac{1}{2}\rho_-\right) e^{-8y\left(\left(n_+ + \frac{1}{2}\rho_+\right)^2 + \alpha\left(n_- + \frac{1}{2}\rho_-\right)^2\right)}}{\left[1 - 4\left(n_+ + n_- + \frac{\rho_+ + \rho_-}{2}\right)^2\right] \left[1 - 4\left(n_+ - n_- + \frac{\rho_+ - \rho_-}{2}\right)^2\right]}$$

$$m_1 = n_+ + n_-, m_2 = n_+ - n_-, \rho_1 = \rho_+ + \rho_-, \rho_2 = \rho_+ - \rho_-$$

$$\begin{aligned} \mathfrak{B}_{\alpha}(y, \rho_1, \rho_2) &= \frac{1}{16} \sum_{m_1, m_2 \in \mathbb{Z}} \left[\frac{1}{1 - 4\left(m_1 + \frac{\rho_1}{2}\right)^2} - \frac{1}{1 - 4\left(m_2 + \frac{\rho_2}{2}\right)^2} \right] \\ &\times e^{-2y\left(\left(m_1 + m_2 + \frac{\rho_1 + \rho_2}{2}\right)^2 + \alpha\left(m_1 - m_2 + \frac{\rho_1 - \rho_2}{2}\right)^2\right)} \end{aligned}$$

$$\begin{aligned} z_{\text{CFT}}^{\text{R}}(\tau, \rho_{\pm}, \bar{\tau}, \bar{\rho}_{\pm})|_{\text{BTZ}} &\sim \frac{4\sqrt{\alpha}}{1+\alpha} \left(\frac{T}{T_{\text{gap}}} \right)^{\frac{1}{2}} \exp(S_{\text{tree}} + 2y(\rho_+^2 + \alpha \rho_-^2 - \gamma)) \\ &\times \cos^2\left(\frac{\pi(\rho_+ + \rho_-)}{2}\right) \cos^2\left(\frac{\pi(\rho_+ - \rho_-)}{2}\right) \int_0^{2\gamma y} du \frac{e^u}{\sqrt{u}} \mathfrak{f}_{\alpha}(u) \end{aligned}$$



$$\begin{aligned} \mathfrak{f}_\alpha(u) = & \sum_{m_1, m_2=0}^{\infty} \left(1 - \frac{\delta_{m_1,0}}{2}\right) \left(1 - \frac{\delta_{m_2,0}}{2}\right) \text{sign}(m_1 - m_2) e^{-\frac{\pi^2 m_2^2}{2y(1+\alpha)}} \\ & \times \left[e^{-\frac{\pi^2(m_1+\eta m_2)^2}{4u}} \chi_{\frac{m_1+m_2-1}{2}}(\rho_+) \chi_{\frac{|m_1-m_2|-1}{2}}(\rho_-) + (m_2 \leftrightarrow -m_2) \right]. \end{aligned}$$

$$\eta=1-2\gamma=\frac{1-\alpha}{1+\alpha}$$

$$\begin{aligned} \hat{z}_{\text{CFT}}^{\text{R}}(T,\rho_+,\rho_-,Q_{\text{U}(1)}=0)|_{\text{BTZ}}\sim & \cos^2\left(\frac{\pi(\rho_++\rho_-)}{2}\right)\cos^2\left(\frac{\pi(\rho_+-\rho_-)}{2}\right) \\ & \times\left(\frac{T}{T_{\text{gap}}}\right)^{\frac{3}{2}}e^{S_{\text{tree}}+\frac{T}{\gamma T_{\text{gap}}}(\rho_+^2+\alpha\rho_-^2-\gamma)-\beta E_{\text{gap}}} \end{aligned}$$

$$z_{\text{CFT}}^{\text{R}}(\tau,\rho_+=1+\rho_-)|_{\text{BTZ}}=\text{Tr}_{\text{R}}\left(q^{L_0}e^{i\pi(J_0^{3,+}-J_0^{3,-})}e^{i\pi\rho_1(J_0^{3,+}+J_0^{3,-})}\right)|_{\text{BTZ}}$$

$$\frac{\cos^2\left(\frac{\pi}{2}\rho_1\right)\cos^2\left(\frac{\pi}{2}\rho_2\right)}{\sin\left(\frac{\pi}{2}(\rho_1+\rho_2)\right)\sin\left(\frac{\pi}{2}(\rho_1-\rho_2)\right)}\rightarrow-\frac{\pi^2}{4}(1-\rho_2)^2+\mathcal{O}((\rho_2-1)^3)$$

$$\mathfrak{C}_\alpha(y,\rho_1)=\lim_{\rho_2\rightarrow 1}\cos\left(\frac{\pi}{2}\rho_2\right)\mathfrak{B}_\alpha(y,\rho_1,\rho_2).$$

$$\begin{aligned} \mathfrak{C}_\alpha(y,\rho_1)= & -\frac{\pi}{64}e^{-\frac{2\alpha}{1+\alpha}y}\sum_{m\in\mathbb{Z}}\left[e^{-2y(1+\alpha)\left(m+\frac{1}{2}\rho_1+\frac{1}{2}\eta\right)^2}-e^{-2y(1+\alpha)\left(m+\frac{1}{2}\rho_1-\frac{1}{2}\eta\right)^2}\right] \\ & =-\frac{\pi^{\frac{3}{2}}e^{-2\gamma y}}{64\sqrt{2y(1+\alpha)}}\left[\vartheta_3\left(e^{i\pi(\rho_1+\eta)},e^{-\frac{\pi^2}{(1+\alpha)y}}\right)-\vartheta_3\left(e^{i\pi(\rho_1-\eta)},e^{-\frac{\pi^2}{(1+\alpha)y}}\right)\right] \\ & =\frac{\pi^{\frac{3}{2}}e^{-2\gamma y}}{16\sqrt{2y(1+\alpha)}}\sum_{n=1}^{\infty}\sin(n\pi\rho_1)\sin(n\pi\eta)e^{-\frac{\pi^2}{2y(1+\alpha)}n^2} \end{aligned}$$

$$\begin{aligned} z_{\text{CFT}}^{\text{R}}(T,\rho_+=1+\rho_-)|_{\text{BTZ}}\sim & 2\pi^2\frac{\sqrt{\pi\alpha}}{1+\alpha}(\rho_2-1)\sqrt{\frac{T}{T_{\text{gap}}}}\sin{(\pi\rho_1)}e^{S_{\text{tree}}+S_1'\frac{T}{T_{\text{gap}}}} \\ & \times\sum_{n=1}^{\infty}\chi_{\frac{n-1}{2}}(\rho_1)\sin\left(n\pi\frac{1-\alpha}{1+\alpha}\right)e^{-\frac{\pi^2\alpha}{(1+\alpha)^2}\beta T_{\text{gap}}n^2} \end{aligned}$$

$$S_1'=-1+\frac{(1+\alpha)^2}{4\alpha}(1+\rho_1^2+2\rho_1\eta)$$

$$E_{\text{R}}(\text{j})=\frac{16\alpha}{(1+\alpha)^2}\Big(\text{j}+\frac{1}{2}\Big)^2E_{\text{gap}}$$

$$\begin{aligned} \hat{z}_{\text{CFT}}^{\text{R}}(T,\rho_+=1+\rho_-,Q_{\text{U}(1)=0})|_{\text{BTZ}}\sim & 2\pi^2\frac{\sqrt{\pi\alpha}}{1+\alpha}\sin{(\pi\rho_1)}e^{S_{\text{tree}}+S_1'\frac{T}{T_{\text{gap}}}} \\ & \times\sum_{\text{j}=0}^{\infty}\chi_{\text{j}}(\rho_1)\sin{((2\text{j}+1)\pi\eta)}e^{-\beta E_{\text{R}}(\text{j})} \end{aligned}$$



$$\tilde{z}_{\text{CFT}}^{\text{R}}(T, \rho_+ = \gamma, \rho_- = \gamma - 1, Q_{\text{U}(1)=0})|_{\text{BTZ}} \sim \frac{2\pi^{\frac{5}{2}}\sqrt{\alpha}e^{S_{\text{tree}}}}{(1+\alpha)} \sum_{j=0}^{\infty} \sin^2((2j+1)\pi\eta) e^{-\beta E_{\text{R}}(j)}.$$

$$A_{ab}^{\pm i}=\frac{1}{2}(\pm\delta_{ia}\delta_{b0}\mp\delta_{ib}\delta_{a0}+\epsilon_{iab})$$

$$c=\frac{6\mathbf{k}_+\mathbf{k}_-}{\mathbf{k}_++\mathbf{k}_-}.$$

$$\begin{aligned}\{G_r^a,G_s^b\} &= \frac{c}{3}\delta^{ab}\left(r^2-\frac{1}{4}\right)\delta_{r+s,0}+2\delta^{ab}L_{r+s}\\ &\quad +4i(r-s)(\gamma A_{ab}^{+i}J_{r+s}^{+,i}+(1-\gamma)A_{ab}^{-i}J_{r+s}^{-,i})\\ \{Q_r^a,G_s^b\} &= 2A_{ab}^{+i}J_{r+s}^{+,i}-2A_{ab}^{-i}J_{r+s}^{-,i}+\delta^{ab}U_{r+s}\\ \{Q_r^a,Q_s^b\} &= \frac{\mathbf{k}_++\mathbf{k}_-}{2}\delta^{ab}\delta_{r+s,0}\\ [J_m^{\pm,i},G_r^a] &= iA_{ab}^{\pm i}G_{m+r}^b\mp\frac{2\mathbf{k}_{\pm}}{\mathbf{k}_++\mathbf{k}_-}mA_{ab}^{pmi}Q_{m+r}^b\\ [U_m,G_r^a] &= mQ_{m+r}^a\\ [J_m^{\pm,i},Q_r^a] &= iA_{ab}^{\pm i}Q_{m+r}^b\\ [J_m^{\pm,i},J_n^{\pm,j}] &= i\epsilon^{ijl}J_{m+n}^{\pm,l}+\frac{\mathbf{k}_{\pm}}{2}m\delta^{ij}\delta_{m+n,0}\\ [U_m,U_n] &= \frac{\mathbf{k}_++\mathbf{k}_-}{2}m\delta_{m+n,0}\end{aligned}$$

$$\mathcal{A}_\gamma = \tilde{\mathcal{A}}_\gamma \oplus \mathcal{A}_{\text{QU}}$$

$$\tilde{\mathbf{k}}_{\pm}=\mathbf{k}_{\pm}-1$$

$$\tilde{c}=c-3=3\frac{2\tilde{\mathbf{k}}_+\tilde{\mathbf{k}}_-+\tilde{\mathbf{k}}_++\tilde{\mathbf{k}}_-}{\tilde{\mathbf{k}}_++\tilde{\mathbf{k}}_-+2}$$

$$\gamma\rightarrow\frac{\mathbf{k}_--1}{\mathbf{k}_++\mathbf{k}_-}, 1-\gamma\rightarrow\frac{\mathbf{k}_+-1}{\mathbf{k}_++\mathbf{k}_-}.$$

$$\tilde{c}'=6\frac{(\mathbf{k}_+-1)(\mathbf{k}_--1)}{\mathbf{k}_++\mathbf{k}_-}$$

$$h\geq \frac{1}{\tilde{\mathbf{k}}_++\tilde{\mathbf{k}}_-+2}\big[(\tilde{\mathbf{k}}_++1)\ell_-+(\tilde{\mathbf{k}}_-+1)\ell_++(\ell_+-\ell_-)^2\big]$$

$$\frac{\Pi_{\pm=\pm}\vartheta_1\left(\frac{\tau+\epsilon_+\rho_++\epsilon_-\rho_-}{2},\mathfrak{t}\right)}{\eta(\mathfrak{t})^6}=\mathfrak{z}^{\frac{1}{4}}q^{-1}\Big[1-q+\chi^{\mathfrak{g}}_{\mathfrak{S}}\Big(\frac{1}{2},\frac{1}{2},\frac{1}{2}\Big)\Big]\sum_{h_{\text{osc}}\geq 0}D_{h_{\text{osc}}}(q,r_+,r_-)\mathfrak{z}^{h_{\text{osc}}}$$

$$\frac{\Pi_{\epsilon_{\pm}=\pm}\vartheta_1\left(\frac{\tau+\epsilon_+\rho_++\epsilon_-\rho_-}{2},\mathfrak{t}\right)}{\eta(\mathfrak{t})^4\mathfrak{z}^{\frac{1}{4}}q^{-1}}=\Big[1-q+\chi^{\mathfrak{g}}_{\mathfrak{S}}\Big(\frac{1}{2},\frac{1}{2},\frac{1}{2}\Big)\Big]\Big(\prod_{n=1}^{\infty}\sum_{m=0}^8A_m(q,r_{\pm})\mathfrak{z}^{mn}\Big)$$



$$A_0 = A_8 = 1$$

$$A_1 = A_7 = -q^{-\frac{1}{2}}(1+q)\chi_{\frac{1}{2}}(r_+)\chi_{\frac{1}{2}}(r_-)$$

$$A_2 = A_6 = q^{-1}[(1+q^2)(\chi_1(r_+) + \chi_1(r_-)) + q(1+\chi_1(r_+))(1+\chi_1(r_-))]$$

$$A_3 = A_5 = -q^{-\frac{3}{2}}\chi_{\frac{1}{2}}(r_+)\chi_{\frac{1}{2}}(r_-)[1+q^3+(q+q^2)(\chi_1(r_+) + \chi_1(r_-))]$$

$$A_4 = q^{-2}\left[1+q^4+(q+q^3)(1+\chi_1(r_+))(1+\chi_1(r_-))+q^2(\chi_1(r_+) + \chi_1(r_-))^2\right]$$

$$\frac{\eta(\mathfrak{t})^3}{\vartheta_1(\tau,\mathfrak{t})}=\sum_{n\in\mathbb{Z}}(-1)^{n+1}\left[\sum_{r\geq n+w+1}(-1)^r\mathfrak{z}^{\frac{r(r+1)}{2}}\right]\mathfrak{z}^{-\frac{n(n+1)}{2}}q^{n+\frac{1}{2}}$$

$$\mathcal{I}_{\text{PF}}=|1-q|^2I_0\left|\sum_{\tilde{}}(-1)^{n+r+1}D_{h_{\text{osc}}}\mathfrak{z}^{\frac{1}{4}+\frac{r(r+1)}{2}-\frac{n(n+1)}{2}+h_{\text{osc}}+p_L^2}q^{n-\frac{1}{2}}\right|^2$$

$$f_{\text{PF}}=|1-q|^2I_0\sum_{w=0}^{\infty}\sum_{\wedge}^{\wedge}q^{n-\frac{1}{2}}\bar{q}^{\bar{n}-\frac{1}{2}}D_{h_{\text{osc}}}\bar{D}_{h_{\text{osc}}}\mathfrak{I}_{k=1}$$

$$\mathfrak{I}_{k=1}\equiv\int_{-\infty}^{\infty}\frac{\zeta d\zeta}{i\pi}\frac{e^{2i\beta\zeta}}{\zeta^2+\Delta_w^2}\Big[e^{-\frac{2\beta}{w+1}(\zeta^2+\Delta_w^2)}-e^{-\frac{2\beta}{w}(\zeta^2+\Delta_w^2)}\Big]$$

$$\Delta_w^2=\frac{1}{4}+\frac{r(r+1)}{2}-\frac{n(n+1)}{2}+h_{\text{osc}}+p_L^2$$

$$\sum_{\wedge}^{\wedge}=\sum_{n,\bar{n}\in\mathbb{Z}}\sum_{r\geq n+w}\sum_{\bar{r}\geq \bar{n}+w}(-1)^{n+r+\bar{n}+\bar{r}}\sum_{h_{\text{osc}},\bar{h}_{\text{osc}}}\sum_{p_L,p_R}\delta_{\text{LM}},$$

$$\delta_{\text{LM}}\colon \frac{r(r+1)}{2}-\frac{n(n+1)}{2}+h_{\text{osc}}+p_L^2=\frac{\bar{r}(\bar{r}+1)}{2}-\frac{\bar{n}(\bar{n}+1)}{2}+\bar{h}_{\text{osc}}+p_R^2,$$

$$\frac{r(r+1)}{2}-\frac{n(n+1)}{2}+h_{\text{osc}}\rightarrow N-\frac{w(w+1)}{2}$$

$$\mathfrak{B}_1(y,\rho_1,\rho_2)=\mathfrak{b}(y,\rho_2)\hat{\mathfrak{b}}(y,\rho_1)-\mathfrak{b}(y,\rho_1)\hat{\mathfrak{b}}(y,\rho_2),$$

$$\mathfrak{b}(y,\rho)\equiv\sum_{m\in\mathbb{Z}}e^{-4y\left(m+\frac{\rho}{2}\right)^2}$$

$$\hat{\mathfrak{b}}(y,\rho)\equiv\sum_{m\in\mathbb{Z}}\frac{e^{-4y\left(m+\frac{\rho}{2}\right)^2}}{1-4\left(m+\frac{\rho}{2}\right)^2}$$

$$\mathfrak{b}(y,\rho)=\sqrt{\frac{\pi}{4y}}\sum_{n\in\mathbb{Z}}e^{i\pi n\rho}$$

$$e^{-y}\frac{\mathrm{d}}{\mathrm{d}y}(e^y\hat{\mathfrak{b}}(y,\rho))=\mathfrak{b}(y,\rho)$$



$$\mathfrak{B}_1(y, \rho_+, \rho_-) \sim \frac{\pi}{8} \sin(\pi \rho_+) \sin(\pi \rho_-) e^{-\frac{\pi^2}{4y}} + \dots$$

$$\frac{1}{x} = \int_C dt e^{-xt}$$

$$\mathfrak{B}_\alpha(y, \rho_1, \rho_2) = \mathfrak{F}_\alpha(y, \rho_1, \rho_2) - \mathfrak{F}_\alpha(y, \rho_2, \rho_1),$$

$$\mathfrak{F}_\alpha(y, \rho_1, \rho_2) = e^{-2y(\alpha\rho_-^2 + \rho_+^2)} \int_C dt e^{-t(1-\rho_1^2)} \sum_{m_1, m_2 \in \mathbb{Z}} e^{-\mathbf{m}^T \cdot \mathbf{A} \cdot \mathbf{m} + 2\pi i \mathbf{z}^T \cdot \mathbf{m}}$$

$$\mathbf{m} = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}, \mathbf{z} = \frac{i(1+\alpha)}{\pi} y \begin{pmatrix} \left(1 - \frac{2t}{y(1+\alpha)}\right) \rho_1 + \eta \rho_2 \\ \rho_2 + \eta \rho_1 \end{pmatrix}$$

$$\mathbf{A} = 2y(1+\alpha) \begin{pmatrix} 1 - \frac{2t}{y(1+\alpha)} & \eta \\ \eta & 1 \end{pmatrix}$$

$$\mathfrak{F}_\alpha(y, \rho_1, \rho_2) = \frac{e^{-2\gamma y}}{64\sqrt{2}\sqrt{y(1+\alpha)}} \int_{C_u} du \frac{e^u}{\sqrt{u}} \tilde{\mathfrak{f}}_\alpha, u = 2\gamma y - t$$

$$\tilde{\mathfrak{f}}_\alpha = \sum_{m_1, m_2 \in \mathbb{Z}} e^{-\frac{\pi^2}{4u}(m_1 + \eta m_2)^2 - \frac{\pi^2}{2y(1+\alpha)} m_2^2} [e^{i\pi(m_1 \rho_1 + m_2 \rho_2)} - e^{i\pi(m_2 \rho_1 + m_1 \rho_2)}]$$

$$\tilde{\mathfrak{f}}_\alpha = -4 \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \left(1 - \frac{\delta_{m_1,0}}{2}\right) \left(1 - \frac{\delta_{m_2,0}}{2}\right) e^{-\frac{\pi^2 m_2^2}{2y(1+\alpha)}} \tilde{F}_\alpha(m_1, m_2)$$

$$\tilde{F}_\alpha(m_1, m_2) = \left[e^{-\frac{\pi^2(m_1 + \eta m_2)^2}{4u}} \sin(\pi(m_1 + m_2)\rho_+) \sin(\pi(m_1 - m_2)\rho_-) \right. \\ \left. + e^{-\frac{\pi^2(m_1 - \eta m_2)^2}{4u}} \sin(\pi(m_1 - m_2)\rho_+) \sin(\pi(m_1 + m_2)\rho_-) \right]$$

$$\mathfrak{B}_\alpha = -\frac{1}{16\sqrt{2}} \frac{e^{-2\gamma y}}{\sqrt{y(1+\alpha)}} \sin(\pi \rho_+) \sin(\pi \rho_-) \int_0^{2\gamma y} du \frac{e^u}{\sqrt{u}} \tilde{\mathfrak{f}}_\alpha$$

$$\tilde{\mathfrak{f}}_\alpha = \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \left(1 - \frac{\delta_{m_1,0}}{2}\right) \left(1 - \frac{\delta_{m_2,0}}{2}\right) e^{-\frac{\pi^2 m_2^2}{2y(1+\alpha)}} F_\alpha(m_1, m_2)$$

$$F_\alpha(m_1, m_2) = \text{sign}(m_1 - m_2) \left[e^{-\frac{\pi^2(m_1 + \eta m_2)^2}{4u}} \chi_{\frac{m_1 + m_2 - 1}{2}}(\rho_+) \chi_{\frac{|m_1 - m_2| - 1}{2}}(\rho_-) \right. \\ \left. + e^{-\frac{\pi^2(m_1 - \eta m_2)^2}{4u}} \chi_{\frac{|m_1 - m_2| - 1}{2}}(\rho_+) \chi_{\frac{m_1 + m_2 - 1}{2}}(\rho_-) \right]$$

En tanto que, la métrica Super de Yang – Mills, en un campo cuántico relativista, en supergravedad, se calcula así:



$$\mathcal{I}_S(\lambda; \alpha) = 4 \sum_{\ell=1}^{\infty} (-1)^{\ell+1} \zeta(2\ell+1) \left(\frac{\lambda}{16\pi^2} \right)^{\ell} \left[\binom{2\ell+1}{\ell}^2 - \alpha^{2(\ell+1)} \binom{2\ell+1}{\ell} {}_2F_1(\ell+1, \ell+2; 1; 1-\alpha) \right],$$

$$\mathcal{I}_{AdS}(\lambda; \beta) = 4 \sum_{\ell=1}^{\infty} (-1)^{\ell} \zeta(2\ell+1) \left(\frac{\lambda}{16\pi^2} \right)^{\ell} \left[\binom{2\ell+1}{\ell}^2 - \beta^{2(\ell+1)} \binom{2\ell+1}{\ell} {}_2F_1(\ell+1, \ell+2; 1; 1+\beta) \right],$$

$$\mathcal{I}_S(\lambda; \alpha) \sim 2\alpha - \frac{4\pi^2}{3\lambda} - \sum_{n=1}^{\infty} \frac{16n\zeta(2n+1)\Gamma\left(n-\frac{1}{2}\right)^2 \Gamma\left(n+\frac{1}{2}\right)}{\lambda^{n+\frac{1}{2}}\pi^{3/2}\Gamma(n)}$$

$$\mathcal{I}_{AdS}(\lambda; \beta) \sim 2\log(1+\beta) - \frac{4\pi^2}{3\lambda} + \sum_{n=1}^{\infty} \frac{16n\zeta(2n+1)\Gamma\left(n-\frac{1}{2}\right)^2 \Gamma\left(n+\frac{1}{2}\right)}{\lambda^{n+\frac{1}{2}}\pi^{3/2}\Gamma(n)}$$

$$\exp\left[-2n\sqrt{\lambda}\right], \exp\left[-n\sqrt{\lambda}(1-\sqrt{1-\alpha})\right], \exp\left[-n\sqrt{\lambda}(1+\sqrt{1-\alpha})\right],$$

$$\exp\left[-2n\sqrt{\lambda}\right], \exp\left[-n\sqrt{\lambda}(\sqrt{\beta+1}-1)\right], \exp\left[-n\sqrt{\lambda}(\sqrt{\beta+1}+1)\right]$$

$$\mathcal{O}_{\vec{p}}(x,Y)=\mathcal{O}_{(p_1\dots p_m)}(x,Y)\!:=\!\frac{p_1\dots p_m}{p}\mathcal{O}_{p_1}(x,Y)\dots\mathcal{O}_{p_m}(x,Y)$$

$$\mathcal{O}_q(x,Y)\!:=\!\frac{1}{q}\mathrm{Tr}(Y\cdot\Phi(x))^q$$

$$\langle \mathcal{O}_{\vec{p}}(x_1,Y_1)\mathcal{O}_{\vec{p}}(x_2,Y_2)\rangle = \mathfrak{R}_{\vec{p}}(N)(d_{12})^p, \; d_{12}\!:=\!\frac{Y_1\cdot Y_2}{x_{12}^2},$$

$$\frac{\langle \mathcal{O}_{\vec{m}}(x_1,Y_1)\mathcal{O}_{\vec{m}}(x_2,Y_2)\mathcal{O}_{\vec{p}}(x_3,Y_3)\rangle}{\langle \mathcal{O}_{\vec{m}}(x_1,Y_1)\mathcal{O}_{\vec{m}}(x_2,Y_2)\rangle}=\mathfrak{C}_{\vec{m}\vec{m}\vec{p}}(N)\times\left(\frac{d_{23}d_{31}}{d_{12}}\right)^{\frac{p}{2}}$$

$$\mathcal{O}_R^{\mathrm{Schur}}(x,Y)\!:=\!\frac{1}{m!}\sum_{\sigma\in S_m}\chi_R(\sigma)\mathrm{Tr}(\sigma\varphi)(x,Y),\text{ with }\varphi(x,Y)\!:=\!Y\cdot\Phi(x)$$

$$\mathrm{Tr}(\sigma\varphi)\!:=\!\sum_{i_1,\dots,i_m=1}^N\varphi_{i_{\sigma(1)}}^{i_1}\dots\varphi_{i_{\sigma(m)}}^{i_m}$$

$$\mathcal{O}_{(\overline{1})_m}^{\mathrm{Schur}}(x,Y)=\det_m\varphi(x,Y)$$

$$\det_m X\!:=\!\frac{1}{m!}\epsilon_{i_1\dots i_m\ell_1\dots\ell_{N-m}}\epsilon^{j_1\dots j_m\ell_1\dots\ell_{N-m}}X_{j_1}^{i_1}\dots X_{j_m}^{i_m}$$

$$ds^2=ds_{AdS_5}^2+ds_{S^5}^2$$

$$ds_{AdS_5}^2=-\cosh^2\rho dt^2+d\rho^2+\sinh^2\rho d\tilde{\Omega}_3^2$$

$$ds_{S^5}^2=d\theta^2+\sin^2\theta d\phi^2+\cos^2\theta d\Omega_3^2$$

$$\cos^2\theta_0=\frac{m}{N}=\alpha$$



$$\sinh^2 \rho_0 = \beta$$

$$\mathcal{G}_S(x,Y;z) \coloneqq \sum_{m=0}^N z^m \mathcal{O}_{(\overline{1})_m}^{\text{Schur}}(x,Y) = \det(\mathbb{1} + zY \cdot \Phi)(x), z \in \mathbb{C}$$

$$\mathcal{O}_{(\overline{1})_m}^{\text{Schur}}(x,Y) = \oint \frac{dz}{2\pi i} \frac{1}{z^{m+1}} \mathcal{G}_S(x,Y;z)$$

$$\mathcal{G}_{S;\alpha}(x,Y)\coloneqq \oint \frac{dz}{2\pi i} \frac{1}{z^{\alpha N+1}} \mathcal{G}_S(x,Y;z)$$

$$\mathcal{G}_{AdS}(x,Y;z)\coloneqq \sum_{m=0}^{\infty} z^m \mathcal{O}_{(m,\overline{0})}^{\text{Schur}}(x,Y) = [\det(\mathbb{1} - zY \cdot \Phi)]^{-1}(x), \; z \in \mathbb{C}$$

$$\mathcal{O}_{(m,\overline{0})}^{\text{Schur}}(x,Y) = \oint \frac{dz}{2\pi i} \frac{1}{z^{m+1}} \mathcal{G}_{AdS}(x,Y;z)$$

$$\mathcal{G}_{AdS;\beta}(x,Y)\coloneqq \oint \frac{dz}{2\pi i} \frac{1}{z^{\beta N+1}} \mathcal{G}_{AdS}(x,Y;z)$$

$$\frac{\langle \mathcal{H}(x_1,Y_1)\mathcal{H}(x_2,Y_2)\mathcal{O}_{\vec{p}}(x_3,Y_3)\rangle}{\langle \mathcal{H}(x_1,Y_1)\mathcal{H}(x_2,Y_2)\rangle}=\mathfrak{C}_{\mathcal{H},\mathcal{H},\vec{p}}(N)\times \left(\frac{d_{23}d_{31}}{d_{12}}\right)^{\frac{p}{2}}.$$

$$\mathfrak{C}_{\mathcal{H},\mathcal{H},\vec{p}}(N)=\sum_{g=0}^{\infty} N^{\frac{p}{2}-g}\mathfrak{C}_{\mathcal{H},\mathcal{H},\vec{p}}^{(g)}$$

$$\begin{aligned}\mathfrak{C}_{\mathcal{G}_{S;\alpha}}^{(0)}\mathcal{G}_{S;\alpha}(2p)&= -2^{-p}\oint_{z=0}\frac{dz}{2\pi i}\frac{1-z^2}{z^{2p+1}\sqrt{1+2z^2(2\alpha-1)+z^4}}\\&= (-2)^{-p+1}{}_2F_1(-p,p;1;1-\alpha)\end{aligned}$$

$$\begin{aligned}\mathfrak{C}_{\mathcal{G}_{AdS;\beta},\mathcal{G}_{AdS;\beta}}^{(0)}(2p)&= 2^{-p}\oint_{z=0}\frac{dz}{2\pi i}\frac{1-z^2}{z^{2p+1}\sqrt{1-2z^2(2\beta+1)+z^4}}\\&= -(-2)^{-p+1}{}_2F_1(-p,p;1;1+\beta)\end{aligned}$$

$$\langle \mathcal{O}_2(x_1,Y_1)\mathcal{O}_2(x_2,Y_2)\mathcal{H}(x_3,Y_3)\mathcal{H}(x_4,Y_4)\rangle = \mathcal{C}_{\text{free}}\left(x_i,Y_i\right) + \mathcal{R}_4(x_i,Y_i)d_{34}^{\Delta_{\mathcal{H}}-2}\mathcal{C}_{\mathcal{H}}(u,v;\tau)$$

$$\tau\coloneqq\tau_1+\mathrm{i}\tau_2=\frac{\theta}{2\pi}+\mathrm{i}\frac{4\pi}{g_{\mathrm{YM}}^2},$$

$$u\coloneqq\frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2},v\coloneqq\frac{x_{14}^2x_{23}^2}{x_{13}^2x_{24}^2}$$

$$N\rightarrow\infty, \text{ with } \lambda=g_{\mathrm{YM}}^2N \text{ fixed}$$

$$\mathcal{I}_{\mathcal{H}}(\tau;N)\coloneqq -\frac{2}{\pi}\int_0^\infty dr\int_0^\pi d\theta \frac{r^3\sin^2\theta}{u^2}\mathcal{C}_{\mathcal{H}}(u,v;\tau)$$



$$\mathcal{Z}(\tau,\tau_p;m)=\int_{\mathbb{R}^N}d\mu(a_i)\left|\exp\left[\mathrm{i}\pi\tau\left(\sum_{i=1}^Na_i^2\right)+\mathrm{i}\sum_{p=3}^{\infty}\pi^{\frac{p}{2}}\tau_p\left(\sum_{i=1}^Na_i^p\right)\right]\right|^2Z_{1\text{-loop}}(a;m)|Z_{\text{inst}}(\tau,\tau_p,a;m)|^2$$

$$d\mu(a_i)=\prod_{i=1}^Nda_i\prod_{i<j}a_{ij}^2\delta\left(\sum_{i=1}^Na_i\right),$$

$$Z_{1\text{-loop}}(a;m)=\frac{1}{H(m)^N}\prod_{i<j}\frac{H^2(a_{ij})}{H(a_{ij}+m)H(a_{ij}-m)}, H(x)\!:=\!e^{-(1+\gamma)x^2}G(1+ix)G(1-ix)$$

$$a\rightarrow \frac{a}{\sqrt{2\pi\tau_2}}$$

$$\left\langle f(a_i)\right\rangle\!:=\!\frac{1}{\mathcal{N}}\int_{\mathbb{R}^N}d\mu(a_i)\exp\left(-\sum_i a_i^2\right)f(a_i)$$

$$\partial_{\tau_{p_1}}\ldots\partial_{\tau_{p_\ell}}\mathcal{Z}(\tau,\tau_p;m)\leftrightarrow T_{\vec{p}}(a)\!:=\!\mathrm{tr}(a^{p_1})\mathrm{tr}(a^{p_2})\cdots\mathrm{tr}(a^{p_\ell})$$

$$\mathcal{O}_{\vec{p}}(a)\!:=\!T_{\vec{p}}(a)+\sum_{\vec{q}\vdash q<p}u_{\vec{p},\vec{q}}(N)T_{\vec{q}}(a),$$

$$\left\langle \langle \mathcal{O}_{\vec{p}}(a) T_{\vec{r}}(a) \rangle \right\rangle = 0, \forall 0 \leq r < p \text{ and } \forall \vec{r} \text{ partitions of } r,$$

$$\left\langle \langle \mathcal{O}_{\vec{p}}(a) \mathcal{O}_{\vec{p}}(a) \rangle \right\rangle = \Re_{\vec{p}}(N), \frac{\left\langle \langle \mathcal{O}_{\vec{m}}(a) \mathcal{O}_{\vec{m}}(a) \mathcal{O}_{\vec{p}}(a) \rangle \right\rangle}{\left\langle \langle \mathcal{O}_{\vec{m}}(a) \mathcal{O}_{\vec{m}}(a) \rangle \right\rangle} = \mathfrak{C}_{\vec{m}\vec{m}\vec{p}}(N)$$

$$T_{\vec{q}}(a)=\mathcal{O}_{\vec{q}}(a)+\sum_{\vec{p}\vdash p<q}v_{\vec{q},\vec{p}}(N)\mathcal{O}_{\vec{p}}(a)$$

$$v_{\vec{q},\vec{p}}(N)=(-1)^{\frac{q-p}{2}}u_{\vec{q},\vec{p}}(N)$$

$$\mathcal{I}_{\mathcal{H}}(\tau;N)=\frac{\partial_{\mathcal{H}}\partial_{\mathcal{H}}\partial_m^2\log\mathcal{Z}(\tau,\tau_p;m)|_{\tau_p,m=0}}{\partial_{\mathcal{H}}\partial_{\mathcal{H}}\log\mathcal{Z}(\tau,\tau_p;m)|_{\tau_p,m=0}}$$

$$\mathcal{Z}(g_{\mathrm{YM}}^2;m)=\int\,d a e^{-\mathrm{tr} a^2} Z_{1-\mathrm{loop}}\left(\frac{a g_{\mathrm{YM}}}{2\sqrt{2}\pi};m\right)$$

$$\partial_m^2\log\mathcal{Z}(g_{\mathrm{YM}}^2;m)|_{m=0}=\left\langle\left\langle\mathbf{M}_2(a,g_{\mathrm{YM}}^2)\right\rangle\right\rangle=\frac{1}{\mathcal{N}}\int\,d a e^{-\mathrm{tr} a^2}\mathbf{M}_2(a,g_{\mathrm{YM}}^2)$$

$$\mathbf{M}_2(a,g_{\mathrm{YM}}^2)=-2\sum_{\ell=1}^{\infty}\sum_{j=0}^{2\ell}(-1)^{j+\ell}(2\ell+1)\binom{2\ell}{j}\zeta(2\ell+1)\left(\frac{g_{\mathrm{YM}}^2}{8\pi^2}\right)^{\ell}T_{(2\ell-j,j)}(a)$$



$$\mathbf{M}_2(a, g_{\text{YM}}^2) = -2 \sum_{\ell=1}^{\infty} \sum_{j=0}^{2\ell} (-1)^{j+\ell} (2\ell+1) \binom{2\ell}{j} \zeta(2\ell+1) \left(\frac{g_{\text{YM}}^2}{8\pi^2} \right)^\ell \left[\mathcal{O}_{(2\ell-j,j)}(a) + \sum_{\vec{p} \vdash p < 2\ell} v_{(2\ell-j,j),\vec{p}} \mathcal{O}_{\vec{p}}(a) \right]$$

$$\mathcal{I}_{\mathcal{H}}(g_{\text{YM}}^2; N) = \frac{\langle \langle \mathcal{H}(a) \mathcal{H}(a) \mathbf{M}_2(a, g_{\text{YM}}^2) \rangle \rangle}{\langle \langle \mathcal{H}(a) \mathcal{H}(a) \rangle \rangle} - \langle \langle \mathbf{M}_2(a, g_{\text{YM}}^2) \rangle \rangle.$$

$$\mathcal{I}_{\mathcal{H}}(g_{\text{YM}}^2; N) = -2 \sum_{\ell=1}^{\infty} \sum_{j=0}^{2\ell} (-1)^{j+\ell} (2\ell+1) \binom{2\ell}{j} \zeta(2\ell+1) \left(\frac{g_{\text{YM}}^2}{8\pi^2} \right)^\ell \times \left(\mathfrak{C}_{\mathcal{H}, \mathcal{H}, (2\ell-j, j)} + \sum_{\vec{p} \vdash p < 2\ell} v_{(2\ell-j, j), \vec{p}} \mathfrak{C}_{\mathcal{H}, \mathcal{H}, \vec{p}} \right)$$

$$\mathcal{I}_{\mathcal{H}}(\lambda; N) = \sum_{g=0}^{\infty} N^{1-g} \mathcal{I}_{\mathcal{H}}^{(g)}(\lambda).$$

$$\mathcal{I}_{\mathcal{H}}(\lambda; N) = -2N \sum_{\ell=1}^{\infty} (-1)^\ell (2\ell+1) \zeta(2\ell+1) \left(\frac{\lambda}{8\pi^2} \right)^\ell \times \left[2\mathfrak{C}_{\mathcal{H}, \mathcal{H}, (2\ell)}^{(0)} + \sum_{j=0}^{\ell} \binom{2\ell}{2j} \sum_{p=1}^{\ell-1} v_{(2\ell-2j, 2j), (2p)}^{(0)} \mathfrak{C}_{\mathcal{H}, \mathcal{H}, (2p)}^{(0)} \right] + O(N^0)$$

$$v_{(2\ell-2j, 2j), (2p)}^{(0)} = 2^{p-\ell} \left[\binom{2\ell-2j}{\ell-j+p} c_j + \binom{2j}{j+p} c_{\ell-j} \right],$$

$$\mathcal{I}_{\mathcal{H}}^{(0)}(\lambda) = -4 \sum_{\ell=1}^{\infty} (-1)^\ell \zeta(2\ell+1) \left(\frac{\lambda}{16\pi^2} \right)^\ell \sum_{p=1}^{\ell} 2^p \binom{2\ell+1}{\ell+p+1} \binom{2\ell+1}{\ell+p} \mathfrak{C}_{\mathcal{H}, \mathcal{H}, (2p)}^{(0)}$$

$$\mathcal{I}_S(\lambda; \alpha) := \mathcal{I}_{g_S; \alpha}^{(0)}(\lambda), \quad \mathcal{I}_{AdS}(\lambda; \beta) := \mathcal{I}_{g_{AdS}; \beta}^{(0)}(\lambda)$$

$$\mathcal{I}_S(\lambda; \alpha) = 8 \sum_{\ell=1}^{\infty} (-1)^\ell \zeta(2\ell+1) \left(\frac{\lambda}{16\pi^2} \right)^\ell \sum_{p=1}^{\ell} (-1)^p \binom{2\ell+1}{\ell+p+1} \binom{2\ell+1}{\ell+p} {}_2F_1(-p, p; 1; 1-\alpha)$$

$$\mathcal{I}_S(\lambda; \alpha) = \mathcal{I}_{S;0}(\lambda) + \mathcal{I}_{S;1}(\lambda; \alpha)$$

$$\mathcal{I}_{S;0}(\lambda) := 4 \sum_{\ell=1}^{\infty} (-1)^{\ell+1} \zeta(2\ell+1) \left(\frac{\lambda}{16\pi^2} \right)^\ell \binom{2\ell+1}{\ell}^2,$$

$$\mathcal{I}_{S;1}(\lambda; \alpha) := 4 \sum_{\ell=1}^{\infty} (-1)^\ell \zeta(2\ell+1) \left(\frac{\lambda}{16\pi^2} \right)^\ell \alpha^{2(\ell+1)} \binom{2\ell+1}{\ell} {}_2F_1(\ell+1, \ell+2; 1; 1-\alpha).$$

$$\mathcal{I}_S(\lambda; 1) = 4 \sum_{\ell=1}^{\infty} (-1)^{\ell+1} \zeta(2\ell+1) \left(\frac{\lambda}{16\pi^2} \right)^\ell \left[\binom{2\ell+1}{\ell}^2 - \binom{2\ell+1}{\ell} \right].$$

$$\mathcal{I}_{AdS}(\lambda; \beta) = \mathcal{I}_{AdS;0}(\lambda) + \mathcal{I}_{AdS;1}(\lambda; \beta)$$



$$\begin{aligned}
\mathcal{I}_{AdS;0}(\lambda) &:= 4 \sum_{\ell=1}^{\infty} (-1)^{\ell} \zeta(2\ell+1) \left(\frac{\lambda}{16\pi^2} \right)^{\ell} \binom{2\ell+1}{\ell}^2 = -\mathcal{I}_S^{(0)}(\lambda) \\
\mathcal{I}_{AdS;1}(\lambda; \beta) &:= 4 \sum_{\ell=1}^{\infty} (-1)^{\ell+1} \zeta(2\ell+1) \left(\frac{\lambda}{16\pi^2} \right)^{\ell} (1+\beta)^{\ell} \binom{2\ell+1}{\ell} (\ell+1) {}_2F_1\left(-\ell, -\ell; 2; \frac{1}{1+\beta}\right) \\
(1+\beta)^{\ell} (\ell+1) {}_2F_1\left(-\ell, -\ell; 2; \frac{1}{1+\beta}\right) &= \beta^{2(\ell+1)} {}_2F_1(\ell+1, \ell+2; 1; 1+\beta), \forall \ell \in \mathbb{N}, \\
\mathcal{I}_{AdS;1}(\lambda; \beta) &= 4 \sum_{\ell=1}^{\infty} (-1)^{\ell+1} \zeta(2\ell+1) \left(\frac{\lambda}{16\pi^2} \right)^{\ell} \beta^{2(\ell+1)} \binom{2\ell+1}{\ell} {}_2F_1(\ell+1, \ell+2; 1; 1+\beta) \\
\alpha^{2(\ell+1)} {}_2F_1(\ell+1, \ell+2; 1; 1-\alpha) &\sim \frac{(1+\sqrt{1-\alpha})^{2(\ell+1)}}{2\sqrt{\pi}\ell^4\sqrt{1-\alpha}}, \ell \gg 1 \\
|\lambda| < R_S(\alpha) &:= \pi^2 \frac{4}{(1+\sqrt{1-\alpha})^2} \\
|\lambda| < R_{AdS}(\beta) &:= \pi^2 \frac{4}{(1+\sqrt{1+\beta})^2}, \\
\mathcal{I}_{S;0}(\lambda) &= - \int_{\text{Res}=\frac{1}{2}} \frac{ds}{2\pi i} \frac{4\pi\zeta(2s+1)}{\sin(\pi s)} \binom{2s+1}{s}^2 \left(\frac{\lambda}{16\pi^2} \right)^s \\
\mathcal{I}_{S;1}(\lambda; \alpha) &= \int_{\text{Res}=\frac{1}{2}} \frac{ds}{2\pi i} \frac{4\pi\zeta(2s+1)}{\sin(\pi s)} \alpha^{2(s+1)} \binom{2s+1}{s} {}_2F_1(s+1, s+2; 1; 1-\alpha) \left(\frac{\lambda}{16\pi^2} \right)^s \\
\mathcal{I}_{AdS;0}(\lambda) &= -\mathcal{I}_{S;0}(\lambda) \\
\mathcal{I}_{AdS;1}(\lambda; \beta) &= - \int_{\text{Res}=\frac{1}{2}} \frac{ds}{2\pi i} \frac{4\pi\zeta(2s+1)}{\sin(\pi s)} (1+\beta)^s \binom{2s+1}{s} (s+1) {}_2F_1\left(-s, -s; 2; \frac{1}{1+\beta}\right) \left(\frac{\lambda}{16\pi^2} \right)^s \\
{}_2F_1(\ell+1, \ell+2; 1; 1+\beta) \ell \in \mathbb{N} &\rightarrow {}_2F_1(s+1, s+2; 1; 1+\beta) s \in \mathbb{C}, \\
\mathcal{I}_S(\lambda; \alpha) &= -\mathcal{I}_{AdS}(\lambda; -\alpha), \alpha < 0 \\
\lim_{\alpha \rightarrow 0} [\alpha^{2(s+1)} {}_2F_1(s+1, s+2; 1; 1-\alpha)] &= \lim_{\beta \rightarrow 0} \left[(1+\beta)^s (s+1) {}_2F_1\left(-s, -s; 2; \frac{1}{1+\beta}\right) \right] = \binom{2s+1}{s} \\
\lim_{\alpha \rightarrow 0} [\mathcal{I}_{S;1}(\lambda; \alpha)] &= -\lim_{\beta \rightarrow 0} [\mathcal{I}_{AdS;1}(\lambda; \beta)] = -\mathcal{I}_{S;0}(\lambda) \\
\mathcal{I}_{S;0}(\lambda) &= -\mathcal{I}_{AdS;0}(\lambda) = \int_{\text{Res}=\frac{1}{2}} \frac{ds}{2\pi i} \frac{8\pi^{s-\frac{3}{2}}(2s-3)\xi(2s-2)\tan(\pi s)\Gamma(s-2)^2}{\Gamma\left(s-\frac{3}{2}\right)} \lambda^{1-s} \\
\mathcal{I}_{S;1}(\lambda; \alpha) &= - \int_{\text{Res}=\frac{1}{2}} \frac{ds}{2\pi i} 4^s \pi^{s-1} (2s-3)\xi(2s-2)\Gamma(s-2) {}_2F_1(s-2, s-1; 1; 1-\alpha) \lambda^{1-s} \\
\mathcal{I}_{AdS;1}(\lambda; \beta) &= - \int_{\text{Res}=\frac{1}{2}} \frac{ds}{2\pi i} 4^s \pi^{s-1} (2s-3)\xi(2s-2)\Gamma(s-1) {}_2F_1\left(s-1, s-1; 2; \frac{1}{\beta+1}\right) (\beta+1)^{1-s} \lambda^{1-s}
\end{aligned}$$



$$\begin{aligned}
& \mathcal{J}_{S;0}(\lambda)|_{\text{strong}} \sim \mathcal{J}_{S;0}^{(p)}(\lambda) \\
& = 2[\log(\lambda) + 2\gamma + 2 - 2\log(4\pi)] + \frac{4\pi^2}{3\lambda} - \sum_{n=1}^{\infty} \frac{16n\zeta(2n+1)\Gamma\left(n-\frac{1}{2}\right)^2 \Gamma\left(n+\frac{1}{2}\right)}{\lambda^{n+\frac{1}{2}}\pi^{3/2}\Gamma(n)} \\
& \mathcal{J}_{S;1}(\lambda; \alpha)|_{\text{strong}} \sim \mathcal{J}_{S;1}^{(p)}(\lambda; \alpha) = 2\alpha - 2[\log(\lambda) + 2\gamma + 2 - 2\log(4\pi)] - \frac{8\pi^2}{3\lambda} \\
& \mathcal{J}_S(\lambda; \alpha)|_{\text{strong}} = \mathcal{J}_{S;0}(\lambda)|_{\text{strong}} + \mathcal{J}_{S;1}(\lambda; \alpha)|_{\text{strong}} \\
& \sim \mathcal{J}_{S;0}^{(p)}(\lambda) + \mathcal{J}_{S;1}^{(p)}(\lambda; \alpha) = 2\alpha - \frac{4\pi^2}{3\lambda} - \sum_{n=1}^{\infty} \frac{16n\zeta(2n+1)\Gamma\left(n-\frac{1}{2}\right)^2 \Gamma\left(n+\frac{1}{2}\right)}{\lambda^{n+\frac{1}{2}}\pi^{3/2}\Gamma(n)} \\
& \mathcal{J}_{AdS;1}(\lambda; \beta)|_{\text{strong}} \sim \mathcal{J}_{AdS;1}^{(p)}(\lambda; \beta) = 2\log(1+\beta) + 2[\log(\lambda) + 2\gamma + 2 - 2\log(4\pi)] \\
& \mathcal{J}_{AdS}(\lambda; \beta)|_{\text{strong}} \sim \mathcal{J}_{AdS;0}^{(p)}(\lambda) + \mathcal{J}_{AdS;1}^{(p)}(\lambda; \beta) \\
& = 2\log(1+\beta) - \frac{4\pi^2}{3\lambda} + \sum_{n=1}^{\infty} \frac{16n\zeta(2n+1)\Gamma\left(n-\frac{1}{2}\right)^2 \Gamma\left(n+\frac{1}{2}\right)}{\lambda^{n+\frac{1}{2}}\pi^{3/2}\Gamma(n)} \\
& \mathcal{J}_{AdS}(\lambda; \beta) = \frac{3\beta\lambda\zeta(3)}{2\pi^2} - \frac{15\beta(\beta+4)\lambda^2\zeta(5)}{32\pi^4} + \frac{35\beta(2\beta^2+15\beta+30)\lambda^3\zeta(7)}{512\pi^6} + \dots \\
& \mathcal{J}_{AdS}(\lambda; \beta)|_{\text{lead}-\beta} = 4 \sum_{\ell=1}^{\infty} (-1)^{\ell+1} \binom{2\ell}{\ell} (\ell+1)\zeta(2\ell+1) \left(\frac{\lambda_\beta}{16\pi^2}\right)^\ell \\
& = 4 \int_0^\infty dw \frac{w}{\sinh^2(w)} \left[1 - J_0\left(\frac{w\sqrt{\lambda_\beta}}{\pi}\right)\right] \\
& 2^{1-s}\zeta(s)\Gamma(s+1) = \int_0^\infty dw \frac{w^s}{\sinh^2(w)} \\
& \mathcal{J}_{AdS}(\lambda; \beta)|_{\text{lead}-\beta} = 4 + 4\gamma + 2\log\left(\frac{\lambda_\beta}{16\pi^2}\right) + 8 \sum_{n=1}^{\infty} \left(n\sqrt{\lambda_\beta} K_1\left(n\sqrt{\lambda_\beta}\right) - K_0\left(n\sqrt{\lambda_\beta}\right)\right) \\
& = 4 + 4\gamma + 2\log\left(\frac{\lambda_\beta}{16\pi^2}\right) \\
& - 8 \sum_{n=1}^{\infty} \left(2\pi n \sqrt{\lambda_\beta}\right)^{\frac{1}{2}} e^{-n\sqrt{\lambda_\beta}} \sum_{k=0}^{\infty} \frac{(4k^2 - 8k - 1)\Gamma\left(k - \frac{1}{2}\right)^2}{\pi\Gamma(k+1)} \left(-2n\sqrt{\lambda_\beta}\right)^{-k} \\
& \langle \Phi_I(x) \rangle_\theta = \frac{2\sqrt{\lambda_\beta}}{4\pi\sqrt{d_{12}}} \left(e^{i\theta} \Omega_2^{(N)} \frac{(Y_1)_I}{(x-x_1)^2} + e^{-i\theta} \Omega_2^{(N)} \frac{(Y_2)_I}{(x-x_2)^2} \right), \text{ with } I = 1, \dots, 6
\end{aligned}$$



$$\Omega_2^{(N)} = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

$$\begin{aligned} \mathcal{J}_{S;0}^{(\text{np})}(\lambda) &= \pm i \frac{64\sqrt{\lambda}}{\pi^2} \sum_{n=1}^{\infty} e^{-2n\sqrt{\lambda}} \int_0^{\infty} dz e^{-2n\sqrt{\lambda}z} \left[(z+1)^3 G_{3,3}^{2,3} \left((z+1)^2 \left| \begin{matrix} -\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ 0, 0, -\frac{3}{2} \end{matrix} \right. \right) - \frac{\pi^3}{4} \right] \\ &= \pm i \sum_{n=1}^{\infty} e^{-2n\sqrt{\lambda}} \left[\frac{32}{\sqrt{\lambda}n^2} + \frac{24}{\lambda n^3} + \frac{5}{\lambda^{3/2}n^4} - \frac{9}{4\lambda^2 n^5} + \frac{123}{64\lambda^{5/2}n^6} + O(\lambda^{-2}) \right] \end{aligned}$$

$$\begin{aligned} \mathcal{J}_{S;1}^{(\text{np})}(\lambda; \alpha) &= - \sum_{n=1}^{\infty} \int_{\text{Res}=\frac{3}{2}} \frac{ds}{2\pi i} 4(2s-1)\Gamma(s)\Gamma(s-1) {}_2F_1(s-1, s; 1; 1-\alpha) \left(\frac{Y_n}{2}\right)^{-2s}, \\ \mathcal{J}_{AdS;1}^{(\text{np})}(\lambda; \beta) &= - \sum_{n=1}^{\infty} \int_{\text{Res}=\frac{3}{2}} \frac{ds}{2\pi i} 4(2s-1)\Gamma(s)^2 {}_2F_1\left(s, s; 2; \frac{1}{1+\beta}\right) \left(\frac{Y_n\sqrt{(\beta+1)}}{2}\right)^{-2s}, \end{aligned}$$

$$\xi(s)=\xi(1-s)$$

$$\begin{aligned} \mathcal{J}_{S;1}^{(\text{np})}(\lambda; \alpha) &= -16\pi(1-\alpha)^{\frac{1}{4}} \sum_{n=1}^{\infty} \left[e^{-n\sqrt{\lambda}(1-\sqrt{1-\alpha})} \sum_{m=0}^{\infty} \frac{1}{m!} c_m(\sqrt{1-\alpha})(2n\sqrt{\lambda(1-\alpha)})^{-m-1} \right. \\ &\quad \left. \pm i e^{-n\sqrt{\lambda}(1+\sqrt{1-\alpha})} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} c_m(-\sqrt{1-\alpha})(2n\sqrt{\lambda(1-\alpha)})^{-m-1} \right] \end{aligned}$$

$$\begin{aligned} c_m(x) &= [m^2+m-1]x_3\tilde{F}_2\left(-\frac{1}{2},\frac{3}{2},-m;\frac{3}{2}-m,\frac{3}{2}-m;x\right) \\ &+ {}_3\tilde{F}_2\left(-\frac{3}{2},\frac{1}{2},-m-1;\frac{1}{2}-m,\frac{1}{2}-m;x\right)-\frac{3}{4}m(m+1)x^2{}_3\tilde{F}_2\left(\frac{1}{2},\frac{5}{2},1-m;\frac{5}{2}-m,\frac{5}{2}-m;x\right) \end{aligned}$$

$$\begin{aligned} \mathcal{J}_{AdS;1}^{(\text{np})}(\lambda; \beta) &= -16\pi(1+\beta)^{\frac{1}{4}} \sum_{n=1}^{\infty} \left[e^{-n\sqrt{\lambda}(\sqrt{\beta+1}-1)} \sum_{m=0}^{\infty} \frac{1}{m!} c_m(\sqrt{1+\beta})(-2n\sqrt{\lambda(1+\beta)})^{-m-1} \right. \\ &\quad \left. \mp i e^{-n\sqrt{\lambda}(\sqrt{\beta+1}+1)} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} c_m(-\sqrt{1+\beta})(2n\sqrt{\lambda(1+\beta)})^{-m-1} \right] \end{aligned}$$

$$\mathcal{J}_{S;0}^{(\text{np})}(\lambda)=-\mathcal{J}_{AdS;0}^{(\text{np})}(\lambda)\propto\exp\left[-2n\sqrt{\lambda}\right]$$

$$\begin{aligned} \mathcal{J}_{S;1}^{(\text{np})}(\lambda; \alpha) \Big|_{\mathbb{R}} &\propto \exp\left[-n\sqrt{\lambda}(1-\sqrt{1-\alpha})\right], \mathcal{J}_{S;1}^{(\text{np})}(\lambda; \alpha) \Big|_{\mathbb{I}} \propto \exp\left[-n\sqrt{\lambda}(1+\sqrt{1-\alpha})\right]. \\ \mathcal{J}_{AdS;1}^{(\text{np})}(\lambda; \beta) \Big|_{\mathbb{R}} &\propto \exp\left[-n\sqrt{\lambda}(\sqrt{\beta+1}-1)\right], \mathcal{J}_{AdS;1}^{(\text{np})}(\lambda; \beta) \Big|_{\mathbb{I}} \propto \exp\left[-n\sqrt{\lambda}(\sqrt{\beta+1}+1)\right]. \end{aligned}$$

$$\mathcal{J}_{S;1}^{(\text{np})}(\lambda; \alpha) \Big|_{\mathbb{R}} \gg \mathcal{J}_{S;1}^{(\text{np})}(\lambda) \Big|_{\mathbb{I}} \gg \mathcal{J}_{S;0}^{(\text{np})}(\lambda)$$

$$\mathcal{J}_{AdS;1}^{(\text{np})}(\lambda; \beta) \Big|_{\mathbb{R}} \gg \mathcal{J}_{AdS;0}^{(\text{np})}(\lambda) \gg \mathcal{J}_{S;1}^{(\text{np})}(\lambda; \beta) \Big|_{\mathbb{I}}$$



$$\mathcal{J}_{AdS;0}^{(\text{np})}(\lambda) \gg \mathcal{J}_{AdS;1}^{(\text{np})}(\lambda; \beta) \Big|_{\mathbb{R}} \gg \mathcal{J}_{S;1}^{(\text{np})}(\lambda; \beta) \Big|_{\mathbb{I}}$$

$$\begin{aligned} \mathcal{J}_{S;1}^{(\text{np})}(\lambda; \alpha) \Big|_{\mathbb{R}/\mathbb{I}} &\propto \exp \left[-2\pi L^2 (1 \mp \sin(\theta_0)) n T_F \right] \\ \mathcal{J}_{AdS;1}^{(\text{np})}(\lambda; \beta) \Big|_{\mathbb{R}/\mathbb{I}} &\propto \exp \left[-2\pi L^2 (\cosh(\rho_0) \mp 1) n T_F \right] \end{aligned}$$

$$\begin{aligned} \frac{\mathcal{J}_{S;1}^{(\text{np})}(\lambda; \alpha) \Big|_{\mathbb{I}}}{\mathcal{J}_{S;1}^{(\text{np})}(\lambda; \alpha) \Big|_{\mathbb{R}}} &\propto \exp \left[-4\pi L^2 \sin(\theta_0) n T_F \right] \\ \frac{\mathcal{J}_{AdS;1}^{(\text{np})}(\lambda; \beta) \Big|_{\mathbb{I}}}{\mathcal{J}_{AdS;1}^{(\text{np})}(\lambda; \beta) \Big|_{\mathbb{R}}} &\propto \exp \left[-4\pi L^2 n T_F \right] \end{aligned}$$

$$\left[N \mathcal{J}_{gS;\alpha}^{(0)}(\lambda) \right]_{N \rightarrow -N} = -N \mathcal{J}_{gS;-\alpha}^{(0)}(\lambda) = +N \mathcal{J}_{gAdS;\beta}^{(0)} \Big|_{\beta = -\alpha}$$

$$\partial_\mu^2 \partial_m^2 \log Z(\tau; m, \mu) \Big|_{m=\mu=0} \sim N F_1(\lambda) + F_2(\lambda) + N^{-1} F_3(\lambda) + \dots,$$

$$F_1(\lambda) = 2^6 \int_0^\infty \frac{w_1^2 dw_1}{\sinh(w_1)^2} \frac{w_2 dw_2}{\sinh(w_2)^2} J_1(x_2) \frac{w_2 J_0(x_2) J_1(x_1) - w_1 J_0(x_1) J_1(x_2)}{w_1^2 - w_2^2}$$

$$\begin{aligned} F_1(\lambda) = & \frac{9\zeta(3)^2}{2\pi^4}\lambda^2 - \frac{105\zeta(3)\zeta(5)}{16\pi^6}\lambda^3 + \frac{105(5\zeta(5)^2 + 12\zeta(3)\zeta(7))}{256\pi^8}\lambda^4 \\ & - \frac{21(65\zeta(5)\zeta(7) + 99\zeta(3)\zeta(9))}{512\pi^{10}}\lambda^5 + \frac{105(427\zeta(7)^2 + 1284\zeta(5)\zeta(9) + 2211\zeta(3)\zeta(11))}{65536\pi^{12}}\lambda^6 + \dots \end{aligned}$$

$$F_1(\lambda)|_{\text{strong}} \sim 8 \left(2 - \frac{4\pi^2}{\lambda} - \sum_{n=1}^{\infty} \frac{16n\zeta(2n+1)\Gamma\left(n-\frac{1}{2}\right)^2\Gamma\left(n+\frac{1}{2}\right)}{\lambda^{n+\frac{1}{2}}\pi^{3/2}\Gamma(n)} \right)$$

$$\frac{x_2J_0(x_2)J_1(x_1)-x_1J_0(x_1)J_1(x_2)}{x_1^2-x_2^2}=4\sum_{\ell=1}^{\infty}\frac{\ell J_{2\ell}(x_1)J_{2\ell}(x_2)}{x_1x_2},$$

$$\begin{aligned} J_\mu(x) &= \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \frac{ds}{2\pi\mathrm{i}} \frac{\Gamma(-s)}{\Gamma(\mu+s+1)} \left(\frac{x}{2}\right)^{\mu+2s} \\ J_\mu(x)J_\nu(x) &= \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \frac{ds}{2\pi\mathrm{i}} \frac{\Gamma(-s)\Gamma(2s+\mu+\nu+1)}{\Gamma(s+\mu+1)\Gamma(s+\nu+1)\Gamma(s+\mu+\nu+1)} \left(\frac{x}{2}\right)^{\mu+\nu+2s} \end{aligned}$$

$$F_1(\lambda) = \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} ds dt 2^{9-2s} \pi^{\frac{5}{2}} \frac{s(2s-1)t(2t-1)\xi(2s)\xi(2t) {}_4\tilde{F}_3(2,s,s+1,t+1;2-s,3-s,2-t;-1)}{\sin(2\pi s)\sin(\pi t)\Gamma\left(s-\frac{1}{2}\right)} \left(\frac{\lambda}{4\pi}\right)^{-s-t}$$

$$\begin{aligned} F_1(\lambda) = & - \sum_{n_1,n_2=1}^{\infty} \sum_{\ell=1}^{\infty} \frac{2^6 \ell}{\pi} \int_{c'-\mathrm{i}\infty}^{c'+\mathrm{i}\infty} ds dt \left[\frac{(2s-1)\Gamma(s-\ell-1)\Gamma(s-\ell)\Gamma(s+\ell-1)\Gamma(s+\ell)}{\Gamma(2s-1)\cot(\pi s)} \right. \\ & \left. \times (2t-1)\Gamma(t-\ell)\Gamma(t+\ell) n_1^{-2s} n_2^{-2t} \left(\frac{\sqrt{\lambda}}{2}\right)^{-2s-2t} \right] \end{aligned}$$



$$\mathrm{iexp}\left[-2n_1\sqrt{\lambda}\right], \mathrm{iexp}\left[-(2n_1+n_2)\sqrt{\lambda}\right], \exp\left[-n_2\sqrt{\lambda}\right]$$

$${}_2F_1(a,b;c;z)=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n}\frac{z^n}{n!}$$

$$\mathcal{J}_{S;1}^{(\mathrm{np})}(\lambda;\alpha)=-\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\frac{4(1-\alpha)^m}{\Gamma(m+1)^2}\int_{\mathrm{Res}=\frac{3}{2}}\frac{ds}{2\pi i}(2s-1)\Gamma(m+s)\Gamma(m+s-1)\left(\frac{Y_n}{2}\right)^{-2s}$$

$$\mathcal{J}_{S;1}^{(\mathrm{np})}(\lambda;\alpha)=\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\frac{8(1-\alpha)^m}{\Gamma(m+1)^2}\left[\left(\frac{Y_n}{2}\right)^{2m-1}K_1(Y_n)+G_{2,3}^{2,1}\left(0;m-1,m,1;\frac{Y_n^2}{4}\right)\right]$$

$$K_1(Y)\sim -\sqrt{\frac{\pi}{2Y}}e^{-Y}\sum_{k=0}^{\infty}\frac{\Gamma\left(k-\frac{1}{2}\right)\Gamma\left(k+\frac{3}{2}\right)}{\pi\Gamma(k+1)}(-2Y)^{-k}$$

$$G_{2,3}^{2,1}\left(0;m-1,m,1;\frac{Y^2}{4}\right)\sim -\sqrt{\frac{\pi}{2Y}}\left(\frac{Y}{2}\right)^{2m}e^{-Y}\sum_{k=0}^{\infty}\frac{\Gamma\left(k-\frac{3}{2}\right)\Gamma\left(k+\frac{1}{2}\right)}{2\pi\Gamma(k+1)}\\ \times [(2k+1)(2k+3)-16km](-2Y)^{-k}$$

$$\mathcal{J}_{S;1}^{(\mathrm{np})}(\lambda;\alpha)=-\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\frac{(1-\alpha)^m}{\Gamma(m+1)^2}\sqrt{\frac{8\pi}{Y_n}}e^{-Y_n}\left(\frac{Y_n}{2}\right)^{2m}\\ \times \left[\sum_{k=0}^{\infty}(-2Y_n)^{-k}\frac{\Gamma(k+1/2)\Gamma(k-3/2)}{\pi\Gamma(k+1)}(4k^2+3-16km)\right]$$

$$\mathcal{J}_{S;1}^{(\mathrm{np})}(\lambda;\alpha)=-\sum_{n=1}^{\infty}\sqrt{\frac{8\pi}{Y_n}}e^{-Y_n}\sum_{k=0}^{\infty}\frac{\Gamma\left(k-\frac{3}{2}\right)\Gamma\left(k+\frac{1}{2}\right)}{\pi\Gamma(k+1)}(-2Y_n)^{-k}\\ \times \left[(4k^2+3)I_0(\sqrt{1-\alpha}Y_n)-8kY_n\sqrt{1-\alpha}I_1(\sqrt{1-\alpha}Y_n)\right]$$

$$\mathcal{J}_{S;1}^{(\mathrm{np})}(\lambda;1)=-\sum_{n=1}^{\infty}e^{-Y_n}\sqrt{\frac{8\pi}{Y_n}}\sum_{k=0}^{\infty}\frac{(4k^2+3)\Gamma\left(k-\frac{3}{2}\right)\Gamma\left(k+\frac{1}{2}\right)}{\pi\Gamma(k+1)}(-2Y_n)^{-k}$$

$$I_\nu(z)=\frac{e^z}{(2\pi z)^{\frac{1}{2}}}\sum_{j=0}^{\infty}\frac{\left(\frac{1}{2}+\nu\right)_j\left(\frac{1}{2}-\nu\right)_j}{j!}(2z)^{-k}\\ \pm \mathrm{i}e^{\pm \nu \pi \mathrm{i}}\frac{e^{-z}}{(2\pi z)^{\frac{1}{2}}}\sum_{j=0}^{\infty}\frac{\left(\frac{1}{2}+\nu\right)_j\left(\frac{1}{2}-\nu\right)_j}{j!}(-2z)^{-k}$$

$$\mathcal{J}_{S;1}^{(\mathrm{np})}(\lambda;\alpha)=-2\sum_{n=1}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^k\left(\frac{1}{2n\sqrt{\lambda}}\right)^{j+k}\frac{\Gamma(j+1/2)^2\Gamma(k+1/2)\Gamma(k-3/2)}{\pi^2\Gamma(j+1)\Gamma(k+1)}\frac{1}{(1-\alpha)^{1/4+j/2}}\\ \times \left[e^{-n\sqrt{\lambda}(1-\sqrt{1-\alpha})}\left(\frac{4k^2+3}{n\sqrt{\lambda}}+8k\frac{2j+1}{2j-1}\sqrt{1-\alpha}\right)\pm \mathrm{i}(-1)^je^{-n\sqrt{\lambda}(1+\sqrt{1-\alpha})}\left(\frac{4k^2+3}{n\sqrt{\lambda}}-8k\frac{2j+1}{2j-1}\sqrt{1-\alpha}\right)\right]$$



$$\mathcal{J}_{S;1}^{(\text{np})}(\lambda; \alpha) = -16\pi(1-\alpha)^{\frac{1}{4}} \sum_{n=1}^{\infty} \left[e^{-n\sqrt{\lambda}(1-\sqrt{1-\alpha})} \sum_{m=0}^{\infty} \frac{1}{m!} c_m(\sqrt{1-\alpha})(2n\sqrt{\lambda(1-\alpha)})^{-m-1} \right. \\ \left. \pm i e^{-n\sqrt{\lambda}(1+\sqrt{1-\alpha})} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} c_m(-\sqrt{1-\alpha})(2n\sqrt{\lambda(1-\alpha)})^{-m-1} \right]$$

$$\mathcal{J}_{S;1}^{(\text{np})}(\lambda; \alpha) \Big|_{\mathbb{R}} \sim \exp[-n\sqrt{\lambda}(1-\sqrt{1-\alpha})], \mathcal{J}_{S;1}^{(\text{np})}(\lambda) \Big|_{\mathbb{I}} \sim i \exp[-n\sqrt{\lambda}(1+\sqrt{1-\alpha})]$$

$$\mathcal{J}_{AdS;1}^{(\text{np})}(\lambda; \beta) = - \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \frac{4}{(1+\beta)^m \Gamma(m+1) \Gamma(m+2)} \int_{\text{Res}=\frac{3}{2}} \frac{ds}{2\pi i} (2s-1) \Gamma(m+s)^2 \left(\frac{Y_n \sqrt{\beta+1}}{2} \right)^{-2s}$$

$$\mathcal{J}_{AdS;1}^{(\text{np})}(\lambda; \beta) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \frac{8(1+\beta)^{-m}}{\Gamma(m+1) \Gamma(m+2)} \left[\left(\frac{Y_n \sqrt{\beta+1}}{2} \right)^{2m} K_0(Y_n \sqrt{\beta+1}) + G_{2,3}^{2,1} \left(0; m, m, 1; \frac{Y_n^2(\beta+1)}{4} \right) \right]$$

$$\mathcal{J}_{AdS;1}^{(\text{np})}(\lambda; \beta) = \sum_{n=1}^{\infty} e^{-Y_n \sqrt{\beta+1}} \sum_{m=0}^{\infty} \frac{\sqrt{2\pi Y_n}}{\Gamma(m+1) \Gamma(m+2)} \left(\frac{Y_n}{2} \right)^{2m} \\ \times \sum_{k=0}^{\infty} (1+\beta)^{1/4-k/2} \frac{\Gamma(k-1/2)^2}{\pi \Gamma(k+1)} (-2Y_n)^{-k} [(4k^2-8k-1)-16km].$$

$$\mathcal{J}_{AdS;1}^{(\text{np})}(\lambda; \beta) = \sum_{n=1}^{\infty} \sqrt{\frac{8\pi}{Y_n}} e^{-Y_n \sqrt{\beta+1}} \sum_{k=0}^{\infty} \frac{\Gamma(k-1/2)^2}{\pi (1+\beta)^{k/2-1/4} \Gamma(k+1)} (-2Y_n)^k \\ \times [(4k^2+8k-1)I_1(Y_n) - 8kY_n I_0(Y_n)].$$

$$\mathcal{J}_{AdS;1}^{(\text{np})}(\lambda; \beta) = \sum_{n=1}^{\infty} \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} (-1)^k \left(\frac{1}{2n\sqrt{\lambda}} \right)^{j+k} \frac{\Gamma(k-1/2)^2 \Gamma(j+1/2) \Gamma(j-1/2)}{\pi^2 \Gamma(j+1) \Gamma(k+1)} \frac{1}{(\beta+1)^{\frac{k}{2}+\frac{1}{4}}} \\ \left[e^{-n\sqrt{\lambda}(\sqrt{\beta+1}-1)} \left(-\frac{(4k^2+8k-1)(2j+1)}{n\sqrt{\lambda}} - 8k(2j-1) \right) \right. \\ \left. \times i(-1)^j e^{-n\sqrt{\lambda}(\sqrt{\beta+1}+1)} \left(\frac{(4k^2+8k-1)(2j+1)}{n\sqrt{\lambda}} - 8k(2j-1) \right) \right]$$

$$\mathcal{J}_{AdS;1}^{(\text{np})}(\lambda; \beta) = -16\pi(1+\beta)^{\frac{1}{4}} \sum_{n=1}^{\infty} \left[e^{-n\sqrt{\lambda}(\sqrt{\beta+1}-1)} \sum_{m=0}^{\infty} \frac{1}{m!} c_m(\sqrt{1+\beta})(-2n\sqrt{\lambda(1+\beta)})^{-m-1} \right. \\ \left. \mp i e^{-n\sqrt{\lambda}(\sqrt{\beta+1}+1)} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} c_m(-\sqrt{1+\beta})(2n\sqrt{\lambda(1+\beta)})^{-m-1} \right]$$

$$\mathcal{J}_{S;1}(\lambda; 1) = \int_{\text{Res}=1/2} \frac{ds}{2\pi i} \frac{4\pi\zeta(2s+1)}{\sin(\pi s)} \binom{2s+1}{s} \left(\frac{\lambda}{16\pi^2} \right)^s$$

$$\binom{2s+1}{s} = \frac{2^{2s+1} \Gamma\left(s + \frac{3}{2}\right)}{\sqrt{\pi} \Gamma(s+2)}$$



$$\begin{aligned}
\tilde{\mathcal{J}}_{S;1}(\lambda; 1; a, b) &:= \int_{\text{Res}=1/2} \frac{ds}{2\pi i} \frac{4\pi \zeta(2s+1)}{\sin(\pi s)} \frac{2^{2s+1} \Gamma\left(s+a+\frac{1}{2}\right)}{\sqrt{\pi} \Gamma(s+b)} \left(\frac{\lambda}{16\pi^2}\right)^s \frac{ds}{2\pi i} \\
\tilde{\mathcal{J}}_{S;1}(\lambda; 1; a, b) &= 16\sqrt{\pi} \int_{\text{Res}=1/2} \frac{ds}{2\pi i} \zeta(2s-2) \frac{\Gamma(2s-2)\Gamma(s-b)}{\Gamma\left(s-a-\frac{1}{2}\right)} \frac{\cos(\pi s) \sin(\pi(s-b))}{\cos(\pi(s-a)) \sin(\pi s)} \lambda^{1-s} \\
\tilde{\mathcal{J}}_{S;1}(\lambda; 1; 1, b) &= - \int_{\text{Res}=1/2} \frac{ds}{2\pi i} 4^s (2s-3) \zeta(2s-2) \Gamma(s-1) \Gamma(s-b) \frac{\sin(\pi(s-b))}{\sin(\pi s)} \lambda^{1-s} \\
\tilde{\mathcal{J}}_{S;1}(\lambda; 1; 1, b) &\sim \tilde{\mathcal{J}}_{S;1}^{(p)}(\lambda; 1; 1, b) = A_1(\lambda; b) + \frac{\sin(\pi b)}{\pi} \sum_{n=1}^{\infty} 4^{n+1} (2n-1) \zeta(2n) \Gamma(n) \Gamma(n+1-b) \lambda^{-n} \\
A_1(\lambda; b) &:= - \frac{2 \left[\log\left(\frac{\lambda}{16\pi^2}\right) + \gamma + 2 - \psi(b) \right]}{\Gamma(b)} \\
\lim_{b \rightarrow 2} A_1(\lambda; b) &= -2 \left[\log\left(\frac{\lambda}{16\pi^2}\right) + 2\gamma + 1 \right], \\
\lim_{b \rightarrow 2} \left[\frac{\sin(\pi b)}{\pi} \sum_{n=1}^{\infty} 4^{n+1} (2n-1) \zeta(2n) \Gamma(n) \Gamma(n+1-b) \lambda^{-n} \right] &= \begin{cases} -\frac{8\pi^2}{3\lambda}, & n=1 \\ 0, & n \geq 2 \end{cases} \\
\mathcal{B}: \sum_{n=1}^{\infty} c_n \lambda^{-n} \rightarrow \hat{\phi}(w) &:= \sum_{n=1}^{\infty} \frac{2c_n}{\zeta(2n) \Gamma(2n+1)} (2w)^{2n} \\
\hat{\phi}(w; b) &= \frac{64w^2}{\Gamma(b-1)} \left[{}_3F_2\left(1, 1, 2-b; \frac{3}{2}, 2; 4w^2\right) - 2 {}_2F_1\left(1, 2-b; \frac{3}{2}; 4w^2\right) \right] \\
\mathcal{S}_{\theta} \left[\tilde{\mathcal{J}}_{S;1}^{(p)} \right] (\lambda; 1; 1, b) &= A_1(\lambda; b) + \sqrt{\lambda} \int_0^{e^{i\theta}\infty} \frac{dw}{4 \sinh^2(\sqrt{\lambda} w)} \hat{\phi}(w) \\
\frac{2^{2s-1}}{\Gamma(2s+1)} \int_0^{\infty} dw \frac{w^{2s}}{\sinh^2(w)} &= \zeta(2s) \\
(\mathcal{S}_+ - \mathcal{S}_-) \left[\tilde{\mathcal{J}}_{S;1}^{(p)} \right] (\lambda; 1; 1, b) &= \Delta \left[\tilde{\mathcal{J}}_{S;1}^{(p)} \right] (\lambda; 1; 1, b) = \lambda \int_0^{\infty} dw \frac{1}{4 \sinh^2(xw)} \text{Disc}_0 \hat{\phi}(w; b) \\
\mathcal{S}_{\pm} \left[\tilde{\mathcal{J}}_{S;1}^{(p)} \right] (\lambda; 1; 1, b) &= \lim_{\theta \rightarrow 0^{\pm}} \mathcal{S}_{\theta} \left[\tilde{\mathcal{J}}_{S;1}^{(p)} \right] (\lambda; 1; 1, b) \\
\text{Disc } {}_2F_1(a, b; c; z) &= \frac{2\pi i \Gamma(c)}{\Gamma(a) \Gamma(b) \Gamma(c-a-b+1)} z^{1-c} (z-1)^{c-a-b} \\
&\quad \times {}_2F_1(1-a, 1-b; c-a-b+1; 1-z), \\
\text{Disc}_3 {}_2F_2\left(1, 1, 2-b; \frac{3}{2}, 2; z\right) &= -i \sin(\pi b) \sqrt{\pi} z^{-\frac{1}{2}} (z-1)^{b-\frac{1}{2}} \Gamma(b-1) {}_2\tilde{F}_1\left(1, b; b+\frac{1}{2}; 1-z\right) \\
\text{Disc}_0 \hat{\phi}(w; b) &= -2i \sin(\pi b) \text{disc}(w; b)
\end{aligned}$$



$$\text{disc}(w; b) := \frac{16\sqrt{\pi}w(4w^2 - 1)^{b-\frac{3}{2}} \left[(4w^2 - 1) {}_2F_1 \left(1, b; b + \frac{1}{2}; 1 - 4w^2 \right) + 1 - 2b \right]}{\Gamma \left(b + \frac{1}{2} \right)}$$

$$\begin{aligned} \mathcal{S}_{\text{med}} \left[\tilde{j}_{S;1}^{(\text{p})} \right] (\lambda; 1; 1, b) &= \mathcal{S}_{\pm} \left[\tilde{j}_{S;1}^{(\text{p})} \right] (\lambda; 1; 1, b) \mp \frac{1}{2} \Delta \left[\tilde{j}_{S;1}^{(\text{p})} \right] (\lambda; 1; 1, b) \\ &= \mathcal{S}_{\pm} \left[\tilde{j}_{S;1}^{(\text{p})} \right] (\lambda; 1; 1, b) \pm i \sin(\pi b) \tilde{j}_{S;1}^{(\text{np})} (\lambda; 1; 1, b) \\ &= \mathcal{S}_{\pm} \left[\tilde{j}_{S;1}^{(\text{p})} \right] (\lambda; 1; 1, b) \pm i \sin(\pi b) \lambda \int_{\frac{1}{2}}^{\infty} dw \frac{1}{4 \sinh^2(\sqrt{\lambda} w)} \text{disc}(w; b) \end{aligned}$$

$$\text{Im} \sigma(b) = \pm i \sin(\pi b) = \begin{cases} \arg(\lambda) > 0 \\ \arg(\lambda) < 0 \end{cases}$$

$$\sigma(b) = \epsilon^{\pm i \pi b}$$

$$\tilde{j}_{S;1}(\lambda; 1; 1, b) = \mathcal{S}_{\pm} \left[\tilde{j}_{S;1}^{(\text{p})} \right] (\lambda; 1; 1, b) + \sigma(b) \tilde{j}_{S;1}^{(\text{np})} (\lambda; 1; 1, b)$$

$$\tilde{j}_{S;1}(\lambda; 1) = j_{S;1}^{(\text{p})}(\lambda; 1) + \tilde{j}_{S;1}^{(\text{np})}(\lambda; 1; 1, 2),$$

$$\begin{aligned} j_{S;1}^{(\text{np})}(\lambda; 1) &= \tilde{j}_{S;1}^{(\text{np})}(\lambda; 1; 1, 2) = \lambda \int_{\frac{1}{2}}^{\infty} dw \frac{1}{4 \sinh^2(\sqrt{\lambda} w)} \text{disc}(w; 2) \\ &= -\lambda \int_{\frac{1}{2}}^{\infty} dw \frac{1}{4 \sinh^2(\sqrt{\lambda} w)} 16 \left(2w\sqrt{4w^2 - 1} + \text{arccosh}(2w) \right) \end{aligned}$$

$$\frac{1}{4 \sinh^2(\sqrt{\lambda} w)} = \sum_{n=1}^{\infty} n e^{-2nw\sqrt{\lambda}}$$

$$j_{S;1}^{(\text{np})}(\lambda; 1) = - \sum_{n=1}^{\infty} e^{-Y_n} \sqrt{\frac{8\pi}{Y_n}} \sum_{k=0}^{\infty} \frac{(4k^2 + 3) \Gamma\left(k - \frac{3}{2}\right) \Gamma\left(k + \frac{1}{2}\right)}{\pi \Gamma(k + 1)} (-2Y_n)^{-k}$$

En este punto, el principio de dualidad holográfica, en supersimetrías de gauge, cobra relevancia, calculándose así:

$$\mathcal{W}_{\mathcal{N}=4} = \text{Tr} \Phi_1 [\Phi_2, \Phi_3],$$

$$\mathcal{W}_{\mathcal{N}=1}^{\text{UV}} = \text{Tr} \Phi_1 [\Phi_2, \Phi_3] + h_1 \text{Tr} \Phi_1 \{ \Phi_2, \Phi_3 \} + h_2 \text{Tr} (\Phi_1^3 + \Phi_2^3 + \Phi_3^3)$$

$$\Delta \mathcal{W} = \frac{m}{2} \text{Tr} \Phi_1^2$$

$$\text{Tr} \Phi_2^4, \text{Tr} \Phi_3^4, \text{Tr} \Phi_2^2 \Phi_3^2, \text{Tr} \Phi_2 \Phi_3 \Phi_2 \Phi_3, \text{Tr} \Phi_2^3 \Phi_3, \text{Tr} \Phi_2 \Phi_3^3$$



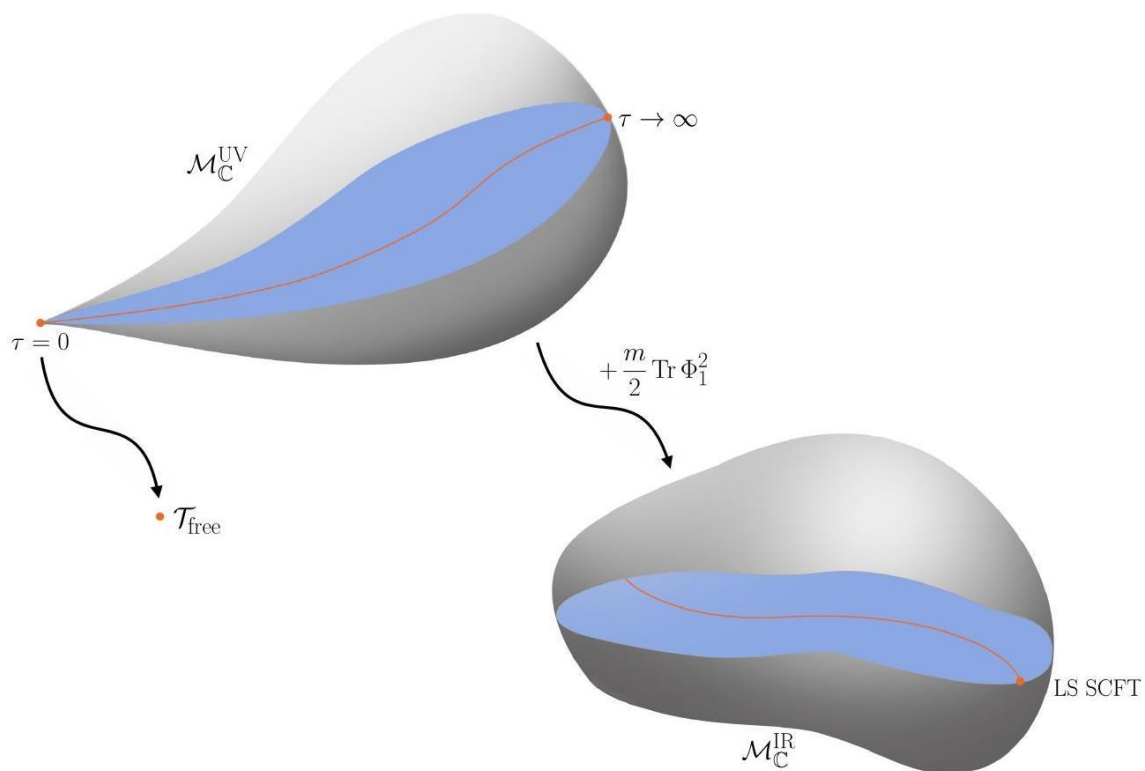


Figura 1. Representación de la dualidad holográfica a escala subatómica.

$$\mathcal{W}_{\mathcal{N}=1}^{\text{IR}} = g_1 \text{Tr}[\Phi_2, \Phi_3]^2 + g_2 \text{Tr}\{\Phi_2, \Phi_3\}^2 + g_3 \text{Tr}(\Phi_2^4 + \Phi_3^4),$$

$$a = \frac{9}{32} \text{Tr} R^3 - \frac{3}{32} \text{Tr} R, c = \frac{9}{32} \text{Tr} R^3 - \frac{5}{32} \text{Tr} R,$$

$$\text{Tr} R_{\mathcal{N}=4}^3 = \frac{8}{9} d_G, \text{Tr} R_{\mathcal{N}=4} = 0$$

$$a_{\mathcal{N}=4} = c_{\mathcal{N}=4} = \frac{1}{4} d_G$$

$$\text{Tr} R_{\text{LS}}^3 = \frac{3}{4} d_G, \text{Tr} R_{\text{LS}} = 0$$

$$\frac{a_{\text{LS}}}{a_{\mathcal{N}=4}} = \frac{c_{\text{LS}}}{c_{\mathcal{N}=4}} = \frac{27}{32}$$

$$\mathcal{K} = - \sum_{a=1}^4 \log (1 - z^a \bar{z}^a)$$

$$\begin{aligned}
z^1 &= \tanh \left[\frac{1}{2} (\alpha_1 + \alpha_2 + \alpha_3 + \varphi - i\phi_1 - i\phi_2 - i\phi_3 + i\phi_4) \right], \\
z^2 &= \tanh \left[\frac{1}{2} (\alpha_1 - \alpha_2 + \alpha_3 - \varphi - i\phi_1 + i\phi_2 - i\phi_3 - i\phi_4) \right], \\
z^3 &= \tanh \left[\frac{1}{2} (\alpha_1 + \alpha_2 - \alpha_3 - \varphi - i\phi_1 - i\phi_2 + i\phi_3 - i\phi_4) \right], \\
z^4 &= \tanh \left[\frac{1}{2} (\alpha_1 - \alpha_2 - \alpha_3 + \varphi - i\phi_1 + i\phi_2 + i\phi_3 + i\phi_4) \right], \\
\bar{z}^1 &= \tanh \left[\frac{1}{2} (\alpha_1 + \alpha_2 + \alpha_3 + \varphi + i\phi_1 + i\phi_2 + i\phi_3 - i\phi_4) \right], \\
\bar{z}^2 &= \tanh \left[\frac{1}{2} (\alpha_1 - \alpha_2 + \alpha_3 - \varphi + i\phi_1 - i\phi_2 + i\phi_3 + i\phi_4) \right], \\
\bar{z}^3 &= \tanh \left[\frac{1}{2} (\alpha_1 + \alpha_2 - \alpha_3 - \varphi + i\phi_1 + i\phi_2 - i\phi_3 + i\phi_4) \right], \\
\bar{z}^4 &= \tanh \left[\frac{1}{2} (\alpha_1 - \alpha_2 - \alpha_3 + \varphi + i\phi_1 - i\phi_2 - i\phi_3 - i\phi_4) \right].
\end{aligned}$$

$$\mathcal{L} = -R + 12(\partial\beta_1)^2 + 4(\partial\beta_2)^2 + 2\mathcal{K}_{a\bar{b}}\partial_\mu z^a\partial^\mu \bar{z}^b + \mathcal{P}$$

$$\begin{aligned}
\mathcal{W} \equiv & \frac{1}{L} e^{2\beta_1+2\beta_2} (1 + z_1 z_2 + z_1 z_3 + z_1 z_4 + z_2 z_3 + z_2 z_4 + z_3 z_4 + z_1 z_2 z_3 z_4) \\
& + \frac{1}{L} e^{2\beta_1-2\beta_2} (1 - z_1 z_2 + z_1 z_3 - z_1 z_4 - z_2 z_3 + z_2 z_4 - z_3 z_4 + z_1 z_2 z_3 z_4) \\
& + \frac{1}{L} e^{-4\beta_1} (1 + z_1 z_2 - z_1 z_3 - z_1 z_4 - z_2 z_3 - z_2 z_4 + z_3 z_4 + z_1 z_2 z_3 z_4)
\end{aligned}$$

$$\mathcal{P} = \frac{1}{2} e^{\mathcal{K}} \left[\frac{1}{6} \partial_{\beta_1} \mathcal{W} \partial_{\beta_1} \overline{\mathcal{W}} + \frac{1}{2} \partial_{\beta_2} \mathcal{W} \partial_{\beta_2} \overline{\mathcal{W}} + \mathcal{K}^{\bar{b}a} \nabla_a \mathcal{W} \nabla_{\bar{b}} \overline{\mathcal{W}} - \frac{8}{3} \mathcal{W} \overline{\mathcal{W}} \right]$$

$$\mathcal{S}_{\text{eff}} = \eta \int d^5x \sqrt{g} \mathcal{L}, \eta = \frac{N^2}{8\pi^2 L^3}$$

$$\varphi=\beta_{1,2}=\alpha_{1,2,3}=\phi_{1,2,3,4}=0, \text{ with } \mathcal{P}=-\frac{12}{L^2},$$

$$\varphi=\alpha_{1,2,3}=\phi_{2,3,4}=0, \phi_1=-\frac{\pi}{6}, \beta_1=\frac{1}{3}\beta_2=\frac{\log{(2)}}{12}, \text{ with } \mathcal{P}=-\frac{12}{L_{\text{LS}}^2},$$

$$\frac{a_{\text{LS}}}{a_{\mathcal{N}=4}}=\frac{c_{\text{LS}}}{c_{\mathcal{N}=4}}\rightarrow\frac{L_{\text{LS}}^3}{L^3}=\frac{27}{32}$$

$$\beta_1 \mapsto \frac{1}{2\sqrt{6}}\beta_1, \beta_2 \mapsto \frac{1}{2\sqrt{2}}\beta_2, \Phi_i \mapsto \frac{1}{2}\Phi_i,$$



$$\mathcal{L}_{\text{kin,UV}} = \frac{1}{2} \left[(\partial_\mu \beta_1)^2 + (\partial_\mu \beta_2)^2 + (\partial_\mu \varphi)^2 + \sum_{i=1}^3 (\partial_\mu \alpha_i)^2 + \sum_{i=1}^4 (\partial_\mu \phi_i)^2 \right] + \mathcal{L}_{\text{kin,UV}}^{(4)} + \dots$$

$$\mathcal{P}_{\text{UV}} = \frac{1}{L^2} \left[-12 - \frac{1}{2} \left(4\beta_1^2 + 4\beta_2^2 + 4 \sum_{i=1}^3 \alpha_i^2 + 3 \sum_{i=1}^4 \phi_i^2 \right) + \mathcal{P}_{\text{UV}}^{(3)} + \mathcal{P}_{\text{UV}}^{(4)} + \dots \right]$$

$$\begin{aligned} \alpha_i, \beta_i: \quad m^2 L^2 = -4 &\leftrightarrow \Delta = 2, \quad (\text{boson bilinears, part of } \mathbf{20}') \\ \phi_i: \quad m^2 L^2 = -3 &\leftrightarrow \Delta = 3, \quad (\text{fermion bilinears, part of } \mathbf{10} \oplus \overline{\mathbf{10}}) \\ \varphi: \quad m^2 L^2 = 0 &\leftrightarrow \Delta = 4. \quad (F_{\mu\nu} F^{\mu\nu}, \mathbf{1}) \end{aligned}$$

$$\phi_1 \mapsto -\frac{\pi}{6} - \phi_1, \beta_1 \mapsto \frac{\log(2)}{12} - \beta_1, \beta_2 \mapsto \frac{\log(2)}{4} - \beta_2,$$

$$\beta_1 \mapsto \frac{1}{2\sqrt{6}}\beta_1, \beta_2 \mapsto \frac{1}{2\sqrt{2}}\beta_2, \Phi_i \mapsto \frac{\sqrt{3}}{4}\Phi_i$$

$$\begin{pmatrix} \hat{\beta} \\ \rho_1 \\ \rho_2 \end{pmatrix} \equiv M \cdot \begin{pmatrix} \beta_1 \\ \beta_2 \\ \phi_1 \end{pmatrix}, M = \frac{1}{2\sqrt{14}} \begin{pmatrix} -\sqrt{42} & \sqrt{14} & 0 \\ \sqrt{7+\sqrt{7}} & \sqrt{3(7+\sqrt{7})} & 2\sqrt{7-\sqrt{7}} \\ -\sqrt{7-\sqrt{7}} & -\sqrt{3(7-\sqrt{7})} & 2\sqrt{7+\sqrt{7}} \end{pmatrix}$$

$$\mathcal{L}_{\text{kin,IR}} = \mathcal{L}_{\text{kin,IR}}^{(2)} + \mathcal{L}_{\text{kin,IR}}^{(3)} + \mathcal{L}_{\text{kin,IR}}^{(4)} + \dots$$

$$\mathcal{P}_{\text{IR}} = \frac{1}{L_{\text{LS}}^2} \left[-12 + \mathcal{P}_{\text{IR}}^{(2)} + \mathcal{P}_{\text{IR}}^{(3)} + \mathcal{P}_{\text{IR}}^{(4)} + \dots \right]$$

$$\mathcal{L}_{\text{kin,IR}}^{(2)} = \frac{1}{2} \left[(\partial_\mu \hat{\beta})^2 + (\partial_\mu \rho_1)^2 + (\partial_\mu \rho_2)^2 + (\partial_\mu \varphi)^2 + \sum_{i=1}^3 (\partial_\mu \alpha_i)^2 + \sum_{i=2}^4 (\partial_\mu \phi_i)^2 \right]$$

$$\mathcal{P}_{\text{IR}}^{(2)} = \frac{1}{2} \left[(4 - 2\sqrt{7})\rho_1^2 + 3\alpha_1^2 + (4 + 2\sqrt{7})\rho_2^2 - 4\hat{\beta}^2 - \frac{15}{4}(\alpha_2^2 + \alpha_3^2 + \phi_2^2 + \phi_3^2) - 3\phi_4^2 \right]$$

	$m^2 L_{\text{LS}}^2$	Δ	$\mathcal{N} = 1$ multiplet	$SU(2)_{\text{F}} \ U(1)_{\text{R}}$
α_2, α_3	$-\frac{15}{4}$	$\frac{3}{2}$	$L\bar{B}_1[\frac{3}{2}; 0, 0; 1] \otimes [1]^{(1)} + \text{c.c.}$	$\mathbf{3}_1 \oplus \mathbf{3}_{-1}$
ϕ_2, ϕ_3	$-\frac{15}{4}$	$\frac{5}{2}$	(chiral)	$\mathbf{3}_{-1} \oplus \mathbf{3}_1$
ϕ_4	-3	3	$L\bar{B}_1[3; 0, 0; 2] \otimes [0]^{(2)} + \text{c.c.}$	$\mathbf{1}_2 \oplus \mathbf{1}_{-2}$
φ	0	4	(chiral)	$\mathbf{1}_0 \oplus \mathbf{1}_0$
$\hat{\beta}$	-4	2	$A\bar{A}[2; 0, 0; 0] \otimes [1]^{(0)}$ (flavour current)	$\mathbf{3}_0$
ρ_1	$4 - 2\sqrt{7}$	$1 + \sqrt{7}$	$LL\bar{[1 + \sqrt{7}; 0, 0; 0] \otimes [0]^{(0)}}$	$\mathbf{1}_0$
α_1	3	$2 + \sqrt{7}$	(long)	$\mathbf{1}_2 \oplus \mathbf{1}_{-2}$
ρ_2	$4 + 2\sqrt{7}$	$3 + \sqrt{7}$		$\mathbf{1}_0$



$$X\bar{Y}[\Delta; j_1, j_2; r] \otimes [k]^{(\mathcal{Y})},$$

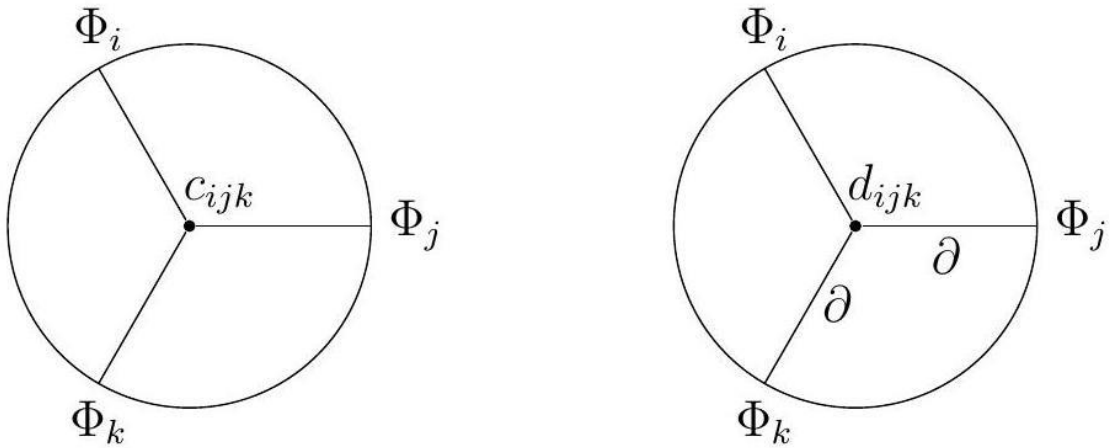
$$\begin{array}{cc} \text{bulk cubic couplings} & \text{CFT 3pt-functions} \\ c_{ijk} \Phi_i \Phi_j \Phi_k + d_{ijk} \Phi_i \partial_\mu \Phi_j \partial^\mu \Phi_k \subset \mathcal{L}_{\text{eff}} & \mapsto \left\langle \mathcal{O}_{\Phi_i} \mathcal{O}_{\Phi_j} \mathcal{O}_{\Phi_k} \right\rangle \end{array} \quad .$$

$$\mathcal{S}_{\text{eff}} = \eta \int_{\text{AdS}_{d+1}} d^{d+1}x \sqrt{g} (-R + \mathcal{L}_{\text{eff}}(\Phi_i))$$

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \frac{1}{2} (\partial_\mu \Phi_i)^2 + d_{ijk} \Phi_i (\partial_\mu \Phi_j) (\partial^\mu \Phi_k) + \mathcal{L}_{\text{kin}}^{(4)} + \dots \\ & + \frac{1}{L^2} [-d(d-1) + m_i^2 \Phi_i^2 + c_{ijk} \Phi_i \Phi_j \Phi_k + \mathcal{P}^{(4)} + \dots] \end{aligned}$$

$$\langle \hat{\mathcal{O}}_\Phi(x_1) \hat{\mathcal{O}}_\Phi(x_2) \rangle = \frac{n_\Delta}{|x_1 - x_2|^{2\Delta}}, \text{ with } n_\Delta = \eta \frac{(2\Delta - d)\Gamma(\Delta)}{\pi^{d/2}\Gamma\left(\Delta - \frac{d}{2}\right)},$$

$$m^2 L^2 = \Delta(\Delta - d)$$



$$\mathcal{O}_\Phi := \frac{\hat{\mathcal{O}}_\Phi}{\sqrt{n_\Delta}}.$$

$$\left\langle \mathcal{O}_{\Phi_i}(x_1) \mathcal{O}_{\Phi_j}(x_2) \mathcal{O}_{\Phi_k}(x_3) \right\rangle = \frac{C_{\Phi_i \Phi_j \Phi_k}}{|x_1 - x_2|^{\Delta_1 + \Delta_2 - \Delta_3} |x_1 - x_3|^{\Delta_1 + \Delta_3 - \Delta_2} |x_2 - x_3|^{\Delta_2 + \Delta_3 - \Delta_1}}.$$

$$C_{\Phi_i \Phi_j \Phi_k} = c_{ijk} A_1 + d_{ijk} A_2,$$

$$\begin{aligned} A_1 = & \frac{1}{\sqrt{\eta}} \frac{\Gamma\left(\frac{\Delta_i + \Delta_j - \Delta_k}{2}\right) \Gamma\left(\frac{\Delta_i + \Delta_k - \Delta_j}{2}\right) \Gamma\left(\frac{\Delta_j + \Delta_k - \Delta_i}{2}\right)}{4\pi^{d/4} \sqrt{2 \prod_{n \in \{i,j,k\}}^3 \Gamma(\Delta_n) \Gamma\left(\Delta_n + 1 - \frac{d}{2}\right)}} \Gamma\left(\frac{\Delta_i + \Delta_j + \Delta_k - d}{2}\right) \\ A_2 = & \left[\Delta_j \Delta_k + \frac{1}{2} (d - \Delta_i - \Delta_j - \Delta_k) (\Delta_j + \Delta_k - \Delta_i) \right] A_1 \end{aligned}$$

$$\partial_\mu (\Phi_i \Phi_j \partial^\mu \Phi_k + \Phi_i \Phi_k \partial^\mu \Phi_j - \Phi_j \Phi_k \partial^\mu \Phi_i) = 2 \Phi_i \partial_\mu \Phi_j \partial^\mu \Phi_k - (m_i^2 - m_j^2 - m_k^2) \Phi_i \Phi_j \Phi_k$$



$$\mathcal{L}'_{\text{eff}} \triangleq \mathcal{L}_{\text{eff}} + \gamma_{ijk} \left(-\Phi_i \partial_\mu \Phi_j \partial^\mu \Phi_k + \frac{m_i^2 - m_j^2 - m_k^2}{2} \Phi_i \Phi_j \Phi_k \right)$$

$$\Phi_i \partial_\mu \Phi_j \partial^\mu \Phi_k \mapsto \frac{m_i^2 - m_j^2 - m_k^2}{2} \Phi_i \Phi_j \Phi_k$$

$$\Delta_j \Delta_k + \frac{1}{2} (d - \Delta_i - \Delta_j - \Delta_k) (\Delta_j + \Delta_k - \Delta_i) = 0.$$

$$\begin{aligned} c_{ijk} &= -\Delta_j \Delta_k d_{ijk}, & \text{for } \Delta_i &= \Delta_j + \Delta_k, \\ c_{ijk} &= -(d - \Delta_j) \Delta_k d_{ijk}, & \text{for } \Delta_j &= \Delta_i + \Delta_k, \\ c_{ijk} &= -(d - \Delta_k) \Delta_j d_{ijk}, & \text{for } \Delta_k &= \Delta_i + \Delta_j. \end{aligned}$$

$$\langle \mathcal{O}_i \mathcal{O}_i \rangle \sim \eta^0, \langle \mathcal{O}_i^2 \mathcal{O}_i^2 \rangle \sim \eta^0, \langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle \sim \eta^{-1/2}, \langle \mathcal{O}_i^2 \mathcal{O}_j \mathcal{O}_k \rangle \sim \eta^0.$$

$$\Phi \leftrightarrow \mathcal{O}'_\Phi = \mathcal{O}_\Phi + c \mathcal{O}_{\Phi_1, \Phi_2} + \cdots$$

$$\left\langle \mathcal{O}_\Phi^{(s)}(x) [\mathcal{O}_{\Phi_1} \cdots \mathcal{O}_{\Phi_k}](y) \right\rangle = 0, k \geq 2$$

$$\begin{aligned} \mathcal{P}^{(3)} = & \frac{9}{8} (2\alpha_1\alpha_2\phi_2 + 2\alpha_1\alpha_3\phi_3 - \alpha_2\alpha_3\phi_4) - \frac{21}{8} \phi_2\phi_3\phi_4 \\ & + \frac{3}{4\sqrt{2}} \hat{\beta} (\alpha_2^2 - \alpha_3^2 - \phi_2^2 + \phi_3^2) \\ & + \frac{\sqrt{3}}{16\sqrt{14}} (\alpha_2^2 + \alpha_3^2) \left(\sqrt{917 + 29\sqrt{7}}\rho_1 - \sqrt{917 - 29\sqrt{7}}\rho_2 \right) \\ & - \frac{\sqrt{3}}{16\sqrt{2}} (\phi_2^2 + \phi_3^2) \left(\sqrt{371 + 107\sqrt{7}}\rho_1 + \sqrt{371 - 107\sqrt{7}}\rho_2 \right) \\ & + \frac{1}{2\sqrt{21}} \hat{\beta}^2 \left(\sqrt{217 + 79\sqrt{7}}\rho_1 - \sqrt{217 - 79\sqrt{7}}\rho_2 \right) \\ & - \frac{3}{56} \phi_4^2 \left(\sqrt{686 + 238\sqrt{7}}\rho_1 - \sqrt{686 - 238\sqrt{7}}\rho_2 \right) \\ & + \frac{\sqrt{3}}{2\sqrt{14}} \rho_1\rho_2 \left(\sqrt{35 + 13\sqrt{7}}\rho_1 - \sqrt{35 - 13\sqrt{7}}\rho_2 \right) \\ & - \frac{\sqrt{3}}{4\sqrt{14}} \alpha_1^2 \left(\sqrt{2891 - 517\sqrt{7}}\rho_1 - \sqrt{2891 + 517\sqrt{7}}\rho_2 \right) \\ & + \frac{1}{6\sqrt{42}} \left(\sqrt{36155 - 13261\sqrt{7}}\rho_1^3 + \sqrt{36155 + 13261\sqrt{7}}\rho_2^3 \right) \end{aligned}$$



$$\begin{aligned}
\mathcal{L}_{\text{kin}}^{(3)} = & -\frac{1}{2}(\phi_2\partial_\mu\alpha_1\partial^\mu\alpha_2 + \phi_3\partial_\mu\alpha_1\partial^\mu\alpha_3 - \phi_4\partial_\mu\alpha_2\partial^\mu\alpha_3) \\
& -\frac{1}{2}(\phi_2\partial_\mu\alpha_3 + \phi_3\partial_\mu\alpha_2 - \phi_4\partial_\mu\alpha_1)\partial^\mu\varphi \\
& +\frac{1}{2}(\phi_2\partial_\mu\phi_3\partial^\mu\phi_4 + \phi_3\partial_\mu\phi_2\partial^\mu\phi_4 + \phi_4\partial_\mu\phi_2\partial^\mu\phi_3) \\
& +\frac{1}{4\sqrt{14}}\left(\sqrt{7-\sqrt{7}}\rho_1 + \sqrt{7+\sqrt{7}}\rho_2\right) \\
& \times \left[(\partial_\mu\alpha_1)^2 + (\partial_\mu\alpha_2)^2 + (\partial_\mu\alpha_3)^2 + (\partial_\mu\phi_2)^2 + (\partial_\mu\phi_3)^2 + (\partial_\mu\varphi)^2 + (\partial_\mu\phi_4)^2\right] \\
& +\frac{1}{2\sqrt{14}}(\phi_2\partial_\mu\phi_2 + \phi_3\partial_\mu\phi_3 + \phi_4\partial_\mu\phi_4)\left(\sqrt{7-\sqrt{7}}\partial^\mu\rho_1 + \sqrt{7+\sqrt{7}}\partial^\mu\rho_2\right) \\
& +\frac{\sqrt{3}}{28}\left(\sqrt{7-\sqrt{7}}\rho_1 + \sqrt{7+\sqrt{7}}\rho_2\right)\partial_\mu\rho_1\partial^\mu\rho_2 \\
& +\frac{1}{56}\rho_1\left(\sqrt{35-11\sqrt{7}}(\partial_\mu\rho_1)^2 + \sqrt{21+3\sqrt{7}}(\partial_\mu\rho_2)^2\right) \\
& +\frac{1}{56}\rho_2\left(\sqrt{21-3\sqrt{7}}(\partial_\mu\rho_1)^2 + \sqrt{35+11\sqrt{7}}(\partial_\mu\rho_2)^2\right)
\end{aligned}$$

	frame 1	frame 2
$\alpha_2\alpha_3\phi_4, \phi_2\phi_3\phi_4$	$c_{ijk} \neq 0, d_{ijk} \neq 0$	$c_{ijk} = d_{ijk} = 0$
$\alpha_2\phi_3\varphi, \alpha_3\phi_2\varphi$	$c_{ijk} = 0, d_{ijk} \neq 0$	$c_{ijk} = d_{ijk} = 0$

$$\begin{aligned}
\mathcal{P}^{(3)}|_{\text{frame 2}} = & \frac{3}{2}\alpha_1(2\alpha_2\phi_2 + 2\alpha_3\phi_3 - \varphi\phi_4) + \frac{3}{4\sqrt{2}}\hat{\beta}(\alpha_2^2 - \alpha_3^2 - \phi_2^2 + \phi_3^2) \\
& +\frac{\sqrt{3}}{4}(\alpha_2^2 + \alpha_3^2 - \phi_2^2 - \phi_3^2)\left(\sqrt{7+\sqrt{7}}\rho_1 + \sqrt{7-\sqrt{7}}\rho_2\right) \\
& +\frac{1}{2\sqrt{21}}\hat{\beta}^2\left(\sqrt{217+79\sqrt{7}}\rho_1 - \sqrt{217-79\sqrt{7}}\rho_2\right) \\
& -\frac{\sqrt{3}}{4\sqrt{14}}\phi_4^2\left(\sqrt{77+29\sqrt{7}}\rho_1 - \sqrt{77-29\sqrt{7}}\rho_2\right) \\
& -\frac{\sqrt{3}}{4\sqrt{14}}\varphi^2\left(\sqrt{35-13\sqrt{7}}\rho_1 - \sqrt{35+13\sqrt{7}}\rho_2\right) \\
& +\frac{\sqrt{3}}{56}\rho_1\rho_2\left(\sqrt{2065+377\sqrt{7}}\rho_1 - \sqrt{2065-377\sqrt{7}}\rho_2\right) \\
& -\frac{\sqrt{3}}{4\sqrt{14}}\alpha_1^2\left(\sqrt{3296-523\sqrt{7}}\rho_1 - \sqrt{3296+523\sqrt{7}}\rho_2\right) \\
& +\frac{1}{168\sqrt{3}}\left(\sqrt{1659343-602999\sqrt{7}}\rho_1^3 + \sqrt{1659343+602999\sqrt{7}}\rho_2^3\right)
\end{aligned}$$

$$C_{\phi_2\phi_3\phi_4} = c_{\phi_2\phi_3\phi_4}A_1 + (d_{\phi_2\phi_3\phi_4} + d_{\phi_3\phi_2\phi_4} + d_{\phi_4\phi_2\phi_3})A_2 = 0.$$



$$\frac{1}{L} \left[3 + \frac{1}{2} (\alpha_{1,2,3}^2 + \beta_{1,2}^2) - \frac{1}{2\sqrt{6}} (\beta_1 (\phi_1^2 + \phi_2^2 - 2\phi_3^2) + \sqrt{3}\beta_2 (\phi_1^2 - \phi_2^2)) + (q) \right]$$

$$\frac{1}{L_{\text{LS}}} \left[3 + \frac{1}{2} \hat{\beta}^2 + \frac{3}{4} \phi_2 \phi_3 \phi_4 + \frac{9}{64} \phi_4^4 \right]$$

$$\{\alpha_2, \alpha_3, \phi_2, \phi_3\}, \{\phi_4, \varphi\} \in \mathcal{C}, \{\hat{\beta}\} \in \mathcal{F}, \{\rho_1, \alpha_1, \rho_2\} \in \mathcal{L}.$$

$$C_{\alpha_2\alpha_3\phi_4} = -\frac{2^{5/6}}{9} \frac{\gamma}{N}, C_{\alpha_2\phi_3\varphi} = C_{\alpha_3\phi_2\varphi} = -\frac{1}{9 \cdot 2^{2/3}} \frac{\gamma}{N}, C_{\phi_2\phi_3\phi_4} = 0.$$

$$C_{\alpha_2\alpha_2\hat{\beta}} = -C_{\alpha_3\alpha_3\hat{\beta}} = \frac{2 \cdot 2^{5/6}}{3} \frac{1}{N}, C_{\phi_2\phi_2\hat{\beta}} = -C_{\phi_3\phi_3\hat{\beta}} = -\frac{2 \cdot 2^{5/6}}{9} \frac{1}{N}.$$

$$C_{\alpha_2\alpha_2\rho_1} = C_{\alpha_3\alpha_3\rho_1} = C_{\alpha_2\phi_2\alpha_1} = C_{\alpha_3\phi_3\alpha_1} = \frac{\sqrt{1+\sqrt{7}}\Gamma\left(\frac{2-\sqrt{7}}{2}\right)\Gamma\left(\frac{1+\sqrt{7}}{2}\right)}{2^{-\frac{10}{3}+\sqrt{7}}3^{\frac{3}{2}}\pi^{\frac{1}{2}}} \frac{1}{N},$$

$$C_{\phi_2\phi_2\rho_1} = C_{\phi_3\phi_3\rho_1} = \frac{7-2\sqrt{7}}{3} \times C_{\alpha_2\alpha_2\rho_1},$$

$$C_{\alpha_2\alpha_2\rho_2} = C_{\alpha_3\alpha_3\rho_2} = -\frac{\sqrt{7-2\sqrt{7}}}{2\sqrt{14}} \times C_{\alpha_2\alpha_2\rho_1},$$

$$C_{\phi_2\phi_2\rho_2} = C_{\phi_3\phi_3\rho_2} = -\frac{\sqrt{7+2\sqrt{7}}}{2\sqrt{6}} \times C_{\alpha_2\alpha_2\rho_1}.$$

$$C_{\phi_4\phi_4\rho_1} = -\frac{\sqrt{29+11\sqrt{7}}\Gamma\left(\frac{5-\sqrt{7}}{2}\right)\Gamma\left(\frac{1+\sqrt{7}}{2}\right)^2\Gamma\left(\frac{3+\sqrt{7}}{2}\right)}{3 \cdot 2^{\frac{1}{6}}\sqrt{21}\Gamma(\sqrt{7})} \frac{1}{N}$$

$$C_{\phi_4\phi_4\rho_2} = -\frac{\sqrt{7-2\sqrt{7}}}{2\sqrt{14}} \times C_{\phi_4\phi_4\rho_1}$$

$$C_{\varphi\varphi\rho_1} = \frac{11-4\sqrt{7}}{12} \times C_{\phi_4\phi_4\rho_1}$$

$$C_{\varphi\varphi\rho_2} = -\frac{\sqrt{133+50\sqrt{7}}}{8\sqrt{42}} \times C_{\phi_4\phi_4\rho_1}$$

$$C_{\phi_4\varphi\alpha_1} = \frac{1}{2\sqrt{2}} \times C_{\phi_4\phi_4\rho_1}.$$

$$C_{\hat{\beta}\hat{\beta}\rho_1} = -\frac{2^{\frac{7}{3}}\sqrt{217+79\sqrt{7}}\Gamma\left(\frac{3-\sqrt{7}}{2}\right)\Gamma\left(\frac{1+\sqrt{7}}{2}\right)^3}{9\sqrt{21}\Gamma(\sqrt{7})\Gamma(1+\sqrt{7})} \frac{1}{N}$$

$$C_{\hat{\beta}\hat{\beta}\rho_2} = \frac{\sqrt{91-34\sqrt{7}}}{2\sqrt{42}} \times C_{\hat{\beta}\hat{\beta}\rho_1}$$



$$C_{\rho_1\rho_1\rho_1} = \frac{2^{\frac{1}{3}}\sqrt{1659343-602999\sqrt{7}}\Gamma\left(\frac{1+\sqrt{7}}{2}\right)^3\Gamma\left(\frac{-1+3\sqrt{7}}{2}\right)}{63\sqrt{3}(\Gamma(\sqrt{7})\Gamma(1+\sqrt{7}))^{3/2}}\frac{1}{N}$$

$$C_{\rho_1\rho_1\rho_2} = \frac{\sqrt{3(2777257+1042598\sqrt{7})}}{1514\sqrt{14}} \times C_{\rho_1\rho_1\rho_1}$$

$$C_{\rho_1\alpha_1\alpha_1} = \frac{\sqrt{1976297+635680\sqrt{7}}}{1514} \times C_{\rho_1\rho_1\rho_1}$$

$$C_{\rho_1\rho_2\rho_2} = -\frac{4963+1156\sqrt{7}}{42392} \times C_{\rho_1\rho_1\rho_1}$$

$$C_{\rho_2\alpha_1\alpha_1} = \frac{\sqrt{325648561+109746134\sqrt{7}}}{3028\sqrt{42}} \times C_{\rho_1\rho_1\rho_1}$$

$$C_{\rho_2\rho_2\rho_2} = \frac{\sqrt{3(7587462463+2704517498\sqrt{7})}}{84784\sqrt{2}} \times C_{\rho_1\rho_1\rho_1}$$

$$\frac{\langle\mathcal{O}_1(y)|\mathcal{C}(x)|\mathcal{O}_2(0)\rangle}{\langle Q^-Q^-\mathcal{O}_1(y)|\mathcal{C}(x)|Q^+Q^+\mathcal{O}_2(0)\rangle},$$

$$\langle\mathcal{O}'_{\pm}\mid\mathcal{O}'_{\pm}\rangle=4\left(\Delta\mp\frac{3}{2}q\right)\left(\Delta\mp\frac{3}{2}q-2\right)$$

$$|\rho_1\rangle=|0,0\rangle,|\phi_4\rangle=\frac{1}{\sqrt{2}}(|0,2\rangle+|0,-2\rangle)$$

$$|\alpha_1\rangle=\xi_1\left(\epsilon^{\dot{\alpha}\dot{\beta}}Q_{\dot{\alpha}}^-Q_{\dot{\beta}}^-|0,0\rangle+\epsilon^{\alpha\beta}Q_{\alpha}^+Q_{\beta}^+|0,0\rangle\right)$$

$$|\varphi\rangle=\xi_2\left(\epsilon^{\dot{\alpha}\dot{\beta}}Q_{\dot{\alpha}}^-Q_{\dot{\beta}}^-|0,2\rangle+\epsilon^{\alpha\beta}Q_{\alpha}^+Q_{\beta}^+|0,-2\rangle\right)$$

$$\langle\alpha_1\mid\alpha_1\rangle=8\xi_1^2\Delta_{\rho_1}(\Delta_{\rho_1}-2)=48\xi_1^2,\langle\varphi\mid\varphi\rangle=64\xi_2^2\Delta_{\phi_4}(\Delta_{\phi_4}-2)=192\xi_2^2$$

$$\xi_1=\frac{1}{\sqrt{48}},\xi_2=\frac{1}{\sqrt{192}}$$

$$\frac{\langle\rho_1|\phi_4|\phi_4\rangle}{\langle\alpha_1|\phi_4|\varphi\rangle}=\frac{1}{24\sqrt{2}\xi_1\xi_2}=2\sqrt{2}.$$

$$\frac{C_{\rho_1\phi_4\phi_4}}{C_{\alpha_1\phi_4\varphi}}=2\sqrt{2}$$

$$\frac{\langle\rho_1|\alpha_2|\alpha_2\rangle}{\langle\alpha_1|\alpha_2|\phi_2\rangle}=\frac{\langle\rho_1|\alpha_3|\alpha_3\rangle}{\langle\alpha_1|\alpha_3|\phi_3\rangle}=1$$

$$\frac{\langle\alpha_3|\alpha_2|\phi_4\rangle}{\langle\phi_3|\alpha_2|\varphi\rangle}=\frac{\langle\alpha_2|\alpha_3|\phi_4\rangle}{\langle\phi_2|\alpha_3|\varphi\rangle}=2\sqrt{2}.$$

$$\langle\phi_2|\phi_4|\phi_3\rangle=0$$

$$g_{\Delta,\ell}=g_{\Delta,\ell}+c_1g_{\Delta+1,\ell+1}+c_2g_{\Delta+1,\ell-1}+c_3g_{\Delta+2,\ell}$$



$$c_3 \equiv c_{\Delta, \ell}^{(\mathcal{M}_1 \mathcal{M}_2 \mathcal{M}_3 \mathcal{M}_4)} = \frac{C_{\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}''} C_{\mathcal{O}'' \mathcal{O}_3 \mathcal{O}_4}}{C_{\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}} C_{\mathcal{O} \mathcal{O}_3 \mathcal{O}_4}},$$

$$c_3 = \frac{1}{16} \frac{(\Delta + \ell)(\Delta - \ell - 2)}{(\Delta^2 - (\ell + 1)^2)}.$$

$$c_{\Delta=1+\sqrt{7}, \ell=0}^{(\bar{\mathcal{C}}\mathcal{C}\mathcal{C}\mathcal{C})} = \frac{7-2\sqrt{7}}{56} \stackrel{!}{=} \left(\frac{C_{\alpha_2 \alpha_2 \rho_2}}{C_{\alpha_2 \alpha_2 \rho_1}} \right)^2$$

$$\frac{C_{\alpha_2 \alpha_2 \rho_2}}{C_{\alpha_2 \alpha_2 \rho_1}} = \frac{C_{\alpha_3 \alpha_3 \rho_2}}{C_{\alpha_3 \alpha_3 \rho_1}} = \frac{C_{\phi_4 \phi_4 \rho_2}}{C_{\phi_4 \phi_4 \rho_1}}$$

$$c_3 = \frac{1}{16} \frac{(\Delta - 2)^2 (\Delta + \ell)(\Delta - \ell - 2)}{\Delta^2 (\Delta^2 - (\ell + 1)^2)}.$$

$$c_{\Delta=1+\sqrt{7}, \ell=0}^{(\mathcal{F}\mathcal{F}\mathcal{F})} = \frac{91-34\sqrt{7}}{168} \stackrel{!}{=} \left(\frac{C_{\hat{\beta} \hat{\beta} \rho_2}}{C_{\hat{\beta} \hat{\beta} \rho_1}} \right)^2.$$

$$c_3^{(\mathcal{L}\mathcal{L}\mathcal{L}\mathcal{L})} = \frac{\left[(\Delta + \ell)^2 - 8(\Delta - 1) \frac{C_{LLL}^{(2)}}{C_{LLL}} \right]^2}{16\Delta^2 (\Delta - \ell - 1)(\Delta - \ell - 2)(\Delta + \ell)(\Delta + \ell + 1)}.$$

$$c_3^{(\bar{\mathcal{C}}\mathcal{C}\mathcal{L}\mathcal{L})} = \frac{(\Delta + \ell)^2 - 8(\Delta - 1) \frac{C_{LLL}^{(2)}}{C_{LLL}}}{16\Delta (\Delta^2 - (\ell + 1)^2)}.$$

$$c_{\Delta=1+\sqrt{7}, \ell=0}^{(\bar{\mathcal{C}}\mathcal{C}\mathcal{L}\mathcal{L})} = \frac{C_{\alpha_2 \alpha_2 \rho_2} C_{\rho_2 \rho_1 \rho_1}}{C_{\alpha_2 \alpha_2 \rho_1} C_{\rho_1 \rho_1 \rho_1}}.$$

$$c_{\Delta=1+\sqrt{7}, \ell=0}^{(\mathcal{L}\mathcal{L}\mathcal{L}\mathcal{L})} = \frac{3(2777257 + 1042598\sqrt{7})}{32090744} \stackrel{!}{=} \left(\frac{C_{\rho_1 \rho_1 \rho_2}}{C_{\rho_1 \rho_1 \rho_1}} \right)^2.$$

$$C_{\alpha_1 \alpha_1 \varphi}, C_{\alpha_1 \phi_{2,3} \phi_{2,3}}, C_{\varphi \varphi \varphi}, C_{\varphi \phi_{2,3} \phi_{2,3}}$$

$$C_{\hat{\beta} \hat{\beta} \varphi}, C_{\hat{\beta} \alpha_{2,3} \phi_{2,3}}, C_{\rho_{1,2} \alpha_{2,3} \phi_{2,3}}, C_{\rho_{1,2} \rho_{1,2} \varphi}, C_{\rho_{1,2} \alpha_1 \phi_4}, C_{\alpha_1 \alpha_{2,3} \alpha_{2,3}}, C_{\varphi \phi_4 \phi_4}, C_{\varphi \alpha_{2,3} \alpha_{2,3}}, C_{\phi_4 \alpha_{2,3} \phi_{2,3}}$$

$$-\langle \mathcal{F}\mathcal{F}\mathcal{F} \rangle: C_{\hat{\beta} \hat{\beta} \hat{\beta}},$$

$$-\langle \mathcal{C}\mathcal{C}\mathcal{F} \rangle: C_{\alpha_2 \alpha_3 \hat{\beta}}, C_{\phi_2 \phi_3 \hat{\beta}},$$

$$-\langle \mathcal{C}\mathcal{C}\mathcal{L} \rangle: C_{\alpha_2 \alpha_3 \rho_{1,2}},$$

$$-\langle \mathcal{C}\mathcal{C}\mathcal{C} \rangle: C_{\alpha_1 \alpha_2 \phi_3}, C_{\alpha_1 \alpha_3 \phi_2}, C_{\alpha_2 \phi_2 \varphi}, C_{\alpha_3 \phi_3 \varphi}, C_{\alpha_2 \alpha_2 \phi_4}, C_{\alpha_3 \alpha_3 \phi_4}, C_{\phi_2 \phi_2 \phi_4}, C_{\phi_3 \phi_3 \phi_4}.$$



$$\begin{aligned}\tilde{\mathcal{L}} = & -R - \left[\rho^4 v^{-4} F_{\mu\nu}^{(1)} F^{(1)\mu\nu} + \rho^4 v^4 F_{\mu\nu}^{(2)} F^{(2)\mu\nu} + \rho^{-8} F_{\mu\nu}^{(3)} F^{(3)\mu\nu} \right] \\ & + 12(\partial_\mu \beta_1)^2 + 4(\partial_\mu \beta_2)^2 + 2 \sum_{j=1}^4 (\partial_\mu \phi_j)^2 \\ & + \frac{1}{2} \sinh^2(2\phi_1) \left(\partial_\mu \theta_1 + \left(A_\mu^{(1)} + A_\mu^{(2)} - A_\mu^{(3)} \right) \right)^2 \\ & + \frac{1}{2} \sinh^2(2\phi_2) \left(\partial_\mu \theta_1 + \left(A_\mu^{(1)} - A_\mu^{(2)} + A_\mu^{(3)} \right) \right)^2 \\ & + \frac{1}{2} \sinh^2(2\phi_3) \left(\partial_\mu \theta_1 + \left(-A_\mu^{(1)} + A_\mu^{(2)} + A_\mu^{(3)} \right) \right)^2 \\ & + \frac{1}{2} \sinh^2(2\phi_4) \left(\partial_\mu \theta_1 - \left(A_\mu^{(1)} + A_\mu^{(2)} + A_\mu^{(3)} \right) \right)^2 + \tilde{\mathcal{P}},\end{aligned}$$

$$\tilde{\mathcal{P}} = \frac{1}{2L^2} \left[\sum_{j=1}^4 \left(\frac{\partial W}{\partial \phi_j} \right)^2 + \frac{1}{6} \left(\frac{\partial W}{\partial \beta_1} \right)^2 + \frac{1}{2} \left(\frac{\partial W}{\partial \beta_2} \right)^2 \right] - \frac{4}{3L^2} W^2,$$

$$W = -\frac{1}{2\rho^2 v^2} [(1 + v^4 - v^2 \rho^6) \cosh(2\phi_1) + (-1 + v^4 + v^2 \rho^6) \cosh(2\phi_2) \\ + (1 - v^4 + v^2 \rho^6) \cosh(2\phi_3) + (1 + v^4 + v^2 \rho^6) \cosh(2\phi_4)].$$

$$\beta_{1,2} = \phi_{1,2,3,4} = 0, \text{ with } \tilde{\mathcal{P}} = -\frac{12}{L^2}$$

$$\beta_{1,2}: m^2 L^2 = -4 \leftrightarrow \Delta = 2$$

$$\phi_{1,2,3,4}: m^2 L^2 = -3 \leftrightarrow \Delta = 3$$

$$A_\mu^{(1,2,3)}: m^2 L^2 = 0 \leftrightarrow \Delta = 3$$

$$C_{\beta_1 \beta_1 \beta_1} = -C_{\beta_1 \beta_2 \beta_2} = -\sqrt{\frac{2}{3}} \frac{1}{N},$$

$$\beta_2 = \phi_{2,3,4} = 0, \beta_1 = -\frac{\log 2}{6}, \phi_1 = \frac{\log 3}{2}, \text{ with } \tilde{\mathcal{P}} = -\frac{12}{L_{\text{LS}}^2},$$

$$\hat{\beta}: m^2 L_{\text{LS}}^2 = -4 \leftrightarrow \Delta = 2,$$

$$\phi_{2,3}: m^2 L_{\text{LS}}^2 = -\frac{15}{4} \leftrightarrow \Delta = \frac{5}{2},$$

$$\phi_4: m^2 L_{\text{LS}}^2 = -3 \leftrightarrow \Delta = 3,$$

$$\rho_1: m^2 L_{\text{LS}}^2 = 4 - 2\sqrt{7} \leftrightarrow \Delta = 1 + \sqrt{7},$$

$$\rho_2: m^2 L_{\text{LS}}^2 = 4 + 2\sqrt{7} \leftrightarrow \Delta = 3 + \sqrt{7}.$$

$$\tilde{A}_\mu^{(1,2)}: m^2 L_{\text{LS}}^2 = 0 \leftrightarrow \Delta = 3,$$

$$\tilde{A}_\mu^{(3)}: m^2 L_{\text{LS}}^2 = 6 \leftrightarrow \Delta = 2 + \sqrt{7},$$



$$\tilde{\mathcal{P}}^{(3)} = \frac{1}{2!} \left(2 \sqrt{\frac{2}{3}} \beta_1 \beta_2^2 \right) + \frac{1}{3!} \left(-2 \sqrt{\frac{2}{3}} \beta_1^3 \right)$$

$$\tilde{\mathcal{P}}^{(4)} \supset \frac{1}{2! \cdot 2!} \left(-\frac{2}{3} \beta_1^2 \beta_2^2 \right) + \frac{1}{4!} \left(-2(\beta_1^4 + \beta_2^4) \right)$$

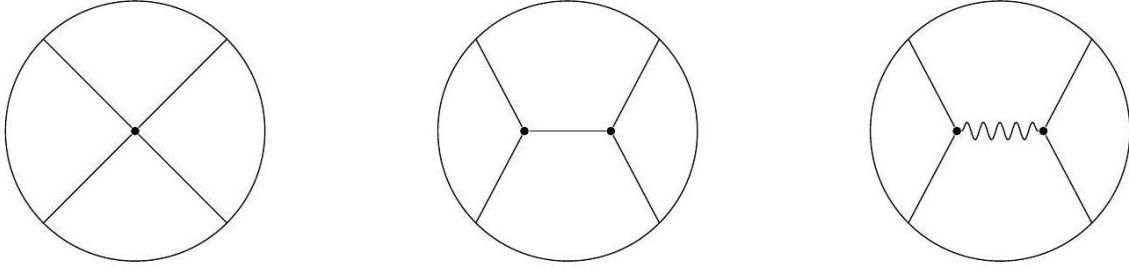


Figura 2. Computación holográfica.

$$\mathcal{P}_{\text{UV}}^{(3)} = \frac{1}{2!} \left(2 \sqrt{\frac{2}{3}} \beta_1 (\alpha_1^2 + \alpha_2^2 - 2\alpha_3^2 + \beta_2^2) + 2\sqrt{2} \beta_2 (\alpha_1^2 - \alpha_2^2) \right) + \frac{1}{3!} \left(-2 \sqrt{\frac{2}{3}} \beta_1^3 \right)$$

$$C_{\alpha_1 \alpha_1 \beta_1} = C_{\alpha_2 \alpha_2 \beta_1} = -C_{\beta_1 \beta_1 \beta_1} = C_{\beta_1 \beta_2 \beta_2} = \sqrt{\frac{2}{3}} \frac{1}{N}$$

$$C_{\alpha_1 \alpha_1 \beta_2} = -C_{\alpha_2 \alpha_2 \beta_2} = \sqrt{2} \frac{1}{N}, C_{\alpha_3 \alpha_3 \beta_1} = -2 \sqrt{\frac{2}{3}} \frac{1}{N}$$

$$\mathcal{O}_{\alpha_i} \sim \text{Tr}(Z_i Z_i + \tilde{Z}_i \tilde{Z}_i), i = 1, 2, 3$$

$$\mathcal{O}_{\beta_1} \sim \text{Tr}(Z_1 \tilde{Z}_1 + Z_2 \tilde{Z}_2 - 2Z_3 \tilde{Z}_3)$$

$$\mathcal{O}_{\beta_2} \sim \text{Tr}(Z_1 \tilde{Z}_1 - Z_2 \tilde{Z}_2)$$

$$Z_1 = \frac{X_1 + iX_2}{\sqrt{2}}, Z_2 = \frac{X_3 + iX_4}{\sqrt{2}}, Z_3 = \frac{X_5 + iX_6}{\sqrt{2}}$$

$$\left\langle (X_I)_a^b(x_1) (X_J)_c^d(x_2) \right\rangle = \delta_{IJ} \left(\delta_a^d \delta_c^b - \frac{1}{N} \delta_a^b \delta_c^d \right) g_{12}, g_{12} \equiv \frac{1}{(x_1 - x_2)^2}$$

$$\left\langle (Z_i)_a^b(x_1) (\tilde{Z}_j)_c^d(x_2) \right\rangle = \delta_{ij} \left(\delta_a^d \delta_c^b - \frac{1}{N} \delta_a^b \delta_c^d \right) g_{12}$$

$$\langle \text{Tr}(Z_i \tilde{Z}_i) \text{Tr}(Z_j \tilde{Z}_j) \rangle = \delta_{ij} (N^2 - 1) g_{12}^2$$

$$\langle \text{Tr}(Z_i Z_i) \text{Tr}(\tilde{Z}_j \tilde{Z}_j) \rangle = 2 \delta_{ij} (N^2 - 1) g_{12}^2$$

$$\langle \text{Tr}(Z_i \tilde{Z}_i) \text{Tr}(Z_j \tilde{Z}_j) \text{Tr}(Z_k \tilde{Z}_k) \rangle = 2 \delta_{ij} \delta_{ik} (N^2 - 1) g_{12} g_{13} g_{23}$$

$$\langle \text{Tr}(Z_i \tilde{Z}_i) \text{Tr}(Z_j Z_j) \text{Tr}(\tilde{Z}_k \tilde{Z}_k) \rangle = 4 \delta_{ij} \delta_{ik} (N^2 - 1) g_{12} g_{13} g_{23}$$

$$\mathcal{O}_{\alpha_i} = \frac{1}{2\sqrt{N^2-1}} \text{Tr}(Z_i Z_i + \tilde{Z}_i \tilde{Z}_i), i = 1, 2, 3$$

$$\mathcal{O}_{\beta_1} = \frac{1}{\sqrt{6(N^2-1)}} \text{Tr}(Z_1 \tilde{Z}_1 + Z_2 \tilde{Z}_2 - 2Z_3 \tilde{Z}_3)$$

$$\mathcal{O}_{\beta_2} = \frac{1}{\sqrt{2(N^2-1)}} \text{Tr}(Z_1 \tilde{Z}_1 - Z_2 \tilde{Z}_2)$$

$$C_{\alpha_1 \alpha_1 \beta_1}^{(\mathcal{N}=4)} = C_{\alpha_2 \alpha_2 \beta_1}^{(\mathcal{N}=4)} = -C_{\beta_1 \beta_1 \beta_1}^{(\mathcal{N}=4)} = C_{\beta_1 \beta_2 \beta_2}^{(\mathcal{N}=4)} = \sqrt{\frac{2}{3}} \frac{1}{\sqrt{N^2-1}}$$

$$C_{\alpha_1 \alpha_1 \beta_2}^{(\mathcal{N}=4)} = -C_{\alpha_2 \alpha_2 \beta_2}^{(\mathcal{N}=4)} = \sqrt{2} \frac{1}{\sqrt{N^2-1}}, C_{\alpha_3 \alpha_3 \beta_1}^{(\mathcal{N}=4)} = -2 \sqrt{\frac{2}{3}} \frac{1}{\sqrt{N^2-1}}.$$

$$\tilde{\mathcal{P}}^{(3)} = \frac{1}{2!} \left(-2 \sqrt{\frac{2}{3}} \beta_1 \beta_2^2 \right) + \frac{1}{3!} \left(2 \sqrt{\frac{2}{3}} \beta_1^3 \right),$$

$$\tilde{\mathcal{P}}^{(4)} \supset \frac{1}{2! \cdot 2!} \left(-\frac{2}{3} \beta_1^2 \beta_2^2 \right) + \frac{1}{4!} \left(-2(\beta_1^4 + \beta_2^4) \right).$$

$$\mathcal{A}^{(c)} = g_{12}^2 g_{34}^2 \cdot \frac{\pi^2}{2} u^2 \bar{D}_{2222}$$

$$\mathcal{A}_s^{(s)} = g_{12}^2 g_{34}^2 \cdot \frac{\pi^2}{8} u \bar{D}_{1122}$$

$$\mathcal{A}_s^{(g)} = g_{12}^2 g_{34}^2 \cdot \frac{2\pi^2}{3} u (-2\bar{D}_{1122} + 3\bar{D}_{2123} + 3\bar{D}_{2132} - 3\bar{D}_{3133})$$

$$\langle \mathcal{O}_{\beta_1} \mathcal{O}_{\beta_1} \mathcal{O}_{\beta_1} \mathcal{O}_{\beta_1} \rangle = \frac{1}{\eta(\sqrt{2\pi^2})^4} \left([\mathcal{A}_s^{(g)} + \mathcal{A}_t^{(g)} + \mathcal{A}_u^{(g)}] + \frac{8}{3} [\mathcal{A}_s^{(s)} + \mathcal{A}_t^{(s)} + \mathcal{A}_u^{(s)}] + 2[\mathcal{A}^{(c)}] \right),$$

$$\langle \mathcal{O}_{\beta_2} \mathcal{O}_{\beta_2} \mathcal{O}_{\beta_2} \mathcal{O}_{\beta_2} \rangle = \langle \mathcal{O}_{\beta_1} \mathcal{O}_{\beta_1} \mathcal{O}_{\beta_1} \mathcal{O}_{\beta_1} \rangle$$

$$\langle \mathcal{O}_{\beta_1} \mathcal{O}_{\beta_1} \mathcal{O}_{\beta_2} \mathcal{O}_{\beta_2} \rangle = \frac{1}{\eta(\sqrt{2\pi^2})^4} \left([\mathcal{A}_s^{(g)}] - \frac{8}{3} [\mathcal{A}_s^{(s)}] + \frac{8}{3} [\mathcal{A}_t^{(s)} + \mathcal{A}_u^{(s)}] + \frac{2}{3} [\mathcal{A}^{(c)}] \right).$$

$$\mathcal{O}_2(x,y)=y^Iy^J\text{Tr}(X_I X_J), y\cdot y\equiv y_Iy^I=0$$

$$\langle \mathcal{O}_2(x_1,y_1)\mathcal{O}_2(x_2,y_2)\mathcal{O}_2(x_3,y_3)\mathcal{O}_2(x_4,y_4)\rangle = \frac{y_{12}^4y_{34}^4}{x_{12}^4x_{34}^4}\times \mathcal{G}(u,v;\sigma,\tau),$$

$$u=\frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2}, v=\frac{x_{14}^2x_{23}^2}{x_{13}^2x_{24}^2}, \sigma=\frac{y_{13}^2y_{24}^2}{y_{12}^2y_{34}^2}, \tau=\frac{y_{14}^2y_{23}^2}{y_{12}^2y_{34}^2}.$$

$$\mathcal{G}_{\text{sugra}}(u,v;\sigma,\tau)=16\left(\sigma u+\tau\frac{u}{v}+\sigma\tau\frac{u^2}{v}\right)+\mathcal{I}\times(-16u^2\bar{D}_{2422}),$$

$$\mathcal{I}=\tau+(1-\sigma-\tau)v+(\tau^2-\tau-\sigma\tau)u+(\sigma^2-\sigma-\sigma\tau)uv+\sigma v^2+\sigma\tau u^2.$$

$$\mathcal{O}_2(x,y_{\{k,l\}})+\mathcal{O}_2(x,\bar{y}_{\{k,l\}})=2\text{Tr}(X_k^2-X_l^2).$$



$$\begin{aligned}\mathcal{O}_{\beta_1} &= \frac{1}{4\sqrt{6(N^2-1)}} \sum_{y=\mathcal{Y}_{\{1,5\}}, \mathcal{Y}_{\{2,5\}}, \mathcal{Y}_{\{3,6\}}, \mathcal{Y}_{\{4,6\}}} (\mathcal{O}_2(x,y) + \mathcal{O}_2(x,\bar{y})), \\ \mathcal{O}_{\beta_2} &= \frac{1}{4\sqrt{2(N^2-1)}} \sum_{y=\mathcal{Y}_{\{1,3\}}, \mathcal{Y}_{\{2,4\}}} (\mathcal{O}_2(x,y) + \mathcal{O}_2(x,\bar{y})).\end{aligned}$$

$$\begin{aligned}\langle \mathcal{O}_{\beta_1} \mathcal{O}_{\beta_1} \mathcal{O}_{\beta_1} \mathcal{O}_{\beta_1} \rangle &= \langle \mathcal{O}_{\beta_2} \mathcal{O}_{\beta_2} \mathcal{O}_{\beta_2} \mathcal{O}_{\beta_2} \rangle = \frac{1}{x_{12}^4 x_{34}^4} \left[\left(u + \frac{u}{v} + \frac{u^2}{v} \right) - (u+v+1)^2 u^2 \bar{D}_{2422} \right], \\ \langle \mathcal{O}_{\beta_1} \mathcal{O}_{\beta_1} \mathcal{O}_{\beta_2} \mathcal{O}_{\beta_2} \rangle &= \frac{1}{x_{12}^4 x_{34}^4} \left[\frac{1}{3} \left(u + \frac{u}{v} + \frac{u^2}{v} \right) - \frac{1}{3} (u^2 - 2uv - 2u + v^2 + 10v + 1) u^2 \bar{D}_{2422} \right],\end{aligned}$$

$$c_{\Delta,\ell}^{(\overline{\mathcal{C}}\mathcal{F}\mathcal{F})} = \sqrt{c_{\Delta,\ell}^{(\overline{\mathcal{C}}\mathcal{C}\mathcal{C})} c_{\Delta,\ell}^{(\mathcal{F}\mathcal{F}\mathcal{F}\mathcal{F})}}$$

$$\langle \mathcal{O}_2(x_1,y_1)\mathcal{O}_2(x_2,y_2)\rangle=2(N^2-1)\frac{(y_1\cdot y_2)^2}{x_{12}^4}$$

$$c_T=\frac{64}{3\pi}\sqrt{2k_{CS}}N^{\frac{3}{2}}+\mathcal{O}(N^{1/2})$$

$$\frac{\ell^6}{\ell_{11}^6} = \left(\frac{3\pi c_T k_{CS}}{2^{11}}\right)^{\frac{2}{3}} + \mathcal{O}(c_T^0) = \frac{N k_{CS}}{8} + \mathcal{O}(N^0)$$

$$\mathcal{O}_k(x,t) \equiv \mathcal{O}_k^{I_1 \dots I_k} t_{I_1} \dots t_{I_k}$$

$$p_j=\frac{1}{\sqrt{2}}\epsilon_j\omega_j(1+z_j\bar{z}_j,z_j+\bar{z}_j,-i(z_j-\bar{z}_j),1-z_j\bar{z}_j)$$

$$\mathcal{C}_n^{\Delta_1,\ldots,\Delta_n}\big(\{u_j,z_j,\bar{z}_j\}^{\epsilon_j}\big)=\int_0^{+\infty}\prod_{j=1}^n\frac{d\omega_j}{2\pi}(-i\epsilon_j\omega_j)^{\Delta_j-1}e^{-i\epsilon_j\omega_ju_j}\mathcal{A}_n\big(\{\omega_j,z_j,\bar{z}_j\}^{\epsilon_j}\big)$$

$$\mathcal{C}_n^{\Delta_1,\ldots,\Delta_n}\big(\{u_j,z_j,\bar{z}_j\}^{\epsilon_j}\big)\equiv\left\langle\prod_{j=1}^n\Phi_{\Delta_j}^{\epsilon_j}(u_j,z_j,\bar{z}_j)\right\rangle$$

$$\mathcal{C}_n^{1,\ldots,1}\big(\{u_j,z_j,\bar{z}_j\}^{\epsilon_j}\big)=\int_0^{+\infty}\prod_{j=1}^n\frac{d\omega_j}{2\pi}e^{-i\epsilon_j\omega_ju_j}\mathcal{A}_n\big(\{\omega_j,z_j,\bar{z}_j\}^{\epsilon_j}\big)$$

$$ds_{AdS_4}^2=-\frac{r^2}{\ell^2}du^2-2dudr+2r^2dzd\bar{z}$$

$$ds_{\mathbb{R}^{3,1}}^2=-2dudr+2r^2dzd\bar{z}$$

$$ds_{\delta AdS_4}^2=-\frac{du^2}{\ell^2}+2dzd\bar{z}$$



$$ds_j^2=0du^2+2dzd\bar{z}$$

$$c_{\rm boundary} \equiv \frac{1}{\ell_{\rm bulk}}$$

$$\langle \mathcal{O}_\Delta(x_1)\mathcal{O}_\Delta(x_2)\rangle=\frac{\mathcal{N}_2}{(x_{12}^2+i\epsilon)^\Delta},$$

$$\lim_{c\rightarrow 0} c^{2\Delta-2}\langle \mathcal{O}_\Delta(x_1)\mathcal{O}_\Delta(x_2)\rangle=\frac{\mathcal{N}_2\delta^2(z_{12})}{2(\Delta-1)(-u_{12}+i\varepsilon)^{2\Delta-2}}\propto \langle \Phi_\Delta^{\epsilon_1}\Phi_\Delta^{\epsilon_2}\rangle,$$

$$\left\langle \mathcal{O}_{\Delta_1}(x_1)\mathcal{O}_{\Delta_2}(x_2)\mathcal{O}_{\Delta_3}(x_3)\right\rangle_K=\frac{\mathcal{N}_3}{c}\frac{1}{(x_{12}^2+i\varepsilon)^{\Delta_{12}}(x_{23}^2+i\varepsilon)^{\Delta_{23}}(x_{13}^2+i\varepsilon)^{\Delta_{13}}}.$$

$$\begin{aligned}&\lim_{c\rightarrow 0} c^{3-\sum_{j=1}^3\Delta_j}\langle \mathcal{O}_{\Delta_1}(u_1,z_1,\bar{z}_1)\mathcal{O}_{\Delta_2}(u_2,z_2,\bar{z}_2)\mathcal{O}_{\Delta_3}(u_3,z_3,\bar{z}_3)\rangle\\&=\frac{\tilde{\mathcal{N}}_3\delta(\bar{z}_{12})\delta(\bar{z}_{23})\Theta(z_{12}z_{31})\Theta(z_{13}z_{23})z_{12}^{\Delta_3-2}z_{23}^{\Delta_1}z_{13}^{\Delta_2-2}}{(u_1z_{23}+u_2z_{31}+u_3z_{12}+i\mathrm{sign}z_{23}\varepsilon)^{\sum_{j=1}^3\Delta_j-4}}\propto \langle \Phi_{\Delta_1}\Phi_{\Delta_2}\Phi_{\Delta_3}\rangle\end{aligned}$$

$$\langle \mathcal{O}_{\Delta_1}(x_1)\mathcal{O}_{\Delta_2}(x_2)\mathcal{O}_{\Delta_3}(x_3)\mathcal{O}_{\Delta_4}(x_4)\rangle\propto \bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4}(U,V)$$

$$U=\frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2}=Z\bar{Z}\,,V=\frac{x_{23}^2x_{14}^2}{x_{13}^2x_{24}^2}=(1-Z)(1-\bar{Z})$$

$$\bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4}(U,V)\stackrel{Z\rightarrow\bar{Z}}{\rightarrow}\hat{\Phi}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4}^{l,s}\equiv\mathcal{K}_\Delta\frac{Z^{\Delta_3+\Delta_4-2}(1-Z)^{\Delta_1+\Delta_4-2}}{(Z-\bar{Z})^{\sum_{i=1}^4\Delta_i-3}}.$$

$$\lim_{c\rightarrow 0}\hat{\Phi}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4}^{l,s}=\mathcal{R}(u_i,z_i)\delta(z-\bar{z})$$

$$\begin{aligned}&\lim_{c\rightarrow 0} c^{4-\Sigma_{\Delta}}\langle \mathcal{O}_{\Delta_1}(x_1)\mathcal{O}_{\Delta_2}(x_2)\mathcal{O}_{\Delta_3}(x_3)\mathcal{O}_{\Delta_4}(x_4)\rangle\\&=\mathcal{N}\left(\frac{|z_{23}|^2}{|z_{34}|^2|z_{24}|^2}\right)^{\frac{4-\Sigma_{\Delta}}{2}}\frac{z^{2-\Delta_1-\Delta_2}(1-z)^{\Delta_1+\Delta_4-2}\delta(z-\bar{z})}{\mathcal{U}^{\sum_{i=1}^4\Delta_i-4}}\propto \left\langle \Phi_{\Delta_1}^{\epsilon_1}\Phi_{\Delta_2}^{\epsilon_2}\Phi_{\Delta_3}^{\epsilon_3}\Phi_{\Delta_4}^{\epsilon_4}\right\rangle,\end{aligned}$$

$$\mathcal{U}=u_4-u_1z\left|\frac{z_{24}}{z_{12}}\right|^2+u_2\frac{1-z}{z}\left|\frac{z_{34}}{z_{23}}\right|^2-u_3\frac{1}{1-z}\left|\frac{z_{14}}{z_{13}}\right|^2$$

$$s=\sum_{k\geq 0}Y_k^{(7)}s_k=\sum_{k\geq 0}\frac{s_k}{\ell^k}\mathcal{C}_{I_1\dots I_k}^{(k)}Z^{I_1}\dots Z^{I_k}$$

$$S=\frac{243}{\kappa^2}\int_{\text{AdS}_4}d^4y\sqrt{-\bar{g}_4}\Bigg\{\sum_{k\geq 2}\frac{(2\ell)^7}{2}A_k s_k(\Box_{AdS}-m_k^2)s_k\langle\mathcal{C}^{(k)}\mathcal{C}^{(k)}\rangle\\+\sum_{k_i\geq 2}\frac{(2\ell)^5}{3}\langle\mathcal{C}^{(k_1)}\mathcal{C}^{(k_2)}\mathcal{C}^{(k_3)}\rangle g_{123}s_{k_1}s_{k_2}s_{k_3}\Bigg\}$$



$$A_k = \frac{4\pi^4 k(k-1)}{3 \times 2^k (k+1)(k+2)^2}, m_k^2 = \frac{k(k-6)}{4\ell^2}$$

$$g_{123} = \frac{192\pi^4(\alpha^2-9)(\alpha^2-1)(\alpha+2)}{(2\alpha+6)!!} \prod_{i=1}^3 \frac{k_i!}{(k_i+2)\Gamma(\alpha_i)}$$

$$\alpha_i=\frac{1}{2}\sum_{j=1}^3k_j-k_i,\alpha=\frac{1}{2}\sum_{i=1}^3k_i$$

$$S_{int}=\int_{\partial\mathrm{AdS}_4}d^3y w_k s_k^{(0)}\mathcal{O}_k$$

$$\mathcal{C}_{I_1\ldots I_k}^k=t_{I_1}\ldots t_{I_k}$$

$$\langle \mathcal{C}^{(k_1)} \mathcal{C}^{(k_2)} \rangle \equiv \mathcal{C}_{I_1 \ldots I_{k_1}}^{(k_1)} \mathcal{C}^{(k_2) I_1 \ldots I_{k_2}} = t_{12}^{k_1} \delta_{k_1, k_2}$$

$$\langle \mathcal{O}_{k_1}(x_1,t_1)\mathcal{O}_{k_2}(x_2,t_2)\rangle=243A_k\frac{(2\ell)^7}{\kappa^2}\frac{(k-3)}{\pi^{\frac{3}{2}}}\frac{\Gamma(\frac{k}{2})}{\Gamma(\frac{k-3}{2})}\frac{w_{k_1}^2t_{12}^{k_1}\delta_{k_1,k_2}}{(x_{12}^2)^{k_1}}$$

$$\langle \mathcal{O}_{k_1}(x_1,t_1)\mathcal{O}_{k_2}(x_2,t_2)\rangle=\frac{\delta_{k_1,k_2}t_{12}^{k_1}}{(x_{12}^2)^{\frac{k_1}{2}}}$$

$$w_k=\frac{(2\pi)^{\frac{3}{2}}\sqrt{2}\ell\left(\frac{\ell_{11}}{2\ell}\right)^{\frac{9}{2}}}{9}\frac{(k+2)}{(k-3)(k-1)}\frac{\sqrt{\Gamma(2+k)}}{\Gamma(\frac{k}{2}+1)}$$

$$\langle \mathcal{O}_{k_1}\mathcal{O}_{k_2}\mathcal{O}_{k_3}\rangle=\left(\frac{\ell_{11}}{\ell}\right)^{\frac{9}{2}}R_{k_1,k_2,k_3}\frac{t_{12}^{\alpha_3}t_{23}^{\alpha_1}t_{13}^{\alpha_2}}{x_{12}^{\alpha_3}x_{23}^{\alpha_1}x_{13}^{\alpha_2}}$$

$$\left(\frac{\ell_{11}}{\ell}\right)^{\frac{9}{2}}R_{k_1,k_2,k_3}=\frac{7776(2\ell)^6}{\ell_{11}^9(2\pi)^8}\Gamma\Big(\frac{\alpha-3}{2}\Big)\prod_{i=1}^3\frac{\Gamma\Big(\frac{\alpha_i}{2}\Big)w_{k_i}}{\Gamma\Big(\frac{k_i-3}{2}\Big)}g_{123}.$$

$$\left(\frac{\ell_{11}}{\ell}\right)^{\frac{9}{2}}R_{k_1,k_2,k_3}=\frac{\pi}{2^{\frac{5}{2}}}\left(\frac{\ell_{11}}{\ell}\right)^{\frac{9}{2}}\frac{2^{-\alpha}}{\Gamma\left(1+\frac{\alpha}{2}\right)}\prod_{i=1}^3\frac{\sqrt{\Gamma(k_i+2)}}{\Gamma\left(\frac{\alpha_i+1}{2}\right)}=\frac{\pi}{N^{\frac{3}{4}}}\frac{2^{-\alpha-\frac{1}{4}}}{\Gamma\left(1+\frac{\alpha}{2}\right)}\prod_{i=1}^3\frac{\sqrt{\Gamma(k_i+2)}}{\Gamma\left(\frac{\alpha_i+1}{2}\right)}.$$

$$\langle \mathcal{C}^{k_1}\mathcal{C}^{k_2}\mathcal{C}^{k_3}\rangle = t_{12}^{\alpha_3}t_{23}^{\alpha_1}t_{13}^{\alpha_2}$$

$$\langle \mathcal{O}_{k_1} \ldots \mathcal{O}_{k_4} \rangle = \prod_{i < j} \left(\frac{t_{ij}^2}{x_{ij}^2} \right)^{\frac{\gamma_{ij}^0}{2}} \left(\frac{t_{12}^2 t_{34}^2}{x_{12}^2 x_{34}^2} \right)^{\frac{\varepsilon}{2}} \mathcal{G}_{k_1, \ldots, k_4}(U, V, \sigma, \tau).$$

$$U=\frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2}=Z\bar{Z},V=\frac{x_{14}^2x_{23}^2}{x_{13}^2x_{24}^2}=(1-Z)(1-\bar{Z}),\sigma=\frac{t_{13}t_{24}}{t_{12}t_{34}},\tau=\frac{t_{14}t_{23}}{t_{12}t_{34}}.$$



$$\varepsilon = \begin{cases} \frac{k_1 + k_2 + k_3 - k_4}{2} & \text{Case I : } k_1 + k_4 \geq k_2 + k_3 \\ k_1 & \text{Case II : } k_1 + k_4 < k_2 + k_3 \end{cases}$$

$$\gamma_{12}^0 = \gamma_{13}^0 = 0, \quad \gamma_{34}^0 = \frac{\kappa_s}{2}, \gamma_{24}^0 = \frac{\kappa_u}{2}$$

$$\text{Case I: } \gamma_{14}^0 = \frac{\kappa_t}{2}, \quad \gamma_{23}^0 = 0, \quad \text{Case II: } \gamma_{14}^0 = 0, \gamma_{23}^0 = \frac{\kappa_t}{2}$$

$$\kappa_s = |k_1 + k_2 - k_3 - k_4|, \kappa_t = |k_1 + k_4 - k_2 - k_3|, \kappa_u = |k_2 + k_4 - k_1 - k_3|.$$

$$\mathcal{G}_{k_1,\dots,k_4} = \mathcal{G}_{k_1,\dots,k_4}^0 + \frac{1}{c_T} \mathcal{G}_{k_1,\dots,k_4}^R + \dots$$

$$\mathcal{G}_{k_1,k_2,k_3,k_4}^R = \mathcal{G}_{k_1,k_2,k_3,k_4,s}^R + \mathcal{G}_{k_1,k_2,k_3,k_4,t}^R + \mathcal{G}_{k_1,k_2,k_3,k_4,u}^R$$

$$\mathcal{G}_{k_1,\dots,k_4}^c(U,V,\sigma,\tau) \equiv \mathcal{G}_{k_1,\dots,k_4}(U,V,\sigma,\tau) - \mathcal{G}_{k_1,\dots,k_4}^0(U,V,\sigma,\tau)$$

$$\mathcal{G}_{k_1,\dots,k_4}^c(U,V,\sigma,\tau) = \int_{-i\infty}^{i\infty} \frac{dsdt}{(4\pi i)^2} U^{\frac{s}{2}-a_s} V^{\frac{t}{2}-a_t} \mathcal{M}_{k_1,\dots,k_4}(s,t;\sigma,\tau) \Gamma_{\{k_i\}}$$

$$\Gamma_{\{k_i\}} = \Gamma\left(\frac{k_1+k_2}{4}-\frac{s}{2}\right)\Gamma\left(\frac{k_3+k_4}{4}-\frac{s}{2}\right)\Gamma\left(\frac{k_1+k_4}{2}-\frac{t}{2}\right)$$

$$\Gamma\left(\frac{k_2+k_3}{4}-\frac{t}{2}\right)\Gamma\left(\frac{k_1+k_3}{4}-\frac{u}{2}\right)\Gamma\left(\frac{k_2+k_4}{4}-\frac{u}{2}\right)$$

$$a_s = \frac{1}{4}(k_1+k_2) - \frac{1}{2}\varepsilon, a_t = \frac{1}{4}\text{Min}\{k_1+k_4, k_2+k_3\}, s+t+u = \frac{1}{2}\sum_{i=1}^4 k_i.$$

$$\bar{D}_{\Delta_1\Delta_2\Delta_3\Delta_4}(U,V) = \int_{-i\infty}^{i\infty} \frac{dj_1dj_2}{(2\pi i)^2} U^{j_1} V^{j_2} \Gamma(j_1+j_2+\Delta_2) \Gamma(j_1+j_2+\Delta-\Delta_4)$$

$$\times \Gamma(-j_1) \Gamma(-j_2) \Gamma(-j_1-\Delta+\Delta_3+\Delta_4) \Gamma(-j_2+\Delta-\Delta_2-\Delta_3)$$

$$\mathcal{G}_{2,2,2,2}(U,V,\sigma,\tau) \equiv \int_{-i\infty}^{i\infty} \frac{dsdt}{(4\pi i)^2} U^{\frac{s}{2}} V^{\frac{t}{2}-1} \mathcal{M}_{2,2,2,2} \Gamma^2\left(\frac{2-s}{2}\right) \Gamma^2\left(\frac{2-t}{2}\right) \Gamma^2\left(\frac{s+t-2}{2}\right)$$

$$\mathcal{G}_{2,2,2,2,s} = -\frac{6}{\sqrt{8N^3\pi^3}} \Big[\Big(3\sqrt{\pi} U \bar{D}_{3,1,0,0} - \sqrt{\pi} U^2 \bar{D}_{4,2,0,0} - 2\sqrt{U} \bar{D}_{\frac{5}{2},\frac{1}{2},0,0} \Big)$$

$$+ \sigma \Big(3\sqrt{\pi} U \bar{D}_{2,1,0,1} - \sqrt{\pi} U^2 \bar{D}_{3,2,0,1} - 2\sqrt{U} \bar{D}_{\frac{3}{2},\frac{1}{2},0,1} \Big)$$

$$+ \tau \Big(3\sqrt{\pi} U \bar{D}_{2,1,1,0} - \sqrt{\pi} U^2 \bar{D}_{3,2,1,0} - 2\sqrt{U} \bar{D}_{\frac{3}{2},\frac{1}{2},1,0} \Big) \Big].$$

$$\mathcal{G}_{2,2,2,2,t}^R(U,V,\sigma,\tau) = \tau^2 \frac{U}{V} \mathcal{G}_{2,2,2,2,s}^R\left(V,U,\frac{\sigma}{\tau},\frac{1}{\tau}\right)$$

$$\mathcal{G}_{2,2,2,2,u}^R(U,V,\sigma,\tau) = \sigma^2 U \mathcal{G}_{2,2,2,2,s}^R\left(\frac{1}{U},\frac{V}{U},\frac{1}{\sigma},\frac{\tau}{\sigma}\right)$$

$$\bar{D}_{\Delta_1\Delta_2\Delta_3\Delta_4}(V,U) = \bar{D}_{\Delta_3\Delta_2\Delta_1\Delta_4}(U,V)$$

$$\bar{D}_{\Delta_1\Delta_2\Delta_3\Delta_4}\left(\frac{1}{U},\frac{V}{U}\right) = U^{\Delta_2} \bar{D}_{\Delta_4\Delta_2\Delta_3\Delta_1}(U,V)$$



$$\mathcal{E} = 2, \kappa_s = \kappa_t = \kappa_u = 0, \gamma_{ij}^0 = 0$$

$$\langle \mathcal{O}_2(x_1, t_1) \dots \mathcal{O}_2(x_4, t_4) \rangle = \frac{t_{12}^2 t_{34}^2}{x_{12}^2 x_{34}^2} \mathcal{G}_{2,2,2,2}(U, V, \sigma, \tau)$$

$$\mathcal{G}_{2,2,k,k}(U,V,\sigma,\tau)=\int_{-i\infty}^{i\infty}\frac{dsdt}{(4\pi i)^2}U^{\frac{s}{2}}V^{\frac{t}{2}-\frac{k}{4}-\frac{1}{2}}\mathcal{M}_{2,2,k,k}\\\times\Gamma\Big(1-\frac{s}{2}\Big)\Gamma\Big(\frac{k-s}{2}\Big)\Gamma^2\Big(\frac{1}{2}+\frac{k}{4}-\frac{t}{2}\Big)\Gamma^2\Big(\frac{s+t-1}{2}-\frac{k}{4}\Big)$$

$$\mathcal{G}_{2,2,k,k,S}=\frac{6}{\sqrt{8\pi^3N^3}}\left[\frac{\sqrt{\pi}}{\Gamma\left(\frac{k}{2}\right)}((1-k)U\partial_U-k)\left(\frac{V}{U}\bar{D}_{-1,1,\frac{k}{2}+1,\frac{k}{2}+1}+\sigma V\bar{D}_{0,1,\frac{k}{2}+1,\frac{k}{2}}+\tau\bar{D}_{0,1,\frac{k}{2},\frac{k}{2}+1}\right)\right.\\\left.+k\left(\frac{V}{U}\bar{D}_{-1,1,\frac{3}{2},\frac{3}{2}}+V\sigma\bar{D}_{0,1,\frac{3}{2},\frac{1}{2}}+\tau\bar{D}_{0,1,\frac{1}{2},\frac{3}{2}}\right)\right]$$

$$\mathcal{G}_{2,2,k,k,t}=(-1)^{\frac{k}{2}}\frac{12k\tau U}{\sqrt{2N^3}}\frac{\Gamma\left(\frac{k}{2}+1\right)}{\Gamma\left(\frac{k}{2}-\frac{1}{2}\right)}\left[\frac{1}{4\pi}\left(\bar{D}_{\frac{1}{2},2-\frac{k}{2},0,\frac{1+k}{2}}+\sigma\bar{D}_{\frac{1}{2},1-\frac{k}{2},1,\frac{1+k}{2}}+\tau\bar{D}_{\frac{1}{2},1-\frac{k}{2},0,\frac{3+k}{2}}\right)\right.\\\left.-\sum_{i=0}^{\left[\frac{k-1}{2}\right]}2^ix_i(k)\left(V\partial_V+\frac{k}{4}+\frac{1}{2}\right)^i\left(\bar{D}_{1,2-\frac{k}{2},0,1+\frac{k}{2}}+\sigma\bar{D}_{1,1-\frac{k}{2},1,1+\frac{k}{2}}+\tau\bar{D}_{1,1-\frac{k}{2},0,2+\frac{k}{2}}\right)\right],$$

$$\mathcal{G}_{2,2,k,k,u}(U,V,\sigma,\tau)=\mathcal{G}_{2,2,k,k,t}\left(\frac{U}{V},\frac{1}{V},\tau,\sigma\right)$$

$$\mathcal{E} = 2, \kappa_s = 2k - 4, \kappa_t = \kappa_u = 0, \gamma_{12}^0 = \gamma_{13}^0 = \gamma_{14}^0 = \gamma_{23}^0 = \gamma_{24}^0 = 0, \gamma_{34}^0 = k - 2$$

