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RELATIVISTIC QUANTUM GRAVITY. AdS/CFT CALIBER SYMMETRIES IN HILBERT–EINSTEIN–RIEMANN SPACES. VOLUME II

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RESUMEN

En este trabajo, abordaré la gravedad cuántica relativista desde la TCCR, esto es, desde la Teoría Cuántica de Campos Relativistas. Es aquí donde se establece una clara distinción entre superpartícula y partícula supermasiva respectivamente, en la medida en que, la primera, refiere a aquellas partículas cuyo centro de masa y energía es extremadamente denso, lo que provoca la deformación intensa del espacio – tiempo cuántico, todo esto, en dimensiones mas altas y en condiciones de supersimetría, causando incluso la formación de agujeros negros cuánticos, esto, sea por razones endógenas o en su defecto, por razones exógenas, esto último, cuando la superpartícula interactúa con el supergravitón, es decir, por permeabilización del espacio – tiempo cuántico a través del campo supergravitónico, en tanto que, la segunda, refiere a las partículas cuyo centro de masa, es extremadamente denso, causando así, la deformación del espacio – tiempo cuántico, en simetrías de calibre compactas, pudiendo provocar o no la formación de agujeros negros, salvo en circunstancias de colisión, en cuyo caso, es inevitable. La partícula supermasiva, por tanto, es la responsable de la gravedad cuántica relativista, sea por razones endógenas, es decir, cuando la partícula supermasiva, por sí misma, distorsiona el espacio – tiempo cuántico o en su defecto, por interacción con el gravitón, esto es, la permeabilización del plano cuántico por intrusión del campo gravitónico.

Palabras Clave: Simetrías de gauge, gravedad cuántica, relatividad general, partícula supermasiva, gravitón, campo gravitónico.

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RELATIVISTIC QUANTUM GRAVITY. AdS/CFT CALIBER SYMMETRIES IN HILBERT–EINSTEIN–RIEMANN SPACES. VOLUME II

ABSTRACT

In this work, I will address relativistic quantum gravity from the TCCR, that is, from the Quantum Theory of Relativistic Fields. It is here that a clear distinction is established between superparticle and supermassive particle respectively, insofar as the former refers to those particles whose center of mass and energy is extremely dense, which causes the intense deformation of quantum space-time, all this, in higher dimensions and under conditions of supersymmetry, even causing the formation of quantum black holes. This, either for endogenous reasons or, failing that, for exogenous reasons, the latter, when the superparticle interacts with the supergraviton, that is, by permeabilization of quantum space-time through the supergravitonic field, while the second refers to particles whose center of mass is extremely dense, thus causing the deformation of quantum space-time. in compact gauge symmetries, which may or may not cause the formation of black holes, except in collision circumstances, in which case, it is inevitable. The supermassive particle, therefore, is responsible for relativistic quantum gravity, either for endogenous reasons, that is, when the supermassive particle, by itself, distorts quantum space-time or, failing that, by interaction with the graviton, that is, the permeabilization of the quantum plane by intrusion of the gravitonic field.

Keywords: Gauge symmetries, quantum gravity, general relativity, supermassive particle, graviton, gravitonic field

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INTRODUCCIÓN.

Las partículas supermasivas, han sido teorizadas en trabajos anteriores, por este autor, sin embargo, existen aclaraciones que precisar. Las partículas supermasivas, o llamadas también “partículas oscuras” o “partículas cosmológicas”, son aquellas, que a propósito de su masa en extremo pesada, son capaces de deformar el espacio – tiempo cuántico, con posibilidad de provocar agujeros negros cuánticos, en casos específicos de colapso o colisión, en tanto que, las superpartículas o llamadas también “partículas estrella” o “partículas blancas”, son aquellas cuyo centro de masa y energía es tan extremo, que no solamente deforma el espacio – tiempo cuántico, sin que además, provoca agujeros negros cuánticos e incluso, agujeros de gusano. La otra gran diferencia, está en cuanto a la gravedad exógena se refiere, esto es, que la partícula supermasiva, distorsiona el espacio – tiempo cuántico en el que interactúa, cuando empata o ancla con el gravitón, lo que supone por tanto, la intrusión y permeo del campo gravitónico en el plano repercutido, en tanto que, cuando se trata de la superpartícula, ésta distorsiona el espacio – tiempo cuántico en el que interactúa, cuando empata o ancla con el supergravitón, lo que supone por tanto, la intrusión y permeo del campo supergravitónico en el plano repercutido. Cabe aclarar que los campos de permeabilización antes referidos, pueden ser fantasmas aunque no por regla general. En cuanto a la gravedad cuántica relativista, no existe la formación de supersimetrías, sino de simetrías de calibre, con propagadores y osciladores armónicos masivos pero no caóticos, es decir, existe un mínimo de compactación. Trabajaremos en un espacio de Hilbert – Einstein para campos cuánticos relativistas, demostrando la acción gravitatoria de las partículas supermasivas en entornos de entropía sin que esto, comporte dimensiones más altas, salvo en casos de colapso o colisión de estas partículas pesadas.

RESULTADOS Y DISCUSIÓN:

A continuación, se expresa el Modelo de Gravedad Cuántica Relativista, en dimensión $\mathbb{R}^4$ para simetrías de gauge puras, con osciladores armónicos y propagadores $\left\langle \psi_{\blacksquare}^{\square} \right\| :: \|\aleph^{\mu\nu}\rangle$, en un espacio de Hilbert – Einstein, tanto con interferencia gravitónica como sin interferencia gravitónica.

En este punto, retomo el desarrollo matemático desplegado en el volumen I de este manuscrito:

Formación de materia y energía oscuras a partir de las interacciones de la partícula supermasiva o partícula oscura con o sin intervención gravitónica. Modelo Inflacionario.

$$\rho(r) = \frac{\rho_0}{r/r_s(1 + r/r_s)^2}$$

$$S = \frac{m_{p(11)}^9}{2} \int d^{11}x\sqrt{-G_{11}} \left[e^{aM} e^{bN} R_{MNab}(\omega) - \bar{\psi}_A \Gamma^{ABC} D_B \left(\frac{\omega + \hat{\omega}}{2}\right) \psi_C - \frac{1}{24} F^{ABCD} F_{ABCD} \right.$$
$$- \frac{\sqrt{2}}{192} \bar{\psi}_M (\Gamma^{ABCDMN} + 12\Gamma^{AB} g^{CM} g^{DN}) \psi_N \left(2F_{ABCD} + \frac{3}{2}\sqrt{2}\bar{\psi}_{[A}\Gamma_{BC}\psi_{D]} \right)$$
$$\left. - \frac{2\sqrt{2}}{(144)^2} \varepsilon^{A'B'C'D'ABCDMNP} F_{A'B'C'D'} F_{ABCD} A_{MNP} \right]$$

$$D_M(\tilde{\omega}) = \nabla_M + \frac{1}{4}\tilde{\omega}_{Mab}\Gamma^{ab}$$

$$\omega_{Mab} = \omega_{Mab}(e) + K_{Mab}$$
$$\hat{\omega}_{Mab} = \omega_{Mab}(e) - \frac{1}{4}(\bar{\psi}_M \Gamma_b \psi_a - \bar{\psi}_a \Gamma_M \psi_b + \bar{\psi}_b \Gamma_a \psi_M)$$
$$\omega_M^{ab}(e) = 2e^{N[a}\nabla_{[M} e_{N]}^{b]} - e^{N[a} e^{b]P} e_{Mc} \nabla_N e_P^c$$
$$K_{Mab} = -\frac{1}{4}(\bar{\psi}_M \Gamma_b \psi_a - \bar{\psi}_a \Gamma_M \psi_b + \bar{\psi}_b \Gamma_a \psi_M) + \frac{1}{8}\bar{\psi}_N \Gamma_{Mab}^{NP} \psi_P$$

$$\delta e_M^a = \frac{1}{2}\bar{\epsilon}\Gamma^a \psi_M$$
$$\delta\psi_M = D_M(\hat{\omega})\epsilon + \frac{\sqrt{2}}{288}\left(\Gamma_M^{ABCD} - 8\Gamma^{BCD}\delta_M^A\right)\left(F_{ABCD} + \frac{3\sqrt{2}}{2}\bar{\psi}_{[A}\Gamma_{BC}\psi_{D]}\right)\epsilon$$
$$\delta A_{MNP} = -\frac{3\sqrt{2}}{4}\bar{\epsilon}\Gamma_{[MN}\psi_{P]}$$

$$ds_{11}^2 = G_{AB} dx^A dx^B$$

$$ds_4^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t)\delta_{ij} dx^i dx^j$$

$$ds_{2(1)}^2 = \Phi(x^\mu) g_{a_1 b_1}(x^{a_1}) dx^{a_1} dx^{b_1}$$
$$ds_{2(2)}^2 = h_{a_2 b_2}(x^{a_2}) dx^{a_2} dx^{b_2}$$
$$ds_{2(3)}^2 = \Phi^2(x^\mu) \gamma_{a_3 b_3}(x^{a_3}) dx^{a_3} dx^{b_3}$$

$$ds_1^2 = \Phi^{-6}(x^\mu) dx^{10} dx^{10}$$

$$ds_{11}^2 = ds_4^2 + ds_{2(1)}^2 + ds_{2(2)}^2 + ds_{2(3)}^2 + ds_1^2$$

$$R_{(11)} = R_{(4)} - \frac{23}{2\Phi^2}\nabla_\mu \Phi \nabla^\mu \Phi + \frac{R_{T_{q_1}^2}}{\Phi} + R_{T_{q_2}^2} + \frac{R_{T_{q_3}^2}}{\Phi^2},$$

$$\mathcal{G}_{\mu\nu}-\frac{23}{2\Phi^2}\nabla_\mu\Phi\nabla_\nu\Phi-\frac{1}{2}g_{\mu\nu}\left(\frac{R_{T^2_{q_1}}}{\Phi}+R_{T^2_{q_2}}+\frac{R_{T^2_{q_3}}}{\Phi^2}-\frac{23}{2\Phi^2}\nabla_\rho\Phi\nabla^\rho\Phi\right)=\frac{1}{m^9_{p(11)}}T^{(A,\psi)}_{\mu\nu}$$

$$\mathcal{G}_{a_1b_1}-\frac{1}{2}g_{a_1b_1}\left(\Phi R_{(4)}+\Phi R_{T^2_{q_2}}+\frac{R_{T^2_{q_3}}}{\Phi}+\nabla_\mu\nabla^\mu\Phi-\frac{25}{2\Phi}\nabla_\mu\Phi\nabla^\mu\Phi\right)=\frac{1}{m^9_{p(11)}}T^{(A,\psi)}_{a_1b_1}$$

$$\mathcal{G}_{a_2b_2}-\frac{1}{2}h_{a_2b_2}\left(R_{(4)}+\frac{R_{T^2_{q_1}}}{\Phi}+\frac{R_{T^2_{q_3}}}{\Phi^2}-\frac{23}{2\Phi^2}\nabla_\mu\Phi\nabla^\mu\Phi\right)=\frac{1}{m^9_{p(11)}}T^{(A,\psi)}_{a_2b_2}$$

$$\mathcal{G}_{a_3b_3}-\frac{1}{2}\gamma_{a_3b_3}\left(\Phi^2R_{(4)}+\Phi R_{T^2_{q_1}}+\Phi^2R_{T^2_{q_2}}+\Phi\nabla_\mu\nabla^\mu\Phi-\frac{27}{2}\nabla_\mu\Phi\nabla^\mu\Phi\right)=\frac{1}{m^9_{p(11)}}T^{(A,\psi)}_{a_3b_3}$$

$$-\frac{1}{2}\left(\frac{R_{(4)}}{\Phi^6}+\frac{R_{T^2_{q_1}}}{\Phi^7}+\frac{R_{T^2_{q_2}}}{\Phi^6}+\frac{R_{T^2_{q_3}}}{\Phi^8}-\frac{6}{\Phi^7}\nabla_\mu\nabla^\mu\Phi-\frac{11}{2\Phi^8}\nabla_\mu\Phi\nabla^\mu\Phi\right)=\frac{1}{m^9_{p(11)}}T^{(A,\psi)}_{10,10}$$

$$S_{EH}=\int d^{11}x\sqrt{-G_{11}}\frac{m^9_{p(11)}}{2}R_{(11)}$$

$$\int d^{11}x\sqrt{-G_{11}}[\cdots]=\int_{V_{\mathrm{FRW}}}d^4x\sqrt{-g_4}\int_{T^2_{q_1}}d^2x\sqrt{g_2}\int_{T^2_{q_2}}d^2x\sqrt{h}\int_{T^2_{q_3}}d^2x\sqrt{\gamma}\int_{S^1}dx[\cdots]$$

$$S_{EH}=\int d^4x\sqrt{-g_4}\left[\frac{m_p^2}{2}R_{(4)}-\frac{1}{2}\frac{23m_p^2}{2\Phi^2}\nabla_\mu\Phi\nabla^\mu\Phi\right.$$
$$\left.+\frac{4\pi(1-q_1)m_p^2}{A_{T^2_{q_1}}\Phi}+\frac{4\pi(1-q_2)m_p^2}{A_{T^2_{q_2}}}+\frac{4\pi(1-q_3)m_p^2}{A_{T^2_{q_3}}\Phi^2}\right]$$

$$\frac{2\pi\tilde{\mathcal{R}}_{10}A_{T^2_{q_1}}A_{T^2_{q_2}}A_{T^2_{q_3}}m^9_{p(11)}}{2}=\frac{m_p^2}{2}.$$

$$\int_{T^2_{q_i}}R_{T^2_{q_i}}=8\pi(1-q_i),i=1,2,3$$

$$\phi=\sqrt{\frac{23}{2}}m_p\ln\Phi$$
$$\Phi=\exp\left(\sqrt{\frac{2}{23}}\frac{\phi}{m_p}\right)$$

$$S_{\mathrm{eff}}=S_{EH}=\int d^4x\sqrt{-g_4}\left[\frac{m_p^2}{2}R_{(4)}-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi-\frac{4\pi m_p^2}{A_0}\left(1-e^{-\sqrt{\frac{2}{23}}\frac{\phi}{m_p}}\right)^2\right]$$

$$V(\Phi(\phi))=\frac{4\pi m_p^2}{A_0}\left(1-e^{-\sqrt{\frac{2}{23}}\frac{\phi}{m_p}}\right)^2,$$

$$ds^2_{2(1)}=\Phi(x^\mu)^{2s_1}g_{a_1b_1}(x^{a_1})dx^{a_1}dx^{b_1}$$
$$ds^2_{2(2)}=\Phi(x^\mu)^{2s_2}h_{a_2b_2}(x^{a_2})dx^{a_2}dx^{b_2}$$
$$ds^2_{2(3)}=\Phi(x^\mu)^{2s_3}\gamma_{a_3b_3}(x^{a_3})dx^{a_3}dx^{b_3}$$

$$ds_1^2 = \Phi^{-2(s_1+s_2+s_3)}dx^{10}dx^{10}$$

$$R_{(11)} = R_{(4)} + \frac{R_{T_{q_1}^2}}{\Phi^{2s_1}} + \frac{R_{T_{q_2}^2}}{\Phi^{2s_2}} + \frac{R_{T_{q_3}^2}}{\Phi^{2s_3}} - \frac{\lambda}{\Phi^2} g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi,$$

$$\lambda = 6\sum_i s_i^2 + 8\sum_{i>j} s_i s_j$$

$$S_{\text{eff}} = \int d^4x\sqrt{-g_4}\left[\frac{m_p^2}{2}R_{(4)} - \frac{1}{2}\frac{\lambda m_p^2}{\Phi^2}\nabla_\mu\Phi\nabla^\mu\Phi - U(\Phi) + \mathcal{L}_{\text{matter}}\right]$$

$$U(\Phi) = -\left(\frac{4\pi(1-q_1)m_p^2}{A_{T_{q_1}^2}\Phi^{2s_1}} + \frac{4\pi(1-q_2)m_p^2}{A_{T_{q_2}^2}\Phi^{2s_2}} + \frac{4\pi(1-q_3)m_p^2}{A_{T_{q_3}^2}\Phi^{2s_3}}\right)$$

$$\phi = \sqrt{\lambda}m_p \ln|\Phi|$$
$$\Phi = \pm\exp\left(\sqrt{\lambda^{-1}}\frac{\phi}{m_p}\right)$$

$$q_1 = \frac{\Phi_{\min}^{2s_1-2s_3}(s_2-s_3)(1-q_3)}{2(s_2-s_1)} + 1$$
$$q_2 = \frac{\Phi_{\min}^{2s_2-2s_3}(s_1-s_3)(1-q_3)}{(s_1-s_2)} + 1$$

$$q_1 = 1 + n^{2s_3-1}(q_3-1)s_3$$
$$q_2 = n^{2s3}[1 + n^{-2s_3} + 2s_3(q_3-1) - q_3]$$

$$U(\Phi) = \frac{4\pi m_p^2(q_3-1)\Phi^{-2s_3-1}}{A_0 n}[n\Phi + (n\Phi)^{2s_3}(2s_3 + (2s_3-1)n\Phi)]$$

$$\epsilon(\phi) = \frac{m_p^2}{2}\left(\frac{U_\phi}{U}\right)^2$$
$$\eta(\phi) = m_p^2\left(\frac{U_{\phi\phi}}{U}\right)$$
$$n_s = 1 - 6\epsilon(\phi_i) + 2\eta(\phi_i)$$
$$r = 16\epsilon(\phi_i)$$

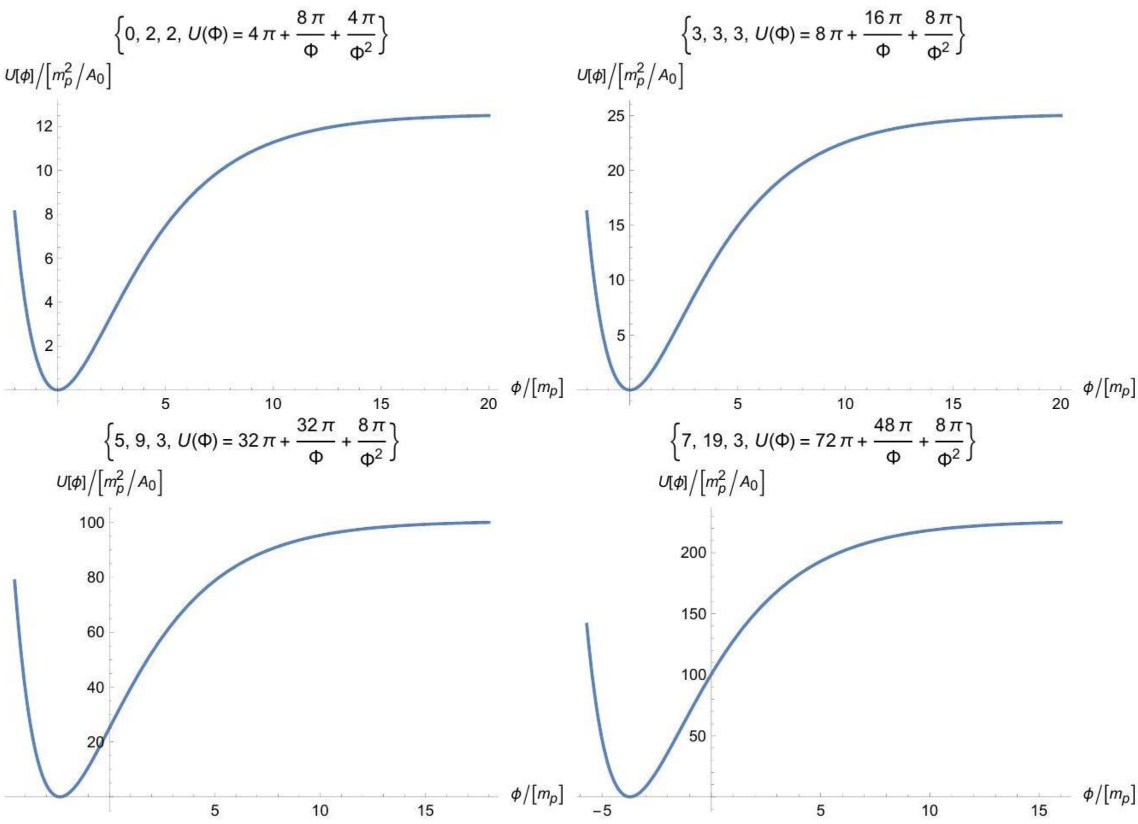


Figura 70. Fluctuaciones inflacionarias de un plano cuántico – relativista oscuro.

$$(q_1, q_2, q_3, n) = (7{,}19{,}3{,}3).$$

$$N = -\frac{1}{m_p^2}\int_{\phi_i}^{\phi_e} \frac{U}{U_\phi} d\phi$$

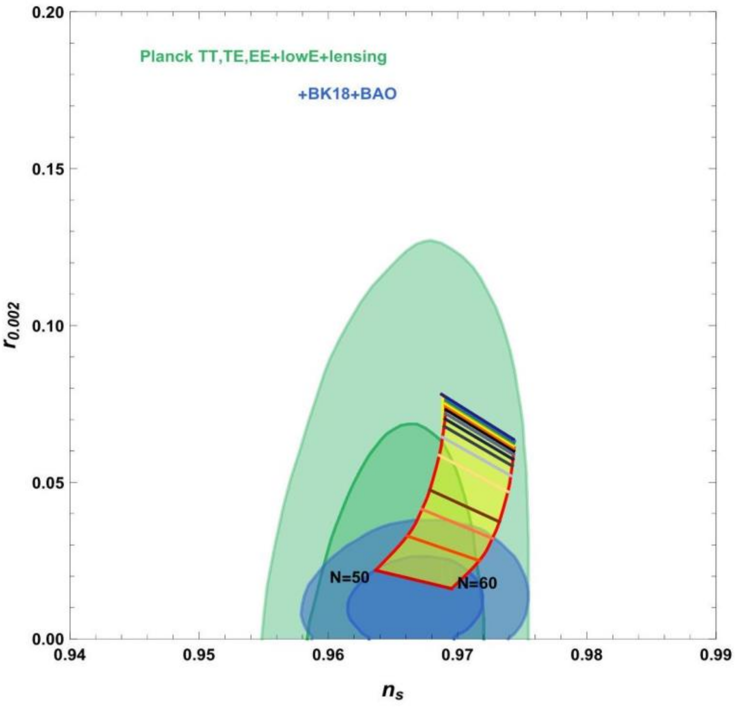


Figura 71. Fluctuaciones de materia y energía oscuras en un plano cuántico – relativista.

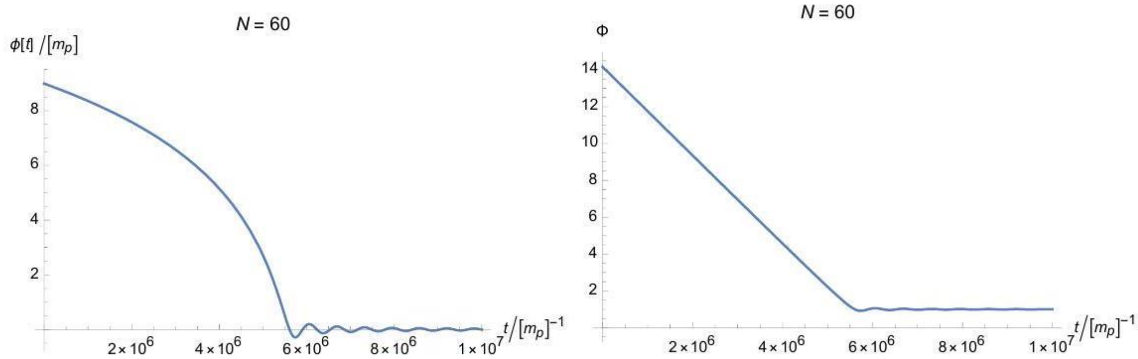


Figura 72. Interacción de la partícula oscura con el gravitón.

$$P_s = \frac{U(\phi_i)}{24\pi^2 m_p^4 \epsilon(\phi_i)} \approx 2.1 \times 10^{-9}.$$

$$A_0 = \frac{4\pi}{\zeta} l_p^2 = 1.43 \times 10^{-58}[\text{ m}]^2$$

$$U(\phi) = \zeta m_p^4 \left(1 - e^{-\sqrt{\frac{3}{23}}\frac{\phi}{m_p}}\right)^2,$$

$$\mathcal{R} = \sqrt{\frac{A_0}{8\pi}} = 29387 l_p = 2.38 \times 10^{-30}[\text{ m}].$$

$$1.417 \leq \Phi \leq 12.271(\text{ for } N = 50)$$
$$1.417 \leq \Phi \leq 14.153(\text{ for } N = 60)$$

$$l_s = \frac{l_{p(11)}}{\sqrt{\frac{\mathcal{R}_{10}}{l_{p(11)}}}}$$

$$G_N^{(11)} = \pi \xi^{-2} l_{p(11)} A_0^3 G$$

$$l_p = \frac{l_{p(11)}^{\frac{9}{2}}}{\left(\pi \xi^{-2} l_{p(11)} A_0^3\right)^{\frac{1}{2}}}$$

$$l_{p(11)} = \left(\pi A_0^3 \xi^{-2}\right)^{\frac{1}{8}} l_p^{\frac{1}{4}} = l_p \left[\frac{(4\pi)^4}{4\zeta^3 \xi^2}\right]^{\frac{1}{8}} = 8676.7 \xi^{-\frac{1}{4}} l_p$$

$$l_s \in \xi\left[1.7 l_{p(11)}, 53.2 l_{p(11)}\right]$$

$$V(\phi) \approx \frac{1}{2} m_\phi^2 \phi^2$$

$$m_\phi = \sqrt{\frac{4\zeta}{23}} m_p = 10^{-5} m_p = 2.4 \times 10^{10} \frac{\mathrm{TeV}}{\mathrm{c}^2}$$

$$\Pi_{\mu\nu\rho\sigma}(x - x') = \int \frac{dk_0}{2\pi} \frac{d^3k}{(2\pi)^3} \frac{-iP_{\mu\nu\rho\sigma}}{k_0^2 - |\mathbf{k}|^2 + i\varepsilon} e^{-ik_0(t-t')} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}$$

$$P_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}\right).$$

$$g^{\mu\nu} = \eta^{\mu\nu} + \frac{2}{m_p} h^{\mu\nu},$$

$$\mathcal{T}^{\mu\nu} = \mathcal{T}_\phi^{\mu\nu} + \mathcal{T}_{(\text{matter})}^{\mu\nu}$$

$$\mathcal{T}_\phi^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - \frac{1}{2}\eta^{\mu\nu}\partial^\alpha\phi\partial_\alpha\phi - \eta^{\mu\nu}V(\phi),$$
$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\partial_i\phi\partial^i\phi + V(\phi).$$

$$iS^{(2)} = \frac{1}{m_p^2}\int d^4x d^4x' \mathcal{T}_1^{\mu\nu}(x)\Pi_{\mu\nu\rho\sigma}(x - x')\mathcal{T}_2^{\rho\sigma}(x')$$

$$\mathcal{T}_{(\text{matter 1})}^{\mu\nu} = M_1 u_1^\mu u_1^\nu \delta^3(\mathbf{x} - \mathbf{q}_1(t)),$$
$$\mathcal{T}_{(\text{matter 2})}^{\mu\nu} = M_2 u_2^\mu u_2^\nu \delta^3(\mathbf{x}' - \mathbf{q}_2(t)).$$

$$iS_{(\text{matter})}^{(2)} = \frac{1}{m_p^2}\int d^4x d^4x' \mathcal{T}_{(\text{matter 1})}^{\mu\nu}(x)\Pi_{\mu\nu\rho\sigma}(x - x')\mathcal{T}_{(\text{matter 2})}^{\rho\sigma}(x')$$

$$iS_\phi^{(2)} = \frac{1}{m_p^2}\int d^4x d^4x' \mathcal{T}_\phi^{\mu\nu}(x)\Pi_{\mu\nu\rho\sigma}(x - x')\mathcal{T}_\phi^{\rho\sigma}(x')$$

$$\begin{aligned} i\left(S^{(2)} - S_{(\text{matter})}^{(2)} - S_\phi^{(2)}\right) =& i\frac{M_1}{m_p^2}\int dt \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{|\mathbf{k}|^2 - i\varepsilon}\int d^3\mathbf{x} F(t,\mathbf{x}) e^{i\mathbf{k}\cdot(\mathbf{q}_1(t)-\mathbf{x})} \\ &+ i\frac{M_2}{m_p^2}\int dt \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{|\mathbf{k}|^2 - i\varepsilon}\int d^3\mathbf{x}' F(t,\mathbf{x}') e^{i\mathbf{k}\cdot(\mathbf{x}'-\mathbf{q}_2(t))} \\ &+ O(\mathbf{v}_1, \mathbf{v}_2) \end{aligned}$$

$$F(t,\mathbf{x}) = \left(\frac{\partial\phi(t,\mathbf{x})}{\partial t}\right)^2 - V(\phi(t,\mathbf{x}))$$

$$\begin{aligned} W(\mathbf{q}_1(t), \mathbf{q}_2(t)) =& -\frac{M_1 M_2}{8\pi m_p^2}\frac{1}{|\mathbf{q}_1(t) - \mathbf{q}_2(t)|} - \frac{M_1}{m_p^2}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{|\mathbf{k}|^2 - i\varepsilon}\int d^3\mathbf{x} F(t,\mathbf{x}) e^{i\mathbf{k}\cdot(\mathbf{q}_1(t)-\mathbf{x})} \\ &- \frac{M_2}{m_p^2}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{|\mathbf{k}|^2 - i\varepsilon}\int d^3\mathbf{x}' F(t,\mathbf{x}') e^{i\mathbf{k}\cdot(\mathbf{x}'-\mathbf{q}_2(t))} + O(\mathbf{v}_1, \mathbf{v}_2) \end{aligned}$$

$$W(\mathbf{q}_1(t),\mathbf{q}_2(t)) = -\frac{M_1M_2}{8\pi m_p^2}\frac{1}{|\mathbf{q}_1(t)-\mathbf{q}_2(t)|} - \frac{M_2}{4\pi m_p^2}\int d^3\mathbf{x}\frac{F(t,\mathbf{x})}{|\mathbf{q}_2(t)-\mathbf{x}|} - \frac{M_1}{4\pi m_p^2}\int d^3\mathbf{x}'\frac{F(t,\mathbf{x}')}{|\mathbf{q}_1(t)-\mathbf{x}'|}$$

$$\ddot{\phi}(t,\mathbf{x}) + 3H\dot{\phi}(t,\mathbf{x}) - \frac{1}{a^2(t)}\partial_i\partial^i\phi(t,\mathbf{x}) + m_\phi^2\phi(t,\mathbf{x}) = 0$$

$$\begin{aligned}\ddot{\phi}_1(t) + 3H\dot{\phi}_1(t) + m_\phi^2\phi_1(t) &= 0\\ \frac{1}{r^2}\partial_r(r^2\partial_r\phi_2(r)) &= 0\end{aligned}$$

$$\begin{aligned}\phi_1(t) &\approx 2\phi_0 a^{-\frac{3}{2}}\cos(m_\phi t),\\ \phi_2(r) &= C_1 + \frac{C_2}{r},\end{aligned}$$

$$\langle F(t,\mathbf{x})\rangle_t = \frac{m_\phi}{2\pi}\int_0^{\frac{2\pi}{m_\phi}}\left(\dot{\phi}^2(t,r) - \frac{1}{2}m_\phi^2\phi^2(t,r)\right)dt = \phi_2^2(r)a^{-3}m_\phi^2\phi_0^2$$

$$\langle F(t,\mathbf{x})\rangle_t = \frac{1}{2}\phi_2^2(r)\overline{\rho_{DM}(t)}$$

$$\overline{\rho_{DM}(t)} = 2a^{-3}m_\phi^2\phi_0^2 = 3m_p^2H^2\times\Omega_{DM}$$

$$F_2(R) = -G\frac{M_1M_2}{R^2} - 4\pi GM_2\overline{\rho_{DM}(t)}\left(\frac{C_1^2}{3}R + \frac{C_2^2}{R} + C_1C_2\right),$$

$$\begin{aligned}\ddot{\phi}_1(t) + 3H\dot{\phi}_1(t) + m_1^2\phi_1(t) &= 0,\\ -\frac{1}{r^2}\partial_r\left(r^2\frac{\partial\phi_2(r)}{\partial r}\right) + a^2m_2^2\phi_2(r) &= 0.\end{aligned}$$

$$\begin{aligned}\phi_1(t) &\approx 2\phi_0 a^{-\frac{3}{2}}\cos(m_1 t)\\ \phi_2(r) &= C\frac{e^{-am_2r}}{r} + D\frac{e^{am_2r}}{r}\end{aligned}$$

$$F_2(R) = -\frac{GM_1M_2}{R^2} - \frac{2\pi GM_2\overline{\rho_{DM}(t)}C^2(1-e^{-2am_2R})}{am_2R^2}.$$

$$\begin{aligned}v_2^2 &\approx \frac{GM_1}{R} + GK,\\ K &= 4\pi C^2\overline{\rho_{DM}(t)},\end{aligned}$$

$$R = \sqrt{\frac{GM_1}{a_N}}.$$

$$a_2 = \frac{v_2^2}{R} = a_N\left(1 + \sqrt{\frac{a_0}{a_N}}\right)$$

$$a_0 = \frac{K^2 G}{M_1}$$

$$v_2^4 = G M_1 a_0.$$

$$C^2 = \frac{\sqrt{\frac{a_0 M_1}{G}}}{4\pi \overline{\rho_{DM}(t)}}$$

$$\rho_{\text{eff}}(r) = \phi_2^2(r)\overline{\rho_{DM}(t)}$$

$$\begin{aligned} \ddot{\phi}_1(t) + 3H\dot{\phi}_1(t) &= 0 \\ -\frac{1}{r^2}\partial_r\left(r^2\frac{\partial\phi_2(r)}{\partial r}\right) + m_\phi^2\phi_2(r) &= 0 \end{aligned}$$

$$\begin{aligned} \phi_1(t) &= C_1 + C_2 a^{-3}(t) \\ \phi_2(r) &= C_3\frac{e^{-m_\phi r}}{r} \end{aligned}$$

$$\langle F(t,\mathbf{x})\rangle_t = -\phi_2^2(r)\overline{\rho_{DE}(t)}$$

$$F_2(R) = -\frac{G M_1 M_2}{R^2} + \frac{4\pi G M_2 \overline{\rho_{DE}(t)} C_3^2(1 - e^{-2m_\phi R})}{m_\phi R^2}$$

$$T_{\mu\nu}^{(A,\psi)}[x^A] = \frac{m_p^2}{m_{p(11)}^9}\mathcal{T}_{\mu\nu}^{(\text{matter})}[x^\mu]$$

$$\begin{aligned} G_{a_i A} &= 0, (A \neq a_i \text{ and } i = 1,2,3) \\ G_{\mu A} &= 0, (A \neq \mu) \\ G_{10,A} &= 0, (A \neq 10) \end{aligned}$$

$$\begin{aligned} \Gamma^\mu &= \hat{\gamma}^\mu \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \\ \Gamma^{a_1} &= \hat{\gamma}^* \otimes \hat{\gamma}^{a_1} \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \\ \Gamma^{a_2} &= \hat{\gamma}^* \otimes \hat{\gamma}_1^* \otimes \hat{\gamma}^{a_2} \otimes \mathbb{1}_2 \\ \Gamma^{a_3} &= \hat{\gamma}^* \otimes \hat{\gamma}_1^* \otimes \hat{\gamma}_2^* \otimes \hat{\gamma}^{a_3} \\ \Gamma^{10} &= \hat{\gamma}^* \otimes \hat{\gamma}_1^* \otimes \hat{\gamma}_2^* \otimes \hat{\gamma}^{10} \end{aligned}$$

$$\delta g_{\mu\nu} = \delta g_{a_1 b_1} = \delta h_{a_2 b_2} = \delta \gamma_{a_3 b_3} = 0$$

$$\delta G_{AB} = \frac{1}{2}\eta_{ab}\bar{\epsilon}\left(\Gamma^a \psi_A e_B^b + e_A^a \Gamma^b \psi_B\right)$$

$$\begin{aligned} \delta e_\mu^{a_0} &= \frac{1}{2}\bar{\epsilon}\Gamma^{a_0}\psi_\mu = 0, a_0 = 1,2,3,4 \\ \delta G_{a_1 b_1} &= g_{a_1 b_1}\delta\Phi \\ \delta G_{a_2 b_2} &= 0 \\ \delta G_{a_3 b_3} &= 2 g_{a_3 b_3}\Phi\delta\Phi \\ \delta G_{10,10} &= -6\Phi^{-7}\delta\Phi \end{aligned}$$

$$\nabla_M \epsilon + \frac{\sqrt{2}}{288}\left(\Gamma_M^{ABCD} - 8\Gamma^{BCD}\delta_M^A\right)F_{ABCD}\epsilon = 0$$

$\mathcal{T}_{00}^{(\text{matter})}[\psi_A, A_{MNP}, a(t), \Phi(x^\mu), x^\mu]$ $\mathcal{T}_{\mu\nu}^{(\text{matter})}[\psi_A, A_{MNP}, a(t), \Phi(x^\mu), x^\mu]$, $\psi_A(a(t), \Phi(x^\mu), x^A)$ and

$A_{MNP}(a(t), \Phi(x^\mu), x^A)$ $\mathcal{T}_{\mu\nu}^{(\text{matter})}[a(t), \Phi(x^\mu), x^\mu]$

$$\mathcal{T}_{\mu\nu} = \frac{23m_p^2}{2\Phi^2}\nabla_\mu\Phi\nabla_\nu\Phi + g_{\mu\nu}\left(-\frac{1}{2}\frac{23m_p^2}{2\Phi^2}\nabla_\rho\Phi\nabla^\rho\Phi - V(\Phi)\right) + \mathcal{T}_{\mu\nu}^{(\text{matter})}$$

$$\mathcal{G}_{\mu\nu} = \frac{1}{m_p^2}\mathcal{T}_{\mu\nu}$$

$$S_{\text{eff}} = \int d^4x\sqrt{-g_4}\left[\frac{m_p^2}{2}R_{(4)} - \frac{1}{2}\frac{23m_p^2}{2\Phi^2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi) + \mathcal{L}_{\text{matter}}\right]$$

$$\mathcal{T}_{\mu\nu}^{(\text{matter})} = -2\frac{\delta\mathcal{L}_{\text{matter}}}{\delta g^{\mu\nu}} + g_{\mu\nu}\mathcal{L}_{\text{matter}}$$

$$\mathcal{L}_{\text{matter}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{DM}} + \mathcal{L}_{\text{DE}}$$

$$\begin{aligned}\tilde{n} &= e^\alpha n \\ \tilde{\Phi} &= e^{-\alpha}\Phi \\ \tilde{\phi} &= -\alpha/\sqrt{\lambda^{-1}} + \phi\end{aligned}$$

$$\tilde{U}(\tilde{\Phi}) = U(\tilde{\Phi}) = e^{2s_3\alpha}U(\Phi).$$

$$\tilde{\epsilon}(\tilde{\phi}) = \frac{m_p^2}{2}\left(\frac{\tilde{U}_{\tilde{\phi}}}{\tilde{U}}\right)^2 = \frac{m_p^2}{2}\left(\frac{e^{2s_3\alpha}U_\phi d\phi/d\tilde{\phi}}{e^{2s_3\alpha}U}\right)^2 = \epsilon(\phi).$$

Modelo Unificado de Gravedad Cuántica Relativista AdS/CFT en simetría de calibre y espacios compactificados, con o sin interferencia gravitónica. Cálculos complementarios.

$$\hbar = 1 = c.$$

$$M_{\text{Pl}} \equiv \frac{1}{\sqrt{8\pi G_N}}$$

$$\mathrm{d}s^2 = -(\mathrm{d}x^0)^2 + (\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2 + (\mathrm{d}x^3)^2 + \cdots \equiv \eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu$$

$$(\eta_{\mu\nu}) = \mathrm{diag}(-1, +1, +1, +1, \ldots).$$

$$f(x) = \int \frac{\mathrm{d}^d x}{(2\pi)^d}\tilde{f}(p)e^{ip\cdot x}, \tilde{f}(p) = \int \mathrm{d}^d x f(x)e^{-ip\cdot x}$$

$$\partial_\mu f(x) = \int \frac{\mathrm{d}^d p}{(2\pi)^d}(ip_\mu)\tilde{f}(p)e^{ip\cdot x} \Rightarrow \partial_\mu \to ip_\mu$$

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}\left(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}\right)$$

$$\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^{\nu}_{\mu\rho} V^\rho \text{ and } \nabla_\mu V_\nu = \partial_\mu V_\nu - \Gamma^{\rho}_{\mu\nu} V_\rho,$$

$$[\nabla_\nu, \nabla_\rho] V^\sigma = V^\mu R^{\sigma}_{\mu\nu\rho} \text{ and } [\nabla_\nu, \nabla_\rho] V_\mu = -V_\sigma R^{\sigma}_{\mu\nu\rho},$$

$$R^{\sigma}{}_{\mu\nu\rho} = \partial_\nu \Gamma^{\sigma}{}_{\mu\rho} - \partial_\rho \Gamma^{\sigma}{}_{\mu\nu} + \Gamma^{\sigma}{}_{\alpha\nu}\Gamma^{\alpha}{}_{\mu\rho} - \Gamma^{\sigma}{}_{\alpha\rho}\Gamma^{\alpha}{}_{\mu\nu}.$$

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\partial_\nu\partial_\rho g_{\mu\sigma} + \partial_\mu\partial_\sigma g_{\nu\rho} - \partial_\sigma\partial_\nu g_{\mu\rho} - \partial_\mu\partial_\rho g_{\nu\sigma}\right) + g_{\alpha\beta}\left(\Gamma^{\alpha}_{\nu\rho}\Gamma^{\beta}_{\mu\sigma} - \Gamma^{\alpha}_{\sigma\nu}\Gamma^{\beta}_{\mu\rho}\right).$$

$$R_{\nu\sigma} = R^{\rho}_{\nu\rho\sigma} = \delta^{\rho}_{\mu} R^{\mu}_{\nu\rho\sigma} = g^{\mu\rho} R_{\mu\nu\rho\sigma}$$

$$R = R^{\nu}_{\nu} = g^{\nu\sigma} R_{\nu\sigma}$$

$$S_{\mathrm{EH}}[g] = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}(R - 2\Lambda)$$

$$\delta(\sqrt{-g}) = -\frac{1}{2}\sqrt{-g}\, g_{\mu\nu}\delta g^{\mu\nu}$$

$$\delta R_{\mu\nu} = \frac{1}{2} g^{\sigma\rho}\left[\nabla_\sigma\nabla_\mu \delta g_{\rho\nu} + \nabla_\sigma\nabla_\nu \delta g_{\mu\rho} - \nabla_\sigma\nabla_\rho \delta g_{\mu\nu} - \nabla_\nu\nabla_\mu \delta g_{\rho\sigma}\right]$$

$$0 = \delta(S_{\mathrm{EH}} + S_m) = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu}\right) - \frac{1}{2} T_{\mu\nu}\right]\delta g^{\mu\nu}$$
$$\Rightarrow G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}}$$

$$x^\mu \to x'^\mu(x)$$

$$x^\mu \to x'^\mu(x) = x^\mu + \zeta^\mu(x)$$

$$\begin{aligned} g'_{\mu\nu}(x') &= \frac{\partial x^\rho}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu} g_{\rho\sigma}(x) \\ &= \left(\delta_\mu{}^\rho - \frac{\partial \zeta^\rho}{\partial x'^\mu}\right)\left(\delta_\nu{}^\sigma - \frac{\partial \zeta^\sigma}{\partial x'^\nu}\right) g_{\rho\sigma}(x) \\ &= g_{\mu\nu}(x) - g_{\mu\rho}(x)\partial_\nu \zeta^\rho(x) - g_{\nu\rho}(x)\partial_\mu \zeta^\rho(x) + \mathcal{O}(\zeta^2) \end{aligned}$$

$$g'_{\mu\nu}(x') = g'_{\mu\nu}(x) + \partial_\rho g_{\mu\nu}(x)\zeta^\rho + \mathcal{O}(\zeta^2)$$

$$\partial_\rho g'_{\mu\nu}(x)\zeta^\rho = \partial_\rho g_{\mu\nu}(x')\zeta^\rho + \mathcal{O}(\zeta^2) = \partial_\rho g_{\mu\nu}(x)\zeta^\rho + \mathcal{O}(\zeta^2)$$

$$\begin{aligned}\delta_\zeta g_{\mu\nu}(x) &\equiv g'_{\mu\nu}(x) - g_{\mu\nu}(x)\\ &= -\zeta^\rho(x)\partial_\rho g_{\mu\nu}(x) - g_{\mu\rho}(x)\partial_\nu\zeta^\rho(x) - g_{\nu\rho}(x)\partial_\mu\zeta^\rho(x)\\ &= -\nabla_\mu\zeta_\nu(x) - \nabla_\nu\zeta_\mu(x)\end{aligned}$$

$$0 = \delta_\zeta S_{\text{EH}} = \frac{1}{2\kappa^2}\int\ \mathrm{d}^4x\frac{\delta(\sqrt{-g}(R-2\Lambda))}{\delta g_{\mu\nu}}\delta_\zeta g_{\mu\nu} = -\frac{1}{\kappa^2}\int\ \mathrm{d}^4x\sqrt{-g}\left(\nabla_\mu G^{\mu\nu}\right)\zeta_\nu$$

$$0 = \nabla_\mu G^{\mu\nu} = \partial_0 G^{0\nu} + \partial_i G^{i\nu} + \Gamma^\mu_{\mu\rho}G^{\rho\nu} + \Gamma^\nu_{\mu\rho}G^{\mu\rho}.$$

$$g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + 2\kappa h_{\mu\nu}(x)$$

$$S_{\text{EH}}[\bar{g}+2\kappa h] = S^{(0)}_{\text{EH}}[\bar{g}] + S^{(1)}_{\text{EH}}[\bar{g},h] + S^{(2)}_{\text{EH}}[\bar{g},h] + \cdots + S^{(n)}_{\text{EH}}[\bar{g},h] + \cdots,$$

$$\begin{aligned}S^{(0)}_{\text{EH}}[\bar{g}] &= \frac{1}{2\kappa^2}\int\ \mathrm{d}^4x\sqrt{-\bar{g}}(\bar{R}-2\Lambda)\\ S^{(1)}_{\text{EH}}[\bar{g},h] &= \frac{1}{1!}2\kappa\int\ \mathrm{d}^4y\frac{\delta S_{\text{EH}}}{\delta g_{\mu\nu}(y)}\bigg|_{g=\bar{g}} h_{\mu\nu}(y)\\ S^{(2)}_{\text{EH}}[\bar{g},h] &= \frac{1}{2!}(2\kappa)^2\int\ \mathrm{d}^4y_1\,\mathrm{d}^4y_2\frac{\delta S_{\text{EH}}}{\delta g_{\mu_1\nu_1}(y_1)\delta g_{\mu_2\nu_2}(y_2)}\bigg|_{g=\bar{g}} h_{\mu_1\nu_1}(y_1)h_{\mu_2\nu_2}(y_2)\\ &\ \vdots\\ S^{(n)}_{\text{EH}}[\bar{g},h] &= \frac{1}{n!}(2\kappa)^n\int\ \mathrm{d}^4y_1\cdots\mathrm{d}^4y_n\frac{\delta S_{\text{EH}}}{\delta g_{\mu_1\nu_1}(y_1)\cdots\delta g_{\mu_n\nu_n}(y_n)}\bigg|_{g=\bar{g}} h_{\mu_1\nu_1}(y_1)\cdots h_{\mu_n\nu_n}(y_n),\\ &\ \vdots\end{aligned}$$

$$\begin{aligned}S^{(1)}_{\text{EH}}[\bar{g},h] &= \frac{1}{\kappa}\int\ \mathrm{d}^4y\left[\frac{\delta}{\delta g_{\mu\nu}(y)}\int\ \mathrm{d}^4x\sqrt{-g(x)}(R(x)-2\Lambda)\right]\bigg|_{g=\bar{g}} h_{\mu\nu}(y)\\ &= -\frac{1}{\kappa}\int\ \mathrm{d}^4y\sqrt{-g(y)}\left[\bar{R}^{\mu\nu}(y) - \frac{1}{2}\bar{g}^{\mu\nu}(y)\bar{R}(y) + \bar{g}^{\mu\nu}(y)\Lambda\right]h_{\mu\nu}(y)\\ &= 0\end{aligned}$$

$$\begin{aligned}S^{(2)}_{\text{EH}}[\bar{g},h] &= \frac{1}{2!}\delta^{(2)}S_{\text{EH}}[\bar{g},h]\\ &= \frac{1}{4\kappa^2}\int\ \mathrm{d}^4x[\delta(\delta(\sqrt{-g})R + \sqrt{-g}\delta R)]_{g=\bar{g}}\\ &= \frac{1}{4\kappa^2}\int\ \mathrm{d}^4x\left[\delta^{(2)}(\sqrt{-g})R + 2\delta(\sqrt{-g})\delta R + \sqrt{-g}\delta^{(2)}R\right]_{g=\bar{g}}\end{aligned}$$

$$\begin{aligned}g^{\mu\nu} &= \bar{g}^{\mu\nu} - 2\kappa h^{\mu\nu} + 4\kappa^2 h^\mu_\rho h^{\nu\rho} + \cdots\\ \Gamma^\mu_{\rho\sigma} &= \bar{\Gamma}^\mu_{\rho\sigma} + \kappa g^{\mu\nu}\left(\bar{\nabla}_\rho h_{\nu\sigma} + \bar{\nabla}_\sigma h_{\nu\rho} - \bar{\nabla}_\nu h_{\rho\sigma}\right)\end{aligned}$$

$$\begin{aligned}R &= \bar{R} + \delta R + \frac{1}{2}\delta^{(2)}R + \cdots,\\ \delta R &= 2\kappa\left(\bar{\nabla}_\mu\bar{\nabla}_\nu h^{\mu\nu} - \bar{\nabla}^2 h - \bar{R}^{\mu\nu}h_{\mu\nu}\right),\\ \delta^{(2)}R &= 4\kappa^2\bigg(\frac{3}{2}\bar{\nabla}_\rho h_{\mu\nu}\bar{\nabla}^\rho h^{\mu\nu} + 2h_{\mu\nu}\bar{\nabla}^2 h^{\mu\nu} - 2\bar{\nabla}_\rho h^\rho_\mu\bar{\nabla}_\sigma h^{\sigma\mu}\\ &\quad + 2\bar{\nabla}_\rho h^\rho_\mu\bar{\nabla}^\mu h - 4h_{\mu\nu}\bar{\nabla}^\mu\bar{\nabla}_\rho h^{\rho\nu} + 2h_{\mu\nu}\bar{\nabla}^\mu\bar{\nabla}^\nu h\\ &\quad - \bar{\nabla}_\mu h_{\nu\rho}\bar{\nabla}^\rho h^{\mu\nu} - \frac{1}{2}\bar{\nabla}_\mu h\bar{\nabla}^\mu h + 2\bar{R}_{\mu\nu\rho\sigma}h^{\mu\rho}h^{\nu\sigma}\bigg).\end{aligned}$$

$$\sqrt{-g} = \sqrt{-\bar{g}} + \delta(\sqrt{-g}) + \frac{1}{2}\delta^{(2)}(\sqrt{-g}) + \cdots$$
$$\delta(\sqrt{-g}) = \sqrt{-\bar{g}}\kappa h, \delta^{(2)}(\sqrt{-g}) = \sqrt{-\bar{g}}\kappa^2\left(h^2 - 2h_{\mu\nu}h^{\mu\nu}\right).$$

$$S_{\text{EH}}^{(2)}[\bar{g},h] = \int \mathrm{d}^4x\sqrt{-\bar{g}}\Big[-\frac{1}{2}\bar{\nabla}_\rho h_{\mu\nu}\bar{\nabla}^\rho h^{\mu\nu} + \bar{\nabla}_\rho h^\rho{}_\mu\bar{\nabla}_\sigma h^{\sigma\mu} - \bar{\nabla}_\mu h\bar{\nabla}_\nu h^{\mu\nu} + \frac{1}{2}\bar{\nabla}_\rho h\bar{\nabla}^\rho h$$
$$-\frac{1}{2}(\bar{R} - 2\Lambda)\left(h_{\mu\nu}h^{\mu\nu} - \frac{1}{2}h^2\right) + \left(h^{\mu\rho}h_\rho{}^\nu - hh^{\mu\nu}\right)\bar{R}_{\mu\nu} + h^{\mu\rho}h^{\nu\sigma}\bar{R}_{\mu\nu\rho\sigma}\Big]$$

$$S^{(n)}[\bar{g},h] \sim \mathcal{O}(\kappa^{n-2}h^n), n \geq 3$$

$$S_{\text{EH}}^{(2)}[\eta,h] = \int \mathrm{d}^4x\left[-\frac{1}{2}\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} + \partial_\rho h_\mu^\rho\partial_\sigma h^{\sigma\mu} - \partial_\mu h\partial_\nu h^{\mu\nu} + \frac{1}{2}\partial_\rho h\partial^\rho h\right]$$

$$S_{\text{EH}}^{(2)}[\eta,h] = \int \mathrm{d}^4x\frac{1}{2}h_{\mu\nu}\mathbb{K}^{\mu\nu\rho\sigma}h_{\rho\sigma}$$

$$\mathbb{K}^{\mu\nu\rho\sigma} \equiv \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho})\,\Box - \eta^{\mu\nu}\eta^{\rho\sigma}\,\Box + \eta^{\mu\nu}\partial^\rho\partial^\sigma + \eta^{\rho\sigma}\partial^\mu\partial^\nu$$
$$-\frac{1}{2}(\eta^{\mu\rho}\partial^\nu\partial^\sigma + \eta^{\mu\sigma}\partial^\nu\partial^\rho + \eta^{\nu\rho}\partial^\mu\partial^\sigma + \eta^{\nu\sigma}\partial^\mu\partial^\rho)$$

$$\mathbb{K}^{\mu\nu\rho\sigma} = \mathbb{K}^{\nu\mu\rho\sigma} = \mathbb{K}^{\mu\nu\sigma\rho} = \mathbb{K}^{\rho\sigma\mu\nu}.$$

$$S_m[\eta + 2\kappa h] = S_m[\eta] + 2\kappa\int \mathrm{d}^4x\frac{\delta S_m}{\delta g_{\mu\nu}}h_{\mu\nu} + \mathcal{O}(\kappa^2h^2)$$
$$= S_m[\eta] + \kappa\int \mathrm{d}^4xT^{\mu\nu}h_{\mu\nu} + \mathcal{O}(\kappa^2h^2)$$

$$\delta_\zeta h_{\mu\nu} = \nabla_\mu\zeta_\nu + \nabla_\nu\zeta_\mu$$
$$= \partial_\mu\zeta_\nu + \partial_\nu\zeta_\mu + 2\kappa\left(h_{\mu\rho}\partial_\nu\zeta^\rho + h_{\nu\rho}\partial_\mu\zeta^\rho + \zeta^\rho\partial_\rho h_{\mu\nu}\right),$$

$$\delta_\zeta h_{\mu\nu} = \partial_\mu\zeta_\nu + \partial_\nu\zeta_\mu \;\Rightarrow\; \delta_\zeta S_{\text{EH}}^{(2)}[\eta,h] = 0$$

$$\mathbb{K}_{\mu\nu}{}^{\rho\sigma}h_{\rho\sigma} = -\kappa T_{\mu\nu}$$
$$\Leftrightarrow \Box\, h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\,\Box\, h + \eta_{\mu\nu}\partial_\rho\left(\partial_\sigma h^{\rho\sigma} - \frac{1}{2}\partial^\rho h\right)$$
$$-\partial_\mu\left(\partial_\rho h^\rho{}_\nu - \frac{1}{2}\partial_\nu h\right) - \partial_\nu\left(\partial_\rho h^\rho{}_\mu - \frac{1}{2}\partial_\mu h\right) = -\kappa T_{\mu\nu}$$

$$-2\,\Box\, h + 2\partial_\rho\partial_\sigma h^{\rho\sigma} = -\kappa T$$

$$\partial_\rho h_\nu^\rho - \frac{1}{2}\partial_\nu h = 0$$

$$\Box\, h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\,\Box\, h = -\kappa T_{\mu\nu}.$$

$$\Box\, h_{\mu\nu} = 0.$$

$$h_{\mu\nu}(x) = \epsilon_{\mu\nu}(p)e^{ip\cdot x} + \epsilon_{\mu\nu}^*(p)e^{-ip\cdot x}, p^2 = -p_0^2 + \vec{p}^2 = 0$$

$$p^{\mu}\epsilon_{\mu\nu} - \frac{1}{2}p_{\nu}\epsilon = 0, \epsilon \equiv \eta^{\mu\nu}\epsilon_{\mu\nu}$$

$$0 = \delta_{\zeta}\left(\partial^{\rho}h_{\rho\nu} - \frac{1}{2}\partial_{\nu}h\right) = \partial^{\rho}(\partial_{\rho}\zeta_{\nu} + \partial_{\nu}\zeta_{\rho}) - \frac{1}{2}\partial_{\nu}(2\partial_{\rho}\zeta^{\rho}) = \Box\, \zeta_{\nu}$$

$$\zeta_{\nu}(x) = r_{\nu}(p)e^{ip\cdot x} + r_{\nu}^{*}(p)e^{-ip\cdot x}, p^2 = -p_0^2 + \vec{p}^2 = 0$$

$$p^{\mu} = (p^0, 0,0, p^3), p^0 = p^3$$

$$\begin{aligned} \nu = 0:&\quad \epsilon_{00} + \epsilon_{30} + \frac{1}{2}\epsilon = 0, \\ \nu = 1:&\quad \epsilon_{01} + \epsilon_{31} = 0, \\ \nu = 2:&\quad \epsilon_{02} + \epsilon_{32} = 0, \\ \nu = 3:&\quad \epsilon_{03} + \epsilon_{33} - \frac{1}{2}\epsilon = 0. \end{aligned}$$

$$r_0: \epsilon_{00} = 0, r_1: \epsilon_{01} = 0, r_2: \epsilon_{02} = 0, r_3: \epsilon_{33} = 0$$

$$\epsilon_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \epsilon_{11} & \epsilon_{12} & 0 \\ 0 & \epsilon_{12} & -\epsilon_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \epsilon_{11}e_{\mu\nu}^{(+)} + \epsilon_{12}e_{\mu\nu}^{(\times)}$$

$$e_{\mu\nu}^{(+)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, e_{\mu\nu}^{(\times)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\partial_{\mu}h_{\nu}^{\mu} = 0, h = \eta^{\mu\nu}h_{\mu\nu} = 0$$

$$R_{\mu}^{\nu}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$e_{\mu\nu}^{(+2)} = \frac{1}{\sqrt{2}}\left(e_{\mu\nu}^{(+)} + ie_{\mu\nu}^{(\times)}\right), e_{\mu\nu}^{(-2)} = \frac{1}{\sqrt{2}}\left(e_{\mu\nu}^{(+)} - ie_{\mu\nu}^{(\times)}\right)$$

$$R_{\mu}^{\rho}(\theta)e_{\rho\sigma}^{(\pm 2)}R_{\nu}^{\sigma}(\theta) = e^{\pm 2i\theta}e_{\mu\nu}^{(\pm 2)}, \lambda = \pm 2$$

$$\epsilon_{\mu\nu} = \frac{1}{\sqrt{2}}(\epsilon_{11} - i\epsilon_{12})e_{\mu\nu}^{(+2)} + \frac{1}{\sqrt{2}}(\epsilon_{11} + i\epsilon_{12})e_{\mu\nu}^{(-2)} \equiv \epsilon_{\mu\nu}^{(+2)} + \epsilon_{\mu\nu}^{(-2)}$$

$$\epsilon_{\mu\nu}^{(+2)} \equiv \frac{1}{\sqrt{2}}(\epsilon_{11} - i\epsilon_{12})e_{\mu\nu}^{(+2)}, \epsilon_{\mu\nu}^{(-2)} \equiv \frac{1}{\sqrt{2}}(\epsilon_{11} + i\epsilon_{12})e_{\mu\nu}^{(-2)}$$

$$\partial_i h_{\mu}^{i} = 0, \mu = 0,1,2,3,$$

$$\Box\, h_{\mu\nu} - \eta_{\mu\nu}\,\Box\, h + \eta_{\mu\nu}\ddot{h}_{00} + \partial_{\mu}\partial_{\nu}h + \partial_{\mu}\dot{h}_{0\nu} + \partial_{\nu}\dot{h}_{0\mu} = 0,$$

$$0 = ip_j h_{\mu}^{j} = ip_3 h_{3\mu} \;\Rightarrow\; h_{30} = h_{31} = h_{32} = h_{33} = 0$$

$$-p_3^2(h_{00}+h)=0 \;\Rightarrow\; h_{00}=-h.$$

$$-\left(\ddot{h}_{11}+p_3^2 h_{11}\right)+p_3^2 h=0, -\left(\ddot{h}_{22}+p_3^2 h_{22}\right)+p_3^2 h=0,$$

$$\ddot{h}_{11}+p_3^2 h_{11}=0.$$

$$\ddot{h}_{12}+p_3^2 h_{12}=0.$$

$$(-p_0^2+p_3^2)\epsilon_{11}=0, (-p_0^2+p_3^2)\epsilon_{12}=0$$

$$\nabla^2\zeta_v+\partial_v\left(\partial^j\zeta_j\right)=0$$

$$V_{\rho\sigma}=\partial_\rho\zeta_\sigma+\partial_\sigma\zeta_\rho, \mathbb{K}^{\mu\nu\rho\sigma}V_{\rho\sigma}=0$$

$$S_{\mathrm{gf}}[\eta,h]=-\frac{1}{\alpha}\int \mathrm{d}^4x\mathcal{F}_\mu\mathcal{F}^\mu, \mathcal{F}_\mu\equiv\partial_v h_\mu^v-\frac{1}{2}\partial_\mu h$$

$$S_{\mathrm{gf}}[\eta,h]=\int \mathrm{d}^4x\frac{1}{2}h_{\mu\nu}\mathbb{K}_{\mathrm{gf}}^{\mu\nu\rho\sigma}h_{\rho\sigma}$$

$$\begin{aligned}\mathbb{K}_{\mathrm{gf}}^{\mu\nu\rho\sigma}\equiv&\frac{1}{2\alpha}\eta^{\mu\nu}\eta^{\rho\sigma}\square-\frac{1}{\alpha}(\eta^{\mu\nu}\partial^\rho\partial^\sigma+\eta^{\rho\sigma}\partial^\mu\partial^\nu)\\&+\frac{1}{2\alpha}(\eta^{\mu\rho}\partial^\nu\partial^\sigma+\eta^{\mu\sigma}\partial^\nu\partial^\rho+\eta^{\nu\rho}\partial^\mu\partial^\sigma+\eta^{\nu\sigma}\partial^\mu\partial^\rho)\end{aligned}$$

$$\tilde{S}^{(2)}[\eta,h]\equiv S_{\mathrm{EH}}^{(2)}[\eta,h]+S_{\mathrm{gf}}[\eta,h]=\int \mathrm{d}^4x\frac{1}{2}h_{\mu\nu}\tilde{\mathbb{K}}^{\mu\nu\rho\sigma}h_{\rho\sigma}$$

$$\begin{aligned}\tilde{\mathbb{K}}^{\mu\nu\rho\sigma}\equiv\mathbb{K}^{\mu\nu\rho\sigma}+\mathbb{K}_{\mathrm{gf}}^{\mu\nu\rho\sigma}=&\frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\nu\rho})\square-\left(1-\frac{1}{2\alpha}\right)\eta^{\mu\nu}\eta^{\rho\sigma}\square\\&+\left(1-\frac{1}{\alpha}\right)(\eta^{\mu\nu}\partial^\rho\partial^\sigma+\eta^{\rho\sigma}\partial^\mu\partial^\nu)\\&-\frac{1}{2}\left(1-\frac{1}{\alpha}\right)(\eta^{\mu\rho}\partial^\nu\partial^\sigma+\eta^{\mu\sigma}\partial^\nu\partial^\rho+\eta^{\nu\rho}\partial^\mu\partial^\sigma+\eta^{\nu\sigma}\partial^\mu\partial^\rho)\end{aligned}$$

$$\tilde{\mathbb{K}}^{\mu\nu\rho\sigma}V_{\rho\sigma}\neq 0.$$

$$\begin{aligned}\tilde{\mathbb{K}}^{\mu\nu\rho\sigma}(p)=&-\frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\nu\rho})p^2+\left(1-\frac{1}{2\alpha}\right)\eta^{\mu\nu}\eta^{\rho\sigma}p^2\\&-\left(1-\frac{1}{\alpha}\right)(\eta^{\mu\nu}p^\rho p^\sigma+\eta^{\rho\sigma}p^\mu p^\nu)\\&+\frac{1}{2}\left(1-\frac{1}{\alpha}\right)(\eta^{\mu\rho}p^\nu p^\sigma+\eta^{\mu\sigma}p^\nu p^\rho+\eta^{\nu\rho}p^\mu p^\sigma+\eta^{\nu\sigma}p^\mu p^\rho)\end{aligned}$$

$$\mathcal{G}_{\mu\nu}{}^{\alpha\beta}(p)\tilde{\mathbb{K}}_{\alpha\beta}{}^{\rho\sigma}(p)=i\mathbb{1}_{\mu\nu}{}^{\rho\sigma},$$

$$\mathcal{G}_{\mu\nu\alpha\beta}(p)\tilde{\mathbb{K}}_{\rho\sigma}^{\alpha\beta}(p)=i\mathbb{1}_{\mu\nu\rho\sigma}$$

$$\mathbb{1}_{\mu\nu}{}^{\rho\sigma}=\frac{1}{2}\left(\delta_\mu{}^\rho\delta_\nu{}^\sigma+\delta_\nu{}^\rho\delta_\mu{}^\sigma\right), \mathbb{1}_{\mu\nu\rho\sigma}=\frac{1}{2}\left(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\nu\rho}\eta_{\mu\sigma}\right)$$

$$B^{(1)}_{\mu\nu\rho\sigma}(p) = \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho},$$
$$B^{(2)}_{\mu\nu\rho\sigma}(p) = \eta_{\mu\nu}\eta_{\rho\sigma},$$
$$B^{(3)}_{\mu\nu\rho\sigma}(p) = \frac{1}{p^2}(\eta_{\mu\nu}p_\rho p_\sigma + \eta_{\rho\sigma}p_\mu p_\nu),$$
$$B^{(4)}_{\mu\nu\rho\sigma}(p) = \frac{1}{p^2}(\eta_{\mu\rho}p_\nu p_\sigma + \eta_{\mu\sigma}p_\nu p_\rho + \eta_{\nu\rho}p_\mu p_\sigma + \eta_{\nu\sigma}p_\mu p_\rho),$$
$$B^{(5)}_{\mu\nu\rho\sigma}(p) = \frac{1}{(p^2)^2}p_\mu p_\nu p_\rho p_\sigma.$$

$$\mathcal{G}_{\mu\nu\rho\sigma}(p) = \sum_{j=1}^{5} c_j(p)B^{(j)}_{\mu\nu\rho\sigma}(p)$$

$$\begin{aligned}\tilde{\mathbb{K}}^{(\alpha=1)\mu\nu\rho\sigma}(p) &= -\frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma})p^2 \\ &= a(p)(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}) + b(p)\eta^{\mu\nu}\eta^{\rho\sigma}\end{aligned}$$

$$\mathcal{G}^{(\alpha=1)}_{\mu\nu\rho\sigma}(p) = A(p)\big(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}\big) + B(p)\eta_{\mu\nu}\eta_{\rho\sigma}$$

$$\begin{aligned}\mathcal{G}^{(\alpha=1)}_{\mu\nu\alpha\beta}(p)\tilde{\mathbb{K}}^{(\alpha=1)\alpha\beta}{}_{\rho\sigma}(p) &= 2aA\big[\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}\big] + [2bA + 2aB + 4bB]\eta_{\mu\nu}\eta_{\rho\sigma} \\ &= \frac{i}{2}\big(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\nu\rho}\eta_{\mu\sigma}\big)\end{aligned}$$

$$\begin{cases}2Aa = \dfrac{i}{2} \\ 2bA + 2aB + 4bB = 0\end{cases} \Leftrightarrow A(p) = -B(p) = -\frac{i}{2p^2}.$$

$$\mathcal{G}^{(\alpha=1)}_{\mu\nu\rho\sigma}(p) = \frac{1}{2}\frac{-i}{p^2 - i\epsilon}\big(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}\big)$$

$$S_{\mathrm{gf}}[\eta, h] = -\frac{1}{\alpha}\int \mathrm{d}^4x\mathcal{F}_\mu\mathcal{F}^\mu, \mathcal{F}_\mu \equiv \partial_i h^i_\mu$$

$$\mathbb{K}^{\mu\nu\rho\sigma}_{\mathrm{gf}} = \frac{1}{2\alpha}(\eta^{\mu\rho}\eta^{\nu i}\eta^{\sigma j} + \eta^{\mu\sigma}\eta^{\nu i}\eta^{\rho j} + \eta^{\nu\rho}\eta^{\mu i}\eta^{\sigma j} + \eta^{\nu\sigma}\eta^{\mu i}\eta^{\rho j})\partial_i\partial_j$$

$$\begin{aligned}\tilde{\mathbb{K}}^{\mu\nu\rho\sigma}(p,\bar{p}) = &-\frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho})p^2 + \eta^{\mu\nu}\eta^{\rho\sigma}p^2 - (\eta^{\mu\nu}p^\rho p^\sigma + \eta^{\rho\sigma}p^\mu p^\nu) \\ &+\frac{1}{2}\Big[\eta^{\mu\rho}\left(p^\nu p^\sigma - \frac{1}{\alpha}\bar{p}^\nu\bar{p}^\sigma\right) + \eta^{\mu\sigma}\left(p^\nu p^\rho - \frac{1}{\alpha}\bar{p}^\nu\bar{p}^\rho\right) \\ &+\eta^{\nu\rho}\left(p^\mu p^\sigma - \frac{1}{\alpha}\bar{p}^\mu\bar{p}^\sigma\right) + \eta^{\nu\sigma}\left(p^\mu p^\rho - \frac{1}{\alpha}\bar{p}^\mu\bar{p}^\rho\right)\Big]\end{aligned}$$

$$\tilde{\mathbb{K}}^{0101} = \tilde{\mathbb{K}}^{0202} = \frac{1}{2}p_3^2, \tilde{\mathbb{K}}^{0303} = \frac{p_3^2}{2\alpha}, \tilde{\mathbb{K}}^{1313} = \tilde{\mathbb{K}}^{2323} = -\frac{p^2}{2} + \frac{1}{2}\left(1 - \frac{1}{\alpha}\right)p_3^2$$
$$\tilde{\mathbb{K}}^{0011} = \tilde{\mathbb{K}}^{0022} = -p_3^2, \tilde{\mathbb{K}}^{0113} = \tilde{\mathbb{K}}^{0223} = -\frac{p_0p_3}{2}, \tilde{\mathbb{K}}^{1103} = \tilde{\mathbb{K}}^{2203} = p_0p_3$$
$$\tilde{\mathbb{K}}^{1122} = p^2, \tilde{\mathbb{K}}^{1212} = -\frac{1}{2}p^2, \tilde{\mathbb{K}}^{1133} = \tilde{\mathbb{K}}^{2233} = p^2 - p_3^2$$

$$\hat{h} \equiv (h_{00}, h_{01}, h_{02}, h_{03}, h_{11}, h_{12}, h_{13}, h_{22}, h_{23}, h_{33}),$$

$$\hat{\mathbb{K}}^{\hat{k}\hat{\ell}} \equiv s^{k\ell}\tilde{\mathbb{K}}^{k\ell}, k,\ell \in \{00,01,02,03,11,12,13,22,23,33\}, \hat{k},\hat{\ell} \in \{1,2,\ldots,10\}$$

$$\tilde{S}^{(2)} = \frac{1}{2}\int \mathrm{d}^4x \sum_{\hat{k},\hat{\ell}=1}^{10} \hat{h}_{\hat{k}}\hat{\mathbb{K}}^{\hat{k}\hat{\ell}}\hat{h}_{\hat{\ell}} = \frac{1}{2}\int \mathrm{d}^4x\hat{h}\cdot\hat{\mathbb{K}}\cdot\hat{h}^{\mathrm{T}}$$

$$\hat{\mathbb{K}} = \begin{pmatrix}
0 & 0 & 0 & 0 & -2p_3^2 & 0 & 0 & -2p_3^2 & 0 & 0 \\
0 & \frac{p_3^2}{2} & 0 & 0 & 0 & 0 & -2p_0p_3 & 0 & 0 & 0 \\
0 & 0 & \frac{p_3^2}{2} & 0 & 0 & 0 & 0 & 0 & -2p_0p_3 & 0 \\
0 & 0 & 0 & \frac{p_3^2}{2\alpha} & 2p_0p_3 & 0 & 0 & 2p_0p_3 & 0 & 0 \\
-2p_3^2 & 0 & 0 & 2p_0p_3 & 0 & 0 & 0 & p^2 & 0 & p^2-p_3^2 \\
0 & 0 & 0 & 0 & 0 & -2p^2 & 0 & 0 & 0 & 0 \\
0 & -2p_0p_3 & 0 & 0 & 0 & 0 & -2p^2+2\left(1-\frac{1}{\alpha}\right)p_3^2 & 0 & 0 & 0 \\
-2p_3^2 & 0 & 0 & 2p_0p_3 & p^2 & 0 & 0 & 0 & 0 & p^2-p_3^2 \\
0 & 0 & -2p_0p_3 & 0 & 0 & 0 & 0 & 0 & -2p^2+2\left(1-\frac{1}{\alpha}\right)p_3^2 & 0 \\
0 & 0 & 0 & 0 & p^2-p_3^2 & 0 & 0 & p^2-p_3^2 & 0 & \frac{-2p_3^2}{\alpha}
\end{pmatrix}.$$

$$\hat{\mathcal{G}} = i\begin{pmatrix}
\frac{\alpha p_0^2\left(p^2+15p_3^2\right)-p_3^2p^2}{8p_3^6} & 0 & 0 & \frac{2\alpha p_0}{p_3^3} & \frac{-1}{4p_3^2} & 0 & 0 & \frac{-1}{4p_3^2} & 0 & \frac{\alpha p_0^2}{4p_3^4} \\
0 & \frac{2\left(p_3^2-\alpha p_0^2\right)}{p_3^2(3\alpha p_0^2+p_3^2)} & 0 & 0 & 0 & 0 & \frac{-2\alpha p_0}{p_3(3\alpha p_0^2+p_3^2)} & 0 & 0 & 0 \\
0 & 0 & \frac{2\left(p_3^2-\alpha p_0^2\right)}{p_3^2(3\alpha p_0^2+p_3^2)} & 0 & 0 & 0 & 0 & 0 & \frac{-2\alpha p_0}{p_3(3\alpha p_0^2+p_3^2)} & 0 \\
\frac{2\alpha p_0}{p_3^3} & 0 & 0 & \frac{2\alpha}{p_3^2} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-1}{4p_3^2} & 0 & 0 & 0 & \frac{-1}{2p^2} & 0 & 0 & \frac{1}{2p^2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{2p^2} & 0 & 0 & 0 & 0 \\
0 & \frac{-2\alpha p_0}{p_3(3\alpha p_0^2+p_3^2)} & 0 & 0 & 0 & 0 & \frac{-\alpha}{6\alpha p_0^2+2p_3^2} & 0 & 0 & 0 \\
\frac{-1}{4p_3^2} & 0 & 0 & 0 & \frac{1}{2p^2} & 0 & 0 & \frac{-1}{2p^2} & 0 & 0 \\
0 & 0 & \frac{-2\alpha p_0}{p_3(3\alpha p_0^2+p_3^2)} & 0 & 0 & 0 & 0 & 0 & \frac{-\alpha}{6\alpha p_0^2+2p_3^2} & 0 \\
\frac{\alpha p_0^2}{4p_3^4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-\alpha}{2p_3^2}
\end{pmatrix}$$

$$\hat{\mathcal{G}}^{(\alpha=0)}=i\begin{pmatrix}\frac{-p^2}{8p_3^4} & 0 & 0 & 0 & \frac{-1}{4p_3^2} & 0 & 0 & \frac{-1}{4p_3^2} & 0 & 0\\ 0 & \frac{2}{p_3^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \frac{2}{p_3^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ -\frac{1}{2p_3^2} & 0 & 0 & 0 & \frac{-1}{2p^2} & 0 & 0 & \frac{1}{2p^2} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{2p^2} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \frac{-1}{4p_3^2} & 0 & 0 & 0 & \frac{1}{2p^2} & 0 & 0 & \frac{-1}{2p^2} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{pmatrix}$$

$$\det\left(\hat{\mathcal{G}}^{(\alpha=0)}-\lambda\mathbb{1}\right)=0\Leftrightarrow\frac{\lambda^4(1+\lambda p^2)(1+2\lambda p^2)(2-\lambda p_3^2)^2(1-\lambda p^2-8\lambda^2p_3^4)}{2(p^2)^2p_3^8}=0$$

$$\lambda_1=\lambda_2=\lambda_3=\lambda_4=0,\lambda_5=\lambda_6=\frac{2}{p_3^2},\lambda_7=-\frac{p^2+\sqrt{(p^2)^2+32p_3^4}}{4p_3^4}$$

$$\lambda_8=\frac{-p^2+\sqrt{(p^2)^2+32p_3^4}}{4p_3^4},\lambda_9=-\frac{2}{p^2},\lambda_{10}=-\frac{1}{p^2}$$

$$\mathcal{G}^{(\alpha=0)}_{\mu\nu\rho\sigma}(p,\bar{p})=\frac{1}{2}\frac{-i}{p^2-i\epsilon}\left(\bar{\delta}_{\mu\rho}\bar{\delta}_{\nu\sigma}+\bar{\delta}_{\mu\sigma}\bar{\delta}_{\nu\rho}-\bar{\delta}_{\mu\nu}\bar{\delta}_{\rho\sigma}\right)+\cdots,$$

$$\bar{\delta}_{\mu\nu}\equiv\mathrm{diag}(0,1,1,0)$$

$$\lim_{p^2\to0}\left[ip^2\mathcal{G}^{(\alpha=0)}_{\mu\nu\rho\sigma}(p,\bar{p})\right]h^{\rho\sigma}=\frac{1}{2}\left(e^{(+)}_{\mu\nu}e^{(+)}_{\rho\sigma}+e^{(\times)}_{\mu\nu}e^{(\times)}_{\rho\sigma}\right)h^{\rho\sigma}$$

$$\bar{\delta}_{\mu\rho}\bar{\delta}_{\nu\sigma}+\bar{\delta}_{\mu\sigma}\bar{\delta}_{\nu\rho}-\bar{\delta}_{\mu\nu}\bar{\delta}_{\rho\sigma}=e^{(+)}_{\mu\nu}e^{(+)}_{\rho\sigma}+e^{(\times)}_{\mu\nu}e^{(\times)}_{\rho\sigma}$$

$$S_A=\int\mathrm{d}^4x\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2\right]=\frac{1}{2}\int\mathrm{d}^4xA_\mu\mathbb{K}^{\mu\nu}A_\nu$$

$$\mathbb{K}^{\mu\nu}\equiv\eta^{\mu\nu}\square-\left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu$$

$$A_\mu\in\mathbf{0}\oplus\mathbf{1}.$$

$$\theta_{\mu\nu}=\eta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2},\omega_{\mu\nu}=\frac{p_\mu p_\nu}{p^2}$$

$$\theta_{\mu\rho}\theta^\rho_\nu=\theta_{\mu\nu},\omega_{\mu\rho}\omega^\rho_\nu=\omega_{\mu\nu},\theta_{\mu\rho}\omega^\rho_\nu=0$$

$$\theta_\mu{}^\nu+\omega_\mu{}^\nu=\delta_\mu{}^\nu\ \Leftrightarrow\ \theta_{\mu\nu}+\omega_{\mu\nu}=\eta_{\mu\nu}.$$

$$\eta^{\mu\nu}\theta_{\mu\nu} = 3 = 2(1) + 1, \eta^{\mu\nu}\omega_{\mu\nu} = 1 = 2(0) + 1$$

$$\tilde{\mathbb{K}}^{\mu\nu} = -p^2\left[\theta^{\mu\nu} + \frac{1}{\xi}\omega^{\mu\nu}\right].$$

$$\mathcal{G}_{\mu\nu}(p) = -\frac{i}{p^2}\left(\theta_{\mu\nu} + \xi\omega_{\mu\nu}\right)$$

$$h^{\mu\nu} \in \mathbf{0} \oplus \mathbf{0} \oplus \mathbf{1} \oplus \mathbf{2}.$$

$$\mathcal{P}^{(2)}{}_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\rho}\right) - \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma},$$
$$\mathcal{P}^{(1)}{}_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\theta_{\mu\rho}\omega_{\nu\sigma} + \theta_{\mu\sigma}\omega_{\nu\rho} + \theta_{\nu\rho}\omega_{\mu\sigma} + \theta_{\nu\sigma}\omega_{\mu\rho}\right),$$
$$\mathcal{P}^{(0,s)}{}_{\mu\nu\rho\sigma} = \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma},$$
$$\mathcal{P}^{(0,w)}{}_{\mu\nu\rho\sigma} = \omega_{\mu\nu}\omega_{\rho\sigma}.$$

$$\mathcal{P}^{(i,a)}{}_{\mu\nu}{}^{\alpha\beta}\mathcal{P}^{(j,b)}{}_{\alpha\beta}{}^{\rho\sigma} = \delta^{ij}\delta^{ab}\mathcal{P}^{(i,a)}{}_{\mu\nu}{}^{\rho\sigma}$$

$$\mathcal{P}^{(2)}{}_{\mu\nu\rho\sigma} + \mathcal{P}^{(1)}{}_{\mu\nu\rho\sigma} + \mathcal{P}^{(0,s)}{}_{\mu\nu\rho\sigma} + \mathcal{P}^{(0,w)}{}_{\mu\nu\rho\sigma} = \mathbb{1}_{\mu\nu\rho\sigma}$$

$$\mathbb{1}^{\mu\nu\rho\sigma}\mathcal{P}^{(2)}{}_{\mu\nu\rho\sigma} = 5 = 2(2) + 1,$$
$$\mathbb{1}^{\mu\nu\rho\sigma}\mathcal{P}^{(1)}{}_{\mu\nu\rho\sigma} = 3 = 2(1) + 1,$$
$$\mathbb{1}^{\mu\nu\rho\sigma}\mathcal{P}^{(0,s)}{}_{\mu\nu\rho\sigma} = 1 = 2(0) + 1,$$
$$\mathbb{1}^{\mu\nu\rho\sigma}\mathcal{P}^{(0,w)}{}_{\mu\nu\rho\sigma} = 1 = 2(0) + 1,$$

$$\mathcal{P}^{(0,x)}_{\mu\nu\rho\sigma} = \mathcal{P}^{(0,sw)}_{\mu\nu\rho\sigma} + \mathcal{P}^{(0,ws)}_{\mu\nu\rho\sigma}$$

$$\mathcal{P}^{(0,sw)}_{\mu\nu\rho\sigma} = \frac{1}{\sqrt{3}}\theta_{\mu\nu}\omega_{\rho\sigma}, \mathcal{P}^{(0,ws)}_{\mu\nu\rho\sigma} = \frac{1}{\sqrt{3}}\omega_{\mu\nu}\theta_{\rho\sigma}$$

$$\mathcal{P}^{(i,ab)}_{\mu\nu}{}^{(j,cd)}_{\mu\beta}{}^{\rho\sigma}_{\alpha\beta} = \delta^{ij}\delta^{bc}\mathcal{P}^{(i,ad)}_{\mu\nu}{}^{\rho\sigma}_{\mu\nu}$$

$$\eta_{\mu\nu}\eta_{\rho\sigma} = \left(3\mathcal{P}^{(0,s)} + \mathcal{P}^{(0,w)} + \sqrt{3}\mathcal{P}^{(0,\times)}\right)_{\mu\nu\rho\sigma}$$
$$\eta_{\mu\nu}\omega_{\rho\sigma} + \eta_{\rho\sigma}\omega_{\mu\nu} = \left(\sqrt{3}\mathcal{P}^{(0,\times)} + 2\mathcal{P}^{(0,w)}\right)_{\mu\nu\rho\sigma}$$
$$\frac{1}{2}\left(\eta_{\mu\rho}\omega_{\nu\sigma} + \eta_{\mu\sigma}\omega_{\nu\rho} + \eta_{\nu\sigma}\omega_{\mu\rho} + \eta_{\nu\rho}\omega_{\mu\sigma}\right) = \left(\mathcal{P}^{(1)} + 2\mathcal{P}^{(0,w)}\right)_{\mu\nu\rho\sigma}$$

$$\tilde{\mathbb{K}}^{\mu\nu\rho\sigma}(p) = -p^2\Bigg[\mathcal{P}^{(2)\mu\nu\rho\sigma} + \frac{1}{\alpha}\mathcal{P}^{(1)\mu\nu\rho\sigma} + \left(\frac{3}{2\alpha} - 2\right)\mathcal{P}^{(0,s)\mu\nu\rho\sigma}$$
$$+\frac{1}{2\alpha}\mathcal{P}^{(0,w)\mu\nu\rho\sigma} - \frac{\sqrt{3}}{2\alpha}\mathcal{P}^{(0,x)\mu\nu\rho\sigma}\Bigg]$$

$$\mathcal{G}_{\mu\nu\rho\sigma}(p) = A(p)\mathcal{P}^{(2)}_{\mu\nu\rho\sigma} + B(p)\mathcal{P}^{(1)}_{\mu\nu\rho\sigma} + C(p)\mathcal{P}^{(0,s)}_{\mu\nu\rho\sigma}$$
$$+D(p)\mathcal{P}^{(0,w)}_{\mu\nu\rho\sigma} + E(p)\mathcal{P}^{(0,\times)}_{\mu\nu\rho\sigma}$$

$$A(p)=-\frac{i}{p^2},B(p)=-\frac{i}{p^2}\alpha,C(p)=\frac{i}{2p^2},D(p)=-\frac{i}{p^2}\left(\frac{4\alpha-3}{2}\right),E(p)=\frac{i}{p^2}\frac{\sqrt{3}}{2},$$

$$\mathcal{G}_{\mu\nu\rho\sigma}(p)=-\frac{i}{p^2}[\mathcal{P}^{(2)}_{\mu\nu\rho\sigma}-\frac{1}{2}\mathcal{P}^{(0,s)}_{\mu\nu\rho\sigma}+\alpha\mathcal{P}^{(1)}_{\mu\nu\rho\sigma}$$
$$+\frac{4\alpha-3}{2}\mathcal{P}^{(0,w)}_{\mu\nu\rho\sigma}-\frac{\sqrt{3}}{2}\mathcal{P}^{(0,x)}_{\mu\nu\rho\sigma}\Big].$$

$$\mathcal{G}^{(\text{gauge-ind})}_{\mu\nu\rho\sigma}(p)=-\frac{i}{p^2}\left[\mathcal{P}^{(2)}_{\mu\nu\rho\sigma}-\frac{1}{2}\mathcal{P}^{(0,s)}_{\mu\nu\rho\sigma}\right]$$

$$h_{\mu\nu}(x)=\sum_{\lambda=+2,-2}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{2\omega_{\vec{p}}}\left(a_{\vec{p},\lambda}\epsilon^{(\lambda)}_{\mu\nu}e^{ip\cdot x}+a^{\dagger}_{\vec{p},\lambda}\epsilon^{(\lambda)*}_{\mu\nu}e^{-ip\cdot x}\right)$$

$$\left[a_{\vec{p},\lambda},a_{\vec{p}',\lambda'}\right]=0=\left[a^{\dagger}_{\vec{p},\lambda},a^{\dagger}_{\vec{p}',\lambda'}\right],\left[a_{\vec{p},\lambda},a^{\dagger}_{\vec{p}',\lambda'}\right]=2\omega_{\vec{p}}(2\pi)^3\delta_{\lambda\lambda'}\delta^{(3)}(\vec{p}-\vec{p}').$$

$$a_{\vec{p},\lambda}|0\rangle=0$$

$$|\vec{p},\lambda\rangle=a^{\dagger}_{\vec{p},\lambda}|0\rangle$$

$$S[g,\eta,h]=\frac{1}{2\kappa^2}\int\mathrm{d}^4x\sqrt{-g}R-\frac{1}{\alpha}\int\mathrm{d}^4x\mathcal{F}_\mu\eta^{\mu\nu}\mathcal{F}_\nu+S_{\text{gh}}[g,\eta,h]$$

$$S_{\text{gh}}[g,\eta,h]=\int\mathrm{d}^4x\bar{c}^\mu\frac{\delta\mathcal{F}_\mu}{\delta\zeta_\nu}c_\nu$$

$$\frac{\delta\mathcal{F}_\mu}{\delta\zeta_\nu}=\frac{\delta\mathcal{F}_\mu}{\delta h_{\rho\sigma}}\frac{\delta h_{\rho\sigma}}{\delta\zeta_\nu}=-\frac{\delta\mathcal{F}_\mu}{\delta h_{\rho\sigma}}(\delta_\sigma{}^\nu\nabla_\rho+\delta_\rho{}^\nu\nabla_\sigma).$$

$$S_{\text{gh}}[g,\eta,h]=-\int\mathrm{d}^4x\bar{c}^\mu\frac{\delta\mathcal{F}_\mu}{\delta h_{\rho\sigma}}(\nabla_\rho c_\sigma+\nabla_\sigma c_\rho)$$

$$\mathcal{F}_\mu=\left[\frac{1}{2}(\delta_\mu{}^\alpha\partial^\beta+\delta_\mu{}^\beta\partial^\alpha)-\frac{1}{2}\eta^{\alpha\beta}\partial_\mu\right]h_{\alpha\beta}.$$

$$\frac{\delta\mathcal{F}_\mu}{\delta h_{\rho\sigma}}=-\left[\frac{1}{2}(\delta_\mu{}^\sigma\partial^\rho+\delta_\mu{}^\rho\partial^\sigma)-\frac{1}{2}\eta^{\rho\sigma}\partial_\mu\right].$$

$$S_{\text{gh}}=\int\mathrm{d}^4x\bar{c}^\mu(\partial^\rho\nabla_\rho c_\mu+\partial^\rho\nabla_\mu c_\rho-\partial_\mu\nabla^\rho c_\rho)$$

$$S_{\text{gh}}=\int\mathrm{d}^4x\bar{c}^\mu\,\square\,c_\mu+\mathcal{O}(\kappa h)$$

$$\mathcal{F}_\mu=\frac{1}{2}\left(\delta^\alpha_\mu\delta^\beta_i+\delta^\beta_\mu\delta^\alpha_i\right)\delta^i_\nu\partial^\nu h_{\alpha\beta}$$

$$\frac{\delta\mathcal{F}_\mu}{\delta h_{\rho\sigma}}=-\frac{1}{2}(\delta_\mu{}^\rho\delta_i{}^\sigma+\delta_\mu{}^\sigma\delta_i{}^\rho)\delta_\nu{}^i\partial^\nu$$

$$S_{\mathrm{gh}} = \int \mathrm{d}^4 x \bar{c}^{\mu} \left(\partial^i \nabla_{\mu} c_i + \partial^i \nabla_i c_{\mu} \right)$$

$$S_{\mathrm{gh}} = \int \mathrm{d}^4 x \bar{c}_{\mu} \left(\delta_i^{\nu} \partial^i \partial^{\mu} + \eta^{\mu\nu} \partial^i \partial_i \right) c_{\nu} + \mathcal{O}(\kappa h)$$

$$\mathcal{A}_{1\to 1}(p^2) = (-i)(-i)^2 \epsilon^{*\mu\nu} \mathcal{G}_{\mu\nu\rho\sigma}(p^2) \epsilon^{\rho\sigma}$$

$$\frac{1}{p^2 - i\epsilon} = \mathrm{P.V.}\left(\frac{1}{p^2}\right) + i\pi\delta(p^2)$$

$$\mathrm{Im}[\mathcal{A}_{1\to 1}(p^2)] = \pi\delta(p^2) \left[\epsilon^{*\mu\nu}\epsilon_{\mu\nu} - \frac{1}{2} \left| \eta^{\mu\nu}\epsilon_{\mu\nu} \right|^2 \right] = \pi\delta(p^2) \epsilon^{*\mu\nu}\epsilon_{\mu\nu} \geq 0$$

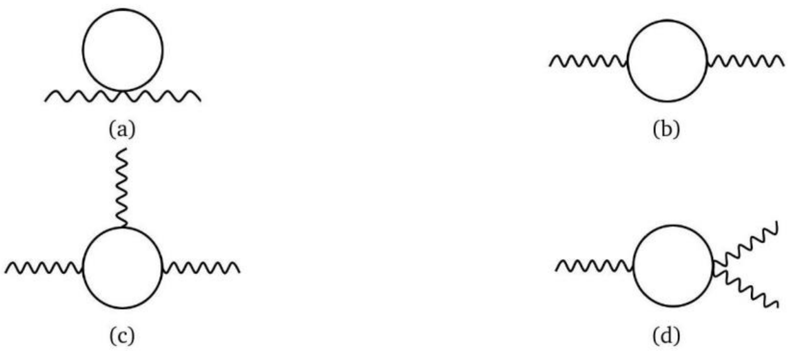
$$S_{\mathrm{gh}} = \int \mathrm{d}^4 x \bar{c}_{\mu} \mathbb{K}_{\mathrm{FP}}^{\mu\nu} c_{\nu} + \mathcal{O}(\kappa h)$$

$$\mathbb{K}_{\mathrm{FP}}^{\mu\nu} = \begin{pmatrix} p_3^2 & 0 & 0 & -\frac{1}{2} p_3^2 \\ 0 & -p_3^2 & 0 & 0 \\ 0 & 0 & -p_3^2 & 0 \\ -\frac{1}{2} p_3^2 & 0 & 0 & -2p_3^2 \end{pmatrix}$$

$$\Delta_n \equiv [\kappa^{n-2}] = 2 - n < 0, n \geq 3$$

$$\delta(G) = 4 - E - \sum_n V_n \Delta_n$$

$$\int \underbrace{d^4k \cdots d^4k}_{L\text{-loops}} \times \underbrace{\frac{1}{k^2} \cdots \frac{1}{k^2}}_{I\text{-internal propagators}} \times \underbrace{k^2 \cdots k^2}_{V\text{-vertices}} \sim k^{2L+2(L-I+V)} = k^{2L+2}$$



$$pppp \int \mathrm{d}^4 k \frac{1}{k^2} \frac{1}{k^2} k^2 k^2 \sim pppp \int \frac{\mathrm{d}k}{k}.$$

$$\Gamma_{\mathrm{div}}^{(1)}[g] = \frac{1}{\varepsilon} \int \mathrm{d}^4 x \sqrt{-g} \left[c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \right.$$
$$\left. + c_4 R_{\mu\nu\rho\sigma} R^{\mu\rho\nu\sigma} + c_5 \nabla_{\mu} \nabla_{\nu} R^{\mu\nu} + c_6 \square R \right],$$

$$\Gamma_{\text{div}}^{(1)}[g] = \frac{1}{\varepsilon}\int \mathrm{d}^4x\sqrt{-g}\left[aR^2 + bR_{\mu\nu}R^{\mu\nu}\right]$$

$$a = -\frac{1}{(4\pi)^2}\frac{1}{120}, b = -\frac{1}{(4\pi)^2}\frac{7}{20}.$$

$$S'[g] = S_{\text{EH}}[g] + \Gamma_{\text{div}}^{(1)}[g]$$

$$g_{\mu\nu} \to g'_{\mu\nu}(g, a, b) = g_{\mu\nu} + \Delta g_{\mu\nu}(a, b)$$

$$S_{\text{EH}}[g] + \Gamma_{\text{div}}^{(1)}[g] = S_{\text{EH}}[g + \Delta g] + \mathcal{O}(a^2, b^2, ab) = S_{\text{EH}}[g'] + \mathcal{O}(a^2, b^2, ab)$$

$$\Delta S_{\text{EH}}[g] \equiv \int \mathrm{d}^4x\sqrt{-g}\frac{\delta S_{\text{EH}}}{\delta g_{\mu\nu}}\Delta g_{\mu\nu} = \Gamma_{\text{div}}^{(1)}[g]$$

$$\begin{aligned}\Delta S_{\text{EH}}[g] &= -\int \mathrm{d}^4x\sqrt{-g}\frac{1}{2\kappa^2}\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right)\left(Ag_{\mu\nu}R + BR_{\mu\nu}\right)\\ &= -\int \mathrm{d}^4x\sqrt{-g}\frac{1}{2\kappa^2}\left[-\left(A + \frac{1}{2}B\right)R^2 + BR_{\mu\nu}R^{\mu\nu}\right]\end{aligned}$$

$$\begin{cases}\dfrac{a}{\varepsilon} = \dfrac{1}{2\kappa^2}(A + \dfrac{B}{2})\\ \dfrac{b}{\varepsilon} = -\dfrac{B}{2\kappa^2}\end{cases} \Leftrightarrow \begin{cases}A = \dfrac{\kappa^2}{\varepsilon}(2a + b)\\ B = -\dfrac{\kappa^2}{\varepsilon}2b\end{cases}$$

$$\begin{aligned}g'_{\mu\nu} &= g_{\mu\nu} + \frac{\kappa^2}{\varepsilon}\left[(2a + b)g_{\mu\nu}R - 2bR_{\mu\nu}\right]\\ &= g_{\mu\nu} + \frac{\kappa^2}{10(4\pi)^2\varepsilon}\left(\frac{11}{3}g_{\mu\nu}R - 7R_{\mu\nu}\right)\end{aligned}$$

$$S_{\text{EH}}^{(\Lambda\neq 0)}[g] + \Gamma_{\text{div}}^{(1)}[g] = S_{\text{EH}}^{(\Lambda\neq 0)}[g'] - \frac{23}{(4\pi)^2 30\varepsilon}\int \mathrm{d}^4x\sqrt{-g'}\Lambda R(g')$$

$$\det(1 + X) = \int \mathcal{D}\bar{q}\mathcal{D}q e^{i\int \mathrm{d}^4x\bar{q}(1+X)q}$$

$$S_{\text{gf}}[\eta, h] = -\frac{1}{\alpha}\int \mathrm{d}^4x\mathcal{F}_\mu\mathcal{F}^\mu, \mathcal{F}_\mu \equiv \partial_\nu h_\mu^\nu - \frac{1+\beta}{4}\partial_\mu h$$

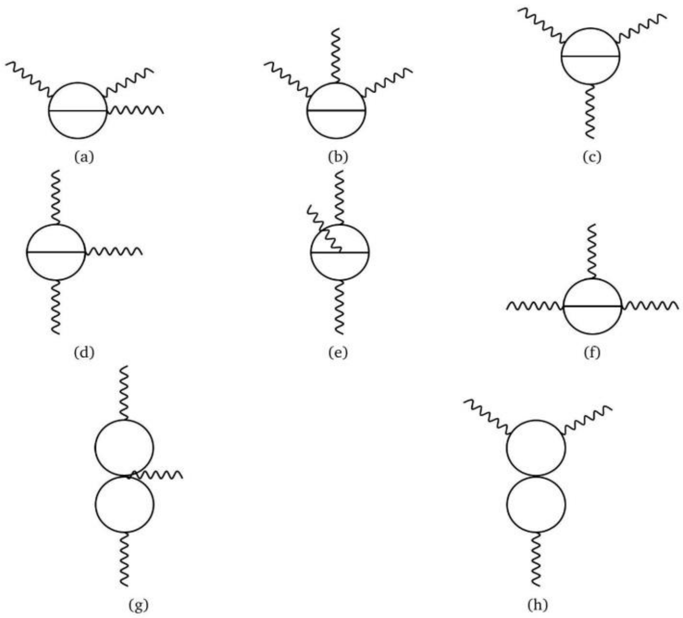
$$\Gamma_{\text{div}}^{(1)}(\alpha_*, \beta_*)\Big|_{\text{off-shell}} = 0$$

$$\Gamma_{\text{div}}^{(1)}(\alpha_1, \beta_1) - \Gamma_{\text{div}}^{(1)}(\alpha_2, \beta_2) = \frac{1}{\varepsilon}\int \mathrm{d}^4x f_{\mu\nu}\frac{\delta S_{\text{EH}}}{\delta g_{\mu\nu}}$$

$$\Gamma_{\text{div}}^{(1)}(\alpha_1, \beta_1) = \frac{1}{\varepsilon}\int \mathrm{d}^4x f_{\mu\nu}\frac{\delta S_{\text{EH}}}{\delta g_{\mu\nu}}$$

$$\frac{\delta(S_{\text{EH}}+S_m)}{\delta g_{\mu\nu}} = -\frac{1}{2\kappa^2}\sqrt{-g}\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R-\kappa^2T^{\mu\nu}\right).$$

$$\kappa^5 h^{\alpha\beta}\partial_\alpha\partial_\mu\partial_\nu h^{\rho\sigma}\partial_\beta\partial_\rho\partial_\sigma h^{\mu\nu}.$$



$$\Gamma^{(2)}_{\text{div}}[g] \propto \kappa^2\int \mathrm{d}^4x\sqrt{-g}\mathcal{R}$$

$$\nabla_\mu R\nabla^\mu R, \nabla_\rho R_{\mu\nu}\nabla^\rho R^{\mu\nu}, R^3, RR_{\mu\nu}R^{\mu\nu}, R_{\mu\nu}R^{\mu\rho}R_\rho{}^\nu, RR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}, R_{\mu\nu}R_{\rho\sigma}R^{\mu\rho\nu\sigma}, R_\alpha{}^\beta R^{\alpha\mu\nu\rho}R_{\beta\mu\nu\rho}, R_{\mu\nu\rho\sigma}R^{\mu\nu}{}_{\alpha\beta}R^{\rho\sigma\alpha\beta}, R_{\mu\nu\rho\sigma}R^\mu_\alpha{}_\alpha R^{\nu\alpha\sigma\beta}.$$

$$R^{[\mu\nu}_{\rho\sigma}R^{\rho\sigma}_{\alpha\beta}R^{\alpha]\beta}_{\mu\nu} = 0$$

$$0 = 4R_{\mu\nu\rho\sigma}R^{\rho\sigma\alpha\beta}R^{\mu\nu}_{\alpha\beta} - 8R_{\mu\nu\rho\sigma}R^\mu_\alpha{}_\beta{}_\beta R^{\nu\alpha\sigma\beta} + (\text{ proportional terms to } R_{\mu\nu} \text{ y } R).$$

$$\Gamma^{(2)}_{\text{div}}[g] \propto \kappa^2\int \mathrm{d}^4x\sqrt{-g}R_{\mu\nu\rho\sigma}R^{\mu\nu}_{\alpha\beta}R^{\rho\sigma\alpha\beta}$$

$$\mu^{2\varepsilon}\left(\frac{C_1}{\varepsilon}+\frac{C_2}{\varepsilon^2}\right)$$

$$\frac{C_1'}{\varepsilon}+\frac{C_2'}{\varepsilon^2}$$

$$(1+2\varepsilon\ln\,\mu+\cdots)\left(\frac{C_1}{\varepsilon}+\frac{C_2}{\varepsilon^2}\right) = \frac{C_1'}{\varepsilon}+\frac{C_2'}{\varepsilon^2}$$

$$\Gamma^{(2)}_{\text{div}}[g] = \frac{209}{2880(4\pi)^4}\frac{\kappa^2}{\varepsilon}\int \mathrm{d}^4x\sqrt{-g}R_{\mu\nu\rho\sigma}R^{\mu\nu}_{\alpha\beta}R^{\rho\sigma\alpha\beta}$$

$$\Gamma^{(L)}_{\text{div}}[g] \propto \kappa^{2L-2}\int \mathrm{d}^4x\sqrt{-g}\mathcal{R}$$

$$\sum_{i=1}^{V}(n_i-2)=\sum_{i=1}^{V}n_i-2V$$

$$\sum_{i=1}^{V}n_i=2I+E$$

$$\sum_{i=1}^{V}(n_i-2)=2I+E-2V=2L-2+E$$

$$S_{\text{EFT}}=\int\ \mathrm{d}^4x\sqrt{-g}\Big[\frac{1}{2\kappa^2}(R-2\Lambda)+a_1R^2+a_2R_{\mu\nu}R^{\mu\nu}$$
$$+a_3\kappa^2R^3+a_4\kappa^2R_{\mu\nu\rho\sigma}R^{\rho\sigma}_{\alpha\beta}R^{\alpha\beta\mu\nu}+\cdots\Big]$$

$$C_{\mu\nu\rho\sigma}=R_{\mu\nu\rho\sigma}+\frac{1}{2}(g_{\mu\sigma}R_{\nu\rho}-g_{\mu\rho}R_{\nu\sigma}+g_{\nu\rho}R_{\mu\sigma}-g_{\nu\sigma}R_{\mu\rho})+\frac{1}{6}(g_{\mu\rho}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\rho})R.$$

$$C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}=R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}-2R_{\mu\nu}R^{\mu\nu}+\frac{1}{3}R^2$$

$$\sqrt{-g}\mathfrak{E}\equiv\sqrt{-g}(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}-4R_{\mu\nu}R^{\mu\nu}+R^2)$$

$$R_{\mu\nu}R^{\mu\nu}=\frac{1}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}+\frac{1}{3}R^2-\frac{1}{2}\mathfrak{E}.$$

$$S_{\text{qg}}=\frac{1}{2}\int\ \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{\kappa^2}(R-2\Lambda)+\frac{c_0}{6}R^2-\frac{c_2}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right]$$

$$\frac{1}{\kappa^2}\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}\right)+\frac{c_0}{3}\left(g_{\mu\nu}\,\Box\,R-\nabla_\mu\nabla_\nu R+RR_{\mu\nu}-\frac{1}{4}g_{\mu\nu}R^2\right)-2c_2B_{\mu\nu}=T_{\mu\nu}$$

$$B_{\mu\nu}=\frac{1}{2}\Box\,R_{\mu\nu}+\frac{1}{6}\left(2\nabla_\mu\nabla_\nu-\frac{1}{2}g_{\mu\nu}\,\Box\right)R-\frac{1}{2}\nabla_\rho\nabla_\mu R^\rho_\nu$$
$$-\frac{1}{2}\nabla_\rho\nabla_\nu R^\rho_\mu-\frac{1}{3}RR_{\mu\nu}+R_{\mu\rho}R^\rho_\nu-\frac{1}{4}\left(R^{\rho\sigma}R_{\rho\sigma}-\frac{1}{3}R^2\right)g_{\mu\nu},$$

$$\frac{1}{\kappa^2}\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}\right)+\frac{2}{3}\left(\frac{c_0}{2}+c_2\right)\left(g_{\mu\nu}\,\Box\,R-\nabla_\mu\nabla_\nu R+RR_{\mu\nu}-\frac{1}{4}g_{\mu\nu}R^2\right)$$
$$-c_2\left(\Box\,R_{\mu\nu}+\frac{1}{2}g_{\mu\nu}\,\Box\,R-\nabla_\rho\nabla_\mu R^\rho_\nu-\nabla_\rho\nabla_\nu R^\rho_\mu+2R^\rho_\mu R_{\rho\nu}-\frac{1}{2}g_{\mu\nu}R_{\rho\sigma}R^{\rho\sigma}\right)=T_{\mu\nu}$$

$$-\frac{1}{\kappa^2}(R-4\Lambda)+c_0\,\Box\,R=T.$$

$$g_{\mu\nu}=\eta_{\mu\nu}+2h_{\mu\nu}$$

$$S_{\text{qg}}[\eta+2h]=S^{(2)}_{\text{qg}}[\eta,h]+S^{(n\geq3)}_{\text{qg}}[\eta,h],$$

$$S_{\mathrm{qg}}^{(2)}[\eta,h]=\int \mathrm{d}^4x\Big[\frac{1}{2}h_{\mu\nu}\,\Box\left(\frac{1}{\kappa^2}-c_2\,\Box\right)h^{\mu\nu}-h_\mu^\rho\left(\frac{1}{\kappa^2}-c_2\,\Box\right)\partial_\rho\partial_\nu h^{\mu\nu}$$
$$+h\left(\frac{1}{\kappa^2}-\frac{1}{3}(2c_0+c_2)\,\Box\right)\partial_\mu\partial_\nu h^{\mu\nu}-\frac{1}{2}h\left(\frac{1}{\kappa^2}-\frac{1}{3}(2c_0+c_2)\,\Box\right)\Box\, h$$
$$+\frac{1}{3}(c_0-c_2)h_{\mu\nu}\partial^\mu\partial^\nu\partial^\rho\partial^\sigma h_{\rho\sigma}\Big]$$

$$\frac{1}{\kappa^2}\partial^2h^n, c_0\partial^4h^n, c_2\partial^4h^n$$

$$S_{\mathrm{qg}}^{(2)}[\eta,h]=\int \mathrm{d}^4x\frac{1}{2}h_{\mu\nu}\mathbb{K}_{\mathrm{qg}}^{\mu\nu\rho\sigma}h_{\rho\sigma}$$

$$\mathbb{K}_{\mathrm{qg}}^{\mu\nu\rho\sigma}\equiv\frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\nu\rho})\left(\frac{1}{\kappa^2}-c_2\,\Box\right)\Box-\eta^{\mu\nu}\eta^{\rho\sigma}\left(\frac{1}{\kappa^2}-\frac{1}{3}(2c_0+c_2)\,\Box\right)\Box$$
$$+(\eta^{\mu\nu}\partial^\rho\partial^\sigma+\eta^{\rho\sigma}\partial^\mu\partial^\nu)\left(\frac{1}{\kappa^2}-\frac{1}{3}(2c_0+c_2)\,\Box\right)$$
$$-\frac{1}{2}(\eta^{\mu\rho}\partial^\nu\partial^\sigma+\eta^{\mu\sigma}\partial^\nu\partial^\rho+\eta^{\nu\rho}\partial^\mu\partial^\sigma+\eta^{\nu\sigma}\partial^\mu\partial^\rho)\left(\frac{1}{\kappa^2}-c_2\,\Box\right)$$
$$+\frac{2}{3}(c_0-c_2)\partial^\mu\partial^\nu\partial^\rho\partial^\sigma$$

$$\mathbb{K}_{\mathrm{qg}}^{\mu\nu\rho\sigma}=\mathbb{K}_{\mathrm{qg}}^{\nu\mu\rho\sigma}=\mathbb{K}_{\mathrm{qg}}^{\mu\nu\sigma\rho}=\mathbb{K}_{\mathrm{qg}}^{\rho\sigma\mu\nu}$$

$$\mathbb{K}_{\mathrm{qg}}^{\mu\nu\rho\sigma}=-\frac{p^2}{\kappa^2}\left[\mathcal{P}^{(2)\mu\nu\rho\sigma}(1+\kappa^2c_2p^2)-2\mathcal{P}^{(0,s)\mu\nu\rho\sigma}(1+\kappa^2c_0p^2)\right]$$

$$S_{\mathrm{gf}}[\eta,h]=-\frac{1}{\alpha\kappa^2}\int \mathrm{d}^4x\mathcal{F}_\mu\mathcal{Y}^{\mu\nu}\mathcal{F}_\nu$$

$$\mathcal{F}_\mu\equiv\partial_\nu h_\mu^\nu-\frac{1+\beta}{4}\partial_\mu h, \mathcal{Y}^{\mu\nu}\equiv\eta^{\mu\nu}(1+\gamma\,\Box)+\omega\partial^\mu\partial^\nu$$

$$\mathcal{G}_{\mathrm{q}g\mu\nu\rho\sigma}(p)=-i\left[\frac{m_2^2\mathcal{P}^{(2)}\mu\nu\rho\sigma}{p^2(p^2+m_2^2)}-\frac{m_0^2\mathcal{P}^{(0,s)}\mu\nu\rho\sigma}{2p^2(p^2+m_0^2)}\right]+\cdots$$
$$=-\frac{i}{p^2}\left[\mathcal{P}^{(2)}{}_{\mu\nu\rho\sigma}-\frac{1}{2}\mathcal{P}^{(0,s)}{}_{\mu\nu\rho\sigma}\right]-\frac{i}{2}\frac{\mathcal{P}^{(0,s)}}{p^2+m_0^2}+i\frac{\mathcal{P}^{(2)}}{p^2+m_2^2}+\cdots,$$

$$m_0^2\equiv\frac{1}{\kappa^2c_0}=\frac{M_{\mathrm{Pl}}^2}{c_0}, m_2^2\equiv\frac{1}{\kappa^2c_2}=\frac{M_{\mathrm{Pl}}^2}{c_2}$$

$$\delta(G)=4-E,$$

$$\int\underbrace{\mathrm{d}^4k\cdots\,\mathrm{d}^4k}_{L\text{-loops}}\times\underbrace{\frac{1}{k^4}\cdots\frac{1}{k^4}}_{I\text{-internal propagators}}\times\underbrace{k^4\cdots k^4}_{V\text{-vertices}}\sim k^{4(L-I+V)}=k^4$$

$$\frac{1}{\Lambda M_{\mathrm{Pl}}^2+M_{\mathrm{Pl}}^2p^2+c_0p^4+c_2p^4}\overset{\mathrm{UV}}{\sim}\frac{1}{c_0p^4+c_2p^4}$$

$$\lambda\mathbb{1}, R, R^2, C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma},$$

$$\left(\frac{c_2}{c_0}\right)^{I_0-V_0}\left(\frac{1}{c_2}\right)^{L-1}=\left(\frac{c_0}{c_2}\right)^{I_2-V_2}\left(\frac{1}{c_0}\right)^{L-1}$$

$$g_0\equiv\frac{1}{c_0},g_2\equiv\frac{1}{c_2}$$

$$\left(\frac{g_0}{g_2}\right)^{I_0-V_0}g_2^{L-1}=\left(\frac{g_2}{g_0}\right)^{I_2-V_2}g_0^{L-1}$$

$$h_{\mu\nu}\to\frac{1}{\sqrt{c_2}}h_{\mu\nu}$$

$$S_{\mathrm{qg}}\sim\int\mathrm{d}^4x\left[h\partial^4h+\frac{c_0}{c_2}h\partial^4h+\frac{M_{\mathrm{Pl}}^2}{c_2}h\partial^2h+\frac{1}{\sqrt{c_2}}\left(h\partial^4h^2+\frac{c_0}{c_2}h\partial^4h^2+\frac{M_{\mathrm{Pl}}^2}{c_2}h\partial^2h^2\right)\right.$$
$$\left.+\cdots+\left(\frac{1}{\sqrt{c_2}}\right)^{n-2}\left(h\partial^4h^{n-1}+\frac{c_0}{c_2}h\partial^4h^{n-1}+\frac{M_{\mathrm{Pl}}^2}{c_2}h\partial^2h^{n-1}\right)+\cdots\right].$$

$$\beta_{g_0}=-\frac{70g_2^2-6g_0g_2-g_0^2}{3},\beta_{g_2}=-\frac{(539g_2+40g_0)g_2}{15}$$

$$g_0=\frac{\sqrt{386761}-569}{90}g_2\simeq0.59g_2.$$

$$\int\mathrm{d}^4x\sqrt{-g}\xi|H|^2R$$

$$S_{\mathrm{EFT}}\simeq\int\mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R+a_1R^2+a_2R_{\mu\nu}R^{\mu\nu}\right]$$

$$\mathcal{G}(p^2)=\frac{-im^2}{p^2(p^2+m^2)}=-i\left[\frac{1}{p^2}-\frac{1}{p^2+m^2}\right],$$

$$i[\langle b|T^\dagger|a\rangle-\langle b|T|a\rangle]=\sum_{|n\rangle\in\mathcal{H}}\langle b|T^\dagger|n\rangle\langle n|T|a\rangle.$$

$$\mathrm{Im}[i\mathcal{G}_{\mathrm{F}}(p^2)]=\mathrm{Im}\left[\frac{1}{p^2-i\epsilon}-\frac{1}{p^2+m^2-i\epsilon}\right]=\pi[\delta(p^2)-\delta(p^2+m^2)]$$

$$i[\langle b|T^\dagger|a\rangle-\langle b|T|a\rangle]=\sum_{|n\rangle\in\mathcal{H}}\sigma_n\langle b|T^\dagger|n\rangle\langle n|T|a\rangle,$$

$$\mathbb{1}=\sum_{|n\rangle\in\mathcal{H}}\sigma_n|n\rangle\langle n|$$

$$\mathcal{G}_{\mathrm{anti-F}}(p^2)=-i\left[\frac{1}{p^2-i\epsilon}-\frac{1}{p^2+m^2+i\epsilon}\right]$$

$$\mathrm{Im}[i\mathcal{G}_{\text{anti-F}}(p^2)]=\mathrm{Im}\left[\frac{1}{p^2-i\epsilon}-\frac{1}{p^2+m^2+i\epsilon}\right]=\pi[\delta(p^2)+\delta(p^2+m^2)]$$

$$\frac{i}{p^2+m^2} \to i\frac{p^2+m^2}{(p^2+m^2)^2+\epsilon^2} = \frac{i}{2}\left[\frac{1}{p^2+m^2+i\epsilon}+\frac{1}{p^2+m^2-i\epsilon}\right]$$

$$\mathcal{G}_{\text{fake}}(p^2) = -i\left[\frac{1}{p^2-i\epsilon}-\frac{1}{2}\left(\frac{1}{p^2+m^2+i\epsilon}+\frac{1}{p^2+m^2-i\epsilon}\right)\right].$$

$$\text{Im}[i\mathcal{G}_{\text{fake}}(p^2)] = \pi\delta(p^2)$$

$$\mathbb{1} = \sum_{|n\rangle\in\mathcal{H}} \sigma_n |n\rangle\langle n| \ \to \ \mathbb{1}_{\text{ph}} = \sum_{|n\rangle\in\mathcal{H}_{\text{ph}}} |n\rangle\langle n|$$

$$r = \frac{24}{N_e^2}\frac{m_2^2}{m_0^2+2m_2^2} = \frac{24}{N_e^2}\frac{c_0}{c_2+2c_0},$$

$$\mathcal{L} = \mathcal{L}(\phi, \partial\phi, \partial^2\phi, \ldots, \partial^{(n)}\phi), n < \infty.$$

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu, \det\Lambda = 1,$$

$$S = \int \mathrm{d}^4x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(i\gamma^\mu\partial_\mu + m)\psi - e\bar{\psi}\gamma^\mu\psi A_\mu\right]$$

$$\psi \to e^{i\alpha(x)}\psi, \bar{\psi} \to e^{-i\alpha(x)}\bar{\psi}, A_\mu \to A_\mu + \frac{1}{e}\partial_\mu\alpha(x).$$

$$S^\dagger S = \mathbb{1}$$

$$-i(T - T^\dagger) = T^\dagger T$$

$$\mathbb{1} = \sum_n \sigma_n |n\rangle\langle n|,$$

$$-i[\langle b|T|a\rangle - \langle b|T^\dagger|a\rangle] = \sum_n \sigma_n \langle b|T^\dagger|n\rangle\langle n|T|a\rangle.$$

$$\mathbb{1} = \sum_{\{n\}}\prod_{l=1}^{n}\int \frac{\mathrm{d}^3k_l}{(2\pi)^3}\frac{1}{2\omega_l}|\{k_l\}\rangle\langle\{k_l\}|,$$

$$i[\langle b|\mathcal{A}^\dagger|a\rangle - \langle b|\mathcal{A}|a\rangle] = \sum_{\{n\}}\sigma_n\prod_{l=1}^{n}\int \frac{\mathrm{d}^3k_l}{(2\pi)^3}\frac{1}{2\omega_l}(2\pi)^4 \times$$

$$\delta^{(4)}\left(P_a - \sum_{l=1}^{n} k_l\right)\langle b|\mathcal{A}^\dagger|\{k_l\}\rangle\langle\{k_l\}|\mathcal{A}|a\rangle$$

$$2\text{Im}[\langle a|\mathcal{A}|a\rangle] = \sum_{\{n\}}\prod_{l=1}^{n}\int \frac{\mathrm{d}^3k_l}{(2\pi)^3}\frac{1}{2\omega_l}(2\pi)^4\delta^{(4)}\left(P_a - \sum_{l=1}^{n}k_l\right)\left|\langle\{k_l\}|\mathcal{A}|a\rangle\right|^2 \geq 0.$$

$$\begin{aligned}\mathcal{G}_\phi(x-y) &\equiv \langle 0|T\{\phi(x)\phi(y)\}|0\rangle \\ &= \theta(x^0-y^0)\langle 0|\phi(x)\phi(y)|0\rangle + \theta(y^0-x^0)\langle 0|\phi(y)\phi(x)|0\rangle \\ &= \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{e^{i\vec{p}\cdot(\vec{x}-\vec{y})}}{2\omega_{\vec{p}}}\left[\theta(x^0-y^0)e^{-i\omega_{\vec{p}}(x^0-y^0)} + \theta(y^0-x^0)e^{i\omega_{\vec{p}}(x^0-y^0)}\right] \\ &= \int \frac{\mathrm{d}^4p}{(2\pi)^4}\tilde{\mathcal{G}}_\phi(p,\epsilon)e^{ip\cdot(x-y)}\end{aligned}$$

$$\theta(x) = \begin{cases} 1, & x>0, \\ 1/2, & x=0, \\ 0, & x<0. \end{cases}$$

$$\tilde{\mathcal{G}}_\phi(p,\epsilon) \equiv \frac{-i}{p^2+m^2-i\epsilon}$$

$$\langle p_3,p_4|\mathcal{A}|p_1,p_2\rangle = (-i)(-i\lambda)\frac{-i}{p^2+m^2-i\epsilon}(-i\lambda) = \frac{\lambda^2}{p^2+m^2-i\epsilon} \equiv \langle p|\mathcal{A}|p\rangle$$

$$2\mathrm{Im}[\langle p|\mathcal{A}|p\rangle] = 2\pi\lambda^2\delta(p^2+m^2)$$

$$\frac{1}{x\pm i\epsilon} = \mathrm{P.V.}\left(\frac{1}{x}\right) \mp i\pi\delta(x)$$

$$\int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{2\omega}(2\pi)^4\delta^{(4)}(p-k)\langle p_3,p_4|\mathcal{A}^\dagger|k\rangle\langle k|\mathcal{A}|p_1,p_2\rangle$$

$$\langle k|\mathcal{A}|p_1,p_2\rangle = (-i)(-i\lambda) = -\lambda, \langle p_3,p_4|\mathcal{A}^\dagger|k\rangle = (\langle k|\mathcal{A}|p_3,p_4\rangle)^* = (-\lambda)^* = -\lambda$$

$$\int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{2\omega} = \int \frac{\mathrm{d}^4k}{(2\pi)^4}2\pi\delta(k^2+m^2)$$

$$\lambda^2\int \frac{\mathrm{d}^4k}{(2\pi)^4}2\pi\delta^{(4)}(p-k)\delta(k^2+m^2) = 2\pi\lambda^2\delta(p^2+m^2)$$

$$I^{(L)}(G) = \int \mathrm{d}^4k_1\cdots\mathrm{d}^4k_L\mathcal{I}(\{k_i\})$$

$$\lim_{\lambda\to\infty}\lambda^{4L}\mathcal{I}(\{\lambda k_i\}) \sim \lambda^{\delta(G)}.$$

$$\int \mathrm{d}^4k\frac{1}{k^2+m^2}$$

$$\int \mathrm{d}^4k\frac{1}{k^2+m^2}\frac{1}{(k-p)^2+m^2}$$

$$\mathcal{K} \sim \sum_f \phi_f\left(\partial^{(2-2s_f)}\right)^{r_f}\phi_f, \mathcal{V} \sim \sum_i g_i\partial^{d_i}\prod_f \phi_f^{n_{i,f}},$$

$$\tilde{\mathcal{G}}_{\phi_f}(p) \sim p^{(2s_f-2)r_f}.$$

$$\int \underbrace{\mathrm{d}^4k \cdots \mathrm{d}^4k}_{L\text{-loops}} \times \underbrace{k^{(2s_f-2)r_f} \ldots k^{(2s_f-2)r_f}}_{I_f\text{-internal propagators}} \times \underbrace{k^{d_i} \ldots k^{d_i}}_{V_i\text{-vertices of type } i}$$

$$k^{4L+I_f(2s_f-2)r_f+d_iV_i} = k^{4(I_f-V_i+1)+I_f(2s_f-2)r_f+d_iV_i}$$

$$\begin{aligned}\delta(G) &= 4\left(\sum_f I_f - \sum_i V_i + 1\right) + \sum_f I_f(2s_f-2)r_f + \sum_i d_iV_i \\ &= 4 + \sum_f I_f[(2s_f-2)r_f+4] + \sum_i (d_i-4)V_i\end{aligned}$$

$$2I_f + E_f = \sum_i V_i n_{i,f} \Leftrightarrow I_f = -\frac{1}{2}E_f + \frac{1}{2}\sum_i V_i n_{i,f}$$

$$\delta(G) = 4 - \sum_f E_f[(s_f-1)r_f+2] - \sum_i V_i\left[4 - d_i - \sum_f n_{i,f}\left((s_f-1)r_f+2\right)\right].$$

$$F_f = (s_f-1)r_f + 2 \text{ and } \Delta_i = 4 - d_i - \sum_f n_{i,f}[(s_f-1)r_f+2].$$

$$\delta(G) = 4 - \sum_f E_fF_f - \sum_i V_i\Delta_i$$

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi).$$

$$\varphi'^{\mu\nu} = \Lambda^\mu{}_\rho\Lambda^\nu{}_\sigma\varphi^{\rho\sigma}.$$

$$\varphi^{\mu\nu} = h^{\mu\nu} + \psi^{\mu\nu}, \begin{cases} h^{\mu\nu} \equiv \frac{1}{2}(\varphi^{\mu\nu} + \varphi^{\nu\mu}) \\ \psi^{\mu\nu} \equiv \frac{1}{2}(\varphi^{\mu\nu} - \varphi^{\nu\mu}) \end{cases}$$

$$h' = \eta_{\mu\nu}h'^{\mu\nu} = \eta_{\mu\nu}\Lambda^\mu_\rho\Lambda^\nu_\sigma h^{\rho\sigma} = \eta_{\rho\sigma}h^{\rho\sigma} = h,$$

$$h^{T\mu\nu} \equiv h^{\mu\nu} - \frac{1}{4}\eta_{\mu\nu}h.$$

$$V^\mu \in \mathbf{0} \oplus \mathbf{1}.$$

$$\begin{aligned}\varphi^{\mu\nu} \in (0 \oplus 1) \otimes (0 \oplus 1) &= (0 \otimes 0) \oplus (0 \otimes 1) \oplus (1 \otimes 0) \oplus (1 \otimes 1) \\ &= 0 \oplus 1 \oplus 1 \oplus (0 \oplus 1 \oplus 2),\end{aligned}$$

$$0 \otimes 0 = 0, 0 \otimes 1 = 1 \otimes 0 = 1\ 1 \otimes 1 = 0 \oplus 1 \oplus 2.$$

$$h \in 0.$$

$$\psi^{\mu\nu} \in \mathbf{1} \oplus \mathbf{1}$$

$$h^{T\mu\nu} \in \mathbf{0} \oplus \mathbf{1} \oplus \mathbf{2}.$$

$$\mathcal{L} = \frac{1}{2}\varphi_{\mu\nu}\mathcal{O}^{\mu\nu\rho\sigma}\varphi_{\rho\sigma}$$

$$\begin{aligned}
h_{\mu\nu} =& (\theta_{\mu\rho} + \omega_{\mu\rho})(\theta_{\nu\sigma} + \omega_{\nu\sigma})h^{\rho\sigma} \\
=& (\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\rho}\omega_{\nu\sigma} + \omega_{\mu\rho}\theta_{\nu\sigma} + \omega_{\mu\rho}\omega_{\nu\sigma})h^{\rho\sigma} \\
=& \frac{1}{2}(\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\rho})h^{\rho\sigma} - \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}h^{\rho\sigma} + \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}h^{\rho\sigma} + \omega_{\mu\nu}\omega_{\rho\sigma}h^{\rho\sigma} \\
&+ \frac{1}{2}(\theta_{\mu\rho}\omega_{\nu\sigma} + \theta_{\mu\sigma}\omega_{\nu\rho} + \theta_{\nu\rho}\omega_{\mu\sigma} + \theta_{\nu\sigma}\omega_{\mu\rho})h^{\rho\sigma} \\
=& \mathcal{P}^{(2)}{}_{\mu\nu\rho\sigma}h^{\rho\sigma} + \mathcal{P}^{(1,m)}{}_{\mu\nu\rho\sigma}h^{\rho\sigma} + \mathcal{P}^{(0,s)}{}_{\mu\nu\rho\sigma}h^{\rho\sigma} + \mathcal{P}^{(0,w)}{}_{\mu\nu\rho\sigma}h^{\rho\sigma},
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}^{(2)}{}_{\mu\nu\rho\sigma} &= \frac{1}{2}(\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\rho}) - \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}, \\
\mathcal{P}^{(1,m)}{}_{\mu\nu\rho\sigma} &= \frac{1}{2}(\theta_{\mu\rho}\omega_{\nu\sigma} + \theta_{\mu\sigma}\omega_{\nu\rho} + \theta_{\nu\rho}\omega_{\mu\sigma} + \theta_{\nu\sigma}\omega_{\mu\rho}), \\
\mathcal{P}^{(0,s)}{}_{\mu\nu\rho\sigma} &= \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}, \\
\mathcal{P}^{(0,w)}{}_{\mu\nu\rho\sigma} &= \omega_{\mu\nu}\omega_{\rho\sigma}.
\end{aligned}$$

$$\mathcal{P}^{(i,a)}{}_{\mu\nu}{}^{\alpha\beta}\mathcal{P}^{(j,b)}{}_{\alpha\beta}{}^{\rho\sigma} = \delta^{ij}\delta^{ab}\mathcal{P}^{(i,a)}{}_{\mu\nu}{}^{\rho\sigma},$$

$$\mathcal{P}^{(2)}_{\mu\nu\rho\sigma} + \mathcal{P}^{(1,m)}_{\mu\nu\rho\sigma} + \mathcal{P}^{(0,s)}_{\mu\nu\rho\sigma} + \mathcal{P}^{(0,w)}_{\mu\nu\rho\sigma} = \mathbb{1}_{\mu\nu\rho\sigma},$$

$$\begin{aligned}
\mathbb{1}^{\mu\nu\rho\sigma}\mathcal{P}^{(2)}{}_{\mu\nu\rho\sigma} &= 5 = 2(2) + 1(\text{spin-two}), \\
\mathbb{1}^{\mu\nu\rho\sigma}\mathcal{P}^{(1,m)}{}_{\mu\nu\rho\sigma} &= 3 = 2(1) + 1\ (\text{spin-one}), \\
\mathbb{1}^{\mu\nu\rho\sigma}\mathcal{P}^{(0,s)}{}_{\mu\nu\rho\sigma} &= 1 = 2(0) + 1\ (\text{spin-zero}), \\
\mathbb{1}^{\mu\nu\rho\sigma}\mathcal{P}^{(0,w)}{}_{\mu\nu\rho\sigma} &= 1 = 2(0) + 1\ (\text{spin-zero}),
\end{aligned}$$

$$\begin{aligned}
\mathbb{1}^{\mu\nu\rho\sigma}\mathcal{P}^{(2)}_{\mu\nu\rho\sigma} &= \mathbb{1}^{\mu\nu\rho\sigma}\theta_{\mu\rho}\theta_{\nu\sigma} - \frac{1}{3}\mathbb{1}^{\mu\nu\rho\sigma}\theta_{\mu\nu}\theta_{\rho\sigma} \\
&= \frac{1}{2}(\theta_{\mu}{}^{\mu}\theta_{\nu}{}^{\nu} + \theta_{\mu}{}^{\mu}) - \frac{1}{6}(2\theta_{\mu}{}^{\mu}) \\
&= \frac{1}{2}(3 \times 3 + 3) - \frac{3}{3} \\
&= 5
\end{aligned}$$

$$\mathcal{P}^{(0,x)}_{\mu\nu\rho\sigma} = \mathcal{P}^{(0,sw)}_{\mu\nu\rho\sigma} + \mathcal{P}^{(0,ws)}_{\mu\nu\rho\sigma}$$

$$\mathcal{P}^{(0,sw)}_{\mu\nu\rho\sigma} = \frac{1}{\sqrt{3}}\theta_{\mu\nu}\omega_{\rho\sigma}, \mathcal{P}^{(0,ws)}_{\mu\nu\rho\sigma} = \frac{1}{\sqrt{3}}\omega_{\mu\nu}\theta_{\rho\sigma}$$

$$\mathcal{P}^{(i,ab)}_{\mu\nu}{}^{(j,cd)}_{\alpha\beta}{}^{\rho\sigma}_{\alpha\beta} = \delta^{ij}\delta^{bc}\mathcal{P}^{(i,ad)}_{\mu\nu}{}^{\rho\sigma}_{\mu\nu}$$

$$\eta^{\mu\nu}\left(\mathcal{P}^{(2)}_{\mu\nu\rho\sigma}h^{\rho\sigma}\right)=\left[\frac{1}{2}\left(\eta^{\mu\nu}\theta_{\mu\rho}\theta_{\nu\sigma}+\eta^{\mu\nu}\theta_{\mu\sigma}\theta_{\nu\rho}\right)-\frac{1}{3}\eta^{\mu\nu}\theta_{\mu\nu}\theta_{\rho\sigma}\right]h^{\rho\sigma}$$
$$=\left[\frac{1}{2}\left(\theta_{\rho}{}^{\nu}\theta_{\nu\sigma}+\theta_{\sigma}{}^{\nu}\theta_{\nu\rho}\right)-\frac{1}{3}(4-1)\theta_{\rho\sigma}\right]h^{\rho\sigma}$$
$$=\left[\frac{1}{2}\left(\theta_{\rho\sigma}+\theta_{\rho\sigma}\right)-\theta_{\rho\sigma}\right]h^{\rho\sigma}=0$$

$$p^{\mu}\left(\mathcal{P}^{(2)}_{\mu\nu\rho\sigma}h^{\rho\sigma}\right)=\left[\frac{1}{2}\left(p^{\mu}\theta_{\mu\rho}\theta_{\nu\sigma}+p^{\mu}\theta_{\mu\sigma}\theta_{\nu\rho}\right)-\frac{1}{3}p^{\mu}\theta_{\mu\nu}\theta_{\rho\sigma}\right]h^{\rho\sigma}=0$$

$$\psi_{\mu\nu}=\left(\theta_{\mu\rho}+\omega_{\mu\rho}\right)(\theta_{\nu\sigma}+\omega_{\nu\sigma})\psi^{\rho\sigma}$$
$$=\left(\theta_{\mu\rho}\theta_{\nu\sigma}+\theta_{\mu\rho}\omega_{\nu\sigma}+\omega_{\mu\rho}\theta_{\nu\sigma}+\omega_{\mu\rho}\omega_{\nu\sigma}\right)\psi^{\rho\sigma}$$
$$=\frac{1}{2}\left(\theta_{\mu\rho}\theta_{\nu\sigma}-\theta_{\mu\sigma}\theta_{\nu\rho}\right)\psi^{\rho\sigma}+\frac{1}{2}\left(\theta_{\mu\rho}\omega_{\nu\sigma}-\theta_{\mu\sigma}\omega_{\nu\rho}-\theta_{\nu\rho}\omega_{\mu\sigma}+\theta_{\nu\sigma}\omega_{\mu\rho}\right)\psi^{\rho\sigma}$$
$$=\mathcal{P}^{(1,b)}{}_{\mu\nu\rho\sigma}\psi^{\rho\sigma}+\mathcal{P}^{(1,e)}{}_{\mu\nu\rho\sigma}\psi^{\rho\sigma}$$

$$\mathcal{P}^{(1,b)}_{\mu\nu\rho\sigma}=\frac{1}{2}\left(\theta_{\mu\rho}\theta_{\nu\sigma}-\theta_{\mu\sigma}\theta_{\nu\rho}\right)$$
$$\mathcal{P}^{(1,e)}_{\mu\nu\rho\sigma}=\frac{1}{2}\left(\theta_{\mu\rho}\omega_{\nu\sigma}-\theta_{\mu\sigma}\omega_{\nu\rho}-\theta_{\nu\rho}\omega_{\mu\sigma}+\theta_{\nu\sigma}\omega_{\mu\rho}\right)$$

$$\mathcal{P}^{(1,c)}_{\mu\nu}{}^{\alpha\beta}_{\alpha\beta}\mathcal{P}^{(1,d)}_{\alpha\beta}=\delta^{cd}\mathcal{P}^{(1,c)}_{\mu\nu}$$

$$\mathcal{P}^{(1,b)}_{\mu\nu\rho\sigma}+\mathcal{P}^{(1,e)}_{\mu\nu\rho\sigma}=\frac{1}{2}\left(\eta_{\mu\rho}\eta_{\nu\sigma}-\eta_{\mu\sigma}\eta_{\nu\rho}\right)$$

$$\frac{1}{2}\left(\eta_{\mu\rho}\eta_{\nu\sigma}-\eta_{\mu\sigma}\eta_{\nu\rho}\right)\mathcal{P}^{(1,b)}_{\mu\nu\rho\sigma}=1=2(1)+1\ \ \text{(spin-one)}\,,$$
$$\frac{1}{2}\left(\eta_{\mu\rho}\eta_{\nu\sigma}-\eta_{\mu\sigma}\eta_{\nu\rho}\right)\mathcal{P}^{(1,e)}_{\mu\nu\rho\sigma}=1=2(1)+1\ \ \text{(spin-one).}$$

$$\left(\mathcal{P}^{(2)}+\mathcal{P}^{(1,m)}+\mathcal{P}^{(0,s)}+\mathcal{P}^{(0,w)}+\mathcal{P}^{(1,b)}+\mathcal{P}^{(1,e)}\right)_{\mu\nu\rho\sigma}=\frac{1}{2}\left(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}\right)$$
$$+\frac{1}{2}\left(\eta_{\mu\rho}\eta_{\nu\sigma}-\eta_{\mu\sigma}\eta_{\nu\rho}\right)$$
$$=\eta_{\mu\rho}\eta_{\nu\sigma}$$

$$\mathcal{P}^{(1,\times)}_{\mu\nu\rho\sigma}=\mathcal{P}^{(1,me)}_{\mu\nu\rho\sigma}+\mathcal{P}^{(1,em)}_{\mu\nu\rho\sigma}$$

$$\mathcal{P}^{(1,me)}_{\mu\nu\rho\sigma}=\frac{1}{2}\left(\theta_{\mu\rho}\omega_{\nu\sigma}-\theta_{\mu\sigma}\omega_{\nu\rho}+\theta_{\nu\rho}\omega_{\mu\sigma}-\theta_{\nu\sigma}\omega_{\mu\rho}\right)$$
$$\mathcal{P}^{(1,em)}_{\mu\nu\rho\sigma}=\frac{1}{2}\left(\theta_{\mu\rho}\omega_{\nu\sigma}+\theta_{\mu\sigma}\omega_{\nu\rho}-\theta_{\nu\rho}\omega_{\mu\sigma}-\theta_{\nu\sigma}\omega_{\mu\rho}\right)$$

$$\left\{\mathcal{O}^{(i)}\right\}\equiv\left\{\mathcal{P}^{(2)},\mathcal{P}^{(1,m)},\mathcal{P}^{(0,s)},\mathcal{P}^{(0,w)},\mathcal{P}^{(0,sw)},\mathcal{P}^{(0,ws)},\mathcal{P}^{(1,b)},\mathcal{P}^{(1,e)},\mathcal{P}^{(1,em)},\mathcal{P}^{(1,me)}\right\}$$

$$\mathcal{P}^{(i,AB)}_{\mu\nu}{}^{\alpha\beta}_{\mu\nu}\mathcal{P}^{(j,CD)}_{\alpha\beta}{}^{\rho\sigma}_{\alpha\sigma}=\delta^{ij}\delta^{BC}\mathcal{P}^{(i,AD)}_{\mu\nu}{}^{\rho\sigma}$$

$$\mathcal{L}=\frac{1}{2}\varphi_{\mu\nu}\mathcal{O}^{\mu\nu\rho\sigma}\varphi_{\rho\sigma}=\frac{1}{2}\varphi_{\mu\nu}\left(\sum_{i=1}^{10}c_i(p)\mathcal{O}^{(i)\mu\nu\rho\sigma}\right)\varphi_{\rho\sigma}$$

$$\mathcal{L}(\varphi,\psi) = -\frac{1}{2}\left(\partial_\mu\varphi\right)^2 - \frac{1}{2}m^2\varphi^2 + \varphi F(\psi) + G(\psi)$$

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\psi e^{i\int \mathrm{d}^4x\mathcal{L}(\varphi,\psi)}$$

$$\mathcal{L}(\varphi,\psi) = \frac{1}{2}\varphi(\square - m^2)\varphi + \varphi F(\psi) + G(\psi).$$

$$\bar{\varphi}(x) = \varphi(x) + \int \mathrm{d}^4y D_F(x-y)F(\psi(y))$$

$$(\square - m^2)D_F(x-y) = \delta^4(x-y).$$

$$\frac{1}{2}\varphi(\square - m^2)\varphi + \varphi F(\psi) = \frac{1}{2}\bar{\varphi}(\square - m^2)\bar{\varphi} - \frac{1}{2}\int \mathrm{d}^4y F(\psi(x))D_F(x-y)F(\psi(y)).$$

$$\mathcal{Z} = \int \mathcal{D}\psi e^{i\int \mathrm{d}^4x G(\psi)}\, e^{-\frac{i}{2}\langle FDF\rangle}$$

$$\langle FDF\rangle = \int \mathrm{d}^4x\, \mathrm{d}^4y F(\psi(x))D_F(x-y)F(\psi(y)).$$

$$D_F(x-y) = -\int \frac{\mathrm{d}^4q}{(2\pi)^4}\frac{e^{-iq\cdot(x-y)}}{q^2+m^2} = -\int \frac{\mathrm{d}^4q}{(2\pi)^4}e^{-iq\cdot(x-y)}\left(\frac{1}{m^2} - \frac{q^2}{m^4} + \cdots\right),$$

$$D_F(x-y) = -\left(\frac{1}{m^2} + \frac{\square}{m^4} + \frac{\square^2}{m^6} + \cdots\right)\delta^4(x-y).$$

$$\mathcal{L}_{\text{eff}}(\psi) = G(\psi) - \frac{1}{2}F(\psi)\frac{1}{m^2}F(\psi) - \frac{1}{2m^4}F(\psi)\,\square\,F(\psi) + \cdots$$

$$\mathcal{Z} = \int \mathcal{D}\psi e^{i\int \mathrm{d}^4x\mathcal{L}_{\text{eff}}(\psi)}$$

$$\nabla_\mu h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}\nabla_\mu h^\alpha_\alpha = 0$$

$$\mathcal{G}_{\mu\nu\rho\sigma}(q) = \frac{1}{2}\frac{-i}{q^2 - i\epsilon}\left(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}\right)$$

$$P^{\alpha\beta\gamma\delta} = \frac{1}{2}\left(\eta^{\alpha\gamma}\eta^{\beta\delta} + \eta^{\alpha\delta}\eta^{\beta\gamma} - \eta^{\alpha\beta}\eta^{\gamma\delta}\right)$$

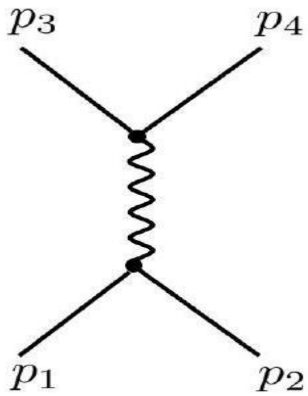
$$\mathcal{G}_{\mu\nu\rho\sigma}(q) = \frac{-i}{q^2 - i\epsilon}P_{\alpha\beta\gamma\delta}$$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{\mathrm{Pl}}^2 R}{2} - \frac{1}{2}\left(\partial_\mu\varphi\right)^2\right)$$

$$V_{\mu\nu}(p_1,p_2) = \frac{i}{2M_{\rm Pl}}\left(p_{1\mu}p_{2\nu} + p_{2\mu}p_{1\nu} - \eta_{\mu\nu}p_1^{\gamma}p_{2\gamma}\right).$$

$$h^{\alpha\beta} \sim h^{\gamma\delta} \equiv \frac{-i}{q^2}P^{\alpha\beta\gamma\delta}$$

$$z_{\varphi}^{q}p_{\varphi} \equiv V_{\mu\nu}(p_1,p_2) = \frac{i}{2M_{\rm Pl}}\left[p_{1\mu}p_{2\nu} + p_{1\nu}p_{2\mu} - \eta_{\mu\nu}p_1^{\gamma}p_{2\gamma}\right]$$



$$\mathcal{A}_s = iV_{\mu\nu}(p_1,p_2)\frac{i}{(p_1+p_2)^2}P^{\mu\nu\alpha\beta}V_{\alpha\beta}(p_3,p_4).$$

$$V_{\mu\nu}(p_1,p_2)P^{\mu\nu\alpha\beta} = \frac{i}{2M_{\rm Pl}}\left(p_1^{\alpha}p_2^{\beta} + p_1^{\beta}p_2^{\alpha}\right).$$

$$2p_{1\alpha}p_{2_\beta}\left(p_3^{\alpha}p_4^{\beta} + p_3^{\beta}p_4^{\alpha} - \eta^{\alpha\beta}(p_3\cdot p_4)\right)$$
$$= 2(p_1\cdot p_3)(p_2\cdot p_4) + 2(p_1\cdot p_4)(p_2\cdot p_3) - 2(p_2\cdot p_1)(p_3\cdot p_4).$$

$$s = -(p_1+p_2)^2 = -(p_3+p_4)^2$$
$$t = -(p_1+p_3)^2 = -(p_2+p_4)^2$$
$$u = -(p_2+p_4)^2 = -(p_2+p_3)^2$$

$$s = -2(p_1\cdot p_2) = -2(p_3\cdot p_4)$$
$$t = -2(p_1\cdot p_3) = -2(p_2\cdot p_4)$$
$$u = -2(p_1\cdot p_4) = -2(p_2\cdot p_3)$$

$$\cos\,\theta = 1 + \frac{2t}{s}$$

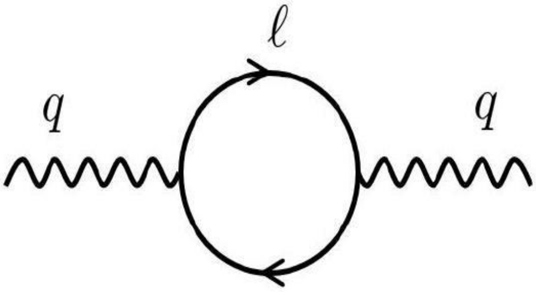
$$\mathcal{A}_s = -\frac{1}{2M_{\rm Pl}^2 s}(t^2+u^2-s^2) = \frac{1}{M_{\rm Pl}^2}\frac{tu}{s}$$

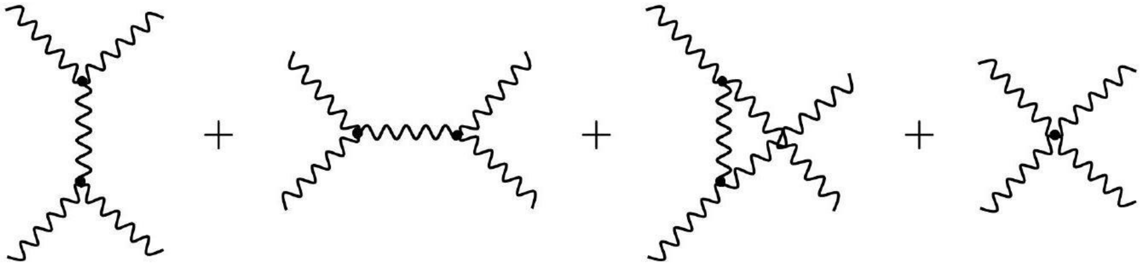
$$\mathcal{A} = \mathcal{A}_s + \mathcal{A}_t + \mathcal{A}_u = \frac{1}{M_{\rm Pl}^2}\left(\frac{tu}{s} + \frac{su}{t} + \frac{ts}{u}\right)$$

$$\frac{1}{t} + \frac{1}{s} + \frac{1}{u}$$

$$\mathcal{A}_{1l} = \int \frac{{\rm d}^4\ell}{(2\pi)^4}\frac{i}{2M_{\rm Pl}}\left(\ell_\alpha(\ell+q)_\beta + \ell_\beta(\ell+q)_\alpha\right)\frac{i}{\ell^2}\frac{i}{(\ell+q)^2}\frac{i}{2M_{\rm Pl}}\left(\ell_\delta(\ell+q)_\gamma + \ell_\gamma(\ell+q)_\delta\right)$$

$$\left(\ln\, q^2 + \frac{1}{\varepsilon}\right) q_\gamma q_\delta q_\alpha q_\beta \propto q^4$$





$$\begin{aligned} p_1^\mu &= (p, 0,0, p) \\ p_2^\mu &= (p, 0,0, -p) \\ p_3^\mu &= (-p, -p\sin\,\theta, 0, -p\cos\,\theta) \\ p_4^\mu &= (-p, p\sin\,\theta, 0, p\cos\,\theta) \end{aligned}$$

$$p = \frac{s}{2}, \cos\,\theta = 1 + \frac{2t}{s}.$$

$$\begin{aligned} e^{\mu\pm}(p_1) &= \frac{1}{\sqrt{2}}(0,1, \pm i, 0) \\ e^{\mu\pm}(p_2) &= \frac{1}{\sqrt{2}}(0, -1, \pm i, 0) \\ e^{\mu\pm}(p_3) &= \frac{1}{\sqrt{2}}(0, \cos\,\theta, \pm i, -\sin\,\theta) \\ e^{\mu\pm}(p_4) &= \frac{1}{\sqrt{2}}(0, -\cos\,\theta, \pm i, \sin\,\theta) \end{aligned}$$

$$e_\pm^{\mu\nu}(p_i) = e^{\mu\pm}(p_i) e^{\nu\pm}(p_i).$$

$$\mathcal{A}_{+-+-}(s,t,u) = \mathcal{A}_{++--}(t,s,u).$$

$$\mathcal{A}_{---+} = \mathcal{A}_{+++-}, \mathcal{A}_{++++} = \mathcal{A}_{----}, \ldots$$

$$\mathcal{A}_{++++}, \mathcal{A}_{+++-}, \mathcal{A}_{++--}.$$

$$\mathcal{A}_{++--} = \frac{1}{M_{\mathrm{Pl}}^2}\frac{s^3}{tu}, \mathcal{A}_{++++} = \mathcal{A}_{+++-} = 0$$

$$S = \int \mathrm{d}^4 x \sqrt{-g}\left(-\frac{1}{2}(\partial_\mu\varphi)^2 + \alpha(\partial_\mu\varphi)^2 \Box\varphi + \beta\left((\partial_\mu\varphi)^2\right)^2 + \gamma(\partial^6(\varphi^4)) + \cdots\right)$$

$$\varphi = \chi - a(\partial_\rho \chi)^2$$

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \chi)^2 + 2a\partial_\mu \chi \partial_\rho \chi \partial^\mu \partial^\rho \chi + 2a^2(\partial_\rho \chi)^2(\partial_\mu \partial_\nu \chi)^2 + \alpha \Box \chi(\partial_\mu \chi)^2$$
$$+ a\alpha \Box (\partial_\rho \chi)^2(\partial_\mu \chi)^2 + 2a\alpha \Box \chi \partial_\mu \chi \partial^\mu \partial_\rho \chi \partial^\rho \chi + \mathcal{O}(\chi^5) + \cdots$$

$$\partial_\mu \chi \partial_\rho \chi \partial^\mu \partial^\rho \chi = -\Box \chi(\partial_\rho \chi)^2 - \partial_\mu \chi \partial^\mu \partial_\rho \chi \partial^\rho \chi = -\frac{1}{2}\Box \chi(\partial_\rho \chi)^2$$

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \varphi)^2 + \frac{g_2}{2}\left((\partial_\mu \varphi)^2\right)^2 + \frac{g_3}{3}(\partial_\mu \varphi)^2(\partial_\rho \partial_\sigma \varphi)^2 + 4g_4(\partial_\rho \partial_\sigma \varphi \partial^\rho \partial^\sigma \varphi)^2 + \cdots$$

$$\mathcal{L} = F[\varphi]\hat{E}\varphi + \frac{1}{2}\varphi \hat{E}\varphi, \hat{E}\varphi = 0$$

$$\varphi = \chi - F[\chi]$$

$$\mathcal{L} = F[\chi]\hat{E}\chi + \frac{1}{2}\chi \hat{E}\chi + F[\chi]\hat{E}(F[\chi]) - F[\chi]\hat{E}\chi + \cdots$$

$$\mathcal{A}(s,t) = g_2(s^2 + t^2 + u^2) + g_3 stu + g_4(s^2 + t^2 + u^2)^2 + \cdots$$

$$S = \frac{M_{\mathrm{Pl}}^2}{2}\int \mathrm{d}^4x\sqrt{-g}R$$

$$\Gamma = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R + aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + d\Box R + \frac{e}{\Lambda_{\mathrm{UV}}^2}\mathrm{Riem}^3 + \cdots\right]$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{M_{\mathrm{Pl}}^2}T_{\mu\nu}$$

$$R_{\mu\nu} = 0, R = 0$$

$$\mathfrak{E} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}.$$

$$R_{\mu\nu\rho\sigma} + R_{\mu\rho\sigma\nu} + R_{\mu\sigma\nu\rho} = 0,$$
$$\nabla_\alpha R_{\mu\nu\rho\sigma} + \nabla_\rho R_{\mu\nu\sigma\alpha} + \nabla_\sigma R_{\mu\nu\alpha\rho} = 0.$$

$$C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - \left(g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu}\right) + \frac{1}{3}g_{\mu[\rho}g_{\sigma]\nu}R.$$

$$A_{[\mu\nu]} = \frac{1}{2}(A_{\mu\nu} - A_{\nu\mu}).$$

$$\nabla^\mu C_{\mu\nu\rho\sigma} = 0, \Box C_{\mu\nu\rho\sigma} \sim C_{\mu\nu\rho\sigma}^2$$

$$C_{\mu\nu\rho\sigma}, \nabla_{(\mu_1}C_{\mu)\nu\rho\sigma}, \nabla_{(\mu_1}\nabla_{\mu_2}C_{\mu)\nu\rho\sigma}, \ldots$$

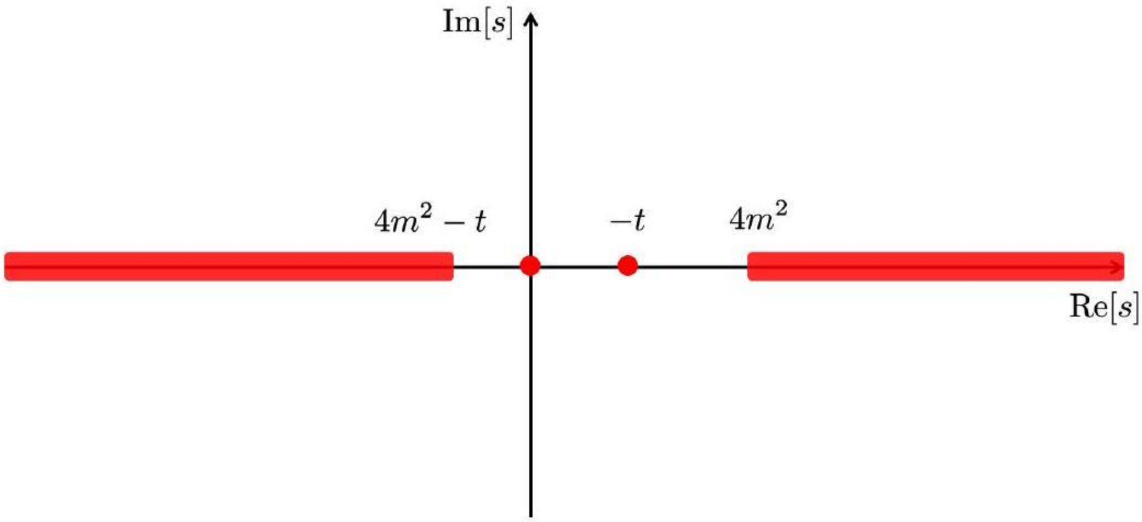
$$C_{L/R} = \frac{1}{2}(C \pm i\tilde{C}), \tilde{C}^{\mu\nu\rho\sigma} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}C_{\alpha\beta}{}^{\rho\sigma}$$

$$S = M_{\mathrm{Pl}}^{d-2}\int \mathrm{d}^d x\sqrt{-g}\left\{\frac{R}{2}+\frac{a_2}{\Lambda_{\mathrm{UV}}^2}\mathfrak{E}+\frac{a_3}{\Lambda_{\mathrm{UV}}^4}C^3+\frac{a_4}{\Lambda_{\mathrm{UV}}^6}C^2+\frac{\tilde{a}_4}{\Lambda_{\mathrm{UV}}^6}\tilde{C}^2\right.$$
$$\left.+\frac{a_5}{\Lambda_{\mathrm{UV}}^8}F^{\alpha}{}_{\alpha}C+\frac{\tilde{a}_5}{\Lambda_{\mathrm{UV}}^8}\tilde{F}^{\alpha}{}_{\alpha}\tilde{C}+\cdots\right\}$$

$$F_{\alpha\beta}=\nabla_\alpha C_{\mu\nu\rho\sigma}\nabla_\beta C^{\mu\nu\rho\sigma}, \tilde{F}_{\alpha\beta}=\nabla_\alpha C_{\mu\nu\rho\sigma}\nabla_\beta \tilde{C}^{\mu\nu\rho\sigma}$$

$$M_{\mathrm{Pl}}^{d-2}\mathcal{A}_{++--}=\frac{s^3}{tu}-\frac{8(d-4)}{d-2}\frac{a_2}{\Lambda_{\mathrm{UV}}^4}s^3-18\frac{a_3^2}{\Lambda_{\mathrm{UV}}^8}s^3\left(\frac{d-4}{d-2}s^2+2st+2t^2\right)+\frac{8s^4}{\Lambda_{\mathrm{UV}}^6}a_{4+}+\frac{4}{\Lambda_{\mathrm{UV}}^8}a_{5+}s^5$$
$$M_{\mathrm{Pl}}^{d-2}\mathcal{A}_{++++}=\frac{12}{\Lambda_{\mathrm{UV}}^2}\left(5a_3-\frac{2(d-4)}{d-2}a_2^2\right)x$$
$$-\frac{2}{\Lambda_{\mathrm{UV}}^8}\left(\frac{9(12-d)}{d-2}a_3^2+10a_{5-}\right)xy+\frac{16}{\Lambda_{\mathrm{UV}}^6}a_{4-}x^2$$
$$M_{\mathrm{Pl}}^{d-2}\mathcal{A}_{+++-}=\frac{6}{\Lambda_{\mathrm{UV}}^4}a_3y+\frac{\gamma y^2}{\Lambda_{\mathrm{UV}}^{10}}$$

$$y=stu, x=st+tu+us$$



$$\mathcal{A}(s,\theta)=32\pi\sum_{l=0}^{\infty}\left(l+\frac{1}{2}\right)f_l(s)P_l(\cos\,\theta).$$

$$\int_{-1}^{1}\mathrm{d}\cos\,\theta P_j(\cos\,\theta)P_k(\cos\,\theta)=\frac{2}{2j+1}\delta_{jk}$$

$$|f_j(s)|<1, \mathrm{Im}f_j(s)>0$$

$$\sigma_{\mathrm{tot}}=\frac{1}{32\pi s}\int_{-1}^{1}\mathrm{d}\cos\,\theta|\mathcal{A}(\theta)|^2$$

$$\sigma_{\text{tot}} = \frac{1}{32\pi s}\sum_{j,k}\int_{-1}^{1} \mathrm{d}\cos\,\theta(32\pi)^2 f_j(s) f_k^*(s) P_j(\cos\,\theta) P_k(\cos\,\theta)\left(j+\frac{1}{2}\right)\left(k+\frac{1}{2}\right)$$

$$= \frac{32\pi}{s}\sum_j \left(j+\frac{1}{2}\right)\left|f_j(s)\right|^2.$$

$$\mathrm{Im}\mathcal{A}(s,\theta=0) \geq \frac{1}{2}\sqrt{s(s-4m^2)}\sigma_{\text{tot}}(s)$$

$$32\pi\sum_j \mathrm{Im}f_j(s)\left(j+\frac{1}{2}\right) \geq \frac{16\pi}{s}\sqrt{s(s-4m^2)}\sum_j \left(j+\frac{1}{2}\right)\left|f_j(s)\right|^2.$$

$$2\mathrm{Im}f_j(s) \geq \left|f_j(s)\right|^2\sqrt{\frac{s-4m^2}{s}}$$

$$\mathcal{A}(s,t) = \frac{1}{2}\sum n(l,d) f_l(s) P_l^{(d)}\left(1+\frac{2t}{s-4m^2}\right)$$

$$P_l^{(d)}(z) = \frac{\Gamma(1+l)\Gamma(d-3)}{\Gamma(l+d-3)} C_l^{\frac{d-3}{2}}(z).$$

$$\frac{1}{2}\int_{-1}^{1} \mathrm{d}z(1-z^2)^{\frac{d-4}{2}} P_l^{(d)}(z) P_{l'}^{(d)}(z) = \frac{\delta_{ll'}}{N(d)n(l,d)}$$

$$N(d) = \frac{(16\pi)^{\frac{2-d}{2}}}{\Gamma\left(\frac{d-2}{2}\right)}, n(l,d) = \frac{(4\pi)^{\frac{d}{2}}(d+2l-3)\Gamma(d+l-3)}{\pi\Gamma\left(\frac{d-2}{2}\right)\Gamma(l+1)}.$$

$$P_l^{(d)}(x) = {}_2F_1\left(-l, l+d-3, \frac{d-2}{2}, \frac{1-x}{2}\right).$$

$$f_l(s) = N(d)\int_{-1}^{1} \mathrm{d}x(1-x^2)^{\frac{d-4}{2}} P_l^{(d)}(x)\mathcal{A}(s,t(x))$$

$$2\mathrm{Im}f_l(s) \geq \frac{(s-4m^2)^{\frac{d-3}{2}}}{\sqrt{s}}|f_l(s)|^2, |f_l(s)| < \frac{\sqrt{2s}}{(s-4m^2)^{\frac{d-3}{2}}}$$

$$\mathcal{A}_{h_1h_2h_3h_4} = 32\pi\sum_l (2l+1) d_{\lambda\mu}^l(\cos\,\theta) f_{h_1h_1h_3h_4}^l(s)$$

$$\lambda = h_2 - h_1, \mu = h_4 - h_3, d_{\lambda\mu}(-\theta) = (-1)^{\lambda-\mu} d_{\lambda\mu}(\theta)$$

$$(-1)^{\lambda-\mu} = 1$$

$$\mathcal{A}_{h_1h_2h_3h_4} = 16\pi\sum_l (2l+1)\left(d_{\lambda\mu}^l(\theta) + d_{\lambda\mu}^l(-\theta)\right) f_{h_1h_1h_3h_4}^l(s).$$

$$d_{\lambda\mu}^l(\theta) + d_{\lambda\mu}^l(-\theta) = 2e^{i\frac{\pi}{2}(\lambda-\mu)}\sum_{\nu=-l}^{l} d_{\lambda\nu}^l\left(\frac{\pi}{2}\right) d_{\mu\nu}^l\left(\frac{\pi}{2}\right)\cos\,\nu\theta$$

$$\mathcal{A}(s,t) = 16\pi \sum_{l=0}^{\infty} (2l+1) f_l(s) P_l\left(1 + \frac{t}{2q^2}\right)$$

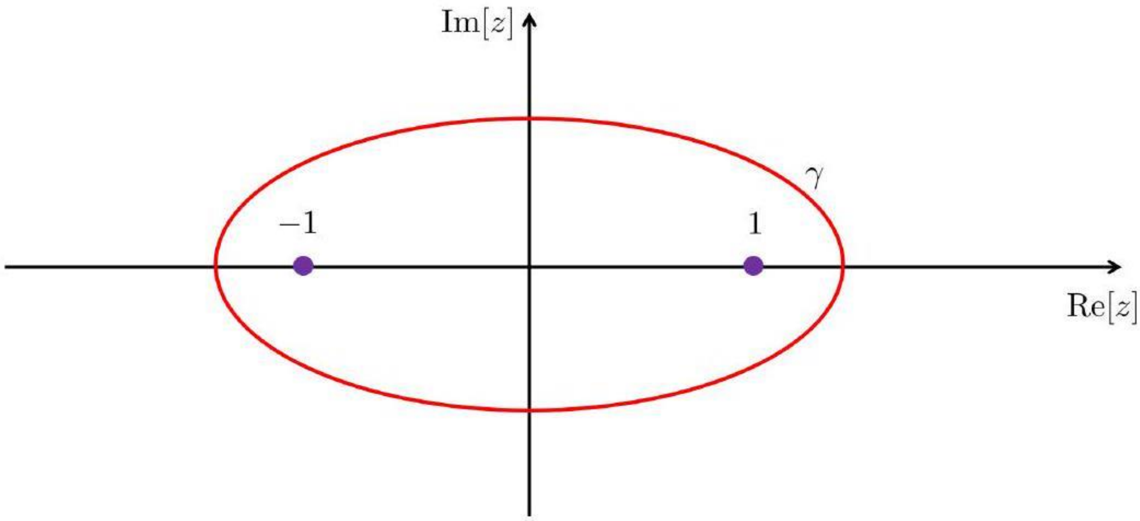
$$q = \frac{1}{2}\sqrt{s - 4m^2}$$

$$Q_l(z) = \frac{1}{2}\int_{-1}^{1} \mathrm{d}\mu \frac{P_l(\mu)}{z - \mu}$$

$$\oiiint_{\gamma} \mathrm{d}z \mathcal{A}(s, t(z)) Q_n(z) = \frac{1}{2}\sum_{l} (2l+1) f_l(s) \int_{-1}^{1} \mathrm{d}\mu \oint_{\gamma} \mathrm{d}z P_l\left(1 + \frac{t(z)}{2q^2}\right)\frac{P_n(\mu)}{z - \mu}$$

$$\oiiint_{\gamma} \mathrm{d}z \mathcal{A}(s, t(z)) Q_n(z) = (2\pi i)\frac{1}{2}\sum_{l} (2l+1) f_l(s) \int_{-1}^{1} \mathrm{d}\mu P_n(\mu) P_l\left(1 + \frac{t(\mu)}{2q^2}\right)$$

$$1 + \frac{t(\mu)}{2q^2} = \mu, t(\mu) = 2q^2(\mu - 1)$$



$$f_l(s) = \frac{1}{2\pi i}\oint_{\gamma} \mathrm{d}z \mathcal{A}(s, 2q^2(z-1)) Q_l(z)$$

$$Q_l(z) \approx K_0(l\sqrt{2(z-1)}) \approx e^{-l\sqrt{2(z-1)}}\sqrt{\frac{\pi}{2l\sqrt{2(z-1)}}}, l \to \infty$$

$$z - 1 = \frac{t}{2q^2}$$

$$|f_l(s)| \propto \frac{q}{2\pi\sqrt{s}} e^{-\frac{\sqrt{t}}{q}\left(l+\frac{1}{2}\right)}\sqrt{\frac{\pi q}{2l\sqrt{t}}} B_0(s), B_0(s) \lesssim s^N.$$

$$|f_l(s)| < 1, l < L$$

$$e^{-\frac{\sqrt{t}}{q}\left(L+\frac{1}{2}\right)}\sqrt{\frac{q}{L\sqrt{t}}}B_0(s) \; = 1$$

$$\frac{\sqrt{t}}{q}\left(L+\frac{1}{2}\right) \; = \ln\,\frac{q}{\sqrt{t}} + \ln\,B_0(s)$$

$$L = \frac{q}{2\sqrt{t}}\ln\,\frac{q}{\sqrt{t}} + \frac{q}{\sqrt{t}}N\ln\,\frac{s}{s_0} \propto \sqrt{s}\ln\,s.$$

$$\mathcal{A}(s,t\to 0^+) = \sum_{l=0}^{L} P_l(1)(2l+1) + \sum_{l=L+1}^{\infty} P_l(1)\frac{(2l+1)}{\sqrt{l}}e^{-\frac{\sqrt{t}}{q}\left(l+\frac{1}{2}\right)}s^N\sqrt{\frac{\pi q}{2\sqrt{t}}}.$$

$$\sum_{l=L+1}^{\infty} \sqrt{l}e^{-\frac{\sqrt{t}}{q}l} \sim \int_L^{\infty} \mathrm{d}y\sqrt{y}e^{-\frac{\sqrt{t}}{q}y} \sim e^{-\frac{\sqrt{t}}{q}L} \sim e^{-\sqrt{t}\sqrt{s}\ln\,s}$$

$$s^N e^{-\sqrt{t}\sqrt{s}\ln\,s} \to 0, s\to\infty$$

$$\mathcal{A}(s,t\to 0^+) = \sum_{1=0}^{L} P_l(1)(2l+1) \sim (L+1)^2 \propto s(\ln\,s)^2$$

$$\mathcal{A}(s,t\to 0) < C|s|(\ln\,|s|)^2$$

$$\mathcal{A}(s,t) = g_2(s^2+t^2+u^2) + g_3stu + g_4(s^2+t^2+u^2)^2 + g_5(s^2+t^2+u^2)stu + \cdots.$$

$$f_l(s) = \frac{1}{16\pi}\int_{-1}^{1} \mathrm{d}xP_l(x)\mathcal{A}\left(s,-\frac{s}{2}(1-x),-\frac{s}{2}(1+x)\right)$$

$$f_0(s) = \frac{5g_2s^2}{48\pi} + \frac{g_3s^3}{96\pi} + \frac{7g_4s^5}{40\pi}, f_2(s) = \frac{g_2s^2}{240\pi} - \frac{g_3s^3}{480\pi} + \frac{g_4s^5}{70\pi}$$

$$\mathrm{Im}f_0 = \left(\frac{5g_2}{48\pi}\right)^2 s^4, \mathrm{Im}f_2 = \left(\frac{g_2}{240\pi}\right)^2 s^4$$

$$\mathrm{Im}\mathcal{A}_{1l} = a_1s^2(s^2+a_2tu), 2\mathrm{Im}f_0 = \frac{a_1s^4}{8\pi} + \frac{a_1a_2s^4}{48\pi}, 2\mathrm{Im}f_2 = -\frac{a_1a_2s^4}{240\pi}$$

$$a_1 = \frac{7g_2^2}{40\pi}, a_2 = -\frac{1}{21}$$

$$\mathcal{A}_{1l} = -\frac{21g_2^2}{240\pi^2}s^2\left(s^2 - \frac{1}{21}tu\right)\ln\,(-s) + (s\to t) + (s\to u)$$

$$\Sigma_{\mathrm{IR}} = \frac{1}{2\pi i}\oiiint_{\Gamma} \mathrm{d}\mu\frac{\mathcal{A}(\mu,0)}{(\mu-\mu_0)^3}$$

$$\Sigma_{\rm IR} = \sum \ \mathrm{Res}\frac{\mathcal{A}(\mu,0)}{(\mu-\mu_0)^3} = \frac{1}{2}\frac{\partial^2\mathcal{A}(\mu,0)}{\partial\mu^2}\bigg|_{\mu=\mu_0}.$$

$$\begin{aligned}\Sigma_{\rm IR} &= \frac{1}{2\pi i}\left(\int_{4m^2}^{\infty}\mathrm{d}\mu + \int_{-\infty}^{0}\mathrm{d}\mu\right)\frac{\mathcal{A}(\mu+i\varepsilon,0)-\mathcal{A}(\mu-i\varepsilon,0)}{(\mu-\mu_0)^3} \\ &= \int_{4m}^{\infty}\frac{\mathrm{d}\mu}{\pi}\left(\frac{\mathrm{Im}\mathcal{A}(\mu,0)}{(\mu-\mu_0)^3}+\frac{\mathrm{Im}\mathcal{A}^*(\mu,0)}{(\mu-4m^2+\mu_0)^3}\right)\end{aligned}$$

$$\mathrm{Im}\mathcal{A}(s,0) = \sqrt{s-4m^2}\sigma_{\rm tot}(s) > 0.$$

$$\frac{\partial^2\mathcal{A}(\mu,0)}{\partial\mu^2}\bigg|_{\mu=\mu_0} > 0.$$

$$\mathcal{L} = -\frac{1}{2}\left(\partial_\mu\varphi\right)^2 + \alpha\left(\partial_\mu\varphi\right)^2\Box\varphi + g_2\left(\left(\partial_\mu\varphi\right)^2\right)^2$$

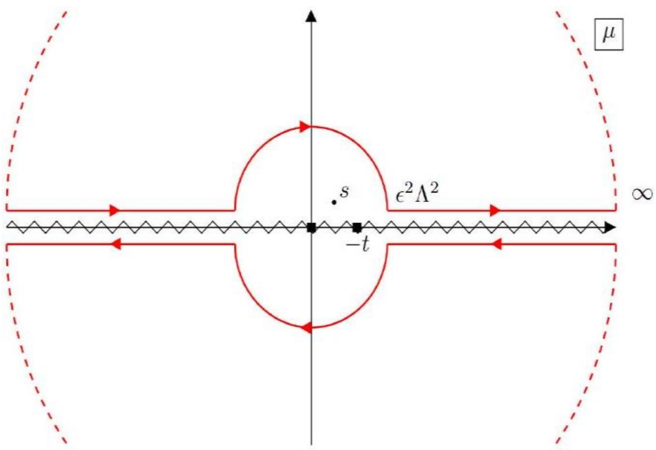
$$\varphi \to q + a_\rho x^\rho, \partial_\mu\varphi \to \partial_\mu\varphi + a_\mu,$$

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}\left(\partial_\mu\varphi + a_\mu\right)^2 + \alpha\Box\varphi\left(\partial_\mu\varphi + a_\mu\right)^2 \\ &= -\frac{1}{2}\left(\partial_\mu\varphi\right)^2 + \alpha\varphi\left(\partial_\mu\varphi\right)^2 + \alpha\varphi a^\mu\partial_\mu\varphi + \alpha\left(a_\mu\right)^2\varphi \\ &= -\frac{1}{2}\left(\partial_\mu\varphi\right)^2 + \alpha\Box\varphi\left(\partial_\mu\varphi\right)^2 + \partial_\mu(\dots)\end{aligned}$$

$$(\Box\varphi)a^\mu\partial_\mu\varphi = -a^\mu\left(\Box\partial_\mu\varphi\right)\varphi = -a^\mu\left(\partial_\mu\varphi\right)\Box\varphi = 0.$$

$$\mathcal{A}(s,t,u) = g_2(s^2+t^2+u^2) + g_3 stu + \cdots.$$

$$g_2 > 0,$$



$$\frac{1}{2\pi i}\left(\oint_{+} + \oint_{-}\right)\mathrm{d}\mu\frac{\mathcal{A}(\mu,t)}{(\mu-s)^3} = 0.$$

$$\mathrm{Disc}_s\mathcal{A}(s,t) = \frac{1}{2i}\left(\mathcal{A}(s+i\varepsilon,t) - \mathcal{A}(s-i\varepsilon,t)\right)$$

$$\frac{1}{2\pi i}\int_{\text{arc}} \mathrm{d}\mu \frac{\mathcal{A}(\mu,t)}{(\mu-s)^3} = \int_{\varepsilon^2\Lambda_{\text{UV}}^2}^{\infty} \frac{\mathrm{d}\mu}{\pi}\frac{\text{Disc}_s\mathcal{A}_s(\mu,t)}{(\mu-s)^3} + \int_{\varepsilon^2\Lambda_{\text{UV}}^2-t}^{\infty} \frac{\mathrm{d}\mu}{\pi}\frac{\text{Disc}_s\mathcal{A}_u(\mu,t)}{(\mu-u)^3}$$

$$\begin{aligned} C(n,m) &= \frac{1}{2}\frac{n!}{2\pi i}\int_{\text{arc}} \mathrm{d}\mu \frac{\partial_t^m \mathcal{A}(\mu,t)}{\mu^{n+1}}\bigg|_{t=0} \\ &= \frac{1}{2}\partial_t^m\partial_s^n \mathcal{A}(s,t)\Big|_{s=0,t=0} \end{aligned}$$

$$I_{s,u}(n,m) = \int_{\varepsilon^2\Lambda_{\text{UV}}^2}^{\infty} \frac{\mathrm{d}\mu}{\pi}\frac{\partial_t^m \text{Disc}_s\mathcal{A}_{s,u}(\mu,t)}{\mu^{n+1}}\bigg|_{t=0}$$

$$\mathcal{A}(s,t) = s^2\int_{\varepsilon^2\Lambda_{\text{UV}}^2}^{\infty} \mathrm{d}\mu \frac{\text{Disc}_s\mathcal{A}(\mu,t)}{\mu^2(\mu-s)} + (s+t)^2\int_{\varepsilon^2\Lambda_{\text{UV}}^2}^{\infty} \mathrm{d}\mu \frac{\text{Disc}_u\mathcal{A}(\mu,t)}{\mu^2(\mu+s+t)} + \frac{\partial\mathcal{A}(s,t)}{\partial s}\bigg|_{s=0} s^2$$

$$\begin{gathered} C(2,0) = I_s(2,0) + I_u(2,0) \\ C(3,0) = 3I_s(3,0) - 3I_u(3,0) \\ C(2,1) = \frac{\text{Disc}_s\mathcal{A}_u(\varepsilon^2\Lambda_{\text{UV}}^2,0)}{\pi(\varepsilon^2\Lambda_{\text{UV}}^2)^3} + I_s(2,1) - 3I_u(3,0) + I_u(2,1) \end{gathered}$$

$$\frac{1}{\mu} < \frac{1}{\varepsilon^2\Lambda_{\text{UV}}^2}, \mu > \varepsilon^2\Lambda_{\text{UV}}^2$$

$$\frac{\partial_t^m \text{Disc}_s\mathcal{A}_{s,u}}{\mu^{n+1}} < \frac{\partial_t^m \text{Disc}_s\mathcal{A}_{s,u}}{\varepsilon^2\Lambda_{\text{UV}}^2\mu^n}, I_{s,u}(n+1,m) < \frac{1}{\varepsilon^2\Lambda_{\text{UV}}^2} I_{s,u}(n,m)$$

$$\begin{aligned} C(2,1) + \frac{3}{2\varepsilon^2\Lambda_{\text{UV}}^2}C(2,0) - \frac{1}{2}C(3,0) > 0, \\ \frac{12}{(\varepsilon^2\Lambda_{\text{UV}}^2)^2}C(2,0) > C(4,0) > 0, \\ C(3,1) + \frac{3}{\varepsilon^2\Lambda_{\text{UV}}^2}C(2,1) - \frac{3}{2\varepsilon^2\Lambda_{\text{UV}}^2}C(3,0) + \frac{9C(2,0)}{2(\varepsilon^2\Lambda_{\text{UV}}^2)^2} > 0, \end{aligned}$$

$$\left(\int_a^b \mathrm{d}x f(x)g(x)\right)^2 \le \left(\int_a^b \mathrm{d}x f^2(x)\right)\left(\int_a^b \mathrm{d}x g^2(x)\right)$$

$$I_{s,u}(3,0)^2 < I_{s,u}(2,0)I_{s,u}(4,0)$$

$$\frac{4}{3}C(3,0)^2 < C(2,0)C(4,0)$$

$$C(2,1) - \frac{1}{2}C(3,0) = I_s(2,1) + I_u(2,1) - \frac{3}{2}(I_s(3,0) + I_u(3,0))$$

$$(I_s(3,0) + I_u(3,0))^2 < (I_s(2,0) + I_u(2,0))(I_s(4,0) + I_u(4,0)).$$

$$C(2,1) - \frac{1}{2}C(3,0) + \frac{\sqrt{3}}{4}\sqrt{C(2,0)C(4,0)} > 0$$

$$C(2n,0)^2 < \alpha(n)C(2n+2,0)C(2n-2,0)$$

$$\mathcal{A} = 2g_2 s^2 + 2g_4 s^4 + \beta s^4 (\ln\ s + \ln(-s)) + 2g_6 s^6 + \cdots$$

$$C(4,0) = \int_{\text{arc}} \frac{\mathrm{d}\mu \mathcal{A}(\mu,0)}{\mu^5} = 2g_2 + 4g_4 \ln\left(\varepsilon^2 \Lambda_{\mathrm{UV}}^2\right)$$

$$C(6,0) = 2g_6 - \frac{\beta}{\varepsilon^4} \Lambda_{\mathrm{UV}}^4$$

$$\frac{21 g_2^2}{240\pi^2} t^2 \left(t^2 + \frac{1}{21} s(s+t)\right) \ln\ t$$

$$C(2,2) \sim \ln\ t$$

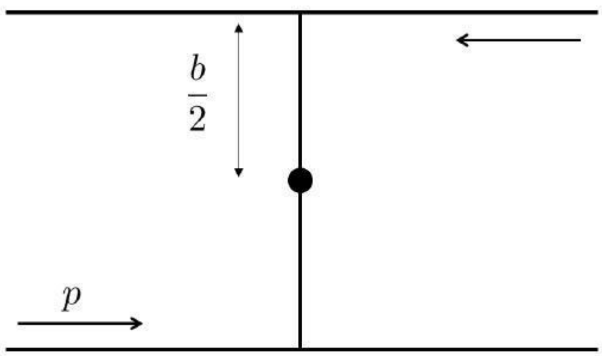
$$\mathcal{A}(s,t) = \frac{1}{2} \sum_{l=0}^{\infty} n(l,d) f_l(s) P_l^{(d)} \left(1 + \frac{2t}{s-4m^2}\right)$$

$$f_l(s) = N(d) \int_{-1}^{1} \mathrm{d}x (1-x^2)^{\frac{d-4}{2}} P_l^{(d)}(x) \mathcal{A}(s,t(x))$$

$$N(d) = \frac{(16\pi)^{\frac{2-d}{2}}}{\Gamma\left(\frac{d-2}{2}\right)}, n(l,d) = \frac{(4\pi)^{\frac{d}{2}} (d+2l-3) \Gamma(d+l-3)}{\pi \Gamma\left(\frac{d-2}{2}\right) \Gamma(l+1)}.$$

$$2\mathrm{Im} f_l(s) \geq \frac{(s-4m^2)^{\frac{d-3}{2}}}{\sqrt{s}} |f_l(s)|^2$$

$$f_l(s) = \frac{\sqrt{s}}{(s-4m^2)^{\frac{d-3}{2}}} i\left(1 - e^{2i\delta_l(s)}\right)$$



$$|\vec{l}| = |\vec{b} \times \vec{p}| = \frac{b\sqrt{s-4m^2}}{2} = \frac{b\sqrt{s}}{2}, b = \frac{2l}{\sqrt{s}}.$$

$$\frac{1}{l^{d-3}} n(l,d) P_l^{(d)} \left(1 - \frac{q^2 b^2}{2l^2}\right) \approx 2^d (2\pi)^{\frac{d-2}{2}} (bq)^{\frac{4-d}{2}} J_{\frac{d-4}{2}}(bq).$$

$$\sum_l \to \frac{\sqrt{s}}{2} \int \mathrm{d}b.$$

$$f_l(s) \to \delta(s,b) = \delta_{\text{tree}}(s,b) = \frac{\Gamma\left(\frac{d-4}{2}\right) G_N s}{(\pi)^{\frac{d-4}{2}} b^{d-4}}, b = \frac{2l}{\sqrt{s}}$$

$$\mathcal{A} = \mathcal{A}_{\text{low}} + \mathcal{A}_{\text{eik}}$$

$$\mathcal{A}_{\text{eik}}(s,t) = 2is(2\pi)^{\frac{d-2}{2}} \int_{b^*}^{\infty} \mathrm{d}b b^{d-3} (bq)^{\frac{4-d}{2}} J_{\frac{4-d}{2}}(bq)\left(1 - e^{2i\delta_{\text{tree}}(s,b)}\right)$$

$$|f_l(s)| < s^{2-\frac{d}{2}}$$

$$\left|P_l^{(d)}\left(1 - \frac{2q^2}{s}\right)\right| < \left(\frac{ql}{\sqrt{s}}\right)^{\frac{3-d}{2}}$$

$$\left|\sum_{l=0}^{l^*} n(l,d) f_l(s) P_l^{(d)}\left(1 - \frac{2q^2}{s}\right)\right| < \sum_{l=0}^{l^*} n(l,d) s^{2-\frac{d}{2}} (ql)^{\frac{3-d}{2}} s^{\frac{d-3}{4}},$$

$$b^* = s^{\frac{1}{d-2}}, l^* = \frac{b^*\sqrt{s}}{2} = s^{\frac{d}{2(d-2)}}$$

$$|\mathcal{A}_{\text{low}}| < s^{\frac{5-d}{4}} (l^*)^{\frac{d-1}{2}} = s^{2-\frac{d-3}{2(d-2)}} < s^2$$

$$\int \mathrm{d}x g(x) e^{if(x)} = g(x_0) e^{if(x_0)} e^{i\frac{\pi}{4}\text{sign}\left(f''(x_0)\right)} \sqrt{\frac{2\pi}{f''(x_0)}}$$

$$J_{\frac{d-4}{2}}(bq) = \sqrt{\frac{2}{\pi bq}} \cos\left(bq + \pi\frac{d-5}{4}\right)$$

$$\frac{\partial}{\partial b}(\pm bq + \delta_{\text{tree}}(b)) = 0.$$

$$\delta_{\text{tree}} = \alpha s b^{4-d}, \alpha = \text{ const.},$$

$$\pm q - (d-4)\alpha s b^{3-d} = 0, b > 0.$$

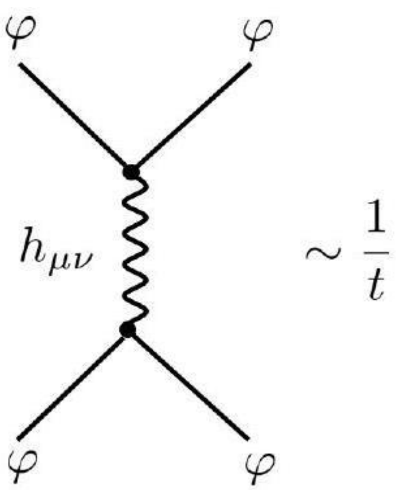
$$b_0 = \left(\frac{q}{(d-4)\alpha s}\right)^{\frac{1}{3-d}} \propto s^{\frac{1}{d-3}}$$

