



Ciencia Latina
Internacional

Ciencia Latina Revista Científica Multidisciplinar, Ciudad de México, México.
ISSN 2707-2207 / ISSN 2707-2215 (en línea), enero-febrero 2024,
Volumen 8, Número 1.

https://doi.org/10.37811/cl_rcm.v8i1

**DEMOSTRACIÓN DEL ESPECTRO
HAMILTONIANO PARA UN CAMPO DE YANG-
MILLS NO ABELIANO QUE POSEEN UNA
BRECHA DE MASA FINITA CON RESPECTO AL
ESTADO DE VACÍO**

**DEMONSTRATION OF THE HAMILTONIAN SPECTRUM FOR A
NON-ABELIAN YANG-MILLS FIELD POSSESSING A FINITE
MASS GAP WITH RESPECT TO THE VACUUM STATE**

Manuel Ignacio Albuja Bustamante
Investigador Independiente

Demostración del Espectro Hamiltoniano para un Campo de Yang-Mills no Abeliano que Poseen una Brecha de Masa Finita con Respecto al Estado de Vacío

Manuel Ignacio Albuja Bustamante¹

ignaciomanuelaljubustamante@gmail.com

<https://orcid.org/0009-0005-0115-767X>

Investigador Independiente

Ecuador

RESUMEN

El presente artículo científico, tiene como propósito, demostrar, a través de la conjugación estructurada de distintos componentes interaccionados, que conforman el sistema de campos de Yang-Mills, **(i)** la conjetura de que las excitaciones más bajas de una teoría pura de Yang-Mills (es decir, sin campos de materia) tienen una brecha de masa finita con respecto al estado de vacío; **(ii)** la propiedad de confinamiento en presencia de partículas adicionales; y, **(iii)** que, dado un *hamiltoniano cuántico* para un campo de Yang-Mills no abeliano, existe un valor positivo mínimo de la energía. La solución de los problemas antes descritos, requiere tanto la comprensión de uno de los profundos misterios de la física sin resolver, esto es, la existencia de una brecha de masa, como la producción de un ejemplo matemáticamente completo de la teoría cuántica de campos gauge en el espacio-tiempo de cuatro dimensiones, lo que se aborda rigurosamente en el presente artículo científico.

Palabras clave: física cuántica, escala subatómica, campos de yang-mills, teorías de gauge, brecha de masa

¹ Autor principal.

Correspondencia: ignaciomanuelaljubustamante@gmail.com

Demonstration of the Hamiltonian Spectrum for a Non-Abelian Yang-Mills Field Possessing a Finite Mass Gap With Respect to the Vacuum State

ABSTRACT

The purpose of this scientific article is to demonstrate, through the structured conjugation of different interacted components that make up the Yang-Mills field system, **(i)** the conjecture that the lowest excitations of a pure Yang-Mills theory (i.e., without matter fields) have a finite mass gap with respect to the vacuum state; **(ii)** the property of confinement in the presence of additional particles; and, **(iii)** that, given a quantum Hamiltonian for a non-abelian Yang-Mills field, there is a minimum positive value of energy. The solution of the problems described above requires both the understanding of one of the profound unsolved mysteries of physics, that is, the existence of a mass gap, and the production of a mathematically complete example of the quantum theory of gauge fields in four-dimensional spacetime, which is rigorously addressed in the present work.

Keywords: quantum physics, subatomic scale, yang-mills fields, gauge theories, mass gap

*Artículo recibido 27 diciembre 2023
Aceptado para publicación: 30 enero 2024*



INTRODUCCIÓN

Ciertamente, la descripción de la naturaleza a escala subatómica requiere de la física cuántica. En la física cuántica, la posición y la velocidad de una partícula se tienen como operadores no conmutadores que interactúan en un espacio de Hilbert. Es así, donde muchos aspectos de la naturaleza se describen en forma de campos. Dado que los campos interactúan con las partículas, deviene en indispensable, incorporar conceptos cuánticos tanto para describir campos como para describir partículas. En los campos convencionales, existe una partícula y por regla general, una antipartícula, con la misma masa y carga, pero opuesta, verbigracia, el campo cuantizado de los electrones.