$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_k \mathcal{O}_k \rangle = \left(\frac{t_{34}^2}{x_{34}^2} \right)^{\frac{k-2}{2}} \left(\frac{t_{12}^2 t_{34}^2}{x_{12}^2 x_{34}^2} \right) \mathcal{G}_{2,2,k,k}(U,V,\sigma,\tau).$$

$$\frac{1}{s}\Gamma\left(\frac{k_1+k_2}{4}-\frac{s}{2}\right)=-\frac{1}{2}\Gamma\left(-\frac{s}{2}\right)\prod_{n=1}^{\frac{k_1+k_2}{4}-1}\left[\frac{k_1+k_2}{4}-\frac{s}{2}-n\right]$$

$$\mathcal{G}_{k_1,k_2,k_3,k_4}^{HE}=-\frac{1}{2}\mathcal{N}_{k_i}P_{k_i}(\sigma,\tau)(-1)^{\frac{k_1+k_2}{2}+\frac{k_1+k_4}{2}+\frac{k_1+k_3}{4}}((1-\alpha)U\partial_U+V\partial_V)^2((1-\bar{\alpha})U\partial_U+V\partial_V)^2\\\sum_r\left(\frac{k_1+k_3}{4}-1\right)^rU^{\frac{k_1+k_2}{4}-1-a_s+r}V^{\frac{2k_1+k_3+k_4}{4}-2-a_t-r}\bar{D}_{a_1+r-1,a_2-3,a_3-r-2,a_4}$$

$$a_1=\frac{k_2-k_4}{4}, a_2=\frac{2k_1+k_2+k_4}{2}, a_3=\frac{k_1-k_2+k_3+k_4}{4}, a_4=\frac{-k_1+k_2+k_3+k_4}{4}.$$



$$\langle \mathcal{O}_{k_1} \dots \mathcal{O}_{k_4} \rangle^{HE} = \prod_{i < j} \left(\frac{t_{ij}^2}{x_{ij}^2} \right)^{\frac{\gamma_{ij}^0}{2}} \left(\frac{t_{12}^2 t_{34}^2}{x_{12}^2 x_{34}^2} \right)^{\frac{\varepsilon}{2}} \mathcal{G}_{k_1, \dots, k_4}^{HE}(U, V, \sigma, \tau).$$

$$ds_{AdS_4 \times S^7}^2 = ds_{AdS_4}^2 + 4\ell^2 ds_{S^7}^2$$

$$\mathcal{O}_k = \sigma_k \ell^{\frac{k}{2}-1} \Phi_k, \sigma_k = \frac{2\pi}{\sqrt{\Gamma(k-1)}}$$

$$\lim_{\ell \rightarrow \infty} \frac{\langle \mathcal{O}_{k_1}(x_1) \mathcal{O}_{k_2}(x_2) \rangle}{\ell^{\frac{k_1+k_2}{2}-2} \sigma_{k_1} \sigma_{k_2}} = \langle \Phi_{k_1} \Phi_{k_2} \rangle = \frac{\delta_{k_1, k_2}}{(2\pi)^2} \frac{(-1)^{\frac{k_1}{2}-1} \Gamma(k_1-2)}{(u_{12}-i\epsilon)^{k_1-2}} t_{12}^{k_1} \delta^2(z_{12})$$

$$\frac{\langle \mathcal{O}_{k_1} \mathcal{O}_{k_2} \mathcal{O}_{k_3} \rangle}{\ell^{\frac{k_1+k_2+k_3}{2}-3}} \stackrel{\ell \rightarrow \infty}{\rightarrow} \mathcal{O}\left(\frac{1}{\ell^{\frac{11}{2}}}\right)$$

$$\lim_{\ell \rightarrow \infty} \ell^{\frac{11}{2}} \frac{\langle \mathcal{O}_{k_1} \mathcal{O}_{k_2} \mathcal{O}_{k_3} \rangle}{\ell^{\frac{k_1+k_2+k_3}{2}-3}} \prod_{i=1}^3 \sigma_{k_i} = \frac{\Gamma\left(\frac{\alpha}{2}-2\right) \Theta(z_{12} z_{31}) \Theta(z_{13} z_{23})}{\Gamma\left(\frac{\alpha_1}{2}\right) \Gamma\left(\frac{\alpha_2}{2}\right) \Gamma\left(\frac{\alpha_3}{2}\right)} \pi^2 R_{k_1, k_2, k_3} \prod_{i=1}^3 \frac{1}{\sigma_{k_i}} \\ \times \delta(\bar{z}_{12}) \delta(\bar{z}_{23}) \frac{t_{12}^{\alpha_3} t_{23}^{\alpha_1} t_{13}^{\alpha_2} z_{12}^{\frac{k_3}{2}-2} z_{23}^{\frac{k_1}{2}-2} z_{13}^{\frac{k_2}{2}-2}}{(u_1 z_{23} + u_2 z_{31} + u_3 z_{12} + i\epsilon)^{\frac{k_1+k_2+k_3}{2}-4}}$$

$$U^a V^b \bar{D}_{\Delta_1, \Delta_2, \Delta_3, \Delta_4} \stackrel{\ell \rightarrow \infty}{\rightarrow} \frac{\ell^{-4+\Sigma_\Delta} \mathcal{K}}{\mathcal{U}^{\Sigma_\Delta-4}} \left(\frac{|z_{23}|^2}{|z_{34}|^2 |z_{24}|^2} \right)^{\frac{4-\Sigma_\Delta}{2}} \frac{(1-z)^{\Delta_1+\Delta_4-2+2b}}{z^{\Delta_1+\Delta_2-2-2a}} \delta(z-\bar{z}) \Theta(z) \Theta(1-z)$$

$$\mathcal{K}=(-1)^{\Delta_1+\Delta_3}2^{\frac{\Sigma_\Delta}{2}}\pi^2\Gamma\left(\frac{\Sigma_\Delta-4}{2}\right)$$

$$\mathcal{G}_{2,2,2,2,s} \stackrel{\ell \rightarrow \infty}{\rightarrow} \frac{48\pi\ell^2}{\sqrt{2N^3}\mathcal{U}^2} \left(\frac{|z_{34}|^2 |z_{24}|^2}{|z_{23}|^2} \right) (1-z)(1-\alpha z)^2 (1-\bar{\alpha} z)^2 \delta(z-\bar{z}) \Theta(z) \Theta(1-z)$$

$$\langle \mathcal{O}_2(x_1,t_1) \dots \mathcal{O}_2(x_4,t_4) \rangle \stackrel{\ell \rightarrow \infty}{\rightarrow} \frac{3\ell_{11}^9 \pi}{8\ell^7 \mathcal{U}^2} \left(\frac{|z_{24}|^2}{|z_{12}|^2 |z_{23}|^2} \right) t_{12}^2 t_{34}^2 (1-\alpha z)^2 (1-\bar{\alpha} z)^2 \delta(z-\bar{z}) \Theta(z) \Theta(1-z)$$

$$\lim_{\ell \rightarrow \infty} \frac{V_7}{(2\pi)^4} \langle \mathcal{O}_1(x_1,t_1) \dots \mathcal{O}_1(x_4,t_4) \rangle = \langle \Phi_2(u_1,z_1,\bar{z}_1) \dots \Phi_2(u_4,z_4,\bar{z}_4) \rangle \\ = \frac{\pi \ell_{11}^9}{2\mathcal{U}^2} \left(\frac{|z_{24}|^2}{|z_{12}|^2 |z_{23}|^2} \right) t_{12}^2 t_{34}^2 (1-\alpha z)^2 (1-\bar{\alpha} z)^2 \delta(z-\bar{z}) \Theta(z) \Theta(1-z)$$

$$\mathcal{G}_{2,2,k,k,s} \stackrel{\ell \rightarrow \infty}{\rightarrow} \frac{-3U}{\sqrt{2N^3}\pi} \frac{(1-k)}{\Gamma\left(\frac{k}{2}\right)} \left(V\bar{D}_{0,2,\frac{k}{2}+1,\frac{k}{2}+1} + \sigma UV\bar{D}_{1,2,\frac{k}{2}+1,\frac{k}{2}} + \tau U\bar{D}_{1,2,\frac{k}{2},\frac{k}{2}+1} \right)$$

$$\mathcal{G}_{2,2,k,k,t} \stackrel{\ell \rightarrow \infty}{\rightarrow} \frac{12k\tau U}{\sqrt{2N^3}} \frac{\Gamma\left(\frac{k}{2}+1\right)}{\Gamma\left(\frac{k}{2}-\frac{1}{2}\right)} 2^{\frac{k}{2}} x_k^{\frac{k}{2}} \left(\bar{D}_{1,2,\frac{k}{2},1+\frac{k}{2}} + \sigma \bar{D}_{1,1,1+\frac{k}{2},1+\frac{k}{2}} + \tau \bar{D}_{1,1,\frac{k}{2},2+\frac{k}{2}} \right),$$



$$\lim_{\ell \rightarrow \infty} V_7 \left[\ell^{2-k} \frac{\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_k \mathcal{O}_k \rangle}{\sigma_k^2} \right] = \frac{\pi \ell_{11}^9}{2} (1 - \alpha z)^2 (1 - \bar{\alpha} z)^2 (-1)^{\frac{k}{2}-1} t_{34}^k t_{12}^2 \\ \times \frac{|z_{24}|^k}{|z_{12}|^2 |z_{23}|^k} \frac{\Gamma(k) (1-z)^{\frac{k}{2}-1} \delta(z - \bar{z})}{\mathcal{U}^k}$$

$$\mathcal{G}_{k_1,k_2,k_3,k_4}^{HE} \stackrel{\ell \rightarrow \infty}{\rightarrow} \frac{\mathcal{N}_{k_i} \mathcal{P}_{k_i}}{N^{\frac{3}{2}} \mathcal{U}^{\frac{\sum_i k_i}{2} - 2}} (-1)^{\frac{k_1+k_3}{2}} z^{-2a_s} (1-z)^{\frac{\sum_i k_i}{2} - 2 - 2a_t} \pi^{\frac{5}{2}} 2^{4 - \frac{\sum_i k_i}{4}} \frac{\Gamma\left(\frac{\sum_i k_i}{2} - 5\right) \Gamma\left(\frac{\sum_i k_i}{4} - 1\right)}{\Gamma\left(\frac{\sum_i k_i}{4} - 4\right) \Gamma\left(\frac{\sum_i k_i}{4} - \frac{1}{2}\right)} \\ \times (1 - \alpha z)^2 (1 - \bar{\alpha} z)^2 \left(\frac{|z_{23}|^2}{|z_{34}|^2 |z_{24}|^2} \right)^{1 - \frac{\sum_i k_i}{4}} \delta(z - \bar{z}) \left(\frac{1}{\ell} \right)^{2 - \frac{\sum_i k_i}{2}}$$

$$\lim_{\ell \rightarrow \infty} V_7 \frac{\langle \mathcal{O}_{k_1} \dots \mathcal{O}_{k_4} \rangle}{\prod_{i=1}^4 \ell^{\frac{k_i}{2}-1} \sigma_k} = \tilde{\mathcal{N}} \left[\prod_{i < j} \left(\frac{t_{ij}}{|z_{ij}|} \right)^{\gamma_{ij}^0} \left(\frac{t_{12} t_{34}}{|z_{12}| |z_{34}|} \right)^{\varepsilon} \left(\frac{|z_{23}|^2}{|z_{34}|^2 |z_{24}|^2} \right)^{1 - \frac{\sum_i k_i}{4}} \right] \\ \times \left[z^{-2a_s} (1-z)^{\frac{\sum_i k_i}{2} - 2 - 2a_t} \frac{\delta(z - \bar{z})}{\mathcal{U}^{\frac{\sum_i k_i}{2} - 2}} \right] \times [(1 - \alpha z)^2 (1 - \bar{\alpha} z)^2 \mathcal{P}_{k_i}(\sigma, \tau)]$$

$$\tilde{\mathcal{N}} = \frac{V_7 \ell_{11}^9}{(2\pi)^4} \mathcal{N}_{k_i} \pi^{\frac{5}{2}} 2^{-\frac{1+\sum_i k_i}{2}} \frac{\Gamma\left(\frac{\sum_i k_i}{2} - 5\right) \Gamma\left(\frac{\sum_i k_i}{4} - 1\right)}{\Gamma\left(\frac{\sum_i k_i}{4} - 4\right) \Gamma\left(\frac{\sum_i k_i}{4} - \frac{1}{2}\right)} (-1)^{\frac{k_1+k_3}{2}}$$

$$\left(Z \partial_Z - \frac{\alpha}{2} \partial_\alpha \right) \mathcal{G}_{k_1,k_2,k_3,k_4}(Z, \bar{Z}, \alpha, \bar{\alpha}) \Big|_{\alpha=\frac{1}{Z}} = \left(\bar{Z} \partial_{\bar{Z}} - \frac{\alpha}{2} \partial_\alpha \right) \mathcal{G}_{k_1,k_2,k_3,k_4}(Z, \bar{Z}, \alpha, \bar{\alpha}) \Big|_{\alpha=\frac{1}{Z}} = 0 \\ \left(Z \partial_Z - \frac{\bar{\alpha}}{2} \partial_{\bar{\alpha}} \right) \mathcal{G}_{k_1,k_2,k_3,k_4}(Z, \bar{Z}, \alpha, \bar{\alpha}) \Big|_{\bar{\alpha}=\frac{1}{\bar{Z}}} = \left(\bar{Z} \partial_{\bar{Z}} - \frac{\bar{\alpha}}{2} \partial_{\bar{\alpha}} \right) \mathcal{G}_{k_1,k_2,k_3,k_4}(Z, \bar{Z}, \alpha, \bar{\alpha}) \Big|_{\bar{\alpha}=\frac{1}{\bar{Z}}} = 0$$

$$\mathcal{G}_{k_1,k_2,k_3,k_4}(Z, \bar{Z}, \alpha, \bar{\alpha}) = \frac{\mathcal{G}^0(Z, \alpha, \bar{\alpha})}{(Z - \bar{Z})^p} + \mathcal{O}\left(\frac{1}{(Z - \bar{Z})^{p-1}}\right)$$

$$\mathcal{G}^0\left(z, \alpha = \frac{1}{z}, \bar{\alpha}\right) = \mathcal{G}^0\left(z, \alpha, \bar{\alpha} = \frac{1}{z}\right) = 0$$

$$S = \int_{\mathbb{R}^{3,1}} d^4x \left\{ \sum_{k=2}^{k_{\max}} \frac{t_{12}^k}{2} s_k \square s_k + \sum_{k_1,k_2,k_3=2}^{k_{\max}} \frac{t_{12}^{\alpha_3} t_{23}^{\alpha_1} t_{13}^{\alpha_2}}{2\ell} \left(\frac{\ell_{11}}{2\ell} \right)^{\frac{9}{2}} \frac{\tilde{g}_{123}}{3} s_{k_1} s_{k_2} s_{k_3} \right\}$$

$$\tilde{g}_{123} = \frac{144\sqrt{3}2^\alpha(\alpha^2-9)(\alpha^2-1)(\alpha+2)}{(2\alpha+6)!!\pi^2} \prod_{i=1}^3 \frac{\Gamma(k_i-1)}{\Gamma(\alpha_i)} \sqrt{(k_i+1)k_i(k_i-1)}$$

$$\mathcal{A}_{k_1,k_2} = \frac{t_{12}^{k_1} \delta_{k_1,k_2}}{\omega_1} \delta_{\epsilon_1,-\epsilon_2} \delta(\omega_1 - \omega_2) \delta^2(z_{12})$$

$$\mathcal{C}_{k_1,k_2}^{\Delta_1,\Delta_2} = \int \prod_{j=1}^2 \frac{d\omega_j}{2\pi} \omega_j^{\Delta_j-1} e^{iu_j \omega_j \epsilon_j} \mathcal{A}_{k_1,k_2} \Bigg|_{\epsilon_1=-\epsilon_2=-1} = \frac{\delta_{k_1,k_2}}{(2\pi)^2} \frac{(-1)^{\Delta_1-1} \Gamma(\Delta_1 + \Delta_2 - 2)}{(u_{12} - i\epsilon)^{\Delta_1 + \Delta_2 - 2}} t_{12}^{k_1} \delta^2(z_{12})$$



$$c_{\frac{k_1}{2}, \frac{k_2}{2}} = \frac{\delta_{k_1, k_2}}{(2\pi)^2} \frac{(-1)^{\frac{k_1}{2}-1} \Gamma(k_1 - 2)}{(u_{12} - i\epsilon)^{k_1-2}} t_{12}^{k_1} \delta^2(z_{12})$$

$$\mathcal{A}_{k_1, k_2, k_3} = \frac{\tilde{g}_{123}}{2\ell} \left(\frac{\ell_{11}}{2\ell}\right)^{\frac{9}{2}} t_{12}^{\alpha_3} t_{23}^{\alpha_1} t_{13}^{\alpha_2} \delta^{(4)}(p_1 + p_2 + p_3)$$

$$c_{k_1, k_2, k_3}^{\Delta_1, \Delta_2, \Delta_3} = \int \prod_{j=1}^3 \frac{d\omega_j}{2\pi} \omega_j^{\Delta_j-1} e^{iu_j \omega_j \epsilon_j} \mathcal{A}_{k_1, k_2, k_3}$$

$$c_{k_1, k_2, k_3}^{\Delta_1, \Delta_2, \Delta_3} = \frac{-i\epsilon_1\epsilon_2\epsilon_3}{(2\pi)^3} \frac{\tilde{g}_{123}}{2\ell} \left(\frac{\ell_{11}}{2\ell}\right)^{\frac{9}{2}} t_{12}^{\alpha_3} t_{23}^{\alpha_1} t_{13}^{\alpha_2} (z_{12})^{\Delta_1-2} (z_{13})^{\Delta_2-2} (z_{23})^{\Delta_3-2} \delta(\bar{z}_{12}) \delta(\bar{z}_{23}) \\ \times \Theta\left(-\frac{z_{13}}{z_{23}} \epsilon_1 \epsilon_2\right) \Theta\left(\frac{z_{12}}{z_{23}} \epsilon_1 \epsilon_3\right) \frac{\Gamma(\sum_{i=1}^3 \Delta_i - 4)}{(z_{23} u_1 + z_{31} u_2 + z_{12} u_3 - i\epsilon \epsilon_1 \text{sign}(z_{23}))^{\sum_{i=1}^3 \Delta_i - 4}}$$

$$A_4 = \frac{-a_4 \ell_{11}^9}{stu} \left(\frac{1}{2} e_2 \cdot e_3 (se_1 \cdot P_3 e_4 \cdot P_2 + te_1 \cdot P_2 e_4 \cdot P_3) + \frac{1}{2} e_1 \cdot e_4 (se_2 \cdot P_4 e_3 \cdot P_1 + te_2 \cdot P_1 e_3 \cdot P_4) \right. \\ \left. + \frac{1}{2} e_2 \cdot e_4 (se_1 \cdot P_4 e_3 \cdot P_2 + ue_1 \cdot P_2 e_3 \cdot P_4) + \frac{1}{2} e_1 \cdot e_3 (se_2 \cdot P_3 e_4 \cdot P_1 + ue_2 \cdot P_1 e_4 \cdot P_3) \right. \\ \left. + \frac{1}{2} e_3 \cdot e_4 (te_1 \cdot P_4 e_2 \cdot P_3 + ue_1 \cdot P_3 e_2 \cdot P_4) + \frac{1}{2} e_1 \cdot e_2 (te_3 \cdot P_2 e_4 \cdot P_1 + ue_3 \cdot P_1 e_4 \cdot P_2) \right. \\ \left. - \frac{1}{4} ste_1 \cdot e_4 e_2 \cdot e_3 - \frac{1}{4} sue_1 \cdot e_3 e_2 \cdot e_4 - \frac{1}{4} tue_1 \cdot e_2 e_3 \cdot e_4 \right)^2 \delta^{(11)} \left(\sum_{i=1}^4 P_i \right)$$

$$P_i^\alpha = (p_i^\mu, \tilde{p}_i^I), p_i \in \mathbb{R}^{1,3}, \tilde{p}_i \in \mathbb{R}^7, p_i^\mu \sim \mathcal{O}(1), \tilde{p}_i^I \sim \mathcal{O}\left(\frac{1}{\ell}\right) \approx 0.$$

$$e_i^\alpha = (0,0,0,0,\xi_i^I)$$

$$t_i = (0,\xi_i)$$

$$e_i \cdot e_j = t_i \cdot t_j \equiv t_{ij}, e_i \cdot p_j \sim \mathcal{O}\left(\frac{1}{\ell}\right) \approx 0$$

$$h_j^{\alpha\beta}(X) = \mathcal{N}_j e_j^\alpha e_j^\beta e^{ip_j \cdot x} (\xi_j \cdot \tilde{x})^{k_j-2},$$

$$\Box \bar{h}_{\alpha\beta} = 0, \bar{h}_{\alpha\beta} = h_{\alpha\beta} - \frac{1}{2} \eta_{\alpha\beta} h^\gamma_\gamma,$$

$$\int_{\mathbb{R}^{1,3}} d^4x \int_{\mathbb{R}^7} d^7\tilde{x} h_1^{\alpha\beta} h_{2,\alpha\beta} = \mathcal{N}_1 \mathcal{N}_2 t_{12}^2 (2\pi)^4 \delta^{(4)}(P_1 + P_2) \int_{\mathbb{R}^7} d^7\tilde{x} (\xi_1 \cdot \tilde{x})^{k_1-2} (\xi_2 \cdot \tilde{x})^{k_2-2} \\ = \mathcal{N}_1 \mathcal{N}_2 t_{12}^2 (2\pi)^4 \delta^{(4)}(P_1 + P_2) \times \frac{2\pi^{\frac{7}{2}} t_{12}^{2k_1-2} \delta_{k_1, k_2}}{\Gamma(\frac{7}{2})} \int_0^\infty d|\tilde{x}| |\tilde{x}|^{k_1+k_2+2}$$

$$h_j^{\alpha\beta}(X) = \frac{1}{\sqrt{V_7}} e_i^\alpha e_i^\beta e^{ip_j \cdot x} \frac{(t_j \cdot Z)^{k_j-2}}{(2\ell)^{k_j-2}}.$$



$$h_j^{\alpha\beta}(X)=\frac{1}{\sqrt{V_7}}e_i^\alpha e_i^\beta e^{ip_j\cdot x}$$

$$\delta^{(11)}\left(\sum_{i=1}^4P_i\right)\rightarrow\frac{1}{(2\pi)^7V_7^2}\int_{\mathbb{R}^{3,1}}\frac{d^4x}{(2\pi)^4}e^{i\sum_{j=1}^4p_j\cdot x}\int_{S^7}d^8Z\delta(Z\cdot Z-4\ell^2)=\frac{\delta^{(4)}(\sum_{i=1}^4p_i)}{(2\pi)^7V_7}$$

$$\mathcal{A}_4^{2,2,2,2}=\frac{-a_4\ell_{11}^9t_{12}^2t_{34}^2}{(2\pi)^7V_7stu}(st\tau+su\sigma+tu)^2\times\delta^{(4)}\left(\sum_{i=1}^4p_i\right)$$

$$V_7\mathcal{C}_4^{1,\dots,1}\Big(\{u_j,z_j,\bar{z}_j\}^{\epsilon_j}\Big)=\frac{a_4\ell_{11}^9}{2(2\pi)^{11}\mathcal{U}^2}\Bigg(\frac{|z_{24}|^2}{|z_{12}|^2|z_{23}|^2}\Bigg)t_{12}^2t_{34}^2(1-\alpha z)^2(1-\bar{\alpha}z)^2\delta(z-\bar{z})\Theta(z)\Theta(1-z),$$

$$\begin{aligned}\delta^{(11)}(P_1+P_2+P_3+P_4)\longrightarrow&\delta^{(4)}\bigg(\sum_{k=1}^4p_k\bigg)\frac{1}{(2\pi)^7V_7^2}\int d^8Z\delta(Z\cdot Z-4\ell^2)\prod_{j=1}^4\frac{(t_j\cdot Z)^{k_j-2}}{(2\ell)^{k_j-2}}\\&=\frac{\tilde{N}_{k_i}}{V_7}\prod_{i<j}t_{ij}^{\gamma_{ij}^0}(t_{12}t_{34})^{\varepsilon-2}\mathcal{P}_{k_i}(\sigma,\tau)\delta^{(4)}\bigg(\sum_{j=1}^4p_j\bigg)\end{aligned}$$

$$\frac{\epsilon_1\cdot\epsilon_3\epsilon_2\cdot\epsilon_4}{\epsilon_1\cdot\epsilon_2\epsilon_3\cdot\epsilon_4}=\frac{t_{13}t_{24}}{t_{12}t_{34}}\equiv\sigma,\frac{\epsilon_2\cdot\epsilon_3\epsilon_1\cdot\epsilon_4}{\epsilon_1\cdot\epsilon_2\epsilon_3\cdot\epsilon_4}=\frac{t_{23}t_{14}}{t_{12}t_{34}}\equiv\tau.$$

$$\mathcal{A}_4^{k_1,k_2,k_3,k_4}=-\frac{\tilde{N}_{k_i}}{V_7}\prod_{i<j}t_{ij}^{-\gamma_{ij}^0}\frac{(t_{12}t_{34})^{2-\varepsilon}}{4\ell_{11}^9stu}(tu+su\sigma+st\tau)^2\mathcal{P}_{k_i}(\sigma,\tau)$$

$$\begin{aligned}\mathcal{C}_{k_1,k_2,k_3,k_4}^{\frac{k_1}{2},\frac{k_2}{2},\frac{k_3}{2},\frac{k_4}{2}}&=\int_0^{+\infty}\prod_{j=1}^4\frac{d\omega_j}{2\pi}(-i\epsilon_j\omega_j)^{\frac{k_j}{2}-1}e^{-i\epsilon_j\omega_ju_j}\mathcal{A}_4^{k_1,k_2,k_3,k_4}\\&=-\frac{\tilde{N}_{k_i}(-1)^{\frac{k_1+k_3}{2}}i^{\sum_{i=1}^4\frac{k_i}{2}}\Gamma\left(-2+\sum_{i=1}^4\frac{k_i}{2}\right)}{V_74\ell_{11}^9(2\pi)^4}\left[(t_{12}t_{34})^{2-\varepsilon}\prod_{i<j}t_{ij}^{-\gamma_{ij}^0}(1-\alpha z)^2(1-\bar{\alpha}z)^2\right] \\&\quad\times\left[\frac{|z_{14}|^{k_3-2}|z_{24}|^{k_1+2}|z_{34}|^{k_2-4}}{|z_{12}|^{k_1+2}|z_{13}|^{k_3-4}|z_{23}|^{k_2}}\right]\times\left[\frac{\delta(z-\bar{z})\Theta(z)\Theta(1-z)z^{\frac{k_1-k_2+4}{2}}}{\mathcal{U}^{-2+\sum_{i=1}^4\frac{k_i}{2}}}(1-z)^{\frac{k_2-k_3}{2}}\right]\end{aligned}$$

$$\begin{aligned}\{Q^{I\alpha},\bar{Q}_{J\beta}\}&=2\delta^I{}_J\gamma^{\mu\alpha}{}_{\beta}P_{\mu},\{S^{I\alpha},\bar{S}_{J\beta}\}=2\delta^I{}_J\gamma^{\mu\alpha}{}_{\beta}K_{\mu} \\ \{Q^{\alpha I},\bar{S}_{\beta J}\}&=-i\delta^I{}_J\Big(2\delta^{\alpha}{}_{\beta}D+(\gamma^{[\mu}\gamma^{\nu]})^{\alpha}{}_{\beta}M_{\mu\nu}\Big)+2i\delta^{\alpha}{}_{\beta}R^I{}_J\end{aligned}$$

$$\bar{Q}_J=(Q^J)^\dagger\gamma^0=-(Q^J)^T\epsilon,\bar{S}_J=(S^J)^\dagger\gamma^0=-(S^J)^T\epsilon.$$

$$\gamma^0=\sigma^3,\gamma^1=i\sigma^1,\gamma^2=i\sigma^2$$

$$[R_{IJ},R_{KL}]=i(\delta^{IK}R^{JL}+\delta^{JL}R^{IK}-\delta^{IL}R^{JK}-\delta^{JK}R^{IL})$$

$$[R_{IJ},Q^K]=i(\delta_I{}^K\delta_{JD}-\delta_J{}^K\delta_{ID})Q^D,[R_{IJ},S^K]=i(\delta_I{}^K\delta_{JD}-\delta_J{}^K\delta_{ID})S^D$$



$$H\rightarrow \frac{1}{c}H, B_i\rightarrow \frac{1}{c}B_i, K\rightarrow \frac{1}{c}K$$

$$Q^{I\alpha}\rightarrow \frac{1}{\sqrt{c}}Q^{I\alpha}, S^{I\alpha}\rightarrow \frac{1}{\sqrt{c}}S^{I\alpha}$$

$$\begin{aligned}\{Q^{I\alpha},\bar{Q}_{J\beta}\}&=-2\delta^I{}_J\gamma^{0\alpha}{}_{\beta}H,\{S^{I\alpha},\bar{S}_{J\beta}\}=-2\delta^I{}_J\gamma^{0\alpha}{}_{\beta}K,\\ \{Q^{\alpha I},\bar{S}_{\beta J}\}&=-2i\delta^I{}_J(\gamma^{[0}\gamma^{i]})^{\alpha}{}_{\beta}B_i,\\ [R_{IJ},R_{KL}]&=i(\delta^{IK}R^{JL}+\delta^{JL}R^{IK}-\delta^{IL}R^{JK}-\delta^{JK}R^{IL}),\\ [R_{IJ},Q^K]&=i(\delta_I{}^KR_{JD}-\delta_J{}^KR_{ID})Q^D,[R_{IJ},S^K]=i(\delta_I{}^KR_{JD}-\delta_J{}^KR_{ID})S^D.\end{aligned}$$

$$\mathfrak{CCarr}_3^{\mathrm{glob}}\simeq \mathrm{iso}(3,1)$$

$$\left\{ \mathcal{Q}^I_\alpha, \overline{\mathcal{Q}}^J_{\dot{\alpha}} \right\} = 2 \delta^{IJ} \sigma^\mu_{\alpha \dot{\alpha}} \mathcal{P}_\mu$$

$$\mathcal{P}_0=-\frac{1}{2}(H+K),,\mathcal{P}_1=-B_1,\mathcal{P}_2=-B_2,\mathcal{P}_3=\frac{1}{2}(H-K)$$

$$Q_1^I=S^{I1}=-\bar{S}_{I2},\overline{Q}_1^I=\bar{S}^{I1}=S^{I2},Q_2^I=\bar{Q}_{I1}=Q^{I2},\overline{Q}_2^I=Q^{I1}=-\bar{Q}_{I2}$$

$$[R_{IJ},Q^K]=i(\delta_I{}^KR_{JD}-\delta_J{}^KR_{ID})Q^D,[R_{IJ},\overline{Q}^K]=i(\delta_I{}^KR_{JD}-\delta_J{}^KR_{ID})\overline{Q}^D$$

$$\mathfrak{sCCaCrr}_3^{\mathrm{glob},\mathcal{N}}\simeq \mathfrak{spoin}(\mathcal{N},4)$$

$$\begin{aligned}[J_{ij},\phi_{\Delta}(0)]&=0,[B_i,\phi_{\Delta}(0)]=0\\ [D,\phi_{\Delta}(0)]&=-i\Delta\phi_{\Delta}(0),[K,\phi_{\Delta}(0)]=0,[K_i,\phi_{\Delta}(0)]=0\end{aligned}$$

$$[S^{I\alpha},\phi_{\Delta}(0)]=0=[\bar{S}_{I\alpha},\phi_{\Delta}(0)]$$

$$[R_{IJ},\phi_{\Delta}(0)]=\mathcal{R}^{(s)}\cdot\phi_{\Delta}(0)$$

$$\phi_k=\phi^{I_1\ldots I_k}t_{I_1}\ldots t_{I_k}$$

$$\phi_{\Delta}(x,\theta)=U\phi_{\Delta}(0,0)U^{-1}, U=e^{-i(-Hu+P_ix^i+\bar{Q}_{I\alpha}\theta^{I\alpha})}$$

$$\begin{aligned}&\delta\phi_{\Delta}(x,\theta)\\&=i\left[aH+b^jB_j+kK+a^jP_j+\frac{1}{2}r^{jk}J_{jk}+\lambda D+k^jK_j+\epsilon^I\bar{Q}_I+\kappa^I\bar{S}_I+\frac{1}{2}\omega^{IJ}R_{IJ},\phi_{\Delta}(x,\theta)\right]\end{aligned}$$

$$\sum_{j=1}^n\left\langle \phi_{\Delta_1}(x_1,\theta_1^I)\ldots\delta\phi_{\Delta_j}(x_j,\theta_j^I)\ldots\phi_{\Delta_n}(x_n,\theta_n^I)\right\rangle=0$$

$$\phi_{\Delta}(x^a,\theta^{I\alpha})=\Phi_{\Delta}(x^a)+\theta^{I\alpha}\bar{\Psi}_{I\alpha}(x^a)+\cdots$$

$$\langle \Phi_{k_1}(x_1)\ldots\Phi_{k_n}(x_n)\rangle=\int_0^{+\infty}\prod_{j=1}^n\frac{d\omega_j}{2\pi}(-i\epsilon_j\omega_k)^{\frac{k_j}{2}-1}e^{-i\epsilon_j\omega_ju_j}\mathcal{A}_n^{k_1,\ldots,k_n}$$



$$\mathcal{M}_{2,2,2,2} = \frac{1}{c_T} \mathcal{M}_{2,2,2,2}^R + \frac{1}{\frac{5}{c_T^3}} B^{R^4} \mathcal{M}_{2,2,2,2}^{(4)} + \frac{1}{\frac{7}{c_T^3}} \left(B_4^{D^6 R^4} \mathcal{M}_{2,2,2,2}^{(4)} + B_6^{D^6 R^4} \mathcal{M}_{2,2,2,2}^{(6)} + B_7^{D^6 R^4} \mathcal{M}_{2,2,2,2}^{(7)} \right) + \dots$$

$$\mathcal{M}_{2,2,2,2}^R(s, t; \sigma, \tau) = \mathcal{M}_{2,2,2,2,s}^R + \mathcal{M}_{2,2,2,2,t}^R + \mathcal{M}_{2,2,2,2,u}^R$$

$$\mathcal{M}_{2,2,2,2,s}^R(s, t; \sigma, \tau) = \sum_{m=0}^{\infty} - \frac{3((t-2)(u-2) + (s+2)((t-2)\sigma + (u-2)\tau))}{\sqrt{2\pi} N^{\frac{3}{2}} \Gamma\left(\frac{1}{2} - m\right)^2 m! \Gamma\left(m + \frac{5}{2}\right) (s-1-2m)}$$

$$\mathcal{M}_{2,2,2,2,s}^R(s, t; \sigma, \tau) = \frac{3}{\sqrt{8\pi^3 N^3}} \frac{1}{s(s+2)} [(t-2)(s+t-2) - \sigma(s+2)(t-2) + \tau(s+2)(s+t-2)]$$

$$\times \left[\sqrt{\pi}(s+4) - 4 \frac{\Gamma\left(\frac{1-s}{2}\right)}{\Gamma\left(1 - \frac{s}{2}\right)} \right]$$

$$\mathcal{M}_{2,2,2,2,t}^R(s, t; \sigma, \tau) = \tau^2 \mathcal{M}_{2,2,2,2,s}^R\left(t, s; \frac{\sigma}{\tau}, \frac{1}{\tau}\right), \mathcal{M}_{2,2,2,2,u}^R(s, t; \sigma, \tau) = \sigma^2 \mathcal{M}_{2,2,2,2,s}^R\left(u, t; \frac{1}{\sigma}, \frac{\tau}{\sigma}\right)$$

$$\mathcal{M}_{2,2,k,k} = \frac{1}{c_T} \mathcal{M}_{2,2,k,k}^R$$

$$\mathcal{M}_{2,2,k,k,s} = \frac{3}{8\sqrt{2}\pi^{3/2} N^{3/2} s(s+2)} [(k-2t+2)(k-2u+2) - 2(s+2)\sigma(k-2t+2) - 2(s+2)\tau(k-2u+2)]$$

$$\times \left(\frac{\sqrt{\pi}(s - k(s+2))}{\Gamma\left(\frac{k}{2}\right)} + \frac{2k\Gamma\left(\frac{1}{2} - \frac{s}{2}\right)}{\Gamma\left(\frac{k-s}{2}\right)} \right)$$

$$\mathcal{M}_{2,2,k,k,t} = \frac{-3k\tau\Gamma\left(\frac{k}{2} + 1\right) [(k+2t+2)(k-2u+2) + 2\sigma(k-s)(k+2t+2) - 2\tau(k-s)(k-2u+2)]}{4\sqrt{2}\pi\sqrt{N^3}(k-2t)\Gamma\left(\frac{k-1}{2}\right)\Gamma\left(\frac{3+k}{2}\right)}$$

$$\times {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \frac{k}{4} - \frac{t}{2}; \frac{k}{2} + \frac{3}{2}, \frac{k}{4} - \frac{t}{2} + 1; 1\right)$$

$${}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \frac{k}{4} - \frac{t}{2}; \frac{k}{2} + \frac{3}{2}, \frac{k}{4} - \frac{t}{2} + 1; 1\right) = \frac{\pi(k-2t)\Gamma\left(\frac{3+k}{2}\right)}{\left(\frac{2-k+2t}{4}\right)^{\frac{2+k}{2}}} \left[\frac{\Gamma\left(\frac{k}{4} - \frac{t}{2}\right)}{4\pi\Gamma\left(\frac{k}{4} + \frac{1}{2} - \frac{t}{2}\right)} - \sum_{i=0}^{\lfloor \frac{k-1}{2} \rfloor} x_i t^i \right]$$

$$x_{\frac{k}{2}} = \frac{\Gamma\left(\frac{1+k}{2}\right)}{4\pi 2^{\frac{k}{2}} \Gamma^2\left(1 + \frac{k}{2}\right)}$$



$$\mathcal{M}_{2,2,k,k,t} = \frac{-3k\tau\Gamma\left(\frac{k}{2}+1\right)[-(k+2t+2)(k-2u+2)+2\sigma(k-s)(k+2t+2)-2\tau(k-s)(k-2u+2)]}{4\sqrt{2}\sqrt{N^3}\Gamma\left(\frac{k-1}{2}\right)\left[\prod_{n=0}^{\frac{k}{2}}\left(\frac{t}{2}+\frac{k}{4}+\frac{1}{2}-n\right)\right]}$$

$$\otimes \langle \frac{\Gamma(\frac{k}{4}-\frac{t}{2})}{4\pi\Gamma(\frac{k}{4}+\frac{1}{2}-\frac{t}{2})}-\sum_{i=0}^{\lfloor \frac{k-1}{2} \rfloor} x_i t^i \rangle$$

$$\lim_{s,t\rightarrow\infty}\mathcal{M}_{k_1,k_2,k_3,k_4}=\frac{\mathcal{N}_{k_i}}{N^{\frac{3}{2}}}\frac{(tu+st\sigma+\tau su)^2}{stu}\mathcal{P}_{k_i}(\sigma,\tau)=\frac{\mathcal{N}_{k_i}}{N^{\frac{3}{2}}}\frac{(s+t-s\alpha)^2(s+t-s\bar{\alpha})^2}{stu}\mathcal{P}_{k_i}(\sigma,\tau),$$

$$\mathcal{P}_{k_i}(\sigma,\tau)=\sum_{\substack{i+j+k=\mathcal{E}-2\\0\leq i,j,k\leq \mathcal{E}-2}}\frac{(\mathcal{E}-2)!\,\sigma^i\tau^j}{i!\,j!\,\big(i+\frac{\kappa_u}{2}\big)!\,\big(j+\frac{\kappa_t}{2}\big)!\,\big(\frac{\kappa_u}{2}\big)!},$$

$$\sigma=\alpha\bar{\alpha},\tau=(1-\alpha)(1-\bar{\alpha})$$

$$\langle O_1(x_1) \ldots O_4(x_4) \rangle = \frac{1}{(x_{12}^2)^{\frac{\Delta_1+\Delta_2}{2}}(x_{34}^2)^{\frac{\Delta_3+\Delta_4}{2}}}\Big(\frac{x_{14}^2}{x_{24}^2}\Big)^a\Big(\frac{x_{14}^2}{x_{13}^2}\Big)^b\mathcal{G}(U,V),$$

$$a=\frac{1}{2}(\Delta_2-\Delta_1), b=\frac{1}{2}(\Delta_3-\Delta_4),$$

$$\mathcal{G}(U,V)=\int_{-i\infty}^{i\infty}\frac{dsdt}{(4\pi i)^2}U^{\frac{s}{2}}V^{\frac{t}{2}-\frac{\Delta_2+\Delta_3}{2}}\mathcal{M}(s,t)_{\Gamma_{\{\Delta_i\}}}$$

$$\Gamma_{\{\Delta_i\}}=\Gamma\Big(\frac{\Delta_1+\Delta_2-s}{2}\Big)\Gamma\Big(\frac{\Delta_3+\Delta_4-s}{2}\Big)\Gamma\Big(\frac{\Delta_1+\Delta_4-t}{2}\Big)\\ \Gamma\Big(\frac{\Delta_2+\Delta_3-t}{2}\Big)\Gamma\Big(\frac{\Delta_1+\Delta_3-u}{2}\Big)\Gamma\Big(\frac{\Delta_2+\Delta_4-u}{2}\Big)$$

$$\mathcal{M}^c(s,t)=\sum_{a=a_0,b=b_0}^{a_1,b_1}\chi_{a,b}s^at^b.$$

$$\mathcal{G}^c(U,V)=\sum_{a=a_0,b=b_0}^{a_1,b_1}\chi_{a,b}(2U\partial_U)^a(2V\partial_V+\Delta_2+\Delta_3)^b\Big(U^{\frac{\Delta_1+\Delta_2}{2}}\bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4}\Big)$$

$$\mathcal{G}^{c,l.s}(U,V)=(-1)^{a_1+b_1}\chi_{a_1,b_1}(2U)^{a_1}(2V)^{b_1}U^{\frac{\Delta_1+\Delta_2}{2}}\phi_{\Delta_1+a_1,\Delta_2+a_1+b_1,\Delta_3+b_1,\Delta_4},$$

$$\mathcal{M}_{\Delta_E,\ell_E}^{ex}(s,t)=\sum_{m=0}^{\infty}\frac{f_{m,\ell_E}Q_{\ell_E}(t,u)}{s-\tau_E-2m}\,,$$

$$\mathcal{M}_{\Delta_E,\ell_E}^{ex}(s,t)=\sum_{m=0}^{m_0}\frac{f_{m,\ell_E}Q_{\ell_E}(t,u)}{s-(\Delta_1+\Delta_2)+2k_m},$$



$$\frac{1}{s-(\Delta_1+\Delta_2)+2k_m}\Gamma\left(\frac{\Delta_1+\Delta_2-s}{2}\right)=-\frac{1}{2}\Gamma\left(\frac{\Delta_1+\Delta_2-s}{2}-k_m\right)\prod_{n=1}^{k_m-1}\left[\frac{\Delta_1+\Delta_2-s}{2}-n\right]$$

$$\mathcal{G}_{\Delta_E,\ell_E}^{ex}(U,V)=-\sum_{m=0}^{m_0}\frac{f_{m,\ell_E}}{2}\hat{Q}_{\ell_E}\prod_{n=1}^{k_m-1}\left[\frac{\Delta_1+\Delta_2}{2}-U\partial_U-n\right]\Big(U^{\frac{\Delta_1+\Delta_2}{2}-k_m}\bar{D}_{\Delta_1-k_m,\Delta_2-k_m,\Delta_3,\Delta_4}\Big)$$

$$t\rightarrow \mathcal{D}_t=2V\partial_V+\Delta_2+\Delta_3,u\rightarrow \mathcal{D}_u=\Delta_1+\Delta_4-2U\partial_U-2V\partial_V$$

$$\mathcal{Q}_{\ell_E}(s,t)=\tilde{\mathcal{Q}}+\sum_{a+b=\ell_E}\chi_{a,b}t^au^b$$

$$\mathcal{G}_{\Delta_E,\ell_E}^{ex,l,s}(U,V)=-\frac{1}{2}\left[\sum_{m=0}^{m_0}f_{m,\ell_E}\right]\sum_{a+b=\ell_E}(-1)^a\chi_{a,b}2^{a+b}V^aU^{\frac{\Delta_1+\Delta_2}{2}-1}\Phi_{\Delta_1-1,\Delta_2-1+a+b,\Delta_3+a,\Delta_4+b}^{l,s}$$

$$\mathcal{M}_{\Delta_E,\ell_E}^{ex}(s,t)\stackrel{s,t\rightarrow\infty}{\rightarrow}\frac{\sum_{a+b=\ell_E}\chi_{a,b}t^au^b}{s}\sum_{m=0}^{\infty}f_{m,\ell_E}$$

$$\frac{1}{s}\Gamma\Big(\frac{\Delta_1+\Delta_2-s}{2}\Big)=-\frac{1}{2}\Gamma\Big(-\frac{s}{2}\Big)\prod_{n=1}^{k-1}\Big[\frac{k-s}{2}-n\Big]$$

$$\mathcal{G}_{\Delta_E,\ell_E}^{ex,l,s}(U,V)=-\frac{1}{2}\left[\sum_{m=0}^{\infty}f_{m,\ell_E}\right]\sum_{a+b=\ell_E}(-1)^a\chi_{a,b}2^{a+b}V^aU^{\frac{\Delta_1+\Delta_2}{2}-1}\Phi_{\Delta_1-1,\Delta_2-1+a+b,\Delta_3+a,\Delta_4+b}^{l,s}$$

$$\mathcal{A}_2=(e_1\cdot e_2)^22P_1^0(2\pi)^{10}\delta^{(10)}(P_1+P_2)$$

$$\mathcal{A}_2=t_{12}^22P_1^0V_7(2\pi)^3\delta^{(3)}(P_1+P_2)$$

$$\begin{aligned}\mathcal{A}_3&=(e_1\cdot e_2e_3\cdot P_1+\text{cyclic})^2\delta^{(11)}\Big(\sum_iP_i\Big)\\&=(e_1\cdot e_2e_3\cdot P_1+e_2\cdot e_3e_1\cdot P_2-e_3\cdot e_1e_2\cdot P_1)^2\delta^{(11)}\Big(\sum_iP_i\Big),\end{aligned}$$

$$\begin{aligned}\mathcal{A}_3&=[(e_1\cdot e_2e_3\cdot P_1)^2+(e_2\cdot e_3e_1\cdot P_2)^2+(e_3\cdot e_1e_2\cdot P_1)^2+2e_1\cdot e_2e_3\cdot P_1e_2\cdot e_3e_1\cdot P_2\\&\quad -2e_2\cdot e_3e_1\cdot P_2e_3\cdot e_1e_2\cdot P_1-2e_1\cdot e_2e_3\cdot P_1e_3\cdot e_1e_2\cdot P_1]\delta^{(11)}\Big(\sum_iP_i\Big)\end{aligned}$$

$$P_i^AP_j^B\rightarrow \frac{1}{\ell^2}\int\,d^6\hat{p}_i^Ad^6\hat{p}_j^B=\frac{1}{\ell^2}\delta_{ij}\delta^{AB}$$



$$\mathcal{A}_3 \rightarrow -\frac{2V_7}{\ell^2} e_1 \cdot e_2 e_2 \cdot e_3 e_3 \cdot e_1 \delta^{(4)} \left(\sum_i P_i \right) = -\frac{2V_7}{\ell^2} t_{12} t_{23} t_{13} \delta^{(4)} \left(\sum_i P_i \right)$$

$$\mathcal{M}_{2,2,2,2} = \frac{1}{c_T} \mathcal{M}_{2,2,2,2}^R + \frac{1}{\frac{c_T^3}{5}} B^{R^4} \mathcal{M}_{2,2,2,2}^{(4)} \\ + \frac{1}{\frac{c_T^3}{7}} \left(B_4^{D^6 R^4} \mathcal{M}_{2,2,2,2}^{(4)} + B_6^{D^6 R^4} \mathcal{M}_{2,2,2,2}^{(6)} + B_7^{D^6 R^4} \mathcal{M}_{2,2,2,2}^{(7)} \right) + \dots$$

$$B_4^{R^4} = 1120 \left(\frac{2}{9\pi^8 k_{CS}^2} \right)^{\frac{1}{3}}, B_4^{D^6 R^4} = -\frac{1352960}{9} \left(\frac{36}{\pi^{10} k_{CS}^4} \right)^{\frac{1}{3}} \\ B_6^{D^6 R^4} = -220528 \left(\frac{36}{\pi^{10} k_{CS}^4} \right)^{\frac{1}{3}}, B_7^{D^6 R^4} = 16016 \left(\frac{36}{\pi^{10} k_{CS}^4} \right)^{\frac{1}{3}}$$

$$\mathcal{M}_{2,2,2,2}^{(4),HE} = (s+t-s\alpha)^2 (s+t-s\bar{\alpha})^2 \\ \mathcal{M}_{2,2,2,2}^{(6),HE} = 2(s^2+t^2+st)(s+t-s\alpha)^2 (s+t-s\bar{\alpha})^2 \\ \mathcal{M}_{2,2,2,2}^{(7),HE} = stu(s+t-s\alpha)^2 (s+t-s\bar{\alpha})^2$$

$$\lim_{\ell \rightarrow \infty} \frac{V_7}{(2\pi)^4} \langle \mathcal{O}_2 \dots \mathcal{O}_2 \rangle = \left[1 + f_1 (\ell_{11} \partial_{u_4})^6 + f_2 (\ell_{11} \partial_{u_4})^{10} + f_3 (\ell_{11} \partial_{u_4})^{11} \right] \tilde{\mathcal{C}}_4$$

$$f_3 = \mathcal{N}_3 f_1^2, f_2 = \mathcal{N}_2 \left| \frac{z_{24}}{z_{12}} \right|^4 \left((1-z)^2 \left| \frac{z_{34}}{z_{23}} \right|^4 - z(1-z) \left| \frac{z_{34}}{z_{23}} \right|^2 + z^2 \right) f_1 \\ f_1 = \mathcal{N}_1 z^2 |z_{24}|^2 |z_{34}|^2 |z_{14}|^4$$

La supersimetría extendida, en un campo cuántico relativista bajo supergravedad, se calcula así:

$$Z_{\text{SUGRA}} = \sum_{m \in \{ \text{Configurations} \}} \prod_{i \in \{ \text{Spectrum} \}} Z_{\text{i-th field in m-th conf.}}$$

$$h\geq j$$

$$\chi^{\mathfrak{g}}(h,j)=\mathrm{Tr}((-1)^Fq^{L_0}zJ_0^3)$$

$$\chi^{\mathfrak{g}}(j)=q^j\chi_j(z)-2q^{j+\frac{1}{2}}\chi_{j-\frac{1}{2}}(z)+q^{j+1}\chi_{j-1}(z)$$

$$\chi_j(z)=\frac{z^{j+1/2}-z^{-j-1/2}}{z^{1/2}-z^{-1/2}}, j\geq 0$$

$$\chi^{\mathfrak{g}}(0)=q^0\chi_0(z)=1, \chi^{\mathfrak{g}}(1/2)=q^{1/2}\chi_{1/2}(z)+2q^1\chi_0(z) \\ \chi^{\mathfrak{g}}(1)=q^1\chi_1(z)-2q^{3/2}\chi_{1/2}(z)+q^2\chi_0(z)$$



$$\begin{aligned}\xi^{(2)}(j) &= \chi^g(j+1)\bar{\chi}^g(j), \quad \bar{\xi}^{(2)}(j) = \chi^g(j)\bar{\chi}^g(j+1) \\ \xi^{(3)}(j) &= \chi^g\left(j+\frac{1}{2}\right)\bar{\chi}^g(j), \quad \bar{\xi}^{(3)}(j) = \chi^g(j)\bar{\chi}^g\left(j+\frac{1}{2}\right) \\ \xi^{(1)}(j) &= \chi^g(j)\bar{\chi}^g(j)\end{aligned}$$

$$\xi^{(2)}(0)+\sum_{j\geq\frac{1}{2}}\left[\xi^{(2)}(j)+\mathbf{21}_{21}\xi^{(1)}(j)+\xi^{(1)}\left(j+\frac{1}{2}\right)\right]+\text{cc.}$$

$$\xi^{(2)}(0) = \left[q^1 \chi_1(z) - 2 q^{3/2} \chi_{1/2}(z) + q^2 \chi_0(z) \right] \times [(\bar{q}^0 \chi_0(\bar{z}))]$$

$$1\times (1,0,1,0)+2\times \left(\frac{3}{2},0,\frac{1}{2},0\right)+1\times (2,0,0,0)$$

$$ds^2_{\rm 6D}=d\rho^2+\cosh^2\rho dt_{\rm E}^2+\sinh^2\rho d\varphi^2+d\theta^2+\sin^2\theta d\phi_1^2+\cos^2\theta d\phi_2^2$$

$$(t_{\rm E},\varphi,\phi_1,\phi_2)\sim (t_{\rm E}+\beta,\varphi+i\beta\mu,\phi_1+i\beta\nu_1,\phi_2+i\beta\nu_2)$$

$$\tau=\frac{\beta\mu+i\beta}{2\pi}, q\equiv e^{2\pi i\tau}$$

$$v=\beta v_2-\beta v_1, z=e^{2\pi i v}$$

$$K^{(s)}(x,y;t)=\sum_n\Psi_n^{(s)}(x)\Psi_n^{*(s)}(y)e^{t\lambda_n^{(s)}}$$

$$K_{G/\Gamma}(x,y;t)=\sum_{\gamma\in\Gamma}K_G(x,\gamma(y);t)$$

$$-\frac{1}{2}\ln\det(-\Delta_{(s)}+M^2)=\frac{1}{2}\int_0^\infty\frac{dt}{t}\int_{G/\Gamma}d\mu(x)K^{(s)}(x,x;t)e^{-M^2t}$$

$$\int_{\mathbf{S}^3/\mathbb{Z}_n}d\mu(x)K_{\mathbf{S}^3/\mathbb{Z}_n}(x,x;t)=\sum_{m\in\mathbb{Z}_n}\sum_{\ell,\bar{\ell}}\chi_\ell(z^m)\chi_{\bar{\ell}}(\bar{z}^m)e^{-C(\ell,\bar{\ell})t}$$

$$\frac{\tau_2}{2\pi}\sum_{m\in\mathbb{Z}}\int_0^\infty d\lambda[\hat{\chi}_{(j_1,j_2)}(e^{i\pi m\tau})(e^{i\pi m\tau})+\hat{\chi}_{(j_2,j_1)}(e^{i\pi m\tau})(e^{i\pi m\tau})]e^{-(\lambda^2+s+1)t}$$

$$\hat{\chi}_{j_1,j_2}(\alpha)=\frac{\alpha^{2j_1+1}\bar{\alpha}^{2j_2+1}+\alpha^{-2j_1-1}\bar{\alpha}^{-2j_2-1}}{|\alpha-\alpha^{-1}|^2}$$

$$\frac{\tau_2}{2\pi}\sum_{m\in\mathbb{Z}}\int_0^\infty d\lambda[\hat{\chi}_{(j_1,j_2)}(e^{i\pi m\tau})\chi_\ell(z^m)\chi_{\bar{\ell}}(\bar{z}^m)+\hat{\chi}_{(j_2,j_1)}(e^{i\pi m\tau})\chi_{\bar{\ell}}(z^m)\chi_\ell(\bar{z}^m)]e^{-(\lambda^2+s+1+M^2)t}$$

$$\sum_{m=1}^\infty \frac{\pi \tau_2 e^{2\pi i s m \tau_1} \chi_\ell(z^m) \chi_{\bar{\ell}}(\bar{z}^m)}{\sqrt{4\pi t} |\sin \pi m \tau|^2} e^{-\frac{4\pi^2 m^2 \tau_2^2}{4t}} e^{-(s+1+M^2)t}$$

$$Z_{1\text{-loop}}=\det^{-1/2}(-\Delta_{(s)}+M^2)$$



$$\ln Z_{1-\text{loop}}^{(3/2)} = \frac{1}{2} \ln \det \left(-\Delta_{(3/2)} + \frac{9}{4} \right) + \frac{1}{2} \ln \det \left(-\Delta_{(1/2)} - \frac{3}{4} \right).$$

$$\frac{1}{4\pi^{1/2}} \int_0^\infty \frac{dt}{t^{3/2}} e^{-\frac{\alpha^2}{4t} - \beta^2 t} = \frac{1}{2\alpha} e^{-\alpha\beta}$$

$$\ln Z_{\text{one helicity}}^{(3/2)} = - \sum_{m=1}^{\infty} \frac{(-1)^m}{m} \frac{q^{\frac{3m}{2}}}{1-q^m} (z^{+1/2} + z^{-1/2})$$

$$\ln Z_{\text{one helicity}}^{(3/2)} = - \sum_{n=2}^{\infty} \left[\ln \left(1 - z^{+1/2} q^{n-1/2} \right) + \ln \left(1 - z^{-1/2} q^{n-1/2} \right) \right]$$

$$Z^{(3/2)} = \prod_{n=2}^{\infty} \left| 1 - z^{+1/2} q^{n-1/2} \right|^2 \left| 1 - z^{-1/2} q^{n-1/2} \right|^2$$

$$Z_1(q,z)=\frac{q^{3/2}}{1-q}\chi_{1/2}(z)$$

$$Z_1(q) \rightarrow \sum_{m=1}^{\infty} \frac{1}{m} Z_1(q^m)$$

$$\begin{array}{l} L_0 \rightarrow L_0 + 2mJ_0^3 + km^2 \\ J_n^3 \rightarrow J_n^3 + km\delta_{n,0} \\ J_n^\pm \rightarrow J_{n\pm w}^\pm \end{array}$$

$$d\theta^2 + \sin^2\theta (d\Phi_1 + A_1)^2 + \cos^2\theta (d\Phi_2 + A_2)^2$$

$$A=A_1+A_2, \bar{A}=A_2-A_1.$$

$$S_{\rm gauge} = -\frac{ik}{4\pi} \int d^3x {\rm Tr} \left[A dA + \frac{2}{3} A^3 \right] + S_{\rm bnd}$$

$$L_0^{\rm gauge} = \frac{1}{8\pi} \oint dw A_w^2, J_0^3 = ik \oint \frac{dw}{2\pi i} A_w$$

$$d\Phi_1 \rightarrow d\Phi_1 + mdw + \bar{m}d\bar{w}, d\Phi_2 \rightarrow d\Phi_2 + mdw - \bar{m}d\bar{w}$$

$$A \rightarrow A + 2mdw, \bar{A} \rightarrow \bar{A} + 2\bar{m}d\bar{w}$$

$$L_0 \rightarrow L_0 + 2mJ_0^3 + km^2, J_0^3 \rightarrow J_0^3 + km,$$

$$A_{\bar{w}} \propto \frac{v}{\tau}, \bar{A}_w \propto \frac{\bar{v}}{\tau}$$

$$\exp\left[i\int_{\gamma}A\right]=-1$$

$$A_w \propto -\frac{v}{\tau} + 2m, \bar{A}_w \propto -\frac{\bar{v}}{\tau} + 2\bar{m},$$



$$Z_{(0,0)}^{1\text{-loop}} = \frac{1}{\underbrace{\prod_{n=1}^{\infty} |1 - z^{+1} q^n|^2 |1 - q^n|^2 |1 - z^{-1} q^n|^2}_{3 \text{ vectors}}} \times$$

$$\underbrace{\prod_{n=2}^{\infty} |1 - z^{1/2} q^{n-1/2}|^4 |1 - z^{-1/2} q^{n-1/2}|^4}_{4 \text{ gravitini}} \times \underbrace{\frac{1}{\prod_{n=2}^{\infty} |1 - q^n|^2}}_{\text{graviton}}$$

$$S_{\text{on-shell}} = -\frac{i\pi k\tau}{2} + \frac{i\pi \bar{k}\bar{\tau}}{2} - \pi\tau_2(A_w^2 + \bar{A}_w^2)$$

$$Z_{\text{SUGRA}}^{1\text{-loop}} = \sum_{m,\bar{m}=0}^{\infty} q^{km^2+k\bar{m}^2} z^{km+k\bar{m}}$$

$$\times \frac{\prod_{n=2}^{\infty} |1 - z^{1/2} q^{m+n-1/2}|^4 |1 - z^{-1/2} q^{-m+n-1/2}|^4}{\prod_{n=1}^{\infty} |1 - zq^{2m+n}|^2 |1 - q^n|^2 |1 - z^{-1} q^{-2m+n}|^2 \prod_{n=2}^{\infty} |1 - q^n|^2}$$

$$\chi(0)|_{m=0} = \prod_{n=1}^{\infty} \frac{(1 - z^{+1/2} q^{n-1/2})^2 (1 - z^{-1/2} q^{n-1/2})^2}{(1 - zq^n)(1 - q^n)^2 (1 - z^{-1} q^{n-1})} \left[\frac{1}{(1 - z^{+1/2} q^{-1/2})^2} - \frac{z^{-1}}{(1 - z^{-1/2} q^{-1/2})^2} \right]$$

$$\frac{(1-z^{-1})(1-q)}{(1-z^{+1/2}q^{1/2})^2(1-z^{-1/2}q^{1/2})^2},$$

$$\mathfrak{f}(q,z) \rightarrow \sum_{m \in \mathbb{Z}} q^{km^2} z^{km} \mathfrak{f}(q, zq^{2m})$$

$$\chi(0)|_{m=0} = q^{\frac{1}{4}} \frac{\vartheta_4^2(z^{1/2},q)}{\eta(q)^3 \vartheta_1(z,q)} [\mu(q,z) - \mu(q,z^{-1})]_{m=0}.$$

$$\frac{\vartheta_4^2(z^{1/2},q)}{\eta(q)^3 \vartheta_1(z,q)} \rightarrow q^{-m^2} z^{-m} \frac{\vartheta_3^2(z^{1/2},q)}{\eta(q)^3 \vartheta_1(z,q)}.$$

$$[\mu(q,z) - \mu(q,z^{-1})]_{m=0} \rightarrow q^{m^2} z^m [\mu(q,z) - \mu(q,z^{-1})]_m.$$

$$\chi(0) = q^{\frac{1}{4}} \frac{\vartheta_4^2(z^{1/2},q)}{\eta(q)^3 \vartheta_1(z,q)} [\mu(q,z) - \mu(q,z^{-1})]$$

$$Z_{\text{SUGRA}}^{1\text{-loop}} = \chi(0) \bar{\chi}(0),$$

$$\xi^{(2)}(0) + 2\xi^{\binom{3}{2}}(0) + \sum_{j \geq \frac{1}{2}} \left[\xi^{(2)}(j) + \mathbf{4}_5 \xi^{\binom{3}{2}}(j) + \mathbf{5}_5 \xi^{(1)}(j) + \xi^{(1)}\left(j + \frac{1}{2}\right) \right] + \text{cc}$$

$$\xi^{\binom{3}{2}}(0) = q^{1/2} \chi_{1/2}(z) - 2q$$

$$1 \times \left(\frac{1}{2},0,\frac{1}{2},0\right) + 2 \times (1,0,0,0)$$



$$\underbrace{\prod_{n=1}^{\infty} |1-z^{+1/2}q^{n-1/2}|^4 |1-z^{-1/2}q^{n-1/2}|^4}_{4\text{ spin-}\frac{1}{2}\text{ fermions}} \times \underbrace{\frac{1}{\prod_{n=1}^{\infty} |1-q^n|^8}}_{4\text{ vectors}}$$

$$\prod_{n=1}^{\infty} \frac{(1-z^{+1/2}q^{n-1/2})^2(1-z^{-1/2}q^{n-1/2})^2}{(1-q^n)^4}$$

$$h\geq \left(\frac{1}{1+\alpha}j_-+\frac{\alpha}{1+\alpha}j_+\right)$$

$$h\geq \frac{1}{k_++k_-} [k_+\ell_-+k_-\ell_++(\ell_+-\ell_-)^2+u^2]$$

$$\chi^{\mathfrak{g}}(h,j_+,j_-)=\mathrm{Tr}\left((-1)^Fq^{L_0}z_+^{J_0^{+,3}}z_-^{J_0^{-,3}}\right)$$

$$\begin{aligned}\chi^{\mathfrak{g}}_{\mathcal{S}}(\ell_+,\ell_-)=& q^h\chi_{\ell_+}(z_+)\chi_{\ell_-}(z_-)\\& +q^{h+\frac{1}{2}}\Big[\chi_{\ell_++\frac{1}{2}}(z_+)\chi_{\ell_--\frac{1}{2}}(z_-)+\chi_{\ell_+-\frac{1}{2}}(z_+)\chi_{\ell_-+\frac{1}{2}}(z_-)+\chi_{\ell_+-\frac{1}{2}}(z_+)\chi_{\ell_--\frac{1}{2}}(z_-)\Big]\\& +q^{h+1}\Big[\chi_{\ell_+}(z_+)\chi_{\ell_--1}(z_-)+\chi_{\ell_+-1}(z_+)\chi_{\ell_-}(z_-)+\chi_{\ell_+}(z_+)\chi_{\ell_-}(z_-)\Big]\\& +q^{h+\frac{3}{2}}\chi_{\ell_+-\frac{1}{2}}(z_+)\chi_{\ell_--\frac{1}{2}}(z_-)\end{aligned}$$

$$\chi^{\mathfrak{g}}_{\mathcal{S}}\Big(\frac{1}{2},\frac{1}{2}\Big)=-q^{\frac{1}{2}}\chi_{\frac{1}{2}}(z_+)\chi_{\frac{1}{2}}(z_-)+q[1+\chi_1(z_+)+\chi_2(z_-)]-q^{\frac{3}{2}}\chi_{\frac{1}{2}}(z_+)\chi_{\frac{1}{2}}(z_-)+q^2$$

$$\sum_{j=0}^{\infty} \left| \chi^{\mathfrak{g}}_{\mathcal{S}}(j,j) + \chi^{\mathfrak{g}}_{\mathcal{S}}\left(j+\frac{1}{2},j+\frac{1}{2}\right) \right|^2 + \text{Long}$$

$$\Big[-q^{\frac{1}{2}}\chi_{\frac{1}{2}}(z_+)\chi_{\frac{1}{2}}(z_-)+q[1+\chi_1(z_+)+\chi_2(z_-)]-q^{\frac{3}{2}}\chi_{\frac{1}{2}}(z_+)\chi_{\frac{1}{2}}(z_-)+q^2\Big]\times [\bar{q}^0]$$

$$\left(\frac{1}{2},0,\frac{1}{2},0,\frac{1}{2},0\right)+(1,0,0,0,0,0)$$

$$(1,0,1,0,0,0)+(1,0,0,0,1,0)+\left(\frac{3}{2},0,\frac{1}{2},0,\frac{1}{2},0\right)+(2,0,0,0,0,0)$$

$$ds^2_{9\mathrm{D}}=d\rho^2+\cosh^2\rho dt_{\mathrm{E}}^2+R_+^2d\Omega_+^2+R_-^2d\Omega_-^2$$

$$\begin{array}{c} (t_{\mathrm{E}},\varphi,\phi_{+,1},\phi_{+,2},\phi_{-,1},\phi_{-,2})\sim\\ (t_{\mathrm{E}}+\beta,\varphi+i\beta\mu,\phi_{+,1}+i\beta\nu_{+,1},\phi_{+,2}+i\beta\nu_{+,2},\phi_{-,1}+i\beta\nu_{-,1},\phi_{-,2}+i\beta\nu_{-,2}) \end{array}$$

$$\nu_+=\beta\nu_{+,2}-\beta\nu_{+,1}\rightarrow z_+=e^{2\pi i\nu_+},\nu_-=\beta\nu_{-,2}-\beta\nu_{-,1}\rightarrow z_-=e^{2\pi i\nu_-}$$

$$\frac{\tau_2}{2\pi}\sum_{m\in\mathbb{Z}}\int_0^\infty d\lambda\left[\hat{\chi}_{(j_1,j_2)}\left(e^{\frac{im\tau}{2}}\right)\chi_{\ell_+}(z_+^m)\chi_{\bar{\ell}_+}(\bar{z}_+^m)\chi_{\ell_-}(z_-^m)\chi_{\bar{\ell}_-}(\bar{z}_-^m)\right]e^{-(\lambda^2+s+1+M^2)t}$$



$$\ln Z^{(3/2)} = - \sum_{m=1}^{\infty} \frac{(-1)^m}{m} \frac{q^{\frac{3m}{2}}}{1-q^m} (z_+^{1/2} + z_+^{-1/2}) (z_-^{1/2} + z_-^{-1/2})$$

$$Z^{(3/2)} = \prod_{n=2}^{\infty} \left| 1 - z_+^{1/2} z_-^{1/2} q^{n-1/2} \right|^2 \left| 1 - z_+^{-1/2} z_-^{1/2} q^{n-1/2} \right|^2 \\ \times \left| 1 - z_+^{1/2} z_-^{-1/2} q^{n-1/2} \right|^2 \left| 1 - z_+^{-1/2} z_-^{-1/2} q^{n-1/2} \right|^2$$

$$Z^{(3/2)} = \prod_{n=2}^{\infty} \prod_{\lambda_{\pm} \in \{\pm\}} \left| 1 - z_+^{\lambda_+/2} z_-^{\lambda_-/2} q^{n-1/2} \right|^2$$

$$d\theta_+^2 + \sin^2 \theta_+ (d\Phi_{+,1} + A_{+,1})^2 + \cos^2 \theta_+ (d\Phi_{+,2} + A_{+,2})^2 \\ + d\theta_-^2 + \sin^2 \theta_- (d\Phi_{-,1} + A_{-,1})^2 + \cos^2 \theta_- (d\Phi_{-,2} + A_{-,2})^2$$

$$A_+ = A_{+,1} + A_{+,2}, \quad \bar{A}_+ = A_{+,2} - A_{+,1} \\ A_- = A_{-,1} + A_{-,2}, \quad \bar{A}_- = A_{-,2} - A_{-,1}$$

$$A_{+,\bar{w}} \propto \frac{v_+}{\tau}, \bar{A}_{+,\bar{w}} \propto \frac{\bar{v}_+}{\tau}, A_{-,\bar{w}} \propto \frac{v_-}{\tau}, \bar{A}_{-,\bar{w}} \propto \frac{\bar{v}_-}{\tau}$$

$$A_{+,w} \propto \frac{v_+}{\tau} + 2m_+, \bar{A}_{+,\bar{w}} \propto \frac{\bar{v}_+}{\tau} + 2\bar{m}_+, A_{-,w} \propto \frac{v_-}{\tau} + 2m_-, \bar{A}_{-,\bar{w}} \propto \frac{\bar{v}_-}{\tau} + 2\bar{m}_-,$$

$$L_0 \rightarrow L_0 + 2m_+ J_0^{+,3} + k_+ m_+^2 + 2m_- J_0^{-,3} + k_- m_-^2 \\ J_n^{+,3} \rightarrow J_n^{+,3} + k_+ m_+ \delta_{n,0} \\ J_n^{-,3} \rightarrow J_n^{-,3} + k_- m_- \delta_{n,0} \\ J_n^{\pm} \rightarrow J_{n \pm m_{\pm}}^{\pm}$$

$$Z_{(0,0)}^{1\text{-loop}} = \underbrace{\prod_{n=1}^{\infty} \prod_{\tilde{\lambda}_{\pm} \in \{\pm\}} \left| 1 - z_+^{\tilde{\lambda}_+/2} z_-^{\tilde{\lambda}_-/2} q^{n-1/2} \right|^2}_{4 \text{ spin-}\frac{1}{2} \text{ fermions}} \times \underbrace{\frac{1}{\prod_{n=1}^{\infty} |1-q^n|^2}}_{\text{u(1) vector}}$$

$$Z_{(0,0)}^{1\text{-loop}} = \frac{1}{\underbrace{\prod_{n=1}^{\infty} \prod_{\mu_{\pm} \in \{0,\pm\}} \left| 1 - z_+^{\mu_{\pm}} q^n \right|^2}_{3\text{su}(2)_+ \text{ vectors}}} \times \frac{1}{\underbrace{\prod_{n=1}^{\infty} \prod_{\mu_{\pm} \in \{0,\pm\}} \left| 1 - z_-^{\mu_{\pm}} q^n \right|^2}_{3\text{su}(2)_- \text{ vectors}}} \\ \times \underbrace{\prod_{n=2}^{\infty} \prod_{\lambda_{\pm} \in \{\pm\}} \left| 1 - z_+^{\lambda_+/2} z_-^{\lambda_-/2} q^{n-\frac{1}{2}} \right|^2}_{4 \text{ gravitini}} \times \underbrace{\frac{1}{\prod_{n=2}^{\infty} |1-q^n|^2}}_{\text{graviton}}$$

$$Z_{\text{SUGRA}}^{(1 \text{ loop})} = \sum_{m_{\pm}, \bar{m}_{\pm}=0}^{\infty} q^{k_+ m_+^2 + k_+ \bar{m}_+^2 + k_- m_-^2 + k_- \bar{m}_-^2} z^{k_+ m_+ + k_+ \bar{m}_+ + k_- m_- + k_- \bar{m}_-} \\ \times \frac{\prod_{n=2}^{\infty} \prod_{\lambda_{\pm} \in \{\pm\}} \left| 1 - z_+^{\lambda_+/2} z_-^{\lambda_-/2} q^{m_+ + m_- - 1/2} \right|^2 \prod_{n=1}^{\infty} \prod_{\tilde{\lambda}_{\pm} \in \{\pm\}} \left| 1 - z_+^{\tilde{\lambda}_+/2} z_-^{\tilde{\lambda}_-/2} q^{m_+ + m_- - 1/2} \right|^2}{\prod_{n=1}^{\infty} \prod_{\mu_{\pm} \in \{0,\pm\}} \left| 1 - z_{\pm}^{\mu_{\pm}} q^{2m_{\pm} + n} \right|^2 \prod_{n=2}^{\infty} |1-q^n|^2 \prod_{n=1}^{\infty} |1-q^n|^2}$$



$$\prod_{n=1}^{\infty} \frac{(1-z_+^{1/2}z_-^{1/2}q^{n-1/2})(1-z_+^{-1/2}z_-^{1/2}q^{n-1/2})(1-z_+^{1/2}z_-^{-1/2}q^{n-1/2})(1-z_+^{-1/2}z_-^{-1/2}q^{n-1/2})}{(1-z_+q^n)(1-z_+^{-1}q^n)(1-z_-q^n)(1-z_-^{-1}q^n)(1-q^n)^3(z_+^{1/2}-z_+^{-1/2})(z_-^{1/2}-z_-^{-1/2})}$$

$$\times \left[\frac{z_+^{1/2}z_-^{1/2}}{1+z_+^{1/2}z_-^{1/2}q^{1/2}} - \frac{z_+^{-1/2}z_-^{1/2}}{1+z_+^{-1/2}z_-^{1/2}q^{1/2}} - \frac{z_+^{1/2}z_-^{-1/2}}{1+z_+^{1/2}z_-^{-1/2}q^{1/2}} + \frac{z_+^{-1/2}z_-^{-1/2}}{1+z_+^{-1/2}z_-^{-1/2}q^{1/2}} \right]$$

$$\frac{(z_+-z_+^{-1})(z_--z_-^{-1})(1-q)}{(1+z_+z_-q^{1/2})(1+z_+^{-1}z_-q^{1/2})(1+z_+z_-^{-1}q^{1/2})(1+z_+^{-1}z_-^{-1}q^{1/2})},$$

$$\chi(0)|_{m_{\pm}=0}=q^{\frac{5}{8}}\frac{\vartheta_4^2\left(z_+^{1/2}z_-^{1/2},q\right)\vartheta_4^2\left(z_+^{1/2}z_-^{-1/2},q\right)}{\eta(q)^3\vartheta_1(z_+,q)\vartheta_1(z_-,q)}\sum_{\epsilon_{\pm}=\pm 1}\epsilon_+\epsilon_-\mu(q,z_+^{\epsilon_+},z_-^{\epsilon_-})\Big|_{m_{\pm}=0}$$

$$\mathfrak{f}(q,z)\rightarrow \sum_{m_{\pm}\in \mathbb{Z}}q^{k_+m_+^2+k_-m_-^2}z^{k_+m_++k_-m_-}\mathfrak{f}(q,z_+q^{2m_+},z_-q^{2m_-})$$

$$\mu(q,z_+,z_-)|_{m_{\pm}=0}\rightarrow \mu(q,z_+,z_-)|_{m_{\pm}}$$

$$\frac{\vartheta_4^2\left(z_+^{1/2}z_-^{1/2},\tau\right)\vartheta_4^2\left(z_+^{1/2}z_-^{-1/2},\tau\right)}{\eta(\tau)^3\vartheta_1(z_+,\tau)\vartheta_1(z_-,\tau)}\rightarrow \frac{\vartheta_4^2\left(z_+^{1/2}z_-^{1/2},\tau\right)\vartheta_4^2\left(z_+^{1/2}z_-^{-1/2},\tau\right)}{\eta(\tau)^3\vartheta_1(z_+,\tau)\vartheta_1(z_-,\tau)}$$

$$Z_{\rm BTZ\;in\;R-R}(\tau,\nu,\bar\tau,\bar\nu)=Z_{\rm AdS\;in\;NS-NS}\left(-\frac{1}{\tau},\nu\tau,-\frac{1}{\bar\tau},\bar\nu\bar\tau\right)$$

$$[L_m,L_n]=(m-n)L_{m+n}+\frac{1}{2}km(m^2-1)\delta_{m+n,0}$$

$$\{G_r^a,G_s^{\dot{a}}\}=2\delta^{a\dot{a}}L_{r+s}-2(r-s)(\sigma^i)^{a\dot{a}}J_{r+s}^i+\frac{1}{2}k(4r^2-1)\delta_{r+s,0}\delta^{a\dot{a}}$$

$$[J_m^i,J_n^j]=i\epsilon^{ijk}J_{m+n}^k+\frac{1}{2}km\delta_{m+n,0}\delta^{ij}$$

$$[L_m,G_r^a]=\left(\frac{1}{2}m-r\right)G_{m+r}^a$$

$$[L_m,J_n^i]=-nJ_{m+n}^i$$

$$[J_m^i,G_r^a]=-\frac{1}{2}(\sigma^i)^a{}_bG_{m+r}^b$$

$$L_0|h,\ell\rangle=h|h,\ell\rangle,J_0^3|h,\ell\rangle=j|h,\ell\rangle,J_0^+|h,\ell\rangle=0$$

$$J_n^i|h,\ell\rangle=G_r^a|h,\ell\rangle=G_r^{\dot{a}}|h,\ell\rangle=L_n|h,\ell\rangle=0,n,r>0$$

$$h\geq \ell$$

$$\chi(h,\ell;\delta)=\mathrm{Tr}_{\mathrm{NS}}(e^{i\pi\delta F}q^{L_0}zJ_0^3)$$

$$\chi(\ell;\delta)=q^{\ell}\frac{\prod_{n=1}\left(1+e^{i\pi\delta}z^{1/2}q^{n-1/2}\right)^2\left(1+e^{i\pi\delta}z^{-1/2}q^{n-1/2}\right)^2}{\prod_{n=1}\left(1-q^n\right)^2\left(1-z^{+1}q^n\right)\left(1-z^{-1}q^{n-1}\right)}z^{-1/2}(\mu(q,z;\delta)-\mu(q,z^{-1};\delta)),$$

$$\mu(q,z;\delta)=\sum_{m\in\mathbb{Z}}\frac{z^{(k+1)m+\ell+1/2}q^{(k+1)m^2+(2\ell+1)m}}{(1+e^{i\pi\delta}z^{1/2}q^{m+1/2})^2}$$



$$\chi(\ell; \delta) = -iq^{\ell+1/4} \frac{\vartheta_{3+\delta}^2(z^{1/2}, q)}{\eta(q)^3 \vartheta_1(z, q)} (\mu(q, z; \delta) - \mu(q, z^{-1}; \delta)),$$

$$\eta(q)=q^{1/24}\prod_{n=1}^{\infty}(1-q^n)$$

$$\vartheta_1(q,z)=-iq^{1/8}(z^{1/2}-z^{-1/2})\prod_{n=1}^{\infty}(1-q^n)(1-zq^m)(1-z^{-1}q^m)$$

$$\vartheta_2(q,z)=q^{1/8}(z^{1/2}+z^{-1/2})\prod_{n=1}^{\infty}(1-q^n)(1+zq^m)(1+z^{-1}q^m)$$

$$\vartheta_3(q,z)=\prod_{n=1}^{\infty}(1-q^n)(1+zq^{m-1/2})(1+z^{-1}q^{m-1/2})$$

$$\vartheta_4(q,z)=\prod_{n=1}^{\infty}(1-q^n)(1-zq^{m-1/2})(1-z^{-1}q^{m-1/2})$$

$$[L_m,L_n]=(m-n)L_{m+n}+\frac{c}{12}m(m^2-1)\delta_{m+n,0},$$

$$\{G_r^a,G_s^b\}=2\delta_{ab}L_{r+s}+4(r-s)i\left(\gamma(\alpha^{+,i})^{ab}J_{r+s}^{+,i}+(1-\gamma)(\alpha^{-,i})^{ab}J_{r+s}^{-,i}\right)\\+\frac{c}{12}(4r^2-1)\delta_{r+s,0}\delta^{ab},$$

$$[J_m^{\pm,i},J_n^{\pm,j}]=i\epsilon^{ijk}J_{m+n}^{\pm,k}+\frac{1}{2}k^{\pm}m\delta_{m+n,0}\delta^{ij},$$

$$[J_m^{\pm,i},G_r^a]=i(\alpha^{\pm,i})^{ab}G_{m+r}^b\mp\frac{2k^{\pm}}{k_++k_-}m(\alpha^{\pm,i})^{ab}Q_{m+r}^b,$$

$$\{Q_r^a,Q_s^b\}=\frac{k_++k_-}{2}\delta^{ab}\delta_{r+s,0}$$

$$[U_m,U_n]=\frac{k_++k_-}{2}m\delta_{m+n,0}$$

$$\{Q_r^a,G_s^b\}=2\alpha_{ab}^{+i}J_{r+s}^{+,i}-2\alpha_{ab}^{-i}J_{r+s}^{-,i}+\delta^{ab}U_{r+s}$$

$$[J_m^{\pm,i},Q_r^a]=i\alpha_{ab}^{\pm i}Q_{m+r}^b$$

$$[U_m,G_r^a]=mQ_{m+r}^a$$

$$(\alpha^{\pm,i})^{ab}=\frac{1}{2}(\pm\delta_{ia}\delta_{b0}\mp\delta_{ib}\delta_{a0}+\epsilon_{iab}).$$

$$c=\frac{6k_+k_-}{k_++k_-},\gamma=\frac{k_-}{k_++k_-}\langle L_0,L_{\pm 1},J_0^{\pm,i},G_{\pm\frac{1}{2}}^a\rangle$$

$$\mathcal{A}_\gamma=\tilde{\mathcal{A}}_\gamma\oplus\mathcal{A}_{\rm QU}$$

$$L_0|h,\ell_+,\ell_-\rangle=h|h,\ell_+,\ell_-\rangle,J_0^{\pm,3}|h,\ell_+,\ell_-\rangle=\ell_{\pm}|h,\ell_+,\ell_-\rangle,J_0^{\pm,+}|h,\ell_+,\ell_-\rangle=0,\\J_n^{\pm,i}|h,\ell_+,\ell_-\rangle=G_r^a|h,\ell_+,\ell_-\rangle=L_n|h,\ell_+,\ell_-\rangle=0,n,r>0.$$

$$h\geq \frac{1}{k_++k_-}[k_+\ell_-+k_-\ell_++(\ell_+-\ell_-)^2]$$



$$\chi(h, \ell_+, \ell_-; \delta) = \text{Tr}_{\text{NS}} \left(e^{i\pi\delta F} q^{L_0} z_+^{J_0^{+,3}} z_-^{J_0^{-,3}} \right)$$

$$\begin{aligned} \tilde{\chi}(\ell_+, \ell_-; \delta) &= q^h \frac{F^{NS}(q, z_+, z_-) B^{+-}(q, z_+, z_-)}{\prod_{n=1}^{\infty} (1 - q^n) \left(z_+^{1/2} - z_+^{-1/2} \right) \left(z_-^{1/2} - z_-^{-1/2} \right)} \\ &\quad \times \sum_{\epsilon_{\pm} = \pm 1} \epsilon_+ \epsilon_- \mu(q, z_+^{\epsilon_+}, z_-^{\epsilon_-}) \end{aligned}$$

$$F^{NS}(q, z_+, z_-) = \prod_{n=1}^{\infty} \prod_{\epsilon_{\pm} \in \pm} \left(1 + e^{i\pi\delta} z_+^{\epsilon_+/2} z_-^{\epsilon_-/2} q^{n-1/2} \right)$$

$$B^{+-}(q, z_+, z_-) = \prod_{n=1}^{\infty} \frac{1}{(1 - q^n)^2 (1 - z_+ q^n) (1 - z_+^{-1} q^n) (1 - z_- q^n) (1 - z_-^{-1} q^n)},$$

$$\mu(q, z_+, z_-) = \sum_{m_{\pm} \in \mathbb{Z}} q^{m_+^2 k_+ + m_-^2 k_- + m_+(2l_+ + 1) + m_-(2l_- + 1)} \frac{z_+^{m_+ k_+ + l_+ + 1/2} z_-^{m_- k_- + l_- + 1/2}}{1 + e^{i\pi\delta} z_+^{1/2} z_-^{1/2} q^{m_+ + m_- + 1/2}},$$

$$\tilde{\chi}(\ell_+, \ell_-; \delta) = -q^{h+1/2} \frac{\vartheta_{3+\delta} \left(q, z_+^{1/2} z_-^{1/2} \right) \vartheta_{3+\delta} \left(q, z_+^{1/2} z_-^{-1/2} \right)}{\eta(\tau)^3 \vartheta_1(q, z_+) \vartheta_1(q, z_-)} \sum_{\epsilon_{\pm} = \pm} \epsilon_+ \epsilon_- \mu(q, z_+^{\epsilon_+}, z_-^{\epsilon_-}).$$

$$\chi_{QU}(\ell_+, \ell_-; \delta) = q^{-1/8} \frac{F^{NS}(q, z_+, z_-)}{\prod_{n=1}^{\infty} (1 - q^n)},$$

$$\chi(\ell_+, \ell_-; \delta) = -q^{h+1/2} \frac{\vartheta_{3+\delta} \left(q, z_+^{1/2} z_-^{1/2} \right) \vartheta_{3+\delta} \left(q, z_+^{1/2} z_-^{-1/2} \right)}{\eta(\tau)^6 \vartheta_1(q, z_+) \vartheta_1(q, z_-)} \sum_{\epsilon_{\pm} = \pm 1} \epsilon_+ \epsilon_- \mu(q, z_+^{\epsilon_+}, z_-^{\epsilon_-}).$$

$$\begin{aligned} &-\frac{1}{2} \ln \det(-\Delta_{(s)} + M^2) = \\ &\sum_{m=1}^{\infty} (-1)^{m(\delta+1)} \frac{q^{sm} \chi_{\ell}(z^m) \chi_{\bar{\ell}}(\bar{z}^m) + \bar{q}^{sm} \chi_{\bar{\ell}}(z^m) \chi_{\ell}(\bar{z}^m)}{m|1 - q^m|^2} |q|^{m(\sqrt{s+1+M^2} - (s-1))} \end{aligned}$$

$$Z_{1-\text{loop}} = \det^{-1/2}(-\Delta_{(s)} + M^2)$$

$$M^2 = m_p^2 + m_s^2$$

$$\text{scalar fields : } h = \frac{1}{2} \left(1 + \sqrt{1 + m_p^2} \right)$$

$$\text{spinor fields : } h = \frac{1}{2} \left(m_p + \frac{3}{2} \right)$$

$$\text{vector fields : } h = 1 + \frac{m_p}{2}$$

$$\text{supergraviton: } h = \frac{1}{2} \left(m_p + \frac{5}{2} \right)$$

$$\text{white particle : } h = \frac{1}{2} \left(3 + \sqrt{1 + m_p^2} \right)$$



$$m_s^2 = \begin{cases} s(s-3) & s = 1, 2 \\ -(s+1) & s = \frac{1}{2}, \frac{3}{2} \end{cases}$$

$$-\frac{1}{2} \ln \det(-\Delta_{(s)} + M^2) = \sum_{m=1}^{\infty} (-1)^{m(\delta+1)} \frac{q^{sm} \chi_{\ell}(z^m) \chi_{\bar{\ell}}(\bar{z}^m) + \bar{q}^{sm} \chi_{\bar{\ell}}(z^m) \chi_{\ell}(\bar{z}^m)}{m|1-q^m|^2} |q|^{2(h-s)}$$

$$Z_{1-\text{loop}} = \prod_{n,\bar{n},p,\bar{p}=0}^{\infty} \frac{(1 + e^{i\pi\delta} z^{\ell+1+p} \bar{z}^{\bar{\ell}-\bar{p}} q^{h+n} \bar{q}^{h-s+\bar{n}})}{(1 + e^{i\pi\delta} z^{\ell+1+p} \bar{z}^{\bar{\ell}+1+\bar{p}} q^{h+n} \bar{q}^{h-s+\bar{n}})} \left| \frac{(1 + e^{i\pi\delta} z^{p-\ell} \bar{z}^{\bar{\ell}+1+\bar{p}} q^{h+n} \bar{q}^{h-s+\bar{n}})}{(1 + e^{i\pi\delta} z^{p-\ell} \bar{z}^{\bar{\ell}-\bar{p}} q^{h+n} \bar{q}^{h-s+\bar{n}})} \right|^{2(-1)^F}$$

$$Z_{1-\text{loop}} = \left[\frac{\det^{\frac{1}{2}}(-\Delta_{(s-1)} + s(s-1))^{TT}}{\det^{\frac{1}{2}}(-\Delta_{(s)} + s(s-3))^{TT}} \right]^{(-1)^F}, s \geq 1$$

$$Z_{1-\text{loop}} = \left| \prod_{n,p,\bar{p}=0}^{\infty} \frac{(1 + e^{i\pi\delta} z^{\ell+1+p} \bar{z}^{\bar{\ell}-\bar{p}} q^{n+s})(1 + e^{i\pi\delta} z^{p-\ell} \bar{z}^{\bar{\ell}+1+\bar{p}} q^{n+s})}{(1 + e^{i\pi\delta} z^{\ell+1+p} \bar{z}^{\bar{\ell}+1+\bar{p}} q^{n+s})(1 + e^{i\pi\delta} z^{p-\ell} \bar{z}^{\bar{\ell}-\bar{p}} q^{n+s})} \right|^{2(-1)^F}$$

$$Z_{1-\text{loop}} = \prod_{n=2}^{\infty} \frac{1}{|1-q^n|^2}$$

$$Z_{1-\text{loop}} = \prod_{n=2}^{\infty} \left| \left(1 + e^{i\pi\delta} z^{-\frac{1}{2}} q^{n-\frac{1}{2}} \right) \left(1 + e^{i\pi\delta} z^{\frac{1}{2}} q^{n-\frac{1}{2}} \right) \right|^2$$

$$Z_{1-\text{loop}} = \prod_{n=1}^{\infty} \left| \frac{1}{(1-z^{-1}q^n)(1-q^n)(1-zq^n)} \right|^2$$

Existen diferentes tipos de supergravedad cuántica relativista, según la geometría de la gravedad y las supersimetría de gauge empleada, a más de sus propagadores y osciladores en relación al espacio – tiempo cuántico en dimensiones altas, siendo éstas:

Supergravedad cuántica relativista tipo I: Modelamiento Matemático.

$$S = \int_X \left(\langle y, \bar{D}y \rangle - \frac{1}{3} \langle y, [y, y] \rangle \right) \wedge \Omega$$

$$Q \simeq \Lambda^{1,0} \oplus \text{End}(V) \oplus T^{1,0}$$



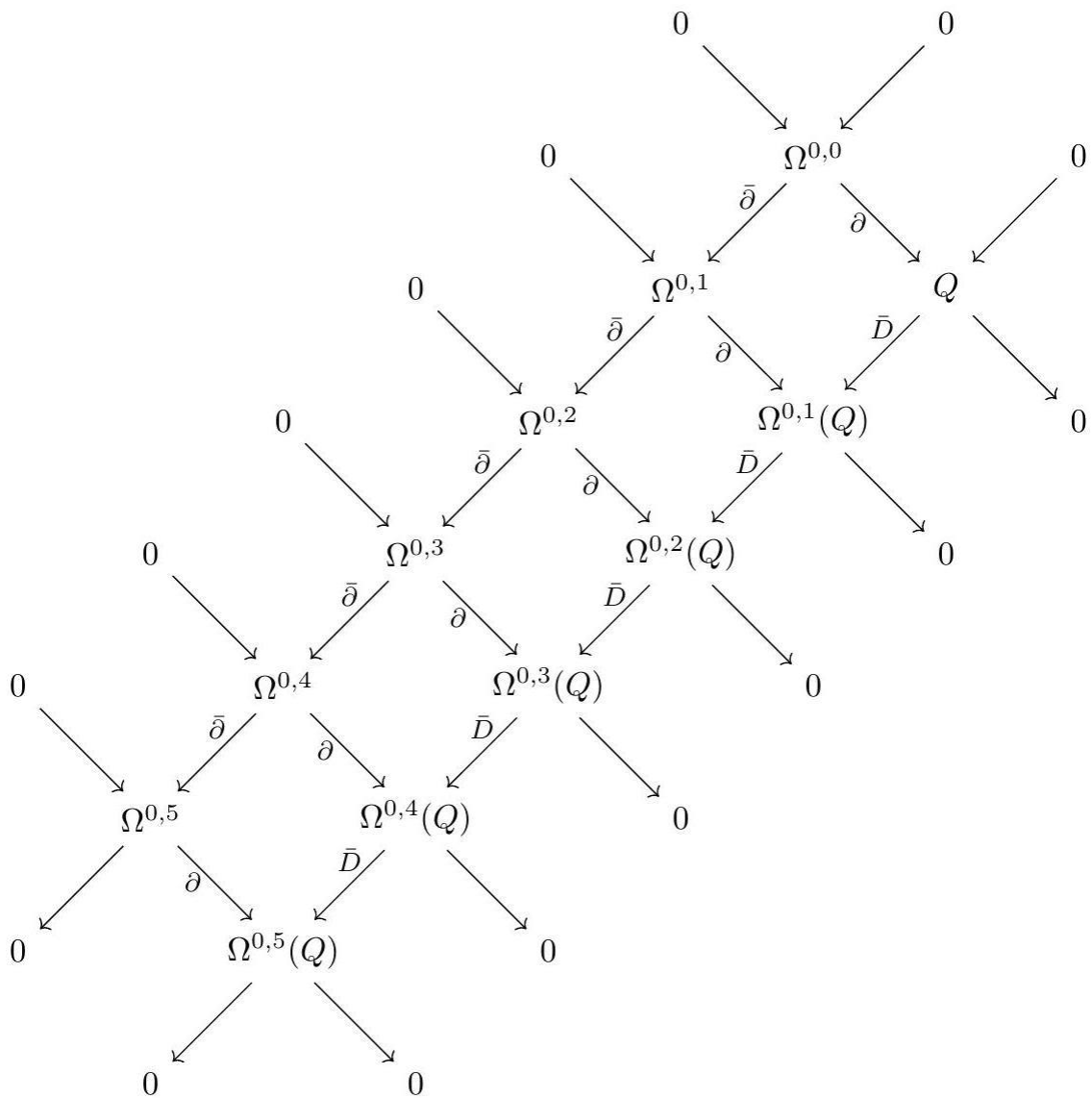
$$\bar{D} = \begin{pmatrix} \bar{\partial} & \alpha' \mathcal{F}^* & \mathcal{T} \\ 0 & \bar{\partial}_{\mathcal{A}} & \mathcal{F} \\ 0 & 0 & \bar{\partial} \end{pmatrix}$$

$$\begin{aligned} \mathcal{F}(\mu) &= F_a \wedge \mu^a \\ (\mathcal{F}^*(\alpha))_a &= \text{tr}(F_a \wedge \alpha) \\ (\mathcal{T}(\mu))_a &= -2T_{ab} \wedge \mu^b \end{aligned}$$

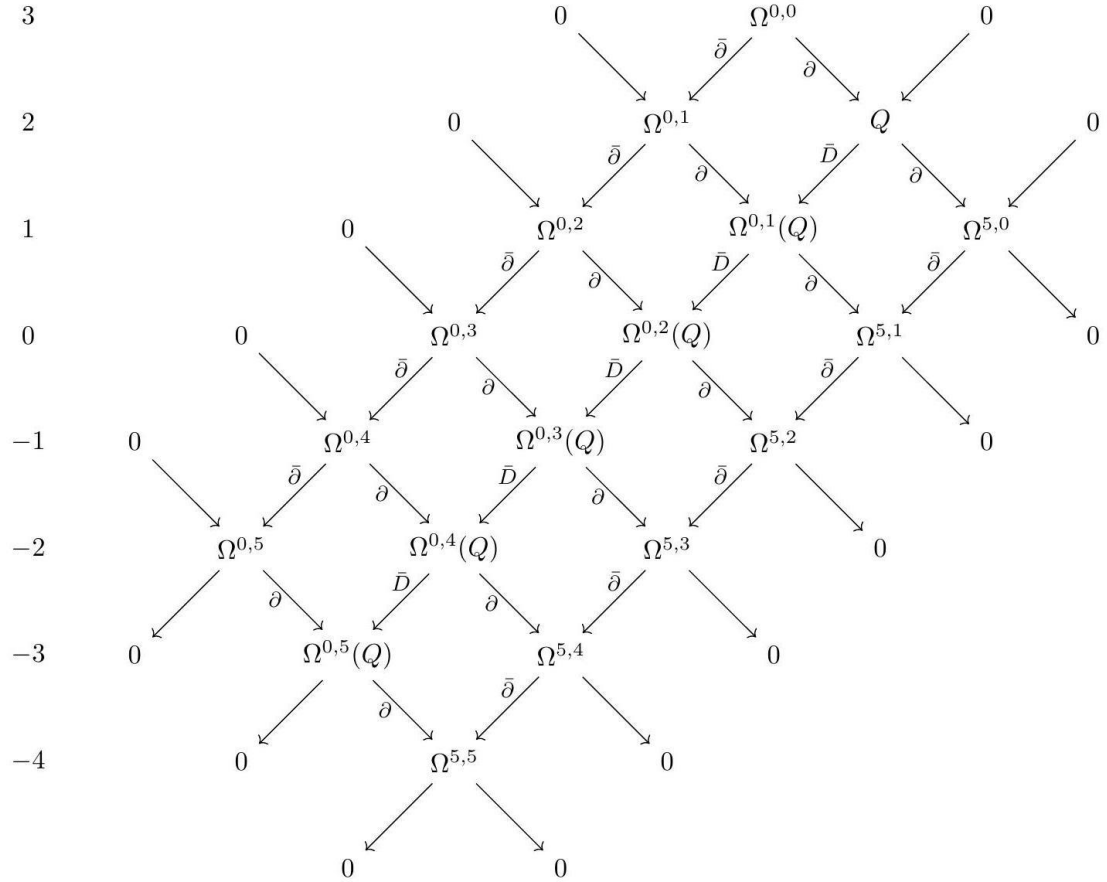
$$i\partial\bar{\partial}\omega = \frac{\alpha'}{4}\text{tr}(F\wedge F),$$

$$i\partial\bar{\partial}\omega = \frac{\alpha'}{4}(\text{tr}(F\wedge F) - \text{tr}(R\wedge R))$$

$$\omega^4\wedge F=0$$



Ghost #



$$Q \simeq \Lambda^{1,0} \oplus \text{End}(V) \oplus T^{1,0} \simeq \Lambda^{1,0} \oplus \text{End}(V) \oplus \Lambda^{4,0}$$

$$\langle b^{0,2}, c^{5,3} \rangle = \int_X b^{0,2} \wedge c^{5,3}$$

$$y^{0,\cdot} \in \Omega^{0,\cdot}(Q), b^{0,\cdot} \in \Omega^{0,\cdot}, c^{5,\cdot} \in \Omega^{5,\cdot}$$

$$S = \langle \Phi, d_{\text{total}} \Phi \rangle = \int_X \langle y, \bar{D} y \rangle \wedge \Omega + \int_X b \wedge \bar{\partial} c + \int_X (y \lrcorner \partial b) \wedge \Omega$$

$$y = (x, \alpha, \mu) \in \Omega^{0,\cdot}(Q) = \Omega^{0,\cdot}(\Lambda^{1,0}) \oplus \Omega^{0,\cdot}(\text{End}(V)) \oplus \Omega^{0,\cdot}(T^{1,0}).$$

$$\langle y_1, y_2 \rangle = (\mu_1)^a \wedge (x_2)_a + (x_1)_a \wedge (\mu_2)^a + \text{tr}(\alpha_1 \wedge \alpha_2)$$

$$\delta y = \bar{D} \gamma + \partial \beta, \delta b = \bar{\partial} \beta, \delta c = \partial (v \lrcorner \Omega) + \bar{\partial} \kappa,$$

$$\delta \Phi = d_{\text{total}} \Lambda,$$



$$\begin{aligned}
S &= \int_X \langle y, \bar{D}y \rangle \wedge \Omega + \int_X b \wedge \bar{\partial}c \\
&= \int_X (\langle y^{0,2}, \bar{D}y^{0,2} \rangle + \langle y^{0,4}, \bar{D}y^{0,0} \rangle) \wedge \Omega + b^{0,3} \wedge \bar{\partial}c^{5,1} + b^{0,1} \wedge \bar{\partial}c^{5,3} \quad (\mathfrak{F}) \\
&\quad + \int_X \langle y^{0,3}, \bar{D}y^{0,1} \rangle \wedge \Omega + b^{0,4} \wedge \bar{\partial}c^{5,0} + b^{0,2} \wedge \bar{\partial}c^{5,2} + b^{0,0} \wedge \bar{\partial}c^{5,4} \quad (\mathfrak{B})
\end{aligned}$$

$$\begin{aligned}
|Z| &= |\bar{D}^{0,2}|^{+1/2} |\bar{D}^{0,1}|^{-1} |\bar{D}^{0,0}|^{+1} |\bar{\partial}^{0,0}|^{-1} |\bar{\partial}^{0,1}|^{+1} |\bar{\partial}^{0,2}|^{-1} |\bar{\partial}^{0,3}|^{+1} |\bar{\partial}^{0,4}|^{-1} \\
&= \left[\prod_{p=0}^5 |\Delta_{\bar{D}}^{0,p}|^{(-1)^{p+1}p} \right]^{1/4} \left[\prod_{q=0}^5 |\Delta_{\bar{\partial}}^{0,q}|^{(-1)^{q+1}q} \right]^{-1/2} \\
&= I_{\bar{D}}(Q)^{1/2} I(\Lambda^{0,0})^{-1}
\end{aligned}$$

$$\begin{aligned}
S &= \int_X \langle y^{0,1}, \bar{D}y^{0,3} \rangle \wedge \Omega + \int_X (b^{0,2} \wedge \bar{\partial}c^{5,2} + b^{0,4} \wedge \bar{\partial}c^{5,0}) \\
&\quad + \int_X (y^{0,1} \lrcorner \partial b^{0,4} + y^{0,3} \lrcorner \partial b^{0,2}) \wedge \Omega
\end{aligned}$$

$$\delta\mu=\bar{\partial}v,\delta c=\partial(v\lrcorner\Omega).$$

$$v^a=\nabla^aw+\tilde{v}^a$$

$$\nabla_a\mu^a=0.$$

$$S=\int_X \langle y^{0,1}, \bar{D}y^{0,3} \rangle \wedge \Omega$$

$$\int_X \langle y,y'\rangle \wedge \Omega$$

$$P_{\sim}^{0,k}=\frac{P^{0,k}}{\Omega_{\bar{\partial}\text{-exact}}^{1,k}} \text{ where } P^{0,k}=\{y\in\Omega^{0,k}(Q)\colon \nabla_a\mu^a=0\}\subset\Omega^{0,\cdot}(Q).$$

$$\delta y = \bar{D}\epsilon \text{ for } \epsilon \in P_{\sim}^{0,\cdot}.$$

$$S=\int_X \langle y, \bar{D}y \rangle \wedge \Omega$$

$$|\bar{D}_{\mathrm{P}}^{0,p}|=\frac{|\bar{D}^{0,p}|}{|\bar{\partial}_{\bar{\partial}\text{-exact}}^{1,p}||\bar{\partial}_{\bar{\partial}\text{-exact}}^{5,p}|}$$

$$|\bar{\partial}_{\bar{\partial}\text{-exact}}^{1,p}|=|\bar{\partial}^{0,p}|,|\bar{\partial}_{\bar{\partial}\text{-exact}}^{5,p}|=|\bar{\partial}^{5,p}|=|\bar{\partial}^{0,p}|.$$

$$|\bar{D}_{\mathrm{P}}^{0,p}|=\frac{|\bar{D}^{0,p}|}{|\bar{\partial}^{0,p}|^2}$$

$$S=\int_X \Big(\langle y, \bar{D}y \rangle - \frac{1}{3}\langle y, [y,y]\rangle\Big)\wedge \Omega$$



$$\begin{aligned}
[y_1, y_2]^a &= \mu_1^b \wedge \partial_b \mu_2^a - (-1)^{|y_1||y_2|} \mu_2^b \wedge \partial_b \mu_1^a \\
[y_1, y_2]_\alpha &= -\alpha_1 \wedge \alpha_2 + (-1)^{|y_1||y_2|} \alpha_2 \wedge \alpha_1 \\
&\quad + \mu_1^b \wedge \nabla_b \alpha_2 - (-1)^{|y_1||y_2|} \mu_2^b \wedge \nabla_b \alpha_1 \\
[y_1, y_2]_a &= 2\mu_1^b \wedge \partial_{[b} x_{2,a]} - (-1)^{|y_1||y_2|} 2\mu_2^b \wedge \partial_{[b} x_{1,a]} \\
&\quad + \frac{1}{2} \text{tr}(\nabla_a \alpha_1 \wedge \alpha_2) - (-1)^{|y_1||y_2|} \frac{1}{2} \text{tr}(\nabla_a \alpha_2 \wedge \alpha_1),
\end{aligned}$$

$$\delta y = \bar{D}\lambda - [y, \lambda]$$

$$\bar{D}y - \frac{1}{2}[y, y] = 0$$

$$\bar{D}y^{0,1} - \frac{1}{2}[y^{0,1}, y^{0,1}] - [y^{0,0}, y^{0,2}] = 0.$$

$$\omega(\delta y, \delta y') = \int_X \langle \delta y', \delta y \rangle \wedge \Omega$$

$$Qy = \bar{D}y - \frac{1}{2}[y, y]$$

$$S(x,\mu)=2\int_X\left(x_a\wedge\bar{\partial}\mu^a-\frac{1}{2}\mu^a\wedge\mu^b\wedge(\partial x)_{ab}\right)\wedge\Omega$$

$$\eta=\partial x$$

$$x=\partial^{-1}\eta=\Delta_{\partial}^{-1}\partial^{\dagger}\eta$$

$$S(x,\mu)=2\int_X\left((\partial^{-1}\eta)_a\wedge\bar{\partial}\mu^a-\frac{1}{2}\eta_{ab}\wedge\mu^a\wedge\mu^b\right)\wedge\Omega$$

$$|Z|^2=\frac{I_{\bar{D}}(Q)}{I(\Lambda^{0,0})^2}$$

$$\mathcal{F}_{\text{Det}}=\frac{1}{2}\left[\int_X\text{td}(X)\wedge\text{ch}(Q)\right]_{(1,1)}-\left[\int_X\text{td}(X)\right]_{(1,1)}:=\left[-\int_X\mathcal{P}\right]_{(1,1)},$$

$$\begin{aligned}
\mathcal{P}_{(6,6)} = & \frac{\dim(G)-496}{252} \text{ch}_6(X) + \frac{\dim(G)-64}{10368} \text{ch}_2(X)^3 \\
& + \frac{\dim(G)+224}{1440} \text{ch}_2(X) \text{ch}_4(X) - \frac{1}{288} \text{ch}_2(X)^2 \text{ch}_2(\text{End}(V)) \\
& - \frac{1}{120} \text{ch}_4(X) \text{ch}_2(\text{End}(V)) + \frac{1}{12} \text{ch}_2(X) \text{ch}_4(\text{End}(V)) - \text{ch}_6(\text{End}(V))
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{(6,6)} = & \frac{1}{24} \text{ch}_2(X)^3 + \frac{1}{2} \text{ch}_2(X) \text{ch}_4(X) - \frac{1}{288} \text{ch}_2(X)^2 \text{ch}_2(\text{End}(V)) \\
& - \frac{1}{120} \text{ch}_4(X) \text{ch}_2(\text{End}(V)) + \frac{1}{12} \text{ch}_2(X) \text{ch}_4(\text{End}(V)) - \text{ch}_6(\text{End}(V))
\end{aligned}$$



$$\begin{aligned}\mathcal{P}_{(6,6)} = & \frac{1}{24} \mathrm{ch}_2(X)^3 + \frac{1}{2} \mathrm{ch}_2(X) \mathrm{ch}_4(X) - \frac{5}{48} \mathrm{ch}_2(X)^2 \mathrm{ch}_2(V) - \frac{1}{4} \mathrm{ch}_4(X) \mathrm{ch}_2(V) \\ & + \frac{1}{24} \mathrm{ch}_2(X) \mathrm{ch}_2(V)^2 + 2 \mathrm{ch}_2(X) \mathrm{ch}_4(V) - \mathrm{ch}_2(V) \mathrm{ch}_4(V)\end{aligned}$$

$$\begin{aligned}\mathcal{P}_{(6,6)} = & \frac{1}{24} \left(\mathrm{ch}_2(X) - \frac{1}{2} \mathrm{ch}_2(V) \right) (\mathrm{ch}_2(X)^2 + 12 \mathrm{ch}_4(X) \\ & - 2 \mathrm{ch}_2(X) \mathrm{ch}_2(V) + 48 \mathrm{ch}_4(V)) \\ = & \frac{1}{24} \left(\mathrm{ch}_2(X) - \frac{1}{2} \mathrm{ch}_2(V) \right) \wedge \mathcal{P}_{(4,4)}^{SO(32)}\end{aligned}$$

$$\begin{aligned}\mathcal{P}_{(6,6)} = & \frac{1}{24} \mathrm{ch}_2(X)^3 + \frac{1}{2} \mathrm{ch}_2(X) \mathrm{ch}_4(X) - \frac{5}{48} \mathrm{ch}_2(X)^2 \mathrm{ch}_2(V_1) - \frac{1}{4} \mathrm{ch}_4(X) \mathrm{ch}_2(V_1) \\ & + \frac{1}{8} \mathrm{ch}_2(X) \mathrm{ch}_2(V_1)^2 - \frac{1}{24} \mathrm{ch}_2(V_1)^3 - \frac{5}{48} \mathrm{ch}_2(X)^2 \mathrm{ch}_2(V_2) \\ & - \frac{1}{4} \mathrm{ch}_4(X) \mathrm{ch}_2(V_2) + \frac{1}{8} \mathrm{ch}_2(X) \mathrm{ch}_2(V_2)^2 - \frac{1}{24} \mathrm{ch}_2(V_2)^3\end{aligned}$$

$$\begin{aligned}\mathcal{P}_{(6,6)} = & \frac{1}{48} \left(\mathrm{ch}_2(X) - \frac{1}{2} \mathrm{ch}_2(V) \right) (2 \mathrm{ch}_2(X)^2 - 24 \mathrm{ch}_4(X) - 4 \mathrm{ch}_2(X) \mathrm{ch}_2(V_1) \\ & + 4 \mathrm{ch}_2(V_1)^2 + 4 \mathrm{ch}_2(X) \mathrm{ch}_2(V_2) - 4 \mathrm{ch}_2(V_1) \mathrm{ch}_2(V_2) + 4 \mathrm{ch}_2(V_2)^2) \\ = & \frac{1}{48} \left(\mathrm{ch}_2(X) - \frac{1}{2} \mathrm{ch}_2(V) \right) \wedge \mathcal{P}_{(4,4)}^{E_8 \times E_8}\end{aligned}$$

$$\mathcal{P}_{12} = \mathrm{d}\mathcal{Q}_{11},$$

$$\delta \mathcal{Q}_{11} = \mathrm{d}\mathcal{P}_{10}(\gamma, \epsilon),$$

$$\delta \log Z = -\mathrm{i} \int_X \mathcal{P}_{10}(\gamma, \epsilon).$$

$$\delta \log Z = \mathrm{i} \beta' \int_X (\mathrm{tr}(R\gamma) - \mathrm{tr}(F\epsilon)) \wedge \mathcal{P}_{(4,4)},$$

$$\delta \mu^a = \bar{\partial} v^a$$

$$S_{\mathrm{counter}}(\mu) = -\beta' \int_X R_b^a \nabla_a (\Delta_{\bar{\partial}}^{-1} \bar{\partial}^{\dagger} \mu^b) \wedge \mathcal{P}_{(4,4)}$$

$$\delta \alpha = \bar{\partial}_A \epsilon$$

$$S_{\mathrm{counter}}(\alpha) = \beta' \int_X \mathrm{tr} \left(F \Delta_{\bar{\partial}_A}^{-1} \bar{\partial}_A^{\dagger} \alpha \right) \wedge \mathcal{P}_{(4,4)}$$

$$\delta x = \beta' (\mathrm{tr}(F\epsilon) - \mathrm{tr}(R\gamma)),$$

$$S_{\mathrm{counter}}(x) = \int_X x \wedge \mathcal{P}_{(4,4)}$$

$$\bar{D} = \begin{pmatrix} \bar{\partial} & \beta' \mathcal{F}^* & \beta' R \cdot \nabla \\ 0 & \bar{\partial}_{\mathcal{A}} & \mathcal{F} \\ 0 & 0 & \bar{\partial} \end{pmatrix}$$



$$\begin{aligned}\mathcal{F}(\mu) &= F_a \wedge \mu^a, \\ (\mathcal{F}^*(\alpha))_a &= \text{tr}(F_a \wedge \alpha), \\ (R \cdot \nabla \mu)_a &= -R_a{}^b{}_c \wedge \nabla_b \mu^c,\end{aligned}$$

$$S=\int_X\langle y,\bar{D}y\rangle\wedge\Omega+\int_Xb\wedge\bar{\partial}c+\int_X(y\lrcorner\partial b)\wedge\Omega$$

$$\bar{D}^2=0\;\Rightarrow\; \text{tr}(F\wedge F)=\text{tr}(R\wedge R).$$

$$\text{tr}(F\wedge F)-\text{tr}(R\wedge R)=i\partial\bar{\partial}\tau,$$

$$\omega=\omega_0+\frac{\beta'}{4}\tau,$$

$$\mathrm{i}\partial\bar{\partial}\omega=\frac{\beta'}{4}(\text{tr}(F\wedge F)-\text{tr}(R\wedge R))$$

$$\bar{D}=\left(\begin{array}{ccc}\bar{\partial} & \beta'\mathcal{F}^* & \mathcal{T}+\beta'R\cdot\nabla\\ 0 & \bar{\partial}_{\mathcal{A}} & \mathcal{F}\\ 0 & 0 & \bar{\partial}\end{array}\right)$$

$$(\mathcal{T}(\mu))_a=-4\mathrm{i}\partial_{[a}\omega_{b]}\wedge\mu^b=-\mathrm{i}\beta'\partial_{[a}\tau_{b]}\wedge\mu^b,$$

$$\delta S_{\text{counter}}(x)=2\mathrm{i}\int_X(v\lrcorner\partial\omega)\wedge\mathcal{P}_{(4,4)}=2\mathrm{i}\int_X\mathcal{L}_v\omega\wedge\mathcal{P}_{(4,4)}$$

$$\mathcal{L}_v\beta=v\lrcorner\partial\beta+\partial(v\lrcorner\beta)$$

$$\mathcal{P}_{(4,4)}=\frac{1}{3!}\omega^3\wedge\mathcal{P}_{(1,1)}^{\mathsf{p}}+\frac{1}{4!}\omega^4\mathcal{P}_0$$

$$\star\mathcal{P}_{(4,4)}=-\mathcal{P}_{(1,1)}^{\mathsf{p}}+\omega\mathcal{P}_0$$

$$\begin{aligned}\delta S_{\text{counter}}(x)&=2\mathrm{i}\int_X\mathcal{L}_v\left(\frac{1}{4!}\omega^4\right)\wedge\left(\mathcal{P}_{(1,1)}^{\mathsf{p}}+\frac{1}{4}\omega\mathcal{P}_0\right)\\&=2\mathrm{i}\int_X\mathcal{L}_v\left(\frac{1}{4!}\omega^4\right)\wedge\left(-\star\mathcal{P}_{(4,4)}+\frac{5}{4}\omega\mathcal{P}_0\right)\end{aligned}$$

$$\omega^4\wedge F=0$$

$$\mathrm{d}(\mathrm{e}^{-2\phi}\omega^4)=0$$

$$\mathrm{d}\star\omega=\mathrm{d}\left(\frac{1}{4!}\omega^4\right)=0$$

$$\mathcal{L}_v\left(\frac{1}{4!}\omega^4\right)=\partial(v\lrcorner\frac{1}{4!}\omega^4)=\partial((v\lrcorner\omega)\wedge\frac{1}{3!}\omega^3)$$



$$\begin{aligned}\delta S_{\text{counter}}(x) &= \frac{5}{2}i \int_X \partial((v \lrcorner \omega) \wedge \frac{1}{3!} \omega^3) \wedge \omega \mathcal{P}_0 = 10i \int_X \partial(v \lrcorner \omega) \wedge \frac{1}{4!} \omega^4 \mathcal{P}_0 \\ &= 10i \int_X \mathcal{L}_v \left(\frac{1}{5!} \omega^5 \right) \mathcal{P}_0 = 10i \int_X \frac{1}{|\Omega|^2} \mathcal{L}_v \Omega \wedge \bar{\Omega} \mathcal{P}_0\end{aligned}$$

$$\bar{\partial}^\dagger \mathcal{P}_{(4,4)} = - \star \partial \star \mathcal{P}_{(4,4)} = 0$$

$$\frac{i}{|\Omega|^2} \Omega \wedge \bar{\Omega} = \frac{1}{5!} \omega^5.$$

$$|\Omega| = \mathrm{e}^{-2\phi},$$

$$\delta c^{5,0} = \partial(v \lrcorner \Omega) \equiv \mathcal{L}_v \Omega$$

$$S_{\text{counter}}(c^{5,0}) = -10i \int_X \frac{1}{|\Omega|^2} c^{5,0} \wedge \bar{\Omega} \mathcal{P}_0$$

$$S_{\text{one-loop}} = S + S_{\text{counter}}(x) + S_{\text{counter}}(c^{5,0})$$

$$\delta \log Z = i\beta' \int_X (\text{tr}(R\gamma) - \text{tr}(F\epsilon)) \wedge \mathcal{P}_{(4,4)}$$

$$\begin{aligned}\frac{1}{4!} \omega^4 \wedge \tilde{F} &= \mathcal{P}_{(4,4)}(\tilde{F}, \tilde{R}) \wedge \tilde{F}, \\ \frac{1}{4!} \omega^4 \wedge \tilde{R} &= \mathcal{P}_{(4,4)}(\tilde{F}, \tilde{R}) \wedge \tilde{R},\end{aligned}$$

$$\begin{aligned}\delta \log Z &= i\beta' \int_X (\text{tr}(\tilde{R}\gamma) - \text{tr}(\tilde{F}\epsilon)) \wedge \mathcal{P}_{(4,4)}(\tilde{F}, \tilde{R}) \\ &= i\beta' \int_X (\text{tr}(\tilde{R}\gamma) - \text{tr}(\tilde{F}\epsilon)) \wedge \frac{1}{4!} \omega^4,\end{aligned}$$

$$S_{\text{counter}}^2(x) = \frac{1}{4!} \int_X x \wedge \omega^4$$

$$\mathrm{e}^{-K(\omega)} = \frac{1}{5!} \int_X \omega^5$$

$$\begin{aligned}\mathcal{F}(\mu) &= \tilde{F}_a \wedge \mu^a, \\ (\mathcal{F}^*(\alpha))_a &= \text{tr}(\tilde{F}_a \wedge \alpha), \\ (\tilde{R} \cdot \tilde{\nabla} \mu)_a &= -\tilde{R}_a{}^b{}_c \wedge \tilde{\nabla}_b^+ \mu^c.\end{aligned}$$

$$\delta S_{\text{counter}}^2(x) = \beta' \int_X (\text{tr}(\tilde{F}\epsilon) - \text{tr}(\tilde{R}\gamma)) \wedge \frac{1}{4!} \omega^4 + 2i \int_X \mathcal{L}_v \omega \wedge \frac{1}{4!} \omega^4$$

$$\int_X \mathcal{L}_v(\omega) \wedge \frac{1}{4!} \omega^4 = \int_X \mathcal{L}_v \left(\frac{1}{5!} \omega^5 \right) = 0$$

$$S_{\text{one-loop}} = S + S_{\text{counter}}^2(x)$$

$$\omega^5 \wedge F(h) = 0$$



$$\Delta_h \delta h = \text{Fifth and higher order in curvatures.}$$

$$\delta \log (Z) = \mathrm{i} \beta' \int_X \left(\mathrm{tr}(R^2) - \mathrm{tr}(F^2) \right) \wedge \mathcal{P}(\gamma, \epsilon)_{(3,3)}$$

$$\mathrm{tr}(R^2) = \partial \omega_{\mathrm{CS}}(\nabla)_{(1,2)}, \mathrm{tr}(F^2) = \partial \omega_{\mathrm{CS}}(A)_{(1,2)}$$

$$S_{\mathrm{counter}}\left(\chi\right)=\int_X\left(\omega_{\mathrm{CS}}(A)_{(1,2)}-\omega_{\mathrm{CS}}(\nabla)_{(1,2)}\right)\wedge\chi$$

$$\delta \chi = \beta' \partial \mathcal{P}(\gamma, \epsilon)_{(3,3)}$$

$$\mathrm{tr}(R^2) - \mathrm{tr}(F^2) = \partial T,$$

$$\delta \log (Z) = \mathrm{i} \beta' \int_X \partial T \wedge \mathcal{P}(\gamma, \epsilon)_{(3,3)}.$$

$$S_{\mathrm{counter}}\left(\chi\right)=-\int_XT\wedge\chi,$$

$$I(V)^2=\prod_{p=0}^n|\Delta_{\bar{\partial}}^{0,p}|^{(-1)^{p+1}p}$$

$$\mathbf{s}_{\mathrm{per.}} \sim \sum_{g,b,c=0}^{\infty} \int d^{10} x e^{(2g+b+c-2)\Phi} \sqrt{-G} \mathcal{L}_{(g,b,c)}(\alpha')$$

$$\mathbf{s}_{\mathrm{per.}} \sim \sum_{g=0}^{\infty} \int d^{10} x e^{2(g-1)\Phi'} \sqrt{-G'} \mathcal{L}_g(\alpha')$$

$$\Phi'=-\Phi, G_{\alpha\beta}'=e^{-\Phi}G_{\alpha\beta}, B_{\alpha\beta}'=C_{\alpha\beta}\equiv B_{\alpha\beta}, A_{\alpha i}'{}^j=A_{\alpha i}{}^j$$

$$\mathbf{s}_{\mathrm{het.}}^{(0)} = -\frac{2}{\kappa^2} \int d^{10} x \sqrt{-G'} e^{-2\Phi'} \left[R' + 4 \nabla_\alpha \Phi' \nabla^\alpha \Phi' - \frac{1}{12} H_{\alpha\beta\gamma}' H'^{\alpha\beta\gamma} + \frac{1}{4} F_{\alpha\beta i}'{}^j F'^{\alpha\beta}{}_j{}^i \right],$$

$$\mathbf{s}_{\mathrm{typeI}}^{(0)} = -\frac{2}{\kappa^2} \int d^{10} x \sqrt{-G} \left[e^{-2\Phi} (R + 4 \nabla_\alpha \Phi \nabla^\alpha \Phi) - \frac{1}{12} H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} + \frac{e^{-\Phi}}{4} F_{\alpha\beta i}{}^j F^{\alpha\beta}{}_j{}^i \right]$$

$$F_{\mu\nu i}^j=\partial_\mu A_{\nu i}^j-\partial_\nu A_{\mu i}^j+\frac{1}{\sqrt{\alpha'}}[A_{\mu i}^k,A_{\nu k}^j]$$

$$H_{\mu\nu\rho}=3\partial_{[\mu}B_{\nu\rho]}+\frac{3}{2}A_{[\mu i}^jF_{\nu\rho]j}^i$$

$$\Omega_{\mu\nu\alpha}=\omega_{[\mu\mu_1}{}^{\nu_1}\partial_\nu\omega_{\alpha]\nu_1}{}^{\mu_1}+\frac{2}{3}\omega_{[\mu\mu_1}{}^{\nu_1}\omega_{\nu\nu_1}{}^{\alpha_1}\omega_{\alpha]\alpha_1}{}^{\mu_1};\omega_{\mu\mu_1}{}^{\nu_1}=e^\nu{}_{\mu_1}\nabla_\mu e_{\nu}{}^{\nu_1},$$

$$\eta_{\mu_1\nu_1}'=e^{-\Phi}\eta_{\mu_1\nu_1}$$

$$\Omega_{\mu\nu\rho}'=\Omega_{\mu\nu\rho}$$



$$H'_{\alpha\beta\gamma} \rightarrow H'_{\alpha\beta\gamma} + \frac{3}{2} \alpha' \Omega'_{\alpha\beta\gamma}$$

$$\Phi' = -\Phi, G'_{\mu\nu} = e^{-2\Phi} G_{\mu\nu}, H' = e^{-2\Phi} * H$$

$$\begin{aligned}\Phi' &= -\Phi + \alpha' \Delta \Phi(\Phi, G, B) \\ G'_{\alpha\beta} &= e^{-\Phi} [G_{\alpha\beta} + \alpha' G_{\alpha\beta} \Delta \Phi(\Phi, G, B) + \alpha' \Delta G_{\alpha\beta}(\Phi, G, B)] \\ B'_{\alpha\beta} &= B_{\alpha\beta} + \alpha' \Delta B_{\alpha\beta}(\Phi, G, B)\end{aligned}$$

$$\begin{aligned}\Delta \Phi(\Phi, G, B) - \Delta \Phi(-\Phi, e^{-\Phi} G, B) &= 0 \\ \Delta G_{\alpha\beta}(\Phi, G, B) + G_{\alpha\beta} \Delta \Phi(-\Phi, e^{-\Phi} G, B) + e^{\Phi} \Delta G_{\alpha\beta}(-\Phi, e^{-\Phi} G, B) &= 0 \\ \Delta B_{\alpha\beta}(\Phi, G, B) + \Delta B_{\alpha\beta}(-\Phi, e^{-\Phi} G, B) &= 0\end{aligned}$$

$$\Delta \Phi = 0, \Delta G_{\alpha\beta} = 0, \Delta B_{\alpha\beta} = 0$$

$$\begin{aligned}\mathbf{S}_{\text{het.}}^{(1)} = & -\frac{2\alpha'}{8\kappa^2} \int d^{10}x \sqrt{-G'} e^{-2\Phi'} \left[\frac{1}{4} F_{\alpha}^{\gamma\kappa l} F'^{\alpha\beta ij} F_{\beta}^{\delta}{}_{\kappa l} F'_{\gamma\delta ij} - \frac{1}{2} F_{\alpha}^{\gamma}{}_{ij} F'^{\alpha\beta ij} F_{\beta}^{\delta\kappa l} F'_{\gamma\delta\kappa l} \right. \\ & - \frac{1}{8} F_{\alpha\beta}^{\kappa l} F'^{\alpha\beta ij} F'_{\gamma\delta\kappa l} F'^{\gamma\delta}{}_{ij} + \frac{1}{4} F'^{\alpha\beta ij} F'^{\gamma\delta}{}_{ij} H'_{\alpha\gamma}{}^{\epsilon} H'_{\beta\delta\epsilon} - \frac{1}{8} F'^{\alpha\beta ij} F'^{\gamma\delta}{}_{ij} H'_{\alpha\beta}{}^{\epsilon} H'_{\gamma\delta\epsilon} \\ & - \frac{1}{2} F_{\alpha}^{\gamma}{}_{ij} F'^{\alpha\beta ij} H_{\beta}^{\delta\epsilon} H'_{\gamma\delta\epsilon} - \frac{1}{8} H_{\alpha\beta}^{\delta} H'^{\alpha\beta\gamma} H_{\gamma}^{\epsilon} H'_{\delta\epsilon\epsilon} + \frac{1}{24} H_{\alpha}^{\delta\epsilon} H'^{\alpha\beta\gamma} H_{\beta\delta}^{\epsilon} H'_{\gamma\epsilon\epsilon} \\ & \left. + R'_{\alpha\beta\gamma\delta} R'^{\alpha\beta\gamma\delta} - \frac{1}{2} H_{\alpha}^{\delta\epsilon} H'^{\alpha\beta\gamma} R'_{\beta\gamma\delta\epsilon} + 2 H'^{\alpha\beta\gamma} \Omega'_{\alpha\beta\gamma} \right]\end{aligned}$$

$$\begin{aligned}2R\nabla_{\alpha}\nabla^{\alpha}\Phi+\frac{1}{4}H_{\beta\gamma\delta}H^{\beta\gamma\delta}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi-3R\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi+9\nabla_{\alpha}\nabla^{\alpha}\Phi\nabla_{\beta}\nabla^{\beta}\Phi-\frac{1}{2}H_{\alpha}^{\gamma\delta}H_{\beta\gamma\delta}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi \\ -2R_{\alpha\beta}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi-7\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla_{\beta}\Phi\nabla^{\beta}\Phi+48\nabla^{\alpha}\Phi\nabla_{\beta}\nabla_{\alpha}\Phi\nabla^{\beta}\Phi-H_{\alpha}^{\gamma\delta}H_{\beta\gamma\delta}\nabla^{\beta}\nabla^{\alpha}\Phi\end{aligned}$$

$$\begin{aligned}\mathbf{S}_{\text{typeI}}^{(1)} = & -\frac{2\alpha'}{8\kappa^2} \int d^{10}x \sqrt{-G} \left(e^{\Phi} \left[\frac{1}{8} F_{\alpha}^{\gamma\kappa l} F^{\alpha\beta ij} F_{\beta}^{\delta}{}_{\kappa l} F_{\gamma\delta ij} \right. \right. \\ & - \frac{1}{16} F_{\alpha\beta}^{\kappa l} F^{\alpha\beta ij} F_{\gamma\delta\kappa l} F^{\gamma\delta}{}_{ij} + \frac{1}{1024} F_{\alpha\beta ij} F^{\alpha\beta ij} F_{\gamma\delta\kappa l} F^{\gamma\delta\kappa l} \Big] \\ & + \left[\frac{1}{4} F^{\alpha\beta ij} F^{\gamma\delta}{}_{ij} \tilde{H}_{\alpha\gamma}{}^{\epsilon} \tilde{H}_{\beta\delta\epsilon} - \frac{1}{8} F_{\alpha}^{\gamma}{}_{ij} F^{\alpha\beta ij} \tilde{H}_{\beta}^{\delta\epsilon} \tilde{H}_{\gamma\delta\epsilon} + \frac{1}{48} F_{\alpha\beta ij} F^{\alpha\beta ij} \tilde{H}_{\gamma\delta\epsilon} \tilde{H}^{\gamma\delta\epsilon} \right. \\ & - \frac{1}{4} F^{\alpha\beta ij} \tilde{H}_{\beta\gamma\delta} \nabla_{\alpha} F^{\gamma\delta}{}_{ij} + \frac{3}{64} F_{\alpha\beta ij} F^{\alpha\beta ij} R - \frac{1}{2} F_{\alpha}^{\gamma ij} F_{\beta\gamma ij} R^{\alpha\beta} \Big] \\ & + e^{-\Phi} \left[\frac{1}{16} R^2 - \frac{1}{16} \tilde{H}_{\alpha\beta}^{\delta} \tilde{H}^{\alpha\beta\gamma} \tilde{H}_{\gamma}{}^{\epsilon\epsilon} \tilde{H}_{\delta\epsilon\epsilon} + \frac{1}{96} \tilde{H}_{\alpha\beta\gamma} \tilde{H}^{\alpha\beta\gamma} \tilde{H}_{\delta\epsilon\epsilon} \tilde{H}^{\delta\epsilon\epsilon} \right. \\ & - R_{\alpha\beta} R^{\alpha\beta} + \frac{1}{24} \tilde{H}_{\alpha}^{\delta\epsilon} \tilde{H}^{\alpha\beta\gamma} \tilde{H}_{\beta\delta}^{\epsilon} \tilde{H}_{\gamma\epsilon\epsilon} + R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} - \frac{1}{2} \tilde{H}_{\alpha}^{\delta\epsilon} \tilde{H}^{\alpha\beta\gamma} R_{\beta\gamma\delta\epsilon} \\ & \left. \left. + 2 \tilde{H}^{\alpha\beta\gamma} \Omega_{\alpha\beta\gamma} \right] \right)\end{aligned}$$

$$ds^2 = G_{\mu\nu}(x)dx^{\mu}dx^{\nu} + g_{ab}(y)dy^ady^b$$



$$\mathbf{s}_{\text{typeI}}^{(0)} = -\frac{2V}{\kappa^2} \int d^6x \sqrt{-G} e^{-2\Phi} \left[R + 4\nabla_\alpha \Phi \nabla^\alpha \Phi - \frac{1}{12} H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} \right]$$

$$\int_{K3} d^4y \sqrt{g} = V$$

$$\mathbf{s}_{\text{typeI}}^{(3)} = \frac{2\alpha'^3 \zeta(3)}{2^6 \kappa^2} \int d^{10}x \sqrt{-G} e^{-2\Phi} \left[R_{\mu\nu\rho\tau} R^{\mu\nu\rho\tau} \left(-\frac{5}{32} H_{\alpha\beta}{}^\delta H^{\alpha\beta\gamma} H_\gamma{}^{\epsilon\epsilon} H_{\delta\epsilon\epsilon} + \frac{5}{4} H_\alpha{}^{\delta\epsilon} H^{\alpha\beta\gamma} R_{\beta\delta\gamma\epsilon} \right. \right. \\ \left. \left. - \frac{5}{24} \nabla_\delta H_{\alpha\beta\gamma} \nabla^\delta H^{\alpha\beta\gamma} \right) + \dots \right]$$

$$\frac{1}{32\pi^2} \int_{K3} d^4y \sqrt{g} R_{abcd} R^{abcd}$$

$$\mathbf{s}_{\text{typeI}}^{(1)} = \frac{48\pi^2 \alpha'^3 \zeta(3)}{\kappa^2} \int d^6x \sqrt{-G} e^{-2\Phi} \left[\left(-\frac{5}{32} H_{\alpha\beta}{}^\delta H^{\alpha\beta\gamma} H_\gamma{}^{\epsilon\epsilon} H_{\delta\epsilon\epsilon} + \frac{5}{4} H_\alpha{}^{\delta\epsilon} H^{\alpha\beta\gamma} R_{\beta\delta\gamma\epsilon} \right. \right. \\ \left. \left. - \frac{5}{24} \nabla_\delta H_{\alpha\beta\gamma} \nabla^\delta H^{\alpha\beta\gamma} \right) \right]$$

$$\delta\Phi D\Phi + \delta G_{\alpha\beta} D G^{\alpha\beta} + \delta B_{\alpha\beta} D B^{\alpha\beta}$$

$$D\Phi \equiv -2V\sqrt{-G}e^{-2\Phi}\left(R+4\nabla_\alpha\nabla^\alpha\Phi-4\nabla_\alpha\Phi\nabla^\alpha\Phi-\frac{1}{12}H^2\right)=0,$$

$$DB^{\alpha\beta}\equiv\frac{V}{2}\sqrt{-G}\nabla_\gamma(e^{-2\Phi}H^{\alpha\beta\gamma})=0,$$

$$DG^{\alpha\beta}\equiv V\sqrt{-G}\left[\frac{1}{4}e^{-2\Phi}H^{\alpha\gamma\delta}H_{\gamma\delta}^\beta-e^{-2\Phi}(R^{\alpha\beta}+2\nabla^\alpha\nabla^\beta\Phi)\right]-\frac{1}{4}D\Phi^{\alpha\beta}=0,$$

$$\delta B_{\alpha\beta}=\frac{5c}{4}\nabla_\gamma H_{\alpha\beta}^\gamma,$$

$$\delta\Phi=\frac{5c}{32}H^2+\frac{5c}{2}\nabla_\alpha\Phi\nabla^\alpha\Phi,$$

$$\delta G_{\alpha\beta}=\frac{5c}{8}H_{\alpha\gamma\delta}H_{\beta}^{\gamma\delta}+5c\nabla_\beta\nabla_\alpha\Phi,$$

$$\mathbf{s}_{\text{exact}} \sim \int d^{10}x e^{-2\Phi} \sqrt{-G} \mathcal{L}_{\text{exact}}(\alpha')$$

$$ds^2=e^{2A}ds^2(\text{AdS}_2)+ds^2(\text{M}_8)$$

$$F=f_{\pm}+e^{2A}\text{vol}(\text{AdS}_2)\wedge\star_8\lambda(f_{\pm})$$

$$H=e^{2A}\text{vol}(\text{AdS}_2)\wedge H_1+H_3$$

$$\chi_1^\dagger \gamma_a \chi_1 = \pm \chi_2^\dagger \gamma_a \chi_2, \chi_1^\dagger \hat{\gamma} \chi_1 = \pm \chi_2^\dagger \hat{\gamma} \chi_2, |\chi_1|^2 = |\chi_2|^2,$$

$$e^A=|\chi_1|^2,e^A\cos\beta=\chi_1^\dagger\hat{\gamma}\chi_1,e^A\sin\beta V=\chi_1^\dagger\gamma_a\chi_1e^a,$$

$$\psi=\frac{1}{16}\sum_{n=0}^8\chi_2^\dagger\gamma_{a_n\dots a_1}\chi_1e^{a_1\dots a_n},\hat{\psi}=\frac{1}{16}\sum_{n=0}^8\chi_2^\dagger\gamma_{a_n\dots a_1}\hat{\gamma}\chi_1e^{a_1\dots a_n},$$



$$e^{2A}H_1 = me^A \sin \beta V - d(e^{2A} \cos \beta), d(e^A \sin \beta V) = 0,$$

$$d_{H_3}(e^{-\Phi}\psi_{\pm}) = \pm \frac{1}{16}e^A \sin \beta V \wedge f_{\pm},$$

$$d_{H_3}(e^{A-\Phi}\hat{\psi}_{\mp}) - me^{-\Phi}\psi_{\pm} = \mp \frac{1}{16}e^{2A}(\star_8 \lambda f_{\pm} + \cos \beta f_{\pm}),$$

$$(\psi_{\pm}, f_{\pm})_8 = \pm \frac{1}{4}e^{-\Phi}\left(m - \frac{1}{2}e^A \sin \beta \iota_V H_1\right) \text{vol}(\text{M}_8),$$

$$dH_3=0, \iota_V(df_{\pm}-H_3\wedge f_{\pm})=0, d(e^{-2\Phi}\star_8 H_1)+\frac{1}{2}(f_{\pm}, f_{\pm})_8=0$$

$$ds^2(\text{M}_8)=e^{2C}ds^2(\mathbb{CP}^3)+ds^2(\text{M}_2), H_3=e^{2C}\tilde{H}_1\wedge J_2$$

$$J_2, J_2\wedge J_2, \text{vol}(\mathbb{CP}^3)=\frac{1}{3!}J_2\wedge J_2\wedge J_2$$

$$\bar{ds}^2(\,S^7)=ds^2(\mathbb{CP}^3)+(e^\tau)^2, e^\tau=(d\tau+\eta), d\eta=2J_2,$$

$$\nabla_a \xi_{\pm} = \pm \frac{i}{2} \gamma_a^{(7)} \xi_{\pm}$$

$$\xi_{\pm}=\sum_{I=1}^8c^I\xi_{\pm}^I$$

$$\xi_{\pm}^{I\dagger}\xi_{\pm}^J=\delta^{IJ}$$

$$\xi_+^I\colon \mathbf{8}\rightarrow \mathbf{6}_0\oplus \mathbf{1}_{-2}\oplus \mathbf{1}_2, \xi_-^I\colon \mathbf{8}\rightarrow \mathbf{4}_{-1}\oplus \overline{\mathbf{4}}_1,$$

$$\Phi_3^I=-\frac{i}{3!}\xi_+^{(I)\dagger}\gamma_{a_1a_2a_3}^{(7)}\xi_+^{(I)}e^{a_1a_2a_3},\star_7\Phi_3^I=-\frac{1}{4!}\xi_+^{(I)\dagger}\gamma_{a_1\dots a_4}^{(7)}\xi_+^{(I)}e^{a_1\dots a_4},\\ d\Phi_3^I=4\star_7\Phi_3^I,\Phi_3^I\wedge\star_7\Phi_3^I=7\text{vol}(S^7),$$

$$\mathbf{6} \text{ of } \text{SO}(6)\colon \xi_6^j, \mathbf{2} \text{ of } \text{SO}(2)\colon \xi_2^a,$$

$$\Phi_3^a=J_2\wedge e^\tau+\text{Im}\Omega^a,\star_7\Phi_3^a=\frac{1}{2}J_2\wedge J_2-\text{Re}\Omega^a\wedge e^\tau$$

$$de^\tau=2J_2, d\Omega_3^a=4ie^\tau\wedge\Omega_3^a$$

$$J_2\wedge J_2\wedge J_2=\frac{3}{4}i\Omega_3^{(a)}\wedge\bar{\Omega}_3^{(a)}, J_2\wedge\Omega_3^a=0$$

$$\Phi_3^j=-\tilde{J}_2^j\wedge e^\tau+\text{Im}\Omega^j,\star_7\Phi_3^j=\frac{1}{2}J_2^{(j)}\wedge J_2^{(j)}-\text{Re}\Omega^j\wedge e^\tau$$

$$d\tilde{J}_2^j=4\text{Re}\Omega_3^j, d\text{Im}\Omega_3^j=6\tilde{J}_2^j\wedge J_2-2J_2\wedge J_2, J_2\wedge\Omega_3^j=0\\ J_2\wedge J_2+\tilde{J}_2^{(j)}\wedge\tilde{J}_2^{(j)}=2\tilde{J}_2\wedge J_2, J_2\wedge J_2\wedge J_2=-\tilde{J}_2^{(j)}\wedge\tilde{J}_2^{(j)}\wedge\tilde{J}_2^{(j)}=\text{vol}(\mathbb{CP}^3)$$



$$\tilde{J}_2^{(J)} \wedge \tilde{J}_2^{(J)} \wedge \tilde{J}_2^{(J)} = -\frac{3}{4} i \Omega_3^{(J)} \wedge \bar{\Omega}_3^{(J)}, \tilde{J}_2^{(J)} \wedge \Omega_3^{(J)} = 0$$

$$\gamma^{(6)}=\gamma_a^{(7)},a=1,\ldots,6,\hat{\gamma}^{(6)}=\gamma_7^{(7)}$$

$$J_2\xi_6^J=-i\hat{\gamma}^{(6)}\xi_6^J,J_2\hat{\gamma}^{(6)}\xi_6^J=-i\xi_6^J.$$

$$\begin{aligned}\tilde{J}_2^J&=\frac{i}{2}\xi_6^{(J)\dagger}\gamma_{a_1a_2}\hat{\gamma}^{(6)}\xi_6^{(J)}e^{a_1a_2},\mathrm{Re}\Omega_3^J=\frac{1}{3!}\xi_6^{(J)\dagger}\gamma_{a_1a_2a_3}\hat{\gamma}^{(6)}\xi_6^{(J)}e^{a_1a_2a_3},\mathrm{Im}\Omega_3^J\\&=-\frac{i}{3!}\xi_6^{(J)\dagger}\gamma_{a_1a_2a_3}\xi_6^{(J)}e^{a_1a_2a_3}\end{aligned}$$

$$\gamma_a=\sigma_3\otimes\gamma_a^{(6)},a=1,\ldots,6,\gamma_{7,8}=\sigma_{1,2}\otimes\mathbb{I}\\\hat{\gamma}=\sigma_3\otimes\hat{\gamma}^{(6)},B=\sigma_1\otimes B^{(6)}$$

$$\chi_{1,2}^J=\frac{e^{\frac{A}{2}}}{2}(\eta_{1,2}\otimes\xi_6^J+i\hat{\eta}_{1,2}\otimes\hat{\gamma}^{(6)}\xi_6^J)$$

$$(\tilde{J}_2^J,\Omega_3^J)\rightarrow(\tilde{J}_2,\Omega_3).$$

$$\eta_1=i\begin{pmatrix}\cos\left(\frac{\alpha-\beta}{2}\right)+i\sin\left(\frac{\alpha+\beta}{2}\right)\\-\left(\cos\left(\frac{\alpha-\beta}{2}\right)-i\sin\left(\frac{\alpha+\beta}{2}\right)\right)\end{pmatrix},\hat{\eta}_1=-\begin{pmatrix}\cos\left(\frac{\alpha+\beta}{2}\right)+i\sin\left(\frac{\alpha-\beta}{2}\right)\\\cos\left(\frac{\alpha+\beta}{2}\right)-i\sin\left(\frac{\alpha-\beta}{2}\right)\end{pmatrix},\\ \eta_2=-i\begin{pmatrix}\cos\left(\frac{\alpha+\beta}{2}\right)+i\cos\left(\frac{\alpha-\beta}{2}\right)\\-\left(\cos\left(\frac{\alpha+\beta}{2}\right)-i\sin\left(\frac{\alpha-\beta}{2}\right)\right)\end{pmatrix},\hat{\eta}_2=-\begin{pmatrix}\cos\left(\frac{\alpha-\beta}{2}\right)+i\sin\left(\frac{\alpha+\beta}{2}\right)\\\cos\left(\frac{\alpha-\beta}{2}\right)-i\sin\left(\frac{\alpha+\beta}{2}\right)\end{pmatrix},$$

$$\hat{\psi}_+=-\frac{e^A\sin\beta}{16}\mathrm{Re}\psi_^{(7)},\psi_-=\frac{e^A}{16}\mathrm{Re}\left(\psi_-^{(7)}+\cos\beta\psi_+^{(7)}\wedge V\right)\\\psi_+^{(7)}=-e^{-i\alpha}e^{ie^{2C}\tilde{J}_2}-e^{3C}i\Omega_3\wedge U,\psi_-^{(7)}=-e^{-i\alpha}e^{ie^{2C}\tilde{J}_2}\wedge U-e^{3C}i\Omega_3,$$

$$\tilde{J}_2,J_2\wedge\tilde{J}_2,\mathrm{Re}\Omega_3,\mathrm{Im}\Omega_3$$

$$e^C=-\frac{4e^A}{\sin\alpha\sin\beta},me^{2C}\tilde{H}=4d(e^{A+C}\cos\alpha\sin\beta)+8e^A\sin\beta U$$

$$d(e^A\sin\beta U)=0,d(e^A\sin\beta V)=0$$

$$d(e^{A+3C-\Phi})+4e^{A+2C-\Phi}(\cos\alpha U-\sin\alpha\cos\beta V)=0$$

$$U+iV=\frac{p}{e^A\sin\beta m}(dx_1+idx_2)$$



$$e^{-\Phi} = e^{-(A+3C)}h$$

$$\frac{m p \cos \beta}{e^{2A} \sin^2 \beta} + \partial_{x_1} \log h = 0, \frac{m p \cos \alpha}{e^{2A} \sin \alpha \sin^2 \beta} + \partial_{x_2} \log h = 0$$

$$\cos \beta = -\frac{\partial_{x_1} h}{2h(\partial_{x_1}^2 + \partial_{x_2}^2)h - \partial_{x_2} h^2}$$

$$p=\frac{L^2}{m}$$

$$(e^{-\Phi},F_-)\rightarrow c(e^{-\Phi},F_-)$$

$$\frac{ds^2}{L^2}=\sqrt{\Delta_1}\left[\frac{h}{\Delta_2}ds^2(\text{AdS}_2)+\frac{1}{m^2}\left(\frac{8}{\Box h}ds^2(\mathbb{CP}^3)+\frac{1}{h}(dx_i)^2\right)\right],e^{-\Phi}=\frac{m^3c\sqrt{h\Delta_2}(\Box h)^{\frac{3}{2}}}{16\sqrt{2}L^4\Delta_1}$$

$$H=dB,\frac{B}{L^2}=\left(\frac{h\partial_1h}{\Delta_2}+x_1\right)\text{vol}(\text{AdS}_2)+\frac{8}{m^2}\left(\frac{\partial_2h}{\Box h}-(x_2-l)\right)J_2$$

$$\Box h = (\partial_{x_1}^2 + \partial_{x_2}^2)h, \partial_i = \partial_{x_i}$$

$$\Delta_1=2h\Box h-(\partial_2h)^2,\Delta_2=2h\Box h-(\partial_1h)^2-(\partial_2h)^2.$$

$$F_- = f_- + e^{2A} \text{vol}(\text{AdS}_2) \wedge \star_8 \lambda(f_-)$$

$$\hat{F}_-=\hat{f}_-+\text{vol}(\text{AdS}_2)\wedge\hat{g}_-$$

$$\begin{aligned}\hat{f}_1 &= F_1 = \frac{m^3 c}{32 L^4} \Big[\partial_2 (\Box h) dx_1 - \partial_1 (\Box h) dx_2 - d \Big(\frac{\partial_1 h \partial_2 h \Box h}{\Delta_1} \Big) \Big] \\ \hat{f}_3 &= \frac{m c}{4 L^2} \Big[((x_2 - l) \partial_2 (\Box h) - \Box h) dx_1 - (x_2 - l) \partial_1 (\Box h) dx_2 + d \Big(\frac{\partial_1 h \Box h (2 h - (x_2 - l) \partial_2 h)}{\Delta_1} \Big) \Big] \wedge J_2 \\ \hat{f}_5 &= \frac{c}{m} \Big[(2 \partial_2 h + (x_2 - l) (-2 \Box h + (x_2 - l) \partial_2 (\Box h))) dx_1 - (2 \partial_1 h + (x_2 - l)^2 \partial_1 (\Box h)) dx_2 \\ &\quad - d \Big(\frac{\partial_1 h ((x_2 - l)^2 \partial_2 h \Box h + 2 h (\partial_2 h - 2 (x_2 - l) \Box h))}{\Delta_1} \Big) \Big] \wedge J_2 \wedge J_2 \\ \hat{f}_7 &= \frac{8 L^2 c}{m^3} \Bigg[\Big(-2 h + (x_2 - l) \Big(2 \partial_2 h + (x_2 - l) \Big(\frac{1}{3} (x_2 - l) \partial_2 (\Box h) - \Box h \Big) \Big) \Big) dx_1 \\ &\quad - 2 \Big((x_2 - l) \partial_1 h + \frac{1}{6} \partial_1 (\Box h) \Big) dx_2 \\ &\quad + d \Big(\frac{\partial_1 h (4 h^2 - (x_2 - l)^3 \partial_2 h \Box h - 6 (x_2 - l) h (\partial_2 h - (x_2 - l) \Box h))}{3 \Delta_1} \Big) \Big] \wedge J_2 \wedge J_2 \wedge J_2.\end{aligned}$$

$$d\hat{f}_{2n+1}\sim (x_2-l)^n\Box^2hdx_1\wedge dx_2\wedge J_2^n$$

$$\Box^2 h = (\partial_1^4 + 2\partial_1^2\partial_2^2 + \partial_2^4)h = 0$$

$$d\hat{g}_{2n+1}\sim x_1(x_2-l)^n\Box^2hdx_1\wedge dx_2\wedge J_2^n$$

$$\Box^2 h \sim \delta(x_2-l)\delta(x_1)$$



$$z = x_1 + ix_2, \partial_z = \frac{1}{2}(\partial_1 - i\partial_2), \partial_{\bar{z}} = \frac{1}{2}(\partial_1 + i\partial_2), \square = 4\partial_z\partial_{\bar{z}}$$

$$\partial_z\partial_{\bar{z}}\partial_z\partial_{\bar{z}}h=0$$

$$\partial_z\partial_{\bar{z}}h=\frac{1}{2}\big(\partial_z\mathcal{A}_1(z)+\partial_{\bar{z}}\overline{\mathcal{A}_1}(\bar{z})\big)$$

$$h=\frac{1}{2}\big(\bar{z}\mathcal{A}_1(z)+z\overline{\mathcal{A}_1}(\bar{z})+\mathcal{A}_2(z)+\overline{\mathcal{A}_2}(\bar{z})\big).$$

$$\mathcal{A}_{1,2}=h_{1,2}+ih_{1,2}^D$$

$$h=x_1h_1+x_2h_1^D+h_2$$

$$ds^2(\text{AdS}_3)=\frac{1}{4}[(2dx_1+\eta)^2+ds^2(\text{AdS}_2)]\text{ with }d\eta=\text{vol}(\text{AdS}_2)$$

$$\frac{ds^2}{L^2}=\frac{h}{\sqrt{\Delta}}ds^2(\text{AdS}_2)+\sqrt{\Delta}\left(\frac{8}{h''}ds^2(\mathbb{CP}^3)+\frac{1}{h}(dx_i)^2\right),e^{-\Phi}=\frac{c\sqrt{h}(h'')^{\frac{3}{2}}}{16\sqrt{2}L^4\sqrt{\Delta}}$$

$$H=dB,\frac{B}{L^2}=x_1\text{vol}(\text{AdS}_2)+8\left(\frac{h'}{h''}-(x_2-l)\right)J_2,\Delta=2hh''-h'^2$$

$$\hat{f}_1=F_1=\frac{c}{32L^4}h'''dx_1,\hat{f}_3=\frac{c}{4L^2}\big((x_2-l)h'''-h''\big)dx_1\wedge J_2$$

$$\hat{f}_5=c\big(2h'+(x_2-l)(-2h''+(x_2-l)h''')\big)dx_1\wedge J_2\wedge J_2$$

$$x_1\rightarrow r,x_2\rightarrow\frac{1}{\pi}y,L^2\rightarrow\frac{\pi}{2},c\rightarrow4\pi^2.$$

$$ds^2(S^7/\mathbb{Z}_k)=\left(\frac{d\tau}{k}+\mathcal{A}\right)^2+ds^2(\mathbb{CP}^3)$$

$$\frac{ds^2}{L^2}=\frac{\sqrt{2}h^{3/2}\sqrt{h''}}{\Delta}ds^2(\text{AdS}_2)+\sqrt{2}\left(\frac{8\sqrt{h}}{\sqrt{h''}}ds^2(\mathbb{CP}^3)+\frac{\sqrt{h''}}{\sqrt{h}}(dx_i)^2\right),e^{-\Phi}=\frac{c\sqrt{h''}\sqrt{\Delta}}{32\sqrt{2}L^4\sqrt{h}}$$

$$H=dB,\frac{B}{L^2}=\left(\frac{hh'}{\Delta}+x_1\right)\text{vol}(\text{AdS}_2)-8(x_2-l)J_2,\Delta=2hh''-h'^2$$

$$\hat{f}_1=F_1=\frac{c}{32L^4}h'''dx_1,\hat{f}_3=-\frac{c}{4L^2}(x_2-l)h'''dx_2\wedge J_2$$

$$\hat{f}_5=c(d(2(x_2-l)h')-2(x_2-l)h''dx_1-(2h+h'''(x_2-l)^2)dx_2)J_2\wedge J_2$$

$$\hat{f}_7=8L^2c\left(dx_1(-(x_2-l)^2h''-2h)-\frac{1}{3}dx_2(x_2-l)(h'''(x_2-l)^2+6h')\right.\\ \left.+d\left(\frac{h'(3(l-x_2)^2h''+2h)}{3h''}\right)\right).$$

$$L\rightarrow\frac{L}{22^{3/4}},c\rightarrow\frac{L^2}{2\sqrt{2}c_0k},x_1\rightarrow-r,x_2\rightarrow\frac{2\sqrt{2}k}{L^2}\tau,$$



$$\hat{h}(x_2),$$

$$\frac{ds^2}{L^2} = \frac{x_1^2 \hat{h} \sqrt{\Delta}}{x_1^2 \Delta - \hat{h}^2} ds^2(\text{AdS}_2) + \frac{8\sqrt{\Delta}}{\hat{h}''} ds^2(\mathbb{CP}^3) + \frac{\sqrt{\Delta}}{\hat{h}} (dx_i^2)$$

$$e^{-\Phi} = \frac{c\sqrt{\hat{h}\hat{h}''^{3/2}}\sqrt{x_1^2\Delta - \hat{h}^2}}{16\sqrt{2}L^4\Delta}, \Delta = -\hat{h}'^2 + 2\hat{h}\hat{h}''$$

$$H = dB, \frac{B}{L^2} = x_1 \left(1 + \frac{\hat{h}^2}{x_1^2 \Delta - \hat{h}^2} \right) \text{vol}(\text{AdS}_2) + 8 \left(\frac{\hat{h}'}{\hat{h}''} - (x_2 - l) \right) \wedge J_2$$

$$\hat{f}_1 = \frac{c}{L^4} \left(\hat{h}''' x_1 dx_1 - \hat{h}'' dx_2 - d \left(\frac{\hat{h}\hat{h}'\hat{h}''}{\Delta} \right) \right)$$

$$\hat{f}_3 = \frac{c}{4L^2} \left[d \left(\frac{\hat{h}\hat{h}''}{\Delta} (2\hat{h} - (x_2 - l)\hat{h}') \right) - (\hat{h}'' - (x_2 - l)\hat{h}''') x_1 dx_1 - (x_2 - l)\hat{h}'' dx_2 \right] \wedge J_2$$

$$\begin{aligned} \hat{f}_5 = & -\frac{c}{m} \left[x_1 \left((x_2 - l)(2\hat{h}'' - (x_2 - l)\hat{h}''') + 2\hat{h}' \right) dx_1 + (2\hat{h} + (x_2 - l)^2 \hat{h}'') dx_2 \right. \\ & \left. + d \left(\frac{\hat{h}}{\Delta} \left((x_2 - l)\hat{h}''(4\hat{h} - (x_2 - l)\hat{h}') - 2\hat{h}\hat{h}' \right) \right) \right] \wedge J_2 \wedge J_2 \end{aligned}$$

$$x_1 \rightarrow \frac{\rho}{\pi}, x_2 \rightarrow r, L^2 \rightarrow \frac{\pi}{2}, c \rightarrow 4\pi^3.$$

$$x_1 \rightarrow r, x_2 \rightarrow \frac{1}{\pi}y, L^2 \rightarrow \frac{\pi}{2}, c \rightarrow 4\pi^2, \hat{h} \rightarrow h.$$

$$\frac{2}{\pi} ds^2 = \frac{h}{\sqrt{2hh'' - (h')^2}} ds^2(\text{AdS}_2) + \frac{8}{h''} \sqrt{2hh'' - (h')^2} ds^2(\mathbb{CP}^3) + \frac{\sqrt{2hh'' - (h')^2}}{h} \left(dr^2 + \frac{1}{\pi^2} dy^2 \right)$$

$$e^{\Phi} = \frac{\sqrt{2(2hh'' - (h')^2)}}{\sqrt{h(h'')^3}}, B = 4\pi \left(-(r-l) + \frac{h'}{h''} \right) J_2 + \frac{y}{2} \text{vol}(\text{AdS}_2)$$

$$\hat{f}_1 = -\frac{h'''}{2\pi} dy, \hat{F}_3 = 2(h'' - (r-l)h''') dy \wedge J_2$$

$$\hat{f}_5 = -4\pi(2h' + (r-l)(-2h'' + (r-l)h''')) dy \wedge J_2 \wedge J_2$$

$$\hat{f}_7 = \frac{16\pi^2}{3} (6h - (r-l)(6h' + (r-l)((r-l)h''' - 3h'')) dy \wedge J_2 \wedge J_2 \wedge J_2$$

$$h'''' = 0.$$

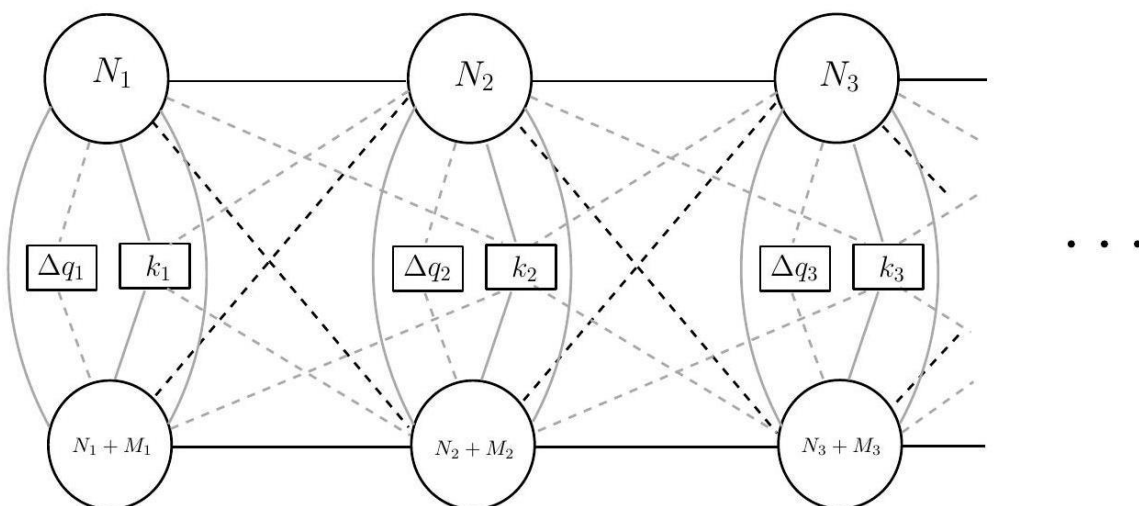
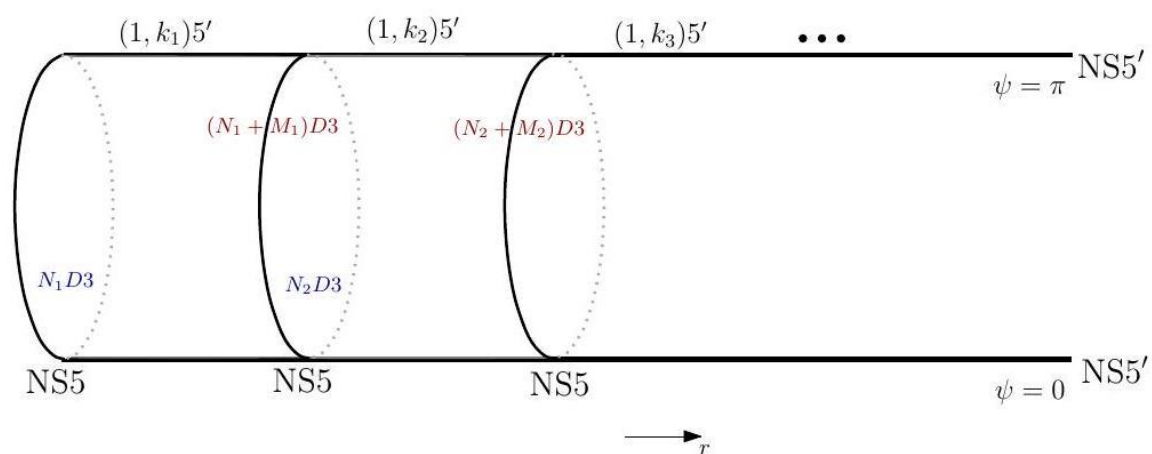
$$\frac{1}{(2\pi)^2} \left| \int_{2\text{-cycle}} B \right| \in [0,1]$$

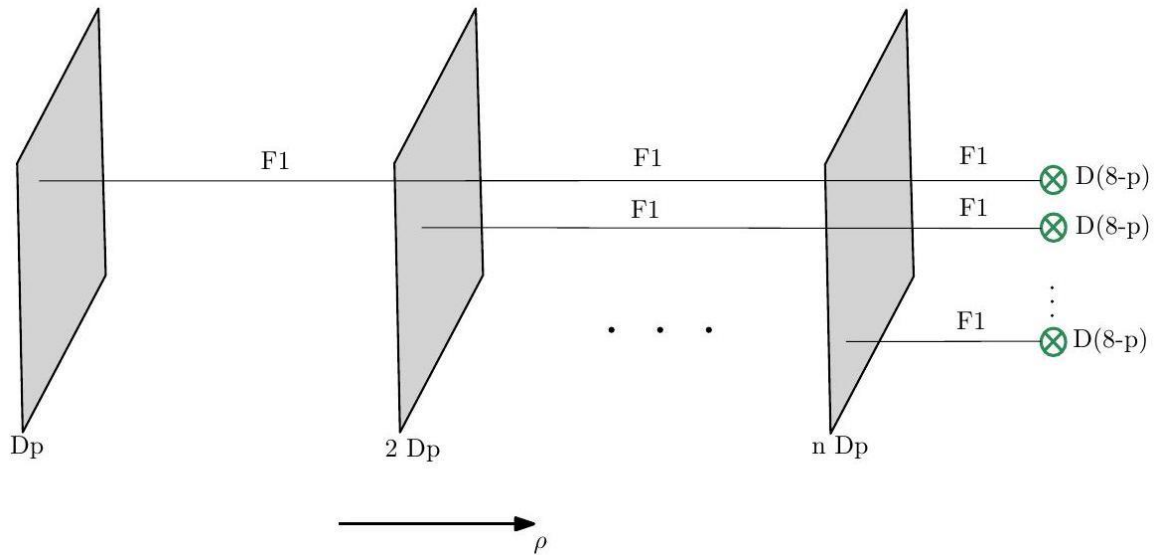


$$h = c_1 + c_2(r - r_0)^3$$

$$h = (c_1 + c_2(r - r_0))(r - r_0)^2$$

$$h = c_1 + c_2(r - r_0) + \frac{c_2^2}{4c_1}(r - r_0)^2 + c_3(r - r_0)^3$$





Figuras 3, 4 y 5. Configuración de Supersimetría.

$$x_1 \rightarrow \frac{\rho}{\pi}, x_2 \rightarrow r, L^2 \rightarrow \frac{\pi}{2}, c \rightarrow 4\pi^3, \hat{h} \rightarrow h$$

$$\frac{2}{\pi} ds^2 = \frac{h}{\sqrt{2hh'' - (h')^2}} \frac{\rho^2}{\Xi} ds^2(\text{AdS}_2) + \frac{8}{h''} \sqrt{2hh'' - (h')^2} ds^2(\mathbb{CP}^3)$$

$$+ \frac{\sqrt{2hh'' - (h')^2}}{h} \left(dr^2 + \frac{1}{\pi^2} d\rho^2 \right), e^\Phi = \frac{1}{\sqrt{\Xi}} \frac{\sqrt{2(2hh'' - (h')^2)}}{\sqrt{h(h'')^3}}$$

$$B = 4\pi \left(-(r-l) + \frac{h'}{h''} \right) J_2 + \left(\frac{\rho^3}{2\Xi} - n\pi \right) \text{vol}(\text{AdS}_2), \Xi = \rho^2 - \frac{\pi^2 h^2}{2hh'' - (h')^2}$$

$$\hat{F}_1 = \frac{h'''}{2\pi} \rho d\rho + \hat{F}_1^r$$

$$\hat{F}_3 = -2(h'' - (r-l)h''') \rho d\rho \wedge J_2 + n\pi \text{vol}(\text{AdS}_2) \wedge \hat{F}_1^r + \hat{F}_3^{r, \mathbb{CP}^1}$$

$$\hat{F}_5 = 4\pi(2h' + (r-l)(-2h'' + (r-l)h''')) \rho d\rho \wedge J_2 \wedge J_2 + n\pi \text{vol}(\text{AdS}_2) \wedge \hat{F}_3^{r, \mathbb{CP}^1} + \hat{F}_5^{r, \mathbb{CP}^2}$$

$$\hat{F}_7 = \frac{16\pi^2}{3} \left(6h - (r-l)(6h' + (r-l)((r-l)h''' - 3h'')) \right) \rho d\rho \wedge J_2 \wedge J_2 \wedge J_2$$

$$+ n\pi \text{vol}(\text{AdS}_2) \wedge \hat{F}_5^{r, \mathbb{CP}^2} + \hat{F}_7^{r, \mathbb{CP}^3}$$

$$\hat{F}_9 = n\pi \text{vol}(\text{AdS}_2) \wedge \hat{F}_7^{r, \mathbb{CP}^3}$$

$$\frac{\pi^2 h^2}{2hh'' - (h')^2} \rightarrow \frac{c_1 \pi^2}{4c_2} (r - r_0)$$

$$S_{Dp} = -T_p \int_{(t, I_r, \mathbb{CP}^{(p-1)/2})} \hat{F}_p \wedge A_t = -T_{F1} Q'_{8-p} \int A_t$$



$$\begin{aligned} Q_1^{(n)} &= \frac{1}{(2\pi)^6} \int \hat{F}_7^{\rho, \mathbb{CP}^3} = (2n+1)Q_1^{(0)} \\ Q_3^{(n)} &= \frac{1}{(2\pi)^4} \int \hat{F}_5^{\rho, \mathbb{CP}^2} = (2n+1)Q_3^{(0)} \\ Q_5^{(n)} &= \frac{1}{(2\pi)^2} \int \hat{F}_3^{\rho, \mathbb{CP}^1} = (2n+1)Q_5^{(0)} \\ Q_7^{(n)} &= \int \hat{F}_1^{\rho} = (2n+1)Q_7^{(0)}, \end{aligned}$$

$$\pi h_l(r) = Q_{1,l}^{(0)} - Q_{3,l}^{(0)}(r-l) + \frac{1}{2}Q_{5,l}^{(0)}(r-l)^2 - \frac{1}{6}Q_{7,l}^{(0)}(r-l)^3 \text{ for } r \in [l, l+1]$$

$$\begin{aligned} Q_{1,l}^{(0)} &= Q_{1,l-1}^{(0)} - Q_{3,l-1}^{(0)} + \frac{1}{2}Q_{5,l-1}^{(0)} - \frac{1}{6}Q_{7,l-1}^{(0)} \\ Q_{3,l}^{(0)} &= Q_{3,l-1}^{(0)} - Q_{5,l-1}^{(0)} + \frac{1}{2}Q_{7,l-1}^{(0)} \\ Q_{5,l}^{(0)} &= Q_{5,l-1}^{(0)} - Q_{7,l-1}^{(0)} \end{aligned}$$

$$\hat{F}_{p+2}^{(\mathrm{AdS}_2,r,\mathbb{CP}^{(p-1)/2})} = n\pi \hat{F}_p^{(r,\mathbb{CP}^{(p-1)/2})} \wedge \mathrm{vol}_{\mathrm{AdS}_2},$$

$$Q_p^{(e)} = nQ'_{8-p} \text{ for } \rho \in [2n\pi, 2(n+1)\pi].$$

$$\begin{aligned} Q_{1,l}^{(n)} &= N_l^{(n)} + \frac{1}{12}k_l^{(n)} \\ Q_{3,l}^{(n)} &= M_l^{(n)} - \frac{1}{2}k_l^{(n)} + \frac{1}{12}q_l^{(n)} \\ Q_{5,l}^{(n)} &= k_l^{(n)} \\ Q_{7,l}^{(n)} &= -q_l^{(n)} \end{aligned}$$

$$c_{hol} = \frac{3}{2^5\pi^8}V_{int} = \frac{3}{2^5\pi^8}\int d^8\xi e^{-2\Phi}\sqrt{\det(g_{ij})}$$

$$c_{hol} = \frac{3}{32\pi^4m^2}\int dx_1dx_2(2h\Box h - (\partial_1h)^2 - (\partial_2h)^2)$$

$$c_R=\frac{k(3k+13)}{k+3}$$

$$c_{1d}=\frac{3}{2^5\pi^4}\int drd\rho(\rho^2(2hh''-h'^2)-\pi^2h^2)$$

$$c_{2d}=\frac{1}{2}\int dr(2hh''-h'^2)$$

$$\begin{aligned} ds^2(S^7) &= ds^2(\mathbb{CP}^3) + (d\tau + \eta)^2, 4\eta = \cos^2 \zeta (d\psi + 2\eta_1) - \sin^2 \zeta (d\psi + 2\eta_2) \\ ds^2(\mathbb{CP}^3) &= d\zeta^2 + \frac{1}{4}\cos^2 \zeta ds^2(S_1^2) + \frac{1}{4}\sin^2 \zeta ds^2(S_2^2) + \frac{1}{4}\cos^2 \zeta \sin^2 \zeta (d\psi + \eta_1 + \eta_2)^2 \\ ds^2(S_i^2) &= d\theta_i^2 + \sin^2 \theta_i d\phi_i^2, \eta_i = \cos \theta_i d\phi_i \end{aligned}$$



$$J_2=\frac{1}{2}\cos\zeta\sin\zeta d\zeta\wedge(d\psi+\eta_1+\eta_2)-\frac{1}{4}\cos^2\zeta\mathrm{vol}(S_1^2)+\frac{1}{4}\sin^2\zeta\mathrm{vol}(S_2^2)$$

$$e^1=d\zeta, e^2=\frac{1}{2}\cos\zeta d\theta_1, e^3=\frac{1}{2}\cos\zeta\sin\theta_1 d\phi_1, e^4=\frac{1}{2}\sin\zeta d\theta_2, e^5=\frac{1}{2}\sin\zeta\sin\theta_2\phi_2$$

$$e^6=\frac{1}{2}\cos\zeta\sin\zeta(d\psi+\eta_1+\eta_2), e^7=d\tau+\eta$$

$$\gamma_1=\sigma_1\otimes\mathbb{I}_2\otimes\mathbb{I}_2,\gamma_{2,3,4}=\sigma_2\otimes\sigma_{1,2,3}\otimes\mathbb{I}_2,\gamma_{5,6,7}=\sigma_3\otimes\mathbb{I}_2\otimes\sigma_{1,2,3}$$

$$B=\sigma_3\otimes\sigma_2\otimes\sigma_2$$

$$\nabla_a \xi_{\pm} = \pm \frac{i}{2} \gamma_a \xi_{\pm}$$

$$\xi_{\pm}=\mathcal{M}_{\pm}\xi_{\pm}^0$$

$$\mathcal{M}_{\pm}=e^{\frac{\zeta}{2}(\pm i\gamma_7+\gamma_{67})}e^{\frac{1}{8}(4\tau+\psi)(\pm\gamma_7-\gamma_{23})}e^{\frac{\theta_1}{4}(\pm i\gamma_2+\gamma_{37})}e^{\frac{\theta_1}{4}(\pm i\gamma_2+\gamma_{23})}e^{\frac{1}{8}(-4\tau+\psi)(\gamma_{16}-\gamma_{45})}e^{\frac{\theta_2}{4}(\gamma_{14}+\gamma_{56})}e^{\frac{\phi_2}{4}(\gamma_{16}+\gamma_{45})}$$

$$\xi_{8\pm}^I=\mathcal{M}_{\pm}\eta^I,\eta^I=\sum_{J=1}^8\delta^{IJ}$$

$$\xi_{4-}^{\rm i}=\frac{1}{\sqrt{2}}\begin{pmatrix}-i(\xi_{8-}^3+\xi_{8-}^8)\\-i(\xi_{8-}^2-\xi_{8-}^5)\\-(\xi_{8-}^1+\xi_{8-}^6)\\(\xi_{8-}^1-\xi_{8-}^6)\end{pmatrix}^{\rm i},\xi_{4+}^{\rm i}=\frac{1}{\sqrt{2}}\begin{pmatrix}i(\xi_{8-}^2+\xi_{8-}^5)\\i(\xi_{8-}^3-\xi_{8-}^8)\\(\xi_{8-}^4+\xi_{8-}^7)\\i(\xi_{8-}^4-\xi_{8-}^7)\end{pmatrix}^{\rm i}$$

$$\partial_\tau \xi_{4\pm}^{\rm i}=\pm i \xi_{4\pm}^{\rm i}, \xi_{4-}^{\rm ic}=\xi_{4+}^{\rm i}$$

$$\xi_-^I=\frac{1}{\sqrt{2}}\Big(\begin{array}{c}\xi_{4-}^{\rm i}+\xi_{4+}^{\rm i}\\i(\xi_{4-}^{\rm i}-\xi_{4+}^{\rm i})\end{array}\Big)$$

$$\partial_\tau(\xi_{8+}^2-\xi_{8+}^5)=-2i(\xi_{8+}^2-\xi_{8+}^5)\,\partial_\tau(\xi_{8+}^3-\xi_{8+}^8)=2i(\xi_{8+}^3-\xi_{8+}^8),$$

$$\xi_2^a=\frac{1}{2}\left(\begin{array}{c}\xi_{8+}^3-\xi_{8+}^8+\xi_{8+}^2-\xi_{8+}^5\\i(\xi_{8+}^3-\xi_{8+}^3-\xi_{8+}^2+\xi_{8+}^5)\end{array}\right),\xi_6^j=\frac{1}{\sqrt{2}}\left(\begin{array}{c}\xi_{8+}^1-\xi_{8+}^4\\i(\xi_{8+}^1+\xi_{8+}^4)\\\xi_{8+}^6-\xi_{7+}^1\\i(\xi_{8+}^6+\xi_{7+}^1)\\\frac{1}{\sqrt{2}}(\xi_{8+}^2+\xi_{8+}^5+\xi_{8+}^3+\xi_{8+}^8)\\i\frac{1}{\sqrt{2}}(\xi_{8+}^2+\xi_{8+}^5-\xi_{8+}^3-\xi_{8+}^8)\end{array}\right).$$

$$\psi_+=\frac{e^A}{16}\mathrm{Re}\left(\psi_+^{(7)}+\cos\beta\psi_-^{(7)}\wedge V\right),\hat{\psi}_-=\frac{e^A\sin\beta}{16}\mathrm{Re}\psi_-^{(7)}\\ \psi_+^{(7)}=e^{i\alpha}e^{ie^{2C}\tilde{J}_2}+ie^{3C}e^{i\delta}\Omega_3\wedge U,\psi_-^{(7)}=e^{i\alpha}e^{ie^{2C}\tilde{J}_2}\wedge U+ie^{3C}e^{i\delta}\Omega_3$$

$$\sin\alpha=0, me^C=6e^A\sin\beta\sin\delta$$



$$\cos \delta U \wedge V = 0$$

$$d(e^{2A+3C}e^{-\Phi}\sin\beta\cos\delta)+e^{A+3C}e^{-\Phi}m\cos\delta\cos\beta V+e^{A+2C}e^{-\Phi}(4e^A\sin\beta-e^Cm\sin\delta)U=0$$

$$\begin{aligned}\hat{g}_1 &= \frac{cm^3}{32L^2} \left(d \left(\frac{\partial_2 h (x_1 \partial_1 h - h) \sqcap h}{\Delta_1} \right) - dx_1 x_1 \partial_2 \sqcap h + dx_2 (x_1 \partial_1 \sqcap h - \sqcap h) \right), \\ \hat{g}_3 &= cm \left(d \left(\frac{(2h - (x_2 - l) \partial_2 h) (h - x_1 \partial_1 h) \sqcap h}{4\Delta_1} \right) \right. \\ &\quad \left. + \frac{1}{4} (dx_2 (x_2 - l) (x_1 \partial_1 \sqcap h - \sqcap h) - dx_1 x_1 ((x_2 - l) \partial_2 \sqcap h - \sqcap h)) \right) \wedge J_2, \\ \hat{g}_5 &= \frac{cL^2}{m} \left[-d \left(\frac{(h - x_1 \partial_1 h) ((l - x_2)^2 \partial_2 h \sqcap h + 2h(\partial_2 h + 2(l - x_2) \sqcap h))}{\Delta_1} \right) \right. \\ &\quad \left. - dx_1 x_1 (-2(x_2 - l) \sqcap h + (x_2 - l)^2 \partial_2 \sqcap h + 2\partial_2 h) \right. \\ &\quad \left. + dx_2 ((x_2 - l)^2 (x_1 \partial_1 \sqcap h - \sqcap h) - 2(h - x_1 \partial_1 h)) \right] \wedge J_2 \wedge J_2, \\ \hat{g}_7 &= \left[-\frac{8cL^4}{3m^3} (dx_2 ((x_2 - l)^3 (\sqcap h - x_1 \partial_1 \sqcap h) - 6(x_2 - l) x_1 \partial_1 h + 4(x_2 - l) h + 2(x_2 - l) h) \right. \\ &\quad \left. + dx_1 (-3(x_2 - l)^2 x_1 \sqcap h + (x_2 - l)^3 x_1 \partial_2 \sqcap h + 6(x_2 - l) x_1 \partial_2 h - 6x_1 h) \right. \\ &\quad \left. + \frac{8cL^4}{3m^3} d \left(\frac{(h - x_1 \partial_1 h) ((x_2 - l)^3 (-\partial_2 h) \sqcap h - 6(x_2 - l) h (\partial_2 h - (x_2 - l) \sqcap h) + 4h^2)}{\Delta_1} \right) \right] \\ &\quad \wedge J_2 \wedge J_2 \wedge J_2.\end{aligned}$$

$$\hat{F}_- = \hat{f}_- + \text{vol}(\text{AdS}_2) \wedge \hat{g}_-$$

$$\begin{aligned}\hat{g}_1 &= -\frac{1}{3}\frac{c}{2^5L^2}\hat{h}''''d(x_1^3) \\ \hat{g}_3 &= \frac{1}{3}\frac{c}{2^2}\left(\hat{h}''-\hat{h}'''(x_2-l)\right)d(x_1^3)\wedge J_2\end{aligned}$$

$$\frac{ds^2}{L^2} = \sqrt{\Delta_1} \left[\frac{x_2 \hat{h}}{\Delta_2} ds^2(\text{AdS}_2) + \frac{1}{m^2} \left(\frac{8}{x_2 \hat{h}''} ds^2(\mathbb{CP}^3) + \frac{1}{x_2 \hat{h}} (dx_i)^2 \right) \right], e^{-\Phi} = \frac{m^3 x_2^2 \sqrt{\hat{h}} \hat{h}''^{3/2}}{16 \sqrt{2} L^4 \Delta_1}$$

$$H = dB, \frac{B}{L^2} = \left(\frac{x_2^2 \hat{h} \hat{h}'}{\Delta_2} + x_1 \right) \text{vol}(\text{AdS}_2) + \frac{8}{m^2} \left(\frac{\hat{h}}{x_2 \hat{h}''} - (x_2 - l) \right) J_2$$

$$\Delta_1=2x_2^2\hat{h}\hat{h}''-\hat{h}^2,\Delta_2=x_2^2(2\hat{h}\hat{h}''-\hat{h}'^2)-\hat{h}^2$$



$$\begin{aligned}
\hat{f}_1 &= F_1 = \frac{m^3}{32L^4} \left[\hat{h}'' dx_1 - x_2 \hat{h}''' dx_2 - d \left(\frac{x_2^2 \hat{h} \hat{h}' \hat{h}''}{\Delta_1} \right) \right] \\
\hat{f}_3 &= \frac{m}{4L^2} \left[-l \hat{h}'' dx_1 - x_2 (x_2 - l) \hat{h}''' dx_2 + d \left(\frac{x_2^2 (x_2 + l) \hat{h} \hat{h}' \hat{h}''}{\Delta_1} \right) \right] \wedge J_2 \\
\hat{f}_5 &= \frac{1}{m} \left[(2\hat{h} - (x_2 - l)(x_2 + l) \hat{h}'') dx_1 - x_2 (2\hat{h}' + (x_2 - l)^2 \hat{h}''') dx_2 \right. \\
&\quad \left. + d \left(\frac{x_2^2 \hat{h} \hat{h}' (-2\hat{h} + (x_2 - l)(3x_2 + l) \hat{h}'')}{\Delta_1} \right) \right] \wedge J_2 \wedge J_2 \\
\hat{f}_7 &= \frac{8L^2}{m^3} \left[\left(-2l\hat{h} - \frac{1}{3} (x_2 - l)^2 (2x_2 + l) \hat{h}'' \right) dx_1 - 2 \left(x_2 (x_2 - l) \hat{h}' + \frac{1}{6} x_2 (x_2 - l)^3 \hat{h}''' \right) dx_2 \right. \\
&\quad \left. + d \left(\frac{x_2^2 \hat{h} \hat{h}' (-2x_2 - 6l) \hat{h} + (x_2 - l)^2 (5x_2 + l) \hat{h}''}{3\Delta_1} \right) \right] \wedge J_2 \wedge J_2 \wedge J_2
\end{aligned}$$

Supergravedad cuántica relativista tipo II: Modelamiento Matemático.

$$\begin{aligned}
S_0 &= \int_M \Re \sigma^2 + \bar{\psi}_\alpha \not{D} \psi^\alpha + \bar{\rho} \not{D} \rho + 2\bar{\rho} D_\alpha \psi^\alpha - \frac{1}{768} \sigma^{-2} (\bar{\psi}_\alpha \gamma_{abc} \psi^\alpha) (\bar{\rho} \gamma^{abc} \rho) \\
&\quad - \frac{1}{384} \sigma^{-2} (\bar{\psi}_\alpha \gamma_{abc} \psi^\alpha) (\bar{\psi} \beta \gamma^{abc} \psi^\beta) \\
F(\delta_1 \mathcal{L}, \delta_2 \mathcal{L}) &\sim [\delta_1 \mathcal{L}, \delta_2 \mathcal{L}]
\end{aligned}$$

$$\begin{aligned}
[\delta_{\epsilon_1}, \delta_{\epsilon_2}] \rho &= \delta_V \rho + \delta_\zeta \rho - \frac{1}{2} VEL[\rho] \\
[\delta_{\epsilon_1}, \delta_{\epsilon_2}] \psi_\alpha &= \delta_V \psi_\alpha + \delta_\zeta \psi_\alpha + \left(\frac{1}{4} \epsilon_{[2} \bar{\epsilon}_{1]} - \frac{1}{2} V \right) EL[\psi]_\alpha
\end{aligned}$$

$$\begin{aligned}
S &= \int_M \Re \sigma^2 + \bar{\psi}_\alpha \not{D} \psi^\alpha + \bar{\rho} \not{D} \rho + 2\bar{\rho} D_\alpha \psi^\alpha \\
&\quad - \frac{1}{768} \sigma^{-2} (\bar{\psi}_\alpha \gamma_{abc} \psi^\alpha) (\bar{\rho} \gamma^{abc} \rho) - \frac{1}{384} \sigma^{-2} (\bar{\psi}_\alpha \gamma_{abc} \psi^\alpha) (\bar{\psi} \beta \gamma^{abc} \psi^\beta) \\
&\quad + \sigma^* \left[\mathcal{L}_\xi \sigma - \frac{1}{8} \sigma^{-1} (\bar{\rho} e) \right] + \mathcal{C}_{a\beta}^* \left[(\mathcal{L}_\xi \mathcal{G})^{a\beta} + \frac{1}{2} \sigma^{-2} (\bar{e} \gamma^a \psi^\beta) \right] \\
&\quad + \bar{\rho}^* \left[\mathcal{L}_\xi \rho + \not{D} e + \frac{1}{192} \sigma^{-2} (\bar{\psi} \beta \gamma_{abc} \psi^\beta) \gamma^{abc} e \right] \\
&\quad + \bar{\psi}_\beta^* \left[(\mathcal{L}_\xi \psi)^\beta + D^\beta e + \frac{1}{8} \sigma^{-2} (\bar{\psi}^\beta \rho) e - \frac{1}{8} \sigma^{-2} (\bar{\psi}^\beta \gamma_a e) \gamma^a \rho \right] \\
&\quad + \bar{e}^* \left[\mathcal{L}_\xi e + \frac{1}{16} \sigma^{-2} (\bar{e} \gamma_a e) \gamma^a \rho \right] + \left\langle \xi^*, \mathcal{D} f + \frac{1}{2} \mathcal{L}_\xi \xi \right\rangle - \frac{1}{8} \xi_a^* \sigma^{-2} (\bar{e} \gamma^a e) \\
&\quad + \frac{1}{2} f^* \left(\mathcal{L}_\xi f + \frac{1}{8} \sigma^{-2} (\bar{e} \gamma_a e) \xi^a - \frac{1}{6} \langle \xi, \mathcal{L}_\xi \xi \rangle \right) \\
&\quad - \frac{1}{64} \sigma^{-2} (\bar{e} \gamma_a e) (\bar{\psi}_\beta^* \gamma^a \psi^{*\beta}) - \frac{1}{32} \sigma^{-2} (\bar{e} \psi_\beta^*) (\bar{e} \psi^{*\beta}) - \frac{1}{64} \sigma^{-2} (\bar{e} \gamma_a e) (\bar{\rho}^* \gamma^a \rho^*)
\end{aligned}$$

$$\{S, S\} = 0$$

$$\langle V, \mathcal{D}f \rangle := a(V)f.$$



$$0\longrightarrow T^*M\overset{a^*}{\rightarrow} E\overset{a}{\rightarrow} TM\longrightarrow 0.$$

$$a([V,W])=[a(V),a(W)].$$

$$\mathcal{L}_V W\!:=\![V,W]$$

$$\mathcal{L}_V f\!:=\!L_{a(V)}f.$$

$$\mathcal{L}_{\mathcal{D}f}=0, \forall f\in C^\infty(M).$$

$$\mathcal{L}_V\mu\!:=\!L_{a(V)}\mu.$$

$$H\rightarrow M$$

$$\mathcal{H}^*:=\{\sigma\in\Gamma(H)\mid\sigma\text{ everywhere nonzero }\}.$$

$$TU\oplus T^*U\oplus (\mathfrak{g}\times U),$$

$$\begin{aligned} a(v+\lambda+s)&:=v\\ \langle v_1+\lambda_1+s_1,v_2+\lambda_2+s_2\rangle&:=\lambda_1(v_2)+\lambda_2(v_1)+\mathrm{Tr}(s_1s_2)\\ [v_1+\lambda_1+s_1,v_2+\lambda_2+s_2]&:=L_{v_1}v_2+\Big(L_{v_1}\lambda_2-i_{v_2}d\lambda_1+\mathrm{Tr}(s_2ds_1)\Big)+\big(L_{v_1}s_2-L_{v_2}s_1+[s_1,s_2]_{\mathfrak{g}}\big) \end{aligned}$$

$$E=C_+\oplus C_-$$

$$C_+=\left\{v+\left(i_vg+i_vB-\frac{1}{2}\mathrm{Tr}(Ai_vA)\right)+i_vA\Bigg|\;v\in TM\right\}$$

$$\sigma^2=\sqrt{|g|}e^{-2\varphi}$$

$$e_A=\{e_a,e_{\alpha}\},\langle e_A,e_B\rangle=\eta_{AB}$$

$$c_{ABC}:=\langle [e_A,e_B],e_C\rangle$$

$$\rho\in\Gamma(\Pi S(C_+)_{+}\otimes H), \psi\in\Gamma(\Pi S(C_+)_{-}\otimes C_{-}\otimes H)$$

$$\rho=\sqrt[4]{2}\sigma\rho,\psi=\sqrt[4]{2}\sigma\psi+\frac{1}{\sqrt[4]{2}}\sigma\chi$$

$$\begin{array}{l} D\colon \Gamma(E)\times \Gamma(E)\rightarrow \Gamma(E)\\ (V,W)\mapsto D_VW, \end{array}$$

$$D_{fV}W=fD_VW,D_V(fW)=fD_VW+(a(V)f)W,D\langle\cdot,\cdot\rangle=0$$

$$D_Vf\!:=\!a(V)f$$

$$D_V\mu\!:=\!L_{a(V)}\mu-v(D_AV^A)\mu$$

$$D_{e_A}e_B\!:=\!\Gamma^C_A\;{}_B e_C$$

$$DG=0,D\sigma=0$$



$$\Gamma_{[ABC]}=-\frac{1}{3}c_{ABC}$$

$$\mathrm{div} V\colon =D_AV^A=\sigma^{-2}\mathcal{L}_V\sigma^2,$$

$$D_{V_+}\colon \Gamma(C_-)\rightarrow \Gamma(C_-), D_{V_-}\colon \Gamma(C_+)\rightarrow \Gamma(C_+)$$

$$\Gamma(S(C_+)\otimes H) \text{ or } \Gamma(S(C_+)\otimes C_-\otimes H),$$

$$\mathbb{D}\colon=\gamma^aD_a$$

$$\mathbb{D}\rho,\mathbb{D}\psi_\alpha,D_\alpha\rho,D_\alpha\psi^\alpha$$

$$[\not{D},D_\alpha]\rho=\frac{1}{4}\mathcal{R}_{\alpha\alpha}\gamma^a\rho$$

$$(\mathbb{D}^2+D_\alpha D^\alpha)\rho=-\frac{1}{8}\mathcal{R}\rho$$

$$\int \sigma^2 \mathcal{R} = \int \sqrt{|g|} e^{-2\varphi} \Big(R + 4 |\nabla \varphi|^2 - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} \Big),$$

$$\mathcal{S}_0\rightarrow \mathcal{M}\times \mathcal{H}^*,$$

$$\Gamma(\Pi\mathcal{S}(C_+)_+\otimes H)\oplus \Gamma(\Pi\mathcal{S}(C_+)_-\otimes C_-\otimes H).$$

$$\begin{aligned} F(\delta_1\mathcal{L}_2,\delta_2\mathcal{L})\rho &= \frac{1}{16}[\delta_1\mathcal{G},\delta_2\mathcal{L}_{ab}\gamma^{ab}\rho\\ F(\delta_1\mathcal{L},\delta_2\mathcal{L})\psi^\alpha &= \frac{1}{16}\Big[\delta_1\mathcal{L},\delta_2\mathcal{L}_{ab}\gamma^{ab}\psi^\alpha + \frac{1}{4}[\delta_1\mathcal{L},\delta_2\mathcal{L}^\alpha{}_\beta\psi^\beta. \end{aligned}$$

$$\mathcal{L}_V\chi=D_V\chi+\frac{1}{2}(D_aV_b)\gamma^{ab}\chi$$

$$\mathcal{L}_V\lambda=D_V\lambda+\frac{1}{2}(D_aV_b)\gamma^{ab}\lambda+\frac{1}{2}(D_AV^A)\lambda$$

$$\begin{aligned} S_0 = \int_M \mathfrak{R} \sigma^2 + \bar{\psi}_\alpha \mathbb{D} \psi^\alpha + \bar{\rho} \mathbb{D} \rho + 2 \bar{\rho} D_\alpha \psi^\alpha - \frac{1}{768} \sigma^{-2} (\bar{\psi}_\alpha \gamma_{abc} \psi^\alpha) (\bar{\rho} \gamma^{abc} \rho) \\ - \frac{1}{384} \sigma^{-2} (\bar{\psi}_\alpha \gamma_{abc} \psi^\alpha) (\bar{\psi} \beta \gamma^{abc} \psi^\beta) \end{aligned}$$

$$\delta_V(\text{ field }) = \mathcal{L}_V(\text{ field }).$$

$$\delta_\epsilon \mathcal{G}_{ab} = \delta_\epsilon \mathcal{G}_{\alpha\beta} = 0, \delta_\epsilon \mathcal{G}_{a\beta} = \delta_\epsilon \mathcal{G}_{\beta a} = \frac{1}{2} \sigma^{-2} \bar{\epsilon} \gamma_a \psi_\beta$$

$$\delta_\epsilon \sigma = \frac{1}{8} \sigma^{-1} (\bar{\rho} \epsilon)$$

$$\delta_\epsilon \rho = \mathbb{D} \epsilon + \frac{1}{192} \sigma^{-2} (\bar{\psi}_\alpha \gamma_{cde} \psi^\alpha) \gamma^{cde} \epsilon$$

$$\delta_\epsilon \psi_\alpha = D_\alpha \epsilon + \frac{1}{8} \sigma^{-2} (\bar{\psi}_\alpha \rho) \epsilon + \frac{1}{8} \sigma^{-2} (\bar{\psi}_\alpha \gamma_c \epsilon) \gamma^c \rho$$



$$\begin{aligned}\delta_\epsilon \rho &= \mathbb{D} \rho + \frac{1}{4} \delta \mathcal{C}_{\alpha\alpha} \gamma^a \psi^\alpha \\ \delta_\epsilon \psi_\alpha &= D_\alpha \epsilon + \frac{1}{8} \sigma^{-2} (\bar{\psi}_\alpha \rho) \epsilon - \frac{1}{4} \delta \mathcal{C}_{\alpha\alpha} \gamma^a \rho\end{aligned}$$

$$T_{\mathcal{G}}\mathcal{M} \cong \Gamma(C_+ \otimes C_-).$$

$$\begin{aligned}EL[\mathcal{L}]_{\alpha\alpha} &:= \mathcal{R}_{\alpha\alpha} + \sigma^{-2} \left(\frac{1}{2} \bar{\psi}_\beta \gamma_a D_\alpha \psi^\beta + \bar{\psi}_\alpha \gamma_a D_\beta \psi^\beta - \bar{\psi}_\beta \gamma_a D^\beta \psi_\alpha + \frac{1}{2} \bar{\rho} \gamma_a D_\alpha \rho - \frac{1}{2} \bar{\psi}_\alpha D_a \rho \right. \\ &\quad \left. + \frac{1}{4} \bar{\rho} \gamma_{ab} D^b \psi_\alpha - \frac{1}{4} \bar{\psi}_\alpha \gamma_{ab} D^b \rho \right) = 0 \\ EL[\sigma] &:= \mathbb{R} + \sigma^{-2} (2 \bar{\psi}^\alpha D_\alpha \rho + 2 \bar{\rho} D_\alpha \psi^\alpha) + \sigma^{-4} \left[\frac{1}{768} (\bar{\psi}_\alpha \gamma_{cde} \psi^\alpha) (\bar{\rho} \gamma^{cde} \rho) \right. \\ &\quad \left. + \frac{1}{384} (\bar{\psi}_\alpha \gamma_{cde} \psi^\alpha) (\bar{\psi}_\beta \gamma^{cde} \psi^\beta) \right] = 0 \\ EL[\rho] &:= \mathbb{D} \rho + D_\alpha \psi^\alpha - \frac{1}{768} \sigma^{-2} (\bar{\psi}_\alpha \gamma_{cde} \psi^\alpha) \gamma^{cde} \rho = 0 \\ EL[\psi]^\alpha &:= \mathbb{D}^\alpha \psi^\alpha - D^\alpha \rho - \sigma^{-2} \left[\frac{1}{768} (\bar{\rho} \gamma_{cde} \rho) \gamma^{cde} \psi^\alpha + \frac{1}{192} (\bar{\psi}_\beta \gamma_{cde} \psi^\beta) \gamma^{cde} \psi^\alpha \right] = 0\end{aligned}$$

$$[\delta_{V_1}, \delta_{V_2}] = \delta_{\mathcal{L}_{V_1} V_2}, [\delta_V, \delta_\epsilon] = \delta_{\mathcal{L}_V \epsilon}$$