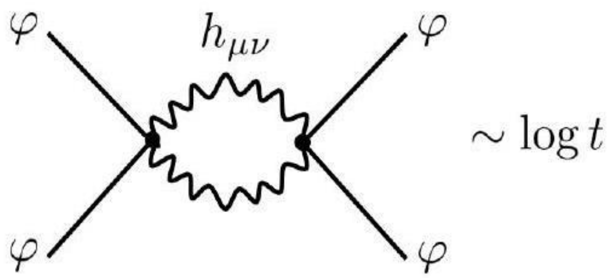
$$\mathcal{A}_{\text{eik}} = -2is(2\pi)^{\frac{d-2}{2}} b_0^{d-3} (b_0 q)^{\frac{4-d}{2}} \sqrt{\frac{2}{\pi b_0 q}} \frac{1}{2} e^{i\left(qb_0 + \alpha s b_0^{4-d}\right)} \sqrt{\frac{2\pi}{\alpha b_0^{2-d} s}} \frac{e^{i\pi\left(\frac{d}{4}-1\right)}}{\sqrt{(d-4)(d-3)}},$$

$$\mathcal{A}_{\text{eik}} \propto e^{i\zeta q^{\frac{4-d}{3-d}} s^{\frac{1}{d-3}}} s^{2-\frac{d-4}{2(d-3)}} q^{\frac{(d-2)^2}{2(3-d)}}$$

$$|\mathcal{A}(s,t)| < s^2$$

$$\mathcal{A}(s,t) = A_0 \frac{s^2}{t} + A_1 s^2 \ln\left(\frac{-t}{\mu^2}\right).$$





$$C(2,0) = \frac{1}{2}\left(\frac{A_0}{t} + A_1 \ln\ t + \cdots\right) = \int_{\epsilon^2 \Lambda_{\mathrm{UV}}^2}^{\infty} \mathrm{d}\mu \frac{\mathrm{Im}\mathcal{A}(\mu,t)}{\mu^3}$$

$$\int_{M^*}^{\infty} \mathrm{d}\mu \frac{\mathrm{Im}\mathcal{A}(s,t)}{\mu^3} = \frac{A_0}{t} + A_1 \ln\ t + (\text{ finite }).$$

$$\mathrm{Im}\mathcal{A} = s^{2+\alpha t}.$$

$$\mathrm{Im}\mathcal{A} = s^{2+\alpha t}\varphi(s,t).$$

$$\varphi(s,t) = \varphi(s,0) + \varphi_t(s,0)t + \frac{1}{2}\varphi_{tt}(s,0)t^2 + \cdots$$

$$s = M^2 e^{\sigma},$$

$$\int_0^{\infty} \frac{\mathrm{d}\mu}{\mu} 2\varphi(\mu,0)\mu^{\alpha t} = \int_0^{\infty} \mathrm{d}\sigma 2M^{2\alpha t}\varphi(\sigma,0)e^{\alpha\sigma t}$$

$$2M^{2\alpha t}L[\varphi(\sigma,0)] = f(t) + \mathcal{O}(t)$$

$$\mathrm{Im}\mathcal{A} = s^{2+\alpha t}\left(1 + \frac{1}{\ln\ s}\right)$$

$$\varphi(\sigma,0) = a_0 L^{-1}[f(t)], \varphi_t(\sigma,0) = a_2 L^{-1}\left[\frac{f(t)}{t}\right], \varphi_{tt}(\sigma,0) = a_2 L^{-1}\left[\frac{f(t)}{t^2}\right], \ldots.$$

$$\varphi(\sigma,t)=\sum_n a_n\sigma^n t^n=\varphi(t\ln\ s),\varphi(0)\neq 0$$

$$\mathcal{A}(s,t)=-\frac{a_0 e^{-i\pi\alpha t}}{\sin\ (\pi\alpha t)}(s^{2+\alpha t}+(-s-t)^{2+\alpha t})+\mathcal{O}(t\ln\ s)$$

$$\mathcal{A}(s,t)=-\frac{s^2}{M_{\text{Pl}}^2 t}-\gamma s^2,\gamma>0$$

$$\mathcal{A}_{\text{UV}}(s,t)=e^{-i\pi\alpha t}\left(-\gamma-\frac{\pi^2}{M_{\text{Pl}}^2\sin\ \pi\alpha t}\right)(s^{2+\alpha t}+u^{2+\alpha t})$$

$$\text{Im}\mathcal{A}_{\text{UV}}=\gamma s^{2+\alpha t}\sin\ \pi\alpha t=\gamma(\pi\alpha t)s^{2+\alpha t}=\mathcal{O}(t\ln\ s)$$

$$S^\dagger S=1$$

$$\mathcal{A}(s,\theta)=32\pi\sum_{l=0}^{\infty}\left(l+\frac{1}{2}\right)f_l(s)P_l(\cos\ \theta).$$

$$\text{Im}f_j\geq 0,\left|f_j(s)\right|\leq 1$$

$$2\text{Im}f_j(s)\geq\left|f_j(s)\right|^2\sqrt{\frac{s-4m^2}{s}}$$

$$\mathcal{L}=-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{1}{2}m^2\phi^2-\sum_n\frac{\tilde{\lambda}_n}{n!}\phi^n$$

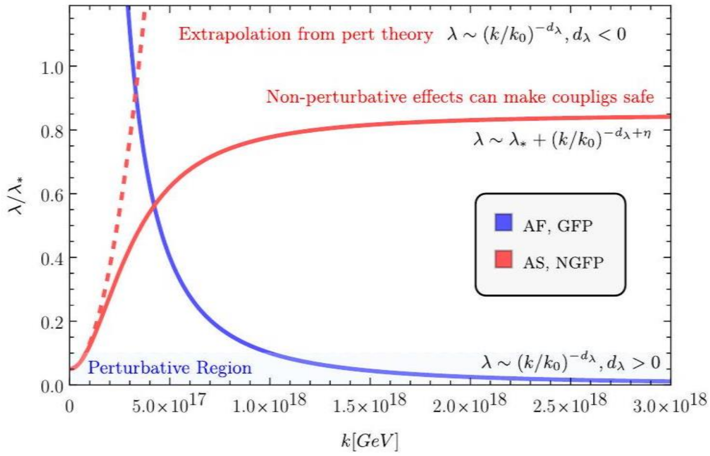
$$[\tilde{\lambda}_n]\cdot M^n=M^4$$

$$k\partial_k\left(\tilde{\lambda}_n(k)k^{-d_{\tilde{\lambda}_n}}\right)=k^{-d_{\tilde{\lambda}_n}}\left(k\partial_k\tilde{\lambda}_n(k)\right)-d_{\tilde{\lambda}_n}k^{-d_{\tilde{\lambda}_n}}\tilde{\lambda}_n(k)$$

$$k\partial_k\lambda_n(k)=\tilde{\lambda}_n(k)k^{-d_{\tilde{\lambda}_n}}\left(\frac{k\partial_k\tilde{\lambda}_n(k)}{\tilde{\lambda}_n(k)}\right)-d_{\tilde{\lambda}_n}\lambda_n(k)=\left(\eta[\lambda_n]-d_{\tilde{\lambda}_n}\right)\cdot\lambda_n$$

$$\eta[\lambda_n]\equiv\frac{\partial\log\ \tilde{\lambda}_n}{\partial\log\ k}$$

$$\lambda_n(k)\simeq\lambda_n(k_0)(k/k_0)^{-d_{\tilde{\lambda}_n}}$$



$$k\partial_k \lambda_n = \beta_{\lambda_n}.$$

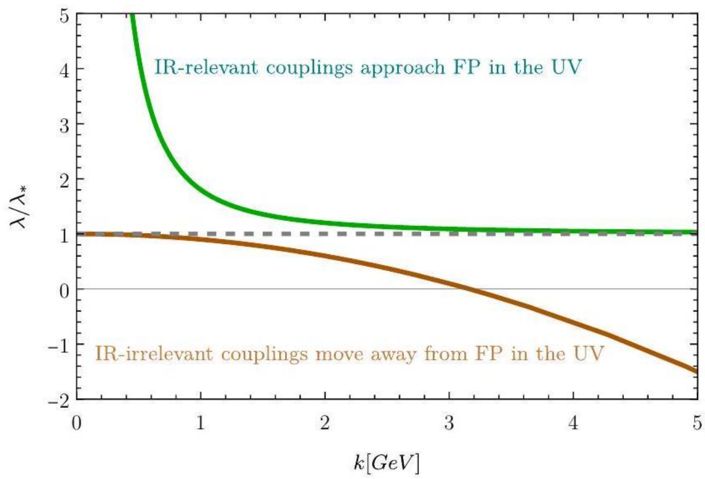
$$\text{Fixed Points: } k\partial_k \lambda_n^* = \beta_{\lambda_n^*} = 0$$

$$\forall \lambda_n : \lambda_n^* = 0,$$
$$\exists n : \lambda_n^* \neq 0.$$

$$k\partial_k \lambda_n = \beta_{\lambda_n} \simeq \beta_{\lambda_n^*} + \sum_m \left.\frac{\partial \beta_{\lambda_n}}{\partial \lambda_m}\right|_{\lambda_m = \lambda_m^*} (\lambda_m - \lambda_m^*),$$

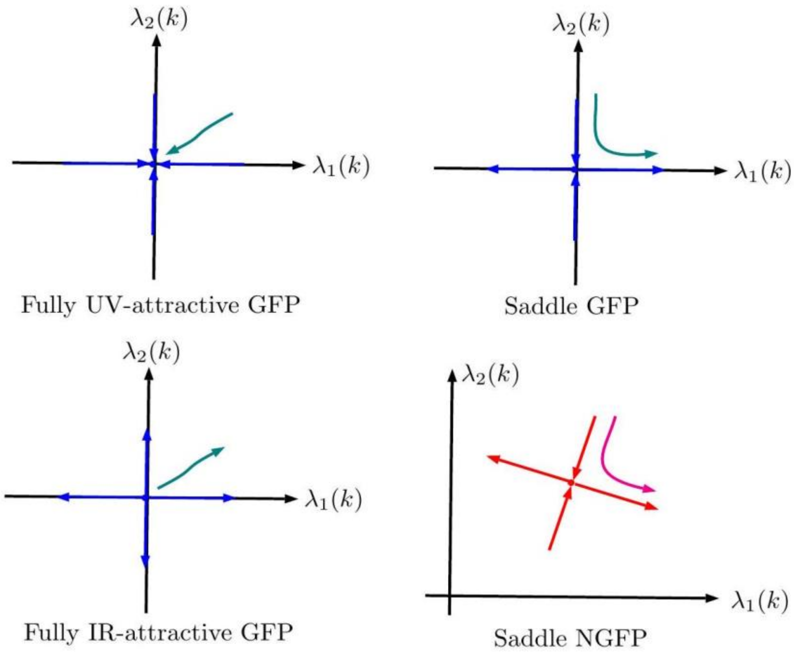
$$\lambda(k) = \lambda^* + (k/k_0)^{-\theta},$$

$$\theta \equiv -\left.\frac{\partial \beta}{\partial \lambda}\right|_{\lambda = \lambda*}$$



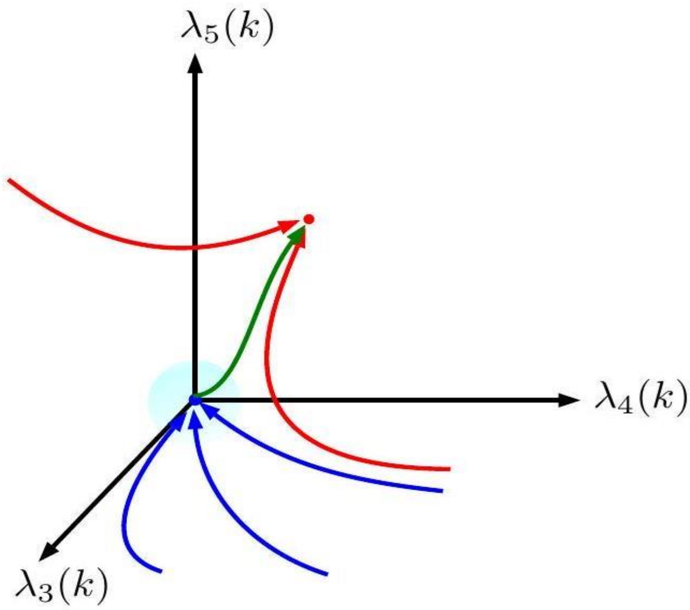
$$\vec{\beta} \simeq \vec{\beta}^* + S_{\text{stab}}(\vec{\lambda} - \vec{\lambda}^*) \Rightarrow \vec{\lambda} = \vec{\lambda}^* + \sum_i c_i \vec{e}_i (k/k_0)^{-\theta_i}$$

$$(S_{\text{stab}})_{nm} \equiv \left.\frac{\partial \beta_n}{\partial \lambda_m}\right|_{\lambda_i = \lambda_i^*}$$



$$\beta_{\lambda_n} = \left(\eta[\lambda_n] - d_{\tilde{\lambda}_n}\right) \cdot \lambda_n = 0$$

$$\eta[\lambda_n] - d_{\tilde{\lambda}_n} = 0$$



$$\mathcal{Z}[J] = \frac{1}{\mathcal{N}} \int \mathcal{D}\varphi e^{-S[\varphi] + \int \mathrm{d}^d x J(x) \varphi(x)}$$

$$\begin{aligned} \langle \varphi(x_1) \cdots \varphi(x_n) \rangle_J &= \frac{\int \mathcal{D}\varphi \varphi(x_1) \cdots \varphi(x_n) e^{-S[\varphi] + \int \mathrm{d}^d x J(x) \varphi(x)}}{\int \mathcal{D}\varphi e^{-S[\varphi] + \int \mathrm{d}^d x J(x) \varphi(x)}} \\ &= \frac{1}{\mathcal{Z}[J]} \frac{\delta^n}{\delta J(x_1) \cdots \delta J(x_n)} \mathcal{Z}[J]. \end{aligned}$$

$$\mathcal{W}[J] = \ln \mathcal{Z}[J].$$

$$\langle\varphi(x_1)\cdots\varphi(x_n)\rangle_{J,c}=\frac{\delta^n}{\delta J(x_1)\cdots\delta J(x_n)}\mathcal{W}[J]\equiv\mathcal{W}^{(n)}[J]$$

$$\begin{aligned}\mathcal{W}^{(2)}[J]&=\frac{\delta^2}{\delta J(x_1)\delta J(x_2)}\ln\,\mathcal{Z}[J]\\&=\frac{\delta}{\delta J(x_1)}\frac{1}{\mathcal{Z}[J]}\frac{\delta\mathcal{Z}[J]}{\delta J(x_2)}\\&=\left[\frac{1}{\mathcal{Z}[J]}\frac{\delta^2\mathcal{Z}[J]}{\delta J(x_1)\delta J(x_2)}\right]-\left[\frac{1}{\mathcal{Z}[J]}\frac{\delta\mathcal{Z}[J]}{\delta J(x_1)}\right]\left[\frac{1}{\mathcal{Z}[J]}\frac{\delta\mathcal{Z}[J]}{\delta J(x_2)}\right]\\&=\langle\varphi(x_1)\varphi(x_2)\rangle_J-\langle\varphi(x_1)\rangle_J\langle\varphi(x_2)\rangle_J=\langle\varphi(x_1)\varphi(x_2)\rangle_{J,c}\end{aligned}$$

$$\Gamma[\phi]=\sup_J\left\{\int\ d^dxJ(x)\phi(x)-\mathcal{W}[J]\right\}$$

$$\phi(x)=\frac{\delta\mathcal{W}[J_{\text{sup}}]}{\delta J_{\text{sup}}}=\frac{1}{\mathcal{Z}[J_{\text{sup}}]}\frac{\delta\mathcal{Z}[J_{\text{sup}}]}{\delta J_{\text{sup}}}=\langle\varphi(x)\rangle_{J_{\text{sup}}}.$$

$$\Gamma^{(1)}[\phi]=J_{\text{sup}}+\frac{\delta J_{\text{sup}}}{\delta\phi}\underbrace{\left[\phi-\frac{\delta\mathcal{W}[J_{\text{sup}}]}{\delta J_{\text{sup}}}\right]}_{=0}=J_{\text{sup}}.$$

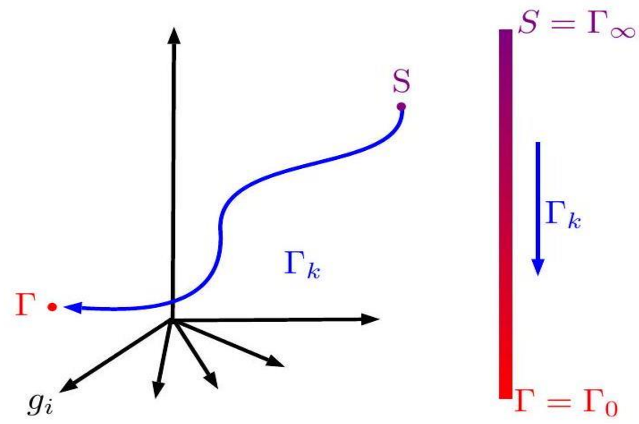
$$\begin{aligned}\int\ d^dy\frac{\delta^2\mathcal{W}}{\delta J(x_1)\delta J(y)}\frac{\delta^2\Gamma}{\delta\phi(y)\delta\phi(x_2)}&=\int\ d^dy\left[\frac{\delta}{\delta J(x_1)}\frac{\delta\mathcal{W}}{\delta J(y)}\right]\left[\frac{\delta}{\delta\phi(y)}\frac{\delta\Gamma}{\delta\phi(x_2)}\right]\\&=\int\ d^dy\left[\frac{\delta}{\delta J(x_1)}\phi(y)\right]\left[\frac{\delta}{\delta\phi(y)}J(x_2)\right]\\&=\frac{\delta J(x_2)}{\delta J(x_1)}\equiv\delta(x_1-x_2)\end{aligned}$$

$$\begin{aligned}e^{-\Gamma[\phi]}&=e^{-\int\ d^dxJ_{\text{sup}}(x)\phi(x)+\mathcal{W}[J_{\text{sup}}]}\\&=e^{-\int\ d^dx\frac{\delta\Gamma[\phi]}{\delta\phi(x)}\phi(x)}e^{\mathcal{W}[J_{\text{sup}}]}\\&=e^{-\int\ d^dx\frac{\delta\Gamma[\phi]}{\delta\phi(x)}\phi(x)}\int\ \mathcal{D}\varphi e^{-S[\varphi]+\int\ d^dxJ_{\text{sup}}(x)\varphi(x)}\end{aligned}$$

$$e^{-\Gamma[\phi]}=\int\ \mathcal{D}\varphi' e^{-S[\phi+\varphi']+\int\ d^dx\frac{\delta\Gamma[\phi]}{\delta\phi(x)}\varphi'(x)}$$

$$\Gamma[\phi]=\sum_{n\geq0}\frac{1}{n!}\int\ d^dx_1\cdots\ d^dx_n\Gamma^{(n)}[\phi=0](x_1,\dots,x_n)\phi(x_1)\cdots\phi(x_n)$$

$$\mathcal{A}_{\phi\phi\to\chi\chi}\simeq\frac{\delta^3\Gamma}{\delta\phi\delta\phi\delta g_{\mu\nu}}\circ\frac{\delta^2\Gamma}{\delta g^{\mu\nu}\delta g\rho\sigma}\circ\frac{\delta^3\Gamma}{\delta g_{\rho\sigma}\delta\chi\delta\chi}+\frac{\delta^4\Gamma}{\delta\phi\delta\phi\delta\chi\delta\chi}$$



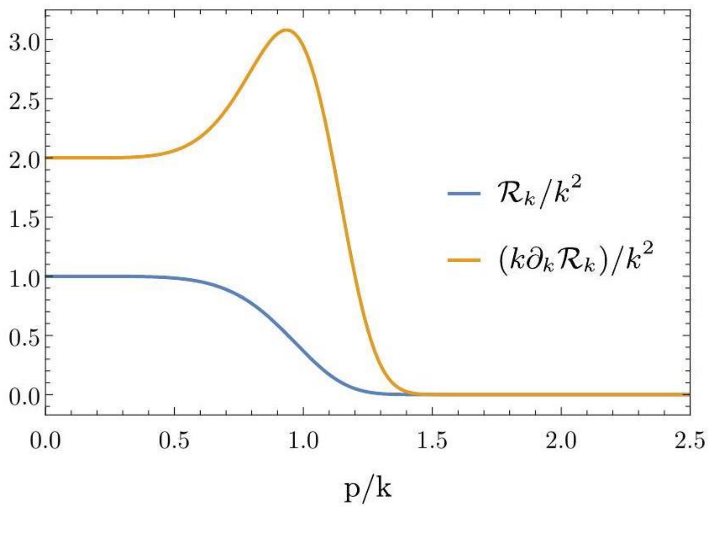
$$\mathcal{Z}_k[J] = \frac{1}{\mathcal{N}} \int \mathcal{D}\varphi e^{-\Delta S_k[\varphi]} e^{-S[\varphi]+\int \mathrm{d}^d x J(x)\varphi(x)}$$

$$\Delta S_k[\varphi] = \frac{1}{2}\int \frac{\mathrm{d}^d p}{(2\pi)^d} \varphi(-p)\mathcal{R}_k(p^2)\varphi(p)$$

$$\lim_{p^2\to 0} \mathcal{R}_k(p^2) > 0.$$

$$\lim_{k\to 0} \mathcal{R}_k(p^2) = 0.$$

$$\lim_{k\to \Lambda_{\mathrm{UV}}\to\infty} \mathcal{R}_k(p^2) \to \infty.$$



$$\mathcal{R}_k(p^2) = (k^2 - p^2)\theta\left(1 - \frac{p^2}{k^2}\right)$$

$$\mathcal{R}_k(p^2) = k^2 e^{-(p^2/k^2)^n},$$

$$\mathcal{R}_k(p^2) = \frac{p^2}{e^{p^2/k^2} - 1}$$

$$k\partial_k \mathcal{Z}_k[J] = -\frac{1}{2}\int \frac{\mathrm{d}^d p}{(2\pi)^d} [k\partial_k \mathcal{R}_k(p^2)] \frac{\delta^2 \mathcal{Z}_k[J]}{\delta J(-p)\delta J(p)}$$

$$k\partial_k\mathcal{Z}_k[J]=\frac{1}{\mathcal{N}}\int\ \mathcal{D}\varphi[-k\partial_k\Delta S_k[\varphi]]e^{-S[\varphi]-\Delta S_k[\varphi]+\int\ \mathrm{d}^dxJ(x)\varphi(x)}$$

$$=-\frac{1}{2\mathcal{N}}\int\ \mathcal{D}\varphi\int\ \frac{\mathrm{d}^dp}{(2\pi)^d}\varphi(-p)[k\partial_k\mathcal{R}_k(p^2)]\varphi(p)e^{-S[\varphi]-\Delta S_k[\varphi]+\int\ \mathrm{d}^dxJ(x)\varphi(x)}$$

$$=-\frac{1}{2\mathcal{N}}\int\ \mathcal{D}\varphi\int\ \frac{\mathrm{d}^dp}{(2\pi)^d}[k\partial_k\mathcal{R}_k(p^2)]\frac{\delta^2}{\delta J(-p)\delta J(p)}e^{-S[\varphi]-\Delta S_k[\varphi]+\int\ \mathrm{d}^dxJ(x)\varphi(x)}$$

$$=-\frac{1}{2}\int\ \frac{\mathrm{d}^dp}{(2\pi)^d}[k\partial_k\mathcal{R}_k(p^2)]\frac{\delta^2\mathcal{Z}_k[J]}{\delta J(-p)\delta J(p)}$$

$$\mathcal{W}_k[J]=\ln\ \mathcal{Z}_k[J]$$

$$k\partial_k\mathcal{W}_k[J]=-\frac{1}{2}\int\ \frac{\mathrm{d}^dp}{(2\pi)^d}[k\partial_k\mathcal{R}_k(p^2)]\left\{\frac{\delta^2\mathcal{W}_k[J]}{\delta J(-p)\delta J(p)}+\frac{\delta\mathcal{W}_k[J]}{\delta J(-p)}\frac{\delta\mathcal{W}_k[J]}{\delta J(p)}\right\}$$

$$\Gamma_k[\phi]=\sup_J\left\{\int\ \mathrm{d}^dxJ(x)\phi(x)-\mathcal{W}_k[J]-\Delta S_k[\phi]\right\}$$

$$J_{\mathrm{sup}}\,[\phi]\ =\frac{\delta(\Gamma_k[\phi]+\Delta S_k[\phi])}{\delta\phi}$$

$$\mathcal{G}_k[\phi]\ \equiv\frac{\delta^2\mathcal{W}_k[J_{\mathrm{sup}}]}{\delta J_{\mathrm{sup}}^2}=\left[\frac{\delta^2}{\delta\phi^2}(\Gamma_k[\phi]+\Delta S_k[\phi])\right]^{-1}\ =\left[\Gamma_k^{(2)}[\phi]+\mathcal{R}_k\right]^{-1}$$

$$k\partial_k\Gamma_k[\phi]=-k\partial_k\mathcal{W}_k[J_{\mathrm{sup}}\,[\phi]]-k\partial_k\Delta S_k[\phi]+\int\ \mathrm{d}^dxk\partial_kJ_{\mathrm{sup}}\,[\phi]\left[\phi-\frac{\delta\mathcal{W}_k[J_{\mathrm{sup}}\,[\phi]]}{\delta\phi}\right]$$

$$=\frac{1}{2}\int\ \frac{\mathrm{d}^dp}{(2\pi)^d}[\mathcal{G}_k(p,-p)+\phi(-p)\phi(p)]k\partial_k\mathcal{R}_k(p^2)-k\partial_k\Delta S_k[\phi]$$

$$=\frac{1}{2}\int\ \frac{\mathrm{d}^dp}{(2\pi)^d}\mathcal{G}_k(p,-p)k\partial_k\mathcal{R}_k(p^2)$$

$$k\partial_k\Gamma_k[\phi]=\frac{1}{2}S\mathrm{Tr}\left[\left(\Gamma_k^{(2)}[\phi]+\mathcal{R}_k\right)^{-1}k\partial_k\mathcal{R}_k\right]$$

$$\Gamma_k^{(2)}+\mathcal{R}_k>0$$

$$\lim_{p^2\to\infty}k\partial_k\mathcal{R}_k(p^2)=0$$

$$\Gamma_k\simeq S+\Delta\Gamma_{k,1l}$$

$$k\partial_k\Delta\Gamma_{k,1l}=\frac{1}{2}S\mathrm{Tr}\left[\left(S^{(2)}+\mathcal{R}_k\right)^{-1}k\partial_k\mathcal{R}_k\right]=\frac{1}{2}k\partial_kS\mathrm{Trln}\left[S^{(2)}+\mathcal{R}_k\right]$$

$$\Gamma\simeq S+\frac{1}{2}S\mathrm{Trln}\ S^{(2)}$$

$$k\partial_k\Gamma_k^{(1)}=-\frac{1}{2}S\mathrm{Tr}\left[\mathcal{G}_k\Gamma_k^{(3)}\mathcal{G}_kk\partial_k\mathcal{R}_k\right]$$

$$k\partial_k\Gamma_k^{(2)}=-\frac{1}{2}S\mathrm{Tr}\left[\mathcal{G}_k\Gamma_k^{(4)}\mathcal{G}_kk\partial_k\mathcal{R}_k\right]+S\mathrm{Tr}\left[\mathcal{G}_k\Gamma_k^{(3)}\mathcal{G}_k\Gamma_k^{(3)}\mathcal{G}_kk\partial_k\mathcal{R}_k\right]$$

$$\Gamma_k[\phi] = \sum_{n=0}^{\infty} \frac{1}{n!}\int \mathrm{d}^d x_1 \cdots \mathrm{d}^d x_n \Gamma_k^{(n)}(x_1, \dots, x_n)\phi(x_1)\cdots\phi(x_n)$$

$$\Gamma_k = \int \mathrm{d}^d x \left[V_k(\phi) + \frac{1}{2} Z_k(\phi)(\partial_\mu\phi)^2 + \mathcal{O}(\partial^4)\right]$$

$$\mathfrak{G}^a_{O(N)} = -f^{abc}\int \mathrm{d}^d x \phi^b(x)\frac{\delta}{\delta\phi^c(x)}$$

$$\mathfrak{G}^a_{SU(N)} = -\mathcal{D}^{ab}_\mu(x)\frac{\delta}{\delta A^b_\mu(x)} = -\left(\partial_\mu\delta^{ab} - g f^{abc} A^c_\mu(x)\right)\frac{\delta}{\delta A^b_\mu(x)}.$$

$$\begin{aligned}
0 &= \frac{1}{\mathcal{Z}[J]}\int \mathcal{D}\varphi \mathfrak{G} e^{-S[\varphi]+\int \mathrm{d}^d x J(x)\varphi(x)} \\
&= \frac{1}{\mathcal{Z}[J]}\int \mathcal{D}\varphi \left[-\mathfrak{G}S[\varphi] + \int \mathrm{d}^d x J(x)\mathfrak{G}\varphi(x)\right] e^{-S[\varphi]+\int \mathrm{d}^d x J(x)\varphi(x)} \\
&= -\langle\mathfrak{G}S\rangle_J + \left\langle\int \mathrm{d}^d x J(x)\mathfrak{G}\varphi(x)\right\rangle_J
\end{aligned}$$

$$\begin{aligned}
0 &= -\langle\mathfrak{G}S\rangle_{J_{\text{sup}}} + \int \mathrm{d}^d x J_{\text{sup}}(x)\langle\mathfrak{G}\varphi(x)\rangle_{J_{\text{sup}}} \\
&= -\langle\mathfrak{G}S\rangle_{J_{\text{sup}}} + \int \mathrm{d}^d x \frac{\delta\Gamma}{\delta\phi(x)}\mathfrak{G}\phi(x) \\
&= -\langle\mathfrak{G}S\rangle_{J_{\text{sup}}} + \mathfrak{G}\Gamma[\phi]
\end{aligned}$$

$$\mathfrak{G}\Gamma = \langle\mathfrak{G}S\rangle_{J_{\text{sup}}}.$$

$$\mathcal{W} = \mathfrak{G}\Gamma - \left\langle\mathfrak{G}\left(S_{\text{gf}} + S_{\text{gh}}\right)\right\rangle_{J_{\text{sup}}} = 0$$

$$\mathcal{W}_k = \mathfrak{G}\Gamma_k + \mathfrak{G}\Delta S_k - \left\langle\mathfrak{G}\left(S_{\text{gf}} + S_{\text{gh}} + \Delta S_k\right)\right\rangle_{J_{\text{sup}}} = 0$$

$$\Gamma_k = \int \mathrm{d}^d x\sqrt{g}\frac{1}{2}(\nabla_\mu\phi)(\nabla^\mu\phi)$$

$$k\partial_k\Gamma_k = \int \mathrm{d}^d x\sqrt{g}\left[c_0 + c_1 R + c_2 R^2 + c_3 R_{\mu\nu}R^{\mu\nu} + \cdots\right]$$

$$\Gamma_k^{(2)} = -g^{\mu\nu}\nabla_\mu\nabla_\nu \equiv -\nabla^2 \equiv \Delta$$

$$\frac{1}{2}\mathrm{STr}[(\Delta + \mathcal{R}_k(\Delta))^{-1}k\partial_k\mathcal{R}_k(\Delta)] \equiv \frac{1}{2}\mathrm{STr}W(\Delta)$$

$$\mathrm{STr}e^{-s\Delta} \equiv \mathrm{STr}H(s)$$

$$H(x,y;s) = \langle y|H(s)|x\rangle$$

$$\begin{aligned}
\partial_s H(x,y;s) &= -\Delta_x H(x,y;s) \\
H(x,y;0) &= \delta(x-y)
\end{aligned}$$

$$H(x,y;s) = \left(\frac{1}{4\pi s}\right)^{d/2} e^{-\frac{(x-y)^2}{4s}}$$

$$\mathrm{STr} e^{-s\Delta} = \mathrm{tr}\int \mathrm{d}^d x\sqrt{g}\langle x|e^{-s\Delta}|x\rangle = \mathrm{tr}\int \mathrm{d}^d x\sqrt{g}H(x,x;s)$$

$$\int \frac{\mathrm{d}^d p}{(2\pi)^d} e^{-sp^2} = \frac{1}{(2\pi)^d}\int \mathrm{d}\Omega \int_0^\infty \mathrm{d}p p^{d-1} e^{-sp^2}$$
$$= \frac{1}{(2\pi)^d}\left[\frac{2\pi^{d/2}}{\Gamma(d/2)}\right]\left[\frac{\Gamma(d/2)}{2s^{d/2}}\right] = \left(\frac{1}{4\pi s}\right)^{d/2}$$

$$H(x,x;s) = \left(\frac{1}{4\pi s}\right)^{d/2}$$

$$H(x,y;s) = \left(\frac{1}{4\pi s}\right)^{d/2} e^{-\frac{\sigma(x,y)}{2s}}\Omega(x,y;s)$$

$$\sigma(x,x) \equiv \bar{\sigma} = 0$$

$$\frac{1}{2}(\nabla_\mu \sigma)(\nabla^\mu \sigma) = \sigma$$

$$\left[\left(-\frac{d}{2s} + \partial_s - \nabla^2 + \frac{1}{2s}(\nabla^2\sigma(x,y))\right)\Omega(x,y;s) + \frac{1}{s}(\nabla_\mu\sigma(x,y))(\nabla^\mu\Omega(x,y;s))\right] = 0.$$

$$\Omega(x,y;s) \sim \sum_{n\geq 0} s^n A_n(x,y), s \to 0$$

$$\left(n - \frac{d}{2} + \frac{1}{2}(\nabla^2\sigma(x,y))\right)A_n(x,y) + (\nabla^\mu\sigma(x,y))\big(\nabla_\mu A_n(x,y)\big) - \nabla^2 A_{n-1}(x,y) = 0$$

$$\left(1 - \frac{d}{2} + \frac{1}{2}\overline{\nabla^2\sigma}\right)\overline{A_1} + \overline{\nabla^\mu\sigma\nabla_\mu A_1} - \overline{\nabla^2 A_0} = 0$$

$$\frac{1}{2}\overline{\nabla^2\nabla^2\sigma}\,\overline{A_0} + \overline{\nabla^\mu\nabla^2\sigma}\,\overline{\nabla_\mu A_0} + \left(-\frac{d}{2} + \frac{1}{2}\overline{\nabla^2\sigma}\right)\overline{\nabla^2 A_0}$$
$$+\overline{\nabla^2\nabla^\mu\sigma}\,\overline{\nabla_\mu A_0} + \overline{\nabla^\mu\sigma}\,\overline{\nabla^2\nabla_\mu A_0} + 2\overline{\nabla^\mu\nabla^\nu\sigma\nabla_\mu\nabla_\nu A_0} = 0.$$

$$\frac{1}{2}\overline{\nabla^\mu\sigma\nabla_\mu\sigma} = \bar{\sigma} = 0 \;\Rightarrow\; \overline{\nabla_\mu\sigma} = 0$$

$$0 = \overline{\nabla^\mu\sigma\nabla_\alpha\nabla_\mu\sigma} = \overline{\nabla_\alpha\sigma} = 0$$

$$(\nabla_\beta\nabla_\alpha\nabla_\mu\sigma(x,y))(\nabla^\mu\sigma(x,y)) + (\nabla_\alpha\nabla_\mu\sigma(x,y))\big(\nabla_\beta\nabla^\mu\sigma(x,y)\big) = \nabla_\beta\nabla_\alpha\sigma(x,y).$$

$$\overline{\nabla_\alpha\nabla_\mu\sigma\nabla_\beta\nabla^\mu\sigma} = \overline{\nabla_\beta\nabla_\alpha\sigma}.$$

$$\overline{\nabla_\mu\nabla_\nu\sigma} = g_{\mu\nu}.$$

$$\overline{\nabla^2\sigma} = d.$$

$$\overline{\nabla_\beta \nabla_\alpha \nabla_\gamma \sigma} + \overline{\nabla_\gamma \nabla_\alpha \nabla_\beta \sigma} = 0.$$

$$[\nabla_\mu, \nabla_\nu] T_{\alpha_1 \ldots \alpha_n} = \sum_{i=1}^{n} R^{\beta}_{\mu\nu\alpha_i} T_{\alpha_1 \ldots \alpha_{i-1} \beta \alpha_{i+1} \ldots \alpha_n}.$$

$$2\overline{\nabla_\gamma \nabla_\beta \nabla_\alpha \sigma} + R_{\alpha\delta\beta\gamma} \overline{\nabla^\delta \sigma} = 0$$

$$\overline{\nabla_\mu \nabla_\nu \nabla_\rho \sigma} = 0.$$

$$\overline{\nabla_\mu \nabla_\nu \nabla_\rho \nabla_\sigma \sigma} = -\frac{1}{3}\left(R_{\mu\rho\nu\sigma} + R_{\mu\sigma\nu\rho}\right).$$

$$\bar{\sigma} = \overline{\nabla_\mu \sigma} = \overline{\nabla_\mu \nabla_\nu \nabla_\rho \sigma} = 0$$
$$\overline{\nabla_\mu \nabla_\nu \sigma} = g_{\mu\nu}$$

$$\overline{\nabla_\mu \nabla_\nu \nabla_\rho \nabla_\sigma \sigma} = -\frac{1}{3}\left(R_{\mu\rho\nu\sigma} + R_{\mu\sigma\nu\rho}\right).$$

$$\overline{\nabla^2 \sigma} = d, \overline{\nabla^2 \nabla^2 \sigma} = -\frac{2}{3} R$$

$$\overline{\nabla^2 A_0} = \frac{1}{6} R$$

$$\overline{A_1} = \frac{1}{6} R$$

$$\bar{\Omega}(s) \sim 1 + \frac{1}{6} sR + \mathcal{O}(s^2)$$

$$\frac{1}{2}\ \mathrm{STr} W(\Delta)$$

$$W(x) = \int_0^\infty \mathrm{d}s \tilde{W}(s) e^{-sx}$$

$$\frac{1}{2}\ \mathrm{STr} W(\Delta) = \frac{1}{2}\ \mathrm{STr} \int_0^\infty \mathrm{d}s \tilde{W}(s) e^{-s\Delta}$$

$$\frac{1}{2}\ \mathrm{STr} W(\Delta) \sim \frac{1}{2} \int \mathrm{d}^d x \sqrt{g} \int_0^\infty \mathrm{d}s \tilde{W}(s) \left(\frac{1}{4\pi s}\right)^{d/2} \left[1 + \frac{1}{6} sR + \mathcal{O}(s^2)\right]$$

$$\int_0^\infty \mathrm{d}s \tilde{W}(s) s^{-n} = \frac{1}{\Gamma(n)} \int_0^\infty \mathrm{d}z z^{n-1} W(z), n > 0$$

$$\int_0^\infty \mathrm{d}s \tilde{W}(s) s^{n} = (-1)^n W^{(n)}(0), n \geq 0$$

$$\frac{1}{\Gamma(n)}\int_0^\infty \mathrm{d}z z^{n-1} W(z) = \frac{1}{\Gamma(n)}\int_0^\infty \mathrm{d}z z^{n-1}\int_0^\infty \mathrm{d}s \tilde{W}(s) e^{-sz}$$

$$= \int_0^\infty \mathrm{d}s \tilde{W}(s) \frac{1}{\Gamma(n)}\int_0^\infty \mathrm{d}z z^{n-1} e^{-sz}$$

$$= \int_0^\infty \mathrm{d}s \tilde{W}(s) \frac{1}{\Gamma(n)}(-\partial_s)^{n-1}\int_0^\infty \mathrm{d}z e^{-sz}$$

$$= \int_0^\infty \mathrm{d}s \tilde{W}(s) \frac{1}{\Gamma(n)}(-\partial_s)^{n-1}\frac{1}{s}$$

$$= \int_0^\infty \mathrm{d}s \tilde{W}(s) s^{-n}$$

$$\int_0^\infty \mathrm{d}s \tilde{W}(s) s^{n} = \left[\int_0^\infty \mathrm{d}s \tilde{W}(s) s^{n} e^{-sz}\right]\Bigg|_{z=0}$$

$$= \left[(-\partial_z)^n \int_0^\infty \mathrm{d}s \tilde{W}(s) e^{-sz}\right]\Bigg|_{z=0}$$

$$= [(-\partial_z)^n W(z)]|_{z=0} = (-1)^n W^{(n)}(0)$$

$$\frac{1}{2} S\mathrm{Tr} W(\Delta) \sim \frac{1}{2}\frac{1}{(4\pi)^{d/2}}\int \mathrm{d}^d x\sqrt{g}\left[\frac{1}{\Gamma\left(\frac{d}{2}\right)}\int_0^\infty \mathrm{d}z z^{\frac{d}{2}-1} W(z)\right.$$

$$\left. + \frac{1}{6}\frac{1}{\Gamma\left(\frac{d}{2}-1\right)} R \int_0^\infty \mathrm{d}z z^{\frac{d}{2}-2} W(z) + \cdots\right]$$

$$\int_0^\infty \mathrm{d}z z^{\frac{d}{2}-1} W(z) = \frac{4}{d} k^d, \int_0^\infty \mathrm{d}z z^{\frac{d}{2}-2} W(z) = \frac{4}{d-2} k^{d-2}$$

$$\Gamma_k = \int \mathrm{d}\tau \left(\frac{1}{2}\dot{x}^2 + V_k(x)\right)$$

$$k\partial_k \Gamma_k \equiv \int \mathrm{d}\tau k \partial_k V_k$$

$$\Gamma_k^{(2)} = (-\partial_\tau^2 + V_k''(x))\delta(\tau - \tau')$$

$$k\partial_k \mathcal{R}_k^{\mathrm{Litim}} = 2k^2\theta(1 - p^2/k^2) - 2\frac{p^2}{k^2}(k^2 - p^2)\delta(1 - p^2/k^2)$$

$$k\partial_k V_k = \frac{1}{2}\int_{-\infty}^{+\infty}\frac{\mathrm{d}p_\tau}{2\pi}\frac{2k^2\theta(1 - p_\tau^2/k^2)}{k^2 + V_k''} = \frac{1}{\pi}\frac{k^3}{k^2 + V_k''}$$

$$V_k = E_k + \frac{1}{2!}\omega_k x^2 + \frac{1}{4!}\lambda_k x^4$$

$$\beta_E + \frac{1}{2!}\beta_\omega x^2 + \frac{1}{4!}\beta_\lambda x^4 = \frac{1}{\pi}\frac{k^3}{k^2 + \omega_k + \lambda_k x^2/2}$$

$$\partial_k E_k = \frac{1}{\pi}\frac{k^2}{k^2+\omega_k}$$
$$\partial_k \omega_k = -\frac{2}{\pi}\frac{k^2}{(k^2+\omega_k)^2}\frac{\lambda_k}{2}$$
$$\partial_k \lambda_k = \frac{24}{\pi}\frac{k^2}{(k^2+\omega_k)^3}\left(\frac{\lambda_k}{2}\right)^2$$

$$\Gamma_k \simeq \frac{1}{16\pi G_k}\int d^4x\sqrt{g}[2\Lambda_k - R]$$

$$\mathfrak{L}_v g_{\mu\nu} = \nabla_\mu v_\nu + \nabla_\nu v_\mu.$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$g_{\mu\nu} = \bar{g}_{\mu\rho}\exp[\bar{g}^{-1}h]^{\rho}_{\nu}$$

$$\bar{g}_{\mu\nu} \to \bar{g}_{\mu\nu},$$
$$h_{\mu\nu} \to h_{\mu\nu} + \mathfrak{L}_v(\bar{g}_{\mu\nu} + h_{\mu\nu}).$$

$$\bar{g}_{\mu\nu} \to \bar{g}_{\mu\nu} + \mathfrak{L}_v\bar{g}_{\mu\nu},$$
$$h_{\mu\nu} \to h_{\mu\nu} + \mathfrak{L}_v h_{\mu\nu}.$$

$$\mathcal{N}_k = \frac{\delta\Gamma_k}{\delta\bar{g}_{\mu\nu}} - \frac{\delta\Gamma_k}{\delta h_{\mu\nu}} - \left\langle\left[\frac{\delta}{\delta\bar{g}_{\mu\nu}} - \frac{\delta}{\delta h_{\mu\nu}}\right](S_{\mathrm{gf}} + S_{\mathrm{gh}})\right\rangle - \frac{1}{2}\mathrm{STr}\left[\frac{1}{\sqrt{\bar{g}}}\frac{\delta\sqrt{\bar{g}}\mathcal{R}_k}{\delta\bar{g}_{\mu\nu}}\mathcal{G}_k\right] = 0$$

$$S_{\mathrm{gf}} = \frac{1}{2}\int d^4x\sqrt{\bar{g}}\mathcal{F}_\mu\bar{g}^{\mu\nu}\mathcal{F}_\nu$$

$$\mathcal{F}_\mu \equiv \mathfrak{F}_\mu^{\alpha\beta}h_{\alpha\beta} = \frac{1}{\sqrt{16\pi G_k\alpha_k}}\left[\delta_\mu^{(\alpha}\bar{\nabla}^{\beta)} - \frac{1+\beta_k}{4}\bar{g}^{\alpha\beta}\bar{\nabla}_\mu\right]h_{\alpha\beta}.$$

$$S_{\mathrm{gh}} = \int d^4x\sqrt{\bar{g}}\bar{c}^\mu\mathfrak{F}_\mu^{\alpha\beta}\mathfrak{L}_c g_{\alpha\beta} = \int d^4x\sqrt{\bar{g}}\bar{c}^\mu\mathfrak{F}_\mu^{\alpha\beta}(\nabla_\alpha c_\beta + \nabla_\beta c_\alpha)$$

$$\Gamma_k \simeq \frac{1}{16\pi G_k}\int d^4x\sqrt{g}[2\Lambda_k - R] + \frac{1}{2}\int d^4x\sqrt{\bar{g}}\mathcal{F}_\mu\bar{g}^{\mu\nu}\mathcal{F}_\nu + \int d^4x\sqrt{\bar{g}}\bar{c}^\mu\mathfrak{F}_\mu^{\alpha\beta}\mathfrak{L}_c g_{\alpha\beta}$$

$$\Delta S_k = \frac{1}{2}\int d^4x\sqrt{\bar{g}}h_{\mu\nu}\mathcal{R}_k^{h,\mu\nu\rho\sigma}h_{\rho\sigma} + \int d^4x\sqrt{\bar{g}}\bar{c}_\mu\mathcal{R}_k^{c,\mu\nu}c_\nu$$

$$k\partial_k\Gamma_k = \frac{1}{2}\mathrm{STr}\left[\begin{pmatrix} \frac{\delta^2\Gamma_k}{\delta h^2} + \mathcal{R}_k^h & \frac{\delta^2\Gamma_k}{\delta h\delta\bar{c}} & \frac{\delta^2\Gamma_k}{\delta h\delta c} \\ \frac{\delta^2\Gamma_k}{\delta\bar{c}\delta h} & \frac{\delta^2\Gamma_k}{\delta\bar{c}^2} & \frac{\delta^2\Gamma_k}{\delta\bar{c}\delta c} + \mathcal{R}_k^c \\ \frac{\delta^2\Gamma_k}{\delta c\delta h} & \frac{\delta^2\Gamma_k}{\delta c\delta\bar{c}} + \mathcal{R}_k^c & \frac{\delta^2\Gamma_k}{\delta c^2} \end{pmatrix}^{-1} k\partial_k\begin{pmatrix} \mathcal{R}_k^h & 0 & 0 \\ 0 & 0 & \mathcal{R}_k^c \\ 0 & \mathcal{R}_k^c & 0 \end{pmatrix}\right]$$

$$k\partial_k\Gamma_k \simeq \frac{1}{2}\mathrm{STr}\left[\left(\frac{\delta^2\Gamma_k}{\delta h^2} + \mathcal{R}_k^h\right)^{-1}k\partial_k\mathcal{R}_k^h\right] + \mathrm{STr}\left[\left(\frac{\delta^2\Gamma_k}{\delta\bar{c}\delta c} + \mathcal{R}_k^c\right)^{-1}k\partial_k\mathcal{R}_k^c\right]\Bigg|_{h=\bar{c}=c=0}$$

$$S_{\mathrm{gh}} \simeq \frac{1}{\sqrt{16\pi G_k \alpha_k}} \int \mathrm{d}^4 x \sqrt{\bar{g}} \bar{c}^{\mu} \left[\delta_{\mu}^{(\alpha \bar{\nabla}^{\beta})} - \frac{1+\beta_k}{4} \bar{g}^{\alpha\beta} \bar{\nabla}_{\mu} \right] (\bar{\nabla}_{\alpha} c_{\beta} + \bar{\nabla}_{\beta} c_{\alpha})$$
$$= \frac{1}{\sqrt{16\pi G_k \alpha_k}} \int \mathrm{d}^4 x \sqrt{\bar{g}} \bar{c}^{\mu} \left[\bar{\nabla}^2 \delta_{\mu}^{\alpha} + \bar{\nabla}^{\alpha} \bar{\nabla}_{\mu} - \frac{1+\beta_k}{2} \bar{\nabla}_{\mu} \bar{\nabla}^{\alpha} \right] c_{\alpha}$$
$$= \frac{1}{\sqrt{16\pi G_k \alpha_k}} \int \mathrm{d}^4 x \sqrt{\bar{g}} \bar{c}^{\mu} \left[\bar{\nabla}^2 \delta_{\mu}^{\alpha} + \frac{1-\beta_k}{2} \bar{\nabla}_{\mu} \bar{\nabla}^{\alpha} + \bar{R}_{\mu}^{\alpha} \right] c_{\alpha}$$

$$S_{\mathrm{gh}} \simeq -\frac{1}{\sqrt{16\pi G_k \alpha_k}} \int \mathrm{d}^4 x \sqrt{\bar{g}} \bar{c}^{\mu} [\bar{\Delta} \delta_{\mu}^{\alpha} - \bar{R}_{\mu}^{\alpha}] c_{\alpha}$$

$$\Delta S_k^{\mathrm{gh}} = -\frac{1}{\sqrt{16\pi G_k \alpha_k}} \int \mathrm{d}^4 x \sqrt{\bar{g}} \bar{c}^{\mu} R_k^c (\bar{\Delta} \mathbb{1} - \overline{\mathrm{Ric}})_{\mu}^{\alpha} c_{\alpha}$$

$$R_k^c (\bar{\Delta} \mathbb{1} - \overline{\mathrm{Ric}})_{\mu}^{\alpha} = \int_0^{\infty} \mathrm{d}s \tilde{R}(s) (\exp[-s(\bar{\Delta}\mathbb{1} - \overline{\mathrm{Ric}})])_{\mu}^{\alpha}$$

$$-\mathrm{STr} W(\bar{\Delta}\mathbb{1} - \overline{\mathrm{Ric}}) = -\int_0^{\infty} \mathrm{d}s \tilde{W}(s) \mathrm{STr} e^{-s(\bar{\Delta}\mathbb{1} - \overline{\mathrm{Ric}})}$$

$$-\frac{1}{16\pi^2} \int \mathrm{d}^4 x \sqrt{\bar{g}} \Bigg[4 \int_0^{\infty} \mathrm{d}z z \frac{k \partial_k R_k^c(z) - \frac{1}{2} \frac{k \partial_k (G_k \alpha_k)}{G_k \alpha_k} R_k^c(z)}{z + R_k^c(z)}$$
$$+ \frac{5}{3} \bar{R} \int_0^{\infty} \mathrm{d}z \frac{k \partial_k R_k^c(z) - \frac{1}{2} \frac{k \partial_k (G_k \alpha_k)}{G_k \alpha_k} R_k^c(z)}{z + R_k^c(z)} + \cdots \Bigg]$$

$$g = \bar{g} + h = \bar{g}(\mathbb{1} + \bar{g}^{-1} h)$$

$$\begin{aligned}
\det g = \det(\bar{g} + h) &= \det[\bar{g}(\mathbb{1} + \bar{g}^{-1} h)] \\
&= (\det \bar{g}) \det[\mathbb{1} + \bar{g}^{-1} h] \\
&= (\det \bar{g}) \exp[\mathrm{tr} \ln(\mathbb{1} + \bar{g}^{-1} h)] \\
&= (\det \bar{g}) \exp\left[\mathrm{tr} \sum_{n \geq 1} -\frac{(-1)^n}{n} (\bar{g}^{-1} h)^n \right] \\
&= (\det \bar{g}) \exp\left[-\sum_{n \geq 1} \frac{(-1)^n}{n} \mathrm{tr}\{(\bar{g}^{-1} h)^n\} \right] \\
&\simeq (\det \bar{g}) \left[1 + h^{\alpha}{}_{\alpha} + \frac{1}{2} h^{\alpha}{}_{\alpha} h^{\beta}{}_{\beta} - \frac{1}{2} h^{\alpha\beta} h_{\alpha\beta} + \cdots \right]
\end{aligned}$$

$$\sqrt{\det g} \simeq \sqrt{\det \bar{g}} \left[1 + \frac{1}{2} h_{\alpha}^{\alpha} + \frac{1}{8} h_{\alpha}^{\alpha} h_{\beta}^{\beta} - \frac{1}{4} h^{\alpha\beta} h_{\alpha\beta} + \cdots \right].$$

$$R = g^{\mu\nu} R_{\mu\nu}$$

$$g^{-1} = (\mathbb{1} + \bar{g}^{-1} h)^{-1} \bar{g}^{-1}$$

$$g^{\mu\nu} \simeq \bar{g}^{\mu\nu} - h^{\mu\nu} + h_{\alpha}^{\mu} h^{\alpha\nu} + \cdots$$

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu}(\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta}).$$

$$\begin{aligned}
\Gamma_{\nu\alpha\beta} &= \frac{1}{2}(\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta}) \\
&= \frac{1}{2}(\partial_\alpha \bar{g}_{\nu\beta} + \partial_\beta \bar{g}_{\nu\alpha} - \partial_\nu \bar{g}_{\alpha\beta}) + \frac{1}{2}(\partial_\alpha h_{\nu\beta} + \partial_\beta h_{\nu\alpha} - \partial_\nu h_{\alpha\beta}) \\
&= \bar{\Gamma}_{\nu\alpha\beta} + \frac{1}{2}(\bar{\nabla}_\alpha h_{\nu\beta} + \bar{\nabla}_\beta h_{\nu\alpha} - \bar{\nabla}_\nu h_{\alpha\beta}) + \bar{\Gamma}^{\mu}_{\alpha\beta} h_{\mu\nu} \\
&= \bar{\Gamma}^{\mu}_{\alpha\beta}(\bar{g}_{\mu\nu} + h_{\mu\nu}) + \frac{1}{2}(\bar{\nabla}_\alpha h_{\nu\beta} + \bar{\nabla}_\beta h_{\nu\alpha} - \bar{\nabla}_\nu h_{\alpha\beta}) \\
&= \bar{\Gamma}^{\mu}_{\alpha\beta} g_{\mu\nu} + \frac{1}{2}(\bar{\nabla}_\alpha h_{\nu\beta} + \bar{\nabla}_\beta h_{\nu\alpha} - \bar{\nabla}_\nu h_{\alpha\beta})
\end{aligned}$$

$$\Gamma^{\mu}_{\alpha\beta} = \bar{\Gamma}^{\mu}_{\alpha\beta} + \frac{1}{2} g^{\mu\nu}(\bar{\nabla}_\alpha h_{\nu\beta} + \bar{\nabla}_\beta h_{\nu\alpha} - \bar{\nabla}_\nu h_{\alpha\beta}).$$

$$\begin{aligned}
\Gamma_k^{h^2} = \frac{1}{32\pi G_k}\int \mathrm{d}^4x\sqrt{\bar{g}}\,h_{\mu\nu}\Big[&\left(\bar{\Delta} - 2\Lambda_k + \frac{2}{3}\bar{R}\right)\mathbb{1}^{\mu\nu\rho\sigma} - 2\bar{C}^{\mu\rho\nu\sigma} \\
&-\left(\left\{1 - \frac{(1+\beta_k)^2}{8\alpha_k}\right\}\bar{\Delta} - \Lambda_k + \frac{1}{6}\bar{R}\right)\bar{g}^{\mu\nu}\bar{g}^{\rho\sigma} \\
&+\frac{1 - 2\alpha_k + \beta_k}{\alpha_k}\bar{g}^{\mu\nu}\bar{\nabla}^\rho\bar{\nabla}^\sigma + 2\left(1 - \frac{1}{\alpha_k}\right)\bar{g}^{\mu\rho}\bar{\nabla}^\nu\bar{\nabla}^\sigma\Big] h_{\rho\sigma}
\end{aligned}$$

$$\mathbb{1}^{\mu\nu\rho\sigma} = \frac{1}{2}(\bar{g}^{\mu\rho}\bar{g}^{\nu\sigma} + \bar{g}^{\mu\sigma}\bar{g}^{\nu\rho})$$

$$\Pi^{\mathrm{Tr}\mu\nu\rho\sigma} = \frac{1}{4}\bar{g}^{\mu\nu}\bar{g}^{\rho\sigma}, \Pi^{\mathrm{TL}\mu\nu\rho\sigma} = \mathbb{1}^{\mu\nu\rho\sigma} - \Pi^{\mathrm{Tr}\mu\nu\rho\sigma}$$

$$\begin{aligned}
\Gamma_k^{h^2} = \frac{1}{32\pi G_k}\int \mathrm{d}^4x\sqrt{\bar{g}}\,h_{\mu\nu}\Big[&\left\{\bar{\Delta} + \frac{2}{3}\bar{R} - 2\mathbb{C} - 2\Lambda_k\right\}\Pi^{\mathrm{TL}} \\
&-\{\bar{\Delta} - 2\Lambda_k\}\Pi^{\mathrm{Tr}}\Big]^{\mu\nu\rho\sigma} h_{\rho\sigma}
\end{aligned}$$

$$\bar{\Delta}_2 = \left\{\bar{\Delta} + \frac{2}{3}\bar{R} - 2\mathbb{C}\right\}\Pi^{\mathrm{TL}}$$

$$\Delta S_k^h = \frac{1}{32\pi G_k}\int \mathrm{d}^4x\sqrt{\bar{g}}\,h_{\mu\nu}[R_k^h(\bar{\Delta}_2)\Pi^{\mathrm{TL}} - R_k^h(\bar{\Delta})\Pi^{\mathrm{Tr}}]h_{\rho\sigma}$$

$$\frac{1}{2}\mathrm{STr}_{\mathrm{TL}}W(\bar{\Delta}_2) + \frac{1}{2}\mathrm{STr}_{\mathrm{Tr}}W(\bar{\Delta})$$

$$\begin{aligned}
\frac{1}{32\pi^2}\int \mathrm{d}^4x\sqrt{\bar{g}}\Big[&10\int_0^\infty \mathrm{d}z\,z\,\frac{k\partial_k R_k^h(z) - \frac{k\partial_k G_k}{G_k}R_k^h(z)}{z + R_k^h(z) - 2\Lambda_k} \\
&-\frac{13}{3}\bar{R}\int_0^\infty \mathrm{d}z\,\frac{k\partial_k R_k^h(z) - \frac{k\partial_k G_k}{G_k}R_k^h(z)}{z + R_k^h(z) - 2\Lambda_k} + \cdots\Big]
\end{aligned}$$

$$k\partial_k\Gamma_k = \int d^4x\sqrt{\bar{g}}\left[\frac{k\partial_k\Lambda_k - \frac{k\partial_k G_k}{G_k}\Lambda_k}{8\pi G_k} + \frac{k\partial_k G_k}{16\pi G_k^2}\bar{R}\right]$$

$$g = G_k k^2, \lambda = \Lambda_k k^{-2}$$

$$\beta_g = k\partial_k g = (k\partial_k G_k + 2G_k)k^2, \beta_\lambda = k\partial_k\lambda = (k\partial_k\Lambda_k - 2\Lambda_k)k^{-2}$$

$$\beta_g = 2g\left(1 - g\frac{23 - 20\lambda}{6\pi(1-2\lambda) + g(-9+5\lambda)}\right)$$

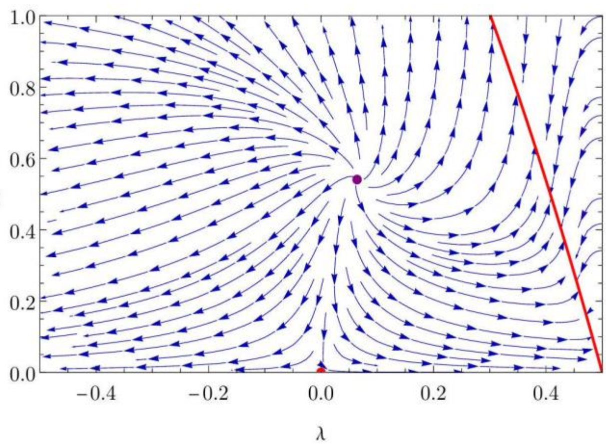
$$\beta_\lambda = \left(-4 + \frac{\beta_g}{g}\right)\lambda + \frac{5g}{12\pi}\frac{8 - \frac{\beta_g}{g}}{1-2\lambda} - \frac{7g}{3\pi}\left(1 - \frac{1}{14}\frac{\beta_g}{g}\right)$$

$$\eta_N \equiv -2g\frac{23 - 20\lambda}{6\pi(1-2\lambda) + g(-9+5\lambda)}$$

$$g_* = \lambda_* = 0$$

$$\theta_1 = 2, \theta_2 = -2$$

$$g_* = \frac{6\pi}{2615}(39\sqrt{19} - 95) \approx 0.541, \lambda_* = \frac{5 - \sqrt{19}}{10} \approx 0.064$$



$$\theta_{1,2} \approx 2.667 \pm 0.958i$$

$$g_* = -\frac{6\pi}{2615}(39\sqrt{19} + 95) \approx -1.910, \lambda_* = \frac{5 + \sqrt{19}}{10} \approx 0.936,$$

$$\theta_1 \approx 7.451, \theta_2 \approx -5.465.$$

$$\Gamma_{\Lambda_{UV}} = S_{\Lambda_{UV}} + \frac{1}{2}\mathrm{STr}_{\Lambda_{UV}}\ln\left(S^{(2)}_{\Lambda_{UV}} + R_{\Lambda_{UV}}\right),$$

$$\lim_{\Lambda_{UV}\to\infty}\Gamma_* = S_{\text{bare}} + ƛ$$

$$\frac{1}{2}\phi f(-\nabla^2)\phi.$$

$$\frac{1}{16\pi G_N}\int d^4x\sqrt{g}(2\Lambda - R) \mapsto \frac{1}{16\pi}\int d^4x\sqrt{g}\frac{1}{G_N(\Delta)}(2\Lambda(\Delta) - R)$$
$$= \frac{1}{16\pi}\int d^4x\sqrt{g}\frac{1}{G_N(0)}(2\Lambda(0) - R)$$

$$Rf_R(\Delta)R + R_{\mu\nu}f_{Ric}(\Delta)R^{\mu\nu}$$

$$\nabla^2 R_{\mu\nu\rho\sigma} = R_\nu{}^\alpha R_{\mu\alpha\rho\sigma} - R_\mu{}^\alpha R_{\nu\alpha\rho\sigma} + 2R_\mu{}^\alpha{}_\sigma{}^\beta R_{\nu\alpha\rho\beta} - 2R_\mu{}^\alpha{}_\rho{}^\beta R_{\nu\alpha\sigma\beta} - 2R_\mu{}^\alpha{}_\nu{}^\beta R_{\rho\sigma\alpha\beta}$$
$$+\nabla_\mu\nabla_\rho R_{\nu\sigma} - \nabla_\mu\nabla_\sigma R_{\nu\rho} - \nabla_\nu\nabla_\rho R_{\mu\sigma} + \nabla_\nu\nabla_\sigma R_{\mu\rho}.$$

$$\nabla_{[\alpha}R_{\mu\nu]\rho\sigma} = 0$$

$$\Gamma[g_{\mu\nu}] = \int d^4x\sqrt{-g}\left[\frac{-2\Lambda + R}{16\pi G_N} + \frac{1}{6}RF_R(\Box)R - \frac{1}{2}C^{\mu\nu\rho\sigma}F_C(\Box)C_{\mu\nu\rho\sigma} + \cdots\right],$$

$$\mathcal{G}(p) \simeq \frac{1}{p^2\left(1 + p^2F(p^2)\right)}$$

$$\Gamma \simeq \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G_N} + \frac{1}{6}RF_R(\Box)R - \frac{1}{2}C^{\mu\nu\rho\sigma}F_C(\Box)C_{\mu\nu\rho\sigma}\right.$$
$$\left.-\frac{1}{2}(\nabla_\mu\phi)(\nabla^\mu\phi) - \frac{1}{2}(\nabla_\mu\chi)(\nabla^\mu\chi)\right].$$

$$\mathcal{A}_s(s,t) = \frac{4\pi G_N}{3}s^2\left[\frac{s^2 + 6t(s+t)}{s^2}\frac{1}{s(1 + sF_C(s))} - \frac{1}{s(1 + sF_R(s))}\right]$$

$$\mathcal{A}_t(s,t) = \mathcal{A}_s(t,s) = \frac{4\pi G_N}{3}t^2\left[\frac{t^2 + 6s(s+t)}{t^2}\frac{1}{t(1 + tF_C(t))} - \frac{1}{t(1 + tF_R(t))}\right]$$

$$\beta_g = 2g - g^2\left(a_{QG} + a_S N_S + a_F N_F + a_V N_V\right) + \mathcal{O}(g^3)$$

$$g_* = \frac{2}{a_{QG} + a_S N_S + a_F N_F + a_V N_V}.$$

$$\beta_c = -f_c c + \beta_{c,1}c^n + \mathcal{O}(c^{n+1})$$

$$\beta_{g_Y} = b_Y g_Y^3 - f_Y g_Y,$$

$$\beta_{\lambda_H} = \frac{3}{2\pi^2}\lambda_H^2 + \tau_H(g_2, g_Y, y_t) + \kappa_H(g_2, g_Y, y_t)\lambda_H - f_H\lambda_H + \mathcal{O}(\lambda_H^3)$$

$$\lambda_H = \frac{1}{2}\left(\frac{m_H}{v}\right)^2$$

$$\mathcal{P}_s(k) \simeq A_s\left(\frac{k}{k_*}\right)^{n_s - 1}, \mathcal{P}_t(k) \simeq A_t\left(\frac{k}{k_*}\right)^{n_t},$$

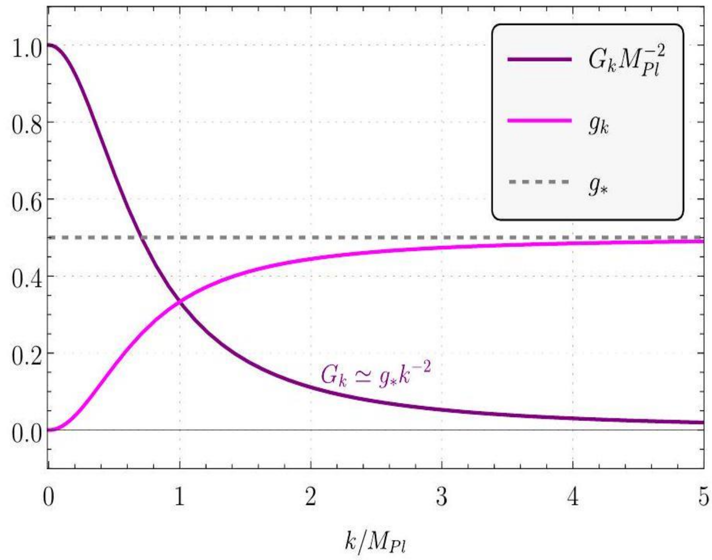
$$\mathcal{G}(p) \simeq \frac{1}{p^{2-\eta_N}}.$$

$$\mathcal{G}(x,y) \simeq \log|x - y|^2$$

$$\xi(\vec{x}) = \langle\delta\rho(\vec{x}+\vec{y})\delta\rho(\vec{y})\rangle \propto \langle\delta R(\vec{x}+\vec{y},t)\delta R(\vec{y},t)\rangle \simeq |\vec{x}|^{-4},$$

$$\left|\delta_{\vec{k}}\right|^2 = V\int \mathrm{d}^3\vec{x}\xi(\vec{x})e^{-i\vec{k}\cdot\vec{x}}$$

$$\left|\delta_{\vec{k}}\right|^2 \propto |\vec{k}|^{n_s}.$$



$$G_k \simeq \frac{G_N}{1+g_*^{-1}G_N k^2}.$$

$$f(r) = 1 - \frac{2G_N M}{r}.$$

$$\frac{\delta\Gamma}{\delta g_{\mu\nu}} = 0.$$

$$S_{\text{eff}} \sim \int \mathrm{d}^d x\sqrt{-g}\left(\mathcal{L}_{\text{stuff}} + \frac{M_{\text{Pl}}^{d-2}}{2}R + M_{\text{Pl}}^{d-2}\sum_k c_k\frac{\mathcal{O}_k(\nabla,\mathcal{R},\ldots)}{\Lambda_{\text{UV}}^{k-2}}\right),$$

$$\Lambda_{\text{sp}} = \frac{M_{\text{Pl}}}{N(\Lambda_{\text{sp}})^{\frac{1}{d-2}}}.$$

$$\Lambda_{\text{UV}} \lesssim \Lambda_{\text{sc}} \lesssim \Lambda_{\text{sp}} \lesssim \frac{M_{\text{Pl}}}{N_{\text{low-energy}}^{\frac{1}{d-2}}} \leq M_{\text{Pl}}$$

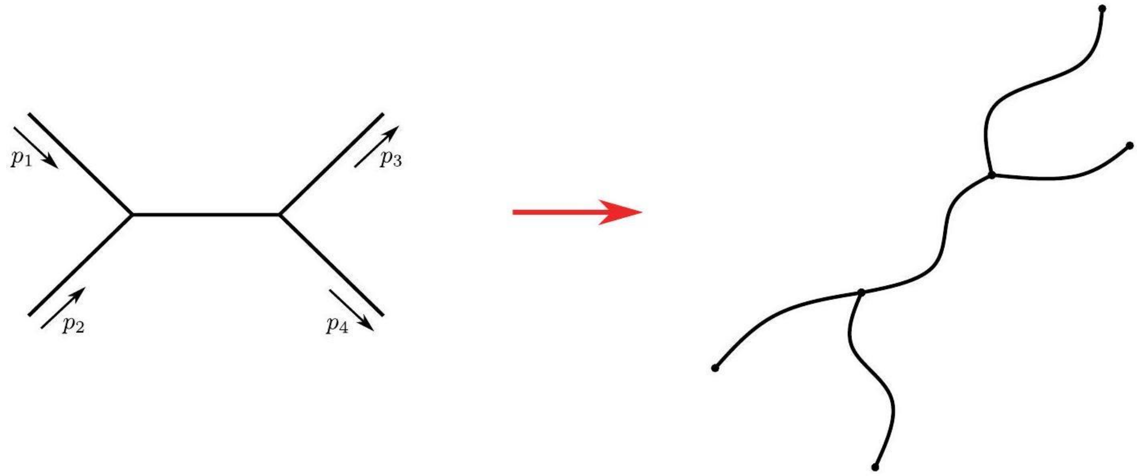
$$S_{\text{BH}} = \frac{\text{Area}(R_{\text{BH}})}{4G_N} + \alpha\ln\frac{\text{Area}(R_{\text{BH}})}{G_N} + \cdots,$$

$$s \equiv -(p_1+p_2)^2, t \equiv -(p_1+p_3)^2, u \equiv -(p_1+p_4)^2.$$

$$\mathcal{A} = \mathbf{K}F(s,t,u),$$

$$\mathcal{A}_{\text{tree}}^{\text{GR}} = \mathbf{K}\frac{G_N}{stu}.$$

$$\mathcal{A}_{\text{tree}}^{\text{UV}} = \mathbf{K}\frac{G_N}{stu}C\left(\frac{s}{\Lambda_{\text{UV}}^2},\frac{t}{\Lambda_{\text{UV}}^2},\frac{u}{\Lambda_{\text{UV}}^2}\right)$$



$$\tilde{g}_{\tau\tau} = \dot{X}^{\mu}\dot{X}^{\nu}g_{\mu\nu}(X(\tau)) \equiv \dot{X}^2,$$

$$S_{\text{wl}}^{\text{tentative}}[X, \dots] = -m\int_W \mathrm{d}\ell + S_{\text{other stuff}}\ .$$

$$S_{\text{wl}}[X,\gamma,\dots] = -\frac{1}{2}\int \mathrm{d}\tau\sqrt{-\gamma_{\tau\tau}}\left(\gamma^{\tau\tau}\dot{X}^{\mu}\dot{X}^{\nu}g_{\mu\nu}(X) + m^2\right) + S_{\text{other stuff}}$$

$$\int_{X(0)=x_i}^{X(T)=x_f}\frac{\mathcal{D}X\mathcal{D}\gamma}{\text{Diff}(W)}e^{-S_{\text{wl}}^E[X,\gamma]}$$

$$1 \equiv \Delta_{\text{FP}}[\gamma]\int \mathrm{d}t\int \mathcal{D}\xi\,\delta\left(\gamma - \hat{\gamma}(t)^{\xi}\right)$$

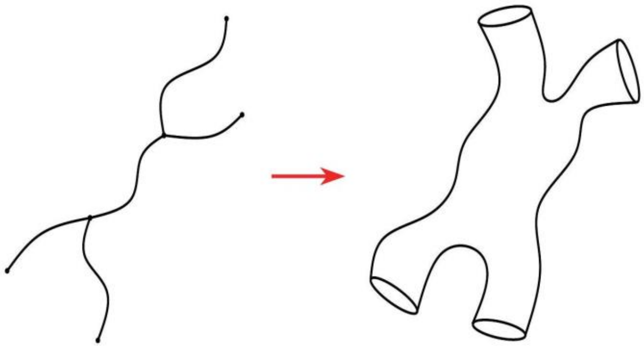
$$\int \mathrm{d}t\Delta_{\text{FP}}[\hat{\gamma}(t)]\int_{X(0)=x_i}^{X(T)=x_f}\mathcal{D}Xe^{-S_{\text{wl}}^E[X,\hat{\gamma}(t)]}$$

$$\det{}'(-\nabla^2) = \exp\left(-\frac{\mathrm{d}}{\mathrm{d}s}\left(\hat{e}^{2s}\zeta(2s)\right)\Big|_{s=0}\right) \propto \hat{e}$$

$$\left(\hat{e}^{\frac{3}{2}},\partial_t\hat{\gamma}\right) = \hat{e}^{-\frac{3}{2}}\partial_t\hat{\gamma}$$

$$\int \mathrm{d}t\frac{\partial_t\hat{\gamma}}{\hat{e}} \propto \int \mathrm{d}\hat{e} = \int \mathrm{d}T$$

$$\int \frac{\mathrm{d}^d p}{(2\pi)^d}e^{ip\cdot(x_f-x_i)}e^{-T(p^2+m^2)}$$

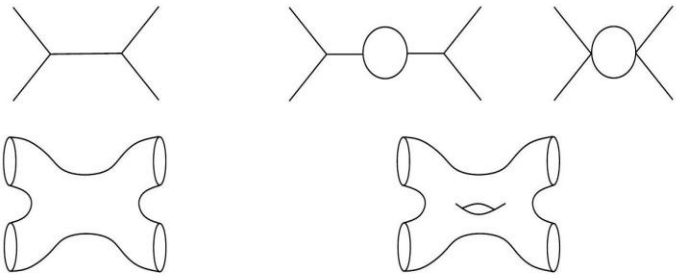


$$S_{\mathrm{NG}} = -T\int_{\Sigma} \mathrm{d}A$$

$$S_{\mathrm{ws}} = -\frac{1}{4\pi\alpha'}\int_{\Sigma} \mathrm{d}^2\sigma\sqrt{-\gamma}\left[\gamma^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}(X)\right] + S_{\text{other stuff}}$$

$$S_{\mathrm{P}} = -\frac{1}{4\pi\alpha'}\int_{\Sigma} \mathrm{d}^2\sigma\sqrt{-\gamma}\left[\left(\gamma^{\alpha\beta}g_{\mu\nu} + \epsilon^{\alpha\beta}B_{\mu\nu}\right)\partial_\alpha X^\mu \partial_\beta X^\nu + \alpha'\mathrm{Ric}(\gamma)\phi\right],$$

$$\gamma \mapsto \Omega^2(\sigma)\gamma.$$



$$\mathrm{MCG}(\Sigma) \equiv \frac{\mathrm{Diff}(\Sigma)}{\mathrm{Diff}_0(\Sigma)}$$

$$\sum_{\text{genus } g}\int_{\mathcal{M}_g}\mathrm{d}\mu(t)\int_{\Sigma_g(t)}\sum_{\text{spin structure } s} C_s \mathcal{D}X\mathcal{D}\psi e^{-S^E_{\mathrm{ws}}}$$

$$\ln\frac{\mathcal{Z}[e^{2\omega}\delta]}{\mathcal{Z}[\delta]} \propto c\int \mathrm{d}^2\sigma(\partial\omega)^2$$

$$c_{\text{spacetime}} = \left(d_{\text{bosonic}} \text{ or } \frac{3}{2}d_{\mathrm{RNS}}\right) + \mathcal{O}(\alpha'\mathcal{R}),$$

$$S_{\mathrm{P}}^{E}\Big|_{\text{dilaton}} = \frac{\phi_0}{4\pi}\int_{\Sigma} \mathrm{d}^2\sigma \mathrm{Ric}(\gamma) + \frac{1}{4\pi}\int_{\Sigma} \mathrm{d}^2\sigma \mathrm{Ric}(\gamma)\tilde{\phi}$$
$$= \chi(\Sigma) + \frac{1}{4\pi}\int_{\Sigma} \mathrm{d}^2\sigma \mathrm{Ric}(\gamma)\tilde{\phi}$$

$$\sum_{\text{genus } g} g_s^{2g-2}\int_{\mathcal{M}_g} \mathrm{d}\mu(t)$$

$$\mathcal{V}_{\delta g}^{\mathrm{wl}} \equiv \gamma^{\tau\tau}\delta g_{\mu\nu}(X)\dot{X}^{\mu}\dot{X}^{\nu}$$

$$\mathcal{V}_{\delta g}^{\mathrm{ws}} \equiv \gamma^{\alpha\beta}\delta g_{\mu\nu}(X)\partial_{\alpha}X^{\mu}\partial_{\beta}X^{\nu} + \mathfrak{F}$$

$$\int \mathrm{d}^2\sigma\sqrt{\hat{\gamma}}\mathcal{V},$$

$$\tilde{\mathcal{O}}(w,\bar{w}) = \left(\frac{\mathrm{d}w}{\mathrm{d}z}\right)^{-h}\left(\frac{\mathrm{d}\bar{w}}{\mathrm{d}\bar{z}}\right)^{-\bar{h}}\mathcal{O}(z,\bar{z}).$$

$$\mathcal{V}_p = Ve^{ip\cdot X},$$

$$h_V = \bar{h}_V = 1 - \frac{\alpha' p^2}{4}$$

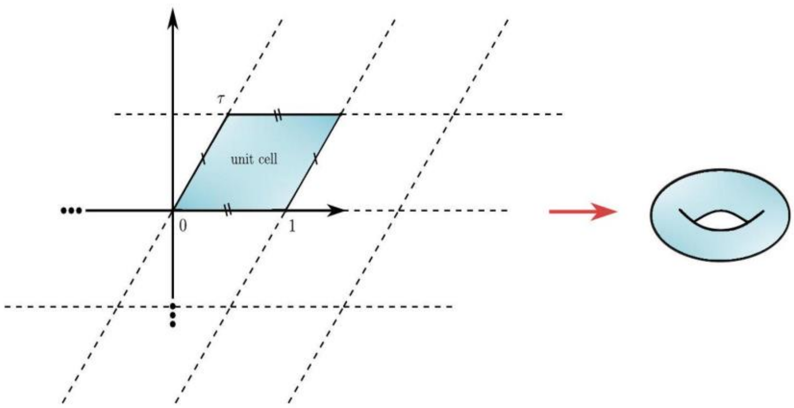
$$m^2 = \frac{4}{\alpha'}(h-1)$$

$$\{\tilde{\psi}_0^{\mu},\tilde{\psi}_0^{\nu}\} \propto \eta^{\mu\nu}$$

$$\mathcal{Z}_{T^2} = \mathrm{tr} q^{L_0 - \frac{c_{\mathrm{L}}}{24}}\bar{q}^{\overline{L_0} - \frac{c_{\mathrm{R}}}{24}}$$

$$\tau = \xi + i\frac{\beta}{2\pi} \equiv \tau_1 + i\tau_2$$

$$\mathcal{Z}_{T^2} = \sum_{\text{spin structures } s} C_s\mathcal{Z}_s = \sum_{s_{\mathrm{L}},s_{\mathrm{R}}} C_{s_{\mathrm{L}},s_{\mathrm{R}}}\mathcal{Z}_{s_{\mathrm{L}},s_{\mathrm{R}}}.$$



$$V = \zeta_{\mu\nu}(p)\partial X^{\mu}\bar{\partial}X^{\nu} + \mathfrak{F},$$

$$\begin{aligned}\mathcal{Z}_{T^2} &= \sum_{s_L,s_R\in\{(R,\pm),(NS,\pm)\}} C_{s_L,s_R}\mathrm{tr}^{(b_L)}_{\mathcal{H}_{a_L}} q^{L_0-\frac{c_L}{24}}\mathrm{tr}_{\mathcal{H}^{(b_R)}_{a_R}}\overline{\bar{q}L_0} - \frac{c_R}{24} \\ &\equiv \sum_{s_L,s_R\in\{(R,\pm),(NS,\pm)\}} C_{s_L,s_R}\mathcal{Z}_{s_L}\overline{\mathcal{Z}_{s_R}}\end{aligned}$$

$$\mathrm{Vol}(M)\int \frac{\mathrm{d}^d p}{(2\pi)^d} e^{-\pi\alpha' p^2\tau_2} = \frac{\mathrm{Vol}(M)}{(4\pi^2\alpha')^{\frac{d}{2}}}\tau_2^{-\frac{d}{2}},$$

$$\sum_{\text{natural tuples }\{n_k\}} q^{\sum_{k>0} k n_k} = \prod_{k>0}\sum_{n\geq 0} q^{kn} = \prod_{k>0}\frac{1}{1-q^k} \equiv \mathcal{Z}_{\text{boson}} \cdot$$

$$\mathcal{Z}_{\text{ghosts}} = \mathcal{Z}_{\text{boson}}^{-2}$$

$$\mathcal{Z}^{\text{single}}_{NS,+} = \sum_{\text{binary tuples }\{n_k\}} q^{\sum_{k>0}\left(k-\frac{1}{2}\right)n_k} = \prod_{k>0}\left(1+q^{k-\frac{1}{2}}\right)$$

$$\mathcal{Z}^{\text{single}}_{R,+} = \dim(R)\sum_{\text{binary tuples }\{n_k\}} q^{\sum_{k>0} k n_k} = \dim(R)\prod_{k>0}\left(1+q^k\right)$$

$$\mathcal{Z}^{\text{single}}_{NS,-} = \prod_{k>0}\left(1-q^{k-\frac{1}{2}}\right), \mathcal{Z}^{\text{single}}_{R,-} = 0$$

$$\mathcal{Z}^{\text{total}}_{R,+} = \prod_{k>0}\left(\frac{1+q^k}{1-q^k}\right)^{d-2}$$

$$d_k = \frac{1}{2\pi i}\oint_{\mathcal{C}_0}\frac{\mathrm{d}q}{q^{k+1}}\mathcal{Z}^{\text{total}}_{R,+}$$

$$\sum_{k>0}\ln\left(1\pm q^k\right) = -\sum_{n,k>0}\frac{\left(\mp q^k\right)^n}{n} = -\sum_{n>0}\frac{(\mp)^n}{n}\frac{q^n}{1-q^n}$$

$$\ln\, \mathcal{Z}^{\text{total}}_{R,+} = 2(d-2)\sum_{n\text{ odd}}\frac{1}{n}\frac{q^n}{1-q^n} \overset{q\to 1}{\sim} \frac{2(d-2)}{1-q}\sum_{n\text{ odd}}\frac{1}{n^2} = \frac{\pi^2}{4}\frac{d-2}{1-q}$$

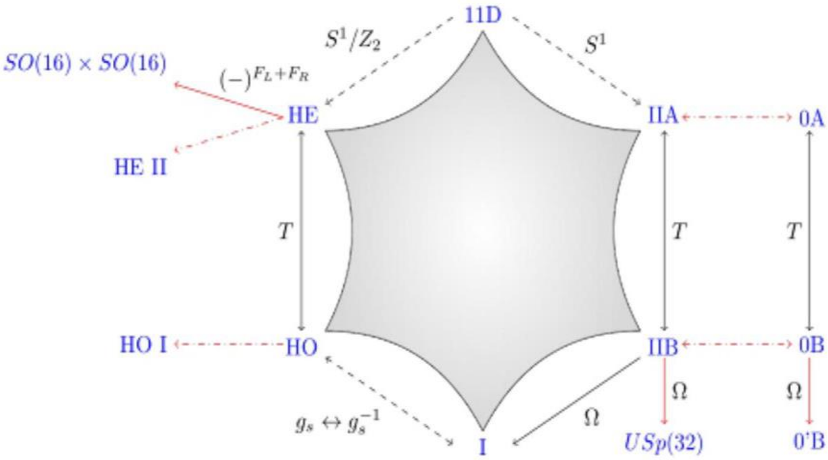
$$1 - q_* \overset{k\gg 1}{} \sqrt{\frac{\pi^2(d-2)}{4k}}$$

$$\ln\, d_k \overset{k\gg1}{\sim} \ln \frac{\mathcal{Z}^{\text{total}}_{\text{R},+}}{q^{k+1}}\bigg|_{q=q_*} \overset{k\gg1}{\sim} 2\pi\sqrt{\frac{(d-2)k}{4}}$$

$$\ln\, \rho(m) \overset{m\gg M_s}{\sim} \sqrt{\frac{\pi(d-2)}{8}}\frac{m}{M_s}$$

$$S^{\text{leading}}_{\text{BH}} \propto \left(\frac{M_{\text{BH}}}{M_{\text{Pl}}}\right)^{\frac{d-2}{d-3}} = \left(g_s^2\,\frac{M_{\text{BH}}^{d-2}}{M_s^{d-2}}\right)^{\frac{1}{d-3}}$$

$$M_{\text{match}} = M_s^{3-d} M_{\text{Pl}}^{d-2}$$



$$M = M_{\text{external}} \times M_{\text{internal}}\,,$$

$$\int \mathcal{D}(\dots)\,\underbrace{e^{-\int \mathrm{d}^2\sigma\sqrt{-\gamma}\mathcal{V}}\, e^{-S^E_{\text{ws}}[\text{ background }]}}_{e^{-S^E_{\text{ws}}[\text{ deformed background }]}}$$

$$g_{\mu\nu}(X)^Y \overset{<1}{\approx} \delta_{\mu\nu} - \frac{\alpha'}{3} R_{\mu\rho\nu\sigma} Y^\rho Y^\sigma.$$

$$\beta^{(g)}_{\mu\nu} \overset{\alpha'\text{Riem}\ll1}{\sim} \alpha' R_{\mu\nu}$$

$$\delta_\omega \Gamma \propto \tilde{\beta}^{(g)}_{\mu\nu}\partial X^\mu\cdot\partial X^\nu + \tilde{\beta}^{(B)}_{\mu\nu}\epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu + \tilde{\beta}^{(\phi)}\alpha'\text{Ric}(\gamma)\phi(X)$$

$$\tilde{\beta}^{(g)}_{\mu\nu} \overset{\alpha'\mathcal{R}\ll1}{\sim} \alpha' R_{\mu\nu} - \frac{\alpha'}{4} H_{\mu\rho\sigma}H_\nu^{\rho\sigma} + 2\alpha'\nabla_\mu\nabla_\nu\phi,$$
$$\tilde{\beta}^{(B)}_{\mu\nu} \overset{\alpha'\mathcal{R}\ll1}{\sim} -\frac{\alpha'}{2}\nabla^\rho H_{\rho\mu\nu} + \alpha'\nabla^\rho\phi H_{\rho\mu\nu},$$
$$\tilde{\beta}^{(\phi)} \overset{\alpha'\mathcal{R}\ll1}{\sim} -\frac{\alpha'}{2}\nabla^2\phi - \frac{\alpha'}{24}H^2 + \alpha'(\nabla\phi)^2.$$

$$S_{\text{eff}}^{\text{NS-NS}} \underset{e^{\phi}\ll 1}{\overset{\alpha'\mathcal{R}\ll 1}{\sim}} \frac{M_s^{d-2}}{2}\int d^d x\sqrt{-g}e^{-2\phi}\left(R+4(\partial\phi)^2-\frac{1}{12}H^2\right).$$

$$M_{\text{Pl}} = M_s g_s^{-\frac{2}{d-2}}$$

$$\mathcal{S}^{\text{tree}}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}(p_1,p_2,p_3,p_4) \propto g_s^2 \int_{\mathbb{C}} d^2 z \langle \mathcal{V}_{\lambda_1,p_1}(0)\mathcal{V}_{\lambda_2,p_2}(1)\mathcal{V}_{\lambda_3,p_3}(\widetilde{\infty})\mathcal{V}_{\lambda_4,p_4}(z)\rangle_{\mathbb{C}P^1}$$

$$\mathcal{A}^{\text{string}}_{\text{tree}} = \mathbf{K}\frac{G_N}{stu}\frac{\Gamma\left(1-\frac{\alpha' s}{4}\right)\Gamma\left(1-\frac{\alpha' t}{4}\right)\Gamma\left(1-\frac{\alpha' u}{4}\right)}{\Gamma\left(1+\frac{\alpha' s}{4}\right)\Gamma\left(1+\frac{\alpha' t}{4}\right)\Gamma\left(1+\frac{\alpha' u}{4}\right)},$$

$$\Gamma(z) = \frac{e^{-\gamma_E z}}{z}\prod_{n>0}\left(1+\frac{z}{n}\right)^{-1} e^{\frac{z}{n}}$$

$$\ln \Gamma(1-z) = \gamma_E z + \sum_{n>1}\frac{\zeta(n)}{n} z^n$$

$$\exp\left(\sum_{n>0}\frac{2\zeta(2n+1)}{2n+1}\left(\frac{\alpha'}{4}\right)^{2n+1}(s^{2n+1}+t^{2n+1}+u^{2n+1})\right)$$

$$\frac{1}{stu} + \alpha M_{\text{Pl}}^{-6} + M_{\text{Pl}}^{2-d} f(s,t,u) + \cdots$$

$$\alpha_{10d}^{\text{II}} \overset{g_s\ll 1}{\sim} \frac{\sqrt{g_s}}{64}\left(\frac{2\zeta(3)}{g_s^2}+\frac{2\pi^2}{3}\right)$$

$$M_s^{d-2}\sum_{k>2}\left(c_k^0(\varphi)e^{-2\phi}+c_k^{(1)}(\varphi)+c_k^{(2)}(\varphi)e^{2\phi}+\cdots\right)\frac{\mathcal{O}_k(\nabla,\mathcal{R},\ldots)}{\Lambda_{\text{UV}}(\varphi)^{k-2}},$$

$$\frac{M_{\text{Pl}}^{d-2}}{2}\int d^d x\sqrt{-g}\left(R-\frac{1}{2}G_{ij}(\varphi)\mathcal{D}_\mu\varphi^i\mathcal{D}^\mu\varphi^j - V(\varphi) - \frac{1}{2}f_{ab}(\varphi)\text{tr}F^a{}_{\mu\nu}F^{b\mu\nu}\right)$$

$$-\sum_p \frac{w^{(p)}_{mn}(\varphi)}{2(p+1)!} dC_p^m \cdot dC_p^n$$

$$\epsilon_\mu(p) J\bar{\partial}X^\mu e^{ip\cdot X}, \epsilon_\mu(p)\bar{J}\partial X^\mu e^{ip\cdot X}$$

$$\exp\left(-\text{const.}\times\left(\frac{\Lambda_{\text{UV}}}{E}\right)^{k>0}\right)$$

$$h_{\text{int}} \mapsto h_{\text{int}} + \frac{1}{2}(Q_{\text{L}}+n_{\text{L}})^2 - \frac{1}{2}Q_{\text{L}}^2$$
$$\bar{h}_{\text{int}} \mapsto \bar{h}_{\text{int}} + \frac{1}{2}(Q_{\text{R}}+n_{\text{R}})^2 - \frac{1}{2}Q_{\text{R}}^2$$

$$h_{\text{int}} = h_{\text{graviton}} + \frac{1}{2} Q_{\text{L}}^2, \bar{h}_{\text{int}} = \bar{h}_{\text{graviton}} + \frac{1}{2} Q_{\text{R}}^2$$

$$\frac{\alpha'}{4} m^2 \leq \frac{1}{2} \max\{Q_{\text{L}}^2, Q_{\text{R}}^2\}$$

$$\alpha \overset{m_{\text{gap}} \ll M_{\text{Pl}}}{\sim} \left(\frac{M_{\text{Pl}}}{M_s}\right)^{8-d} \left(\frac{m_{\text{gap}}}{M_s}\right)^{-\hat{c}},$$

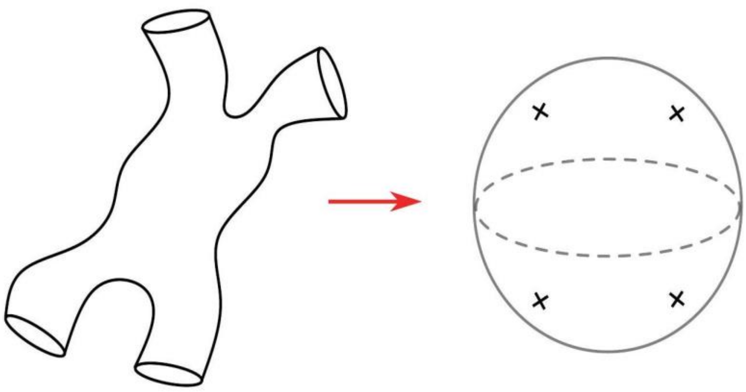
$$\Lambda_{\text{UV}} \overset{m_{\text{gap}} \ll M_{\text{Pl}}}{\sim} M_{\text{Pl}} \left(\frac{m_{\text{gap}}}{M_{\text{Pl}}}\right)^{\frac{\hat{c}}{d+\hat{c}-2}},$$

$$\mathcal{M}_{g,n} \equiv \frac{(\text{ metrics on } \Sigma_g) \times \Sigma_g^n}{\text{Diff } \times \text{ Weyl}},$$

$$\text{CKG} \subset \text{Diff}_0 \times \text{ Weyl},$$

$$\#\text{moduli} - \#\text{CKVs} = -3\,\chi(\Sigma_g) = 6g - 6.$$

$$\mathcal{S}_{\lambda_i \ldots \lambda_n}(p_1, \ldots, p_n)^{g_s \ll 1} \sum_{\text{genus } g} g_s^{2g-2} \int_{\mathcal{M}_{g,n}(t)} \mathrm{d}\mu(t) \left\langle \prod_{i=1}^{n} \mathcal{V}_{\lambda_i}(p_i) \right\rangle_{\Sigma_g(t)}$$



$$\left\langle \prod_{k=1}^{n} e^{i p_k \cdot X(\sigma_k)} \right\rangle = \left\langle e^{i \sum_{k=1}^{n} p_k \cdot X_0} \right\rangle_{\text{zero-modes}} \left\langle e^{i \int \mathrm{d}^2 z J \cdot X} \right\rangle_{\text{no zero-modes}}$$
$$= \int \mathrm{d}^d X_0 e^{i \sum_{k=1}^{n} p_k \cdot X_0} \left\langle e^{i \int \mathrm{d}^2 z J \cdot X} \right\rangle_{\text{no zero-modes}}$$
$$= (2\pi)^d \delta^{(d)} \left(\sum_{k=1}^{n} p_k \right) \left\langle e^{i \int \mathrm{d}^2 z J \cdot X} \right\rangle_{\text{no zero-modes}}$$

$$\exp\left(\frac{\pi\alpha'}{2} \int \mathrm{d}^2 z J \cdot (\mathcal{G} \star J)\right) = \exp\left(\frac{\alpha'}{4} \sum_{i,j=1}^{n} p_i \cdot p_j \ln |z_i - z_j|^2\right)$$
$$= \prod_{i<j} |z_i - z_j|^{\alpha' p_i \cdot p_j}$$

$$|z|^{\alpha' p_1 \cdot p_4 - 2}|1-z|^{\alpha' p_2 \cdot p_4 - 2} = |z|^{-\frac{\alpha' u}{2} - 2}|1-z|^{-\frac{\alpha' t}{2} - 2}.$$

$$\int_{\mathbb{C}} \mathrm{d}^2 z |z|^{2a-2}|1-z|^{2b-2} = 2\pi \frac{\Gamma(a)\Gamma(b)\Gamma(1-a-b)}{\Gamma(1-a)\Gamma(1-b)\Gamma(a+b)}$$

$$\mathcal{A}_{\text{tree}}^{\text{open}} \propto \frac{\Gamma\left(-\frac{\alpha' s}{4}\right)\Gamma\left(-\frac{\alpha' t}{4}\right)}{\Gamma\left(1-\frac{\alpha'(s+t)}{4}\right)}$$

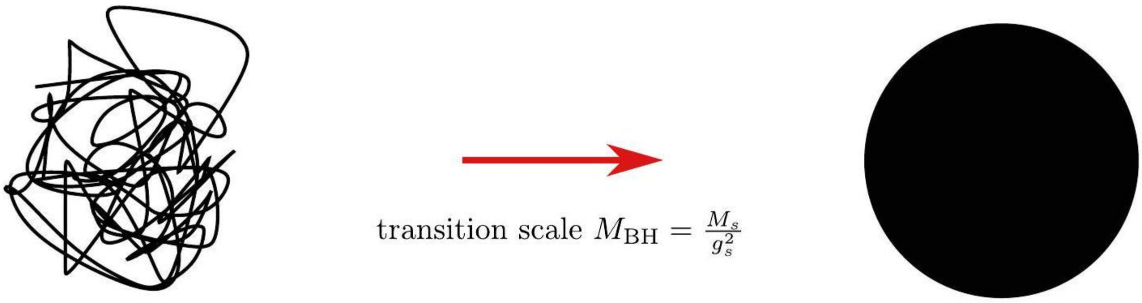
$$\alpha' s \gg 1, s/t \text{ fixed.}$$

$$\ln F_{\text{tree}} \overset{\text{hard}}{\sim} -\frac{1}{2}(s\ln s + t\ln t + u\ln u),$$

$$\ln F_g \overset{\text{hard}}{\sim} \frac{1}{g+1} \ln F_{\text{tree}}$$

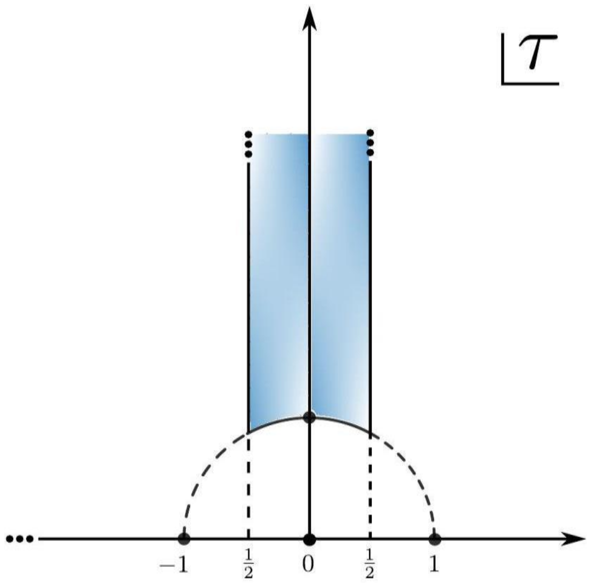
$$g_s^2 \exp(-As\ln s) = \exp\left(-\frac{A}{2} s\ln s\right)$$

$$M_{\text{th}} = M_s^{3-d} M_{\text{Pl}}^{d-2} = \frac{M_s}{g_s^2}$$



$$\tau \mapsto \tau + 1, \tau \mapsto -\frac{1}{\tau}.$$

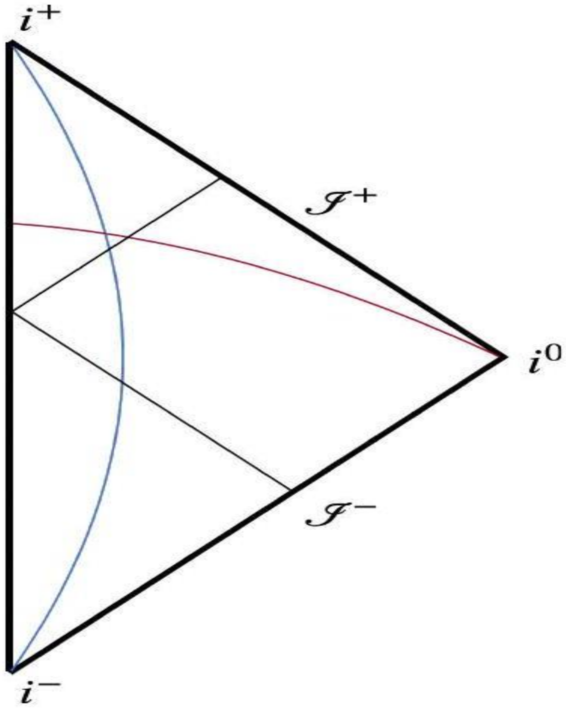
$$\begin{aligned} \mathrm{MCG}(T^2) &= \left\{\tau \mapsto \frac{a\tau + b}{c\tau + d} : ad - bc = 1\right\} / \{(a,b,c,d) \sim (-a,-b,-c,-d)\} \\ &\simeq \mathrm{PSL}(2,\mathbb{Z}) \equiv \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2 \end{aligned}$$



$$\int_{\mathcal{F}} \frac{\mathrm{d}^2\tau}{\tau_2^2} \frac{1}{\tau_2^{\frac{d-2}{2}}} \equiv \int_{\mathcal{F}} \mathrm{d}\mu_{T^2}(\tau) \mathcal{Z}(\tau) \equiv \mathcal{T}.$$

$$\Lambda \overset{e^{\phi} \ll 1}{\approx} - e^{\frac{2d}{d-2}\phi} M_s^d \mathcal{T},$$

$$m^2 = \frac{4}{\alpha'}(h-1)$$



$$\mathrm{d}s^2 = -\left(1 - \frac{2G_N M}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2G_N M}{r}\right)^{-1} \mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + r^2 \sin^2\,\theta\,\mathrm{d}\phi^2.$$

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{48 G_N^2 M^2}{r^6}.$$

$$v = t + r^*, u = t - r^*.$$

$$\frac{\mathrm{d}r^*}{\mathrm{d}r} = \sqrt{\frac{g_{rr}}{g_{tt}}} \Longrightarrow r^* = r + 2G_N M \ln\left|\frac{r}{2G_N M} - 1\right|.$$

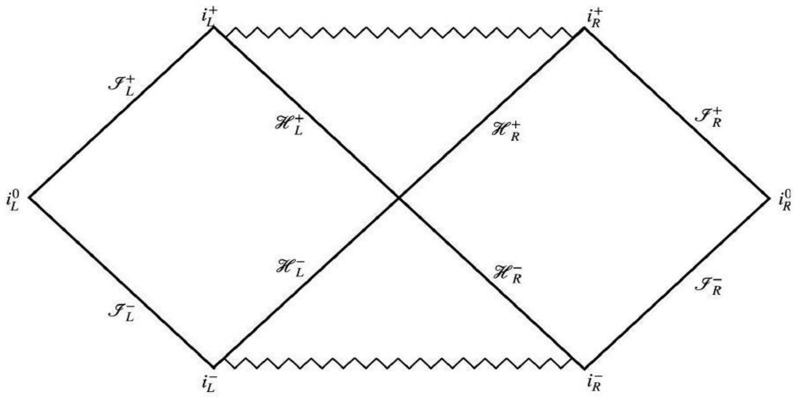
$$\mathrm{d}s^2 = -\left(1 - \frac{2G_N M}{r}\right)\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}r + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\,\theta\,\mathrm{d}\phi^2$$
$$\mathrm{d}s^2 = -\left(1 - \frac{2G_N M}{r}\right)\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\,\theta\,\mathrm{d}\phi^2$$
$$\mathrm{d}s^2 = -\left(1 - \frac{2G_N M}{r}\right)\mathrm{d}v\,\mathrm{d}u + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\,\theta\,\mathrm{d}\phi^2$$

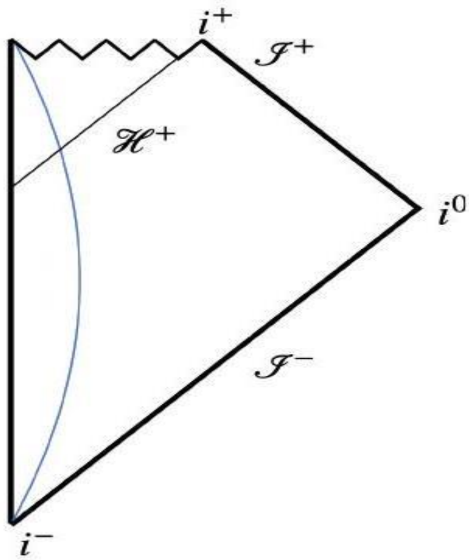
$$1 - \frac{2G_N M}{r} \approx e^{(v-u)/4G_N M}$$

$$U = \mp e^{-u/4G_N M}, V = e^{v/4G_N M}$$

$$\mathrm{d}s^2 = -\frac{32G_N^3 M^3}{r} e^{-r/2G_N M}\,\mathrm{d}U\,\mathrm{d}V + r^2\,\mathrm{d}\Omega^2$$

$$U \in (-\infty, 0), V \in (0, +\infty)$$





$$\partial_\mu \partial^\mu \phi = 0.$$

$$\phi = \sum_i \left(a_i f_i + a_i^\dagger f_i^*\right).$$

$$f_{\vec{k}} = \frac{1}{\sqrt{16\pi\omega_k}} e^{-i\omega_k t \pm i\vec{k}\cdot\vec{x}}, \omega_k = |\vec{k}| > 0$$

$$\left(f_i, f_j\right) \equiv -i\int \mathrm{d}^3\vec{x}\left(f_i\partial_t f_j^* - f_j^*\partial_t f_i\right) = \delta_{ij}$$

$$\left[a_{\vec{k}}, a_{\vec{k}'}\right] = \left[a_{\vec{k}}^\dagger, a_{\vec{k}'}^\dagger\right] = 0, \left[a_{\vec{k}}, a_{\vec{k}'}^\dagger\right] = \delta^3\left(\vec{k} - \vec{k}'\right).$$

$$a_{\vec{k}}|0\rangle = 0$$

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = 0$$

$$\phi = \sum_i a_i f_i + a_i^\dagger f_i^*$$

$$\xi^\mu\nabla_\mu f_i = -i\omega_i f_i, \text{ with } \omega_i > 0$$

$$\phi = \sum_i a_i^{in} f_i^{in} + a_i^{\dagger in} f_i^{in*}$$

$$\phi = \sum_i a_i^{\text{out}} f_i^{\text{out}} + a_i^{\dagger\text{out}} f_i^{\text{out}\,*}$$

$$f_i^{\text{out}} = \sum_j \left(\alpha_{ij} f_j^{\text{in}} + \beta_{ij} f_j^{\text{in}\,*}\right)$$

$$\alpha_{ij} = \left(f_i^{\text{out}}, f_j^{\text{in}}\right), \beta_{ij} = -\left(f_i^{\text{out}}, f_j^{\text{in}\,*}\right)$$

$$\sum_k \left(\alpha_{ik}\alpha_{jk}^* - \beta_{ik}\beta_{jk}^*\right) = \delta_{ij}, \sum_k \left(\alpha_{ik}\beta_{jk} - \beta_{ik}\alpha_{jk}\right) = 0,$$

$$\int \mathrm{d}\omega\alpha_{\omega_1\omega'}\alpha_{\omega_2\omega'} - \beta_{\omega_1\omega'}\beta_{\omega_2\omega'^*} = \delta(\omega_1 - \omega_2)$$

$$a_i^{\text{in}} \mid \text{in}\rangle = 0\; a_i^{\text{out}} \mid \text{out}\rangle = 0$$

$$\langle \text{in}| N_i^{\text{out}} \mid \text{in}\rangle = \langle \text{in}| a_i^{\text{out}\,\dagger} a_i^{\text{out}} \mid \text{in}\rangle.$$

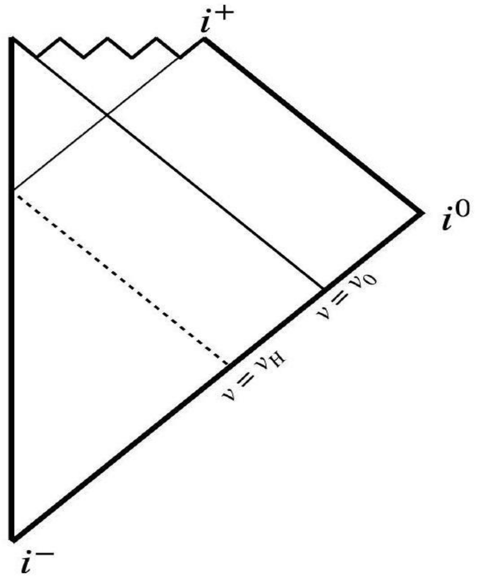
$$a_i^{\text{out}} = \sum_j \alpha_{ij} a_j^{\text{in}} - \beta_{ij}^* a_j^{\text{in}\,\dagger}.$$

$$\langle \text{in}| a_i^{\text{out}\,\dagger} a_i^{\text{out}} \mid \text{in}\rangle = \sum_j \left|\beta_{ij}\right|^2.$$

$$\phi = \sum_{l,m} \frac{\phi_{lm}(t,r)}{r} Y_m^l(\theta,\varphi)$$

$$\mathrm{d}s^2 = -\mathrm{d}u^{in}\,\mathrm{d}v + r^2\,\mathrm{d}\Omega^2,$$

$$\frac{\partial\phi}{\partial t^2} + \frac{\partial\phi}{\partial r^{*2}} + \frac{l(l+1)}{r^2}\phi = 0.$$



$$\mathrm{d}s^2 = -\left(1 - \frac{2G_N M}{r}\right)\mathrm{d}u^{\mathrm{out}}\,\mathrm{d}v + r^2\,\mathrm{d}\Omega^2,$$

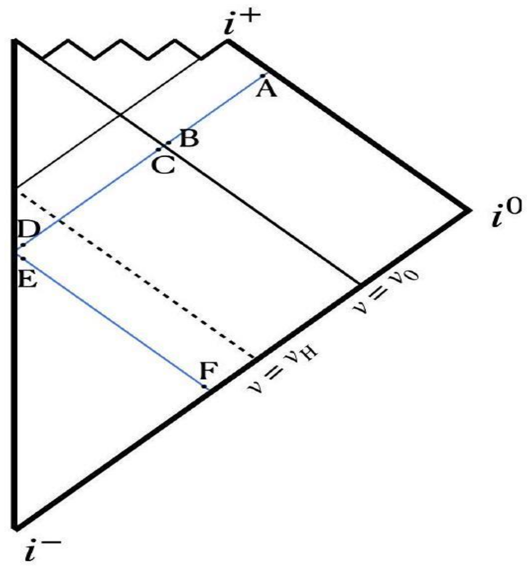
$$\frac{\partial\phi}{\partial t^2} + \frac{\partial\phi}{\partial r^{*2}} + \left(1 - \frac{2G_N M}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{2G_N M}{r^3}\right)\phi = 0.$$

$$f_\omega^{in} = \frac{1}{4\pi\sqrt{\omega}}e^{-i\omega v}.$$

$$f_\omega^{out} = \frac{1}{4\pi\sqrt{\omega}}e^{-i\omega u^{out}}.$$

$$\alpha_{\omega\omega'} = \left(f_{\omega'}^{\mathrm{out}}, f_\omega^{\mathrm{in}}\right) = -i\int_{\mathcal{J}^-}\mathrm{d}r r^2\left(f_\omega^{\mathrm{out}}\,\partial_v f_{\omega'}^{\mathrm{in}\,*} - f_\omega^{\mathrm{in}\,*}\partial_v f_{\omega'}^{\mathrm{out}}\right),$$
$$\beta_{\omega\omega'} = -\left(f_\omega^{\mathrm{out}}, f_{\omega'}^{\mathrm{in}\,*}\right) = i\int_{\mathcal{J}^-}\mathrm{d}r r^2\left(f_\omega^{\mathrm{out}}\,\partial_v f_{\omega'}^{\mathrm{in}} - f_\omega^{\mathrm{out}}\,\partial_v f_{\omega'}^{\mathrm{in}}\right).$$

$$\alpha_{\omega\omega'} = -2i\int_{\mathcal{J}^-}\mathrm{d}r r^2 f_\omega^{in}\partial_v f_{\omega'}^{out}, \beta_{\omega\omega'} = 2i\int_{\mathcal{J}^-}\mathrm{d}r r^2 f_\omega^{in}\partial_v f_{\omega'}^{out}.$$



$$f_\omega^{out} = \frac{1}{4\pi\sqrt{\omega}} e^{-i\omega u^{out}(u^{in})}$$

$$r^* = r + 2G_N M \ln\left|\frac{r}{2G_N M} - 1\right| = \frac{v_0 - u^{\text{out}}}{2}$$

$$r = \frac{v_0 - u^{in}}{2}$$

$$u^{\text{out}} = u^{\text{in}} - 4G_N M \ln\left|\frac{v_0 - 4G_N M - u^{\text{in}}}{4G_N M}\right|$$

$$u_H^{in} = v_0 - 4G_N M$$

$$v_H = v_0 - 4G_N M$$

$$\phi(r = 0) = 0$$

$$f_\omega^{out} = -\frac{1}{4\pi\sqrt{\omega}} e^{-i\omega u^{out}(v)} \theta(v - v_H)$$

$$u^{out} = v - 4G_N M \ln\left|\frac{v_H - v}{4G_N M}\right|$$

$$u^{out} \approx v_H - 4G_N M \ln\left|\frac{v_H - v}{4G_N M}\right|$$

$$|\alpha_{\omega\omega'}| = e^{4G_N M\omega} |\beta_{\omega\omega'}|.$$

$$\sum_{\omega'} \alpha_{\omega\omega'} \alpha^*_{\omega''\omega'} - \beta_{\omega\omega'} \beta^*_{\omega''\omega'} = \delta_{\omega\omega''}$$

$$\sum_{\omega'} |\alpha_{\omega\omega'}|^2 - |\beta_{\omega\omega'}|^2 = (e^{8G_N M\omega} - 1) \sum_{\omega'} |\beta_{\omega\omega'}|^2 = 1,$$

$$N_\omega^{\text{out}} = \frac{1}{e^{8\pi G_N M\omega} - 1}.$$

$$\frac{1}{e^{\omega/k_B T} - 1},$$

$$T = \frac{\hbar c^3}{8\pi k_B G_N M}.$$

$$T \approx 10^{-7}\left(\frac{M_\odot}{M}\right)\mathrm{K},$$

$$\langle \text{ in } | N_\omega^{\text{out}} N_{\omega'}^{\text{out}} \mid \text{ in } \rangle = \langle \text{ in } | N_\omega^{\text{out}} \mid \text{ in } \rangle \langle \text{ in } | N_{\omega'}^{\text{out}} \mid \text{ in } \rangle, \text{ for } \omega \neq \omega'.$$

$$N_\omega^{\text{out}} = \frac{\Gamma_{\omega l}}{e^{8\pi G_N M\omega} - 1}.$$

$$T_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S_\phi}{\delta g^{\mu\nu}}$$

$$\langle \psi | \hat{T}_{\mu\nu} | \psi \rangle.$$

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}^{\text{cl}} + 8\pi G_N \langle \psi | \hat{T}_{\mu\nu} | \psi \rangle.$$

$$S = \int \mathrm{d}^4 x \sqrt{-g^{(4)}} \left[-\frac{1}{2} (\nabla\phi)^2 \right]$$

$$\mathrm{d}s^2 = g_{ab}\, \mathrm{d}x^a\, \mathrm{d}x^b + r^2\, \mathrm{d}\Omega^2$$

$$S^{(4)} = 4\pi \int \mathrm{d}^2 x \sqrt{-g^{(2)}} r^2 \left[-\frac{1}{2} (\nabla\phi)^2 \right]$$

$$S^{(2)} = \int \mathrm{d}^2 x \sqrt{-g^{(2)}} \left[-\frac{1}{2} (\nabla\phi)^2 \right]$$

$$T_{ab}^{(4)} = \frac{1}{4\pi r^2} T_{ab}^{(2)}$$

$$g_{\mu\nu} \to \Omega^2(x) g_{\mu\nu}$$

$$\delta g_{\mu\nu} = \omega g_{\mu\nu}, 0 = \delta S = \int \mathrm{d}^n x \sqrt{-g}\, \omega T^{\mu\nu} g_{\mu\nu} \Leftrightarrow T^{\mu\nu} g_{\mu\nu} = 0$$

$$\langle T^{(2)} \rangle = aR$$

$$g_{\mu\nu} = \Omega^2(x) \eta_{\mu\nu}$$

$$\langle \hat{T}_{\mu\nu} \rangle = \frac{-2}{\sqrt{-g}} \frac{\delta\Gamma}{\delta g^{\mu\nu}}.$$

$$\Gamma[g] = \Gamma[\eta] + \int \mathrm{d}^2 x \sqrt{-g} \langle \hat{T}_\mu^\mu \rangle \delta\Omega^2$$

$$\langle \hat{T}_{UU} \rangle = \langle \hat{T}_{uu} \rangle \left(\frac{\partial U}{\partial u}\right)^{-2} \propto U^{-2} \langle \hat{T}_{uu} \rangle$$
$$\langle \hat{T}_{UV} \rangle = \langle \hat{T}_{uv} \rangle \left(\frac{\partial U}{\partial u}\right)\left(\frac{\partial V}{\partial v}\right) \propto U^{-1}V^{-1} \langle \hat{T}_{uv} \rangle$$
$$\langle \hat{T}_{VV} \rangle = \langle \hat{T}_{vv} \rangle \left(\frac{\partial V}{\partial v}\right)^{-2} \propto V^{-2} \langle \hat{T}_{vv} \rangle$$

$$\left(1 - \frac{2G_N M}{r}\right)^{-2} \langle \hat{T}_{uu} \rangle < \infty,$$
$$\left(1 - \frac{2G_N M}{r}\right) \langle \hat{T}_{uv} \rangle < \infty, (6.78)$$
$$\langle \hat{T}_{vv} \rangle < \infty. (6.78)$$

$$\langle \hat{T}_{uu} \rangle < \infty,$$
$$\left(1 - \frac{2G_N M}{r}\right) \langle \hat{T}_{uv} \rangle < \infty, (6.79)$$
$$\left(1 - \frac{2G_N M}{r}\right)^{-2} \langle \hat{T}_{vv} \rangle < \infty. (6.79)$$

$$f_L \propto e^{-i\omega v}$$

$$f_R \propto e^{-i\omega u}$$

$$\phi = \sum_{\omega} \left[\frac{1}{4\pi\sqrt{\omega}} e^{-i\omega v} a_\omega + \frac{1}{4\pi\sqrt{\omega}} e^{-i\omega u} a_\omega\right] + h.c..$$

$$\langle B|\hat{T}_{uu}|B\rangle = \langle B|\hat{T}_{vv}|B\rangle = \frac{1}{24\pi}\left(-\frac{G_N M}{r^3} + \frac{3}{2}\frac{G_N^2 M^2}{r^4}\right)$$
$$\langle B|\hat{T}_{uv}|B\rangle = -\frac{1}{24\pi}\left(1 - \frac{2G_N M}{r}\right)\frac{G_N M}{r^3}$$

$$\tilde{u}(u), \text{ and } \tilde{v}(v).$$

$$\phi = \sum_{\omega} \left[\frac{1}{4\pi\sqrt{\omega}} e^{-i\omega \tilde{v}} a_\omega + \frac{1}{4\pi\sqrt{\omega}} e^{-i\omega \tilde{u}} a_\omega\right] + h.c..$$

$$\langle \tilde{0}|\hat{T}_{uu}|\tilde{0}\rangle = \langle B|\hat{T}_{uu}|B\rangle - \frac{1}{24\pi}\{\tilde{u}, u\},$$
$$\langle \tilde{0}|\hat{T}_{vv}|\tilde{0}\rangle = \langle B|\hat{T}_{vv}|B\rangle - \frac{1}{24\pi}\{\tilde{v}, v\},$$
$$\langle \tilde{0}|\hat{T}_{uv}|\tilde{0}\rangle = \langle B|\hat{T}_{uv}|B\rangle,$$

$$\{f(x), x\} \equiv \frac{f'''(x)}{f'(x)} - \frac{3}{2}\left(\frac{f''(x)}{f'(x)}\right)$$

$$\phi = \sum_{\omega} \left[\frac{1}{4\pi\sqrt{\omega}} e^{-i\omega v} a_\omega + \frac{1}{4\pi\sqrt{\omega}} e^{-i\omega U} a_\omega\right] + h.c..$$

$$-\frac{1}{24\pi}\{\tilde{u}, u\} = \frac{k^2}{48\pi},$$

$$\langle U|\hat{T}_{uu}|U\rangle = \langle B|\hat{T}_{uu}|B\rangle - \frac{1}{24\pi}\{U,u\}$$
$$= \frac{1}{32G_N^2M^2}\left(1-\frac{2G_NM}{r}\right)^2\left(1+4\frac{G_NM}{r}+12\left(\frac{G_NM}{r}\right)^2\right),$$
$$\langle U|\hat{T}_{vv}|U\rangle = \langle B|\hat{T}_{vv}|B\rangle,$$
$$\langle U|\hat{T}_{uv}|U\rangle = \langle B|\hat{T}_{uv}|B\rangle.$$

$$\phi = \sum_{\omega}\left[\frac{1}{4\pi\sqrt{\omega}}e^{-i\omega V}a_\omega + \frac{1}{4\pi\sqrt{\omega}}e^{-i\omega U}a_\omega\right] + h.c..$$

$$\phi = \sum_{\omega}\left[\frac{1}{4\pi\sqrt{\omega}}e^{-i\omega v}a_\omega + \frac{1}{4\pi\sqrt{\omega}}e^{-i\omega u^{in}}a_\omega\right] + h.c..$$

$$\langle in|\hat{T}_{vv}|in\rangle\big|_{r=2G_NM} = -\frac{1}{648G_N^2M^2\pi}$$

$$\langle in|\hat{T}_{uu}|in\rangle\big|_{r\to\infty} = \frac{1}{648G_N^2M^2\pi}$$

$$\rho = \sum_i \rho_i|\psi_i\rangle\langle\psi_i|.$$

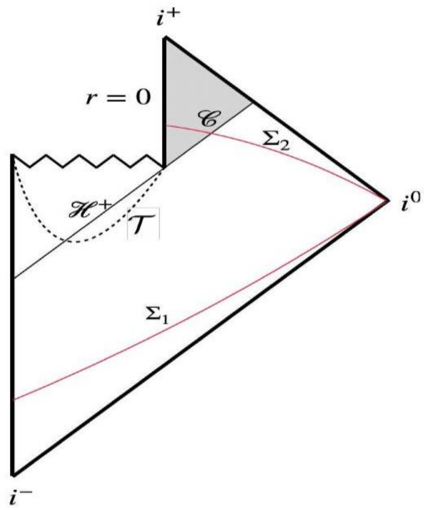
$$\rho = \frac{1}{2}(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|)$$

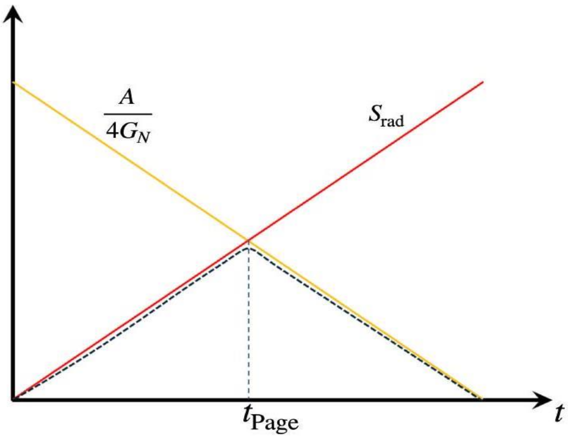
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle), \rho = |\psi\rangle\langle\psi| = \frac{1}{2}(|\psi_1\rangle\langle\psi_1| + |\psi_1\rangle\langle\psi_2| + |\psi_2\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|)$$

$$S_{\mathrm{vN}} = -\mathrm{Tr}(\rho\ln\,\rho)$$

$$S_{\mathrm{Th}} \geq S_{\mathrm{vN}}.$$

$$S_{\mathrm{vN}}(A\cup B) = 0, S_{\mathrm{vN}}(A) = S_{\mathrm{vN}}(B) \neq 0$$





$$S_{\rm BH} = \frac{A}{4G_N}.$$

$$S \leq 2\pi E R$$

$$S \leq \frac{A}{4G_N} = S_{\rm BH}$$

$$\tau_\Phi \sim 4\pi \frac{M_P^2}{m_\Phi^3}$$

$$\frac{t_{\rm end}}{t_{\rm start}} \sim 4\pi \left(\frac{M_P}{m_\Phi}\right)^2$$

$$\ddot{\phi} + 3H\dot{\phi} + \nabla^2\phi = -\frac{\partial V}{\partial \phi},$$

$$\phi = \phi_0 + \sqrt{\frac{2}{3}} M_P \ln\left(\frac{t}{t_0}\right),$$

$$\rho_{KE} \propto \frac{1}{a(t)^6}$$

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{1/3}, H = \frac{1}{3t},$$

$$\eta(t) = \frac{3}{2a_0}(t^2 t_0)^{1/3} = \eta_0 \left(\frac{t}{t_0}\right)^{2/3},$$

$$\phi = \phi_0 + \sqrt{\frac{3}{2}} M_P \ln\left(\frac{\eta}{\eta_0}\right) \text{ with } a(\eta) = a_0 \left(\frac{\eta}{\eta_0}\right)^{\frac{1}{2}}.$$

$$\mathcal{H} \equiv \frac{a'}{a} = \frac{1}{2\eta}.$$

$$\frac{\mathcal{V}}{\mathcal{V}_0} \sim \frac{t}{t_0}.$$

$$\frac{M_P^2}{3t_0^2} \sim V(\phi_0) \sim \Lambda_{\text{inf}}^4.$$

$$x = \frac{\dot{\phi}}{M_P}\frac{1}{\sqrt{6}H}, y = \sqrt{\frac{V(\phi)}{3}}\frac{1}{M_P H},$$

$$\left|\begin{aligned} x'(N) &= -3x - \frac{M_P V'(\phi)}{V(\phi)}\sqrt{\frac{3}{2}}y^2 + \frac{3}{2}x[2x^2 + \gamma(1 - x^2 - y^2)] \\ y'(N) &= \frac{M_P V'(\phi)}{V(\phi)}\sqrt{\frac{3}{2}}xy + \frac{3}{2}y[2x^2 + \gamma(1 - x^2 - y^2)] \\ H'(N) &= -\frac{3}{2}H\left(2x^2 + \gamma(1 - x^2 - y^2)\right) \\ \phi'(N) &= \sqrt{6}M_P x \end{aligned}\right\rangle$$

$$V(\Phi) = M_P^4 e^{-\sqrt{\frac{27}{2}}\frac{\Phi}{M_P}} = \Lambda_{\text{inf}}^4\left(\frac{\mathcal{V}_0}{\mathcal{V}}\right)^3$$

$$(x, y) = \left(\sqrt{\frac{3}{2}}\frac{\gamma}{\lambda}, \sqrt{\frac{3(2-\gamma)\gamma}{2\lambda^2}}\right)$$

$$\Omega_\gamma(t_0) \equiv \frac{\rho_\gamma(t_0)}{3H_{\text{inf}}^2 M_P^2} = \frac{\rho_\gamma(t_0)}{\Lambda_{\text{inf}}^4} = \varepsilon \ll 1,$$

$$\frac{dx}{dN} = x^3 - x$$

$$x(a) = \frac{1}{\sqrt{1 + \frac{\varepsilon}{1-\varepsilon}\left(\frac{a}{a_0}\right)^2}} \simeq \frac{1}{\sqrt{1 + \varepsilon\left(\frac{a}{a_0}\right)^2}}$$

$$\frac{dH}{dN} = -2H - \frac{H}{1 + \frac{\varepsilon}{1-\varepsilon}e^{2(N-N_0)}}$$

$$H(a) = H_0\sqrt{1-\varepsilon}\left(\frac{a}{a_0}\right)^{-3}\sqrt{1 + \frac{\varepsilon}{1-\varepsilon}\left(\frac{a}{a_0}\right)^2}.$$

$$a(\eta) = a_0\sqrt{1 + 2\mathcal{H}_0(\eta - \eta_0) + \varepsilon\mathcal{H}_0^2(\eta - \eta_0)^2},$$

$$\mathcal{H}(\eta) = \mathcal{H}_0\frac{1 + \varepsilon\mathcal{H}_0(\eta - \eta_0)}{1 + \mathcal{H}_0(\eta - \eta_0)\left(2 + \mathcal{H}_0\varepsilon(\eta - \eta_0)\right)}$$

$$\phi''(\eta) + 2\mathcal{H}_0 \frac{1 + \varepsilon\mathcal{H}_0(\eta - \eta_0)}{1 + \mathcal{H}_0(\eta - \eta_0)\big(2 + \mathcal{H}_0\varepsilon(\eta - \eta_0)\big)} \phi'(\eta) = 0$$

$$\phi(\eta) = \phi(\eta_0) + \sqrt{\frac{3}{2}} M_P \log\left(\frac{\big(\varepsilon\mathcal{H}_0(\eta - \eta_0) + 1 - \sqrt{1-\varepsilon}\big)(1 + \sqrt{1-\varepsilon})}{\big(\varepsilon\mathcal{H}_0(\eta - \eta_0) + 1 + \sqrt{1-\varepsilon}\big)(1 - \sqrt{1-\varepsilon})}\right)$$

$$\phi'(\eta) = \frac{\sqrt{6(1-\varepsilon)}\mathcal{H}_0 M_P}{1 + \mathcal{H}_0(\eta - \eta_0)\big(2 + \mathcal{H}_0\varepsilon(\eta - \eta_0)\big)}.$$

$$\mathcal{H}_0\eta_0 = \frac{1 - \sqrt{1-\varepsilon}}{\varepsilon},$$

$$\phi(\eta) = \phi(\eta_0) + \sqrt{\frac{3}{2}} M_P \ln\left(\frac{(1 + \sqrt{1-\varepsilon})\eta}{(1 - \sqrt{1-\varepsilon})\eta + 2\eta_0\sqrt{1-\varepsilon}}\right)$$

$$\Delta\phi = \sqrt{\frac{3}{2}} M_P \ln\left(\frac{1 + \sqrt{1-\varepsilon}}{1 - \sqrt{1-\varepsilon}}\right) \simeq \sqrt{\frac{3}{2}} M_P \log\left(\frac{4}{\varepsilon}\right)$$

$$a(\eta) = a_0\sqrt{\frac{\eta}{\eta_0} + \frac{\varepsilon}{4}\left(\frac{\eta}{\eta_0}\right)^2}$$

$$\frac{a(\eta)}{a_0} \underset{\eta \ll \frac{\eta_0}{\varepsilon}}{\simeq} \left(\frac{\eta}{\eta_0}\right)^{1/2} \quad \text{and} \quad \frac{a(\eta)}{a_0} \underset{\eta \gg \frac{\eta_0}{\varepsilon}}{\simeq} \frac{\sqrt{\varepsilon}}{2\eta_0}\left(\frac{\eta}{\eta_0}\right).$$

$$\mathcal{H}(\eta) = \frac{1}{2\eta} \frac{1 + \frac{\varepsilon\eta}{2\eta_0}}{1 + \frac{\varepsilon\eta}{4\eta_0}},$$

$$\frac{1}{2}\dot{\phi}^2 \equiv \frac{1}{2a(\eta)^2}\phi'^2 \gtrsim V(\phi) = V_0 e^{-\frac{\lambda\phi}{M_P}}$$

$$w = \left(\frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}\right) \sim 1$$

$$\frac{3M_P^2}{4a_0^2\eta_0^2\tilde{x}^3\left(1 + \frac{\varepsilon\tilde{x}}{4}\right)\left(1 + \frac{\varepsilon\tilde{x}}{2} + \frac{\varepsilon^2\tilde{x}^2}{16}\right)} \sim V_0 e^{-\frac{\lambda\phi_0}{M_P}}\left(\frac{4\tilde{x}}{4 + \varepsilon\tilde{x}}\right)^{-\lambda\sqrt{\frac{3}{2}}}.$$

$$\frac{3M_P^2}{4a_0^2\eta_0^2} \sim V_0 e^{-\frac{\lambda\phi_0}{M_P}}$$

$$\eta_t = \eta_0\varepsilon^{-\frac{\lambda}{6}\sqrt{\frac{3}{2}}-\frac{1}{2}}$$

$$\tilde{x} \sim \varepsilon^{-5/4}$$

$$\eta \sim \left(\frac{\eta_0}{\varepsilon}\right)\left(\varepsilon^{-3/2}\right)^{1/6} \sim \eta_0 \varepsilon^{-5/4}$$

$$\eta \sim \left(\frac{\eta_0}{\varepsilon}\right)\left(\varepsilon^{-3/2}\right)^{1/4} \sim \eta_0 \varepsilon^{-11/8}$$

$$\tilde{x}_{\text{tracker}} \sim \varepsilon^{-11/8}$$

$$(x, y) = \left(\sqrt{\frac{3}{2}\frac{\gamma}{\lambda}}, \sqrt{\frac{3(2-\gamma)\gamma}{2\lambda^2}}\right)$$

$$H'(N) = -\frac{3}{2}H\gamma, \phi'(N) = \frac{3\gamma}{\lambda}M_P$$

$$\phi = \phi_t + \frac{3\gamma M_P}{\lambda}\log\left(\frac{a}{a_t}\right)$$

$$H = H_t\left(\frac{a}{a_t}\right)^{-\frac{3}{2}\gamma}, a = a_t\left(\frac{t}{t_t}\right)^{2/3\gamma}, \eta(a) = \eta_t\left(\frac{a}{a_t}\right)^{3\gamma/2-1}$$

$$\phi = \phi_t + \frac{4M_P}{\lambda}\ln\left(\frac{a}{a_t}\right) = \phi_t + \frac{2M_P}{\lambda}\ln\left(\frac{t}{t_t}\right)$$

$$\Phi = \Phi_t + \frac{4\sqrt{2}M_P}{3\sqrt{3}}\ln\left(\frac{a}{a_t}\right) = \Phi_t + \left(\frac{2}{3}\right)^{3/2} M_P \ln\left(\frac{t}{t_t}\right)$$

$$a(\eta) \propto \eta$$

$$\mathcal{H} = \frac{1}{\eta}$$

$$\Phi(\eta) = \Phi_t + \frac{4M_P}{\lambda}\ln\left(\frac{\eta}{\eta_t}\right) = \Phi_t + \frac{4\sqrt{2}M_P}{3\sqrt{3}}\ln\left(\frac{\eta}{\eta_t}\right)$$

$$\phi = \phi_t + \frac{3M_P}{\lambda}\ln\left(\frac{a}{a_t}\right) = \phi_t + \frac{2M_P}{\lambda}\ln\left(\frac{t}{t_t}\right).$$

$$\Phi = \Phi_t + \sqrt{\frac{2}{3}}M_P \ln\left(\frac{a}{a_t}\right) = \Phi_t + \left(\frac{2}{3}\right)^{3/2} M_P \ln\left(\frac{t}{t_t}\right)$$

$$a(\eta) \propto \eta^2$$

$$\mathcal{H} = \frac{2}{\eta}$$

$$\Phi(\eta) = \Phi_t + \frac{6M_P}{\lambda}\ln\left(\frac{\eta}{\eta_t}\right) = \Phi_t + \frac{2\sqrt{2}M_P}{\sqrt{3}}\ln\left(\frac{\eta}{\eta_t}\right)$$

$$\rho_{\delta\Phi} \sim n_{\delta\Phi}(t) m_{\delta\Phi}(t) \sim \frac{1}{a^3(t)}\frac{1}{a^2(t)} \sim \frac{1}{a^5(t)}.$$

$$\ddot{\phi} + \frac{3}{2t}\dot{\phi} = -\frac{\partial V}{\partial \phi},$$

$$m_{\delta\Phi}^2(t) \equiv \frac{\partial^2 V}{\partial\Phi^2} = \frac{1}{t^2} = 4H^2$$

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} = -\frac{\mu^2}{t^2}\delta\phi,$$

$$\delta\phi(t) = t^\alpha(A\cos(\tilde{\omega}\ln t) + B\sin(\tilde{\omega}\ln t))$$

$$\rho_{\delta\phi} = \frac{\delta\dot{\phi}^2}{2} + \frac{m_{\delta\phi}^2(t)\delta\phi^2}{2},$$

$$\tilde{\omega}^2 = \frac{2-\gamma}{4\gamma^2}(9\gamma - 2).$$

$$\frac{d}{dN}\begin{pmatrix} f \\ g \end{pmatrix} = \begin{pmatrix} \frac{3}{2}\left(\gamma - 2 + \frac{6\gamma^2}{\lambda^2} - 3\frac{\gamma^3}{\lambda^2}\right) & 3\sqrt{(2-\gamma)\gamma}\left(1 - \frac{3}{2}\frac{\gamma^2}{\lambda^2}\right) \\ 3\sqrt{(2-\gamma)\gamma}\left(\frac{3\gamma}{\lambda^2} - \frac{3\gamma^2}{2\lambda^2} - \frac{1}{2}\right) & \frac{9\gamma^2}{2\lambda^2}(\gamma - 2) \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}$$
$$= \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}$$

$$x(N) = x_0 + e^{\frac{3}{4}(\gamma-2)N}\left[c_1\cos(\omega N) + \frac{1}{2\omega}[2\beta c_2 + (\alpha - \delta)c_1]\sin(\omega N))\right]$$
$$y(N) = y_0 + e^{\frac{3}{4}(\gamma-2)N}\left[c_2\cos(\omega N) + \frac{1}{2\omega}[2\gamma c_1 - (\alpha - \delta)c_2]\sin(\omega N))\right]$$

$$\omega^2 = \left(\frac{3\gamma}{2}\right)^2\tilde{\omega}^2 - \frac{27}{2\lambda^2}\gamma^2(2-\gamma)$$

$$\delta H = H\left[c_3 + A' e^{\frac{3}{4}(\gamma-2)N}\sin(\omega N + \alpha')\right]$$

$$V(\phi) = \frac{1}{2}m^2(\tilde{\phi} - \phi)^2$$

$$\phi \approx \tilde{\phi} + A(a)\sin(mt)$$

$$\langle V\rangle = \frac{1}{4}m^2A^2, \langle K\rangle = \left\langle\frac{1}{2}\dot{\phi}^2\right\rangle = \frac{1}{4}m^2A^2,$$

$$\langle V\rangle \equiv \frac{1}{T}\int_0^T V dt$$

$$\rho = \langle K\rangle + \langle V\rangle = \frac{1}{2}m^2A^2, P = \langle K\rangle - \langle V\rangle = 0,$$

$$\eta \propto t^{1/3} \propto a^{1/2}, \mathcal{H} = \frac{2}{\eta}.$$

$$\tau_\Phi \sim \frac{4\pi M_P^2}{m_\Phi^3}, T_{\text{reheat}} \sim 1\text{GeV}\left(\frac{M_\Phi}{10^6\text{GeV}}\right)^{3/2}.$$

$$f_k'' + \left(k^2 - \frac{z''}{z}\right) f_k = 0, \text{ with } z \equiv \frac{a(\eta)\bar{\phi}'}{\mathcal{H}},$$

$$f_k'' + \left(k^2 + \frac{1}{4\eta^2}\right) f_k = 0$$

$$f_k(\eta) = \sqrt{\eta}[C_1 J_0(k\eta) + C_2 Y_0(k\eta)],$$

$$\delta\phi_k(\eta) = [C_1 J_0(k\eta) + C_2 Y_0(k\eta)],$$

$$\mathcal{R}_k(\eta) = \frac{\delta\phi_k(\eta)}{\sqrt{6}M_P} = [C_1 J_0(k\eta) + C_2 Y_0(k\eta)],$$

$$\Phi'' + \frac{3}{\eta}\Phi' - \nabla^2\Phi = 0$$

$$\Phi_k(\eta) = \frac{C_1}{k\eta} J_1(k\eta) + \frac{C_2}{k\eta} Y_1(k\eta)$$

$$\mathcal{R} \equiv \Phi - \mathcal{H}v = \Phi - \frac{\mathcal{H}q}{\bar{P} + \bar{\rho}}$$

$$\partial_i \partial^j \Pi = \Pi_j^i = T_j^i - \delta_j^i(\bar{P} + \delta P) = \partial^i \delta\phi \partial_j \delta\phi = \mathcal{O}(\delta\phi^2)$$

$$\Phi - \Psi = \frac{\Pi a^2}{M_P^2}$$

$$\frac{a^2 q}{2M_P^2} = -(\Phi' + \mathcal{H}\Phi)$$

$$\mathcal{R}_k = \frac{4}{3}\Phi_k + \frac{\Phi_k'}{3\mathcal{H}} = \frac{2C_1}{3} J_0(k\eta) + \frac{2C_2}{3} Y_0(k\eta),$$

$$f_k'' + \left(k^2 - \frac{2}{\eta^2}\right) f_k = 0$$

$$f_k(\eta) = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$$

$$\delta\varphi_k = \lim_{\eta \to 0} \frac{f_k(\eta)}{a(\eta)} = \frac{iH_{\text{inf}}}{\sqrt{2k^3}},$$

$$\mathcal{R}_k = \lim_{\eta \to 0} \frac{f_k(\eta)}{z(\eta)} = \frac{H}{\dot{\varphi}}\bigg|_{\text{inf}} \delta\varphi_k$$

$$\mathcal{R}_k = \frac{1}{\sqrt{2\varepsilon_V}} \frac{\delta\varphi_k}{M_P}, \text{ where } \varepsilon_V = \frac{M_P^2}{2}\left(\frac{V_{,\varphi}}{V}\right)^2$$

$$\mathcal{R}_k(\eta) = \frac{\delta\phi_k(\eta)}{\sqrt{6}M_P},$$

$$\delta\phi_k(\eta) = iH_{\text{inf}}\sqrt{\frac{3}{2\varepsilon_V k^3}}\frac{J_0(k\eta)}{J_0(k\eta_0)}.$$

$$\delta\phi_k(\eta) = \begin{cases} iH_{\text{inf}}\sqrt{\dfrac{3}{2\varepsilon_V k^3}} \\ iH_{\text{inf}}\sqrt{\dfrac{3}{\pi\varepsilon_V \eta}}\dfrac{\cos\left(k\eta - \frac{\pi}{4}\right)}{J_0(k\eta_0)k^2} \end{cases}$$

$$C_1 = \frac{3iH_{\text{inf}}}{4M_P}\frac{1}{J_0(k\eta_0)\sqrt{\varepsilon_V k^3}} \quad \text{and} \quad C_2 = 0.$$

$$\bar{\rho}\Delta \equiv \delta\bar{\rho} + \bar{\rho}'(v + B),$$

$$\nabla^2\Phi = \frac{\bar{\rho}a^2}{2M_P^2}\Delta = \frac{3\mathcal{H}^2}{2}\Delta.$$

$$\Delta_k = -\frac{8k^2\eta^2}{3}\Phi_k,$$

$$\Delta_k(\eta) = Ak\eta J_1(k\eta) + Bk\eta Y_1(k\eta),$$

$$\Delta_k(\eta) = -\frac{2iH_{\text{inf}}}{M_P\sqrt{\varepsilon_V k^3}}\frac{k\eta J_1(k\eta)}{J_0(k\eta_0)}.$$

$$J_1(k\eta) \sim \frac{\sin(k\eta)}{\sqrt{k\eta}}$$

$$\Delta_k \sim \sqrt{k\eta}\sin(k\eta)$$

$$\lim_{k\eta\to\infty} Y_1(k\eta) \sim \frac{\cos(k\eta)}{\sqrt{k\eta}}$$

$$\Delta_k \sim \frac{\delta\rho}{\bar{\rho}} \sim a^2 \sim \eta$$

$$\int d^4x\,\partial_\mu\phi\,\partial^\mu\phi$$

$$\phi(x,t) = \phi_0 + vt + \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}\left(\alpha_k e^{i(kx-\omega t)} + \alpha_k^* e^{-i(kx-\omega t)}\right)$$

$$\dot{\phi}^2 = v^2 + 2v\int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}\left(-i\omega\alpha_k e^{i(kx-\omega t)} + i\omega\alpha_k^* e^{-i(kx-\omega t)}\right) + \mathcal{O}\left(\alpha_k^2\right)$$

$$A = \frac{\varepsilon\mathcal{H}}{\phi'}\delta\phi = \frac{\phi'\delta\phi}{2\mathcal{H}M_P^2}$$

$$\Delta_k = 3\sqrt{\frac{3}{2}\frac{M_P}{\eta}}\mathcal{H}^2\delta\phi_k' = -\frac{2iH_{\text{inf}}}{M_P\sqrt{\varepsilon_V k^3}}\frac{k\eta J_1(k\eta)}{J_0(k\eta_0)},$$

$$\phi_k(\eta) = \phi_0 + M_P\left(\frac{2}{3}\right)^{3/2}\ln\,\eta + \sum_k\left(A_k\frac{e^{ik\eta}}{\sqrt{k\eta}} + A_k^*\frac{e^{-ik\eta}}{\sqrt{k\eta}}\right).$$

$$\frac{\dot{\phi}^2}{2} + \frac{1}{a^2}\frac{(\nabla\phi)^2}{2} \equiv \frac{1}{a^2}\frac{\phi'(\eta)^2}{2} + \frac{1}{a^2}\frac{(\nabla\phi)^2}{2}$$

$$\phi'(\eta) = \frac{M_P}{\eta}\left(\frac{2}{3}\right)^{3/2} + \sum_k\left(ikA_k\frac{e^{ik\eta}}{\sqrt{k\eta}} - ikA_k^*\frac{e^{-ik\eta}}{\sqrt{k\eta}}\right),$$

$$\rho_{\text{cross term}} \propto \frac{\sin\,(k\eta+\theta)}{\eta^{5/2}},$$

$$\frac{\rho_{\text{cross term}}}{\rho_{KE}} \sim \sqrt{k\eta}\sin\,(k\eta+\theta)$$

$$\rho_\phi \sim a^{-2}\phi'^2 \sim a^{-6}, \rho_{\delta\phi} \sim a^{-2}(\delta\phi')^2 \sim a^{-4}$$

$$\delta\rho_\phi = a^{-2}(\phi'\delta\phi' + \Phi\phi'^2)$$

$$a^{-2}(\phi'\delta\phi') \sim a^{-5}, a^{-2}(\Phi\phi'^2) \sim a^{-9}.$$

$$\rho_{\delta\phi} \sim a^{-2}(\phi'\delta\phi') \sim \sqrt{\rho_\phi\cdot\rho_{\delta\phi}} \sim a^{-5}$$

$$\Delta_k \propto (k\eta)^2 \sim a^4$$

$$L = \int d^4x\sqrt{-g}\frac{1}{\mathcal{V}^{4/3}}\partial_\mu\chi\partial^\mu\chi$$

$$L = \int d^4x\frac{1}{t^{1/3}}\partial_\mu\chi\partial^\mu\chi$$

$$\partial_\mu\left(t^{-1/3}\partial^\mu\chi\right) = 0$$

$$\ddot{\chi} - \frac{1}{3t}\dot{\chi} + \frac{\nabla^2\chi}{t^{2/3}} = 0$$

$$\chi(t) = A + Bt^{4/3}$$

$$\chi_k'' - \frac{1}{\eta}\chi_k' + k^2\chi_k = 0$$

$$\chi_k = A(k\eta)J_1(k\eta) + B(k\eta)Y_1(k\eta).$$

$$\chi = A + B\eta^2$$

$$\chi_k \sim \alpha\sqrt{k\eta}\sin\,(k\eta+\beta),$$

$$\rho_\chi = \frac{1}{\mathcal{V}^{4/3}}\frac{\dot{\chi}^2}{2} + \frac{1}{\mathcal{V}^{4/3}}\frac{(\nabla\chi)^2}{2a^2}.$$

$$\rho_{\chi_k} \sim \frac{|\alpha|^2}{\eta^2} \propto a^{-4}$$

$$L = \int d^4x\sqrt{-g}\left(\frac{1}{2}\partial_\mu U\partial^\mu U - V(U)\right)$$

$$L = \int d^4x\sqrt{-g}\left(\frac{1}{2}\partial_\mu U\partial^\mu U - \frac{\mu_U^2}{2t^2}U^2\right)$$

$$\ddot{U} + 3H\dot{U} = -\frac{\mu_U^2}{t^2}U,$$

$$U(t) = \tilde{\alpha}\cos\left(\mu_U \ln\, t + \tilde{\beta}\right)$$

$$\frac{\dot{U}^2}{2} + \frac{\mu_U^2 U^2}{2t^2} \propto \frac{1}{t^2} = \frac{1}{a^6}$$

$$ds^2 = -a^2(\eta)(1+2\Psi)d\eta^2 + a^2(\eta)(1-2\Phi)(dx_1^2 + dx_2^2 + dx_3^2)$$

$$\delta\rho \equiv -\delta T_0^0 = \delta\left(-\frac{1}{2}g^{00}\phi^2 + V(\phi)\right) = \frac{1}{a^2}(\phi'\delta\phi' - \Phi\phi^2) + \frac{dV(\phi)}{d\phi}\delta\phi$$

$$\delta P \equiv \delta T_i^i = \delta\left(-\frac{1}{2}g^{00}\phi'^2 - V(\phi)\right) = \frac{1}{a^2}(\phi'\delta\phi' - \Phi\phi'^2) - \frac{dV(\phi)}{d\phi}\delta\phi.$$