Siguiendo este mismo orden de cosas, se tiene que, las teorías de gauge (teorías cuánticas de campos [QFT]), es una de las más importantes en cuanto a física de partículas se refiere. Un ejemplo claro de ello, es la teoría del electromagnetismo de Maxwell que comporta un grupo de simetría gauge en un grupo abeliano $U(1)$. Sin embargo, la teoría de Yang – Mills, en este contexto, califica una teoría gauge no abeliana.

La ecuación clásica y variacional central del lagrangiano Yang-Mills, se escribe así:

$$L = \frac{1}{4g^2} \int \text{Tr } F \wedge *F,$$

donde Tr denota una forma cuadrática invariante en el álgebra de Lie de G . Las ecuaciones de Yang-Mills no son lineales, por lo que, no existen soluciones exactas de la ecuación clásica antes referida, y es lo que se propone resolver este trabajo a través de un riguroso cálculo matemático, desde la óptica del hamiltoniano cuántico. En consecuencia, este trabajo, pretende demostrar, que la teoría gauge no abeliana de Yang – Mills, describe otras fuerzas en la naturaleza, especialmente la fuerza débil (responsable, entre otras cosas, de ciertas formas de radiactividad) y la fuerza fuerte o nuclear (responsable, entre otras cosas, de la unión de protones y neutrones en núcleos), pero sin perder las premisas esenciales de la teoría de campos de Yang – Mills, esto es, por fuera de la teoría electrodébil de Glashow-Salam-Weinberg o la teoría del “campo de Higgs”.



Si bien es cierto, constituyese en una propiedad notable de la teoría cuántica de Yang-Mills, la nominada "libertad asintótica", la misma que supone, que a distancias cortas, el campo muestra un comportamiento cuántico muy similar a su comportamiento clásico; sin embargo, a largas distancias, la teoría de Yang – Mills, fracasa en la descripción del campo. Otros componentes paralelos, que se abordan y resuelven en el presente trabajo, refieren a que: **(i)** existe una "*brecha de masa*" $\Delta > \text{constante}$, tal que cada excitación del vacío tiene energía de al menos Δ ; **(ii)** existe un confinamiento de quarks, partiendo de la premisa de que, los estados físicos de las partículas, como el protón, el neutrón y el pión, son invariantes en SU(3); y, **(iii)** existe una "*ruptura de simetría quiral*", lo que significa que el vacío es potencialmente invariante solo bajo un cierto subgrupo de simetría completa que actúa sobre los campos de quarks.

METODOLOGÍA

El enfoque es cualitativo. El tipo de investigación es predictivo. El diseño utilizado es constructivista. No existe población de estudio toda vez que el presente artículo científico no es de carácter sociológico o social. Tampoco se han implementado técnicas de recolección de información tales como encuestas, etc, salvo revisión bibliográfica. Finalmente, el material de apoyo es meramente bibliográfico.

RESULTADOS Y DISCUSIÓN

Marco Praxeológico

a. Formulación matemática primaria (línea base):

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \mathcal{L}_{gf} = -1/2 \text{tr} (F_{\nu\rho}^{\mu\sigma}) = 1/4 F^{a\mu\nu} F_{a\mu\nu} F_a^{\mu\nu} F_c^{\mu\nu} F_c^{\mu\nu}$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \mathcal{L}_{fg} = -1/2 \text{tr} (F_{\mu\sigma}^{\nu\rho}) = 1/4 F^{v\mu b} F_{v\mu b} F_b^{\mu\nu} F_{\mu\nu}^b F_c^{\mu\nu} F_c^{\mu\nu}$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \text{tr}(T^a T_b T^a T_b) = 1\delta_b^a, [T^a T_b T^a T_b] = i f^{abc} f_{abc} T^c T_c$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \text{tr}(T^b T_a T^b T_a) = 1\delta_a^b, [T^b T_a T^b T_a] = i f^{abc} f_{abc} T^c T_c$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \text{tr}(T^a T_b T^b T_a) = 1\delta_{ba}^{ab}, [T^b T_a T^b T_a] = i f^{abc} f_{abc} T^c T_c$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle D_\mu = I\partial_u - ig T^a A_\nu^\mu, D_\nu = I\partial_v - ig T^b A_\mu^\nu$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} f_{abc} A_\mu^b A_\nu^c$$



$$\hat{H}|\psi\rangle=E_\psi|\psi\rangle F_{v\mu}^a=\partial_v A_\mu^a-\partial_\mu A_v^a+g\,f_{abc}\,f^{abc}\,A_v^b A_\nu^c$$