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \delta_V + \delta_\zeta$$

$$V^a:=\frac{1}{4}\sigma^{-2}\bar{\epsilon}_2\gamma^a\epsilon_1,\zeta:=-\frac{1}{2}V\rho+\delta_{\epsilon_1}\epsilon_2-\delta_{\epsilon_2}\epsilon_1$$

$$\begin{aligned}[\delta_{\epsilon_1}, \delta_{\epsilon_2}] \rho &= \delta_V \rho + \delta_\zeta \rho - \frac{1}{2} V EL[\rho] \\ [\delta_{\epsilon_1}, \delta_{\epsilon_2}] \psi_\alpha &= \delta_V \psi_\alpha + \delta_\zeta \psi_\alpha + \left(\frac{1}{4} \sigma^{-2} \epsilon_{[2} \bar{\epsilon}_{1]} - \frac{1}{2} V \right) EL[\psi]_\alpha\end{aligned}$$

$$\mathcal{S} \rightarrow \mathcal{M} \times \mathcal{H}^*,$$

$$\Gamma(\mathrm{PS}(C_+)_+ \otimes H) \times \Gamma(\mathrm{PS}(C_+)_- \otimes C_- \otimes H) \times \Gamma(\mathrm{PS}(C_+)_- \otimes H)[1] \times \Gamma(E)[1] \times C^\infty(M)[2]$$

$$\begin{aligned}\mathcal{C}_{\mathcal{L}} &\in \mathcal{M}, \\ \sigma &\in \mathcal{H}^* \\ \rho &\in \Gamma(\mathrm{PS}(C_+)_+ \otimes H) \\ \psi &\in \Gamma(\mathrm{PS}(C_+)_- \otimes C_- \otimes H) \\ \xi &\in \Gamma(E)[1] \\ e &\in \Gamma(\mathrm{PS}(C_+)_- \otimes H)[1] \\ f &\in C^\infty(M)[2]\end{aligned}$$

$$\mathcal{F}_{BV}:=T^*[-1]\mathcal{S}.$$

$$T\mathcal{S} \cong \pi^*T(\mathcal{M} \times \mathcal{H}^*) \oplus \pi^*\mathcal{S}$$

$$T^*[-1]\mathcal{S} \cong \pi^*T^*[-1](\mathcal{M} \times \mathcal{H}^*) \oplus \pi^*\mathcal{S}^*[-1]$$

$$(\mathcal{G},\sigma,\psi,\rho,\xi,e,f)\in\mathcal{S}$$



$$\begin{aligned}
\mathcal{G}^* &\in T_{\mathcal{G}}^*[-1]\mathcal{M} \cong \Gamma(C_+ \otimes C_- \otimes H^2)[-1], \\
\sigma^* &\in T_{\sigma}^*[-1]\mathcal{H}^* \cong \Gamma(H)[-1], \\
\rho^* &\in \Gamma(\Pi S(C_+)_- \otimes H)[-1], \\
\psi^* &\in \Gamma(\Pi S(C_+)_+ \otimes C_- \otimes H)[-1], \\
\xi^* &\in \Gamma(E \otimes H^2)[-2], \\
e^* &\in \Gamma(\Pi S(C_+)_+ \otimes H)[-2], \\
f^* &\in \Gamma(H^2)[-3].
\end{aligned}$$

$$E^* \cong E, C_{\pm}^* \cong C_{\pm}, S(C_+)^*_{\pm} \cong S(C_+)^*_{\mp}.$$

$$\begin{aligned}
\frac{1}{2}\{S, S\} = & \int_M \frac{\delta S}{\delta \mathcal{G}_{a\alpha}^*} \frac{\delta S}{\delta \mathcal{G}_{\alpha}^{a\alpha}} + \frac{\delta S}{\delta \sigma^*} \frac{\delta S}{\delta \sigma} + \frac{\delta S}{\delta \bar{\rho}^*} \frac{\delta S}{\delta \rho} + \frac{\delta S}{\delta \bar{\psi}^*} \frac{\delta S}{\delta \psi^\alpha} + \frac{\delta S}{\delta \xi^*} \frac{\delta S}{\delta \xi} + \frac{\delta S}{\delta \bar{e}^*} \frac{\delta S}{\delta e} + \frac{\delta S}{\delta f^*} \frac{\delta S}{\delta f} \\
& + \frac{1}{16} \frac{\delta S}{\delta \mathcal{G}_{a\alpha}^*} \frac{\delta S}{\delta \mathcal{G}_b^{*a}} (\bar{\rho}^* \gamma_{ab} \rho + \bar{\psi}^* \gamma_{ab} \psi^\alpha + \bar{e}^* \gamma_{ab} e) + \frac{1}{4} \frac{\delta S}{\delta \mathcal{G}_{a\alpha}^*} \frac{\delta S}{\delta \mathcal{G}_\beta^{*a}} (\bar{\psi}^* \psi^\beta)
\end{aligned}$$

$$\frac{1}{2}\{S, S\} = \int_M \frac{\delta S}{\delta \phi_i^*} \frac{\delta S}{\delta \phi^i} + \frac{1}{2} \sum_{\phi \in \{\rho, \psi, e\}} \bar{\phi}^* F\left(\frac{\delta S}{\delta \mathcal{G}^*}, \frac{\delta S}{\delta \mathcal{G}^*}\right) \phi$$

$$\begin{aligned}
S = & \int_M \mathcal{R} \sigma^2 + \bar{\psi}_\alpha \not{D} \psi^\alpha + \bar{\rho} \not{D} \rho + 2 \bar{\rho} D_\alpha \psi^\alpha \\
& - \frac{1}{768} \sigma^{-2} (\bar{\psi}_\alpha \gamma_{abc} \psi^\alpha) (\bar{\rho} \gamma^{abc} \rho) - \frac{1}{384} \sigma^{-2} (\bar{\psi}_\alpha \gamma_{abc} \psi^\alpha) (\bar{\psi}_\beta \gamma^{abc} \psi^\beta) \\
& + \sigma^* \left[\mathcal{L}_\xi \sigma - \frac{1}{8} \sigma^{-1} (\bar{\rho} e) \right] + \mathcal{G}_{a\beta}^* \left[(\mathcal{L}_\xi \mathcal{G})^{a\beta} + \frac{1}{2} \sigma^{-2} (\bar{e} \gamma^a \psi^\beta) \right] \\
& + \bar{\rho}^* \left[\mathcal{L}_\xi \rho + \not{D} e + \frac{1}{192} \sigma^{-2} (\bar{\psi}_\beta \gamma_{abc} \psi^\beta) \gamma^{abc} e \right] \\
& + \bar{\psi}_\beta^* \left[(\mathcal{L}_\xi \psi)^\beta + D^\beta e + \frac{1}{8} \sigma^{-2} (\bar{\psi}^\beta \rho) e - \frac{1}{8} \sigma^{-2} (\bar{\psi}^\beta \gamma_a e) \gamma^a \rho \right] \\
& + \bar{e}^* \left[\mathcal{L}_\xi e + \frac{1}{16} \sigma^{-2} (\bar{e} \gamma_a e) \gamma^a \rho \right] + \left\langle \xi^*, \mathcal{D} f + \frac{1}{2} \mathcal{L}_\xi \xi \right\rangle - \frac{1}{8} \xi_a^* \sigma^{-2} (\bar{e} \gamma^a e) \\
& + \frac{1}{2} f^* \left(\mathcal{L}_\xi f + \frac{1}{8} \sigma^{-2} (\bar{e} \gamma_a e) \xi^a - \frac{1}{6} \langle \xi, \mathcal{L}_\xi \xi \rangle \right) \\
& - \frac{1}{64} \sigma^{-2} (\bar{e} \gamma_a e) (\bar{\psi}_\beta^* \gamma^a \psi^{*\beta}) - \frac{1}{32} \sigma^{-2} (\bar{e} \psi_\beta^*) (\bar{e} \psi^{*\beta}) - \frac{1}{64} \sigma^{-2} (\bar{e} \gamma_a e) (\bar{\rho}^* \gamma^a \rho^*)
\end{aligned}$$

$$\{S, S\} = 0$$

$$\sigma^*, \mathcal{L}^*, \rho^*, \psi^*, e^*, \xi^*, f^*$$

$$S = S_0 + S_{\text{lin}} + S_{\text{quad}}$$

$$\begin{aligned}
\frac{1}{2}\{S, S\} = & \int_M \frac{\delta S_{\text{lin}}}{\delta \phi_i^*} \frac{\delta S_0}{\delta \phi^i} \\
& + \int_M \frac{\delta S_{\text{lin}}}{\delta \phi_i^*} \frac{\delta S_{\text{lin}}}{\delta \phi^i} + \frac{\delta S_{\text{quad}}}{\delta \phi_i^*} \frac{\delta S_0}{\delta \phi^i} + \frac{1}{2} \sum_{\phi \in \{\rho, \psi, e\}} \bar{\phi}^* F\left(\frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*}, \frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*}\right) \phi \\
& + \int_M \frac{\delta S_{\text{lin}}}{\delta \phi_i^*} \frac{\delta S_{\text{quad}}}{\delta \phi^i} + \frac{\delta S_{\text{quad}}}{\delta \phi_i^*} \frac{\delta S_{\text{lin}}}{\delta \phi^i} \\
& + \int_M \frac{\delta S_{\text{quad}}}{\delta \phi_i^*} \frac{\delta S_{\text{quad}}}{\delta \phi^i}
\end{aligned}$$



$$X^a = X^a(\sigma, e) := \frac{1}{4} \sigma^{-2} (\bar{e} \gamma^a e)$$

$$\begin{aligned} \{S, X^a\} &= \int_M \frac{\delta S_{\text{lin}}}{\delta \phi_i^*} \frac{\delta X^a}{\delta \phi^i} = \int_M \frac{\delta S_{\text{lin}}}{\delta \sigma^*} \frac{\delta X^a}{\delta \sigma} + \frac{\delta S_{\text{lin}}}{\delta \bar{e}^*} \frac{\delta X^a}{\delta e} \\ &= -\frac{1}{2} \sigma^{-3} \left(\mathcal{L}_\xi \sigma - \frac{1}{8} \sigma^{-1} (\bar{\rho} e) \right) (\bar{e} \gamma^a e) + \frac{1}{2} \sigma^{-2} \left(\mathcal{L}_\xi \bar{e} - \frac{1}{16} (\bar{e} \gamma_c e) \bar{\rho} \gamma^c \right) \gamma^a e \\ &= \frac{1}{4} (\mathcal{L}_\xi \sigma^{-2}) (\bar{e} \gamma^a e) + \frac{1}{2} \sigma^{-2} (\mathcal{L}_\xi \bar{e}) \gamma^a e + \frac{1}{16} \sigma^{-4} \left[(\bar{\rho} e) (\bar{e} \gamma^a e) - \frac{1}{2} (\bar{\rho} \gamma^c \gamma^a e) (\bar{e} \gamma_c e) \right] \\ &= (\mathcal{L}_\xi X)^a + \frac{1}{32} \sigma^{-4} (\bar{\rho} \gamma^a \gamma^c e) (\bar{e} \gamma_c e) \\ &\stackrel{(\bar{e} \gamma_a e) \bar{e} \gamma^a = 0}{=} (\mathcal{L}_\xi X)^a. \end{aligned}$$

$$XDe = 2\mathcal{L}_X e.$$

$$\mathcal{L}_{[V_1, V_2]} \lambda = (\mathcal{L}_{V_1} \mathcal{L}_{V_2} - \mathcal{L}_{V_2} \mathcal{L}_{V_1}) \lambda - F(\mathcal{L}_{V_1} \mathcal{L}, \mathcal{L}_{V_2} \mathcal{L}) \lambda$$

$$\mathcal{L}_{[\xi, \xi]} e = 2\mathcal{L}_\xi \mathcal{L}_\xi e - F(\mathcal{L}_\xi \mathcal{L}_\mathcal{L}, \mathcal{L}_\xi \mathcal{L}_\mathcal{L}) e.$$

$$\int_M \frac{\delta S_{\text{lin}}}{\delta \phi_i^*} \frac{\delta}{\delta \phi^i} \left(\int_M \bar{e}^* \left(\mathcal{L}_\xi e + \frac{1}{4} X \rho \right) \right) + \frac{1}{2} \int_M \bar{e}^* F \left(\frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*}, \frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*} \right) e =: - \int_M \bar{e}^* \kappa_e$$

$$\begin{aligned} \kappa_e &:= \int_M \frac{\delta S_{\text{lin}}}{\delta \phi_i^*} \frac{\delta \mathcal{L}_\xi e}{\delta \phi^i} + \frac{1}{4} \left(\int_M \frac{\delta S_{\text{lin}}}{\delta \phi_i^*} \frac{\delta X^a}{\delta \phi^i} \right) \gamma_a \rho + \frac{1}{4} X \left(\int_M \frac{\delta S_{\text{lin}}}{\delta \phi_i^*} \frac{\delta \rho}{\delta \phi^i} \right) - \frac{1}{2} F \left(\frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*}, \frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*} \right) e \\ &\stackrel{(\hat{\psi}_{\mu\nu}^+)}{=} \mathcal{L}_{\mathcal{D}f + \frac{1}{2} \mathcal{L}_\xi \xi - \frac{1}{2} X} e - \mathcal{L}_\xi \left(\mathcal{L}_\xi e + \frac{1}{4} X \rho \right) + F \left(\frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*}, \mathcal{L}_\xi \mathcal{G} \right) e + \frac{1}{4} (\mathcal{L}_\xi X)^a \gamma_a \rho \\ &+ \frac{1}{4} X \left(\mathcal{L}_\xi \rho + D e + \frac{1}{192} \sigma^{-2} (\bar{\psi}_\alpha \gamma_{abc} \psi^\alpha) \gamma^{abc} e \right) - \frac{1}{2} F \left(\frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*}, \frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*} \right) e \end{aligned}$$

$$\begin{aligned} \kappa_e &= -\frac{1}{2} F(\mathcal{L}_\xi \mathcal{G}, \mathcal{L}_\xi \mathcal{G}) e + F(\mathcal{L}_\xi \mathcal{L} + \delta_e \mathcal{G}, \mathcal{L}_\xi \mathcal{G}) e + \frac{1}{768} \sigma^{-2} X (\bar{\psi}_\alpha \gamma_{abc} \psi^\alpha) \gamma^{abc} e \\ &\quad - \frac{1}{2} F(\mathcal{L}_\xi \mathcal{G} + \delta_e \mathcal{L}, \mathcal{L}_\xi \mathcal{G} + \delta_e \mathcal{L}) e \\ &= \frac{1}{768} \sigma^{-2} X (\bar{\psi}_\alpha \gamma_{abc} \psi^\alpha) \gamma^{abc} e - \frac{1}{2} F(\delta_e \mathcal{L}, \delta_e \mathcal{G}) e. \end{aligned}$$



$$\begin{aligned}
& \frac{1}{768} \sigma^{-2} X (\bar{\psi}_a \gamma_{abc} \psi^a) \gamma^{abc} e^{\langle (\bar{\lambda}_1 \gamma_a \lambda_2) (\bar{\lambda}_3 \gamma^a \lambda_4) = \frac{1}{2} (\bar{\lambda}_1 \gamma_a \lambda_3) (\bar{\lambda}_2 \gamma^a \lambda_4) + \frac{1}{24} (\bar{\lambda}_1 \gamma_{(3) \lambda_3} (\bar{\lambda}_2 \gamma^{(3) \lambda_4}) \rangle} - \frac{1}{32} \sigma^{-2} X_a \psi_a (\bar{e} \gamma^a \psi^a) = -\frac{1}{128} \sigma^{-4} \gamma_b \\
& \|(\bar{e} \gamma_a e) \bar{e} \gamma^a = 0\| \frac{1}{128} \sigma^{-4} \gamma_a \gamma_b \psi_a (\bar{e} \gamma^b e) (\bar{e} \gamma^a \psi^a) \gamma_a \psi_a (\bar{e} \gamma^b e) (\bar{e} \gamma^a \psi^a) \\
& \quad \overbrace{\phi_{\mu\nu}''^* \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(i, j)}^{(\bar{e} \gamma_a e_2) (\bar{e} \gamma^a e_4) = -\frac{1}{2} (\bar{e} \gamma_a e) (e_2 \gamma^a e_4)} - \frac{1}{64} \sigma^{-4} \gamma_a \gamma_b e (\bar{e} \gamma^b \psi_a) (\bar{e} \gamma^a \psi^a) \\
& \quad = -\frac{1}{16} \delta_e \mathcal{L}_a^b \delta_e \mathcal{C}^{aa} \gamma_a \gamma_b e = \frac{1}{32} [\delta_e \mathcal{L}, \delta_e \mathcal{L}^{ab} \gamma_{ab} e \\
& \quad \quad \quad \frac{1}{2} F(\delta_e \mathcal{L}, \delta_e \mathcal{L}) e].
\end{aligned}$$

$$\int_M \frac{\delta S_{\text{lin}}}{\delta \phi_i^*} \frac{\delta}{\delta \phi^i} \left(\int_M \left\langle \xi^*, \mathcal{D}f + \frac{1}{2} \mathcal{L}_\xi \xi - \frac{1}{2} X \right\rangle \right) =: \int_M \langle \xi^*, \kappa_\xi \rangle$$

$$\begin{aligned}
\kappa_\xi &= \frac{1}{2} \mathcal{D} \left[\mathcal{L}_\xi f + \frac{1}{2} \langle X, \xi \rangle - \frac{1}{6} \langle \xi, \mathcal{L}_\xi \xi \rangle \right] + \frac{1}{2} \mathcal{L}_{\mathcal{D}f + \frac{1}{2} \mathcal{L}_\xi \xi - \frac{1}{2} X} \xi - \frac{1}{2} \mathcal{L}_\xi \left(\mathcal{D}f + \frac{1}{2} \mathcal{L}_\xi \xi - \frac{1}{2} X \right) - \frac{1}{2} \mathcal{L}_\xi X \\
&= \frac{1}{2} (\mathcal{D} \mathcal{L}_\xi - \mathcal{L}_\xi \mathcal{D}) f + \frac{1}{4} \left(\mathcal{D} \langle X, \xi \rangle - \frac{1}{4} \mathcal{L}_X \xi - \frac{1}{4} \mathcal{L}_\xi X \right) - \frac{1}{12} \mathcal{D} \langle \xi, \mathcal{L}_\xi \xi \rangle + \frac{1}{4} \mathcal{L}_{\mathcal{L}_\xi \xi} \xi - \frac{1}{4} \mathcal{L}_\xi \mathcal{L}_\xi \xi \\
&= -\frac{1}{12} (\mathcal{L}_\xi \mathcal{L}_\xi \xi + \mathcal{L}_{\mathcal{L}_\xi \xi} \xi) + \frac{1}{4} \mathcal{L}_{\mathcal{L}_\xi \xi} \xi - \frac{1}{4} \mathcal{L}_\xi \mathcal{L}_\xi \xi = \frac{1}{6} \mathcal{L}_{\mathcal{L}_\xi \xi} \xi - \frac{1}{3} \mathcal{L}_\xi \mathcal{L}_\xi \xi = 0
\end{aligned}$$

$$\int_M \frac{\delta S_{\text{lin}}}{\delta \phi_i^*} \frac{\delta}{\delta \phi^i} \left(\int_M \frac{1}{2} f^* \left(\mathcal{L}_\xi f + \frac{1}{2} \langle X, \xi \rangle - \frac{1}{6} \langle \xi, \mathcal{L}_\xi \xi \rangle \right) \right) =: -\frac{1}{2} \int_M f^* \kappa_f,$$

$$\begin{aligned}
\kappa_f &= \mathcal{L}_{\mathcal{D}f + \frac{1}{2} \mathcal{L}_\xi \xi - \frac{1}{2} X} f - \mathcal{L}_\xi \left(\frac{1}{2} \mathcal{L}_\xi f + \frac{1}{4} \langle X, \xi \rangle - \frac{1}{12} \langle \xi, \mathcal{L}_\xi \xi \rangle \right) + \frac{1}{2} \langle \mathcal{L}_\xi X, \xi \rangle + \frac{1}{2} \left\langle X, \mathcal{D}f + \frac{1}{2} \mathcal{L}_\xi \xi - \frac{1}{2} X \right\rangle \\
&\quad - \frac{1}{6} \left\langle \mathcal{D}f + \frac{1}{2} \mathcal{L}_\xi \xi - \frac{1}{2} X, \mathcal{L}_\xi \xi \right\rangle + \frac{1}{6} \left\langle \xi, \mathcal{L}_{\mathcal{D}f + \frac{1}{2} \mathcal{L}_\xi \xi - \frac{1}{2} X} \xi \right\rangle - \frac{1}{6} \left\langle \xi, \mathcal{L}_\xi \left(\mathcal{D}f + \frac{1}{2} \mathcal{L}_\xi \xi - \frac{1}{2} X \right) \right\rangle \\
&= \left(\frac{1}{2} - \frac{1}{6} \right) \mathcal{L}_{\mathcal{L}_\xi \xi} f - \left(\frac{1}{2} + \frac{1}{6} \right) \mathcal{L}_\xi \mathcal{L}_\xi f \\
&\quad - \frac{1}{4} \mathcal{L}_\xi \langle X, \xi \rangle + \left(\frac{1}{2} - \frac{1}{12} \right) \langle \mathcal{L}_\xi X, \xi \rangle + \left(\frac{1}{4} + \frac{1}{12} \right) \langle X, \mathcal{L}_\xi \xi \rangle + \frac{1}{12} \langle \mathcal{L}_X \xi, \xi \rangle - \frac{1}{4} \langle X, X \rangle \\
&\quad + \frac{1}{12} (\mathcal{L}_\xi \langle \xi, \mathcal{L}_\xi \xi \rangle - \langle \mathcal{L}_\xi \xi, \mathcal{L}_\xi \xi \rangle + \langle \xi, \mathcal{L}_{\mathcal{L}_\xi \xi} \xi \rangle - \langle \xi, \mathcal{L}_\xi \mathcal{L}_\xi \xi \rangle) \\
&= \frac{1}{3} (\mathcal{L}_{\mathcal{L}_\xi \xi} f - 2 \mathcal{L}_\xi \mathcal{L}_\xi f) - \frac{1}{4} \mathcal{L}_\xi \langle X, \xi \rangle + \frac{5}{12} \langle \mathcal{L}_\xi X, \xi \rangle + \frac{1}{3} \langle X, \mathcal{L}_\xi \xi \rangle + \frac{1}{12} \langle \mathcal{D} \langle X, \xi \rangle - \mathcal{L}_\xi X, \xi \rangle \\
&\quad - \frac{1}{64} \sigma^{-4} (\bar{e} \gamma_a e) (\bar{e} \gamma^a e) + \frac{1}{12} (\mathcal{L}_\xi \langle \xi, \mathcal{L}_\xi \xi \rangle - \langle \mathcal{L}_\xi \xi, \mathcal{L}_\xi \xi \rangle + \langle \xi, \mathcal{L}_\xi \mathcal{L}_\xi \xi \rangle)
\end{aligned}$$

$$\begin{aligned}
& \overbrace{\phi_{\mu\nu}''^* \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(i, j)}^{(\bar{e} \gamma_a e_2) (\bar{e} \gamma^a e_4) = -\frac{1}{2} (\bar{e} \gamma_a e) (e_2 \gamma^a e_4)} - \frac{1}{3} (\mathcal{L}_\xi \langle X, \xi \rangle - \langle \mathcal{L}_\xi X, \xi \rangle - \langle X, \mathcal{L}_\xi \xi \rangle) = 0.
\end{aligned}$$

$$\begin{aligned}
\frac{1}{2} \{S, S\} &= \int_M \frac{\delta S_{\text{lin}}}{\delta \phi_i^*} \frac{\delta S_{\text{quad}}}{\delta \phi^i} + \frac{\delta S_{\text{quad}}}{\delta \phi_i^*} \frac{\delta S_{\text{lin}}}{\delta \phi^i} \\
&= \int_M \frac{\delta S_{\text{lin}}}{\delta \sigma^*} \frac{\delta S_{\text{quad}}}{\delta \sigma} + \frac{\delta S_{\text{lin}}}{\delta \bar{e}^*} \frac{\delta S_{\text{quad}}}{\delta e} + \frac{\delta S_{\text{quad}}}{\delta \bar{\rho}^*} \frac{\delta S_{\text{lin}}}{\delta \rho} + \frac{\delta S_{\text{quad}}}{\delta \bar{\psi}_\alpha^*} \frac{\delta S_{\text{lin}}}{\delta \psi^\alpha}
\end{aligned}$$



$$\begin{aligned} \frac{1}{2}\{S, S\} = & \int_M \left[\mathcal{L}_\xi \sigma - \frac{1}{8} \sigma^{-1} (\bar{\rho} e) \right] \sigma^{-3} \left[\frac{1}{32} \bar{\psi}_\beta^* \psi \psi^{*\beta} + \frac{1}{16} (\bar{e} \psi_\beta^*) (\bar{e} \psi^{*\beta}) + \frac{1}{32} \bar{\rho}^* \psi \rho^* \right] \\ & + \left[\mathcal{L}_\xi \bar{e} + \frac{1}{16} \sigma^{-2} \bar{\psi} \rho \right] \sigma^{-2} \left[-\frac{1}{32} \gamma_a e (\bar{\psi}_\beta^* \gamma^a \psi^{*\beta}) - \frac{1}{16} \psi_\beta^* (\bar{e} \psi^{*\beta}) - \frac{1}{32} \gamma_a e (\bar{\rho}^* \gamma^a \rho^*) \right] \\ & - \left[\frac{1}{32} \sigma^{-2} \bar{\psi} \rho^* \right] \left[\frac{1}{8} \sigma^* \sigma^{-1} e - \mathcal{L}_\xi \rho^* + \frac{1}{8} \sigma^{-2} \psi^\beta (\bar{\psi}_\beta^* e) + \frac{1}{8} \sigma^{-2} \gamma^c \psi_\beta^* (\bar{\psi}^\beta \gamma_c e) - \frac{1}{16} \sigma^{-2} \psi e^* \right] \\ & + \left[-\frac{1}{32} \sigma^{-2} \bar{\psi} \psi^{*\alpha} + \frac{1}{16} \sigma^{-2} (\bar{e} \psi^{*\alpha}) \bar{e} \right] \left[-\frac{1}{2} \sigma^{-2} \mathcal{G}_{a\alpha} \gamma^a e + \frac{1}{96} \sigma^{-2} \gamma_{abc} \psi_\alpha (\bar{\rho}^* \gamma^{abc} e) \right. \\ & \left. - (\mathcal{L}_\xi \psi^*)_\alpha + \frac{1}{8} \sigma^{-2} \rho (\bar{\psi}_\alpha^* e) - \frac{1}{8} \sigma^{-2} \gamma_a e (\bar{\psi}_\alpha^* \gamma^a \rho) \right] \end{aligned}$$

$$\frac{1}{32} \sigma^{-2} (\bar{e} \gamma_e e) (\bar{\psi}^{*\alpha} \gamma^e \gamma_a e) + \frac{1}{16} \sigma^{-2} (\bar{e} \psi^{*\alpha}) (\bar{e} \gamma_a e) = -\frac{1}{32} \sigma^{-2} (\bar{e} \gamma_e e) (\bar{\psi}^{*\alpha} \gamma_a \gamma^e e) \quad \overbrace{\phi_{\mu\nu}^{**} \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}^{(\bar{e} \gamma_a e_2) (\bar{e} \gamma^a e_4) = -\frac{1}{2} (\bar{e} \gamma_a e) (e_2 \gamma^a e_4)}$$

$$\frac{1}{2}\{S, S\} =: \frac{1}{128} \int_M \sigma^{-4} \kappa_{\text{quad}}$$

$$\begin{aligned} \kappa_{\text{quad}} = & -\frac{1}{2} (\bar{\rho} e) (\bar{\rho}^* \psi \rho^*) - \frac{1}{2} (\bar{\rho} e) (\bar{\psi}^* \psi \psi^*) - (\bar{\rho} e) (\bar{e} \psi^*) (\bar{e} \psi^*) + \frac{1}{4} (\bar{\rho} \psi \gamma_a e) (\bar{\rho}^* \gamma^a \rho^*) \\ & + \frac{1}{4} (\bar{\rho} \psi \gamma_a e) (\bar{\psi}^* \gamma^a \psi^*) + \frac{1}{2} (\bar{\rho} \psi \psi^*) (\bar{e} \psi^*) + \frac{1}{2} (\bar{\psi}^* e) (\bar{\psi} \psi \rho^*) - \frac{1}{2} (\bar{\psi} \gamma_a e) (\bar{\psi}^* \gamma^a \psi \rho^*) \\ & + \frac{1}{24} (\bar{\rho}^* \gamma_{abc} e) (\bar{\psi} \gamma^{abc} \psi \psi^*) - \frac{1}{12} (\bar{\rho}^* \gamma^{abc} e) (\bar{\psi} \gamma_{abc} e) (\bar{e} \psi^*) + \frac{1}{2} (\bar{\rho} \psi \psi^*) (\bar{\psi}^* e) \\ & - (\bar{\rho} e) (\bar{\psi}^* e) (\bar{e} \psi^*) + \frac{1}{2} (\bar{\psi}^* \gamma^a \rho) (\bar{e} \gamma_a \psi \psi^*) - (\bar{\psi}^* \psi \rho) (\bar{e} \psi^*) \\ = & -\frac{1}{2} (\bar{\rho} e) (\bar{\rho}^* \psi \rho^*) - \frac{1}{2} (\bar{\rho} e) (\bar{\psi}^* \psi \psi^*) + \frac{1}{4} (\bar{\rho} \psi \gamma_a e) (\bar{\rho}^* \gamma^a \rho^*) + \frac{1}{4} (\bar{\rho} \psi \gamma_a e) (\bar{\psi}^* \gamma^a \psi^*) \\ & + \frac{1}{2} (\bar{\psi}^* e) (\bar{\psi} \psi \rho^*) - \frac{1}{2} (\bar{\psi} \gamma_a e) (\bar{\psi}^* \gamma^a \psi \rho^*) + \frac{1}{24} (\bar{\rho}^* \gamma_{abc} e) (\bar{\psi} \gamma^{abc} \psi \psi^*) \\ & - \frac{1}{12} (\bar{\rho}^* \gamma^{abc} e) (\bar{\psi} \gamma_{abc} e) (\bar{e} \psi^*) + \frac{1}{2} (\bar{\psi}^* \gamma^a \rho) (\bar{e} \gamma_a \psi \psi^*) - (\bar{\psi}^* \psi \rho) (\bar{e} \psi^*). \end{aligned}$$

$$\psi e = \bar{e} \psi = 0$$

$$\begin{aligned} \kappa_{\text{quad}} = & \frac{1}{2} (\bar{\psi}^* e) (\bar{\psi} \psi \rho^*) - \frac{1}{2} (\bar{\psi} \gamma_a e) (\bar{\psi}^* \gamma^a \psi \rho^*) + \frac{1}{24} (\bar{\rho}^* \gamma_{abc} e) (\bar{\psi} \gamma^{abc} \psi \psi^*) - \frac{1}{12} (\bar{\rho}^* \gamma^{abc} e) (\bar{\psi} \gamma_{abc} e) (\bar{e} \psi^*) \\ & \underbrace{\phi_{mn}^{**} \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}_{(\bar{e}_1 \gamma_{(3)} e) (\bar{e}_3 \gamma^{(3)} e) = -18 \ell (\bar{e}_1 \gamma_a e_3) (\bar{e} \gamma^a e)} - (\bar{\psi}^* e) (\bar{\psi} \psi \rho^*) - \frac{1}{2} (\bar{\psi} \gamma_a e) (\bar{\psi}^* \gamma^a \psi \rho^*) + \frac{1}{24} (\bar{\rho}^* \gamma_{abc} e) (\bar{\psi} \gamma^{abc} \psi \psi^*) \\ & \underbrace{\phi_{mn}^{**} \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}_{(\bar{e}_1 \gamma_{(3)} e) (\bar{e}_3 \gamma^{(3)} e) = -18 \ell (\bar{e}_1 \gamma_a e_3) (\bar{e} \gamma^a e)} - (\bar{\psi}^* e) (\bar{\psi} \psi \rho^*) - \frac{1}{2} (\bar{\psi} \gamma_a e) (\bar{\psi}^* \gamma^a \psi \rho^*) - \frac{1}{2} (\bar{\rho}^* \gamma_a \psi) (\bar{e} \gamma^a \psi \psi^*) + \frac{1}{2} (\bar{e} \gamma_a \psi) (\bar{\rho}^* \gamma^a \psi \psi^*) \\ = & -\frac{1}{2} (\bar{\psi} \gamma_a e) (\bar{\psi}^* \gamma^a \psi \rho^*) + \frac{1}{2} (\bar{e} \gamma^a \psi) (\bar{\rho}^* \gamma_a \psi \psi^*) \\ = & -\frac{1}{2} (\bar{\psi} \gamma_a e) (\bar{\psi}^* (\gamma^a \psi + \psi \gamma^a) \rho^*) = -(\bar{\psi} \psi e) (\bar{\psi}^* \rho^*) = 0. \end{aligned}$$

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] \Phi_{(0)} = \delta_{[\epsilon_1, \epsilon_2]} \Phi_{(0)} + M(\Phi_{(0)}, \epsilon_1, \epsilon_2) EL[\Phi_{(0)}]$$



$$Q_B\Phi_{(0)}:=\delta_{\Phi_{(1)}}\Phi_{(0)}$$

$$Q_B\Phi_{(1)}:=\delta_{\Phi_{(1)}}\Phi_{(1)}+\delta_{\Phi_{(2)}}\Phi_{(1)}=\frac{1}{2}[\Phi_{(1)},\Phi_{(1)}]+\delta_{\Phi_{(2)}}\Phi_{(1)}$$

$$Q_B\Phi^i=\delta_\Phi\Phi^i$$

$$Q_B^2\Phi^i=\tilde{M}(\Phi)^{ij}\frac{\partial S_0}{\partial\Phi^j}$$

$$Q_{BV}|_{\Phi^*=0}\Phi^i=Q_B\Phi^i$$

$$Q_{BV}=\{S,\cdot\}=\frac{\partial S}{\partial\Phi_i^*}\frac{\partial}{\partial\Phi^i}+\frac{\partial S}{\partial\Phi^i}\frac{\partial}{\partial\Phi_i^*},\deg S=0$$

$$S(\Phi,\Phi^*)=S_0(\Phi)+\sum_{k=1}^N\frac{1}{k!}\Phi_{i_1}^*\cdots\Phi_{i_k}^*S_k^{i_1\ldots i_k}(\Phi)$$

$$\{S,S\}=0$$

$$S(\Phi,\Phi^*)=S_0(\Phi)+\Phi_i^*Q_B\Phi^i$$

$$S(\Phi,\Phi^*)=S_0(\Phi)+\Phi_i^*S_1^i(\Phi)+\frac{1}{2}\Phi_i^*\Phi_j^*S_2^{ij}(\Phi)$$

$$\delta_\Phi\Phi^i=Q_B\Phi^i=(Q_{BV}\Phi^i)|_{\Phi^*=0}=\{S,\Phi^i\}|_{\Phi^*=0}=S_1^i(\Phi)$$

$$Q_BS_0=0,$$

$$Q_B^2\Phi^i+(\partial_jS_0)S_2^{ij}=0,$$

$$Q_BS_2^{ij}+S_2^{k[i}\partial_k(Q_B\Phi^{j]})=0,$$

$$S_2^{i[j}\partial_iS_2^{kl]}=0,$$

$$S_2^{ij}\sim \tilde{M}(\Phi)^{ij}$$

$$\epsilon_{0\dots 9}=-\epsilon^{0\dots 9}=1.$$

$$\{\gamma_a,\gamma_b\}=2g_{ab}$$

$$C\gamma_aC^{-1}=-\gamma_a^T,C^T=-C$$

$$\bar{\psi}\colon=\psi^TC$$

$$\bar{\psi}\chi=\bar{\chi}\psi.$$

$$C\gamma_{a_1\dots a_k}C^{-1}=(-1)^{\lfloor\frac{k+1}{2}\rfloor}\gamma_{a_1\dots b_k}^T$$

$$\bar{\psi}\gamma_{a_1\dots a_k}\chi=(-1)^{\lfloor\frac{k+1}{2}\rfloor}\bar{\chi}\gamma_{a_1\dots a_k}\psi$$

$$\gamma_{a_1\dots a_k}\gamma^*_k=(-1)^{\lfloor\frac{k}{2}\rfloor}\frac{1}{(10-k)!}\sqrt{-g}\epsilon_{a_1\dots a_kb_1\dots b_{10-k}}\gamma^{b_1\dots b_{10-k}}$$



$$\bar{\psi}\chi = 0 \text{ if } \text{ch}\psi = \text{ch}\chi$$

$$\text{ch}(\gamma_{(p)}\psi) = (-1)^p \text{ch}\psi$$

$$\gamma_{(p)}\dot{}:=\gamma_{a_1\dots a_p}$$

$$\bar{\lambda}\gamma_{(3)}\lambda,$$

$$\bar{\lambda}\gamma_{(1)}\lambda=\bar{\lambda}\gamma_{(5)}\lambda=0$$

$$\frac{1}{p!}\Big(\bar{\lambda}_1\gamma_{a_1\dots a_nb_1\dots b_p}\lambda_2\Big)\Big(\bar{\lambda}_3\gamma^{b_1\dots b_p}\lambda_4\Big)=(\text{ch}\lambda_2)(\text{ch}\lambda_4)(-1)^{1+\lfloor\frac{n}{2}\rfloor}\frac{1}{q!}\Big(\bar{\lambda}_1\gamma^{c_1\dots c_q}\lambda_2\Big)\Big(\bar{\lambda}_3\gamma_{a_1\dots a_nc_1\dots c_q}\lambda_4\Big)$$

$$\frac{1}{(10-p)!}(\bar{\lambda}_1\gamma_{(10-p)}\lambda_2)(\bar{\lambda}_3\gamma^{(10-p)}\lambda_4)=-\frac{1}{p!}(\text{ch}\lambda_2)(\text{ch}\lambda_4)(\bar{\lambda}_1\gamma_{(p)}\lambda_2)(\bar{\lambda}_3\gamma^{(p)}\lambda_4)$$

$$\gamma_a\gamma^{b_1\dots b_p}=\gamma_a^{b_1\dots b_p}+p\delta_a^{[b_1}\gamma^{b_2\dots b_p]}$$

$$\{\gamma_a,\gamma^{b_1\dots b_{2p+1}}\}=2(2p+1)\delta_a^{[b_1}\gamma^{b_2\dots b_{2p+1}]}$$

$$\lambda\bar{\psi}=\frac{1}{32}\sum_{p=0}^{10}\frac{(-1)^{p+1}}{p!}\gamma_{(p)}\psi\bar{\lambda}\gamma^{(p)}$$

$$(\bar{\lambda}_1\psi_1)(\bar{\lambda}_2\psi_2)=\frac{1}{16}(\bar{\lambda}_1\gamma_{(1)}\lambda_2)(\bar{\psi}_1\gamma^{(1)}\psi_2)+\frac{1}{96}(\bar{\lambda}_1\gamma_{(3)}\lambda_2)(\bar{\psi}_1\gamma^{(3)}\psi_2)+\frac{1}{3840}(\bar{\lambda}_1\gamma_{(5)}\lambda_2)(\bar{\psi}_1\gamma^{(5)}\psi_2)$$

$$\begin{aligned}(\bar{\lambda}_1\gamma_a\lambda_2)(\bar{\lambda}_3\gamma^a\lambda_4)&=\frac{1}{2}(\bar{\lambda}_1\gamma_a\lambda_3)(\bar{\lambda}_2\gamma^a\lambda_4)+\frac{1}{24}(\bar{\lambda}_1\gamma_{(3)}\lambda_3)(\bar{\lambda}_2\gamma^{(3)}\lambda_4)\\(\bar{\lambda}_1\gamma_{(3)}\lambda_2)(\bar{\lambda}_3\gamma^{(3)}\lambda_4)&=18\ell(\bar{\lambda}_1\gamma_a\lambda_3)(\bar{\lambda}_2\gamma^a\lambda_4)-\frac{1}{2}(\bar{\lambda}_1\gamma_{(3)}\lambda_3)(\bar{\lambda}_2\gamma^{(3)}\lambda_4)\end{aligned}$$

$$(\bar{\lambda}_{[1}\gamma_{|a}\lambda_{2|})(\bar{\lambda}_{3]}\gamma^a\lambda_4)=\frac{1}{2}(\bar{\lambda}_1\gamma_a\lambda_3)(\bar{\lambda}_2\gamma^a\lambda_4)$$

$$(\bar{\lambda}\gamma^{abc}\lambda)\bar{\lambda}\gamma_{abc}=0$$

$$(\bar{e}_1\gamma_a e_2)(\bar{e}_3\gamma^a e_4)=-\frac{1}{2}(\bar{e}_1\gamma_a e_3)(\bar{e}_2\gamma^a e_4)-\frac{1}{24}(\bar{e}_1\gamma_{(3)}e_3)(\bar{e}_2\gamma^{(3)}e_4)$$

$$(\bar{e}\gamma_a e_2)(\bar{e}\gamma^a e_4)=-\frac{1}{2}(\bar{e}\gamma_a e)(e_2\gamma^a e_4)$$

$$(\bar{e}\gamma_a e)\bar{e}\gamma^a=0$$

$$\frac{1}{24}(\bar{e}_1\gamma_{(3)}e_3)(\bar{e}_2\gamma^{(3)}e_4)=-\frac{1}{2}(\bar{e}_1\gamma_a e_2)(\bar{e}_3\gamma^a e_4)+\frac{1}{2}(\bar{e}_3\gamma_a e_2)(\bar{e}_1\gamma^a e_4)$$

$$(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)$$

$$|A|\in\mathbb{Z}_2=\{0,1\}$$



$$|d|\equiv 1, |i_X|\equiv -1+|X|$$

$$L_X=[d,i_X],$$

$$L_Xf=Xf$$

$$i_{X_H}\omega=-dH$$

$$\omega = dp_i \wedge dq^i$$

$$\{H,H'\}=X_HH'$$

$$df=dx^i\frac{\partial f}{\partial x^i}$$

$$i_Xdf=(-1)^{|X|}L_Xf=(-1)^{|X|}Xf$$

$$|q|\equiv |p|+1\,\mathrm{mod}2$$

$$i_{X_H}\omega=dp\left(i_{A\partial_q}dq\right)+\left(i_{B\partial_p}dp\right)dq\quad \blacksquare \frac{\odot d\hbar''}{\prod_{(4)}\boxplus''\otimes\frac{\partial\mathfrak{G}}{\partial\mathfrak{M}}d^4\chi}-dpA-Bdq$$

$$dH=dq(\partial_qH)+dp(\partial_pH)=(\partial_qH)dq+dp(\partial_pH)$$

$$X_H=\frac{\partial H}{\partial p_i}\frac{\partial}{\partial q^i}+\frac{\partial H}{\partial q^i}\frac{\partial}{\partial p_i}$$

$$\{H,H'\}=\frac{\partial H}{\partial p_i}\frac{\partial H'}{\partial q^i}+\frac{\partial H}{\partial q^i}\frac{\partial H'}{\partial p_i}$$

$$\{F,F'\}=(-1)^{|F|\cdot |p_i|}\frac{\partial F}{\partial p_i}\frac{\partial F'}{\partial q^i}+(-1)^{|F|\cdot |q^i|}\frac{\partial F}{\partial q^i}\frac{\partial F'}{\partial p_i}$$

$$\{H,H'\}=\int_M\frac{\delta H}{\delta\bar\lambda^*}\frac{\delta H'}{\delta\lambda}+\frac{\delta H'}{\delta\bar\lambda^*}\frac{\delta H}{\delta\lambda}$$

$$\frac{\delta H}{\delta(\bar\lambda^*)_y}=\int_M\frac{\delta H}{\delta(\lambda^*)^x}\frac{\delta(\lambda^*)^x}{\delta(\bar\lambda^*)_y}=\frac{\delta H}{\delta(\lambda^*)^x}(C^{-1})^{yx}$$

$$\int_M\frac{\delta H}{\delta(\bar\lambda^*)_y}\frac{\delta H'}{\delta\lambda^y}=(C^{-1})^{yx}\int_M\frac{\delta H}{\delta(\lambda^*)^x}\frac{\delta H'}{\delta\lambda^y}=(C^{-1})^{yx}\int_M(C_{xz}\chi^z)(C_{yw}\varphi^w)=\int_M\chi^yC_{yw}\varphi^w$$

$$\left\{\int_M\bar\lambda^*\chi,\int_M\bar\lambda\varphi\right\}=\int_M\bar\chi\varphi$$

$$F^+(\delta_1\mathcal{L},\delta_2\mathcal{L})=\frac{1}{4}[\delta_1\mathcal{L},\delta_2\mathcal{L}]\colon\Gamma(C_+)\rightarrow\Gamma(C_+)$$

$$F^-(\delta_1\mathcal{L},\delta_2\mathcal{L})=\frac{1}{4}[\delta_1\mathcal{G},\delta_2\mathcal{L}]\colon\Gamma(C_-)\rightarrow\Gamma(C_-)$$



$$\begin{aligned} (F^+(\delta_1\mathcal{L},\delta_2\mathcal{L})u_+)^a &= \frac{1}{4}(\delta_1\mathcal{L}^{a\alpha}\delta_2\mathcal{L}_{b\alpha} - \delta_2\mathcal{L}^{a\alpha}\delta_1\mathcal{L}_{b\alpha})u_+^b \\ (F^-(\delta_1\mathcal{L},\delta_2\mathcal{L})u_-)^\alpha &= \frac{1}{4}(\delta_1\mathcal{G}^{a\alpha}\delta_2\mathcal{G}_{a\beta} - \delta_2\mathcal{L}^{a\alpha}\delta_1\mathcal{G}_{a\beta})u_-^\beta \end{aligned}$$

$$\begin{aligned} F(\delta_1\mathcal{L}_2,\delta_2\mathcal{L})\rho &= \frac{1}{16}[\delta_1\mathcal{L}_2,\delta_2\mathcal{L}_{ab}\gamma^{ab}\rho \\ F(\delta_1\mathcal{L}_2,\delta_2\mathcal{L})\psi^\alpha &= \frac{1}{16}\Big[\delta_1\mathcal{L}_2,\delta_2\mathcal{L}_{ab}\gamma^{ab}\psi^\alpha + \frac{1}{4}[\delta_1\mathcal{L}_2,\delta_2\mathcal{L}^\alpha{}_\beta\psi^\beta. \end{aligned}$$

$$C_+=\{\hat{u}_+ + \varphi(\hat{u}_+) \mid \hat{u}_+ \in \hat{C}_+\}$$

$$\hat{p}\circ (p|_{C'_+})^{-1}\circ (\hat{p}|_{\hat{C}_+})^{-1}:\hat{C}_+\rightarrow \hat{C}_+,$$

$$\hat{u}_+\mapsto \hat{u}_+-\varphi^*(1+\varphi\varphi^*)^{-1}\delta\varphi(\hat{u}_+),$$

$$A^+=\varphi^*(1+\varphi\varphi^*)^{-1}d\varphi,$$

$$F^+(\delta_1\varphi,\delta_2\varphi)=\delta_1\varphi^*\delta_2\varphi-\delta_2\varphi^*\delta_1\varphi\colon\hat{C}_+\rightarrow\hat{C}_+$$

$$F^+(\delta_1\mathcal{G},\delta_2\mathcal{L})=\frac{1}{4}[\delta_1\mathcal{G},\delta_2\mathcal{G}]\colon\hat{C}_+\rightarrow\hat{C}_+.$$

$$T_{\mathcal{G}}\mathcal{M}\cong\Gamma(C_+\otimes C_-).$$

$$T_{\mathcal{G}}\mathcal{M}\cong\Gamma(C_+\otimes C_-)\ni u,v\mapsto\int_M\sigma^2\langle u,v\rangle$$

$$TE=T_{\mathrm{hor}}E\oplus T_{\mathrm{ver}}E\cong\pi^*TM\oplus\pi^*E$$

$$B\in\Gamma(\pi^*TM),C\in\Gamma(\pi^*E).$$

$$B=[B_1,B_2]+C_1B_2-C_2B_1, C=[C_1,C_2]+\nabla_{B_1}C_2-\nabla_{B_2}C_1-F(B_1,B_2),$$

$$\nabla u=du+Au,\,\,\text{i.e.}\,\,\left(\nabla_{\partial_{x^i}}u\right)^a=\partial_{x^i}u^a+A_i{}^a{}_bu^b,$$

$$u(x_0+\Delta x)=u(x_0)-\varepsilon_aA_i{}^a{}_b(x_0)u^b(x_0)\Delta x^i.$$

$$\partial_{x^i}-A_i{}^a{}_b(x_0)u^b\partial_{y^a}.$$

$$\text{ver}=\text{span}\{e_a\},\,\text{hor}=\text{span}\{e_i\},\,\,e_a:=\partial_{y^a},e_i:=\partial_{x^i}-A_i{}^a{}_b(x)y^b\partial_{y^a}.$$

$$V(x,y)=B^i(x,y)e_i+C^a(x,y)e_a.$$

$$[e_a,e_b]=0,[e_a,e_i]=-A_i{}^b{}_a(x)e_b,[e_i,e_j]=-F_{ij}{}^a{}_b(x)e_a,$$

$$\begin{aligned} [V_1,V_2]&= (B_1^je_jB_2^i-B_2^je_jB_1^i-C_2^ae_aB_1^i+C_1^ae_aB_2^i)e_i \\ &+ (B_1^ie_iC_2^a-B_2^ie_iC_1^a+C_1^be_bC_2^a-C_2^be_bC_1^a+A_i{}^a{}_bB_1^iC_2^b-A_i{}^a{}_bB_2^iC_1^b-B_1^iB_2^jF_{ij}{}^a{}_by^b)e_a, \end{aligned}$$

$$[\delta_1,\delta_2]:=[\delta_{\epsilon_1},\delta_{\epsilon_2}]$$



$$V^a := \frac{1}{4} \sigma^{-2} (\bar{\epsilon}_2 \gamma^a \epsilon_1)$$

$$[\delta_1, \delta_2] = \delta_V + \delta_\zeta$$

$$\zeta := -\frac{1}{2} V \rho.$$

$$\begin{aligned} [\delta_1, \delta_2] \mathcal{L}_{\alpha\alpha} = & -\frac{1}{8} \sigma^{-4} (\bar{\rho} \epsilon_1) (\bar{\epsilon}_2 \gamma_a \psi_\alpha) + \frac{1}{2} \sigma^{-2} (\bar{\epsilon}_2 \gamma_a D_\alpha \epsilon_1) + \frac{1}{16} \sigma^{-4} (\bar{\psi}_\alpha \rho) (\bar{\epsilon}_2 \gamma_a \epsilon_1) \\ & + \frac{1}{16} \sigma^{-4} (\bar{\epsilon}_2 \gamma_a \gamma^b \rho) (\bar{\psi}_\alpha \gamma_b \epsilon_1) - 1 \leftrightarrow 2 \\ & \frac{\psi_{mn}''^\dagger \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}{\underbrace{(\bar{e}_1 \gamma_{(3)} e) (\bar{e}_3 \gamma^{(3)} e) = -18 \ell (\bar{e}_1 \gamma_a e_3) (\bar{e} \gamma^a e)}} \frac{1}{2} \sigma^{-2} (\bar{\epsilon}_2 \gamma_a D_\alpha \epsilon_1) + \frac{1}{4} \sigma^{-2} (\bar{\psi}_\alpha \rho) V_a + \frac{1}{16} \sigma^{-4} (\bar{\epsilon}_1 \gamma_b \psi_\alpha) (\bar{\epsilon}_2 \gamma^b \gamma_a \rho) - 1 \leftrightarrow 2 \\ & \frac{(\bar{e}_1 \gamma_{(3)} e) (\bar{e}_3 \gamma^{(3)} e) = -18 \ell (\bar{e}_1 \gamma_a e_3) (\bar{e} \gamma^a e)}{2} = \frac{1}{2} \sigma^{-2} [(\bar{\epsilon}_2 \gamma_a D_\alpha \epsilon_1) + (D_\alpha \bar{\epsilon}_2 \gamma_a \epsilon_1)] + \frac{1}{2} \sigma^{-2} (\bar{\psi}_\alpha \rho) V_a + \frac{1}{8} \sigma^{-4} (\bar{\epsilon}_{[1} \gamma_{|b} \psi_{\alpha]}) (\bar{\epsilon}_{2]} \gamma^b \gamma_a \rho) \\ & \frac{\psi_{mn}''^* \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}{\underbrace{(\bar{e}_1 \gamma_{(3)} e) (\bar{e}_3 \gamma^{(3)} e) = -18 \ell (\bar{e}_1 \gamma_a e_3) (\bar{e} \gamma^a e)}} 2 D_\alpha V_a + \frac{1}{2} \sigma^{-2} (\bar{\psi}_\alpha \rho) V_a - \frac{1}{4} \sigma^{-2} V_b (\bar{\psi}_\alpha \gamma^b \gamma_a \rho) \\ & \frac{\Gamma_{mn}''^\dagger \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}{\underbrace{(\bar{e}_1 \gamma_{(3)} e) (\bar{e}_3 \gamma^{(3)} e) = -18 \ell (\bar{e}_1 \gamma_a e_3) (\bar{e} \gamma^a e)}} 2 D_\alpha V_a + \frac{1}{4} \sigma^{-2} (\bar{\psi}_\alpha \gamma_a V \rho) \stackrel{(2.41)}{=} \delta_V \mathcal{C}_{\alpha\alpha} + \delta_\zeta \mathcal{C}_{\alpha\alpha}, \end{aligned}$$

$$(\mathcal{L}_X \mathcal{L})_{\alpha\alpha} = 2(D_\alpha X_\alpha - D_\alpha X_\alpha),$$

$$\begin{aligned} [\delta_1, \delta_2] \sigma = & \frac{1}{8} \delta_1 [\sigma^{-1} (\bar{\rho} \epsilon_2)] - 1 \leftrightarrow 2 \\ = & -\frac{1}{64} \sigma^{-3} (\bar{\rho} \epsilon_1) (\bar{\rho} \epsilon_2) + \frac{1}{8} \sigma^{-1} (\overline{\not{D} \epsilon_1} \epsilon_2) + \frac{1}{8 \times 192} \sigma^{-3} (\psi_\alpha \gamma_{(3)} \psi^\alpha) (\bar{\epsilon}_1 \gamma^{(3)} \epsilon_2) - 1 \leftrightarrow 2 \\ = & \frac{1}{8} \sigma^{-1} (\bar{\epsilon}_2 \not{D} \epsilon_1 - \bar{\epsilon}_1 \not{D} \epsilon_2) = \frac{1}{2} \sigma D_\alpha \left(\frac{1}{4} \sigma^{-2} \bar{\epsilon}_2 \gamma^a \epsilon_1 \right) \stackrel{(2.26)}{=} \mathcal{L}_V \sigma = \delta_V \sigma = \delta_V \sigma + \delta_\zeta \sigma \end{aligned}$$

$$\delta_\zeta \sigma = -\frac{1}{16} \sigma^{-1} \bar{\rho} V \rho = 0,$$

$$\begin{aligned} (\delta \not{D}) \rho = & \frac{1}{2} \delta \mathcal{C}^\alpha{}_a \gamma^a D_\alpha \rho + \frac{1}{4} (D_\alpha \delta \mathcal{C}^\alpha{}_a) \gamma^a \rho, \\ (\delta \not{D}) \psi^\alpha = & \frac{1}{2} \delta \mathcal{C}^\gamma{}_a \gamma^a D_\gamma \psi^\alpha + \frac{1}{4} (D_\gamma \delta \mathcal{C}^\gamma{}_a) \gamma^a \psi^\alpha + (D^{[\alpha} \delta \mathcal{C}^{\gamma]}{}_a) \gamma^a \psi_\gamma, \\ (\delta D_\alpha) \rho = & -\frac{1}{2} \delta \mathcal{C}_\alpha{}^a D_a \rho - \frac{1}{4} (D_b \delta \mathcal{C}_{\alpha c}) \gamma^{bc} \rho - \left(D_\alpha \frac{\delta \sigma}{\sigma} \right) \rho, \end{aligned}$$



$$\begin{aligned}
& \frac{1}{32} \sigma^{-2} (\bar{\psi}_\alpha \rho) \delta_2 \mathcal{G}_a^\alpha (\bar{\chi} \gamma^a \epsilon_1) - 1 \leftrightarrow 2 = -\frac{1}{32} \sigma^{-4} (\bar{\psi}_\alpha \rho) (\bar{\epsilon}_{[2} \gamma_a \psi^\alpha) (\bar{\epsilon}_1] \gamma^a \chi) \\
& \frac{\Gamma_{mn}^{\prime\prime\prime} \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}{(\bar{e}_1 \gamma_{(3)} e) (\bar{e}_3 \gamma^{(3)} e) = -18 \ell (\bar{e}_1 \gamma_a e_3) (\bar{e} \gamma^a e)} \frac{1}{64} \sigma^{-4} (\bar{\psi}_\alpha \rho) (\bar{\epsilon}_2 \gamma_a \epsilon_1) (\bar{\chi} \gamma^a \psi^\alpha) = \frac{1}{16} \sigma^{-2} (\bar{\psi}_\alpha \rho) (\bar{\chi} V \psi^\alpha) \\
& \frac{\Gamma_{mn}^{\prime\prime\prime} \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}{(\bar{e}_1 \gamma_{(3)} e) (\bar{e}_3 \gamma^{(3)} e) = -18 \ell (\bar{e}_1 \gamma_a e_3) (\bar{e} \gamma^a e)} \frac{1}{1536} \sigma^{-2} (\bar{\chi} V \gamma_{(3)} \rho) (\bar{\psi}_\alpha \gamma^{(3)} \psi^\alpha) \\
& [\delta_1, \delta_2] \rho = \delta_V \rho + \delta_\zeta \rho - \frac{1}{2} V \left[\not{D} \rho + D_\alpha \psi^\alpha - \frac{1}{768} \sigma^{-2} (\bar{\psi}_\alpha \gamma_{(3)} \psi^\alpha) \gamma^{(3)} \rho \right] \\
& = \delta_V \rho + \delta_\zeta \rho - \frac{1}{2} VEL[\rho] \\
& [\delta_1, \delta_2] \psi_\alpha = \delta_V \psi_\alpha + \delta_\zeta \psi_\alpha + \left(\frac{1}{4} \sigma^{-2} \epsilon_{[2} \bar{\epsilon}_1] - \frac{1}{2} V \right) EL[\psi]_\alpha \\
& [\delta_1, \delta_2] \psi_\alpha = \delta_1 \left(D_\alpha \epsilon_2 + \frac{1}{8} \sigma^{-2} (\bar{\psi}_\alpha \rho) \epsilon_2 - \frac{1}{4} \delta_2 \mathcal{C}_{aa} \gamma^a \rho \right) - 1 \leftrightarrow 2 - F(\delta_1 \mathcal{C}, \delta_2 \mathcal{C}) \psi_\alpha \\
& \frac{\zeta_{mn}^{\prime\prime\prime} \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}{(\bar{e}_1 \gamma_{(3)} e) (\bar{e}_3 \gamma^{(3)} e) = -18 \ell (\bar{e}_1 \gamma_a e_3) (\bar{e} \gamma^a e)} \left[(\delta_1 D_\alpha) \epsilon_2 - \frac{1}{32} \sigma^{-4} (\bar{\rho} \epsilon_1) (\bar{\psi}_\alpha \rho) \epsilon_2 + \frac{1}{8} \sigma^{-2} \bar{\rho} \left[D_\alpha \epsilon_1 + \frac{1}{8} \sigma^{-2} (\bar{\psi}_\alpha \rho) \epsilon_1 - \frac{1}{4} \delta_1 \mathcal{C}_{aa} \gamma^a \rho \right] \epsilon_2 \right. \\
& + \frac{1}{8} \sigma^{-2} \bar{\psi}_\alpha \left(\not{D} \epsilon_1 + \frac{1}{4} \delta_1 \mathcal{G}_{a\beta} \gamma^a \psi^\beta \right) \epsilon_2 - \frac{1}{4} \delta_1 \delta_2 \mathcal{G}_{aa} \gamma^a \rho - \frac{1}{4} \delta_2 \mathcal{G}_{aa} \gamma^a \left(\not{D} \epsilon_1 + \frac{1}{4} \delta_1 \mathcal{G}_{b\beta} \gamma^b \psi^\beta \right) \\
& \left. - 1 \leftrightarrow 2 - \frac{1}{16} [\delta_1 \mathcal{L}_2, \delta_2 \mathcal{L}_2]_{ab} \gamma^{ab} \psi_\alpha - \frac{1}{4} [\delta_1 \mathcal{L}_2, \delta_2 \mathcal{L}_{\alpha\beta} \psi^\beta \right. \\
& \left. \frac{\zeta_{mn}^{\prime\prime\prime} \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}{(\bar{e}_1 \gamma_{(3)} e) (\bar{e}_3 \gamma^{(3)} e) = -18 \ell (\bar{e}_1 \gamma_a e_3) (\bar{e} \gamma^a e)} \left[(\delta_1 D_\alpha) \epsilon_2 + \frac{1}{8} \sigma^{-2} (\bar{\rho} D_\alpha \epsilon_1) \epsilon_2 + \frac{1}{8} \sigma^{-2} (\bar{\psi}_\alpha \not{D} \epsilon_1) \epsilon_2 - \frac{1}{4} \delta_2 \mathcal{C}_{aa} \gamma^a \not{D} \epsilon_1 - 1 \leftrightarrow 2 \right] \right. \\
& \left. + \left[\frac{1-2}{64} \sigma^{-4} (\bar{\psi}_\alpha \rho) (\bar{\rho} \epsilon_1) \epsilon_2 + \frac{1}{32} \sigma^{-2} \delta_1 \mathcal{L}_{a\beta} (\bar{\psi}_\alpha \gamma^a \psi^\beta) \epsilon_2 + \frac{1}{16} \delta_1 \mathcal{L}_{aa} \delta_2 \mathcal{G}_{b\beta} \gamma^a \gamma^b \psi^\beta - 1 \leftrightarrow 2 \right] \right. \\
& \left. - \frac{1}{4} [\delta_1, \delta_2] \mathcal{G}_{aa} \gamma^a \rho - \frac{1}{16} [\delta_1 \mathcal{G}, \delta_2 \mathcal{G}]_{ab} \gamma^{ab} \psi_\alpha - \frac{1}{4} [\delta_1 \mathcal{G}, \delta_2 \mathcal{G}]_{\alpha\beta} \psi^\beta \right. \\
& \left. \frac{\zeta_{mn}^{\prime\prime\prime} \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}{(\bar{e}_1 \gamma_{(3)} e) (\bar{e}_3 \gamma^{(3)} e) = -18 \ell (\bar{e}_1 \gamma_a e_3) (\bar{e} \gamma^a e)} \left[(\delta_1 D_\alpha) \epsilon_2 + \frac{1}{8} \sigma^{-2} (\bar{\rho} D_\alpha \epsilon_1) \epsilon_2 + \frac{1}{8} \sigma^{-2} (\bar{\psi}_\alpha \not{D} \epsilon_1) \epsilon_2 - \frac{1}{4} \delta_2 \mathcal{C}_{aa} \gamma^a \not{D} \epsilon_1 - 1 \leftrightarrow 2 \right. \right. \\
& \left. \left. - \frac{1}{4} \mathcal{L}_V \mathcal{G}_{aa} \gamma^a \epsilon_2 \right] \frac{\zeta_{mn}^{\prime\prime\prime} \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}{(\bar{e}_1 \gamma_{(3)} e) (\bar{e}_3 \gamma^{(3)} e) = -18 \ell (\bar{e}_1 \gamma_a e_3) (\bar{e} \gamma^a e)} \right. \\
& \left. + \left[\frac{1}{32} \sigma^{-2} \delta_1 \mathcal{G}_{a\beta} (\bar{\psi}_\alpha \gamma^a \psi^\beta) \epsilon_2 + \frac{1}{16} \delta_1 \mathcal{G}_{aa} \delta_2 \mathcal{G}_{b\beta} \gamma^a \gamma^b \psi^\beta - 1 \leftrightarrow 2 \right] - \frac{1}{4} \delta_\zeta \mathcal{G}_{aa} \gamma^a \rho \right. \\
& \left. - \frac{1}{3072} \sigma^{-4} \epsilon_{[2} \bar{\epsilon}_1] (\bar{\rho} \gamma_{(3)} \rho) \gamma^{(3)} \psi_\alpha - \frac{1}{16} [\delta_1 \mathcal{L}, \delta_2 \mathcal{L}]_{ab} \gamma^{ab} \psi_\alpha - \frac{1}{4} [\delta_1 \mathcal{G}, \delta_2 \mathcal{G}]_{\alpha\beta} \psi^\beta \right. \\
& [\delta_1, \delta_2] \psi_\alpha = \text{Der} + \text{Alg}
\end{aligned}$$



$$\begin{aligned}
\text{Der} \stackrel{\circledast dh''}{\mathbb{M}_{(4)}'' \otimes \frac{\partial \mathbb{G}}{\partial \mathbb{M}} d^4 \chi} &= \frac{1}{2} \delta_1 \mathcal{L}_\alpha{}^a D_a \epsilon_2 - \frac{1}{4} D_a \delta_1 \mathcal{G}_{ab} \gamma^{ab} \epsilon_2 - \frac{1}{8} \sigma^{-2} D_\alpha (\bar{\rho} \epsilon_1) \epsilon_2 + \frac{1}{8} \sigma^{-2} (\bar{\rho} D_\alpha \epsilon_1) \epsilon_2 \\
&\quad + \frac{1}{8} \sigma^{-2} (\bar{\psi}_\alpha \not{D} \epsilon_1) \epsilon_2 - \frac{1}{4} \delta_2 \mathcal{G}_{a\alpha} \gamma^a \not{D} \epsilon_1 - 1 \leftrightarrow 2 - \frac{1}{4} \mathcal{L}_V \mathcal{G}_{a\alpha} \gamma^a \rho \\
\zeta &= -\frac{1}{2} \delta_1 \mathcal{L}_\alpha{}^a D_a \epsilon_2 - \frac{1}{4} \delta_2 \mathcal{L}_{a\alpha} \gamma^a \gamma^b D_b \epsilon_1 - \frac{1}{4} D_a \delta_1 \mathcal{L}_{b\alpha} \gamma^{ab} \epsilon_2 + \frac{1}{8} \sigma^{-2} (\bar{\psi}_\alpha \not{D} \epsilon_1) \epsilon_2 \\
&\quad - \frac{1}{8} \sigma^{-2} \epsilon_2 (\bar{\epsilon}_1 D_\alpha \rho) - 1 \leftrightarrow 2 - \frac{1}{2} D_\alpha V_a \gamma^a \rho \\
\delta &= -\frac{1}{2} \delta_1 \mathcal{L}_\alpha{}^a D_a \epsilon_2 + \frac{1}{4} \delta_1 \mathcal{L}_{a\alpha} \gamma^a \gamma^b D_b \epsilon_2 - \frac{1}{4} D_a \delta_1 \mathcal{G}_{b\alpha} \gamma^{ab} \epsilon_2 - \frac{1}{4} D_a \delta_1 \mathcal{L}_\alpha{}^a \epsilon_2 + \frac{1}{8} \sigma^{-2} (\bar{\epsilon}_1 \not{D} \psi_\alpha) \epsilon_2 \\
&\quad - \frac{1}{8} \sigma^{-2} \epsilon_2 (\bar{\epsilon}_1 D_\alpha \rho) - 1 \leftrightarrow 2 - \frac{1}{2} D_\alpha (V \rho) + \frac{1}{2} V D_\alpha \rho \\
\xi &= -\frac{1}{4} \delta_1 \mathcal{G}_{a\alpha} \gamma^b \gamma^a D_b \epsilon_2 - \frac{1}{4} D_a \delta_1 \mathcal{G}_{b\alpha} \gamma^a \gamma^b \epsilon_2 + \frac{1}{8} \sigma^{-2} \epsilon_2 \bar{\epsilon}_1 (\not{D} \psi_\alpha - D_\alpha \rho) - 1 \leftrightarrow 2 + D_\alpha \zeta + \frac{1}{2} V D_\alpha \rho \\
\varphi &= -\frac{1}{4} D_\alpha (\delta_1 \mathcal{G}_{b\alpha} \gamma^a \gamma^b \epsilon_2) - 1 \leftrightarrow 2 + \frac{1}{4} \sigma^{-2} \epsilon_{[2} \bar{\epsilon}_{1]} (\not{D} \psi_\alpha - D_\alpha \rho) + D_\alpha \zeta + \frac{1}{2} V D_\alpha \rho. \\
\delta_1 \mathcal{G}_{b\alpha} \gamma^a \gamma^b \epsilon_2 - 1 \leftrightarrow 2 &= -\sigma^{-2} \gamma^a \gamma^b \epsilon_{[2} (\bar{\psi}_\alpha \gamma_b \epsilon_{1]}) \underbrace{\zeta_{mn}'' \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}_{(\bar{\epsilon}_1 \gamma_{(3)} e)(\bar{\epsilon}_3 \gamma^{(3)} e) = -18\ell(\bar{\epsilon}_1 \gamma_a e_3)(\bar{e} \gamma^a e)} - \frac{1}{2} \sigma^{-2} \gamma^a \gamma_b \psi_\alpha (\bar{\epsilon}_2 \gamma^b \epsilon_1) = -2\gamma^a V \psi_\alpha, \\
\text{Der} &= \frac{1}{2} \not{D} V \psi_\alpha + \frac{1}{4} \sigma^{-2} \epsilon_{[2} \bar{\epsilon}_{1]} (\not{D} \psi_\alpha - D_\alpha \rho) + D_\alpha \zeta + \frac{1}{2} V D_\alpha \rho \\
&\quad \underbrace{\zeta_{mn}'' \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}_{(\bar{\epsilon}_1 \gamma_{(3)} e)(\bar{\epsilon}_3 \gamma^{(3)} e) = -18\ell(\bar{\epsilon}_1 \gamma_a e_3)(\bar{e} \gamma^a e)} \mathcal{L}_V \psi_\alpha + D_\alpha \zeta + \left(\frac{1}{4} \sigma^{-2} \epsilon_{[2} \bar{\epsilon}_{1]} - \frac{1}{2} V \right) (\not{D} \psi_\alpha - D_\alpha \rho) \\
&\quad \underbrace{\Lambda_{mn}'' \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}_{(\bar{\epsilon}_1 \gamma_{(3)} e)(\bar{\epsilon}_3 \gamma^{(3)} e) = -18\ell(\bar{\epsilon}_1 \gamma_a e_3)(\bar{e} \gamma^a e)} \frac{1}{8} \sigma^{-2} (\bar{\psi}_\alpha \rho) \zeta = -\frac{1}{16} \sigma^{-2} (\bar{\psi}_\alpha \rho) V \rho \stackrel{(\text{B.19})}{=} -\frac{1}{1536} \sigma^{-2} V (\bar{\rho} \gamma_{(3)} \rho) \gamma^{(3)} \psi_\alpha \\
&\quad \frac{1}{8} \sigma^{-2} (\bar{\psi}_\alpha \rho) \zeta + \frac{1}{1536} \sigma^{-2} V (\bar{\rho} \gamma_{(3)} \rho) \gamma^{(3)} \psi_\alpha = 0 \\
\text{Alg} &= \frac{1}{8} \sigma^{-2} (\bar{\psi}_\alpha \rho) \zeta - \frac{1}{4} \delta_\zeta \mathcal{G}_{a\alpha} \gamma^a \rho + \left(\frac{1}{4} \sigma^{-2} \epsilon_{[2} \bar{\epsilon}_{1]} - \frac{1}{2} V \right) \left[-\frac{1}{768} \sigma^{-2} (\bar{\rho} \gamma_{(3)} \rho) \gamma^{(3)} \psi_\alpha \right] \\
&\quad + \left[\frac{1}{16} \delta_1 \mathcal{G}_{a\alpha} \delta_2 \mathcal{G}_{b\beta} \gamma^a \gamma^b \psi^\beta + \frac{1}{32} \sigma^{-2} \delta_1 \mathcal{G}_{a\beta} (\bar{\psi}_\alpha \gamma^a \psi^\beta) \epsilon_2 - 1 \leftrightarrow 2 \right] \\
&\quad - \frac{1}{16} \left[\delta_1 \mathcal{G}_2, \delta_2 \mathcal{G}_{ab} \gamma^{ab} \psi_\alpha - \frac{1}{4} [\delta_1 \mathcal{L}, \delta_2 \mathcal{G}_{a\beta} \psi^\beta \right. \\
&\quad \left. \frac{1}{16} \delta_1 \mathcal{G}_{a\alpha} \delta_2 \mathcal{G}_{b\beta} \gamma^a \gamma^b \psi^\beta + \frac{1}{32} \sigma^{-2} \delta_1 \mathcal{G}_{a\beta} (\bar{\psi}_\alpha \gamma^a \psi^\beta) \epsilon_2 - 1 \leftrightarrow 2 - \frac{1}{16} [\delta_1 \mathcal{G}, \delta_2 \mathcal{G}_{ab} \gamma^{ab} \psi_\alpha \right. \\
&\quad \left. - \frac{1}{4} [\delta_1 \mathcal{G}, \delta_2 \mathcal{G}_{a\beta} \psi^\beta = \left(\frac{1}{4} \sigma^{-2} \epsilon_{[2} \bar{\epsilon}_{1]} - \frac{1}{2} V \right) \left[-\frac{1}{192} \sigma^{-2} (\bar{\psi}_\beta \gamma_{(3)} \psi^\beta) \gamma^{(3)} \psi_\alpha \right] \right]
\end{aligned}$$



$$\begin{aligned}
-(\bar{\epsilon}_1 \gamma_a \psi_\alpha)(\bar{\lambda} \gamma^b \gamma^a \psi^\beta) &= (\bar{\epsilon}_1 \gamma_a \psi_\alpha) \left(\overline{\gamma^b \lambda \gamma^a \psi^\beta} \right) \stackrel{\square}{\text{Iff}}_{(4)}^{\odot dh''} \stackrel{\partial \mathbb{G}}{\partial \mathfrak{M}} d^4 \chi \left(\bar{\psi}_\alpha \gamma_a \gamma^b \lambda \right) (\bar{\epsilon}_1 \gamma^a \psi^\beta) - (\bar{\epsilon}_1 \gamma_a \gamma^b \lambda) (\bar{\psi}_\alpha \gamma^a \gamma^b \psi^\beta) \\
&= \bar{\lambda} [2\sigma^2 \gamma^b \gamma^a \psi_\alpha \delta_1 \mathcal{G}_a{}^\beta - \gamma^b \gamma_a \epsilon_1 (\bar{\psi}_\alpha \gamma^a \psi^\beta)],
\end{aligned}$$

$$\begin{aligned}
\frac{1}{4} [\delta_1 \mathcal{G}, \delta_2 \mathcal{G}]_{\alpha\beta} &= \frac{1}{16} \sigma^{-4} (\bar{\epsilon}_1 \gamma^a \psi_\alpha) (\bar{\epsilon}_2 \gamma_a \psi_\beta) - 1 \leftrightarrow 2 \quad \frac{\Lambda_{mn}^{\prime\prime+} \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}{(\bar{\epsilon}_1 \gamma_{(3)} e)(\bar{\epsilon}_3 \gamma^{(3)} e) = -18\ell(\bar{\epsilon}_1 \gamma_a e_3)(\bar{e} \gamma^a e)} - \frac{1}{4} \sigma^{-2} V^a (\bar{\psi}_\alpha \gamma_a \psi_\beta)
\end{aligned}$$

$$\begin{aligned}
\text{LHS} \stackrel{\square}{\text{Iff}}_{(4)}^{\odot dh''} \stackrel{\partial \mathbb{G}}{\partial \mathfrak{M}} d^4 \chi &= \frac{1}{32} \delta_2 \mathcal{L}_{b\beta} \sigma^{-2} (\bar{\epsilon}_1 \gamma_a \psi_\alpha) \gamma^b \gamma^a \psi^\beta + \frac{1}{8} \delta_1 \mathcal{G}_{a\alpha} \delta_2 \mathcal{G}_\beta^a \psi^\beta - \frac{1}{32} \sigma^{-2} \delta_2 \mathcal{G}_b^\beta (\bar{\psi}_\alpha \gamma^b \psi_\beta) \epsilon_1 - 1 \leftrightarrow 2 \\
&\quad - \frac{1}{16} \left[\delta_1 \mathcal{G}, \delta_2 \mathcal{G}_{ab} \gamma^{ab} \psi_\alpha - \frac{1}{4} [\delta_1 \mathcal{G}, \delta_2 \mathcal{G}_{\alpha\beta} \psi^\beta] \right]
\end{aligned}$$

$$\begin{aligned}
&\frac{\Lambda_{mn}^{\prime\prime+} \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}{(\bar{\epsilon}_1 \gamma_{(3)} e)(\bar{\epsilon}_3 \gamma^{(3)} e) = -18\ell(\bar{\epsilon}_1 \gamma_a e_3)(\bar{e} \gamma^a e)} \frac{1}{32} \delta_2 \mathcal{C}_{b\beta} \sigma^{-2} [2\sigma^2 \gamma^{ba} \psi_\alpha \delta_1 \mathcal{C}_a{}^\beta - \gamma^b \gamma_a \epsilon_1 (\bar{\psi}_\alpha \gamma^a \psi^\beta)] + \frac{1}{8} \delta_1 \mathcal{C}_{a\alpha} \delta_2 \mathcal{C}^a{}_\beta \psi^\beta \\
&\quad - \frac{1}{32} \sigma^{-2} \delta_2 \mathcal{C}^\beta{}_b (\bar{\psi}_\alpha \gamma^b \psi_\beta) \epsilon_1 - 1 \leftrightarrow 2 - \frac{1}{16} [\delta_1 \mathcal{L}, \delta_2 \mathcal{G}]_{ab} \gamma^{ab} \psi_\alpha - \frac{1}{4} [\delta_1 \mathcal{C}, \delta_2 \mathcal{C}]_{\alpha\beta} \psi^\beta \\
&= \frac{1}{32} \sigma^{-2} \delta_2 \mathcal{G}_{b\beta} \gamma^b \gamma_a \epsilon_1 (\bar{\psi}_\alpha \gamma^a \psi^\beta) - \frac{1}{32} \sigma^{-2} \delta_2 \mathcal{G}_b^\beta (\bar{\psi}_\alpha \gamma^b \psi_\beta) \epsilon_1 - 1 \leftrightarrow 2 - \frac{1}{8} [\delta_1 \mathcal{G}, \delta_2 \mathcal{G}]_{\alpha\beta} \psi^\beta \\
&= -\frac{1}{32} \sigma^{-2} \delta_2 \mathcal{G}_{b\beta} \gamma_a \gamma^b \epsilon_1 (\bar{\psi}_\alpha \gamma^a \psi^\beta) + \frac{1}{32} \sigma^{-2} \delta_2 \mathcal{G}_{a\beta} (\bar{\psi}_\alpha \gamma^a \psi^\beta) \epsilon_1 - 1 \leftrightarrow 2 - \frac{1}{8} [\delta_1 \mathcal{G}, \delta_2 \mathcal{G}]_{\alpha\beta} \psi^\beta \\
&= \frac{1}{64} \sigma^{-4} \gamma^a \gamma^b \epsilon_1 (\bar{\psi}_\beta \gamma_b \epsilon_2) (\bar{\psi}_\alpha \gamma_a \psi^\beta) + \frac{1}{64} \sigma^{-4} (\bar{\epsilon}_2 \gamma_a \psi^\beta) (\bar{\psi}_\alpha \gamma^a \psi_\beta) \epsilon_1 - 1 \leftrightarrow 2 - \frac{1}{8} [\delta_1 \mathcal{L}, \delta_2 \mathcal{L}]_{\alpha\beta} \psi^\beta \\
&\quad \frac{\Lambda_{mn}^{\prime\prime+} \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}{(\bar{\epsilon}_1 \gamma_{(3)} e)(\bar{\epsilon}_3 \gamma^{(3)} e) = -18\ell(\bar{\epsilon}_1 \gamma_a e_3)(\bar{e} \gamma^a e)} \frac{1}{128} \sigma^{-4} \gamma^a \gamma^b \psi_\beta (\bar{\epsilon}_1 \gamma_b \epsilon_2) (\bar{\psi}_\alpha \gamma_a \psi^\beta) + \frac{1}{64} \sigma^{-4} (\bar{\epsilon}_2 \gamma_a \psi^\beta) (\bar{\psi}_\alpha \gamma^a \psi_\beta) \epsilon_1 - 1 \leftrightarrow 2 - \frac{1}{8} [\delta_1 \mathcal{L}, \delta_2 \mathcal{G}]_{\alpha\beta} \psi^\beta \\
&\quad \frac{\Lambda_{mn}^{\prime\prime+} \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}{(\bar{\epsilon}_1 \gamma_{(3)} e)(\bar{\epsilon}_3 \gamma^{(3)} e) = -18\ell(\bar{\epsilon}_1 \gamma_a e_3)(\bar{e} \gamma^a e)} \frac{1}{16} \sigma^{-2} \gamma^a V \psi_\beta (\bar{\psi}_\alpha \gamma_a \psi^\beta) - \frac{1}{768} \sigma^{-4} \epsilon_{[2} \bar{\epsilon}_{1]} (\bar{\psi}_\beta \gamma_{(3)} \psi^\beta) \gamma^{(3)} \psi_\alpha - \frac{1}{8} [\delta_1 \mathcal{L}_1, \delta_2 \mathcal{C}_1]_{\alpha\beta} \psi^\beta \\
&\quad \frac{\Lambda_{mn}^{\prime\prime+} \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}{(\bar{\epsilon}_1 \gamma_{(3)} e)(\bar{\epsilon}_3 \gamma^{(3)} e) = -18\ell(\bar{\epsilon}_1 \gamma_a e_3)(\bar{e} \gamma^a e)} \frac{1}{16} \sigma^{-2} V \gamma^a \psi_\beta (\bar{\psi}_\alpha \gamma_a \psi^\beta) - \frac{1}{8} \sigma^{-2} \psi_\beta (\bar{\psi}_\alpha V \psi^\beta) - \frac{1}{768} \sigma^{-4} \epsilon_{[2} \bar{\epsilon}_{1]} (\bar{\psi}_\beta \gamma_{(3)} \psi^\beta) \gamma^{(3)} \psi_\alpha \\
&\quad - \frac{1}{8} [\delta_1 \mathcal{C}, \delta_2 \mathcal{C}]_{\alpha\beta} \psi^\beta \\
&\quad \frac{\Lambda_{mn}^{\prime\prime+} \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}{(\bar{\epsilon}_1 \gamma_{(3)} e)(\bar{\epsilon}_3 \gamma^{(3)} e) = -18\ell(\bar{\epsilon}_1 \gamma_a e_3)(\bar{e} \gamma^a e)} \frac{1}{384} \sigma^{-2} V (\bar{\psi}_\beta \gamma_{(3)} \psi^\beta) \gamma^{(3)} \psi_\alpha - \frac{1}{8} \sigma^{-2} \psi_\beta (\bar{\psi}_\alpha V \psi^\beta) - \frac{1}{768} \sigma^{-4} \epsilon_{[2} \bar{\epsilon}_{1]} (\bar{\psi}_\beta \gamma_{(3)} \psi^\beta) \gamma^{(3)} \psi_\alpha \\
&\quad - \frac{1}{8} [\delta_1 \mathcal{L}, \delta_2 \mathcal{G}]_{\alpha\beta} \psi^\beta \\
&\quad \frac{\Lambda_{mn}^{\prime\prime+} \sum_{\substack{0 \leq l \leq m \\ 0 < j < n}} \int P(l, j)}{(\bar{\epsilon}_1 \gamma_{(3)} e)(\bar{\epsilon}_3 \gamma^{(3)} e) = -18\ell(\bar{\epsilon}_1 \gamma_a e_3)(\bar{e} \gamma^a e)} \left(\frac{1}{4} \sigma^{-2} \epsilon_{[2} \bar{\epsilon}_{1]} - \frac{1}{2} V \right) \left[-\frac{1}{192} \sigma^{-2} (\bar{\psi}_\beta \gamma_{(3)} \psi^\beta) \gamma^{(3)} \psi_\alpha \right],
\end{aligned}$$

$$T^*E \cong \pi^*(T^*M \oplus E^*)$$

$$m \in M, e \in E_m, p \in T_m^*M, e^* \in E_m^*.$$



$$\begin{aligned}
T_{(m,e,p,e^*)}T^*E &\cong T_{(m,e,p,e^*)}\pi^*(T^*M \oplus E^*) \\
&\cong T_{(m,e)}E \oplus \pi^*(T^*M \oplus E^*)_{(m,e)} \\
&\cong (T_mM \oplus E_m) \oplus (T^*M \oplus E^*)_m \\
&\cong T_mM \oplus E_m \oplus T_m^*M \oplus E_m^*
\end{aligned}$$

$$\pi = \varepsilon^i \wedge \varepsilon_i + \varepsilon^a \wedge \varepsilon_a - \frac{1}{2} \bar{T}^k{}_{ij}(m) p_k \varepsilon^i \wedge \varepsilon^j - \frac{1}{2} F_{ij}{}^a{}_b(m) e^b e_a^* \varepsilon^i \wedge \varepsilon^j,$$

$$\pi^*(T^*M \oplus E^*) \ni (m, e, p, e^*) \mapsto (m, e, \pi^*p + ie^*) \in T^*E,$$

$$\alpha(\dot{\gamma}) = \langle \gamma, \pi_*^X \dot{\gamma} \rangle$$

$$\alpha(\dot{\gamma}) = \langle (m, e)^{\cdot}; \pi^*p + ie^* \rangle = p(\dot{m}) + e^*(\text{ver}(m, e)^{\cdot})$$

$$\dot{\gamma} = \dot{\gamma}^i \partial_{x^i} + \dot{\gamma}^a \partial_{y^a} + \dot{\gamma}_i \partial_{x_i^*} + \dot{\gamma}_a \partial_{y_a^*}.$$

$$\text{ver} \partial_{x^i} = A_i^a{}_b(x) y^b \partial_{y^a}, \text{ver} \partial_{y^a} = \partial_{y^a}$$

$$\alpha(\dot{\gamma}) = x_i^* \dot{\gamma}^i + y_a^* (A_i^a{}_b(x) y^b \dot{\gamma}^i + \dot{\gamma}^a),$$

$$\begin{aligned}
\alpha &= (x_i^* + y_a^* A_i^a{}_b(x) y^b) dx^i + y_a^* dy^a \\
&= x_i^* dx^i + y_a^* (dy^a + A^a{}_b(x) y^b)
\end{aligned}$$

$$\begin{aligned}
\omega &= d\alpha = dx_i^* \wedge dx^i + dy_a^* \wedge dy^a + d(y_a^* A_i^a{}_b(x) y^b) \wedge dx^i \\
&= dx_i^* \wedge dx^i + (dy_a^* - A^b{}_a(x) y_b^*) \wedge (dy^a + A^a{}_c(x) y^c) + y_a^* F^a{}_b y^b.
\end{aligned}$$

$$\pi = \partial_{x_i^*} \wedge \partial_{x^i} + \partial_{y_a^*} \wedge \partial_{y^a} + y_b^* A_i^b{}_a \partial_{x_i^*} \wedge \partial_{y_a^*} - A_i^a{}_b y^b \partial_{x_i^*} \wedge \partial_{y^a} - \frac{1}{2} y_a^* y^b F_{ij}{}^a{}_b \partial_{x_i^*} \wedge \partial_{x_j^*}$$

$$T_mM \oplus E_m \oplus T_m^*M \oplus E_m^* \rightarrow T_{(m,e,p,e^*)}\pi^*(T^*M \oplus E^*)$$

$$T_mM \ni v \mapsto \lambda_{(m,e)}^{\pi^*(T^*M \oplus E^*) \rightarrow E} \lambda_e^{E \rightarrow M} v$$

$$E_m \ni \tilde{e} \mapsto \lambda_{(m,e)}^{\pi^*(T^*M \oplus E^*) \rightarrow E} \left. \frac{d}{dt} \right|_{t=0} (e + t\tilde{e})$$

$$T_m^*M \ni \beta \mapsto \left. \frac{d}{dt} \right|_{t=0} (m, e, p + t\beta, e^*)$$

$$E_m^* \ni \tilde{e}^* \mapsto \left. \frac{d}{dt} \right|_{t=0} (m, e, p, e^* + t\tilde{e}^*)$$

$$\lambda_e^{E \rightarrow M}: \partial_{x^i} \mapsto \partial_{x^i} - A_i^a{}_b(x) y^b \partial_{y^a}$$

$$\lambda_{(p,e^*)}^{T^*M \oplus E^* \rightarrow M}: \partial_{x^i} \mapsto \partial_{x^i} + A_i^a{}_b y_a^* \partial_{y_b^*} + \bar{A}_i{}^j{}_k x_j^* \partial_{x_k^*}, \partial_{y^a} \mapsto \partial_{y^a}$$

$$\lambda_{(p,e^*)}^{\pi^*(T^*M \oplus E^*) \rightarrow E}: \partial_{x^i} \mapsto \partial_{x^i} + A_i^a{}_b y_a^* \partial_{y_b^*} + \bar{A}_i{}^j{}_k x_j^* \partial_{x_k^*}, \partial_{y^a} \mapsto \partial_{y^a}$$

$$T_mM \ni \partial_{x^i} \mapsto \partial_{x^i} + A_i^a{}_b y_a^* \partial_{y_b^*} + \bar{A}_i{}^j{}_k x_j^* \partial_{x_k^*} - A_i^a{}_b(x) y^b \partial_{y^a}$$

$$E_m \ni \varepsilon_a \mapsto \partial_{y^a}$$

$$T_m^*M \ni dx^i \mapsto \partial_{x_i^*}$$

$$E_m^* \ni \varepsilon^a \mapsto \partial_{y_a^*}$$



$$\pi = \varepsilon^i \wedge \varepsilon_i + \varepsilon^a \wedge \varepsilon_a - \bar{A}_i{}^j{}_k x_j^* \varepsilon^i \wedge \varepsilon^k - \frac{1}{2} y_a^* y^b F_{ij}{}^a{}_b \varepsilon^i \wedge \varepsilon^j$$

$$\bar{A}_{[i}{}^j{}_k] = \frac{1}{2} \bar{T}^j{}_{ik},$$

$$\delta(\mathcal{L}_V) = \frac{1}{16} [\delta \mathcal{L}, \mathcal{L}_V \mathcal{G}]_{ab} \gamma^{ab} = F(\delta \mathcal{L}, \mathcal{L}_V \mathcal{G}) \text{ on } \Gamma(S(C_+) \otimes H)$$

$$\Gamma(S(C_+)) \cong C^\infty(M) \otimes S_0$$

$$\delta e_a = \frac{1}{2} \delta \mathcal{G}_a^\alpha e_\alpha, \delta e_\alpha = -\frac{1}{2} \delta \mathcal{G}_\alpha^a e_a$$

$$\delta(\Gamma_{abc}) = -\frac{1}{2} \delta \mathcal{G}_\alpha{}^a \Gamma_{abc} - \partial_{[b} \delta \mathcal{G}_{c]\alpha} - \Gamma_{[bc]}{}^a \mathcal{G}_{a\alpha} - \Gamma_{[b\alpha}{}^\beta \mathcal{G}_{c]\beta},$$

$$\delta(\Gamma_{[abc]}) = \frac{1}{2} \delta \mathcal{G}_{[a\mid}{}^\alpha \Gamma_{\alpha\mid bc]}.$$

$$\begin{aligned} \delta(\mathcal{L}_V) &= \delta \left(D_V + \frac{1}{2} (D_a V_b) \gamma^{ab} \right) \\ &= \delta \left(V^A a(e_A) + \frac{1}{4} V^A \Gamma_{Abc} \gamma^{bc} + \frac{1}{2} (a(e_a) V_b + \Gamma_{ab}{}^c V_c) \gamma^{ab} \right) \\ &= \delta \left(V^A a(e_A) + \frac{1}{2} (a(e_a) V_b) \gamma^{ab} + \frac{1}{4} V^a \Gamma_{abc} \gamma^{bc} + \frac{3}{4} V^a \Gamma_{[abc]} \gamma^{bc} \right) \\ &= \frac{1}{2} (a(\delta e_a) V_b + a(e_a) \delta V_b) \gamma^{ab} + \frac{1}{4} (\delta V^a \Gamma_{abc} + V^a \delta \Gamma_{abc}) \gamma^{bc} + \frac{3}{4} (\delta V^a \Gamma_{[abc]} + V^a \delta \Gamma_{[abc]}) \gamma^{bc} \\ &= \frac{1}{4} (\delta \mathcal{G}_a{}^\alpha \partial_\alpha V_b + \partial_a (\delta \mathcal{G}_b{}^\alpha V_\alpha)) \gamma^{ab} - \frac{1}{8} \delta \mathcal{G}_\alpha{}^a V^a \Gamma_{abc} \gamma^{bc} \\ &\quad + \frac{1}{4} V^a \left(-\frac{1}{2} \delta \mathcal{G}_\alpha{}^a \Gamma_{abc} - \partial_b \delta \mathcal{G}_{c\alpha} - \Gamma_{bc}{}^a \delta \mathcal{G}_{a\alpha} - \Gamma_{b\alpha}{}^\beta \delta \mathcal{G}_{c\beta} \right) \gamma^{bc} \\ &\quad + \frac{3}{8} (\delta \mathcal{G}_\alpha{}^a V^a \Gamma_{[abc]} + V^a \delta \mathcal{G}_{[a\mid}{}^\alpha \Gamma_{\alpha\mid bc]}) \gamma^{bc}. \end{aligned}$$

$$\begin{aligned} \delta(\mathcal{L}_V) &= \frac{1}{4} (\delta \mathcal{G}_a{}^\alpha \partial_\alpha V_b + \delta \mathcal{G}_b{}^\alpha \partial_a V_\alpha) \gamma^{ab} \\ &\quad + \frac{1}{4} \left(-\frac{1}{2} \delta \mathcal{G}_\alpha{}^a V^a \Gamma_{abc} - \frac{1}{2} V^a \delta \mathcal{G}_\alpha{}^a \Gamma_{abc} - V^a \Gamma_{bc}{}^a \delta \mathcal{G}_{a\alpha} - V^a \Gamma_{b\alpha}{}^\beta \delta \mathcal{G}_{c\beta} \right. \\ &\quad \left. + \frac{1}{2} \delta \mathcal{G}_\alpha{}^a V^a \Gamma_{abc} + \delta \mathcal{G}_\alpha{}^a V^a \Gamma_{bca} + \frac{1}{2} V^a \delta \mathcal{G}_\alpha{}^a \Gamma_{abc} + V^a \delta \mathcal{G}_b{}^\alpha \Gamma_{aca} \right) \gamma^{bc} \\ &= \frac{1}{4} (\delta \mathcal{G}_b{}^\alpha \partial_\alpha V_c - \delta \mathcal{G}_b{}^\alpha \partial_c V_\alpha - V^a \Gamma_{b\alpha}{}^\beta \delta \mathcal{G}_{c\beta} + V^a \delta \mathcal{G}_b{}^\alpha \Gamma_{aca}) \gamma^{bc} \\ &= \frac{1}{4} \delta \mathcal{G}_b{}^\alpha (D_\alpha V_c - D_c V_\alpha) \gamma^{bc} = \frac{1}{8} \delta \mathcal{G}_b{}^\alpha (\mathcal{L}_V \mathcal{G}_l)_{c\alpha} \gamma^{bc} = \frac{1}{16} [\delta \mathcal{G}, \mathcal{L}_V \mathcal{G}_{ab}] \gamma^{ab} \end{aligned}$$

$$\mathrm{d}s^2 = H_1^{\alpha_1} H_2^{\alpha_2} \, \mathrm{d}x^2 + H_1^{\alpha_1} H_2^{\beta_2} \, \mathrm{d}y_1^2 + H_1^{\beta_1} H_2^{\alpha_2} \, \mathrm{d}y_2^2 + H_1^{\beta_1} H_2^{\beta_2} \, \mathrm{d}z^2$$

$$\mathrm{d}\hat{s}_{10}^2 = H(y)^{\frac{p-7}{8}} \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + H(y)^{\frac{p+7}{8}} \delta_{IJ} \, \mathrm{d}y^I \, \mathrm{d}y^J, e^{\hat{\Phi}} = H(y)^{\frac{3-p}{4}}$$

$$\hat{F}_{(p+2)}^{\rm ele} = \partial_I H^{-1} \, \mathrm{d}x^0 \wedge \cdots \wedge \, \mathrm{d}x^p \wedge \, \mathrm{d}y^I, \hat{F}_{(8-p)}^{\rm mag} = \frac{1}{(8-p)!} \, \varepsilon_{I_1 \cdots I_{9-p}} \partial_{I_1} H \, \mathrm{d}y^{I_2} \wedge \cdots \wedge \, \mathrm{d}y^{I_{9-p}}$$



$$\partial_I^2 H(y) = 0$$

$$H(y) = 1 + \frac{\alpha N}{r^{7-p}}$$

$$\begin{aligned} d\hat{s}_{10}^2 &= -H_1^{-\frac{3}{4}}H_2^{-\frac{1}{4}} dt^2 + H_1^{-\frac{3}{4}}H_2^{\frac{3}{4}} dx_1^2 + H_1^{\frac{1}{4}}H_2^{-\frac{1}{4}}(dx_2^2 + \dots + dx_6^2) + H_1^{\frac{1}{4}}H_2^{\frac{3}{4}}(dy_1^2 + dy_2^2 + dy_3^2) \\ \hat{H}_{(3)} &= H_1^{-2}\partial_I H_1 dt \wedge dx_1 \wedge dy^I, \tilde{F}_{(3)} = \frac{1}{2}\varepsilon_{IJK}\partial_I H_2 dx_1 \wedge dy^J \wedge dy^K, e^\Phi = H_1^{-\frac{1}{2}}H_2^{-\frac{1}{2}} \end{aligned}$$

$$\partial_I^2 H_1(y) = \partial_I^2 H_2(y) = 0$$

$$\Gamma_{\underline{012345}}\varepsilon = c_1\varepsilon, \Gamma_{\underline{016789}}\varepsilon = c_2\varepsilon$$

$$\Gamma_{\underline{01,10}}\varepsilon = c_1c_2\varepsilon$$

$$\begin{aligned} d\hat{s}_{10}^2 &= H_0^{-\frac{1}{2}}[-h^{-1} dt^2 + h(dx_1^2 + dx_2^2 + dx_3^2)] + H_0^{\frac{1}{2}}(dy_1^2 + \dots + dy_6^2), \\ \tilde{F}_{(5)} &= hH_0^{-2}\partial_I H_0 dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dy^I - \frac{1}{5!}\varepsilon_{I_1\dots I_6}\partial_{I_1}H_0 dy^{I_2}\dots \wedge dy^{I_6}, \\ \hat{H}_{(3)} &= \partial_a h^{-1} dt \wedge dx^a \wedge dy^1, \tilde{F}_{(3)} = -\frac{1}{2!}\varepsilon_{abc}\partial_a h dx^b \wedge dx^c \wedge dy^1, \\ e^\Phi &= h^{-1}, h = h(x_1, x_2, x_3), H_0 = H_0(y_1, \dots, y_6). \end{aligned}$$

$$\partial_I^2 H_0(y) = 0, \partial_a^2 h(x) = 0$$

$$\tilde{F}_{(5)} = d\hat{C}_{(4)} - \frac{1}{2}\hat{C}_{(2)} \wedge \hat{H}_{(3)} + \frac{1}{2}\hat{B}_{(2)} \wedge d\hat{C}_{(2)}$$

$$\begin{aligned} d\hat{s}_{10}^2 &= -H_0^{-\frac{1}{2}}H_1^{-\frac{3}{4}}H_2^{-\frac{1}{4}} dt^2 + H_0^{-\frac{1}{2}}H_1^{\frac{1}{4}}H_2^{\frac{3}{4}}(dx_1^2 + dx_2^2 + dx_3^2) \\ &\quad + H_0^{\frac{1}{2}}H_1^{-\frac{3}{4}}H_2^{\frac{3}{4}} dy_1^2 + H_0^{\frac{1}{2}}H_1^{\frac{1}{4}}H_2^{-\frac{1}{4}}(dy_2^2 + \dots + dy_6^2), \end{aligned}$$

$$d\tilde{F}_{(4)} = \hat{F}_{(2)} \wedge \hat{H}_{(3)}.$$

$$\begin{aligned} d\hat{s}_{10}^2 &= H_0^{-\frac{1}{4}}\left[-h^{-\frac{5}{4}}dt^2 + h^{\frac{3}{4}}(dx_1^2 + \dots + dx_5^2)\right] + H_0^{\frac{3}{4}}h^{-\frac{1}{4}}(dy_1^2 + \dots + dy_4^2) \\ \tilde{F}_{(5)} &= H_0 h^{-2}\partial_a h dt \wedge dy_2 \wedge dy_3 \wedge dy_4 \wedge dx^a - \frac{1}{4!}\varepsilon_{abcde}\partial_a h dy^1 \wedge dx^b \wedge dx^c \wedge dx^d \wedge dx^e \\ \hat{H}_{(3)} &= h^{-2}\partial_a h dt \wedge dy^1 \wedge dx^a, \tilde{F}_{(3)} = \frac{1}{3!}\varepsilon_{IJKL}\partial_I H_0 dy^J \wedge dy^K \wedge dy^L \\ e^\Phi &= h^{-\frac{1}{2}}H_0^{-\frac{1}{2}}, h = h(x_1, \dots, x_5), H_0 = H_0(y_1, \dots, y_4). \end{aligned}$$

$$\begin{aligned} d\hat{s}_{11}^2 &= H_0^{-\frac{1}{3}}[h^{-1}(-dt^2 + dx^2) + h(dr^2 + r^2 d\Omega_3^2)] + H_0^{\frac{2}{3}}(dy_1^2 + \dots + dy_5^2) \\ \hat{F}_{(4)} &= -\frac{1}{4!}\varepsilon_{I_1\dots I_5}\partial_{I_1}H_0 dy^{I_2} \wedge \dots \wedge dy^{I_5} - h^{-2}\partial_r h dt \wedge dt \wedge dr \wedge dy^1 - r^3\partial_r h d\Omega_3 \wedge dy^1 \\ h &= h(r), H_0 = H_0(y_1, \dots, y_5) \end{aligned}$$



$$d\hat{s}_{10}^2 = H_0^{-\frac{3}{8}} \left[-h^{-\frac{9}{8}} dt^2 + h^{\frac{7}{8}} (dr^2 + r^2 d\Omega_3^2) \right] + H_0^{\frac{5}{8}} h^{-\frac{1}{8}} (dy_1^2 + \dots + dy_5^2),$$

$$\tilde{F}_{(4)} = r^3 \partial_r h dy_1 \wedge d\Omega_3 - \frac{1}{4!} \varepsilon_{I_1 \dots I_5} \partial_{I_1} H_0 dy^{I_2} \wedge \dots \wedge dy^{I_5},$$

$$\hat{H}_{(3)} = h^{-2} \partial_r h dt \wedge dy_1 \wedge dr,$$

$$e^{\hat{\Phi}} = h^{-\frac{3}{4}} H_0^{-\frac{1}{4}}, h = h(r), H_0 = H_0(y_1, \dots, y_5),$$

$$d\hat{s}_{10}^2 = H_0^{-\frac{1}{4}} \left[-h^{-\frac{5}{4}} dt^2 + h^{\frac{3}{4}} (dr^2 + r^2 d\Omega_4^2) \right] + H_0^{\frac{3}{4}} h^{-\frac{1}{4}} (dy_1^2 + \dots + dy_4^2)$$

$$\tilde{F}_{(5)} = H_0 h^{-2} \partial_r h dt \wedge dy_2 \wedge dy_3 \wedge dy_4 \wedge dr - r^4 \partial_r h dy_1 \wedge d\Omega_4$$

$$\hat{H}_{(3)} = -\frac{1}{3!} \varepsilon_{IJKL} \partial_I H_0 dy^J \wedge dy^K \wedge dy^L, \tilde{F}_{(3)} = h^{-2} \partial_r h dt \wedge dy_1 \wedge dr$$

$$e^{\hat{\Phi}} = h^{\frac{1}{2}} H_0^{\frac{1}{2}}, h = h(r), H_0 = H_0(y_1, \dots, y_4)$$

$$d\hat{s}^2 = H(y)^{\frac{p-7}{8}} g_{\mu\nu} dx^\mu dx^\nu + H(y)^{\frac{p+7}{8}} \delta_{IJ} dy^I dy^J, \mathcal{R}_{\mu\nu}(g) = 0$$

$$ds_{p+1}^2 = h(r)^\alpha (-dt^2 + dx_1^2 + \dots + dx_s^2) + h(r)^\beta (dr^2 + r^2 d\Omega_{p-s-1}^2),$$

$$d\hat{s}_{10}^2 = H_0(y)^{-\frac{1}{2}} g_{\mu\nu} dx^\mu dx^\nu + H_0(y)^{\frac{1}{2}} \delta_{IJ} dy^I dy^J, \hat{\Phi} = \phi, \hat{C}_0 = -\chi$$

$$\tilde{F}_{(5)} = -\text{vol}_g \wedge dH_0^{-1} - *_{\delta} dH_0, \tilde{F}_{(3)} = -\frac{e^{-\phi}}{\sqrt{2}} *_{\mathcal{G}} \mathcal{F}_{(2)}^I \wedge dy^I, \hat{H}_{(3)} = \frac{1}{\sqrt{2}} \mathcal{F}_{(2)}^I \wedge dy^I.$$

$$ds_4^2 = -h^{-1} dt^2 + h \delta_{ab} dX^a dX^b, \mathcal{A}_{(1)}^1 = -\sqrt{2} h^{-1} dt, \phi = -\ln h,$$

$$\partial_X^2 h(X) = 0$$

$$\mathcal{F}_{(2)}^1 = \sqrt{2} dt \wedge dh^{-1}, * \mathcal{F}_{(2)}^1 = -\frac{\sqrt{2}}{h} *_X dh,$$

$$d\hat{s}_{10}^2 = H_0^{-\frac{1}{2}}(y) [-h^{-1}(X) dt^2 + h(X) \delta_{ab} dX^a dX^b] + H_0^{\frac{1}{2}}(y) \delta_{IJ} dy^I dy^J, \hat{\Phi} = -\ln h,$$

$$\tilde{F}_{(5)} = -\text{vol}_g \wedge dH_0^{-1} - *_{\delta} dH_0, \tilde{F}_{(3)} = -*_X (dh) \wedge dy^1, \hat{H}_{(3)} = dt \wedge dh^{-1} \wedge dy^1.$$

$$d\hat{s}_{10}^2 = H_0^{-\frac{1}{2}}(y) g_{\mu\nu}(x) dx^\mu dx^\nu + H_0^{\frac{1}{2}}(y) \delta_{IJ} dy^I dy^J, \hat{\Phi} = \phi(x), \hat{C}_{(0)} = \alpha \chi(x)$$

$$\tilde{F}_{(5)} = H_0^{-2} \text{vol}_g \wedge dH_0 - *_{\delta} dH_0, \tilde{F}_{(3)} = -\frac{e^{-\phi}}{\sqrt{2}} *_{\mathcal{G}} \mathcal{F}_{(2)}^I \wedge dy^I, \hat{H}_{(3)} = \frac{1}{\sqrt{2}} \mathcal{F}_{(2)}^I \wedge dy^I.$$

$$d\hat{s}_{10} = H_0^{-\frac{1}{2}} [-dt^2 + dx_0^2 + h(dx_1^2 + dx_2^2)] + H_0^{\frac{1}{2}} (dy_1^2 + \dots + dy_6^2)$$

$$\tilde{F}_{(5)} = h H_0^{-2} \partial_I H_0 dt \wedge dx_0 \wedge dx_1 \wedge dx_2 \wedge dy^I - \frac{1}{5!} \varepsilon_{I_1 \dots I_6} \partial_{I_1} H_0 dy^{I_2} \wedge \dots \wedge dy^{I_6}$$

$$e^{\hat{\Phi}} = h^{-1}, \hat{F}_{(1)} = -\varepsilon_{ab} \partial^a h dx^b, h = h(x_1, x_2) H_0 = H_0(y), a, b = 1, 2$$

$$ds^2 = -dt^2 + dx_0^2 + h(dx_1^2 + dx_2^2), e^{\Phi} = h^{-1},$$

$$F_{(1)} = -\varepsilon_{ab} \partial^a h dx^b, a, b = 1, 2.$$



$$d\hat{s}_{11}^2 = H_0^{-\frac{1}{3}}(y)g_{\mu\nu}dx^\mu dx^\nu + H_0^{\frac{2}{3}}(y)\delta_{IJ}dy^I dy^J, \hat{F}_{(4)} = \delta_{IJ}G_{(3)}^I \wedge dy^J - *_\delta dH_0, G_{(3)}^I = - *_g G_{(3)}^I.$$

$$ds_6^2 = h^{-1}(-dt^2 + dx_1^2) + h\delta_{ab} dX^a dX^b, G_{(3)}^1 = -\frac{1}{h^2} dt \wedge dx^1 \wedge dh - *_X dh,$$

$$d\hat{s}_{11}^2 = H_0^{-\frac{1}{3}}(y)[h^{-1}(X)(-dt^2 + dx_1^2) + h(X)\delta_{ab} dX^a dX^b] + H_0^{\frac{2}{3}}(y)\delta_{IJ}dy^I dy^J$$

$$\tilde{F}_{(4)} = -\left[\frac{1}{h^2} dt \wedge dx^1 \wedge dh + *_X dh\right] \wedge dy^1 - *_\delta dH_0$$

$$d\hat{s}_{10}^2 = H_0^{-\frac{3}{8}}(y)e^{\frac{5}{8\sqrt{6}}\sigma}g_{\mu\nu}(x)dx^\mu dx^\nu + H_0^{\frac{5}{8}}(y)e^{-\frac{3}{8\sqrt{6}}\sigma}\delta_{IJ}dy^I dy^J, e^\Phi = H_0^{-\frac{1}{4}}e^{-\frac{9}{4\sqrt{6}}\sigma}$$

$$\tilde{F}_{(4)} = - *_\delta dH_0 - \frac{1}{\sqrt{2}}e^{\frac{2}{\sqrt{6}}\sigma} *_g F_{(2)}^1 \wedge dy^1, \hat{H}_{(3)} = \frac{1}{\sqrt{2}}F_{(2)}^1 \wedge dy^1$$

$$\hat{F}_{(2)} = \alpha G_{(2)}.$$

$$d\hat{s}_{10}^2 = H_0^{-\frac{3}{8}}(y)e^{\frac{5}{8\sqrt{6}}\sigma}g_{\mu\nu}(x)dx^\mu dx^\nu + H_0^{\frac{5}{8}}(y)e^{-\frac{3}{8\sqrt{6}}\sigma}\delta_{IJ}dy^I dy^J, e^\Phi = H_0^{-\frac{1}{4}}e^{-\frac{9}{4\sqrt{6}}\sigma}$$

$$\tilde{F}_{(4)} = - *_\delta dH_0 - \frac{1}{\sqrt{2}}e^{\frac{2}{\sqrt{6}}\sigma} *_g F_{(2)}^I \wedge dy^I, \hat{H}_{(3)} = \frac{1}{\sqrt{2}}F_{(2)}^I \wedge dy^I, \hat{F}_{(2)} = -G_{(2)}.$$

$$ds^2 = -H_0^{-\frac{3}{8}}\left[h^{-\frac{7}{8}}dt^2 + h^{\frac{1}{8}}(dr^2 + r^2 d\Omega_3^2)\right] + H_0^{\frac{5}{8}}h^{\frac{1}{8}}(dy_1^2 + \dots + dy_5^2),$$

$$\tilde{F}_{(4)} = - *_\delta dH_0, \hat{F}_{(2)} = -h^{-2}\partial_r h dt \wedge dr, e^\Phi = H_0^{-\frac{1}{4}}h^{\frac{3}{4}},$$

$$H_0 = H_0(y), h = h(r),$$

$$d\hat{s}_{10}^2 = H_0^{-\frac{1}{4}}(y)e^{-\frac{\sqrt{2}}{4}\varphi}g_{\mu\nu}(x)dx^\mu dx^\nu + H_0^{\frac{3}{4}}(y)e^{\frac{\sqrt{2}}{4}\varphi}\delta_{IJ}dy^I dy^J$$

$$\hat{H}_{(3)} = - *_\delta dH_0, \tilde{F}_{(3)} = -\frac{1}{\sqrt{2}}F_{(2)}^1(x) \wedge dy^1, e^\Phi = H_0^{\frac{1}{2}}e^{-\frac{\varphi}{\sqrt{2}}}$$

$$\tilde{F}_{(5)} = \frac{1}{\sqrt{2}}\left[-\frac{1}{3!}H_0\varepsilon_{1I_2\dots I_4}F_{(2)}^1 \wedge dy^{I_2} \wedge dy^{I_3} \wedge dy^{I_4} + e^{-\frac{\sqrt{2}}{2}\varphi} *_g F_{(2)}^1 \wedge dy^1\right].$$

$$\hat{H}_{(3)} = \alpha H_{(3)}(y).$$

$$d\hat{s}_{10}^2 = H_0^{-\frac{1}{4}}(y)e^{-\frac{\sqrt{2}}{4}\varphi}g_{\mu\nu}(x)dx^\mu dx^\nu + H_0^{\frac{3}{4}}(y)e^{\frac{\sqrt{2}}{4}\varphi}\delta_{IJ}dy^I dy^J$$

$$\hat{H}_{(3)} = - *_\delta dH_0 - H_{(3)}(x), \tilde{F}_{(3)} = -\frac{1}{\sqrt{2}}F_{(2)}^I(x) \wedge dy^I, e^\Phi = H_0^{\frac{1}{2}}e^{-\frac{\varphi}{\sqrt{2}}}$$

$$\tilde{F}_{(5)} = \frac{1}{\sqrt{2}}\left[-\frac{1}{3!}H_0\varepsilon_{I_1\dots I_4}F_{(2)}^{I_1} \wedge dy^{I_2} \wedge dy^{I_3} \wedge dy^{I_4} + e^{-\frac{\sqrt{2}}{2}\varphi} *_g F_{(2)}^I \wedge dy^I\right]$$

$$d\hat{s}_{10}^2 = H_0^{-\frac{1}{4}}h^{-\frac{1}{4}}\left[h^{-\frac{1}{2}}(-dt^2 + dx^2) + h^{\frac{1}{2}}(dr^2 + r^2 d\Omega_3^2)\right] + H_0^{\frac{3}{4}}h^{\frac{1}{4}}\delta_{IJ}dy^I dy^J,$$

$$\hat{H}_{(3)} = - *_\delta dH_0 - h^{-2}\partial_r h dt \wedge dx \wedge dr, e^\Phi = H_0^{\frac{1}{2}}h^{-\frac{1}{2}}(r),$$

$$H_0 = H_0(y), h = h(r), I, J = 1, 2, 3, 4,$$



$$\begin{aligned} d\hat{s}_{10}^2 &= H_0^{-\frac{1}{4}}(y)e^{-\frac{\sqrt{2}}{4}\varphi}g_{\mu\nu}(x)dx^\mu dx^\nu + H_0^{\frac{3}{4}}(y)e^{\frac{\sqrt{2}}{4}\varphi}\delta_{IJ}dy^I dy^J \\ \hat{H}_{(3)} &= \frac{1}{\sqrt{2}}F_{(2)}^I(x)\wedge dy^I, \tilde{F}_{(3)} = -*_\delta dH_0 - H_{(3)}(x), e^{\hat{\Phi}} = H_0^{-\frac{1}{2}}e^{\frac{\varphi}{\sqrt{2}}} \\ \tilde{F}_{(5)} &= \frac{1}{\sqrt{2}}\left[-\frac{1}{3!}H_0\varepsilon_{I_1\dots I_4}F_{(2)}^{I_1}\wedge dy^{I_2}\wedge dy^{I_3}\wedge dy^{I_4} + e^{-\frac{\sqrt{2}}{2}\varphi}*_g F_{(2)}^I\wedge dy^I\right]. \end{aligned}$$

$$\begin{aligned} S_{IIA} &= \frac{1}{2\kappa^2}\int\left(*\hat{R}-\frac{1}{2}d\hat{\Phi}\wedge*d\hat{\Phi}-\frac{1}{2}e^{-\hat{\Phi}}\hat{H}_{(3)}\wedge*\hat{H}_{(3)}-\frac{1}{2}e^{\frac{3}{2}\hat{\Phi}}\hat{F}_{(2)}\wedge*\hat{F}_{(2)}\right. \\ &\quad \left.-\frac{1}{2}e^{\frac{\hat{\Phi}}{2}}\tilde{F}_{(4)}\wedge*\tilde{F}_{(4)}\right)-\frac{1}{4\kappa^2}\int\hat{F}_{(4)}\wedge\hat{F}_{(4)}\wedge\hat{B}_{(2)} \end{aligned}$$

$$\begin{aligned} d* d\hat{\Phi} &= \frac{3}{4}e^{\frac{3}{2}\hat{\Phi}}\hat{F}_{(2)}\wedge*\hat{F}_{(2)}-\frac{1}{2}e^{-\hat{\Phi}}\hat{H}_{(3)}\wedge*\hat{H}_{(3)}+\frac{1}{4}e^{\frac{\hat{\Phi}}{2}}\tilde{F}_{(4)}\wedge*\tilde{F}_{(4)} \\ d\left(e^{\frac{\hat{\Phi}}{2}}*\tilde{F}_{(4)}\right) &= -\hat{H}_{(3)}\wedge\tilde{F}_{(4)} \\ d(e^{-\hat{\Phi}}*\hat{H}_{(3)}) &= e^{\frac{\hat{\Phi}}{2}}\hat{F}_{(2)}\wedge*\tilde{F}_{(4)}+\frac{1}{2}\tilde{F}_{(4)}\wedge\tilde{F}_{(4)} \\ d\left(e^{\frac{3\hat{\Phi}}{2}}*\hat{F}_{(2)}\right) &= -e^{\frac{\hat{\Phi}}{2}}\hat{H}_{(3)}\wedge*\tilde{F}_{(4)} \end{aligned}$$

$$\begin{aligned} \hat{R}_{MN} &= \frac{1}{2}\partial_M\hat{\Phi}\partial_N\hat{\Phi}+\frac{1}{2}e^{\frac{3\hat{\Phi}}{2}}\left(\hat{F}_{MA}\hat{F}_N^A-\frac{1}{8\cdot 2!}g_{MN}\hat{F}_{(2)}^2\right) \\ &\quad +\frac{1}{4}e^{-\hat{\Phi}}\left(\hat{H}_M^{P_1P_2}\hat{H}_{NP_1P_2}-\frac{1}{2\cdot 3!}\hat{g}_{MN}\hat{H}_{(3)}^2\right) \\ &\quad +\frac{1}{12}e^{\frac{\hat{\Phi}}{2}}\left(\tilde{F}_M^{P_1P_2P_3}\tilde{F}_{NP_1P_2P_3}-\frac{9}{4\cdot 4!}\hat{g}_{MN}\tilde{F}_{(4)}^2\right) \end{aligned}$$

$$\begin{aligned} \delta\hat{\Psi}_M &= \left(\hat{D}_M-\frac{1}{4\cdot 2!}\hat{H}_{MPQ}\Gamma^{PQ}\Gamma^{(10)}\right. \\ &\quad \left.-\frac{1}{16}e^{\hat{\Phi}}\hat{F}_{PQ}\Gamma^{PQ}\Gamma_M\Gamma^{(10)}+\frac{1}{8\cdot 4!}e^{\hat{\Phi}}\tilde{F}_{NPQL}\Gamma^{NPQL}\Gamma_M\right)\hat{\epsilon} \\ \delta\hat{\lambda} &= \left(-\frac{1}{3}\Gamma^M\partial_M\hat{\Phi}\Gamma^{(10)}+\frac{1}{6\cdot 3!}\hat{H}_{MNP}\Gamma^{MNP}\right. \\ &\quad \left.-\frac{1}{2\cdot 4}e^{\hat{\Phi}}\hat{F}_{MN}\Gamma^{MN}+\frac{1}{12\cdot 4!}e^{\hat{\Phi}}\tilde{F}_{MNPQ}\Gamma^{MNPQ}\Gamma^{(10)}\right)\hat{\epsilon} \end{aligned}$$

$$\hat{\epsilon}=\hat{\epsilon}_L+\hat{\epsilon}_R,\Gamma^{(10)}\hat{\epsilon}_L=\hat{\epsilon}_L,\Gamma^{(10)}\hat{\epsilon}_R=-\hat{\epsilon}_R$$

$$\begin{aligned} ds_{10}^2 &= H^{-\frac{3}{8}}g_{\mu\nu}(x)dx^\mu dx^\nu + H^{\frac{5}{8}}\delta_{IJ}dy^I dy^J, e^{\Phi} = H^{-\frac{1}{4}}(y), \\ \tilde{F}_{(4)} &= -\frac{1}{4!}\partial_{I_1}H\varepsilon_{I_1\dots I_5}dy^{I_2}\wedge\dots\wedge dy^{I_5}, I,J=1,\dots,5. \end{aligned}$$

$$\Gamma_{\underline{01234}}\epsilon=-\Gamma^{(10)}\epsilon,\epsilon=H^{-3/32}\epsilon_0$$

$$\Gamma_{\underline{\mu}}=\sigma_1\otimes\gamma_{\underline{\mu}}\otimes\mathbf{1},\Gamma_{\underline{l}}=\sigma_2\otimes\mathbf{1}\otimes\Sigma_{\underline{l}}$$

$$\epsilon=\begin{pmatrix}1\\-i\end{pmatrix}\otimes\varepsilon^i\otimes\eta_i,i=1,2,3,4$$

$$(\sigma_2\otimes\mathbf{1}\otimes\mathbf{1})\epsilon=-\epsilon$$



$$S_{IIB} = \frac{1}{2\kappa_{10}^2} \int \left(*R - \frac{1}{2} d\hat{\Phi} \wedge * d\hat{\Phi} - \frac{1}{2} e^{2\hat{\Phi}} \hat{F}_{(1)} \wedge * \hat{F}_{(1)} - \frac{1}{2} e^{-\hat{\Phi}} \hat{H}_{(3)} \wedge * \hat{H}_{(3)} \right. \\ \left. - \frac{1}{2} e^{\hat{\Phi}} \tilde{F}_{(3)} \wedge * \tilde{F}_{(3)} - \frac{1}{4} \tilde{F}_{(5)} \wedge * \tilde{F}_{(5)} \right) - \frac{1}{4\kappa_{10}^2} \int \hat{C}_{(4)} \wedge \hat{H}_{(3)} \wedge \tilde{F}_{(3)}$$

$$d * d\hat{\Phi} = e^{2\hat{\Phi}} \hat{F}_{(1)} \wedge * \hat{F}_{(1)} - \frac{1}{2} e^{-\hat{\Phi}} \hat{H}_{(3)} \wedge * \hat{H}_{(3)} + \frac{1}{2} e^{\hat{\Phi}} \tilde{F}_{(3)} \wedge * \tilde{F}_{(3)}$$

$$d(e^{2\hat{\Phi}} * \hat{F}_{(1)}) = -e^{\hat{\Phi}} \hat{H}_{(3)} \wedge * \tilde{F}_{(3)}$$

$$d(e^{\hat{\Phi}} * \tilde{F}_{(3)}) = \tilde{F}_{(5)} \wedge \hat{H}_{(3)}$$

$$d(e^{-\hat{\Phi}} * \hat{H}_{(3)}) = e^{\hat{\Phi}} \hat{F}_{(1)} \wedge * \tilde{F}_{(3)} - \tilde{F}_{(5)} \wedge \tilde{F}_{(3)}$$

$$d * \tilde{F}_{(5)} = -\tilde{F}_{(3)} \wedge \hat{H}_{(3)}$$

$$\hat{R}_{MN} = \frac{1}{2} \partial_M \hat{\Phi} \partial_N \hat{\Phi} + \frac{1}{2} e^{2\hat{\Phi}} \hat{F}_M \hat{F}_N + \frac{1}{4 \cdot 4!} \tilde{F}_{MPQRS} \tilde{F}_N P Q R S \\ + \frac{1}{4} e^{\hat{\Phi}} \left(\hat{F}_{MPQ} \hat{F}_N{}^{PQ} - \frac{1}{12} G_{MN} \hat{F}_{PQR} \hat{F}^{PQR} \right) \\ + \frac{1}{4} e^{-\hat{\Phi}} \left(\hat{H}_{MPQ} \hat{H}_N{}^{PQ} - \frac{1}{12} G_{MN} \hat{H}_{PQR} \hat{H}^{PQR} \right)$$

$$\delta \hat{\lambda} = \frac{1}{2} (\partial_M \hat{\Phi} + i e^{\hat{\Phi}} \hat{F}_M) \Gamma^M \hat{\epsilon} - \frac{1}{4 \cdot 3!} (e^{-\hat{\Phi}/2} \hat{H}_{MNP} + i e^{\hat{\Phi}/2} \hat{F}_{MNP}) \Gamma^{MNP} \hat{\epsilon}^c$$

$$\delta \hat{\Psi}_M = \hat{D}_M \hat{\epsilon} - \frac{i}{4} e^{\hat{\Phi}} \hat{F}_M \hat{\epsilon} + \frac{1}{96} (e^{-\hat{\Phi}/2} \hat{H}_{NPQ} - i e^{\hat{\Phi}/2} \hat{F}_{NPQ}) (\Gamma_M^{NPQ} - 9 \delta_M^N \Gamma^{PQ}) \hat{\epsilon}^c \\ - \frac{i}{16 \cdot 5!} \hat{F}_{NPQRT} \Gamma^{NPQRT} \Gamma_M \hat{\epsilon}$$

$$\Gamma^{(10)} \hat{\Psi}_M = \hat{\Psi}_M \Gamma^{(10)} \hat{\epsilon} = \hat{\epsilon}$$

$$\Gamma^{(10)} \hat{\lambda} = -\hat{\lambda}$$

$$\hat{\epsilon} = \hat{\epsilon}_L + i \hat{\epsilon}_R, \hat{\epsilon}^c = \hat{\epsilon}_L - i \hat{\epsilon}_R, \Gamma^{(10)} \hat{\epsilon}_{L,R} = \hat{\epsilon}_{L,R}$$

$$ds_{10}^2 = H^{-\frac{1}{4}} \eta_{\mu\nu} dx^\mu dx^\nu + H^{\frac{3}{4}} \delta_{IJ} dy^I dy^J,$$

$$\hat{H}_{(3)} = - *_\delta dH, e^\Phi = H^{\frac{1}{2}}, I, J = 1, 2, 3, 4.$$

$$\epsilon = -\Gamma_{\underline{012345}} \epsilon^c, \epsilon = H^{-\frac{1}{16}} \epsilon_0.$$

$$\Gamma_{\underline{\mu}} = \gamma_{\underline{\mu}} \otimes 1, \Gamma_{\underline{m}} = \gamma^{(6)} \otimes \Sigma_{\underline{m}},$$

$$\epsilon = \varepsilon^A \otimes \eta_A(y) + i \varepsilon^{\dot{A}} \otimes \eta_{\dot{A}}(y)$$

$$\varepsilon^A = -\gamma^{(6)} \varepsilon^A, \varepsilon^{\dot{A}} = \gamma^{(6)} \varepsilon^{\dot{A}}$$

$$\eta_A = -\Sigma^{(4)} \eta_A, \eta_{\dot{A}} = \Sigma^{(4)} \eta_{\dot{A}}$$

$$S = \frac{1}{2\kappa_6^2} \int *R - \frac{1}{2} e^{-\frac{\varphi}{\sqrt{2}}} \delta_{IJ} F_{(2)}^I \wedge * F_{(2)}^J - \frac{1}{2} e^{\sqrt{2}\varphi} H_{(3)} \wedge * H_{(3)} \\ - \frac{1}{2} d\varphi \wedge * d\varphi + \frac{1}{2} \delta_{IJ} B_{(2)} \wedge F_{(2)}^I \wedge F_{(2)}^J.$$



$$\begin{aligned}
d * d\varphi &= -\frac{1}{2\sqrt{2}} e^{-\frac{\varphi}{\sqrt{2}}} \delta_{IJ} F_{(2)}^I \wedge * F_{(2)}^J + \frac{\sqrt{2}}{2} e^{\sqrt{2}\varphi} H_{(3)} \wedge * H_{(3)} \\
d\left(e^{-\frac{\varphi}{\sqrt{2}}} * F_{(2)}^I\right) &= H_{(3)} \wedge F_{(2)}^I \\
d\left(e^{\sqrt{2}\varphi} * H_{(3)}\right) &= -\frac{1}{2} \delta_{IJ} F_{(2)}^I \wedge F_{(2)}^J \\
R_{\mu\nu} &= \frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi + \frac{1}{2} e^{-\frac{\varphi}{\sqrt{2}}} \delta_{IJ} \left(F_{\mu\rho}^I F_\nu^{J\rho} - \frac{1}{8} g_{\mu\nu} F_{\rho\sigma}^I F^{J\rho\sigma} \right) \\
&\quad + \frac{1}{4} e^{\sqrt{2}\varphi} \left(H_{\mu\rho\sigma} H_\nu^{\rho\sigma} - \frac{1}{6} g_{\mu\nu} H_{\rho\sigma\lambda} H^{\rho\sigma\lambda} \right) \\
\delta\psi_{\mu i} &= D_\mu \varepsilon_i + \frac{1}{16\sqrt{2}} e^{-\frac{\varphi}{2\sqrt{2}}} [F_{\rho\sigma}^0 \varepsilon_{ij} \gamma^{(6)} - i F_{\rho\sigma}^a \varepsilon_{ik} (\sigma^a)_j^k] (\gamma_\mu^{\nu\lambda} - 6\delta_\mu^\nu \gamma^\lambda) \varepsilon^j \\
&\quad + \frac{1}{48} e^{\frac{\varphi}{\sqrt{2}}} H_{\nu\lambda\rho} \gamma^{(6)} (\gamma_\mu^{\nu\lambda\rho} - 3\delta_\mu^\nu \gamma^{\lambda\rho}) \varepsilon_i \\
\delta\chi_i &= -\frac{1}{4} \gamma^\mu \partial_\mu \varphi \varepsilon_i - \frac{1}{16} e^{-\frac{\varphi}{2\sqrt{2}}} [F_{\rho\sigma}^0 \varepsilon_{ij} \gamma^{(6)} + i F_{\rho\sigma}^a \varepsilon_{ik} (\sigma^a)_j^k] \gamma^{\rho\sigma} \varepsilon^j \\
&\quad + \frac{\sqrt{2}}{48} e^{\frac{\varphi}{\sqrt{2}}} H_{\mu\nu\lambda} \gamma^{(6)} \gamma^{\mu\nu\lambda} \varepsilon_i
\end{aligned}$$

$$\begin{aligned}
\psi_\mu^i &= \delta_A^i \psi_\mu^A + i \delta_A^i \psi_\mu^{\dot{A}}, \chi^i = \delta_A^i \chi^A + i \delta_A^i \chi^{\dot{A}}, \varepsilon^i = \delta_A^i \varepsilon^A + i \delta_A^i \varepsilon^{\dot{A}} \\
\varepsilon^A &= -\gamma^{(6)} \varepsilon^{\dot{A}}, \varepsilon^{\dot{A}} = \gamma^{(6)} \varepsilon^A, \psi_\mu^A = -\gamma^{(6)} \psi_\mu^{\dot{A}}, \psi_\mu^{\dot{A}} = \gamma^{(6)} \psi_\mu^A
\end{aligned}$$

$$\chi^A = \gamma^{(6)} \chi^{\dot{A}}, \chi^{\dot{A}} = -\gamma^{(6)} \chi^A$$

$$\begin{aligned}
ds^2 &= -H^{-\frac{3}{2}} dt^2 + H^{\frac{1}{2}} (dr^2 + r^2 d\Omega_4^2) \\
F_{(2)} &= \sqrt{2} \partial_r H^{-1} dt \wedge dr, e^\varphi = H^{-\frac{1}{\sqrt{2}}}
\end{aligned}$$

$$\begin{aligned}
ds^2 &= H^{-\frac{1}{2}} (-dt^2 + dx_1^2 + dx_2^2) + H^{\frac{3}{2}} (dr^2 + r^2 d\Omega_2^2), \\
F_{(2)} &= \sqrt{2} r^2 \partial_r h d\Omega_2, e^\varphi = H^{\frac{1}{\sqrt{2}}}.
\end{aligned}$$

$$\begin{aligned}
ds^2 &= H^{-\frac{1}{2}} (-dt^2 + dx^2) + H^{\frac{1}{2}} (dr^2 + r^2 d\Omega_3^2), \\
H_{(3)} &= H^{-2} \partial_r h dt \wedge dx \wedge dr, e^\varphi = H^{\frac{1}{\sqrt{2}}},
\end{aligned}$$

$$A_\mu^{ij} = -A_\mu^{ji}, A_\mu^{ij} \Omega_{ij} = 0$$

$$T^i = \Omega^{ij} T_j, T_i = T^j \Omega_{ji}$$

$$Q^{ij} = L^{lik} dL_{lk}^j$$



$$S = \frac{1}{2\kappa_5^2} \int *R - \frac{1}{2} e^{\frac{2}{\sqrt{6}}\sigma} a_{IJ} F_{(2)}^I \wedge *F_{(2)}^J - \frac{1}{2} e^{-\frac{4}{\sqrt{6}}\sigma} G_{(2)} \wedge *G_{(2)} \\ - \frac{1}{2} d\sigma \wedge *d\sigma + \frac{1}{2} C_{IJ} F_{(2)}^I \wedge F_{(2)}^J \wedge a_{(1)}$$

$$\delta\psi_{\mu i} = D_\mu \varepsilon_i + \frac{i}{6\sqrt{2}} e^{\frac{1}{\sqrt{6}}\sigma} L_{lij} F_{\rho\sigma}^I (\Gamma_\mu^{\rho\sigma} - 4\delta_\mu^\rho \Gamma^\sigma) \varepsilon^j \\ + \frac{i}{24} e^{-\frac{2}{\sqrt{6}}\sigma} G_{\rho\sigma} (\Gamma_\mu^{\rho\sigma} - 4\delta_\mu^\rho \Gamma^\sigma) \varepsilon_i \\ \delta\chi_i = -\frac{i}{2\sqrt{2}} \Gamma^\mu \partial_\mu \sigma \varepsilon_i + \frac{1}{2\sqrt{6}} e^{\frac{1}{\sqrt{6}}\sigma} L_{lij} F_{\rho\sigma}^I \Gamma^{\rho\sigma} \varepsilon^j - \frac{1}{4\sqrt{3}} e^{-\frac{2}{\sqrt{6}}\sigma} G_{\rho\sigma} \Gamma^{\rho\sigma} \varepsilon_i$$

$$a_{IJ} = C_{IJ} = L_I^{ij} L_{Jij},$$

$$L_I^{ij} = a_{IJ} L^{Jij}$$

$$(L_I)^i_j = \frac{1}{2} (\Sigma_I)^i_j,$$

$$\{\Sigma_I, \Sigma_J\} = 2\delta_{IJ}.$$

$$a_{IJ} = C_{IJ} = \delta_{IJ}$$

$$d * d\sigma = \frac{1}{\sqrt{6}} e^{\frac{2}{\sqrt{6}}\sigma} \delta_{IJ} F_{(2)}^I \wedge *F_{(2)}^J - \frac{2}{\sqrt{6}} e^{-\frac{4}{\sqrt{6}}\sigma} G_{(2)} \wedge *G_{(2)} \\ d\left(e^{-\frac{4}{\sqrt{6}}\sigma} *G_{(2)}\right) = \frac{1}{2} \delta_{IJ} F_{(2)}^I \wedge F_{(2)}^J \\ d\left(e^{\frac{2}{\sqrt{6}}\sigma} *F_{(2)}^I\right) = F_{(2)}^I \wedge G_{(2)} \\ R_{\mu\nu} = \frac{1}{2} \partial_\mu \sigma \partial_\nu \sigma + \frac{1}{2} \delta_{IJ} e^{\frac{2}{\sqrt{6}}\sigma} \left(F_{\mu\rho}^I F_\nu^{J\rho} - \frac{1}{6} g_{\mu\nu} F_{\rho\sigma}^I F^{J\rho\sigma}\right) \\ + \frac{1}{2} e^{-\frac{4}{\sqrt{6}}\sigma} \left(G_{\mu\rho} G_\nu^\rho - \frac{1}{6} g_{\mu\nu} G_{\rho\sigma} G^{\rho\sigma}\right)$$

$$ds_5^2 = -h^{-4/3} dt^2 + h^{2/3} (dx_1^2 + \cdots + dx_4^2), e^\sigma = h^{2/\sqrt{6}} \\ F_{(2)}^1 = \sqrt{2} \partial_a h^{-1} dt \wedge dx^a, \partial^a \partial_a h(x_1, \cdots, x_4) = 0, a, b = 1, 2, \cdots, 4,$$

$$ds_5^2 = -h^{-2/3} dt^2 + h^{1/3} (dx_1^2 + \cdots + dx_4^2), e^\sigma = h^{-2/\sqrt{6}} \\ G_{(2)} = h^{-2} \partial_a h dt \wedge dx^a, \partial^a \partial_a h(x_1, \cdots, x_4) = 0, a, b = 1, 2, \cdots, 4,$$

$$\epsilon = \Gamma_{\underline{012}}\epsilon, \\ \epsilon = \Gamma_{\underline{012345}}\epsilon, \\ \epsilon_L = \Gamma_{\underline{01}}\epsilon_L, \quad \epsilon_R = -\Gamma_{\underline{01}}\epsilon_R, \\ \epsilon_L = \Gamma_{\underline{012345}}\epsilon_L, \quad \epsilon_R = \Gamma_{\underline{012345}}\epsilon_R, \\ \epsilon_L = \Gamma_{\underline{012345}}\epsilon_L, \quad \epsilon_R = -\Gamma_{\underline{012345}}\epsilon_R, \\ \epsilon_L = \Gamma_{\underline{01\cdots p}}\epsilon_R.$$



$$\epsilon = \varepsilon^A \otimes \eta_A$$

$$\begin{aligned}\hat{\Psi}_\mu &= e^{\frac{5}{32\sqrt{6}}\sigma} \left\{ \begin{pmatrix} 1 \\ -i \end{pmatrix} \otimes \left(\psi_\mu^i + \frac{5i}{16\sqrt{3}} \gamma_\mu \chi^i \right) \otimes \eta_i \right\} \\ \hat{\Psi}_I &= -\frac{\sqrt{3}}{16} H_0^{\frac{3}{16}} e^{-\frac{5}{16\sqrt{6}}\sigma} e_I^J (\sigma_3 \otimes \mathbf{1} \otimes \Sigma_I) \left\{ \begin{pmatrix} 1 \\ -i \end{pmatrix} \otimes \chi^i \otimes \eta_i \right\} \\ \hat{\lambda} &= -\frac{\sqrt{3}}{2} H_0^{\frac{3}{16}} e^{-\frac{5}{32\sqrt{6}}\sigma} \begin{pmatrix} 1 \\ -i \end{pmatrix} \otimes \chi^i \otimes \eta_i, \hat{\epsilon} = e^{\frac{5}{32\sqrt{6}}\sigma} \begin{pmatrix} 1 \\ -i \end{pmatrix} \otimes \varepsilon^i(x) \otimes \eta_i\end{aligned}$$

$$\Sigma_{\underline{I}}\eta_i=2\eta_j(L_{\underline{I}})^j_i$$

$$\begin{aligned}\hat{\Psi}_\mu &= e^{-\frac{\sqrt{2}}{16}\varphi} \left\{ (\psi_\mu^A \otimes \eta_A + i\psi_\mu^{\dot{A}} \otimes \eta_{\dot{A}}) + \frac{\sqrt{2}}{4} \gamma_\mu (\chi^A \otimes \eta_A + i\chi^{\dot{A}} \otimes \eta_{\dot{A}}) \right\} \\ \hat{\Psi}_I &= -\frac{1}{2\sqrt{2}} H_0^{\frac{1}{8}} e^{\frac{1}{8\sqrt{2}}\varphi} e_I^J \left(\gamma^{(6)} \otimes \Sigma_I \right) (\chi^A \otimes \eta_A + i\chi^{\dot{A}} \otimes \eta_{\dot{A}}) \\ \hat{\lambda} &= \sqrt{2} H_0^{\frac{1}{8}} e^{\frac{\sqrt{2}}{16}\varphi} (\chi^A \otimes \eta_A + i\chi^{\dot{A}} \otimes \eta_{\dot{A}}) \\ \hat{\epsilon} &= e^{-\frac{\sqrt{2}}{16}\varphi} \varepsilon, \varepsilon = \varepsilon^A \otimes \eta_A + i\varepsilon^{\dot{A}} \otimes \eta_{\dot{A}}\end{aligned}$$

$$\begin{aligned}*\tilde{F}_{(5)} &= \frac{1}{\sqrt{2}} \left[e^{-\frac{\sqrt{2}}{2}\varphi} *_g F_{(2)}^I \wedge \mathrm{d}y^I - \frac{1}{3!} H_0 \varepsilon_{I_1 \dots I_4} F_{(2)}^{I_1} \wedge \mathrm{d}y^{I_2} \wedge \mathrm{d}y^{I_3} \wedge \mathrm{d}y^{I_4} \right] \\ *\hat{H}_{(3)} &= H_0^{-\frac{3}{2}} e^{-\sqrt{2}\varphi} \partial_I H_0 \mathrm{d}y^I \wedge \mathrm{vol}_g - H_0^{\frac{3}{2}} e^{\frac{\sqrt{2}}{2}\varphi} *_g H_{(3)} \wedge \mathrm{vol}_\delta \\ *\tilde{F}_{(3)} &= -\frac{1}{\sqrt{2}} \left(\frac{1}{3!} H_0^{\frac{1}{2}} \varepsilon_{I_1 I_2 I_3 I_4} *_g F_{(2)}^{I_1}(x) \wedge \mathrm{d}y^{I_2} \wedge \dots \wedge \mathrm{d}y^{I_4} \right)\end{aligned}$$

$$\mathrm{d}H_{(3)}=0$$

$$\mathrm{d}F_{(2)}^I=0$$

$$\begin{aligned}\mathrm{d}\tilde{F}_{(5)} &= -\frac{1}{\sqrt{2}}\partial_I H_0 F_{(2)}^I \wedge \mathrm{vol}_\delta + \frac{1}{\sqrt{2}}\mathrm{d}\left(e^{-\frac{\sqrt{2}}{2}\varphi} *_g F_{(2)}^I\right) \wedge \mathrm{d}y^I \\ -\tilde{F}_{(3)} \wedge \hat{H}_{(3)} &= -\frac{1}{\sqrt{2}}\partial_I H_0 F_{(2)}^I(x) \wedge \mathrm{vol}_\delta + \frac{1}{\sqrt{2}}H_{(3)}(x) \wedge F_{(2)}^I(x) \wedge \mathrm{d}y^I \\ \mathrm{d}\left(e^{-\frac{\sqrt{2}}{2}\varphi} *_g F_{(2)}^I\right) - H_{(3)}(x) \wedge F_{(2)}^I(x) &= 0\end{aligned}$$

$$\begin{aligned}\mathrm{d}*\mathrm{d}\hat{\Phi} &= -\frac{1}{2}e^{-\frac{\sqrt{2}}{2}\varphi}H_0^{-2}\partial_I H_0\partial_I H_0\mathrm{vol}_\delta\wedge\mathrm{vol}_g-\frac{1}{\sqrt{2}}H_0\mathrm{d}*\mathrm{d}\varphi\wedge\mathrm{vol}_\delta \\ \frac{1}{2}e^{\hat{\Phi}}\tilde{F}_{(3)}\wedge*\tilde{F}_{(3)} &= \frac{1}{4}H_0e^{-\frac{\varphi}{\sqrt{2}}}\delta_{IJ}F_{(2)}^I(x)\wedge*_gF^J(x)\wedge\mathrm{vol}_\delta \\ -\frac{1}{2}e^{-\hat{\Phi}}\hat{H}_{(3)}\wedge*\hat{H}_{(3)} &= -\frac{1}{2}H_0^{-2}e^{-\frac{\varphi}{\sqrt{2}}}(\partial_I H_0\partial_I H_0\mathrm{vol}_\delta\wedge\mathrm{vol}_g) \\ &\quad -\frac{1}{2}H_0e^{\sqrt{2}\varphi}H_{(3)}(x)\wedge*_gH_{(3)}\wedge\mathrm{vol}_\delta\end{aligned}$$



$$\begin{aligned}
0 &= d * d\varphi + \frac{1}{2\sqrt{2}} e^{-\frac{\varphi}{\sqrt{2}}} \delta_{IJ} F_{(2)}^I(x) \wedge *_g F_{(2)}^J(x) - \frac{\sqrt{2}}{2} e^{\sqrt{2}\varphi} H_{(3)}(x) \wedge *_g H_{(3)} \\
d(e^{\hat{\Phi}} * \tilde{F}_{(3)}) &= -\frac{1}{\sqrt{2}} e^{-\frac{\varphi}{\sqrt{2}}} \partial_I H_0 *_g F_{(2)}^I(x) \wedge \text{vol}_y \\
&\quad - \frac{1}{\sqrt{2}} \frac{1}{3!} H_0 \varepsilon_{I_1 I_2 I_3 I_4} d\left(e^{-\frac{\varphi}{\sqrt{2}}} *_g F_{(2)}^{I_1}(x)\right) \wedge dy^{I_2} \wedge \cdots \wedge dy^{I_4} \\
\tilde{F}_{(5)} \wedge \hat{H}_{(3)} &= -\frac{1}{\sqrt{2}} e^{-\frac{\sqrt{2}}{2}\varphi} \partial_I H_0 *_g F_{(2)}^I \wedge \text{vol}_y \\
&\quad + \frac{1}{\sqrt{2}} \left[\frac{1}{3!} H_0 \varepsilon_{I_1 \dots I_4} F_{(2)}^{I_1} \wedge dy^{I_2} \wedge dy^{I_3} \wedge dy^{I_4} \right] \wedge H_{(3)}(x) \\
d(e^{-\hat{\Phi}} * \hat{H}_{(3)}) &= -H_0 d\left(e^{\sqrt{2}\varphi} *_g H_{(3)}\right) \wedge \text{vol}_y \\
-\tilde{F}_{(5)} \wedge \tilde{F}_{(3)} &= \frac{1}{2} H_0 \delta_{IJ} F_{(2)}^I \wedge F_{(2)}^J(x) \wedge \text{vol}_y \\
d\left(e^{\sqrt{2}\varphi} *_g H_{(3)}\right) &+ \frac{1}{2} \delta_{IJ} F_{(2)}^I \wedge F_{(2)}^J(x) = 0 \\
\hat{e}^\mu &= H_0^{-\frac{1}{8}} e^{-\frac{\sqrt{2}}{8}\varphi} e^\mu, \hat{e}^I = H_0^{\frac{3}{8}} e^{\frac{\sqrt{2}}{8}\varphi} dy^I \\
\hat{R}_{\mu\nu} &= R_{\mu\nu} - \frac{1}{4} \partial_\mu \varphi \partial_\nu \varphi + \frac{\sqrt{2}}{8} \nabla_\rho (g^{\rho\eta} \partial_\eta \varphi) g_{\mu\nu} + \frac{1}{8} g_{\mu\nu} H_0^{-1} e^{-\frac{\sqrt{2}}{2}\varphi} \partial_K H_0^{-1} \partial_K H_0, \\
\hat{R}_{IJ} &= -\frac{\sqrt{2}}{8} H_0 e^{\frac{\sqrt{2}}{2}\varphi} \nabla_\rho (\partial_\sigma \varphi g^{\rho\sigma}) \delta_{IJ} + \frac{3}{8} \partial_J H_0^{-1} \partial_I H_0 - \frac{3}{8} \delta_{IJ} \partial_K H_0^{-1} \partial_K H_0, \\
\hat{R}_{\mu I} &= -\frac{\sqrt{2}}{8} H_0^{-1} \partial_I H_0 \partial_\mu \varphi. \\
\text{RHS}_{\mu\nu} &= \frac{1}{4} \partial_\mu \varphi \partial_\nu \varphi + \frac{1}{4} e^{\sqrt{2}\varphi} \left(H_{\mu\rho\sigma} H_\nu^{\rho\sigma} - \frac{1}{12} g_{\mu\nu} H_{\rho\sigma\lambda} H^{\rho\sigma\lambda} \right) \\
&\quad + \frac{1}{2} e^{-\frac{\varphi}{\sqrt{2}}} \delta_{IJ} \left(F_{\mu\rho I}^I F_\nu^{J\rho} - \frac{3}{16} g_{\mu\nu} F_{\rho\sigma}^I F^{I\rho\sigma} \right) + \frac{1}{8} H^{-1} e^{-\frac{\varphi}{\sqrt{2}}} g_{\mu\nu} \partial_K H_0^{-1} \partial_K H_0, \\
\text{RHS}_{IJ} &= \left(\frac{1}{32} H_0 F_{\rho\sigma}^K F^{K\rho\sigma} - \frac{1}{48} H_0 e^{\frac{3\sqrt{2}}{2}\varphi} H_{\rho\sigma\lambda} H^{\rho\sigma\lambda} \right) \delta_{IJ} + \frac{3}{8} \partial_I H_0^{-1} \partial_J H_0 - \frac{3}{8} \delta_{IJ} \partial_M H_0^{-1} \partial_M H_0, \\
\text{RHS}_{\mu I} &= -\frac{1}{4\sqrt{2}} H_0^{-1} \partial_I H_0 \partial_\mu \varphi. \\
R_{\mu\nu} &= \frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi + \frac{1}{2} e^{-\frac{\varphi}{\sqrt{2}}} \delta_{IJ} \left(F_{\mu\rho}^I F_\nu^{J\rho} - \frac{1}{8} g_{\mu\nu} F_{\rho\sigma}^I F^{I\rho\sigma} \right) \\
&\quad + e^{\sqrt{2}\varphi} \frac{1}{4} \left(H_{\mu\rho\sigma} H_\nu^{\rho\sigma} - \frac{1}{6} g_{\mu\nu} H_{\rho\sigma\lambda} H^{\rho\sigma\lambda} \right).
\end{aligned}$$



$$\begin{aligned}\delta\hat{\lambda} &= \sqrt{2}H_0^{\frac{1}{8}}e^{\frac{\sqrt{2}}{16}\varphi}(\delta\chi^A \otimes \eta_A + i\delta\chi^{\dot{A}} \otimes \eta_{\dot{A}}) \\ &= \sqrt{2}H_0^{\frac{1}{8}}e^{\frac{\sqrt{2}}{16}\varphi}\left\{-\frac{1}{4}\partial_\mu\varphi(\gamma^\mu \otimes \mathbf{1})(\varepsilon^A \otimes \eta_A + i\varepsilon^{\dot{A}} \otimes \eta_{\dot{A}}) \right. \\ &\quad \left. + \left(\frac{1}{24\sqrt{2}}e^{\frac{\varphi}{\sqrt{2}}}H_{\mu\nu\rho}(\gamma^{\mu\nu\rho} \otimes \mathbf{1}) + \frac{i}{16}e^{-\frac{\varphi}{2\sqrt{2}}}F_{\mu\nu}^I\delta_I^J(\gamma^{\mu\nu}\gamma^{(6)} \otimes \Sigma_{\underline{J}})\right)(\varepsilon^A \otimes \eta_A - i\varepsilon^{\dot{A}} \otimes \eta_{\dot{A}})\right\}.\end{aligned}$$

$$\begin{aligned}\delta\hat{\Psi}_\mu &= e^{-\frac{\sqrt{2}}{16}\varphi}\left\{(\psi_\mu^A \otimes \eta_A + i\psi_\mu^{\dot{A}} \otimes \eta_{\dot{A}}) + \frac{\sqrt{2}}{4}\gamma_\mu(\chi^A \otimes \eta_A + i\chi^{\dot{A}} \otimes \eta_{\dot{A}})\right\} \\ &= e^{-\frac{\sqrt{2}}{16}\varphi}\left\{D_\mu\hat{\varepsilon} + \frac{i}{16\sqrt{2}}e^{-\frac{\varphi}{2\sqrt{2}}}F_{\rho\sigma}^I\delta_I^J((\gamma_\mu^{\rho\sigma} - 6\delta_\mu^\rho\gamma^\sigma)\gamma^{(6)} \otimes \Sigma_{\underline{J}})\hat{\varepsilon}^c \right. \\ &\quad \left. - \frac{1}{48}e^{\frac{\varphi}{\sqrt{2}}}H_{\nu\rho\sigma}[(\gamma_\mu^{\nu\rho\sigma} - 3\delta_\mu^\nu\gamma^{\rho\sigma}) \otimes \mathbf{1}]\hat{\varepsilon}^c\right\} \\ &\quad + e^{-\frac{\sqrt{2}}{16}\varphi}\frac{\sqrt{2}}{4}(\gamma_\mu \otimes \mathbf{1})\left\{-\frac{1}{4}\partial_\lambda\varphi(\gamma^\lambda \otimes \mathbf{1})\hat{\varepsilon} \right. \\ &\quad \left. + \left(\frac{1}{24\sqrt{2}}e^{\frac{\varphi}{\sqrt{2}}}H_{\rho\sigma\lambda}(\gamma^{\rho\sigma\lambda} \otimes \mathbf{1}) + \frac{i}{16}e^{-\frac{\varphi}{2\sqrt{2}}}F_{\rho\sigma}^I\delta_I^J(\gamma^{\rho\sigma}\gamma^{(6)} \otimes \Sigma_{\underline{J}})\right)\hat{\varepsilon}^c\right\}\end{aligned}$$

$$\begin{aligned}\delta\hat{\Psi}_I &= -\frac{1}{2\sqrt{2}}H_0^{\frac{1}{8}}e^{\frac{1}{8\sqrt{2}}\varphi}e_I^J(\gamma^{(6)} \otimes \Sigma_{\underline{J}})\delta(\chi^A \otimes \eta_A + i\chi^{\dot{A}} \otimes \eta_{\dot{A}}) \\ &= -\frac{1}{2\sqrt{2}}H_0^{\frac{1}{8}}e^{\frac{1}{8\sqrt{2}}\varphi}e_I^J(\gamma^{(6)} \otimes \Sigma_{\underline{J}})\left\{-\frac{1}{4}\partial_\nu\varphi(\gamma^\nu \otimes \mathbf{1})\hat{\varepsilon} \right. \\ &\quad \left. + \left(\frac{1}{24\sqrt{2}}\left(e^{\frac{1}{\sqrt{2}}\varphi}H_{\mu\nu\rho}(\gamma^{\mu\nu\rho} \otimes \mathbf{1})\right) + \frac{i}{16}e^{-\frac{1}{2\sqrt{2}}\varphi}F_{\mu\nu}^K\delta_K^L(\gamma^{\mu\nu}\gamma^{(6)} \otimes \Sigma_{\underline{L}})\right)\hat{\varepsilon}^c\right\}\end{aligned}$$

$$\Sigma^0 = \begin{pmatrix} & \delta_B^A \\ \delta_B^{\dot{A}} & \end{pmatrix}, \Sigma^a = \begin{pmatrix} & (\sigma^a)_B^A \\ (\sigma^a)_C^{\dot{A}} & \end{pmatrix}.$$

$$\begin{aligned}\varepsilon^A \otimes \Sigma^0\eta_A &= \varepsilon^A \otimes \eta_A, & \varepsilon^{\dot{A}} \otimes \Sigma^a\eta_{\dot{A}} &= \varepsilon^A \otimes \eta_A, \\ \varepsilon^A \otimes \Sigma^a\eta_A &= (\sigma^a)_C^A\varepsilon^C \otimes \eta_{\dot{A}}, & \varepsilon^{\dot{A}} \otimes \Sigma^a\eta_{\dot{A}} &= (\sigma^a)_C^{\dot{A}}\varepsilon^{\dot{C}} \otimes \eta_A.\end{aligned}$$

$$\begin{aligned}\delta\chi^A &= -\frac{1}{4}\partial_\mu\varphi\gamma^\mu\varepsilon^A + \frac{1}{24\sqrt{2}}e^{\frac{\varphi}{\sqrt{2}}}H_{\mu\nu\rho}\gamma^{(6)}\gamma^{\mu\nu\rho}\varepsilon^A \\ &\quad - \frac{1}{16}e^{-\frac{\varphi}{2\sqrt{2}}}F_{\mu\nu}^0\gamma^{(6)}\gamma^{\mu\nu}\varepsilon^A + \frac{1}{16}e^{-\frac{\varphi}{2\sqrt{2}}}F_{\mu\nu}^a\gamma^{\mu\nu}(\sigma^a)_C^A\varepsilon^{\dot{C}} \\ \delta\chi^{\dot{A}} &= -\frac{1}{4}\partial_\mu\varphi\gamma^\mu\varepsilon^{\dot{A}} + \frac{1}{24\sqrt{2}}e^{\frac{\varphi}{\sqrt{2}}}H_{\mu\nu\rho}\gamma^{(6)}\gamma^{\mu\nu\rho}\varepsilon^{\dot{A}} \\ &\quad - \frac{1}{16}e^{-\frac{\varphi}{2\sqrt{2}}}F_{\mu\nu}^I\gamma^{(6)}\gamma^{\mu\nu}\varepsilon^{\dot{A}} - \frac{1}{16}e^{-\frac{\varphi}{2\sqrt{2}}}F_{\mu\nu}^a\gamma^{\mu\nu}(\sigma^a)_C^{\dot{A}}\varepsilon^{\dot{C}} \\ \delta\chi^i &= -\frac{1}{4}\partial_\mu\varphi\gamma^\mu\varepsilon^i + \frac{1}{24\sqrt{2}}e^{\frac{\varphi}{\sqrt{2}}}H_{\mu\nu\rho}\gamma^{(6)}\gamma^{\mu\nu\rho}\varepsilon^i \\ &\quad - \frac{1}{16}e^{-\frac{\varphi}{2\sqrt{2}}}[F_{\mu\nu}^0\gamma^{(6)}\delta_j^i + iF_{\mu\nu}^a(\sigma^a)_j^i]\gamma^{\mu\nu}\varepsilon^j,\end{aligned}$$



$$\delta\psi_\mu^i = D_\mu \varepsilon^i + \frac{1}{48} e^{\frac{\varphi}{\sqrt{2}}} H_{\nu\rho\sigma} \gamma^{(6)} (\gamma_\mu^{\nu\rho\sigma} - 3\delta_\mu^\nu \gamma^{\rho\sigma}) \varepsilon^i$$

$$+ \frac{1}{16\sqrt{2}} e^{-\frac{\varphi}{2\sqrt{2}}} [F_{\rho\sigma}^0 \gamma^{(6)} \delta_j^i - i F_{\rho\sigma}^a (\sigma^a)_j^i] (\gamma_\mu^{\rho\sigma} - 6\delta_\mu^\rho \gamma^\sigma) \varepsilon^j,$$

$$\lambda_{ijk} = \beta_{ijk} \frac{\pi^{d/2} \sqrt{\mathcal{C}_{\Delta_1} \mathcal{C}_{\Delta_2} \mathcal{C}_{\Delta_3}} \Gamma\left(\frac{\Delta_i + \Delta_j - \Delta_k}{2}\right) \Gamma\left(\frac{\Delta_k + \Delta_i - \Delta_j}{2}\right) \Gamma\left(\frac{\Delta_j + \Delta_k - \Delta_i}{2}\right) \Gamma\left(\frac{\Delta_i + \Delta_j + \Delta_k - d}{2}\right)}{2\Gamma(\Delta_i) \Gamma(\Delta_j) \Gamma(\Delta_k)},$$

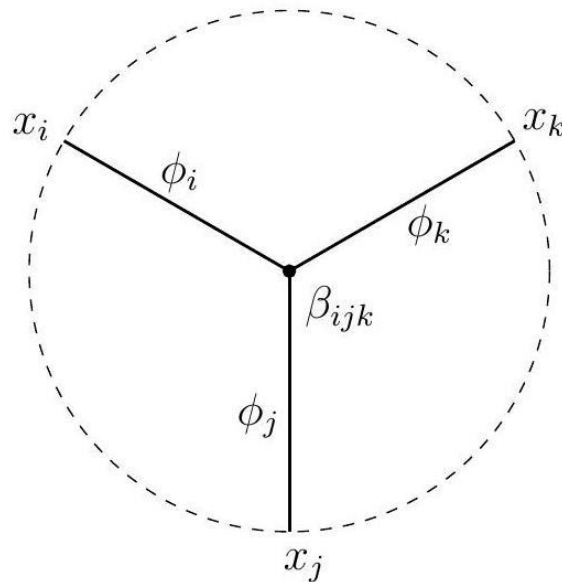


Figura 6. Diagramas de Witten.

$$M = \frac{1}{N} M_{F^2} + \frac{1}{N^2} \left[M_R + M_{F^2|F^2}(s, t) + \sum_i (b_{F^4, s}^i M_{F^4, s}^i + b_{F^4, tu}^i M_{F^4, tu}^i) \right]$$

$$+ \frac{\log N}{N^2} (b_{\log, s} M_{\log, s} + b_{\log, tu} M_{\log, tu}) + O(N^{-5/2}),$$

$$M_R = -\frac{p}{(p-2)! \Delta} \left[\left[(p+1) H_{1-\frac{s}{2}} + \frac{4}{s-2} \right] \delta^{AB} \delta^{CD} + \left[(p+1) H_{\frac{2+p-u}{2}} + \frac{2p}{u-p} \right] \delta^{AC} \delta^{BD} \right.$$

$$\left. + \left[(p+1) H_{\frac{2+p-t}{2}} + \frac{2p}{t-p} \right] \delta^{AD} \delta^{BC} \right]$$

$$\mathcal{A}_R^{ABCD} = -32 \Delta \pi^7 \delta^{AB} \delta^{CD} \int \frac{d^2 p_\perp}{(2\pi^2)} \frac{1}{-s + p_\perp^2} + \text{crossed}$$

$$= -8 \Delta \pi^6 (\log(-s) + \dots) \delta^{AB} \delta^{CD} + \text{crossed}$$

$$ds^2 = L^2 \big(ds_{\text{AdS}_5}^2 + d\theta^2 + \cos^2 \theta \, d\phi^2 + \sin^2 \theta \, ds_{S^3}^2 \big),$$

$$F_5 = dC_4 = 4L^4(1+\star)\text{vol}_{M_5}, \text{ with } \frac{1}{(2\pi\ell_s)^4 g_s} \int_{M_5} F_5 = N$$



$$(L/\ell_s)^4 = 4\pi\Delta g_s N$$

$$\mathrm{SO}(6) \rightarrow \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_R$$

$$S_{\mathrm{IIB}}=\frac{1}{2\kappa^2}\int\mathrm{d}^{10}X\sqrt{g}\Big(R+\frac{1}{5!}F_5^2\Big),\frac{1}{2\kappa^2}=\frac{2\pi}{(2\pi\ell_s)^8g_s^2}\equiv\frac{2\pi}{(2\pi\ell_p)^8}$$

$$S_7=T_7\left(\int\mathrm{d}^8\hat{X}\sqrt{\det(\hat{g}+2\pi\ell_s^2F)}+\int\hat{C}_4\wedge\mathrm{e}^{2\pi\ell_s^2F}\right),T_7=\frac{2\pi}{(2\pi\ell_s)^8g_s}$$

$$S_{\mathrm{IIB}}=\frac{1}{2\kappa_5^2}\int\mathrm{d}^5x\sqrt{g}R+\cdots, \text{ where } \frac{1}{2\kappa_5^2}=\frac{\mathrm{vol}_{M_5}}{2\kappa^2}=\frac{\Delta N^2}{8\pi^2L^3}$$

$$\langle T_{\hat{\mu}\hat{\nu}}(z)T_{\hat{\rho}\hat{\sigma}}(0)\rangle=\frac{c}{640\pi^2z^8}\big(I_{\hat{\mu}(\hat{\rho}I_{\hat{\sigma}})\hat{\nu}}-\text{trace}\big),c\approx\frac{\pi^2L^3}{\kappa_5^2}=\frac{\Delta}{4}N^2$$

$$S_7\approx-\int\mathrm{d}^5x\frac{1}{4\,g_{5d}^2}\big(F_{\mu\nu}^A\big)^2+\cdots, \text{ where } \frac{1}{4\,g_{5d}^2}=\frac{N\Delta}{8\pi^2L}$$

$$\mathrm{tr} T^A T^B = 2 \delta^{AB}$$

$$\langle J^A_{\hat{\mu}}(x)J^B_{\hat{\nu}}(0)\rangle=\frac{3\mathbf{k}}{2\pi^2}\frac{1}{z^6}\delta^{AB}I_{\hat{\mu}\hat{\nu}},\mathbf{k}=\frac{8\pi^2L}{g_{5d}^2}=2\Delta N$$

$$\delta g_{ab}=h_{ab}=h'_{ab}+\frac{h}{5}g_{ab}, \text{ with } g^{ab}h'_{ab}=0, h'_{ab}=h'_{ab},$$

$$\delta g_{\mu\nu}=h_{\mu\nu}=h'_{\mu\nu}-\frac{3h}{25}g_{\mu\nu}, \text{ with } g^{\mu\nu}h'_{\mu\nu}=0, h'_{\mu\nu}=h'_{\nu\mu},$$

$$\delta C_4 = \big((\star \mathrm{d})_{X_5} - (\star \mathrm{d})_{\mathrm{AdS}_5}\big)a.$$

$$\begin{aligned} h(x,y)&=\sum_k h_k(x)\mathcal{Y}^{I_k}(y),\\ h'_{(\mu\nu)}(x,y)&=\sum_k h'_{(\mu\nu),k}(x)\mathcal{Y}^{I_k}(y),\\ a(x,y)&=\sum_k a_k(x)\mathcal{Y}^{I_k}(y), \end{aligned}$$

$$\Delta_{S^5}\mathcal{Y}^{I_{k_p}}=-k_p(k_p+4), \text{ and } \Delta_{S^3}\mathcal{Y}^{I_{k_p}}=-p(p-2)$$

$$k_p=\begin{cases} p,p+2,p+4,\ldots & p=2,3\\ p-2,p,p+2,\ldots & p>3 \end{cases}$$

$$\mathcal{Y}^{I_{k_p}}=T_{i_1i_2\dots i_{p-2}}\mathcal{Y}_{i_1}\mathcal{Y}_{i_2}\cdots\mathcal{Y}_{i_{p-2}}\,{}_2F_1\left(\frac{p-2-k_p}{2},\frac{p+2+k_p}{2},1,\rho^2\right),$$

$$h_{k_p}(x)=10k_p s_{k_p}(x)+20(k_p+4)t_{k_p}(x), a_{k_p}(x)=t_{k_p}(x)-s_{k_p}(x),$$

$$\left(\square-k_p(k_p-4)\right)s_{k_p}=\left(\square-(k_p+8)(k_p+4)\right)t_{k_p}=0$$



$$S^{(2)}\left(s_{k_p}\right)=\frac{1}{2 \kappa^2} \frac{64 k_p\left(k_p-1\right)}{\left(k_p+1\right)(p-1)} \frac{\pi^3}{\Delta} \int d^5 x \sqrt{g}\left(\left(\partial_\mu s_{k_p}\right)^2+m_{k_p}^2 s_{k_p}^2\right)$$

$$h'_{(\mu\nu),k_p}=H'_{(\mu\nu),k_p}+\frac{4\nabla_{(\mu}\nabla_{\nu)}S_{k_p}}{k_p+1},$$

$$S_{\rm brane}^{(3)}=-2\pi^2\ell_s^4T_7\int\left[{\rm d}^8\hat{X}\sqrt{\hat{g}}(F^{A,\hat{M}\hat{N}}F_{\hat{M}\hat{N}}^A-2F^{A,\hat{M}\hat{N}}F_{\hat{N}\hat{P}}^A\hat{h}^{\hat{P}}{}_{\hat{M}})-2\hat{C}_4\wedge F^A\wedge F^A\right]$$

$$A^A_\mu=\sum_Ia^{A,I}_\mu(x)\hat{\mathcal{Y}}^I(\hat{y}), A^A_{\hat{a}}=\varphi^A(x)\cdot\mathcal{X}_{\hat{a}}(\hat{y})$$

$$\mathcal{X}^p_{\hat{a}}(\hat{y})\!:\!(\frac{p}{2}\!-\!1,\frac{p}{2})_0\text{ and }\tilde{\mathcal{X}}^p_{\hat{a}}(\hat{y})\!:\!(\frac{p}{2},\frac{p}{2}\!-\!1)_0$$

$$\nabla^{\hat{b}}\nabla_{\hat{b}}\mathcal{X}^p_{\hat{a}}=(2-p^2)\mathcal{X}^p_{\hat{a}},\epsilon^{\hat{b}\hat{c}}_{\hat{a}}\nabla_{\hat{b}}\mathcal{X}^p_{\hat{c}}=-p\mathcal{X}^p_{\hat{a}},\nabla^{\hat{a}}\mathcal{X}^p_{\hat{a}}=0$$

$$S^{(2)}(\varphi_p)=-\frac{4\pi^4\ell_s^4T_7}{p-1}\int d^5x\sqrt{g}\left((\partial_\mu\varphi^A_{(p)})^2+m_p^2(\varphi^A_{(p)})^2\right),m_p^2=p(p-4)$$

$$S^{(3)}=\int d^5x\sqrt{g}\Bigg(\sum_{p=2}^{\infty}\Big(-\frac{1}{2}(\partial_\mu\varphi^A_{(p)})^2-\frac{1}{2}m_p^2(\varphi^A_{(p)})^2\Big)+\sum_{p=2}^{\infty}\sum_{k_p}\Big(-\frac{1}{2}(\partial_\mu s_{(k_p)})^2-\frac{1}{2}m_{k_p}^2s_{(k_p)}^2\Big)\\ +\sum_{p,q=2}^{\infty}\sum_r\sum_{k_r}\frac{1}{2}\beta_{pqk_r}S_{(k_r)}\varphi^A_{(p)}\varphi^A_{(q)}\Bigg)$$

$$r=|p-q|+2,|p-q|+4,\ldots,p+q-2$$

$$\beta_{pqk_r}=\frac{\pi}{2N}\sqrt{\frac{(p-1)(q-1)(r-1)}{2\Delta k_r(k_r-1)(k_r+1)}}(k_r^2-(p-q)^2)(k_r+p+q-4)(k_r+p+q-2)\\ \times \frac{\Gamma(p-1)\Gamma(q-1)\Gamma(r-1)}{\Gamma\left(\frac{p+q-r}{2}\right)\Gamma\left(\frac{p-q+r}{2}\right)\Gamma\left(\frac{q-p+r}{2}\right)\Gamma\left(\frac{p+q+r-2}{2}\right)}$$

$$\lambda_{pqk_r}=\frac{1}{N}\sqrt{\frac{r-1}{\Delta k_r(k_r+1)}}\frac{\Gamma\left(\frac{2+k_r+p-q}{2}\right)\Gamma\left(\frac{2+k_r-p+q}{2}\right)\Gamma\left(\frac{p+q-k_r}{2}\right)\Gamma\left(\frac{k_r+p+q}{2}\right)\Gamma(r-1)}{\Gamma(k_r)\Gamma\left(\frac{p+q-r}{2}\right)\Gamma\left(\frac{p-q+r}{2}\right)\Gamma\left(\frac{q-p+r}{2}\right)\Gamma\left(\frac{p+q+r-2}{2}\right)}+O(1/N^2)$$

$$\lambda_{pp(k_2=2)}=\frac{p}{N\sqrt{6\Delta}}+O(1/N^2)$$

$$\begin{aligned} &\langle \phi_2^A(y_1,x_1)\phi_2^B(y_2,x_2)\phi_p^C(y_3,\bar{y}_3,x_3)\phi_p^D(y_4,\bar{y}_4,x_4)\rangle \\ &= \frac{\langle y_1,y_2\rangle^2\langle y_3,y_4\rangle^p\langle \bar{y}_3,\bar{y}_4\rangle^{p-2}}{x_{12}^4x_{34}^{2p}}\sum_{\mathbf{r}\in \mathbf{g}\otimes \mathbf{g}}G_{\mathbf{r}}(U,V;w)P_{\mathbf{r}}^{ABCD}\\ &\langle \phi_2^A(y_1,x_1)\phi_p^B(y_2,x_2)\phi_2^C(y_3,\bar{y}_3,x_3)\phi_p^D(y_4,\bar{y}_4,x_4)\rangle \\ &= \frac{\langle y_1,y_2\rangle^2\langle y_3,y_4\rangle^2\langle y_2,y_4\rangle^{p-2}}{x_{12}^4x_{34}^4x_{24}^{2(p-2)}}\sum_{\mathbf{r}\in \mathbf{g}\otimes \mathbf{g}}\tilde{G}_{\mathbf{r}}(U,V;w)P_{\mathbf{r}}^{ABCD} \end{aligned}$$



$$G_{\mathbf{r}}(U,V;w)=\frac{z(w-\bar{z})f_{\mathbf{r}}(\bar{z})-\bar{z}(w-z)f_{\mathbf{r}}(z)}{w(z-\bar{z})}+\left(1-\frac{z}{w}\right)\left(1-\frac{\bar{z}}{w}\right)G_{\mathbf{r}}(U,V),$$

$$\mathcal{G}_{\mathbf{r}}(U,V)=U^{-1}\left[\sum_{\ell}\sum_{\Delta\geq\ell+2}\lambda_{\Delta,\ell,\mathbf{r}}^{(2,p)}g_{\Delta+2,\ell}^{0,0}(U,V)+\sum_{\ell}\lambda_{\ell+3,\ell+1,\mathbf{r}}^{(2,p)}g_{\ell+5,\ell+1}^{0,0}(U,V)+\lambda_{2,0,\mathbf{r}}^{(2,p)}g_{4,0}^{0,0}(U,V)\right]$$

$$f_{\mathbf{r}}(z)=\sum_{\ell}\lambda_{\ell+3,\ell+1,\mathbf{r}}^{(2,p)}k_{2\ell+6}^{0,0}(z)+\lambda_{2,0,\mathbf{r}}^{(2,p)}k_4^{0,0}(z)+\delta_{\mathbf{r},1}\dim(G)-\delta_{\mathbf{r},\text{Adj}}\frac{4h^{\vee}}{\mathbf{k}}k_2^{0,0}(z)-\delta_{\mathbf{r},1}\frac{p\dim(G)}{12c}k_4^{0,0}(z),$$

$$g_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}(U,V)=\frac{z\bar{z}}{z-\bar{z}}(k_{\Delta+\ell}^{\Delta_{12},\Delta_{34}}(z)k_{\Delta-\ell-2}^{\Delta_{12},\Delta_{34}}(\bar{z})-k_{\Delta+\ell}^{\Delta_{12},\Delta_{34}}(\bar{z})k_{\Delta-\ell-2}^{\Delta_{12},\Delta_{34}}(z)),\\ k_h^{\Delta_{12},\Delta_{34}}(z)\equiv z^{\frac{h}{2}}{}_2F_1(h/2+\Delta_{12}/2,h/2-\Delta_{34}/2,h,z).$$

$$c=\frac{\Delta}{4}N^2+\frac{3(\Delta-1)}{4}N-\frac{1}{12},\mathbf{k}=2\Delta N.$$

$$\lambda_{2,0,1}^{(2,p)}=2\delta_{2,p}+\frac{p\dim(G_F)}{12c}-\frac{4h^{\vee}}{\mathbf{k}}.$$

$$\tilde{\mathcal{G}}_{\mathbf{r}}(U,V)=U^{-\frac{p}{2}}\sum_{\ell}\sum_{\Delta\geq\ell+p}\lambda_{2p(\Delta,\ell,\mathbf{r})}^2g_{\Delta+2,\ell}^{2-p,2-p}(U,V)+\cdots,$$

$$\mathcal{G}_{\text{conn}}^{ABCD}(U,V)=\int\frac{dsdt}{(4\pi i)^2}U^{\frac{s}{2}}V^{\frac{t-p-2}{2}}M^{ABCD}(s,t)\Gamma\left[2-\frac{s}{2}\right]\Gamma\left[p-\frac{s}{2}\right]\Gamma\left[\frac{2+p-t}{2}\right]^2\Gamma\left[\frac{2+p-u}{2}\right]^2\\ \tilde{\mathcal{G}}_{\text{conn}}^{ABCD}(U,V)=\int\frac{dsdt}{(4\pi i)^2}U^{\frac{s+2-p}{2}}V^{\frac{t-p-2}{2}}M^{ABCD}(s,t)\Gamma\left[\frac{2+p-s}{2}\right]^2\Gamma\left[\frac{2+p-t}{2}\right]^2\Gamma\left[2-\frac{u}{2}\right]\Gamma\left[p-\frac{u}{2}\right]$$

$$\mathcal{G}_{\text{conn}}^{ABCD}(U,V)=\mathcal{G}^{ABCD}(U,V)-\left(\delta^{AB}\delta^{CD}+\delta_{2,p}U\delta^{AC}\delta^{BD}+\delta_{2,p}\frac{U}{V^2}\delta^{AD}\delta^{BC}\right),\\ \tilde{\mathcal{G}}_{\text{conn}}^{ABCD}(U,V)=\tilde{\mathcal{G}}^{ABCD}(U,V)-\left(\delta_{2,p}\delta^{AB}\delta^{CD}+U^{\frac{p}{2}}\delta^{AC}\delta^{BD}+\delta_{2,p}\frac{U}{V^2}\delta^{AD}\delta^{BC}\right).$$

$$M_{F^2}(s,t)=-\frac{4}{(p-2)!\,\Delta}\Big[\frac{c_s}{(s-2)(u-p)}-\frac{c_t}{(t-p)(u-p)}\Big],$$

$$c_s^{ABCD}=f^{ABJ}f^{JCD},c_t^{ABCD}=f^{ADJ}f^{JBC},c_u^{ABCD}=f^{ACJ}f^{JDB}$$

$$M_{F^2|F^2}(s,t)=-\frac{p}{4(p-2)!\,\Delta^2}\big[\,\mathrm{d}_{st}\mathcal{B}_{st}^p+\mathrm{d}_{su}\mathcal{B}_{su}^p+\mathrm{d}_{tu}\mathcal{B}_{tu}^p\big]$$

$$\mathrm{d}_{st}^{ABCD}=f^{IAJ}f^{JBK}f^{KCL}f^{LDI}\\ \mathrm{d}_{su}^{ABCD}=f^{IAJ}f^{JBK}f^{KDL}f^{LCI}\\ \mathrm{d}_{tu}^{ABCD}=f^{IAJ}f^{JCK}f^{KBL}f^{LDI}$$



$$\mathcal{B}_{st}^p = \sum_{m,n=0}^{\infty} \frac{(m-n)(m+2n+3) - m(n+1)(p+1)(m+n+2)}{54(m+n)(m+n+1)(m+n+2)(s-4-2m)(t-2-p-2n)}$$

$$\mathcal{B}_{su}^p = \sum_{m,n=0}^{\infty} \frac{(m-n)(m+2n+3) - m(n+1)(p+1)(m+n+2)}{54(m+n)(m+n+1)(m+n+2)(s-4-2m)(u-2-p-2n)}$$

$$\mathcal{B}_{tu}^p = \sum_{m,n=0}^{\infty} \frac{(m-n)^2 - (m+1)(n+1)(p+1)(m+n)}{54(m+n)(m+n+1)(m+n+2)(t-2-p-2m)(u-2-p-2n)}$$

$$\mathcal{B}_{st}^2 = R_0(s,t) \left[\psi^{(1)}\left(2 - \frac{s}{2}\right) + \psi^{(1)}\left(2 - \frac{t}{2}\right) - \left(\psi\left(2 - \frac{s}{2}\right) - \psi\left(2 - \frac{t}{2}\right) \right)^2 \right]$$

$$+ R_1(s,t) \psi^{(0)}\left(2 - \frac{s}{2}\right) + R_1(t,s) \psi^{(0)}\left(2 - \frac{t}{2}\right) + \pi^2 R_2(s,t) + \frac{4}{27(s+t-8)} - \frac{\gamma}{9}$$

$$R_0(s,t) = \frac{3s^2t - 8s^2 + 3st^2 - 32st + 60s - 8t^2 + 60t - 96}{54(s+t-8)(s+t-6)(s+t-4)}$$

$$R_1(s,t) = -\frac{3s^2 + 3st - 26s - 10t + 48}{27(s+t-8)(s+t-4)}, R_2(s,t) = -R_0(s,t)$$

$$M_{F^4,s}^1(s,t) = M_{\log,s}(s,t) = \mathfrak{t}_1, M_{F^4,tu}^1(s,t) = M_{\log,tu}(s,t) = \mathfrak{t}_2 + \mathfrak{t}_3$$

$$\mathfrak{t}_1^{ABCD} = \delta^{AB} \delta^{CD}, \mathfrak{t}_2^{ABCD} = \delta^{AC} \delta^{DB}, \mathfrak{t}_3^{ABCD} = \delta^{AD} \delta^{BC}$$

$$p=2: M_{F^4}^1(s,t) = M_{\log}(s,t) = \mathfrak{t}_1 + \mathfrak{t}_2 + \mathfrak{t}_3$$

$$b_{\log} = \frac{6}{\Delta}.$$

$$-\lambda_{22k_2}\lambda_{ppk_2}\sum_{m=0}^{\infty}\frac{1}{s-2m-k_2}\frac{4^{k_2+1}\Gamma\left(\frac{k_2+1}{2}\right)\Gamma\left(\frac{k_2+3}{2}\right)}{\pi(k_2/2)!^2m!(1-m-k_2/2)!(p-1-m-k_2/2)!(m+k_2)!},$$

$$-\lambda_{2pk_p}^2\sum_{m=0}^{\infty}\frac{1}{s-2m-k_p}\frac{2k_p!(k_p+1)!}{m!(k_p+m)!\Gamma\left(\frac{k_p}{2}-\frac{p}{2}+2\right)^2\Gamma\left(\frac{k_p}{2}+\frac{p}{2}\right)^2\Gamma\left(1-\frac{k_p}{2}-m+\frac{p}{2}\right)^2},$$

$$\lambda_{22k_2}\lambda_{ppk_2}=\beta_{22k_2}\beta_{ppk_2}\frac{\Gamma(2-k_2/2)\Gamma(k_2/2)^5\Gamma(p-k_2/2)\Gamma(k_2/2+p-2)}{32\pi^2\Gamma(k_2)\Gamma(k_2-1)\Gamma(p-1)\Gamma(p)}$$

$$\lambda_{2pk_p}^2=\beta_{2pk_p}^2\frac{\Gamma\left(\frac{1}{2}(k_p-p+2)\right)^2\Gamma\left(\frac{1}{2}(-k_p+p+2)\right)^2\Gamma\left(\frac{1}{2}(k_p+p-2)\right)^4}{32\pi^2\Gamma(k_p-1)\Gamma(k_p)\Gamma(p-1)\Gamma(p)}.$$



$$M_R \approx \frac{2}{\Delta} \sum_{m=0} \left[\sum_{k_2} \frac{k_2^2(k_2^2 - 4) \Gamma\left(\frac{k_2}{2} + m - 1\right) \Gamma\left(\frac{k_2}{2} + p\right) \Gamma\left(\frac{k_2}{2} + m - p + 1\right)}{8(k_2 + 2m - s) \Gamma(m + 1) \Gamma(p - 1) \Gamma(p) \Gamma(k_2 + m + 1) \Gamma\left(\frac{k_2}{2} - p + 1\right)} t_1 \right. \\ \left. + \sum_{k_p} \frac{k_p(p - 1) \Gamma\left(\frac{1}{2}(k_p + p + 2)\right)^2 \Gamma\left(\frac{k_p}{2} + m - \frac{p}{2}\right)^2}{(k_p + 2m - u) \Gamma(m + 1) \Gamma(p)^2 \Gamma(k_p + m + 1) \Gamma\left(\frac{k_p - p}{2}\right)^2} t_2 \right. \\ \left. + \sum_{k_p} \frac{k_p(p - 1) \Gamma\left(\frac{1}{2}(k_p + p + 2)\right)^2 \Gamma\left(\frac{k_p}{2} + m - \frac{p}{2}\right)^2}{(k_p + 2m - t) \Gamma(m + 1) \Gamma(p)^2 \Gamma(k_p + m + 1) \Gamma\left(\frac{k_p - p}{2}\right)^2} t_3 \right]$$

$$\mathcal{A}(s,t) = (u+s/w)^2 \Big[\ell_P^4 \mathcal{A}_{F^2} + \ell_P^8 \big[\mathcal{A}_{F^4}(\tau) + \mathcal{A}_{F^2|F^2} + \mathcal{A}_R + \mathcal{A}_{\log\log} \ell_P^2 + O(\ell_P^{10}) \big] \Big]$$

$$\mathcal{A}^{ABCD}(s,t)=\lim_{L\rightarrow\infty}8\pi^4(p-1)!(p-2)!L^4\int\frac{d\beta}{2\pi i}\frac{e^\beta}{\beta^{2+p}}\frac{L^4(u+s/w)^2}{16}M^{ABCD}\left(\frac{L^2}{2\beta}s,\frac{L^2}{2\beta}t\right)$$

$$\mathcal{A}_{F^2}(s,t)=-\frac{(2\pi)^5}{stu}(tc_s-sc_t)$$

$$\mathcal{A}_{\log}(s,t)=-16\pi^6\mathbf{\Delta}(t_1+t_2+t_3),$$

$$\mathcal{A}_{F^2|F^2}(s,t)=-\frac{2\pi^6}{3}[\,\mathrm{d}_{st}f_{\mathrm{box}}(s,t)+\mathrm{d}_{su}f_{\mathrm{box}}(s,u)+\mathrm{d}_{tu}f_{\mathrm{box}}(t,u)],$$

$$f_{\mathrm{box}}(s,t)\sim \mathcal{B}_{st}^2\sim \frac{st\log^2\left(\frac{-s}{-t}\right)}{(s+t)^2}+2\frac{s\log\left(-s/2\right)+t\log\left(-t/2\right)}{s+t}+\pi^2\frac{st}{(s+t)^2}+2\gamma.$$

$$\mathcal{A}_R=-8\Delta\pi^6[t_1(\log{(-s/2)}+\gamma)+t_2(\log{(-t/2)}+\gamma)+t_3(\log{(-u/2)}+\gamma)]$$

$$:\phi_2^A\Box^{n-2}\phi_2^A:,\,: \phi_3^A\Box^{n-3}\phi_3^A:,\ldots,\,: \phi_n^A\phi_n^A: ,$$

$$\Delta_a=4+\frac{1}{N}\gamma_a^{(1)}+\frac{1}{N^2}\gamma_a^{(2)}+\mathcal{O}\left(\frac{1}{N^3}\right),$$

$$\lambda_{2,2,\mathcal{O}_a}=\lambda_{2,2,\mathcal{O}_a}^{(0)}+\frac{1}{N}\lambda_{2,2,\mathcal{O}_a}^{(1)}+\frac{1}{N^2}\lambda_{2,2,\mathcal{O}_a}^{(2)}+\mathcal{O}\left(\frac{1}{N^3}\right)$$



$$\begin{aligned}
\mathcal{G}_1^{2,2}(U, V) = U^{-1} & \left[\left\langle (\lambda^{(0)})^2 \right\rangle + \frac{1}{N} \left(2 \left\langle \lambda^{(0)} \lambda^{(1)} \right\rangle + \frac{1}{2} \left\langle (\lambda^{(0)})^2 \gamma^{(1)} \right\rangle \left(\log U + 2 \partial_{\Delta}^{\text{no log}} \right) \right) \right. \\
& + \frac{1}{N^2} \left(\left\langle 2 \lambda^{(0)} \lambda^{(2)} + (\lambda^{(1)})^2 \right\rangle + \frac{1}{2} \left\langle 2 \lambda^{(0)} \lambda^{(1)} \gamma^{(1)} + (\lambda^{(0)})^2 \gamma^{(2)} \right\rangle \left(\log U + 2 \partial_{\Delta}^{\text{no log}} \right) \right) \\
& + \frac{1}{8} \left\langle (\lambda^{(0)})^2 (\gamma^{(1)})^2 \right\rangle \left(\log^2 U + 4 \log U \partial_{\Delta} + 4 \left(\partial_{\Delta}^{\text{no log}} \right)^2 \right) \\
& \left. + \mathcal{O} \left(\frac{1}{N^3} \right) \right] g_{\Delta+2,0}(U, V) \Big|_{\Delta=4} \\
& + \dots
\end{aligned}$$

$$\left\langle (\lambda^{(0)})^2 \right\rangle = \sum_{a=1}^2 \left(\lambda_a^{(0)} \right)^2.$$

$$\begin{aligned}
\left\langle (\lambda^{(0)})^2 \right\rangle &= \frac{2}{3} \\
\left\langle (\lambda^{(0)})^2 \gamma^{(1)} \right\rangle &= -\frac{4}{3} \frac{h^V}{\Delta} = \frac{8(1-\Delta)}{\Delta} \\
\left\langle (\lambda^{(0)})^2 (\gamma^{(1)})^2 \right\rangle &= \left\langle (\lambda^{(0)})^2 (\gamma^{(1)})^2 \right\rangle_{\text{superparticle}} + \left\langle (\lambda^{(0)})^2 (\gamma^{(1)})^2 \right\rangle_{\text{grav}}
\end{aligned}$$

$$\begin{aligned}
\left\langle (\lambda^{(0)})^2 (\gamma^{(1)})^2 \right\rangle_{\text{superparticle}} &= \frac{3}{2} \left(-\frac{4}{3} \frac{h^V}{\Delta} \right)^2 = \frac{96(1-\Delta)^2}{\Delta^2} \\
\left\langle (\lambda^{(0)})^2 (\gamma^{(1)})^2 \right\rangle_{\text{grav}} &= \frac{16 \dim(G_F)}{5 \Delta} = \frac{32(6-5\Delta)(5-6\Delta)}{5 \Delta^2}
\end{aligned}$$

$$h^V = 6(\Delta - 1), \dim(G_F) = \frac{2(6-5\Delta)(5-6\Delta)}{\Delta}$$

$$\begin{aligned}
& \begin{pmatrix} \langle \rho_4(x) \rho_4(y) \rangle & \frac{1}{N} \langle \rho_4(x) : \phi_2 \phi_2 : (y) \rangle \\ \frac{1}{N} \langle : \phi_2 \phi_2 : (x) \rho_4(y) \rangle & \frac{1}{N^2} \langle : \phi_2 \phi_2 : (x) : \phi_2 \phi_2 : (y) \rangle \end{pmatrix} \\
&= \frac{1}{|x-y|^8} \left[\delta_{ab} - \frac{1}{N} M_{ab} \log(|x-y|^2) + \mathcal{O} \left(\frac{1}{N^2} \right) \right]
\end{aligned}$$

$$\begin{aligned}
M &= \frac{1}{\langle (\lambda^{(0)})^2 \rangle} \begin{pmatrix} 0 & \sqrt{\langle (\lambda^{(0)})^2 (\gamma^{(1)})^2 \rangle_{\text{grav}} \langle (\lambda^{(0)})^2 \rangle} \\ \sqrt{\langle (\lambda^{(0)})^2 (\gamma^{(1)})^2 \rangle_{\text{grav}} \langle (\lambda^{(0)})^2 \rangle} & \langle (\lambda^{(0)})^2 \gamma^{(1)} \rangle \end{pmatrix} \\
&= \begin{pmatrix} 0 & \frac{4}{\Delta} \sqrt{\frac{3}{5}} \sqrt{(6-5\Delta)(5-6\Delta)} \\ \frac{4}{\Delta} \sqrt{\frac{3}{5}} \sqrt{(6-5\Delta)(5-6\Delta)} & \frac{12(1-\Delta)}{\Delta} \end{pmatrix}
\end{aligned}$$

$$\gamma_1^{(1)} = \frac{6}{\Delta} \left(1 - \Delta - \frac{1}{\sqrt{15}} \sqrt{135 - 274\Delta + 135\Delta^2} \right)$$

$$\gamma_2^{(1)} = \frac{6}{\Delta} \left(1 - \Delta + \frac{1}{\sqrt{15}} \sqrt{135 - 274\Delta + 135\Delta^2} \right)$$

$$\left(\lambda_{2,2,\mathcal{O}_1}^{(0)}\right)^2 = \frac{2}{3} \left(\frac{-\sqrt{15}(1-\Delta) + \sqrt{135 - 274\Delta + 135\Delta^2}}{2\sqrt{135 - 274\Delta + 135\Delta^2}} \right)$$

$$\left(\lambda_{2,2,\mathcal{O}_2}^{(0)}\right)^2 = \frac{2}{3} \left(\frac{\sqrt{15}(1-\Delta) + \sqrt{135 - 274\Delta + 135\Delta^2}}{2\sqrt{135 - 274\Delta + 135\Delta^2}} \right)$$

$$\gamma_1^{(1)} = -106.132 \dots, \gamma_2^{(1)} = 72.274 \dots, \gamma_3^{(1)} = -4.542 \dots$$

$$\left(\lambda_{2,2,\mathcal{O}_1}^{(0)}\right)^2 = 0.0086 \dots, \quad \left(\lambda_{2,2,\mathcal{O}_2}^{(0)}\right)^2 = 0.0016 \dots, \quad \left(\lambda_{2,2,\mathcal{O}_3}^{(0)}\right)^2 = 0.0898 \dots,$$

$$\left(\lambda_{3,3,\mathcal{O}_1}^{(0)}\right)^2 = 0.1283 \dots, \quad \left(\lambda_{3,3,\mathcal{O}_2}^{(0)}\right)^2 = 0.0985 \dots, \quad \left(\lambda_{3,3,\mathcal{O}_3}^{(0)}\right)^2 = 0.0232 \dots$$

$$28 \otimes 28 = 1 \oplus 28 \oplus 35_v \oplus 35_c \oplus 35_s \oplus 300 \oplus 350$$

$$P_1 = \begin{pmatrix} \frac{1}{28} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot t$$

$$P_{28} = \begin{pmatrix} 0 & 0 & 0 & -\frac{1}{12} & \frac{1}{12} & 0 & 0 \end{pmatrix} \cdot t$$

$$P_{35_c} = \begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & 0 & \frac{1}{12} & -\frac{1}{12} & \frac{1}{2} \end{pmatrix} \cdot t$$

$$P_{35_s} = \begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & 0 & \frac{1}{12} & -\frac{1}{12} & -\frac{1}{2} \end{pmatrix} \cdot t$$

$$P_{35_v} = \begin{pmatrix} -\frac{1}{12} & 0 & 0 & \frac{1}{12} & -\frac{1}{12} & \frac{1}{6} & 0 \end{pmatrix} \cdot t$$

$$P_{350} = \begin{pmatrix} 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{12} & -\frac{1}{12} & 0 & 0 \end{pmatrix} \cdot t$$

$$P_{300} = \begin{pmatrix} \frac{1}{21} & \frac{1}{3} & \frac{1}{3} & -\frac{1}{12} & -\frac{1}{12} & 0 & 0 \end{pmatrix} \cdot t$$

$$t_i = \left(\delta^{AB} \delta^{CD}, \delta^{AC} \delta^{BD}, \delta^{AD} \delta^{BC}, f^{ACE} f^{BDE}, f^{ADE} f^{BCE} \right. \\ \left. \text{tr}(T^A T^B T^C T^D), \frac{1}{4 \cdot 4!} \epsilon_{a_1 a_2 b_1 b_2 c_1 c_2 d_1 d_2} T_{a_1 a_2}^A T_{b_1 b_2}^B T_{c_1 c_2}^C T_{d_1 d_2}^D \right).$$

$$M_{F^4}^2 = t_6 + \frac{1}{3}t_4 - \frac{2}{3}t_5, M_{F^4}^3 = t_7$$

$$-\partial_{\mu_1}^4 F|_{\mu=0} = k^2 I[M_1 + M_2 + M_3 + 2M_6],$$

$$-\partial_{\mu_1}^2 \partial_{\mu_2}^2 F|_{\mu=0} = \frac{k^2}{3} I[M_1 + M_2 + M_3],$$

$$-\partial_{\mu_1} \partial_{\mu_2} \partial_{\mu_3} \partial_{\mu_4} F|_{\mu=0} = \frac{k^2}{6} I[M_7],$$



$$I[M_r] \equiv - \int \frac{dsdt}{(4\pi i)^2} [M_r(s,t) \Gamma[2-s/2] \Gamma[s/2] \Gamma[2-t/2] \Gamma[t/2] \Gamma[2-u/2] \Gamma[u/2] \\ \times \left(\frac{H_{\frac{s}{2}-1} + H_{1-\frac{s}{2}}}{(t-2)(u-2)} + \frac{H_{\frac{t}{2}-1} + H_{1-\frac{t}{2}}}{(s-2)(u-2)} + \frac{H_{\frac{u}{2}-1} + H_{1-\frac{u}{2}}}{(s-2)(t-2)} \right)]$$

$$\mathcal{F}_v \equiv -4\partial_{\mu_1}^2 \partial_{\mu_2}^2 F|_{\mu=0}, \\ \mathcal{F}_c \equiv -\partial_{\mu_1}^4 F|_{\mu=0} - \partial_{\mu_1}^2 \partial_{\mu_2}^2 F|_{\mu=0} + 2\partial_{\mu_1} \partial_{\mu_2} \partial_{\mu_3} \partial_{\mu_4} F|_{\mu=0}, \\ \mathcal{F}_s \equiv -\partial_{\mu_1}^4 F|_{\mu=0} - \partial_{\mu_1}^2 \partial_{\mu_2}^2 F|_{\mu=0} - 2\partial_{\mu_1} \partial_{\mu_2} \partial_{\mu_3} \partial_{\mu_4} F|_{\mu=0},$$

$$S: \quad \tau \rightarrow -1/\tau \quad \Leftrightarrow \quad \mathbf{35}_v \leftrightarrow \mathbf{35}_c \quad \Leftrightarrow \quad \mathcal{F}_v \leftrightarrow \mathcal{F}_c, \\ T: \quad \tau \rightarrow \tau + 1 \quad \Leftrightarrow \quad \mathbf{35}_c \leftrightarrow \mathbf{35}_s \quad \Leftrightarrow \quad \mathcal{F}_c \leftrightarrow \mathcal{F}_s.$$

$$\mathcal{F}_v = -8\log [\tau_2|\theta_3(\tau)\theta_4(\tau)|^2] + 8\log [8\pi N] + 4f(N) + O(e^{-N}), \\ \mathcal{F}_c = -8\log [\tau_2|\theta_2(\tau)\theta_3(\tau)|^2] + 8\log [8\pi N] + 4f(N) + O(e^{-N}), \\ \mathcal{F}_s = -8\log [\tau_2|\theta_2(\tau)\theta_4(\tau)|^2] + 8\log [8\pi N] + 4f(N) + O(e^{-N}),$$

$$f(N) = \frac{16}{3} - 4\zeta(3) + 4\gamma - 4\log(4\pi) - 2\log(2N) + 2\psi(2N + 5/2) \\ + (N + 1/4)\psi^{(1)}(N + 5/4) + (N + 3/4)\psi^{(1)}(N + 7/4) \\ = \frac{22}{3} - 4\zeta(3) + 4\gamma - 4\log(4\pi) + \frac{1}{N} - \frac{7}{48N^2} - \frac{1}{48N^3} + \frac{67}{2560N^4} + O(N^{-5})$$

$$I[1] = \frac{1}{24}, I[\mathcal{B}(s,t)] = \frac{11}{1296} - \frac{\zeta(3)}{108} I\left[3H_{1-\frac{s}{2}} + \frac{4}{s-2}\right] = -\frac{11}{96},$$

$$b_{F^4}^1 = 6(\gamma - \log(4\pi)) - 3\log[\tau_2|\theta_3(\tau)\theta_4(\tau)|^2], \\ b_{F^4}^2 = -3\log[\tau_2|\theta_2(\tau)\theta_3(\tau)|^2] - 3\log[\tau_2|\theta_2(\tau)\theta_4(\tau)|^2] + 6\log[\tau_2|\theta_3(\tau)\theta_4(\tau)|^2], \\ b_{F^4}^3 = 18\log[\tau_2|\theta_2(\tau)\theta_3(\tau)|^2] - 18\log[\tau_2|\theta_2(\tau)\theta_4(\tau)|^2],$$

$$\int \frac{d\beta}{2\pi i} \frac{e^\beta}{\beta^4} \log(1/(2\beta)) = -\frac{11}{36} + \frac{\gamma}{6} - \frac{\log(2)}{6}$$

$$\mathcal{A}_{F^4} = a_{F^4}^1(t_1+t_2+t_3) + a_{F^4}^2\left(t_6+\frac{1}{3}t_4-\frac{2}{3}t_5\right) + a_{F^4}^1t_7$$

$$a_{F^4}^1 = -\frac{16}{3}\pi^6(6\log(2\pi)-11)-16\pi^6\log[\tau_2|\theta_3(\tau)\theta_4(\tau)|^2] \\ a_{F^4}^2 = -16\pi^6\log[\tau_2|\theta_2(\tau)\theta_3(\tau)|^2]-16\pi^6\log[\tau_2|\theta_2(\tau)\theta_4(\tau)|^2]+32\pi^6\log[\tau_2|\theta_3(\tau)\theta_4(\tau)|^2] \\ a_{F^4}^3 = 96\pi^6\log[\tau_2|\theta_2(\tau)\theta_3(\tau)|^2]-96\pi^6\log[\tau_2|\theta_2(\tau)\theta_4(\tau)|^2]$$

$$\gamma_{1,\ell}^{(2)} = -\frac{8(\ell^6+15\ell^5+56\ell^4+7\ell^3+357\ell^2+2360\ell+1536)}{\ell(\ell+1)^3(\ell+4)^3(\ell+5)} \\ \gamma_{35,*,\ell}^{(2)} = -\frac{8(\ell^6+15\ell^5+82\ell^4+203\ell^3+295\ell^2+400\ell+192)}{\ell(\ell+1)^3(\ell+4)^3(\ell+5)} \\ \gamma_{300,\ell}^{(2)} = -\frac{8(\ell^6+15\ell^5+91\ell^4+287\ell^3+511\ell^2+505\ell+192)}{\ell(\ell+1)^3(\ell+4)^3(\ell+5)}$$



$$\gamma_{28,\ell}^{(2)} = -\frac{8(\ell^6 + 15\ell^5 + 77\ell^4 + 163\ell^3 + 273\ell^2 + 665\ell + 384)}{\ell(\ell+1)^3(\ell+4)^3(\ell+5)}$$

$$\gamma_{350,\ell}^{(2)} = -\frac{8(\ell+2)(\ell+3)}{\ell(\ell+1)(\ell+4)(\ell+5)}$$

$$\gamma_{300,0}^{(2)} = \frac{12}{5} \left(2\log [\sqrt{\tau_2} |\eta(\tau)|^2] - 2\gamma + 2\log \pi + 5\log 2 \right) + \frac{723}{200},$$

$$\gamma_{35,0}^{(2)} = -\frac{6}{5} \left(8\log [\sqrt{\tau_2} |\eta(\tau)|^2] - 12\log [\sqrt{\tau_2} |\theta_2(\tau)|^2] + 4\gamma - 4\log \pi + 2\log 2 \right) + \frac{126}{25},$$

$$\gamma_{35,0}^{(2)} = -\frac{6}{5} \left(8\log [\sqrt{\tau_2} |\eta(\tau)|^2] - 12\log [\sqrt{\tau_2} |\theta_4(\tau)|^2] + 4\gamma - 4\log \pi + 2\log 2 \right) + \frac{126}{25},$$

$$\gamma_{35,0}^{(2)} = -\frac{6}{5} \left(8\log [\sqrt{\tau_2} |\eta(\tau)|^2] - 12\log [\sqrt{\tau_2} |\theta_3(\tau)|^2] + 4\gamma - 4\log \pi + 2\log 2 \right) + \frac{126}{25},$$

$$y_1^2+y_2^2+y_3^2+y_4^2=1-\rho^2, y_5^2+y_6^2=\rho^2$$

$$SO(6) \cong SU(4) \rightarrow SO(4) \times SO(2) \cong SU(2)_L \times SU(2)_R \times U(1)$$

$$\frac{\int_{S^D} d\Omega_D y_{I_1} y_{I_2} \dots y_{I_{2n}}}{\text{Vol}(S^D)} (\delta_{I_1 I_2} \dots \delta_{I_{2n-1} I_{2n}} + \not\!\!\!\!\!\! \delta_{\text{all other contractions}})$$

$$y^{\bar{\alpha}\alpha}=(\sigma_i)^{\bar{\alpha}\alpha}y_i$$

$$(\sigma_i)^{\bar{\alpha}\alpha}(\sigma_i)^{\bar{\beta}\beta}=2\epsilon^{\bar{\alpha}\bar{\beta}}\epsilon^{\alpha\beta}$$

$$\int_{\mathbb{S}^5} d\Omega_5 y^{\bar{\alpha}_1\alpha_1} \dots y^{\bar{\alpha}_{2n}\alpha_{2n}} f(\rho^2)$$

$$= \frac{2\pi^3}{(n+1)! \Delta} \left(\int_0^1 dt (1-t)^{n+1} f(t) \right) (\epsilon^{\bar{\alpha}_1\bar{\alpha}_2} \epsilon^{\alpha_1\alpha_2} \dots \epsilon^{\bar{\alpha}_{2n-1}\bar{\alpha}_{2n}} \epsilon^{\alpha_{2n-1}\alpha_{2n}} + \not\!\!\!\!\!\! \delta_{\text{all other contractions}})$$

$$\epsilon^{\bar{\alpha}_1\bar{\alpha}_2}\epsilon^{\alpha_1\alpha_2}\epsilon^{\bar{\alpha}_3\bar{\alpha}_4}\epsilon^{\alpha_3\alpha_4}+\epsilon^{\bar{\alpha}_1\bar{\alpha}_3}\epsilon^{\alpha_1\alpha_3}\epsilon^{\bar{\alpha}_2\bar{\alpha}_4}\epsilon^{\alpha_2\alpha_4}+\epsilon^{\bar{\alpha}_1\bar{\alpha}_4}\epsilon^{\alpha_1\alpha_4}\epsilon^{\bar{\alpha}_2\bar{\alpha}_3}\epsilon^{\alpha_2\alpha_3}$$

$$\int_{S^3} d\Omega_3 y^{\bar{\alpha}_1\alpha_1} \dots y^{\bar{\alpha}_{2n}\alpha_{2n}} = \frac{2\pi^2}{(n+1)!} (\epsilon^{\bar{\alpha}_1\bar{\alpha}_2} \epsilon^{\alpha_1\alpha_2} \dots \epsilon^{\bar{\alpha}_{2n-1}\bar{\alpha}_{2n}} \epsilon^{\alpha_{2n-1}\alpha_{2n}} + \not\!\!\!\!\!\! \delta_{\text{all other contractions}})$$

$$k_p=\begin{cases} p,p+2,p+4,\ldots & p=2,3\\ p-2,p,p+2,\ldots & p>3 \end{cases}$$

$$S_{I_1I_2\ldots I_k}y_{I_1}y_{I_2}\cdots y_{I_k}$$

$$k_p\colon s_{i_1i_2\ldots i_{p-2}}y_{i_1}y_{i_2}\cdots y_{i_{p-2}}(1-\rho^2)^{(k_p-p+2)/2}-(SO(6)\mathfrak{I}),$$

$$k_p-(p-2)=0,2,4,\ldots$$

$$k_p\colon s_{i_1i_2\ldots i_{p-2}}y_{i_1}y_{i_2}\cdots y_{i_{p-2}}(1-\rho^2)^{(k_p-p+2)/2}f_{k_p}\left(\frac{\rho^2}{1-\rho^2}\right)$$



$$(k_p - p + 2)(k_p + p)f_{k_p}(z) + 4(1 - k_p z)f'_{k_p}(z) + 4z(1 + z)f''_{k_p}(z) = 0$$

$$f_{k_p}(z) = {}_2F_1\left(-\frac{k_p + p}{2}, \frac{p - k_p - 2}{2}, 1; -z\right)$$

$$s_{(k_p)}(x, y) = \frac{1}{\mathcal{N}(k_p)} (s_{(k_p)})_{\tilde{\alpha}_1 \dots \tilde{\alpha}_{p-2}, \alpha_1 \dots \alpha_{p-2}}(x) y^{\tilde{\alpha}_1 \alpha_1} \dots y^{\tilde{\alpha}_{p-2} \alpha_{p-2}} {}_2F_1\left(\frac{p + k_p + 2}{2}, \frac{p - k_p - 2}{2}, 1; \rho^2\right),$$

$$\delta g_{\mu\nu} = \frac{2}{k_p + 1} \left(\nabla_\mu \nabla_\nu + \nabla_\nu \nabla_\mu - \frac{2}{5} g_{\mu\nu} \square \right) s_{(k_p)} - \frac{6}{5} k_p g_{\mu\nu} s_{(k_p)}$$

$$\delta g_{ab} = 2k_p g_{ab} s_{(k_p)}$$

$$\delta g_{\mu a} = 0$$

$$\delta C = \frac{1}{6} \left(\varepsilon_{\mu\nu\rho\sigma\tau} \left(\nabla^\tau s_{(k_p)} \right) dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma - \varepsilon_{abcde} \left(\nabla^e s_{(k_p)} \right) dy^a \wedge dy^b \wedge dy^c \wedge dy^d \right)$$

$$\mathcal{N}(k_p)^2 \int_{\tilde{S}^5} d\Omega_5 \left(s_{(k_p)} \right)^2 = \frac{2\pi^3}{(p-1)\Delta k_p + 2} \left(s_{(k_p)} \right)_{\tilde{\alpha}_1 \dots \tilde{\alpha}_{p-2}, \alpha_1 \dots \alpha_{p-2}} \left(s_{(k_p)} \right)^{\tilde{\alpha}_1 \dots \tilde{\alpha}_{p-2}, \alpha_1 \dots \alpha_{p-2}}$$

$$\int_0^1 dt (1-t)^{p-1} \left[{}_2F_1\left(\frac{p+k_p+2}{2}, \frac{p-k_p-2}{2}; 1; t\right) \right]^2 = \frac{1}{k_p+2}$$

$$\mathcal{N}(k_p)^2 = \frac{32\pi^3}{\Delta\kappa^2} \frac{k_p(k_p-1)}{(k_p+1)(p-1)},$$

$$S_{\text{kinetic}}^{(k_p)} = \int_{\text{AdS}_5} d^5x \sqrt{-g_{\text{AdS}}} \left(-\frac{1}{2} \nabla_\mu \left(s_{(k_p)} \right)_{\tilde{\alpha}_1 \dots \tilde{\alpha}_{p-2}, \alpha_1 \dots \alpha_{p-2}} \nabla^\mu \left(s_{(k_p)} \right)^{\tilde{\alpha}_1 \dots \tilde{\alpha}_{p-2}, \alpha_1 \dots \alpha_{p-2}} \right. \\ \left. - \frac{1}{2} k_p (k_p - 4) \left(s_{(k_p)} \right)_{\tilde{\alpha}_1 \dots \tilde{\alpha}_{p-2}, \alpha_1 \dots \alpha_{p-2}} \left(s_{(k_p)} \right)^{\tilde{\alpha}_1 \dots \tilde{\alpha}_{p-2}, \alpha_1 \dots \alpha_{p-2}} \right)$$

$$\delta g_{\mu\nu} = \frac{2}{k_p + 1} \left(\nabla_\mu \nabla_\nu + \nabla_\nu \nabla_\mu - \frac{2}{5} g_{\mu\nu} \square \right) s_{(k_p)} - \frac{6}{5} k_p g_{\mu\nu} s_{(k_p)}$$

$$\delta g_{\hat{a}\hat{b}} = 2k_p g_{\hat{a}\hat{b}} s_{(k_p)}$$

$$\delta g_{\mu\hat{a}} = 0$$

$$\delta C = \frac{1}{6} \varepsilon_{\mu\nu\rho\sigma\tau} \left(\nabla^\tau s_{(k_p)} \right) dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma$$

$$s_{(k_p)}(x, y) = \frac{1}{\mathcal{N}(k_p)} (s_{(k_p)})_{\tilde{\alpha}_1 \dots \tilde{\alpha}_{p-2}, \alpha_1 \dots \alpha_{p-2}}(x) y^{\tilde{\alpha}_1 \alpha_1} \dots y^{\tilde{\alpha}_{p-2} \alpha_{p-2}}$$

$$g^{\mu\nu} \delta g_{\mu\nu} + g^{\hat{a}\hat{b}} \delta g_{\hat{a}\hat{b}} = 0$$

$$\nabla_{\hat{a}} \nabla^{\hat{a}} s_{k_p} = -p(p-2) s_{k_p}$$

$$V_p = \varphi_{\tilde{\alpha}_1 \dots \tilde{\alpha}_{p-2}, \beta \gamma \alpha_1 \dots \alpha_{p-2}}^{(p)}(x) y^{\tilde{\alpha}_1 \alpha_1} \dots y^{\tilde{\alpha}_{p-2} \alpha_{p-2}} \left((\sigma_j)^{\bar{\delta}(\beta} (\sigma_k)^{\gamma)}_{\bar{\delta}} \right) y_j dy_k$$

$$= \varphi_{\tilde{\alpha}_1 \dots \tilde{\alpha}_{p-2}, \beta \gamma \alpha_1 \dots \alpha_{p-2}}^{(p)}(x) y^{\tilde{\alpha}_1 \alpha_1} \dots y^{\tilde{\alpha}_{p-2} \alpha_{p-2}} (\sigma_j)^{\bar{\delta} \beta} (\sigma_k)^{\gamma}_{\bar{\delta}} y_j dy_k$$



$$\left((\sigma_i)^{\bar{\delta}(\beta}(\sigma_j)^{\gamma)}_{\bar{\delta}}\right)=-\left((\sigma_j)^{\bar{\delta}(\beta}(\sigma_i)^{\gamma\gamma)}_{\bar{\delta}}\right)=-\frac{1}{2}\epsilon_{ijkl}\left((\sigma_k)^{\bar{\delta}(\beta}(\sigma_l)^{\gamma)}_{\bar{\delta}}\right).$$

$$A=\frac{1}{\mathcal{B}(p)}V_p^*$$

$$\begin{array}{l} \nabla_{\hat{b}}\nabla^{\hat{b}}A_{\hat{a}}=(2-p^2)A_{\hat{a}}\\ \epsilon_{\hat{a}\hat{b}\hat{c}}\nabla^{\hat{b}}A^{\hat{c}}=-pA_{\hat{a}}\\ \nabla_{\hat{a}}A^{\hat{a}}=0 \end{array}$$

$$\begin{aligned} g^{\hat{a}\hat{b}}(V_p^*)_{\hat{a}}(V_q^*)_{\hat{b}} &= (V_p)_i(V_q)_i \\ &= 2\varphi_{\bar{\alpha}_1\ldots\bar{\alpha}_{p-2},\gamma_1\gamma_2\alpha_1\ldots\alpha_{p-2}}^{(p)}\varphi_{\bar{\beta}_1\ldots\bar{\beta}_{q-2},\frac{\gamma_1\gamma_2}{\beta_1\ldots\beta_{q-2}}}^{(q)}y^{\bar{\alpha}_1\alpha_1}\ldots y^{\bar{\alpha}_{p-2}\alpha_{p-2}}y^{\bar{\beta}_1\beta_1}\ldots y^{\bar{\beta}_1\beta_{q-2}} \end{aligned}$$

$$\begin{aligned} S_{\text{kinetic}}^{(p)} &= \mu \text{Tr} \int_{\text{AdS}_5 \times S^3} \left(-\frac{1}{2} F \wedge \star_8 F + \frac{1}{2} C_4 \wedge F \wedge F \right) \Bigg|_{\text{quadratic}} \\ &= \text{Tr} \int_{\text{AdS}_5} d^5x \sqrt{-g_{\text{AdS}}} \left(-\frac{1}{2} \nabla_\mu (\varphi_{(p)})_{\bar{\alpha}_1 \ldots \bar{\alpha}_{p-2}, \alpha_1 \ldots \alpha_p} \nabla^\mu (\varphi_{(p)})^{\bar{\alpha}_1 \ldots \bar{\alpha}_{p-2}, \alpha_1 \ldots \alpha_p} \right. \\ &\quad \left. - \frac{1}{2} p(p-4) (\varphi_{(p)})_{\bar{\alpha}_1 \ldots \bar{\alpha}_{p-2}, \alpha_1 \ldots \alpha_p} (\varphi_{(p)})^{\bar{\alpha}_1 \ldots \bar{\alpha}_{p-2}, \alpha_1 \ldots \alpha_p} \right) \end{aligned}$$

$$\mathcal{B}(p)^2=\frac{4\pi^2}{p-1}\mu$$

$$pqk_r$$

$$r=|p-q|+2,|p-q|+4,\ldots,p+q-2.$$

$$\begin{aligned}\nabla_\mu\nabla_\nu f_1\nabla^\mu f_2\nabla^\nu f_3&=\frac{1}{4}((\Box^2f_1)f_2f_3-f_1(\Box^2f_2)f_3-f_1f_2(\Box^2f_3)+2f_1(\Box f_2)(\Box f_3))+\nabla_\mu(\ldots),\\ f_1\nabla^\mu f_2\nabla_\mu f_3&=\frac{1}{2}((\Box f_1)f_2f_3-f_1(\Box f_2)f_3-f_1f_2(\Box f_3))+\nabla_\mu(\ldots),\end{aligned}$$

$$\begin{aligned} S_{\text{cubic}}^{(p,q,k_r)} &= \text{Tr} \int_{\text{AdS}_5} d^5x \sqrt{-g_{\text{AdS}}} \\ &\quad \times \frac{1}{1+\delta_{p,q}} \beta_{pqk_r}(\varphi_{(p)})_{\bar{\alpha}_1\ldots\bar{\alpha}_{(p+q-r-2)/2}\bar{\beta}_1\ldots\bar{\beta}_{(p-q+r-2)/2},\gamma\delta\alpha_1\ldots\alpha_{(p+q-r-2)/2}\beta_1\ldots\beta_{(p-q+r-2)/2}} \\ &\quad \times (\varphi_{(q)})^{\bar{\alpha}_1\ldots\bar{\alpha}_{(p+q-r-2)/2}\bar{\beta}_{(p-q+r)/2}\ldots\bar{\beta}_{r-2},\,\gamma\delta\alpha_1\ldots\alpha_{(p+q-r-2)/2}\beta_{(p-q+r)/2}\ldots\beta_{r-2}} \\ &\quad \times (s_{(k_r)})^{\bar{\beta}_1\ldots\bar{\beta}_{r-2},\beta_1\ldots\beta_{r-2}} \end{aligned}$$

$$\begin{aligned} \beta_{pqk_r} &= \frac{4\pi^2\mu}{\mathcal{N}(k_r)\mathcal{B}(p)\mathcal{B}(q)}\frac{(k_r+p-q)(k_r+q-p)(k_r+p+q-2)(k_r+p+q-4)}{k_r+1} \\ &\quad \times \frac{\Gamma(p-1)\Gamma(q-1)\Gamma(r-1)}{\Gamma\left(\frac{p+q-r}{2}\right)\Gamma\left(\frac{r+p-q}{2}\right)\Gamma\left(\frac{r+q-p}{2}\right)\Gamma\left(\frac{p+q+r-2}{2}\right)} \\ &\quad : \phi_p\,\Box^{n-p}\,\phi_p:, p=2,3,\ldots,n, \end{aligned}$$



$$\Delta_{n,a} = 2n + \frac{1}{N} \gamma_{n,a}^{(1)} + \frac{1}{N^2} \gamma_{n,a}^{(2)} + \mathcal{O}\left(\frac{1}{N^3}\right)$$

$$\lambda_{p,p,\mathcal{O}_{n,a}} = \lambda_{p,p,\mathcal{O}_{n,a}}^{(0)} + \frac{1}{N} \lambda_{p,p,\mathcal{O}_{n,a}}^{(1)} + \frac{1}{N^2} \lambda_{p,p,\mathcal{O}_{n,a}}^{(2)} + \mathcal{O}\left(\frac{1}{N^3}\right)$$

$$\begin{aligned} & \langle \phi_p^A(y_1, x_1) \phi_p^B(y_2, x_2) \phi_q^C(y_3, \bar{y}_3, x_3) \phi_q^D(y_4, \bar{y}_4, x_4) \rangle \\ &= \frac{\langle y_1, y_2 \rangle^p \langle y_3, y_4 \rangle^q \langle \bar{y}_1, \bar{y}_2 \rangle^{p-2} \langle \bar{y}_3, \bar{y}_4 \rangle^{q-2}}{x_{12}^{2p} x_{34}^{q q}} \sum_{\mathbf{r} \in \mathfrak{g} \otimes \mathfrak{g}} G_{\mathbf{r}}^{p,q}(U, V; w) P_{\mathbf{r}}^{ABCD} \end{aligned}$$

$$G_{\mathbf{r}}^{p,q}(U,V;w)=\frac{z(w-\bar{z})f_r(\bar{z})-\bar{z}(w-z)f_r(z)}{w(z-\bar{z})}+\left(1-\frac{z}{w}\right)\left(1-\frac{\bar{z}}{w}\right)\mathcal{G}_{\mathbf{r}}^{p,q}(U,V).$$

$$\begin{aligned} \mathcal{G}_1^{p,q}(U,V) &= \sum_{n=2,3,\dots} U^{-1} \left[\left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)} \lambda_{q,q,\mathcal{O}_n}^{(0)} \right\rangle \right. \\ &\quad + \frac{1}{N} \left(\left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)} \lambda_{q,q,\mathcal{O}_n}^{(1)} + \lambda_{p,p,\mathcal{O}_n}^{(1)} \lambda_{q,q,\mathcal{O}_n}^{(0)} \right\rangle + \frac{1}{2} \left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)} \gamma_n^{(1)} \lambda_{q,q,\mathcal{O}_n}^{(0)} \right\rangle (\log U + 2\partial_\Delta) \right) \\ &\quad + \frac{1}{N^2} \left(\left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)} \lambda_{q,q,\mathcal{O}_n}^{(2)} + \lambda_{p,p,\mathcal{O}_n}^{(2)} \lambda_{q,q,\mathcal{O}_n}^{(0)} + \lambda_{p,p,\mathcal{O}_n}^{(1)} \lambda_{q,q,\mathcal{O}_n}^{(1)} \right\rangle \right. \\ &\quad + \frac{1}{2} \left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)} \gamma_n^{(1)} \lambda_{q,q,\mathcal{O}_n}^{(1)} + \lambda_{p,p,\mathcal{O}_n}^{(1)} \gamma_n^{(1)} \lambda_{q,q,\mathcal{O}_n}^{(0)} + \lambda_{p,p,\mathcal{O}_n}^{(0)} \gamma_n^{(2)} \lambda_{q,q,\mathcal{O}_n}^{(0)} \right\rangle (\log U + 2\partial_\Delta) \\ &\quad \left. + \frac{1}{8} \left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)} \left(\gamma_n^{(1)} \right)^2 \lambda_{q,q,\mathcal{O}_n}^{(0)} \right\rangle (\log^2 U + 4 \log U \partial_\Delta + 4 \partial_\Delta^2) \right) \\ &\quad + \mathcal{O}\left(\frac{1}{N^3}\right) \Big] g_{\Delta+2,0}(U,V) \Big|_{\Delta=2n} \\ &\quad + \dots \end{aligned}$$

$$\left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)} \gamma_n^{(1)} \lambda_{q,q,\mathcal{O}_n}^{(0)} \right\rangle = \sum_{a=1}^n \lambda_{p,p,\mathcal{O}_{n,a}}^{(0)} \gamma_{n,a}^{(1)} \lambda_{q,q,\mathcal{O}_{n,a}}^{(0)}$$

$$\left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)} \lambda_{q,q,\mathcal{O}_n}^{(0)} \right\rangle = a_n^p a_n^q \delta_{p,q}$$

$$\left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)} \gamma^{(1)} \lambda_{q,q,\mathcal{O}_n}^{(0)} \right\rangle = a_n^p a_n^q b_n^{p,q}$$

$$\left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)} \left(\gamma^{(1)} \right)^2 \lambda_{q,q,\mathcal{O}_n}^{(0)} \right\rangle = a_n^p a_n^q \left(\sum_{s=2}^n b_n^{p,s} b_n^{q,s} + c_n^p c_n^q \right).$$

$$\begin{aligned} & \left(\begin{array}{cc} \langle \rho_{2n}(x) \rho_{2n}(y) \rangle & \langle \rho_{2n}(x) : \phi_q \square^{n-q} \phi_q : (y) \rangle \\ \langle : \phi_p \square^{n-p} \phi_p : (x) \rho_{2n}(y) \rangle & \langle : \phi_p \square^{n-p} \phi_p : (x) : \phi_q \square^{n-q} \phi_q : (y) \rangle \end{array} \right) \\ &= \frac{1}{|x-y|^{4n}} \left[\delta_{ab} - \frac{1}{N} (M_n)_{ab} \log(|x-y|^2) + \mathcal{O}\left(\frac{1}{N^2}\right) \right] \end{aligned}$$

$$M_n = \begin{pmatrix} d_n & c_n^2 & c_n^3 & \dots & c_n^n \\ c_n^2 & b_n^{2,2} & b_n^{2,3} & \dots & b_n^{2,n} \\ c_n^3 & b_n^{3,2} & b_n^{3,3} & \dots & b_n^{3,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_n^n & b_n^{n,2} & b_n^{n,3} & \dots & b_n^{n,n} \end{pmatrix}$$



$$d_n = 0$$

$$\langle \gamma_n^{(1)} \rangle = \sum_{p=2}^n \frac{\langle \lambda_{p,p,\mathcal{O}_n}^{(0)} \gamma_n^{(1)} \lambda_{p,p,\mathcal{O}_n}^{(0)} \rangle}{\langle (\lambda_{p,p,\mathcal{O}_n}^{(0)})^2 \rangle}$$

$$\gamma_{n,a}^{(1)} = \mu_{n,a}$$

$$\lambda_{p,p,\mathcal{O}_{n,a}}^{(0)} = a_n^p (v_{n,a})^p$$

$$\langle \lambda_{p,p,\mathcal{O}_n}^{(0)} \lambda_{p,p,\mathcal{O}_n}^{(0)} \rangle = (a_n^p)^2 = \frac{2(2n-1)\Gamma(n)^4}{(n^2-1)\Gamma(2n)^2} \frac{\Gamma(n+p-1)\Gamma(n+p)}{\Gamma(p)^4\Gamma(n-p+2)\Gamma(n-p+1)}.$$

$$\begin{aligned} \langle \lambda_{2,2,\mathcal{O}_n}^{(0)} \gamma^{(1)} \lambda_{q,q,\mathcal{O}_n}^{(0)} \rangle &= a_n^2 a_n^q b_n^{2,q} = -8 \frac{h^\vee}{(\mathbf{k}/N)} \frac{(-1)^q (q-1)n(2n-1)\Gamma(n)^4\Gamma(n+q-1)}{\Gamma(2n)^2\Gamma(q)^2\Gamma(n-q+1)} \\ &= \frac{24(1-\Delta)}{\Delta} \frac{(-1)^q (q-1)n(2n-1)\Gamma(n)^4\Gamma(n+q-1)}{\Gamma(2n)^2\Gamma(q)^2\Gamma(n-q+1)} \end{aligned}$$

$$a_2^2 = \sqrt{\frac{2}{3}}$$

$$\langle \lambda_{2,2,\mathcal{O}_2}^{(0)} \gamma^{(1)} \lambda_{2,2,\mathcal{O}_2}^{(0)} \rangle = (a_2^2)^2 b_2^{2,2} = 8 \frac{(1-\Delta)}{\Delta}.$$

$$\langle \lambda_{2,2,\mathcal{O}_2}^{(0)} (\gamma^{(1)})^2 \lambda_{2,2,\mathcal{O}_2}^{(0)} \rangle_{\text{grav}} = (a_2^2)^2 (c_2^2)^2 = \frac{16 \dim(G_F)}{5} \frac{1}{\Delta} = \frac{32}{5} \frac{(6-5\Delta)(5-6\Delta)}{\Delta^2},$$

$$a_3^2 = \sqrt{\frac{1}{10}}, a_3^3 = \sqrt{\frac{1}{4}}$$

$$\begin{aligned} \langle \lambda_{2,2,\mathcal{O}_3}^{(0)} \gamma^{(1)} \lambda_{2,2,\mathcal{O}_3}^{(0)} \rangle &= (a_3^2)^2 b_3^{2,2} = \frac{12}{5} \frac{(1-\Delta)}{\Delta} \\ \langle \lambda_{2,2,\mathcal{O}_3}^{(0)} \gamma^{(1)} \lambda_{3,3,\mathcal{O}_3}^{(0)} \rangle &= a_3^2 a_3^3 b_3^{2,3} = -\frac{24}{5} \frac{(1-\Delta)}{\Delta}. \end{aligned}$$

$$\langle \lambda_{3,3,\mathcal{O}_3}^{(0)} \gamma^{(1)} \lambda_{3,3,\mathcal{O}_3}^{(0)} \rangle = (a_3^3)^2 b_3^{3,3} = \frac{66}{5} \frac{(1-\Delta)}{\Delta}.$$

$$\begin{aligned} \langle \lambda_{2,2,\mathcal{O}_3}^{(0)} (\gamma^{(1)})^2 \lambda_{2,2,\mathcal{O}_3}^{(0)} \rangle_{\text{grav}} &= (a_3^2)^2 (c_3^2)^2 = \frac{864 \dim(G_F)}{175} \frac{1}{\Delta} = \frac{1728}{175} \frac{(6-5\Delta)(5-6\Delta)}{\Delta^2}, \\ \langle \lambda_{2,2,\mathcal{O}_3}^{(0)} (\gamma^{(1)})^2 \lambda_{3,3,\mathcal{O}_3}^{(0)} \rangle_{\text{grav}} &= a_3^2 a_3^3 c_3^2 c_3^3 = -\frac{864 \dim(G_F)}{35} \frac{1}{\Delta} = -\frac{1728}{35} \frac{(6-5\Delta)(5-6\Delta)}{\Delta^2}. \end{aligned}$$



$$M_3 = \begin{pmatrix} 0 & \frac{24}{\Delta} \sqrt{\frac{6}{35}} \sqrt{(6-5\Delta)(5-6\Delta)} & -\frac{48}{\Delta} \sqrt{\frac{3}{7}} \sqrt{(6-5\Delta)(5-6\Delta)} \\ \frac{24}{\Delta} \sqrt{\frac{6}{35}} \sqrt{(6-5\Delta)(5-6\Delta)} & 24 \frac{(1-\Delta)}{\Delta} & -48 \sqrt{\frac{2}{5}} \frac{(1-\Delta)}{\Delta} \\ -\frac{48}{\Delta} \sqrt{\frac{3}{7}} \sqrt{(6-5\Delta)(5-6\Delta)} & -48 \sqrt{\frac{2}{5}} \frac{(1-\Delta)}{\Delta} & \frac{264}{5} \frac{(1-\Delta)}{\Delta} \end{pmatrix}$$