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + \frac{d^2V(\phi)}{d\phi^2}a^2\delta\phi - 4\phi'\Phi' + 2\frac{dV(\phi)}{d\phi}a^2\Phi = 0,$$

$$\delta_r'' - \frac{1}{3}\nabla^2\delta_r - \frac{4}{3}\nabla^2\Phi - 4\Phi'' = 0$$

$$\nabla^2\Phi - 3\mathcal{H}(\Phi' + \mathcal{H}\Phi) = \frac{a^2}{2M_P^2}(\delta\rho_k + \delta\rho_p + \delta\rho_r)$$

$$\Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2)\Phi = \frac{a^2}{2M_P^2}\left(\delta\rho_k - \delta\rho_p + \frac{1}{3}\delta\rho_r\right)$$

$$\delta\phi_k'' + 2\mathcal{H}\delta\phi_k' + k^2\delta\phi_k + \lambda^2 a^2\frac{V(\phi)}{M_P^2}\delta\phi_k = 4\phi'\Phi_k' + 2\lambda a^2\frac{V(\phi)}{M_P}\Phi_k,$$

$$\Phi_k'' + 4\mathcal{H}\Phi_k' + 2\left(\mathcal{H}' + \mathcal{H}^2 + \frac{k^2}{6}\right)\Phi_k = \frac{1}{3M_P^2}\left(\phi'\delta\phi_k' - \Phi_k\phi'^2 + 2\lambda a^2\frac{V(\phi)}{M_P}\delta\phi_k\right).$$

$$y = \frac{\rho_r}{\rho_\phi} = \frac{a^2}{a_{eq}^2} = \varepsilon a^2$$

$$\frac{dy}{d\eta} = 2y\mathcal{H} \text{ and } \mathcal{H} = \frac{\varepsilon}{2\eta_0}\frac{\sqrt{1+y}}{y},$$

$$\mathcal{H}' = -\mathcal{H}^2\left(1 + \frac{y}{1+y}\right)$$

$$F'(\eta) = 2y\mathcal{H}\frac{\mathrm{d}F}{\mathrm{d}y}\ F''(\eta) = 4y^2\mathcal{H}^2\frac{\mathrm{d}^2F}{\mathrm{d}y^2} + \frac{2y^2\mathcal{H}^2}{1+y}\frac{\mathrm{d}F}{\mathrm{d}y}.$$

$$\begin{cases} 2y\frac{\mathrm{d}^2\Phi}{\mathrm{d}y^2} + \left(4 + \frac{y}{1+y}\right)\frac{\mathrm{d}\Phi}{\mathrm{d}y} + \frac{2\eta_0^2k^2}{3\varepsilon^2}\frac{y}{1+y}\Phi = \frac{\sqrt{6}}{3M_P\sqrt{1+y}}\frac{\mathrm{d}\delta\phi}{\mathrm{d}y} \\ 2y\frac{\mathrm{d}^2\delta\phi}{\mathrm{d}y^2} + \left(2 + \frac{y}{1+y}\right)\frac{\mathrm{d}\delta\phi}{\mathrm{d}y} + \frac{2\eta_0^2k^2}{\varepsilon^2}\frac{y}{1+y}\delta\phi = \frac{4\sqrt{6}M_P}{\sqrt{1+y}}\frac{\mathrm{d}\Phi}{\mathrm{d}y} \end{cases}$$

$$\begin{cases} 2y\frac{\mathrm{d}^2\Phi}{\mathrm{d}y^2} + \left(4 + \frac{y}{1+y}\right)\frac{\mathrm{d}\Phi}{\mathrm{d}y} = \frac{\sqrt{6}}{3M_P\sqrt{1+y}}\frac{\mathrm{d}\delta\phi}{\mathrm{d}y} \\ 2y\frac{\mathrm{d}^2\delta\phi}{\mathrm{d}y^2} + \left(2 + \frac{y}{1+y}\right)\frac{\mathrm{d}\delta\phi}{\mathrm{d}y} = \frac{4\sqrt{6}M_P}{\sqrt{1+y}}\frac{\mathrm{d}\Phi}{\mathrm{d}y} \end{cases}$$

$$q(y) \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}y}$$

$$2y(1+y)\frac{\mathrm{d}^2q}{\mathrm{d}y} + (8+11y)\frac{\mathrm{d}q}{\mathrm{d}y} + 10q = 0$$

$$q(y) = -\frac{9c_1(y(\sqrt{y+1}-3) + 4(\sqrt{y+1}-1))}{24y^3\sqrt{y+1}} - \frac{8c_3(y\sqrt{y+1} - 3y + 4\sqrt{y+1} - 4) + 9c_4(3y+4)}{24y^3\sqrt{y+1}}$$

$$\Phi(y) = \frac{8c_3\left(2y^2 + y - 2\sqrt{y+1} + 2\right) + 9c_1(y - 2\sqrt{y+1} + 2) + 18c_4\sqrt{y+1}}{24y^2}.$$

$$c_1 = c_4 = 0$$

$$\frac{\Phi(y\to\infty)}{\Phi(y\to 0)} = \frac{8}{9}$$

$$\mathcal{R} = \Phi - \frac{y+1}{2y+3}\left(\Phi + 2y\frac{\mathrm{d}\Phi}{\mathrm{d}y}\right) = \frac{3c_1}{8(2y+3)} + c_3.$$

$$\delta\phi(y) = \frac{(9c_1 + 8c_3)(y-2)\sqrt{y+1} + 2(9c_1 + 8c_3 - 9c_4)}{4\sqrt{6}y^2} + c_2$$

$$\delta_\phi = 2\frac{\delta\phi'}{\phi'} - 2\Phi = \frac{-3c_1\left(y^2 - 3y + 6\sqrt{y+1} - 6\right) + 8c_3\left(-y^2 + y - 2\sqrt{y+1} + 2\right) + 18c_4\sqrt{y+1}}{4y^2}$$

$$\delta_r = -\frac{\delta_\phi}{y} - \frac{2(y+1)}{y}\left(\Phi + 2y\frac{\mathrm{d}\Phi}{\mathrm{d}y}\right)$$
$$= \frac{8c_3\left(-y^2 + y - 2\sqrt{y+1} + 2\right) + 9c_1\left(y - 2\sqrt{y+1} + 2\right) + 18c_4\sqrt{y+1}}{6y^2}.$$

$$\begin{cases} 2y\frac{\mathrm{d}^2\Phi}{\mathrm{d}y^2} + 5\frac{\mathrm{d}\Phi}{\mathrm{d}y} + \frac{2\eta_0^2k^2}{3\varepsilon^2}\Phi = \frac{\sqrt{6}}{3M_P\sqrt{y}}\frac{\mathrm{d}\delta\phi}{\mathrm{d}y} \\ 2y\frac{\mathrm{d}^2\delta\phi}{\mathrm{d}y^2} + 3\frac{\mathrm{d}\delta\phi}{\mathrm{d}y} + \frac{2\eta_0^2k^2}{\varepsilon^2}\phi = \frac{4\sqrt{6}M_P}{\sqrt{y}}\frac{\mathrm{d}\Phi}{\mathrm{d}y} \end{cases}$$

$$\frac{\Phi(x)}{\delta\phi(x)} \sim \mathcal{O}\left(x^{-b}\right) \text{ with } b > 0$$

$$\delta\phi = \frac{c_1}{\sqrt{y}}\cos\left(\frac{2\eta_0 k\sqrt{y}}{\varepsilon}\right) + \frac{c_2}{\sqrt{y}}\sin\left(\frac{2\eta_0 k\sqrt{y}}{\varepsilon}\right),$$

$$\Phi = \frac{\left(\sqrt{6}\pi c_2 + c_3\right)}{y}\cos\left(\frac{2\eta_0 k\sqrt{y}}{\sqrt{3}\varepsilon}\right) + \frac{c_4}{y}\cos\left(\frac{2\eta_0 k\sqrt{y}}{\sqrt{3}\varepsilon}\right) + \mathcal{O}\left(\frac{1}{y^2}\right).$$

$$\sqrt{y} \simeq \frac{\varepsilon\eta}{2\eta_0}$$

$$\Phi \simeq \frac{c_3'}{a^2(\eta)}\cos\left(\frac{k\eta}{\sqrt{3}}\right) + \frac{c_4'}{a^2(\eta)}\sin\left(\frac{k\eta}{\sqrt{3}}\right)$$

$$\delta\phi \simeq \frac{c_1'}{a(\eta)}\cos(k\eta) + \frac{c_2'}{a(\eta)}\sin(k\eta).$$

$$\Delta_r \simeq c_5\cos\left(\frac{k\eta}{\sqrt{3}}\right) + c_6\sin\left(\frac{k\eta}{\sqrt{3}}\right)$$

$$y = \frac{\rho_m}{\rho_\phi} = \frac{a^3}{a_{eq}^3} = \varepsilon a^3$$

$$\frac{dy}{d\eta} = 3y\mathcal{H} \text{ and } \mathcal{H} = \frac{\varepsilon^{2/3}}{2\eta_0}\frac{\sqrt{1+y}}{y^{2/3}}.$$

$$\mathcal{H}' = -\mathcal{H}^2\frac{y+4}{2y+2},$$

$$G'(\eta) = 3y\mathcal{H}\frac{\mathrm{d}G}{\mathrm{d}y}\ G''(\eta) = 9\mathcal{H}^2y^2\frac{\mathrm{d}^2G}{\mathrm{d}y^2} + 3y\mathcal{H}^2\frac{5y+2}{2y+2}\frac{\mathrm{d}G}{\mathrm{d}y}.$$

$$\Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2)\Phi = \frac{a^2}{2M_P^2}\left(\delta\rho_k - \delta\rho_p\right)$$

$$\begin{cases} 3y\dfrac{\mathrm{d}^2\Phi}{\mathrm{d}y^2}+\dfrac{11y+8}{2y+2}\dfrac{\mathrm{d}\Phi}{\mathrm{d}y}=\dfrac{\sqrt{6}}{2M_P\sqrt{1+y}}\dfrac{\mathrm{d}\delta\phi}{\mathrm{d}y} \\ 3y\dfrac{\mathrm{d}^2\delta\phi}{\mathrm{d}y^2}+\dfrac{9y+6}{2y+2}\dfrac{\mathrm{d}\delta\phi}{\mathrm{d}y}+\dfrac{4k^2\eta_0^2}{3\varepsilon^{4/3}}\dfrac{y^{1/3}}{y+1}\delta\phi=\dfrac{4\sqrt{6}M_P}{\sqrt{1+y}}\dfrac{\mathrm{d}\Phi}{\mathrm{d}y}. \end{cases}$$

$$\begin{cases} 3y\dfrac{\mathrm{d}^2\Phi}{\mathrm{d}y^2}+\dfrac{11y+8}{2y+2}\dfrac{\mathrm{d}\Phi}{\mathrm{d}y}=\dfrac{\sqrt{6}}{2M_P\sqrt{1+y}}\dfrac{\mathrm{d}\delta\phi}{\mathrm{d}y} \\ 3y\dfrac{\mathrm{d}^2\delta\phi}{\mathrm{d}y^2}+\dfrac{9y+6}{2y+2}\dfrac{\mathrm{d}\delta\phi}{\mathrm{d}y}=\dfrac{4\sqrt{6}M_P}{\sqrt{1+y}}\dfrac{\mathrm{d}\Phi}{\mathrm{d}y} \end{cases}$$

$$r(y)\equiv\frac{\mathrm{d}\Phi(y)}{\mathrm{d}y}$$

$$6y(1+y)\frac{\mathrm{d}^2r(y)}{\mathrm{d}y^2}+(20+29y)\frac{\mathrm{d}r(y)}{\mathrm{d}y}+22r(y)=0$$

$$r(y)=\frac{8\left(c_1+3c_3(y+2)-\dfrac{c_4\sqrt{y+1}(5y+8)}{y^{4/3}}\right)}{48y(y+1)}$$
$$+\frac{\frac{1}{5}(5y+8)\left({}_2F_1\left(\frac{5}{6},1;\frac{7}{3};-y\right)(3c_3(y-4)-5c_1)\right)-18c_3(y+1)\,{}_2F_1\left(\frac{5}{6},2;\frac{7}{3};-y\right)}{48y(y+1)}$$

$$\Phi(y)=-\frac{1}{24}(y+1)(5c_1+6c_3)_2F_1\left(1,\frac{11}{6};\frac{7}{3};-y\right)+\frac{c_1}{3}+c_3+\frac{c_4\sqrt{y+1}}{y^{4/3}}$$

$$\frac{\Phi(y\to\infty)}{\Phi(y\to 0)}=\frac{4}{5}$$

$$\begin{cases} 3y\dfrac{\mathrm{d}^2\Phi}{\mathrm{d}y^2}+\dfrac{11}{2}\dfrac{\mathrm{d}\Phi}{\mathrm{d}y}=\dfrac{\sqrt{6}}{2M_P\sqrt{y}}\dfrac{\mathrm{d}\delta\phi}{\mathrm{d}y} \\ 3y\dfrac{\mathrm{d}^2\delta\phi}{\mathrm{d}y^2}+\dfrac{9}{2}\dfrac{\mathrm{d}\delta\phi}{\mathrm{d}y}+\dfrac{4k^2\eta_0^2}{3\varepsilon^{4/3}y^{2/3}}\delta\phi=\dfrac{4\sqrt{6}M_P}{\sqrt{1+y}}\dfrac{\mathrm{d}\Phi}{\mathrm{d}y} \end{cases}$$

$$\Phi(y)=c_1+\frac{c_2}{y^{5/6}}+\mathcal{O}\left(\frac{1}{y}\right)$$

$$\delta\phi(y)=\frac{c_3}{y^{1/3}}\sin\left(\frac{4k\eta_0y^{1/6}}{\varepsilon^{2/3}}\right)+\frac{c_4}{y^{1/3}}\cos\left(\frac{4k\eta_0y^{1/6}}{\varepsilon^{2/3}}\right)+\mathcal{O}\left(\frac{1}{\sqrt{y}}\right).$$

$$\Phi(a)=c_1+c_2a^{-5/2}$$

$$\begin{cases} \delta\phi''+\dfrac{2}{\eta}\delta\phi'+\left(k^2+\dfrac{4}{\eta^2}\right)\delta\phi=\dfrac{8M_P}{\lambda\eta}\left(2\Phi'+\dfrac{\Phi}{\eta}\right) \\ \Phi''+\dfrac{4}{\eta}\Phi'+\dfrac{\Phi}{3}\left(k^2+\dfrac{16}{\lambda^2\eta^2}\right)=\dfrac{4}{3M_P\lambda\eta}\left(\delta\phi'+\dfrac{2\delta\phi}{\eta}\right). \end{cases}$$

$$x \equiv k\eta \; \delta\tilde{\phi} \equiv \frac{\delta\phi_k(k\eta)}{M_P} \; \tilde{\Phi}(x) \equiv \Phi_k(k\eta).$$

$$\begin{cases} x^2\delta\tilde{\phi}''(x) + 2x\delta\tilde{\phi}'(x) + (x^2+4)\delta\tilde{\phi}(x) = \frac{8}{\lambda}\left(2x\tilde{\Phi}'(x) + \tilde{\Phi}(x)\right) \\ x^2\tilde{\Phi}''(x) + 4x\tilde{\Phi}'(x) + \frac{\tilde{\Phi}(x)}{3}\left(x^2 + \frac{16}{\lambda^2}\right) = \frac{4}{3\lambda}\left(x\delta\tilde{\phi}'(x) + 2\delta\tilde{\phi}(x)\right) \end{cases}$$

$$\delta\tilde{\phi}(x) = x^\alpha \; \tilde{\Phi}(x) = Kx^\beta.$$

$$\begin{cases} (4+\alpha+\alpha^2)\lambda - 8K(1+2\alpha) = 0 \\ (16 + 9\alpha\lambda^2 + 3\alpha^2\lambda^2)K - 4(2+\alpha)\lambda = 0 \end{cases}$$

$$\alpha = \left\{0, -3, \frac{-1 \pm \sqrt{\frac{64}{\lambda^2} - 15}}{2}\right\}.$$

$$\delta\phi(\eta) = c_a\eta^{-\frac{1}{2}}\cos\left(\frac{\sqrt{15 - \frac{64}{\lambda^2}}}{2}\log(\eta)\right) + c_b\eta^{-\frac{1}{2}}\sin\left(\frac{\sqrt{15 - \frac{64}{\lambda^2}}}{2}\log(\eta)\right)$$

$$\begin{cases} x^2\delta\tilde{\phi}''(x) + 2x\delta\tilde{\phi}'(x) + x^2\delta\tilde{\phi}(x) = \frac{8}{\lambda}\left(2x\tilde{\Phi}'(x) + \tilde{\Phi}(x)\right) \\ x^2\tilde{\Phi}''(x) + 4x\tilde{\Phi}'(x) + \frac{x^2}{3}\tilde{\Phi}(x) = \frac{4}{3\lambda}\left(x\delta\tilde{\phi}'(x) + 2\delta\tilde{\phi}(x)\right) \end{cases}$$

$$\frac{\tilde{\Phi}(x)}{\delta\tilde{\phi}(x)} \sim \mathcal{O}\left(x^{-b}\right) \text{ with } b > 0$$

$$\delta\tilde{\phi}(x) = \frac{c_1\cos(x) + c_2\sin(x)}{x},$$

$$\tilde{\Phi}(x) = \frac{c_3\sin\left(\frac{x}{\sqrt{3}}\right) + c_4\cos\left(\frac{x}{\sqrt{3}}\right)}{x^2} + \frac{8\pi c_1\cos\left(\frac{x}{\sqrt{3}}\right) + 2c_1\sin(x) - 2c_2\cos(x)}{\lambda x^2} + \mathcal{O}\left(\frac{1}{x^3}\right)$$

$$\mathcal{R}(\eta) = \frac{3}{2}\Phi + \frac{\Phi'}{2\mathcal{H}}$$

$$\mathcal{R}(\eta) = \frac{c_3\cos\left(\frac{k\eta}{\sqrt{3}}\right) - c_4\sin\left(\frac{k\eta}{\sqrt{3}}\right)}{\sqrt{3}k\eta} + \frac{6c_1\cos(k\eta) + 6c_2\sin(k\eta) - 8\sqrt{3}\pi c_1\sin\left(\frac{k\eta}{\sqrt{3}}\right)}{3\lambda k\eta}$$

$$\Delta = -\frac{2}{3}(k\eta)^2\Phi(\eta) = -\frac{2}{3}\left(c_4\cos\left(\frac{k\eta}{\sqrt{3}}\right) + c_3\sin\left(\frac{k\eta}{\sqrt{3}}\right)\right) + \frac{4c_2\cos(k\eta) - 4c_1\left(\sin(k\eta) + 4\pi\cos\left(\frac{k\eta}{\sqrt{3}}\right)\right)}{3\lambda}$$

$$\delta_r \equiv \frac{\delta\rho_r}{\rho_r} = \frac{3\lambda^2}{2(\lambda^2-4)}\left(\Phi''\eta^2 + 4\Phi\frac{4-\lambda^2}{\lambda^2} - (k\eta)^2\Phi - \frac{4}{\lambda}\delta\phi'\eta\right)$$

$$\delta_r = \frac{4(c_4\lambda + 8\pi c_1)\cos\left(\frac{k\eta}{\sqrt{3}}\right)}{3\lambda} - \frac{4}{3}c_3\sin\left(\frac{k\eta}{\sqrt{3}}\right)$$

$$\phi(\eta) = \phi_t + \frac{4}{\lambda}\log(\eta) + \sum_k a_{\pm k}\frac{e^{\pm ik\eta}}{(k\eta)},$$

$$\phi'(\eta) = \frac{4}{\lambda\eta} + \sum_k (\pm ik)a_{\pm k}\frac{e^{\pm ik\eta}}{(k\eta)}$$

$$\frac{1}{a^2}(\phi'\delta\phi') \sim a^{-4}, \frac{1}{a^2}(\Phi\phi^2) \sim a^{-6}, \frac{dV(\phi)}{d\phi}\delta\phi \sim a^{-5}$$

$$\left\langle \begin{aligned} \Phi'' + \frac{6}{\eta}\Phi' + \frac{18}{\lambda^2\eta^2}\Phi &= \frac{3}{\lambda\eta M_p}\left(\frac{3}{\eta}\delta\phi + \delta\phi'\right) \\ \delta\phi'' + \frac{4}{\eta}\delta\phi' + k^2\delta\phi &= \frac{12}{\lambda\eta}M_p\left(2\Phi' + \frac{3}{\eta}\Phi\right) \end{aligned} \right|$$

$$\delta\phi(\eta) = \eta^\alpha, \Phi(\eta) = K\eta^\alpha$$

$$\alpha = \left\{0, -\frac{5}{2}, \frac{3\left(\lambda \pm \sqrt{24-7\lambda^2}\right)}{4\lambda}\right\}.$$

$$\delta\phi(\eta) = c_a\eta^{-\frac{3}{2}}\cos\left(\frac{3\sqrt{7\lambda^2-24}}{2\lambda}\log\eta\right) + c_b\eta^{-\frac{3}{2}}\sin\left(\frac{3\sqrt{7\lambda^2-24}}{2\lambda}\log\eta\right)$$

$$\rho_{\delta\phi_1} \sim a^{-3}, \rho_{\delta\phi_2} \sim a^{-8}, \rho_{\delta\phi_{3,4}} \sim a^{-9/2}$$

$$\Phi(\eta) = A(k\eta)^{-a}, \delta\phi(\eta) = B(k\eta)^{-b}f(k\eta),$$

$$a = \frac{5}{2} \pm \frac{\sqrt{25\lambda^2-72}}{2\lambda}$$

$$b = 2, f(k\eta) = \cos(k\eta) \text{ or } f(k\eta) = \sin(k\eta)$$

$$\begin{aligned} \Phi(\eta) &= A_1\eta^{\frac{1}{2}\left(\frac{\sqrt{25\lambda^2-72}}{\lambda}-5\right)} + A_2\eta^{\frac{1}{2}\left(-\frac{\sqrt{25\lambda^2-72}}{\lambda}-5\right)} \\ &= A_1(k\eta)^{\frac{\sqrt{\frac{59}{3}}}{2}-\frac{5}{2}} + A_2(k\eta)^{-\frac{5}{2}-\frac{\sqrt{\frac{59}{3}}}{2}} \approx A_1(k\eta)^{-0.28} + A_2(k\eta)^{-4.72} \end{aligned}$$

$$\delta\phi(\eta) = \frac{B_1\cos(k\eta) + B_2\sin(k\eta)}{(k\eta)^2}$$

$$\mathcal{R}(\eta) = \left(-\frac{A_1\sqrt{25\lambda^2-72}}{6\lambda} + \frac{5}{6}A_1\right)(k\eta)^{\frac{\sqrt{25\lambda^2-72}}{2\lambda}-\frac{5}{2}} + \left(\frac{A_2\sqrt{25\lambda^2-72}}{6\lambda} + \frac{5}{6}A_2\right)(k\eta)^{-\frac{\sqrt{25\lambda^2-72}}{2\lambda}-\frac{5}{2}}.$$

$$\delta_m'' + \mathcal{H}\delta_m' = 3\Phi'' + 3\mathcal{H}\Phi' - k^2\Phi,$$

$$\delta_m \approx \frac{k^2\lambda^2\eta^{\frac{\sqrt{25\lambda^2-72}}{2\lambda}} - \frac{1}{2}}{18 - 6\lambda^2} A_1,$$

$$\Delta \sim \mathcal{H}^{-2}\Phi \sim \eta^2\Phi \sim \eta^{\frac{1}{2}\left(\frac{\sqrt{25\lambda^2-72}}{\lambda}-1\right)}.$$

$$\Delta \sim \eta^2 \sim a,$$

$$\Delta \sim \eta^{\sqrt{\frac{59}{3}+\frac{1}{6}(-3-\sqrt{177})}} \approx \eta^{1.72} \sim a^{0.86},$$

$$\phi'' + 2\mathcal{H}\phi' + k^2\phi = -a^2\frac{dV}{d\phi}(\phi)$$

$$\phi(\eta) = \phi_t + \frac{6}{\lambda}\log(\eta) + \sum_k a_{\pm k}\frac{e^{\pm ik\eta}}{(k\eta)^2},$$

$$\phi'(\eta) = \frac{6}{\lambda\eta} + \sum_k (\pm ik)a_{\pm k}\frac{e^{\pm ik\eta}}{(k\eta)^2}.$$

$$x = \frac{1}{2}\left(5 - \frac{\sqrt{25\lambda^2 - 72}}{\lambda}\right)$$

$$\delta\rho_\phi = a^{-2}(\phi'\delta\phi' - \Phi\phi'^2) + (dV(\phi)/d\phi)\delta\phi$$

$$\frac{1}{a^2}(\phi'\delta\phi') \sim a^{-\frac{7}{2}}, \frac{1}{a^2}(\Phi\phi'^2) \sim a^{-3-\frac{x}{2}}, \frac{dV(\phi)}{d\phi}\delta\phi \sim a^{-4}$$

$$\delta\rho_\phi \sim a^{-3-\frac{x}{2}} \approx a^{-3.14}$$

$$\rho \sim a^{-3}$$

$$\Phi'' + \frac{6}{\eta}\Phi' = 0$$

$$\Phi = c_1 + c_2\eta^{-5}$$

$$\Delta \sim \eta^2 \sim a.$$

$$\Phi(\eta) = \frac{C_1}{k\eta}J_1(k\eta) + \frac{C_2}{k\eta}Y_1(k\eta),$$

$$C_1 = \frac{3iH_{\text{inf}}}{4M_P}\frac{1}{J_0(k\eta_0)\sqrt{\varepsilon_V k^3}} \quad \text{and} \quad C_2 = 0.$$

$$\Delta(\eta) = -\frac{2iH_{\text{inf}}}{M_P\sqrt{\varepsilon_V k^3}}\frac{k\eta J_1(k\eta)}{J_0(k\eta_0)}$$

$$\Phi(\eta) \simeq \frac{C_3 \sin\left(\frac{k\eta}{\sqrt{3}}\right) + C_4 \sin(k\eta + C_5)}{a(\eta)^2}$$

$$\Delta \simeq C_3 \sin\left(\frac{k\eta}{\sqrt{3}}\right) + C_4 \sin(k\eta + C_5)$$

$$\Phi \simeq C_6 a(\eta)^{\frac{1}{4}\left(\frac{\sqrt{25\lambda^2-72}}{\lambda}-5\right)}$$

$$\Delta \simeq C_7 a(\eta)^{\frac{1}{4}\left(\frac{\sqrt{25\lambda^2-72}}{\lambda}-1\right)},$$

$$\frac{2\pi}{g_{YM}^2} = \int_{\Sigma_i} e^{-\varphi}\sqrt{g},$$

$$\mathcal{L}_{SM} = \sum_i \frac{1}{g_i^2(\Phi)} \mathrm{Tr}\left(F_{i,\mu\nu}F^{i,\mu\nu}\right) + \sum_i K_f(\Phi)\bar{\psi}\gamma^i\partial_i\psi + K_h(\Phi)\partial_\mu H \partial^\mu H^* + \sum_i Y_{ab}(\Phi) H \bar{\psi}^a \psi^b,$$

$$V(\Phi) = V_0 e^{-\sqrt{\frac{27}{2}}\Phi}\left(1 - \alpha\Phi^{3/2}\right) + \varepsilon e^{-\sqrt{6}\Phi},$$

$$n_\psi(t_{\mathrm{reheat}}) = n_\psi(t_{\mathrm{moduli}})\left(\frac{a(t_{\mathrm{moduli}})}{a(t_{\mathrm{reheat}})}\right)^3$$

$$n_\psi(t_{\mathrm{reheat}}) = n_\psi(t_{\mathrm{moduli}})\left(\frac{t_{\mathrm{moduli}}}{t_{\mathrm{reheat}}}\right)^2$$

$$n_\psi(t_{\mathrm{reheat}}) = n_\psi(t_{\mathrm{moduli}})\left(\frac{\Gamma_\Phi}{m_\Phi}\right)^2$$

$$T^4 \sim 3H^2 M_P^2, T \sim (M_P m_\Phi)^{1/2} \sim 10^{12}\mathrm{GeV}\left(\frac{m_\Phi}{10^6\mathrm{GeV}}\right)^{\frac{1}{2}}$$

$$s_{\mathrm{moduli}} \sim T^3 \sim M_P^{3/2} m_\Phi^{3/2}$$

$$T_{\mathrm{reheat}} \sim \Gamma_\Phi^{1/2} M_P^{1/2} \text{ with } s_{\mathrm{reheat}} \sim \Gamma_\Phi^{3/2} M_P^{3/2},$$

$$s_{\mathrm{reheat}} = s_{\mathrm{moduli}}\left(\frac{\Gamma_\Phi}{m_\Phi}\right)^{3/2}$$

$$\frac{n_\psi(t_{\mathrm{reheat}})}{s(t_{\mathrm{reheat}})} = \frac{n_\psi(t_{\mathrm{moduli}})}{s(t_{\mathrm{moduli}})}\left(\frac{\Gamma_\Phi}{m_\Phi}\right)^{1/2}$$

$$y=\left(\frac{\Gamma_{\Phi}}{m_{\Phi}}\right)^{1/2}$$

$$\Gamma\sim\frac{m_{\Phi}^{3}}{M_{P}^{2}}$$

$$y_{\text{dilution}}\sim\frac{m_{\Phi}}{M_{P}}\sim10^{-12}\left(\frac{m_{\Phi}}{10^{6}\text{GeV}}\right)$$

$$\Gamma_{\Phi}\sim\frac{m_{3/2}^{4}\alpha_{\text{loop}}^{2}}{m_{\Phi}M_{P}^{2}}\sim\left(\alpha_{\text{loop}}\mathcal{V}\right)^{2}\frac{m_{\Phi}^{3}}{M_{P}^{2}}\gg\frac{m_{\Phi}^{3}}{M_{P}^{2}},$$

$$y_{\text{dilution}}\sim\left(\alpha_{\text{loop}}\mathcal{V}\right)\frac{m_{\Phi}}{M_{P}}\sim10^{-12}\left(\alpha_{\text{loop}}\mathcal{V}\right)\left(\frac{m_{\Phi}}{10^{6}\text{GeV}}\right)$$

$$\tau_{\text{Hawking}}\sim\frac{M_{PBH}^{3}}{M_{P}^{4}},$$

$$M_{PBH}\sim\frac{M_{P}^{2}}{H_{\text{formation}}}$$

$$x(N)=x_{0}+e^{\frac{3}{4}(\gamma-2)N}\eta_{1}\sin\left(\omega N+\tilde{\alpha}_{1}\right),$$
$$y(N)=y_{0}+e^{\frac{3}{4}(\gamma-2)N}\eta_{2}\sin\left(\omega N+\tilde{\alpha}_{2}\right),$$

$$\eta_{1}=\sqrt{c_{1}^{2}+\frac{1}{4\omega^{2}}\left[2\beta c_{2}+(\alpha-\delta)c_{1}\right]^{2}}$$
$$\eta_{2}=\sqrt{c_{2}^{2}+\frac{1}{4\omega^{2}}\left[2\gamma c_{1}-(\alpha-\delta)c_{2}\right]^{2}}$$

$$\tan\ \alpha_{1}=\frac{2\omega c_{1}}{2\beta c_{2}+(\alpha-\delta)c_{1}},\tan\ \alpha_{2}=\frac{2\omega c_{2}}{2\gamma c_{1}-(\alpha-\delta)c_{2}}$$

$$\phi(N)=\phi_{0}+\sqrt{6}M_{P}x_{0}N+\frac{\sqrt{6}M_{P}\eta_{1}}{\sqrt{\omega^{2}+\frac{9}{16}(\gamma-2)^{2}}}e^{\frac{3}{4}(\gamma-2)N}\sin\left(\omega N+\tilde{\alpha}_{1}+\alpha'\right)$$

$$\tan\ \alpha'=-\frac{4}{3}\frac{\omega}{\gamma-2}$$

$$\delta H'=-\frac{3}{2}\gamma\delta H-3\sqrt{\frac{3}{2}\frac{\gamma}{\lambda}}H_{0}e^{-3\gamma N/2}\left[(2-\gamma)(x(N)-x_{0})-\sqrt{(2-\gamma)\gamma}(y(N)-y_{0})\right].$$

$$\delta H'=-\frac{3}{2}\gamma\delta H-3\sqrt{\frac{3}{2}\frac{\gamma}{\lambda}}H_{0}e^{-3(\gamma+2)N/4}\left[(2-\gamma)\eta_{1}\sin\left(\omega N+\tilde{\alpha}_{1}\right)-\sqrt{(2-\gamma)\gamma}\eta_{2}\sin\left(\omega N+\tilde{\alpha}_{2}\right)\right]$$

$$\delta H = H\left[c_3 + \frac{12}{9(\gamma-2)^2+16\omega^2}\sqrt{\frac{3}{2}\frac{\gamma}{\lambda}}H_0 e^{-3(\gamma+2)N/4}h(N)\right],$$

$$h(N) = 3(2-\gamma)\left[(2-\gamma)\eta_1\sin(\omega N+\tilde{\alpha}_1) - \sqrt{(2-\gamma)\gamma}\eta_2\sin(\omega N+\tilde{\alpha}_2)\right] + 4\omega\left[(2-\gamma)\eta_1\cos(\omega N+\tilde{\alpha}_1) - \sqrt{(2-\gamma)\gamma}\eta_2\cos(\omega N+\tilde{\alpha}_2)\right].$$

$$\delta H = H\left[c_3 + A' e^{\frac{3}{4}(\gamma-2)N}\sin(\omega N+\alpha')\right],$$

$$\rho_\gamma = \frac{N_\gamma}{V}\langle E_\gamma(t)\rangle = \frac{2N_0\left(1-e^{-\alpha(t-t_0)}\right)}{V_0}\frac{a_0^3}{a^3}\langle E_\gamma(t)\rangle,$$

$$\langle E_\gamma(t)\rangle = \frac{1}{n_{\gamma,\,\mathrm{tot}}}\int_{4\pi/m}^{\frac{4\pi a(t)}{m a(t_0)}} d\lambda\,\tilde{n}(t,\lambda)E_\gamma(\lambda)$$

$$n_{\gamma,\,\mathrm{tot}} = \int_{4\pi/m}^{\frac{4\pi a(t)}{m a(t_0)}} d\lambda\,\tilde{n}_\gamma(t,\lambda) = 2n_0\left(1-e^{-\alpha(t-t_0)}\right)$$

$$d\lambda = \frac{4\pi}{m}\frac{a(t)}{a_c(t_c)} - \frac{4\pi}{m}\frac{a(t)}{a_c(t_c+dt)} \approx \frac{4\pi}{m}\frac{aH_c}{a_c}dt,$$

$$\tilde{n}_\gamma d\lambda = \dot{n}_\gamma(a_c)\frac{m}{4\pi H_c}\frac{a_c d\lambda}{a(t)}.$$

$$da_c/d\lambda = -m a_c^2/4\pi a$$

$$\langle E_\gamma(t)\rangle = \frac{1}{n_{\gamma,\,\mathrm{tot}}}\frac{m}{2a(t)}\int_{a(t_0)}^{a(t)} da_c\frac{\dot{n}_\gamma(a_c)}{H_c} = \frac{1}{n_{\gamma,\,\mathrm{tot}}}\frac{m}{2a(t)}\int_{t_0}^{t} dt_c a(t_c)\dot{n}_\gamma(t_c).$$

$$\dot{n}_\gamma(t_c) = 2\alpha n_0 e^{-\alpha(t_c-t_0)}.$$

$$H^2 = \frac{H_0^2 a_0^3}{a^3}e^{\alpha t_0}\left(e^{-\alpha t} + \frac{\alpha}{a}\int_{t_0}^{t} dt_c a(t_c)e^{-\alpha t_c}\right),$$

$$\ddot{a} + \frac{\dot{a}^2}{a} = \frac{H_0^2 a_0^3}{2a^2}e^{-\alpha(t-t_0)}.$$

$$-3\mathcal{H}(\Phi' + \mathcal{H}\Psi) = 4\pi G a^2(\rho_1\delta_1 + \rho_2\delta_2).$$

$$y \equiv \frac{\rho_1}{\rho_2} \sim a^{-3(\gamma_1-\gamma_2)},$$

$$\frac{\delta_1}{\gamma_1} = \frac{\delta_2}{\gamma_2}$$

$$-3\mathcal{H}(\Phi' + \mathcal{H}\Phi) = 4\pi G a^2\rho_2\delta_2\left(1+\frac{1}{y}\frac{\gamma_1}{\gamma_2}\right) = \frac{3}{2}\mathcal{H}^2\frac{y}{y+1}\delta_2\left(1+\frac{1}{y}\frac{\gamma_1}{\gamma_2}\right),$$

$$8\pi G\rho_2/3 = (\mathcal{H}^2 y)/(a^2(y+1))$$

$$-3\left(3(\gamma_1-\gamma_2)y\frac{d\Phi}{dy}+\Phi\right)=\frac{3}{2}\frac{y}{y+1}\left(1+\frac{1}{y}\frac{\gamma_1}{\gamma_2}\right)\delta_2$$

$$-(\gamma_1(9\gamma_1-6\gamma_2+2)+(6\gamma_1-3\gamma_2+2)\gamma_2 y^2+2(6\gamma_1^2-3\gamma_2\gamma_1+\gamma_1+\gamma_2)y)\Phi'(y)$$
$$-6(\gamma_1-\gamma_2)y(y+1)(\gamma_1+\gamma_2 y)\Phi''(y)+2(\gamma_2-\gamma_1)\Phi(y)=0.$$

$$\Phi(y)=\frac{2(\gamma_1-\gamma_2)\left(3\gamma_1-2\sqrt{y+1}\,{}_2F_1\left(\frac{1}{2},\frac{3\gamma_1+2}{6\gamma_1-6\gamma_2};\frac{3\gamma_1+2}{6\gamma_1-6\gamma_2}+1;-y\right)+2\right)}{3\gamma_1+2}$$

$$\lim_{y\to 0}\Phi(y)=\frac{6\gamma_1(\gamma_1-\gamma_2)}{3\gamma_1+2}$$

$$\sqrt{y+1}\,{}_2F_1\left(\frac{1}{2},b;b+1,-y\right)=b\sqrt{y+1}\int_0^1\frac{t^{b-1}}{(1+ty)^{1/2}}\to b\int_0^1 t^{b-3/2}=\frac{b}{b-1/2}$$

$$\lim_{y\to\infty}\Phi(y)=\frac{6(\gamma_1-\gamma_2)\gamma_2}{3\gamma_2+2}$$

$$\lim_{y\to\infty}\Phi(y)/\Phi(0)=\frac{\gamma_2(3\gamma_1+2)}{\gamma_1(3\gamma_2+2)}$$

$$\mathcal{R}\simeq\frac{3\gamma_i+2}{3\gamma_i}\Phi\ \text{ for }\ k\eta\ll 1$$

$$a(\eta)=a_0\eta^{\frac{2}{3\gamma-2}},$$

$$\phi=\phi_t+\frac{6\gamma}{(3\gamma-2)\lambda}\log(\eta)$$

$$V=V_0e^{-\lambda\phi}=-\frac{18(\gamma-2)\gamma\eta^{\frac{6\gamma}{2-3\gamma}}}{(2-3\gamma)^2\lambda^2}.$$

$$\nabla^2\Phi-3\mathcal{H}(\Phi'+\mathcal{H}\Phi)\ =\frac{a^2}{2M_P^2}(\delta\rho_\phi+\delta\rho_\gamma)$$
$$\Phi''+3\mathcal{H}\Phi'+(2\mathcal{H}'+\mathcal{H}^2)\Phi\ =\frac{a^2}{2M_P^2}(\delta P_\phi+(\gamma-1)\delta\rho_\gamma),$$

$$\delta\rho_\phi=\frac{1}{a^2}(\phi'\delta\phi'-\Phi\phi'^2)+\frac{dV(\phi)}{d\phi}\delta\phi$$
$$=\frac{6\gamma\eta^{\frac{6\gamma}{2-3\gamma}}((3\gamma-2)\eta\lambda\delta\phi'(\eta)+3(\gamma-2)\lambda\delta\phi(\eta)-6\gamma\Phi(\eta))}{(2-3\gamma)^2\lambda^2}$$
$$\delta P_\phi=\frac{1}{a^2}(\phi'\delta\phi'-\Phi\phi'^2)-\frac{dV(\phi)}{d\phi}\delta\phi$$
$$=-\frac{6\gamma\eta^{\frac{6\gamma}{2-3\gamma}}((2-3\gamma)\eta\lambda\delta\phi'(\eta)+3(\gamma-2)\lambda\delta\phi(\eta)+6\gamma\Phi(\eta))}{(2-3\gamma)^2\lambda^2}.$$

$$\begin{cases} \Phi''(\eta) + \frac{6\gamma\Phi'(\eta)}{(3\gamma-2)\eta} + \Phi(\eta)\left((\gamma-1)k^2 - \frac{18(\gamma-2)\gamma^2}{(3\gamma-2)^2\eta^2\lambda^2}\right) = -\frac{3(\gamma-2)\gamma((3\gamma-2)\eta\delta\phi'(\eta)+3\gamma\delta\phi(\eta))}{(3\gamma-2)^2\eta^2\lambda}. \\ \delta\phi''(\eta) + \frac{4\delta\phi'(\eta)}{(3\gamma-2)\eta} + \delta\phi(\eta)\left(k^2 - \frac{18(\gamma-2)\gamma}{(3\gamma-2)^2\eta^2}\right) = \frac{12\gamma(2(3\gamma-2)\eta\Phi'(\eta) - 3(\gamma-2)\Phi(\eta))}{(3\gamma-2)^2\eta^2\lambda}. \end{cases}$$

$$\Phi(\eta) = c_1 + c_2\eta^{\frac{3\gamma+2}{2-3\gamma}} + \gamma\eta^{\frac{3(\gamma-2)}{2(3\gamma-2)}}(c_3\cos(\omega\log\eta) + c_4\sin(\omega\log\eta)),$$
$$\delta\phi(\eta) = \frac{2c_1}{\lambda} - 3\gamma c_2\eta^{\frac{3\gamma+2}{2-3\gamma}} + \frac{4(3\gamma-2)\lambda\omega}{9(\gamma-2)}\eta^{\frac{3(\gamma-2)}{2(3\gamma-2)}}(c_3\cos(\omega\log\eta) + c_4\sin(\omega\log\eta)),$$

$$\omega = \frac{3\sqrt{-(3\gamma-2)^2(\gamma-2)((9\gamma-2)\lambda^2 - 24\gamma^2)}}{2(3\gamma-2)^2\lambda}.$$

$$\Phi(\eta) = c_1 + c_2\eta^{\frac{3\gamma+2}{2-3\gamma}} + \gamma\eta^{\frac{3(\gamma-2)}{2(3\gamma-2)}}(c_3\eta^{\omega} + c_4\eta^{-\omega}),$$

$$\delta\phi(\eta) = \frac{2c_1}{\lambda} - 3\gamma c_2\eta^{\frac{3\gamma+2}{2-3\gamma}} + \frac{4(3\gamma-2)\lambda\omega}{9(\gamma-2)}\eta^{\frac{3(\gamma-2)}{2(3\gamma-2)}}(c_3\eta^{\omega} + c_4\eta^{-\omega}),$$

$$\omega = \frac{3\sqrt{(3\gamma-2)^2(\gamma-2)((9\gamma-2)\lambda^2 - 24\gamma^2)}}{2(3\gamma-2)^2\lambda}$$

$$\delta\phi(\eta) = (k\eta)^{\frac{-2}{3\gamma-2}}f(k\eta), \Phi(\eta) = (k\eta)^{\frac{-3\gamma}{3\gamma-2}}g(k\eta),$$

$$f''(k\eta) + f(k\eta) = 0,$$

$$\delta\phi(\eta) = A_1(k\eta)^{\frac{-2}{3\gamma-2}}\cos(k\eta).$$

$$(2-3\gamma)^2\eta k\lambda(g''(k\eta) + (\gamma-1)g(k\eta)) = 3A_1(\gamma-2)\gamma\sin(k\eta)$$

$$g(k\eta) = A_1\frac{3\gamma\sin(k\eta)}{(3\gamma-2)\lambda} + A_2\cos(k\sqrt{\gamma-1}\eta),$$

$$\Phi(\eta) = (k\eta)^{\frac{-3\gamma}{3\gamma-2}}\left[A_1\frac{3\gamma\sin(k\eta)}{(3\gamma-2)\lambda} + A_2\cos(k\sqrt{\gamma-1}\eta)\right],$$

$$\phi'' + 2\mathcal{H}\phi' + k^2\phi = -a^2V'(\phi)$$

$$\phi(\eta) = \phi_t + \frac{6\gamma}{(3\gamma-2)\lambda}\log(\eta) + \sum_k a_{\pm k}\frac{e^{\pm ik\eta}}{(k\eta)^{\frac{2}{3\gamma-2}}},$$

$$\phi'(\eta) = \frac{6\gamma}{(3\gamma-2)\lambda}\frac{1}{\eta} + \sum_k (\pm ik)a_{\pm k}\frac{e^{\pm ik\eta}}{(k\eta)^{\frac{2}{3\gamma-2}}},$$

$$\delta\rho_\phi = a^{-2}(\phi'\delta\phi' - \Phi\phi'^2) + (dV(\phi)/d\phi)\delta\phi$$

$$\frac{1}{a^2}(\phi'\delta\phi') \sim a^{-\frac{4+3\gamma}{2}}, \frac{1}{a^2}(\Phi\phi'^2) \sim a^{-\frac{9\gamma}{2}}, \frac{dV(\phi)}{d\phi}\delta\phi \sim a^{-1-3\gamma},$$

$$\Phi(\eta) = f(k\eta)e^{\sqrt{1-\gamma}k\eta}, \delta\phi(\eta) = g(k\eta)e^{\sqrt{1-\gamma}k\eta}$$

$$f(k\eta) \gg f'(k\eta), f''(k\eta), g(k\eta), g'(k\eta), g''(k\eta)$$

$$f(k\eta) = (\sqrt{1-\gamma}(3\gamma-2)k\eta)^{\frac{3\gamma}{2-3\gamma}}c_1$$

$$g(k\eta) = -\frac{24\gamma\left((\sqrt{1-\gamma}(3\gamma-2)k\eta)^{\frac{2}{2-3\gamma}} - 3\gamma(\sqrt{1-\gamma}(3\gamma-2)k\eta)^{\frac{3\gamma}{2-3\gamma}}\right)}{(2-3\gamma)^2(\gamma-2)\lambda k\eta^2}c_1 + e^{-\sqrt{1-\gamma}(k\eta)}(c_2\cos(k\eta) + c_3\sin(k\eta))$$

$$\Phi(\eta) = c_1 e^{k\eta} + c_2 e^{-k\eta}$$
$$\delta\phi(\eta) = (c_3 k\eta + c_4)\cos(k\eta) + (c_4 k\eta - c_3)\sin(k\eta)$$

$$\chi_k''(\eta) + \left[\frac{2a'}{a} + \frac{f'}{f}\right]\chi_k'(\eta) + k^2\chi_k(\eta) = 0$$

$$a(\eta) = \eta^{\frac{2}{3\gamma-2}}, f(\eta) = \exp\left(\sqrt{\frac{8}{3}\frac{\Phi}{M_p}}\right) \sim \eta^{-\frac{4\sqrt{6}\gamma}{(3\gamma-2)\lambda}}$$

$$k^2\chi_k(\eta) + \chi_k''(\eta) + \frac{4(\lambda - \sqrt{6}\gamma)\chi_k'(\eta)}{(3\gamma-2)\eta\lambda} = 0.$$

$$\chi_k(\eta) = \eta^{-\frac{3\gamma\lambda + 4\sqrt{6}\gamma - 6\lambda}{4\lambda - 6\gamma\lambda}}(c_1 J_\alpha(k\eta) + c_2 Y_\alpha(k\eta)),$$

$$\alpha = -\frac{\sqrt{96\gamma^2 + 9(\gamma-2)^2\lambda^2 + 24\sqrt{6}(\gamma-2)\gamma\lambda}}{2(3\gamma-2)\lambda}.$$

$$\rho = \frac{f(\eta)\chi_k'(\eta)^2}{a(\eta)} \sim a^{-4}.$$

$$\mathcal{S} = \frac{3\mathcal{H}\gamma_r\gamma_\phi\Omega_r}{\gamma}\left(\frac{\delta\rho_\phi}{\rho_\phi'} - \frac{\delta\rho_r}{\rho_r'}\right) = \Omega_r\frac{\gamma_\phi\delta_r - \gamma_r\delta_\phi}{\gamma},$$

$$\gamma = \Omega_\phi\gamma_\phi + \Omega_r\gamma_r$$

$$\Gamma = \frac{3\mathcal{H}\gamma_\phi c_\phi^2}{1 - c_\phi^2}\left(\frac{\delta\rho_\phi}{\rho_\phi'} - \frac{\delta P_\phi}{P_\phi'}\right),$$

$$c_\phi^2 = P_\phi'/\rho_\phi'$$

$$\frac{d\mathbf{v}}{dN} = (M_0 + k^2\mathcal{H}^{-2}M_1 + k^4\mathcal{H}^{-4}M_2)\mathbf{v}, \mathbf{v} = (\Phi, \mathcal{R}, \mathcal{S}, \Gamma)^T,$$

$$M_0 = \begin{pmatrix} -\frac{3\gamma}{2} - 1 & \frac{3\gamma}{2} & 0 & 0 \\ 0 & 0 & \frac{\Omega_\phi(\omega_r - c_\phi^2)}{\gamma} & \frac{(1-c_\phi^2)\Omega_r}{\gamma} \\ 0 & 0 & \frac{3\gamma_r\Omega_r(\omega_r - c_\phi^2)}{\gamma} + 3(\omega_\phi - \omega_r) & \frac{3(1-c_\phi^2)\gamma_r\Omega_r}{\gamma} \\ 0 & 0 & -\frac{3\gamma}{2} & 3\left(\omega_\phi - \frac{\gamma}{2}\right) \end{pmatrix},$$

$$M_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -\frac{2c^2}{3\gamma} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & -\gamma_\phi & -\frac{1}{3} & -\frac{1}{3} \end{pmatrix}, M_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{2\gamma_\phi}{9\gamma} & 0 & 0 & 0 \\ -\frac{2\gamma_\phi}{9\gamma} & 0 & 0 & 0 \end{pmatrix}$$

$$c^2 = \frac{(\rho_\phi + P_\phi)c_\phi^2 + (\rho_r + P_r)c_r^2}{(\rho_\phi + P_\phi) + (\rho_r + P_r)}$$

$$y = \frac{\rho_\gamma}{\rho_\phi} = \frac{a^{3(2-\gamma)}}{a_{eq}^{3(2-\gamma)}}$$

$$3(2-\gamma)y\frac{d\mathbf{v}}{dy} = \frac{d\mathbf{v}}{dN} = (M_0 + k^2\mathcal{H}^{-2}M_1 + k^4\mathcal{H}^{-4}M_2)\mathbf{v} \equiv \mathcal{M}\mathbf{v}, \mathbf{v} = (\Phi, \mathcal{R}, \mathcal{S}, \Gamma)^T$$

$$\Omega_\phi(y) = \frac{1}{1+y}, \Omega_r(y) = \frac{y}{1+y}$$

$$M_0 = \begin{pmatrix} -\frac{3(\gamma y+2)}{2(y+1)} - 1 & \frac{3\gamma y + 6}{2y+2} & 0 & 0 \\ 0 & 0 & \frac{\gamma - 2}{\gamma y + 2} & 0 \\ 0 & 0 & -\frac{6(\gamma-2)}{\gamma y + 2} & 0 \\ 0 & 0 & -\frac{3(\gamma y + 2)}{2(y+1)} & -\frac{3(\gamma-2)y}{2(y+1)} \end{pmatrix}$$

$$S(y) = \frac{c_1 y}{\gamma y + 2}, \Gamma(y) = \sqrt{2}c_2\sqrt{y+1} - \frac{c_1}{\gamma - 2}, \mathcal{R}(y) = \frac{c_1}{6\gamma + 3\gamma^2 y} + c_3$$

$$\Phi(y) = \frac{\sqrt{y+1}}{8\gamma}\Bigg(3\gamma^2 c_3\, {}_2F_1\left(\frac{1}{2}, \frac{4}{6-3\gamma}; 1 + \frac{4}{6-3\gamma}; -y\right)$$
$$+ (c_1 - 3(\gamma-2)\gamma c_3)\, {}_2F_1\left(\frac{3}{2}, \frac{4}{6-3\gamma}; 1 + \frac{4}{6-3\gamma}; -y\right) + 8\gamma c_4 y^{\frac{4}{3(\gamma-2)}}\Bigg)$$

$$\Phi(y) = \frac{c_3\left(2y^2 + y - 2\sqrt{y+1} + 2\right)}{3y^2}$$

$$\Phi(y) = c_3 - \frac{1}{4} c_3 (y+1)_2 F_1\left(1, \frac{11}{6}; \frac{7}{3}; -y\right)$$

$$M_0 = \begin{pmatrix} -\frac{3\gamma}{2} - 1 & \frac{3\gamma}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{3(2-\gamma)}{\lambda^2} \\ 0 & 0 & 0 & 3(2-\gamma)\left(1 - \frac{3\gamma}{\lambda^2}\right) \\ 0 & 0 & -\frac{3\gamma}{2} & 3\left(\frac{\gamma}{2} - 1\right) \end{pmatrix}$$

$$\mathbf{v} = \sum_i c_i \mathbf{v}_i a^{\lambda_i}$$

$$\lambda_1, \lambda_2, \lambda_{3,4} = \left\{0, -\frac{3\gamma}{2} - 1, \frac{3\left((\gamma-2) \pm \sqrt{(\gamma-2)(-24\gamma^2 + 9\gamma\lambda^2 - 2\lambda^2)}\lambda\right)}{4\lambda}\right\}$$

$$\mathbf{v}_1 = \left(\frac{3\gamma}{3\gamma+2}, 1,0,0\right), \mathbf{v}_2 = (1,0,0,0)$$

$$M_0 = \begin{pmatrix} -3 & 2 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{\lambda^2} \\ 0 & 0 & 0 & 2\left(1 - \frac{4}{\lambda^2}\right) \\ 0 & 0 & -2 & -1 \end{pmatrix}$$

$$\mathbf{v} = \sum_i c_i \mathbf{v}_i a^{\lambda_i}, \mathbf{v} = (\Phi, \mathcal{R}, \mathcal{S}, \Gamma)^T$$

$$\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_{3,4} = \left\{(2,3,0,0), (1,0,0,0), \left(-\frac{4}{5\lambda^2 \pm i\lambda\nu - 16}, -\frac{4}{\lambda^2 \pm i\lambda\nu}, \frac{-\lambda \pm i\nu}{4\lambda}, 1\right)\right\}$$

$$\lambda_1, \lambda_2, \lambda_{3,4} = \left\{0, -3, \frac{-1 \pm \sqrt{\frac{64}{\lambda^2} - 15}}{2}\right\}$$

$$M_0 = \begin{pmatrix} -\frac{5}{2} & \frac{3}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{\lambda^2} \\ 0 & 0 & 0 & 3\left(1 - \frac{3}{\lambda^2}\right) \\ 0 & 0 & -\frac{3}{2} & -\frac{3}{2} \end{pmatrix}.$$

$$\mathbf{v} = \sum_i c_i \mathbf{v}_i a^{\lambda_i}$$

$$\lambda_1, \lambda_2, \lambda_{3,4} = \left\{0, -\frac{5}{2}, \frac{3\left(\lambda \pm \sqrt{24 - 7\lambda^2}\right)}{4\lambda}\right\},$$

$$\mathbf{v}_1 = \left(\frac{3}{5}, 1,0,0\right), \mathbf{v}_2 = (1,0,0,0), \mathbf{v}_{3,4} = \left(\frac{6}{-7\lambda^2 \pm i\lambda\nu + 18}, \frac{4}{\lambda(-\lambda \pm i\nu)}, -\frac{\lambda \pm i\nu}{2\lambda}, 1\right)$$

$$\frac{dZ_1(x)}{dx} = Z_0(x) - \frac{Z_1(x)}{x} \text{ for } Z = J, Y.$$

$$R_k = -\mathcal{H}(v + B)$$

$$\Delta_k^\phi = \delta\rho - \bar{\rho}' \frac{\delta\phi}{\phi'}$$

$$\delta\rho \equiv -\delta T_0^0 = \delta\left(-\frac{1}{2} g^{00} \phi'^2\right) = \frac{1}{a^2} (\phi' \delta\phi' - A\phi'^2),$$

$$y(\mu) = y(M_s) + \beta \ln\left(\frac{M_s}{\mu}\right) \equiv y\left(\frac{M_P}{\sqrt{\mathcal{V}}}\right) + \beta \ln\left(\frac{M_s}{\mu}\right)$$

$$S_{\text{eff}} = -\int \mathrm{d}^4x \sqrt{g} \left(\frac{1}{8\pi G}\Lambda + \frac{1}{16\pi G} R + aR_{\mu\nu}R^{\mu\nu} + bR^2 + cR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \cdots\right)$$

$$S_{\mathrm{B}} = -\int_M \mathrm{d}^{d+1}x \sqrt{-g} \left(\frac{1}{8\pi G}\Lambda + \frac{1}{16\pi G} R\right) + \frac{1}{8\pi G} \int_{\partial M^-}^{\partial M^+} \mathrm{d}^d x \sqrt{h} K.$$

$$l_\Lambda = \sqrt{\frac{(d-1)(d-2)}{2\Lambda}},$$

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + l_\Lambda^2 \cosh^2 (t/l_\Lambda) \mathrm{d}\Omega_{d-1}^2$$

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \exp(2\tau/l_\Lambda) \mathrm{d}\vec{x}^2 = \frac{l_\Lambda^2}{\eta^2} (-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2)$$

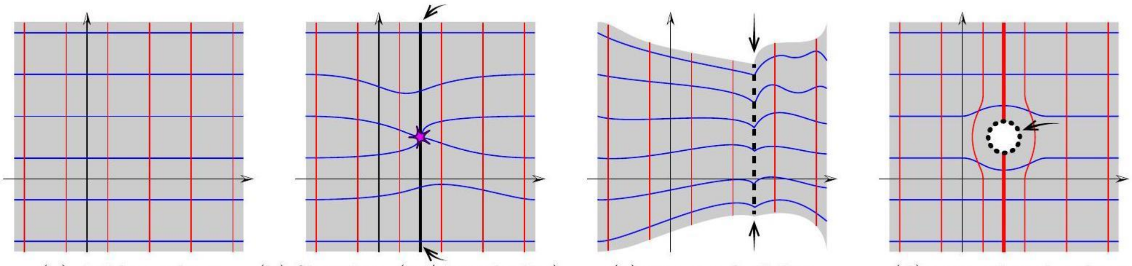
$$\mathrm{d}s^2 = -(1 - r^2/l_\Lambda^2)\mathrm{d}t^2 + \left(1 - r^2/l_\Lambda^2\right)^{-1} \mathrm{d}r^2 + r^2\, \mathrm{d}\Omega_{d-2}^2$$

$$S_{\text{eff}} = \int \sqrt{|g|} \left(\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} G_{ij}(\phi) \partial_\mu \phi^i \partial_\nu \phi^j - V(\phi) + \cdots\right)$$

$$S_{\text{eff}} = \int \sqrt{|g|} \left(\frac{1}{2} R - \frac{1}{2} \|\partial\phi\|^2 - \sum_p c_p e^{a_p \phi} F_p^2\right) + CS$$

$$V = -\int \sqrt{g_6} R_6 + \int \sqrt{g_6} c_p e^{a_p \phi} F_p^2$$

$$V_{\text{src}} = \mu \int \sqrt{|g_n|}$$



(a) rigid product (b) fibration (w/singularity) (c) non-analyticity (d) exceptional subset

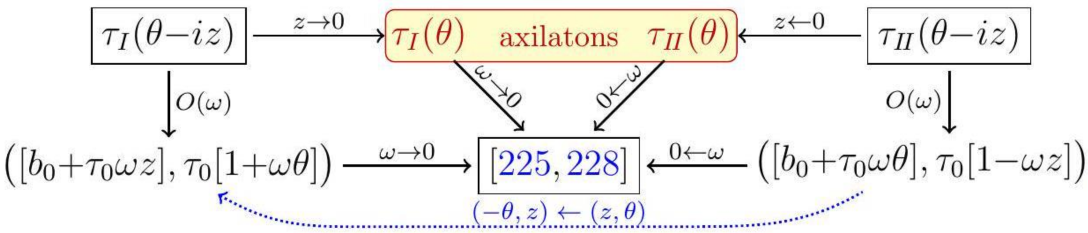
$$S_{\text{eff}} = \frac{1}{2\kappa^2}\int d^D x\sqrt{-g}\left(R - \mathcal{G}_{\tau\bar{\tau}}g^{\mu\nu}\partial_\mu\tau\partial_\nu\bar{\tau} + \cdots\right)$$

$$\tilde{T}_{\mu\nu} \overset{\text{def}}{=} T_{\mu\nu} - \frac{1}{D-2}g_{\mu\nu}T, \text{ so } R_{\mu\nu} = \kappa^2\tilde{T}_{\mu\nu}(\tau,\bar{\tau}),$$
$$g^{\mu\nu}\left[(\nabla_\mu\nabla_\nu\tau) + \Gamma^{\tau}_{\tau\tau}(\nabla_\mu\tau)(\nabla_\nu\tau)\right] = 0,$$

$$ds^2 = A^2(z)\bar{g}_{ab}\,dx^a\,dx^b + \ell^2 B^2(z)(dz^2 + d\theta^2)$$
$$\bar{g}_{ab}\,dx^a\,dx^b = -dx_0^2 + e^{2\sqrt{\Lambda_b}x_0}(\,dx_1^2 + \cdots + dx_{D-3}^2)$$

$$\tau_I(\theta) = b_0 + ig_s^{-1}e^{\omega\theta}$$

$$\tau_{II}(\theta) = (b_0 \pm g_s^{-1}\tanh(\omega\theta)) \pm ig_s^{-1}\text{sech}(\omega\theta).$$



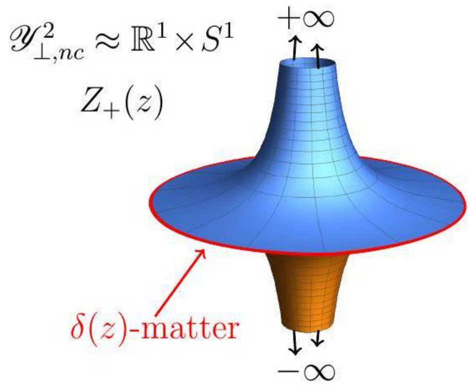
$$g_s^{(\text{eff})}[\tau_I(\pi - \epsilon)] \ll 1 \text{ whereas } g_s^{(\text{eff})}[\tau_I(\pi + \epsilon)] \gg 1.$$

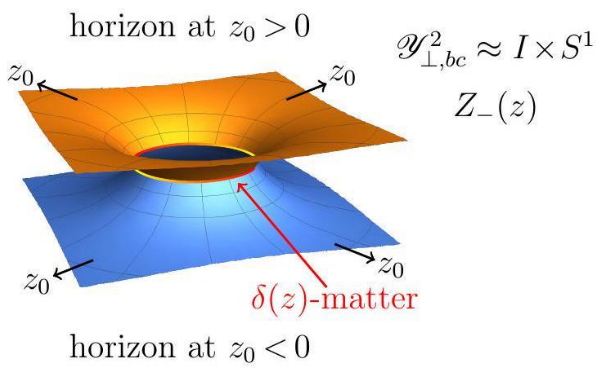
$$\mathcal{W}^{D-3,1}_{z=0}\colon g_s^{D-2} = g_s^D\sqrt{\alpha'/V_\perp} \ll 1$$

$$R_{\mu\nu} \propto \tilde{T}_{\mu\nu}(\tau,\bar{\tau}) = \text{diag}\left[0,\cdots,0,\frac{1}{4}\omega^2\ell^{-2}\right]$$

$$A(z) = Z(z)\left(1 - \frac{\omega^2 z_0^2(D-3)}{24(D-1)(D-2)}Z(z)^2 + O(\omega^4)\right)$$
$$B(z) = \frac{1}{\ell z_0\sqrt{\Lambda_b}}\left(1 - \frac{\omega^2 z_0^2}{8(D-1)}Z(z)^2 + O(\omega^4)\right)$$
$$Z(z) = Z_\pm(z) := 1 \pm |z|/z_0, z_0 > 0, \text{ and } \Lambda_b := \frac{\omega^2}{2(D-2)\ell^2} \geqslant 0.$$

$$\tilde{A}(z) = Z(z)^{\frac{1}{(D-2)}} \text{ and } \tilde{B}(z) = Z(z)^{\frac{-(D-3)}{2(D-2)}} e^{2\xi\left(1-Z(z)^2\right)/z_0}, \text{ when } \Lambda_b = 0.$$





$$z_0 = -\left.\frac{h}{h'}\right|_{z=0}, \xi := \left.\left(\frac{h''}{2h'} - \frac{\omega^2 h}{8h'}\right)\right|_{z=0}, \ell = \Lambda_b^{-1/2} \left.\sqrt{\frac{h'' h^{-(D-4)/(D-2)}}{(D-2)(D-3)}}\right|_{z=0},$$

$$\Lambda_b = \frac{\left(\omega^2 - \omega_{GCB}^2 A^2\big|_{z=0}\right)}{4\ell^2 (D-2)(D-3)} \stackrel{\text{def}}{=} \frac{\Delta\omega^2}{4\ell^2 (D-2)(D-3)}$$

$$\omega_{GCB}^2 \stackrel{\text{def}}{=} 8\xi/z_0$$

$$G_N^{(D-2)} = \left(M_P^{(D-2)}\right)^{-(D-4)}$$

$$G_N^{(D-2)} = \left(M_P^{(D)}\right)^{-(D-2)} V_\perp^{-1}$$

$$V_\perp \approx \frac{\pi \ell^2}{2z_0}\left(\frac{2z_0}{\xi}\right)^{\frac{D-3}{2(D-2)}} e^{\xi/2z_0}\gamma\left(\frac{D-3}{4(D-2)};\frac{\xi}{2z_0}\right) \sim \frac{\pi}{D-3}\frac{\ell}{\sqrt{\Lambda_b}}.$$

$$\left(M_P^{(D-2)}\right)^{D-4} = \left(M_P^{(D)}\right)^{D-2} 2\pi\ell^2 z_0^{-\frac{D-1}{2(D-2)}} e^{z_0}\gamma\left(\frac{D-3}{2(D-2)};\frac{1}{|z_0|}\right),\ \text{for}\ Z(z) = 1 \pm |z|/z_0$$

$$M_P^{(4)} = \sqrt{\zeta_0}|z_0|^{-\frac{5}{16}} e^{z_0/2}\frac{\left(M_P^{(6)}\right)^2}{M_\ell}, M_\ell \overset{\text{def}}{=} 1/\ell$$

$$0 \leqslant \zeta_0 \overset{\text{def}}{=} 2\pi\Gamma\left(\frac{3}{8};\frac{1}{|z_0|}\right) \leqslant 2\pi\Gamma\left(\frac{3}{8}\right) \approx 14.89$$

$$\Lambda_{D-2} = \Lambda_b / G_N^{(D-2)}$$

$$\Lambda_{D-2} \sim \left(\frac{\pi}{D-3}\right)^2 \left(M_P^{(D-2)}\right)^{D-2} \left(\ell M_P^{(D-2)}\right)^2 \left(\frac{M_P^{(D)}}{M_P^{(D-2)}}\right)^{2D-4}$$

$$\Lambda_4 \sim \left(M_P^{(4)}\right)^4 \left(\ell M_P^{(4)}\right)^2 \left(\frac{M_P^{(6)}}{M_P^{(4)}}\right)^8 \sim \left(M_P^{(4)}\right)^4 \left(\frac{M_P^{(6)}}{M_P^{(4)}}\right)^8$$

$$\ell \sim \left(M_P^{(4)}\right)^{-1}$$

$$M_\Lambda \sim \frac{\pi^2}{9\zeta_0}|z_0|^{5/4} e^{-2z_0}\frac{\left(M_P^{(4)}\right)^2}{\ell^2} = \frac{\pi^2}{9\zeta_0}|z_0|^{5/4} e^{-2z_0}\left(M_P^{(4)}\right)^2 M_\ell^2$$

$$\left(M_P^{(10)}\right)^{-1} = \left(M_P^{(6)}\right)^{-1} \sim (10\text{TeV})^{-1} \sim 10^{-19}\ \text{nm}$$

$$M_P^{(4)} \sim 10^{19}\text{GeV}$$

$$L \overset{\text{def}}{=} \Lambda_b^{-1/2} \sim 10^{41}\text{GeV}^{-1} \sim 10^{25}\ \text{nm}$$

$$S \overset{\text{def}}{=} \int_0^1 \mathrm{d}\tau \left[p\cdot\dot{x} - \frac{1}{2}N(p^2+m^2)\right]$$

$$Z_{\text{vac}}(m^2) = \exp\left[Z_{S^1}(m^2)\right].$$

$$Z_{\text{vac}}(m^2) = \langle 0|\exp(-iHT)|0\rangle = \exp(-i\rho_0 V_D)$$

$$\rho_0 = \frac{i}{V_D} Z_{S^1}(m^2)$$

$$Z_{S^1}(m^2) = V_D \int \frac{\mathrm{d}^D p}{(2\pi)^D}\int_0^\infty \frac{\mathrm{d}l}{2l} e^{-(p^2+m^2)l/2} = iV_D\int_0^\infty \frac{\mathrm{d}l}{2l}(2\pi l)^{-D/2} e^{-m^2 l/2}$$

$$\int_0^\infty \frac{\mathrm{d}l}{2l} e^{-(p^2+m^2)l/2} \to -\frac{1}{2}\log\left(p^2+m^2\right)$$

$$i\int_0^\infty \frac{\mathrm{d}l}{2l}\frac{\mathrm{d}p^0}{(2\pi)} e^{-(p^2+m^2)l/2} \to \frac{1}{2}E_p$$

$$\rho_0 = \int \mathrm{d}^3 p \frac{1}{2}E_p$$

$$\rho_0 = \int^\Lambda \mathrm{d}^3\, p \frac{1}{2}E_p$$

$$\rho_0 \sim \Lambda^4$$

$$Z_{S^1}(m^2) \stackrel{\text{def}}{=} iV_D \int_0^\infty \frac{\mathrm{d}l}{2l}(2\pi l)^{-D/2} e^{-m^2 l/2} = V_D \int \frac{\mathrm{d}^D p}{(2\pi)^D}\int_0^\infty \frac{\mathrm{d}l}{2l} e^{-(p^2+m^2)l/2}$$

$$m^2 = \frac{2}{\alpha'}(h+\tilde{h}-2)$$

$$\delta_{h,\tilde{h}} = \int_{-\pi}^{\pi}\frac{\mathrm{d}\theta}{2\pi} e^{i(h-\tilde{h})\theta}$$

$$\sum_i Z_{S^1}\left(m_i^2\right) = iV_D \int_0^\infty \frac{\mathrm{d}l}{2l}\int_{-\pi}^{\pi}\frac{\mathrm{d}\theta}{2\pi}(2\pi l)^{-D/2}\sum_i e^{-i(h_i+\tilde{h}_i-2)+i(h_i-\tilde{h}_i)\theta}$$

$$2\pi\tau \stackrel{\text{def}}{=} \theta + \frac{il}{\alpha'} \stackrel{\text{def}}{=} \tau_1 + i\tau_2, q \stackrel{\text{def}}{=} \exp\left(2\pi i\tau\right)$$

$$Z_{\text{string}} = \sum_i Z_{S^1}\left(m_i^2\right) = iV_D \int_R \frac{\mathrm{d}\tau\, \mathrm{d}\bar{\tau}}{4\tau_2}(4\pi^2\alpha'\tau_2)^{-D/2}\sum_i q^{h_i-1}\bar{q}^{\tilde{h}_i-1}$$

$$R\colon \tau_2 > 0, |\tau_1| < \frac{1}{2}$$

$$F\colon |\tau| > 1, |\tau_1| < \frac{1}{2}$$

$$S_{mp} \stackrel{\text{def}}{=} \int_0^1 \mathrm{d}\tau \left[p\cdot\dot{x} + \tilde{p}\cdot\dot{\tilde{x}} + \alpha' p\cdot\dot{\tilde{p}} + \frac{1}{2}N(p^2+\tilde{p}^2+\mathrm{m}^2) + \tilde{N}(p\cdot\tilde{p}-\mu)\right]$$

$$\{p_\mu, x^\nu\} = \delta_\mu^\nu, \{\tilde{p}_\mu, \tilde{x}^\nu\} = \delta_\mu^\nu, \{\tilde{x}_\mu, x^\nu\} = \pi\alpha'\delta_\mu^\nu$$

$$G(p,\tilde{p};p_i,\tilde{p}_i) \sim \delta^{(d)}(p-p_i)\delta^{(d)}(\tilde{p}-\tilde{p}_i)\frac{\delta(p\cdot\tilde{p}-\mu)}{p^2+\tilde{p}^2+\mathrm{m}^2-i\varepsilon}$$

$$\mathcal{H} \stackrel{\text{def}}{=} p^2+\tilde{p}^2+\mathrm{m}^2 = 0,$$
$$\mathcal{D} \stackrel{\text{def}}{=} p\cdot\tilde{p}-\mu = 0.$$

$$g^{\mu\nu}p_\mu p_\nu + g_{\mu\nu}\tilde{p}^\mu\tilde{p}^\nu = -\mathrm{m}^2$$

$$ds^2 = -dt^2 + a^2 d\mathbf{x}^2$$

$$S_{\text{str}}^{\text{ch}} = \frac{1}{4\pi}\int_\Sigma \mathrm{d}^2\sigma[\partial_\tau \mathbb{X}^A(\eta_{AB}(\mathbb{X}) + \omega_{AB}(\mathbb{X})) - \partial_\sigma \mathbb{X}^A H_{AB}(\mathbb{X})]\partial_\sigma \mathbb{X}^B$$

$$\mathbb{X}^A \stackrel{\text{def}}{=} (x^a/\lambda, \tilde{x}_a/\lambda)^T$$

$$\eta_{AB} \stackrel{\text{def}}{=} \begin{pmatrix} 0 & \delta \\ \delta^T & 0 \end{pmatrix}, H_{AB} \stackrel{\text{def}}{=} \begin{pmatrix} h & 0 \\ 0 & h^{-1} \end{pmatrix}, \omega_{AB} = \begin{pmatrix} 0 & \delta \\ -\delta^T & 0 \end{pmatrix},$$

$$\mathcal{H} = H_{AB}\partial_\sigma \mathbb{X}^A \partial_\sigma \mathbb{X}^A, \mathcal{D} = \eta_{AB}\partial_\sigma \mathbb{X}^A \partial_\sigma \mathbb{X}^A$$

$$[\mathbb{X}^A(\sigma), \mathbb{X}^B(\sigma')] = 2i\lambda^2[\pi\omega^{AB} - \eta^{AB}\theta(\sigma - \sigma')]$$

$$\begin{aligned} K(x_f, \tilde{p}_f, \ell_f, \tilde{\ell}_f; x_i, \tilde{p}_i, \ell_i, \tilde{\ell}_i) &= \langle x_f, \tilde{p}_f; \ell_f, \tilde{\ell}_f \mid x_i, \tilde{p}_i; \ell_i, \tilde{\ell}_i \rangle \\ &= \langle x_f, \tilde{p}_f | e^{-i(\ell_f - \ell_i)\hat{\mathcal{H}} - i(\tilde{\ell}_f - \tilde{\ell}_i)\hat{\mathcal{D}}} | x_i, \tilde{p}_i \rangle. \end{aligned}$$

$$\ell = \int_{\mathcal{C}} |e|(\tau), \tilde{\ell} = \int_{\mathcal{C}} \tilde{e}(\tau)$$

$$G(x_f, \tilde{p}_f; x_i, \tilde{p}_i) = \int [\mathrm{d}e\ \mathrm{d}\tilde{e}] \int_{x_i, \tilde{p}_i}^{x_f, \tilde{p}_f} [\mathrm{d}^d x\ \mathrm{d}^d \tilde{p}] \int [\mathrm{d}^d p\ \mathrm{d}^d \tilde{x}] e^{i\int_{\mathcal{C}} (p\cdot \mathrm{d}x - \tilde{x}\cdot \mathrm{d}\tilde{p} + \pi\alpha' p\cdot \mathrm{d}\tilde{p} - |e|\mathcal{H} - \tilde{e}\mathcal{D})}$$

$$G(x_f, \tilde{p}; x_i, \tilde{p}_i) \sim \delta^{(d)}(\tilde{p} - \tilde{p}_i) \int \frac{\mathrm{d}^d p}{(2\pi)^d} \int \mathrm{d}\ell\ \mathrm{d}\tilde{\ell} e^{-i\ell\mathcal{H} - i\tilde{\ell}\mathcal{D}} e^{ip\cdot(x_f - x_i)}$$

$$G(x, \tilde{p}; 0, \tilde{p}_i) \sim \delta^{(d)}(\tilde{p} - \tilde{p}_i) \int \frac{\mathrm{d}^d p}{(2\pi)^d} \frac{e^{ip\cdot x}}{p^2 + \tilde{p}^2 + m^2 - i\varepsilon} \delta(p \cdot \tilde{p} - \mu)$$

$$\int_0^l \mathrm{d}a \int_0^l \frac{\mathrm{d}\tau}{l} \delta(f(\tau) - a) = 1$$

$$V_{\tilde{p}} Z_{mp}(m^2, \mu) = \left(\int_{-\pi}^{\pi} \frac{\mathrm{d}\theta}{2\pi} e^{i(p\cdot\tilde{p} - \mu)l_s^2\theta/2}\right) \int \frac{\mathrm{d}^D \tilde{p}\ \mathrm{d}^D p}{(2\pi)^{2D}} \int_0^\infty \frac{\mathrm{d}l}{2l} e^{i(p^2 + \tilde{p}^2 + m^2)l/2}$$

$$Z_{mp}(m^2, \mu) = a_D \int \frac{\mathrm{d}^D p}{(2\pi)^D}$$

$$\Lambda\tilde{\Lambda} = \mu_{\text{vac}}$$

$$Z_{mp}(m^2, \mu) = a_D \int \frac{\mathrm{d}^D p}{(2\pi)^D} = a_D \Lambda^D = a_D \left(\frac{\mu_{\text{vac}}}{\tilde{\Lambda}}\right)^D$$

$$Z_{mp}(m^2, \mu) = a_D \left(\frac{\mu_{\text{vac}}}{\Lambda}\right)^D$$

$$\rho_0 = a_D \left(\frac{\mu_{\text{vac}}}{\tilde{\Lambda}}\right)^D$$

$$m^2 = \frac{2}{\alpha'}(N_L + N_R - 2) \text{ and } \mu = \frac{2}{\alpha'}(N_L - N_R)$$

$$V_{\tilde{p}} \sum_{L,R} Z_{mp}(m^2, \mu) \overset{\text{def}}{=} V_{\tilde{p}} Z_{ms}$$

$$V_{\tilde{p}} Z_{ms} = \sum_{L,R} \left(\int_{-\pi}^{\pi} \frac{\mathrm{d}\theta}{2\pi} e^{i[p\cdot\tilde{p} - (N_L - N_R)]l_s^2\theta/2}\right) \int \frac{\mathrm{d}^D\tilde{p}\, \mathrm{d}^D p}{(2\pi)^{2D}} \int_0^\infty \frac{\mathrm{d}l}{2l} e^{i[p^2 + \tilde{p}^2 + (N_L + N_R)]l/2}$$

$$Z_{ms} = b_D \int \frac{\mathrm{d}^D p}{(2\pi)^D} = b_D \Lambda^D = b_D \left(\frac{\mu_{\text{vac}}}{\Lambda}\right)^D$$

$$\rho_{ms} = b_D \Lambda^D = b_D \left(\frac{\mu_{\text{vac}}}{\tilde{\Lambda}}\right)^D$$

$$\int \mathrm{d}^4 x \int \mathrm{d}^4 p = c_4 l^4 \Lambda^4 = N$$

$$\lambda_{cc} = 8\pi\rho l_P^2 = 8\pi b_4 \Lambda^4 l_P^2.$$

$$\lambda_{cc} = 8\pi \frac{b_4}{c_4} N \left(\frac{l_P^2}{l^2}\right) \frac{1}{l^2}.$$

$$N \overset{\text{def}}{=} S = \frac{d_4 l^2}{l_P^2} \to N \frac{l_P^2}{d_4 l^2} = 1.$$

$$S_{4d} = -\iint \sqrt{-g(x)}\sqrt{-\tilde{g}(\tilde{x})}[R(x) + \tilde{R}(\tilde{x})],$$

$$S_{\text{eff}} = \frac{-1}{8\pi G} \int_X \sqrt{-g}\left(\lambda_{cc} + \frac{1}{2} R + O(R^2)\right)$$

$$\lambda_{cc} = 8\pi \left(\frac{b_4 d_4}{c_4}\right) \frac{1}{l^2}$$

$$\lambda_{cc} = 8\pi \left(\frac{b_4 d_4}{c_4}\right) \frac{1}{l^2} \overset{\text{def}}{=} 8\pi b_4 \Lambda^4 l_P^2 \to \Lambda = \left(\frac{d_4}{c_4}\right)^{1/4} \frac{1}{\sqrt{l l_P}}$$

$$\Lambda\tilde{\Lambda} = \mu_{\text{vac}} \to \mu_{\text{vac}} = \left(\frac{d_4}{c_4}\right)^{1/4} \frac{1}{\sqrt{l l_P} l_P}$$

$$l^{D+1}\Lambda^{D+1} \sim N, \qquad N \frac{l_P^{D-1}}{l^{D-1}} \sim 1, \qquad \lambda_{cc} \sim \frac{1}{l^2}$$

$$\Lambda^{D+1} \sim \frac{1}{l^2 l_P^{D-1}}, \qquad \tilde{\Lambda} \sim l_P^{-1}, \qquad \mu_{\text{vac}} \sim \Lambda\tilde{\Lambda}$$

$$m \stackrel{\text{def}}{=} \frac{l}{l_P}, n \stackrel{\text{def}}{=} \left(\frac{l}{l_P}\right)^{1/2}$$

$$2\pi V_{26} \int \frac{\mathrm{d}\tau^2}{\tau_2} (4\pi^2 \alpha' \tau_2)^{-13} \sum_i e^{-\pi\alpha' m_i^2 \tau_2}$$

$$l^4 \rightarrow \int_l \sqrt{-g}\, \mathrm{d}^4x$$

$$\Lambda^4 \rightarrow \frac{N}{\int_l \sqrt{-g}\, \mathrm{d}^4x}$$

$$\frac{1}{8\pi G} \int \sqrt{-h}\, \mathrm{d}^3xK$$

$$\int_l \sqrt{-h}\, \mathrm{d}^3x = 4\pi l^2 \int \mathrm{d}t(1 + O(1/l))$$

$$\frac{1}{8\pi G} \int \sqrt{-h}\, \mathrm{d}^3xK = \frac{1}{8\pi G} \frac{\partial}{\partial n} \int \sqrt{-h}\, \mathrm{d}^3x$$

$$4GN = 4\pi l^2 = \frac{\int_l \sqrt{-h}\, \mathrm{d}^3x}{\int \mathrm{d}t(1 + O(1/l))}$$

$$\lambda_{cc} = 8\pi G \frac{N}{\int_l \sqrt{-g}\, \mathrm{d}^4x}$$

$$\lambda_{cc} = \frac{8\pi^2 l^2}{\int_l \sqrt{-g}\, \mathrm{d}^4x}$$

$$N = \frac{\lambda_{cc} V_4}{8\pi G}$$

$$V_4 = al^4$$

$$N = \left(\frac{8\pi^2}{a}\right) \frac{1}{\lambda_{cc} G}$$

$$\lambda_{cc} \int_l \sqrt{-g}\, \mathrm{d}^4x = 8\pi GN = 2\pi N(4G)$$

$$\frac{1}{8\pi G} \lambda_{cc} \int_l \sqrt{-g}\, \mathrm{d}^4x$$

$$\frac{1}{2} \int_l R\sqrt{-g}\, \mathrm{d}^4x = 2\pi N(4G)$$

$$\int_l K\sqrt{-h} d^3x = 2\pi N(4G)$$

$$\text{Area}(l) \stackrel{\text{def}}{=} 4\pi l^2 = 4GN$$

$$N \to \infty, l \to \infty, \frac{N}{l^4} \stackrel{\text{def}}{=} 1/l_\Lambda^4 = \text{ד}$$

$$l \to \infty, l_P \to 0, l^2/l_P^2 \stackrel{\text{def}}{=} N \to \infty$$

$$N \stackrel{\text{def}}{=} l^4/l_\Lambda^4 \stackrel{\text{def}}{=} l^2/l_P^2, N \to \infty, l \to \infty, l_P \to 0, l_\Lambda \to \text{ג},$$

$$l \to \infty, l_P \to 0, l_\Lambda \stackrel{\text{def}}{=} \sqrt{l l_P} = \text{אב}.$$

$$\rho_{\text{vac}} \stackrel{\text{def}}{=} \Lambda^4 = 1/l_\Lambda^4$$

$$l_\Lambda \tilde{l} = l_s^2 N_L^{1/4} = l_s^2 \left(\frac{l_\Lambda^2}{l_P^2}\right)^{1/4} = l_s^2 \left(\frac{l_\Lambda}{l_P}\right)^{1/2}$$

$$g_s l_s = l_P, M_s = g_s M_P$$

$$\tilde{l} = \frac{l_P}{g_s^2}\left(\frac{l_P}{l_\Lambda}\right)^{1/2}$$

$$g_s = \left(\frac{l_P}{l_\Lambda}\right)^{1/4} \stackrel{\text{def}}{=} \left(\frac{M_\Lambda}{M_P}\right)^{1/4} \to g_s^2 = \left(\frac{M_\Lambda}{M_P}\right)^{1/2}$$

$$m_H^2 = \frac{\xi \Lambda^4}{M_P^2} - \frac{g_s^2 M_s^2}{8\pi^2}\langle \mathcal{X} \rangle$$

$$m_H \sim g_s M_s \sqrt{\frac{\langle \mathcal{X} \rangle}{8\pi^2}} = g_s^2 M_P \sqrt{\frac{\langle \mathcal{X} \rangle}{8\pi^2}}$$

$$g_s^2 = \left(\frac{M_\Lambda}{M_P}\right)^{1/2}$$

$$m_H \sim \sqrt{M_\Lambda M_P}\sqrt{\frac{\langle \mathcal{X} \rangle}{8\pi^2}}$$

$$M_s \sim \sqrt{m_H M_P}$$

$$\mathrm{d}s^2 = \mathrm{d}r^2 + e^{Ar}\,\mathrm{d}s^2_{\text{curve}}, (\text{ AdS })$$

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + e^{Ht}\,\mathrm{d}s^2_{\text{curve}}, (\text{dS})$$

$$-x_0^2 + x_1^2 + \cdots + x_{D+1}^2 = l^2, |\Lambda| = \frac{D(D-1)}{2l^2},$$

$$\psi(Y) = \int \mathrm{d}X K(Y,X)\phi(X)$$

$$K(gX, gY) = K(X, Y)$$

$$Y(\xi) \overset{\text{def}}{=} (Y_0, \cdots, Y_{D+1}) = \left(\sqrt{1+\xi^2}, 0, \cdots, 0, \xi\right).$$

$$K(E, Y(\xi)) \overset{\text{def}}{=} K(\xi)$$

$$P(X, X) = 1$$

$$P(X, Y) = -X_0 Y_0 + X_1 Y_1 + \cdots + X_{D+1} Y_{D+1}$$

$$K(X, Y) = \int \mathrm{d}\xi \delta(P(X, Y) - \xi) K(\xi)$$

$$\psi(Y, U) = \int \mathrm{d}X \delta(P(X, Y)) |P(X, U)|^{-1-i\rho/2}$$

$$S = c[I_{CS}(A) - I_{CS}(\bar{A})]$$

$$I_{CS}(A) = \int \mathrm{d}^3 x \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A^3\right)$$

$$2ie = A - \bar{A}, 2\omega = A + \bar{A}.$$

$$\Psi_{BCS} = \prod_k \psi_k$$

$$\psi_k = (u_k + v_k a_k^\dagger a_{-k}^\dagger) \phi_0$$

$$H_{BCS} = \sum_k (e_k a_k^\dagger a_k + e_{-k} a_{-k}^\dagger a_{-k}) + \sum_{k,k'} V_{k,k'} a_{k'}^\dagger a_{-k'}^\dagger a_k a_{-k}$$

$$E_k = \sqrt{e_k^2 + \Delta_k^2}$$

$$\Delta_k = -\sum_{k'} V_{kk'} u_{k'} v_{k'}$$

$$u_k^2 = \frac{1}{2}\left(1 - \frac{e_k}{E_k}\right), v_k^2 = \frac{1}{2}\left(1 + \frac{e_k}{E_k}\right)$$

$$\Delta_k = -\sum_{k'} V_{kk'} \frac{\Delta_{k'}}{2\sqrt{e_{k'}^2 + \Delta_{k'}^2}}$$

$$1 = -V \sum_k \frac{1}{2\sqrt{e_k^2 + \Delta^2}}$$

$$1 = \frac{|V|}{2} \int_{-e_c}^{e_c} \frac{D(e)\mathrm{d}e}{\sqrt{e^2 + \Delta^2}} \sim |V| D(0) \int_0^{e_c} \frac{\mathrm{d}e}{\sqrt{e^2 + \Delta^2}}$$

$$\Delta = \frac{e_c}{\sinh\left(\frac{1}{|V|D(0)}\right)}$$

$$a_{\mathrm{dS}} = \sqrt{a^2 + a_0^2}$$

$$\Psi_{\mathrm{dS}} = \prod_k \psi_k$$

$$\psi_k = \prod_n (u_{k,n} + v_{k,n} L^\dagger_{k,n} L^\dagger_{-k,-n})\phi_0$$

$$H_{\mathrm{dS}} = \sum_{k,n} (e_{k,n} L^\dagger_{k,n} L_{k,n} + e_{-k,-n} L^\dagger_{-k,-n} L_{-k,-n}) + \sum_{k,k';n,n'} V_{k,k';n,n'} L^\dagger_{k',n'} L^\dagger_{-k',-n'} L_{k,n} L_{-k,-n}$$

$$M_\Lambda = M_{c\mathrm{AdS}} \exp\left(-\frac{1}{|V|D_{\mathrm{AdS}}}\right)$$

$$\left[\hat{x}^a, \hat{\tilde{x}}_b\right] = 2\pi i \lambda^2 \delta^a{}_b$$

$$S^{\mathrm{ch}}_{\mathrm{str}} = \frac{1}{4\pi} \int_\Sigma \mathrm{d}^2\sigma [\partial_\tau \mathbb{X}^A (\eta_{AB}(\mathbb{X}) + \omega_{AB}(\mathbb{X})) - \partial_\sigma \mathbb{X}^A H_{AB}(\mathbb{X})] \partial_\sigma \mathbb{X}^B$$

$$(I := \omega^{-1} H)^A_B = \begin{pmatrix} 0 & -h^{ab} \\ h_{ab} & 0 \end{pmatrix}, I^2 = -1;$$
$$(J := \eta^{-1} H)^A_B = \begin{pmatrix} 0 & h^{ab} \\ h_{ab} & 0 \end{pmatrix}, J^2 = +1;$$
$$(K := \eta^{-1} \omega)^A_B = \begin{pmatrix} -\delta^a{}_b & 0 \\ 0 & \delta_a{}^b \end{pmatrix}, K^2 = +1.$$
$$[I, J] = 2K, [K, I] = 2J, [J, K] = -2I;\ \{J, I\} = \{I, K\} = \{K, J\} = 0.$$

$$\left[\hat{\mathbb{X}}^A, \hat{\mathbb{X}}^B\right] = i\omega^{AB}$$

$$\left[\hat{x}^a, \hat{\tilde{x}}_b\right] = 2\pi i \lambda^2 \delta^a_b, [\hat{x}^a, \hat{x}^b] = 0 = \left[\hat{\tilde{x}}_a, \hat{\tilde{x}}_b\right]$$

$$\partial_\sigma \mathbb{X}^A H_{AB} \partial_\sigma \mathbb{X}^B = \lambda^{-2}\left[g_{ab}(x, \tilde{x})(\partial_\sigma x^a)\left(\partial_\sigma x^b\right) + g^{ab}(x, \tilde{x})(\partial_\sigma \tilde{x}_a)(\partial_\sigma \tilde{x}_b)\right] = 0,$$
$$\partial_\sigma \mathbb{X}^A \eta_{AB} \partial_\sigma \mathbb{X}^B = 2\lambda^{-2}[(\partial_\sigma x^a)(\partial_\sigma \tilde{x}_a)] = 0,$$

$$[\hat{x}^a, \hat{x}^b] = 0, \left[\hat{x}^a, \hat{\tilde{x}}_b\right] = 2\pi i \lambda^2 \delta^a{}_b, \left[\hat{\tilde{x}}_a, \hat{\tilde{x}}_b\right] = -4\pi i \lambda^2 B_{ab}$$

$$[\hat{x}^a, \hat{x}^b] = 4\pi i \lambda^2 \beta^{ab}, \left[\hat{x}^a, \hat{\tilde{x}}_b\right] = 2\pi i \lambda^2 \delta^a{}_b, \left[\hat{\tilde{x}}_a, \hat{\tilde{x}}_b\right] = 0$$

$$\int \mathrm{Tr}[\partial_\tau \mathbb{X}^A \partial_\sigma \mathbb{X}^B (\omega_{AB} + \eta_{AB}) - \partial_\sigma \mathbb{X}^A H_{AB} \partial_\sigma \mathbb{X}^B] \mathrm{d}\tau\, \mathrm{d}\sigma$$

$$\partial_\sigma \mathbb{X}^A \to [\mathbb{X}^{26}, \mathbb{X}^A]$$

$$\int \mathrm{Tr}[\partial_\tau \mathbb{X}^a[\mathbb{X}^b,\mathbb{X}^c]\eta_{abc} - H_{ac}[\mathbb{X}^a,\mathbb{X}^b][\mathbb{X}^c,\mathbb{X}^d]H_{bd}]\mathrm{d}\tau$$

$$\partial_\tau \mathbb{X}^C = [\mathbb{X},\mathbb{X}^C],$$

$$\mathbb{S}_{\mathrm{ncF}} = \frac{1}{4\pi}[\mathbb{X}^a,\mathbb{X}^b][\mathbb{X}^c,\mathbb{X}^d]f_{abcd},$$

$$\mathbb{S}_{\mathrm{ncM}} = \frac{1}{4\pi}\int_\tau \left(\partial_\tau \mathbb{X}^i[\mathbb{X}^j,\mathbb{X}^k]g_{ijk} - [\mathbb{X}^i,\mathbb{X}^j][\mathbb{X}^k,\mathbb{X}^l]h_{ijkl}\right),$$

$$S_{\mathrm{eff}}^{nc} = \iint \mathrm{Tr}\sqrt{g(x,\tilde{x})}[R(x,\tilde{x}) + L_m(x,\tilde{x}) + \cdots]$$

$$\bar{S} = \frac{\int_X \sqrt{-g(x)}[R(x) + \cdots]}{\int_X \sqrt{-g(x)}} + \cdots$$

$$\iint \sqrt{g(x)\tilde{g}(\tilde{x})}\left[R(x) + \tilde{R}(\tilde{x}) + L_m(A(x,\tilde{x})) + \tilde{L}_{dm}(\tilde{A}(x,\tilde{x}))\right]$$

$$\bar{S}' = \frac{\int_X \sqrt{-g(x)}\left[R(x) + L_m(x) + \tilde{L}_{dm}(x)\right]}{\int_X \sqrt{-g(x)}} + \cdots$$

$$S_0 = -\iint \sqrt{g(x)\tilde{g}(\tilde{x})}\left[L_m(A(x,\tilde{x})) + \tilde{L}_{dm}(\tilde{A}(x,\tilde{x}))\right]$$

$$\bar{S}_0 = \frac{\int_X \sqrt{-g(x)}\left[L_m(x) + \tilde{L}_{dm}(x)\right]}{\int_X \sqrt{-g(x)}} + \cdots$$

$$M_\Lambda M_P \sim M_H^2$$

$$\rho_0 \sim M_\Lambda^4 = \left(\frac{M_H^2}{M_P}\right)^4$$

$$\int_X \sqrt{-g}\left[\frac{R}{2G} + s^4 L\left(s^{-2}g^{ab}\right) + \frac{\Lambda}{G}\right] + \sigma\left(\frac{\Lambda}{s^4\mu_s^4}\right)$$

$$S_{mp} = \int \mathrm{d}\tau\left(p\cdot\dot{x} + \tilde{p}\cdot\dot{\tilde{x}} + \lambda^2 p\cdot\dot{\tilde{p}} + Nh + \tilde{N}d\right)$$

$$p_a \to p_a + A_a(x,\tilde{x}), \tilde{p}_a \to \tilde{p}_a + \tilde{A}_a(x,\tilde{x})$$

$$\int_{x,\tilde{x}} \left[F^2 - a\lambda^2[\![A,\tilde{A}]\!] + \tilde{F}^2 + F\tilde{F} + \cdots\right]$$

$$[\![A,\tilde{A}]\!] \overset{\mathrm{def}}{=} \int_{\tau_*}^{\tau} \mathrm{d}\tau' A\left(x(\tau')\right)\frac{\mathrm{d}\tilde{A}\left(\tilde{x}(\tau')\right)}{\mathrm{d}\tau'}$$
$$= \frac{1}{2}[A(x)\tilde{A}(\tilde{x})]_{\tau_*}^{\tau} + \frac{1}{2}\int_{\tau_*}^{\tau} \mathrm{d}\tau'\left[A\left(x(\tau')\right)\frac{\mathrm{d}\tilde{A}\left(\tilde{x}(\tau')\right)}{\mathrm{d}\tau'} - \frac{\mathrm{d}A\left(x(\tau')\right)}{\mathrm{d}\tau'}\tilde{A}\left(\tilde{x}(\tau')\right)\right]$$

$$\int_{x,\tilde{x}} \left[(\partial A)^2 - a\lambda^2 [\![A, \tilde{A}]\!] + (\tilde{\partial}\tilde{A})^2 + \partial A \tilde{\partial}\tilde{A} + \cdots\right]$$

$$\int_{x,\tilde{x}} \left[(\partial A)^2 + (\tilde{\partial} A)^2 - a\lambda^2 [\![A, \tilde{A}]\!] + (\tilde{\partial}\tilde{A})^2 + (\partial \tilde{A})^2 + \partial A \tilde{\partial}\tilde{A} + \cdots\right]$$

$$\int_x \left[\left(\partial_{[a}A_{b]}\right)^2 - \frac{\lambda_4^2}{L^4}\mathcal{F}^{ab}[\![A_a, \tilde{A}_b]\!] + \left(\partial_{[a}\tilde{A}_{b]}\right)^2 + \cdots\right], \tilde{A}_a \stackrel{\text{def}}{=} \eta_{ab}\tilde{A}^b$$

$$\int_x \left[\|\partial\phi\|^2 + \|\partial\tilde{\phi}\|^2 - \frac{\lambda_4^2}{L^4}\mathfrak{F}_{ij}[\![\phi^i, \tilde{\phi}^j]\!] + \cdots\right]$$

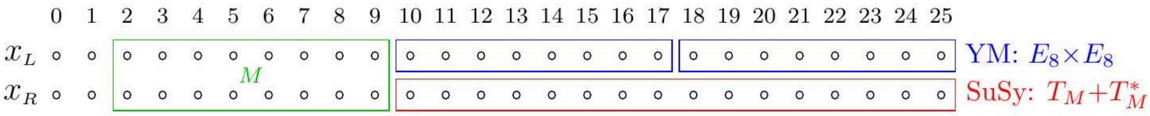
$$\int_x \left[(\partial a)^2 - \frac{\lambda_4^2}{L^4}[\![a, \tilde{a}]\!] + (\partial\tilde{a})^2 + \cdots\right]$$

$$\int_x \left[i(\bar{\psi}\,\partial\psi) + i(\bar{\tilde{\psi}}\,\partial\tilde{\psi}) - \frac{\lambda_4^3}{L^4}\mathfrak{F}_{ij}\left([\![\bar{\psi}^i, \tilde{\psi}^j]\!] + \text{ h.c. }\right) + \cdots\right]$$

$$[\![\bar{\psi}^i, \tilde{\psi}^j]\!] = \frac{1}{2}\left(\bar{\psi}^i\tilde{\psi}^j - \bar{\psi}^j\tilde{\psi}^i\right)$$

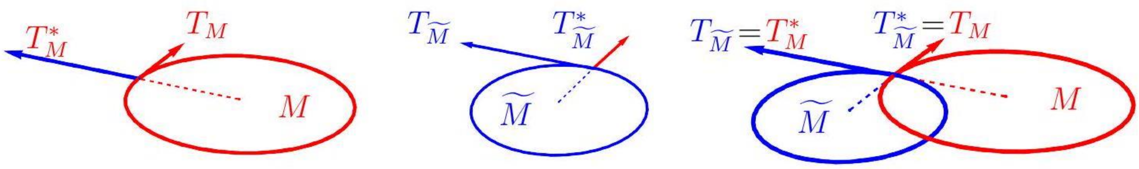
$$\begin{bmatrix} m & {\lambda_4}^3/L^4 \\ {\lambda_4}^3/L^4 & \tilde{m} \end{bmatrix} = \frac{1}{2}\left[(m+\tilde{m}) \pm \sqrt{(m-\tilde{m})^2 + 4({\lambda_4}^3/L^4)^2}\right]$$

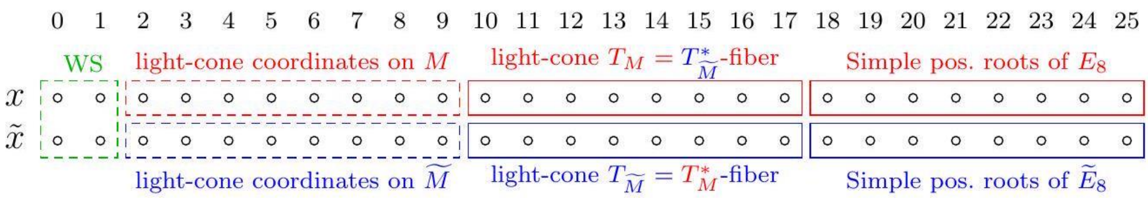
$$\approx \begin{cases} m + \delta m \\ \tilde{m} - \delta m \end{cases}, \delta m \stackrel{\text{def}}{=} \frac{{\lambda_4}^6}{L^8(m-\tilde{m})} + \cdots$$



$$\left(\begin{Bmatrix} x_L^2, \cdots, x_L^9 \\ x_R^2, \cdots, x_R^9 \end{Bmatrix} = \begin{Bmatrix} x^2, \cdots, x^9 \\ \tilde{x}^2, \cdots, \tilde{x}^9 \end{Bmatrix}\right) \stackrel{\text{Susy}}{\longleftrightarrow} \left(\begin{Bmatrix} x_R^{10}, \cdots, x_R^{25} \\ x_R^{10}, \cdots, x_R^{25} \end{Bmatrix} \begin{matrix} B \\ F \end{matrix} \begin{Bmatrix} \psi_R^2, \cdots, \psi_R^9 \\ \bar{\psi}_R^2, \cdots, \bar{\psi}_R^9 \end{Bmatrix}\right).$$

$$\underbrace{\left(x^\mu \stackrel{\text{"Susy"}}{\longleftrightarrow} \left(x_R^{\mu+8}, x_R^{\mu+16}\right)\right)}_{M \leftrightarrow T_M + T_M^*} \quad \underset{\text{boson/fermion-ization}}{\overset{B}{\underset{F}{\leftrightarrows}}} \quad \left(x^\mu \stackrel{\text{SuSy}}{\longleftrightarrow} \left(\psi_R^\mu[x_R^{\mu+8}, x_R^{\mu+16}], \bar{\psi}_R^\mu[x_R^{\mu+8}, x_R^{\mu+16}]\right)\right),$$





$$\frac{\lambda_4{}^2}{L^4}\int_x \mathfrak{F}^{\mu\nu}\left[\!\left[A_\mu^\alpha(x), \tilde{A}_\nu^{\tilde{\beta}}(\tilde{x})\right]\!\right]\eta_{\alpha\tilde{\beta}}$$

$$\frac{\lambda_4{}^2}{L^4}\int_x \mathfrak{F}^{ab}[\![A_a, \Gamma_b]\!]$$

$$A_8 = \frac{1}{2}Y \wedge Y$$

$$dH = Y$$

$$\frac{1}{2}\int \; B \wedge Y$$

$$\frac{1}{2}\int \; A \wedge Y$$

$$1 \to G_1 \to G \to \pi_0(G) \to 1$$

$$G_1 \cong \left(\tilde{G}_{\mathrm{ss}} \times G_{\mathrm{a}}\right)/\Gamma$$

$$\mathfrak{g} = \mathfrak{g}_{\mathrm{ss}} \oplus \mathfrak{g}_{\mathrm{a}} = \bigoplus_i \; \mathfrak{g}_i \oplus \bigoplus_I \; \mathfrak{u}(1)_I$$

$$* H = \theta H$$

$$A_8 = \frac{1}{2}Y \wedge Y$$

$$dH = Y$$

$$\exp \; 2\pi i\left(\frac{1}{2}\int \; B \wedge Y\right)$$

$$2\pi i\frac{1}{2}\int \; B \wedge Y$$

$$2\pi i\frac{1}{2}\int_U A \wedge Y$$

$$Y = \frac{1}{4}ap_1 - \sum_i \; b_i c_2^i + \frac{1}{2}\sum_{IJ} \; b_{IJ} c_1^I c_1^J$$

$$a, b_i, \frac{1}{2}b_{II}, b_{IJ} \in \Lambda$$

$$(a, x) = (x, x) \bmod 2 \; \forall x \in \Lambda$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\frac{1}{2\pi i}\ln \; \mathrm{An}_{\mathrm{Wf},R}(U) = \xi_R(U) = \frac{\eta_R + h_R}{2},$$

$$\xi_R(U) = \int_W I_R - \text{index}\left(D_R^{(W)}\right)$$

$$\xi_R(U) = \int_W I_R \text{mod} 1$$

$$\frac{1}{2\pi i}\ln \text{An}_{\text{MWf},R}(U) = \frac{1}{2}\xi_R(U).$$

$$\frac{1}{2\pi i}\ln \text{An}_{\text{SMWf},R}(U) = \frac{1}{2}\xi_R(U).$$

$$\frac{1}{2\pi i}\ln \text{An}_{\text{SD0}}(U) = \frac{1}{4}\xi_\sigma(U) = \frac{1}{8}\left(\int_W L_{TW} - \sigma_W\right)$$

$$-\frac{1}{2g^2}\int_M (dB - qA) \wedge * (dB - qA) + i\pi p \int_M A \wedge dB$$

$$-\frac{i}{g^2}\int_M ((dB)^- \wedge (dB)^+ + qA^-A^+ + (\pi g^2 p + q)A^+ \wedge (dB)^- + (\pi g^2 p - q)A^- \wedge (dB)^+)$$

$$\frac{1}{2\pi i}\ln \text{An}_{\text{SDk}}(U) = \frac{1}{4}\xi_\sigma(U) - k\text{Arf}(q),$$

$$\frac{1}{2\pi i}\ln \text{An}_{\text{SDk}}(U) = \frac{1}{8}\left(\int_W L_{TW} - \sigma_W\right) - k\left(\frac{1}{2}\int_W Y_W^2 - \frac{1}{8}\sigma_W\right)$$

$$\frac{1}{2\pi i}\ln \text{An}_{\text{SDgen}}(U) = \frac{\text{sgn}(\Lambda)}{8}\left(\int_W L_{TW} - \sigma_W\right) - \left(\frac{1}{2}\int_W Y_W^2 - t\right),$$

$$\frac{1}{2\pi i}\ln \text{An}_{\Lambda,G,R}(U) = \frac{1}{2}\xi_{R'}(U) + \frac{\text{sgn}(\Lambda)}{4}\xi_\sigma(U)$$

$$R' = ((\text{VecSpin}(7) \ominus 1) \otimes 1) \ominus (T - 1)(1 \otimes 1) \oplus (1 \otimes \text{Ad}G) \ominus (1 \otimes R)$$

$$\frac{1}{2\pi i}\ln \text{An}_{\text{bare}}(U) = \frac{1}{2}\int_W Y_W \wedge Y_W - \frac{\sigma_{H^4(W,\partial W;\Lambda)}}{8}.$$

$$t = \frac{\sigma_{H^4(W,\partial W;\Lambda)}}{8}$$

$$\frac{1}{2\pi i}\ln \text{An}_{\text{GS}}(U) = -\frac{1}{2}\int_W Y_W \wedge Y_W + \frac{\sigma_{H^4(W,\partial W;\Lambda)}}{8}$$

$$\check{C}^p(M;\Lambda_\mathbb{R}) = C^p(M;\Lambda_\mathbb{R}) \times C^{p-1}(M;\Lambda_\mathbb{R}) \times \Omega^p(M;\Lambda_\mathbb{R}).$$

$$d\check{c} = (da, \omega - a - dh, d\omega), d^2 = 0$$

$$\check{x} \simeq \check{x} + d\check{y}, \check{y} = (b, g, 0), b \in C^{p-1}(M;\Lambda), g \in C^{p-2}(M;\Lambda_\mathbb{R}).$$

$$\cup: \check{C}^p(M;\Lambda) \times \check{C}^q(M;\Lambda) \to \check{C}^{p+q}(M;\mathbb{Z})$$

$$\check{c}_1 \cup \check{c}_2 = \left(a_1 \cup a_2, (-1)^{\text{deg}a_1} a_1 \cup h_2 + h_1 \cup \omega_2 + H_\cup^\wedge(\omega_1,\omega_2), \omega_1 \wedge \omega_2\right)$$

$$dH_{\cup}^{\wedge}(\omega_1,\omega_2)+H_{\cup}^{\wedge}(d\omega_1,\omega_2)+(-1)^{\deg\omega_1}H_{\cup}^{\wedge}(\omega_1,d\omega_2)=\omega_1\wedge\omega_2-\omega_1\cup\omega_2$$

$$d(\check{c}_1\cup\check{c}_1)=d\check{c}_1\cup c_2+(-1)^{\deg\check{c}_1}\check{c}_1\cup d\check{c}_2$$

$$H=dB+A$$

$$2\pi i\frac{1}{2}\int_M B\wedge Y$$

$$\check{X}=\check{Y}-\frac{1}{2}\check{v},$$

$$d\check{H}=\check{Y}$$

$$dh=y,H-h-dB=A,dH=Y$$

$$\check{F}=\check{H}-\frac{1}{2}\check{\eta},d\check{F}=\check{X}$$

$$\check{H}\mapsto f^*\check{H},\check{Y}\mapsto f^*\check{Y},\check{\eta}\mapsto f^*\check{\eta},$$

$$\begin{aligned}&\check{Y}\mapsto\check{Y}\\&\check{H}\mapsto\check{H}+d\check{W}\\&\check{\eta}\mapsto\check{\eta}\end{aligned}$$

$$h\mapsto h+dw,B\mapsto B-w-dW,H\mapsto H$$

$$\begin{aligned}&\check{Y}\mapsto\check{Y}+d\check{V},\\&\check{H}\mapsto\check{H}+\check{V},\\&\check{\eta}\mapsto\check{\eta},\end{aligned}$$

$$\begin{aligned}y\mapsto y+dv,A\mapsto A-v-dV,Y\mapsto Y,\\h\mapsto h+v,B\mapsto B+V,H\mapsto H.\end{aligned}$$

$$\begin{aligned}&\check{Y}\mapsto\check{Y}+\frac{1}{2}d\check{\rho},\\&\check{H}\mapsto\check{H}+\frac{1}{2}\check{\rho},\\&\check{\eta}\mapsto\check{\eta}+\check{\rho},\end{aligned}$$

$$\begin{aligned}&\eta\mapsto\eta+\rho,y\mapsto y+\frac{1}{2}d\rho,A\mapsto A-\frac{1}{2}\rho,Y\mapsto Y,\\&h\mapsto h+\frac{1}{2}\rho,B\mapsto B,H\mapsto H,\end{aligned}$$

$$\tilde{L}(x_1,x_2):=\frac{1}{k}\int_U x_1\cup y\mathrm{mod}1$$

$$L(\check{X}_1,\check{X}_2):=\int_U x_1\cup A_2=\int_U\check{X}_1\cup\check{X}_2$$

$$\frac{1}{2}x\cup(x+v_\Lambda)$$

$$\check{v} := (d\eta_\Lambda, -\eta_\Lambda, 0) = d(\eta_\Lambda, 0,0) \in \check{Z}_0^4(M;\Lambda).$$

$$l(\check{X}) := \frac{1}{2}[\check{X} \cup (\check{X} + \check{v})]_{\text{hol}}$$
$$= \frac{1}{2} x \cup (A - \eta_\Lambda) + \frac{1}{2} A \cup X + \frac{1}{2} H_\Lambda^\cup (X, X) \text{mod} 1,$$

$$S_\omega(U;\check{X}) := \int_{U,\omega}^{\text{E}} \left(l(\check{X}), x_2\right)$$

$$\begin{aligned}
S_\omega(U;\check{X}+\check{Z}) &= \int_{U,\omega}^{\text{E}} \bar{l}(\check{X}+\check{Z}) \\
&= \int_{U,\omega}^{\text{E}} \left(l(\check{X}+\check{Z}), (x)_2 + (z)_2\right) \\
&= \int_{U,\omega}^{\text{E}} \left(l(\check{X}) + l(\check{Z}) + \frac{1}{2}(x \cup Z + z \cup A + Z \cup X), (x)_2 + (z)_2\right) \\
&= \int_{U,\omega}^{\text{E}} \left(l(\check{X}) + l(\check{Z}) + \frac{1}{2}(x \cup Z + Z \cup x - d(Z \cup A)), (x)_2 + (z)_2\right) \\
&= \int_{U,\omega}^{\text{E}} \left(l(\check{X}) + l(\check{Z}) + x \cup Z + \frac{1}{2} x \cup_1 z - \frac{1}{2} d(Z \cup A - x \cup_1 Z), (x)_2 + (z)_2\right) \\
&= S_\omega(U;\check{X}) + \int_U \check{X} \cup \check{Z} + S_\omega(U;\check{Z})
\end{aligned}$$

$$S_\omega(U;\check{X}) = \frac{1}{2}\int_W X_W \wedge (X_W + \lambda')\text{mod} 1$$

$$\text{WCS}_\omega^{\text{PQ}}(U;\check{X}) = \exp\ 2\pi i S_\omega(U;\check{X})$$

$$\check{X}_{12} = \check{X}_1 + dW_I, W_I = (\tilde{\rho} w, \rho W, 0),$$

$$f\left(U_2, \check{X}_{U_2}\right) / f\left(U_1, \check{X}_{U_1}\right) = \exp\ 2\pi i S_\omega\left(U_{12}; \check{X}_{U_{12}}\right)$$

$$S_\omega(U;\check{X}+\check{Z}) - S_\omega(U;\check{X}) = S_\omega(U;\check{Z}) + \int_U \check{X} \cup \check{Z} = q_\omega(z) + \int_U x \cup Z$$

$$T := H_{\text{tors}}^3(M;\Lambda) \cup \theta \subset H_{\text{tors}}^4(M;\Lambda)$$

$$-\int_U v \cup z = -\tilde{L}(x,z)$$

$$q_\omega(z) = q_{\mathbb{Z},\omega}(z') \cdot (\alpha, \alpha)$$

$$\tilde{L}(x_0, z) = \langle z_2 \cup (\delta \otimes \gamma), [M \times S^1] \rangle = \langle z_2' \cup \delta, [M \times S^1] \rangle (\alpha, \gamma)$$

$$q_{\omega'}(z) = q_\omega(z) - \langle z_2 \cup (\delta \otimes \gamma), [M \times S^1] \rangle$$

$$\text{Arf}(q_\omega) = \arg\left(\sum_{z \in H^4(U,\partial U;\Lambda)} \exp\ 2\pi i q_\omega(z)\right)$$

$$\mathrm{WCS}_\omega(U;\check{X}) := N(U) \sum_{[z]\in H^4_{\mathrm{tors}}(U,\partial U;\Lambda)} \exp\ 2\pi i\big(S_\omega(U;\check{X}) - S_\omega(U;\check{Z})\big)$$
$$= \exp\ 2\pi i\left(S_\omega(U;\check{X}) - \mathrm{Arf}(q_\omega)\right)$$

$$N(U) := \frac{1}{\sqrt{|H^4_{\mathrm{tors}}(U,\partial U;\Lambda)||R(U)|}}$$

$$\mathrm{Arf}(q_\omega) = \frac{1}{8}\left(\sigma_{H^4(W,U;\Lambda)} - \int_W \lambda'^2\right)$$

$$\mathrm{WCS}_\omega(U;\check{X}) = \exp\ \frac{2\pi i}{8}\left(4\int_W X_W \wedge (X_W + \lambda') + \int_W \lambda'^2 - \sigma_{H^4(W,U;\Lambda)}\right)$$
$$= \exp\ 2\pi i\left(\int_W \frac{1}{2}(X'_W)^2 - \frac{\sigma_{H^4(W,U;\Lambda)}}{8}\right)$$

$$S_{\omega'}(U;\check{X}) = S_\omega(U;\check{X}) - \langle x_2 \cup \delta_{\Lambda/2\Lambda}, [U,\partial U]\rangle,$$

$$q_{\omega'}(x) = q_\omega(x) - \langle x_2 \cup \delta_{\Lambda/2\Lambda}, [U,\partial U]\rangle.$$

$$q_{\omega'}(x) = q_\omega(x) - \tilde{L}\left(x, \beta(\delta_{\Lambda/2\Lambda})\right)$$

$$\mathrm{Arf}(q_{\omega'}) = \mathrm{Arf}(q_\omega) - q_\omega\left(\beta(\delta_{\Lambda/2\Lambda})\right)$$

$$S_{\omega'}(U;\check{X}) - \mathrm{Arf}(q_{\omega'}) = S_\omega(U;\check{X}) - \mathrm{Arf}(q_\omega) + L(\check{X},\check{\Delta}/2) + S_\omega(U;\check{\Delta}/2)$$
$$= S_\omega(U;\check{X} + \check{\Delta}/2) - \mathrm{Arf}(q_\omega)\mathrm{mod}1$$

$$\mathrm{WCS}_{\omega'}(U;\check{X}) = \mathrm{WCS}_\omega(U;\check{X} + \check{\Delta}/2).$$

$$\mathrm{WCS}_\omega(M;\check{X}) := \mathrm{WCS}^{\mathrm{PQ}}_\omega(M;\check{X}) \otimes \mathrm{WCS}^{\mathrm{PQ}}_\omega(M;\check{0}).$$

$$\mathrm{WCS}_\omega(M;\check{X}) := \mathrm{WCS}^{\mathrm{PQ}}_\omega(M;\check{X}) \otimes \bigoplus_{[z]\in H^4_{\mathrm{tors}}(M;\Lambda)} \mathrm{WCS}^{\mathrm{PQ}}_\omega(M;\check{Z}),$$

$$X_W = Y_W - \frac{1}{2}\lambda'$$

$$\check{X} = \check{Y} - \frac{1}{2}\check{\nu}$$

$$\mathrm{WCS}^{\mathrm{s}}(N;\check{Y}) := \mathrm{WCS}^{\dagger}\left(N;\check{Y} - \frac{1}{2}\check{\nu}\right),$$

$$\mathrm{GST}(M;\check{Y},\check{H}) \in \mathrm{WCS}^{\mathrm{s}}(M;\check{Y})$$

$$\mathrm{GST}(M,\check{Y},\check{H}) := \exp - 2\pi i \int^{\mathrm{E}}_{M,\omega} \overline{\mathrm{gst}}(M,\check{Y},\check{H})$$

$$\overline{\mathrm{gst}}(M,\check{Y},\check{H}) = \left(\frac{1}{2}\left[\left(\check{H} - \frac{1}{2}\check{\eta}\right) \cup \left(\check{Y} + \frac{1}{2}\check{\nu}\right)\right]_{\mathrm{hol}}, h_2 - \frac{1}{2}\eta\right) = \left(\frac{1}{2}[\check{F} \cup (\check{X} + \check{\nu})]_{\mathrm{hol}}, f_2\right)$$

$$\left[\left(\check{H}-\tfrac{1}{2}\check{\eta}\right)\cup\left(\check{Y}+\tfrac{1}{2}\check{\nu}\right)\right]_{\mathrm{hol}}\ \overline{\mathrm{gst}}(M,\check{Y},\check{H})$$

$$\int_{M,\omega'}^{\mathrm{E}}(s,y)=\int_{M,\omega}^{\mathrm{E}}(s,y)+\frac{1}{2}\int_{M}y\cup\delta_{\Lambda/2\Lambda}$$

$$\begin{aligned}
\frac{1}{2\pi i}\ln\ \mathrm{GST}_{\omega'}(M,\check{Y},\check{H}) &=-\int_{M,\omega'}^{\mathrm{E}}\left(\frac{1}{2}[\check{F}\cup(\check{X}+\check{\nu}+\check{\Delta})]_{\mathrm{hol}},f_2\right)\\
&=-\int_{M,\omega}^{\mathrm{E}}\left(\left(\frac{1}{2}[\check{F}\cup(\check{X}+\check{\nu})]_{\mathrm{hol}},f_2\right)\boxplus\left(\frac{1}{2}[\check{F}\cup\check{\Delta}]_{\mathrm{hol}},0\right)\right)\\
&\quad-\frac{1}{2}\int_{M}f_2\cup\delta_{\Lambda/2\Lambda}\\
&=\frac{1}{2\pi i}\ln\ \mathrm{GST}_{\omega}(M,\check{Y},\check{H})+\frac{1}{2}\int_{M}f\cup\delta_{\Lambda}-\frac{1}{2}\int_{M}f_2\cup\delta_{\Lambda/2\Lambda}\\
&=\frac{1}{2\pi i}\ln\ \mathrm{GST}_{\omega}(M,\check{Y},\check{H})\mathrm{mod}1
\end{aligned}$$

$$\begin{aligned}
&\Delta_{\check{W}}\left(\frac{1}{2\pi i}\ln\ \mathrm{GST}(M;\check{Y},\check{H})\right)=-\int_{M,\omega}^{\mathrm{E}}\left(\frac{1}{2}[(\check{F}+d\check{W})\cup(\check{X}+\check{\nu})]_{\mathrm{hol}},f_2+dw_2\right)\\
&+\int_{M,\omega}^{\mathrm{E}}\left(\frac{1}{2}[\check{F}\cup(\check{X}+\check{\nu})]_{\mathrm{hol}},f_2\right)\\
&=-\int_{M,\omega}^{\mathrm{E}}\left(\left(\frac{1}{2}[(\check{F}+d\check{W})\cup(\check{X}+\check{\nu})]_{\mathrm{hol}},f_2+dw_2\right)\right.\\
&\left.\boxminus\left(\frac{1}{2}[\check{F}\cup(\check{X}+\check{\nu})],f_2\right)\right)\\
&=-\int_{M,\omega}^{\mathrm{E}}\left(\frac{1}{2}[d\check{W}\cup(\check{X}+\check{\nu})]_{\mathrm{hol}}+\frac{1}{2}dw\cup f,dw_2\right)\\
&=-\int_{M,\omega}^{\mathrm{E}}\left(\frac{1}{2}[d\check{W}\cup(\check{X}+\check{\nu})]_{\mathrm{hol}}+\frac{1}{2}(dw\cup f+w\cup\nu_{\Lambda}+dw\cup dw),0\right)\\
&=-\int_{M}\frac{1}{2}(-dw\cup(A-\eta_{\Lambda})+(-w-dW)\cup X+dw\cup f\\
&=-\int_{M}\frac{1}{2}(-w\cup x+dw\cup\eta_{\Lambda}+dw\cup f+w\cup\nu_{\Lambda})\\
&=0\mathrm{mod}1.
\end{aligned}$$

$$\Delta_{\check{V}}\left(\frac{1}{2\pi i}\ln\ \mathrm{GST}(M;\check{Y},\check{H})\right)=-\int_{M,\omega}^{\mathrm{E}}\left(\frac{1}{2}[(\check{F}+\check{V})\cup(\check{X}+d\check{V}+\check{v})]_{\mathrm{hol}},f_2+v_2\right)$$
$$+\int_{M,\omega}^{\mathrm{E}}\left(\frac{1}{2}[\check{F}\cup(\check{X}+\check{v})]_{\mathrm{hol}},f_2\right)$$
$$=-\int_{M,\omega}^{\mathrm{E}}\left(\frac{1}{2}[\check{F}\cup d\check{V}+\check{V}\cup(\check{X}+\check{v})+\check{V}\cup d\check{V}]_{\mathrm{hol}}\right.$$
$$\left.+\frac{1}{2}(dv\cup_1 f+v\cup f),v_2\right)$$
$$=-\int_{M,\omega}^{\mathrm{E}}\left(\frac{1}{2}(-f\cup(-v-dV)-v\cup(A-\eta_\Lambda)+V\cup X\right.$$
$$=\int_{M,\omega}^{\mathrm{E}}\left(\frac{1}{2}(-v\cup_1 x-x\cup V+v\cup A-V\cup X+v\cup\eta_\Lambda\right.$$
$$-v\cup v-v\cup dV),v_2)\mathrm{mod}1,$$

$$v\cup f+f\cup v=-dv\cup_1 f+v\cup_1 x+d(v\cup_1 f)$$

$$\Delta_{\check{V}}\left(\frac{1}{2\pi i}\ln\ \mathrm{WCS}^{s}(N;\check{Y})\right)=\int_{N,\omega}^{\mathrm{E}}\left(\frac{1}{2}[(\check{X}+d\check{V})\cup(\check{X}+d\check{V}+\check{v})]_{\mathrm{hol}},x_2+dv_2\right)$$
$$-\int_{N,\omega}^{\mathrm{E}}\left(\frac{1}{2}[\check{X}\cup(\check{X}+\check{v})]_{\mathrm{hol}},x_2\right)$$
$$=\int_{N,\omega}^{\mathrm{E}}\left(\frac{1}{2}(x\cup(-v-dV)+dv\cup(A-\eta_\Lambda)+(-v-dV)\cup X\right.$$
$$+dv\cup(-v-dV)+dv\cup_1 x),dv_2)$$
$$=\int_{N,\omega}^{\mathrm{E}}\left(\frac{1}{2}d(v\cup A+v\cup_1 x-V\cup X-x\cup V+v\cup\eta_\Lambda\right.$$
$$-v\cup dV-v\cup v)+\frac{1}{2}(-v\cup dv+v\cup\hat{v}_\Lambda),dv_2)$$
$$=\int_{M,\omega}^{\mathrm{E}}\left(\frac{1}{2}(v\cup A+v\cup_1 x-V\cup X-x\cup V+v\cup\eta_\Lambda\right.$$
$$-v\cup dV-v\cup v),v_2)\mathrm{mod}1,$$

$$\mathrm{ch}\left(R^{(1)}\right)=\mathrm{ch}\left(R^{(2)}\right)$$

$$\dim R^{(1)}=\dim R^{(2)}$$
$$\mathrm{Tr}_{R^{(1)}}F^2=\mathrm{Tr}_{R^{(2)}}F^2$$
$$\mathrm{Tr}_{R^{(1)}}F^4=\mathrm{Tr}_{R^{(2)}}F^4$$

$$\exp\left[\pi i\left(\xi_{R^{(1)}}(U)-\xi_{R^{(2)}}(U)\right)\right]$$

$$\xi_{R_s}(U)=\frac{1}{360\cdot n}(-11+10n^2+n^4-60ns+60s^2-30n^2s^2+60ns^3-30s^4)$$

$$R^{(i)} = \bigoplus x_s^{(i)} R_s, i = 1,2,$$

$$\sum_{s=0}^{n-1} \Delta x_s p(n,s) = 0 \bmod 2$$

$$p(n,s) = \frac{1}{12n}(-2ns + 2s^2 - n^2s^2 + 2ns^3 - s^4)$$

$$n = 2: \frac{1}{16}\Delta x_1 = 0 \bmod 1$$
$$n = 3: \frac{1}{9}(\Delta x_1 + \Delta x_2) = 0 \bmod 1$$
$$n = 4: \frac{1}{32}(5\Delta x_1 + 8\Delta x_2 + 5\Delta x_3) = 0 \bmod 1$$
$$n = 5: \frac{1}{5}(\Delta x_1 + 2\Delta x_2 + 2\Delta x_3 + \Delta x_4) = 0 \bmod 1$$
$$n = 6: \frac{1}{144}(35\Delta x_1 + 80\Delta x_2 + 99\Delta x_3 + 80\Delta x_4 + 35\Delta x_5) = 0 \bmod 1$$
...

$$n = 2: \frac{1}{4}\Delta x_1 = 0 \bmod 1$$
$$n = 3: \frac{1}{3}(\Delta x_1 + \Delta x_2) = 0 \bmod 1$$
$$n = 4: \frac{1}{4}(5\Delta x_1 + 8\Delta x_2 + 5\Delta x_3) = 0 \bmod 1$$
$n = 5$: No apparent constraint
$$n = 6: \frac{1}{12}(35\Delta x_1 + 80\Delta x_2 + 99\Delta x_3 + 80\Delta x_4 + 35\Delta x_5) = 0 \bmod 1$$
…

$$\Omega_6^{\text{spin}}(BU(1)) = \Omega_6^{\text{spin}}(K(\mathbb{Z},2)) \cong \Omega_4^{\text{spin}^c}(pt) \cong \mathbb{Z} \oplus \mathbb{Z}$$

$$x + \frac{1}{2}v_\Lambda = y = \frac{1}{2}\lambda \otimes a + v$$

$$v = -\sum_i b_i c_2^i + \frac{1}{2}\sum_{IJ} b_{IJ} c_1^I \cup c_1^J$$

$$\frac{1}{2}b \in H_{\text{free}}^4(BG_1; \mathbb{Z}) \otimes \Lambda \subset H^4(BG_1; \Lambda_{\mathbb{R}})$$

$$b_i, \frac{1}{2}b_{II}, b_{IJ} \in \Lambda$$

$$\frac{1}{2}b \in H^4(BG; \Lambda)$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\mathcal{T} := \mathcal{A} \otimes \text{WCS}^{\text{s}}$$

$$\check{Y}_{\mathrm{U}} = (\hat{y}_{\mathrm{U}}, \hat{A}_{\mathrm{U}}, Y_{\mathrm{U}})$$

$$\tau_\rho(\theta_2, \theta_1) := \int_{[0,1]} \rho(\Theta).$$

$$\tau_\rho(\theta_3, \theta_2) + \tau_\rho(\theta_2, \theta_1) = \tau_\rho(\theta_3, \theta_1).$$

$$\check{Y} = \check{Y}(\theta, \gamma) = \left(\bar{\gamma}^* \hat{y}_{\mathrm{U}}, \tau_\rho\big(\theta, \gamma^*(\theta_{\mathrm{U}})\big) + \bar{\gamma}^* \hat{A}_{\mathrm{U}}, \rho(\theta)\right).$$

$$\frac{1}{2} a \lambda_{BSpin} - \sum_i b_i c^i_{2,BG} + \frac{1}{2} \sum_{IJ} b_{IJ} c^I_{1,BG} c^J_{1,BG}$$

$$Y = \rho(\theta) = \frac{1}{4} a p_1 - \sum_i b_i c^i_2 + \frac{1}{2} \sum_{IJ} b_{IJ} c^I_1 c^J_1$$

$$\hat{y}_{\mathrm{U}} := \frac{1}{2} a \hat{\lambda}_{BSpin} - \sum_i b_i \hat{c}^i_{2,BG} + \frac{1}{2} \sum_{IJ} b_{IJ} \hat{c}^I_{1,BG} \hat{c}^J_{1,BG}$$

$$\frac{1}{2} \bar{\gamma}^* \big(\hat{\lambda}_{\mathrm{BSpin}}\big) \otimes a = \frac{1}{2} \bar{\gamma}^*(\hat{v}) \otimes a \bmod \Lambda.$$

$$\hat{y}_{\mathrm{U}} := \frac{1}{2} a \hat{\lambda}_{BSpin} - \sum_i b_i \hat{c}^i_{2,BG} + \frac{1}{2} \sum_{IJ} b_{IJ} \hat{c}^I_{1,BG} \hat{c}^J_{1,BG} + \hat{t}_{4,BG}$$

$$\hat{t}_{4,BG} = \sum_k b^T_k \hat{t}^k_{4,BG}$$

$$\hat{y}_{\mathrm{U}} := \frac{1}{2} a \hat{\lambda}_{BSpin} - \sum_i b_i \hat{c}^i_{2,BG} + \frac{1}{2} \sum_{IJ} b_{IJ} \hat{c}^I_{1,BG} \hat{c}^J_{1,BG} + b_T \hat{u}^2_{2,BG}.$$

$$\theta \to f^*(\theta)$$

$$\check{Y} \mapsto \bar{f}^* \check{Y}, \check{H} \mapsto \bar{f}^* \check{H}, \check{\eta} \mapsto \bar{f}^*(\check{\eta}),$$

$$\hat{x}_{\mathrm{U}} = \hat{y}_{\mathrm{U}} - \frac{1}{2} d\eta_{\Lambda,\mathrm{U}}, \hat{C}_{\mathrm{U}} = \hat{A}_{\mathrm{U}} + \frac{1}{2} \eta_{\Lambda,\mathrm{U}}, X_{\mathrm{U}} = Y_{\mathrm{U}}$$

$$\check{X} \mapsto \check{X} + \left((\bar{\gamma}'^* - \bar{\gamma}^*) \hat{x}_{\mathrm{U}}, \tau_\rho\big(\gamma^*(\theta_{\mathrm{U}}), \gamma'^*(\theta_{\mathrm{U}})\big) + (\bar{\gamma}'^* - \bar{\gamma}^*) \hat{C}_{\mathrm{U}}, 0\right).$$

$$\check{X} \mapsto \check{X} + d\check{V}, \check{V} = (v, V, 0).$$

$$\check{Y} \mapsto \check{Y} + d\check{V} + \frac{1}{2} d\check{\rho},$$

$$\check{H} \mapsto \check{H} + \check{V} + \frac{1}{2} \check{\rho},$$

$$d\tau_\rho\big(\gamma^*(\theta_{\mathrm{U}}), \gamma'^*(\theta_{\mathrm{U}})\big) = \gamma^*(X_{\mathrm{U}}) - \gamma'^*(X_{\mathrm{U}})$$

$$d\Delta\hat{C}_{\mathrm{U}} = (\gamma'^* - \gamma^*)\big(d\hat{C}_{\mathrm{U}} - X_{\mathrm{U}}\big) = -(\gamma'^* - \gamma^*) \hat{x}_{\mathrm{U}} = -\Delta\hat{x}_{\mathrm{U}}$$

$$\int_{\Sigma} \Delta\hat{C} = \int_{\Sigma\times I} \left(-\Gamma^* X_{\mathrm{U}} + d\Gamma^*\hat{C}_{\mathrm{U}}\right) = \int_{\Sigma\times I} \Gamma^*\hat{x}_{\mathrm{U}}$$

$$\cup_i : C^p(M;\Xi_1) \times C^q(M;\Xi_2) \to C^{p+q-i}(M;\Xi_3).$$

$$d(u \cup_i v) - du \cup_i v - (-1)^p u \cup_i dv = (-1)^{p+q-i} u \cup_{i-1} v + (-1)^{pq+p+q} v \cup_{i-1} u$$

$$\bar{s} = (s,y) \in \bar{C}^p(M;\Lambda) := C^p(M;\mathbb{R}/\mathbb{Z}) \times C^{p-3}(M;\Lambda/2\Lambda)$$

$$\cup : C^{\cdot}(M;\Lambda/2\Lambda) \otimes C^{\cdot}(M;\Lambda/2\Lambda) \to C^{\cdot}(M;\mathbb{Z}_2)$$

$$\frac{1}{2} y_1 \cup y_2 \in C^{\cdot}(M;\mathbb{R}/\mathbb{Z})$$

$$(s_1,y_1) \boxplus (s_2,y_2) = \left(s_1 + s_2 + \frac{1}{2} dy_1 \cup_{p-5} y_2 + \frac{1}{2} y_1 \cup_{p-6} y_2, y_1 + y_2\right)$$

$$\boxminus (s,y) = \left(-s + \frac{1}{2} dy \cup_{p-5} y + \frac{1}{2} y \cup_{p-6} y, y\right).$$

$$d(s,y) = \left(ds + y \cup_{p-6} dy + \frac{1}{2} y \cup_{p-7} y + \frac{1}{2} y \cup \hat{v}_{\Lambda/2\Lambda}, dy\right)$$

$$\ldots H^p(M;\mathbb{R}/\mathbb{Z}) \xrightarrow{i} E[\Lambda/2\Lambda,3]^p(M) \xrightarrow{j} H^{p-3}(M;\Lambda/2\Lambda)$$
$$\xrightarrow{\mathrm{Sq}^4} H^{p+1}(M;\mathbb{R}/\mathbb{Z}) \xrightarrow{i} E[\Lambda/2\Lambda,3]^{p+1}(M) \xrightarrow{j} H^{p-2}(M;\Lambda/2\Lambda) \ldots$$

$$I^{\mathrm{E}}_{U,\omega} : E[\Lambda/2\Lambda,3]^p(U,\partial U) \to \mathbb{R}/\mathbb{Z}$$
$$I^{\mathrm{E}}_{M,\omega_M} : E[\Lambda/2\Lambda,3]^{p-1}(M) \to \mathbb{R}/\mathbb{Z}$$

$$\int_{U,\omega}^{\mathrm{E}} : \bar{C}^p(U;\Lambda) \to \mathbb{R}/\mathbb{Z}$$
$$\int_{M,\omega}^{\mathrm{E}} : \bar{C}^{p-1}(M;\Lambda) \to \mathbb{R}/\mathbb{Z}$$

$$\int_{M,\omega}^{\mathrm{E}} \bar{x} = \int_{U,\omega}^{\mathrm{E}} d\bar{x}'$$

$$\int_{U,\omega}^{\mathrm{E}} (s,0) = \int_U s$$

$$2\int_{U,\omega}^{\mathrm{E}} (s,y) = \int_{U,\omega}^{\mathrm{E}} (s,y) \boxplus (s,y) = \int_U 2s + \int_U \frac{1}{2} y \cup \hat{v}_{\Lambda/2\Lambda} \bmod 1$$

$$\int_{U,\omega}^{E} (s,y) = \int_U s + f(y)$$

$$f(y_1 + y_2) - f(y_1) - f(y_2) = \int_U y_1 \cup_{p-6} y_2$$

$$\mathrm{Tr}_{\mathrm{WCS}(M;\check{X}_M)}\mathrm{WCS}(U_M;\check{X}_{U_M}) \simeq \mathrm{WCS}(U;\check{X}_U)$$

$$\mathrm{WCS}(\partial U_M;\check{X}_{\partial U_M}) \simeq \mathrm{WCS}(\partial U;\check{X}_{\partial U}) \otimes \mathrm{WCS}(M;\check{X}_M) \otimes \left(\mathrm{WCS}(M;\check{X}_M)\right)^{\dagger},$$

$$\mathrm{Tr}_{\mathrm{WCS^{PQ}}(M;\check{X}_M)}\mathrm{WCS^{PQ}}(U_M;\check{X}_{U_M}) \otimes \sum_{z\in H^4_{\mathrm{tors}}(U_M,\partial U_M;\Lambda)} \mathrm{Tr}_{\mathrm{WCS^{PQ}}(M;0)}\mathrm{WCS^{PQ}}(U_M;\check{Z}_{U_M}),$$

$$\mathrm{WCS^{PQ}}(U;\check{X}_U) \otimes \sum_{z\in H^4_{\mathrm{tors}}(U_M,\partial U_M;\Lambda)} \mathrm{WCS^{PQ}}(U;\check{Z}_U)$$

$$H^3_{\mathrm{tors}}(M;\Lambda) \xrightarrow{h} H^4_{\mathrm{tors}}(U,\partial U;\Lambda)$$

$$q(x+y)-q(x)=\tilde{L}(x,y)$$

$$E^2_{p,q} = H_p\left(BG,\Omega^{\mathrm{spin}}_q(\mathrm{pt.})\right) \Rightarrow \Omega^{\mathrm{Spin}}_{p+q}(BG)$$

$$\Omega^{\mathrm{spin}}_{\cdot}(\mathrm{pt.}) = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \\ \mathbb{Z} & \mathbb{Z}_2 & \mathbb{Z}_2 & 0 & \mathbb{Z} & 0 & 0 & 0 & \mathbb{Z}^2 & \cdots \end{pmatrix}$$

8	$\mathbb{Z}^2$	0	$\mathbb{Z}^2$	0	$\mathbb{Z}^2$	0	$\mathbb{Z}^2$	0	$\mathbb{Z}^2$
7	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0
4	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
3	0	0	0	0	0	0	0	0	0
2	$\mathbb{Z}_2$	0	$\mathbb{Z}_2$	0	$\mathbb{Z}_2$	0	$\mathbb{Z}_2$	0	$\mathbb{Z}_2$
1	$\mathbb{Z}_2$	0	$\mathbb{Z}_2$	0	$\mathbb{Z}_2$	0	$\mathbb{Z}_2$	0	$\mathbb{Z}_2$
0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
q/p	0	1	2	3	4	5	6	7	8

$$E^2_{8,0} \xrightarrow{d^2_{8,0}} E^2_{6,1} \xrightarrow{d^2_{6,1}} E^2_{4,2}$$

$$d^2_{p,0} = (\mathrm{Sq}^2)^* \circ \rho_2, d^2_{p,1} = (\mathrm{Sq}^2)^*$$

$$\mathrm{Sq}^2(w_2^2) = 2w_2^3 + w_3^2 = 0$$

$$\mathrm{Sq}^2(w_2^3) = w_2^4 = \rho_2(c_1^4)$$

$$\Omega^{\mathrm{spin}}_7(BU(1)) = 0$$

$$H_{\cdot}(BU(2);\mathbb{Z}) = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \mathbb{Z} & 0 & \mathbb{Z} & 0 & \mathbb{Z}^2 & 0 & \mathbb{Z}^2 & 0 & \mathbb{Z}^3 \\ 1 & - & c_1^* & - & (c_1^2)^*, c_2^* & 0 & (c_1^3)^*, (c_1c_2)^* & - & (c_1^4)^*, (c_1^2c_2)^*, (c_2^2)^* \end{pmatrix}$$

$$\mathrm{Sq}^2(w_4) = w_2w_4 + w_6 = w_2w_4$$

$$\Omega_7^{\text{spin}}(BU(2)) = 0$$

$$E^2_{6,1} = \text{span}_{\mathbb{Z}_2}((w_2^3)^*, (w_2w_4)^*, (w_6)^*)$$

$$\begin{gathered}
\text{Sq}^2(w_2^3) = w_2^4 = \rho_2(c_1^4) \\
\text{Sq}^2(w_2w_4) = w_2w_6 = \rho_2(c_1c_3) \\
\text{Sq}^2(w_6) = w_2w_6 = \rho_2(c_1c_3) \\
\text{Sq}^2(w_4) = w_2w_4 + w_6
\end{gathered}$$

$$\Omega_7^{\text{spin}}(BU(n)) = 0.$$

$$\Omega_7^{\text{spin}}(BSp(n)) = 0$$

$$H.(K(\mathbb{Z},4);\mathbb{Z}) = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \cdots \\ \mathbb{Z} & 0 & 0 & 0 & \mathbb{Z} & 0 & \mathbb{Z}_2 & 0 & \mathbb{Z} & \end{pmatrix}$$

$$H.(K(\mathbb{Z},4);\mathbb{Z}_2) = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \cdots \\ \mathbb{Z}_2 & 0 & 0 & 0 & \mathbb{Z}_2 & 0 & \mathbb{Z}_2 & \mathbb{Z}_2 & \mathbb{Z}_2 & \end{pmatrix}$$

8	$\mathbb{Z}^2$	0	0	0	$\mathbb{Z}^2$	0	0	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$
7	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0
4	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	0	$\mathbb{Z}$
3	0	0	0	0	0	0	0	0	0
2	$\mathbb{Z}_2$	0	0	0	$\mathbb{Z}_2$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
1	$\mathbb{Z}_2$	0	0	0	$\mathbb{Z}_2$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	0	$\mathbb{Z}$
q/p	0	1	2	3	4	5	6	7	8

$$\Omega_7^{\text{spin}}(BE_8) = 0$$

8	$\mathbb{Z}^2$	$\mathbb{Z}_n^2$	0	$\mathbb{Z}_n^2$	0	$\mathbb{Z}_n^2$	0	$\mathbb{Z}_n^2$	0
7	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0
4	$\mathbb{Z}$	$\mathbb{Z}_n$	0	$\mathbb{Z}_n$	0	$\mathbb{Z}_n$	0	$\mathbb{Z}_n$	0
3	0	0	0	0	0	0	0	0	0
2	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
0	$\mathbb{Z}$	$\mathbb{Z}_n$	0	$\mathbb{Z}_n$	0	$\mathbb{Z}_n$	0	$\mathbb{Z}_n$	0
q/p	0	1	2	3	4	5	6	7	8

$$\xi_R(U) = \frac{1}{|G|}\sum_{g\in G-\{1\}} \mathrm{Tr}(R(g))\frac{\sqrt{\det(\tau(g))}}{\det(\tau(g)-I)}$$

$$\tau = \rho_1^{\oplus 4}$$

$$\det(\tau(z)) = z^4$$

$$\sqrt{\det(\tau(z))} = z^2$$

$$\det(\tau(z)-I) = (z-1)^4$$

$$\frac{\sqrt{\det(\tau(z))}}{\det(\tau(z)-I)} = \frac{1}{16\left(\sin\left(\frac{\pi}{n}j\right)\right)^4}$$

$$R_s := \rho_s \oplus \rho_{-s}$$

$$\xi_{R_{s_1}\ominus R_{s_2}}(U) = f(s_1) - f(s_2)$$

$$f(s) = \frac{1}{8n}\sum_{j=1}^{n-1}\frac{\cos\left(\frac{2\pi}{n}js\right)}{\left(\sin\frac{\pi}{n}j\right)^4}$$

$$f(s) = \frac{1}{8\cdot 45\cdot n}(-11+10n^2+n^4-60ns+60s^2-30n^2s^2+60ns^3-30s^4)$$

$$f(s+n) = f(s) - \frac{1}{3}s(s^2-1)$$

$$(1,\phi)\colon \mathrm{Spin}(d-1,1) \to \mathrm{Spin}(d-1,1)\times G_R$$

$$q \in C^\infty(\mathbb{R}^n, S),$$

$$\int \psi^* \nabla_g q$$

$$\int V^* \Gamma(q,q)$$

$$\mathcal{O} \mapsto \sum f^i Q_{\Psi_i}\mathcal{O}$$

$$\mathrm{PV}^{i,j}(X) = \Omega^{0,j}\left(X, \wedge^j TX\right)$$

$$\mathrm{PV}_c^{i,j}(X) \subset \mathrm{PV}^{i,j}(X), \Omega_c^{i,j}(X) \subset \Omega^{i,j}(X)$$

$$\mathrm{PV}^{d,d}(X) \cong \Omega_c^{0,d}(X) \cong \Omega_c^{d,d}(X)$$

$$\mathrm{Ker}\partial \subset \bigoplus \mathrm{PV}^{i,j}(X)[2].$$

$$\frac{1}{2}\int \alpha\bar{\partial}\partial^{-1}\alpha + \frac{1}{6}\int \alpha^3.$$

$$\mathrm{PV}^{*,*}(\mathrm{X})[\![t]\!]$$

$$(\partial \otimes 1)\delta_{\mathrm{Diag}} \in \bigoplus_{\substack{i_1+i_2=d+1\\ j_1+j_2=d}} \Omega^{i_1,j_1}(X) \otimes \Omega^{i_2,j_2}(X)$$

$$(\partial \otimes 1)\delta_{\mathrm{Diag}} \in \bigoplus_{\substack{i_1+i_2=d-1\\ j_1+j_2=d}} t^0\mathrm{PV}^{i_1,j_1}(X) \otimes t^0\mathrm{PV}^{i_2,j_2}(X)$$

$$(\partial \otimes 1)\delta_{\mathrm{Diag}} : (\alpha \otimes \beta) \to \int_X \alpha\partial\beta$$

$$P \in \mathrm{PV}(X)[[t]] \,\widehat{\otimes}\, \mathrm{PV}(X)[[t]]$$

$$(\bar{\partial} + t\partial)P = (\partial \otimes 1)\delta_{\mathrm{Diag}} + \mathfrak{K}_{\text{something smooth (the regularized Poisson kernel)}}$$

$$\begin{aligned} P =& \bar{\partial}_z^*\partial_z \frac{(\partial_{z_1} - \partial_{w_1}) \dots (\partial_{z_n} - \partial_{w_n})(\mathrm{d}\bar{z}_1 - \mathrm{d}\bar{w}_1) \dots (\mathrm{d}\bar{z}_n - \mathrm{d}\bar{w}_n))}{\|z-w\|^{2n-2}} \\ =& \sum (-1)^i(-1)^{j+n}(\partial_{z_1} - \partial_{w_1}) \dots \left(\widehat{\partial z_i}^{-\partial_{w_i}}\right) \dots (\partial_{z_n} - \partial_{w_n}) \\ & \times (\mathrm{d}\bar{z}_1 - \mathrm{d}\bar{w}_1) \dots \left(\widehat{\mathrm{d}\bar{z}_j - \mathrm{d}\bar{w}_j}\right)(\mathrm{d}\bar{z}_n - \mathrm{d}\bar{w}_n)\big) \frac{\partial}{\partial z_i}\frac{\partial}{\partial z_j}\|z-w\|^{2-2n} \end{aligned}$$

$$I_n : \mathrm{PV}_c^{*,*}(X)[\![t]\!] \to \mathbb{C}$$

$$I_n(\alpha) = \sum_{k_1,\dots,k_n \text{ with } \sum k_i = n-3} \frac{(n-3)!}{k_1! \dots k_n!}\int \alpha_{k_1} \wedge \dots \wedge \alpha_{k_n}.$$

$$I(\alpha) = \sum_{n\geq 3} \frac{1}{n!} I_n(\alpha)$$

$$QI + \frac{1}{2}\{I, I\} = 0$$

$$AdS_5 \times S_5 \simeq (\mathbb{R}^{10} \setminus \mathbb{R}^4, g)$$

$$F = N\frac{3}{4i\pi^3}\,\mathrm{d}z_1\,\mathrm{d}z_2\,\mathrm{d}z_3 r^{-6}(\bar{z}_1\,\mathrm{d}\bar{z}_2\,\mathrm{d}\bar{z}_3 - \bar{z}_2\,\mathrm{d}\bar{z}_1\,\mathrm{d}\bar{z}_3 + \bar{z}_3\,\mathrm{d}\bar{z}_1\,\mathrm{d}\bar{z}_2)$$

$$\bar{\partial}F = N\delta_{\mathbb{C}^2}$$

$$\mathcal{L}_V g = Q(\Psi)$$

$$\Omega_+^5 = \Omega^{5,0} \oplus \Omega^{3,2} \oplus \Omega^{1,4}$$

$$\begin{aligned} \operatorname{Sym}^2 S_+ &\cong V \oplus \Omega^5_{+,\mathrm{const}} \\ \wedge^2 S_+ &\cong \Omega^3_{\mathrm{const}} \\ S_+ \otimes S_- &\cong \Omega^0_{\mathrm{const}} \oplus \Omega^2_{\mathrm{const}} \oplus \Omega^4_{\mathrm{const}} \\ \operatorname{Sym}^2 S_- &\cong V \oplus \Omega^5_{-,\mathrm{const}} . \end{aligned}$$

$$\begin{aligned} \Gamma = \Gamma_{\Omega^1} &: S_+ \otimes S_+ \to V = \Omega^1_{\mathrm{const}} \\ \Gamma_{\Omega^5_+} &: S_+ \otimes S_+ \to \Omega^5_{+,\mathrm{const}} . \end{aligned}$$

$$\begin{aligned} S_+ &\cong \Omega^{0,ev}_{const} \cong \Omega^{odd,0}_{const} \\ S_- &\cong \Omega^{0,odd}_{const} \cong \Omega^{ev,0}_{const}. \end{aligned}$$

$$\mathcal{T}^{2,0} = V \oplus \Pi(S_+ \otimes \mathbb{C}^2).$$

$$\mathcal{T}^{1,1} = V \oplus \Pi S_+ \oplus \Pi S_-$$

$$S_+ \otimes S_+ = \Omega^1_{\mathrm{const}} \oplus \Omega^3_{\mathrm{const}} \oplus \Omega^5_{+,\mathrm{const}} .$$

$$[\psi_1 \otimes e_i, \psi_2 \otimes e_j] = \Gamma_{\Omega^1}(\psi_1 \otimes \psi_2)\omega(e_i, e_j)$$

$$[\psi_1 \otimes e_i, \psi_2 \otimes e_j] = \Gamma_{\Omega^5_+}(\psi_1 \otimes \psi_2)\omega(e_i, e_j).$$

$$[\psi_1 \otimes e_i, \psi_2 \otimes e_j] = \Gamma_{\Omega^3}(\psi_1 \otimes \psi_2)\eta(e_i, e_j).$$

$$[\mathcal{C}(\psi_1 \otimes e_i), \mathcal{C}(\psi_2 \otimes e_j)] = \mathcal{C}(\Gamma(\psi_1 \otimes \psi_2)\delta_{ij})$$

$$[\mathcal{C}(\psi_1 \otimes e_i), \mathcal{C}(\psi_2 \otimes e_j)] = \mathcal{C}(\Gamma(\psi_1 \otimes \psi_2)\delta_{ij}) + \int \Gamma_{\Omega^1}(\psi_1 \otimes \psi_2)\omega(e_i, e_j).$$

$$\mu_1(\psi_1 \otimes e_i, \psi_2 \otimes e_j, v) = \Gamma_{\Omega^1}(\psi_1 \otimes \psi_2)(v)\omega(e_i, e_j)$$

$$l_k : \left(\mathcal{T}^{(2,0)}\right)^{\otimes k} \to \mathbb{C}[k-2]$$

$$\begin{aligned} C_{Dk} &\in C^2\left(\mathcal{T}^{(2,0)}, \Omega^k_{\mathrm{closed}}(\mathbb{R}^{10})\right) \\ C_{FS} &\in C^2\left(\mathcal{T}^{(2,0)}, \Omega^1_{\mathrm{closed}}(\mathbb{R}^{10})\right) \\ C_{NS5} &\in C^2\left(\mathcal{T}^{(2,0)}, \Omega^5_{\mathrm{closed}}(\mathbb{R}^{10})\right), \end{aligned}$$

$$C^*\left(\mathcal{T}^{(2,0)}, \Omega^0(\mathbb{R}^{10})\right) \overset{\mathrm{d}_{dR}}{\to} C^*\left(\mathcal{T}^{(2,0)}, \Omega^1(\mathbb{R}^{10}\right) \overset{\mathrm{d}_{dR}}{\to} \dots$$

$$C^2(\mathcal{T}^{(2,0)}, \Omega^k_{\mathrm{closed}}) \to C^{2+k}\left(\mathcal{T}^{(2,0)}, \Omega^*(\mathbb{R}^{10})\right)$$

$$C^*(\mathcal{T}^{(2,0)}, \mathbb{C}) \to C^*\left(\mathcal{T}^{(2,0)}, \Omega^*(\mathbb{R}^{10})\right)$$

$$\begin{aligned} \tilde{C}_{Dk} &\in C^{k+2}(\mathcal{T}^{(2,0)}, \mathbb{C}) \\ \tilde{C}_{FS} &\in C^3(\mathcal{T}^{(2,0)}, \mathbb{C}) \\ \tilde{C}_{NS5} &\in C^7(\mathcal{T}^{(2,0)}, \mathbb{C}) \end{aligned}$$

$$V \oplus \mathfrak{so}(10,\mathbb{C}) \oplus \Pi(S_+ \otimes \mathbb{C}^2) \oplus \Omega^*(\mathbb{R}^{10})$$

$$Q = \Psi \otimes e_1 \in \mathfrak{S}.$$

$$V = V^{1,0} \oplus V^{0,1}$$

$$V^{1,0} \oplus \mathrm{Stab}(Q) \oplus \Pi(S_+ \otimes e_2) \oplus \pi\mathbb{C} \cdot c$$

$$\psi_1, \psi_2 \in S_+ = S_+ \otimes e_2$$

$$[\psi_1, \psi_2] = \pi_{V^{1,0}}\Gamma(\psi_1 \otimes \psi_2)$$

$$[v, \psi] = c\langle\Gamma(\Psi \otimes \psi), v\rangle_V$$

$$\mathfrak{G}_{\mathfrak{cuantica}} = \mathfrak{so}(10,\mathbb{C}) \oplus V \oplus \Pi(S_+ \otimes \mathbb{C}^2)$$

$$[Q, -]: S_+ \otimes e_1 \to V^{0,1}$$

$$[Q, -]: \mathfrak{so}(10,\mathbb{C}) \to S_+ \otimes e_1.$$

$$\mathfrak{G}_{\mathfrak{cuantica}} = \mathfrak{so}(10,\mathbb{C}) \oplus V \oplus \Pi(S_+ \otimes \mathbb{C}^2) \oplus \Omega^*(\mathbb{R}^{10})$$

$$[Q, -]: S_+ \otimes e_2 \to \Omega^1(\mathbb{R}^{10})$$
$$\psi \otimes e_2 \mapsto \Gamma_{\Omega^1}(\Psi, \psi).$$

$$H: \Omega^1(\mathbb{R}^{10}) \to \Omega^0(\mathbb{R}^{10})$$

$$(H\omega)(x) = \int_0^x \omega$$

$$\mathrm{d}_{dR} H\omega = \omega.$$

$$L(\psi) = \psi \otimes e_2 - H\Gamma_{\Omega^1}(\Psi \otimes \psi) \in S_+ \otimes e_2 \oplus \Omega^0[1].$$

$$[v, L(\psi)] = -\langle v, \Gamma(\Psi \otimes \psi)\rangle = c\langle v, \Gamma(\Psi \otimes \psi)\rangle$$

$$\mathrm{Stab}(Q) \cong \mathfrak{sl}(5,\mathbb{C}) \oplus \mathrm{PV}^{3,0}_{\mathrm{const}} \cong \mathfrak{sl}(5,\mathbb{C}) \oplus \Omega^{2,0}_{\mathrm{const}}$$
$$S_+ \cong \Omega^{\mathrm{odd},0}_{\mathrm{const}} \cong \Omega^{1,0}_{\mathrm{const}} \oplus \mathrm{PV}^{2,0}_{\mathrm{const}} \oplus \Omega^{0,0}_{\mathrm{const}}$$

$$\mathfrak{G}_{\mathfrak{cuantica}} = \mathfrak{sl}(5,\mathrm{C}) \oplus \mathrm{PV}^{1,0}_{\mathrm{const}} \oplus \mathrm{PV}^{3,0}_{\mathrm{const}} \oplus \Pi\left(\Omega^{1,0}_{\mathrm{const}} \oplus \mathrm{PV}^{2,0}_{\mathrm{const}} \oplus \Omega^{0,0}_{\mathrm{const}} \oplus \mathrm{C} \cdot c\right)$$

$$A_{ij} \in \mathfrak{sl}(5, \mathrm{C}) \mapsto \sum A_{ij} z_i \frac{\partial}{\partial z_j} \in \mathrm{PV}^{1,0} \subset \mathrm{PV}[\![t]\!]$$
$$\frac{\partial}{\partial z_i}\frac{\partial}{\partial z_j}\frac{\partial}{\partial z_k} \in \mathrm{PV}^{3,0} \mapsto \frac{\partial}{\partial z_i}\frac{\partial}{\partial z_j}\frac{\partial}{\partial z_k} \in \mathrm{PV}^{3,0} \subset \mathrm{PV}[\![t]\!]$$
$$\frac{\partial}{\partial z_i} \in \mathrm{PV}^{1,0}_{\mathrm{const}} = V^{1,0} \mapsto \frac{\partial}{\partial z_i} \in \mathrm{PV}[\![t]\!]$$
$$\mathrm{d}z_i \in \Omega^{1,0}_{\mathrm{const}} \mapsto z_i \in \mathrm{PV}^{0,0} \subset \mathrm{PV}[\![t]\!]$$
$$\frac{\partial}{\partial z_i}\frac{\partial}{\partial z_j} \in \mathrm{PV}^{2,0}_{\mathrm{const}} \mapsto \frac{\partial}{\partial z_i}\frac{\partial}{\partial z_j} \in \mathrm{PV}^{2,0} \subset \mathrm{PV}[\![t]\!]$$
$$1 \in \Omega^{0,0}_{\mathrm{const}} \mapsto 0$$
$$c \mapsto 1 \in \mathrm{PV}^{0} \subset \mathrm{PV}[\![t]\!].$$

$$\mathrm{Ker}\partial \subset \mathrm{PV}_{hol}(\mathbb{C}^5)$$

$$\{z_i, \partial_{z_j}\partial_{z_k}\} = \delta_{ij}\partial_{z_k} - \delta_{ik}\partial_{z_j}$$

$$\left\{\frac{\partial}{\partial z_i}, z_j\right\} = \delta_{ij}$$

$$\mathrm{Ext}_{\mathcal{O}(\mathbb{C}^5)}\left(\mathcal{O}^N_{\mathbb{C}^k}, \mathcal{O}^N_{\mathbb{C}^k}\right) \simeq \Omega^{0,*}(\mathbb{C}^k)[\epsilon_1, \dots, \epsilon_{5-k}] \otimes \mathfrak{gl}_N$$

$$S(A) = \int_{\mathbb{C}^{k|5-k}} \left(\frac{1}{2}\mathrm{Tr}(A\bar{\partial}A) + \frac{1}{3}\mathrm{Tr}(A^3)\right) \prod \mathrm{d}z_i \prod \mathrm{d}\epsilon_i$$

$$\Omega^{0,*}(\mathbb{C}^k)[\mathrm{d}\bar{z}_1 \dots \mathrm{d}\bar{z}_{5-k}].$$

$$HH^*(\mathcal{O}_X) \to HH^*(\mathrm{RHom}(E, E))$$

$$\mathrm{PV}(X) \to HH^*(\mathrm{RHom}(E, E)).$$

$$\mathrm{PV}(X)[\![t]\!] \to HC^*(\mathrm{RHom}(E, E))$$

$$\mathrm{PV}(\mathbb{C}^5)[\![t]\!] \to HC^*\left(\Omega^{0,*}(\mathbb{C}^k)[\epsilon_i]\right)$$

$$\mathrm{PV}^{*,*}(\mathbb{C}^{k|5-k})[\![t]\!] = \mathrm{PV}^{*,*}(\mathbb{C}^k)\left[\!\left[\epsilon_i, \frac{\partial}{\partial \epsilon_i}\right]\!\right][\![t]\!]$$

$$\mathrm{PV}_{hol}(\mathbb{C}^5) \to \mathrm{PV}_{hol}(\mathbb{C}^{k|5-k}) = \mathrm{PV}_{hol}(\mathbb{C}^k)\left[\!\left[\epsilon_\alpha, \frac{\partial}{\partial \epsilon_\alpha}\right]\!\right]$$

$$HH^*\left(\Omega^{0,*}(\mathbb{C}^5)\right) \to HH^*\left(\Omega^{0,*}(\mathbb{C}^{k|5-k})\right)$$

$$w_i \mapsto w_i$$
$$\frac{\partial}{\partial w_i} \mapsto \frac{\partial}{\partial w_i}$$
$$z_\alpha \mapsto \frac{\partial}{\partial \epsilon_\alpha}$$
$$\frac{\partial}{\partial z_\alpha} \mapsto \epsilon_\alpha.$$

$$\mathbb{C}\left[z_\alpha, w_i, \frac{\partial}{\partial z_\alpha}, \frac{\partial}{\partial w_i}\right]$$

$$\mathrm{PV}(V^\vee[-1]) = \wedge^* V \otimes \widehat{\mathrm{Sym}}^* V^\vee$$

$$\mathrm{PV}(V) = \mathrm{Sym}^* V^\vee \otimes \wedge^* V.$$

$$\mathrm{Sym}^* V^\vee \otimes \wedge^* V \to \wedge^* V \otimes \widehat{\mathrm{Sym}}^* V^\vee.$$

$$\mathrm{PV}_{hol}(V) = \mathcal{O}(V) \otimes \wedge^* V \to \widehat{\mathrm{Sym}}^* V^\vee \otimes \wedge^* V$$

$$z_\alpha, w_i, \frac{\partial}{\partial z_\alpha}\frac{\partial}{\partial z_\beta}, \frac{\partial}{\partial w_i}\frac{\partial}{\partial w_j}, \text{ and } \frac{\partial}{\partial z_\alpha}\frac{\partial}{\partial w_j}$$

$$\frac{\partial}{\partial \epsilon_\alpha}, w_i, \epsilon_\alpha \epsilon_\beta, \frac{\partial}{\partial w_i}\frac{\partial}{\partial w_j}, \text{ and } \epsilon_\alpha \frac{\partial}{\partial w_j}$$

$$\int_{\mathbb{C}^{k|5-k}} \prod \mathrm{d}w_i \prod \mathrm{d}\epsilon_\alpha \mathrm{Tr}\left(A \frac{\partial}{\partial \epsilon_\alpha} A\right)$$

$$A \mapsto \int_{\mathbb{C}^{k|5-k}} \prod \mathrm{d}w_i \prod \mathrm{d}\epsilon_\alpha \mathrm{Tr}(A) w_i$$
$$A \mapsto \int_{\mathbb{C}^{k|5-k}} \prod \mathrm{d}w_i \prod \mathrm{d}\epsilon_\alpha \mathrm{Tr}(A) \epsilon_\alpha \epsilon_\beta.$$

$$\{f, g\} = \frac{\partial}{\partial w_i} f \frac{\partial}{\partial w_j} g$$

$$A \mapsto \int_{\mathbb{C}^{k|5-k}} \prod \mathrm{d}w_i \prod \mathrm{d}\epsilon_\alpha \mathrm{Tr}\left(A \frac{\partial}{\partial w_i} A \frac{\partial}{\partial w_j} A\right).$$

$$\int_{\mathbb{C}^{k|5-k}} \prod \mathrm{d}w_i \prod \mathrm{d}\epsilon_\alpha \mathrm{Tr}\left(A \epsilon_\alpha \frac{\partial}{\partial w_j} A\right).$$

$$\bar{\partial} A + \frac{1}{2}[A, A] + w_i \mathrm{Id} = 0$$
$$\bar{\partial} A + \frac{1}{2}[A, A] + \epsilon_\alpha \epsilon_\beta \mathrm{Id} = 0$$

$$s\left(\epsilon_1 \frac{\partial}{\partial w_1} + \epsilon_2 \frac{\partial}{\partial w_2}\right) + t \frac{\partial}{\partial \epsilon_3}$$

$$s\left(\frac{\partial}{\partial z_1}\frac{\partial}{\partial w_1} + \frac{\partial}{\partial z_2}\frac{\partial}{\partial w_2}\right) + t z_3$$

$$\mathrm{d}^{-1}\delta_{\mathbb{R}^{2k}} \in \Omega^{10-2k-1}(\mathbb{R}^{10})$$

$$F_{10-2k-1} = \mathrm{d}^{-1}\delta_{\mathbb{R}^{2k}}$$

$$F_l = * F_{10-l}$$

$$\int_{\mathbb{R}^{2k}} A_{2k} + \int \mathrm{d}A_{2k} * \mathrm{d}A_{2k} = \int_{\mathbb{R}^{2k}} A_{2k} - \int A_{2k}\, \mathrm{d}F_{10-2k-1}$$

$$\begin{aligned} \mathrm{PV}_c^{*,*}(\mathbb{C}^5) &\mapsto \mathbb{C} \\ \alpha^{i,j} &\mapsto 0 \text{ if } (i,j) \neq (5-k,k) \\ \alpha^{5-k,k} &\mapsto \int_{\mathbb{C}^k} \alpha^{5-k,k} \vee \Omega \end{aligned}$$

$$L: \mathrm{PV}_c^{*,*}(\mathbb{C}^5) \to \mathbb{C}$$

$$L(\partial\alpha) = \int_{\mathbb{C}^k} \alpha \vee \Omega$$

$$L(\alpha) = \int_{\mathbb{C}^k} (\partial^{-1}\alpha) \vee \Omega$$

$$\int_{\mathbb{C}^5} \alpha^{k,4-k} \wedge \bar{\partial}\partial^{-1}\alpha 4 - k, k + \int_{\mathbb{C}^k} \left(\partial^{-1}\alpha^{4-k,k}\right) \vee \Omega$$

$$\bar{\partial}\alpha^{k,4-k} = \delta_{\mathbb{C}^k}$$

$$* \;\mathrm{d}A_k = \mathrm{d}A_{8-k}.$$

$$\left(\oplus_{i+j\leq 2} t^j \mathrm{PV}^{i,*}(X)\right) \bigoplus \left(\oplus_{l-k\geq 2} t^{-k} \mathrm{PV}^{l,*}(X)\right)$$

$$\begin{aligned} \left(\oplus_{i+j\leq 2} t^j \mathrm{PV}^{i,*}(X)\right) \bigoplus \left(\oplus_{l-k\geq 2} t^{-k} \mathrm{PV}^{l,*}(X)\right) &\to \oplus_{i+j\leq 4} t^j \mathrm{PV}^{i,*}(X) \\ t^j \mathrm{PV}^{i,*}(X) \ni \alpha &\mapsto \alpha \in t^j \mathrm{PV}^{i,*}(X) \\ t^{-k} \mathrm{PV}^{l,*}(X) \ni \alpha &\mapsto \delta_{k=0}\partial\alpha \in \mathrm{PV}^{l+1,*}(X). \end{aligned}$$

$$\mathfrak{iso}(4) = (\mathfrak{sl}(2) \oplus \mathfrak{sl}(2)) \ltimes \mathbb{C}^4$$

$$V_C \otimes V_R^* \oplus V_C^* \otimes V_R = S_+ \otimes V_R \oplus S_- \otimes V_R \oplus S_+ \otimes V_R^* \oplus S_- \otimes V_R^*$$

$$F \in \bar{\Omega}^{3,2}(\mathbb{C}^5)$$

$$\bar{\partial}F = \delta_{\mathbb{C}^2}$$

$$r = \sqrt{|z_1|^2 + |z_2|^2 + |z_3|^2}$$

$$F = \frac{3}{4i\pi^3}\, \mathrm{d}z_1\, \mathrm{d}z_2\, \mathrm{d}z_3 r^{-6}(\bar{z}_1\, \mathrm{d}\bar{z}_2\, \mathrm{d}\bar{z}_3 - \bar{z}_2\, \mathrm{d}\bar{z}_1\, \mathrm{d}\bar{z}_3 + \bar{z}_3\, \mathrm{d}\bar{z}_1\, \mathrm{d}\bar{z}_2).$$

$$\int_{\Sigma|z_i|^2=1} F = 1$$

$$F = \frac{3}{4i\pi^3}\frac{\partial}{\partial w_1}\frac{\partial}{\partial w_2} r^{-6}(\bar{z}_1\, \mathrm{d}\bar{z}_2\, \mathrm{d}\bar{z}_3 - \bar{z}_2\, \mathrm{d}\bar{z}_1\, \mathrm{d}\bar{z}_3 + \bar{z}_3\, \mathrm{d}\bar{z}_1\, \mathrm{d}\bar{z}_2).$$

$$\bar{\partial}F + t\partial F + \frac{1}{2}\{F, F\} = 0.$$

$$\alpha = NF \in \mathrm{PV}^{2,2}(\mathbb{C}^5 \setminus \mathbb{C}^2)$$

$$\sum w_i \frac{\partial}{\partial w_i} - \frac{2}{3} \sum z_i \frac{\partial}{\partial z_i}.$$

$$w_i \left(\sum_j w_j \frac{\partial}{\partial w_j} - \sum_k z_k \frac{\partial}{\partial z_k} \right)$$

$$\frac{\partial}{\partial z_i} \left(\sum_l w_l \frac{\partial}{\partial w_l} - \sum_k z_k \frac{\partial}{\partial z_k} \right) \in \mathrm{PV}^{2,0}(\mathbb{C}^5 \setminus \mathbb{C}^2).$$

$$\mathcal{T}^{(1,1)} = V \oplus \Pi(S_+ \oplus S_-).$$

$$\Gamma: S_\pm \otimes S_\pm \to V.$$

$$\Gamma_{\Omega^1}: S_- \otimes S_- \to \Omega^1 \subset \Omega^*(\mathbb{R}^{10}).$$

$$S_+ = \Omega^{0,ev}_{\mathrm{const}}$$
$$S_- = \Omega^{0,\,\mathrm{odd}}_{\mathrm{const}}\ .$$

$$Q = 1 + \mathrm{d}\bar{z}_1 \in \Omega^{0,0}_{const} \oplus \Omega^{0,1}_{const} \subset S_+ \oplus S_-.$$

$$\Omega^*(\mathbb{R}^2) \mathbin{\widehat{\otimes}} \mathrm{PV}(\mathbb{C}^4)$$

$$\Omega^*(\mathbb{R}^2) \mathbin{\widehat{\otimes}} \mathrm{PV}(\mathbb{C}^4)[\,[t]\,]$$

$$\mathrm{d}^{\mathbb{R}^2}_{dR} + \bar{\partial}^{\mathbb{C}^4} + t\partial^{\mathbb{C}^4}$$

$$\pi = \left(\partial^{\mathbb{C}^4} \otimes 1\right)\delta_{\mathrm{Diag}}$$

$$\Omega^*(\mathbb{R}^2) \mathbin{\widehat{\otimes}} \mathrm{PV}(\mathbb{C}^4) \cong \Omega^*(\mathbb{R}^2) \mathbin{\widehat{\otimes}} \Omega^{*,*}(\mathbb{C}^4)$$

$$\mathrm{PV}^{4,4}(\mathbb{C}^4) = \Omega^{0,4}(\mathbb{C}^4) = \Omega^{4,4}(\mathbb{C}^4).$$

$$\alpha = \sum \alpha_k t^k \in \Omega^*(\mathbb{R}^2) \mathbin{\widehat{\otimes}} \mathrm{PV}(\mathbb{C}^4)[\![t]\!]$$

$$I(\alpha) = \int \alpha_0^3 + \mathbb{H}$$

$$\mathfrak{G}_{\mathfrak{cuantica}} = \mathfrak{so}(10,\mathbb{C}) \ltimes (V \oplus \Pi S_+ \oplus \Pi S_-)$$

$$\mathfrak{sl}(4) \oplus W \oplus \wedge^2 W^\vee \oplus \Pi(W^\vee \oplus \wedge^2 W^\vee \oplus \mathbb{C} \cdot c)$$

$$W^\vee \otimes \wedge^2 W^\vee \xrightarrow{\wedge} \wedge^3 W^\vee = W$$

$$S_+ = \Omega^{0,ev}_{const}$$
$$S_- = \Omega^{0,odd}_{const}.$$

$$[1,-]:\Omega^{0,4}\to\mathbb{C}^5\oplus\overline{\mathbb{C}}^5$$

$$\mathrm{d}\bar{z}_1\wedge\cdots\wedge\widehat{\mathrm{d}}_i\cdots\wedge\ \mathrm{d}\bar{z}_5\mapsto\frac{\partial}{\partial\bar{z}_i}.$$

$$[\mathrm{d}\bar{z}_1,-]:\Omega^{0,3}_{\mathrm{const}}\to\overline{\mathbb{C}}^5$$

$$\mathrm{d}\bar{z}_2\wedge\cdots\wedge\widehat{\mathrm{d}}_{\bar{z}}\cdots\wedge\ \mathrm{d}\bar{z}_5\mapsto\frac{\partial}{\partial\bar{z}_i}$$

$$[\ \mathrm{d}\bar{z}_1,-]:\Omega^{0,5}_{\mathrm{const}}\to\mathbb{C}^5$$

$$\mathrm{d}\bar{z}_1\wedge\cdots\wedge\ \mathrm{d}\bar{z}_5\mapsto\frac{\partial}{\partial z_1}.$$

$$\overline{\mathbb{C}}^4=W^\vee\subset\Omega^{0,4}_{\mathrm{const}}\oplus\Omega^{0,3}_{\mathrm{const}}$$

$$\frac{\partial}{\partial z_i}\vee(\ \mathrm{d}\bar{z}_1\wedge\cdots\wedge\ \mathrm{d}\bar{z}_5-\mathrm{d}\bar{z}_2\wedge\cdots\wedge\ \mathrm{d}\bar{z}_5)$$

$$\mathfrak{so}(10,\mathbb{C})=\mathfrak{sl}(5,\mathbb{C})\oplus\wedge^2\mathbb{C}^5\oplus\wedge^2\overline{\mathbb{C}}^5$$

$$[Q,-]:\mathfrak{so}(10,\mathbb{C})\to\Omega^{0,*}_{\mathrm{const}}=S_+\oplus S_-$$

$$\wedge^2\mathbb{C}^4=\wedge^2 W\subset\Omega^{0,2}_{\mathrm{const}}\oplus\Omega^{0,3}_{\mathrm{const}}$$

$$\mathrm{d}\bar{z}_i\wedge\ \mathrm{d}\bar{z}_j-\mathrm{d}\bar{z}_1\ \mathrm{d}\bar{z}_i\ \mathrm{d}\bar{z}_j$$

$$\frac{\partial}{\partial z_i}\vee(\ \mathrm{d}\bar{z}_1\wedge\cdots\wedge\ \mathrm{d}\bar{z}_5-\mathrm{d}\bar{z}_2\wedge\cdots\wedge\ \mathrm{d}\bar{z}_5)\text{ for } i=1,\dots 4$$

$$\mathrm{d}\bar{z}_i\ \mathrm{d}\bar{z}_j\text{ for } 2\le i<j\le 4.$$

$$\mathrm{Stab}_{\mathfrak{sl}(5,\mathbb{C})}(Q)=\mathfrak{sl}(4,\mathbb{C})\oplus W^\vee$$

$$\mathfrak{sl}(5,\mathbb{C})=\mathfrak{sl}(4,\mathbb{C})\oplus W\oplus W^\vee$$

$$\mathrm{Stab}(Q)=\mathfrak{sl}(4,\mathbb{C})\oplus W^\vee\oplus\wedge^2\overline{\mathbb{C}^5}=\mathfrak{sl}(4,\mathbb{C})\oplus W^\vee\oplus W^\vee\oplus\wedge^2 W^\vee.$$

$$W^\vee\otimes\wedge^2 W^\vee\to\wedge^3 W^\vee=W$$

$$W^\vee\otimes W^\vee\to\wedge^2 W^\vee\subset\mathrm{Stab}(Q).$$

$$\mathfrak{G}_{\mathrm{cuantica}}\to\Omega^*(\mathbb{R}^2)\mathbin{\widehat{\otimes}}\mathrm{PV}(\mathbb{C}^4)[1][\![t]\!]$$

$$\Omega^*(\mathbb{R}^6)\mathbin{\widehat{\otimes}}\mathrm{PV}(\mathbb{C}^2)[\![t]\!]$$

$$\Omega^*(\mathbb{R})\mathbin{\widehat{\otimes}}\mathrm{PV}(\mathbb{C}^4)[\![t]\!][\epsilon]$$

$$\Omega^*(\mathbb{R})\mathbin{\widehat{\otimes}}\Omega^{0,*}(\mathbb{C}^k)[\epsilon_1,\dots,\epsilon_{4-k}]\otimes\mathfrak{gl}_N$$

$$\int_{\mathbb{R}\times\mathbb{C}^{k|4-k}}\mathrm{d}w_1\dots\mathrm{d}w_k\ \mathrm{d}\epsilon_1\dots\ \mathrm{d}\epsilon_{4-k}\left(\frac{1}{2}\mathrm{Tr}\left(A\left(\bar{\partial}^{\mathbb{C}^4}+\mathrm{d}^{\mathbb{R}^2}_{dR}\right)A\right)+\frac{1}{3}\mathrm{Tr}A^3\right)$$

$$\Omega^*(\mathbb{R})\,\widehat{\otimes}\,\mathrm{PV}^{*,*}\left(\mathbb{C}^{k|4-k}\right)$$

$$\Omega^*(\mathbb{R})\,\widehat{\otimes}\,\mathrm{PV}^{*,*}\left(\mathbb{C}^{k|4-k}\right)[\![t]\!]$$

$$\mathrm{PV}^{*,*}(\mathbb{C}^5)[\,[t]\,] \to \mathrm{PV}^{*,*}\left(\mathbb{C}^{k|5-k}\right)[\![t]\!].$$

$$\Omega^*(\mathbb{R}^2)\,\widehat{\otimes}\,\mathrm{PV}(\mathbb{C}^4)[\![t]\!] \to \Omega^*(\mathbb{R})\,\widehat{\otimes}\,\mathrm{PV}^{\mathbb{C}^{k|4-k}}\big)\big)\,[\![t]\!]$$

$$\mathcal{T}_{2k} = \mathbb{C}^{2k} \oplus \Pi S$$

$$\mathfrak{siso}^R(2k) = (\mathfrak{so}(2k,\mathbb{C}) \oplus \mathfrak{so}(10-2k,\mathbb{C})) \ltimes \mathcal{T}_{2k}$$

$$\mathrm{Stab}(Q) \subset \mathfrak{so}(2k,\mathbb{C}) \oplus \mathfrak{so}(10-2k,\mathbb{C})$$

$$[w^\vee, w\otimes v] = \langle w^\vee, w\rangle v$$

$$S = S_+^{(2)} \otimes S_+^{(8)} \oplus S_-^{(2)} \otimes S_-^{(8)}$$

$$[u^\vee, u] = \langle u^\vee, u\rangle \frac{\partial}{\partial w}$$

$$\wedge^* V \otimes \widehat{\mathrm{Sym}}^* V^*[\![t]\!] = \mathbb{C}\left[\!\!\left[\epsilon_\alpha, \frac{\partial}{\partial \epsilon_\alpha}, t\right]\!\!\right]$$

$$\mathrm{Ker}\partial \subset \wedge^* V \otimes \widehat{\mathrm{Sym}}^* V^*.$$

$$V \otimes \wedge^* V \otimes \widehat{\mathrm{Sym}}^* V^*.$$

$$\mathrm{Ker}(\partial) \subset \mathrm{Hol}\left(\mathbb{C}^k\right)\left[\!\!\left[\frac{\partial}{\partial w_i}, \epsilon_\alpha, \frac{\partial}{\partial \epsilon_\beta}\right]\!\!\right]$$

$$\epsilon_\alpha \frac{\partial}{\partial w_j} + A_{\alpha j}$$

$$\mathrm{C}\left[\!\!\left[\frac{\partial}{\partial w_i}, \frac{\partial}{\partial \epsilon_\alpha}\right]\!\!\right].$$

$$A_{\alpha j} = \frac{\partial}{\partial w_i}\frac{\partial}{\partial \epsilon_1}\cdots\frac{\hat{\partial}}{\partial \epsilon_\alpha}\cdots\frac{\partial}{\partial \epsilon_{5-k}}.$$

$$A_{\alpha 1} = \frac{\partial}{\partial \epsilon_1}\cdots\frac{\hat{\partial}}{\partial \epsilon_\alpha}\cdots\frac{\partial}{\partial \epsilon_{5-k}}$$

$$z_1^{-1}z_2^{-1}z_3^{-1}\mathbb{C}[z_1^{-1}, z_2^{-1}z_3^{-1}] \hookrightarrow H^2_{\bar{\partial}}(\mathbb{C}^3 \setminus 0).$$

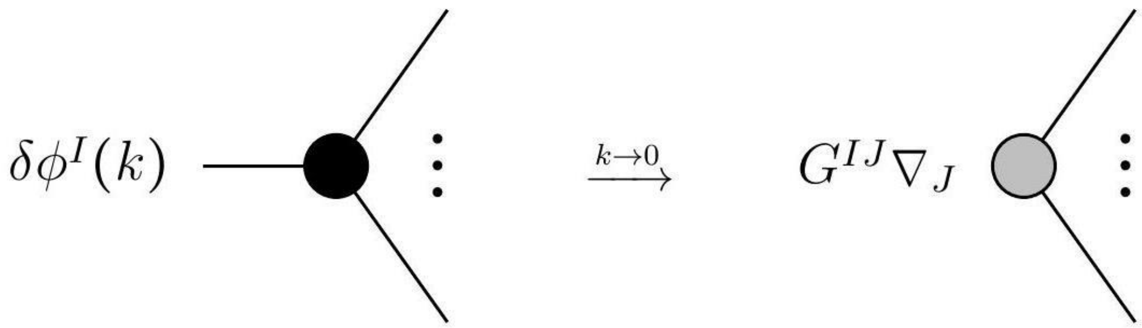
$$[F] = z_1^{-1}z_2^{-1}z_3^{-1}\frac{\partial}{\partial w_1}\frac{\partial}{\partial w_2}$$

$$H^*\left(\mathrm{PV}(\mathbb{C}^5 \setminus \mathbb{C}^2), \bar{\partial}\right) = H^*\left(\mathrm{PV}(\mathbb{C}^5)\right) \oplus H^2_{\bar{\partial}}(\mathbb{C}^3 \setminus 0)\left[w_i, \partial_{w_i}, \partial_{z_i}\right].$$

$$(\dagger) H^*\left(\mathrm{PV}(\mathbb{C}^5 \setminus \mathbb{C}^2)[[t]], \bar{\partial} + t\partial\right) \simeq H^*(\mathrm{PV}(\mathbb{C}^5)[[t]]) \oplus H^2_{\bar{\partial}}(\mathbb{C}^3 \setminus 0)\left[w_i, \partial_{w_i}, \partial_{z_i}\right][[t]]$$

$$\begin{aligned}\{[F], w_1 z_1\} &= \{z_1^{-1} z_2^{-1} z_3^{-1} \partial_{w_1} \partial_{w_2}, w_1 z_1\} \\ &= z_1 (z_1^{-1} z_2^{-1} z_3^{-1}) \partial_{w_2} \\ &= 0\end{aligned}$$

$$\lim_{k \to 0} \mathcal{A}(\delta\phi^I(k), \{\psi_i(p_i)\}) = G^{IJ}(\phi_0) \nabla_J \mathcal{A}(\{\psi_i(p_i)\}).$$



$$\mathcal{L} = \mathcal{L}_{(2)} + \mathcal{O},$$

$$\frac{1}{2}\left(\lim_{k_1 \to 0} \lim_{k_2 \to 0} + \lim_{k_2 \to 0} \lim_{k_1 \to 0}\right) \mathcal{A}(\delta\phi^I(k_1), \delta\phi^J(k_2), \{\psi_i(p_i)\}) = G^{(IK}(\phi_0) G^{J)L}(\phi_0) \nabla_K \nabla_L \mathcal{A}(\{\psi_i(p_i)\})$$

$$p^m \delta_{AB} = \Gamma^m_{\alpha\beta} \zeta_{\alpha A} \zeta_{\beta B}, \zeta_{\alpha A} \zeta_{\beta A} = \frac{1}{2} p_m \Gamma^m_{\alpha\beta}$$

$$q_\alpha = \zeta_{\alpha A} \eta_A, \text{ and } \bar{q}_\alpha = \zeta_{\alpha A} \frac{\partial}{\partial \eta_A}$$

$$\{q_\alpha, \bar{q}_\beta\} = \frac{1}{2} p_m \Gamma^m_{\alpha\beta}, \{q_\alpha, q_\beta\} = \{\bar{q}_\alpha, \bar{q}_\beta\} = 0$$

$$\mathcal{A} = \delta^{10}(P) \delta^{16}(Q) \mathcal{F}(\zeta_i, \eta_i)$$

$$\delta^{16}(Q) \bar{Q}^{16} \mathcal{P}(\zeta_i, \eta_i)$$

$$\delta^{16}(Q) f(s_{ij})$$

$$f(s_{ij}) \bar{Q}^{16} \prod_{i=1}^{n} \eta_i^8$$

$$R = -\frac{1}{4} \sum_i \left(\eta_{iA} \frac{\partial}{\partial \eta_{iA}} - 4\right)$$

$$\gamma^m_{\alpha\beta} \lambda_{\alpha A} \lambda_{\beta B} = \delta_{AB} p^m, \lambda_{\alpha A} \lambda_{\alpha B} = 0$$

$$\lambda_{\alpha A} \lambda_{\beta A} = \frac{1}{2} \gamma^m_{\alpha\beta} p_m$$

$$q_\alpha = \lambda_{\alpha A}\eta_A, \bar{q}_\alpha = \lambda_{\alpha A}\frac{\partial}{\partial \eta_A}$$

$$p^m\delta_I^J = \gamma^m_{A\dot{B}}\lambda_{AI}\tilde{\lambda}^J_{\dot{B}}, \lambda_{AI}\tilde{\lambda}^I_{\dot{B}} = \frac{1}{2}p_m\gamma^m_{A\dot{B}}$$

$$q_A = \lambda_{AI}\eta^I, \tilde{q}_{\dot{A}} = \tilde{\lambda}^I_{\dot{A}}\tilde{\eta}_I$$
$$\bar{\tilde{q}}_A = \lambda_{AI}\frac{\partial}{\partial\tilde{\eta}_I}, \bar{q}_{\dot{A}} = \tilde{\lambda}^I_{\dot{A}}\frac{\partial}{\partial\eta^I}$$

$$R_+ = \eta\tilde{\eta}, R_- = \partial_\eta\partial_{\tilde{\eta}}, R_3 = -\frac{1}{2}\left(\eta\partial_\eta + \tilde{\eta}\partial_{\tilde{\eta}}\right) + 2$$
$$R' = \frac{1}{4}\left(\eta\partial_\eta - \tilde{\eta}\partial_{\tilde{\eta}}\right)$$

$$V_n^- = \delta^8(Q_A)\prod_{A=1}^{8}\bar{\tilde{Q}}_A\prod_{i=1}^{n}\tilde{\eta}_i^4 = \delta^8(Q_A)\prod_{A=1}^{8}\left(\sum_{j=1}^{n}\lambda_{iAI}\frac{\partial}{\partial\tilde{\eta}_{iI}}\right)\prod_{i=1}^{n}\tilde{\eta}_i^4$$
$$V_n^+ = \delta^8(\tilde{Q}_{\dot{A}})\prod_{A=1}^{8}\bar{Q}_{\dot{A}}\prod_{i=1}^{n}\eta_i^4 = \delta^8(\tilde{Q}_{\dot{A}})\prod_{A=1}^{8}\left(\sum_{j=1}^{n}\tilde{\lambda}_{i\dot{A}}{}^{I}\frac{\partial}{\partial\eta_i{}^I}\right)\prod_{i=1}^{n}\eta_i^4$$

$$\delta^{16}(Q)\bar{Q}^8\mathcal{P}(\eta_i^4) = \delta^{16}(Q)\prod_{A=1}^{8}\left(\sum_{j=1}^{n}\tilde{\lambda}_{iA}{}^{I}\frac{\partial}{\partial\eta_i{}^I}\right)\mathcal{P}(\eta_i^4),$$
$$\delta^{16}(Q)\bar{\tilde{Q}}^8\mathcal{P}(\tilde{\eta}_i^4) = \delta^{16}(Q)\prod_{A=1}^{8}\left(\sum_{j=1}^{n}\lambda_{iAI}\frac{\partial}{\partial\tilde{\eta}_{iI}}\right)\mathcal{P}(\tilde{\eta}_i^4)$$

$$\delta^{16}(Q)\bar{Q}^8\sum_{1\le i<j<k\le 6}\eta_i^4\eta_j^4\eta_k^4$$
$$\delta^{16}(Q)\bar{\tilde{Q}}^8\sum_{1\le i<j<k\le 6}\tilde{\eta}_i^4\tilde{\eta}_j^4\tilde{\eta}_k^4$$

$$\delta^{16}(Q)\bar{Q}^8\sum_{i<j}s_{ij}\mathcal{P}_{ij}(\eta_k^4)$$
$$\delta^{16}(Q)\bar{\tilde{Q}}^8\sum_{i<j}s_{ij}\mathcal{P}_{ij}(\tilde{\eta}_k^4)$$

$$\delta^{16}(Q)\bar{Q}^8\sum_{1\le i<j<k\le 6}s_{ijk}\eta_i^4\eta_j^4\eta_k^4$$
$$\delta^{16}(Q)\bar{\tilde{Q}}^8\sum_{1\le i<j<k\le 6}s_{ijk}\tilde{\eta}_i^4\tilde{\eta}_j^4\tilde{\eta}_k^4$$

$$\gamma^m_{\alpha\beta}\lambda_{\alpha I}\lambda_{\beta J} = p^m\Omega_{IJ}, \lambda_{\alpha I}\lambda_{\alpha J} = 0$$

$$\lambda_{\alpha I}\lambda_{\beta J}\Omega^{IJ} = \frac{1}{2}p_m\gamma^m_{\alpha\beta}.$$

$$q_{\alpha a} = \lambda_{\alpha I}\eta^I_a, \bar{q}^a_\alpha = \lambda_{\alpha I}\frac{\partial}{\partial \eta_{Ia}}.$$

$$1, \Omega_{IJ}\eta^I_{(a}\eta^J_{b)}, \Omega_{IJ}\eta^I_{(a}\eta^J_{b)}\Omega_{KL}\eta^K_{(c}\eta^L_{d)}, \Omega_{IJ}\frac{\partial^2}{\partial\eta^{(a}_I\partial\eta^{b)}_J}\eta^8, \eta^8$$

$$R^+_{(ab)} = \sum_i \Omega_{IJ}\eta^I_{i(a}\eta^J_{ib)}, R^{-(ab)} = \sum_i \Omega_{IJ}\frac{\partial^2}{\partial\eta_{iI(a}\partial\eta_{iJb)}}, R^0_a{}^b = \sum_i \left(\eta^I_{ia}\frac{\partial}{\partial\eta^I_{ib}} - 2\delta^b_a\right).$$

$$\mathrm{Sym}^2[2,0] = [0,0] \oplus [0,4] \oplus [2,0] \oplus [4,0]$$

$$\delta^{16}(Q)\sum_{1\le i<j\le 6} s^2_{ij},$$

$$\delta^{16}(Q)\bar{Q}^8_- \sum_{1\le i<j<k<\ell\le 6} \eta^4_{i+}\eta^4_{j+}\eta^4_{k+},$$

$$\bar{Q}^8_- \equiv \prod_{\alpha=1}^{8}\bar{Q}_{\alpha-} = \prod_{\alpha=1}^{8}\left(\sum_{i=1}^{6}\lambda_{i\alpha I}\frac{\partial}{\partial\eta_{I+}}\right), \eta^4_{i+} \equiv \frac{1}{4!}\epsilon_{IJKL}\eta^I_{i+}\eta^J_{i+}\eta^K_{i+}\eta^L_{i+}.$$

$$\delta^{16}(Q)\left(\bar{Q}^8_- \sum_{1\le i<j<k<\ell\le 6}\eta^4_{i+}\eta^4_{j+}\eta^4_{k+} - \frac{2^2\times 35}{12!\,8!}\sum_{1\le i\le j\le 6} s^2_{ij}\left(R^+_{(++)}\right)^2\right)$$

$$p_{AB} = \lambda_{Aa}\lambda_{Bb}\epsilon^{ab}, p^{AB} = \frac{1}{2}\epsilon^{ABCD}p_{CD} = \tilde{\lambda}^A_{\dot{a}}\tilde{\lambda}^B_{\dot{b}}\epsilon^{\dot{a}\dot{b}}$$

$$q_{Aa'} = \lambda_{Aa}\eta^a_{a'}, \tilde{q}^A_{\dot{a}'} = \tilde{\lambda}^A{}_{\dot{a}}\tilde{\eta}_{\dot{a}'}$$
$$\bar{q}_{Aa'} = \lambda_{Aa}\frac{\partial}{\partial\eta_a{}^{a'}}, \bar{\tilde{q}}^A{}_{\dot{a}'} = \tilde{\lambda}^A{}_{\dot{a}}\frac{\partial}{\partial\tilde{\eta}_{\dot{a}}{}^{a'}}$$

$$(\eta^2)_{a'b'},\quad \left(\partial^2_\eta\right)_{a'b'},\quad \eta_{a'}\partial_{\eta_{b'}} - \delta^{b'}_{a'}$$
$$(\tilde{\eta}^2)_{\dot{a}'b'},\quad \left(\partial^2_{\tilde{\eta}}\right)_{\dot{a}'b'},\quad \tilde{\eta}_{\dot{a}'}\partial_{\tilde{\eta}_{b'}} - \delta^{b'}_{\dot{a}'}$$

$$\mathrm{Sym}^2([1,0;1,0]) = [0,0;0,0] \oplus [0,2;0,2] \oplus [0,0;2,0] \oplus [2,0;0,0] \oplus [2,0;2,0].$$

$$\delta^{16}(Q)\bar{Q}^8\bar{\tilde{Q}}^4_-\mathcal{P}\left(\eta^2_{ia'b'}, \tilde{\eta}^2_{i++}\right)$$

$$\bar{Q}^8 \equiv \prod_{A=1}^{4}\bar{Q}_{A+}\bar{Q}_{A-},\qquad \bar{\tilde{Q}}^4_- \equiv \prod_{A=1}^{4}\bar{\tilde{Q}}^A_- = \prod_{A=1}^{4}\left(\sum_{i=1}^{n}\tilde{\lambda}^A_i{}^A_{\dot{a}}\frac{\partial}{\partial\tilde{\eta}_{i\dot{a}+}}\right)$$
$$\eta^2_{ia'b'} \equiv \epsilon_{ab}\eta^a_i{}_{a'}\eta^b_i{}_{b'},\quad \tilde{\eta}^2_{i++} \equiv \epsilon_{\dot{a}\dot{b}}\tilde{\eta}^{\dot{a}}_i\tilde{\eta}^{\dot{b}}_i +$$

$$p_{AB} = \lambda_{Aa}\lambda_{Bb}\epsilon^{ab}, \Omega^{AB}\lambda_{Aa}\lambda_{Bb}\epsilon^{ab} = 0$$

$$q_{AI} = \lambda_{Aa}\eta^a_I, \bar{q}^I_A = \lambda_{Aa}\frac{\partial}{\partial\eta_{aI}}$$

$$R_{IJ}^{+} = \eta_{aI}\eta_{bJ}\epsilon^{ab}, R^{-IJ} = \frac{\partial}{\partial\eta_{aI}}\frac{\partial}{\partial\eta_{bJ}}\epsilon_{ab}, R_I^{0\ J} = \eta_{aI}\frac{\partial}{\partial\eta_{aJ}} - \delta_I^J.$$

$$\mathrm{Sym}^2[0,0,0,1] = [0,0,0,0] \oplus [0,2,0,0] \oplus [0,0,0,2].$$

$$\delta^{16}(Q)\prod_{A=1}^{4}\bar{Q}_{A--}\bar{Q}_{A+-}\sum_{1\le i_1<i_2<i_3\le 6}\prod_{k=1}^{3}(\eta_{i_k}{}^{a}{}_{++}\eta_{i_k}{}^{b}{}_{++}\epsilon_{ab})(\eta_{i_k}{}^{c}{}_{-+}\eta_{i_k}{}^{d}{}_{-+}\epsilon_{cd}).$$

$$\delta^{16}(Q)\sum_{1\le i<j\le 6}s_{ij}^2 R_{++++}^{+}R_{-+-+}^{+}, \delta^{16}(Q)\sum_{1\le i<j\le 6}s_{ij}^2(R_{++-+}^{+})^2.$$

$$p_{\alpha\dot{\beta}} = \lambda_\alpha\tilde{\lambda}_{\dot{\beta}}$$

$$q_{\alpha I} = \lambda_\alpha\eta_I, \tilde{q}_{\dot{\alpha}}^I = \tilde{\lambda}_{\dot{\alpha}}\tilde{\eta}^I$$
$$\bar{\tilde{q}}_{\alpha I} = \lambda_\alpha\frac{\partial}{\partial\tilde{\eta}^I}, \bar{q}_{\dot{\alpha}}^I = \tilde{\lambda}_{\dot{\alpha}}\frac{\partial}{\partial\eta_I}$$

$$R_I^{+\ J}{}_J = \eta_I\tilde{\eta}^J, R_J^{-I} = \frac{\partial}{\partial\eta_I}\frac{\partial}{\partial\tilde{\eta}^J}, M_I^J = \eta_I\frac{\partial}{\partial\eta_J} + \tilde{\eta}^J\frac{\partial}{\partial\tilde{\eta}^I} - \delta_I^J$$
$$N_I^J = \eta_I\frac{\partial}{\partial\eta_J} - \tilde{\eta}^J\frac{\partial}{\partial\tilde{\eta}^I} - \frac{1}{4}\delta_I^J(\eta\partial_\eta - \tilde{\eta}\partial_{\tilde{\eta}}).$$

$$[R^{+}{}_I{}^J, R^{-K}{}_L] = -\delta_I^K\tilde{\eta}^J\frac{\partial}{\partial\tilde{\eta}^L} + \delta_L^J\frac{\partial}{\partial\eta_K}\eta_I = -\frac{1}{2}\delta_L^J(M_I{}^K + N_I^K) - \frac{1}{2}\delta_I^K(M_L{}^J - N_L{}^J).$$

$$\mathrm{Sym}^2[0001000] = [0000000] \oplus [0100010] \oplus [0002000].$$

$$\delta^{16}(Q)\bar{Q}^8\sum_{1\le i<j<k\le 6}\eta_i^4\eta_j^4\eta_k^4$$

$$p_{\alpha\beta} = \lambda_\alpha\lambda_\beta$$

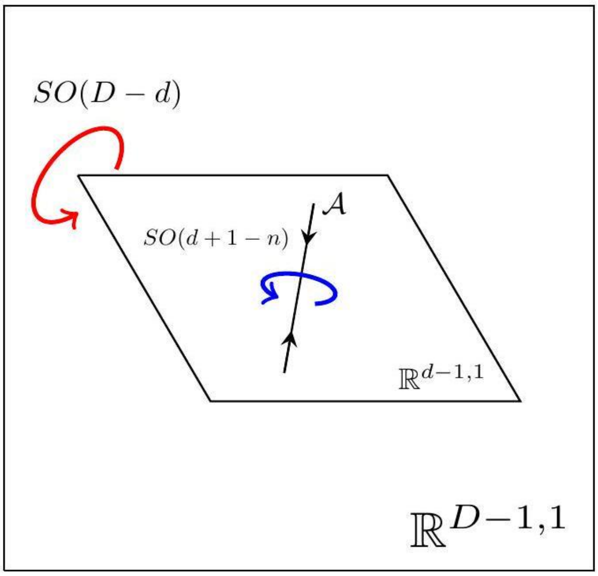
$$q_{\alpha A} = \lambda_\alpha\eta_A, \bar{q}_{\alpha A} = \lambda_\alpha\frac{\partial}{\partial\eta_A}.$$

$$R_{[AB]}^{+} = \eta_A\eta_B, R_{[AB]}^{-} = \frac{\partial}{\partial\eta_A}\frac{\partial}{\partial\eta_B}, R_{AB}^{0} = \eta_A\frac{\partial}{\partial\eta_B} - \frac{1}{2}\delta_{AB}$$

$$\mathrm{Sym}^2[00000001] = [00000000] \oplus [00010000] \oplus [00000002].$$

$$\delta^{16}(Q)\prod_{\alpha=1,2}\bar{Q}_{\alpha+++}\bar{Q}_{\alpha+-+}\bar{Q}_{\alpha-++}\bar{Q}_{\alpha--+}\sum_{1\le i_1<i_2<i_3\le 6}\prod_{k=1}^{3}\eta_{i_k+++}\eta_{i_k+-+}\eta_{i_k-++}\eta_{i_k--+}$$

$$\delta^{16}(Q)\sum_{1\le i<j\le 6}s_{ij}^2 R_{[+++,+-+]}^{+}R_{[-++,--+]}^{+}$$



$$\zeta_{\alpha A} \to \left(\lambda_{AI}, \lambda_A^I = 0, \tilde{\lambda}_{\dot{A}I} = 0, \tilde{\lambda}_{\dot{A}}^I\right),$$
$$\eta_A \to \left(\theta^I, \tilde{\theta}_I\right).$$

$$\not{p}_{iA\dot{B}} \tilde{\lambda}_{i\dot{B}}^I = 0, p_{iA\dot{B}} \lambda_{iAI} = 0,$$

$$q_i = \not{p}_i u_i,$$

$$q_i \gamma \bar{\tilde{u}}_i = u_i p_i \gamma \, \mathrm{V} \bar{\tilde{u}}_i = -u_i \gamma p_i \bar{\tilde{u}}_i = -u_i \gamma \, \mathrm{V} \bar{\tilde{q}}_i,$$

$$M^+(\mathbf{v}) = \sum_{i=1}^{n} q_i \boldsymbol{\gamma} \bar{\tilde{u}}_i = -\sum_i u_i \boldsymbol{\gamma} \bar{\tilde{q}}_i$$
$$M^-(\mathbf{v}) = \sum_{i=1}^{n} \tilde{q}_i \boldsymbol{\gamma} \bar{u}_i = -\sum_i \tilde{u}_i \boldsymbol{v} \bar{q}_i.$$

$$[M^+(\mathrm{v}), \bar{Q}_{\dot{A}}] = -\frac{1}{2} \sum_{i=1}^{n} \left(\not{p}_i \gamma \bar{\tilde{u}}_i\right)_{\dot{A}} = \frac{1}{2} (\gamma \bar{\tilde{Q}})_{\dot{A}}$$

$$\left[M^+(\mathrm{v}), \tilde{Q}_{\dot{A}}\right] = -\frac{1}{2} \sum_{i=1}^{n} \left(\not{p}_i \gamma u_i\right)_{\dot{A}} = \frac{1}{2} (\gamma Q)_{\dot{A}}$$

$$\left[M^-(\mathrm{v}), \bar{\tilde{Q}}_A\right] = -\frac{1}{2} \sum_{i=1}^{n} \left(\not{p}_i \gamma \bar{u}_i\right)_A = \frac{1}{2} (\gamma \bar{Q})_A$$

$$[M^-(\mathrm{v}), Q_A] = -\frac{1}{2} \sum_{i=1}^{n} \left(\not{p}_i \gamma \tilde{u}_i\right)_A = \frac{1}{2} (\gamma \tilde{Q})_A$$

$$M^0 \equiv 2[M^+(\mathbf{v}), M^-(\mathbf{v})] = -\sum_i \left(u_i \vee p_i \vee \bar{u}_i - \tilde{u}_i \vee p_i \vee \bar{\tilde{u}}_i\right)$$
$$= \sum_i \left(u_i \not{p}_i \bar{u}_i - \tilde{u}_i p_i \bar{\tilde{u}}_i\right) = \sum_i \left(q_i \bar{u}_i - \tilde{q}_i \bar{\tilde{u}}_i\right) = \sum_i \left(u_i \bar{q}_i - \tilde{u}_i \bar{\tilde{q}}_i\right).$$

$$[M^0, M^{\pm}(\mathbf{v})] = \pm M^{\pm}(\mathbf{v})$$

$$M(\mathbf{v}) = M^{+}(\mathbf{v}) + M^{-}(\mathbf{v})$$

$$\begin{aligned} M(\mathbf{v})\delta^{16}(Q) &= \left[-\frac{1}{2}\sum_{i} (u_i \vee p_i)_{\dot{A}} \frac{\partial}{\partial \tilde{Q}_{\dot{A}}} - \frac{1}{2}\sum_{i} (\tilde{u}_i \vee p_i)_{A} \frac{\partial}{\partial Q_{A}}\right]\delta^{16}(Q) \\ &= \left[\frac{1}{2}(Q \vee)_{\dot{A}} \frac{\partial}{\partial \tilde{Q}_{\dot{A}}} + \frac{1}{2}(\tilde{Q} \vee)_{A} \frac{\partial}{\partial Q_{A}}\right]\delta^{16}(Q) = 0 \end{aligned}$$

$$M^{+}(\mathbf{v})V_5^0 = 2\delta^{16}(Q)R_{+}\sum_{i=1}^{5} u_i \forall q_i$$

$$(M^{+}(\mathbf{v}))^2 V_5^0 = 2\delta^{16}(Q)\left(\sum_{i=1}^{5} u_i \vee q_i\right)^2$$

$$V_6^{+} \sum_{1\le i<j\le 6} s_{ij}^2,$$

$$\delta^{16}(Q)\bar{Q}^8 R_{+}^2 \sum_{1\le i<j<k\le 6} \eta_i^4 \eta_j^4 \eta_k^4,$$

$$\delta^{16}(Q)R_{+}^4 \sum_{1\le i<j\le 6} s_{ij}^2,$$

$$\delta^{16}(Q)\overline{\tilde{Q}}^8 R_{+}^2 \sum_{1\le i<j<k\le 6} \tilde{\eta}_i^4 \tilde{\eta}_j^4 \tilde{\eta}_k^4,$$

$$V_6^{-} \sum_{1\le i<j\le 6} s_{ij}^2,$$

$$\delta_{A\dot{B}}\zeta_{AI}\tilde{\zeta}_{\dot{B}}^{J} = 0$$

$$\zeta_{AI} = \delta_{A\dot{B}}\Omega_{IJ}\tilde{\zeta}_{\dot{B}}{}^{J} = \lambda_{AI}$$

$$\eta^I = \theta_{+}^I, \tilde{\eta}_I = \Omega_{IJ}\theta_{-}^J.$$

$$\begin{aligned} (Q_A, \tilde{Q}_{\dot{A}}) &\sim (Q_{A+}, \delta_{B\dot{A}}Q_{B-}), \\ (\tilde{Q}_A, \bar{Q}_{\dot{A}}) &\sim (\bar{Q}_{A-}, \delta_{B\dot{A}}\bar{Q}_{B+}). \end{aligned}$$