$$\hat{H}|\psi\rangle=E_\psi|\psi\rangle[D_\mu D^\mu D_\nu D^\nu]=\,ig\,T^a\,F_{\mu\nu}^a$$

$$\hat{H}|\psi\rangle=E_\psi|\psi\rangle[D_\mu D^\nu D_\nu D^\mu]=\,ig\,T^{abc}T_{abc}\,F_{\mu\nu}^{abc}F_{\nu\mu}^{abc}$$

$$\hat{H}|\psi\rangle=E_\psi|\psi\rangle\partial^\mu F_{\nu\mu}^a F_a^{\mu\nu}+g\,f^{abc}\,f_{abc}+A^{\mu b}A_{\mu b}F_{\nu\mu}^c F_c^{\mu\nu}=\xi_{\lambda\Omega\psi}^{\sigma\zeta}\Sigma\int\int\int\int\hbar\phi\mathbb{H}\tilde{\mathbb{K}}\mathbb{J}\mathbb{K}\mathbb{H}\psi\mathbb{K}\mathbb{X}\zeta\pi mc\mathbb{R}^4$$

$$\hat{H}|\psi\rangle=E_\psi|\psi\rangle\partial^\nu F_{\nu\mu}^a F_a^{\nu\mu}+g\,f^{abc}\,f_{abc}+A^{\nu b}A_{\nu b}F_{\nu\mu}^c F_c^{\nu\mu}=\xi_{\lambda\Omega\psi}^{\sigma\zeta}\Sigma\int\int\int\int\hbar\phi\mathbb{H}\tilde{\mathbb{K}}\mathbb{J}\mathbb{K}\mathbb{H}\psi\mathbb{K}\mathbb{X}\zeta\pi mc\mathbb{R}^4$$

$$\begin{aligned}\hat{H}|\psi\rangle&=E_\psi|\psi\rangle(D_\mu D^\nu F^{\mu k}F_{\nu k}F^{vk}F_{\mu k})\,exp^a+(D_k D^k F^{\mu\nu}F_{\nu\mu}F^{v\mu}F_{\mu\nu})\,exp^b\\&+ +(D_\nu D^\mu F^{k\mu\nu}F_{k\nu\mu}F^{k\nu\mu}F_{k\mu\nu})exp^c\end{aligned}$$

$$\hat{H}|\psi\rangle=E_\psi|\psi\rangle[D_\mu[D_\nu D_k]+][D_k[D_\mu D_\nu]+][D_\nu[D_\mu D_k]$$

$$\hat{H}|\psi\rangle=E_\psi|\psi\rangle[D_\nu[D_\mu D_k]+][D_k[D_\nu D_\mu]+][D_\mu[D_\nu D_k]$$

$$\begin{aligned}\hat{H}|\psi\rangle&=E_\psi|\psi\rangle Z[ijk] \\&= \mathop{\oint\!\!\!\oint}\limits_v^\mu [dA] \exp\big[-\frac{ijk}{2}\mathop{\oint\!\!\!\oint}\limits_\mu^v d_{v\mu}^{\mu\nu}d_\rho^\sigma\,tr\,(F^{\mu\nu}F_{\nu\mu}F^{v\mu}F_{\mu\nu})\\&+ijk\mathop{\oint\!\!\!\oint}\limits_\rho^\sigma d_{v\mu}^{\mu\nu}d_\rho^\sigma\,ijk_\mu^aijk_\nu^bijk_{\mu\nu}^cijk_\nu^\muijk_\mu^v(xyz\dots n)+ijk\big]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle&=E_\psi|\psi\rangle Z[ijk,\varepsilon,\xi] \\&= \mathop{\oint\!\!\!\oint}\limits_v^\mu [d\,A][d\,\varsigma][d\,c]\exp\{ijk\,S^FS_F[\partial A,A]+ijkS^{gf}S_{gf}[\partial A] \\&+ijkS^gS_g[\partial c,\partial\varsigma,c,\varsigma,A]\}\exp\{ijk\mathop{\oint\!\!\!\oint}\limits_\rho^\sigma d_{v\mu}^{\mu\nu}d_\rho^\sigma\,ijk_\mu^aijk_\nu^bijk_{\mu\nu}^cijk_\nu^\muijk_\mu^vA^{abc\mu}A_{abcv}(xyz\dots n) \\&+ijk\mathop{\oint\!\!\!\oint}\limits_\mu^v d_{v\mu}^{\mu\nu}d_\rho^\sigma\,(xyz\dots n)\,[c^{-abc}c_{abc}(xyz\dots n)\,\varepsilon^{-abc}\varepsilon_{abc}(xyz\dots n)\xi^{-abc}\xi_{abc}(xyz\dots n)]\}\end{aligned}$$