Supergravedad cuántica relativista Tipo IIA: Modelamiento Matemático.

$$S_0 = -\frac{2}{\kappa_{11}^2} \left[\int d^{11}x \sqrt{-g} \left(R - \frac{1}{2 \cdot 4!} F_{abcd} F^{abcd} \right) - \frac{1}{6} \int A^{(3)} \wedge F^{(4)} \wedge F^{(4)} \right],$$

$$g_{ab} = e^{-2\Phi/3} \begin{pmatrix} G_{\mu\nu} + e^{2\Phi} C_\mu C_\nu & e^{2\Phi} C_\mu \\ e^{2\Phi} C_\nu & e^{2\Phi} \end{pmatrix}; g^{ab} = e^{2\Phi/3} \begin{pmatrix} G^{\mu\nu} & -C^\mu \\ -C^\nu & e^{-2\Phi} + C_\alpha C^\alpha \end{pmatrix}$$

$$A_{\mu\nu\alpha} = C_{\mu\nu\alpha}; A_{\mu\nu\gamma} = B_{\mu\nu}$$

$$\sqrt{-g} = e^{-8\Phi/3} \sqrt{-G}$$

$$R = e^{2\Phi/3} \left(R - \frac{16}{3} \nabla_\mu \Phi \nabla^\mu \Phi + \frac{14}{3} \nabla_\mu \nabla^\mu \Phi - \frac{1}{2 \cdot 2!} e^{2\Phi} F_{\mu\nu} F^{\mu\nu} \right)$$

$$\sqrt{-g} R = e^{-2\Phi} \sqrt{-G} \left(R + 4 \nabla_\mu \Phi \nabla^\mu \Phi - \frac{1}{2 \cdot 2!} e^{2\Phi} F_{\mu\nu} F^{\mu\nu} \right)$$

$$-\frac{1}{2 \cdot 4!} \sqrt{-g} F_{abcd} F^{abcd} = e^{-2\Phi} \sqrt{-G} \left(-\frac{1}{2 \cdot 3!} H_{\mu\nu\alpha} H^{\mu\nu\alpha} - \frac{1}{2 \cdot 4!} e^{2\Phi} \bar{F}_{\mu\nu\alpha\beta} \bar{F}^{\mu\nu\alpha\beta} \right)$$

$$-\frac{1}{6} A^{(3)} \wedge F^{(4)} \wedge F^{(4)} = -\frac{1}{2} e^{-2\Phi} [B^{(2)} \wedge e^\Phi F^{(4)} \wedge e^\Phi F^{(4)}]$$

$$S_0 = -\frac{2}{\kappa_{10}^2} \left[\int d^{10}x \sqrt{-G} \left(e^{-2\Phi} \left(R + 4 \nabla_\mu \Phi \nabla^\mu \Phi - \frac{1}{2} |H|^2 \right) - \frac{1}{2} |F^{(2)}|^2 - \frac{1}{2} |\bar{F}^{(4)}|^2 \right) - \frac{1}{2} \int B^{(2)} \wedge F^{(4)} \wedge F^{(4)} \right]$$

$$S_6 = -\frac{2}{\kappa_{11}^2} \frac{\pi^2 \ell_p^6}{2^{11} \cdot 3^2} \int d^{11}x \sqrt{-g} \left[\left(t_8 t_8 - \frac{1}{4!} \epsilon_{11} \epsilon_{11} \right) R^4 + \dots \right]$$

$$t_8^{abcdefgh} = -2g^{af}g^{be}g^{gd}g^{ch} + 8g^{ad}g^{be}g^{gf}g^{ch} + 8g^{ah}g^{be}g^{gd}g^{cf} \\ + 8g^{ah}g^{bc}g^{gf}g^{de} - 2g^{ah}g^{bg}g^{cf}g^{de} - 2g^{ad}g^{bc}g^{gf}g^{eh}$$

$$S_6 = -\frac{2}{\kappa_{11}^2} \frac{\pi^2 \ell_p^6}{2^3 \cdot 3} \int d^{11}x \sqrt{-g} \left[\frac{1}{4} R_{ab}{}^{de} R^{abcf} R_{cd}{}^{gh} R_{fegh} - R_{ab}{}^{de} R^{abcf} R_c{}^g{}_d{}^h R_{fgeh} \right. \\ \left. + \frac{1}{16} R_{ab}{}^{de} R^{abcf} R_{cf}{}^{gh} R_{degh} + \frac{1}{2} R_a{}^d{}_c{}^e R^{abcf} R_b{}^g{}_f{}^h R_{dgeh} \right. \\ \left. - R_{abc}{}^d R^{abcf} R_f{}^{egh} R_{dgeh} + \frac{1}{32} R_{abcf} R^{abcf} R_{degh} R^{degh} \right]$$



$$S_6 = -\frac{2}{\kappa^2} \frac{\pi^2 \ell_s^6}{2^3 \cdot 3} \int d^{10}x \sqrt{-G} \left[\frac{1}{4} R_{\alpha\beta}{}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_{\gamma\epsilon}{}^{\mu\nu} R_{\delta\epsilon\mu\nu} - R_{\alpha\beta}{}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_{\gamma}{}^{\mu}{}_{\epsilon}{}^{\nu} R_{\delta\mu\epsilon\nu} \right. \\ \left. + \frac{1}{16} R_{\alpha\beta}{}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_{\gamma\delta}{}^{\mu\nu} R_{\epsilon\epsilon\mu\nu} + \frac{1}{2} R_{\alpha}{}^{\epsilon}{}_{\gamma}{}^{\epsilon} R^{\alpha\beta\gamma\delta} R_{\beta}{}^{\mu}{}_{\delta}{}^{\nu} R_{\epsilon\mu\epsilon\nu} \right. \\ \left. - R_{\alpha\beta\gamma}{}^{\epsilon} R^{\alpha\beta\gamma\delta} R_{\delta}{}^{\epsilon\mu\nu} R_{\epsilon\mu\epsilon\nu} + \frac{1}{32} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} R_{\epsilon\epsilon\mu\nu} R^{\epsilon\epsilon\mu\nu} + \dots \right].$$

$$S_6 \sim -\frac{2}{\kappa^2} \frac{\pi^2 \ell_s^6}{2^3 \cdot 3} \int d^{10}x \sqrt{-G} \mathcal{L}$$

$$\begin{aligned} c_1 &= 2341/4608, c_2 = -209747/921600, c_3 = -1027/7200, c_4 = -880957/1843200 \\ c_5 &= 4625129/29491200, c_6 = -66793/28800, c_7 = -817/960, c_8 = 385121/230400 \\ c_9 &= -3433/46080, c_{10} = 194693/368640, c_{11} = 24653/19200, c_{12} = -979/1600 \\ c_{13} &= 4307/3840, c_{14} = 2979/1600, c_{15} = -43241/9600, c_{16} = 3351/3200, c_{17} = -1151/800 \\ c_{18} &= -30283/4800, c_{19} = 4, c_{20} = -7167/3200, c_{21} = -7/2, c_{22} = -3/2, c_{23} = 1/2 \\ c_{24} &= 89527/115200, c_{25} = 9721/115200, c_{26} = 1, c_{27} = 1027/640, c_{28} = -3849/3200 \\ c_{29} &= -5873/3840, c_{30} = -75041/38400, c_{31} = 17/128, c_{32} = 16717/20480, c_{33} = 3 \\ c_{34} &= 1/2, c_{35} = 7167/6400, c_{36} = -13/2, c_{37} = -3/4, c_{38} = -3/8, c_{39} = 0, c_{40} = -1 \\ c_{41} &= -1, c_{42} = 1/32, c_{43} = 1119/6400, c_{44} = 1693/3840, c_{45} = -4451/12800 \\ c_{46} &= -647/3200, c_{47} = -4193/7680, c_{48} = -3/2, c_{49} = 11/4, c_{50} = 11/2, c_{51} = -7/8 \\ c_{52} &= 3/2, c_{53} = -5/4, c_{54} = 1/16, c_{55} = 1/4, c_{56} = 0, c_{57} = 3391/11520 \\ c_{58} &= -173339/115200, c_{59} = 3449/800, c_{60} = -5/2, c_{61} = 73027/115200 \\ c_{62} &= 43031/57600, c_{63} = -267211/115200, c_{64} = -29399/230400, c_{65} = 246391/921600 \\ c_{66} &= -6197/3200, c_{67} = 1559/9600, c_{68} = 2159/9600, c_{69} = -8857/12800, c_{70} = 3/4 \\ c_{71} &= -1/4, c_{72} = 5/4, c_{73} = 14741/800, c_{74} = 19219/1600, c_{75} = 41/4, c_{76} = -6767/600 \\ c_{77} &= 6767/1200, c_{78} = -5497/400, c_{79} = -1217/400, c_{80} = 14781/3200, c_{81} = -57/16 \\ c_{82} &= -45/8, c_{83} = -186973/28800, c_{84} = 144301/115200, c_{85} = 58361/7680 \\ c_{86} &= -1711343/921600, c_{87} = -16457/1600, c_{88} = 23339/1600, c_{89} = -2259/1600 \\ c_{90} &= 12251/800, c_{91} = 181/128, c_{92} = -5167/400, c_{93} = 5, c_{94} = 3567/400, c_{95} = 3, c_{96} = 2 \end{aligned}$$



$c_{97} = 6367/200, c_{98} = 14377/1600, c_{99} = -681/128, c_{100} = -1, c_{101} = 1, c_{102} = -145/64,$
 $c_{103} = -5471/640, c_{104} = -973/1600, c_{105} = -5667/400, c_{106} = 3/2, c_{107} = 6367/400,$
 $c_{108} = -6167/3200, c_{109} = 4367/400, c_{110} = -3/4, c_{111} = 1, c_{112} = -6367/200,$
 $c_{113} = 141/100, c_{114} = -2761/800, c_{115} = -131/10, c_{116} = 9/2, c_{117} = 8/5,$
 $c_{118} = -125353/230400, c_{119} = 293867/1843200, c_{120} = -22253/19200, c_{121} = 5/16,$
 $c_{122} = 37/1152, c_{123} = 21661/57600, c_{124} = -5213/9600, c_{125} = -3827/1920, c_{126} = 1,$
 $c_{127} = 1129/800, c_{128} = -10889/14400, c_{129} = -849/3200, c_{130} = 1049/400,$
 $c_{131} = -2879/800, c_{132} = 2929/800, c_{133} = -7/2, c_{134} = -11/16, c_{135} = -3587/1920,$
 $c_{136} = 89/40, c_{137} = 2, c_{138} = 2081/800, c_{139} = -13/8, c_{140} = -1/5, c_{141} = -357/200,$
 $c_{142} = 0, c_{143} = -1/4, c_{144} = -5/16, c_{145} = -3/4, c_{146} = -1/2, c_{147} = -5, c_{148} = 0,$
 $c_{149} = 3, c_{150} = 1, c_{151} = 0, c_{152} = 3/2, c_{153} = 3/4, c_{154} = 0, c_{155} = 1/4, c_{156} = 0, c_{157} = 0,$
 $c_{158} = -14331/6400, c_{159} = -26579/800, c_{160} = 5647/160, c_{161} = -1501/60,$
 $c_{162} = 7041/100, c_{163} = -563/60, c_{164} = -2927/200, c_{165} = 125/4, c_{166} = 2127/200,$
 $c_{167} = -61/2, c_{168} = 85/2, c_{169} = 6927/100, c_{170} = 2271/80, c_{171} = -1543/80, c_{172} = 0,$
 $c_{173} = -77/4, c_{174} = -1531/80, c_{175} = -1255/64, c_{176} = 319/15, c_{177} = -3727/200,$
 $c_{178} = -49/8, c_{179} = 6927/200, c_{180} = -4527/1600, c_{181} = 2427/200, c_{182} = 61/8, c_{183} = 2,$
 $c_{184} = -6927/100, c_{185} = -552/5, c_{186} = 0, c_{187} = 65, c_{188} = 4/5, c_{189} = -727/16,$
 $c_{190} = -52291/3200, c_{191} = 467/20, c_{192} = -3/10, c_{193} = -1/5, c_{194} = 342/5,$
 $c_{195} = -1/16, c_{196} = 3/8, c_{197} = 4, c_{198} = -1/4, c_{199} = -39/5, c_{200} = 553/200,$
 $c_{201} = -13/5, c_{202} = 0, c_{203} = -27/16, c_{204} = -31/4, c_{205} = -5/2, c_{206} = -1779/25,$
 $c_{207} = -26781/800, c_{208} = 161/4, c_{209} = 173/8, c_{210} = -4559/100, c_{211} = -147/2,$
 $c_{212} = 129/2, c_{213} = -29, c_{214} = -2599/50, c_{215} = 353/40, c_{216} = -23/2,$
 $c_{217} = -811/50, c_{218} = 294/5, c_{219} = -979/50, c_{220} = -17383/200, c_{221} = -107,$
 $c_{222} = 157/2, c_{223} = 8/5, c_{224} = 53/10, c_{225} = -392/5, c_{226} = -499/100, c_{227} = 499/100,$
 $c_{228} = 557/100, c_{229} = -657/400, c_{230} = 1573/800, c_{231} = 5/2, c_{232} = -5/2,$
 $c_{233} = -121/5, c_{234} = -8/5, c_{235} = 8/5, c_{236} = -1/2, c_{237} = 13/4, c_{238} = 4/5, c_{239} = 133/5,$
 $c_{240} = -197/80, c_{241} = 2953/400, c_{242} = -512/5, c_{243} = 106, c_{244} = 168/5, c_{245} = -38,$
 $c_{246} = 19, c_{247} = 84/5, c_{248} = -342/5, c_{249} = -128/5, c_{250} = 5/4, c_{251} = 63/20, c_{252} = 15/4,$
 $c_{253} = 5/4, c_{254} = 0, c_{255} = -5/2, c_{256} = 71/40, c_{257} = -261/200, c_{258} = 252/5,$
 $c_{259} = -41/100, c_{260} = 39/160, c_{261} = -8/5, c_{262} = -32/5, c_{263} = 4/5, c_{264} = 16,$
 $c_{265} = 1071/200, c_{266} = -10, c_{267} = 807/20, c_{268} = 617/10, c_{269} = -103/5, c_{270} = -7,$
 $c_{271} = 13, c_{272} = -316/5, c_{273} = -264/5, c_{274} = 15029/14400, c_{275} = -6749/14400,$
 $c_{276} = 3389/9600, c_{277} = -1177/2304, c_{278} = 967/3840, c_{279} = -1667/960, c_{280} = 2387/960,$
 $c_{281} = -389/320, c_{282} = 1/4, c_{283} = -3/2, c_{284} = -3827/960, c_{285} = 5/2, c_{286} = 17/32,$
 $c_{287} = 3587/1920, c_{288} = -5/4, c_{289} = 1/4, c_{290} = -119/40, c_{291} = 3, c_{292} = 0,$
 $c_{293} = -11/10, c_{294} = -57/20, c_{295} = 5, c_{296} = -31, c_{297} = -9/10, c_{298} = 1/4, c_{299} = -4,$
 $c_{300} = 1403/1600, c_{301} = 6449/28800, c_{302} = -5769/3200, c_{303} = 343/128, c_{304} = -1327/1920,$
 $c_{305} = 1427/1920, c_{306} = 1427/1920, c_{307} = -11/16, c_{308} = 2987/960, c_{309} = -3827/960,$
 $c_{310} = -3, c_{311} = 3347/1920, c_{312} = -5/2, c_{313} = -1/3, c_{314} = 2827/960, c_{315} = 3667/960,$
 $c_{316} = 6, c_{317} = -8, c_{318} = -67/40, c_{319} = 2, c_{320} = -3, c_{321} = -5/4, c_{322} = -283/5,$
 $c_{323} = -1/128, c_{324} = -41/40, c_{325} = -13/40, c_{326} = 235/4, c_{327} = 85, c_{328} = 112/5,$
 $c_{329} = -2168/5, c_{330} = -3844/5, c_{331} = -502, c_{332} = -4, c_{333} = 4/15, c_{334} = 0, c_{335} = -32,$
 $c_{336} = -333/400, c_{337} = 0, c_{338} = 84, c_{339} = -368/5, c_{340} = 102, c_{341} = -112/5, c_{342} = 0,$
 $c_{343} = 6, c_{344} = 2, c_{345} = 11/20, c_{346} = -2, c_{347} = 2, c_{348} = 2, c_{349} = -10, c_{350} = 5, c_{351} = -1,$
 $c_{352} = 4, c_{353} = -2, c_{354} = 0, c_{355} = -3/2, c_{356} = 7/4, c_{357} = 6, c_{358} = -7/2, c_{359} = -5/4,$
 $c_{360} = -5/2, c_{361} = -5/2, c_{362} = 5/4, c_{363} = 7/2, c_{364} = -3/4, c_{365} = 7, c_{366} = -9/2,$
 $c_{367} = 4, c_{368} = 2, c_{369} = -9/2, c_{370} = -1/2, c_{371} = 639/200, c_{372} = -4/5, c_{373} = -3,$
 $c_{374} = 4, c_{375} = 1/16, c_{376} = 3/32, c_{377} = 0$



$$[\Phi''^4]_2 = -\frac{8}{5} \nabla_\alpha \nabla^\gamma \Phi \nabla^\alpha \nabla^\beta \Phi \nabla_\beta \nabla^\delta \Phi \nabla_\delta \nabla_\gamma \Phi + \frac{4}{5} \nabla^\alpha \nabla^\beta \Phi \nabla_\beta \nabla_\alpha \Phi \nabla^\gamma \nabla^\delta \Phi \nabla_\delta \nabla_\gamma \Phi.$$

$$S_{hhCC} = -\frac{2}{\kappa^2} \frac{\pi^2 \ell_s^6}{2^6 \cdot 3} \int d^{10} x \sqrt{-G} \left[\frac{1}{4} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \partial_\gamma F_{\alpha\beta} \partial^\gamma F^{\alpha\beta} + \frac{1}{2} R^{\alpha\beta}_{\tau\kappa} R^{\tau\kappa\mu\nu} \partial_\gamma F_{\alpha\beta} \partial^\gamma F_{\mu\nu} \right. \\ \left. - R^{\mu\nu}_{\tau\gamma} R^{\tau\kappa\alpha\beta} \partial_\gamma F_{\mu\nu} \partial_\kappa F_{\alpha\beta} + 2R^{\beta\mu}_{\kappa\rho} R^{\tau\kappa}_{\alpha\beta} \partial^\alpha F^{\rho\gamma} \partial_\mu F_{\tau\gamma} + 2R_{\beta\mu\kappa\rho} R^{\tau\kappa\alpha\beta} \partial_\alpha F_{\tau\gamma} \partial^\mu F^{\rho\gamma} \right. \\ \left. - 4R_{\alpha}{}^{\nu\rho\gamma} R^{\tau\kappa\alpha\beta} \partial_\beta F_{\kappa\rho} \partial_\nu F_{\tau\gamma} + 2R^{\alpha\beta\kappa\rho} R^{\tau}_{\kappa\alpha\beta} \partial_\nu F_{\tau\gamma} \partial^\nu F_{\rho}{}^\gamma + R^{\alpha\beta\rho\gamma} R^{\tau\kappa}_{\alpha\beta} \partial_\nu F_{\kappa\rho} \partial^\nu F_{\tau\gamma} \right. \\ \left. + 2R_{\beta}{}^{\mu}_{\kappa\rho} R^{\tau\kappa\alpha\beta} \partial^\rho F_{\mu}{}^\nu \partial_\tau F_{\alpha\nu} + R^{\mu\nu}_{\kappa}{}^{\rho} R^{\tau\kappa\alpha\beta} \partial_\rho F_{\alpha\beta} \partial_\tau F_{\mu\nu} + R^{\alpha\beta\kappa}_{\rho} R^{\tau}_{\kappa\alpha\beta} \partial^\rho F^{\mu\nu} \partial_\tau F_{\mu\nu} \right. \\ \left. + 2R^{\beta\mu\kappa\rho} R_{\tau\kappa}{}^{\alpha}_{\beta} \partial_\rho F_{\alpha\nu} \partial^\tau F_{\mu}{}^\nu \right]$$

$$S_{R^2 F'^2} = -\frac{2}{\kappa^2} \frac{\pi^2 \ell_s^6}{2^3 \cdot 3} \int d^{10} x \sqrt{-G} [R^2 F^2]_{13}$$

$$[R^2 F^2]_{13} = -\frac{5}{2} e^{2\Phi} R_{\gamma}{}^{\epsilon\epsilon\mu} R_{\epsilon\mu\delta\epsilon} \nabla_\alpha F_{\beta}{}^{\delta} \nabla^\alpha F^{\beta\gamma} + \frac{3}{4} e^{2\Phi} R_{\epsilon\mu\delta\epsilon} R^{\epsilon\mu}_{\beta\gamma} \nabla_\alpha F^{\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \\ - \frac{1}{4} e^{2\Phi} R_{\epsilon\mu\gamma\epsilon} R^{\epsilon\mu}_{\beta\delta} \nabla_\alpha F^{\delta\epsilon} \nabla^\alpha F^{\beta\gamma} + \frac{5}{16} e^{2\Phi} R^{\delta\epsilon\mu}_{\epsilon\mu\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_{\alpha\gamma} \\ + e^{2\Phi} R_{\gamma}{}^{\epsilon\mu} R_{\epsilon\mu\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_{\alpha}{}^{\delta} - \frac{7}{2} e^{2\Phi} R_{\delta}{}^{\epsilon}_{\alpha}{}^{\mu} R_{\epsilon\mu\gamma\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F^{\delta\epsilon} \\ - \frac{3}{2} e^{2\Phi} R_{\alpha}{}^{\epsilon}_{\delta}{}^{\mu} R_{\epsilon\mu\gamma\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F_{\beta}{}^{\epsilon} + \frac{5}{2} e^{2\Phi} R_{\gamma}{}^{\epsilon}_{\alpha}{}^{\mu} R_{\epsilon\mu\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F_{\beta}{}^{\epsilon} \\ - \frac{5}{4} e^{2\Phi} R_{\epsilon\mu\gamma\delta} R^{\epsilon\mu}_{\alpha\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F_{\beta}{}^{\epsilon} - 3e^{2\Phi} R_{\beta}{}^{\mu}_{\alpha\gamma} R_{\epsilon\mu\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F^{\epsilon\epsilon} \\ - \frac{5}{2} e^{2\Phi} R_{\alpha}{}^{\mu}_{\delta\epsilon} R_{\epsilon\mu\beta\gamma} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F^{\epsilon\epsilon} + 6e^{2\Phi} R_{\beta\epsilon\alpha}{}^{\mu} R_{\epsilon\mu\gamma\delta} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F^{\epsilon\epsilon} \\ - 8e^{2\Phi} R_{\beta}{}^{\mu}_{\alpha\epsilon} R_{\epsilon\mu\gamma\delta} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F^{\epsilon\epsilon}$$

$$S_{h\Phi CC} = -\frac{2}{\kappa^2} \frac{\pi^2 \ell_s^6}{2^6 \cdot 3} \int d^{10} x \sqrt{-G} [-8R_{\beta\mu\kappa\nu} \partial^\beta \partial^\alpha \Phi \partial_\gamma F^{\mu\nu} \partial^\kappa F_{\alpha}{}^\gamma + 24R_{\beta\mu\gamma\nu} \partial^\beta \partial^\alpha \Phi \partial_\kappa F^{\mu\nu} \partial^\kappa F_{\alpha}{}^\gamma \\ - 8R_{\alpha\kappa\beta\gamma} \partial^\beta \partial^\alpha \Phi \partial_\gamma F_{\mu}{}^\nu \partial^\mu F^{\gamma\kappa} + 8R_{\beta\mu\gamma\nu} \partial^\beta \partial^\alpha \Phi \partial^\kappa F_{\alpha}{}^\gamma \partial_\nu F_{\kappa}{}^\mu]$$

$$S_{RF'^2\Phi''} = -\frac{2}{\kappa^2} \frac{\pi^2 \ell_s^6}{2^3 \cdot 3} \int d^{10} x \sqrt{-G} [RF^2\Phi'']_8$$

$$[RF'^2\Phi'']_8 = -10e^{2\Phi} R_{\epsilon\epsilon\alpha\gamma} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F^{\delta\epsilon} \nabla_\delta \nabla^\epsilon \Phi + 3e^{2\Phi} R_{\delta\epsilon\gamma\epsilon} \nabla_\alpha \nabla^\epsilon \Phi \nabla^\alpha F^{\beta\gamma} \nabla_\delta F_{\beta}{}^{\epsilon} \\ + 5e^{2\Phi} R_{\epsilon\epsilon\alpha\delta} \nabla^\alpha F^{\beta\gamma} \nabla_\gamma \nabla^\epsilon \Phi \nabla_\delta F_{\beta}{}^{\epsilon} + 2e^{2\Phi} R_{\epsilon\epsilon\gamma\delta} \nabla^\alpha F^{\beta\gamma} \nabla_\beta \nabla_\alpha \Phi \nabla_\delta F^{\epsilon\epsilon} \\ + 2e^{2\Phi} R_{\delta\epsilon\beta\gamma} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F^{\epsilon\epsilon} \nabla_\epsilon \nabla_\alpha \Phi - 2e^{2\Phi} R_{\gamma\epsilon\alpha\delta} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F^{\epsilon\epsilon} \nabla_\epsilon \nabla_\beta \Phi \\ - 3e^{2\Phi} R_{\gamma\epsilon\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_{\alpha}{}^{\delta} \nabla^\epsilon \nabla^\epsilon \Phi + 4e^{2\Phi} R_{\delta\epsilon\alpha\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_{\gamma}{}^{\delta} \nabla^\epsilon \nabla^\epsilon \Phi.$$

$$S_{\Phi\Phi CC} = -\frac{2}{\kappa^2} \frac{\pi^2 \ell_s^6}{2^6 \cdot 3} \int d^{10} x \sqrt{-G} [8\partial^\theta \partial^\eta \Phi \partial^\kappa \partial_\eta \Phi \partial_\nu F_{\kappa\mu} \partial^\nu F_{\theta}{}^\mu - 2\partial_\theta \partial_\eta \Phi \partial^\theta \partial^\eta \Phi \partial_\nu F_{\kappa\mu} \partial^\nu F^{\kappa\mu}]$$

$$S_{RF'^2\Phi''} = -\frac{2}{\kappa^2} \frac{\pi^2 \ell_s^6}{2^3 \cdot 3} \int d^{10} x \sqrt{-G} [F^2\Phi''^2]_6$$

$$[F'^2\Phi''^2]_6 = 2e^{2\Phi} \nabla_\alpha F^{\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\delta \nabla_\beta \Phi \nabla_\epsilon \nabla_\gamma \Phi + 2e^{2\Phi} \nabla^\alpha F^{\beta\gamma} \nabla_\delta \nabla_\alpha \Phi \nabla_\delta F_{\beta}{}^{\epsilon} \nabla_\epsilon \nabla_\gamma \Phi \\ + 5e^{2\Phi} \nabla_\alpha F_{\beta}{}^{\delta} \nabla^\alpha F^{\beta\gamma} \nabla_\gamma \nabla^\epsilon \Phi \nabla_\epsilon \nabla_\delta \Phi - e^{2\Phi} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_{\alpha}{}^{\delta} \nabla_\gamma \nabla^\epsilon \Phi \nabla_\epsilon \nabla_\delta \Phi \\ - 2e^{2\Phi} \nabla^\alpha F^{\beta\gamma} \nabla_\gamma \nabla_\alpha \Phi \nabla_\delta F_{\beta}{}^{\epsilon} \nabla_\epsilon \nabla_\delta \Phi - \frac{3}{2} e^{2\Phi} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_{\alpha\gamma} \nabla^\delta \nabla^\epsilon \Phi \nabla_\epsilon \nabla_\delta \Phi.$$



$$ds^2 = G_{\mu\nu}(x)dx^\mu dx^\nu + g_{ab}(y)dy^a dy^b$$

$$\mathbf{S}^{(0)} = -\frac{2V}{\kappa_{10}^2} \int d^6x \sqrt{-G} e^{-2\Phi} \left[R + 4\nabla_\mu \Phi \nabla^\mu \Phi - \frac{1}{4} e^{2\Phi} F_{\mu\nu} F^{\mu\nu} \right]$$

$$ds'^2 = G'_{MN}(x)dx^M dx^N + g'_{ij}(z)dz^i dz^j$$

$$G'_{MN} = \begin{pmatrix} G'_{\mu\nu} + R_1^{-2} C'_\mu C'_\nu & C'_\mu \\ C'_\nu & R_1^2 \end{pmatrix}$$

$$\mathbf{S}'^{(0)} = -\frac{2V'}{\kappa_{10}'^2} \int d^6x \sqrt{-G'} e^{-2\Phi'} \left[R' + 4\nabla_\mu \Phi' \nabla^\mu \Phi' - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} \right],$$

$$\kappa_{10}^2/V = \kappa_{10}'^2/V'.$$

$$G'_{\mu\nu} = e^{-2\Phi} G_{\mu\nu}, \Phi' = -\Phi, C'_\mu = C_\mu,$$

$$\frac{1}{32\pi^2} \int_{K3} d^4y \sqrt{g} R_{abcd} R^{abcd}$$

$$S_{R^2} = -\frac{2}{\kappa^2} \frac{4\pi^4 \ell_s^6}{3} \int d^6x \sqrt{-G} \left[\frac{3}{2} R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \right].$$

$$S'_{R^2} = -\frac{2}{\kappa_{10}'^2} \frac{\ell_s'^2}{8} \int d^{10}x \sqrt{-G'} e^{-2\Phi'} R'_{\mu\nu\alpha\beta} R'^{\mu\nu\alpha\beta}.$$

$$S'_{R^2} = -\frac{2}{\kappa_{10}'^2} \frac{\ell_s'^2 V'}{8} \int d^6x \sqrt{-G'} e^{-2\Phi'} R'_{\mu\nu\alpha\beta} R'^{\mu\nu\alpha\beta}$$

$$\frac{2\pi V'}{(2\pi \ell'_s)^6 g_s'^2} = \frac{1}{2\pi \ell_s^2}$$

$$\begin{aligned} S_{6D} = & -\frac{2}{\kappa^2} \frac{4\pi^4 \ell_s^6}{3} \int d^6x \sqrt{-G} \left[\frac{3}{2} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} + \frac{51}{16} e^{4\Phi} F_\alpha^\gamma F^{\alpha\beta} F_\beta^\delta F_{\gamma\delta} + \frac{50151}{2560} e^{4\Phi} F_{\alpha\beta} F^{\alpha\beta} F_{\gamma\delta} F^{\gamma\delta} \right. \\ & - 3e^{2\Phi} F^{\alpha\beta} F^{\gamma\delta} R_{\alpha\beta\gamma\delta} - 3e^{2\Phi} F^{\alpha\beta} F^{\gamma\delta} R_{\alpha\gamma\beta\delta} + \frac{44343}{400} e^{2\Phi} F_{\beta\gamma} F^{\beta\gamma} \nabla_\alpha \Phi \nabla^\alpha \Phi \\ & - \frac{13581}{200} e^{2\Phi} F_\alpha^\gamma F_{\beta\gamma} \nabla^\alpha \Phi \nabla^\beta \Phi - \frac{36}{5} \nabla_\alpha \Phi \nabla^\alpha \Phi \nabla_\beta \Phi \nabla^\beta \Phi - \frac{24}{5} \nabla^\alpha \Phi \nabla_\beta \nabla_\alpha \Phi \nabla^\beta \Phi \\ & - \frac{24}{5} \nabla_\beta \nabla_\alpha \Phi \nabla^\beta \nabla^\alpha \Phi + \frac{15}{2} e^{2\Phi} F^{\beta\gamma} \nabla^\alpha \Phi \nabla_\gamma F_{\alpha\beta} - \frac{18501}{400} e^{2\Phi} F_\alpha^\gamma F^{\alpha\beta} \nabla_\gamma \nabla_\beta \Phi \\ & \left. + \frac{15}{2} e^{2\Phi} \nabla_\beta F_{\alpha\gamma} \nabla^\gamma F^{\alpha\beta} \right]. \end{aligned}$$



$$S_{6D} = -\frac{2}{\kappa^2} \frac{4\pi^4 \ell_s^6}{3} \int d^6x \sqrt{-G} \left[\frac{3}{2} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} + a_1 e^{4\Phi} F_\alpha^\gamma F^{\alpha\beta} F_\beta^\delta F_{\gamma\delta} + a_2 e^{4\Phi} F_{\alpha\beta} F^{\alpha\beta} F_{\gamma\delta} F^{\gamma\delta} \right. \\ \left. + \left(-\frac{9}{8} - 2a_1 + 2a_{19} - \frac{1}{2} a_3 \right) e^{2\Phi} F_\alpha^\gamma F^{\alpha\beta} R_{\beta\gamma} + \frac{1}{4800} (-65403 + 800a_1 + 1800a_{10} \right. \\ \left. + 900a_{11} + 150a_{12} - 825a_{13} - 450a_{14} + 450a_{15} - 525a_{16} + 300a_{17} + 1200a_{19} \right. \\ \left. + 3200a_2 - 1200a_3 \right) e^{2\Phi} F_{\alpha\beta} F^{\alpha\beta} R + \frac{1}{16} (-12 + 4a_{10} + 4a_{11} - 2a_{13} - a_{14} + a_{15} \\ - 2a_{16} - 8a_3) R^2 + \left(-\frac{3}{4} - a_{19} \right) e^{2\Phi} F^{\alpha\beta} F^{\gamma\delta} R_{\alpha\beta\gamma\delta} + a_3 R_{\alpha\beta} R^{\alpha\beta} + \left(\frac{20001}{400} + \frac{2}{3} a_1 \right. \\ \left. - \frac{3}{8} a_{12} + \frac{5}{16} a_{13} + \frac{3}{8} a_{14} - \frac{3}{16} a_{15} + \frac{1}{16} a_{16} - \frac{3}{4} a_{17} - 3a_{19} + \frac{8}{3} a_2 \right) e^{2\Phi} F_{\beta\gamma} F^{\beta\gamma} \nabla_\alpha \Phi \nabla^\alpha \Phi \\ + a_{10} R \nabla_\alpha \Phi \nabla^\alpha \Phi + a_{11} R^{\alpha\beta} \nabla_\beta \nabla_\alpha \Phi + a_{12} e^{2\Phi} F_\alpha^\gamma F_{\beta\gamma} \nabla^\alpha \Phi \nabla^\beta \Phi + a_{13} R_{\alpha\beta} \nabla^\alpha \Phi \nabla^\beta \Phi \\ + a_{14} \nabla_\alpha \Phi \nabla^\alpha \Phi \nabla_\beta \Phi \nabla^\beta \Phi + a_{15} \nabla^\alpha \Phi \nabla_\beta \nabla_\alpha \Phi \nabla^\beta \Phi + a_{16} \nabla_\beta \nabla_\alpha \Phi \nabla^\beta \nabla^\alpha \Phi \\ + a_{17} e^{2\Phi} F^{\beta\gamma} \nabla^\alpha \Phi \nabla_\gamma F_{\alpha\beta} + \frac{1}{4} (-3 - 16a_1 + 2a_{12} + a_{13} - a_{16}) e^{2\Phi} F_\alpha^\gamma F^{\alpha\beta} \nabla_\gamma \nabla_\beta \Phi \\ \left. + a_{19} e^{2\Phi} \nabla_\gamma F_{\alpha\beta} \nabla^\gamma F^{\alpha\beta} \right]$$

$$S'_{6D} = -\frac{2}{\kappa'^2} \frac{\ell'^2 V'}{8} \int d^6x \sqrt{-G'} e^{-2\Phi'} \left[R'_{\alpha\beta\gamma\delta} R'^{\alpha\beta\gamma\delta} + \frac{5}{8} F_\alpha'^\gamma F'^{\alpha\beta} F_\beta'^\delta F'_{\gamma\delta} + \frac{3}{8} F'_{\alpha\beta} F'^{\alpha\beta} F'_{\gamma\delta} F'^{\gamma\delta} \right. \\ \left. - F'^{\alpha\beta} F'^{\gamma\delta} R'_{\alpha\beta\gamma\delta} - F'^{\alpha\beta} F'^{\gamma\delta} R'_{\alpha\gamma\beta\delta} + \nabla_\gamma F'_{\alpha\beta} \nabla^\gamma F'^{\alpha\beta} \right]$$

$$S'_{6D} = -\frac{2}{\kappa^2} \frac{4\pi^4 \ell_s^6}{3} \int d^6x \sqrt{-G'} e^{-2\Phi'} \left[\frac{3}{2} R'_{\alpha\beta\gamma\delta} R'^{\alpha\beta\gamma\delta} + b_1 F_\alpha'^\gamma F'^{\alpha\beta} F_\beta'^\delta F'_{\gamma\delta} + b_2 F'_{\alpha\beta} F'^{\alpha\beta} F'_{\gamma\delta} F'^{\gamma\delta} \right. \\ \left. + \frac{1}{4} (2b_{11} + 2b_{12} + b_{13} - b_{16}) R'_{\alpha\beta} R'^{\alpha\beta} + \frac{1}{8} (-9 - 16b_1 - 2b_{11} - 2b_{12} - b_{13} + b_{16} \right. \\ \left. + 16b_{19}) F_\alpha'^\gamma F'^{\alpha\beta} R'_{\beta\gamma} + \frac{1}{192} (162 + 32b_1 - 36b_{10} - 12b_{12} - 6b_{13} + 9b_{14} + 2b_{16} + 8b_{17} \right. \\ \left. + 8b_{18} - 32b_{19} - 256b_2) F'_{\alpha\beta} F'^{\alpha\beta} R + \frac{1}{16} (4b_{10} - b_{14}) R'^2 + \left(-\frac{3}{4} - b_{19} \right) F'^{\alpha\beta} F'^{\gamma\delta} R'_{\alpha\beta\gamma\delta} + \right. \\ \left. + \frac{1}{48} (162 + 32b_1 - 12b_{12} - 9b_{14} - 3b_{15} + 2b_{16} + 8b_{17} + 8b_{18} - 32b_{19} \right. \\ \left. - 256b_2) F'_{\beta\gamma} F'^{\beta\gamma} \nabla_\alpha \Phi' \nabla^\alpha \Phi' + b_{10} R' \nabla_\alpha \Phi' \nabla^\alpha \Phi' + b_{11} R'^{\alpha\beta} \nabla_\beta \nabla_\alpha \Phi' + b_{12} F_\alpha'^\gamma F_\beta'^\delta \nabla^\alpha \Phi' \nabla^\beta \Phi' \right. \\ \left. + b_{13} R'_{\alpha\beta} \nabla^\alpha \Phi' \nabla^\beta \Phi' + b_{14} \nabla_\alpha \Phi' \nabla^\alpha \Phi' \nabla_\beta \Phi' \nabla^\beta \Phi' + b_{15} \nabla^\alpha \Phi' \nabla_\beta \nabla_\alpha \Phi' \nabla^\beta \Phi' \right. \\ \left. + b_{16} \nabla_\beta \nabla_\alpha \Phi' \nabla^\beta \nabla^\alpha \Phi' + b_{17} F'^{\beta\gamma} \nabla^\alpha \Phi' \nabla_\gamma F'_{\alpha\beta} + b_{18} F_\alpha'^\gamma F'^{\alpha\beta} \nabla_\gamma \nabla_\beta \Phi' + b_{19} \nabla_\gamma F'_{\alpha\beta} \nabla^\gamma F'^{\alpha\beta} \right]$$

$$a_2 = 65403/6400 - a_1/4 + (3a_{13})/64 + (3a_{14})/64 - (3a_{15})/128, a_3 = -(20901/200) + a_{10} \\ + a_{11}/2 - (3a_{13})/4 - a_{14}/2 + (3a_{15})/8 - a_{16}/4, b_1 = (4a_1)/3, b_{10} = 13734/25 - 4a_{10} \\ - (4a_{11})/3 + 4a_{13} + (8a_{14})/3 - 2a_{15} + (4a_{16})/3, b_{11} = 27268/25 + (8a_{10})/3 + (20a_{11})/3 \\ + (4a_{13})/3 + 2a_{14} - (2a_{15})/3 - 4a_{16}, b_{12} = 6767/25 - (8a_{10})/3 - (4a_{11})/3 + (4a_{13})/3 \\ + (4a_{14})/3 - a_{15} + (4a_{16})/3, b_{13} = -(13834/25) + 12a_{10} + (16a_{11})/3 - 6a_{13} - (13a_{14})/3 \\ + (11a_{15})/3 - (14a_{16})/3, b_{14} = 27468/25 - (32a_{10})/3 - (16a_{11})/3 + (32a_{13})/3 + (20a_{14})/3 \\ - (16a_{15})/3 + (16a_{16})/3, b_{15} = 26468/25 + (88a_{10})/3 + (64a_{11})/3 - (68a_{13})/3 - (34a_{14})/3 \\ + (34a_{15})/3 - 20a_{16}, b_{16} = 13634/5 + (20a_{10})/3 + (40a_{11})/3 + (10a_{13})/3 + 5a_{14} - (5a_{15})/3 \\ - (26a_{16})/3, b_{17} = 130273/100 - (32a_1)/3 + 4a_{10} + 2a_{11} + (5a_{12})/3 + (9a_{13})/2 + (14a_{14})/3 \\ - (11a_{15})/6 - (11a_{16})/6 + 2a_{17} + (8a_{19})/3, b_{18} = 6742/25 - (16a_1)/3 - (8a_{10})/3 - (4a_{11})/3 \\ - (2a_{12})/3 + (5a_{13})/3 + (4a_{14})/3 - a_{15} + a_{16} + (32a_{19})/3, b_{19} = (4a_{19})/3, b_2 = 21801/1600 \\ - a_1/3 + a_{13}/16 + a_{14}/16 - a_{15}/32$$



$$\mathcal{L}' = c_1' R_{\beta}{}^{\delta\epsilon\nu} R_{\gamma\epsilon}{}^{\mu\zeta} R_{\delta\nu\mu\zeta} F_{\alpha}{}^{\beta} F^{\alpha\gamma} e^{2\Phi} + \dots$$

$$\mathcal{J} \equiv \nabla_{\alpha}(\mathcal{J}^{\alpha})$$

$$\begin{aligned} \mathcal{K} \equiv & e^{\Phi} \nabla_{\beta} F^{\beta\alpha} \delta C_{\alpha} - \frac{1}{8} e^{2\Phi} F_{\alpha\beta} F^{\alpha\beta} \delta G^{\mu}{}_{\mu} - \left(R^{\alpha\beta} - \frac{1}{2} e^{2\Phi} F^{\alpha\gamma} F^{\beta}{}_{\gamma} + 2 \nabla^{\beta} \nabla^{\alpha} \Phi \right) \delta G_{\alpha\beta} \\ & - 2(R + 4 \nabla_{\alpha} \nabla^{\alpha} \Phi - 4 \nabla_{\alpha} \Phi \nabla^{\alpha} \Phi) \left(\delta \Phi - \frac{1}{4} \delta G^{\mu}{}_{\mu} \right) \end{aligned}$$

$$G_{\mu\nu} \rightarrow G_{\mu\nu} + \ell_s^6 e^{2\Phi} \delta G_{\mu\nu}$$

$$C_{\mu} \rightarrow C_{\mu} + \ell_s^6 e^{\Phi} \delta C_{\mu}$$

$$\Phi \rightarrow \Phi + \ell_s^6 e^{2\Phi} \delta \Phi$$

$$\Delta - \mathcal{J} - \mathcal{K} = 0$$

$$R_{\alpha[\beta\gamma\delta]}=0,$$

$$\nabla_{[\mu}R_{\alpha\beta]\gamma\delta}=0,$$

$$\nabla_{[\mu}F_{\alpha\beta]}=0,$$

$$[\nabla,\nabla]\mathcal{O}-R\mathcal{O}=0.$$

$$[R^2R'']_1=c_{56}R^{\alpha\beta\gamma\delta}R^{\mu\nu}{}_{\gamma\delta}\nabla_{\alpha}\nabla_{\mu}R_{\beta\nu}$$

$$\begin{aligned} \mathcal{L} = & [R^4]_7 + [F'^4]_5 + [R\Phi'^2F'^2]_{11} + [RF'^2\Phi'']_9 + [R^2F'^2]_{13} + [F^2R^3]_{16} + [R^3\Phi'']_3 + [R^3\Phi'^2]_5 \\ & + [F^2R\Phi''\Phi'^2]_8 + [R^2\Phi''^2]_5 + [R^2\Phi''\Phi'^2]_4 + [R^2\Phi'^4]_3 + [F^3F'\Phi'^3]_9 + [F^4R^2]_{20} + [F^2R^2\Phi'']_{14} \\ & + [F^2R^2\Phi'^2]_{19} + [F^4R\Phi'^2]_{12} + [F^2RF'^2]_{33} + [F^6R]_7 + [F^6\Phi'']_4 + [F^6\Phi'^2]_1 + [FR^2F'\Phi']_2 \\ & + [FR^2F'\Phi']_{15} + [F^4\Phi'^4]_5 + [F^2R\Phi'^4]_4 + [FR\Phi'^3F']_{10} + [F^4R\Phi'']_9 + [F^2\Phi'^6]_2 + [F^4\Phi''^2]_8 \\ & + [F^2R\Phi''^2]_9 + [F^2F'^2\Phi'']_{21} + [F^3R\Phi'F']_1 + [FR\Phi''\Phi'F']_2 + [F^2\Phi''^2\Phi'^2]_4 + [F^2\Phi'^2F'^2]_{17} \\ & + [FF'\Phi'\Phi''^2]_1 + [FF'\Phi'^3\Phi'']_5 + [R\Phi'^2\Phi''^2]_2 + [\Phi'^4F'^2]_4 + [\Phi''^4]_2 + [FF'\Phi'^5]_2 + [F^2\Phi'^4\Phi'']_2 \\ & + [R\Phi''^3]_1 + [R\Phi'^4\Phi'']_1 + [F'^2\Phi''^2]_6 + [F'^2\Phi'^2\Phi'']_4 + [FF'^3\Phi']_1 + [\Phi'^4\Phi''^2]_2 + [\Phi'^6\Phi'']_1 \\ & + [F^8]_5 + [F^4F'^2]_{20} + [R^2R'']_1 \end{aligned}$$

$$\begin{aligned} [R^4]_7 = & c_{23} R^{\alpha\beta\gamma\delta} R_{\gamma}{}^{\epsilon}{}_{\epsilon}{}^{\epsilon} R_{\delta}{}^{\mu}{}_{\beta}{}^{\nu} R_{\epsilon\nu\epsilon\mu} + c_{39} R_{\alpha}{}^{\epsilon\epsilon\mu} R^{\alpha\beta\gamma\delta} R_{\gamma}{}^{\nu}{}_{\beta\epsilon} R_{\mu\nu\delta\epsilon} + c_{40} R_{\alpha}{}^{\epsilon}{}_{\gamma}{}^{\epsilon} R^{\alpha\beta\gamma\delta} R_{\beta\epsilon}{}^{\mu\nu} R_{\mu\nu\delta\epsilon} \\ & + c_{41} R_{\alpha\gamma\beta}{}^{\epsilon} R^{\alpha\beta\gamma\delta} R_{\delta}{}^{\epsilon\mu\nu} R_{\mu\nu\epsilon\epsilon} + c_{42} R^{\alpha\beta\gamma\delta} R_{\gamma\delta\alpha\beta} R^{\epsilon\epsilon\mu\nu} R_{\mu\nu\epsilon\epsilon} + c_{54} R_{\alpha\beta}{}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_{\mu\nu\epsilon\epsilon} R^{\mu\nu}{}_{\gamma\delta} \\ & + c_{55} R_{\alpha\beta}{}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_{\mu\nu\delta\epsilon} R^{\mu\nu}{}_{\gamma\epsilon} \end{aligned}$$



$$\begin{aligned}
[F'^4]_5 &= e^{4\Phi} c_{254} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_\alpha{}^\delta \nabla_\gamma F^{\epsilon\epsilon} \nabla_\delta F_{\epsilon\epsilon} + e^{4\Phi} c_{323} \nabla_\alpha F_{\beta\gamma} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F_{\epsilon\epsilon} \nabla^\delta F^{\epsilon\epsilon} \\
&+ e^{4\Phi} c_{342} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_\gamma{}^\delta \nabla_\delta F^{\epsilon\epsilon} \nabla_\epsilon F_{\alpha\epsilon} + e^{4\Phi} c_{375} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_\alpha{}^\delta \nabla^\epsilon F_\gamma{}^\epsilon \nabla_\epsilon F_{\delta\epsilon} \\
&+ e^{4\Phi} c_{376} \nabla_\alpha F^{\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\epsilon F_{\gamma\epsilon} \nabla^\epsilon F_{\beta\delta}, \\
[R\Phi'^2 F'^2]_{11} &= e^{2\Phi} c_{198} R_{\epsilon\epsilon\beta\delta} \nabla_\alpha \Phi \nabla^\alpha \Phi \nabla^\beta F^{\gamma\delta} \nabla_\gamma F^{\epsilon\epsilon} + e^{2\Phi} c_{253} R_{\alpha\epsilon\beta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\gamma F^{\delta\epsilon} \nabla_\delta F_\gamma{}^\epsilon \\
&+ e^{2\Phi} c_{255} R_{\alpha\epsilon\beta\gamma} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\gamma F^{\delta\epsilon} \nabla_\delta F_\epsilon{}^\epsilon + e^{2\Phi} c_{321} R_{\epsilon\epsilon\gamma\delta} \nabla_\alpha F_\beta{}^\gamma \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\delta F^{\epsilon\epsilon} \\
&+ e^{2\Phi} c_{359} R_{\epsilon\epsilon\gamma\delta} \nabla_\alpha F^{\gamma\delta} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\epsilon F_\beta{}^\epsilon + e^{2\Phi} c_{362} R_{\delta\epsilon\gamma\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\gamma F_\alpha{}^\delta \nabla^\epsilon F_\beta{}^\epsilon \\
&+ e^{2\Phi} c_{364} R_{\delta\epsilon\beta\epsilon} \nabla_\alpha \Phi \nabla^\alpha \Phi \nabla^\beta F^{\gamma\delta} \nabla^\epsilon F_\gamma{}^\epsilon + e^{2\Phi} c_{365} R_{\delta\epsilon\beta\epsilon} \nabla_\alpha F^{\gamma\delta} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\epsilon F_\gamma{}^\epsilon \\
&+ e^{2\Phi} c_{366} R_{\epsilon\epsilon\beta\delta} \nabla_\alpha F^{\gamma\delta} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\epsilon F_\gamma{}^\epsilon + e^{2\Phi} c_{369} R_{\delta\epsilon\beta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\gamma F_\alpha{}^\delta \nabla^\epsilon F_\gamma{}^\epsilon \\
&+ e^{2\Phi} c_{370} R_{\epsilon\epsilon\beta\delta} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\gamma F_\alpha{}^\delta \nabla^\epsilon F_\gamma{}^\epsilon, \\
[RF'^2 \Phi'']_9 &= e^{2\Phi} c_{266} R_{\epsilon\epsilon\alpha\gamma} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F^{\delta\epsilon} \nabla_\delta \nabla^\epsilon \Phi + e^{2\Phi} c_{291} R_{\delta\epsilon\gamma\epsilon} \nabla_\alpha \nabla^\epsilon \Phi \nabla^\alpha F^{\beta\gamma} \nabla_\delta F_\beta{}^\epsilon \\
&+ e^{2\Phi} c_{292} R_{\epsilon\epsilon\gamma\delta} \nabla_\alpha \nabla^\epsilon \Phi \nabla^\alpha F^{\beta\gamma} \nabla_\delta F_\beta{}^\epsilon + e^{2\Phi} c_{295} R_{\epsilon\epsilon\alpha\delta} \nabla^\alpha F^{\beta\gamma} \nabla_\gamma \nabla^\epsilon \Phi \nabla^\delta F_\beta{}^\epsilon \\
&+ e^{2\Phi} c_{319} R_{\epsilon\epsilon\gamma\delta} \nabla^\alpha F^{\beta\gamma} \nabla_\beta \nabla_\alpha \Phi \nabla^\delta F^{\epsilon\epsilon} + e^{2\Phi} c_{344} R_{\delta\epsilon\beta\gamma} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F^{\epsilon\epsilon} \nabla_\epsilon \nabla_\alpha \Phi \\
&+ e^{2\Phi} c_{346} R_{\gamma\epsilon\alpha\delta} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F^{\epsilon\epsilon} \nabla_\epsilon \nabla_\beta \Phi + e^{2\Phi} c_{373} R_{\gamma\epsilon\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_\alpha{}^\delta \nabla^\epsilon \nabla^\epsilon \Phi \\
&+ e^{2\Phi} c_{374} R_{\delta\epsilon\alpha\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_\gamma{}^\delta \nabla^\epsilon \nabla^\epsilon \Phi, \\
[R^2 F'^2]_{13} &= e^{2\Phi} c_{60} R_\gamma{}^{\epsilon\epsilon\mu} R_{\epsilon\mu\delta\epsilon} \nabla_\alpha F_\beta{}^\delta \nabla^\alpha F^{\beta\gamma} + e^{2\Phi} c_{70} R_{\epsilon\mu\delta\epsilon} R^{\epsilon\mu}{}_\beta \nabla_\alpha F^{\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \\
&+ e^{2\Phi} c_{71} R_{\epsilon\mu\gamma\epsilon} R^{\epsilon\mu}{}_\beta \nabla_\alpha F^{\delta\epsilon} \nabla^\alpha F^{\beta\gamma} + e^{2\Phi} c_{121} R^{\delta\epsilon\epsilon\mu} R_{\epsilon\mu\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_{\alpha\gamma} \\
&+ e^{2\Phi} c_{126} R_\gamma{}^{\epsilon\epsilon\mu} R_{\epsilon\mu\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_\alpha{}^\delta + e^{2\Phi} c_{133} R_\delta{}^\epsilon{}_\alpha{}^\mu R_{\epsilon\mu\gamma\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F^{\delta\epsilon} \\
&+ e^{2\Phi} c_{283} R_\alpha{}^\epsilon{}_\delta{}^\mu R_{\epsilon\mu\gamma\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F_\beta{}^\epsilon + e^{2\Phi} c_{285} R_\gamma{}^\epsilon{}_\alpha{}^\mu R_{\epsilon\mu\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F_\beta{}^\epsilon \\
&+ e^{2\Phi} c_{288} R_{\epsilon\mu\gamma\delta} R^{\epsilon\mu}{}_\alpha \nabla^\alpha F^{\beta\gamma} \nabla_\delta F_\beta{}^\epsilon + e^{2\Phi} c_{310} R_\beta{}^\mu{}_\alpha \gamma R_{\epsilon\mu\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F^{\epsilon\epsilon} \\
&+ e^{2\Phi} c_{312} R_\alpha{}^\mu{}_\delta \epsilon R_{\epsilon\mu\beta\gamma} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F^{\epsilon\epsilon} + e^{2\Phi} c_{316} R_{\beta\epsilon\alpha}{}^\mu R_{\epsilon\mu\gamma\delta} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F^{\epsilon\epsilon} \\
&+ e^{2\Phi} c_{317} R_\beta{}^\mu{}_\alpha \epsilon R_{\epsilon\mu\gamma\delta} \nabla^\alpha F^{\beta\gamma} \nabla_\delta F^{\epsilon\epsilon}, \\
[F^2 R^3]_{16} &= e^{2\Phi} c_{19} F^{\alpha\beta} F^{\gamma\delta} R_\alpha{}^\epsilon{}_\gamma{}^\epsilon R_\epsilon{}^\mu{}_\beta{}^\nu R_{\epsilon\nu\delta\mu} + e^{2\Phi} c_{20} F_{\alpha\beta} F^{\alpha\beta} R^{\gamma\delta\epsilon\epsilon} R_\epsilon{}^\mu{}_\gamma{}^\nu R_{\epsilon\nu\delta\mu} \\
&+ e^{2\Phi} c_{21} F^{\alpha\beta} F^{\gamma\delta} R_\alpha{}^\epsilon{}_\gamma{}^\epsilon R_\delta{}^\mu{}_\beta{}^\nu R_{\epsilon\nu\epsilon\mu} + e^{2\Phi} c_{22} F_\alpha{}^\gamma F^{\alpha\beta} R_\delta{}^\mu{}_\gamma{}^\nu R^{\delta\epsilon}{}_\beta{}^\epsilon R_{\epsilon\nu\epsilon\mu} \\
&+ e^{2\Phi} c_{33} F^{\alpha\beta} F^{\gamma\delta} R_\alpha{}^{\epsilon\epsilon\mu} R_\epsilon{}^\nu{}_\beta \epsilon R_{\mu\nu\gamma\delta} + e^{2\Phi} c_{34} F^{\alpha\beta} F^{\gamma\delta} R_\alpha{}^{\epsilon\epsilon\mu} R_\epsilon{}^\nu{}_\beta \gamma R_{\mu\nu\delta\epsilon} \\
&+ e^{2\Phi} c_{35} F_{\alpha\beta} F^{\alpha\beta} R_{\gamma\epsilon}{}^{\mu\nu} R^{\gamma\delta\epsilon\epsilon} R_{\mu\nu\delta\epsilon} + e^{2\Phi} c_{36} F^{\alpha\beta} F^{\gamma\delta} R_\alpha{}^\epsilon{}_\beta \gamma R_\delta{}^{\epsilon\mu\nu} R_{\mu\nu\epsilon\epsilon} \\
&+ e^{2\Phi} c_{37} F_\alpha{}^\gamma F^{\alpha\beta} R_\beta{}^\delta{}_\gamma{}^\epsilon R_\delta{}^{\epsilon\mu\nu} R_{\mu\nu\epsilon\epsilon} + e^{2\Phi} c_{38} F^{\alpha\beta} F^{\gamma\delta} R_{\alpha\gamma\beta\delta} R^{\epsilon\epsilon\mu\nu} R_{\mu\nu\epsilon\epsilon} \\
&+ e^{2\Phi} c_{48} F^{\alpha\beta} F^{\gamma\delta} R_\alpha{}^\epsilon{}_\gamma{}^\epsilon R_{\mu\nu\epsilon\epsilon} R^{\mu\nu}{}_\beta \delta + e^{2\Phi} c_{49} F^{\alpha\beta} F^{\gamma\delta} R^{\epsilon\epsilon}{}_\alpha \gamma R_{\mu\nu\delta\epsilon} R^{\mu\nu}{}_\beta \epsilon \\
&+ e^{2\Phi} c_{50} F^{\alpha\beta} F^{\gamma\delta} R_\alpha{}^\epsilon{}_\gamma{}^\epsilon R_{\mu\nu\delta\epsilon} R^{\mu\nu}{}_\beta \epsilon + e^{2\Phi} c_{51} F^{\alpha\beta} F^{\gamma\delta} R_\alpha{}^\epsilon{}_\beta{}^\epsilon R_{\mu\nu\epsilon\epsilon} R^{\mu\nu}{}_\gamma \delta \\
&+ e^{2\Phi} c_{52} F_\alpha{}^\gamma F^{\alpha\beta} R_\beta{}^{\delta\epsilon\epsilon} R_{\mu\nu\delta\epsilon} R^{\mu\nu}{}_\gamma \epsilon + e^{2\Phi} c_{53} F^{\alpha\beta} F^{\gamma\delta} R^{\epsilon\epsilon}{}_\alpha \beta R_{\mu\nu\delta\epsilon} R^{\mu\nu}{}_\gamma \epsilon, \\
[R^3 \Phi'']_3 &= c_{97} R_\gamma{}^\epsilon{}_\beta{}^\mu R^{\gamma\delta}{}_\alpha{}^\epsilon R_{\epsilon\mu\delta\epsilon} \nabla^\alpha \nabla^\beta \Phi + c_{107} R_\alpha{}^\gamma{}_\beta{}^\delta R_\gamma{}^{\epsilon\epsilon\mu} R_{\epsilon\mu\delta\epsilon} \nabla^\alpha \nabla^\beta \Phi \\
&+ c_{112} R_\alpha{}^{\gamma\delta\epsilon} R_{\epsilon\mu\gamma\epsilon} R^{\epsilon\mu}{}_\beta \delta \nabla^\alpha \nabla^\beta \Phi,
\end{aligned}$$



$$\begin{aligned}
[R^3\Phi'^2]_5 &= c_{76}R^{\beta\gamma\delta\epsilon}R_{\delta}{}^{\epsilon}{}_{\beta}{}^{\mu}R_{\epsilon\mu\gamma\epsilon}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi + c_{77}R_{\beta\delta}{}^{\epsilon\mu}R^{\beta\gamma\delta\epsilon}R_{\epsilon\mu\gamma\epsilon}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi \\
&+ c_{169}R_{\gamma}{}^{\epsilon}{}_{\beta}{}^{\mu}R^{\gamma\delta}{}_{\alpha}{}^{\epsilon}R_{\epsilon\mu\delta\epsilon}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi + c_{179}R_{\alpha}{}^{\gamma}{}_{\beta}{}^{\delta}R_{\gamma}{}^{\epsilon\epsilon\mu}R_{\epsilon\mu\delta\epsilon}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi \\
&+ c_{184}R_{\alpha}{}^{\gamma\delta\epsilon}R_{\epsilon\mu\gamma\epsilon}R^{\epsilon\mu}{}_{\beta\delta}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi, \\
[F^2R\Phi''\Phi'^2]_8 &= e^{2\Phi}c_{185}F_{\delta}^{\epsilon}F^{\delta\epsilon}R_{\gamma\epsilon\beta\epsilon}\nabla_{\alpha}\nabla^{\gamma}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi + e^{2\Phi}c_{186}F_{\gamma}^{\delta}F^{\epsilon\epsilon}R_{\epsilon\epsilon\beta\delta}\nabla_{\alpha}\nabla^{\gamma}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi \\
&+ e^{2\Phi}c_{187}F_{\beta}{}^{\delta}F^{\epsilon\epsilon}R_{\epsilon\epsilon\gamma\delta}\nabla_{\alpha}\nabla^{\gamma}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi + e^{2\Phi}c_{241}F_{\epsilon\epsilon}F^{\epsilon\epsilon}R_{\alpha\gamma\beta\delta}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi \\
&+ e^{2\Phi}c_{242}F_{\alpha}{}^{\epsilon}F_{\epsilon}{}^{\epsilon}R_{\gamma\epsilon\beta\delta}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi + e^{2\Phi}c_{243}F_{\alpha}{}^{\epsilon}F_{\gamma}{}^{\epsilon}R_{\delta\epsilon\beta\epsilon}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi \\
&+ e^{2\Phi}c_{245}F_{\alpha}{}^{\epsilon}F_{\gamma}{}^{\epsilon}R_{\epsilon\epsilon\beta\delta}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi + e^{2\Phi}c_{246}F_{\alpha\gamma}F^{\epsilon\epsilon}R_{\epsilon\epsilon\beta\delta}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi, \\
[R^2\Phi''^2]_5 &= c_{117}R_{\beta}{}^{\delta\epsilon\epsilon}R_{\epsilon\epsilon\gamma\delta}\nabla_{\alpha}\nabla^{\gamma}\Phi\nabla^{\alpha}\nabla^{\beta}\Phi + c_{140}R^{\gamma\delta\epsilon\epsilon}R_{\epsilon\epsilon\gamma\delta}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla_{\beta}\nabla_{\alpha}\Phi \\
&+ c_{234}R_{\alpha}{}^{\epsilon}{}_{\beta}{}^{\epsilon}R_{\gamma\epsilon\delta\epsilon}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi + c_{235}R_{\alpha}{}^{\epsilon}{}_{\gamma}{}^{\epsilon}R_{\delta\epsilon\beta\epsilon}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi \\
&+ c_{238}R_{\epsilon\epsilon\beta\delta}R^{\epsilon\epsilon}{}_{\alpha\gamma}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi, \\
[R^2\Phi''\Phi'^2]_4 &= c_{188}R_{\beta}{}^{\delta\epsilon\epsilon}R_{\epsilon\epsilon\gamma\delta}\nabla_{\alpha}\nabla^{\gamma}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi + c_{193}R^{\gamma\delta\epsilon\epsilon}R_{\epsilon\epsilon\gamma\delta}\nabla^{\alpha}\Phi\nabla_{\beta}\nabla_{\alpha}\Phi\nabla^{\beta}\Phi \\
&+ c_{244}R_{\gamma}{}^{\epsilon}{}_{\alpha}{}^{\epsilon}R_{\delta\epsilon\beta\epsilon}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi + c_{247}R_{\epsilon\epsilon\beta\delta}R^{\epsilon\epsilon}{}_{\alpha\gamma}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi, \\
[R^2\Phi'^4]_3 &= c_{192}R^{\gamma\delta\epsilon\epsilon}R_{\epsilon\epsilon\gamma\delta}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla_{\beta}\Phi\nabla^{\beta}\Phi + c_{223}R_{\beta}{}^{\delta\epsilon\epsilon}R_{\epsilon\epsilon\gamma\delta}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi \\
&+ c_{328}R_{\alpha}{}^{\epsilon}{}_{\beta}{}^{\epsilon}R_{\gamma\epsilon\delta\epsilon}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi, \\
[F^3F'\Phi'^3]_9 &= e^{4\Phi}c_{206}F_{\gamma}{}^{\epsilon}F_{\delta}{}^{\epsilon}F_{\epsilon\epsilon}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}F_{\beta}{}^{\delta} + e^{4\Phi}c_{207}F_{\gamma\delta}F_{\epsilon\epsilon}F^{\epsilon\epsilon}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}F_{\beta}{}^{\delta} \\
&+ e^{4\Phi}c_{209}F_{\beta}{}^{\epsilon}F_{\gamma\delta}F_{\epsilon\epsilon}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}F^{\delta\epsilon} + e^{4\Phi}c_{210}F_{\beta\delta}F_{\gamma}{}^{\epsilon}F_{\epsilon\epsilon}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}F^{\delta\epsilon} \\
&+ e^{4\Phi}c_{214}F_{\gamma}{}^{\epsilon}F_{\delta}{}^{\epsilon}F_{\epsilon\epsilon}\nabla_{\alpha}F_{\beta}{}^{\delta}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi + e^{4\Phi}c_{215}F_{\gamma\delta}F_{\epsilon\epsilon}F^{\epsilon\epsilon}\nabla_{\alpha}F_{\beta}{}^{\delta}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi \\
&+ e^{4\Phi}c_{217}F_{\beta\delta}F_{\gamma}{}^{\epsilon}F_{\epsilon\epsilon}\nabla_{\alpha}F^{\delta\epsilon}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi + e^{4\Phi}c_{267}F_{\beta}{}^{\epsilon}F_{\gamma\epsilon}F_{\delta\epsilon}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi\nabla^{\delta}F_{\alpha}{}^{\epsilon} \\
&+ e^{4\Phi}c_{268}F_{\beta\delta}F_{\gamma}{}^{\epsilon}F_{\epsilon\epsilon}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi\nabla^{\delta}F_{\alpha}{}^{\epsilon}, \\
[F^4R^2]_{20} &= e^{4\Phi}c_{11}F^{\alpha\beta}F^{\gamma\delta}F^{\epsilon\epsilon}F^{\mu\nu}R_{\alpha\gamma\beta\delta}R_{\epsilon\mu\epsilon\nu} + e^{4\Phi}c_{12}F_{\alpha}{}^{\gamma}F^{\alpha\beta}F_{\delta}{}^{\epsilon}F^{\delta\epsilon}R_{\beta}{}^{\mu}{}_{\gamma}{}^{\nu}R_{\epsilon\nu\epsilon\mu} \\
&+ e^{4\Phi}c_{13}F^{\alpha\beta}F^{\gamma\delta}F^{\epsilon\epsilon}F^{\mu\nu}R_{\alpha\gamma\epsilon\mu}R_{\epsilon\nu\beta\delta} + e^{4\Phi}c_{14}F_{\alpha}{}^{\gamma}F^{\alpha\beta}F_{\delta}{}^{\epsilon}F^{\delta\epsilon}R_{\beta}{}^{\mu}{}_{\epsilon}{}^{\nu}R_{\epsilon\nu\gamma\mu} \\
&+ e^{4\Phi}c_{15}F_{\alpha}{}^{\gamma}F^{\alpha\beta}F^{\delta\epsilon}F^{\epsilon\mu}R_{\delta}{}^{\nu}{}_{\beta\epsilon}R_{\epsilon\nu\gamma\mu} + e^{4\Phi}c_{16}F_{\alpha\beta}F^{\alpha\beta}F^{\gamma\delta}F^{\epsilon\epsilon}R_{\gamma}{}^{\mu}{}_{\epsilon}{}^{\nu}R_{\epsilon\nu\delta\mu} \\
&+ e^{4\Phi}c_{17}F_{\alpha}{}^{\gamma}F^{\alpha\beta}F_{\beta}{}^{\delta}F^{\epsilon\epsilon}R_{\epsilon}{}^{\mu}{}_{\gamma}{}^{\nu}R_{\epsilon\nu\delta\mu} + e^{4\Phi}c_{18}F_{\alpha}{}^{\gamma}F^{\alpha\beta}F^{\delta\epsilon}F^{\epsilon\mu}R_{\beta}{}^{\nu}{}_{\gamma\delta}R_{\epsilon\nu\epsilon\mu} \\
&+ e^{4\Phi}c_{26}F_{\alpha}{}^{\gamma}F^{\alpha\beta}F^{\delta\epsilon}F^{\epsilon\mu}R_{\beta\delta\epsilon}{}^{\nu}R_{\mu\nu\gamma\epsilon} + e^{4\Phi}c_{27}F_{\alpha}{}^{\gamma}F^{\alpha\beta}F^{\delta\epsilon}F^{\epsilon\mu}R_{\delta}{}^{\nu}{}_{\beta\epsilon}R_{\mu\nu\gamma\epsilon} \\
&+ e^{4\Phi}c_{28}F^{\alpha\beta}F^{\gamma\delta}F^{\epsilon\epsilon}F^{\mu\nu}R_{\alpha\beta\gamma\epsilon}R_{\mu\nu\delta\epsilon} + e^{4\Phi}c_{29}F_{\alpha}{}^{\gamma}F^{\alpha\beta}F_{\beta}{}^{\delta}F_{\gamma}{}^{\epsilon}R_{\delta}{}^{\epsilon\mu\nu}R_{\mu\nu\epsilon\epsilon} \\
&+ e^{4\Phi}c_{30}F_{\alpha\beta}F^{\alpha\beta}F_{\gamma}{}^{\epsilon}F^{\gamma\delta}R_{\delta}{}^{\epsilon\mu\nu}R_{\mu\nu\epsilon\epsilon} + e^{4\Phi}c_{31}F_{\alpha}{}^{\gamma}F^{\alpha\beta}F_{\beta}{}^{\delta}F_{\gamma\delta}R^{\epsilon\epsilon\mu\nu}R_{\mu\nu\epsilon\epsilon} \\
&+ e^{4\Phi}c_{32}F_{\alpha\beta}F^{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}R^{\epsilon\epsilon\mu\nu}R_{\mu\nu\epsilon\epsilon} + e^{4\Phi}c_{43}F_{\alpha}{}^{\gamma}F^{\alpha\beta}F_{\delta}{}^{\epsilon}F^{\delta\epsilon}R_{\mu\nu\gamma\epsilon}R^{\mu\nu}{}_{\beta\epsilon} \\
&+ e^{4\Phi}c_{44}F_{\alpha}{}^{\gamma}F^{\alpha\beta}F_{\beta}{}^{\delta}F^{\epsilon\epsilon}R_{\mu\nu\epsilon\epsilon}R^{\mu\nu}{}_{\gamma\delta} + e^{4\Phi}c_{45}F_{\alpha\beta}F^{\alpha\beta}F^{\gamma\delta}F^{\epsilon\epsilon}R_{\mu\nu\epsilon\epsilon}R^{\mu\nu}{}_{\gamma\delta} \\
&+ e^{4\Phi}c_{46}F_{\alpha}{}^{\gamma}F^{\alpha\beta}F_{\beta}{}^{\delta}F^{\epsilon\epsilon}R_{\mu\nu\delta\epsilon}R^{\mu\nu}{}_{\gamma\epsilon} + e^{4\Phi}c_{47}F_{\alpha\beta}F^{\alpha\beta}F^{\gamma\delta}F^{\epsilon\epsilon}R_{\mu\nu\delta\epsilon}R^{\mu\nu}{}_{\gamma\epsilon},
\end{aligned}$$

$$\begin{aligned}
[F^2 R^2 \Phi'']_{14} &= e^{2\Phi} c_{92} F_\gamma^\epsilon F^{\gamma\delta} R_\alpha^\epsilon{}_\beta{}^\mu R_{\delta\mu\epsilon\epsilon} \nabla^\alpha \nabla^\beta \Phi + e^{2\Phi} c_{93} F^{\gamma\delta} F^{\epsilon\epsilon} R_\gamma{}^\mu{}_\alpha{}_\delta R_{\epsilon\mu\beta\epsilon} \nabla^\alpha \nabla^\beta \Phi \\
&+ e^{2\Phi} c_{94} F_\gamma^\epsilon F^{\gamma\delta} R_\delta^\epsilon{}_\alpha{}^\mu R_{\epsilon\mu\beta\epsilon} \nabla^\alpha \nabla^\beta \Phi + e^{2\Phi} c_{95} F_\alpha{}^\gamma F^{\delta\epsilon} R_\delta^\epsilon{}_\beta{}^\mu R_{\epsilon\mu\gamma\epsilon} \nabla^\alpha \nabla^\beta \Phi \\
&+ e^{2\Phi} c_{96} F^{\gamma\delta} F^{\epsilon\epsilon} R_\alpha{}^\mu{}_\beta{}_\gamma R_{\epsilon\mu\delta\epsilon} \nabla^\alpha \nabla^\beta \Phi + e^{2\Phi} c_{100} F^{\gamma\delta} F^{\epsilon\epsilon} R_{\alpha\gamma\epsilon}{}^\mu R_{\epsilon\mu\beta\delta} \nabla^\alpha \nabla^\beta \Phi \\
&+ e^{2\Phi} c_{101} F^{\gamma\delta} F^{\epsilon\epsilon} R_\gamma{}^\mu{}_\alpha{}_\epsilon R_{\epsilon\mu\beta\delta} \nabla^\alpha \nabla^\beta \Phi + e^{2\Phi} c_{103} F_{\gamma\delta} F^{\gamma\delta} R_\alpha{}^\epsilon{}_\mu R_{\epsilon\mu\beta\epsilon} \nabla^\alpha \nabla^\beta \Phi \\
&+ e^{2\Phi} c_{105} F_\alpha{}^\gamma F_\gamma{}^\delta R_\beta{}^\epsilon{}_\mu R_{\epsilon\mu\delta\epsilon} \nabla^\alpha \nabla^\beta \Phi + e^{2\Phi} c_{106} F_\alpha{}^\gamma F_\beta{}^\delta R_\gamma{}^\epsilon{}_\mu R_{\epsilon\mu\delta\epsilon} \nabla^\alpha \nabla^\beta \Phi \\
&+ e^{2\Phi} c_{108} F_\alpha{}^\gamma F_{\beta\gamma} R^{\delta\epsilon\epsilon\mu} R_{\epsilon\mu\delta\epsilon} \nabla^\alpha \nabla^\beta \Phi + e^{2\Phi} c_{109} F_\gamma^\epsilon F^{\gamma\delta} R_{\epsilon\mu\beta\epsilon} R^{\epsilon\mu}{}_\alpha{}_\delta \nabla^\alpha \nabla^\beta \Phi \\
&+ e^{2\Phi} c_{110} F_\alpha{}^\gamma F^{\delta\epsilon} R_{\epsilon\mu\delta\epsilon} R^{\epsilon\mu}{}_\beta{}_\gamma \nabla^\alpha \nabla^\beta \Phi + e^{2\Phi} c_{111} F_\alpha{}^\gamma F^{\delta\epsilon} R_{\epsilon\mu\gamma\epsilon} R^{\epsilon\mu}{}_\beta{}_\delta \nabla^\alpha \nabla^\beta \Phi, \\
[F^2 R^2 \Phi'^2]_{19} &= e^{2\Phi} c_{75} F^{\beta\gamma} F^{\delta\epsilon} R_\beta{}^\epsilon{}_\delta R_{\epsilon\mu\gamma\epsilon} \nabla_\alpha \Phi \nabla^\alpha \Phi + e^{2\Phi} c_{79} F_\beta{}^\delta F^{\beta\gamma} R_\gamma{}^\epsilon{}_\mu R_{\epsilon\mu\delta\epsilon} \nabla_\alpha \Phi \nabla^\alpha \Phi \\
&+ e^{2\Phi} c_{80} F_{\beta\gamma} F^{\beta\gamma} R^{\delta\epsilon\epsilon\mu} R_{\epsilon\mu\delta\epsilon} \nabla_\alpha \Phi \nabla^\alpha \Phi + e^{2\Phi} c_{81} F^{\beta\gamma} F^{\delta\epsilon} R_{\epsilon\mu\delta\epsilon} R^{\epsilon\mu}{}_\beta{}_\gamma \nabla_\alpha \Phi \nabla^\alpha \Phi \\
&+ e^{2\Phi} c_{82} F^{\beta\gamma} F^{\delta\epsilon} R_{\epsilon\mu\gamma\epsilon} R^{\epsilon\mu}{}_\beta{}_\delta \nabla_\alpha \Phi \nabla^\alpha \Phi + e^{2\Phi} c_{164} F_\gamma^\epsilon F^{\gamma\delta} R_\alpha^\epsilon{}_\beta{}^\mu R_{\delta\mu\epsilon\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \\
&+ e^{2\Phi} c_{165} F^{\gamma\delta} F^{\epsilon\epsilon} R_\gamma{}^\mu{}_\alpha{}_\delta R_{\epsilon\mu\beta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi + e^{2\Phi} c_{166} F_\gamma^\epsilon F^{\gamma\delta} R_\delta^\epsilon{}_\alpha{}^\mu R_{\epsilon\mu\beta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \\
&+ e^{2\Phi} c_{167} F_\alpha{}^\gamma F^{\delta\epsilon} R_\delta^\epsilon{}_\beta{}^\mu R_{\epsilon\mu\gamma\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi + e^{2\Phi} c_{168} F^{\gamma\delta} F^{\epsilon\epsilon} R_\alpha{}^\mu{}_\beta{}_\gamma R_{\epsilon\mu\delta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \\
&+ e^{2\Phi} c_{172} F^{\gamma\delta} F^{\epsilon\epsilon} R_{\alpha\gamma\epsilon}{}^\mu R_{\epsilon\mu\beta\delta} \nabla^\alpha \Phi \nabla^\beta \Phi + e^{2\Phi} c_{173} F^{\gamma\delta} F^{\epsilon\epsilon} R_\gamma{}^\mu{}_\alpha{}_\epsilon R_{\epsilon\mu\beta\delta} \nabla^\alpha \Phi \nabla^\beta \Phi \\
&+ e^{2\Phi} c_{175} F_{\gamma\delta} F^{\gamma\delta} R_\alpha{}^\epsilon{}_\mu R_{\epsilon\mu\beta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi + e^{2\Phi} c_{177} F_\alpha{}^\gamma F_\gamma{}^\delta R_\beta{}^\epsilon{}_\mu R_{\epsilon\mu\delta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \\
&+ e^{2\Phi} c_{178} F_\alpha{}^\gamma F_\beta{}^\delta R_\gamma{}^\epsilon{}_\mu R_{\epsilon\mu\delta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi + e^{2\Phi} c_{180} F_\alpha{}^\gamma F_{\beta\gamma} R^{\delta\epsilon\epsilon\mu} R_{\epsilon\mu\delta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \\
&+ e^{2\Phi} c_{181} F_\gamma^\epsilon F^{\gamma\delta} R_{\epsilon\mu\beta\epsilon} R^{\epsilon\mu}{}_\alpha{}_\delta \nabla^\alpha \Phi \nabla^\beta \Phi + e^{2\Phi} c_{182} F_\alpha{}^\gamma F^{\delta\epsilon} R_{\epsilon\mu\delta\epsilon} R^{\epsilon\mu}{}_\beta{}_\gamma \nabla^\alpha \Phi \nabla^\beta \Phi \\
&+ e^{2\Phi} c_{183} F_\alpha{}^\gamma F^{\delta\epsilon} R_{\epsilon\mu\gamma\epsilon} R^{\epsilon\mu}{}_\beta{}_\delta \nabla^\alpha \Phi \nabla^\beta \Phi, \\
[F^4 R \Phi'^2]_{12} &= e^{4\Phi} c_{73} F_\beta{}^\delta F^{\beta\gamma} F_\epsilon{}^\mu F^{\epsilon\epsilon} R_{\gamma\epsilon\delta\mu} \nabla_\alpha \Phi \nabla^\alpha \Phi + e^{4\Phi} c_{74} F_{\beta\gamma} F^{\beta\gamma} F^{\delta\epsilon} F^{\epsilon\mu} R_{\delta\epsilon\epsilon\mu} \nabla_\alpha \Phi \nabla^\alpha \Phi \\
&+ e^{4\Phi} c_{78} F_\beta{}^\delta F^{\beta\gamma} F_\gamma^\epsilon F^{\epsilon\mu} R_{\epsilon\mu\delta\epsilon} \nabla_\alpha \Phi \nabla^\alpha \Phi + e^{4\Phi} c_{159} F_\gamma^\epsilon F^{\gamma\delta} F_\delta^\epsilon F_\epsilon{}^\mu R_{\alpha\epsilon\beta\mu} \nabla^\alpha \Phi \nabla^\beta \Phi \\
&+ e^{4\Phi} c_{160} F_{\gamma\delta} F^{\gamma\delta} F_\epsilon{}^\mu F^{\epsilon\epsilon} R_{\alpha\epsilon\beta\mu} \nabla^\alpha \Phi \nabla^\beta \Phi + e^{4\Phi} c_{161} F_\alpha{}^\gamma F_\beta{}^\delta F_\epsilon{}^\mu F^{\epsilon\epsilon} R_{\gamma\epsilon\delta\mu} \nabla^\alpha \Phi \nabla^\beta \Phi \\
&+ e^{4\Phi} c_{162} F_\alpha{}^\gamma F_\gamma{}^\delta F_\epsilon{}^\mu F^{\epsilon\epsilon} R_{\delta\epsilon\beta\mu} \nabla^\alpha \Phi \nabla^\beta \Phi + e^{4\Phi} c_{163} F_\alpha{}^\gamma F_{\beta\gamma} F^{\delta\epsilon} F^{\epsilon\mu} R_{\delta\epsilon\epsilon\mu} \nabla^\alpha \Phi \nabla^\beta \Phi \\
&+ e^{4\Phi} c_{170} F_\alpha{}^\gamma F_\delta{}^\epsilon F^{\delta\epsilon} F_\epsilon{}^\mu R_{\epsilon\mu\beta\gamma} \nabla^\alpha \Phi \nabla^\beta \Phi + e^{4\Phi} c_{171} F_\alpha{}^\gamma F_{\delta\epsilon} F^{\delta\epsilon} F^{\epsilon\mu} R_{\epsilon\mu\beta\gamma} \nabla^\alpha \Phi \nabla^\beta \Phi \\
&+ e^{4\Phi} c_{174} F_\alpha{}^\gamma F_\gamma{}^\delta F_\delta{}^\epsilon F^{\epsilon\mu} R_{\epsilon\mu\beta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi + e^{4\Phi} c_{176} F_\alpha{}^\gamma F_\beta{}^\delta F_\gamma^\epsilon F^{\epsilon\mu} R_{\epsilon\mu\delta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi, \\
[F^2 R F'^2]_{33} &= e^{4\Phi} c_{59} F_\gamma^\epsilon F^{\epsilon\mu} R_{\epsilon\mu\delta\epsilon} \nabla_\alpha F_\beta{}^\delta \nabla^\alpha F^{\beta\gamma} + e^{4\Phi} c_{66} F_\beta{}^\epsilon F_\delta{}^\mu R_{\epsilon\mu\gamma\epsilon} \nabla_\alpha F^{\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \\
&+ e^{4\Phi} c_{67} F_{\beta\delta} F^{\epsilon\mu} R_{\epsilon\mu\gamma\epsilon} \nabla_\alpha F^{\delta\epsilon} \nabla^\alpha F^{\beta\gamma} + e^{4\Phi} c_{68} F_\beta{}^\epsilon F_\gamma{}^\mu R_{\epsilon\mu\delta\epsilon} \nabla_\alpha F^{\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \\
&+ e^{4\Phi} c_{69} F_{\beta\gamma} F^{\epsilon\mu} R_{\epsilon\mu\delta\epsilon} \nabla_\alpha F^{\delta\epsilon} \nabla^\alpha F^{\beta\gamma} + e^{4\Phi} c_{120} F^{\delta\epsilon} F^{\epsilon\mu} R_{\delta\epsilon\epsilon\mu} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_{\alpha\gamma} \\
&+ e^{4\Phi} c_{124} F_\epsilon{}^\mu F^{\epsilon\epsilon} R_{\gamma\epsilon\delta\mu} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_\alpha{}^\delta + e^{4\Phi} c_{125} F_\gamma^\epsilon F^{\epsilon\mu} R_{\epsilon\mu\delta\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_\alpha{}^\delta \\
&+ e^{4\Phi} c_{127} F_\epsilon{}^\mu F^{\epsilon\epsilon} R_{\delta\epsilon\alpha\mu} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F_\gamma{}^\delta + e^{4\Phi} c_{129} F_{\epsilon\mu} F^{\epsilon\mu} R_{\delta\epsilon\alpha\gamma} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F^{\delta\epsilon} \\
&+ e^{4\Phi} c_{130} F_\delta{}^\epsilon F_\epsilon{}^\mu R_{\epsilon\mu\alpha\gamma} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F^{\delta\epsilon} + e^{4\Phi} c_{131} F_\gamma^\epsilon F_\delta{}^\mu R_{\epsilon\mu\alpha\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F^{\delta\epsilon} \\
&+ e^{4\Phi} c_{132} F_\alpha{}^\epsilon F_\delta{}^\mu R_{\epsilon\mu\gamma\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F^{\delta\epsilon} + e^{4\Phi} c_{134} F_\alpha{}^\epsilon F_\delta{}^\mu R_{\epsilon\mu\gamma\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F^{\delta\epsilon} \\
&+ e^{4\Phi} c_{135} F_{\alpha\delta} F^{\epsilon\mu} R_{\epsilon\mu\gamma\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla_\beta F^{\delta\epsilon} + e^{4\Phi} c_{278} F_{\epsilon\mu} F^{\epsilon\mu} R_{\gamma\epsilon\alpha\delta} \nabla^\alpha F^{\beta\gamma} \nabla^\delta F_\beta{}^\epsilon \\
&+ e^{4\Phi} c_{279} F_\alpha{}^\epsilon F_\epsilon{}^\mu R_{\delta\mu\gamma\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla^\delta F_\beta{}^\epsilon + e^{4\Phi} c_{280} F_\alpha{}^\epsilon F_\epsilon{}^\mu R_{\delta\mu\gamma\epsilon} \nabla^\alpha F^{\beta\gamma} \nabla^\delta F_\beta{}^\epsilon
\end{aligned}$$

$$\begin{aligned}
& +e^{4\Phi}c_{281}F_{\gamma}{}^{\varepsilon}F_{\varepsilon}{}^{\mu}R_{\varepsilon\mu\alpha\delta}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F_{\beta}{}^{\varepsilon}+e^{4\Phi}c_{282}F_{\alpha}{}^{\varepsilon}F_{\varepsilon}{}^{\mu}R_{\varepsilon\mu\gamma\delta}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F_{\beta}{}^{\varepsilon} \\
& +e^{4\Phi}c_{284}F_{\alpha}{}^{\varepsilon}F_{\gamma}{}^{\mu}R_{\varepsilon\mu\delta\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F_{\beta}{}^{\varepsilon}+e^{4\Phi}c_{286}F_{\alpha}{}^{\varepsilon}F_{\delta}{}^{\mu}R_{\varepsilon\mu\gamma\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F_{\beta}{}^{\varepsilon} \\
& +e^{4\Phi}c_{287}F_{\alpha\delta}F^{\varepsilon\mu}R_{\varepsilon\mu\gamma\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F_{\beta}{}^{\varepsilon}+e^{4\Phi}c_{304}F_{\beta}{}^{\mu}F_{\varepsilon\mu}R_{\gamma\varepsilon\alpha\delta}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F^{\varepsilon\varepsilon} \\
& +e^{4\Phi}c_{305}F_{\alpha}{}^{\mu}F_{\varepsilon\mu}R_{\delta\varepsilon\beta\gamma}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F^{\varepsilon\varepsilon}+e^{4\Phi}c_{306}F_{\alpha}{}^{\mu}F_{\beta\mu}R_{\varepsilon\varepsilon\gamma\delta}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F^{\varepsilon\varepsilon} \\
& +e^{4\Phi}c_{307}F_{\alpha\delta}F_{\beta}{}^{\mu}R_{\varepsilon\mu\gamma\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F^{\varepsilon\varepsilon}+e^{4\Phi}c_{308}F_{\alpha\beta}F_{\delta}{}^{\mu}R_{\varepsilon\mu\gamma\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F^{\varepsilon\varepsilon} \\
& +e^{4\Phi}c_{309}F_{\alpha\beta}F_{\gamma}{}^{\mu}R_{\varepsilon\mu\delta\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F^{\varepsilon\varepsilon}+e^{4\Phi}c_{311}F_{\alpha\varepsilon}F_{\delta}{}^{\mu}R_{\varepsilon\mu\beta\gamma}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F^{\varepsilon\varepsilon} \\
& +e^{4\Phi}c_{313}F_{\alpha}{}^{\mu}F_{\beta\varepsilon}R_{\varepsilon\mu\gamma\delta}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F^{\varepsilon\varepsilon}+e^{4\Phi}c_{314}F_{\alpha\varepsilon}F_{\beta}{}^{\mu}R_{\varepsilon\mu\gamma\delta}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F^{\varepsilon\varepsilon} \\
& +e^{4\Phi}c_{315}F_{\alpha\beta}F_{\varepsilon}{}^{\mu}R_{\varepsilon\mu\gamma\delta}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F^{\varepsilon\varepsilon}, \\
[F^6R]_7 & =e^{6\Phi}c_6F_{\alpha}{}^{\gamma}F^{\alpha\beta}F_{\beta}{}^{\delta}F_{\varepsilon}{}^{\mu}F^{\varepsilon\varepsilon}F_{\varepsilon}{}^{\nu}R_{\gamma\mu\delta\nu}+e^{6\Phi}c_7F_{\alpha}{}^{\gamma}F^{\alpha\beta}F_{\beta}{}^{\delta}F_{\gamma}{}^{\varepsilon}F_{\varepsilon}{}^{\nu}F^{\varepsilon\mu}R_{\delta\mu\varepsilon\nu} \\
& +e^{6\Phi}c_8F_{\alpha\beta}F^{\alpha\beta}F_{\gamma}{}^{\varepsilon}F^{\gamma\delta}F_{\varepsilon}{}^{\nu}F^{\varepsilon\mu}R_{\delta\mu\varepsilon\nu}+e^{6\Phi}c_9F_{\alpha}{}^{\gamma}F^{\alpha\beta}F_{\beta}{}^{\delta}F_{\gamma\delta}F^{\varepsilon\varepsilon}F^{\mu\nu}R_{\varepsilon\mu\varepsilon\nu} \\
& +e^{6\Phi}c_{10}F_{\alpha\beta}F^{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}F^{\varepsilon\varepsilon}F^{\mu\nu}R_{\varepsilon\mu\varepsilon\nu}+e^{6\Phi}c_{24}F_{\alpha}{}^{\gamma}F^{\alpha\beta}F_{\beta}{}^{\delta}F_{\gamma}{}^{\varepsilon}F_{\delta}{}^{\nu}F^{\varepsilon\mu}R_{\mu\nu\varepsilon\varepsilon} \\
& +e^{6\Phi}c_{25}F_{\alpha\beta}F^{\alpha\beta}F_{\gamma}{}^{\varepsilon}F^{\gamma\delta}F_{\delta}{}^{\nu}F^{\varepsilon\mu}R_{\mu\nu\varepsilon\varepsilon}, \\
[F^6\Phi'']_4 & =e^{6\Phi}c_{83}F_{\alpha}{}^{\gamma}F_{\beta}{}^{\delta}F_{\gamma}{}^{\varepsilon}F_{\delta}{}^{\nu}F_{\varepsilon}{}^{\mu}F_{\varepsilon\mu}\nabla^{\alpha}\nabla^{\beta}\Phi+e^{6\Phi}c_{84}F_{\alpha}{}^{\gamma}F_{\beta\gamma}F_{\delta}{}^{\varepsilon}F^{\delta\varepsilon}F_{\varepsilon}{}^{\mu}F_{\varepsilon\mu}\nabla^{\alpha}\nabla^{\beta}\Phi \\
& +e^{6\Phi}c_{85}F_{\alpha}{}^{\gamma}F_{\beta}{}^{\delta}F_{\gamma}{}^{\varepsilon}F_{\delta\varepsilon}F_{\varepsilon\mu}F^{\varepsilon\mu}\nabla^{\alpha}\nabla^{\beta}\Phi+e^{6\Phi}c_{86}F_{\alpha}{}^{\gamma}F_{\beta\gamma}F_{\delta\varepsilon}F^{\delta\varepsilon}F_{\varepsilon\mu}F^{\varepsilon\mu}\nabla^{\alpha}\nabla^{\beta}\Phi, \\
[F^6\Phi'^2]_1 & =e^{6\Phi}c_{158}F_{\alpha}^{\gamma}F_{\beta\gamma}F_{\delta\varepsilon}F^{\delta\varepsilon}F_{\varepsilon\mu}F^{\varepsilon\mu}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi, \\
[F^4\Phi'^2\Phi'']_2 & =e^{4\Phi}c_{239}F_{\alpha\gamma}F_{\beta}{}^{\varepsilon}F_{\delta}{}^{\nu}F_{\varepsilon\varepsilon}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi+e^{4\Phi}c_{240}F_{\alpha\gamma}F_{\beta\delta}F_{\varepsilon\varepsilon}F^{\varepsilon\varepsilon}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi, \\
[FR^2F'\Phi']_{15} & =e^{2\Phi}c_{72}F_{\beta}{}^{\delta}R_{\gamma}{}^{\varepsilon\varepsilon\mu}R_{\varepsilon\mu\delta\varepsilon}\nabla_{\alpha}F^{\beta\gamma}\nabla^{\alpha}\Phi+e^{2\Phi}c_{143}F_{\beta}{}^{\delta}R_{\gamma}{}^{\varepsilon\varepsilon\mu}R_{\varepsilon\mu\delta\varepsilon}\nabla^{\alpha}\Phi\nabla^{\beta}F_{\alpha}{}^{\gamma} \\
& +e^{2\Phi}c_{144}F_{\beta\gamma}R^{\delta\varepsilon\varepsilon\mu}R_{\varepsilon\mu\delta\varepsilon}\nabla^{\alpha}\Phi\nabla^{\beta}F_{\alpha}{}^{\gamma}+e^{2\Phi}c_{145}F^{\delta\varepsilon}R_{\varepsilon\mu\delta\varepsilon}R^{\varepsilon\mu}{}_{\beta\gamma}\nabla^{\alpha}\Phi\nabla^{\beta}F_{\alpha}{}^{\gamma} \\
& +e^{2\Phi}c_{146}F^{\delta\varepsilon}R_{\varepsilon\mu\gamma\varepsilon}R^{\varepsilon\mu}{}_{\beta\delta}\nabla^{\alpha}\Phi\nabla^{\beta}F_{\alpha}{}^{\gamma}+e^{2\Phi}c_{147}F^{\varepsilon\varepsilon}R_{\beta}{}^{\mu}{}_{\alpha\gamma}R_{\varepsilon\mu\delta\varepsilon}\nabla^{\alpha}\Phi\nabla^{\beta}F^{\gamma\delta} \\
& +e^{2\Phi}c_{148}F_{\alpha}{}^{\varepsilon}R_{\gamma}{}^{\nu}{}_{\beta}{}^{\mu}R_{\varepsilon\mu\delta\varepsilon}\nabla^{\alpha}\Phi\nabla^{\beta}F^{\gamma\delta}+e^{2\Phi}c_{149}F^{\varepsilon\varepsilon}R_{\gamma}{}^{\mu}{}_{\alpha\beta}R_{\varepsilon\mu\delta\varepsilon}\nabla^{\alpha}\Phi\nabla^{\beta}F^{\gamma\delta} \\
& +e^{2\Phi}c_{150}F^{\varepsilon\varepsilon}R_{\gamma}{}^{\mu}{}_{\beta\varepsilon}R_{\varepsilon\mu\alpha\delta}\nabla^{\alpha}\Phi\nabla^{\beta}F^{\gamma\delta}+e^{2\Phi}c_{151}F^{\varepsilon\varepsilon}R_{\beta}{}^{\mu}{}_{\alpha\varepsilon}R_{\varepsilon\mu\gamma\delta}\nabla^{\alpha}\Phi\nabla^{\beta}F^{\gamma\delta} \\
& +e^{2\Phi}c_{152}F^{\varepsilon\varepsilon}R_{\varepsilon}{}^{\mu}{}_{\alpha\beta}R_{\varepsilon\mu\gamma\delta}\nabla^{\alpha}\Phi\nabla^{\beta}F^{\gamma\delta}+e^{2\Phi}c_{153}F_{\alpha\gamma}R_{\beta}{}^{\varepsilon\varepsilon\mu}R_{\varepsilon\mu\delta\varepsilon}\nabla^{\alpha}\Phi\nabla^{\beta}F^{\gamma\delta} \\
& +e^{2\Phi}c_{154}F_{\gamma}{}^{\varepsilon}R_{\varepsilon\mu\beta\varepsilon}R^{\varepsilon\mu}{}_{\alpha\delta}\nabla^{\alpha}\Phi\nabla^{\beta}F^{\gamma\delta}+e^{2\Phi}c_{155}F_{\beta}{}^{\varepsilon}R_{\varepsilon\mu\gamma\delta}R^{\varepsilon\mu}{}_{\alpha\varepsilon}\nabla^{\alpha}\Phi\nabla^{\beta}F^{\gamma\delta} \\
& +e^{2\Phi}c_{156}F_{\alpha}{}^{\varepsilon}R_{\varepsilon\mu\gamma\delta}R^{\varepsilon\mu}{}_{\beta\varepsilon}\nabla^{\alpha}\Phi\nabla^{\beta}F^{\gamma\delta}, \\
[F^4\Phi'^4]_5 & =e^{4\Phi}c_{189}F_{\gamma}{}^{\varepsilon}F^{\gamma\delta}F_{\delta}{}^{\nu}F_{\varepsilon\varepsilon}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla_{\beta}\Phi\nabla^{\beta}\Phi+e^{4\Phi}c_{190}F_{\gamma\delta}F^{\gamma\delta}F_{\varepsilon\varepsilon}F^{\varepsilon\varepsilon}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla_{\beta}\Phi\nabla^{\beta}\Phi \\
& +e^{4\Phi}c_{219}F_{\beta}{}^{\delta}F_{\gamma}{}^{\varepsilon}F_{\delta}{}^{\nu}F_{\varepsilon\varepsilon}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi+e^{4\Phi}c_{220}F_{\beta}{}^{\delta}F_{\gamma\delta}F_{\varepsilon\varepsilon}F^{\varepsilon\varepsilon}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi \\
& +e^{4\Phi}c_{326}F_{\alpha}{}^{\varepsilon}F_{\beta\varepsilon}F_{\gamma}{}^{\nu}F_{\delta\varepsilon}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi\nabla^{\delta}\Phi, \\
[F^2R\Phi'^4]_4 & =e^{2\Phi}c_{191}F^{\gamma\delta}F^{\varepsilon\varepsilon}R_{\gamma\varepsilon\delta\varepsilon}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla_{\beta}\Phi\nabla^{\beta}\Phi+e^{2\Phi}c_{221}F_{\delta}{}^{\varepsilon}F^{\delta\varepsilon}R_{\beta\varepsilon\gamma\varepsilon}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi \\
& +e^{2\Phi}c_{222}F_{\beta}{}^{\delta}F^{\varepsilon\varepsilon}R_{\varepsilon\varepsilon\gamma\delta}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi+e^{2\Phi}c_{327}F_{\alpha}{}^{\varepsilon}F_{\beta}{}^{\nu}R_{\gamma\varepsilon\delta\varepsilon}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi\nabla^{\delta}\Phi,
\end{aligned}$$



$$\begin{aligned}
[FR\Phi'^3F']_{10} &= e^{2\Phi}c_{208}F^{\epsilon\epsilon}R_{\epsilon\epsilon\gamma\delta}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F_\beta{}^\delta + e^{2\Phi}c_{211}F_\gamma{}^\epsilon R_{\delta\epsilon\beta\epsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F^{\delta\epsilon} \\
&+ e^{2\Phi}c_{212}F_\beta{}^\epsilon R_{\delta\epsilon\gamma\epsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F^{\delta\epsilon} + e^{2\Phi}c_{213}F_\delta{}^\epsilon R_{\epsilon\epsilon\beta\gamma}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F^{\delta\epsilon} \\
&+ e^{2\Phi}c_{216}F^{\epsilon\epsilon}R_{\epsilon\epsilon\gamma\delta}\nabla_\alpha F_\beta{}^\delta\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi + e^{2\Phi}c_{218}F_\delta{}^\epsilon R_{\beta\epsilon\gamma\epsilon}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi \\
&+ e^{2\Phi}c_{269}F_\delta{}^\epsilon R_{\beta\epsilon\gamma\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla^\delta F_\alpha{}^\epsilon + e^{2\Phi}c_{270}F_\beta{}^\epsilon R_{\delta\epsilon\gamma\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla^\delta F_\alpha{}^\epsilon \\
&+ e^{2\Phi}c_{271}F_\beta{}^\epsilon R_{\epsilon\epsilon\gamma\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla^\delta F_\alpha{}^\epsilon + e^{2\Phi}c_{322}F_{\alpha\epsilon}R_{\beta\epsilon\gamma\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla^\delta F^{\epsilon\epsilon}, \\
[F^4R\Phi'']_9 &= e^{4\Phi}c_{87}F_\gamma{}^\epsilon F^{\gamma\delta}F_\delta{}^\epsilon F_\epsilon{}^\mu R_{\alpha\epsilon\beta\mu}\nabla^\alpha\nabla^\beta\Phi + e^{4\Phi}c_{88}F_{\gamma\delta}F^{\gamma\delta}F_\epsilon{}^\mu F^{\epsilon\epsilon}R_{\alpha\epsilon\beta\mu}\nabla^\alpha\nabla^\beta\Phi \\
&+ e^{4\Phi}c_{89}F_\alpha{}^\gamma F_\beta{}^\delta F_\epsilon{}^\mu F^{\epsilon\epsilon}R_{\gamma\epsilon\delta\mu}\nabla^\alpha\nabla^\beta\Phi + e^{4\Phi}c_{90}F_\alpha{}^\gamma F_\gamma{}^\delta F_\epsilon{}^\mu F^{\epsilon\epsilon}R_{\delta\epsilon\beta\mu}\nabla^\alpha\nabla^\beta\Phi \\
&+ e^{4\Phi}c_{91}F_\alpha{}^\gamma F_{\beta\gamma}F^{\delta\epsilon}F^{\epsilon\mu}R_{\delta\epsilon\epsilon\mu}\nabla^\alpha\nabla^\beta\Phi + e^{4\Phi}c_{98}F_\alpha{}^\gamma F_\delta{}^\epsilon F^{\delta\epsilon}F_\epsilon{}^\mu R_{\epsilon\mu\beta\gamma}\nabla^\alpha\nabla^\beta\Phi \\
&+ e^{4\Phi}c_{99}F_\alpha{}^\gamma F_{\delta\epsilon}F^{\delta\epsilon}F^{\epsilon\mu}R_{\epsilon\mu\beta\gamma}\nabla^\alpha\nabla^\beta\Phi + e^{4\Phi}c_{102}F_\alpha{}^\gamma F_\gamma{}^\delta F_\delta{}^\epsilon \\
&+ e^{4\Phi}c_{104}F_\alpha{}^\gamma F_\gamma{}^\delta F_\gamma{}^\epsilon F^{\epsilon\mu}R_{\epsilon\mu\delta\epsilon}\nabla^\alpha\nabla^\beta\Phi, F^{\epsilon\mu}R_{\epsilon\mu\beta\epsilon}\nabla^\alpha\nabla^\beta\Phi \\
[F^2\Phi'^6]_2 &= e^{2\Phi}c_{224}F_{\delta\epsilon}F^{\delta\epsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\Phi\nabla^\beta\Phi\nabla_\gamma\Phi\nabla^\gamma\Phi + e^{2\Phi}c_{330}F_\gamma{}^\epsilon F_{\delta\epsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\Phi\nabla^\beta\Phi\nabla_\gamma\Phi\nabla^\delta\Phi, \\
[F^4\Phi''^2]_8 &= e^{4\Phi}c_{113}F_\beta{}^\delta F_\gamma{}^\epsilon F_\delta{}^\epsilon F_{\epsilon\epsilon}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\nabla^\beta\Phi + e^{4\Phi}c_{114}F_\beta{}^\delta F_{\gamma\delta}F_{\epsilon\epsilon}F^{\epsilon\epsilon}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\nabla^\beta\Phi \\
&+ e^{4\Phi}c_{137}F_\gamma{}^\epsilon F^{\gamma\delta}F_\delta{}^\epsilon F_{\epsilon\epsilon}\nabla^\alpha\nabla^\beta\Phi\nabla_\beta\nabla_\alpha\Phi + e^{4\Phi}c_{138}F_{\gamma\delta}F^{\gamma\delta}F_{\epsilon\epsilon}F^{\epsilon\epsilon}\nabla^\alpha\nabla^\beta\Phi\nabla_\beta\nabla_\alpha\Phi \\
&+ e^{4\Phi}c_{226}F_\alpha{}^\epsilon F_\beta{}^\epsilon F_{\gamma\epsilon}F_{\delta\epsilon}\nabla^\alpha\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi + e^{4\Phi}c_{227}F_\alpha{}^\epsilon F_{\beta\epsilon}F_\gamma{}^\epsilon F_{\delta\epsilon}\nabla^\alpha\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi \\
&+ e^{4\Phi}c_{228}F_{\alpha\gamma}F_\beta{}^\epsilon F_\delta{}^\epsilon F_{\epsilon\epsilon}\nabla^\alpha\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi + e^{4\Phi}c_{229}F_{\alpha\gamma}F_{\beta\delta}F_{\epsilon\epsilon}F^{\epsilon\epsilon}\nabla^\alpha\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi, \\
[F^2R\Phi''^2]_9 &= e^{2\Phi}c_{115}F_\delta{}^\epsilon F^{\delta\epsilon}R_{\beta\epsilon\gamma\epsilon}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\nabla^\beta\Phi + e^{2\Phi}c_{116}F_\beta{}^\delta F^{\delta\epsilon}R_{\epsilon\epsilon\gamma\delta}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\nabla^\beta\Phi \\
&+ e^{2\Phi}c_{139}F^{\gamma\delta}F^{\epsilon\epsilon}R_{\gamma\epsilon\delta\epsilon}\nabla^\alpha\nabla^\beta\Phi\nabla_\beta\nabla_\alpha\Phi + e^{2\Phi}c_{230}F_{\epsilon\epsilon}F^{\epsilon\epsilon}R_{\alpha\gamma\beta\delta}\nabla^\alpha\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi \\
&+ e^{2\Phi}c_{231}F_\alpha{}^\epsilon F_\gamma{}^\epsilon R_{\beta\epsilon\delta\epsilon}\nabla^\alpha\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi + e^{2\Phi}c_{232}F_\alpha{}^\epsilon F_\beta{}^\epsilon R_{\gamma\epsilon\delta\epsilon}\nabla^\alpha\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi \\
&+ e^{2\Phi}c_{233}F_\alpha{}^\epsilon F_\epsilon{}^\epsilon R_{\gamma\epsilon\beta\delta}\nabla^\alpha\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi + e^{2\Phi}c_{236}F_\alpha{}^\epsilon F_\gamma{}^\epsilon R_{\epsilon\epsilon\beta\delta}\nabla^\alpha\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi \\
&+ e^{2\Phi}c_{237}F_{\alpha\gamma}F^{\epsilon\epsilon}R_{\epsilon\epsilon\beta\delta}\nabla^\alpha\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi, \\
[F^2F'^2\Phi'']_{21} &= e^{4\Phi}c_{136}F_{\gamma\delta}F_{\epsilon\epsilon}\nabla_\alpha\nabla^\epsilon\Phi\nabla^\alpha F^{\beta\gamma}\nabla_\beta F^{\delta\epsilon} + e^{4\Phi}c_{141}F_{\gamma\delta}F_{\epsilon\epsilon}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta\nabla^\epsilon\Phi \\
&+ e^{4\Phi}c_{200}F_\delta{}^\epsilon F_{\epsilon\epsilon}\nabla_\alpha F_\beta{}^\delta\nabla^\alpha F^{\beta\gamma}\nabla_\gamma\nabla^\epsilon\Phi + e^{4\Phi}c_{201}F_\delta{}^\epsilon F_{\epsilon\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta\nabla_\gamma\nabla^\epsilon\Phi \\
&+ e^{4\Phi}c_{256}F_\gamma{}^\epsilon F_{\epsilon\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F^{\delta\epsilon}\nabla_\delta\nabla_\alpha\Phi + e^{4\Phi}c_{257}F_\gamma{}^\epsilon F_{\epsilon\epsilon}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\delta\nabla_\beta\Phi \\
&+ e^{4\Phi}c_{259}F_{\epsilon\epsilon}F^{\epsilon\epsilon}\nabla_\alpha F_\beta{}^\delta\nabla^\alpha F^{\beta\gamma}\nabla_\delta\nabla_\gamma\Phi + e^{4\Phi}c_{260}F_{\epsilon\epsilon}F^{\epsilon\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta\nabla_\delta\nabla_\gamma\Phi \\
&+ e^{4\Phi}c_{265}F_{\alpha\gamma}F_{\epsilon\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F^{\delta\epsilon}\nabla_\delta\nabla^\epsilon\Phi + e^{4\Phi}c_{289}F_{\gamma\epsilon}F_{\delta\epsilon}\nabla_\alpha\nabla^\epsilon\Phi\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon \\
&+ e^{4\Phi}c_{290}F_{\gamma\delta}F_{\epsilon\epsilon}\nabla_\alpha\nabla^\epsilon\Phi\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon + e^{4\Phi}c_{293}F_\delta{}^\epsilon F_{\epsilon\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\gamma\nabla_\alpha\Phi\nabla^\delta F_\beta{}^\epsilon \\
&+ e^{4\Phi}c_{294}F_{\alpha\delta}F_{\epsilon\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\gamma\nabla^\epsilon\Phi\nabla^\delta F_\beta{}^\epsilon + e^{4\Phi}c_{297}F_\gamma{}^\epsilon F_{\epsilon\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\delta\nabla_\alpha\Phi\nabla^\delta F_\beta{}^\epsilon \\
&+ e^{4\Phi}c_{318}F_{\gamma\delta}F_{\epsilon\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta\nabla_\alpha\Phi\nabla^\delta F^{\epsilon\epsilon} + e^{4\Phi}c_{324}F_{\beta\epsilon}F_{\gamma\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\delta\nabla_\alpha\Phi\nabla^\delta F^{\epsilon\epsilon} \\
&+ e^{4\Phi}c_{325}F_{\beta\gamma}F_{\epsilon\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\delta\nabla_\alpha\Phi\nabla^\delta F^{\epsilon\epsilon} + e^{4\Phi}c_{336}F_\delta{}^\epsilon F_{\epsilon\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_{\alpha\gamma}\nabla^\delta\nabla^\epsilon\Phi \\
&+ e^{4\Phi}c_{345}F_{\alpha\delta}F_{\gamma\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\epsilon}\nabla_\epsilon\nabla_\beta\Phi + e^{4\Phi}c_{371}F_{\gamma\epsilon}F_{\delta\epsilon}\nabla_\alpha F_\beta{}^\delta\nabla^\alpha F^{\beta\gamma}\nabla^\epsilon\nabla^\epsilon\Phi \\
&+ e^{4\Phi}c_{372}F_{\gamma\epsilon}F_{\delta\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta\nabla^\epsilon\nabla^\epsilon\Phi,
\end{aligned}$$

$$\begin{aligned}
[F^3 R \Phi' F']_1 &= e^{4\Phi} c_{142} F_\beta{}^\delta F_\gamma{}^\epsilon F^{\epsilon\mu} R_{\epsilon\mu\delta\epsilon} \nabla^\alpha \Phi \nabla^\beta F_\alpha{}^\gamma, \\
[FR\Phi''\Phi'F']_2 &= e^{2\Phi} c_{157} F_\beta{}^\epsilon R_{\epsilon\epsilon\gamma\delta} \nabla_\alpha \nabla^\epsilon \Phi \nabla^\alpha \Phi \nabla^\beta F_\gamma{}^\delta + e^{2\Phi} c_{337} F_\delta{}^\epsilon R_{\epsilon\epsilon\beta\gamma} \nabla^\alpha \Phi \nabla^\beta F_\alpha{}^\gamma \nabla^\delta \nabla^\epsilon \Phi, \\
[F^2 \Phi''^2 \Phi'^2]_4 &= e^{2\Phi} c_{194} F_\gamma{}^\epsilon F_{\delta\epsilon} \nabla_\alpha \nabla^\gamma \Phi \nabla^\alpha \Phi \nabla_\beta \nabla^\delta \Phi \nabla^\beta \Phi + e^{2\Phi} c_{199} F_{\delta\epsilon} F^{\delta\epsilon} \nabla_\alpha \nabla^\gamma \Phi \nabla^\alpha \Phi \nabla^\beta \Phi \nabla_\gamma \nabla_\beta \Phi \\
&\quad + e^{2\Phi} c_{248} F_\gamma{}^\epsilon F_{\delta\epsilon} \nabla^\alpha \Phi \nabla_\beta \nabla_\alpha \Phi \nabla^\beta \Phi \nabla^\gamma \nabla^\delta \Phi + e^{2\Phi} c_{338} F_{\beta\delta} F_{\gamma\epsilon} \nabla_\alpha \nabla^\gamma \Phi \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\delta \nabla^\epsilon \Phi, \\
[F^2 \Phi'^2 F'^2]_{17} &= e^{4\Phi} c_{195} F_{\epsilon\epsilon} F^{\epsilon\epsilon} \nabla_\alpha \Phi \nabla^\alpha \Phi \nabla^\beta F_\gamma{}^\delta \nabla_\gamma F_{\beta\delta} + e^{4\Phi} c_{196} F_\delta{}^\epsilon F_{\epsilon\epsilon} \nabla_\alpha \Phi \nabla^\alpha \Phi \nabla^\beta F_\gamma{}^\delta \nabla_\gamma F_{\beta\epsilon}{}^\epsilon \\
&\quad + e^{4\Phi} c_{197} F_\delta{}^\epsilon F_{\epsilon\epsilon} \nabla_\alpha F_\gamma{}^\delta \nabla^\alpha \Phi \nabla^\beta \Phi \nabla_\gamma F_{\beta\epsilon}{}^\epsilon + e^{4\Phi} c_{203} F_{\epsilon\epsilon} F^{\epsilon\epsilon} \nabla_\alpha \Phi \nabla^\beta \Phi \nabla_\gamma F_{\beta\delta} \nabla^\gamma F_\alpha{}^\delta \\
&\quad + e^{4\Phi} c_{204} F_\delta{}^\epsilon F_{\epsilon\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla_\gamma F_{\beta\epsilon}{}^\epsilon \nabla^\gamma F_\alpha{}^\delta + e^{4\Phi} c_{205} F_{\beta\epsilon} F_{\delta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla_\gamma F^{\epsilon\epsilon} \nabla^\gamma F_\alpha{}^\delta \\
&\quad + e^{4\Phi} c_{250} F_{\epsilon\epsilon} F^{\epsilon\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\gamma F_\alpha{}^\delta \nabla_\delta F_{\beta\gamma} + e^{4\Phi} c_{252} F_\beta{}^\epsilon F_{\epsilon\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\gamma F_\alpha{}^\delta \nabla_\delta F_\gamma{}^\epsilon \\
&\quad + e^{4\Phi} c_{298} F_\delta{}^\epsilon F_{\epsilon\epsilon} \nabla_\alpha F_\beta{}^\gamma \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\delta F_\gamma{}^\epsilon + e^{4\Phi} c_{320} F_{\gamma\delta} F_{\epsilon\epsilon} \nabla_\alpha F_\beta{}^\gamma \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\delta F^{\epsilon\epsilon} \\
&\quad + e^{4\Phi} c_{356} F_\delta{}^\epsilon F_{\epsilon\epsilon} \nabla_\alpha F_\gamma{}^\delta \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\epsilon F_{\beta\gamma} + e^{4\Phi} c_{358} F_{\gamma\epsilon} F_{\delta\epsilon} \nabla_\alpha F_\gamma{}^\delta \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\epsilon F_{\beta\epsilon}{}^\epsilon \\
&\quad + e^{4\Phi} c_{360} F_{\gamma\epsilon} F_{\delta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\gamma F_\alpha{}^\delta \nabla^\epsilon F_{\beta\epsilon}{}^\epsilon + e^{4\Phi} c_{361} F_{\gamma\delta} F_{\epsilon\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\gamma F_\alpha{}^\delta \nabla^\epsilon F_{\beta\epsilon}{}^\epsilon \\
&\quad + e^{4\Phi} c_{363} F_\beta{}^\epsilon F_{\epsilon\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\gamma F_\alpha{}^\delta \nabla^\epsilon F_{\gamma\delta} + e^{4\Phi} c_{367} F_{\beta\epsilon} F_{\delta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\gamma F_\alpha{}^\delta \nabla^\epsilon F_\gamma{}^\epsilon \\
&\quad + e^{4\Phi} c_{368} F_{\beta\delta} F_{\epsilon\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\gamma F_\alpha{}^\delta \nabla^\epsilon F_\gamma{}^\epsilon, \\
[FF'\Phi'\Phi''^2]_1 &= e^{2\Phi} c_{202} F_{\delta\epsilon} \nabla^\alpha \Phi \nabla_\beta \nabla_\alpha \Phi \nabla^\beta F_\gamma{}^\delta \nabla_\gamma \nabla^\epsilon \Phi, \\
[FF'\Phi'^3 \Phi'']_5 &= e^{2\Phi} c_{225} F_{\delta\epsilon} \nabla_\alpha F_\beta{}^\delta \nabla^\alpha \Phi \nabla^\beta \Phi \nabla_\gamma \nabla^\epsilon \Phi \nabla^\gamma \Phi + e^{2\Phi} c_{258} F_{\gamma\epsilon} \nabla_\alpha F^{\delta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla_\gamma \Phi \nabla_\delta \nabla_\beta \Phi \\
&\quad + e^{2\Phi} c_{264} F_{\gamma\epsilon} \nabla_\alpha F_\beta{}^\delta \nabla^\alpha \Phi \nabla^\beta \Phi \nabla_\gamma \Phi \nabla_\delta \nabla^\epsilon \Phi + e^{2\Phi} c_{272} F_{\delta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla_\gamma \nabla_\beta \Phi \nabla^\gamma \Phi \nabla^\delta F_\alpha{}^\epsilon \\
&\quad + e^{2\Phi} c_{273} F_{\gamma\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla_\gamma \Phi \nabla_\delta \nabla_\beta \Phi \nabla^\delta F_\alpha{}^\epsilon, \\
[R\Phi'^2 \Phi''^2]_2 &= c_{249} R_{\alpha\delta\beta\epsilon} \nabla^\alpha \Phi \nabla^\beta \Phi \nabla_\gamma \nabla^\epsilon \Phi \nabla^\gamma \nabla^\delta \Phi + c_{339} R_{\gamma\delta\beta\epsilon} \nabla_\alpha \nabla^\gamma \Phi \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\delta \nabla^\epsilon \Phi, \\
[\Phi'^4 F'^2]_4 &= e^{2\Phi} c_{251} \nabla_\alpha \Phi \nabla^\alpha \Phi \nabla_\beta \Phi \nabla^\beta \Phi \nabla^\gamma F^{\delta\epsilon} \nabla_\delta F_{\gamma\epsilon} + e^{2\Phi} c_{296} \nabla_\alpha \Phi \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\gamma \Phi \nabla_\delta F_{\gamma\epsilon} \nabla^\delta F_\beta{}^\epsilon \\
&\quad + e^{2\Phi} c_{332} \nabla_\alpha F_\beta{}^\epsilon \nabla^\alpha \Phi \nabla^\beta \Phi \nabla_\gamma F_{\delta\epsilon} \nabla^\gamma \Phi \nabla^\delta \Phi + e^{2\Phi} c_{343} \nabla_\alpha \Phi \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\gamma \Phi \nabla^\delta F_\beta{}^\epsilon \nabla_\epsilon F_{\gamma\delta}, \\
[\Phi''^4]_2 &= c_{261} \nabla_\alpha \nabla^\gamma \Phi \nabla^\alpha \nabla^\beta \Phi \nabla_\beta \nabla^\delta \Phi \nabla_\delta \nabla_\gamma \Phi + c_{263} \nabla^\alpha \nabla^\beta \Phi \nabla_\beta \nabla_\alpha \Phi \nabla^\gamma \nabla^\delta \Phi \nabla_\delta \nabla_\gamma \Phi, \\
[FF'\Phi'^5]_2 &= e^{2\Phi} c_{299} F_{\delta\epsilon} \nabla_\alpha \Phi \nabla^\alpha \Phi \nabla_\beta \Phi \nabla^\beta \Phi \nabla^\gamma \Phi \nabla^\delta F_\gamma{}^\epsilon + e^{2\Phi} c_{329} F_{\delta\epsilon} \nabla_\alpha \Phi \nabla^\alpha \Phi \nabla_\beta F_\gamma{}^\epsilon \nabla^\beta \Phi \nabla^\gamma \Phi \nabla^\delta \Phi, \\
[F^2 \Phi'^4 \Phi'']_2 &= e^{2\Phi} c_{331} F_\gamma{}^\epsilon F_{\delta\epsilon} \nabla^\alpha \Phi \nabla_\beta \nabla_\alpha \Phi \nabla^\beta \Phi \nabla^\gamma \Phi \nabla^\delta \Phi + e^{2\Phi} c_{340} F_{\beta\delta} F_{\gamma\epsilon} \nabla_\alpha \Phi \nabla^\alpha \Phi \nabla^\beta \Phi \nabla^\gamma \Phi \nabla^\delta \nabla^\epsilon \Phi,
\end{aligned}$$



$$\begin{aligned}
[R\Phi'^3]_1 &= c_{335}R_{\beta\delta\gamma\epsilon}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\nabla^\beta\Phi\nabla^\delta\nabla^\epsilon\Phi, \\
[R\Phi'^4\Phi'']_1 &= c_{341}R_{\beta\delta\gamma\epsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla^\delta\nabla^\epsilon\Phi, \\
[F'^2\Phi''^2]_6 &= e^{2\Phi}c_{347}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\delta\nabla_\beta\Phi\nabla_\epsilon\nabla_\gamma\Phi + e^{2\Phi}c_{348}\nabla^\alpha F^{\beta\gamma}\nabla_\delta\nabla_\alpha\Phi\nabla^\delta F_\beta{}^\epsilon\nabla_\epsilon\nabla_\gamma\Phi \\
&\quad + e^{2\Phi}c_{350}\nabla_\alpha F_\beta{}^\delta\nabla^\alpha F^{\beta\gamma}\nabla_\gamma\nabla^\epsilon\Phi\nabla_\epsilon\nabla_\delta\Phi + e^{2\Phi}c_{351}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta\nabla_\gamma\nabla^\epsilon\Phi\nabla_\epsilon\nabla_\delta\Phi \\
&\quad + e^{2\Phi}c_{353}\nabla^\alpha F^{\beta\gamma}\nabla_\gamma\nabla_\alpha\Phi\nabla^\delta F_\beta{}^\epsilon\nabla_\epsilon\nabla_\delta\Phi + e^{2\Phi}c_{355}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_{\alpha\gamma}\nabla^\delta\nabla^\epsilon\Phi\nabla_\epsilon\nabla_\delta\Phi, \\
[F'^2\Phi'^2\Phi'']_4 &= e^{2\Phi}c_{349}\nabla_\alpha F^{\gamma\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla_\gamma F_\beta{}^\epsilon\nabla_\epsilon\nabla_\delta\Phi + e^{2\Phi}c_{352}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla_\gamma F_\beta{}^\epsilon\nabla_\gamma F_\alpha{}^\delta\nabla_\epsilon\nabla_\delta\Phi \\
&\quad + e^{2\Phi}c_{354}\nabla_\alpha F_\beta{}^\gamma\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\delta F_\gamma{}^\epsilon\nabla_\epsilon\nabla_\delta\Phi + e^{2\Phi}c_{357}\nabla_\alpha F^{\gamma\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla_\epsilon\nabla_\delta\Phi\nabla^\epsilon F_{\beta\gamma}, \\
[FF'^3\Phi']_1 &= e^{4\Phi}c_{377}F_{\epsilon\epsilon}\nabla^\alpha\Phi\nabla^\beta F_\alpha{}^\gamma\nabla^\delta F_\beta{}^\epsilon\nabla^\epsilon F_{\gamma\delta}, \\
[\Phi'^4\Phi''^2]_2 &= c_{262}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\nabla^\delta\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla_\delta\nabla_\gamma\Phi + c_{334}\nabla^\alpha\Phi\nabla_\beta\nabla_\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla_\delta\nabla_\gamma\Phi\nabla^\delta\Phi, \\
[\Phi'^6\Phi'']_1 &= c_{333}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\Phi\nabla^\beta\Phi\nabla_\gamma\Phi\nabla^\gamma\Phi\nabla_\delta\Phi\nabla^\delta\Phi, \\
[F^8]_5 &= e^{8\Phi}c_1F_\alpha{}^\gamma F^{\alpha\beta}F_\beta{}^\delta F_\gamma{}^\epsilon F_\delta{}^\epsilon F_\epsilon{}^\mu F_\epsilon{}^\nu F_{\mu\nu} + e^{8\Phi}c_2F_{\alpha\beta}F^{\alpha\beta}F_\gamma{}^\epsilon F^{\gamma\delta}F_\delta{}^\epsilon F_\epsilon{}^\mu F_\epsilon{}^\nu F_{\mu\nu} \\
&\quad + e^{8\Phi}c_3F_\alpha{}^\gamma F^{\alpha\beta}F_\beta{}^\delta F_{\gamma\delta}F_\epsilon{}^\mu F^{\epsilon\epsilon}F_\epsilon{}^\nu F_{\mu\nu} + e^{8\Phi}c_4F_{\alpha\beta}F^{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}F_\epsilon{}^\mu F^{\epsilon\epsilon}F_\epsilon{}^\nu F_{\mu\nu} \\
&\quad + e^{8\Phi}c_5F_{\alpha\beta}F^{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}F_\epsilon{}^\mu F^{\epsilon\epsilon}F_{\mu\nu}F^{\mu\nu}, \\
[F^4F^2]_{20} &= e^{6\Phi}c_{57}F_\gamma{}^\epsilon F_\delta{}^\epsilon F_\epsilon{}^\mu F_{\epsilon\mu}\nabla_\alpha F_\beta{}^\delta\nabla^\alpha F^{\beta\gamma} + e^{6\Phi}c_{58}F_\gamma{}^\epsilon F_{\delta\epsilon}F_{\epsilon\mu}F^{\epsilon\mu}\nabla_\alpha F_\beta{}^\delta\nabla^\alpha F^{\beta\gamma} \\
&\quad + e^{6\Phi}c_{61}F_\beta{}^\epsilon F_\gamma{}^\mu F_{\delta\epsilon}F_{\epsilon\mu}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma} + e^{6\Phi}c_{62}F_{\beta\delta}F_\gamma{}^\epsilon F_\epsilon{}^\mu F_{\epsilon\mu}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma} \\
&\quad + e^{6\Phi}c_{63}F_{\beta\gamma}F_\delta{}^\epsilon F_\epsilon{}^\mu F_{\epsilon\mu}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma} + e^{6\Phi}c_{64}F_{\beta\delta}F_\gamma{}^\epsilon F_{\epsilon\mu}F^{\epsilon\mu}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma} \\
&\quad + e^{6\Phi}c_{65}F_{\beta\gamma}F_{\delta\epsilon}F_{\epsilon\mu}F^{\epsilon\mu}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma} + e^{6\Phi}c_{118}F_\delta{}^\epsilon F_\epsilon{}^\mu F_{\epsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_{\alpha\gamma} \\
&\quad + e^{6\Phi}c_{119}F_{\delta\epsilon}F_\epsilon{}^\mu F_{\epsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_{\alpha\gamma} + e^{6\Phi}c_{122}F_\gamma{}^\epsilon F_\delta{}^\epsilon F_\epsilon{}^\mu F_{\epsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta \\
&\quad + e^{6\Phi}c_{123}F_\gamma{}^\epsilon F_{\delta\epsilon}F_{\epsilon\mu}F^{\epsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta + e^{6\Phi}c_{128}F_{\alpha\delta}F_\gamma{}^\epsilon F_\epsilon{}^\mu F_{\epsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F^{\delta\epsilon} \\
&\quad + e^{6\Phi}c_{274}F_\alpha{}^\epsilon F_\gamma{}^\mu F_{\delta\epsilon}F_{\epsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon + e^{6\Phi}c_{275}F_\alpha{}^\epsilon F_\gamma{}^\epsilon F_\delta{}^\mu F_{\epsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon \\
&\quad + e^{6\Phi}c_{276}F_{\alpha\delta}F_\gamma{}^\epsilon F_\epsilon{}^\mu F_{\epsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon + e^{6\Phi}c_{277}F_{\alpha\delta}F_\gamma{}^\epsilon F_{\epsilon\mu}F^{\epsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon \\
&\quad + e^{6\Phi}c_{300}F_{\alpha\epsilon}F_{\beta\delta}F_\gamma{}^\mu F_{\epsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\epsilon} + e^{6\Phi}c_{301}F_{\alpha\delta}F_{\beta\epsilon}F_\gamma{}^\mu F_{\epsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\epsilon} \\
&\quad + e^{6\Phi}c_{302}F_{\alpha\beta}F_\gamma{}^\mu F_{\delta\epsilon}F_{\epsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\epsilon} + e^{6\Phi}c_{303}F_{\alpha\beta}F_\gamma{}^\epsilon F_\delta{}^\mu F_{\epsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\epsilon},
\end{aligned}$$

$$\begin{aligned}
ds^2 &= \frac{u}{\sqrt{h_4 h_8}} \left(ds^2(\text{AdS}_3) + \frac{h_8 h_4}{g_1} ds^2(S^2) \right) + \frac{\sqrt{h_4 h_8}}{u} d\rho^2 + \sqrt{\frac{h_4}{h_8}} ds^2(\text{CY}_2) \\
e^\Phi &= \frac{2h_4^{1/4}}{h_8^{3/4}} \sqrt{\frac{u}{g_1}}, B_2 = (g_2 + 2\pi k) \text{vol}(S^2), g_1 = 4h_4 h_8 + u'^2, g_2 = -\frac{1}{2} \left(\rho - \frac{uu'}{g_1} \right)
\end{aligned}$$

$$F_0 = h'_8, F_2 = -\frac{1}{2} \left(h_8 - \frac{h'_8 uu'}{g_1} \right) \text{vol}(S^2), F_8 = \star_{10} F_2, F_{10} = -\star_{10} F_0$$

$$F_4 = - \left(\left(\frac{uu'}{2h_4} \right)' + 2h_8 \right) d\rho \wedge \text{vol}(\text{AdS}_3) - h'_4 \text{vol}(\text{CY}_2), F_6 = -\star_{10} F_4$$

$$\begin{aligned}
h_{4,8}(\rho) &= h_{4,8}^{(k)} \Theta[k, k+1], \\
h_{4,8}(\rho) \Big|_{\rho=\rho_i, \rho_f} &= 0, \\
h_{4,8}^{(k-1)}(\rho) \Big|_{\rho=k} &= h_{4,8}^{(k)}(\rho) \Big|_{\rho=k}, \\
u^{(k-1)}(\rho) \Big|_{\rho=k} &= u^{(k)}(\rho) \Big|_{\rho=k}, \\
u^{(k-1)}(\rho) \Big|_{\rho=k} &= u^{(k)}(\rho) \Big|_{\rho=k}
\end{aligned}$$



$$ds^2 = e^{2A} ds^2(\text{AdS}_3) + ds^2(M_7), F = f + e^{3A} \text{vol}(\text{AdS}_3) \wedge \star_7 \lambda(f),$$

$$\epsilon_1 = \sum_{I=1}^4 \xi^I \otimes \binom{1}{0} \otimes \chi_1^I, \epsilon_2 = \sum_{I=1}^4 \xi^I \otimes \binom{0}{1} \otimes \chi_2^I$$

$$ds^2=e^{2A}ds(\text{AdS}_3)+e^{2C}ds^2(\text{ }S^2)+ds^2(M_5)$$

$$d_H(e^{A-\Phi}\Psi_-)=0$$

$$d_H(e^{2A-\Phi}\Psi_+)-2me^{A-\Phi}\Psi_-=\frac{e^{3A}}{8}\star_7\lambda(f)$$

$$(\Psi_-,f)_7=\mp\frac{m}{2}e^{-\Phi}\text{vol}(M_7)$$

$$\Psi_++i\Psi_-=\frac{1}{8}\sum_{j=0}^{d=7}\frac{1}{j!}\big(\chi_2^\dagger\gamma_{a_1,\dots,a_j}\chi_1\big)e^{a_1}\wedge\dots\wedge e^{a_j}$$

$$\chi_1^I=\frac{1}{2}e^{\frac{A}{2}}\mathcal{M}_{\delta\sigma}^I\left(\left(\cos\frac{\beta}{2}+\sin\frac{\beta}{2}\right)\xi^\sigma+\left(\cos\frac{\beta}{2}-\sin\frac{\beta}{2}\right)\xi^\delta\right)\otimes\eta_1^\sigma\\ \chi_2^I=\frac{1}{2}e^{\frac{A}{2}}\mathcal{M}_{\delta\sigma}^I\left(\left(\cos\frac{\beta}{2}-\sin\frac{\beta}{2}\right)\xi^\delta+\left(\cos\frac{\beta}{2}+\sin\frac{\beta}{2}\right)\xi^\delta\right)\otimes\eta_2^\sigma$$

$$\eta_2=a_1\eta+a_2\eta^c+\frac{a_3}{2}\bar{w}\eta, |\eta|=1,$$

$$\psi_+^1=(\eta\otimes\eta^\dagger)_+=\frac{1}{4}e^{-ij},\qquad\psi_+^2=(\eta\otimes\eta^{c\dagger})_+=\frac{1}{4}\omega\\ \psi_-^1=(\eta\otimes\eta^\dagger)_-=\frac{1}{4}v\wedge e^{-ij},\quad\psi_-^2=(\eta\otimes\eta^{c\dagger})_-=\frac{1}{4}v\wedge\omega$$

$$j=e^7\wedge e^8+e^9\wedge e^{10}, \omega=(e^7+ie^8)\wedge(e^9+ie^{10})$$

$$j\wedge\omega=\omega\wedge\omega=0, \omega\wedge\bar{\omega}=2j\wedge j=\text{vol}(M_4),\\ \iota_v v=1, \iota_v j=\iota_v\omega=0,$$

$$e^\rho=\frac{(h_8h_4)^{1/4}}{\sqrt{u}}d\rho, e^i=\left(\frac{h_4}{h_8}\right)^{1/4}dz_i, i=7,\dots,10$$

$$\Psi_+^{(2)}=\frac{1}{2}\text{Re}\Big[\psi_+^1\wedge\Big((1+ie^{2C}y_3\text{vol}(\text{ }S^2))\cos\beta+i(y_3+ie^{2C}\text{vol}(\text{ }S^2))\sin\beta\Big)+i\psi_-^1\wedge K_3e^C\\ +\psi_+^2\wedge(e^{2C}\text{vol}(\text{ }S^2))\cos\beta+\sin\beta)(iy_1+y_2)e^{2C}+\psi_-^2\wedge(K_2+iK_1)e^C\Big]\\\Psi_-^{(2)}=-\frac{1}{2}\text{Im}\Big[\psi_+^1\wedge dy_3e^C-\psi_-^1\wedge\Big((y_3+ie^{2C}\text{vol}(\text{ }S^2))\cos\beta+(i-y_3e^{2C}\text{vol}(\text{ }S^2))\sin\beta\Big)\\ +\psi_+^2\wedge(dy_1-idy_2)e^C-\psi_-^2\wedge(\cos\beta-e^{2C}\text{vol}(\text{ }S^2))\sin\beta)(y_1-iy_2)\Big]$$

$$S_{\rm DBI}=-T_p\int\,e^{-\Phi}\sqrt{|\mathcal{P}[g+\mathcal{F}]|}dX^{p+1}, S_{\rm WZ}=\pm T_p\int\,\mathcal{P}[C\wedge e^{\mathcal{F}}+F_0\sigma]$$



$$ds^2(\text{AdS}_3) = r^2(-dt^2 + dx^2) + \frac{dr^2}{r^2}$$

$$C_1 = \frac{1}{2}(h_8 - (\rho - 2\pi k)h'_8)\eta, d\eta = -\text{vol}(\text{S}^2)$$

$$C_3 = \frac{1}{2h_4^2} dt \wedge d\rho \wedge \left((h_4 g_1 - uu'h'_4) r x dr + \sqrt{h_4^3 h_8 g_1} d(r^2 x) \right) - z_7 h'_4 \text{vol}(\text{T}^3)$$

$$C_5 = -\frac{u^2}{4h_4} \text{vol}(\text{AdS}_3) \wedge \text{vol}(\text{S}^2) + \frac{1}{2}(h_4 - \rho h'_4) \text{vol}(\text{T}^4) \wedge \eta \\ - \left(\frac{2h_4 u' u g_1 - h'_4 u^2 (g_1 + 4h_4 h_8)}{8h_4^2 g_1} r^2 dt \wedge dx \wedge d\rho - g_2 z_7 h'_4 \text{vol}(\text{T}^3) \right) \wedge \text{vol}(\text{S}^2)$$

$$C_7 = \frac{h_4}{h_8} \left(C_3 + \frac{2g_1 h_4 h_8 - uu' \partial_\rho (h_4 h_8)}{2h_4^2} r x dt \wedge dr \wedge d\rho \right) \wedge \text{vol}(\text{T}^4)$$

$$C_9 = \frac{h_4}{h_8} \left(C_5 - \frac{2f_1 h'_4 h_8 - u'^2 \partial_\rho (h_4 h_8)}{8h_4^2 h_8 f_1} u^2 r^2 dt \wedge dx \wedge d\rho \right) \wedge \text{vol}(\text{S}^2) \wedge \text{vol}(\text{T}^4)$$

$$e^{-\Phi} \sqrt{|\mathcal{P}[g + B_2]|} \Big|_{\Sigma^{(3)}} = \frac{1}{2} r^2 \sqrt{\frac{h_8}{h_4}} g_1$$

$$C_3 - C_1 \wedge B_2 \Big|_{\Sigma^{(3)}} = \frac{1}{2} r^2 \sqrt{\frac{h_8}{h_4}} g_1$$

$$e^{-\Phi} \sqrt{|\mathcal{P}[g + B]|} \Big|_{\Sigma^{(7)}} = \frac{1}{2} r^2 \sqrt{\frac{h_4}{h_8}} g_1$$

$$C_7 - C_5 \wedge B_2 \Big|_{\Sigma^{(7)}} = -\frac{1}{2} r^2 \sqrt{\frac{h_4}{h_8}} g_1$$

$$e^{-\Phi} \sqrt{|\mathcal{P}[g + B]|} \Big|_{\Sigma^{(5)}} = \frac{1}{4} \frac{u \sqrt{4h_4 h_8 u^2 + (uu' - g_1(\rho - 2\pi k))^2} r \sin \theta}{h_4 \sqrt{g_1}}$$

$$C_5 - C_3 \wedge B_2 \Big|_{\Sigma^{(5)}} = -\frac{1}{4} \frac{u^2}{h_4} r \sin \theta$$

$$e^{-\Phi} \sqrt{|\mathcal{P}[g + B]|} \Big|_{\Sigma^{(9)}} = \frac{1}{4} \frac{u \sqrt{4h_4 h_8 u^2 + (uu' - g_1(\rho - 2\pi k))^2} r \sin \theta}{h_8 \sqrt{g_1}}$$

$$C_9 - C_7 \wedge B_2 \Big|_{\Sigma^{(9)}} = -\frac{1}{4} \frac{u^2}{h_8} r \sin \theta$$

$$e^{-\Phi} \sqrt{|\mathcal{P}[g + \mathcal{F}]|} \Big|_{\Sigma} = \pm \Psi_{Dp}^{(\text{cal})} \Big|_{\Sigma} = \pm 8e^{3A-\Phi} \text{vol}(\text{AdS}_3) \wedge \Psi_- \wedge e^{-\mathcal{F}} \Big|_{\Sigma}$$

$$\Psi_{D4}^{(\text{cal})} = \frac{u(4h_4 h_8 u + u'(uu' - g_1(\rho - 2\pi k)))}{4h_4 g_1} \text{vol}(\text{AdS}_3) \wedge \text{vol}(\text{S}^2)$$

$$\Psi_{D8}^{(\text{cal})} = \frac{u(4h_4 h_8 u + u'(uu' - g_1(\rho - 2\pi k)))}{4h_8 g_1} \text{vol}(\text{AdS}_3) \wedge \text{vol}(\text{S}^2) \wedge \text{vol}(\text{T}^4)$$



$$\frac{ds^2}{2\pi} = \frac{|h|}{\sqrt{\Delta}} ds^2(\text{AdS}_3) + \sqrt{\Delta} \left[\frac{1}{4|h|} d\rho^2 + \frac{2}{|h''|} ds^2(\mathbb{CP}^3) \right]$$

$$e^{-\Phi} = \frac{(|h''|)^{\frac{3}{2}}}{2\sqrt{\pi}\Delta^{\frac{1}{4}}}, B_2 = c(\rho)J, c(\rho) = 4\pi \left(-(\rho - k) + \frac{h'}{h''} \right)$$

$$ds^2(\mathbb{CP}^3) = d\xi^2 + \frac{1}{4} \cos^2 \xi ds^2(S_1^2) + \frac{1}{4} \sin^2 \xi ds^2(S_2^2) + \frac{1}{4} \sin^2 \xi \cos^2 \xi (d\psi + \eta_1 - \eta_2)^2$$

$$d\eta_i = -\text{vol}(S_i^2)$$

$$J = -\frac{1}{4} \sin^2 \xi \text{vol}(S_2^2) - \frac{1}{4} \cos^2 \xi \text{vol}(S_1^2) - \frac{1}{2} \sin \xi \cos \xi d\xi \wedge (d\psi + \eta_1 - \eta_2)$$

$$e^1 = 2\sqrt{\pi} \frac{\Delta^{1/4}}{\sqrt{h''}} d\xi, e^2 = \sqrt{\pi} \frac{\Delta^{1/4}}{\sqrt{h''}} d\theta_1, e^3 = \sqrt{\pi} \frac{\Delta^{1/4}}{\sqrt{h''}} \sin \theta_1 d\phi_1, e^4 = \sqrt{\pi} \frac{\Delta^{1/4}}{\sqrt{h''}} d\theta_2$$

$$e^5 = \sqrt{\pi} \frac{\Delta^{1/4}}{\sqrt{h''}} \sin \theta_2 d\phi_2, e^6 = \sqrt{\pi} \frac{\Delta^{1/4}}{\sqrt{h''}} (d\psi + \eta_1 - \eta_2), e^7 = \sqrt{\frac{\pi}{2}} \frac{\Delta^{1/4}}{\sqrt{h}} d\rho$$

$$F_0 = -\frac{1}{2\pi} h''', F_2 = B_2 F_0 + 2(h'' - (\rho - k)h''')J$$

$$F_4 = \pi d \left(h' + \frac{hh'h''}{\Delta} \right) \wedge \text{vol}(\text{AdS}_3) + B_2 \wedge F_2 - \frac{1}{2} B_2 \wedge B_2 F_0$$

$$-4\pi(2h' + (\rho - k)(-2h'' + (\rho - k)h'''))J \wedge J$$

$$h = Q_{\text{D}_2} - Q_{\text{D}_4} \rho + \frac{1}{2} Q_{\text{D}_6} \rho^2$$

$$\zeta_6^j = \frac{1}{\sqrt{2}} \begin{pmatrix} i(\zeta_+^1 + \zeta_+^4) \\ \zeta_+^1 + \zeta_+^4 \\ i(\zeta_+^6 + \zeta_+^7) \\ \zeta_+^6 - \zeta_+^7 \\ \frac{1}{\sqrt{2}}(\zeta_+^2 - \zeta_+^5 + (\zeta_+^3 - \zeta_+^8)) \\ \frac{i}{\sqrt{2}}(\zeta_+^2 - \zeta_+^5 - (\zeta_+^3 - \zeta_+^8)) \end{pmatrix}^j, \hat{\zeta}_6^j = i\gamma_7 \zeta_6^j$$

$$\chi_1^j = \cos\left(\frac{\beta_1 + \beta_2}{2}\right) \zeta_6^j + \sin\left(\frac{\beta_1 + \beta_2}{2}\right) \hat{\zeta}_6^j$$

$$\chi_2^j = \cos\left(\frac{\beta_1 - \beta_2}{2}\right) \zeta_6^j + \sin\left(\frac{\beta_1 - \beta_2}{2}\right) \hat{\zeta}_6^j$$

$$\Psi_+^{(5)} = \frac{1}{8} \text{Re}[e^{i\beta_2} e^{-i\mathcal{I}} - e^{i\beta_1} \Omega \wedge V], \Psi_-^{(5)} = \frac{1}{8} \text{Im}[e^{i\beta_2} e^{-i\mathcal{I}} \wedge V + e^{i\beta_1} \Omega]$$



$$\begin{aligned}
J = & -\left(e^1 \wedge e^6 + c_{2\xi}(e^2 \wedge e^3 - e^4 \wedge e^5) + c_\psi s_{2\xi}(e^2 \wedge e^4 + e^3 \wedge e^5)\right) \\
& + s_{2\xi} s_\psi (e^3 \wedge e^4 - e^2 \wedge e^5) \\
V = & e^7 \\
\Omega = & s_{2\xi}(e^1 \wedge e^2 \wedge e^3 - e^1 \wedge e^4 \wedge e^5) - c_\psi c_{2\xi}(e^1 \wedge e^2 \wedge e^4 + e^1 \wedge e^3 \wedge e^5) \\
& + c_{2\xi} s_\psi (e^1 \wedge e^2 \wedge e^5 - e^1 \wedge e^3 \wedge e^4) - s_\psi (e^2 \wedge e^4 \wedge e^6 + e^3 \wedge e^5 \wedge e^6) \\
& - c_\psi (e^2 \wedge e^5 \wedge e^6 - e^3 \wedge e^4 \wedge e^6) + i[s_{2\xi}(e^4 \wedge e^5 \wedge e^6 - e^2 \wedge e^3 \wedge e^6) \\
& + c_\psi c_{2\xi}(e^2 \wedge e^4 \wedge e^6 + e^3 \wedge e^5 \wedge e^6) + c_{2\xi} s_\psi (e^3 \wedge e^4 \wedge e^6 - e^2 \wedge e^5 \wedge e^6) \\
& - s_\psi (e^1 \wedge e^2 \wedge e^4 + e^1 \wedge e^3 \wedge e^5) + c_\psi (e^1 \wedge e^3 \wedge e^4 - e^1 \wedge e^2 \wedge e^5)]
\end{aligned}$$

$$\iota_V J = \iota_V \Omega = 0, J \wedge \Omega = 0, J \wedge J \wedge J = \frac{3i}{4} \Omega \wedge \bar{\Omega}$$

$$\begin{aligned}
\zeta_6^5 = & \frac{e^{-i\left(\xi+\frac{\psi}{2}\right)}}{4} \left(1 - e^{2i\xi}, 1 + e^{2i\xi}, e^{i\psi}(1 + e^{2i\xi}), e^{i\psi}(1 - e^{2i\xi}), -(1 + e^{2i\xi}), -(1 - e^{2i\xi})\right. \\
& \left. - e^{i\psi}(1 - e^{2i\xi}), -e^{i\psi}(1 + e^{2i\xi})\right)^T \\
\zeta_6^6 = & \frac{ie^{-i\left(\xi+\frac{\psi}{2}\right)}}{4} \left(1 - e^{2i\xi}, 1 + e^{2i\xi}, -e^{i\psi}(1 + e^{2i\xi}), -e^{i\psi}(1 - e^{2i\xi}), -(1 + e^{2i\xi}), -(1 - e^{2i\xi})\right. \\
& \left. e^{i\psi}(1 - e^{2i\xi}), e^{i\psi}(1 + e^{2i\xi})\right)^T
\end{aligned}$$

$$\begin{aligned}
ds^2 = & \frac{u}{\sqrt{h_4 h_8}} \left(ds^2(\text{AdS}_3) + \frac{h_8 h_4}{g_1} ds^2(S^2) \right) + \frac{\sqrt{h_4 h_8}}{u} d\rho^2 + \sqrt{\frac{h_4}{h_8}} (dz_{7,10}^2 + G dz_{9,8}^2) \\
e^{\tilde{\Phi}} = & \sqrt{G} e^{\Phi}, \tilde{B}_2 = B_2 - \gamma G g dz_9 \wedge dz_8, G = (1 + \gamma^2 g)^{-1}, g = \frac{h_4}{h_8}
\end{aligned}$$

$$\begin{aligned}
\tilde{F}_0 = & h'_8, \tilde{F}_2 = F_2 - \gamma h'_4 dz_7 \wedge dz_{10} - \gamma G g h'_8 dz_9 \wedge dz_8 \\
\tilde{F}_4 = & F_4 + \gamma^2 h'_4 G g \text{vol}(T^4) + \\
& \frac{\gamma}{2} \left(G g \left(h_8 - \frac{uu'h'_8}{g_1} \right) dz_9 \wedge dz_8 + \left(h_4 - \frac{uu'h'_4}{f_1} \right) dz_7 \wedge dz_{10} \right) \wedge \text{vol}(S^2) \\
\tilde{F}_6 = & -\star_{10} \tilde{F}_4, \tilde{F}_8 = \star_{10} \tilde{F}_2, \tilde{F}_{10} = -\star_{10} \tilde{F}_0
\end{aligned}$$

$$\begin{aligned}
\tilde{C}_1 = & C_1 - \gamma h'_4 z_7 dz_{10} \\
\tilde{C}_3 = & C_3 + \tilde{C}_1 \wedge \tilde{B}_2 + \frac{\gamma}{2} (h_4 - \rho h'_4) dz_7 \wedge dz_{10} \wedge \eta + \gamma k \pi z_7 dz_{10} \wedge \text{vol}(S^2)
\end{aligned}$$

$$\begin{aligned}
\tilde{C}_5 = & C_5 - \frac{\gamma}{2} \sqrt{\frac{h_4}{h_8}} g_1 r^2 dt \wedge dx \wedge d\rho \wedge (dz_7 \wedge dz_{10} - G dz_9 \wedge dz_8) \\
& - \frac{\gamma}{2h_8} r x dt \wedge dr \wedge d\rho \wedge \left(\left(-g_1 + \frac{uu'h'_8}{h_8} + 2\sqrt{h_4 h_8 g_1} \right) dz_7 \wedge dz_{10} \right. \\
& \left. - G \left(g_1 - \frac{uu'h'_4}{h_4} + 2\sqrt{h_4 h_8 g_1} \right) dz_9 \wedge dz_8 \right) \\
& - \frac{\gamma^2}{2} (h_4 - \rho h'_4) G g \text{vol}(T^4) \wedge \eta + \gamma^2 G g h'_4 g_2 z_7 dz_{10} \wedge \text{vol}(T^2) \wedge \text{vol}(S^2)
\end{aligned}$$



$$Q_{\tilde{D}6}^{(k)} = \gamma Q_{D4}^{(k)}, \in \mathbb{Z} \quad Q_{\tilde{D}4}^{(k)} = \gamma Q_{D2}^{(k)}, \in \mathbb{Z}$$

$$ds^2 = \frac{c_0}{\sqrt{c_4 c_8}} \left(ds^2(\text{AdS}_3) + \frac{1}{4} \left(ds^2(S^2) + \gamma^2 \sqrt{\frac{c_4}{c_8}} G(d\psi + \cos \theta d\phi)^2 \right) \right) \\ + \sqrt{\frac{c_4}{c_8}} ds^2(T^3) + \frac{\sqrt{c_4 c_8}}{c_0} G d\rho^2 \\ \tilde{B}_2 = \frac{G}{2} (d\psi + \cos \theta d\phi) \wedge d\rho, e^{\tilde{\Phi}} = \frac{\sqrt{G c_0}}{(c_8^5 c_4)^{1/4}}, G = (1 + \gamma^2 g)^{-1}, g = \frac{c_4}{c_0}$$

$$\tilde{F}_2 = -\frac{1}{2} c_8 \text{vol}(S^2), \tilde{F}_4 = 2c_8 \text{vol}(\text{AdS}_3) \wedge d\rho + \tilde{F}_2 \wedge \tilde{B}_2 \\ \tilde{F}_6 = -\star_{10} \tilde{F}_4, \tilde{F}_8 = \star_{10} \tilde{F}_2$$

$$\frac{ds^2}{2\pi} = \frac{|h|}{\sqrt{\Delta}} ds^2(\text{AdS}_3) + \sqrt{\Delta} \left[\frac{1}{4|h|} d\rho^2 + \frac{2}{|h''|} ds^2(\widetilde{\mathbb{CP}}^3) \right] \\ e^{\tilde{\Phi}} = 2\Delta^{1/4} (h'')^{-3/2} \sqrt{\pi G}, \tilde{B}_2 = B_2 + \gamma \left(\frac{c(\rho)^2}{16} Z_1 - \frac{\pi^2 \Delta}{(h'')^2} W \right)$$

$$ds^2(\widetilde{\mathbb{CP}}^3) = d\xi^2 + \frac{1}{4} \cos^2 \xi d\theta_1^2 + \frac{1}{4} \sin^2 \xi d\theta_2^2 + \gamma^2 G \frac{\Delta \cos^4 \xi \sin^4 \xi \sin^2 \theta_1 \sin^2 \theta_2}{4h''^2} d\psi^2 \\ + \frac{1}{4} \sin^2 \xi \cos^2 \xi G \left(d\psi + \eta_1 - \eta_2 - \frac{\gamma}{4} c(\rho) \cos \theta_1 \sin \theta_2 \sin^2 \xi d\theta_2 - \frac{\gamma}{4} c(\rho) \cos \theta_2 \sin \theta_1 \cos^2 \xi d\theta_1 \right)^2 \\ + \frac{1}{4} \cos^2 \xi \sin^2 \theta_1 G \left(d\phi_1 + \frac{\gamma}{4} c(\rho) \cos \theta_2 \sin 2\xi d\xi - \frac{\gamma}{4} c(\rho) \sin \theta_2 \sin^2 \xi d\theta_2 \right)^2 \\ + \frac{1}{4} \sin^2 \xi \sin^2 \theta_2 G \left(d\phi_2 + \frac{\gamma}{4} c(\rho) \cos \theta_1 \sin 2\xi d\xi + \frac{\gamma}{4} c(\rho) \sin \theta_1 \cos^2 \xi d\theta_1 \right)^2 \\ G = \left(1 + \gamma^2 \frac{\pi^2 \Delta}{h''^2} M \right)^{-1}, M = s_{\theta_1}^2 c_{\theta_2}^2 c_{\xi}^2 + s_{\theta_1}^2 s_{\theta_2}^2 + c_{\theta_1}^2 s_{\theta_2}^2 s_{\xi}^2$$

$$\tilde{F}_0 = -\frac{h'''}{2\pi}, \\ \tilde{F}_2 = \frac{2((h'')^2 - h'h''')}{h''} J + \frac{\gamma}{2} \left(\frac{\pi \Delta h'''}{(h'')^2} W + \left(\frac{\pi h'''(h')^2}{(h'')^2} + \frac{c(\rho)}{2h''} ((h'')^2 - h'h''') \right) Z_1 \right) \\ \tilde{F}_4 = \pi d \left(h' + \frac{h'h''}{\Delta} \right) \wedge \text{vol}(\text{AdS}_3) - \frac{4\pi h'''(h')^2}{(h'')^2} J \wedge J, \\ - \frac{\gamma \pi \Delta}{2(h'')^3 M} \left[\left(3(h'')^2 + \frac{h'^2 h'''}{\Delta} (h' - 2(\rho - k)h'') \right) Z_1 \wedge Z_2 \right. \\ \left. + \left(((h'')^2 - h'h''') Z_2 + \gamma \pi^2 M \frac{(h')^2 h'''}{h''} Z_1 \right) \wedge W \right].$$



$$\begin{aligned}
Z_1 &= c_\xi s_\xi (c_\xi s_\xi s_{\theta_1} s_{\theta_2} d\theta_1 \wedge d\theta_2 - 2(s_{\theta_1} c_{\theta_2} c_\xi^2 d\theta_1 + c_{\theta_1} s_{\theta_2} s_\xi^2 d\theta_2) \wedge d\xi) \\
Z_2 &= \pi c_\xi s_\xi s_{\theta_1} s_{\theta_2} (c_{\theta_1} s_{\theta_2} s_\xi c_\xi d\theta_1 - s_{\theta_1} (c_{\theta_2} c_\xi s_\xi d\theta_2 + 2s_{\theta_2} d\xi)) \wedge d\psi \\
W &= G c_\xi^2 s_\xi^2 \left(M d\phi_1 \wedge d\phi_2 - (c_{\theta_2} s_{\theta_1}^2 c_\xi^2 d\phi_1 + c_{\theta_1} s_{\theta_2}^2 s_\xi^2 d\phi_2) \wedge d\psi + \gamma c(\rho) \left(MJ - \frac{Z_2}{4\pi} \right) \right. \\
&\quad \left. + \gamma^2 \frac{c(\rho)^2}{16} M Z_1 \right)
\end{aligned}$$

$$\begin{aligned}
\tilde{\psi}_+^1 &= \sqrt{G}(\psi_+^1 + ib\psi_+^2), & \tilde{\psi}_+^2 &= \sqrt{G}(\psi_+^2 + ib\psi_+^1), \\
\tilde{\psi}_-^1 &= \sqrt{G}(\psi_-^1 + ib\psi_-^2), & \tilde{\psi}_-^2 &= \sqrt{G}(\psi_-^2 + ib\psi_-^1).
\end{aligned}$$

$$\begin{aligned}
\tilde{\psi}_+^1 &= \sqrt{G}\psi_+^1 \wedge e^{4ib\psi_+^2}, & \tilde{\psi}_+^2 &= ib\sqrt{G}\psi_+^1 \wedge e^{(-4ib^{-1}\psi_+^2)} \\
\tilde{\psi}_-^1 &= \sqrt{G}\psi_-^1 \wedge e^{4ib\psi_+^2}, & \tilde{\psi}_-^2 &= ib\sqrt{G}\psi_-^1 \wedge e^{(-4ib^{-1}\psi_+^2)}
\end{aligned}$$

$$\begin{aligned}
\tilde{\psi}_+^1 &= \frac{\sqrt{G}}{4} e^{-ij-bw\wedge v}, & \tilde{\psi}_+^2 &= -\frac{\sqrt{G}}{4} u \wedge e^{-ij} \wedge (w-bv) \\
\tilde{\psi}_-^1 &= \frac{\sqrt{G}}{4} (v+bw) \wedge e^{-ij-bw\wedge v}, & \tilde{\psi}_-^2 &= -\frac{\sqrt{G}}{4} u \wedge (w\wedge v + be^{-ij})
\end{aligned}$$

$$v = e^\rho, w = e^{10} - ie^9, u = e^8 - ie^7$$

$$j = \frac{i}{2}(w \wedge \bar{w} + u \wedge \bar{u}), \omega = w \wedge u$$

$$\begin{aligned}
O &= \begin{pmatrix} \cos \kappa_1 & 0 & \sin \kappa_1 \\ 0 & 1 & 0 \\ -\sin \kappa_1 & 0 & \cos \kappa_1 \end{pmatrix}, O' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \kappa_2 & -\sin \kappa_2 \\ 0 & \sin \kappa_2 & \cos \kappa_2 \end{pmatrix} \\
\kappa_1 &= \arctan (\sin \xi \cot \theta_1) \kappa_2 = \arctan \left(\frac{\sin \theta_1 \cos \xi \cot \theta_2}{\sqrt{\sin \theta_1^2 + \cos \theta_1^2 \sin^2 \xi}} \right)
\end{aligned}$$

$$\begin{aligned}
\hat{e}^3 &= \frac{c_\xi}{\sqrt{s_{\theta_1}^2 + c_{\theta_1}^2 s_\xi^2}} \left(\frac{1}{2} d\phi_1 (s_{\theta_1}^2 + c_{\theta_1}^2 s_\xi^2) - c_{\theta_1} s_\xi^2 (d\psi - \eta_2) \right) \\
\hat{e}^5 &= \frac{s_\xi}{4c_\xi \sqrt{M(s_{\theta_1}^2 + c_{\theta_1}^2 s_\xi^2)}} (c_\xi s_{\theta_1} M d\phi_2 - s_{\theta_1}^3 c_{\theta_2}^2 c_\xi^3 d\psi) \\
\hat{e}^6 &= \frac{s_{2\xi} s_{\theta_1} s_{\theta_2}}{8\sqrt{M}} d\psi
\end{aligned}$$

$$\Omega_o = \cos \frac{\kappa_1}{2} + \sin \frac{\kappa_1}{2} \Gamma^{36}, \Omega_{o'} = \cos \frac{\kappa_2}{2} - \sin \frac{\kappa_2}{2} \Gamma^{56},$$

$$\Psi_{\pm}^{(I)} \rightarrow \Omega_{o'} \Omega_o \Psi_{\pm}^{(I)} (\Omega_{o'} \Omega_o)^{-1}, I=5,6$$



$$\begin{aligned}\tilde{\Psi}_+^{(5)} &= \Psi_+^{(5)} + \frac{bc_\alpha}{8} \left[s_{\beta_2} - c_{\beta_2} \mathcal{J}_b - \frac{c_\alpha^2 c_{\beta_2}^2}{\varkappa} \left(\frac{s_{\beta_2}}{2} - \frac{c_{\beta_2}}{6} \mathcal{J}_b \right) \wedge \mathcal{J}_b \wedge \mathcal{J}_b - \frac{c_\alpha^2 c_{\beta_2}^3}{\varkappa^{3/2}} \text{Re}[\Omega_b] \wedge V_b \right] \\ \tilde{\Psi}_-^{(5)} &= \Psi_-^{(5)} - \frac{bc_\alpha}{8} \left[\left(c_{\beta_2} + s_{\beta_2} \mathcal{J}_b - \frac{c_\alpha^2 c_{\beta_2}^2}{\varkappa} \left(\frac{c_{\beta_2}}{2} + \frac{s_{\beta_2} c_\alpha^2 c_{\beta_2}^2}{6\varkappa} \mathcal{J}_b \right) \wedge \mathcal{J}_b \wedge \mathcal{J}_b \right) \wedge V_b - \frac{c_{\beta_2}}{\sqrt{\varkappa}} \text{Im}[\Omega_b] \right] \\ c_\alpha &= \frac{s_2 \xi \mathcal{C} \psi s_{\theta_1} s_{\theta_2}}{\sqrt{M}}, \varkappa = 1 - c_\alpha^2 s_{\beta_2}^2\end{aligned}$$

$$\tilde{\Psi}_+ = \frac{1}{8} \text{Re}[e^{i\beta_2} e^{-i(j+U_1 \wedge U_2)} \wedge (1 - ibc_\alpha e^{-\tan \alpha \omega}) - (e^{i\beta_1} U \wedge \omega - ibs_\alpha e^{-i\beta_1} \bar{U} \wedge e^{ij+\cot \alpha \bar{\omega}}) \wedge V],$$

$$\tilde{\Psi}_- = \frac{1}{8} \text{Im}[e^{i\beta_2} e^{-i(j+U_1 \wedge U_2)} \wedge (1 - ibc_\alpha e^{-\tan \alpha \omega}) \wedge V + e^{i\beta_1} U \wedge \omega + ibs_\alpha e^{-i\beta_1} \bar{U} \wedge e^{ij+\cot \alpha \bar{\omega}}]$$

$$\begin{aligned}\mathcal{J} &= j + U_1 \wedge U_2, & \Omega &= U \wedge \omega, & \omega &= \omega_R + i\omega_I, \\ U &= U_1 + iU_2, & \iota_{U_1} U_2 &= 0, & \iota_{U_2} U_2 &= \iota_{U_1} U_1 = 1,\end{aligned}$$

$$\begin{aligned}V_b &= V - \frac{\tan \alpha}{c_{\beta_2}} \text{Re}[e^{i\beta_1} U] \\ \Omega_b &= \frac{\sqrt{\varkappa}}{c_{\beta_2}} \left[\frac{\tan \alpha (s_\alpha^2 + c_\alpha^2 c_{\beta_2}^2)}{\varkappa c_{\beta_2}} \text{Im}[e^{i\beta_1} U] \wedge \left(s_{\beta_2} j + s_{\beta_2} \tan \alpha \omega_I - \frac{c_{\beta_2}}{c_\alpha s_\alpha} \omega_R \right) \right. \\ &\quad \left. + (\sec \beta_2 \tan \alpha V + \text{Re}[e^{i\beta_1} U]) \wedge (\tan \alpha j - \omega_I) + \frac{i}{c_\alpha^2 c_{\beta_2}^2} (s_\alpha V \wedge (c_\alpha c_{\beta_2} \omega_R + s_\alpha s_{\beta_2} U_1 \wedge U_2) \right. \\ &\quad \left. + c_\alpha c_{\beta_2}^2 \text{Im}[e^{i\beta_1} U] \wedge (s_\alpha j - c_\alpha \omega_I) - \text{Re}[e^{i\beta_1} U] \wedge ((s_\alpha^2 - c_{\beta_2}^2) \omega_R + s_\alpha c_{\beta_2} s_{\beta_2} (c_\alpha j + s_\alpha \omega_I))) \right] \\ \mathcal{J}_b &= \mathcal{J} + \tan \alpha (\omega_I + \tan \beta_2 \omega_R - \sec \beta_2 \text{Im}[e^{i\beta_1} U] \wedge V)\end{aligned}$$

$$(\mathrm{N}_{\mathrm{D}8},\,\mathrm{N}_{\mathrm{D}6},\,\mathrm{N}_{\mathrm{D}4},\,\mathrm{N}_{\mathrm{D}2})\stackrel{\mathrm{TST}}{\rightarrow}(\mathrm{N}_{\mathrm{D}8},\,\mathrm{N}_{\mathrm{D}6}+\gamma\mathrm{N}_{\mathrm{D}4},\,\mathrm{N}_{\mathrm{D}4}+\gamma\mathrm{N}_{\mathrm{D}2},\,\mathrm{N}_{\mathrm{D}2})$$

$$e^{-\tilde{\Phi}}\sqrt{|\tilde{g}+\tilde{B}_2|}\Big|_{\Sigma^{(3)}}=\frac{1}{2}r^2\sqrt{\frac{h_8g_1}{h_4G}},\tilde{\mathcal{C}}_3-\tilde{\mathcal{C}}_1\wedge\tilde{B}_2\Big|_{\Sigma^{(3)}}=\frac{1}{2}r^2\sqrt{\frac{h_8}{h_4}}g_1,$$

$$e^{-\tilde{\Phi}}\sqrt{|\tilde{g}+\tilde{B}_2|}\Big|_{\Sigma^{(7)}}=\frac{1}{2}r^2\sqrt{\frac{h_4}{h_8}}g_1,\tilde{\mathcal{C}}_7-\tilde{\mathcal{C}}_5\wedge\tilde{B}_2\Big|_{\Sigma^{(7)}}=-\frac{1}{2}r^2\sqrt{\frac{h_4}{h_8}}g_1,$$

$$e^{-\tilde{\Phi}}\sqrt{|\tilde{g}+\tilde{B}_2|}\Big|_{\Sigma^{(3)}\times\mathrm{T}^2}=\frac{1}{2}r^2\sqrt{\frac{h_8g_1}{h_4G}},\tilde{\mathcal{C}}_5-\tilde{\mathcal{C}}_3\wedge\tilde{B}_2+\frac{1}{2}\tilde{\mathcal{C}}_1\wedge\tilde{B}_2\wedge\tilde{B}_2\Big|_{\Sigma^{(3)}\times\mathrm{T}^2}$$

$$e^{-\tilde{\Phi}}\sqrt{|\tilde{g}+\tilde{B}_2|}\Big|_{\Sigma^{(3)}\times\mathrm{T}^2}=\frac{1}{\gamma}\frac{r^2}{2}\sqrt{\frac{h_8}{h_4}}g_1,-\tilde{\mathcal{C}}_3\wedge f_2\Big|_{\Sigma^{(3)}\times\mathrm{T}^2}=\frac{1}{\gamma}\frac{r^2}{2}\sqrt{\frac{h_8}{h_4}}g_1,$$



$$e^{-\Phi}\sqrt{|\tilde{g}+\tilde{B}_2|}\Big|_{\Sigma^{(5)}}=\frac{1}{\sqrt{G}}\frac{u^2}{4h_4}r\sin\theta$$

$$\tilde{\Psi}_{D4}^{(\text{cal})}=\frac{u^2}{4h_4}\text{vol}(\text{AdS}_3)\wedge\text{vol}(\text{S}^2)$$

$$e^{-\tilde{\Phi}}\sqrt{|\tilde{g}+\tilde{B}_2|}\Big|_{\Sigma^{(9)}}=\frac{u^2}{4h_8}r\sin\theta$$

$$\tilde{\Psi}_{D8}^{(\text{cal})}=\frac{u^2}{4h_8}\text{vol}(\text{AdS}_3)\wedge\text{vol}(\text{S}^2)$$

$$e^{-\tilde{\Phi}}\sqrt{|\tilde{g}+\tilde{B}_2|}\Big|_{\Sigma^{(5)}\times\mathbb{T}^2}=\frac{1}{\gamma}\frac{u^2}{4h_4}r\sin\theta=\tilde{\Psi}_{\tilde{\text{D}}6}^{(\text{cal})}=\frac{1}{\gamma}\Psi_{D4}^{(\text{cal})},$$

$$2Q_{\tilde{\text{D}}4}^{(k)}=\Delta Q_{\tilde{\text{D}}6}^{(k)},2Q_{\text{D}6}^{(k)}=\Delta Q_{\text{D}8}^{(k)}$$

$$e^A=\left(e^a{}_{\mu}dx^{\mu},e^{c_1}(du+A_1),e^{c_2}(dv+A_2)\right),a=1,\ldots,8$$

$$\hat{e}^m_v=\frac{1}{g_{vv}}e^m_v,\hat{e}^m_{\sigma}=e^m_{\sigma}-\frac{1}{g_{vv}}(g_{\sigma v}+B_{\sigma v})e^m_v,\sigma\neq v$$

$$e^{\hat{\Phi}}=e^{\Phi-C_2}$$

$$\frac{1}{k!}F_{a_1,\ldots,a_k}e^{a_1}\wedge\ldots\wedge e^{a_k}\leftrightarrow\frac{1}{k!}F_{a_1,\ldots,a_k}\Gamma^{a_1\ldots a_k}\equiv F_k$$

$$e^{\hat{\Phi}}\hat{\mathbb{F}}=e^{\Phi}\mathbb{F}\Gamma^v,\mathbb{F}\Gamma^a=(-1)^k(e^a\wedge-\iota^a)\mathbb{F}.$$

$$e^a=O_b^ae^b,O=\frac{1}{|O|}\begin{pmatrix}1&-\gamma\sqrt{g}\\ \gamma\sqrt{g}&1\end{pmatrix},$$

$$\hat{F}\rightarrow\Omega^{-1}\hat{F}\Omega.$$

$$e^{\tilde{\Phi}}\tilde{\mathbb{F}}=e^{\Phi}\mathbb{F}\Omega,\Omega=\sqrt{G}(1+\gamma\sqrt{g}\Gamma^{vu}),$$

$$e^{\tilde{\Phi}}\tilde{\phi}=e^{\Phi}\not\subset\Omega$$

$$\Psi\rightarrow\tilde{\Psi}=\Omega\Psi,\Gamma^a\Psi=(e^a\wedge+\iota^a)\Psi$$

$$Q_{Dp}=\frac{1}{(2\pi)^{7-p}}\int_{\Sigma_{8-p}}\hat{f}_{8-p}d\vec{x}$$

	$0\leq \rho\leq 2\pi$	$2\pi j\leq \rho\leq 2\pi(j+1)$	$2\pi P\leq \rho\leq 2\pi(P+1)$
h_8	$\frac{\nu_0}{2\pi}\rho$	$\mu_j+\frac{\nu_j}{2\pi}(\rho-2\pi j)$	$\mu_P-\frac{\mu_P}{2\pi}(\rho-2P\pi)$

h_4	$\frac{\beta_0}{2\pi}\rho$	$\alpha_j + \frac{\beta_j}{2\pi}(\rho - 2\pi j)$	$\alpha_P - \frac{\alpha_P}{2\pi}(\rho - 2P\pi)$
u	$\frac{b_0}{2\pi}\rho$		

$$\alpha_k=\sum_{j=0}^{k-1}\beta_j,\mu_k=\sum_{j=0}^{k-1}\nu_j.$$

$$\hat{f}_0=h_8',\hat{f}_2=-\frac{1}{2}(h_8-h_8'(\rho-2\pi k))\mathrm{vol}(S^2)$$

$$\hat{f}_4=h_4'\mathrm{vol}(\mathrm{T}^4),\hat{f}_6=\frac{1}{2}(h_4-h_4'(\rho-2\pi k))\mathrm{vol}(S^2)\wedge\mathrm{vol}(\mathrm{T}^4)$$

$$Q_{\mathrm{NS5}}^{(k)}=\frac{1}{(2\pi)^2}\int_{(\rho,S^2)}H_3=1,Q_{D8}^{(k)}=2\pi h_8'=\nu_k,Q_{D6}^{(k)}=\frac{1}{2\pi}\int_{S^2}\hat{f}_2=\mu_k$$

$$Q_{D4}^{(k)}=\frac{1}{(2\pi)^3}\int_{T^4}\hat{f}_4=\beta_k,Q_{D2}^{(k)}=\frac{1}{(2\pi)^5}\int_{(S^2,T^4)}\hat{f}_6=\alpha_k$$

$$\begin{array}{ll}d\hat{f}_0=2\pi h_8''d\rho,& d\hat{f}_2=0\\d\hat{f}_4=h_4''d\rho\wedge\mathrm{vol}(\mathrm{T}^4),& d\hat{f}_6=0\end{array}$$

$$\hat{\hat{f}}_2=-\frac{1}{2}(h_8-h_8'(\rho-2\pi k))\mathrm{vol}(S^2)+\gamma h_4'dz_7\wedge dz_{10}$$

$$\hat{\hat{f}}_4=h_4'\mathrm{vol}(\mathrm{T}^4)-\frac{\gamma}{2}(h_4-h_4'(\rho-2\pi k))\mathrm{vol}(S^2)\wedge dz_7\wedge dz_{10}$$

$$\hat{\hat{f}}_6=-\frac{1}{2}(h_4-h_4'(\rho-2\pi k))\mathrm{vol}(S^2)\wedge\mathrm{vol}(\mathrm{T}^4)$$

$$Q_{\tilde{\mathrm{D}}6}^{(k)}=\frac{1}{2\pi}\int_{z_7z_{10}}\hat{\hat{f}}_2=\gamma Q_{D4}^{(k)},Q_{\tilde{\mathrm{D}}4}^{(k)}=\frac{1}{(2\pi)^3}\int_{(S^2,z_7z_{10})}\hat{\hat{f}}_4=\gamma Q_{D2}^{(k)}$$

$$d\hat{\hat{f}}_2=\gamma h_4''d\rho\wedge dz_7\wedge dz_{10},d\hat{\hat{f}}_4=0$$

$$d\hat{\hat{f}}\Big|_{(z_7,z_{10})}=(2\pi)^3N_{D6}\delta(\rho-\rho_0)e^{\tilde{f}_2}d\rho\wedge dz_7\wedge dz_{10}$$

$$\gamma\Delta h_4'=(2\pi)^3N_{D6}=(2\pi)^3\gamma N_{D4},\tilde{f}_2=\frac{1}{\gamma}dz_9\wedge dz_8$$

$$ds^2(S^7)=d\xi^2+\cos^2\xi ds^2(S_1^3)+\sin^2\xi ds^2(S_2^3)$$

$$\nabla_a \xi_{\pm} = \pm \frac{i}{2} \gamma_a \xi_{\pm}$$

$$\begin{aligned}
e^1 &= d\xi, e^2 = \frac{\cos \xi}{2} d\theta_1, e^3 = \frac{\cos \xi}{2} \sin \theta_1 d\phi_1 \\
e^4 &= \frac{\sin \xi}{2} d\theta_2, e^5 = \frac{\sin \xi}{2} \sin \theta_2 d\phi_2 \\
e^6 &= -\frac{\cos \xi \sin \xi}{2} (d[\psi_1 - \psi_2] + \eta_1 - \eta_2) \\
e^7 &= \frac{1}{2} (\cos^2 \xi (d\psi_1 + \eta_1) + \sin^2 \xi (d\psi_2 + \eta_2))
\end{aligned}$$

$$\begin{aligned}
\xi_{\pm} &= \mathcal{M}_{\pm} \xi_{\pm}^0 \\
\mathcal{M}_{\pm} &= e^{\frac{\xi}{2}(\pm i\gamma_1 - \gamma_{67})} e^{\frac{\psi_1}{4}(\pm i\gamma_7 - \gamma_{23})} e^{\frac{\theta_1}{4}(\pm i\gamma_2 + \gamma_{37})} e^{\frac{\phi_1}{4}(\pm i\gamma_7 + \gamma_{23})} \\
&\quad \times e^{\frac{\psi_2}{4}(\gamma_{16} - \gamma_{45})} e^{\frac{\theta_2}{4}(\gamma_{14} + \gamma_{56})} e^{\frac{\phi_2}{4}(\gamma_{16} + \gamma_{45})}
\end{aligned}$$

$$\xi_+^A = \mathcal{M}_+ \eta^A, A = 1, \dots, 8$$

$$ds^2(S^7) = ds^2(\mathbb{CP}^3) + \frac{1}{16} \left(d\tilde{\psi} + \cos^2 \xi (d\psi + 2\eta_1) + \sin^2 \xi (d\psi - 2\eta_2) \right)^2$$

$$\tilde{\psi} = \frac{1}{2}(\psi_1 + \psi_2), \psi = \frac{1}{2}(\psi_1 - \psi_2).$$



$$\begin{aligned}
j &= \frac{1}{p\sqrt{M}} \left(c_{\theta_2} c_{\xi} s_{\theta_1}^2 (e^1 \wedge \hat{e}^5 - s_{2\xi} c_{\psi} \hat{e}^3 \wedge \hat{e}^6) + (p_1 e^4 \wedge \hat{e}^5 - p_2 c_{\xi} s_{\xi} e^2 \wedge \hat{e}^5) \right) \\
&- \frac{1}{p} \left(s_{\theta_1} (c_{2\xi} e^2 \wedge \hat{e}^3 + s_{2\xi} s_{\psi} \hat{e}^3 \wedge e^4) + c_{\theta_1} s_{\xi} (e^1 \wedge \hat{e}^3 + s_{2\xi} c_{\psi} \hat{e}^5 \wedge \hat{e}^6) \right) \\
&- c_{\alpha} \hat{e}^3 \wedge \hat{e}^5 + \frac{c_{\alpha}}{s_{\alpha}^2 \sqrt{M}} \left(c_{\xi} p_3 (e^1 \wedge e^2 - c_{\alpha} e^4 \wedge \hat{e}^6) - s_{\xi} p_4 \sqrt{M} (e^1 \wedge e^4 + c_{\alpha} e^2 \wedge \hat{e}^6) \right) \\
&- s_{\theta_1} s_{\theta_2} \sqrt{M} (1 - s_{2\xi}^2 c_{\psi}^2) e^2 \wedge e^4 - s_{2\xi} c_{\psi} (c_{\theta_2}^2 s_{\theta_1}^2 c_{\xi}^2 + c_{\theta_1}^2 s_{\theta_2}^2 s_{\xi}^2) e^1 \wedge \hat{e}^6, \\
U &= \frac{1}{M s_{\alpha}^2} \left(s_{\theta_1} s_{\theta_2} s_{\psi} e^1 - s_{\xi} (2 c_{\theta_2} s_{\theta_1} c_{\xi}^2 + c_{\theta_1} s_{\theta_2} s_{\psi} c_{2\xi}) e^2 + c_{\xi} (2 c_{\theta_1} s_{\theta_2} s_{\xi}^2 - c_{\theta_2} s_{\theta_1} s_{\psi} c_{2\xi}) e^4 - c_{\psi} c_{2\xi} \sqrt{M} \hat{e}^6 \right) \\
&+ i \left(c_{\psi} s_{\theta_1} s_{\theta_2} c_{2\xi} e^1 - c_{\theta_1} c_{\psi} s_{\theta_2} s_{\xi} e^2 - c_{\theta_2} c_{\xi} c_{\psi} s_{\theta_1} e^4 + s_{\psi} \sqrt{M} \hat{e}^6 \right), \\
\omega &= \frac{1}{2 p s_{\alpha} \sqrt{M}} \left[2 c_{\theta_2} c_{\xi} s_{\theta_1}^2 (e^1 \wedge \hat{e}^3 + s_{2\xi} c_{\psi} \hat{e}^5 \wedge \hat{e}^6) + s_{2\xi} (2 s_{\theta_2} s_{\psi} (1 - c_{\theta_1}^2 c_{\xi}^2) - c_{\theta_1} c_{\theta_2} s_{\theta_1} c_{2\xi}) e^2 \wedge \hat{e}^3 \right. \\
&- (2 s_{\theta_2} c_{2\xi} (1 - c_{\theta_1}^2 c_{\xi}^2) + c_{\theta_1} c_{\theta_2} s_{\theta_1} s_{2\xi}^2 s_{\psi}) \hat{e}^3 \wedge e^4 + 2 \sqrt{M} (c_{\theta_1} s_{\xi} (e^1 \wedge \hat{e}^5 - s_{2\xi} c_{\psi} \hat{e}^3 \wedge \hat{e}^6) \\
&+ s_{\theta_1} (c_{2\xi} e^2 \wedge \hat{e}^5 - s_{2\xi} s_{\psi} e^4 \wedge \hat{e}^5)) \left. \right] + i \left[-s_{\alpha} \hat{e}^3 \wedge \hat{e}^5 + \frac{\cot \alpha}{p} (c_{\theta_1} s_{\xi} e^1 \wedge \hat{e}^3 + s_{\theta_1} c_{2\xi} e^2 \wedge \hat{e}^3 \right. \\
&+ s_{\theta_1} s_{2\xi} s_{\psi} \hat{e}^3 \wedge e^4) + \frac{\cot \alpha}{2 p \sqrt{M}} \left(s_{2\xi} (c_{\theta_1} c_{\theta_2} s_{\theta_1} c_{2\xi} - 2 s_{\theta_2} s_{\psi} (1 - c_{\theta_1}^2 c_{\xi}^2)) e^2 \wedge \hat{e}^5 \right. \\
&+ 2 s_{2\xi} c_{\psi} (c_{\theta_2} c_{\xi} s_{\theta_1}^2 \hat{e}^3 \wedge \hat{e}^6 + c_{\theta_1} s_{\xi} \sqrt{M} \hat{e}^5 \wedge \hat{e}^6) - 2 s_{\theta_1}^2 c_{\xi} c_{\theta_2} e^1 \wedge \hat{e}^5 - (2 s_{\theta_2} c_{2\xi} (1 - c_{\theta_1}^2 c_{\xi}^2) \\
&+ s_{2\xi}^2 c_{\theta_1} c_{\theta_2} s_{\theta_1} s_{\psi}) e^4 \wedge \hat{e}^5) - \frac{1}{M s_{\alpha}} (s_{2\xi} c_{\psi} (c_{\theta_2}^2 c_{\xi}^2 s_{\theta_1}^2 + s_{\theta_1}^2 s_{\theta_2}^2 s_{\psi}^2) e^1 \wedge \hat{e}^6 + s_{\theta_1} s_{\theta_2} \sqrt{M} (1 - c_{\psi}^2 s_{2\xi}^2) e^2 \wedge e^4) \\
&\left. + \frac{1}{s_{\alpha} \sqrt{H}} (p_3 c_{\xi} (e^1 \wedge e^2 - c_{\alpha} e^4 \wedge \hat{e}^6) - p_4 s_{\xi} (e^1 \wedge e^4 + c_{\alpha} e^2 \wedge \hat{e}^6)) \right], \\
p &= \sqrt{s_{\xi}^2 c_{\theta_1}^2 + s_{\theta_1}^2}, M = c_{\theta_2}^2 c_{\xi}^2 s_{\theta_1}^2 + s_{\theta_2}^2 p^2, \\
p_1 &= \frac{s_{2\xi}^2 c_{\theta_1} s_{\theta_1} c_{\theta_2} s_{\psi}}{2} + c_{2\xi} s_{\theta_2} p^2, p_2 = c_{\theta_1} s_{\theta_1} c_{\theta_2} c_{2\xi} - 2 s_{\theta_2} s_{\psi} p^2, \\
p_3 &= c_{\theta_2} s_{\theta_1} c_{2\xi} - 2 c_{\theta_1} s_{\theta_2} s_{\psi} s_{\xi}^2, p_4 = c_{\theta_1} s_{\theta_2} c_{2\xi} + 2 c_{\theta_2} s_{\theta_1} s_{\psi} c_{\xi}^2.
\end{aligned}$$