$R_{(+-)}^{+}, R_{(+-)}^{-}, R_{(+-)}^{0}$, and $R_{ab}^{0}\epsilon^{ab}$

$$\eta^I = \theta_{+}^I, \tilde{\eta}_I = \frac{\partial}{\partial \theta_{-}^I}$$

$$\mathcal{A}(\eta_i, \tilde{\eta}_i) = \int \prod_i d^4\theta_{i-} e^{\sum_i \tilde{\eta}_i \theta_{i-}^I} \mathcal{A}(\theta_{i+} = \eta_i, \theta_{i-})$$

$$\begin{aligned} (Q_A, \tilde{Q}_{\dot{A}}) &\sim (Q_{A+}, \delta_{B\dot{A}}\bar{Q}_{B-}), \\ (\tilde{Q}_A, \bar{Q}_{\dot{A}}) &\sim (Q_{A-}, \delta_{B\dot{A}}\bar{Q}_{B+}), \end{aligned}$$

$$M_{+}(\mathbf{v}) = \sum_{i=1}^{n} q_i \psi u_i$$
$$M_{-}(\mathbf{v}) = \sum_{i=1}^{n} \bar{q}_i \psi \bar{u}_i$$

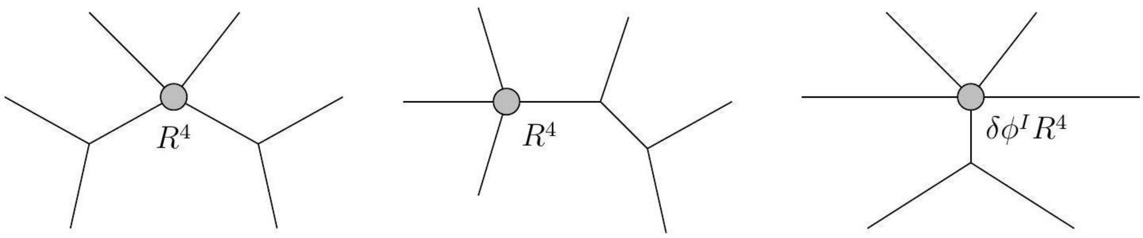
$$q_i = p_i u_i, \bar{q}_i = p_i \bar{u}_i.$$

$$[M_{+}(\mathbf{v}), \bar{Q}_\alpha] = \mathfrak{t} Q_\alpha, [M_{-}(\mathbf{v}), Q_\alpha] = \mathfrak{t} \bar{Q}_\alpha$$

$$M_{-}(\mathbf{v})\delta^{16}(Q) = \frac{1}{4}\sum_{i} \left(\not{p}_i \boldsymbol{v}\right)_{\alpha\beta} \frac{\partial}{\partial Q_\alpha}\frac{\partial}{\partial Q_\beta}\delta^{16}(Q) = \frac{1}{4}(P\boldsymbol{\psi})_{\alpha\beta}\frac{\partial}{\partial Q_\alpha}\frac{\partial}{\partial Q_\beta}\delta^{16}(Q) = 0$$

$$M_{+}(\mathbf{v})\delta^{16}(Q) = M_{+}(\mathbf{v})\bar{Q}^{16}\prod_{i=1}^{4}\eta_i^8 = 0$$

$$\mathrm{Sym}^2[2,0] = [0,0] \oplus [0,4] \oplus [2,0] \oplus [4,0].$$



$$\left[\nabla_I \nabla_J f_0(\phi)\right]_{[0,0]} \sim f_0(\phi)$$
$$\left[\nabla_I \nabla_J f_0(\phi)\right]_{[2,0]} \sim \frac{\partial}{\partial \phi^K} f_0(\phi)$$
$$\left[\nabla_I \nabla_J f_0(\phi)\right]_{[0,4]} = 0$$

$$g^{ij}\delta g_{ij} - \frac{1}{2} g^{ij}\delta g_{jk} g^{k\ell}\delta g_{\ell i} = \mathcal{O}((\delta g)^3)$$

$$\delta f(g) = \delta g_{ij} f^{ij}(g) + \delta g_{ij}\delta g_{k\ell} f^{ij,k\ell}(g) + \mathcal{O}((\delta g)^3)$$

$$g_{ij} g_{k\ell} f^{ik,j\ell}(g) = a f(g)$$
$$g_{k\ell} f^{ik,j\ell}(g) - \frac{1}{5} g^{ij} g_{k\ell} g_{mn} f^{mk,n\ell}(g) = b f^{ij}(g)$$
$$f^{ij,k\ell}(g) - f^{i\ell,kj}(g) = 0$$

$$g_{ij} = g_7^{-2/5}\begin{pmatrix} g_7^2 & 0 & 0 \\ 0 & v_3^{-1} & v_3^{-1} B_{\mathrm{i}}^{NS} \\ 0 & v_3^{-1} B_{\mathrm{i}}^{NS} & v_3^{\frac{1}{3}} \tilde{g}_{\mathrm{ij}} - v_3^{-1} B_{\mathrm{i}}^{NS} B_{\mathrm{j}}^{NS} \end{pmatrix}$$

$$f(g) = \mathbf{E}^{SL(5)}_{[1000];\frac{3}{2}}(g)$$

$$\mathbf{E}^{SL(5)}_{[1000];s} = \sum_{(m_1,\cdots,m_5)\in\mathbb{Z}^5\backslash\{0\}} \frac{1}{(m^i g_{ij} m^j)^s}.$$

$$f^{ij}_{(m)} = -\frac{s}{(m^k g_{kl} m^l)^{s+1}}\left(m^i m^j - \frac{1}{5}(m^k g_{kl} m^l) g^{ij}\right)$$

$$g_{kl} f^{ik,jl}_{(m)} = \frac{s}{50(m^k g_{kl} m^l)^{s+1}}\left(15(s+1) m^i m^j + (s-13)(m^k g_{kl} m^l) g^{ij}\right)$$

$$M = \begin{pmatrix} g^{ij} & -g^{ik} b_{kj} \\ b_{ik} g^{kj} & g_{ij} - b_{ik} g^{k\ell} b_{\ell j} \end{pmatrix}$$

$$M\eta M = \eta, \eta = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}$$

$$M \to \Omega M \Omega^T.$$

$$\delta M \eta M_0 + M_0 \eta \delta M = -\delta M \eta \delta M.$$

$$\mathcal{O} M_0 \mathcal{O}^T = M_0$$

$$\delta M \to \mathcal{O} \delta M \mathcal{O}^T.$$

$$f(M) = f(M_0) + \delta M_{ij} f^{ij}_{(1)}(M_0) + \delta M_{ij} \delta M_{k\ell} f^{ij,k\ell}_{(2)}(M_0) + \cdots$$

$$f^{ij}_{(1)} \to f^{ij}_{(1)} + (\eta M_0 N)^{ji} + (N M_0 \eta)^{ji}$$

$$f^{ij,k\ell}_{(2)} \to f^{ij,k\ell}_{(2)} + \frac{1}{4}(\eta^{ik} N^{j\ell} + \eta^{jk} N^{i\ell} + \eta^{i\ell} N^{jk} + \eta^{j\ell} N^{ik})$$
$$+[(\eta M_0 P)^{ji} + (P M_0 \eta)^{ji}] Q^{k\ell} + [(\eta M_0 P)^{\ell k} + (P M_0 \eta)^{\ell k}] Q^{ij}$$

$$S = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathbb{I} & -\mathbb{I} \\ \mathbb{I} & \mathbb{I} \end{pmatrix}$$

$$S^T \eta S = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{pmatrix} \equiv \tilde{\eta}$$

$$\widetilde{\delta M} \tilde{\eta} + \tilde{\eta} \widetilde{\delta M} = -\widetilde{\delta M} \tilde{\eta} \widetilde{\delta M}$$

$$f(M) = f(M_0) + \widetilde{\delta M}_{ij} \tilde{f}^{ij}_{(1)}(M_0) + \widetilde{\delta M}_{ij} \widetilde{\delta M}_{k\ell} \tilde{f}^{ij,k\ell}_{(2)}(M_0) + \cdots$$

$$\mathcal{O} = S \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} S^T = \frac{1}{2}\begin{pmatrix} A+B & A-B \\ A-B & A+B \end{pmatrix}, A, B \in SO(5)$$

$$\delta_{ab} \delta_{\dot{a}\dot{b}} \tilde{f}^{a\dot{a},b\dot{b}}_{(2)}(\mathbb{I}) = c f(\mathbb{I}),$$
$$\delta_{ab} \tilde{f}^{a\dot{a},b\dot{b}}(\mathbb{I}) = \delta_{\dot{a}\dot{b}} \tilde{f}^{a\dot{a},b\dot{b}}(\mathbb{I}) = 0,$$
$$\tilde{f}^{a\dot{a},b\dot{b}}_{(2)}(\mathbb{I}) = f^{a\dot{b},\dot{b}}_{(2)}(\mathbb{I}).$$

$$\widetilde{\delta M}_{ab} = \frac{1}{2} \widetilde{\delta M}_{a\dot{c}} \widetilde{\delta M}_{b\dot{c}}, \widetilde{\delta M}_{\dot{a}\dot{b}} = \frac{1}{2} \widetilde{\delta M}_{c\dot{a}} \widetilde{\delta M}_{c\dot{b}}$$

$$v^T\delta M v = u^T\widetilde{\delta M}u = \widetilde{\delta M}_{ab}u^a u^b + \widetilde{\delta M}_{\dot a\dot b}u^{\dot a}u^{\dot b} + 2\widetilde{\delta M}_{a\dot b}u^a u^{\dot b}$$
$$= 2\widetilde{\delta M}_{a\dot b}u^a u^{\dot b} + \frac{1}{2}\widetilde{\delta M}_{a\dot c}\widetilde{\delta M}_{b\dot c}u^a u^b + \frac{1}{2}\widetilde{\delta M}_{c\dot a}\widetilde{\delta M}_{c\dot b}u^{\dot a}u^{\dot b} + \mathcal{O}((\delta M)^3)$$

$$F_s(M) \equiv (v^T M v)^{-s}$$

$$F_s(\mathbb{I}+\delta M) = (u^T u)^{-s} - 2s(u^T u)^{-s-1}\widetilde{\delta M}_{a\dot b}u^a u^{\dot b} + 2s(s+1)(u^T u)^{-s-2}\widetilde{\delta M}_{a\dot b}\widetilde{\delta M}_{c\dot d}u^a u^{\dot b}u^c u^{\dot d}$$
$$-\frac{s}{2}(u^T u)^{-s-1}\left(\widetilde{\delta M}_{a\dot c}\widetilde{\delta M}^{a}_{b\dot c}u^b + \widetilde{\delta M}_{c\dot a}\widetilde{\delta M}_{c\dot b}u^{\dot a}u^{\dot b}\right) + \mathcal{O}((\delta M)^3)$$

$$\tilde F^{a\dot b,c\dot d}_{s(2)} = F_s(\mathbb{I})\left[\frac{2s(s+1)}{(u^T u)^2}u^a u^{\dot b}u^c u^{\dot d} - \frac{s}{2u^T u}\left(u^a u^c\delta^{\dot b\dot d} + u^{\dot b}u^{\dot d}\delta^{ac}\right)\right].$$

$$\delta_{ac}\tilde F^{a\dot b,c\dot d}_{s(2)} - \frac{1}{5}\delta_{ac}\delta^{\dot b\dot d}\delta_{\dot e\dot f}\tilde F^{a\dot e,c\dot f}_{s(2)}$$
$$= \left[\frac{s(2s-3)\delta_{ac}u^a u^c}{(u^T u)^2} + \frac{5s\left(\delta_{ac}u^a u^c - \delta_{\dot a\dot c}u^{\dot a}u^{\dot c}\right)}{2(u^T u)^2}\right]\left(u^{\dot b}u^{\dot d} - \frac{1}{5}\delta^{\dot b\dot d}\delta_{\dot e\dot f}u^{\dot e}u^{\dot f}\right)F_s(\mathbb{I})$$

$$\delta_{ac}u^a u^c - \delta_{\dot a\dot c}u^{\dot a}u^{\dot c} = u^T\tilde\eta u = 0 \iff v^T\eta v = 0.$$

$$\delta_{ac}\delta_{\dot b\dot d}\tilde F^{a\dot b,c\dot d}_{\frac{3}{2},(2)} = -\frac{15}{8}F_{\frac{3}{2}}(\mathbb{I}),$$
$$\tilde F^{a\dot b,c\dot d}_{s(2)} = F^{a\dot d,c\dot b}_{s(2)}.$$

$$f(\Omega,U) = f_{SL(3)}(\Omega) + f_{SL(2)}(U).$$

$$4U_2^2\partial_U\partial_{\bar U}f_{SL(2)}(U) = af_{SL(2)}(U),$$

$$\mathrm{Sym}^2\mathbf{5} = \mathbf{1}\oplus\mathbf{5}\oplus\mathbf{9}$$

$$\delta f_{SL(3)}(\Omega) = \delta\Omega_{ij}f^{ij}(\Omega) + \delta\Omega_{ij}\delta\Omega_{k\ell}f^{ij,k\ell}(\Omega) + \mathcal{O}((\delta\Omega)^3)$$

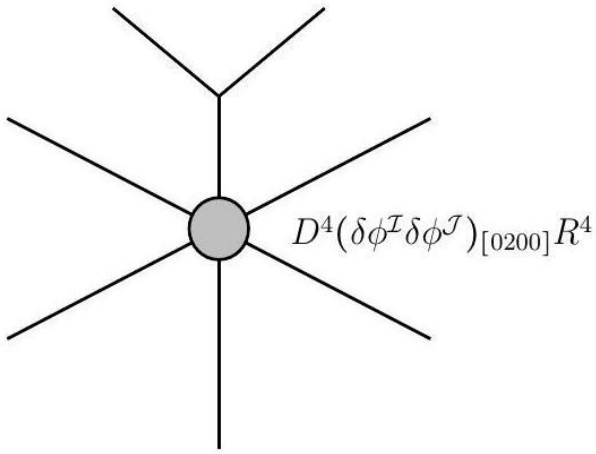
$$\Omega_{ij}\Omega_{k\ell}f^{ik,j\ell}(\Omega) = bf_{SL(3)}(\Omega)$$
$$\Omega_{k\ell}f^{ik,j\ell}(\Omega) - \frac{1}{3}\Omega^{ij}\Omega_{k\ell}\Omega_{mn}f^{mk,n\ell}(\Omega) = c\left[f^{ij}(\Omega) - \frac{1}{3}\Omega^{ij}\Omega_{mn}f^{mn}(\Omega)\right].$$

$$4\tau_2^2\partial_\tau\partial_{\bar\tau}f = a_1\partial_\sigma f + a_2 f$$
$$\partial_\sigma^2 f = a_3\partial_\sigma f + a_4 f$$
$$\partial_\tau\partial_\sigma f = a_5\partial_\tau f$$

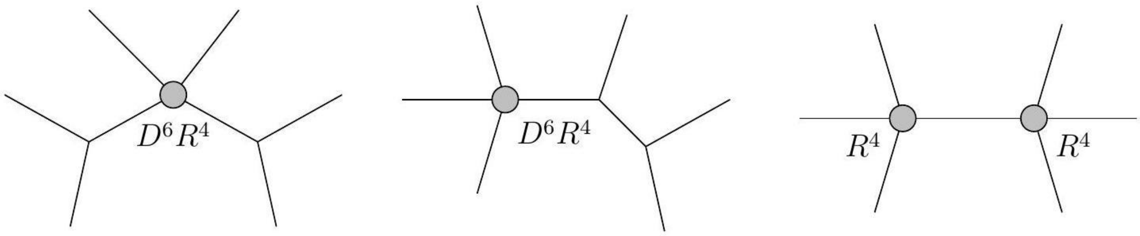
$$a_1 = -\frac{1}{2}, a_2 = \frac{3}{7}, a_3 = \frac{3}{14}, a_4 = \frac{27}{49}, a_5 = -\frac{9}{14}.$$

$$f(\tau,\bar\tau,\sigma) = e^{-\frac{9}{14}\sigma}F_{\frac{3}{4}}(\tau,\bar\tau) + e^{\frac{6}{7}\sigma}C$$

$$\mathfrak{D} = \nabla_{(\mathcal{I}}\nabla_{\mathcal{J}}\nabla_{\mathcal{K})}f_4\big|_{[0200]} \sim \nabla_{(\mathcal{I}}\nabla_{\mathcal{J})}f_4\big|_{[0200]}, \nabla_{(\mathcal{I}}\nabla_{\mathcal{J}}\nabla_{\mathcal{K})}f_4\big|_{[2001]} = 0,$$
$$\mathfrak{D} = \nabla_{(\mathcal{I}}\nabla_{\mathcal{J}}\nabla_{\mathcal{K})}f_4\big|_{[0200000]} = \nabla_{(\mathcal{I}}\nabla_{\mathcal{J}}\nabla_{\mathcal{K})}f_4\big|_{[0000020]} = \nabla_{(\mathcal{I}}\nabla_{\mathcal{J}}\nabla_{\mathcal{K})}f_4\big|_{[1001001]} = 0,$$
$$\mathfrak{D} = \nabla_{(\mathcal{I}}\nabla_{\mathcal{J}}\nabla_{\mathcal{K})}f_4\big|_{[01000001]} = 0.$$



$$\Delta f_6 = a f_6 + b f^2,$$

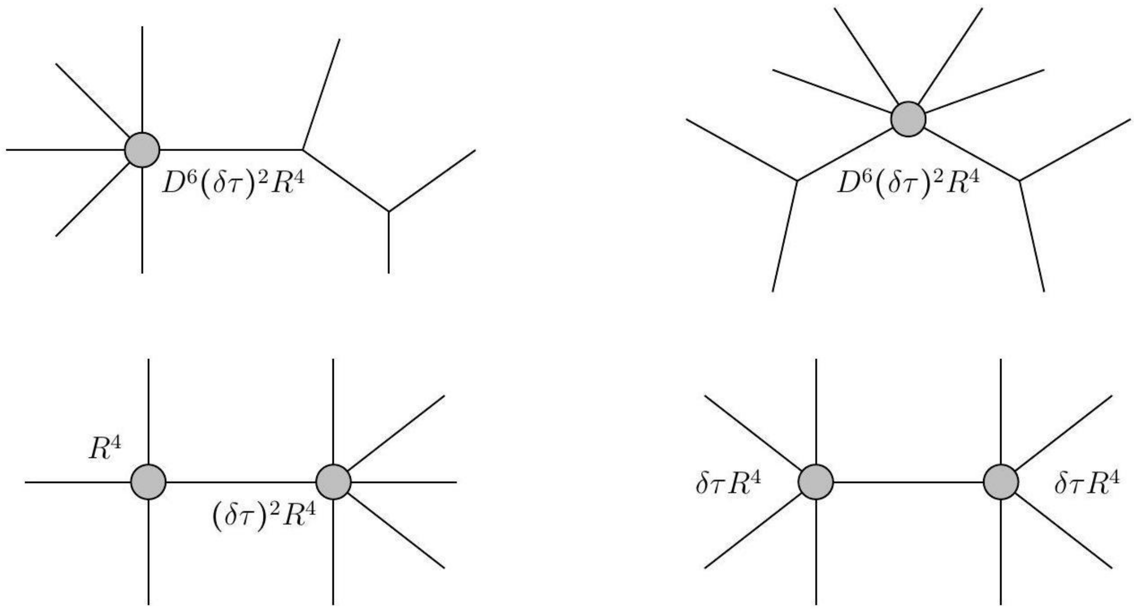


$$\frac{\partial}{\partial \phi^I} f(\phi)\hat{e}^I \cdot \left[\hat{v}\delta^{16}\left(\sum_{i=1}^{5} Q_i\right)\right]^{SO(5)_R}$$

$$\delta^{16}(Q)\left[F_1(\tau,\bar{\tau}) \sum_{1\le i<j\le 6} s_{ij}^3 + F_2(\tau,\bar{\tau}) \sum_{1\le i<j<k\le 6} s_{ijk}^3\right]$$

$$\delta^{16}(Q)[F_1(\tau,\bar{\tau}) + 2F_2(\tau,\bar{\tau})] \sum_{1\le i<j\le 5} s_{ij}^3$$

$$F_1(\tau,\bar{\tau}) + 2F_2(\tau,\bar{\tau}) = \nabla_\tau^2 f_6(\tau,\bar{\tau}) = \left(\partial_\tau^2 - \frac{i}{\tau_2}\partial_\tau\right) f_6(\tau,\bar{\tau})$$



$$\Delta F_1 = aF_1 + b\nabla_\tau^2 f_6 + cf_0\nabla_\tau^2 f_0 + d(\partial_\tau f_0)^2,$$

$$g_{ij}g_{kl}f^{ik,jl} = \frac{1}{4}\Delta_{SL(5)}f$$

$$\mathcal{L}_{kin} = -\frac{1}{2}\sum_p M^{(p)}_{I_pJ_p}(\phi)F^{(p)I_p} \wedge * F^{(p)J_p}$$

$$M^{(p)}_{I_pJ_p} = \delta_{\alpha_p\beta_p}\mathcal{V}^{\alpha_p}_{I_p}\mathcal{V}^{\beta_p}_{J_p}$$

$$F^{\alpha_p} = \mathcal{V}^{\alpha_p}_{I_p}F^{I_p}$$

$$\alpha_p = \left(\hat{\alpha}_p, \tilde{\alpha}_p\right)$$

$$\delta\psi^i_M = \mathrm{D}_M\epsilon^i + (\mathcal{F}_M)^i_j\epsilon^j + A^i_{0j}\Gamma_M\epsilon^j$$

$$\mathrm{D}_M\epsilon^i = \nabla_M\epsilon^i - (\mathcal{Q}_M)^i_j\epsilon^j$$

$$(\mathcal{F}_M)^i_j = \sum_{p\geq 2}\sum_{\hat{\alpha}_p}\left(B^{(p)}_{\hat{\alpha}_p}\right)^i_j F^{\hat{\alpha}_p}_{N_1\dots N_p}T^{N_1\dots N_p}_{(p)}{}_M$$

$$T^{N_1\dots N_p}_{(p)}{}_M = \Gamma^{N_1\dots N_p}{}_M + \beta_{(p)}\Gamma^{[N_1\dots N_{p-1}}\delta^{N_p]}_M,$$

$$\beta_{(p)} = \frac{p(D-p-1)}{p-1}$$

$$\delta\chi^a = \sum_{p\geq 1}\sum_{\hat{\alpha}_p}\left(C^{(p)}_{\hat{\alpha}_p}\right)^a_i F^{\hat{\alpha}_p}_{N_1\dots N_p}\Gamma^{N_1\dots N_p}\epsilon^i + A^a_{1i}\epsilon^i$$

$$\delta\lambda^s = \sum_{p\geq 1}\sum_{\tilde{\alpha}_p}\left(D^{(p)}_{\tilde{\alpha}_p}\right)^s_i F^{\tilde{\alpha}_p}_{N_1\dots N_p}\Gamma^{N_1\dots N_p}\epsilon^i + A^s_{2i}\epsilon^i$$

$$V = -c_0\mathrm{tr}\left(A_0^\dagger A_0\right) + c_1\mathrm{tr}\left(A_1^\dagger A_1\right) + c_2\mathrm{tr}\left(A_2^\dagger A_2\right),$$

$$\mathcal{Q}_M = \mathcal{Q}_M^{\text{scalar}} + \mathcal{Q}_M^{\text{gauge}}$$

$$\mathcal{Q}_M^{\text{gauge}} = A_M^{\alpha_2}t_{\alpha_2}.$$

$$[\mathrm{D}_M, \mathrm{D}_N]\epsilon^i = \frac{1}{4}R_{MNPQ}\Gamma^{PQ}\epsilon^i - (\mathcal{H}_{MN})^i_j\epsilon^j$$

$$\mathcal{H}_{MN} = \mathcal{H}_{MN}^{\text{scalar}} + \mathcal{H}_{MN}^{\text{gauge}}$$

$$\mathcal{H}_{MN}^{\text{gauge}} = F_{MN}^{\alpha_2}t_{\alpha_2}$$

$$\mathcal{H}_{MN}^{\text{scalar}} = h_1 C^\dagger_{\hat{\alpha}_1}C_{\hat{\beta}_1}F^{\hat{\alpha}_1}_{[M}F^{\hat{\beta}_1}_{N]} + h_2 D^\dagger_{\tilde{\alpha}_1}D_{\tilde{\beta}_1}F^{\tilde{\alpha}_1}_{[M}F^{\tilde{\beta}_1}_{N]}$$

$$\delta\psi^i_M = \delta\chi^a = \delta\lambda^s = 0$$

$$A_1 = A_2 = 0$$

$$\delta\psi^i_M = \mathrm{D}_M\epsilon^i + A^i_{0j}\Gamma_M\epsilon^j = 0$$

$$\left[\left(\frac{1}{4}R_{MN}{}^{PQ}\delta^i_k + 2A^i_{0j}A^j_{0k}\delta^P_M\delta^Q_N\right)\Gamma_{PQ} + 2\left(\mathrm{D}_{[M}A_0\right)^i_k\Gamma_{N]}\right]\epsilon^k = 0$$

$$\partial_M A_0^2 = \mathrm{D}_M A_0^2 = 0$$

$$R_{MNPQ} = -\frac{4}{\mathcal{N}}\mathrm{tr}(A_0^2)\left(g_{MP}g_{NQ} - g_{MQ}g_{NP}\right)$$

$$\Lambda = -\frac{2}{\mathcal{N}}(D-1)(D-2)\mathrm{tr}(A_0^2)$$

$$\mathcal{M}_D = \mathrm{AdS}_D \ \text{ or } \ \mathcal{M}_D = M_d \times T^{(D-d)}, 1 \leq d \leq D,$$

$$A_0^2 = -\frac{\Lambda}{2(D-1)(D-2)}\mathbb{1}, A_1 = A_2 = 0$$

$$A_1 = A_2 = 0 \ \text{ and } \ F^{(p)} = 0$$

$$* F^{(p)}\cdot\Gamma = -(-1)^{p(p-1)/2}i^{D/2+1}\left(F^{(p)}\cdot\Gamma\right)\Gamma_*$$

$$F_\pm\cdot\Gamma = (F_\pm\cdot\Gamma)P_\pm$$

$$\Big(\frac{1}{4}R_{MNPQ}\,\Gamma^{PQ}\delta^i_j - (\mathcal{H}_{MN})^i_j + 2\big(\mathrm{D}_{[M}\mathcal{F}_{N]} + \mathrm{D}_{[M}A_0\Gamma_{N]}\big)^i_j$$
$$+\big[(\mathcal{F}_M + A_0\Gamma_M)^i_k(\mathcal{F}_N + A_0\Gamma_N)^k_j - (M \leftrightarrow N)\big]\Big)\epsilon^j = 0$$

$$\mathcal{H}_{MN} = 0.$$

$$\mathcal{H}^{\text{gauge}}_{MN} = 0.$$

$$\mathcal{H}^{\text{gauge}}_{MN} \sim F^{\hat{\alpha}_2}_{MN}\{A_0, B_{\hat{\alpha}_2}\},$$

$$\mathcal{H}^{\text{gauge}}_{MN} \sim F_{MN}A_0,$$

$$\frac{1}{4}R_{MNPQ}\Gamma^{PQ}\delta^i_j + 2\big(\nabla_{[M}\mathcal{F}_{N]}\big)^i_j + 2\big(\mathcal{F}_{[M}\big)^i_k\big(\mathcal{F}_{N]}\big)^k_j = 0$$

$$\nabla F^{\hat{\alpha}_p} = 0$$

$$F^{\hat{\alpha}_p} = v^{\hat{\alpha}_p}F \ \text{ or } \ F^{\hat{\alpha}_p} = v^{\hat{\alpha}_p}(F + *F)$$

$$\mathcal{M}_D = \mathrm{AdS}_p \times S^{(D-p)} \ \text{ or } \ \mathcal{M}_D = \mathrm{AdS}_{(D-p)} \times S^p$$

$$\Gamma^M\Gamma^N + \Gamma^N\Gamma^M = 2g^{MN}\mathbb{1}$$

$$\Gamma^{M_1\ldots M_p} = \Gamma^{[M_1}\ldots\Gamma^{M_p]}$$

$$\Gamma_* = (-i)^{m+1}\Gamma_0\Gamma_1\ldots\Gamma_{D-1}.$$

$$\Gamma^{M_1\ldots M_p}\Gamma_* = -(-i)^{m+1}\frac{1}{(D-p)!}\epsilon^{M_p\ldots M_1}{}_{N_1\ldots N_{D-p}}\Gamma^{N_1\ldots N_{D-p}},$$

$$\Gamma^{M_1\ldots M_p} = i^{m+1}\frac{1}{(D-p)!}\epsilon^{M_1\ldots M_p}{}_{N_{D-p}\ldots N_1}\Gamma^{N_1\ldots N_{D-p}}.$$

$$F\cdot\Gamma = F_{M_1\ldots M_p}\Gamma^{M_1\ldots M_p}$$

$$\big\{\Gamma^{M_1\ldots M_r}, \Gamma_{N_1\ldots N_r}\big\} = p!\,\delta^{[M_1}_{N_r}\ldots\delta^{M_r]}_{N_1} + \cdots$$

$$\begin{aligned}
\big((\mathcal{F}_M + A_0\Gamma_M)(\mathcal{F}_N + A_0\Gamma_N) - (M \leftrightarrow N)\big) &= \\
&= [\mathcal{F}_M, \mathcal{F}_N] + [\mathcal{F}_M, A_0\Gamma_N] - A_0[\mathcal{F}_N, A_0\Gamma_M] + 2A_0A_0\Gamma_{MN} \\
&= \frac{1}{2}(p-1)!\,\big(\beta^2_{(p)} - p^2\big)\Big[B_{\hat{\alpha}_p}, B_{\hat{\beta}_p}\Big]F^{\hat{\alpha}_p}_{MP_1\ldots P_{p-1}}F^{\hat{\beta}_pP_{p-1}\ldots P_p}_N \\
&\quad + \delta_{p,2}4(D-3)\big[B_{\hat{\alpha}_2}, A_0\big]F^{\hat{\alpha}_2}_{MN} + \cdots,
\end{aligned}$$

$$\begin{aligned}
&\Big[B_{\hat{\alpha}_p}\Gamma^{M_1\ldots M_r}, B_{\hat{\beta}_p}\Gamma^{N_1\ldots N_r}\Big] \\
&= \frac{1}{2}\Big(\Big[B_{\hat{\alpha}_p}, B_{\hat{\beta}_p}\Big]\{\Gamma^{M_1\ldots M_r}, \Gamma^{N_1\ldots N_r}\} + \Big\{B_{\hat{\alpha}_p}, B_{\hat{\beta}_p}\Big\}[\Gamma^{M_1\ldots M_r}, \Gamma^{N_1\ldots N_r}]\Big)
\end{aligned}$$

$$\left(Q_M^{\text{gauge}}\right)^i_{\ j} = A_M^{\alpha_2}\left(t_{\alpha_2}\right)^i_{\ j}$$

$$e^{-1}\mathcal{L}_{\bar{\psi}\partial\psi} = -\frac{1}{2}\bar{\psi}_{iM}\Gamma^{MNP}\mathrm{D}_N\psi_P^i$$

$$e^{-1}\delta\mathcal{L}_{\bar{\psi}\partial\psi} = \frac{1}{2}\left(\mathcal{H}_{MN}^{\text{gauge}}\right)^i_{\ j}\bar{\psi}_{iP}\Gamma^{MNP}\epsilon^j + \cdots$$

$$e^{-1}\mathcal{L}_{\bar{\psi}\psi} = \frac{1}{2}d_0 A^i_{0j}\bar{\psi}_{iM}\Gamma^{MN}\psi_N^j$$

$$e^{-1}\mathcal{L}_{F\bar{\psi}\psi} = \frac{1}{2}e_0 F_{MN}^{\hat{\alpha}_2}\left(B_{\hat{\alpha}_2}\right)^i_{\ j}\bar{\psi}_i^P\Gamma_{[P}\Gamma^{MN}\Gamma_{R]}\psi^{jR}$$

$$d_0 = -e_0 = (D-2)$$

$$e^{-1}\delta\mathcal{L}_{\bar{\psi}\psi} = d_0 F_{MN}^{\hat{\alpha}_2}A^i_{0j}\left(B_{\hat{\alpha}_2}\right)^j_{\ k}\bar{\psi}_i^P\left(-(D-3)\Gamma^{MN}{}_P + 2\delta_P^{[M}\Gamma^{N]}\right)\epsilon^k + \cdots$$

$$e^{-1}\delta\mathcal{L}_{F\bar{\psi}\psi} = e_0 F_{MN}^{\hat{\alpha}_2}\left(B_{\hat{\alpha}_2}\right)^i_{\ j}A^j_{0k}\bar{\psi}_i^P\left((D-3)\Gamma^{MN}{}_P + 2\delta_P^{[M}\Gamma^{N]}\right)\epsilon^k + \cdots$$

$$t_{\hat{\alpha}_2} = 2(D-2)(D-3)\left\{A_0, B_{\hat{\alpha}_2}\right\}, t_{\tilde{\alpha}_2} = 0$$

$$\left[t_{\hat{\alpha}_2}, A_0\right] = 0$$

$$\left(T_A\right)_{\hat{\alpha}_2}{}^{\hat{\beta}_2}B_{\hat{\beta}_2} - \left[T_A, B_{\hat{\alpha}_2}\right] = 0$$

$$t_{\alpha_2} = \Theta_{\alpha_2}{}^A T_A$$

$$\begin{aligned}\left[t_{\hat{\alpha}_2}, t_{\hat{\beta}_2}\right] &= 2(D-2)(D-3)\left\{A_0, \left[t_{\hat{\alpha}_2}, B_{\hat{\beta}_2}\right]\right\} \\ &= 2(D-2)(D-3)\left\{A_0, \left(t_{\hat{\alpha}_2}\right)_{\hat{\beta}_2}^{\hat{\gamma}^2}B_{\hat{\gamma}_2}\right\} \\ &= \left(t_{\hat{\alpha}_2}\right)_{\hat{\beta}_2}^{\hat{\gamma}_2}t_{\hat{\gamma}_2}\end{aligned}$$

$$R_{\alpha\beta ab} = 0$$

$$X_{12} = \frac{62}{945}\mathrm{tr}R^6 - \frac{7}{180}\mathrm{tr}R^4\mathrm{tr}R^2 + \frac{1}{216}\left(\mathrm{tr}R^2\right)^3$$

$$\Omega_L A_L = 0$$

$$\begin{aligned}\Omega_L A_L &= 0 \\ \Omega_S A_L + \Omega_L A_S &= 0 \\ \Omega_S A_S &= 0\end{aligned}$$

$$X_{12} = dY.$$

$$\begin{aligned}dH_4 &= 0 \\ d\tilde{H}_7 &= 0.\end{aligned}$$

$$\phi_p = \frac{1}{p!}E^{A_1} - E^{A_p}\phi_{A_p - A_1}(z)$$

$$\Gamma^{a_1 - a_k} \equiv \Gamma^{[a_1} - \Gamma^{a_k]}$$

$$(\Gamma^a)_{\alpha\beta}(\Gamma^b)^{\beta\gamma} + (\Gamma^b)_{\alpha\beta}(\Gamma^a)^{\beta\gamma} = 2\eta^{ab}\delta_\alpha^\gamma$$

$$T^A = DE^A = \frac{1}{2}E^B E^C T_{CB}{}^A$$
$$R_A{}^B = d\Omega_A{}^B + \Omega_A{}^C\Omega_C{}^B = \frac{1}{2}E^C E^D R_{DCA}{}^B$$

$$DT^A = E^B R_B^A$$
$$DR_A^B = 0$$

$$D_{[A}T_{BC)}{}^D + T_{[AB}^G T_{GC)}{}^D = R_{[ABC)}{}^D$$

$$T_{\alpha\beta}^a = 2\Gamma_{\alpha\beta}^a.$$

$$T_{\alpha\beta}{}^\gamma = (1440 \oplus 560 \oplus 144 \oplus 2\cdot 16)$$
$$T_{\alpha a}{}^b = (720 \oplus 560 \oplus 2\cdot 144 \oplus 2\cdot 16)$$

$$E'^\alpha = E^\alpha + E^b H_b^\alpha$$
$$\Omega'_{\alpha a}{}^b = \Omega_{\alpha a}{}^b + X_{\alpha a}{}^b,$$

$$(\Gamma^a)_{\delta(\alpha}T_{\beta\gamma)}{}^\delta = (\Gamma^g)_{(\alpha\beta}T_{\gamma)g}{}^a$$

$$T_{\alpha\beta}^\gamma = 2\delta_{(\alpha}^\gamma V_{\beta)} - (\Gamma^g)_{\alpha\beta}(\Gamma_g)^{\gamma\varphi}V_\varphi.$$

$$R_{\alpha\beta ab} = 0$$

$$T_{\alpha\beta}^a = 2\Gamma_{\alpha\beta}^a$$
$$T_{\alpha\beta}^\gamma = 2\delta_{(\alpha}^\gamma V_{\beta)} - (\Gamma^g)_{\alpha\beta}(\Gamma_g)^{\gamma\varphi}V_\varphi$$
$$T_{\alpha a}^b = 0 = T_{a\alpha}^b$$
$$R_{\alpha\beta ab} = 0.$$

$$V_\alpha = D_\alpha\phi$$

$$D_A D_B - (-)^{AB} D_B D_A = -T_{AB}{}^C D_C - R_{AB\#}{}^\#.$$

$$2T_{a(\alpha}^\gamma(\Gamma^b)_{\beta)\gamma} + (\Gamma^g)_{\alpha\beta}T_{ga}^b = 0$$
$$D_{(\alpha}T_{\beta\gamma)}^\delta - 2(\Gamma^g)_{(\alpha\beta}T_{\gamma)g}^\delta + T_{(\alpha\beta}^\varepsilon T_{\gamma)\varepsilon}^\delta = 0$$
$$D_\alpha T_{ab}^c + 2(\Gamma^c)_{\alpha\gamma}T_{ab}^\gamma = 2R_{\alpha[ab]}^c$$

$$D_a T_{\alpha\beta}^\gamma + 2D_{(\alpha}T_{\beta)a}^\gamma + 2T_{a(\alpha}^\delta T_{\beta)\delta}^\gamma + 2\Gamma_{\alpha\beta}^g T_{ga}^\gamma - T_{\alpha\beta}^\varepsilon T_{a\varepsilon}^\gamma = 2R_{a(\alpha\beta)}^\gamma$$
$$D_{[a}T_{bc]}^d - T_{[ab}^g T_{c]g}^d = R_{[abc]}^d$$
$$D_\alpha T_{ab}{}^\beta + 2D_{[a}T_{b]\alpha}{}^\beta - 2T_{\alpha[a}{}^\delta T_{b]\delta}{}^\beta + T_{ab}{}^g T_{g\alpha}{}^\beta + T_{ab}{}^\delta T_{\delta\alpha}{}^\beta = R_{ab\alpha}{}^\beta$$

$$T_{abc} \equiv T_{ab}{}^{d}\eta_{dc} = T_{[abc]}$$

$$T_{a\alpha}^{\beta} = \frac{1}{4}(\Gamma^{bc})_{\alpha}^{\beta}T_{abc}$$

$$D_{\alpha}D_{\beta}\phi = -\Gamma_{\alpha\beta}^{g}D_{g}\phi + V_{\alpha}V_{\beta} + \frac{1}{12}(\Gamma_{abc})_{\alpha\beta}T^{abc}$$

$$D_{\alpha}T_{abc} = -6T_{[ab}{}^{\varepsilon}(\Gamma_{c]})_{\varepsilon\alpha}$$

$$R_{a\alpha bc} = 2(\Gamma_{a})_{\alpha\varepsilon}T_{bc}^{\varepsilon}$$

$$(\Gamma^{b})_{\alpha\varepsilon}T_{ba}^{\varepsilon} = D_{a}V_{\alpha} + \frac{1}{4}T_{abc}(\Gamma^{bc})_{\alpha}^{\beta}V_{\beta}.$$

$$F_{a\alpha} = (\Gamma_{a})_{\alpha\varepsilon}\chi^{\varepsilon}$$

$$R_{[ab]} = -\frac{1}{2}D^{c}T_{cab}$$

$$R_{bc} = 2D_{c}D_{b}\phi - D_{a}T^{a}{}_{bc}$$

$$R_{(ab)} = 2D_{(a}D_{b)}\phi$$

$$D_{(\varepsilon}D_{\alpha)}D_{\beta}\phi = -\Gamma_{\varepsilon\alpha}^{g}D_{g}D_{\beta}\phi - \frac{1}{2}T_{\varepsilon\alpha}{}^{\delta}D_{\delta}V_{\beta}$$

$$(\Gamma^{a})^{\alpha\beta}D_{a}V_{\beta} = 2(\Gamma^{a})^{\alpha\beta}V_{\beta}D_{a}\phi - \frac{1}{12}(\Gamma_{abc})^{\alpha\beta}T^{abc}V_{\beta}.$$

$$D^{a}D_{a}\phi = 2D_{a}\phi D^{a}\phi - \frac{1}{12}T_{abc}T^{abc}$$

$$D_{(\beta}D_{\alpha)}T_{abc} = -\Gamma_{\alpha\beta}^{g}D_{g}T_{abc} - \frac{1}{2}T_{\alpha\beta}{}^{\delta}D_{\delta}T_{abc}$$

$$D_{d}T_{abc} - 6D_{[a}T_{bc]d} + 3R_{[abc]d} - 9T_{[abc]d}^{2} = 0$$

$$D_{[a}T_{bcd]} + \frac{3}{2}T_{[abcd]}^{2} = 0$$

$$H_{\alpha\beta\gamma} = 0 = H_{ab\alpha}$$
$$H_{a\alpha\beta} = 2(\Gamma_{a})_{\alpha\beta}$$
$$H_{abc} = T_{abc}$$

$$dH_{3} = 0$$

$$H_{3} = dB_{2}$$

$$D_{c}T_{ab}^{c} = -4D_{[a}D_{b]}\phi$$

$$D_{c}(e^{-2\phi}T_{ab}^{c}) = 2e^{-2\phi}T_{ab}^{\alpha}V_{\alpha}$$

$$D^{c}(e^{-2\phi}V_{abc}) = e^{-2\phi}\left((\Gamma_{ab}^{hg})_{\varepsilon}{}^{\beta}T_{hg}{}^{\varepsilon}V_{\beta} + 2T_{ab}{}^{\alpha}V_{\alpha} + T^{c_1c_2}{}_{[a}V_{b]c_1c_2}\right)$$

$$D^c\left(e^{-2\phi}(T_{abc}-V_{abc})\right)=-e^{-2\phi}\left(\left(\Gamma_{ab}^{hg}\right)_\varepsilon^{\ \beta}T_{hg}^{\varepsilon}V_\beta+T^{c_1c_2}{}_{[a}V_{b]c_1c_2}\right)$$

$$H_{a_1-a_7}\equiv\frac{1}{3!}\varepsilon_{a_1-a_7b_1b_2b_3}e^{-2\phi}\left(T^{b_1b_2b_3}-V^{b_1b_2b_3}\right)$$
$$H_{\alpha a_1-a_6}\equiv-2e^{-2\phi}\left(\Gamma_{a_1-a_6}\right)_\alpha{}^\beta V_\beta$$

$$D_{[a_1}H_{a_2-a_8]}+\frac{7}{2}T_{[a_1a_2}{}^bH_{a_3-a_8]b}+\frac{7}{2}T_{[a_1a_2}{}^\delta H_{\delta a_3-a_8]}=0$$

$$H_{\alpha\beta a_1-a_5}\equiv-2e^{-2\phi}\left(\Gamma_{a_1-a_5}\right)_{\alpha\beta}$$
$$H_{\alpha_1\alpha_2\alpha_3a_1-a_4}=\cdots=H_{\alpha_1\alpha_2\ldots\alpha_7}=0$$

$$H_7\equiv\frac{1}{7!}E^{A_1}-E^{A_7}H_{A_7-A_1}$$

$$dH_7=0$$

$$H_7=dB_6$$

$$D_bH^b_{a_1-a_6}=-2D_g\phi H^g_{a_1-a_6}-6V_{b_1b_2[a_1}H^{b_1b_2}{}_{a_2-a_6]}$$
$$+\frac{1}{3!}\varepsilon^{b_1-b_4}_{a_1-a_6}e^{-2\phi}\left(D_{b_1}V_{b_2-b_4}+\frac{3}{2}V^2_{b_1b_2b_3b_4}\right)$$
$$-\frac{1}{180}e^{2\phi}\varepsilon_{a_1-a_6b_1-b_4}H^{b_1b_2}{}_{f_1-f_5}H^{b_3b_4f_1-f_5}$$

$$\tilde{\Omega}_{ab}{}^c=\Omega_{ab}{}^c-\frac{1}{2}T_{ab}{}^c.$$

$$\tilde{D}_{[a}T_{bcd]}=\tilde{D}_{[a}H_{bcd]}=0$$
$$\tilde{D}_g\left(e^{2\phi}H^g_{a_1-a_6}\right)=\frac{1}{3!}\varepsilon^{b_1-b_4}_{a_1-a_6}\tilde{D}_{b_1}V_{b_2b_3b_4}.$$

$$B_6\to B_6+d\phi_5$$
$$B_2\to B_2+d\phi_1$$

$$T_{\alpha\beta}{}^a=2\Gamma^a_{\alpha\beta}$$

$$T_{\alpha\beta}{}^\gamma=5280\oplus4224\oplus3520\oplus2\cdot1408\oplus3\cdot320\oplus3\cdot32$$
$$T_{\alpha a}{}^b=1760\oplus1408\oplus2\cdot320\oplus2\cdot32.$$

$$(\Gamma^a)_{\delta(\alpha}T^\delta_{\beta\gamma)}=(\Gamma^g)_{(\alpha\beta}T_{\gamma)g}{}^a.$$

$$36960\oplus10240\oplus5280\oplus4224\oplus3520\oplus1760\oplus2\cdot1408\oplus3\cdot320\oplus2\cdot32$$

$$T^\gamma_{\alpha\beta}=16\delta^\gamma_{(\alpha}V_{\beta)}-6(\Gamma^g)_{\alpha\beta}\left(\Gamma_g\right)^{\gamma\delta}V_\delta+\left(\Gamma^{ab}\right)_{\alpha\beta}(\Gamma_{ab})^{\gamma\delta}V_\delta$$

$$T^\gamma_{\alpha\beta}=0\Leftrightarrow V_\alpha=0.$$

$$T_{\alpha\beta}^{a} = 2\Gamma_{\alpha\beta}^{a}$$
$$T_{\alpha a}^{b} = T_{\alpha\beta}^{\gamma} = T_{ab}^{c} = 0.$$

$$4T_{a(\alpha}{}^{\gamma}\Gamma_{\beta)\gamma}^{b} = R_{\alpha\beta a}{}^{b}$$
$$2\Gamma_{(\alpha\beta}^{g}T_{g\gamma)}{}^{\delta} = R_{(\alpha\beta\gamma)}{}^{\delta}$$

$$T_{a\alpha}^{\beta} = 8\left(\Gamma^{b_1b_2b_3}\right)_{\alpha\beta}W_{ab_1b_2b_3} + \left(\Gamma_{ab_1-b_4}\right)_{\alpha\beta}W^{b_1-b_4}$$
$$R_{\alpha\beta ab} = 96(\Gamma^{c_1c_2})_{\alpha\beta}W_{c_1c_2ab} + 4\left(\Gamma_{abc_1-c_4}\right)_{\alpha\beta}W^{c_1-c_4}.$$

$$R_{\alpha abc} = 2T_{a[b}^{\delta}\Gamma_{c]\delta\alpha} - T_{bc}^{\delta}(\Gamma_a)_{\delta\alpha}$$
$$D_\alpha W_{a_1-a_4} = \frac{1}{24}\left(\Gamma_{[a_1a_2}\right)_{\alpha\gamma}T_{a_3a_4]}^{\gamma}$$

$$\left(\Gamma^{abc}\right)_{\alpha\beta}T_{bc}^{\beta} = 0$$

$$T_{ab}{}^{\alpha}\left(\Gamma^{b}\right)_{\alpha\beta} = 0.$$

$$D_\alpha T_{ab}{}^{\beta} = -2D_{[a}T_{b]\alpha}{}^{\beta} - 2T_{[a\alpha}{}^{\gamma}T_{b]\gamma}{}^{\beta} + \frac{1}{4}R_{abcd}\left(\Gamma^{cd}\right)_{\alpha}{}^{\beta}.$$

$$R_{ab} - \frac{1}{2}\eta_{ab}R = -288\cdot 4!\left(W_{ab}^{2} - \frac{1}{8}\eta_{ab}W^{2}\right)$$

$$R = R^{a}{}_{a}, W_{ab}^{2} \equiv W_{ac_1c_2c_3}W_{b}{}^{c_1c_2c_3}, W^{2} \equiv W_{a_1-a_4}W^{a_1-a_4}$$

$$\left(\Gamma_{a_5}\right)^{\beta\alpha}D_\beta D_\alpha W_{a_1-a_4} = -32D_{a_5}W_{a_1-a_4}.$$

$$D_{[a_1}W_{a_2a_3a_4a_5]} = 0$$

$$H_{a_1-a_4} = W_{a_1-a_4}$$
$$H_{ab\alpha\beta} = -\frac{1}{144}(\Gamma_{ab})_{\alpha\beta}$$
$$H_{\alpha\beta\gamma\delta} = H_{\alpha\beta\gamma a} = H_{\alpha abc} = 0$$

$$dH_4 = 0$$

$$H_4 = dB_3.$$

$$D_\alpha T_{ag}{}^{\beta}(\Gamma^{g}\Gamma_{bc})_{\beta}{}^{\alpha} = 0$$

$$D_dW_{abc}^{d} = -\frac{1}{4}\varepsilon_{abcf_1-f_4g_1-g_4}W^{f_1-f_4}W^{g_1-g_4}$$

$$H_{a_1-a_7} = \frac{1}{4!}\varepsilon_{a_1-a_7b_1-b_4}W^{b_1-b_4}$$
$$H_{\alpha\beta a_1-a_5} = \frac{1}{144}\left(\Gamma_{a_1-a_5}\right)_{\alpha\beta}$$

$$dH_7 = \frac{1}{144} H_4 \wedge H_4$$

$$d\left(H_7 - \frac{1}{144} B_3 \wedge H_4\right) = 0$$

$$d\tilde{H}_7 = 0 \;\Rightarrow\; \tilde{H}_7 = dB_6$$

$$H_7 = dB_6 + \frac{1}{144} B_3 \wedge H_4$$

$$H_3 = dB_2 + kA \wedge F$$

$$B_2 \to B_2 - k\phi \wedge F.$$

$$B_6 \to B_6 - \frac{1}{144} \phi_2 \wedge H_4$$

$$D^b H_{ba_1 - a_6} = 0.$$

$$R_{\alpha\beta ab} = \left(\Gamma_{abc_1c_2c_3}\right)_{\alpha\beta} J^{c_1c_2c_3}$$

$$T_{a\alpha}{}^{\beta} = \frac{1}{4}\left(\Gamma^{bc}\right)_{\alpha}{}^{\beta} T_{abc} - \frac{1}{4}\left(\Gamma_{abcd}\right)_{\alpha}{}^{\beta} J^{bcd}.$$

$$\left[D_\alpha\left(e^{2\phi} J_{abc}\right)\right]^{1200} = 0$$

$$dH_3 = K$$

$$K_{\alpha\beta\gamma\delta} = K_{\alpha\beta\gamma a} = 0$$
$$K_{\alpha\beta ab} = 2\left(\Gamma_{abc_1c_2c_3}\right)_{\alpha\beta} J^{c_1c_2c_3}$$
$$dK = 0$$

$$H_3 = dB_2 + \Omega$$

$$dH_7 = 0$$

$$H_{a_1 - a_7} = \frac{1}{3!} e^{-2\phi} \varepsilon_{a_1 - a_7}^{b_1 - b_3} \left(T_{b_1 - b_3} - V_{b_1 - b_3} - 6J_{b_1 - b_3}\right)$$
$$H_{\alpha a_1 - a_6} = -2e^{-2\phi}\left(\Gamma_{a_1 - a_6}\right)_{\alpha}{}^{\varepsilon} V_{\varepsilon}$$
$$H_{\alpha\beta a_1 - a_5} = -2e^{-2\phi}\left(\Gamma_{a_1 - a_5}\right)_{\alpha\beta},$$

$$d\omega_{YM} = \mathrm{Tr}F^2 = dX_{YM} + K_{YM}$$

$$X_{YM} = -\frac{1}{48} E^c E^b E^a \left(\Gamma_{abc}\right)_{\alpha\beta} \mathrm{Tr}\left(\chi^\alpha \chi^\beta\right).$$

$$d\omega_L = \mathrm{tr}R^2 \equiv R_a{}^b R_b{}^a = dX_L + K_L,$$

$$\Omega = \left(\omega_{YM} - \omega_L\right) - \left(X_{YM} - X_L\right)$$

$$H_3 = dB_2 + (\omega_{YM} - X_{YM}) - (\omega_L - X_L)$$
$$dH_3 = \mathrm{Tr}F^2 - \mathrm{tr}R^2 - d(X_{YM} - X_L)$$

$$dH_7 = \frac{1}{24}\mathrm{Tr}F^4 - \frac{1}{7200}(\mathrm{Tr}F^2)^2 - \frac{1}{240}\mathrm{Tr}F^2\mathrm{tr}R^2 + \frac{1}{8}\mathrm{tr}R^4 + \frac{1}{32}(\mathrm{tr}R^2)^2$$
$$\equiv X_8 = d\omega_7$$

$$(\Gamma^a)_{\alpha\beta} = (\gamma^a)_\alpha{}^{\varepsilon} C_{\underline{\varepsilon}\beta}$$
$$(\Gamma^a)^{\alpha\beta} = C^{\alpha\underline{\varepsilon}}(\gamma^a)_{\underline{\varepsilon}}^{\alpha},$$

$$(\Gamma^a)_{\alpha\beta}(\Gamma^b)^{\beta\gamma} + (\Gamma^b)_{\alpha\beta}(\Gamma^a)^{\beta\gamma} = 2\eta^{ab}\delta_\alpha^\gamma$$

$$\Gamma^{a_1\cdots a_k} = \Gamma^{[a_1}\dots\Gamma^{a_k]}$$

$$\Gamma^{a_1\cdots a_k} = \pm(-1)^{\frac{k}{2}(k+1)}\frac{1}{(D-k)!}\varepsilon^{a_1\cdots a_D}\Gamma_{a_{k+1}\cdots a_D}$$

$$(\Gamma^g)_{(\alpha\beta}(\Gamma_g)_{\gamma)\delta} = 0$$

$$(\Gamma^g)_{(\alpha\beta}(\Gamma_g{}^{a_1\cdots a_4})_{\gamma\delta)} = 0.$$

$$(\Gamma^a)_\alpha^\beta = (\gamma^a)_\alpha^\beta$$
$$(\Gamma^a)_{\alpha\beta} = (\gamma^a)_\alpha{}^\varepsilon C_{\varepsilon\beta}$$
$$(\Gamma^a)^{\alpha\beta} = C^{\alpha\varepsilon}(\gamma^a)_\varepsilon{}^\beta$$
$$(\Gamma^a)^\alpha{}_\beta = C^{\alpha\varepsilon}(\gamma^a)_\varepsilon{}^\lambda C_{\lambda\beta}.$$

$$\Gamma^a, \Gamma^{a_1a_2}, \Gamma^{a_1\cdots a_5}, \Gamma^{a_1\cdots a_6}, \Gamma^{a_1\cdots a_9}, \Gamma^{a_1\cdots a_{10}}$$

$$C, \Gamma^{a_1\cdots a_3}, \Gamma^{a_1\cdots a_4}, \Gamma^{a_1\cdots a_7}, \Gamma^{a_1\cdots a_8}, \Gamma^{a_1\cdots a_{11}}.$$

$$\Gamma^{a_1\cdots a_k} = -(-1)^{\frac{k}{2}(k-1)}\frac{1}{(D-k)!}\varepsilon^{a_1\cdots a_D}\Gamma_{a_{k+1}\cdots a_D}$$

$$(\Gamma^{ga})_{(\alpha\beta}(\Gamma_g)_{\gamma\delta)} = 0$$

$$(\Gamma^g)_{(\alpha\beta}(\Gamma_g{}^{a_1\cdots a_4})_{\gamma\delta)} = 3(\Gamma^{[a_1a_2})_{(\alpha\beta}(\Gamma^{a_3a_4]})_{\gamma\delta)},$$

$$4T_{a(\alpha}{}^\varepsilon(\Gamma_b)_{\beta)\varepsilon} = R_{\alpha\beta ab}$$
$$2(\Gamma^g)_{(\alpha\beta}T_{g\gamma)}{}^\delta = R_{(\alpha\beta\gamma)}{}^\delta.$$

$$\begin{aligned} T_{a\alpha\beta} &= T_{a(\alpha\beta)} + T_{a[\alpha\beta]} \\ &= (11 \oplus 55 \oplus 462) \otimes 11 \oplus (1 \oplus 165 \oplus 330) \otimes 11 \\ &= 4290 \oplus 3003 \oplus 1430 \oplus 2\cdot 462 \oplus 429 \\ &\quad \oplus 2\cdot 330 \oplus 2\cdot 165 \oplus 65 \oplus 2\cdot 55 \oplus 2\cdot 11 \oplus 1. \end{aligned}$$

$$T^{(a}{}_{(\alpha}{}^\varepsilon\Gamma^{b)}_{\beta)\varepsilon} = 0$$

$$(ab)(\alpha\beta)= (1\oplus 65)\otimes(11\oplus 55\oplus 462)$$
$$= 22275\oplus 4290\oplus 3003\oplus 2025\oplus 1430\oplus 2\cdot 462$$
$$\oplus 429\oplus 275\oplus 65\oplus 2\cdot 55\oplus 2\cdot 11$$

$$T_{a\alpha}{}^{\beta} = 2\cdot 330\oplus 2\cdot 165\oplus 1.$$

$$g_{11} = e^{2\Delta}\left(g_{\mathrm{AdS}_4} + g_7\right), G_{(4)} = m\mathrm{vol}(\mathrm{AdS}_4) + F_{(4)}$$

$$\frac{1}{2}\partial_n\Delta\gamma^n\chi_i - \frac{ime^{-3\Delta}}{6}\chi_i + \frac{e^{-3\Delta}}{288}F_{bcde}\gamma^{bcde}\chi_i + \chi_i^c = 0$$
$$\nabla_m\chi_i + \frac{ime^{-3\Delta}}{4}\gamma_m\chi_i - \frac{e^{-3\Delta}}{24}\gamma^{cde}F_{mcde}\chi_i - \gamma_m\chi_i^c = 0$$

$$E_1 = \frac{1}{4}\|\xi\|(\mathrm{d}\psi + \mathcal{A}), E_2 = \frac{e^{-3\Delta}}{4\sqrt{1-\|\xi\|^2}}\,\mathrm{d}\rho, E_3 = \frac{6}{m}\frac{\rho\|\xi\|}{4\sqrt{1-\|\xi\|^2}}(\,\mathrm{d}\tau + \mathcal{A})$$

$$\|\xi\|^2 = \frac{e^{-6\Delta}}{36}(m^2 + 36\rho^2)$$

$$g_7 = g_{\mathrm{SU}(2)} + E_1^2 + E_2^2 + E_3^2$$

$$J_3 = e^{45} + e^{67}, \Omega = J_1 + iJ_2 = (e^4 + ie^5)\wedge(e^6 + ie^7)$$

$$e^{-3\Delta}\,\mathrm{d}\left[\|\xi\|^{-1}\left(\frac{m}{6}E_1 + e^{3\Delta}|S|\sqrt{1-\|\xi\|^2}E_3\right)\right] = 2(J_3 - \|\xi\|E_2\wedge E_3)$$
$$\mathrm{d}\left(\|\xi\|^2 e^{9\Delta}J_2\wedge E_2\right) - e^{3\Delta}|S|\mathrm{d}\left(\|\xi\|e^{6\Delta}|S|^{-1}J_1\wedge E_3\right) = 0$$
$$\mathrm{d}\left(e^{6\Delta}J_1\wedge E_2\right) + e^{3\Delta}|S|\mathrm{d}\left(\|\xi\|e^{3\Delta}|S|^{-1}J_2\wedge E_3\right) = 0$$

$$F_{(4)} = \frac{1}{\|\xi\|}E_1\wedge\,\mathrm{d}\left(e^{3\Delta}\sqrt{1-\|\xi\|^2}J_1\right) - m\frac{\sqrt{1-\|\xi\|^2}}{\|\xi\|}J_1\wedge E_2\wedge E_3$$

$$\mathrm{d}\mathcal{A} = \frac{4me^{-3\Delta}}{3\|\xi\|^2}\left[J_3 + \left(3\|\xi\| - \frac{4}{\|\xi\|}\right)E_2\wedge E_3\right]$$

$$g_{11} = e^{2\Delta}(g_4 + \hat{g}_7), G_{(4)} = m\mathrm{vol}_4 + \hat{F}_{(4)} - \alpha\wedge g\bar{F} - \beta\wedge g\star_4\bar{F}.$$

$$\hat{E}_1 = \frac{1}{4}\|\xi\|(\mathrm{d}\psi + \mathcal{A} - g\bar{A})$$

$$\hat{g}_7 = g_{SU(2)} + \hat{E}_1^2 + E_2^2 + E_3^2$$
$$\hat{F}_{(4)} = \frac{1}{\|\xi\|}\hat{E}_1\wedge\,\mathrm{d}\left(e^{3\Delta}\sqrt{1-\|\xi\|^2}J_1\right) - m\frac{\sqrt{1-\|\xi\|^2}}{\|\xi\|}J_1\wedge E_2\wedge E_3$$

$$\bar{F}\wedge\bar{F}{:}i_\xi\alpha = 0, \qquad \bar{F}{:}\frac{1}{4}i_\xi F_{(4)} + \mathrm{d}\tilde{\alpha} = 0$$
$$\star_4\bar{F}\wedge\bar{F}{:}i_\xi\beta = 0, \qquad \star_4\bar{F}{:}\mathrm{d}\tilde{\beta} = 0$$

$$\bar{F}\wedge\bar{F}\text{:}\frac{1}{4}e^{3\Delta}i_\xi\star_7\beta+\frac{1}{2}(\beta\wedge\beta-\alpha\wedge\alpha)=0$$

$$\star_4\bar{F}\wedge\bar{F}\text{:}\frac{1}{4}e^{3\Delta}i_\xi\star_7\alpha+\alpha\wedge\beta=0$$

$$\bar{F}\text{:}\frac{m}{8}\|\xi\|e^{3\Delta}J_3\wedge J_3\wedge E_2\wedge E_3-\frac{1}{4}e^{3\Delta}\,\mathrm{d}\mathcal{A}\wedge i_\xi\star_7\beta$$

$$+\frac{1}{4}\hat{e}\wedge\,\mathrm{d}\left(e^{3\Delta}i_\xi\star_7\beta\right)+\alpha\wedge\hat{F}_{(4)}=0$$

$$\star_4\bar{F}\text{:}\frac{1}{4}e^{3\Delta}\,\mathrm{d}\mathcal{A}\wedge i_\xi\star_7\alpha-\frac{1}{4}\hat{e}\wedge\,\mathrm{d}\left(e^{3\Delta}i_\xi\star_7\alpha\right)+\beta\wedge\hat{F}_{(4)}=0$$

$$\mathrm{Ric}_{\alpha\beta}-\frac{g^2}{32}\|\xi\|^2\bar{F}_{\alpha\gamma}\bar{F}_\beta^\gamma-9(\partial_a\Delta\partial^a\Delta+\nabla_a\nabla^a\Delta)\eta_{\alpha\beta}$$

$$=-e^{-6\Delta}\left\{\frac{1}{3}m^2\eta_{\alpha\beta}-\frac{g^2}{4}(\alpha^2+\beta^2)\bar{F}_{\alpha\gamma}\bar{F}_\beta^\gamma+\frac{g^2}{24}\eta_{\alpha\beta}\bar{F}^2(\alpha^2+2\beta^2)\right.$$

$$\left.+\frac{g^2}{4}\bar{F}_{\gamma(\alpha}\epsilon_{\beta)}^{\gamma\mu\nu}\bar{F}_{\mu\nu}\alpha_{cd}\beta^{cd}+\frac{g^2}{24}\eta_{\alpha\beta}\epsilon_{\mu\nu\rho\sigma}\bar{F}^{\mu\nu}\bar{F}^{\rho\sigma}\alpha_{cd}\beta^{cd}\right\}$$

$$\frac{g}{8}\|\xi\|\delta_{8b}\nabla_\gamma\bar{F}_\alpha^\gamma=0$$

$$\mathrm{Ric}_{ab}+\frac{g^2}{64}\|\xi\|^2\delta_{8a}\delta_{8b}\bar{F}_{\gamma\delta}\bar{F}^{\gamma\delta}+9[\partial_a\Delta\partial_b\Delta-\nabla_a\nabla_b\Delta-(\partial_c\Delta\partial^c\Delta-\nabla_c\nabla^c\Delta)$$

$$=e^{-6\Delta}\left\{\frac{1}{2}\left[F_{acde}F_b^{cde}-\frac{1}{12}\eta_{ab}F^2\right]+\frac{g^2}{24}\bar{F}^2[6(\alpha_{ac}\alpha_b^c-\beta_{ac}\beta_b^c)-\eta_{ab}(\alpha^2-\beta^2)]\right.$$

$$\left.+\frac{g^2}{24}\epsilon_{\mu\nu\rho\sigma}\bar{F}^{\mu\nu}\bar{F}^{\rho\sigma}[3(\alpha_{ac}\beta_b^c+\beta_{ac}\alpha_b^c)-\eta_{ab}\alpha_{cd}\beta^{cd}]+\frac{1}{2}m^2\eta_{ab}\right\}$$

$$\alpha=-\frac{1}{4}e^{3\Delta}\sqrt{1-\|\xi\|^2}J_1,\beta=-\frac{1}{4}e^{3\Delta}(J_3-\|\xi\|E_2\wedge E_3)$$

$$\epsilon=\psi_i^+\otimes e^{\Delta/2}\chi_i+(\psi_i^+)^c\otimes e^{\Delta/2}\chi_i^c$$

$$\left[\|\xi\|(3\gamma^8+i\gamma^{910})+\sqrt{1-\|\xi\|^2}(\gamma^{46}-\gamma^{57})-i(\gamma^{45}+\gamma^{67})\right]\chi_i=0$$

$$(\gamma^{46}+\gamma^{57})\chi_i=0,(\gamma^{45}-\gamma^{67})\chi_i=0$$

$$\left[-\sqrt{1-\|\xi\|^2}\gamma^{46}+i(\gamma^{45}+\|\xi\|\gamma^{910})\right]\chi_i=0$$

$$i\gamma^{45}\chi_i=-\epsilon_{ij}\chi_j^c$$

$$\Psi_\mu=\psi_{i\mu}^+\otimes e^{\Delta/2}\chi_i+\left(\psi_{i\mu}^+\right)^c\otimes e^{\Delta/2}\chi_i^c$$

$$g_{11}\;=e^{2\Delta}(g_4+\hat{g}_7)$$

$$G_{(4)}\;=m\mathrm{vol}_4+\hat{F}_{(4)}+\frac{1}{4}e^{3\Delta}\sqrt{1-\|\xi\|^2}J_1\wedge g\bar{F}+\frac{1}{4}e^{3\Delta}(J_3-\|\xi\|E_2\wedge E_3)\wedge g\star_4\bar{F}$$

$$J_I=\frac{m}{24}e^{-3\Delta}f(r)\mathbb{J}_I,I=1,2,3,(1+r^2)(\mathrm{d}\tau+\mathcal{A})=f(r)(\mathrm{d}\tau+A_{\mathrm{KE}})$$

$$\mathrm{d}A_{\mathrm{KE}}=2\mathbb{J}_3,\;\mathrm{d}(\mathbb{J}_1+i\mathbb{J}_2)=3i(\mathbb{J}_1+i\mathbb{J}_2)\wedge(\mathrm{d}\tau+A_{\mathrm{KE}}),$$

$$f' = -\frac{1}{2} r\Omega^2 f, \frac{(r\Omega' - r^2\Omega^3)f}{\sqrt{1+(1+r^2)\Omega^2}} = -3$$

$$e^{6\Delta} = \left(\frac{m}{6}\right)^2 (1 + r^2 + \Omega^{-2})$$

$$E_1 = \frac{\Omega\sqrt{1+r^2}}{4\sqrt{1+(1+r^2)\Omega^2}} \left[\mathrm{d}\psi - \mathrm{d}\tau + \frac{f}{1+r^2}(\mathrm{d}\tau + A_{\mathrm{KE}}) \right]$$
$$E_2 = \frac{1}{4}\Omega\, \mathrm{d}r, E_3 = \frac{1}{4}\frac{r\Omega f}{\sqrt{1+r^2}}(\mathrm{d}\tau + A_{\mathrm{KE}})$$

$$g_{11} = e^{2\Delta}\left\{ g_4 + \frac{\Omega f}{4\sqrt{1+(1+r^2)\Omega^2}} g_{\mathrm{KE}} + \frac{\Omega^2}{16}\left[\mathrm{d}r^2 + \frac{r^2 f^2}{1+r^2}(\mathrm{d}\tau + A_{\mathrm{KE}})^2 + \frac{1+r^2}{1+(1+r^2)\Omega^2}\left(\mathrm{D}\psi - \mathrm{d}\tau + \frac{f}{1+r^2}(\mathrm{d}\tau + A_{\mathrm{KE}}) \right)^2 \right] \right\}$$
$$\hat{F}_{(4)} = h_1(r)(\mathrm{D}\psi - \mathrm{d}\tau) \wedge \mathrm{d}r \wedge \mathbb{J}_1 + h_2(r)(\mathrm{D}\psi - \mathrm{d}\tau) \wedge (\mathrm{d}\tau + A_{\mathrm{KE}}) \wedge \mathbb{J}_2 + h_3(r)(\mathrm{d}\tau + A_{\mathrm{KE}}) \wedge \mathrm{d}r \wedge \mathbb{J}_1 - \alpha \wedge g\bar{F} - \beta \wedge g \star_4 \bar{F}$$

$$h_1(r) = \frac{m^2}{3^2 \cdot 2^6}(\Omega^{-1}e^{-3\Delta}f)', h_2(r) = -\frac{m^2}{3 \cdot 2^6}(\Omega^{-1}e^{-3\Delta}f)$$
$$h_3(r) = \frac{m^2}{3^2 \cdot 2^7}\frac{f}{1+r^2}\left[2(\Omega^{-1}e^{-3\Delta}f)' - 3r\Omega^2(\Omega^{-1}e^{-3\Delta}f)\right]$$

$$\alpha = -\frac{m^2}{576}(\Omega^{-1}e^{-3\Delta}f)\mathbb{J}_1, \beta = -\frac{mf}{96}\left[\mathbb{J}_3 - \frac{1}{4}r\Omega^2\, \mathrm{d}r \wedge (\mathrm{d}\tau + A_{\mathrm{KE}})\right]$$

$$f(r) = 3\left(2 - \frac{r}{\sqrt{2}}\right), \Omega(r) = \sqrt{\frac{2}{2\sqrt{2}r - r^2}},$$

$$r_{\text{here}} = 2\sqrt{2}\sin^2 \alpha_{\text{there}}, (\mathrm{d}\psi - \mathrm{d}\tau)_{\text{here}} = -2\, \mathrm{d}\psi'_{\text{there}}, (\mathrm{d}\tau + A_{\mathrm{KE}})_{\text{here}} = \boldsymbol{\eta}'_{\text{there}},$$
$$\mathbb{J}_{3\ \text{here}} = \boldsymbol{J}'_{\text{there}}, (\mathbb{J}_1 + i\mathbb{J}_2)_{\text{here}} = \boldsymbol{\Omega}'_{\text{there}},$$

$$\mathcal{L}_{11} = R\mathrm{vol}_{11} - \frac{1}{2}G_{(4)} \wedge \star_{11} G_{(4)} - \frac{1}{6}A_{(3)} \wedge G_{(4)} \wedge G_{(4)},$$

$$\mathrm{d}G_{(4)} = 0,$$

$$\mathrm{d} \star_{11} G_{(4)} + \frac{1}{2}G_{(4)} \wedge G_{(4)} = 0,$$

$$R_{MN} - \frac{1}{12}\left[G_{MPQR}G_N PQR - \frac{1}{12}G^2 g_{MN}\right] = 0.$$

$$\delta_\epsilon \Psi_M = \nabla_M \epsilon + \frac{1}{288}(\Gamma_M^{SPQR} - 8\delta_M^S \Gamma^{PQR})G_{SPQR}\epsilon = 0$$

$$\mathcal{L} = \bar{R}\overline{\mathrm{vol}}_4 - \frac{1}{2}\bar{F} \wedge \bar{x}_4 \bar{F} + 6g^2\overline{\mathrm{vol}}_4$$

$$\mathrm{d}\bar{F}=0,\ \mathrm{d}\bar{\star}_4\bar{F}=0,\bar{R}_{\mu\nu}=-3g^2\bar{g}_{\mu\nu}+\frac{1}{2}\left(\bar{F}_{\mu\sigma}\bar{F}_\nu^\sigma-\frac{1}{4}\bar{g}_{\mu\nu}\bar{F}_{\rho\sigma}\bar{F}^{\rho\sigma}\right)$$

$$\delta\psi_{i\mu}^+=\nabla_\mu\psi_i^++\frac{ig}{2}\epsilon_{ij}\bar{A}_\mu\psi_j^+-\frac{g}{2}\bar{\rho}_\mu(\psi_i^+)^c+\frac{g^2}{32}\bar{F}_{\delta\epsilon}\bar{\rho}^{\delta\epsilon}\bar{\rho}_\mu\epsilon_{ij}(\psi_j^+)^c$$

$$\{\Gamma_A,\Gamma_B\}=2\eta_{AB},\{\rho_\alpha,\rho_\beta\}=2\eta_{\alpha\beta},\{\gamma_a,\gamma_b\}=2\delta_{ab}$$

$$\Gamma_0\ldots\Gamma_{10}=1,\rho_5=i\rho_0\rho_1\rho_2\rho_3$$

$$\epsilon_{\alpha\beta\gamma\delta}\rho^\delta=-i\rho_{\alpha\beta\gamma}\rho_5,\epsilon_{\alpha\beta\gamma\delta}\rho^{\gamma\delta}=-2i\rho_{\alpha\beta}\rho_5,\epsilon_{\alpha\beta\gamma\delta}\rho^{\beta\gamma\delta}=6i\rho_\alpha\rho_5$$

$$\rho_\alpha{}^{\delta\epsilon}=\rho^{\delta\epsilon}\rho_\alpha-2\rho^{[\delta}\delta_\alpha^{\epsilon]}$$

$$\Gamma_\alpha=\rho_\alpha\otimes\mathbb{1},\Gamma_a=\rho_5\otimes\gamma_a$$

$$\hat{F}_{(4)}=F_{(4)}-\frac{g}{4}\bar{A}\wedge i_\xi F_{(4)}.$$

$$\alpha=\hat{e}\wedge i_{\hat{e}^*}\alpha+\tilde{\alpha},\beta=\hat{e}\wedge i_{\hat{e}^*}\beta+\tilde{\beta},\ \text{with}\ \ i_{\hat{e}^*}\tilde{\alpha}=i_{\hat{e}^*}\tilde{\beta}=0,$$

$$\mathrm{d}\alpha=(\mathrm{d}\mathcal{A}-g\bar{F})\wedge i_{\hat{e}^*}\alpha-\hat{e}\wedge\ \mathrm{d}i_{\hat{e}^*}\alpha+\mathrm{d}\tilde{\alpha},$$

$$\begin{aligned}\mathrm{d}G_{(4)}=&-\frac{g}{4}\bar{F}\wedge i_\xi F_{(4)}-\frac{g}{4}\bar{A}\wedge\ \mathrm{d}i_\xi F_{(4)}-g\bar{F}\wedge[(\ \mathrm{d}\mathcal{A}-g\bar{F})\wedge i_{\hat{e}^*}\alpha-\hat{e}\wedge\ \mathrm{d}i_{\hat{e}^*}\alpha+\mathrm{d}\tilde{\alpha}]\\&-g\star_4\bar{F}\wedge[(\ \mathrm{d}\mathcal{A}-g\bar{F})\wedge i_{\hat{e}^*}\beta-\hat{e}\wedge\ \mathrm{d}i_{\hat{e}^*}\beta+\mathrm{d}\tilde{\beta}]\end{aligned}$$

$$\mathrm{vol}_7=-e^4\wedge\cdots\wedge e^{10}=-E_1\wedge E_2\wedge E_3\wedge\mathrm{vol}(g_{\mathrm{SU}(2)}),$$

$$\begin{aligned}\mathrm{d}\star_{11}G_{(4)}=&\mathrm{vol}_4\wedge\ \mathrm{d}(e^{3\Delta}\star_7F_{(4)})-g\bar{F}\wedge\left(\frac{m}{4}\|\xi\|e^{3\Delta}\mathrm{vol}(g_{\mathrm{SU}(2)})\wedge E_2\wedge E_3\right)\\&-\frac{g}{4}\star_4\bar{F}\wedge[(\ \mathrm{d}\mathcal{A}-g\bar{F})e^{3\Delta}\wedge i_\xi\star_7\alpha-\hat{e}\wedge\ \mathrm{d}(e^{3\Delta}\wedge i_\xi\star_7\alpha)]\\&+\frac{g}{4}\bar{F}\wedge[(\ \mathrm{d}\mathcal{A}-g\bar{F})e^{3\Delta}\wedge i_\xi\star_7\beta-\hat{e}\wedge\ \mathrm{d}(e^{3\Delta}\wedge i_\xi\star_7\beta)].\end{aligned}$$

$$\begin{aligned}G_{(4)}\wedge G_{(4)}=&2m\mathrm{vol}_4\wedge F_{(4)}-2g\hat{F}_{(4)}\wedge(\bar{F}\wedge\alpha+\star_4\bar{F}\wedge\beta)\\&+2g^2\bar{F}\wedge\star_4\bar{F}\wedge\alpha\wedge\beta+g^2\bar{F}\wedge\bar{F}\wedge(\alpha\wedge\alpha-\beta\wedge\beta)\end{aligned}$$

$$\begin{aligned}\tilde{\mathrm{Ric}}_{\alpha\beta}=&e^{-2\Delta}\left\{\mathrm{Ric}_{\alpha\beta}-\frac{g^2}{32}\|\xi\|^2\bar{F}_{\alpha\gamma}\bar{F}_\beta^\gamma-9(\partial_a\Delta\partial^a\Delta+\nabla_a\nabla^a\Delta)\eta_{\alpha\beta}\right\}\\\tilde{\mathrm{Ric}}_{\alpha b}=&e^{-2\Delta}\left\{-\frac{g}{8}\|\xi\|\delta_{8b}\nabla_\gamma\bar{F}_\alpha^\gamma\right\}\\\tilde{\mathrm{Ric}}_{ab}=&e^{-2\Delta}\left\{\mathrm{Ric}_{ab}+\frac{g^2}{64}\|\xi\|^2\delta_{8a}\delta_{8b}\bar{F}_{\gamma\delta}\bar{F}^{\gamma\delta}\right.\\&\left.+9[\partial_a\Delta\partial_b\Delta-\nabla_a\nabla_b\Delta-(\partial_c\Delta\partial^c\Delta-\nabla_c\nabla^c\Delta)\delta_{ab}]\right\}\end{aligned}$$

$$G_{\alpha\beta\gamma\delta}=me^{-4\Delta}\epsilon_{\alpha\beta\gamma\delta},G_{abcd}=e^{-4\Delta}F_{abcd},G_{\alpha\beta ab}=-ge^{-4\Delta}\left[\bar{F}_{\alpha\beta}\alpha_{ab}+\frac{1}{2}\epsilon_{\alpha\beta\gamma\delta}\bar{F}^{\gamma\delta}\beta_{ab}\right]$$

$$T_{AB} \equiv \frac{1}{12}\left(G_{ACDE}G_B{}^{CDE} - \frac{1}{12}\eta_{AB}G^2\right)$$

$$T = T_{AB}\tilde{e}^A \otimes \tilde{e}^B$$

$$\begin{aligned}
e^{8\Delta}T_{\alpha\beta} =& -\frac{1}{3}m^2\eta_{\alpha\beta} + \frac{g^2}{4}(\alpha^2+\beta^2)\bar{F}_{\alpha\gamma}\bar{F}^{\gamma}_{\beta} - \frac{g^2}{24}\eta_{\alpha\beta}\bar{F}^2(\alpha^2+2\beta^2) \\
& - \frac{g^2}{4}\bar{F}_{\gamma(\alpha}\epsilon_{\beta)}{}^{\gamma\mu\nu}\bar{F}_{\mu\nu}\alpha_{cd}\beta^{cd} - \frac{g^2}{24}\eta_{\alpha\beta}\epsilon_{\mu\nu\rho\sigma}\bar{F}^{\mu\nu}\bar{F}^{\rho\sigma}\alpha_{cd}\beta^{cd} \\
e^{8\Delta}T_{\alpha b} =& 0 \\
e^{8\Delta}T_{ab} =& \frac{1}{2}\left[F_{acde}F_b^{cde} - \frac{1}{12}\eta_{ab}F^2\right] + \frac{g^2}{24}\bar{F}^2[6(\alpha_{ac}\alpha_b^c - \beta_{ac}\beta_b^c) - \eta_{ab}(\alpha^2-\beta^2)] \\
& + \frac{g^2}{24}\epsilon_{\mu\nu\rho\sigma}\bar{F}^{\mu\nu}\bar{F}^{\rho\sigma}[3(\alpha_{ac}\beta_b^c + \beta_{ac}\alpha_b^c) - \eta_{ab}\alpha_{cd}\beta^{cd}] + \frac{1}{2}m^2\eta_{ab}
\end{aligned}$$

$$\mathrm{d}\alpha = -\frac{1}{4}i_\xi F_{(4)} = -\frac{1}{4}\,\mathrm{d}\left(e^{3\Delta}\sqrt{1-\|\xi\|^2}J_1\right)$$

$$\alpha = -\frac{1}{4}e^{3\Delta}\sqrt{1-\|\xi\|^2}J_1 + \delta$$

$$\beta = ke^{3\Delta}(J_3 - \|\xi\|E_2\wedge E_3)$$

$$\begin{aligned}
i_\xi \star_7 \alpha &= \frac{\|\xi\|}{4}e^{3\Delta}\sqrt{1-\|\xi\|^2}J_1\wedge E_2\wedge E_3 + i_\xi\star_7\delta \\
i_\xi \star_7 \beta &= -k\|\xi\|e^{3\Delta}(E_2\wedge E_3 - \|\xi\|J_3)\wedge J_3
\end{aligned}$$

$$\begin{aligned}
& e^{6\Delta}\left\{\frac{k\|\xi\|}{4}(1+4k)J_3\wedge E_2\wedge E_3 + \left[\frac{\|\xi\|^2}{32}(1+4k) + \frac{1}{2}\left(k^2 - \frac{1}{16}\right)\right]J_1\wedge J_1\right\} \\
& + \frac{1}{2}\delta\wedge\left(\delta - \frac{1}{2}e^{3\Delta}\sqrt{1-\|\xi\|^2}J_1\right) = 0 \\
& \frac{1}{4}\left(k+\frac{1}{4}\right)e^{6\Delta}\|\xi\|\sqrt{1-\|\xi\|^2}J_1\wedge E_2\wedge E_3 \\
& + e^{3\Delta}\left[\frac{1}{4}i_\xi\star_7\delta + k(J_3 - \|\xi\|E_2\wedge E_3)\wedge\delta\right] = 0 \\
& me^{3\Delta}\left[-\|\xi\|\left(\frac{k}{2}+\frac{1}{8}\right) + \frac{1}{\|\xi\|}\left(k+\frac{1}{4}\right)\right]J_3\wedge J_3\wedge E_2\wedge E_3 - \delta\wedge\hat{F}_{(4)} - \frac{1}{8}\left(k+\frac{1}{4}\right)\hat{e}\wedge\,\mathrm{d}\left[e^{6\Delta}(1-\|\xi\|^2)J_1\wedge J_1\right] = 0 \\
& \frac{m}{3\|\xi\|^2}i_\xi\star_7\delta\wedge\left[J_3 + \left(3\|\xi\| - \frac{4}{\|\xi\|}\right)E_2\wedge E_3\right] - \frac{1}{4}\hat{e}\wedge\,\mathrm{d}\left(e^{3\Delta}i_\xi\star_7\delta\right) = 0
\end{aligned}$$

$$9(\partial_a\Delta\partial^a\Delta + \nabla_a\nabla^a\Delta) - \frac{1}{3}e^{-6\Delta}m^2 = -12$$

$$\begin{aligned}
\alpha_{ac}\beta_b^c &= -\frac{1}{16}\sqrt{1-\|\xi\|^2}e^{6\Delta}[\delta_a^6\delta_b^5 - \delta_a^7\delta_b^4 + \delta_a^4\delta_b^7 - \delta_a^5\delta_b^6] \\
\alpha_{ac}\alpha^{bc} &= \frac{1}{16}(1-\|\xi\|^2)e^{6\Delta}[\delta_a^4\delta_4^b + \delta_a^5\delta_5^b + \delta_a^6\delta_6^b + \delta_a^7\delta_7^b] \\
\beta_{ac}\beta^{bc} &= \frac{1}{16}e^{6\Delta}[\delta_a^4\delta_4^b + \delta_a^5\delta_5^b + \delta_a^6\delta_6^b + \delta_a^7\delta_7^b + \|\xi\|^2(\delta_a^9\delta_9^b + \delta_a^{10}\delta_{10}^b)].
\end{aligned}$$

$$\mathrm{Ric}_{\alpha\beta} + 12\eta_{\alpha\beta} = \frac{g^2}{8}\left(\bar{F}_{\alpha\gamma}\bar{F}^{\gamma}_{\beta} - \frac{1}{4}\eta_{\alpha\beta}\bar{F}^2\right)$$

$$\bar{g}_4 = 4g^{-2}g_4$$

$$\begin{aligned}\delta\Psi_a =&\delta^0\Psi_a - ge^{-\Delta/2}\Big\{\bar{F}_{\beta\gamma}(\rho^{\beta\gamma}\otimes\mathbb{1})\Big[-\frac{1}{8}k_a\psi_i^+\otimes\chi_i - \frac{1}{8}k_a(\psi_i^+)^c\otimes\chi_i^c \\ &+\frac{e^{-3\Delta}}{48}\alpha_{de}\psi_i^+\otimes\gamma_a^{de}\chi_i - \frac{e^{-3\Delta}}{48}\alpha_{de}(\psi_i^+)^c\otimes\gamma_a^{de}\chi_i^c \\ &-\frac{e^{-3\Delta}}{12}\alpha_{ae}\psi_i^+\otimes\gamma^e\chi_i + \frac{e^{-3\Delta}}{12}\alpha_{ae}(\psi_i^+)^c\otimes\gamma^e\chi_i^c\Big] \\ &+\bar{F}^*_{\beta\gamma}(\rho^{\beta\gamma}\otimes\mathbb{1})\Big[\frac{e^{-3\Delta}}{48}\beta_{de}\psi_i^+\otimes\gamma_a^{de}\chi_i - \frac{e^{-3\Delta}}{48}\beta_{de}(\psi_i^+)^c\otimes\gamma_a^{de}\chi_i^c \\ &-\frac{e^{-3\Delta}}{12}\beta_{ae}\psi_i^+\otimes\gamma^e\chi_i + \frac{e^{-3\Delta}}{12}\beta_{ae}(\psi_i^+)^c\otimes\gamma^e\chi_i^c\Big]\Big\}\end{aligned}$$

$$\bar{F}^*_{\delta\epsilon} \equiv \frac{1}{2}\epsilon_{\delta\epsilon\kappa\lambda}\bar{F}^{\kappa\lambda}$$

$$k_a = \frac{1}{4}\xi_a = \frac{1}{4}\|\xi\|\delta_{a8}$$

$$\begin{aligned}\delta\Psi_a =& -ge^{-\Delta/2}\bar{F}_{\beta\gamma}(\rho^{\beta\gamma}\otimes\mathbb{1})\Big[-\frac{1}{8}k_a\psi_i^+\otimes\chi_i - \frac{1}{8}k_a(\psi_i^+)^c\otimes\chi_i^c \\ &+\frac{e^{-3\Delta}}{48}\alpha_{de}\psi_i^+\otimes\gamma_a^{de}\chi_i - \frac{e^{-3\Delta}}{48}\alpha_{de}(\psi_i^+)^c\otimes\gamma_a^{de}\chi_i^c \\ &-\frac{e^{-3\Delta}}{12}\alpha_{ae}\psi_i^+\otimes\gamma^e\chi_i + \frac{e^{-3\Delta}}{12}\alpha_{ae}(\psi_i^+)^c\otimes\gamma^e\chi_i^c \\ &-\frac{ie^{-3\Delta}}{48}\beta_{de}\psi_i^+\otimes\gamma_a^{de}\chi_i - \frac{ie^{-3\Delta}}{48}\beta_{de}(\psi_i^+)^c\otimes\gamma_a^{de}\chi_i^c \\ &+\frac{ie^{-3\Delta}}{12}\beta_{ae}\psi_i^+\otimes\gamma^e\chi_i + \frac{ie^{-3\Delta}}{12}\beta_{ae}(\psi_i^+)^c\otimes\gamma^e\chi_i^c\Big]\end{aligned}$$

$$P_\pm = \frac{1}{2}(\mathbb{1}\pm\rho_5)\otimes\mathbb{1}$$

$$\left(-6k_a + e^{-3\Delta}(\alpha_{de} - i\beta_{de})(\gamma_a^{de} - 4\delta_a^d\gamma^e)\right)\chi_i = 0$$

$$\begin{aligned}&\bar{\chi}_+^c\Big[\|\xi\|(3\gamma^8 + i\gamma^{910}) + \sqrt{1-\|\xi\|^2}(\gamma^{46}-\gamma^{57}) - i(\gamma^{45}+\gamma^{67})\Big]\chi_- \\ &\quad = \|\xi\|(3(-i\|\xi\|) + i\|\xi\|) + \sqrt{1-\|\xi\|^2}\left(-2i\sqrt{1-\|\xi\|^2}\right) - i(-2)\end{aligned}$$

$$\begin{aligned}\delta\Psi_\mu =& e^{\Delta/2}\Big\{\nabla_\mu\psi_i^+\otimes\chi_i - \rho_\mu\psi_i^+\otimes\chi_i^c - \frac{g\|\xi\|}{16}\bar{F}_{\mu\beta}\rho^\beta\psi_i^+\otimes\gamma^8\chi_i \\ &+\frac{g}{4}\nabla_b k_c\bar{A}_\mu\psi_i^+\otimes\gamma^{bc}\chi_i + \frac{g\|\xi\|}{4}\bar{A}_\mu\psi_i^+\otimes\nabla_8\chi_i \\ &-\frac{g^2\|\xi\|^2}{128}\bar{A}_\mu\bar{F}_{\beta\gamma}\rho^{\beta\gamma}\psi_i^+\otimes\chi_i - \frac{ge^{-3\Delta}}{48}(\bar{F}_{\delta\epsilon}\alpha_{bc} + \bar{F}^*_{\delta\epsilon}\beta_{bc})\rho_\mu{}^{\delta\epsilon}\psi_i^+\otimes\gamma^{bc}\chi_i \\ &+\frac{g^2\|\xi\|e^{-3\Delta}}{192}\bar{A}_\mu(\bar{F}_{\delta\epsilon}\alpha_{bc} + \bar{F}^*_{\delta\epsilon}\beta_{bc})\rho^{\delta\epsilon}\psi_i^+\otimes\gamma_8{}^{bc}\chi_i \\ &+\frac{ge^{-3\Delta}}{12}(\bar{F}_{\mu\gamma}\alpha_{de} + \bar{F}^*_{\mu\gamma}\beta_{de})\rho^\gamma\psi_i^+\otimes\gamma^{de}\chi_i\Big\} + \text{m.c.}\end{aligned}$$

$$\mathcal{L}_\xi \chi = \nabla_\xi \chi + \frac{1}{4}\nabla_a \xi_b \gamma^{ab}\chi$$

$$\|\xi\|\nabla_8 \chi_1 + \nabla_a k_b \gamma^{ab}\chi_1 = -2\chi_2, \|\xi\|\nabla_8 \chi_2 + \nabla_a k_b \gamma^{ab}\chi_2 = 2\chi_1$$

$$\delta\Psi_\mu = e^{\Delta/2}\Big\{\nabla_\mu \psi_i^+ \otimes \chi_i - \rho_\mu (\psi_i^+)^c \otimes \chi_i - \frac{g\|\xi\|}{16}\bar{F}_{\mu\beta}\rho^\beta \psi_i^+ \otimes \gamma^8 \chi_i - \frac{ig}{2}\epsilon_{ij}\bar{A}_\mu \psi_i^+ \otimes \chi_j$$
$$- \frac{ge^{-3\Delta}}{48}\bar{F}_{\delta\epsilon}\left[\alpha_{bc}\left(\rho^{\delta\epsilon}\rho_\mu + 2\rho^\delta e_\mu^\epsilon\right)\psi_i^+ + 2i\beta_{bc}\left(\rho^{\delta\epsilon}\rho_\mu - \rho^\delta e_\mu^\epsilon\right)\psi_i^+\right] \otimes \gamma^{bc}\chi_i\Big\} + m.c.$$

$$\delta\Psi_\mu = e^{\Delta/2}\Big\{\nabla_\mu \psi_i^+ \otimes \chi_i - \rho_\mu (\psi_i^+)^c \otimes \chi_i - \frac{ig}{2}\epsilon_{ij}\bar{A}_\mu \psi_i^+ \otimes \chi_j$$
$$+ \frac{ig}{16}\bar{F}_{\delta\epsilon}\rho^{\delta\epsilon}\rho_\mu \psi_i^+ \otimes \gamma^{45}\chi_i\Big\} + \text{ m.c.}$$

$$\left(\rho_{(n)}\psi_i^+\right)^c = \rho_{(n)}(\psi_i^+)^c$$

$$\delta\Psi_\mu = e^{\Delta/2}\Big\{\nabla_\mu \psi_i^+ - \rho_\mu (\psi_i^+)^c + \frac{ig}{2}\epsilon_{ij}\bar{A}_\mu \psi_j^+ + \frac{g}{16}\bar{F}_{\delta\epsilon}\rho^{\delta\epsilon}\rho_\mu \epsilon_{ij}\left(\psi_j^+\right)^c\Big\} \otimes \chi_i + m.c.$$

$$g_4 + g_7 = -e^0 \otimes e^0 + \textstyle\sum_{i=1}^{10} e^i \otimes e^i$$

$$(n^a) = \left(0{,}0, \cdots, 0, +\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \quad (n_a) = \left(0{,}0, \cdots, 0, +\frac{1}{\sqrt{2}}, +\frac{1}{\sqrt{2}}\right)$$
$$(m^a) = \left(0{,}0, \cdots, 0, +\frac{1}{\sqrt{2}}, +\frac{1}{\sqrt{2}}\right), \quad (m_a) = \left(0{,}0, \cdots, 0, +\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right).$$

$$V_\pm \equiv \frac{1}{\sqrt{2}}\left(V_{(11)} \pm V_{(12)}\right)$$

$$n^a n_a = m^a m_a = 0\,, m^a n_a = m^+ n_+ = m_- n^- = +1$$

$$P_\uparrow \equiv \frac{1}{2}\not{n}\not{m} = \frac{1}{2}\gamma^+\gamma^- \quad , \qquad P_\downarrow \equiv \frac{1}{2}\not{m}\not{n} = \frac{1}{2}\gamma^-\gamma^+ \quad ,$$
$$P_\uparrow P_\uparrow = +P_\uparrow \quad , \qquad P_\downarrow P_\downarrow = +P_\downarrow \quad , \qquad P_\uparrow + P_\downarrow = +I \quad ,$$
$$P_{\uparrow\downarrow} \equiv P_\uparrow - P_\downarrow = \gamma^{+-} \quad ,$$

$$(\not{n})_{\alpha\dot{\beta}} = -(\not{n})_{\dot{\beta}\alpha} \quad , \quad (\not{m})_{\alpha\dot{\beta}} = -(\not{m})_{\dot{\beta}\alpha} \quad ,$$
$$(P_{\uparrow})_{\alpha\beta} = -(P_{\downarrow})_{\beta\alpha} \quad , \quad (P_{\uparrow\downarrow})_{\alpha\beta} = +(P_{\uparrow\downarrow})_{\beta\alpha} \quad .$$

$$(\widetilde{\mathcal{M}}_{ab})^{cd} \equiv +\tilde{\delta}_{[a}{}^{c}\,\tilde{\delta}_{b]}{}^{d} \quad ,$$
$$(\widetilde{\mathcal{M}}_{ab})_{\alpha}{}^{\beta} \equiv +\frac{1}{2}(\gamma_{ab}P_{\uparrow})_{\alpha}{}^{\beta} \quad , \quad (\widetilde{\mathcal{M}}_{ab})_{\dot{\alpha}}{}^{\dot{\beta}} \equiv +\frac{1}{2}(P_{\downarrow}\gamma_{ab})_{\dot{\alpha}}{}^{\dot{\beta}}$$

$$\tilde{\delta}_{a}{}^{b} \equiv \delta_{a}{}^{b} - m_{a}n^{b} = \begin{cases} \delta_{i}{}^{j} & (\text{ for } a = i, b = j) \\ \delta_{+}{}^{+} = 1 & (\text{ for } a = +, b = +) \\ 0 & (\text{ otherwise }) \end{cases}$$

$$\nabla_{M}n^{-} = \partial_{M}n^{-} + \frac{1}{2}\phi_{M}^{ab}(\widetilde{\mathcal{M}}_{ba})^{-+}n_{+} = 0$$
$$\nabla_{M}m^{+} = \partial_{M}m^{+} + \frac{1}{2}\phi_{M}^{ab}(\widetilde{\mathcal{M}}_{ba})^{+-}m_{-} = 0$$

$$\widetilde{\mathcal{M}}_{ab}\Psi_{\alpha} = +(\widetilde{\mathcal{M}}_{ab})_{\alpha}{}^{\beta}\Psi_{\beta} \quad , \quad \widetilde{\mathcal{M}}_{ab}\Psi^{\alpha} = -\Psi^{\beta}(\widetilde{\mathcal{M}}_{ab})_{\beta}{}^{\alpha} \quad ,$$
$$\widetilde{\mathcal{M}}_{ab}\overline{\Psi}_{\dot{\alpha}} = +(\widetilde{\mathcal{M}}_{ab})_{\dot{\alpha}}{}^{\dot{\beta}}\Psi_{\dot{\beta}} \quad , \quad \widetilde{\mathcal{M}}_{ab}\overline{\Psi}^{\dot{\alpha}} = -\overline{\Psi}^{\dot{\beta}}(\widetilde{\mathcal{M}}_{ab})_{\dot{\beta}}{}^{\dot{\alpha}}$$

$$[\widetilde{\mathcal{M}}_{ij}, C_{\alpha\beta}] = +\frac{1}{2}(\gamma_{ij}P_{\uparrow})_{\alpha\beta} \quad , \quad [\widetilde{\mathcal{M}}_{ij}, C^{\alpha\beta}] = -\frac{1}{2}(\gamma_{ij}P_{\uparrow})^{\alpha\beta} \quad ,$$
$$[\widetilde{\mathcal{M}}_{ij}, C_{\dot{\alpha}\dot{\beta}}] = +\frac{1}{2}(\gamma_{ij}P_{\uparrow})_{\alpha\beta} \quad , \quad [\widetilde{\mathcal{M}}_{ij}, C^{\dot{\alpha}\dot{\beta}}] = -\frac{1}{2}(\gamma_{ij}P_{\uparrow})^{\dot{\alpha}\dot{\beta}} \quad ,$$
$$[\widetilde{\mathcal{M}}_{-i}, C_{\alpha\beta}] = [\widetilde{\mathcal{M}}_{-i}, C^{\alpha\beta}] = [\widetilde{\mathcal{M}}_{-i}, C_{\dot{\alpha}\dot{\beta}}] = [\widetilde{\mathcal{M}}_{-i}, C^{\dot{\alpha}\dot{\beta}}] = 0 \quad ,$$

$$[\widetilde{\mathcal{M}}_{ab}, (\gamma^{c})_{\gamma}{}^{\dot{\delta}}] = \tilde{\delta}_{[a}{}^{c}(\gamma_{b]})_{\gamma}{}^{\dot{\delta}} + \frac{1}{2}(\gamma_{ab}P_{\uparrow}\gamma^{c})_{\gamma}{}^{\dot{\delta}} - \frac{1}{2}(\gamma_{ab}P_{\downarrow}\gamma^{c})_{\gamma}{}^{\dot{\delta}} \quad ,$$
$$[\widetilde{\mathcal{M}}_{ab}, (\gamma^{c})_{\dot{\gamma}}{}^{\delta}] = \tilde{\delta}_{[a}{}^{c}(\gamma_{b]})_{\dot{\gamma}}{}^{\delta} + \frac{1}{2}(\gamma_{ab}P_{\uparrow}\gamma^{c})_{\dot{\gamma}}{}^{\delta} - \frac{1}{2}(\gamma_{ab}P_{\downarrow}\gamma^{c})_{\dot{\gamma}}{}^{\delta} \quad ,$$

$$[\widetilde{\mathcal{M}}_{ab}, (\not{n})_{\gamma}{}^{\dot{\delta}}] = [\widetilde{\mathcal{M}}_{ab}, (\not{n})_{\dot{\gamma}}{}^{\delta}] = 0 \quad ,$$

$$[\widetilde{\mathcal{M}}_{ab}, (\not{m})_{\gamma}{}^{\dot{\delta}}] = [\widetilde{\mathcal{M}}_{ab}, (\not{m})_{\dot{\gamma}}{}^{\delta}] = 0 \quad .$$

$$[\nabla_A, (\not{n})_{\alpha}{}^{\dot{\beta}}] = [\nabla_A, (\not{m})_{\alpha}{}^{\dot{\beta}}] = [\nabla_A, (P_{\uparrow})_{\alpha}{}^{\beta}] = [\nabla_A, (P_{\downarrow})_{\alpha}{}^{\beta}] = 0 \quad ,$$

$$[\nabla_A, (\gamma_{cd} P_{\uparrow})_{\alpha}{}^{\beta}] = [\nabla_A, (P_{\downarrow}\gamma_{cd})_{\dot{\alpha}}{}^{\dot{\beta}}] = 0 \quad ,$$

$$[\nabla_A, (\not{n}\gamma^a P_{\downarrow})_{\alpha\beta}] = [\nabla_A, (P_{\uparrow}\gamma^a \not{n})_{\alpha\beta}] = 0 \quad .$$

$$[\nabla_A, T^c_{\alpha\beta}] = 0$$

$$\nabla_a\nabla_b\varphi = 0, \quad \nabla_a\nabla_b\tilde{\varphi} = 0, \quad \nabla_{\underline{\alpha}}\varphi = 0, \nabla_{\underline{\alpha}}\tilde{\varphi} = 0$$
$$(\nabla_a\varphi)^2 = 0, \quad (\nabla_a\tilde{\varphi})^2 = 0, \quad (\nabla_a\varphi)(\nabla^a\tilde{\varphi}) = 1$$

$$\nabla_a\varphi = n_a, \nabla_a\tilde{\varphi} = m_a$$

$$P_{\uparrow} \equiv \frac{1}{2}(\gamma^a\gamma^b)(\nabla_a\varphi)(\nabla_b\tilde{\varphi})\,, P_{\downarrow} \equiv \frac{1}{2}(\gamma^a\gamma^b)(\nabla_a\tilde{\varphi})(\nabla_b\varphi)\,, P_{\uparrow\downarrow} \equiv P_{\uparrow} - P_{\downarrow}$$

$$\nabla_M\nabla^-\varphi = \partial_M\nabla^-\varphi + \frac{1}{2}\phi_M^{ab}(\widetilde{\mathcal{M}}_{ba})^{-+}\nabla_+\varphi = 0$$
$$\nabla_M\nabla^+\tilde{\varphi} = \partial_M\nabla^+\tilde{\varphi} + \frac{1}{2}\phi_M^{ab}(\widetilde{\mathcal{M}}_{ba})^{+-}\nabla_-\tilde{\varphi} = 0$$

$$\tilde{\delta}_a{}^b \equiv \delta_a{}^b - (\nabla_a\tilde{\varphi})(\nabla^b\varphi)$$

$$\frac{1}{2}\nabla_{[A}T^D_{BC)} - \frac{1}{2}T^E_{[AB|}T_{E|C)} - \frac{1}{4}R^f_{[AB|e}(\mathcal{M}_f^e)^D_{|C)} \equiv 0$$
$$\frac{1}{6}\nabla_{[A}G_{BCD)} - \frac{1}{4}T^E_{[AB|}G_{E|CD)} \equiv 0$$
$$\frac{1}{2}\nabla_{[A}R_{BC)d}{}^e - \frac{1}{2}T^E_{[AB|}R_{E|C)d}{}^e \equiv 0$$

$$
\begin{aligned}
&T_{\alpha\beta}{}^{c} = (\gamma^{cd})_{\alpha\beta}\nabla_d\varphi + (\gamma^{de})_{\alpha\beta}(\nabla^c\varphi)(\nabla_d\varphi)(\nabla_e\widetilde{\varphi}) = (\gamma^{cd})_{\alpha\beta}\nabla_d\varphi + (P_{\uparrow\downarrow})_{\alpha\beta}\nabla^c\varphi \ , \\
&G_{\alpha\beta c} = T_{\alpha\beta c} \ , \\
&T_{\alpha\beta}{}^{\gamma} = (P_\uparrow)_{(\alpha|}{}^{\gamma}(\gamma^c\overline{\chi})_{|\beta)}\nabla_c\varphi - (\gamma^{ab})_{\alpha\beta}(P_\downarrow\gamma_a\overline{\chi})^{\gamma}\nabla_b\varphi \\
&\nabla_\alpha\Phi = (\gamma^c\overline{\chi})_\alpha\nabla_c\varphi \ , \\
&\nabla_\alpha\overline{\chi}_{\dot\beta} = -\frac{1}{24}(\gamma^{cde}P_\uparrow)_{\alpha\dot\beta}G_{cde} + \frac{1}{2}(\gamma^c P_\uparrow)_{\alpha\dot\beta}\nabla_c\Phi - (\gamma^c\overline{\chi})_\alpha\overline{\chi}_{\dot\beta}\nabla_c\varphi \ , \\
&T_{\alpha b}{}^{c} = 0 \ , \quad T_{\alpha b}{}^{\gamma} = 0 \ , \quad G_{\alpha bc} = 0 \ , \\
&T_{ab}{}^{c} = -G_{ab}{}^{c} \ , \\
&R_{\alpha\beta cd} = +(\gamma^{ef})_{\alpha\beta}G_{fcd}\nabla_e\varphi \ , \\
&\nabla_\alpha G_{bcd} = +\frac{1}{2}(\gamma^e\gamma_{[b}T_{cd]})_\alpha\nabla_e\varphi = -\nabla_\alpha T_{bcd} \ , \\
&R_{\alpha bcd} = +(\gamma^e\gamma_{[c}T_{d]b})_\alpha\nabla_e\varphi \ , \\
&\nabla_\alpha T_{bc}{}^{\delta} = -\frac{1}{4}(\gamma^{de}P_\uparrow)_\alpha{}^{\delta}R_{bcde} + T_{bc}{}^{\delta}(\gamma^e\overline{\chi})_\alpha\nabla_e\varphi + (P_\uparrow)_\alpha{}^{\delta}(\overline{\chi}\gamma^e T_{bc})\nabla_e\varphi \\
&\qquad\qquad + (\gamma^{de}T_{bc})_\alpha(P_\downarrow\gamma_d\overline{\chi})^{\delta}\nabla_e\varphi \ , \\
&\nabla_{\underline{\alpha}}\varphi = \nabla_{\underline{\alpha}}\widetilde{\varphi} = 0 \ , \quad (\nabla_a\varphi)^2 = (\nabla_a\widetilde{\varphi})^2 = 0 \ , \quad (\nabla_a\varphi)(\nabla^a\widetilde{\varphi}) = 1 \ , \\
&\nabla_a\nabla_b\varphi = \nabla_a\nabla_b\widetilde{\varphi} = 0 \ .
\end{aligned}
$$

$$
\begin{aligned}
&T_{AB}{}^{c}\nabla_c\varphi = 0 \ , \quad G_{ABc}\nabla^c\varphi = 0 \ , \quad T_{aB}{}^{C}\nabla^a\varphi = 0 \ , \\
&R_{ABc}{}^{d}\nabla_d\varphi = R_{aBc}{}^{d}\nabla^a\varphi = 0 \ , \\
&(\nabla^a\varphi)\nabla_a\Phi = 0 \ , \quad (\nabla^a\varphi)\nabla_a\overline{\chi}_{\dot\beta} = 0 \ , \\
&(\gamma^c)_\alpha{}^{\dot\beta}\overline{\chi}_{\dot\beta}\nabla_c\widetilde{\varphi} = 0 \ , \quad T_{ab}{}^{\gamma}(\gamma^d)_\gamma{}^{\dot\alpha}\nabla_d\widetilde{\varphi} = 0 \ , \\
&\phi_{Ab}{}^{c}\nabla_c\varphi = \phi_{ab}{}^{c}\nabla^a\varphi = 0 \ .
\end{aligned}
$$

$$(\sigma^{ab})_{(\alpha\beta|}(\sigma^{fc})_{|\gamma)}{}^{\delta}G_{acd}(\nabla_f\varphi)(\nabla_b\varphi)(\nabla^d\tilde{\varphi})$$

$$R_{(\alpha\beta\gamma)}{}^{\delta} = -\frac{1}{4}R_{(\alpha\beta|}{}^{cd}(\gamma_{cd}P_\uparrow)_{|\gamma)}{}^{\delta}$$

$$
\begin{aligned}
&T_{\alpha\beta}{}^{\gamma} = a_1(P_\uparrow)_{(\alpha|}{}^{\gamma}(\gamma^d\overline{\chi})_{|\beta)}\nabla_b\varphi + a_2(\gamma^{ab})_{\alpha\beta}(P_\downarrow\gamma_a\overline{\chi})^{\gamma}\nabla_b\varphi \ , \\
&\nabla_\alpha\overline{\chi}_{\dot\beta} = c_1(\gamma^{cde}P_\uparrow)_{\alpha\dot\beta}G_{cde} + c_2(\gamma^c P_\uparrow)_{\alpha\dot\beta}\nabla_c\Phi + g(\gamma^c\overline{\chi})_\alpha\overline{\chi}_{\dot\beta}\nabla_c\varphi \ ,
\end{aligned}
$$

$$\{\nabla_\alpha,\nabla_\beta\}\Phi = \nabla_{(\alpha|}[(\gamma^c\bar{\chi})_{|\beta)}\nabla_c\varphi] = +2c_2(\gamma^{cd})_{\alpha\beta}(\nabla_c\Phi)(\nabla_d\varphi)$$

$$X_{\alpha\beta\gamma d} = 2(a_1 + a_2)[+2(\gamma_d^a)_{(\alpha\beta|}(\gamma^b\bar{\chi})_{|\gamma)}(\nabla_a\varphi)(\nabla_b\varphi)$$
$$-(\gamma^{ab})_{(\alpha\beta|}(\gamma^c\bar{\chi})_{|\gamma)}(\nabla_b\varphi)(\nabla_c\varphi)(\nabla_d\varphi)(\nabla_a\tilde{\varphi})\Big]$$

$$a_1 = -a_2$$

$$(\nabla\Phi\text{-terms}) = -c_2(a_1 + a_2)[(\gamma^{ab})_{(\alpha\beta|}(\gamma^{cd})^{\delta}_{|\gamma)}(\nabla_a\Phi)(\nabla_b\varphi)(\nabla_c\varphi)(\nabla_d\tilde{\varphi})$$
$$+(\gamma^{ab})_{(\alpha\beta}\delta^{\delta}_{\gamma)}(\nabla_a\Phi)(\nabla_b\varphi)\Big]$$

$$(G\text{-terms}) = \left(-3c_1a_2 + \frac{1}{8}\right)$$
$$\times\Big[(\gamma^{ab})_{(\alpha\beta|}(\gamma^{cd})_{|\gamma)}{}^{\delta}G_{cda}(\nabla_b\varphi) - 2(\gamma^{ab})_{(\alpha\beta|}(\gamma^{fc})_{|\gamma)}{}^{\delta}G_{cda}(\nabla_f\varphi)(\nabla_b\varphi)(\nabla_d\tilde{\varphi})$$
$$+(\gamma^{ab})_{(\alpha\beta|}(\gamma^{fcdg})_{|\gamma)}{}^{\delta}G_{cda}(\nabla_f\varphi)(\nabla_b\varphi)(\nabla_g\tilde{\varphi})\Big]$$

$$c_1a_2 = +\frac{1}{24}$$

$$(\chi^2\text{-terms}) = a_2(g - a_1 - 2a_2)(\gamma^{ab})_{(\alpha\beta|}[(\gamma^c\bar{\chi})_{|\gamma)}(\gamma^d\bar{\chi})^{\delta}(\nabla_c\varphi)(\nabla_b\varphi)(\nabla_d\varphi)(\nabla_a\tilde{\varphi})$$
$$-(\gamma^c\bar{\chi})_{|\gamma)}(\gamma_a\bar{\chi})^{\delta}(\nabla_c\varphi)(\nabla_b\varphi)]$$

$$g = a_1 + 2a_2$$

$$a_1 = -a_2\,, c_1a_2 = +\frac{1}{24}\,, g = a_1 + 2a_2$$

$$a_1 = -a_2\,, c_1 = \frac{1}{24}a_2^{-1}\,, g = -a_1$$

$$(\gamma^{bc})_{\alpha\beta}T_{ab}{}^{\beta}\nabla_c\varphi - 2(\gamma^c)_{\alpha}{}^{\dot{\beta}}(\nabla_a\overline{\chi}_{\dot{\beta}})\nabla_c\varphi = 0 \quad ,$$
$$R_{a[b|}\nabla_{|c]}\varphi + 4(\nabla_a\nabla_{[b|}\Phi)\nabla_{|c]}\varphi - 4(\overline{\chi}\gamma^d T_{a[b|})(\nabla_{|c]}\varphi)\nabla_d\varphi = 0 \quad ,$$
$$R_{[ab]} = -\nabla_c G_{ab}{}^{c} \quad .$$

$$X^{\beta}_{\alpha\beta\gamma} = -\frac{7}{2}[\gamma^{bc}T_{ab}\nabla_c\varphi + 2a_1(\gamma^c\nabla_a\bar{\chi})\nabla_c\varphi]_{\gamma} = 0$$

$$0 = +(\gamma_{de})^{\beta\gamma}\nabla_{\beta}\left[(\gamma^{bc})_{\gamma}{}^{\delta}T_{ab\delta}\nabla_c\varphi + 2a_1(\gamma^b)_{\gamma}{}^{\dot{\delta}}(\nabla_a\overline{\chi}_{\dot{\delta}})\nabla_b\varphi\right]$$
$$= +8\left[R_{a[d}n_{e]} + 8a_1c_2(\nabla_a\nabla_{[d}\Phi)\nabla_{e]}\varphi - 4a_1(\overline{\chi}\gamma^b T_{a[d})(\nabla_{e]}\varphi)\nabla_b\varphi\right]$$

$$0 = X^c_{abc} = -R_{[ab]} - \nabla_c G^c_{ab}$$

$$\delta_Q e_m{}^a = +(\epsilon\gamma^{ab}\psi_m)D_b\varphi + (\epsilon P_{\uparrow\downarrow}\psi_m)\,D^a\varphi \quad , \qquad \delta_Q\Phi = -\,(\epsilon\gamma^m\overline{\chi})\,\partial_m\varphi \quad ,$$

$$\delta_Q\psi_m{}^\alpha = D_m\epsilon^\alpha + (P_\downarrow\epsilon)^\alpha\,(\overline{\chi}\gamma^n\psi_m)\,\partial_n\varphi + (P_\downarrow\psi_m)^\alpha\,(\epsilon\gamma^n\overline{\chi})\,\partial_n\varphi$$
$$- \,(P_\downarrow\gamma_a\overline{\chi})^\alpha\,(\epsilon\gamma^{an}\psi_m)\partial_n\varphi \quad ,$$

$$\delta_Q B_{mn} = +(\epsilon\gamma_{[m}{}^r\psi_{n]})\partial_r\varphi - \left(\epsilon P_{\uparrow\downarrow}\psi_{[m}\right)\partial_{n]}\varphi \quad ,$$

$$\delta_Q\overline{\chi}_{\dot\alpha} = +\frac{1}{24}\,(P_\downarrow\gamma^{mnr}\epsilon)_{\dot\alpha}\,G_{mnr} + \frac{1}{2}\,(P_\downarrow\gamma^m\epsilon)_{\dot\alpha}\,\partial_m\Phi - \overline{\chi}_{\dot\alpha}\,(\epsilon\gamma^m\overline{\chi})\,\partial_m\varphi \quad ,$$

$$\delta_Q\varphi = 0 \quad .$$

$$\hat{\gamma}_{\hat{a}} = \begin{cases} \hat{\gamma}_a = \gamma_a \otimes \tau_3 \\ \hat{\gamma}_{(11)} = I \otimes \tau_1 \\ \hat{\gamma}_{(12)} = -I \otimes i\tau_2 \end{cases}$$

$$\hat{C} = C \otimes \tau_1 \,, \hat{\gamma}_{13} = \gamma_{11} \otimes \tau_3$$

$$(\hat{\not{p}})_{\hat\alpha}{}^{\dot{\hat\beta}} = (\hat\gamma^+)_{\hat\alpha}{}^{\dot{\hat\beta}} = \sqrt{2}I \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad , \qquad (\hat{\not{n}})_{\hat\alpha}{}^{\dot{\hat\beta}} = (\hat\gamma^-)_{\hat\alpha}{}^{\dot{\hat\beta}} = \sqrt{2}I \otimes \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\hat{P}_\uparrow = I \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad , \qquad \hat{P}_\downarrow = I \otimes \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad .$$

$$\left(\hat{\chi}_{\dot{\hat\alpha}}\right) = \begin{pmatrix} \hat{\chi}_{\alpha\uparrow} \\ \hat{\chi}_{\alpha\downarrow} \end{pmatrix} = \begin{pmatrix} 0 \\ \chi_\alpha \end{pmatrix} \quad , \qquad \left(\hat{T}_{\hat a\hat b}{}^{\hat\gamma}\right) = \left(\hat{T}_{\hat a\hat b}{}^{\gamma\uparrow}, \hat{T}_{\hat a\hat b}{}^{\gamma\downarrow}\right) = (T_{\hat a\hat b}{}^\gamma, 0)$$

$$\hat{T}_{\alpha\uparrow\beta\uparrow}{}^c = (\hat\gamma^{c+})_{\alpha\uparrow\beta\uparrow} = (\hat\gamma^c\hat{\not{p}})_{\alpha\uparrow\beta\uparrow} = (\hat\gamma^c)_{\alpha\uparrow}{}^{\dot{\hat\gamma}}\,(\hat{\not{p}})_{\dot{\hat\gamma}}{}^{\hat\delta}\hat{C}_{\hat\delta\beta\uparrow}$$
$$= \left[(\gamma^c)_\alpha{}^\gamma \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\delta_\gamma{}^\delta \otimes \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix} C_{\delta\beta} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right]_{\uparrow\uparrow}$$
$$= \sqrt{2}(\gamma^c)_{\alpha\beta} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_{\uparrow\uparrow} = \sqrt{2}(\gamma^c)_{\alpha\beta} \equiv T_{\alpha\beta}{}^c \quad ,$$

$$\hat{T}_{\alpha\uparrow\beta\uparrow}{}^{\gamma\uparrow} \rightarrow T_{\alpha\beta}{}^\gamma = \sqrt{2}\left[\,\delta_{(\alpha}{}^\gamma\chi_{\beta)} - (\gamma^a)_{\alpha\beta}(\gamma_a\chi)^\gamma\,\right] \quad .$$

$$(\hat\gamma^{\hat\beta}\hat\gamma^+)\hat{T}_{\hat a\hat b}{}^{\hat\beta} + 2(\hat\gamma^+)_{\hat\alpha\dot{\hat\beta}}\hat\nabla_{\hat a}\hat\chi^{\dot{\hat\beta}} \quad = 0 \quad .$$

$$\gamma^b T_{ab} + \nabla_a\chi = 0$$

$$0 = +\hat{R}_{ab}\hat{n}_+ + 4\hat{\nabla}_a\hat{\nabla}_b\hat{\Phi}\hat{n}_+ - 4(\hat{\bar{\chi}}\hat{\gamma}^+\hat{T}_{ab})\hat{n}_+$$
$$= +R_{ab} + 4\nabla_a\nabla_b\Phi - 4\hat{T}_{ab}^{\gamma\uparrow}(\hat{\gamma}^+)_{\gamma\uparrow}^{\delta\downarrow}\hat{\chi}_{\delta\downarrow}$$

$$R_{ab} + 4\nabla_a\nabla_b\Phi - 4\sqrt{2}(\bar{T}_{ab}\chi) = 0$$

$$R_{[ab]} = -2\nabla_c G^c_{ab},$$

$$S \equiv S_\sigma + S_B + S_\Lambda$$
$$S_\sigma \equiv \int d^2\sigma[V^{-1}\eta_{ab}\Pi^{\underline{a}}_+\Pi^{\underline{b}}_-]$$
$$S_B \equiv \int d^2\sigma[V^{-1}\Pi^{\underline{A}}_+\Pi^{\underline{B}}_- B_{BA}]$$
$$S_\Lambda \equiv \int d^2\sigma\big[V^{-1}\Lambda_{++}(\Pi^{\underline{a}}_-\nabla_a\varphi)\big(\Pi^{\underline{b}}_-\nabla_b\tilde{\varphi}\big)$$
$$+V^{-1}\tilde{\Lambda}_{++}\{(\Pi^{\underline{a}}_-\nabla_a\varphi)^2 + (\Pi^{\underline{a}}_-\nabla_a\tilde{\varphi})^2\}\big]$$

$$\delta V_+{}^i = \overline{\kappa}_+{}^{\dot{\alpha}}(\gamma^c)_{\dot{\alpha}}{}^{\beta}(\nabla_c\varphi)\Pi_{+\beta}V_-{}^i \equiv (\overline{\kappa}_+\gamma^c\Pi_+)\,V_-{}^i\nabla_c\varphi \quad ,$$
$$(\gamma^c)_\alpha{}^{\dot{\beta}}\overline{\kappa}_{+\dot{\beta}}\nabla_c\varphi \equiv (\gamma^c\overline{\kappa}_+)_\alpha\nabla_c\varphi = 0 \quad ,$$
$$\delta V_-{}^i = 0 \quad , \qquad \delta(V^{-1}) = 0 \quad , \qquad \delta\overline{E}^{\dot{\alpha}} = \delta E^a = 0 \quad ,$$
$$\delta E^\alpha = \frac{1}{2}(\gamma_a)^{\alpha\dot{\beta}}\,\overline{\kappa}_{+\dot{\beta}}\Pi_-{}^a + (P_\uparrow)^{\alpha\beta}\eta_\beta \equiv \frac{1}{2}(\not{\Pi}_-\overline{\kappa}_+)^\alpha + (P_\uparrow\eta)^\alpha \quad ,$$
$$\delta\Lambda_{++} = -2\,(\overline{\kappa}_+\gamma^c\Pi_+)\,\nabla_c\varphi \quad , \qquad \delta\widetilde{\Lambda}_{++} = 0 \quad , \qquad \delta\varphi = 0 \quad , \qquad \delta\widetilde{\varphi} = 0 \quad ,$$

$$\delta_\kappa\Pi^{\underline{A}}_\pm = V_\pm{}^iD_i(\delta_\kappa E^A) + \big(\delta_\kappa V_\pm{}^i\big)\Pi_i{}^A - \Pi_\pm{}^D(\delta_\kappa E^C)(T_{CD}{}^A - \phi_{CD}{}^A)$$

$$\delta_\kappa(S_\sigma + S_B) = +(\bar{\kappa}_+\gamma^c\Pi_+)(\Pi^{\underline{a}}_-)^2\nabla_c\varphi - \frac{1}{2}\big(\bar{\kappa}_+\gamma^d\gamma^a\gamma^e\gamma^{bc}\Pi_+\big)\Pi_{-e}\Pi_{-b}(\nabla_c\varphi)(\nabla_d\varphi)(\nabla_a\tilde{\varphi})$$
$$-\frac{1}{2}(\bar{\kappa}_+\gamma^c\gamma^a\gamma^e P_{\uparrow\downarrow}\Pi_+)\Pi_{-e}\Pi^{\underline{b}}_-(\nabla_b\varphi)(\nabla_c\varphi)(\nabla_a\tilde{\varphi})$$
$$= +2V^{-1}(\bar{\kappa}_+\gamma^c\Pi_+)(\Pi^{\underline{a}}_-\nabla_a\varphi)\big(\Pi^{\underline{b}}_-\nabla_b\tilde{\varphi}\big)\nabla_c\varphi$$

$$\bar{\kappa}_+\gamma^a\gamma^b(\nabla_a\tilde{\varphi})(\nabla_b\varphi) = +2\bar{\kappa}_+$$

$$\delta_\kappa S_\Lambda = -2V^{-1}(\bar{\kappa}_+\gamma^c\Pi_+)(\Pi_-{}^a\nabla_a\varphi)\big(\Pi_-{}^b\nabla_b\tilde{\varphi}\big)\nabla_c\varphi,$$

$$\delta_\kappa(\Pi_-{}^a\nabla_a\varphi) = \delta_\kappa(\Pi_-{}^a\nabla_a\tilde{\varphi}) = 0$$

$$T_{\alpha\beta}{}^c\nabla_c\tilde{\varphi} = 0$$

$$\big(\widetilde{M}_{bc}\big)^{-d} = 0$$

$$\delta_\kappa S = \delta_\kappa(S_\sigma + S_B + S_\Lambda) = 0$$

$$\delta_\eta S = 0$$

$$(\Pi_-{}^a\nabla_a\varphi)(\Pi_-{}^b\nabla_b\tilde{\varphi}) = 0, (\Pi_-{}^a\nabla_a\varphi)^2 + (\Pi_-{}^a\nabla_a\tilde{\varphi})^2 = 0$$

$$\Pi_-{}^a\nabla_a\varphi = 0, \Pi_-{}^a\nabla_a\tilde{\varphi} = 0$$

$$\delta V_\pm^\alpha = m_\beta^\alpha V_\pm^\beta \pm i\Sigma V_\pm^\alpha$$

$$\begin{pmatrix} V_-^1 & V_+^1 \\ V_-^2 & V_+^2 \end{pmatrix} = \exp\begin{pmatrix} i\varphi & A \\ A^* & -i\varphi \end{pmatrix} = \begin{pmatrix} \cosh\rho + i\varphi\dfrac{\sinh\rho}{\rho} & A\dfrac{\sinh\rho}{\rho} \\ A^*\dfrac{\sinh\rho}{\rho} & \cosh\rho - i\varphi\dfrac{\sinh\rho}{\rho} \end{pmatrix}$$

$$(m^\alpha{}_\beta) = \begin{pmatrix} i\gamma & \alpha \\ \alpha^* & -i\gamma \end{pmatrix}$$

$$\epsilon_{\alpha\beta}V_-^\alpha V_+^\beta = \det V = 1, V_-^\alpha V_+^\beta - V_+^\alpha V_-^\beta = \epsilon^{\alpha\beta}$$

$$Q_\mu = -i\epsilon_{\alpha\beta}V_-^\alpha\partial_\mu V_+^\beta$$

$$\delta A_{\mu\nu}{}^\alpha = m^\alpha{}_\beta A_{\mu\nu}{}^\beta, \delta\psi_\mu = \frac{i}{2}\Sigma\psi_\mu, \delta\lambda = \frac{3i}{2}\Sigma\lambda.$$

$$\gamma^{[6]}\psi_+ S_{[6]} \equiv 0, \gamma^{[6]}\psi_- A_{[6]} \equiv 0, \gamma_{13}\psi_\pm \equiv \pm\psi_\pm$$
$$S_{[6]} = +\frac{1}{6!}\epsilon_{[6]}^{[6]'}S_{[6]'}, A_{[6]} = -\frac{1}{6!}\epsilon_{[6]}^{[6]'}A_{[6]'}$$

$$(\bar{\psi}_1\gamma^{\mu_1\cdots\mu_N}\psi_2) = +(\bar{\psi}_2\gamma^{\mu_N\cdots\mu_1}\psi_1) = (-)^{N(N-1)/2}(\bar{\psi}_2\gamma^{\mu_1\cdots\mu_N}\psi_1)$$
$$(\bar{\psi}_1\gamma^{\mu_1\cdots\mu_N}\psi_2)^\dagger = \psi_2^\dagger(\gamma^{\mu_1\cdots\mu_N})^\dagger\bar{\psi}_1^\dagger = +(\bar{\psi}_2\gamma^{\mu_N\cdots\mu_1}\psi_1)$$
$$= (-1)^{N(N-1)/2}(\bar{\psi}_2\gamma^{\mu_1\cdots\mu_N}\psi_1) = +(\bar{\psi}_1^*\gamma^{\mu_1\cdots\mu_N}\psi_2^*)$$

$$\delta_Q e_\mu^m = \left[(\bar{\epsilon}\gamma^{mn}\psi_\mu)D_n\varphi + (\bar{\epsilon}P_{\uparrow\downarrow}\psi_\mu)D^m\varphi\right] + \text{ c.c.}$$
$$\delta_Q\psi_\mu = \widehat{D}_\mu\epsilon - \frac{i}{480}(P_\downarrow\gamma^{[5]}\gamma_\mu\epsilon)\widehat{F}_{[5]} + \frac{1}{96}P_\downarrow\left(\gamma_\mu^{[3]}\widehat{G}_{[3]} - 9\gamma^{[2]}\widehat{G}_{\mu[2]}\right)\epsilon^*$$
$$\delta_Q A_{\mu\nu}^\alpha = V_+^\alpha(\bar{\epsilon}^*\gamma_{\mu\nu}^\rho\lambda^*)\partial_\rho\varphi + V_-^\alpha(\bar{\epsilon}\gamma_{\mu\nu}^\rho\lambda)\partial_\rho\varphi$$
$$-4V_+^\alpha\left(\bar{\epsilon}\gamma_{[\mu|}^\rho\psi_{|\nu]}^*\right)\partial_\rho\varphi - 4V_-^\alpha\left(\bar{\epsilon}^*\gamma_{[\mu|}^\rho\psi_{|\nu]}\right)\partial_\rho\varphi$$
$$\delta_Q A_{\mu\nu\rho\sigma} = i(\bar{\epsilon}\gamma_{[\mu\nu\rho|}^\tau\psi_{|\sigma]})\partial_\tau\varphi - i(\bar{\epsilon}^*\gamma_{[\mu\nu\rho|}^\tau\psi_{|\sigma]}^*)\partial_\tau\varphi - \frac{3i}{8}\epsilon_{\alpha\beta}A_{[\mu\nu}^\alpha\delta_Q A_{\rho\sigma]}^\beta$$
$$\delta_Q\lambda = (P_\downarrow\gamma^\mu\epsilon^*)\widehat{P}_\mu - \frac{1}{24}(P_\downarrow\gamma^{\mu\nu\rho}\epsilon)\widehat{G}_{\mu\nu\rho}$$
$$\delta_Q V_+^\alpha = V_-^\alpha(\bar{\epsilon}^*\gamma^\mu\lambda)\partial_\mu\varphi, \delta_Q V_-^\alpha = V_+^\alpha(\bar{\epsilon}\gamma^\mu\lambda^*)\partial_\mu\varphi$$
$$(4\cdot 2\cdot 1\ g)\delta_Q\varphi = 0\delta_Q\tilde{\varphi} = 0$$

$$\delta_Q e_\mu{}^m = [\,(\bar\epsilon\gamma^{mn}\psi_\mu)D_n\varphi + (\bar\epsilon P_{\uparrow\downarrow}\psi_\mu)D^m\varphi\,] + \text{c.c.} \quad ,$$

$$\delta_Q\psi_\mu = \widehat{D}_\mu\epsilon - \frac{i}{480}\,(P_\downarrow\gamma^{[5]}\gamma_\mu\epsilon)\widehat{F}_{[5]} + \frac{1}{96}\,P_\downarrow\left(\gamma_\mu{}^{[3]}\widehat{G}_{[3]} - 9\gamma^{[2]}\widehat{G}_{\mu[2]}\right)\epsilon^* \quad ,$$

$$\begin{aligned}\delta_Q A_{\mu\nu}{}^\alpha = {} & V_+{}^\alpha(\bar\epsilon^*\gamma_{\mu\nu}{}^\rho\lambda^*)\partial_\rho\varphi + V_-{}^\alpha(\bar\epsilon\gamma_{\mu\nu}{}^\rho\lambda)\partial_\rho\varphi \\ & - 4V_+{}^\alpha(\bar\epsilon\gamma_{[\mu|}{}^\rho\psi^*_{|\nu]})\partial_\rho\varphi - 4V_-{}^\alpha(\bar\epsilon^*\gamma_{[\mu|}{}^\rho\psi_{|\nu]})\partial_\rho\varphi \quad ,\end{aligned}$$

$$\delta_Q A_{\mu\nu\rho\sigma} = i(\bar\epsilon\gamma_{[\mu\nu\rho|}{}^\tau\psi_{|\sigma]})\partial_\tau\varphi - i(\bar\epsilon^*\gamma_{[\mu\nu\rho|}{}^\tau\psi^*_{|\sigma]})\partial_\tau\varphi - \frac{3i}{8}\,\epsilon_{\alpha\beta}A_{[\mu\nu}{}^\alpha\delta_Q A_{\rho\sigma]}{}^\beta \quad ,$$

$$\delta_Q\lambda = -\,(P_\downarrow\gamma^\mu\epsilon^*)\,\widehat{P}_\mu - \frac{1}{24}\,(P_\downarrow\gamma^{\mu\nu\rho}\epsilon)\,\widehat{G}_{\mu\nu\rho} \quad ,$$

$$\delta_Q V_+{}^\alpha = V_-{}^\alpha(\bar\epsilon^*\gamma^\mu\lambda)\partial_\mu\varphi \quad , \qquad \delta_Q V_-{}^\alpha = V_+{}^\alpha(\bar\epsilon\gamma^\mu\lambda^*)\partial_\mu\varphi \quad ,$$

$$\delta_Q\varphi = 0 \quad , \qquad \delta_Q\tilde\varphi = 0 \quad ,$$

$$G_{\mu\nu\rho} \equiv -\epsilon_{\alpha\beta}V_+^\alpha F_{\mu\nu\rho}^\beta\,, P_\mu \equiv -\epsilon_{\alpha\beta}V_+^\alpha\partial_\mu V_+^\beta\,, Q_\mu \equiv -i\epsilon_{\alpha\beta}V_-^\alpha\partial_\mu V_+^\beta$$

$$F_{\mu\nu\rho}^\alpha \equiv 3\partial_{[\mu}A_{\nu\rho]}^\alpha, F_{\mu\nu\rho\sigma\tau} \equiv 5\partial_{[\mu}A_{\nu\rho\sigma\tau]} + \frac{5i}{8}\epsilon_{\alpha\beta}A_{[\mu\nu}^\alpha F_{\rho\sigma\tau]}^\beta$$

$$D_{[\mu}P_{\nu]} = 0, D_{[\mu}G_{\nu\rho\sigma]} = +P_{[\mu}G_{\nu\rho\sigma]}^*$$

$$\partial_{[\mu_1}F_{\mu_2\cdots\mu_6]} \equiv \frac{5i}{12}G_{[\mu_1\mu_2\mu_3}G_{\mu_4\mu_5\mu_6]}^*$$

$$\partial_{[\mu}Q_{\nu]} = -iP_{[\mu}P_{\nu]}^*$$

$$\hat{G}_{\mu\nu}^\rho\partial_\rho\varphi = 0\,, \hat{F}_{\mu\nu\rho\sigma}^\tau\partial_\tau\varphi = 0\,, \hat{R}_\mu^{\nu mn}\partial_\nu\varphi = 0\,, \hat{R}_{\mu\nu}^{mn}D_m\varphi = 0$$

$$\begin{aligned} &\hat{P}^\mu\partial_\mu\varphi = 0, && Q^\mu\partial_\mu\varphi = 0 \\ &\hat{\mathcal{R}}_\mu{}^\nu\partial_\nu\varphi = 0, && \hat{\hat{\mathcal{R}}}_{\mu\nu}\gamma^m D_m\tilde\varphi = 0 \\ &\gamma^\mu\lambda\partial_\mu\tilde\varphi = 0, && (\widehat{D}_m\lambda)(D^m\varphi) = 0 \\ &D_m D_n\varphi = 0, && D_m D_n\tilde\varphi = 0, (D_m\varphi)(D^m\tilde\varphi) = 1 \\ &(D_m\varphi)^2 = 0, && (D_m\tilde\varphi)^2 = 0 \end{aligned}$$

$$\delta_{\mathrm{E}}\phi_{\mu_1\cdots\mu_m}{}^{r_1\cdots r_n} = \Omega_{[\mu_1\cdots\mu_{m-1}}{}^{r_1\cdots r_n}\partial_{\mu_m]}\varphi + \Omega'{}_{\mu_1\cdots\mu_m}^{[r_1\cdots r_{n-1}}D^{r_n]}\varphi$$

$$\delta_{\mathrm{E}}A_{\mu\nu\rho\sigma} = \Omega_{[\mu\nu\rho}\partial_{\sigma]}\varphi$$

$$\widehat{D}_\mu \widehat{P}^\mu - \frac{1}{24}\widehat{G}^2_{\mu\nu\rho} + \mathcal{O}(\psi^2) = 0$$

$$(\widehat{D}_\mu \widehat{G}^\mu{}_{[\nu\rho})\partial_{\sigma]}\varphi + \widehat{P}^\mu \widehat{G}^*_{\mu[\nu\rho}\partial_{\sigma]}\varphi + \frac{2i}{3}\widehat{F}^{\tau\omega\lambda}_{[\nu\rho|}\widehat{G}_{\tau\omega\lambda}\partial_{|\sigma]}\varphi + \mathcal{O}(\psi^2) = 0$$

$$\Big(\widehat{R}_{\rho[\mu|} - \widehat{P}_\rho \widehat{P}^*_{[\mu|} - \widehat{P}_{[\mu|}\widehat{P}^*_\rho - \frac{1}{6}\widehat{F}_{[4]\rho}\widehat{F}^{[4]}_{[4]}$$

$$-\frac{1}{8}\widehat{G}^{\sigma\tau}_\rho \widehat{G}^*_{\sigma\tau[\mu|} - \frac{1}{8}\widehat{G}_{\sigma\tau[\mu|}\widehat{G}^{*\sigma\tau}_\rho + \frac{1}{48}g_{\rho[\mu|}\widehat{G}^{[3]}\widehat{G}^*_{[3]}\Big)\partial_{|\nu]}\varphi + \mathcal{O}(\psi^2) = 0$$

$$\widehat{F}_{[\mu_1\cdots\mu_5}\partial_{\mu_6]}\varphi = -\frac{1}{6!}e^{-1}\epsilon^{\nu_1\cdots\nu_6}_{\mu_1\cdots\mu_6}\widehat{F}_{\nu_1\cdots\nu_5}\partial_{\nu_6}\varphi$$

$$\gamma^\sigma\left(\gamma^\rho\widehat{\mathcal{R}}_{\rho[\mu|} + \lambda^*\widehat{P}_{[\mu|} - \frac{1}{48}\gamma^{[3]}\gamma_{[\mu|}\lambda\widehat{G}^*_{[3]} - \frac{1}{96}\gamma_{[\mu|}\gamma^{[3]}\lambda\widehat{G}^*_{[3]}\right)(\partial_{|\nu]}\varphi)(\partial_\sigma\varphi) = 0$$

$$\gamma^\sigma\left(\gamma^\mu\widehat{D}_\mu\lambda - \frac{i}{240}\gamma^{[5]}\lambda\widehat{F}_{[5]}\right)\partial_\sigma\varphi = 0$$

$$\widehat{D}_\mu \widehat{P}^\mu - \frac{1}{24}\widehat{G}^2_{\mu\nu\rho} + \mathcal{O}(\psi^2) = 0 \quad ,$$

$$(\widehat{D}_\mu \widehat{G}^\mu{}_{[\nu\rho})\partial_{\sigma]}\varphi + \widehat{P}^\mu \widehat{G}^*_{\mu[\nu\rho}\partial_{\sigma]}\varphi + \frac{2i}{3}\widehat{F}_{[\nu\rho|}{}^{\tau\omega\lambda}\widehat{G}_{\tau\omega\lambda}\partial_{|\sigma]}\varphi + \mathcal{O}(\psi^2) = 0 \quad ,$$

$$\Big(\widehat{R}_{\rho[\mu|} - \widehat{P}_\rho \widehat{P}^*_{[\mu|} - \widehat{P}_{[\mu|}\widehat{P}^*_\rho - \frac{1}{6}\widehat{F}_{[4]\rho}\widehat{F}^{[4]}{}_{[\mu|}$$

$$-\frac{1}{8}\widehat{G}_\rho{}^{\sigma\tau}\widehat{G}^*_{\sigma\tau[\mu|} - \frac{1}{8}\widehat{G}_{\sigma\tau[\mu|}\widehat{G}^{*\sigma\tau}_\rho + \frac{1}{48}g_{\rho[\mu|}\widehat{G}^{[3]}\widehat{G}^*_{[3]}\Big)\partial_{|\nu]}\varphi + \mathcal{O}(\psi^2) = 0 \quad ,$$

$$\widehat{F}_{[\mu_1\cdots\mu_5}\partial_{\mu_6]}\varphi = -\frac{1}{6!}e^{-1}\epsilon_{\mu_1\cdots\mu_6}{}^{\nu_1\cdots\nu_6}\widehat{F}_{\nu_1\cdots\nu_5}\partial_{\nu_6}\varphi \quad ,$$

$$\gamma^\sigma\left(\gamma^\rho\widehat{\mathcal{R}}_{\rho[\mu|} + \lambda^*\widehat{P}_{[\mu|} - \frac{1}{48}\gamma^{[3]}\gamma_{[\mu|}\lambda\widehat{G}^*_{[3]} - \frac{1}{96}\gamma_{[\mu|}\gamma^{[3]}\lambda\widehat{G}^*_{[3]}\right)(\partial_{|\nu]}\varphi)(\partial_\sigma\varphi) = 0 \quad ,$$

$$\gamma^\sigma\left(\gamma^\mu\widehat{D}_\mu\lambda - \frac{i}{240}\gamma^{[5]}\lambda\widehat{F}_{[5]}\right)\partial_\sigma\varphi = 0 \quad .$$

$$[\delta_Q(\epsilon_1),\delta_Q(\epsilon_2)]A^\alpha_{\mu\nu} = \Big[-\frac{3}{4}V^\alpha_+(\bar\epsilon_2\gamma^{\sigma\rho}\epsilon_1)G^*_{\sigma\mu\nu}\partial_\rho\varphi + \frac{1}{4}V^\alpha_-(\bar\epsilon_2\gamma^{\tau\rho}\epsilon_1)G_{\rho\mu\nu}\partial_\tau\varphi$$

$$+\frac{1}{24}V^\alpha_-(\bar\epsilon_2\gamma_{\mu\nu}^{\rho\sigma\tau\omega}\epsilon_1)G_{\rho\sigma\tau}\partial_\omega\varphi$$

$$-\frac{1}{24}V^\alpha_+(\bar\epsilon_2\gamma_{\mu\nu}^{\rho\sigma\tau\omega}\epsilon_1)G^*_{\rho\sigma\tau}\partial_\omega\varphi\Big] - (1\leftrightarrow 2) + \text{ c.c.}$$

$$= (\bar\epsilon_1\gamma^{\rho\sigma}\epsilon_2)F^\alpha_{\rho\mu\nu}\partial_\sigma\varphi = \xi^\rho F^\alpha_{\rho\mu\nu}$$

$$\left[\frac{1}{4}V^\alpha_-(\bar\epsilon_2\gamma^{\tau\rho}\epsilon_1)G_{\rho\mu\nu}\partial_\tau\varphi - \frac{3}{4}V^\alpha_+(\bar\epsilon_2\gamma^{\sigma\rho}\epsilon_1)G^*_{\sigma\mu\nu}\partial_\rho\varphi\right] - (1\leftrightarrow 2) + \text{ c.c. } = +\xi^\rho F^\alpha_{\rho\mu\nu}$$

$$P_\downarrow\gamma^\mu\widehat{D}_\mu\lambda - ia_1P_\downarrow\gamma^{[5]}\lambda\widehat{F}_{[5]} = 0$$

$$(FG\text{-terms }) = +i\left(\frac{1}{320} - \frac{3}{4}a_1\right)(\partial\tilde\varphi)\gamma^{\rho\sigma\nu_1\cdots\nu_5}\epsilon F_{[\nu_1\cdots\nu_5|}G^\tau_{\rho\sigma}\partial_{|\tau]}\varphi$$

$$-i\left(\frac{1}{16} + 5a_1\right)P_\downarrow\gamma^{\mu\nu}\epsilon F^{[3]}_{\mu\nu}G_{[3]}$$

$$a_1 = +\frac{1}{240}$$

$$(\epsilon\text{-terms}) = -\frac{1}{8}P_{\downarrow}\gamma^{\mu\nu}\epsilon\left[D_\tau G^{\tau}_{\mu\nu} + P_\tau G^{*\tau}_{\mu\nu} + \frac{2i}{3}F^{[3]}_{\mu\nu}G_{[3]}\right]$$

$$= -\frac{1}{4}\gamma^{\sigma}\gamma^{\rho\mu\nu}\epsilon\Big[D_\tau G^{\tau}_{[\mu\nu}(\partial_{\rho]}\varphi) + P^{\tau}G^{*}_{\tau[\mu\nu}(\partial_{\rho]}\varphi) + \frac{2i}{3}F^{[3]}_{[\mu\nu|}G_{[3]}(\partial_{|\rho]}\varphi)\Big]\partial_\sigma\varphi = 0$$

$$(PF\text{-terms}) = i\left(\frac{1}{24} - 10a_1\right)P_{\downarrow}\gamma^{[4]}\epsilon^{*}P^{\mu}F_{[4]\mu}$$

$$a_1 = +\frac{1}{240}$$

$$(\epsilon^{*}\text{-terms}) = P_{\downarrow}\epsilon^{*}\left(D_\mu P^{\mu} - \frac{1}{24}G^{2}_{\rho\sigma\tau}\right)$$

$$\gamma^{\lambda}[\gamma^{\rho}\hat{\mathcal{R}}_{\rho[\mu|} + b_2(\gamma^{\rho}\gamma_{[\mu|}\lambda^{*})\hat{P}_{\rho} + b_3(\gamma_{[\mu|}\gamma^{\rho}\lambda^{*})\hat{P}_{\rho} + b_4(\gamma^{\rho\sigma\tau}\gamma_{[\mu|}\lambda)\hat{G}^{*}_{\rho\sigma\tau} + b_5(\gamma_{[\mu|}\gamma^{\rho\sigma\tau}\lambda)\hat{G}^{*}_{\rho\sigma\tau}](\partial_{|\nu]}\varphi)(\partial_\lambda\varphi) = 0$$

$$\gamma_{[\mu_1\mu_2\mu_3}{}^{\nu}\mathcal{R}_{\mu_4\mu_5}(\partial_{\mu_6]}\varphi)(\partial_\nu\varphi) + \frac{1}{720}\epsilon_{\mu_1\cdots\mu_6}{}^{\nu_1\cdots\nu_6}\gamma_{\nu_1\nu_2\nu_3}{}^{\rho}\mathcal{R}_{\nu_4\nu_5}(\partial_{\nu_6}\varphi)(\partial_\rho\varphi) + \mathcal{O}(\phi^2)$$

$$\begin{aligned}(DG\text{-terms}) + (PG^{*}\text{-terms}) =& +\left(-\frac{1}{96} - b_5\right)\gamma^{\lambda}\gamma^{\rho\sigma\tau\omega}_{[\mu|}\epsilon^{*}P_\rho G^{*}_{\sigma\tau\omega}(\partial_{|\nu]}\varphi)(\partial_\lambda\varphi)\\
&+\left(\frac{1}{32} - \frac{1}{12}b_2 - b_5\right)\gamma^{\lambda}\gamma^{\rho\sigma\tau}\epsilon^{*}P_{[\mu|}G^{*}_{\rho\sigma\tau}(\partial_{|\nu]}\varphi)(\partial_\lambda\varphi)\\
&+\left(-\frac{3}{32} - 9b_5\right)\gamma^{\lambda}\gamma^{\rho\sigma\tau}\epsilon^{*}P_\rho G^{*}_{\sigma\tau[\mu}(\partial_{\nu]}\varphi)(\partial_\lambda\varphi)\\
&+\left(+\frac{3}{16} + 18b_5\right)\gamma^{\lambda}\gamma^{\rho}\epsilon^{*}P^{\rho}G^{*}_{\sigma\rho[\mu}(\partial_{\nu]}\varphi)(\partial_\lambda\varphi)\\
&+\left(+\frac{1}{32} + 3b_5\right)\gamma^{\lambda}\gamma^{\rho\sigma}_{[\mu|}\epsilon^{*}P^{\tau}G^{*}_{\rho\sigma\tau}(\partial_{|\nu]}\varphi)(\partial_\lambda\varphi)\\
&-\frac{i}{8}\gamma^{\lambda}\gamma^{\rho}\epsilon^{*}F_{\rho[\mu|}{}^{[3]}G_{[3]}(\partial_{|\nu]}\varphi)(\partial_\lambda\varphi)\\
&+\frac{i}{48}\gamma^{\lambda}\gamma^{\rho\sigma}_{[\mu|}\epsilon^{*}F_{\rho\sigma}{}^{[3]}G_{[3]}(\partial_{|\nu]}\varphi)(\partial_\lambda\varphi)\\
&+[(b_2 - b_3)\text{-terms}] + [(b_4 - 2b_5)\text{-terms}]\end{aligned}$$

$$b_2 = +\frac{1}{2}, b_3 = +\frac{1}{2}, b_4 = -\frac{1}{48}, b_5 = -\frac{1}{96}$$

$$\begin{aligned}&\gamma^{\lambda}\gamma^{\rho\sigma\tau}\gamma_{[\mu|}P_{\downarrow}\gamma^{\omega}\epsilon^{*}P_\omega G^{*}_{\rho\sigma\tau}(\partial_{|\nu]}\varphi)(\partial_\lambda\varphi)\\
&= -\gamma^{\lambda}\gamma^{\rho\sigma\tau\omega}_{[\mu|}\epsilon^{*}P_\omega G^{*}_{\rho\sigma\tau}(\partial_{|\nu]}\varphi)(\partial_\lambda\varphi) - 3\gamma^{\lambda}\gamma^{\rho\sigma}_{[\mu|}\epsilon^{*}P^{\tau}G^{*}_{\rho\sigma\tau}(\partial_{|\nu]}\varphi)(\partial_\lambda\varphi)\\
&\quad+\gamma^{\lambda}\gamma^{\rho\sigma\tau}\epsilon^{*}P_{[\mu|}G^{*}_{\rho\sigma\tau}(\partial_{|\nu]}\varphi)(\partial_\lambda\varphi)\\
&\quad+3\gamma^{\lambda}\gamma^{\rho\sigma\tau}\epsilon^{*}P_\tau G^{*}_{\rho\sigma[\mu|}(\partial_{|\nu]}\varphi)(\partial_\lambda\varphi) + 6\gamma^{\lambda}\gamma^{\rho}\epsilon^{*}P^{\sigma}G^{*}_{\rho\sigma[\mu|}(\partial_{|\nu]}\varphi)(\partial_\lambda\varphi)\end{aligned}$$

$$\mathcal{F}_{[6]} = -\frac{1}{6!}\epsilon^{[6]'}_{[6]}\mathcal{F}_{[6]'}, \mathcal{F}_{\mu_1\cdots\mu_6} \equiv F_{[\mu_1\cdots\mu_5}\partial_{\mu_6]}\varphi$$

$$(PP^*\text{-terms}) = +\gamma^{\lambda}\gamma^{\rho}\epsilon\left[-\frac{1}{2}P_{\rho}P^*_{[\mu|} + \left(\frac{1}{2} - 2b_2\right)P_{[\mu|}P^*_{\rho}\right](\partial_{|\nu]}\varphi)(\partial_{\lambda}\varphi)$$
$$+(b_2 - b_3)\gamma^{\lambda}\gamma_{[\mu|}\gamma^{\rho}P_{\downarrow}\gamma^{\sigma}\epsilon P_{\rho}P^*_{\sigma}(\partial_{|\nu]}\varphi)(\partial_{\lambda}\varphi)$$

$$b_2 = b_3 = +\frac{1}{2}$$

$$(DF\text{-terms}) = -\frac{1}{192}\gamma^{\lambda}\gamma^{[3][2]}\epsilon\left[G_{[3]}G^*_{[2][\mu|}(\partial_{|\nu]}\varphi) - G_{[2][\mu|}G^*_{[3]}(\partial_{|\nu]}\varphi)\right](\partial_{\lambda}\varphi)$$
$$-\frac{1}{576}\gamma^{\lambda}\gamma_{[\mu|}^{[3][3]'}\epsilon G_{[3]}G^*_{[3]'}(\partial_{|\nu]}\varphi)(\partial_{\lambda}\varphi)$$

$$(F^2\text{-terms}) = -\frac{1}{12}\gamma^{\lambda}\gamma^{\rho}\epsilon F_{[4][\mu|}F_{\rho}^{[4]}{}_{\rho}(\partial_{|\nu]}\varphi)(\partial_{\lambda}\varphi),$$

$$F_{i_1\cdots i_5} = +\frac{1}{5!}\epsilon_{i_1\cdots i_5}^{j_1\cdots j_5}F_{j_1\cdots j_5}$$

$$\gamma^{i[2]}\epsilon F^{[3]}{}_{ij}F_{[2][3]} \equiv 0, \gamma^{[3][4]}\epsilon F_{[3]ij}F^{j}_{[4]} \equiv 0$$

$$(\partial\varphi\gamma^{[7]}\epsilon\text{-terms}) = \frac{1}{768}\left[32(-b_4 + b_5) + \frac{1}{3}\right]\gamma^{\lambda}\gamma_{[\mu|}^{[3][3]'}\epsilon G_{[3]}G^*_{[3]'}(\partial_{|\nu]}\varphi)(\partial_{\lambda}\varphi)$$

$$b_4 - b_5 = -\frac{1}{96}$$

$$(\partial\varphi\gamma^{[5]}\epsilon\text{-terms}) = +\frac{1}{256}[96(b_4 - b_5) + 1]\gamma^{\psi}\gamma_{[\mu|}^{\rho\sigma\lambda\omega}\epsilon G_{\rho\sigma\tau}G^*_{\lambda\omega}(\partial_{|\nu]}\varphi)(\partial_{\psi}\varphi)$$
$$+\frac{1}{3072}[-12 - 384(b_4 + b_5)]\gamma^{\lambda}\gamma^{[3][2]}\epsilon G_{[3]}G^*_{[2][\mu}(\partial_{\nu]}\varphi)(\partial_{\lambda}\varphi)$$
$$+\frac{1}{3072}[-4 - 384(b_4 - b_5)]\gamma^{\omega}\gamma^{\rho\sigma\tau\lambda\omega}\epsilon G_{\lambda\omega[\mu|}G^*_{\rho\sigma\tau}(\partial_{|\nu]}\varphi)(\partial_{\omega}\varphi).$$

$$(\partial\varphi\gamma^{[3]}\epsilon\text{-terms}) = +\frac{1}{512}[4 + 384(b_4 - b_5)]\gamma^{\psi}\gamma_{[\mu|}^{\rho\omega}\epsilon G_{\rho\sigma\tau}G^{*\sigma\tau}_{\omega}(\partial_{|\nu]}\varphi)(\partial_{\psi}\varphi)$$
$$+\frac{1}{512}[-12 - 384(b_4 + b_5)]\gamma^{\psi}\gamma^{\sigma\tau\lambda}\epsilon G_{\sigma\tau\omega}G^{*\omega}_{\lambda}{}_{[\mu}(\partial_{\nu]}\varphi)(\partial_{\psi}\varphi)$$
$$+\frac{1}{512}[+4 + 384(b_4 - b_5)]\gamma^{\omega}\gamma^{\rho\sigma\lambda}\epsilon G_{\lambda\tau[\mu|}G^*_{\rho\sigma}(\partial_{|\nu]}\varphi)(\partial_{\omega}\varphi)$$

$$(\partial\varphi\gamma^{[1]}\epsilon\text{-terms})$$
$$= \frac{1}{16}\gamma^{\lambda}\gamma^{\tau}\epsilon\left[-G_{\rho\sigma\tau}G^*_{\rho\sigma[\mu|} - G_{\rho\sigma[\mu|}G^*_{\rho\sigma\tau} + \frac{1}{6}g_{\tau[\mu|}|G_{\rho\sigma\omega}|^2\right](\partial_{|\nu]}\varphi)(\partial_{\lambda}\varphi)$$

$$\frac{1}{2}\gamma^{\lambda}\gamma^{\rho}\epsilon\Big[R_{[\mu|\rho} - P_{\rho}P^*_{[\mu|} - P_{[\mu|}P^*_{\rho} - \tfrac{1}{6}F_{[4]\rho}F^{[4]}{}_{[\mu|}$$
$$-\tfrac{1}{8}G_{[2]\rho}G^*_{[2][\mu|} - \tfrac{1}{8}G_{[2][\mu|}G^*_{[2]\rho} + \tfrac{1}{48}g_{\rho[\mu|}G_{\sigma\tau\omega}G^{*\sigma\tau\omega}\Big](\partial_{|\nu]}\varphi)(\partial_{\lambda}\varphi) = 0 \quad .$$

$$\overline{\hat{\mathcal{R}}}_{\hat{\mu}\hat{\nu}} = (\overline{\mathcal{R}}_{\hat{\mu}\hat{\nu}}, 0), \hat{\lambda} = \begin{pmatrix} 0 \\ \lambda \end{pmatrix}$$

$$S = S_\sigma + S_A$$
$$S_\sigma \equiv \int d^4\sigma \left(\frac{1}{2}\sqrt{g}g^{ij}\eta_{ab}\Pi_i^a\Pi_j^b - \sqrt{g}\right)$$
$$S_A \equiv \int d^4\sigma \left(-\frac{i}{6}\epsilon^{i_1\cdots i_4}\Pi_{i_1}^{B_1}\cdots\Pi_{i_4}^{B_4}A_{B_4\cdots B_1}\right)$$

$$T_{\alpha\beta}^c = \left(\gamma^{cd}\right)_{\alpha\beta}\nabla_d\varphi + (P_{\uparrow\downarrow})_{\alpha\beta}\nabla^c\varphi, T_{\bar\alpha\bar\beta}^c = \left(\gamma^{cd}\right)_{\alpha\beta}\nabla_d\varphi + (P_{\uparrow\downarrow})_{\alpha\beta}\nabla^c\varphi$$
$$F_{\alpha\beta cde} = -\frac{i}{4}\left(\gamma_{cde}^f\right)_{\alpha\beta}\nabla_f\varphi, F_{\bar\alpha\bar\beta cde} = +\frac{i}{4}\left(\gamma_{cde}^f\right)_{\alpha\beta}\nabla_f\varphi$$
$$\nabla_a\nabla_b\varphi = 0, \nabla_a\nabla_b\tilde\varphi = 0, (\nabla_a\varphi)(\nabla^a\tilde\varphi) = 1,$$
$$(\nabla_a\varphi)^2 = 0, (\nabla_a\tilde\varphi)^2 = 0, \nabla_\alpha\varphi = 0, \nabla_\alpha\tilde\varphi = 0$$

$$\delta E^\alpha = (I+\Gamma)^\alpha{}_\beta\kappa^\beta + \frac{1}{2}\left[(\nabla\varphi)(\nabla\widetilde\varphi)\right]^{\alpha\beta}\eta_\beta \equiv -\left[(I+\Gamma)\kappa\right]^\alpha + (P_\uparrow\eta)^\alpha \quad ,$$
$$\delta\overline{E}^{\overline\alpha} = (I+\Gamma)^\alpha{}_\beta\overline\kappa^{\overline\beta} + \frac{1}{2}\left[(\nabla\varphi)(\nabla\widetilde\varphi)\right]^{\alpha\beta}\overline\eta_{\overline\beta} \equiv -\left[(I+\Gamma)\overline\kappa\right]^{\overline\alpha} + (P_\uparrow\overline\eta)^{\overline\alpha} \quad ,$$
$$\delta E^a = 0 \quad , \qquad \delta\varphi = 0 \quad , \qquad \delta\widetilde\varphi = 0 \quad ,$$

$$\Gamma \equiv \frac{1}{24\sqrt{g}}\epsilon^{ijkl}\Pi_i^a\Pi_j^b\Pi_k^c\Pi_l^d(\gamma_{abcd})$$

$$\Gamma^2 = I$$
$$\epsilon_i^{jkl}\Pi_j^a\Pi_k^b\Pi_l^c\gamma_{abc}\Gamma = +6\sqrt{g}\Pi_i^a\gamma_a$$

$$\delta(S_\sigma + S_A) = [+\sqrt{g}g^{ij}\Pi_i^\gamma\left(\gamma_a\gamma^b\right)_{\gamma\beta}(\nabla_b\varphi)\left(\delta E^\beta\right)\Pi_j^a$$
$$+\frac{1}{6}\epsilon^{ijkl}\Pi_i^\gamma(\gamma_{bcd}\gamma^a)_{\gamma\alpha}(\nabla_a\varphi)(\delta E^\alpha)\Pi_j^b\Pi_k^c\Pi_l^d\Big] + (\delta E^\alpha \to \delta\bar E^{\bar\alpha})$$

$$\Pi_i^a\nabla_a\varphi = 0$$

$$(n_m) = \left(0,0,\cdots,0,+\frac{1}{\sqrt{2}},+\frac{1}{\sqrt{2}}\right), (m_m) = \left(0,0,\cdots,0,+\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$$

$$V_\pm \equiv \frac{1}{\sqrt{2}}\left(V_{(12)} \pm V_{(13)}\right)$$

$$n_\mu \equiv \partial_\mu\varphi\, , m_\mu \equiv \partial_\mu\tilde\varphi\, , D_mD_n\varphi = 0\, , D_mD_n\tilde\varphi = 0$$

$$\delta_Q e_\mu^m = \left(\bar\epsilon\gamma^{m\nu}\psi_\mu\right)\partial_\nu\varphi$$
$$\delta_Q\psi_\mu = D_\mu\epsilon + \frac{1}{144}P_\downarrow\left(\gamma_\mu^{[4]}\epsilon\hat F_{[4]} - 8\gamma^{[3]}\epsilon\hat F_{\mu[3]}\right)$$
$$\delta_Q A_{\mu\nu\rho} = +\frac{3}{2}\left(\bar\epsilon\gamma_{[\mu\nu}^\sigma\psi_{\rho]}\right)\partial_\sigma\varphi$$
$$\delta_Q\varphi = 0, \delta_Q\tilde\varphi = 0$$

$$\hat F_{\mu\nu\rho\sigma}^\tau\partial_\tau\varphi = 0, \quad \hat R_\mu^{\nu mn}\partial_\nu\varphi = 0, \hat R_{\mu\nu}^{mn}D_m\varphi = 0$$
$$\hat{\mathcal{R}}_\mu^\nu\partial_\nu\varphi = 0, \qquad \hat{\hat{\mathcal{R}}}_{\mu\nu}\gamma^m D_m\tilde\varphi = 0$$
$$(D_m\varphi)^2 = 0, \qquad (D_m\tilde\varphi)^2 = 0, (D_m\varphi)(D^m\tilde\varphi) = 1$$

$$\delta_Q \psi_\mu = P_\downarrow\left(a_1 \gamma_\mu{}^{[4]} \epsilon F_{[4]} + a_2 \gamma^{[3]} \epsilon F_{\mu[3]}\right)$$
$$\delta_Q A_{\mu\nu\rho} = a_3\left(\bar{\epsilon}\gamma_{[\mu\nu}{}^{\sigma}\psi_{\rho]}\right)\partial_\sigma \varphi$$

$$\delta_{\mathrm{E}} e_\mu^m = \alpha_\mu (D^m \varphi) + \tilde{\alpha}^m \partial_\mu \varphi$$
$$\delta_{\mathrm{E}} A_{\mu\nu\rho} = \beta_{[\mu\nu}\partial_{\rho]}\varphi$$

$$\begin{aligned}[\delta_1, \delta_2] A_{\mu\nu\rho} = & -12 a_2 a_3 \xi^\sigma F_{\sigma\mu\nu\rho} + \partial_{[\mu}\Lambda_{\nu\rho]} \\ & -2a_3(8a_1 + a_2)\left(\bar{\epsilon}_2 \gamma_{[\mu\nu|}{}^{[3]m}\epsilon_1\right) F_{|\rho][3]} D_m \varphi \\ & + a_1\left[-2\left(\bar{\epsilon}_2 \gamma_{[\mu|} P_\downarrow \gamma_{|\nu|}{}^{[4]}\epsilon_1\right) F_{[4]} - \left(\bar{\epsilon}_2 \gamma_{[\mu\nu|}\gamma^{[4]}\epsilon_1\right) F_{[4]}\right]\partial_{|\rho]}\varphi - (1 \leftrightarrow 2) \\ & + a_2\left[-2\left(\bar{\epsilon}_2 \gamma_{[\mu|} P_\downarrow \gamma^{[3]}\epsilon_1\right) F_{|\nu|[3]} + 6\left(\bar{\epsilon}_2 \gamma^{[2]}\epsilon_1\right) F_{[\mu\nu|[2]}\right]\partial_{|\rho]}\varphi - (1 \leftrightarrow 2)\end{aligned}$$

$$a_2 a_3 = -\frac{1}{12}$$
$$8a_1 + a_2 = 0$$

$$a_1 = +\frac{1}{144}, a_2 = -\frac{1}{18}, a_3 = +\frac{3}{2}$$

$$\left[\hat{R}_{\rho[\mu|} + \frac{1}{3}\hat{F}_{\rho[3]}\hat{F}^{[3]}_{[\mu|} - \frac{1}{36} g_{\rho[\mu|}\hat{F}^2_{[4]}\right]\partial_{|\nu]}\varphi = \mathcal{O}(\psi^2)$$
$$\left(\hat{D}_\mu \hat{F}^\mu_{[\nu\rho\sigma}\right)\partial_{\tau]}\varphi = +\frac{1}{2304}\epsilon_{\nu\rho\sigma\tau}{}^{[4][4]'\mu}\hat{F}_{[4]}\hat{F}_{[4]'}\partial_\mu\varphi$$
$$\gamma^\sigma \gamma^\rho \hat{\mathcal{R}}_{\rho[\mu}\left(\partial_{\nu]}\varphi\right)(\partial_\sigma \varphi) = 0$$

$$\left(D_\mu \hat{F}^\mu_{[\nu\rho\sigma}\right)\partial_{\tau]}\varphi = \alpha e^{-1}\epsilon^{[4][4]'\mu}_{\nu\rho\sigma\tau}\hat{F}_{[4]}\hat{F}_{[4]'}\partial_\mu \varphi$$
$$\left[\hat{R}_{\rho[\mu|} + \delta F_{\rho[3]}F^{[3]}_{[\mu|} + \beta g_{\rho[\mu|}F_{[4]}{}^2\right]\partial_{|\nu]}\varphi = 0$$

$$\begin{aligned}-3a_2(\partial\varphi)\gamma^{[2]}\epsilon\left(D_\rho F^\rho_{[2][\mu}\right)\partial_{\nu]}\varphi = & -24 a_2 \alpha (\partial\varphi)\gamma^{[4][4]'}{}_{[\mu|}\epsilon F_{[4]}F_{[4]'}\partial_{|\nu]}\varphi \\ & -6a_2(\partial\varphi)\gamma^{\rho\sigma}\epsilon\left[\left(D_\tau F^\tau_{[\rho\sigma\mu}\right)\partial_{\nu]}\varphi - \alpha\epsilon_{\rho\sigma\mu\nu}{}^{[4][4]'\tau}F_{[4]}F_{[4]'}\partial_\tau\varphi\right]\end{aligned}$$

$$\begin{aligned}-4a_1(\partial\varphi)\gamma^{[3]}_{[\mu|}\epsilon\left(D_\rho F^\rho_{[3]}\right)\partial_{|\nu]}\varphi = & +768 a_1 \alpha (\partial\varphi)\gamma^{[4][3]}\epsilon F_{[4]}F_{[3][\mu}\partial_{\nu]}\varphi \\ & -8a_1(\partial\varphi)\gamma_\mu{}^{\sigma\tau\omega}\epsilon\left[\left(D_\rho F^\rho_{[\sigma\tau\omega}\right)\partial_{\nu]}\varphi - \alpha\epsilon_{\sigma\tau\omega\nu}{}^{[4][4]'\lambda}F_{[4]}F_{[4]'}\partial_\lambda\varphi\right] - (\mu \leftrightarrow \nu)\end{aligned}$$

$$\begin{aligned}\frac{1}{2}(\partial\varphi)\gamma^\rho\epsilon R_{\rho[\mu}\partial_{\nu]}\varphi = & +\frac{1}{2}(\partial\varphi)\gamma^\rho\epsilon\left(\delta F_{\rho[3]}F^{[3]}_{[\mu|}\partial_{|\nu]}\varphi - \beta g_{\rho[\mu|}F^2_{[4]}\partial_{|\nu]}\varphi\right) \\ & +\frac{1}{2}(\partial\varphi)\gamma^\rho\epsilon\left(R_{\rho[\mu}\partial_{\nu]}\varphi + \delta F_{\rho[3]}F^{[3]}_{[\mu|}\partial_{|\nu]}\varphi + \beta g_{\rho[\mu|}F^2_{[4]}\partial_{|\nu]}\varphi\right)\end{aligned}$$

$$\begin{aligned}&\delta\left[(\partial\varphi)\gamma^\rho\hat{\mathcal{R}}_{\rho[\mu|}\partial_{|\nu]}\varphi\right] = \left(768\alpha a_1 + 32a_1^2 - 6a_1a_2 - 2a_2^2\right)N_{\mu\nu} \\ &+\left(-24a_2\alpha + 4a_1^2 + 2a_1a_2\right)W_{\mu\nu} + \left(\frac{1}{2}\beta + 288a_1^2\right)S_{\mu\nu} + \left(-\frac{1}{2}\delta + 1152a_1^2 + 36a_2^2\right)P_{\mu\nu} \\ &-36(8a_1 + a_2)(2a_1 - a_2)Q_{\mu\nu} - 6(8a_1 + a_2)^2 T_{\mu\nu} - 72a_1(8a_1 + a_2)U_{\mu\nu}\end{aligned}$$

$N_{\mu\nu} \equiv (\partial\varphi)\gamma^{[4][3]}\epsilon F_{[4]}F_{[3][\mu}\partial_{\nu]}\varphi, P_{\mu\nu} \equiv (\partial\varphi)\gamma^{\rho}\epsilon F_{\rho}^{[3]}F_{[3][\mu}\partial_{\nu]}\varphi$
$Q_{\mu\nu} \equiv (\partial\varphi)\gamma^{[2]\rho}\epsilon F_{[2]}{}^{[2]'}F_{[2]'\rho[\mu}\partial_{\nu]}\varphi, S_{\mu\nu} \equiv (\partial\varphi)\gamma_{[\mu|}\epsilon F_{[4]}{}^{2}\partial_{|\nu]}\varphi$
$T_{\mu\nu} \equiv (\partial\varphi)\gamma^{[3][2]}\epsilon F_{[3]}{}^{\rho}F_{\rho[2][\mu}\partial_{\nu]}\varphi, U_{\mu\nu} \equiv (\partial\varphi)\gamma^{[2][2]'}{}_{[\mu|}\epsilon F_{[2][2]''}F_{[2]'}{}^{[2]''}\partial_{|\nu]}\varphi$
$W_{\mu\nu} \equiv (\partial\varphi)\gamma^{[4][4]'}{}_{[\mu|}\epsilon F_{[4]}F_{[4]'}\partial_{|\nu]}\varphi.$

$$\alpha = +\frac{1}{2304}$$

$$\beta = -\frac{1}{36}, \delta = +\frac{1}{3}$$

$$T^{c}_{\alpha\beta} = \left(\gamma^{cd}\right)_{\alpha\beta}\nabla_d\varphi + (P_{\uparrow\downarrow})_{\alpha\beta}\nabla^c\varphi$$

$$F_{\alpha\beta cd} = -\frac{1}{2}(\gamma^{e}_{cd})_{\alpha\beta}\nabla_e\varphi - \frac{1}{2}\left(P_{\downarrow}\gamma_{[c]}\right)_{(\alpha\beta)}\nabla_{|d]}\varphi$$

$$(\nabla_a\varphi)(\nabla^a\varphi) = 0, (\nabla_a\tilde{\varphi})(\nabla^a\tilde{\varphi}) = 0, (\nabla_a\varphi)(\nabla^a\tilde{\varphi}) = 1, \nabla_\alpha\varphi = \nabla_\alpha\tilde{\varphi} = 0$$

$$\tilde{V}_a \equiv V_a - (\nabla_a\varphi)(\nabla^b\tilde{\varphi})V_b.$$

$$\left(\gamma^{ab}\right)_{(\alpha\beta|}(\gamma^{c}_{ad})_{|\gamma\delta)}(\nabla_b\varphi)(\nabla_c\varphi) - 2\left(\gamma^{ab}\right)_{(\alpha\beta|}(P_{\downarrow}\gamma_a)_{|\gamma\delta)}(\nabla_b\varphi)(\nabla_d\varphi) = 0$$

$$\left(\gamma^{i}\right)_{(\alpha\beta|}\left(\gamma_{ij}\right)_{|\gamma\delta)} \equiv 0$$

$$S = S_\sigma + S_A$$

$$S_\sigma \equiv \int d^3\sigma\left(-\frac{1}{2}\sqrt{-g}g^{ij}\eta_{ab}\Pi_i^a\Pi_j^b + \frac{1}{2}\sqrt{-g}\right)$$

$$S_A \equiv \int d^3\sigma\left(+\frac{1}{3}\epsilon^{ijk}\Pi_i^A\Pi_j^B\Pi_k^C A_{CBA}\right)$$

$$\delta E^\alpha = (I + \Gamma)^\alpha{}_\beta\kappa^\beta + (P_\uparrow)^{\alpha\beta}\eta_\beta$$

$$\delta E^a = 0$$

$$\Gamma \equiv \frac{1}{6\sqrt{-g}}\epsilon^{ijk}\Pi_i^a\Pi_j^b\Pi_k^c(\gamma_{abc})$$

$$\Gamma^2 = I$$

$$\epsilon_i{}^{jk}\Pi_j{}^a\Pi_k{}^b\gamma_{ab}\Gamma = +2\sqrt{-g}\Pi_i{}^a\gamma_a$$

$$\Pi_i{}^a\nabla_a\varphi = 0$$

$$\delta_\eta S = \sqrt{-g}\Pi_{ia}\left(\bar{\eta}P_\downarrow\gamma^{ba}\right)_\beta\Pi^{i\beta}\nabla_b\varphi + \frac{1}{4}\epsilon^{ijk}(\bar{\eta}P_\downarrow\gamma_{dcb})_\beta\Pi_i{}^\beta\Pi_j{}^b\Pi_k{}^c\nabla^d\varphi.$$

$$\delta_{(\alpha}{}^\beta\delta_{\gamma)}{}^\delta = \frac{1}{32}\left(\gamma^{ab}\right)_{\alpha\gamma}(\gamma_{ab})^{\beta\delta} + \frac{1}{32(6!)}\left(\gamma^{[6]}\right)_{\alpha\gamma}\left(\gamma_{[6]}\right)^{\beta\delta}$$

$$\delta_{[\alpha}{}^\beta\delta_{\gamma]}{}^\delta = +\frac{1}{16}C_{\alpha\gamma}C^{\beta\delta} + \frac{1}{16(4!)}\left(\gamma^{[4]}\right)_{\alpha\gamma}\left(\gamma_{[4]}\right)^{\beta\delta}$$

$$(\gamma^a)_{(\alpha|\dot\beta}(\gamma_a)_{|\gamma)}{}^{\dot\delta} = \frac{1}{4}(\gamma^{bc})_{\alpha\gamma}(\gamma_{bc})_{\dot\beta}{}^{\dot\delta} \quad ,$$

$$(\gamma^{ab})_{(\alpha\beta|}(\gamma_a)_{|\gamma)}{}^{\dot\delta} = +\frac{1}{12}(\gamma^{cd})_{(\alpha\beta|}(\gamma_{cd}\gamma^b)_{|\gamma)}{}^{\dot\delta} = \frac{1}{8}(\gamma^{cd})_{(\alpha\beta|}(\gamma^b\gamma_{cd})_{|\gamma)}{}^{\dot\delta} \quad ,$$

$$(\gamma^{ab})_{(\alpha\beta|}(\gamma_{ac})_{|\gamma)}{}^{\delta} = \frac{1}{10}(\gamma^{de})_{(\alpha\beta|}(\gamma_{de}{}^b{}_c)_{|\gamma)}{}^{\delta} + (\gamma^b{}_c)_{(\alpha\beta|}\delta_{|\gamma)}{}^{\delta}$$
$$+\frac{1}{10}\delta_c{}^b(\gamma^{de})_{(\alpha\beta|}(\gamma_{de})_{|\gamma)}{}^{\delta} - \frac{1}{5}(\gamma^d{}_c)_{(\alpha\beta|}(\gamma_d{}^b)_{|\gamma)}{}^{\delta} \quad ,$$

$$(\gamma^{ab})_{\dot\alpha\dot\beta}(\gamma_a)_{\gamma\dot\delta} = -(\gamma_a)_{\gamma(\dot\alpha|}(\gamma_a{}^b)_{|\dot\beta)\dot\delta} - \frac{1}{20}(\gamma^{cdb})_{\gamma(\dot\alpha|}(\gamma_{cd})_{|\dot\beta\dot\delta)} \quad ,$$

$$(\gamma_{ab})_{(\alpha\beta|}(\gamma^{abcdef})_{|\gamma)}{}^{\delta} = -(\gamma^{abcdef})_{(\alpha\beta|}(\gamma_{ab})_{|\gamma)}{}^{\delta} - 2(\gamma^{[cd})_{(\alpha\beta}(\gamma^{ef]})_{|\gamma)}{}^{\delta} \quad ,$$

$$(\gamma_{ab})_{(\alpha\beta|}(\gamma^{abcdef})_{|\gamma\delta)} = -(\gamma^{[cd})_{(\alpha\beta|}(\gamma^{ef]})_{|\gamma\delta)} \quad ,$$

$$(\gamma^{[5]b})_{(\alpha\beta|}(\gamma_{[5]})_{|\gamma)}{}^{\dot\delta} = -720(\gamma^{ab})_{(\alpha\beta|}(\gamma_a)_{|\gamma)}{}^{\dot\delta} \quad ,$$

$$(\gamma^{[5]})_{(\alpha|}{}^{\dot\beta}(\gamma_{[5]})_{|\gamma)}{}^{\dot\delta} = -180(\gamma^{ab})_{\alpha\gamma}(\gamma_{ab})^{\dot\beta\dot\delta} \quad ,$$

$$(\gamma^{[5](a|})_{(\alpha\beta|}(\gamma_{[5]}{}^{|b)})_{|\gamma)}{}^{\delta} = -120\eta^{ab}(\gamma^{cd})_{(\alpha\beta|}(\gamma_{cd})_{|\gamma)}{}^{\delta} \quad ,$$

$$(\gamma^{[4]ab})_{(\alpha\beta|}(\gamma_{[4]})_{|\gamma)}{}^{\delta} = +12(\gamma^{cd})_{(\alpha\beta|}(\gamma_{cd}{}^{ab})_{|\gamma)}{}^{\delta} + 360(\gamma^{ab})_{(\alpha\beta}\delta_{\gamma)}{}^{\delta} \quad ,$$

$$(\gamma^{cd})_{(\alpha\beta|}(\gamma_{cd}{}^{ab})_{|\gamma)}{}^{\delta} = +4(\gamma^{[a|c})_{(\alpha\beta|}(\gamma^{|b]}{}_c)_{|\gamma)}{}^{\delta} - 10(\gamma^{ab})_{(\alpha\beta}\delta_{\gamma)}{}^{\delta} \quad ,$$

$$(\gamma^a)_\alpha{}^{\dot\beta}(\gamma_a)_\gamma{}^{\dot\delta} = +\frac{3}{8}C_{\alpha\gamma}C^{\dot\beta\dot\delta} + \frac{1}{8}(\gamma^{[2]})_{\alpha\gamma}(\gamma_{[2]})^{\dot\beta\dot\delta} + \frac{1}{192}(\gamma^{[4]})_{\alpha\gamma}(\gamma_{[4]})^{\dot\beta\dot\delta} \quad ,$$

$$(\gamma^{[3]})_\alpha{}^{\dot\beta}(\gamma_{[3]})_\gamma{}^{\dot\delta} = +\frac{165}{4}C_{\alpha\gamma}C^{\dot\beta\dot\delta} + \frac{15}{4}(\gamma^{[2]})_{\alpha\gamma}(\gamma_{[2]})^{\dot\beta\dot\delta} - \frac{3}{32}(\gamma^{[4]})_{\alpha\gamma}(\gamma_{[4]})^{\dot\beta\dot\delta} \quad ,$$

$$(\gamma^{[5]})_\alpha{}^{\dot\beta}(\gamma_{[5]})_\gamma{}^{\dot\delta} = +2970C_{\alpha\gamma}C^{\dot\beta\dot\delta} - 90(\gamma^{[2]})_{\alpha\gamma}(\gamma_{[2]})^{\dot\beta\dot\delta} + \frac{5}{4}(\gamma^{[4]})_{\alpha\gamma}(\gamma_{[4]})^{\dot\beta\dot\delta} \quad .$$

$$(\gamma^c\not{n})_{\alpha\beta}(\gamma_c\not{n})_\gamma{}^\delta = -\frac{1}{2}(\gamma^{ca})_{\alpha\gamma}(\gamma_c{}^b)_\beta{}^\delta n_a n_b - \frac{1}{24}(\gamma^{[3]a})_{\alpha\gamma}(\gamma_{[3]}{}^b)_\beta{}^\delta n_a n_b \quad ,$$

$$(\gamma^{[3]}\not{n})_{\alpha\beta}(\gamma_{[3]}\not{n})_\gamma{}^\delta = -15(\gamma^{ca})_{\alpha\gamma}(\gamma_c{}^b)_\beta{}^\delta n_a n_b + \frac{3}{4}(\gamma^{[3]a})_{\alpha\gamma}(\gamma_{[3]}{}^b)_\beta{}^\delta n_a n_b \quad ,$$

$$(\gamma^{[5]}\not{n})_{\alpha\beta}(\gamma_{[5]}\not{n})_\gamma{}^\delta = +360(\gamma^{ca})_{\alpha\gamma}(\gamma_c{}^b)_\beta{}^\delta n_a n_b - 10(\gamma^{[3]a})_{\alpha\gamma}(\gamma_{[3]}{}^b)_\beta{}^\delta n_a n_b \quad .$$

$$\gamma_{[n]} = \frac{(-1)^{n(n-1)/2}}{(12-n)!}\epsilon_{[n]}^{[12-n]}\gamma_{[12-n]}\gamma_{13} \ (0 \le n \le 12)$$

$$S^{[6]}A_{[6]} \equiv 0$$

$\bar\psi = \psi^\dagger A$ (for Dirac conjugate)
$\psi = C\bar\psi^T$ (for Majorana-Weyl condition)
$\gamma_\mu^\dagger = -A\gamma_\mu A^{-1}, A \equiv \gamma_{(1)}\gamma_{(12)}, A^\dagger = -A$
$A^T = -CAC^{-1}, \gamma^\mu = -B^{-1}\gamma^{\mu *}B$ ($\eta = -1$ for Majorana spinors)
$B \equiv (A^T)^{-1}C^T = -CA, C^T = -C$ ($\epsilon = +1, \eta = -1$)
$\gamma_\mu^T = +C\gamma_\mu C^{-1}, C^\dagger C = +I, \psi^* = B\psi$

$$\gamma_{\mu}\gamma^{[n]}\gamma^{\mu} = (-1)^{n}(12-2n)\gamma^{[n]}, \gamma_{\mu}\gamma^{[6]}\gamma^{\mu} = 0,$$

$$\gamma^{\mu\nu}\gamma_{\rho\sigma}\gamma_{\mu\nu} = -52\gamma_{\rho\sigma}, \gamma^{\mu\nu}\gamma_{\rho}\gamma_{\mu\nu} = -88\gamma_{\rho},$$

$$\gamma^{\nu}\gamma_{\mu}\gamma_{\nu\rho} = -9\gamma_{\mu\rho} - 11g_{\mu\rho}, \gamma_{\nu\rho}\gamma_{\mu}\gamma^{\nu} = -9\gamma_{\mu\rho} + 11g_{\mu\rho},$$

$$\gamma_{\mu\nu}\gamma_{\sigma\tau}\gamma^{\mu\nu\rho} = -38\gamma_{\sigma\tau}{}^{\rho} + 140\delta_{[\sigma}{}^{\rho}\gamma_{\tau]},$$

$$\gamma_{\mu}{}^{\nu}\gamma^{\sigma\tau}\gamma_{\nu\rho} = +6\gamma_{\mu\rho}{}^{\sigma\tau} + 32\gamma_{(\rho}{}^{[\sigma}\gamma^{\tau]}{}_{\mu)} + 7g_{\mu\rho}\gamma^{\sigma\tau} + 20\delta_{\mu}{}^{[\sigma}\delta_{\rho}{}^{\tau]},$$

$$\gamma^{\rho}\gamma^{\mu\nu}\gamma_{\rho\sigma} = +7\gamma_{\sigma}{}^{\mu\nu} - 18\delta_{\sigma}{}^{[\mu}\gamma^{\nu]}, \gamma^{\rho\sigma}\gamma_{\mu\nu}\gamma_{\rho} = -7\gamma_{\mu\nu}{}^{\sigma} - 18\delta_{[\mu}{}^{\sigma}\gamma_{\nu]},$$

$$\gamma^{\rho\sigma\tau}\gamma_{\mu\nu}\gamma_{\rho} = +6\gamma_{\mu\nu}{}^{\sigma\tau} + 32\delta_{[\mu}{}^{[\sigma}\gamma_{\nu]}{}^{\tau]} - 20\delta_{[\mu}{}^{\sigma}\delta_{\nu]}{}^{\tau},$$

$$\gamma^{[2]\sigma\tau\omega}\gamma_{\mu}\gamma_{[2]} = +40\gamma_{\mu}{}^{\sigma\tau\omega} - 216\delta_{\mu}{}^{[\sigma}\gamma^{\tau\omega]},$$

$$\gamma^{[2]\tau\lambda\omega}\gamma_{\mu\nu\rho}\gamma_{[2]} = -144\delta_{[\mu}^{[\tau}\gamma_{\nu\rho]}{}^{\lambda\omega]} - 720\delta_{[\mu}{}^{[\tau}\delta_{\nu}{}^{\lambda}\gamma_{\rho]}{}^{\omega]} + 432\delta_{[\mu}{}^{[\tau}\delta_{\nu}{}^{\lambda}\delta_{\rho]}{}^{\omega]},$$

$$\gamma^{[2]\mu\nu\rho}\gamma_{\sigma\tau}\gamma_{[2]} = -16\gamma_{\sigma\tau}{}^{\mu\nu\rho} - 240\delta_{[\sigma}{}^{[\mu}\gamma_{\tau]}{}^{\nu\rho]} + 432\delta_{[\sigma}{}^{[\mu}\delta_{\tau]}{}^{\nu}\gamma^{\rho]},$$

$$\gamma^{\mu\nu\rho\sigma\tau}\gamma_{\lambda\omega}\gamma_{\tau} = +4\gamma_{\lambda\omega}{}^{\mu\nu\rho\sigma} + 48\delta_{[\lambda}{}^{[\mu}\gamma_{\omega]}{}^{\nu\rho\sigma]} - 96\delta_{[\lambda}{}^{[\mu}\delta_{\omega]}{}^{\nu}\gamma^{\rho\sigma]},$$

$$\gamma^{\mu\nu\rho}\gamma_{\lambda\omega}\gamma_{\mu}{}^{\sigma} = +5\gamma_{\lambda\omega}{}^{\nu\rho\sigma} + 12g^{\sigma[\rho}\gamma_{\lambda\omega}{}^{\nu]} - 14\delta_{[\lambda}{}^{\sigma}\gamma_{\omega]}{}^{\nu\rho} + 28\delta_{[\lambda}{}^{[\nu}\gamma_{\omega]}{}^{\rho]\sigma}$$

$$-18\delta_{[\lambda}{}^{[\nu}\delta_{\omega]}{}^{\rho]}\gamma^{\sigma} + 32\delta_{[\lambda}{}^{[\nu}g^{\rho]\sigma}\gamma_{\omega]} - 36\delta_{[\lambda}{}^{[\nu|}\delta_{\omega]}{}^{\sigma}\gamma^{|\rho]},$$

$$\gamma^{\nu\rho\sigma\tau}\gamma_{\lambda\omega}\gamma_{\sigma\tau} = -26\gamma_{\lambda\omega}{}^{\nu\rho} - 216\delta_{[\lambda}{}^{[\nu}\gamma_{\omega]}{}^{\rho]} + 180\delta_{[\lambda}{}^{\nu}\delta_{\omega]}{}^{\rho},$$

$$\gamma^{\rho}\gamma_{\sigma\tau}\gamma_{\rho}{}^{\lambda\mu\nu} = +5\gamma_{\sigma\tau}{}^{\lambda\mu\nu} - 42\delta_{[\sigma}{}^{[\lambda}\gamma_{\tau]}{}^{\mu\nu]} - 54\delta_{\sigma}{}^{[\lambda}\delta_{\tau}{}^{\mu}\gamma^{\nu]},$$

$$\gamma^{[\mu\nu}\gamma_{\lambda\omega}\gamma^{\rho\sigma]} = \gamma_{\lambda\omega}{}^{\mu\nu\rho\sigma} + 4\delta_{[\lambda}{}^{[\mu}\delta_{\omega]}{}^{\nu}\gamma^{\rho\sigma]},$$

$$\gamma_{[\mu}\gamma^{\rho\sigma}\gamma_{\nu]} = +\gamma_{\mu\nu}{}^{\rho\sigma} + 2\delta_{[\mu}{}^{\rho}\delta_{\nu]}{}^{\sigma},$$

$$\gamma_{[\mu}\gamma^{\rho\sigma\tau\omega}\gamma_{\nu]} = \gamma_{\mu\nu}{}^{\rho\sigma\tau\omega} + 12\delta_{[\mu}{}^{[\rho}\delta_{\nu]}{}^{\sigma}\gamma^{\tau\omega]},$$

$$\gamma^{[2]}\gamma_{\lambda\omega}\gamma_{[2]}{}^{\mu\nu\rho\sigma} = -8\gamma_{\lambda\omega}{}^{\mu\nu\rho\sigma} + 224\delta_{[\lambda}{}^{[\mu}\gamma_{\omega]}{}^{\nu\rho\sigma]} + 672\delta_{[\lambda}{}^{[\mu}\delta_{\omega]}{}^{\nu}\gamma^{\rho\sigma]},$$

$$\gamma^{\rho\sigma}\gamma_{\mu\nu}\gamma_{\rho\sigma}{}^{\tau\lambda} = -26\gamma_{\mu\nu}{}^{\tau\lambda} + 216\delta_{[\mu}{}^{[\tau}\gamma_{\nu]}{}^{\lambda]} + 180\delta_{[\mu}{}^{\tau}\delta_{\nu]}{}^{\lambda},$$

$$\gamma^{[3]\mu\nu\rho}\gamma_{\sigma\tau}\gamma_{[3]} = +1008\delta_{[\sigma}{}^{[\mu}\gamma_{\tau]}{}^{\nu\rho]} - 3024\delta_{[\sigma}{}^{[\mu}\delta_{\tau]}{}^{\nu}\gamma^{\rho]},$$

$$\gamma^{[2]\mu\nu\rho\sigma}\gamma_{\lambda\omega}\gamma_{[2]} = -8\gamma_{\lambda\omega}{}^{\mu\nu\rho\sigma} - 224\delta_{[\lambda}{}^{[\mu}\gamma_{\omega]}{}^{\nu\rho\sigma]} + 672\delta_{[\lambda}{}^{[\mu}\delta_{\omega]}{}^{\nu}\gamma^{\rho\sigma]},$$

$$\gamma^{[3][\mu\nu\rho}\gamma_{\lambda\omega}\gamma_{[3]}{}^{\sigma]} = -24\gamma_{\lambda\omega}{}^{\mu\nu\rho\sigma} + 336\delta_{[\lambda}{}^{[\mu}\gamma_{\omega]}{}^{\nu\rho\sigma]},$$

$$\gamma_{[3]}\gamma_{[2]}\gamma^{[3]} = -240\gamma_{[2]}, \gamma_{[4]}\gamma_{[2]}\gamma^{[4]} = +360\gamma_{[3]},$$

$$\gamma_{[2]}\gamma_{[3]}\gamma^{[2]} = -24\gamma_{[3]}, \gamma_{[3]}\gamma_{[3]}\gamma^{[3]'} = +12\gamma_{[3]},$$

$$\gamma_{[4]}\gamma_{[3]}\gamma^{[4]} = -648\gamma_{[3]}, \gamma_{[5]}\gamma_{[3]}\gamma^{[5]} = +4320\gamma_{[3]}, \gamma_{[6]}\gamma_{[3]}\gamma^{[6]} = 0,$$

$$\gamma_{[2]}\gamma_{[4]}\gamma^{[2]} = -4\gamma_{[4]}, \gamma_{[3]}\gamma_{[4]}\gamma^{[3]} = +72\gamma_{[4]}, \gamma_{[4]'}\gamma_{[4]}\gamma^{[4]'} = -408\gamma_{[4]},$$

$$\gamma_{[5]}\gamma_{[4]}\gamma^{[5]} = +960\gamma_{[4]}, \gamma_{[6]}\gamma_{[4]}\gamma^{[6]} = -20160\gamma_{[4]}$$

$$\gamma_{[2]}\gamma_{[5]}\gamma^{[2]} = +8\gamma_{[5]}, \gamma_{[3]}\gamma_{[5]}\gamma^{[3]} = -60\gamma_{[5]}, \gamma_{[4]}\gamma_{[5]}\gamma^{[4]} = +120\gamma_{[5]}$$

$$\gamma_{[5]}\gamma_{[5]}\gamma^{[5]'} = -2400\gamma_{[5]}, \gamma_{[6]}\gamma_{[5]}\gamma^{[6]} = 0,$$

$$\gamma_{[2]}\gamma_{[6]}\gamma^{[2]} = +12\gamma_{[6]}, \gamma_{[3]}\gamma_{[6]}\gamma^{[3]} = 0, \gamma_{[4]}\gamma_{[6]}\gamma^{[4]} = +360\gamma_{[6]},$$

$$\gamma_{[5]}\gamma_{[6]}\gamma^{[5]} = 0, \gamma_{[6]}\gamma_{[6]}\gamma^{[6]'} = +14400\gamma_{[6]}.$$

$$\gamma_\mu \gamma^{[n]} \gamma^\mu = (-1)^n (12-2n) \gamma^{[n]} \quad , \qquad \gamma_\mu \gamma^{[6]} \gamma^\mu = 0 \quad ,$$

$$\gamma^{\mu\nu} \gamma_{\rho\sigma} \gamma_{\mu\nu} = -52 \gamma_{\rho\sigma} \quad , \qquad \gamma^{\mu\nu} \gamma_\rho \gamma_{\mu\nu} = -88 \gamma_\rho \quad ,$$

$$\gamma^\nu \gamma_\mu \gamma_{\nu\rho} = -9 \gamma_{\mu\rho} - 11 g_{\mu\rho} \quad , \qquad \gamma_{\nu\rho} \gamma_\mu \gamma^\nu = -9 \gamma_{\mu\rho} + 11 g_{\mu\rho} \quad ,$$

$$\gamma_{\mu\nu} \gamma_{\sigma\tau} \gamma^{\mu\nu\rho} = -38 \gamma_{\sigma\tau}{}^\rho + 140 \delta_{[\sigma}{}^\rho \gamma_{\tau]} \quad ,$$

$$\gamma_\mu{}^\nu \gamma^{\sigma\tau} \gamma_{\nu\rho} = +6 \gamma_{\mu\rho}{}^{\sigma\tau} + 32 \gamma_{(\rho}{}^{[\sigma} \gamma^{\tau]}{}_{\mu)} + 7 g_{\mu\rho} \gamma^{\sigma\tau} + 20 \delta_\mu{}^{[\sigma} \delta_\rho{}^{\tau]} \quad ,$$

$$\gamma^\rho \gamma^{\mu\nu} \gamma_{\rho\sigma} = +7 \gamma_\sigma{}^{\mu\nu} - 18 \delta_\sigma{}^{[\mu} \gamma^{\nu]} \quad , \qquad \gamma^{\rho\sigma} \gamma_{\mu\nu} \gamma_\rho = -7 \gamma_{\mu\nu}{}^\sigma - 18 \delta_{[\mu}{}^\sigma \gamma_{\nu]} \quad ,$$

$$\gamma^{\rho\sigma\tau} \gamma_{\mu\nu} \gamma_\rho = +6 \gamma_{\mu\nu}{}^{\sigma\tau} + 32 \delta_{[\mu}{}^{[\sigma} \gamma_{\nu]}{}^{\tau]} - 20 \delta_{[\mu}{}^\sigma \delta_{\nu]}{}^\tau \quad ,$$

$$\gamma^{[2]\sigma\tau\omega} \gamma_\mu \gamma_{[2]} = +40 \gamma_\mu{}^{\sigma\tau\omega} - 216 \delta_\mu{}^{[\sigma} \gamma^{\tau\omega]} \quad ,$$

$$\gamma^{[2]\tau\lambda\omega} \gamma_{\mu\nu\rho} \gamma_{[2]} = -144 \delta_{[\mu}{}^{[\tau} \gamma_{\nu\rho]}{}^{\lambda\omega]} - 720 \delta_{[\mu}{}^{[\tau} \delta_\nu{}^\lambda \gamma_{\rho]}{}^{\omega]} + 432 \delta_{[\mu}{}^{[\tau} \delta_\nu{}^\lambda \delta_{\rho]}{}^{\omega]} \quad ,$$

$$\gamma^{[2]\mu\nu\rho} \gamma_{\sigma\tau} \gamma_{[2]} = -16 \gamma_{\sigma\tau}{}^{\mu\nu\rho} - 240 \delta_{[\sigma}{}^{[\mu} \gamma_{\tau]}{}^{\nu\rho]} + 432 \delta_{[\sigma}{}^{[\mu} \delta_{\tau]}{}^\nu \gamma^{\rho]} \quad ,$$

$$\gamma^{\mu\nu\rho\sigma\tau} \gamma_{\lambda\omega} \gamma_\tau = +4 \gamma_{\lambda\omega}{}^{\mu\nu\rho\sigma} + 48 \delta_{[\lambda}{}^{[\mu} \gamma_{\omega]}{}^{\nu\rho\sigma]} - 96 \delta_{[\lambda}{}^{[\mu} \delta_{\omega]}{}^\nu \gamma^{\rho\sigma]} \quad ,$$

$$\gamma^{\mu\nu\rho} \gamma_{\lambda\omega} \gamma_\mu{}^\sigma = +5 \gamma_{\lambda\omega}{}^{\nu\rho\sigma} + 12 g^{\sigma[\rho} \gamma_{\lambda\omega}{}^{\nu]} - 14 \delta_{[\lambda}{}^\sigma \gamma_{\omega]}{}^{\nu\rho} + 28 \delta_{[\lambda}{}^{[\nu} \gamma_{\omega]}{}^{\rho]\sigma}$$

$$- 18 \delta_{[\lambda}{}^{[\nu} \delta_{\omega]}{}^{\rho]} \gamma^\sigma + 32 \delta_{[\lambda}{}^{[\nu} g^{\rho]\sigma} \gamma_{\omega]} - 36 \delta_{[\lambda}{}^{[\nu|} \delta_{\omega]}{}^\sigma \gamma^{|\rho]} \quad ,$$

$$\gamma^{\nu\rho\sigma\tau} \gamma_{\lambda\omega} \gamma_{\sigma\tau} = -26 \gamma_{\lambda\omega}{}^{\nu\rho} - 216 \delta_{[\lambda}{}^{[\nu} \gamma_{\omega]}{}^{\rho]} + 180 \delta_{[\lambda}{}^\nu \delta_{\omega]}{}^\rho \quad ,$$

$$\gamma^\rho \gamma_{\sigma\tau} \gamma_\rho{}^{\lambda\mu\nu} = +5 \gamma_{\sigma\tau}{}^{\lambda\mu\nu} - 42 \delta_{[\sigma}{}^{[\lambda} \gamma_{\tau]}{}^{\mu\nu]} - 54 \delta_\sigma{}^{[\lambda} \delta_\tau{}^\mu \gamma^{\nu]} \quad ,$$

$$\gamma^{[\mu\nu} \gamma_{\lambda\omega} \gamma^{\rho\sigma]} = \gamma_{\lambda\omega}{}^{\mu\nu\rho\sigma} + 4 \delta_{[\lambda}{}^{[\mu} \delta_{\omega]}{}^\nu \gamma^{\rho\sigma]} \quad ,$$

$$\gamma_{[\mu} \gamma^{\rho\sigma} \gamma_{\nu]} = + \gamma_{\mu\nu}{}^{\rho\sigma} + 2 \delta_{[\mu}{}^\rho \delta_{\nu]}{}^\sigma \quad ,$$

$$\gamma_{[\mu} \gamma^{\rho\sigma\tau\omega} \gamma_{\nu]} = \gamma_{\mu\nu}{}^{\rho\sigma\tau\omega} + 12 \delta_{[\mu}{}^{[\rho} \delta_{\nu]}{}^\sigma \gamma^{\tau\omega]} \quad ,$$

$$\gamma^{[2]} \gamma_{\lambda\omega} \gamma_{[2]}{}^{\mu\nu\rho\sigma} = -8 \gamma_{\lambda\omega}{}^{\mu\nu\rho\sigma} + 224 \delta_{[\lambda}{}^{[\mu} \gamma_{\omega]}{}^{\nu\rho\sigma]} + 672 \delta_{[\lambda}{}^{[\mu} \delta_{\omega]}{}^\nu \gamma^{\rho\sigma]} \quad ,$$

$$\gamma^{\rho\sigma} \gamma_{\mu\nu} \gamma_{\rho\sigma}{}^{\tau\lambda} = -26 \gamma_{\mu\nu}{}^{\tau\lambda} + 216 \delta_{[\mu}{}^{[\tau} \gamma_{\nu]}{}^{\lambda]} + 180 \delta_{[\mu}{}^\tau \delta_{\nu]}{}^\lambda \quad ,$$

$$\gamma^{[3]\mu\nu\rho} \gamma_{\sigma\tau} \gamma_{[3]} = +1008 \delta_{[\sigma}{}^{[\mu} \gamma_{\tau]}{}^{\nu\rho]} - 3024 \delta_{[\sigma}{}^{[\mu} \delta_{\tau]}{}^\nu \gamma^{\rho]} \quad ,$$

$$\gamma^{[2]\mu\nu\rho\sigma} \gamma_{\lambda\omega} \gamma_{[2]} = -8 \gamma_{\lambda\omega}{}^{\mu\nu\rho\sigma} - 224 \delta_{[\lambda}{}^{[\mu} \gamma_{\omega]}{}^{\nu\rho\sigma]} + 672 \delta_{[\lambda}{}^{[\mu} \delta_{\omega]}{}^\nu \gamma^{\rho\sigma]} \quad ,$$

$$\gamma^{[3][\mu\nu\rho} \gamma_{\lambda\omega} \gamma_{[3]}{}^{\sigma]} = -24 \gamma_{\lambda\omega}{}^{\mu\nu\rho\sigma} + 336 \delta_{[\lambda}{}^{[\mu} \gamma_{\omega]}{}^{\nu\rho\sigma]} \quad ,$$

$$\gamma_{[3]} \gamma_{[2]} \gamma^{[3]} = -240 \gamma_{[2]} \quad , \qquad \gamma_{[4]} \gamma_{[2]} \gamma^{[4]} = +360 \gamma_{[3]} \quad ,$$

$$\gamma_{[2]} \gamma_{[3]} \gamma^{[2]} = -24 \gamma_{[3]} \quad , \qquad \gamma_{[3]'} \gamma_{[3]} \gamma^{[3]'} = +12 \gamma_{[3]} \quad ,$$

$$\gamma_{[4]} \gamma_{[3]} \gamma^{[4]} = -648 \gamma_{[3]} \quad , \qquad \gamma_{[5]} \gamma_{[3]} \gamma^{[5]} = +4320 \gamma_{[3]} \quad , \qquad \gamma_{[6]} \gamma_{[3]} \gamma^{[6]} = 0 \quad ,$$

$$\gamma_{[2]} \gamma_{[4]} \gamma^{[2]} = -4 \gamma_{[4]} \quad , \qquad \gamma_{[3]} \gamma_{[4]} \gamma^{[3]} = +72 \gamma_{[4]} \quad , \qquad \gamma_{[4]'} \gamma_{[4]} \gamma^{[4]'} = -408 \gamma_{[4]} \quad ,$$

$$\gamma_{[5]}\gamma_{[4]}\gamma^{[5]} = +960\gamma_{[4]} \quad , \quad \gamma_{[6]}\gamma_{[4]}\gamma^{[6]} = -20160\gamma_{[4]} \quad ,$$

$$\gamma_{[2]}\gamma_{[5]}\gamma^{[2]} = +8\gamma_{[5]} \quad , \quad \gamma_{[3]}\gamma_{[5]}\gamma^{[3]} = -60\gamma_{[5]} \quad , \quad \gamma_{[4]}\gamma_{[5]}\gamma^{[4]} = +120\gamma_{[5]} \quad ,$$

$$\gamma_{[5]'}\gamma_{[5]}\gamma^{[5]'} = -2400\gamma_{[5]} \quad , \quad \gamma_{[6]}\gamma_{[5]}\gamma^{[6]} = 0 \quad ,$$

$$\gamma_{[2]}\gamma_{[6]}\gamma^{[2]} = +12\gamma_{[6]} \quad , \quad \gamma_{[3]}\gamma_{[6]}\gamma^{[3]} = 0 \quad , \quad \gamma_{[4]}\gamma_{[6]}\gamma^{[4]} = +360\gamma_{[6]} \quad ,$$

$$\gamma_{[5]}\gamma_{[6]}\gamma^{[5]} = 0 \quad , \quad \gamma_{[6]'}\gamma_{[6]}\gamma^{[6]'} = +14400\gamma_{[6]} \quad .$$

$$\gamma_{[n]} = \frac{(-1)^{(n+1)(n+2)/2}}{(13-n)!}\epsilon_{[n]}^{[13-n]}\gamma_{[13-n]} \; (0 \le n \le 13)$$

$$\delta_{(\alpha}{}^{\beta}\delta_{\gamma)}{}^{\delta} = +\frac{1}{64}\left(\gamma^{[2]}\right)_{\alpha\gamma}\left(\gamma_{[2]}\right)^{\beta\delta} + \frac{1}{192}\left(\gamma^{[3]}\right)_{\alpha\gamma}\left(\gamma_{[3]}\right)^{\beta\delta} + \frac{1}{32(6!)}\left(\gamma^{[6]}\right)_{\alpha\gamma}\left(\gamma_{[6]}\right)^{\beta\delta}$$

$$(\gamma^{a})_{(\alpha|}^{\beta}(\gamma_{a})_{|\gamma)}^{\delta} = \frac{9}{64}\left(\gamma^{[2]}\right)_{\alpha\gamma}\left(\gamma_{[2]}\right)^{\beta\delta} - \frac{7}{192}\left(\gamma^{[3]}\right)_{\alpha\gamma}\left(\gamma_{[3]}\right)^{\beta\delta} + \frac{1}{32(6!)}\left(\gamma^{[6]}\right)_{\alpha\gamma}\left(\gamma_{[6]}\right)^{\beta\delta}$$

$$= \delta_{(\alpha}{}^{\beta}\delta_{\gamma)}{}^{\delta} + \frac{1}{8}\left(\gamma^{[2]}\right)_{\alpha\gamma}\left(\gamma_{[2]}\right)^{\beta\delta} - \frac{1}{24}\left(\gamma^{[3]}\right)_{\alpha\gamma}\left(\gamma_{[3]}\right)^{\beta\delta}$$

$$\left(\gamma_{[2]}\right)_{(\alpha\beta|}\left(\gamma^{[2]}\right)_{|\gamma\delta)} = +\frac{1}{3}\left(\gamma_{[3]}\right)_{(\alpha\beta|}\left(\gamma^{[3]}\right)_{|\gamma\delta)} = -\frac{1}{6!}\left(\gamma_{[6]}\right)_{(\alpha\beta|}\left(\gamma^{[6]}\right)_{|\gamma\delta)}$$

$$\gamma^{a}\gamma^{[n]}\gamma_{a} = (-1)^{n}(13-2n)\gamma^{[n]}$$

$$\gamma^{ab}\gamma^{cd}\gamma_{ab} = -68\gamma^{cd}, \gamma_{abc}\gamma^{de}\gamma^{abc} = -396\gamma^{de}$$

$$\gamma_{ad}\gamma^{bc}\gamma^{d} = +8\gamma_{a}^{bc} + 10\delta_{a}^{[b}\gamma^{c]}, \gamma^{d}\gamma^{bc}\gamma_{ad} = -8\gamma_{a}^{bc} + 10\delta_{a}^{[b}\gamma^{c]}$$

$$\gamma_{ad}\gamma^{bcd} = +10\gamma_{a}^{bc} + 11\delta_{a}^{[b}\gamma^{c]},$$

$$\gamma_{ae}\gamma^{cd}\gamma_{b}^{e} = -7\gamma_{ab}^{cd} - 8\eta_{ab}\gamma^{cd} + 9\delta_{(a}^{[c}\gamma_{b)}^{d]} - 11\delta_{[a}^{c}\delta_{b]}^{d},$$

$$\gamma_{e}\gamma_{ab}\gamma^{cde} = +7\gamma_{ab}{}^{cd} - 9\delta_{[a}{}^{[c}\gamma_{b]}{}^{d]} - 11\delta_{[a}{}^{c}\delta_{b]}^{d},$$

$$\gamma^{cde}\gamma_{ab}\gamma_{e} = +7\gamma_{ab}{}^{cd} + 9\delta_{[a}{}^{[c}\gamma_{b]}{}^{d]} - 11\delta_{[a}{}^{c}\delta_{b]}{}^{d},$$

$$\gamma^{d}\gamma^{abc}\gamma_{de} = +6\gamma_{e}{}^{abc} - 4\delta_{e}{}^{[a}\gamma^{bc]},$$

$$\gamma^{ab}\gamma^{de}\gamma_{abc} = -52\gamma^{de}{}_{c} + 88\delta_{c}{}^{[d}\gamma^{e]}, \gamma_{dea}\gamma^{bc}\gamma^{de} = -52\gamma_{a}{}^{bc} - 88_{a}{}^{[b}\gamma^{c]}$$

$$\gamma_{[3]}\gamma^{bc}\gamma^{[3]}{}_{a} = -240\gamma_{a}{}^{bc} + 660\delta_{a}{}^{[b}\gamma^{c]}$$

$$\gamma_{[2]a}\gamma^{cd}\gamma^{[2]}{}_{b} = -38\gamma_{ab}{}^{cd} + 70\delta_{(a}{}^{[c}\gamma_{b)}{}^{d]} - 110\delta_{a}{}^{[c}\delta_{b}{}^{d]} - 52\eta_{ab}\gamma^{cd}$$

$$\gamma_{[2]}\gamma^{cd}\gamma_{ab}{}^{[2]} = -38\gamma_{ab}{}^{cd} - 70\gamma\delta_{[a}{}^{[c}\gamma_{b]}{}^{d]} + 110\delta_{a}{}^{[c}\delta_{b}{}^{d]}$$

$$\gamma_{[3]}\gamma^{ab}\gamma^{[3]}{}_{cd} = -126\gamma^{ab}{}_{cd} - 450\delta_{[c}{}^{[a}\gamma_{d]}{}^{b]} + 990\delta_{c}{}^{[a}\delta_{d}{}^{b]}$$

$$\gamma^{b}\gamma^{a_1\cdots a_6}\gamma_{bc} = -\frac{1}{60}\delta_{c}{}^{[a_1}\gamma^{a_2\cdots a_6]}$$

$$
\begin{aligned}
&\gamma_{[n]} = \frac{(-1)^{(n+1)(n+2)/2}}{(13-n)!}\,\epsilon_{[n]}{}^{[13-n]}\gamma_{[13-n]} \qquad (0 \le n \le 13) \quad , \\
&\delta_{(\alpha}{}^{\beta}\delta_{\gamma)}{}^{\delta} = +\tfrac{1}{64}(\gamma^{[2]})_{\alpha\gamma}(\gamma_{[2]})^{\beta\delta} + \tfrac{1}{192}(\gamma^{[3]})_{\alpha\gamma}(\gamma_{[3]})^{\beta\delta} + \tfrac{1}{32(6!)}(\gamma^{[6]})_{\alpha\gamma}(\gamma_{[6]})^{\beta\delta} \quad , \\
&(\gamma^{a})_{(\alpha|}{}^{\beta}(\gamma_{a})_{|\gamma)}{}^{\delta} = \tfrac{9}{64}(\gamma^{[2]})_{\alpha\gamma}(\gamma_{[2]})^{\beta\delta} - \tfrac{7}{192}(\gamma^{[3]})_{\alpha\gamma}(\gamma_{[3]})^{\beta\delta} + \tfrac{1}{32(6!)}(\gamma^{[6]})_{\alpha\gamma}(\gamma_{[6]})^{\beta\delta} \\
&\qquad\qquad = \delta_{(\alpha}{}^{\beta}\delta_{\gamma)}{}^{\delta} + \tfrac{1}{8}(\gamma^{[2]})_{\alpha\gamma}(\gamma_{[2]})^{\beta\delta} - \tfrac{1}{24}(\gamma^{[3]})_{\alpha\gamma}(\gamma_{[3]})^{\beta\delta} \quad , \\
&(\gamma_{[2]})_{(\alpha\beta|}(\gamma^{[2]})_{|\gamma\delta)} = +\tfrac{1}{3}(\gamma_{[3]})_{(\alpha\beta|}(\gamma^{[3]})_{|\gamma\delta)} = -\tfrac{1}{6!}(\gamma_{[6]})_{(\alpha\beta|}(\gamma^{[6]})_{|\gamma\delta)} \quad , \\
&\gamma^{a}\gamma^{[n]}\gamma_{a} = (-1)^{n}(13-2n)\gamma^{[n]} \quad , \\
&\gamma^{ab}\gamma^{cd}\gamma_{ab} = -68\gamma^{cd} \quad , \qquad \gamma_{abc}\gamma^{de}\gamma^{abc} = -396\gamma^{de} \quad , \\
&\gamma_{ad}\gamma^{bc}\gamma^{d} = +8\gamma_{a}{}^{bc} + 10\delta_{a}{}^{[b}\gamma^{c]} \quad , \qquad \gamma^{d}\gamma^{bc}\gamma_{ad} = -8\gamma_{a}{}^{bc} + 10\delta_{a}{}^{[b}\gamma^{c]} \quad , \\
&\gamma_{ad}\gamma^{bcd} = +10\gamma_{a}{}^{bc} + 11\delta_{a}{}^{[b}\gamma^{c]} \quad , \\
&\gamma_{ae}\gamma^{cd}\gamma_{b}{}^{e} = -7\gamma_{ab}{}^{cd} - 8\eta_{ab}\gamma^{cd} + 9\delta_{(a}{}^{[c}\gamma_{b)}{}^{d]} - 11\delta_{[a}{}^{c}\delta_{b]}{}^{d} \quad , \\
&\gamma_{e}\gamma_{ab}\gamma^{cde} = +7\gamma_{ab}{}^{cd} - 9\delta_{[a}{}^{[c}\gamma_{b]}{}^{d]} - 11\delta_{[a}{}^{c}\delta_{b]}{}^{d} \quad ,
\end{aligned}
$$

$$
\begin{aligned}
&\gamma^{cde}\gamma_{ab}\gamma_{e} = +7\gamma_{ab}{}^{cd} + 9\delta_{[a}{}^{[c}\gamma_{b]}{}^{d]} - 11\delta_{[a}{}^{c}\delta_{b]}{}^{d} \quad , \\
&\gamma^{d}\gamma^{abc}\gamma_{de} = +6\gamma_{e}{}^{abc} - 4\delta_{e}{}^{[a}\gamma^{bc]} \quad ,
\end{aligned}
$$

$$
\begin{aligned}
&\gamma^{ab}\gamma^{de}\gamma_{abc} = -52\gamma^{de}{}_{c} + 88\delta_{c}{}^{[d}\gamma^{e]} \quad , \qquad \gamma_{dea}\gamma^{bc}\gamma^{de} = -52\gamma_{a}{}^{bc} - 88_{a}{}^{[b}\gamma^{c]} \quad , \\
&\gamma_{[3]}\gamma^{bc}\gamma^{[3]}{}_{a} = -240\gamma_{a}{}^{bc} + 660\delta_{a}{}^{[b}\gamma^{c]} \quad , \\
&\gamma_{[2]a}\gamma^{cd}\gamma^{[2]}{}_{b} = -38\gamma_{ab}{}^{cd} + 70\delta_{(a}{}^{[c}\gamma_{b)}{}^{d]} - 110\delta_{a}{}^{[c}\delta_{b}{}^{d]} - 52\eta_{ab}\gamma^{cd} \quad , \\
&\gamma_{[2]}\gamma^{cd}\gamma_{ab}{}^{[2]} = -38\gamma_{ab}{}^{cd} - 70\gamma\delta_{[a}{}^{[c}\gamma_{b]}{}^{d]} + 110\delta_{a}{}^{[c}\delta_{b}{}^{d]} \quad , \\
&\gamma_{[3]}\gamma^{ab}\gamma^{[3]}{}_{cd} = -126\gamma^{ab}{}_{cd} - 450\delta_{[c}{}^{[a}\gamma_{d]}{}^{b]} + 990\delta_{c}{}^{[a}\delta_{d}{}^{b]} \\
&\gamma^{b}\gamma^{a_1\cdots a_6}\gamma_{bc} = -\tfrac{1}{60}\delta_{c}{}^{[a_1}\gamma^{a_2\cdots a_6]} \quad .
\end{aligned}
$$

$$
R_{AB\gamma}{}^{\delta} = -\tfrac{1}{2}R_{AB}{}^{ij}(\widetilde{\mathcal{M}}_{ij})_{\gamma}{}^{\delta} \quad , \qquad R_{AB\dot{\gamma}}{}^{\dot{\delta}} = -\tfrac{1}{2}R_{AB}{}^{ij}(\widetilde{\mathcal{M}}_{ij})_{\dot{\gamma}}{}^{\dot{\delta}} \quad ,
$$

$$
\phi_{M}{}^{ij}(\widetilde{\mathcal{M}}_{ij})_{\underline{\underline{\alpha}}}\ \phi_{M}{}^{\pm i}(\widetilde{\mathcal{M}}_{\pm i})_{\underline{\underline{\alpha}}}\ \phi_{M}{}^{+-}(\widetilde{\mathcal{M}}_{+-})_{\underline{\underline{\alpha}}}{}^{\underline{\beta}}\ \mathbb{Y}^{[4]}
$$

$\widetilde{\mathcal{M}}'$ s$[\widetilde{\mathcal{M}}_{ab}, \widetilde{\mathcal{M}}_{cd}]_{\gamma}{}^{\delta}$ $\pm$: (i) $[\widetilde{\mathcal{M}}_{ij}, \widetilde{\mathcal{M}}_{kl}]_{\underline{\gamma}}^{\underline{\delta}}$, (ii) $[\widetilde{\mathcal{M}}_{ij}, \widetilde{\mathcal{M}}_{+k}]_{\underline{\gamma}}^{\underline{\delta}}$, (iii) $[\widetilde{\mathcal{M}}_{ij}, \widetilde{\mathcal{M}}_{+-}]_{\underline{\gamma}}^{\underline{\delta}}$, (iv) $[\widetilde{\mathcal{M}}_{+i}, \widetilde{\mathcal{M}}_{+j}]_{\underline{\gamma}}^{\underline{\delta}}$, (v)

$[\widetilde{\mathcal{M}}_{+i}, \widetilde{\mathcal{M}}_{+-}]_{\underline{\gamma}}\underline{\underline{\delta}}$, (vi) $[\widetilde{\mathcal{M}}_{+-}, \widetilde{\mathcal{M}}_{+-}]_{\underline{\gamma}}\underline{\underline{\delta}}$.

$$[\mathcal{M}_{ab}, \mathcal{M}^{cd}] = -\delta^{[c|}_{[a|}\mathcal{M}_{|b]}{}^{|d]}$$

$$\left[\widetilde{\mathcal{M}}_{ab}, [\widetilde{\mathcal{M}}_{cd}, \widetilde{\mathcal{M}}_{ef}]\right] + \left[\widetilde{\mathcal{M}}_{cd}, [\widetilde{\mathcal{M}}_{ef}, \widetilde{\mathcal{M}}_{ab}]\right] + \left[\widetilde{\mathcal{M}}_{ef}, [\widetilde{\mathcal{M}}_{ab}, \widetilde{\mathcal{M}}_{cd}]\right] \equiv 0$$

(i) $[ij][kl][mn]$, (ii) $[ij][kl][+m]$, (iii) $[ij][+k][+l]$, (iv) $[+i][+j][+k]$, (v) $[ij][kl][+-]$, (vi) $[ij][+k][+-]$, (vii) $[ij][+-][+-]$, (viii) $[+i][+-][+-]$, (ix) $[+i][+j][+-]$, (x) $[+-][+-][+-]$.

(I) $\widetilde{\mathcal{M}}\widetilde{\mathcal{M}}\widetilde{\mathcal{M}}$, (II) PPP, (III) QQQ, (IV) $\widetilde{\mathcal{M}}\widetilde{\mathcal{M}}P$, (V) $\widetilde{\mathcal{M}}\widetilde{\mathcal{M}}Q$, (VI) $PP\widetilde{\mathcal{M}}$, (VII) PPQ, (VIII) $QQ\widetilde{\mathcal{M}}$, (IX) QQP, (X) $\widetilde{\mathcal{M}}PQ$

$$\text{(V)}\ \left[\widetilde{\mathcal{M}}_{+i}, [\widetilde{\mathcal{M}}_{jk}, Q_\alpha]\right] + (2\text{ perms.}) \neq 0$$

$$\text{(VIII)}\ \left[\widetilde{\mathcal{M}}_{+i}, \{Q_\alpha, Q_\beta\}\right] + (2\text{ perms.}) \neq 0$$

$$[\nabla_A, [\nabla_B, \nabla_C\}\} + (2\text{ perms.}) \equiv 0$$

$$\begin{aligned}\delta_Q\left(R_\mu{}^\nu{}_{rs}\partial_\nu\varphi\right) &= +(D^\nu\varphi)D_\mu(\delta\omega_{\nu rs}) - (D^\nu\varphi)D_\nu(\delta\omega_{\mu rs}) \\ &= -2(D^\nu\varphi)\left(\bar{\epsilon}\gamma_{[r|}{}^\tau D_{|s]}\mathcal{R}_{\mu\nu}\right)\partial_\tau\varphi + \text{ c.c. } = 0\end{aligned}$$

$$\begin{aligned}\delta_Q\left(P^\mu\partial_\mu\varphi\right) &= -\epsilon_{\alpha\beta}(D^\mu\varphi)\left(\delta_Q V_+^\alpha\right)\partial_\mu V_+^\beta - \epsilon_{\alpha\beta}V_+^\alpha(D^\mu\varphi)\partial_\mu\left(\delta_Q V_+^\alpha\right) \\ &= -\epsilon_{\alpha\beta}V_+^\alpha V_-^\beta(D^\mu\varphi)\left(\bar{\epsilon}^*\gamma^\nu D_\mu\lambda\right)\partial_\nu\varphi = 0.\end{aligned}$$

$$R(P)_{\mu\nu}{}^m = 0$$

$$\delta S \sim \int \epsilon^{\mu\nu\rho\sigma}\epsilon_{mnrs}R(P)_{\mu\nu}{}^m\left[\delta\omega_\rho{}^{nr} - \Omega(e,\psi)_\rho{}^{nr}\right]e_\sigma{}^s$$

$$\begin{aligned}S = \int d^4x\epsilon^{\mu\nu\rho\sigma}\big[&R(M)^{mn}_{\mu\nu}R(M)_{\rho\sigma}{}^{rs}\epsilon_{mnrs} \\ &+8\lambda R(Q)^\alpha_{\mu\nu}\gamma^5_{\alpha\beta}R(Q)_{\rho\sigma}{}^\beta\big]\end{aligned}$$

$$\begin{array}{cccc} P_m\left(e_\mu{}^m\right) & Q_\alpha\left(\psi_\mu{}^\alpha\right) & M_{mn}\left(\omega_\mu{}^{mn}\right) & S_\alpha\left(\phi_\mu{}^\alpha\right) \\ & D\left(b_\mu\right) & & \\ & A\left(a_\mu\right) & & \end{array}$$

$$\int d^4x\big[aR(P)R(K) + bR(Q)\gamma_5R(S) + cR(M)\tilde{R}(M) + dR(D)R(A) + \alpha e g^{\mu\rho}g^{\nu\sigma}R(A)_{\mu\nu}R(A)_{\rho\sigma}\big].$$

$${}_*R(D)_{\mu\nu} = \frac{1}{2}e\epsilon_{\mu\nu\rho\sigma}R(D)_{\rho'\sigma'}g^{\rho\rho'}g^{\sigma\sigma'}\Big)$$

$$R(Q)_{\mu\nu} + \frac{1}{2}\gamma^5 e\epsilon_{\mu\nu\rho\sigma}R(Q)^{\rho\sigma} = 0$$

$$\gamma^\mu R(Q)_{\mu\nu} = 0$$

$$R(A)_{\mu\nu} = \frac{1}{2} e\epsilon_{\mu\nu\rho\sigma} R^{\rho\sigma}(D)$$

$$J^{\mu_1 \dots \mu_{p+1}}(x) = \frac{1}{\sqrt{g}} \int d\tau \int d^p \sigma \delta^d(x - X(\tau, \sigma))$$
$$\epsilon^{i_1 \dots i_{p+1}} \partial_{i_1} X^{\mu_1}(\tau, \sigma) \dots \partial_{i_{(p+1)}} X^{\mu_{(p+1)}}(\tau, \sigma)$$

$$\partial_\nu \left(\sqrt{g} J^{\nu \mu_1 \dots \mu_p}(x) \right) = 0$$

$$Z^{\mu_1 \dots \mu_p} = \int d^{d-1} x J^{0 \mu_1 \dots \mu_p}(x)$$

$$\{Q_a, Q_b\} = \Gamma^\mu_{ab} P_\mu + \Gamma^\mu_{ab} Z_\mu + \Gamma^{\mu_1 \dots \mu_5}_{ab} Z^+_{\mu_1 \dots \mu_5}$$

$$\{Q^a, Q^b\} = \Gamma^{\mu ab} P_\mu - \Gamma^{\mu ab} Z_\mu + \Gamma^{\mu_1 \dots \mu_5 ab} Z^-_{\mu_1 \dots \mu_5}$$
$$\{Q_a, Q^b\} = \delta^b_a Z + \Gamma^{\mu\nu}{}_a{}^b Z_{\mu\nu} + \Gamma^{\mu_1 \dots \mu_4}{}_a{}^b Z_{\mu_1 \dots \mu_4}$$

$$\{Q_\alpha, Q_\beta\} = \Gamma^M_{\alpha\beta} P_M + \Gamma^{MN}_{\alpha\beta} Z_{MN} + \Gamma^{M_1 \dots M_5}_{\alpha\beta} Z_{M_1 \dots M_5}$$

$$\{Q_\alpha, Q_\beta\} = \Gamma^{MN}_{\alpha\beta} M_{MN} + \Gamma^{M_1 \dots M_6}_{\alpha\beta} Z_{M_1 \dots M_6}$$

$$\{Q_{ai}, Q_{bj}\} = \Gamma^\mu_{ab} \Sigma^J_{ij} Z_{J\mu} + \Gamma^{\mu_1\mu_2\mu_3}_{ab} \epsilon_{ij} Z_{\mu_1\mu_2\mu_3} + \Gamma^{\mu_1 \dots \mu_5}_{ab} \Sigma^J_{ij} Z_{J\mu_1 \dots \mu_5},$$

$$\Sigma^J_{(ij)} = \epsilon_{il} \Sigma^{Jl}{}_j, \Sigma_0 = -i\sigma^2, \Sigma_1 = -\sigma^1 \text{ and } \Sigma_2 = \sigma^3.$$

$$\Sigma_I \Sigma_J = \eta_{IJ} + \epsilon_{IJ}{}^K \Sigma_K = \eta_{IJ} + \epsilon_{IJL} \eta^{LK} \Sigma_K, \text{ with } \eta_{IJ} = (-++), \epsilon_{012} = 1$$

$$\begin{aligned}
\{Q_\alpha, Q_\beta\} &= -\frac{1}{128} \Gamma^{MN}_{\alpha\beta} J_{MN} - \frac{1}{128 \cdot 6!} \Gamma^{M_1 \dots M_6}_{\alpha\beta} J_{M_1 \dots M_6} \\
[J_{MN}, Q_\alpha] &= -(\Gamma_{MN})_\alpha{}^\beta Q_\beta \\
[J_{M_1 \dots M_6}, Q_\alpha] &= -(\Gamma_{M_1 \dots M_6})_\alpha{}^\beta Q_\beta \\
[J_{MN}, J^{KL}] &= 8\delta^{[K}_{[N} J_{M}{}^{L]} \\
[J_{MN}, J^{M_1 \dots M_6}] &= 24\delta^{[M_1}_{[N} J^{M_2 \dots M_6]}_{M]} \\
[J_{N_1 \dots N_6}, J^{M_1 \dots M_6}] &= -12 \cdot 6!\, \delta^{[M_1 \dots M_5}_{[N_1 \dots N_5} J^{M_6]}_{N_6]} \\
&\quad + 12\epsilon_{N_1 \dots N_6}{}^{[M_1 \dots M_5 |R|} J_R{}^{M_6]}.
\end{aligned}$$

$$\begin{array}{lll}
\Psi^\mu_{-1/2} \mid k > & \mu = 0,1 & \text{2-d gauge fields} \\
\Psi^m_{-1/2} \mid k > & m = 2, \cdots, 9 & \text{transverse fluctuations,}
\end{array}$$

$$\begin{array}{lll}
P_m = \frac{1}{\sqrt{2}} \gamma_m \otimes \sigma^+ & M_{mn} = \frac{1}{2} \gamma_{mn} \otimes \mathbf{1} & K_m = -\frac{1}{\sqrt{2}} \gamma_m \otimes \sigma^- \\
E_{mn} = \frac{1}{\sqrt{2}} \gamma_{mn} \otimes \sigma^+ & D = \mathbf{1} \otimes \sigma^3 & F_{mn} = \frac{1}{\sqrt{2}} \gamma_{mn} \otimes \sigma^- \\
 & A = -\gamma^5 \otimes \sigma^3 & \\
 & V_m = -\gamma_m \otimes \mathbf{1} & \\
 & Z_m = \gamma^5 \gamma_m \otimes \sigma^3 &
\end{array}$$

$$\{\gamma_m, \gamma_n\} = 2\eta_{mn} = 2\mathrm{diag}(-1,1,1,1)_{mn}$$

$$\gamma^5 \equiv \gamma^0\gamma^1\gamma^2\gamma^3 \; (\gamma^5)^2 = -1, \gamma^5 = -(\gamma^5)^\top.$$

$$\gamma^{mn} \equiv \gamma^{[m}\gamma^{n]} = \tfrac{1}{2}(\gamma^m\gamma^n - \gamma^n\gamma^m).$$

$$M^\top C + CM = 0$$

$$C = \gamma^0 \otimes \sigma^1 = -C^\top$$

$$C_4 = \gamma^0 \; (C_4\gamma_m = -\gamma_m^\top C_4\;), C_2 = (-i\sigma^2)\sigma^3 \; (C_2\sigma^\pm = (\sigma^\pm)^\top C_2\;), \mathrm{Sp}(8, \mathbf{R}).$$

$$\left[P_m, \begin{pmatrix} S \\ Q \end{pmatrix}\right] = -\sqrt{2}\begin{pmatrix} \gamma_m Q \\ 0 \end{pmatrix}.$$

$$T_A = \{P_m, E_{mn}, Q, M_{mn}, D, A, V_m, Z_m, S, K_m, F_{mn}\}$$

$$[T_A, T_B\} = f_{AB}^C T_C$$

$$h^A = \{e^m, E^{mn}, \psi, \omega^{mn}, b, a, v^m, z^m, \phi, f^m, F^{mn}\}$$

$$R = dh + hh = \left(dh^A - \frac{1}{2}h^C h^B f_{BC}^A\right) T_A = R^A T_A \equiv \frac{1}{2} R_{\mu\nu}^A T_A dx^\mu dx^\nu,$$

$$R(P)^m = de^m + \omega^m{}_n e^n + 2be^m - 2E^m{}_n v^n - 2\tilde{E}^m{}_n z^n - \frac{1}{4\sqrt{2}}\bar{\psi}\gamma^m\psi$$

$$R(E)^{mn} = dE^{mn} - 2\omega^{[m}{}_k E^{n]k} + 2bE^{mn} + 2\tilde{E}^{mn}a - 4e^{[m}v^{n]}$$
$$+2\epsilon^{mnpq}e_p z_q + \frac{1}{4\sqrt{2}}\bar{\psi}\gamma^{mn}\psi$$

$$R(Q) = d\psi + \left(-a\gamma^5 + b + \frac{1}{4}\omega^{mn}\gamma_{mn} - v^m\gamma_m + z^m\gamma^5\gamma_m\right)\psi$$
$$+\left(\sqrt{2}e^m\gamma_m + \frac{1}{\sqrt{2}}E^{mn}\gamma_{mn}\right)\phi$$

$$R(M)^{mn} = d\omega^{mn} - \omega^{[m}{}_k\omega^{n]k} + 4v^{[m}v^{n]} + 4z^{[m}z^{n]} - 8e^{[m}f^{n]}$$
$$-8E^{[m}{}_k F^{n]k} + \frac{1}{2}\bar{\psi}\gamma^{mn}\phi$$

$$R(D) = db - 2e^m f_m - E^{mn}F_{mn} + \frac{1}{4}\bar{\psi}\phi$$

$$R(A) = da - 2v^m z_m - \tilde{E}^{mn}F_{mn} + \frac{1}{4}\bar{\psi}\gamma^5\phi$$

$$R(V)^m = dv^m + \omega^m{}_n v^n + 2z^m a + 2E^m{}_n f^n - 2F^m{}_n e^n + \frac{1}{4}\bar{\psi}\gamma^m\phi$$

$$R(Z)^m = dz^m + \omega^m{}_n z^n - 2v^m a + 2\tilde{E}^{mn}f_n + 2\tilde{F}^{mn}e_n + \frac{1}{4}\bar{\psi}\gamma^5\gamma^m\phi$$

$$R(S) = d\phi + \left(a\gamma^5 - b + \frac{1}{4}\omega^{mn}\gamma_{mn} - v^m\gamma_m - z^m\gamma^5\gamma_m\right)\phi$$
$$+\left(-\sqrt{2}f^m\gamma_m + \frac{1}{\sqrt{2}}F^{mn}\gamma_{mn}\right)\psi$$

$$R(K)^m = df^m + \omega^m{}_n f^n - 2bf^m + 2F^m{}_n v^n - 2\tilde{F}^{mn}z_n + \frac{1}{4\sqrt{2}}\bar{\phi}\gamma^m\phi$$

$$R(F)^{mn} = dF^{mn} - 2\omega^{[m}{}_k F^{n]k} - 2bF^{mn} - 2\tilde{F}^{mn}a + 4f^{[m}v^{n]}$$
$$+2\epsilon^{mnpq}f_p z_q + \frac{1}{4\sqrt{2}}\bar{\phi}\gamma^{mn}\phi.$$

$$dR^A = -R^C h^B f_{BC}{}^A$$

$$\{Q_a, Q_b\} = a_{(a}a_{b)}$$

$$\delta^c_{(a}\delta^d_{b)} = -\frac{1}{8}\left\{\Gamma^7_{ab}\Gamma^{7cd} + \frac{1}{2}\Gamma^{MN}_{ab}\Gamma^{cd}_{MN} + \frac{1}{6}\Gamma^{LMN}_{ab}\Gamma_{LMN}{}^{cd}\right\}$$

$$\{Q_a, Q_b\} = \frac{1}{8}\left\{\Gamma^7_{ab}a^c\Gamma^{7d}_c a_d + \frac{1}{2}\Gamma^{MN}_{ab}a^c\Gamma_{MNc}{}^d a_d\right.$$
$$\left.+\frac{1}{6}\Gamma^{LMN}_{ab}a^c\Gamma_{LMNc}{}^d a_d\right\}$$
$$\equiv \frac{1}{4}\left\{\Gamma^7_{ab}J_7 + \frac{1}{2}\Gamma^{MN}_{ab}J_{MN} + \frac{1}{6}\Gamma^{LMN}_{ab}J_{LMN}\right\}$$

$$\Gamma^m = -\gamma^m \otimes \sigma^3, \Gamma^{\oplus} = \frac{1}{\sqrt{2}}\mathbf{1}\otimes\sigma^+, \Gamma^{\ominus} = \frac{1}{\sqrt{2}}\mathbf{1}\otimes\sigma^-$$
$$\{\Gamma^M, \Gamma^N\} = 2\eta^{MN} = \mathrm{diag}(--++++)^{MN}, \Gamma^7 = -\gamma^5\otimes\sigma^3$$

$$a^a = C^{ab}a_b, \Gamma^{*ab} = \Gamma^{*a}{}_c C^{cb} = C^{ac}\Gamma^*{}_c{}^b, \Gamma^*{}_{ab} = \Gamma^*{}_a{}^c C_{cb} = C_{ac}\Gamma^{*c}{}_b, C^{ac}C_{cb} = \delta^a_b. C^{ab} =$$

$$(\gamma^0\otimes\sigma^1)^{ab}$$

$$[J^*, Q_a] = -\Gamma^*{}_a{}^b Q_b$$

$$[J^7, J^{MNP}] = \frac{1}{3}\epsilon^{MNPRST} J_{RST}$$

$$[J^{MN}, J_{RS}] = 8\delta^{[N}_{[R} J^{M]}{}_{S]}$$

$$[J^{MN}, J_{RST}] = 12\delta^{[N}_{[R} J^{M]}{}_{ST]}$$

$$[J^{MNP}, J_{RST}] = 2\epsilon^{MNP}{}_{RST} J^7 - 36\delta^{[M}_{[R}\delta^N_S J^{P]}T]$$

$$P^m = \Gamma^{\oplus m} \qquad M^{mn} = \frac{1}{2}\Gamma^{mn} \qquad K^m = \Gamma^{\ominus m}$$

$$E^{mn} = \Gamma^{\oplus mn} \qquad D = \Gamma^{\oplus\ominus} \qquad F^{mn} = \Gamma^{\ominus mn}$$

$$A = \Gamma^7$$

$$V^m = \Gamma^{\oplus\ominus m}$$

$$Z^m = -\frac{1}{3!}\epsilon^{mnpq}\Gamma_{npq}$$

$$\begin{aligned} R = & \left\{dh_7 + \tfrac{1}{36}\epsilon^{MNPRST}h_{MNP}h_{RST} + \tfrac{1}{8}\psi^a\Gamma_{ab}\psi^b\right\}J^7 \\ & + \tfrac{1}{2}\left\{dh_{MN} + 2h_{MK}h^K{}_N - h^{RS}{}_M h_{RSN} + \tfrac{1}{8}\psi^a\Gamma_{MNab}\psi^b\right\}J^{MN} \\ & + \tfrac{1}{6}\left\{dh_{MNP} + \tfrac{1}{3}\epsilon_{MNPRST}h^{RST}h_7 + 6h_M{}^K h_{KNP} + \tfrac{1}{8}\psi^a\Gamma_{MNPab}\psi^b\right\}J^{MNP} \\ & + \left\{d\psi^a + h_7\Gamma^{7a}{}_b\psi^b + \tfrac{1}{2}h_{MN}\Gamma^{MNa}{}_b\psi^b + \tfrac{1}{6}h_{MNP}\Gamma^{MNPa}{}_b\psi^b\right\}Q_a \end{aligned}$$

$$\begin{aligned} \delta R = & \left\{\tfrac{1}{18}\epsilon^{MNPRST}R_{MNP}\lambda_{RST} - \tfrac{1}{4}R^a\Gamma^7{}_{ab}\lambda^b\right\}J^7 \\ & + \tfrac{1}{2}\left\{4R_{MK}\lambda^K{}_N - 2R^{RS}{}_M\lambda_{RSN} - \tfrac{1}{4}R^a\Gamma_{MNab}\lambda^b\right\}J^{MN} \\ & + \tfrac{1}{6}\left\{-\tfrac{1}{3}\epsilon_{MNPRST}R_7\lambda^{RST} + 6\mathrm{R}_M{}^K\lambda_{KNP} + \tfrac{1}{3}\epsilon_{MNPRST}R^{RST}\lambda_7 - 6\mathrm{R}_{MN}{}^K\lambda_P{}^K - \tfrac{1}{4}R^a\Gamma_{MNPab}\lambda^b\right\}J^{MNP} \\ & + \left\{R_7\Gamma^{7a}{}_b\lambda^b + \tfrac{1}{2}R_{MN}\Gamma^{MNa}{}_b\lambda^b + \tfrac{1}{6}R_{MNP}\Gamma^{MNPa}{}_b\lambda^b + \lambda_7\Gamma^{7a}{}_bR^b + \tfrac{1}{2}\lambda_{MN}\Gamma^{MNa}{}_bR^b + \tfrac{1}{6}\lambda_{MNP}\Gamma^{MNPa}{}_bR^b\right\}Q_a \end{aligned}$$

$$\delta h^A = d\epsilon^A + \epsilon^C h^B f_{BC}{}^A$$

$$\begin{aligned} -\mathcal{L} = & \alpha_0\epsilon_{mnpq}R(M)^{mn}R(M)^{pq} + \alpha_1 R(A)R(D) + \alpha_2 R(V)^m R(Z)_m \\ & + \alpha_3\epsilon_{mnpq}R(E)^{mn}R(F)^{pq} + \beta\overline{R(Q)}\gamma^5 R(S) \end{aligned}$$

$$\alpha_0 = 1, \alpha_1 = -32, \alpha_2 = 0, \alpha_3 = 8, \beta = -8$$

$$\delta R^A = -R^C \epsilon^B f_{BC}{}^A.$$

$$\begin{aligned} -\delta_K \mathcal{L} = & -32 R(P)_m \left[\tilde{R}(M)^{mn} + \frac{\alpha_1}{16} R(A) \eta^{mn}\right] \epsilon_n + \sqrt{2}\beta \overline{R(Q)} \gamma^5 \gamma^m R(Q) \epsilon_m \\ & + 2[(\alpha_2 + 4\alpha_3) \tilde{R}(E)^{mn} R(V)_m + (\alpha_2 - 4\alpha_3) R(E)^{mn} R(Z)_m] \epsilon_n \\ -\delta_F \mathcal{L} = & \tilde{R}(E)_{mn} [(4\alpha_3 - 32) R(M)^m{}_k \epsilon^{kn} - (4\alpha_3 + \alpha_1)(R(D)\epsilon^{mn} - R(A)\tilde{\epsilon}^{mn})] \\ & -2\alpha_2 R(P)_m [\epsilon^{mn} R(Z)_n - \tilde{\epsilon}^{mn} R(V)_n] \\ -\delta_S \mathcal{L} = & \overline{R(Q)} \left[\left(\frac{\alpha_1}{4} - \beta\right)(R(A) + \gamma^5 R(D)) + \left(\frac{\alpha_2}{4} + \beta\right) \gamma^m R(Z)_m \right. \\ & \left. + \left(\frac{\alpha_2}{4} - \beta\right) \gamma^5 \gamma^m R(V)_m + \left(2 + \frac{\beta}{4}\right) \tilde{R}(M)^{mn} \gamma_{mn}\right] \epsilon \\ & + \overline{R(S)} \left[\sqrt{2}\beta \gamma^5 \gamma^m R(P)_m + \left(\frac{\beta}{\sqrt{2}} + \frac{\alpha_3}{\sqrt{2}}\right) \gamma^{mn} \tilde{R}(E)_{mn}\right] \epsilon, \end{aligned}$$

$$\epsilon_{mnpq} X_r = \epsilon_{rnpq} X_m + \epsilon_{mrpq} X_n + \epsilon_{mnrq} X_p + \epsilon_{mnpr} X_q$$

$$\widetilde{XY}^{mn} \equiv \frac{1}{2} \epsilon^{mnpq} X_{pk} Y^k{}_q = \frac{1}{2} [X\tilde{Y} - \tilde{Y}X]^{mn} \equiv X_k{}^{[m} \tilde{Y}^{n]k}$$

$$R(Q) = -\gamma^5 {}_*R(Q),$$

$$R(P)^m = 0.$$

$$\delta_S [R(Q) + \gamma_*^5 R(Q)] = \frac{1}{\sqrt{2}} [R(E)^{mn} + {}_*\tilde{R}(E)^{mn}] \gamma_{mn} \epsilon$$

$$R(E)^{mn} = - {}_*\tilde{R}(E)^{mn}.$$

$$2R(E)^{mn} [\alpha_2 ({}_*R(V)_m + R(Z)_m) + 4\alpha_3 ({}_*R(V)_m - R(Z)_m)] \epsilon_n.$$

$${}_*R(V)^m = R(Z)^m.$$

$$\overline{R(Q)} \gamma^m \left\{ \frac{\alpha_2}{4} [R(Z)_m + {}_*R(V)_m] + \beta [R(Z)_m - {}_*R(V)_m] \right\}.$$

$$\begin{aligned} & -\mathcal{L} = \epsilon_{mnpq} R(M)^{mn} R(M)^{pq} - 32 R(A) R(D) \\ & +8 \epsilon_{mnpq} R(E)^{mn} R(F)^{pq} - 8 \overline{R(Q)} \gamma^5 R(S) \\ & R(P)^m = 0 \\ & R(E)^{mn} = - {}_*\tilde{R}(E)^{mn} \\ & R(Z)^m = {}_*R(V)^m \\ & R(Q) = -\gamma^5 {}_*R(Q). \end{aligned}$$

$$\begin{aligned} \omega_{\mu mn} = & \frac{1}{2} \left(-\hat{R}(P)_{mn\mu} + \hat{R}(P)_{\mu mn} - \hat{R}(P)_{\mu nm}\right); \\ & \hat{R}(P)_{\mu\nu}{}^m \equiv R\left(P; \omega_\mu{}^{mn} = 0\right)_{\mu\nu}{}^m \end{aligned}$$

$R(P)_{\mu\nu}{}^{m}$	$=$	$\underline{24}$	$=$	$\underline{16}$	$+$	$\underline{\tilde{4}}$	$+$	$\underline{4}$						
				$\surd$		$\surd$		$\surd$						
$R(E)_{\mu\nu}{}^{mn}$	$=$	$\underline{36}$	$=$	$\underline{\tilde{1}}$	$+$	$\underline{10}$	$+$	$\underline{\tilde{9}}$	$+$	$\underline{9}$	$+$	$\underline{6}$	$+$	$\underline{1}$
				$\surd$		$\times$		$\surd$		$\surd$		$\surd$		$\surd$
$R(Q)_{\mu\nu}$	$=$	$\underline{24}$	$=$	$\underline{8}$	$+$	$\underline{12}$	$+$	$\underline{4}$						
				$\times$		$\surd$		$\surd$						
$R(M)_{\mu\nu}{}^{mn}$	$=$	$\underline{36}$	$=$	$\underline{\tilde{1}}$	$+$	$\underline{10}$	$+$	$\underline{\tilde{9}}$	$+$	$\underline{9}$	$+$	$\underline{6}$	$+$	$\underline{1}$
				$\times$		$\times$		$\times$		$\surd$		$\surd$		$\surd$
$R(D)_{\mu\nu}$	$=$	$\underline{6}$												
		$\surd$												
$R(V)_{\mu\nu}{}^{m} + {}_{*}R(Z)_{\mu\nu}{}^{m}$	$=$	$\underline{24}$	$=$	$\underline{16}$	$+$	$\underline{\tilde{4}}$	$+$	$\underline{4}$						
				$\surd$		$\surd$		$\surd$						
$R(V)_{\mu\nu}{}^{m} - {}_{*}R(Z)_{\mu\nu}{}^{m}$	$=$	$\underline{24}$	$=$	$\underline{16}$	$+$	$\underline{\tilde{4}}$	$+$	$\underline{4}$						
				$\times$		$\surd$		$\surd$						

$$\gamma^{\nu} R(Q)_{\mu\nu} - \frac{1}{4}\gamma_{\mu}\gamma^{\alpha\beta} R(Q)_{\alpha\beta} = \underline{12}$$

$$\gamma^{\alpha\beta} R(Q)_{\alpha\beta} = \underline{4}$$

$$\gamma^{\mu} R(Q)_{\mu\nu} = 0$$

$$\gamma^{\mu\nu} R(Q)_{\mu\nu} = 0.$$

$$z_{[\mu\nu]} + {}_{*}v_{[\mu\nu]}, v_{\mu}{}^{\mu} \text{ and } v_{(\mu\nu)} - \tfrac{1}{4} g_{\mu\nu} v_{\rho}{}^{\rho}$$

$$z_{(\mu\nu)} - \tfrac{1}{4} g_{\mu\nu} z_{\rho}{}^{\rho} \text{ and } z_{\mu}{}^{\mu}$$

$$R(E)^{mn} = -{}_{*}\tilde{R}(E)^{mn}$$

$$R(E)_{\nu[\mu m]}{}^{\nu} = 0$$
$$R(E)_{\mu\nu}{}^{\nu\mu} = 0$$
$$\epsilon^{\mu\nu\rho\sigma} R(E)_{\mu\nu\rho\sigma} = 0$$

$$
\begin{aligned}
R(P)^m &= de^m + \omega_n^m e^n + 2be^m - \frac{1}{4\sqrt{2}}\bar{\psi}\gamma^m\psi \\
R(Q) &= d\psi + \left(3a\gamma^5 + b + \frac{1}{4}\omega^{mn}\gamma_{mn}\right)\psi + \sqrt{2}e^m\gamma_m\phi \\
R(M)^{mn} &= d\omega^{mn} - \omega_k^{[m}\omega^{n]k} - 8e^{[m}f^{n]} + \frac{1}{2}\bar{\psi}\gamma^{mn}\phi \\
R(D) &= db - 2e^m f_m + \frac{1}{4}\bar{\psi}\phi \\
R(A) &= da + \frac{1}{4}\bar{\psi}\gamma^5\phi \\
R(S) &= d\phi + \left(-3a\gamma^5 - b + \frac{1}{4}\omega^{mn}\gamma_{mn}\right)\phi - \sqrt{2}f^m\gamma_m\psi \\
R(K)^m &= df^m + \omega_n^m f^n - 2bf^m + \frac{1}{4\sqrt{2}}\bar{\phi}\gamma^m\phi
\end{aligned}
$$

$$Q_a^{(+)} = a_a a = (a^K a, \bar{a}_K a) \text{ and } Q_a^{(-)} = a_a \bar{a} = (a^K \bar{a}, \bar{a}_K \bar{a})$$

$$Q_a = a_a(a + \bar{a})/\sqrt{2}$$

$$J^K{}_L = \tfrac{1}{2}\{a^K, \bar{a}_L\} - \tfrac{1}{8}\delta_L^K\{a^N, \bar{a}_N\} \text{ and } J = \tfrac{1}{2}\{a^K, \bar{a}_K\} = \tfrac{1}{2}[a, \bar{a}]$$

$$\{Q^K, \bar{Q}_L\} = \frac{1}{2}\{a^K, \bar{a}_L\} - \frac{1}{2}\delta_L^K[a, \bar{a}] = J^K{}_L - \frac{3}{4}\delta_L^K J,$$

$$\{Q^K, \bar{Q}_L\} = \frac{1}{2}\{a^K, \bar{a}_L\} = J_L^K + \frac{1}{4}\delta_L^K J,$$

$$J = \tfrac{1}{2}\{a^K, \bar{a}_K\}\, J^{KL} = a^{(K}a^{L)} \text{ and } \bar{J}_{KL} = \bar{a}_{(K}\bar{a}_{L)}$$

$$
\begin{aligned}
-\mathcal{L} = \epsilon_{mnpq}R(M)^{mn}R(M)^{pq} &+ 32R(A)R(D) - 8\overline{R(Q)}\gamma^5 R(S), \\
R(P)^m &= 0 \\
R(Q) &= -\gamma^5\, {}_*R(Q), \\
R(A) &= {}_*R(D).
\end{aligned}
$$

$$\mathcal{L}_{\text{non-affine}} = \mathcal{L} + 64R(A)(R(D) + {}_*R(A))$$

$$0 = \tilde{R}(M)^{mn}e_n + 2R(A)e^m - \frac{1}{2\sqrt{2}}\bar{\psi}\gamma^5\gamma^m R(Q)$$

$$\gamma^\mu R(Q)_{\mu\nu} = 0$$

$$0 = dR(P)^m = R(M)^{mn}e_n + 2R(D)e^m + \frac{1}{2\sqrt{2}}\bar{\psi}\gamma^m R(Q)$$

$$
\begin{aligned}
0 &= R(M)^\rho_{\rho[\mu\nu]} + 2{}_*R(A)_{\mu\nu} + \frac{1}{4\sqrt{2}}\overline{R(Q)}_{\mu\nu}\gamma\cdot\psi \; \{f^m \text{ equation}\} \\
0 &= R(M)^\rho_{\rho[\mu\nu]} - 2R(D)_{\mu\nu} + \frac{1}{4\sqrt{2}}\overline{R(Q)}_{\mu\nu}\gamma\cdot\psi \; \{\text{Bianchi}\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\text{non-affine}} = \mathcal{L} &+ 32(1-2\alpha)R(V)^m R(Z)_m + 32\alpha R(V)^m\, {}_*R(V)_m \\
&+ 32(1-\alpha)R(Z)^m\, {}_*R(Z)_m.
\end{aligned}
$$

$$0 = \tilde{R}(M)^{mn} e_n + 2R(A) e^m - \frac{1}{2\sqrt{2}} \bar{\psi} \gamma^5 \gamma^m R(Q) - 2\tilde{R}(E)^{mn} v_n$$
$$-2\tilde{E}^{mn} R(V)_n + 2R(E)^{mn} z_n + 2E^{mn} R(Z)_n$$

$$0 = dR(P)^m = R(M)^{mn} e_n + 2R(D) e^m + \frac{1}{2\sqrt{2}} \bar{\psi} \gamma^m R(Q) - 2R(E)^{mn} v_n$$
$$+2E^{mn} R(V)_n - 2\tilde{R}(E)^{mn} z_n + 2\tilde{E}^{mn} R(Z)_n$$

$$0 = {}_*\tilde{R}(M)_{\rho[\mu\nu]\rho'} g^{\rho\rho'} - 2{}_*R(A)_{\mu\nu} + \frac{1}{4\sqrt{2}} \bar{\psi} \cdot \gamma R(Q)_{\mu\nu}$$
$$-R(E)_{\mu\nu\rho\sigma} v^{\rho\sigma} + 2\tilde{E}^{\rho\sigma}{}_{[\mu} R(Z)_{\nu]\rho\sigma} - {}_*R(E)_{\mu\nu\rho\sigma} z^{\rho\sigma} + 2E^{\rho\sigma}{}_{[\mu} R(V)_{\nu]\rho\sigma}$$
$$0 = {}_*R(M)_{\rho[\mu\nu]}{}^{\rho} - 2{}_*R(D)_{\mu\nu} - \frac{1}{4\sqrt{2}} \bar{\psi} \cdot \gamma_* R(Q)_{\mu\nu}$$
$$+ {}_*R(E)_{\mu\nu\rho\sigma} v^{\rho\sigma} - 2E^{\rho\sigma}{}_{[\mu} R(Z)_{\nu]\rho\sigma} - R(E)_{\mu\nu\rho\sigma} z^{\rho\sigma} + 2\tilde{E}^{\rho\sigma}{}_{[\mu} R(V)_{\nu]\rho\sigma}.$$

$$R(E)_{[\mu\nu\rho]\sigma} = 0$$

$${}_*R(E)_{[\mu\nu\rho]\sigma} = 0$$

$$\gamma_{[\mu} R(Q)_{\rho\sigma]} = 0$$

$$0 = R(A)_{\mu\nu} - {}_*R(D)_{\mu\nu} + \tilde{E}^{\rho\sigma}{}_{[\mu} R(V)_{\nu]\rho\sigma} - E^{\rho\sigma}{}_{[\mu} R(Z)_{\nu]\rho\sigma}$$
$$+\frac{1}{2} e\epsilon_{\mu\nu\alpha\beta} \left[E^{\rho\sigma\alpha} R(V)^{\beta}{}_{\rho\sigma} + \tilde{E}^{\rho\sigma\alpha} R(Z)^{\beta}{}_{\rho\sigma} \right].$$

$$R(V)^m = - {}_*R(Z)^m$$

$$0 = R(P)_{\mu\nu}{}^m$$
$$0 = R(E)_{\rho[\mu\nu]}{}^{\rho}$$
$$0 = R(E)_{\mu\nu}{}^{\nu\mu}$$
$$0 = \epsilon^{\mu\nu\rho\sigma} R(E)_{\mu\nu\rho\sigma}$$
$$0 = R(E)_{\mu\nu}{}^{mn} + {}_*\tilde{R}(E)_{\mu\nu}{}^{mn}$$
$$0 = R(Z)_{\mu\nu}{}^m - {}_*R(V)_{\mu\nu}{}^m$$
$$0 = \gamma^{\mu} R(Q)_{\mu\nu}$$
$$0 = R(M)_{\rho\mu\nu}{}^{\rho} - \frac{1}{2} g_{\mu\nu} R(M)_{\rho\sigma}{}^{\sigma\rho} + 2{}_*R(A)_{\mu\nu} + \frac{1}{2\sqrt{2}} \overline{R(Q)}_{\rho\nu} \gamma_{\mu} \psi^{\rho}$$
$$-2{}_*R(E)_{\rho\nu\sigma\mu} z^{\rho\sigma} - 2R(E)_{\rho\nu\sigma\mu} v^{\rho\sigma} + 2R(V)_{\rho\nu\sigma} E^{\rho\sigma}{}_{\mu} + 2R(Z)_{\rho\nu\sigma} \tilde{E}^{\rho\sigma}{}_{\mu}.$$

$$\delta h^A_{\text{Indept.}} = d\epsilon^A + \epsilon^C h^B f_{BC}{}^A = \delta_{\text{Group}}\, h^A_{\text{Indept.}}$$

$$\delta h^A_{\text{Dept.}} = d\epsilon^A + \epsilon^C h^B f^A_{BC} + \hat{\delta} h^A_{\text{Dept.}} \equiv \delta_{\text{Group}}\, h^A_{\text{Dept.}} + \hat{\delta} h^A_{\text{Dept.}}$$

$$0 = \delta R(P)^m = \delta_{\text{Group}}\, R(P)^m + \hat{\delta} R(P)^m$$
$$= \delta_{\text{Group}}\, R(P)^m + \hat{\delta}\omega^{mn} e_n - 2E^{mn} \hat{\delta} v_n - 2\tilde{E}^{mn} \hat{\delta} z_n$$

$$\hat{\delta}\omega_{\mu mn} = \frac{1}{2} \left(-\delta_{\text{Group}}\, R(P)_{mn\mu} + \delta_{\text{Group}}\, R(P)_{\mu mn} - \delta_{\text{Group}}\, R(P)_{\mu nm} \right) + \cdots$$

$$\hat{\delta} h^A_{\text{Dept.}} \sim (\text{ curvatures }) + (\text{ curvatures }) \times (\text{ fields }) + (\text{ curvatures }) \times (\text{ fields })^2 + \cdots.$$

$$-\delta\mathcal{L} = -\delta_{\text{Group}}\,\mathcal{L} + 32R(V)^m\delta_{\text{Group}}\left[R(Z)_m - {}_*R(V)_m\right] - 16\sqrt{2}\overline{R(S)}\gamma^5\gamma^m e_m\hat{\delta}\phi$$
$$+32R(K)^m e^n\hat{\delta}\widetilde{\omega}_{mn} - 128\tilde{R}(F)_{mn}e^m\hat{\delta}v^n - 128R(F)_{mn}e^m\hat{\delta}z^n$$
$$+O([\text{ curvature }]^2 \times (\text{ field }))$$

$$\begin{aligned}
\hat{\delta}v_{(\mu\nu)} &= -\frac{1}{4}\delta_{\text{Group}}\left[R(E)_{\rho(\mu\nu)}{}^{\rho} - \frac{1}{6}g_{\mu\nu}R(E)^{\sigma\rho}_{\rho\sigma}\right]\\
\hat{\delta}z_{(\mu\nu)} &= -\frac{1}{4}\delta_{\text{Group}}\left[\tilde{R}(E)_{\rho(\mu\nu)\rho'}g^{\rho\rho'} - \frac{1}{6}g_{\mu\nu}\tilde{R}(E)^{\rho\rho'}_{\rho\sigma'\rho'}g^{\sigma\sigma'}\right]\\
\hat{\delta}z_{[\mu\nu]} &= -\frac{1}{4}\alpha_*\left(\delta_{\text{Group}}\,R(E)^{\rho}_{\rho[\mu\nu]}\right)\\
\hat{\delta}v_{[\mu\nu]} &= -\frac{1}{4}(1-\alpha)\delta_{\text{Group}}\,R(E)^{\rho}_{\rho[\mu\nu]}\\
\hat{\delta}\omega^0_{\mu mn} &= -\frac{1}{2}\delta_{\text{Group}}\left[R(P)^0_{mn\mu} - R(P)^0_{\mu mn} + R(P)^0_{\mu nm}\right]\\
\hat{\delta}\omega_m &= (1-\beta)\delta_{\text{Group}}\,R(P)_m\\
\hat{\delta}b_\mu &= \frac{1}{6}\beta\delta_{\text{Group}}\,R(P)_\mu\\
\hat{\delta}\phi_\mu &= \frac{1}{2\sqrt{2}}\delta_{\text{Group}}\left[\gamma^\rho R(Q)_{\mu\rho} + \frac{1}{6}\gamma_\mu\gamma^{\sigma\rho}R(Q)_{\rho\sigma}\right].
\end{aligned}$$

$R(P)_{\mu\nu m}$ and $\omega_{\mu mn}$ by $R(P)^0_{\mu\nu m} = R(P)_{\mu\nu m} + \frac{2}{3}R(P)_{[\mu}e_{\nu]m}$ and $\omega^0_{\mu mn} = \omega_{\mu mn} + \frac{2}{3}e_{\mu[m}\omega_{n]}$

$$R(P)_\mu = R(P)_{\rho\mu}{}^{\rho} \text{ and } \omega_m = \omega_{\rho m}{}^{\rho}.$$

$z_{[\mu\nu]} + {}_*v_{[\mu\nu]}$ and $b_\mu + \frac{1}{6}\omega_\mu\ \alpha v_{[\mu\nu]} + (1-\alpha)_*z_{[\mu\nu]}$ and $(1-\beta)b_\mu - \frac{1}{6}\beta\omega_\mu$

$$\begin{aligned}
\hat{\delta}_V\omega^0_{\mu mn} &= 2R(E)_{mn\mu k}\epsilon^k &&; \hat{\delta}_Z\omega^0_{\mu mn} = 2\tilde{R}(E)_{mn\mu k}\epsilon^k\\
\hat{\delta}_V\omega_m &= 0 = \hat{\delta}_V b_\mu &&; \hat{\delta}_Z\omega_m = 0 = \hat{\delta}_Z b_\mu\\
\hat{\delta}_V v_\mu{}^m &= 0 = \hat{\delta}_V z_\mu{}^m &&; \hat{\delta}_Z v_\mu{}^m = 0 = \hat{\delta}_Z z_\mu{}^m\\
\hat{\delta}_V\phi_\mu &= \frac{1}{\sqrt{2}}R(Q)_{\mu\nu}\epsilon^\nu &&; \hat{\delta}_Z\phi_\mu = \frac{1}{\sqrt{2}}\gamma^5R(Q)_{\mu\nu}\epsilon^\nu
\end{aligned}$$

$$\begin{aligned}
e^n\hat{\delta}_V\widetilde{\omega}_{mn} &= -2\tilde{R}(E)_{mn}\epsilon^n\\
e^n\hat{\delta}_Z\widetilde{\omega}_{mn} &= 2R(E)_{mn}\epsilon^n\\
e^m\gamma_m\hat{\delta}_V\phi &= -\frac{1}{\sqrt{2}}\gamma_m R(Q)\epsilon^m\\
e^m\gamma_m\hat{\delta}_Z\phi &= +\frac{1}{\sqrt{2}}\gamma^5\gamma_m R(Q)\epsilon^m
\end{aligned}$$

$$\begin{aligned}
-\delta_V\mathcal{L} &= 32\left[\tilde{R}(M)^{mn}R(V)_m - 2R(D)R(Z)^n\right]\epsilon_n\\
&\quad +32R(V)^m\delta_{V,\text{ Group}}\left[R(Z)_m - {}_*R(V)_m\right]\\
-\delta_Z\mathcal{L} &= 32\left[\tilde{R}(M)^{mn}R(Z)_m + 2R(D)R(V)^n\right]\epsilon_n\\
&\quad +32R(V)^m\delta_{Z,\text{ Group}}\left[R(Z)_m - {}_*R(V)_m\right].
\end{aligned}$$

$$\begin{aligned}
\delta_{V,\text{ Group}}\left[R(Z)_m - {}_*R(V)_m\right] &= -\tilde{R}(M)_{mn}\epsilon^n + 2{}_*R(D)\epsilon_m\\
\delta_{Z,\text{ Group}}\left[R(Z)_m - {}_*R(V)_m\right] &= R(M)_{mn}\epsilon^n - 2R(D)\epsilon_m
\end{aligned}$$

$$R(M)^{mn} = -\, {}_*\tilde{R}(M)^{mn}$$
$$R(A) = {}_*R(D)$$

$$-\delta_Q\mathcal{L} = -16\sqrt{2}\overline{R(S)}\gamma^5\gamma^m e_m\hat{\delta}_Q\phi - 128\tilde{R}(F)_{mn}e^m\hat{\delta}_Q v^n - 128R(F)_{mn}e^m\hat{\delta}_Q z^n$$
$$-\delta_E\mathcal{L} = 164R(K)^m\left[\tilde{\epsilon}_{mn}R(V)^n - \epsilon_{mn}R(Z)^n + \frac{1}{2}e^n\hat{\delta}_E\tilde{\omega}_{mn}\right]$$
$$-128\tilde{R}(F)_{mn}e^m\hat{\delta}_E v^n - 128R(F)_{mn}e^m\hat{\delta}_E z^n - 16\sqrt{2}\overline{R(S)}\gamma^5\gamma^m e_m\hat{\delta}\phi$$

$$\begin{aligned}
\hat{\delta}_Q\phi_\mu &= \frac{1}{3\sqrt{2}}[R(D)_{\mu\rho} + R(A)_{\mu\rho}\gamma^5 + \frac{1}{2}(R(V)_\rho + R(Z)_\rho\gamma^5)\gamma_\mu]\gamma^\rho \\
\hat{\delta}_Q v_{\mu\nu} &= \frac{1}{8\sqrt{2}}(1-\alpha)\overline{R(Q)}\epsilon \\
\hat{\delta}_Q z_{\mu\nu} &= \frac{1}{8\sqrt{2}}\alpha\overline{R(Q)}\gamma^5\epsilon \\
\hat{\delta}_E\phi_\mu &= -\frac{1}{4}[\gamma^\rho\gamma^{mn}R(S)_{\mu\rho} + \frac{1}{6}\gamma_\mu\gamma^{\sigma\rho}\gamma^{mn}R(S)_{\rho\sigma}]\epsilon_{mn} \\
\hat{\delta}_E\omega^0_{\mu mn} &= R(V)_{mn}{}^r\epsilon_{r\mu} - R(V)_{\mu m}{}^r\epsilon_{rn} + R(V)_{\mu n}{}^r\epsilon_{rm} \\
&\quad + R(Z)_{mn}{}^\rho{}_*\epsilon_{\rho\mu} - R(Z)_{\mu m}{}^r\tilde{\epsilon}_{rn} + R(Z)_{\mu n}{}^r\tilde{\epsilon}_{rm} - \frac{2}{3}e_{\mu[m}\hat{\delta}\omega_{n]} \\
\hat{\delta}_E\omega_\mu &= 2(1-\beta)(\tilde{\epsilon}_{mn}R(Z)^n + \epsilon_{mn}R(V)^n) \\
\hat{\delta}_E v_{[\mu\nu]} &= \frac{1}{8}(1-\alpha)[R(M)_{\rho\sigma\mu\nu}\epsilon^{\rho\sigma} + 8R(D)_{\rho[\mu}\epsilon_{\nu]}{}^\rho] \\
\hat{\delta}_E z_{[\mu\nu]} &= \frac{1}{8}\alpha[R(M)_{\rho\sigma\mu\nu\,*}\epsilon^{\rho\sigma} + 8R(D)_{\rho[\mu\,*}\epsilon_{\nu]}{}^\rho] \\
\hat{\delta}_E v_{(\mu\nu)} &= \frac{1}{6}g_{\mu\nu}R(D)_{\rho\sigma}\epsilon^{\rho\sigma} \\
\hat{\delta}_E z_{(\mu\nu)} &= \frac{1}{6}g_{\mu\nu}R(D)_{\rho\sigma\,*}\epsilon^{\rho\sigma},
\end{aligned}$$

$$R(V)_\nu = R(V)_{\rho\nu}{}^\rho \text{ and } R(Z)_\nu = R(Z)_{\rho\nu}{}^\rho$$

$$R(Z)^m = {}_*R(V)^m$$

$$R(V)_{\mu\rho\sigma} + R(V)_{\rho\sigma\mu} + R(V)_{\sigma\mu\rho} = -e\epsilon_{\mu\rho\sigma\tau}R(Z)^\tau$$
$$R(Z)_{\mu\rho\sigma} + R(Z)_{\rho\sigma\mu} + R(Z)_{\sigma\mu\rho} = e\epsilon_{\mu\rho\sigma\tau}R(V)^\tau$$

$$-\delta_Q\mathcal{L} = 8\sqrt{2}e\overline{R(Q)}_{\mu\nu}\epsilon\left[g_{\rho\sigma*}\tilde{R}(F)^{\rho\mu\nu\sigma}\right] + \cdots$$

$z_{[\mu\nu]}$ ${}_*v_{[\mu\nu]}$ $g_{\rho\sigma*}\tilde{R}(F)^{\rho\mu\nu\sigma}$ $\delta_Q\mathcal{L}$ $R(F)_{\mu\nu}{}^{mn}$ ${}_*\tilde{R}(F)_{\mu\nu}{}^{mn}$$\hat{\delta}z_{[\mu\nu]}$ $\hat{\delta}_* v_{[\mu\nu]}$ $(1-\alpha)_*\hat{\delta}z_{[\mu\nu]} = -\alpha\hat{\delta}v_{[\mu\nu]}$.

$$-\delta_E\mathcal{L} = -16e\left(R(M)_{\mu\nu\alpha\beta}\epsilon^{\alpha\beta} + 4R(D)_{\alpha[\mu}\epsilon^\alpha_{\nu]}\right)\left[g_{\rho\sigma*}\tilde{R}(F)^{\rho\mu\nu\sigma}\right] + \cdots$$

$R(K)^{\rho\mu}{}_\rho \equiv R(K)^\mu$ $\delta_E\mathcal{L}$

$$-\delta_E \mathcal{L} = -\frac{64}{3}(2\beta - 3)eR(K)^{\mu}\left[\epsilon_{\mu\nu}R(V)^{\nu} + {}_*\epsilon_{\mu\nu}R(Z)^{\nu}\right] + \cdots,$$

$$\left(b_\mu + \tfrac{1}{2}\omega_\mu\right)\delta_E \mathcal{L}\, E_{mn}$$

$$b_\mu \to b_\mu - 2\epsilon_\mu\ a_\mu, b_\mu\ z_{[\mu\nu]} - {}_*v_{[\mu\nu]} e_\mu{}^m, E_\mu{}^{mn}, \psi_\mu, b_\mu, a_\mu z_{[\mu\nu]} - {}_*v_{\mu\nu}$$

$$\mathcal{L} = \mathcal{L}\left(e_\mu{}^m, E_\mu{}^{mn}, \psi_\mu, b_\mu, a_\mu, z_{[\mu\nu]} - {}_*v_{[\mu\nu]}\right).$$

$$b_\mu \ \to b_\mu - 2\epsilon_\mu$$
$$z_{[\mu\nu]} - {}_*v_{[\mu\nu]} \ \to z_{[\mu\nu]} - {}_*v_{[\mu\nu]} + 2e_{m[\nu}\tilde{E}^{mn}_{\mu]}\epsilon_n - 2_*[E^n_{[\mu\nu]}\epsilon_n].$$

$$a_\mu \ \to a_\mu - \tilde{E}_\mu{}^{mn}\epsilon_{mn}$$
$$b_\mu \ \to b_\mu - E_\mu{}^{mn}\epsilon_{mn}$$
$$z_{[\mu\nu]} - {}_*v_{[\mu\nu]} \ \to z_{[\mu\nu]} - {}_*v_{[\mu\nu]} + 4_*\epsilon_{\mu\nu}.$$

$$z_{[\mu\nu]} - {}_*v_{\mu\nu}\ a'_\mu = a_\mu + \Delta a_\mu(b, z - {}_*v)\ \mathcal{L}\left(a_\mu, b_\mu, z_{[\mu\nu]} - {}_*v_{\mu\nu}\right) = \mathcal{L}\left(a'_\mu, 0{,}0\right)$$

$$\delta_{\text{Gen. coord.}} h_\mu{}^A = \partial_\mu \xi^\rho h_\rho{}^A + \xi^\rho \partial_\rho h_\mu{}^A,$$

$$\delta_{\text{Group},\text{P}^m}\left(\epsilon^m = \xi^\rho e_\rho{}^m\right)h_\mu{}^A + \xi^\rho R_{\rho\mu}{}^A$$
$$= \delta_{\text{Gen. coord.}}(\xi^\rho)h_\mu{}^A - \delta_{\text{Group}}\left(\epsilon^{B\neq P_m} = \xi^\rho h_\rho{}^B\right)h_\mu{}^A.$$

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)]e_\mu{}^m = \delta_P\left(\frac{1}{2\sqrt{2}}\bar{\epsilon}_2\gamma^r\epsilon_1\right)e_\mu{}^m + \delta_E\left(-\frac{1}{2\sqrt{2}}\bar{\epsilon}_2\gamma^{rs}\epsilon_1\right)e_\mu{}^m$$

$$R(P)_{\mu\nu}{}^m = 0$$

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)]E_\mu{}^{mn} = \delta_P\left(\frac{1}{2\sqrt{2}}\bar{\epsilon}_2\gamma^r\epsilon_1\right)E_\mu{}^{mn} + \delta_E\left(-\frac{1}{2\sqrt{2}}\bar{\epsilon}_2\gamma^{rs}\epsilon_1\right)E_\mu{}^{mn}$$

$$R(E)_{\mu\nu}{}^{mn} = 0$$

$$\{Q_A, Q_B\} = E_{(AB)}$$
$$\{Q_A, Q_{\dot{B}}\} = P_{A\dot{B}}$$
$$\{Q_{\dot{A}}, Q_{\dot{B}}\} = E_{(\dot{A}\dot{B})}$$

$$-\delta_{V,\,\text{Group}}\,\mathcal{L} = 16[2\tilde{R}(M)^{mn}R(V)_m - 4R(D)R(Z)^n$$
$$+4\tilde{R}(E)^{mn}R(K)_m - \overline{R(Q)}\gamma^5\gamma^n R(S)]\epsilon_n$$
$$-\delta_{Z,\,\text{Group}}\,\mathcal{L} = 16[[2\tilde{R}(M)^{mn}R(Z)_m + 4R(D)R(V)^n$$
$$-4R(E)^{mn}R(K)_m + \overline{R(Q)}\gamma^n R(S)]\epsilon_n$$
$$-\delta_{Q,\,\text{Group}}\,\mathcal{L} = 8\big[\overline{R(S)}\{\gamma^5\gamma_m R(V)^m + \gamma_m R(Z)^m\} + \sqrt{2}\overline{R(Q)}\gamma^5\gamma_m R(K)^m\big]\epsilon$$
$$-\delta_{P,\,\text{Group}}\,\mathcal{L} = -32[\tilde{R}(M)^{mn}R(K)_m + 2R(A)R(K)^n + 2\tilde{R}(F)^{mn}R(V)_m$$
$$+2R(F)^{mn}R(Z)_m + 4\sqrt{2}\overline{R(S)}\gamma^5\gamma^n R(S)]\epsilon_n$$
$$-\delta_{E,\,\text{Group}}\,\mathcal{L} = 0$$

$$-\hat{\delta}\mathcal{L} = 32[v^m R(V)^n + z^m R(Z)^n - e^m R(K)^n]\hat{\delta}\tilde{\omega}_{mn}$$
$$+32[\tilde{R}(M)^{mn}v_m - 2\tilde{R}(F)^{mn}e_m + 2\tilde{R}(E)^{mn}f_m$$
$$-2R(D)z^n - \frac{1}{4}\bar{\psi}\gamma^5\gamma^n R(S) - \frac{1}{4}\overline{R(Q)}\gamma^5\gamma^n\phi\Big]\,\hat{\delta}v_n$$
$$+32[\tilde{R}(M)^{mn}z_m - 2R(F)^{mn}e_m - 2R(E)^{mn}f_m$$
$$+2R(D)v^n + \frac{1}{4}\bar{\psi}\gamma^n R(S) + \frac{1}{4}\overline{R(Q)}\gamma^n\phi\Big]\,\hat{\delta}z_n$$
$$+16\Big[\overline{R(Q)}(\gamma^5 v^m\gamma_m - z^m\gamma_m) - \frac{1}{2}\bar{\psi}(\gamma^5 R(V)^m\gamma_m - R(Z)^m\gamma_m)$$
$$-\sqrt{2}\overline{R(S)}\gamma^5 e^m\gamma_m]\hat{\delta}\phi$$

$$+32[\tilde{R}(M)^{mn}e_n - 2\tilde{R}(E)^{mn}v_n + 2R(E)^{mn}z_n$$
$$+2R(A)e^m - \frac{1}{2\sqrt{2}}\bar{\psi}\gamma^5\gamma^m R(Q)\Big]\,\hat{\delta}f_m$$
$$+64\big[R(V)^m e^n\hat{\delta}\tilde{F}_{mn} + R(Z)^m e^n\hat{\delta}F_{mn}\big]$$
$$-32[R(V)^m z_m - R(Z)^m v_m]\hat{\delta}b + 64R(K)^m e_m\hat{\delta}a$$
$$-8\big[\bar{\phi}(R(V)^m\gamma^5\gamma_m + R(Z)^m\gamma_m) - 2\overline{R(S)}(v^m\gamma^5\gamma_m + z^m\gamma_m)$$
$$+\sqrt{2}\bar{\psi}R(K)^m\gamma^5\gamma_m - 2\sqrt{2}\overline{R(Q)}f^m\gamma^5\gamma_m]\hat{\delta}\psi$$
$$-64\big[(R(K)^m v^n - f^m R(V)^n)\hat{\delta}\tilde{E}_{mn} - (R(K)^m z^n - f^m R(Z)^n)\hat{\delta}E_{mn}\big]$$
$$+32[\tilde{R}(M)^{mn}f_n - 2R(A)f^m + 2\tilde{R}(F)^{mn}v_n$$
$$+2R(F)^{mn}z_n + \frac{1}{2\sqrt{2}}\overline{R(S)}\gamma^5\gamma^m\phi\Big]\,\hat{\delta}e_m.$$

$$0 = \hat{\delta}a = \hat{\delta}\psi = \hat{\delta}e^m = \hat{\delta}E^{mn}$$

$$h^A \to h^A + \hat{\delta}h^A\; d\hat{\delta}h^A$$

$$-64R(V)_m\tilde{E}^{mn}\hat{\delta}f_n + 64R(Z)_m E^{mn}\hat{\delta}f_n R(Z)^m = {}_*R(V)^m\; R(Z)^m\; R(V)^m\; R(V)^m$$

$$-32R(V)^m\left\{\begin{matrix} 2\hat{\delta}\tilde{F}_{mn}e^n + 2\tilde{E}_{mn}\hat{\delta}f^n + \frac{1}{4}\bar{\psi}\gamma^5\gamma_m\hat{\delta}\phi - \hat{\delta}\tilde{\omega}_{mn}v^n + z_m\hat{\delta}b \\ - {}_*\Big[-2\hat{\delta}F_{mn}e^n + 2E_{mn}\hat{\delta}f^n + \frac{1}{4}\bar{\psi}\gamma_m\hat{\delta}\phi + \hat{\delta}\tilde{\omega}_{mn}z^n + v_m\hat{\delta}b\Big] \end{matrix}\right\}.$$

$$0 = \delta[R(Z)_m - {}_*R(V)_m]$$
$$= \delta_{\text{Group}}\,[R(Z)_m - {}_*R(V)_m]$$

$$+\begin{Bmatrix} d\delta\hat{z}_m+\hat{\delta}(\omega_{mn}z^n)-2\hat{\delta}v_m a+2\hat{\delta}\tilde{F}_{mn}e^n+2\tilde{E}_{mn}\hat{\delta}f^n+\frac{1}{4}\bar{\psi}\gamma^5\gamma_m\hat{\delta}\phi \\ -*\left[d\hat{\delta}v_m+\hat{\delta}(\omega_{mn}v^n)+2\hat{\delta}z_m a-2\hat{\delta}F_{mn}e^n+2E_{mn}\hat{\delta}f^n+\frac{1}{4}\bar{\psi}\gamma_m\hat{\delta}\phi\right]\end{Bmatrix}.$$

$$\begin{aligned}-\hat{\delta}\mathcal{L}=&32R(V)^m\{\delta_{\text{Group}}\,[R(Z)_m-{}_*R(V)_m]\\ &+\hat{\delta}\widetilde{\omega}_{mn}v^n+{}_*\big(\hat{\delta}\widetilde{\omega}_{mn}z^n\big)-z_m\hat{\delta}b+{}_*(v_m\hat{\delta}b)\}\\ &-128\tilde{R}(F)_{mn}e^m\hat{\delta}v^n-128R(F)_{mn}e^m\hat{\delta}z^n\\ &-64R(K)^m\left[E_{mn}\hat{\delta}z^n-\tilde{E}_{mn}\hat{\delta}v^n-\frac{1}{2}e^n\hat{\delta}\widetilde{\omega}_{mn}\right]\\ &+64R(D)\big[\hat{\delta}v^m z_m+v^m\hat{\delta}z_m\big]+64R(A)\big[z^m\hat{\delta}z_m+v^m\hat{\delta}v_m\big]\\ &+32R(M)^{mn}\left[-\hat{\delta}v_m z_n+v_m\hat{\delta}z_n+\frac{1}{2}\epsilon_{mnpq}\big(v^p\hat{\delta}v^q+z^p\hat{\delta}z^q\big)\right]\\ &+16\overline{R(Q)}[\gamma^5\gamma^m v_m-\gamma^m z_m]\hat{\delta}\phi-16\sqrt{2R(S)}\gamma^5\gamma^m e_m\hat{\delta}\phi\\ &+16\bar{\psi}[\gamma^m\hat{\delta}z_m-\gamma^5\gamma^m\hat{\delta}v_m]R(S),\end{aligned}$$

$$[e_\mu{}^m]=[E_\mu{}^{mn}]=0,[\psi_\mu]=1/2,[\omega_\mu{}^{mn}]=[b_\mu]=[a_\mu]=[v_\mu{}^m]=[z_\mu{}^m]=1,[\phi_\mu]=$$

$$3/2,[f_\mu{}^m]=[F_\mu{}^{mn}]=2。$$

$$\epsilon_{\mu\nu\rho\sigma}\epsilon^{mnrs}=-e_\mu{}^m e_\nu{}^n e_\rho{}^r e_\sigma{}^s+\cdots$$

$$-\mathcal{L}_{\text{Kin}}=-2\left[R(\omega)_{\mu\nu}R(\omega)^{\nu\mu}-\frac{1}{3}R(\omega)^2\right]+24R(A)_{\mu\nu}R(A)^{\mu\nu}$$

$$R(\omega)_{\mu\nu}\equiv R(\omega)_{\alpha\mu\nu}{}^{\alpha}\quad R(\omega)\equiv R(\omega)_\mu{}^\mu$$

$$R(\omega)_{\mu\nu}{}^{mn}=2\partial_{[\mu}\omega_{\nu]}{}^{mn}+2\omega_{[\mu}{}^{k[m}\omega_{\nu]}{}^{n]}k$$

$$-\mathcal{L}_{\text{Kin}}=-2\left[R(\omega)_{\mu\nu}R(\omega)^{\nu\mu}-\frac{1}{3}R(\omega)^2\right]-8R(A)_{\mu\nu}R(A)^{\mu\nu}$$

$$\delta_{\text{Group}}\,\tilde{R}(E)_{\mu\nu\rho\sigma}\equiv\frac{1}{2}e\epsilon_{\rho\sigma\alpha\beta}\delta_{\text{Group}}\,R(E)_{\mu\nu}{}^{\alpha\beta}\neq\delta_{\text{Group}}\,{}_*R(E)_{\mu\nu\rho\sigma}\equiv\frac{1}{2}e\epsilon_{\mu\nu\alpha\beta}\delta_{\text{Group}}\,R(E)^{\alpha\beta}{}_{\rho\sigma}$$

$$\partial_i\tilde{\partial}^i\mathcal{E}_{jk}=\partial_i\tilde{\partial}^i d=0$$

$$\mathcal{H}_{MN}\eta^{NP}\mathcal{H}_{PQ}\eta^{QR}=\mathcal{H}_{MN}\mathcal{H}^{NR}=\delta_M^R$$

$$\partial_M\partial^M\mathcal{H}_{NP}=\partial^M\mathcal{H}_{NP}\partial_M\mathcal{H}_{QR}=\partial^M\mathcal{H}_{NP}\partial_M d=\partial^M\mathcal{H}_{NP}\partial_M\zeta^Q=\partial^M d\partial_M d=\cdots=0$$

$$X_I=\Theta_I{}^{\Lambda}T_\Lambda$$

$$[X_I,X_J]=-X_{IJ}{}^K X_K$$

$$\begin{aligned}\xi_+{}^A\xi_{+A}&=0\\ \xi_+{}^A f_{+ABC}&=0\\ f_{+[AB}{}^E f_{+C]DE}&=\frac{2}{3}f_{+[ABC}\xi_{+D]}\end{aligned}$$

$$w^i p_i = \frac{1}{2}\eta^{MN} P_M P_N = N - \bar{N}$$

$$\eta = (\eta^{MN}) = \begin{pmatrix} \eta^{ij} & \eta^{i}{}_{j} \\ \eta_{i}{}^{j} & \eta_{ij} \end{pmatrix} = \begin{pmatrix} 0 & \delta^{i}{}_{j} \\ \delta_{i}{}^{j} & 0 \end{pmatrix}$$

$$\mathcal{H} = (\mathcal{H}_{MN}) = \begin{pmatrix} \mathcal{H}_{ij} & \mathcal{H}_{i}{}^{j} \\ \mathcal{H}^{i}{}_{j} & \mathcal{H}^{ij} \end{pmatrix} = \begin{pmatrix} g_{ij} - B_{ik} g^{kl} B_{lj} & B_{ik} g^{kj} \\ -g^{ik} B_{kj} & g^{ij} \end{pmatrix}.$$

$$P_M \longrightarrow O_M^N P_N, O_M^N \in O(D, D, \mathbb{Z}), O_M^P \eta_{PQ} O_N^Q = \eta_{MN}$$

$$e^{-2d} = e^{-2\phi}\sqrt{|g|}$$

$$\partial_i \tilde{\partial}^i \mathcal{E}_{jk} = \partial_i \tilde{\partial}^i d = \partial_i \tilde{\partial}^i \epsilon_j = \partial_i \tilde{\partial}^i \tilde{\epsilon}_j = 0$$

$$S = \int d^{2D} X e^{-2d} \Big(\frac{1}{8}\mathcal{H}^{MN}\partial_M \mathcal{H}^{PQ}\partial_N \mathcal{H}_{PQ} - \frac{1}{2}\mathcal{H}^{MN}\partial_M \mathcal{H}^{PQ}\partial_Q \mathcal{H}_{PN} \\ -2\mathcal{H}^{MN}\partial_M d \partial_N d + 4\partial_M \mathcal{H}^{MN}\partial_N d)$$

$$\mathcal{H}_{MN}\eta^{NP}\mathcal{H}_{PQ} = \eta_{MQ}.$$

$$\partial_M \partial^M \mathcal{H}_{NP} = \partial_M \partial^M d = \partial_M \partial^M \zeta^N = \partial_M \mathcal{H}_{NP}\partial^M \mathcal{H}_{QR} = \partial_M \mathcal{H}_{NP}\partial^M d = \\ \partial_M \mathcal{H}_{NP}\partial^M \zeta^Q = \partial_M d \partial^M d = \partial_M d \partial^M \zeta^N = \partial_M \zeta^N \partial^M \zeta^Q = 0.$$

$$\hat{\delta}\mathcal{H}_{MN} = \hat{\mathcal{L}}_\zeta \mathcal{H}_{MN} = \zeta^P \partial_P \mathcal{H}_{MN} + (\partial_M \zeta^P - \partial^P \zeta_M)\mathcal{H}_{PN} + (\partial_N \zeta^P - \partial^P \zeta_N)\mathcal{H}_{MP}$$

$$\zeta_3^M = [\zeta_1, \zeta_2]_{(C)}^M = 2\zeta_{[1}^N \partial_N \zeta_{2]}^M - \zeta_{[1}^N \partial^M \zeta_{2]N}$$

$$\hat{\delta} e^{-2d} = \partial_M (\zeta^M e^{-2d})$$

$$S = \int d^{2D} X e^{-2d} \mathcal{R}$$

$$\mathcal{R} = \frac{1}{8}\mathcal{H}^{MN}\partial_M \mathcal{H}^{PQ}\partial_N \mathcal{H}_{PQ} - \frac{1}{2}\mathcal{H}^{MN}\partial_M \mathcal{H}^{PQ}\partial_Q \mathcal{H}_{PN} - \partial_M \partial_N \mathcal{H}^{MN} \\ +4\mathcal{H}^{MN}\partial_M \partial_N d - 4\mathcal{H}^{MN}\partial_M d \partial_N d + 4\partial_M \mathcal{H}^{MN}\partial_N d$$

$$\mathcal{H}_{MN} = E^A{}_M \mathcal{H}_{AB} E^B{}_N$$
$$\eta_{MN} = E^A{}_M \eta_{AB} E^B{}_N$$

$$(\mathcal{H}_{AB}) = \begin{pmatrix} h_{ab} & 0 \\ 0 & h^{ab} \end{pmatrix}.$$

$$(E_A^M) = \begin{pmatrix} e_a^i & e_a^j B_{ij} \\ 0 & e_i^a \end{pmatrix}$$

$$\delta E_A{}^M = \Lambda_A{}^B E_B{}^M$$

$$\Omega_{ABC} = E_A^M (\partial_M E_B^N) E_{CN} = -\Omega_{ACB}$$

$$\hat{\delta}_\zeta E_A{}^M = \hat{\mathcal{L}}_\zeta E_A{}^M = \zeta^N \partial_N E_A{}^M + \partial^M \zeta_N E_A{}^N - \partial_N \zeta^M E_A{}^N$$

$$[E_A, E_B]^M_{(C)} = [E_A, E_B]^M_{(D)} = F_{AB}{}^C E_C{}^M$$

$$F_{ABC} = \Omega_{ABC} + \Omega_{CAB} + \Omega_{BCA} = 3\Omega_{[ABC]}$$

$$\tilde{\Omega}_A = 2E_A{}^M \partial_M d + \Omega^B{}_{BA}$$

$$\mathcal{R} = 2\mathcal{H}^{AB} E_A{}^M \partial_M \tilde{\Omega}_B - \mathcal{H}^{AB} \tilde{\Omega}_A \tilde{\Omega}_B + \frac{1}{4}\mathcal{H}^{AB} F_{ACD} F_B{}^{CD}$$
$$-\frac{1}{12}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF} F_{ACE} F_{BDF} - \frac{1}{2}\mathcal{H}^{AB}\Omega^{CD}{}_A \Omega_{CDB}.$$

$$S = \int d^{2D} X e^{-2d} \left(\frac{1}{4}\mathcal{H}^{AB} F_{ACD} F_B^{CD} - \frac{1}{12}\mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF} F_{ACE} F_{BDF} + \mathcal{H}^{AB}\tilde{\Omega}_A \tilde{\Omega}_B \right).$$

$$\delta S = \int d^{2D} X e^{-2d} K_{AB} \Delta^{AB}$$

$$K_{[AB]} = 0.$$

$$K_{[AB]} = \frac{1}{2}(\tilde{\Omega}_C - E_C{}^M \partial_M) Z^C{}_{AB} + \frac{1}{2} Z_{[A}{}^{CD} F_{B]CD} - 2\mathcal{H}_{[A}{}^C E_{B]}{}^M \partial_M \tilde{\Omega}_C,$$

$$Z_{ABC} = 3\mathcal{H}_{[A}{}^D F_{BC]D} - \mathcal{H}_A{}^D \mathcal{H}_B{}^E \mathcal{H}_C{}^F F_{DEF}.$$

$$F = \sum_p F_p = \sum_p \frac{1}{p!} F_{i_1 \dots i_p} dx^{i_1} \wedge \dots \wedge dx^{i_p},$$

$$S = \frac{1}{4}\int * F \wedge F, F = * \sigma(F)$$

$$F = (d + H \wedge)C, (d + H \wedge)F = 0$$

$$\delta \mathcal{F} = \frac{1}{2}\Lambda_{AB} \Gamma^{AB} \mathcal{F}$$

$$\{\Gamma^A, \Gamma^B\} = \eta^{AB}$$

$$\mathcal{F} = H\mathcal{F}, H = (\Gamma^0 - \Gamma_0)(\Gamma^1 + \Gamma_1) \dots (\Gamma^D + \Gamma_D),$$

$$\mathcal{F} = \sum_p \frac{e^\phi}{p!} F_{i_1 \dots i_p} e^{i_1}_{a_1} \dots e^{i_p}_{a_p} \Gamma^{a_1 \dots a_p} |0\rangle$$

$$\nabla_A = e^d \left(E_A^M \partial_M - \frac{1}{2}\Omega_{ABC}\Gamma^{BC} \right) e^{-d}$$

$$\not\nabla = /0 - \not{F} - \frac{1}{2}\tilde{\mathring{A}} = \Gamma^A E_A{}^M \partial_M - \frac{1}{6}\Gamma^{ABC} F_{ABC} - \frac{1}{2}\Gamma^A \tilde{\Omega}_A$$

$$\mathcal{F} = \not{/}\mathcal{A}, \not\nabla \mathcal{F} = 0$$

$$S=\frac{1}{4}\int\ d^{2D}Xe^{-2d}\mathcal{F}^{T}B\mathcal{F},B=(\Gamma^{0}+\Gamma_{0})(\Gamma^{0}-\Gamma_{0})$$

$$\nabla^{2}\mathcal{A}=e^{d}\partial^{M}\partial_{M}\left(e^{-d}\mathcal{A}\right)-e^{d}\Gamma^{BC}\,\Omega^{A}{}_{BC}E_{A}{}^{M}\partial_{M}\left(e^{-d}\mathcal{A}\right)$$
$$+\left(-\frac{1}{4}\Gamma^{AB}\,(\partial^{M}\partial_{M}E_{A}{}^{N})E_{BN}+\frac{1}{16}\Gamma^{BCDE}\,\Omega^{A}{}_{BC}\Omega_{ADE}-\frac{1}{16}\Omega^{ABC}\Omega_{ABC}\right)\mathcal{A}=0,$$

$$\frac{SL(2,\mathbb{R})}{SO(2)}\times\frac{SO(6,n)}{SO(6)\times SO(n)}$$

$$V=\frac{1}{4}\Big(f_{\alpha ABC}f_{\beta DEF}\mathcal{M}^{\alpha\beta}\left[\frac{1}{3}\mathcal{M}^{AD}\mathcal{M}^{BE}\mathcal{M}^{CF}+\left(\frac{2}{3}\eta^{AD}-\mathcal{M}^{AD}\right)\eta^{BE}\eta^{CF}\right]$$
$$-\frac{4}{9}f_{\alpha ABC}f_{\beta DEF}\epsilon^{\alpha\beta}\mathcal{M}^{ABCDEF}+3\xi_{\alpha}^{M}\xi_{\beta}^{N}\mathcal{M}^{\alpha\beta}\mathcal{M}_{AB}\Big)$$

$$\xi^{A}\xi_{A}=0$$
$$\xi^{A}f_{ABC}=0$$
$$f^{E}_{[AB}f_{C]DE}=\frac{2}{3}f_{[ABC}\xi_{D]}$$

$$S=\int\ \Big(R*1-\frac{*\mathcal{D}\tau\wedge\mathcal{D}\bar{\tau}}{2(\mathrm{Im}\tau)^{2}}+\frac{1}{8}*\mathcal{D}\mathcal{M}_{AB}\wedge\mathcal{D}\mathcal{M}^{AB}$$
$$-\frac{\mathrm{Im}\tau}{2}\mathcal{M}_{AB}*F^{A}\wedge F^{B}+\frac{\mathrm{Re}\tau}{2}\eta_{AB}F^{A}\wedge F^{B}$$
$$+\frac{1}{2}A^{A}\wedge dA_{A}\wedge X+\frac{\hat{f}_{ABE}\hat{f}^{E}_{CD}}{8}A^{A}\wedge A^{B}\wedge A^{C}\wedge X^{D}$$
$$+\frac{1}{2}\xi_{A}B\wedge\left(dX^{A}-\hat{f}_{BC}A^{A}A^{B}\wedge X^{C}\right)-V(\mathcal{M})*1\Big)$$

$$\hat{f}_{ABC}=f_{ABC}-\xi_{[A}\eta_{C]B}-\frac{3}{2}\xi_{B}\eta_{AC}$$

$$\mathcal{D}\tau=d\tau+X+A\tau$$
$$\mathcal{D}\mathcal{M}_{AB}=d\mathcal{M}_{AB}+2A^{C}f^{D}_{C(A}\mathcal{M}_{B)D}+A_{(A}\mathcal{M}_{B)C}\xi^{C}-\xi_{(A}\mathcal{M}_{B)C}A^{C}$$
$$F^{A}=dA^{A}-\frac{1}{2}f_{BC}{}^{A}A^{B}\wedge A^{C}-\frac{1}{2}A\wedge A^{A}+\xi^{A}B$$

$$\mathcal{D}a\qquad\qquad=da+X+Aa$$
$$\mathcal{D}\phi=d\phi-\frac{1}{2}A(3.7)$$

$$S=\int\ \left(-\frac{e^{4\phi}}{2}*da\wedge da+\frac{a}{2}F^{A}\wedge F_{A}+\cdots\right)$$

$$dH=-\frac{1}{2}F^{A}\wedge F_{A}$$

$$S=\int\left(R*1-2*d\phi\wedge d\phi-\frac{e^{-4\phi}}{2}*H\wedge H\right.$$
$$\left.+\frac{1}{8}*\mathcal{D}\mathcal{M}_{AB}\wedge\mathcal{D}\mathcal{M}^{AB}-\frac{e^{-2\phi}}{2}\mathcal{M}_{AB}*F^{A}\wedge F^{B}-V(\mathcal{M})*1\right)$$

$$S=\int\left(-\frac{e^{4\phi}}{2}*X\wedge X-H\wedge X+\cdots\right)$$

$$H=dB-A\wedge B-\frac{1}{2}A^{A}\wedge dA_{A}+\frac{1}{6}f_{ABC}A^{A}\wedge A^{B}\wedge A^{C}$$

$$S=\int\left(R*1-2*\mathcal{D}\phi\wedge\mathcal{D}\phi-\frac{e^{-4\phi}}{2}*H\wedge H\right.$$
$$\left.+\frac{1}{8}*\mathcal{D}\mathcal{M}_{AB}\wedge\mathcal{D}\mathcal{M}^{AB}-\frac{e^{-2\phi}}{2}\mathcal{M}_{AB}*F^{A}\wedge F^{B}-V(\mathcal{M})*1\right)$$

$$\delta\phi=\frac{1}{2}\Lambda$$
$$\delta A^{A}=d\Lambda^{A}-f_{BC}{}^{A}A^{B}\Lambda^{C}-\xi^{A}\lambda+\frac{1}{2}(A^{A}\Lambda-A\Lambda^{A}+\xi^{A}A^{B}\Lambda_{B})$$
$$\delta B=d\lambda-\frac{1}{2}A\wedge\lambda-\frac{1}{2}d\Lambda^{A}\wedge A_{A}+\Lambda B$$
$$\delta\mathcal{M}_{AB}=-2\Lambda^{C}f_{C(A}{}^{D}\mathcal{M}_{B)D},+\xi_{(A}\mathcal{M}_{B)C}\mathcal{L}^{C}-\mathcal{L}_{(A}\mathcal{M}_{B)C}\xi^{C}$$

$$\hat{X}^{\hat{M}}=\left(x^{\mu},\tilde{x}_{\mu},Y^{M}\right)$$

$$\hat{\zeta}^{\hat{M}}=\left(\xi^{\mu}(x),e^{\gamma}\lambda_{\mu}(x),e^{\frac{\gamma}{2}}\Lambda^{A}(x)E_{A}^{M}(Y)\right)$$

$$\hat{E}_{a}{}^{\mu}=e^{-\phi-\frac{\gamma}{2}}e_{a}{}^{\mu}(x)$$
$$\hat{E}^{a\mu}=0$$
$$\hat{E}_{A}{}^{\mu}=0$$
$$\hat{E}_{a\mu}=e^{-\phi+\frac{\gamma}{2}}e_{a}{}^{\nu}\left(B_{\mu\nu}(x)-\frac{1}{2}A_{\mu}{}^{A}A_{\nu A}\right)$$
$$\hat{E}^{a}{}_{\mu}=e^{\phi+\frac{\gamma}{2}}e^{a}{}_{\mu}(x)$$
$$\hat{E}_{A\mu}=e^{\frac{\gamma}{2}}\Phi_{A}{}^{B}(x)A_{\mu B}(x)$$
$$\hat{E}_{a}{}^{M}=-e^{-\phi}e_{a}{}^{\mu}(x)A_{\mu}{}^{A}E_{A}{}^{M}(Y)$$
$$\hat{E}^{aM}=0$$
$$\hat{E}_{A}{}^{M}=\Phi_{A}{}^{B}(x)E_{B}{}^{M}(Y)$$

$$\hat{d}=-\frac{1}{4}\log\,\det g-\phi(x)+d(Y)$$

$$\hat{E}_{\hat{A}}\hat{M}\hat{E}_{\hat{B}\hat{M}}=\hat{\eta}_{\hat{A}\hat{B}}$$

$$\hat{\hat{\Omega}}_a = e^{-\phi-\frac{c}{2}}\left(\tau_{ab}^{b} - e_a^{\mu}\left(\partial_\mu\phi + e^{\frac{\gamma}{2}}A_\mu^A\tilde{\Omega}_A\right)\right),$$
$$\hat{\hat{\Omega}}^a = 0,$$
$$\hat{\hat{\Omega}}_A = \Phi_A{}^B\tilde{\Omega}_B,$$

$$\tilde{\Omega}_A = 2E_A^M\partial_M d + \Omega_{BA}^B$$

$$\hat{F}_{abc} = -e^{-3\phi-\frac{\gamma}{2}}e_a^\mu e_b^\nu e_c^\rho\{3\partial_{[\mu}B_{\nu\rho]} - 3A_{[\mu}^A B_{\nu\rho]}e^{\frac{\gamma}{2}}E_A\gamma$$
$$-3\partial_{[\mu}A_\nu^A A_{\rho]A} + A_\mu^A A_\nu^B A_\rho^C e^{\frac{\gamma}{2}}F_{ABC}\}$$

$$F_{ABC} = 3\left(E_{[A}^M\partial_M E_B^N\right)E_{C]}$$

$$\hat{F}_{abc} = -e^{-3\phi-\frac{\gamma}{2}}e_a^\mu e_b^\nu e_c^\rho H_{\mu\nu\rho}$$

$$e^{\frac{\gamma}{2}}E_A\gamma = \xi_A$$
$$e^{\frac{\gamma}{2}}F_{ABC} = f_{ABC}$$

$$-\frac{1}{12}\int d^{2D}X\sqrt{-g}e^{-4\phi-\gamma-2d}g^{\mu\nu}g^{\rho\sigma}g^{\lambda\tau}H_{\mu\rho\lambda}H_{\nu\sigma\tau}$$

$$\hat{F}_{abC} = -e^{-2\phi-\frac{\gamma}{2}}e_a^\mu e_b^\nu\Phi_C^D F_{\mu\nu D}$$
$$= -e^{-2\phi-\frac{\gamma}{2}}e_a^\mu e_b^\nu\Phi_C^D\{2\partial_{[\mu}A_{\nu]D} - f_{DAB}A_\mu^A A_\nu^B - A_{[\mu}A_{\nu]C}^A + \xi_C B_{\mu\nu}\}$$

$$\mathcal{M}^{AB} = \mathcal{H}^{CD}\Phi_C{}^A\Phi_D{}^B$$

$$\hat{F}_{aBC} = e^{-\phi-\frac{\gamma}{2}}e_a^\mu(\mathcal{D}_\mu\Phi_{BA})\Phi_C^A,$$
$$\mathcal{D}_\mu\Phi_{BA} = \partial_\mu\Phi_{BA} + A_\mu^C f_{CA}^D\Phi_{BD} + \frac{1}{2}A_{\mu A}\Phi_{BD}\xi^D - \frac{1}{2}\xi_A\Phi_{BD}A_\mu^D.$$

$$\frac{1}{4}\int d^{2D}Xe^{2\phi-\gamma-2d}\left\{\mathcal{M}^{AB}f_{ACD}f_B^{CD} - \frac{1}{3}\mathcal{M}^{AB}\mathcal{M}^{CD}\mathcal{M}^{EF}f_{ACE}f_{BDF} - 3\mathcal{M}^{AB}\xi_A\xi_B\right\}$$

$$\tilde{\Omega}_A = -\frac{e^{-\frac{\gamma}{2}}}{2}\xi_A$$

$$\hat{\hat{\Omega}}_a = e^{-\phi-\frac{c}{2}}(\tau_{ab}^b - \mathcal{D}_a\phi)$$
$$\hat{F}_{ab}^c = e^{-\phi-\frac{\gamma}{2}}(\tau_{ab}^c + 2\delta_{[a}^c\mathcal{D}_{b]}\phi)$$
$$\mathcal{D}_a\phi = e_a^\mu\left(\partial_\mu\phi - \frac{1}{2}A_\mu\right)$$
$$\tau_{ab}^c = \left(e_a^\mu\partial_\mu e_b^\nu - e_b^\mu\partial_\mu e_a^\nu\right)e_\nu^c.$$

$$f_{ABC} = e^{\frac{\gamma}{2}}F_{ABC} = cst.,$$
$$F_{ABC} = \Omega_{ABC} + \Omega_{CAB} + \Omega_{BCA} = F_{[ABC]}$$
$$\xi_A = e^{\frac{\gamma}{2}}E_A{}^M\partial_M\gamma = -2e^{\frac{\gamma}{2}}(2E_A{}^M\partial_M d + \Omega_{BA}^B) = cst.$$

$$\int d^{12}Ye^{-2d-\gamma}$$

$$E_{[A}{}^{M}\partial_M F_{BCD]} = 3\Omega_{[AB}{}^{E}\Omega_{CD]E} + 3\Omega^{E}{}_{[AB}\Omega_{CD]E}.$$

$$f^{E}_{[AB}f_{C]DE} = \frac{2}{3}f_{[ABC}\xi_{D]}$$

$$\Omega_{E[AB}\Omega^{E}_{C]D} = 0$$

$$\Omega^{E}{}_{AB}\Omega_{ECD} = 0.$$

$$\xi^{A}f_{ABC} = 0$$

$$\Omega_{ABC}E^{AM}\partial_M d = \Omega_{ABC}E^{AM}\partial_M\gamma = 0$$

$$(\partial^M\partial_M E^{N}_{[A})E_{B]N} = 0$$

$$\partial^M\gamma\partial_M\gamma = \tilde{\Omega}^A\tilde{\Omega}_A = 0$$

$$\tilde{V} = \frac{e^{2\phi}}{6}f_{ABC}f^{ABC}$$

$$\tilde{S} = -\frac{1}{6}\int d^{2D}Xe^{-2\hat{d}}\hat{F}_{\hat{A}\hat{B}\hat{C}}\hat{F}^{\hat{A}\hat{B}\hat{C}}$$

$$\tilde{S} = \int d^{2D}Xe^{-2\hat{d}}\left\{-2\partial_{\hat{M}}\partial^{\hat{M}}\hat{d} + \hat{\tilde{\Omega}}^{\hat{A}}\hat{\tilde{\Omega}}_{\hat{A}} - \frac{1}{2}\hat{\Omega}^{\hat{A}\hat{B}\hat{C}}\hat{\Omega}_{\hat{A}\hat{B}\hat{C}}\right\}$$

$$\mathcal{H}_{[A}{}^{G}F_{B]CD}F_{EFG}(\eta^{CE}\eta^{DF} - \mathcal{H}^{CE}\mathcal{H}^{DF}) = 0.$$

$$P_{\pm ABCD} = \frac{1}{2}(\eta_{A(C}\eta_{D)B} \pm \mathcal{H}_{A(C}\mathcal{H}_{D)B})$$

$$W^{AB} = P_{-}^{ABCD}Z_{CD} = 0$$

$$Z_{AB} = \frac{1}{4}F_{ACD}F_{BEF}P_{-}^{CEDF}$$

$$-\frac{1}{12}Z^{ABC}F_{ABC} = R^{ext}$$

$$V(\mathcal{M}) = -\frac{1}{12}Z^{ABC}(\mathcal{M})F_{ABC}$$

$$P_{\pm ABCD}(\mathcal{M}) = \frac{1}{2}(\eta_{A(C}\eta_{D)B} \pm \mathcal{M}_{A(C}\mathcal{M}_{D)B})$$

$$\hat{V} = V + L^{AB}(\mathcal{M}_{AB} - (\mathcal{M}^{-1})^{CD}\eta_{CA}\eta_{DB})$$

$$\frac{\partial\hat{V}}{\partial\mathcal{M}^{AB}} = -Z_{AB}(\mathcal{M}) + 2P_{+ABCD}(\mathcal{M})L^{CD} = 0$$

$$-Z_{AB}(\mathcal{M}) = \frac{\partial V}{\partial \mathcal{M}^{AB}} = -\frac{1}{4} F_{ACD} F_{BEF} (\eta^{CE}\eta^{DF} - \mathcal{M}^{CE}\mathcal{M}^{DF}) = F_{ACD} F_{BEF} P_{-}^{CEDF}(\mathcal{M}) = 0$$

$$P_{-}^{ABCD}(\mathcal{M}) Z_{CD}(\mathcal{M}) = 0$$

$$e^a T_a = g^{-1} dg, de^a = -\frac{1}{2} \tau_{bc}{}^a e^b \wedge e^c, [T_a, T_b] = \tau_{ab}{}^c T_c$$

$$\tau_{[ab}{}^d \tau_{c]d}{}^e = 0$$
$$H_{e[ab}\tau_{cd]}{}^e = 0.$$

$$E_A^N = \begin{pmatrix} e_a^n(y) & e_a^m(y) B_{mn}(y) \\ 0 & e^a{}_n(y) \end{pmatrix}_A^N$$

$$[E_a, E_b] = H_{abc}E^c + \tau_{ab}{}^c E_c$$
$$[E^a, E_b] = \tau_{bc}{}^a E^c$$
$$[E^a, E^b] = 0$$

$$H_{abc} = 3 e_{[a}{}^m e_b{}^n e_{c]}{}^p \partial_m B_{np}.$$

$$e^{-2d} = \det(e^a{}_m)$$

$$E_A{}^N = \begin{pmatrix} \delta_a{}^n & 0 \\ \beta^{ab}(y)\delta_b{}^m & \delta^a{}_n \end{pmatrix}_A^N$$

$$\Omega_a^{bc} = \partial_a \beta^{bc}$$
$$\Omega^{abc} = \beta^{ad}\partial_d \beta^{bc}$$

$$\beta^{ab}(y) = y^c \beta_c{}^{ab}$$

$$Q_d{}^{[ab} Q_e{}^{c]d} = 0$$
$$R^{e[ab} Q_e{}^{cd]} = 0$$

$$Q_a{}^{bc} = \beta_a{}^{bc}$$
$$R^{abc} = 3\beta^{[a|d}\beta_d{}^{|bc]} = 0.$$

$$\beta^{ab} = y^c \beta_c^{ab} + \tilde{y}_c \beta^{cab}$$

$$\beta^{a[bc}\beta_a^{de]} = 0$$

$$E_A{}^M = (e^{y\mathcal{N}})_A{}^B \delta_B^M$$

$$\Omega_{y\bar{A}\bar{B}} = \mathcal{N}_{\bar{A}\bar{B}}$$

$$[T_{\bar{A}}, T_{\bar{B}}] = \mathcal{N}_{\bar{A}\bar{B}} T$$
$$[\tilde{T}, T_{\bar{A}}] = \mathcal{N}_{\bar{A}\bar{B}} T^{\bar{B}}$$

$$E_A^M = \left(e^{Y^I T_I}\right)_A^B \delta_B^M$$

$$\Omega_{I\bar{A}\bar{B}} = T_{I\bar{A}\bar{B}}$$

$$[T_I, T_J] = 0$$
$$[T_I, T_{\bar{A}}] = T_{I\bar{A}}{}^{\bar{B}} T_{\bar{B}}$$
$$[T_{\bar{A}}, T_{\bar{B}}] = T_{I\bar{A}\bar{B}} T^I$$

$$T^I{}_{[\bar{A}\bar{B}} T_{I\bar{C}]\bar{D}} = 0$$

$$T_{Ia}{}^{b} = -T_I{}^{b}{}_{a} = \alpha_{Ia}\delta_a^b$$

$$\alpha^I{}_a \alpha_{Ib} = 0, a \neq b$$

$$\alpha_{Ia} = \pm \mathcal{H}_{IJ}\alpha^J{}_a$$

$$V(\mathcal{H}) = -e^{2\phi}(1 \mp 1)\alpha^I{}_a \alpha_{Ib}\delta^{ab},$$

$$S^{conf} = \int d^4x \sqrt{g}\left(\tfrac{1}{2}(\partial_\mu\phi)(\partial_\nu\phi)g^{\mu\nu} - \tfrac{1}{12}\phi^2 R - \tfrac{1}{4}\mathrm{Tr}\, F_{\mu\nu}g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma} - \tfrac{1}{2}\bar{\lambda}\,\not{D}\lambda\right)$$

$$g'_{\mu\nu} = \mathrm{e}^{-2\sigma(x)}g_{\mu\nu}, \phi' = \mathrm{e}^{\sigma(x)}\phi, W'_\mu = W_\mu, \lambda' = \mathrm{e}^{\frac{3}{2}\sigma(x)}\lambda.$$

$$S^{conf}_{gauge\text{-}fixed} \sim \int d^4x \sqrt{g}\left(-\tfrac{1}{2}M_P^2 R - \tfrac{1}{4}F_{\mu\nu}g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma} + \tfrac{1}{2}\bar{\lambda}\,\not{D}\lambda\right)$$

$$S^{grav} = \int d^4x\left\{\sqrt{g}(\mathcal{D}_\mu\phi)(\mathcal{D}_\nu\phi^*)g^{\mu\nu} - \frac{1}{6}|\phi|^2\left[\sqrt{g}R + \sqrt{g}\bar{\psi}_\mu R^\mu + \partial_\mu\left(\sqrt{g}\bar{\psi}\cdot\gamma\psi^\mu\right)\right]\right\},$$

$$R^\mu = e^{-1}\varepsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\mathcal{D}_\rho\psi_\sigma = \gamma^{\mu\rho\sigma}\mathcal{D}_\rho\psi_\sigma,$$

$$\mathcal{D}_\mu\psi_\nu = \left(\left(\partial_\mu + \frac{1}{4}\omega_\mu^{mn}\gamma_{mn} + \frac{1}{2}\mathrm{i}\gamma_5 A_\mu\right)\delta_\nu^\lambda - \Gamma_{\mu\nu}^\lambda\right)\psi_\lambda$$

$$g'_{\mu\nu} = \mathrm{e}^{-2\sigma(x)}g_{\mu\nu}, \phi' = \mathrm{e}^{\sigma(x)-\frac{1}{3}\mathrm{i}\Lambda(x)}\phi,$$
$$\psi'_\mu = \mathrm{e}^{-\frac{1}{2}[\sigma(x)+\mathrm{i}\gamma_5\Lambda(x)]}\psi_\mu, \quad A'_\mu = A_\mu + \partial_\mu\Lambda(x).$$

$$\phi = \phi^* = \sqrt{3}M_P.$$

$$-\frac{1}{6}\int d^4x\left[\sqrt{g}\bar{\tilde{\psi}}_\mu\tilde{R}^\mu - 2(\partial_\mu \ln\,\phi)\left(\sqrt{g}\bar{\tilde{\psi}}\cdot\gamma\tilde{\psi}^\mu\right)\right].$$

$$\tilde{R}^\mu - \gamma^\mu\tilde{\psi}^\nu\partial_\nu \ln\,\phi + \gamma\cdot\tilde{\psi}\partial_\mu \ln\,\phi = 0$$

$$\gamma\cdot\psi \neq 0 \ \text{ or } \ \psi_0 \neq 0$$

$$\Phi' = \mathrm{e}^{w\sigma(x)+\mathrm{i}c\Lambda(x)}\Phi$$

$$\mathcal{L} = [\mathcal{N}(X,X^*)]_D + [\mathcal{W}(X)]_F + \left[f_{\alpha\beta}(X)\bar{\lambda}_L^\alpha \lambda_L^\beta\right]_F$$

$$\mathcal{N} = X^I \mathcal{N}_I = X_I \mathcal{N}^I = X_I \mathcal{N}^I{}_J X^J, \quad \mathcal{N}_I = X_J \mathcal{N}^J{}_I$$
$$X_J \mathcal{N}^{JI} = 0, X_I \mathcal{N}^I{}_{JK} = \mathcal{N}_{JK}, \qquad X_K \mathcal{N}_J^{IK} = 0$$

$$\begin{aligned}
[\mathcal{N}]_D e^{-1} &= \tfrac{1}{6}\mathcal{N}(X,X^*)\left[R + \bar{\psi}_\mu R^\mu + e^{-1}\partial_\mu(e\bar{\psi}\cdot\gamma\psi^\mu) + \mathcal{L}_{SG,torsion}\right] \\
&- \mathcal{N}_I{}^J(X,X^*)\left[(\mathcal{D}_\mu X^I)(\mathcal{D}^\mu X_J) + \bar{\Omega}_J \not{\mathcal{D}}\Omega^I + \bar{\Omega}^I \not{\mathcal{D}}\Omega_J - h_J h^I\right] \\
&+ \Big\{-\mathcal{N}_J{}^{IK}\bar{\Omega}_I\Omega_K h^J + \mathcal{N}_K{}^{IJ}\bar{\Omega}_I(\not{\mathcal{D}}X_J)\Omega^K \\
&+ \mathcal{N}_J{}^I\bar{\psi}_{\mu L}(\not{\mathcal{D}}X^J)\gamma^\mu\Omega_I - \tfrac{2}{3}\mathcal{N}^I\bar{\Omega}_I\gamma^{\mu\nu}\hat{\mathcal{D}}_\mu\psi_{\nu L} - \tfrac{1}{8}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_\nu\psi_\rho\mathcal{N}^I\mathcal{D}_\sigma X_I \\
&+ \tfrac{1}{2}\mathcal{N}^I k_{\alpha I}(-\mathrm{i}D^\alpha + \bar{\psi}_L\cdot\gamma\lambda_R^\alpha) - 2\mathcal{N}_I{}^J k_{\alpha J}\bar{\lambda}_R^\alpha\Omega^I + \text{h.c.}\Big\} \\
&+ \mathcal{N}_J{}^I\left(\tfrac{1}{8}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_\nu\psi_\rho\bar{\Omega}^J\gamma_\sigma\Omega_I - \bar{\psi}_\mu\Omega^J\,\bar{\psi}_\mu\Omega_I\right) + \mathcal{N}_{KL}^{IJ}\bar{\Omega}_I\Omega_J\,\bar{\Omega}^K\Omega^L
\end{aligned}$$

$$[\mathcal{W}]_F e^{-1} = \mathcal{W}^I h_I - \mathcal{W}^{IJ}\bar{\Omega}_I\Omega_J + \mathcal{W}^I\bar{\psi}_R\cdot\gamma\Omega_I + \tfrac{1}{2}\mathcal{W}\bar{\psi}_{\mu R}\gamma^{\mu\nu}\psi_{\nu R} + \text{h.c.}$$

$$[\mathcal{W}]_F e^{-1} = \mathcal{W}^I h_I - \mathcal{W}^{IJ}\bar{\Omega}_I\Omega_J + \mathcal{W}^I\bar{\psi}_R\cdot\gamma\Omega_I + \tfrac{1}{2}\mathcal{W}\bar{\psi}_{\mu R}\gamma^{\mu\nu}\psi_{\nu R} + \text{h.c.}$$

$$\begin{aligned}
&\left[f_{\alpha\beta}\bar{\lambda}_L^\alpha\lambda_L^\beta\right]_F e^{-1} = \\
&\qquad \mathrm{Re}\, f_{\alpha\beta}(X)\left[-\tfrac{1}{4}F_{\mu\nu}^\alpha F^{\mu\nu\,\beta} - \tfrac{1}{2}\bar{\lambda}^\alpha\,\hat{\not{\mathcal{D}}}\lambda^\beta + \tfrac{1}{2}D^\alpha D^\beta + \tfrac{1}{8}\bar{\psi}_\mu\gamma^{\nu\rho}\left(F_{\nu\rho}^\alpha + \hat{F}_{\nu\rho}^\alpha\right)\gamma^\mu\lambda^\beta\right] \\
&\quad + \mathrm{i}\tfrac{1}{4}\,\mathrm{Im}\, f_{\alpha\beta}F_{\mu\nu}^\alpha\tilde{F}^{\mu\nu\,\beta} + \mathrm{i}\tfrac{1}{4}\left(\mathcal{D}_\mu\,\mathrm{Im}\, f_{\alpha\beta}\right)\bar{\lambda}^\alpha\gamma_5\gamma^\mu\lambda^\beta \\
&\quad + \Big\{\tfrac{1}{2}f_{\alpha\beta}^I(X)\left[\bar{\Omega}_I\left(-\tfrac{1}{2}\gamma^{\mu\nu}\hat{F}_{\mu\nu}^{-\alpha} + \mathrm{i}D^\alpha\right)\lambda_L^\beta - \tfrac{1}{2}\left(h_I + \bar{\psi}_R\cdot\gamma\Omega_I\right)\bar{\lambda}_L^\alpha\lambda_L^\beta\right] \\
&\qquad + \tfrac{1}{4}f_{\alpha\beta}^{IJ}\bar{\Omega}_I\Omega_J\bar{\lambda}_L^\alpha\lambda_L^\beta + \text{h.c.}\Big\}
\end{aligned}$$

$$\mathcal{D}_\mu = \partial_\mu - \mathrm{i}cA_\mu - W_\mu^\alpha\delta_\alpha + \frac{1}{4}\omega_\mu^{ab}(e)\gamma_{ab}$$

$$\hat{\mathcal{D}}_\mu = \partial_\mu - \mathrm{i}cA_\mu - W_\mu^\alpha\delta_\alpha + \frac{1}{4}\hat{\omega}_\mu^{ab}(e,\psi)\gamma_{ab}$$

$$\hat{\omega}_\mu^{ab}(e,\psi) = \omega_\mu^{ab}(e) + \frac{1}{4}\left(2\bar{\psi}_\mu\gamma^{[a}\psi^{b]} + \bar{\psi}^a\gamma_\mu\psi^b\right)$$

$$\hat{F}_{\mu\nu}^\alpha = F_{\mu\nu}^\alpha + \bar{\psi}_{[\mu}\gamma_{\nu]}\lambda^\alpha, F_{\mu\nu}^\alpha = 2\partial_{[\mu}W_{\nu]}^\alpha + W_\mu^\beta W_\nu^\gamma f_{\beta\gamma}^\alpha$$

$$\tilde{F}^{\mu\nu\alpha} = \frac{1}{2}e^{-1}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}^\alpha$$

$$\hat{F}_{\mu\nu}^{-\alpha} = \frac{1}{2}\left(\hat{F}_{\mu\nu}^\alpha - \tilde{\hat{F}}_{\mu\nu}^\alpha\right) = F_{\mu\nu}^{-\alpha} - \frac{1}{4}\bar{\psi}_{\rho L}\gamma_{\mu\nu}\gamma^\rho\lambda_R^\alpha + \frac{1}{4}\bar{\psi}_R\cdot\gamma\gamma_{\mu\nu}\lambda_R^\alpha$$

$$\mathcal{L}_{SG,\,\text{torsion}} = \frac{1}{16}\left[2(\bar{\psi}_\mu\gamma_\nu\psi_\rho)(\bar{\psi}^\nu\gamma^\mu\psi^\rho) + (\bar{\psi}_\mu\gamma_\nu\psi_\rho)^2 - 4(\bar{\psi}\cdot\gamma\psi_\mu)^2\right]$$

$$\delta_\alpha X_I = k_{\alpha I}(X) \quad , \qquad \delta_\alpha X^I = k_\alpha{}^I(X^*)\,,$$
$$\delta_\alpha \Omega_I = k_{\alpha I}{}^J\Omega_J \quad , \qquad \delta_\alpha \Omega^I = k_\alpha{}^I{}_J\Omega^J\,.$$

$$k_{\alpha IJ} = k_\alpha{}^{IJ} = 0, k_{\alpha I}{}^J X_J = k_{\alpha I}, k_\alpha{}^I{}_J X^J = k_\alpha{}^I$$

$$k_{\beta I}{}^J k_{\alpha J} - k_{\alpha I}{}^J k_{\beta J} = f_{\alpha\beta}^\gamma k_{\gamma I}$$

$$\delta_\gamma \lambda^\alpha = \lambda^\beta f_{\beta\gamma}^\alpha$$

$$\mathcal{N}^I k_{\alpha I} + \mathcal{N}_I k_\alpha{}^I = \mathcal{W}^I k_{\alpha I} = 0, f_{\alpha\beta}^I k_{\gamma I} + 2 f_{\delta(\alpha} f_{\beta)\gamma}^\delta = \mathrm{i} c_{\alpha\beta}$$

$$\begin{aligned}
\delta e_\mu^a &= \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu \\
\delta\psi_\mu &= \left(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}(e,\psi)\gamma_{ab} + \frac{1}{2}\mathrm{i}A_\mu\gamma_5\right)\epsilon - \gamma_\mu\eta \\
\delta X_I &= \bar{\epsilon}_L\Omega_I \\
\delta\Omega_I &= \frac{1}{2}\gamma^\mu(\mathcal{D}_\mu X_I - \bar{\psi}_\mu\Omega_I)\epsilon_R + \frac{1}{2}h_I\epsilon_L + X_I\eta_L \\
\delta W_\mu^\alpha &= -\frac{1}{2}\bar{\epsilon}\gamma_\mu\lambda^\alpha \\
\delta\lambda^\alpha &= \frac{1}{4}\gamma^{\mu\nu}\hat{F}_{\mu\nu}^\alpha\epsilon + \frac{1}{2}\mathrm{i}\gamma_5\epsilon D^\alpha
\end{aligned}$$

$$\begin{aligned}
-h^J\mathcal{N}_J^I &= \mathcal{W}^I - \mathcal{N}^I{}_{JK}\bar{\Omega}^J\Omega^K - \frac{1}{4}f_{\alpha\beta}^I\bar{\lambda}_L^\alpha\lambda_L^\beta \\
(\mathrm{Re} f_{\alpha\beta})D^\beta &= \mathcal{P}_\alpha + \mathcal{P}_\alpha^F \\
\mathcal{P}_\alpha &= \mathrm{i}\frac{1}{2}[\mathcal{N}^I k_{\alpha I} - \mathcal{N}_I k_\alpha^I] = \mathrm{i}\mathcal{N}^I k_{\alpha I} = -\mathrm{i}\mathcal{N}_I k_\alpha^I \\
\mathcal{P}_\alpha^F &= -\mathrm{i}\frac{1}{2}f_{\alpha\beta}^I\bar{\Omega}_I\lambda^\beta + \mathrm{i}\frac{1}{2}f_{\alpha\beta I}\bar{\Omega}^I\lambda^\beta \\
A_\mu &= A_\mu^B + A_\mu^F \\
A_\mu^B &= \frac{3\mathrm{i}}{2\mathcal{N}}[\mathcal{N}^I\hat{\partial}_\mu X_I - \mathcal{N}_I\hat{\partial}_\mu X^I] = \frac{3\mathrm{i}}{2\mathcal{N}}[\mathcal{N}^I\partial_\mu X_I - \mathcal{N}_I\partial_\mu X^I] - \frac{3}{\mathcal{N}}W_\mu^\alpha\mathcal{P}_\alpha \\
A_\mu^F &= \frac{3\mathrm{i}}{2\mathcal{N}}\left[\mathcal{N}_I\bar{\psi}_{\mu R}\Omega^I - \mathcal{N}^I\bar{\psi}_{\mu L}\Omega_I - \mathcal{N}_I{}^J\bar{\Omega}_J\gamma_\mu\Omega^I - \frac{3}{4}(\mathrm{Re} f_{\alpha\beta})\bar{\lambda}^\alpha\gamma_\mu\gamma_5\lambda^\beta\right]
\end{aligned}$$

$$\begin{aligned}
e^{-1}\mathcal{L} &= \tfrac{1}{6}\mathcal{N}\left[R + \bar{\psi}_\mu R^\mu + e^{-1}\partial_\mu(e\bar{\psi}\cdot\gamma\psi^\mu) + \mathcal{L}_{SG,torsion}\right] \\
&- \mathcal{N}_I{}^J\left[(\mathcal{D}_\mu X^I)(\mathcal{D}^\mu X_J) + \bar{\Omega}_J\,\not{\mathcal{D}}\Omega^I + \bar{\Omega}^I\,\not{\mathcal{D}}\Omega_J\right] \\
&+ (\mathrm{Re}\,f_{\alpha\beta})\left[-\tfrac{1}{4}F_{\mu\nu}^\alpha F^{\mu\nu\,\beta} - \tfrac{1}{2}\bar{\lambda}^\alpha\,\hat{\not{\mathcal{D}}}\lambda^\beta\right] + \mathrm{i}\tfrac{1}{4}(\mathrm{Im}\,f_{\alpha\beta})\left[F_{\mu\nu}^\alpha\tilde{F}^{\mu\nu\,\beta} - \hat{\partial}_\mu\left(\bar{\lambda}^\alpha\gamma_5\gamma^\mu\lambda^\beta\right)\right] \\
&- \mathcal{N}_I{}^J h_J h^I - \tfrac{1}{2}(\mathrm{Re}\,f_{\alpha\beta})D^\alpha D^\beta \\
&+ \tfrac{1}{8}(\mathrm{Re}\,f_{\alpha\beta})\bar{\psi}_\mu\gamma^{\nu\rho}\left(F_{\nu\rho}^\alpha + \hat{F}_{\nu\rho}^\alpha\right)\gamma^\mu\lambda^\beta
\end{aligned}$$

$$+\ \Big\{\mathcal{N}_K{}^{IJ}\bar{\Omega}_I(\hat{\not{\partial}}X_J)\Omega^K+\mathcal{N}_J{}^I\bar{\psi}_{\mu L}(\not{\mathcal{D}}X^J)\gamma^\mu\Omega_I-\tfrac{1}{4}f^I_{\alpha\beta}\bar{\Omega}_I\gamma^{\mu\nu}\hat{F}^{-\alpha}_{\mu\nu}\lambda^\beta_L$$
$$-\tfrac{2}{3}\mathcal{N}^I\bar{\Omega}_I\gamma^{\mu\nu}\hat{\mathcal{D}}_\mu\psi_{\nu L}+\tfrac{1}{2}\bar{\psi}_R\cdot\gamma\left(\mathcal{N}_I k_\alpha{}^I\lambda^\alpha_L+2\mathcal{W}^I\Omega_I\right)$$
$$+\tfrac{1}{2}\mathcal{W}\bar{\psi}_{\mu R}\gamma^{\mu\nu}\psi_{\nu R}-\mathcal{W}^{IJ}\bar{\Omega}_I\Omega_J-2\mathcal{N}_I{}^J k_{\alpha J}\bar{\lambda}^\alpha_R\Omega^I$$
$$-\tfrac{1}{4}f^I_{\alpha\beta}\bar{\psi}_R\cdot\gamma\Omega_I\bar{\lambda}^\alpha_L\lambda^\beta_L+\tfrac{1}{4}f^{IJ}_{\alpha\beta}\bar{\Omega}_I\Omega_J\bar{\lambda}^\alpha_L\lambda^\beta_L+\text{h.c.}\Big\}$$

$$+\ \mathcal{N}_J{}^I\left(\tfrac{1}{8}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_\nu\psi_\rho\bar{\Omega}^J\gamma_\sigma\Omega_I-\bar{\psi}_\mu\Omega^J\,\bar{\psi}_\mu\Omega_I\right)+\mathcal{N}^{IJ}_{KL}\bar{\Omega}_I\Omega_J\,\bar{\Omega}^K\Omega^L+\tfrac{1}{9}\mathcal{N}(A^F_\mu)^2$$

$$\mathcal{D}_\mu X_I=\hat{\partial}_\mu X_I+\frac{1}{3}\mathrm{i}A^B_\mu X_I,\hat{\partial}_\mu X_I=\partial_\mu X_I-W^\alpha_\mu k_{\alpha I}$$
$$\mathcal{D}_\mu\Omega_I=\left(\partial_\mu+\frac{1}{4}\omega^{ab}_\mu(e)\gamma_{ab}-\frac{1}{6}\mathrm{i}A^B_\mu\right)\Omega_I-W^\alpha_\mu k^J_{\alpha I}\Omega_J$$
$$\mathcal{D}_\mu\lambda^\alpha=\left(\partial_\mu+\frac{1}{4}\omega^{ab}_\mu(e)\gamma_{ab}+\frac{1}{2}\mathrm{i}A^B_\mu\gamma_5\right)\lambda^\alpha-W^\gamma_\mu\lambda^\beta f^\alpha_{\beta\gamma}$$
$$R^\mu=\gamma^{\mu\rho\sigma}\mathcal{D}_\rho\psi_\sigma,\mathcal{D}_{[\mu}\psi_{\nu]}=\left(\partial_{[\mu}+\frac{1}{4}\omega^{ab}_{[\mu}(e)\gamma_{ab}+\frac{1}{2}\mathrm{i}A^B_{[\mu}\gamma_5\right)\psi_{\nu]}$$

$$\mathcal{N}^I\mathcal{D}_\mu X_I=\mathcal{N}_I\mathcal{D}_\mu X^I=\frac{1}{2}\left(\mathcal{N}^I\partial_\mu X_I+\mathcal{N}_I\partial_\mu X^I\right)$$

$$[F]\equiv\int F_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu=2\pi n,n\in\mathbb{Z}$$

D-gauge: $\mathcal{N}=-3M_P^2.$

$$X_I=Yx_I(z_i),$$

$$\mathcal{K}(z,z^*)=-3\ln\left[-\frac{1}{3}x^I(z^*)\mathcal{N}_I^J(z,z^*)x_J(z)\right]$$
$$g^i_j\equiv\partial^i\partial_j\mathcal{K}=-3(\partial^iX_I)(\partial_jX^J)\partial^I\partial_J\ln\ \mathcal{N}.$$

$$YY^*\exp\left(-\frac{1}{3}\mathcal{K}\right)=M_P^2=-\frac{1}{3}\mathcal{N}$$

$$Y'=Y\mathrm{e}^{\frac{1}{3}\Lambda_Y(z)},x_I'=x_I\mathrm{e}^{-\frac{1}{3}\Lambda_Y(z)}$$

$$\mathcal{K}'=\mathcal{K}+\Lambda_Y(z)+\Lambda_Y^*(z^*)$$

$$\mathcal{W}=Y^3M_P^{-3}W(z)$$

$$W'=W\mathrm{e}^{-\Lambda_Y(z)}$$

Kähler symmetric $U(1)$-gauge: $\mathcal{W}=\mathcal{W}^*.$

non-singular at $\mathcal{W}=0U(1)$-gauge: $Y=Y^*.$

S-gauge: $\mathcal{N}^I\Omega_I=0$

$$\delta z_i = M_P^{-1}\bar{\epsilon}\chi_i, \delta z^i = M_P^{-1}\bar{\epsilon}\chi^i$$

$$\Omega_I = M_P^{-1} Y \chi_i \mathcal{D}^i x_I = M_P^{-1}\chi_i \frac{1}{Y^*}\partial^i(YY^* x_i) \Rightarrow \chi_i = M_P^{-1} g^{-1j}{}_i{}^j Y^* (\mathcal{D}_j x^J)\mathcal{N}_J^I \Omega_I,$$

$$\mathcal{D}^i x_I = \partial^i x_I + \frac{1}{3}(\partial^i \mathcal{K}) x_I = \frac{1}{YY^*}\partial^i(YY^* x_I), \mathcal{N}^I \mathcal{D}^i x_I = 0$$

$$\begin{aligned} F_{\mu\nu}^{quant} &= \frac{1}{2}(\partial_\mu A_\nu - \partial_\nu A_\mu) = 3\mathrm{i}(\partial_{[\mu} X^I)(\partial_{\nu]} X_J)\partial_I \partial^J \ln\ \mathcal{N} \\ &= 3\mathrm{i}(\partial_{[\mu} z^i)(\partial_i X^I)(\partial_{\nu]} z_j)(\partial^j X_J)\partial_I \partial^J \ln\ \mathcal{N} \\ &= -\mathrm{i}(\partial_{[\mu} z^i)(\partial_{\nu]} z_j)\partial^i \partial_j \mathcal{K} \end{aligned}$$

$$\frac{\mathrm{i}}{2\pi} g_i^j\ \mathrm{d}z_j \wedge\ \mathrm{d}z^i$$

$$c_1 \in 2\mathbb{Z}.$$

$$0 = g_k^j \partial_i \xi_\alpha{}^k + g_i^k \partial^j \xi_{\alpha k} + (\xi_{\alpha k}\partial^k + \xi_\alpha{}^k \partial_k) g_i^j = \partial_i \partial^j (\xi_\alpha{}^k \partial_k \mathcal{K} + \xi_{\alpha k}\partial^k \mathcal{K}).$$

$$\delta_\alpha Y = Y r_\alpha(z), \delta_\alpha z_i = \xi_{\alpha i}(z)$$

$$k_{\alpha I} = Y[r_\alpha(z) x_I(z) + \xi_{\alpha i}(z)\partial^i x_I(z)]$$

$$0 = \mathcal{N}^I k_{\alpha I} + \mathcal{N}_I k_\alpha{}^I = \mathcal{N}\left[r_\alpha(z) + r_\alpha^*(z^*) - \frac{1}{3}\Big(\xi_{\alpha i}\partial^i \mathcal{K}(z,z^*) + \xi_\alpha{}^i \partial_i \mathcal{K}(z,z^*)\Big)\right]$$

$$\delta_\alpha \mathcal{K} = \xi_{\alpha i}(z)\partial^i \mathcal{K}(z,z^*) + \xi_\alpha{}^i \partial_i \mathcal{K}(z,z^*) = 3(r_\alpha(z) + r_\alpha^*(z^*))$$

$$\begin{aligned} \mathcal{P}_\alpha(z,z^*) &= \frac{1}{2}\mathrm{i}M_P^2\left[\Big(\xi_{\alpha i}(z)\partial^i \mathcal{K}(z,z^*) - \xi_\alpha{}^i \partial_i \mathcal{K}(z,z^*)\Big) - 3r_\alpha(z) + 3r_\alpha^*(z^*)\right] \\ &= \mathrm{i}M_P^2(\xi_{\alpha i}(z)\partial^i \mathcal{K}(z,z^*) - 3r_\alpha(z)) = \mathrm{i}M_P^2\left(-\xi_\alpha{}^i \partial_i \mathcal{K}(z,z^*) + 3r_\alpha^*(z^*)\right) \end{aligned}$$

$$\partial_i \mathcal{P}_\alpha(z,z^*) = \mathrm{i}M_P^2 \xi_{\alpha j} g_i^j$$

$$Y\mathcal{W}^I \xi_{\alpha i}\partial^i x_I = -3r_\alpha \mathcal{W}, Y f_{\alpha\beta}^I \xi_{\gamma i}\partial^i x_I + 2f_{\delta(\alpha} f_{\beta)\gamma}^\delta = \mathrm{i}c_{\alpha\beta,\gamma},$$

$$\xi_{\alpha i}\partial^i W = -3r_\alpha W$$

$$x_0 = 1, x_i = z_i.$$

$$q_i z_i \partial^i W(z) = -M_P^{-2}\varpi W(z)$$

$$\mathcal{P} = -q_i M_P^2 z_i z^i + \varpi$$

$$\mathcal{K} = -3\nu \ln\left[-\frac{1}{3}(1 + zz^*)^{-1/3}\right],$$

$$g_z^z = \frac{\nu}{(1 + zz^*)^2}$$

$$\mathcal{P}_1 = \frac{\nu}{2}M_P^2\frac{z+z^*}{1+zz^*}, \mathcal{P}_2 = -\frac{\mathrm{i}\nu}{2}M_P^2\frac{z-z^*}{1+zz^*}, \mathcal{P}_3 = M_P^2\frac{\nu zz^*}{1+zz^*} + \varpi$$

$$\delta_1 z = \frac{1}{2}\mathrm{i}(z^2-1), \delta_2 z = \frac{1}{2}(z^2+1), \delta_3 z = -\mathrm{i}z.$$

$$r_1 = \frac{1}{6}\mathrm{i}\nu z, r_2 = \frac{1}{6}\nu z, r_3 = \frac{1}{3}\mathrm{i}\varpi M_P^{-2} = -\frac{1}{6}\mathrm{i}\nu.$$

$$\begin{aligned} g_i{}^j &= \mathrm{e}^{\mathcal{K}/3}\mathcal{D}_i x^I \mathcal{N}_I{}^J \mathcal{D}^j x_J \\ -3 &= \mathrm{e}^{\mathcal{K}/3} x^I \mathcal{N}_I{}^J x_J \end{aligned}$$

$$\begin{pmatrix} -3 & 0 \\ 0 & g_i{}^j \end{pmatrix} = \mathrm{e}^{\mathcal{K}/3}\begin{pmatrix} x^I \\ \mathcal{D}_i x^I \end{pmatrix}\mathcal{N}_I^J\begin{pmatrix} x_J & \mathcal{D}^j x_J \end{pmatrix}$$

$$\begin{aligned} V &= V_F + V_D, \\ V_F &= h^I \mathcal{N}_I{}^J h_J\big|_{\mathrm{bos}} = \mathcal{W}^I \mathcal{N}^{-1}{}_I{}^J \mathcal{W}_J \\ &= \mathrm{e}^{\mathcal{K}/3}\left[-\frac{1}{3}\mathcal{W}^I x_I x^J \mathcal{W}_J + \mathcal{W}^I \mathcal{D}^i x_I g^{-1}{}_i{}^j \mathcal{D}_j x^J \mathcal{W}_J\right] \\ &= M_P^{-2}\mathrm{e}^{\mathcal{K}}\left[-3WW^* + (\mathcal{D}^i W) g^{-1}{}_i{}^j (\mathcal{D}_j W^*)\right], \\ V_D &= \frac{1}{2}(\mathrm{Re} f_{\alpha\beta}) D^\alpha D^\beta\Big|_{\mathrm{bos}} = \frac{1}{2}(\mathrm{Re} f)^{-1\alpha\beta}\mathcal{P}_\alpha \mathcal{P}_\beta. \end{aligned}$$

$$\mathcal{D}^i W = \partial^i W + (\partial^i \mathcal{K}) W.$$

$$V_{F,+} = V + 3\mathcal{W}\mathcal{W}^* = M_P^{-2}\mathrm{e}^{\mathcal{K}}(\mathcal{D}^i W) g^{-1}{}_i{}^j (\mathcal{D}_j W^*) \geq 0, V_D \geq 0, V_+ = V_{F,+} + V_D \geq 0.$$

$$-2\mathcal{N}\eta_L = 6M_P^2\eta_L = \mathcal{N}^I \not{D} X_I \epsilon_R + \mathcal{N}^I h_I \epsilon_L.$$

$$\eta_L = -\frac{1}{2}M_P^{-2}\mathcal{W}^*\epsilon_L$$

$$e^{-1}\mathcal{L}_{mix} = \mathcal{N}_J{}^I \bar{\psi}_{\mu R}(\not{D} X_I)\gamma^\mu \Omega^J + \bar{\psi}_R \cdot \gamma\left(\frac{1}{2}\mathcal{N}_I k_\alpha{}^I \lambda_L^\alpha + \mathcal{W}^I \Omega_I\right) + \text{ h.c.}$$

$$v_L^1 = \frac{1}{2}\mathcal{N}_I k_\alpha{}^I \lambda_L^\alpha + \mathcal{W}^I \Omega_I, v_L^2 = \not{D} X_I \mathcal{N}^I{}_J \Omega^J$$

$$\delta v_L^1 = \frac{1}{2}\not{D}\mathcal{W}\epsilon_R - \frac{1}{2}V\epsilon_L + 3\mathcal{W}\eta_L, \delta v_L^2 = -\frac{1}{2}\not{D}\mathcal{W}\epsilon_R + \frac{1}{2}\not{D} X_I \mathcal{N}_J^I{}_J \not{D} X^J \epsilon_L + \not{D} X_I \mathcal{N}^I \eta_R,$$

$$v = v^1 + v^2$$

$$v_L = \mathrm{i}\frac{1}{2}\lambda_L^\alpha \mathcal{P}_\alpha + \chi_i M_P^{-4} Y^3 \mathcal{D}^i W + M_P \hat{\not{\partial}} z_i \chi^j g_j^i$$

$$\delta v_L = \frac{1}{2}M_P^2 g^i{}_j \hat{\not{\partial}} z_i \hat{\partial} z^j \epsilon_L - \frac{1}{2}V_+ \epsilon_L,$$

$$\begin{aligned} e^{-1}\mathcal{L}_{\mathrm{mix}} &= 2\mathcal{N}_J{}^I \bar{\psi}_{\mu R}\gamma^{\nu\mu}\Omega^J \mathcal{D}_\nu X_I + \bar{\psi}_R \cdot \gamma v_L + \text{ h.c.} \\ &= 2M_P g_j{}^i \bar{\psi}_{\mu R}\gamma^{\nu\mu}\chi^j \hat{\partial}_\nu z_i + \bar{\psi}_R \cdot \gamma v_L + \text{ h.c.} \end{aligned}$$

$$
\begin{aligned}
e^{-1}\mathcal{L} &= -\tfrac{1}{2}M_P^2\left[R+\bar{\psi}_\mu R^\mu+\mathcal{L}_{SG,torsion}\right]-g_i{}^j\left[M_P^2(\hat{\partial}_\mu z^i)(\hat{\partial}^\mu z_j)+\bar{\chi}_j\,\not{\mathcal{D}}\chi^i+\bar{\chi}^i\,\not{\mathcal{D}}\chi_j\right]\\
&+ (\mathrm{Re}\,f_{\alpha\beta})\left[-\tfrac{1}{4}F^\alpha_{\mu\nu}F^{\mu\nu\,\beta}-\tfrac{1}{2}\bar{\lambda}^\alpha\,\hat{\not{\mathcal{D}}}\lambda^\beta\right]+\tfrac{1}{4}\mathrm{i}(\mathrm{Im}\,f_{\alpha\beta})\left[F^\alpha_{\mu\nu}\tilde{F}^{\mu\nu\,\beta}-\hat{\partial}_\mu\left(\bar{\lambda}^\alpha\gamma_5\gamma^\mu\lambda^\beta\right)\right]\\
&- M_P^{-2}\mathrm{e}^{\mathcal{K}}\left[-3WW^*+(\mathcal{D}^iW)g^{-1}{}_i{}^j(\mathcal{D}_jW^*)\right]-\tfrac{1}{2}(\mathrm{Re}\,f)^{-1\,\alpha\beta}\mathcal{P}_\alpha\mathcal{P}_\beta
\end{aligned}
$$

$$
\begin{aligned}
&+ \tfrac{1}{8}(\mathrm{Re}\,f_{\alpha\beta})\bar{\psi}_\mu\gamma^{\nu\rho}\left(F^\alpha_{\nu\rho}+\hat{F}^\alpha_{\nu\rho}\right)\gamma^\mu\lambda^\beta\\
&+ \Big\{M_Pg_j{}^i\bar{\psi}_{\mu L}(\hat{\not{\partial}}z^j)\gamma^\mu\chi_i+\bar{\psi}_R\cdot\gamma\left[\tfrac{1}{2}\mathrm{i}\lambda^\alpha_L\mathcal{P}_\alpha+\chi_iY^3M_P^{-4}\mathcal{D}^iW\right]\\
&\quad +\tfrac{1}{2}Y^3M_P^{-3}W\bar{\psi}_{\mu R}\gamma^{\mu\nu}\psi_{\nu R}-\tfrac{1}{4}M_P^{-1}f^i_{\alpha\beta}\bar{\chi}_i\gamma^{\mu\nu}\hat{F}^{-\alpha}_{\mu\nu}\lambda^\beta_L\\
&\quad -Y^3M_P^{-5}(\mathcal{D}^i\mathcal{D}^jW)\bar{\chi}_i\chi_j+\tfrac{1}{2}\mathrm{i}(\mathrm{Re}\,f)^{-1\,\alpha\beta}\mathcal{P}_\alpha M_P^{-1}f^i_{\beta\gamma}\bar{\chi}_i\lambda^\gamma-2M_P\xi_\alpha{}^ig_i{}^j\bar{\lambda}^\alpha\chi_j\\
&\quad +\tfrac{1}{4}M_P^{-5}Y^3(\mathcal{D}^jW)g^{-1}{}_j{}^if_{\alpha\beta i}\bar{\lambda}^\alpha_R\lambda^\beta_R
\end{aligned}
$$

$$
-\tfrac{1}{4}M_P^{-1}f^i_{\alpha\beta}\bar{\psi}_R\cdot\gamma\chi_i\bar{\lambda}^\alpha_L\lambda^\beta_L+\tfrac{1}{4}M_P^{-2}(\mathcal{D}^i\partial^jf_{\alpha\beta})\bar{\chi}_i\chi_j\bar{\lambda}^\alpha_L\lambda^\beta_L+\mathrm{h.c.}\Big\}
$$

$$
\begin{aligned}
&+ g_j{}^i\left(\tfrac{1}{8}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_\nu\psi_\rho\bar{\chi}^j\gamma_\sigma\chi_i-\bar{\psi}_\mu\chi^j\,\bar{\psi}^\mu\chi_i\right)\\
&+ M_P^{-2}\left(R^{k\ell}_{ij}-\tfrac{1}{2}g_i{}^kg_j{}^\ell\right)\bar{\chi}^i\chi^j\bar{\chi}_k\chi_\ell\\
&+ \tfrac{3}{64}M_P^{-2}\left((\mathrm{Re}\,f_{\alpha\beta})\bar{\lambda}^\alpha\gamma_\mu\gamma_5\lambda^\beta\right)^2-\tfrac{1}{16}M_P^{-2}f^i_{\alpha\beta}\bar{\lambda}^\alpha_L\lambda^\beta_Lg^{-1}{}_i{}^jf_{\gamma\delta j}\bar{\lambda}^\gamma_R\lambda^\delta_R\\
&+ \tfrac{1}{8}(\mathrm{Re}\,f)^{-1\,\alpha\beta}M_P^{-2}\left(f^i_{\alpha\gamma}\bar{\chi}_i\lambda^\gamma-f_{\alpha\gamma i}\bar{\chi}^i\lambda^\gamma\right)\left(f^j_{\beta\delta}\bar{\chi}_j\lambda^\delta-f_{\beta\delta j}\bar{\chi}^j\lambda^\delta\right).
\end{aligned}
$$

$$\hat{\partial}_\mu z_i=\partial_\mu z_i-W^\alpha_\mu(\delta_\alpha z_i)$$

$$A^B_\mu=\frac{1}{2}\mathrm{i}\left[(\partial_i\mathcal{K})\partial_\mu z^i-(\partial^i\mathcal{K})\partial_\mu z_i\right]+\frac{3}{2}\mathrm{i}\partial_\mu\ln\frac{Y}{Y^*}+\frac{1}{M_P^2}W_\mu^\alpha\mathcal{P}_\alpha$$

$$\mathcal{D}_\mu\chi_i=\left(\partial_\mu+\frac{1}{4}\omega^{ab}_\mu(e)\gamma_{ab}-\frac{1}{2}\mathrm{i}A^B_\mu\right)\chi_i+\Gamma^{jk}_i\chi_j\hat{\partial}_\mu z_k-W^\alpha_\mu(\partial^j\xi_{\alpha i})\chi_j,$$

$$\Gamma^{jk}_i=g^{-1\ell}{}_i^\ell\partial^jg^k_\ell$$

$$R^{k\ell}_{ij}\equiv g^m_i\partial_j\Gamma^{k\ell}_m$$

Kähler symmetric $U(1)$-gauge: $Y^3W=(Y^*)^3W^*$.

$$\mathrm{e}^{-\mathcal{G}}=M_P^{-6}\mathrm{e}^{\mathcal{K}}|W|^2$$

$$Y^3W=M_P^6\mathrm{e}^{-\mathcal{G}/2},Y^3\mathcal{D}^iW=-M_P^6\mathcal{G}^i\mathrm{e}^{-\mathcal{G}/2}$$

non-singular at $W=0U(1)$-gauge: $Y^3=(Y^*)^3=M_P^3\mathrm{e}^{\mathcal{K}/2}$

$$A^B_\mu=\frac{1}{2}\mathrm{i}\left(\mathcal{G}^i\hat{\partial}_\mu z_i-\mathcal{G}_i\hat{\partial}_\mu z^i\right)$$

$$A_\mu^B = \frac{1}{2}\mathrm{i}\left[(\partial_i\mathcal{K})\partial_\mu z^i - (\partial^i\mathcal{K})\partial_\mu z_i\right] + M_P^{-2} W_\mu^\alpha \mathcal{P}_\alpha$$

$$\mathrm{i}\Lambda = \frac{1}{2}\left(\Lambda_Y(z) - \Lambda_Y^*(z^*)\right)$$

$$Y' = Y\exp\frac{1}{6}(\Lambda_Y + \Lambda_Y^*),$$

$$Q' = Q\mathrm{e}^{\mathrm{i}w_K\Lambda},$$

$$m = \mathrm{e}^{\mathcal{K}/2}W,$$

$$m_{3/2} = |m|M_P^{-2},$$

	m	m^*	$\psi_{\mu L}$	$\psi_{\mu R}$	χ_i	χ^i	λ_L^α	λ_R^α
w_K	-1	1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

$$\mathcal{D}_iQ = \partial_iQ + \frac{1}{2}w_K(\partial_i\mathcal{K})Q, \mathcal{D}^iQ = \partial^iQ - \frac{1}{2}w_K(\partial^i\mathcal{K})Q.$$

$$m^i \equiv \mathcal{D}^i m = \mathrm{e}^{\mathcal{K}/2}\mathcal{D}^iW = \partial^i m + \frac{1}{2}(\partial^i\mathcal{K})m, \qquad \mathcal{D}_i m = \partial_i m - \frac{1}{2}(\partial_i\mathcal{K})m = 0,$$
$$m_i \equiv \mathcal{D}_i m^* = \mathrm{e}^{\mathcal{K}/2}\mathcal{D}_iW^* = \partial_i m^* + \frac{1}{2}(\partial_i\mathcal{K})m^*, \quad \mathcal{D}^i m^* = \partial^i m^* - \frac{1}{2}(\partial^i\mathcal{K})m^* = 0.$$

$$z_i = \overset{0}{z}_i + M_P^{-1}\phi_i$$

$$\mathcal{K} = K_0 + M_P^{-1}\left(K_i\phi^i + K^i\phi_i\right) + M_P^{-2}K(\phi,\phi^*,M_P^{-1}),$$

$$K_i \propto x_I \mathcal{N}_J^I \partial_i\partial_i x^J\big|_{z=\overset{0}{z}}$$

$$x_0 = 1, x_i = M_P^{-1}\phi_i$$

$$g_j^i = \frac{\partial}{\partial z_i}\frac{\partial}{\partial z^j}\mathcal{K} = \frac{\partial}{\partial\phi_i}\frac{\partial}{\partial\phi^j}K,$$

$$X_0 = Y = M_P + \mathcal{O}(M_P^{-1}), X_i = Yx_i = YM_P^{-1}\phi_i = \phi_i\exp\left[K/(6M_P^2)\right] = \phi_i + \mathcal{O}(M_P^{-2}).$$

$$\Omega_0 = \mathcal{O}(M_P^{-1}), \Omega_i = \chi_i + \mathcal{O}(M_P^{-2})$$

$$f_{\alpha\beta}^i = \frac{\partial}{\partial\phi_i}f_{\alpha\beta} = M_P^{-1}\frac{\partial}{\partial z_i}f_{\alpha\beta}.$$

$$\xi_{\alpha i} = \delta_\alpha z_i = M_P^{-1}\delta_\alpha\phi_i$$
$$\frac{\partial}{\partial z_i}\mathcal{K} = M_P^{-1}\frac{\partial}{\partial\phi_i}K$$

$$
\begin{aligned}
e^{-1}\mathcal{L} &= -\tfrac{1}{2}M_P^2 R - g_i{}^j(\hat{\partial}_\mu\phi^i)(\hat{\partial}^\mu\phi_j) - V \\
&- \tfrac{1}{2}M_P^2\bar{\psi}_\mu R^\mu + \tfrac{1}{2}m\,\bar{\psi}_{\mu R}\gamma^{\mu\nu}\psi_{\nu R} + \tfrac{1}{2}m^*\bar{\psi}_{\mu L}\gamma^{\mu\nu}\psi_{\nu L} \\
&- g_i{}^j\left[\bar{\chi}_j\,\not{D}\chi^i + \bar{\chi}^i\,\not{D}\chi_j\right] - m^{ij}\bar{\chi}_i\chi_j - m_{ij}\bar{\chi}^i\chi^j + e^{-1}\mathcal{L}_{mix} \\
&- 2m_{i\alpha}\bar{\chi}^i\lambda^\alpha - 2m^i{}_\alpha\bar{\chi}_i\lambda^\alpha - m_{R,\alpha\beta}\bar{\lambda}_R^\alpha\lambda_R^\beta - m_{L,\alpha\beta}\bar{\lambda}_L^\alpha\lambda_L^\beta \\
&+ (\mathrm{Re}\,f_{\alpha\beta})\left[-\tfrac{1}{4}F_{\mu\nu}^\alpha F^{\mu\nu\,\beta} - \tfrac{1}{2}\bar{\lambda}^\alpha\,\not{D}\lambda^\beta\right] + \tfrac{1}{4}\mathrm{i}(\mathrm{Im}\,f_{\alpha\beta})\left[F_{\mu\nu}^\alpha\tilde{F}^{\mu\nu\,\beta} - \hat{\partial}_\mu\left(\bar{\lambda}^\alpha\gamma_5\gamma^\mu\lambda^\beta\right)\right]
\end{aligned}
$$

$$
+ \quad \frac{1}{4}\left\{(\mathrm{Re}\,f_{\alpha\beta})\bar{\psi}_\mu\gamma^{\nu\rho}F_{\nu\rho}^\alpha\gamma^\mu\lambda^\beta - \left[f_{\alpha\beta}^i\bar{\chi}_i\gamma^{\mu\nu}F_{\mu\nu}^{-\alpha}\lambda_L^\beta + \mathrm{h.c.}\right]\right\}
$$

$$
\begin{aligned}
m^{ij} &= \mathcal{D}^i\mathcal{D}^j m = \left(\partial^i + \frac{1}{2}(\partial^i\mathcal{K})\right)m^j - \Gamma_k^{ij}m^k \\
m_{i\alpha} &= -\mathrm{i}\left[\partial_i\mathcal{P}_\alpha - \frac{1}{4}(\mathrm{Re}f)^{-1\beta\gamma}\mathcal{P}_\beta f_{\gamma\alpha i}\right] \\
m_{R,\alpha\beta} &= -\frac{1}{4}f_{\alpha\beta i}g^{-1i}{}_j m^j
\end{aligned}
$$

$$
\begin{aligned}
e^{-1}\mathcal{L}_{\mathrm{mix}} &= g_j{}^i\bar{\psi}_{\mu L}(\hat{\partial}\phi^j)\gamma^\mu\chi_i + \bar{\psi}_R\cdot\gamma v_L^1 + \text{ h.c.} \\
&= 2g_j{}^i\bar{\psi}_{\mu R}\gamma^{\nu\mu}\chi^j\hat{\partial}_\nu\phi_i + \bar{\psi}_R\cdot\gamma v_L + \text{ h.c.},
\end{aligned}
$$

$$
\begin{aligned}
v_L &= v_L^1 + v_L^2 \\
v_L^1 &= \frac{1}{2}\mathrm{i}\mathcal{P}_\alpha\lambda_L^\alpha + m^i\chi_i, v_L^2 = (\hat{\partial}\phi_i)\chi^j g_j{}^i.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{D}_\mu\chi_i &= \left(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}(e)\gamma_{ab}\right)\chi_i - W_\mu^\alpha\chi_j\partial^j\xi_{\alpha i} - \frac{\mathrm{i}}{2M_P^2}W_\mu^\alpha\mathcal{P}_\alpha\chi_i \\
&+ \frac{1}{4}\left[(\partial_j\mathcal{K})\partial_\mu\phi^j - (\partial^j\mathcal{K})\partial_\mu\phi_j\right]\chi_i + \Gamma_i^{jk}\chi_j\hat{\partial}_\mu\phi_k,
\end{aligned}
$$

$$
\partial^j\xi_{\alpha i} = \frac{\partial}{\partial z_j}\delta_\alpha z_i = \frac{\partial}{\partial\phi_j}\delta_\alpha\phi_i
$$

$$
\begin{aligned}
\delta e_\mu^a &= \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu, \delta\phi_i = \bar{\epsilon}_L\chi_i, \delta W_\mu^\alpha = -\frac{1}{2}\bar{\epsilon}\gamma_\mu\lambda^\alpha \\
\delta\psi_{\mu L} &= \left(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}(e)\gamma_{ab} + \frac{1}{2}\mathrm{i}A_\mu^B\right)\epsilon_L + \frac{1}{2}M_P^{-2}m\gamma_\mu\epsilon_R \\
\delta\chi_i &= \frac{1}{2}\hat{\partial}\phi_i\epsilon_R - \frac{1}{2}g^{-1j}{}_i m_j\epsilon_L \\
\delta\lambda^\alpha &= \frac{1}{4}\gamma^{\mu\nu}F_{\mu\nu}^\alpha + \frac{1}{2}\mathrm{i}\gamma_5(\mathrm{Re}f)^{-1\alpha\beta}\mathcal{P}_\beta\epsilon.
\end{aligned}
$$

$$
\begin{aligned}
S_{\mu\nu} &= M_P^2 G_{\mu\nu} + \partial_\mu\phi^i g_i{}^j\partial_\nu\phi_j + \partial_\nu\phi^i g_i{}^j\partial_\mu\phi_j - g_{\mu\nu}\left(\partial_\rho\phi^i g_i{}^j\partial^\rho\phi_j + V\right) \\
S_i &= g_i{}^j\mathcal{D}_\mu\partial^\mu\phi_j - \partial_i V \\
\Sigma_R^\mu &= M_P^2 R_R^\mu - \gamma^{\mu\nu}\left[m\psi_{\nu R} - 2\chi^j g_j{}^i\partial_\nu\phi_i\right] - \gamma^\mu\upsilon_L \\
\Sigma^i &= g^i{}_j\,\not{D}\chi^j + m^{ij}\chi_j + m^i{}_\alpha\lambda_L^\alpha - \tfrac{1}{2}\gamma^\mu\;\not{\partial}\phi^j g_j{}^i\psi_{\mu L} + \tfrac{1}{2}m^i\,\gamma\cdot\psi_R
\end{aligned}
$$

$$\Sigma_{\alpha R} = (\mathrm{Re}\, f_{\alpha\beta})\, \not{\mathcal{D}}\lambda_L^\beta + 2m_{i\alpha}\chi^i + 2m_{R\alpha\beta}\lambda_R^\beta - \tfrac{1}{4}\left(f_{\alpha\beta}^i\, \not{\partial}\phi_i - f_{\alpha\beta\, i}\, \not{\partial}\phi^i\right)\lambda_L^\beta - \tfrac{1}{2}\mathrm{i}\mathcal{P}_\alpha\gamma\cdot\psi_L$$

$$V = -3M_P^{-2}|m|^2 + m_i g^{-1i}{}_j m^j + \frac{1}{2}\mathcal{P}_\alpha(\mathrm{Re} f)^{-1\alpha\beta}\mathcal{P}_\beta = -3M_P^{-2}|m|^2 + V_+$$
$$\partial_i V = -2M_P^{-2} m m_i + m_{ij} g^{-1j}{}_k m^k + \mathrm{i} m_{i\alpha}(\mathrm{Re} f)^{-1\alpha\beta}\mathcal{P}_\beta.$$

$$\frac{1}{2}S_{\mu\nu}\gamma^\mu\psi_L^\nu + S_i\chi^i + \mathcal{D}_\mu\Sigma_R^\mu + \frac{1}{2}M_P^{-2} m\gamma_\mu\Sigma_L^\mu$$
$$+(\not{\partial}\phi_i)\Sigma^i + m^j g_j^{-1i}\Sigma_i + \frac{1}{2}\mathrm{i}\left(\mathrm{Re} f_{\alpha\beta}\right)^{-1}\mathcal{P}_\beta\Sigma_{\alpha R} = 0$$

$$\gamma^\mu R_\mu = 2\gamma^{\mu\nu}\mathcal{D}_\mu\psi_\nu$$
$$\mathcal{D}_\mu R^\mu = -\frac{1}{2}G_{\mu\nu}\gamma^\mu\psi^\nu + \mathrm{i}\tilde{F}_{\mu\nu}^{\mathrm{quant}}\gamma^\mu\psi^\nu$$
$$R_\mu - \frac{1}{2}\gamma_\mu\gamma\cdot R = \not{\mathcal{D}}\psi_\mu - \mathcal{D}_\mu\gamma\cdot\psi$$

$$\mathcal{D}_\mu\psi_\nu = \left(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}(e)\gamma_{ab} + \frac{1}{2}\mathrm{i}A_\mu^B\gamma_5\right)\psi_\nu - \Gamma_{\mu\nu}^\lambda\psi_\lambda$$

$$\Upsilon_\mu \equiv g_j{}^i(\chi_i\partial_\mu\phi^j + \chi^j\partial_\mu\phi_i)$$

$$\upsilon^2 = \gamma^\mu\Upsilon_\mu,$$

$$\begin{aligned} \mathbf{m} &= \mathrm{Re}\, m - \mathrm{i}\gamma_5\, \mathrm{Im}\, m\,, & \mathbf{m}^\dagger &= \mathrm{Re}\, m + \mathrm{i}\gamma_5\, \mathrm{Im}\, m \\ &= P_R m + P_L m^*\,, & &= P_R m^* + P_L m \\ m &= P_R\mathbf{m} + P_L\mathbf{m}^\dagger & m^* &= P_R\mathbf{m}^\dagger + P_L\mathbf{m}\,. \end{aligned}$$

$$\begin{aligned} 0 &= \mathcal{D}_\mu\Sigma^\mu = M_P^2\mathcal{D}_\mu R^\mu - \gamma^{\mu\nu}\mathcal{D}_\mu\left(\mathbf{m}\psi_\nu - 2\Upsilon_\nu\right) - \not{\mathcal{D}}\upsilon\,, \\ 0 &= \gamma_\mu\Sigma^\mu = M_P^2\gamma_\mu R^\mu - 3\gamma^\mu\left(\mathbf{m}\psi_\mu - 2\Upsilon_\mu\right) - 4\upsilon\,. \end{aligned}$$

$$\mathcal{D}_\mu\mathbf{m} \equiv (\partial_\mu - \mathrm{i}\gamma_5 A_\mu^B)\mathbf{m} = P_R m^i\partial_\mu\phi_i + P_L m_i\partial_\mu\phi^i - \mathrm{i}\gamma_5 M_P^{-2} W_\mu^\alpha\mathcal{P}_\alpha\mathbf{m}$$
$$\mathcal{D}_\mu\mathbf{m}^\dagger \equiv (\partial_\mu + \mathrm{i}\gamma_5 A_\mu^B)\mathbf{m}^\dagger = P_R m_i\partial_\mu\phi^i + P_L m^i\partial_\mu\phi_i + \mathrm{i}\gamma_5 M_P^{-2} W_\mu^\alpha\mathcal{P}_\alpha\mathbf{m}^\dagger$$

$$\begin{aligned} 0 = \mathcal{D}_\mu\Sigma^\mu + \tfrac{1}{2}M_P^{-2}\mathbf{m}\gamma_\mu\Sigma^\mu = &-\tfrac{1}{2}M_P^2\mathbf{G}_{\mu\nu}\gamma^\mu\psi^\nu - \gamma^{\mu\nu}(\mathcal{D}_\mu\mathbf{m})\psi_\nu - \tfrac{3}{2}M_P^{-2}|m|^2\gamma^\mu\psi_\mu \\ &+2\gamma^{\mu\nu}\mathcal{D}_\mu\Upsilon_\nu + 3M_P^{-2}\mathbf{m}\gamma^\mu\Upsilon_\mu - \not{\mathcal{D}}\upsilon - 2M_P^{-2}\mathbf{m}\upsilon\,, \end{aligned}$$

$$\mathbf{G}_{\mu\nu} = G_{\mu\nu} - 2\mathrm{i}\tilde{F}_{\mu\nu}^{\mathrm{quant}}$$

$$0 = \Sigma_\mu - \tfrac{1}{2}\gamma_\mu\gamma^\nu\Sigma_\nu = M_P^2\,\not{\mathcal{D}}\psi_\mu + \mathbf{m}\psi_\mu - \left(M_P^2\mathcal{D}_\mu - \tfrac{1}{2}\mathbf{m}\gamma_\mu\right)\gamma^\nu\psi_\nu - 2\Upsilon_\mu - \gamma_\mu\gamma\cdot\Upsilon + \gamma_\mu\upsilon\,.$$

possible Q-gauge: $\upsilon = 0$.