Supergravedad cuántica relativista Tipo IIB: Modelamiento Matemático.

$$\begin{aligned}
&\mathfrak{so}(3; \mathbb{R}) \oplus \mathfrak{so}(1,6; \mathbb{R}) \\
&\mathfrak{so}(3; \mathbb{R}) \oplus \mathfrak{so}(2,5; \mathbb{R}) \\
&\mathfrak{so}(1,2; \mathbb{R}) \oplus \mathfrak{so}(7; \mathbb{R}) \\
&\mathfrak{so}(1,2; \mathbb{R}) \oplus \mathfrak{so}(3,4; \mathbb{R})
\end{aligned}$$

$$\begin{aligned}
\Lambda''^{(+)} &= \nabla_M \varepsilon - \frac{1}{8} e^{\phi} (\Gamma \cdot \partial \chi) \Gamma_M s^2 \varepsilon - \frac{1}{480} e^{\phi} (\Gamma \cdot F_{(5)}) \Gamma_M s^2 \varepsilon \\
&\quad - \frac{1}{8} H_{(3)MNP} \Gamma^{NP} s^3 \varepsilon - \frac{1}{48} e^{\phi} (\Gamma \cdot \tilde{F}_{(3)}) \Gamma_M s^1 \varepsilon \\
\Lambda'^{(*)} &= (\Gamma \cdot \partial \phi) \varepsilon + e^{\phi} (\Gamma \cdot \partial \chi) s^2 \varepsilon - \frac{1}{12} (\Gamma \cdot H_{(3)}) s^3 \varepsilon + \frac{1}{12} e^{\phi} (\Gamma \cdot \tilde{F}_{(3)}) s^1 \varepsilon \\
H_{(3)} &= dB_{(2)} \quad \tilde{F}_{(3)} = F_{(3)} - \chi H_{(3)} \\
F_{(3)} &= dC_{(2)} \quad F_{(5)} = dC_{(4)} + \frac{1}{8} (B_{(2)} \wedge dC_{(2)} - C_{(2)} \wedge dB_{(2)}) \\
\varepsilon &= \begin{pmatrix} \varepsilon_+ \\ \varepsilon_- \end{pmatrix} \quad s^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad s^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad s^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\end{aligned}$$



$$g_{MN}^* = g_{MN} \, \phi^* = \phi$$

$$(\Gamma_A)^* = \mathcal{B} \Gamma_A \mathcal{B}^{-1} \, \mathcal{B} = \mathcal{C} \Gamma^0$$

$$(e^A)^* = \eta_{AA} e^A$$

$$(e^A\Gamma_A)^* = \mathcal{B}_\eta(e^A\Gamma_A)\mathcal{B}_\eta^{-1}$$

$$\mathcal{B}_\eta = u\mathcal{C}\prod_{i\in\mathcal{I}}\Gamma^i\,\mathcal{B}_\eta^*\mathcal{B}_\eta = (-)^{\frac{1}{2}t(t-1)}I_{32}$$

$$\mathcal{B}_\eta^{-1}\varepsilon^*=S\varepsilon$$

$$\mathcal{B}_\eta^{-1}\lambda^*=S\lambda$$

$$\mathcal{B}_\eta^{-1}\psi^*=S\psi$$

$$S=\begin{cases} I_2 & \text{for} & \text{Type IIB}_{\mathbb{R}} \\ s^1 & \text{for} & \text{Type IIB}' \\ s^2 & \text{for} & \text{Type IIB}_3 \\ s^3 & \text{for} & \text{Type IIB}^* \end{cases}$$

$$\begin{array}{ll} t\equiv 1(\mathrm{mod}4) & S=I_2,s^1,s^3 \\ t\equiv 3(\mathrm{mod}4) & S=s^2 \end{array}$$

$$ds^2=f_6^2ds_{\mathcal{M}_{6\mathbb{C}}}^2+f_2^2ds_{\mathcal{M}_{2\mathbb{C}}}^2+ds_{\Sigma_{\mathbb{C}}}^2$$

$$e^m=f_6\hat{e}^m\,ds_{\mathcal{M}_{6\mathbb{C}}}^2=\eta_{mn}\hat{e}_m\hat{e}_n\,m,n=0,\ldots,5$$

$$e^i=f_2\hat{e}^i\,ds_{\mathcal{M}_{2\mathbb{C}}}^2=\delta_{ij}\hat{e}^i\hat{e}^j\,i,j=6,7$$

$$e^a\,ds_{\Sigma_{\mathbb{C}}}^2=\delta_{ab}e^ae^b\,a,b=8,9$$

$$F_{(5)}=0\,\tilde{F}_{(3)}=g_ae^a\wedge e^{67}\,H_{(3)}=h_ae^a\wedge e^{67}$$

$$d\phi=\varphi_a e^a\,d\chi=\chi_a e^a$$

$$\mathcal{M}_{n\mathbb{C}} = \{(z_1,\cdots,z_{n+1}) \in \mathbb{C}^{n+1} \text{ such that } z_1^2 + \cdots + z_{n+1}^2 = 1\}$$

$$\mathfrak{so}(n+1;\mathbb{C})=\mathfrak{so}(n;\mathbb{C})\oplus\mathfrak{M}_{n\mathbb{C}}$$

$$[t_{AB},t_{CD}]=\eta_{AD}t_{BC}+\eta_{BC}t_{AD}-\eta_{AC}t_{BD}-\eta_{BD}t_{AC}$$

$$[t_{AB},u_C]=\eta_{BC}u_A-\eta_{AC}u_B$$

$$[u_A,u_B]=-t_{AB}$$

$$\Omega=\frac{1}{2}\hat{\omega}^{AB}t_{AB}+\hat{e}^Au_A$$

$$d\hat{e}^A+\eta_{BC}\hat{\omega}^{AB}\wedge\hat{e}^C=0$$

$$d\hat{\omega}^{AB}+\eta_{CD}\hat{\omega}^{AC}\wedge\hat{\omega}^{DB}=\hat{e}^A\wedge\hat{e}^B$$



$$\begin{aligned} \left(d + \frac{1}{4}\hat{\omega}^{mn}\gamma_{mn} + \frac{k_1}{2}\hat{e}^m\gamma_m\gamma_{(1)}\right)\chi_6 &= 0 \\ \left(d + \frac{1}{4}\hat{\omega}^{ij}\gamma_{ij} + \frac{k_2}{2}\hat{e}^i\gamma_i\gamma_{(2)}\right)\chi_2 &= 0 \end{aligned}$$

$$\begin{aligned} \left(\hat{\nabla}_m - \frac{k_1}{2}\eta_1\gamma_m \otimes I_2\right)\chi^{\eta_1,\eta_2} &= 0 \\ \left(\hat{\nabla}_i - \frac{k_2}{2}\eta_2 I_8 \otimes \gamma_i\right)\chi^{\eta_1,\eta_2} &= 0 \end{aligned}$$

$$\varepsilon = \sum_{\eta_1,\eta_2=\pm} \chi^{\eta_1,\eta_2} \otimes \zeta_{\eta_1,\eta_2} \quad \varepsilon = \begin{pmatrix} \varepsilon_+ \\ \varepsilon_- \end{pmatrix} \zeta_{\eta_1,\eta_2} = \begin{pmatrix} \zeta_{+,\eta_1,\eta_2} \\ \zeta_{-,\eta_1,\eta_2} \end{pmatrix}$$

$$(\tau^{(ij)}\zeta)_{\eta_1,\eta_2} = \sum_{\eta'_1,\eta'_2=\pm} (\tau^i)_{\eta_1\eta'_1} (\tau^j)_{\eta_2\eta'_2} \zeta_{\eta'_1\eta'_2}$$

$$(I_2 \otimes \tau^{(11)} \otimes \gamma_{(3)})\zeta = \zeta \, (I_2 \otimes \gamma_{(3)})\zeta_{-\eta_1,-\eta_2} = \zeta_{\eta_1,\eta_2}$$

$$\nabla_m \varepsilon = \hat{\nabla}_m \varepsilon + \frac{D_a f_6}{2 f_6} \Gamma_m \Gamma^a \varepsilon \, \nabla_i \varepsilon = \hat{\nabla}_i \varepsilon + \frac{D_a f_2}{2 f_2} \Gamma_i \Gamma^a \varepsilon$$

$$\begin{aligned} (m) \quad 0 &= -\frac{ik_1}{2f_6}\tau^{(20)}\zeta + \left(\frac{D_af_6}{2f_6} - \frac{1}{8}e^\phi\chi_a s^2\right)\tau^{(01)}\gamma^a\zeta - \frac{i}{8}e^\phi g_a s^1\gamma^a\zeta \\ (i) \quad 0 &= \frac{k_2}{2f_2}\tau^{(03)}\zeta + \left(\frac{D_af_2}{2f_2} - \frac{1}{8}e^\phi\chi_a s^2\right)\tau^{(01)}\gamma^a\zeta + \left(-\frac{i}{4}h_a s^3 + \frac{i}{8}e^\phi g_a s^1\right)\gamma^a\zeta \\ (a) \quad 0 &= \left(D_a + \frac{i}{2}\hat{\omega}_a\gamma_{(3)}\right)\zeta + \frac{1}{8}e^\phi\chi_b s^2\gamma^b\gamma_a\zeta + \left(-\frac{i}{4}h_a s^3 + \frac{i}{8}e^\phi g_b s^1\gamma^b\gamma_a\right)\tau^{(01)}\zeta \\ (\lambda) \quad 0 &= (\varphi_a - e^\phi\chi_a s^2)\tau^{(01)}\gamma^a\zeta - \frac{i}{2}(h_a s^3 + e^\phi g_a s^1)\gamma^a\zeta \end{aligned}$$

$$\begin{aligned} I_2 \otimes \tau^{(11)} \otimes \gamma_{(3)} \quad \gamma_{(3)} &= \sigma^3 \\ i s^2 \otimes \tau^{(32)} \otimes I_2 \\ i l_2 \otimes \tau^{(20)} \otimes I_2 \end{aligned}$$

$$\begin{aligned} \mathcal{I} &= I_2 \otimes \tau^{(11)} \otimes \gamma_{(3)} \\ \mathcal{J} &= i s^2 \otimes \tau^{(32)} \otimes I_2 \\ \mathcal{J} \mathcal{J} &= i s^2 \otimes \tau^{(23)} \otimes \gamma_{(3)} \end{aligned}$$

$$\begin{aligned} (I_2 \otimes \tau^{(11)} \otimes \gamma_{(3)})\zeta &= \zeta \\ (i s^2 \otimes \tau^{(32)} \otimes I_2)\zeta &= \nu \zeta \end{aligned}$$

$$\begin{aligned} (I_2 \otimes \tau^{(11)} \otimes \gamma_{(3)})\zeta_{s,\eta_1,\eta_2,\sigma} &= \sigma \zeta_{s,-\eta_1,-\eta_2,\sigma} = \zeta_{s,\eta_1,\eta_2,\sigma} \\ (i s^2 \otimes \tau^{(32)} \otimes I_2)\zeta_{s,\eta_1,\eta_2,\sigma} &= s \eta_1 \eta_2 \zeta_{-s,\eta_1,-\eta_2,\sigma} = \nu \zeta_{s,\eta_1,\eta_2,\sigma} \end{aligned}$$

$$\begin{aligned} \alpha &= \zeta_{++++} = +\zeta_{+---} = +\nu\zeta_{-++-} = +\nu\zeta_{----} \\ \beta &= \zeta_{+---} = -\zeta_{+++-} = -\nu\zeta_{-+-} = +\nu\zeta_{---} \\ \gamma &= \zeta_{-++-} = +\zeta_{----} = -\nu\zeta_{++++} = -\nu\zeta_{++-} \\ \delta &= \zeta_{----} = -\zeta_{+++-} = +\nu\zeta_{-++-} = -\nu\zeta_{+-} \end{aligned}$$



$$e^z = \frac{1}{2}(e^8 + ie^9) e^{\bar{z}} = \frac{1}{2}(e^8 - ie^9) \delta_{z\bar{z}} = 2 \delta^{z\bar{z}} = \frac{1}{2}$$

$$\gamma^z = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \frac{1}{2} \gamma_{\bar{z}} \gamma^{\bar{z}} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \frac{1}{2} \gamma_z$$

$$0 = \varphi_z \beta - e^\phi \chi_z \delta + \frac{iv}{2} (h_z \delta - e^\phi g_z \beta)$$

$$0 = \varphi_z \delta + e^\phi \chi_z \beta + \frac{iv}{2} (h_z \beta + e^\phi g_z \delta)$$

$$0 = \varphi_{\bar{z}} \alpha - e^\phi \chi_{\bar{z}} \gamma + \frac{iv}{2} (h_{\bar{z}} \gamma - e^\phi g_{\bar{z}} \alpha)$$

$$0 = \varphi_{\bar{z}} \gamma + e^\phi \chi_{\bar{z}} \alpha + \frac{iv}{2} (h_{\bar{z}} \alpha + e^\phi g_{\bar{z}} \gamma)$$

$$\Lambda''^{(\dagger)} = + \frac{k_1}{2f_6} \alpha + \frac{D_z f_6}{2f_6} \beta - \frac{1}{8} e^\phi \chi_z \delta - \frac{iv}{8} e^\phi g_z \beta$$

$$\Lambda'^{(*)} = + \frac{k_1}{2f_6} \gamma + \frac{D_z f_6}{2f_6} \delta + \frac{1}{8} e^\phi \chi_z \beta + \frac{iv}{8} e^\phi g_z \delta$$

$$\Lambda''^{(\dagger)} = - \frac{k_1}{2f_6} \beta + \frac{D_{\bar{z}} f_6}{2f_6} \alpha - \frac{1}{8} e^\phi \chi_{\bar{z}} \gamma - \frac{iv}{8} e^\phi g_{\bar{z}} \alpha$$

$$\Lambda'^{(*)} = - \frac{k_1}{2f_6} \delta + \frac{D_{\bar{z}} f_6}{2f_6} \gamma + \frac{1}{8} e^\phi \chi_{\bar{z}} \alpha + \frac{iv}{8} e^\phi g_{\bar{z}} \gamma$$

$$\Lambda''^{(\dagger)} = - \frac{k_2 v}{2f_2} \gamma + \frac{D_z f_2}{2f_2} \beta - \frac{1}{8} e^\phi \chi_z \delta + \frac{iv}{8} e^\phi g_z \beta + \frac{iv}{4} h_z \delta$$

$$\Lambda'^{(*)} = + \frac{k_2 v}{2f_2} \alpha + \frac{D_z f_2}{2f_2} \delta + \frac{1}{8} e^\phi \chi_z \beta - \frac{iv}{8} e^\phi g_z \delta + \frac{iv}{4} h_z \beta$$

$$\Lambda''^{(\dagger)} = - \frac{k_2 v}{2f_2} \delta + \frac{D_{\bar{z}} f_2}{2f_2} \alpha - \frac{1}{8} e^\phi \chi_{\bar{z}} \gamma + \frac{iv}{8} e^\phi g_{\bar{z}} \alpha + \frac{iv}{4} h_{\bar{z}} \gamma$$

$$\Lambda'^{(*)} = + \frac{k_2 v}{2f_2} \beta + \frac{D_{\bar{z}} f_2}{2f_2} \gamma + \frac{1}{8} e^\phi \chi_{\bar{z}} \alpha - \frac{iv}{8} e^\phi g_{\bar{z}} \gamma + \frac{iv}{4} h_{\bar{z}} \alpha$$

$$\Lambda''^{(\dagger)} = D_{\bar{z}} \alpha + \frac{i}{2} \hat{\omega}_{\bar{z}} \alpha + \frac{iv}{4} h_{\bar{z}} \gamma$$

$$\Lambda'^{(*)} = D_{\bar{z}} \gamma + \frac{i}{2} \hat{\omega}_{\bar{z}} \gamma + \frac{iv}{4} h_{\bar{z}} \alpha$$

$$\Lambda''^{(\dagger)} = D_z \beta - \frac{i}{2} \hat{\omega}_z \beta + \frac{iv}{4} h_z \delta$$

$$\Lambda'^{(*)} = D_z \delta - \frac{i}{2} \hat{\omega}_z \delta + \frac{iv}{4} h_z \beta$$

$$\Lambda''^{(\dagger)} = D_z \alpha + \frac{i}{2} \hat{\omega}_z \alpha + \frac{1}{4} e^\phi \chi_z \gamma + \frac{iv}{4} e^\phi g_z \alpha + \frac{iv}{4} h_z \gamma$$

$$\Lambda'^{(*)} = D_z \gamma + \frac{i}{2} \hat{\omega}_z \gamma - \frac{1}{4} e^\phi \chi_z \alpha - \frac{iv}{4} e^\phi g_z \gamma + \frac{iv}{4} h_z \alpha$$

$$\Lambda''^{(\dagger)} = D_{\bar{z}} \beta - \frac{i}{2} \hat{\omega}_{\bar{z}} \beta + \frac{1}{4} e^\phi \chi_{\bar{z}} \delta + \frac{iv}{4} e^\phi g_{\bar{z}} \beta + \frac{iv}{4} h_{\bar{z}} \delta$$

$$\Lambda'^{(*)} = D_{\bar{z}} \delta - \frac{i}{2} \hat{\omega}_{\bar{z}} \delta - \frac{1}{4} e^\phi \chi_{\bar{z}} \beta - \frac{iv}{4} e^\phi g_{\bar{z}} \delta + \frac{iv}{4} h_{\bar{z}} \beta$$



$$f_6 = c_6(\alpha\delta - \beta\gamma)$$

$$f_2 = c_2(\alpha\beta + \gamma\delta)$$

$$\Lambda''^{(\dagger)} = \frac{k_1}{2c_6} - \frac{1}{8}e^\phi\chi_z(\beta^2 + \delta^2) - \frac{iv}{4}e^\phi g_z\beta\delta$$

$$\Lambda'^{(\star)} = \frac{k_1}{2c_6} - \frac{1}{8}e^\phi\chi_{\bar{z}}(\alpha^2 + \gamma^2) - \frac{iv}{4}e^\phi g_{\bar{z}}\alpha\gamma$$

$$\Lambda''^{(\dagger)} = \frac{k_2\nu}{2c_2} + \frac{1}{8}e^\phi\chi_z(\beta^2 + \delta^2) - \frac{iv}{4}e^\phi g_z\beta\delta + \frac{iv}{4}h_z(\beta^2 - \delta^2)$$

$$\Lambda'^{(\star)} = \frac{k_2\nu}{2c_2} + \frac{1}{8}e^\phi\chi_{\bar{z}}(\alpha^2 + \gamma^2) - \frac{iv}{4}e^\phi g_{\bar{z}}\alpha\gamma + \frac{iv}{4}h_{\bar{z}}(\alpha^2 - \gamma^2)$$

$$\Lambda''^{(\dagger)} = \frac{D_z f_6}{2f_6}(\alpha\delta - \beta\gamma) + \frac{1}{8}e^\phi\chi_z(\alpha\beta + \gamma\delta) + \frac{iv}{8}e^\phi g_z(\alpha\delta + \beta\gamma)$$

$$\Lambda'^{(\star)} = \frac{D_{\bar{z}} f_6}{2f_6}(\alpha\delta - \beta\gamma) - \frac{1}{8}e^\phi\chi_{\bar{z}}(\alpha\beta + \gamma\delta) - \frac{iv}{8}e^\phi g_{\bar{z}}(\alpha\delta + \beta\gamma)$$

$$D_z(\alpha\delta - \beta\gamma) = -\frac{1}{4}e^\phi\chi_z(\alpha\beta + \gamma\delta) - \frac{iv}{4}e^\phi g_z(\alpha\delta + \beta\gamma)$$

$$D_{\bar{z}}(\alpha\delta - \beta\gamma) = \frac{1}{4}e^\phi\chi_{\bar{z}}(\alpha\beta + \gamma\delta) + \frac{iv}{4}e^\phi g_{\bar{z}}(\alpha\delta + \beta\gamma)$$

$$\frac{D_z f_6}{f_6}(\alpha\delta - \beta\gamma) - D_z(\alpha\delta - \beta\gamma) = 0$$

$$\frac{D_{\bar{z}} f_6}{f_6}(\alpha\delta - \beta\gamma) - D_{\bar{z}}(\alpha\delta - \beta\gamma) = 0$$

$$\Lambda''^{(\dagger)} = e^\phi\chi_z(\beta^2 + \delta^2) + \frac{iv}{2}h_z(\beta^2 - \delta^2) + ive^\phi g_z\beta\delta$$

$$\Lambda'^{(\star)} = e^\phi\chi_{\bar{z}}(\alpha^2 + \gamma^2) + \frac{iv}{2}h_{\bar{z}}(\alpha^2 - \gamma^2) + ive^\phi g_{\bar{z}}\alpha\gamma$$

$$\Lambda''^{(\dagger)} = \frac{k_2\nu}{2c_2} - \frac{3}{8}e^\phi\chi_z(\beta^2 + \delta^2) - \frac{3iv}{4}e^\phi g_z\beta\delta$$

$$\Lambda'^{(\star)} = \frac{k_2\nu}{2c_2} - \frac{3}{8}e^\phi\chi_{\bar{z}}(\alpha^2 + \gamma^2) - \frac{3iv}{4}e^\phi g_{\bar{z}}\alpha\gamma$$

$$\frac{k_2\nu}{c_2} = \frac{3k_1}{c_6}$$

$$\Lambda''^{(\dagger)} = \frac{k_1}{c_6} + \frac{1}{4}e^\phi\chi_z(\beta^2 + \delta^2) + \frac{iv}{4}h_z(\beta^2 - \delta^2)$$

$$\Lambda'^{(\star)} = \frac{k_1}{c_6} + \frac{1}{4}e^\phi\chi_{\bar{z}}(\alpha^2 + \gamma^2) + \frac{iv}{4}h_{\bar{z}}(\alpha^2 - \gamma^2)$$

$$e^z = \rho dw \quad D_z = \rho^{-1}\partial_w \quad \hat{\omega}_z = +i\rho^{-2}\partial_w \rho$$

$$e^{\bar{z}} = \rho d\tilde{w} \quad D_{\bar{z}} = \rho^{-1}\partial_{\tilde{w}} \quad \hat{\omega}_{\bar{z}} = -i\rho^{-2}\partial_{\tilde{w}} \rho$$

$$\Lambda''^{(\dagger)} = D_{\bar{z}}(\alpha^2 - \gamma^2) + i\hat{\omega}_{\bar{z}}(\alpha^2 - \gamma^2)$$

$$\Lambda'^{(\star)} = D_z(\beta^2 - \delta^2) - i\hat{\omega}_z(\beta^2 - \delta^2)$$



$$\begin{aligned}\partial_{\bar{w}}(\rho(\alpha^2 - \gamma^2)) &= 0 \\ \partial_w(\rho(\beta^2 - \delta^2)) &= 0\end{aligned}$$

$$\begin{aligned}\rho(\alpha^2 - \gamma^2) &= 2\kappa_1 \\ \rho(\beta^2 - \delta^2) &= 2\tilde{\kappa}_1\end{aligned}$$

$$\begin{aligned}\Lambda''^{(\dagger)} &= D_z(\beta\delta) - i\hat{\omega}_z\beta\delta - \varphi_z\beta\delta - \frac{1}{2}e^\phi\chi_z(\beta^2 - \delta^2) \\ \Lambda'^{(\star)} &= D_{\bar{z}}(\alpha\gamma) + i\hat{\omega}_{\bar{z}}\alpha\gamma - \varphi_{\bar{z}}\alpha\gamma - \frac{1}{2}e^\phi\chi_{\bar{z}}(\alpha^2 - \gamma^2)\end{aligned}$$

$$\begin{aligned}\varphi_z &= \rho^{-1}\partial_w\phi & \chi_z &= \rho^{-1}\partial_w\chi \\ \varphi_{\bar{z}} &= \rho^{-1}\partial_{\bar{w}}\phi & \chi_{\bar{z}} &= \rho^{-1}\partial_{\bar{w}}\chi\end{aligned}$$

$$\begin{aligned}\partial_w(e^{-\phi}\rho\beta\delta + \chi\tilde{\kappa}_1) &= \Lambda''^{(\dagger)} \\ \partial_{\bar{w}}(e^{-\phi}\rho\alpha\gamma + \chi\kappa_1) &= \Lambda'^{(\star)}\end{aligned}$$

$$\begin{aligned}\rho\beta\delta &= e^\phi(\tilde{\kappa}_1\chi - \tilde{\kappa}_2) \\ \rho\alpha\gamma &= e^\phi(\kappa_1\chi - \kappa_2)\end{aligned}$$

$$\tau_{\pm} = \chi \pm ie^{-\phi}\xi = \frac{\kappa_2}{\kappa_1}\xi = \frac{\tilde{\kappa}_2}{\tilde{\kappa}_1}$$

$$\begin{aligned}\rho(\alpha \pm i\gamma)^2 &= \mp 4\kappa_1 \frac{\tau_{\mp} - \xi}{\tau_{+} - \tau_{-}} \\ \rho(\beta \pm i\delta)^2 &= \mp 4\tilde{\kappa}_1 \frac{\tau_{\mp} - \tilde{\xi}}{\tau_{+} - \tau_{-}}\end{aligned}$$

$$\begin{aligned}\rho f_6 &= -2c_6 \frac{(\kappa_1\tilde{\kappa}_1)^{\frac{1}{2}}}{\tau_{+} - \tau_{-}} \left\{ (\tau_{-} - \xi)^{\frac{1}{2}}(\tau_{+} - \tilde{\xi})^{\frac{1}{2}} - (\tau_{+} - \xi)^{\frac{1}{2}}(\tau_{-} - \tilde{\xi})^{\frac{1}{2}} \right\} \\ \rho f_2 &= 2ic_2 \frac{(\kappa_1\tilde{\kappa}_1)^{\frac{1}{2}}}{\tau_{+} - \tau_{-}} \left\{ (\tau_{-} - \xi)^{\frac{1}{2}}(\tau_{+} - \tilde{\xi})^{\frac{1}{2}} + (\tau_{+} - \xi)^{\frac{1}{2}}(\tau_{-} - \tilde{\xi})^{\frac{1}{2}} \right\}\end{aligned}$$

$$\begin{aligned}\rho(h_z \pm ie^\phi g_z) &= 2ive^\phi \frac{\beta \pm i\delta}{\beta \mp i\delta} \partial_w \tau_{\pm} = \pm 4iv \left(\frac{\tau_{\mp} - \tilde{\xi}}{\tau_{\pm} - \xi} \right)^{\frac{1}{2}} \frac{\partial_w \tau_{\pm}}{\tau_{+} - \tau_{-}} \\ \rho(h_{\bar{z}} \pm ie^\phi g_{\bar{z}}) &= 2ive^\phi \frac{\alpha \pm i\gamma}{\alpha \mp i\gamma} \partial_{\bar{w}} \tau_{\pm} = \pm 4iv \left(\frac{\tau_{\mp} - \tilde{\xi}}{\tau_{\pm} - \xi} \right)^{\frac{1}{2}} \frac{\partial_{\bar{w}} \tau_{\pm}}{\tau_{+} - \tau_{-}}\end{aligned}$$

$$\begin{aligned}\Lambda''^{(\dagger)} &= \frac{(D_z + i\hat{\omega}_z)(\alpha^2 - \gamma^2)}{\alpha^2 - \gamma^2} + \frac{\chi - \xi}{2}e^{2\phi}\chi_z + \frac{1}{X}(\varphi_z - e^{2\phi}(\chi - \tilde{\xi})\chi_z) \\ \Lambda'^{(\star)} &= \frac{(D_{\bar{z}} - i\hat{\omega}_{\bar{z}})(\beta^2 - \delta^2)}{\beta^2 - \delta^2} + \frac{\chi - \tilde{\xi}}{2}e^{2\phi}\chi_{\bar{z}} + X(\varphi_{\bar{z}} - e^{2\phi}(\chi - \xi)\chi_{\bar{z}}) \\ \Lambda''^{(\dagger)} &= \frac{(D_z + i\hat{\omega}_z)(\alpha\gamma)}{\alpha^2 - \gamma^2} - \frac{1}{4}e^\phi\chi_z - \frac{1}{2X}e^\phi((\chi - \tilde{\xi})\varphi_z + \chi_z) \\ \Lambda'^{(\star)} &= \frac{(D_{\bar{z}} - i\hat{\omega}_{\bar{z}})(\beta\delta)}{\beta^2 - \delta^2} - \frac{1}{4}e^\phi\chi_{\bar{z}} - \frac{1}{2}Xe^\phi((\chi - \xi)\varphi_{\bar{z}} + \chi_{\bar{z}})\end{aligned}$$



$$X = \frac{\beta^2 + \delta^2 \kappa_1}{\alpha^2 + \gamma^2 \tilde{\kappa}_1}$$

$$\Lambda''^{(\dagger)} = -\frac{k_1 \beta^2 + \delta^2}{c_6 \tilde{\kappa}_1^2} e^{-\phi} \rho^2 + (1 - e^{2\phi} (\chi - \xi)^2) \chi_z + 2(\chi - \xi) \varphi_z$$

$$\Lambda'^{(\star)} = -\frac{k_1 \alpha^2 + \gamma^2}{c_6 \kappa_1^2} e^{-\phi} \rho^2 + (1 - e^{2\phi} (\chi - \xi)^2) \chi_{\bar{z}} + 2(\chi - \xi) \varphi_{\bar{z}}$$

$$\alpha^2 + \gamma^2 = \frac{4\epsilon_\kappa \kappa_1}{\rho(\tau_+ - \tau_-)} \sqrt{-(\tau_+ - \xi)(\tau_- - \xi)} \quad \epsilon_\kappa = \frac{\sqrt{\kappa_1^2}}{\kappa_1}$$

$$\beta^2 + \delta^2 = \frac{4\tilde{\epsilon}_\kappa \tilde{\kappa}_1}{\rho(\tau_+ - \tau_-)} \sqrt{-(\tau_+ - \tilde{\xi})(\tau_- - \tilde{\xi})} \quad \tilde{\epsilon}_\kappa = \frac{\sqrt{\tilde{\kappa}_1^2}}{\tilde{\kappa}_1}$$

$$(\tau_{\mp} - \xi) \partial_w \ln \hat{\rho}^2 + \partial_w \xi \mp \frac{\partial_w \tau_{\mp}}{\tau_+ - \tau_-} \left\{ \tau_{\pm} - \xi - \frac{2}{X} (\tau_{\pm} - \xi) \right\} = \Lambda''^{(\dagger)}$$

$$(\tau_{\mp} - \tilde{\xi}) \partial_{\tilde{w}} \ln \hat{\rho}^2 + \partial_{\tilde{w}} \tilde{\xi} \mp \frac{\partial_{\tilde{w}} \tau_{\mp}}{\tau_+ - \tau_-} \{ \tau_{\pm} - \tilde{\xi} - 2X(\tau_{\pm} - \xi) \} = \Lambda'^{(\star)}$$

$$X = \frac{\tilde{\epsilon}_\kappa}{\epsilon_\kappa} \sqrt{\frac{\tau_+ - \tilde{\xi}}{\tau_+ - \xi} \cdot \frac{\tau_- - \tilde{\xi}}{\tau_- - \xi}} \rho^2 = \frac{\kappa_1 \tilde{\kappa}_1 \hat{\rho}^2}{(\tau_+ - \tau_-)^{\frac{1}{2}}}$$

$$\frac{k_1}{c_6} \tilde{\epsilon}_\kappa \kappa_1 \hat{\rho}^2 \left(\frac{1}{\tau_- - \tilde{\xi}} - \frac{1}{\tau_+ - \tilde{\xi}} \right)^{\frac{3}{2}} + \partial_w \left(\frac{1}{\tau_- - \tilde{\xi}} + \frac{1}{\tau_+ - \tilde{\xi}} \right) = \Lambda''^{(\dagger)}$$

$$\frac{k_1}{c_6} \epsilon_\kappa \tilde{\kappa}_1 \hat{\rho}^2 \left(\frac{1}{\tau_- - \xi} - \frac{1}{\tau_+ - \xi} \right)^{\frac{3}{2}} + \partial_{\tilde{w}} \left(\frac{1}{\tau_- - \xi} + \frac{1}{\tau_+ - \xi} \right) = \Lambda'^{(\star)}$$

$$Z_{\pm}^2 = \frac{\tau_{\pm} - \tilde{\xi}}{\tau_{\pm} - \xi} \frac{1}{R^2} = -\frac{k_1}{c_6} \frac{(Z_- + \epsilon_\kappa Z_+)^{\frac{3}{2}} \hat{\rho}^2}{X^{\frac{1}{2}} (Z_- - \epsilon_\kappa Z_+)^{\frac{1}{2}} (\xi - \tilde{\xi})^{\frac{1}{2}}} \epsilon_\kappa = \frac{\tilde{\epsilon}_\kappa}{\epsilon_\kappa}$$

$$X = \epsilon_\kappa Z_+ Z_- Y = -\frac{(Z_- + \epsilon_\kappa Z_+)^2}{X}$$

$$\partial_w \left(2R^2 X^{\frac{1}{2}} (\tilde{\xi} - \xi) \right) = -\epsilon_\kappa (\kappa_2 - \kappa_1 \tilde{\xi})$$

$$\partial_{\tilde{w}} \left(2R^2 X^{-\frac{1}{2}} (\xi - \tilde{\xi}) \right) = +\epsilon_\kappa (\tilde{\kappa}_2 - \tilde{\kappa}_1 \xi)$$

$$\partial_w \ln \left(R^2 X^{-\frac{1}{2}} \right) - X \partial_w \ln (\tilde{\xi} - \xi) = \Lambda''^{(\dagger)}$$

$$\partial_{\tilde{w}} \ln \left(R^2 X^{\frac{1}{2}} \right) - X^{-1} \partial_{\tilde{w}} \ln (\xi - \tilde{\xi}) = \Lambda'^{(\star)}$$

$$\partial_w Y + 2(Y + 3)X \partial_w \ln (\tilde{\xi} - \xi) + (Y + 6) \partial_w \ln (X(\tilde{\xi} - \xi)) = \Lambda''^{(\dagger)}$$

$$\partial_{\tilde{w}} Y + 2(Y + 3)/X \partial_{\tilde{w}} \ln (\xi - \tilde{\xi}) + (Y + 6) \partial_{\tilde{w}} \ln (X^{-1}(\xi - \tilde{\xi})) = \Lambda'^{(\star)}$$



$$\begin{aligned} -\epsilon_{\kappa}\kappa_1 &= \partial_w \mathcal{A}_1 & \epsilon_{\kappa}\tilde{\kappa}_1 &= \partial_{\tilde{w}} \tilde{\mathcal{A}}_1 \\ -\epsilon_{\kappa}\kappa_2 &= \partial_w \mathcal{A}_2 & \epsilon_{\kappa}\tilde{\kappa}_2 &= \partial_{\tilde{w}} \tilde{\mathcal{A}}_2 \\ \xi &= \frac{\kappa_2}{\kappa_1} & \tilde{\xi} &= \frac{\tilde{\kappa}_2}{\tilde{\kappa}_1} \end{aligned}$$

$$\begin{aligned} 2R^2 X^{+\frac{1}{2}}(\tilde{\xi} - \xi) &= \tilde{\mathcal{A}}_0 + \mathcal{A}_2 - \tilde{\xi}\mathcal{A}_1 \\ 2R^2 X^{-\frac{1}{2}}(\xi - \tilde{\xi}) &= \mathcal{A}_0 + \tilde{\mathcal{A}}_2 - \xi\tilde{\mathcal{A}}_1 \end{aligned}$$

$$\begin{aligned} \mathcal{A}_0 &= -\xi\mathcal{A}_1 + \mathcal{A}_2 \\ \tilde{\mathcal{A}}_0 &= -\tilde{\xi}\tilde{\mathcal{A}}_1 + \tilde{\mathcal{A}}_2 \end{aligned}$$

$$\begin{aligned} 2R^2 X^{+\frac{1}{2}}(\tilde{\xi} - \xi) &= \tilde{\mathcal{L}} & \tilde{\mathcal{L}} &= -\tilde{\xi}(\tilde{\mathcal{A}}_1 + \mathcal{A}_1) + \tilde{\mathcal{A}}_2 + \mathcal{A}_2 \\ 2R^2 X^{-\frac{1}{2}}(\xi - \tilde{\xi}) &= \mathcal{L} & \mathcal{L} &= -\xi(\mathcal{A}_1 + \tilde{\mathcal{A}}_1) + \mathcal{A}_2 + \tilde{\mathcal{A}}_2 \end{aligned}$$

$$R^4 = -\frac{\mathcal{L}\tilde{\mathcal{L}}}{4(\xi - \tilde{\xi})^2} X = -\frac{\tilde{\mathcal{L}}}{\mathcal{L}}$$

$$\begin{aligned} \partial_w(Y(\tilde{\xi} - \xi)R^4) &= \frac{3}{2}(-\mathcal{A}_2\partial_w\mathcal{A}_1 + \mathcal{A}_1\partial_w\mathcal{A}_2 + \tilde{\mathcal{A}}_1\partial_w\mathcal{A}_2 - \tilde{\mathcal{A}}_2\partial_w\mathcal{A}_1) \\ \partial_{\tilde{w}}(Y(\xi - \tilde{\xi})R^4) &= \frac{3}{2}(-\tilde{\mathcal{A}}_2\partial_{\tilde{w}}\tilde{\mathcal{A}}_1 + \tilde{\mathcal{A}}_1\partial_{\tilde{w}}\tilde{\mathcal{A}}_2 + \mathcal{A}_1\partial_{\tilde{w}}\tilde{\mathcal{A}}_2 - \mathcal{A}_2\partial_{\tilde{w}}\tilde{\mathcal{A}}_1) \end{aligned}$$

$$\begin{aligned} \partial_w \mathcal{B} &= \mathcal{A}_2\partial_w\mathcal{A}_1 - \mathcal{A}_1\partial_w\mathcal{A}_2 \\ \partial_{\tilde{w}} \tilde{\mathcal{B}} &= \tilde{\mathcal{A}}_2\partial_{\tilde{w}}\tilde{\mathcal{A}}_1 - \tilde{\mathcal{A}}_1\partial_{\tilde{w}}\tilde{\mathcal{A}}_2 \end{aligned}$$

$$Y(\tilde{\xi} - \xi)R^4 = \frac{3}{2}(-\mathcal{B} + \tilde{\mathcal{B}} + \tilde{\mathcal{A}}_1\mathcal{A}_2 - \mathcal{A}_1\tilde{\mathcal{A}}_2)$$

$$Y = -6\frac{\tilde{\xi} - \xi}{\mathcal{L}\tilde{\mathcal{L}}}(-\mathcal{B} + \tilde{\mathcal{B}} + \tilde{\mathcal{A}}_1\mathcal{A}_2 - \mathcal{A}_1\tilde{\mathcal{A}}_2)$$

$$\tau_{\pm} - \xi = \frac{\xi - \tilde{\xi}}{Z_{\pm}^2 - 1} \quad \tau_{\pm} - \tilde{\xi} = \frac{Z_{\pm}^2(\xi - \tilde{\xi})}{Z_{\pm}^2 - 1}$$

$$Z_{\pm}^2 = -\frac{1}{2}X(Y+2) \pm \frac{1}{2}XYT$$

$$T^2 = \frac{Y+4}{Y}$$

$$\rho^2 = \frac{c_6}{k_1} \frac{\kappa_1 \tilde{\kappa}_1}{R^2 Y} \left(X + \frac{1}{X} + Y + 2 \right)^{\frac{1}{2}}$$

$$\left(\frac{f_6}{c_6} \pm i \frac{f_2}{c_2} \right)^2 = -(\alpha \pm i\gamma)^2 (\beta \mp i\delta)^2 = \frac{16\kappa_1 \tilde{\kappa}_1 Z_{\pm}^2}{\rho^2 XY(Y+4)} \left(X + \frac{1}{X} + Y + 2 \right)$$

$$\rho^2 f_2 f_6 = 2c_2 c_6 e^{\phi} (\tilde{\kappa}_1 \kappa_2 - \kappa_1 \tilde{\kappa}_2)$$



$$\frac{f_6^2}{c_6^2} - \frac{f_2^2}{c_2^2} = -\frac{8\kappa_1\tilde{\kappa}_1}{\rho^2 Y} \frac{Y+2}{Y+4} \left(X + \frac{1}{X} + Y + 2\right)$$

$$\frac{f_6^2}{c_6^2} + \frac{f_2^2}{c_2^2} = \frac{16\eta\kappa_1\tilde{\kappa}_1}{\rho^2 Y(Y+4)} \left(X + \frac{1}{X} + Y + 2\right)$$

$$f_6^2 = -4c_6^2 \frac{\kappa_1\tilde{\kappa}_1}{\rho^2 Y} \frac{Y+2-2\eta}{Y+4} \left(X + \frac{1}{X} + Y + 2\right)$$

$$f_2^2 = +4c_2^2 \frac{\kappa_1\tilde{\kappa}_1}{\rho^2 Y} \frac{Y+2+2\eta}{Y+4} \left(X + \frac{1}{X} + Y + 2\right)$$

$$(\rho^E)^2 = +\frac{c_6}{k_1} \kappa_1 \tilde{\kappa}_1 \left(-\frac{i(\tilde{\xi} - \xi)}{2R^4 Y} T \right)^{\frac{1}{2}}$$

$$(f_6^E)^2 = +2c_6^2 \frac{i\kappa_1\tilde{\kappa}_1(\tilde{\xi} - \xi)}{(\rho^E)^2} T^{-\eta}$$

$$(f_2^E)^2 = -2c_2^2 \frac{i\kappa_1\tilde{\kappa}_1(\tilde{\xi} - \xi)}{(\rho^E)^2} T^{\eta}$$

$$\mathcal{G} = i(-\mathcal{B} + \tilde{\mathcal{B}} + \tilde{\mathcal{A}}_1 \mathcal{A}_2 - \mathcal{A}_1 \tilde{\mathcal{A}}_2)$$

$$\kappa^2 = i(\partial_w \mathcal{A}_1 \partial_{\bar{w}} \tilde{\mathcal{A}}_2 - \partial_{\bar{w}} \tilde{\mathcal{A}}_1 \partial_w \mathcal{A}_2) = -\partial_w \partial_{\bar{w}} \mathcal{G}$$

$$Y = \frac{6\kappa^2 \mathcal{G}}{\partial_w \mathcal{G} \partial_{\bar{w}} \mathcal{G}} \quad R^4 Y = -\frac{3i\mathcal{G}}{2(\tilde{\xi} - \xi)} X = \frac{\partial_w \mathcal{A}_1 \partial_{\bar{w}} \mathcal{G}}{\partial_{\bar{w}} \tilde{\mathcal{A}}_1 \partial_w \mathcal{G}}$$

$$T^2 = 1 + \frac{2}{3} \frac{\partial_w \mathcal{G} \partial_{\bar{w}} \mathcal{G}}{\kappa^2 \mathcal{G}}$$

$$(\rho^E)^2 = -\frac{c_6}{k_1} \kappa^2 \sqrt{-\frac{T}{3\mathcal{G}}} \quad (f_6^E)^2 = -\frac{2c_6^2 \kappa^2}{(\rho^E)^2} \frac{1}{T^\eta} \quad (f_2^E)^2 = +\frac{2c_2^2 \kappa^2}{(\rho^E)^2} T^\eta$$

$$\tau_{\pm} = \frac{\partial_{\bar{w}} \tilde{\mathcal{A}}_2 (T \pm 1) \partial_w \mathcal{G} + \partial_w \mathcal{A}_2 (T \mp 1) \partial_{\bar{w}} \mathcal{G}}{\partial_{\bar{w}} \tilde{\mathcal{A}}_1 (T \pm 1) \partial_w \mathcal{G} + \partial_w \mathcal{A}_1 (T \mp 1) \partial_{\bar{w}} \mathcal{G}}$$

$$\begin{pmatrix} \mathcal{A}'_2 \\ \mathcal{A}'_1 \end{pmatrix} = M \begin{pmatrix} \mathcal{A}_2 \\ \mathcal{A}_1 \end{pmatrix} \quad \begin{pmatrix} \tilde{\mathcal{A}}'_2 \\ \tilde{\mathcal{A}}'_1 \end{pmatrix} = M \begin{pmatrix} \tilde{\mathcal{A}}_2 \\ \tilde{\mathcal{A}}_1 \end{pmatrix} \quad M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\tau'_{\pm} = \frac{a\tau_{\pm} + b}{c\tau_{\pm} + d}$$

$$\kappa'_1 = (c\xi + d)\kappa_1 \quad \xi' = \frac{a\xi + b}{c\xi + d} \quad \mathcal{L}' = \frac{\mathcal{L}}{c\xi + d}$$

$$\tilde{\kappa}'_1 = (c\tilde{\xi} + d)\tilde{\kappa}_1 \quad \tilde{\xi}' = \frac{a\tilde{\xi} + b}{c\tilde{\xi} + d} \quad \tilde{\mathcal{L}}' = \frac{\tilde{\mathcal{L}}}{c\tilde{\xi} + d}$$

$$\rho'(h'_z \pm ie^{\phi'} g'_z) = (c\tau_{\mp} + d)^{\frac{1}{2}} (c\tau_{\pm} + d)^{-\frac{1}{2}} \rho(h_z \pm ie^{\phi} g_z)$$

$$\rho'(h'_z \pm ie^{\phi'} g'_z) = (c\tau_{\pm} + d)^{\frac{1}{2}} (c\tau_{\mp} + d)^{-\frac{1}{2}} \rho(h_z \pm ie^{\phi} g_z)$$



$$\mathcal{B}_\eta^{-1}\varepsilon^* = S\varepsilon S = I_2, s^1, s^2, s^3$$

$$\varepsilon = \sum_{\eta_1, \eta_2 = \pm} \chi^{\eta_1, \eta_2} \otimes \zeta_{\eta_1, \eta_2}$$

$$\mathcal{B}_\eta^{-1}\varepsilon^* = \sum_{\eta_1, \eta_2 = \pm} \chi^{\eta_1, \eta_2} \otimes (\mathfrak{b}_\eta^{-1}\zeta^*)_{\eta_1, \eta_2}$$

$$\mathcal{K}\zeta^* = \zeta\mathcal{K} = S^{-1} \otimes \mathfrak{b}_\eta^{-1}$$

$$\zeta \rightarrow \mathcal{K}\zeta^* \mathcal{K} = S^{-1} \otimes \mathfrak{b}_\eta^{-1}$$

$$\tilde{F}_{(3)}=g_ae^a\wedge e^{67}\,H_{(3)}=h_ae^a\wedge e^{67}\,d\phi=\varphi_ae^a\,d\chi=\chi_ae^a$$

$$(e^A)^* = \eta_{AA} e^A$$

$$\mathfrak{b}_\eta^{-1} = K_1 K_2 K_3 \tau^{(02)} \otimes \sigma^2$$

$$K_1 = \begin{cases} \tau^{(00)} \otimes I_2 & \text{for } (\mathbb{k}_1, \mathbb{k}_2) = (1, i) \\ \tau^{(01)} \otimes I_2 & \text{for } (\mathbb{k}_1, \mathbb{k}_2) = (1, 1) \\ \tau^{(10)} \otimes I_2 & \text{for } (\mathbb{k}_1, \mathbb{k}_2) = (i, i) \\ \tau^{(11)} \otimes I_2 & \text{for } (\mathbb{k}_1, \mathbb{k}_2) = (i, 1) \end{cases}$$

$$\mathbb{k}_1=k_1\frac{|f_6|}{f_6}\,\mathbb{k}_2=k_2\frac{|f_2|}{f_2}$$

$$K_2 = \left\{ \begin{array}{ll} \tau^{(00)} \otimes I_2 & \text{for } \mathcal{M}_{0,2}, \mathcal{M}_{2,0} \\ -i\tau^{(32)} \otimes I_2 & \text{for } \mathcal{M}_{1,1} \end{array} \right. K_3 = \left\{ \begin{array}{ll} \tau^{(00)} \otimes I_2 & \text{for } \Sigma_{0,2} \\ i\tau^{(20)} \otimes \sigma^1 & \text{for } \Sigma_{1,1} \\ \tau^{(00)} \otimes \sigma^3 & \text{for } \Sigma_{2,0} \end{array} \right.$$

$$(\hat{e}^m\gamma_m)^*=B_{\eta(1)}(\hat{e}^m\gamma_m)B_{\eta(1)}^{-1}$$

$$\left(\hat{e}^i\gamma_i\right)^*=-B_{\eta(2)}\big(\hat{e}^i\gamma_i\big)B_{\eta(2)}^{-1}$$

$$(e^a\gamma_a)^*=-B_{\eta(3)}(e^a\gamma_a)B_{\eta(3)}^{-1}$$

$$\mathcal{M}_{0,6} \quad B_{\eta(1)}^{-1} = \gamma_{(1)} \gamma^0 B_{(1)}^{-1} \qquad \alpha_1 = 1$$

$$\mathcal{M}_{1,5} \quad B_{\eta(1)}^{-1} = B_{(1)}^{-1} \qquad \alpha_1 = 0$$

$$\mathcal{M}_{2,4} \quad B_{\eta(1)}^{-1} = \gamma_{(1)} \gamma^1 B_{(1)}^{-1} \qquad \alpha_1 = 1$$

$$\mathcal{M}_{3,3} \quad B_{\eta(1)}^{-1} = \gamma^1 \gamma^2 B_{(1)}^{-1} \qquad \alpha_1 = 0$$

$$\mathcal{M}_{4,2} \quad B_{\eta(1)}^{-1} = \gamma_{(1)} \gamma^1 \gamma^2 \gamma^3 B_{(1)}^{-1} \qquad \alpha_1 = 1$$

$$\mathcal{M}_{5,1} \quad B_{\eta(1)}^{-1} = \gamma^1 \gamma^2 \gamma^3 \gamma^4 B_{(1)}^{-1} \qquad \alpha_1 = 0$$

$$\mathcal{M}_{0,2} \quad B_{\eta(2)}^{-1} = B_{(2)}^{-1} = \sigma^2 \qquad \alpha_2 = 1$$

$$\mathcal{M}_{1,1} \quad B_{\eta(2)}^{-1} = \gamma_{(2)} \gamma^7 B_{(2)}^{-1} = \sigma^3 \qquad \alpha_2 = 0$$

$$\mathcal{M}_{2,0} \quad B_{\eta(2)}^{-1} = \gamma^6 \gamma^7 B_{(2)}^{-1} = \sigma^1 \qquad \alpha_2 = 1$$



$$\begin{array}{lll} \Sigma_{0,2} & B_{\eta(3)}^{-1} = B_{(3)}^{-1} = \sigma^2 & \alpha_3 = 1 \\ \Sigma_{1,1} & B_{\eta(3)}^{-1} = \gamma_{(3)} \gamma^9 B_{(3)}^{-1} = \sigma^3 & \alpha_3 = 0 \\ \Sigma_{2,0} & B_{\eta(3)}^{-1} = i \gamma^8 \gamma^9 B_{(3)}^{-1} = i \sigma^1 & \alpha_3 = 1 \end{array}$$

$$\begin{aligned} \gamma_{(1)} B_{\eta(1)} &= (-)^{\alpha_1} B_{\eta(1)} \gamma_{(1)} \\ \gamma_{(2)} B_{\eta(2)} &= (-)^{\alpha_2} B_{\eta(2)} \gamma_{(2)} \\ \gamma_{(3)} B_{\eta(3)} &= (-)^{\alpha_3} B_{\eta(3)} \gamma_{(3)} \end{aligned}$$

$$\mathcal{B}_\eta = \tilde{B}_{\eta(1)} \otimes \tilde{B}_{\eta(2)} \otimes \tilde{B}_{\eta(3)}$$

$$\mathcal{B}_\eta(\hat{e}^i \Gamma_i) \mathcal{B}_\eta^{-1} = B_{\eta(1)} \gamma_{(1)} B_{\eta(1)}^{-1} \otimes \tilde{B}_{\eta(2)}(\hat{e}^i \gamma_i) \tilde{B}_{\eta(2)}^{-1} \otimes I_2 = \gamma_{(1)} \otimes (\hat{e}^i \gamma_i)^* \otimes I_2$$

$$\tilde{B}_{\eta(2)}(\hat{e}^i \gamma_i) \tilde{B}_{\eta(2)}^{-1} = (-)^{\alpha_1} (\hat{e}^i \gamma_i)^* = (-)^{1-\alpha_1} B_{\eta(2)}(\hat{e}^i \gamma_i) B_{\eta(2)}^{-1}$$

$$\mathcal{B}_\eta = B_{\eta(1)} \otimes \left(B_{\eta(2)} \gamma_{(2)}^{\alpha_1+1} \right) \otimes \left(B_{\eta(3)} \gamma_{(3)}^{\alpha_1+\alpha_2+1} \right)$$

$$\alpha_1 + \alpha_2 + \alpha_3 \equiv 0 \pmod{2}$$

$$\mathcal{B}_\eta = \left(B_{\eta(1)} \gamma_{(1)}^{1+\alpha_3} \right) \otimes \left(B_{\eta(2)} \gamma_{(2)}^{\alpha_2} \right) \otimes B_{\eta(3)}$$

$$(\mathcal{B}_{\eta(1)}^{-1} \otimes \mathcal{B}_{\eta(2)}^{-1})(\chi^{+,+})^* = \chi^{k_1^2, -k_2^2}$$

$$\begin{aligned} (\gamma_{(1)} \otimes I_2) \chi^{\eta_1, \eta_2} &= \chi^{-\eta_1, \eta_2} \\ (I_8 \otimes \gamma_{(2)}) \chi^{\eta_1, \eta_2} &= \chi^{\eta_1, -\eta_2} \end{aligned}$$

$$(\mathcal{B}_{\eta(1)}^{-1} \otimes \mathcal{B}_{\eta(2)}^{-1})(\chi^{\eta_1, \eta_2})^* = \eta_1^{\alpha_1} \eta_2^{\alpha_2} \chi^{k_1^2 \eta_1, -k_2^2 \eta_2}$$

$$\mathcal{B}_\eta^{-1} \varepsilon^* = \sum_{\eta_1, \eta_2} \eta_1^{\alpha_2+\alpha_3} \eta_2^{\alpha_2} \gamma_{(1)}^{1+\alpha_3} \otimes \gamma_{(2)}^{\alpha_2} \chi^{k_1^2 \eta_1, -k_2^2 \eta_2} \otimes B_{\eta(3)}^{-1} \zeta_{\eta_1, \eta_2}^*$$

$$\begin{aligned} (\alpha_2, \alpha_3) = (0,0) \quad \mathcal{B}_\eta^{-1} \varepsilon^* &= \sum \chi^{\eta_1, \eta_2} \otimes B_{\eta(3)}^{-1} \zeta_{-k_1^2 \eta_1, -k_2^2 \eta_2}^* \\ (\alpha_2, \alpha_3) = (0,1) \quad \mathcal{B}_\eta^{-1} \varepsilon^* &= \sum_{\eta_1, \eta_2} \chi^{\eta_1, \eta_2} \otimes (k_1^2 \eta_1) B_{\eta(3)}^{-1} \zeta_{k_1^2 \eta_1, -k_2^2 \eta_2}^* \\ (\alpha_2, \alpha_3) = (1,0) \quad \mathcal{B}_\eta^{-1} \varepsilon^* &= \sum_{\eta_1, \eta_2} \chi^{\eta_1, \eta_2} \otimes (-k_1^2 \eta_1 k_2^2 \eta_2) B_{\eta(3)}^{-1} \zeta_{-k_1^2 \eta_1, k_2^2 \eta_2}^* \\ (\alpha_2, \alpha_3) = (1,1) \quad \mathcal{B}_\eta^{-1} \varepsilon^* &= \sum_{\eta_1, \eta_2} \chi^{\eta_1, \eta_2} \otimes (k_2^2 \eta_2) B_{\eta(3)}^{-1} \zeta_{k_1^2 \eta_1, k_2^2 \eta_2}^* \end{aligned}$$

$$\hat{K}_1 = \begin{cases} \tau^{(00)} \otimes I_2 & \text{for } (k_1, k_2) = (1, i) \\ \tau^{(01)} \otimes I_2 & \text{for } (k_1, k_2) = (1, 1) \\ \tau^{(10)} \otimes I_2 & \text{for } (k_1, k_2) = (i, i) \\ \tau^{(11)} \otimes I_2 & \text{for } (k_1, k_2) = (i, 1) \end{cases}$$



$$\begin{aligned}
(\alpha_2, \alpha_3) &= (0,0) & \mathfrak{b}_\eta^{-1} &= \hat{K}_1 \tau^{(10)} \otimes \sigma^3 \\
(\alpha_2, \alpha_3) &= (0,1) & \mathfrak{b}_\eta^{-1} &= -i \hat{K}_1 \tau^{(30)} \otimes B_{\eta(3)}^{-1} \\
(\alpha_2, \alpha_3) &= (1,0) & \mathfrak{b}_\eta^{-1} &= -\hat{K}_1 \tau^{(22)} \otimes \sigma^3 \\
(\alpha_2, \alpha_3) &= (1,1) & \mathfrak{b}_\eta^{-1} &= \hat{K}_1 \tau^{(02)} \otimes B_{\eta(3)}^{-1}
\end{aligned}$$

$$\mathcal{K} \zeta^* = \zeta \mathcal{K} = S^{-1} \otimes \mathfrak{b}_\eta^{-1} \mathcal{K}^* = \mathcal{K}$$

$$J \mathcal{K} J \zeta = \mathcal{K} \zeta \mathcal{K}^* \mathcal{K} \zeta = \mathcal{K}^2 \zeta = \zeta$$

$$\begin{aligned}
J S J &= S \cdot \begin{cases} +1 & \text{IIB}_{\mathbb{R}} \quad \text{or IIB}_3 \\ -1 & \text{IIB}^* \quad \text{or IIB}' \end{cases} \\
J K_1 J &= K_1 \cdot \begin{cases} +1 & (\mathbb{k}_1, \mathbb{k}_2) \in \{(1, i), (i, 1)\} \\ -1 & (\mathbb{k}_1, \mathbb{k}_2) \in \{(1, 1), (i, i)\} \end{cases} \\
J K_2 J &= K_2 \\
J K_3 J &= K_3 \cdot \begin{cases} +1 & \Sigma_{2,0} \text{ or } \Sigma_{0,2} \\ -1 & \Sigma_{1,1} \end{cases}
\end{aligned}$$

$$\begin{aligned}
\text{IIB}_{\mathbb{R}}, \text{IIB}_3: & \quad (\mathbb{k}_1, \mathbb{k}_2) \in \{(1, i), (i, 1)\} & \quad \{\Sigma_{2,0} \text{ or } \Sigma_{0,2}\} \times \{\mathcal{M}_{0,2} \text{ or } \mathcal{M}_{1,1}\} \\
& \quad (\mathbb{k}_1, \mathbb{k}_2) \in \{(1, 1), (i, i)\} & \quad \Sigma_{1,1} \times \mathcal{M}_{2,0} \\
& & \quad \Sigma_{1,1} \times \{\mathcal{M}_{0,2} \text{ or } \mathcal{M}_{1,1}\} \\
& & \quad \{\Sigma_{2,0} \text{ or } \Sigma_{0,2}\} \times \mathcal{M}_{2,0} \\
\text{IIB}^*, \text{IIB}': & \quad (\mathbb{k}_1, \mathbb{k}_2) \in \{(1, 1), (i, i)\} & \quad \{\Sigma_{2,0} \text{ or } \Sigma_{0,2}\} \times \{\mathcal{M}_{0,2} \text{ or } \mathcal{M}_{1,1}\} \\
& & \quad \Sigma_{1,1} \times \mathcal{M}_{2,0} \\
& \quad (\mathbb{k}_1, \mathbb{k}_2) \in \{(1, i), (i, 1)\} & \quad \Sigma_{1,1} \times \{\mathcal{M}_{0,2} \text{ or } \mathcal{M}_{1,1}\} \\
& & \quad \{\Sigma_{2,0} \text{ or } \Sigma_{0,2}\} \times \mathcal{M}_{2,0}
\end{aligned}$$

$$S^{-2} = \begin{cases} +I_2 & \text{for IIB}_{\mathbb{R}}, \text{IIB}^*, \text{IIB}' \\ -I_2 & \text{for IIB}_3 \end{cases}$$

$$\mathcal{K}^2 \zeta = \zeta \begin{Bmatrix} \mathbb{k}_2^2, & \mathcal{M}_{0,2}, \mathcal{M}_{2,0} \\ \mathbb{k}_1^2, & \mathcal{M}_{1,1} \end{Bmatrix} \begin{Bmatrix} -1, \Sigma_{0,2} \\ 1, \Sigma_{2,0} \end{Bmatrix} \begin{Bmatrix} +1, \text{IIB}_{\mathbb{R}}, \text{IIB}^*, \text{IIB}' \\ -1, \text{IIB}_3 \end{Bmatrix}$$

$$\mathcal{K}^2 \zeta = \zeta \begin{Bmatrix} -\mathbb{k}_1^2 \mathbb{k}_2^2, & \{\mathcal{M}_{0,2} \text{ or } \mathcal{M}_{2,0}\} \times \Sigma_{1,1} \\ 1, & \mathcal{M}_{1,1} \times \Sigma_{1,1} \end{Bmatrix} \begin{Bmatrix} +1, & \text{IIB}_{\mathbb{R}}, \text{IIB}^*, \text{IIB}' \\ -1, & \text{IIB}_3 \end{Bmatrix}$$

$$\begin{aligned}
S^n &= \{\mu_{ij} x^i x^j = +1; ds^2 = \mu_{ij} dx^i dx^j; \mu = (+_{n+1})\} \\
H_n &= \{\mu_{ij} x^i x^j = -1; ds^2 = \mu_{ij} dx^i dx^j; \mu = (-_1, +_n)\} \\
AdS_{r,s} &= \{\mu_{ij} x^i x^j = -1; ds^2 = \mu_{ij} dx^i dx^j; \mu = (-_{r+1}, +_s)\} \\
dS_{r,s} &= \{\mu_{ij} x^i x^j = +1; ds^2 = \mu_{ij} dx^i dx^j; \mu = (-_r, +_{s+1})\}
\end{aligned}$$

$$-dS_{p,q} = AdS_{q,p} - AdS_{p,q} = dS_{q,p}$$

k_1	k_2	$f_2^2 \mathcal{M}_2$	Σ	real form	geometry	bosonic subalgebra
1	i	(0,2)	(0,2)	$\text{IIB}_{\mathbb{R}}$	$AdS_{1,5} \times S^2 \times \Sigma_{0,2}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$



i	1	(1,1)	(0,2)	IIB $_{\mathbb{R}}$	$S^6 \times AdS_{1,1} \times \Sigma_{0,2}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
i	i	(0,2)	(0,2)	IIB * , IIB $^{'}$	$dS_{1,5} \times S^2 \times \Sigma_{0,2}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
1	i	(0,2)	(1,1)	IIB * , IIB $^{'}^{'}$	$H^6 \times S^2 \times \Sigma_{1,1}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
i	1	(0,2)	(1,1)	IIB * , IIB $^{'}^{'}$	$S^6 \times H^2 \times \Sigma_{1,1}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
i	i	(1,1)	(0,2)	IIB * , IIB $^{'}^{'}$	$S^6 \times dS_{1,1} \times \Sigma_{0,2}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
i	1	(0,2)	(0,2)	IIB $_3$	$dS_{3,3} \times H^2 \times \Sigma_{0,2}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	1	(0,2)	(1,1)	IIB $_3$	$AdS_{2,4} \times H^2 \times \Sigma_{1,1}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
i	i	(0,2)	(1,1)	IIB $_3$	$dS_{2,4} \times S^2 \times \Sigma_{1,1}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$
1	i	(0,2)	(2,0)	IIB $_3$	$AdS_{1,5} \times S^2 \times \Sigma_{2,0}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$
1	i	(1,1)	(0,2)	IIB $_3$	$AdS_{2,4} \times dS_{1,1} \times \Sigma_{0,2}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
i	1	(1,1)	(2,0)	IIB $_3$	$S^6 \times AdS_{1,1} \times \Sigma_{2,0}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
i	1	(2,0)	(0,2)	IIB $_3$	$dS_{1,5} \times (-S^2) \times \Sigma_{0,2}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
1	1	(2,0)	(1,1)	IIB $_3$	$H^6 \times (-S^2) \times \Sigma_{1,1}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
i	i	(2,0)	(1,1)	IIB $_3$	$S^6 \times (-H^2) \times \Sigma_{1,1}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$

$$\mathcal{M}_2 = S^2 \rightarrow \mathcal{M}_2 = AdS_{2,0} \quad \mathcal{M}_2 = H^2 \rightarrow \mathcal{M}_2 = dS_{2,0}$$

$$(AdS_{p,q}, dS_{p,q}, \Sigma_{r,2-r}) \rightarrow (-AdS_{p,q}, -dS_{p,q}, -\Sigma_{r,2-r}) = (dS_{q,p}, AdS_{q,p}, \Sigma_{2-r,r})$$

$\mathbb{k}_1$	$\mathbb{k}_2$	$f_2^2 \mathcal{M}_2$	Σ	real form	geometry	bosonic subalgebra
1	i	(0,2)	(0,2)	IIB $_5$	$AdS_{5,1} \times S^2 \times \Sigma_{0,2}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
i	1	(0,2)	(2,0)	IIB $_5$	$dS_{3,3} \times H_2 \times \Sigma_{2,0}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$



i	1	(1,1)	(0,2)	IIB ₅	$dS_{4,2} \times AdS_{1,1} \times \Sigma_{0,2}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	i	(1,1)	(2,0)	IIB ₅	$AdS_{2,4} \times dS_{1,1} \times \Sigma_{2,0}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	1	(1,1)	(1,1)	IIB ₅	$AdS_{3,3} \times AdS_{1,1} \times \Sigma_{1,1}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
i	i	(1,1)	(1,1)	IIB ₅	$dS_{3,3} \times dS_{1,1} \times \Sigma_{1,1}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	i	(2,0)	(0,2)	IIB ₅	$AdS_{3,3} \times (-H_2) \times \Sigma_{0,2}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
i	1	(2,0)	(2,0)	IIB ₅	$dS_{1,5} \times (-S^2) \times \Sigma_{2,0}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
i	i	(0,2)	(0,2)	IIB ₅ [*] , IIB ₅ '	$dS_{5,1} \times S^2 \times \Sigma_{0,2}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$
1	i	(0,2)	(1,1)	IIB ₅ [*] , IIB ₅ '	$AdS_{4,2} \times S^2 \times \Sigma_{1,1}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$
i	1	(0,2)	(1,1)	IIB ₅ [*] , IIB ₅ '	$dS_{4,2} \times H^2 \times \Sigma_{1,1}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	1	(0,2)	(2,0)	IIB ₅ [*] , IIB ₅ '	$AdS_{3,3} \times H^2 \times \Sigma_{2,0}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
i	i	(1,1)	(0,2)	IIB ₅ [*] , IIB ₅ '	$dS_{4,2} \times dS_{1,1} \times \Sigma_{0,2}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	1	(1,1)	(2,0)	IIB ₅ [*] , IIB ₅ '	$AdS_{2,4} \times AdS_{1,1} \times \Sigma_{2,0}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	i	(1,1)	(1,1)	IIB ₅ [*] , IIB ₅ '	$AdS_{3,3} \times dS_{1,1} \times \Sigma_{1,1}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
i	1	(1,1)	(1,1)	IIB ₅ [*] , IIB ₅ '	$dS_{3,3} \times AdS_{1,1} \times \Sigma_{1,1}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	i	(2,0)	(1,1)	IIB ₅ [*] , IIB ₅ '	$AdS_{2,4} \times (-H^2) \times \Sigma_{1,1}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	1	(2,0)	(2,0)	IIB ₅ [*] , IIB ₅ '	$AdS_{1,5} \times (-S^2) \times \Sigma_{2,0}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$
i	i	(2,0)	(0,2)	IIB ₅ [*] , IIB ₅ '	$dS_{3,3} \times (-H^2) \times \Sigma_{0,2}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
i	1	(2,0)	(1,1)	IIB ₅ [*] , IIB ₅ '	$dS_{2,4} \times (-S^2) \times \Sigma_{1,1}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$

$$(AdS_{5,1} \times S^2 \times \Sigma_{2,0}, dS_{5,1} \times (-S^2) \times \Sigma_{0,2})$$



k_1	k_2	$f^2\mathcal{M}_2$	Σ	real form	geometry	bosonic subalgebra
1	i	(0,2)	(2,0)	IIB ₇	$AdS_{5,1} \times S^2 \times \Sigma_{2,0}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
1	1	(0,2)	(1,1)	IIB ₇	$(-S^6) \times H^2 \times \Sigma_{1,1}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
i	i	(0,2)	(1,1)	IIB ₇	$(-H^6) \times S^2 \times \Sigma_{1,1}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
i	1	(1,1)	(2,0)	IIB ₇	$dS_{4,2} \times AdS_{1,1} \times \Sigma_{2,0}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	i	(1,1)	(0,2)	IIB ₇	$(-S^6) \times dS_{1,1} \times \Sigma_{0,2}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
1	i	(2,0)	(2,0)	IIB ₇	$AdS_{3,3} \times (-H^2) \times \Sigma_{2,0}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	1	(2,0)	(1,1)	IIB ₇	$AdS_{4,2} \times (-S^2) \times \Sigma_{1,1}$	$\mathfrak{so}(5,2) \oplus \mathfrak{so}(3)$
i	i	(2,0)	(1,1)	IIB ₇	$dS_{4,2} \times (-H^2) \times \Sigma_{1,1}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
i	1	(2,0)	(0,2)	IIB ₇	$dS_{5,1} \times (-S^2) \times \Sigma_{0,2}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$
1	i	(1,1)	(2,0)	IIB ₉	$(-S^6) \times dS_{1,1} \times \Sigma_{2,0}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
i	1	(2,0)	(2,0)	IIB ₉	$dS_{5,1} \times (-S^2) \times \Sigma_{2,0}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$
1	1	(1,1)	(2,0)	IIB ₉ [*] , IIB ₉ [']	$(-S^6) \times AdS_{1,1} \times \Sigma_{2,0}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
1	1	(2,0)	(1,1)	IIB ₉ [*] , IIB ₉ [']	$(-S^6) \times (-H^2) \times \Sigma_{1,1}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
1	i	(2,0)	(2,0)	IIB ₉ [*] , IIB ₉ [']	$AdS_{5,1} \times (-S^2) \times \Sigma_{2,0}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
i	i	(2,0)	(1,1)	IIB ₉ [*] , IIB ₉ [']	$(-H^6) \times (-S^2) \times \Sigma_{1,1}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$

$$\rho^* = \Lambda_\rho \rho$$

geometry	real form	$\{\alpha, \beta, \gamma, \delta\}^*$	$\Lambda_\rho \{\tilde{\kappa}_1, \kappa_1, \tilde{\kappa}_2, \kappa_2\}^*$
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$AdS_{1,5} \times S^2 \times \Sigma_{0,2}$	IIB $_{\mathbb{R}}$	$\{-\delta v, \gamma v, \beta v, -\alpha v\}$	$\{-\kappa_1, -\tilde{\kappa}_1, -\kappa_2, -\tilde{\kappa}_2\}$
$S^6 \times AdS_{1,1} \times \Sigma_{0,2}$	IIB $_{\mathbb{R}}$	$\{\beta, \alpha, \delta, \gamma\}$	$\{+\kappa_1, +\tilde{\kappa}_1, +\kappa_2, +\tilde{\kappa}_2\}$
$dS_{1,5} \times S^2 \times \Sigma_{0,2}$	IIB*	$\{-\beta, -\alpha, \delta, \gamma\}$	$\{+\kappa_1, +\tilde{\kappa}_1, -\kappa_2, -\tilde{\kappa}_2\}$
	IIB'	$\{-\delta, -\gamma, -\beta, -\alpha\}$	$\{-\kappa_1, -\tilde{\kappa}_1, +\kappa_2, +\tilde{\kappa}_2\}$
$H^6 \times S^2 \times \Sigma_{1,1}$	IIB*	$\{\alpha, \beta, -\gamma, -\delta\}$	$\{+\tilde{\kappa}_1, +\kappa_1, -\tilde{\kappa}_2, -\kappa_2\}$
	IIB'	$\{\gamma, \delta, \alpha, \beta\}$	$\{-\tilde{\kappa}_1, -\kappa_1, +\tilde{\kappa}_2, +\kappa_2\}$
$S^6 \times H^2 \times \Sigma_{1,1}$	IIB*	$\{\alpha, -\beta, -\gamma, \delta\}$	$\{+\tilde{\kappa}_1, +\kappa_1, -\tilde{\kappa}_2, -\kappa_2\}$
	IIB'	$\{\gamma, -\delta, \alpha, -\beta\}$	$\{-\tilde{\kappa}_1, -\kappa_1, +\tilde{\kappa}_2, +\kappa_2\}$
$S^6 \times dS_{1,1} \times \Sigma_{0,2}$	IIB*	$\{-\delta v, -\gamma v, -\beta v, -\alpha v\}$	$\{-\kappa_1, -\kappa_1, +\kappa_2, +\tilde{\kappa}_2\}$
	IIB'	$\{\beta v, \alpha v, -\delta v, -\gamma v\}$	$\{+\kappa_1, +\tilde{\kappa}_1, -\kappa_2, -\tilde{\kappa}_2\}$
$dS_{3,3} \times H^2 \times \Sigma_{0,2}$	IIB $_3$	$\{\beta v, \alpha v, \delta v, \gamma v\}$	$\{+\kappa_1, +\tilde{\kappa}_1, +\kappa_2, +\tilde{\kappa}_2\}$
$AdS_{2,4} \times H^2 \times \Sigma_{1,1}$	IIB $_3$	$\{-\alpha v, -\beta v, -\gamma v, -\delta v\}$	$\{+\tilde{\kappa}_1, +\kappa_1, +\tilde{\kappa}_2, +\kappa_2\}$
$dS_{2,4} \times S^2 \times \Sigma_{1,1}$	IIB $_3$	$\{-\alpha v, \beta v, -\gamma v, \delta v\}$	$\{+\tilde{\kappa}_1, +\kappa_1, +\tilde{\kappa}_2, +\kappa_2\}$
$AdS_{1,5} \times S^2 \times \Sigma_{2,0}$	IIB $_3$	$\{\beta v, \alpha v, \delta v, \gamma v\}$	$\{+\kappa_1, +\tilde{\kappa}_1, +\kappa_2, +\tilde{\kappa}_2\}$
$AdS_{2,4} \times dS_{1,1} \times \Sigma_{0,2}$	IIB $_3$	$\{\delta, -\gamma, -\beta, \alpha\}$	$\{-\kappa_1, -\tilde{\kappa}_1, -\kappa_2, -\tilde{\kappa}_2\}$
$S^6 \times AdS_{1,1} \times \Sigma_{2,0}$	IIB $_3$	$\{\delta, -\gamma, -\beta, \alpha\}$	$\{-\kappa_1, -\tilde{\kappa}_1, -\kappa_2, -\tilde{\kappa}_2\}$
$dS_{1,5} \times (-S^2) \times \Sigma_{0,2}$	IIB $_3$	$\{\beta v, \alpha v, \delta v, \gamma v\}$	$\{+\kappa_1, +\tilde{\kappa}_1, +\kappa_2, +\tilde{\kappa}_2\}$
$H^6 \times (-S^2) \times \Sigma_{1,1}$	IIB $_3$	$\{-\alpha v, -\beta v, -\gamma v, -\delta v\}$	$\{+\tilde{\kappa}_1, +\kappa_1, +\tilde{\kappa}_2, +\kappa_2\}$
$S^6 \times (-H^2) \times \Sigma_{1,1}$	IIB $_3$	$\{-\alpha v, \beta v, -\gamma v, \delta v\}$	$\{+\tilde{\kappa}_1, +\kappa_1, +\tilde{\kappa}_2, +\kappa_2\}$

$$AdS_6 \times S^2 \times \Sigma: \quad (\alpha^*, \beta^*) = v(-\delta, \gamma) \quad \kappa_{1/2}^* = -\tilde{\kappa}_{1/2} \quad c_6 \in \mathbb{R}$$

$$S^6 \times AdS_2 \times \Sigma: \quad (\alpha^*, \gamma^*) = (\beta, \delta) \quad \kappa_{1/2}^* = +\tilde{\kappa}_{1/2} \quad c_6 \in i\mathbb{R}$$

$$\epsilon_{\kappa} = \mp \tilde{\epsilon}_{\kappa} \epsilon_{\kappa} = \mp 1$$

$$(\mathcal{A}_1)^{\star} = -\varepsilon_{\kappa} \tilde{\mathcal{A}}_1 \, (\mathcal{A}_2)^{\star} = -\varepsilon_{\kappa} \tilde{\mathcal{A}}_2$$

$$h_z = \frac{2i\left(2\beta\delta\phi_z + e^{\phi}\chi_z(\beta^2 - \delta^2)\right)}{v(\beta^2 + \delta^2)} \quad h_{\bar{z}} = \frac{2i\left(2\alpha\gamma\phi_{\bar{z}} + e^{\phi}\chi_{\bar{z}}(\alpha^2 - \gamma^2)\right)}{v(\alpha^2 + \gamma^2)}$$

$$g_z = \frac{2i\left((\delta^2 - \beta^2)e^{-\phi}\phi_z + 2\beta\delta\chi_z\right)}{v(\beta^2 + \delta^2)} \quad g_{\bar{z}} = \frac{2i\left((\gamma^2 - \alpha^2)e^{-\phi}\phi_{\bar{z}} + 2\alpha\gamma\chi_{\bar{z}}\right)}{v(\alpha^2 + \gamma^2)}$$

$$(h_z)^{\star} = \pm h_{\bar{z}} \, (g_z)^{\star} = \pm g_{\bar{z}}$$

$$\hat{f}_2^2 = \frac{c^2 \hat{\kappa}^2}{9 \hat{\rho}^2} \frac{1}{\hat{T}} \quad \hat{f}_6^2 = \frac{c^2 \hat{\kappa}^2}{\hat{\rho}^2} \hat{T} \quad \hat{\rho}^2 = c \hat{\kappa}^2 \sqrt{\frac{\hat{T}}{6 \Lambda \hat{\mathcal{G}}}}$$

$$\hat{\mathcal{G}}=|\mathcal{A}_+|^2-|\mathcal{A}_-|^2+\hat{B}+\hat{B}^*\partial_w\hat{B}=\mathcal{A}_+\partial_w\mathcal{A}_--\mathcal{A}_-\partial_w\mathcal{A}_+$$

$$\hat{\kappa}^2=-|\partial_w\mathcal{A}_+|^2+|\partial_w\mathcal{A}_-|^2\hat{T}=\frac{1+\Lambda\hat{R}}{1-\Lambda\hat{R}}=\sqrt{1+\frac{2|\partial_w\hat{\mathcal{G}}|^2}{3\hat{\kappa}^2\hat{\mathcal{G}}}}$$

$$B=\frac{1+i\hat{t}}{1-i\hat{t}}=\frac{\partial_w\mathcal{A}_+\partial_{\bar{w}}\hat{\mathcal{G}}-\Lambda\hat{R}\partial_{\bar{w}}\overline{\mathcal{A}}_-\partial_w\hat{\mathcal{G}}}{\Lambda\hat{R}\partial_{\bar{w}}\overline{\mathcal{A}}_+\partial_w\hat{\mathcal{G}}-\partial_w\mathcal{A}_-\partial_{\bar{w}}\hat{\mathcal{G}}}$$

$$C=\frac{4ic}{9}\Lambda\left[\frac{\partial_{\bar{w}}\overline{\mathcal{A}}-\partial_w\hat{\mathcal{G}}}{\hat{\kappa}^2}-2\Lambda\hat{R}\frac{\partial_{\bar{w}}\hat{\mathcal{G}}\partial_w\mathcal{A}_++\partial_w\hat{\mathcal{G}}\partial_{\bar{w}}\overline{\mathcal{A}}_-}{(1+\Lambda\hat{R})^2\hat{\kappa}^2}-\overline{\mathcal{A}}_- -2\mathcal{A}_+\right]$$

$$\mathcal{A}_1=\sqrt{-\varepsilon_{\kappa}}(a_1\mathcal{A}_+-a_1^*\mathcal{A}_-)\quad \mathcal{A}_2=\sqrt{-\varepsilon_{\kappa}}(a_2\mathcal{A}_+-a_2^*\mathcal{A}_-)$$

$$\tilde{\mathcal{A}}_1=\sqrt{-\varepsilon_{\kappa}}(a_1^*\mathcal{A}_+^*-a_1\mathcal{A}_-^*)\quad \tilde{\mathcal{A}}_2=\sqrt{-\varepsilon_{\kappa}}(a_2^*\mathcal{A}_+^*-a_2\mathcal{A}_-^*)$$

$$\kappa^2 = -\varepsilon_{\kappa} i (a_1^* a_2 - a_1 a_2^*) \hat{\kappa}^2 \, \mathcal{G} = -\varepsilon_{\kappa} i (a_1^* a_2 - a_1 a_2^*) \hat{\mathcal{G}}$$

$$\tau_+=\frac{\partial_{\bar{w}}\mathcal{G}(T-1)(a_2\partial_w\mathcal{A}_+-a_2^*\partial_w\mathcal{A}_-)+\partial_w\mathcal{G}(T+1)(a_2^*\partial_{\bar{w}}\tilde{\mathcal{A}}_+-a_2\partial_{\bar{w}}\tilde{\mathcal{A}}_-)}{\partial_{\bar{w}}\mathcal{G}(T-1)(a_1\partial_w\mathcal{A}_+-a_1^*\partial_w\mathcal{A}_-)+\partial_w\mathcal{G}(T+1)(a_1^*\partial_{\bar{w}}\tilde{\mathcal{A}}_+-a_1\partial_{\bar{w}}\tilde{\mathcal{A}}_-)}$$

$$T=-\hat{T} \, a_1 \in \mathbb{R} \, a_2=-ia_1$$

$$(\rho^{\rm E})^2=\frac{2a_1c_6}{k_1}\varepsilon_{\kappa}\hat{\kappa}^2\sqrt{\frac{\hat{T}}{6(-\varepsilon_{\kappa})\hat{G}}}\, (f_6^{\rm E})^2=-\frac{4\varepsilon_{\kappa}c_6^2a_1^2}{(\rho^{\rm E})^2}\frac{\hat{\kappa}^2}{\hat{T}\eta}\, (f_2^{\rm E})^2=\frac{4c_2^2a_1^2\varepsilon_{\kappa}}{9(\rho^{\rm E})^2}\hat{\kappa}^2\hat{T}\eta$$

$$\tilde{c}=\frac{2a_1\varepsilon_{\kappa}c_6}{k_1}$$

$$(\rho^{\rm E})^2=\tilde{c}\hat{\kappa}^2\sqrt{\frac{\hat{T}}{6(-\varepsilon_{\kappa})\hat{G}}}\, (f_6^{\rm E})^2=\frac{\tilde{c}^2}{(\rho^{\rm E})^2}\frac{\hat{\kappa}^2}{\hat{T}\eta}\, (f_2^{\rm E})^2=\frac{\tilde{c}^2\hat{\kappa}^2}{9(\rho^{\rm E})^2}\hat{T}\eta$$



$$\begin{array}{llll} \text{IIB}^*: & (\alpha^*, \gamma^*) = (-\beta, +\delta) & (\kappa_1, \kappa_2)^* = (+\tilde{\kappa}_1, -\tilde{\kappa}_2) & c_6 \in \mathbb{R} \quad c_2 \in \mathbb{R} \\ \text{IIB}': & (\alpha^*, \beta^*) = (-\delta, -\gamma) & (\kappa_1, \kappa_2)^* = (-\tilde{\kappa}_1, +\tilde{\kappa}_2) & c_6 \in i\mathbb{R} \quad c_2 \in i\mathbb{R} \end{array}$$

$$\epsilon_{\kappa} = \pm \tilde{\epsilon}_{\kappa} \quad \epsilon_{\kappa} = \pm 1 \quad \xi^* = -\tilde{\xi}$$

$$\mathcal{A}_1^* = \mp \tilde{\mathcal{A}}_1 \quad \mathcal{A}_2^* = \pm \tilde{\mathcal{A}}_2$$

$$(h_z)^* = \pm h_{\bar{z}} \quad (g_z)^* = \mp g_{\bar{z}}$$

$$dS_{1,5} \rightarrow S^6$$

$$\begin{array}{llll} \{\xi, \tilde{\xi}\}^* = \{\tilde{\xi}, \xi\} & \text{IIB}_{\mathbb{R}}, \text{IIB}_3 & \Sigma_{(0,2)}, \Sigma_{(2,0)} & M^* = \pm M \\ \{\xi, \tilde{\xi}\}^* = \{-\tilde{\xi}, -\xi\} & \text{IIB}^*, \text{IIB}' & \Sigma_{(0,2)}, \Sigma_{(2,0)} & M^* = \pm \sigma^3 M \sigma^3 \\ \{\xi, \tilde{\xi}\}^* = \{\xi, \tilde{\xi}\} & \text{IIB}_3 & \Sigma_{(1,1)} & M^* = \pm M \\ \{\xi, \tilde{\xi}\}^* = \{-\xi, -\tilde{\xi}\} & \text{IIB}^*, \text{IIB}' & \Sigma_{(1,1)} & M^* = \pm \sigma^3 M \sigma^3 \end{array}$$

$$S_{\text{IKKT}} = \text{tr} \left(-\frac{1}{4} [X_{\mu}, X_{\nu}] [X^{\mu}, X^{\nu}] + \frac{1}{2} \Psi_{\alpha} (\mathcal{C} \Gamma^{\mu})_{\alpha\beta} [X_{\mu}, \Psi_{\beta}] \right)$$

$$\delta X^{\mu} = -\Psi_{\alpha} (\mathcal{C} \Gamma^{\mu})_{\alpha\beta} \varepsilon_{\beta}$$

$$\delta \Psi_{\alpha} = \frac{1}{2} \Gamma_{\alpha\beta}^{\mu\nu} [X_{\mu}, X_{\nu}] \varepsilon_{\beta} + v_{\alpha} \mathbf{1}$$

$$S_{\Omega} = S_{\text{IKKT}} + \text{tr} \left(\frac{\Omega^2}{4^3} X^A X_A + \frac{3\Omega^2}{4^3} X_a X^a + i\Omega X_8 [X_9, X_{10}] + i\frac{\Omega}{8} \Psi_{\alpha} (\mathcal{C} \mathfrak{N})_{\alpha\beta} \Psi_{\beta} \right)$$

$$\delta X^{\mu} = -\Psi_{\alpha} (\mathcal{C} \Gamma^{\mu})_{\alpha\beta} \varepsilon_{\beta}$$

$$\delta \Psi_{\alpha} = \frac{1}{2} \Gamma_{\alpha\beta}^{\mu\nu} [X_{\mu}, X_{\nu}] \varepsilon_{\beta} + i\frac{\Omega}{8} (\Gamma^{\mu} \mathfrak{N} + 2\mathfrak{N} \Gamma^{\mu}) X_{\mu} \varepsilon_{\beta}$$

$$\begin{array}{ll} \Gamma^m &= \gamma^m \otimes I_2 \otimes I_2 \quad m = 0,1,2,3,4,5 \\ \Gamma^i &= \gamma_{(1)} \otimes \gamma^i \otimes I_2 \quad i = 6,7 \\ \Gamma^a &= \gamma_{(1)} \otimes \gamma_{(2)} \otimes \gamma^a \quad a = 8,9 \end{array}$$

$$\begin{array}{ll} \gamma^0 = -i\sigma^2 \otimes I_2 \otimes I_2 & \\ \gamma^1 = \sigma^1 \otimes I_2 \otimes I_2 & \\ \gamma^2 = \sigma^3 \otimes \sigma^2 \otimes I_2 & \gamma^6 = \sigma^1 \\ \gamma^3 = \sigma^3 \otimes \sigma^1 \otimes I_2 & \gamma^7 = \sigma^2 \\ \gamma^4 = \sigma^3 \otimes \sigma^3 \otimes \sigma^1 & \gamma^8 = \sigma^1 \\ \gamma^5 = \sigma^3 \otimes \sigma^3 \otimes \sigma^2 & \gamma^9 = \sigma^2 \end{array}$$

$$\gamma_{(1)} = \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \quad \gamma_{(2)} = \sigma^3 \quad \gamma_{(3)} = \sigma^3$$

$$\begin{array}{ll} \Gamma^{012345} = -\gamma_{(1)} \otimes I_2 \otimes I_2 & \Gamma^{67} = iI_6 \otimes \gamma_{(2)} \otimes I_2 \\ & \Gamma^{89} = iI_6 \otimes I_2 \otimes \gamma_{(3)} \end{array}$$

$$\Gamma^{11} = \Gamma^{0123456789} = \gamma_{(1)} \otimes \gamma_{(2)} \otimes \gamma_{(3)} = \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \sigma^3$$

$$(\Gamma^A)^t = -\mathcal{C} \Gamma^A \mathcal{C}^{-1}$$



$$\mathcal{C} = i\sigma^2 \otimes \sigma^1 \otimes \sigma^2 \otimes \sigma^1 \otimes \sigma^2$$

$$\mathcal{C}^* = \mathcal{C} \mathcal{C}^t = -\mathcal{C} \mathcal{C}^2 = -I_{32} \{\mathcal{C}, \Gamma^{11}\} = 0$$

$$\begin{array}{lll} (\gamma^m)^* = +B_{(1)}\gamma^m B_{(1)}^{-1} & (B_{(1)})^* B_{(1)} = -I_6 & B_{(1)} = -i\gamma^2\gamma^5 = I_2 \otimes \sigma_1 \otimes \sigma_2 \\ (\gamma^i)^* = -B_{(2)}\gamma^i B_{(2)}^{-1} & (B_{(2)})^* B_{(2)} = -I_2 & B_{(2)} = \gamma^7 = \sigma^2 \\ (\gamma^a)^* = -B_{(3)}\gamma^a B_{(3)}^{-1} & (B_{(3)})^* B_{(3)} = -I_2 & B_{(3)} = \gamma^9 = \sigma^2 \end{array}$$

$$(\Gamma^M)^* = \mathcal{B}\Gamma^M\mathcal{B}^{-1}\mathcal{B}^*\mathcal{B} = I_{32}[\mathcal{B},\Gamma^{11}] = 0$$

$$\mathcal{B} = -iB_{(1)} \otimes (B_{(2)}\gamma_{(2)}) \otimes B_{(3)} = I_2 \otimes \sigma^1 \otimes \sigma^2 \otimes \sigma^1 \otimes \sigma^2$$

$$\begin{array}{ll} dP - 2iQ \wedge P = 0 & dG - iQ \wedge G + P \wedge \bar{G} = 0 \\ dQ + iP \wedge \bar{P} = 0 & dF_{(5)} - \frac{i}{8}G \wedge \bar{G} = 0 \end{array}$$

$$\begin{aligned} \delta\psi_M &= \nabla_M \varepsilon - \frac{i}{2} Q_M \varepsilon + \frac{i}{480} (\Gamma \cdot F_{(5)}) \Gamma_M \varepsilon - \frac{1}{96} (\Gamma_M (\Gamma \cdot G) + 2(\Gamma \cdot G) \Gamma_M) \mathcal{B}^{-1} \varepsilon^* \\ \delta\lambda &= i(\Gamma \cdot P) \mathcal{B}^{-1} \varepsilon^* - \frac{i}{24} (\Gamma \cdot G) \varepsilon \end{aligned}$$

$$\begin{array}{ll} P \rightarrow e^{2i\theta} P & \lambda \rightarrow e^{3i\theta/2} \lambda \\ Q \rightarrow Q + d\theta & \psi_M \rightarrow e^{i\theta/2} \psi_M \\ G \rightarrow e^{i\theta} G & \varepsilon \rightarrow e^{i\theta/2} \varepsilon \end{array}$$

$$P = \frac{1}{2}d\phi + \frac{i}{2}e^{\phi}d\chi \, Q = -\frac{1}{2}e^{\phi}d\chi \, G = e^{-\phi/2}H_{(3)} + ie^{\phi/2}\tilde{F}_{(3)}$$

$$\begin{array}{ll} \varepsilon = \varepsilon_+ + i\varepsilon_- & \mathcal{B}^{-1}\varepsilon^* = \varepsilon_+ - i\varepsilon_- \\ \lambda = \lambda_+ + i\lambda_- & \mathcal{B}^{-1}\lambda^* = \lambda_+ - i\lambda_- \\ \psi = \psi_+ + i\psi_- & \mathcal{B}^{-1}\psi^* = \psi_+ - i\psi_- \end{array}$$

$$\begin{aligned} \delta\psi_{\pm M} &= \nabla_M \varepsilon_{\pm} \mp \frac{e^{\phi}}{4} \partial_M \chi \varepsilon_{\mp} \mp \frac{e^{-\phi/2}}{96} \{ \Gamma_M (\Gamma \cdot H_{(3)}) + 2(\Gamma \cdot H_{(3)}) \Gamma_M \} \varepsilon_{\pm} \\ &\quad \mp \frac{1}{480} (\Gamma \cdot F_{(5)}) \Gamma_M \varepsilon_{\mp} - \frac{e^{\phi/2}}{96} \{ \Gamma_M (\Gamma \cdot \tilde{F}_{(3)}) + 2(\Gamma \cdot \tilde{F}_{(3)}) \Gamma_M \} \varepsilon_{\mp} \\ \delta\lambda_{\pm} &= \frac{1}{2} (\Gamma \cdot \partial\phi) \varepsilon_{\mp} \mp \frac{e^{\phi}}{2} (\Gamma \cdot \partial\chi) \varepsilon_{\pm} \pm \frac{e^{-\phi/2}}{24} (\Gamma \cdot H_{(3)}) \varepsilon_{\mp} + \frac{e^{\phi/2}}{24} (\Gamma \cdot \tilde{F}_{(3)}) \varepsilon_{\pm} \end{aligned}$$

$$\lambda = \begin{pmatrix} \lambda_+ \\ \lambda_- \end{pmatrix} \, \psi_M = \begin{pmatrix} \psi_{+M} \\ \psi_{-M} \end{pmatrix} \, \varepsilon = \begin{pmatrix} \varepsilon_+ \\ \varepsilon_- \end{pmatrix}$$

$$s^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \, s^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \, s^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{aligned} \delta\psi_M &= \nabla_M \varepsilon - \frac{e^{\phi}}{4} \partial_M \chi s^2 \varepsilon - \frac{e^{-\phi/2}}{96} \{ \Gamma_M (\Gamma \cdot H_{(3)}) + 2(\Gamma \cdot H_{(3)}) \Gamma_M \} s^3 \varepsilon \\ &\quad - \frac{1}{480} (\Gamma \cdot F_{(5)}) \Gamma_M s^2 \varepsilon - \frac{e^{\phi/2}}{96} \{ \Gamma_M (\Gamma \cdot \tilde{F}_{(3)}) + 2(\Gamma \cdot \tilde{F}_{(3)}) \Gamma_M \} s^1 \varepsilon \\ \delta\lambda &= \frac{1}{2} (\Gamma \cdot \partial\phi) s^1 \varepsilon - \frac{e^{\phi}}{2} (\Gamma \cdot \partial\chi) s^3 \varepsilon + \frac{e^{-\phi/2}}{24} (\Gamma \cdot H_{(3)}) s^2 \varepsilon + \frac{e^{\phi/2}}{24} (\Gamma \cdot \tilde{F}_{(3)}) \varepsilon \end{aligned}$$



$$g_{MN}^E = e^{-\phi/2} g_{MN}^S$$

$$\Gamma^M \rightarrow e^{\phi/4} \Gamma^M \Gamma_M \rightarrow e^{-\phi/4} \Gamma_M$$

$$\omega_{Mab}^E \Gamma^{ab} = \omega_{Mab}^S \Gamma^{ab} - \frac{1}{4} \Gamma_M (\Gamma \cdot \partial \phi) + \frac{1}{4} (\Gamma \cdot \partial \phi) \Gamma_M$$

$$\begin{aligned} \delta \psi_M &= \nabla_M \varepsilon - \frac{1}{8} (\Gamma_M (\Gamma \cdot \partial \phi) - (\Gamma \cdot \partial \phi) \Gamma_M) \varepsilon - \frac{e^\phi}{4} \partial_M \chi s^2 \varepsilon \\ &\quad - \frac{1}{96} \{ \Gamma_M (\Gamma \cdot H_{(3)}) + 2 (\Gamma \cdot H_{(3)}) \Gamma_M \} s^3 \varepsilon \\ &\quad - \frac{e^\phi}{480} (\Gamma \cdot F_{(5)}) \Gamma_M s^2 \varepsilon - \frac{e^\phi}{96} \{ \Gamma_M (\Gamma \cdot \tilde{F}_{(3)}) + 2 (\Gamma \cdot \tilde{F}_{(3)}) \Gamma_M \} s^1 \varepsilon \\ e^{-\phi/4} \delta \lambda &= \frac{1}{2} (\Gamma \cdot \partial \phi) s^1 \varepsilon + \frac{1}{24} (\Gamma \cdot H_{(3)}) s^2 \varepsilon - \frac{e^\phi}{2} (\Gamma \cdot \partial \tau_1) s^3 \varepsilon + \frac{e^\phi}{24} (\Gamma \cdot \tilde{F}_{(3)}) \varepsilon \end{aligned}$$

$$\delta \tilde{\psi}_M = \delta \psi_M + \frac{1}{4} e^{-\phi/4} s^1 \Gamma_M \delta \lambda$$

$$\begin{aligned} \Gamma \cdot \partial \phi &= \varphi_a \Gamma^a & \Gamma \cdot H_{(3)} &= 3! h_a \Gamma^{67a} & \Gamma^a &= \gamma_{(1)} \otimes \gamma_{(2)} \otimes \gamma^a \\ \Gamma \cdot \partial \tau_1 &= \chi_a \Gamma^a & \Gamma \cdot \tilde{F}_{(3)} &= 3! g_a \Gamma^{67a} & \Gamma^{67} &= i I_8 \otimes \gamma_{(2)} \otimes I_2 \end{aligned}$$

$$\begin{aligned} \Gamma^{67} \chi^{\eta_1, \eta_2} &= i \chi^{\eta_1, -\eta_2} \\ \Gamma^{67a} \chi^{\eta_1, \eta_2} &= i \gamma^a \chi^{\eta_1, -\eta_2} \end{aligned}$$

$$0 = (\varphi_a - e^\phi \chi_a s^2) \Gamma^a \varepsilon - \frac{1}{2} (h_a s^3 - e^\phi g_a s^1) \Gamma^{a67} \varepsilon$$

$$0 = \sum_{\eta_1, \eta_2} \chi^{\eta_1, \eta_2} \otimes \left[(\varphi_a - e^\phi \chi_a s^2) \gamma^a \tau^{(11)} \zeta - \frac{1}{2} (h_a s^3 + e^\phi g_a s^1) i \tau^{(10)} \gamma^a \zeta \right]$$

$$\begin{aligned} H_{(3)mNP} \Gamma^{NP} &= 0 \\ H_{(3)iNP} \Gamma^{NP} &= 2 h_a \Gamma_i \Gamma^{67} \Gamma^a \\ H_{(3)aNP} \Gamma^{NP} &= 2 h_a \Gamma^{67} \end{aligned}$$

$$\begin{aligned} (m) \quad 0 &= \hat{\nabla}_m \varepsilon + \left(\frac{D_a f_6}{2 f_6} - \frac{1}{8} e^\phi \chi_a s^2 - \frac{1}{8} e^\phi g_a \Gamma^{67} s^1 \right) \Gamma_m \Gamma^a \varepsilon \\ (i) \quad 0 &= \hat{\nabla}_i \varepsilon + \left(\frac{D_a f_2}{2 f_2} - \frac{1}{4} h_a \Gamma^{67} s^3 - \frac{1}{8} e^\phi \chi_a s^2 - \frac{1}{8} e^\phi g_a \Gamma^{67} s^1 \right) \Gamma_i \Gamma^a \varepsilon \\ (a) \quad 0 &= \nabla_a \varepsilon - \frac{1}{4} h_a \Gamma^{67} s^3 \varepsilon + \frac{1}{8} e^\phi \chi_b \Gamma^b \Gamma_a s^2 \varepsilon + \frac{1}{8} e^\phi g_b \Gamma^{67} \Gamma^b \Gamma_a s^1 \varepsilon \end{aligned}$$

$$\begin{aligned} 0 &= \Gamma_m \sum_{\eta_1, \eta_2} \chi^{\eta_1, \eta_2} \otimes \left[\frac{k_1}{2 f_6} \eta_1 \zeta_{\eta_1, \eta_2} + \left(\frac{D_a f_6}{2 f_6} - \frac{1}{8} e^\phi \chi_a s^2 \right) \gamma^a \zeta_{-\eta_1, -\eta_2} - \frac{i}{8} e^\phi g_a \gamma^a s^1 \zeta_{-\eta_1, \eta_2} \right] \\ 0 &= \Gamma_i \sum_{\eta_1, \eta_2} \chi^{\eta_1, \eta_2} \otimes \left[\frac{k_2}{2 f_2} \eta_2 \zeta_{-\eta_1, \eta_2} + \left(\frac{D_a f_2}{2 f_2} - \frac{1}{8} e^\phi \chi_a s^2 \right) \gamma^a \zeta_{-\eta_1, -\eta_2} \right. \\ &\quad \left. - \frac{i}{4} h_a \gamma^a s^3 \zeta_{-\eta_1, \eta_2} + \frac{i}{8} e^\phi g_a \gamma^a s^1 \zeta_{-\eta_1, \eta_2} \right] \end{aligned}$$



$$0 = \sum_{\eta_1, \eta_2} \chi^{\eta_1, \eta_2} \otimes \left[\left(D_a + \frac{i}{2} \hat{\omega}_a \sigma^3 \right) \zeta_{\eta_1, \eta_2} - \frac{i}{4} h_a s^3 \zeta_{\eta_1, -\eta_2} \right. \\ \left. + \frac{1}{8} e^\phi \chi_b \gamma^b \gamma_a s^2 \zeta_{\eta_1, \eta_2} + \frac{i}{8} e^\phi g_b \gamma^b \gamma_a s^1 \zeta_{\eta_1, -\eta_2} \right]$$

$$g_{mn} = e_m^a e_n^b \delta_{ab} \; a, b = 1, 2$$

$$g_{mn} = 2e_m^ze_n^{\bar{z}} + 2e_m^{\bar{z}}e_n^z$$

$$g_{mn}(\xi^1,\xi^2)d\xi^md\xi^n=4\rho(w,\tilde{w})^2dwd\tilde{w}$$

$$e_m{}^zd\xi^m=\rho dw$$

$$e_m{}^{\bar{z}}d\xi^m=\tilde{\rho}d\tilde{w}$$

$$D_zf=e_z{}^m\partial_mf=\rho^{-1}\partial_wf\quad D_zf=0\text{ is solved by }\quad f(\tilde{w})\\ D_{\bar{z}}f=e_{\bar{z}}{}^m\partial_mf=\rho^{-1}\partial_{\tilde{w}}f\quad D_{\bar{z}}f=0\text{ is solved by }\quad f(w)$$

$$Z_{\pm}^2=\frac{\tau_{\pm}-\tilde{\xi}}{\tau_{\pm}-\xi}\;X=\varepsilon_{\kappa}Z_+Z_-\;\varepsilon_{\kappa}=\frac{\tilde{\epsilon}_{\kappa}}{\epsilon_{\kappa}}$$

$$\tau_{\pm}-\xi=\frac{\xi-\tilde{\xi}}{Z_{\pm}^2-1}\;\tau_{\pm}-\tilde{\xi}=\frac{Z_{\pm}^2(\xi-\tilde{\xi})}{Z_{\pm}^2-1}$$

$$\frac{\partial_w\tau_{\pm}}{\tau_+-\tau_-}=\frac{Z_{\pm}^2(Z_{\mp}^2-1)}{Z_-^2-Z_+^2}\partial_w\ln{(\xi-\tilde{\xi})}-\frac{Z_{\mp}^2-1}{Z_{\pm}^2-1}\cdot\frac{\partial_wZ_{\pm}^2}{Z_-^2-Z_+^2}$$

$$\tau_{\pm}-\xi+\frac{2}{Z_+Z_-}(\tau_{\pm}-\tilde{\xi})=\frac{\xi-\tilde{\xi}}{Z_{\mp}^2(Z_{\pm}^2-1)}(Z_{\mp}^2+2Z_+Z_-)$$

$$\tau_{\pm}-\tilde{\xi}+2Z_+Z_-(\tau_{\pm}-\xi)=\frac{\xi-\tilde{\xi}}{Z_{\pm}^2-1}(Z_{\pm}^2+2Z_+Z_-)$$

$$\partial_w\ln{\hat{\rho}^2}\mp\frac{Z_{\pm}^2-2X}{Z_-^2-Z_+^2}(Z_{\mp}^2-1)\partial_w\ln{(\xi-\tilde{\xi})}\pm\frac{Z_{\mp}^2-2X}{Z_-^2-Z_+^2}\cdot\frac{\partial_wZ_{\mp}^2}{Z_{\mp}^2}=0$$

$$\partial_{\tilde{w}}\ln{\hat{\rho}^2}\mp\frac{Z_{\mp}^2-2X}{Z_{\mp}^2(Z_{\mp}^2-Z_+^2)}(Z_{\mp}^2-1)\partial_{\tilde{w}}\ln{(\xi-\tilde{\xi})}\pm\frac{Z_{\pm}^2-2X}{Z_-^2-Z_+^2}\cdot\frac{\partial_{\tilde{w}}Z_{\mp}^2}{Z_{\mp}^2}=0$$

$$0=2\partial_w\ln{\hat{\rho}^2}+\partial_w\ln{\frac{(Z_-+\varepsilon_{\kappa}Z_+)^3}{(Z_--\varepsilon_{\kappa}Z_+)}}-(1-2X)\partial_w\ln{(\xi-\tilde{\xi})}$$

$$0=2\partial_{\tilde{w}}\ln{\hat{\rho}^2}+\partial_{\tilde{w}}\ln{\frac{(Z_-+\varepsilon_{\kappa}Z_+)^3}{(Z_--\varepsilon_{\kappa}Z_+)}}-2\partial_{\tilde{w}}\ln{X}-\left(1-\frac{2}{X}\right)\partial_{\tilde{w}}\ln{(\xi-\tilde{\xi})}$$

$$\frac{1}{R^2}=-\frac{k_1}{c_6}\frac{(Z_-+\varepsilon_{\kappa}Z_+)^3\hat{\rho}^2}{X^{\frac{1}{2}}(Z_--\varepsilon_{\kappa}Z_+)^{\frac{1}{2}}(\xi-\tilde{\xi})^{\frac{1}{2}}}$$

$$\partial_w\ln{R^2}-\frac{1}{2}\partial_w\ln{X}-X\partial_w\ln{(\xi-\tilde{\xi})}=0$$

$$\partial_{\tilde{w}}\ln{R^2}+\frac{1}{2}\partial_w\ln{X}-\frac{1}{X}\partial_{\tilde{w}}\ln{(\xi-\tilde{\xi})}=0$$



$$\begin{aligned}
0 &= (2X^2 + 4X - (Z_+^2 + Z_-^2)(1 + 2X))\partial_w \ln(\xi - \tilde{\xi}) - \partial_w(Z_+^2 + Z_-^2 - 4X) \\
0 &= (2 + 4X - (Z_+^2 + Z_-^2)(1 + 2X^{-1}))\partial_{\tilde{w}} \ln(\xi - \tilde{\xi}) - \partial_{\tilde{w}}(Z_+^2 + Z_-^2) \\
&\quad + 2(Z_+^2 + Z_-^2 - 2X)\partial_{\tilde{w}} \ln X
\end{aligned}$$

$$\begin{aligned}
0 &= \frac{\tilde{\epsilon}_\kappa \kappa_1}{\epsilon_\kappa R^2} (Z_+^2 + Z_-^2 - 2X)X^{-\frac{1}{2}} - \partial_w(Z_+^2 + Z_-^2) + 2(Z_+^2 + Z_-^2)\partial_w \ln X \\
&\quad - (2X^2 - Z_+^2 - Z_-^2)\partial_w \ln(\xi - \tilde{\xi}) \\
0 &= \frac{\epsilon_\kappa \tilde{\kappa}_1}{R^2} (Z_+^2 + Z_-^2 - 2X)X^{\frac{1}{2}} - \partial_{\tilde{w}}(Z_+^2 + Z_-^2) + (Z_+^2 + Z_-^2 - 2)\partial_{\tilde{w}} \ln(\xi - \tilde{\xi})
\end{aligned}$$

$$0 = -\frac{\epsilon_\kappa \kappa_1}{R^2 X^{\frac{1}{2}}} + 2\partial_w \ln X + 2(1 + X)\partial_w \ln(\xi - \tilde{\xi})$$

$$0 = \frac{\epsilon_\kappa \tilde{\kappa}_1}{R^2} X^{\frac{1}{2}} - 2\partial_{\tilde{w}} \ln X + 2\left(1 + \frac{1}{X}\right)\partial_{\tilde{w}} \ln(\xi - \tilde{\xi})$$

$$0 = -\frac{\epsilon_\kappa \kappa_1}{R^2 X^{\frac{1}{2}}} + 2\partial_w \ln R^2 + \partial_w \ln X + 2\partial_w \ln(\xi - \tilde{\xi})$$

$$0 = \frac{\epsilon_\kappa \tilde{\kappa}_1}{R^2} X^{\frac{1}{2}} + 2\partial_{\tilde{w}} \ln R^2 - \partial_{\tilde{w}} \ln X + 2\partial_{\tilde{w}} \ln(\xi - \tilde{\xi})$$

$$0 = -\epsilon_\kappa(\kappa_2 - \kappa_1 \tilde{\xi}) + \partial_w \left(2R^2 X^{\frac{1}{2}}(\xi - \tilde{\xi})\right)$$

$$0 = \epsilon_\kappa(\tilde{\kappa}_2 - \tilde{\kappa}_1 \xi) + \partial_{\tilde{w}} \left(2R^2 X^{-\frac{1}{2}}(\tilde{\xi} - \xi)\right)$$

$$\partial_w \left(R^2 X^{-\frac{1}{2}}(\tilde{\xi} - \xi)\right) - \left(R^2 X^{\frac{1}{2}} + R^2 X^{-\frac{1}{2}}\right)\partial_w(\xi - \tilde{\xi}) = 0$$

$$\partial_{\tilde{w}} \left(R^2 X^{\frac{1}{2}}(\xi - \tilde{\xi})\right) - \left(R^2 X^{-\frac{1}{2}} + R^2 X^{\frac{1}{2}}\right)\partial_{\tilde{w}}(\tilde{\xi} - \xi) = 0$$

$$(\xi - \tilde{\xi})\frac{\partial_w \mathcal{A}_0}{\partial_w \xi} + \mathcal{A}_0 - \tilde{\mathcal{A}}_0 - \mathcal{A}_2 + \tilde{\mathcal{A}}_2 + \xi(\mathcal{A}_1 - \tilde{\mathcal{A}}_1) = 0$$

$$(\tilde{\xi} - \xi)\frac{\partial_{\tilde{w}} \tilde{\mathcal{A}}_0}{\partial_{\tilde{w}} \tilde{\xi}} + \mathcal{A}_0 - \tilde{\mathcal{A}}_0 - \mathcal{A}_2 + \tilde{\mathcal{A}}_2 + \xi(\mathcal{A}_1 - \tilde{\mathcal{A}}_1) = 0$$

$$\frac{\partial_w \mathcal{A}_0}{\partial_w \xi} + \mathcal{A}_1 = \frac{\partial_{\tilde{w}} \tilde{\mathcal{A}}_0}{\partial_{\tilde{w}} \tilde{\xi}} + \tilde{\mathcal{A}}_1$$

$$\partial_w \mathcal{A}_0 = -\mathcal{A}_1 \partial_w \xi$$

$$\partial_{\tilde{w}} \tilde{\mathcal{A}}_0 = -\tilde{\mathcal{A}}_1 \partial_{\tilde{w}} \tilde{\xi}$$

$$\tilde{\mathcal{A}}_0 - \tilde{\mathcal{A}}_2 + \xi \tilde{\mathcal{A}}_1 = \mathcal{A}_0 - \mathcal{A}_2 + \xi \mathcal{A}_1$$

$$\eta_{ab} = (-)^{s_a} \delta_{ab} \tilde{\eta}_{mn} = (-)^{\tilde{s}_m} \delta_{mn}$$

$$\begin{aligned}
\{\gamma_a, \gamma_b\} &= 2\eta_{ab} & \gamma_{ab} &= \frac{1}{2}[\gamma_a, \gamma_b] & \sigma_{ab} &= \frac{1}{2}\gamma_{ab} \\
\{\tilde{\gamma}_m, \tilde{\gamma}_n\} &= 2\tilde{\eta}_{mn} & \tilde{\gamma}_{mn} &= \frac{1}{2}[\tilde{\gamma}_m, \tilde{\gamma}_n] & \tilde{\sigma}_{mn} &= \frac{1}{2}\tilde{\gamma}_{mn}
\end{aligned}$$



$$\begin{aligned}\gamma_a^t &= -C\gamma_a C^{-1} & C^t &= \varepsilon_C C & (C\gamma_{ab})^t &= -\varepsilon_C C\gamma_{ab} \\ \tilde{\gamma}_m^t &= -\tilde{C}\tilde{\gamma}_m C^{-1} & \tilde{C}^t &= \varepsilon_{\tilde{C}} \tilde{C} & (\tilde{C}\tilde{\gamma}_{mn})^t &= -\varepsilon_{\tilde{C}} \tilde{C}\tilde{\gamma}_{mn}\end{aligned}$$

$$\begin{aligned}C &= \gamma_2 & \varepsilon_C &= -1 \\ \tilde{C} &= \tilde{\gamma}_2 \tilde{\gamma}_4 \tilde{\gamma}_6 & \varepsilon_{\tilde{C}} &= +1\end{aligned}$$

$$\begin{aligned}[\mathcal{A}_{ab}, \mathcal{A}_{cd}] &= \eta_{ad}\mathcal{A}_{bc} + \eta_{bc}\mathcal{A}_{ad} - \eta_{ac}\mathcal{A}_{bd} - \eta_{bd}\mathcal{A}_{ac} \\ [\tilde{\mathcal{A}}_{mn}, \tilde{\mathcal{A}}_{pq}] &= \tilde{\eta}_{mq}\tilde{\mathcal{A}}_{np} + \tilde{\eta}_{np}\tilde{\mathcal{A}}_{mq} - \tilde{\eta}_{mp}\tilde{\mathcal{A}}_{nq} - \tilde{\eta}_{nq}\tilde{\mathcal{A}}_{mp}\end{aligned}$$

$$\begin{aligned}[\mathcal{A}_{ab}, \mathcal{S}_{\alpha\mu}] &= \mathcal{S}_{\beta\mu}(\sigma_{ab})_{\beta\alpha} \\ [\tilde{\mathcal{A}}_{mn}, \mathcal{S}_{\alpha\mu}] &= \mathcal{S}_{\alpha\nu}(\tilde{\sigma}_{mn})_{\nu\mu}\end{aligned}$$

$$\{\mathcal{S}_{\alpha\mu}, \mathcal{S}_{\beta\nu}\} = s\tilde{C}_{\mu\nu}(C\gamma^{ab})_{\alpha\beta}\mathcal{A}_{ab} + tC_{\alpha\beta}(\tilde{C}\tilde{\gamma}^{mn})_{\mu\nu}\tilde{\mathcal{A}}_{mn}$$

$$[\{\mathcal{S}_{\alpha\mu}, \mathcal{S}_{\beta\nu}\}, \mathcal{S}_{\gamma\rho}] + \text{cycl}(\alpha\mu, \beta\nu, \gamma\rho) = 0$$

$$s\tilde{C}_{\mu\nu}(C\gamma^{ab})_{\alpha\beta}[\mathcal{A}_{ab}, \mathcal{S}_{\gamma\rho}] + tC_{\alpha\beta}(\tilde{C}\tilde{\gamma}^{mn})_{\mu\nu}[\tilde{\mathcal{A}}_{mn}, \mathcal{S}_{\gamma\rho}] + \text{cycl}(\alpha\mu, \beta\nu, \gamma\rho) = 0$$

$$s(C\gamma^{ab})_{\alpha\beta}(\gamma_{ab})_{\delta\gamma}\tilde{C}_{\mu\nu}\delta_{\sigma\rho} + tC_{\alpha\beta}\delta_{\gamma\delta}(\tilde{C}\tilde{\gamma}^{mn})_{\mu\nu}(\tilde{\gamma}_{mn})_{\sigma\rho} + \text{cycl}(\alpha\mu, \beta\nu, \gamma\rho) = 0$$

$$(C\gamma^{ab})_{\alpha\beta}(\gamma_{ab})_{\delta\gamma} = 2C_{\alpha\beta}\delta_{\gamma\delta} - 4C_{\alpha\gamma}\delta_{\beta\delta}$$

$$(\tilde{C}\tilde{\gamma}^{mn})_{\mu\nu}(\tilde{\gamma}_{mn})_{\rho\sigma} + (\tilde{C}\tilde{\gamma}^{mn})_{\mu\sigma}(\tilde{\gamma}_{mn})_{\rho\nu} = 6\delta_{\mu\rho}\tilde{C}_{\nu\sigma} - 3\delta_{\nu\rho}\tilde{C}_{\mu\sigma} - 3\delta_{\rho\sigma}\tilde{C}_{\mu\nu}$$

$$\{\mathcal{S}_{\alpha\mu}, \mathcal{S}_{\beta\nu}\} = -\frac{1}{2}\tilde{C}_{\mu\nu}(C\gamma^{ab})_{\alpha\beta}\mathcal{A}_{ab} + \frac{1}{3}C_{\alpha\beta}(\tilde{C}\tilde{\gamma}^{mn})_{\mu\nu}\tilde{\mathcal{A}}_{mn}$$

$$(\gamma_a^E)^* = -C\gamma_a^E C^{-1} \quad (\tilde{\gamma}_m^E)^* = -\tilde{C}\tilde{\gamma}_m^E \tilde{C}^{-1}$$

$$\begin{aligned}\eta_{ab} &= (-)^{s_a} \delta_{ab} & \gamma_a &= i^{s_a} \gamma_a^E & (\gamma_a)^* &= -(-)^{s_a} C\gamma_a C^{-1} \\ \tilde{\eta}_{mn} &= (-)^{\tilde{s}_m} \delta_{mn} & \tilde{\gamma}_m &= i^{\tilde{s}_m} \tilde{\gamma}_m^E & (\tilde{\gamma}_m)^* &= -(-)^{\tilde{s}_m} \tilde{C}\tilde{\gamma}_m \tilde{C}^{-1}\end{aligned}$$

$$\begin{aligned}\gamma_1^E &= \sigma_1 & \tilde{\gamma}_1^E &= \sigma_1 \otimes I_2 \otimes I_2 & \tilde{\gamma}_2^E &= \sigma_2 \otimes I_2 \otimes I_2 \\ \gamma_2^E &= \sigma_2 & \tilde{\gamma}_3^E &= \sigma_3 \otimes \sigma_1 \otimes I_2 & \tilde{\gamma}_4^E &= \sigma_3 \otimes \sigma_2 \otimes I_2 \\ \gamma_3^E &= \sigma_3 & \tilde{\gamma}_5^E &= \sigma_3 \otimes \sigma_3 \otimes \sigma_1 & \tilde{\gamma}_6^E &= \sigma_3 \otimes \sigma_3 \otimes \sigma_2 \\ C &= \sigma_2 & \tilde{\gamma}_7^E &= \sigma_3 \otimes \sigma_3 \otimes \sigma_3 & \tilde{C} &= \sigma_2 \otimes \sigma_1 \otimes \sigma_2\end{aligned}$$

$$\mathcal{A}_{ab}^* = \mathcal{A}_{ab} \quad \tilde{\mathcal{A}}_{mn}^* = \tilde{\mathcal{A}}_{mn} \quad \mathcal{S}_{\alpha\mu}^* = S_{\beta\nu} X_{\beta\alpha} Y_{\nu\mu}$$

$$XX^* \otimes YY^* = I_2 \otimes I_8$$

$$\gamma_{ab}X = X\gamma_{ab}^* \quad \tilde{\gamma}_{mn}Y = Y\tilde{\gamma}_{mn}^*$$

$$XX^* = x_0 I_2 \quad YY^* = y_0 I_8$$



$$\begin{aligned}\tilde{C}_{\mu\nu}^*(C\gamma^{ab})_{\alpha\beta}^* &= \tilde{C}_{\rho\sigma}(C\gamma^{ab})_{\gamma\delta}X_{\gamma\alpha}X_{\delta\beta}Y_{\rho\mu}Y_{\sigma\nu} \\ C_{\alpha\beta}^*(\tilde{C}\tilde{\gamma}^{mn})_{\mu\nu}^* &= C_{\gamma\delta}(\tilde{C}\tilde{\gamma}^{mn})_{\rho\sigma}X_{\gamma\alpha}X_{\delta\beta}Y_{\rho\mu}Y_{\sigma\nu}\end{aligned}$$

$$\begin{aligned}\tilde{C}^* &= \lambda Y^t \tilde{C} Y & (C\gamma^{ab})^* &= \lambda^{-1} X^t C \gamma^{ab} X \\ C^* &= \mu X^t C X & (\tilde{C}\tilde{\gamma}^{mn})^* &= \mu^{-1} Y^t \tilde{C} \tilde{\gamma}^{mn} Y\end{aligned}$$

$$\begin{aligned}\gamma_{ab}^E X &= (-)^{s_a+s_b+a+b} X \gamma_{ab}^E \\ \tilde{\gamma}_{mn}^E X &= (-)^{\tilde{s}_m+\tilde{s}_n+m+n} X \tilde{\gamma}_{mn}^E\end{aligned}$$

$$\begin{array}{lll} (+++) & \mathfrak{so}(3;\mathbb{R}) & X=x\sigma^2 \quad XX^*=-x^2I_2 \\ (++-) & \mathfrak{so}(2,1;\mathbb{R}) & X=x\sigma^3 \quad XX^*=+x^2I_2 \end{array}$$

$$\begin{array}{llll} (++++++) & \mathfrak{so}(7;\mathbb{R}) & Y=y\sigma^2\otimes\sigma^1\otimes\sigma^2 & YY^*=+y^2I_8 \\ (+++++-) & \mathfrak{so}(6,1;\mathbb{R}) & Y=y\sigma^3\otimes\sigma^1\otimes\sigma^2 & YY^*=-y^2I_8 \\ (+++++--) & \mathfrak{so}(5,2;\mathbb{R}) & Y=y\sigma^1\otimes\sigma^1\otimes\sigma^2 & YY^*=-y^2I_8 \\ (+++++---) & \mathfrak{so}(4,3;\mathbb{R}) & Y=y\sigma^2\otimes\sigma^1\otimes\sigma^2 & YY^*=+y^2I_8 \end{array}$$

$$\begin{array}{l} \mathfrak{so}(3;\mathbb{R})\oplus\mathfrak{so}(1,6;\mathbb{R}) \\ \mathfrak{so}(3;\mathbb{R})\oplus\mathfrak{so}(2,5;\mathbb{R}) \\ \mathfrak{so}(1,2;\mathbb{R})\oplus\mathfrak{so}(7;\mathbb{R}) \\ \mathfrak{so}(1,2;\mathbb{R})\oplus\mathfrak{so}(3,4;\mathbb{R}) \end{array}$$

$$\mathrm{d}_\xi = \mathrm{d} - \xi \lrcorner$$

$$\int_{\Gamma} \Phi = \sum_{\Sigma} \frac{1}{d_{2k}} \int_{\Sigma} \frac{f^* \Phi}{e_{\xi}(\mathcal{N})}$$

$$\int_{\Gamma} \Phi = \sum_{\Sigma} \frac{1}{d_{2k}} \frac{(2\pi)^k}{\prod_{i=1}^k \epsilon_i} \int_{\Sigma} \frac{f^* \Phi}{\left[1 + \frac{2\pi}{\epsilon_i} c_1(L_i)\right]}$$

$$\int_{\Gamma_2} \Phi_2 = \sum_{\Sigma_0} \frac{2\pi}{d_0} \frac{\Phi_0}{\epsilon_1} \Bigg|_{\Sigma_0}$$

$$\int_{\Gamma_4} \Phi_4 = \sum_{\Sigma_0} \frac{(2\pi)^2}{d_0} \frac{\Phi_0}{\epsilon_1 \epsilon_2} \Bigg|_{\Sigma_0} + \sum_{\Sigma_2} \frac{2\pi}{d_2} \int_{\Sigma_2} \left[\frac{\Phi_2}{\epsilon_1} - \frac{2\pi \Phi_0}{\epsilon_1^2} c_1(L) \right]$$

$$\begin{aligned}\int_{M_6} \Phi_6 &= \sum_{\Sigma_0} \frac{(2\pi)^3}{d_0} \frac{\Phi_0}{\epsilon_1 \epsilon_2 \epsilon_3} \Bigg|_{\Sigma_0} + \sum_{\Sigma_2} \frac{(2\pi)^2}{d_2} \frac{1}{\epsilon_1 \epsilon_2} \int_{\Sigma_2} \left[\Phi_2 - 2\pi \Phi_0 \left(\frac{c_1(L_1)}{\epsilon_1} + \frac{c_1(L_2)}{\epsilon_2} \right) \right] \\ &+ \sum_{\Sigma_4} \frac{2\pi}{d_4} \int_{\Sigma_4} \left[\frac{\Phi_4}{\epsilon_1} - \frac{2\pi \Phi_2}{\epsilon_1^2} \wedge c_1(L) + \frac{(2\pi)^2 \Phi_0}{\epsilon_1^3} c_1(L) \wedge c_1(L) \right]\end{aligned}$$

$$\int_{\Gamma} \Phi = \text{BV} - \text{AB} - \int_{\partial \Gamma} f^* \frac{\xi \wedge \Phi}{\mathrm{d}_{\xi} \xi}$$



$$\frac{1}{d_{\xi}\xi} = -\frac{1}{|\xi|^2(1-|\xi|^{-2}d\xi)} = -\frac{1}{|\xi|^2}\left[1+\sum_{a=1}^{\infty}\left(\frac{d\xi}{|\xi|^2}\right)^a\right].$$

$$ds^2 = d\alpha^2 + \sin^2 \alpha (d\theta^2 + \sin^2 \theta d\phi_1^2 + \cos^2 \theta d\phi_2^2)$$

$$\xi=b_1\partial_{\phi_1}+b_2\partial_{\phi_2}$$

$$|\xi|^2=\sin^2\alpha(b_1^2\sin^2\theta+b_2^2\cos^2\theta),$$

$$\mathrm{Vol}(X_{\alpha_0})=\frac{4\pi^2}{3}+\frac{2\pi^2\cos\alpha_0(\cos\alpha_0^2-3)}{3}$$

$$\Phi_4\!=\!\frac{1}{b_1b_2}\mathrm{vol}\!=\!\frac{1}{b_1b_2}\sin^3\alpha\sin\theta\cos\theta\,d\alpha\wedge d\theta\wedge d\phi_1\wedge d\phi_2$$

$$\Phi_2=-\frac{1}{2}\sin^3\alpha\Big(\frac{1}{b_1}\sin^2\theta\,d\phi_1+\frac{1}{b_2}\cos^2\theta\,d\phi_2\Big)\wedge d\alpha$$

$$\Phi_0=\frac{1}{6}(3\cos\alpha-\cos^3\alpha)$$

$$\begin{aligned}\int_{X_{\alpha_0}}\frac{1}{b_1b_2}\mathrm{vol}(X_{\alpha_0})&=\frac{(2\pi)^2}{b_1b_2}\Phi_0\Big|_{\alpha=0}+\int_{\partial X_{\alpha_0}}f^*\frac{1}{|\xi|^2}\Big[\xi\wedge\Phi_2+\frac{\Phi_0}{|\xi|^2}\xi\wedge d\xi\Big] \\ &=\frac{4\pi^2}{3b_1b_2}+\int_{\partial X_{\alpha_0}}\frac{b_1b_2\cos\alpha_0(3-\cos^2\alpha_0)\cos\theta\sin\theta}{3(b_1^2\sin^2\theta+b_2^2\cos^2\theta)^2}d\theta\wedge d\phi_1\wedge d\phi_2\end{aligned}$$

$$\int_{X_{\alpha_0}}\frac{1}{b_1b_2}\mathrm{vol}(X_{\alpha_0})=\frac{2\pi^2(2-3\cos\alpha_0+\cos^3\alpha_0)}{3b_1b_2}$$

$$ds_{10}^2 = e^{2A} \big[ds_{\mathrm{AdS}_4}^2 + ds_{M_6}^2 \big]$$

$$ds_{M_6}^2=\frac{1}{e^{4A}-y^2}\,dy^2+\frac{1}{4}(1-e^{-4A}y^2)D\psi^2+\frac{y}{F_0^2e^{4A}y^2+l^2}g_{ij}^{(4)}(y,x)dx^i\,dx^j$$

$$\partial_\psi j=0,\quad \partial_y j=\frac{1}{2}(F_0^2y^2+l^2y^{-2})d_4\rho,\quad d_4j=0$$

$$\partial_\psi \omega=0,\quad \partial_y \omega=-\frac{1}{2}(F_0^2y^2+l^2y^{-2})T\omega,\quad d_4\omega=iP\wedge\omega$$

$$P\,\equiv\,-\rho + \mathrm{i} \frac{2\mathrm{e}^{4A}(F_0^2y^4+l^2)}{(\mathrm{e}^{4A}-y^2)(F_0^2\mathrm{e}^{4A}y^2+l^2)}\mathrm{d}_4A$$

$$T\,\equiv\,\frac{\partial_y(\mathrm{e}^{4A}y^2)}{(\mathrm{e}^{4A}-y^2)(F_0^2\mathrm{e}^{4A}y^2+l^2)}$$

$$n_0=2\pi\ell_sF_0\in\mathbb{Z}$$



$$F_2 = l \left(\frac{1}{2y} d\rho - \frac{1}{2} d(e^{-4A} y D\psi) + \frac{1}{F_0^2 e^{4A} y^2 + l^2} j \right)$$

$$F_4 = -\frac{F_0}{2} \left[\frac{e^{4A} y^2}{(F_0^2 e^{4A} y^2 + l^2)^2} j \wedge j \right. \\ \left. - (d(e^{-4A} y D\psi) - y^{-1} d\rho) \wedge \left(\frac{e^{4A} y^2}{F_0^2 e^{4A} y^2 + l^2} j + \frac{y^2}{2} dy \wedge D\psi \right) \right]$$

$$F_6 = \frac{3l}{4} \frac{y^2}{(F_0^2 e^{4A} y^2 + l^2)^2} dy \wedge D\psi \wedge j \wedge j$$

$$B_1 = -\frac{F_0}{l} \left(\frac{e^{4A} y^2}{F_0^2 e^{4A} y^2 + l^2} j + \frac{y^2}{2} dy \wedge D\psi \right)$$

$$B_2 = \frac{l}{F_0} \left(\frac{1}{F_0^2 e^{4A} y^2 + l^2} j + \frac{1}{2y^2} dy \wedge D\psi \right)$$

$$e^{2\phi} = \frac{e^{6A}}{F_0^2 e^{4A} y^2 + l^2}$$

$$\frac{1}{(2\pi\ell_s)^{p-1}} \int_{\Sigma_p} f_p \in \mathbb{Z}$$

$$f_2 = \frac{l}{2} d[y^{-1}(1 - e^{-4A} y^2) D\psi] - F_0(B_1 - B_2)$$

$$f_4 = \frac{F_0}{2l^2} \left[\frac{e^{4A} y^2}{F_0^2 e^{4A} y^2 + l^2} j \wedge j + y^2 dy \wedge D\psi \wedge j \right]$$

$$f_6 = \frac{y^2}{4l^3} \frac{F_0^2 e^{4A} y^2 + 3l^2}{F_0^2 e^{4A} y^2 + l^2} dy \wedge D\psi \wedge j \wedge j$$

$$ds^2 \sim \frac{1}{\sqrt{r}} ds_9^2 + \sqrt{r} dr^2$$

$$e^{2\phi} \sim r^{-5/2}$$

$$e^{2A} \sim y^{-1/2}$$

$$ds^2 \sim \frac{1}{\sqrt{y}} \left(ds_{\text{AdS}_4}^2 + \frac{1}{4} D\psi^2 + \frac{1}{F_0^2} g_{ij}^{(4)}(0,x) dx^i dx^j \right) + \sqrt{y} dy^2,$$

$$e^{2\phi} \sim y^{-5/2}.$$

$$e^{2A} \sim \frac{a_1}{\sqrt{y}} + a_2 \sqrt{y}$$

$$\mathcal{F} = \frac{16\pi^3}{(2\pi\ell_s)^8} \int_{M_6} e^{8A-2\phi} \text{vol}_{M_6}$$

$$\Phi^{\mathcal{F}} = e^{8A-2\phi} \star_1 - \frac{1}{12} \left(\frac{y^3}{F_0^2 e^{4A} y^2 + l^2} j \wedge j + y dy \wedge D\psi \wedge j \right) \\ + \frac{1}{24} \left(y^2 j + \frac{1}{2} (F_0^2 y^4 + l^2) dy \wedge D\psi \right) - \frac{y}{48} \left(\frac{F_0^2 y^4}{5} + l^2 \right)$$



$$\begin{aligned}\Phi^{f_2} &= f_2 - \frac{y}{6l} (F_0^2 y^2 - 3l^2 e^{-4A}) \\ \Phi^{f_4} &= f_4 - \frac{F_0}{6l^2} \left(y^3 j + \frac{F_0^2 y^5 + l^2 y}{2} dy \wedge D\psi \right) + \frac{F_0 y^2}{72l^2} (F_0^2 y^4 + 3l^2) \\ \Phi^{f_6} &= f_6 - \frac{1}{12l^3} \left(y^3 \frac{F_0^2 e^{4A} y^2 + 3l^2}{F_0^2 e^{4A} y^2 + l^2} j \wedge j + (F_0^2 y^5 + 3l^2 y) dy \wedge D\psi \wedge j \right) \\ &\quad + \frac{1}{144l^3} (2y^2 (F_0^2 y^4 + 9l^2) j + (F_0^2 y^4 + 9l^2) (F_0^2 y^4 + l^2) dy \wedge D\psi) \\ &\quad - \frac{y}{1296l^3} (F_0^2 y^4 + 9l^2)^2\end{aligned}$$

$$Y = j + \frac{F_0^2 y^2 + l^2 y^{-2}}{2} dy \wedge D\psi$$

$$\Phi^Y = Y - \frac{1}{6y} (F_0^2 y^4 - 3l^2)$$

$$Z = j \wedge j + (F_0^2 y^2 + l^2 y^{-2}) dy \wedge D\psi \wedge j,$$

$$\Phi^Z = Z - \frac{1}{6} \left(F_0^2 y^3 - \frac{3l^2}{y} \right) (2j + (F_0^2 y^2 + l^2 y^{-2}) dy \wedge D\psi) + \frac{(F_0^2 y^4 - 3l^2)^2}{36y^2}$$

$$F_4 = -\frac{F_0}{2} \left[\frac{1}{F_0^4 e^{4A} y^2} j \wedge j - (d(e^{-4A} y D\psi) - y^{-1} d\rho) \wedge \left(\frac{1}{F_0^2} j + \frac{y^2}{2} dy \wedge D\psi \right) \right]$$

$$\Phi^{F_4} = F_4 - \frac{F_0}{2} \left[\frac{y}{F_0^2 e^{4A}} j + \frac{y}{2} dy \wedge D\psi \right] + \frac{F_0 y^2}{8}$$

$$\begin{aligned}\mathcal{F} &= \frac{1}{24\pi^2 \ell_s^8} \left\{ - \sum_{\Sigma_0} \frac{1}{d_0} \frac{F_0^2}{20} \frac{y^5}{\epsilon_1 \epsilon_2 \epsilon_3} \right\}_{\Sigma_0} + \sum_{\Sigma_2} \frac{1}{d_2} \frac{1}{\epsilon_1 \epsilon_2} \int_{\Sigma_2} \left[\frac{y^2}{4\pi} j + \frac{F_0^2 y^5}{20} \left(\frac{c_1(L_1)}{\epsilon_1} + \frac{c_1(L_2)}{\epsilon_2} \right) \right] \\ &\quad - \sum_{\Sigma_4} \frac{1}{d_4} \int_{\Sigma_4} \left[\frac{1}{F_0^2} \frac{1}{(2\pi)^2 \epsilon_1} \frac{1}{y} j \wedge j + \frac{y^2}{4\pi \epsilon_1^2} j \wedge c_1(L) + \frac{F_0^2 y^5}{20 \epsilon_1^3} c_1(L) \wedge c_1(L) \right] \end{aligned}$$

$$N^I = \frac{1}{(2\pi \ell_s)^3} \int_{\Gamma_4^I} F_4 = \frac{F_0}{4\pi \ell_s^3} \left[\sum_{\Sigma_0} \frac{1}{d_0} \frac{y^2}{4\epsilon_1 \epsilon_2} \right]_{\Sigma_0} - \sum_{\Sigma_2} \frac{1}{d_2} \int_{\Sigma_2} \left[\frac{1}{2\pi \epsilon_1} \frac{1}{F_0^2 y} j + \frac{y^2}{4\epsilon_1^2} c_1(L) \right]$$

$$N = \frac{1}{(2\pi \ell_s)^3} \int_{\Sigma_4} F_4 = -\frac{1}{(2\pi \ell_s)^3} \int_{\Sigma_4} \frac{1}{2F_0^3 y^4} j \wedge j$$

$$\Delta(\Sigma_2) = \frac{1}{(2\pi)^2 \ell_s^3} \int_{\Sigma_2} F_0 e^{2A} y \text{vol}_{\Sigma_2} = \frac{1}{(2\pi)^2 \ell_s^3} \int_{\Sigma_2} Y'$$

$$Y' = F_0 \left[\frac{y}{F_0^2 e^{4A}} j + \frac{y}{2} dy \wedge D\psi \right] = -2\Phi_2^{F_4}$$



$$\Delta(\Sigma_2)=-\frac{F_0}{2\pi\ell_s^3}\sum_{\Sigma_0}\frac{1}{d_0}\frac{y^2}{4\epsilon_1},\Delta(\Sigma_2)=\frac{1}{(2\pi)^2\ell_s^3}\int_{\Sigma_2}\frac{1}{F_0y}j$$

$$\mathcal{O}(-p_1)\oplus\mathcal{O}(-p_2)\rightarrow\Sigma_g$$

$$\xi=\sum_{i=1}^2b_i\partial_{\varphi_i}$$

$$\int_{\Sigma_g}c_1(\mathcal{L}_i)=-p_i$$

$$N=\frac{1}{(2\pi\ell_s)^3}\int_{HS^4}F_4=\frac{1}{2\pi\ell_s^3}\frac{1}{b_1b_2}\frac{F_0y_p^2}{8}$$

$$y_p=4\sqrt{\frac{\pi l_s^3}{F_0}b_1b_2N}$$

$$N_i=\frac{1}{(2\pi\ell_s)^3}\int_{c_4^i}F_4=\frac{1}{(2\pi\ell_s)^3}\left(\int_{\Sigma_g^p}-\frac{2\pi}{b_i}\frac{1}{2F_0y_p}j+\left(\frac{2\pi}{b_i}\right)^2\frac{F_0y_p^2}{8}p_i\right)$$

$$\int_{\Sigma_g^p}j=32\sqrt{\pi^5\ell_s^9F_0b_1b_2N^3(b_1p_2+b_2p_1)}$$

$$\begin{aligned}\mathcal{F} &= -\frac{16\pi^3}{(2\pi\ell_s)^8}\frac{(2\pi)^2}{b_1b_2}\left(\frac{y_p^2}{24}\int_{\Sigma_g^p}j-\frac{2\pi}{b_1b_2}\frac{F_0^2y_p^5}{240}\left(\frac{p_1}{b_1}+\frac{p_2}{b_2}\right)\right)\\ &= -\frac{16\sqrt{2}\pi}{5n_0^{1/2}}\sqrt{b_1b_2}(b_1p_2+b_2p_1)N^{5/2}\end{aligned}$$

$$b_1=\frac{1}{2}(1+\varepsilon), b_2=\frac{1}{2}(1-\varepsilon)$$

$$p_1=(1-g)(1+\kappa^{-1}z), p_2=(1-g)(1-\kappa^{-1}z)$$

$$\mathcal{F}=-\frac{8\sqrt{2}\pi}{5\kappa n_0^{1/2}}\sqrt{(1-\varepsilon^2)}(g-1)(\varepsilon z-\kappa)N^{5/2}$$

$$\varepsilon_{\pm}=\frac{\kappa\pm\sqrt{\kappa^2+8z^2}}{4z}$$

$$\mathcal{F}=\frac{2\pi(1-g)N^{5/2}}{5\kappa n_0^{1/2}}\Big(\sqrt{\kappa^2+8z^2}-3\kappa\Big)\sqrt{1-\frac{\kappa(\sqrt{\kappa^2+8z^2}+\kappa)}{4z^2}}.$$



$$\begin{aligned}\Delta(HS^{2,1}) &= -\frac{F_0}{2\pi\ell_s^3} \frac{y_p^2}{4b_1} = 2b_2N = (1-\varepsilon)N \\ \Delta(HS^{2,2}) &= -\frac{F_0}{2\pi\ell_s^3} \frac{y_p^2}{4b_2} = 2b_1N = (1+\varepsilon)N \\ \Delta(\Sigma_g^p) &= -\frac{1}{(2\pi)^2\ell_s^3} \int_{\Sigma_g^p} \frac{1}{F_0 y} j = -2(b_1p_2 + b_2p_1)N = \frac{2(1-g)}{\kappa} (z\varepsilon - \kappa)N\end{aligned}$$

$$\begin{aligned}\Delta(HS^{2,i}) &= \left(1 \mp \frac{\sqrt{\kappa^2 + 8z^2} + \kappa}{4z}\right)N \\ \Delta(\Sigma_g^p) &= \frac{(1-g)}{2\kappa} \left(\sqrt{\kappa^2 + 8z^2} - 3\kappa\right)N\end{aligned}$$

$$\begin{aligned}\Delta(HS^{2,i}) &= \left(1 \mp \frac{\kappa - \sqrt{\kappa^2 + 8z^2}}{4z}\right)N \\ \Delta(\Sigma_g^p) &= \left(\frac{(g-1)(3\kappa + \sqrt{\kappa^2 + 8z^2})}{2\kappa}\right)N\end{aligned}$$

$$\xi = \sum_{i=1}^2 b_i \partial_{\varphi_i} + \varepsilon \partial_{\varphi_0}$$

$$N_{\pm} = \frac{1}{(2\pi\ell_s)^3} \int_{HS^4/\mathbb{Z}_{n_{\pm}}} F_4 = \frac{1}{2\pi\ell_s^3} \frac{1}{n_{\pm}} \frac{1}{b_1^{\pm} b_2^{\pm}} \frac{F_0 y_{\pm}^2}{8}$$

$$n_+N_+ = n_-N_- \equiv N$$

$$y_+ = 4\sqrt{\frac{\pi l_s^3}{F_0} b_1^+ b_2^+} N, y_- = 4\sigma\sqrt{\frac{\pi l_s^3}{F_0} b_1^- b_2^-} N$$

$$\begin{aligned}\mathcal{F} &= -\frac{1}{24\pi^2\ell_s^8} \frac{F_0^2}{20} \left(\frac{1}{n_+} \frac{y_+^5}{b_1^+ b_2^+ (-\varepsilon/n_+)} + \frac{1}{n_-} \frac{y_-^5}{b_1^- b_2^- (\varepsilon/n_-)} \right) \\ &= \frac{32\sqrt{2}\pi}{15n_0^{1/2}} \frac{(b_1^+ b_2^+)^{3/2} - \sigma (b_1^- b_2^-)^{3/2}}{\varepsilon} N^{5/2}.\end{aligned}$$

$$b_i^{\pm} = b_i + \frac{\varepsilon}{2} \left(\pm \frac{p_i}{n_+ n_-} + \frac{n_+ - \sigma n_-}{2 n_+ n_-} \right), b_1 + b_2 = 1$$

$$b_1^{\pm} + b_2^{\pm} = 1 \pm \frac{\varepsilon}{n_{\pm}}, b_i^+ - b_i^- = \frac{p_i}{n_+ n_-} \varepsilon$$

$$N_i = \frac{1}{(2\pi\ell_s)^3} \int_{C_i^4} F_4 = \frac{1}{2\pi\ell_s^3} \frac{F_0}{8} \left(\frac{1}{n_+} \frac{y_+^2}{b_i^+ (-\varepsilon/n_+)} + \frac{1}{n_-} \frac{y_-^2}{b_i^- (\varepsilon/n_-)} \right)$$

$$N_1 = \frac{b_2^- - b_2^+}{\varepsilon} N = \frac{p_2}{n_+ n_-} N, N_2 = \frac{b_1^- - b_1^+}{\varepsilon} N = \frac{p_1}{n_+ n_-} N$$



$$\Delta(\Sigma^p) = -\frac{F_0}{2\pi\ell_s^3} \left(\frac{1}{n_+} \frac{y_+^2}{4(-\varepsilon/n_+)} + \frac{1}{n_-} \frac{y_-^2}{4(\varepsilon/n_-)} \right) = -2 \frac{b_1^- b_2^- - b_1^+ b_2^+}{\varepsilon} N.$$

$$\Delta(HS_{\pm}^{2,i}) = \frac{F_0}{2\pi\ell_s^3} \frac{y_{\pm}^2}{4b_i^{\pm}} = 2 \frac{b_1^{\pm} b_2^{\pm}}{b_i^{\pm}} N.$$

$$HS^2 \hookrightarrow M_6 \rightarrow B_4$$

$$c_\alpha = \int_{\Gamma_\alpha} j, n_\alpha = \int_{\Gamma_\alpha} c_1(\mathcal{L})$$

$$N_\alpha = \frac{1}{(2\pi\ell_s)^3} \int_{C_\alpha^4} F_4 = \frac{F_0}{4\pi\ell_s^3} \left[\frac{1}{2\pi} \frac{1}{F_0^2 y_p} c_\alpha - \frac{y_p^2}{4} n_\alpha \right]$$

$$c_\alpha = \frac{\pi F_0 y_p}{2} (F_0 y_p^2 n_\alpha - 16\pi\ell_s^3 N_\alpha)$$

$$\begin{aligned} N_p &= \frac{1}{(2\pi\ell_s)^3} \int_{B_p^4} F_4 = -\frac{1}{(2\pi\ell_s)^3} \frac{1}{2F_0^3 y_p^4} \langle c, c \rangle \\ &= \frac{1}{2} \langle N, n \rangle - \frac{F_0 y_p^2}{64\pi\ell_s^3} \langle n, n \rangle - \frac{4\pi\ell_s^3}{F_0 y_p^2} \langle N, N \rangle \end{aligned}$$

$$N_p = -\langle N, n \rangle + M$$

$$y_p = 4 \sqrt{\frac{\pi\ell_s^3}{F_0} \frac{3\langle N, n \rangle - 2M \pm \sqrt{(3\langle N, n \rangle - 2M)^2 - \langle N, N \rangle \langle n, n \rangle}}{\langle n, n \rangle}}$$

$$\begin{aligned} \mathcal{F} &= -\frac{1}{24\pi^2\ell_s^8} \left[\frac{1}{(2\pi F_0)^2 y_p} \langle c, c \rangle - \frac{y_p^2}{4\pi} \langle c, n \rangle + \frac{F_0^2 y_p^5}{20} \langle n, n \rangle \right] \\ &= \frac{2y_p}{3\ell_s^2} \langle N, N \rangle - \frac{F_0^2 y_p^5}{1920\pi^2\ell_s^8} \langle n, n \rangle \\ &= \frac{16\sqrt{2}\pi}{15n_0^{1/2}} \frac{1}{\langle n, n \rangle^{3/2}} \sqrt{3\langle N, n \rangle - 2M \pm \sqrt{(3\langle N, n \rangle - 2M)^2 - \langle N, N \rangle \langle n, n \rangle}} \\ &\times \left(3\langle N, N \rangle \langle n, n \rangle + (3\langle N, n \rangle - 2M) \left(3\langle N, n \rangle \pm \sqrt{(3\langle N, n \rangle - 2M)^2 - \langle N, N \rangle \langle n, n \rangle} \right) \right) \end{aligned}$$

$$\Delta(HS^2) = \frac{F_0}{2\pi\ell_s^3} \frac{y_p^2}{4} = \frac{2}{\langle n, n \rangle} \left(3\langle N, n \rangle - 2M \pm \sqrt{(3\langle N, n \rangle - 2M)^2 - \langle N, N \rangle \langle n, n \rangle} \right)$$

$$\begin{aligned} \Delta(\Gamma_\alpha) &= \frac{1}{(2\pi)^2\ell_s^3 F_0 y_p} c_\alpha \\ &= \frac{2}{\langle n, n \rangle} \left((3\langle N, n \rangle - 2M) n_\alpha + \left(\langle n, n \rangle \pm \sqrt{(3\langle N, n \rangle - 2M)^2 - \langle N, N \rangle \langle n, n \rangle} \right) N_\alpha \right) \end{aligned}$$

$$\langle n, n \rangle = \bar{M}, \langle N, n \rangle = k\bar{M}, \langle N, N \rangle = k^2\bar{M}$$

$$k = \frac{N}{h}.$$



$$\mathcal{F}=\frac{32\sqrt{2}\pi}{5n_0^{1/2}}(3\pm2\sqrt{2})\bar{M}\left(\frac{N}{h}\right)^{5/2}$$

$$\Delta(HS^2)=\frac{2(3\pm2\sqrt{2})}{h}N,\Delta(\Gamma_\alpha)=\frac{4(1\pm\sqrt{2})}{h}Nn_\alpha,$$

$$\langle n,n\rangle=2\chi_1\chi_2,\langle N,n\rangle=\chi_1N_2+\chi_2N_1,\langle N,N\rangle=2N_1N_2.$$

$$\mathcal{F}=\frac{32\pi}{5n_0^{1/2}}\sqrt{3\pm\sqrt{8+z^2}}\Big(\sqrt{8+z^2}\pm(2+z^2)\Big)N^{5/2}$$

$$\Delta(HS^2)=\left(3\pm\sqrt{8+z^2}\right)N\\ \Delta(S_1^2)=2\left(2-z\pm\sqrt{8+z^2}\right)N,\Delta(S_2^2)=2\left(2+z\pm\sqrt{8+z^2}\right)N,$$

$$L^N=\mathcal{L}^{-1}, L^S=\mathcal{L}$$

$$[\Gamma_\alpha^S]-[\Gamma_\alpha^N]=n_\alpha[S_{\rm fibre}^2]\in H^2(M_6,\mathbb{Z})$$

$$n_\alpha\equiv\int_{\Gamma_\alpha}c_1(\mathcal{L})\in\mathbb{Z}$$

$$[B_4^S]-[B_4^N]=\sum_{\alpha=1}^{b_2}n_\alpha[C_\alpha]$$

$$c_\alpha^{N,S}\equiv\int_{\Gamma_\alpha^{N,S}}j$$

$$c_\alpha^S-c_\alpha^N=\frac{l^2\pi}{\epsilon}\Big(\frac{1}{y^N}-\frac{1}{y^S}\Big)n_\alpha$$

$$k_2\!=\!\frac{1}{2\pi\ell_s}\!\int_{S^2_{\rm fibre}}\!f_2\!=\!\frac{l}{2\epsilon\ell_s}\!\Big[\frac{1}{y_N}\!-\!\frac{1}{y_S}\Big] \\ k^{N,S}_\alpha\!=\!\frac{1}{2\pi\ell_s}\!\int_{\Gamma^{N,S}_\alpha}\!f_2\!=\!\frac{c^{N,S}_\alpha}{2l\ell_s\pi}$$

$$c_\alpha^S-c_\alpha^N=2l\ell_s\pi k_2n_\alpha\Leftrightarrow k_\alpha^S-k_\alpha^N=k_2n_\alpha$$

$$y_N=\frac{ly_S}{l+2\ell_sk_2y_S\epsilon}$$

$$N\!=\!-\frac{1}{(2\pi\ell_s)^5}\!\int_{M_6}\!f_6\\ \!=\!\frac{l^4k_2^2\pi^2(y_S-y_N)+l^2k_2\epsilon\pi(c_Ny_N^2-c_Sy_S^2)+\epsilon^2(c_S^2y_S^3-c_N^2y_N^3)}{64l^3\epsilon^3\pi^4\ell_s^5}$$

$$l=-Ly_sk_2\ell_s$$



$$N = \frac{y_S^2(\langle k^N, k^N \rangle L^2 + \langle k^S, k^S \rangle (L-2)^2 + \langle k^N, k^S \rangle L(L-2))}{8L(L-2)^3 \ell_s^4 k_2 \pi^2}$$

$$\mathcal{F} = \frac{4\pi}{3} \sqrt{\frac{2k_2^3 L^3 (L-2)^3}{\langle k^N, k^N \rangle L^2 + \langle k^S, k^S \rangle (L-2)^2 + \langle k^N, k^S \rangle L(L-2)}} N^{3/2}$$

$$\langle k^N, k^N \rangle L^2 (2L-1) + \langle k^S, k^S \rangle (L-L)^2 (2L-3) + 2\langle k^N, k^S \rangle L(2-3L+L^2) = 0$$

$$\mathcal{F} = \sqrt{\frac{2\pi^6}{27\mathrm{Vol}(Y^{p,k}(\mathrm{KE}_4))}} N^{3/2}$$

$$\mathrm{Vol}\left(Y^{p,k}(\mathrm{KE}_4)\right)=\mathrm{Vol}(\mathrm{KE}_4)\frac{2\pi^2}{3\cdot 4^4}\frac{x_2-x_1}{p(x_2-1)(1-x_1)}(x_2^3-x_1^3),$$

$$\begin{aligned} 0 &= 3p^3x_1^3 + 2p^2(6b-5p)x_1^2 + p(18b^2-28bp+11p^2)x_1 + 4(3b^3+4bp^2-6b^2p-p^2) \\ 0 &= 3p^3x_2^3 + 2p^2(p-6b)x_2^2 + p(18b^2-8bp+p^2)x_2 + 4b(3bp-3b^2-p^2) \end{aligned}$$

$$k_2\equiv p, k^S_\alpha=\frac{k}{h}n_\alpha\equiv bn_\alpha, k^N_\alpha=(b-p)n_\alpha, \mathrm{Vol}(\mathrm{KE}_4)=\frac{\pi^2}{2}\langle n,n\rangle,$$

$$\xi=\sum_{i=1}^3\epsilon_i\partial_{\varphi_i}$$

$$0=\int_{S^4_i}\Phi^Z=\frac{(2\pi)^2\epsilon_i}{\epsilon_1\epsilon_2\epsilon_3}\Big[\frac{(F_0^2y_N^4-3l^2)^2}{36y_N^2}+\frac{(F_0^2y_S^4-3l^2)^2}{36y_S^2}\Big]$$

$$|y_N|=|y_S|=\left(\frac{3l^2}{F_0^2}\right)^{1/4}$$

$$0=\int_{S^2_i}\Phi^Y=-\frac{\pi}{3\epsilon_i}\Big[\frac{1}{y_N}(F_0^2y_N^4-3l^2)-\frac{1}{y_S}(F_0^2y_N^S-3l^2)\Big]$$

$$N=\frac{1}{(2\pi\ell_s)^5}\int_{M_6}f_6=\frac{(2\pi)^3}{1296l^3(2\pi\ell_s)^5\epsilon_1\epsilon_2\epsilon_3}[y_N(F_0^2y_N^4+9l^2)^2-y_S(F_0^2y_S^4+9l^2)^2]$$

$$\ell_s^{-1}=\frac{2^{1/9}3^{7/18}n_0^{1/9}N^{2/9}\pi^{1/3}(\epsilon_1\epsilon_2\epsilon_3)^{2/9}}{l^{1/3}}, F_0=\frac{n_0}{2\pi\ell_s}$$

$$\begin{aligned}\mathcal{F}&=\frac{(2\pi)^3}{240\epsilon_1\epsilon_2\epsilon_3}\frac{16\pi^3}{(2\pi\ell_s)^8}[y_N(F_0^2y_N^4+5l^2)-y_S(F_0^2y_S+5l^2)]\\&=\frac{9}{5}2^{1/3}3^{1/6}n_0^{1/3}N^{5/3}\pi(\epsilon_1\epsilon_2\epsilon_3)^{2/3}\end{aligned}$$

$$\epsilon_i=\frac{1}{3}$$



$$\mathcal{F} = \frac{2^{1/3}3^{1/6}n_0^{1/3}\pi^3}{5\mathrm{Vol}(S^5)^{2/3}}N^{5/3}$$

$$b_i=3\epsilon_i$$

$$\mathrm{Vol}(S^5)[b_i]=\frac{\pi^3}{b_1b_2b_3},$$

$$\mathcal{F}=\frac{2^{1/3}3^{1/6}n_0^{1/3}\pi^3}{5(\mathrm{Vol}(S^5)[b_i])^{2/3}}N^{5/3}$$

$$|y_N|=|y_S|=\left(\frac{3l^2}{F_0^2}\right)^{1/4}$$

$$\begin{aligned} N &= \frac{1}{(2\pi\ell_s)^2} \int_{\mathcal{S}(\mathrm{SE}_5)} f_6 \\ &= \frac{1}{1296(2\pi\ell_s)^5} \left[\frac{y_N(F_0^2y_N^4+9l^2)^2}{e(\mathcal{N}_N)} + \frac{y_S(F_0^2y_S^4+9l^2)^2}{e(\mathcal{N}_S)} \right] \\ &= \frac{1}{9(2\pi\ell_s)^5} \left[\frac{y_N}{e(\mathcal{N}_N)} + \frac{y_S}{e(\mathcal{N}_S)} \right] \end{aligned}$$

$$\ell_s^{-1}=\frac{2\cdot 3^{7/18}\pi n_0^{1/9}N^{2/9}}{\mathcal{E}}, F_0=\frac{n_0}{2\pi\ell_s}$$

$$\mathcal{E}=\frac{1}{e(\mathcal{N}_N)}-\frac{1}{e(\mathcal{N}_S)}$$

$$\begin{aligned}\mathcal{F}&=\frac{\pi^3}{15(2\pi\ell_s)^8}\left[\frac{y_N(F_0^2y_N^4+5l^2)}{e(\mathcal{N}_N)}+\frac{y_S(F_0^2y_S^4+5l^2)}{e(\mathcal{N}_S)}\right] \\ &=\frac{8\pi^3y_N}{15(2\pi\ell_s)^8}\left[\frac{1}{e(\mathcal{N}_N)}-\frac{1}{e(\mathcal{N}_S)}\right]\end{aligned}$$

$$\mathcal{F}=\frac{72\times 3^{1/6}n_0^{1/3}\pi^3}{5\mathcal{E}^{2/3}}N^{5/3}$$

$$\mathcal{E}=2^43^3\mathrm{Vol}(\mathrm{SE}_5)$$

$$\mathcal{F}=\frac{2^{1/3}3^{1/6}\pi^3}{5\mathrm{Vol}(\mathrm{SE}_5)^{2/3}}n_0^{1/3}N^{5/3}$$

$$\Phi^{\mathrm{Thom}}=\frac{1}{2\pi}\,\mathrm{d}(\rho(r)\eta)$$

$$s^*\Phi^{\mathrm{Thom}}=-e$$

$$\Psi_p\equiv \mathrm{PD}[B_4]=\Phi^{\mathrm{Thom}}$$



$$\tau = \sum_{\alpha=1}^{b_2} \hat{C}_\alpha \Psi_\alpha$$

$$C_\alpha \equiv \int_{\Gamma_\alpha} \tau$$

$$C_\alpha = \int_{B_4} \tau \wedge \Psi_\alpha = \sum_{\beta=1}^{b_2} Q_{\alpha\beta} \hat{C}_\beta$$

$$Q_{\alpha\beta} \equiv \int_{B_4} \Psi_\alpha \wedge \Psi_\beta = [\Gamma_\alpha] \cap [\Gamma_\beta]$$

$$\int_{B_4} \tau \wedge \tau = \sum_{\alpha,\beta=1}^{b_2} Q_{\alpha\beta} \hat{C}_\alpha \hat{C}_\beta = \sum_{\alpha,\beta=1}^{b_2} I_{\alpha\beta} C_\alpha C_\beta$$

$$c_1(\mathcal{L}) = \sum_{\alpha=1}^{b_2} \hat{n}_\alpha \Psi_\alpha$$

$$\tau = \sum_{\alpha=1}^{b_2} \hat{C}_\alpha \Psi_\alpha, \omega = \sum_{\alpha=1}^{b_2} \hat{N}_\alpha \Psi_\alpha$$

$$\int_{B_4} \tau \wedge \omega = \sum_{\alpha,\beta=1}^{b_2} Q_{\alpha\beta} \hat{C}_\alpha \hat{N}_\beta = \sum_{\alpha,\beta=1}^{b_2} I_{\alpha\beta} C_\alpha N_\beta \equiv \langle C, N \rangle$$

$$F_4 = \Phi^{\text{Thom}} \wedge \sum_{\alpha=1}^{b_2} \hat{N}_\alpha \Psi_\alpha + M \widetilde{\text{vol}}(B_4)$$

$$\begin{aligned} (2\pi\ell_s)^3 N &= \int_{B_4} F_4 \\ &= M + \hat{N}_\alpha \int_{B_4} s^*(\Phi^{\text{Thom}}) \wedge \Psi_\alpha \\ &= M - \hat{N}_\alpha \int_{B_4} e \wedge \Psi_\alpha \\ &= M - \langle N, n \rangle \end{aligned}$$

$$\text{PD}[\Gamma_\alpha] = \pi^* \Psi_\alpha \wedge \Phi^{\text{Thom}} \in H^4(M_6)$$

$$\int_{B_4} \pi^* \Psi_\alpha \wedge \Phi^{\text{Thom}} = n_\alpha$$

$$[\Gamma_\alpha] = n_\alpha [HS^2_{\text{fibre}}]$$



$$t_8 t_8 \hat{R}^4.$$

$$\hat{e}^a = e^{-\Phi/3} e^a, \hat{e}^{10} = e^{2\Phi/3} (dx^{10} + A), \hat{F} \equiv d\hat{C} = F_4 + H \wedge (dx^{10} + A)$$

$$\hat{e}^a = e^{2\Phi/3} \hat{e}'^a$$

$$\hat{e}'^a = e'^a, \hat{e}'^{10} = dx^{10} + A$$

$$e'^a = e^{-\Phi} e^a$$

$$\hat{F}_{abc10} = H_{abc}, \hat{R}'_{abc10} = -\frac{1}{2} \nabla_c F_{ab}$$

$$e^a = e^{k\Phi} e'^a$$

$$R_{abcd} = e^{-2k\Phi} (R'_{abcd} + D_{abcd} + \dots) \text{ with } D^{ab}{}_{cd} = 4k\delta_{[c}^{[a} \nabla_{d]} \nabla^{b]} \Phi$$

$$t_8 t_8 R^4 = t_8 t_8 R'^4 + 96k(2l - k(D - 2)) \nabla^a \nabla^b \Phi \nabla^c \nabla^d \Phi X_{abcd} + \dots$$

$$X_{ab}^{cd} = 8R'_{aghb} R'^{cghd} + 8R'_{agh}{}^c R_b'^{hgd} - 8\delta_a^d R'_{begh} R'^{cegh} + \delta_a^c \delta_b^d R'_{efgh} R'^{efgh} \\ + 2k(2l - k(D - 2)) [3\nabla_a \nabla_b \Phi \nabla^c \nabla^d \Phi - 2\nabla_a \nabla^c \Phi \nabla_b \nabla^d \Phi]$$

$$R_{ab} = l \nabla_a \nabla_b \Phi.$$

$$t_8 t_8 \hat{R}^4 = t_8 t_8 \hat{R}'^4 - 2^7 3 \nabla^a \nabla^b \Phi \nabla^c \nabla^d \Phi \hat{X}_{abcd} + \dots,$$

$$\hat{X}_{abcd} = 8\hat{R}'_{aghb} \hat{R}_c'^{gh}{}_d + 8\hat{R}'_{aghc} \hat{R}_b'^{hg}{}_d - 8\eta_{ad} \hat{R}'_{begh} \hat{R}_c'^{egh} + \eta_{ac} \eta_{bd} \hat{R}'_{efgh} \hat{R}^{efgh} \\ - 24 \nabla_a \nabla_b \Phi \nabla_c \nabla_d \Phi + 16 \nabla_a \nabla_c \Phi \nabla_b \nabla_d \Phi$$

$$t_8 t_8 R^4 = 12 \left(16 R_{efab} R^{fgbc} R_{ghcd} R^{heda} + 32 R_{efab} R^{hebc} R^{fg}{}_{cd} R_{gh}{}^{da} - 16 R_{efab} R^{fgab} R_{ghcd} R^{hecd} \right. \\ \left. - 8 R_{efab} R_{gh}{}^{ab} R^{hecd} R^{fg}{}_{cd} + R_{abcd} R^{abcd} R_{efgh} R^{efgh} + 2 R_{efab} R^{ghab} R^{efcd} R_{ghcd} \right)$$

$$t_8 t_8 R'^4 + L_{R'^2 F_2^2} + L_{F_2^4} - 2^7 3 \nabla^a \nabla^b \Phi \nabla^c \nabla^d \Phi (X_{abcd} + \tilde{X}_{abcd})$$

$$L_{R'^2 F_2^2} = 12 \left(-32 R'_{efab} R'^{heda} \nabla^f F^{bc} \nabla_h F_{cd} - 32 R'_{fgcd} R'^{ghda} \nabla^f F_{ab} \nabla_h F^{bc} \right. \\ \left. - 32 R'_{hebc} R'^{gh}{}_{da} \nabla^e F^{ab} \nabla_g F^{cd} - 16 R'_{ghcd} R'^{hecd} \nabla_a F_{ef} \nabla^a F^{fg} \right. \\ \left. + 8 R'_{ghcd} R'^{hecd} \nabla_e F_{ab} \nabla^g F^{ab} + 8 R'_{efab} R'^{hecd} \nabla^f F^{ab} \nabla_h F_{cd} \right. \\ \left. - 8 R'_{efab} R_{gh}{}^{ab} \nabla_c F^{he} \nabla^c F^{fg} + 8 R'_{ghab} R'^{fgcd} \nabla_f F^{ab} \nabla^h F_{cd} \right. \\ \left. + 4 R'_{efab} R'^{efcd} \nabla^g F^{ab} \nabla_g F_{cd} + 2 R'_{efab} R'^{efab} \nabla_g F_{cd} \nabla^g F^{cd} \right),$$

$$\tilde{X}_{abcd} = 4 \nabla_a F_{be} \nabla_c F_d{}^e + 4 \nabla_a F_{ce} \nabla_d F_b{}^e - 4 \eta_{ad} \nabla_e F_{fb} \nabla^e F^f{}_c - 2 \eta_{ad} \nabla_b F_{ef} \nabla_c F^{ef} \\ + \eta_{ac} \eta_{bd} \nabla_g F_{ef} \nabla^g F^{ef}$$

$$k = -1, l = -\frac{2}{3}(11 - 2) = -6$$

$$t_8 t_8 R^4 + L_{R'^2 F_2^2} + L_{F_2^4} - 2^7 3 \nabla^a \nabla^b \Phi \nabla^c \nabla^d \Phi \tilde{X}_{abcd} + \dots$$



$$R'_{abcd} = R_{abcd} - 4\delta_{[c}^{[a}\nabla_{d]}\nabla^{b]}\Phi$$

$$t_8 t_8 \hat{R}^4 \rightarrow t_8 t_8 R^4 + L_{R^2 F_2^2} + L_{F_2^4} + L_{\Phi R F_2^2} + L_{\Phi^2 F_2^2} + \dots$$

$$\begin{aligned} L_{\Phi R F_2^2} &= 2^8 3 \nabla_a \nabla^b \Phi (2R^{aec d} \nabla^f F_{bc} \nabla_e F_{df} + 2R_{cd}^{ae} \nabla^b F^{cf} \nabla^d F_{ef} + R_{db}^{ac} \nabla_e F_{cf} \nabla^e F^{fd}) \\ &= -2^9 3 \Phi R_{cd}^{ab} \nabla_a \nabla^e F^{fc} \nabla^d \nabla_f F_{eb} + \dots, \end{aligned}$$

$$\begin{aligned} L_{\Phi^2 F_2^2} &= 2^7 3 \nabla_a \nabla^b \Phi \nabla_c \nabla_d \Phi (2\nabla^a F^{ce} \nabla^d F_{be} - 2\nabla^a F_{be} \nabla^c F^{de} + 2\nabla_e F^{ac} \nabla^e F_b^d \\ &\quad + 4\eta^{ad} \nabla^e F_{bf} \nabla^f F^{ce} + 3\eta^{ad} \nabla_b F^{ef} \nabla^c F_{ef} - \eta^{ad} \delta_b^c \nabla_g F_{ef} \nabla^g F^{ef}) \\ &= -2^8 3 \Phi \nabla_a \nabla^b \Phi \nabla^c \nabla^d F^{ea} \nabla_c \nabla_d F_{eb} + \dots \end{aligned}$$

$$\hat{L}_{\hat{R}'\hat{F}^2} + \hat{L}_{\Phi\hat{R}'\hat{F}^2} + \hat{L}_{\Phi^2\hat{F}^2} + \dots$$