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

$$g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$$

$$\overline{\not{\partial}} = \gamma^a \delta_a^\mu \partial_\mu\,, \qquad \not{\partial} = \gamma^\mu \partial_\mu = a^{-1}\overline{\not{\partial}}\,, \qquad \overline{\gamma}_\mu = \delta_\mu^a \gamma_a\,, \qquad \overline{\gamma}^\mu = \delta_a^\mu \gamma^a\,.$$

$$\gamma^\mu \psi_\mu = a^{-1}\bar{\gamma}^\mu \psi_\mu = a^{-1}(\gamma^0\psi_0 + \vec{\gamma}\cdot\vec{\psi})$$

$$\vec{\Upsilon} = 0, v^2 = a^{-1}\gamma^0\Upsilon_0,$$
$$A_0^B = \frac{1}{2}\mathrm{i}\left((\partial_i\mathcal{K})\partial_0\phi^i - (\partial^i\mathcal{K})\partial_0\phi_i\right), \vec{A}^B = 0, F_{\mu\nu}^{\text{quant}} = 0,$$

$$G_0^0 = M_P^{-2}\rho, G = -\mathbb{1}_3 M_P^{-2}p$$

$$\rho = |\dot{\phi}|^2 + V, \qquad p = |\dot{\phi}|^2 - V$$
$$\dot{\phi} \equiv a^{-1}\partial_0\phi, \quad |\dot{\phi}|^2 \equiv g_i{}^j\dot{\phi}_j\dot{\phi}^i$$

$$-S_i = g_i^j(\ddot{\phi}_j + 3H\dot{\phi}_j) + g_i^{jk}\dot{\phi}_j\dot{\phi}_k + \partial_i V = 0$$

$$\dot{\rho} = -6H|\dot{\phi}|^2 = -3H(\rho + p).$$

$$\rho = 3M_P^2H^2, p = -M_P^2(3H^2 + 2\dot{H}),$$

$$v_L^1 = \frac{1}{2}\mathrm{i}\mathcal{P}_\alpha\lambda_L^\alpha + m^i\chi_i, v_L^2 = \gamma^0 n_i\chi^i$$
$$v_R^1 = -\frac{1}{2}\mathrm{i}\mathcal{P}_\alpha\lambda_R^\alpha + m_i\chi^i, v_R^2 = \gamma^0 n^i\chi_i$$
$$\delta\chi_i = -\frac{1}{2}P_L g^{-1j}{}_i\xi_j\epsilon, \delta\lambda^\alpha = \frac{1}{2}\mathrm{i}\gamma_5(\mathrm{Re}f)^{-1\alpha\beta}\mathcal{P}_\beta\epsilon.$$

$$n_i = g_i{}^j\dot{\phi}_j, n^i = g_j{}^i\dot{\phi}^j$$

$$\xi^i \equiv m^i + \gamma_0 n^i\,, \qquad \xi^{\dagger i} \equiv m^i - \gamma_0 n^i$$
$$\xi_i \equiv m_i + \gamma_0 n_i\,, \qquad \xi_i^\dagger \equiv m_i - \gamma_0 n_i\,.$$

$$v = v^1 + v^2 = \xi^{\dagger i}\chi_i + \xi_i^\dagger\chi^i + \frac{1}{2}\mathrm{i}\gamma_5\mathcal{P}_\alpha\lambda^\alpha$$

$$-2\delta v = \left(\xi^{\dagger i}P_L g^{-1j}{}_i^j\xi_j + \xi_j^\dagger P_R g^{-1j}{}_i^j\xi^i + \frac{1}{2}\mathcal{P}_\alpha(\mathrm{Re}f)^{-1\alpha\beta}\mathcal{P}_\beta\right)\epsilon = \alpha\epsilon$$

$$\begin{aligned}\alpha &= \xi^{\dagger i} P_L g^{-1j}\xi_j + \xi_j^{\dagger} P_R g^{-1j}{}_i \xi^i + V_D \\ &= \frac{1}{2}\left(\xi^{\dagger i} g^{-1j}{}_i \xi_j + \xi^i g^{-1j}{}_i \xi_j^{\dagger}\right) + V_D \\ &= m^i g^{-1j} m_j + n^i g^{-1j} n_j + V_D \\ &= |\dot{\phi}|^2 + m^i g^{-1j} m_j + V_D = |\dot{\phi}|^2 + V_+ \\ &= \rho + 3M_P^{-2}|m|^2 = 3(M_P^2 H^2 + M_P^{-2}|m|^2) \\ &= 3M_P^2\left(H^2 + m_{3/2}^2\right)\end{aligned}$$

$$\varpi_\Lambda + \delta\varpi_\Lambda\left(\frac{2}{\alpha}v\right)$$

$$\begin{aligned} g_i^j \chi_j &= \hat{\Pi}_i^j \chi_j + \hat{\Pi}_{ij}\chi^j + \hat{\Pi}_{i\alpha}\lambda^\alpha + \frac{1}{\alpha} P_L \xi_i v \\ g_j^i \chi^j &= \hat{\Pi}^{ij}\chi_j + \hat{\Pi}_j^i \chi^j + \hat{\Pi}_\alpha^i \lambda^\alpha + \frac{1}{\alpha} P_R \xi^i v \\ (\mathrm{Re} f_{\alpha\beta})\lambda^\beta &= \hat{\Pi}_\alpha^j \chi_j + \hat{\Pi}_{\alpha j}\chi^j + \hat{\Pi}_{\alpha\beta}\lambda^\beta - \frac{i}{\alpha}\gamma_5 \mathcal{P}_\alpha v \end{aligned}$$

$$\hat{\Pi}_i^j = P_L\left(g_i^j - \frac{1}{\alpha}\xi_i \xi^{\dagger j}\right) P_L, \hat{\Pi}_{ij} = -\frac{1}{\alpha} P_L \xi_i \xi_j^{\dagger} P_R, \hat{\Pi}_{i\alpha} = -\frac{\mathrm{i}}{2\alpha} P_L \xi_i^{\dagger} \mathcal{P}_\alpha$$

$$\begin{aligned} \hat{\Pi}^{ij} &= -\frac{1}{\alpha} P_R \xi^i \xi^{\dagger j} P_L, \quad \hat{\Pi}_j^i = P_R\left(g_j^i - \frac{1}{\alpha}\xi^i \xi_j^{\dagger}\right) P_R, \hat{\Pi}_\alpha^i = \frac{\mathrm{i}}{2\alpha} P_R \xi^{\dagger i} \mathcal{P}_\alpha \\ \hat{\Pi}_\alpha{}^j &= \frac{\mathrm{i}}{\alpha}\mathcal{P}_\alpha \xi^j P_L, \qquad \hat{\Pi}_{\alpha j} = -\frac{\mathrm{i}}{\alpha}\mathcal{P}_\alpha \xi_j P_R, \hat{\Pi}_{\alpha\beta} = \mathrm{Re} f_{\alpha\beta} - \frac{1}{2\alpha}\mathcal{P}_\alpha \mathcal{P}_\beta \end{aligned}$$

$$\begin{aligned} \hat{\Pi}_i{}^j &= P_L \Pi_i{}^j \qquad \text{with} \qquad \Pi_i{}^j = g_i{}^j - \frac{1}{\alpha}\left(m_i m^j + n_i n^j\right), \\ \hat{\Pi}_{ij} &= P_L \gamma_0 \Pi_{ij} \qquad \text{with} \qquad \Pi_{ij} = \frac{1}{\alpha}\left(m_i n_j - n_i m_j\right). \end{aligned}$$

$$\Pi_i{}^j g^{-1i}{}_j = \frac{1}{\alpha} V_D, \Pi_{ij} = -\Pi_{ji},$$

$$\Upsilon = a\left(n_i \chi^i + n^i \chi_i\right) = -\frac{1}{2} a\gamma_0\left(\xi_i \chi^i + \xi^i \chi_i\right).$$

$$P_L \xi^{\dagger i}\Upsilon = \alpha a \Pi^{ij}\chi_j$$

$$\begin{aligned} 0 &= -\alpha\gamma^0\psi_0 + (\alpha_1 + \gamma_0\alpha_2)\theta + 4\left(a^{-1}\mathrm{i}\vec{\gamma}\cdot\vec{k} + \frac{3}{2}M_P^{-2}\widehat{m}\right)\gamma^0\Upsilon \\ &\alpha_1 \equiv p - 3M_P^{-2}|m|^2, \alpha_2 \equiv 2\dot{\mathbf{m}}^{\dagger} \\ &\theta \equiv \vec{\gamma}\cdot\vec{\psi}, \text{ and } \widehat{m} \equiv \mathbf{m} + M_P^2 H\gamma_0, \widehat{m}^{\dagger} \equiv \mathbf{m}^{\dagger} - M_P^2 H\gamma_0, \end{aligned}$$

$$a^2\gamma^{\mu\nu}\mathcal{D}_\mu\Upsilon_\nu = \left(\mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma^0 + \frac{3}{2}\dot{a}\right)\Upsilon_0$$

$$\dot{\mathbf{m}}^{\dagger} = a^{-1}\mathcal{D}_0\mathbf{m}^{\dagger} = a^{-1}(\partial_0 + \mathrm{i}\gamma_5 A_0^B)\mathbf{m}^{\dagger}$$

$$\alpha_1 = -m^i g^{-1j} m_j + n^i g^{-1j}{}_i n_j - V_D = -\frac{1}{2}\left(\xi^{\dagger i} g^{-1j}{}_i \xi_j + \xi^i g^{-1j}{}_i \xi_j^\dagger\right) - V_D,$$
$$\alpha_2 = 2m^i g^{-1j}{}_i n_j P_L + 2n^i g^{-1j}{}_i m_j P_R = \gamma_0\left(\xi^{\dagger i} P_R g^{-1j}{}_i^j \xi_j^\dagger - \xi^i P_R g^{-1j}{}_i^j \xi_j\right),$$
$$\alpha_2^\dagger = 2m^i g^{-1j}{}_i^j n_j P_R + 2n^i g^{-1j}{}_i^j m_j P_L = \gamma^0 \alpha_2 \gamma_0.$$

$$\begin{pmatrix} m^i \\ n^i \end{pmatrix} g_i^{-1j} \begin{pmatrix} m_j & n_j \end{pmatrix} + \begin{pmatrix} \mathcal{P}_\alpha \\ 0 \end{pmatrix} \frac{1}{2} (\mathrm{Re} f)^{-1\alpha\beta} \begin{pmatrix} \mathcal{P}_\beta & 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \alpha - \alpha_1 & \alpha_2 P_R + \alpha_2^\dagger P_L \\ \alpha_2 P_L + \alpha_2^\dagger P_R & \alpha + \alpha_1 \end{pmatrix}.$$

$$\begin{aligned}\frac{1}{4}\alpha^2\Delta^2 &\equiv \frac{1}{4}\left(\alpha^2 - \alpha_1^2 - |\alpha_2|^2\right) \\ &= n^i n_j \left[m_k m^\ell \left(g^{-1}{}_i{}^j g^{-1}{}_\ell{}^k - g^{-1}{}_i{}^k g^{-1}{}_\ell{}^j\right) + \frac{1}{2} g^{-1}{}_i{}^j \mathcal{P}_\alpha (\mathrm{Re} f)^{-1\alpha\beta} \mathcal{P}_\beta\right] \\ &= \dot{\phi}^i \dot{\phi}_j m_k m^\ell \left(g^{-1}{}_\ell{}^k g_i{}^j - \delta_i{}^k \delta_\ell{}^j\right) + \frac{1}{2} |\dot{\phi}|^2 \mathcal{P}_\alpha (\mathrm{Re} f)^{-1\alpha\beta} \mathcal{P}_\beta \geq 0\end{aligned}$$

$$\Pi^{ij} g^{-1k}{}_i^k g^{-1\ell} \Pi_{k\ell} = \frac{1}{2}\Delta^2 - \frac{2}{\alpha^2} V_D n^i g^{-1j}{}_i^j n_j.$$

$$\gamma^0 \psi_0 = \hat{A}\theta + \hat{C}\Upsilon$$

$$\hat{A} \equiv \frac{1}{\alpha}(\alpha_1 + \gamma_0 \alpha_2), \hat{A}^\dagger \equiv \frac{1}{\alpha}(\alpha_1 - \gamma_0 \alpha_2),$$
$$\hat{C} \equiv \frac{4}{\alpha}\left(a^{-1} \mathrm{i}\vec{\gamma} \cdot \vec{k} + \frac{3}{2} M_P^{-2} \widehat{m}\right)\gamma^0.$$

$$\hat{A} = -\frac{\xi^{\dagger i} P_R g^{-1j} \xi_j^\dagger + \xi_j^\dagger P_L g^{-1j}{}_i^j \xi^{\dagger i} + V_D}{\xi^{\dagger i} P_L g^{-1j}{}_i^j \xi_j + \xi_j^\dagger P_R g^{-1j}{}_i^j \xi^i + V_D}$$

$$1 - |\hat{A}|^2 = \Delta^2 \geq 0, \text{ and } |\hat{A}|^2 = 1$$

$$\mathrm{i}\vec{k} \cdot \vec{\psi} = \left(\mathrm{i}\vec{\gamma} \cdot \vec{k} - M_P^{-2} a \mathbf{m}^\dagger - \gamma_0 \dot{a}\right)\theta$$

$$\vec{\psi} = \vec{\psi}^T + \left(\frac{1}{2}\vec{\gamma} - \frac{1}{2\vec{k}^2}\vec{k}(\vec{k} \cdot \vec{\gamma})\right)\theta + \left(\frac{3}{2\vec{k}^2}\vec{k} - \frac{1}{2\vec{k}^2}\vec{\gamma}(\vec{k} \cdot \vec{\gamma})\right)\vec{k} \cdot \vec{\psi}$$

$$\mathbf{P} = \mathbb{1}_3 - \left(\frac{1}{2}\vec{\gamma} - \frac{1}{2\vec{k}^2}\vec{k}(\vec{k} \cdot \vec{\gamma})\right)\vec{\gamma}^t - \left(\frac{3}{2\vec{k}^2}\vec{k} - \frac{1}{2\vec{k}^2}\vec{\gamma}(\vec{k} \cdot \vec{\gamma})\right)\vec{k}^t$$

$$\mathbf{P}\vec{\gamma} = \mathbf{P}\vec{k} = \vec{\gamma}^t \mathbf{P} = \vec{k}^t \mathbf{P} = 0, \mathbf{P}\gamma_0 = \gamma_0 \mathbf{P}, \vec{\gamma} \cdot \vec{k}\mathbf{P} = \mathbf{P}\vec{\gamma} \cdot \vec{k}$$

$$\vec{\psi} = \vec{\psi}^T + \frac{1}{\vec{k}^2}\left[\vec{k}(\vec{\gamma} \cdot \vec{k}) + \frac{1}{2}\mathrm{i}(3\vec{k} - \vec{\gamma}(\vec{k} \cdot \vec{\gamma}))\left(\dot{a}\gamma_0 + M_P^{-2} a \mathbf{m}^\dagger\right)\right]\theta.$$

$$\begin{aligned} a\,\vec{\not{D}}\vec{\psi} &= \left(\vec{\not{\partial}} + \tfrac{1}{2}\dot{a}\gamma^0 + \tfrac{1}{2}\gamma^0 \mathrm{i}\gamma_5 A_0^B\right)\vec{\psi} - \tfrac{1}{2}\dot{a}\vec{\gamma}\psi_0\,, \\ a\vec{D}\gamma^\mu \psi_\mu &= \mathrm{i}\vec{k}\vec{\gamma}^\mu \psi_\mu - \tfrac{1}{2}\dot{a}\vec{\gamma}\left(\psi_0 - \gamma_0 \theta\right)\,. \end{aligned}$$

$$\left(\overline{\partial}+\frac{1}{2}\dot{a}\gamma^0+\frac{1}{2}\gamma^0\mathrm{i}\gamma_5A_0^B+M_P^{-2}\mathbf{m}a\right)\vec{\psi}^T=0$$

$$a\,\overline{\not{D}}\psi_0=\left(\overline{\not{\partial}}+\mathrm{i}\tfrac{1}{2}\gamma^0\gamma_5A_0^B+\tfrac{1}{2}\dot{a}\gamma^0\right)\psi_0-\dot{a}\theta\,,$$
$$a\mathcal{D}_0\gamma^\mu\psi_\mu=\overline{\gamma}^\mu a\dot{\psi}_\mu-\dot{a}\overline{\gamma}^\mu\psi_\mu\,,$$

$$\left(\frac{3}{2}(M_P^{-2}\mathbf{m}a-\dot{a}\gamma_0)+\mathrm{i}\vec{\gamma}\cdot\vec{k}\right)\psi_0=\dot{\theta}-\frac{1}{2}M_P^{-2}\mathbf{m}a\gamma_0\theta+3M_P^{-2}a\Upsilon_0$$

$$\left[\hat{\partial}_0-\frac{1}{2}M_P^{-2}\mathbf{m}a\gamma_0-\gamma_0\left(\frac{3}{2}M_P^{-2}a\widehat{m}^\dagger+\mathrm{i}\vec{\gamma}\cdot\vec{k}\right)\hat{A}\right]\theta-\frac{4}{\alpha a}\vec{k}^2\Upsilon=0$$

$$\widehat{m}^\dagger\widehat{m}=|m|^2+M_P^4H^2=\frac{1}{3}M_P^2\alpha$$

$$\hat{B}=-\frac{3}{2}\dot{a}\hat{A}-\frac{1}{2}M_P^{-2}\mathbf{m}a\gamma_0(1+3\hat{A})=-\frac{1}{2}M_P^{-2}\mathbf{m}a\gamma_0-\frac{3}{2}M_P^{-2}\gamma_0a\widehat{m}^\dagger\hat{A},$$
$$\hat{B}^\dagger=-\frac{3}{2}\dot{a}\hat{A}^\dagger+\frac{1}{2}M_P^{-2}(1+3\hat{A}^\dagger)a\mathbf{m}\gamma_0,$$

$$(\hat{\partial}_0+\hat{B}-\mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A})\theta-\frac{4}{\alpha a}\vec{k}^2\Upsilon=0$$

$$\gamma^0\dot{\phi}_i\Sigma^i=-a^{-2}\left(\hat{\partial}_0+\mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma^0+\frac{1}{2}\dot{a}\right)\Upsilon_R+\chi^j\left(g_j{}^i\ddot{\phi}_i+g_j^{ik}\dot{\phi}_i\dot{\phi}_k\right)$$
$$+\gamma^0\dot{\phi}_i\left(m^{ij}\chi_j+m^i{}_\alpha\lambda_L^\alpha\right)-\frac{1}{2}|\dot{\phi}|^2a^{-1}(-\gamma^0\psi_{0L}+\theta_R)+\frac{1}{2}\gamma^0\dot{\phi}_im^i\gamma\cdot\psi_R.$$

$$\gamma^0\dot{\phi}_i\Sigma^i=-a^{-2}\left(\hat{\partial}_0+\mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma^0+\frac{7}{2}\dot{a}\right)\Upsilon_R-(\partial_iV)\chi^i+\gamma^0\dot{\phi}_im^{ij}\chi_j+\gamma^0\dot{\phi}_im_\alpha^i\lambda_L^\alpha$$
$$+\frac{1}{4}a^{-1}[(\alpha+\alpha_1)(\gamma^0\psi_{0L}-\theta_R)+\gamma^0\alpha_2(\gamma^0\psi_{0R}+\theta_L)]$$
$$=-a^{-2}\left(\hat{\partial}_0+\mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma^0+\frac{7}{2}\dot{a}\right)\Upsilon_R-2M_P^{-2}a^{-1}m\gamma^0\Upsilon_L+\Xi_R$$
$$+\frac{1}{4}a^{-1}P_R\alpha[(1+\hat{A}^\dagger)\gamma^0\psi_0-(1+\hat{A})\theta].$$

$$\Xi_R=-m^kg^{-1}{}_k{}^jm_{ji}\chi^i-\mathrm{i}\mathcal{P}_\alpha(\mathrm{Re}f)^{-1\alpha\beta}m_{\beta i}\chi^i+\gamma^0\dot{\phi}_j\left(m^{ji}\chi_i+m^j{}_\alpha\lambda_L^\alpha\right)+\mathrm{i}M_P^{-2}m\mathcal{P}_\alpha\lambda_R^\alpha$$

$$\left[\hat{\partial}_0+\mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma^0+\frac{7}{2}\dot{a}-\frac{1}{4}a\alpha(1+\hat{A}^\dagger)\hat{C}+2\mathbf{m}a\gamma^0\right]\Upsilon-a^2\Xi+\frac{1}{4}a\alpha\Delta^2\theta=0$$

$$\left[\hat{\partial}_0+\mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A}+\dot{a}\left(2-\frac{3}{2}\hat{A}^\dagger\right)-\frac{1}{2}M_P^{-2}(1-3\hat{A}^\dagger)a\mathbf{m}\gamma_0\right]\Upsilon-a^2\Xi+\frac{1}{4}a\alpha\Delta^2\theta=0.$$

$$\hat{A}=\frac{p-3M_P^{-2}|m|^2+2\gamma_0\dot{\mathbf{m}}^\dagger}{\rho+3M_P^{-2}|m|^2}$$

$$(\hat{\partial}_0 + \hat{B} - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A})\theta - \frac{4}{\alpha a}\vec{k}^2\Upsilon = 0$$

$$[\hat{\partial}_0 + \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A} + \hat{B}^\dagger + 2\dot{a} - M_P^{-2}a\mathbf{m}\gamma_0]\Upsilon - a^2\Xi + \frac{1}{4}a\alpha\Delta^2\theta = 0$$

$$\Xi = -\xi^k g^{-1j}{}_k m_{ji}\chi^i - \xi_k g^{-1k}{}_j^k m^{ji}\chi_i + \mathfrak{B}$$

$$2B_1 \equiv \hat{B} + \hat{B}^\dagger = -3\dot{a}\frac{\alpha_1}{\alpha} + \frac{3a}{2\alpha}M_P^{-2}(\mathbf{m}\alpha_2 + \alpha_2^\dagger\mathbf{m}^\dagger) = a\frac{\dot{\alpha}}{\alpha} + 3\dot{a}.$$

$$\begin{aligned} 0 =& \frac{1}{\alpha a}[\hat{\partial}_0 + \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A} + \hat{B}^\dagger + 2\dot{a} - M_P^{-2}a\mathbf{m}\gamma_0]\alpha a[\hat{\partial}_0 + \hat{B} - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A}]\theta \\ &- \frac{4\vec{k}^2}{\alpha a}\left[a^2\Xi - \frac{1}{4}a\alpha\Delta^2\theta\right] \\ =& \left[\hat{\partial}_0\hat{\partial}_0 + \vec{k}^2 + |\hat{B}|^2 + 2B_1\hat{\partial}_0 + a\dot{\hat{B}} - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0 a\dot{\hat{A}}\right]\theta \\ &- \frac{4a\vec{k}^2}{\alpha}\Xi + (2B_1 - M_P^{-2}a\mathbf{m}\gamma_0)[\hat{\partial}_0 + \hat{B} - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A}]\theta. \end{aligned}$$

$$g^i{}_j \not{D}\chi^j + m^{ij}\chi_j + m^i{}_\alpha\lambda_L^\alpha - \tfrac{1}{2}\gamma^\mu \not{\partial}\phi^j g_j{}^i\psi_{\mu L} + \tfrac{1}{2}m^i\gamma\cdot\psi_R = 0\,.$$

$$\begin{aligned} &g^i{}_j \not{D}\chi^j + m^{ij}\chi_j + m^i{}_\alpha\lambda_L^\alpha - \dot{\phi}^j g_j{}^i\psi_{0L} = 0\,, \\ &(\mathrm{Re}\, f_{\alpha\beta})\not{D}\lambda_L^\beta + 2m_{i\alpha}\chi^i + 2m_{R\alpha\beta}\lambda_R^\beta - \tfrac{1}{4}\left(f^i_{\alpha\beta}\not{\partial}\phi_i - f_{\alpha\beta i}\not{\partial}\phi^i\right)\lambda_L^\beta = 0\,. \end{aligned}$$

$$M_P^2\not{D}\psi_\mu + \mathbf{m}\psi_\mu - 2\Upsilon_\mu - \gamma_\mu\gamma\cdot\Upsilon + \gamma_\mu\upsilon = 0\,.$$

$$\not{\partial}\psi_0 + M_P^{-2}(\mathbf{m}\psi_0 - 3\Upsilon + a\gamma_0\upsilon) = 0$$

$$\begin{aligned} &g^i_j\not{\partial}\chi^j + m^{ij}\chi_j + m^i_\alpha\lambda_L^\alpha - \dot{\phi}^j g_j{}^i\psi_{0L} = 0 \\ &(\mathrm{Re} f_{\alpha\beta})\not{\partial}\lambda_L^\beta + 2m_{i\alpha}\chi^i + 2m_{R\alpha\beta}\lambda_R^\beta - \frac{1}{4}(f^i_{\alpha\beta}\not{\partial}\phi_i - f_{\alpha\beta i}\not{\partial}\phi^i)\lambda_L^\beta = 0 \end{aligned}$$

$$\begin{aligned} &g^i_j\not{\partial}\chi^j + m^{ij}\chi_j + m^i_\alpha\lambda_L^\alpha = 0 \\ &(\mathrm{Re} f_{\alpha\beta})\not{\partial}\lambda_L^\beta + 2m_{i\alpha}\chi^i + 2m_{R\alpha\beta}\lambda_R^\beta - \frac{1}{4}(f^i_{\alpha\beta}\not{\partial}\phi_i - f_{\alpha\beta i}\not{\partial}\phi^i)\lambda_L^\beta = 0 \end{aligned}$$

$$(\gamma^0\partial_0 + \mathrm{i}\vec{\gamma}\cdot\vec{k} + \Omega_T)\vec{\Psi}^T = 0, \vec{\Psi}^T \equiv a^{1/2}\vec{\psi}^T.$$

$$(\partial_0^2 + k^2 + \Omega_T^2 - \gamma^0\Omega_T')\vec{\Psi}_T = 0.$$

$$(\hat{\partial}_0 + \hat{B} - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A})\theta = 0,$$

$$\gamma^0\psi_0 = \hat{A}\theta.$$

$$\left[\hat{\partial}_0\hat{\partial}_0 + \vec{k}^2 + |\hat{B}|^2 + 2B_1\hat{\partial}_0 + a\dot{\hat{B}} - a\dot{\hat{A}}^\dagger\hat{A}(\hat{\partial}_0 + \hat{B})\right]\theta = 0.$$

$$\hat{B} = B_1 + \gamma_0 B_2 = -\frac{3}{2}\dot{a}\hat{A} - \frac{1}{2}\gamma_0 m_{3/2} a(1+3\hat{A}),$$

$$\hat{B}^{\dagger} = B_1 - \gamma_0 B_2 = -\frac{3}{2}\dot{a}\hat{A}^{\dagger} + \frac{1}{2} m_{3/2} a\gamma_0 (1+3\hat{A}^{\dagger})$$

$$B_1 = \frac{3}{2\alpha}(-\dot{\alpha}\alpha_1 + m_{3/2} a\alpha_2), B_2 = -\frac{1}{2\alpha}\left(3\dot{\alpha}\alpha_2 + m_{3/2} a(\alpha + 3\alpha_1)\right)$$

$$\theta = \theta_+ + \theta_-, \theta_\pm = \frac{1}{2}(1 \mp \mathrm{i}\gamma_0)\theta$$

$$\theta_\pm(\vec{k})^* = \mp \mathcal{C}\theta_\mp(-\vec{k})$$

$$\theta_+ = \sum_{\alpha,\beta=1}^{2} a^{\alpha\beta}(k) f_\alpha(k,\eta) u_\beta(k), \theta_- = -\mathcal{C}^{-1} \sum_{\alpha,\beta=1}^{2} a^{*\alpha\beta}(-k) f_\alpha^*(-k,\eta) u_\beta^*(-k)$$

$$\left[\partial_0^2 + 2(B_1 + \mathrm{i}a\mu)\partial_0 + \left(\vec{k}^2 + |B|^2 + (\partial_0 B) + 2\mathrm{i}Ba\mu\right)\right] f(k,\eta) = 0$$

$$A = \frac{1}{\alpha}(\alpha_1 + \mathrm{i}\alpha_2), \mu \equiv \frac{\mathrm{i}}{2}\dot{A}^* A = \frac{\mathrm{i}}{2}\frac{\dot{A}^*}{A^*}$$

$$\mu = (m_{11} + m_{3/2}) + 3(H\dot{\phi} - m_{3/2} m_1)\frac{m_1}{\dot{\phi}^2 + m_1^2}$$

$$f(k,\eta) = E(\eta) y(k,\eta)\ E(\eta) = (-A^*)^{1/2} \exp\left(-\int^{\eta} \mathrm{d}\,\mathrm{d}\eta B_1(\eta)\right).$$

$$\left(\partial_0^2 + k^2 + \Omega^2 - \mathrm{i}(\partial_0 \Omega)\right) y = 0$$

$$A^* = -\exp\left(-2\mathrm{i}\int_{-\infty}^{t} \mathrm{d}t\mu(\eta)\right)$$

$$\begin{aligned}\tilde{m} \equiv \frac{\Omega}{a} &= \mu + \frac{3}{2\alpha} H\alpha_2 + \frac{1}{2\alpha} m_{3/2}(\alpha + 3\alpha_1) \\ &= \mu - \frac{3}{2} H \sin 2\int \mathrm{d}t\mu + \frac{1}{2} m_{3/2}\left(1 - 3\cos 2\int \mathrm{d}t\mu\right)\end{aligned}$$

$$\Xi = -a^{-1}\hat{F}\Upsilon$$

$$|\Pi_{12}|^2 = \frac{1}{4}\Delta^2 \det g.$$

$$\Pi_{ij} P_L \xi^{\dagger j}\Upsilon = -\frac{1}{4} a\alpha\chi_i \Delta^2 \det g$$

$$\hat{F} = -\frac{4}{\alpha\Delta^2 \det g}\left[\xi^k P_R g_k^{-1\ell} m_{\ell i}\Pi^{ij}\xi_j^{\dagger} + \xi_k P_L g_\ell^{-1k} m^{\ell i}\Pi_{ij}\xi^{\dagger j}\right]$$

$$(\hat{\partial}_0 + \hat{B} - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A})\theta - \frac{4}{\alpha a}\vec{k}^2\Upsilon = 0$$
$$(\hat{\partial}_0 + \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A} + \hat{B}^\dagger + a\hat{F} + 2\dot{a} - M_P^{-2}a\mathbf{m}\gamma_0)\Upsilon + \frac{1}{4}a\alpha\Delta^2\theta = 0$$

$$0 = \left[\hat{\partial}_0\hat{\partial}_0 + \vec{k}^2 + |\hat{B}|^2 + 2B_1\hat{\partial}_0 + a\dot{\hat{B}} - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0 a\dot{\hat{A}}\right]\theta$$
$$+(2B_1 + a\hat{F} - M_P^{-2}a\mathbf{m}\gamma_0)[\hat{\partial}_0 + \hat{B} - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A}]\theta$$

$$g_i^j = \delta_i^j, m_i = W_i = \frac{\partial W}{\partial\phi^i}, n_i = \dot{\phi}_i, m_{ij} = W_{ij} = \frac{\partial^2 W}{\partial\phi^i\partial\phi^j}$$

$$\xi_k = W_k + \gamma_0\dot{\phi}_k, \xi_k^\dagger = W_k - \gamma_0\dot{\phi}_k$$

$$\gamma_0\dot{\xi}_i = W_{ij}\xi^j, \gamma_0\dot{\xi}_i^\dagger = -W_{ij}\xi^{\dagger j}$$

$$\alpha = \rho, \alpha_1 = p, \alpha_2 = \dot{W} + \dot{W}^* + \gamma_5(\dot{W} - \dot{W}^*),$$
$$\hat{A} = \frac{p}{\rho} + \gamma_0\frac{\alpha_2}{\rho}.$$

$$\hat{A} = \frac{p}{\rho} + \frac{2\gamma_0\dot{W}}{\rho}, \Delta^2 \equiv 1 - |\hat{A}|^2$$

$$(\partial_0 - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A})\theta - \frac{4}{\rho}\vec{k}^2\Upsilon = 0,$$
$$[\partial_0 + \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A}]\Upsilon - \Xi + \frac{1}{4}\rho\Delta^2\theta = 0. (9.27)$$

$$\Upsilon = -\frac{1}{2}\gamma_0(\xi_i\chi^i + \xi^i\chi_i), \Xi = -\xi^j W_{ji}\chi^i - \xi_j W^{ji}\chi_i$$

$$v = \xi^{\dagger i}\chi_i + \xi_i^\dagger\chi^i = 0$$

$$\ddot{\theta} + \left[\vec{k}^2 - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\dot{\hat{A}}\right]\theta - \frac{4\vec{k}^2}{\rho}\Xi = 0$$

$$\Omega = \tilde{m} = \mu = m_{11} = \partial_\phi^2 W$$

$$f(k,t) = \mathrm{e}^{-\mathrm{i}\int \mathrm{d}t\mu} y(k,t)$$

$$\xi^{\dagger 1} = \xi_1^\dagger = \rho^{1/2}\mathrm{e}^{\gamma_0\int dt\mu}$$

$$v = \rho^{1/2}\mathrm{e}^{\gamma_0\int \mathrm{d}t\mu}(\chi_1 + \chi^1)$$

$$\ddot{\theta} + \left[\vec{k}^2 - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\dot{\hat{A}} + \hat{F}(\hat{\partial}_0 - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A})\right]\theta = 0$$

$$\Delta = 2\Pi_{12} = \frac{2}{\rho}(W_1\dot{\phi}_2 - W_2\dot{\phi}_1)$$

$$\hat{F} = -\frac{2}{\rho\Delta}\xi^k W_{ki}\varepsilon^{ij}\xi_j^\dagger = -\frac{\gamma_0(\dot{\xi}_1\xi_2^\dagger - \dot{\xi}_2\xi_1^\dagger)}{W_1\dot{\phi}_2 - W_2\dot{\phi}_1}$$

$$v = \sum_i \rho_i^{1/2} \mathrm{e}^{\gamma_0 \int \mathrm{d}t\mu_i}(\chi_i + \chi^i)$$

$$\rho_i = \xi_i^\dagger \xi_i, \rho = \sum_i \rho_i$$

$$\xi_1 = \mu C \mathrm{e}^{-\gamma_0\mu t}, \rho_1 = (\mu C)^2, \xi_2 = \zeta, \rho_2 = \zeta^2$$

$$\Delta = \frac{2(\rho_1\rho_2)^{1/2}\sin\,\mu t}{\rho}, \hat{F} = -\frac{\mu \mathrm{e}^{\gamma_0\mu t}}{\sin\,\mu t}$$

$$\Delta = \frac{2(\rho_1\rho_2)^{1/2}\sin\,(\mu_1-\mu_2)t}{\rho}, \hat{F} = -\frac{\mu_1 \mathrm{e}^{\gamma_0(\mu_2-\mu_1)t} - \mu_2 \mathrm{e}^{\gamma_0(\mu_1-\mu_2)t}}{\sin\,(\mu_1-\mu_2)t}.$$

$$(\partial_0 - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A})\theta - \frac{4}{\rho}\vec{k}^2\Upsilon = 0$$

$$(\partial_0 + \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A} + \hat{F})\Upsilon + \frac{1}{4}\rho\Delta^2\theta = 0$$

$$\Delta = \frac{2}{\rho}(W_1\dot{\phi}_2 - W_2\dot{\phi}_1) \text{ and } |W_i|, |\dot{\phi}_i| \leq \sqrt{\rho}_i.$$

$$\Delta^2 \leq \frac{16\rho_1\rho_2}{\rho^2}$$

$$\rho \approx \rho_1 \gg \rho_2\Delta^2 \leq \frac{16\rho_2}{\rho_1} \ll 1$$

$$(\partial_0 + \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A} + \hat{F})(\partial_0 - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A})\theta + \vec{k}^2\Delta^2\theta = 0$$

$$(\partial_0 + \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A} + \hat{F})(\partial_0 - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A})\theta = 0$$

$$(\partial_0 - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A})\theta = 0$$

$$v = \rho_1^{1/2} \mathrm{e}^{\gamma_0 \int \mathrm{d}t\mu_1}(\chi_1 + \chi^1)$$

$$(\hat{\partial}_0 - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\hat{A})\theta + \hat{F}^{-1}\left[\ddot{\theta} + \left(\vec{k}^2 - \mathrm{i}\vec{\gamma}\cdot\vec{k}\gamma_0\dot{\hat{A}}\right)\theta\right] = 0$$

$$(\hat{\partial}_0 - \mathrm{i}\vec{\gamma}\cdot\vec{k}\,\gamma_0\,\hat{A})\theta + \hat{F}^{-1}\left[\ddot{\theta} + (\vec{k}^2 - \mathrm{i}\vec{\gamma}\cdot\vec{k}\,\gamma_0\,\dot{\hat{A}})\theta\right] = 0\,.$$

$$\left(\partial_0\theta + \vec{\gamma}\cdot\vec{\partial} + \frac{3}{2}\dot{a} + m_{3/2}a\gamma_0\right)\theta = 0$$

$$\left(\partial\!\!\!/ + \frac{3}{2}\dot{a}\gamma_0 + m_{3/2}a\right)\psi_0 = 0$$

$$(\partial\!\!\!/ + m_{3/2}a)\Psi_0 = 0.$$

$$\partial_0^2 y + \left(k^2 + m_{3/2}^2 a^2 + \mathrm{i}m_{3/2}a\dot{a}\right)y = 0.$$

$$y(k,\eta) = \frac{1}{2}\sqrt{\pi k|\eta|}\exp\left(\frac{\pi m_{3/2}}{2H}\right)\mathcal{H}^{(1)}_{\frac{1}{2}-\mathrm{i}m_{3/2}/H}(|k\eta|),$$

$$\frac{n_{3/2}}{s} \sim 10^{-2}\left(\frac{m_\phi}{M_P}\right)^{3/2} \sim 10^{-10}$$

$$\rho = m_\phi^2\phi_0^2(t)/2$$

$$W = \sqrt{\lambda}\phi^3/3$$

$$\phi \to \phi/\sqrt{2}$$

$$\phi \ll M_P \text{ is } \lambda\phi^4/4$$

$$\phi(\eta) = \frac{\phi_0}{a}\,\mathrm{cn}\left(\sqrt{\lambda}\phi_0, \frac{1}{\sqrt{2}}\right)$$

$$\phi_0 \simeq M_P$$

$$\mu = \sqrt{2\lambda}\phi\sqrt{2\lambda}\phi_0\ m_{3/2}\lambda\phi^4/4n_k \simeq \tfrac{1}{2}\sqrt{\lambda}\phi_0$$

$$\rho_{3/2} \sim \left(\sqrt{\lambda}\phi_0\right)^4 \sim \lambda V(\phi_0),$$

$$n_{3/2} \sim \lambda^{3/4}V^{3/4}(\phi_0)$$

$$\frac{n_{3/2}}{s} \sim \lambda^{3/4} \sim 10^{-10}$$

$$m_{3/2} = \mathrm{e}^{\mathcal{K}/2}WM_P^{-2} \sim 10^2\mathrm{GeV}$$

$$V \sim -M_P^2 m_{3/2}^2 \sim -10^{-32}M_P^4$$

$$\rho_{\mathrm{vac}} \sim 10^{-122}M_P^4$$

$$W = \tfrac{1}{2}m_\phi\phi^2 \text{ with } m_\phi \sim 10^{13}\mathrm{GeV}$$

$$W_{\phi\phi} = m_\phi$$

$$W_{\phi\phi} \text{ at } \phi \ll M_P$$

$$\mu \approx m_\phi \gg m_{3/2}$$

$$|\rho| \ll M_P^2 m_{3/2}^2\ M_P^2 m_{3/2}^2\ \hat{A} = M_P^2 m_{3/2}^2$$

$$W = \frac{1}{2}m_\phi\phi^2$$

$$W = \zeta(\phi + 2 - \sqrt{3})\ \mathrm{Re}\phi$$

$$W = \zeta(\phi + 2 - \sqrt{3})$$

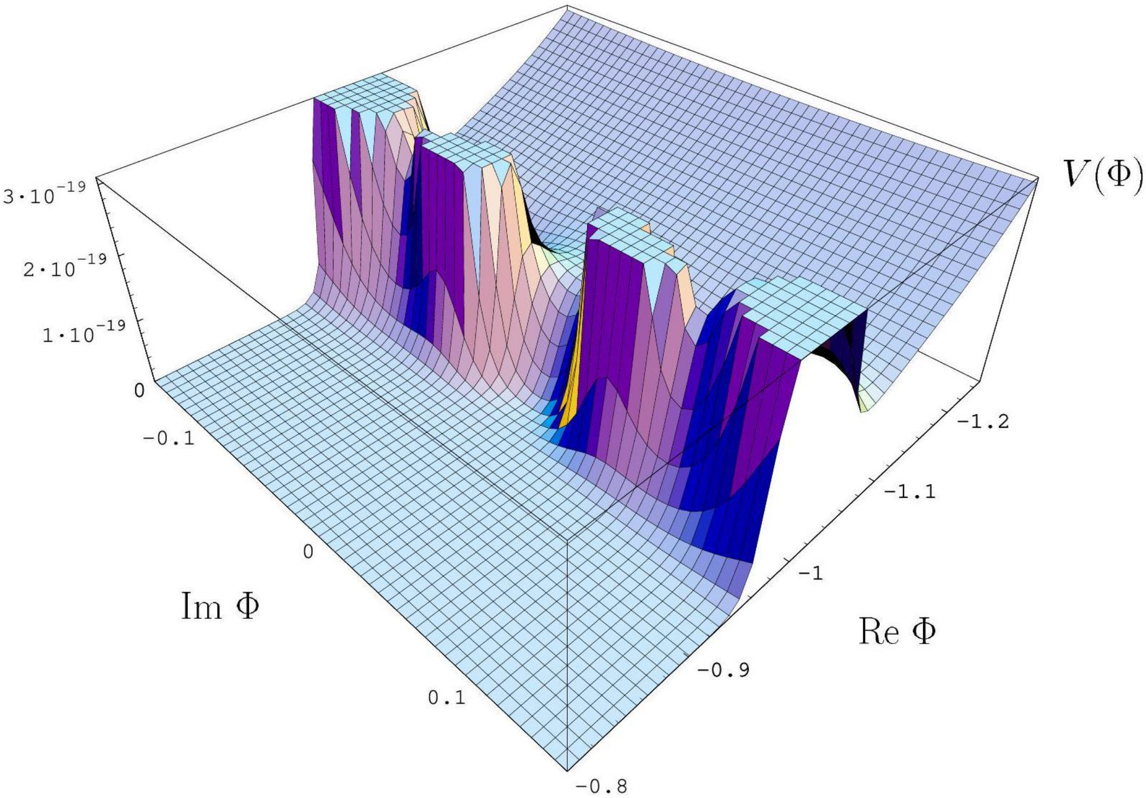
$$\phi = M_P(\sqrt{3} - 1)$$

$$\mathrm{e}^{\mathcal{K}} = \exp\left(\phi^2/M_P^2\right)$$

$$W = \zeta(\phi + 2 - \sqrt{3}) + C_1(\phi + C_2)(1 - \tanh\left(C_3(\phi + C_2)\right))\ \mathrm{Re}\phi$$

$$\zeta(\phi + 2 - \sqrt{3})\ \phi = M_P(\sqrt{3} - 1)$$

$$W = \zeta(\phi + 2 - \sqrt{3}) + C_1(\phi + C_2)\left(1 - \tanh\left(C_3(\phi + C_2)\right)\right)$$



$$W = \zeta(\phi + 2 - \sqrt{3}) + C_1(\phi + C_2)(1 - \tanh(C_3(\phi + C_2)))$$

$$W = \zeta(\phi + 2 - \sqrt{3})$$

$$W = m_\phi \phi_1^2/2 + \zeta\left(\phi_2 + 2 - \sqrt{3}\right)$$

$$W = \sqrt{\lambda}\phi_1^3/3 + \zeta\left(\phi_2 + 2 - \sqrt{3}\right)$$

$$(1 + \phi_2^2/M_P^2)\rho_1 \approx (\rho_1 + 3\phi_2^2 H^2)$$

$$\phi_2 = M_P(\sqrt{3} - 1)$$

$$v = \rho_1^{1/2} \mathrm{e}^{\gamma_0 \int \mathrm{d}t\mu_1}(\chi_1 + \chi^1)$$

$$v=\rho_2^{1/2}\mathrm{e}^{\gamma_0\int \mathrm{d}t\mu_2}(\chi_2+\chi^2)$$

$$\mathrm{O}(H^{-1})\sim M_p/\sqrt{\rho}$$

$$\rho_1^{1/2}\mathrm{e}^{\gamma_0\int \mathrm{d}t\mu_1}(\chi_1+\chi^1)\text{ to }\rho_2^{1/2}\mathrm{e}^{\gamma_0\int \mathrm{d}t\mu_2}(\chi_2+\chi^2)$$

$$W=m_\phi\phi_1^2/2+\zeta\left(\phi_2+2-\sqrt{3}\right)\text{ with }m_\phi\sim 10^{13}\mathrm{GeV}$$

$$\mu_1\sim m_\phi\sim 10^{13}\mathrm{GeV}$$

$$H^{-1}\sim m_{3/2}^{-1}\,\mathrm{O}\left(m_{3/2}\right)\sim 10^2\mathrm{GeV}$$

$$n_{3/2}\sim 10^{-2}\left(m_\phi m_{3/2}\right)^{3/2}$$

$$s\sim\left(M_P m_{3/2}\right)^{3/2}\text{ and }\frac{n_{3/2}}{s}\sim 10^{-2}\left(\frac{m_\phi}{M_P}\right)^{3/2}$$

$$\mathcal{D}_\mu\psi_\nu=\left(\partial_\mu+\frac{1}{4}\omega_\mu^{ab}(e)\gamma_{ab}+\frac{1}{2}\mathrm{i}A_\mu^B\gamma_5\right)\psi_\nu-\Gamma_{\mu\nu}^\lambda\psi_\lambda$$

$$A_\mu^B=\frac{1}{2}\mathrm{i}\left[(\partial_i\mathcal{K})\partial_\mu z^i-\left(\partial^i\mathcal{K}\right)\partial_\mu z_i\right]+M_P^{-2}W_\mu^\alpha\mathcal{P}_\alpha$$

$$J_\mu=z_i\partial_\mu z^i-z^i\partial_\mu z_i$$

$$\gamma_0=\begin{pmatrix}\mathrm{i}\mathbb{1}_2 & 0\\ 0 & -\mathrm{i}\mathbb{1}_2\end{pmatrix};\ \vec{\gamma}=\begin{pmatrix}0 & -\mathrm{i}\vec{\sigma}\\ \mathrm{i}\vec{\sigma} & 0\end{pmatrix};\ \gamma_5=\begin{pmatrix}0 & -\mathbb{1}_2\\ -\mathbb{1}_2 & 0\end{pmatrix}.$$

$$P_L=\frac{1}{2}(1+\gamma_5),P_R=\frac{1}{2}(1-\gamma_5)$$

$$\lambda^*=\begin{pmatrix}0 & -\sigma_2\\ \sigma_2 & 0\end{pmatrix}\lambda$$

$$\bar{\lambda}_L\equiv(\lambda_L)^T C=\bar{\lambda}P_L=-\mathrm{i}(\lambda_R)^\dagger\gamma_0$$

$$R_{\mu\nu}^{ab}=2\partial_{[\mu}\omega_{\nu]}^{ab}+2\omega_{[\mu}^{ac}\omega_{\nu]c}^{\ b}$$
$$R_{\mu\nu}=R_{\mu\rho}^{ab}e_a^\rho e_\nu^b,R=g^{\mu\nu}R_{\mu\nu}$$
$$G_{\mu\nu}=e^{-1}\frac{\delta}{\delta g^{\mu\nu}}\int \mathrm{d}^4xeR=R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R$$
$$T_{\mu\nu}=-e^{-1}e_\nu^a\frac{\delta}{\delta e_a^\mu}M_P^{-2}\int \mathrm{d}^4x\mathcal{L}^{(m)}$$

$$
\begin{array}{llll}
X_I^C = X^I\,, & Y^C = Y^*\,, & z_i^C = z^i\,, & \phi_i^C = \phi^i\,,\\
\gamma_\mu^C = \gamma_\mu\,, & \gamma_5^C = -\gamma_5\,, & P_L^C = P_R & \\
\Omega_I^C = \Omega^I\,, & \chi_i^C = \chi^i\,, & \bar{\chi}_i^C = \bar{\chi}^i\,, & \\
\lambda^{\alpha\,C} = \lambda^\alpha\,, & \lambda_L^{\alpha\,C} = \lambda_R^\alpha\,, & \mathcal{P}_\alpha{}^C = \mathcal{P}_\alpha\,. &
\end{array}
$$

$$e_\mu^a = a(\eta)\delta_\mu^a, g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$$

$$\omega_\mu^{ab} = 2\delta_\mu^{[a}\partial^{b]}\ln\, a, \Gamma_{\mu\nu}^\rho = 2\delta_{(\nu}^\rho\partial_{\mu)}\ln\, a - \eta_{\mu\nu}\partial^\rho\ln\, a.$$

$$
\begin{aligned}
&R^\mu_{\nu\rho\sigma} = \eta_{\nu\nu'}\left[4\delta^{[\mu}_{[\sigma}\partial_{\rho]}\partial^{\nu']}\ln\, a + 4\delta^{[\mu}_{[\rho}(\partial^{\nu']}\ln\, a)(\partial_{\sigma]}\ln\, a) - 2\delta^{[\mu}_{[\rho}\delta^{\nu']}_{\sigma]}(\partial\ln\, a)^2\right]\\
&R_{\mu\nu} = 2\partial_\mu\partial_\nu\ln\, a - 2(\partial_\mu\ln\, a)(\partial_\nu\ln\, a) + \eta_{\mu\nu}[\Box\ln\, a + 2(\partial\ln\, a)^2]\\
&a^2R = 6\,\Box\ln\, a + 6(\partial\ln\, a)^2\\
&G_{\mu\nu} = 2\partial_\mu\partial_\nu\ln\, a - 2(\partial_\mu\ln\, a)(\partial_\nu\ln\, a) - \eta_{\mu\nu}[2\,\Box\ln\, a + (\partial\ln\, a)^2].
\end{aligned}
$$

$$+\frac{1}{4}\omega_\mu^{ab}\gamma_{ab} = +\frac{1}{2}\delta_\mu^a\gamma_a^b\delta_b^\nu\partial_\nu\ln\, a.$$

$$a\,\not{\mathcal{D}}\psi_\mu \equiv \overline{\not{\partial}}\psi_\mu + \tfrac{1}{2}\left(\overline{\not{\partial}}\ln a\right)\psi_\mu + \overline{\gamma}_\mu(\partial_\sigma\ln a)\eta^{\nu\sigma}\psi_\nu - \overline{\gamma}\cdot\psi\partial_\mu\ln a\,.$$

$$\dot{a} \equiv \frac{\partial_0 a}{a},$$

$$H = \frac{\dot{a}}{a} = \frac{\partial_0 a}{a^2}.$$

$$
\begin{aligned}
\mathcal{D}_\mu\chi &= \partial_\mu\chi - \tfrac{1}{2}\overline{\gamma}_{\mu0}\dot{a}\chi\\
a\,\not{\mathcal{D}}\chi &= \overline{\not{\partial}}\chi + \tfrac{3}{2}\gamma^0\dot{a}\chi\\
\mathcal{D}_\mu\psi^\mu &= \partial^\mu\psi_\mu - 2a^{-2}\dot{a}\psi_0 + \tfrac{1}{2}a^{-2}\dot{a}\gamma_0\vec{\gamma}\cdot\vec{\psi}\\
a\,\not{\mathcal{D}}\psi_\mu &= \overline{\not{\partial}}\tilde{\psi}_\mu - \dot{a}\left(\overline{\gamma}_\mu\psi_0 + \tfrac{1}{2}\gamma_0\psi_\mu + \overline{\gamma}\cdot\psi\delta_\mu^0\right)\,.
\end{aligned}
$$

$$
\begin{aligned}
&-M_P^{-2}\rho \equiv G_0^0 \;=\; -3a^{-2}(\partial_0\ln\, a)^2 = -3H^2\\
&-M_P^{-2}p\mathbb{1}_3 \equiv G = a^{-2}\,\mathbb{1}_3[2\partial_0^2\ln\, a + (\partial_0\ln\, a)^2] = \mathbb{1}_3(3H^2 + 2\dot{H}),
\end{aligned}
$$

$$a^{-1} = H(C-\eta)$$
$$R_{\mu\nu}^{\rho\sigma} = 2\delta_{\mu}^{[\rho}\delta_{\nu}^{\sigma]}H^2$$
$$R = -12H^2 = -4VM_P^{-2}$$
$$G_{\mu\nu} = -\frac{1}{4}g_{\mu\nu}R = g_{\mu\nu}VM_P^{-2},$$

$$\mathcal{L} = -\mathcal{N}_I{}^J\mathcal{D}_\mu X^I\mathcal{D}^\mu X_J$$

$$\mathcal{D}_\mu X_I = \partial_\mu X_I + \frac{1}{3}\mathrm{i}A_\mu X_I, \mathcal{D}_\mu X^I = \partial_\mu X^I - \frac{1}{3}\mathrm{i}A_\mu X^I.$$

$$X^I\frac{\partial}{\partial X^K}\mathcal{N}_I^J = X^K\frac{\partial}{\partial X^K}\mathcal{N}_I^J = 0$$

$$X_I = Yx_I(z), X^I = Y^*x^I(z^*)$$

$$\mathcal{L} = -\frac{1}{4}\mathcal{N}^{-1}(\partial_\mu\mathcal{N})^2 - \frac{1}{9}\mathcal{N}\left(A_\mu + \frac{\mathrm{i}}{2}(\partial^i\mathcal{K}\partial_\mu z_i - \partial_i\mathcal{K}\partial_\mu z^i) - \frac{3\mathrm{i}}{2}\partial_\mu\ln\frac{Y}{Y^*}\right)^2$$
$$+\frac{1}{3}\mathcal{N}(\partial^j\partial_i\mathcal{K})\partial_\mu z^i\partial^\mu z_j$$

$$\mathcal{N} \equiv X^I\mathcal{N}_I^J X_J$$
$$\mathcal{K}(z,z^*) \equiv -3\ln\left[-\frac{1}{3}x^I(z^*)\mathcal{N}_I^J(z,z^*)x_J(z)\right]$$

$$\mathcal{N} = -3M_P^2$$

$$\mathcal{L}(A_\mu) = -\frac{1}{9}\mathcal{N}(A_\mu - \tilde{A}_\mu)^2 + \mathcal{L}(\tilde{A}_\mu)$$
$$\tilde{A}_\mu \equiv \frac{3\mathrm{i}}{2\mathcal{N}}\left[X^I\mathcal{N}_I^J(\partial_\mu X_J) - (\partial_\mu X^I)\mathcal{N}_I^J X_J\right]$$

$$\partial^i\mathcal{K} = -3\frac{x^I\mathcal{N}_I^J\partial^i x_J}{x^K\mathcal{N}_K{}^L x_L}$$
$$\tilde{A}_\mu = -\frac{\mathrm{i}}{2}(\partial^i\mathcal{K}\partial_\mu z_i - \partial_i\mathcal{K}\partial_\mu z^i) + \frac{3\mathrm{i}}{2}\partial_\mu\ln\frac{Y}{Y^*}$$

$$\ln(-\mathcal{N}) = -\frac{1}{3}\mathcal{K} + \ln(3YY^*)$$

$$\tilde{\mathcal{D}}_\mu X_I = Y\partial_\mu z_i\mathcal{D}^i x_I + \frac{1}{2}X_I\partial_\mu\ln\mathcal{N}, \mathcal{D}^i x_I \equiv \left[\partial^i + \frac{1}{3}(\partial^i\mathcal{K})\right]x_I$$

$$\mathcal{L} = -\frac{1}{4}\mathcal{N}^{-1}(\partial_\mu\mathcal{N})^2 - \frac{1}{9}\mathcal{N}(A_\mu - \tilde{A}_\mu)^2 - YY^*\mathcal{N}_I^J\mathcal{D}^i x_I\mathcal{D}_j x^J\partial_\mu z_i\partial^\mu z^j$$

$$\partial^j\partial_i(x^I\mathcal{N}_I^J x_J) = (\partial_i x^I)\mathcal{N}_I^J(\partial^j x_J)$$

$$\mathcal{L} = -M_P^2(\partial^j\partial_i\mathcal{K})\partial_\mu z^i\partial^\mu z_j$$

$$x^I\mathcal{N}_I{}^J x_J < 0$$

$$\partial^j\partial_i\mathcal{K} \propto -\mathcal{N}\left(\partial_i x^I \mathcal{N}_I{}^J \partial^j x_J\right) + (\partial_i x^I \mathcal{N}_I{}^K x_K)\left(x^L \mathcal{N}_L{}^J \partial^j x_J\right)$$

$$V'(z,z^*) = V(z,z^*)\exp\left[-\frac{1}{3}(w_+\Lambda_Y + w_-\Lambda_Y^*)\right].$$

$$\mathcal{D}^i V = \partial^i V + \frac{1}{3}w_+\left(\partial^i\mathcal{K}\right)V$$

$$(\partial^I\mathcal{V})\mathcal{D}^i x_I = Y^{w_+-1}(Y^*)^{w_-}\mathcal{D}^i V$$

$$\mathcal{D}^i\mathcal{D}^j x_I \equiv \partial^i\mathcal{D}^j x_I + \frac{1}{3}\left(\partial^i\mathcal{K}\right)\mathcal{D}^j x_I - \Gamma_k^{ij}\mathcal{D}^k x_I$$

$$= -Y\mathcal{N}^{-1}{}_I{}^J\mathcal{N}_J{}^{KL}\left(\mathcal{D}^i x_K\right)\left(\mathcal{D}^j x_L\right)$$
$$\mathcal{D}^i\mathcal{D}^j W \equiv \partial^i\mathcal{D}^j W + \left(\partial^i\mathcal{K}\right)\mathcal{D}^j W - \Gamma_k^{ij}\mathcal{D}^k W$$
$$= Y^{-1}M_P^3\mathcal{W}^{IJ}\left(\mathcal{D}^i x_I\right)\left(\mathcal{D}^j x_J\right) + Y^{-2}M_P^3\mathcal{W}^I\mathcal{D}^i\mathcal{D}^j x_I$$
$$M_P^2\left(R_{k\ell}^{ij} - \frac{2}{3}g_k^{(i}g_\ell^{j)}\right) = -YY^*(\mathcal{D}_\ell x^L)\mathcal{N}_L{}^I\partial_k\mathcal{D}^i\mathcal{D}^j x_I$$
$$= \left(\mathcal{N}_{KL}^{IJ} - \mathcal{N}_{KL}^M\mathcal{N}^{-1}{}_M{}^N\mathcal{N}_N^{IJ}\right)(\mathcal{D}_\ell x^L)(\mathcal{D}_k x^K)\left(\mathcal{D}^i x_I\right)\left(\mathcal{D}^j x_J\right)(YY^*)^2$$

$$\mu = \frac{1}{2}\mathrm{i}A\dot{A}^* = -\frac{\dot{A}_1}{2A_2} = \frac{1}{2\alpha^2}(\dot{\alpha}_2\alpha_1 - \dot{\alpha}_1\alpha_2).$$

$$m = \mathrm{e}^{\mathcal{K}/2}|W|, \dot{m} = (\mathcal{D}_1 m)\dot{\phi} \equiv m_1\dot{\phi}$$
$$\dot{m}_1 = (\mathcal{D}_1\mathcal{D}_1 m + M_P^{-2}m)\dot{\phi} = \left(m_{11} + m_{3/2}\right)\dot{\phi}$$

$$\alpha = \dot{\phi}^2 + m_1^2, \alpha_1 = \dot{\phi}^2 - m_1^2, \alpha_2 = 2m_1\dot{\phi}$$

$$\mu = \frac{-\ddot{\phi}m_1 + \left(m_{11} + m_{3/2}\right)\dot{\phi}^2}{\dot{\phi}^2 + m_1^2}.$$

$$\ddot{\phi} = -3\frac{\dot{a}}{a}\dot{\phi} - m_1\left(-2m_{3/2} + m_{11}\right),$$

$$\mu = \left(m_{11} + m_{3/2}\right) + 3\left(H\dot{\phi} - m_{3/2}m_1\right)\frac{m_1}{\dot{\phi}^2 + m_1^2}$$

$$\mathcal{D}_\mu\partial^\mu\phi_i = \partial_\mu\partial^\mu\phi_i + \Gamma_{\mu\nu}^\mu\partial^\nu\phi_i + \Gamma_i^{jk}\partial_\mu\phi_j\partial_\mu\phi_k$$

$$\mathbf{m}' = \exp\,(\mathrm{i}\gamma_5\Lambda)\mathbf{m}, \psi_\mu' = \exp\left(-\frac{1}{2}\mathrm{i}\gamma_5\Lambda\right)\psi_\mu, \Upsilon_\mu' = \exp\left(\frac{1}{2}\mathrm{i}\gamma_5\Lambda\right)\Upsilon_\mu$$

$$(P_L)^\dagger = P_L, (P_L)^C = P_R$$

$$\theta' = \mathrm{e}^{\mathrm{i}\gamma_5\Lambda/2}\theta, \hat{A}' = \mathrm{e}^{\mathrm{i}\gamma_5\Lambda/2}\hat{A}\mathrm{e}^{-\mathrm{i}\gamma_5\Lambda/2}, \hat{B}' = \mathrm{e}^{\mathrm{i}\gamma_5\Lambda/2}\hat{B}\mathrm{e}^{-\mathrm{i}\gamma_5\Lambda/2}$$

$$\hat{\partial}_0\theta = \partial_0\theta - \mathrm{i}\gamma_5 A_0^B/2$$

$$\dot{\theta} = a^{-1}\partial_0\theta + \tfrac{1}{4}\gamma_5\left(\dot{\phi}^i\partial_i\mathcal{K} - \dot{\phi}_i\partial^i\mathcal{K}\right)$$

$$V = -\frac{1}{2}g^2(\cosh{(a|\phi|)} + 2)$$

$$\Lambda = -\frac{3}{2}g^2$$

$$V = -\frac{1}{2}g^2(\cosh{(a|\phi|)} - 2)$$

$$\Lambda = \frac{1}{2}g^2$$

$$\eta_{AB}z^A z^B = R^2$$

$$ds^2 = f_1^2(y)dS_d^2(x) + f_2^2(y)d\Omega_{p,q,0}^2(y)$$

$$\eta_{AB} = \mathrm{diag}\left(\mathbb{1}_p, -c^2\mathbb{1}_q\right)$$

$$L^2 = R^{-2}\left[\sum_{i=1}^{p} \left(z^i\right)^2 + c^4\sum_{i=p+1}^{p+q} \left(z^i\right)^2\right]$$

$$f_1 = L^a, f_2 = L^b$$

$$p = 4, q = 4, c^2 = 1, a = 2/3, b = -1/3$$

$$p = 5, q = 3, c^2 = 3, a = 2/3, b = -1/3$$

$$p = 3, q = 3, c^2 = 1, a = 1/2, b = -1/2$$

$$S_{IIA^*} = \int d^{10}x\sqrt{-g}[e^{-2\Phi}(R + 4(\partial\Phi)^2 - H^2) \\ +G_2^2 + G_4^2] + \cdots$$

$$S_{IIB^*} = \int d^{10}x\sqrt{-g}[e^{-2\Phi}(R + 4(\partial\Phi)^2 - H^2) \\ +G_1^2 + G_3^2 + G_5^2] + \cdots$$

$$H^d = \frac{SO(d,1)}{SO(d)}$$

$$dS_d = \frac{SO(d,1)}{SO(d-1,1)}, AdS_d = \frac{SO(d-1,2)}{SO(d-1,1)}$$

$$E = \frac{2\pi n}{R}$$

$$E^2 - \mathbf{p}^2 = m_S^2 N$$

$$\Delta\Phi = -m^2\Phi$$

$$\Delta_N f_n = \lambda_n f_n$$

$$\Delta\Phi^a = -M^a{}_b\Phi^b + c_{bc}^a\Phi^b\Phi^c + O(\Phi^3)$$

$$\Phi^a(x,y) = \sum_n \phi_n^a(x)f_n$$

$$f_m(y)f_n(y) = \sum_p d_{mn}^p f_p(y)$$

$$\Delta_M \phi_n^a = M^a{}_b \phi_n^b - \lambda_n \phi_n^a + c_{bc}^a d_{pq}^n \phi_p^b \phi_q^c + O(\phi^3)$$

$$ds^2 = H^{-\delta}(-dt^2 + dx_1^2 + \cdots + dx_m^2) + H^{\alpha}(dr^2 + r^2 d\Omega_N^2)$$

$$H = c + \frac{a^{n-1}}{r^{n-1}}$$

$$e^{\phi} = H^{\gamma}$$

$$ds^2 = H^A d\tilde{s}^2, A = \alpha - \frac{2}{n-1}$$

$$d\tilde{s}^2 = \frac{r^{\beta}}{a^{\beta}} dx_{\parallel}^2 + \frac{a^2}{r^2} dr^2 + a^2 d\Omega_N^2$$

$$ds_{AdS}^2 = \frac{U^2}{a^{\beta}} dx_{\parallel}^2 + \frac{4a^2}{\beta^2} \frac{1}{U^2} dU^2$$

$$e^{\phi} \propto U^{-2(n-1)\gamma/\beta}$$

$$ds^2 = H^{-\delta}(dx_1^2 + \cdots + dx_m^2) + H^{\alpha}(-dt^2 + dr^2 + r^2 d\Omega_N^2),$$

$$e^{\phi} = H^{\gamma}$$

$$H = c + \frac{q}{r^{n-2}}$$

$$H = mt + b$$

$$H = c + \frac{a^{n-1}}{\tau^{n-1}}$$

$$t = \tau \cosh\ \rho, r = \tau \sinh\ \rho$$

$$d\tilde{s}^2 = \frac{\tau^{\beta}}{a^{\beta}} dx_{\parallel}^2 - \frac{a^2}{\tau^2} d\tau^2 + a^2 d\tilde{\Omega}_{\tilde{N}}^2$$

$$d\Omega_{\tilde{N}}^2 = d\rho^2 + \sinh^2\ \rho d\Omega_N^2$$

$$ds^2 = \frac{\tau^{\beta}}{a^{\beta}} dx_{\parallel}^2 - \frac{a^2}{\tau^2} d\tau^2$$

$$ds_{dS}^2 = \frac{T^2}{a^2} dx_{\parallel}^2 - \frac{a^2}{T^2} dT^2$$

$$e^{\phi} \propto T^{-2(n-1)\gamma/\beta}$$

$$c' + \frac{b^{n-1}}{\sigma^{n-1}}$$

$$r = \sigma \cosh\ \xi, t = \sigma \sinh\ \xi$$

$$ds^2 = \frac{\sigma^{\beta}}{a^{\beta}} dx_{\parallel}^2 + \frac{b^2}{\sigma^2} d\sigma^2 + b^2 d\hat{\Omega}_{\tilde{N}}^2$$

$$d\bar{\Omega}_{\tilde{N}}^2 = -d\xi^2 + \cosh^2\ \xi d\Omega_N^2$$

$$ds^2 = \frac{\sigma^\beta}{b^\beta} dx_\parallel^2 + \frac{b^2}{\sigma^2} d\sigma^2$$

$$ds_H^2 = \frac{X^2}{b^2} dx_\parallel^2 + \frac{b^2}{X^2} dX^2$$

$$e^\phi \propto X^{-2(n-1)\gamma/\beta}$$

Morfología y fenomenología de la partícula supermasiva u oscura. Modelo matemático.

$$\mathcal{G} = R^{\mu\nu\rho\tau} R_{\mu\nu\rho\tau} - 4R^{\mu\nu} R_{\mu\nu} + R^2,$$

$$\lim_{D\to 4} (D-4)\alpha \to \alpha$$

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[R + \alpha \left(\phi \mathcal{G} + 4G_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi - 4(\nabla\phi)^2 \Box \phi + 2(\nabla\phi)^4 \right) \right] + S_m$$

$$\mathcal{E}_\phi = -\mathcal{G} + 8G^{\mu\nu} \nabla_\nu \nabla_\mu \phi + 8R^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - 8(\Box \phi)^2 + 8(\nabla\phi)^2 \Box \phi + 16 \nabla^\mu \phi \nabla^\nu \phi \nabla_\nu \nabla_\mu \phi$$
$$+ 8 \nabla_\nu \nabla_\mu \phi \nabla^\nu \nabla^\mu \phi = 0.$$

$$\begin{aligned}
\mathcal{E}_{\mu\nu} = & G_{\mu\nu} + \alpha \{ \phi H_{\mu\nu} - 2R \left[(\nabla_\mu \phi)(\nabla_\nu \phi) + \nabla_\nu \nabla_\mu \phi \right] + 8R^\sigma_{(\mu} \nabla_{\nu)} \nabla_\sigma \phi + 8R^\sigma_{(\mu} (\nabla_{\nu)} \phi)(\nabla_\sigma \phi) \\
& -2G_{\mu\nu} [(\nabla\phi)^2 + 2 \Box \phi] - 4 \left[(\nabla_\mu \phi)(\nabla_\nu \phi) + \nabla_\nu \nabla_\mu \phi \right] \Box \phi - \left[g_{\mu\nu} (\nabla\phi)^2 - 4(\nabla_\mu \phi)(\nabla_\nu \phi) \right] (\nabla\phi)^2 \\
& +8 (\nabla_{(\mu} \phi)(\nabla_{\nu)} \nabla_\sigma \phi) \nabla^\sigma \phi - 4 g_{\mu\nu} R^{\sigma\rho} [\nabla_\sigma \nabla_\rho \phi + (\nabla_\sigma \phi)(\nabla_\rho \phi)] + 2 g_{\mu\nu} (\Box \phi)^2 \\
& -4 g_{\mu\nu} (\nabla^\sigma \phi)(\nabla^\rho \phi)(\nabla_\sigma \nabla_\rho \phi) + 4(\nabla_\sigma \nabla_\nu \phi)(\nabla^\sigma \nabla_\mu \phi) \\
& -2 g_{\mu\nu} (\nabla_\sigma \nabla_\rho \phi)(\nabla^\sigma \nabla^\rho \phi) + 4 R_{\mu\nu\sigma\rho} [(\nabla^\sigma \phi)(\nabla^\rho \phi) + \nabla^\rho \nabla^\sigma \phi] \} = \kappa T_{\mu\nu}
\end{aligned}$$

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}}$$

$$H_{\mu\nu} = 2 \left[R R_{\mu\nu} - 2 R_{\mu\alpha\nu\beta} R^{\alpha\beta} + R_{\mu\alpha\beta\sigma} R_\nu^{\alpha\beta\sigma} - 2 R_{\mu\alpha} R_\nu^\alpha - \frac{1}{4} g_{\mu\nu} \mathcal{G} \right]$$

$$\kappa g^{\mu\nu} T_{\mu\nu} = g^{\mu\nu} \mathcal{E}_{\mu\nu} + \frac{\alpha}{2} \mathcal{E}_\phi = -R - \frac{\alpha}{2} \mathcal{G}$$

$$ds^2 = -e^{2\Phi(r)} dt^2 + e^{2\Psi(r)} dr^2 + r^2 d\Omega^2$$

$$T_{\mu\nu} = (\rho + p_\perp) u_\mu u_\nu + p_\perp g_{\mu\nu} - \sigma \chi_\mu \chi_\nu$$

$$\frac{2}{r} \left[1 + \frac{2\alpha(1 - e^{-2\Psi})}{r^2} \right] \frac{d\Psi}{dr} = e^{2\Psi} \left[8\pi\rho - \frac{1 - e^{-2\Psi}}{r^2} \left(1 - \frac{\alpha(1 - e^{-2\Psi})}{r^2} \right) \right]$$
$$\frac{2}{r} \left[1 + \frac{2\alpha(1 - e^{-2\Psi})}{r^2} \right] \frac{d\Phi}{dr} = e^{2\Psi} \left[8\pi p_r + \frac{1 - e^{-2\Psi}}{r^2} \left(1 - \frac{\alpha(1 - e^{-2\Psi})}{r^2} \right) \right]$$

$$\frac{dp_r}{dr} = -(\rho + p_r) \frac{d\Phi}{dr} + \frac{2}{r} \sigma$$

$$e^{-2\Psi} = 1 + \frac{r^2}{2\alpha} \left[1 - \sqrt{1 + \frac{8\alpha m}{r^3}} \right]$$

$$\lim_{\alpha\to 0} e^{-2\Psi} = 1 - \frac{2m}{r} + \frac{4m^2}{r^4}\alpha + \cdots$$

$$\frac{d\Phi}{dr} = \frac{r^3\left(\sqrt{1+\frac{8\alpha m}{r^3}} + 8\pi\alpha p_r - 1\right) - 2\alpha m}{r^2(2\alpha + r^2)\sqrt{1+\frac{8\alpha m}{r^3}} - 8\alpha m r - r^4}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

$$\frac{dp_r}{dr} = \frac{(\rho + p_r)[2\alpha m + r^3(1 - \mathcal{A} - 8\pi\alpha p_r)]}{r^2\mathcal{A}(r^2 + 2\alpha - r^2\mathcal{A})} + \frac{2}{r}\sigma$$

$$p_r(k_F) = \frac{1}{3\pi^2\hbar^3}\int_0^{k_F} \frac{k^4}{\sqrt{k^2 + m_e^2}} dk$$

$$p_r(k_F) = \frac{\pi m_e^4}{3h^3}\left[x_F(2x_F^2 - 3)\sqrt{x_F^2 + 1} + 3\sinh^{-1} x_F\right]$$

$$\rho = \frac{8\pi\mu_e m_H m_e^3}{3h^3} x_F^3$$

$$\sigma \equiv p_\perp - p_r = \beta p_r \mu$$

$$p_r(r\to R) = 0, p_\perp(r\to R) = 0$$

$$S = \frac{1}{2\kappa}\int f(\mathbb{Q},\mathcal{T})\sqrt{-g}d^4x + \int (\mathbb{L}_m + \mathbb{L}_e)\sqrt{-g}d^4x$$

$$\mathbb{L}_e = \frac{-1}{16\pi}F^{\varsigma\nu}F_{\varsigma\nu}.$$

$$\mathbb{Q} = -g^{\xi\varrho}\left(\mathrm{L}^{\rho}_{\mu\xi}\mathrm{L}^{\mu}_{\varrho\rho} - \mathrm{L}^{\rho}_{\mu\rho}\mathrm{L}^{\mu}_{\xi\varrho}\right),$$

$$\mathrm{L}^{\rho}_{\mu\varpi} = -\frac{1}{2}g^{\rho\lambda}(\nabla_\varpi g_{\mu\lambda} + \nabla_\mu g_{\lambda\varpi} - \nabla_\lambda g_{\mu\varpi})$$

$$\mathbb{Q}_\rho = \mathbb{Q}_\rho{}^{\xi}{}_{\xi}, \tilde{\mathbb{Q}}_\rho = \mathbb{Q}^{\xi}_{\rho\xi}.$$

$$P^{\rho}_{\xi\varrho} = -\frac{1}{2}\mathrm{L}^{\rho}_{\xi\varrho} + \frac{1}{4}(\mathbb{Q}^\rho - \tilde{\mathbb{Q}}^\rho)g_{\xi\varrho} - \frac{1}{4}\delta^{\rho}{}_{(\xi\mathbb{Q}_\varrho)}.$$

$$\mathbb{Q} = -\mathbb{Q}_{\rho\xi\varrho}P^{\rho\xi\varrho} = -\frac{1}{4}\left(-\mathbb{Q}^{\rho\xi\varrho}\mathbb{Q}_{\rho\varrho\xi} + 2\mathbb{Q}^{\rho\varrho\xi}\mathbb{Q}_{\xi\rho\varrho} - 2\mathbb{Q}^{\varrho}\tilde{\mathbb{Q}}_\varrho + \mathbb{Q}^\varrho\mathbb{Q}_\varrho\right)$$

$$\delta S = 0 = \int \frac{1}{2}\delta[f(\mathbb{Q},\mathcal{T})\sqrt{-g}] + \delta[\mathbb{L}_m\sqrt{-g}]d^4x$$

$$0 = \int \frac{1}{2}\left(\frac{-1}{2}f g_{\xi\varrho}\sqrt{-g}\delta g^{\xi\varrho} + f_{\mathbb{Q}}\sqrt{-g}\delta\mathbb{Q} + f_{\mathcal{T}}\sqrt{-g}\delta\mathcal{T}\right)$$
$$-\frac{1}{2}\mathcal{T}_{\xi\varrho}\sqrt{-g}\delta g^{\xi\varrho}d^4x$$

$$\mathcal{T}_{\xi\varrho} = \frac{-2}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g}\mathbb{L}_m\right)}{\delta g^{\xi\varrho}}, \Theta_{\xi\varrho} = g^{\rho\mu} \frac{\delta \mathcal{T}_{\rho\mu}}{\delta g^{\xi\varrho}}.$$

$$\delta\mathcal{T} = \delta\left(\mathcal{T}_{\xi\varrho} g^{\xi\varrho}\right) = \left(\mathcal{T}_{\xi\varrho} + \Theta_{\xi\varrho}\right)\delta g^{\xi\varrho}$$

$$\delta S = 0 = \int \frac{1}{2}\left\{\frac{-1}{2} f g_{\xi\varrho}\sqrt{-g}\delta g^{\xi\varrho} + f_{\mathcal{T}}\left(\mathcal{T}_{\xi\varrho} + \Theta_{\xi\varrho}\right)\sqrt{-g}\delta g^{\xi\varrho}\right.$$
$$-f_{\mathbb{Q}}\sqrt{-g}\left(P_{\xi\rho\mu}\mathbb{Q}_{\varrho}{}^{\rho\mu} - 2\mathbb{Q}^{\rho\varrho}{}_{\xi}P_{\rho\mu\varrho}\right)\delta g^{\xi\varrho} + 2f_{\mathbb{Q}}\sqrt{-g}P_{\rho\xi\varrho}\nabla^{\rho}\delta g^{\xi\varrho}$$

$$\left.+2f_{\mathbb{Q}}\sqrt{-g}P_{\rho\xi\varrho}\nabla^{\rho}\delta g^{\xi\varrho}\right\} - \frac{1}{2}\mathcal{T}_{\xi\varrho}\sqrt{-g}\delta g^{\xi\varrho} d^4x$$

$$\mathcal{T}_{\xi\varrho} + E_{\xi\varrho} = \frac{-2}{\sqrt{-g}}\nabla_{\rho}\left(f_{\mathbb{Q}}\sqrt{-g}P^{\rho}_{\xi\varrho}\right) - \frac{1}{2} f g_{\xi\varrho} + f_{\mathcal{T}}\left(\mathcal{T}_{\xi\varrho} + \Theta_{\xi\varrho}\right)$$
$$-f_{\mathbb{Q}}\left(P_{\xi\rho\mu}\mathbb{Q}_{\varrho}{}^{\rho\mu} - 2\mathbb{Q}^{\rho\mu}{}_{\xi}P_{\rho\mu\varrho}\right)$$

$$ds^2 = -e^{\alpha(r)}dt^2 + e^{\beta(r)}dr^2 + r^2(d\theta^2 + \sin^2 \, d\phi^2)$$

$$\mathcal{T}_{\xi\varrho} = (\rho + p_t)u_{\xi}u_{\varrho} + p_t g_{\xi\varrho} - \sigma k_{\xi}k_{\varrho}.$$

$$\mathcal{T} = 3p_r - \rho + 2\sigma$$

$$E_{\xi\varrho} = \frac{1}{4\pi}\left(F^{\nu}_{\xi}F_{\nu\varrho} - \frac{1}{4}g_{\xi\varrho}F_{\varsigma\nu}F^{\varsigma\nu}\right).$$

$$\left(\sqrt{-g}F_{\xi_{\varrho}}\right)_{;\varrho} = 4\pi J_{\xi}\sqrt{-g}F_{[\xi\varrho;\delta]} = 0$$

$$E(r) = \frac{e^{\frac{a+b}{2}}}{r^2} q(r)$$

$$q(r) = 4\pi \int_0^r \sigma r^2 e^b dr$$

$$\sigma = \frac{e^{-b}}{4\pi r^2}\frac{dq(r)}{dr}$$

$$q = q_0 r^3$$

$$\rho = \frac{1}{2r^2 e^{\beta}}\left[f_{\mathbb{Q}\mathbb{Q}} 2r\mathbb{Q}'(r)\left(e^{\beta} - 1\right) + f_{\mathbb{Q}}\left(\left(e^{\beta} - 1\right)(2 + r\alpha'(r)) + \left(e^{\beta} + 1\right)r\beta'(r)\right)\right.$$

$$+fr^2e^{\beta}\Big] - \frac{1}{3}f_{\mathcal{T}}(3\rho + p_r + 2p_t) - \frac{q^2}{r^4}$$

$$p_r = -\frac{1}{2r^2e^{\beta}}\Big[2r\mathbb{Q}'(r)f_{\mathbb{Q}\mathbb{Q}}(e^{\beta}-1) + f_{\mathbb{Q}}\Big(\big((e^{\beta}-1)(2+r\alpha'(r)) + rb'\big) - 2r\alpha'(r)\Big)\Big)$$

$$+fr^2e^{\beta}\Big] + \frac{q^2}{r^4} + \frac{2}{3}f_{\mathcal{T}}(p_t - p_r)$$

$$p_t = -\frac{1}{4re^{\beta}}\big[-2r\mathbb{Q}'(r)\alpha' f_{\mathbb{Q}\mathbb{Q}} + f_{\mathbb{Q}}(2\alpha'(e^{\alpha}-2) - r\beta'^2 + b'(2e^{\alpha} + r\alpha') - 2r\alpha')$$

$$+2fre^{\beta}\big] - \frac{q^2}{r^4} + \frac{1}{3}f_{\mathcal{T}}(p_r - p_t)$$

$$f(\mathbb{Q},\mathcal{T}) = A\mathbb{Q} + B\mathcal{T}$$

$$\rho = \Big[e^{-\beta(r)}\Big(\Big(\alpha'(r)B\Big(\big(e^{\beta(r)}-1\big)\big(2r^2-1+e^{\beta(r)}\big) + r\beta'(r)\Big) + \Big(B\big(e^{\beta(r)}\big)$$
$$\times\Big(2r^2\big(e^{5+\beta(r)}\big)-1\Big)\Big)\Big)\beta'(r) - 2Br\alpha''(r) + B(-r)\alpha'(r)^2Ar^3)$$
$$-2\Big(B\big(6r^2e^{\beta(r)}-7\big)-6\Big)\Big(e^{\beta(r)}(q^2-Ar^2)+Ar^2\Big)\Big)\Big]\big[2r^4(B(-8B$$
$$+2(4B+3)r^2e^{\beta(r)}-15)-6)\big]^{-1}$$

$$p_r = \Big[e^{-\beta(r)}\Big(4Br^2e^{2\beta(r)}((7B+3)q^2 + A(B+3)r^2) - 2e^{\beta(r)}((B(3B+13)+6)q^2$$
$$+Ar^2(B(13B+2(B+3)r^2+17)+6)\Big) + 2A(B(13B+17)+6)r^2 + Ar^3$$
$$\times\Big(-2Br\alpha''(r)\big(3B+2(4B+3)e^{\beta(r)}r^2+2\big) + \alpha'(r)(B(27B+47) + Br(3B$$
$$+2(4B+3)r^2e^{\beta(r)}+2\big)\beta'(r) + 2(B+1)(11B+6)r^2e^{2\beta(r)} - e^{\beta(r)}((B+1)$$
$$\times(11B+6) + 2(B(19B+23)+6)r^2) + 18) + \alpha'(r)^2B(-r)(3B+2(4B+3)$$
$$\times r^2e^{\beta(r)}+2\big) + \big(B(21+13B) + 2(B+1)(11B+6)r^2e^{2\beta(r)} - e^{\beta(r)}((B+1)$$
$$\times(11B+6) + 2(B+2)(B+3)r^2) + 6)\beta'(r)))\Big]\big[2(B+1)r^4(B(-8B$$
$$+2(4B+3)r^2e^{\beta(r)}\big)\big)\Big]^{-1},$$

$$p_t = \Big[e^{-\beta(r)}\Big(8Br^2e^{2\beta(r)}(ABr^2 - (5B+3)q^2) + 4e^{\beta(r)}((B(9B+17)+6)q^2 - ABr^2$$
$$\big(-2(B+2Br^2+2)\big) + 4AB(B+2)r^2 + Ar^3(2(r\alpha''(r)(B(-6B+2(4B+3)$$
$$r^2e^{\beta(r)}\big)\big) + \Big(B\Big(B + e^{\beta(r)}\Big(B - 2r^2\big(B+(B+1)e^{\beta(r)}\big)\Big)\Big)\Big)$$
$$\times\beta'(r)) + \alpha'(r)\big(r\big(B\big(6B-2(4B+3)r^2e^{\beta(r)}+13\big)+6\big)\beta'(r) + 2B(-9B$$
$$+e^{\beta(r)}\big(B - 2r^2\big(-5B+(B+1)e^{\beta(r)}-4\big)+1\big)-16\big)-12\big) + r\alpha'(r)^2(B$$
$$\times\big(-6B+2(4B+3)r^2e^{\beta(r)}-13\big)-6\big)\big)\big)\Big]\big[4(B+1)r^4(B(-8B+2$$
$$\times(4B+3)r^2e^{\beta(r)}\big)\big)\Big].$$

$$\alpha(r) = \gamma_0 r^2 + \gamma_1, \beta(r) = \gamma_2 r^2$$

$$ds^2 = -\left(1 + \frac{\mathbf{Q}^2}{r^2} - \frac{2\mathbf{M}}{r}\right)dt^2 + \left(1 + \frac{\mathbf{Q}^2}{r^2} - \frac{2\mathbf{M}}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$g_{tt} = e^{\gamma_0\mathbf{R}^2+\gamma_1} = \left(1+\frac{\mathbf{Q}^2}{\mathbf{R}^2}-\frac{2\mathbf{M}}{\mathbf{R}}\right),$$
$$g_{rr} = e^{\gamma_2\mathbf{R}^2} = \left(1+\frac{\mathbf{Q}^2}{\mathbf{R}^2}-\frac{2\mathbf{M}}{\mathbf{R}}\right)^{-1},$$
$$g_{tt,r} = 2\gamma_0\mathbf{R}e^{\gamma_0\mathbf{R}^2+\gamma_1} = \frac{2\mathbf{M}}{\mathbf{R}^2}-\frac{2\mathbf{Q}^2}{\mathbf{R}^3}.$$

$$\gamma_0 = \left(-\frac{2\mathbf{Q}^2}{\mathbf{R}^4}+\frac{2\mathbf{M}}{\mathbf{R}^3}\right)\left(-\frac{2\mathbf{M}}{\mathbf{R}}+1+\frac{\mathbf{Q}^2}{\mathbf{R}^2}\right)^{-1},$$

$$\gamma_1 = \ln\left(1-\frac{2\mathbf{M}}{\mathbf{R}}+\frac{\mathbf{Q}^2}{\mathbf{R}^2}\right)-\left(\frac{2\mathbf{M}}{\mathbf{R}^3}-\frac{2\mathbf{Q}^2}{\mathbf{R}^4}\right)\left(1-\frac{2\mathbf{M}}{R}+\frac{\mathbf{Q}^2}{\mathbf{R}^2}\right)^{-1}$$
$$\gamma_2 = \frac{\ln\left(\frac{\mathbf{R}^2}{-2\mathbf{MR}+\mathbf{Q}^2+\mathbf{R}^2}\right)}{\mathbf{R}^2}$$

$$\begin{aligned}\rho= &\left[e^{-\gamma_2 r^2}\big(2Br^2e^{2\gamma_2 r^2}(A(r^2(\gamma_0+\gamma_2)+3)-3q_0^2r^4)+e^{\gamma_2 r^2}(A(B(-(2r^4(\gamma_0\right.\\ &-5\gamma_2)+r^2(\gamma_0+\gamma_2+6)+7))-6)+(7B+6)q_0^2r^4)+A(r^2(-B(2\gamma_0 r^2\\ &\times(a-\gamma_2)+\gamma_0+9\gamma_2)-\gamma_2)+B+6))\big]\left[r^2\left(B\big(2(4B+3)r^2e^{\gamma_2 r^2}-8B\big)\right)\right]^{-1},\end{aligned}$$

$$\begin{aligned}p_r = &\left[e^{-\gamma_2 r^2}\big(e^{\gamma_2 r^2}((B(3B+13)+6)(-q_0^2)r^4-A(4\gamma_0^2B(4B+3)r^6+\gamma_0 r^2(-4\right.\\ &\times(4B+3)r^4+(B+1)(11B+6)+2(B(27B+29)+6)r^2)+\gamma_2 r^2((B+1)\\ &\times(11B+6)+2(B+2)(B+3)r^2)+B(13B+2(B+3)r^2+17)+6))\\ &+A(-2\gamma_0^2B(3B+2)r^4+\gamma_0 r^2(B(2\gamma_2(3B+2)r^2+21B+43)+18)+\gamma_2(B\\ &\times(13B+21)+6)r^2+B(13B+17)+6)+2r^2e^{2\gamma_2 r^2}(\gamma_0A(B+1)(11B+6)r^2\\ &+\gamma_2A(B+1)(11B+6)r^2+B(A(B+3)+(7B+3)q_0^2r^4))\big)\big][(B+1)r^2(B(2(4B\\ &+3)r^2e^{\gamma_2 r^2}-8B-15)-6)]^{-1}\end{aligned}$$

$$\begin{aligned}p_t = &\left[e^{-\gamma_2 r^2}\big(e^{\gamma_2 r^2}(AB(r^2(2\gamma_0^2(4B+3)r^4+\gamma_0(2r^2(-\gamma_2(4B+3)r^2+9B+7)\right.\\ &+B+1)+\gamma_2(B-2(B+2)r^2+1)-2B)-B-2)+(B(9B+17)+6)q_0^2r^4)\\ &+A(-\gamma_0^2(2B+3)(3B+2)r^4+\gamma_0 r^2(\gamma_2(2B+3)(3B+2)r^2-(3B+4))\\ &+\gamma_2(B(B+6)+6)r^2+B(B+2))-2Br^2e^{2\gamma_2 r^2}(A((B+1)r^2(\gamma_0+\gamma_2)-B)\\ &+(5B+3)q_0^2r^4)\big)\big]\left[(B+1)r^2\big(B\big(2(4B+3)r^2e^{\gamma_2 r^2}-8B-15\big)-6\big)\right]^{-1}.\end{aligned}$$

$$\chi = \left[e^{-\gamma_2 r^2}\big(e^{\gamma_2 r^2}((B(3B+13)+6)(-q_0^2)r^4-A(4\gamma_0^2B(4B+3)r^6+\gamma_0 r^2(-4\right.$$

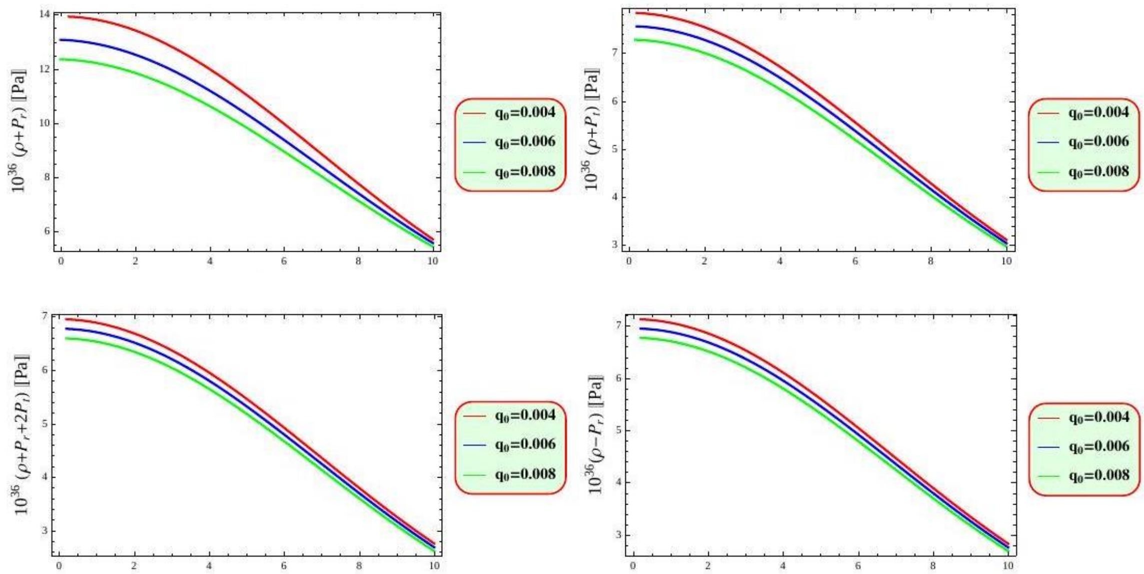
$$\times\gamma_2B(4B+3)r^4+(B+1)(11B+6)+2(B(27B+29)+6)r^2)+\gamma_2r^2((B+1)$$
$$+2(B+2)(B+3)r^2)+\eta(13B+2(B+3)r^2+17)+6)\big)+A(-2\gamma_0^2B(3B+2)$$
$$\times r^4+\gamma_0r^2(B(2\gamma_2(3B+2)r^2+21B+43)+18)+\gamma_2(B(13B+21)+6)r^2+B$$
$$\times(13B+17)+6)+2r^2e^{2\gamma_2r^2}(\gamma_0A(B+1)(11B+6)r^2+\gamma_2A(B+1)(11B+6)r^2$$
$$+B(A(B+3)+(7B+3)q_0^2r^4)\big)\big)\Big]\Big[(B+1)r^2\left(B\big(2(4B+3)r^2e^{\gamma_2r^2}-8B-15\big)\right)\Big]^{-1}$$
$$-\Big[e^{-\gamma_2r^2}\left(e^{\gamma_2r^2}(AB(r\gamma_2^2(2\gamma_0^2(4B+3)r^4+\gamma_0(2r^2(-\gamma_2(4B+3)r^2+9B+7)\right.$$
$$+B+1)+\gamma_2(B-2(B+2)r^2+1)-2B)-B-2)+(B(9B+17)+6)q_0^2r^4)$$
$$+A(-\gamma_0^2(2B+3)(3B+2)r^4+\gamma_0r^2(\gamma_2(2B+3)(3B+2)r^2-(3B+4)(5B+3))$$
$$+\gamma_2(B(B+6)+6)r^2+B(B+2))-2Br^2e^{2\gamma_2r^2}(A((B+1)r^2(a+\gamma_2)-B)$$
$$+(5B+3)q_0^2r^4)\big)\Big]\Big[(B+1)r^2\big(B\big(2(4B+3)r^2e^{\gamma_2r^2}-8B-15\big)-6\big)\Big]^{-1}.$$

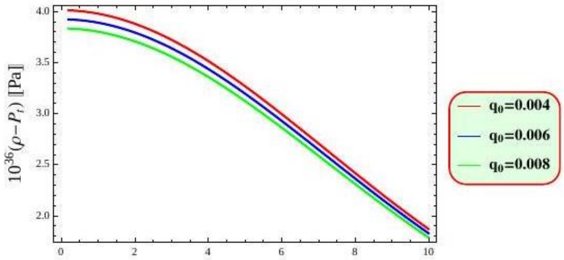
$$M(r)=4\pi\int_0^r\varsigma^2\rho(\varsigma)d\varsigma$$

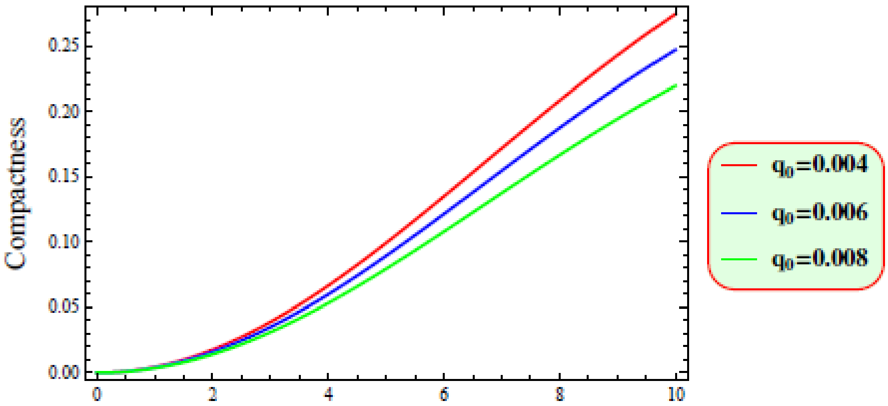
$$Z(r)=-1+\frac{1}{\sqrt{-g_{tt}}}$$

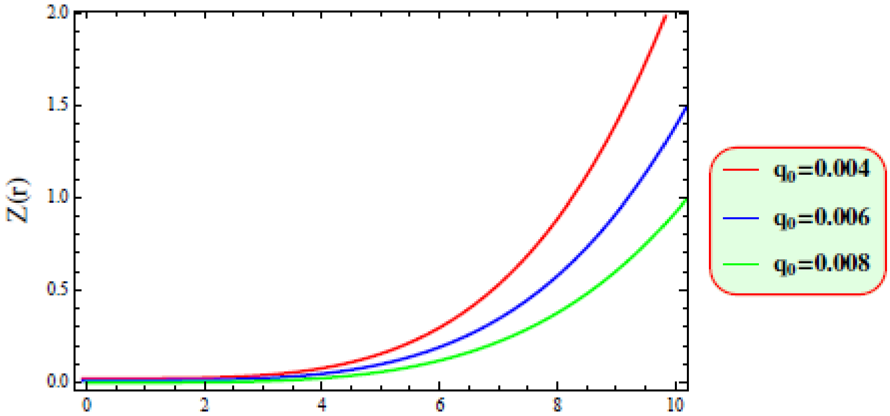
$$Z(r)=-1+\frac{1}{\sqrt{1-\mu}}$$

$$\rho(0)=\frac{A(-2\gamma_0B-17B\gamma_2-18\gamma_2)}{-8B^2-15B-6}$$
$$p_r(0)=\frac{A(10\gamma_0B^2+26\gamma_0B+12\gamma_0-11B^2\gamma_2-13B\gamma_2-6\gamma_2)}{(B+1)(-8B^2-15B-6)}$$
$$p_t(0)=\frac{A(-14\gamma_0B^2-28\gamma_0B-12\gamma_0+2B^2\gamma_2+7B\gamma_2+6\gamma_2)}{(B+1)(-8B^2-15B-6)}$$

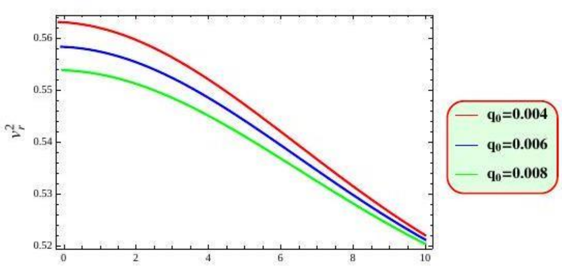


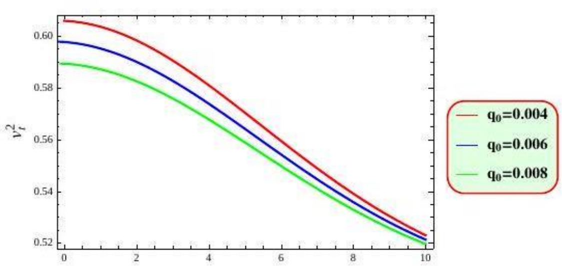


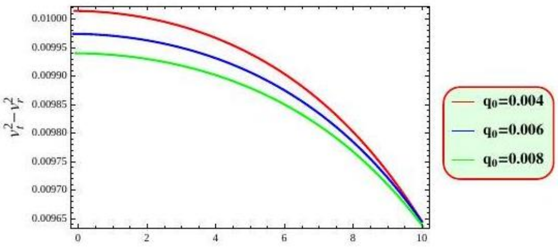




$$v_r^2 = \frac{dp_r}{d\rho}, v_t^2 = \frac{dp_t}{d\rho}$$





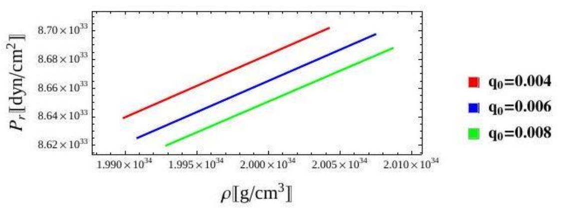


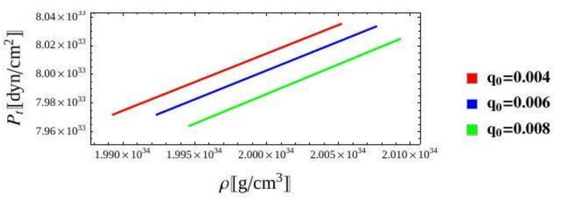
$$p_r(\rho) \approx v_r^2(\rho - \rho_I), p_t(\rho) \approx v_t^2(\rho - \rho_{II}),$$

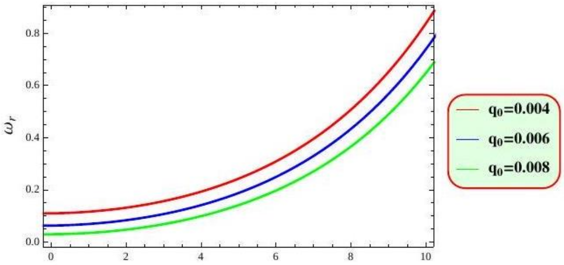
$$\omega_r = \frac{p_r}{\rho}, \omega_t = \frac{p_t}{\rho}$$

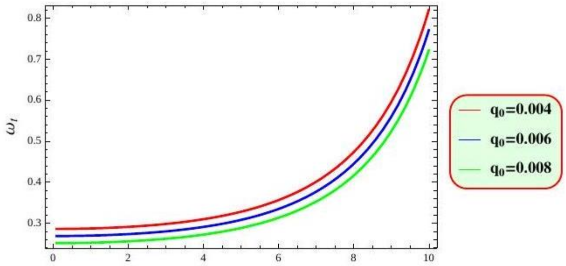
$$\begin{aligned}
\omega_r = {} & \big[e^{\gamma_2 r^2}\big((B(3B+13)+6)(-q_0^2)r^4 - A\big(4a^2B(4B+3)r^6 + \gamma_0 r^2(-4\gamma_2 B \\
& \times (4B+3)r^4 + (1+B)(B+6) + 2(B(27B)+6)r^2\big) + \gamma_2 r^2\big((B+1) \\
& \times (B+6) + 2(2+B)(3+B)r^2\big) + \big(13B + (B+3)r^2 + 17\big) + 6\big)\big) \\
& + \big(-2a^2B(3B+2)r^4 + \gamma_0 r^2\big(B\big(2\gamma_2(3B+2)r^2 + 21B + 43\big) + 18\big) \\
& + \gamma_2(B(13B+21)+6)r^2 + B(13B+17)+6\big) + 2r^2 e^{2\gamma_2 r^2}\big(\gamma_0 A(B+1) \\
& \times (11B+6)r^2\big) + \gamma_2 A(B+1)(11B+6)r^2 + B\big(A(B+3+(7B+3)q_0^2 r^4)\big)\big] \\
& \times \big[(B+1)\big(2Br^2 e^{2\gamma_2 r^2}\big(A\big(r^2(\gamma_0+\gamma_2)+3\big) - 3q_0^2 r^4\big) + e^{\gamma_2 r^2}\big((7B+6)q_0^2 r^4 \\
& - A\big(B\big(2r^4(\gamma_0 - 5\gamma_2) + r^2(\gamma_0+\gamma_2+6)+7\big)+6\big)\big)A\big(r^2\big(-B\big(2\gamma_0 r^2(\gamma_0-\gamma_2) \\
& + \gamma_0 + 9\gamma_2\big) - 12\gamma_2\big) + 7B + 6\big)\big)\big]^{-1}, \\
\omega_t = {} & \big[e^{\gamma_2 r^2}\big(AB\big(r^2\big(2a^2(4B+3)r^4 + \gamma_0\big(-2\gamma_2(4B+3)r^4 + B + 2(9B+7)r^2 + 1\big) \\
& + \gamma_2\big(-2 + B(2+B)r^2 + 1\big) - 2B\big) - 2 - B\big) + \big(B(9B+17)+6\big)q_0^2 r^4\big) + A\big(-a^2 \\
& \times (2B+3)(3B+2)r^4 + \gamma_0 r^2\big(\gamma_2(2B+3)(3B+2)r^2 - (3B+4)(5B+3)\big) \\
& + \gamma_2(B(B+6)+6)r^2 + B(B+2)\big) - 2Br^2 e^{2\gamma_2 r^2}\big(A\big((B+1)r^2(a+\gamma_2) - B\big) \\
& + (5B+3)q_0^2 r^4\big)\big]\big[(B+1)\big(2Br^2 e^{2\gamma_2 r^2}\big(A\big(r^2(\gamma_0+\gamma_2)+3\big) - 3q_0^2 r^4\big) \\
& + e^{\gamma_2 r^2}\big((7B+6)q_0^2 r^4 - A\big(B\big(2r^4(\gamma_0-5\gamma_2) + r^2(\gamma_0+\gamma_2+6)
\end{aligned}$$

$$+ 7\big) + 6\big)\big) + A\big(r^2\big(-B\big(2\gamma_0 r^2(\gamma_0-\gamma_2) + \gamma_0 + 9\gamma_2\big) - 12\gamma_2\big) + 7B + 6\big)\big)\big].$$







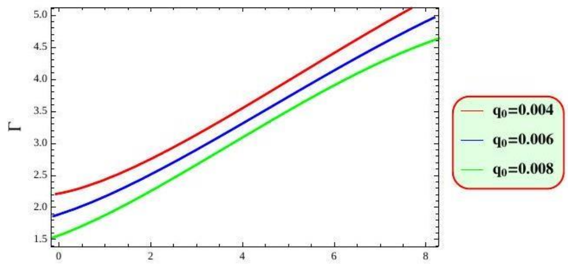


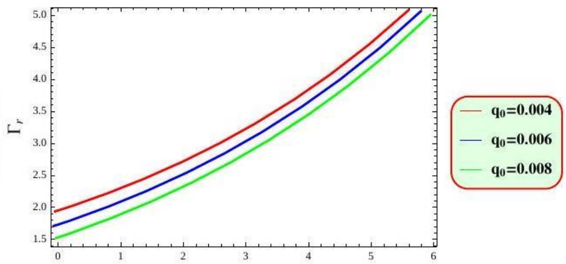
$$\Gamma = \frac{4}{3}\left(1 + \frac{\sigma}{r|\dot{p}_r|}\right)_{\max}$$

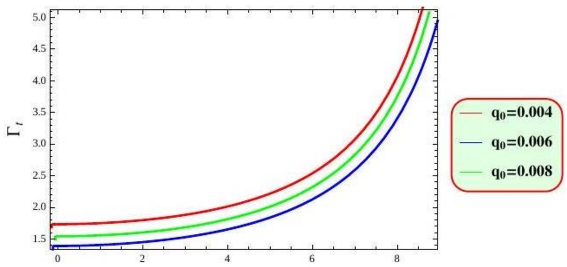
$$\Gamma_r = \frac{p_r + \rho}{p_r} v_r^2, \Gamma_t = \frac{p_t + \rho}{p_t} v_t^2$$

$$\frac{dp_r}{dr} + \frac{1}{r^2} e^{\frac{\alpha-\beta}{2}} \mathbf{M}_G(r)(\rho + p_r) - \frac{2}{r}(p_t - p_r) = 0$$

$$\mathbf{M}_G(r) = 4\pi \int \left(\mathcal{T}_t^t - \mathcal{T}_r^r - \mathcal{T}_\phi^\phi - \mathcal{T}_\theta^\theta\right) r^2 e^{\frac{\alpha+\beta}{2}} dr$$







$$\mathbf{M}_G(r) = \frac{1}{2} r^2 e^{\frac{\beta-\alpha}{2}} \alpha'.$$

$$\frac{1}{2}\alpha'(\rho + p_r) + \frac{dp_r}{dr} - \frac{2}{r}(p_t - p_r) = 0.$$

$$F_g + F_a + F_h + F_{(\mathbb{Q},\mathcal{T})} = 0.$$

$$F_g = \frac{(\rho + p_r)\alpha'}{2}, F_a = \frac{2\sigma}{r},$$

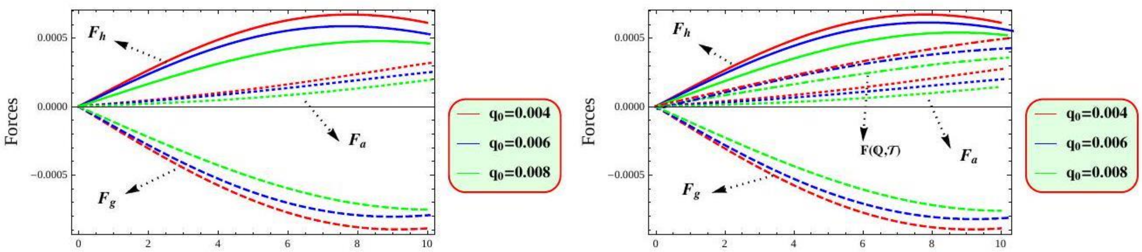
$$F_h = -p_r', F_{(\mathbb{Q},\mathcal{T})} = p_r' + \frac{1}{2}(\rho - p_r)\beta'(r) - 2r^{-1}(p_t - p_r).$$

$$F_a=-\Big[6e^{-\gamma_2 r^2}\big(e^{\gamma_2 r^2}(-A(2\gamma_0^2 B(4B+3)r^6+2\gamma_0 r^2(-\gamma_2 B(4B+3)r^4+2B^2$$
$$+3B+2(6B^2+6B+1)r^2+1)+2\gamma_2 r^2(2B^2+3B+(\eta+2)r^2+1)+4B^2$$
$$+5B+2Br^2+2)-2(2\eta^2+5B+2)q_0^2 r^4)+A(\gamma_0^2(3B+2)r^4+\gamma_0 r^2(2(6\eta^2$$
$$+12B+5)-\gamma_2(3B+2)r^2)+4\eta^2(\gamma_2 r^2+1)+5B(\gamma_2 r^2+1)+2)+2r^2$$
$$\times e^{2\gamma_2 r^2}(2\gamma_0 A(2\eta^2+3B+1)r^2+2\gamma_2 A(2B^2+3B+1)r^2+B(A+2(2B+1)$$
$$\times q_0^2 r^4)\big)\Big)\Big]\Big[(B+1)r^3\big(8B^2\big(r^2 e^{\gamma_2 r^2}-1\big)+3B\big(2r^2 e^{\gamma_2 r^2}-5\big)-6\big)\Big]^{-1},$$

$$F_g=-\Big[\gamma_0 e^{-\gamma_2 r^2}\Big(A\big(e^{\gamma_2 r^2}(2\gamma_0^2 B(4B+3)r^6+\gamma_0 r^2(-2\gamma_2 B(4B+3)r^4+6B^2$$
$$+9B+(28B^2+30B+6)r^2+3)+\gamma_2 r^2(6B^2+9B+(6-4\eta^2)r^2+3)$$
$$+10B^2+15B+4B^2r^2+6Br^2+6)+\gamma_0^2 B(4B+3)r^4-\gamma_0 r^2(2B^2(2\gamma_2 r^2$$
$$+5)+3\eta(\gamma_2 r^2+7)+9)-2r^2 e^{2\gamma_2 r^2}(3\gamma_0(2B^2+3\eta+1)r^2+3\gamma_2(2\eta^2$$
$$+3B+1)r^2+B(2B+3))-2\gamma_2 B^2 r^2+3\gamma_2 r^2-10B^2-15B-6)-2B^2$$
$$+q_0^2 r^4 e^{\gamma_2 r^2}\big(r^2 e^{\gamma_2 r^2}+1\big)\Big)\Big]\Big[(B+1)r\left(\eta^2\big(r^2 e^{\gamma_2 r^2}-1\big)+\eta\big(2r^2 e^{\gamma_2 r^2}\big)\right)\Big]^{-1},$$

$$F_h=\Big[2e^{-\gamma_0 r^2}\Big(-4Br^2 e^{2\gamma_0 r^2}(A(-\gamma_2^2\eta(4B+3)^2 r^6(\gamma_0 r^2-)+(\gamma_0^2\eta(4B+3)^2$$
$$\times\gamma_2 r^2 r^6+\gamma_0 r^2(\eta^3(2-52r^2)+B^2(8-79r^2)+B(9-30r^2)+3)$$
$$+(2B^3+8\eta^2\gamma_2+9B+3))+(\gamma_0^2 r^4(B^3(4r^2+2)+B^2(11r^2+8)$$
$$+\eta(6r^2+9)+3)+\gamma_0 r^2(9B+4B^3r^2+3B^2(r^2+2)+3)+B(4B+3)$$
$$\times(B+Br^2+2))\Big)+q_0^2 r^4(2\gamma_0 B^2(B+1)r^2+(40B^3+99B^2+75B$$
$$+18)))+e^{\gamma_0 r^2}(AB(-4\gamma_2^2(4B+3)r^6(\gamma_0(6B^2+13B+6)r^2-(8B^2$$
$$+15B+6))+4\gamma_2 r^4(\gamma_0^2(24B^3+70B^2+63B+18)r^4-\gamma_0(92B^3+245B^2$$
$$+204B+54)r^2+3(B+1)^2(2B+1))+(4\gamma_0^2(4B^3+27B^2+42B)r^6$$
$$+4\gamma_2(2B^3+9B^2+9B+3)r^4+(B+2)(8B^2(2r^2+1)+3B(4r^2+5)$$
$$+6)))+(72B^4+271B^3+357B^2+192B+36)q_0^2 r^4)+A(8B^2+15B+6)$$
$$\times(\gamma_2^2(6B^2+13B+6)r^4(\gamma_2 r^2-)+\gamma_2\gamma_0 r^4(3(7B^2+14B+6)-\gamma_0(6B^2$$
$$+13B+6)r^2)-(\gamma_0^2(B^2+6B+6)r^4+\gamma_0 B(B+2)r^2+B(B+2)R^4)\Big)$$
$$+4B^2(4B+3)r^4 e^{3\gamma_0 r^2}(AB+(5B+3)q_0^2 r^4)\Big)\Big]\Big[(B+1)r^3\big(8B^2\big(r^2 e^{\gamma_0 r^2}\big)$$

$$+3B\big(2r^2 e^{\gamma_0 r^2}-5\big)-6\big)^2\Big]^{-1}$$



$$
\begin{aligned}
v_r^2 = &-\big[(B+1)\big(12e^{3\gamma_0 r^2} r^4 (q_0^2 r^4 + A)B^2(4B+3) + 4e^{2\gamma_0 r^2} r^2 B(q_0^2(2\gamma_0 \\
&\times B^2 - 3(8B^2+15B+6))r^4 + A\big(\gamma_0^2\big((20r^2+2)B^2 + 3(5r^2+2)B\big)r^4 \\
&+3\gamma_0(-4r^2B^2 + (2-3r^2)B + 1)r^2 + a\big(\gamma_0\big((2-4r^2)B^2 - (r^2-2)B \\
&+3)r^2 + (2B^2+6B+3)\big)r^2 - (4B+3)\big((3r^2+7)B+6\big)\big)\big) - A \\
&\times (8B^2+15B+6)((7B+6) + 2a^2r^4B + \gamma_0^2 r^4(2ar^2B - 3(3B+4)) \\
&+\gamma_0 r^2(-2a^2Br^4 - 3aBr^2 + (7B+6))) + e^{\gamma_0 r^2}(q_0^2 r^4(56B^3 + 153B^2 \\
&+132B + 36) + A(-8a(a-\gamma_0)\gamma_0 B^2(4B+3)r^8 - 4\gamma_0 B(4B+3)(aB \\
&+3\gamma_0(3B+4))r^6 - 12(a-5\gamma_0)\eta(2B^2+3B+1)r^4 + (7B+6)(8(2r^2 \\
&+1)B^2 + 3(4r^2+5)B + 6))\big)\big)\big]\big[4e^{3\gamma_0 r^2} r^4 B^2(4\eta+3)(q_0^2 r^4(7B+3) - A \\
&-(B+3)) - A - (8B^2+15B+6)(2a^2(\gamma_0 r^2 -)B(3B+2)r^4 - a\gamma_0 \\
&-\big(2\gamma_0 B(3B+2)r^2 + 3(5B^2+13B+6)\big)r^4 - (\gamma_0^2(13B^2+21B+6)r^4 \\
&+\gamma_0(13B^2+17B+6)r^2 + (13B^2+17B+6))\big) + e^{\gamma_0 r^2}(q_0^2 r^4(24B^4 \\
&+149B^3 + 261B^2 + 168B + 36) - A(-8a^2B(4B+3)(\gamma_0 B(3B+2)r^2 \\
&+R^2(8B^2+15B+6)\big)r^6 + 4a(2\gamma_0^2 B^2(12B^2+17B+6)r^4 + \gamma_0 B(148B^3 \\
&+403B^2 + 339B + 90)r^2 - 3(B+1)^2(22B^2+23B+6)\big)r^4 + (4\gamma_0^2 B \\
&\times (52B^3 + 123B^2 + 87B + 18)r^6 + 4\gamma_0(74B^4 + 141B^3 + 54B^2 - 33B \\
&-18)r^4 + (13B^2+17B+6)(8(2r^2+1)B^2 + 3(4r^2+5)B + 6))\big)\big) \\
&\times e^{2\gamma_0 r^2} r^2(-2a^2(\gamma_0 r^2 -)AB^2(4B+3)^2 r^6 + aA(2\gamma_0^2 B^2(4B+3)^2 r^6 \\
&+\gamma_0\big((22-140r^2)B^4 - 5(49r^2-20)B^3 - 3(43r^2-49)B^2 + (87 \\
&-18r^2)B + 18)r^2 + (22B^4 + B^3 + 147B^2 + 87B + 18)\big)r^2 + (\gamma_0^2 A \\
&+\big((22-4r^2)B^4 + (100-23r^2)B^3 + (147-39r^2)B^2 + (87-18r^2)B \\
&+18)r^4 + \gamma_0(2q_0^2 B^2(11B^2 + 17\eta + 6)r^4 + A(-4r^2B^4 + (-15r^2)B^3
\end{aligned}
$$

$$-9(r^2-15)B^2+87B+18)\big)r^2B(q_0^2r^4(56B^3+B^2+B+18)-A$$
$$\times(4B+3)\big((r^2+13)B^2+(3r^2+17)B+6)\big)\big)\Big]^{-1},$$
$$v_t^2=\big[(B+1)\big(12e^{3\gamma_0r^2}r^4(q_0^2r^4+A)B^2(4B+3)+4e^{2\gamma_0r^2}r^2B(q_0^2(2\gamma_0r^2B^2$$
$$-3(8B^2+15B+6)\big)r^4+A\big(\gamma_0^2\big((20r^2+2)B^2+3(5r^2+2)B+3\big)r^4$$
$$+3\gamma_0(-4r^2B^2+(2-3r^2)B+1)r^2+a\big(\gamma_0\big((2-4r^2)B^2-3(r^2-2)B$$
$$+3)r^2+(2B^2+6B+3)\big)r^2-(4B+3)\big((3r^2+7)B+)\big)\Big)-A(B^2$$
$$+15B+6)((7B+6)+2a^2r^4B+\gamma_0^2r^4(2ar^2B-3(3B+4))+\gamma_0r^2$$
$$\times(-2a^2Br^4-3aBr^2+(7B+6))\big)+e^{\gamma_0r^2}(q_0^2r^4(56B^3+B^2+132B$$
$$+36)+A(-8a(a-\gamma_0)\gamma_0B^2(4B+3)r^8-4\gamma_0B(4B+3)(aB+3\gamma_0$$
$$\times(3B+4))r^6-12(a-5\gamma_0)B(2B^2+3B+1)r^4+(7B+6)(8$$
$$\times(2r^2+1)B^2+3(4r^2+5)B+6)\big)\big)\Big)\Big]\big[4e^{3\gamma_0r^2}r^4B^2(4B+3)(q_0^2(5B+3)r^4$$
$$+AB)+A(8B^2+15B+6)(a^2(\gamma_0r^2-)(6B^2+13B+6)r^4+a\gamma_0$$
$$\times\big(3(7B^2+14B+6)-r^2(6B^2+13B+6)\big)r^4-(\gamma_0^2(B^2+6B+6)r^4$$
$$+\gamma_0B(B+2)r^2+B(B+2))\big)+e^{\gamma_0r^2}(q_0^2r^4(72B^4+271B^3+357B^2$$
$$+192B+36)R^6+AB(-4a^2(4B+3)(\gamma_0r^2(6B^2+13B+6)-(8B^2$$
$$+15B+6))r^6+4a(\gamma_0^2(24B^3+70B^2+63B+18)r^4-\gamma_0(92B^3+245B^2$$
$$+204B+54)r^2+3(B+1)^2(2B+1))r^4+(4\gamma_0^2(4B^3+27B^2+42B$$
$$+18)r^6+4f(2B^3+9B^2+9B+3)r^4+(B+2)(8(2r^2+1)B^2+3$$
$$+(4r^2+5)B+6)\big)\big)-4e^{2\gamma_0r^2}r^2B\big(q_0^2r^4\big((40B^3+99B^2+75B+18)$$
$$+2\gamma_0r^2B^2(B+1)\big)+A(-a^2(\gamma_0r^2-R^2)B(4B+3)^2r^6+a(\gamma_0^2B$$
$$+(4B+3)^2+r^6+\gamma_0\big((2-52r^2)B^3+(8-79r^2)B^2+(9-30r^2)B$$
$$+3)r^2+(2B^3+8B^2+9B+3)\big)r^2+\big(\gamma_0^2\big((4r^2+2)B^3+(r^2+8)B^2$$
$$+(6r^2+9)B+3)r^4+\gamma_0(4r^2B^3+3(r^2+2)B^2+9B+3)r^2+B(4B$$
$$+3)(Br^2+B+2)\big)\big)\big)\Big]^{-1}$$

$$
\begin{aligned}
\Gamma &= \frac{4}{3}\Big(\Big[1-3\big(8\big(e^{\gamma_0 r^2}r^2-1\big)B^2+3\big(2e^{\gamma_0 r^2}r^2-5\big)B6\big)\big(2e^{2\gamma_0 r^2}r^2 \\
&\times \big(2\gamma_2 A\big(2B^2+3B+1\big)r^2+2\gamma_0 A\big(2B^2+3B+1\big)r^2+R^2B\big(2q_0^2(2B+1)r^4 \\
&+ A\big)\big)+A\big(\gamma_2^2(3B+2)r^4+\gamma_2\big(2R^2\big(6B^2+12B+5\big)-\gamma_0 r^2(3B+2)\big)r^2 \\
&+ \big(\gamma_0 B(4B+5)r^2+\big(4\eta^2+5B+2\big)\big)\big)+e^{\frac{\gamma_0 r^2}{R^2}}\big(-2q_0^2r^4\big(2B^2+5B\big) \\
&- A\big(2\gamma_2^2\eta(4B+3)r^6+2\gamma_2\big(\big(2\big(6\eta^2+6B+1\big)r^2+2\eta^2+3B+1\big)-\gamma_0 r^4\eta \\
&\times (4B+3)\big)r^2+\big(2\gamma_0\big((B+2)r^2+2B^2+\eta+1\big)r^2+\big(B^2+\big(r^2+5\big) \\
&\times B+2\big)\big)\big)\big)\big)\Big]\Big[2\big(4e^{3\gamma_0 r^2}r^4\eta^2(4B+3)\big(q_0^2r^4(7B+3)-A(\eta+3)\big)-A8B^2 \\
&\times \big(+15B+6\big)\big(2\gamma_2^2\big(\gamma_0 r^2-R^2\big)\eta(3B+2)r^4-\gamma_2\gamma_0\big(2\gamma_0 B(3\eta+2)r^2+3 \\
&\times \big(+6\big)\big)r^4-\big(\gamma_0^2\big(13\eta^2+21B+6\big)r^4+\gamma_2\big(13\eta^2+17B+6\big)r^2+ \\
&\times \big(13\eta^2+6\big)\big)\big)+e^{\gamma_0 r^2}\big(q_0^2r^4R^6\big(24B^4+149\eta^3+261B^2+168B+36\big)-A\big(\\
&+ R^2\big(8\eta^2+15B+6\big)\big)r^6+4a\big(2\gamma_0^2B^2\big(12B^2+17B+6\big)r^4+\gamma_0 B\big(148B^3 \\
&- 3(B+1)^2\big(22B^2+23B+6\big)\big)r^4+\big(4_0^{\gamma}2B\big(52B^3+123B^2+87B+18\big)r^6 \\
&+ 4fR^2\big(74\eta^4+141B^3+54B^2-33B\big)r^4\big(8\big(2r^2+1\big)\eta^2+3\big(4r^2+5\big)B+6\big)\big)\big)\big) \\
&- 4e^{2\gamma_0 r^2}r^2\big(-2\gamma_2^2\big(\gamma_0 r^2-R^2\big)AB^2(4\eta+3)^2r^6+\gamma_2 A\big(2\gamma_0^2B^2(4B+3)^2r^6 \\
&+ \gamma_0\big(\big(22-140r^2\big)B^4-5\big(49r^2-20\big)B^3-3\big(43r^2-49\big)B^2+\big(87-18r^2\big) \\
&+ 18\big)r^2+\big(22B^4+100B^3+147B^2+87B+18\big)\big)r^2+\big(\gamma_0^2A\big(\big(22-4r^2\big)B^4 \\
&+ \big(147-39r^2\big)B^2+\big(87-18r^2\big)B+18\big)r^4+\gamma_0\big(2q_0^2B^2\big(11B^2+17B+6\big)r^4 \\
&+ \big(66-15r^2\big)B^3-9\big(r^2-15\big)B^2+87B+18\big)\big)r^2+B\big(j^2r^4\big(56B^3+129B^2 \\
&- A(4B+3)\big(\big(r^2+13\big)B^2+\big(3r^2+17\big)B+6\big)\big)\big)\big)\big)\Big]^{-1}
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_r &= -\Big[2(B+1)\big(B\big(2e^{\gamma_0 r^2}(4B+3)r^2-8B-15\big)-6\big)\big(-\gamma_2^2\big(2e^{\gamma_0 r^2}r^2+1\big)AB \\
&\times (4B+3)r^4+\gamma_2 A\big(\gamma_0\big(2e^{\gamma_0 r^2}r^2+1\big)B(4B+3)r^2\big(6e^{2\gamma_0 r^2}\big(B^2+B\big) \\
&\times r^2+10B^2+21B-e^{\gamma_0 r^2}\big(\big(28\eta^2+30B+6\big)r^2+6B^2+9B\big)+9\big)\big)r^2 \\
&+ R^2\big(\gamma_0 A\big(6e^{2\gamma_0 r^2}\big(2B^2+3B+1\big)r^2+2B^2+e^{\gamma_0 r^2}\big(r^2\big(4B^2-6\big)-3\big(2\eta^2 \\
&+ 3B+1\big)\big)-3\big)r^2+\big(2e^{\frac{\gamma_0 r^2}{R^2}}q_0^2\big(2e^{\gamma_0 r^2}r^2+1\big)\eta^2r^4+\big(-1+e^{\gamma_0 r^2}\big)
\end{aligned}
$$

$$
\begin{aligned}
\times\ & A\big(2\big(2e^{\gamma_0 r^2}r^2-5\big)B^2+3\big(2e^{\gamma_0 r^2}r^2-5\big)B-6\big)\big)\big)\big)\big(12e^{3\gamma_0 r^2}r^4\big(q_0^2r^4\\
+\ & A\big)\eta^2(4B+3)+4e^{2\gamma_0 r^2}r^2B\big(q_0^2\big(2\gamma_0 r^2B^2-3\big(B^2+B+6\big)\big)r^4\\
+\ & A\big(\gamma_0^2\big(\big(20r^2+2\big)B^2+3\big(5r^2+2\big)\eta 3\big)r^4+3\gamma_0\big(-4r^2B^2+\big(2-3+r^2\big)\\
\times\ & B+1\big)-r^2+\gamma_2\big(\gamma_0\big(\big(4r^2\big)B^2-3\big(r^2-2\big)\eta+3\big)r^2+\big(2B^2+B+3\big)\big)\\
\times\ & r^2-(4B+3)\big(\big(3r^2+7\big)B+6\big)\big)\big)R^2-A\big(8\eta^2+15B+6\big)\big((7B+6)\\
+\ & 2a^2r^4\eta+\gamma_0^2r^4\big(2ar^2B-33B+4\big)\big)+\gamma_0 r^2\big(-2\gamma_2^2\eta r^4-3\gamma_2R^2Br^2\\
+\ & (7B+6)\big)\big)+e^{\gamma_0 r^2}\big(q_0^2r^4\big(56B^3+153B^2+132B+36\big)+A\big(-8\gamma_2(\gamma_2\\
-\ & \gamma_0)\gamma_0B^2(4B+3)r^8-4\gamma_0B(4B+3)(aB+3\gamma_0(3\eta+4))r^6-12(\gamma_2-5)\\
-\ & \big(2B^2+3B+1\big)r^4+(7B+6)\big(8\big(2r^2+1\big)\eta^2+3\big(4r^2+5\big)B+6\big)\big)\big)\big)\Big]\\
\times\ & \Big[\big(8\big(e^{\gamma_0 r^2}r^2-1\big)B^2+3\big(2e^{\gamma_0 r^2}r^2-5\big)B-6\big)\big(4e^{3\gamma_0 r^2}r^4B^2(4B+3)\big(q_0^2r^4\\
\times\ & (7B+3)-A(\eta+3)\big)-A\big(8B^2+15B+6\big)\big(2\gamma_2^2\big(\gamma_0 r^2-\big)\eta(3\eta+2)r^4\\
-\ & a\gamma_0\big(2\gamma_0B(3B+2)r^2+3\big(5\eta^2+13B+6\big)\big)r^4-\big(\gamma_0^2\big(13\eta^2+21B+6\big)r^4\\
+\ & \gamma_0R^2\big(13\eta^2+17B+6\big)r^2+\big(13B^2+17\eta+6\big)\big)\big)+e^{\gamma_0 r^2}\big(q_0^2r^4\big(24\eta^4\\
+\ & 149B^3+261B^2+168B+36\big)-A\big(-8\gamma_2^2\eta(4B+3)\big(\gamma_0B(3B+2)r^2\\
+\ & \big(8B^2+15\eta+6\big)\big)r^6+4\gamma_2\big(2\gamma_0^2B^2\big(12\eta^2+17B+6\big)r^4+\gamma_0B\big(B^3\\
+\ & 403B^2+339B+90\big)r^2-3(\eta+1)^2\big(22B^2+23\eta+6\big)\big)r^4+\big(4\gamma_0^2B\\
\times\ & \big(52B^3+123B^2+87B+18\big)r^6+4\gamma_0\big(74B^4+141B^3+54B^2-33B\\
-\ & 18\big)r^4+\big(13B^2+17B+6\big)\big(8\big(2r^2+1\big)\eta^2+3\big(4r^2+5\big)B+6\big)\big)\big)\big)\\
-\ & 4e^{2\gamma_0 r^2}r^2\big(-2\gamma_2^2\big(\gamma_0 r^2-\big)AB^2(4B+3)^2r^6+aA\big(2\gamma_0^2B^2(4B+3)^2r^6+\gamma_0\\
-\ & 5\big(49r^2-20\big)B^3-3\big(43r^2-49\big)B^2\big(87-18r^2\big)\eta+18\big)r^2+\big(22B^4\\
+\ & 100B^3+147\eta^2+87B+18\big)\big)r^2+\big(^{\gamma}_{0}2A\big(\big(22-4r^2\big)\eta^4+\big(100-23r^2\big)B^3\\
+\ & \big(147-39r^2\big)\eta^2+\big(87-18r^2\big)B+18\big)r^4+\gamma_0\big(2q_0^2B^2\big(11B^2+17\eta+6\big)r^4\\
+\ & A\big(-4r^2B^4+\big(66-15r^2\big)\eta^3-9\big(r^2-15\big)B^2+87\eta+18\big)\big)r^2+B\big(q_0^2r^4\\
\times\ & \big(56B^3+129B^2+87B+18\big)-A(4B+3)\big(\big(r^2+13\big)B^2+\big(3r^2+17\big)\big)\big)\big)\big)\big)\\
\times\ & \big(2e^{\frac{2\gamma_0 r^2}{R^2}}r^2\big(\gamma_2A(B+1)r^2+\gamma_0A(B+1)(11B+6)r^2+R^2B\big(q_0^2(7B+3)r^4\\
+\ & A(B+3)\big)\big)+A\big(-2\gamma_2^2\eta(3B+2)r^4+\gamma_2\big(2\gamma_0\eta(3B+2)r^2+\big(B(21B\\
+\ & 43)+18\big)\big)r^2+\big(\gamma_0(B(13\eta+21)+6)r^2+(B(13B+17)+6)\big)\big)\\
-\ & e^{\gamma_0 r^2}\big(q_0^2r^4(B(3B+13)+6)+A\big(4\gamma_2^2B(4B+3)r^6+\gamma_2\big(\big(2(B(27B
\end{aligned}
$$

$$
\begin{aligned}
+ & \quad 29)+6)r^2+(B+1)(11B+6))-4\gamma_0 r^4 B(4B+3))r^2+\big(\gamma_0\big(2(B+2) \\
+ & \quad (B+1)(11B+6)\big)r^2+\big(B\big(2(B+3)r^2+13B+17\big)+6\big)\big)\big)\big)\Big]^{-1},
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_t \;=\; & -\Big[2(B+1)\big(\gamma_0^2\big(2\gamma_2\eta\big(8\big(2e^{\gamma_2 r^2}r^2-1\big)B^2+3\big(4e^{\gamma_0 r^2}r^2-5\big)B-6\big)r^2 \\
+ & \quad R^2\big(8\big(-18e^{\gamma_0 r^2}r^2+e^{2\gamma_0 r^2}\big(10r^4+r^2\big)+9\big)B^3+3\big(-100e^{\gamma_0 r^2}r^2 \\
+ & \quad 4e^{\frac{2\gamma_0 r^2}{R^2}}\big(5r^2+2\big)r^2+77\big)B^2+6\big(-24e^{\gamma_0 r^2}r^2+2e^{2\gamma_0 r^2}r^2+39\big)\eta+72\big)\big)r^4 \\
+ & \quad \gamma_0\big(-2a^2B\big(8\big(2e^{\gamma_0 r^2}r^2-1\big)\eta^2+3\big(4e^{\gamma_0 r^2}r^2-5\big)B-6\big)r^4-a\eta\big(4e^{\gamma_0 r^2} \\
\times & \quad B(4\eta+3)r^2+4e^{2\gamma_0 r^2}\big(\big(4r^2-2\big)B^2+3\big(r^2-2\big)B-3\big)r^2-3\big(8B^2 \\
+ & \quad 15B+6\big)\big)r^2+\big(-8\big(6e^{2\gamma_0 r^2}r^4-15e^{\gamma_0 r^2}r^2+7\big)B^3-3\big(-60e^{\gamma_0 r^2}r^2 \\
+ & \quad 4e^{2\gamma_0 r^2}\big(3r^2-2\big)r^2+51\big)B^2+12\big(5e^{\gamma_0 r^2}r^2+e^{2\gamma_0 r^2}r^2-11\big)B-36\big)\big)r^2 \\
+ & \quad R^2\big(-2\gamma_2^2B\big(8B^2+15B+6\big)r^4+4\gamma_2 e^{\gamma_0 r^2}\eta\big(e^{\gamma_0 r^2}\big(2B^2+6B+3\big) \\
- & \quad 3\big(2B^2+3B+1\big)\big)r^4+\big(-1+e^{\gamma_0 r^2}\big)R^4\big(8\big(6e^{2\gamma_0 r^2}r^4-14e^{\gamma_0 r^2}r^2+7\big)B^3 \\
+ & \quad 9\big(4e^{2\gamma_0 r^2}r^4-20e^{\gamma_0 r^2}r^2+17\big)B^2+\big(132-72e^{\gamma_0 r^2}r^2\big)B+36\big)\big)\big)\big(-a^2 \\
\times & \quad \big(2e^{\gamma_0 r^2}r^2+1\big)B(4B+3)r^4+a\big(\gamma_0\big(2e^{\gamma_0 r^2}r^2+1\big)B(4B+3)r^2+\big(6e^{2fr^2} \\
\times & \quad \big(2B^2+3B+1\big)r^2+10B^2+21B-e^{\gamma_0 r^2}\big(\big(28B^2+30B+6\big)r^2+6B^2 \\
+ & \quad 9B+3\big)+9\big)\big)r^2+\big(\gamma_0\big(6e^{2\gamma_0 r^2}\big(2\eta^2+3B+1\big)r^2+2B^2+e^{\gamma_0 r^2}\big(r^2 \\
\times & \quad \big(4B^2-6\big)-3\big(2B^2+3B+1\big)\big)-3\big)r^2+\big(-1+e^{\gamma_0 r^2}\big)R^2\big(2\big(2e^{\gamma_0 r^2}r^2 \\
- & \quad 5\big)B^2+3\big(2e^{\gamma_0 r^2}r^2-5\big)B-6\big)\big)\big)\Big]\Big[\big(a^2\big(\big(16\big(4e^{2\gamma_0 r^2}r^4-8e^{\gamma_0 r^2}r^2+3\big)B^4 \\
+ & \quad 2\big(48e^{2\gamma_0 r^2}r^4-168e^{\gamma_0 r^2}r^2+97\big)B^3+3\big(12e^{2\gamma_0 r^2}r^4-92e^{\gamma_0 r^2}r^2+93\big)B^2 \\
- & \quad 24\big(3e^{\gamma_0 r^2}r^2-7\big)B+36\big)-\gamma_0 r^2\big(16\big(4e^{2\gamma_0 r^2}r^4-6e^{\gamma_0 r^2}r^2+3\big)B^4+2\big(48 \\
\times & \quad e^{2\gamma_0 r^2}r^4-140e^{\gamma_0 r^2}r^2+97\big)\eta^3+9\big(4e^{2\gamma_0 r^2}r^4-28e^{\gamma_0 r^2}r^2+31\big)B^2 \\
- & \quad 24\big(3e^{\gamma_0 r^2}r^2-7\big)\eta+36\big)\big)r^4+a\big(4e^{\gamma_0 r^2}B(\eta+1)\big(e^{\gamma_0 r^2}\big(2B^2+6\eta+3\big) \\
- & \quad 3\big(2B^2+3B+1\big)\big)-\gamma_0\big(8\big(-46e^{\gamma_0 r^2}r^2+e^{2\gamma_0 r^2}\big(26r^2-1\big)r^2+21\big)B^4 \\
\times & \quad \big(-980e^{\gamma_0 r^2}r^2+4e^{2\gamma_0 r^2}\big(79r^2-8\big)r^2+651\big)B^3+12\big(-68e^{\gamma_0 r^2}r^2 \\
+ & \quad e^{2\gamma_0 r^2}\big(10r^2-3\big)r^2+75\big)B^2-6\big(36e^{\gamma_0 r^2}r^2+2e^{2\gamma_0 r^2}r^2-87\big)\eta+108\big)R^2 \\
+ & \quad f^2r^2\big(16\big(4e^{2\gamma_0 r^2}r^4-6e^{\gamma_0 r^2}r^2+3\big)B^4+2\big(48e^{2\gamma_0 r^2}r^4-140e^{\gamma_0 r^2}r^2+97\big)B^3
\end{aligned}
$$

$$
\begin{aligned}
+ &\ 9\big(4e^{2\gamma_0 r^2}r^4 - 28e^{\gamma_0 r^2}r^2 + 31\big)B^2 - 24\big(3e^{\gamma_0 r^2}r^2 - 7\big)\eta + 36\big)\big)r^4 + \big(\gamma_0^2 \\
\times &\ \big(8\big(-2e^{\gamma_0 r^2}r^2 + e^{2\gamma_0 r^2}\big(2r^4 + r^2\big) + 1\big)B^4 + \big(-108e^{A_1 r^2}r^2 + 4e^{2\gamma_0 r^2} \\
\times &\ \big(11r^2 + 8\big)r^2 + 63\big)\eta^3 + 12\big(-14e^{\gamma_0 r^2}r^2 + e^{2\gamma_0 r^2}\big(2r^2 + 3\big)r^2 + 12\big)\eta^2 \\
+ &\ 6\big(-12e^{\gamma_0 r^2}r^2 + 2e^{2\gamma_0 r^2}r^2 + 21\big)\eta + 36\big)r^4 + \gamma_0 B\big(8B^3 + 31B^2 + 36\eta \\
- &\ 4e^{\gamma_0 r^2}r^2\big(2B^3 + 9B^2 + 9\eta + 3\big) + 4e^2\gamma_0 r^2 r^2\big(4r^2B^3 + 3\big(r^2 + 2\big)B^2 + 9\eta + 3\big) \\
+ &\ 12\big)r^2 - \big(-1 + e^{\gamma_0 r^2}\big)R^4B\big(8\big(2e^{2r^2}r^4 - 2e^{\gamma_0 r^2}r^2 + 1\big)B^3 + \big(12e^{2\gamma_0 r^2}r^4 \\
- &\ 44e^{\gamma_0 r^2}r^2 + 31\big)B^2 + \big(36 - 24e^{\gamma_0 r^2}r^2\big)B + 12\big)\big)\big)\big(-2a^2B\big(2e^{\gamma_0 r^2}(4B + 3)r^2 \\
+ &\ 3B + 2\big)r^4 + a\big(2\gamma_0 B\big(2e^{\gamma_0 r^2}(4B + 3)r^2 + 3B + 2\big)r^2 + \big(2e^{2fr^2}\big(11B^2 \\
+ &\ 17B + 6\big)r^2 + 21\eta^2 + 43B - e^{\gamma_0 r^2}\big(2\big(27B^2 + 29\eta + 6\big)r^2 + 11B^2 + 17\eta + 6\big) \\
+ &\ 18\big)\big)r^2 + \big(\gamma_0\big(2e^{2\gamma_0 r^2}\big(11B^2 + 17\eta + 6\big)r^2 + 13B^2 + 21B - e^{\gamma_0 r^2}\big(2\big(\eta^2 \\
+ &\ 5B + 6\big)r^2 + 11\eta^2 + 17B + 6\big) + 6\big)r^2 + \big(-1 + e^{\gamma_0 r^2}\big)\big(\big(2e^{\gamma_0 r^2}r^2 \\
- &\ 13\big)\eta^2 + \big(6e^{\gamma_0 r^2}r^2 - 17\big)B - 6\big)\big)\big)\Big].
\end{aligned}
$$

$$\delta l = l\big(\nabla_\eta h_1 - \nabla_1 h_\eta\big)\delta s^{1\eta}.$$

$$\bar{\Gamma}^{\tau}{}_{1\eta} = \Gamma^{\tau}{}_{1\eta} + \mathrm{g}_{1\eta}h^{\tau} - \delta^{\tau}{}_{1}h_{\eta} - \delta^{\tau}{}_{\eta}h_{1},$$

$$\overline{\mathbb{C}}_{1\eta\mu\chi} = \overline{\mathbb{C}}_{(1\eta)\mu\chi} + \overline{\mathbb{C}}_{[1\eta]\mu\chi}.$$

$$\overline{\mathbb{C}}^{1}{}_{\eta} = \overline{\mathbb{C}}^{\mu}{}_{\mu\eta} = \mathbb{C}^{1}{}_{\eta} + 2h^{1}h_{\eta} + 3\nabla_\eta h^{1} - \nabla_1 h^{\eta} + g^{1}{}_{\eta}(\nabla_\tau h^{\tau} - 2h_\tau h^{\tau}).$$

$$\overline{\mathbb{C}} = \overline{\mathbb{C}}^{\tau}{}_{\tau} = \mathbb{C} + 6(\nabla_1 h^{1} - h_1 h^{1}).$$

$$\tilde{\Gamma}^{\tau}{}_{1\eta} = \Gamma^{\tau}{}_{1\eta} + \mathbb{W}^{\tau}{}_{1} + \mathbb{L}^{\tau}{}_{1\eta},$$

$$\mathbb{L}^{\tau}{}_{1\eta} = \frac{1}{2}\,\mathrm{g}^{\mu\chi}\big(\mathbb{Q}_{\eta^{1}\chi} + \mathbb{Q}_{1\eta\chi} - \mathbb{Q}_{\mu 1\eta}\big),$$

$$\mathbb{W}^{\tau}{}_{1\eta} = \tilde{\Gamma}^{\tau}{}_{[1\eta]} + \mathrm{g}^{\mu\chi}\mathrm{g}_{1\nu}\tilde{\Gamma}^{\nu}{}_{[\eta\chi]} + \mathrm{g}^{\mu\chi}\mathrm{g}_{\eta\nu}\tilde{\Gamma}^{\nu}{}_{[1\chi]}$$

$$\mathbb{Q}_{\mu 1\eta} = \nabla_\tau \mathrm{g}_{1\eta} = -\partial \mathrm{g}_{1\eta,\tau} + \mathrm{g}_{\eta\chi}\tilde{\Gamma}^{\chi}_{1\mu} + \mathrm{g}_{\chi_1}\tilde{\Gamma}^{\chi}_{\eta\mu}$$

$$
\begin{aligned}
\tilde{\Gamma}^{\tau}{}_{1\eta} &= \Gamma^{\tau}{}_{1\eta} + \mathrm{g}_{1\eta}h^{\tau} - \delta^{\tau}h_{\eta} - \delta^{\tau}{}_{\eta}h_{1} + \mathbb{W}^{\tau}{}_{1\eta} \\
\mathbb{W}^{\tau}{}_{1\eta} &= \mathcal{T}^{\tau}{}_{1\eta} - \mathrm{g}^{\mu\chi}\mathrm{g}_{\nu 1}\mathcal{T}^{\nu\mu}_{\chi\eta} - \mathrm{g}^{\mu\chi}\mathrm{g}_{\nu\eta}\mathcal{T}^{\nu\mu}{}_{\chi 1}
\end{aligned}
$$

$$\mathcal{T}^{\tau}{}_{1\eta} = \frac{1}{2}\big(\tilde{\Gamma}^{\tau}{}_{1\eta} - \tilde{\Gamma}^{\tau}{}_{\eta 1}\big).$$

$$\tilde{\mathbb{C}}^{\tau}{}_{1\eta\chi} = \tilde{\Gamma}^{\tau}{}_{1\chi,\eta} - \tilde{\Gamma}^{\tau}{}_{1\eta,\chi} + \tilde{\Gamma}^{\tau}{}_{1\chi}\tilde{\Gamma}^{\nu}{}_{\mu\eta} - \tilde{\Gamma}^{\tau}{}_{1\eta}\tilde{\Gamma}^{\nu}{}_{\mu\chi}.$$

$$
\begin{aligned}
\tilde{\mathbb{C}} = \tilde{\mathbb{C}}^{1\eta}_{1\eta} = \mathbb{C} + 6\nabla_\eta h^{\eta} - 4\nabla_\eta \mathcal{T}^{\eta} - 6h_\eta h^{\eta} + 8h_\eta \mathcal{T}^{\eta} + \mathcal{T}^{1\mu\eta}\mathcal{T}_{1\mu\eta} \\
+2\mathcal{T}^{1\mu\eta}\mathcal{T}_{\eta\mu_1} - 4\mathcal{T}^{\eta}\mathcal{T}_{\eta}.
\end{aligned}
$$

$$\mathcal{J} = \frac{1}{2\kappa}\int \mathrm{g}^{1\eta}\left(\Gamma^{\tau}{}_{\chi 1}\Gamma^{\chi}{}_{\mu\eta} - \Gamma^{\tau}{}_{\chi\mu}\Gamma^{\chi}{}_{1\eta}\right)\sqrt{-\mathrm{g}}d^4x.$$

$$\mathcal{J} = \frac{1}{2\kappa}\int -\mathrm{g}^{1\eta}\left(\mathbb{L}^{\tau}{}_{\chi 1}\mathbb{L}^{\chi}{}_{\mu\eta} - \mathbb{L}^{\tau}{}_{\chi\mu}\mathbb{L}^{\chi}{}_{1\eta}\right)\sqrt{-\mathrm{g}}d^4x$$

$$\mathbb{Q} \equiv -\mathrm{g}^{1\eta}\left(\mathbb{L}^{\tau}{}_{\chi 1}\mathbb{L}^{\chi}{}_{\mu\eta} - \mathbb{L}^{\tau}{}_{\chi\mu}\mathbb{L}^{\chi}{}_{1\eta}\right),$$

$$\mathbb{L}^{\tau}_{\chi^1} \equiv -\frac{1}{2}\,\mathrm{g}^{\mu\nu}\left(\nabla_1\,\mathrm{g}_{\chi\nu} + \nabla_\chi \mathrm{g}_{\nu\mu} - \nabla_{\nu\mu}\mathrm{g}_{\chi^1}\right)$$

$$\mathcal{J} = \int \frac{\sqrt{-\mathrm{g}}}{2\kappa} f(\mathbb{Q})d^4x$$

$$\mathcal{J} = \int \frac{\sqrt{-\mathrm{g}}}{2\kappa} f(\mathbb{Q})d^4x + \int \mathcal{L}_m\sqrt{-\mathrm{g}}d^4x$$

$$\mathcal{J} = \int \frac{\sqrt{-\mathrm{g}}}{2\kappa} f(\mathbb{Q},\mathbb{T})d^4x + \int \mathcal{L}_m\sqrt{-\mathrm{g}}d^4x$$

$$\mathbb{P}^{\tau}_{1\eta} = -\frac{1}{2}\mathbb{L}^{\tau}_{1\eta} + \frac{1}{4}\left(\mathbb{Q}^{\tau} - \tilde{\mathbb{Q}}^{\tau}\right)\mathrm{g}_{1\eta} - \frac{1}{4}\delta^{\tau}_{(1\mathbb{Q}_\eta)}$$

$$\mathbb{Q}_\tau \equiv \mathbb{Q}_\tau{}^{1}{}_{1}, \tilde{\mathbb{Q}}_\tau \equiv \mathbb{Q}^{1}{}_{\mu 1}.$$

$$\mathbb{Q} = -\mathbb{Q}_{\mu i\eta}\mathbb{P}^{\mu i\eta} = -\frac{1}{4}\left(-\mathbb{Q}^{\mu\eta\chi}\mathbb{Q}_{\mu\eta\chi} + 2\mathbb{Q}^{\mu\eta\chi}\mathbb{Q}_{\chi\mu\eta} - 2\mathbb{Q}^{\chi}\tilde{\mathbb{Q}}_\chi + \mathbb{Q}^{\chi}\mathbb{Q}_\chi\right)$$

$$\begin{aligned}\delta\mathcal{J} &= \int \frac{1}{2\kappa}\delta\left(f(\mathbb{Q},\mathbb{T})\sqrt{-\mathrm{g}}\right)d^4x + \int \delta\left(\mathcal{L}_m\sqrt{-\mathrm{g}}\right)d^4x \\ &= \int \frac{1}{2\kappa}\Big(-\frac{1}{2}f\,\mathrm{g}_{1\eta}\sqrt{-\mathrm{g}}\delta\,\mathrm{g}^{1\eta} + f_{\mathbb{Q}}\sqrt{-\mathrm{g}}\delta\mathbb{Q} + f_{\mathbb{T}}\sqrt{-\mathrm{g}}\delta\mathbb{T} - \kappa\mathbb{T}_{1\eta} \\ &\times\sqrt{-\mathrm{g}}\delta\,\mathrm{g}^{1\eta}\Big)d^4x\end{aligned}$$

$$\mathbb{T}_{1\eta} \equiv \frac{-2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_m\right)}{\delta\mathrm{g}^{1\eta}}, \Theta_{1\eta} \equiv \mathrm{g}^{\mu\chi}\frac{\delta\mathbb{T}_{\mu\chi}}{\delta\mathrm{g}^{1\eta}},$$

$$\delta\mathbb{T} = \delta\left(\mathbb{T}_{1\eta}\,\mathrm{g}^{1\eta}\right) = \left(\mathbb{T}_{1\eta} + \Theta_{1\eta}\right)\delta\mathrm{g}^{1\eta}$$

$$\begin{aligned}\delta\mathcal{J} = \int \frac{1}{2\kappa}\Big(\frac{-1}{2}f\,\mathrm{g}_{1\eta}\sqrt{-\mathrm{g}}\delta\mathrm{g}^{1\eta} + f_{\mathbb{T}}\left(\mathbb{T}_{1\eta} + \Theta_{1\eta}\right)\sqrt{-\mathrm{g}}\delta\mathrm{g}^{1\eta} - f_{\mathbb{Q}}\sqrt{-\mathrm{g}}\big(\mathbb{P}_{1\mu\chi}\mathbb{Q}_\eta{}^{\mu\chi} \\ -2\mathbb{Q}^{\mu\chi}{}_{1}\mathbb{P}_{\mu\chi\eta}\big)\delta\mathrm{g}^{1\eta} + 2f_{\mathbb{Q}}\sqrt{-\mathrm{g}}\mathbb{P}_{\mu 1\eta}\nabla^{\tau}\delta\mathrm{g}^{1\eta} - \kappa\mathbb{T}_{1\eta}\sqrt{-\mathrm{g}}\delta\,\mathrm{g}^{1\eta}\Big)d^4x\end{aligned}$$

$$\begin{aligned}\mathbb{T}_{1\eta} = \frac{-2}{\sqrt{-g}}\nabla_\tau\left(f_{\mathbb{Q}}\sqrt{-g}\mathbb{P}^{\tau}_{1\eta}\right) - \frac{1}{2}f\,\mathrm{g}_{1\eta} + f_{\mathbb{T}}\left(\mathbb{T}_{1\eta} + \Theta_{1\eta}\right) - f_{\mathbb{Q}}\big(\mathbb{P}_{1\mu\chi}\mathbb{Q}^{\mu\chi}_{\eta} \\ -2\mathbb{Q}^{\mu\chi}_{1}\mathbb{P}_{\mu\chi\eta}\big)\end{aligned}$$

$$ds^2 = dt^2e^{\xi(r)} - dr^2e^{\eta(r)} - r^2d\Omega^2$$

$$\mathbb{T}_{\tau v} = \mathbb{U}_\tau \mathbb{U}_v \varrho + \mathbb{V}_\tau \mathbb{V}_v p_r - p_t g_{\tau v} + \mathbb{U}_\tau \mathbb{U}_v p_t - \mathbb{V}_\tau \mathbb{V}_v p_t.$$

$$\varrho = \frac{1}{2r^2 e^\eta}\Big(2rf_{\mathbb{Q}\mathbb{Q}}(e^\eta - 1)\mathbb{Q}' + f_{\mathbb{Q}}\big((e^\eta - 1)(2 + r\xi') + (e^\eta + 1)r\eta'\big)$$
$$+ fr^2 e^\eta\Big) - \frac{1}{3} f_{\mathbb{T}}(3\varrho + p_r + 2p_t)$$
$$p_r = \frac{-1}{2r^2 e^\eta}\Big(2rf_{\mathbb{Q}\mathbb{Q}}(e^\eta - 1)\mathbb{Q}' + f_{\mathbb{Q}}\big((e^\eta - 1)(2 + r\xi' + r\eta') - 2r\xi'\big)$$
$$+ fr^2 e^\eta\Big) + \frac{2}{3} f_{\mathbb{T}}(p_t - p_r)$$
$$p_t = \frac{-1}{4re^\eta}\Big(-2r\mathbb{Q}'\xi' f_{\mathbb{Q}\mathbb{Q}} + f_{\mathbb{Q}}(2\xi'(e^\eta - 2) - r\xi'^2 + \eta'(2e^\eta + r\xi')$$
$$-2r\xi'') + 2fre^\eta\Big) + \frac{1}{3} f_{\mathbb{T}}(p_r - p_t)$$

$$f(\mathbb{Q}, \mathbb{T}) = h\mathbb{Q} + k\mathbb{T}$$

$$\varrho = \frac{he^{-\eta}}{12r^2(2h^2 + k - 1)}\Big(k\big(2r\big(-\eta'(r\xi' + 2) + 2r\xi'' + \xi'(r\xi' + 4)\big) - 4e^\eta$$
$$+4\big) + 3kr(\xi'(4 - r\eta' + r\xi') + 2r\xi'') + 12(k - 1)(r\eta' + e^\eta - 1)\Big),$$
$$p_r = \frac{he^{-\eta}}{12r^2(2k^2 + k - 1)}\Big(2k(r\eta'(r\xi' + 2) + 2(e^\eta - 1) - r(2r\xi'' + \xi'(r\xi'$$
$$+4))) + 3\big(r\big(k\eta'(r\xi' + 4) - 2kr\xi'' - \xi'(-4k + kr\xi' + 4)\big) - 4(k - 1)$$
$$\times (e^\eta - 1)\big)\Big),$$
$$p_t = \frac{he^{-\eta}}{12r^2(2k^2 + k - 1)}\Big(2k(r\eta'(r\xi' + 2) + 2(e^\eta - 1) - r(2r\xi'' + \xi'$$
$$\times (r\xi' + 4))\big) + 3\big(r\big(2(k - 1)r\xi'' - ((k - 1)r\xi' - 2)(\eta' - \xi')\big)$$
$$+4k(e^\eta - 1)\big)\Big).$$

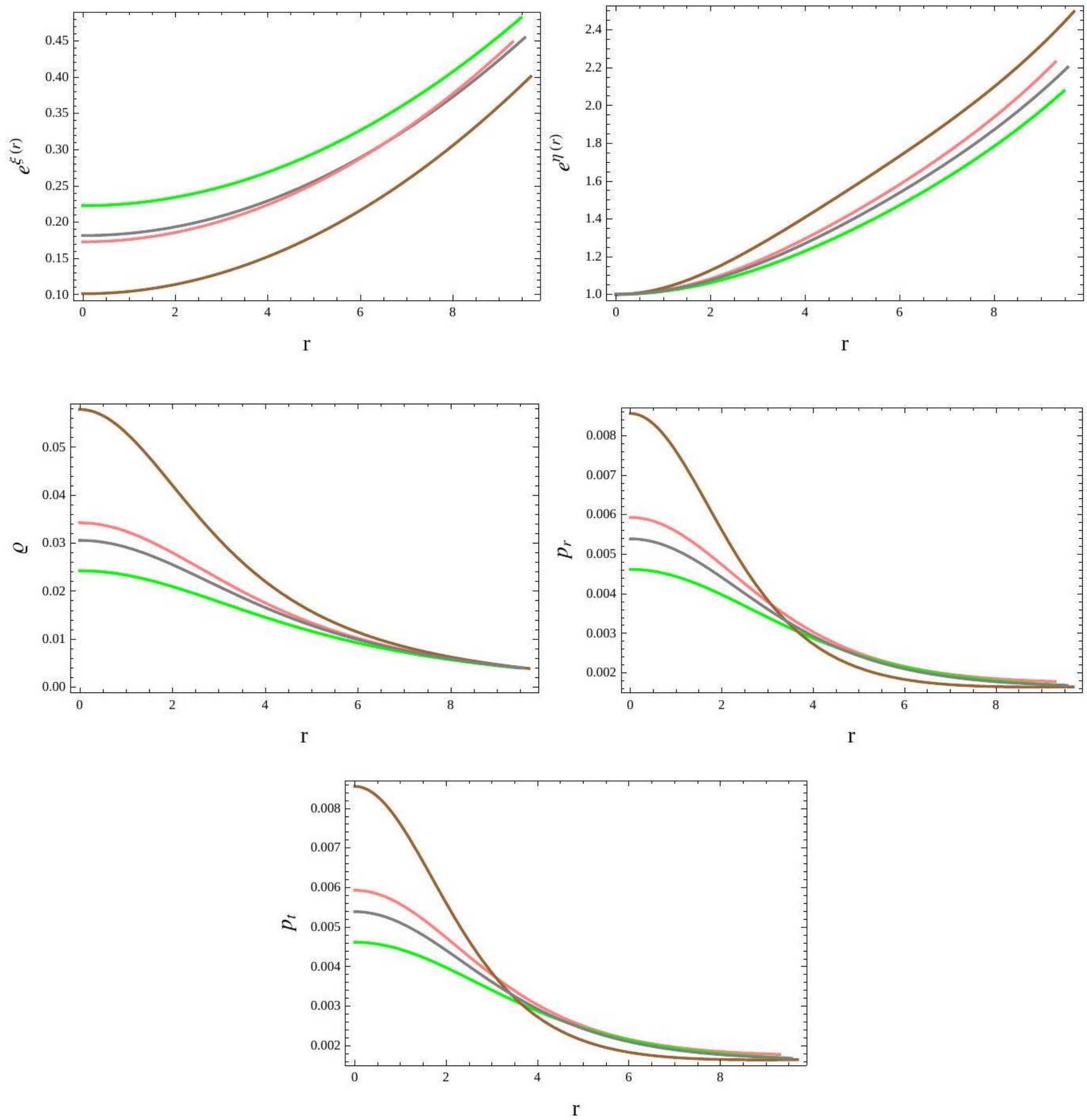
$$e^{\xi(r)} = a\left(\frac{r^2}{b} + 1\right), e^{\eta(r)} = \frac{\frac{2r^2}{b} + 1}{\left(\frac{r^2}{b} + 1\right)\left(1 - \frac{r^2}{c}\right)}$$

$$ds_+^2 = dt^2 \aleph - dr^2 \aleph^{-1} - r^2 d\Omega^2$$

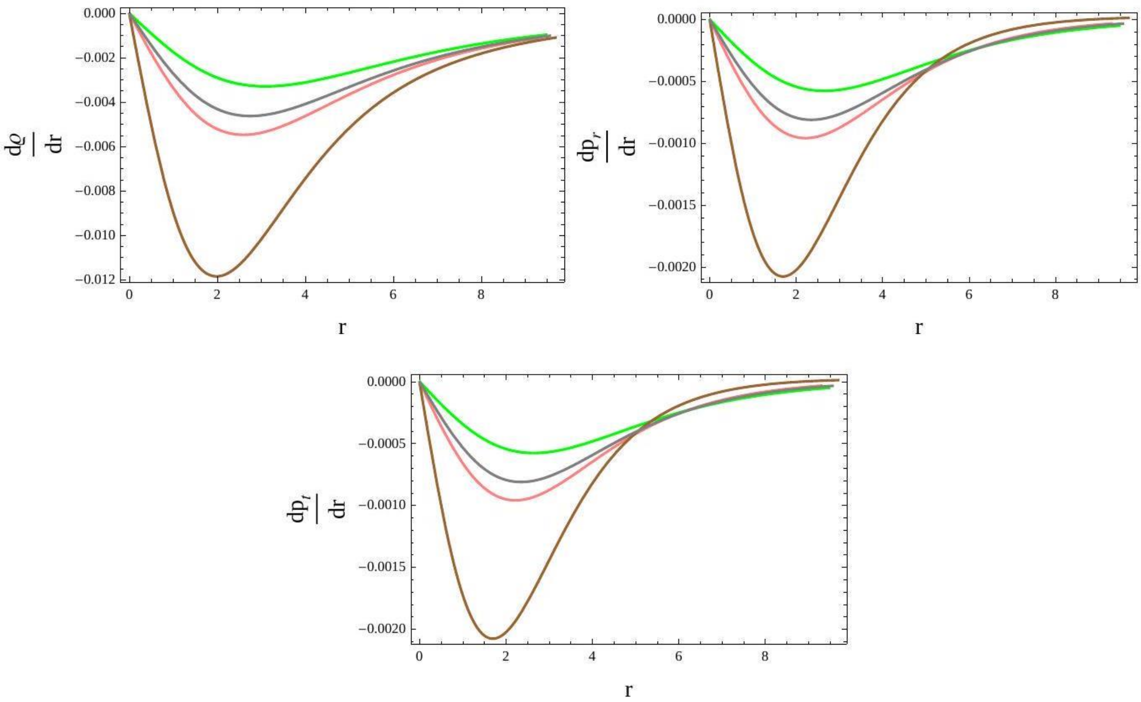
$$\aleph = \left(1 - \frac{2m}{r}\right)$$