$$L_{\Phi R' F^2} = 48k \Phi R'_{abcd} (-3\nabla^4 (F^{abef} F^{cd}_{ef}) + 8\nabla^2 (\nabla^c F^{aefg} \nabla^d F^b_{efg}) - 4\nabla^a \nabla^c F^{efgh} \nabla^b \nabla^d F_{efgh})$$

$$\begin{aligned} L_{\Phi^2 F^2} &= 16k(2l - k(D - 2)) \nabla^a \nabla^b \Phi \nabla^c \nabla^d \Phi (-12\nabla_g F_{acef} \nabla^g F^{ef}_{bd} + 8\nabla_a F_{befg} \nabla_c F^{efg}_d \\ &\quad - 8\eta_{ac} \nabla_h F_{befg} \nabla^h F_d{}^{efg} - 2\eta_{ac} \nabla_b F_{efgh} \nabla_d F^{efgh} + \eta_{ac} \eta_{bd} \nabla_k F_{efgh} \nabla^k F^{efgh}) \\ &\quad - 288k^2 \Phi \nabla_a \nabla^b \Phi \nabla^4 (F_{befg} F^{aefg}). \end{aligned}$$

$$\hat{L}_{\Phi\hat{R}'\hat{F}^2} = 32\Phi\hat{R}'_{abcd} (-3\nabla^4 (\hat{F}^{abef} \hat{F}^{cd}_{ef}) + 8\nabla^2 (\nabla^c \hat{F}^{aefg} \nabla^d \hat{F}^b_{efg}) - 4\nabla^a \nabla^c \hat{F}^{efgh} \nabla^b \nabla^d \hat{F}_{efgh})$$

$$\begin{aligned} \hat{L}_{\Phi^2\hat{F}^2} &= 64\nabla^a \nabla^b \Phi \nabla^c \nabla^d \Phi (12\nabla_g \hat{F}_{acef} \nabla^g \hat{F}^{ef}_{bd} - 8\nabla_a \hat{F}_{befg} \nabla_c \hat{F}^{efg}_d \\ &\quad + 8\eta_{ac} \nabla_h \hat{F}_{befg} \nabla^h \hat{F}_d{}^{efg} + 2\eta_{ac} \nabla_b \hat{F}_{efgh} \nabla_d \hat{F}^{efgh} - \eta_{ac} \eta_{bd} \nabla_k \hat{F}_{efgh} \nabla^k \hat{F}^{efgh}) \\ &\quad - 128\Phi \nabla_a \nabla^b \Phi \nabla^4 (\hat{F}_{befg} \hat{F}^{aefg}) \end{aligned}$$

$$L_{R'^2 H^2} + L_{\Phi R' H^2} + L_{\Phi^2 H^2} + L_{R'^2 F_4^2} + L_{\Phi R' F_4^2} + L_{\Phi^2 F_4^2} + L_{R' H F_2 F_4} + L_{\Phi H F_2 F_4} + L_{H^2 F_2^2} + L_{F_2^2 F_4^2},$$

$$\begin{aligned} L_{R'^2 H^2} &= 16(48R'_{abcd} R'_{efg}{}^d \nabla^a H^c{}_g \nabla^b H^{efh} - 12R'_{abcd} R'_{efg}{}^d \nabla^c H^{ab}{}_h \nabla^g H^{efh} \\ &\quad - 12R'_{abcd} R'_{efg}{}^d \nabla_i H^{efc} \nabla^i H^{abg} + 12R'_{abcd} R'_{efg}{}^d \nabla^c H^{ef}{}_h \nabla^g H^{abh} \\ &\quad + 24R'_{abcd} R'^{eafc} \nabla_i H^b{}_{fg} \nabla^i H_e{}^{dg} + 24R'_{abcd} R'^{eafc} \nabla_i H^{bdg} \nabla^i H_{efg} \\ &\quad + 24R'_{abcd} R'^{eafc} \nabla^b H^{dgh} \nabla_e H_{fgh} - 12R'_{abcd} R'^{efcd} \nabla^a H_{egh} \nabla^b H_f{}^{gh} \\ &\quad + 12R'_{abcd} R'^{eacd} \nabla_i H^{bgh} \nabla^i H_{egh} + 4R'_{abcd} R'_{eacd} \nabla^b H_{fgh} \nabla^e H^{fgh} \\ &\quad + 3R'_{abcd} R'^{abcd} \nabla_e H_{fgh} \nabla^f H^{egh}) \end{aligned}$$

$$L_{\Phi R' H^2} = 64\Phi R'_{abcd} (-3\nabla^4 (H^{abe} H^{cd}{}_e) + 12\nabla^2 (\nabla^c H^{aef} \nabla^d H^b{}_{ef}) - 8\nabla^a \nabla^c H^{efg} \nabla^b \nabla^d H_{efg})$$

$$\begin{aligned} L_{\Phi^2 H^2} &= 2^8 \nabla^a \nabla^b \Phi \nabla^c \nabla^d \Phi (6\nabla_g H_{ace} \nabla^g H^e_{bd} - 6\nabla_a H_{bef} \nabla_c H^{ef}_d \\ &\quad + 6\eta_{ac} \nabla_h H_{bef} \nabla^h H^{ef}_d + 2\eta_{ac} \nabla_b H_{efg} \nabla_d H^{efg} - \eta_{ac} \eta_{bd} \nabla_k H_{efg} \nabla^k H^{efg}) \\ &\quad - 2^7 3 \Phi \nabla_a \nabla^b \Phi \nabla^4 (H_{bef} H^{aef}) \end{aligned}$$



$$L_{R'HF_2F_4} = 16(24R'_{abcd}\nabla^h H^{cef}\nabla_e F^{gd}\nabla_h F^{ab}{}_{fg} - 12R'^{ab}{}_{cd}\nabla^f H_{abe}\nabla^c F_{gh}\nabla_f F^{degh} \\ - 12R'_{abcd}\nabla^e H_{fgh}\nabla^f F^{ec}\nabla^d F^{abgh} - 12R'_{abcd}\nabla^e H^{cgh}\nabla^d F_{ef}\nabla^f F^{ab}{}_{gh} \\ + 12R'_{abcd}\nabla^c H_{egh}\nabla^e F^{df}\nabla_f F^{abgh} + 12R'_{abcd}\nabla^a H^{cef}\nabla^d F^{gh}\nabla^b F_{efgh} \\ - 24R'_{abcd}\nabla^a H^{egh}\nabla_e F_f{}^c\nabla^b F^{dfgh} + 24R'^{cd}{}_{ab}\nabla^e H^{agh}\nabla_c F^{bf}\nabla_e F_{fdgh} \\ - 6R'_{abcd}\nabla^e H_{fgh}\nabla^f F^{ab}\nabla_e F^{cdgh} - 8R'_{abcd}\nabla_e H_{fgh}\nabla^c F^{ea}\nabla^b F^{dfgh} \\ + 4R'_{abcd}\nabla^a H_{fgh}\nabla^e F^{cd}\nabla^b F_{efgh} - 4R'_{abcd}\nabla_e H_{fgh}\nabla^a F^{cd}\nabla^e F^{bfgh})$$

$$L_{\Phi HF_2F_4} = 64\Phi[3\nabla^e F_{ab}\nabla^4(F^{abcd}H_{ecd}) + 4\nabla_f F_{ab}\nabla^2(\nabla^a F^{fcde}\nabla^b H_{cde})].$$

$$L_{R^2H^2} + L_{R^2F_4^2} + L_{RH_2F_4} + L_{\Phi RF_4^2} + L_{\Phi^2F_4^2} + L_{\Phi HF_2F_4} + L_{H^2F_2^2} + L_{F_2^2F_4^2},$$

$$L_{\Phi RF_4^2} = 16\Phi R_{abcd}(3\nabla^4(F^{abef}F^{cd}{}_{ef}) - 8\nabla^2(\nabla^c F^{aefg}\nabla^d F^b{}_{efg}) + 4\nabla^a\nabla^c F^{efgh}\nabla^b\nabla^d F_{efgh}),$$

$$L_{\Phi^2F_4^2} = -32\Phi\nabla_a\nabla^b\Phi\nabla^4(F^{acde}F_{bcde})$$

$$L_{\Phi HF_2F_4} = 64\Phi[3\nabla_a H_{bef}\nabla^2(\nabla^b F_{gh}\nabla^a F^{efgh}) - 2\nabla_a H_{cde}\nabla^2(\nabla^f F^{ag}\nabla_g F_{fcde})].$$

$$L_{\text{NSNSRR}} = c\left(A_1 + \frac{1}{2}A_2 + \frac{1}{4}A_3\right)$$

$$A_1 = \mathcal{R}_{aefb}\mathcal{R}^{cef}{}_d\text{tr}(\gamma^a\nabla_c\mathbf{F}^{(b}\nabla^d)\mathbf{F}^T)$$

$$A_2 = \mathcal{R}_{abeg}\mathcal{R}_{cdf}{}^g\text{tr}(\gamma^{abc}\nabla^d\mathbf{F}^{(e}\nabla^f)\mathbf{F}^T) + \mathcal{R}_{egab}\mathcal{R}_f{}^g{}_{cd}\text{tr}(\gamma^{abc}\nabla^d\mathbf{F}^T\gamma^{(e}\nabla^f)\mathbf{F}')$$

$$A_3 = \mathcal{R}_{abcd}\mathcal{R}_{efgh}\text{tr}(\gamma^{abe}\nabla^f\mathbf{F}^{[cdg}\nabla^h]\mathbf{F}^T)$$

$$\mathcal{R}_{abcd} = R_{abcd} + \nabla_{[a}H_{b]cd}$$

$$\mathbb{F} = \frac{1}{2}F_{ab}\gamma^{ab} + \frac{1}{4!}F_{abcd}\gamma^{abcd}, \mathbb{F}^T = -\frac{1}{2}F_{ab}\gamma^{ab} + \frac{1}{4!}F_{abcd}\gamma^{abcd}.$$

$$[A_1]_{\Phi} = -\frac{1}{2}R^{ef}\mathcal{R}_{cefd}\text{tr}(\nabla^c\mathbb{F}^dF^T)$$

$$[A_2]_{\Phi} = R_{ag}\mathcal{R}_{cdf}{}^g\text{tr}(\gamma^{ac}\nabla^d\mathbb{F}^fF^T) + R_{ag}\mathcal{R}_f{}^g{}_{cd}\text{tr}(\gamma^{ac}\nabla^d\mathbb{F}^{Tf}\nabla^f) \\ + \frac{1}{2}R_{dg}\mathcal{R}_{abe}{}^g\text{tr}(\gamma^{ab}\nabla^d\nabla^eF^T) + \frac{1}{2}R_{dg}\mathcal{R}_e{}^g{}_{ab}\text{tr}(\gamma^{ab}\nabla^dF^T\nabla^e\mathbb{F}')$$

$$[A_3]_{\Phi} = 2R_{ad}\mathcal{R}_{efgh}\text{tr}(\gamma^{ae}\nabla^f\mathbb{F}^{dg}\nabla^hF^T) + \frac{1}{2}R_{fh}\mathcal{R}_{abcd}\text{tr}(\gamma^{ab}\nabla^f\mathbb{F}^{cd}\nabla^hF^T) \\ + R_{ad}\mathcal{R}_{efgh}\text{tr}(\gamma^{ae}\nabla^f\mathbb{F}^{gh}\nabla^dF^T) + R_{fh}\mathcal{R}_{abcd}\text{tr}(\gamma^{ab}\nabla^f\mathbb{F}^{hc}\nabla^dF^T) \\ + 2R_{ad}R_{fh}\text{tr}(\gamma^a\nabla^f\mathbb{F}^d\gamma^hF^T) - 2R_{ad}R_{fh}\text{tr}(\gamma^a\nabla^f\mathbb{F}^h\nabla^dF^T)$$

$$[A_1]_{\Phi F_2^2} = 8\nabla^a\nabla^b\Phi R_{acbd}\nabla^c F_{ef}\nabla^d F^{ef}$$

$$[A_2]_{\Phi F_2^2} = 32\nabla_a\nabla^b\Phi(R_{becd}\nabla^e F^fa\nabla_f F^{cd} + 2R_{becd}\nabla_f F^{ac}\nabla^e F^{fd})$$

$$[A_3]_{\Phi F_2^2} = 32\nabla^a\nabla^b\Phi(R_{acbd}\nabla^c F_{ef}\nabla^d F^{ef} - 4R^{cdef}\nabla_a F_{be}\nabla_f F_{cd} + 4R^{cdef}\nabla_e F_{ca}\nabla_d F_{fb} \\ - R_{cdef}\nabla_a F^{cd}\nabla_b F^{ef} - 2R_{becd}\nabla^e F_{af}\nabla^f F^{cd} + 4R_{be}{}^{cd}\nabla^f F_{ac}\nabla^e F_{df} \\ + 8R_{be}{}^{cd}\nabla_a F_{fc}\nabla_d F^{fe}) + 64\nabla^a\nabla^b\Phi\nabla^c\nabla_d\Phi(4\nabla_a F_{ce}\nabla_b F^{de} \\ - 2\nabla_a F_{be}\nabla_c F^{de} - 2\nabla_c F_{ae}\nabla_b F^{de} + \delta_a^d\nabla_b F_{ef}\nabla_c F^{ef})$$



$$\begin{aligned}
[A_1]_{\Phi F_4^2} &= \frac{2}{3} \nabla^a \nabla^b \Phi R_{acbd} \nabla^c F_{efgh} \nabla^d F^{efgh} \\
[A_2]_{\Phi F_4^2} &= \frac{32}{3} \nabla^a \nabla^b \Phi R_{becd} (\nabla^e F_{afgh} \nabla^c F^{dfgh} - \nabla^c F_{afgh} \nabla^e F^{dfgh} + \nabla_a F^{cfgh} \nabla^e F_{fgh}^d) \\
[A_3]_{\Phi F_4^2} &= \frac{8}{3} \nabla^a \nabla^b \Phi (R_{acbd} \nabla^c F_{efgh} \nabla^d F^{efgh} - 12 R_{cdef} \nabla_a F^{cdgh} \nabla_b F^{ef}_{gh} - 24 R^{cdef} \nabla_e F_{cagh} \nabla_d F_{bf}^{gh} \\
&\quad - 24 R^{cdef} \nabla_c F_{dagh} \nabla_e F_{fb}^{gh} - 24 R^{cdef} \nabla_e F_{fagh} \nabla_b F_{cd}^{gh} - 24 R^{cdef} \nabla_a F_{begh} \nabla_f F_{cd}^{gh} \\
&\quad + 8 R_{becd} \nabla^e F_{afgh} \nabla^c F^{dfgh} + 8 R_{becd} \nabla^c F_{afgh} \nabla^e F^{dfgh} + 8 R_{becd} \nabla_a F^{cfgh} \nabla^e F_{fgh}^d \\
&\quad - 16 R_{becd} \nabla_a F^{efgh} \nabla^c F_{fgh}^d) + \frac{16}{3} \nabla^a \nabla_b \Phi \nabla^c \nabla^d \Phi (8 \nabla_c F_{aefg} \nabla_d F^{befg} \\
&\quad - 4 \nabla_a F_{cefg} \nabla_d F^{befg} - 4 \nabla_a F^{befg} \nabla_c F_{defg} + \eta_{ad} \nabla_c F_{efgh} \nabla^b F^{efgh})
\end{aligned}$$

$$\begin{aligned}
[A_1]_{\Phi F_2 F_4} &= 0, \\
[A_2]_{\Phi F_2 F_4} &= 16 \nabla_a \nabla^b \Phi (\nabla H)_{becd} (2 \nabla^c F_{gh} \nabla^e F^{adgh} - 2 \nabla^e F_{gh} \nabla^c F^{adgh} - \nabla^a F_{gh} \nabla^e F^{cdgh} \\
&\quad + \nabla^e F_{gh} \nabla^a F^{cdgh})
\end{aligned}$$

$$\begin{aligned}
[A_3]_{\Phi F_2 F_4} &= 64 \nabla^a \nabla^b \Phi \left(\frac{1}{3} \nabla_f H_{cde} \nabla_a F_{bg} \nabla^f F^{gcde} - \frac{2}{3} \nabla^f H_{cde} \nabla_a F_{fg} \nabla_b F^{gcde} \right. \\
&\quad + \frac{1}{3} \nabla_f H_{cde} \nabla^f F_{ag} \nabla_b F^{gcde} + 4 (\nabla H)^{cdef} \nabla_e F_{ag} \nabla^c F_{bfd}^g \\
&\quad - 2 (\nabla H)^{cdef} \nabla_a F_{cg} \nabla^e F_{bfd}^g - 2 (\nabla H)^{cdef} \nabla_e F_{cg} \nabla^a F_{bfd}^g \\
&\quad + (\nabla H)^{cdef} \nabla_a F_{eg} \nabla_f F_{bcd}^g - (\nabla H)^{cdef} \nabla_e F_{fg} \nabla_a F_{bcd}^g \\
&\quad + (\nabla H)_{becd} \nabla^e F^{gh} \nabla^c F_a^{dgh} + (\nabla H)_{becd} \nabla^c F_{gh} \nabla^e F_a^{dgh} \\
&\quad \left. + (\nabla H)_{becd} \nabla_a F_{gh} \nabla^c F^{degh} + (\nabla H)_{becd} \nabla^c F_{gh} \nabla_a F^{degh} \right)
\end{aligned}$$

$$-24 \left(A_1 + \frac{1}{2} A_2 + \frac{1}{4} A_3 \right)_{\Phi} = L_{\Phi R F_2^2} + L_{\Phi^2 F_2^2} + L_{\Phi R F_4^2} + L_{\Phi^2 F_4^2} + L_{\Phi H F_2 F_4} + \dots,$$

$$-24 \left(A_1 + \frac{1}{2} A_2 + \frac{1}{4} A_3 \right)_{R^2 F_2^2} = L_{R^2 F_2^2} + 12 \cdot 6! R_{[ab}{}^{ab} R_{cd}{}^{cd} \nabla_e F^{eg} \nabla_f F^f{}_g + 24 \cdot 6! R_{[ab}{}^{ag} R_{cd}{}^{bc} \nabla_e F^{de} \nabla_f F^f{}_g,$$

$$\mathcal{R}_{abcd} = R_{abcd} + \nabla_{[a} H_{b]cd} + \frac{1}{2} H_{a[c}{}^e H_{d]be}$$

$$t_8 t_8 \mathcal{R}^4 \equiv t_{a_1 \dots a_8} t^{b_1 \dots b_8} \mathcal{R}^{a_1 a_2}{}_{b_1 b_2} \dots \mathcal{R}^{a_7 a_8}{}_{b_7 b_8}$$

$$t_8 M_1 M_2 M_3 M_4 = 8 \text{tr}(M_1 M_2 M_3 M_4) - 2 \text{tr}(M_1 M_2) \text{tr}(M_3 M_4) + \text{cycl}(2,3,4).$$

$$\mathcal{R} = \frac{1}{4} R_{abcd} \Gamma^{ab} \otimes \Gamma^{cd} + \frac{1}{4} \nabla_a H_{bcd} \Gamma^{ab} \otimes \Gamma^{cd} \Gamma_{11}.$$

$$\begin{aligned}
t_{8s} M_1 M_2 M_3 M_4 &= -\frac{3}{4} \text{str}(M_1 \dots M_4) + \frac{1}{64} [\text{tr}(M_1 M_2) \text{tr}(M_3 M_4) + \text{cycl}(2,3,4)] \\
&\quad + \frac{1}{256} [\text{tr}(M_1 \{M_2, \hat{\Gamma}^a\}) \text{tr}(\{\hat{\Gamma}_a, M_3\} M_4) + \text{cycl}(2,3,4)]
\end{aligned}$$

$$\text{str}(ABCD) = \frac{1}{6} \text{tr}(ABCD + \text{perm}(B, C, D))$$

$$t_{8s} M_1 M_2 M_3 M_4 = t_8 M_1 M_2 M_3 M_4$$

$$t_{8s} t_{8s} \mathbb{R}^4.$$



$$\hat{\mathcal{R}} = \frac{1}{4} \hat{R}_{abcd} \hat{f}^{ab} \otimes \hat{f}^{cd} + \nabla_a \hat{F}_{bcde} \left(\frac{1}{12} \hat{f}^{ab} \otimes \hat{f}^{cde} + \frac{a}{48} \hat{f}^a \otimes \hat{f}^{bcde} \right),$$

$$\hat{\mathcal{R}} \rightarrow \mathbb{R} + \mathcal{R}' \text{ with } \mathbb{R}' = \frac{1}{12} \nabla_a H_{bcd} (-\Gamma^a \Gamma_{11} \otimes \Gamma^{bcd} + a \Gamma^a \otimes \Gamma^{bcd} \Gamma_{11}).$$

$$t_{8s}t_{8s}\hat{\mathbb{R}}^4 \rightarrow t_{8s}t_{8s}\mathbb{R}^4 + 6t_{8s}t_{8s}\mathbb{R}^2\mathbb{R}'^2 + t_{8s}t_{8s}\mathbb{R}'^4$$

$$\begin{aligned} 6t_{8s}t_{8s}\mathbb{R}^2\mathbb{R}'^2 = & -\frac{3}{32}(1+a^2)R_{abcd}R_{efgh}t_{8s}(\Gamma^{ab}\Gamma^{ef}\Gamma^i\Gamma^j)t_{8s}(\Gamma^{cd}\Gamma^{gh}\nabla_i\mathbb{H}\nabla_j\mathbb{H}) \\ & -\frac{3}{32}(1+a^2)(\nabla H)_{abcd}(\nabla H)_{efgh}t_{8s}(\Gamma^{ab}\Gamma^{ef}\Gamma^i\Gamma^j)t_{8s}(\Gamma^{cd}\Gamma_{11}\Gamma^{gh}\Gamma_{11}\nabla_i\mathbb{H}\nabla_j\mathbb{H}) \\ & +192a^2R^a{}_{bcd}R^{bgef}\nabla_{(a}H^{bcd}\nabla_{g)}H_{hef}+48a^2R_{abcd}R^{abef}\nabla^gH^{bcd}\nabla_gH_{hef}, \end{aligned}$$

$$a=i$$

$$\begin{aligned} \hat{L}_1 = & 96\hat{R}_{bcd}^a\hat{R}^{bgef}\nabla_{(a}\hat{F}^{cdij}\nabla_{g)}\hat{F}_{efij}+24\hat{R}_{abcd}\hat{R}^{abef}\nabla^g\hat{F}^{cdij}\nabla_g\hat{F}_{efij} \\ = & -\frac{9}{8}t_{8s}(\hat{f}^{ab}\hat{f}^{cd}\hat{f}^e\hat{f}^f)\hat{R}_{abgh}\hat{R}_{cdij}\nabla_e\hat{F}^{ghkl}\nabla_f\hat{F}^{ij}{}_{kl}, \end{aligned}$$

$$\begin{aligned} t_{8s}t_{8s}\mathbb{R}'^4 = & -3\mathrm{tr}(\nabla_a\mathbb{H}^a\mathbb{H}\nabla_b\mathbb{H}\nabla^b\mathbb{H})+\frac{3}{2}\mathrm{tr}(\nabla_a\mathbb{H}\nabla_b\mathbb{H}\nabla^a\mathbb{H}\nabla^b\mathbb{H}) \\ & +\frac{3}{32}\mathrm{tr}(\nabla_a\mathbb{H}\nabla^a\mathbb{H})\mathrm{tr}(\nabla_b\mathbb{H}\nabla^b\mathbb{H}). \end{aligned}$$

$$\hat{L}_2=3\mathrm{tr}(\nabla_a\hat{F}\nabla^a\hat{F}\nabla_b\hat{F}\nabla^b\hat{F})-\frac{3}{2}\mathrm{tr}(\nabla_a\hat{F}\nabla_b\hat{F}\nabla^a\hat{F}\nabla^b\hat{F})-\frac{3}{32}\mathrm{tr}(\nabla_a\hat{F}\nabla^a\hat{F})\mathrm{tr}(\nabla_b\hat{F}\nabla^b\hat{F}),$$

$$\begin{aligned} \hat{L} = & t_{8s}t_{8s}\hat{\mathbb{R}}^4 - \frac{9}{8}t_{8s}(\hat{f}^{ab}\hat{f}^{cd}\hat{f}^e\hat{f}^f)\hat{R}_{abgh}\hat{R}_{cdij}\nabla_e\hat{F}^{ghkl}\nabla_f\hat{F}^{ij}{}_{kl} + 3\mathrm{tr}(\nabla_a\hat{F}\nabla^a\hat{F}\nabla_b\hat{F}\nabla^b\hat{F}) \\ & - \frac{3}{2}\mathrm{tr}(\nabla_a\hat{F}\nabla_b\hat{F}\nabla^a\hat{F}\nabla^b\hat{F}) - \frac{3}{32}\mathrm{tr}(\nabla_a\hat{F}\nabla^a\hat{F})\mathrm{tr}(\nabla_b\hat{F}\nabla^b\hat{F}) + \hat{L}', \end{aligned}$$

$$\hat{L}'=b\hat{R}_{abcd}\hat{R}_{efgh}\nabla^k\hat{F}^{abef}\nabla_k\hat{F}^{cdgh}+\sum_ic_iY_i$$

$$\begin{aligned} Y_1 = & C_2+12C_7+4C_9-24C_{10}+6C_{14}+48C_{17}-6C_{18}+24C_{19}-6C_{21}+6C_{23} \\ Y_2 = & 3C_4-3C_5+C_6+9C_7-6C_8+12C_{17}-3C_{18} \\ Y_3 = & 3C_{10}+C_{11}-3C_{13}-3C_{15}-6C_{17}+6C_{20}+3C_{21} \\ Y_4 = & C_4+3C_7+C_9-6C_{15}+12C_{19} \\ Y_5 = & C_1+4C_2-16C_3-16C_6-144C_7+144C_8 \\ Y_6 = & C_2-6C_4-2C_9+6C_{11}+36C_{12}-18C_{13}+18C_{15} \\ Y_7 = & C_2-12C_4+12C_5+4C_9 \\ Y_8 = & C_9-9C_{15}-C_{22}+9C_{23} \\ Y_9 = & C_{21}+C_{24} \\ Y_{10} = & C_{16}-36C_{20}-9C_{21}-C_{22} \end{aligned}$$

$$b=-48,c_2=12,c_3=24,c_5=\frac{1}{3},c_7=-\frac{10}{3},c_8=\frac{68}{3},c_9=12$$



$$\begin{aligned}
& \frac{9}{8} t_{8s} (\hat{f}^{ab} \hat{f}^{cd} \hat{f}^e \hat{f}^f) \nabla_a \hat{f}_b^{ghm} \nabla_c \hat{f}_{dijm} \nabla_e \hat{f}_{ghkl} \nabla_f \hat{f}^{ijkl} \\
&= 96 \nabla_{[a} \hat{f}_{b]ecd} \nabla^{[a} \hat{f}^{f]egh} \nabla_{(b} \hat{f}^{cdij} \nabla_{f)} \hat{f}_{ghij} - 24 \nabla_a \hat{f}_{becd} \nabla^{[a} \hat{f}^{b]egh} \nabla^f \hat{f}^{cdij} \nabla_f \hat{f}_{ghij} \\
&= -16C_4 + 8C_5 - 24C_7 + 24C_8 - 48C_{12} - 24C_{13} + 12C_{14} - 12C_{18} - 48C_{19} \\
&\quad - 36C_{21} - 60C_{23} \\
& \frac{9}{8} t_{8s} (\hat{f}^{ab} \hat{f}^{cd} \hat{f}^e \hat{f}^f) \nabla_m \hat{f}_{abgh} \nabla_m \hat{f}_{cdij} \nabla_e \hat{f}^{ghkl} \nabla_f \hat{f}^{ij}_{kl} \\
&= 96 \nabla_a \hat{f}^{be}_{gh} \nabla^a \hat{f}_b^{fij} \nabla_{(e} \hat{f}^{ghkl} \nabla_{f)} \hat{f}_{ijkl} - 24 \nabla_a \hat{f}_{efcd} \nabla^a \hat{f}^{efgh} \nabla^b \hat{f}^{cdij} \nabla_b \hat{f}_{ghij} \\
&= -32C_4 + 32C_5 + 96C_7 - 96C_8 - 192C_{12} - 96C_{13} + 48C_{14} - 48C_{18} - 192C_{19} \\
&\quad + 48C_{21} - 48C_{23}
\end{aligned}$$

$$t_{8s} (\hat{f}^{ab} \hat{f}^{cd} \hat{f}^{ef} \hat{f}^{gh}) \nabla_a \hat{f}_{bijk} \nabla_c \hat{f}_d^{ijk} \nabla_e \hat{f}_{flmn} \nabla_g \hat{f}_h^{lmn} = -\frac{9}{32} C_1 + C_2 + C_3 - 3C_4 - 9C_7 - 4C_9 - 9C_{15} - 9C_{24}$$

$$\begin{aligned}
\sum_i c_i Y_i = & \sum_i d'_i \tilde{Z}_i - \frac{3}{2} \hat{L}_2 - \frac{9}{8} t_{8s} (\hat{f}^{ab} \hat{f}^{cd} \hat{f}^e \hat{f}^f) \nabla_a \hat{f}_{bghm} \nabla_c \hat{f}_d^{ijm} \nabla_e \hat{f}^{ghkl} \nabla_f \hat{f}_{ijkl} \\
& - \frac{9}{8} t_{8s} (\hat{f}^{ab} \hat{f}^{cd} \hat{f}^e \hat{f}^f) \nabla_m \hat{f}_{abgh} \nabla_m \hat{f}_{cdij} \nabla_e \hat{f}^{ghkl} \nabla_f \hat{f}^{ij}_{kl} \\
& - \frac{4}{3} t_{8s} (\hat{f}^{ab} \hat{f}^{cd} \hat{f}^{ef} \hat{f}^{gh}) \nabla_a \hat{f}_{bijk} \nabla_c \hat{f}_d^{ijk} \nabla_e \hat{f}_{flmn} \nabla_g \hat{f}_h^{lmn} \\
& - 15 \nabla_a \hat{f}_{cdef} \nabla^a \hat{f}^{efgh} \nabla_b \hat{f}^{cdij} \nabla^b \hat{f}_{ghij} + \frac{9}{2} \nabla_a \hat{f}_{cdef} \nabla^a \hat{f}_{ghij} \nabla_b \hat{f}^{cdij} \nabla^b \hat{f}^{efgh} \\
& + \frac{64}{3} \nabla_a \hat{f}_{bcde} \nabla^g \hat{f}^{hcde} \nabla^a \hat{f}^{bijk} \nabla_{(g} \hat{f}_{h)ijk} - 2 \nabla_a \hat{f}_{bcde} \nabla^a \hat{f}^{fcde} \text{tr}(\nabla^b \hat{f} \nabla_f \hat{f}) \\
& + \frac{3}{32} \text{tr}(\nabla_a \hat{f} \nabla^a \hat{f}) \text{tr}(\nabla_b \hat{f} \nabla^b \hat{f}) - \frac{15}{32} \text{tr}(\nabla_a \hat{f} \nabla_b \hat{f}) \text{tr}(\nabla^a \hat{f} \nabla^b \hat{f})
\end{aligned}$$

$$\begin{aligned}
\hat{L} = & t_{8s} t_{8s} \hat{\mathbb{R}}^4 - \frac{9}{8} t_{8s} (\hat{f}^{ab} \hat{f}^{cd} \hat{f}^e \hat{f}^f) \hat{R}_{abgh} \hat{R}_{cdij} \nabla_e \hat{f}^{ghkl} \nabla_f \hat{f}^{ij}_{kl} - 48 \hat{R}_{abcd} \hat{R}_{efgh} \nabla^i \hat{f}^{abef} \nabla_i \hat{f}^{cdgh} \\
& - \frac{9}{8} t_{8s} (\hat{f}^{ab} \hat{f}^{cd} \hat{f}^e \hat{f}^f) \nabla_a \hat{f}_{bghm} \nabla_c \hat{f}_d^{ijm} \nabla_e \hat{f}^{ghkl} \nabla_f \hat{f}_{ijkl} \\
& - \frac{9}{8} t_{8s} (\hat{f}^{ab} \hat{f}^{cd} \hat{f}^e \hat{f}^f) \nabla_m \hat{f}_{abgh} \nabla_m \hat{f}_{cdij} \nabla_e \hat{f}^{ghkl} \nabla_f \hat{f}^{ij}_{kl} \\
& - \frac{4}{3} t_{8s} (\hat{f}^{ab} \hat{f}^{cd} \hat{f}^{ef} \hat{f}^{gh}) \nabla_a \hat{f}_{bijk} \nabla_c \hat{f}_d^{ijk} \nabla_e \hat{f}_{flmn} \nabla_g \hat{f}_h^{lmn} \\
& - \frac{3}{2} \text{tr}(\nabla_a \hat{f} \nabla_a \hat{f} \nabla_b \hat{f} \nabla_b \hat{f}) + \frac{3}{4} \text{tr}(\nabla_a \hat{f} \nabla_b \hat{f} \nabla_a \hat{f} \nabla_b \hat{f}) \\
& - \frac{15}{32} \text{tr}(\nabla_a \hat{f} \nabla_b \hat{f}) \text{tr}(\nabla^a \hat{f} \nabla^b \hat{f}) + \frac{9}{64} \text{tr}(\nabla_a \hat{f} \nabla^a \hat{f}) \text{tr}(\nabla_b \hat{f} \nabla^b \hat{f}) \\
& - 15 \nabla_a \hat{f}_{cdef} \nabla^a \hat{f}^{efgh} \nabla_b \hat{f}^{cdij} \nabla^b \hat{f}_{ghij} + \frac{9}{2} \nabla_a \hat{f}_{cdef} \nabla^a \hat{f}_{ghij} \nabla_b \hat{f}^{cdij} \nabla^b \hat{f}^{efgh} \\
& + \frac{64}{3} \nabla_a \hat{f}_{bcde} \nabla^g \hat{f}^{hcde} \nabla^a \hat{f}^{bijk} \nabla_{(g} \hat{f}_{h)ijk} - 2 \nabla_a \hat{f}_{bcde} \nabla^a \hat{f}^{fcde} \text{tr}(\nabla^b \hat{f} \nabla_f \hat{f}).
\end{aligned}$$

$$\mathbb{M}^{(n)} = \frac{1}{n!} M_{a_1 \dots a_n} \Gamma^{a_1 \dots a_n}, M^{(n')} = \frac{1}{n!} M_{a_1 \dots a_n} \Gamma^{a_1 \dots a_n} \Gamma_{11}$$



$$\begin{aligned}
t_{8s} \left(M_1^{(1')} M_2^{(1')} M_3^{(1')} M_4^{(1')} \right) &= t_{8s} \left(M_1^{(1)} M_2^{(1)} M_3^{(1)} M_4^{(1)} \right) \\
t_{8s} \left(M_1^{(1)} M_2^{(1)} M_3^{(1')} M_4^{(1')} \right) &= -t_{8s} \left(M_1^{(1)} M_2^{(1)} M_3^{(1)} M_4^{(1)} \right) + 16 M_1^a M_{2a} M_3^b M_{4b} \\
t_{8s} \left(M_1^{(2)} M_2^{(2)} M_3^{(1')} M_4^{(1')} \right) &= -t_{8s} \left(M_1^{(2)} M_2^{(2)} M_3^{(1)} M_4^{(1)} \right) \\
t_{8s} \left(M_1^{(2)} M_2^{(2)} M_3^{(3')} M_4^{(3')} \right) &= -t_{8s} \left(M_1^{(2)} M_2^{(2)} M_3^{(3)} M_4^{(3)} \right) + 8 M_1^{ab} M_{2cd} M_{3abe} M_4^{cde} \\
t_{8s} \left(M_1^{(2')} M_2^{(2')} M_3^{(3')} M_4^{(3')} \right) &= -t_{8s} \left(M_1^{(2')} M_2^{(2')} M_3^{(3)} M_4^{(3)} \right) \\
t_{8s} \left(M_1^{(3')} M_2^{(3')} M_3^{(3')} M_4^{(3')} \right) &= t_{8s} \left(M_1^{(3)} M_2^{(3)} M_3^{(3)} M_4^{(3)} \right) \\
t_{8s} \left(M_1^{(3)} M_2^{(3)} M_3^{(3')} M_4^{(3')} \right) &= -t_{8s} \left(M_1^{(3)} M_2^{(3)} M_3^{(3)} M_4^{(3)} \right) - \frac{1}{2} \text{tr} \left(M_1^{(3)} M_4^{(3)} M_2^{(3)} M_3^{(3)} \right) \\
&\quad + \frac{1}{64} \text{tr} \left(M_1^{(3)} M_3^{(3)} \right) \text{tr} \left(M_4^{(3)} M_2^{(3)} \right) \\
&\quad + \frac{1}{64} \text{tr} \left(M_1^{(3)} M_4^{(3)} \right) \text{tr} \left(M_2^{(3)} M_3^{(3)} \right) \\
t_{8s} \left(\Gamma^a \Gamma^b \Gamma^c \Gamma^d \right) &= 24 \eta^{a(b} \eta^{cd)}
\end{aligned}$$

$$\begin{aligned}
B_1 &= \hat{R}_{abcd} \hat{R}_{efgh} \nabla^e \hat{F}^{agh}{}_i \nabla^c \hat{F}^{bdfi} B_{13} \\
B_2 &= \hat{R}_{abcd} \hat{R}_{efgh} \nabla^e \hat{F}^{acg}{}_i \nabla^h \hat{R}^{bdfi}{}_a{}_f{}^c \nabla_i \hat{F}^{bf}{}_{gh} \nabla^i \hat{F}^{degh} \\
B_3 &= \hat{R}_{abcd} \hat{R}_{efgh} \nabla^e \hat{F}^{acg}{}_i \nabla^f \hat{F}^{bdhi} \\
B_4 &= \hat{R}_{abcd} \hat{R}_{efgh} \nabla_i \hat{F}^{cdgh} \nabla^f \hat{F}^{iabe} \\
B_5 &= \hat{R}_{abcd} \hat{R}_{efg}{}^a \nabla^a \hat{F}^{bc}{}_{hi} \nabla^e \hat{F}^{fghi} \\
B_6 &= \hat{R}_{abcd} \hat{R}_{efg}{}^a \nabla^a \hat{F}^{be}{}_{hi} \nabla^c \hat{F}^{fghi} \\
B_7 &= \hat{R}_{abcd} \hat{R}_{efg}{}^a \nabla^a \hat{F}^{be}{}_{hi} \nabla^g \hat{F}^{cfhi} \\
B_8 &= \hat{R}_{abcd} \hat{R}_{efg}{}^a \nabla^a F^{ce}{}_{hi} \nabla^b \hat{F}^{fghi} \\
B_9 &= \hat{R}_{abcd} \hat{R}_{efg}{}^a \nabla^a \hat{F}^{ce}{}_{hi} \nabla^f \hat{F}^{bghi} \\
B_{10} &= \hat{R}_{abcd} \hat{R}_{efg}{}^a \nabla_i \hat{F}^{cegh} \nabla^i \hat{F}^{abfh} \\
B_{11} &= \hat{R}_{abcd} \hat{R}_{efg}{}^a \nabla_h \hat{F}^{abf}{}_i \nabla^i \hat{F}^{cegh} \\
B_{12} &= \hat{R}_{abcd} \hat{R}_{efg}{}^a \nabla^c \hat{F}^{ef}{}_{hi} \nabla^g \hat{F}^{bahi} \\
B_{14} &= \hat{R}_{abcd} \hat{R}_e{}^a{}_f{}^c \nabla_i \hat{F}^{bd}{}_{gh} \nabla^i \hat{F}^{efgh} \\
B_{15} &= \hat{R}_{abcd} \hat{R}_e{}^a{}_f{}^c \nabla^b \hat{F}^f{}_{ghi} \nabla^e \hat{F}^{dghi} \\
B_{16} &= \hat{R}_{abcd} \hat{R}_e{}^a{}_f{}^c \nabla^b \hat{F}^d{}_{ghi} \nabla^e \hat{F}^{fghi} \\
B_{17} &= \hat{R}_{abcd} \hat{R}_e{}^a{}_f{}^c \nabla^b F^e{}_{ghi} \nabla^d \hat{F}^{fghi} \\
B_{18} &= \hat{R}_{abcd} \hat{R}_e{}^a{}_f{}^c \nabla_i \hat{F}^{ef}{}_{gh} \nabla^d \hat{F}^{bghi} \\
B_{19} &= \hat{R}_{abcd} \hat{R}_{ef}{}^{cd} \nabla_i \hat{F}^{ae}{}_{gh} \nabla^i \hat{F}^{bfgh} \\
B_{20} &= \hat{R}_{abcd} \hat{R}_{ef}{}^{cd} \nabla^a \hat{F}^e{}_{ghi} \nabla^b \hat{F}^{fghi} \\
B_{21} &= \hat{R}_{abcd} \hat{R}_{ef}{}^{cd} \nabla^a \hat{F}^e{}_{ghi} \nabla^f \hat{F}^{bghi} \\
B_{22} &= \hat{R}_{abcd} \hat{R}_e{}^{acd} \nabla^b \hat{F}^f{}_{ghi} \nabla^e \hat{F}^{fghi} \\
B_{23} &= \hat{R}_{abcd} \hat{R}_e{}^{acd} \nabla_i \hat{F}^b{}_{fgh} \nabla^i \hat{F}^{efgh} \\
B_{24} &= \hat{R}_{abcd} \hat{R}^{abcd} \nabla_e \hat{F}^f{}_{ghi} \nabla^f \hat{F}^{eghi}
\end{aligned}$$

$$\begin{aligned}
Z_1 &= 48B_1 + 48B_2 - 48B_3 + 36B_4 + 96B_6 + 48B_7 - 48B_8 + 96B_{10} \\
&\quad + 12B_{12} + 24B_{13} - 12B_{14} + 8B_{15} + 8B_{16} - 16B_{17} + 6B_{19} + 2B_{22} + B_{24}, \\
Z_2 &= -48B_1 - 48B_2 - 24B_4 - 24B_5 + 48B_6 - 48B_8 - 24B_9 - 72B_{10} \\
&\quad - 24B_{13} + 24B_{14} - B_{22} + 4B_{23}, \\
Z_3 &= 12B_1 + 12B_2 - 24B_3 + 9B_4 + 48B_6 + 24B_7 - 24B_8 + 24B_{10} + 6B_{12} + 6B_{13} \\
&\quad + 4B_{15} - 4B_{17} + 3B_{19} + 2B_{21}, \\
Z_4 &= 12B_1 + 12B_2 - 12B_3 + 9B_4 + 24B_6 + 12B_7 - 12B_8 + 24B_{10} + 3B_{12} \\
&\quad + 6B_{13} + 4B_{15} - 4B_{17} + 2B_{20}, \\
Z_5 &= 4B_3 - 8B_6 - 4B_7 + 4B_8 - B_{12} - 2B_{14} + 4B_{18}, \\
Z_6 &= B_4 + 2B_{11}.
\end{aligned}$$



$$\begin{aligned}
C_1 &= \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{bcde} \nabla_f \hat{F}^{ghij} \nabla^f \hat{F}^{ghij} \\
C_2 &= \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{bcd}{}_f \nabla^e \hat{F}^{ghij} \nabla^f \hat{F}^{ghij} \\
C_3 &= \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{bcd}{}_f \nabla_g \hat{F}^e{}_{hij} \nabla^g \hat{F}^{f hij} \\
C_4 &= \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{bc}{}_{fg} \nabla^d \hat{F}^e{}_{hij} \nabla^f \hat{F}^{ghij} \\
C_5 &= \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^b{}_{fgh} \nabla_i \hat{F}^{cde}{}_j \nabla^f \hat{F}^{ghij} \\
C_6 &= \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^b{}_{fgh} \nabla_i \hat{F}^{cde}{}_j \nabla^j \hat{F}^{fghi} \\
C_7 &= \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^{ac}{}_{fg} \nabla^d \hat{F}^e{}_{hij} \nabla^h \hat{F}^{fgij} \\
C_8 &= \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^{ac}{}_{fg} \nabla_h \hat{F}^{de}{}_{ij} \nabla^i \hat{F}^{fghj} \\
C_9 &= \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^{fghi} \nabla^f \hat{F}^{ghi}{}_j \nabla^j \hat{F}^{acde} \\
C_{10} &= \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{bc}{}_{fg} \nabla_h \hat{F}^{df}{}_{ij} \nabla^i \hat{F}^{eghj} \\
C_{11} &= \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{bc}{}_{fg} \nabla^d \hat{F}^f{}_{hij} \nabla^g \hat{F}^{ehij} \\
C_{12} &= \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^b{}_{fgh} \nabla^c \hat{F}^{df}{}_{ij} \nabla^i \hat{F}^{eghj}
\end{aligned}$$

$$\begin{aligned}
C_{13} &= \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^b{}_{fgh} \nabla^c \hat{F}^{fg}{}_{ij} \nabla^d \hat{F}^{ehij} \\
C_{14} &= \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^b{}_{fgh} \nabla_i \hat{F}^{cdf}{}_j \nabla^i \hat{F}^{eghj} \\
C_{15} &= \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{fghi} \nabla^b \hat{F}^{cdf}{}_j \nabla^g \hat{F}^{ehij} \\
C_{16} &= \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{fghi} \nabla^b \hat{F}^{fgh}{}_j \nabla^i \hat{F}^{cdej} \\
C_{17} &= \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^{ac}{}_{fg} \nabla_h \hat{F}^{df}{}_{ij} \nabla^i \hat{F}^{eghj} \\
C_{18} &= \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^a{}_{fgh} \nabla_i \hat{F}^{cdf}{}_j \nabla^j \hat{F}^{eghi} \\
C_{19} &= \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^c{}_{fgh} \nabla^f \hat{F}^{dg}{}_{ij} \nabla^i \hat{F}^{aehj} \\
C_{20} &= \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^c{}_{fgh} \nabla^f \hat{F}^{ad}{}_{ij} \nabla^i \hat{F}^{eghj} \\
C_{21} &= \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^c{}_{fgh} \nabla^d \hat{F}^{fg}{}_{ij} \nabla^h \hat{F}^{aeij} \\
C_{22} &= \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^{fghi} \nabla^f \hat{F}^{cde}{}_j \nabla^j \hat{F}^{aghi} \\
C_{23} &= \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^{fghi} \nabla^f \hat{F}^{cdg}{}_j \nabla^j \hat{F}^{aehi} \\
C_{24} &= \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^{fghi} \nabla^f \hat{F}^{acd}{}_j \nabla^g \hat{F}^{ehij}
\end{aligned}$$

$$\begin{aligned}
\tilde{Z}_1 &= -C_3 + 12C_4 - 6C_5 + 72C_7 - 9C_8 - C_9 + 54C_{10} - 6C_{11} - 144C_{12} + 18C_{14} - 27C_{18} + 18C_{21}, \\
\tilde{Z}_2 &= C_3 - 6C_5 - 18C_7 + 9C_8 + C_9 + 6C_{11} + 9C_{18} + 18C_{23}, \\
\tilde{Z}_3 &= C_1 + 96C_4 - 96C_5 + 32C_6 + 288C_7 + 64C_9 + 32C_{22}, \\
\tilde{Z}_4 &= -C_{10} + 2C_{12} + 2C_{20}, \\
\tilde{Z}_5 &= C_7 + C_{10} + 4C_{19}, \\
\tilde{Z}_6 &= -C_7 - C_{10} + 2C_{17}, \\
\tilde{Z}_7 &= C_1 - 8C_2 + 32C_6 + 32C_9 + 32C_{16}, \\
\tilde{Z}_8 &= -C_2 - 12C_4 + 12C_5 - 4C_9 - 12C_{11} + 36C_{15}, \\
\tilde{Z}_9 &= C_{10} - 2C_{12} + C_{13}.
\end{aligned}$$

$$t_{8s} t_{8s} \hat{\mathcal{R}}^4 = t_{8s} t_{8s} \hat{R}^4 + 6 t_{8s} t_{8s} \hat{\mathcal{R}}^2 \hat{\mathcal{R}}'^2 + t_{8s} t_{8s} \hat{\mathcal{R}}'^4,$$

$$\hat{\mathbb{R}}' = \nabla_a \hat{F}_{bcde} \left(\frac{1}{12} \hat{\Gamma}^{ab} \otimes \hat{\Gamma}^{cde} + \frac{i}{48} \hat{\Gamma}^a \otimes \hat{\Gamma}^{bcde} \right)$$

$$t_{8s} t_{8s} \hat{R}^4 = t_8 t_8 \hat{R}^4$$



$$\begin{aligned}
6t_{8s}t_{8s}\hat{R}^2\hat{R}'^2 &= \frac{1}{24}t_{a_1\cdots a_8}\hat{R}^{a_1a_2}{}_{ab}\hat{R}^{a_3a_4}{}_{cd}\nabla^{a_5}\hat{F}^{a_6}{}_{efg}\nabla^{a_7}\hat{F}^{a_8}{}_{hij}t_{8s}(\hat{\Gamma}^{ab}\hat{\Gamma}^{cd}\hat{\Gamma}^{efg}\hat{\Gamma}^{hij}) \\
&\quad - \frac{3}{32}\hat{R}^{ab}{}_{ef}\hat{R}^{cd}{}_{gh}t_{8s}(\hat{\Gamma}_{ab}\hat{\Gamma}_{cd}\hat{\Gamma}^i\hat{\Gamma}^j)t_{8s}(\hat{\Gamma}^{ef}\hat{\Gamma}^{gh}\nabla_i\hat{F}\nabla_j\hat{F}). \\
t_{8s}(\hat{\Gamma}^{ab}\hat{\Gamma}^{cd}M_1M_2) &= -\frac{1}{4}\text{tr}(\hat{\Gamma}^{abcd}M_1M_2) - \frac{1}{8}\text{tr}(\hat{\Gamma}^{ab}M_1\hat{\Gamma}^{cd}M_2) + \frac{1}{64}\text{tr}(\hat{\Gamma}^{ab}M_1)\text{tr}(\hat{\Gamma}^{cd}M_2) \\
&\quad + \frac{1}{256}\text{tr}(\hat{\Gamma}^{ab}\{M_1,\hat{\Gamma}^e\})\text{tr}(\{\hat{\Gamma}_e,M_2\}\hat{\Gamma}^{cd}) + (1 \leftrightarrow 2) \\
6t_{8s}t_{8s}\hat{R}^2\hat{R}'^2 &= 8(24B_1 + 24B_2 - 6B_4 - 48B_6 - 48B_8 + 24B_9 - 24B_{10} + 24B_{11} \\
&\quad - 18B_{12} - 12B_{13} + 8B_{16} + 8B_{17} - 8B_{20} + 4B_{21} + B_{22} + 4B_{23} + B_{24}), \\
&\quad 16(-12B_5 + 3B_{12} + 6B_{14}) - 4bB_4, \\
16(-24B_5 - 48B_8 - 24B_{10} - 6B_{12} - 12B_{13} + 12B_{14} + 8B_{16} - 4B_{20} + B_{22} + 4B_{23} + B_{24}). \\
t_{8s}t_{8s}\hat{R}'^4 &= \frac{1}{6^4}t_8^{a_1a_2b_1b_2c_1c_2d_1d_2}\nabla_{a_1}\hat{F}_{a_2a_3a_4a_5}\nabla_{b_1}\hat{F}_{b_2b_3b_4b_5}\nabla_{c_1}\hat{F}_{c_2c_3c_4c_5}\nabla_{d_1}\hat{F}_{d_2d_3d_4d_5} \\
&\quad \times t_{8s}(\hat{\Gamma}^{a_3a_4a_5}\hat{\Gamma}^{b_3b_4b_5}\hat{\Gamma}^{c_3c_4c_5}\hat{\Gamma}^{d_3d_4d_5}) \\
&\quad + \frac{2}{3}\nabla_{[a}\hat{F}_{c]a_3a_4a_5}\nabla^{[a}\hat{F}^{d]b_3b_4b_5}t_{8s}(\hat{\Gamma}^{a_3a_4a_5}\hat{\Gamma}_{b_3b_4b_5}\nabla^c\hat{F}\nabla_d\hat{F}) \\
&\quad - \frac{1}{6}\nabla_{[a}\hat{F}_{b]a_3a_4a_5}\nabla^a\hat{F}^{bb_3b_4b_5}t_{8s}(\hat{\Gamma}^{a_3a_4a_5}\hat{\Gamma}_{b_3b_4b_5}\nabla_c\hat{F}\nabla^c\hat{F}) \\
&\quad + \frac{3}{2}t_{8s}(\nabla_a\hat{F}\nabla^a\hat{F}\nabla_b\hat{F}\nabla^b\hat{F}). \\
-36t_8^{a_1a_2b_1b_2c_1c_2d_1d_2}\nabla_{a_1}\hat{F}_{a_2cd}{}^a\nabla_{b_1}\hat{F}_{b_2efa}\nabla_{c_1}\hat{F}_{c_2}{}^{b[cd}\nabla_{d_1}\hat{F}_{d_2b}{}^{ef]} \\
&\quad + \frac{2}{3}t_8^{a_1a_2b_1b_2c_1c_2d_1d_2}\nabla_{a_1}\hat{F}_{a_2efg}\nabla_{b_1}\hat{F}_{b_2}{}^{efg}\nabla_{c_1}\hat{F}_{c_2hij}\nabla_{d_1}\hat{F}_{d_2}{}^{hij} \\
&= -\frac{3}{16}C_1 + \frac{4}{3}C_2 + \frac{10}{3}C_3 - 2C_4 - 2C_5 - 6C_6 + 6C_7 - 18C_8 - \frac{10}{3}C_9 - 72C_{10} \\
&\quad + 12C_{11} - 144C_{12} - 48C_{13} + 66C_{14} + 60C_{15} + 4C_{16} + 84C_{17} - 96C_{19} + 48C_{20} \\
&\quad + 18C_{21} + 2C_{22} + 18C_{23} + 36C_{24} \\
\frac{2}{3}\nabla_{[a}\hat{F}_{c]a_3a_4a_5}\nabla^{[a}\hat{F}^{d]b_3b_4b_5}t_{8s}(\hat{\Gamma}^{a_3a_4a_5}\hat{\Gamma}_{b_3b_4b_5}\nabla^c\hat{F}\nabla_d\hat{F}) \\
&= C_2 - \frac{4}{3}C_3 - 4C_4 + 8C_5 - 4C_6 - 72C_7 + 72C_8 - \frac{4}{3}C_9 - 24C_{10} + 4C_{11} - 96C_{12} - 24C_{13} \\
&\quad - 6C_{14} + 36C_{15} + \frac{4}{3}C_{16} - 30C_{18} - 120C_{19} + 48C_{20} - 30C_{21} + \frac{8}{3}C_{22} - 90C_{23} - 24C_{24} \\
&\quad - \frac{1}{6}\nabla_{[a}\hat{F}_{b]a_3a_4a_5}\nabla^a\hat{F}^{bb_3b_4b_5}t_{8s}(\hat{\Gamma}^{a_3a_4a_5}\hat{\Gamma}_{b_3b_4b_5}\nabla_c\hat{F}\nabla^c\hat{F}) \\
&= -\frac{1}{8}C_1 + \frac{1}{3}C_2 + \frac{8}{3}C_3 - 24C_4 + 4C_5 + 36C_7 - 36C_8 + 36C_{10} + 6C_{14} - 24C_{15} \\
&\quad - 48C_{17} - 6C_{18} - 24C_{19} + 6C_{21} - 6C_{23} - 24C_{24}
\end{aligned}$$



$$\begin{aligned}
& \frac{3}{2} t_{8s} (\nabla_a \hat{F} \nabla^a \hat{F} \nabla_b \hat{F} \nabla^b \hat{F}) \\
&= -\frac{1}{48} C_1 + \frac{2}{3} C_3 - 6C_4 + 2C_5 + \frac{2}{3} C_6 + 18C_7 - 18C_8 + \frac{2}{3} C_9 + 12C_{10} + 6C_{14} \\
&\quad - 12C_{15} - 12C_{17} + 18C_{21} - \frac{2}{3} C_{22} + 18C_{23} \\
&\quad - \frac{1}{12} C_1 + \frac{8}{3} C_3 + 8C_4 + 8C_5 + \frac{8}{3} C_6 - 24C_7 + 24C_8 - \frac{8}{3} C_9 - 144C_{10} + 24C_{14} \\
&\quad + 144C_{15} + 144C_{17} - 24C_{21} + 8C_{22} - 24C_{23} \\
&\hat{L}_{\hat{F}^4} + \sum_i d_i \tilde{Z}_i = \frac{8}{3} (3C_5 + C_6 - 9C_8 + C_9 - 72C_{12} + 9C_{14} + 18C_{17} - 9C_{18} - 72C_{19} - C_{22})
\end{aligned}$$

$$c_2 = 12, c_3 = 24, c_5 = \frac{1}{3}, c_7 = -\frac{10}{3}, c_8 = \frac{68}{3}, c_9 = 12$$

$$d_1 = -4, d_2 = -\frac{20}{3}, d_3 = \frac{1}{4}, d_4 = -120, d_5 = 12, d_6 = -60, d_7 = -\frac{1}{6}$$

$$d_8 = 2, d_9 = 144$$

$$t_8 t_8 R^4$$

$$e^a = e^{k\Phi} e'^a$$

$$R_{abcd} = e^{-2k\Phi} (R'_{abcd} + D_{abcd} + \cdots) D^{ab}{}_{cd} = 4k\delta_{[c}^{[a}\nabla_{d]}\nabla^{b]} \Phi$$

$$t_8 t_8 R^4 = t_8 t_8 R^4 + 4 t_8 t_8 D R^3 + 6 t_8 t_8 D^2 R^2 + 4 t_8 t_8 D^3 R' + t_8 t_8 D^4 + \cdots$$

$$t_8 t_8 D R^{(1)} R^{(2)} R^{(3)},$$

$$4k\Phi t_8 t_8 \nabla^2 (R^{(1)} R^{(2)} R^{(3)})$$

$$\begin{aligned}
t_8 t_8 \nabla^2 (R^{(1)} R^{(2)} R^{(3)}) = & 32 \delta_{[a}^e \nabla_{b]} \nabla^f \left(R^{(1)bc}{}_{fg} R_{cd}^{(2)gh} R^{(3)da}{}_{he} \right) + 64 \delta_{[a}^{[e} \nabla_{b]} \nabla^f \left(R_{cdfg}^{(1)} R^{(2)dagh} R^{(3)bc}{}_{he} \right) \\
& - 32 \delta_{[a}^e \nabla_{b]} \nabla^f \left(R^{(1)bc}{}_{ef} R_{cdgh}^{(2)} R^{(3)dagh} \right) - 16 \delta_{[a}^e \nabla_{b]} \nabla^f \left(R^{(1)bcgh} R_{cdef}^{(2)} R^{(3)da}{}_{gh} \right) \\
& + 4 \delta_a^e \nabla_b \nabla^f \left(R^{(1)abgh} R_{cdef}^{(2)} R^{(3)cd}{}_{gh} \right) + 2 \delta_a^e \nabla_b \nabla^f \left(R^{(1)ab}{}_{ef} R_{cdgh}^{(2)} R^{(3)cdgh} \right) \\
& + \text{perm}(1,2,3).
\end{aligned}$$

$$\begin{aligned}
& 32 \delta_{[a}^e \nabla_{b]} \nabla^f \left(R_{fg}^{(1)bc} R_{cd}^{(2)gh} R_{he}^{(3)da} \right) + \text{perm}(1,2,3) \\
& = 16 \nabla_b \nabla_f \left(R^{(1)bcfg} R_{cdgh}^{(2)} R^{(3)dh} \right) - 16 \nabla_a \nabla^f \left(R_{fg}^{(1)bc} R_{cd}^{(2)gh} R_{hb}^{(3)da} \right) + \text{perm}(1,2,3),
\end{aligned}$$

$$\begin{aligned}
& -16 \nabla_a \nabla^f \left(R^{(1)bc}{}_{fg} R_{cd}^{(2)gh} R^{(3)da}{}_{hb} \right) + \text{perm}(1,2,3) \\
& = 8 \nabla_a \left(R^{(1)bc}{}_{fg} \nabla^h R_{cd}^{(2)fg} R^{(3)da}{}_{hb} \right) + 8 \nabla_a \left(R^{(1)bc}{}_{fg} R_{cd}^{(2)gh} \nabla^a R^{(3)fd}{}_{hb} \right) + \text{perm}(1,2,3) + \cdots,
\end{aligned}$$



$$\begin{aligned}
& 8\nabla_a \nabla^h \left(R^{(1)bc} {}_f g R_{cd}^{(2)fg} R^{(3)da} {}_{hb} \right) + 4\nabla^a \left(\nabla_f R^{(1)cd} {}_g h R^{(2)fbgh} R_{bacd}^{(3)} \right) \\
& - 8\nabla_a \left(R_f^{(1)cb} {}_g R_{cd}^{(2)gh} \nabla^a R^{(3)df} {}_{hb} \right) + 2\nabla_a \left(R_{cdef}^{(1)} R^{(2)cdgh} \nabla^a R^{(3)ef} {}_{gh} \right) + \text{perm}(1,2,3) + \dots \\
& = 8\nabla_a \nabla^h \left(R^{(1)bc} {}_f g R_{cd}^{(2)fg} R^{(3)da} {}_{hb} \right) + 4\nabla^a \nabla_f \left(R^{(1)cd} {}_g h R^{(2)fbgh} R_{bacd}^{(3)} \right) \\
& - \frac{8}{3} \nabla^2 \left(R_f^{(1)cb} {}_g R_{cd}^{(2)gh} R^{(3)df} {}_{hb} \right) + \frac{4}{3} \nabla^2 \left(R_{cdef}^{(1)} R^{(2)cdgh} R^{(3)ef} {}_{gh} \right) + \text{perm}(1,2,3) + \dots
\end{aligned}$$

$$\begin{aligned}
& 32\delta_{[a}^e \nabla_{b]} \nabla^f \left(R^{(1)bc} {}_f g R_{cd}^{(2)gh} R^{(3)da} {}_{he} \right) + \text{perm}(1,2,3) \\
& = 16\nabla_b \nabla^f \left(R^{(1)bc} {}_f g R_{cdgh}^{(2)} R^{(3)dh} \right) + 8\nabla_a \nabla^h \left(R^{(1)bc} {}_f g R_{cd}^{(2)fg} R^{(3)da} {}_{hb} \right) \\
& + 4\nabla^a \nabla_f \left(R^{(1)cd} {}_g h R^{(2)fbgh} R_{bacd}^{(3)} \right) - \frac{8}{3} \nabla^2 \left(R_f^{(1)cb} {}_g R_{cd}^{(2)gh} R^{(3)df} {}_{hb} \right) \\
& + \frac{4}{3} \nabla^2 \left(R_{cdef}^{(1)} R^{(2)cdgh} R^{(3)ef} {}_{gh} \right) + \text{perm}(1,2,3) + \dots
\end{aligned}$$

$$\begin{aligned}
& 64\delta_{[a}^{[e} \nabla_{b]} \nabla^f] \left(R_{cdfg}^{(1)} R^{(2)dagh} R_{he}^{(3)bc} \right) \\
& = 16\nabla_b \nabla^f \left(R_{cdfg}^{(1)} R^{(2)dagh} R_{ha}^{(3)bc} \right) - 16\nabla_b \nabla^e \left(R_{cdag}^{(1)} R^{(2)dagh} R^{(3)bc} {}_{he} \right) \\
& + 16\nabla_a \nabla^e \left(R_{cdbg}^{(1)} R^{(2)dagh} R^{(3)bc} {}_{he} \right) + 16\nabla_a \nabla^f \left(R_{cdfg}^{(1)} R^{(2)dagh} R^{(3)c} {}_h \right) \\
& = 8\nabla_b \nabla^e \left(R_{cdag}^{(1)} R^{(2)dagh} R^{(3)bc} {}_{he} \right) + 4\nabla_b \nabla^f \left(R_{cfdg}^{(1)} R^{(2)dagh} R^{(3)bc} {}_{ha} \right) \\
& - 4\nabla_a \nabla^e \left(R_{cdgh}^{(1)} R^{(2)ghab} R^{(3)cd} {}_{be} \right) + 16\nabla_a \nabla^f \left(R_{cdfg}^{(1)} R^{(2)dagh} R^{(3)c} {}_h \right)
\end{aligned}$$

$$\begin{aligned}
t_8 t_8 \nabla^2 (R^{(1)} R^{(2)} R^{(3)}) &= 16\nabla_b \nabla_f \left(R^{(1)dh} R^{(2)bcfg} R_{cdgh}^{(3)} \right) + 16\nabla_b \nabla^f \left(R^{(1)d} {}_h R_{cdfg}^{(2)} R^{(3)bcgh} \right) \\
& + 16\nabla_b \nabla_f \left(R^{(1)cf} R_{cdgh}^{(2)} R^{(3)dbgh} \right) + 2\nabla^b \nabla^f \left(R_{bf}^{(1)} R_{cdgh}^{(2)} R^{(3)cdgh} \right) \\
& - \frac{8}{3} \nabla^2 \left(R_f^{(1)cb} {}_g R_{cd}^{(2)gh} R^{(3)df} {}_{hb} \right) + \frac{4}{3} \nabla^2 \left(R_{cdef}^{(1)} R^{(2)cdgh} R^{(3)ef} {}_{gh} \right) \\
& + \text{perm}(1,2,3) + \dots
\end{aligned}$$

$$\begin{aligned}
t_8 t_8 D R^{(1)} R^{(2)} R^{(3)} &= 4k \Phi t_8 t_8 \nabla^2 (R^{(1)} R^{(2)} R^{(3)}) + \dots \\
& = 8k \nabla^a \nabla^b \Phi \left[8R^{(1)cd} R_{aghb}^{(2)} R^{(3)cghd} + 8R_{cd}^{(1)} R_{ag}^{(2)hc} R_{bh}^{(3)gd} \right. \\
& \quad \left. - 8R_{ac}^{(1)} R^{(2)cdgh} R_{bdgh}^{(3)} + R_{ab}^{(1)} R_{cdgh}^{(2)} R^{(3)cdgh} + \text{perm}(1,2,3) \right] + \dots,
\end{aligned}$$

$$\begin{aligned}
& 8k \nabla^a \nabla^b \Phi \left(8R^{(1)cd} D_{aghb} R^{(3)cghd} + 8R_{cd}^{(1)} D_{ag}^{hc} R_{bh}^{(3)gd} - 8R_{ac}^{(1)} D_{cdgh} R_{bdgh}^{(3)} + R_{ab}^{(1)} D_{cdgh} R^{(3)cdgh} \right) \\
& = 32k^2 \nabla^a \nabla^b \Phi \left(6R^{(1)cd} \nabla_a \nabla^e \Phi R_{bcde}^{(3)} + 3R_{ab}^{(1)} \nabla^c \nabla^d \Phi R_{cd}^{(3)} - 2R^{(1)cd} \nabla_a \nabla_c \Phi R_{bd}^{(3)} \right) + \dots \\
& = 32k^2 \nabla^a \nabla^b \Phi \left(3R_{ab}^{(1)} \nabla^c \nabla^d \Phi R_{cd}^{(3)} - 2R^{(1)cd} \nabla_a \nabla_c \Phi R_{bd}^{(3)} \right) + \dots
\end{aligned}$$

$$R_{ef} = R'_{ef} + D_{ef} = l \nabla_e \nabla_f \Phi + \dots,$$

$$D_{ef} = k(D-2) \nabla_e \nabla_f \Phi + \dots,$$

$$t_8 t_8 R^4 = t_8 t_8 R^4 + 96k(2l - k(D-2)) \nabla^a \nabla^b \Phi \nabla^c \nabla^d \Phi X_{abcd} + \dots$$



$$X_{ab}^{cd} = 8R'_{aghb}R'^{cghd} + 8R'_{ag}{}^{hc}R'_{bh}{}^{gd} - 8\delta_a^c R'^{dfgh}R'_{bfg h} + \delta_a^c \delta_b^d R'_{efgh}R'^{efgh} \\ + 2k(2l - k(D - 2))[3\nabla_a \nabla_b \Phi \nabla^c \nabla^d \Phi - 2\nabla_a \nabla^c \Phi \nabla_b \nabla^d \Phi]$$

$$d'_1=d'_2=\frac{16}{3}, d'_3=-\frac{1}{3}, d'_4=72, d'_5=-60, d'_6=108, d'_7=0, d'_8=-2, d'_9=-192$$

$$\begin{aligned}\nabla^a\nabla^b\Phi\nabla^c\nabla^d\Phi\nabla_a\nabla^e\Phi R_{bcde}^{(3)} &= -\nabla^d\nabla^a\nabla^b\Phi\nabla^c\Phi\nabla_a\nabla^e\Phi R_{bcde}^{(3)}+\cdots \\ &= -\frac{1}{2}\nabla^d\nabla^a\nabla^b\Phi\nabla_a(\nabla^c\Phi\nabla^e\Phi)R_{bcde}^{(3)}+\cdots \\ &= -\frac{1}{4}\nabla^2(\nabla^d\nabla^b\Phi\nabla^c\Phi\nabla^e\Phi)R_{bcde}^{(3)}+\frac{1}{4}\nabla^d\nabla^b\Phi\nabla^2(\nabla^c\Phi\nabla^e\Phi)R_{bcde}^{(3)}+\cdots \\ &= -\frac{1}{2}\nabla^d\nabla^c\Phi\nabla^a\nabla^b\Phi\nabla_a\nabla^e\Phi R_{bcde}^{(3)}+\cdots,\end{aligned}$$

Supergravedad cuántica relativista en D=11.

$$\langle T_{\mu\nu}\rangle_{\mathcal{M}}.$$

$$G_{\mu\nu}=8\pi G\big[T_{\mu\nu}^{\rm Cl}+\langle T^{\mu\nu}\rangle_{\mathcal{M}}\big]$$

$$T_{\mu\nu}^{\rm Cl}\sim \langle T^{\mu\nu}\rangle_{\mathcal{M}}\sim \epsilon M_P^D, \epsilon\ll 1.$$

$$g^{(0)}\rightarrow g^{(0)}+\epsilon g^{(1)}+\cdots, T_{\mu\nu}=\epsilon T_{\mu\nu}^{(0)}+\epsilon^2 T_{\mu\nu}^{(1)}+\cdots$$

$$G_{\mu\nu}^{(n)}=8\pi GT_{\mu\nu}^{(n-1)},$$

$$\mathcal{M}=\mathbb{R}^{D-k}\times \mathcal{N}_k$$

$$ds_{\mathcal{M}}^2=ds_{\Lambda}^2+ds_{\mathcal{N}_k}^2$$

$$\langle T_{\mu\nu}\rangle_{\mathbb{R}^{D-k}\times\mathcal{N}_k}=\hat{\rho}g_{\mu\nu}^{\mathbb{R}^{D-k},\Lambda}+T_{\mu\nu}^{\mathcal{N}_k}$$

$$V_{\rm Cas}=\int\,d^kz\sqrt{-g_0}\hat{\rho}$$

$$V_{\rm Cas}\sim -\frac{\mathcal{C}}{R^d},$$

$$V^{(d)}=V_{\rm Fluxes}+V_{\rm Cas}\,,$$

$$\frac{n_p^2}{R^{2p-k}}$$

$$\frac{V^{(d)}}{M_P^d}\sim \frac{1}{R^{\bar{d}-2}k}\left(\sum_p\frac{n_p^2}{R^{2p-k}}-\frac{\mathcal{C}}{R^{\bar{d}}}\right)$$



$$(R,-R,0,0), (0,R,-R,0), (0,0,R,-R) \text{ and } (0,0,0,2R)$$

$$\frac{V^{(7d)}}{M_7^7}=\left(\frac{\ell_{11}^9}{2R^4}\right)^{\frac{7}{5}}\left[\left(\frac{\pi}{2}\right)^{\frac{2}{3}}\frac{n_4^2}{2R^4\ell_{11}^3}-\frac{\mathcal{C}}{R^7}\right], \mathcal{C}\approx 8.827$$

$$M_7^5\equiv\frac{\mathrm{Vol}(T^4)}{\ell_{11}^9}=\frac{2R^4}{\ell_{11}^9}$$

$$R\approx \frac{2.58\ell_{11}}{n_4^{2/3}}, \frac{V^{(7d)}}{M_7^7}\approx 6.85\cdot 10^{-6}n_4^{42/5}$$

$$\frac{1}{H_0}=\sqrt{\frac{15}{V^{(7d)}}}\approx 1480 \text{ (7 d Planck lengths)},$$

$$\int |G_4|^{2n}$$

$$\int R^k_{\mu\nu\rho\sigma}|G_4|^{2n}$$

$$\mathcal{F}_k=\mathbb{R}^k/\mathcal{B}$$

$$b\in ISO(k)\colon \vec{z}\rightarrow \mathbf{D}\vec{z}+\vec{b}, \mathbf{D}\in SO(k)$$

$$T^k=\mathbb{R}^k/\Lambda$$

$$\Gamma\equiv\mathcal{B}/\sim, b\sim b'\text{ iff }b'\circ b^{-1}\in\Lambda.$$

$$0\longrightarrow \Lambda \longrightarrow \mathcal{B} \longrightarrow \Gamma \longrightarrow 1$$

$$\mathbf{D}_\gamma \vec{z} + \vec{b}_\gamma = \vec{z} + \vec{l}, \vec{l} \in \Lambda,$$

$$(\mathbf{D}_\gamma - \mathbf{I})\vec{z} + \vec{b}_\gamma = 0 \mathrm{mod} \Lambda$$

$$z\rightarrow -\bar{z}+\frac{i}{2}, z\rightarrow z+\beta$$

$$b_1\colon \vec{z}\rightarrow \begin{pmatrix}-1&0\\0&1\end{pmatrix}\vec{z}+\begin{pmatrix}0\\1/2\end{pmatrix}, b_2\colon \vec{z}\rightarrow \vec{z}+\begin{pmatrix}\beta\\0\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\0&-1&0\\0&0&1\end{pmatrix}\text{ or }\begin{pmatrix}1&0&1\\0&-1&0\\0&0&1\end{pmatrix}$$

$$\mathbf{D}_\gamma \sim \begin{pmatrix} \mathbf{B}_1 & 0 & 0 & \vdots & \vdots & \vdots \\ 0 & \ddots & 0 & \vec{v}_1 & \cdots & \vec{v}_l \\ 0 & 0 & \mathbf{B}_i & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \ddots & \vdots & \\ \vdots & 0 & \vdots & \cdots & \mathbf{I}_{l\times l} & \cdots \\ 0 & \cdots & 0 & \vdots & \ddots & \end{pmatrix}$$



$$\mathbf{D}_\gamma \vec{z} + \vec{b}_\gamma \rightarrow \mathbf{U} \cdot \mathbf{D}_\gamma \cdot \mathbf{U}^{-1} \vec{z} - \mathbf{U} \cdot \mathbf{D}_\gamma \cdot \mathbf{U}^{-1} \vec{u} + \mathbf{U} \vec{b}_\gamma + \vec{u}$$

$$\vec{b}_\gamma \rightarrow \mathbf{D}_\gamma^q \vec{b}_\gamma + (\mathbf{I} - \mathbf{D}_\gamma) \vec{u} = \mathbf{D}_\gamma^q \vec{b}_\gamma + \vec{u}_\perp$$

$$\vec{b}_\gamma = \mathbf{D}_\gamma \vec{b}_\gamma \bmod \Lambda_{\vec{u}_\perp}$$

$$\frac{k(k+1)}{2}$$

$$\mathbf{D}_\gamma^T \cdot \mathbf{G} \cdot \mathbf{D}_\gamma = \mathbf{G}, \forall \mathbf{D}_\gamma$$

$$\delta \mathbf{G} \cdot \mathbf{D}_\gamma = \left(\mathbf{D}_\gamma^T\right)^{-1} \cdot \delta \mathbf{G}$$

$$\mathcal{F}_k = T^{l-1} \times \left(\frac{T^{k-l+1} \times S^1}{\mathbb{Z}_n} \right)$$

$$\psi(\vec{z}+\vec{n})=e^{2\pi i\vec{h}\cdot\vec{n}}\psi(\vec{z})$$

$$\psi(\vec{z}) \rightarrow \mathcal{D}_{\mathbf{g}} \psi(\mathbf{D}_{\mathbf{g}} \vec{z})$$

$$\mathcal{D}_{\mathbf{g}}\psi(\mathbf{D}_{\mathbf{g}}(\vec{z}+\vec{n}))=e^{2\pi i\vec{h}\cdot\mathbf{D}_{\mathbf{g}}\vec{n}}\mathcal{D}_{\mathbf{g}}\psi(\mathbf{D}_{\mathbf{g}}\vec{z})$$

$$(\mathbf{D}_g^T-\mathbf{I})\cdot\vec{h}\in\mathbb{Z}^{k-l+1}$$

$$s_{\mathbf{g}} \equiv 2p\vec{h} \cdot \vec{b}_g \bmod 2\mathbb{Z}$$

$$S=\int\,\sqrt{-g}\mathcal{L}=\frac{1}{2}\int\,\sqrt{-g}\phi\cdot\mathcal{D}\cdot\phi=0$$

$$\mathcal{D}\phi=0$$

$$T_{\mu\nu}\equiv\frac{-2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}=\phi\cdot\left[\frac{\delta\mathcal{D}}{\delta g^{\mu\nu}}-\frac{1}{2}g_{\mu\nu}\mathcal{D}\right]\phi$$

$$\langle T_{\mu\nu}(x)\rangle=\lim_{y\rightarrow x}\left[\frac{\delta\mathcal{D}}{\delta g^{\mu\nu}}-\frac{1}{2}g_{\mu\nu}\mathcal{D}\right]\langle\phi(x)\phi(y)\rangle$$

$$T_{\mu\nu}^{\rm sc}=\frac{1}{2}\big(2\nabla_\mu\phi\nabla_\nu\phi-g_{\mu\nu}\nabla^\rho\phi\nabla_\rho\phi\big),$$

$$\langle T_{\mu\nu}^{\rm sc}\rangle\rightarrow\frac{1}{2}\lim_{x\rightarrow y}(\nabla_\mu\nabla'_\nu+\nabla'_\mu\nabla_\nu-g_{\mu\nu}\nabla^\rho\nabla'_\rho)G(x,y),$$

$$\hat{\rho} = -\langle T_{00} \rangle = -\sum_{\text{Polarizations}} (-1)^F \langle T_{00}^{\text{sc}} \rangle,$$

$$\mathcal{N}_k=\frac{\tilde{\mathcal{N}}_k}{\mathcal{B}}$$



$$G_{\mathcal{N}_k}(x,y)=\sum_{b\in\mathcal{B}}G_{\tilde{\mathcal{N}}_k}(x,b(y))$$

$$G_{\mathcal{N}_k}(b(x),y)=\sum_{b\in\mathcal{B}}G_{\tilde{\mathcal{N}}_k}(b(x),b(y))=\sum_{b\in\mathcal{B}}G_{\tilde{\mathcal{N}}_k}(x,b^{-1}\circ b(y))$$

$$G_{\mathcal{N}_k}^{\mathrm{ren.}}(x,y)=\sum_{b\neq \mathrm{Id.}\in\mathcal{B}}G_{\tilde{\mathcal{N}}_k}(x,b(y)),$$

$$G_{\mathcal{N}_k,\varphi}^{\mathrm{ren.}}(x,y)=\sum_{b\neq \mathrm{Id.}\in\mathcal{B}}e^{i\varphi(b)}G_{\tilde{\mathcal{N}}_k}(x,b(y)),$$

$$\begin{aligned} G_{\mathcal{N}_k,\varphi}^{\mathrm{ren.}}(b_0(x),b_0(y))&=\sum_{b\neq \mathrm{Id.}\in\mathcal{B}}e^{i\varphi(b)}G_{\tilde{\mathcal{N}}_k}(b_0(x),b\circ b_0(y))\\ &=\sum_{b\neq \mathrm{Id.}\in\mathcal{B}}e^{i\varphi(b)}G_{\tilde{\mathcal{N}}_k}(x,b_0^{-1}\circ b\circ b_0(y)) \end{aligned}$$

$$e^{i\varphi(b_0^{-1}\circ b\circ b_0(y))}=e^{i\varphi(b)},\forall b_0,b$$

$$G_{\mathcal{N}_k}^{\mathrm{ren.}ab}(x,y)=\sum_{b\neq \mathrm{Id.}\in\mathcal{B}}R_c^a(b)G_{\tilde{\mathcal{N}}_k}^{cb}(x,b(y)),$$

$$1\longrightarrow \mathcal{Z}\longrightarrow \hat{\mathcal{B}}\longrightarrow \mathcal{B}\longrightarrow 1$$

$$\phi^a(\vec{z})\rightarrow R_c^a(b)\phi^c(b(\vec{z}))=R_c^a(b)\phi^c(\mathbf{D}_\gamma\vec{z}+\vec{b}_\gamma+\vec{n})$$

$$\hat{\rho}(\vec{z})=-\frac{\Gamma\left(\frac{D}{2}\right)}{2\pi^{D/2}}\sum_{\substack{\gamma\in\Gamma\\ \vec{n}\in\mathbb{Z}^k}}^{\sim}\frac{\mathrm{Tr}(\mathbf{R})}{\left|\vec{z}-\left(\mathbf{D}_\gamma\vec{z}+\vec{b}_\gamma+\vec{n}\right)\right|^D}$$

$$SO(D-1,1)\rightarrow SO(D-k-1,1)\times SO(k)$$

$$\mathbf{r}_{SO(D-1,1)}\rightarrow\sum_i\mathbf{r}_{SO(D-k-1,1)}^i\otimes\tilde{\mathbf{r}}_{SO(k)}^i$$

$$\mathrm{Tr}_{\mathbf{r}_{SO(D-1,1)}}=\sum_i g_{\mathbf{r}_{SO(D-k-1,1)}^i}\mathrm{Tr}_{\tilde{\mathbf{r}}_{SO(k)}^i}$$

$$T_{\vec{n}}\rightarrow e^{2\pi i\vec{h}\cdot\vec{n}}\cdot\mathbf{I}$$



$$\hat{\rho}(\vec{z}) = -\frac{\Gamma\left(\frac{D}{2}\right)}{2\pi^{D/2}} \sum_{\substack{\gamma \in \Gamma \\ \vec{n} \in \mathbb{Z}^k}}^{\sim} \frac{\text{Tr}_{\mathbf{r}}(\mathbf{D}_{\gamma}) e^{2\pi i \vec{h} \cdot \vec{n}}}{|\vec{z} - (\mathbf{D}_{\gamma} \vec{z} + \vec{b}_{\gamma} + \vec{n})|^D}$$

$$V_{\text{Cas}} = \int_{\mathcal{F}_k} d^k \vec{z} \sqrt{G} \hat{\rho}(\vec{z}) = -\frac{\Gamma(s)}{2\pi^s} \frac{1}{|\Gamma|} \sum_{\substack{\gamma \in \Gamma \\ \vec{n} \in \mathbb{Z}^k}}^{\sim} \int_{[0,1]^k} d^k \vec{z} \sqrt{G} \frac{\text{Tr}_{\mathbf{r}}(\mathbf{D}_{\gamma}) e^{2\pi i \vec{h} \cdot \vec{n}}}{|\vec{z} - (\mathbf{D}_{\gamma} \vec{z} + \vec{b}_{\gamma} + \vec{n})|^D}$$

$$\mathcal{E}(\gamma) \equiv -\frac{\Gamma(s)}{2\pi^s} \frac{1}{|\Gamma|} \sum_{\vec{n} \in \mathbb{Z}^k}^{\sim} \int_{[0,1]^k} dV \frac{e^{2\pi i \vec{h} \cdot \vec{n}}}{|(\mathbf{I} - \mathbf{D}_{\gamma}) \vec{z} + \vec{b}_{\gamma} + \vec{n}|^{2s}}$$

$$dV = \sqrt{G} dz^1 \wedge \dots \wedge dz^k$$

$$V_{\text{Cas}} = \sum_{\mathbf{r}} \sum_{\gamma \in \Gamma} \text{Tr}_{\mathbf{r}}(\mathbf{D}_{\gamma}) \mathcal{E}(\gamma)$$

$$M_s[f] \equiv \frac{1}{\Gamma(s)} \int_0^\infty dt t^{s-1} f$$

$$\frac{1}{|(\mathbf{I} - \mathbf{D}_{\gamma}) \vec{z} - \vec{b}_{\gamma} + \vec{n}|^{2s}} = \frac{1}{\Gamma(s)} \int_0^\infty dt t^{s-1} e^{-|(\mathbf{I} - \mathbf{D}_{\gamma}) \vec{z} - \vec{b}_{\gamma} + \vec{n}|^2 t}$$

$$\mathcal{E}(\gamma) = -\frac{\Gamma(s)}{2\pi^s} \cdot \frac{\sqrt{G}}{|\Gamma|} \frac{1}{\Gamma(s)} \int_0^\infty dt t^{s-1} \int_{[0,1]^k} d^k \vec{z} \sum_{\vec{n} \in \mathbb{Z}^k} e^{-|(\mathbf{I} - \mathbf{D}_{\gamma}) \vec{z} + \vec{b}_{\gamma} + \vec{n}|^2 t + 2\pi i \vec{h} \cdot \vec{n}}$$

$$\begin{aligned} -|\vec{n} + \vec{\alpha}|^2 t + 2\pi i \vec{h} \cdot \vec{n} &= -(\vec{n} + \vec{\alpha})^T (t\mathbf{G})(\vec{n} + \vec{\alpha}) + 2\pi i \vec{h} \cdot \vec{n} \\ &= -\left(\vec{n} + \vec{\alpha} - \frac{\pi i}{t} \mathbf{G}^{-1} \vec{h}\right)^T (t\mathbf{G}) \left(\vec{n} + \vec{\alpha} - \frac{\pi i}{t} \mathbf{G}^{-1} \vec{h}\right) \\ &\quad -2\pi i \vec{\alpha} \cdot \vec{h} - \frac{\pi^2}{t} \vec{h}^T \mathbf{G}^{-1} \vec{h} \end{aligned}$$

$$\sum_{\vec{n} \in \mathbb{Z}^k}^{\prime} e^{-|\vec{\alpha} + \vec{n}|^2 t + 2\pi i \vec{h} \cdot \vec{n}} = e^{-2\pi i \vec{\alpha} \cdot \vec{h} - \frac{\pi^2}{t} \vec{h}^T \mathbf{G}^{-1} \vec{h}} \sum_{\vec{n} \in \mathbb{Z}^k} e^{-\left|\vec{n} + \vec{\alpha} - \frac{\pi i}{t} \mathbf{G}^{-1} \vec{h}\right|^2 t}$$

$$\sum_{\vec{n} \in \Lambda} f(\vec{n}) = \sum_{\vec{q} \in \Lambda^*} \mathcal{F}[f](\vec{q}),$$

$$\Lambda^* \equiv \{\vec{q} \in \mathbb{R}^k \mid \vec{n} \cdot \vec{q} \in \mathbb{Z}, \forall \vec{n} \in \Lambda\}$$

$$\mathcal{F}[f(\mathbf{A}\vec{z} + \vec{b})](\vec{\xi}) = \frac{e^{2\pi i \vec{b} \cdot (\mathbf{A}^{-1})^T \vec{\xi}}}{|\det \mathbf{A}|} \mathcal{F}[f(\vec{z})](\mathbf{A}^T)^{-1} \vec{\xi})$$

$$\vec{v}^T (t\mathbf{G}) \vec{v} = (\sqrt{t}\mathbf{M}\vec{v})^T (\sqrt{t}\mathbf{M}\vec{v})$$



$$\mathcal{F}[f((\sqrt{t}\mathbf{M})(\vec{n} + \vec{\sigma}))](\vec{\xi}) = \frac{e^{2\pi i \vec{\sigma} \cdot \vec{q}}}{|\det \sqrt{t}\mathbf{M}|} \cdot \mathcal{F}[f(\vec{n})] \left((\sqrt{t}\mathbf{M}^T)^{-1} \vec{q} \right)$$

$$\begin{aligned} \mathcal{F}\left[e^{-|\vec{n}+\vec{\alpha}-\frac{\pi i}{t}\mathbf{G}^{-1}\vec{h}|^2 t}\right] &= \frac{\pi^{\frac{k}{2}}}{t^{\frac{k}{2}}|\det\mathbf{M}|} \cdot e^{2\pi i(\vec{\alpha}-\frac{\pi i}{t}\mathbf{G}^{-1}\vec{h})\cdot\vec{q}} \cdot e^{-\frac{\pi^2\vec{q}^T\mathbf{G}^{-1}\vec{q}}{t}} \\ &= \frac{\pi^{\frac{k}{2}}}{t^{\frac{k}{2}}\sqrt{G}} \cdot e^{-\frac{\pi^2}{t}(\vec{q}^T\mathbf{G}^{-1}\vec{q}-2\mathbf{G}^{-1}\vec{h}\cdot\vec{q})} \cdot e^{2\pi i\vec{\alpha}\cdot\vec{q}} \end{aligned}$$

$$\mathcal{E}(\gamma) = -\frac{\Gamma(s)}{2\pi^s} \cdot \frac{\sqrt{G}}{|\Gamma|} \frac{\pi^{\frac{k}{2}}}{\Gamma(s)} \int_0^\infty dt \frac{t^{s-\frac{k}{2}-1}}{\sqrt{G}} \sum_{\vec{q} \in \mathbb{Z}^k} e^{-\frac{\pi^2}{t}(\vec{q}-\vec{h})^T \mathbf{G}^{-1}(\vec{q}-\vec{h}) + 2\pi i \vec{b}_\gamma \cdot (\vec{q}-\vec{h})} \cdot \mathcal{J}_k(\mathbf{D}_\gamma)$$

$$\mathcal{J}_k(\mathbf{D}_\gamma) = \int_{[0,1]^k} \mathrm{d}^k \vec{z} e^{2\pi i [(\mathbf{I}-\mathbf{D}_\gamma)\vec{z}] \cdot (\vec{q}-\vec{h})} = \int_{[0,1]^k} \mathrm{d}^k \vec{z} e^{2\pi i \vec{z} \cdot [(\mathbf{I}-\mathbf{D}_\gamma)^T \cdot (\vec{q}-\vec{h})]}$$

$$\mathcal{J}_k(\mathbf{D}_\gamma) = \prod_{j=1}^k \frac{e^{2\pi i [(\mathbf{I}-\mathbf{D}_\gamma)^T \cdot (\vec{q}-\vec{h})]_j} - 1}{[(\mathbf{I}-\mathbf{D}_\gamma)^T \cdot (\vec{q}-\vec{h})]_j}$$

$$(\mathbf{I}-\mathbf{D}_\gamma)^T \cdot \vec{h} \in \mathbb{Z}^k$$

$$\mathcal{J}_k(\mathbf{D}_\gamma) = \prod_{j=1}^k \delta_{0,[(\mathbf{I}-\mathbf{D}_\gamma)^T \cdot (\vec{q}-\vec{h})]_j}$$

$$(\mathbf{I}-\mathbf{D}_\gamma)^T(\vec{q}-\vec{h})=0$$

$$(\mathbf{I}-\mathbf{D}_\gamma)^T\vec{\eta}=(\mathbf{I}-\mathbf{D}_\gamma)^T\vec{h}$$

$$\mathcal{E}(\gamma) = -\hat{\delta}_{\vec{h}} \frac{\Gamma(s)}{2\pi^s} \cdot \frac{\sqrt{G}}{|\Gamma|} \frac{\pi^{\frac{k}{2}}}{\Gamma(s)} \int_0^\infty dt \frac{t^{s-\frac{k}{2}-1}}{\sqrt{G}} \sum_{\vec{\kappa} \in \Lambda_{\parallel}} e^{-\frac{\pi^2}{t}(\vec{\kappa}-\vec{\beta})^T \mathbf{G}_{\parallel}^{-1}(\vec{\kappa}-\vec{\beta}) + 2\pi i \vec{b}_{\gamma}^{\parallel} \cdot (\vec{\kappa}-\vec{\beta})}$$

$$\mathcal{E}(\gamma) = -\hat{\delta}_{\vec{h}} \frac{\Gamma(s)}{2\pi^s} \cdot \frac{\sqrt{G}}{|\Gamma|} \pi^{\frac{k-k'}{2}} \int_0^\infty dt t^{s-\frac{k-k'}{2}-1} \frac{\sqrt{G_{\parallel}}}{\sqrt{G}} \sum_{\vec{\xi} \in \Xi} e^{-|\vec{\xi} + \vec{b}_{\gamma}^{\parallel}|^2 t + 2\pi i \vec{\beta} \cdot \vec{\xi}}$$

$$(\mathbf{D}_\gamma^T \vec{\kappa}) \cdot \vec{\xi} = \vec{\kappa} \cdot (\mathbf{D}_\gamma \vec{\xi})$$

$$\Xi = (\Lambda^{\parallel})^* \equiv \{ \vec{\xi} \in \mathbb{R}^k \mid \vec{\xi} \cdot \vec{\kappa} \in \mathbb{Z}, \mathbf{D}_\gamma \vec{\xi} = \vec{\xi}, \forall \vec{\kappa} \in \Lambda^{\parallel} \}$$

$$\vec{\xi} = \vec{\xi}^\perp + \vec{\xi}^{\mathrm{Inv}}$$

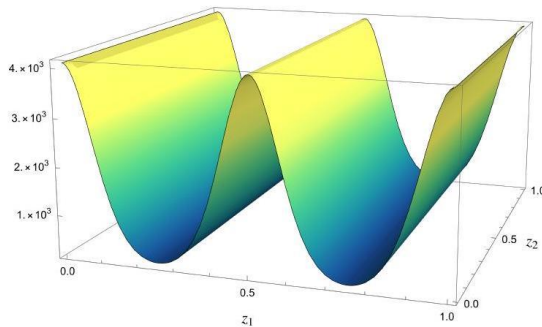
$$\vec{\kappa} \cdot \vec{\xi} = \vec{\kappa} \cdot \vec{\xi}^{\mathrm{Inv}}$$



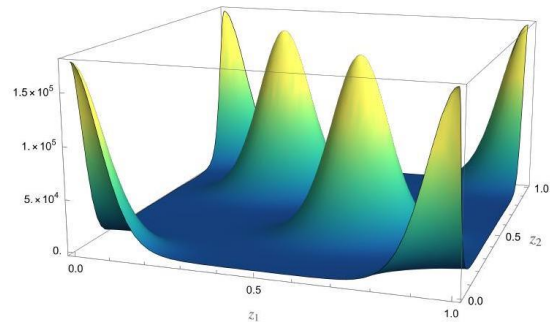
$$V_{\text{Cas}} = \sum_{\mathbf{r}} \sum_{\gamma \in \Gamma} \text{Tr}_{\mathbf{r}}(\mathbf{D}_{\gamma}) \mathcal{E}(\gamma), \mathcal{E}(\gamma) = -\hat{\delta}_{\vec{h}} \frac{\Gamma(s_{\gamma})}{2\pi^{s_{\gamma}}} \cdot \frac{\sqrt{G_{\parallel}}}{|\Gamma|} \sum_{\vec{\xi} \in \Xi_{\gamma}} \frac{e^{2\pi i \vec{\beta}_{\gamma} \cdot \vec{\xi}}}{\left| \vec{\xi} + \vec{b}_{\gamma}^{\parallel} \right|^{2s_{\gamma}}},$$

$$\mathbf{D}_{\mathbf{g}} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \vec{b}_{\mathbf{g}} = \begin{pmatrix} 0 \\ 1/2 \end{pmatrix}$$

$$\mathbf{G} = R^2 \begin{pmatrix} x & 0 \\ 0 & x^{-1} \end{pmatrix}$$



(a) $T^2/\mathbb{Z}_2$ (Klein bottle)



(b) $T^3/\mathbb{Z}_3$

Figura 1. Superespacios de Klein.

$$(\mathbf{I} - \mathbf{D}_{\mathbf{g}})\vec{z} = \begin{pmatrix} 2z_1 \\ 0 \end{pmatrix} \stackrel{!}{=} 0 \bmod \mathbb{Z}^2 \Rightarrow z_1 \in \left\{0, \frac{1}{2}\right\}, z_2 \in [0, 1).$$

$$2h_1 \in \mathbb{Z}, s_g = 2h_2 \bmod 2\mathbb{Z}.$$

$$\mathcal{E}(\mathbf{g}) = -\frac{\Gamma(s_{\mathbf{g}})}{2\pi^{s_{\mathbf{g}}}} \cdot \frac{x^{s_{\mathbf{g}}-\frac{1}{2}}}{2R^{2s_{\mathbf{g}}-1}} \sum_{\xi \in \mathbb{Z}} \frac{e^{2\pi i h_2 \xi}}{\left| \xi + \frac{1}{2} \right|^{2s_{\mathbf{g}}}} = -\frac{\Gamma(s_{\mathbf{g}})}{2\pi^{s_{\mathbf{g}}}} \cdot \frac{x^{s_{\mathbf{g}}-\frac{1}{2}}}{R^{2s_{\mathbf{g}}-1}} (2^{2s_{\mathbf{g}}} - 1) \zeta(2s_{\mathbf{g}}) \delta_{h_2,0}$$

$$\mathcal{E}(\mathbf{g}) = -\frac{124\pi^5}{945} \frac{x^{9/2}}{R^9} \approx -\frac{40.2x^{9/2}}{R^9}$$

$$\text{Tr}_{44}(\mathbf{D}_{\mathbf{g}}) = 16, \text{Tr}_{84}(\mathbf{D}_{\mathbf{g}}) = 0, \text{Tr}_{128}(\mathbf{D}_{\mathbf{g}}) = 0$$

$$V_{\text{Cas}}^{(\mathbf{l})} = -\frac{\Gamma(s)}{2\pi^s} \cdot \frac{128x^{2s}}{2R^{2s-2}} \sum_{\vec{n} \in \mathbb{Z}^2} \frac{1 - e^{2\pi i \vec{h} \cdot \vec{n}}}{|x^2 n_1^2 + n_2^2|^{2s}}$$

$$V_{\text{Cas}}^{(\mathbf{l})} = 0, \text{ for } \vec{h} = (0,0)$$

$$V_{\text{Cas}}^{(\mathbf{l})} = -\frac{\Gamma(s)}{2\pi^s} \cdot \frac{128x^{2s}}{2R^{2s-2}} \cdot 4\beta(s)[\zeta(s) - \eta(s)] \approx -\frac{0.56}{R^9}, \text{ for } \vec{h} = \left(\frac{1}{2}, 0\right)$$



$$\mathbf{D}_g = \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \vec{b}_g = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{3} \end{pmatrix}.$$

$$\mathbf{G} = R^2 x^{4/3} \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & x^{-2} \end{pmatrix},$$

$$(\mathbf{I} - \mathbf{D}_g) \vec{z} = \begin{pmatrix} z_1 + z_2 \\ -z_1 + 2z_2 \\ 0 \end{pmatrix} \stackrel{!}{=} 0 \bmod \mathbb{Z}^3$$

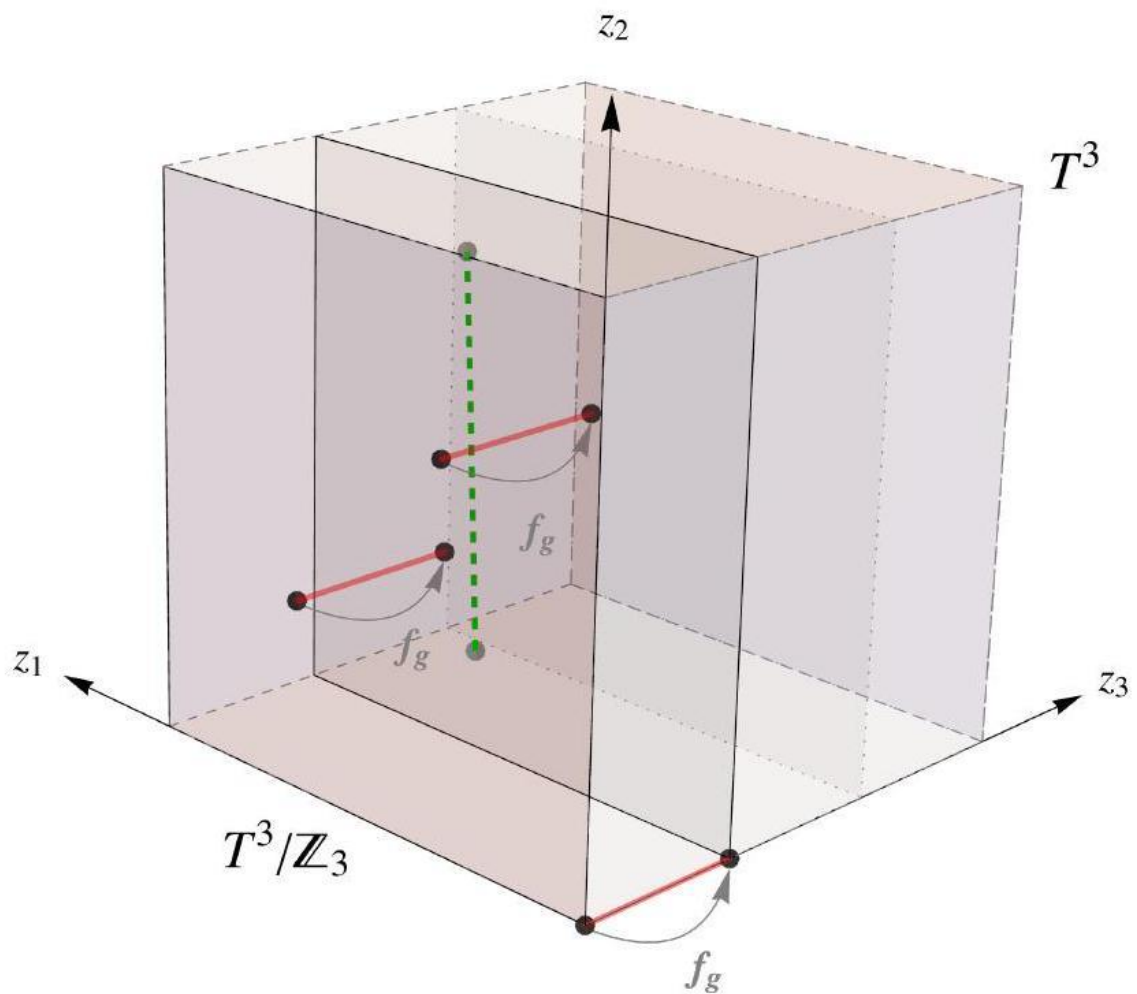


Figura 2. Multidimensiones.

$$\Rightarrow (z_1, z_2) \in \left\{ (0,0), \left(\frac{1}{3}, \frac{2}{3}\right), \left(\frac{2}{3}, \frac{1}{3}\right) \right\}, z_3 \in [0,1)$$

$$s_g = 2h_3 \bmod 2\mathbb{Z}$$

$$\mathcal{E}(\mathbf{g}) = -\frac{\Gamma(s_g)}{2\pi^{s_g}} \cdot \frac{x^{\frac{2s_g-1}{3}}}{3R^{2s_g-1}} \sum_{\xi \in \mathbb{Z}} \frac{e^{2\pi i h_3 \xi}}{\left| \xi + \frac{1}{3} \right|^{2s_g}}$$

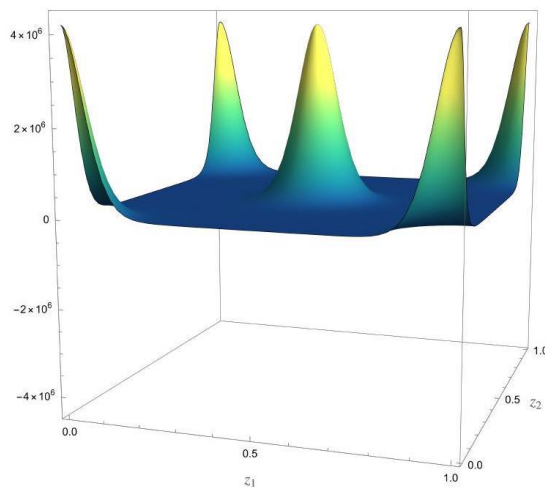
$$\mathcal{E}(\mathbf{g}) = -\frac{\Gamma(s_{\mathbf{g}})}{2\pi^{s_{\mathbf{g}}}} \cdot \frac{x^{\frac{2s_{\mathbf{g}}-1}{3}}}{3R^{2s_{\mathbf{g}}-1}} \left[\zeta\left(2s_{\mathbf{g}}, \frac{1}{3}\right) + \zeta\left(2s_{\mathbf{g}}, \frac{2}{3}\right) \right] \approx -\frac{221.4x^{\frac{8}{3}}}{R^8}$$

$$\mathcal{E}(\mathbf{g}) = -\frac{\Gamma(s_{\mathbf{g}})}{2\pi^{s_{\mathbf{g}}}} \cdot \frac{x^{s_{\mathbf{g}}-\frac{1}{3}}}{3R^{2s_{\mathbf{g}}-1}} \frac{1}{2^{2s}} \left[\zeta\left(2s_{\mathbf{g}}, \frac{1}{6}\right) - \zeta\left(2s_{\mathbf{g}}, \frac{1}{3}\right) - \zeta\left(2s_{\mathbf{g}}, \frac{2}{3}\right) + \zeta\left(2s_{\mathbf{g}}, \frac{5}{6}\right) \right]$$

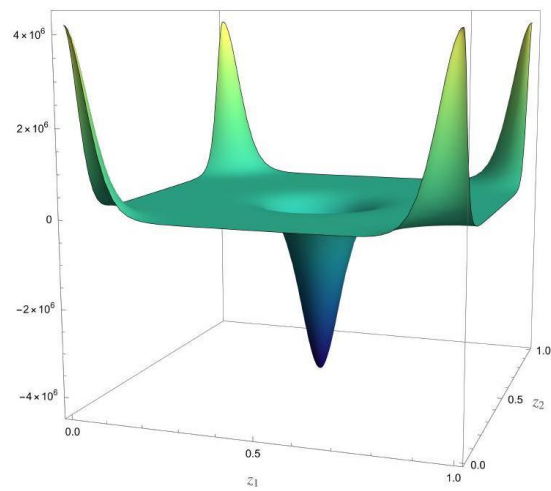
$$\text{Tr}_{44}(\mathbf{D}_{\mathbf{g}}) = 20, \text{Tr}_{84}(\mathbf{D}_{\mathbf{g}}) = 21, \text{Tr}_{128}(\mathbf{D}_{\mathbf{g}}) = 40$$

$$\mathcal{E}(\mathbf{g}^j) = -\mathcal{E}(\mathbf{g}^{p-j})$$

$$\mathbf{D}_{\mathbf{g}} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \vec{b}_{\mathbf{g}} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{4} \end{pmatrix}$$



(a) Periodic $\vec{h} = (0, 0, 0)$.



(b) Anti-periodic $\vec{h} = (\frac{1}{2}, \frac{1}{2}, 0)$.

Figura 3. Deformación intensa del espacio – tiempo cuántico.

$$s_{\mathbf{g}} = 2h_3 \text{mod} 2\mathbb{Z}.$$

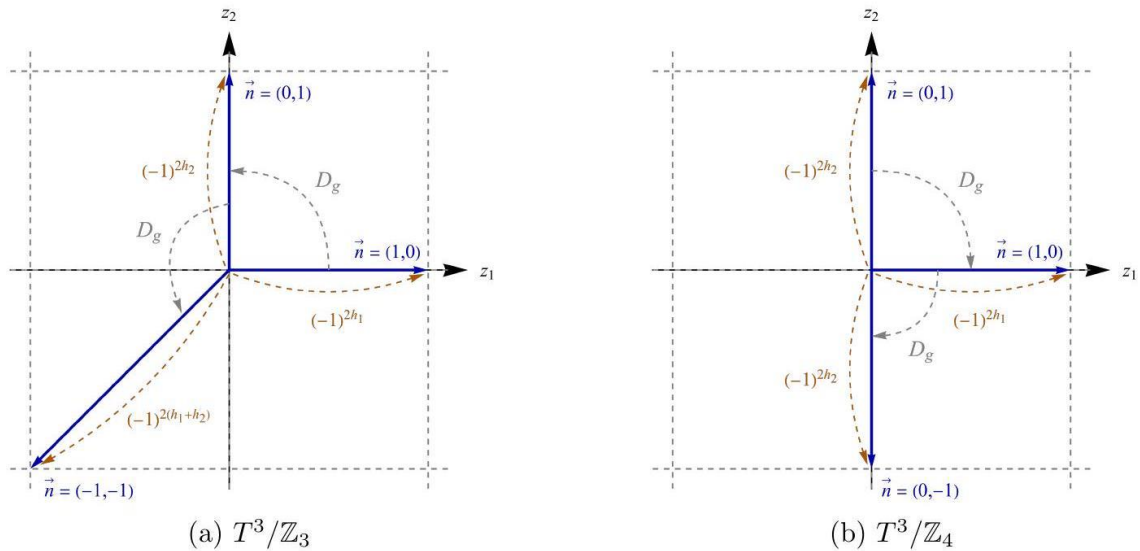


Figura 4. Difeomorfismos.

$$V_{\text{Cas}}^{\text{M-theory on } S^1/\mathbb{Z}_2} = -\frac{\mathcal{J}}{L^{10}}, \mathcal{J} = \frac{\Gamma\left(\frac{11}{2}\right) 2^{11} - 1}{2\pi^{11/2} 2^{13}} \zeta(11) \approx 1.206 \cdot 10^{-2}.$$

$$V_{\text{Cas}}^{\text{M-theory on } T^4} = -\frac{\Gamma\left(\frac{11}{2}\right)}{2\pi^{11/2}} \sqrt{G} \sum_{\vec{n} \in \mathbb{Z}^4} \frac{\text{Tr}_{44}(\mathbf{I}) + \text{Tr}_{84}(\mathbf{I}) - \text{Tr}_{128}(\mathbf{I}) e^{2\pi i \vec{h} \cdot \vec{n}}}{|\vec{n}|^{11}},$$

$$\mathbf{G} = R^2 \begin{pmatrix} 2 & -1 & 0 & 1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 1 & 0 & -1 & 2 \end{pmatrix}$$

$$V_{\text{Cas}}^{\text{M-theory on } T^4} = -128 \cdot \frac{945}{64\pi^5} \cdot 2R^4 \sum_{\vec{n} \in \mathbb{Z}^4} \frac{1 - (-1)^{n_1+n_2+n_3+n_4}}{|\vec{n}|^{11}} \approx -\frac{8.827}{R^7},$$

$$\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -2 \\ 0 & 0 & -2 & 4 \end{pmatrix}$$

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}$$

$$(\mathbf{I}-\mathbf{D}_\gamma)\cdot\vec{e}\in\mathbb{Z}^k$$

$$e^{2\pi i[(\mathbf{I}-\mathbf{D}_{\mathbf{g}})^T\vec{h}]\cdot\vec{e}}$$

$$\vec{h}\cdot(\mathbf{I}-\mathbf{D}_\gamma)\vec{e}\in\mathbb{Z}, \text{ whenever } (\mathbf{I}-\mathbf{D}_\gamma)\vec{e}\in\mathbb{Z}^k$$

$$(\mathbf{I}-\mathbf{D}_\gamma)^T\vec{\eta}+\vec{\xi}_{\text{Inv}},\vec{\eta}\in\mathbb{Z}^k,\mathbf{D}_\gamma^T\vec{\xi}_{\text{Inv}}=\vec{\xi}_{\text{Inv}}$$



$$V_{\rm Cas} \sim \int_{\rm RFM} \hat{\rho}(\vec{z}) dV$$

$$\hat{\rho}(\vec{z}+\vec{e})=e^{i\varphi(\vec{e})}\hat{\rho}(\vec{z}),$$

$$V^{1\text{-loop}} \sim \int_{\mathcal{F}} \mathcal{Z}(\tau) \frac{d^2\tau}{\tau_2}$$

$$\mathcal{Z}\left(-\frac{1}{2\tau}\right)=e^{i\omega\mathcal{A}}\mathcal{Z}(\tau)$$

$$\sum_{\vec{n}\neq-\vec{c}}\frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}}.$$

$$\sum_{\vec{n}\neq-\vec{c}}\frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}}=\sum_{\vec{n}\neq-\vec{c}}\frac{F(\vec{n})e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}}+\sum_{\vec{n}}\frac{(1-F(\vec{n}))e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}},$$

$$\lim_{\vec{y}\rightarrow-\vec{c}}\frac{1-F(\vec{y})}{|\vec{y}+\vec{c}|^{2s}}\,F(\vec{y})\rightarrow 0\text{ as }|\vec{y}|\rightarrow\infty$$

$$\begin{aligned} \sum_{\vec{n}+\vec{c}\neq 0} \frac{e^{2\pi i \vec{h}\cdot \vec{n}}}{|\vec{n}+\vec{c}|^{2s}} = \\ \sum_{\vec{n}+\vec{c}\neq 0} \frac{e^{2\pi i \vec{h}\cdot \vec{n}}}{|\vec{n}+\vec{c}|^{2s}} \frac{\Gamma(s,\alpha|\vec{n}+\vec{c}|^2)}{\Gamma(s)} + \frac{\pi^{2s-\frac{k}{2}}}{\Gamma(s)\sqrt{G}} \sum_{\vec{k}-\vec{h}\neq 0} \Gamma\left(\frac{k}{2}-s,\frac{\pi^2|\vec{h}-\vec{k}|_D^2}{\alpha}\right) \frac{e^{-2\pi i(\vec{h}-\vec{k})\cdot \vec{c}}}{|\vec{h}-\vec{k}|_D^{k-2s}} \\ + \delta_{\vec{h},\vec{0}} \frac{\pi^{\frac{k}{2}}\alpha^{s-k/2}}{\sqrt{G}\Gamma(s)\left(s-\frac{k}{2}\right)} - \frac{\alpha^s e^{-2\pi i \vec{h}\cdot \vec{c}}}{\Gamma(s+1)} \chi_{\mathbb{Z}^k}(\vec{c}) \end{aligned}$$

$$\Gamma(s,z)\equiv \int_z^\infty t^{s-1}e^{-t}dt$$

$$\begin{aligned}\Gamma(s,z)&\approx \Gamma(s)-\frac{z^s}{s}+\mathcal{O}(z^{s+1}),\quad |\vec{z}|\ll 1\\ \Gamma(s,z)&\approx e^{-z}z^{s-1}[1+\mathcal{O}(z^{-1})]\quad |\vec{z}|\gg 1\end{aligned}$$

$$\sum_{\text{KK spectrum}} m_{\text{KK}}^d,$$

$$V^{(5d)}=V_{\rm Cas}+V_{G_4}$$

$$z_i \sim z_i + 1$$



$$\vec{z} \rightarrow \iota_{\mathbf{g}}(\vec{z}) = \mathbf{D}_{\mathbf{g}}\vec{z} + \vec{b}_{\mathbf{g}}, \text{ with } \mathbf{D}_{\mathbf{g}} \equiv \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \vec{b}_{\mathbf{g}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{8} \end{pmatrix}$$

$$\mathcal{F}_6 \equiv T^6/\mathbb{Z}_8$$

$$ds^2 = G_{ij} dz^i dz^j$$

$$\mathbf{G} = \begin{pmatrix} (\gamma+2)R_2^2 & -(\gamma+1)R_2^2 & 0 & (\gamma+1)R_2^2 & 0 & 0 \\ -(\gamma+1)R_2^2 & (\gamma+2)R_2^2 & -(\gamma+1)R_2^2 & 0 & 0 & 0 \\ 0 & -(\gamma+1)R_2^2 & (\gamma+2)R_2^2 & -(\gamma+1)R_2^2 & 0 & 0 \\ (\gamma+1)R_2^2 & 0 & -(\gamma+1)R_2^2 & (\gamma+2)R_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta R_1^2 & \alpha R_1^2 \\ 0 & 0 & 0 & 0 & \alpha R_1^2 & \beta^{-1}R_1^2 \end{pmatrix}.$$

$$f(\iota_{\mathbf{g}}(\vec{z})) = \iota_{\mathbf{g}}^{n_1+1}(f(\vec{z})).$$

$$\mathbf{A} \cdot \mathbf{D}_{\mathbf{g}} \cdot \mathbf{A}^{-1} = \mathbf{D}_{\mathbf{g}}^{n_1}, n_1 \in \mathbb{Z}$$

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\mathbf{G} \rightarrow \mathbf{A}^T \cdot \mathbf{G} \cdot \mathbf{A} = \begin{pmatrix} (\gamma+2)R_2^2 & -(\gamma+1)R_2^2 & 0 & (\gamma+1)R_2^2 & 0 & 0 \\ -(\gamma+1)R_2^2 & (\gamma+2)R_2^2 & -(\gamma+1)R_2^2 & 0 & 0 & 0 \\ 0 & -(\gamma+1)R_2^2 & (\gamma+2)R_2^2 & -(\gamma+1)R_2^2 & 0 & 0 \\ (\gamma+1)R_2^2 & 0 & -(\gamma+1)R_2^2 & (\gamma+2)R_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta^{-1}R_1^2 & -\alpha R_1^2 \\ 0 & 0 & 0 & 0 & -\alpha R_1^2 & \beta R_1^2 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{G} = R_1^2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} + R_2^2 \begin{pmatrix} 2 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}$$

$$e_1 \rightarrow e_1, e_2 \leftrightarrow e_4, e_3 \rightarrow -e_3$$

$$(0,0,0,0,0,0) \text{ or } \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0\right)$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{D}_{\mathbf{g}} \vec{b}_{\mathbf{g}} = \vec{b}_{\mathbf{g}} \bmod \mathbb{Z}$$

$$\vec{b}_{\mathbf{g}} = \left(0,0,0,0,\frac{a}{8},\frac{b}{8}\right) \text{ or } \left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{a}{8},\frac{b}{8}\right)$$

$$\mathrm{Vol}(T^6) = 2R_2^4R_1^2$$

$$R^6 \equiv R_2^4 R_1^2, x \equiv \frac{R_2}{R_1}$$

$$R_1=\frac{R}{x^{2/3}}, R_2=R x^{1/3}$$

$$V_{\mathrm{Cas}}=-\frac{\mathcal{C}(x)}{4R^5},$$

$$V_{\mathrm{Cas}}=-\frac{\Gamma(s)}{2\pi^s}\cdot\int_{\mathcal{F}_6}d^6z\sqrt{G}\sum_{j=0}^7\sum_{\vec{n}\in\mathbb{Z}^6}\frac{\mathrm{Tr}_{\mathbf{B}}(\mathbf{D}_{\mathbf{g}}^j)-\mathrm{Tr}_{\mathbf{F}}(\mathbf{D}_{\mathbf{g}}^j)\cdot e^{2\pi i\vec{h}\cdot\vec{n}}}{|(\mathbf{I}-\mathbf{D}_{\mathbf{g}}^j)\vec{z}-j\vec{b}_{\mathbf{g}}+\vec{n}|^{2s}},$$

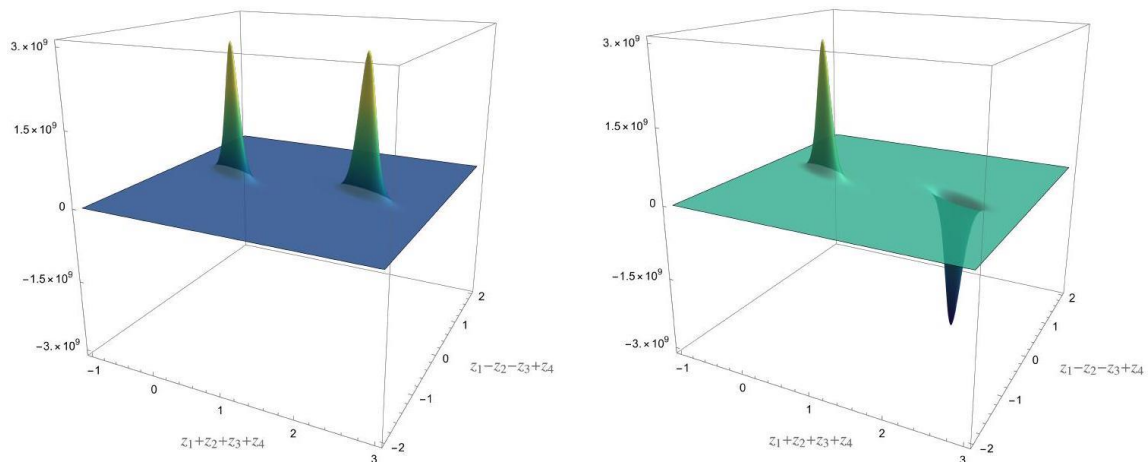


Figura 5. Torsión del espacio – tiempo cuántico.

$$V_{\text{Cas}} = -\frac{\Gamma(s)}{2\pi^s} \cdot \frac{2R_1^2 R_2^4}{|\mathbb{Z}_8|} \cdot \sum_{j=0}^7 \int_{T^6} d^4 z \sum_{\substack{\vec{n} \in \mathbb{Z}^4 \\ \vec{m} \in \mathbb{Z}^2}} \frac{\text{Tr}_{\mathbf{B}}(\mathbf{D}_{\mathbf{g}}^j) - \text{Tr}_{\mathbf{F}}(\mathbf{D}_{\mathbf{g}}^j) \cdot e^{2\pi i \vec{h} \cdot (\vec{n}, \vec{m})}}{\left[R_2^2 |(\mathbf{I} - \mathbf{D}_{\mathbf{g}}^j) \vec{z} + \vec{n}|^2 + R_1^2 |\vec{m} - j \vec{b}_{\mathbf{g}}|^2 \right]^s}.$$

$$V_{\text{Cas}} = -\frac{\Gamma(s)}{2\pi^s} \cdot \frac{x^{\frac{4s}{3}}}{4 \cdot R^{2s-6}} \cdot \sum_{j=0}^7 \int_{T^6} d^4 z \sum_{\substack{\vec{n} \in \mathbb{Z}^4 \\ \vec{m} \in \mathbb{Z}^2}} \frac{\text{Tr}_{\mathbf{B}}(\mathbf{D}_{\mathbf{g}}^j) - \text{Tr}_{\mathbf{F}}(\mathbf{D}_{\mathbf{g}}^j) \cdot e^{2\pi i \vec{h} \cdot (\vec{n}, \vec{m})}}{\left[x^2 |(\mathbf{I} - \mathbf{D}_{\mathbf{g}}^j) \vec{z} + \vec{n}|^2 + |\vec{m} - j \vec{b}_{\mathbf{g}}|^2 \right]^s}$$

$$\mathcal{C}(x) = \frac{\Gamma(s)}{2\pi^s} \cdot x^{\frac{22}{3}} \sum_{j=0}^7 \int_{T^6} d^4 z \sum_{\substack{\vec{n} \in \mathbb{Z}^4 \\ \vec{m} \in \mathbb{Z}^2}} \frac{\text{Tr}_{\mathbf{B}}(\mathbf{D}_{\mathbf{g}}^j) - \text{Tr}_{\mathbf{F}}(\mathbf{D}_{\mathbf{g}}^j) \cdot e^{\pi i (n_1 + n_2 + n_3 + n_4)}}{\left[x^2 |(\mathbf{I} - \mathbf{D}_{\mathbf{g}}^j) \vec{z} + \vec{n}|^2 + |\vec{m} - j \vec{b}_{\mathbf{g}}|^2 \right]^s}$$

$$\begin{aligned} \mathcal{C}_{\text{twisted}}(x) &= \frac{\Gamma(s-2)}{2\pi^{s-2}} \cdot x^{\frac{10}{3}} \sum_{j=1}^7 \sum_{\vec{m} \in \mathbb{Z}^2} \frac{\text{Tr}_{\mathbf{B}}(\mathbf{D}_{\mathbf{g}}^j)}{\left[m_1^2 + \left(m_2 - \frac{j}{8} \right)^2 \right]^{s-2}} \\ &= 1.538 \times 10^6 \cdot x^{\frac{10}{3}} \end{aligned}$$

k	0	1	2	3	4	5	6	7
$\text{Tr}_{44}(\mathbf{D}_{\mathbf{g}}^j)$	44	14	12	14	4	14	12	14
$\text{Tr}_{84}(\mathbf{D}_{\mathbf{g}}^j)$	84	10	20	10	-4	10	20	10
$\text{Tr}_{128}(\mathbf{D}_{\mathbf{g}}^j)$	128	$16\sqrt{2}$	-32	$-16\sqrt{2}$	0	$-16\sqrt{2}$	-32	$16\sqrt{2}$

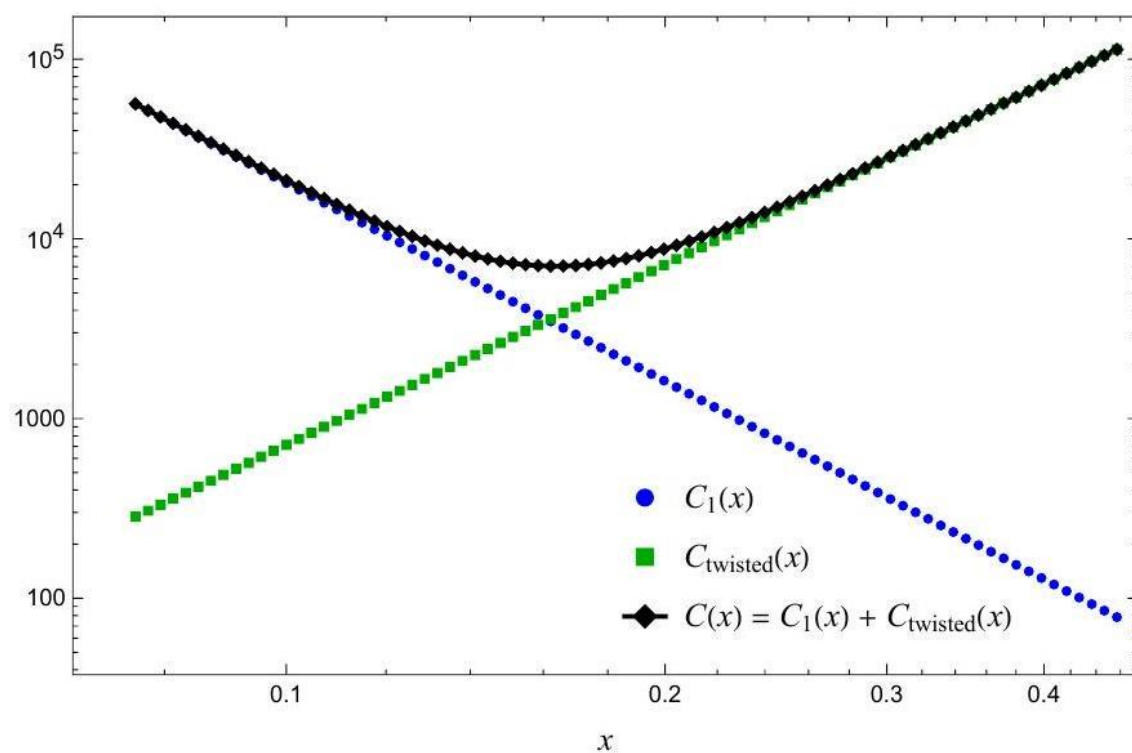
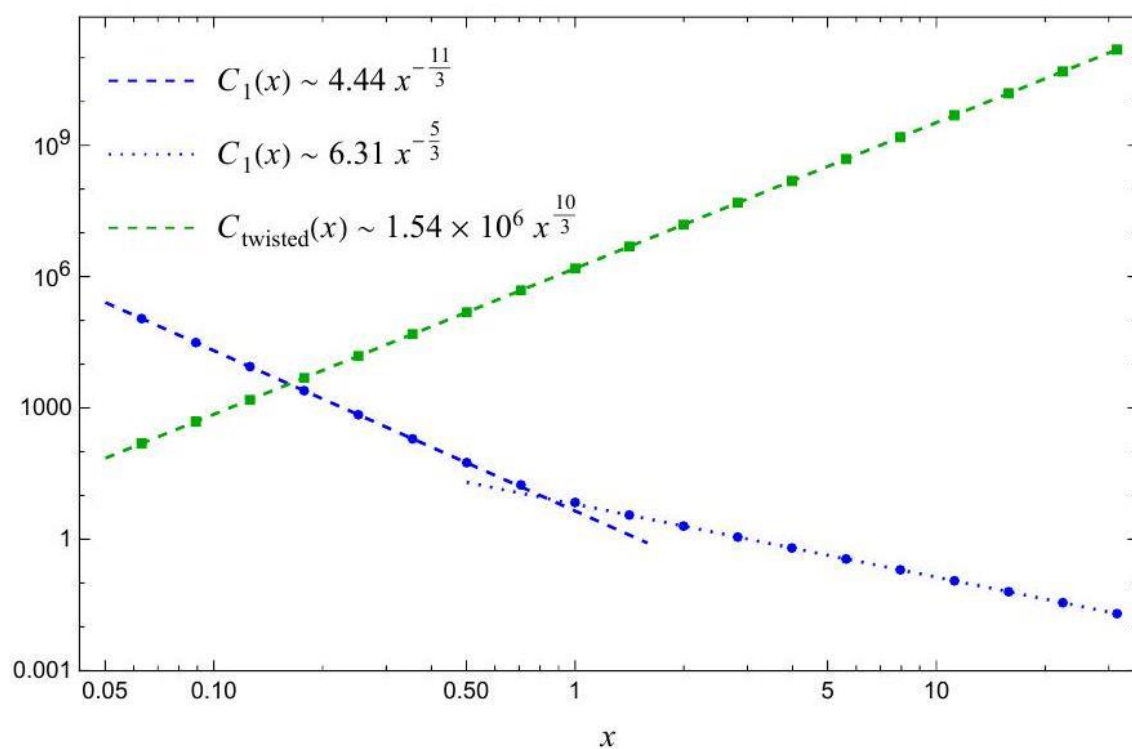
$$\mathcal{C}_{\mathbf{I}}(x) = \frac{\Gamma(s)}{2\pi^s} \cdot 128 \cdot x^{\frac{22}{3}} \sum_{\substack{\vec{n} \in \mathbb{Z}^4 \\ \vec{m} \in \mathbb{Z}^2}} \frac{1 - e^{\pi i (n_1 + n_2 + n_3 + n_4)}}{[x^2 |\vec{n}|^2 + |\vec{m}|^2]^s}$$

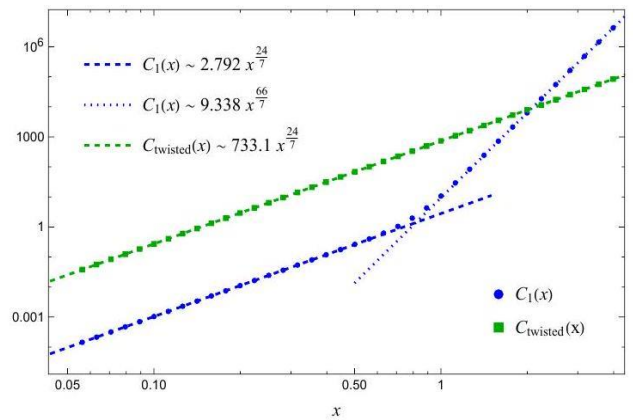
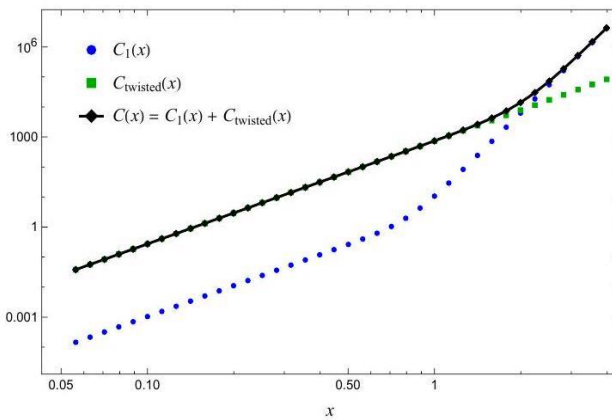
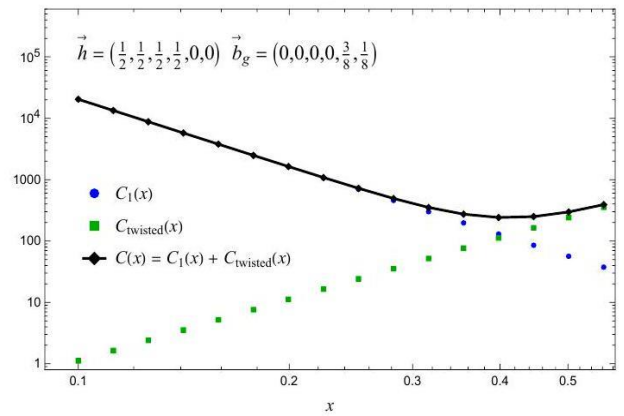
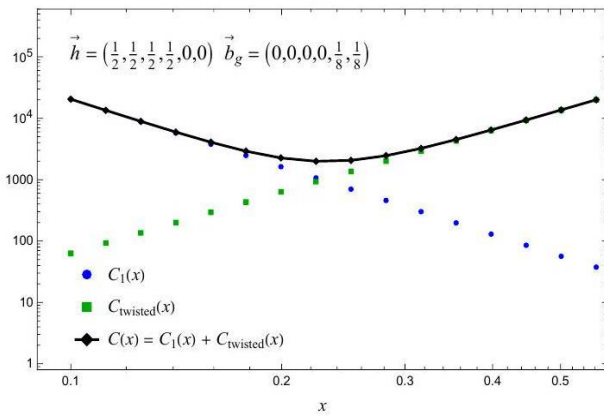
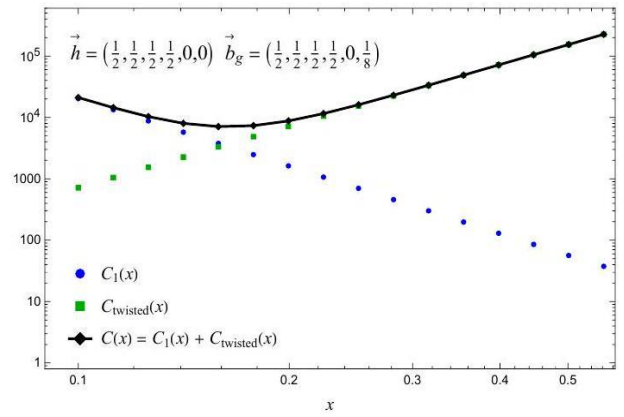
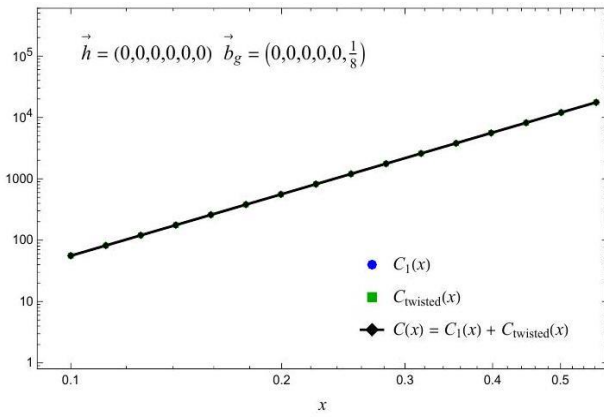
$$\mathcal{C}_{\mathbf{I}}(x) \sim 6.31 \cdot x^{-5/3}, \text{ as } x \rightarrow \infty$$

$$\mathcal{C}_{\mathbf{I}}(x) \sim \frac{\Gamma(s)}{2\pi^s} \cdot 128 \cdot x^{\frac{22}{3}-2s} \sum_{\vec{n} \in \mathbb{Z}^4} \frac{1 - e^{\pi i (n_1 + \dots + n_4)}}{|\vec{n}|^{2s}} \approx 4.414 \cdot x^{-\frac{11}{3}}, \text{ as } x \rightarrow 0$$

$$\mathcal{C}(x_{\min} = 0.1637) \approx 7046.54.$$







Figuras 6, 7, 8, 9 y 10. Paquetes de energía en dimensiones altas.

$$\int_{\mathcal{C}_4} G_4 \in \mathbb{Z}, \forall \mathcal{C}_4 \in H_4(\mathcal{F}_6, \mathbb{Z})$$

$$\int_{\mathcal{F}_6} \alpha \wedge G_4 \in \mathbb{Z}, \forall \alpha \in H^2(\mathcal{F}_6, \mathbb{Z})$$

$$G_4 = \sum_{i,j} n^i g_{ij}^{-1} * \alpha_j, n^i \in \mathbb{Z}$$



$$\mathcal{G}_{ij} \equiv \int_{\mathcal{F}^6} \alpha_i \wedge^* \alpha_j$$

$$\int_{\mathcal{F}^6} |G_4|^2 = \sum_{i,j} \mathcal{G}_{ij}^{-1} n^i n^j$$

$$\int_{\mathcal{F}^6} \alpha_i \wedge^* \alpha_j = \frac{1}{8} \int_{T^6} p^*(\alpha_i) \wedge^* p^*(\alpha_j)$$

$$\begin{aligned}\omega_1 &\equiv dz_1 \wedge dz_2 + dz_1 \wedge dz_4 + dz_2 \wedge dz_3 + dz_3 \wedge dz_4 - dz_1 \wedge dz_3 - dz_2 \wedge dz_4 \\ \omega_2 &\equiv dz_1 \wedge dz_2 + dz_1 \wedge dz_4 + dz_2 \wedge dz_3 + dz_3 \wedge dz_4 - 2dz_1 \wedge dz_3 - 2dz_2 \wedge dz_4 \\ \omega_3 &\equiv dz_5 \wedge dz_6\end{aligned}$$

$$\omega_1 \rightarrow \omega_1, \omega_2 \rightarrow -\omega_2, \omega_3 \rightarrow \omega_3$$

$$W(\ell)=\exp\left(2\pi i\int_{\ell}A\right)$$

$$\vec{n}\cdot\mathbf{F}\cdot\vec{m}\in\mathbb{Z}$$

$$\ell_1=(1-t)\vec{z}_0+t(\mathbf{D}_{\mathbf{g}}^j\vec{z}_0+j\vec{b}_{\mathbf{g}})\,t\in[0,1)$$

$$[(\mathbf{D}_{\mathbf{g}}^l\vec{z}_0+l\vec{b}_{\mathbf{g}}+\vec{n})]\cdot\mathbf{F}\cdot[(\mathbf{D}_{\mathbf{g}}^j-\mathbf{I})\vec{z}_0+j\vec{b}_{\mathbf{g}}]$$

$$\vec{n}\cdot\mathbf{F}\cdot\vec{b}_{\mathbf{g}}\in\mathbb{Z}$$

$$\kappa_1=\frac{1}{R_2^4}, \kappa_2=\frac{2}{R_2^4}, \kappa_3=\frac{64}{R_1^4}$$

$$\int_{\mathcal{F}^6} |G_4|^2 = \frac{1}{R^2} \bigg(4n_1^2 x^{4/3} + \frac{n_3^2}{16x^{8/3}} \bigg)$$

$$V_{G_4}=\frac{(2\pi^2)^{\frac{1}{3}}}{2\ell_{11}^3}\int_{\mathcal{F}_6}|G_4|^2=\frac{(2\pi^2)^{\frac{1}{3}}}{2R^2\ell_{11}^3}\bigg(4n_1^2x^{4/3}+\frac{n_3^2}{16x^{8/3}}\bigg)$$

$$V^{(5d)}=\frac{2^{\frac{10}{3}}}{R^{10}}\bigg[\frac{(2\pi^2)^{\frac{1}{3}}}{2R^2}\bigg(4n_1^2x^{4/3}+\frac{n_3^2}{16x^{8/3}}\bigg)-\frac{\mathcal{C}(x)}{4R^5}\bigg]$$

$$\left(\begin{array}{cc} \frac{2(\gamma^2+2)}{(\gamma^2-2)^2R_2^4} & -\frac{8\gamma}{(\gamma^2-2)^2R_2^4} \\ -\frac{8\gamma}{(\gamma^2-2)^2R_2^4} & \frac{4(\gamma^2+2)}{(\gamma^2-2)^2R_2^4} \end{array}\right)$$

$$\begin{aligned}R^3 &= \frac{\mathcal{C}}{(2\pi^2)^{1/3}} \cdot \frac{10x^{8/3}}{64n_1^2x^4+n_3^2} \\ \frac{x\mathcal{C}'}{\mathcal{C}} &= \frac{10}{3} \cdot \frac{32n_1^2x^4-n_3^2}{64n_1^2x^4+n_3^2}\end{aligned}$$



$$\frac{x\mathcal{C}'(x)}{\mathcal{C}(x)} = -\frac{10}{3} \quad \text{and} \quad \frac{x\mathcal{C}'(x)}{\mathcal{C}(x)} = \frac{5}{3}$$

$$R^3 \approx \frac{\mathcal{C}(x_{\max})}{(2\pi^2)^{\frac{1}{3}}} \frac{10}{96n_1^2 x_{\max}^{4/3}} \approx \frac{3034}{n_1^2}.$$

n_1	n_3	R	x	$\frac{1}{8}R_1$	$\sqrt{2}R_2$	$\text{vol}(\mathcal{F}_6)$	V_{saddle}
0	1	5.45	0.105	3.06	3.64	6.59×10^3	9.99×10^{-8}
1	0	16.2	0.191	6.11	13.18	4.50×10^6	3.68×10^{-15}
7	1	4.00	0.166	1.65	3.12	1.03×10^3	4.06×10^{-6}



Property	Value
5d vacuum energy	$V^{(5d)} \approx 3.68 \cdot 10^{-15} \ell_5^{-3}$
Hubble radius	$H_0^{-1} \sim 4.04 \cdot 10^7 \ell_5 = 2.45 \cdot 10^5 \ell_{11}$
Internal space volume	$\mathcal{V} = 4.50 \times 10^6 \ell_{11}^6 = 9.1 \cdot 10^{19} \ell_5^6$
Internal manifold radii	$\frac{1}{8} R_1 \sim 6.1 \ell_{11} = 1009 \ell_5,$ $\sqrt{2} R_2 \sim 13.2 \ell_{11} = 2176 \ell_5$
Scale separation	Yes: $\frac{1}{8} R_1 = 2.50 \cdot 10^{-5} H_0^{-1}$
Geometric moduli	Five, with masses $m_i^2 =$ $\{90.45, 23.82, -309.55, -47.43, -45.16\} \cdot H_0^2$
Axions	Five, with negligible masses
Vectors	Seven: two KK photons, five from C_3
Light fermions	None
11d average energy density	$\rho_{11d} = 1.00 \cdot 10^{-10} \ell_{11}^{-11}$
Flux & Casimir contributions to ρ_{11d}	$\rho_{\text{Cas}} \approx -4.02 \cdot 10^{-10} \ell_{11}^{-11}, \quad \rho_{G_4} \approx 5.02 \cdot 10^{-10} \ell_{11}^{-11}$
Internal gradients	$\langle \rho_{11d} \rangle^2 \sim 10^{-9}$

$$\frac{R_1}{8} \text{ and } \sqrt{2} R_2$$

$$\frac{1}{2\kappa_d^2} \int d^d x \sqrt{-g_d} \left[\left(\frac{D-2}{k(d-2)} \right) (d \log \sqrt{\det \mathbf{G}})^2 + \frac{1}{4} \text{Tr}((\mathbf{G}^{-1} \partial \mathbf{G})^2) \right]$$

$$\{90.45, 23.82, -309.55, -47.43, -45.16\} \times H_0^2,$$

$$\begin{array}{cccccccc} p & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ H^p(\mathcal{F}_6, \mathbb{R}) & \mathbb{R} & 2\mathbb{R} & 3\mathbb{R} & 4\mathbb{R} & 3\mathbb{R} & 2\mathbb{R} & \mathbb{R} \end{array}$$

$$m^2 \gtrsim \frac{gq}{\sqrt{G}} H$$

$$\frac{g}{\sqrt{G}} \sim \frac{\text{Vol}(\omega_2)}{\ell_{11}^3} \sim \frac{R^2}{\ell_{11}^3} \gg H$$



$$\frac{g}{\sqrt{G}}\sim \frac{\mathrm{Vol}(\omega_5)}{\ell_{11}^6}\sim \frac{R^5}{\ell_{11}^6}\gg H$$

$$H_0^{-1}=\sqrt{\frac{6}{V_{\mathrm{Saddle}}}}\approx 4.04\cdot 10^7\ell_5$$

$$\frac{\ell_{11}}{\ell_5}=\frac{R^2}{4^{1/3}}\approx 165.14$$

$$\rho_{11d}=1.00\cdot 10^{-10}\ell_{11}^{-11}$$

$$\rho_{\mathrm{Cas}}\approx -4.02\cdot 10^{-10}\ell_{11}^{-11} \; \rho_{G_4}\approx 5.02\cdot 10^{-10}\ell_{11}^{-11}.$$

$$\int P(R_{\mu\nu\rho\sigma},|G_4|)$$

$$\hat{\rho}(0)\approx 1.28\cdot 10^{-9}\ell_{11}^{-11}$$

$$V_{\mathrm{Cas}}\sim \frac{1}{4R^5}\Bigg[\mathcal{C}+\sum_{l=1}^{\infty}\mathcal{C}_i\Big(\frac{\ell_{11}}{R}\Big)^{9l}\Bigg]$$

$$\left(\frac{\ell_{11}}{R}\right)^9\sim 1.31\cdot 10^{-11}$$

$$\mathcal{C}_i\gtrsim 10^{6+11l}$$

$$e^{-S_{\mathrm{M}2}}\sim e^{-T_{\mathrm{M}2}\left(\frac{R_1R_2^2}{4}\right)}\approx e^{-716.9},e^{-S_{\mathrm{M}5}}\sim e^{-T_{\mathrm{M}5}\frac{R^6}{4}}\approx e^{-5.24\cdot 10^6}$$

$$\delta V^{(5d)}=\frac{2^{\frac{3}{10}}}{R^{10}}\frac{R^6}{4}\left[\sum_{n=1}^{\infty}\frac{(2\mu^{11})K_{\frac{11}{2}}(\mu nR_1)}{(2\pi)^{\frac{11}{2}}(\mu nR_1)^{\frac{11}{2}}}\right]$$

$$\sqrt{T}\sim (2\pi^2)^{1/3}\frac{R_1}{8}\approx 16.48$$

$$\mathcal{F}_6=T^6/\mathbb{Z}_8$$

$$\mathcal{F}_7=\frac{T^7}{\Gamma}$$

$$\left\{\mathbf{g}\colon \mathbf{D}_{\mathbf{g}}=\begin{pmatrix} \mathbf{A} & 0 \\ 0 & 1 \end{pmatrix}, \vec{b}_{\mathbf{g}}=\begin{pmatrix} \vec{0} \\ |\mathbf{A}|^{-1} \end{pmatrix}\right\},$$



$$\mathbf{D}_{\mathbf{g}} = \left(\begin{array}{cccccc|c} 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right), \quad \vec{b}_{\mathbf{g}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{7} \end{pmatrix}$$

$$\mathbf{D}_{\mathbf{g}} = \left(\begin{array}{cccc|cc|c} 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right), \quad \vec{b}_{\mathbf{g}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{30} \end{pmatrix}$$

$$\mathbf{G} = \left(\begin{array}{cccccc|c} 2R_2^2 & \gamma - R_2^2 & \alpha & -\alpha - \gamma & -\alpha - \gamma & \alpha & 0 \\ \gamma - R_2^2 & 2R_2^2 & \gamma - R_2^2 & \alpha & -\alpha - \gamma & -\alpha - \gamma & 0 \\ \alpha & \gamma - R_2^2 & 2R_2^2 & \gamma - R_2^2 & \alpha & -\alpha - \gamma & 0 \\ -\alpha - \gamma & \alpha & \gamma - R_2^2 & 2R_2^2 & \gamma - R_2^2 & \alpha & 0 \\ -\alpha - \gamma & -\alpha - \gamma & \alpha & \gamma - R_2^2 & 2R_2^2 & \gamma - R_2^2 & 0 \\ \alpha & -\alpha - \gamma & -\alpha - \gamma & \alpha & \gamma - R_2^2 & 2R_2^2 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & R_1^2 \end{array} \right)$$

$$\mathbf{Q} = \left(\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & -1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right)$$

$$\mathbf{G} = R_1^2 \left(\begin{array}{cccccc|c} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right) + R_2^2 \left(\begin{array}{cccccc|c} 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$V_{\text{Cas}} = -\frac{\Gamma(s)}{2\pi^s} \cdot \int_{\mathcal{F}_7} d^7z \sqrt{G} \sum_{j=0}^6 \sum_{\vec{n} \in \mathbb{Z}^7} \frac{\text{Tr}_{\mathbf{B}}(\mathbf{D}_{\mathbf{h}}^j) - \text{Tr}_{\mathbf{F}}(\mathbf{D}_{\mathbf{g}}^j) \cdot e^{2\pi i \vec{h} \cdot \vec{n}}}{|(\mathbf{I} - \mathbf{D}_{\mathbf{g}}^j)\vec{z} - j\vec{b}_{\mathbf{g}} + \vec{n}|^{2s}}$$

$$(0,0,0,0,0,0), \left(\frac{6}{7}, \frac{5}{7}, \frac{4}{7}, \frac{3}{7}, \frac{2}{7}, \frac{1}{7}\right), \left(\frac{5}{7}, \frac{3}{7}, \frac{1}{7}, \frac{6}{7}, \frac{4}{7}, \frac{2}{7}\right), \left(\frac{4}{7}, \frac{1}{7}, \frac{5}{7}, \frac{2}{7}, \frac{6}{7}, \frac{3}{7}\right) \\ \left(\frac{3}{7}, \frac{6}{7}, \frac{2}{7}, \frac{5}{7}, \frac{1}{7}, \frac{4}{7}\right), \left(\frac{2}{7}, \frac{4}{7}, \frac{6}{7}, \frac{1}{7}, \frac{3}{7}, \frac{5}{7}\right), \left(\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}\right)$$

$$V_{\text{Cas}} = -\frac{\Gamma(s)}{2\pi^s} \cdot \frac{\sqrt{7}R_1R_2^6}{|\mathbb{Z}_7|} \cdot \sum_{j=0}^6 \int_{T^7} d^6z \sum_{\substack{\vec{n} \in \mathbb{Z}^6 \\ m \in \mathbb{Z}}} \frac{\text{Tr}_{\mathbf{B}}(\mathbf{D}_{\mathbf{g}}^j) - \text{Tr}_{\mathbf{F}}(\mathbf{D}_{\mathbf{g}}^j) \cdot e^{2\pi i m h_7}}{\left[R_2^2 |(\mathbf{I} - \mathbf{D}_{\mathbf{g}}^j)\vec{z} + \vec{n}|^2 + R_1^2 \left|m - \frac{j}{7}\right|^2\right]^s},$$

$$R_1 = \frac{R}{x^{6/7}}, R_2 = Rx^{1/7}$$



$$V_{\text{Cas}} = -\frac{\Gamma(s)}{2\pi^s} \cdot \frac{x^{\frac{66}{7}}}{\sqrt{7} \cdot R^4} \cdot \sum_{j=0}^6 \int_{T^6} d^6z \sum_{\substack{\vec{n} \in \mathbb{Z}^6 \\ m \in \mathbb{Z}}} \frac{\text{Tr}_{\mathbf{B}}(\mathbf{D}_{\mathbf{g}}^j) - \text{Tr}_{\mathbf{F}}(\mathbf{D}_{\mathbf{g}}^j) \cdot e^{2\pi i m h_7}}{\left[x^2 |(\mathbf{I} - \mathbf{D}_{\mathbf{g}}^j)\vec{z} + \vec{n}|^2 + \left| m - \frac{j}{7} \right|^2 \right]^s}$$

$$s_{\mathbf{g}} = 2 \cdot 7 \vec{h} \cdot \vec{b}_{\mathbf{g}} \bmod 2\mathbb{Z} = 2h_7 \bmod 2\mathbb{Z}$$

j	0	1	2	3	4	5	6
$\text{Tr}_{\mathbf{B}}(\mathbf{D}_{\mathbf{g}}^j)$	128	2	2	2	2	2	2
$\text{Tr}_{\mathbf{F}}(\mathbf{D}_{\mathbf{g}}^j)$	128	± 2	2	± 2	2	± 2	2

$$\begin{aligned} V_{\text{Cas}}^{\text{twisted}} &= -\frac{\Gamma(s-3)}{2\pi^{s-3}} \cdot \frac{x^{\frac{24}{7}}}{7R^4} 2 \sum_{j=1}^6 \sum_{m \in \mathbb{Z}} \frac{1 - (-1)^{j+m}}{\left(m - \frac{j}{7}\right)^{2s-6}} \\ &= -\frac{\Gamma(s-3)}{2\pi^{s-3}} \cdot \frac{x^{\frac{24}{7}}}{7R^4} \cdot \frac{2^{2s-6} - 1}{2^{2s-7}} (7^{2s-6} - 1) \zeta(2s-6) \end{aligned}$$

$$V_{\text{Cas}} = -\frac{\mathcal{C}_1(x)}{R^4} - \underbrace{\frac{\Gamma\left(\frac{5}{2}\right)}{2\pi^{\frac{5}{2}}} \cdot \frac{2^5 - 1}{2^4} \cdot \frac{7^5 - 1}{7} \zeta(5)}_{\approx 733.1} \cdot \frac{x^{\frac{24}{7}}}{R^4}$$

$$\mathcal{C}_1(x) = \frac{\Gamma(s)}{2\pi^s} \cdot \frac{128x^{\frac{66}{7}}}{\sqrt{7}} \sum_{\substack{\vec{n} \in \mathbb{Z}^6 \\ m \in \mathbb{Z}}} \frac{1 - (-1)^m}{[x^2 |\vec{n}|^2 + m^2]^s}$$

$$\mathcal{C}_1(x) \sim \frac{\Gamma(s)}{2\pi^s} \cdot \frac{x^{\frac{66}{7}}}{\sqrt{7} \cdot 2^{2s-8}} \sum_{m \in \mathbb{Z}} \frac{1}{\left|m + \frac{1}{2}\right|^{2s}} = \underbrace{\frac{\Gamma\left(\frac{11}{2}\right)}{2\pi^{11/2}} \cdot \frac{(2^{11} - 1)\zeta(11)}{4\sqrt{7}}}_{\approx 9.337} x^{\frac{66}{7}} \text{ as } x \rightarrow \infty$$

$$\mathcal{C}_1(x) \sim \frac{186\zeta(5)}{7\pi^2} x^{\frac{24}{7}} \approx 2.791 x^{\frac{24}{7}} \text{ as } x \rightarrow 0$$

$$\frac{x\mathcal{C}'}{\mathcal{C}} = \frac{d(k+4)-8}{k(k+(d-2)p)} \frac{(k-p)n_p^2x^2 - p\tilde{n}_p^2}{n_p^2x^2 + \tilde{n}_p^2},$$

$$\frac{x\mathcal{C}'}{\mathcal{C}} = \frac{12}{35} \cdot \frac{3n_p^2x^2 - 4\tilde{n}_p^2}{n_p^2x^2 + \tilde{n}_p^2} \in \left[-\frac{48}{35}, \frac{36}{35}\right]$$

$$\frac{x\mathcal{C}'}{\mathcal{C}} \in \left[\frac{24}{7}, \frac{66}{7}\right] = \left[\frac{120}{35}, \frac{330}{35}\right]$$



$$\mathbf{D}_{\mathbf{g}} = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \vec{b}_{\mathbf{g}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{8} \end{pmatrix}$$

$$2\kappa_{11}^2S_{\rm TCCR}=\int d^{11}x\sqrt{-g}\left[R-\frac{1}{2}|\tilde{G}_4|^2\right]-\frac{1}{6}\int \tilde{G}_4\wedge\tilde{G}_4\wedge\tilde{C}_3$$

$$\frac{1}{2\kappa_{11}^2}\Big(\int\tilde{G}_4\Big)^3=2\pi n, n\in\mathbb{Z}$$

$$S_{\rm TCCR}=\int d^{11}x\sqrt{-g}\Big[\frac{1}{2\kappa_{11}^2}R-\frac{1}{2g_3^2}|G_4|^2\Big]-\frac{1}{24\pi^2}\int G_4\wedge G_4\wedge C_3$$

$$g_3^2\equiv 2\cdot (2\pi^2)^{2/3}\kappa_{11}^{2/3}=2\cdot (2\pi^2)^{2/3}\ell_{11}^6\approx (3.822\ell_{11}^3)^2$$

$$T_{\rm M2}=\sqrt{\frac{1}{2}\frac{g_3^2}{\kappa_{11}^2}}=\frac{(2\pi^2)^{1/3}}{\ell_{11}^3}, T_{\rm M5}=\sqrt{\frac{1}{2}\frac{4\pi^2}{g_3^2\kappa_{11}^2}}=\left(\frac{\pi}{2}\right)^{\frac{1}{3}}\frac{1}{\ell_{11}^6}.$$

$$\int \frac{1}{2g_3^2}|G_4|^2=\left(\frac{\pi}{2}\right)^{\frac{2}{3}}\frac{n_4^2}{\ell_{11}^3}\int |\omega_4|^2$$

$$\mathcal{E}(\gamma)=-\frac{\Gamma(s_\gamma)}{2\pi^{s_\gamma}}\cdot\frac{G_{\parallel}^{-\left(s_\gamma-\frac{1}{2}\right)}}{|\Gamma|}\sum_{n\in\mathbb{Z}}\frac{e^{2\pi i\beta n}}{\left|n-b_{\gamma}^{\parallel}\right|^{2s_\gamma}}$$

$$\sum_{n\in\mathbb{Z}}\frac{e^{2\pi i\beta n}}{\left|n-b_{\gamma}^{\parallel}\right|^{2s_\gamma}}=\begin{cases}\zeta\left(2s_\gamma,b_{\gamma}^{\parallel}\right)+\zeta\left(2s_\gamma,1-b_{\gamma}^{\parallel}\right)&\text{if }\beta=0\text{mod}\mathbb{Z}\\2^{2s_\gamma}\left[\zeta\left(2s_\gamma,\frac{b_{\gamma}^{\parallel}}{2}\right)-\zeta\left(2s_\gamma,\frac{1-b_{\gamma}^{\parallel}}{2}\right)\right.\\ \left.+\zeta\left(2s_\gamma,1-\frac{b_{\gamma}^{\parallel}}{2}\right)-\zeta\left(2s_\gamma,1-\frac{1-b_{\gamma}^{\parallel}}{2}\right)\right]&\text{if }\beta=\frac{1}{2}\text{mod}\mathbb{Z}\end{cases}$$

$$\left(\mathbf{I}-\mathbf{D}_{\gamma}^{-1}\right)\vec{n}=0\text{mod}\mathbb{Z}\Leftrightarrow\left(\mathbf{I}-\mathbf{D}_{\gamma}\right)\vec{n}=0\text{mod}\mathbb{Z},$$

$$V_{\rm Cas}=\sum_{\mathbf{r}}\sum_{\gamma\in\Gamma}\mathrm{Tr}_{\mathbf{r}}(\mathbf{D}_{\gamma})\mathcal{E}(\gamma)=\frac{1}{2}\sum_{\mathbf{r}}\sum_{\gamma\in\Gamma}\left(\mathrm{Tr}_{\mathbf{r}}(\mathbf{D}_{\gamma})+(-1)^{2\beta_{\mathbf{r}}}\mathrm{Tr}_{\mathbf{r}}(\mathbf{D}_{\gamma}^{-1})\right)\mathcal{E}(\gamma).$$

$$\mathrm{Tr}_{\mathrm{Spinor}}\left(\mathbf{D}_{\gamma}^{-1}\right)=(-1)^{\sum n_i}\mathrm{Tr}_{\mathrm{Spinor}}\left(\mathbf{D}_{\gamma}\right),\,\,\text{with}\,\,p\cdot\theta_i=2\pi n_i,$$

$$\sum_{\vec{n}+\vec{c}\neq 0}\frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}}$$



$$|\vec{n}|^2 = G_{ij} n^i n^j$$

$$\frac{e^{2\pi i \vec{h} \cdot \vec{n}}}{|\vec{n} + \vec{c}|^{2s}}$$

$$\mathcal{F}\left[\frac{e^{2\pi i \vec{h} \cdot \vec{n}}}{|\vec{n} + \vec{c}|^{2s}}\right](\vec{k}) \equiv \int d^k \vec{n} \frac{e^{2\pi i (\vec{h} - \vec{k}) \cdot \vec{n}}}{|\vec{n} + \vec{c}|^{2s}} = \frac{2^{k-2s} \pi^{k/2} \Gamma\left(\frac{k}{2} - s\right)}{\Gamma(s)} \frac{e^{-2\pi i \vec{h} \cdot \vec{c}}}{|\vec{h} - \vec{k}|^{k-2s}}$$

$$F(\vec{n}) \equiv \frac{\Gamma(s, \alpha |\vec{n} + \vec{c}|^2)}{\Gamma(s)}, \Gamma(s, x) \equiv \int_x^\infty t^{s-1} e^{-t} dt$$

$$f(\vec{n}) \equiv 1 - F(\vec{n}) = \frac{\gamma(s, \alpha |\vec{n} + \vec{c}|^2)}{\Gamma(s)}$$

$$\lim_{\vec{n} + \vec{c} \rightarrow 0} f(\vec{n}) \frac{e^{2\pi i \vec{h} \cdot \vec{n}}}{|\vec{n} + \vec{c}|^{2s}} = \frac{\alpha^s e^{-2\pi i \vec{h} \cdot \vec{c}}}{\Gamma(s+1)} \chi_{\mathbb{Z}^k}(\vec{c})$$

$$\begin{aligned} \mathcal{F}\left[\frac{e^{2\pi i \vec{h} \cdot \vec{n}}}{|\vec{n} + \vec{c}|^{2s}} \cdot \frac{\gamma(s, \alpha |\vec{n} + \vec{c}|^2)}{\Gamma(s)}\right](\vec{k}) &\equiv \int d^k \vec{n} e^{-2\pi i \vec{k} \cdot \vec{n}} \frac{e^{2\pi i \vec{h} \cdot \vec{n}}}{|\vec{n} + \vec{c}|^{2s}} \cdot \frac{\gamma(s, \alpha |\vec{n} + \vec{c}|^2)}{\Gamma(s)} \\ &= \frac{1}{\Gamma(s)} \int d^k \vec{n} \frac{e^{2\pi i (\vec{h} - \vec{k}) \cdot \vec{n}}}{|\vec{n} + \vec{c}|^{2s}} \gamma(s, \alpha |\vec{n} + \vec{c}|^2) = \frac{e^{-2\pi i (\vec{h} - \vec{k}) \cdot \vec{c}}}{\Gamma(s)} \int d^k \vec{n}' \frac{e^{2\pi i (\vec{h} - \vec{k}) \cdot \vec{n}'}}{|\vec{n}'|^{2s}} \gamma(s, \alpha |\vec{n}'|^2) \end{aligned}$$

$$\mathbf{G} = \mathbf{M}^T \mathbf{M}, \vec{m} = \mathbf{M} \vec{n'}$$

$$\vec{q} \equiv \mathbf{G}^{-1}(\vec{h} - \vec{k})$$

$$\begin{aligned} \mathcal{F}\left[\frac{e^{2\pi i \vec{h} \cdot \vec{n}}}{|\vec{n} + \vec{c}|^{2s}} \cdot \frac{\gamma(s, \alpha |\vec{n} + \vec{c}|^2)}{\Gamma(s)}\right](\vec{k}) &= \frac{e^{-2\pi i (\vec{h} - \vec{k}) \cdot \vec{c}}}{\Gamma(s) \sqrt{G}} \int d^k \vec{m} \frac{e^{2\pi i ((\mathbf{M}^{-1})^T (\vec{h} - \vec{k})) \cdot \vec{m}}}{|\vec{m}|^{2s}} \gamma(s, \alpha |\vec{m}|^2) \\ &= \frac{e^{-2\pi i (\vec{h} - \vec{k}) \cdot \vec{c}}}{\Gamma(s) \sqrt{G}} \mathcal{S}_{k-2} \int_0^\infty dr \frac{\gamma(s, \alpha r^2)}{r^{2s-k+1}} \int_0^\pi d\theta (\sin \theta)^{k-2} e^{-2\pi i |\vec{q}| r \cos \theta} \\ &= \frac{(2\pi)^{\frac{k}{2}} e^{-2\pi i (\vec{h} - \vec{k}) \cdot \vec{c}}}{\Gamma(s) \sqrt{G}} \frac{1}{(2\pi |\vec{q}|)^{k-1-2s}} \int_0^\infty dr \frac{\gamma(s, \alpha r^2)}{(2\pi |\vec{q}| r)^{2s-\frac{k}{2}}} J_{\frac{k-2}{2}}(2\pi |\vec{q}| r) \\ &= \frac{(2\pi)^{\frac{k}{2}} e^{-2\pi i (\vec{h} - \vec{k}) \cdot \vec{c}}}{\Gamma(s) \sqrt{G}} \frac{1}{(2\pi |\vec{q}|)^{k-2s}} \int_0^\infty d\tau \frac{\gamma\left(s, \frac{\alpha}{(2\pi |\vec{q}|)^2} \tau^2\right)}{\tau^{2s-\frac{k}{2}}} J_{\frac{k-2}{2}}(\tau) \\ &= \frac{2^{k-2s} \pi^{\frac{k}{2}}}{\Gamma(s)} \Gamma\left(\frac{k}{2} - s, \frac{\pi^2 |\vec{q}|^2}{\alpha}\right) \frac{e^{-2\pi i (\vec{h} - \vec{k}) \cdot \vec{c}}}{\sqrt{G} (2\pi |\vec{q}|)^{k-2s}} \end{aligned}$$

$$J_\nu(t) = \frac{t^\nu}{(2\pi)^{\nu+1}} \mathcal{S}_{2\nu} \int_0^\pi e^{-it \cos \theta} \sin^{2\nu} \theta d\theta$$

$$\frac{\mathcal{S}_{k-2}}{\Gamma(s) \sqrt{G}} \int_0^\infty dr \frac{\gamma(s, \alpha r^2)}{r^{2s-k+1}} \int_0^\pi d\theta (\sin \theta)^{k-2} = \frac{\pi^{\frac{k}{2}} \alpha^{s-k/2}}{\sqrt{G} \Gamma(s) \left(s - \frac{k}{2}\right)}$$



$$\sum_{\vec{n}+\vec{c}\neq 0}\frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}}=$$

$$\sum_{\vec{n}+\vec{c}\neq 0}\frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}}\frac{\Gamma(s,\alpha|\vec{n}+\vec{c}|^2)}{\Gamma(s)}+\frac{\pi^{2s-\frac{k}{2}}}{\Gamma(s)\sqrt{G}}\sum_{\vec{k}-\vec{h}\neq 0}\Gamma\left(\frac{k}{2}-s,\frac{\pi^2|\vec{h}-\vec{k}|_D^2}{\alpha}\right)\frac{e^{-2\pi i(\vec{h}-\vec{k})\cdot\vec{c}}}{|\vec{h}-\vec{k}|_D^{k-2s}}$$

$$+\delta_{\vec{h},\vec{0}}\frac{\pi^{\frac{k}{2}}\alpha^{s-k/2}}{\sqrt{G}\Gamma(s)\left(s-\frac{k}{2}\right)}-\frac{\alpha^se^{-2\pi i\vec{h}\cdot\vec{c}}}{\Gamma(s+1)}\chi_{\mathbb{Z}^k}(\vec{c})$$

$$\sum_{\vec{n}\neq\vec{0}}\frac{(-1)^m}{m^2+mn+n^2}=-\frac{4\pi\ln 2}{3\sqrt{3}},\sum_{\vec{n}\neq\vec{0}}\frac{(-1)^{m+n}}{m^2+5n^2}=-\frac{\pi}{\sqrt{5}}\ln\left(1+\sqrt{5}\right),$$

$$\sum_{\vec{n}\in\mathbb{R}^3}\frac{(-1)^{n_1+n_2+n_3}}{\left[\left(n_1+\frac{1}{6}\right)^2+\left(n_2+\frac{1}{6}\right)^2+\left(n_3+\frac{1}{6}\right)^2\right]^s}=12^s\beta(2s-1),$$

$$\sum_{\vec{n}\in\mathbb{R}^8-\{\vec{0}\}}\frac{(-1)^{\sum_i n_i}}{|\vec{n}|^{2s}}=-16\zeta(s)\eta(s-3),\sum_{\vec{n}\in\mathbb{R}^8}\frac{1}{|\vec{n}+\vec{1}/2|^{2s}}=\frac{256}{2^s}\zeta(s-3)\lambda(s),$$

$$\frac{\Gamma\left(\frac{k}{2}-s,\frac{\pi^2\lambda}{\alpha}\right)}{\lambda^{k/2-s}}\leq \frac{\Gamma(k/2)}{2\pi^{k/2}}\frac{\epsilon}{\lambda^{\frac{k-1}{2}}}.$$

$$\frac{\Gamma\left(\frac{k}{2}-s,\frac{\pi^2\lambda}{\alpha}\right)}{\left(\frac{\pi^2}{\alpha}\lambda\right)^{1/2-s}}\leq \frac{\Gamma(k/2)}{2\pi^{k/2}}\left(\frac{\pi^2}{\alpha}\right)^{s-1/2}\epsilon$$

$$\frac{\Gamma(a,z)}{z^a}\leq \frac{e^{-z}}{z}$$

$$\frac{e^{-\frac{\pi^2\lambda}{\alpha}}}{\frac{\pi^2\lambda}{\alpha}}\leq \frac{\Gamma(k/2)}{2\pi^{k/2}}\left(\frac{\pi^2}{\alpha}\right)^{s-1/2}\epsilon$$

$$\lambda\geq \frac{\alpha}{\pi^2}W\left(\frac{2\pi^{k/2}}{\Gamma(k/2)\epsilon}\left(\frac{\alpha}{\pi^2}\right)^{s-1/2}\right)$$

$$\int_{[0,1]^k}dV\sum_{n\in\mathbb{Z}^k}\frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|(\mathbf{I}-\mathbf{D}_\gamma)\vec{z}+\vec{b}_\gamma+\vec{n}|^{2s}}$$

$$\Lambda^\perp\equiv(\mathbf{I}-\mathbf{D}_\gamma)\mathbb{Z}^k\subset\mathbb{Z}^k$$

$$\Lambda^\parallel\equiv\{\vec{n}\in\mathbb{Z}^k\mid(\mathbf{I}-\mathbf{D}_\gamma)\vec{n}=\vec{0}\}$$

$$\Lambda\equiv\Lambda^\parallel\oplus\Lambda^\perp\supset\mathbb{Z}^k$$

$$\Gamma_\Lambda\equiv\mathbb{Z}^k/\Lambda$$



$$\vec{n} = \vec{k} + \vec{l} + \vec{\gamma}$$

$$\sum_{\substack{\vec{k} \in \Lambda^\perp \\ \vec{l} \in \Lambda^\parallel, \gamma \in \Gamma_\Lambda}} \int_{[0,1]^k} dV \frac{e^{2\pi i \vec{h} \cdot [\vec{k} + \vec{l} + \vec{\gamma}]}}{|(\mathbf{I} - \mathbf{D}_\gamma) \vec{x} + \vec{k} + \vec{b}_\gamma + \vec{l} + \vec{\gamma}|^{2s}}$$

$$\vec{h} \cdot (\mathbf{I} - \mathbf{D}_\gamma) \vec{l} = 0 \text{ mod } \mathbb{Z}$$

$$\vec{y} \equiv (\mathbf{I} - \mathbf{D}_\gamma) \vec{x}_\perp + \vec{k}$$

$$\frac{V_\parallel}{|\det(\mathbf{I} - \mathbf{D}_\gamma)|} \sum_{\substack{\vec{l} \in \Lambda^\parallel \\ \gamma \in \Gamma_\Lambda}} \int_{\mathbb{R}^{k''}} dV \frac{e^{2\pi i \vec{h} \cdot (\vec{\gamma} + \vec{l})}}{|\vec{y} + \vec{b}_\gamma + \vec{l} + \vec{\gamma}|^{2s}}$$

$$\vec{y} + \vec{b}_\gamma + \vec{l} + \vec{\gamma} \rightarrow \vec{y} + \vec{b}_\gamma^\parallel + \vec{l} + \vec{\gamma}^\parallel$$

$$\left| \vec{y} + \vec{b} \right|_\gamma^\parallel + \vec{l} + \vec{\gamma}^\parallel|^2 = |\vec{y}|^2 + \alpha^2, \alpha^2 \equiv \left| \vec{b} \right|_\gamma^\parallel + \vec{l} + \vec{\gamma}^\parallel|^2$$

$$\int_{\mathbb{R}^{k''}} \frac{dV}{(|\vec{y}|^2 + \alpha^2)^s} = \frac{2\pi^{\frac{k''}{2}}}{\Gamma(k''/2)} \int_0^\infty \frac{y^{k''-1} dy}{(y^2 + \alpha^2)^s} = \frac{2\pi^{\frac{k''}{2}}}{\Gamma(k''/2)} = \pi^{\frac{k-k'}{2}} \frac{\Gamma(s')}{\Gamma(s)} \frac{1}{\alpha^{2s'}}$$

$$\pi^{\frac{k-k'}{2}} \frac{\Gamma(s')}{\Gamma(s)} \frac{V_\parallel}{|\det(\mathbf{I} - \mathbf{D}_\gamma)|} \sum_{\substack{\vec{l} \in \Lambda^\parallel \\ \gamma \in \Gamma_\Lambda}} \frac{e^{2\pi i \vec{h} \cdot (\vec{\gamma} + \vec{l})}}{\left| \vec{b}_\gamma^\parallel + \vec{l} + \vec{\gamma}^\parallel \right|^{2s'}}$$

$$\sum_{\gamma \in \Gamma_\Sigma} e^{2\pi i \vec{h} \cdot \vec{\gamma}} = |\det(\mathbf{I} - \mathbf{D}_\gamma)| \hat{\delta}_{\vec{h}}$$

$$\hat{\delta}_{\vec{h}} \equiv 1 \text{ iff } \vec{h} \cdot \vec{l} \in \mathbb{Z}, \forall \vec{l} \in \Sigma^\perp, \text{ and } 0 \text{ otherwise.}$$

$$(\mathbf{I} - \mathbf{D}_\gamma)^T \cdot \vec{h} = (\mathbf{I} - \mathbf{D}_\gamma)^T \cdot \vec{\eta}$$

$$\vec{h} \cdot \vec{w} = \vec{\beta} \cdot \vec{w} + \vec{\eta} \cdot \vec{w} = \vec{\beta} \cdot \vec{w} \text{ mod } \mathbb{Z}$$

$$\sqrt{G_\parallel} \pi^{\frac{k-k'}{2}} \frac{\Gamma(s')}{\Gamma(s)} \sum_{\vec{w} \in \Xi} \frac{e^{2\pi i \vec{\beta} \cdot \vec{w}}}{\left| \vec{w} + \vec{b}_\gamma^\parallel \right|^{2s'}},$$

$$\vec{c} = (\mathbf{I} - \mathbf{D}_\gamma) \vec{z} + \vec{b}_\gamma$$

$$\begin{aligned} \sqrt{G} \int_{[0,1]^k} d^k z e^{-2\pi i (\vec{h} - \vec{k}) \cdot [(\mathbf{I} - \mathbf{D}_\gamma) \vec{z} + \vec{b}_\gamma]} &= \sqrt{G} e^{-2\pi i (\vec{h} - \vec{k}) \cdot \vec{b}_\gamma} \int_{[0,1]^k} d^k z e^{-2\pi i (\vec{h} - \vec{k}) \cdot [(\mathbf{I} - \mathbf{D}_\gamma) \vec{z}]} \\ &= \sqrt{G} e^{-2\pi i (\vec{h} - \vec{k}) \cdot \vec{b}_\gamma} \prod_{l=1}^k \frac{e^{2\pi i \vec{q}_l} - 1}{2\pi i \vec{q}_l}, \vec{q} \equiv -(\mathbf{I} - \mathbf{D}_\gamma)^T (\vec{h} - \vec{k}) \end{aligned}$$



$$\vec{h} \cdot (\mathbf{I} - \mathbf{D}_\gamma) \vec{l} \in \mathbb{Z}, \forall \vec{l} \in \mathbb{Z}^k$$

$$\frac{e^{2\pi i \vec{q}_l} - 1}{2\pi i \vec{q}_l} \rightarrow \delta_{q_l,0} \text{ for } q_l \in \mathbb{Z}$$

$$(\mathbf{I}-\mathbf{D}_\gamma)^T(\vec{h}-\vec{k})=0,\Rightarrow \vec{k}=\vec{\eta}+\vec{\kappa}$$

$$(\mathbf{I}-\mathbf{D}_\gamma)^T\vec{\eta}=(\mathbf{I}-\mathbf{D}_\gamma)^T\vec{h}$$

$$(\vec{\beta}-\vec{k})\cdot\vec{b}_\gamma=(\vec{\beta}-\vec{k})\cdot\vec{b}_\gamma^\parallel$$

$$\begin{aligned} \mathcal{S}_{\text{position}}^\alpha &+ \frac{\pi^{2s-\frac{k}{2}}}{\Gamma(s)} \sum_{\substack{k \in \Lambda \\ \kappa - \vec{\beta} \neq \vec{0}}} \Gamma\left(\frac{k}{2} - s, \frac{\pi^2 |\vec{\beta} - \vec{\kappa}|_D^2}{\alpha}\right) \frac{e^{-2\pi i (\vec{\beta} - \vec{\kappa}) \cdot \vec{b}_\gamma^\parallel}}{|\vec{\beta} - \vec{\kappa}|_D^{k-2s}} \\ &+ \delta_{\vec{h}, \vec{0}} \frac{\pi^{\frac{k}{2}} \alpha^{s-k/2}}{\Gamma(s) \left(s - \frac{k}{2}\right)} - \frac{\sqrt{G} \alpha^s}{\Gamma(s+1)} \delta_{(\mathbf{D}_\gamma, \vec{b}_\gamma)} \end{aligned}$$

$$|\vec{v}|_I \equiv \vec{v}^T \cdot \mathbf{G}_I \cdot \vec{v}, \mathbf{G}_I \equiv \mathbf{V}^T \mathbf{G}_D \mathbf{V}, \vec{d} \equiv \mathbf{V}^{-1} \vec{\beta}, \vec{v}_\gamma \equiv \mathbf{V}^T \vec{b}_\gamma^\parallel, s' = \frac{k' - k}{2} + s$$

$$\mathcal{S}_{\text{position}}^\alpha + \frac{\pi^{2s'-k'+\frac{k}{2}}}{\Gamma(s)} \sum_{\substack{\vec{l} \in \mathbb{Z}^{k'} \\ \vec{l} - \vec{d} \neq \vec{0}}} \Gamma\left(\frac{k'}{2} - s', \frac{\pi^2 |\vec{d} - \vec{l}|_I^2}{\alpha}\right) \frac{e^{-2\pi i (\vec{d} - \vec{l}) \cdot \vec{v}_\gamma}}{|\vec{d} - \vec{l}|_I^{k'-2s'}} + \delta_{\vec{d}, \vec{0}} \frac{\pi^{\frac{k}{2}} \alpha^{s'-k'/2}}{\Gamma(s) \left(s' - \frac{k'}{2}\right)}$$

$$\begin{aligned} &\frac{\Gamma(s)}{\pi^{k/2}} \frac{\pi^{k'/2}}{\Gamma(s')} \frac{1}{\sqrt{G_I}} \mathcal{S}_{\text{position}}^\alpha + \frac{\pi^{2s'-\frac{k'}{2}}}{\Gamma(s') \sqrt{G_I}} \sum_{\substack{\vec{l} \in \mathbb{Z}^{k'} \\ \vec{l} - \vec{d} \neq \vec{0}}} \Gamma\left(\frac{k'}{2} - s', \frac{\pi^2 |\vec{d} - \vec{l}|_I^2}{\alpha}\right) \frac{e^{-2\pi i (\vec{d} - \vec{l}) \cdot \vec{v}_\gamma}}{|\vec{d} - \vec{l}|_I^{k'-2s'}} \\ &+ \delta_{\vec{d}, \vec{0}} \frac{\pi^{\frac{k'}{2}} \alpha^{s'-k'/2}}{\sqrt{G_I} \Gamma(s') \left(s' - \frac{k'}{2}\right)} \end{aligned}$$

$$\mathcal{E}(\gamma)=-\hat{\delta}_{\vec{h}}\frac{\Gamma(s')}{2\pi^{s'}}\cdot\frac{\sqrt{G_{\parallel}}}{|\Gamma|}\sum_{\vec{m}}\frac{e^{2\pi i\vec{d}\cdot\vec{m}}}{|\vec{m}+\vec{v}_{\gamma}|^{2s'}},$$

$$\mathbf{G}_{\parallel}=\mathbf{V}^{-1}\mathbf{G}(\mathbf{V}^{-1})^T, \vec{v}_{\gamma}=\mathbf{V}^T\cdot\vec{b}_{\gamma}^{\parallel}, \vec{d}\equiv\mathbf{V}^{-1}\vec{\beta}, s'=\frac{k'-k}{2}+s$$

$$\mathcal{E}(\gamma)=-\hat{\delta}_{\vec{h}}\frac{\Gamma(s')}{2\pi^{s'}}\cdot\frac{\sqrt{G_{\parallel}}}{|\Gamma|}\sum_{\vec{\xi}\in\Xi}\frac{e^{2\pi i\vec{\beta}\cdot\vec{\xi}}}{\left|\vec{\xi}+\vec{b}_{\gamma}^{\parallel}\right|^{2s'}}$$

$$SO(D-2)\rightarrow SO(k)\times SO(d-2)$$



$$v^a w^b \rightarrow M_c^a M_d^b v^c w^d$$

$$\mathrm{Tr}(\mathbf{M}^{\mathrm{Tens.}}) = \delta_a^c \delta_b^d M_c^a M_d^b = (\mathrm{Tr}(\mathbf{M}))^2$$

$$\mathbb{P}^{\mathrm{Sym.}} T^{ab} = \frac{1}{2} (T^{ab} + T^{ba}) - \frac{T}{n} \delta^{ab}$$

$$\mathbb{P}^{\mathrm{ASym.}} T^{ab} = \frac{1}{2} (T^{ab} - T^{ba})$$

$$\mathbb{P}^{\mathrm{Sing.}} T^{ab} = \frac{T}{n} \delta^{ab}$$

$$\mathrm{Tr}(\mathbf{M}^{\mathrm{Sym.}}) = \frac{1}{2} (\mathrm{Tr}(\mathbf{M})^2 + \mathrm{Tr}(\mathbf{M}^2)) - 1$$

$$\mathrm{Tr}(\mathbf{I}^{\mathrm{Sym.}}) = \frac{n^2}{2} \left(\frac{1}{2} - \frac{1}{n} \right) \cdot n = \frac{n(n+1)}{2} - 1$$

$$\mathbb{P}^{3-\mathrm{Antisym.}} T^{abc} = \frac{1}{6} (T^{abc} + T^{bca} + T^{cab} - T^{bac} - T^{acb} - T^{cba}),$$

$$\mathrm{Tr}(\mathbf{M}^{3-\mathrm{Antisym.}}) = \frac{1}{6} (\mathrm{Tr}(\mathbf{M})^3 - 3\mathrm{Tr}(\mathbf{M}^2) \cdot \mathrm{Tr}(\mathbf{M}) + 2\mathrm{Tr}(\mathbf{M}^3))$$

$$\mathrm{Tr}(\mathbf{I}^{3-\mathrm{Antisym.}}) = \frac{1}{6} (n^3 - 3n^2 + 2n) = \frac{n(n-1)(n-2)}{6} = \binom{n}{3}$$

$$\sum_{k=0}^{\infty} x^k \mathrm{Tr}(\mathbf{M}^{k-\mathrm{Antisym.}}) = \prod_{l=1}^{\infty} \exp \left(-(-x)^l \frac{\mathrm{Tr}(\mathbf{M}^l)}{l} \right)$$

$$\mathbb{P}^{\mathrm{R.S.}} \rho_a^\alpha = \rho_a^\alpha - \frac{1}{n^2} \delta^{ab} (\gamma_b)^\alpha_\beta \rho_a^\beta$$

$$\mathrm{Tr}(\mathbf{M}^{\mathrm{R.S.}}) = (\mathrm{Tr}(\mathbf{M}) - 1) \mathrm{Tr}(\mathbf{M}^{\mathrm{Spinor}}),$$

$$\mathbf{2^k} \otimes \mathbf{2^k} = \sum_{l=1}^k \binom{\mathbf{n}}{\mathbf{l}}$$

$$\begin{aligned} \mathrm{Tr}(\mathbf{M}^{\mathrm{Spinor}})^2 &= 1 + \mathrm{Tr}(\mathbf{M}) + \frac{1}{2} (\mathrm{Tr}(\mathbf{M})^2 - \mathrm{Tr}(\mathbf{M}^2)) \\ &+ \frac{1}{6} (\mathrm{Tr}(\mathbf{M})^3 - 3\mathrm{Tr}(\mathbf{M})\mathrm{Tr}(\mathbf{M}^2) + 2\mathrm{Tr}(\mathbf{M}^3)) \\ &+ \frac{1}{24} (\mathrm{Tr}(\mathbf{M})^4 - 6\mathrm{Tr}(\mathbf{M})^2\mathrm{Tr}(\mathbf{M}^2) + 3\mathrm{Tr}(\mathbf{M}^2)^2 + 8\mathrm{Tr}(\mathbf{M})\mathrm{Tr}(\mathbf{M}^3) - 6\mathrm{Tr}(\mathbf{M}^4)) \end{aligned}$$



$$\mathbf{M} = \begin{pmatrix} \cos \theta_1 & \sin \theta_1 & 0 & 0 & 0 & 0 \\ -\sin \theta_1 & \cos \theta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \cos \theta_4 & \sin \theta_4 & 0 \\ 0 & 0 & \cdots & -\sin \theta_4 & \cos \theta_4 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{pmatrix} = \exp (i \theta_{\alpha} E_{\alpha}^3)$$

$$\mathrm{Tr}_{\mathbf{R}}(\mathbf{M}) = \sum_{\vec{\mu} \in R} \exp (i \vec{\theta} \cdot \vec{\mu})$$

$$\mathrm{Tr}_9(\mathbf{M}) = 2 \sum_{i=1}^4 \cos \theta_i + 1$$

$$\mathrm{Tr}_{\mathrm{Spinor}}(\mathbf{M}) = \prod_{i=1}^4 2 \cos \frac{\theta_i}{2}$$

$$\begin{aligned} \mathrm{Tr}_{\mathrm{Spinor}}(-\mathcal{M}) &= \prod_{i=1}^4 2 \cos \left(\frac{\theta'_i}{2} + \pi \eta_i \right) \\ &= (-1)^{\sum \eta_i} \prod_{i=1}^4 2 \cos \frac{\theta'_i}{2} = (-1)^{\sum \eta_i} \mathrm{Tr}_{\mathrm{Spinor}}(\mathcal{M}) \end{aligned}$$

$$\mathrm{Tr}_9(\mathbf{D}_{\mathbf{g}}^j) = 2 \sum_{i=1}^4 \cos (j \cdot \theta_i^{\mathbf{g}}) + 1$$

$$\mathrm{Tr}_{\mathrm{Spinor}}(\mathbf{D}_{\mathbf{g}}^p) = \prod_{i=1}^4 2 \cos (\pi n_i) = (-1)^{\sum n_i} \cdot 16$$

$$\mathrm{Tr}_{44}(\mathbf{D}_{\mathbf{g}}^j) = \sum_{a=1}^4 \left[2 \cos (2j \theta_a^{\mathbf{g}}) + 2 \cos (j \theta_a^{\mathbf{g}}) + 1 + 2 \sum_{\substack{b=1 \\ j \neq i}}^4 \cos (j \theta_a^{\mathbf{g}}) \cos (j \theta_b^{\mathbf{g}}) \right]$$

$$\begin{aligned} \mathrm{Tr}_{84}(\mathbf{D}_{\mathbf{g}}) &= \sum_{a=1}^4 \left[1 + 6 \cos (j \theta_a^{\mathbf{g}}) + 2 \sum_{\substack{b=1 \\ j \neq i}}^4 \cos (j \theta_a^{\mathbf{g}}) \cos (j \theta_b^{\mathbf{g}}) \right. \\ &\quad \left. + \frac{4}{3} \sum_{\substack{b,c=1 \\ b \neq c \neq a}}^4 \cos (j \theta_a^{\mathbf{g}}) \cos (j \theta_b^{\mathbf{g}}) \cos (j \theta_c^{\mathbf{g}}) \right] \end{aligned}$$

$$\mathrm{Tr}_{128}(\mathbf{D}_{\mathbf{g}}^j) = \left(2 \sum_{a=1}^4 \cos (j \theta_a^{\mathbf{g}}) \right) \prod_{a=1}^4 2 \cos \left(j \frac{\theta_a^{\mathbf{g}}}{2} \right)$$



$$\text{Tr}_{128}(\mathbf{D}_g^{p-j}) = (-1)^{\sum n_i} \left(2 \sum_{a=1}^4 \cos(j\theta_a^g) \right) \prod_{a=1}^4 2 \cos\left(j \frac{\theta_a^g}{2}\right), \text{ with } p \cdot \theta_i = 2\pi n_i$$

$\mathbf{D}_g$	$\vec{b}_g$	$ \Gamma $	Moduli	$\vec{h}$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \hline 30 \end{pmatrix}$	30	4	$(0, 0, 0, 0, 0, 0, 0)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \hline 30 \end{pmatrix}$	30	4	$(0, 0, 0, 0, 0, 0, 0)$
$\begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \hline 30 \end{pmatrix}$	30	4	$(0, 0, 0, 0, 0, 0, 0)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \hline 24 \end{pmatrix}$	24	4	$(0, 0, h_3, h_3, h_3, h_3, 0)$

$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{24} \end{pmatrix}$	24	4	$(0,0,h_3,h_3,h_3,h_3,0)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{20} \\ 0 \end{pmatrix}$	20	4	$(h_1,h_1,0,0,0,0,0)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{20} \\ 0 \end{pmatrix}$	20	4	$(h_1,h_1,0,0,0,0,0)$

$\mathbf{D_g}$	$\vec{b_g}$	$ \Gamma $	Moduli	$\vec{h}$
$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{18} \end{pmatrix}$	18	4	$(0,0,0,0,0,0,0)$



$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(h_1, h_2, 0, 0, 0, 0, 0)$
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$\mathbf{D}_g$	$\vec{b}_g$	$ \Gamma $	Moduli	$\vec{h}$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(h_1, h_2, 0, 0, 0, 0, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{12} \end{pmatrix}$	12	8	$(h_1, h_2, 0, 0, 0, 0, 0)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(h_1, h_1, h_3, h_3, 0, 0, 0)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(h_1, h_1, 0, 0, 0, 0, 0)$