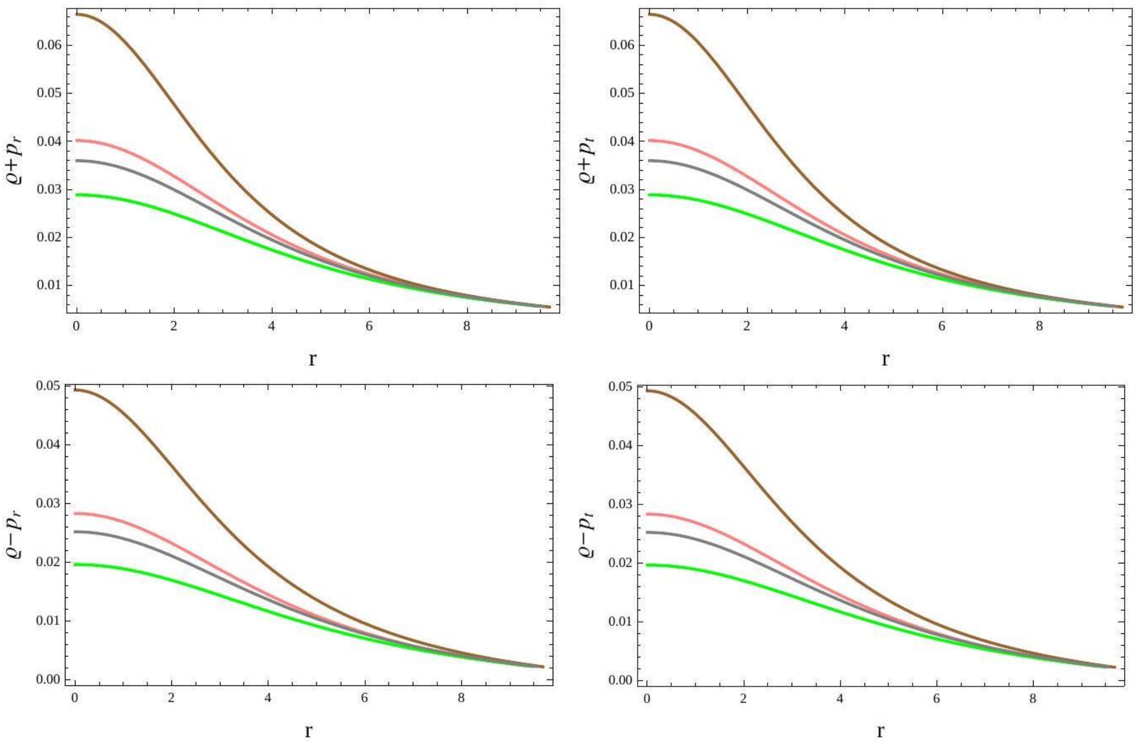
$$g_{tt} = a\left(\frac{R^2}{b} + 1\right) = \aleph,$$
$$g_{rr} = \frac{\frac{2R^2}{b} + 1}{\left(\frac{R^2}{b} + 1\right)\left(1 - \frac{R^2}{c}\right)} = \aleph^{-1}$$
$$g_{tt,r} = \frac{2aR}{b} = \frac{m}{R^2}.$$

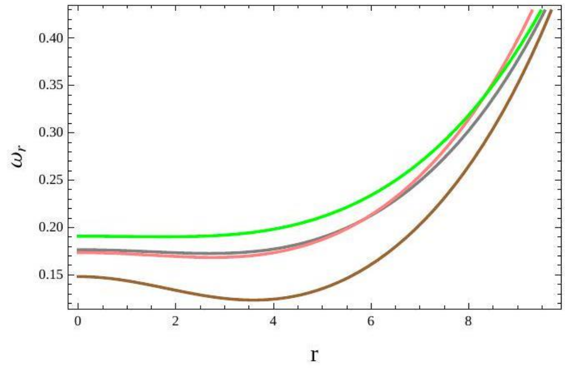
$$a = 1 - \frac{3m}{R}, b = \frac{R^3 - 3mR^2}{m}, c = \frac{R^3}{m}.$$

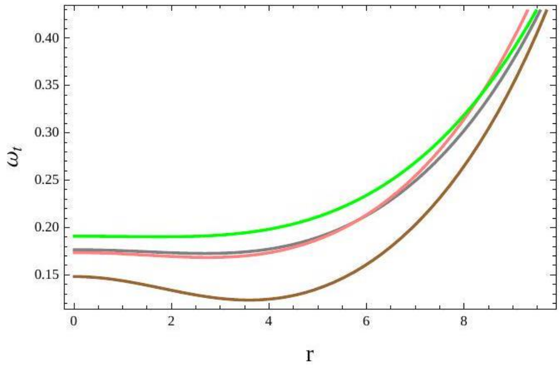


e^ξ(r)
e^η(r)
ϱ
p_r
p_t
r


dϱ/dr
dp_r/dr
dp_t/dr
r

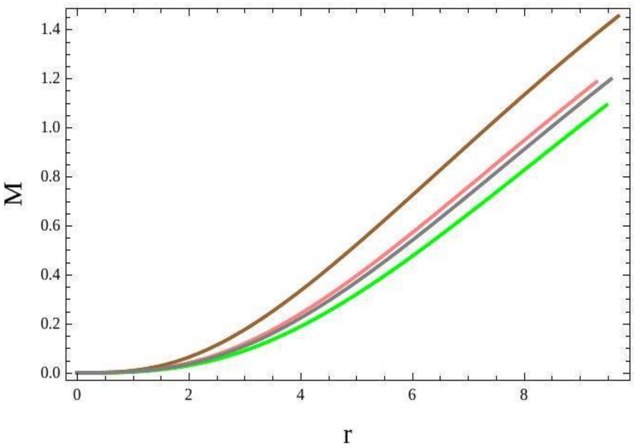

ϱ+p_r
ϱ+p_t
ϱ−p_r
ϱ−p_t
r

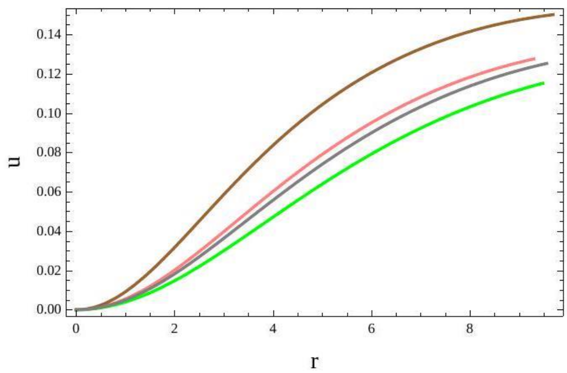


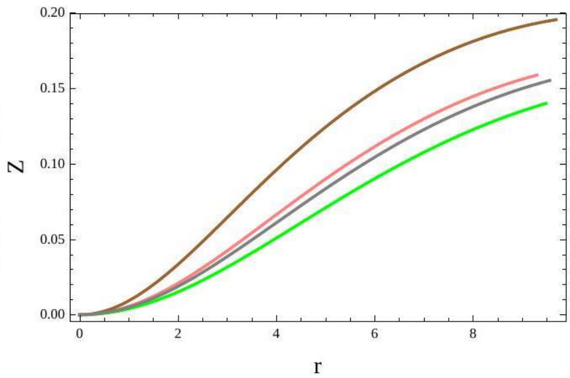


$$\omega_r = \frac{(b+2r^2)(b-c+3r^2)+(b(2b+c)+6br^2+6r^4)k}{b^2(2k-3)-2r^2(c-4ck+3r^2(1+k))+b(c(7k-3)-r^2(2k+7))},$$
$$\omega_t = \frac{(b+2r^2)(b-c+3r^2)+(b(2b+c)+6br^2+6r^4)k}{b^2(2k-3)-2r^2(c-4ck+3r^2(1+k))+b(c(7k-3)-r^2(2k+7))}.$$

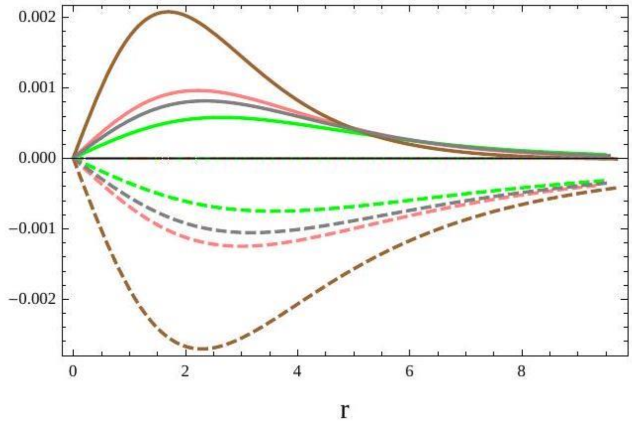
$$M = 4\pi \int_0^R r^2 \varrho dr$$







$$Z_s = -1 + \frac{1}{\sqrt{1-2u}}.$$

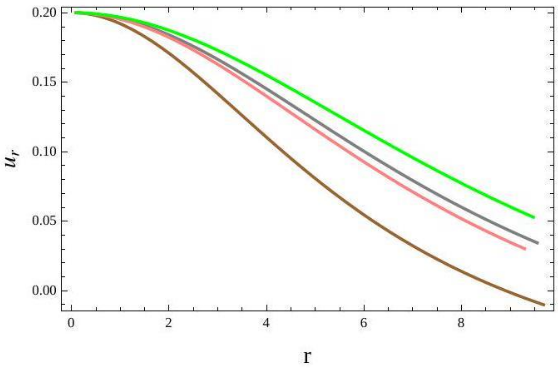


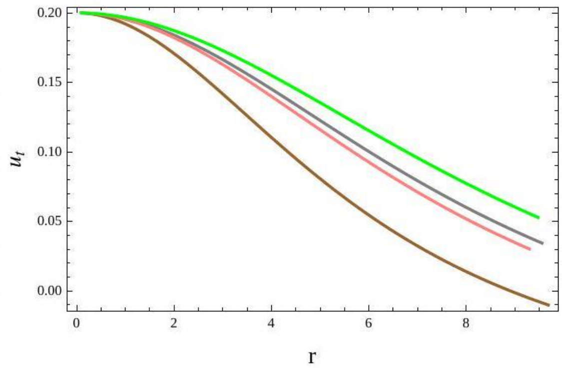
$$\frac{M_G(r)}{r^2}(\varrho + p_r)e^{\frac{\xi-\eta}{2}} + p_r' - \frac{2\Delta}{r},$$

$$M_G(r) = 4\pi \int (\mathbb{T}_0^0 - \mathbb{T}_1^1 - \mathbb{T}_2^2 - \mathbb{T}_3^3) r^2 e^{\frac{\xi+\eta}{2}} dr$$

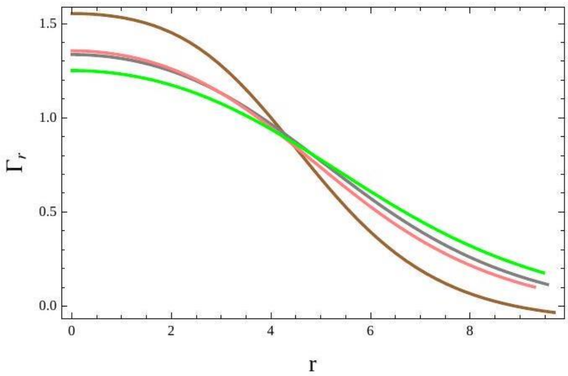
$$M_G(r) = \frac{1}{2} r^2 e^{\frac{\eta-\xi}{2}} \xi'.$$

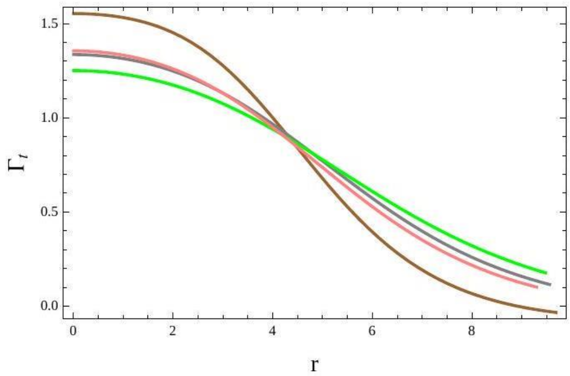
$$\frac{1}{2}\xi'(\varrho + p_r) + p_r' - \frac{2\Delta}{r} = 0.$$





$$\Gamma_r = \frac{\varrho + p_r}{p_r} u_r, \Gamma_t = \frac{\varrho + p_t}{p_t} u_t.$$





$$\Gamma_r = -[2(b+2c)(b+r^2)(2k-1)(b+2r^2-2bk)]\big[\big((b+2r^2)(b-c+3r^2)$$
$$+(b(2b+c)+6br^2+6r^4)k\big)(5b(2k-1)+2r^2(4k-1))\big]^{-1},$$
$$\Gamma_t = -[2(b+2c)(b+r^2)(2k-1)(b+2r^2-2bk)]\big[\big((b+2r^2)(b-c+3r^2)$$
$$+(b(2b+c)+6br^2+6r^4)k\big)(5b(2k-1)+2r^2(4k-1))\big]^{-1}.$$

$$\mathbb{Q} \equiv -g^{1\nu}\left(\mathbb{L}^{\chi}{}_{\tau 1}\mathbb{L}^{\tau}{}_{\nu\chi} - \mathbb{L}^{\chi}{}_{\tau\chi}\mathbb{L}^{\tau}{}_{1\nu}\right),$$

$$\mathbb{L}^{\chi}_{\tau 1} = -\frac{1}{2}\, g^{\chi\eta}\left(\mathbb{Q}_{1\tau\eta} + \mathbb{Q}_{\tau\eta 1} - \mathbb{Q}_{\eta 1\tau}\right)$$
$$\mathbb{L}^{\tau}_{\nu\chi} = -\frac{1}{2}\, g^{\tau\eta}\left(\mathbb{Q}_{\chi\nu\eta} + \mathbb{Q}_{\nu\eta\chi} - \mathbb{Q}_{\eta\chi\nu}\right)$$
$$\mathbb{L}^{\chi}_{\tau 1} = -\frac{1}{2}\, g^{\chi\eta}\left(\mathbb{Q}_{\chi\tau\eta} + \mathbb{Q}_{\tau\eta\chi} - \mathbb{Q}_{\eta\chi\tau}\right)$$
$$= -\frac{1}{2}\left(\overline{\mathbb{Q}}_{\tau} + \mathbb{Q}_{\tau} - \overline{\mathbb{Q}}_{\tau}\right) = -\frac{1}{2}\mathbb{Q}_{\tau}$$
$$\mathbb{L}^{\tau}_{1\nu} = -\frac{1}{2}\, g^{\tau\eta}\left(\mathbb{Q}_{\nu 1\eta} + \mathbb{Q}_{1\eta\nu} - \mathbb{Q}_{\eta\nu 1}\right)$$

$$-g^{1\nu}\mathbb{L}^{\chi}_{\tau 1}\mathbb{L}^{\tau}_{\nu\chi} = -\frac{1}{4}\, g^{1\nu}\, g^{\chi\eta} g^{\tau\eta}\left(\mathbb{Q}_{1\tau\eta} + \mathbb{Q}_{\tau\eta 1} - \mathbb{Q}_{\eta 1\tau}\right)$$
$$\times\left(\mathbb{Q}_{\chi\nu\eta} + \mathbb{Q}_{\nu\eta\chi} - \mathbb{Q}_{\eta\chi\nu}\right)$$
$$= -\frac{1}{4}\left(\mathbb{Q}^{\nu\eta\chi} + \mathbb{Q}^{\eta\chi\nu} - \mathbb{Q}^{\chi\nu\eta}\right)$$
$$\times\left(\mathbb{Q}_{\chi\nu\eta} + \mathbb{Q}_{\nu\eta\chi} - \mathbb{Q}_{\eta\chi\nu}\right)$$
$$= -\frac{1}{4}\left(2\mathbb{Q}^{\nu\eta\chi}\mathbb{Q}_{\eta\chi\nu} - \mathbb{Q}^{\nu\eta\chi}\mathbb{Q}_{\nu\eta\chi}\right)$$
$$g^{1\nu}\mathbb{L}^{\chi}_{\tau\chi}\mathbb{L}^{\tau}{}_{1\nu} = \frac{1}{4}\, g^{1\nu}\, g^{\tau\eta}\mathbb{Q}_{\eta}\left(\mathbb{Q}_{\nu 1\eta} + \mathbb{Q}_{1\eta\nu} - \mathbb{Q}_{\eta\nu 1}\right)$$
$$= \frac{1}{4}\mathbb{Q}^{\eta}\left(2\overline{\mathbb{Q}}_{\eta} - \mathbb{Q}_{\eta}\right)$$
$$\mathbb{Q} = -\frac{1}{4}\left(\mathbb{Q}^{\chi\nu\eta}\mathbb{Q}_{\chi\nu\eta} + 2\mathbb{Q}^{\chi\nu\eta\chi}\mathbb{Q}_{\eta\chi\nu}\right.$$
$$\left.-2\mathbb{Q}^{\eta}\overline{\mathbb{Q}}_{\eta} + \mathbb{Q}^{\eta}\mathbb{Q}_{\eta}\right)$$

$$\mathbb{P}^{\chi^{1\nu}} = \frac{1}{4}\Big[-\mathbb{Q}^{\chi^{1\nu}} + \mathbb{Q}^{1\chi^{\nu}} + \mathbb{Q}^{\nu\chi^{1}} + \mathbb{Q}^{\chi}g^{1\nu}$$
$$\left.-\mathbb{Q}^{\chi}g^{1\nu} - \frac{1}{2}\left(g^{\chi^{1}}\mathbb{Q}^{\nu} + g^{\chi\nu}\mathbb{Q}^{1}\right)\right]$$
$$-\mathbb{Q}_{\chi^{1\nu}}\mathbb{P}^{\chi^{1\nu}} = -\frac{1}{4}\Big[-\mathbb{Q}_{\chi^{1\nu}}\mathbb{Q}^{\chi^{1\nu}}$$
$$+\mathbb{Q}_{\chi^{1\nu}}\mathbb{Q}^{1\chi^{\nu}} + \mathbb{Q}^{\nu\chi^{1}}\mathbb{Q}_{\chi^{1\nu}} + \mathbb{Q}_{\chi^{1\nu}}\mathbb{Q}^{\chi}g^{1\nu}$$
$$\left.-\mathbb{Q}_{\chi^{1\nu}}\overline{\mathbb{Q}^{\chi}}g^{1\nu} - \frac{1}{2}\mathbb{Q}_{\chi^{1\nu}}\left(g^{\chi^{1}}\mathbb{Q}^{\nu} + g^{\chi\nu}\mathbb{Q}^{1}\right)\right],$$

$$= -\frac{1}{4}\left(-\mathbb{Q}_{\chi^{1\nu}}\mathbb{Q}^{\chi^{1\nu}} + 2\mathbb{Q}_{\chi_{1\nu}}\mathbb{Q}^{1\chi\nu} + \mathbb{Q}^{\chi}\mathbb{Q}_{\chi} - 2\tilde{\mathbb{Q}}^{\chi}\chi\mathbb{Q}_{\chi}\right) = \mathbb{Q}$$

$$\mathbb{Q}_{\chi 1\nu} = \nabla_\chi g_{1\nu},$$
$$\mathbb{Q}^{\chi}{}_{1\nu} = g^{\chi\tau}\mathbb{Q}_{\tau 1\nu} = g^{\chi\tau}\nabla_\tau g_{1\nu} = \nabla^\chi g_{1\nu},$$
$$\mathbb{Q}_{\chi}{}^{1}{}_{\nu} = g^{1\eta}\mathbb{Q}_{\chi\eta\nu} = g^{1\eta}\nabla_\chi g_{\eta\nu} = -g_{1\eta}\nabla_\chi g^{1\eta},$$
$$\mathbb{Q}_{\chi\eta}{}^{\nu} = g^{\nu\eta}\mathbb{Q}_{\chi 1\eta} = g^{\nu\eta}\nabla_\chi g_{1\eta} = -g_{1\eta}\nabla_\chi g^{\nu\eta},$$
$$\mathbb{Q}^{\chi^1}{}_{\nu} = g^{1\eta}\, g^{\chi\tau}\nabla_\tau g_{\eta\nu} = g^{1\eta}\nabla^\chi g_{\nu\eta} = -g_{\eta\nu}\nabla^\chi g^{1\eta},$$
$$\mathbb{Q}_1^{\chi\ \nu} = g^{\nu\eta}g^{\chi\tau}\nabla_\tau g_{1\eta} = g^{\nu\eta}\nabla^\chi g_{1\eta} = -g_{1\eta}\nabla^\chi g^{\nu\eta},$$
$$\mathbb{Q}_{\chi}{}^{1\nu} = g^{1\eta}\, g^{\nu\tau}\nabla_\chi g_{\eta\tau} = -g^{1\eta}\, g_{\eta\tau}\nabla_\chi g^{\nu\eta} = -\nabla_\chi g^{1\nu}.$$

$$\begin{aligned}\delta\mathbb{Q} &= -\frac{1}{4}\delta\left(-\mathbb{Q}^{\chi\nu\eta}\mathbb{Q}_{\chi\nu\eta} + 2\mathbb{Q}^{\chi\nu\eta}\mathbb{Q}_{\eta\chi\nu} - 2\mathbb{Q}^{\eta}\overline{\mathbb{Q}}_\eta + \mathbb{Q}^\eta\mathbb{Q}_\eta\right)\\
&= -\frac{1}{4}\big(-\delta\mathbb{Q}^{\chi\nu\eta}\mathbb{Q}_{\chi\nu\eta} - \mathbb{Q}^{\chi\nu\eta}\delta\mathbb{Q}_{\chi\nu\eta} + 2\delta\mathbb{Q}_{\chi\nu\eta}\mathbb{Q}^{\eta\chi\nu}\\
&+2\mathbb{Q}^{\chi\nu\eta}\delta\mathbb{Q}_{\eta\chi\nu} - 2\delta\mathbb{Q}^{\eta}\overline{\mathbb{Q}}_\eta + \delta\mathbb{Q}^\eta\mathbb{Q}_\eta - 2\mathbb{Q}^\eta\delta\bar{Q}_\eta + \mathbb{Q}^\eta\delta\mathbb{Q}_\eta\big)\\
&= -\frac{1}{4}\big[\mathbb{Q}_{\chi\nu\eta}\nabla^\chi\delta g^{\nu\eta} - \mathbb{Q}^{\chi\nu\varsigma}\nabla_\chi\delta g_{\nu\eta} - 2\mathbb{Q}_{\eta\chi\nu}\nabla^\chi\delta g^{\nu\eta}\\
&+2\mathbb{Q}^{\chi\nu\eta}\nabla_\eta\delta g_{\chi\nu} + 2\overline{\mathbb{Q}}_\eta g^{1\nu}\nabla^\eta\delta g_{1\nu} + 2\mathbb{Q}^\eta\nabla^\tau\delta g_{\eta\tau}\\
&+2\overline{\mathbb{Q}}_\eta g_{1\nu}\nabla^\eta\delta g^{1\nu} - \mathbb{Q}_\eta\nabla^\tau g^{1\nu}\delta\, g_{1\nu} - \mathbb{Q}_\eta g_{1\nu}\nabla^\eta\delta g^{1\nu}\\
&-\mathbb{Q}_\eta g^{1\nu}\nabla_\eta\delta g_{1\nu} - \mathbb{Q}^\eta g_{1\nu}\nabla_\eta\delta g_{1\nu}\big].\end{aligned}$$

$$\begin{aligned}\delta g_{1\nu} &= -g_{1\chi}\delta\, g^{\chi\tau}g_{\tau\nu} - \mathbb{Q}^{\chi\nu\eta}\nabla_\chi\delta g_{\nu\eta},\\
&= -\mathbb{Q}^{\chi\nu\eta}\nabla_\chi\left(-g_{\nu 1}\delta\, g^{1\tau}\, g_{\tau\eta}\right),\\
&= 2\mathbb{Q}^{\chi\nu}{}_{\eta}\mathbb{Q}_{\chi\nu 1}\delta\, g^{1\theta} + \mathbb{Q}_{\chi\tau\eta}\nabla^\chi g^{1\eta},\\
&= 2\mathbb{Q}^{\chi\tau}{}_{\nu}\mathbb{Q}_{\chi\tau\nu}\delta g^{1\nu} + \mathbb{Q}_{\chi\nu\eta}\nabla^\chi g^{\nu\eta},\\
2\mathbb{Q}^{\chi\nu\eta}\nabla_\eta\delta g_{\chi\nu} &= -4\mathbb{Q}_1{}^{\tau\eta}\mathbb{Q}_{\eta\tau\nu}\delta g^{1\nu} - 2\mathbb{Q}_{\nu\eta\chi}\nabla^\chi\delta g^{\nu\eta},\end{aligned}$$

$$\begin{aligned}-2\mathbb{Q}^\eta\nabla^\tau\delta g_{\eta\tau} &= 2\mathbb{Q}^\chi\mathbb{Q}_{\nu\chi 1}\delta\, g^{1\nu} + 2\mathbb{Q}_1\overline{\mathbb{Q}}_\nu\delta g^{1\nu}\\
&+2\mathbb{Q}_\nu g_{\chi\eta}\nabla^\chi g^{\nu\eta}\end{aligned}$$

$$\delta\mathbb{Q} = 2\mathbb{P}_{\chi\nu\eta}\nabla^\chi\delta g^{\nu\eta} - \left(\mathbb{P}_{1\chi\tau}\mathbb{Q}_\nu^{\chi\tau} - 2\mathbb{P}_{\chi\tau\nu}\mathbb{Q}_\nu^{\chi\tau}\right)\delta g^{1\nu}$$

$$\mathcal{P}(\mathcal{D}) = \sum_j \mathcal{P}(\mathcal{D}\mid\mathcal{M}_j)\mathcal{P}(\mathcal{M}_j) = \int \mathcal{P}(\mathcal{D}\mid\mathcal{M})\mathcal{P}(\mathcal{M})d^N\mathcal{M}$$

$$\mathcal{P}(\mathcal{D}\mid\mathcal{M}) \propto \Pi_i\mathcal{D}_i$$

$$\mathcal{P}(\mathcal{M}\mid\mathcal{D}) = \frac{\mathcal{P}(\mathcal{D}\mid\mathcal{M})\mathcal{P}(\mathcal{M})}{\mathcal{P}(\mathcal{D})} = \frac{\Pi_i\mathcal{D}_i}{\sum_j\ \Pi_i\mathcal{D}_i(\mathcal{M}_j)}$$

$$Q = \sqrt{\frac{c^4}{4\pi G\mathcal{E}_c}}$$

$$\hat{r} = \frac{r}{Q}, \hat{m} = \frac{Gm}{Qc^2}, \hat{\mathcal{E}} = \frac{\mathcal{E}}{\mathcal{E}_c}, \hat{P} = \frac{P}{\mathcal{E}_c}$$

$$\frac{d\hat{P}}{d\hat{r}} = -\frac{(\hat{\mathcal{E}} + \hat{P})(\hat{m} + \hat{r}^3\hat{P})}{\hat{r}(\hat{r} - 2\hat{m})}, \frac{d\hat{m}}{d\hat{r}} = \hat{\mathcal{E}}\hat{r}^2$$

$$\hat{\mathcal{E}} = \sum_i a_i \hat{r}^i, \hat{P} = \sum_i b_i \hat{r}^i, \hat{m} = \sum_i c_i \hat{r}^i$$

$$a_0 = 1, a_2 = -\frac{1}{\hat{R}^2}, b_0 = \hat{P}_c, b_2 = -\frac{1 + 4\hat{P}_c + 3\hat{P}_c^2}{6}, c_3 = \frac{a_0}{3}, c_5 = \frac{a_2}{5}$$

$$\hat{M}_{\max} = \frac{GM_{\max}}{c^2}\sqrt{\frac{4\pi G\mathcal{E}_{\max}}{c^4}} \simeq \frac{2\hat{R}_{\max}^3}{15}$$

$$\hat{R}_{\max} = R_{\max}\sqrt{\frac{4\pi G\mathcal{E}_{\max}}{c^4}} \simeq \sqrt{\frac{6\hat{P}_{\max}}{1 + 4\hat{P}_{\max} + 3\hat{P}_{\max}^2}} \equiv \sqrt{6\phi_{\max}}$$

$$\frac{c_{s,\max}^2}{c^2} = \left(\frac{dP}{d\mathcal{E}}\right)_{\max} = \frac{b_2}{a_2} \simeq \hat{P}_{\max}$$

$$M_{\max} = \alpha_M + \beta_M \left(\frac{\mathcal{E}_{\max}}{\text{GeVfm}^3}\right) \nu_{\max}^3, R_{\max} = \alpha_R + \beta_R \nu_{\max}$$

$$\nu_{\max} = \sqrt{\frac{\phi_{\max}\text{GeVfm}^{-3}}{\mathcal{E}_{\max}}}$$

$$\nu_{\max} = a_\nu + b_\nu R_{\max}, \phi_{\max} = a_\phi + b_\phi\left(\frac{GM_{\max}}{R_{\max}c^2}\right), \frac{c_{s,\max}}{c} = \alpha_c + \beta_c\sqrt{\frac{GM_{\max}}{R_{\max}c^2}}$$

$$\hat{P}_{\max} = \frac{1}{3\phi_{\max}}\left[\left(\frac{1}{2} - 2\phi_{\max}\right) - \sqrt{\phi_{\max}^2 - 2\phi_{\max} + \frac{1}{4}}\right]$$

$$G = a_G \left(\frac{P_{\max}}{\text{MeVfm}^{-3}}\right)^{b_G} \left(\frac{\mathcal{E}_{\max}}{\text{GeVfm}^{-3}}\right)^{c_G}$$

$$\chi_G^2 = N^{-1}\sum_i^N \left[\ln\left(\frac{G_i}{a_G}\right) - b_G \ln\left(\frac{P_{\max,i}}{\text{MeVfm}^{-3}}\right) - c_G \ln\left(\frac{\mathcal{E}_{\max,i}}{\text{GeVfm}^{-3}}\right)\right]^2,$$

$$\delta G_i = \frac{a_G}{G_i}\left(\frac{P_{\max,i}}{\text{MeVfm}^{-3}}\right)^{b_G} \left(\frac{\mathcal{E}_{\max,i}}{\text{GeVfm}^{-3}}\right)^{c_G} - 1$$

$$< \delta G >= \sqrt{N^{-1}\sum_i^N (\delta G_i)^2}$$

$$G = a_G \left(\frac{M_{\max}}{M_\odot}\right)^{b_G} \left(\frac{R_{\max}}{10\text{ nm}}\right)^{c_G}$$

$$\nu_f = a_{\nu f} + b_{\nu f} R_f, \phi_f = a_{\phi f} + b_{\phi f}\left(\frac{GM_f}{R_f c^2}\right), \frac{c_{s,f}}{c} = \alpha_{cf} + \beta_{cf}\sqrt{\frac{GM_f}{R_f c^2}}.$$

$$\hat{P}_f = \frac{1}{3\phi_f^2}\left[\left(\frac{1}{2} - 2\phi_f\right) - \sqrt{\phi_f^2 - 2\phi_f + \frac{1}{4}}\right]$$

$$G_f = a_{Gf}\left(\frac{M_{\max}}{M_\odot}\right)^{b_{Gf}}\left(\frac{R_f}{10\ \text{nm}}\right)^{c_{Gf}}$$

$$G_f = a_{Gf}\left(\frac{M_{\max}}{M_\odot}\right)^{b_{Gf}}\left(\frac{R_g}{10\ \text{nm}}\right)^{c_{Gf}}\left(\frac{R_h}{10\ \text{nm}}\right)^{d_{Gf}},$$

$$\chi_{Gf}^2 = \sum_i^N \left[\ln\left(\frac{G_{fi}}{a_{Gf}}\right) - b_{Gf}\ln\left(\frac{M_{\max,i}}{M_\odot}\right) - c_{Gf}\ln\left(\frac{R_{gi}}{10\ \text{nm}}\right) - d_{Gf}\ln\left(\frac{R_{hi}}{10\ \text{nm}}\right)\right]^2$$

$$\Delta P_i = \frac{P_{fit,i}(\mathcal{E}_{fit,i})}{P_i(\mathcal{E}_{fit,i})} - 1,$$

$$G_i = c_i\left(\frac{P_{\max}}{\mathcal{E}_{\max}}\right)^{p_i}\left(\frac{\mathcal{E}_{\max}}{\mathcal{E}_0}\right)^{q_i} + d_i,$$

$$\nu_j = a_\nu + b_\nu R_j, \phi_j = a_\phi + b_\phi\left(\frac{GM_j}{R_j c^2}\right), \frac{c_s}{c} = \alpha_c + \beta_c\sqrt{\frac{GM_j}{R_j c^2}}$$

$$\frac{\mathcal{E}_j}{\text{GeVfm}^{-3}} = \frac{\phi_j}{\nu_j^2}, \hat{P}_j = \frac{1}{3\phi_j}\left[\left(\frac{1}{2} - 2\phi_j\right) - \sqrt{\phi_j^2 - 2\phi_j + \frac{1}{4}}\right]$$

$$G_j = a_G\left(\frac{R_j}{10\ \text{nm}}\right)^{b_G}$$

$$\ln\ G = a_G + b_G\ln\left(\frac{M}{M_\odot}\right) + c_G\ln\left(\frac{R}{\text{nm}}\right) + d_G\left[\ln\left(\frac{M}{M_\odot}\right)\right]^2 + e_G\ln\left(\frac{M}{M_\odot}\right)\ln\left(\frac{R}{\text{nm}}\right) + f_G\left[\ln\left(\frac{R}{\text{nm}}\right)\right]^2$$

$$\ln\ G = a_G + b_G\ln\left(\frac{M}{M_\odot}\right) + c_G\ln\left(\frac{R}{\text{nm}}\right) + d_G\left[\ln\left(\frac{M}{M_\odot}\right)\right]^2 + e_G\ln\left(\frac{M}{M_\odot}\right)\ln\left(\frac{R}{\text{nm}}\right)$$
$$+ f_G\left[\ln\left(\frac{R}{\text{nm}}\right)\right]^2 + g_G\left(\frac{dR}{dM}\right)\left(\frac{M_\odot}{\text{nm}}\right)$$

$$\mathcal{E}_{bag} = 3P_{bag} + 4B = \frac{3}{4}\mu_{bag}n_{bag} + B$$

$$\mu_{u,d} = \hbar c\left(\frac{3}{N_c g}\pi^2 n_{u,d}\right)^{1/3}$$

$$\mu_{bag} = 2\mu_d + \mu_u = \hbar c(2^{4/3} + 1)(\pi^2 n_{bag})^{1/3}$$

$$\mu_0 = \left[\hbar c\left(2^{4/3}+1\right)\right]^{3/4}\pi^{1/2}(4B)^{1/4}, n_0 = \frac{4B}{\mu_0}$$

$$n_{bag} = n_0\left(\frac{P_{bag}+B}{B}\right)^{\frac{3}{4}}, P_{bag} = B\left[\left(\frac{n_{bag}}{n_0}\right)^{\frac{4}{3}} - 1\right]$$

$$\left[\gamma^\alpha, \gamma^\beta\right]_+ \equiv \gamma^\alpha\gamma^\beta + \gamma^\beta\gamma^\alpha = 2\eta^{\alpha\beta}\mathbf{1}_4, \alpha, \beta = 0,1,2,3$$

$$\gamma^5\gamma^\alpha = -i\delta^\alpha{}_{\beta\gamma\delta}\gamma^\beta\gamma^\gamma\gamma^\delta, \quad \text{with}$$
$$\delta^0{}_{123} = \delta^1{}_{023} = \delta^2{}_{031} = \delta^3{}_{012} = 1 \quad \text{otherwise } \delta^\alpha{}_{\beta\gamma\delta} = 0$$

$$\gamma^0 = \beta \equiv \begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & -\mathbf{1}\end{pmatrix}, \gamma^a = \beta\alpha_a, \text{ with } \alpha_a \equiv \begin{pmatrix}\mathbf{0} & \sigma_a\\ \sigma_a & \mathbf{0}\end{pmatrix}$$

$$\sigma^{ab} = \begin{pmatrix}\sigma_c & \mathbf{0}\\ \mathbf{0} & \sigma_c\end{pmatrix} \equiv \Sigma^c, a, b, c = 1,2,3 \text{ in cyclic order,}$$

$$\sigma^{0a} = i\begin{pmatrix}\mathbf{0} & \sigma_a\\ \sigma_a & \mathbf{0}\end{pmatrix} = i\gamma^5\Sigma^a \equiv {}^*\Sigma^a \text{ with } \gamma^5 = \begin{pmatrix}\mathbf{0} & \mathbf{1}\\ \mathbf{1} & \mathbf{0}\end{pmatrix}$$

$$[\Sigma^a, \Sigma^b]_- = 2i\Sigma^c, [\Sigma^a, {}^*\Sigma^b]_- = 2i{}^*\Sigma^c, [{}^*\Sigma^a, {}^*\Sigma^b]_- = -2i\Sigma^c$$
$$a, b, c = 1,2,3 \text{ in cyclic order}$$

$$\Sigma^\rho = i\delta^\rho{}_{\alpha\beta}\gamma^\alpha\gamma^\beta \text{ with}$$
$$\delta^1{}_{23} = \delta^2{}_{31} = \delta^3{}_{12} = \delta^4{}_{01} = \delta^5{}_{02} = \delta^6{}_{03} = 1 \text{ otherwise } \delta^\rho{}_{\alpha\beta} = 0$$

$$\hat\theta^\alpha(x) = e^\alpha{}_i(x)dx^i \text{ and } \hat\omega^{\alpha\beta}(x) = \omega^{\alpha\beta}{}_i(x)dx^i$$

$$g_{ij}(x) = e_i^\alpha(x)e_j^\beta(x)\eta_{\alpha\beta}$$

$$g^{ij}(x) = e_\alpha{}^i(x)e_\beta{}^j(x)\eta^{\alpha\beta}$$
$$\text{with } e_\alpha{}^i(x)e^\beta{}_i(x) = \delta_\alpha^\beta \text{ and } e_\alpha{}^i(x)e^\alpha{}_j(x) = \delta^i{}_j$$

$$\check\theta_\alpha(x) = e_\alpha{}^i(x)\partial_i \text{ and } \check\gamma(x) = \gamma^\alpha\check\theta_\alpha(x).$$

$$\hat\gamma(x) = \gamma_\alpha\hat\theta^\alpha(x) = \gamma_i(x)dx^i \text{ and } \hat\Gamma(x) = \frac{1}{2}\sigma_{\alpha\beta}\hat\omega^{\alpha\beta}(x) \equiv \Gamma_i(x)dx^i$$

$$\gamma_i(x) = \gamma_\alpha e_i^\alpha(x) \text{ and } \Gamma_i(x) = \sum_{\rho=1}^{6}\delta_{\alpha\beta}^\rho\Sigma_\rho\omega_i^{\alpha\beta}(x).$$

$$\gamma_i(x)\gamma_j(x) + \gamma_j(x)\gamma_i(x) = 2g_{ij}(x)\mathbf{1}_4$$

$$\hat\Theta = \hat d\hat\gamma + \frac{1}{2}[\hat\Gamma, \hat\gamma]_- \equiv \hat D\hat\gamma \text{ and } \hat\Omega = \hat d\hat\Gamma + \frac{1}{2}[\hat\Gamma, \hat\Gamma]_- \equiv \hat D\hat\Gamma$$

$$\hat\Theta = \gamma_\alpha\hat\Theta^\alpha \text{ and } \hat\Omega = \frac{1}{2}\sigma_{\alpha\beta}\hat\Omega^{\alpha\beta} = \sum_{\rho=1}^{6}\delta_{\alpha\beta}^\rho\Sigma_\rho\hat\Omega^{\alpha\beta}$$

$$\hat{\eta} = \tilde{e}\hat{d}^4x = \frac{i\gamma_5}{4!}\hat{\gamma}^4 \text{ with } \tilde{e} = \det|e_i^\alpha| \text{ and } \hat{d}^4x \equiv dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$$

$$\frac{\hat{\gamma}^4}{4!} = \gamma_0\gamma_1\gamma_2\gamma_3\varepsilon_{\alpha\beta\gamma\delta}e_0^\alpha e_1^\beta e_2^\gamma e_3^\delta \hat{d}^4x = -i\gamma^5 \det|e_i^\alpha| \hat{d}^4x$$

$$\frac{\hat{\gamma}^3}{3!} = -i\gamma^5\gamma^\alpha \tilde{e}_\alpha{}^i (\hat{d}^3x)^i \text{ with } (\hat{d}^3x)^i \equiv \delta^i{}_{jk\ell} dx^j \wedge dx^k \wedge dx^\ell$$

$$\tilde{e}_\alpha{}^i = \tilde{e}e_\alpha{}^i.$$

$$\frac{\hat{\gamma}^2}{2!} = 2\sum_{\rho,r=1}^{6} \Sigma_\rho d_r^\rho (\hat{d}^2)^r \text{ with } (\hat{d}^2)^r = \delta_{ij}^r dx^i \wedge dx^j$$

$$i\gamma_5 \exp\hat{\gamma} = {}^\star(\exp\hat{\gamma})$$

$$i\hbar_* {}^\star\hat{\gamma}\hat{D}_\mathrm{F}\Psi + {}^\star 1 mc\Psi = 0$$

$${}^\star\hat{\gamma} = \gamma^\alpha \tilde{e} e_\alpha^i (\hat{d}^3x)^i, \text{ and } {}^\star\hat{1} = \hat{\eta}$$

$$\hat{D}_\mathrm{F}\Psi = \hat{d}\Psi + \frac{i}{2}\hat{\Gamma}\Psi$$

$$\hat{T} = \frac{i\hbar_* c_0}{2}\left(\bar{\Psi}{}^\star\hat{\gamma}\hat{D}_\mathrm{F}\Psi - (\hat{D}_\mathrm{F}\bar{\Psi})^\star\hat{\gamma}\Psi\right) \text{ and } \hat{S} = \frac{\hbar_*}{2}\hat{\gamma}^2\bar{\Psi}\hat{\gamma}\gamma_5\Psi$$

$$\hat{\Omega} = \kappa_0\hat{T} \text{ and } \hat{\Theta} = \kappa_0 c_0\hat{S}$$

$$\hat{D}\hat{\Gamma} = \underline{\hat{\Omega}} \text{ and } \hat{D}\hat{\gamma} = \underline{\hat{\Theta}}$$

$$\hat{\gamma}(x) = \gamma_0\hat{\theta}^0(x) + \sum_{a=1}^{3}\gamma_a\hat{\theta}^a(x) = \begin{pmatrix} \hat{f}(x) & -\hat{h}(x) \\ \hat{h}(x) & -\hat{f}(x) \end{pmatrix}$$

$$\hat{\Gamma}(x) = \sum_{(abc)=1}^{3}\Sigma_c\hat{\omega}^{ab}(x) + \sum_{a=1}^{3}{}^*\Sigma_a\hat{\omega}^{0a}(x) = \begin{pmatrix} \hat{F}(x) & -i\hat{H}(x) \\ -i\hat{H}(x) & \hat{F}(x) \end{pmatrix},$$

$$\begin{pmatrix} \hat{f}(x) \\ \hat{h}(x) \end{pmatrix} \equiv \begin{pmatrix} 1\hat{\theta}^0(x) \\ \sum_{a=1}^{3}\sigma_a\hat{\theta}^a(x) \end{pmatrix}, \text{ and } \begin{pmatrix} \hat{F}(x) \\ \hat{H}(x) \end{pmatrix} \equiv \begin{pmatrix} \sum_{(abc)=1}^{3}\sigma_a\hat{\omega}^{bc}(x) \\ \sum_{a=1}^{3}\sigma_a\hat{\omega}^{0a}(x) \end{pmatrix}$$

$$[\circ,\cdot]_\tau \equiv \circ\cdot\cdot - (-1)^\tau \cdot\cdot\circ \text{ with } (\tau = 0 \text{ or } 1)$$

$$\hat{\Omega}(x) = \hat{D}\hat{\Gamma}(x) = \begin{pmatrix} \hat{d}\hat{F}(x) + \hat{F}_1(x) & -i\hat{d}\hat{H}(x) - i\hat{H}_1(x) \\ -i\hat{d}\hat{H}(x) - i\hat{H}_1(x) & \hat{d}\hat{F}(x) + \hat{F}_1(x) \end{pmatrix},$$

$$\begin{pmatrix} \hat{F}_1(x) \\ \hat{H}_1(x) \end{pmatrix} = \frac{1}{2}\begin{pmatrix} [\hat{F}(x),\hat{F}(x)]_0 - [\hat{H}(x),\hat{H}(x)]_0 \\ 2[\hat{F}(x),\hat{H}(x)]_1 \end{pmatrix}$$

$$\hat{\Theta}(x) = \hat{D}\hat{\gamma}(x) = \begin{pmatrix} \hat{d}\hat{f}(x) + \hat{f}_1(x) & \hat{d}\hat{h}(x) + \hat{h}_1(x) \\ -\hat{d}\hat{h}(x) - \hat{h}_1(x) & -\hat{d}\hat{f}(x) - \hat{f}_1(x) \end{pmatrix},$$

$$\begin{pmatrix}\hat{f}_1(x)\\ \hat{h}_1(x)\end{pmatrix}=\frac{1}{2}\begin{pmatrix}[\hat{F}(x),\hat{f}(x)]_0-i[\hat{H}(x),\hat{h}(x)]_1\\ [\hat{F}(x),\hat{h}(x)]_0+i[\hat{H}(x),\hat{f}(x)]_1\end{pmatrix}$$

$$\hat{f}(x)=\sum_{\lambda}\hat{f}_\lambda(tr)\mathcal{Z}^{(J_\lambda)}_{0J_\lambda M_\lambda}(\Omega),$$

$$\begin{pmatrix}\hat{h}(x)\\ \hat{F}(x)\\ \hat{H}(x)\end{pmatrix}=\sum_{\lambda}\begin{pmatrix}\hat{h}_\lambda(tr)\\ \hat{F}_\lambda(tr)\\ \hat{H}_\lambda(tr)\end{pmatrix}\mathcal{Z}^{(J_\lambda)}_{1L_\lambda M_\lambda}(\Omega),$$

$$\mathcal{Z}^{(J)}_{sLM}(\Omega)=\sum_{\mu}\langle s\mu L(M-\mu)\mid JM\rangle\sigma^{(s)}_\mu C^{(L)}_{M-\mu}(\Omega)$$

$$C^{(L)}_M(\Omega)=\sqrt{\frac{4\pi}{2L+1}}Y_{LM}(\Omega),$$

$$\sigma^{(0)}_0=\mathbf{1},\sigma^{(1)}_{\pm1}=\mp\frac{1}{\sqrt{2}}(\sigma_1\pm i\sigma_2),\sigma^{(1)}_0=\sigma_3$$

$$\hat{d}\begin{pmatrix}\hat{f}_\lambda(tr)\\ \hat{h}_\lambda(tr)\end{pmatrix}+\sum_{\lambda_1\lambda_2}D(\lambda_1\lambda_2;\lambda)\begin{pmatrix}\delta^{\{\lambda\}}_0\hat{H}_{\lambda_1}(tr)\hat{h}_{\lambda_2}(tr)\\ -i\delta^{\{\lambda\}}_1\left(\hat{F}_{\lambda_1}(tr)\hat{h}_{\lambda_2}(tr)+\hat{H}_{\lambda_1}(tr)\hat{f}_{\lambda_2}(tr)\right)\end{pmatrix}=\begin{pmatrix}\hat{\imath}_\lambda(tr)\\ \hat{\jmath}_\lambda(tr)\end{pmatrix}$$

$$\hat{d}\begin{pmatrix}\hat{F}_\lambda(tr)\\ \hat{H}_\lambda(tr)\end{pmatrix}+\sum_{\lambda_1\lambda_2}D(\lambda_1\lambda_2;\lambda)\left(\delta^{\{\lambda\}}_0\begin{pmatrix}\hat{F}_{\lambda_1}(tr)\hat{F}_{\lambda_2}(tr)-\hat{H}_{\lambda_1}(tr)\hat{H}_{\lambda_2}(tr)\\ -2i\delta^{\{\lambda\}}_1\hat{H}_{\lambda_1}(tr)\hat{F}_{\lambda_2}(tr)\end{pmatrix}\right)=\begin{pmatrix}\hat{I}_\lambda(tr)\\ \hat{J}_\lambda(tr)\end{pmatrix}$$

$$\delta^{\lambda_1\lambda_2\lambda}_{q_1q_2\tau}=\frac{1}{2}\left(1-(-1)^{s_1+s_2+s+L_1+L_2+L+J_1+J_2+J+q_1q_2+\tau}\right)$$

$$\delta^{\{\lambda\}}_\tau=\delta^{\lambda_1\lambda_2\lambda}_{11\tau}$$

$$\hat{\Omega}(x)=\begin{pmatrix}\hat{I}(x)&-i\hat{J}(x)\\ -i\hat{J}(x)&\hat{I}(x)\end{pmatrix}\text{ and }\hat{\Theta}(x)=\begin{pmatrix}\hat{\imath}(x)&-\hat{\jmath}(x)\\ \hat{\jmath}(x)&-\hat{\imath}(x)\end{pmatrix}$$

$$\begin{pmatrix}\hat{I}(x)\\ \hat{J}(x)\end{pmatrix}=\sum_{\lambda}\begin{pmatrix}\hat{I}_\lambda(tr)\\ \hat{J}_\lambda(tr)\end{pmatrix}\mathcal{Z}^{(J_\lambda)}_{1J_\lambda M_\lambda}(\Omega)\text{ and }\begin{pmatrix}\hat{I}(x)\\ \hat{J}(x)\end{pmatrix}=\sum_{\lambda}\begin{pmatrix}\hat{\imath}_\lambda(tr)\mathcal{Z}^{(J_\lambda)}_{0J_\lambda}(\Omega)\\ \hat{\jmath}_\lambda(tr)\mathcal{Z}^{(J_\lambda)}_{1J_\lambda M_\lambda}(\Omega)\end{pmatrix}$$

$$\left\{i\hbar_*c\gamma^i(x)\eta^{ij}\left(\partial_j+\frac{i}{2}\Gamma_j(x)\right)-mc^2\right\}\Psi(x\sigma)=0$$

$$\gamma_i(x)\gamma^j(x)=\mathbf{1}_4\delta_i{}^j-i\sigma_i{}^j(x)\text{ with }\sigma_i{}^j(x)=e^\alpha{}_i(x)\sigma_\alpha{}^\beta e_\beta{}^j(x),$$

$$i\hbar_*(\mathbf{1}_4-i\sigma_0{}^0(x))\frac{\partial\Psi}{\partial t}=\{c_0\boldsymbol{\alpha}(x)\cdot\mathbf{\Pi}(x)+V_0(x)+mc_0^2\beta(x)\}\Psi(x\sigma)$$

$$\boldsymbol{\alpha}(x)=\gamma_0(x)\boldsymbol{\gamma}(x),\text{ and }\beta(x)=\gamma_0(x)$$

$$\mathbf{\Pi}(x)=-i\hbar_*\boldsymbol{\nabla}+\frac{\mathbf{\Gamma}(x)}{2}$$

$$\nabla = \sum_{a=1}^{3} n_a(\Omega)\frac{\partial}{\partial x^a}$$

$$\gamma(x) = \sum_{a=1}^{3} \gamma_a(x)n_a(\Omega),\ \mathbf{\Gamma}(x) = \sum_{a=1}^{3} \Gamma_a(x)n_a(\Omega)$$

$$n_a(\Omega) = (\sin\ \theta\cos\ \phi, \sin\ \theta\sin\ \phi, \cos\ \theta)$$

$$V_0(x) = \frac{\hbar_* c_0}{2}(\mathbf{1}_4 - i\sigma_0^0(x))\Gamma_0(x)$$

$$\Psi_\nu(x) = e^{-iE_\nu(t)t/\hbar_*}\psi_\nu(x)$$

$$i\hbar_*\frac{\partial\Psi}{\partial t} = E(t)\Psi,$$

$$H\psi_\nu(x) = (\mathbf{1}_4 - i\sigma_0{}^0(x))E_\nu(t)\psi_\nu(x)$$

$$\gamma^i(x) = \begin{pmatrix} f^i(x) & h^i(x) \\ -h^i(x) & -f^i(x) \end{pmatrix} \text{ and } \gamma_i(x) = \begin{pmatrix} f_i(x) & -h_i(x) \\ h_i(x) & -f_i(x) \end{pmatrix}$$

$$\Psi(x) = \sum_{\kappa\mu} \begin{pmatrix} g_{\kappa\mu}^{(u)}(tr)\mathcal{Y}_{\kappa\mu}(\theta\phi\sigma) \\ g_{\kappa\mu}^{(\ell)}(tr)\mathcal{Y}_{\bar{\kappa}\mu}(\theta\phi\sigma) \end{pmatrix}$$

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2d\theta^2 - r^2\sin^2\ \theta d\phi^2$$

$$\left.\begin{aligned} \kappa p(r) &= e^{-\lambda(r)}\left(\frac{\nu'(r)}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}, \\ \kappa\rho(r) &= e^{-\lambda(r)}\left(\frac{\lambda'(r)}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} \end{aligned}\right\} \text{ with } p'(r) = -\frac{p(r)+\rho(r)}{2}\nu'(r)$$

$$\frac{dp}{dr} = \frac{p+\rho(p)}{r(r-2u)}[4\pi pr^3 + u],\ \text{with } \frac{du}{dr} = 4\pi\rho(p)r^2$$

$$T_{M_{12}}^{(J_{12})} = \sum_{m_1(m_2)} \langle j_1m_1j_2m_2 \mid J_{12}M_{12}\rangle T_{m_1}^{(j_1)}T_{m_2}^{(j_2)} \equiv \left[T^{(j_1)} \times T^{(j_2)}\right]_{M_{12}}^{(J_{12})}$$

$$T_M^{(J)} = \left[\left[T^{(j_1)} \times T^{(j_2)}\right]^{(J_{12})} \times \left[T^{(j_3)} \times T^{(j_4)}\right]^{(J_{34})}\right]_M^{(J)}$$

$$T_M^{(J)} = \left[\left[T^{(j_1)} \times T^{(j_3)}\right]^{(J_{13})} \times \left[T^{(j_2)} \times T^{(j_4)}\right]^{(J_{24})}\right]_M^{(J)}$$

$$\left[\left[T_1^{(j_1)}\times T_2^{(j_2)}\right]^{(J_{12})}\times\left[T_3^{(j_3)}\times T_4^{(j_4)}\right]^{(J_{34})}\right]_M^{(J)}=\sum_{J_{13}J_{24}}U\begin{pmatrix}j_1 & j_2 & J_{12}\\ j_3 & j_4 & J_{34}\\ J_{13} & j_{24} & J\end{pmatrix}$$

$$\left[\left[T_1^{(j_1)}\times T_3^{(j_3)}\right]^{(J_{13})}\times\left[T_2^{(j_2)}\times T_4^{(j_4)}\right]^{(J_{24})}\right]_M^{(J)}$$

$$U\begin{pmatrix}j_1 & j_2 & J_{12}\\ j_3 & j_4 & J_{34}\\ J_{13} & J_{24} & J\end{pmatrix}=\sqrt{(2J_{12}+1)(2J_{34}+1)(2J_{13}+1)(2J_{24}+1)}\begin{Bmatrix}j_1 & j_2 & J_{12}\\ j_3 & j_4 & J_{34}\\ J_{13} & J_{24} & J\end{Bmatrix}$$

$$|j_1-j_2|\le J_{12}\le j_1+j_2, |j_3-j_4|\le J_{34}\le j_3+j_4, |j_1-j_3|\le J_{13}\le j_1+j_3$$
$$|j_2-j_4|\le J_{24}\le j_2+j_4, |J_{12}-J_{34}|\le J\le J_{12}+J_{34}, |J_{13}-J_{24}|\le J\le J_{13}+J_{24}$$

$$\left[\mathcal{Z}_{s_1L_1}^{(J_1)}(\Omega)\times\mathcal{Z}_{s_2L_2}^{(J_2)}(\Omega)\right]_M^{(J)}\equiv\sum_{(M_1M_2)}\langle J_1M_1J_2M_2\mid JM\rangle\mathcal{Z}_{s_1L_1M_1}^{(J_1)}(\Omega)\mathcal{Z}_{s_2L_2M_2}^{(J_2)}(\Omega)$$
$$=\sum_{sL}\langle L_10L_20\mid L0\rangle U\begin{pmatrix}s_1 & L_1 & J_1\\ s_2 & L_2 & J_2\\ s & L & J\end{pmatrix}\left[\left[\sigma^{(s_1)}\times\sigma^{(s_2)}\right]^{(s)}\times\left[C^{(L_1)}(\Omega)\times C^{(L_2)}(\Omega)\right]^{(L)}\right]^{(J)}$$
$$=\sum_{sL}D(\lambda_1\lambda_2;\lambda)\mathcal{Z}_{sLM}^{(J)}(\Omega)$$

$$\left[\sigma^{(0)}\times\sigma^{(s)}\right]_\mu^{(s)}=\left[\sigma^{(s)}\times\sigma^{(0)}\right]_\mu^{(s)}=\sigma_\mu^{(s)},\left[\sigma^{(1)}\times\sigma^{(1)}\right]_\mu^{(2)}=\mathbf{0}$$
$$\left[\sigma^{(1)}\times\sigma^{(1)}\right]_\mu^{(1)}=-\sqrt{2}\sigma_\mu^{(1)}\ \left[\sigma^{(1)}\times\sigma^{(1)}\right]_0^{(0)}=-\sqrt{3}\sigma_0^{(0)}$$

$$\left[C^{(L_1)}(\Omega)\times C^{(L_2)}(\Omega)\right]_M^{(L)}=\langle L_10L_20\mid L0\rangle C_M^{(L)}(\Omega)$$

$$D(\lambda_1\lambda_2;\lambda)=-\sqrt{6(2L+1)(2J_1+1)(2J_2+1)}\langle L_10L_20\mid L0\rangle\begin{Bmatrix}1 & L_1 & J_1\\ 1 & L_2 & J_2\\ 1 & L & J\end{Bmatrix}$$

$$D(\lambda_1\lambda_2;\lambda)=(-1)^{J_1+L_2+L}\sqrt{(2J_1+1)(2J_2+1)}\langle L_10L_20\mid L0\rangle\begin{Bmatrix}L_1 & J_1 & 1\\ J_2 & L_2 & L\end{Bmatrix}$$

$$D(\lambda_2\lambda_1;\lambda)=(-1)^{s_1+s_2+s+L_1+L_2+L+J_1+J_2+J}D(\lambda_1\lambda_2;\lambda)$$

$$D(\lambda_1,\lambda_2;\lambda_3)=(-1)^{J_{i+1}+L_{i+1}+1}\sqrt{(2J_{i+1}+1)(2L_{i+1}+1)}\langle L_10L_20\mid L0\rangle$$

$$i\hbar\frac{\partial}{\partial t}\Psi(x)=(c_0\boldsymbol{\alpha}\cdot\boldsymbol{p}+U(r)\mathbf{1}_4+\beta mc^2)\Psi(x)=H\Psi(x)\ \text{ with }\boldsymbol{p}=-i\hbar_*\boldsymbol{\nabla}$$

$$\Psi(x)=\begin{pmatrix}\psi^u(\mathrm{tr}\sigma)\\ \psi^\ell(\mathrm{tr}\sigma)\end{pmatrix}=\begin{pmatrix}\sum_{\kappa\mu}\ g_{\kappa\mu}^{(u)}(\mathrm{tr})\mathcal{Y}_{\kappa\mu}(\theta\phi\sigma),\\ \sum_{\kappa\mu}\ g_{\kappa\mu}^{(\ell)}(\mathrm{tr})\mathcal{Y}_{\bar{\kappa}\mu}(\theta\phi\sigma)\end{pmatrix},$$

$$\mathcal{Y}_{\kappa\mu}(\theta\phi\sigma)=\sum_{\sigma=\pm\frac{1}{2}}\left\langle\ell_\kappa m\frac{1}{2}\sigma\middle|\ j_\kappa\mu\right\rangle(i)^{\ell_\kappa}Y_{\ell_\kappa m}(\theta\phi)\chi_\sigma^{\left(\frac{1}{2}\right)}$$

$$|\kappa|=k=j_\kappa+\frac{1}{2},\ell_\kappa\pm\frac{1}{2}\ \text{for }\kappa=\pm k$$

$$K = \beta(\boldsymbol{\sigma}\cdot\boldsymbol{\ell} + \mathbf{1})$$

$$H\Psi_{\kappa}^{(\nu)} = \begin{pmatrix} (U(r)+mc_0^2)\mathbf{1} & c_0\boldsymbol{\sigma}\cdot\boldsymbol{p} \\ c_0\boldsymbol{\sigma}\cdot\boldsymbol{p} & (U(r)-mc_0^2)\mathbf{1} \end{pmatrix} \begin{pmatrix} g_{\kappa\mu}^{(u)}(r)\mathcal{Y}_{\kappa\mu}(\theta\phi\sigma), \\ g_{\kappa\mu}^{(\ell)}(r)\mathcal{Y}_{\bar{\kappa}\mu}(\theta\phi\sigma) \end{pmatrix} = E_\nu\Psi_{\kappa}^{(\nu)}$$

$$\sigma_r = \boldsymbol{\sigma}\cdot\boldsymbol{n}_r = -\sqrt{3}\mathcal{Z}_{110}^{(0)}(\theta\phi), \boldsymbol{n}_r = (\sin\,\theta\cos\,\phi, \sin\,\theta\sin\,\phi, \cos\,\theta),$$

$$\sigma_r\mathcal{Y}_{\kappa\mu} = -iS_\kappa\mathcal{Y}_{\bar{\kappa}\mu}$$

$$\boldsymbol{\sigma}\cdot\boldsymbol{p} = \frac{\sigma_r}{r}(\boldsymbol{\sigma}\cdot\boldsymbol{r})(\boldsymbol{\sigma}\cdot\boldsymbol{p}) = \frac{\sigma_r}{r}(\boldsymbol{r}\cdot\boldsymbol{p} + i\boldsymbol{\sigma}\cdot(\boldsymbol{r}\times\boldsymbol{p})) = i\hbar_*\sigma_r\left(-\mathbf{1}\frac{\partial}{\partial r} + \boldsymbol{\sigma}\cdot\boldsymbol{\ell}\right)$$

$$\left(G_{\kappa\mu}^{(u)}(r), G_{\kappa\mu}^{(\ell)}(r)\right) = \left(rg_{\kappa\mu}^{(u)}(r), rg_{\kappa\mu}^{(\ell)}(r)\right)$$

$$\frac{d}{dr}\begin{pmatrix} G_{\kappa\mu}^{(u)}(r) \\ G_{\kappa\mu}^{(\ell)}(r) \end{pmatrix} = \begin{pmatrix} -\frac{\kappa}{r} & -\frac{1}{\hbar_*}(U(r)-E_\nu-mc_0^2) \\ \frac{1}{\hbar_*}(U(r)-E_\nu+mc_0^2) & \frac{\kappa}{r} \end{pmatrix} \begin{pmatrix} G_{\kappa\mu}^{(u)}(r) \\ G_{\kappa\mu}^{(\ell)}(r) \end{pmatrix}$$

$$S_B = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa} + \frac{\xi}{2\kappa}B^\mu B^\nu R_{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - V\left(B^\mu B_\mu \pm b^2\right) + \mathcal{L}_M\right]$$

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$\begin{aligned} G_{\mu\nu} =& R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa\left(T_{\mu\nu}^B + T_{\mu\nu}^M\right) \\ =& \kappa\left[2V'B_\mu B_\nu + B_\mu^\alpha B_{\nu\alpha} - \left(V + \frac{1}{4}B_{\alpha\beta}B^{\alpha\beta}\right)g_{\mu\nu}\right] + \xi\left[\frac{1}{2}B^\alpha B^\beta R_{\alpha\beta}g_{\mu\nu} - B_\mu B^\alpha R_{\alpha\nu} - B_\nu B^\alpha R_{\alpha\mu}\right. \\ &\left. + \frac{1}{2}\nabla_\alpha\nabla_\mu\left(B^\alpha B_\nu\right) + \frac{1}{2}\nabla_\alpha\nabla_\nu\left(B^\alpha B_\mu\right) - \frac{1}{2}\nabla^2\left(B_\mu B_\nu\right) - \frac{1}{2}g_{\mu\nu}\nabla_\alpha\nabla_\beta\left(B^\alpha B^\beta\right)\right] + \kappa T_{\mu\nu}^M \end{aligned}$$

$$\nabla_\mu B^{\mu\nu} = J_B^\nu + J_M^\nu$$

$$J_B^\nu = 2\left(V'B^\nu - \frac{\xi}{2\kappa}B_\mu R^{\mu\nu}\right)$$

$$g_{\mu\nu} = \mathrm{diag}\left(-e^{2\alpha(r)}, e^{2\beta(r)}, r^2, r^2\sin^2\,\theta\right)$$

$$B_\mu = b_\mu = (0, b_r(r), 0, 0)$$

$$b_r(r) = |b|e^{\beta(r)}$$

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + (1+\ell)\left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\,\theta d\phi^2\right)$$

$$T_{\mu\nu}^M = (\rho + p)u_\mu u_\mu + pg_{\mu\nu},$$

$$\frac{e^{-2\beta}}{r^2}\left[e^{2\beta}-(1+\ell)(1-2r\beta')\right]=\kappa\rho$$

$$\frac{e^{-2\beta}}{r^2}\left[(1+\ell)(1+2r\alpha')-e^{2\beta}-\ell r^2\left(\alpha''+\alpha'^2-\alpha'\beta'-\frac{2}{r}\beta'\right)\right]=\kappa p$$

$$(1+\ell)e^{-2\beta}\left[\alpha''+\alpha'^2-\alpha'\beta'+\frac{1}{r}(\alpha'-\beta')\right]=\kappa p$$

$$g_{rr}=e^{2\beta}=(1+\ell)\left(1-\frac{2m(r)}{r}\right)^{-1},$$

$$\frac{dm}{dr}=4\pi r^2\rho$$

$$\frac{dp}{dr}=-\left(\frac{\rho+p+\ell(\rho+2p)}{1+2\ell}\right)\alpha'-\frac{\ell(8\pi rp-m'')}{(1+2\ell)8\pi r^2}$$

$$\frac{d\alpha}{dr}=\frac{(1+2\ell)8\pi r^3p+(2+3\ell)m-\ell rm'}{(2+3\ell)r(r-2m)}.$$

$$\frac{dp}{dr}=q_0+q_1p+q_2p^2$$

$$u''-q_3u'+q_4u=0$$

$$p=-\frac{1}{q_2}\frac{u'}{u}=\frac{(2+3\ell)(r-2m)}{8\pi r^2(1+2\ell)}\frac{u'}{u}.$$

$$\alpha=\int\frac{(2+3\ell)m-\ell rm'}{(2+3\ell)r(r-2m)}dr+\ln u,\ \text{for } r<R$$

$$\rho(r)=\rho_0$$

$$u(r)=(3-8\pi r^2\rho_0)^{\frac{2-3\ell}{8}}\left(C_1(3-8\pi r^2\rho_0)^{\frac{1}{2}}+C_2\right),$$

$$C_1=-\frac{(2-3\ell)}{4\sqrt{3}}$$

$$C_2=\frac{\sqrt{3}}{4}(2-\ell)(3-8\pi R^2\rho_0)^{\frac{1}{2}}.$$

$$p(r)=\rho_0\left(\frac{\sqrt{1-\frac{2M}{R}}-\sqrt{1-\frac{2Mr^2}{R^3}}}{\sqrt{1-\frac{2Mr^2}{R^3}}-3\sqrt{1-\frac{2M}{R}}}\right)+\ell\rho_0\left(\frac{7-\frac{2M}{R}\left(\frac{r^2}{R^2}+6\right)-7\sqrt{\left(1-\frac{2Mr^2}{R^3}\right)\left(1-\frac{2M}{R}\right)}}{\left(\sqrt{1-\frac{2Mr^2}{R^3}}-3\sqrt{1-\frac{2M}{R}}\right)^2}\right),$$

$$\alpha(r)=\ln\left[\frac{u(r)}{(3-8\pi r^2\rho_0)^{\frac{1}{4+6\ell}}}\right]$$

$$e^{\alpha(r)} = \frac{3}{2}\sqrt{1-\frac{2M}{R}} - \frac{1}{2}\sqrt{1-\frac{2Mr^2}{R^3}} + \frac{3\ell}{4}\left(\sqrt{1-\frac{2Mr^2}{R^3}} - \sqrt{1-\frac{2M}{R}}\right).$$

$$p = \frac{1}{3}(\rho - 4\mathcal{B})$$

$$\gamma = \left(1+\frac{\rho}{p}\right)\left(\frac{dp}{d\rho}\right)_S$$

$$D_\mu\psi = \partial_\mu\psi + \frac{1}{4}\omega_{ij\mu}\gamma^i\gamma^j\psi$$

$$\mathcal{L}_\psi = -\frac{i}{2}\bar{\psi}[\not{D} - \overleftarrow{\not{D}}]\psi$$

$$R^{ij}{}_{\mu\nu} = \partial_\mu\omega^{ij}{}_\nu - \partial_\nu\omega^{ij}{}_\mu + \omega^i{}_{k\mu}\omega^{kj}{}_\nu - \omega^i{}_{k\nu}\omega^{kj}{}_\mu.$$

$$R = e_i{}^\mu e_j{}^\nu R^{ij}{}_{\mu\nu}(\omega, \partial\omega).$$

$$R = R_E + T - 2\nabla_{E\mu}T^\mu,$$

$$T = \frac{1}{4}T^{\rho\mu\nu}T_{\rho\mu\nu} - \frac{1}{4}T^{\rho\mu\nu}T_{\mu\nu\rho} - \frac{1}{4}T^{\rho\mu\nu}T_{\nu\rho\mu} - T^\mu T_\mu.$$

$$S = \int d^4xe\left(\frac{M_{\rm Pl}^2}{2}F(R) + \mathcal{L}_\psi\right)$$

$$M_{\rm Pl}^2[(T^\nu{}_{kl} - e_l{}^\nu T_k + e_k{}^\nu T_l)F'(R) + (e_k{}^\alpha e_l{}^\nu - e_k{}^\nu e_l{}^\alpha)\partial_\alpha F'(R)] + S^\nu{}_{kl} = 0,$$

$$S^\mu{}_{kl} \equiv -\frac{\delta\mathcal{L}_\psi}{e\delta\omega^{kl}{}_\mu} = \frac{i}{2}\bar{\psi}\gamma^i\gamma^j\gamma^k\psi = \frac{1}{2}\bar{\psi}\epsilon^{ijkl}\gamma^5\gamma_l\psi.$$

$$T_{ij}{}^k = \frac{1}{2}\left(\delta_j^k e_i{}^\lambda - \delta_i^k e_j{}^\lambda\right)\partial_\lambda \ln F'(R) - \frac{S^k{}_{ij}}{F'(R)M_{\rm Pl}}$$

$$R = R_E - \frac{3}{2}\partial_\lambda \ln F'(R)\partial^\lambda \ln F'(R) - 3\nabla_E^2 \ln F'(R) - \frac{1}{4F'(R)^2M_{\rm Pl}^2}S^{\mu\nu\rho}S_{\mu\nu\rho}$$

$$\phi \equiv -\sqrt{\frac{3}{2}}M_{\rm Pl}\ln F'(R)$$

$$S = \int d^4xe\left(\frac{M_{\rm Pl}^2}{2}R_E - \frac{1}{2}\partial_\lambda\phi\partial^\lambda\phi - V(\phi) + \mathcal{L}_\psi\right)$$

$$V(\phi) \equiv -\frac{M_{\rm Pl}^2}{2}f(R)\Big|_{R=R(\phi)}$$

$$f(R) = -\alpha R\ln\left(1+\frac{R}{R_0}\right)$$

$$V(\phi) = \frac{M_{\rm Pl}^2 R_0}{2e}\left\{\frac{e}{1+\alpha}\left(1 - e^{-\sqrt{2/3}\phi/M_{\rm Pl}}\right)\right\}^{\frac{1+\alpha}{\alpha}}$$

$$R_0 = 2\Lambda_{\rm Inf} e^{-\frac{1}{\alpha}}$$

$$V(\phi) = -\sqrt{\frac{2}{3}}\alpha\Lambda_{\rm Inf} e^{-1/\alpha} M_{\rm Pl}\phi$$

$$\sqrt{\frac{2}{3}}\alpha\Lambda_{\rm Inf} e^{-\frac{1}{\alpha}} \sim \frac{\Lambda_{\rm Inf}}{10^{114}} \sim \Lambda_{\rm DE}$$

$$\omega_{ijk} = \omega_{\diamond_{ijk}} + \frac{1}{2}(T_{ijk} + T_{jki} + T_{kji})$$

$$\omega_{\diamond ijk} = \frac{1}{2}(\Delta_{kij} - \Delta_{ijk} + \Delta_{jik})$$
$$\Delta_{kij} = \left(e_i^\mu e_j^\nu - e_j^\mu e_i^\nu\right)\partial_\nu e_{k\mu}$$

$$S = \int d^4x e\left(\frac{M_{\rm Pl}^2}{2}R_E - \frac{1}{2}\partial_\lambda\phi\partial^\lambda\phi - V(\phi) - i\bar{\psi}\frac{1}{2}[\not{D}_\diamond - \overleftarrow{\not{D}}_\diamond]\psi - \frac{\Omega(\phi)}{4}(\bar{\psi}\gamma^5\gamma_l\psi)^2\right)$$

$$\Omega(\phi) = \frac{3}{4M_{\rm Pl}^2}\left(2e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}} - e^{2\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}\right)$$

$$(\bar{\psi}\gamma^5\gamma_l\psi)^2 = 2(\bar{\psi}\psi)^2 + 2(\bar{\psi}i\gamma^5\psi)^2 - (\bar{\psi}\gamma^\mu\psi)^2.$$

$$S = \int d^4x e\Big(\frac{{M_{\rm Pl}}^2}{2}R_E - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$
$$- i\bar{\psi}\frac{1}{2}\left[\not{D}_\diamond - \overleftarrow{\not{D}}_\diamond\right]\psi + \frac{\lambda}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2]\Big),$$

$$S = \int d^4x e\Big(\frac{{M_{\rm Pl}}^2}{2}R_E - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$
$$- i\bar{\psi}\frac{1}{2}\left[\not{D}_\diamond - \overleftarrow{\not{D}}_\diamond\right]\psi + \left(\frac{1}{2\lambda}(\sigma^2 + \pi^2) + \bar{\psi}(\sigma + i\gamma^5\pi)\psi\right)\Big)$$

$$\mathcal{V} = \frac{1}{2\lambda}\sigma^2 - \frac{1}{16\pi^2}\left\{2\Lambda_{\text{cut-off}}^2\sigma^2 + \sigma^4\left(\ln\frac{\sigma^2}{\Lambda_{\text{cut-off}}^2} - \frac{1}{2}\right)\right\}$$
$$- \frac{R_E}{16\pi^2}\frac{2}{3}\left\{\ln\left(\frac{\Lambda_{\text{cut-off}}^2}{\sigma^2}\right) - 1\right\}\sigma^2.$$

$$\frac{1}{\lambda} - \frac{M^2}{4\pi^2}\left(\frac{\Lambda_{\text{cut-off}}^2}{v^2} - \ln\frac{\Lambda_{\text{cut-off}}^2}{v^2}\right) + \frac{R_E}{24\pi^2}\left(2 - \ln\frac{\Lambda_{\text{cut-off}}^2}{v^2}\right) = 0$$

$$S_{\text{eff}}=\int d^4x\sqrt{-g}\left\{\frac{M_{\text{Pl}}^2}{2}(1+\Pi)R_E-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi\right.$$
$$\left.+M_{\text{Pl}}^2\Lambda_{\text{Inf}}\alpha e^{-1/\alpha}\left(\frac{\phi}{M_{\text{Pl}}}\right)-\frac{3m^6}{8M_{\text{Pl}}^2}\left\{2e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_{\text{Pl}}}}-e^{2\sqrt{\frac{2}{3}}\frac{\phi}{M_{\text{Pl}}}}\right)\right\}$$

$$\Pi=\frac{M^2}{12\pi^2M_{\text{pl}}^2}\left\{\ln\left(\frac{\Lambda_{\text{cut-off}}^2}{M^2}\right)-1\right\}$$

$$U(\phi)=-U_1\left(\frac{\phi}{M_{\text{Pl}}}\right)+U_2\left(2e^{\sqrt{\frac{2}{3}}(1+\Pi)^{1/2}\frac{\phi}{M_{\text{Pl}}}}-e^{2\sqrt{\frac{2}{3}}(1+\Pi)^{1/2}\frac{\phi}{M_{\text{Pl}}}}\right),$$

$$U_1=M_{\text{pl}}^2\Lambda_{\text{Inf}}\alpha e^{-1/\alpha}(1+\Pi)^{-3/2},\qquad U_2=\frac{3m^6}{8M_{\text{pl}}^2}(1+\Pi)^{-2}$$

$$\phi_{\min}=\sqrt{\frac{3}{2}}M_{\text{Pl}}\ln\left(\sqrt{\frac{3}{8}}\frac{U_1}{U_2(1+\Pi)^{1/2}}\right)$$

$$d^2s=-\varphi(r)dt^2+h(r)^{-1}d^2r+r^2(d^2\theta+\sin^2\ \theta d^2\vartheta)$$

$$\frac{\varphi'}{\varphi}=-\frac{8\pi G}{h(r)r}\left[\frac{h(r)-1}{8\pi G}-r^2(P+P_s)\right]$$
$$\frac{dP}{dr}+\frac{\varphi'}{2\varphi}(\rho+P)=0$$
$$\frac{dM}{dr}=4\pi(\rho+\rho_s)r^2$$
$$\frac{d^2\phi}{d^2r}+\left(\frac{2}{r}+\frac{1}{2}\frac{d}{dr}\ln\left(\varphi(r)h(r)\right)\right)\frac{d\phi}{dr}-h(r)^{-1}U_\phi=0$$

$$\rho_s=\frac{h(r)}{16\pi G}\left(\frac{d\phi}{dr}\right)^2+U(\phi)$$
$$P_s=\frac{h(r)}{16\pi G}\left(\frac{d\phi}{dr}\right)^2-U(\phi)$$

$$h(r)=1-\frac{2GM(r)}{r},$$

$$\tilde{P}\equiv P/\rho_0c^2,\tilde{\rho}\equiv\rho/\rho_0,\tilde{M}\equiv M/(\rho_0r_0^3),s\equiv r/r_0,\tilde{\phi}\equiv\phi/M_{\text{Pl}},\tilde{U}\equiv U/(\rho_0c^2)$$

$$\tilde{G}\equiv G\rho_0r_0^2/c^2=0.996861$$

$$\zeta(\xi)=\frac{a_1+a_2\xi+a_3\xi^3}{1+a_4\xi}f_0\big(a_5(\xi-a_6)\big)+(a_7+a_8\xi)f_0\big(a_9(a_{10}-\xi)\big)$$
$$+(a_{11}+a_{12}\xi)f_0\big(a_{13}(a_{14}-\xi)\big)+(a_{15}+a_{16}\xi)f_0\big(a_{17}(a_{18}-\xi)\big)$$

$$\zeta=\log_{10}\ P/\text{dyn}\cdot\ \text{nm}^{-2}=\frac{\ln\ \rho_0c^2/\text{dyn}\cdot\ \text{nm}^{-2}}{\ln\ 10}+\frac{\ln\ \tilde{P}}{\ln\ 10}$$
$$\xi=\log_{10}\ \rho/\text{g}\cdot\ \text{nm}^{-3}=\frac{\ln\ \rho_0/\text{g}\cdot\ \text{nm}^{-3}}{\ln\ 10}+\frac{\ln\ \tilde{\rho}}{\ln\ 10}$$

$$\frac{\varphi'}{\varphi} = -\frac{8\pi\tilde{G}}{h(s)s}\left[\frac{h(s)-1}{8\pi\tilde{G}} - s^2\left(\tilde{P}+\tilde{P}_s\right)\right]$$
$$\tilde{\rho}' + \frac{\tilde{\rho}}{2\tilde{P}}\left(\frac{d\zeta}{d\xi}\right)^{-1}\frac{\varphi'}{\varphi}(\tilde{\rho}+\tilde{P}) = 0$$
$$\tilde{\phi}'' + \left(\frac{2}{s}+\frac{1}{2}\frac{d}{ds}\ln\left(\varphi(s)h(s)\right)\right)\tilde{\phi}' - h(s)^{-1}8\pi\tilde{G}_{\tilde{G}}\tilde{\phi}_{\tilde{\phi}} = 0$$
$$\tilde{M}' = 4\pi(\tilde{\rho}+\tilde{\rho}_s)s^2$$

$$\tilde{U}(\phi) = \frac{U(\phi)}{\rho_0 c^2} = -\tilde{U}_1\left(\frac{\phi}{M_{\mathrm{Pl}}}\right) + \tilde{U}_2\left(2e^{\sqrt{\frac{2}{3}\frac{\phi}{M_{\mathrm{Pl}}}}} - e^{2\sqrt{\frac{2}{3}\frac{\phi}{M_{\mathrm{Pl}}}}}\right)$$

$$\tilde{U}_1 = \left(\frac{M_{\mathrm{Pl}}^2\Lambda}{\rho_0 c^2}\right) \sim 8.2\times 10^{21}\left(\frac{\Lambda^{1/2}}{1\mathrm{eV}}\right)^2 \sim 1.38\times 10^{33}\left(\frac{U_1}{1\mathrm{eV}}\right)^4$$
$$\tilde{U}_2 = \frac{3m^6}{8M_{\mathrm{Pl}}^2\rho_0 c^2} \sim 2.3\times 10^{-40}\left(\frac{\langle\bar{\psi}\psi\rangle}{(100\mathrm{MeV})^3}\right)^2$$

$$U(\phi) = -4.7\times 10^{-44}\phi + 5.7\times 10^{-38}\left(2e^{\sqrt{\frac{2}{3}}\phi} - e^{2\sqrt{\frac{2}{3}}\phi}\right)$$

$$U(\phi) = -5\times 10^{-2}\phi + 5\times 10^{-1.4}\left(2e^{\sqrt{\frac{2}{3}}\phi} - e^{2\sqrt{\frac{2}{3}}\phi}\right)$$

$$\phi_{\min} = \ln\left(\sqrt{\frac{3}{8}\frac{U_1}{U_2}}\right)$$

$\rho(s=10^{-6}) = \rho_c, \varphi(s=10^{-6}) = 1, M(s=10^{-6}) = 4\pi/3(10^{-6})^3\rho_c$ and $\phi'(s=10^{-6}) = 0.$

$$\Delta \equiv \frac{\phi_{\mathrm{surface}}}{\phi_{\min}} - 1.$$

$$ds^2 = -\exp(2\phi)c^2dt^2 + \frac{dr^2}{1-\frac{2Gm(r)}{rc^2}} + r^2d\Omega^2$$

$$\frac{dm}{dr} = 4\pi r^2\rho(r)$$
$$\frac{d\phi}{dr} = \frac{G(m(r)+4\pi r^3P/c^2)}{r(rc^2-2Gm(r))}$$
$$\frac{dP}{dr} = -c^2\left(\rho(r)+\frac{P}{c^2}\right)\frac{d\phi}{dr}$$

$$\frac{dm}{dr} = 4\pi r^2 \left(\rho(r) + \frac{B^2}{8\pi c^2}\right)$$

$$\frac{d\phi}{dr} = \frac{G\left(m(r) + 4\pi r^3 \frac{P}{c^2}\right)}{r(rc^2 - 2Gm(r))}$$

$$\frac{dP}{dr} = -c^2 \left(\rho(r) + \frac{B^2}{8\pi c^2} + \frac{P}{c^2}\right)\left(\frac{d\phi}{dr} - L(r)\right)$$

$$L(r) = B_c^2[-3.8x + 8.1x^3 - 1.6x^5 - 2.3x^7] \times 10^{-41},$$

$$S = \int \sqrt{-g}\left[\frac{1}{16\pi G} f(R,T) + \mathcal{L}_m\right] d^4x$$

$$G_{\mu\nu} + (f_R - 1)R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} f + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) f_R = 8\pi G T_{\mu\nu} + f_T T_{\mu\nu} + f_T \Theta_{\mu\nu}$$

$$f_R = \frac{\partial f}{\partial R},$$

$$f_T = \frac{\partial f}{\partial T} \; \Box = g^{\mu\nu} \nabla_\mu \nabla_\nu,$$

$$\Theta_{\mu\nu} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}}.$$

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 d\Omega^2$$

$$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu}$$

$$\frac{dm}{dr} = 4\pi r^2 \left(\rho + \frac{B^2}{8\pi}\right) - \frac{2\lambda T r^2}{4}$$

$$\frac{d\phi}{dr} = \frac{\left[m + 4\pi r^3 P + \frac{r^3}{2}\left(\frac{2\lambda T}{2} + \left(\rho + \frac{B^2}{8\pi} + P\right) 2\lambda\right)\right]}{r^2\left(1 - \frac{2m}{r}\right)}$$

$$\frac{dP}{dr} = -\left(\rho + \frac{B^2}{8\pi} + P\right)\left(\frac{d\phi}{dr} - \mathrm{L}\right)$$

$$T = -\left(\rho + \frac{B^2}{8\pi}\right) + 3P$$

$$C_v \frac{dT_b^\infty}{dt} = -L_\nu^\infty(T_b^\infty) - L_\gamma^\infty(T_s) + H$$

$$L_\gamma^\infty = 4\pi\sigma R^2 (T_s^\infty)^4$$

$$T_s^{\infty} = T_s\sqrt{1 - \frac{2GM}{c^2R}}$$

$$n + n \to [nn] + \nu + \bar{\nu}$$
$$p + p \to [pp] + \nu + \bar{\nu}$$

$$\epsilon_\nu^s = \frac{5G_F^2}{14\pi^3}\nu_N(0)v_F(N)^2T^7I_\nu^s$$

$$I_\nu^s = z_N^7\left(\int_1^\infty \frac{y^5}{\sqrt{y^2-1}}[f_F(z_Ny)]^2dy\right)$$

$$\epsilon_{vee,\text{ brem}} = 7.42\times 10^{-2}G_F^2Z^2a_e^4n_IT^6L.$$

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu},$$

$$ds^2 = -e^{2\phi}dt^2 + e^{2\psi}dr^2 + r^2(d\theta^2 + \sin^2\ \theta d\phi^2)$$

$$\begin{aligned}8\pi\rho\ &= \frac{2}{r}e^{-2\psi}\psi_r + \frac{1-e^{-2\psi}}{r^2}\\ 8\pi p\ &= \frac{2}{r}e^{-2\psi}\phi_r + \frac{e^{-2\psi}-1}{r^2}\\ 8\pi p\ &= e^{-2\psi}\left[(\phi_r)^2 + \phi_{rr} - \phi_r\psi_r + \frac{\phi_r - \psi_r}{r}\right]\end{aligned}$$

$$\phi_r = \frac{\partial\phi}{\partial r}$$

$$A(r) := e^{2\psi(r)} = \left(1 - \frac{2m(r)}{r}\right)^{-1}, m(r) := 4\pi\int_0^r \rho(r')r'^2dr'$$

$$\frac{d\phi}{dr} = \frac{m(r) + 4\pi r^3p(r)}{r(r-2m(r))}$$

$$p_r = -(p + \rho)\phi_r$$

$$\frac{dp}{dr} = -(p(r) + \rho(r))\frac{m(r) + 4\pi r^3p(r)}{r(r-2m(r))}.$$

$$\rho(r) = \sum_{i=0}^{n} c_ir^i$$

$$\rho(r) = \rho_c - c_1r - c_2r^2 - c_3r^3 - c_4r^4$$

$$\rho(r) = \begin{cases}\rho_c - c_2r^2 - c_4r^4, & r < R\\ 0, & r > R\end{cases}$$

$$\rho_c = c_4R^4 + c_2R^2$$

$$m(r) = \begin{cases} 4\pi r^3 \left[\frac{\rho_c}{3} - \frac{c_2}{5} r^2 - \frac{c_4}{7} r^4\right], & r < R \\ 4\pi R^3 \left[\frac{\rho_c}{3} - \frac{c_2}{5} R^2 - \frac{c_4}{7} R^4\right] \equiv M, & r > R \end{cases}$$

$$A(r) = \left(1 - \frac{2m(r)}{r}\right)^{-1} = \begin{cases} \left(1 - 8\pi r^2 \left[\frac{\rho_c}{3} - \frac{c_2 r^2}{5} - \frac{c_4 r^4}{7}\right]\right)^{-1}, & r < R \\ \left(1 - \frac{2M}{r}\right)^{-1}, & r > R \end{cases}$$

$$de = -pd\left(\frac{1}{\rho_0}\right) = \frac{p}{\rho_0^2} d\rho_0$$

$$\rho(r) = \rho_0(r)(1 + e(r))$$

$$\frac{d\rho_0}{dr} = \frac{\rho_0}{[p(r) + \rho(r)]} \frac{d\rho}{dr},$$

$$\frac{d\rho}{dr} = \begin{cases} -2r(c_2 + 2c_4 r^2), & r < R \\ 0, & r > R \end{cases}$$

$$v_s^2 = \frac{\partial p}{\partial \rho}$$

$$v_s^2 = \frac{\Delta p}{\Delta \rho}$$

$$\left.\frac{dp}{dr}\right|_{r=0} = 0, \left.\frac{d\rho_0}{dr}\right|_{r=0} = 0$$

$$v_s^2|_{r=0} = \left[\frac{dp}{dr}\left(\frac{d\rho}{dr}\right)^{-1}\right]_{r=0} = \frac{2\pi}{c_2}(p_c + \rho_c)\left(p_c + \frac{\rho_c}{3}\right)$$

$$\frac{2\pi}{c_2}(p_c + \rho_c)\left(p_c + \frac{\rho_c}{3}\right) < 1$$

$$\frac{2m(r)}{r} < \frac{8}{9}.$$

$$A(r) = \left(1 - \frac{2m(r)}{r}\right)^{-1} < 9$$

$$e_c = \frac{\rho_c}{\rho_{0_c}} - 1 \geq 0$$

$$M = 4\pi R^3 \left[\frac{\rho_c}{3} - \frac{c_2 R^2}{5} - \frac{c_4 R^4}{7}\right].$$

$$M(R) = \frac{8\pi R^3}{7}\left[\frac{2}{3}\rho_c - \frac{1}{5} c_2 R^2\right].$$

$$c_2 = 3.27 \times 10^{-6}, c_4 = 1.42 \times 10^{-9}.$$

$$\frac{dA}{dr} = A\left[\frac{1-A}{r} + 8\pi r A\rho\right],$$

$$\frac{dA}{dr} = \begin{cases} 16\pi r A(r)^2 \left[\frac{1}{3}\rho_c - \frac{2}{5}c_2 r^2 - \frac{3}{7}c_4 r^4\right], & r < R \\ -\frac{2M}{(r-2M)^2}, & r > R. \end{cases}$$

$$P \lesssim \varepsilon/3 \leftrightarrow \phi \lesssim 1/3$$

$$\Delta \equiv 1/3 - P/\varepsilon \gtrsim 0, \rho \gtrsim 40\rho_0.$$

$$s^2 = \phi f(\phi), \phi = P/\varepsilon$$

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM\varepsilon}{r^2}\left(1+\frac{P}{\varepsilon}\right)\left(1+\frac{4\pi r^3 P}{M}\right)\left(1-\frac{2GM}{r}\right)^{-1}, \frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2\varepsilon$$

$$W = \frac{1}{G}\frac{1}{\sqrt{4\pi G\varepsilon_{\mathrm{c}}}} = \frac{1}{\sqrt{4\pi\varepsilon_{\mathrm{c}}}}, Q = \frac{1}{\sqrt{4\pi G\varepsilon_{\mathrm{c}}}} = \frac{1}{\sqrt{4\pi\varepsilon_{\mathrm{c}}}},$$

$$\frac{\mathrm{d}\hat{P}}{\mathrm{d}\hat{r}} = -\frac{\hat{\varepsilon}\hat{M}}{\hat{r}^2}\frac{(1+\hat{P}/\hat{\varepsilon})(1+\hat{r}^3\hat{P}/\hat{M})}{1-2\hat{M}/\hat{r}}, \frac{\mathrm{d}\hat{M}}{\mathrm{d}\hat{r}} = \hat{r}^2\hat{\varepsilon}$$

$$\mathrm{X} \equiv \phi_{\mathrm{c}} \equiv \hat{P}_{\mathrm{c}} \equiv P_{\mathrm{c}}/\varepsilon_{\mathrm{c}},$$

$$\mu \equiv \hat{\varepsilon} - \hat{\varepsilon}_{\mathrm{c}} = \hat{\varepsilon} - 1$$

$$\mathcal{U}/\mathcal{U}_{\mathrm{c}} \approx 1 + \sum_{i+j\geq 1} u_{ij}\mathrm{X}^i\mu^j,$$

$$P(R) = 0 \leftrightarrow \hat{P}(\hat{R}) = 0,$$

$$M_{\mathrm{NS}} = \hat{M}_{\mathrm{NS}}W, \text{ with } \hat{M}_{\mathrm{NS}} \equiv \hat{M}(\hat{R}) = \int_0^{\hat{R}} \mathrm{d}\hat{r}\hat{r}^2\hat{\varepsilon}(\hat{r}).$$

$$\hat{\varepsilon}(\hat{r}) \approx 1 + a_2\hat{r}^2 + a_4\hat{r}^4 + a_6\hat{r}^6 + \cdots$$
$$\hat{P}(\hat{r}) \approx \mathrm{X} + b_2\hat{r}^2 + b_4\hat{r}^4 + b_6\hat{r}^6 + \cdots$$
$$\hat{M}(\hat{r}) \approx \frac{1}{3}\hat{r}^3 + \frac{1}{5}a_2\hat{r}^5 + \frac{1}{7}a_4\hat{r}^7 + \frac{1}{9}a_6\hat{r}^9 + \cdots$$

$$b_2 = -\frac{1}{6}\left(1 + 3\hat{P}_{\mathrm{c}}^2 + 4\hat{P}_{\mathrm{c}}\right),$$
$$b_4 = \frac{\hat{P}_{\mathrm{c}}}{12}\left(1 + 3\hat{P}_{\mathrm{c}}^2 + 4\hat{P}_{\mathrm{c}}\right) - \frac{a_2}{30}\left(4 + 9\hat{P}_{\mathrm{c}}\right),$$
$$b_6 = -\frac{1}{216}\left(1 + 9\hat{P}_{\mathrm{c}}^2\right)\left(1 + 3\hat{P}_{\mathrm{c}}^2 + 4\hat{P}_{\mathrm{c}}\right) - \frac{a_2^2}{30} + \left(\frac{2}{15}\hat{P}_{\mathrm{c}}^2 + \frac{1}{45}\hat{P}_{\mathrm{c}} - \frac{1}{54}\right)a_2 - \frac{5 + 12\hat{P}_{\mathrm{c}}}{63}a_4,$$

$$s^2 = \frac{\mathrm{d}\hat{P}}{\mathrm{d}\hat{\varepsilon}} = \frac{\mathrm{d}\hat{P}}{\mathrm{d}\hat{r}} \cdot \frac{\mathrm{d}\hat{r}}{\mathrm{d}\hat{\varepsilon}} = \frac{b_2 + 2b_4\hat{r}^2 + \cdots}{a_2 + 2a_4\hat{r}^2 + \cdots}$$

$$R = \hat{R}Q \approx \left(\frac{3}{2\pi G}\right)^{1/2} v_c, \text{ with } v_c \equiv \frac{1}{\sqrt{\varepsilon_c}}\left(\frac{X}{1+3X^2+4X}\right)^{1/2}$$

$$M_{NS} \approx \frac{1}{3}\hat{R}^3\hat{\varepsilon}_c W = \frac{1}{3}\hat{R}^3 W \approx \left(\frac{6}{\pi G^3}\right)^{1/2} \Gamma_c, \text{ with } \Gamma_c \equiv \frac{1}{\sqrt{\varepsilon_c}}\left(\frac{X}{1+3X^2+4X}\right)^{3/2}$$

$$\xi \equiv \frac{M_{NS}}{R} \approx \frac{2}{G}\frac{X}{1+3X^2+4X} = \frac{2\Pi_c}{G}, \text{ with } \Pi_c \equiv \frac{X}{1+3X^2+4X}$$

$$\left.\frac{dM_{NS}}{d\varepsilon_c}\right|_{M_{NS}=M_{NS}^{max}=M_{TOV}} = 0.$$

$$\frac{dM_{NS}}{d\varepsilon_c} = \frac{1}{2}\frac{M_{NS}}{\varepsilon_c}\left[3\left(\frac{s_c^2}{X}-1\right)\frac{1-3X^2}{1+3X^2+4X}-1\right], \text{ where } s_c^2 \equiv \frac{dP_c}{d\varepsilon_c}.$$

$$s_c^2 = X\left(1+\frac{1+\Psi}{3}\frac{1+3X^2+4X}{1-3X^2}\right),$$

$$\Psi = 2\frac{d\ln M_{NS}}{d\ln \varepsilon_c} \geq 0.$$

$$s_c^2 = X\left(1+\frac{1}{3}\frac{1+3X^2+4X}{1-3X^2}\right).$$

$$\frac{dR}{d\varepsilon_c} \sim \frac{d}{d\varepsilon_c}\left(\frac{\hat{R}}{\sqrt{\varepsilon_c}}\right)_{R_{max}\leftrightarrow M_{NS}^{max}} = \left(\frac{s_c^2}{X}-1\right)\frac{1-3X^2}{1+3X^2+4X}-1 = -\frac{2}{3},$$

$$R_{max}/\text{nm} \approx 1050^{+30}_{-30} \times \left(\frac{v_c}{\text{fm}^{3/2}/\text{MeV}^{1/2}}\right) + 0.64^{+0.25}_{-0.25},$$

$$M_{NS}^{max}/M_\odot \approx 1730^{+30}_{-30} \times \left(\frac{\Gamma_c}{\text{fm}^{3/2}/\text{MeV}^{1/2}}\right) - 0.106^{+0.035}_{-0.035},$$

$$s_c^2 \leq 1 \leftrightarrow X = \hat{P}_c \lesssim 0.374 \equiv X_+^{GR}.$$

$$\phi = P/\varepsilon = \hat{P}/\hat{\varepsilon} \approx \hat{P}_c/\hat{\varepsilon}_c + \left(1-\frac{\hat{P}_c}{s_c^2}\right)b_2\hat{r}^2 = \hat{P}_c + \left(1-\frac{\hat{P}_c}{s_c^2}\right)b_2\hat{r}^2 \approx \hat{P}_c - \left(\frac{1+7\hat{P}_c}{24}\right)\hat{r}^2 < \hat{P}_c$$

$$\phi = \hat{P}/\hat{\varepsilon} \approx \hat{P}_c - \frac{1}{24}\frac{1+\Psi}{(1+\Psi/4)^2}\left[1+7\hat{P}_c+\Psi\left(\hat{P}_c+\frac{1}{4}\right)\right]\hat{r}^2 < \hat{P}_c$$

$$\phi = P/\varepsilon = \hat{P}/\hat{\varepsilon} \leq X \leq 0.374.$$

$$s_c^2 \approx 4X/3,$$

$$E(\rho) = B_{\mathrm{FFG}}\left(\frac{\rho}{\rho_0}\right)^{2/3} + B\left(\frac{\rho}{\rho_0}\right)^{\sigma},$$

$$\sigma\left(\mathrm{X}_{+}^{\mathrm{GR}}\sigma - 1\right) + \frac{\ell^{2/3}}{3}\left(\frac{B_{\mathrm{FFG}}}{M_{\mathrm{N}}}\right)\left(\sigma - \frac{2}{3}\right)\left[(3\sigma + 2)\mathrm{X}_{+}^{\mathrm{GR}} - 2\sigma - 3\right] = 0.$$

$$\sigma = \frac{1}{2}\left(\mathrm{X}_{+}^{\mathrm{GR}} + \Lambda\left(\mathrm{X}_{+}^{\mathrm{GR}} - \frac{2}{3}\right)\right)^{-1}\left\{1 + \frac{5}{9}\Lambda \pm \sqrt{1 + \frac{16\Lambda}{9}\left[\left(\mathrm{X}_{+}^{\mathrm{GR},2} - \frac{3\mathrm{X}_{+}^{\mathrm{GR}}}{2} + \frac{5}{8}\right) + \Lambda\left(\mathrm{X}_{+}^{\mathrm{GR}} - \frac{13}{12}\right)^{2}\right]}\right\}$$

$$\Lambda \equiv \ell^{2/3}\left(\frac{B_{\mathrm{FFG}}}{M_{\mathrm{N}}}\right) \ll 1$$

$$B = \left(\frac{1 + 5\Lambda/9}{\sigma^2 - 1}\frac{1}{\ell^{\sigma}}\right)M_{\mathrm{N}}$$

$$\sigma_{\mathrm{small}} \rightarrow \frac{2}{3}\frac{1}{1 + 3/\Lambda} = \frac{2}{3}\left(1 + \frac{3}{\ell^{2/3}}\left(\frac{M_{\mathrm{N}}}{B_{\mathrm{FFG}}}\right)\right)^{-1} \ll 1, \text{ and } \sigma_{\mathrm{large}} \rightarrow 1 \text{ from above.}$$

$$\sum_{j=1}^{J}\left(\frac{B_j}{M_{\mathrm{N}}}\right)\left(\sigma_j - \mathrm{X}_{+}^{\mathrm{GR}}\right)\ell^{\sigma_j} + \ell^{2/3}\left(\frac{B_{\mathrm{FFG}}}{M_{\mathrm{N}}}\right)\left(\frac{2}{3} - \mathrm{X}_{+}^{\mathrm{GR}}\right) - \mathrm{X}_{+}^{\mathrm{GR}} = 0$$
$$\sum_{j=1}^{J}\left(\frac{B_j}{M_{\mathrm{N}}}\right)\left(1 - \sigma_j^2\right)\ell^{1/3+\sigma_j} - \ell^{1/3}\left(1 + \frac{5}{9}\ell^{2/3}\left(\frac{B_{\mathrm{FFG}}}{M_{\mathrm{N}}}\right)\right) = 0$$

$$\Delta \geq \Delta_{\mathrm{GR}} \approx -0.04.$$

$$M_{\mathrm{NS}} \approx \left(\frac{1}{3}\hat{R}^3 + \frac{1}{5}a_2\hat{R}^5\right)W = \frac{1}{3}\hat{R}^3W\left(1 + \frac{3}{5}a_2\hat{R}^2\right) = \frac{1}{3}\hat{R}^3W\left(1 - \frac{3}{5}\frac{\mathrm{X}}{s_c^2}\right) \sim \Gamma_{\mathrm{c}}\left(1 - \frac{3}{5}\frac{\mathrm{X}}{s_c^2}\right),$$

$$\langle\hat{\varepsilon}\rangle = \int_0^{\hat{R}} \mathrm{d}\hat{r}\hat{r}^2\hat{\varepsilon}(\hat{r}) \Big/ \int_0^{\hat{R}} \mathrm{d}\hat{r}\hat{r}^2 = 1 + \frac{3}{5}a_2\hat{R}^2, \hat{\varepsilon}(\hat{r}) \approx 1 + a_2\hat{r}^2$$

$$s_c^2 \approx \mathrm{X}\left(1 + \frac{1}{3}\frac{1 + 3\mathrm{X}^2 + 4\mathrm{X}}{1 - 3\mathrm{X}^2}\right)(1 + \kappa_1\mathrm{X}) \approx \frac{4}{3}\mathrm{X} + \frac{4}{3}(1 + \kappa_1)\mathrm{X}^2 + \mathcal{O}(\mathrm{X}^3),$$

$$s_c^2 \approx \frac{4}{3}\mathrm{X} + \frac{1}{11}\left(\frac{38}{3} - 2\kappa_1\right)\mathrm{X}^2 + \mathcal{O}(\mathrm{X}^3).$$

$$M_{\mathrm{NS}} \sim \frac{1}{\sqrt{\varepsilon_{\mathrm{c}}}}\left(\frac{\mathrm{X}}{1 + 3\mathrm{X}^2 + 4\mathrm{X}}\right)^{3/2} \cdot \left(1 + \frac{18}{25}\mathrm{X}\right)$$

$$Q_{\alpha ab} = \nabla_{\alpha}g_{ab} = \partial_{\alpha}g_{ab} - \Gamma_{\alpha a}^{\sigma}g_{\sigma b} - \Gamma_{\alpha b}^{\sigma}g_{a\sigma}.$$

$$\Gamma_{ab}^{\sigma} = \{{}^{\sigma}{}_{ab}\} + \zeta_{ab}^{\sigma} + L_{ab}^{\sigma},$$

$$\left\{{}_{ab}\ {}_{ab} \equiv \frac{1}{2}g^{\sigma\beta}\left(\partial_a g_{\beta b} + \partial_b g_{\beta b} - \partial_{\beta}g_{ab}\right),\right.$$

$$\zeta^{\sigma}_{ab} \equiv \frac{1}{2} T^{\sigma} ab + T^{\sigma}_{(a\ b)}$$

$$L^{\sigma}_{ab} \equiv \frac{1}{2} Q^{\sigma}_{ab} - Q^{\sigma}_{(a}\ b\Big)$$

$$F^{\alpha}_{ab} = \frac{-1}{4} Q^{\alpha}_{ab} + \frac{1}{2} Q^{\alpha}_{(ab)} + \frac{1}{4}\left(Q^{\alpha} - \tilde{Q}^{\alpha}\right) g_{ab} - \frac{1}{4} \delta^{\alpha}_{(ab)}.$$

$$Q_{\alpha} \equiv Q^{a}_{\alpha a}\ \tilde{Q}_{\alpha} \equiv Q^{a}_{\alpha a},$$

$$Q = -Q_{\alpha b a} F^{\alpha a b}$$

$$S = \int \sqrt{-g} d^4 x \left[\frac{1}{2} F(Q) + \sigma^{\beta a b}_{\alpha} \mathcal{R}^{z}_{\beta a b} + \sigma^{ab}_{\alpha} T^{\alpha}_{ab} + \mathcal{L}_n\right]$$

$$T_{ab} = \frac{2}{\sqrt{-g}} \Delta_{\alpha}\left(\sqrt{-g} F_Q H^{\alpha}_{ab}\right) + \frac{1}{2} g_{ab} F + F_Q \left(H_{a\alpha\beta} Q^{\alpha\beta}_{b} - 2 Q_{\alpha\beta a} H^{\alpha\beta}_{b}\right)$$

$$T_{ab} \equiv \frac{2}{\sqrt{g}} \frac{\delta(\sqrt{-g})\mathcal{L}_n}{\delta g^{ab}}$$

$$\nabla_{\rho} \sigma^{ba\rho}_{\alpha} + \sigma^{ab}_{\alpha} = \sqrt{-g F_Q} H^{\alpha}_{ab} + J^{ab}_{\alpha}$$

$$J^{ab}_{\alpha} = \frac{-1}{2} \frac{\delta \mathcal{L}_n}{\delta T^{\alpha}_{ab}}$$

$$\nabla_a \nabla_b \left(\sqrt{-g} F_Q H^{ab}_{\alpha} + J^{ab}_{\alpha}\right) = 0.$$

$$\nabla_a \nabla_b \left(\sqrt{-g} F_Q H^{ab}_{\alpha}\right) = 0$$

$$\Gamma^{\alpha}_{ab} = \left(\frac{\partial u^{u}}{\partial \xi^{\sigma}}\right) \partial_a \partial b \xi^{\sigma}$$

$$Q_{\alpha a b} = \partial_{\alpha} g_{ab}$$

$$ds^2 = -e^{v(r)} dt^2 + e^{\sigma(r)} dr^2 + r^2 dw^2$$

$$Q(r) = -\frac{2e^{-\sigma}}{r}\left(v' + \frac{1}{r}\right)$$

$$T_{ab} = (\rho + p_t) a_a a_b + p_t g_{ab} + (p_r - p_t) b_a b_b$$

$$\rho = -\frac{F}{2} + F_Q \left[Q + \frac{1}{r^2} + \frac{e^{-\sigma}}{r}(v' + \sigma')\right],$$
$$p_r = \frac{F}{2} - F_Q \left[Q + \frac{1}{r^2}\right],$$
$$p_t = \frac{F}{2} - F_Q \left[\frac{Q}{2} - e^{-\sigma}\frac{Q}{2} - e^{-\sigma}\left[\frac{v''}{2} + \left(\frac{v'}{4} + \frac{1}{2r}\right)(v' - \sigma')\right]\right],$$
$$0 = \frac{\cot\theta}{2} Q' F_{QQ}.$$

$$\frac{\cot\theta}{2} Q' F_{QQ} = 0$$

$$F_{QQ} = 0 \Rightarrow F(Q) = \beta_1 Q + \beta_2$$

$$\rho = \frac{1}{2r^2}[2\beta_1 + 2e^{-\sigma}\beta_1(r\sigma' - 1) - r^2\beta_2]$$
$$p_r = \frac{1}{2r^2}[-2\beta_1 + 2e^{-\sigma}\beta_1(rv' + 1) + r^2\beta_2]$$
$$p_t = \frac{e^{-\sigma}}{4r}[2e^{\sigma} r\beta_2 + \beta_1(2 + rv')(v' - \sigma') + 2r\beta_1 v'']$$

$$\Delta(r) = \frac{\beta_1}{8\pi}\left[e^{-\sigma}\left(\frac{v''}{2} - \frac{\sigma' v'}{4} + \frac{(v')^2}{4} - \frac{v' + \sigma'}{2r} - \frac{1}{2r^2}\right) + \frac{1}{r^2}\right]$$

$$\sigma = \ln(\zeta(1 + x)) - \ln(\zeta + x)$$

$$v = 2\ln\,\eta$$

$$\Delta = \frac{\beta_1}{8\pi}\left[\frac{\zeta + x}{\zeta(1 + x)}\left[\frac{\eta''}{\eta} - \sqrt{\frac{\xi}{x}}\frac{\eta'}{\eta} + \frac{\sqrt{\xi x}(\zeta - 1)}{(\zeta + x)(1 + x)}\left(\sqrt{\xi x} - \frac{\eta'}{\eta}\right)\right]\right].$$

$$X'' + \frac{1}{Y_1^2 - 1}\left[1 - \zeta + \frac{8\pi\Delta\zeta(1 + x)^2}{\xi x} + \frac{3Y_1^2 + 2}{4(1 - Y_1^2)}\right]X = 0$$

$$\Delta = \frac{\xi x}{8\pi\zeta(1 + x)^2}\beta_1\left[\frac{5}{4}\frac{1}{(1 - Y_1^2)} - \frac{2\alpha(1 - Y_1^2)}{Y_1^2(\alpha + \beta Y_1)} + \zeta - \frac{7}{4}\right]$$

$$X'' - \frac{2\alpha}{Y_1^2(\alpha + \beta Y_1)}X = 0$$

$$X = A_1\frac{\alpha + \beta Y_1}{Y_1}\left[\frac{\alpha}{\beta^3}f(Y_1) + \frac{B_1}{A_1}\right]$$

$$\eta = A_1(1 - Y_1^2)^{1/4}\frac{\alpha + \beta Y_1}{Y_1}\left[\frac{\alpha}{\beta^3}f(Y_1) + \frac{B_1}{A_1}\right],$$

$$f(Y_1) = \frac{\sec^2 v - \cos^2 v}{2} + \log\cos^2 v$$

$$v = \tan^{-1}\sqrt{\frac{\beta Y_1}{\alpha}}, A_1 B_1$$

$$\rho = \frac{\beta_1 \xi(\zeta - 1)(3 + x)}{8\pi\zeta(1 + x)^2} - \frac{\beta_2}{16\pi}$$

$$p_r = \frac{-2\xi\beta_1 + \beta_2(1 + x) + 2(1 + x)\xi\beta_1\psi_1}{8\pi 2(1 + x)\psi_2\psi_3} + \frac{2\xi^2 r^2 \beta_1 \psi_4}{8\pi\zeta(1 + x)^2\psi_5\psi_6}$$

$$p_t = \frac{2B_1\beta^3(\zeta - 1)\psi_7 + A_1\beta\zeta^2\beta_2\psi_8 + 2A_1(\zeta - 1)\alpha\psi_9\psi_{10}}{8\pi\psi_{11}\psi_{12}}$$

$$0 < \frac{p}{\rho} < 1.$$

$$\text{i:e } 0 < \left(\frac{dp}{d\rho}\right)_{r=0} \leq 1$$

$$\left(\frac{d}{dr}\left(\frac{dp}{d\rho}\right)\right)_{r=0} < 0$$

$$\left(\frac{d}{dr}\left(\frac{p}{\rho}\right)\right)_{r=0} < 0$$

$$ds^2 = -\left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right)}$$
$$+ r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right) = e^{\nu(R)}$$
$$\left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right) = e^{-\sigma(R)}$$
$$p_r(R) = 0$$

$$A_1 = -\frac{2B_1(-1 + \zeta)\beta^3\psi_{13}}{(\zeta + \xi R^2)\beta\psi_{14} + 2(-1 + \zeta)\alpha Z}$$
$$B_1 = \frac{\psi_{15}}{(1 + \xi R^2)(\alpha + Y_2\beta)\psi_{16}}$$

$$\omega_r = \frac{p_r}{\rho}, \omega_t = \frac{p_t}{\rho}.$$

$$V_r^2 = \frac{dp_r}{d\rho} < 1, V_t^2 = \frac{dp_t}{d\rho} < 1,$$

$$m(r) = \frac{\kappa}{2}\int_0^r \rho r^2 dr$$
$$u(r) = \frac{\kappa}{2r}\int_0^r \rho r^2 dr$$

$$m(r) = -\frac{44r^3(-6\beta 1\xi(\zeta-1)+\beta_2\xi\zeta r^2+\beta_2\zeta)}{7(\zeta(3\xi r^2+3))}$$

$$u(r) = -\frac{44r^2(-6\beta_1\xi(\zeta-1)+\beta_2\xi\zeta r^2+\beta_2\zeta)}{7(\zeta(3\xi r^2+3))}$$

$$\begin{aligned} NEC&: \rho \geq 0, \\ WEC&: \rho + p_r \geq 0, \rho + p_t \geq 0, \\ DEC&: \rho - p_r \geq 0, \rho - p_t \geq 0, \\ SEC&: \rho + p_r + 2p_t \geq 0, \\ TEC&: \rho - p_r - 2p_t \geq 0. \end{aligned}$$

$$\Gamma = \frac{p_r+\rho}{p_r} \cdot \frac{dp_r}{d\rho}$$

$$Z_G = \frac{1}{\sqrt{|e^v(r)|}} - 1,$$

$$Z_S = \frac{1}{\sqrt{1-2u(r)}} - 1 = \frac{1}{\sqrt{1+2\left(\frac{44r^2(-6\beta_1\xi(\zeta-1)+\beta_2\xi\zeta r^2+\beta_2\zeta)}{7(\zeta(3\xi r^2+3))}\right)}} - 1.$$

$$F_g + F_h + F_a = 0$$

$$F_g = -\frac{v'}{2}(\rho+p_r), F_h = -\frac{dp_r}{dr}, F_a = \frac{2}{r}(p_t - p_r).$$

$$Y_1 = \sqrt{\frac{\zeta+x}{\zeta-1}}, Y_2 = \sqrt{\frac{\zeta+\xi R^2}{\zeta-1}}, Y_3 = \sqrt{\frac{\zeta+\xi R^2}{\zeta+\xi\zeta R^2}}$$

$$\psi_1 = 2A_1\beta^3Y_1 + \frac{2\beta}{(\zeta-1)(x+1)Y_1(\alpha+\beta Y_1)} - \frac{2}{(x+1)(\zeta+x)} + \frac{1}{\zeta x+\zeta} + \frac{4}{(x+1)^2}, \psi_2 = (1+x)(\alpha+Y_1\beta)$$

$$\psi_3 = \beta(2A_1(-1+\zeta)Y_1\alpha + A_1(\zeta+x)\beta + 2B_1(-1+\zeta)\beta^2(\alpha+Y_1\beta)) + 2A_1(-1+\zeta)\alpha(\alpha+Y_1\beta)\log\left[\frac{\alpha}{\alpha+Y_1\beta}\right]$$

$$\psi_4 = A_1\beta^3(x+1)Y_1 + \frac{\beta(x+1)}{(\zeta-1)Y_1(\alpha+\beta Y_1)} - \frac{x+1}{\zeta+x} + 2, \psi_5 = \alpha + \beta Y_1$$

$$\psi_6 = \beta(A_1\beta(\zeta+x) + 2\alpha A_1(\zeta-1)Y_1 + 2\beta^2B_1(\zeta-1)(\alpha+\beta Y_1)) + 2\alpha A_1(\zeta-1)(\alpha+\beta Y1)\log\left(\frac{\alpha}{\alpha+\beta Y_1}\right)$$

$$\psi_7 = \xi\zeta\psi_{74} + \beta^2\xi x^4\psi_{72} + \xi x^3\psi_{73} + \xi x^2\psi_{75} + \xi x\psi_{76} + \beta 2\zeta^2\psi_{71}$$

$$\psi_{71} = \alpha^3(\zeta-1) + 3\alpha\beta^2\zeta + 3\alpha^2\beta(\zeta-1)Y_1 + \beta^3\zeta Y_1, \psi_{72}$$
$$= 3\alpha(6\beta_1 + \beta_2\zeta r^2) + \beta Y_1(8\beta_1 + \beta_2\zeta r^2)$$

$$\psi_{73} = \alpha^3(\zeta-1)(2\beta_1 + \beta_1\zeta r^2) + \alpha\beta^2(\beta_1(38\zeta+36) + 3\beta_2\zeta(2\zeta+3)r^2) + 3\alpha^2\beta(\zeta-1)Y_1(4\beta_1 + \beta_2\zeta r^2) + \beta^3 Y_1(2\beta_1(9\zeta+7) + \beta_2\zeta(2\zeta+3)r^2)$$

$$\psi_{74} = \alpha^3(\zeta-1)(\beta_1(6\zeta-2) + \beta_1(3\zeta+1)r^2) + \alpha\beta^2\zeta(2\beta_1(9\zeta+1) + 3\beta_2(3\zeta+2)r^2) + \alpha^2\beta(\zeta-1)Y_1(2\beta_1(9\zeta-1) + 3\beta_2(3\zeta+1)r^2) + \beta^3\zeta Y_1(\beta_1(6\zeta+2) + \beta_2(3\zeta+2)r^2)$$

$$\psi_{75} = \alpha\beta^2(\beta_1(26\zeta^2+82\zeta+6) + 3\beta_2\zeta(\zeta^2+6\zeta+3)r^2) + \alpha^2\beta(\zeta^2+2\zeta-3)Y_1(10\beta_1 + 3\beta_2\zeta r^2) + \beta^3 Y_1(2\beta_1(6\zeta^2+17\zeta+1) + \beta_2\zeta(\zeta^2+6\zeta+3)r^2) + \alpha^3(\zeta-1)(8\beta_1 + \beta_2\zeta(\zeta+3)r^2)$$

$$\psi_{76} = \alpha^3(\zeta-1)(2\beta_1(\zeta^2+3\zeta+1) + 3\beta_1\zeta(\zeta+1)r^2) + \alpha\beta^2\zeta(\beta_1(6\zeta\zeta^2+64\zeta+8) + 3\beta_2(3\zeta^2+6\zeta+1)r^2) + \alpha^2\beta(\zeta-1)Y_1(\beta_1(6\zeta^2+32\zeta+6) + 9\beta_2\zeta(\zeta+1)r^2) + \beta^3\zeta Y_1(2\beta_1(\zeta^2+13\zeta+2) + \beta_2(3\zeta^2+6\zeta+1)r^2)$$

$$\psi_8 = \psi_{81} + \xi x^5\beta^3(18\beta_1 + \zeta r^2\beta_2) + \xi x^4\beta\psi_{82} + \xi\zeta\psi_{83} + \xi x^3\psi_{84} + \xi x\psi_{85} + \xi x^2\psi_{86}$$

$$\psi_{81} = \beta^3\zeta^2 + 5\alpha^2\beta(\zeta-1)\zeta + 2\alpha^3(\zeta-1)^2 Y_1 + 4\alpha\beta^2(\zeta-1)\zeta Y_1$$

$$\psi_{82} = \alpha^2(\zeta-1)(22\beta_1 + 5\beta_2\zeta r^2) + \beta^2(28\beta_1(2\zeta+1) + 3\beta_1\zeta(\zeta+1)r^2) + 4\alpha\beta(\zeta-1)Y_1(9\beta_1 + \beta_2\zeta r^2)$$

$$\psi_{83} = 3\beta^3\zeta^2(\zeta+1)(2\beta_1 + \beta_2 r^2) + \alpha^2\beta(\zeta-1)\zeta(\beta_1(30\zeta-2) + 5\beta_2(3\zeta+2)r^2) + 2\alpha^3(\zeta-1)^2 Y_1(\beta_1(6\zeta-2) + \beta_2(3\zeta+1)r^2) + 4\alpha\beta^2(\zeta-1)\zeta Y_1(\beta_1(6\zeta+2) + \beta_2(3\zeta+2)r^2)$$

$$\psi_{84} = 3\beta^3(\beta_1(20\zeta^2+30\zeta+2) + \beta_2\zeta(\zeta^2+3\zeta+1)r^2) + \alpha^2\beta(\zeta-1)(\beta_1(42\zeta+52) + 5\beta_2\zeta(2\zeta+3)r^2) + 2\alpha^3(\zeta-1)^2 Y_1(2\beta_1 + \beta_2\zeta r^2) + 4\alpha\beta^2(\zeta-1)Y_1(\beta_1(19\zeta+16) + \beta_2\zeta(2\zeta+3)r^2)$$

$$\psi_{85} = \alpha^2\beta(\zeta-1)\zeta(2\beta_1(5\zeta^2+44\zeta+4)+5\beta_2(3\zeta^2+6\zeta+1)r^2)$$
$$+\beta^3\zeta^2(2\beta_1(\zeta^2+23\zeta+9)+3\beta_2(\zeta^2+3\zeta+1)r^2)$$
$$+2\alpha^3(\zeta-1)^2Y_1(2\beta_1(\zeta^2+3\zeta+1)+3\beta_2\zeta(\zeta+1)r^2)$$
$$+4\alpha\beta^2(\zeta-1)\zeta Y_1(\beta_1(2\zeta^2+26\zeta+5)+\beta_2(3\zeta^2+6\zeta+1)r^2)$$
$$\psi_{86} = 5\alpha^2\beta(\zeta-1)(\beta_1(6\zeta^2+22\zeta+2)+\beta_2\zeta(\zeta^2+6\zeta+3)r^2)$$
$$+4\alpha\beta^2(\zeta-1)Y_1(3\beta_1(4\zeta^2+12\zeta+1)+\beta_2\zeta(\zeta^2+6\zeta+3)r^2))$$
$$+\beta^3\zeta(6\beta_1(4\zeta^2+17\zeta+3)+\beta_2(\zeta^3+9\zeta^2+9\zeta+1)r^2)+2\alpha^3(\zeta-1)^2Y_1(8\beta_1+\beta_2\zeta(\zeta+3)r^2)$$
$$\psi_9 = \zeta^2\beta 2\psi_{91}+\xi x^4\beta^2\psi_{92}+\xi x^3\psi_{93}+\xi\zeta\psi_{94}+\xi x^2\psi_{95}+\xi x\psi_{96}$$
$$\psi_{91} = \alpha^3(\zeta-1)+3\alpha\beta^2\zeta+3\alpha^2\beta(\zeta-1)Y_1+\beta^3\zeta Y_1, \psi_{92} = 3\alpha(6\beta_1+\beta_2\zeta r^2)+\beta Y_1(8\beta_1+\beta_2\zeta r^2)$$
$$\psi_{93} = \alpha^3(\zeta-1)(2\beta_1+\beta_2\zeta r^2)+\alpha\beta^2(\beta_1(38\zeta+36)+3\beta_2\zeta(2\zeta+3)r^2)+3\alpha^2\beta(\zeta-1)Y1_1(4\beta_1+\beta_1\zeta r^2)$$
$$+\beta^3Y_1(2\beta_1(9\zeta+7)+\beta_2\zeta(2\zeta+3)r^2)$$
$$\psi_{94} = \alpha^3(\zeta-1)(\beta_1(6\zeta-2)+\beta_2(3\zeta+1)r^2)+\alpha\beta^2\zeta(2\beta_1(9\zeta+1)+3\beta_1(3\zeta+2)r^2)$$
$$+\alpha^2\beta(\zeta-1)Y_1(2\beta_1(9\zeta-1)+3\beta_2(3\zeta+1)r^2)+\beta^3\zeta Y_1(\beta_1(6\zeta+2)+\beta_2(3\zeta+2)r^2)$$
$$\psi_{95} = \alpha\beta^2(\beta_1(26\zeta^2+82\zeta+6)+3\beta_2\zeta(\zeta^2+6\zeta+3)r^2)+\alpha^2\beta(\zeta^2+2\zeta-3)Y_1(10\beta_1+3\beta_2\zeta r^2)$$
$$+\beta^3Y_1\beta_1(6\zeta^2+17\zeta+1)+\beta_2\zeta(\zeta^2+6\zeta+3)r^2)+\alpha^3(\zeta-1)(8\beta_1+\beta_2\zeta(\zeta+3)r^2)$$
$$\psi_{96} = \alpha^3(\zeta-1)(2\beta_1(\zeta^2+3\zeta+1)+3\beta_2\zeta(\zeta+1)r^2)+\alpha\beta^2\zeta(\beta_1(6\zeta^2+64\zeta+8)+3\beta_2(3\zeta^2+6\zeta+1)r^2)$$
$$+\alpha^2\beta(\zeta-1)Y_1(\beta_1(6\zeta^2+32\zeta+6)+9\beta_2\zeta(\zeta+1)r^2)+\beta^3\zeta Y_1(2\beta_1(\zeta^2+13\zeta+2)+\beta_2(3\zeta^2+6\zeta+1)r^2)$$
$$\psi_{10} = \log\left(\frac{\alpha}{\alpha+\beta Y_1}\right), \psi_{11} = 2(\zeta-1)\zeta(x+1)^3(\zeta+x)(\alpha+\beta Y_1)^2$$
$$\psi_{12} = \beta\big(A_1\beta(\zeta+x)+2\alpha A_1(\zeta-1)Y_1+2\beta^2B_1(\zeta-1)(\alpha+\beta Y_1)\big)+2\alpha A_1(\zeta-1)(\alpha+\beta Y_1)\log\left(\frac{\alpha}{\alpha+\beta Y_1}\right)$$
$$\psi_{13} = (\zeta\beta_2\psi_{131}+\xi^3R^4\beta\psi_{132}+\xi^2R^2\psi_{133}+\xi\psi_{134})$$
$$\psi_{131} = 2\alpha\beta\zeta+\beta^2\zeta Y_2+(\zeta-1)Y_2\alpha^2, \psi_{132} = \alpha(2\beta_2\zeta R^2-4\beta_1(\zeta-4))+\beta Y_2(\beta_2\zeta R^2-2\beta_1(\zeta-5))$$
$$\psi_{133} = \beta^2Y_2\big(\beta_2\zeta(\zeta+2)R^2-2\beta_1(\zeta^2-8\zeta-1)\big)+2\alpha\beta\zeta(\beta_2(\zeta+2)R^2-2\beta_1(\zeta-7))$$
$$+\alpha^2(\zeta-1)Y_2(\beta_2\zeta R^2-2\beta_1(\zeta-3))$$
$$\psi_{134} = 2\alpha\beta\zeta(6\beta_1\zeta+2\beta_2\zeta R^2+\beta_2R^2)+2\alpha^2(\zeta-1)Y_2(\beta_1(3\zeta-1)+\beta_2\zeta R^2)$$
$$+\beta^2\zeta Y_2(\beta_1(6\zeta+2)+\beta_2(2\zeta+1)R^2)$$
$$\psi_{14} = (\zeta\beta 2\psi_{141}+\xi^3R^4\beta^2(-2(-7+\zeta)\beta 1+\zeta\beta^2\beta 2)+\xi^2R^2\psi_{142}+\xi\psi_{143})$$

$$\psi_{141} = 2\alpha\beta\zeta+\alpha^2(\zeta-1)Y_2+\beta^2\zeta Y_2$$

$$\psi_{142} = \beta^2(\beta_1(-2\zeta^2+20\zeta+6)+\beta_2\zeta(\zeta+2)R^2)-2\alpha^2(\zeta-1)(2\beta_1(\zeta-3)-\beta_2\zeta R^2)$$
$$+\alpha\beta(\zeta-1)Y_2(\beta_1(26-6\zeta)+3\beta_2\zeta R^2)$$

$$\psi_{143} = 4\alpha^2(\zeta-1)(\beta_1(3\zeta-1)+\beta_2\zeta R^2)+\beta^2\zeta(6\beta_1(\zeta+1)+\beta_2(2\zeta+1)R^2)$$
$$+2\alpha\beta(\zeta-1)Y_2(\beta_1+9\beta_1\zeta+3\beta_2\zeta R^2)$$

$$Z = (\zeta\beta 2Z_1+\xi^3R^4\beta Z_2+\xi^2R^2Z_3+\xi Z_4)\left(\log\left[\frac{\alpha}{\alpha+Y2}\right]\right), Z_1 = 2\alpha\beta\zeta+\alpha^2(\zeta-1)Y_2+\beta^2\zeta Y_2$$

$$Z_2 = \alpha(2\beta_2\zeta R^2-4\beta_1(\zeta-4))+\beta Y_2(\beta_2\zeta R^2-2\beta_1(\zeta-5))$$

$$Z_3 = \beta^2Y_2\big(\beta_2\zeta(\zeta+2)R^2-2\beta_1(\zeta^2-8\zeta-1)\big)+2\alpha\beta\zeta(\beta_2(\zeta+2)R^2-2\beta_1(\zeta-7))$$
$$+\alpha^2(\zeta-1)Y_2(\beta_2\zeta R^2-2\beta_1(\zeta-3))$$

$$Z_4 = 2\alpha\beta\zeta(6\beta_1\zeta+2\beta_2\zeta R^2+\beta_2R^2)$$
$$+2\alpha^2(\zeta-1)Y_2(\beta 1(3\zeta-1)+\beta 2\zeta R^2)+\beta^2\zeta Y_2(\beta_1(6\zeta+2)+\beta_2(2\zeta+1)R^2)$$

$$\psi_{15} = 4(1-\zeta)Y_2Y_3, \psi_{16} = \left(1 - \frac{\psi_{17}}{\psi_{18}}\right)$$

$$\psi_{17} = 2(-1+\zeta)\alpha\frac{1}{2}\left(1 + \frac{Y_2\beta}{2} - \frac{\alpha}{\alpha + Y_2\beta}\right)$$
$$+2(-1+\zeta)\log\left[\frac{\alpha}{\alpha + Y_2\beta}\right](\zeta\beta_2\psi_{171} + \xi^3R^4\beta\psi_{172} + \xi^2R^2\psi_{173} + \xi\psi_{174})$$

$$\psi_{171} = 2\alpha\beta\zeta + \alpha^2(\zeta-1)Y_2 + \beta^2\zeta Y_2, \psi_{172}$$
$$= \alpha(2\beta_2\zeta R^2 - 4\beta_1(\zeta-4)) + \beta Y_2(\beta_2\zeta R^2 - 2\beta_1(\zeta-5))$$

$$\psi_{173} = \beta^2Y_2\left(\beta_2\zeta(\zeta+2)R^2 - 2\beta_1(\zeta^2 - 8\zeta - 1)\right) + 2\alpha\beta\zeta(\beta_2(\zeta+2)R^2 - 2\beta_1(\zeta-7))$$
$$+\alpha^2(\zeta-1)Y_2(\beta_2\zeta R^2 - 2\beta_1(\zeta-3))$$

$$\psi_{174} = 2\alpha\beta\zeta(6\beta_1\zeta + 2\beta_2\zeta R^2 + \beta_2R^2) + 2\alpha^2(\zeta-1)Y_2(\beta_1(3\zeta-1) + \beta_2\zeta R^2)$$
$$+\beta^2\zeta Y_2(\beta_1(6\zeta+2) + \beta_2(2\zeta+1)R^2)$$

$$\psi_{18} = (\zeta + \xi R^2)\beta(\zeta\beta_2\psi_{181} + \xi^3R^4\beta^2\psi_{182} + \xi^2R^2\psi_{183} + \xi\psi_{184})$$
$$+2(-1+\zeta)\alpha\left(\zeta\beta_2\psi_{185} + \xi^3R^3\beta\psi_{186} + \xi^2R^2\psi 187 + \xi\psi_{188}\log\left[\frac{\alpha}{\alpha + Y2\beta}\right]\right)$$

$$\psi_{181} = 2\alpha^2(\zeta-1) + \beta^2\zeta + 3\alpha\beta(\zeta-1)Y_2, \psi_{182} = \beta_2\zeta R^2 - 2\beta_1(\zeta-7)$$
$$\psi_{183} = \beta^2(\beta_1(-2\zeta^2 + 20\zeta + 6) + \beta_2\zeta(\zeta+2)R^2) - 2\alpha^2(\zeta-1)(2\beta_1(\zeta-3) - \beta_2\zeta R^2)$$
$$+\alpha\beta(\zeta-1)Y_2(\beta 1(26 - 6\zeta) + 3\beta_2\zeta R^2) - 24 -$$

$$\psi_{184} = 4\alpha^2(\zeta-1)(\beta_1(3\zeta-1) + \beta_2\zeta R^2) + \beta^2\zeta(6\beta_1(\zeta+1) + \beta_2(2\zeta+1)R^2)$$
$$+2\alpha\beta(\zeta-1)Y_2(\beta_1 + 9\beta_1\zeta + 3\beta_2\zeta R^2)$$

$$\psi_{185} = 2\alpha\beta\zeta + \alpha^2(\zeta-1)Y_2 + \beta^2\zeta Y_2, \psi_{186}$$
$$= \alpha(2\beta_2\zeta R^2 - 4\beta_1(\zeta-4)) + \beta Y_2(\beta_2\zeta R^2 - 2\beta_1(\zeta-5))$$

$$\psi_{187} = \beta^2Y_2\left(\beta_2\zeta(\zeta+2)R^2 - 2\beta_1(\zeta^2 - 8\zeta - 1)\right) + 2\alpha\beta\zeta(\beta_2(\zeta+2)R^2 - 2\beta_1(\zeta-7))$$
$$+\alpha^2(\zeta-1)Y_2(\beta_2\zeta R^2 - 2\beta_1(\zeta-3))$$

$$\psi_{188} = 2\alpha\beta\zeta(6\beta_1\zeta + 2\beta_2\zeta R^2 + \beta_2R^2)$$
$$+2\alpha^2(\zeta-1)Y_2(\beta_1(3\zeta - 1 + \beta_2\zeta R^2) + \beta^2\zeta Y_2(\beta_1(6\zeta+2) + \beta_2(2\zeta+1)R^2)$$

$$e^{v}(r) = \left((1 - Y_1^2)^{\frac{1}{4}}\left(\frac{\alpha + \beta \times Y_1}{Y1}\left(\frac{\alpha}{\beta^3}A_1\left(\frac{\sec\ v \times \sec\ v - \cos\ v \times \cos\ v}{2}\right) + \log\left[\cos\ v \times \cos\ v\right]\right)\right.\right.$$
$$\left.\left. + B_1\right)\right)^2$$

$$Y_1=\sqrt{\frac{\zeta+x}{\zeta-1}},\quad Y_2=\sqrt{\frac{\zeta+\xi R^2}{\zeta-1}},\quad Y_3=\sqrt{\frac{\zeta+\xi R^2}{\zeta+\xi\zeta R^2}}$$

$$\psi_1=2A_1\beta^3Y_1+\frac{2\beta}{(\zeta-1)(x+1)Y_1(\alpha+\beta Y_1)}-\frac{2}{(x+1)(\zeta+x)}+\frac{1}{\zeta x+\zeta}+\frac{4}{(x+1)^2},\quad \psi_2=(1+x)(\alpha+Y_1\beta)$$

$$\psi_3=\beta\left(2A_1(-1+\zeta)Y_1\alpha+A_1(\zeta+x)\beta+2B_1(-1+\zeta)\beta^2(\alpha+Y_1\beta)\right)+2A_1(-1+\zeta)\alpha(\alpha+Y_1\beta)\log[\frac{\alpha}{\alpha+Y_1\beta}]$$

$$\psi_4=A_1\beta^3(x+1)Y_1+\frac{\beta(x+1)}{(\zeta-1)Y_1(\alpha+\beta Y_1)}-\frac{x+1}{\zeta+x}+2,\quad \psi_5=\alpha+\beta Y_1$$

$$\psi_6=\beta\left(A_1\beta(\zeta+x)+2\alpha A_1(\zeta-1)Y_1+2\beta^2B_1(\zeta-1)(\alpha+\beta Y_1)\right)+2\alpha A_1(\zeta-1)(\alpha+\beta Y1)\log\left(\frac{\alpha}{\alpha+\beta Y_1}\right)$$

$$\psi_7=\xi\zeta\psi_{74}+\beta^2\xi x^4\psi_{72}+\xi x^3\psi_{73}+\xi x^2\psi_{75}+\xi x\psi_{76}+\beta 2\zeta^2\psi_{71}$$

$$\psi_{71}=\alpha^3(\zeta-1)+3\alpha\beta^2\zeta+3\alpha^2\beta(\zeta-1)Y_1+\beta^3\zeta Y_1,\quad \psi_{72}=3\alpha\left(6\beta_1+\beta_2\zeta r^2\right)+\beta Y_1\left(8\beta_1+\beta_2\zeta r^2\right)$$

$$\psi_{73}=\alpha^3(\zeta-1)\left(2\beta_1+\beta_1\zeta r^2\right)+\alpha\beta^2\left(\beta_1(38\zeta+36)+3\beta_2\zeta(2\zeta+3)r^2\right)+3\alpha^2\beta(\zeta-1)Y_1\left(4\beta_1+\beta_2\zeta r^2\right)$$
$$+\beta^3Y_1\left(2\beta_1(9\zeta+7)+\beta_2\zeta(2\zeta+3)r^2\right)$$

$$\psi_{74}=\alpha^3(\zeta-1)\left(\beta_1(6\zeta-2)+\beta_1(3\zeta+1)r^2\right)+\alpha\beta^2\zeta\left(2\beta_1(9\zeta+1)+3\beta_2(3\zeta+2)r^2\right)$$
$$+\alpha^2\beta(\zeta-1)Y_1\left(2\beta_1(9\zeta-1)+3\beta_2(3\zeta+1)r^2\right)+\beta^3\zeta Y_1\left(\beta_1(6\zeta+2)+\beta_2(3\zeta+2)r^2\right)$$

$$\psi_{75}=\alpha\beta^2\left(\beta_1\left(26\zeta^2+82\zeta+6\right)+3\beta_2\zeta\left(\zeta^2+6\zeta+3\right)r^2\right)+\alpha^2\beta\left(\zeta^2+2\zeta-3\right)Y_1\left(10\beta_1+3\beta_2\zeta r^2\right)$$
$$+\beta^3Y_1\left(2\beta_1\left(6\zeta^2+17\zeta+1\right)+\beta_2\zeta\left(\zeta^2+6\zeta+3\right)r^2\right)+\alpha^3(\zeta-1)\left(8\beta_1+\beta_2\zeta(\zeta+3)r^2\right)$$

$$\psi_{76}=\alpha^3(\zeta-1)\left(2\beta_1\left(\zeta^2+3\zeta+1\right)+3\beta_1\zeta(\zeta+1)r^2\right)+\alpha\beta^2\zeta\left(\beta_1\left(6\zeta\zeta^2+64\zeta+8\right)+3\beta_2\left(3\zeta^2+6\zeta+1\right)r^2\right)$$
$$+\alpha^2\beta(\zeta-1)Y_1\left(\beta_1\left(6\zeta^2+32\zeta+6\right)+9\beta_2\zeta(\zeta+1)r^2\right)+\beta^3\zeta Y_1\left(2\beta_1\left(\zeta^2+13\zeta+2\right)+\beta_2\left(3\zeta^2+6\zeta+1\right)r^2\right)$$

$$\psi_8=\psi_{81}+\xi x^5\beta^3(18\beta_1+\zeta r^2\beta_2)+\xi x^4\beta\psi_{82}+\xi\zeta\psi_{83}+\xi x^3\psi_{84}+\xi x\psi_{85}+\xi x^2\psi_{86}$$

$$\psi_{81}=\beta^3\zeta^2+5\alpha^2\beta(\zeta-1)\zeta+2\alpha^3(\zeta-1)^2Y_1+4\alpha\beta^2(\zeta-1)\zeta Y_1$$

$$\psi_{82}=\alpha^2(\zeta-1)\left(22\beta_1+5\beta_2\zeta r^2\right)+\beta^2\left(28\beta_1(2\zeta+1)+3\beta_1\zeta(\zeta+1)r^2\right)+4\alpha\beta(\zeta-1)Y_1\left(9\beta_1+\beta_2\zeta r^2\right)$$

$$\psi_{83}=3\beta^3\zeta^2(\zeta+1)\left(2\beta_1+\beta_2r^2\right)+\alpha^2\beta(\zeta-1)\zeta\left(\beta_1(30\zeta-2)+5\beta_2(3\zeta+2)r^2\right)$$
$$+2\alpha^3(\zeta-1)^2Y_1\left(\beta_1(6\zeta-2)+\beta_2(3\zeta+1)r^2\right)+4\alpha\beta^2(\zeta-1)\zeta Y_1\left(\beta_1(6\zeta+2)+\beta_2(3\zeta+2)r^2\right)$$

$$\psi_{84}=3\beta^3\left(\beta_1\left(20\zeta^2+30\zeta+2\right)+\beta_2\zeta\left(\zeta^2+3\zeta+1\right)r^2\right)+\alpha^2\beta(\zeta-1)\left(\beta_1(42\zeta+52)+5\beta_2\zeta(2\zeta+3)r^2\right)$$
$$+2\alpha^3(\zeta-1)^2Y_1\left(2\beta_1+\beta_2\zeta r^2\right)+4\alpha\beta^2(\zeta-1)Y_1\left(\beta_1(19\zeta+16)+\beta_2\zeta(2\zeta+3)r^2\right)$$

$$\psi_{85}=\alpha^2\beta(\zeta-1)\zeta\left(2\beta_1\left(5\zeta^2+44\zeta+4\right)+5\beta_2\left(3\zeta^2+6\zeta+1\right)r^2\right)$$
$$+\beta^3\zeta^2\left(2\beta_1\left(\zeta^2+23\zeta+9\right)+3\beta_2\left(\zeta^2+3\zeta+1\right)r^2\right)$$
$$+2\alpha^3(\zeta-1)^2Y_1\left(2\beta_1\left(\zeta^2+3\zeta+1\right)+3\beta_2\zeta(\zeta+1)r^2\right)$$
$$+4\alpha\beta^2(\zeta-1)\zeta Y_1\left(\beta_1\left(2\zeta^2+26\zeta+5\right)+\beta_2\left(3\zeta^2+6\zeta+1\right)r^2\right)$$

$$\psi_{86}=5\alpha^2\beta(\zeta-1)\left(\beta_1\left(6\zeta^2+22\zeta+2\right)+\beta_2\zeta\left(\zeta^2+6\zeta+3\right)r^2\right)$$
$$+4\alpha\beta^2(\zeta-1)Y_1\left(3\beta_1\left(4\zeta^2+12\zeta+1\right)+\beta_2\zeta\left(\zeta^2+6\zeta+3\right)r^2\right))$$
$$+\beta^3\zeta\left(6\beta_1\left(4\zeta^2+17\zeta+3\right)+\beta_2\left(\zeta^3+9\zeta^2+9\zeta+1\right)r^2\right)+2\alpha^3(\zeta-1)^2Y_1\left(8\beta_1+\beta_2\zeta(\zeta+3)r^2\right)$$

$$\psi_9=\zeta^2\beta2\psi_{91}+\xi x^4\beta^2\psi_{92}+\xi x^3\psi_{93}+\xi\zeta\psi_{94}+\xi x^2\psi_{95}+\xi x\psi_{96}$$

$$\psi_{91}=\alpha^3(\zeta-1)+3\alpha\beta^2\zeta+3\alpha^2\beta(\zeta-1)Y_1+\beta^3\zeta Y_1,\quad \psi_{92}=3\alpha\left(6\beta_1+\beta_2\zeta r^2\right)+\beta Y_1\left(8\beta_1+\beta_2\zeta r^2\right)$$

$$\psi_{93}=\alpha^3(\zeta-1)\left(2\beta_1+\beta_2\zeta r^2\right)+\alpha\beta^2\left(\beta_1(38\zeta+36)+3\beta_2\zeta(2\zeta+3)r^2\right)+3\alpha^2\beta(\zeta-1)Y1_1\left(4\beta_1+\beta_1\zeta r^2\right)$$
$$+\beta^3Y_1\left(2\beta_1(9\zeta+7)+\beta_2\zeta(2\zeta+3)r^2\right)$$

$$\psi_{94}=\alpha^3(\zeta-1)\left(\beta_1(6\zeta-2)+\beta_2(3\zeta+1)r^2\right)+\alpha\beta^2\zeta\left(2\beta_1(9\zeta+1)+3\beta_1(3\zeta+2)r^2\right)$$
$$+\alpha^2\beta(\zeta-1)Y_1\left(2\beta_1(9\zeta-1)+3\beta_2(3\zeta+1)r^2\right)+\beta^3\zeta Y_1\left(\beta_1(6\zeta+2)+\beta_2(3\zeta+2)r^2\right)$$

$$\psi_{95}=\alpha\beta^2\left(\beta_1\left(26\zeta^2+82\zeta+6\right)+3\beta_2\zeta\left(\zeta^2+6\zeta+3\right)r^2\right)+\alpha^2\beta\left(\zeta^2+2\zeta-3\right)Y_1\left(10\beta_1+3\beta_2\zeta r^2\right)$$
$$+\beta^3Y_1\beta_1\left(6\zeta^2+17\zeta+1\right)+\beta_2\zeta\left(\zeta^2+6\zeta+3\right)r^2\right)+\alpha^3(\zeta-1)\left(8\beta_1+\beta_2\zeta(\zeta+3)r^2\right)$$

$$\psi_{96}=\alpha^3(\zeta-1)\left(2\beta_1\left(\zeta^2+3\zeta+1\right)+3\beta_2\zeta(\zeta+1)r^2\right)+\alpha\beta^2\zeta\left(\beta_1\left(6\zeta^2+64\zeta+8\right)+3\beta_2\left(3\zeta^2+6\zeta+1\right)r^2\right)$$
$$+\alpha^2\beta(\zeta-1)Y_1\left(\beta_1\left(6\zeta^2+32\zeta+6\right)+9\beta_2\zeta(\zeta+1)r^2\right)+\beta^3\zeta Y_1\left(2\beta_1\left(\zeta^2+13\zeta+2\right)+\beta_2\left(3\zeta^2+6\zeta+1\right)r^2\right)$$

$$\psi_{10}=\log\left(\frac{\alpha}{\alpha+\beta Y_1}\right),\quad \psi_{11}=2(\zeta-1)\zeta(x+1)^3(\zeta+x)(\alpha+\beta Y_1)^2$$

$$\psi_{12}=\beta\left(A_1\beta(\zeta+x)+2\alpha A_1(\zeta-1)Y_1+2\beta^2B_1(\zeta-1)(\alpha+\beta Y_1)\right)+2\alpha A_1(\zeta-1)(\alpha+\beta Y_1)\log\left(\frac{\alpha}{\alpha+\beta Y_1}\right)$$

$$\psi_{13}=\left(\zeta\beta_2\psi_{131}+\xi^3R^4\beta\psi_{132}+\xi^2R^2\psi_{133}+\xi\psi_{134}\right)$$

$$\psi_{131}=2\alpha\beta\zeta+\beta^2\zeta Y_2+(\zeta-1)Y_2\alpha^2,\quad \psi_{132}=\alpha\left(2\beta_2\zeta R^2-4\beta_1(\zeta-4)\right)+\beta Y_2\left(\beta_2\zeta R^2-2\beta_1(\zeta-5)\right)$$

$$\psi_{133}=\beta^2Y_2\left(\beta_2\zeta(\zeta+2)R^2-2\beta_1\left(\zeta^2-8\zeta-1\right)\right)+2\alpha\beta\zeta\left(\beta_2(\zeta+2)R^2-2\beta_1(\zeta-7)\right)$$
$$+\alpha^2(\zeta-1)Y_2\left(\beta_2\zeta R^2-2\beta_1(\zeta-3)\right)$$

$$\psi_{134}=2\alpha\beta\zeta\left(6\beta_1\zeta+2\beta_2\zeta R^2+\beta_2R^2\right)+2\alpha^2(\zeta-1)Y_2\left(\beta_1(3\zeta-1)+\beta_2\zeta R^2\right)$$
$$+\beta^2\zeta Y_2\left(\beta_1(6\zeta+2)+\beta_2(2\zeta+1)R^2\right)$$

$$\psi_{141} = 2\alpha\beta\zeta + \alpha^2(\zeta-1)Y_2 + \beta^2\zeta Y_2$$

$$\psi_{142} = \beta^2\left(\beta_1\left(-2\zeta^2+20\zeta+6\right)+\beta_2\zeta(\zeta+2)R^2\right) - 2\alpha^2(\zeta-1)\left(2\beta_1(\zeta-3)-\beta_2\zeta R^2\right)$$
$$+\alpha\beta(\zeta-1)Y_2\left(\beta_1(26-6\zeta)+3\beta_2\zeta R^2\right)$$

$$\psi_{143} = 4\alpha^2(\zeta-1)\left(\beta_1(3\zeta-1)+\beta_2\zeta R^2\right)+\beta^2\zeta\left(6\beta_1(\zeta+1)+\beta_2(2\zeta+1)R^2\right)$$
$$+2\alpha\beta(\zeta-1)Y_2\left(\beta_1+9\beta_1\zeta+3\beta_2\zeta R^2\right)$$

$$Z = \left(\zeta\beta 2Z_1 + \xi^3R^4\beta Z_2 + \xi^2R^2Z_3 + \xi Z_4\right)\left(\log\left[\frac{\alpha}{\alpha+Y2}\right]\right), \quad Z_1 = 2\alpha\beta\zeta + \alpha^2(\zeta-1)Y_2 + \beta^2\zeta Y_2$$

$$Z_2 = \alpha\left(2\beta_2\zeta R^2 - 4\beta_1(\zeta-4)\right) + \beta Y_2\left(\beta_2\zeta R^2 - 2\beta_1(\zeta-5)\right)$$

$$Z_3 = \beta^2Y_2\left(\beta_2\zeta(\zeta+2)R^2 - 2\beta_1\left(\zeta^2-8\zeta-1\right)\right) + 2\alpha\beta\zeta\left(\beta_2(\zeta+2)R^2 - 2\beta_1(\zeta-7)\right)$$
$$+\alpha^2(\zeta-1)Y_2\left(\beta_2\zeta R^2 - 2\beta_1(\zeta-3)\right)$$

$$Z_4 = 2\alpha\beta\zeta\left(6\beta_1\zeta + 2\beta_2\zeta R^2 + \beta_2R^2\right)$$
$$+2\alpha^2(\zeta-1)Y_2\left(\beta 1(3\zeta-1)+\beta 2\zeta R^2\right) + \beta^2\zeta Y_2\left(\beta_1(6\zeta+2)+\beta_2(2\zeta+1)R^2\right)$$

$$\psi_{15} = 4(1-\zeta)Y_2Y_3, \quad \psi_{16} = \left(1-\frac{\psi_{17}}{\psi_{18}}\right)$$

$$\psi_{17} = 2(-1+\zeta)\alpha\frac{1}{2}\left(1+\frac{Y_2\beta}{2}-\frac{\alpha}{\alpha+Y_2\beta}\right)$$
$$+2(-1+\zeta)\log\left[\frac{\alpha}{\alpha+Y_2\beta}\right]\left(\zeta\beta_2\psi_{171}+\xi^3R^4\beta\psi_{172}+\xi^2R^2\psi_{173}+\xi\psi_{174}\right)$$

$$\psi_{171} = 2\alpha\beta\zeta + \alpha^2(\zeta-1)Y_2 + \beta^2\zeta Y_2, \quad \psi_{172} = \alpha(2\beta_2\zeta R^2 - 4\beta_1(\zeta-4)) + \beta Y_2(\beta_2\zeta R^2 - 2\beta_1(\zeta-5))$$

$$\psi_{173} = \beta^2Y_2\left(\beta_2\zeta(\zeta+2)R^2 - 2\beta_1\left(\zeta^2-8\zeta-1\right)\right) + 2\alpha\beta\zeta\left(\beta_2(\zeta+2)R^2 - 2\beta_1(\zeta-7)\right)$$
$$+\alpha^2(\zeta-1)Y_2\left(\beta_2\zeta R^2 - 2\beta_1(\zeta-3)\right)$$

$$\psi_{174} = 2\alpha\beta\zeta\left(6\beta_1\zeta + 2\beta_2\zeta R^2 + \beta_2R^2\right) + 2\alpha^2(\zeta-1)Y_2\left(\beta_1(3\zeta-1)+\beta_2\zeta R^2\right)$$
$$+\beta^2\zeta Y_2\left(\beta_1(6\zeta+2)+\beta_2(2\zeta+1)R^2\right)$$

$$\psi_{18} = (\zeta+\xi R^2)\beta\left(\zeta\beta_2\psi_{181}+\xi^3R^4\beta^2\psi_{182}+\xi^2R^2\psi_{183}+\xi\psi_{184}\right)$$
$$+2(-1+\zeta)\alpha\left(\zeta\beta_2\psi_{185}+\xi^3R^3\beta\psi_{186}+\xi^2R^2\psi 187+\xi\psi_{188}\log\left[\frac{\alpha}{\alpha+Y2\beta}\right]\right)$$

$$\psi_{181} = 2\alpha^2(\zeta-1)+\beta^2\zeta+3\alpha\beta(\zeta-1)Y_2, \quad \psi_{182} = \beta_2\zeta R^2 - 2\beta_1(\zeta-7)$$

$$\psi_{183} = \beta^2\left(\beta_1\left(-2\zeta^2+20\zeta+6\right)+\beta_2\zeta(\zeta+2)R^2\right) - 2\alpha^2(\zeta-1)\left(2\beta_1(\zeta-3)-\beta_2\zeta R^2\right)$$

$$+\alpha\beta(\zeta-1)Y_2\left(\beta 1(26-6\zeta)+3\beta_2\zeta R^2\right)$$

$$\psi_{184} = 4\alpha^2(\zeta-1)\left(\beta_1(3\zeta-1)+\beta_2\zeta R^2\right)+\beta^2\zeta\left(6\beta_1(\zeta+1)+\beta_2(2\zeta+1)R^2\right)$$
$$+2\alpha\beta(\zeta-1)Y_2\left(\beta_1+9\beta_1\zeta+3\beta_2\zeta R^2\right)$$

$$\psi_{185} = 2\alpha\beta\zeta+\alpha^2(\zeta-1)Y_2+\beta^2\zeta Y_2, \quad \psi_{186} = \alpha\left(2\beta_2\zeta R^2-4\beta_1(\zeta-4)\right)+\beta Y_2\left(\beta_2\zeta R^2-2\beta_1(\zeta-5)\right)$$

$$\psi_{187} = \beta^2 Y_2\left(\beta_2\zeta(\zeta+2)R^2-2\beta_1\left(\zeta^2-8\zeta-1\right)\right)+2\alpha\beta\zeta\left(\beta_2(\zeta+2)R^2-2\beta_1(\zeta-7)\right)$$
$$+\alpha^2(\zeta-1)Y_2\left(\beta_2\zeta R^2-2\beta_1(\zeta-3)\right)$$

$$\psi_{188} = 2\alpha\beta\zeta\left(6\beta_1\zeta+2\beta_2\zeta R^2+\beta_2R^2\right)$$
$$+2\alpha^2(\zeta-1)Y_2\left(\beta_1(3\zeta-1+\beta_2\zeta R^2\right)+\beta^2\zeta Y_2\left(\beta_1(6\zeta+2)+\beta_2(2\zeta+1)R^2\right)$$

$$e^v(r) = \left((1-Y_1^2)^{\frac{1}{4}}\left(\frac{\alpha+\beta\times Y_1}{Y1}\left(\frac{\alpha}{\beta^3}A_1\left(\frac{\sec v\times\sec v-\cos v\times\cos v}{2}\right)+\log\left[cosv\times\cos v\right]\right)+B_1\right)\right)^2$$

SUPLEMENTO. ECUACIONES COMPLEMENTARIAS.

$$E^a = dx^a - \frac{i}{2}d\theta^\alpha(\Gamma^a)_{\alpha\beta}\theta^\beta;\ E^\alpha = d\theta^\alpha$$

$$D_\alpha = \partial_\alpha + \frac{i}{2}(\Gamma^a\theta)_\alpha\partial_a$$

$$\nabla_\alpha\Lambda^\beta = -\frac{i}{4}\left(\Gamma^{ab}\right)_\alpha{}^\beta F_{ab}$$

$$\Gamma^a\nabla_a\Lambda = 0$$
$$\nabla^b F_{ab} = 2i\Lambda\Gamma_a\Lambda,$$

$$J_{abc} = \mathrm{tr}(\Lambda\Gamma_{abc}\Lambda)$$

$$I = \int d^Dx\varepsilon^{m_1\ldots m_D}L_{m_1\ldots m_D}(x,\theta=0)$$

$$K_{8,2} \sim i\Gamma_{5,2}J_{3,0} + 3i\Gamma_{1,2}\star J_{3,0}$$

$$P_{2,2} \sim it_0J_{3,0} + \frac{1}{6}\Gamma_{2abc,2}J^{abc}$$

$$K_{6,4} = i\Gamma_{1,2}J_{3,0}P_{2,2} + \frac{i}{2}\Gamma_{3ab,2}J^{ab}_{1,0}P_{2,2}$$

$$Z'_{4,6} \sim \Gamma_{4a,2}N^a_{,4}$$

$$N^a_{,4} = \hat{\Xi}^a_{,1}\hat{Q}_{0,3} + \frac{7}{96}\Gamma^b_{,2}\Gamma_b^{\alpha\beta}\hat{\Xi}^a_\alpha\hat{Q}_{0,2\beta} + \frac{1}{96}\Gamma_{b,2}\Gamma^{a,\alpha\beta}\hat{\Xi}^b_\alpha\hat{Q}_{0,2\beta} + \frac{1}{16}[\Gamma^a\Gamma_b]^\alpha_{,1}\hat{\Xi}^b_{,1}\hat{Q}_{0,2\alpha}$$
$$+\frac{1}{192}[\Gamma^a\Gamma_b]^\alpha_{,1}\Gamma^c_{,2}\Gamma_{c,}^{\beta\gamma}\hat{\Xi}^b_\gamma\hat{Q}_{0,1\alpha\beta}$$

$$\hat{\Xi}^a \equiv \frac{i}{10}\mathrm{tr}\left([-8F^{ab}\Gamma_b + F_{bc}\Gamma^{abc}]\Lambda\right),$$

$$\hat{Q}_{\alpha\beta\gamma} \equiv Q_{\alpha\beta\gamma} - \frac{3}{20}\Gamma^{a}_{(\alpha\beta}\Gamma_{a}^{\delta\eta}Q_{\gamma)\delta\eta}.$$

$$d_1 Z'_{4,6}(N) \sim \Gamma_{4a,2}O^{a}{}_{,5}(N)$$

$$d_0 Z'_{4,6}(N) + d_1 Z'_{5,5}(N) \sim \Gamma_{5,2}O_{0,4}(N)$$

$$[O_{0,4}] \sim [D^2]_{abc}[\Gamma^{ab}]_1{}^{\alpha}N^{c}{}_{,3\alpha} + c\partial_a N^{a}{}_{,4}$$

$$[d_1\partial_a N^{a}{}_{,4}] \sim \Gamma^{5}{}_{,2}\hat{P}_{4,0}d_0\hat{Q}_{0,3}$$

$$\hat{P}_{abcd} \equiv \mathrm{tr}F_{[ab}F_{cd]} - \frac{i}{2}\partial_{[a}J_{bcd]}$$

$$D_\delta\hat{Q}_{\alpha\beta\gamma} = \frac{15i}{8}\Gamma^{a}_{\delta(\alpha}\hat{Q}'_{a,\beta\gamma)} - \frac{3i}{8}\Gamma^{a}_{(\alpha\beta}\hat{Q}'_{a,\gamma)\delta} + \frac{1}{4}\Gamma^{abc}_{\delta(\alpha}\hat{S}_{abc,\beta\gamma)};$$

$$\begin{aligned}\hat{Q}'_{1,2} \equiv \frac{2}{5}\Big(Q_{1,2} + \frac{1}{96}\Gamma_{1,2}\Gamma^{a,\alpha\beta}Q_{a,\alpha\beta} - \frac{1}{96}\Gamma^{a}_{,2}\Gamma^{\alpha\beta}_{1,}Q_{a,\alpha\beta} - \frac{5}{96}\Gamma_{a,2}\Gamma^{a,\alpha\beta}Q_{1,\alpha\beta}\\ - \frac{1}{12}[\Gamma_1\Gamma^{a}]_{,1}{}^{\alpha}Q_{a,1\alpha}\Big) - \frac{i}{20}\Big(\Gamma^{ab}_{,1}{}^{\alpha}_{,1}S_{1ab,1\alpha} + \frac{1}{6}\Gamma^{b}_{,2}\Gamma^{a,\alpha\beta}S_{1ab,\alpha\beta}\\ - \frac{1}{12}[\Gamma_1\Gamma^{abc}]_{,1}{}^{\alpha}S_{abc,1\alpha} + \frac{1}{12}\Gamma^{ab}_{,1}{}^{\alpha}_{,1}[\Gamma_1\Gamma^{c}]_{,1}{}^{\beta}S_{abc,\alpha\beta}\Big)\end{aligned}$$

$$\begin{aligned}[O_{0,4}] \sim{}& c_1\hat{\Xi}^{a}_{,1}\partial_a\hat{Q}_{0,3} + c_2\partial_a\hat{\Xi}^{a}_{,1}\hat{Q}_{0,3} + c_3[\Gamma^{ab}]_1{}^{\alpha}\partial_a\hat{\Xi}_{b,1}\hat{Q}_{0,2\alpha} + c_4[\Gamma^{ab}]_1{}^{\alpha}\partial^{c}(d_1J_{abc})\hat{Q}_{0,2\alpha}\\ &+\Gamma^{abcde}_{,2}\big(c_5\hat{P}_{abcd}\hat{Q}'_{e,2} + c_6\partial^{f}J_{abf}\hat{S}_{cde,2} + c_7\partial^{f}J_{abc}\hat{S}_{def,2} + c_8\hat{P}_{abc}{}^{f}\hat{S}_{def,2} + c_9\hat{\Xi}_{a,1}\hat{S}_{bcde,1}\\ &+c_{10}[\Gamma^{fg}]_1{}^{\alpha}\hat{\Xi}_{a\alpha}\hat{S}_{bcde,fg,1}\big) + c_{11}[\Gamma^{ab}]_1{}^{\alpha}\hat{\Xi}_{a,1}\partial_b\hat{Q}_{0,2\alpha} + c_{12}[\Gamma^{a}]_{1\alpha}\Xi^{\alpha}\partial_a\hat{Q}_{0,3}\\ &\sim -d_1M_{0,3}\end{aligned}$$

$$\begin{aligned}M_{0,3} \sim{}& c'_1TQ_{0,3} + c'_2[\Gamma^{ab}]_1{}^{\alpha}\partial^{c}J_{abc}\hat{Q}_{0,2\alpha} + c'_3[\Gamma^{abcd}]_1{}^{\alpha}\hat{P}_{abcd}\hat{Q}_{0,2\alpha} + c'_4\hat{\Xi}^{a}{}_{,1}Q_{a,2}\\ &+c'_5\hat{\Xi}^{a}{}_{,1}\hat{Q}'_{a,2} + c'_6\Gamma^{a}{}_{,1\alpha}\Xi^{\alpha}\hat{Q}'_{a,2} + c'_7[\Gamma^{ab}]_1{}^{\alpha}\hat{\Xi}^{c}_{\alpha}\hat{S}_{abc,2} + c'_8[\Gamma^{abc}]_{1\alpha}\Xi^{\alpha}\hat{S}_{abc,2}\end{aligned}$$

$$Z'_{5,5} = Z'_{5,5}(N) + \Gamma_{5,2}M_{0,3}$$

$$\Gamma_{4a,2}N^{a}_{,4} = \Gamma_{4a,2}\hat{\Xi}^{a}_{,1}\hat{Q}_{0,3} + t_0V_{5,4}$$

$$\begin{aligned}V_{5,4} \equiv \frac{7i}{96}\Gamma_{4a,2}\Gamma_{1,}{}^{\alpha\beta}\hat{\Xi}^{a}_{\alpha}\hat{Q}_{0,2\beta} + \frac{i}{96}\Gamma_{4a,2}\Gamma^{a,\alpha\beta}\hat{\Xi}_{1,\alpha}\hat{Q}_{0,2\beta} - \frac{i}{16}[\Gamma_5\Gamma_a]_{,1}{}^{\alpha}\hat{\Xi}^{a}_{,1}\hat{Q}_{0,2\alpha}\\ - \frac{i}{192}\Gamma_{4a,2}[\Gamma^{a}\Gamma_b]_{,}{}^{\alpha}\Gamma_{1,}{}^{\beta\gamma}\hat{\Xi}^{b}_{\gamma}\hat{Q}_{0,1\alpha\beta}\end{aligned}$$

$$\begin{aligned}d_1(\Gamma_{4a,2}N^{a}_{,4}) = -t_0\Big(\Gamma_{4a,2}\hat{\Xi}^{a}_{,1}\hat{Q}'_{1,2} - i\Gamma_{4a,2}\Big(\hat{T}^{a}_1 + \frac{i}{5}\partial_bJ^{ab}_1\Big)\hat{Q}_{0,3}\\ - \frac{6i}{5}\Gamma_{1,2}\hat{P}_4\hat{Q}_{0,3} + \frac{3i}{5}\Gamma^{ab}_{3}{}_{,2}\hat{P}_{2ab}\hat{Q}_{0,3} + d_1V_{5,4}\Big)\end{aligned}$$

$$d_0(\Gamma_{4a,2}N^a_{,4}) + d_1(\Gamma_{4a,2}\hat{\Xi}^a_{,1}\hat{Q}'_{1,2} + \cdots + d_1V_{5,4})$$
$$+t_0\left(-i\left[\Gamma_{4a,2}\left(\hat{T}^a_1 + \frac{i}{5}\partial_b J^{ab}_1\right) + \frac{6}{5}\Gamma_{1,2}\hat{P}_4 - \frac{3}{5}\Gamma^{ab}_3{}_{,2}\hat{P}_{2ab}\right]\hat{Q}'_{1,2} + d_0V_{5,4}\right)$$
$$=\Gamma_{4a,2}(\hat{\Xi}^a_{,1}(d_0\hat{Q}_{0,3} + d_1\hat{Q}'_{1,2}) - d_0\hat{\Xi}^a_{,1}\hat{Q}_{0,3})$$
$$+id_1\left[\Gamma_{4a,2}\left(\hat{T}^a_1 + \frac{i}{5}\partial_b J^{ab}_1\right) + \frac{6}{5}\Gamma_{1,2}\hat{P}_4 - \frac{3}{5}\Gamma^{ab}_3{}_{,2}\hat{P}_{2ab}\right]\hat{Q}_{0,3}$$

$$d_1[\Gamma^{ab}]_{,1}{}^{\alpha}\hat{\Xi}^c_\alpha\hat{S}_{abc,2} \sim \Gamma^{abcde}{}_{,2}\Gamma^{fg}{}_{,1}{}^{\alpha}\hat{\Xi}_{a,\alpha}\hat{S}_{bcde,fg,1} + \cdots$$

$$d_1M_{0,3} \sim -[D^2]_{abc}[\Gamma^{ab}]_1{}^{\alpha}N^c{}_{,3\alpha} - c\partial_a N^a{}_{,4}$$

$$L_{9,1} \sim \Gamma_{9,1\alpha}[D^4]^{\alpha\beta\gamma\delta}M_{\beta\gamma\delta} + [D^5]^{\alpha\beta\gamma}N_{9,1\alpha\beta\gamma}$$

$$Z_{1,9} \sim \Gamma_{1,2}Q_{0,7}$$

$$Z'_{2,8} \sim \frac{1}{6}\Gamma_{2abc,2}\hat{S}^{abc}_{,6}$$

$$M_{0,3} \sim \iota_\theta\Gamma_{a,2}Y^a_{,2}$$

$$Y_{\alpha\beta\gamma} = \left(\Gamma^{bcdef}\right)_{\beta\gamma}J_{abc}J_{def}$$

$$\int d^{10}x[D^5]^{\alpha\beta\gamma}M_{\alpha\beta\gamma} = \int d^{10}x[D^4]^{a\alpha\beta}[DM]_{a\alpha\beta}$$

$$I = \int d^{10}x[D^4]^{a\alpha\beta}Y_{a\alpha\beta}$$

$$\mathcal{L}_a P = \iota_a dP + d\iota_a P = d\iota_a P,$$

$$\hat{W}_{11} := H_3\mathcal{L}_a P_4\mathcal{L}^a P_4 = H_3(d\iota_a P_4)\mathcal{L}^a P_4$$
$$= d(H_3\iota_a P_4\mathcal{L}^a P_4) := d\hat{V}_{10}$$

$$\hat{V}_{4,6} = \Gamma_{1,2}\iota_a P_{2,2}\mathcal{L}^a P_{2,2} = -t_0 N_{5,4},$$

$$N_{5,4} = \iota_a P_{2,2}X^a_{4,2}$$

$$\Gamma_{1,2}\mathcal{L}^a P_{2,2} = t_0 X^a_{4,2}$$

$$M^{\text{inv}}_3 \sim \Gamma^{abcde}{}_{,2}\partial_a\hat{\Xi}_{b,1}J_{cde}$$

$$I = \int d^{10}x d^{16}\theta(\Gamma^a)^{\alpha\beta}Q_{a\alpha\beta}$$

$$\underline{A} := A + E^r W_r,$$

$$\underline{F} = (F + G^r W_r) + E^r DW_r - \frac{1}{2}E^s E^r[W_r, W_s]$$

$$\underline{\omega_p} = \omega_p + E^r \omega_{p,r} + \cdots \frac{1}{n!} E^{r_n} \ldots E^{r_1} \omega_{p-n,r_1 \ldots r_n}$$

$$\underline{W_{11}} = W_{11} + E^9 W_{10} = (H_3 + E^9 H_2)\underline{P_8}$$

$$W_{10} = H_2(P_8 + G^9 X_6) - H_3 dX_6 = H_2 P_8 - d(H_3 X_6)$$

$$Q_3(\underline{A}) = Q_3(A + E^9 W) = Q_3(A) + E^9 \mathrm{tr}(2WF(A) + G^9 W^2) + d\big(\mathrm{tr}(AE^9 W)\big)$$

$$F_{\alpha\beta} = i(\Gamma^9)_{\alpha\beta} W$$

$$(\Gamma^{\underline{a}})^{\alpha\beta} Q_{\underline{a}\alpha\beta}(\underline{A}) = (\Gamma^a)^{\alpha\beta} Q_{a\alpha\beta}(A) + (\Gamma^9)^{\alpha\beta} Q_{9\alpha\beta},$$

$$I = \int d^{10}x d^{16}\theta (\Gamma^a)^{\alpha\beta} Q_{a\alpha\beta} \to \aleph \int d^9 x d^{16}\theta \mathrm{tr} W^2$$

	$\mathrm{tr}\, F^2$	$\mathrm{tr}\, F^4$, $\mathrm{tr}^2\, F^4$	$d^2 \mathrm{tr}^2\, F^4$	$d^2 \mathrm{tr}\, F^4$
$D=10$	$\int L^{CS}_{D,0}$	$\int L^{CS}_{D,0}$	$\int L^{\mathrm{inv}}_{D,0}$	$\int d^{10}x\, d^{16}\theta\, Q$
$D=9$	"	"	"	$\int d^D x\, d^{16}\theta\, K$
$D=7,8$	"	$\frac{1}{2}$ BPS	"	"
$D=5,6$	"	"	$\frac{1}{4}$ BPS	"
$D=4$	$\frac{1}{2}$ BPS	"	"	"

$$K = -\frac{2}{g_5^2} \int \sqrt{g} |d^2 z| \mathrm{Tr}\big(g^{z\bar{z}} A_{\bar{z}} A_z + h^{i\bar{j}} \Phi_i \bar{\Phi}_{\bar{j}}\big)$$

$$W = -\frac{\sqrt{2} i}{g_5^2} \int dz \wedge d\bar{z} \epsilon^{ij} \mathrm{Tr}\big(\Phi_i D_{\bar{z}} \Phi_j\big)$$

$$g^{z\bar{z}} F_{z\bar{z}} + h^{i\bar{j}} [\Phi_i, \Phi_{\bar{j}}^\dagger] = 0, D_{\bar{z}} \Phi_i = 0, \epsilon^{ij} [\Phi_i, \Phi_j] = 0$$
$$D_{\bar{z}} \sigma = 0, [\sigma, \Phi_i] = 0$$

$$W \supset -\sqrt{2} \left(\sum_{a \in A} \mathrm{Tr}\left(\mu_{a,H}^{(3d)} \Phi_1(z_a)\right) - \sum_{b \in B} \mathrm{Tr}\left(\mu_{b,H}^{(3d)} \Phi_2(z_b)\right) \right),$$

$$\Phi_2 \to \frac{c_1 \mu_{a,H}^{(3d)}}{z - z_a} \ (z \to z_a), \Phi_1 \to \frac{c_1 \mu_{b,H}^{(3d)}}{z - z_b} \ (z \to z_b),$$

$$\det(x - c_2 \Phi_1) \to \det\left(x - \mu_{a,C}^{(3d)}\right), \Phi_2 \to \frac{c_3 m_a}{z - z_a} \ (z \to z_a)$$

$$W|_{\text{solution}} = c_4 \sum_{a\in A} \oint_{|z-z_b|=\epsilon} dz \mathrm{Tr}(\Phi_1\Phi_2) = -c_4 \sum_{b\in B} \oint_{|z-z_a|=\epsilon} dz \mathrm{Tr}(\Phi_1\Phi_2)$$

$$0 = \frac{1}{N!}\left(x_{i_1} - \Phi_{i_1}\right)^{\alpha_1}_{\beta_1} \cdots \left(x_{i_N} - \Phi_{i_N}\right)^{\alpha_N}_{\beta_N} \epsilon_{\alpha_1\cdots\alpha_N}\epsilon^{\beta_1\cdots\beta_N}.$$

$$z = \frac{x^4 + ix^5}{\sqrt{2}}, u^1 = \frac{x^6 + ix^7}{\sqrt{2}}, u^2 = \frac{x^8 + ix^9}{\sqrt{2}}.$$

$$(\Gamma_I)^T = -C\Gamma_I C^\dagger, C^T = -C$$

$$(\Gamma_{01\cdots9})\epsilon = \epsilon$$
$$\epsilon^\dagger\Gamma_0 = \epsilon^T C$$

$$I_{10} = \frac{1}{e_{10}^2}\int d^{10}x \mathrm{Tr}\left(\frac{1}{2}F_{IJ}F^{IJ} - i\lambda^T C\Gamma^I D_I\lambda\right)$$

$$\delta A_I = i\epsilon^T C\Gamma_I\lambda, \delta\lambda = \frac{1}{2}\Gamma^{IJ}F_{IJ}\epsilon$$

$$\mathbf{16} \to \sum_{\pm}\left[(\mathbf{2},\mathbf{1})^{\pm\frac{1}{2},+1} \oplus (\mathbf{2},\mathbf{2})^{\pm\frac{1}{2},0} \oplus (\mathbf{2},\mathbf{1})^{\pm\frac{1}{2},-1}\right]$$

$$\omega + \mathrm{tr}\omega_u = 0$$

$$\mathbf{2}_\ell \equiv (\mathbf{2},\mathbf{1})^{+\frac{1}{2},+1}, \mathbf{2}_r \equiv (\mathbf{2},\mathbf{1})^{-\frac{1}{2},-1}$$

$$\Gamma_{z\bar{z}}\epsilon_\ell = \Gamma_{u^1\bar{u}^1}\epsilon_\ell = \Gamma_{u^2\bar{u}^2}\epsilon_\ell = \epsilon_\ell, \quad \Gamma_z\epsilon_\ell = \Gamma_u\epsilon_\ell = 0$$
$$\Gamma_{z\bar{z}}\epsilon_r = \Gamma_{u^1\bar{u}^1}\epsilon_r = \Gamma_{u^2\bar{u}^2}\epsilon_r = -\epsilon_r, \quad \Gamma_{\bar{z}}\epsilon_r = \Gamma_{\bar{u}}\epsilon_r = 0$$

$$z^1 = u^1, z^2 = u^2, z^3 = z$$

$$\Omega = \frac{1}{3!}\Omega_{ijk}dz^i \wedge dz^j \wedge dx^k = du^1 \wedge du^2 \wedge dz$$

$$\lambda = -\lambda_\ell - \lambda_r + \frac{1}{4}\Omega^{\bar{i}\bar{j}\bar{k}}\Gamma_{\bar{i}\bar{j}}\psi_{\ell,\bar{k}} + \frac{1}{4}\bar{\Omega}^{ijk}\Gamma_{ij}\psi_{r,k}$$

$$\lambda_\ell^\dagger\Gamma_0 = \lambda_r^T C, \psi_{\ell,i}^\dagger\Gamma_0 = \psi_{r,i}^T C$$

$$\begin{aligned}\delta_\ell\lambda &= \left(\frac{1}{2}F_{\mu\nu}\Gamma^{\mu\nu} + F_{i\bar{\imath}}\Gamma^{i\bar{\imath}} + \frac{1}{2}F_{ij}\Gamma^{ij} + F_{\mu i}\Gamma^{\mu i}\right)\epsilon_\ell \\ &= \left(\frac{1}{2}F_{\mu\nu}\Gamma^{\mu\nu} - F_{i\bar{\imath}} + \frac{1}{2}F_{ij}\Gamma_{\bar{\imath}\bar{\jmath}} - \frac{1}{8}\bar{\Omega}^{ijk}F_{\mu i}\Gamma_{jk}\Gamma^\mu\Gamma_{\overline{123}}\right)\epsilon_\ell,\end{aligned}$$

$$\delta_\ell \lambda_\ell \ = \left(-\frac{1}{2}F_{\mu\nu}\Gamma^{\mu\nu} + F_{i\bar{\imath}}\right)\epsilon_\ell$$
$$\delta_\ell \lambda_r \ = 0$$
$$\delta_\ell \psi_{\ell,\bar{\imath}} \ = \bar{\Omega}_{\bar{\imath}\bar{\jmath}\bar{k}} F_{jk}\epsilon_\ell$$
$$\delta_\ell \psi_{r,i} \ = -\frac{1}{2}F_{\mu i}\Gamma^\mu \Gamma_{\overline{123}}\epsilon_\ell$$

$$\delta_\ell A_I = i\epsilon_\ell^T C\Gamma_I \left(-\lambda_r + \frac{1}{4}\Omega^{\overline{ij}\bar{k}}\Gamma_{\bar{\imath}\bar{\jmath}}\psi_{\ell,\bar{k}}\right),$$

$$\delta_\ell A_\mu \ = -i\epsilon_\ell^T C\Gamma_\mu \lambda_r$$
$$\delta_\ell A_{\bar{\imath}} \ = \frac{i}{2}\epsilon_\ell^T C\Gamma_{\overline{123}}\psi_{\ell,\bar{\imath}}$$
$$\delta_\ell A_i \ = 0$$

$$\left(\Gamma_{123} + \Gamma_{\overline{123}}\right)^2 \epsilon_{\ell,r} = -8\epsilon_{\ell,r}.$$

$$\left(\Gamma_{123} + \Gamma_{\overline{123}}\right)^2 \cong -8$$

$$\gamma_\mu = \frac{1}{2\sqrt{2}}\left(\Gamma_{123} + \Gamma_{\overline{123}}\right)\Gamma_\mu, C_4 = \frac{i}{2\sqrt{2}}C\left(\Gamma_{123} + \Gamma_{\overline{123}}\right).$$

$$\{\gamma_\mu, \gamma_\nu\} \cong 2g_{\mu\nu}, \qquad \gamma_{\mu\nu} \cong \Gamma_{\mu\nu}$$
$$(\gamma_I)^T \cong -C_4\gamma_I C_4^\dagger, \quad C^T = -C$$

$$\lambda_\ell^\dagger(-i\gamma_0) = \lambda_r^T C_4, \psi_{\ell,\bar{\imath}}^\dagger(-i\gamma_0) = \psi_{r,i}^T C_4$$

$$D \ = -iF_{i\bar{\imath}}$$
$$F_{\bar{\imath}} \ = \frac{1}{\sqrt{2}}\bar{\Omega}_{\bar{\imath}\bar{\jmath}\bar{k}} F_{jk}$$

$$\delta_\ell A_\mu \ = \epsilon_\ell^T C_4 \gamma_\mu \lambda_r$$
$$\delta_\ell \lambda_\ell \ = \left(-\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu} + iD\right)\epsilon_\ell$$
$$\delta_\ell \lambda_r \ = 0$$

$$\delta_\ell A_{\bar{\imath}} \ = \sqrt{2}\epsilon_\ell^T C_4 \psi_{\ell,\bar{\imath}},$$
$$\delta_\ell A_i \ = 0,$$
$$\delta_\ell \psi_{\ell,\bar{\imath}} \ = \sqrt{2}F_{\bar{\imath}}\epsilon_\ell,$$
$$\delta_\ell \psi_{r,i} \ = \sqrt{2}D_\mu A_i \gamma^\mu \epsilon_\ell,$$

$$\Phi_1 = A_{\bar{u}^1}, \quad \psi_1 = \psi_{\ell,\bar{u}^1}, \quad F_1 = F_{\bar{u}^1}$$
$$\Phi_2 = A_{\bar{u}^2}, \quad \psi_2 = \psi_{\ell,\bar{u}^2}, \quad F_2 = F_{\bar{u}^2}.$$

$$D \ = -i\left(g^{z\bar{z}}F_{z\bar{z}} - h^{i\bar{\jmath}}\left[\Phi_i, \bar{\Phi}_{\bar{\jmath}}\right]\right),$$
$$F_{\bar{z}} \ = \frac{1}{\sqrt{2}}\left[\bar{\Phi}_{\bar{\imath}}, \bar{\Phi}_{\bar{\jmath}}\right]\epsilon^{\bar{\imath}\bar{\jmath}},$$
$$g_{z\bar{z}}h^{i\bar{\jmath}}F_i \ = -\sqrt{2}\epsilon^{\bar{\jmath}\bar{k}}D_z\bar{\Phi}_{\bar{k}},$$

$$K_{3d} = -\frac{2}{g_5^2}\int \sqrt{g}|d^2z|\mathrm{Tr}\left(g^{z\bar{z}}A_{\bar{z}}A_z + h^{i\bar{j}}\Phi_i\bar{\Phi}_{\bar{j}}\right)$$
$$W_{3d} = -\frac{2}{g_5^2}\int |d^2z|\frac{1}{\sqrt{2}}\epsilon^{ij}\mathrm{Tr}\left(\Phi_i D_{\bar{z}}\Phi_j\right)$$

$$\delta_\alpha A_{\bar{z}} = -D_{\bar{z}}\alpha, \delta_\alpha\Phi_i = [\alpha, \Phi_i]$$

$$\omega = -\int \sqrt{g}|d^2z|\mathrm{Tr}\left(ig^{z\bar{z}}\delta A_{\bar{z}} \wedge \delta A_z + ih^{i\bar{j}}\delta\Phi_i \wedge \delta\bar{\Phi}_{\bar{j}}\right)$$

$$\iota_{V(\alpha)}\omega = -\int \sqrt{g}|d^2z|\mathrm{Tr}\left(-ig^{z\bar{z}}\left((D_{\bar{z}}\alpha)\delta A_z - (D_z\alpha)\delta A_{\bar{z}}\right) + ih^{i\bar{j}}\left([\alpha,\Phi_i]\delta\bar{\Phi}_{\bar{j}} - [\alpha,\bar{\Phi}_{\bar{i}}]\delta\Phi_j\right)\right)$$

$$\mu(\alpha) = i\int \sqrt{g}|d^2z|\mathrm{Tr}\alpha\left(g^{z\bar{z}}F_{z\bar{z}} - h^{i\bar{j}}[\Phi_i, \bar{\Phi}_j]\right)$$
$$\equiv -\int \sqrt{g}|d^2z|\mathrm{Tr}\alpha\mu$$

$$-\int d^2\theta \int \sqrt{g}|d^2z|\frac{1}{2g_5^2}\mathrm{Tr}(W^\alpha W_\alpha) \equiv \frac{1}{4g_5^2}\int d^2\theta[W^\alpha W_\alpha]$$

$$\mathcal{L}_{3d} = \int d^2\theta d^2\bar{\theta}K_{3d} + \int d^2\theta W_{3d} + \int d^2\theta \frac{1}{4g_5^2}[W^\alpha W_\alpha] + \text{ h.c.}$$

$$\Phi_i^{(6\text{ d})} = R^{-1}\Phi_i$$

$$\frac{1}{g_5^2} = \frac{1}{8\pi^2 R}$$

$$W_{4d} \equiv \frac{1}{2\pi R}W_{3d}$$
$$= \frac{1}{\sqrt{2}(2\pi)^3 i}\int dz \wedge d\bar{z}\epsilon^{ij}\mathrm{Tr}\left(\Phi_i^{(6\text{ d})}D_{\bar{z}}\Phi_j^{(6\text{ d})}\right)$$

$$0 = g^{z\bar{z}}F_{z\bar{z}} - h^{i\bar{j}}[\Phi_i, \bar{\Phi}_{\bar{j}}]$$
$$0 = D_{\bar{z}}\Phi_i$$
$$0 = \epsilon^{ij}[\Phi_i, \Phi_j]$$
$$0 = D_{\bar{z}}\sigma$$
$$0 = [\sigma, \Phi_i]$$

$$0 = \det(x - \Phi(z)) = x^N + \sum_{k=2}^{N}\phi_k(z)x^{N-k}$$

$$B_H = \bigoplus_{k=2}^{N} H^0(C, K^k),$$

$$\pi: (A_{\bar{z}}, \Phi) \mapsto \{\phi_k\}_{2\le k\le N}$$

$$0 = P_{i_1\cdots i_N} \equiv \frac{1}{N!}\left(x_{i_1} - \Phi_{i_1}\right)_{\beta_1}^{\alpha_1}\cdots\left(x_{i_N} - \Phi_{i_N}\right)_{\beta_N}^{\alpha_N}\epsilon_{\alpha_1\cdots\alpha_N}\epsilon^{\beta_1\cdots\beta_N},$$

$$\Phi_i \to \mathrm{diag}\big(\lambda_{i,1}(z), \cdots, \lambda_{i,N}(z)\big)$$

$$\begin{aligned}\Sigma &= \{(z, x_1, x_2) \in F : P_{i_1\cdots i_N}(z, x_1, x_2) = 0\} \\ &= \{(z, x_1, x_2) \in F : (x_1, x_2) = \big(\lambda_{1,k}(z), \lambda_{2,k}(z)\big), k = 1, \cdots, N\}\end{aligned}$$

$$\phi^{(k)}_{i_1\cdots i_k} = \frac{(-1)^k}{k!\,(N-k)!}\left(\Phi_{i_1}\right)^{\alpha_1}_{\beta_1} \cdots \left(\Phi_{i_k}\right)^{\alpha_k}_{\beta_k} \epsilon_{\alpha_1\cdots\alpha_k\alpha_{k+1}\cdots\alpha_N} \epsilon^{\beta_1\cdots\beta_k\alpha_{k+1}\cdots\alpha_N}.$$

$$0 = (x^N)_{i_1\cdots i_N} + \sum_{k=2}^{N} \phi^{(k)}_{(i_1\cdots i_k}\left(x^{N-k}\right)_{i_{k+1}\cdots i_N)},$$

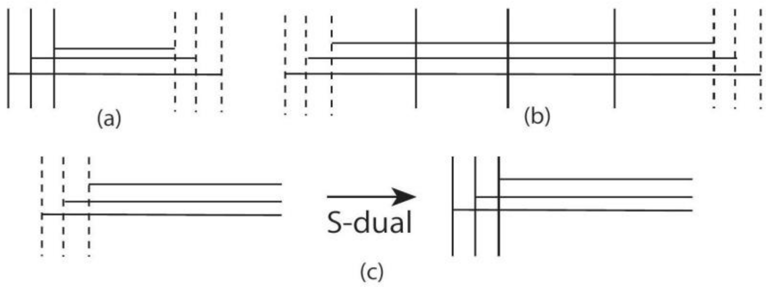
$$\pi : (A_{\bar z}, \Phi_i) \mapsto \left\{\phi^{(k)}_{i_1\cdots i_k}\right\}_{2\le k\le N}$$

$$B_{GH} = \pi(M_{GH}) \subset \bigoplus_{k=2}^{N} H^0\big(C, \mathrm{Sym}^k F\big).$$

$$0 = \det(x - \Phi) = x^N + \sum_{k=2}^{N} \tilde\phi_k(z) x^{N-k}$$

$$\pi : (A_{\bar z}, \Phi) \mapsto \{\tilde\phi_k\}_{2\le k\le N}$$

$$\mathrm{U}(1) - \mathrm{U}(2) - \cdots - \mathrm{U}(N-1) - \mathrm{SU}(N)_H|_{\text{flavor}}.$$



$$\mu_H^{(3\,\mathrm{d})} = A_{N-1}B_{N-1} - \frac{1}{N}\mathrm{tr}(A_{N-1}B_{N-1}),$$

$$\rho : \mathrm{SU}(2) \to \mathrm{SU}(N).$$

$$\left\langle \mu_C^{(3\,\mathrm{d})} \right\rangle \propto \rho(\sigma^+),$$

$$\mathrm{adj} \to \bigoplus_{a\in A} (\mathbf{2j_a} + \mathbf{1})$$

$$\mu_C^{(3\,\mathrm{d})} = \rho(\sigma^+) + \sum_{a\in A} \sum_{m=-j_a}^{j_a} T_{a,m}\mu_C^{a,m}$$

$$\mu_C^{(3\,\mathrm{d})} = \rho(\sigma^+) + \sum_{a\in A} T_{a,-j_a}\mu_C^{a,-j_a}$$

$$g_{\mathrm{hk}} = -\frac{1}{g_5^2}\int |d^2z|\mathrm{Tr}(\delta A_{\bar z}\otimes\delta A_z + \delta A_z\otimes\delta A_{\bar z} + \delta\Phi_2\otimes\delta\bar\Phi_2 + \delta\bar\Phi_2\otimes\delta\Phi_2)$$

$$\begin{aligned} I^T(\delta A_{\bar z},\delta\Phi_2,\delta A_z,\delta\bar\Phi_2) &= (i\delta A_{\bar z}, i\delta\Phi_2, -i\delta A_z, -i\delta\bar\Phi_2)\\ J^T(\delta A_{\bar z},\delta\Phi_2,\delta A_z,\delta\bar\Phi_2) &= (-\delta\bar\Phi_2, \delta A_z, -\delta\Phi_2, \delta A_{\bar z})\\ K^T(\delta A_{\bar z},\delta\Phi_2,\delta A_z,\delta\bar\Phi_2) &= (-i\delta\bar\Phi_2, i\delta A_z, i\delta\Phi_2, -i\delta A_{\bar z}) \end{aligned}$$

$$\omega_J + i\omega_K = -\int |d^2z|2\mathrm{Tr}[\delta A_{\bar z}\wedge\delta\Phi_2]$$

$$\begin{aligned} \mu_{\mathbb{C}}^{(5\,\mathrm{d})}(\alpha) &= -\int |d^2z|2\mathrm{Tr}[\alpha(D_{\bar z}\Phi_2)]\\ &\equiv -\int \sqrt{g}|d^2z|\mathrm{Tr}\left[\alpha\mu_{\mathbb{C}}^{(5\,\mathrm{d})}\right] \end{aligned}$$

$$\mu_{\mathbb{C}} = g_5^{-2}\mu_{\mathbb{C}}^{(5\,\mathrm{d})} + \sqrt{g^{-1}}\delta^2(z-z_p)\mu_H^{(3d)}$$

$$\begin{aligned} W_{3d} &= -\int \sqrt{g}|d^2z|\sqrt{2}\mathrm{Tr}(\Phi_1\mu_{\mathbb{C}})\\ &= -\sqrt{2}\left(\mathrm{Tr}\left(\mu_H^{(3d)}\Phi_1(z_p)\right) + \int |d^2z|\frac{2}{g_5^2}\mathrm{Tr}(\Phi_1 D_{\bar z}\Phi_2)\right) \end{aligned}$$

$$\begin{aligned} &B'_kA'_k - A'_{k-1}B'_{k-1} = \xi_k\ (2\le k\le N-1)\\ &B'_1A'_1 = \xi_1 \end{aligned}$$

$$\begin{aligned} M'(M'-\xi_{N-1}) &= (A'_{N-1}B'_{N-1}A'_{N-1}B'_{N-1} - \xi_{N-1}A'_{N-1}B'_{N-1})\\ &= A'_{N-1}A'_{N-2}B'_{N-2}B'_{N-1} \end{aligned}$$

$$\begin{aligned} &B'_kB'_{k+1}\cdots B'_{N-1}\left(A'_{N-1}B'_{N-1} - \sum_{i=k}^{N-1}\xi_i\right) = B'_kB'_{k+1}\cdots B'_{N-2}\left(A'_{N-2}B'_{N-2} - \sum_{i=k}^{N-2}\xi_i\right)B'_{N-1}\\ &= \cdots = A'_{k-1}B'_{k-1}B'_{k+1}\cdots B'_{N-1} \end{aligned}$$

$$M'(M'-\xi_{N-1})(M'-\xi_{N-1}-\xi_{N-2})\cdots\left(M' - \sum_{k=1}^{N-1}\xi_k\right) = 0$$

$$\begin{aligned} P(x) &\equiv \det\left(x - \mu_C^{(3d)}\right) = \prod_{k=1}^{N}(x-\lambda_k)\\ \lambda_k &= \sum_{i=k}^{N-1}\xi_i - \frac{1}{N}\sum_{i=1}^{N-1} i\xi_i \end{aligned}$$

$$\det\left(x - \mu_C^{(3d)}\right) = \det\left(x - \frac{\sqrt{2}}{4\pi}\Phi_1(z_p)\right)$$

$$\mu_C^{(3d)} \approx \frac{\sqrt{2}}{4\pi}\Phi_1(z_p)$$

$$\mathcal{O}_\lambda = \{g_{\mathbb{C}}\lambda g_{\mathbb{C}}^{-1}; g_{\mathbb{C}} \in \mathrm{SU}(N)_{\mathbb{C}}\}$$

$$(2\pi R)K_{4d} = K_{3d}, (2\pi R)W_{4d} = W_{3d}$$

$$\mu^{(4d)} = \frac{1}{2\pi R}\mu_C^{(3d)} \approx \frac{\sqrt{2}}{8\pi^2 R}\Phi_1(z_p) = \frac{\sqrt{2}}{8\pi^2}\Phi_1^{(6d)}(z_p),$$

$$W_{3d} = -\sqrt{2}\left(\sum_{a\in A}\mathrm{Tr}\left(\mu_{a,H}^{(3d)}\Phi_1(z_a)\right) - \sum_{b\in B}\mathrm{Tr}\left(\mu_{b,H}^{(3d)}\Phi_2(z_b)\right) + \int |d^2z|\frac{2}{g_5^2}\mathrm{Tr}(\Phi_1 D_{\bar{z}}\Phi_2)\right),$$

$$0 = \sum_{a\in A}\mu_{a,H}^{(3d)}\delta^2(z-z_a) + \frac{2}{g_5^2}D_{\bar{z}}\Phi_2$$

$$0 = \sum_{b\in B}\mu_{b,H}^{(3d)}\delta^2(z-z_b) + \frac{2}{g_5^2}D_{\bar{z}}\Phi_1$$

$$\Phi_2 \to -\frac{g_5^2}{4\pi}\frac{\mu_{a,H}^{(3d)}}{z-z_a} \quad (z \to z_a)$$

$$\Phi_1 \to -\frac{g_5^2}{4\pi}\frac{\mu_{b,H}^{(3d)}}{z-z_b} \quad (z \to z_b)$$

$$W_{3d}|_{\mathrm{solution}} = \sqrt{2}\sum_{b\in B}\mathrm{Tr}\left(\mu_{b,H}^{(3d)}\Phi_2(z_b)\right)$$

$$= \frac{2\sqrt{2}i}{g_5^2}\sum_{b\in B}\oint_{|z-z_b|=\epsilon} dz\mathrm{Tr}(\Phi_1\Phi_2)$$

$$W_{4d}|_{\mathrm{solution}} = \frac{1}{2\pi R}W_{3d}\Big|_{\mathrm{solution}}$$

$$= -\sum_{b\in B}\oint_{|z-z_b|=\epsilon}\frac{dz}{2\pi i}\frac{\sqrt{2}}{4\pi^2}\mathrm{Tr}\left(\Phi_1^{(6d)}\Phi_2^{(6d)}\right)$$

$$W_{4d}|_{\mathrm{solution}} = \sum_{a\in A}\oint_{|z-z_a|=\epsilon}\frac{dz}{2\pi i}\frac{\sqrt{2}}{4\pi^2}\mathrm{Tr}\left(\Phi_1^{(6d)}\Phi_2^{(6d)}\right)$$

$$W_{4d} \supset \mathrm{tr}\left(m\mu^{(4d)}\right) \leftrightarrow W_{3d} \supset \mathrm{tr}\left(m\mu_C^{(3d)}\right).$$

$$\mu_H^{(3d)} \approx \frac{1}{4\pi}m$$

$$\Phi_2^{(6d)} = R^{-1}\Phi_2 \to \frac{1}{2}\frac{m}{z-z_p}$$

$$W_{4d}|_{\text{solution}} = \sum_{a\in A} \oint_{|z-z_a|=\epsilon} \frac{dz}{2\pi i} 2\text{Tr}\left(\Phi_1^{(6d)}\mu_a^{(4d)}\right)$$
$$= \sum_{a\in A} \text{tr}\left(m_a \mu_a^{(4d)}\right)$$

$$\tilde{\Phi}_1 = \frac{\sqrt{2}}{8\pi^2}\Phi_1^{(6d)}, \tilde{\Phi}_2 = 2\Phi_2^{(6d)}$$

$$\tilde{\Phi}_1 \to \mu_a^{(4d)}, \quad \tilde{\Phi}_2 \to \frac{m_a}{z - z_a} \quad (z \to z_a)$$
$$\tilde{\Phi}_2 \to \mu_b^{(4d)}, \quad \tilde{\Phi}_1 \to \frac{m_b}{z - z_b} \quad (z \to z_a)$$

$$W_{4d}|_{\text{solution}} = \sum_{a\in A} \oint_{|z-z_a|=\epsilon} \frac{dz}{2\pi i} \text{Tr}(\tilde{\Phi}_1\tilde{\Phi}_2)$$

$$\langle \Phi_{\mathcal{N}=4}^{\vee} \rangle = \frac{(e^{\vee})^2}{4\pi} \langle \Phi_{\mathcal{N}=4} \rangle$$

$$\mu_C^{(3d)} = \frac{\sqrt{2}}{(e^{\vee})^2}\Phi_{\mathcal{N}=4}^{\vee}(x^3 = L) \approx \frac{\sqrt{2}}{(e^{\vee})^2}\langle \Phi_{\mathcal{N}=4}^{\vee}\rangle = \frac{\sqrt{2}}{4\pi}\Phi_1(z_p),$$

$$\mathcal{L}_{\text{eff}}^{(4d)} = \int d^2\theta d^2\bar{\theta}^2 K_{\text{eff}}^{(4d)}(u, u^{\dagger}) + \int d^2\theta W_{\text{eff}}^{(4d)}(u) + \int d\theta^2 \frac{\tau_{IJ}(u)}{8\pi i} W^{\alpha I} W_{\alpha}^{J} + \text{ h.c.}$$

$$\int d^4x \mathcal{L}_{\text{eff}}^{(4d)} \supset \int \left(\frac{\tau_{IJ}}{4\pi i} F_+^I \wedge * F_+^J - \frac{\bar{\tau}_{IJ}}{4\pi i} F_-^I \wedge * F_-^J\right)$$

$$2\pi R \int d^3x \mathcal{L}_{\text{eff}}^{(4d)} \supset 2\pi R \int \left(e_{IJ}^{-2}(F'^I \wedge * F'^J + R^{-2} da^I \wedge * da^J) - \frac{i\theta_{IJ}}{4\pi^2} R^{-1} F'^I \wedge da^J\right)$$

$$\int \left(2\pi R e_{IJ}^{-2}(F'^I \wedge * F'^J + R^{-2} da^I \wedge * da^J) - \frac{i\theta_{IJ}}{2\pi} F^I \wedge da^J + i b_I dF'^I\right)$$

$$\frac{1}{2R}\int \left(\left(\frac{4\pi}{e^2}\right)_{IJ} da^I \wedge * da^J + \left(\frac{e^2}{4\pi}\right)^{IJ} \left(db_I + \frac{\theta_{IK}}{2\pi} da^K\right) \wedge * \left(db_J + \frac{\theta_{JL}}{2\pi} da^L\right)\right)$$
$$= \int \left(\frac{e^2}{8\pi R}\right)^{IJ} d\varphi_I \wedge * \varphi_J^{\dagger} + \cdots$$

$$\varphi_I = b_I + \tau_{IJ} a^J$$

$$\varphi_I \cong \varphi_I + m_I + \tau_{IJ} n^J$$

$$K_{\text{eff}}^{(3d)} = 2\pi R K_{\text{eff}}^{(4d)}(u, u^{\dagger}) + \frac{1}{R}((\text{Im}\tau)^{-1})^{IJ} \text{Im}\varphi_I \text{Im}\varphi_J$$
$$W_{\text{eff}}^{(3d)} = 2\pi R W_{\text{eff}}^{(4d)}(u)$$

$$W_{\mathrm{UV}}^{(4d)} = \sum_a \xi_a O_a$$

$$\begin{aligned} A_{\bar{z}}(x,\theta,z) &= \sum_n A_{\bar{z}}^{(n)}(x,\theta)\psi_{\bar{z}}^{(n)}(z) \\ \Phi_i(x,\theta,z) &= \sum_n \Phi_i^{(n)}(x,\theta)\psi_i^{(n)}(z) \\ V(x,\theta,z) &= \sum_n V^{(n)}(x,\theta)\psi_V^{(n)}(z) \end{aligned}$$

$$\begin{aligned} A_{\bar{z}}(x,\theta,z) &\to \sum_{n=1}^{g} A_{\bar{z}}^{(n)}(x,\theta)\left(s_K^{(n)}\right)^*(z) \\ \Phi_i(x,\theta,z) &\to \sum_{n=1}^{g_i} \Phi_i^{(n)}(x,\theta)s_i^{(n)}(z) \\ V(x,\theta,z) &\to V(x,\theta) \end{aligned}$$

$$P_X(x) = \det(x-\mu_X)\ (X = A,B,C)$$

$$\begin{gathered} P_A(x) = P_B(x) = P_C(x) \equiv P(x), \\ (\mu_A)^{i_A}{}_{j_A} Q^{j_A i_B i_C} = (\mu_B)^{i_B}{}_{j_B} Q^{i_A j_B i_C} = (\mu_C)^{i_C}{}_{j_C} Q^{i_A i_B j_C}, \\ (\mu_A)^{j_A}{}_{i_A} Q_{j_A i_B i_C} = (\mu_B)^{j_B} Q_{i_B} Q_{i_A j_B i_C} = (\mu_C)^{j_C} Q_{i_A i_B j_C}, \\ \left(Q^{i_A i_B i_C} Q_{j_A j_B i_C}\right) = \left[\left(\frac{P(x)-P(y)}{x-y}\right)(x = \mu_A \otimes \mathbf{1}, y = \mathbf{1}\otimes \mu_B)\right]^{i_A i_B}_{j_A j_B}, \\ \frac{1}{N!} Q^{i_{A,1} i_{B,1} i_{C,1}} \ldots Q^{i_{A,N} i_{B,N} i_{C,N}} \epsilon_{i_{B,1}\cdots i_{B,N}} \epsilon_{i_{C,1}\cdots i_{C,N}} = (\mu_A^0)^{(i_{A,1}} \ldots (\mu_A^{N-1})^{i_{A,N}}{}_{j_{A,N}} \epsilon^{j_{A,1}\cdots j_{A,N}}, \end{gathered}$$

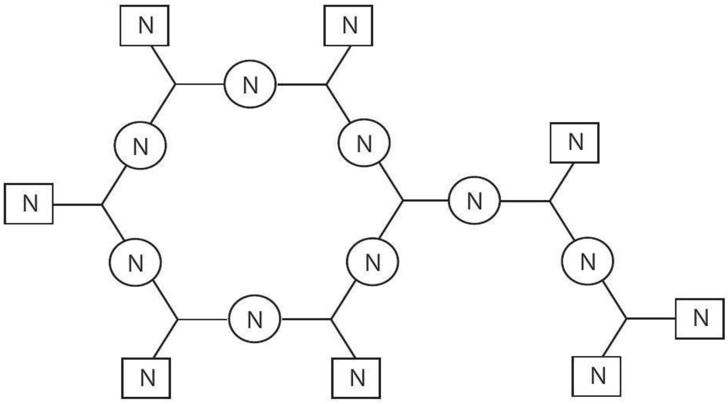
$$U_X \mu_X U_X^{-1} = \mathrm{diag}(\lambda_1,\cdots,\lambda_N) \equiv \lambda\ (X = A,B,C),$$

$$\begin{gathered} \tilde{Q}^{i_A i_B i_C} = (U_A)^{i_A}_{j_A}(U_B)^{i_B}_{j_B}(U_C)^{i_C}_{j_C} Q^{j_A j_B j_C}, \\ \tilde{Q}_{i_A i_B i_C} = (U_A^{-1})^{j_A}_{i_A}(U_B^{-1})^{j_B}_{i_B}(U_C^{-1})^{j_C}_{i_C} Q_{j_A j_B j_C}. \end{gathered}$$

$$\tilde{Q}^{kkk} = q^k, \tilde{Q}_{kkk} = q_k,$$

$$q^k q_k = \prod_{\ell \neq k} (\lambda_k - \lambda_\ell)$$

$$\prod_{k=1}^{N} q^k = \prod_{1\le k<\ell\le N} (\lambda_\ell - \lambda_k).$$



$$\mathrm{SU}(N)_{(I)} \ni g \mapsto (g,\ {}^t g^{-1}) \in \mathrm{SU}(N)_{(V,I)} \times \mathrm{SU}(N)_{(V',I)},$$

$$W \supset \sqrt{2}\mathrm{tr}\phi_{(I)}\left(\mu_{(V,I)} - {}^t\mu_{(V',I)}\right).$$

$$U_{(V,I)}\mu_{(V,I)}U_{(V,I)}^{-1} = U_{(V,E)}\mu_{(V,E)}U_{(V,E)}^{-1} = \mathrm{diag}(\lambda_1, \cdots, \lambda_N) \equiv \lambda$$

$$\tilde{Q}_{(V)}^{kkk} = q_{(V)}^k, \left(\tilde{Q}_{(V)}\right)_{kkk} = \left(q_{(V)}\right)_k.$$

$$(-1)^{h(V,I)}V_{(I)} + (-1)^{h(V,J)}V_{(J)} + (-1)^{h(V,K)}V_{(K)} = 0$$

$$q_{\mathrm{tot}}^k = \prod_V q_{(V)}^k$$

$$\mu_{(V,E)} \in \mathcal{O}_\lambda^{(E)} \equiv \left\{U_{(V,E)}^{-1}\lambda U_{(V,E)}\right\}$$

$$\prod_{k=1}^{N} q_{\mathrm{tot}}^k = \left(\prod_{1\le k<\ell\le N} (\lambda_\ell - \lambda_k)\right)^{N_V}$$

$$\begin{gathered} 0 = F_{z\bar{z}} - [\Phi_2, \bar{\Phi}_2] \\ 0 = D_{\bar{z}}\Phi_2 \\ 0 = D_{\bar{z}}\sigma = D_{\bar{z}}\Phi_1 = D_{\bar{z}}\bar{\Phi}_1 \\ 0 = [\sigma, \Phi_2] = [\Phi_1, \Phi_2] = [\bar{\Phi}_1, \Phi_2] \\ 0 = [\sigma, \Phi_1] = [\Phi_1, \bar{\Phi}_1] \end{gathered}$$

$$\begin{aligned} 0 &= \sqrt{g}\mathrm{Tr}(g^{z\bar{z}}F_{z\bar{z}} - g^{z\bar{z}}[\Phi_2, \bar{\Phi}_2] - [\Phi_1, \bar{\Phi}_1])^2 \\ &= \sqrt{g^{-1}}\mathrm{Tr}(F_{z\bar{z}} - [\Phi_2, \bar{\Phi}_2])^2 + \sqrt{g}\mathrm{Tr}([\Phi_1, \bar{\Phi}_1])^2 - 2\mathrm{Tr}\big((F_{z\bar{z}} - [\Phi_2, \bar{\Phi}_2])[\Phi_1, \bar{\Phi}_1]\big) \end{aligned}$$

$$\begin{gathered} \mathrm{Tr}(F_{z\bar{z}}[\Phi_1, \bar{\Phi}_1]) = \mathrm{Tr}\big(\bar{\Phi}_1([D_z, D_{\bar{z}}]\Phi_1)\big) \\ \mathrm{Tr}([\Phi_2, \bar{\Phi}_2][\Phi_1, \bar{\Phi}_1]) = \mathrm{Tr}([\Phi_1, \bar{\Phi}_2][\Phi_2, \bar{\Phi}_1]) + \mathrm{Tr}([\Phi_1, \Phi_2][\bar{\Phi}_1, \bar{\Phi}_2]) \end{gathered}$$

$$\begin{aligned} 0 = &\int |d^2z|\sqrt{g^{-1}}\mathrm{Tr}(F_{z\bar{z}} - [\Phi_2, \bar{\Phi}_2])^2 + \int |d^2z|\sqrt{g}\mathrm{Tr}([\Phi_1, \bar{\Phi}_1])^2 \\ &-2\int |d^2z|\mathrm{Tr}(D_{\bar{z}}\bar{\Phi}_1 D_z\Phi_1) + 2\int |d^2z|\mathrm{Tr}([\Phi_1, \bar{\Phi}_2][\Phi_2, \bar{\Phi}_1]) \end{aligned}$$

$$\lambda = \mathrm{diag}(\lambda_1, \cdots, \lambda_N) = \frac{\sqrt{2}}{8\pi^2 R}\Phi_1$$

$$\mu_{(E)}^{(4d)} = \left(\mu_C^{(3d)}\right)_{(E)} / 2\pi R$$

$$\mu_{(E)}^{(4d)} = U_{(E)}^{-1} \lambda U_{(E)},$$

$$\sigma = \mathrm{diag}(\sigma_1, \cdots, \sigma_N), A_\mu = \mathrm{diag}\big((A_1)_\mu, \cdots, (A_N)_\mu\big).$$

$$\varphi_k = \rho_k + \frac{2i\mathcal{A}}{g_5^2}\sigma_k = \rho_k + \frac{i\mathcal{A}}{4\pi^2}\sigma_k^{(6d)}$$

$$\mathcal{A} = \int \sqrt{g}|d^2 z|$$

$$\tilde{q}^k = \exp\,(2\pi i \varphi_k).$$

$$\prod_{k=1}^{N} \tilde{q}^k = 1.$$

$$\tilde{q}^k \sim q_{\mathrm{tot}}^k.$$

$$q_{\mathrm{tot}}^k = \tilde{q}^k \prod_{\ell \neq k} (\lambda_k - \lambda_\ell)^{\frac{N_V}{2}}$$

$$p\left(\mu_{(E)}^{(4d)}\right) = p\left(\mu_{(E')}^{(4d)}\right),$$

$$N_f = 0 \colon \Phi \to \frac{\zeta}{\left(z - z_p\right)^{1+1/N}} \mathrm{diag}(1, \omega_N, \cdots, \omega_N^{N-1})$$
$$N_f < N \colon \Phi \to \frac{\zeta}{\left(z - z_p\right)^{1+1/(N-N_f)}} \mathrm{diag}\left(0, \cdots, 0, 1, \omega_{N-N_f}, \cdots, \omega_{N-N_f}^{N-N_f-1}\right)$$
$$+ \frac{1}{\left(z - z_p\right)} \mathrm{diag}\left(m_1, \cdots, m_{N_f}, m, \cdots, m\right) - (\mathfrak{T})$$
$$N_f = N \colon \Phi \to \frac{1}{\left(z - z_p\right)} \mathrm{diag}(m_1, \cdots, m_N)$$

$$N_f = 0 \colon \det(x - \Phi) \to x^N - \frac{\zeta^N}{\left(z - z_p\right)^{N+1}} + (\mathfrak{L})$$
$$N_f < N - 1 \colon \det(x - \Phi) \to x^N - \frac{\zeta^{N-N_f}}{\left(z - z_p\right)^{N-N_f+1}} \prod_{k=1}^{N_f} \left(x - \frac{m_k}{z - z_p}\right) + (\mathfrak{L})$$
$$N_f = N \colon \det(x - \Phi) \to \prod_{k=1}^{N} \left(x - \frac{m_k}{z - z_p}\right) + (\mathfrak{L})$$

$$\text{If } \Phi_2 \to \frac{\zeta}{(z-z_p)^{1+1/N}} \mathrm{diag}(1, \omega_N, \cdots, \omega_N^{N-1})$$

$$\Phi_2 \approx \begin{pmatrix} \mu - \frac{1}{N}(\mathrm{tr}\mu)\mathbf{1}_{N_f} & 0 \\ 0 & -\frac{1}{N}(\mathrm{tr}\mu)\mathbf{1}_{N-N_f} \end{pmatrix}$$

$$\oiiint \frac{dz}{2\pi i} \mathrm{tr}(\Phi_1\Phi_2) = \mathrm{tr}(m\mu) + \cdots$$

$$s_1 = \prod_{b\in B} (z - z_b),$$

$$(s_1)_\infty \equiv z^{-n_1} s_1 \to 1 \; (z \to \infty)$$

$$\Phi_1' = s_1 \Phi_1$$

$$L_1' = L_1 \otimes L_B$$

$$\mathrm{deg}L_1' = \mathrm{deg}L_1 + n_1 = p$$

$$\Phi_1' \to \frac{\zeta}{(z-z_b)^{1/(N-N_f)}} \mathrm{diag}\left(0, \cdots, 0, 1, \omega_{N-N_f}, \cdots, \omega_{N-N_f}^{N-N_f-1}\right)$$
$$+\mathrm{diag}\left(m_1, \cdots, m_{N_f}, m, \cdots, m\right) - (\mathfrak{T})$$

$$W = \mathrm{tr}mM + (N - N_f)\left(\frac{\Lambda^{3N-N_f}}{\mathrm{det}M}\right)^{\frac{1}{N-N_f}}$$

$$M = \left(\Lambda^{3N-N_f}\mathrm{det}m\right)^{\frac{1}{N}} m^{-1}$$
$$W = N\left(\Lambda^{3N-N_f}\mathrm{det}m\right)^{\frac{1}{N}}$$

$$\det(x_1' - \Phi_1') = x_1'^N - z\zeta_1^{N-N_f} \prod_{k=1}^{N_f} (x_1' - m_k) + \sum_{k=2}^{N} u_k x_1'^{N-k}$$
$$\det(x_2' - \Phi_2') = x_2'^N - \frac{\zeta_2^N}{z} + \sum_{k=2}^{N} u_k' x_1'^{N-k}$$

$$0 = \det(x_1' - \Phi_1') = x_1'^N - z\zeta_1^{N-N_f} \prod_{k=1}^{N_f} (x_1' - m_k)$$

$$\Phi_1' \to \left((-1)^{N_f} \zeta_1^{N-N_f} \prod_{k=1}^{N_f} m_k\right)^{\frac{1}{N}} z^{\frac{1}{N}} \mathrm{diag}(1, \omega_N, \cdots, \omega_N^{N-1})$$

$$\Phi_2' \sim \left(\Lambda_{\mathrm{eff}}^{3N}\right)^{\frac{1}{N}} (\Phi_1')^{-1}$$

$$\Lambda_{\text{eff}}^{3N} = (-1)^{N_f} \zeta_1^{N-N_f} \zeta_2^N \prod_{k=1}^{N_f} m_k$$

$$\Phi_2' = \Lambda_{\text{eff}}^3 \left((\Phi_1')^{-1} - \frac{\mathbf{1}_N}{N} \sum_{k=1}^{N_f} \frac{1}{m_k} \right)$$

$$x_1' x_2' = \Lambda_{\text{eff}}^3 \left(1 - \frac{x_1'}{N} \sum_{k=1}^{N_f} m_k^{-1} \right).$$

$$\Phi_2' \to \Lambda_{\text{eff}}^3 \left(\operatorname{diag}\left(m_1^{-1}, \cdots, m_{N_f}^{-1}, 0, \cdots, 0 \right) - \frac{\mathbf{1}_N}{N} \sum_{k=1}^{N_f} m_k^{-1} \right)$$

$$M \approx \Lambda_{\text{eff}}^3 \operatorname{diag}\left(m_1^{-1}, \cdots, m_{N_f}^{-1} \right)$$

$$(-1)^{N_f} \zeta_1^{N-N_f} \zeta_2^N = \Lambda^{3N-N_f}$$

$$W = \oiiint_{z\sim 0} \frac{dz}{2\pi i} \operatorname{Tr}(\Phi_1 \Phi_2) = N \Lambda_{\text{eff}}^3$$

$$M = \begin{pmatrix} \tilde{q}^i \\ \tilde{p}^\ell \end{pmatrix} (q_j, p_m) = \begin{pmatrix} (M_1)^i_j & L^i_m \\ \tilde{L}^\ell_j & (M_2)^\ell_m \end{pmatrix}$$

$$\begin{aligned} W &= c \left(q_i^\alpha \tilde{q}^i_\beta - \frac{\delta^\alpha_\beta}{N} q_i^\gamma \tilde{q}^i_\gamma \right) \left(p_\ell^\beta \tilde{p}^\ell_\alpha - \frac{\delta^\beta_\alpha}{N} p_\ell^\gamma \tilde{p}^\ell_\gamma \right) \\ &= c \left(\operatorname{tr}(L\tilde{L}) - \frac{1}{N} (\operatorname{tr} M_1)(\operatorname{tr} M_2) \right) \end{aligned}$$

$$W = X\left(M_1 M_2 - L\tilde{L} - B\tilde{B} - \Lambda^4 \right) + c \left(L\tilde{L} - \frac{1}{2} M_1 M_2 \right)$$

$$\begin{aligned} &(1): X = c/2, \quad M_1 M_2 = \Lambda^4, \quad L = \tilde{L} = B = \tilde{B} = 0 \\ &(2): X = c, \quad L\tilde{L} = -\Lambda^4, \quad M_1 = M_2 = B = \tilde{B} = 0 \\ &(3): X = 0, \quad B\tilde{B} = -\Lambda^4, \quad M_1 = M_2 = L = \tilde{L} \end{aligned}$$

$$x_1'^2 = \frac{1}{2} \operatorname{Tr} \Phi_1'^2, x_2'^2 = \frac{1}{2} \operatorname{Tr} \Phi_1'^2, x_1' x_2' = \frac{1}{2} \operatorname{Tr} \Phi_1' \Phi_2'$$

$$x_1'^2 = \frac{1}{4} \zeta_1^2 z^2 + u_1, x_2'^2 = \frac{1}{4} \frac{\zeta_2^2}{z^2} + u_2, x_1' x_2' = h(z)$$

$$u_1 u_2 = \frac{1}{16}\zeta_1^2\zeta_2^2, \quad x_1' x_2' = \frac{\zeta_1\sqrt{u_2}}{2z}\left(z^2 + \frac{\zeta_2^2}{4u_2}\right),$$
$$u_1 = u_2 = 0, \qquad x_1' x_2' = +\frac{1}{4}\zeta_1\zeta_2$$
$$u_1 = u_2 = 0, \qquad x_1' x_2' = -\frac{1}{4}\zeta_1\zeta_2.$$

$$\Phi_1' = \frac{1}{2}\begin{pmatrix} -\zeta_1 z & 0 \\ 0 & +\zeta_1 z \end{pmatrix}, \quad \Phi_2' = \frac{1}{2}\begin{pmatrix} -\zeta_2/z & 0 \\ 0 & +\zeta_2/z \end{pmatrix}$$
$$\Phi_1' = \frac{1}{2}\begin{pmatrix} -\zeta_1 z & 0 \\ 0 & +\zeta_1 z \end{pmatrix}, \quad \Phi_2' = \frac{1}{2}\begin{pmatrix} +\zeta_2/z & 0 \\ 0 & -\zeta_2/z \end{pmatrix}$$

$$\sigma = \begin{pmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{pmatrix},$$

$$W = X\left(\mathrm{det}M - B\tilde{B} - \Lambda^{2N}\right) + c\left(\mathrm{tr}(L\tilde{L}) - \frac{1}{N}(\mathrm{tr}M_1)(\mathrm{tr}M_2)\right)$$

$$\Phi_1' = \begin{pmatrix} \Phi_{1,1}' & 0 \\ 0 & \Phi_{1,2}' \end{pmatrix}, \Phi_2' = \begin{pmatrix} \Phi_{2,1}' & 0 \\ 0 & \Phi_{2,2}' \end{pmatrix}, \sigma = \begin{pmatrix} \sigma_0 \mathbf{1}_{N_1}/N_1 & 0 \\ 0 & -\sigma_0 \mathbf{1}_{N_2}/N_2 \end{pmatrix},$$

$$\Phi_{1,1}' \to \zeta_1 z^{1/N_1}\mathrm{diag}\left(1, \omega_{N_1}, \cdots, \omega_{N_1}^{N_1-1}\right), \quad z \to \infty$$
$$\Phi_{2,2}' \to \frac{\zeta_2}{z^{1/N_2}}\mathrm{diag}\left(1, \omega_{N_2}, \cdots, \omega_{N_2}^{N_2-1}\right), \qquad z \to 0$$

$$\det(x_1' - \Phi_1') \;= x_1'^{N_2}\left(x_1'^{N_1} + \sum_{k=2}^{N_1} u_{1,k} x_1'^{N_1-k} - z\zeta_1^{N_1}\right)$$
$$\det(x_2' - \Phi_2') \;= x_2'^{N_1}\left(x_2'^{N_2} + \sum_{k=2}^{N_2} u_{2,k} x_2'^{N_2-k} - z^{-1}\zeta_2^{N_2}\right)$$

$$\det(x_1' - \mu_1) = x_1'^{N_1} + \sum_{k=2}^{N_1} u_{1,k} x_1'^{N_1-k}$$
$$\det(x_2' - \mu_2) = x_2'^{N_2} + \sum_{k=2}^{N_2} u_{2,k} x_2'^{N_2-k}$$

$$\mathrm{deg}L_1' = \mathrm{deg}L_1 + n_1 = 0, \mathrm{deg}L_2' = \mathrm{deg}L_2 + n_2 = 1$$

$$\det(x - \mu_A) = \det(x - \mu_B) = \det(x - \mu_C)$$

$$\det(x - \mu_A) = \det(x - \mu_B) - \Lambda_C^{2N},$$

$$\Phi_1' \to \zeta_C z^{1/N}\mathrm{diag}(1, \omega_N, \cdots, \omega_N^{N-1})$$

$$\det(x_1' - \Phi_1') = x_1'^N + \sum_{k=2}^{N} u_k x_1'^{N-k} - \zeta_C^N z$$

$$\det(x_1' - \mu_A) = \det(x_1' - \Phi_1')|_{z=0} = x_1'^N + \sum_{k=2}^{N} u_k x_1'^{N-k}$$

$$\det(x_1' - \mu_B) = \det(x_1' - \Phi_1')|_{z=1} = x_1'^N + \sum_{k=2}^{N} u_k x_1'^{N-k} - \zeta_C^N$$

$$\det(x_1' - \mu_A) = \det(x_1' - \mu_B) + \zeta_C^N$$

$$0 = \det(x_1' - \Phi_1') = x_1'^N + \sum_{k=2}^{N} u_k x_1'^{N-k} - \frac{\zeta_B^N z}{z-1} - \zeta_C^N z,$$

$$\det(x_1' - \mu_A) = \det(x_1' - \Phi_1')|_{z=0} = x_1'^N + \sum_{k=2}^{N} u_k x_1'^{N-k}$$

$$y^2 = w^3 + (\zeta_C^2 + \zeta_B^2 - \det\mu_A)w^2 + \zeta_C^2\zeta_B^2 w$$

$$0 = \det(x_1' - \Phi_1') = x_1'^N + \sum_{k=2}^{N} u_k x_1'^{N-k} - \frac{\zeta_A^N}{z} - \frac{\zeta_B^N z}{z-1} - \zeta_C^N z$$

$$W = \sqrt{2}\mathrm{tr}\phi_B\mu_B + \sum_{k=2}^{N} X_k\left(\mathrm{tr}\mu_A^k - \mathrm{tr}\mu_B^k - N\Lambda_C^{2N}\delta^{k,N}\right)$$

$$\mu_A = \Lambda_C^2 \mathrm{diag}(1, \omega_N, \cdots, \omega_N^{N-1}), \mu_B = 0$$

$$\det\left(x - \frac{\mu_A}{\Lambda_C}\right) - \frac{\Lambda_A^{2N}}{z} + z \sim 0$$

$$x^N - \Lambda_C^{2N} - \frac{\Lambda_A^{2N}}{z} + \Lambda_C^{2N} z \sim 0$$

$$x^N - \frac{\Lambda_A^{2N}}{z} + \Lambda_C^{2N} z - (\Lambda_C^{2N} - \Lambda_A^{2N}) = 0$$

$$\Lambda_{B,\,\mathrm{low}}^{3N} \sim \Lambda_B^N \Lambda_C^{2N}$$

$$\Lambda_{B,\,\mathrm{low}}^{3N} = \Lambda_B^N(\Lambda_C^{2N} + \Lambda_A^{2N})$$

$$W_{\mathrm{condense}} = N\Lambda_{B,\,\mathrm{low}}^3 = N[\Lambda_B^N(\Lambda_C^{2N} + \Lambda_A^{2N})]^{\frac{1}{N}}.$$

$$\det(x_1' - \Phi_1') = x_1'^N + \sum_{k=2}^{N} u_k x_1'^{N-k} - \frac{\zeta_A^N}{z} + \zeta_C^N z,$$

$$0 = \det(x_1' - \Phi_1') = x_1'^N - \frac{\zeta_A^N(1-z)}{z} - \zeta_C^N(1-z)$$

$$\Phi_2' \sim (\Phi_1')^{-1}.$$

$$\Phi_1' \to \text{ const. } (z + \zeta_A^N/\zeta_C^N)^{1/N} \text{diag}(1, \omega_N, \cdots, \omega_N^{N-1})$$

$$\lambda = \frac{\zeta_B}{(\zeta_A^N + \zeta_C^N)^{1-1/N}} (\zeta_C^N z + \zeta_A^N)$$

$$\Phi_2' = \lambda(\Phi_1')^{-1}.$$

$$x_1'^N = \frac{(\zeta_C^N z + \zeta_A^N)(1 - z)}{z}$$

$$x_1' x_2' = \frac{\zeta_B}{(\zeta_C^N + \zeta_A^N)^{1-1/N}} (\zeta_C^N z + \zeta_A^N)$$

$$s_1 = z, s_2 = (z - 1)$$

$$x_1 x_2 = \frac{\zeta_B}{(\zeta_C^N + \zeta_A^N)^{1-1/N}} \left(\frac{\zeta_C^N z + \zeta_A^N}{z(z-1)} \right)$$

$$W = \oint_{z\sim 1} \frac{dz}{2\pi i} x_1 x_2 N$$
$$= N\zeta_B (\zeta_C^N + \zeta_A^N)^{1/N}$$

$$\Phi_2(z = 0) \approx M_A.$$

$$\Phi_2'' = s\Phi_2 = \frac{\Phi_2}{z}$$

$$0 = \det(x'' - \Phi_2'')$$

$$\Phi_2'' \to \frac{M_A}{z}$$

$$\text{tr}\big(\phi(\mu_1 - {}^t\mu_2)\big),$$

$$W = c\text{tr}(\mu_1\, {}^t\mu_2)$$

$$W = c'\text{tr}(\mu_1\, {}^t\mu_2) + \text{tr}(\mu_B M_B) + \text{tr}(\mu_C M_C),$$

$$W = c''\text{tr}(\mu_1\, {}^t\mu_2) + \text{tr}(\mu_A M_A) + \text{tr}(\mu_B M_B) + \text{tr}(\mu_C M_C) + \text{tr}(\mu_D M_D).$$

$$c\text{tr}(\mu_1\, {}^t\mu_2).$$

$$\text{tr}(\mu_A M_A).$$

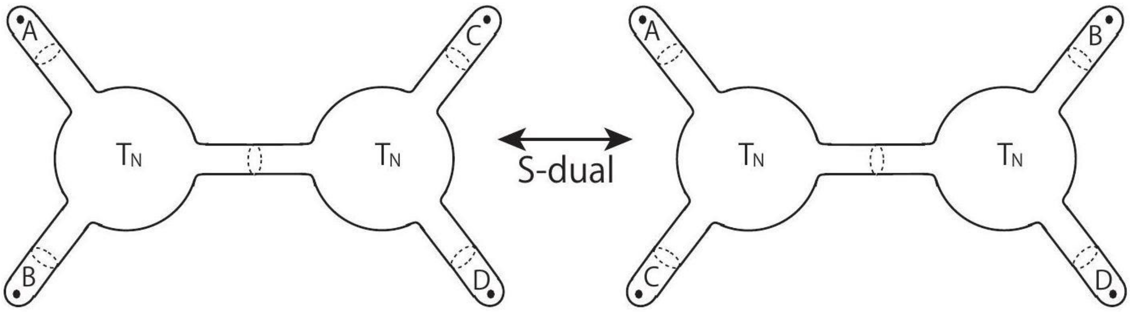
$$(\mu_1)_\beta^\alpha = q_i^\alpha \tilde{q}_\beta^i - \frac{\delta_\beta^\alpha}{N} q_i^\gamma \tilde{q}_\gamma^i$$

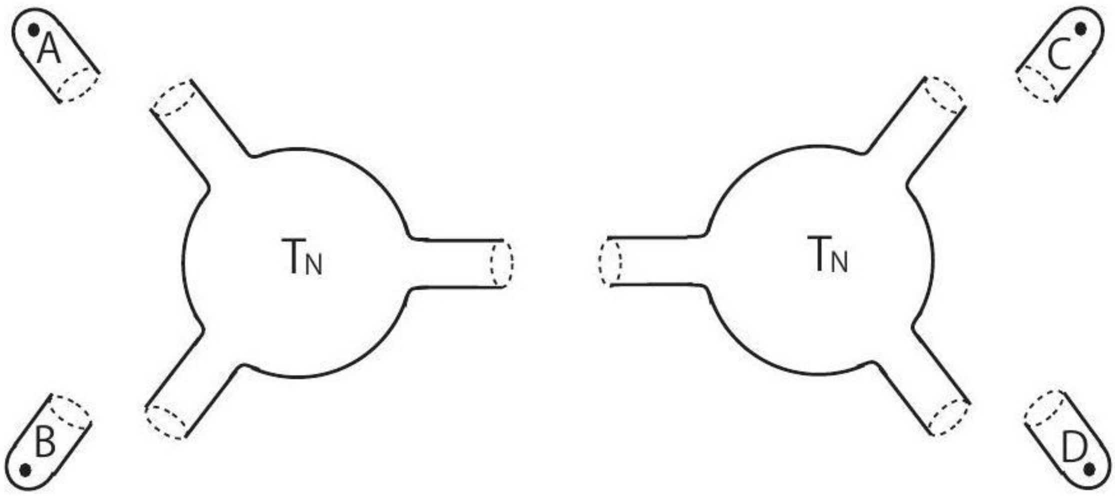
$$({}^t\mu_2)_\alpha^\beta = p_\ell^\beta \tilde{p}_\alpha^\ell - \frac{\delta_\alpha^\beta}{N} p_\ell^\gamma \tilde{p}_\gamma^\ell$$

$$W \supset \text{tr}\phi_B\big(\mu_B + q_i \tilde{q}^i\big)$$

$$W \supset \text{tr}\phi_B\big(M_B + q_i \tilde{q}^i\big) + \text{tr}(M_B \mu_B)$$

$$W \supset -\mathrm{tr}\left(\mu_B q_i \tilde{q}^i\right)$$





$$\deg L_1 = p - n_+, \deg L_2 = q - n_-.$$

$$0 = P_{i_1 \cdots i_N}(x_1, x_2) \equiv \frac{1}{N!}\left(x_{i_1} - \Phi_{i_1}\right)^{\alpha_1} \cdots \left(x_{i_N} - \Phi_{i_N}\right)^{\alpha_N} \epsilon_{\beta_N} \epsilon_{\alpha_1 \cdots \alpha_N} \epsilon^{\beta_1 \cdots \beta_N}.$$

$$\phi_{k,\ell}(z) = (-1)^{k+\ell} \frac{(\Phi_1)^{\alpha_1}_{\beta_1} \cdots (\Phi_1)^{\alpha_k}_{\beta_k} (\Phi_2)^{\alpha_{k+1}}_{\beta_{k+1}} \cdots (\Phi_2)^{\alpha_{k+\ell}}_{\beta_{k+\ell}} \delta^{\beta_1 \cdots \beta_k \beta_{k+1} \cdots \beta_{k+\ell} \gamma_{k+\ell+1} \cdots \gamma_N}_{\alpha_1 \cdots \alpha_k \alpha_{k+1} \cdots \alpha_{k+\ell} \gamma_{k+\ell+1} \cdots \gamma_N}}{k!\, \ell!\, (N-k-\ell)!},$$

$$P_{1 \cdots 12 \cdots 2} = x_1^{N-m} x_2^m + \sum_{k,\ell} \frac{(N-m)!\, m!\, (N-k-\ell)!}{N!\, (N-m-k)!\, (m-\ell)!} \phi_{k,\ell}(z) x_1^{N-m-k} x_2^{m-\ell},$$

$$0 = P_1 \equiv x_1^N + \sum_{k=2}^{N} \phi_{k,0}(z) x_1^{N-k}$$

$$0 = \frac{\partial P_1(x_1, z)}{\partial x_1} x_2 + \sum_{k=1}^{N-1} \phi_{k,1}(z) x_1^{N-1-k}$$

$$\left(x_2 - \lambda_{2,k}\right) \prod_{\ell \neq k} \left(\lambda_{1,k} - \lambda_{1,\ell}\right) = 0$$

$$\Phi_2 = -\left[\frac{\partial P_1}{\partial x_1}(x_1 = \Phi_1)\right]^{-1} \sum_{k=1}^{N-1} \phi_{k,1} \Phi_1^{N-1-k}$$

$$\Phi_1 = \begin{pmatrix} 0 & 1 \\ z & 0 \end{pmatrix}$$

$$\Phi_2(z=0) = \begin{pmatrix} 0 & c \\ 0 & 0 \end{pmatrix}$$

$$x_1^2 = f(z), x_2^2 = g(z), x_1 x_2 = h(z),$$

$$\Phi_2 = \sum_{k=0}^{N-1} f_k \Phi_1^k$$

$$\sum_{k=0}^{N-1} \mathrm{tr}\left(\Phi_1^{m+k}\right) f_k = \mathrm{tr}(\Phi_1^m \Phi_2), (m = 0, \cdots, N-1)$$

$$\det(x_1 - \Phi_1) \sim \prod_{\ell} \left[(x_1 - a_\ell)^{n_\ell} - b_\ell^{n_\ell} z^{m_\ell}\right]$$

$$\Phi_1 \sim \mathrm{diag}\left[\bigoplus_{\ell} \left(a_\ell + b_\ell z^{m_\ell/n_\ell}, \cdots, a_\ell + \left(\omega_{n_\ell}\right)^{n_\ell - 1} b_\ell z^{m_\ell/n_\ell}\right)\right]$$

$$\Phi_2 \sim \mathrm{diag}\left[\bigoplus_{\ell} \left(c_\ell z^{r_\ell} + d_\ell z^{p_\ell + q_\ell m_\ell/n_\ell}, \cdots, c_\ell z^{r_\ell} + \left(\omega_{n_\ell}^{q_\ell}\right)^{n_\ell - 1} d_\ell z^{p_\ell + q_\ell m_\ell/n_\ell}\right)\right]$$

$$\Phi_1 \to z^{m_\ell/N} \mathrm{diag}(1, \omega_N, \cdots, \omega_N^{N-1})$$

$$\Phi_1' \to c_1 z^{1/N} \mathrm{diag}(1, \omega_N, \cdots, \omega_N^{N-1})$$
$$\Phi_2' \to \frac{\zeta_2}{z^{1/N}} \mathrm{diag}(1, \omega_N^{-1}, \cdots, \omega_N^{-N+1})$$

$$\begin{aligned} \Phi_1' \to & \zeta_1 z^{1/(N-N_f)} \mathrm{diag}\left(0, \cdots, 0, 1, \omega_{N-N_f}, \cdots, \omega_{N-N_f}^{N-N_f-1}\right) \\ & + \mathrm{diag}\left(m_1, \cdots, m_{N_f}, m', \cdots, m'\right) \\ \Phi_2' \to & \mathrm{diag}\left(c_1', \cdots, c_{N_f}', c', \cdots, c'\right) \end{aligned}$$

$$\sum_{i=1}^{N_f} m_i + (N - N_f) m = 0, \sum_{i=1}^{N_f} c_i' + (N - N_f) c' = 0$$

$$\phi_{k,\ell}' \to \begin{cases} O(z^1) & k > \ell \\ O(z^0) & \ell \neq N \\ O(z^{-1}) & (k, \ell) = (0, N) \end{cases}$$

$$\phi_{k,\ell}' \to \begin{cases} O(z^0) k < N - N_f \\ O(z^1) k \geq N - N_f \end{cases}$$

$$0 = P_1 = x_1'^N - \zeta_1^{N-N_f} z Q_1(x_1')$$

$$Q_1(x_1') = \prod_{i=1}^{N_f} (x_1' - m_i)$$

$$\sum_{k=1}^{N-1} \phi_{k,1}' x_1'^{N-1-k} = a x_1'^{N-2} + z \sum_{k=N-N_f}^{N-1} b_{N-1-k} x_1'^{N-1-k}$$

$$0 = \left(N x_1'^{N-1} x_2' - z \zeta_1^{N-N_f} \frac{\partial Q_1}{\partial x_1'} x_2' \right) + \left(a x_1'^{N-2} + z \sum_{k=0}^{N_f-1} b_k x_1'^k \right)$$

$$0 = \left(N Q_1 - x_1' \frac{\partial Q_1}{\partial x_1'} \right) x_1' x_2' + \left(a Q_1 + \sum_{k=0}^{N_f-1} b_k' x_1'^{k+2} \right)$$

$$a Q_1 + \sum_{k=0}^{N_f-1} b_k' x_1'^{k+2}$$

$$a Q_1 + \sum_{k=0}^{N_f-1} b_k' x_1'^{k+2} = c'(1 + c x_1') \left(Q_1 - \frac{x_1'}{N} \frac{\partial Q_1}{\partial x_1'} \right)$$

$$c' = a, c = -\frac{1}{N} \sum_{i=1}^{N_f} \frac{1}{m_i}.$$

$$x_1' x_2' + \frac{a}{N} \left(1 - \frac{x_1'}{N} \sum_{i=1}^{N_f} \frac{1}{m_i} \right) = 0.$$

$$x_1'^N \to (-1)^{N_f} \zeta_1^{N-N_f} z \prod_{i=1}^{N_f} m_i$$

$$(x_1' x_2')^N \to (-1)^{N_f} \zeta_1^{N-N_f} \zeta_2^N \prod_{i=1}^{N_f} m_i$$

$$0 \ = x_1' x_2' - \Lambda_{\text{eff}}^3 \left(1 - \frac{x_1'}{N} \sum_{i=1}^{N_f} \frac{1}{m_i} \right),$$

$$\Lambda_{\text{eff}}^3 \ = \left[(-1)^{N_f} \zeta_1^{N-N_f} \zeta_2^N \prod_{i=1}^{N_f} m_i \right]^{\frac{1}{N}}.$$

$$\Phi_1' \to \frac{\zeta_A}{z^{1/N}} \operatorname{diag}(1, \omega_N, \cdots, \omega_N^{N-1})$$
$$\Phi_2' \to c z^{1/N} \operatorname{diag}(1, \omega_N^{-1}, \cdots, \omega_N^{-N+1})$$

$$\Phi_1' \to \zeta_C z^{1/N} \mathrm{diag}(1, \omega_N, \cdots, \omega_N^{N-1})$$
$$\Phi_2' \to z \frac{c}{z^{1/N}} \mathrm{diag}(1, \omega_N^{-1}, \cdots, \omega_N^{-N+1})$$

$$\Phi_1' \to c(z-1)^{1/N} \mathrm{diag}(1, \omega_N, \cdots, \omega_N^{N-1})$$
$$\Phi_2' \to \frac{\zeta_B}{(z-1)^{1/N}} \mathrm{diag}(1, \omega_N^{-1}, \cdots, \omega_N^{-N+1}).$$

$$\phi_{k,\ell}' \to \begin{cases} O(z^1) & \ell > k \\ O(z^0) & k \neq N \\ O(z^{-1}) & (k,\ell) = (N,0) \end{cases}$$

$$\phi_{k,\ell}' \to \begin{cases} O\left(z^{\ell-1}\right) & \ell > k \\ O\left(z^{\ell}\right) & k \neq N \\ O(z^1) & (k,\ell) = (N,0) \end{cases}$$

$$\phi_{k,\ell}' \to \begin{cases} O((z-1)^1) & k > \ell \\ O((z-1)^0) & \ell \neq N \\ O((z-1)^{-1}) & (k,\ell) = (0,N) \end{cases}$$

$$P_1 = x_1'^N - \left(\frac{\zeta_A^N}{z} + \zeta_C^N\right)(1-z)$$

$$0 = x_1'^{N-1} x_2' + (a_1 z + a_2) x_1'^{N-2} + (1-z) \sum_{k=2}^{N-1} b_k x_1'^{N-1-k}$$

$$0 = (\zeta_C^N z + \zeta_A^N)(x_1' x_2' + a_1 z + a_2) + z \sum_{k=2}^{N-1} b_k x_1'^{N+1-k}$$

$$x_1' x_2' = \left(\frac{\zeta_B^N}{(\zeta_C^N + \zeta_A^N)^{N-1}}\right)^{1/N} (\zeta_C^N z + \zeta_A^N)$$

$$\gamma_\mu = i \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}.$$

$$\mathcal{L} = \mathrm{Tr}\left(-\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2}\left(D_\mu X^i\right)^2 + i\chi^T \not{D}\chi + \frac{g^2}{4}\left[X^i, X^j\right]^2 - \sqrt{2} g \chi_L^T \gamma_i \left[X^i, \chi_R\right]\right)$$

$$e^{i\theta} = e^{i\frac{2\pi M}{N}}$$

$$X^i = I_{N/d} \otimes \begin{pmatrix} x_1^i & & \\ & \ddots & \\ & & x_d^i \end{pmatrix}, \text{ with } \mathrm{Tr} X^i = 0$$

$$\mathcal{I}^\theta(\tau \mid \xi) = \sum_k e^{i\theta k} \mathrm{Tr}_{\mathcal{H}_k} (-1)^F a^f q^{H_L} \bar{q}^{H_R}$$

$$\mathcal{I}^{\theta(M)}_{SU(N)/\mathbb{Z}_N}(\tau \mid \xi) = \frac{\mathcal{I}_D}{\mathcal{I}_1}(\tau \mid \xi)$$

$$\mathcal{I}^{\theta(M)}_{SU(N)/\mathbb{Z}_K}(\tau \mid \xi) = \sum_{m \equiv M} \sum_{(\mathrm{mod} K)} \frac{\mathcal{I}_{\gcd(m,N)}}{\mathcal{I}_1}(\tau \mid \xi),$$

$$\mathcal{I}_{U(N)}(\tau \mid \xi) = \mathcal{I}_{U(1)}\mathcal{I}_{SU(N)}(\tau \mid \xi) = \sum_{m=1}^{N} \mathcal{I}_{\gcd(m,N)}(\tau \mid \xi)$$

$$\mathcal{I}^{M=0}_{U(N)+B}(\tau \mid \xi) = \mathcal{I}_{U(1)}\mathcal{I}^{\theta=0}_{SU(N)/\mathbb{Z}_N}(\tau \mid \xi) = \mathcal{I}_N(\tau \mid \xi)$$

$$\mathcal{I}^{M}_{U(N)+B}(\tau \mid \xi) = \mathcal{I}_D(\tau \mid \xi)$$

$$\mathcal{I}_{U(N)+B}(\tau \mid \xi) = \sum_{M \in \mathbb{Z}} e^{iM\phi}\mathcal{I}^{M}_{U(N)+B}(\tau \mid \xi) = \sum_{M \in \mathbb{Z}} e^{iM\phi}\mathcal{I}_D(\tau \mid \xi)$$

$$w_2(P) \in H^2\left(\Sigma, \pi_1\left(G_{adj}\right)\right) = H^2(\Sigma, \mathbb{Z}_N)$$

$$\left\langle e^{i2\pi Mk/N} \middle| e^{iM\int_\Sigma d\hat{A}/N} \right\rangle$$

$$-\frac{1}{4}\mathrm{Tr}\left(F_{\mu\nu} + B_{\mu\nu}\mathbf{1}_N\right)^2,$$

$$A = \frac{1}{N}\hat{A}\mathbf{1}_N + A'$$

$$e^{iM\oint_{\partial C}\hat{A}/N}$$

$$\left(\mathrm{Tr}_R\,\mathcal{P}\exp\oint_{\partial\Sigma'} A\right) e^{iN_R\int_{\Sigma'} B} = \left(\mathrm{Tr}_R\,\mathcal{P}\exp\oint_{\partial\Sigma'} A'\right) e^{iN_R\int_{\Sigma'} \frac{d\hat{A}}{N} + B}.$$

$$e^{iM\oint_{\partial C}\frac{\hat{A}}{N}} e^{iM\int_C B} = e^{iM\int_C \frac{d\hat{A}}{N}} e^{iM\int_C B}$$

$$e^{iM\int_{T^2} d\hat{A}/N} e^{iM\int_{T^2} B}$$

$$\mathcal{I} = \mathrm{Tr}(-1)^F \prod_{J_L} y^{J_L} \prod_f x^f q^{H_L}\bar{q}^{H_R}$$

$$\mathcal{M}_{\text{curvature}} = \mathrm{Hom}\left(\pi_1(T^2), G_{\text{adj}}\right)/G_{\text{adj}}$$

$$[A][B][A]^{-1}[B]^{-1} = 1$$

$$ABA^{-1}B^{-1} = \omega_N^k$$

$$SDS^\dagger = \omega_N^k D$$

$$D_N = \begin{pmatrix} 1 & & & \\ & \omega_N & & \\ & & \ddots & \\ & & & \omega_N^{N-1} \end{pmatrix}, \text{ and } S_N = \begin{pmatrix} 0 & 1 & & \\ & 0 & 1 & \\ & & & \ddots \\ 1 & & & \end{pmatrix}$$

$$S_N^k D_N \left(S_N^k\right)^\dagger = \omega_N^k D_N$$

$$\mathcal{M}_N = \bigsqcup_{k=0}^{N-1} \mathcal{M}_{N,k}$$

$$k = \int_{T^2} w_2\left(P_{N,k}\right)$$

$$\begin{aligned} S_N\left(\omega_N^a S_N^k, \omega_N^b D_N\right) S_N^\dagger &= \left(\omega_N^a S_N S_N^k S_N^\dagger, \omega_N^b S_N D_N S_N^\dagger\right) = \left(\omega_N^a S_N^k, \omega_N^{b+1} D_N\right) \\ D_N\left(\omega_N^a S_N^k, \omega_N^b D_N\right) D_N^\dagger &= \left(\omega_N^a D_N S_N^k D_N^\dagger, \omega_N^b D_N D_N D_N^\dagger\right) = \left(\omega_N^{a-1} S_N^k, \omega_N^b D_N\right) \end{aligned}$$

$$\begin{aligned} [S_N]\left([S_N^k],[D_N]\right)[S_N]^\dagger &= \left([S_N^k],[D_N]\right) \\ [D_N]\left([S_N^k],[D_N]\right)[D_N]^\dagger &= \left([S_N^k],[D_N]\right) \end{aligned}$$

$$\mathcal{M}_{N,k} = \left\{\left([S_N^k],[D_N]\right)\right\}/\mathbb{Z}_N^2$$

$$\mathcal{M}_{N,d=1} = \left\{\left([S_N],[D_N]\right)\right\}/\mathbb{Z}_N^2$$

$$S_N^d = S_{N/d} \otimes I_d, \text{ and } D_N^d = D_{N/d} \otimes D_d^{d/N}$$

$$\left(e^{i\mathfrak{h}_{N,d}(\theta_s)} S_N^k, e^{i\mathfrak{h}_{N,d}(\theta_t)} D_N\right) = \left(S_{N/d} \otimes e^{i\mathfrak{h}_d(\theta_s)}, D_{N/d} \otimes e^{i\mathfrak{h}_d(\theta_t)} D_N^d\right)$$

$$e^{i\mathfrak{h}_{N,d}(\theta)} := I_{N/d} \otimes e^{i\mathfrak{h}_d(\theta)} := I_{N/d} \otimes \begin{pmatrix} e^{2\pi i\theta_1} & & & \\ & e^{2\pi i\theta_2} & & \\ & & \ddots & \\ & & & e^{2\pi i\theta_d} \end{pmatrix}$$

$$\begin{aligned} &\left(e^{i\mathfrak{h}_{N,d}(\theta_s)} S_N^k\right)\left(e^{i\mathfrak{h}_{N,d}(\theta_t)} D_N\right)\left(e^{i\mathfrak{h}_{N,d}(\theta_s)} S_N^k\right)^\dagger \\ &\qquad = S_{N/d}^{k/d} D_{N/d} \left(S_{N/d}^{k/d}\right)^\dagger \otimes e^{i\mathfrak{h}_d(\theta_s)} \left(e^{i\mathfrak{h}_d(\theta_t)} D_d^{d/N}\right) e^{-i\mathfrak{h}_d(\theta_s)} \\ &\qquad = \omega_{N/d}^{k/d} D_{N/d} \otimes e^{i\mathfrak{h}_d(\theta_t)} D_d^{d/N} \\ &\qquad = \omega_N^k \left(e^{i\mathfrak{h}_{N,d}(\theta_t)} D_N\right) \end{aligned}$$

$$\widetilde{\mathfrak{M}}_{N,k} := \left\{\left(S_{N/d}^{k/d} \otimes e^{i\mathfrak{h}_d(\theta_s \cdot N/d)}, D_{N/d} \otimes e^{i\mathfrak{h}_d(\theta_t \cdot N/d)}\right)\right\} \cong \left(T^2/\mathbb{Z}_{N/d}^2\right)^{d-1}$$

$$\mathcal{M}_{N,k} \cong \left\{\left(\left[S_{N/d}^{k/d} \otimes e^{i\mathfrak{h}_d(N\theta_s)}\right], \left[D_{N/d} \otimes e^{i\mathfrak{h}_d(N\theta_t)}\right]\right)\right\}/\mathbb{Z}_{N/d}^2 \times \mathbb{S}_d$$

$$\mathcal{M}_{N,d} \cong \left(\mathfrak{M}_{N,d}/\mathbb{S}_d\right)/\mathbb{Z}_{N/d}^2$$

$$\mathfrak{M}_{N,d} = \{(S_{N/d} \otimes e^{i\mathfrak{h}_d(N\theta_s)}, D_{N/d} \otimes e^{i\mathfrak{h}_d(N\theta_t)})\} \cong (T^2/\mathbb{Z}_N^2)^{d-1}$$

$$\delta\Theta = \Gamma^{MN}\mathcal{F}_{MN}\epsilon_1 + \mathbf{1}_N\epsilon_2$$

$$\mathcal{I}^\theta = \sum_P e^{i\theta\int w(P)} Z_P$$

$$\mathcal{I}^\theta_{SU(N)/\mathbb{Z}_N} = \sum_{k=0}^{N-1} e^{i\theta k} Z_{N,k}$$

$$\theta = 0{,}2\pi\frac{1}{N}, 2\pi\frac{2}{N}, \ldots, 2\pi\frac{N-1}{N}$$

$$\mathcal{I}_{U(N)} = \mathcal{I}_{U(1)}\mathcal{I}_{SU(N)}$$

$$\mathcal{I}^{\tilde{c}_1=0}_{U(N)+B} = \mathcal{I}_{U(1)}\mathcal{I}^{\theta=0}_{SU(N)/\mathbb{Z}_N}$$

$$e^{iM\int_\Sigma \frac{d\hat{A}}{N}+B}$$

$$e^{iM\phi}\mathcal{I}^M_{U(N)+B} = e^{iM\phi}\mathcal{I}_1\mathcal{I}^{\theta=2\pi M/N}_{SU(N)},$$

$$\mathcal{I}_{U(N)+B} = \sum_{M\in\mathbb{Z}} e^{iM\phi}\mathcal{I}^M_{U(N)+B}$$

$$Z_P = \frac{1}{\mathrm{Vol}(\mathcal{G}(P))}\int_{A\in\Omega^1(T^2,\mathrm{ad}_P)} \mathcal{D}AZ(A)$$

$$Z_{N,1} = Z_{1-\text{ loop}}(u)\Big|_{u\in\mathcal{M}_{N,1}} = \frac{1}{N^2} Z_{1-\text{ loop}}(u)\Big|_{u\in\mathfrak{M}_{N,1}}$$

$$\frac{\mathrm{Vol}\left(\mathcal{G}(\tilde{P}_N)\right)}{\mathrm{Vol}\left(\mathcal{G}(P_{N,0})\right)} = |\pi_1(SU(N)/\mathbb{Z}_N)|^{1-2g}$$

$$Z_{N,0} = \frac{1}{N}\tilde{Z}_{\tilde{P}_N} = \frac{1}{N}\frac{1}{|\mathbb{S}_N|}\oint_{\tilde{\mathfrak{M}}_N} Z_{1-\text{loop}}$$

$$Z_{N,0} = N\frac{1}{|\mathbb{S}_N|}\oint_{\mathfrak{M}_{N,0}} Z_{1-\text{loop}}$$

$$Z_{N,d} = \frac{1}{(N/d)^2} d \frac{1}{|\mathbb{S}_d|}\oint_{\mathfrak{M}_{N,d}} Z_{1-\text{loop}}$$

$$\mathcal{I}^{\theta}_{SU(N)/\mathbb{Z}_N} = \sum_{k=0}^{N-1} e^{i\theta k} \gcd(N,k) \frac{1}{|W_{N,k}|} \oint_{\mathfrak{M}_{N,k}} Z_{1-\text{loop}}(u)$$

$$= \sum_{k \not\perp N} e^{i\theta k} \gcd(N,k) \frac{1}{|W_{N,k}|} \sum_{u_* \in \mathfrak{M}^*_{N,d}} \underset{u=u_*}{\text{JK-Res}}(\mathsf{Q}(u_*),\eta) Z_{1-\text{loop}}(u)$$

$$+ \sum_{k \perp N} e^{i\theta k} \frac{1}{|W_{N,k}|} \sum_{u \in \mathfrak{M}_{N,1}} Z_{1-\text{loop}}(u)$$

$$S_N D_N A (S_N D_N)^\dagger = \omega_N D_N S_N A (\omega_N D_N S_N)^\dagger = D_N S_N A (D_N S_N)^\dagger$$

$$\mathfrak{C}_N = \begin{cases} \left\{-\frac{N-1}{2}, -\frac{N-1}{2}+1, \dots, \frac{N-1}{2}\right\} & \text{for } N \text{ odd} \\ \left\{-\frac{N}{2}, -\frac{N}{2}+1, \dots, \frac{N}{2}, \frac{N}{2}+1\right\} & \text{for } N \text{ even} \end{cases}$$

$$Z_{1-\text{ loop}}(u)\big|_{u \in \mathcal{M}_{N,1}} = \frac{1}{N^2} \prod_{\substack{a,b \in \mathfrak{C}_N \\ (a,b) \neq (0,0)}} \Xi\left(\frac{a + (-1)^a b\tau}{N}\right)$$

$$S^a_{N/d} D^b_{N/d} \otimes \left(E_{(d)}\right)_{i,j}$$

$$\left(\omega^b_{N/d} e^{2\pi i\left((\theta_s)_i - (\theta_s)_j\right)}, \omega^{-a}_{N/d} e^{2\pi i\left((\theta_t)_i - (\theta_t)_j\right)}\right)$$

$$\int_{\mathcal{M}_{N,d}} Z_{1-\text{loop}} = d \frac{1}{(N/d)^2} \frac{1}{d!} \oint_{\mathfrak{M}_d} \left(\prod_i du_i\right) \frac{1}{\Xi(0)} \prod_{a,b \in \mathfrak{C}_{N/d}} \prod_{i,j=1}^{d} \Xi\left(\frac{a+(-1)^a b\tau}{N/d} + u_i - u_j\right)$$

$$\begin{aligned} \mathbf{8}_s &\to \mathbf{1}_{+1} \oplus \mathbf{6}_0 \oplus \mathbf{1}_{-1} \\ \mathbf{8}_c &\to \mathbf{4}_{-\frac{1}{2}} \oplus \bar{\mathbf{4}}_{+\frac{1}{2}} \\ \mathbf{8}_v &\to \mathbf{4}_{+\frac{1}{2}} \oplus \bar{\mathbf{4}}_{-\frac{1}{2}}. \end{aligned}$$

$$\text{Tr}_{\mathcal{H}} (-1)^F q^{H_L} \bar{q}^{H_R} \prod_A a_A^{f_A}$$

$$\text{Tr}_{\mathcal{H}} e^{i\theta \int w_2} (-1)^F q^{H_L} \bar{q}^{H_R} \prod_A a_A^{f_A} = \sum_k \text{Tr}_{\mathcal{H}_k} e^{i\theta k} (-1)^F q^{H_L} \bar{q}^{H_R} \prod_A a_A^{f_A}.$$

$$\begin{aligned} \{X^i\} &\to \{\phi^A, \bar{\phi}_A\} \\ \{\chi^\alpha_L\} &\to \{\lambda_-, \bar{\lambda}_-, \psi^{AB}_-\} \\ \{\chi^{\dot\alpha}_R\} &\to \{\psi^A_+, \bar{\psi}_{+A}\} \end{aligned}$$

$$\begin{aligned} \Phi^A &= \phi^A + \theta^+ \psi^A_+ + \theta^+ \bar{\theta}^+ D_+ \phi^A \\ \Lambda &= \lambda_- + \theta^+ \frac{1}{\sqrt{2}} (D + iF_{09}) + \theta^+ \bar{\theta}^+ D_+ \lambda_- \\ \Psi^{A4} &= \psi^{A4}_+ + \theta^+ G^{A4} + \bar{\theta}^+ E^{A4}(\Phi) + \theta^+ \bar{\theta}^+ D_+ \psi^{A4}_+ \end{aligned}$$

$$ig\mathrm{Tr}\int d\theta^{+}\Psi^{A4}J_{A}(\Phi)\Big|_{\bar{\theta}^{+}=0}+h.c.=ig\frac{\epsilon_{ABC4}}{3!}\mathrm{Tr}\int d\theta^{+}\Psi^{A4}[\Phi^{B},\Phi^{C}]\Big|_{\bar{\theta}^{+}=0}+h.c.$$

$$ig\mathrm{Tr}\int d\theta^{2}\tilde{\Phi}^{1}[\tilde{\Phi}^{2},\tilde{\Phi}^{3}]+h.c..$$

$$\mathcal{I}_{U(1)}=Z_{\Lambda}\prod_{A}Z_{\Phi^{A}}Z_{\Psi^{A4}}=\eta(\tau)^{3}\frac{\prod_{A=1}^{3}\theta_{1}(\tau\mid\xi_{A}+\xi_{4})}{\prod_{A=1}^{4}\theta_{1}(\tau\mid\xi_{A})}$$

$$\sum_{A}\xi_{A}=0$$

$$\eta(\tau)=q^{1/24}\prod_{n=1}^{\infty}(1-q^{n})$$

$$\theta_{1}(\tau\mid u)=-iq^{1/8}z^{1/2}\prod_{n=1}^{\infty}(1-q^{n})(1-zq^{n})(1-z^{-1}q^{n-1})$$

$$Z_{1-\text{ loop}}(\tau\mid u;\xi)=\prod_{\alpha}\Xi(\tau\mid\alpha(u);\xi)$$

$$\Xi(\tau\mid u;\xi):=\frac{\theta_{1}(\tau\mid u)\prod_{A=1}^{3}\theta_{1}(\tau\mid\xi_{A}+\xi_{4}+u)}{\prod_{A=1}^{4}\theta_{1}(\tau\mid\xi_{A}+u)}$$

$$\mathcal{I}_{U(1)}(\tau\mid\xi)=-\frac{\partial}{\partial u}\Big|_{u=0}\Xi(\tau\mid u;\xi)$$

$$\begin{aligned}\Xi(\tau\mid u+a+b\tau;\xi)&=e^{-2\pi ib(2\xi_{4})}\Xi(\tau\mid u;\xi)\\ \Xi(\tau\mid u;\xi_{1}+a+b\tau,\xi_{2},\xi_{3})&=e^{2\pi ib(2u)}\Xi(\tau\mid u;\xi)\end{aligned}$$

$$\begin{aligned}&\Xi(\tau+1\mid u;\xi)=\Xi(\tau\mid u;\xi),\\ &\Xi\left(-\frac{1}{\tau}\Big|\frac{u}{\tau};\frac{\xi}{\tau}\right)=e^{\frac{\pi i}{\tau}(4u\xi_{4})}\Xi(\tau\mid u;\xi).\end{aligned}$$

$$Z_{1-\text{ loop}}\big|_{\mathcal{M}_{N,1}}=\frac{1}{N^{2}}\prod_{\substack{a,b\in\mathfrak{C}_{N}\\(a,b)\neq(0,0)}}\Xi\left(\tau\Big|\frac{a+(-1)^{a}b\tau}{N};\xi_{A}\right)$$

$$\prod_{a,b\in\mathfrak{C}_{N}}\Xi\left(\tau\Big|u+\frac{a+(-1)^{a}b\tau}{N};\xi_{A}\right)=\Xi(\tau\mid Nu;N\xi_{A}).$$

$$Z_{1-\text{ loop}}\big|_{\mathcal{M}_{N,1}}=\frac{1}{N^{2}}\lim_{u\to0}\frac{\Xi(\tau\mid Nu;N\xi_{A})}{\Xi(\tau\mid u;\xi_{A})}=\frac{1}{N}\frac{\mathcal{I}_{U(1)}(\tau\mid N\xi_{A})}{\mathcal{I}_{U(1)}(\tau\mid\xi_{A})}.$$

$$\oint_{\mathcal{M}_{N,N}}Z_{1-\text{loop}}(u)=\frac{1}{|\pi_{1}(SU(N)/\mathbb{Z}_{N})|}\frac{1}{|\mathbb{S}_{N}|}\sum_{u_{*}\in\tilde{\mathfrak{M}}_{\text{sing}^{*}}}\underset{u=u_{*}}{\text{JK-Res}}(\mathsf{Q}(u_{*}),\eta)Z_{1-\text{loop}}(u)$$

$$Z_{1-\text{ loop}} = \left(\mathcal{I}_{U(1)}\right)^{N-1} \prod_{i\neq j} \frac{\theta_1(\tau \mid u_i - u_j) \prod_{A=1}^{3} \theta_1(\tau \mid \xi_A + \xi_4 + u_i - u_j)}{\prod_{A=1}^{4} \theta_1(\tau \mid \xi_A + u_i - u_j)} \bigwedge_{i=2}^{N} du_i$$

$$H_{ij}^{A} = \{u_i - u_j + \xi_A = 0 \bmod \mathbb{Z} + \tau\mathbb{Z}\} \subset \tilde{\mathfrak{M}}$$

$$\underset{u=u_*}{\text{JK} - \text{Res}}(\text{Q}(u_*), \eta) \frac{du_1 \wedge \cdots \wedge du_r}{Q_{j_1}(u - u_*) \cdots Q_{j_r}(u - u_*)} = \begin{cases} \dfrac{1}{\left|\det\left(Q_{j_1} \cdots Q_{j_r}\right)\right|} & \text{if } \eta \in \text{Cone}\left(Q_{j_1} \cdots Q_{j_r}\right) \\ 0 & \text{otherwise.} \end{cases}$$

$$N_{ij} = \{u_i - u_j = 0 \bmod \mathbb{Z} + \tau\mathbb{Z}\}$$
$$N_{ij}^{B4} = \{u_i - u_j + \xi_B + \xi_4 = 0 \bmod \mathbb{Z} + \tau\mathbb{Z}\}$$

$$N_{ij}^{AB} = \{u_i - u_j + \xi_A + \xi_B = 0 \bmod \mathbb{Z} + \tau\mathbb{Z}\}$$

$$(u_2, u_3, u_4) = \frac{1}{2}(\xi_A - \xi_B, -\xi_A + \xi_B, \xi_A + \xi_B) + \frac{a + b\tau}{4}(1,1,1)$$

$$(v_2, v_3, v_4) = (\xi_A, \xi_B, \xi_A + \xi_B) + (a + b\tau)(1,1,1)$$

$$\underset{v=0}{\text{JK} - \text{Res}}(\text{Q}_*, \eta) \frac{\epsilon(A, B) v_4}{v_2 v_3 (v_4 - v_2)(v_4 - v_3)} \frac{dv_2 \wedge dv_3 \wedge dv_4}{4}$$

$$Q_{12} = (-1,0,0), Q_{13} = (0,-1,0), Q_{24} = (1,0,-1), Q_{34} = (0,1,-1)$$

$$\omega_{234} = \left(\frac{a}{v_2 v_3 (v_4 - v_2)} + \frac{b}{v_2 v_3 (v_4 - v_3)} + \frac{c}{v_2 (v_4 - v_2)(v_4 - v_3)} + \frac{d}{v_3 (v_4 - v_2)(v_4 - v_3)}\right)$$

$$(v_i)_{i=2,\ldots,8} = (\xi_1, \xi_2, \xi_3, \xi_1 + \xi_2, \xi_1 + \xi_3, \xi_2 + \xi_3, \xi_1 + \xi_2 + \xi_3)$$

$$H_{12}^{1}, H_{13}^{2}, H_{14}^{3}, H_{25}^{1}, H_{35}^{2}, H_{26}^{3}, H_{46}^{1}, H_{37}^{3}, H_{47}^{2}, H_{58}^{3}, H_{68}^{2}, H_{78}^{1}, H_{81}^{4}$$

$$\underset{v=0}{\text{JK} - \text{Res}}(Q_*, \eta) \frac{(v_5 + \epsilon)(v_6 + \epsilon)(v_7 + \epsilon)}{v_2 v_3 v_4 (v_5 - v_2)(v_5 - v_3)(v_6 - v_2)(v_6 - v_4)(v_7 - v_3)(v_7 - v_4)}$$
$$\times \frac{(v_8 - v_2 + \epsilon)(v_8 - v_3 + \epsilon)(v_8 - v_4 + \epsilon)}{(v_8 - v_5)(v_8 - v_6)(v_8 - v_7)(-\epsilon - v_8)} \frac{\Lambda_{i=2}^{8} dv_i}{8}$$

$$\underset{v=0}{\text{JK} - \text{Res}}(\text{Q}_*, \eta) \bigwedge_{i=2}^{8} dv_i = \underset{v_8=0}{\text{Res}} \underset{v_7=0}{\text{Res}} \ldots \underset{v_2=0}{\text{Res}}$$

$$y_i \leq y_j \ \text{ if } y_i^A \leq y_j^A$$

$$\underset{u=u_*}{\text{JK} - \text{Res}(\mathbf{Q}_*, \eta) \bigwedge du_i} = \frac{1}{N} \underset{v_N = y_N^A \xi_A}{\text{Res}} \cdots \underset{v_3 = y_3^A \xi_A}{\text{Res}} \underset{v_2 = y_2^A \xi_A}{\text{Res}} .$$

$$\oint_{\mathcal{M}_{N,N}} Z_{1-\text{loop}} = \frac{1}{N}\sum_{Y\in\mathcal{Y}_N} N^2 \underset{v_i=y_i^A\xi_A}{\text{JK-Res}}(\mathsf{Q}_*,\eta)\ Z_{1-\text{loop}}(u)$$
$$= \frac{1}{N}\sum_{Y\in\mathcal{Y}_N} \epsilon(Y)\lim_{\delta\to 0}\frac{1}{\Xi(\tau|\delta;\xi)}\prod_{i,j}\Xi(\tau|y_i^A\xi_A - y_j^A\xi_A+\delta;\xi)$$

$$c_3(Y) = \#\{y_i \in Y \mid y_i^B = 0$$
$$c_4(Y) = \#\{y_i \in Y \mid y_i^A = 0$$

$$\epsilon(Y) = (-1)^{c_3(Y)+c_4(Y)}$$

$$\frac{1}{N}\sum_{|Y|=N}\epsilon(Y)\lim_{\delta\to 0}\frac{1}{\Xi(\tau\mid\delta;\xi)}\prod_{i,j}\Xi(\tau\mid y_i^A\xi_A - y_j^A\xi_A+\delta;\xi) = \frac{1}{N}\sum_{s|N} s\,\frac{\mathcal{I}_{U(1)}\left(\tau\middle|\frac{N}{s}\xi\right)}{\mathcal{I}_{U(1)}(\tau\mid\xi)}$$

$$\int_{\mathcal{M}_{N,d}} Z_{1-\text{loop}} = \frac{d^2}{N}\frac{\mathcal{I}_{U(1)}(\tau|\frac{N}{d}\xi)}{\mathcal{I}_{U(1)}(\tau|\xi)}\frac{1}{d!}\oint_{\mathfrak{M}_d}\left(\prod_i du_i\right)\left(\frac{N}{d}\mathcal{I}_{U(1)}\left(\tau|\tfrac{N}{d}\xi\right)\right)^{d-1}\prod_{\substack{i,j=1\\ i\neq j}}^{d}\Xi(\tau|\tfrac{N}{d}(u_i-u_j);\tfrac{N}{d}\xi)$$

$$\int_{\mathcal{M}_{N,d}} Z_{1-\text{ loop}} = \frac{1}{N}\frac{\mathcal{I}_{U(1)}\left(\tau\middle|\frac{N}{d}\xi\right)}{\mathcal{I}_{U(1)}(\tau\mid\xi)}\sum_{s|d} s\,\frac{\mathcal{I}_{U(1)}\left(\tau\middle|\frac{d}{s}\frac{N}{d}\xi\right)}{\mathcal{I}_{U(1)}\left(\tau\middle|\frac{N}{d}\xi\right)} = \frac{1}{N}\sum_{s|d} s\,\frac{\mathcal{I}_{U(1)}\left(\tau\middle|\frac{N}{s}\xi\right)}{\mathcal{I}_{U(1)}(\tau\mid\xi)}$$

$$\mathcal{I}^{\theta}_{SU(N)/\mathbb{Z}_N}(\tau\mid\xi) = \frac{1}{N}\sum_{k=1}^{N} e^{i\theta k}\sum_{s|\gcd(k,N)} s\,\frac{\mathcal{I}_{U(1)}\left(\tau\middle|\frac{N}{s}\xi\right)}{\mathcal{I}_{U(1)}(\tau\mid\xi)}$$

$$\mathcal{I}^{\theta=\frac{2\pi M}{N}}_{SU(N)/\mathbb{Z}_N}(\tau\mid\xi) = \sum_{s|D}\frac{\mathcal{I}_{U(1)}(\tau\mid s\xi)}{\mathcal{I}_{U(1)}(\tau\mid\xi)} = \frac{\mathcal{I}_D}{\mathcal{I}_1}(\tau\mid\xi)$$

$$\mathcal{I}_{SU(N)} = \sum_{s|N} s\,\frac{\mathcal{I}_{U(1)}\left(\tau\middle|\frac{N}{s}\xi\right)}{\mathcal{I}_{U(1)}(\tau\mid\xi)} = \sum_{k=1}^{N}\frac{\mathcal{I}_{\gcd(k,N)}}{\mathcal{I}_1}(\tau\mid\xi)$$

$$\mathcal{I}^{\theta=\frac{2\pi M}{N}}_{SU(N)/\mathbb{Z}_K}(\tau\mid\xi) = \frac{1}{K}\sum_{k=1}^{N/K} e^{i\theta kK}\sum_{s|\gcd(kK,N)} s\,\frac{\mathcal{I}_{U(1)}\left(\tau\middle|\frac{N}{s}\xi\right)}{\mathcal{I}_{U(1)}(\tau\mid\xi)}$$
$$= \sum_{k\equiv M}\ \sum_{(\text{mod}K)}\frac{\mathcal{I}_{U(1)}(\tau\mid s\xi)}{\mathcal{I}_{U(1)}(\tau\mid\xi)}$$
$$= \sum_{k\equiv M(\text{mod}K)}\frac{\mathcal{I}_{\gcd(k,N)}}{\mathcal{I}_1}(\tau\mid\xi)$$

$$\mathcal{I}_{U(N)}(\tau\mid\xi) = \mathcal{I}_{U(1)}\mathcal{I}_{SU(N)}(\tau\mid\xi) = \sum_{k=1}^{N}\mathcal{I}_{\gcd(k,N)}(\tau\mid\xi)$$

$$\mathcal{I}_{U(N)+B}^{M}(\tau \mid \xi) = \mathcal{I}_{U(1)}^{\tilde{c}_1=M} \mathcal{I}_{SU(N)}^{\theta=\frac{2\pi M}{N}}(\tau \mid \xi) = \mathcal{I}_D(\tau \mid \xi)$$

$$\mathcal{I}_{U(N)+B}(\tau \mid \xi) = \sum_{M\in\mathbb{Z}} e^{iM\phi} \mathcal{I}_D(\tau \mid \xi)$$

$$(\mathbf{8}_v, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8}_s, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{8}_c).$$

$$Z_1(\tau \mid \xi_A, \tilde{\zeta}_{\tilde{A}}) = \mathrm{Tr}_{RR}(-1)^F q^{H_L} \bar{q}^{H_R} \prod_{A=1}^{4} a_A^{K_{b,A}} \prod_{\tilde{A}=1}^{4} b_{\tilde{A}}^{K_{l,\tilde{A}}} = \frac{\theta_1(\tilde{\zeta}_1)\theta_1(\tilde{\zeta}_2)\theta_1(\tilde{\zeta}_3)\theta_1(\tilde{\zeta}_4)}{\theta_1(\xi_1)\theta_1(\xi_2)\theta_1(\xi_3)\theta_1(\xi_4)}$$

$$a_A = e^{2\pi i \xi_A}, \tilde{b}_{\tilde{A}} = e^{2\pi i \tilde{\zeta}_{\tilde{A}}}$$

$$K_{l,\tilde{A}} = M_{\tilde{A}}^{A} K_{l,A}, \text{ where } M_{\tilde{A}}^{A} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 \end{pmatrix}$$

$$Z_1(\tau \mid \xi_B') = \mathrm{Tr}_{RR}(-1)^F q^{H_L} \bar{q}^{H_R} \prod_{B=1}^{3} a_B'^{K_B'}$$

$$Z_1(\tau \mid \xi_A) = \frac{\theta_1\left(\frac{\xi_1+\xi_2+\xi_3+\xi_4}{2}\right)\theta_1\left(\frac{\xi_1-\xi_2-\xi_3+\xi_4}{2}\right)\theta_1\left(\frac{-\xi_1+\xi_2-\xi_3+\xi_4}{2}\right)\theta_1\left(\frac{-\xi_1-\xi_2+\xi_3+\xi_4}{2}\right)}{\theta_1(\xi_1)\theta_1(\xi_2)\theta_1(\xi_3)\theta_1(\xi_4)}.$$

$$K_B' = M_{B+1}^{A} K_A, B = 1,2,3$$

$$\xi_1 + \xi_2 + \xi_3 + \xi_4 = 0$$

$$b_1 = \exp\left(2\pi i \tilde{\zeta}_1\right) = \exp\left(2\pi i \frac{\xi_1+\xi_2+\xi_3+\xi_4}{2}\right) = \sqrt{a_1 a_2 a_3 a_4}$$

$$\mathcal{I}_1(\tau \mid \xi_A) := -\frac{\partial}{\partial b_1} Z_1(\tau \mid \xi_A)\Big|_{b_1=1}$$
$$= \frac{\eta^3(\tau)\theta_1(\tau \mid \xi_1+\xi_4)\theta_1(\tau \mid \xi_2+\xi_4)\theta_1(\tau \mid \xi_3+\xi_4)}{\theta_1(\tau \mid \xi_1)\theta_1(\tau \mid \xi_2)\theta_1(\tau \mid \xi_3)\theta_1(\tau \mid \xi_4)}$$

$$\mathcal{I}(\tau \mid \xi_A) = -\frac{\partial}{\partial b_1}\Big|_{b_1=1} \mathrm{Tr}_{RR}(-1)^F q^{H_L} \bar{q}^{H_R} \prod_{A=1}^{4} a_A^{K_A}$$
$$= \mathrm{Tr}_{RR}(-1)^F J_R q^{H_L} \bar{q}^{H_R} \prod_{B=1}^{3} a_B^{K_B'}.$$

$$\mathcal{I}_1(\tau \mid \xi + \Omega \cdot n) = \mathcal{I}_1(\tau \mid \xi)$$

$$\Omega = \begin{pmatrix} 1 & \tau & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \tau & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \tau \end{pmatrix}$$

$$\mathcal{I}_1\left(\frac{a\tau+b}{c\tau+d} \,\middle|\, \frac{\xi_A}{c\tau+d}\right) = \mathcal{I}_1(\tau \mid \xi_A), \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

$$\mathcal{J}_1(\tau \mid \xi_A) = \sum_m q^m f_m(\xi) = \sum_{m\geq 0,l} c(m,l) q^m \prod_A a_A^{l_A}$$

$$\mathcal{J}_1|_{q=0}(\xi) := \mathcal{J}_1(\tau = i\infty \mid \xi) = \frac{(1-a_1a_2)(1-a_1a_3)(1-a_2a_3)}{(1-a_1)(1-a_2)(1-a_3)(1-a_1a_2a_3)}$$
$$= 1 + \frac{a_1}{1-a_1} + \frac{a_2}{1-a_2} + \frac{a_3}{1-a_3} - \frac{a_1a_2a_3}{1-a_1a_2a_3}$$

$$\mathcal{J}_1(\tau \mid \xi) = \mathcal{J}_1|_{q=0}(\xi) + \sum_{m=1}^{\infty} q^m \sum_{s|m} \chi(s\xi)$$

$$\chi(\xi_A) = \chi_{\square}(\xi_A) - \chi_{\wedge^3\square}(\xi_A) = a_1 + a_2 + a_3 + a_4 - \frac{1}{a_1} - \frac{1}{a_2} - \frac{1}{a_3} - \frac{1}{a_4}.$$

$$\sum_{m\geq 0,l_A} c(m,l_1,l_2,l_3) q^m a_1^{l_1} a_2^{l_2} a_3^{l_3} = \sum_{m\geq 0,l_A} c(m,l_A) q^{m+l_1} a_1^{l_1} a_2^{l_2} a_3^{l_3}$$

$$c(m,l_1,l_2,l_3) = \begin{cases} 0 & \text{if } m > 0 \text{ and, } m + l_A < 0 \text{ or } m - l_A < 0 \\ c(m + l_A, l_1, l_2, l_3) & \text{if } m + l_A > 0 \end{cases}$$

$$\mathcal{J}_1|_{q=0}(\xi) = \sum_{l_1\leq 0,l_2,l_3} c(-l_1,l_1,l_2,l_3) a_1^{l_1} a_2^{l_2} a_3^{l_3}$$

$$c(m,l_1,l_2,l_3) = \begin{cases} c(0,l_1,l_2,l_3) & \\ \tilde{c}(0,l_1,l_2,l_3) & \\ c(m,0,0,0) & \\ 0 & \text{otherwise.} \end{cases}$$

$$Z_N = \frac{1}{|\mathbb{S}_N|} \sum_{gh=hg} (Z_1^N)^{g,h}$$

$$\mathcal{Z} := 1 + \sum_{N\geq 1} p^N Z_N(q,\vec{a}) = \prod_{n>0,m\geq 0,\vec{l}} \frac{1}{(1 - p^n q^m \vec{a})^{c(nm,\vec{l})}}$$

$$\log \mathcal{Z} = \sum_{M=1}^{\infty} p^M T_M Z_1$$

$$Z_N = T_N Z_1 + \cdots + \frac{1}{N!} (T_1 Z_1)^N$$

$$T_M Z_1(\tau \mid \vec{\xi}) := \frac{1}{M} \sum_{d|M,d=1}^{M} \sum_{b=0}^{M/d-1} Z_1\left(\frac{d\tau + b}{M/d}\middle| d\vec{\xi}\right)$$

$$\mathcal{J}_N := -\frac{\partial}{\partial b_1}\bigg|_{b_1=1} Z_N$$

$$\mathcal{I}_N = -\frac{\partial}{\partial b_1}\Big|_{b_1=1} T_N Z_1 = \frac{1}{N}\sum_{d|N}\sum_{b=0}^{N/d-1} d\mathcal{I}_1\left(\frac{d\tau+b}{N/d}\Big| d\xi_A\right)$$

$$\mathcal{I}_N = \sum_{d|N} \mathcal{I}_1(\tau \mid d\xi_A)$$

$$\frac{1}{N}\sum_{d|N}\sum_{b=0}^{N/d-1} d\mathcal{I}_1\left(\frac{d\tau+b}{N/d}\Big| d\xi_A\right) = \sum_{d|N} \mathcal{I}_1\Big|_{q=0}(d\xi_A) + \sum_{d|N}\sum_{m=1}^{\infty} q^{dm} \sum_{s|\frac{N}{d}m} \chi(sd\xi_A)$$
$$= \sum_{d|N} \mathcal{I}_1(\tau = i\infty \mid d\xi_A) + \sum_{k=1}^{\infty} q^k \sum_{d'|N}\sum_{s'|k} \chi(s'd'\xi_A)$$
$$= \sum_{d|N} \mathcal{I}_1(\tau \mid d\xi_A)$$

$$\mathcal{Z}_0(\tau,\sigma \mid \xi) = 1 + \sum_{N\geq 1} p^N \mathcal{I}_N(\tau \mid \xi)$$

$$\int d^{10}x \operatorname{Tr}\left(-\frac{1}{4}F_{MN}F^{MN} + \frac{i}{2}\bar{\Theta}\Gamma^M D_M \Theta\right)$$

$$D_M = \partial_M + ig[A_M,\cdot]$$
$$F_{MN} = \frac{1}{ig}[D_M, D_N] = \partial_M A_N - \partial_N A_M + ig[A_M, A_N]$$

$$\mathcal{L} = \operatorname{Tr}\left(-\frac{1}{2}(D_\mu X^i)^2 + i\chi^T \not{D}\chi - \frac{1}{4}F_{\mu\nu}^2 + \frac{g^2}{4}[X^i, X^j]^2 - \sqrt{2}g\chi_L^T\gamma_i[X^i, \chi_R]\right)$$

$$g_{MN} = \eta_{\mu\nu} \oplus \delta_{ij}$$

$$\Gamma^0 = \sigma_2 \otimes I_{16} \qquad \gamma^i = \begin{pmatrix} 0 & \beta_i \\ \beta_i^T & 0 \end{pmatrix}$$
$$\Gamma^i = i\sigma_1 \otimes \gamma^i$$
$$\Gamma^9 = i\sigma_1 \otimes \gamma^9, \quad \gamma^9 = \begin{pmatrix} I_8 & 0 \\ 0 & -I_8 \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S_{\mathrm{MSYM}_2} = \int dx^2 \operatorname{Tr}\Big(-\frac{1}{2}(D_\mu X^i)^2 + \frac{i}{2}\chi_L^T(D_0 + D_9)\chi_L + \frac{i}{2}\chi_R^T(D_0 - D_9)\chi_R - \frac{1}{4}F_{\mu\nu}^2$$
$$+ \frac{g^2}{4}[X^i, X^j]^2 - g\chi_L^\alpha \gamma^i_{\alpha\dot{\beta}}\left[X^i, \chi_R^{\dot{\beta}}\right]\Big)$$

$$\delta A_M = i\bar{\varepsilon}\Gamma_M\Theta$$
$$\delta\Theta = \Gamma_{MN}F^{MN}\varepsilon.$$

$$\begin{aligned} \delta A_\mu &= i\varepsilon^T \Gamma^0 \Gamma_\mu \chi \\ \delta X^i &= i\varepsilon_L^\alpha \gamma^i_{\alpha\dot{\alpha}} \chi_R^{\dot{\alpha}} + i\varepsilon_R^{\dot{\alpha}} \gamma^i_{\dot{\alpha}\alpha} \chi_L^\alpha \\ \delta \chi_L^\alpha &= 4c\left[\left(+F_{09}\delta_{\alpha\beta} - \frac{ig}{2}\left[X_i, X_j\right]\gamma^i_{\alpha\dot{\rho}}\gamma^j_{\dot{\rho}\beta}\right)\varepsilon_L^\beta + (D_0 + D_9) X_i \gamma^i_{\alpha\dot{\beta}} \varepsilon_R^{\dot{\beta}}\right] \\ \delta \chi_R^{\dot{\alpha}} &= 4c\left[\left(-F_{09}\delta_{\dot{\alpha}\dot{\beta}} - \frac{ig}{2}\left[X_i, X_j\right]\gamma^i_{\dot{\alpha}\rho}\gamma^j_{\rho\dot{\beta}}\right)\varepsilon_R^{\dot{\beta}} + (D_0 - D_9) X_i \gamma^i_{\dot{\alpha}\beta} \varepsilon_L^\beta\right] \end{aligned}$$

$$r := \frac{1}{2}(e_1 + e_2 + e_3 + e_4)$$

$$\begin{aligned} \mathbf{8}_s &\to \mathbf{1}_{+1} \oplus \mathbf{6}_0 \oplus \mathbf{1}_{-1} \\ \mathbf{8}_c &\to \mathbf{4}_{-\frac{1}{2}} \oplus \bar{\mathbf{4}}_{+\frac{1}{2}} \\ \mathbf{8}_v &\to \mathbf{4}_{+\frac{1}{2}} \oplus \bar{\mathbf{4}}_{-\frac{1}{2}} \end{aligned}$$

$$\left(\varepsilon_L^{\dot{\alpha}}, \varepsilon_R^\alpha\right) = \left(\varepsilon_L^A, (\varepsilon_L)_A, \varepsilon_R^\pm, \varepsilon_R^{AB}\right)$$

$$l := \frac{1}{2}(e_1 + e_2 + e_3 - e_4)$$

$$R_V = r + l = e_1 + e_2 + e_3, R_A = r - l = e_4$$

$$\begin{aligned} \mathbf{8}_s &\to \mathbf{1}_{+1,+\frac{1}{2}} \oplus \mathbf{3}_{0,+\frac{1}{2}} \oplus \bar{\mathbf{3}}_{0,-\frac{1}{2}} \oplus \mathbf{1}_{-1,-\frac{1}{2}} \\ \mathbf{8}_c &\to \mathbf{3}_{-\frac{1}{2},0} \oplus \mathbf{1}_{-\frac{1}{2},-1} \oplus \bar{\mathbf{3}}_{+\frac{1}{2},0} \oplus \mathbf{1}_{+\frac{1}{2},+1} \\ \mathbf{8}_v &\to \mathbf{3}_{+\frac{1}{2},+\frac{1}{2}} \oplus \mathbf{1}_{+\frac{1}{2},-\frac{1}{2}} \oplus \bar{\mathbf{3}}_{-\frac{1}{2},-\frac{1}{2}} \oplus \mathbf{1}_{-\frac{1}{2},+\frac{1}{2}} \end{aligned}$$

$$\mathcal{O}_M(U) \cong C^\infty(\hat{U}) \otimes \wedge (\mathbb{R}^n)^\vee$$

$$\mathbf{sMfd}_A \to \mathbf{sMfd}_{\tilde{A}}, (M, \sigma, A) \mapsto (M, \sigma \circ \varphi, \tilde{A})$$

$$X(f) \circ \phi_t^X = \frac{\mathrm{d}}{\mathrm{d}t} f \circ \phi_t^X$$

$$\mathcal{T}_U = \mathrm{span}_{\mathcal{O}_U}\left\{\frac{\partial}{\partial x^a}, \frac{\partial}{\partial \xi^\alpha}\right\} = \mathrm{span}_{\mathcal{O}_U}\left\{\frac{\partial}{\partial X_A}\right\}$$

$$X_A = \begin{cases} x^a & \text{if } A = a \\ \xi^\alpha & \text{if } A = \alpha \end{cases}$$

$$\phi_t^{\frac{\partial}{\partial X^B}} =: \phi_t^B$$

$$\phi_t^B : U \to U, (X^1, \ldots, X^{B-1}, X^B, X^{B+1}, \ldots) \mapsto (X^1, \ldots, X^{B-1}, X^B + t, X^{B+1}, \ldots)$$

$$\mathcal{T}_M = \mathcal{B} \oplus \mathcal{F},$$

$$\wedge^2_{\mathcal{O}_M} \mathcal{F} \to \mathcal{B}, (X \wedge Y) \mapsto [X, Y] \bmod \mathcal{F}$$

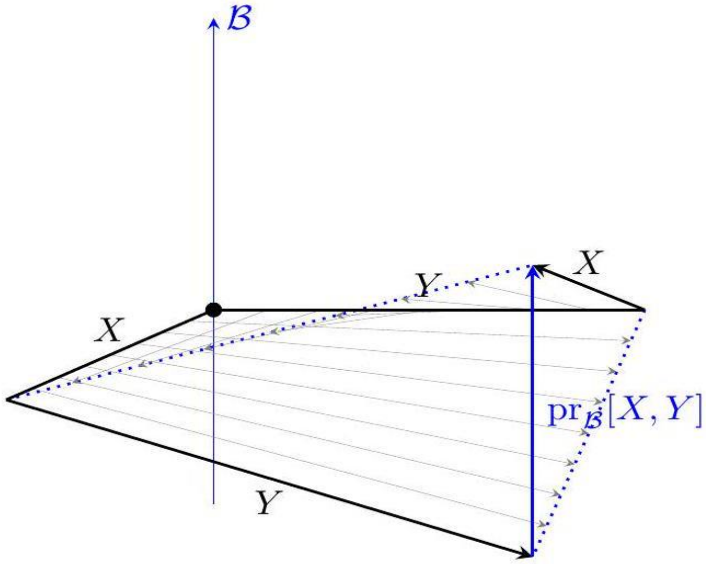
$$\mathbf{lfVB}_M \overset{\Gamma(M,-)}{\to} \mathbf{lfMod}_{\mathcal{O}_M}, E \mapsto \Gamma(M, E) =: \mathcal{E}$$

$$\mathcal{B} = \Gamma(M, B), \mathcal{F} = \Gamma(M, F).$$

$$\wedge^2_{\mathcal{O}_M} \mathcal{F} \to \mathcal{T}_M/\mathcal{F}, X \wedge Y \mapsto [X, Y] \bmod \mathcal{F}.$$

$$\mathcal{T}_M|_{N_\alpha} \cong \mathcal{O}_M\big|_{N_\alpha} \otimes \mathcal{T}_{N_\alpha}.$$

$$f_F: \wedge^2_{\mathcal{O}_M} \mathcal{F} \to \mathcal{B}, X \wedge Y \mapsto \mathrm{pr}_{\mathcal{B}}([X, Y]).$$



$$\Omega^r(M) = \bigoplus_{p+q=r} \Omega^{p|q}(M), \Omega^{p|q}(M) = \wedge^p_{\mathcal{O}_M} \mathcal{B}^\vee \hat{\otimes} \wedge^q_{\mathcal{O}_M} \mathcal{F}^\vee,$$

$$Q^B \times_M Q^F \to \mathrm{Fr}(TM) \to M.$$

$$p_B: \mathbb{R}^{m|0} \times U \to B|_U, p_F: \mathbb{R}^{0|n} \times U \to F|_U.$$

$$\begin{aligned} \theta_{(p_B, p_F)} &= (p_B, p_F)^{-1} \circ \,\mathrm{d}\pi = p_B^{-1} \circ \mathrm{pr}_B^* \,\mathrm{d}\pi \oplus p_{\mathcal{F}}^{-1} \circ \mathrm{pr}_F^* \,\mathrm{d}\pi \\ &= \theta^B_{p_B} \oplus \theta^F_{p_F} \in \Omega^1\big(Q^B \times_M Q^F, \mathbb{R}^{m|0} \oplus \mathbb{R}^{0|n}\big)^{\mathrm{GL}(m)\times\mathrm{GL}(n)}_{\mathrm{hor}} \end{aligned}$$

$$\mathrm{pr}_{\mathbb{R}^{m|0}}(\Theta^\varphi(X, Y)) = \iota_{[X,Y]}\theta^B$$

$$\mathrm{pr}_{\mathbb{R}^{m|0}}\Theta^\varphi(X, Y) = \iota_X \iota_Y \,\mathrm{d}^\varphi \theta^B = \iota_X L_Y \theta^B = \iota_{[X,Y]}\theta^B$$

$$\begin{gathered} \tilde{\gamma}(v) \circ \gamma(w) + \tilde{\gamma}(w) \circ \gamma(v) = q(v, w)\mathrm{id}_{S^\vee}, \\ \gamma(v) \circ \tilde{\gamma}(w) + \gamma(w) \circ \tilde{\gamma}(v) = q(v, w)\mathrm{id}_S. \end{gathered}$$

$$[v, w] = 0, [v, s] = 0, [s, t] = \Gamma(s, t)$$

$$Q^F \to Q^B \times_M Q^F \to \mathrm{Fr}(TM)$$

$$\mathrm{Spin}(V, q) \overset{\alpha\times\mathrm{id}}{\to} \mathrm{SO}(V, q) \times \mathrm{Spin}(V, q) \to \mathrm{GL}(m \mid n),$$

$$f_F: \wedge^2 F \overset{[-,-]}{\to} TM \overset{\mathrm{pr}_B}{\to} B,$$

$$p{:}\,\Pi S\times U\xrightarrow{\sim}F\big|_U,\alpha(p){:}\,V\times U\xrightarrow{\sim}B\big|_U.$$

$$\wedge^2\,(F|_U)\overset{f_F}{\to}B\Big|_U\quad\downarrow^{\hat{\alpha}(p)^{-1}}$$
$$U\times\wedge^2\,(\Pi S)^{\hat{\alpha}(p)^{-1}(f)}U\times V.$$

$$C_F\in\Omega^2(Q,V)_{\text{hor}}^{\text{Spin}(V,q)}$$

$$\text{Hom}(\wedge^2\,(\mathcal{F}),\mathcal{B})\cong\wedge^2\,(\mathcal{F})^\vee\otimes_{\mathcal{O}_M}\mathcal{B}$$

$$T\check{M}\cong\iota^*B\cong B_{\iota(\check{M})}$$

$$S\cong\bigoplus_{i=1}^{k}S_0$$

$$S\cong\bigoplus_{i=1}^{k^+}S_0^+\oplus\bigoplus_{i=1}^{k^-}S_0^-$$

$$\gamma(V)\circ\tilde{\gamma}(W)+(-1)^{p(V)p(W)}\gamma(W)\circ\tilde{\gamma}(V)=2g(V,W)\cdot\text{id}_{\mathcal{F}},$$
$$\tilde{\gamma}(V)\circ\gamma(W)+(-1)^{p(V)p(W)}\tilde{\gamma}(W)\circ\gamma(V)=2g(V,W)\cdot\text{id}_{\mathcal{F}}\vee.$$

$$\text{Spin}^{\mathbb{F}}(V,q){:}=\big(\text{Spin}(V,q)\times K^{\mathbb{F}}\big)/\{\pm1\}.$$

$$1\longrightarrow K\longrightarrow\text{Spin}^{\mathbb{F}}(V,q)\overset{\alpha}{\to}\text{SO}(V,q)\longrightarrow 1.$$

$$\hat{Q}\to Q^B\times_M\hat{Q}\to\text{Fr}(TM)$$

$$\text{Spin}^{\mathbb{F}}(V,q)\overset{\alpha\times\text{id}}{\to}\text{SO}(V,q)\times\text{Spin}^{\mathbb{F}}(V,q)\to\text{GL}(m\mid n).$$

$$\Gamma\in\Omega^{0|2}(\hat{Q},V)_{\text{hor}}^{\text{Spin}^{\mathbb{F}}(V,q)}.$$

$$\hat{Q}=\big(Q\times_M K^{\mathbb{F}}\big)/\{\pm1\}\to M.$$

$$\hat{Q}=\big(Q\times_M Q^{\mathbb{F}}\big)/\{\pm1\}\to M$$

$$\mathcal{A}_{\hat{Q}}\cong\Omega^{2|0}(\hat{Q},V)_{\text{hor}}^{\text{Spin}^{\mathbb{F}}(V,q)}\times\mathcal{A}_{Q^{\mathbb{F}}},$$

$$\mathcal{A}_Q\xrightarrow{\sim}\Omega^{2|0}(\hat{Q},V)^{\text{Spin}^{\mathbb{F}}(V,q)}\cong\Omega^{2|0}(Q,V)^{\text{SO}(V,q)}.$$

$$0\longrightarrow\hat{K}\longrightarrow\Omega^1(Q^B,\mathfrak{so}(V,q))_{\text{hor}}^{\text{SO}(V,q)}\overset{\hat{\alpha}}{\to}\Omega^2(Q^B,V)_{\text{hor}}^{\text{SO}(V,q)}\longrightarrow\hat{C}\longrightarrow 0,$$

$$0\longrightarrow K\longrightarrow V^\vee\otimes\mathfrak{so}(V,q)\overset{\alpha}{\to}\wedge^2\,(V^\vee)\otimes V\longrightarrow C\longrightarrow 0,$$

$$\text{id}_{V^\vee}\otimes\rho_*{:}\,V^\vee\otimes\mathfrak{so}(V,q)\to V^\vee\otimes\text{End}(V)=V^\vee\otimes V^\vee\otimes V$$

$$\mathcal{A}_{Q^B}\to\Omega^2(Q^B,V)_{\text{hor}}^{\text{SO}(V,q)},\tilde{\varphi}\mapsto\text{d}^{\tilde{\varphi}}\theta_{\mathcal{B}}$$

$$\Psi: \mathcal{A}_Q \to \Omega^{2|0}(Q,V)_{\text{hor}}^{\text{SO}(V,q)}, \varphi \mapsto (\widetilde{\text{pr}}_{\mathcal{B}})^* \, \text{d}^{\varphi}\theta_{\mathcal{B}}$$

$$\mathcal{A}_Q^s \cong \mathcal{A}_{Q^{\mathbb{F}}}$$

$$\text{vol} \in \mathcal{B}er_M$$

$$\mathcal{F}^{\mathbb{C}} = \mathcal{F}^{1,0} \oplus \mathcal{F}^{0,1}$$

$$\Omega^{p|q,r}(M) = \wedge^p \left(\mathcal{B}^{\mathbb{C}}\right)^{\vee} \otimes_{\mathcal{O}_M} \wedge^q (\mathcal{F}^{1,0})^{\vee} \otimes_{\mathcal{O}_M} \wedge^r (\mathcal{F}^{0,1})^{\vee}$$

$$\text{d}: \mathcal{O}_M \otimes \mathbb{C} \to \Omega^1(M,\mathbb{C}) = \left(\mathcal{B}^{\mathbb{C}}\right)^{\vee} \oplus (\mathcal{F}^{1,0})^{\vee} \oplus (\mathcal{F}^{0,1})^{\vee}$$
$$\text{d} = \text{d}_{\mathcal{B}} \oplus \partial \oplus \bar{\partial}$$

$$\text{d}^{\varphi^s} = \text{d}_B^{\varphi^s} \oplus \partial^{\varphi^s} \oplus \bar{\partial}^{\varphi^s}$$

$$\partial^{\varphi^s}: \Omega^{\cdot}(M, F^{1,0}) \to \Omega^{\cdot+1}(M, F^{1,0})$$

$$\mathcal{O}_M^{\omega} := \{f \in \mathcal{O}_M \otimes \mathbb{C}: Xf = 0 \text{ for all } X \in \Omega^0(M, \mathcal{F}^{0,1})\}$$

$$\mathcal{O}_M^{\bar{\omega}} := \{f \in \mathcal{O}_M \otimes \mathbb{C}: Xf = 0 \text{ for all } X \in \Omega^0(M, \mathcal{F}^{1,0})\}$$

$$\bar{\partial}^{\varphi^s}(fX) = \bar{\partial}^{\varphi^s}(f)X + f\bar{\partial}^{\varphi^s}X = 0,$$

$$\mathcal{B}|_{i(\check{M})} = \mathcal{T}_{\check{M}}.$$

$$i^*Q^B \times_{\check{M}} i^*Q^F \to \check{M}.$$

$$i^*Q^F \to i^*Q^B \to \check{M}.$$

$$S\check{M} = i^*Q^F \times_{\text{Spin}^{\mathbb{F}}(V,q)} \Pi S \to \check{M}.$$

$$\Omega^0(\check{M}, S\check{M}) \xrightarrow{\sim} \Omega^0(i^*Q, \Pi S)^{\text{Spin}(V,q)} \xrightarrow{\langle i^*\theta^{\mathcal{F}}, \cdot \rangle} i^*\mathcal{F}.$$

$$\check{Q} \longrightarrow \text{Fr}^{\text{SO}}(T\check{M}) \longrightarrow \check{M},$$

$$M := \Pi S\check{M},$$

$$\tau: \Pi S\check{M} = \check{Q} \times_{\text{Spin}(V,q)} \Pi S \to \check{M}$$

$$0 \longrightarrow V\check{Q} \xrightarrow{\varphi} T\check{Q} \xrightarrow{\text{d}\check{\pi}} \check{\pi}^* T\check{M} \longrightarrow 0.$$

$$i: \check{M} \to M.$$

$$\psi: Q \to \check{Q} \times \Pi S$$

$$R'_g(q,s) = (qg, g^{-1}s), g \in \text{Spin}(V,q), q \in \check{Q}, s \in \Pi S$$

$$\pi': \check{Q} \times \Pi S \to \Pi S\check{M}, (q,s) \mapsto [(q,s)], (q,s) \sim (q \cdot g, g^{-1}s).$$

$$
\begin{array}{ccc}
\tau^*\check{Q} & \longrightarrow & \check{Q} \\
\downarrow & & \downarrow \\
\Pi S\check{M} & \xrightarrow{\ \tau\ } & \check{M}
\end{array},
$$

$$\check{Q} \times_{\check{M}} \Pi SM \cong \check{Q} \times_{\check{M}} \left(\check{Q} \times_{\mathrm{Spin}(V,q)} \Pi S\right).$$

$$\psi : \check{Q} \times_{\check{M}} \left(\check{Q} \times_{\mathrm{Spin}(V,q)} \Pi SM\right) \to \check{Q} \times \Pi S, (q, [(q', s')]) = (q, [(q, s)]) \mapsto (q, s).$$

$$R_g\left((q, [(q', s')]) = (q \cdot g, [(q, s)]) = \left(q \cdot g, [(q \cdot g, g^{-1}s]\right) \mapsto (q \cdot g, g^{-1}s) =: R'_g(q, s).$$

$$\check{Q} \times \Pi S \to \check{Q} \times\ _{\mathrm{Spin}(V,q)} \Pi S = \Pi S\check{M}, (q, s) \mapsto [(q, s)]$$

$$
\begin{array}{ccccc}
\check{Q} \times_{\check{M}} \Pi S\check{M} \times \mathrm{Spin}(V,q) & \xrightarrow{\ R\ } & \check{Q} \times_{\check{M}} \Pi S\check{M} & \longrightarrow & \Pi S\check{M}. \\
\Big\downarrow{\scriptstyle \psi\times\mathrm{id}} & & \Big\downarrow{\scriptstyle \psi} & \nearrow{\scriptstyle \mathrm{mod}\ \mathrm{Spin}(V,q)} & \\
\check{Q} \times \Pi S \times \mathrm{Spin}(V,q) & \xrightarrow{\ R'\ } & \check{Q} \times \Pi S & &
\end{array}
$$

$$
\begin{array}{ccc}
\check{Q} \times \Pi S & \xrightarrow{\ \mathrm{pr}_{\check{Q}}\ } & \check{Q} \\
\Big\downarrow{\scriptstyle q} & & \Big\downarrow{\scriptstyle \pi} \\
\Pi S\check{M} & \xrightarrow{\ \tau\ } & \check{M}.
\end{array}
$$

$$\check{Q} \overset{\mathrm{pr}_{\check{Q}}}{\longleftrightarrow} \check{Q} \times \Pi S \overset{\mathrm{pr}_{\Pi S}}{\to} \Pi S,$$

$$
\begin{array}{ccccccccc}
0 & \longrightarrow & \check{Q} \times T\Pi S & \longrightarrow & T\check{Q} \times T\check{\Pi} S & \longrightarrow & T\check{Q} \times \Pi S & \longrightarrow & 0 \\
 & & \Big\downarrow{\scriptstyle \mathrm{pr}^*_{\Pi S}\,\mathrm{d}q} & & \Big\downarrow{\scriptstyle \mathrm{d}q} & & {\scriptstyle \mathrm{d}\pi\times\mathrm{id}}\Big\downarrow \ \Big\uparrow{\scriptstyle \varphi\times\mathrm{id}} & & \\
 & & & & & & \pi^*T\check{M} \times \Pi S & & \\
 & & & & & & \Big\downarrow{\scriptstyle q} & & \\
0 & \longrightarrow & \tau^*\Pi S\check{M} & \longrightarrow & T\Pi S\check{M} & \xrightarrow{\ \mathrm{d}\tau\ } & \tau^*T\check{M} & \longrightarrow & 0.
\end{array}
$$

$$\nabla^{\varphi} : \tau^*T\check{M} \to T\Pi S\check{M}$$

$$\mathcal{E}_S = \sum_i s^i \frac{\partial}{\partial s^i}.$$

$$\check{Q} \times T\Pi S \overset{\mathrm{pr}^*_{\Pi S}\,\mathrm{d}q}{\to} \tau^*\Pi S\check{M} \hookrightarrow T\Pi S\check{M}.$$

$$\delta_{\mathcal{F}}: \tau^*\Pi S\check{M} \to T\Pi S\check{M}, X \mapsto X + \frac{1}{2}\nabla^{\varphi}_{\Gamma(\mathcal{E},X)},$$

$$\mathcal{F} := \Omega^0\left(\Pi S\check{M}, \delta_{\mathcal{F}}(\tau^*\Pi S\check{M})\right) \to \mathcal{T}_{\Pi S\check{M}}.$$

$$[\delta_{\mathcal{F}}(X), \delta_{\mathcal{F}}(Y)]|_{\check{M}} = \left[X + \frac{1}{2}\nabla^{\varphi}_{\Gamma(\mathcal{E},X)}, Y + \frac{1}{2}\nabla^{\varphi}_{\Gamma(\mathcal{E},Y)}\right]\Big|_{\check{M}}$$

$$f_\mu = \frac{\partial}{\partial s^\mu} =: \partial_\mu$$

$$\left[\partial_\mu + \frac{1}{2}\nabla^{\varphi}_{\Gamma(\mathcal{E},\partial_\mu)}, \partial_\nu + \frac{1}{2}\nabla^{\varphi}_{\Gamma(\mathcal{E},\partial_\nu)}\right] = \frac{1}{2}\left[\partial_\mu, \nabla^{\varphi}_{\Gamma(\mathcal{E},\partial_\nu)}\right] + \frac{1}{2}\left[\nabla^{\varphi}_{\Gamma(\mathcal{E},\partial_\mu)}, \partial_\nu\right] + \frac{1}{4}\left[\nabla^{\varphi}_{\Gamma(\mathcal{E},\partial_\mu)}, \nabla^{\varphi}_{\Gamma(\mathcal{E},\partial_\nu)}\right]$$

$$\mathcal{E} = \sum_\lambda s^\lambda \partial_\lambda$$

$$\left[\partial_\mu, \frac{1}{2}\nabla^{\varphi}_{\Gamma(\mathcal{E},\partial_\nu)}\right] = \frac{1}{2}\sum_\lambda \left[\partial_\mu, s^\lambda \nabla^{\varphi}_{\Gamma(\partial_\lambda,\partial_\nu)}\right]$$

$$= \frac{1}{2}\nabla_{\Gamma(\partial_\mu,\partial_\nu)} \,\mathrm{mod}\Pi S$$

$$\left[\partial_\mu, \nabla^{\varphi}_{\Gamma(\mathcal{E},\partial_\nu)}\right]\Big|_{\check{M}} = (\psi_S^*\Gamma)\left(\partial_\mu|_{\check{M}}, \partial_\nu|_{\check{M}}\right)$$

$$\left[\nabla^{\varphi}_{\Gamma(\mathcal{E},\partial_\mu)}, \nabla^{\varphi}_{\Gamma(\mathcal{E},\partial_\nu)}\right]$$

$$[\delta_{\mathcal{F}}(\partial_\mu), \delta_{\mathcal{F}}(\partial_\nu)]|_{\check{M}} = (\psi_S^*\Gamma)\left(\partial_\mu|_{\check{M}}, \partial_\nu|_{\check{M}}\right),$$

$$M \times A \overset{\mathrm{pr}_A}{\to} A$$

$$T\check{M} \cong \check{M} \times V \overset{\mathrm{pr}}{\to} \check{M},$$

$$\mathrm{Fr}^{\mathrm{SO}(V,q)}(T\check{M}) \cong \check{M} \times \mathrm{SO}(V,q) \overset{\mathrm{pr}}{\to} M.$$

$$M = \check{M} \times \Pi S$$

$$f = \sum_I f_I \xi^I \in \mathcal{O}_M, \quad f_I \in C^\infty(\check{M}), \quad \xi^I \in \wedge^\bullet(S^\vee),$$

$$TM \cong M \times (V \oplus \Pi S) \to M$$

$$\mathcal{F} = \mathrm{span}_{\mathcal{O}_M}\left\{D_\mu = \partial_\mu + \frac{1}{2}\xi^\nu \Gamma^i_{\mu\nu}\partial_i : \mu = 1, \ldots, \dim(S)\right\}$$

$$q = (\tau^*(e_1, \dots, e_m), \delta_{\mathcal{F}}(\partial_1)^\vee, \dots, \delta_{\mathcal{F}}(\partial_n)^\vee)$$

$$\Psi: (\tau^*(e_1, \dots, e_n), \partial_1, \dots, \partial_n) \mapsto \big(e_1, \dots, e_n, \delta_{\mathcal{F}}(\partial_1), \dots, \delta_{\mathcal{F}}(\partial_n)\big)$$

$$\Psi = \left(\begin{array}{c|c} \mathrm{id} & \Gamma(\mathcal{E}, \cdot) \\ \hline 0 & \mathrm{id} \end{array}\right)$$

$$\int_M f[q] = \int_{\check{M}} i^*(\partial_1 \dots \partial_n f) e^1 \wedge \dots \wedge e^m$$

$$i^*(\partial_1 \partial_2 f) = \frac{1}{2} i^* \Big(\big(\delta_{\mathcal{F}}(\partial_1) \circ \delta_{\mathcal{F}}(\partial_2) - \delta_{\mathcal{F}}(\partial_2) \circ \delta_{\mathcal{F}}(\partial_1)\big) f\Big)$$

$$i^*(\delta_{\mathcal{F}}(\partial_1) \circ \delta_{\mathcal{F}}(\partial_2) f) = i^* \left(\partial_1 \circ \left(\partial_2 + \frac{1}{2} \nabla^{\varphi}_{\Gamma(\mathcal{E}, \partial_2)} f\right) = i^* \left(\left(\partial_1 \circ \partial_2 + \frac{1}{2} \nabla^{\varphi}_{\Gamma(\partial_1, \partial_2)}\right) f\right)\right.$$

$$\int_M f \mathrm{vol} = \frac{1}{2} \int_{\check{M}} i^* \Big(\big(\delta_{\mathcal{F}}(\partial_1) \circ \delta_{\mathcal{F}}(\partial_2) - \delta_{\mathcal{F}}(\partial_1) \circ \delta_{\mathcal{F}}(\partial_2)\big) f\Big) \check{\mathrm{vol}}$$

$$\check{\mathcal{L}}(a, \lambda) = -\frac{1}{2} \big(\mu \|F^a\|^2 - \langle \lambda, \not{D}^a \lambda \rangle\big) \check{\mathrm{vol}}\,,$$

$$\mathfrak{F}^{sYM} = \mathfrak{F}_P = \{\mathrm{a} \in \mathcal{A}_P : \langle \mathcal{F} \wedge \mathcal{F}, F^{\mathrm{a}} \rangle = 0\},$$

$$F^{\mathrm{a}} = \mathrm{da} + \frac{1}{2} [\mathrm{a} \wedge \mathrm{a}] \in \Omega^2(P, \mathfrak{g})\big) \cong \Omega^2(M, \mathrm{ad}(P))$$

$$F^{\mathrm{a}} = f^{\mathrm{a}} \oplus \phi^{\mathrm{a}} \in \Omega^{2|0}(P, \mathfrak{g})^G_{\mathrm{hor}} \oplus \Omega^{1|1}(P, \mathfrak{g})^G_{\mathrm{hor}}$$

$$\psi^{\mathrm{a}} := \frac{1}{\sqrt{n}} \langle \phi, \tilde{\gamma} \rangle \in \Omega^0(M, \mathrm{ad}(P)) \otimes \mathcal{F}^\vee \cong \Omega^0(Q \times_M P, \Pi S \otimes \mathfrak{g})^{\mathrm{Spin}(V,q) \times G}_{\mathrm{hor}}$$

$$\mathcal{L}: \mathfrak{F}^{\mathrm{sYM}} \to \mathcal{B}er_M, \mathrm{a} \mapsto \frac{1}{2} \|\psi^{\mathrm{a}}\|^2_{\kappa \otimes \epsilon} \mathrm{vol}_g$$

$$\check{\mathcal{L}}(a, \lambda) = \left(-\frac{1}{4} \|F^a\|^2 + \frac{1}{2} \langle \lambda, \not{D}^a \lambda \rangle\right) \check{\mathrm{vol}}$$

$$a = i^* \mathrm{a} \in \mathcal{A}_{\check{P}}, \lambda = (\psi_S^{-1} \otimes \mathrm{id})(\psi|_{\check{M}}) \in \Omega^0(\check{M}, S\check{M} \otimes \mathrm{ad}(\check{P})).$$

$$\Pi S \check{M} = \check{Q} \times_{\mathrm{Spin}(V,q)} \Pi S$$

$$\check{\mathcal{L}} = \frac{1}{2} i^* \left(\sum_{X,Y} \tilde{\epsilon}(X^\vee, Y^\vee) X \circ Y\big((\epsilon \otimes \kappa)(\psi^{\mathrm{a}}, \psi^{\mathrm{a}})\big)\right) \check{\mathrm{vol}}$$

$$\frac{1}{2} X \circ Y((\epsilon \otimes \kappa)(\psi, \psi)) = X\big((\epsilon \otimes \kappa)(\nabla_Y \psi, \psi)\big) = (\epsilon \otimes \kappa)(\nabla_X \nabla_Y \psi, \psi) - (\epsilon \otimes \kappa)(\nabla_Y \psi, \nabla_X \psi)$$

$$\frac{1}{4}i^*\left(\sum_{X,Y}\tilde{\epsilon}(\tilde{X},\tilde{Y})\big((\epsilon\otimes\kappa)(\nabla_X\nabla_Y\psi,\psi)-(\epsilon\otimes\kappa)(\nabla_Y\psi,\nabla_X\psi)\big)\right)=\frac{1}{2}\langle\lambda,\not{D}^a\lambda\rangle-\frac{1}{4}\|F^a\|^2$$

$$S^{\mathbb{C}}=S^{1,0}\oplus S^{0,1}$$

$$(\psi^{\mathrm{a}})^{\mathbb{C}}=\chi^{\mathrm{a}}\oplus\overline{\chi^{\mathrm{a}}}\in\Omega^0\big(M,\mathrm{ad}(P)^{\mathbb{C}}\big)\otimes_{\mathcal{O}_M}((\mathcal{F}^{1,0})^\vee\oplus(\mathcal{F}^{0,1})^\vee).$$

$$\mathcal{L}:\mathfrak{F}^{\mathrm{sYM}}\to\mathcal{B}er_M^{1,0}\oplus\mathcal{B}er_M^{0,1},$$
$$\mathcal{L}(\mathrm{a})=\frac{1}{4}\left(\|\chi^{\mathrm{a}}\|^2_{\epsilon\otimes\kappa}\mathrm{vol}^{1,0}+\|\overline{\chi^{\mathrm{a}}}\|^2_{\overline{\epsilon\otimes\kappa}^2}\mathrm{vol}^{0,1}\right)=\frac{1}{2}\mathrm{Re}\big(\|\chi^{\mathrm{a}}\|^2_{\epsilon\otimes\kappa}\mathrm{vol}^{1,0}\big)$$

$$\check{\mathcal{L}}(a,\psi,E)=-\frac{1}{2}\big(\|F^a\|^2-\langle\lambda,\not{D}^a\lambda\rangle\big)\,\check{\mathrm{vol}}\,,$$

$$a=i^*\mathrm{a}\in\mathcal{A}_{\check{P}},\lambda=(\psi_S^{-1}\otimes\mathrm{id})(\psi|_{\check{M}})\in\Omega^0(\check{M},S\check{M}\otimes\mathrm{ad}(\check{P})).$$

$$\mathrm{d}^{\mathrm{a}}f+\mathrm{d}^{\mathrm{a}}\zeta+\mathrm{d}^{\mathrm{a}}\bar{\zeta}=0.$$

$$\chi=\langle\bar{\zeta},\tilde{\gamma}\rangle,\bar{\chi}=\langle\zeta,\tilde{\gamma}\rangle.$$

$$\check{\mathcal{L}}=\frac{1}{4}i^*(\epsilon^{XY}X\circ Y(\epsilon\otimes\kappa)(\chi,\chi))\check{\mathrm{vol}}+c.c.,$$

$$(\epsilon^{XY}X\circ Y)(\epsilon\otimes\kappa)(\chi,\chi)=-2\epsilon^{XY}\big((\epsilon\otimes\kappa)(\nabla_X\chi,\nabla_Y\chi)-(\epsilon\otimes\kappa)(\nabla_X\nabla_Y\chi,\chi)\big)$$

$$\epsilon\left(\chi|_{\check{M}},\not{D}^a(\chi_{\check{M}})\right)$$

$$\langle\bar{\chi}|_{\check{M}},\not{D}^a(\chi_{\check{M}})\rangle.$$

$$\check{\mathcal{L}}=-\frac{1}{2}\big(\|F^a\|^2-\langle\lambda,\not{D}^a\lambda\rangle\big)\check{\mathrm{vol}}$$

$$M_1=\big(\underline{M},\mathcal{O}_{M_1}\big),\mathcal{O}_{M_1}=\{s\in\mathcal{O}_M\otimes\mathbb{C}:Xs=0\text{ for all }X\in\Omega^0(M,D_2)\}.$$

$$X(st)=(Xs)t+(-1)^{p(X)p(s)}s(Xt)=0$$

$$A^b=\epsilon(\cdot,A)\Leftrightarrow\epsilon^{XY}A_Y=A^X.$$

$$A_Z=\delta_Z^XA_X=\epsilon^{XY}\epsilon_{YZ}A_X=-\epsilon_{YZ}A^Y$$

$$\iota_X\iota_Y\,\mathrm{d}^{\mathrm{a}}\phi=-\iota_{[X,Y]}f$$

$$\begin{aligned}\iota_X\iota_Y\,\mathrm{d}^{\mathrm{a}}\phi&=-\iota_X\iota_Y\,\mathrm{d}^{\mathrm{a}}f\\&=-\iota_XL_Yf\\&=-\iota_{[X,Y]}f,\end{aligned}$$

$$(\nabla_X\psi)|_{\check{M}}=\frac{1}{\sqrt{n}}\langle f,\tilde{\gamma}\circ\gamma(X)\rangle\Big|_{\check{M}}$$

$$\sqrt{n}\nabla_X\psi=\iota_X\,\mathrm{d}^{\mathrm{a}}\langle\phi,\tilde{\gamma}\rangle=\langle\iota_X\,\mathrm{d}^{\mathrm{a}}\phi,\tilde{\gamma}\rangle,$$

$$\tilde{\gamma} = \sum_Y \tilde{\gamma}(Y^\vee) \otimes Y$$

$$\begin{aligned}\sqrt{n}\nabla_X\psi &= \left\langle \iota_X \, \mathrm{d}^{\mathrm{a}}\phi, \sum_Y \tilde{\gamma}(Y^\vee) \otimes Y \right\rangle \\ &= \sum_Y \langle \iota_Y \iota_X \, \mathrm{d}^{\mathrm{a}}\phi, \tilde{\gamma}(Y^\vee)\rangle \\ &= -\sum_Y \left\langle \iota_{[X,Y]} f, \tilde{\gamma}(Y^\vee)\right\rangle\end{aligned}$$

$$\begin{aligned}\sqrt{n} \cdot (\psi_S^{-1} \otimes \mathrm{id})((\nabla_X\psi)|_{\check{M}}) &= \sum_Y \langle f, \tilde{\gamma}(Y^\vee) \otimes \Gamma(X,Y)\rangle\Big|_{\check{M}} \\ &= \langle f, \tilde{\gamma} \circ \gamma(X)\rangle|_{\check{M}}\end{aligned}$$

$$i^*\big(\epsilon^{XY}(\epsilon \otimes \kappa)(\nabla_X\psi, \nabla_Y\psi)\big) = \|F^a\|^2$$

$$\mathrm{tr}(\gamma \circ \tilde{\gamma}) = n \cdot g, Q := \mathrm{tr}(\tilde{\gamma} \circ \gamma \circ \tilde{\gamma} \circ \gamma)|_{\mathrm{Sym}^2(\Omega^{2|0}(M))} = -n\| \cdot \|_g^2,$$

$$\begin{aligned}n \cdot i^*\big(\epsilon^{XY}(\epsilon \otimes \kappa)(\nabla_X\psi, \nabla_Y\psi)\big) &= i^*(\epsilon^{XY}(\epsilon \otimes \kappa)(\langle f, \tilde{\gamma} \circ \gamma(X)\rangle, \langle f, \tilde{\gamma} \circ \gamma(Y)\rangle)) \\ &= i^*\big(\epsilon^{XY}\epsilon_{WZ}\kappa(\langle f, \tilde{\gamma}(W^\vee) \circ \gamma(X)\rangle, \langle f, \tilde{\gamma}(Z^\vee) \circ \gamma(Y)\rangle)\big)\end{aligned}$$

$$i^*\left(\delta_W^X\delta_Z^Y\kappa(\langle f, \tilde{\gamma}(W^\vee) \circ \gamma(X)\rangle, \langle f, \tilde{\gamma}(Z^\vee) \circ \gamma(Y)\rangle)\right) = i^*(\kappa(\langle f, g\rangle, \langle f, g\rangle)) = 0$$

$$\begin{aligned}i^*\left(\delta_Z^X\delta_W^Y\kappa(\langle f, \tilde{\gamma}(W^\vee) \circ \gamma(X)\rangle, \langle f, \tilde{\gamma}(Z^\vee) \circ \gamma(Y)\rangle)\right) &= i^*(\langle \kappa(f \wedge f), \mathrm{tr}(\tilde{\gamma} \circ \gamma \circ \tilde{\gamma} \circ \gamma)\rangle) \\ &= n\|i^*f\|^2 = n\|F^a\|^2\end{aligned}$$

$$\not{D}^a\lambda = \frac{1}{\sqrt{n}}\langle \, \mathrm{d}^{\mathrm{a}}\phi, \tilde{\gamma} \circ \gamma \circ \tilde{\epsilon}\rangle\Big|_{\check{M}}$$

$$\not{D}^a\check{\psi} = \left\langle \mathrm{d}^a\check{\psi}, \gamma \circ \tilde{\epsilon}\right\rangle (T\check{M} \otimes S\check{M}),$$

$$\begin{aligned}\sqrt{n} \cdot \not{D}^a\check{\psi} &= \langle \mathrm{d}^a i^*\langle \psi, \tilde{\gamma}\rangle, \gamma \circ \tilde{\epsilon}\rangle \\ &= i^*\langle \, \mathrm{d}^{\mathrm{a}}\phi, \tilde{\gamma} \circ \gamma \circ \tilde{\epsilon}\rangle\end{aligned}$$

$$\Delta_\epsilon^{\mathrm{a}} = \epsilon^{XY}\nabla_X \circ \nabla_Y$$

$$(\Delta_\epsilon^{\mathrm{a}}\psi)|_{\check{M}} = 2\not{D}^a\lambda$$

$$\begin{aligned}\sqrt{n} \cdot \Delta_\epsilon^{\mathrm{a}}\psi &= \epsilon^{XY}\iota_X \, \mathrm{d}^{\mathrm{a}}\iota_Y \, \mathrm{d}^{\mathrm{a}}\langle \phi, \tilde{\gamma}\rangle \\ &= \epsilon^{XY}\langle \iota_Z\iota_X \, \mathrm{d}^{\mathrm{a}}\iota_Y \, \mathrm{d}^{\mathrm{a}}\phi, \tilde{\gamma}(Z^\vee)\rangle \\ &= \epsilon^{XY}\langle \iota_X\iota_Z L_Y \, \mathrm{d}^{\mathrm{a}}\phi, \tilde{\gamma}(Z^\vee), \rangle \\ &= \epsilon^{XY}\left\langle \iota_X\big(\iota_{[Z,Y]} - \mathrm{d}^{\mathrm{a}}\iota_Y\iota_Z\big)\mathrm{d}^{\mathrm{a}}\phi, \tilde{\gamma}(Z^\vee)\right\rangle \\ &= -\epsilon^{XY}\left\langle \iota_{[Z,Y]}\iota_X \, \mathrm{d}^{\mathrm{a}}\phi, \tilde{\gamma}(Z^\vee)\right\rangle - \epsilon^{XY}\langle \iota_X \, \mathrm{d}^{\mathrm{a}}\iota_Y\iota_Z \, \mathrm{d}^{\mathrm{a}}\phi, \tilde{\gamma}(Z^\vee)\rangle\end{aligned}$$

$$\epsilon^{XY}\left\langle \iota_{[Z,Y]}\iota_X \, \mathrm{d}^{\mathrm{a}}\phi, \tilde{\gamma}(Z^{\vee})\right\rangle\Big|_{\check{M}} = \epsilon^{XY}\langle \iota_X \, \mathrm{d}^{\mathrm{a}}\phi, \tilde{\gamma}(Z^{\vee}) \otimes \Gamma(Z,Y)\rangle|_{\check{M}}$$
$$= \epsilon^{XY}\langle \iota_X \, \mathrm{d}^{\mathrm{a}}\phi, \tilde{\gamma} \circ \gamma(Y)\rangle|_{\check{M}}$$
$$= -\langle \mathrm{d}^{\mathrm{a}}\phi, \tilde{\gamma} \circ \gamma \circ \tilde{\epsilon}\rangle|_{\check{M}}$$
$$= -\sqrt{n} \cdot \not{D}^{a}\lambda$$

$$-\epsilon^{XY}\langle \iota_X \, \mathrm{d}^{\mathrm{a}}\iota_Y\iota_Z \, \mathrm{d}^{\mathrm{a}}\phi, \tilde{\gamma}(Z^{\vee})\rangle|_{\check{M}} = \epsilon^{XY}\left\langle \iota_X \, \mathrm{d}^{\mathrm{a}}\iota_{[Y,Z]}f, \tilde{\gamma}(Z^{\vee})\right\rangle\Big|_{\check{M}}$$
$$= \epsilon^{XY}\left\langle L_X\iota_{[Y,Z]}f, \tilde{\gamma}(Z^{\vee})\right\rangle\Big|_{\check{M}}$$
$$= \epsilon^{XY}\left\langle \iota_{[X,[Y,Z]]}f + \iota_{[Y,Z]}\iota_X \, \mathrm{d}^{\mathrm{a}}\phi, \tilde{\gamma}(Z^{\vee})\right\rangle\Big|_{\check{M}}$$
$$= \epsilon^{XY}\left\langle \iota_{[X,[Y,Z]]}f, \tilde{\gamma}(Z^{\vee})\right\rangle\Big|_{\check{M}} + \sqrt{n} \cdot D^{a}\lambda$$

$$\Delta_{\epsilon}^{\mathrm{a}}\psi|_{\check{M}} = 2\not{D}^{a}\lambda$$

$$\sum_{XY} \epsilon^{XY}(\epsilon \otimes \kappa)(\nabla_X\chi, \nabla_Y\chi)\Bigg|_{\check{M}} = \|F^{a}\|^2$$

$$F^{\mathrm{a}} = f^{\mathrm{a}} \oplus \zeta^{\mathrm{a}} \oplus \bar{\zeta}^{\mathrm{a}}$$

$$\begin{array}{ccc} \zeta & & \bar{\zeta} \\ \Big\downarrow \frac{1}{\sqrt{n}}\langle \cdot, \tilde{\gamma}\rangle & & \Big\downarrow \frac{1}{\sqrt{n}}\langle \cdot, \tilde{\gamma}\rangle \\ \chi & & \bar{\chi}. \end{array}$$

$$\sqrt{n}\nabla_X\chi = \iota_X \, \mathrm{d}^{\mathrm{a}}\langle \zeta, \tilde{\gamma}\rangle$$
$$= \sum_{Y} \langle \iota_Y\iota_X \, \mathrm{d}^{\mathrm{a}}\zeta, \tilde{\gamma}(Y^{\vee})\rangle$$
$$= -\sum_{Y} \langle \iota_Y\iota_X \, \mathrm{d}^{\mathrm{a}}(f + \bar{\zeta}), \tilde{\gamma}(Y^{\vee})\rangle$$
$$= -\sum_{Y} \left\langle \iota_{[Y,X]}\mathrm{d}^{\mathrm{a}}(f + \bar{\zeta}), \tilde{\gamma}(Y^{\vee})\right\rangle$$

$$\sqrt{n}\nabla_X\chi = \iota_X \, \mathrm{d}^{\mathrm{a}}\langle \zeta, \tilde{\gamma}\rangle|_{\check{M}} = -\langle f + \bar{\zeta}, \tilde{\gamma} \circ \gamma(X)\rangle|_{\check{M}} = -\langle f, \tilde{\gamma} \circ \gamma(X)\rangle|_{\check{M}}$$

$$\epsilon^{XY}(\epsilon \otimes \kappa)(\nabla_X\chi, \nabla_Y\chi)|_{\check{M}} = \epsilon^{XY}(\epsilon \otimes \kappa)(\langle f, \tilde{\gamma} \circ \gamma(X)\rangle, \langle f, \tilde{\gamma} \circ \gamma(Y)\rangle)|_{\check{M}} = \|F^{a}\|^2,$$

$$(\Delta_{\epsilon}^{\mathrm{a}}\chi)|_{\check{M}} := (\epsilon^{XY}\nabla_X\nabla_Y\chi)|_{\check{M}} = -\not{D}^{a}(\chi|_{\check{M}})$$

$$\begin{aligned}\sqrt{n}\cdot\Delta^{\mathrm{a}}_{\epsilon}\chi &= \epsilon^{XY}\iota_X\,\mathrm{d}^{\mathrm{a}}\iota_Y\,\mathrm{d}^{\mathrm{a}}\langle\zeta,\tilde{\gamma}\rangle\\ &= \epsilon^{XY}\langle\iota_Z\iota_X\,\mathrm{d}^{\mathrm{a}}\iota_Y\,\mathrm{d}^{\mathrm{a}}\zeta,\tilde{\gamma}(Z^\vee)\rangle\\ &= \epsilon^{XY}\langle\iota_X\iota_Z L_Y\,\mathrm{d}^{\mathrm{a}}\zeta,\tilde{\gamma}(Z^\vee),\rangle\\ &= \epsilon^{XY}\big\langle\iota_X\big(\iota_{[Z,Y]}-\mathrm{d}^{\mathrm{a}}\iota_Y\iota_Z\big)\mathrm{d}^{\mathrm{a}}\zeta,\tilde{\gamma}(Z^\vee)\big\rangle\\ &= -\epsilon^{XY}\big\langle\iota_{[Z,Y]}\iota_X\,\mathrm{d}^{\mathrm{a}}\zeta,\tilde{\gamma}(Z^\vee)\big\rangle-\epsilon^{XY}\langle\iota_X\,\mathrm{d}^{\mathrm{a}}\iota_Y\iota_Z\,\mathrm{d}^{\mathrm{a}}\zeta,\tilde{\gamma}(Z^\vee)\rangle\end{aligned}$$

$$\begin{aligned}\epsilon^{XY}\big\langle\iota_{[Z,Y]}\iota_X\,\mathrm{d}^{\mathrm{a}}\zeta,\tilde{\gamma}(Z^\vee)\big\rangle\big|_{\check{M}} &= \epsilon^{XY}\langle\iota_X\,\mathrm{d}^{\mathrm{a}}\zeta,\tilde{\gamma}(Z^\vee)\otimes\Gamma(Z,Y)\rangle|_{\check{M}}\\ &= \epsilon^{XY}\langle\iota_X\,\mathrm{d}^{\mathrm{a}}\zeta,\tilde{\gamma}\circ\gamma(Y)\rangle|_{\check{M}}\\ &= -\langle\mathrm{d}^{\mathrm{a}}\zeta,\tilde{\gamma}\circ\gamma\circ\tilde{\epsilon}\rangle|_{\check{M}}\\ &= -\sqrt{n}\cdot\not{D}^{a}(\chi|_{\check{M}})\end{aligned}$$

$$\begin{aligned}-\epsilon^{XY}\langle\iota_X\,\mathrm{d}^{\mathrm{a}}\iota_Y\iota_Z\,\mathrm{d}^{\mathrm{a}}\zeta,\tilde{\gamma}(Z^\vee)\rangle|_{\check{M}} &= \epsilon^{XY}\big\langle\iota_X\,\mathrm{d}^{\mathrm{a}}\iota_{[Y,Z]}(f+\bar{\zeta}),\tilde{\gamma}(Z^\vee)\big\rangle\big|_{\check{M}}\\ &= \epsilon^{XY}\big\langle L_X\iota_{[Y,Z]}(f+\bar{\zeta}),\tilde{\gamma}(Z^\vee)\big\rangle\big|_{\check{M}}\\ &= \epsilon^{XY}\big\langle\iota_{[X,[Y,Z]]}(f+\bar{\zeta})+\iota_{[Y,Z]}\iota_X\,\mathrm{d}^{\mathrm{a}}\zeta,\tilde{\gamma}(Z^\vee)\big\rangle\big|_{\check{M}}\\ &= \epsilon^{XY}\big\langle\iota_{[X,[Y,Z]]}(f+\bar{\zeta}),\tilde{\gamma}(Z^\vee)\big\rangle\big|_{\check{M}}+\sqrt{n}\cdot\not{D}^{a}(\chi|_{\check{M}})\end{aligned}$$

$$\Delta^{\mathrm{a}}_{\epsilon}\chi|_{\check{M}} = 2\not{D}^{a}(\chi|_{\check{M}})$$

$$m_i:\otimes^i\ \mathrm{A}\to\ \mathrm{A}$$

$$\sum_{\substack{r+s+t=i\\ r,t\in\mathbb{N}_0\\ s\in\mathbb{N}}}(-1)^{rs+t}m_{r+1+t}\circ(\underbrace{\mathbb{1}\otimes\cdots\otimes\mathbb{1}}_{r-\text{ times}}\otimes m_s\otimes\underbrace{\mathbb{1}\otimes\cdots\otimes\mathbb{1}}_{t-\text{ times}})=0$$

$$\begin{aligned}i=1:&\quad m_1\big(m_1(a_1)\big)=0\\ i=2:&\quad m_1\big(m_2(a_1,a_2)\big)-m_2(m_1(a_1),a_2)-(-1)^{|a_1|}m_2\big(a_1,m_1(a_2)\big)=0\\ i=3:&\quad m_1\big(m_3(a_1,a_2,a_3)\big)-m_2(m_2(a_1,a_2),a_3)+m_2\big(a_1,m_2(a_2,a_3)\big)+\\ &\quad +m_3(m_1(a_1),a_2,a_3)+(-1)^{|a_1|}m_3(a_1,m_1(a_2),a_3)+\\ &\quad +(-1)^{|a_1|+|a_2|}m_3\big(a_1,a_2,m_1(a_3)\big)=0\end{aligned}$$

$$\langle -,-\rangle_{\mathrm{A}}:\mathrm{A}\odot\ \mathrm{A}\to\mathbb{R}$$

$$\langle a_1,m_i(a_2,\dots,a_{i+1})\rangle_{\mathrm{A}}=(-1)^{i+i(|a_1|+|a_{i+1}|)+|a_{i+1}|\sum_{j=1}^{i}|a_j|}\langle a_{i+1},m_i(a_1,a_2,\dots,a_i)\rangle_{\mathrm{A}}$$

$$\mu_i:\wedge^i\ \mathrm{L}\to\ \mathrm{L}$$

$$\sum_{r+s=i}\sum_{\sigma}\chi(\sigma;\ell_1,\dots,\ell_{r+s})(-1)^s\mu_{s+1}\big(\mu_r\big(\ell_{\sigma(1)},\dots,\ell_{\sigma(r)}\big),\ell_{\sigma(r+1)},\dots,\ell_{\sigma(r+s)}\big)=0$$

$$\ell_1\wedge\dots\wedge\ell_i=\chi(\sigma;\ell_1,\dots,\ell_i)\ell_{\sigma(1)}\wedge\dots\wedge\ell_{\sigma(i)}$$

$$\mu_i(\ell_1,\dots,\ell_i)=\sum_{\sigma}\chi(\sigma;\ell_1,\dots,\ell_i)m_i\big(\ell_{\sigma(1)},\dots,\ell_{\sigma(i)}\big)$$

$$\langle -,-\rangle_{\mathbf{L}}:\mathbf{L}\odot\mathbf{L}\to\mathbb{R}$$

$$\langle \ell_1, \mu_i(\ell_2, \dots, \ell_{i+1}) \rangle_{\mathbf{L}} = (-1)^{i+i(|\ell_1|+|\ell_{i+1}|)+|\ell_{i+1}| \sum_{j=1}^{i} |\ell_j|_{\langle \ell_{i+1}}, \mu_i(\ell_1, \dots, \ell_i) \rangle_{\mathbf{L}}}$$

$$\sum_{i \in \mathbb{N}} m_i(a, \dots, a) = m_1(a) + m_2(a, a) + \cdots = 0.$$

$$\hat{\mathsf{A}} := V \otimes \mathsf{A} := \bigoplus_{p \in \mathbb{Z}} \hat{\mathsf{A}}_p \text{ with } \hat{\mathsf{A}}_p := \bigoplus_{i+j=p} V_i \otimes \mathsf{A}_j$$

$$\hat{m}_1(v_1 \otimes a_1) := \mathrm{d}v_1 \otimes a_1 + (-1)^{|v_1|} v_1 \otimes m_1(a_1)$$

$$\hat{m}_i(v_1 \otimes a_1, \dots, v_i \otimes a_i) := (-1)^{i \sum_{j=1}^{i} |v_j| + \sum_{j=0}^{i-2} |v_{i-j}| \sum_{k=1}^{i-j-1} |a_k|}$$

$$\times (v_1 \wedge \dots \wedge v_i) \otimes m_i(a_1, \dots, a_i)$$

$$\langle \omega_1 \otimes a_1, \omega_2 \otimes a_2 \rangle_{\hat{\mathsf{A}}} := (-1)^{|\omega_2||a_1|} \int_M \omega_1 \wedge \omega_2 \langle a_1, a_2 \rangle_{\mathsf{A}}$$

$$\hat{a} = a + \mathrm{d}t\lambda,$$

$$\delta a := \sum_{i \in \mathbb{N}} [(-1)^{i-1} m_i(\lambda, \underbrace{a, \dots, a}_{(i-1)-\text{ times}}) +$$

$$+(-1)^{i-2} m_i(a, \lambda, \underbrace{a, \dots, a}_{(i-2)-\text{ times}}) + \cdots + m_i(\underbrace{a, \dots, a}_{(i-1)-\text{ times}}, \lambda))].$$

$$S := \sum_{i \in \mathbb{N}} \frac{1}{i+1} \langle a, m_i(a, \dots, a) \rangle_{\mathsf{A}},$$

$$S := \sum_{i \in \mathbb{N}} \frac{1}{(i+1)!} \langle \ell, \mu_i(\ell, \dots, \ell) \rangle_{\mathsf{L}}, \sum_{i \in \mathbb{N}} \frac{1}{i!} \mu_i(\ell, \dots, \ell) = 0$$

$$\delta \ell := \sum_{i \in \mathbb{N}} \frac{1}{(i-1)!} \mu_i(\ell, \dots, \ell, \lambda)$$

$$\mathcal{C}^\infty(M) \xrightarrow{\mathrm{d}} \Omega^1(M) \xrightarrow{\mathrm{d}} \Omega^2(M) \xrightarrow{\mathrm{d}} \cdots$$

$$\mathcal{C}^\infty(X) \xrightarrow{\bar{\partial}} \Omega^{0,1}(X) \xrightarrow{\bar{\partial}} \Omega^{0,2}(X) \xrightarrow{\bar{\partial}} \cdots,$$

$$m_2(a_1, a_2) := \begin{cases} a_1 a_2 \in \mathsf{A}_{|a_1|+|a_2|} & \text{for } |a_1| + |a_2| > -n \\ 0 & \text{else} \end{cases}$$

$$\langle a_1, a_2 \rangle_{\mathsf{A}} := \begin{cases} \mathrm{tr}(a_1^\dagger a_2) & \text{for } |a_1| + |a_2| = -n+1 \\ 0 & \text{else} \end{cases}$$

$$\Omega^\bullet(M, \mathsf{A}) := \bigoplus_{p \in \mathbb{N}_0} \Omega_p^\bullet(M, \mathsf{A}) \quad \text{with} \quad \Omega_p^\bullet(M, \mathsf{A}) := \bigoplus_{\substack{i+j=p \\ 0 \leqslant i \leqslant d \\ -n+1 \leqslant j \leqslant 0}} \Omega^i(M, \mathsf{A}_j),$$

$$\Omega^{0,\bullet}(X, \mathsf{A}) := \bigoplus_{p \in \mathbb{N}_0} \Omega_p^{0,\bullet}(X, \mathsf{A}) \quad \text{with} \quad \Omega_p^{0,\bullet}(X, \mathsf{A}) := \bigoplus_{\substack{i+j=p \\ 0 \leqslant i \leqslant d \\ -n+1 \leqslant j \leqslant 0}} \Omega^{0,i}(X, \mathsf{A}_j),$$

$$\hat{m}_1(\omega_1 \otimes a_1) := \mathrm{d}\omega_1 \otimes a_1$$
$$\hat{m}_2(\omega_1 \otimes a_1, \omega_2 \otimes a_2) := (-1)^{|a_1||\omega_2|}(\omega_1 \wedge \omega_2) \otimes a_1 a_2$$

$$\hat{m}_1(\omega_1 \otimes a_1) := \bar{\partial}\omega_1 \otimes a_1$$
$$\hat{m}_2(\omega_1 \otimes a_1, \omega_2 \otimes a_2) := (-1)^{|a_1||\omega_2|}(\omega_1 \wedge \omega_2) \otimes a_1 a_2$$

$$\langle \omega_1 \otimes a_1, \omega_2 \otimes a_2 \rangle_{\hat{\mathsf{A}}} := (-1)^{|a_1||\omega_2|} \int_M \omega_1 \wedge \omega_2 \mathrm{tr}\left(a_1^\dagger a_2\right)$$

$$\langle \omega_1 \otimes a_1, \omega_2 \otimes a_2 \rangle_{\hat{\mathsf{A}}} := (-1)^{|a_1||\omega_2|} \int_X \Omega^{d,0} \wedge \omega_1 \wedge \omega_2 \mathrm{tr}\left(a_1^\dagger a_2\right),$$

$$S = \int_M \mathrm{tr}\left\{\frac{1}{2}\hat{a}^\dagger \wedge \mathrm{d}\hat{a} + \frac{1}{3}\hat{a}^\dagger \wedge \hat{a} \wedge \hat{a}\right\}$$
$$= \int_M \mathrm{tr}\left\{\frac{1}{2}\sum_{i+j=d-1} \hat{a}_i^\dagger \wedge \mathrm{d}\hat{a}_j + \frac{1}{3}\sum_{i+j+k=d} \hat{a}_i^\dagger \wedge \hat{a}_j \wedge \hat{a}_k\right\}$$

$$\Omega^\bullet(M, \mathsf{A}) := \bigoplus_{p \in \mathbb{N}_0} \Omega_p^\bullet(M, \mathsf{A}) \quad \text{with} \quad \Omega_p^\bullet(M, \mathsf{A}) := \bigoplus_{\substack{i+j=p \\ 0 \leqslant i \leqslant d \\ -n+1 \leqslant j \leqslant 0}} \Omega^i(M, \mathsf{A}_j)$$

$$\hat{m}_1(\omega_1 \otimes a_1) := \mathrm{d}\omega_1 \otimes a_1 + (-1)^{|\omega_1|}\omega_1 \otimes m_1(a_1)$$
$$\hat{m}_i(\omega_1 \otimes a_1, \dots, \omega_i \otimes a_i) := (-1)^{i\sum_{j=1}^{i}|\omega_j| + \sum_{j=0}^{i-2}|\omega_{i-j}|\sum_{k=1}^{i-j-1}|a_k|}$$
$$\times (\omega_1 \wedge \dots \wedge \omega_i) \otimes m_i(a_1, \dots, a_i)$$

$$S = \int_M \mathrm{tr}\left\{\frac{1}{2}A \wedge \mathrm{d}A + \frac{1}{3}A \wedge A \wedge A\right\}$$

$$\mathsf{A} = \mathsf{A}_{-2} \oplus \mathsf{A}_{-1} \oplus \mathsf{A}_0 := \mathfrak{g}^* \oplus \mathfrak{h} \oplus \mathfrak{g}$$

$$\langle a_{-2} + a_{-1} + a_0, b_{-2} + b_{-1} + b_0 \rangle_{\mathsf{A}} = b_{-2}(a_0) + \langle a_{-1}, b_{-1} \rangle_{\mathfrak{h}} + a_{-2}(b_0),$$

$$\begin{aligned}
\Omega_0^\bullet(M, \mathsf{A}) &= \Omega^0(M, \mathfrak{g}) \oplus \Omega^1(M, \mathfrak{h}) \oplus \Omega^2(M, \mathfrak{g}^*) , \\
\Omega_1^\bullet(M, \mathsf{A}) &= \Omega^1(M, \mathfrak{g}) \oplus \Omega^2(M, \mathfrak{h}) \oplus \Omega^3(M, \mathfrak{g}^*) , \\
\Omega_2^\bullet(M, \mathsf{A}) &= \Omega^2(M, \mathfrak{g}) \oplus \Omega^3(M, \mathfrak{h}) \oplus \Omega^4(M, \mathfrak{g}^*) , \\
\Omega_3^\bullet(M, \mathsf{A}) &= \Omega^3(M, \mathfrak{g}) \oplus \Omega^4(M, \mathfrak{h}) \oplus \Omega^5(M, \mathfrak{g}^*) , \\
\Omega_4^\bullet(M, \mathsf{A}) &= \Omega^4(M, \mathfrak{g}) \oplus \Omega^5(M, \mathfrak{h}) , \\
\Omega_5^\bullet(M, \mathsf{A}) &= \Omega^5(M, \mathfrak{g}) .
\end{aligned}$$

$$\hat{a} := A + B + C \text{ with } A \in \Omega^1(M, \mathfrak{g}), B \in \Omega^2(M, \mathfrak{h}), C \in \Omega^3(M, \mathfrak{g}^*).$$

$$S=\int_M \Big\{\langle A,\mathrm{d}C\rangle_{\mathrm{A}}+\langle B,m_1(C)\rangle_{\mathrm{A}}-\frac{1}{2}\langle B,\mathrm{d}B\rangle_{\mathrm{A}}+\langle A,m_2(A,C)\rangle_{\mathrm{A}}$$
$$+\langle A,m_2(B,B)\rangle_{\mathrm{A}}+\langle A,m_3(A,A,B)\rangle_{\mathrm{A}}+\frac{1}{5}\langle A,m_4(A,A,A,A)\rangle_{\mathrm{A}}\Big\}$$

$$m_i(a_1,\ldots,a_i)+\text{ graded cyclic }=\mu_i(a_1,\ldots,a_i),a_1,\ldots,a_i\in\{A,B,C\}$$

$$\mathcal{F}:=\mathrm{d}A+\frac{1}{2}\mu_2(A,A)+\mu_1(B)=0$$
$$\mathcal{H}:=\mathrm{d}B+\mu_2(A,B)-\frac{1}{3!}\mu_3(A,A,A)-\mu_1(C)=0$$
$$\mathcal{G}:=\mathrm{d}C+\mu_2(A,C)+\frac{1}{2}\mu_3(A,A,B)+\frac{1}{2}\mu_2(B,B)+\frac{1}{4!}\mu_4(A,A,A,A)=0$$

$$\lambda:=X+\Lambda+\Sigma\ \text{ with }\ X\in\mathcal{C}^\infty(M,\mathfrak{g}),\Lambda\in\Omega^1(M,\mathfrak{h}),\Sigma\in\Omega^2(M,\mathfrak{g}^*)$$

$$\delta A=\mathrm{d}X-\mu_1(\Lambda)+\mu_2(A,X)$$
$$\delta B=\mathrm{d}\Lambda+\mu_1(\Sigma)+\mu_2(B,X)+\mu_2(A,\Lambda)+\frac{1}{2}\mu_3(A,A,X)$$
$$\delta C=\mathrm{d}\Sigma+\mu_2(C,X)-\mu_2(B,\Lambda)+\mu_2(A,\Sigma)-$$
$$-\mu_3(A,B,X)-\frac{1}{2}\mu_3(A,A,\Lambda)+\frac{1}{3!}\mu_4(A,A,A,X)$$

$$x^{\alpha\dot{\alpha}}=x_0^{\alpha\dot{\alpha}}+t\mu^\alpha\lambda^{\dot{\alpha}}+t^i\mu^\alpha\eta_i^{\dot{\alpha}}-t_i\theta^{i\alpha}\lambda^{\dot{\alpha}}$$
$$\theta^{i\alpha}=\theta_0^{i\alpha}+t^i\mu^\alpha,\eta_i^{\dot{\alpha}}=\eta_{0i}^{\dot{\alpha}}+t_i\lambda^{\dot{\alpha}}$$

$$V:=\mu^\alpha\lambda^{\dot{\alpha}}\partial_{\alpha\dot{\alpha}},V_i:=\mu^\alpha D_{i\alpha},V^i:=\lambda^{\dot{\alpha}}D_{\dot{\alpha}}^i$$

$$D_{i\alpha}:=\partial_{i\alpha}+\eta_i^{\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\ \text{ and }\ D_{\dot{\alpha}}^i:=\partial_{\dot{\alpha}}^i+\theta^{i\alpha}\partial_{\alpha\dot{\alpha}}$$

$$z^\alpha\mu_\alpha-w^{\dot{\alpha}}\lambda_{\dot{\alpha}}+2\theta^i\eta_i=0$$

$$z^\alpha=\left(x^{\alpha\dot{\alpha}}-\theta^{i\alpha}\eta_i^{\dot{\alpha}}\right)\lambda_{\dot{\alpha}},\eta_i=\eta_i^{\dot{\alpha}}\lambda_{\dot{\alpha}},w^{\dot{\alpha}}=\left(x^{\alpha\dot{\alpha}}+\theta^{i\alpha}\eta_i^{\dot{\alpha}}\right)\mu_\alpha,\theta^i=\theta^{i\alpha}\mu_\alpha,$$

$$\begin{array}{ccc} & F^{6|12} & \\ {\scriptstyle \pi_1}\swarrow & & \searrow{\scriptstyle \pi_2} \\ L^{5|6} & & \mathbb{C}^{4|12} \end{array}$$

$$\Omega^{5|6,0}:=\oint_\gamma \frac{\mathrm{d}z^\alpha\wedge\mathrm{d}z_\alpha\wedge\mathrm{d}\lambda^{\dot{\alpha}}\lambda_{\dot{\alpha}}\wedge\mathrm{d}w^{\dot{\beta}}\wedge\mathrm{d}w_{\dot{\beta}}\wedge\mathrm{d}\mu^\beta\mu_\beta\otimes\mathrm{d}\eta_1\mathrm{d}\eta_2\mathrm{d}\eta_3\mathrm{d}\theta^1\mathrm{d}\theta^2\mathrm{d}\theta^3}{z^\alpha\mu_\alpha-w^{\dot{\alpha}}\lambda_{\dot{\alpha}}+2\theta^i\eta_i}$$

$$A_b^{0,1} = g_{ab}^{-1} A_a^{0,1} g_{ab} + g_{ab}^{-1} \bar{\partial} g_{ab} \text{ on } U_a \cap U_b$$

$$F_a^{0,2} := \bar{\partial} A_a^{0,1} + \frac{1}{2}\left[A_a^{0,1}, A_a^{0,1}\right]$$

$$F_b^{0,2} = g_{ab}^{-1} F_a^{0,2} g_{ab} \text{ on } U_a \cap U_b.$$

$$g_b = g_{ab}^{-1} g_a g_{ab} \Leftrightarrow g_{ab} = g_a g_{ab} g_b^{-1} \text{ on } U_a \cap U_b.$$

$$A_a^{0,1} \mapsto g_a^{-1} A_a^{0,1} g_a + g_a^{-1} \bar{\partial} g_a \text{ and } F_a^{0,2} \mapsto g_a^{-1} F_a^{0,2} g_a$$

$$F_a^{0,2} = 0 \text{ on } U_a$$

$$A_a^{0,1} = \psi_a^{-1} \bar{\partial} \psi_a$$

$$\check{g}_{ab} := \psi_a g_{ab} \psi_b^{-1} \text{ with } \bar{\partial} \check{g}_{ab} = 0$$

$$S = \int_X \Omega^{3,0} \wedge \operatorname{tr}\left\{A^{0,1} \wedge \bar{\partial} A^{0,1} + \frac{2}{3} A^{0,1} \wedge A^{0,1} \wedge A^{0,1}\right\}$$

$$A^{0,1|0} \in \Omega^{0,1}\left(L^{5|6}, \mathrm{L}_0\right), B^{0,2|0} \in \Omega^{0,2}\left(L^{5|6}, \mathrm{L}_{-1}\right), \text{ and } C^{0,3|0} \in \Omega^{0,3}\left(L^{5|6}, \mathrm{L}_{-2}\right).$$

$$\begin{aligned} S = \int_{L^{5|6}} \Omega^{5|6,0} \wedge \Big\{ & \left\langle A^{0,1|0}, \bar{\partial} C^{0,3|0}\right\rangle_{\mathrm{L}} + \left\langle B^{0,2|0}, \mu_1\left(C^{0,3|0}\right)\right\rangle_{\mathrm{L}} - \frac{1}{2}\left\langle B^{0,2|0}, \bar{\partial} B^{0,2|0}\right\rangle_{\mathrm{L}} \\ & + \frac{1}{2}\left\langle A^{0,1|0}, \mu_2\left(A^{0,1|0}, C^{0,3|0}\right)\right\rangle_{\mathrm{L}} + \frac{1}{2}\left\langle A^{0,1|0}, \mu_2\left(B^{0,2|0}, B^{0,2|0}\right)\right\rangle_{\mathrm{L}} \\ & + \frac{1}{3!}\left\langle A^{0,1|0}, \mu_3\left(A^{0,1|0}, A^{0,1|0}, B^{0,2|0}\right)\right\rangle_{\mathrm{L}} \\ & + \frac{1}{5!}\left\langle A^{0,1|0}, \mu_4\left(A^{0,1|0}, A^{0,1|0}, A^{0,1|0}, A^{0,1|0}\right)\right\rangle_{\mathrm{L}} \Big\} \end{aligned}$$

$$\mathcal{F}_a^{0,2|0} := \bar{\partial} A_a^{0,1|0} + \frac{1}{2}\mu_2\left(A_a^{0,1|0}, A_a^{0,1|0}\right) + \mu_1\left(B_a^{0,2|0}\right) = 0$$

$$\mathcal{H}_a^{0,3|0} := \bar{\partial} B_a^{0,2|0} + \mu_2\left(A_a^{0,1|0}, B_a^{0,2|0}\right) - \frac{1}{3!}\mu_3\left(A_a^{0,1|0}, A_a^{0,1|0}, A_a^{0,1|0}\right) - \mu_1\left(C_a^{0,3|0}\right) = 0$$

$$\begin{aligned} \mathcal{G}_a^{0,4|0} := & \bar{\partial} C_a^{0,3|0} + \mu_2\left(A_a^{0,1|0}, C_a^{0,3|0}\right) + \frac{1}{2}\mu_2\left(B_a^{0,2|0}, B_a^{0,2|0}\right) + \\ & + \frac{1}{2}\mu_3\left(A_a^{0,1|0}, A_a^{0,1|0}, B_a^{0,2|0}\right) + \frac{1}{4!}\mu_4\left(A_a^{0,1|0}, A_a^{0,1|0}, A_a^{0,1|0}, A_a^{0,1|0}\right) = 0 \end{aligned}$$

$$\mathcal{F}_a^{0,2|0} = \bar{\partial} A_a^{0,1|0} + \frac{1}{2}\mu_2^{\min}\left(A_a^{0,1|0}, A_a^{0,1|0}\right) = 0$$

CONCLUSIONES.

De los resultados obtenidos en el apartado anterior, se concluye que: 1) el espacio – tiempo cuántico es susceptible de deformación, no necesariamente extrema, lo que aparece como un curvatura hipergeométrica; 2) la gravedad cuántica, en escenarios entrópicos, puede formar membranas que no necesariamente se despliegan en dimensiones más altas, sino que, se tratan de desdoblamientos dentro de un mismo espacio – tiempo cuántico, por lo que, no se forman dimensiones en sí mismas, sino distorsiones de realidad espacial y temporal; 3) en un escenario de gravedad cuántica, la formación de agujeros negros cuánticos es posible, especialmente cuando la partícula supermasiva colapsa o colisiona con otra, en tanto que, es imposible la formación de agujeros cuánticos de gusano, pues la gravedad no es extrema aunque entrópica; 4) la partícula supermasiva o llamada también partícula oscura, a propósito de su interacción, produce gravedad cuántica. Cuando la partícula supermasiva, por sí misma, tuerce el espacio – tiempo cuántico, esto se denomina gravedad cuántica endógena, en tanto que, cuando la gravedad cuántica se produce por la interacción de la partícula supermasiva y el gravitón, entonces se vuelve exógena por permeabilización del espacio – tiempo cuántico por un campo gravitónico; 5) la simetría de calibre, se torna esencial para efectos de conciliar la relatividad general y especial y la mecánica cuántica, aunque no en circunstancias extremas o primitivas; 6) la formación de materia oscura, es inevitable en gravedad cuántica, fundamentalmente por interacción de la partícula supermasiva y la formación de agujeros negros cuánticos que devoran materia y energía a escala microscópica, volviéndola en materia pura y oscura, extremadamente densa, en la que, la intervención de la gravedad es unitaria.

ACLARACIONES FINALES:

Algunas aclaraciones finales a tener en consideración y aplicar, a propósito de la Teoría Cuántica de Campos Relativistas o Curvos (TCCR) propuesta por este autor:

1. En todos los casos, este símbolo † será reemplazado por este símbolo†o por este símbolo †, equivaliendo lo mismo.

Símbolo a ser reemplazado.	**Símbolos de reemplazo.**
†	†
	†

2. En todos los casos, este símbolo ‡, será reemplazado por este símbolo ‡ o por este símbolo ‡.

Símbolo a ser reemplazado.	**Símbolos de reemplazo.**
‡	‡
	‡

3. En todos los casos, se añadirá y por ende, se calculará la magnitud§que equivale a un campo de Yang – Mills y por ende, a la teoría de Yang – Mills en sentido amplio, en relación a la Teoría Cuántica de Campos Relativistas propuesta por este autor.

4. Este símbolo • podrá usarse como exponente u operador, según sea el caso.

Las aclaraciones antes referidas aplican tanto a este trabajo como a todos los trabajos previos y posteriores publicados por este autor, según corresponda.

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