$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(0,0,h_3,h_3,h_5,h_5,0)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{12} \end{pmatrix}$	12	4	$(0,0,h_3,h_3,0,0,0)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(0,0,0,0,h_5,h_5,0)$

$\mathbf{D_g}$	$\vec{b_g}$	$ \Gamma $	Moduli	$\vec{h}$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(h_1,h_2,h_3,h_3,0,0,0)$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(h_1,h_2,0,0,h_5,h_5,0)$



$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(h_1, h_2, h_3, h_3, 0, 0, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(h_1, h_2, 0, 0, h_5, h_5, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	8	$(h_1, h_2, h_3, h_3, 0, 0, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	8	$(h_1, h_2, 0, 0, h_5, h_5, 0)$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{10} \end{pmatrix}$	10	6	$(h_1, h_2, 0, 0, 0, 0, 0)$

D_g	$\vec{b}_g$	$ \Gamma $	Moduli	$\vec{h}$
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$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{8} \end{pmatrix}$	8	4	$(h_1, h_1, h_3, h_3, h_3, h_3, 0)$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{8} \end{pmatrix}$	8	6	$(h_1, h_2, h_3, h_3, h_3, h_3, 0)$

$\mathbf{D}_g$	$\vec{b}_g$	$ \Gamma $	Moduli	$\vec{h}$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{8} \end{pmatrix}$	8	6	$(h_1, h_2, h_3, h_3, h_3, h_3, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{8} \end{pmatrix}$	8	8	$(h_1, h_2, h_3, h_3, h_3, h_3, 0)$
$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{7} \end{pmatrix}$	7	4	$(0, 0, 0, 0, 0, 0, h_7)$



$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{6} \end{pmatrix}$	6	6	(0,0,0,0,0,0,0)
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{6} \end{pmatrix}$	6	6	(0, 0, 0, 0, 0, 0, 0)
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{6} \end{pmatrix}$	6	10	(0, 0, 0, 0, 0, 0, 0)
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{6} \end{pmatrix}$	6	8	($h_1, h_2, 0, 0, 0, 0, 0$)

$\mathbf{D_g}$	$\vec{b_g}$	$ \Gamma $	Moduli	$\vec{h}$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{6} \end{pmatrix}$	6	6	($h_1, h_2, 0, 0, 0, 0, 0$)



$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{6} \end{pmatrix}$	6	10	$(h_1, h_2, 0, 0, 0, 0, 0)$
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$\mathbf{D_g}$	$\vec{b_g}$	$ \Gamma $	Moduli	$\vec{h}$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{6} \end{pmatrix}$	6	12	$(h_1, h_2, h_3, h_4, 0, 0, 0)$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{6} \end{pmatrix}$	6	12	$(h_1, h_2, h_3, h_4, 0, 0, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{6} \end{pmatrix}$	6	10	$(h_1, h_2, h_3, h_4, 0, 0, 0)$



$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{6} \end{pmatrix}$	6	16	$(h_1, h_2, h_3, h_4, 0, 0, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{5} \end{pmatrix}$	5	8	$(h_1, h_2, 0, 0, 0, 0, h_7)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{4} \end{pmatrix}$	4	10	$(h_1, h_1, h_3, h_3, h_5, h_5, 0)$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{4} \end{pmatrix}$	4	8	$(h_1, h_2, h_3, h_3, h_5, h_5, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{4} \end{pmatrix}$	4	8	$(h_1, h_2, h_3, h_3, h_5, h_5, 0)$

$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{4} \end{pmatrix}$	4	16	$(h_1, h_2, h_3, h_4, h_5, 0)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{3} \end{pmatrix}$	3	10	$(0,0,0,0,0,0,h_7)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{3} \end{pmatrix}$	3	10	$(h_1, h_2, 0,0,0,0,h_7)$

$\mathbf{D_g}$	$\vec{b_g}$	$ \Gamma $	Moduli	$\vec{h}$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{3} \end{pmatrix}$	3	16	$(h_1, h_2, h_3, h_4, 0,0,h_7)$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{2} \end{pmatrix}$	2	22	$(h_1, h_2, h_3, h_4, h_5, h_6, 0)$

$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$	2	18	$(h_1, h_2, h_3, h_4, h_5, h_6, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$	2	16	$(h_1, h_2, h_3, h_4, h_5, h_6, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$	2	16	$(h_1, h_2, h_3, h_4, h_5, h_6, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$	2	18	$(h_1, h_2, h_3, h_4, h_5, h_6, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$	2	22	$(h_1, h_2, h_3, h_4, h_5, h_6, 0)$

$$\mathbf{D}_g(\mathbf{D}_g^{-1}\vec{z} - \mathbf{D}_g^{-1}\vec{b}_g + \vec{k}) + \vec{b}_g = \vec{z} + \mathbf{D}_g \cdot \vec{k}$$

$$\Phi_n(x) = \prod_{1 \leq k \leq n} \left(x - e^{2\pi i \frac{k}{n}} \right)$$



Supergravedad cuántica relativista en D=8.

$$\begin{aligned}
 S &= \int_{\Sigma} Z^a \cdot \bar{\partial} Z_a + A_{ab} Z^a \cdot Z^b + a \langle \lambda_A \lambda_B \rangle \Omega^{AB} + \tilde{a} (\langle \lambda_A \lambda_B \rangle \Omega^{AB} + \langle \eta_J \eta_J \rangle \Omega^{JJ}) \\
 &\quad + S_{\rho_1} + S_{(\rho_2, \tau)} + S_{\text{aux}} \\
 S_{\rho} &= \int_{\Sigma} \frac{1}{2} \langle \rho_A \bar{\partial} \rho_B \rangle \Omega^{AB} + B_{ab} \lambda_A^a \rho_B^b \Omega^{AB} + b \langle \lambda_A \rho_B \rangle \Omega^{AB} \\
 S_{(\rho, \tau)} &= \int_{\Sigma} \frac{1}{2} (\langle \rho_A \bar{\partial} \rho_B \rangle \Omega^{AB} + \langle \tau_J \bar{\partial} \tau_J \rangle \Omega^{JJ}) + D_{ab} (\lambda_A^a \rho_B^b \Omega^{AB} + \eta_J^a \tau_J^b \Omega^{JJ}) \\
 &\quad + d (\langle \lambda_A \rho_B \rangle \Omega^{AB} + \langle \eta_J \tau_J \rangle \Omega^{JJ}) \\
 S_{\text{aux}} &= \sum_{i=3}^4 \int_{\Sigma} Z_i^a \cdot \bar{\partial} Z_{ia} + \mathcal{A}_{iab} Z_i^a \cdot Z_i^b + \sum_{j=1}^2 \left(\frac{1}{2} \rho_{ija}^A \bar{\partial} \rho_{ijA}^a + \mathcal{B}_{ij}^{ab} \lambda_{ia}^A \rho_{ijA}^b + b_{ij} \lambda_{ia}^a \rho_{ijA}^A + \tilde{\tau}_{ij}^A \bar{\partial} \tau_{ijA} \right) \\
 &\quad + a_i \lambda_{ia}^{\alpha} \lambda_{i\alpha}^a + \tilde{a}_i \tilde{\lambda}_{ia}^{\dot{\alpha}} \tilde{\lambda}_{i\dot{\alpha}}^a
 \end{aligned}$$

$$\begin{aligned}
 Z &= (\lambda_A, \mu^A, \eta_{\iota}): \lambda_A = (\lambda_{\alpha}, \tilde{\lambda}_{\dot{\alpha}}), \mu^A = (\tilde{\mu}^{\alpha}, \mu^{\dot{\alpha}}), \eta_{\iota} = (\eta_J, \tilde{\eta}^J), \eta_J = (\eta_I, \tilde{\eta}_I), \tilde{\eta}^J = (\tilde{\eta}'^I, \eta'^I), \\
 &\quad I, \dot{I} = 1 \dots \frac{\mathcal{N}}{2}, J = 1, \dots, \mathcal{N}, \iota = 1, \dots, 2\mathcal{N}, \mathcal{N} = 8,
 \end{aligned}$$

$$Z_a \cdot Z_b = \frac{1}{2} (\tilde{Z}_a \cdot Z_b + \tilde{Z}_b \cdot Z_a + \tilde{\eta}_a^J \eta_{bJ} + \tilde{\eta}_b^J \eta_{aJ}), \tilde{Z}_a \cdot Z_b = \tilde{\mu}_a^{\alpha} \lambda_{b\alpha} + \tilde{\lambda}_{a\dot{\alpha}} \mu_b^{\dot{\alpha}} = \mu_a^A \lambda_{bA},$$

$$\Omega^{AB} = \Omega^{(\alpha, \dot{\alpha})(\beta, \dot{\beta})} \stackrel{\text{e.g.}}{=} \varepsilon^{\alpha\beta} \oplus \varepsilon^{\dot{\beta}\dot{\alpha}}, \Omega_{AB} \stackrel{\text{e.g.}}{=} \varepsilon_{\beta\alpha} \oplus \varepsilon_{\dot{\alpha}\dot{\beta}}, \Omega^{AB} \Omega_{BC} = \delta_C^A.$$

$$\Omega^{JJ} = \begin{pmatrix} 0 & 1^{IJ} \\ \mathbb{1}^{IJ} & 0 \end{pmatrix}$$

$$\rho_{rA}^a (r=1,2)$$

$$\begin{aligned}
 &\left| Z_{ia} = \left(\mu_{ia}^A, \lambda_{iaA}, \eta_{iaJ} \otimes \| \lambda_{ia}^a, \mu_{ia}^A, \| \odot \| \rho_{ijA}^a, \tau_{ijA}, \tilde{\tau}_{ij}^A \tilde{\tau}_{ij}^A \widehat{\text{SL}(2, \mathbb{C})}, \| \odot \| [\rho_{ijA}^a, \tau_{ijA}, \tilde{\tau}_{ij}^A \varepsilon_{ab} \varepsilon^{ab}] \right] \boxplus (\alpha, \dot{\alpha}) \eta_{ij}^a \Omega_{\iota}^{JJ} \cdot \tau_{ijA} \right| \rangle \\
 &\langle (B_{ab}, b, D_{ab}, d, \mathcal{B}_{ij}^{ab}, b_{ij}) \boxtimes \| \lambda_A^{(a} \rho_{1B}^{b)} \Omega^{AB} \langle \lambda_A \rho_{1B} \rangle \Omega^{AB} \lambda_A^{(a} \rho_{2B}^{b)} \Omega^{AB} \| \boxtimes [\eta_J^{(a} \tau_J^{b)} \Omega^{JJ} \langle \lambda_A \rho_{2B} \rangle \Omega^{AB}] \boxplus (\eta_J \tau_J) \Omega^{JJ} \lambda_{iA}^{(a} \rho_{ijB}^{b)} \Omega^{AB} \lambda_{iaA} \rho_{ijB}^a \Omega^{AB} A_{ab} A_{(ab)} Z_{ia} \mathcal{A}_{iab} \mathcal{A}_{\iota(ab)} a \tilde{a} a_i \tilde{a} \lambda_{iA}^a \rangle
 \end{aligned}$$

$$Z^A_a(z) \cdot Z^b_B(0) = \frac{\delta^A_B \delta^b_a}{z} + \cdots \tilde{\tau}^J_a(z) \tau^b_J(0) = \frac{\delta^J_J \delta^b_a}{z} + \cdots$$

$$\{ (M^{ab}, N_{ab}), (\mathcal{M}_i^{ab}, \mathcal{N}_{iab}), (m, n), (\tilde{m}, \tilde{n}), (m_i, n_i), (\tilde{m}_i, \tilde{n}_i) \}$$

$$\{ (\beta^{ab}, \gamma_{ab}), (\beta, \gamma), (\tilde{\beta}^{ab}, \tilde{\gamma}ab), (\tilde{\beta}, \tilde{\gamma}), (\beta_{ij}^{ab}, \gamma_{ijab}), (\beta_{ij}, \gamma_{ij}) \}.$$

$$\begin{aligned}
 J_{BRST} &= cT + N_{ab} (J^{ab} + M^a_c N^{bc}) + n \langle \lambda_A \lambda^A \rangle + \tilde{n} (\langle \lambda_A \lambda^A \rangle + \langle \eta_J \eta^J \rangle) + \gamma_{ab} \lambda^{aA} \rho_{1A}^b + \gamma \langle \lambda^A \rho_{1A} \rangle \\
 &\quad + m \gamma \gamma + \tilde{\gamma}_{ab} (\lambda^{aA} \rho_{2A}^b + \eta^{aJ} \tau_J^b) + \tilde{\gamma} (\langle \lambda^A \rho_{2A} \rangle + \langle \eta^J \tau_J \rangle) + \tilde{m} \tilde{\gamma} \tilde{\gamma} + \sum_{i=3}^4 (\mathcal{N}_{iab} (J_i^{ab} + \mathcal{M}_{ic}^a \mathcal{N}_i^{bc}) \\
 &\quad + \sum_{j=1}^2 (\gamma_{ijab} \lambda_i^{aA} \rho_{jA}^b + \gamma_{ij} \lambda_{iA}^a \rho_{jA}^A + (m_i - \tilde{m}_i) \gamma_{ij} \gamma_{ij}) + n_i \lambda_{ia}^{\alpha} \lambda_{i\alpha}^a + \tilde{n}_i \tilde{\lambda}_{ia}^{\dot{\alpha}} \tilde{\lambda}_{i\dot{\alpha}}^a \Bigg), \\
 T &= Z^a \cdot \partial Z_a + \frac{1}{2} \sum_{r=1,2} \langle \rho_{rA} \partial \rho_r^A \rangle + \beta^{ab} \partial \gamma_{ab} + \beta \partial \gamma + \frac{1}{2} \langle \tau_J \partial \tau^J \rangle + \tilde{\beta}^{ab} \partial \tilde{\gamma}_{ab} + \tilde{\beta} \partial \tilde{\gamma}
 \end{aligned}$$



$$\begin{aligned}
& + M^{ab} \partial N_{ab} + m \partial n + \tilde{m} \partial \tilde{n} + \frac{1}{2} (b \partial c + \partial (bc)) + \sum_{i=3}^4 (\mathcal{M}_i^{ab} \partial \mathcal{N}_{iab} + Z_i^a \cdot \partial Z_{ia} \\
& + \sum_{j=1}^2 \left(\frac{1}{2} (\rho_{ijA}^A \partial \rho_{ijA}^a + \beta_{ij}^{ab} \partial \gamma_{ijab} + \beta_{ji} \partial \gamma_{ij} + \tilde{\tau}_{ij}^A \partial \tau_{ijA}) + m_i \partial n_i + \tilde{m}_i \partial \tilde{n}_i \right), \\
J^{ab} & = Z^a \cdot Z^b + \frac{1}{2} \sum_{r=1,2} \rho_{rA}^{(a} \rho_r^{b)A} + \beta^{c(a} \gamma_c^{b)} + \frac{1}{2} \tau_j^{(a} \tau^{b)j} + \tilde{\beta}^{c(a} \tilde{\gamma}_c^{b)}, \\
J_i^{ab} & = Z_i^a \cdot Z_i^b + \sum_{j=1}^2 \left(\frac{1}{2} \rho_{ijA}^{(a} \rho_{ij}^{b)A} + \beta_{ij}^{c(a} \gamma_{ijc}^{b)} \right),
\end{aligned}$$

$$c_{\text{aux}} = \sum_{i=3}^4 \left(-6_{\mathcal{M}_i \mathcal{N}_i} - (8 - \mathcal{N})_{Z_i} - 2_{m_i n_i} - 2_{\tilde{m}_i \tilde{n}_i} + 2 \left(4_{\rho_{ij}} + 8_{\beta_{ij} \gamma_{ij}} - 4_{\tau_{ij}} \right) \right) = 2(\mathcal{N} - 2)$$

$$\begin{aligned}
& \left(\text{tr}_{\text{adj}}(t^k t^k) = 6, \text{tr}_F(t^k t^k) = \frac{3}{2} \right) a_i^{sl2} \\
& = \frac{1}{2} \text{tr}_F(t^k t^k) ((8 - \mathcal{N})_{Z_i} - 4_{\rho_{i1}} - 4_{\rho_{i2}}) \\
& + \text{tr}_{\text{adj}}(t^k t^k) (-1_{\mathcal{M}_i \mathcal{N}_i} + 1_{\beta_{i1} \gamma_{i1}} + 1_{\beta_{i2} \gamma_{i2}}) = \frac{3}{4} (8 - \mathcal{N})
\end{aligned}$$

$$\mathcal{V} = \int \mathrm{d}^2 u \, \mathrm{d}^2 v \mathcal{W}(u) \mathcal{W}_{\text{aux}} \bar{\delta}(\langle v \epsilon \rangle - 1) \bar{\delta}^4(\langle u \lambda_A \rangle - \langle v \kappa_A \rangle) \bar{\delta}^{\mathcal{N}}(\langle u \eta_j \rangle - \langle v \zeta_j \rangle) e^{\langle u \mu^A \rangle \epsilon_A + \langle u \tilde{\eta}^j \rangle q_j}$$

$$\mathcal{W}(u) = \text{cn}\tilde{n}\delta(\gamma)\delta(\gamma_{ab}u^au^b)\delta(\tilde{\gamma})\delta(\tilde{\gamma}_{ab}u^au^b)$$

$$\mathcal{W}_{\text{aux}} = \prod_{i=3}^4 n_i \tilde{n}_i \int \mathrm{d} u_i'^2 \bar{\delta}(u_i'^a - \chi_i^A \lambda_{iA}^a) \prod_{j=1}^2 \delta(\gamma_{ij}) \delta(\gamma_{ijab} u_i'^a u_i'^b) \frac{1}{2} (\chi_i^A \rho_{ijA}^a) (\chi_i^A \rho_{ijAa})$$

$$\begin{aligned}
Y_{\beta}(z) &= \delta(\beta)[Q_B, \beta], & Y_{\hat{u}\beta}(z) &= \delta(\beta^{ab}\hat{u}_a\hat{u}_b)[Q_B, \beta^{ab}\hat{u}_a\hat{u}_b] \\
Y_{\tilde{\beta}}(z) &= \delta(\tilde{\beta})[Q_B, \tilde{\beta}], & Y_{\hat{u}\tilde{\beta}}(z) &= \delta(\tilde{\beta}^{ab}\hat{u}_a\hat{u}_b)[Q_B, \tilde{\beta}^{ab}\hat{u}_a\hat{u}_b] \\
Y_m(z) &= m\delta(\{Q_B, m\}), & Y_{\tilde{m}}(z) &= \tilde{m}\delta(\{Q_B, \tilde{m}\}), Y_{M^{ab}}(z) = M^{ab}\delta(\{Q_B, M^{ab}\}) \\
Y_{m_i}(z) &= m_i\delta(\{Q_B, m_i\}), & Y_{\tilde{m}_i}(z) &= \tilde{m}_i\delta(\{Q_B, \tilde{m}_i\}), Y_{\mathcal{M}_i^{ab}}(z) = \mathcal{M}_i^{ab}\delta(\{Q_B, \mathcal{M}_i^{ab}\}), \\
Y_{\beta_{ij}}(z) &= \delta(\beta_{ij})[Q_B, \beta_{ij}], & Y_{\hat{u}'\beta_{ij}}(z) &= \delta(\beta_{ij}^{ab}\hat{u}'_{ia}\hat{u}'_{ib})[Q_B, \beta_{ij}^{ab}\hat{u}'_{ia}\hat{u}'_{ib}].
\end{aligned}$$



$$\begin{aligned}
& \lim_{x,y \rightarrow \sigma} \int du_i'^2 \bar{\delta}(u_i'^a - \chi_i^A \lambda_{iA}^a) Y_{\beta_{ij}}(x) Y_{u_i' \beta_{ij}}(y) \delta(\gamma_{ij}(\sigma)) \delta(\gamma_{ijab}(\sigma) u_i'^a u_i'^b) \frac{1}{2} (\chi_i^A \rho_{ijA}^a(\sigma)) (\chi_i^A \rho_{ijAa}(\sigma)) \\
&= \lim_{x,y \rightarrow \sigma} \int du_i'^2 \bar{\delta}(u_i'^a - \chi_i^A \lambda_{iA}^a) \delta(\beta_{ij}(x)) \delta(\beta_{ij}^{ab}(y) \hat{u}_{ia}' \hat{u}_{ib}') \delta(\gamma_{ij}(\sigma)) \delta(\gamma_{ijab}(\sigma) u_i'^a u_i'^b) \\
&\quad (\lambda_{ia}^A(x) \rho_{ijA}^a(x)) (\lambda_{ib}^A(y) \rho_{ij}^{bB}(y) \hat{u}_{ia}' \hat{u}_{ib}') \frac{1}{2} (\chi_i^A \rho_{ijA}^a(\sigma)) (\chi_i^A \rho_{ijAa}(\sigma)) \\
&= \int du_i'^2 \bar{\delta}(u_i'^a - \chi_i^A \lambda_{iA}^a) \frac{(\chi_i^A \lambda_{iA}^a(\sigma) \hat{u}_{ia}')^2}{(u_{ia}' \hat{u}_{ia}')^2} = 1
\end{aligned}$$

$$\lim_{x,y \rightarrow \sigma} n_i Y_{m_i}(x) \tilde{n}_i Y_{\tilde{m}_i}(y) = \delta(\text{Res}_\sigma(\lambda_i^\alpha \lambda_{i\alpha})) \delta(\text{Res}_\sigma(\tilde{\lambda}_i^{\dot{\alpha}} \tilde{\lambda}_{i\dot{\alpha}}))$$

$$V = \int \mathrm{d}\sigma \, \mathrm{d}^2u \, \mathrm{d}^2v w(u) \bar{\delta}(\langle v\epsilon \rangle - 1) \bar{\delta}^4(\langle u\lambda_A \rangle - \langle v\kappa_A \rangle) \bar{\delta}^N(\langle u\eta_J \rangle - \langle v\zeta_J \rangle) e^{\langle u\mu^A \rangle \epsilon_A + \langle u\eta^J \rangle q_J}$$

$$\begin{aligned}
w(u) &= \delta(\text{Res}_\sigma(\langle \lambda_A \lambda_B \rangle \Omega^{AB})) \delta(\text{Res}_\sigma(\langle \eta_J \eta_J \rangle \Omega^{JJ})) \left(\frac{\langle \hat{u} \lambda_A \rangle \epsilon^A}{\langle u \hat{u} \rangle} + \frac{\langle \rho_{1A} \rho_{1B} \rangle}{2} \epsilon^A \epsilon^B \right) \\
&\quad \left(\frac{\langle \hat{u} \lambda_A \rangle \epsilon^A}{\langle u \hat{u} \rangle} + \frac{\langle \rho_{2A} \rho_{2B} \rangle}{2} \epsilon^A \epsilon^B + \frac{\langle \hat{u} \eta_J \rangle q^J}{\langle u \hat{u} \rangle} + \frac{\langle \tau_J \mathcal{T}_J \rangle}{2} q^J q^J \right)
\end{aligned}$$

$$\begin{aligned}
& \int du_1'^2 du_2'^2 du_3'^2 \frac{\bar{\delta}(u_1'^a - \chi_{i1}^A \lambda_{iA}^a(\sigma_1)) \bar{\delta}(u_2'^a - \chi_{i2}^A \lambda_{iA}^a(\sigma_2)) \bar{\delta}(u_3'^a - \chi_{i3}^A \lambda_{iA}^a(\sigma_3))}{[u_1' u_2']^2 [u_2' u_3']^2 [u_3' u_1']^2} \sigma_{12}^2 \sigma_{31}^2 \\
& \prod_{j=1}^2 \frac{1}{2} (\chi_{i1}^A [\rho_{ijA}(\sigma_1) \rho_{ijB}(\sigma_1)] \chi_{i1}^B) (\chi_{i2}^A [\rho_{ijA}(\sigma_2) \lambda_{iB}(\sigma_2)] \chi_{i2}^B) (\chi_{i3}^A [\rho_{ijA}(\sigma_3) \lambda_{iB}(\sigma_3)] \chi_{i3}^B) \\
&= \int du_1'^2 du_2'^2 du_3'^2 \frac{\prod_{k=1}^3 \bar{\delta}(u_k'^a - \chi_{ik}^A \lambda_{iA}^a(\sigma_k))}{[u_1' u_2']^2 [u_2' u_3']^2 [u_3' u_1']^2} (\chi_{i1} \chi_{i2})^2 (\chi_{i1} \chi_{i3})^2 (\chi_{i2}^A [\lambda_{iA}(\sigma_2) \lambda_{iB}(\sigma_3)] \chi_{i3}^B)^2 \\
&= \frac{(\chi_{i1} \chi_{i2})^2 (\chi_{i1} \chi_{i3})^2}{(\chi_{i1}^A [\lambda_{iA}(\sigma_1) \lambda_{iB}(\sigma_2)] \chi_{i2}^B)^2 (\chi_{i3}^A [\lambda_{iA}(\sigma_3) \lambda_{iB}(\sigma_1)] \chi_{i1}^B)^2} \\
&= \frac{(\chi_{i1} \chi_{i2})^2 (\chi_{i1} \chi_{i3})^2}{(\chi_{i1}^A [\kappa'_{i1A} \kappa'_{i2B}] \chi_{i2}^B)^2 (\chi_{i1}^A [\kappa'_{i1A} \kappa'_{i3B}] \chi_{i3}^B)^2}
\end{aligned}$$

$$\lim_{\kappa'_{ij}, \chi_{ij} \rightarrow \kappa'_i, \chi_i} \frac{(\chi_{i1} \chi_{i2})^2 (\chi_{i1} \chi_{i3})^2}{(\chi_{i1}^A [\kappa'_{i1A} \kappa'_{i2B}] \chi_{i2}^B)^2 (\chi_{i1}^A [\kappa'_{i1A} \kappa'_{i3B}] \chi_{i3}^B)^2} = \frac{1}{16(k_i'^0)^4}$$

$$\bigg\bigg\bigg\limits_{a_i} \omega_j = \delta_{ij}, \bigg\bigg\bigg\limits_{b_i} \omega_i = \tau_{ij}.$$

$$M=\begin{bmatrix} A & B \\ C & D \end{bmatrix} \in \operatorname{Sp}(2g,\mathbb{Z})$$

$$\theta[\alpha](\mathbf{z})=\sum_{m\in\mathbb{Z}^g}\exp\left[i\pi(m+\alpha')^T\tau(m+\alpha')+2\pi i(m+\alpha')(\mathbf{z}+\alpha'')\right],$$

$$\theta[\alpha](-\mathbf{z})=(-1)^{4\alpha'\cdot\alpha''}\theta[\alpha](\mathbf{z}), \alpha=\begin{pmatrix}\alpha'\\\alpha''\end{pmatrix}, \alpha',\alpha''\in(\mathbb{Z}/2\mathbb{Z})^g,$$



$$\sum_{i=j}^n c_j z_j \rightarrow \mathbf{z} = \sum_{i=j}^n c_j \int_{z_0}^{z_j} \omega_i \in \mathbb{C}^g$$

$$\Delta \in \mathbb{C}^g, \Delta_i = \frac{1+\tau_{ii}}{2} - \sum_{j \neq i} \oint_{a_j} \omega_j(z) \int_{z_0}^z \omega_i$$

$$\theta(z)=0 \Leftrightarrow \exists z_1,\ldots,z_{g-1} \text{ with } z=\Delta-\sum_{i=1}^{g-1} z_i.$$

$$E(z,w)=\frac{\theta[\alpha](z-w)}{h_\alpha(z)h_\alpha(w)}, h_\alpha(z)=\left(\sum_{i=1}^g\partial_i\theta[\alpha](0)\omega_i(z)\right)^{\frac{1}{2}},$$

$$S_\alpha(z,w\mid \tau)=\frac{\theta[\alpha](z-w)}{E(z,w)\theta[\alpha](0)},\hspace{10em}\alpha\text{ even}$$

$$S_\alpha(z,w\mid \tau)=\frac{1}{E(z,w)}\frac{\sum_{i=1}^g\partial_i\theta[\alpha](z-w)\omega_i(y)}{\sum_{i=1}^g\partial_i\theta[\alpha](0)\omega_i(y)},\hspace{10em}\alpha\text{ odd}$$

$$Z_\lambda(z_1,\ldots,z_G)=\int \mathrm{D} b \mathrm{D} c b(z_1)\ldots b(z_G) \mathrm{exp}\left(-\frac{1}{2\pi}\int \mathrm{d}^2z \sqrt{g} b(z) \bar{\nabla}_{1-\lambda} c(z)\right),\hspace{10em}\lambda>1,$$

$$Z_1(z_1,\ldots,z_g,w)=\int \mathrm{D} \beta \mathrm{D} \gamma \beta(z_1)\ldots \beta(z_g) \gamma(w) \mathrm{exp}\left(-\frac{1}{2\pi}\int \mathrm{d}^2z \sqrt{g} \beta(z) \bar{\nabla}_{1-\lambda} \gamma(z)\right),$$

$$Z_{\frac{1}{2},\alpha}^{\rm even}=\int \mathrm{D}\chi \mathrm{D}\psi \mathrm{exp}\left(-\frac{1}{2\pi}\int \mathrm{d}^2z \sqrt{g} \chi(z) \bar{\nabla}_{\frac{1}{2}} \psi(z)\right),$$

$$Z_{\frac{1}{2},\alpha}^{\rm odd}(w)=\int \mathrm{D}\chi \mathrm{D}\psi \chi(w) \psi(w) \mathrm{exp}\left(-\frac{1}{2\pi}\int \mathrm{d}^2z \sqrt{g} \chi(z) \bar{\nabla}_{\frac{1}{2}} \psi(z)\right),$$

$$Z_{\lambda>1}(z_1,\ldots,z_G)=\tilde{Z}_1^{-\frac{1}{2}}\theta[\alpha]\left(\sum_{i=1}^Gz_i-Q\Delta\right)\prod_{j>i=1}^GE(z_i,z_j)\prod_{i=1}^G\sigma(z_i)^Q$$

$$Z_1(z_1,\ldots,z_g,w)=\tilde{Z}_1^{-\frac{1}{2}}\theta\left(\sum_{i=1}^gz_i-w-\Delta\right)\frac{\prod_{j>i=1}^gE(z_i,z_j)\prod_{i=1}^g\sigma(z_i)}{\prod_{i=1}^gE(z_i,w)\sigma(w)}$$

$$Z_{\frac{1}{2},\alpha}^{\rm even}=\tilde{Z}_1^{-\frac{1}{2}}\theta[\alpha](0), Z_{\frac{1}{2},\alpha}^{\rm odd}(z)=\left(\tilde{Z}_1^{\frac{1}{2}}\mathrm{det}_\omega\right)^{-1}h_\alpha(z)^2$$

$$\tilde{Z}_1=\frac{Z_1(z_1,\ldots,z_g,w)}{\det_\omega}, \det_\omega\equiv \det\|\omega_i(z_j)\|, \int_\Sigma \mathrm{d}^2z \bar{\omega}_i \omega_j(z)=<\omega_i\mid \omega_j>=2i\mathrm{Im}\tau_{ij}$$

$$\frac{\sigma(z)}{\sigma(w)}=\frac{\theta(z-\sum_{i=1}^gr_i-\Delta)}{\theta(w-\sum_{i=1}^gr_i-\Delta)}\prod_{i=1}^g\frac{E(w,r_i)}{E(z,r_i)}$$



$$\det \bar{V}_{1-\lambda} = \frac{Z_\lambda(z_1, \dots, z_G)}{\det \|\phi_i(z_j)\|} \cdot \phi_1 \wedge \dots \wedge \phi_G \quad \lambda > 1$$

$$\det \bar{V}_0 = \frac{Z_1(z_1, \dots, z_g, w)}{\det_\omega e} \cdot \omega_1 \wedge \dots \wedge \omega_g \otimes e = \tilde{Z}_1 \cdot \omega_1 \wedge \dots \wedge \omega_g \otimes e$$

$$\det \bar{V}_{\frac{1}{2}, \alpha}^{\text{ev}} = Z_{\frac{1}{2}, \alpha}^{\text{even}}, \det \bar{V}_{\frac{1}{2}, \alpha}^{\text{odd}} = \frac{Z_{\frac{1}{2}, \alpha}^{\text{odd}}(z)}{h_\alpha(z)^2} \cdot h_\alpha \otimes h_\alpha$$

$$\det_{[\text{bos}]} \bar{V}_{1-\lambda} = (\det_{[\text{ferm}]} \bar{V}_{1-\lambda})^{-1}$$

$$Z_\lambda(\tilde{z}_1, \dots, \tilde{z}_G) = \epsilon^{\frac{2}{3}} Z_\lambda(z_1, \dots, z_G), (\lambda \in \mathbb{Z}),$$

$$Z_{\frac{1}{2}, \tilde{\alpha}}^{\text{odd}}(\tilde{z}) = \epsilon^{\frac{2}{3}} e^{i\pi\phi(\alpha)} \det \|C\tau + D\| Z_{\frac{1}{2}, \alpha}^{\text{odd}}(z),$$

$$Z_{\frac{1}{2}, \tilde{\alpha}}^{\text{even}} = \epsilon^{\frac{2}{3}} e^{i\pi\phi(\alpha)} Z_{\frac{1}{2}, \alpha}^{\text{even}},$$

$$\text{with } \tilde{z}_i = (\tau C^T + D^T)_{ij}^{-1} z_j, \tilde{\omega}_i = \omega_j (C\tau + D)_{ji}^{-1}, \tilde{\Delta} - \tilde{\tau} \delta_1 - \delta_2 = (\tau C^T + D^T)^{-1} \Delta,$$

$$\tilde{\alpha} = \alpha' + \delta, \begin{bmatrix} \alpha'_1 \\ \alpha'_2 \end{bmatrix} = \begin{bmatrix} D & -C \\ -B & A \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}, \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \text{diag} CD^T \\ \text{diag} AB^T \end{bmatrix},$$

$$\epsilon^8 = 1, \phi(\alpha) = \alpha'_1 \cdot \alpha'_2 - \alpha_1 \cdot \alpha_2 + 2\alpha'_1 \cdot \delta_2 \in \frac{\mathbb{Z}_2}{4}.$$

$$\tilde{Z}_2 \det_\omega = \frac{Z_2}{\det_\omega} = Z_1 = \tilde{Z}_1 \det_\omega = \tilde{Z}_1^{-\frac{1}{2}} \frac{\theta(z-w)}{E(z,w)} = \tilde{Z}_1^{-\frac{1}{2}} \theta'(0) \det_\omega = Z_{\frac{1}{2}, \alpha}^{\text{odd}}(z) \det_\omega,$$

$$\tilde{Z}_2 = \frac{Z_2}{\det_\omega^2} = \frac{Z_1}{\det_\omega} = \tilde{Z}_1 = Z_{\frac{1}{2}}^{\text{odd}} = \eta(\tau)^2, Z_{\frac{1}{2}, \alpha}^{\text{even}} = \frac{\theta[\alpha](0)}{\eta(\tau)}$$

$$\mathrm{d}(\mathrm{WP})(\det \hat{P}_1^\dagger \hat{P}_1)^{\frac{1}{2}} = W \wedge \bar{W}, W = dm^{3(g-1)} \int db \, dc \prod_{i=1}^{3(g-1)} (\mu_i, b) \exp \left[\int d^2 z b(z) \bar{V}_{-1} c(z) \right]$$

$$W = dm^{3(g-1)} \frac{\det \|(\mu_i, \phi_j)\|}{\det \|\phi_i(z_j)\|} Z_2(z_1, \dots, z_{3(g-1)}) = \det \bar{V}_{-1}$$

$$\mathrm{d}\mu_g = \frac{\phi_1 \wedge \dots \wedge \phi_{3(g-1)}}{\det \|\phi_i(z_j)\|} = dm^{3(g-1)} \frac{\det \|(\mu_i, \phi_j)\|}{\det \|\phi_i(z_j)\|}$$



$$\begin{aligned}
Z_{g \geq 2} &= \left\langle \prod_{i=1}^{3(g-1)} b(y_i) \prod_{j=1}^g Y_m(x_j) Y_{\tilde{m}}(\tilde{x}_j) Y_{M_{00}}(w_{1j}) Y_{M_{11}}(w_{2j}) Y_{M_{(01)}}(w_{3j}) \right. \\
&\quad \left(\prod_{k=3}^4 Y_{m_k}(x_{kj}) Y_{\tilde{m}_k}(\tilde{x}_{kj}) Y_{M_{k00}}(w_{1kj}) Y_{M_{i11}}(w_{2kj}) Y_{M_{i(01)}}(w_{3kj}) \right. \\
&\quad \left. \prod_{l=1}^2 Y_{\beta_{kl}}(r_{klj}) Y_{(0,1)\beta_{kl}}(r_{1klj}) Y_{(-1,1)\beta_{kl}}(r_{2klj}) Y_{(-1,0)\beta_{kl}}(r_{3klj}) \right) \\
&\quad Y_{\beta}(z_j) Y_{(0,1)\beta}(z_{1j}) Y_{(-1,1)\beta}(z_{2j}) Y_{(-1,0)\beta}(z_{3j}) Y_{\tilde{\beta}}(\tilde{z}_j) Y_{(0,1)\tilde{\beta}}(\tilde{z}_{1j}) Y_{(-1,1)\tilde{\beta}}(\tilde{z}_{2j}) Y_{(-1,0)\tilde{\beta}}(\tilde{z}_{3j}) \Big\rangle \\
&= \sum_{\alpha} \int d\mu_g \frac{Z_2((y)) Z_1((x))^6 Z_1((w))^9}{Z_1((z))^8 Z_1((r))^{16}} \left\langle \prod_{j=1}^g (\bar{\delta}_{\text{tot}} j_{\text{tot}}) \right\rangle_{\alpha} \\
\bar{\delta}_{\text{tot}} &= \bar{\delta}(\langle \lambda^A \lambda_A \rangle + \langle \eta^j \eta_j \rangle) \dots, j_{\text{tot}} = \langle \lambda^A \rho_{2A} + \eta^j \tau_j \rangle \dots
\end{aligned}$$

$$\begin{aligned}
Z_{g=1}^{\text{ev}} &= \sum_{\alpha} \int \frac{d\mu_1 Z_2 Z_1^{-9} \left(Z_{\frac{1}{2}}^{\text{even}} \right)^8}{\text{Vol}(\text{CKV})_{\text{chir}}} \langle \bar{\delta}_{\text{tot}} j_{\text{tot}} \rangle_{\alpha} = \sum_{\alpha} \int \frac{d\mu_1 \det_{\omega}^{-7}}{\text{Vol}(\text{CKV})_{\text{chir}}} \left(\frac{\theta_{\alpha}(0)}{\eta(\tau)} \right)^8 \frac{\langle \bar{\delta}_{\text{tot}} j_{\text{tot}} \rangle_{\alpha}}{\eta(\tau)^{16}}, \\
Z_{g=1}^{\text{odd}} &= \int \frac{d\mu_1 Z_2 Z_1^{-9} \left(Z_{\frac{1}{2}}^{\text{odd}} \right)^8}{\text{Vol}(\text{CKV})_{\text{chir}}} d^{16} \psi \langle \bar{\delta}_{\text{tot}} j_{\text{tot}} \rangle = \int \frac{d\mu_1 \det_{\omega}^{-7} d^{16} \psi}{\text{Vol}(\text{CKV})_{\text{chir}}} \langle \bar{\delta}_{\text{tot}} j_{\text{tot}} \rangle
\end{aligned}$$

$$d\mu_1 = d\tau \det_{\omega}^{-2}$$

$$\begin{aligned}
Z_{g=1}^{\text{ev}} &= \sum_{\tilde{\alpha}} \int d\tilde{\tau} \frac{\langle \bar{\delta}_{\text{tot}} j_{\text{tot}} \rangle_{\alpha}}{\det_{\omega}^{10}} \left(\frac{\theta_{\tilde{\alpha}}(0)}{\eta(\tilde{\tau})^3} \right)^8 = \sum_{\alpha} \int \frac{d\tau}{M_{\tau}^2} \frac{\langle \bar{\delta}_{\text{tot}} j_{\text{tot}} \rangle_{\alpha}}{(\det_{\omega} M_{\tau}^{-1})^{10}} \left(\frac{\theta_{\alpha}(0)}{\eta(\tau)^3 M_{\tau}} \right)^8 \\
Z_{g=1}^{\text{odd}} &= \int d\tilde{\tau} \frac{d^{16} \tilde{\psi}}{\det_{\omega}^{10}} \langle \bar{\delta}_{\text{tot}} j_{\text{tot}} \rangle = \int \frac{d\tau}{M_{\tau}^2} \frac{d^{16} \psi M_{\tau}^{-8}}{(\det_{\omega} M_{\tau}^{-1})^{10}} \langle \bar{\delta}_{\text{tot}} j_{\text{tot}} \rangle \\
\tilde{\tau} &= \frac{A\tau + B}{C\tau + D}, M_{\tau} = \det \| C\tau + D \|
\end{aligned}$$

$$\begin{aligned}
A_{g,n} &= \langle V_1 \dots V_n \rangle = \sum_{\alpha} \int d\mu_g Z_g^{\text{chir}}((z), (y), (x), (w), (r)) A_g^{\alpha}((z), (x), (w), (r)) \\
Z_g^{\text{chir}} &= \frac{Z_2((y)) Z_1((x))^6 Z_1((w))^9}{Z_1((z))^8 Z_1((r))^{16} \tilde{Z}_1^4} = \frac{Y_3(y) Y(x) Y(\tilde{x}) \prod_{k=3}^4 Y(x_k) Y(x_k) \prod_{j=1}^3 Y(w_j) \prod_{k=3}^4 Y(w_{jk})}{Y(z) Y(\tilde{z}) \prod_{j=1}^3 Y(z_j) Y(\tilde{z}_j) \prod_{k=3}^4 \prod_{l=1}^2 Y(r_{kl}) \prod_{j=1}^3 Y(r_{jkl})} \\
A_g^{\alpha}((z), (x), (w), (r)) &= \int dm_0 \tilde{Z}_1^4 \left\langle \prod_{j=1}^g (\bar{\delta}_{x_j} \bar{\delta}'_{w_j} j_{z_j} \tilde{j}_{\tilde{z}_j} j'_{r_j}) V_1 \dots V_n \right\rangle_{\alpha} \\
Y_Q(y) &\equiv \theta(\Delta_y^Q) \prod_{j>i=1}^{Q(g-1)} E(y_i, y_j) \prod_{i=1}^{Q(g-1)} \sigma(y_i)^Q, Y(a) \equiv \theta(\Delta_a) \frac{\prod_{j>i=1}^g E(a_i, a_j) \prod_{i=1}^g \sigma(a_i)}{\prod_{i=1}^g E(a_i, a_0) \sigma(a_0)}
\end{aligned}$$



$$\begin{aligned}
\Delta_a^Q &\equiv \sum_{i=1}^{Q(g-1)} a_i - Q\Delta, \Delta_a \equiv \sum_{i=1}^g a_i - a_0 - \Delta \\
\bar{\delta}_{x_j} &= \bar{\delta}(\langle \lambda^A(x_j) \lambda_A(x_j) \rangle) \bar{\delta}(\langle \lambda^A(\tilde{x}_j) \lambda_A(\tilde{x}_j) \rangle + \langle \eta^j(\tilde{x}_j) \eta_j(\tilde{x}_j) \rangle) \\
&\prod_{k=3}^4 \bar{\delta}(\lambda_k^\alpha(x_{kj}) \lambda_{k\alpha}(x_{kj})) \bar{\delta}(\tilde{\lambda}_k^\alpha(\tilde{x}_{kj}) \tilde{\lambda}_{k\alpha}(\tilde{x}_{kj})) \\
\bar{\delta}'_{w_j} &= \bar{\delta}(\{Q_B, M^{00}(w_{1j})\}) \bar{\delta}(\{Q_B, M^{01}(w_{2j})\}) \bar{\delta}(\{Q_B, M^{11}(w_{3j})\}) \\
&\prod_{k=3}^4 \bar{\delta}(\{Q_B, \mathcal{M}^{00}(w_{1kj})\}) \bar{\delta}(\{Q_B, \mathcal{M}^{01}(w_{2kj})\}) \bar{\delta}(\{Q_B, \mathcal{M}^{11}(w_{3kj})\}) \\
j_{z_j} &= \langle \lambda^A(z_j) \rho_{1A}(z_j) \rangle \left(\lambda^{A0}(z_{1j}) \rho_{1A}^0(z_{1j}) \right) \left(\lambda^{A0}(z_{2j}) \rho_{1A}^1(z_{2j}) \right) \left(\lambda^{A1}(z_{3j}) \rho_{1A}^1(z_{3j}) \right), \\
\tilde{j}_{\tilde{z}_j} &= (\langle \lambda^A(\tilde{z}_j) \rho_{2A}(\tilde{z}_j) \rangle + \langle \eta^j(\tilde{z}_j) \tau_j(\tilde{z}_j) \rangle) \left(\lambda^{A0}(\tilde{z}_{1j}) \rho_{2A}^0(\tilde{z}_{1j}) + \eta^{j0}(\tilde{z}_{1j}) \tau_j^0(\tilde{z}_{1j}) \right) \\
&\left(\lambda^{A0}(\tilde{z}_{2j}) \rho_{2A}^1(\tilde{z}_{2j}) + \eta^{j0}(\tilde{z}_{2j}) \tau_j^1(\tilde{z}_{2j}) \right) \left(\lambda^{A1}(\tilde{z}_{3j}) \rho_{2A}^1(\tilde{z}_{3j}) + \eta^{j1}(\tilde{z}_{3j}) \tau_j^1(\tilde{z}_{3j}) \right) \\
j'_{r_j} &= \prod_{k=3}^4 \prod_{l=1}^2 \left(\left(\lambda_{ka}^A(r_{klj}) \rho_{kla}^a(r_{klj}) \right) \left(\lambda^{A0}(r_{1klj}) \rho_{kla}^0(r_{1klj}) \right) \left(\lambda_k^{A0}(r_{2klj}) \rho_{kla}^1(r_{2klj}) \right) \right. \\
&\left. \left(\lambda_k^{A1}(r_{3klj}) \rho_{kla}^1(r_{3klj}) \right) \right) \\
&\delta'_{w_j} j_{z_j} \tilde{j}_{\tilde{z}_j} j'_{r_j} \rightarrow j_{z_j}^u \tilde{j}_{\tilde{z}_j}^u j_{r_j}'^u \\
j_{z_j}^u &= \langle \lambda^A(z_j) \rho_{1A}(z_j) \rangle \int \frac{d^2 u_{1j} d^2 u_{2j} d^2 u_{3j} e^{-\sum_{k=1}^3 \|u_{kj}\|^2}}{\langle u_{1j} u_{2j} \rangle \langle u_{2j} u_{3j} \rangle \langle u_{3j} u_{1j} \rangle} \prod_{k=1}^3 (\langle u_{kj} \lambda^A(z_{kj}) \rangle \langle u_{kj} \rho_{1A}(z_{kj}) \rangle) \\
\tilde{j}_{\tilde{z}_j}^u &= (\langle \lambda^A(\tilde{z}_j) \rho_{2A}(\tilde{z}_j) \rangle + \langle \eta^j(\tilde{z}_j) \tau_j(\tilde{z}_j) \rangle) \int \frac{d^2 \tilde{u}_{1j} d^2 \tilde{u}_{2j} d^2 \tilde{u}_{3j}}{\langle \tilde{u}_{1j} \tilde{u}_{2j} \rangle \langle \tilde{u}_{2j} \tilde{u}_{3j} \rangle \langle \tilde{u}_{3j} \tilde{u}_{1j} \rangle} e^{-\sum_{k=1}^3 \|\tilde{u}_{kj}\|^2} \\
&\prod_{k=1}^3 (\langle \tilde{u}_{kj} \lambda^A(\tilde{z}_{kj}) \rangle \langle \tilde{u}_{kj} \rho_{2A}(\tilde{z}_{kj}) \rangle + \langle \tilde{u}_{kj} \eta^j(\tilde{z}_{kj}) \rangle \langle \tilde{u}_{kj} \tau_j(\tilde{z}_{kj}) \rangle) \\
j_{r_j}'^u &= \prod_{k=3}^4 \prod_{l=1}^2 \left(\left[\lambda_k^A(r_{klj}) \rho_{kla}(r_{klj}) \right] \int \frac{d^2 u'_{1kj} d^2 u'_{2kj} d^2 u'_{3kj}}{[u'_{1kj} u'_{2kj}] [u'_{2kj} u'_{3kj}] [u'_{3kj} u'_{1kj}]} e^{-\sum_{s=1}^3 \|u'_{skj}\|^2} \right. \\
&\left. \prod_{s=1}^3 [u'_{skj} \lambda_{kj}^A(r_{sklj})] [u'_{skj} \rho_{kljA}(r_{sklj})] \right) \\
\langle v_r \kappa_r^A \rangle &= \langle u_r \lambda^A(\sigma_r) \rangle, r = 1, \dots, n, \lambda_A^a(\sigma) = \lambda_{0A}^a \frac{h_a(\sigma)}{\sqrt{d\sigma}} + \sum_{l=1}^n u_l^a \epsilon_{lA} \frac{S_a(\sigma, \sigma_l | \tau)}{\sqrt{d\sigma} \sqrt{d\sigma_l}} \\
\langle v_r \zeta_r^j \rangle &= \langle u_r \eta^j(\sigma_r) \rangle, r = 1, \dots, n, \eta_j^a(\sigma) = \eta_{0A}^a \frac{h_a(\sigma)}{\sqrt{d\sigma}} + \sum_{l=1}^n u_l^a q_{lj} \frac{S_a(\sigma, \sigma_l | \tau)}{\sqrt{d\sigma} \sqrt{d\sigma_l}}
\end{aligned}$$



$$\mathcal{B} = \begin{pmatrix} A & C \\ -C^T & H \end{pmatrix}$$

$$A_{ij} = \langle \Lambda_{iA}(z'_i) \Lambda_j^A(z'_j) \rangle \frac{S_\alpha(z'_i, z'_j | \tau)}{\sqrt{dz'_i} \sqrt{dz'_j}}, A = 4g \times 4g \text{ antisymmetric matrix,}$$

$$C_{ij}^a = \Lambda_{iA}^a(z'_i) \epsilon_j^A \frac{S_\alpha(z'_i, \sigma_j | \tau)}{\sqrt{dz'_i} \sqrt{d\sigma_j}},$$

$$H_{ij} = \epsilon_{iA} \epsilon_j^A \frac{S_\alpha(\sigma_i, \sigma_j | \tau)}{\sqrt{d\sigma_i} \sqrt{d\sigma_j}}, (i \neq j = 1, \dots, n), u_i^a H_{ii} = -\lambda_A^a(\sigma_i) \epsilon_i^A$$

$$\Lambda_{iA}^a(z'_i) = \begin{cases} \lambda_A^a(z_k), i = 4k, \\ u_{lk}^a \langle u_{lk} \lambda_A(z_{lk}) \rangle, i = 4k + l, l = 1, 2, 3, \end{cases} k = 1, \dots, g$$

$$\tilde{\mathcal{B}} = \begin{pmatrix} \tilde{A} & \tilde{C} \\ -\tilde{C}^T & \tilde{G} \end{pmatrix}$$

$$\tilde{A}_{ij} = (\langle \tilde{\Lambda}_{iA}(\tilde{z}'_i) \tilde{\Lambda}_j^A(\tilde{z}'_j) \rangle + \langle \tilde{H}_{i\mathcal{J}}(\tilde{z}'_i) \tilde{H}_j^{\mathcal{J}}(\tilde{z}'_j) \rangle) \frac{S_\alpha(\tilde{z}'_i, \tilde{z}'_j | \tau)}{\sqrt{d\tilde{z}'_i} \sqrt{d\tilde{z}'_j}}, \tilde{A} = 4g \times 4g \text{ antisymmetric matrix,}$$

$$\tilde{C}_{ij}^a = (\tilde{\Lambda}_{iA}^a(\tilde{z}'_i) \epsilon_j^A + \tilde{H}_{i\mathcal{J}}^a(\tilde{z}'_i) q_j^{\mathcal{J}}) \frac{S_\alpha(\tilde{z}'_i, \sigma_j | \tau)}{\sqrt{d\tilde{z}'_i} \sqrt{d\sigma_j}},$$

$$\langle \tilde{C}_{ij} \tilde{C}_{kl} \rangle \equiv (\epsilon_j^A \langle \tilde{\Lambda}_{iA}(\tilde{z}'_i) \tilde{\Lambda}_{kB}(\tilde{z}'_k) \rangle \epsilon_l^B + q_j^{\mathcal{J}} \langle \tilde{H}_{i\mathcal{J}}(\tilde{z}'_i) \tilde{H}_{k\mathcal{J}}(\tilde{z}'_k) \rangle q_l^{\mathcal{J}}) \frac{S_\alpha(\tilde{z}'_i, \sigma_j | \tau)}{\sqrt{d\tilde{z}'_i} \sqrt{d\sigma_j}} \frac{S_\alpha(\tilde{z}'_k, \sigma_l | \tau)}{\sqrt{d\tilde{z}'_k} \sqrt{d\sigma_l}}$$

$$G_{ij} = (\epsilon_{iA} \epsilon_j^A + q_{i\mathcal{J}} q_j^{\mathcal{J}}) \frac{S_\alpha(\sigma_i, \sigma_j | \tau)}{\sqrt{d\sigma_i} \sqrt{d\sigma_j}}, u_i^a G_{ii} = -\lambda_A^a(\sigma_i) \epsilon_i^A - \eta_{\mathcal{J}}^a(\sigma_i) q_i^{\mathcal{J}},$$

$$\tilde{\Lambda}_{iA}^a(\tilde{z}'_i) = \begin{cases} \lambda_A^a(\tilde{z}_k), i = 4k, \\ \tilde{u}_{lk}^a \langle \tilde{u}_{lk} \lambda_A(\tilde{z}_{lk}) \rangle, \end{cases} \tilde{H}_{i\mathcal{J}}^a(\tilde{z}'_i) = \begin{cases} \eta_{\mathcal{J}}^a(\tilde{z}_k), i = 4k, \\ \tilde{u}_{lk} \eta_{\mathcal{J}}(\tilde{z}_{lk}), \end{cases} i = 4k + l, l = 1, 2, 3$$



$$\begin{aligned}
A_{g,v,n}^\alpha &= \left\langle \prod_{j=1}^g \left(j_{z_j}^u \tilde{j}_{z_j}^u j_{r_j}^{\prime u} \right) V_1 \dots V_n \right\rangle_\alpha \\
&= \prod_{l=1}^n \frac{d\sigma_l d^2 u_l d^2 v_l}{\text{vol}(\text{SL}(2, \mathbb{C}))_u} \prod_{r=1}^n \bar{\delta}(\langle v_r \epsilon_r \rangle - 1) \bar{\delta}^4(\langle u_r \lambda^A \rangle - \langle v_r \kappa_r^A \rangle) \bar{\delta}^{\mathcal{N}}(\langle u_r \eta^J \rangle - \langle v_r \zeta_r^J \rangle) \mathcal{H} \mathcal{G} \bar{\delta}_r j_r', \\
\bar{\delta}_r &= \prod_{r=2}^n \bar{\delta} \left(\text{Res}_{\sigma_r}(\langle \lambda_A \lambda_B \rangle \Omega^{AB}) \right) \bar{\delta} \left(\text{Res}_{\sigma_r}(\langle \eta_J \eta_J \rangle \Omega^{JJ}) \right) \\
j_r' &= \left\langle \prod_{j=1}^g j_{r_j}^{\prime u} \prod_{i=3}^4 \prod_{k=s,t} \int du_{ki}'^2 \bar{\delta}(u_{ki}'^a - \chi_{ki}^A \lambda_{iA}^a(\sigma_k)) \frac{\prod_{r=1}^2 \prod_{p=s,t} \frac{1}{2} (\chi_{pi}^A \rho_{irA}^a(\sigma_p)) (\chi_{pi}^A \rho_{irAa}(\sigma_p))}{\text{vol}(\text{SL}(2, \mathbb{C}))_{u_{si}'} [u_{si}' u_{ti}']^4} \right\rangle_\alpha \\
\mathcal{H} &= \prod_{j=1}^g \int \frac{d^2 u_{1j} d^2 u_{2j} d^2 u_{3j}}{\langle u_{1j} u_{2j} \rangle \langle u_{2j} u_{3j} \rangle \langle u_{3j} u_{1j} \rangle} e^{-\sum_{k=1}^3 \|u_{kj}\|^2} \sum_{I,J} \text{Pf}(A_J^I) \sum_D \det' H_{DJ}^I, \\
\mathcal{G} &= \prod_{j=1}^g \int \frac{d^2 \tilde{u}_{1j} d^2 \tilde{u}_{2j} d^2 \tilde{u}_{3j}}{\langle \tilde{u}_{1j} \tilde{u}_{2j} \rangle \langle \tilde{u}_{2j} \tilde{u}_{3j} \rangle \langle \tilde{u}_{3j} \tilde{u}_{1j} \rangle} e^{-\sum_{k=1}^3 \|\tilde{u}_{kj}\|^2} \sum_{I,J} \text{Pf}(\tilde{A}_J^I) \sum_D \det' G_{DJ}^I, \\
\det' H_{DJ}^I &= \frac{\det(H_{[kl]}^{[ij]})_{DJ}^I}{\langle u_i u_j \rangle \langle u_k u_l \rangle}, \quad \det' G_{DJ}^I = \frac{\det(G_{[st]}^{[pr]})_{DJ}^I}{\langle u_p u_r \rangle \langle u_s u_t \rangle}, \\
H_{ij} &= \epsilon_{iA} \epsilon_j^A \frac{S_\alpha(\sigma_i, \sigma_j | \tau)}{\sqrt{d\sigma_i} \sqrt{d\sigma_j}}, \quad u_i^a H_{ii} = -\lambda_A^a(\sigma_i) \epsilon_i^A, \\
G_{ij} &= (\epsilon_{iA} \epsilon_j^A + q_{ij} q_j^j) \frac{S_\alpha(\sigma_i, \sigma_j | \tau)}{\sqrt{d\sigma_i} \sqrt{d\sigma_j}}, \quad u_i^a G_{ii} = -\lambda_A^a(\sigma_i) \epsilon_i^A - \eta_j^a(\sigma_i) q_i^j. \\
0 &= \sum_i \epsilon_i^{[A} (\langle v_i \kappa_i^{B]} \rangle - \langle u_i \lambda(\sigma_i)^{B]} \rangle) = \sum_i \langle \kappa_i^A \kappa_i^B \rangle - \sum_{ij} \langle u_i u_j \rangle \epsilon_i^{[A} \epsilon_j^{B]} S_\alpha(\sigma_i, \sigma_j | \tau) = \sum_i k_i^{AB} \\
&\sum_i \langle \kappa_i^A \zeta_i^J \rangle = \sum_i (\langle \kappa_i^A v_i \rangle \langle \epsilon_i \zeta_i^J \rangle - \langle \kappa_i^A \epsilon_i \rangle \langle v_i \zeta_i^J \rangle) = \sum_i (\langle u_i \lambda(\sigma_i)^A \rangle q_i^J - \epsilon_i^A \langle u_i \eta(\sigma_i)^J \rangle) \\
&= \sum_{ij} (\epsilon_j^A q_i^J - \epsilon_i^A q_j^J) \langle u_i u_j \rangle S_\alpha(\sigma_i, \sigma_j | \tau) = 0 \\
A_{g,n}^{\text{even}} &= \sum_\alpha \int d\mu_g Z_g^\alpha((z), (y), (x), (w), (r)) \left(\prod_{j=1}^g \bar{\delta}_{x_j} \right) A_{g,v,n}^\alpha \\
Z_g^\alpha &= \frac{Z_2((y)) Z_1((x))^6 Z_1((w))^9 \left(Z_{\frac{1}{2}, \alpha}^{\text{even}} \right)^8}{Z_1((z))^8 Z_1((r))^{16}} \\
&= \frac{Y_3(y) Y(x) Y(\tilde{x}) \prod_{k=3}^4 Y(x_k) Y(x_k) \prod_{j=1}^3 Y(w_j) \prod_{k=3}^4 Y(w_{jk}) \theta[\alpha](0)^8}{Y(z) Y(\tilde{z}) \prod_{j=1}^3 Y(z_j) Y(\tilde{z}_j) \prod_{k=3}^4 Y(r_{kl}) \prod_{l=1}^2 Y(r_{kl}) \prod_{j=1}^3 Y(r_{jkl})} \\
A_{1,n}^{\text{even}} &= \sum_\alpha \int \frac{d\tau}{\det_\omega^{10}} \left(\frac{\theta[\alpha](0)}{\eta(\tau)^3} \right)^8 \bar{\delta}_{x_1} \frac{A_{1,v,n}^\alpha}{\text{vol}(\text{GL}(1, \mathbb{C}))_\sigma}
\end{aligned}$$



$$\begin{aligned}
\mu_0^{a\dot{\beta}} \frac{h_\alpha}{\sqrt{d\sigma}} &= x_0^{\beta\dot{\beta}} \lambda_\beta^a - \Theta_0^{j\dot{\beta}} \eta_j^a, \tilde{\mu}_0^{a\beta} \frac{h_\alpha}{\sqrt{d\sigma}} = -x_0^{\beta\dot{\beta}} \tilde{\lambda}_\beta^a - \Theta_0^{j\dot{\beta}} \eta_j^a, \tilde{\eta}_0^{a\dot{j}} \frac{h_\alpha}{\sqrt{d\sigma}} = \Theta_0^{jA} \lambda_A^a \\
&\int \frac{d^8 \mu_0^{aA} d^{2N} \tilde{\eta}_0^{a\dot{j}}}{\text{vol}(\text{SL}(2, \mathbb{C}) \times \mathbb{C})_{\mu_0}} \exp \left[- \sum_{i=1}^n U_i \right] \\
U_i &= (\langle u_i \mu_0^A \rangle \epsilon_{iA} + \langle u_i \tilde{\eta}_0^{\dot{j}} \rangle q_{i\dot{j}}) \frac{h_\alpha}{\sqrt{d\sigma}} \\
&= x_0^{\beta\dot{\beta}} (\epsilon_{i\dot{\beta}} \langle u_i \lambda_{i\beta} \rangle - \epsilon_{i\beta} \langle u_i \tilde{\lambda}_{i\dot{\beta}} \rangle) - \Theta_0^{jA} \langle u_i \eta_j \rangle \epsilon_{iA} + \Theta_0^{jA} \langle u_i \lambda_{iA} \rangle q_{i\dot{j}} \\
&= x_0^{\beta\dot{\beta}} (\langle v_i \kappa_{i\beta} \rangle \langle \epsilon_i \tilde{\kappa}_{i\dot{\beta}} \rangle - \langle \epsilon_i \kappa_{i\beta} \rangle \langle v_i \tilde{\kappa}_{i\dot{\beta}} \rangle) + \Theta_0^{jA} (-\langle \epsilon_i \kappa_{iA} \rangle \langle v_i \zeta_{i\dot{j}} \rangle + \langle v_i \kappa_{iA} \rangle \langle \epsilon_i \zeta_{i\dot{j}} \rangle) \\
&= x_0^{\beta\dot{\beta}} \langle \kappa_{i\beta} \tilde{\kappa}_{i\dot{\beta}} \rangle + \Theta_0^{jA} \langle \kappa_{iA} \zeta_{i\dot{j}} \rangle = x_0 \cdot k_i + \Theta_0^{jA} \langle \kappa_{iA} \zeta_{i\dot{j}} \rangle \\
A_{g,n}^{\text{odd}} &= \delta \left(\sum_{i=1}^n k_i \right) \delta \left(\sum_{i=1}^n \langle \kappa_{iA} \zeta_{i\dot{j}} \rangle \right) \sum_\alpha \int d\mu_g Z_g^\alpha((z), (y), (x), (w), (r)) \int dm_0 \left(\prod_{j=1}^g \bar{\delta}_{x_j} \right) A_{g,v,n}^\alpha \\
Z_g^\alpha &= \frac{Z_2((y)) Z_1((x))^6 Z_1((w))^9 \left(Z_{\frac{1}{2}, \alpha}^{\text{odd}} \right)^8}{Z_1((z))^8 Z_1((r))^{16}} \\
&= \frac{Y_3(y) Y(x) Y(\tilde{x}) \prod_{k=3}^4 Y(x_k) Y(x_k) \prod_{j=1}^3 Y(w_j) \prod_{k=3}^4 Y(w_{jk})}{Y(z) Y(\tilde{z}) \prod_{j=1}^3 Y(z_j) Y(\tilde{z}_j) \prod_{k=3}^4 \prod_{l=1}^2 Y(r_{kl}) \prod_{j=1}^3 Y(r_{jkl})} \left(\frac{h_\alpha(z)^2}{\det_\omega} \right)^8 \\
dm_0 &= \frac{d^8 \lambda_{0A}^a d^{2N} \eta_{0\dot{j}}^a}{\text{vol}(\text{SL}(2, \mathbb{C}) \times \mathbb{C})_{\lambda_0}} \\
A_{1,n}^{\text{odd}} &= \delta \left(\sum_{i=1}^n k_i \right) \delta \left(\sum_{i=1}^n \langle \kappa_{iA} \zeta_{i\dot{j}} \rangle \right) \int \frac{d\tau}{\det_\omega^{10} \text{vol}(\text{SL}(2, \mathbb{C}) \times \mathbb{C})_{\lambda_0}} \bar{\delta}_{x_1} \frac{A_{1,v,n}}{\text{vol}(\text{GL}(1, \mathbb{C}))_\sigma} \\
A_{1,n}^{\text{odd}} &= \delta \left(\sum_{i=1}^n k_i \right) \delta \left(\sum_{i=1}^n \langle \kappa_{iA} \zeta_{i\dot{j}} \rangle \right) \int \frac{d\tau}{d^2 z} \frac{d^8 (\lambda_{0A}^a \sqrt{dz}) d^{2N} (\eta_{0\dot{j}}^a \sqrt{dz})}{(\text{vol}(\text{SL}(2, \mathbb{C}) \times \mathbb{C})_{\lambda_0} d^4 z)} \bar{\delta}_{x_1} \frac{A_{1,v,n}^\alpha}{\text{vol}(\text{GL}(1, \mathbb{C}))_\sigma} \\
\tau &= \begin{pmatrix} \tau_{ij}^{(g-1)} & \int_{\sigma_-}^{\sigma_+} \omega_i^{(g-1)} \\ \int_{\sigma_-}^{\sigma_+} \omega_j^{(g-1)} & \frac{1}{\pi i} \ln q_{gg} + \text{const} \end{pmatrix} + O(q_{gg}^2) \\
\omega_i &= \omega_i^{(g-1)} + O(q_{gg}^2) \text{ for } i < g \\
\omega_g &= \omega_{+-}^{(g-1)} + O(q_{gg}^2) \\
S_\alpha(\sigma, \sigma_* | \tau) &= S_{\alpha^{(g-1)}}(\sigma, \sigma_* | \tau^{(g-1)}) + O(q_{gg}) \equiv S_\alpha^{(g-1)}(\sigma, \sigma_* | \tau) + O(q_{gg}) \\
P^\mu &= \sum_{l=1}^g c_l^\mu \omega_l + \sum_{j=1}^n k_j^\mu \omega_{j*}, \omega_{j*} = S_\alpha(\sigma, \sigma_j | \tau) - S_\alpha(\sigma, \sigma_* | \tau),
\end{aligned}$$



$$\begin{aligned}
P^\mu &\rightarrow \sum_{l=1}^{g-1} c_l^\mu \omega_l^{(g-1)} + k_+^\mu S_\alpha^{(g-1)}(\sigma, \sigma_+ | \tau) + k_-^\mu S_\alpha^{(g-1)}(\sigma, \sigma_+ | \tau) + \sum_{j=1}^n k_j^\mu S_\alpha^{(g-1)}(\sigma, \sigma_j | \tau) \\
&= \sum_{l=1}^{g-1} c_l^\mu \omega_l^{(g-1)} + \sum_{j=1}^{n+2} k_j^\mu S_\alpha^{(g-1)}(\sigma, \sigma_j | \tau)
\end{aligned}$$

$$\begin{aligned}
\lambda_A^a(\sigma) &\rightarrow \sum_{i=1}^{n+2} u_i^a \epsilon_{iA} \frac{S_\alpha^{(g-1)}(\sigma, \sigma_i | \tau)}{\sqrt{d\sigma} \sqrt{d\sigma_i}}, \kappa_{n+1A}^a = \kappa_{+A}^a = (-1)^{\delta_{a2}} \kappa_{-A}^a = (-1)^{\delta_{a2}} \kappa_{n+2A}^1, \\
\eta_j^a(\sigma) &\rightarrow \sum_{i=1}^{n+2} u_i^a q_{ij} \frac{S_\alpha^{(g-1)}(\sigma, \sigma_i | \tau)}{\sqrt{d\sigma} \sqrt{d\sigma_i}}, \zeta_{n+1j}^a = \zeta_{+j}^a = (-1)^{\delta_{a2}} \zeta_{-j}^a = (-1)^{\delta_{a2}} \zeta_{n+2j}^1,
\end{aligned}$$

$$P^{AB}(\sigma) = \langle \lambda^A(\sigma) \lambda^B(\sigma) \rangle,$$

$$P^{AB}(\sigma) = \epsilon_+^A \langle u_+ \lambda^B(\sigma_+) \rangle \frac{S_\alpha(\sigma, \sigma_+ | \tau)}{\sqrt{d\sigma} \sqrt{d\sigma_+}} + \epsilon_-^A \langle u_- \lambda^B(\sigma_-) \rangle \frac{S_\alpha(\sigma, \sigma_- | \tau)}{\sqrt{d\sigma} \sqrt{d\sigma_-}} + \sum_{i=1}^n k_i^{AB} \frac{S_\alpha(\sigma, \sigma_i | \tau)}{\sqrt{d\sigma} \sqrt{d\sigma_i}}, P_A^A(\sigma) =$$

$$0, P^2(\sigma) = -P_A^A(\sigma) P_B^B(\sigma) = 0$$

$$\langle v_\pm \kappa_\pm^A \rangle = \langle u_\pm \lambda^A(\sigma_\pm) \rangle, k_\pm \cdot P(\sigma_\pm) = 0$$

$$\begin{aligned}
&\delta^4\left(\sum_{i=1}^n k_i^\mu\right)\delta^{4\mathcal{N}}\left(\sum_{i=1}^n \langle \kappa_{iA} \zeta_{i\mathcal{I}} \rangle\right) \frac{\mathrm{d}^8\lambda_{0A}^a \, \mathrm{d}^{2\mathcal{N}}\eta_{0\mathcal{I}}^a}{\mathrm{vol}(\mathrm{SL}(2,\mathbb{C})\times \mathbb{C})_{\lambda_0}} \frac{1}{(2\pi i)^3} \frac{\mathrm{d}q_{gg}}{q_{gg}} \frac{\mathrm{d}q_1}{q_1} \frac{\mathrm{d}q_2}{q_2} \bar{\delta}(\langle \lambda^A(x_j) \lambda_A(x_j) \rangle) \\
&\bar{\delta}(\langle \lambda^A(\tilde{x}_j) \lambda_A(\tilde{x}_j) \rangle) \bar{\delta}(\langle \eta^{\mathcal{I}}(\tilde{x}_j) \eta_{\mathcal{I}}(\tilde{x}_j) \rangle) \\
&= \int \frac{\prod_{l=n+1}^{n+2} \mathrm{d}^4\epsilon_l^A \, \mathrm{d}^{\mathcal{N}}q_l^{\mathcal{I}}}{\mathrm{vol}(\mathrm{SL}(2,\mathbb{C})\times \mathbb{C})_\epsilon} \frac{\prod_{l=n+1}^{n+2} \mathrm{d}\sigma_l \, \mathrm{d}^2u_l \, \mathrm{d}^2v_l}{\mathrm{vol}(\mathrm{SL}(2,\mathbb{C})_\sigma \times \mathrm{SL}(2,\mathbb{C})_u)} \frac{1}{\langle u_{n+1} u_{n+2} \rangle^4} \\
&\prod_{r=n+1}^{n+2} \bar{\delta}(\langle v_r \epsilon_r \rangle - 1) \bar{\delta}^4(\langle u_r \lambda^A \rangle - \langle v_r \kappa_r^A \rangle) \bar{\delta}^{\mathcal{N}}(\langle u_r \eta^{\mathcal{I}} \rangle - \langle v_r \zeta_r^{\mathcal{I}} \rangle) \\
&\bar{\delta}(\langle \lambda^A(x_j) \lambda_A(x_j) \rangle) \bar{\delta}(\langle \lambda^A(\tilde{x}_j) \lambda_A(\tilde{x}_j) \rangle + \langle \eta^{\mathcal{I}}(\tilde{x}_j) \eta_{\mathcal{I}}(\tilde{x}_j) \rangle)
\end{aligned}$$

$$1 = \int \frac{\prod_{l=n+1}^{n+2} \mathrm{d}^4\epsilon_l^A \, \mathrm{d}^{\mathcal{N}}q_l^{\mathcal{I}}}{\mathrm{vol}(\mathrm{SL}(2,\mathbb{C})\times \mathbb{C})_\epsilon} \frac{\mathcal{W}_{\mathrm{vac}}}{\mathrm{d}\sigma_* \, \mathrm{d}^3u} \prod_{l=n+1}^{n+2} \mathrm{d}^2u_l \, \mathrm{d}^2v_l \bar{\delta}(\langle v_l \epsilon_l \rangle - 1) \bar{\delta}^4(\langle u_l \lambda_A(\sigma_l) \rangle - \langle v_l \kappa_{lA} \rangle)$$

$$\bar{\delta}^{\mathcal{N}}(\langle u_l \eta_{\mathcal{I}}(\sigma_l) \rangle - \langle v_l \zeta_{l\mathcal{I}} \rangle) e^{\langle u_l \mu^A(\sigma_l) \rangle \epsilon_{lA} + \langle u_l \tilde{\eta}^{\mathcal{I}}(\sigma_l) \rangle q_{l\mathcal{I}}}$$

$$\prod_{i=3}^4 \frac{\prod_{l=n+1}^{n+2} \mathrm{d}u_{il}'^2}{\mathrm{d}^3u_i'} \bar{\delta}\left(u_{il}'^a - \chi_{il}^A \lambda_{iA}^a(\sigma_l)\right) \prod_{j=1}^2 \frac{1}{2} \left(\chi_{il}^A \rho_{ijA}^a(\sigma_l)\right) \left(\chi_{il}^A \rho_{ijAa}(\sigma_l)\right),$$

$$\kappa_{n+1A}^1 = \kappa_{n+2A}^1, \kappa_{n+1A}^2 = -\kappa_{n+2A}^2, \zeta_{n+1\mathcal{I}}^1 = \zeta_{n+2\mathcal{I}}^1, \zeta_{n+1\mathcal{I}}^2 = -\zeta_{n+2\mathcal{I}}^2$$

$$\mathcal{W}_{\mathrm{vac}}(\sigma_{n+1},\sigma_{n+2},\sigma_*)=c(\sigma_{n+1})c(\sigma_{n+2})c(\sigma_*)n\tilde{n}\prod_{i=3}^4(n_i\tilde{n}_i)\delta(\gamma)\delta(\tilde{\gamma})\prod_{i=3}^4\prod_{j=1}^2\delta(\gamma_{ij})$$

$$\begin{aligned}
&\prod_{a,b}\left(\delta(\gamma_{ab})\delta(\tilde{\gamma}_{ab})N_{ab}\prod_{i=3}^4\mathcal{N}_{iab}\prod_{j=1}^2\delta(\gamma_{ijab})\right), \\
&\langle 0|\mathcal{W}_{\mathrm{vac}}(0,0,0)|0\rangle=1,\langle\epsilon_{n+1}\epsilon_{n+2}\rangle=1,
\end{aligned}$$



$$\begin{aligned}
& (\delta(\gamma_{ab}u_1^a u_1^b)\delta(\beta_{ab}\hat{u}^a \hat{u}^b)\langle\lambda_A(\sigma_1)\hat{u}\rangle\langle\rho_1^A\hat{u}\rangle)(\delta(\gamma)\delta(\beta)\langle\lambda_A(\sigma_1)\rho_1^A\rangle)e^{\langle u_1\mu^A(\sigma_1)\rangle\epsilon_{1A}} \Rightarrow \\
& \left(\frac{\langle\hat{u}\lambda_A(\sigma_1)\rangle\epsilon_1^A}{\langle u_1\hat{u}\rangle} + \frac{\langle\rho_1^A\rho_1^B\rangle}{2}\epsilon_{1A}\epsilon_{1B}\right)e^{\langle u_1\mu^A(\sigma_1)\rangle\epsilon_{1A}} \\
& (\delta(\gamma_{ab}u_2^a u_2^b)\delta(\beta_{ab}\hat{u}^a \hat{u}^b)\langle\lambda_A(\sigma_2)\hat{u}\rangle\langle\rho_1^A\hat{u}\rangle)(\langle u_2\hat{u}\rangle\delta(\gamma_{ab}u_2^a \hat{u}^b)\delta(\beta_{ab}\hat{u}^a u_2^b)\langle\lambda_A(\sigma_2)\hat{u}\rangle\langle\rho_1^A u_2\rangle) \\
& e^{\langle u_2\mu^A(\sigma_2)\rangle\epsilon_{2A}} \Rightarrow \\
& \left(\frac{\langle\hat{u}\lambda_A(\sigma_2)\rangle\epsilon_2^A}{\langle u_2\hat{u}\rangle} + \frac{\langle\rho_1^A\rho_1^B\rangle}{2}\epsilon_{2A}\epsilon_{2B}\right)e^{\langle u_2\mu^A(\sigma_2)\rangle\epsilon_{2A}}
\end{aligned}$$

$$\begin{aligned}
A_{g,n}^{\text{odd}} &= A_{g-1,n+2}^{\text{even}} = \sum_{\alpha} \int d\mu_{g-1} Z_{g-1}^{\alpha}((z), (y), (x), (w), (r)) \int dm_{\text{fw}} \prod_{j=1}^{g-1} \bar{\delta}_{x_j} A_{g-1,V,n+2}^{\alpha} \\
dm_{\text{fw}} &= \frac{\prod_{l=n+1}^{n+2} d^4\epsilon_l^A d^{\mathcal{N}}q_l^j}{\text{vol}(\text{SL}(2, \mathbb{C}) \times \mathbb{C})_{\epsilon}}
\end{aligned}$$

$$\begin{aligned}
A_{g,n}^{\text{odd}} &= A_{g-1,n+2}^{\text{odd}} = \delta\left(\sum_{i=1}^{n+2} k_i\right) \delta\left(\sum_{i=1}^{n+2} \langle\kappa_{iA}\zeta_{ij}\rangle\right) \sum_{\alpha} \int d\mu_{g-1} Z_{g-1}^{\alpha}((z), (y), (x), (w), (r)) \\
& \int dm_0 dm_{\text{fw}} \prod_{j=1}^{g-1} \bar{\delta}_{x_j} A_{g-1,V,n+2}^{\alpha} \\
dm_0 &= \frac{d^8\lambda_{0A}^a d^{2\mathcal{N}}\eta_{0j}^a}{\text{vol}(\text{SL}(2, \mathbb{C}) \times \mathbb{C})_{\lambda_0}}, \quad dm_{\text{fw}} = \frac{\prod_{l=n+1}^{n+2} d^4\epsilon_l^A d^{\mathcal{N}}q_l^j}{\text{vol}(\text{SL}(2, \mathbb{C}) \times \mathbb{C})_{\epsilon}}
\end{aligned}$$

$$A_{g,n}^{\text{even}} = A_{g-1,n+2}^{\text{even}} = \sum_{\alpha} \int d\mu_{g-1} Z_{g-1}^{\alpha}((z), (y), (x), (w), (r)) \int dm_{\text{fw}} \prod_{j=1}^{g-1} \bar{\delta}_{x_j} A_{g-1,V,n+2}^{\alpha}$$

$$dm_{\text{fw}} \delta(\langle\kappa_{n+1}^A \kappa_{n+1A}\rangle) \rightarrow \frac{d^4 p^{\mu}}{p^2} d^{\mathcal{N}}q_{n+1}^j d^{\mathcal{N}}q_{n+2}^j$$

$$\det' H_{DJ}^I \det' G_{DJ}^I = \frac{\det\left(H_{[+-]}^{[+-]}\right)_{DJ}^I \det\left(G_{[+-]}^{[+-]}\right)_{DJ}^I}{\langle u_+ u_- \rangle^4}$$

$$\frac{1}{\langle\lambda^A(x)\lambda_A(x)\rangle}\frac{1}{\langle\lambda^A(x)\lambda_A(x)\rangle}=\frac{1}{P_A^A(x)^2}=-\frac{1}{P^{AB}P_{BA}(x)}=-\frac{1}{P^2(x)}.$$

$$\begin{aligned}
\epsilon_j^A(u_{+a}\epsilon_{+A}+u_{-a}\epsilon_{-A})\sum_i u_i^a\left(H_{[+-]}^{[+-]}\right)_{ij}&=\epsilon_j^A(u_{+a}\epsilon_{+A}+u_{-a}\epsilon_{-A})\left(-\frac{u_+^a\epsilon_+^B\epsilon_{jB}}{\sigma_+-\sigma_j}-\frac{u_-^a\epsilon_-^B\epsilon_{jB}}{\sigma_--\sigma_j}\right) \\
&=\epsilon_j^A\epsilon_{+A}\epsilon_j^B\epsilon_{-A}\langle u_+u_- \rangle\left(\frac{1}{\sigma_+-\sigma_j}-\frac{1}{\sigma_--\sigma_j}\right)=O(\Lambda^{-1})
\end{aligned}$$

$$\text{SO}(10) \rightarrow \text{SO}(8) \times \text{U}(1) \rightarrow \text{SU}(2) \times \text{SU}(2) \times \text{SU}(2) \times \text{SU}(2) \times \text{U}(1)$$



$$S = \sum_{i=1}^4 \int_{\Sigma} Z_i^a \cdot \bar{\partial} Z_{ia} + \mathcal{A}_{iab} Z_i^a \cdot Z_i^b + \sum_{j=1}^2 \left(\frac{1}{2} \rho_{ija}^A \bar{\partial} \rho_{ija}^a + \mathcal{B}_{ij}^{ab} \lambda_{ia}^A \rho_{ija}^b + b_{ij} \lambda_{ia}^a \rho_{ija}^A + \tilde{\tau}_{ij}^A \bar{\partial} \tau_{ija} \right) \\ + \begin{cases} a_i \lambda_{ia}^A \lambda_{ia}^a & i = 1 \\ a_i \lambda_{ia}^a \lambda_{ia}^a + \tilde{a}_i \tilde{\lambda}_{ia}^{\dot{\alpha}} \tilde{\lambda}_{ia}^a & i > 1 \end{cases} \\ Z_i^a = (\lambda_{iA}^a, \mu_i^{aA}, \eta_{ij}^a), A = 1, \dots, 4, I = 1, \dots, 8, a = 1, 2$$

$$S = \int_{\Sigma} Z^a \cdot \bar{\partial} Z_a + A_{ab} Z^a \cdot Z^b + a \langle \lambda_A \lambda_B \rangle \Omega^{AB} + \tilde{a} (\langle \lambda_A \lambda_B \rangle \Omega^{AB} + \langle \eta_J \eta_J \rangle \Omega^{JJ}) \\ + S_{\rho_1} + S_{(\rho_2, \tau)} + S_{\text{aux}} \\ S_{\rho} = \int_{\Sigma} \frac{1}{2} \langle \rho_A \bar{\partial} \rho_B \rangle \Omega^{AB} + B_{ab} \lambda_A^a \rho_B^b \Omega^{AB} + b \langle \lambda_A \rho_B \rangle \Omega^{AB} \\ S_{(\rho, \tau)} = \int_{\Sigma} \frac{1}{2} (\langle \rho_A \bar{\partial} \rho_B \rangle \Omega^{AB} + \langle \tau_J \bar{\partial} \tau_J \rangle \Omega^{JJ}) + D_{ab} (\lambda_A^a \rho_B^b \Omega^{AB} + \eta_J^a \tau_J^b \Omega^{JJ}) \\ + d (\langle \lambda_A \rho_B \rangle \Omega^{AB} + \langle \eta_J \tau_J \rangle \Omega^{JJ}) \\ S_{\text{aux}} = \sum_{i=3}^4 \int_{\Sigma} Z_i^a \cdot \bar{\partial} Z_{ia} + \mathcal{A}_{iab} Z_i^a \cdot Z_i^b + \sum_{j=1}^2 \left(\frac{1}{2} \rho_{ija}^A \bar{\partial} \rho_{ija}^a + \mathcal{B}_{ij}^{ab} \lambda_{ia}^A \rho_{ija}^b + b_{ij} \lambda_{ia}^a \rho_{ija}^A + \tilde{\tau}_{ij}^A \bar{\partial} \tau_{ija} \right) \\ + a_i \lambda_{ia}^a \lambda_{ia}^a + \tilde{a}_i \tilde{\lambda}_{ia}^{\dot{\alpha}} \tilde{\lambda}_{ia}^a$$

$$p^{\alpha\dot{\alpha}} = \sigma_{\mu}^{\alpha\dot{\alpha}} p^{\mu} = \begin{pmatrix} p^0 + p^3 & p^1 - ip^2 \\ p^1 + ip^2 & p^0 - p^3 \end{pmatrix} = \lambda^{\alpha} \tilde{\lambda}^{\dot{\alpha}}$$

$$(z, \bar{z}) \mapsto \left(\frac{az+b}{cz+d}, \frac{\bar{a}\bar{z}+\bar{b}}{\bar{c}\bar{z}+\bar{d}} \right), \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{SL}(2, \mathbb{C})$$

$$p^{\mu}=\omega q^{\mu}, q^{\mu}=\frac{1}{2}(1+|z|^2, z+\bar{z}, -i(z-\bar{z}), 1-|z|^2)$$

$$\omega \mapsto (cz+d)(\bar{c}\bar{z}+\bar{d})\omega, q^{\mu} \mapsto q'^{\mu} = (cz+d)^{-1}(\bar{c}\bar{z}+\bar{d})^{-1}\Lambda^{\mu}{}_{\nu}q^{\nu}.$$

$$q^{\alpha\dot{\alpha}} = \sigma_{\mu}^{\alpha\dot{\alpha}} q^{\mu} = \begin{pmatrix} 1 & \bar{z} \\ z & z\bar{z} \end{pmatrix} = \begin{pmatrix} 1 \\ z \end{pmatrix} \begin{pmatrix} 1 & \bar{z} \end{pmatrix}$$

$$\lambda^{\alpha} \equiv \left\langle p |^{\alpha} = \sqrt{\omega} \begin{pmatrix} 1 \\ z \end{pmatrix} = \sqrt{\omega} \langle q |^{\alpha}, \tilde{\lambda}^{\dot{\alpha}} \equiv | p \right\rangle^{\dot{\alpha}} = \sqrt{\omega} \begin{pmatrix} 1 \\ \bar{z} \end{pmatrix} = \sqrt{\omega} \left| q \right\rangle^{\dot{\alpha}},$$

$$\left\langle q |^{\alpha} = \begin{pmatrix} 1 \\ z \end{pmatrix}, \left| q \right\rangle^{\dot{\alpha}} = \begin{pmatrix} 1 \\ \bar{z} \end{pmatrix}.$$

$$\langle ij \rangle = -\sqrt{\omega_i \omega_j} z_{ij}, [ij] = \sqrt{\omega_i \omega_j} \bar{z}_{ij}$$

$$\varphi_{\Delta}^{\pm}(X^{\mu},z,\bar{z})=\int_0^{\infty}d\omega\omega^{\Delta-1}e^{\pm i\omega q\cdot X-\epsilon\omega}=\frac{(\mp i)^{\Delta}\Gamma(\Delta)}{(-q\cdot X\mp i\epsilon)^{\Delta}}$$



$$\psi_{\Delta,\ell=-1/2;\alpha}^{\pm}(X,z,\bar{z})=|q\rangle_{\alpha}\varphi_{\Delta+\frac{1}{2}}^{\pm}(X,z,\bar{z})$$

$$\psi_{\Delta,\ell=1/2}^{\pm\dot{\alpha}}(X,z,\bar{z})=|q]^{\dot{\alpha}}\varphi_{\Delta+\frac{1}{2}}^{\pm}(X,z,\bar{z})$$

$$V_{\Delta,\ell=\pm1}^{\mu\pm}(X,z,\bar{z})=\epsilon_{\ell=\pm1}^{\mu}(q,r)\varphi_{\Delta}^{\pm}(X,z,\bar{z})$$

$$H_{\Delta,\ell=\pm2}^{\mu\nu\pm}(X,z,\bar{z})=\epsilon_{\ell=\pm1}^{\mu}(q,r)V_{\Delta,\ell=\pm1}^{\nu\pm}(X,z,\bar{z})$$

$$\psi_{\Delta,\ell=-3/2}^{\mu\pm}(X,z,\bar{z})=\epsilon_{\ell=-1}^{\mu}(q,r)\psi_{\Delta,\ell=-1/2}^{\pm}(X,z,\bar{z})$$

$$\bar{\psi}_{\Delta,\ell=+3/2}^{\mu\pm}(X,z,\bar{z})=\epsilon_{\ell=+1}^{\mu}(q,r)\bar{\psi}_{\Delta,\ell=+1/2}^{\pm}(X,z,\bar{z})$$

$$\epsilon_{\ell=+1}^{\mu}(q,r)=\frac{\langle r|\sigma^{\mu}|q\rangle}{\sqrt{2}\langle rq\rangle},\epsilon_{\ell=-1}^{\mu}(q,r)=\frac{[r|\bar{\sigma}^{\mu}|q]}{\sqrt{2}[qr]}$$

$$\left\langle \prod_{n=1}^N \mathcal{O}_{\Delta_n,\ell_n}(z_n,\bar{z}_n) \right\rangle \equiv \left(\prod_{n=1}^N \int d\omega_n \omega_n^{\Delta_n-1} \right) \delta^{(4)} \left(\sum_{n=1}^N \omega_n q_n \right) A_{\ell_1 \dots \ell_N}(\omega_n, z_n, \bar{z}_n),$$

$$\left\langle \prod_{n=1}^N \mathcal{O}_{\Delta_n,\ell_n}\left(\frac{az_n+b}{cz_n+d},\frac{\bar{a}\bar{z}_n+\bar{b}}{\bar{c}\bar{z}_n+\bar{d}}\right) \right\rangle = \prod_{i=1}^N (cz_i+d)^{\Delta_i+\ell_i}(\bar{c}\bar{z}_i+\bar{d})^{\Delta_i-\ell_i} \left\langle \prod_{n=1}^N \mathcal{O}_{\Delta_n,\ell_n}(z_n,\bar{z}_n) \right\rangle.$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{C}).$$

$$\begin{aligned}\Psi(p,\eta)=&H^+(p)+\eta_A\psi^A_+(p)+\eta_{AB}G^{AB}_+(p)+\eta_{ABC}\chi^{ABC}_+(p)\\&+\eta_{ABCD}\Phi^{ABCD}(p)+\tilde{\eta}^{ABC}\chi^-_{ABC}(p)+\tilde{\eta}^{AB}G^-_{AB}(p)+\tilde{\eta}^A\psi^-_A(p)+\tilde{\eta}H^-(p)\end{aligned}$$

$$\begin{aligned}\eta_{A_1\dots A_n}&\equiv\frac{1}{n!}\eta_{A_1}\dots\eta_{A_2}\\ \tilde{\eta}^{A_1\dots A_n}&\equiv\epsilon^{A_1\dots A_nB_1\dots B_{8-n}}\eta_{B^1\dots B^{8-n}}\\ \tilde{\eta}&\equiv\prod_{A=1}^8\eta^A\end{aligned}$$

$$\mathcal{M}_n(\{p_1,\eta^1\},\ldots\{p_n,\eta^n\})=\langle \Psi_1(p_1,\eta^1)\ldots\Psi_n(p_n,\eta^n)\rangle$$

$$\begin{aligned}\Psi_{\Delta}(z,\bar{z},\eta)=&H_{\Delta}^{+}(z,\bar{z})+\eta_A\psi_{\Delta}^A(z,\bar{z})+\eta_{AB}G_{\Delta}^{AB}(z,\bar{z})+\eta_{ABC}\chi_{\Delta}^{ABC}(z,\bar{z})\\&+\eta_{ABCD}\Phi_{\Delta}^{ABCD}(z,\bar{z})+\tilde{\eta}^{ABC}\bar{\chi}_{ABC}\Delta(z,\bar{z})+\tilde{\eta}^{AB}\bar{G}_{AB}\Delta(z,\bar{z})\\&+\tilde{\eta}^A\bar{\psi}_A\Delta(z,\bar{z})+\tilde{\eta}H_{\Delta}^{-}(z,\bar{z})\end{aligned}$$

$$\begin{aligned}\langle \mathcal{O}_{\Delta_1,+2}\mathcal{O}_{\Delta_2,+2}\ldots\mathcal{O}_{\Delta_n,\ell_n}\rangle=&\left(\prod_{j=1}^n\int_0^\infty d\omega_j\omega_j^{\Delta_j-1}\right)\delta^4\left(\sum_i\omega_iq_i\right)M_n(1^{+2},2^{+2},\ldots,n)\\=&\left(\prod_{j=3}^n\int_0^\infty d\omega_j\omega_j^{\Delta_j-1}\int_0^\infty d\omega_1\int_0^\infty d\omega_2\omega_1^{\Delta_1-1}\omega_2^{\Delta_2-1}\right)\\&\times\delta^4\left(\sum_{i=3}^n\omega_iq_i+\omega_pq_p\right)\frac{\omega_p^2}{\omega_1\omega_2}\frac{\bar{z}_{12}}{z_{12}}M_{n-1}(p^{+2},\ldots,n)\end{aligned}$$



$$M_n(1^{+2}, 2^{+2}, \dots, n) = \frac{\omega_p^2}{\omega_1 \omega_2} \frac{\bar{z}_{12}}{z_{12}} M_{n-1}(p^{+2}, \dots, n)$$

$$\int_0^\infty d\omega_2 \omega_2^{\Delta_2-1} \int_0^\infty d\omega_1 \omega_1^{\Delta_1-1} \omega_1^\alpha \omega_2^\beta \omega_p^\gamma f(\omega_p) = B(\Delta_1 + \alpha, \Delta_2 + \beta) \int_0^\infty d\omega_p \omega_p^{\Delta_1+\Delta_2-1} f(\omega_p)$$

$$B(x,y)=\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

$$\begin{aligned} \langle \mathcal{O}_{\Delta_1,+2} \mathcal{O}_{\Delta_2,+2} \dots \mathcal{O}_{\Delta_n,\ell_n} \rangle &= \frac{\bar{z}_{12}}{z_{12}} B(\Delta_1 - 1, \Delta_2 - 1) \left(\prod_{j=3}^n \int_0^\infty d\omega_j \omega_j^{\Delta_j-1} \int_0^\infty d\omega_p \omega_p^{\Delta_1+\Delta_2-1} \right) \\ &\times \delta^4 \left(\sum_{i=3}^n \omega_i q_i + \omega_p q_p \right) M_{n-1}(p^{+2}, 3, \dots, n) \\ &= \frac{\bar{z}_{12}}{z_{12}} B(\Delta_1 - 1, \Delta_2 - 1) \langle \mathcal{O}_{\Delta_1+\Delta_2,+2} \mathcal{O}_{\Delta_3,\ell_3} \dots \mathcal{O}_{\Delta_n,\ell_n} \rangle \end{aligned}$$

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,+2}(z_2,\bar{z}_2)\sim \frac{\bar{z}_{12}}{z_{12}}B(\Delta_1-1,\Delta_2-1)\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2)$$

$$M_n(1^{-2}, 2^{-2}, \dots) = \frac{\omega_p^2}{\omega_1 \omega_2} \frac{z_{12}}{\bar{z}_{12}} M_{n-1}(p^{-2}, \dots, n)$$

$$\mathcal{O}_{\Delta_1,-2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,-2}(z_2,\bar{z}_2)\sim \frac{z_{12}}{\bar{z}_{12}}B(\Delta_1-1,\Delta_2-1)\mathcal{O}_{\Delta_1+\Delta_2,-2}(z_2,\bar{z}_2)$$

$$\begin{aligned} M_n(1^{+2}, 2^{-2}, 3, \dots, n) &= \frac{\omega_1^3}{\omega_p^2 \omega_2} \frac{\bar{z}_{12}}{z_{12}} M_{n-1}(p^{-2}, 3, \dots, n) \\ &+ \frac{\omega_2^3}{\omega_p^2 \omega_1} \frac{z_{12}}{\bar{z}_{12}} M_{n-1}(p^{+2}, 3, \dots, n) \end{aligned}$$

$$\begin{aligned} \mathcal{O}_{\Delta_1,+2}(z_1,z_1)\mathcal{O}_{\Delta_2,-2}(z_2,\bar{z}_2) &= B(\Delta_1+3,\Delta_2-1)\frac{\bar{z}_{12}}{z_{12}}\mathcal{O}_{\Delta_1+\Delta_2,-2}(z_2,\bar{z}_2) \\ &+ B(\Delta_1-1,\Delta_2+3)\frac{z_{12}}{\bar{z}_{12}}\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2) \end{aligned}$$

$$\left\langle \prod_{n=1}^N \mathcal{O}_{\Delta_n}(z_n, \bar{z}_n, \eta^n) \right\rangle \equiv \left(\prod_{n=1}^N \int d\omega_n \omega_n^{\Delta_n-1} \right) \delta^{(4)} \left(\sum_{n=1}^N \omega_n q_n \right) \times \mathcal{M}_N(\{\omega_1, z_1, \bar{z}_1, \eta^1\}, \dots, \{\omega_N, z_N, \bar{z}_N, \eta^N\})$$

$$\mathcal{M}_N(\cdots,j-1,j,j+1,\cdots) \stackrel{\omega_j \rightarrow 0}{\rightarrow} \frac{1}{\omega_j} \sum_{\substack{i=1 \\ i \neq j}}^N \omega_i z_{ji} \bar{z}_{ji} \left[\frac{z_{j-1,i}^2}{z_{j-1,j}^2 z_{j,i}^2} + \frac{\bar{z}_{j-1,i}^2}{\bar{z}_{j-1,j}^2 \bar{z}_{j,i}^2} \delta^4(\eta^j) \right] \times \mathcal{M}_{N-1}(\cdots,j-1,j+1,\cdots)$$



$$\begin{aligned}
\left\langle \prod_{n=1}^N \lim_{\Delta_j \rightarrow 1} (\Delta_j - 1) \mathcal{O}_{\Delta_n}(z_n, \bar{z}_n, \eta^n) \right\rangle &= \left(\prod_{\substack{n=1 \\ n \neq j}}^N \int d\omega_n \omega_n^{\Delta_n-1} \right) \lim_{\Delta_j \rightarrow 1} \int_0^\infty d\omega_j (\Delta_j - 1) \omega_j^{\Delta_j-1} \\
&\quad \times \delta^{(4)} \left(\sum_{\substack{k=1 \\ k \neq j}}^N \omega_k q_k \right) \mathcal{M}_N(1, \dots, n, \dots, N) \\
&= \left(\prod_{\substack{n=1 \\ n \neq j}}^N \int d\omega_n \omega_n^{\Delta_n-1} \right) \int_0^\infty d\omega_j \frac{d}{d\omega_j} \left(\lim_{\Delta_j \rightarrow 1} \omega_j^{\Delta_j-1} \right) \delta^{(4)} \left(\sum_{\substack{k=1 \\ k \neq j}}^N \omega_k q_k \right) \omega_j \mathcal{M}_N(1, \dots, n, \dots, N) \\
&\quad \frac{d}{d\omega_j} \left(\lim_{\Delta_j \rightarrow 1} \omega_j^{\Delta_j-1} \right) = \frac{d}{d\omega_j} \theta(\omega_j) = \delta(\omega_j)
\end{aligned}$$

$$\begin{aligned}
\left\langle \prod_{n=1}^N \lim_{\Delta_j \rightarrow 1} (\Delta_j - 1) \mathcal{O}_{\Delta_n}(z_n, \bar{z}_n, \eta^n) \right\rangle &= \left(\prod_{\substack{n=1 \\ n \neq j}}^N \int d\omega_n \omega_n^{\Delta_n-1} \right) \delta^{(4)} \left(\sum_{\substack{k=1 \\ k \neq j}}^N \omega_k q_k \right) \\
&\quad \times \lim_{\omega_j \rightarrow 0} \omega_j \mathcal{M}_N(1, \dots, n, \dots, N)
\end{aligned}$$

$$\begin{aligned}
\left\langle \prod_{n=1}^N \lim_{\Delta_j \rightarrow 1} (\Delta_j - 1) \mathcal{O}_{\Delta_n}(z_n, \bar{z}_n, \eta^n) \right\rangle &= \sum_{\substack{i=1 \\ i \neq j}}^N \omega_i \left\{ \frac{z_{j-1,i}^2 \bar{z}_{ji}}{z_{j-1,j}^2 z_{ji}} + \frac{\bar{z}_{j-1,i}^2 z_{ji}}{\bar{z}_{j-1,j}^2 \bar{z}_{ji}} \delta^4(\eta^j) \right\} \\
&\quad \times \left(\prod_{\substack{n=1 \\ n \neq j}}^N \int_0^\infty d\omega_k \omega_k^{\Delta_k-1} \right) \delta^{(4)} \left(\sum_{\substack{k=1 \\ k \neq j}}^N \omega_k q_k \right) \mathcal{M}_{N-1}(1, \dots, i, \dots, N) \\
&= \sum_{\substack{i=1 \\ i \neq j}}^n \left\{ \frac{z_{j-1,i}^2 \bar{z}_{ji}}{z_{j-1,j}^2 z_{ji}} + \frac{\bar{z}_{j-1,i}^2 z_{ji}}{\bar{z}_{j-1,j}^2 \bar{z}_{ji}} \delta^8(\eta^j) \right\} \left[\prod_{\substack{n=1 \\ n \neq i}}^N \int_0^\infty d\omega_k \omega_k^{\Delta_k-1} \int_0^\infty d\omega_i \omega_i^{\Delta_i} \right. \\
&= \sum_{\substack{i=1 \\ i \neq j}}^n \left\{ \frac{z_{j-1,i}^2 \bar{z}_{ji}}{z_{j-1,j}^2 z_{ji}} + \frac{\bar{z}_{j-1,i}^2 z_{ji}}{\bar{z}_{j-1,j}^2 \bar{z}_{ji}} \delta^8(\eta^j) \right\} \langle \mathcal{O}_{\Delta_1}(z_1, \bar{z}_1, \eta_1), \dots, \mathcal{O}_{\Delta_{i+1}}(z_i, \bar{z}_i, \eta^i), \dots \rangle
\end{aligned}$$

$$\begin{aligned}
&\langle J_1(z, \bar{z}, \eta) \mathcal{O}_{\Delta_1}(z_1, \bar{z}_1, \eta^1) \dots \mathcal{O}_{\Delta_N}(z_N, \bar{z}_N, \eta^N) \rangle \\
&= \sum_{i=1}^N \left\{ \frac{(\bar{z} - \bar{z}_i)(z_N - z_i)^2}{(z - z_i)(z_N - z)^2} + \frac{(z - z_i)(\bar{z}_N - \bar{z}_i)^2}{(\bar{z} - \bar{z}_i)(\bar{z}_N - \bar{z})^2} \delta^8(\eta) \right\} \\
&\quad \times \langle \mathcal{O}_{\Delta_1}(z_1, \bar{z}_1, \eta^1), \dots, \mathcal{O}_{\Delta_{i+1}}(z_i, \bar{z}_i, \eta^i), \dots, \mathcal{O}_{\Delta_N}(z_N, \bar{z}_N, \eta^N) \rangle
\end{aligned}$$

$$J_1(z, \bar{z}, \eta) = \lim_{\Delta \rightarrow 1} (\Delta - 1) \mathcal{O}_\Delta(z, \bar{z}, \eta)$$



$$\begin{aligned}
& \langle J_1(z, \bar{z}, \eta) \mathcal{O}_{\Delta_1}(z_1, \bar{z}_1, \eta^1) \cdots \mathcal{O}_{\Delta_N}(z_N, \bar{z}_N, \eta^N) \rangle \\
&= \sum_{i=1}^N \left\{ \frac{(\bar{z} - \bar{z}_i)(\xi - z_i)^2}{(z - z_i)(\bar{\xi} - \bar{z})^2} + \frac{(z - z_i)(\bar{\xi} - \bar{z}_i)^2}{(\bar{z} - \bar{z}_i)(\xi - \bar{z})^2} \delta^8(\eta) \right\} \\
&\quad \times \langle \mathcal{O}_{\Delta_1}(z_1, \bar{z}_1, \eta^1), \cdots \mathcal{O}_{\Delta_{i+1}}(z_i, \bar{z}_i, \eta^i), \cdots, \mathcal{O}_{\Delta_N}(z_N, \bar{z}_N, \eta^N) \rangle \\
&\left\langle J_1(z, \bar{z}) \prod_{n=1}^N \mathcal{O}_{\Delta_n, \ell_n}(z_n, \bar{z}_n) \right\rangle = \sum_{i=1}^N \frac{(\bar{z} - \bar{z}_i)(\xi - z_i)^2}{(z - z_i)(\bar{\xi} - \bar{z})^2} \langle \mathcal{O}_{\Delta_1, \ell_1}(z_1, \bar{z}_1) \\
&\quad \cdots \mathcal{O}_{\Delta_{i+1}, \ell_i}(z_i, \bar{z}_i), \cdots, \mathcal{O}_{\Delta_N, \ell_N}(z_N, \bar{z}_N) \rangle \\
&\left\langle \bar{J}_1(z, \bar{z}) \prod_{n=1}^N \mathcal{O}_{\Delta_n, \ell_n}(z_n, \bar{z}_n) \right\rangle = \sum_{i=1}^N \frac{(z - z_i)(\bar{\xi} - \bar{z}_i)^2}{(\bar{z} - \bar{z}_i)(\xi - \bar{z})^2} \langle \mathcal{O}_{\Delta_1, \ell_1}(z_1, \bar{z}_1) \cdots \mathcal{O}_{\Delta_{i+1}, \ell_i}(z_i, \bar{z}_i), \cdots, \mathcal{O}_{\Delta_N, \ell_N}(z_N, \bar{z}_N) \rangle
\end{aligned}$$

$$J_1(z, \bar{z}) = \lim_{\Delta \rightarrow 1} (\Delta - 1) \mathcal{O}_{\Delta+2}(z, \bar{z}), \bar{J}_1(z, \bar{z}) = \lim_{\Delta \rightarrow 1} (\Delta - 1) \mathcal{O}_{\Delta-2}(z, \bar{z})$$

$$\begin{aligned}
& \langle J_0(z, \bar{z}, \eta) \mathcal{O}_{\Delta_1}(z, \bar{z}, \eta^1) \cdots \mathcal{O}_{\Delta_N}(z_N, \bar{z}_N, \eta^N) \rangle \\
&= \sum_{i=1}^N \left\{ \frac{(\bar{z} - \bar{z}_i)(\xi - z_i)}{(z - z_i)(\bar{\xi} - \bar{z})} \left((\bar{z} - \bar{z}_i) \partial_{\bar{z}_i} - 2\bar{h}_i \right) + \frac{(z - z_i)(\bar{\xi} - \bar{z}_i)}{(\bar{z} - \bar{z}_i)(\xi - \bar{z})} \delta^8(\eta) \left((z - z_i) \partial_{z_i} - 2h_i \right) \right\} \\
&\quad \times \langle \mathcal{O}_{\Delta_1}(z_1, \bar{z}_1, \eta^1), \cdots \mathcal{O}_{\Delta_i}(z_i, \bar{z}_i, \eta^i) \cdots, \mathcal{O}_{\Delta_N}(z_N, \bar{z}_N, \eta^N) \rangle
\end{aligned}$$

$$J_0(z, \bar{z}, \eta) = \lim_{\Delta \rightarrow 0} \Delta [\mathcal{O}_{\Delta+2}(z, \bar{z}) + \delta^8(\eta) \mathcal{O}_{\Delta-2}(z, \bar{z})].$$

$$\begin{aligned}
& \left\langle J_0(z, \bar{z}) \prod_{n=1}^N \mathcal{O}_{\Delta_n, \ell_n}(z_n, \bar{z}_n) \right\rangle = \sum_{i=1}^N \frac{(\bar{z} - \bar{z}_i)(\xi - z_i)}{(z - z_i)(\bar{\xi} - \bar{z})} \left((\bar{z} - \bar{z}_i) \partial_{\bar{z}_i} - 2\bar{h}_i \right) \\
&\quad \times \langle \cdots \mathcal{O}_{\Delta_i, \ell_i}(z_i, \bar{z}_i) \cdots \rangle
\end{aligned}$$

$$\begin{aligned}
& \left\langle \bar{J}_0(z, \bar{z}) \prod_{n=1}^N \mathcal{O}_{\Delta_n, \ell_n}(z_n, \bar{z}_n) \right\rangle = \sum_{i=1}^N \frac{(z - z_i)(\bar{\xi} - \bar{z}_i)}{(\bar{z} - \bar{z}_i)(\xi - \bar{z})} \left((z - z_i) \partial_{z_i} - 2h_i \right) \\
&\quad \times \langle \cdots \mathcal{O}_{\Delta_i, \ell_i}(z_i, \bar{z}_i) \cdots \rangle
\end{aligned}$$

$$J_0(z, \bar{z}) = \lim_{\Delta \rightarrow 0} \Delta \mathcal{O}_{\Delta+2}(z, \bar{z}), \bar{J}_0(z, \bar{z}) = \lim_{\Delta \rightarrow 0} \Delta \mathcal{O}_{\Delta-2}(z, \bar{z})$$

$$\mathcal{M}_{N+1}(\psi_{s+}^A, \{p_1, \eta^1\}, \dots \{p_N, \eta^N\}) = \sum_{i=1}^N \frac{[si]\langle ri \rangle}{\langle si \rangle \langle rs \rangle} \frac{\partial}{\partial \eta_{iA}} \mathcal{M}_N(\{p_1, \eta^1\}, \dots \{p_N, \eta^N\})$$

$$\ell_i \in \{-3/2, -1, -1/2, 0, +1/2, +1, +3/2, +2\}$$

$$\ell_i^c \in \{-2, -3/2, -1, -1/2, 0, +1/2, +1, +3/2\}$$



$$\left\langle J_{1/2}^A(z, \bar{z}) \prod_{n=1}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle = \sum_{i=1}^N f(A, \ell_i, *_{i'}, *'_i) (-1)^{\sigma_i} \frac{(\bar{z} - \bar{z}_i)(\xi - z_i)}{(z - z_i)(\bar{\xi} - \bar{z})} \left\langle \dots \mathcal{O}_{\Delta_i + \frac{1}{2}, \ell_i^c}^{*'_i}(z_i, \bar{z}_i), \dots \right\rangle$$

$$\left\langle \bar{J}_{1/2A}(z, \bar{z}) \prod_{n=1}^N \mathcal{O}_{\Delta_n, \ell_n^c}^{*n}(z_n, \bar{z}_n) \right\rangle = \sum_{i=1}^N \bar{f}(A, \ell_i^c, *_{i'}, *'_i) (-1)^{\sigma_i} \frac{(z - z_i)(\bar{\xi} - \bar{z}_i)}{(\bar{z} - \bar{z}_i)(\xi - z)} \left\langle \dots \mathcal{O}_{\Delta_i + \frac{1}{2}, \ell_i}^{*'_i}(z_i, \bar{z}_i), \dots \right\rangle$$

$$J_{1/2}^A(z, \bar{z}) = \lim_{\Delta \rightarrow \frac{1}{2}} \left(\Delta - \frac{1}{2} \right) \mathcal{O}_{\Delta, +\frac{3}{2}}^A(z, \bar{z}), \bar{J}_{1/2A}(z, \bar{z}) = \lim_{\Delta \rightarrow \frac{1}{2}} \left(\Delta - \frac{1}{2} \right) \mathcal{O}_{\Delta, -\frac{3}{2}, A}(z, \bar{z})$$

$$T_0(z, \bar{z}) = \lim_{\Delta \rightarrow 0} \frac{3! \Delta}{2\pi} \int d^2 z' \frac{1}{(z - z')^4} \mathcal{O}_{\Delta, -2}(z', \bar{z}')$$

$$\bar{T}_0(z, \bar{z}) = \lim_{\Delta \rightarrow 0} \frac{3! \Delta}{2\pi} \int d^2 z' \frac{1}{(\bar{z} - \bar{z}')^4} \mathcal{O}_{\Delta, +2}(z', \bar{z}')$$

$$\bar{\partial} T_0 = -\frac{1}{2} \partial^3 \bar{J}_0(z, \bar{z})$$

$$T_{\text{mod}} := T_0 + \frac{1}{2} \partial^3 \epsilon_{\bar{J}_0}$$

$$\epsilon_{\bar{J}_0} := \int_{\bar{z}_0}^{\bar{z}} d\bar{w} \bar{J}_0(z, \bar{w})$$

$$T(z) \mathcal{O}_{\Delta, \ell}(w, \bar{w}) = \frac{h}{(z - w)^2} \mathcal{O}_{\Delta, \ell}(w, \bar{w}) + \frac{1}{z - w} \partial_w \mathcal{O}_{\Delta, \ell}(w, \bar{w})$$

$$\bar{T}(\bar{z}) \mathcal{O}_{\Delta, \ell}(w, \bar{w}) = \frac{\bar{h}}{(\bar{z} - \bar{w})^2} \mathcal{O}_{\Delta, \ell}(w, \bar{w}) + \frac{1}{\bar{z} - \bar{w}} \partial_{\bar{w}} \mathcal{O}_{\Delta, \ell}(w, \bar{w})$$

$$P(z) = \lim_{\Delta \rightarrow 1} \frac{(\Delta - 1)}{4} \partial_{\bar{z}} \mathcal{O}_{\Delta, +2}(z, \bar{z})$$

$$\bar{P}(\bar{z}) = \lim_{\Delta \rightarrow 1} \frac{(\Delta - 1)}{4} \partial_z \mathcal{O}_{\Delta, -2}(z, \bar{z})$$

$$P(z) \mathcal{O}_{\Delta, \ell}(w, \bar{w}) = \frac{1}{z - w} \mathcal{O}_{\Delta+1, \ell}(w, \bar{w})$$

$$P(z) = \frac{1}{2\pi i} \oint d\bar{z} \mathcal{P}(z, \bar{z}), \bar{P}(\bar{z}) = \frac{1}{2\pi i} \oint dz \mathcal{P}(z, \bar{z})$$

$$\mathcal{P}(z, \bar{z}) \mathcal{O}_{\Delta, \ell}(w, \bar{w}) = \frac{1}{z - w} \frac{1}{\bar{z} - \bar{w}} \mathcal{O}_{\Delta+1, \ell}(w, \bar{w})$$



$$S_A(z) = \lim_{\Delta \rightarrow \frac{1}{2}} \frac{\Delta - \frac{1}{2}}{\pi} \int d^2 z' \frac{1}{(z - z')^3} \mathcal{O}_{A, \Delta, -\frac{3}{2}}(z', \bar{z}')$$

$$\bar{S}^A(\bar{z}) = \lim_{\Delta \rightarrow \frac{1}{2}} \frac{\Delta - \frac{1}{2}}{\pi} \int d^2 z' \frac{1}{(\bar{z} - \bar{z}')^3} \mathcal{O}_{\Delta, +\frac{3}{2}}^A(z', \bar{z}')$$

$$\bar{\partial} S_A(z, \bar{z}) = \lim_{\Delta \rightarrow 1/2} \left(\Delta - \frac{1}{2} \right) \partial_z^2 \mathcal{O}_{A, \Delta, -\frac{3}{2}}(z, \bar{z}) = \partial^2 \bar{J}_{1/2A}(z, \bar{z}) \neq 0$$

$$\epsilon_{\bar{J}_{1/2}A}(z, \bar{z}) := \int_{\bar{z}_0}^z d\bar{w} \bar{J}_{1/2A}(w, \bar{w})$$

$$S_{\text{mod}}^A = S^A - \partial^2 \epsilon_{f_{1/2}}^A$$

$$T(z)S_A(w) = \frac{3}{2} \frac{S_A(w)}{(z-w)^2} + \frac{\partial S_A(w)}{z-w}$$

$$\bar{T}(\bar{z})\bar{S}^A(\bar{w}) = \frac{3}{2} \frac{\bar{S}^A(\bar{z})}{(\bar{z}-\bar{w})^2} + \frac{\partial \bar{S}^A(\bar{w})}{\bar{z}-\bar{w}}$$

$$:\{S_B(z), \bar{S}^A(\bar{z})\}: = S_B(z) \bar{S}^A(\bar{z}) + \bar{S}^A(\bar{z}) S_B(z) =: \delta_B^A \mathcal{P}(z, \bar{z})$$

$$\begin{aligned} & \left\langle S_B(z) \bar{S}^A(\bar{w}) \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \\ &= \delta_B^A \sum_{i=3}^N \left[\frac{1}{(\bar{w} - \bar{z})^2} \frac{\bar{z} - \bar{z}_i}{z - z_i} + \frac{1}{\bar{z} - \bar{w}} \frac{1}{z - z_i} + \frac{1}{\bar{w} - \bar{z}_i} \frac{1}{z - z_i} \right] \left\langle \cdots \mathcal{O}_{\Delta_i+1, \ell_i}^{*i}(z_i, \bar{z}_i), \cdots \right\rangle \\ & - \sum_{i=3}^N f(A, \ell_i, *i, *'_i) \bar{f}(B, \ell_i - 1/2, *'_i, *''_i) \frac{1}{z - z_i} \frac{1}{\bar{w} - \bar{z}_i} \left\langle \cdots \mathcal{O}_{\Delta_i+1, \ell_i}^{*'_i}(z_i, \bar{z}_i), \cdots \right\rangle \\ & + \sum_{\substack{i,j=3 \\ i \neq j}}^N (-1)^{\sigma_i + \sigma_j} f(A, \ell_i, *i, *'_i) \bar{f}(B, \ell_j, *j, *'_j) \frac{1}{z - z_i} \frac{1}{\bar{w} - \bar{z}_j} \left\langle \cdots \mathcal{O}_{\Delta_i+\frac{1}{2}, \ell_i-\frac{1}{2}}^{*'_i}(z_i, \bar{z}_i), \cdots \right. \\ & \left. \mathcal{O}_{\Delta_j+\frac{1}{2}, \ell_j+\frac{1}{2}}^{*'_j}(z_j, \bar{z}_j) \cdots \right\rangle \end{aligned}$$

$$\begin{aligned}
& \left\langle \bar{S}^A(\bar{z}) S_B(w) \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \\
&= \delta_B^A \sum_{i=3}^N \left[\frac{1}{(w-z)^2} \frac{z-z_i}{\bar{z}-\bar{z}_i} + \frac{1}{z-w} \frac{1}{\bar{z}-\bar{z}_i} + \frac{1}{\bar{z}-\bar{z}_i} \frac{1}{w-z_i} \right] \left\langle \cdots \mathcal{O}_{\Delta_i+1, \ell_i}^{*i}(z_i, \bar{z}_i), \cdots \right\rangle \\
&\quad - \sum_{i=3}^N \bar{f}(B, \ell_i, *_i, *_i') f(A, \ell_i + 1/2, *_i', *_i'') \frac{1}{w-z_i} \frac{1}{\bar{z}-\bar{z}_i} \left\langle \cdots \mathcal{O}_{\Delta_i+1, \ell_i}^{*i'}(z_i, \bar{z}_i), \cdots \right\rangle \\
&\quad - \sum_{\substack{i,j=3 \\ i \neq j}}^N (-1)^{\sigma_i + \sigma_j} \bar{f}(B, \ell_i, *_i, *_i') f(A, \ell_j, *_j, *_j') \frac{1}{w-z_i} \frac{1}{\bar{z}-\bar{z}_j} \left\langle \cdots \mathcal{O}_{\Delta_i+\frac{1}{2}, \ell_i+\frac{1}{2}}^{*i'}(z_i, \bar{z}_i), \cdots \right. \\
&\quad \left. \mathcal{O}_{\Delta_j+\frac{1}{2}, \ell_j-\frac{1}{2}}^{*j'}(z_j, \bar{z}_j), \cdots \right\rangle
\end{aligned}$$

$$\begin{aligned}
& \left\langle (\bar{S}^A(\bar{z}) S_B(w) + S_B(z) \bar{S}^A(\bar{w})) \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \\
&= \delta_B^A \sum_{i=3}^N \left[\frac{1}{(w-z)^2} \frac{z-z_i}{\bar{z}-\bar{z}_i} + \frac{1}{z-w} \frac{1}{\bar{z}-\bar{z}_i} + \frac{1}{\bar{z}-\bar{z}_i} \frac{1}{w-z_i} \right] \left\langle \cdots \mathcal{O}_{\Delta_i+1, \ell_i}^{*i}(z_i, \bar{z}_i), \cdots \right\rangle \\
&\quad + \delta_B^A \sum_{i=3}^N \left[\frac{1}{(\bar{w}-\bar{z})^2} \frac{\bar{z}-\bar{z}_i}{z-z_i} + \frac{1}{\bar{z}-\bar{w}} \frac{1}{z-z_i} + \frac{1}{\bar{w}-\bar{z}_i} \frac{1}{z-z_i} \right] \left\langle \cdots \mathcal{O}_{\Delta_i+1, \ell_i}^{*i}(z_i, \bar{z}_i), \cdots \right\rangle \\
&\quad - \sum_{i=3}^N f(A, \ell_i, *_i, *_i') \bar{f}(B, \ell_i - 1/2, *_i', *_i'') \frac{1}{z-z_i} \frac{1}{\bar{w}-\bar{z}_i} \left\langle \cdots \mathcal{O}_{\Delta_i+1, \ell_i}^{*i''}(z_i, \bar{z}_i), \cdots \right\rangle \\
&\quad - \sum_{i=3}^N \bar{f}(B, \ell_i, *_i, *_i') f(A, \ell_i + 1/2, *_i', *_i'') \frac{1}{w-z_i} \frac{1}{\bar{z}-\bar{z}_i} \left\langle \cdots \mathcal{O}_{\Delta_i+1, \ell_i}^{*i''}(z_i, \bar{z}_i), \cdots \right\rangle
\end{aligned}$$

$$\begin{aligned}
& \left\langle \{S_B(z), \bar{S}^A(\bar{z})\} : \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \\
&= 2\delta_B^A \sum_{i=3}^N \frac{1}{\bar{z}-\bar{z}_i} \frac{1}{z-z_i} \left\langle \cdots \mathcal{O}_{\Delta_i+1, \ell_i}^{*i}(z_i, \bar{z}_i), \cdots \right\rangle \\
&\quad - \sum_{i=3}^N f(A, \ell_i, *_i, *_i') \bar{f}(B, \ell_i - 1/2, *_i', *_i'') \frac{1}{z-z_i} \frac{1}{\bar{z}-\bar{z}_i} \left\langle \cdots \mathcal{O}_{\Delta_i+1, \ell_i}^{*i''}(z_i, \bar{z}_i), \cdots \right\rangle \\
&\quad - \sum_{i=3}^N \bar{f}(B, \ell_i, *_i, *_i') f(A, \ell_i + 1/2, *_i', *_i'') \frac{1}{z-z_i} \frac{1}{\bar{z}-\bar{z}_i} \left\langle \cdots \mathcal{O}_{\Delta_i+1, \ell_i}^{*i''}(z_i, \bar{z}_i), \cdots \right\rangle.
\end{aligned}$$

$$f(A, +2, -, *_i') \bar{f}(B, +3/2, *_i', -) \mathcal{O}_{\Delta_i+1, +2}(z_i, \bar{z}_i) = \delta_B^A \mathcal{O}_{\Delta_i+1, +2}(z_i, \bar{z}_i).$$

$$f(A, +3/2, C, *_i') \bar{f}(B, +1, *_i', *_i'') \mathcal{O}_{\Delta_i+1, +3/2}^{*i''}(z_i, \bar{z}_i) = 2! \delta_B^{[A} \mathcal{O}_{\Delta_i+1, +3/2}^{C]}(z_i, \bar{z}_i)$$

$$\bar{f}(B, +3/2, C, -) f(A, +2, -, *_i'') \mathcal{O}_{\Delta_i+1, +3/2}^{*i''}(z_i, \bar{z}_i) = \delta_B^C \mathcal{O}_{\Delta_i+1, +3/2}^A(z_i, \bar{z}_i)$$



$$\begin{aligned}
f(A, +1, CD, *_i') \bar{f}(B, +1/2, *_i', *_i'') \mathcal{O}_{\Delta_i+1, +2}^{*i''}(z_i, \bar{z}_i) \\
= \bar{f}(B, +1/2, *_i', ACD) \mathcal{O}_{\Delta_i+\frac{1}{2}, +1/2}^{ACD}(z_i, \bar{z}_i) \\
= 3\delta_B^{[A} \mathcal{O}_{\Delta_i+1, +1}^{CD]}(z_i, \bar{z}_i)
\end{aligned}$$

$$\bar{f}(B, +1, CD, *_i') f(A, +3/2, *_i', *_i'') \mathcal{O}_{\Delta_i+1, +3/2}^{*i''}(z_i, \bar{z}_i) = -2! \delta_B^{[C} \mathcal{O}_{\Delta_i+1, +1}^{D]A}(z_i, \bar{z}_i)$$

$$\begin{aligned}
3\delta_B^{[A} \mathcal{O}^{CD]} - 2! \delta_B^{[C} \mathcal{O}^{D]A} &= \frac{1}{2} [(\delta_B^A \mathcal{O}^{CD} - \delta_B^A \mathcal{O}^{DC}) + (\delta_B^C \mathcal{O}^{DA} - \delta_B^C \mathcal{O}^{AD}) + (\delta_B^D \mathcal{O}^{AC} - \delta_B^D \mathcal{O}^{CA})] \\
&\quad - \delta_B^C \mathcal{O}^{DA} + \delta_B^D \mathcal{O}^{CA} \\
&= \delta_B^A \mathcal{O}^{CD}
\end{aligned}$$

$$\left\langle : \{S_B(z), \bar{S}^A(\bar{z})\} : \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle = \delta_B^A \sum_{i=3}^N \frac{1}{\bar{z} - \bar{z}_i} \frac{1}{z - z_i} \left\langle \cdots \mathcal{O}_{\Delta_i+1, \ell_i}^{*i}(z_i, \bar{z}_i), \cdots \right\rangle$$

$$: \{S_B(z), \bar{S}^A(\bar{z})\} : \mathcal{O}_{\Delta, \ell}(w, \bar{w}) = \frac{1}{z - w} \frac{1}{\bar{z} - \bar{w}} \mathcal{O}_{\Delta+1, \ell}(w, \bar{w})$$

$$: \{S_B(z), \bar{S}^A(\bar{z})\} : = \delta_B^A \mathcal{P}(z, \bar{z}).$$

$$[T_B^A, T_D^C] = \delta_D^A T_B^C - \delta_B^C T_D^A,$$

$$(T_B^A)^C{}_D = \delta_D^A \delta_B^C - \frac{1}{8} \delta_B^A \delta_D^C$$

$$[T_B^A, Q_\alpha^C] = (T_B^A)_D^C Q_\alpha^D, [T_B^A, \bar{Q}_{\dot{\alpha}C}] = -(T_B^A)_C^D \bar{Q}_{\dot{\alpha}D}.$$

$$\begin{aligned}
\tilde{\mathcal{G}}_B^A(z, \bar{z}) S_C(w) &\sim ((z - w) \text{ singularity}) (T_B^A)_C^D S_D(w) \\
\tilde{\mathcal{G}}_B^A(z, \bar{z}) \bar{S}^C(w) &\sim -((\bar{z} - \bar{w}) \text{ singularity}) (T_B^A)_D^C \bar{S}^D(w).
\end{aligned}$$

$$\begin{aligned}
G_{AB}(z, \bar{z}) &= \lim_{\Delta \rightarrow 0} \frac{\Delta}{\pi} \int d^2 z' \frac{1}{(z - z')^a} \frac{1}{(\bar{z} - \bar{z}')^b} \mathcal{O}_{AB; \Delta, -1}(z', \bar{z}') \\
\bar{G}^{CD}(z, \bar{z}) &= \lim_{\Delta \rightarrow 0} \frac{\Delta}{\pi} \int d^2 z' \frac{1}{(\bar{z} - \bar{z}')^{a'}} \frac{1}{(z - z')^{b'}} \mathcal{O}_{\Delta, +1}^{CD}(z', \bar{z}')
\end{aligned}$$

$$\begin{aligned}
&\left\langle G_{AB}(z, \bar{z}) \prod_{n=2}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \\
&= \lim_{\Delta_1 \rightarrow 0} \frac{\Delta_1}{\pi} \int d^2 z_1 \frac{1}{(z - z_1)^a} \frac{1}{(\bar{z} - \bar{z}_1)^b} \left\langle \mathcal{O}_{AB\Delta_1, -1}(z_1, \bar{z}_1) \prod_{n=2}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \\
&= \frac{1}{\pi} \int d^2 z_1 \frac{1}{(z - z_1)^a} \frac{1}{(\bar{z} - \bar{z}_1)^b} \left[\sum_{n=2}^N f(A, B, \ell_i, *_n, *_n') \frac{z_1 - z_n}{\bar{z}_1 - \bar{z}_n} \left\langle \cdots \mathcal{O}_{\Delta_n, \ell_n+1}^{*n'}(z_n, \bar{z}_n) \right\rangle \right]
\end{aligned}$$



$$\int d^2 z_1 \frac{1}{(z - z_1)^A} \frac{1}{(\bar{z} - \bar{z}_1)^B} \frac{(\bar{z}_1 - \bar{z}_j)^s}{z_1 - z_j} = C_s(A, B) \frac{1}{(z_j - z)^A (\bar{z}_j - \bar{z})^{B-s-1}}$$

$$\int d^2 z_1 \frac{1}{(\bar{z} - \bar{z}_1)^A} \frac{1}{(z_1 - z_1)^B} \frac{(z_1 - z_j)^s}{\bar{z}_1 - \bar{z}_j} = C_s(A, B) \frac{1}{(\bar{z}_j - \bar{z})^A} \frac{1}{(z_j - z)^{B-s-1}}$$

$$C_s(A, B) = \frac{(-1)^{s+A+B} (-\pi) s!}{(-B+1)(-B+2) \cdots (-B+s+1)}$$

$$\int d^2 z_1 \frac{1}{(z - z_1)^a} \frac{1}{(\bar{z} - \bar{z}_1)^b} \frac{z_1 - z_n}{\bar{z}_1 - \bar{z}_n} = C_1(b, a) \frac{1}{(\bar{z}_n - \bar{z})^b} \frac{1}{(z_n - z)^{a-2}}$$

$$\left\langle G_{AB}(z, \bar{z}) \prod_{n=2}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle$$

$$= \sum_{i=2}^N f(A, B, \ell_i, *, *_i) C_1(b, a) \frac{1}{(\bar{z}_i - \bar{z})^b} \frac{1}{(z_i - z)^{a-2}} \left\langle \cdots \mathcal{O}_{\Delta_i, \ell_i+1}^{*i}(z_i, \bar{z}_i) \right\rangle$$

$$\ell_n \in \left\{ -2, -\frac{3}{2}, -1, -\frac{1}{2}, 0, +\frac{1}{2}, +1 \right\}$$

$$\ell'_n \in \left\{ -1, -\frac{1}{2}, 0, +\frac{1}{2}, +1, +\frac{3}{2}, +2 \right\}$$

$$\left\langle \bar{G}^{CD}(z, \bar{z}) \prod_{n=2}^N \mathcal{O}_{\Delta_n, \ell'_n}^{*n}(z_n, \bar{z}_n) \right\rangle$$

$$= \sum_{n=2}^N \bar{f}(C, D, \ell'_i, *, *_n) C(b', a') \frac{1}{(\bar{z}_n - \bar{z})^{b'}} \frac{1}{(z_n - z)^{a'-2}} \left\langle \cdots \mathcal{O}_{\Delta_n, \ell'_n-1}^{*n}(z_n, \bar{z}_n) \right\rangle$$

$$G_{AB}(z) \mathcal{O}_{\Delta, \ell}^*(w, \bar{w}) \sim f(A, B, \ell, *, *) C_1(b, a) \frac{1}{(\bar{w} - \bar{z})^b} \frac{1}{(w - z)^{a-2}} \mathcal{O}_{\Delta, \ell'}^{*'}(z, \bar{w})$$

$$\bar{G}^{CD}(z) \mathcal{O}_{\Delta, \ell'}^*(w, \bar{w}) \sim \bar{f}(C, D, \ell', *, *) C_1(b', a') \frac{1}{(\bar{w} - \bar{z})^{b'}} \frac{1}{(z - z)^{a'-2}} \mathcal{O}_{\Delta, \ell}^{*'}(w, \bar{w})$$

$$\mathcal{G}_{AB}^{CD}(z, \bar{z}; w, \bar{w}) := G_{AB}(z) \bar{G}^{CD}(\bar{w}) - \bar{G}^{CD}(\bar{w}) G_{AB}(z) \equiv [G_{AB}(z), \bar{G}^{CD}(\bar{w})].$$

$$\mathcal{G}_{AB}^{CD}(z, \bar{z}) =: \mathcal{G}_{AB}^{CD}(z, \bar{z}; z, \bar{z}) =: G_{AB}(z) \bar{G}^{CD}(\bar{z}) - \bar{G}^{CD}(\bar{z}) G^{AB}(z):$$

$$\equiv: [G_{AB}(z), \bar{G}^{CD}(\bar{z})]:$$



$$\begin{aligned}
& \left\langle \mathcal{G}_{AB}^{CD}(z, \bar{z}) \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \\
&= (-1)^{a+b+a'+b'} C_1(b, a) C_1(b', a') [f(A, B, \ell_j, *, *, *) \bar{f}(C, D, \ell_j + 1, *, *, *)] \\
&\times \frac{1}{(z - z_j)^{a+b'-2}} \frac{1}{(\bar{z} - \bar{z}_j)^{a'+b-2}} \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_j, \ell_j}^{*j''}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \\
&- \bar{f}(C, D, \ell_j, *, *, *) f(A, B, \ell_j - 1, *, *, *) \\
&\times \frac{1}{(z - z_j)^{a'+b-2}} \frac{1}{(\bar{z} - \bar{z}_j)^{a+b'-2}} \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_j, \ell_j}^{*j''}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \Big]
\end{aligned}$$

$$\mathcal{G}_{AB}^{CD}(z, \bar{z}) \mathcal{O}_{E\Delta, -\frac{3}{2}}(w, \bar{w}) \sim -\delta_{AB}^{CD} \frac{(-1)^{a+b+a'+b'} C_1(b, a) C_1(b', a')}{(z-w)^{a+b'-2} (\bar{z}-\bar{w})^{a'+b-2}} \mathcal{O}_{E\Delta, -\frac{3}{2}}(w, \bar{w})$$

$$\mathcal{G}_{AB}^{CD}(z, \bar{z}) \mathcal{O}_{\Delta, +\frac{3}{2}}^E(w, \bar{w}) \sim \delta_{AB}^{CD} \frac{(-1)^{a+b+a'+b'} C_1(b, a) C_1(b', a')}{(z-w)^{a'+b-2} (\bar{z}-\bar{w})^{a+b'-2}} \mathcal{O}_{\Delta, +\frac{3}{2}}^E(w, \bar{w})$$

$$\lim_{\Delta_1 \rightarrow 0} \Delta_1 \mathcal{O}_{AB; \Delta_1, -1}(z, \bar{z}) \mathcal{O}_{E\Delta, -\frac{3}{2}}(z_1, \bar{z}_1) = \frac{z - z_1}{\bar{z} - \bar{z}_1} \mathcal{O}_{ABE; \Delta, -\frac{1}{2}}(z_1, \bar{z}_1)$$

$$\lim_{\Delta_2 \rightarrow 0} \Delta_2 \mathcal{O}_{\Delta_2, +1}^{cd}(w, \bar{w}) \mathcal{O}_{abr; \Delta, -\frac{1}{2}}(z_1, \bar{z}_1) = -\delta_{ab}^{cd} \frac{\bar{w} - \bar{z}_1}{w - z_1} \mathcal{O}_{r\Delta, -\frac{3}{2}}(z_1, \bar{z}_1)$$

$$(\tilde{\mathcal{G}}_B^A)_D^C(z, \bar{z}) := - \left(\frac{1}{7} \delta_D^A \sum_{E=1}^8 \mathcal{G}_{BE}^{EC}(z, \bar{z}) + \frac{1}{56} \delta_B^A \sum_{E=1}^8 \mathcal{G}_{ED}^{EC}(z, \bar{z}) \right).$$

$$\delta_{b_1 \dots b_n}^{a_1 \dots a_n} = \sum_{\sigma \in S_n} \text{sign}(\sigma) \delta_{b_{\sigma(1)}}^{a_1} \dots \delta_{b_{\sigma(n)}}^{a_n}$$

$$\sum_{E=1}^8 \delta_{BE}^{EC} = -7\delta_B^C, \sum_{E=1}^8 \delta_{ED}^{EC} = 7\delta_D^C.$$

$$(\tilde{\mathcal{G}}_A^C)_B^D(z, \bar{z}) \mathcal{O}_{D\Delta, -\frac{3}{2}}(w, \bar{w}) \sim \frac{(-1)^{a+b+a'+b'} C_1(b, a) C_1(b', a')}{(z-w)^{a+b'-2} (\bar{z}-\bar{w})^{a'+b-2}} (T_A^C)_B^D \mathcal{O}_{D\Delta, -\frac{3}{2}}(w, \bar{w})$$

$$(\tilde{\mathcal{G}}_A^C)_B^D(z, \bar{z}) \mathcal{O}_{\Delta, +\frac{3}{2}}^B(w, \bar{w}) \sim -\frac{(-1)^{a+b+a'+b'} C_1(b, a) C_1(b', a')}{(z-w)^{a'+b-2} (\bar{z}-\bar{w})^{a+b'-2}} (T_A^C)_B^D \mathcal{O}_{\Delta, +\frac{3}{2}}^B(w, \bar{w})$$

$$\begin{aligned}
(\tilde{\mathcal{G}}_A^C)_B^D(z, \bar{z}) S_D(w) &\sim (T_A^C)_B^D \lim_{\Delta \rightarrow \frac{1}{2}} \frac{\Delta - \frac{1}{2}}{\pi} (-1)^{a+b+a'+b'} C_1(b, a) C_1(b', a') \\
&\times \int d^2 z_1 \frac{1}{(w - z_1)^3} \frac{1}{(z - z_1)^{a+b'-2}} \frac{1}{(\bar{z} - \bar{z}_1)^{a'+b-2}} \mathcal{O}_{D\Delta, -\frac{3}{2}}(z_1, \bar{z}_1)
\end{aligned}$$

$$\begin{aligned}
(\tilde{\mathcal{G}}_A^C)_B^D(z, \bar{z}) \bar{S}^B(\bar{w}) &\sim -(T_A^C)_B^D \lim_{\Delta \rightarrow \frac{1}{2}} \frac{\Delta - \frac{1}{2}}{\pi} (-1)^{a+b+a'+b'} C_1(b, a) C_1(b', a') \\
&\times \int d^2 z_1 \frac{1}{(\bar{w} - \bar{z}_1)^3} \frac{1}{(z - z_1)^{a'+b-2}} \frac{1}{(\bar{z} - \bar{z}_1)^{a+b'-2}} \mathcal{O}_{\Delta, +\frac{3}{2}}^B(z_1, \bar{z}_1)
\end{aligned}$$



$$a + b' - 2 = 0 \text{ and } a' + b - 2 = 0$$

$$(\tilde{\mathcal{G}}_A^C)_B^D(z, \bar{z}) S_D(w) \sim \varsigma, (\tilde{\mathcal{G}}_A^C)_B^D(z, \bar{z}) \bar{S}^B(\bar{w}) \sim \mathfrak{q}$$

$$T(z)=\sum_{n\in\mathbb{Z}}L_nz^{-n-2},\bar{T}(\bar{z})=\sum_{n\in\mathbb{Z}}\bar{L}_n\bar{z}^{-n-2};$$

$$\mathcal{P}(z,\bar{z})\equiv\sum_{n,m\in\mathbb{Z}}P_{n-\frac{1}{2},m-\frac{1}{2}}z^{-n-1}\bar{z}^{-m-1}$$

$$P(z)=\sum_{n\in\mathbb{Z}}P_{n-\frac{1}{2}}z^{-n-1},\bar{P}(\bar{z})=\sum_{m\in\mathbb{Z}}\bar{P}_{m-\frac{1}{2}}\bar{z}^{-m-1}$$

$$P_{n-\frac{1}{2},-\frac{1}{2}}=P_{n-\frac{1}{2}},P_{-\frac{1}{2},m-\frac{1}{2}}=\bar{P}_{m-\frac{1}{2}}$$

$$\begin{aligned} P_{n-\frac{1}{2},m-\frac{1}{2}} &= \frac{1}{i\pi(m+1)} \oint d\bar{w} \bar{w}^{m+1} \left[\bar{T}(\bar{w}), P_{n-\frac{1}{2},-\frac{1}{2}} \right] \\ &= \frac{1}{i\pi(m+1)} \oint dw w^{n+1} \left[T(w), \bar{P}_{-\frac{1}{2},m-\frac{1}{2}} \right]. \end{aligned}$$

$$[L_m,L_n]=(m-n)L_{m+n},[\bar{L}_m,\bar{L}_n]=(m-n)\bar{L}_{m+n}\\[L_n,P_{kl}]=\left(\frac{1}{2}n-k\right)P_{n+k,l},[\bar{L}_n,P_{kl}]=\left(\frac{1}{2}n-l\right)P_{k,n+l}$$

$$S_A(z)=\sum_{k\in\mathbb{Z}+\frac{1}{2}}\frac{(S_A)_k}{z^{k+\frac{3}{2}}},\text{ with } (S_A)_k=\frac{1}{2\pi i}\oint dz z^{k+\frac{1}{2}}S_A(z)$$

$$\bar{S}^A(\bar{z})=\sum_{l\in\mathbb{Z}+\frac{1}{2}}\frac{\bar{S}_l^A}{\bar{z}^{l+\frac{3}{2}}},\text{ with } \bar{S}_l^A=\frac{1}{2\pi i}\oint d\bar{z} \bar{z}^{l+\frac{1}{2}}\bar{S}^A(\bar{z})$$

$$[L_n,(S_A)_m]=\left(\frac{n}{2}-m\right)(S_A)_{m+n},[\bar{L}_n,(S_A)_m]=0$$

$$[L_n,\bar{S}_m^A]=0,[\bar{L}_n,\bar{S}_m^A]=\left(\frac{n}{2}-m\right)\bar{S}_{m+n}^A$$

$$\{(S_B)_m,\bar{S}_n^A\}=\delta_B^AP_{mn},m,n\in\mathbb{Z}+\frac{1}{2}$$

$$(\tilde{\mathcal{G}}_B^A)_D^C(z,\bar{z})=\sum_{n,m\in\mathbb{Z}}\left\{(\tilde{\mathcal{G}}_B^A)_D^C\right\}_{mn}z^{-m-h}\bar{z}^{-n-\bar{h}}$$

$$\left\{(\tilde{\mathcal{G}}_B^A)_D^C\right\}_{mn}=\frac{1}{(2\pi i)^2}\oint dz\oint d\bar{z}z^{m+h-1}\bar{z}^{n+\bar{h}+1}(\tilde{\mathcal{G}}_B^A)_D^C(z,\bar{z})$$

$$(\tilde{\mathcal{G}}_B^A)_D^C(z,\bar{z})S_C(w)\sim\frac{1}{z-w}\frac{1}{\bar{z}-\bar{w}}(T_B^A)_D^CS_C(w)$$

$$\left[(\tilde{\mathcal{G}}_B^A)_D^C\right]_{mn},(S_C)_k\Big]=\bar{w}^n(T_B^A)_D^C\{S_C\}_{m+k}$$



$$\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n}, [\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n} \\
[L_n, P_{kl}] &= \left(\frac{1}{2}n - k\right)P_{n+k,l}, [\bar{L}_n, P_{kl}] = \left(\frac{1}{2}n - l\right)P_{k,n+l} \\
[L_n, (S_A)_m] &= \left(\frac{n}{2} - m\right)(S_A)_{m+n}, [\bar{L}_n, (S_A)_m] = 0 \\
[L_n, \bar{S}_m^A] &= 0, [\bar{L}_n, \bar{S}_m^A] = \left(\frac{n}{2} - m\right)\bar{S}_{m+n}^A \\
\{(S_B)_m, \bar{S}_n^A\} &= \delta_B^A P_{mn}
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_{\Delta_1, +\frac{3}{2}}^A(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2, +\frac{3}{2}}^B(z_2, \bar{z}_2) &\sim \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 - \frac{1}{2}, \Delta_2 - \frac{1}{2}\right) \mathcal{O}_{\Delta_1+\Delta_2, +1}^{AB}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1, +\frac{3}{2}}^A(z_1, \bar{z}_1) \mathcal{O}_{B\Delta_2, -\frac{3}{2}}(z_2, \bar{z}_2) &\sim \frac{\bar{z}_{12}}{z_{12}} \delta_B^A B\left(\Delta_1 - \frac{1}{2}, \Delta_2 + \frac{5}{2}\right) \mathcal{O}_{\Delta_1+\Delta_2, -2}(z_2, \bar{z}_2) \\
&\quad + \frac{z_{12}}{\bar{z}_{12}} \delta_B^A B\left(\Delta_1 + \frac{5}{2}, \Delta_2 - \frac{1}{2}\right) \mathcal{O}_{\Delta_1+\Delta_2, +2}(z_2, \bar{z}_2)
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_{\Delta_1, +1}^{AB}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2, +1}^{CD}(z_2, \bar{z}_2) &\sim \frac{\bar{z}_{12}}{z_{12}} B(\Delta_1, \Delta_2) \mathcal{O}_{\Delta_1+\Delta_2, 0}^{ABCD}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1, +1}^{AB}(z_1, \bar{z}_1) \mathcal{O}_{CD; \Delta_2, -1}(z_2, \bar{z}_2) &\sim -\delta_{CD}^{AB} \left[\frac{\bar{z}_{12}}{z_{12}} B(\Delta_1, \Delta_2 + 2) \mathcal{O}_{\Delta_1+\Delta_2, -2}(z_2, \bar{z}_2) \right. \\
&\quad \left. + \frac{z_{12}}{\bar{z}_{12}} B(\Delta_1 + 2, \Delta_2) \mathcal{O}_{\Delta_1+\Delta_2, +2}(z_2, \bar{z}_2) \right]
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_{\Delta_1, +\frac{1}{2}}^{ars}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2, +\frac{1}{2}}^{btu}(z_2, \bar{z}_2) &\sim \epsilon^{rstu} \epsilon^{abcd} \frac{z_{12}}{\bar{z}_{12}} B\left(\Delta_1 + \frac{1}{2}, \Delta_2 + \frac{1}{2}\right) \mathcal{O}_{cd; \Delta_1+\Delta_2, -1}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1, +\frac{1}{2}}^{ars}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2, +\frac{1}{2}}^{bct}(z_2, \bar{z}_2) &\sim \epsilon^{rstu} \epsilon^{abcd} \frac{z_{12}}{\bar{z}_{12}} B\left(\Delta_1 + \frac{1}{2}, \Delta_2 + \frac{1}{2}\right) \mathcal{O}_{ud; \Delta_1+\Delta_2, -1}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1, +\frac{1}{2}}^{rst}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2, +\frac{1}{2}}^{abc}(z_2, \bar{z}_2) &\sim \epsilon^{rstu} \epsilon^{abcd} \frac{z_{12}}{\bar{z}_{12}} B\left(\Delta_1 + \frac{1}{2}, \Delta_2 + \frac{1}{2}\right) \mathcal{O}_{ud; \Delta_1+\Delta_2, -1}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1, +\frac{1}{2}}^{ars}(z_1, \bar{z}_1) \mathcal{O}_{btu; \Delta_2, -\frac{1}{2}}(z_2, \bar{z}_2) &\sim \epsilon_{tuvw} \epsilon^{rs w} \delta_b^a \left[\frac{z_{12}}{\bar{z}_{12}} B\left(\Delta_1 + \frac{3}{2}, \Delta_2 + \frac{1}{2}\right) \mathcal{O}_{\Delta_1+\Delta_2, +2}(z_2, \bar{z}_2) \right. \\
&\quad \left. + \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 + \frac{1}{2}, \Delta_2 + \frac{3}{2}\right) \mathcal{O}_{\Delta_1+\Delta_2, -2}(z_2, \bar{z}_2) \right]
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_{\Delta_1, 0}^{abrs}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2, 0}^{cdtu}(z_2, \bar{z}_2) &\sim \epsilon^{abcd} \epsilon^{rstu} B(\Delta_1 + 1, \Delta_2 + 1) \left[\frac{z_{12}}{\bar{z}_{12}} \mathcal{O}_{\Delta_1+\Delta_2, +2}(z_2, \bar{z}_2) \right. \\
&\quad \left. + \frac{\bar{z}_{12}}{z_{12}} \mathcal{O}_{\Delta_1+\Delta_2, -2}(z_2, \bar{z}_2) \right]
\end{aligned}$$



$$\begin{aligned}
\mathcal{O}_{\Delta_1+2}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2+\frac{3}{2}}^A(z_2, \bar{z}_2) &\sim \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 - 1, \Delta_2 - \frac{1}{2}\right) \mathcal{O}_{\Delta_1+\Delta_2+\frac{3}{2}}^A(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+2}(z_1, \bar{z}_1) \mathcal{O}_{A;\Delta_2-\frac{3}{2}}(z_2, \bar{z}_2) &\sim \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 - 1, \Delta_2 + \frac{5}{2}\right) \mathcal{O}_{A;\Delta_1+\Delta_2-\frac{3}{2}}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+2}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2+1}^{AB}(z_2, \bar{z}_2) &\sim \frac{\bar{z}_{12}}{z_{12}} B(\Delta_1 - 1, \Delta_2) \mathcal{O}_{\Delta_1+\Delta_2+1}^{AB}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+2}(z_1, \bar{z}_1) \mathcal{O}_{AB;\Delta_2-1}(z_2, \bar{z}_2) &\sim \frac{\bar{z}_{12}}{z_{12}} B(\Delta_1 - 1, \Delta_2 + 2) \mathcal{O}_{AB;\Delta_1+\Delta_2-1}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+2}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2+\frac{1}{2}}^{abr}(z_2, \bar{z}_2) &\sim \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 - 1, \Delta_2 + \frac{1}{2}\right) \mathcal{O}_{\Delta_1+\Delta_2+\frac{1}{2}}^{abr}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+2}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2+\frac{1}{2}}^{abc}(z_2, \bar{z}_2) &\sim \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 - 1, \Delta_2 + \frac{1}{2}\right) \mathcal{O}_{\Delta_1+\Delta_2+\frac{1}{2}}^{abc}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+2}(z_1, \bar{z}_1) \mathcal{O}_{abc;\Delta_2-\frac{1}{2}}(z_2, \bar{z}_2) &\sim -\frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 - 1, \Delta_2 + \frac{3}{2}\right) \mathcal{O}_{abc;\Delta_1+\Delta_2-\frac{1}{2}}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+2}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2,0}^{ABCD}(z_2, \bar{z}_2) &\sim \frac{\bar{z}_{12}}{z_{12}} B(\Delta_1 - 1, \Delta_2 + 1) \mathcal{O}_{\Delta_1+\Delta_2,0}^{ABCD}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+\frac{3}{2}}^A(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2+1}^{BC}(z_2, \bar{z}_2) &\sim \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 - \frac{1}{2}, \Delta_2\right) \mathcal{O}_{\Delta_1+\Delta_2+\frac{1}{2}}^{ABC}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+\frac{3}{2}}^A(z_1, \bar{z}_1) \mathcal{O}_{BC;\Delta_2-1}(z_2, \bar{z}_2) &\sim 2! \delta_{[B}^A \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 - \frac{1}{2}, \Delta_2\right) \mathcal{O}_{C];\Delta_1+\Delta_2-\frac{3}{2}}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+\frac{3}{2}}^A(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2+\frac{1}{2}}^{BCD}(z_2, \bar{z}_2) &\sim \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 - \frac{1}{2}, \Delta_2 + \frac{1}{2}\right) \mathcal{O}_{\Delta_1+\Delta_2,0}^{ABCD}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+\frac{3}{2}}^A(z_1, \bar{z}_1) \mathcal{O}_{BCD\Delta_2-\frac{1}{2}}(z_2, \bar{z}_2) &\sim 3 \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 - \frac{1}{2}, \Delta_2 + \frac{3}{2}\right) \delta_{[B}^A \mathcal{O}_{CD];\Delta_1+\Delta_2-1}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+\frac{3}{2}}^A(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2,0}^{BCDE}(z_2, \bar{z}_2) &\sim -\frac{1}{6} \epsilon^{ABCDEFGH} \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 - \frac{1}{2}, \Delta_2 + 1\right) \mathcal{O}_{FGH;\Delta_1+\Delta_2-\frac{1}{2}}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+\frac{3}{2}}^A(z_1, \bar{z}_1) \mathcal{O}_{BCDE\Delta_2,0}(z_2, \bar{z}_2) &\sim 3! \delta_{[B}^A \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 - \frac{1}{2}, \Delta_2 + 1\right) \mathcal{O}_{CDE];\Delta_1+\Delta_2-\frac{1}{2}}(z_2, \bar{z}_2) \\
\\
\mathcal{O}_{\Delta_1+1}^{ab}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2+\frac{1}{2}}^{cdr}(z_2, \bar{z}_2) &\sim \frac{1}{3!} \epsilon^{rstu} \epsilon^{abcd} \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1, \Delta_2 + \frac{1}{2}\right) \mathcal{O}_{stu;\Delta_1+\Delta_2-\frac{1}{2}}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+1}^{AB}(z_1, \bar{z}_1) \mathcal{O}_{CDE;\Delta_2-\frac{1}{2}}(z_2, \bar{z}_2) &\sim -\delta_{CD}^{AB} \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1, \Delta_2 + \frac{3}{2}\right) \mathcal{O}_{E;\Delta_1+\Delta_2-\frac{3}{2}}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+1}^{ab}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2,0}^{cdrs}(z_2, \bar{z}_2) &\sim \epsilon^{rstu} \epsilon^{abcd} \frac{\bar{z}_{12}}{z_{12}} B(\Delta_1, \Delta_2 + 1) \mathcal{O}_{tu;\Delta_1+\Delta_2-1}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+1}^{ab}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2,0}^{cdef}(z_2, \bar{z}_2) &\sim -\epsilon^{cdef} \epsilon^{abgh} \frac{\bar{z}_{12}}{z_{12}} B(\Delta_1, \Delta_2 + 1) \mathcal{O}_{gh;\Delta_1+\Delta_2-1}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+\frac{1}{2}}^{abr}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2,0}^{cdst}(z_2, \bar{z}_2) &\sim \epsilon^{rstu} \epsilon^{abcd} \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 + \frac{1}{2}, \Delta_2 + 1\right) \mathcal{O}_{u;\Delta_1+\Delta_2-\frac{3}{2}}(z_2, \bar{z}_2) \\
\mathcal{O}_{\Delta_1+\frac{1}{2}}^{abr}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2,0}^{cstu}(z_2, \bar{z}_2) &\sim -\epsilon^{rstu} \epsilon^{abcd} \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 + \frac{1}{2}, \Delta_2 + 1\right) \mathcal{O}_{d;\Delta_1+\Delta_2-\frac{3}{2}}(z_2, \bar{z}_2)
\end{aligned}$$



$$\begin{aligned}
& \left\langle G_{AB}(z, \bar{z}) \bar{G}^{CD}(w, \bar{w}) \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \\
&= \lim_{\substack{\Delta_1 \rightarrow 0 \\ \Delta_2 \rightarrow 0}} \frac{\Delta_1 \Delta_2}{\pi^2} \int d^2 z_1 \frac{1}{(z - z_1)^a} \frac{1}{(\bar{z} - \bar{z}_1)^b} \frac{1}{(\bar{w} - \bar{z}_2)^{a'}} \frac{1}{(w - z_2)^{b'}} \\
&\quad \times \left\langle \mathcal{O}_{AB} \Delta_1, -1(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2, +1}^{CD}(z_2, \bar{z}_2) \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \\
& \left\langle G_{AB}(z) \bar{G}^{CD}(\bar{w}) \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \\
&= \lim_{\Delta_2 \rightarrow 0} \frac{\Delta_2}{\pi^2} \int d^2 z_1 \int d^2 z_2 \frac{1}{(z - z_1)^a} \frac{1}{(\bar{z} - \bar{z}_1)^b} \frac{1}{(\bar{w} - \bar{z}_2)^{a'}} \frac{1}{(w - z_2)^{b'}} \\
&\quad \times \left[-\delta_{AB}^{CD} \frac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2} \left\langle \mathcal{O}_{\Delta_2, +2}(z_2, \bar{z}_2) \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \right. \\
&\quad \left. + \sum_{j=3}^N f(A, B, \ell_j, *j, *'_j) \frac{z_1 - z_j}{\bar{z}_1 - \bar{z}_j} \left\langle \mathcal{O}_{\Delta_2, +1}^{CD}(z_2, \bar{z}_2) \cdots \mathcal{O}_{\Delta_j, \ell_j + 1}^{*'_j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \right]
\end{aligned}$$

$$\begin{aligned}
& \left\langle G_{AB}(z) \bar{G}^{CD}(\bar{w}) \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \\
&= \lim_{\Delta_2 \rightarrow 0} \frac{\Delta_2}{\pi} \int d^2 z_2 \frac{1}{(\bar{w} - \bar{z}_2)^{a'}} \frac{1}{(w - z_2)^{b'}} \\
&\quad \times \left[-\delta_{AB}^{CD} C_1(b, a) \frac{1}{(z_2 - z)^{a-2} (\bar{z}_2 - \bar{z})^b} \left\langle \mathcal{O}_{\Delta_2, +2}(z_2, \bar{z}_2) \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \right. \\
&\quad + \sum_{j=3}^N f(A, B, \ell_j, *j, *'_j) C_1(b, a) \frac{1}{(z_j - z)^{a-2}} \frac{1}{(\bar{z}_j - \bar{z})^b} \\
&\quad \left. \times \left\langle \mathcal{O}_{\Delta_2, +1}^{CD}(z_2, \bar{z}_2) \cdots \mathcal{O}_{\Delta_j, \ell_j + 1}^{*'_j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \right]
\end{aligned}$$

$$\begin{aligned}
& \lim_{\Delta_2 \rightarrow 0} \frac{\Delta_2}{\pi} \int d^2 z_2 \frac{1}{(\bar{w} - \bar{z}_2)^{a'}} \frac{1}{(w - z_2)^{b'}} \left[-\delta_{AB}^{CD} C_1(b, a) \frac{1}{(z_2 - z)^{a-2} (\bar{z}_2 - \bar{z})^b} \right. \\
& \times \left. \left\langle \mathcal{O}_{\Delta_2, +2}(z_2, \bar{z}_2) \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \right] = -\delta_{AB}^{CD} C_1(b, a) \lim_{\Delta_2 \rightarrow 0} \frac{\Delta_2}{\pi} \int d^2 z_2 \frac{1}{(\bar{w} - \bar{z}_2)^{a'}} \\
& \frac{1}{(w - z_2)^{b'}} \frac{1}{(z_2 - z)^{a-2} (\bar{z}_2 - \bar{z})^b} \\
& \times \sum_{i=3}^N B(\Delta_2 - 1, f(\Delta_i)) \frac{\bar{z}_2 - \bar{z}_i}{z_2 - z_i} \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_i, \ell_i}^{*i}(z_i, \bar{z}_i) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \\
& = -\delta_{AB}^{CD} C_1(b, a) \frac{1}{\pi} \int d^2 z_2 \frac{1}{(\bar{w} - \bar{z}_2)^{a'}} \frac{1}{(w - z_2)^{b'}} \frac{1}{(z_2 - z)^{a-2} (\bar{z}_2 - \bar{z})^b} \\
& \times \sum_{i=3}^N (1 - f(\Delta_i)) \frac{\bar{z}_2 - \bar{z}_i}{z_2 - z_i} \times \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_i, \ell_i}^{*i}(z_i, \bar{z}_i) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle
\end{aligned}$$

$$\lim_{\Delta_2 \rightarrow 0} \Delta_2 B(\Delta_2 - 1, f(\Delta_i)) = 1 - f(\Delta_i)$$

$$\begin{aligned}
& \lim_{\Delta_2 \rightarrow 0} \frac{\Delta_2}{\pi} \sum_{j=3}^N f(A, B, \ell_j, *'_j, *'_j) \frac{1}{(z_j - z)^{a-2}} \frac{1}{(\bar{z}_j - \bar{z})^b} \int d^2 z_2 \frac{1}{(\bar{w} - \bar{z}_2)^{a'}} \frac{1}{(w - z_2)^{b'}} \\
& \times \left\langle \mathcal{O}_{\Delta_2, +1}^{CD}(z_2, \bar{z}_2) \cdots \mathcal{O}_{\Delta_j, \ell_j + 1}^{*j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle = \frac{1}{\pi} \sum_{j=3}^N f(A, B, \ell_j, *'_j, *'_j) \bar{f}(C, D, \ell_j + 1, *'_j, *'_j) \\
& C_1(b, a) \frac{1}{(z_j - z)^{a-2}} \frac{1}{(\bar{z}_j - \bar{z})^b} \times \int d^2 z_2 \frac{1}{(\bar{w} - \bar{z}_2)^{a'}} \frac{1}{(w - z_2)^{b'}} \\
& \frac{\bar{z}_2 - \bar{z}_j}{z_2 - z_j} \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_j, \ell_j}^{*j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle + \frac{1}{\pi} \sum_{i,j=3}^N f(A, B, \ell_j, *'_j, *'_j) \\
& \bar{f}(C, D, \ell_i, *'_i, *'_i) C_1(b, a) \frac{1}{(z_j - z)^{a-2}} \frac{1}{(\bar{z}_j - \bar{z})^b} \times \int d^2 z_2 \frac{1}{(\bar{w} - \bar{z}_2)^{a'}} \frac{1}{(w - z_2)^{b'}} \\
& \frac{\bar{z}_2 - \bar{z}_i}{z_2 - z_i} \left\langle \cdots \mathcal{O}_{\Delta_j, \ell_j + 1}^{*j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_i, \ell_i - 1}^{*i}(z_i, \bar{z}_i) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle = \sum_{j=3}^N f(A, B, \ell_j, *'_j, *'_j) \bar{f} \\
& (C, D, \ell_j + 1, *'_j, *'_j) C_1(b', a') \frac{1}{(z_j - z)^{a-2}} \frac{1}{(\bar{z}_j - \bar{z})^b} \times \frac{1}{(z_j - w)^{b'}} \frac{1}{(\bar{z}_j - \bar{w})^{a'-2}} \\
& \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_j, \ell_j}^{*j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle + \sum_{i,j=3}^N f(A, B, \ell_j, *'_j, *'_j) \\
& \bar{f}(C, D, \ell_i, *'_i, *'_i) C_1(b, a) C_1(b', a') \frac{1}{(z_j - z)^{a-2}} \frac{1}{(\bar{z}_j - \bar{z})^b} \\
& \times \frac{1}{(z_i - w)^{b'}} \frac{1}{(\bar{z}_i - \bar{w})^{a'-2}} \left\langle \cdots \mathcal{O}_{\Delta_j, \ell_j + 1}^{*j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_i, \ell_i - 1}^{*i}(z_i, \bar{z}_i) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle
\end{aligned}$$



$$\begin{aligned}
& \left\langle G_{AB}(z, \bar{z}) \bar{G}^{CD}(w, \bar{w}) \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \\
&= -\delta_{AB}^{CD} C_1(b, a) \frac{1}{\pi} \int d^2 z_2 \frac{1}{(\bar{w} - \bar{z}_2)^{a'}} \frac{1}{(w - z_2)^{b'}} \frac{1}{(z_2 - z)^{a-2} (\bar{z}_2 - \bar{z})^b} \\
&\times \sum_{i=3}^N (1 - f(\Delta_i)) \frac{\bar{z}_2 - \bar{z}_i}{z_2 - z_i} \times \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_i, \ell_i}^{*i}(z_i, \bar{z}_i) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \\
&+ \sum_{j=3}^N f(A, B, \ell_j, *'_j, *'_j) \bar{f}(C, D, \ell_j + 1, *'_j, *''_j) C_1(b', a') \frac{1}{(z_j - z)^{a-2}} \frac{1}{(\bar{z}_j - \bar{z})^b} \\
&\times \frac{1}{(z_j - w)^{b'}} \frac{1}{(\bar{z}_j - \bar{w})^{a'-2}} \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_j, \ell_j}^{*''j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \\
&+ \sum_{i,j=3}^N f(A, B, \ell_j, *'_j, *'_j) \bar{f}(C, D, \ell_i, *'_i, *'_i) C_1(b, a) C_1(b', a') \frac{1}{(z_j - z)^{a-2}} \frac{1}{(\bar{z}_j - \bar{z})^b} \\
&\times \frac{1}{(z_i - w)^{b'}} \frac{1}{(\bar{z}_i - \bar{w})^{a'-2}} \left\langle \cdots \mathcal{O}_{\Delta_j, \ell_j+1}^{*'_j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_i, \ell_i-1}^{*'_i}(z_i, \bar{z}_i) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \\
&\int d^2 z_2 \frac{1}{(\bar{w} - \bar{z}_2)^{a'}} \frac{1}{(w - z_2)^{b'}} \frac{1}{(z_2 - z)^{a-2}} \frac{1}{(\bar{z}_2 - \bar{z})^b} \frac{\bar{z}_2 - \bar{z}_i}{z_2 - z_i} \xrightarrow{z=w} \\
&= (-1)^{a+b-2} \int d^2 z_2 \frac{1}{(\bar{z} - \bar{z}_2)^{a'+b}} \frac{1}{(z - z_2)^{b'+a-2}} \frac{\bar{z}_2 - \bar{z}_i}{z_2 - z_i} \\
&= (-1)^{a+b} C_1(a + b' - 2, a' + b) \frac{1}{(z_i - z)^{a'+b-2}} \frac{1}{(\bar{z}_i - \bar{z})^{a+b'-2}}
\end{aligned}$$

$$\begin{aligned}
& \left\langle : G_{AB}(z, \bar{z}) \bar{G}^{CD}(z, \bar{z}) : \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \\
&= \frac{(-1)^{a+b+1}}{\pi} \delta_{AB}^{CD} C_1(b, a) C(a + b' - 2, a' + b) \sum_{i=3}^N (1 - f(\Delta_i)) \\
&\times \frac{1}{(z_i - z)^{a'+b-2}} \frac{1}{(\bar{z}_i - \bar{z})^{a+b'-2}} \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_i, \ell_i}^{*i}(z_i, \bar{z}_i) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \\
&+ \sum_{j=3}^N f(A, B, \ell_j, *'_j, *'_j) \bar{f}(C, D, \ell_j + 1, *'_j, *''_j) C_1(b, a) C_1(b', a') (-1)^{a+b+a'+b'} \\
&\times \frac{1}{(z - z_j)^{a+b'-2}} \frac{1}{(\bar{z} - \bar{z}_j)^{a'+b-2}} \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_j, \ell_j}^{*''j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \\
&+ \sum_{\substack{i,j=3 \\ i \neq j}}^N f(A, B, \ell_j, *'_j, *'_j) \bar{f}(C, D, \ell_i, *'_i, *'_i) C_1(b, a) C_1(b', a') (-1)^{a+b+a'+b'} \frac{1}{(z - z_j)^{a-2}} \frac{1}{(\bar{z} - \bar{z}_j)^b} \\
&\times \frac{1}{(z - z_i)^{b'}} \frac{1}{(\bar{z} - \bar{z}_i)^{a'-2}} \left\langle \cdots \mathcal{O}_{\Delta_j, \ell_j+1}^{*'_j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_i, \ell_i-1}^{*'_i}(z_i, \bar{z}_i) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle
\end{aligned}$$



$$\begin{aligned}
& \left\langle : \bar{G}^{CD}(z, \bar{z}) G_{AB}(z, \bar{z}) : \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \\
&= \frac{(-1)^{a'+b'+1}}{\pi} \delta_{AB}^{CD} C_1(b', a') C(a' + b - 2, a + b') \sum_{i=3}^N (1 - f(\Delta_i)) \\
&\times \frac{1}{(z_i - z)^{a+b'-2}} \frac{1}{(\bar{z}_i - \bar{z})^{a'+b-2}} \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_i, \ell_i}^{*i}(z_i, \bar{z}_i) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \\
&+ \sum_{j=3}^N \bar{f}(C, D, \ell_j, *j, *'_j) f(A, B, \ell_j - 1, *'_j, *''_j) C_1(b', a') C_1(b, a) (-1)^{a+b+a'+b'} \\
&\times \frac{1}{(z - z_j)^{a'+b-2}} \frac{1}{(\bar{z} - \bar{z}_j)^{a+b'-2}} \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_j, \ell_j}^{*j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \\
&+ \sum_{\substack{i,j=3 \\ i \neq j}}^N \bar{f}(C, D, \ell_j, *j, *'_j) f(A, B, \ell_i, *i, *'_i) C_1(b, a) C_1(b', a') (-1)^{a+b+a'+b'} \frac{1}{(\bar{z} - \bar{z}_j)^{a'-2}} \frac{1}{(z - z_j)^{b'}} \\
&\times \frac{1}{(\bar{z} - \bar{z}_i)^b} \frac{1}{(z - z_i)^{a-2}} \left\langle \cdots \mathcal{O}_{\Delta_j, \ell_j-1}^{*j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_i, \ell_i+1}^{*i}(z_i, \bar{z}_i) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \\
&\left\langle \mathcal{G}_{AB}^{CD}(z, \bar{z}) \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle \\
&= -\delta_{AB}^{CD} \sum_{i=3}^N (1 - f(\Delta_i)) \left[\frac{(-1)^{a+b}}{\pi} C_1(b, a) C(a + b' - 2, a' + b) \frac{1}{(z_i - z)^{a'+b-2}} \frac{1}{(\bar{z}_i - \bar{z})^{a+b'-2}} \right. \\
&\quad \times \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_i, \ell_i}^{*i}(z_i, \bar{z}_i) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \\
&\quad - \frac{(-1)^{a'+b'}}{\pi} C_1(b', a') C_1(a' + b - 2, a + b') \frac{1}{(z_i - z)^{a+b'-2}} \frac{1}{(\bar{z}_i - \bar{z})^{a'+b-2}} \\
&\quad \times \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_i, \ell_i}^{*i}(z_i, \bar{z}_i) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \Big] \\
&+ (-1)^{a+b+a'+b'} C_1(b, a) C_1(b', a') [f(A, B, \ell_j, *j, *'_j) \bar{f}(C, D, \ell_j + 1, *'_j, *''_j) \\
&\times \frac{1}{(z - z_j)^{a+b'-2}} \frac{1}{(\bar{z} - \bar{z}_j)^{a'+b-2}} \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_j, \ell_j}^{*j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \\
&\times \frac{-\bar{f}(C, D, \ell_j, *j, *'_j) f(A, B, \ell_j - 1, *'_j, *''_j)}{(z - z_j)^{a'+b-2}} \frac{1}{(\bar{z} - \bar{z}_j)^{a+b'-2}} \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_j, \ell_j}^{*j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \Big]
\end{aligned}$$

$$C_1(b', a') C_1(a' + b - 2, a + b') (-1)^{a+b} = C_1(b, a) C_1(a + b' - 2, a' + b) (-1)^{a'+b'}$$

$$a' + b - 2 = a + b' - 2$$

$$(-a' - b + 1)(-a' - b + 2)(-a + 1)(-a + 2)$$

$$= (-a' + 1)(-a' + 2)(-a - b' + 1)(-a - b' + 2)$$

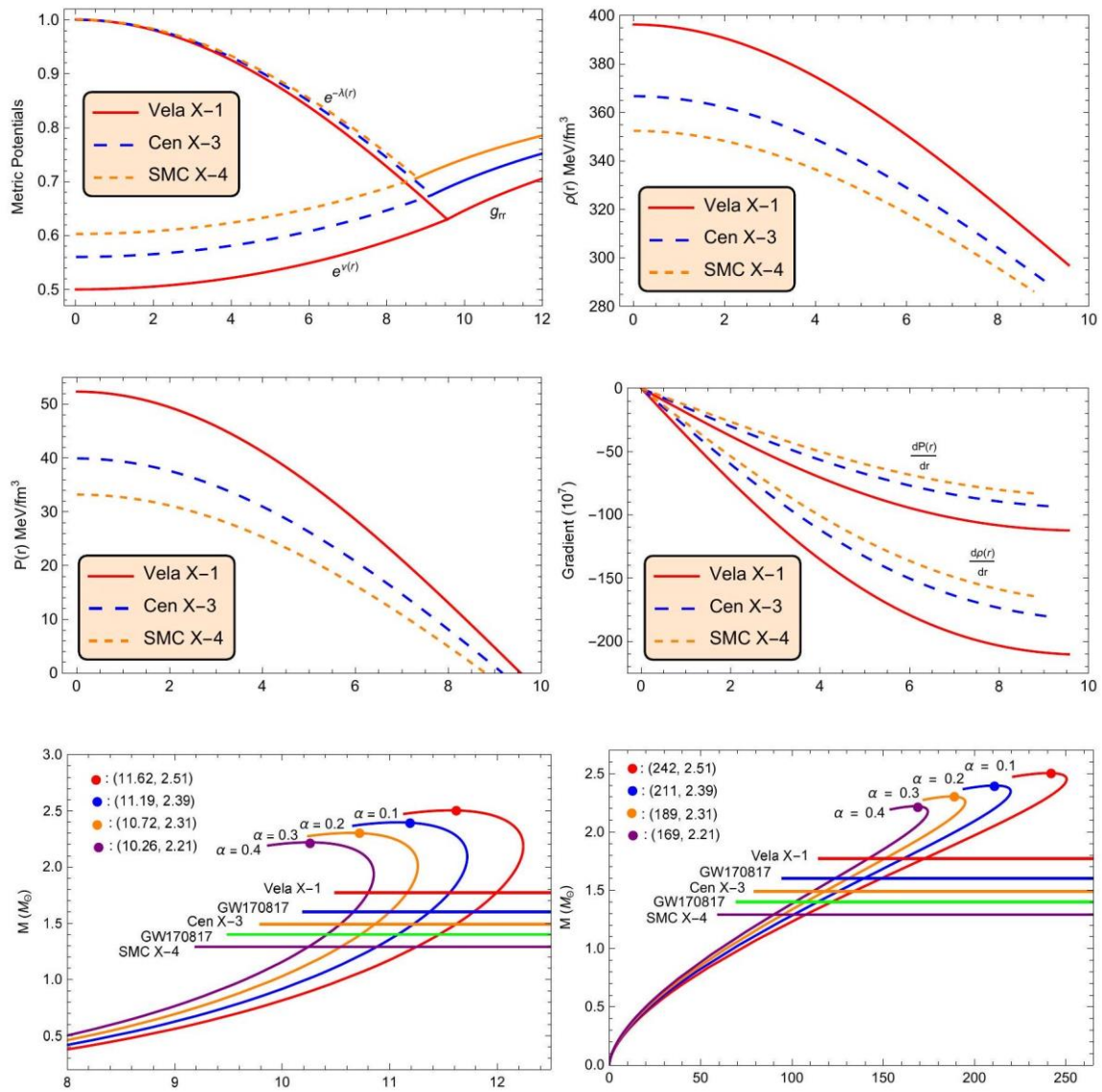
$$(-a + 1)(-a + 2) = (-a' + 1)(-a' + 2)$$



$$\begin{aligned}
& \left\langle \mathcal{G}_{AB}^{CD}(z, \bar{z}) \prod_{n=3}^N \mathcal{O}_{\Delta_n, \ell_n}^{*n}(z_n, \bar{z}_n) \right\rangle = (-1)^{a+b+a'+b'} C_1(b, a) C_1(b', a') [f(A, B, \ell_j, *'_j, *''_j) \bar{f}(C, D, \ell_j + 1, *'_j, *''_j) \\
& \times \frac{1}{(z - z_j)^{a+b'-2}} \frac{1}{(\bar{z} - \bar{z}_j)^{a'+b-2}} \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_j, \ell_j}^{*j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \\
& - \bar{f}(C, D, \ell_j, *'_j, *''_j) f(A, B, \ell_j - 1, *'_j, *''_j) \\
& \times \frac{1}{(z - z_j)^{a'+b-2}} \frac{1}{(\bar{z} - \bar{z}_j)^{a+b'-2}} \left\langle \mathcal{O}_{\Delta_3, \ell_3}^{*3}(z_3, \bar{z}_3) \cdots \mathcal{O}_{\Delta_j, \ell_j}^{*j}(z_j, \bar{z}_j) \cdots \mathcal{O}_{\Delta_N, \ell_N}^{*N}(z_N, \bar{z}_N) \right\rangle \Bigg]
\end{aligned}$$

Partícula Blanca o Partícula Estrella. Modelo TOV.

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2).$$



Figuras 1, 2 y 3. Densidad del centro de masa – energía de una partícula blanca o estrella.



$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu},$$

$$T_0^0 = \rho^{\text{eff}}(r) = \rho(r) + \rho^{de}(r)$$

$$T_1^1 = T_2^2 = T_3^3 = -P^{\text{eff}}(r) = -[P(r) + P^{de}(r)]$$

$$\rho(r) + \rho^{de}(r) = \frac{e^{-\lambda}}{8\pi} \left\{ \frac{\lambda'}{r} - \frac{1}{r^2} \right\} + \frac{1}{8\pi r^2},$$

$$P(r) + P^{de}(r) = \frac{e^{-\lambda}}{8\pi} \left\{ \frac{v'}{r} + \frac{1}{r^2} \right\} - \frac{1}{8\pi r^2},$$

$$P(r) + P^{de}(r) = \frac{e^{-\lambda}}{32\pi} \left\{ 2v'' + v'^2 + \frac{2(v' - \lambda')}{r} - v'\lambda' \right\}.$$

$$e^{\lambda(r)} = \left[1 - \frac{3ar^2}{2} \left(\frac{1 + C(1 + 4ar^2)^{-\frac{1}{2}}}{1 + ar^2} \right) \right]^{-1}$$

$$r^2(2v''(r) + v'^2(r) - v'(r)\lambda'(r)) - 2r(v'(r) + \lambda'(r)) + 4(e^{\lambda(r)} - 1) = 0.$$

$$e^{v(r)} = A^2(1 + ar^2)^3,$$

$$\rho(r) + \rho^{de}(r) = \frac{3a[C(3 + 9ar^2) + (3 + ar^2)(1 + 4ar^2)^{3/2}]}{16\pi(1 + ar^2)^2(1 + 4ar^2)^{3/2}}$$

$$P(r) + P^{de}(r) = \frac{3a[3(1 - ar^2)\sqrt{1 + 4ar^2} - C - 7aCr^2]}{16\pi(1 + ar^2)^2\sqrt{1 + 4ar^2}}$$

$$P^{de}(r) = -\rho^{de}(r),$$

$$\rho^{de}(r) = \alpha\rho(r),$$

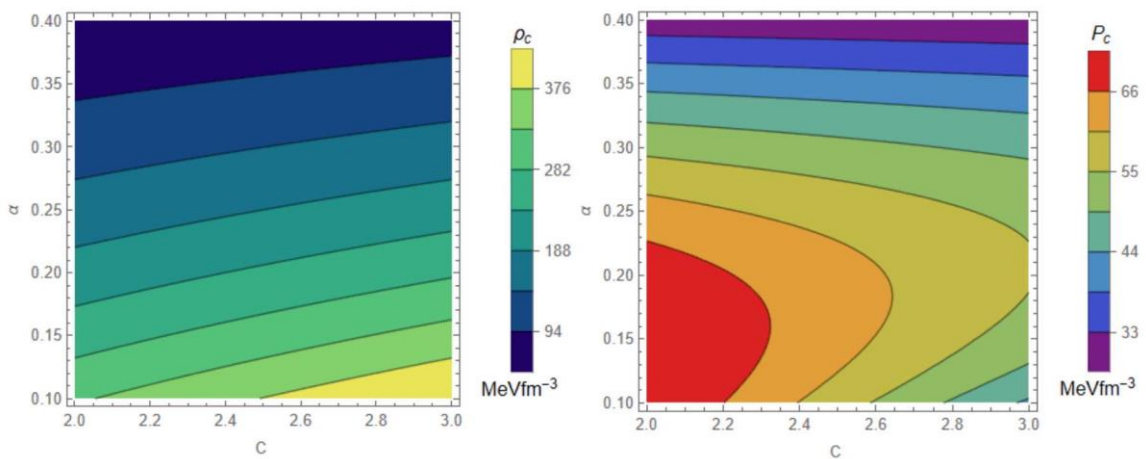


Figura 4. Fluctuaciones de radiación de una partícula estrella o blanca.

$$\rho(r) = \frac{3a[C(3 + 9ar^2) + (3 + ar^2)(1 + 4ar^2)^{3/2}]}{16\pi(1 + \alpha)(1 + ar^2)^2(1 + 4ar^2)^{3/2}},$$

$$P(r) = \frac{3a[(1 + 4ar^2)^{3/2}(3 + 6\alpha - ar^2(3 + 2\alpha)) - C(1 - 2\alpha + 28a^2r^4(1 + \alpha) + ar^2(11 + 2\alpha))]}{16\pi(1 + \alpha)(1 + ar^2)^2(1 + 4ar^2)^{3/2}}.$$

$$\rho^{de}(r) = \frac{3a\alpha[C(3 + 9ar^2) + (3 + ar^2)(1 + 4ar^2)^{3/2}]}{16\pi(1 + \alpha)(1 + ar^2)^2(1 + 4ar^2)^{3/2}},$$

$$P^{de}(r) = -\frac{3a\alpha[C(3 + 9ar^2) + (3 + ar^2)(1 + 4ar^2)^{3/2}]}{16\pi(1 + \alpha)(1 + ar^2)^2(1 + 4ar^2)^{3/2}}.$$

$$\rho^{\text{eff}}(r) = \rho(r) + \rho^{de}(r) = \frac{3a[C(3 + 9ar^2) + (3 + ar^2)(1 + 4ar^2)^{3/2}]}{16\pi(1 + ar^2)^2(1 + 4ar^2)^{3/2}},$$

$$P^{\text{eff}}(r) = P(r) + P^{de}(r) = \frac{3a[3(1 - ar^2)\sqrt{1 + 4ar^2} - C - 7aCr^2]}{16\pi(1 + ar^2)^2\sqrt{1 + 4ar^2}}.$$

$$ds^2 = g_{tt}(R)dt^2 - g_{rr}(R)dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

$$A^2(1 + aR^2)^3 = 1 - \frac{2M}{R},$$

$$1 - \frac{3aR^2[1 + C(1 + 4aR^2)^{-1/2}]}{2(1 + aR^2)} = 1 - \frac{2M}{R}.$$

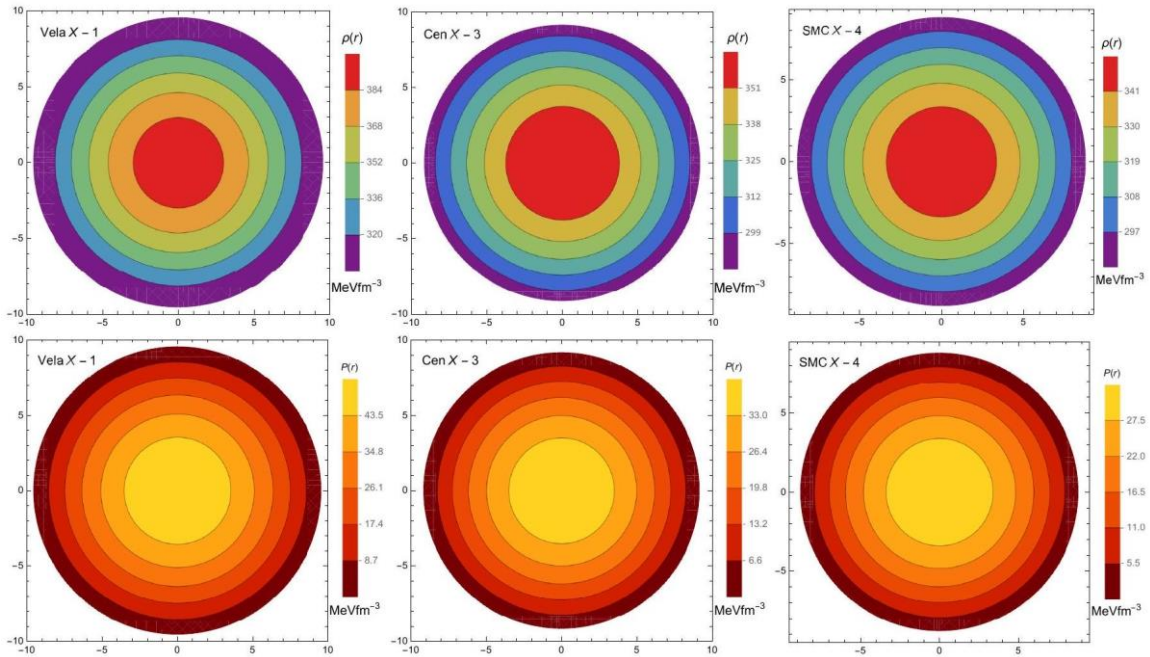


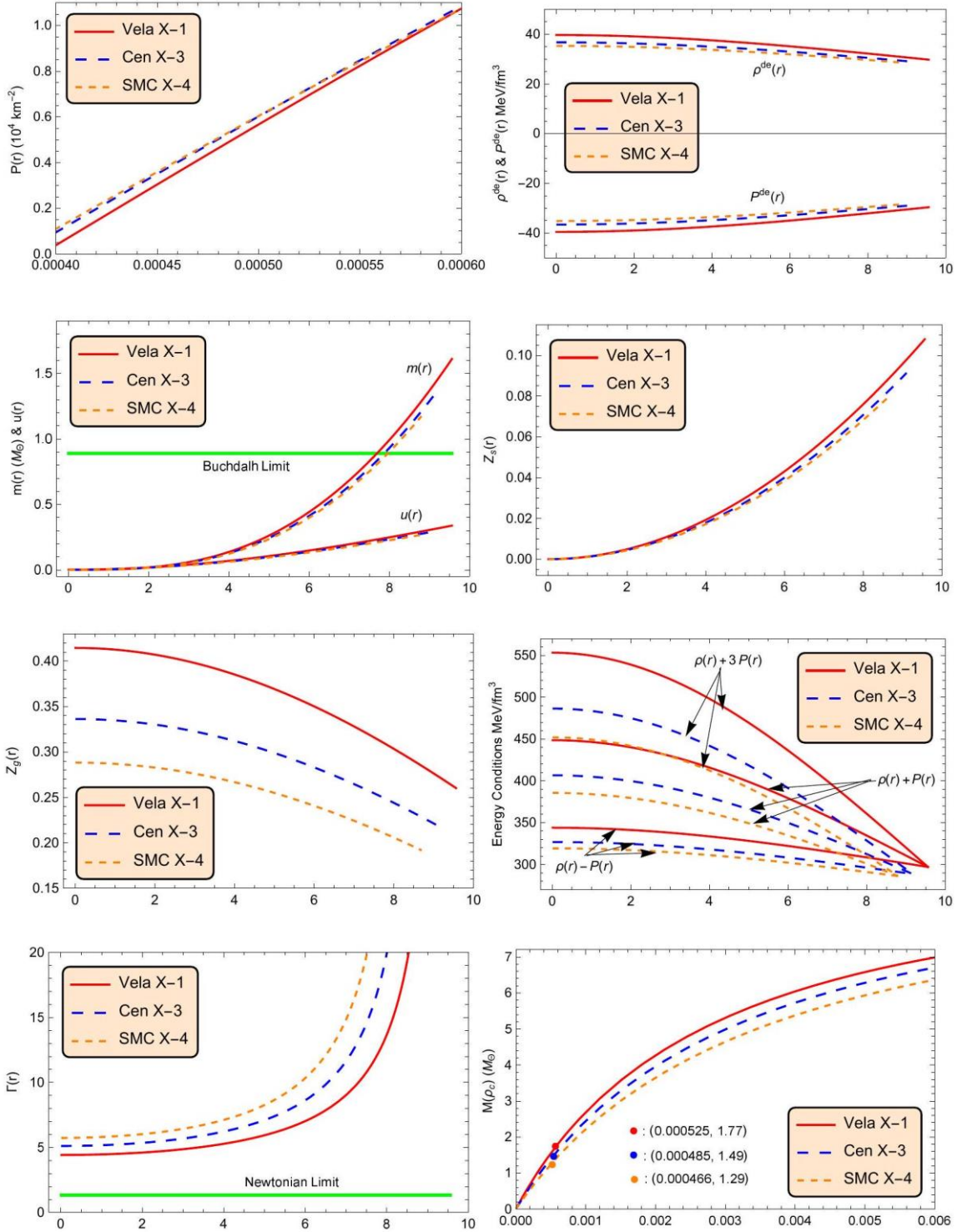
Figura 5. Interior de una partícula estrella o blanca.

$$C[1 - 2\alpha + 28a^2R^4(1 + \alpha) + aR^2(11 + 2\alpha)] + (1 + 4aR^2)^{3/2}[aR^2(3 + 2\alpha) - 6\alpha - 3] = 0.$$

$$a = \frac{M(11 + 2\alpha) - 3R(1 + \alpha) + \sqrt{3M^2(3 + 52\alpha + 76\alpha^2) - 6MR(3 + 19\alpha + 22\alpha^2) + 9R^2(1 + \alpha)^2}}{2R^2[3R(4 + 5\alpha) - 28M(1 + \alpha)]},$$

$$C = \frac{\sqrt{1 + 4aR^2}[4M(1 + aR^2) - 3aR^3]}{3aR^3},$$

$$A = \frac{\sqrt{R - 2M}}{\sqrt{R(1 + aR^2)^3}}.$$



Figuras 6, 7, 8 y 9. Formación de materia oscura por interacción de la partícula estrella con el campo supergravitónico. Ondas cuánticas gravitacionales extremas.

$$(e^{\nu(r)})' = \frac{12ar[C - 2a^2Cr^4 + 2aCr^2 + (4ar^2 + 1)^{3/2}]}{\sqrt{4ar^2 + 1}[ar^2(\sqrt{4ar^2 + 1} + 3C) - 2\sqrt{4ar^2 + 1}]^2}$$

$$(e^{\lambda(r)})' = 6aA^2r(1 + ar^2)^2$$

$$\rho_c = \rho(0) = \frac{9a(1 + C)}{16\pi(1 + \alpha)} > 0$$

$$P_c = P(0) = \frac{3a[3 + 6\alpha - C(1 - 2\alpha)]}{16\pi(1 + \alpha)} > 0$$

$$\frac{d\rho(r)}{dr} = -\frac{3a^2r[3C(30a^2r^4 + 23ar^2 + 5) + (ar^2 + 5)(4ar^2 + 1)^{5/2}]}{8\pi(\alpha + 1)(ar^2 + 1)^3(4ar^2 + 1)^{5/2}}$$

$$\frac{dP(r)}{dr} = \frac{3a^2r[(4ar^2 + 1)^{5/2}(a(2\alpha + 3)r^2 - 14\alpha - 9) + 3c\xi_1]}{8\pi(\alpha + 1)(ar^2 + 1)^3(4ar^2 + 1)^{5/2}}$$

$$\xi_1 = 56a^3(\alpha + 1)r^6 - 6a^2(2\alpha - 3)r^4 - a(26\alpha + 3)r^2 - 6\alpha - 1.$$

$$\rho_{\max}^{de} = \rho^{de}(0) = \frac{9a\alpha(1 + C)}{16\pi(1 + \alpha)} > 0.$$

$$P_{\min}^{de} = -\rho^{de}(0) = -\frac{9a\alpha(1 + C)}{16\pi(1 + \alpha)} < 0.$$

$$m(r) = 4\pi \int_0^r r^{*2} \rho(r^*) dr^* = \frac{3ar^3(1 + 4ar^2 + C\sqrt{4ar^2 + 1})}{4(1 + \alpha)(1 + 5ar^2 + 4a^2r^4)},$$

$$u(r) = \frac{2m(r)}{r} = \frac{3ar^2(1 + 4ar^2 + C\sqrt{4ar^2 + 1})}{2(1 + \alpha)(1 + 5ar^2 + 4a^2r^4)}$$

$$Z_s(r) = [1 - u(r)]^{-1/2} - 1 = \left[1 - \frac{3ar^2(1 + 4ar^2 + C\sqrt{1 + 4ar^2})}{2(1 + \alpha)(1 + 5ar^2 + 4a^2r^4)}\right]^{-1/2} - 1.$$

$$Z_g(r) = e^{-\nu(r)/2} - 1 = \frac{1}{A}(ar^2 + 1)^{-3/2} - 1.$$



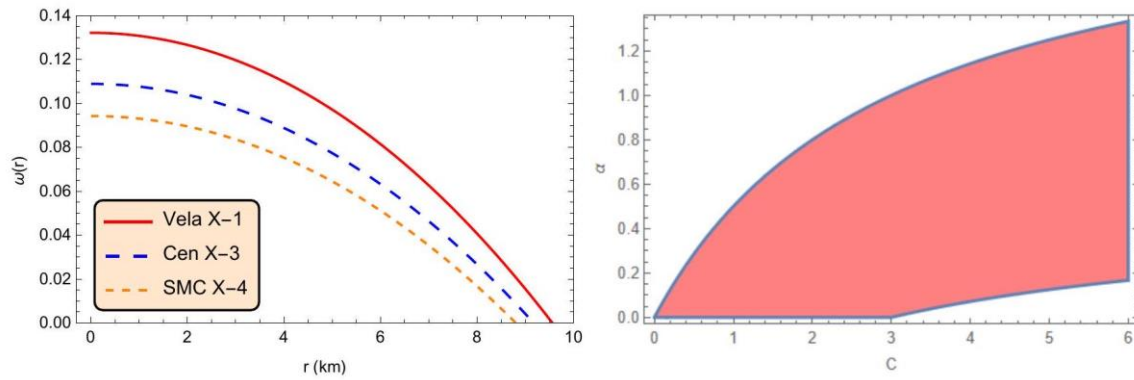


Figura 10. Fluctuaciones gravitacionales extremas provocadas por la partícula estrella o blanca.

$$\begin{aligned} \text{NEC: } \rho(r) + P(r) &\geq 0, \quad \text{WEC: } \rho(r) \geq 0, \rho(r) + P(r) \geq 0, \\ \text{DEC: } \rho(r) &\geq 0, \rho(r) - P(r) \geq 0, \quad \text{SEC: } \rho(r) + 3P(r) \geq 0. \end{aligned}$$

$$\begin{aligned} \rho(r) + P(r) &= \frac{3a[C(1 - 14a^2r^4 - ar^2) + (3 - 4a^2r^4 + 11ar^2)\sqrt{4ar^2 + 1}]}{8\pi(1 + ar^2)^2(1 + 4ar^2)^{3/2}}, \\ \rho(r) - P(r) &= \frac{3a[C(14a^2(\alpha + 1)r^4 + a(\alpha + 10)r^2 - \alpha + 2) + (4ar^2 + 1)^{3/2}(a(\alpha + 2)r^2 - 3\alpha)]}{8\pi(\alpha + 1)(ar^2 + 1)^2(4ar^2 + 1)^{3/2}}, \\ \rho(r) + 3P(r) &= \frac{3a}{8\pi(1 + \alpha)(1 + ar^2)^2(1 + 4ar^2)^{3/2}} \left[3 \left((2 + 3\alpha)\sqrt{1 + 4ar^2} + \alpha C \right) - 2a^2r^4 \left(2(4 + 3\alpha)\sqrt{1 + 4ar^2} \right. \right. \\ &\quad \left. \left. + 21(1 + \alpha)C \right) - ar^2 \left(3(4 + \alpha)C - (33\alpha + 20)\sqrt{1 + 4ar^2} \right) \right]. \end{aligned}$$

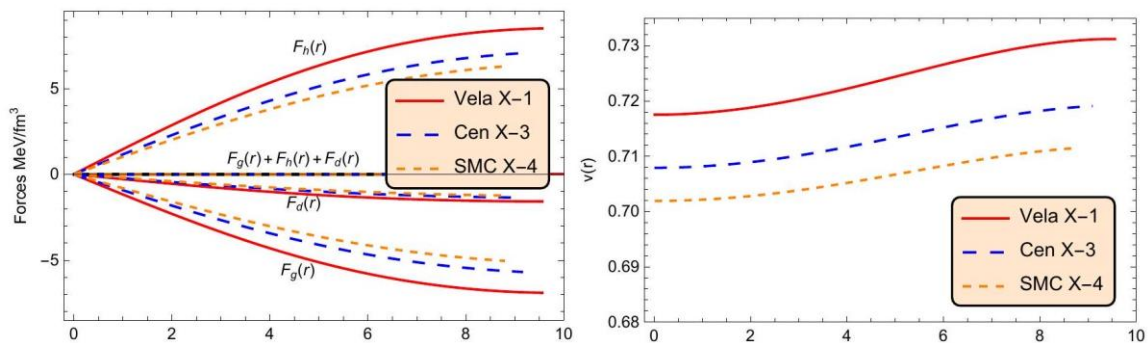


Figura 11. Cinética y coordenadas radiales de una partícula blanca o estrella.

$$\alpha < \frac{2C}{3 + C}.$$

$$\alpha > \frac{C - 3}{2(C + 3)}.$$

$$\frac{C - 3}{2(C + 3)} < \alpha < \frac{2C}{3 + C}.$$

$$-\frac{v'(r)}{2}[\rho(r) + P(r)] - \frac{dP(r)}{dr} - \frac{dP^{de}(r)}{dr} = 0.$$

$$F_g(r) + F_h(r) + F_d(r) = 0,$$

$$F_g(r) = -\frac{v'(r)}{2} [\rho(r) + P(r)],$$

$$F_h(r) = -\frac{dP(r)}{dr},$$

$$F_d(r) = -\frac{dP^{de}(r)}{dr},$$

$$F_g(r) = \frac{9a^2r[C(14a^2r^4 + ar^2 - 1) + (4a^2r^4 - 11ar^2 - 3)\sqrt{1 + 4ar^2}]}{8\pi(1 + ar^2)^3(1 + 4ar^2)^{3/2}},$$

$$F_h(r) = \frac{3a^2r}{8\pi(1 + \alpha)(1 + ar^2)^3(1 + 4ar^2)^{5/2}} [3C(-56a^3r^6(1 + \alpha) + 6a^2r^4(2\alpha - 3) + ar^2(3 + 26\alpha) + 6\alpha + 1) + (1 + 4ar^2)^{5/2}(9 + 14\alpha - ar^2(3 + 2\alpha))],$$

$$F_d(r) = -\frac{3a^2ar[3C(5 + 23ar^2 + 30a^2r^4) + (5 + ar^2)(1 + 4ar^2)^{5/2}]}{8\pi(1 + \alpha)(1 + ar^2)^3(1 + 4ar^2)^{5/2}}.$$

$$V(r) = \left[\frac{dP(r)}{d\rho(r)} \right]^{1/2} = \left[\frac{3C(1 + 6\alpha + 6a^2r^4(2\alpha - 3) - 56a^3r^6(1 + \alpha) + ar^2(3 + 26\alpha)) + \xi_2}{3C(30a^2r^4 + 23ar^2 + 5) + (5 + ar^2)(1 + 4ar^2)^{5/2}} \right]^{\frac{1}{2}},$$

$$\xi_2 = (1 + 4ar^2)^{\frac{5}{2}}(9 + 14\alpha - ar^2(2\alpha + 3))$$

$$\Gamma(r) = \left(1 + \frac{\rho(r)}{P(r)}\right) V^2(r) = \frac{[3C(ar^2(3 - 2ar^4(9 + 28ar^2(\alpha + 1) - 6\alpha) + 26\alpha) + 6\alpha + 1) + \xi_2]\xi_3}{3C(5 + ar^2(30ar^2 + 23)) + (5 + ar^2)(1 + 4ar^2)^{5/2}},$$

$$\xi_3 = 1 - \frac{C(3 + 9ar^2) + (3 + ar^2)(1 + 4ar^2)^{3/2}}{C(ar^2(28ar^2(\alpha + 1) + 2\alpha + 11) - 2\alpha + 1) + (1 + 4ar^2)^{3/2}(ar^2(2\alpha + 3) - 6\alpha - 3)}.$$

$$M(\rho_c) = \frac{\pi\rho_c R^3 [C\sqrt{82944\pi\rho_c R^2(1 + \alpha)(1 + C) + 11664(1 + C)^2} + 108(1 + C) + 768\pi\rho_c R^2(1 + \alpha)]}{16\pi\rho_c R^2(1 + \alpha)(45C + 64\pi\rho_c R^2(1 + \alpha) + 45) + 81(1 + C)^2}.$$

$$I = \frac{8\pi}{3} \int_0^R r^4 [\rho(r) + P(r)] e^{[\lambda(r) - \nu(r)]/2} \frac{\bar{\omega}}{\Omega} dr$$

$$\frac{d}{dr} \left(r^4 \mathcal{H} \frac{d\bar{\omega}}{dr} \right) = -4r^3 \bar{\omega} \frac{d\mathcal{H}}{dr}$$

$$I = \frac{2}{5}(1+x)MR^2$$

$$p = \frac{1}{3}(\rho - 4B_g)$$

$$\mathcal{S} = \frac{1}{16\pi} \int \sqrt{-g} f(\tilde{R}, T) d^4x + \int \sqrt{-g} \mathcal{L}_{\mathcal{M}} d^4x$$



$$f_R R_{ij} - \frac{1}{2} g_{ij} f(\tilde{R}, T) + f_R [g_{ij} \square - \nabla_i \nabla_j] = 8\pi T_{ij} - f_T T_{ij} - f_T \Theta_{ij}.$$

$$f_R = \frac{df(\tilde{R}, T)}{dR}, f_T = \frac{df(\tilde{R}, T)}{dT}, \square \equiv \frac{\partial_i (\sqrt{-g} g^{ij} \partial_j)}{\sqrt{-g}}, \Theta_{ij} = g^{ij} \frac{\delta T_{ij}}{\delta g_{ij}}$$

$$G_{ij} + 2\alpha R R_{ij} - \frac{1}{2} g_{ij} \alpha R^2 + 2\alpha [g_{ij} \square - \nabla_i \nabla_j] R = 8\pi T_{ij} - f_T T_{ij} - f_T \Theta_{ij} + g_{ij} \beta T,$$

$$ds^2 = -e^{2\nu(r)} dt^2 + e^{2\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$T_{ij} = (p + \rho) u_i u_j + p g_{ij},$$

$$\nabla^i T_{ij} = \frac{f_T(\tilde{R}, T)}{8\pi - f_T(\tilde{R}, T)} \left[(T_{ij} + \Theta_{ij}) \nabla^i \ln f_T(\tilde{R}, T) + \nabla^i \Theta_{ij} - \frac{1}{2} g_{ij} \nabla^i T \right]$$

$$\nabla^i T_{ij} = \frac{2\beta}{8\pi + 2\beta} \left[\nabla^i \left(p g_{ij} - \frac{1}{2} g_{ij} \nabla^i T \right) \right].$$

$$\frac{2\lambda' e^{-2\lambda}}{r} + \frac{1 - e^{-2\lambda}}{r^2} + 2\alpha R \left[e^{-2\lambda} \left\{ -v'' - v'^2 + \lambda' v' - \frac{2v'}{r} \right\} \right] + \frac{1}{2} \alpha R^2 - 2\alpha \left[e^{-2\lambda} \left\{ R'' + \left(v' - \lambda' + \frac{2}{r} \right) R' \right\} \right] = 8\pi \rho + 3\beta \rho - \beta p$$

$$\frac{2v' e^{-2\lambda}}{r} - \frac{1 - e^{-2\lambda}}{r^2} + 2\alpha R \left[e^{-2\lambda} \left\{ v'' + v'^2 - \lambda' v' - \frac{2\lambda'}{r} \right\} \right] - \frac{1}{2} \alpha R^2 + 2\alpha \left[e^{-2\lambda} \left\{ R'' + \left(v' - \lambda' + \frac{2}{r} \right) R' \right\} \right] - 2\alpha [R'' - \lambda' R'] = 8\pi p + 2\beta p - \beta \rho$$

$$\frac{dm}{dr} = 4\pi r^2 \rho + \frac{\beta r^2}{2} [3\rho - p] + \frac{r^2}{2} \left[-2\alpha R \left[e^{-2\lambda} \left\{ -v'' - v'^2 + \lambda' v' - \frac{2v'}{r} \right\} \right] - \frac{1}{2} \alpha R^2 + 2\alpha \left[e^{-2\lambda} \left\{ R'' + \left(v' - \lambda' + \frac{2}{r} \right) R' \right\} \right] \right]$$

$$\frac{dp}{dr} = \left[\frac{\beta}{8\pi + 3\beta} \right] \rho' - \left[\frac{8\pi + 2\beta}{8\pi + 3\beta} \right] (\rho + p) \left[\frac{m(r) + 4\pi r^3 p_{eff}}{r[r - 2m(r)]} \right]$$

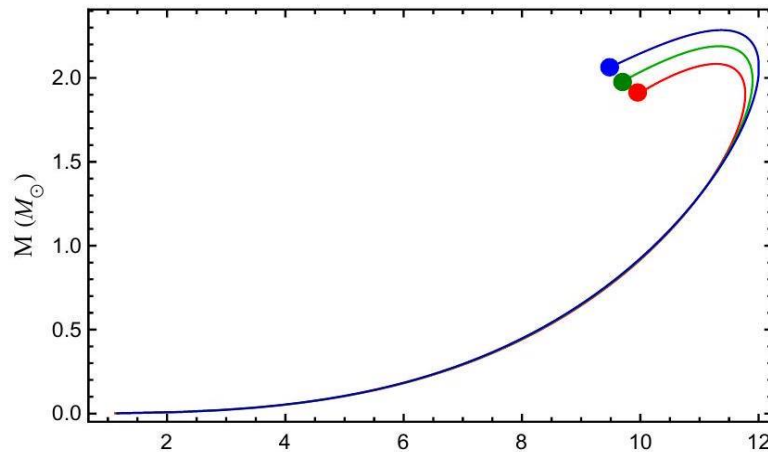


Figura 12. Trayectorias escalares y vectoriales de una partícula blanca o estrella en relación a su centro de masa – energía extremadamente denso.

β	α	Maximum mass $M_{\odot}$	Radius (nm)
	10	2.08	11.08
0	30	2.23	11.35
	50	2.41	11.64

β	α	Maximum mass $M_{\odot}$	Radius (nm)
	0	1.94	10.67
0.5	2.5	1.87	10.66
	5	1.81	10.64

β	α	Maximum mass $M_{\odot}$	Radius (nm)
	0	1.62	8.78
-0.5	2.5	1.76	8.84
	5	1.90	8.88



β	α	Maximum mass $M_{\odot}$	Radius (nm)
	0	1.56	8.53
0	2.5	1.58	8.57
	5	1.61	8.61

β	α	Maximum mass $M_{\odot}$	Radius (nm)
	10	1.65	8.69
0	30	1.87	9.05
	50	2.16	9.54

β	α	Maximum mass $M_{\odot}$	Radius (nm)
	0	1.51	8.30
0.5	2.5	1.43	8.28
	5	1.35	8.24



α	β	Maximum mass $M_{\odot}$	Radius (nm)
	-0.82	2.13	11.50
0	0	2.012	10.96
	0.82	1.90	10.49

α	β	Maximum mass $M_{\odot}$	Radius (nm)
	-0.61	2.17	11.39
1.5	0	2.02	10.98
	0.61	1.87	10.59

α	β	Maximum mass $M_{\odot}$	Radius (nm)
	-4.1	2.13	11.32
0	0	1.56	8.53
	4.1	1.21	7.00

α	β	Maximum mass $M_{\odot}$	Radius (nm)
----------	---------	-----------------------------	----------------



	-2.2	2.17	9.73
1.5	0	1.58	8.55
	1.9	1.16	7.57

$$\mathcal{S}(\Psi, g) = \int d^4x \sqrt{-g} \left\{ \frac{c^3}{16\pi G} f(R) + \mathcal{L}_m(\Psi, g) \right\}$$

$$T_{\mu\nu} = c g_{\mu\nu} \mathcal{L}_m - 2c \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$$

$$f_R R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} f + (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) f_R = \kappa T_{\mu\nu}$$

$$\kappa = \frac{8\pi G}{c^4}, f_R = \frac{\partial f}{\partial R}$$

$$\nabla_\mu \left[f_R R^\mu_\nu - \frac{1}{2} \delta^\mu_\nu f + (\delta^\mu_\nu \square - \nabla^\mu \nabla_\nu) f_R \right] = 0$$

$$f(R) = R + \alpha R^2$$

$$G_{\mu\nu} + 2\alpha R \left(R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R \right) + 2\alpha (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) R = \kappa T_{\mu\nu}$$

$$6\alpha \square R = R + \kappa T,$$

$$ds^2 = -e^{\nu(r)} c^2 dt^2 + e^{\lambda(r)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2$$

$$T_{\mu\nu} = \frac{(\varepsilon + P)}{c^2} u_\mu u_\nu + g_{\mu\nu} P$$

$$\lambda' = \frac{1-e^\lambda}{r} + \frac{re^\lambda}{6} \left(\frac{2R+3\alpha R^2}{1+2\alpha R} \right) - \left(\frac{\alpha r R'}{1+2\alpha R} \right) v' + \frac{\kappa r e^\lambda}{3} \left(\frac{T+3\varepsilon}{1+2\alpha R} \right)$$

$$\gamma v' = \frac{e^\lambda - 1}{r} + \frac{\kappa r e^\lambda P}{1+2\alpha R} - \frac{r \alpha e^\lambda R^2}{2(1+2\alpha R)} - \frac{4}{r} \left(\frac{\alpha r R'}{1+2\alpha R} \right)$$

$$\gamma = 1 + \frac{\alpha r R'}{1+2\alpha R}$$

$$R'' = \left(\frac{\lambda' - \nu'}{2} - \frac{2}{r} \right) R' + \frac{\kappa}{6\alpha} e^{\lambda T} + \frac{e^{\lambda} R}{6\alpha}$$

$$P' = -\frac{\nu'}{2}(P + \varepsilon)$$

$$m(r) = \frac{c^2}{2G} r (1 - e^{-\lambda(r)})$$

$$R'' = \left(\frac{\lambda' - \nu'}{2} - \frac{2}{r} \right) R' + \frac{e^{\lambda} R}{6\alpha}$$

$$R'' + \frac{2}{r} R' - \frac{R}{6\alpha} = 0$$

$$R(r) = \frac{c_1}{r} e^{-\frac{r}{\sqrt{6\alpha}}} + \frac{c_2}{2r} \sqrt{6\alpha} e^{\frac{r}{\sqrt{6\alpha}}}$$

$$R(r) = \frac{c_1}{r} e^{-\frac{r}{\sqrt{6\alpha}}}$$

$$\zeta = \frac{a_1 + a_2 \xi + a_3 \xi^3}{(a_4 \xi + 1)(e^{a_5(\xi - a_6)} + 1)} + \frac{a_7 + a_8 \xi}{e^{a_9(a_{10} - \xi)} + 1} + \frac{a_{11} + a_{12} \xi}{e^{a_{13}(a_{14} - \xi)} + 1} + \frac{a_{15} + a_{16} \xi}{e^{a_{17}(a_{18} - \xi)} + 1}$$

$$\begin{aligned} \zeta = & \frac{a_1 + a_2 \xi + a_3 \xi^3}{(a_4 \xi + 1)(e^{a_5(\xi - a_6)} + 1)} + \frac{a_7 + a_8 \xi}{e^{a_9(a_6 - \xi)} + 1} + \frac{a_{10} + a_1 \xi}{e^{a_{12}(a_{13} - \xi)} + 1} \\ & + \frac{a_{14} + a_{15} \xi}{e^{a_{16}(a_{17} - \xi)} + 1} + \frac{a_{18}}{[a_{19}(\xi - a_{20})]^2 + 1} + \frac{a_{21}}{[a_{22}(\xi - a_{23})]^2 + 1} \end{aligned}$$

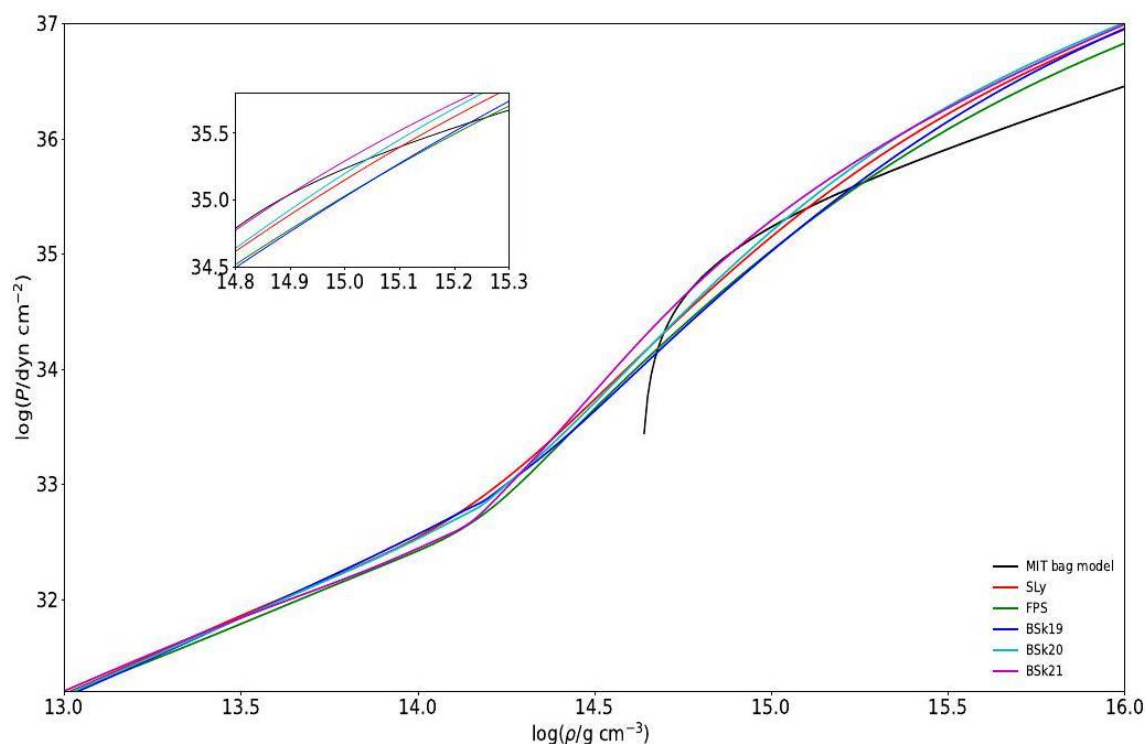


Figura 13. MIT bag model aplicable a un partícula blanca o estrella. Parametrizaciones.

$$P = k(\rho c^2 - 4B),$$

$$\eta = \frac{r}{r_g}, \tilde{P} = \frac{P}{P_0}, \tilde{\rho} = \frac{\rho}{\rho_0}, \chi = Rr_g^2, \tilde{T} = \frac{T}{P_0}, \Omega = \frac{d\chi}{d\eta},$$

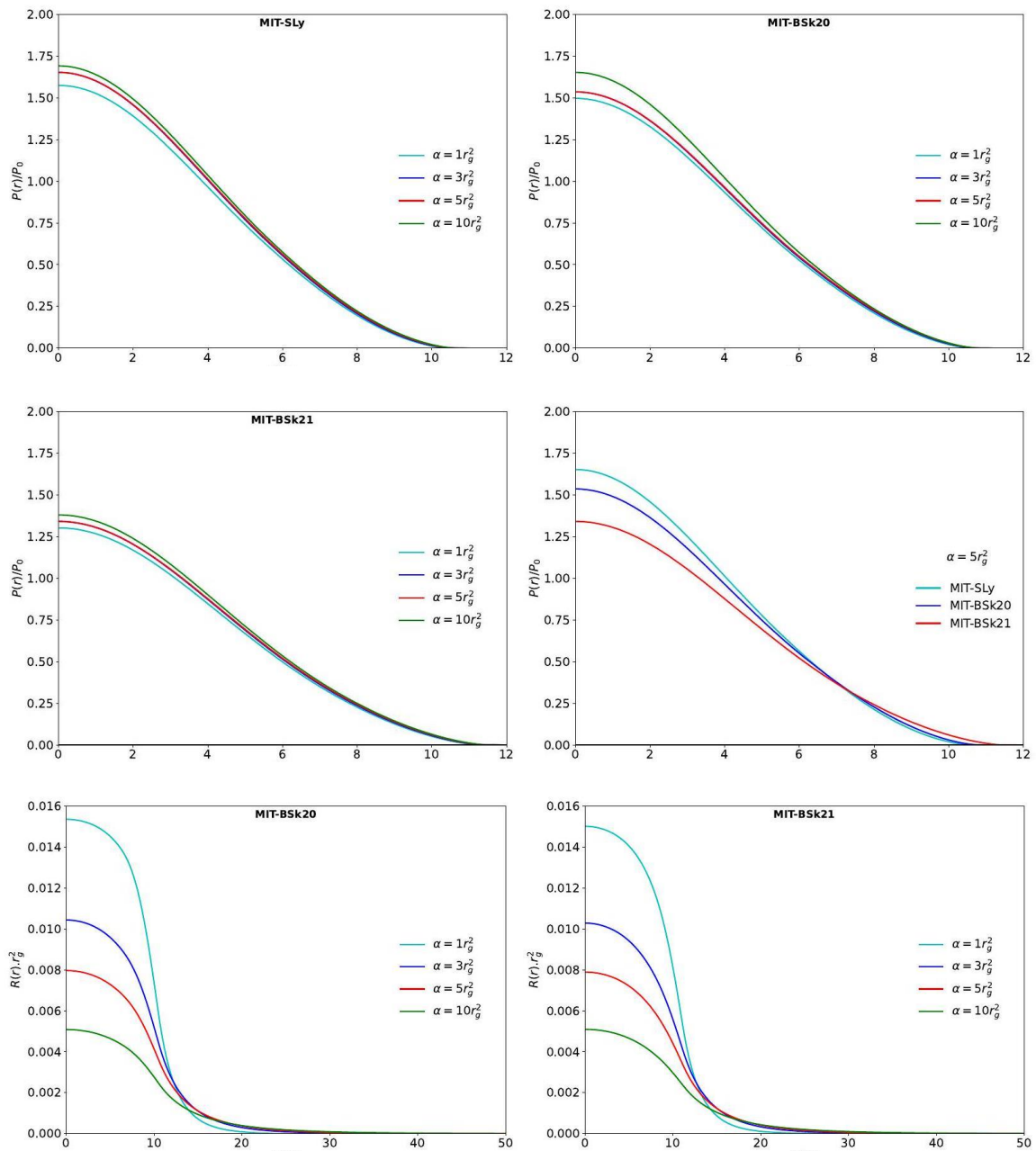
$$M = \frac{c^2}{2G} r_\infty (1 - e^{-\lambda(r_\infty)})$$

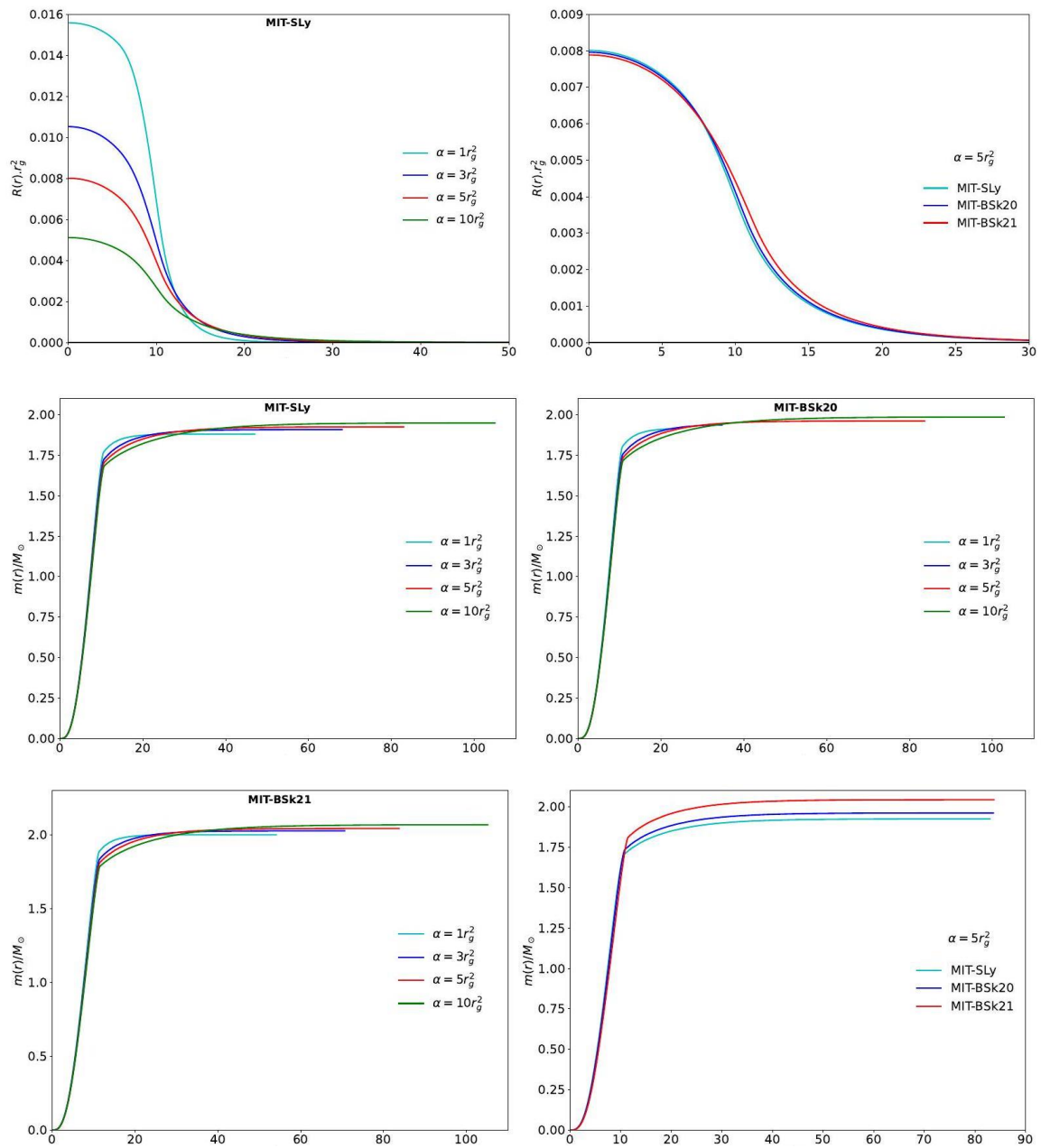
$$M_s = m(r_s) = \frac{c^2}{2G} r_s (1 - e^{-\lambda(r_s)})$$

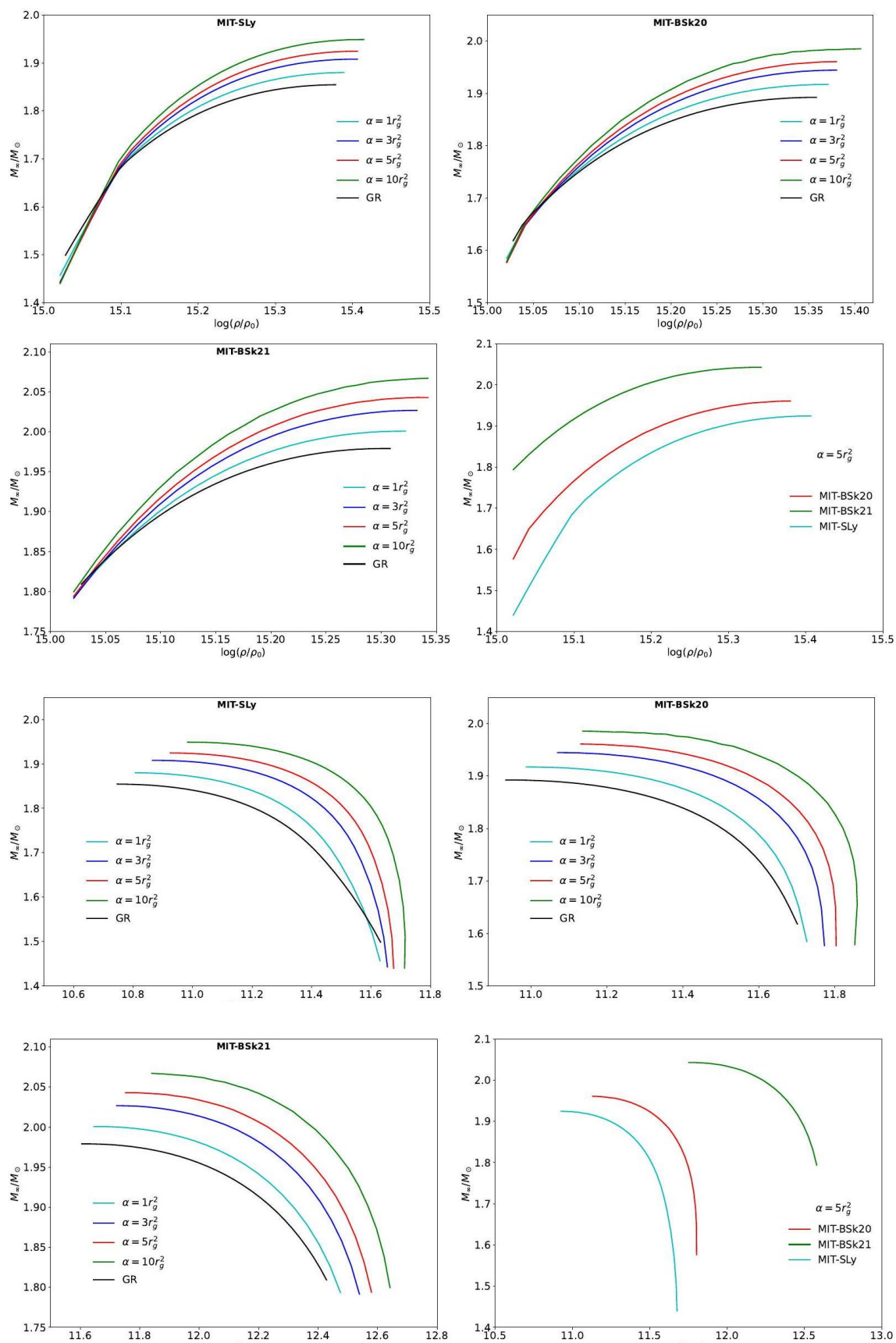
Equation of state	α	$\rho_c \times 10^{15}$	R_c	$m(r_s)$	r_s	M_∞
	r_g^2	g. nm^{-3}	r_g^2	$M_\odot$	nm	$M_\odot$
MIT-SLy	GR	2.39	-	1.85	10.75	1.85
	1.0	2.45	0.016	1.78	10.81	1.88
	3.0	2.50	0.010	1.73	10.89	1.91
	5.0	2.55	0.008	1.71	10.92	1.92
	10.0	2.55	0.005	1.69	11.01	1.95
MIT-BSk20	GR	2.28	-	1.89	10.93	1.89
	1.0	2.35	0.015	1.81	10.98	1.92
	3.0	2.40	0.010	1.76	11.07	1.94
	5.0	2.40	0.008	1.74	11.13	1.96
	10.0	2.55	0.005	1.72	11.14	1.98
MIT-BSk21	GR	2.03	-	1.98	11.60	1.98
	1.0	2.10	0.015	1.90	11.65	2.00



	3.0	2.15	0.010	1.84	11.72	2.03
	5.0	2.20	0.008	1.82	11.75	2.04
	10.0	2.20	0.005	1.79	11.84	2.07







Figuras 14, 15, 16, 17 y 18. Perfiles de presión radial de una superpartícula o partícula blanca o estrella en gravedad extrema o supergravedad.

$$p(k_F) = \frac{1}{3\pi^2\hbar^3} \int_0^{k_F} \frac{k^4}{\sqrt{k^2 + m_e^2}} dk + p_L(Z)$$

$$p_L(Z) = \frac{1}{3} C e^2 n_e^{4/3} Z^{2/3}$$

$$\begin{aligned} \epsilon(k_F) &= \epsilon_i + \epsilon_e + \epsilon_L - \epsilon_\epsilon \\ &= n_i M(Z, A) c^2 + \frac{1}{\pi^2 \hbar^3} \int_0^{k_F} \sqrt{k^2 + m_e^2} k^2 dk \\ &\quad + C e^2 n_e^{4/3} Z^{2/3} - n_e m_e c^2 \end{aligned}$$

$$A(N, Z) + e^- \rightarrow A(N + 1, Z - 1) + \nu_e$$

$$\epsilon_e + p_e = n_e \mu_e$$

$$g(A, Z) = m_n c^2 + \frac{M(Z, A) c^2}{A} + \gamma_e \left[\mu_e - m_e c^2 + \frac{4}{3} \frac{\epsilon_L}{n_e} \right],$$

$$g(A, Z) \geq g(A, Z - 1)$$

$$\mu_e + C e^2 n_e^{1/3} f(Z, Z - 1) \geq \mu_e^\beta$$

$$\mu_e^\beta(A, Z) \equiv M(Z - 1, A) c^2 - M(Z, A) c^2 + m_e c^2$$

$$f(Z, Z - 1) = Z^{5/3} - (Z - 1)^{5/3} + \frac{1}{3} Z^{2/3}$$

$$\begin{aligned} n_e &= \frac{k_F^3}{3\pi^2\hbar^3} \\ \rho &= \frac{1}{\gamma_e} m n_e \end{aligned}$$

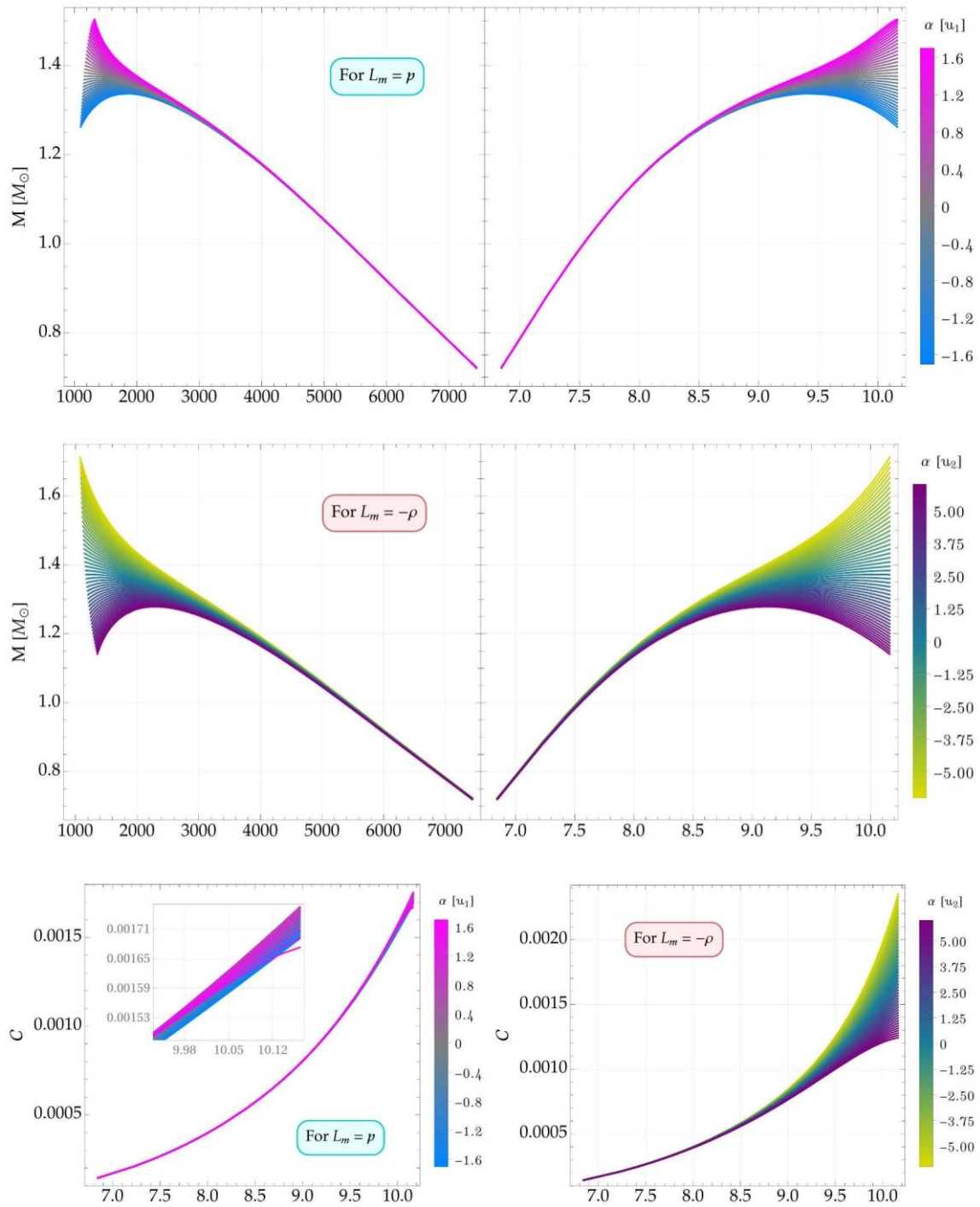
$$k_F = \hbar \left(\frac{3\pi^2 \rho}{m_n} \frac{Z}{A} \right)^{1/3}$$

$$\begin{aligned} \frac{dm}{dr} &= 4\pi r^2 \rho + \frac{\alpha r^2}{2} \left[\frac{\rho}{2} (5p - \rho) + p^2 \right] \\ \frac{dp}{dr} &= - \frac{(\rho + p) \left[4\pi r p + \frac{m}{r^2} + \frac{\alpha r}{4} (3p - \rho) p \right]}{\left(1 - \frac{2m}{r} \right) \left[1 + \frac{\alpha p}{16\pi + \alpha(5p - \rho)} \left(1 - \frac{d\rho}{dp} \right) \right]} \end{aligned}$$

$$\begin{aligned} \frac{dm}{dr} &= 4\pi r^2 \rho + \frac{\alpha r^2}{4} (3p - \rho) \rho \\ \frac{dp}{dr} &= - \frac{(\rho + p) \left[4\pi r p + \frac{m}{r^2} + \frac{3\alpha r}{4} (p - \rho) p - \frac{\alpha r}{2} \rho^2 \right]}{\left(1 - \frac{2m}{r} \right) \left\{ 1 + \frac{\alpha [3p(1 - d\rho/dp) - 4\rho(d\rho/dp)]}{16\pi + 3\alpha(p - \rho)} \right\}} \end{aligned}$$

$$m(0) = 0, \rho(0) = \rho_c$$





$$S_{\text{grav}} [g_{\mu\nu}, \varphi] = \int d^4x \sqrt{-g} \left(K + Q \nabla^\mu \nabla_\mu \varphi + FR + \sum_{i=1}^5 A_i L_i \right)$$

$$L_1 = \nabla^\nu \nabla^\mu \varphi \nabla_\nu \nabla_\mu \varphi \quad L_2 = (\nabla^\mu \nabla_\mu \varphi)^2 \quad L_3 = \nabla^\alpha \nabla_\alpha \varphi \nabla_\nu \nabla_\mu \varphi \nabla^\nu \varphi \nabla^\mu \varphi$$

$$L_4 = \nabla^\nu \varphi \nabla_\nu \nabla_\mu \varphi \nabla^\mu \nabla^\alpha \varphi \nabla_\alpha \varphi, \quad L_5 = (\nabla^\nu \varphi \nabla_\nu \nabla_\mu \varphi \nabla^\mu \varphi)^2$$

$$K = Q = A_1 = A_2 = A_5 = 0, A_3 = -A_4 = -4 \frac{F_X}{X}$$



$$S_{\text{grav}} = \int d^4x \sqrt{-g} \left(FR - 4 \frac{F_X}{X} (L_3 - L_4) \right)$$

$$T_{\alpha\beta} = (P + \rho)u_\alpha u_\beta + P g_{\alpha\beta},$$

$$S_{\text{m}} = \int d^4x \sqrt{-g} P(\mu)$$

$$\mu = \sqrt{-g^{\alpha\beta} \mu_\alpha \mu_\beta} \,)$$

$$\mu_\alpha = \mu u_\alpha = \partial_\alpha L + A \partial_\alpha B$$

$$ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\varphi(t,r)=qt+\psi(r)$$

$$S_{\text{bgd}} = 4\pi \int dr dt \left[r^2 \sqrt{f h} \left(\frac{2F h'}{h^2 r} + \frac{2F(h-1)}{h r^2} - \frac{4F_X X' (fX + q^2)}{f h r X} + P \right) + \lambda_0 (\mu \sqrt{f})' \right]$$

$$X=\psi'^2/h-q^2/f$$

$$\frac{\mu'}{\mu}=-\frac{f'}{2f},$$

$$\begin{aligned} & h' \left(\frac{F_X(-4f^2(\psi')^4 + 8fhq^2(\psi')^2 + 4h^2q^4)}{fh^2q^2 - f^2h(\psi')^2} + 2F \right) - \frac{8F_X(f(\psi')^3 - 2hq^2\psi')}{f(\psi')^2 - hq^2} \psi'' \\ & + \frac{4hq^2F_X(f(h+1)(\psi')^2 - (h-1)hq^2)}{fr(f(\psi')^2 - hq^2)} + \frac{2F(h-1)h}{r} - h^2r\rho = 0 \\ & f' \left(\frac{F_X(-4f^2(\psi')^4 + 8fhq^2(\psi')^2 + 4h^2q^4)}{f^2h^2q^2 - f^3h(\psi')^2} + \frac{2F}{f} \right) + \frac{8q^2F_X\psi'\psi''}{hq^2 - f(\psi')^2} \\ & + \frac{F_X(4(h-1)hq^2(\psi')^2 - 4f(h+1)(\psi')^4)}{hr(hq^2 - f(\psi')^2)} + \frac{F(2-2h)}{r} - hrP = 0 \end{aligned}$$

$$\lambda_0' = r^2 \sqrt{h} \frac{dP}{d\mu},$$

$$\rho = \mu \frac{dP}{d\mu} - P.$$

$$\frac{d}{dr}(\sqrt{f h} r^2 J^r) = 0$$

$$J^r = \frac{2\sqrt{fX+q^2}}{f^3h^3r^2X} \{-2F_X[hrf'(fX-q^2) + f(fh(h+1)X - q^2rh' + 2hq^2)]\},$$

$$J^r=0$$

$$P'=-\frac{f'(P+\rho)}{2f}$$



$$F = \frac{\kappa}{2} + \sigma X, \kappa \equiv (8\pi G)^{-1}$$

$$ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \\ + 2r^2 \sin \theta \partial_\theta Y_{l0}(\theta)(H_0(r,t)dtd\phi + H_1(r,t)drd\phi)$$

$$u^\mu = \bar{u}^\mu + \delta u^\mu, \bar{u}^\mu = \frac{\delta_0^\mu}{f^{1/2}}, \delta u^\mu = \left\{ 0, 0, 0, v(r,t) \frac{\partial_\theta Y_{l0}(\theta)}{f^{\frac{1}{2}} \sin \theta} \right\}.$$

$$\delta T^{t\phi} = \frac{\partial_\theta Y_{l0}(\theta)}{\sin \theta f} [PH_0 + (\rho + P)v]$$

$$\delta T^{r\phi} = -\frac{\partial_\theta Y_{l0}(\theta)}{\sin \theta h} PH_1$$

$$v = -H_0.$$

$$S_{\rm axial} = l(l+1) \int dr dt [Q_2 H_0^2 - Q_1 H_1^2 + Q_3 H_1 H_0 + Q_0 (H_0' - \dot{H}_1)^2]$$

$$Q_0 = \frac{Fr^4}{2\sqrt{fh}}, Q_1 = \frac{\sqrt{f}Fr^2(l(l+1)-2)}{2\sqrt{h}}, Q_2 = \frac{h}{f}Q_1$$

$$Q_3 = -\frac{2\sqrt{f}\sqrt{h}qr^2F_X\psi'(f(\psi')^2(4F+Pr^2)-q^2(4F(h-1)+hr^2(2P+\rho)))}{fFh(f(\psi')^2-hq^2)+2F_X(f^2(\psi')^4-2fhq^2(\psi')^2-h^2q^4)}$$

$$\chi = \dot{H}_1 - H_0'.$$

$$S'_{\rm axial} = l(l+1) \int dr dt [Q_2 H_0^2 - Q_1 H_1^2 + Q_0 \chi^2 + \lambda(\chi + H_0' - \dot{H}_1)].$$

$$\lambda = -2Q_0\chi, \lambda' = 2Q_2H_0, \dot{\lambda} = 2Q_1H_1.$$

$$\dot{\chi} = 0, (Q_0\chi)' = 0,$$

$$\chi = \chi_0 \frac{\sqrt{fh}}{Fr^4},$$

$$S'_{\rm axial} = \int dr dt \frac{l(l+1)}{4Q_1} \left[\dot{\lambda}^2 - \frac{f}{h} \lambda'^2 - \frac{Q_1}{Q_0} \lambda^2 \right]$$

$$\ddot{\lambda} - Q_1 \frac{d}{dr} \left[\frac{1}{Q_1} \frac{f}{h} \lambda' \right] + \frac{Q_1}{Q_0} \lambda = 0.$$

$$S''_{\rm axial} = \frac{1}{2} \int dt dr_* \frac{1}{Fr^2} \left[\dot{\lambda}^2 - (\partial_* \lambda)^2 - \frac{Q_1}{Q_0} \lambda^2 \right]$$

$$dr_* = \sqrt{\frac{h}{f}} dr$$

$$\ddot{\lambda} - Fr^2 \frac{\partial}{\partial r_*} \left[\frac{1}{Fr^2} \frac{\partial \lambda}{\partial r_*} \right] + \frac{f}{r^2} (l(l+1)-2)\lambda$$



$$\lambda(t,r)=\int\,d\omega e^{i\omega t}\lambda(\omega,r)$$

$$U\equiv\frac{\lambda}{r\sqrt{F}}$$

$$-\frac{d^2U}{dr_*^2}+V_lU=\omega^2U$$

$$V_l=\frac{f}{r^2}\left(l(l+1)-2\right)-r\sqrt{F}\partial_*\left(\frac{\partial_*(r\sqrt{F})}{r^2F}\right)$$

$$\mathrm{d}\tilde{s}^2=F[-f\mathrm{d}t^2+h\mathrm{d}r^2+r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2)]$$

$$F=\frac{\kappa}{2}+\sigma X$$

$$p\equiv q^2\sigma$$

$$\log_{10}\left(\frac{P}{\text{g nm}^{-3}}\right)=\frac{b_1+b_2\xi+b_3\xi^3}{1+b_4\xi}u[b_5(\xi-b_6)]+(b_7+b_8\xi)u[b_9(b_{10}-\xi)]+(b_{11}+b_{12}\xi)u[b_{13}(b_{14}-\xi)]\\ +(b_{15}+b_{16}\xi)u[b_{17}(b_{18}-\xi)]+\frac{b_{19}}{b_{20}^2(b_{21}-\xi)^2+1}+\frac{b_{22}}{b_{23}^2(b_{24}-\xi)^2+1}$$

$$u[x]\equiv\frac{1}{e^x+1}, \xi=\log_{10}\left(\rho/g\text{ nm}^{-3}\right)$$

$$b_i=a_i^{\rm HP} \text{ for } 1\leq i\leq 18, b_j=0 \text{ for } 19\leq j\leq 24,$$

$$b_i=a_i^{\rm PFCPG} \text{ for } 1\leq i\leq 9, b_{10}=a_6^{\rm PFCPG}, b_j=a_{j-1}^{\rm PFCPG} \text{ for } 11\leq j\leq 24,$$

$$f=\frac{1}{h}=1-\frac{2M}{r},$$

$$\psi'=\frac{q\sqrt{2Mr}}{r-2M},$$

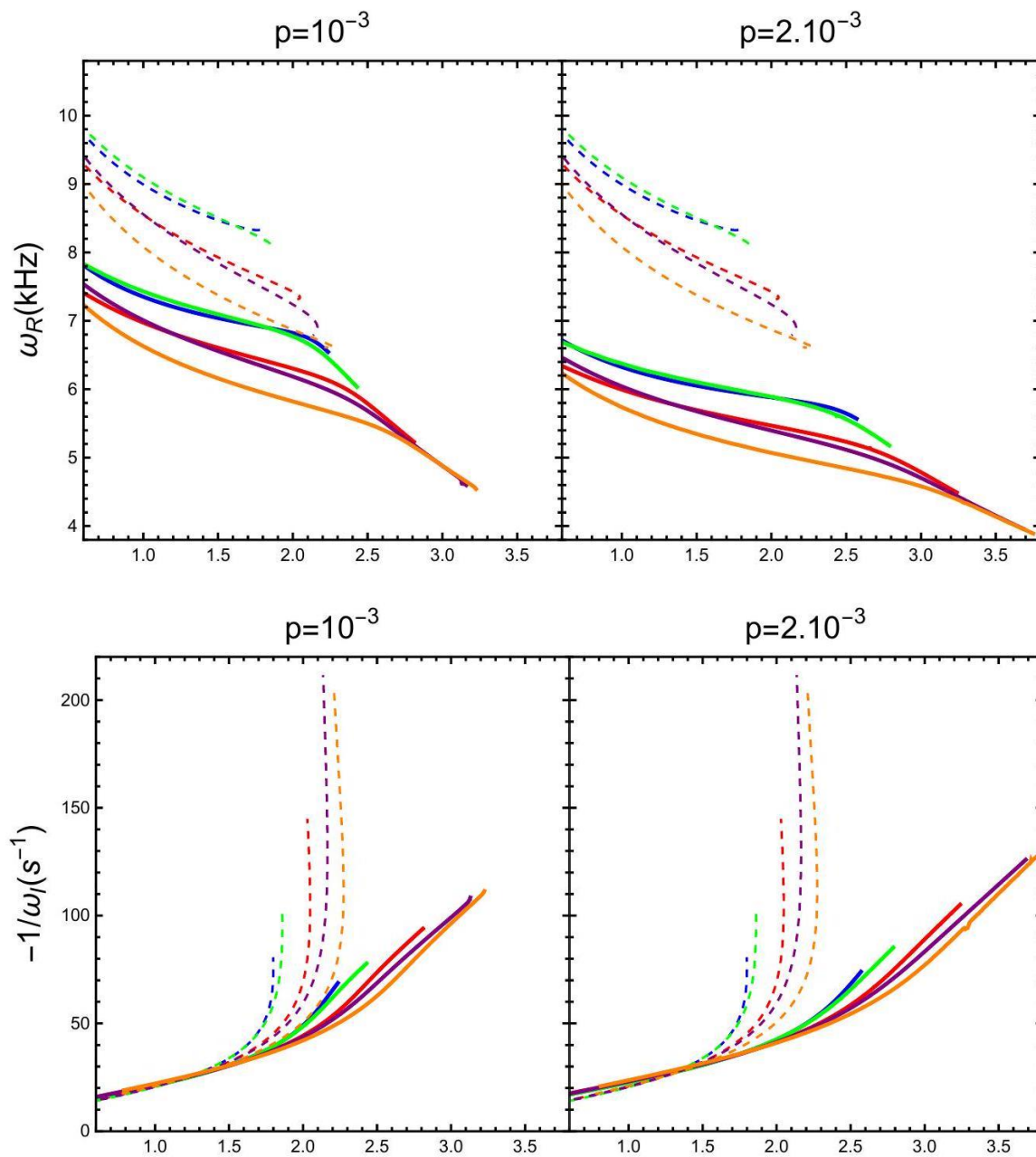
$$Q_1\frac{d}{dr}\left[\frac{1}{Q_1}\frac{f}{h}\lambda'\right]+\left(\omega^2-\frac{Q_1}{Q_0}\right)\lambda=0.$$

$$\lambda(\omega,r)\sim \lambda_-r^{1-l}+\lambda_+r^{l+2}\;(r\rightarrow 0),$$

$$\lambda(\omega,r)\sim \lambda_{\rm in}\,e^{i\omega r_*}+\lambda_{\rm out}\,e^{-i\omega r_*}\;(r_*\rightarrow \infty),$$

$$r=r_s+\eta e^{i\Theta}$$





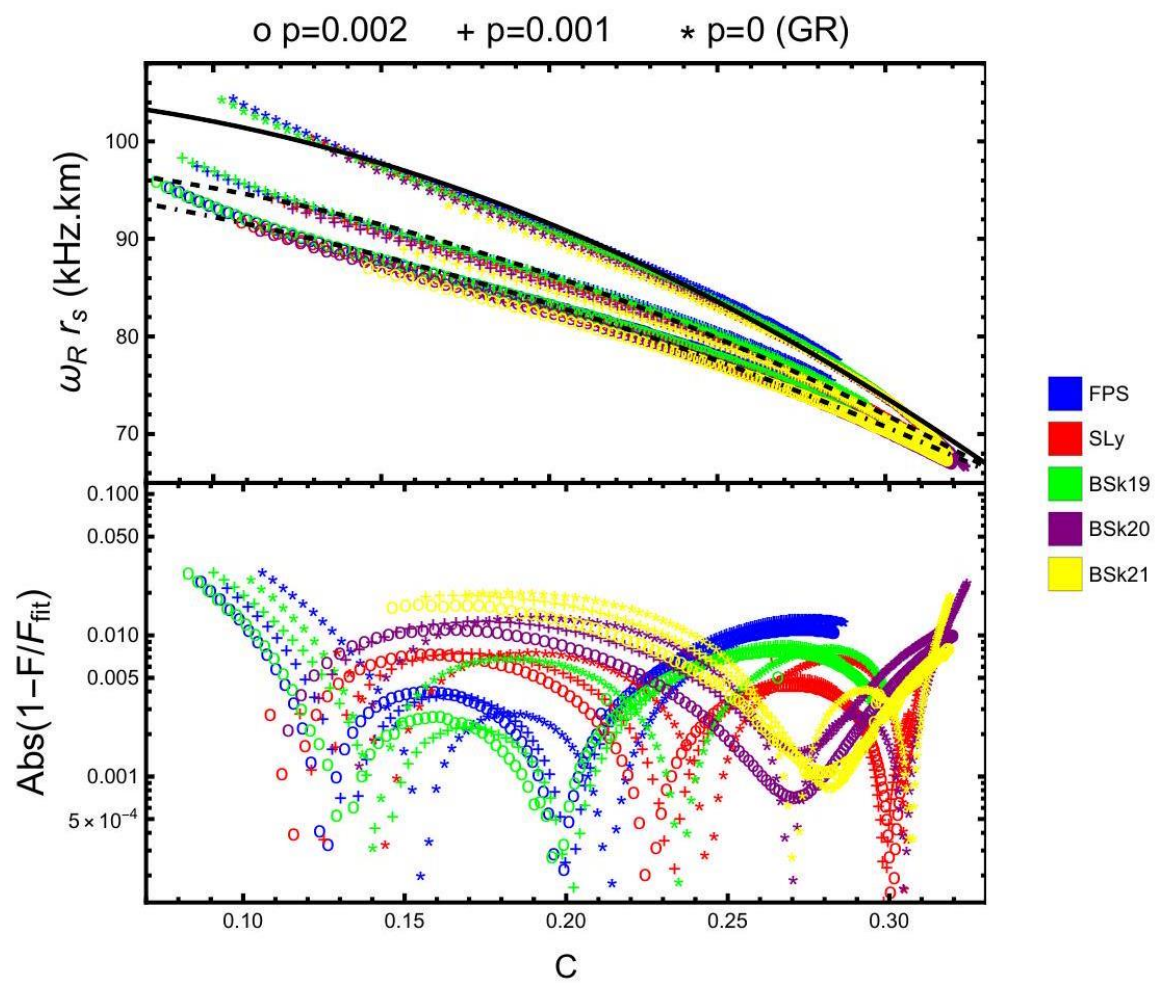
Figuras 22 y 23. Densidad de masa extrema de una partícula blanca o estrella.

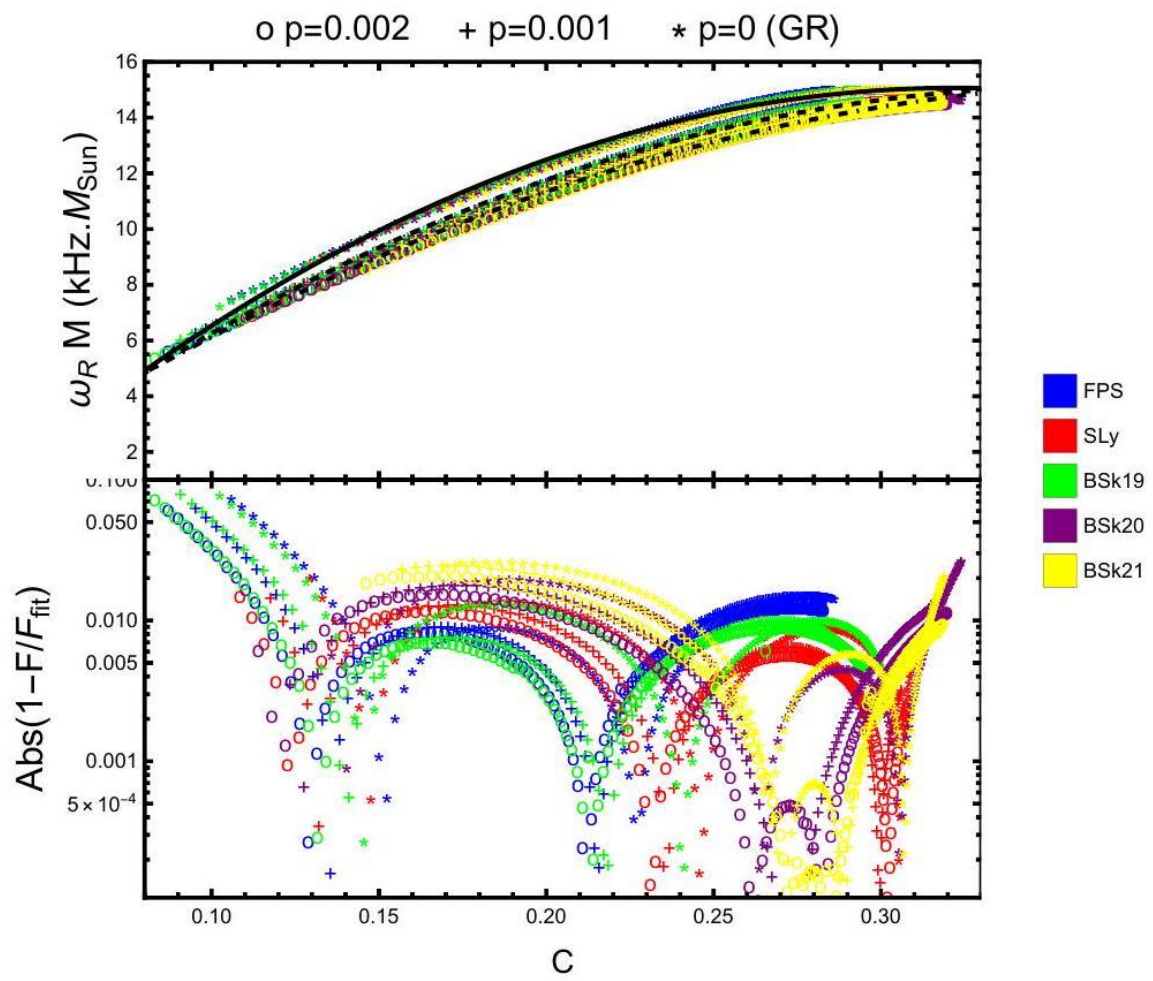
$$\lambda(\omega, r) \sim \lambda_{\text{in}} e^{i(\omega r_s + \eta(\omega_R \cos(\Theta) - \omega_I \sin(\Theta)))} + \lambda_{\text{out}} e^{-i(\omega r_s + \eta(\omega_R \cos(\Theta) - \omega_I \sin(\Theta)))}$$

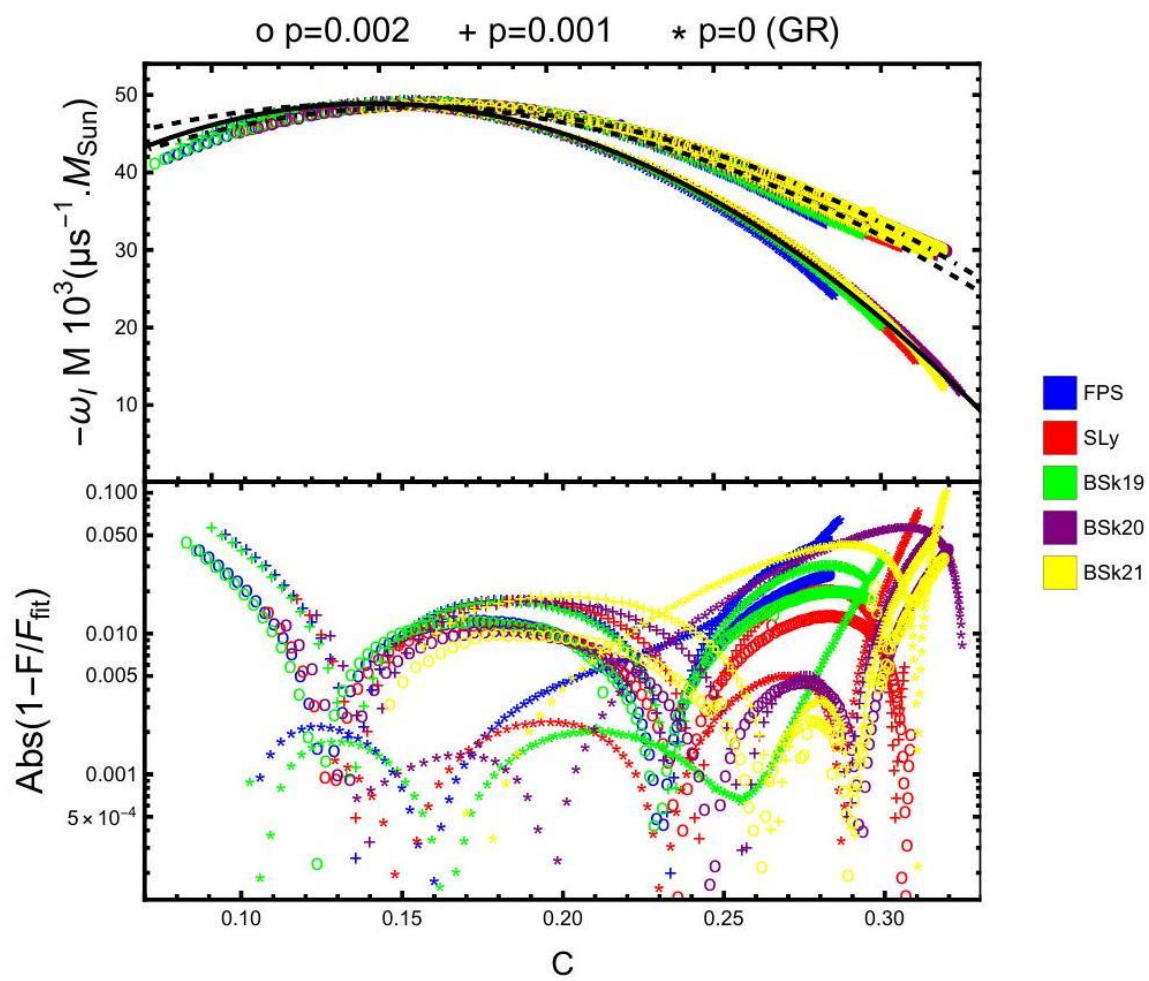
$$\frac{d\lambda}{dr}(r = r_s) = e^{-i\Theta} \frac{d\lambda}{d\eta}(\eta = 0)$$

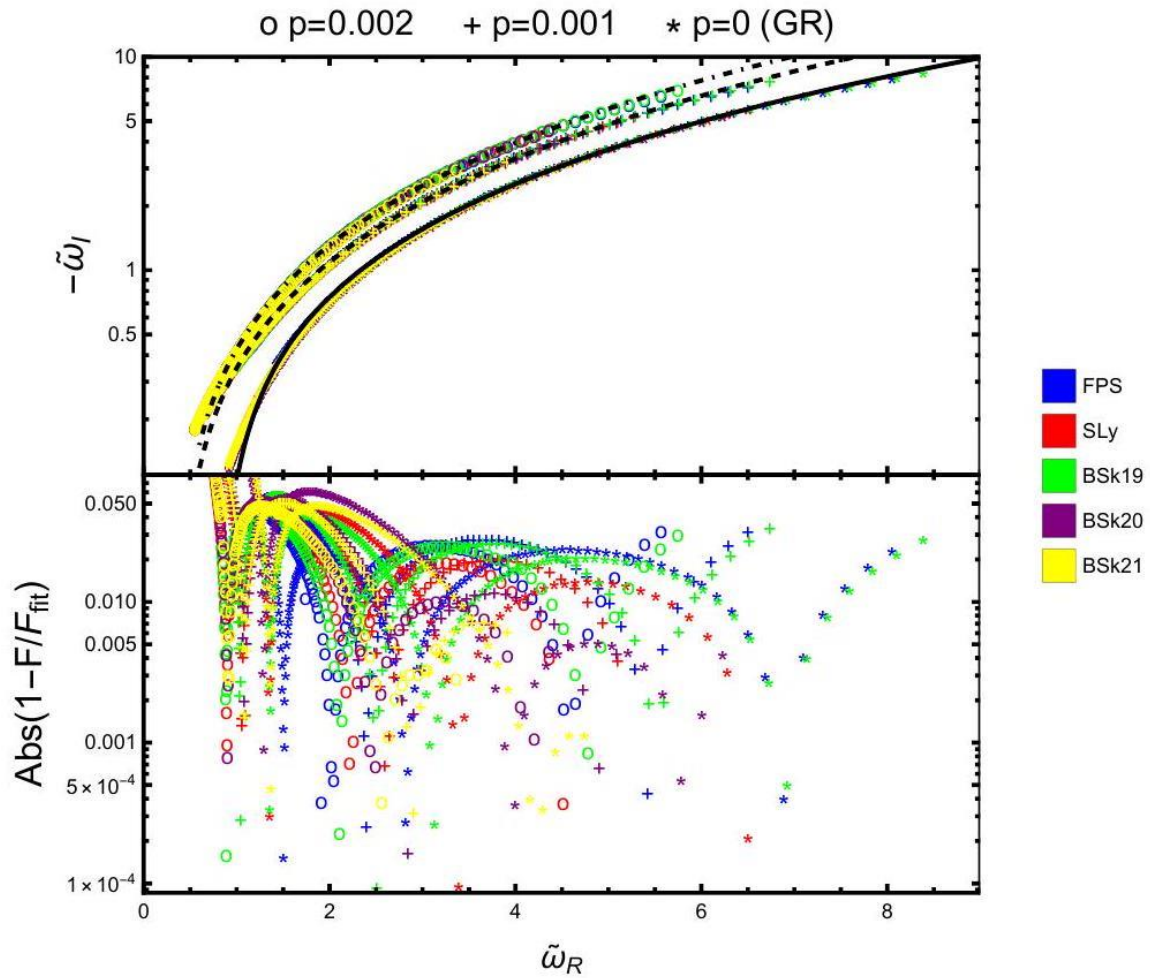
$$C \equiv \frac{M}{r_s},$$

$$\omega_R r_s = \begin{cases} 105.1(\pm 0.5) + 6C(\pm 10) - 369(\pm 22)C^2, & p = 0 \text{ (GR)} \\ 98.6(\pm 0.9) + 7(\pm 8)C - 273(\pm 18)C^2 & p = 10^{-3} \\ 97.5(\pm 0.7) - 34(\pm 6)C - 182.6(\pm 14.2)C^2, & p = 2 \times 10^{-3}. \end{cases}$$









Figuras 24, 25, 26 y 27. Densidad de energía extrema de una partícula blanca o estrella.

$$\omega_R M = \begin{cases} -2.9(\pm 0.2) + 111.5(\pm 2.1)C - 173.2(\pm 4.7)C^2, & p = 0 \text{ (GR)} \\ -1.9(\pm 0.2) + 94.4(\pm 1.6)C - 131.3(\pm 3.5)C^2, & p = 10^{-2} \\ -1.2(\pm 0.1) + 84.4(\pm 1.2)C - 108.0(\pm 2.6)C^2, & p = 2 \times 10^{-2}. \end{cases}$$

$$\omega_I M = \begin{cases} 22.5(\pm 1.1) + 356.4(\pm 9.7)C - 1203(\pm 21)C^2, & p = 0 \text{ (GR)} \\ 32.9(\pm 1.1) + 216.5(\pm 10.7)C - 733(\pm 23)C^2, & p = 10^{-3} \\ 27.0(\pm 0.6) + 262.7(\pm 6.1)C - 803(\pm 14)C^2, & p = 2 \times 10^{-3} \end{cases}$$

$$\tilde{\omega}_I = \begin{cases} -0.379(\pm 0.016) + 0.400(\pm 0.011)\tilde{\omega}_R + 0.082(\pm 0.002)\tilde{\omega}_R^2, & p = 0 \text{ (GR)} \\ -0.151(\pm 0.017) + 0.374(\pm 0.015)\tilde{\omega}_R + 0.124(\pm 0.003)\tilde{\omega}_R^2, & p = 10^{-3} \\ -0.174(\pm 0.015) + 0.456(\pm 0.015)\tilde{\omega}_R + 0.144(\pm 0.003)\tilde{\omega}_R^2, & p = 2 \times 10^{-3} \end{cases}$$

$$S_m[L, A, B] = \int d^4x \sqrt{-g} P(\mu)$$

$$\mu = \sqrt{-g^{\alpha\beta}(\partial_\alpha L + A\partial_\alpha B)(\partial_\beta L + A\partial_\beta B)}$$

$$u_\alpha = \frac{1}{\mu} (\partial_\alpha L + A \partial_\alpha B).$$

$$\nabla_\alpha \left(\frac{dP}{d\mu} u^\alpha \right) = 0, u^\alpha \partial_\alpha A = 0, u^\alpha \partial_\alpha B = 0$$

$$\nabla_{[\alpha}(\mu u_{\beta]}) = \partial_{[\alpha} A \partial_{\beta]} B$$

$$u^\alpha \nabla_{[\alpha}(\mu u_{\beta]}) = 0$$

$$T_{\alpha\beta} = (\rho + P)u_\alpha u_\beta + P g_{\alpha\beta}$$

$$\bar{u}^\alpha = \left\{ \frac{1}{\sqrt{f}}, 0, 0, 0 \right\}, \bar{u}_\alpha = \{-\sqrt{f}, 0, 0, 0\}$$

$$L(t,r)=-\sqrt{f}\mu t$$

$$(\mu\sqrt{f})'=0 \text{ or } \frac{\mu'}{\mu}=-\frac{f'}{2f}$$

$$\delta^2\mu(t,r,\theta)=-\frac{r^2\mu}{2f}v^2(\partial_\theta Y_{l0})^2$$

$$\delta^2\sqrt{-g}=\frac{1}{2}r^4\left(\sqrt{\frac{h}{f}}H_0^2-\sqrt{\frac{f}{h}}H_1^2\right)\sin\theta(\partial_\theta Y_{l0})^2.$$

$$\begin{aligned} S_{\rm m, axial} &= \int d^4x \left(P \delta^2 \sqrt{-g} + \sqrt{-g} \frac{dP}{d\mu} \delta^2 \mu \right) \\ &= -\frac{l(l+1)}{2} \int dr dt \sqrt{f h} r^4 \left(\frac{P}{h} H_1^2 + \frac{\rho}{f} H_0^2 \right) \end{aligned}$$

$$g_{\mu\nu}=g^0_{\mu\nu}+h_{\mu\nu}$$

$$h_{at}=r^2\sum_{lm}H_0^{lm}(r,t)\sin\theta\epsilon_{ab}\nabla^bY_{lm}$$

$$h_{ar}=r^2\sum_{lm}H_1^{lm}(r,t)\sin\theta\epsilon_{ab}\nabla^bY_{lm}$$

$$h_{ab}=\frac{1}{2}r^2\sum_{lm}Q^{lm}(r,t)\sin\theta(\epsilon_a^c\nabla_c\nabla_bY_{lm}+\epsilon_b^c\nabla_c\nabla_aY_{lm})$$

$$\xi^a=r^2\sum_{lm}\Lambda^{lm}(r,t)\sin\theta\epsilon_b^a\nabla^bY_{lm},$$

$$H_0\rightarrow H_0+\dot{\Lambda}, H_1\rightarrow H_1+\Lambda', Q\rightarrow Q+2\Lambda,$$

$$\delta u^a=r^2f^{-1/2}\sum_{lm}v^{lm}(r,t)\sin\theta\epsilon_b^a\nabla^bY_{lm}$$



$$\lambda'' - \frac{2}{r}\lambda' - \frac{(l^2 + l - 2)}{r^2}\lambda$$

$$\mathcal{L}_{NL} = \mathcal{L}_{nm} + \mathcal{L}_{\sigma} + \mathcal{L}_{\omega} + \mathcal{L}_{\rho} + \mathcal{L}_{int},$$

$$\mathcal{L}_{nm} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - M)\psi + g_{\sigma}\sigma\bar{\psi}\psi - g_{\omega}\bar{\psi}\gamma^{\mu}\omega_{\mu}\psi - \frac{g_{\rho}}{2}\bar{\psi}\gamma^{\mu}\vec{\rho}_{\mu}\vec{\tau}\psi$$

$$\mathcal{L}_{\sigma} = \frac{1}{2}(\partial^{\mu}\sigma\partial_{\mu}\sigma - m_{\sigma}^2\sigma^2) - \frac{A}{3}\sigma^3 - \frac{B}{4}\sigma^4$$

$$\mathcal{L}_{\omega} = -\frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_{\omega}^2\omega^{\mu}\omega_{\mu} + \frac{C}{4}(g_{\omega}^2\omega_{\mu}\omega^{\mu})^2$$

$$\mathcal{L}_{\rho} = -\frac{1}{4}\vec{B}^{\mu\nu}\vec{B}_{\mu\nu} + \frac{1}{2}m_{\rho}^2\vec{\rho}_{\mu}\vec{\rho}^{\mu}$$

$$\mathcal{L}_{int} = \frac{1}{2}\Lambda_v g_{\omega}^2 g_{\rho}^2 \omega_{\mu}\omega^{\mu}\vec{\rho}_{\mu}\vec{\rho}^{\mu}$$

$$\Omega_{\mu\nu} = \partial_{\nu}\omega_{\mu} - \partial_{\mu}\omega_{\nu}, \vec{B}_{\mu\nu} = \partial_{\nu}\vec{\rho}_{\mu} - \partial_{\mu}\vec{\rho}_{\nu} - g_{\rho}(\vec{\rho}_{\mu} \times \vec{\rho}_{\nu})$$

$$\mathcal{E}_{HM} = \frac{1}{\pi^2} \int_0^{k_p, k_n} dk k^2 \sqrt{k^2 + (M^*)^2} + \frac{1}{2} m_{\sigma}^2 \sigma^2 - \frac{1}{2} m_{\omega}^2 \omega^2 - \frac{1}{2} m_{\rho}^2 \rho^2 + \frac{A}{3} \sigma^3 + \frac{B}{4} \sigma^4 - \frac{C}{4} g_{\omega}^4 \omega^4 + g_{\omega} \omega (\rho_p + \rho_n) - \frac{\Lambda_v}{2} (g_{\rho} g_{\omega} \rho \omega)^2 + \frac{g_{\rho}}{2} \rho (\rho_p - \rho_n)$$

$$P_{HM} = \frac{1}{3\pi^2} \int_0^{k_p, k_n} dk \frac{k^4}{\sqrt{k^2 + (M^*)^2}} - \frac{1}{2} m_{\sigma}^2 \sigma^2 + \frac{1}{2} m_{\omega}^2 \omega^2 + \frac{1}{2} m_{\rho}^2 \rho^2 - \frac{A}{3} \sigma^3 - \frac{B}{4} \sigma^4 + \frac{C}{4} g_{\omega}^4 \omega^4 + \frac{\Lambda_v}{2} (g_{\rho} g_{\omega} \rho \omega)^2$$

Model	Interaction	g_{σ}	g_{ω}	g_{ρ}	A	B	C	Λ_v
BITSH-E	with ω^4 ($C \neq 0$)	10.0	12.52	10.32	10.62	-14.07	0.0027	0.08
BITSH-I	with ω^4 ($C \neq 0$)	9.05	10.69	9.95	15.29	-13.56	0.0006	0.06
BITSH-E	without ω^4 ($C = 0$)	9.58	11.58	10.44	15.13	-38.21	-	0.1702
BITSH-I	without ω^4 ($C = 0$)	8.96	10.35	9.95	19.71	-46.44	-	0.0726

$$\mathcal{L}_{\chi} = \bar{\chi}_D [\gamma_{\mu} (i\partial^{\mu} - g_{vd} V_D^{\mu}) - M_D] \chi_D - \frac{1}{4} V_{\mu\nu,D} V_D^{\mu\nu} + \frac{1}{2} m_{vd}^2 V_{\mu,D} V_D^{\mu}.$$



$$\begin{aligned}\varepsilon_{DM} &= \frac{1}{\pi^2} \int_0^k dk k^2 \sqrt{k^2 + (M_D)^2} + \frac{g_{vd}^2}{2m_{vd}^2} \rho_D^2, \\ P_{DM} &= \frac{1}{3\pi^2} \int_0^k dk \frac{k^4}{\sqrt{k^2 + (M_D)^2}} + \frac{g_{vd}^2}{2m_{vd}^2} \rho_D^2,\end{aligned}$$

$$\begin{aligned}\frac{dP_{HM}}{dr} &= -(P_{HM} + \varepsilon_{HM}) \frac{4\pi r^3 (P_{HM} + P_{DM}) + m(r)}{r(r - 2m(r))}, \\ \frac{dP_{DM}}{dr} &= -(P_{DM} + \varepsilon_{DM}) \frac{4\pi r^3 (P_{HM} + P_{DM}) + m(r)}{r(r - 2m(r))}, \\ \frac{dm(r)}{dr} &= 4\pi (\varepsilon_{HM}(r) + \varepsilon_{DM}(r)) r^2.\end{aligned}$$

$$f_{DM} = \frac{M_{DM}}{M_{\text{Total}}}$$

$$M_{DM}(R_{DM}) = 4\pi \int_0^{R_{DM}} r^2 \varepsilon_{DM}(r) dr$$

$$M_{\text{Total}}(R) = 4\pi \int_0^R r^2 (\varepsilon_{HM}(r) + \varepsilon_{DM}(r)) dr$$

$$\lambda = \frac{2}{3} k_2 R^5, \Lambda = \frac{2}{3} k_2 \mathcal{C}^{-5}, \mathcal{C} = M/R$$

$$\begin{aligned}k_2 = \frac{8\mathcal{C}^5}{5} (1 - 2\mathcal{C})^2 [2 + 2\mathcal{C}(y_R - 1) - y_R] \times & (2\mathcal{C}(6 - 3y_R + 3\mathcal{C}(5y_R - 8)) \\ & + 4\mathcal{C}^3[13 - 11y_R + \mathcal{C}(3y_R - 2) + 2\mathcal{C}^2(1 + y_R)] \\ & + 3(1 - 2\mathcal{C})^2(2 - y_R + 2\mathcal{C}(y_R - 1)) \log(1 - 2\mathcal{C}))^{-1}\end{aligned}$$

$$r\frac{dy(r)}{dr}+y^2(r)+y(r)F(r)+r^2Q(r)=0.$$

$$F(r)=\frac{r-4\pi r^3(\varepsilon_{HM}(r)+\varepsilon_{DM}(r))-(P_{HM}(r)+P_{DM}(r))}{r-2m(r)},$$

$$\begin{aligned}Q(r) = \frac{4\pi r \left(5(\varepsilon_{HM}(r) + \varepsilon_{DM}(r)) + 9(P_{HM}(r) + P_{DM}(r)) + \frac{\varepsilon_{HM}(r) + P_{HM}(r)}{\partial P_{HM}(r)/\partial \varepsilon_{HM}(r)} + \frac{\varepsilon_{DM}(r) + P_{DM}(r)}{\partial P_{DM}(r)/\partial \varepsilon_{DM}(r)} - \frac{6}{4\pi r^2} \right)}{r - 2m(r)} \\ - 4 \left[\frac{m(r) + 4\pi r^3 (P_{HM}(r) + P_{DM}(r))}{r^2 (1 - 2m(r)/r)} \right]^2\end{aligned}$$

$$c_s^2 = \frac{1}{3} - c_1 \exp \left[-\frac{(n - c_2)^2}{n_{bl}^2} \right] + h_p \exp \left[\frac{(n - n_p)^2}{w_p^2} \right] \left(1 + \operatorname{erf} \left[s_p \frac{(n - n_p)}{w_p} \right] \right).$$

$$P(\boldsymbol{\Theta} \mid D) = \frac{\mathcal{L}(D \mid \boldsymbol{\Theta})P(\boldsymbol{\Theta})}{\mathcal{Z}},$$

$$\mathcal{L}(D \mid \boldsymbol{\Theta}) = \mathcal{L}_{obs}(D \mid \boldsymbol{\Theta}) \times \mathcal{L}_{GW}(D \mid \boldsymbol{\Theta}).$$



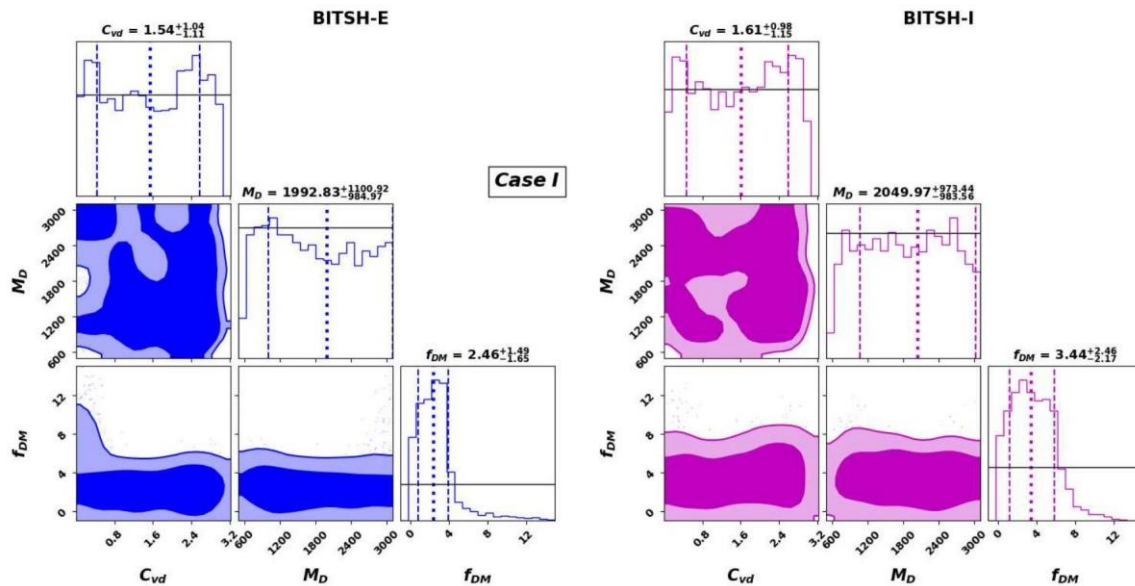
$$\mathcal{L}_{\sigma}(D | \Theta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left(-\frac{(D(\Theta) - D_{Obs})^2}{2\sigma^2} \right)$$

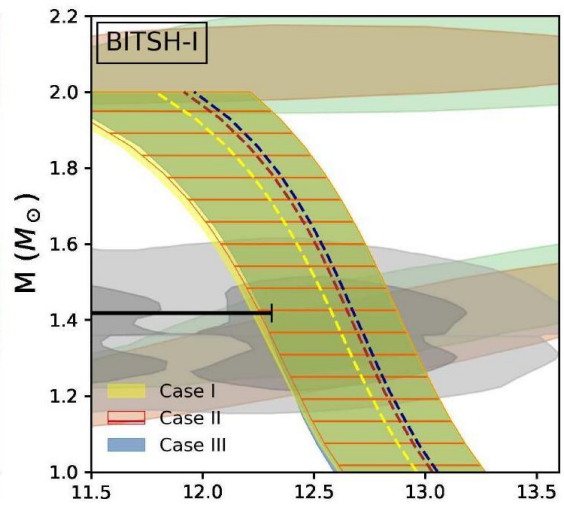
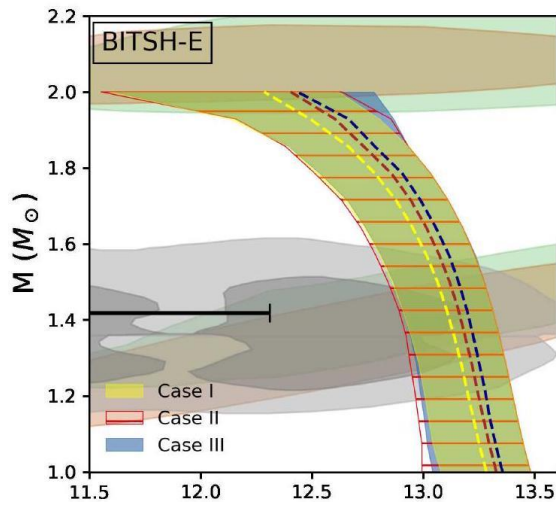
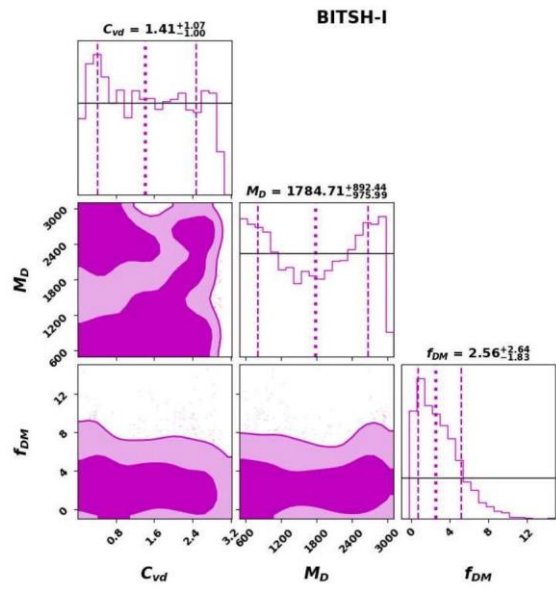
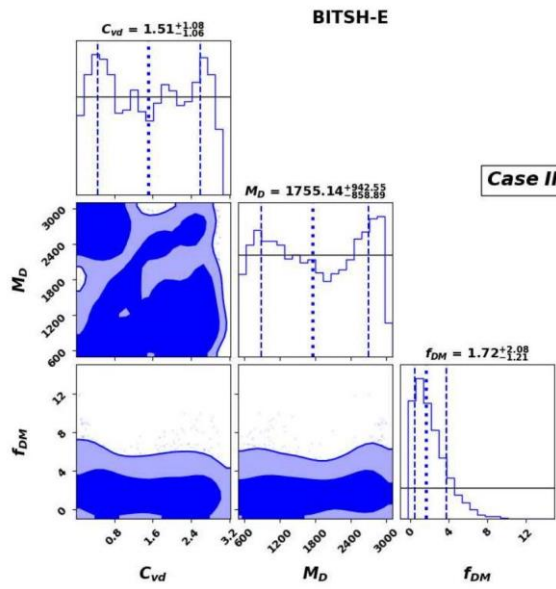
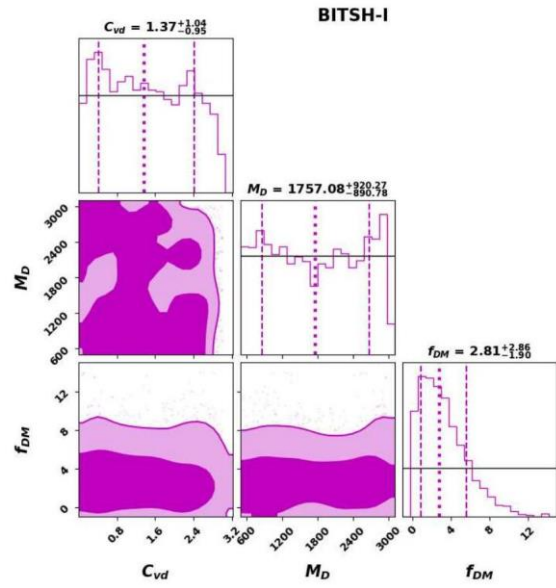
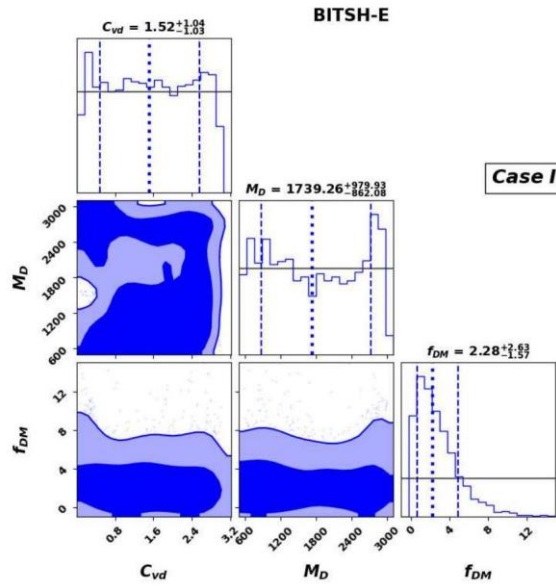
$$\mathcal{L}_{Obs}(D | \Theta) = \mathcal{L}_{NICER}^{high} \times \mathcal{L}_{NICER}^{low}$$

$$\mathcal{L}_{NICER}(D | \Theta) = \int_{M_0}^{M_{max}} dm P(m | \Theta) \times P(D | m, R(m, \Theta))$$

$$\mathcal{L}_{GW}(D | \Theta) = \int_{m_l}^{M_u} dm_1 \int_{M_l}^{m_1} dm_2 P(m_1, m_2 | \Theta) \\ \times P(d_{GW} | m_1, m_2, \Lambda_1(m_1, \Theta), \Lambda_2(m_2, \Theta))$$

$$P(m | \Theta) = \begin{cases} \frac{1}{M_{max} - M_0} & \text{if } M_0 \leq m \leq M_{max} \\ 0 & \text{else} \end{cases}$$





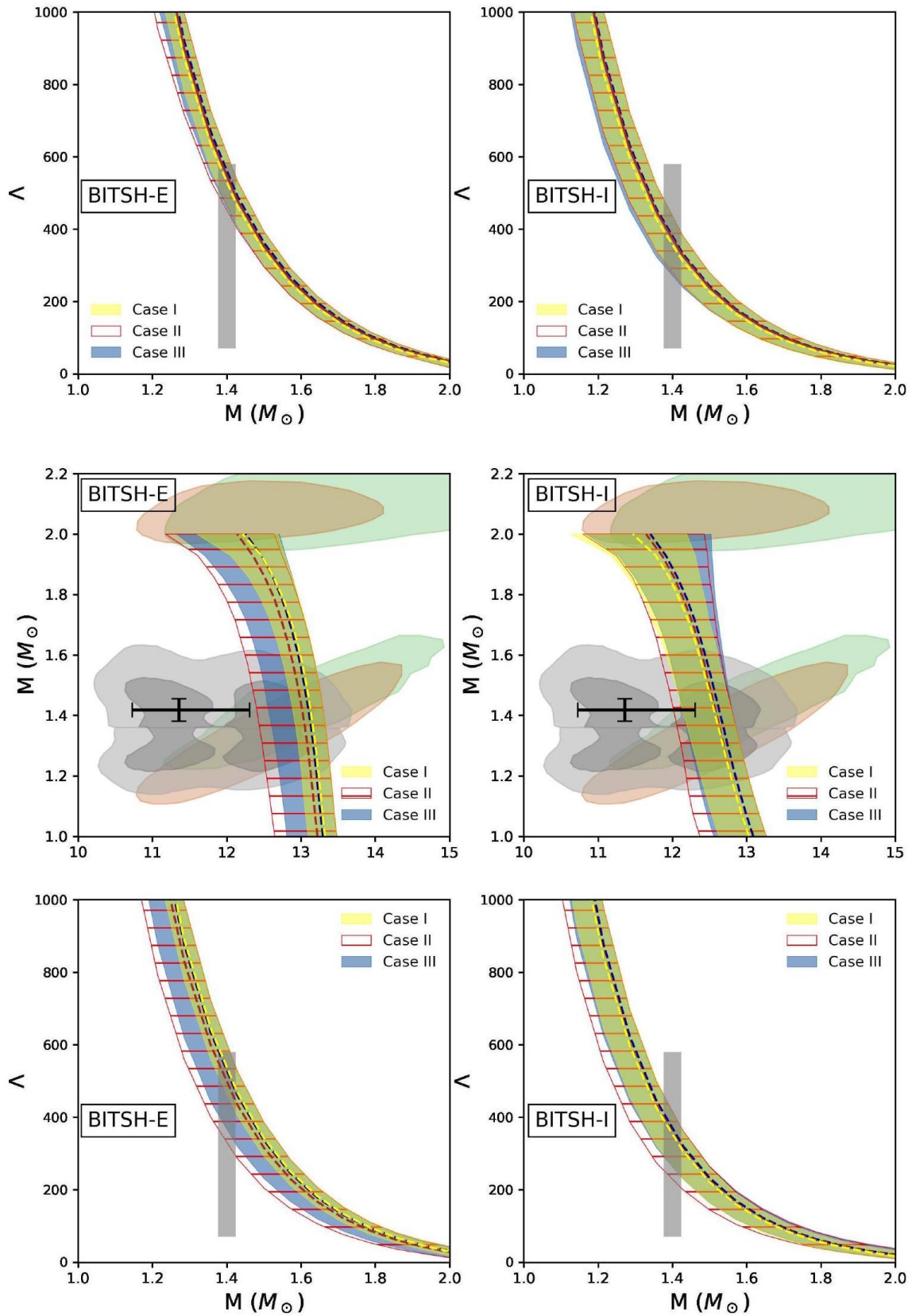


Figura 28, 29, 30, 31, 32, 33 y 34. Formación de materia oscura por colapso de una partícula estrella o blanca.

Models	BITSH-E			BITSH-I		
DM parameter	$C_{vd}(\text{MeV}^{-1}) (\times 10^{-2})$	$M_D(\text{MeV})$	$f_{DM}(\%)$	$C_{vd}(\text{MeV}^{-1}) (\times 10^{-2})$	$M_D(\text{MeV})$	$f_{DM}(\%)$

Fixed HM EoS						
Case I	$1.54^{+1.04}_{-1.11}$	$1992.83^{+1100.91}_{-984.97}$	$2.46^{+1.49}_{-1.65}$	$1.61^{+0.98}_{-1.15}$	$2049.96^{+973.43}_{-983.56}$	$3.44^{+2.46}_{-2.17}$
Case II	$1.52^{+1.04}_{-1.03}$	$1739.25^{+979.92}_{-862.07}$	$2.28^{+2.63}_{-1.57}$	$1.37^{+1.04}_{-0.95}$	$1757.07^{+920.26}_{-890.77}$	$2.81^{+2.86}_{-1.90}$
Case III	$1.51^{+1.08}_{-1.06}$	$1755.13^{+942.55}_{-858.89}$	$1.72^{+2.08}_{-1.21}$	$1.41^{+1.07}_{-1.00}$	$1784.70^{+892.43}_{-975.98}$	$2.56^{+2.64}_{-1.83}$

HM EoS with HDU						
Case I	$1.53^{+0.97}_{-1.04}$	$1962.62^{+1069}_{-938.55}$	$2.36^{+1.38}_{-1.52}$	$1.48^{+0.97}_{-0.96}$	$2099.68^{+935.17}_{-986.67}$	$3.65^{+2.22}_{-2.28}$
Case II	$1.50^{+0.95}_{-0.94}$	$1706.20^{+836.3}_{-740.55}$	$3.36^{+3.37}_{-2.19}$	$1.48^{+0.97}_{-0.99}$	$1832.88^{+767.58}_{-836.21}$	$2.85^{+3.11}_{-1.91}$
Case III	$1.49^{+0.94}_{-1.00}$	$1849.82^{+774.37}_{-888.29}$	$2.40^{+2.73}_{-1.57}$	$1.43^{+1.06}_{-0.95}$	$1818.82^{+779.63}_{-846.20}$	$2.28^{+2.75}_{-1.57}$

HM EoS ($C = 0$) with HDU						
Case I	$1.58^{+0.91}_{-0.96}$	$1936.31^{+939.70}_{-812.02}$	$8.12^{+5.74}_{-4.86}$	$1.57^{+0.94}_{-1.01}$	$2056.07^{+968.79}_{-972.54}$	$4.37^{+3.52}_{-2.83}$



Case II	$1.58^{+0.93}_{-0.95}$	$1783.23^{+826}_{-762.95}$	$4.82^{+4.73}_{-2.97}$	$1.54^{+0.93}_{-1.04}$	$1833.26^{+834.36}_{-903.34}$	$2.92^{+3.51}_{-1.99}$
Case III	$1.53^{+0.95}_{-1.01}$	$1689.08^{+858.60}_{-782.98}$	$3.58^{+4.00}_{-2.41}$	$1.58^{+0.92}_{-1.10}$	$1820.58^{+800.18}_{-883.46}$	$2.35^{+2.92}_{-1.63}$

Interaction terms	Models	$C_{\nu d}(\text{MeV}^{-1}) (\times 10^{-2})$	$M_D(\text{MeV})$	$f_{DM}(\%)$
$C = 0, \Lambda_\nu = 0$	GM1	$1.63^{+0.85}_{-0.92}$	$1835.18^{+732.33}_{-787.64}$	$5.34^{+4.12}_{-3.28}$
	NL3	$1.41^{+0.89}_{-0.76}$	$1841.52^{+697}_{-700.78}$	$15.39^{+3.08}_{-4.38}$
$C \neq 0, \Lambda_\nu = 0$	MS1	$1.71^{+0.84}_{-1.02}$	$1704.71^{+759.38}_{-704.62}$	$5.63^{+4.89}_{-2.95}$
	TM1	$1.69^{+0.86}_{-0.94}$	$1580.95^{+865}_{-668.57}$	$5.19^{+3.87}_{-2.68}$
$C \neq 0, \Lambda_\nu \neq 0$	BITSH-E	$1.50^{+0.95}_{-0.94}$	$1706.20^{+836.3}_{-740.55}$	$3.36^{+3.37}_{-2.19}$
	BITSH-I	$1.48^{+0.97}_{-0.99}$	$1832.88^{+767.58}_{-836.21}$	$2.85^{+3.11}_{-1.91}$
$C = 0, \Lambda_\nu \neq 0$	BITSH-E	$1.58^{+0.93}_{-0.95}$	$1783.23^{+826}_{-762.95}$	$4.82^{+4.73}_{-2.97}$
	BITSH-I	$1.54^{+0.93}_{-1.04}$	$1833.26^{+834}_{-903.34}$	$2.92^{+3.51}_{-1.99}$

$$\mathcal{Q}_{\psi\gamma\gamma} = \nabla_{\psi} g_{\gamma\gamma} = \partial_{\psi} g_{\gamma\gamma} - \Gamma_{\psi\gamma}^{\sigma} g_{\sigma\gamma} - \Gamma_{\psi\gamma}^{\sigma} g_{\gamma\sigma}.$$

$$\Gamma_{\gamma\gamma}^{\sigma} = \{ \sigma_{\gamma\gamma} \} + \zeta_{\gamma\gamma}^{\sigma} + L_{\gamma\gamma}^{\sigma},$$

$$\{ \sigma_{\gamma\gamma} \} \equiv \frac{1}{2} g^{\sigma\Psi} (\partial_{\gamma} g_{\Psi\gamma} + \partial_{\gamma} g_{\Psi\gamma} - \partial_{\Psi} g_{\gamma\gamma}),$$

$$\zeta_{\gamma\gamma}^{\sigma} \equiv \frac{1}{2} \mathcal{T}^{\sigma}{}_{\gamma\gamma} + \mathcal{T}_{(\gamma}{}^{\sigma}{}_{\gamma)}$$



$$L^\sigma_{\gamma\Upsilon}\equiv\frac{1}{2}Q^\sigma_{\gamma\Upsilon}-Q_{(\gamma}{}^\sigma{}_{\Upsilon)}$$

$$H_{\gamma\Upsilon}^\psi=\frac{-1}{4}Q_{\gamma\Upsilon}^\psi+\frac{1}{2}Q_{(\gamma\Upsilon)}^\psi+\frac{1}{4}(Q^\psi-\tilde{Q}^\psi)g_{\gamma\Upsilon}-\frac{1}{4}\delta_{(\gamma\Upsilon)}^\psi$$

$$\mathcal{Q}_\psi\equiv\mathcal{Q}^\gamma_{\psi\gamma}\;\tilde{\mathcal{Q}}_\psi\equiv\mathcal{Q}^\gamma{}^\gamma_{\psi\gamma},$$

$$\mathcal{Q}=-\mathcal{Q}_{\psi\Upsilon\gamma}F^{\psi\gamma\Upsilon}.$$

$$S=\int\sqrt{-g}d^4x\left[\frac{1}{2}\mathcal{F}(\mathcal{Q})+\sigma_\psi^{\Psi\Upsilon}\mathcal{R}^\psi_{\Psi\gamma\Upsilon}+\sigma_\psi^{\gamma\Upsilon}\mathcal{T}^\psi_{\gamma\Upsilon}+\mathcal{L}_n\right]$$

$$\begin{aligned}\mathcal{T}_{\gamma\Upsilon}=&\frac{2}{\sqrt{-g}}\Delta_\psi\left(\sqrt{-g}\mathcal{F}_\mathcal{Q}H^\psi_{\gamma\Upsilon}\right)\\&+\frac{1}{2}g_{\gamma\Upsilon}\mathcal{F}+\mathcal{F}_\mathcal{Q}\left(H_{\gamma\psi\Psi}\mathcal{Q}^{\psi\Psi}_\Upsilon-2\mathcal{Q}_{\psi\Psi\gamma}H^\psi_{\Upsilon}{}^{\psi\Psi}\right).\end{aligned}$$

$$\mathcal{F}_\mathcal{Q}=\frac{\partial \mathcal{F}}{\partial \mathcal{Q}}$$

$$\mathcal{T}_{\gamma\Upsilon}\equiv\frac{2}{\sqrt{g}}\frac{\delta(\sqrt{-g})\mathcal{L}_n}{\delta g^{\gamma\Upsilon}}.$$

$$\nabla_\rho\sigma_\psi^{\Upsilon\rho\gamma}+\sigma_\psi^{\gamma\Upsilon}=\sqrt{-g\mathcal{F}_\mathcal{Q}}H^\psi_{\gamma\Upsilon}+J_\psi^{\gamma\Upsilon}.$$

$$J_\psi^{\gamma\Upsilon}=\frac{-1}{2}\frac{\delta\mathcal{L}_n}{\delta T_{\gamma\Upsilon}^\psi}.$$

$$\nabla_\gamma\nabla_\Upsilon(\sqrt{-g}\mathcal{F}_\mathcal{Q}H^{\gamma\Upsilon}{}_\psi+J_\psi{}^{\gamma\Upsilon})=0.$$

$$\nabla_\gamma\nabla_\Upsilon(\sqrt{-g}\mathcal{F}_\mathcal{Q}H^{\gamma\Upsilon}{}^\gamma_\psi)=0.$$

$$\Gamma_{\gamma\Upsilon}^\psi=\left(\frac{\partial x^a}{\partial \eta^\sigma}\right)\partial_\gamma\partial_\Upsilon\eta^\sigma.$$

$$\mathcal{Q}_{\psi\gamma\Upsilon}=\partial_\psi g_{\gamma\Upsilon}.$$

$$ds^2=-e^{\Upsilon(r)}dt^2+e^{\sigma(r)}dr^2+r^2dw^2.$$

$$\mathcal{Q}(r)=-\frac{2e^{-\sigma}}{r}\Big(\Upsilon'+\frac{1}{r}\Big),$$

$$\mathcal{T}_{\gamma\Upsilon}=(\rho+p_t)u_\gamma u_\Upsilon+p_t g_{\gamma\Upsilon}+(p_r-p_t)v_\gamma v_\Upsilon,$$

$$\begin{aligned}\rho &= -\frac{\mathcal{F}}{2} + \mathcal{F}_Q \left[Q + \frac{1}{r^2} + \frac{e^{-\sigma}}{r} (Y' + \sigma') \right], \\ p_r &= \frac{\mathcal{F}}{2} - \mathcal{F}_Q \left[Q + \frac{1}{r^2} \right], \\ p_t &= \frac{\mathcal{F}}{2} - \mathcal{F}_Q \frac{Q}{2} - \mathcal{F}_Q e^{-\sigma} \frac{Q}{2} \\ &\quad - \mathcal{F}_Q e^{-\sigma} \left[\frac{Y''}{2} + \left(\frac{Y'}{4} + \frac{1}{2r} \right) (Y' - \sigma') \right], \\ 0 &= \frac{\cot \theta}{2} Q' \mathcal{F}_{QQ}. \\ \frac{\cot \theta}{2} Q' \mathcal{F}_{QQ} &= 0.\end{aligned}$$

$$\mathcal{F}_{QQ} = 0 \Rightarrow \mathcal{F}(Q) = \delta_1 Q + \delta_2.$$

$$\rho = \frac{1}{16\pi r^2} [2\delta_1 + 2e^{-\sigma} \delta_1 (r\sigma' - 1) - r^2 \delta_2],$$

$$p_r = \frac{1}{16\pi r^2} [-2\delta_1 + 2e^{-\sigma} \delta_1 (rY' + 1) + r^2 \delta_2],$$

$$p_t = \frac{e^{-\sigma}}{32\pi r} [2e^{\sigma} r \delta_2 + \delta_1 (2 + rY') (Y' - \sigma') + 2r \delta_1 Y'']$$

$$\Delta(r) = \frac{\delta_1}{8\pi} \left[e^{-\sigma} \left(\frac{Y''}{2} - \frac{\sigma' Y'}{4} + \frac{(Y')^2}{4} - \frac{Y' + \sigma'}{2r} - \frac{1}{r^2} \right) + \frac{1}{r^2} \right].$$

$$R_{1414} = \frac{R_{1212} R_{3434} - R_{1224} R_{1334}}{R_{2323}},$$

$$R_{2323} = \frac{\sin^2 \theta (e^{\sigma} - 1) r^2}{e^{-\sigma}}, R_{1212} = \frac{\sigma' r}{2}$$

$$R_{2424} = -\frac{Y' r e^{Y-\sigma}}{4}, R_{1334} = R_{1224} \sin^2 \theta = 0$$

$$R_{1414} = -\frac{1}{4e^Y} [2Y'' + (Y')^2 - \sigma' Y'], R_{3434} = \sin^2 \theta R_{2424}$$

$$\frac{2Y'' + (Y')^2}{Y'} = \frac{\sigma' e^{\sigma}}{(e^{\sigma} - 1)}.$$

$$e^Y = \left[A_1 + B_1 \int \sqrt{(e^{\sigma(r)} - 1)} dr \right]^2$$

$$e^{\sigma} = b^2 r^2 \operatorname{csch}(fr^2 + h) \operatorname{csch}(fr^2 + h) + 1,$$

$$e^Y = \left[A_1 + \frac{B_1 b}{2f} \log \left(\frac{e^{(fr^2+h)} - 1}{e^{(fr^2+h)} + 1} \right) \right]^2.$$



$$8\pi\rho = -\frac{b^4r^2(\delta_2r^2 - 2\delta_1)\text{csch}^4(fr^2 + h) + \delta_2 + \phi_1}{2(b^2r^2\text{csch}^2(fr^2 + h) + 1)^2}$$

$$8\pi p_r = \frac{8B_1b\delta_1fr^2\log(e)e^{fr^2+h} + \delta_1\phi_2}{r^2\phi_3(e^{fr^2+h} - 1)(e^{fr^2+h} + 1)} + \frac{\delta_2}{2} - \frac{\delta_1}{r^2},$$

$$8\pi p_t = \frac{\delta_2\phi_5(b^4r^4\text{csch}^4(fr^2 + h) + 1) + 2b^2\phi_6 + \phi_4}{2\phi_7(e^{fr^2+h} - 1)^2(e^{fr^2+h} + 1)^2}$$

$$8\pi\Delta = \frac{\delta_2\phi_5(b^4r^4\text{csch}^4(fr^2 + h) + 1) + 2b^2\phi_6 + \phi_4}{2\phi_7(e^{fr^2+h} - 1)^2(e^{fr^2+h} + 1)^2} - \frac{8B_1b\delta_1fr^2\log(e)e^{fr^2+h} + \delta_1\phi_2}{r^2\phi_3(e^{fr^2+h} - 1)(e^{fr^2+h} + 1)} - \frac{\delta_2}{2} + \frac{\delta_1}{r^2}$$

Compact particle star	$M(M_0)$	R(nm)	δ_1	$b(nm^{-1})$	$f(nm^{-1})$	h	$\frac{M}{R}$	A_1	B_1
Cen X-3	1.48	11.6	0.2	0.107	-0.0011	1.39	0.128291379	0.0269800	0.0263286
Cen X-3	1.49	11.6	0.4	0.107	-0.0011	1.39	0.128481897	0.0235791	0.0264463
Cen X-3	1.49	11.6	0.6	0.107	-0.0011	1.39	0.128544828	0.0224455	0.0264855
Cen X-3	1.49	11.6	0.8	0.107	-0.0011	1.39	0.128576724	0.0218787	0.0265051
Cen X-3	1.49	11.6	1.0	0.107	-0.0011	1.39	0.128671552	0.0201782	0.026564

Expression	Remarks
ρ, p_r, p_t	> 0 , acceptable



$\underline{p}_{rc}$	≤ 0
	> 0 , acceptable
$\frac{d\rho}{dr}, \frac{dp_r}{dr}, \frac{dp_t}{dr}$	< 0 , acceptable
F_a, F_h, F_g	balanced
$\rho \pm p_r$	> 0 , acceptable
$\rho \pm p_t$	> 0 , acceptable
$\rho + p_r$ $+ 2p_t$	> 0 , acceptable
$m(r)$	> 0 , acceptable
$u(r)$	$0 < u(r) < \frac{8}{9}$, acceptable
Z_S	$0 < Z_S < 5$, acceptable
ω_r, ω_t	$0 < \omega_r, \omega_t < 1$, acceptable
V_r^2, V_t^2	$0 < V_r^2, V_t^2 < 1$, acceptable
$V_t^2 - V_r^2$	$-1 < V_t^2 - V_r^2 < 1$, acceptable
Γ	$> \frac{4}{3}$, acceptable

$$ds^2 = -\left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$



$$\begin{aligned}\left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right) &= e^{\Upsilon(R)} \\ \left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right) &= e^{-\sigma(R)} \\ p_r(R) &= 0\end{aligned}$$

$$\left(\frac{\partial w_{ss}^-}{\partial r}\right)_{r=R} = \left(\frac{\partial w_{ss}^+}{\partial r}\right)_{r=R}$$

$$w_{ss}^- = e^{\Upsilon}, w_{ss}^+ = \left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right).$$

$$\Upsilon'(R) = \frac{6M - 2\Lambda R^3}{3R^2 - 6MR - \Lambda R^4}$$

$$\Upsilon'(R) = \frac{6M - 2\Lambda R^3}{3R^2 - 6MR - \Lambda R^4}$$

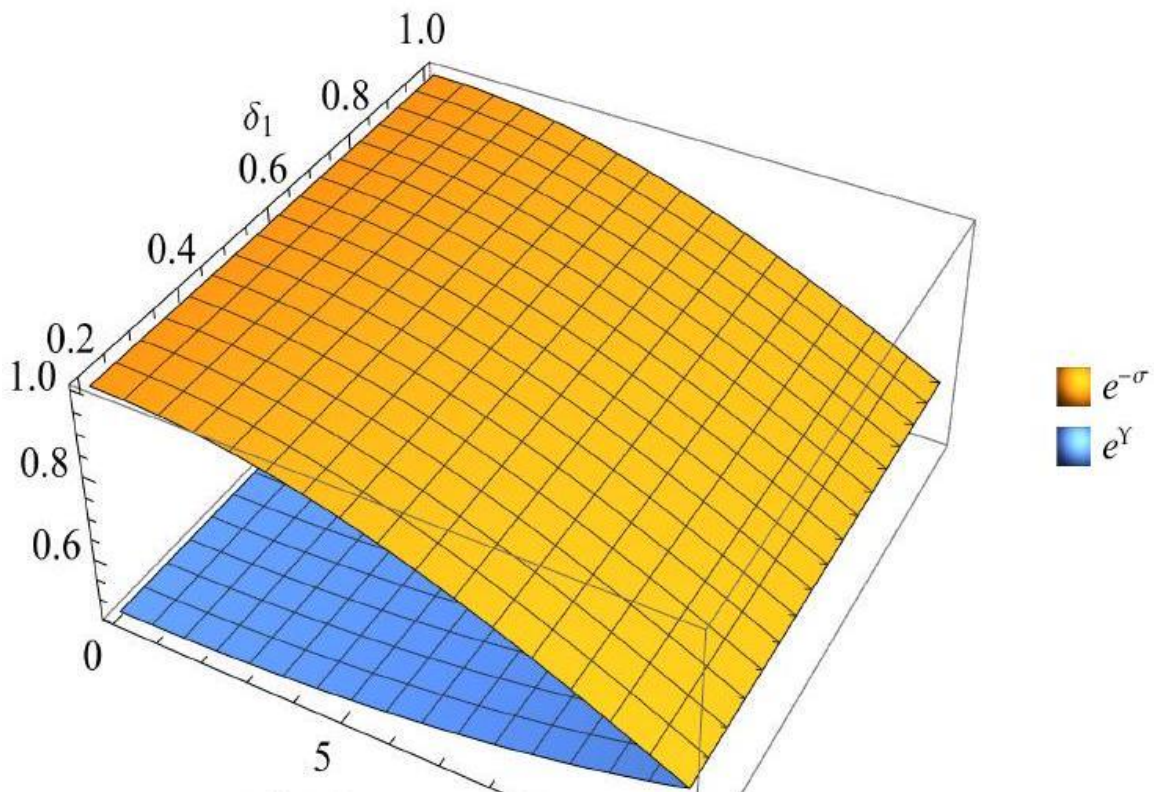


Figura 35. Deformación extrema del espacio – tiempo cuántico por interacción de una partícula blanca o estrella.

$$\begin{aligned}A_1 &= \frac{B_1 b (-\phi_{10}(\delta_2 + \phi_{11}) - 16\delta_1 f \log(e) e^{fR^2+h})}{2f\phi_{12}(e^{2(fR^2+h)} - 1)} \\ B_1 &= -\frac{\phi_{13} e^{-fR^2-h} (e^{2(fR^2+h)} - 1)}{8b\delta_1 \log(e)}\end{aligned}$$

$$\omega_r = \left(\frac{8B_1 b \delta_1 f r^2 \log(e) e^{f r^2 + h} + \delta_1 \phi_2}{r^2 \phi_3 (e^{f r^2 + h} - 1)(e^{f r^2 + h} + 1)} + A \right) B$$

$$\omega_t = \left(\frac{\delta_2 \phi_5 (b^4 r^4 \operatorname{csch}^4(f r^2 + h) + 1) + C}{2 \phi_7 (e^{f r^2 + h} - 1)^2 (e^{f r^2 + h} + 1)^2} \right) B$$

$$\begin{aligned} NEC: \rho &\geq 0, \\ WEC: \rho + p_r &\geq 0, \rho + p_t \geq 0, \\ DEC: \rho - p_r &\geq 0, \rho - p_t \geq 0, \\ SEC: \rho + p_r + 2p_t &\geq 0, \\ TEC: \rho - p_r - 2p_t &\geq 0. \end{aligned}$$

$$\rho + p_r, \rho + p_t, \rho - p_r, \rho - p_t, \rho + p_r + 2p_t, \rho - p_r - 2p_t$$

$$\Gamma < \frac{4}{3} - \left[\frac{4}{3} \frac{p_r - p_t}{|p_r'| r} \right]_{\max}$$

$$\frac{dp_r}{dr} + \frac{Y'(\rho + p_r)}{2} - \frac{2(p_t - p_r)}{r} = 0$$

$$F_g + F_h + F_a = 0; F_g = -\frac{Y'(\rho + p_r)}{2}, F_h = -\frac{dp_r}{dr}, F_a = \frac{2(p_t - p_r)}{r}$$

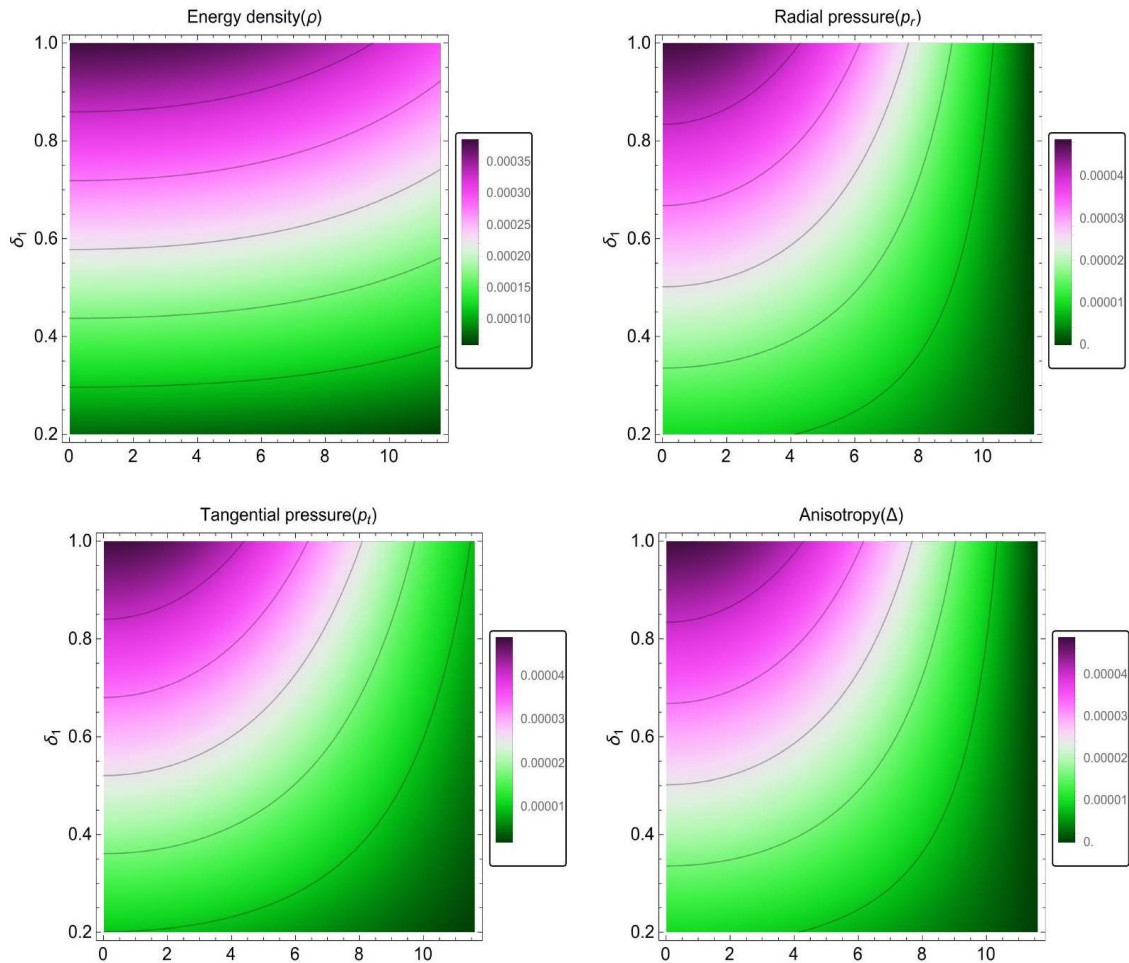
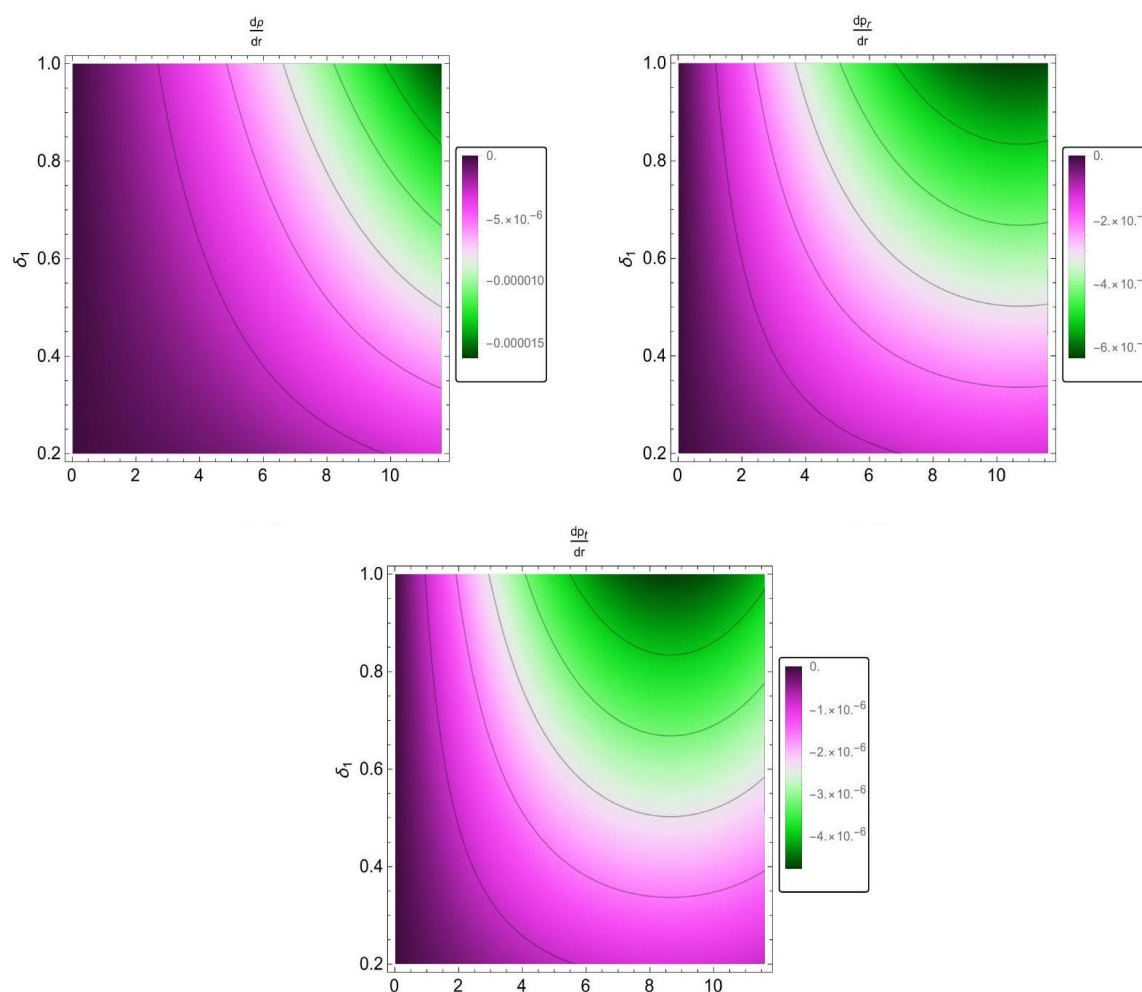


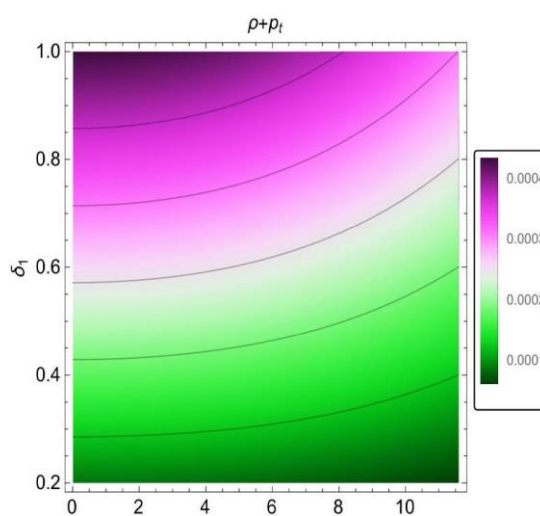
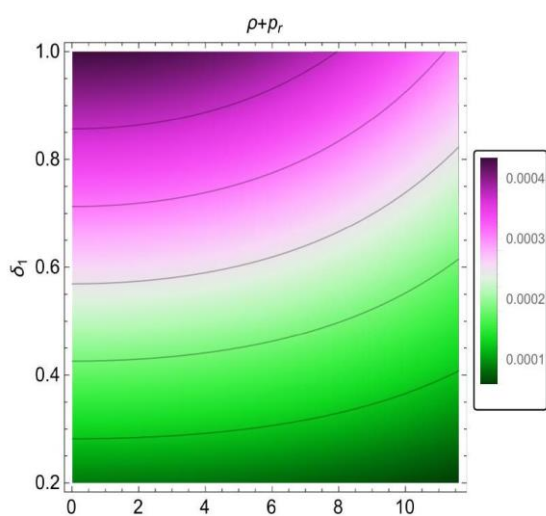
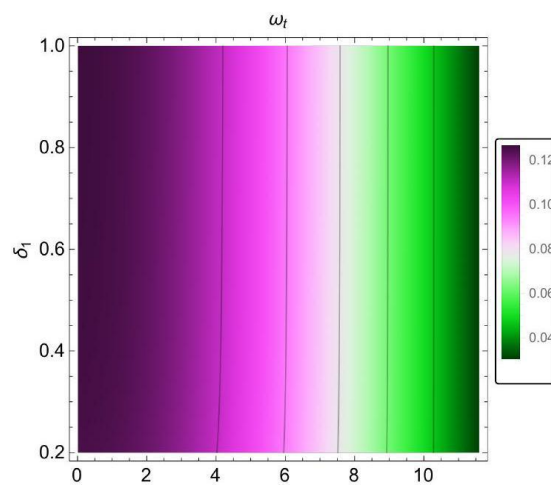
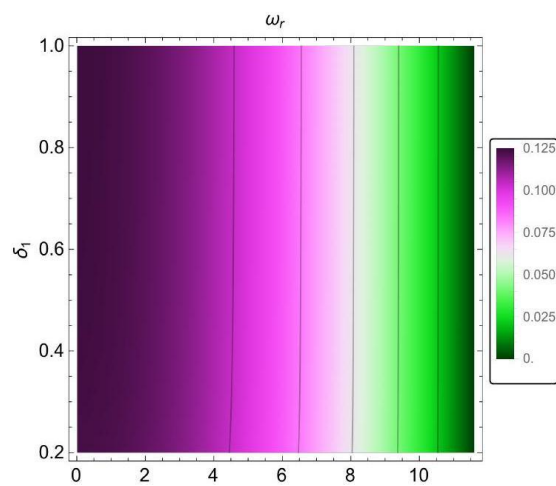
Figura 36. Densidad de energía, presión radial, presión tangencial y asintropía de una partícula blanca o estrella.

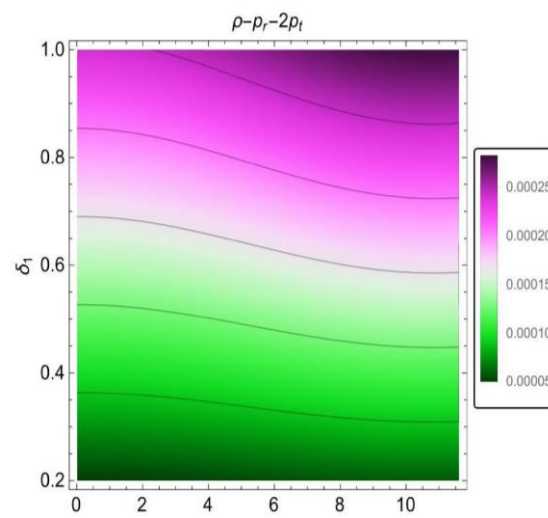
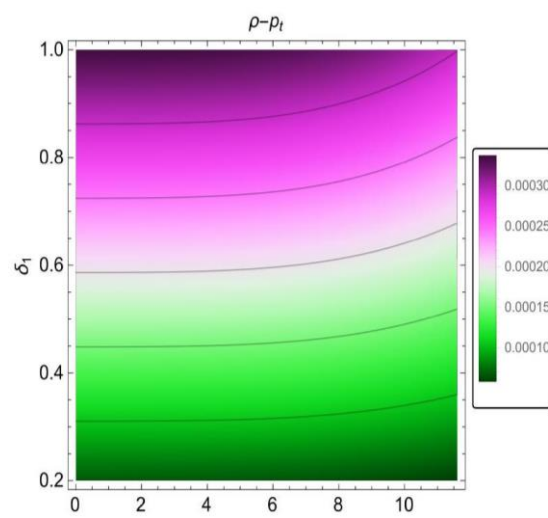
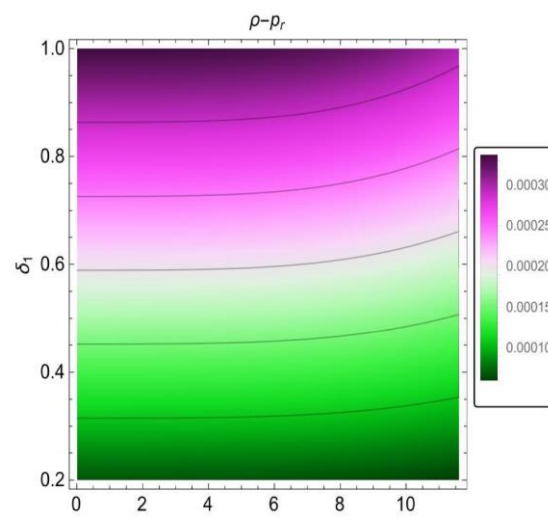
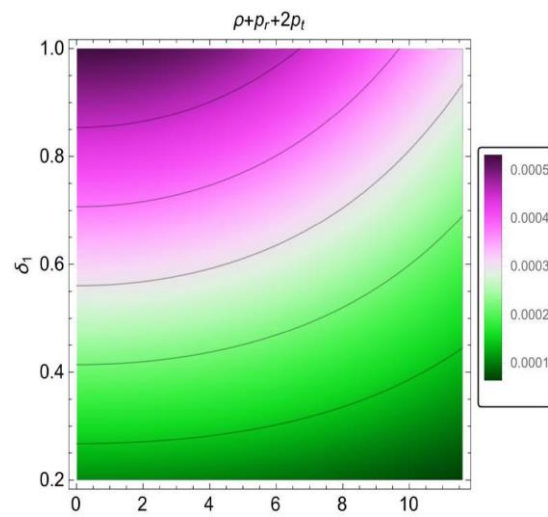
$$V_r^2 = \frac{dp_r}{d\rho}, V_t^2 = \frac{dp_t}{d\rho}$$

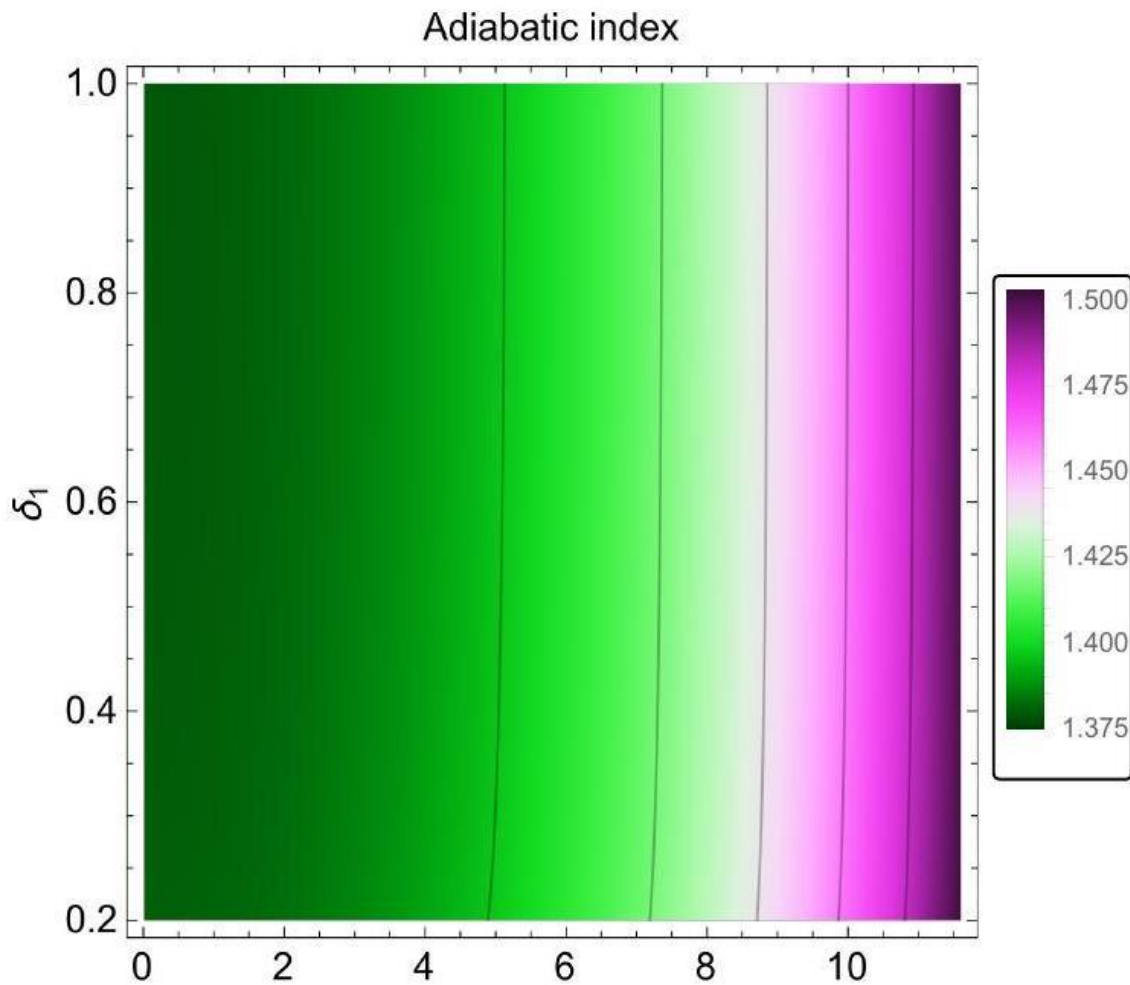
$$V_r^2 = \frac{dp_r}{d\rho} V_t^2 = \frac{dp_r}{d\rho}$$

$$\rho''(r) \leq 0$$









Figuras. 37, 38, 39, 40, 41, 42 y 43. Gradiente de la densidad de energía $\left(\frac{d\rho}{dr}\right)$, gradiente de la presión radial $\left(\frac{dp_r}{dr}\right)$, gradiente de la presión tangencial $\left(\frac{dp_t}{dr}\right)$ y la indexación adiabática $\left(\frac{dp_{index\ adiabatic}}{dr}\right)$ de una partícula blanca o estrella.

$$M(\rho_c) = \frac{R}{2} \left(-\frac{D}{E(2\rho_c + \delta_2) + D} - \frac{\delta_2 R^2}{6\delta_1} + 1 \right).$$

$$\frac{dM}{d\rho_c} = \frac{4EDR}{(4E\rho_c + 2E\delta_2 + 12\delta_1 \text{csch}^2(h))^2}$$

$$Z_G = \frac{1}{\sqrt{|e^{Y(r)}|}} - 1 = \frac{1}{A_1 + \frac{B_1 b}{2f} \log \left(\frac{e^{(fr^2+h)} - 1}{e^{(fr^2+h)} + 1} \right)} - 1$$

$$Z_S = \frac{1}{\sqrt{1 - 2u(r)}} - 1$$

$$m(r) = \frac{\kappa}{2} \int_0^r \rho r^2 dr$$

$$u(r) = \frac{\kappa}{2r} \int_0^r \rho r^2 dr$$

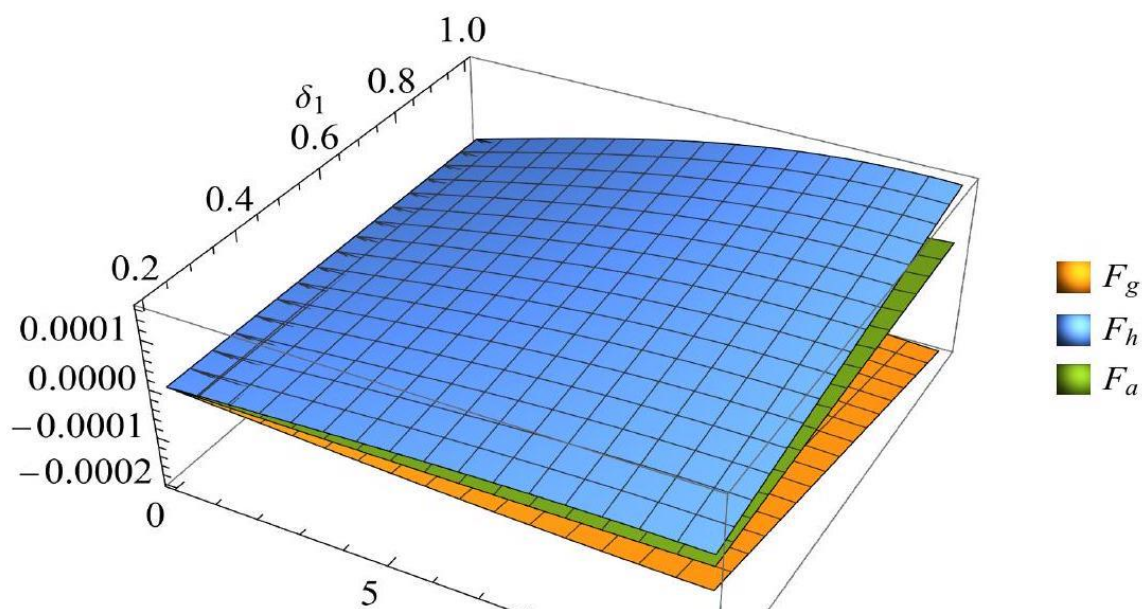
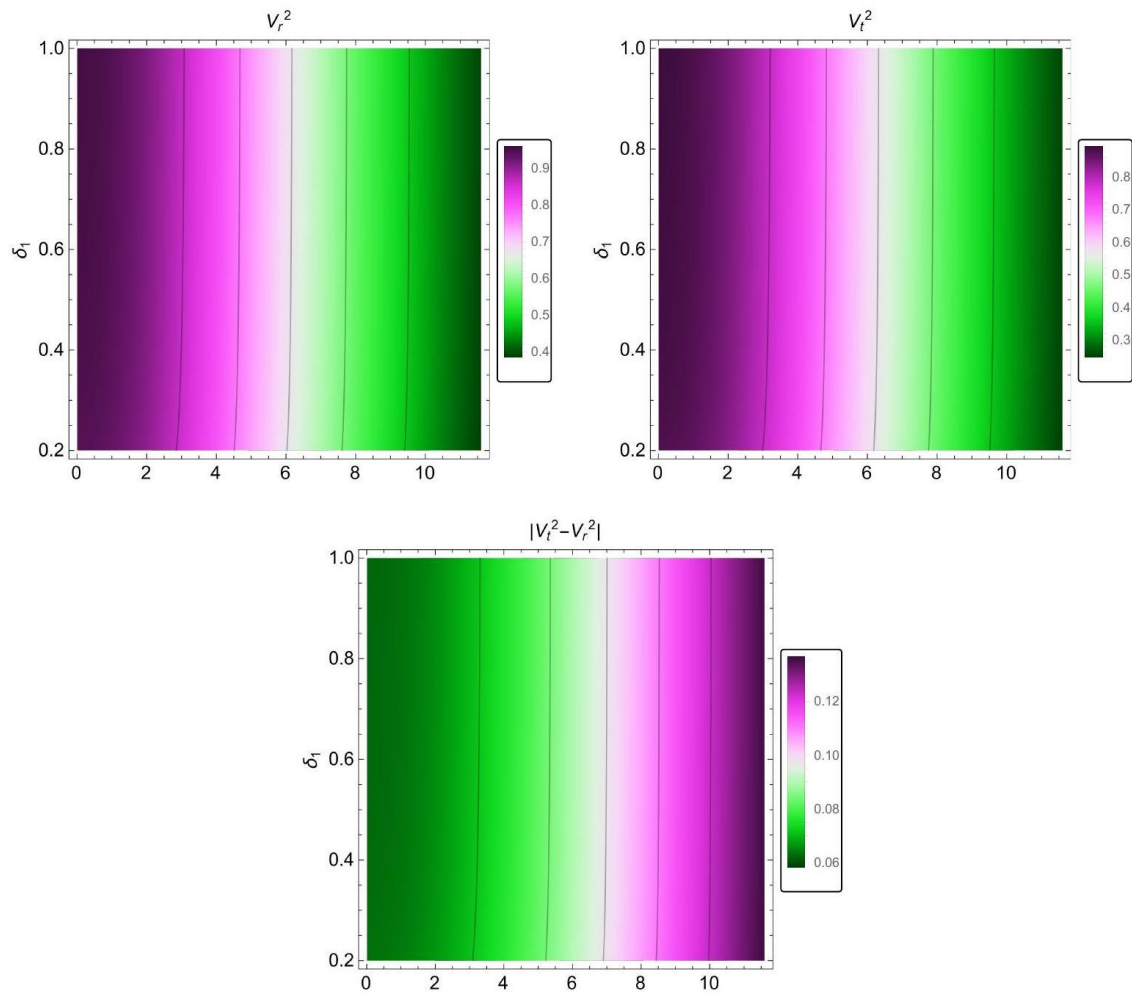


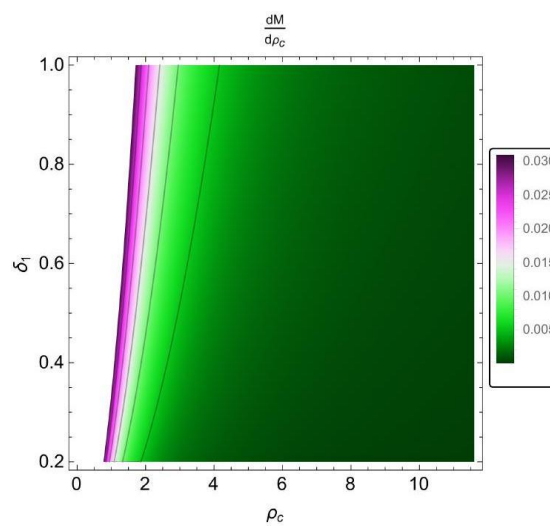
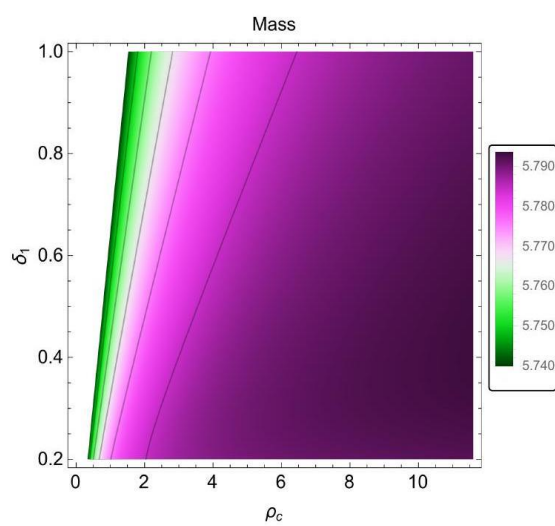
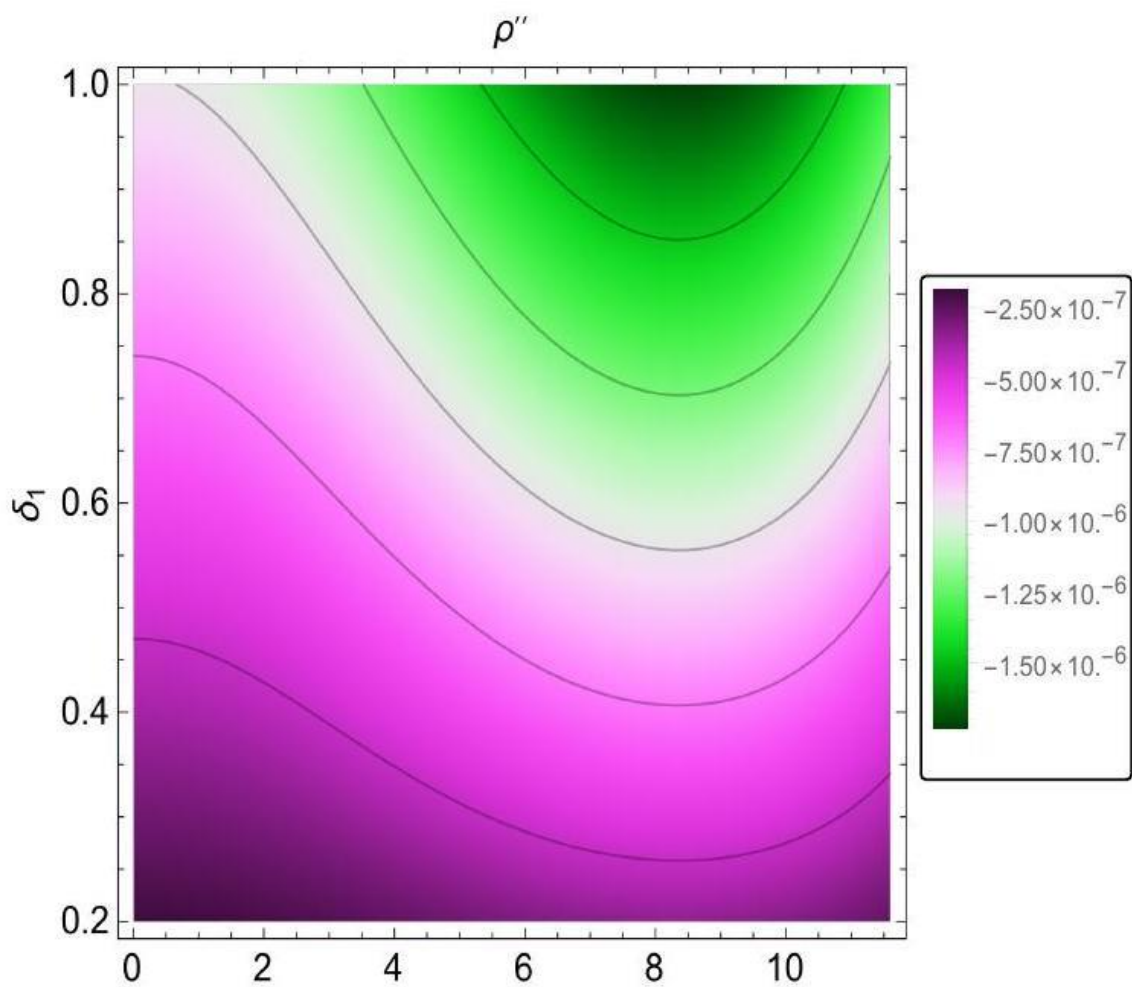
Figura. 44. Fuerza gravitacional (F_g), fuerza hidrostática (F_h) y fuerza anisotrópica (F_a) de una partícula blanca o estrella.

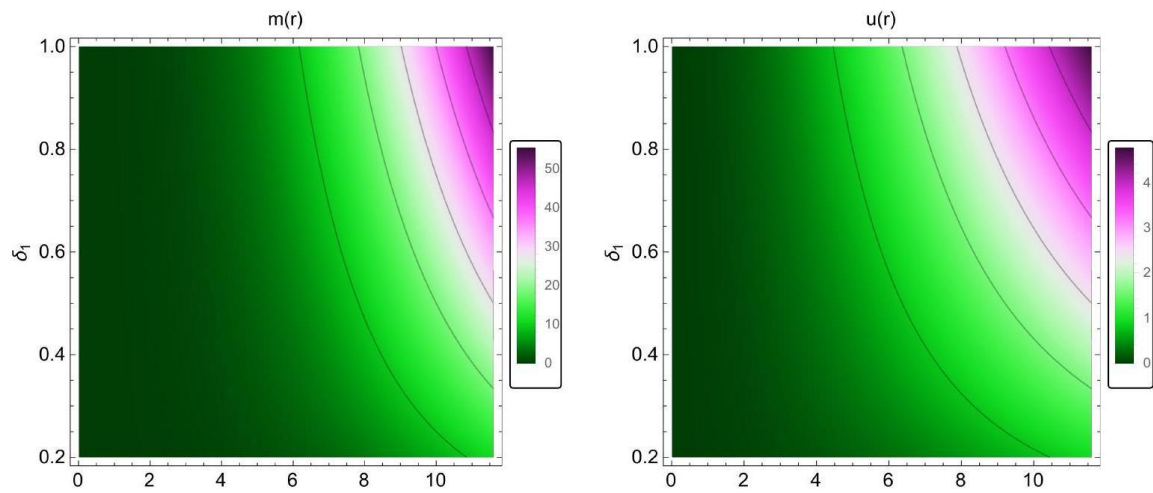
Compact particle star	δ_1	density value at $r = 0$ (gm/nm ³)	density value at $r = R$ (gm/nm ³)	pressure value at $r = 0$ (dyne /nm ²)
Cen X-3	0.2	0.041349×10^{14}	0.03222×10^{14}	0.043506×10^{34}
Cen X-3	0.4	0.082698×10^{14}	0.06444×10^{14}	0.091846×10^{34}
Cen X-3	0.6	0.124047×10^{14}	0.09666×10^{14}	0.140186×10^{34}
Cen X-3	0.8	0.165396×10^{14}	0.12888×10^{14}	0.188526×10^{34}
Cen X-3	1.0	0.206745×10^{14}	0.16110×10^{14}	0.236866×10^{34}

$$m(r) = -\frac{2\pi r^3(2b^2(\delta_2 r^2 - 6\delta_1) + \delta_2(\phi_{14} - 1))}{3(2b^2 r^2 + \phi_{14} - 1)}$$

$$u(r) = -\frac{2\pi r^2(2b^2(\delta_2 r^2 - 6\delta_1) + \delta_2(\phi_{14} - 1))}{3(2b^2 r^2 + \phi_{14} - 1)}$$







Figuras 45, 46, 47 y 48. Velocidad radial (V_r^2), velocidad tangencial (V_t^2) y velocidad resultante ($|V_t^2 - V_r^2|$) y comportamiento de la masa $\frac{dM}{d\rho_c}$ de una partícula estrella o blanca.

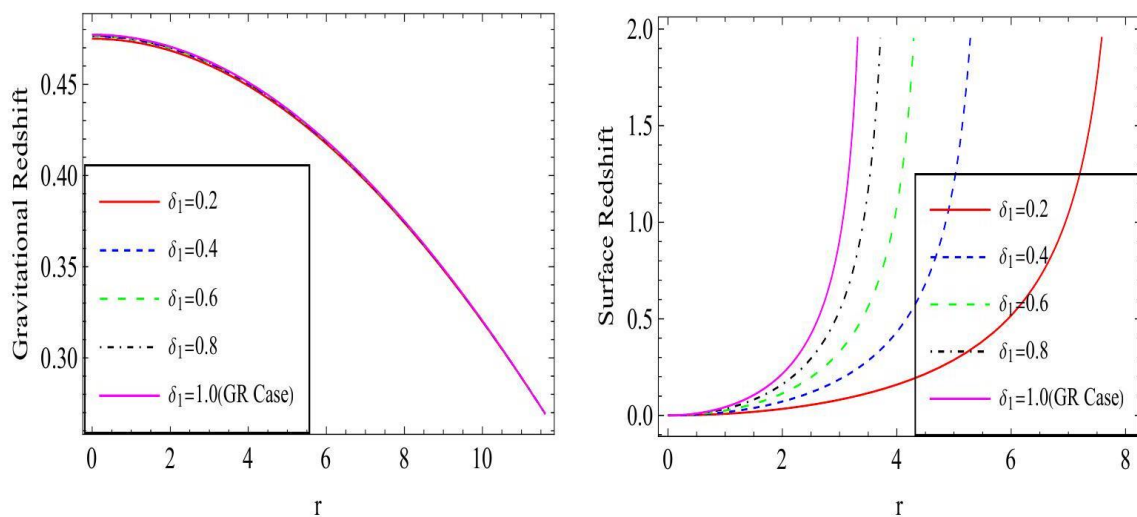


Figura 49. Gravitational Redshift (Z_G) y Surface Redshift (Z_S) de una partícula blanca o estrella.

$$\begin{aligned}
\phi_1 &= 2b^2 \operatorname{csch}^2(fr^2 + h)(-3\delta_1 + 4\delta_1 fr^2 \coth(fr^2 + h) + \delta_2 r^2) \\
\phi_2 &= (e^{2(fr^2+h)} - 1) \left(2A_1 f + B_1 b \log \left(\frac{e^{fr^2+h} - 1}{e^{fr^2+h} + 1} \right) \right) \\
\phi_3 &= (b^2 r^2 \operatorname{csch}^2(fr^2 + h) + 1) \left(2A_1 f + B_1 b \log \left(\frac{e^{fr^2+h} - 1}{e^{fr^2+h} + 1} \right) \right) \\
\phi_4 &= 16B_1 b \delta_1 f \log(e) e^{fr^2+h} (e^{2(fr^2+h)} - 1) - 16B_1 b \delta_1 f^2 r^2 \log^2(e) e^{fr^2+h} (e^{2(fr^2+h)} + 1) \\
\phi_5 &= (e^{2(fr^2+h)} - 1)^2 \left(2A_1 f + B_1 b \log \left(\frac{e^{fr^2+h} - 1}{e^{fr^2+h} + 1} \right) \right) \\
\phi_6 &= \operatorname{csch}^2(fr^2 + h) (4B_1 b \delta_1 fr^2 \log(e) (e^{2(fr^2+h)} - 1) e^{fr^2+h} - 8B_1 \phi_8 + 2\delta_1 \phi_9 - \phi_5 (\delta_1 - \delta_2 r^2)) \\
\phi_7 &= (b^2 r^2 \operatorname{csch}^2(fr^2 + h) + 1)^2 \left(2A_1 f + B_1 b \log \left(\frac{e^{fr^2+h} - 1}{e^{fr^2+h} + 1} \right) \right) \\
\phi_8 &= b \delta_1 f^2 r^4 \log^2(e) e^{fr^2+h} (e^{2(fr^2+h)} + 1) \\
\phi_9 &= 2\delta_1 fr^2 (e^{2(fr^2+h)} - 1) \coth(fr^2 + h) (4B_1 b fr^2 \log(e) e^{fr^2+h} + \phi_2) \\
\phi_{10} &= (e^{2(fR^2+h)} - 1) \log \left(\frac{e^{fR^2+h} - 1}{e^{fR^2+h} + 1} \right) \\
\phi_{11} &= b^2 (2\delta_1 - \delta_2 R^2) \operatorname{csch}^2(fR^2 + h) \\
\phi_{12} &= \delta_1 - b^2 (2\delta_1 - \delta_2 R^2) \operatorname{csch}^2(fR^2 + h) \\
\phi_{13} &= \sqrt{\frac{1}{b^2 R^2 \operatorname{csch}^2(fR^2 + h) + 1}} (\delta_2 - b^2 (2\delta_1 - \delta_2 R^2) \operatorname{csch}^2(fR^2 + h)) \\
\phi_{14} &= \cosh(2(fr^2 + h)) \\
A &= \frac{\delta_2}{2} - \frac{\delta_1}{r^2} \\
B &= -\frac{2(b^2 r^2 \operatorname{csch}^2(fr^2 + h) + 1)^2}{b^4 r^2 (\delta_2 r^2 - 2\delta_1) \operatorname{csch}^4(fr^2 + h) + \delta_2 + \phi_1} \\
C &= 2b^2 \phi_6 + \phi_4 \\
D &= 6\delta_1 \operatorname{csch}^2(h) \\
E &= R^2 \operatorname{csch}(fR^2 + h) \operatorname{csch}(fR^2 + h) \\
\phi_{15} &= 1 + b^2 r^2 \operatorname{csch}(fr^2 + h)^2 \\
\phi_{16} &= aB_1 \delta_1 \left(\frac{2fr \log(e) e^{fr^2+h}}{e^{fr^2+h} + 1} - \frac{2fr \log(e) e^{fr^2+h} (e^{fr^2+h} - 1)}{(e^{fr^2+h} + 1)^2} \right) (8bB_1 fr^2 \log(e) e^{fr^2+h} + \phi_2) \\
\phi_{18} &= \frac{2\delta_1 f \log(e) e^{fr^2+h} (8bB_1 fr^2 \log(e) e^{fr^2+h} + \phi_2)}{r \phi_3 (e^{fr^2+h} - 1) (e^{fr^2+h} + 1)^2} \\
\phi_{19} &= \delta_1 (2b^2 fr \operatorname{csch}^2(fr^2 + h) - 4b^2 fr^3 \coth(fr^2 + h) \operatorname{csch}^2(fr^2 + h)) (8bB_1 fr^2 \log(e) e^{fr^2+h} + \phi_2) \\
\phi_{20} &= A - \frac{b^4 r^2 (\delta_2 r^2 - 2\delta_1) \operatorname{csch}^4(fr^2 + h) + \delta_2 + \phi_1}{2\phi_{15}^2} + \frac{\delta_1 (8bB_1 fr^2 \log(e) e^{fr^2+h} + \phi_2)}{r^2 \phi_3 (e^{fr^2+h} - 1) (e^{fr^2+h} + 1)} \\
\phi_{21} &= A + \frac{\delta_1 (8bB_1 fr^2 \log(e) e^{fr^2+h} + \phi_2)}{r^2 \phi_3 (e^{fr^2+h} - 1) (e^{fr^2+h} + 1)} \\
\phi_{22} &= \frac{2fr \log(e) e^{fr^2+h}}{e^{fr^2+h} + 1} - \frac{2fr \log(e) e^{fr^2+h} (e^{fr^2+h} - 1)}{(e^{fr^2+h} + 1)^2}
\end{aligned}$$



$$\begin{aligned}
\phi_{23} &= \frac{bB_1\phi_{22}(e^{fr^2+h}+1)(e^{2(fr^2+h)}-1)}{e^{fr^2+h}-1} \\
\phi_{24} &= \frac{2\delta_1(8bB_1fr^2\log(e)e^{fr^2+h}+\phi_2)}{r^3\phi_3(e^{br^2+h}+1)(e^{fr^2+h}-1)} \\
\phi_{25} &= \frac{4fr\log(e)e^{2(fr^2+h)}\left(2A_1f+bB_1\log\left(\frac{e^{br^2+c}-1}{e^{b^2+c}+1}\right)\right)}{r^2\phi_3(e^{fr^2+h}-1)(e^{fr^2+h}+1)} \\
\phi_{26} &= b^4r^2(\delta_2r^2-2\delta_1)\operatorname{csch}^4(fr^2+h)+2b^2\operatorname{csch}^2(fr^2+h)(-3\delta_1+4\delta_1fr^2\coth(fr^2+h)+\delta_2r^2)+\delta_2 \\
\phi_{27} &= 2b^4r(\delta_2r^2-2\delta_1)\operatorname{csch}^4(fr^2+h)+2b^4\delta_2r^3\operatorname{csch}^4(fr^2+h) \\
&\quad -8b^4fr^3(\delta_2r^2-2\delta_1)\coth(fr^2+h)\operatorname{csch}^4(fr^2+h) \\
&\quad +2b^2\operatorname{csch}^2(fr^2+h)(-8\delta_1f^2r^3\operatorname{csch}^2(fr^2+h)+8\delta_1fr\coth(fr^2+h)+2\delta_2r) \\
&\quad -8b^2fr\coth(fr^2+h)\operatorname{csch}^2(fr^2+h)(-3\delta_1+4\delta_1fr^2\coth(fr^2+h)+\delta_2r^2) \\
\phi_{28} &= 2b^2r\operatorname{csch}^2(fr^2+h)-4b^2fr^3\coth(fr^2+h)\operatorname{csch}^2(fr^2+h) \\
\phi_{29} &= -\frac{\phi_{19}}{r^2\phi_7(e^{fr^2+h}-1)(e^{fr^2+h}+1)} \\
\phi_{30} &= \frac{2\delta_1}{r^3}-\frac{\phi_{16}}{r^2\phi_5\phi_{15}}-2\phi_{18}-\phi_{24}-\phi_{29} \\
\Gamma &= \frac{\phi_{20}\phi_{28}\phi_{26}(\delta_1(16bB_1f^2r^3\log^2(e)e^{fr^2+h}+16bB_1fr\log(e)e^{fr^2+h}+\phi_{23}+\phi_{25})+\phi_{30})}{\phi_{21}\phi_{15}^3}-\frac{\phi_{27}}{2\phi_{15}^2} \\
\rho+p_r &= A-\frac{b^4r^2(\delta_2r^2-2\delta_1)\operatorname{csch}^4(fr^2+h)+\delta_2+\phi_1}{2\phi_{15}^2}+\frac{\delta_1(8bB_1fr^2\log(e)e^{fr^2+h}+\phi_2)}{r^2\phi_3(e^{fr^2+h}-1)(e^{fr^2+h}+1)} \\
W_1 &= b^4\delta_2r^4\phi_5\operatorname{csch}^4(fr^2+h)-16bB_1\delta_1f^2r^2\log^2(e)(e^{2(fr^2+h)}+1)e^{fr^2+h} \\
&\quad +16bB_1\delta_1f\log(e)(e^{2(fr^2+h)}-1)e^{fr^2+h}+\delta_2\phi_5 \\
W_2 &= -8bB_1\delta_1f^2r^4\log^2(e)(e^{2(fr^2+h)}+1)e^{fr^2+h} \\
&\quad +2\delta_1fr^2(e^{2(fr^2+h)}-1)\coth(fr^2+h)(4bB_1fr^2\log(e)e^{fr^2+h}+\phi_2) \\
&\quad +4bB_1\delta_1fr^2\log(e)(e^{2(fr^2+h)}-1)e^{fr^2+h}-\phi_5(\delta_1-\delta_2r^2) \\
\rho+p_t &= -\frac{b^4r^2(\delta_2r^2-2\delta_1)\operatorname{csch}^4(br^2+c)+\delta_2+\phi_1}{2\phi_{15}^2}+\frac{2b^2W_2\operatorname{csch}^2(fr^2+h)}{2\phi_7(e^{br^2+c}-1)^2(e^{br^2+c}+1)^2}+W_1 \\
\rho+p_r+2p_t &= -\frac{b^4r^2(\delta_2r^2-2\delta_1)\operatorname{csch}^4(fr^2+h)+\delta_2+\phi_1}{2\phi_{15}^2}+\frac{2b^2W_2\operatorname{csch}^2(fr^2+h)}{\phi_7(e^{fr^2+h}-1)^2(e^{fr^2+h}+1)^2} \\
&\quad +\frac{\delta_1(8bB_1fr^2\log(e)e^{fr^2+h}+\phi_2)}{r^2\phi_3(e^{fr^2+h}-1)(e^{fr^2+h}+1)}+A+W_1 \\
\rho-p_r &= -\frac{b^4r^2(\delta_2r^2-2\delta_1)\operatorname{csch}^4(fr^2+h)+\delta_2+\phi_1}{2\phi_{15}^2}+A-\frac{\delta_1(8bB_1fr^2\log(e)e^{fr^2+h}+\phi_2)}{r^2\phi_3(e^{fr^2+h}-1)(e^{fr^2+h}+1)} \\
\rho-p_t &= -\frac{b^4r^2(\delta_2r^2-2\delta_1)\operatorname{csch}^4(fr^2+h)+\delta_2+\phi_1}{2\phi_{15}^2}+A+\frac{2b^2W_2\operatorname{csch}^2(fr^2+h)}{\phi_7(e^{fr^2+h}-1)^2(e^{fr^2+h}+1)^2} \\
&\quad +\frac{\delta_1(8bB_1fr^2\log(e)e^{fr^2+h}+\phi_2)}{r^2\phi_3(e^{fr^2+h}-1)(e^{fr^2+h}+1)}+W_1 \\
W_3 &= b^4\delta_2r^4\phi_5\operatorname{csch}^4(fr^2+h)-16bB_1\delta_1f^2r^2\log^2(e)(e^{2(fr^2+h)}+1)e^{fr^2+h} \\
&\quad -16bB_1\delta_1f\log(e)(e^{2(fr^2+h)}-1)e^{fr^2+h}+\delta_2\phi_5 \\
\rho-p_r-2p_t &= A-\frac{b^4r^2(\delta_2r^2-2\delta_1)\operatorname{csch}^4(fr^2+h)+\delta_2+\phi_1}{2\phi_{15}^2}+b^4\delta_2r^4\phi_5\operatorname{csch}^4(fr^2+h) \\
&\quad +\frac{2b^2W_2\operatorname{csch}^2(fr^2+h)}{\phi_7(e^{fr^2+h}-1)^2(e^{fr^2+h}+1)^2}-\frac{\delta_1(8bB_1fr^2\log(e)e^{fr^2+h}+\phi_2)}{r^2\phi_3(e^{fr^2+h}-1)(e^{fr^2+h}+1)}-W_3
\end{aligned}$$



$$\begin{aligned}
Z_1 &= A - \frac{b^4 r^2 (\delta_2 r^2 - 2\delta_1) \operatorname{csch}^4(fr^2 + h) + \delta_2 + \phi_1}{2\phi_{15}^2} + \frac{\phi_{24}}{2} \\
Z_2 &= \frac{2\delta_1 f \log(e) e^{fr^2+h} (8bB_1 fr^2 \log(e) e^{fr^2+h} + \phi_2)}{r\phi_3 (e^{fr^2+h} - 1)^2 (e^{fr^2+h} + 1)} \\
Z_3 &= 16bB_1 f^2 r^3 \log^2(e) e^{fr^2+h} + 16bB_1 fr \log(e) e^{fr^2+h} \\
Z_4 &= \left(A_1 + \frac{bB_1 \log\left(\frac{e^{fr^2+h} - 1}{e^{fr^2+h} + 1}\right)}{2f} \right) \\
Z_5 &= \frac{bB_1 \phi_{22} (e^{fr^2+h} + 1) (e^{2(fr^2+h)} - 1)}{e^{fr^2+h} - 1} \\
Z_6 &= 2b^4 \delta_2 r^3 \operatorname{csch}^4(fr^2 + h) - 8b^2 fr \coth(fr^2 + h) \operatorname{csch}^2(fr^2 + h) (-3\delta_1 + 4\delta_1 fr^2 \coth(fr^2 + h) + \delta_2 r^2) \\
Z_7 &= 2b^4 r (\delta_2 r^2 - 2\delta_1) \operatorname{csch}^4(fr^2 + h) - 8b^4 fr^3 (\delta_2 r^2 - 2\delta_1) \coth(fr^2 + h) \operatorname{csch}^4(fr^2 + h) \\
&\quad + 2b^2 \operatorname{csch}^2(fr^2 + h) (-8\delta_1 f^2 r^3 \operatorname{csch}^2(fr^2 + h) + 8\delta_1 fr \coth(fr^2 + h) + 2\delta_2 r) \\
Z_8 &= -8b^2 f \coth(fr^2 + h) \operatorname{csch}^2(fr^2 + h) (-3\delta_1 + 4\delta_1 fr^2 \coth(fr^2 + h) + \delta_2 r^2) \\
Z_9 &= 32b^2 f^2 r^2 \coth^2(fr^2 + h) \operatorname{csch}^2(fr^2 + h) (-3\delta_1 + 4\delta_1 fr^2 \coth(fr^2 + h) + \delta_2 r^2) \\
Z_{10} &= 10b^4 \delta_1 r^2 \operatorname{csch}^4(fr^2 + h) + 2b^4 (\delta_2 r^2 - 2\delta_1) \operatorname{csch}^4(fr^2 + h) - 32b^4 \delta_2 fr^4 \coth(fr^2 + h) \operatorname{csch}^4(fr^2 + h) \\
Z_{11} &= 16b^4 f^2 r^4 (\delta_2 r^2 - 2\delta_1) \operatorname{csch}^6(fr^2 + h) + 64b^4 f^2 r^4 (\delta_2 r^2 - 2\delta_1) \coth^2(fr^2 + h) \operatorname{csch}^4(fr^2 + h) \\
&\quad + 40b^4 fr^2 (\delta_2 r^2 - 2\delta_1) \coth(fr^2 + h) \operatorname{csch}^4(fr^2 + h) + 16b^2 f^2 r^2 \operatorname{csch}^4(fr^2 + h) (4b\delta_1 r^2 \coth(fr^2 + h) - 3\delta_1 + \delta_2 r^2) \\
Z_{12} &= 16b^2 fr \coth(fr^2 + h) \operatorname{csch}^2(fr^2 + h) (-8\delta_1 f^2 r^3 \operatorname{csch}^2(fr^2 + h) + 8\delta_1 fr \coth(fr^2 + h) + 2\delta_1 r) \\
Z_{13} &= 2b^2 \operatorname{csch}^2(fr^2 + h) (2\delta_2 - 40\delta_2 f^2 r^2 \operatorname{csch}^2(fr^2 + h) + 8\delta_1 f \coth(fr^2 + h)) \\
Z_{14} &= b^4 r^2 (\delta_2 r^2 - 2\delta_1) \operatorname{csch}^4(fr^2 + h) + \delta_2 + \phi_1 \\
Z_{15} &= 8b^2 f^2 r^4 \operatorname{csch}^4(fr^2 + h) + 16b^2 f^2 r^4 \coth^2(fr^2 + h) \operatorname{csch}^2(fr^2 + h) + 2b^2 \operatorname{csch}^2(fr^2 + h) - 20b^2 fr^2 \coth(fr^2 + h) \\
&\quad \times \operatorname{csch}^2(fr^2 + h) \\
F_g &= -\frac{bB_1 Z_1 \phi_{22} (e^{fr^2+h} + 1)}{2f (e^{fr^2+h} - 1) Z_4} \\
F_h &= \frac{aB_1 \delta_1 \phi_{22} (8bB_1 fr^2 \log(e) e^{fr^2+h} + \phi_2)}{r^2 \phi_2 \phi_5} \\
&\quad + \frac{\delta_1 (2b^2 r \operatorname{csch}^2(fr^2 + h) - 4b^2 br^3 \coth(fr^2 + h) \operatorname{csch}^2(fr^2 + h) (8bB_1 fr^2 \log(e) e^{fr^2+h} + \phi_2))}{r^2 \phi_7 (e^{fr^2+h} - 1) (e^{fr^2+h} + 1)} \\
&\quad - \frac{\delta_1 (4fr Z_4 \log(e) e^{2(fr^2+h)} + Z_3 + Z_5)}{r^2 \phi_3 (e^{fr^2+h} - 1) (e^{fr^2+h} + 1)} - \frac{2\delta_1}{r^3} + Z_2 + \phi_{18} + \phi_{24} \\
F_a &= \frac{2}{r} \left(\frac{2a^2 W_2 \operatorname{csch}^2(br^2 + c)}{2\phi_7 (e^{br^2+c} - 1)^2 (e^{br^2+c} + 1)^2} + A - \frac{r\phi_{24}}{2} + W_1 \right) \\
\rho'' &= \frac{-Z_{15} (a^4 r^2 (\delta_2 r^2 - 2\delta_1) \operatorname{csch}^4(br^2 + c) + \delta_2 + \phi_1)}{\phi_{15}^7} \\
&\quad - \frac{3Z_{14} (2a^2 r \operatorname{csch}^2(br^2 + c) - 4a^2 br^3 \coth(br^2 + c) \operatorname{csch}^2(br^2 + c))^2}{\phi_{15}^4} \\
&\quad - \frac{2(Z_6 + Z_7) (2a^2 r \operatorname{csch}^2(br^2 + c) - 4a^2 br^3 \coth(br^2 + c) \operatorname{csch}^2(br^2 + c))}{\phi_{15}^7} \\
&\quad + \frac{Z_8 + Z_9 + Z_{10} - Z_{11} - Z_{12} + Z_{13} + Z_{14}}{2\phi_{15}^6} \\
T_1 &= b^3 (e^{2(fr^2+h)} - 1)^2 \operatorname{csch}^4(fr^2 + h) \left(2A_1 f + bB_1 \log\left(\frac{e^{fr^2+h} - 1}{e^{fr^2+h} + 1}\right) \right)^2
\end{aligned}$$



$$\begin{aligned}
T_2 &= 2A_1f(e^{2(fr^2+h)} + 1) + 2bB_1e^{fr^2+h} + bB_1(e^{2(fr^2+h)} + 1)\log\left(\frac{e^{fr^2+h}-1}{e^{fr^2+h}+1}\right) \\
T_3 &= 2bf\phi_2\coth(fr^2+h)\operatorname{csch}^2(fr^2+h)(8bB_1fr^2\log(e)e^{fr^2+h} + \phi_2) \\
T_4 &= fr^2\log(e)\left(2A_1f(e^{2(fr^2+h)} + 1) + 2bB_1e^{fr^2+h} + bB_1(e^{2(fr^2+h)} + 1)\log\left(\frac{e^{fr^2+h}-1}{e^{fr^2+h}+1}\right)\right) + \phi_2 \\
T_5 &= \operatorname{csch}^2(fr^2+h)(b^2r^2(b^2-4f^2r^2)\operatorname{csch}^2(fr^2+h) + 5b^2-4f^2r^2) \\
&\quad + 8f^2r^2\coth^2(fr^2+h)(b^2r^2\operatorname{csch}^2(fr^2+h)-1) + 2f\coth(fr^2+h)(5-3b^2r^2\operatorname{csch}^2(fr^2+h)) \\
D_1 &= b^4\delta_2r^4\phi_5\operatorname{csch}^4(fr^2+h) - 16b^3B_1\delta_1f^2r^4\log^2(e)(e^{2(fr^2+h)} + 1)e^{fr^2+h}\operatorname{csch}^2(fr^2+h) \\
&\quad + 4b^2\delta_1fr^2(e^{2(fr^2+h)} - 1)\coth(fr^2+h)\operatorname{csch}^2(fr^2+h)(4bB_1fr^2\log(e)e^{fr^2+h} + \phi_2) \\
&\quad + 8bb^2B_1\delta_1fr^2\log(e)(e^{2(fr^2+h)} - 1)e^{fr^2+h}\operatorname{csch}^2(fr^2+h) - 2b^2\phi_5(\delta_1 - \delta_2r^2)\operatorname{csch}^2(fr^2+h) \\
&\quad - 16bB_1\delta_1f^2r^2\log^2(e)(e^{2(fr^2+h)} + 1)e^{fr^2+h} + 16bB_1\delta_1f\log(e)(e^{2(fr^2+h)} - 1)e^{fr^2+h} + \delta_2\phi_5 \\
D_2 &= 4b^5B_1\delta_2fr^5\log(e)(-e^{fr^2+h})(e^{fr^2+h} + 1)e^{fr^2+h}\operatorname{csch}^4(fr^2+h) + 8b^4\delta_2fr^5\phi_2\log(e)e^{2(fr^2+h)}\operatorname{csch}^4(fr^2+h) \\
&\quad - 8b^4\delta_2fr^5\phi_5\coth(fr^2+h)\operatorname{csch}^4(fr^2+h) + 4b^4\delta_2r^3\phi_5\operatorname{csch}^4(fr^2+h) \\
&\quad - 64b^3B_1\delta_1f^3r^4\log^2(e)(e^{2(fr^2+h)} + 1)e^{fr^2+h}\coth(fr^2+h)\operatorname{csch}^2(fr^2+h) \\
&\quad - 32b^3B_1\delta_1f^2r^2\log(e)(e^{2(fr^2+h)} - 1)e^{fr^2+h}\coth(fr^2+h)\operatorname{csch}^2(fr^2+h) \\
&\quad + 16b^2\delta_1f^2r^3(e^{2(fr^2+h)} - 1)\coth^2(fr^2+h)\operatorname{csch}^3(fr^2+h)(4bB_1fr^2\log(e)e^{fr^2+h} + \phi_2) \\
&\quad - 8b^2fr\phi_5(\delta_1 - \delta_2r^2)\coth(fr^2+h)\operatorname{csch}^2(fr^2+h) - 64bB_1\delta_1f^3r^3\log^3(e)e^{3(br^2+c)} \\
&\quad - 32bB_1\delta_1f^3r^3\log^3(e)(e^{2(fr^2+h)} + 1)e^{fr^2+h} - 32bB_1\delta_1f^2r\log^2(e)(e^{2(fr^2+h)} + 1)e^{fr^2+h} \\
&\quad + 32bB_1\delta_1f^2r\log^2(e)(e^{2(fr^2+h)} - 1)e^{fr^2+h} + 64bB_1\delta_1f^2r\log^2(e)e^{3(fr^2+h)} \\
&\quad + 4bB_1\delta_2fr\log(e)(e^{fr^2+h} - 1)(e^{fr^2+h} + 1)e^{fr^2+h} + 8b\delta_2r\phi_2\log(e)e^{2(fr^2+h)} \\
D_3 &= 8\delta_1f^2r^3\log(e)e^{fr^2+h}(e^{2(fr^2+h)} - 1)\coth(fr^2+h) \\
D_4 &= D_3\left(2A_1fe^{fr^2+h} + bB_1e^{fr^2+h}\log\left(\frac{e^{fr^2+h}-1}{e^{fr^2+h}+1}\right) + 2bB_1fr^2\log(e) + 3B_1b\right) \\
&\quad - 16bB_1\delta_1f^3r^5\log^3(e)(e^{2(fr^2+h)} + 1)e^{fr^2+h} - 32bB_1\delta_1f^3r^5\log^3(e)e^{3(fr^2+h)} \\
&\quad + 8\delta_1f^2r^3\log(e)e^{2(fr^2+h)}\coth(fr^2+h)(4bB_1fr^2\log(e)e^{fr^2+h} + \phi_2) \\
&\quad - 4\delta_1f^2r^3(e^{2(fr^2+h)} - 1)\operatorname{csch}^2(fr^2+h)(4bB_1fr^2\log(e)e^{fr^2+h} + \phi_2) \\
&\quad - 32bB_1\delta_1f^2r^3\log^2(e)(e^{2(fr^2+h)} + 1)e^{fr^2+h} + 8bB_1\delta_1f^2r^3\log^2(e)(e^{2(fr^2+h)} - 1)e^{fr^2+h} \\
&\quad + 16bB_1\delta_1f^2r^3\log^2(e)e^{3(fr^2+h)} + 4\delta_1fr(e^{2(fr^2+h)} - 1)\coth(fr^2+h)(4bB_1fr^2\log(e)e^{fr^2+h} + \phi_2) \\
&\quad + 8bB_1\delta_1fr\log(e)(e^{2(fr^2+h)} - 1)e^{fr^2+h} + 4bB_1fr\log(e)(\delta_2r^2 - \delta_1)(e^{fr^2+h} - 1)(e^{fr^2+h} + 1)e^{fr^2+h} \\
&\quad + 8fr\phi_2\log(e)(\delta_2r^2 - \delta_1)e^{2(br^2+c)} + 2\delta_2r\phi_5 \\
D_5 &= -4b^4r^2 - 2(b^2(8f^2r^4 + 5) - 4f^2r^2)\cosh(2(fr^2+h)) + 12b^2fr^2\sinh(2(fr^2+h)) + 10b^2 \\
&\quad - 5f\sinh(4(br^2+c)) + 4f^2r^2\cosh(4(fr^2+h)) - 12f^2r^2 + 10f\sinh(2(fr^2+h)) \\
D_6 &= 4b^2r(e^{fr^2+h} - 1)(e^{fr^2+h} + 1)(1 - 2fr^2\coth(fr^2+h))\operatorname{csch}^2(fr^2+h) \\
V_r^2 &= -\frac{\phi_{15}\sinh^2(fr^2+h)(-8b^2fB_1T_4\log(e)e^{fr^2+h}\operatorname{csch}^2(fr^2+h) - 8b^2B_1T_2\log^2(e)e^{fr^2+h} + T_1 + T_3)}{aT_5(e^{fr^2+h} - 1)^2(e^{fr^2+h} + 1)^2\left(fB_1\log\left(\frac{e^{fr^2+h}-1}{e^{fr^2+h}+1}\right) + 2A_1b\right)^2}
\end{aligned}$$



$$V_t^2 = \frac{\phi_3(e^{bfr^2+h} - 1)(e^{bfr^2+h} + 1)\sinh^6(bfr^2 + h)(2b^2D_3\operatorname{csch}^2(bfr^2 + h) + D_2)}{b^2\delta_1D_5r(e^{2(bfr^2+h)} - 1)^3\left(2A_1f + bB_1\log\left(\frac{e^{bfr^2+h} - 1}{e^{bfr^2+h} + 1}\right)\right)^2}$$

$$-4D_1fr\phi_{15}\log(e)(e^{bfr^2+h} + 1)e^{bfr^2+h}\sinh^6(bfr^2 + h)\left(2A_1f + bB_1\log\left(\frac{e^{bfr^2+h} - 1}{e^{bfr^2+h} + 1}\right)\right)$$

$$-D_6D_1\sinh^6(bfr^2 + h)\left(2A_1f + bB_1\log\left(\frac{e^{bfr^2+h} - 1}{e^{bfr^2+h} + 1}\right)\right) - 4bB_1D_1fr\phi_{15}\log(e)e^{bfr^2+h}\sinh^6(bfr^2 + h)$$

$$-4D_1fr\phi_3\log(e)(e^{bfr^2+h} - 1)e^{bfr^2+h}\sinh^6(bfr^2 + h)$$

En el volumen 2 de este trabajo, continúa el desarrollo matemático.

CONCLUSIONES.

De los resultados obtenidos en el apartado anterior, se concluye: 1) que la supergravedad cuántica relativista consiste en la deformación extrema del espacio – tiempo cuántico; 2) que la deformación extrema del espacio – tiempo cuántico, se desarrolla en dimensiones altas; 3) que las dimensiones o supermembranas formadas por la referida distorsión, comportan otros puntos lejanos del espacio – tiempo cuántico, en los que, la dimensión temporal es diferente para cada caso; 4) la formación de dimensiones altas, supone la existencia de múltiples regiones que no convergen en un mismo espacio – tiempo cuántico, sino en distintos sectores espaciales, por lo que, la transdimensionalidad y la multidimensionalidad ocurren a escalas infinitas y en ramificación, pues parten de una dimensión cero o de origen; 5) la supergravedad cuántica relativista, obedece a la interacción de una superpartícula, llamada también partícula blanca o estrella, es decir, aquella cuyo centro de masa y energía es extremadamente denso, capaz de deformar el espacio – tiempo cuántico en forma extrema, formando así, la supergravedad. Sin embargo, esta partícula también puede alcanzar la velocidad de la luz, por lo que, se la denomina suprapartícula, con las mismas propiedades que la aquí referida. Asimismo, la superpartícula o suprapartícula, según sea el caso, también produce supergravedad cuando interactúa con el supergravitón, esto es, con la supercompañera del gravitón, por lo que el espacio – tiempo cuántico distorsionado, se ve permeado y por ende, mutado por el campo supergravitónico vinculante; y, 6) la supersimetría de gauge y su modelamiento matemático, permite conciliar la relatividad general y especial y la mecánica cuántica, en la medida en que, estamos ante campos cuánticos en los que interfiere la dualidad holográfica y el principio cosmológico.



ACLARACIONES FINALES:

Algunas aclaraciones finales a tener en consideración y aplicar, a propósito de la Teoría Cuántica de Campos Relativistas o Curvos (TCCR) propuesta por este autor:

1. En todos los casos, este símbolo $\dagger$ será reemplazado por este símbolo $\dagger$ o por este símbolo $\dagger$, equivaliendo lo mismo.

Símbolo a ser reemplazado.	Símbolos de reemplazo.
$\dagger$	$\dagger$
	$\dagger$

2. En todos los casos, este símbolo $\ddagger$, será reemplazado por este símbolo $\ddagger$ o por este símbolo $\ddagger$.

Símbolo a ser reemplazado.	Símbolos de reemplazo.
$\ddagger$	$\ddagger$
	$\ddagger$

3. En todos los casos, se añadirá y por ende, se calculará la magnitud $\S$ que equivale a un campo de Yang – Mills y por ende, a la teoría de Yang – Mills en sentido amplio, en relación a la Teoría Cuántica de Campos Relativistas propuesta por este autor.

4. Este símbolo $\bullet$ podrá usarse como exponente u operador, según sea el caso.

Las aclaraciones antes referidas aplican tanto a este trabajo como a todos los trabajos previos y posteriores publicados por este autor, según corresponda.



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