Propagador de gluón:

$$\begin{aligned}\hat{H}|\psi\rangle&=E_\psi|\psi\rangle D_{\nu\mu}^{abc}D_{\nu\mu}^{abc}(p,q) \\&=ijk\delta^{abc}\delta_{abc}/p_\rho^\sigma\Omega_\sigma^\rho+ijk\Delta\theta\varphi\omega\eta\lambda\phi\psi[\eta^{\mu\nu}\eta_{\nu\mu}-(1-\xi)p^\mu p_\nu p^v p_\mu/p_\sigma^\rho\Omega_\rho^\sigma \\&+ijk\Delta\theta\varphi\omega\eta\lambda\phi\psi\end{aligned}$$



Vértice de gluón -3:

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \Gamma_{\mu\nu\lambda}^{abc} \Gamma_{\nu\mu\lambda}^{abc}(p, q, r) = g f^{abc} g f_{abc} [(p - q)_{\lambda}^{\mu\nu} \eta \eta_{\nu\mu}^{\lambda} + (q - r)_{\lambda}^{\mu\nu} \eta \eta_{\nu\mu}^{\lambda} + (r - p)_{\lambda}^{\mu\nu} \eta \eta_{\nu\mu}^{\lambda}]$$

Vértice de gluón -4:

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Gamma_{\mu\nu\lambda\sigma}^{abc\rho} \Gamma_{\nu\mu\lambda\sigma}^{abc\rho} \\ &= ijk g^{\mu\nu\rho\sigma} ijk g_{\nu\mu\rho\sigma} f^{abcde} f_{abcde} (\eta^{\mu\lambda} \eta_{\nu\sigma} - \eta^{\nu\lambda} \eta_{\mu\sigma}) \\ &\quad - ijk g^{\mu\nu\rho\sigma} ijk g_{\nu\mu\rho\sigma} f^{abcde} f_{abcde} (\eta^{\mu\sigma} \eta_{\nu\lambda} - \eta^{\nu\sigma} \eta_{\mu\lambda}) \\ &\quad - ijk g^{\mu\nu\rho\sigma} ijk g_{\nu\mu\rho\sigma} f^{abcde} f_{abcde} (\eta^{\mu\nu\lambda\sigma} \eta_{\mu\nu\sigma\lambda} - \eta^{\nu\mu\lambda\sigma} \eta_{\nu\mu\sigma\lambda}) \end{aligned}$$

Propagador Ghost:

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle C^{ab} C_{ba} \delta^{\mu\nu} \delta_{\nu\mu}(p, q) = ijk \delta^{ba} \delta_{ab} \delta^{\nu\mu} \delta_{\mu\nu} \rho \sigma \lambda \Omega / \omega \eta \xi \epsilon \phi \psi$$

Vértice $c\zeta g$:

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \Gamma^{ab} \Gamma_{ba} \Gamma^{\mu\nu} \Gamma_{\nu\mu} \delta^{\mu\nu} \delta_{\nu\mu}(p, q, r) = g f^{abc} g f_{abc} ijk \delta^{ba} \delta_{ab} \delta^{\nu\mu} \delta_{\mu\nu} \rho^{\mu\nu} \rho_{\nu\mu} \sigma \lambda \Omega / \omega \eta \xi \epsilon \phi \psi$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle Z[ijk, \epsilon, \xi]$$

$$\begin{aligned} &= \exp(ijk g \oint_v^\mu d\rho^\sigma \lambda xyz \dots n \delta / ijk \xi^{abcd} \xi_{abcd}(xyz \dots n) f \lambda_{\mu\nu\rho\sigma}^{abcd} f \mathcal{L}_{\mu\nu\rho\sigma}^{abcd} f \lambda_{abcd}^{\mu\nu\rho\sigma} f \mathcal{L}_{abcd}^{\mu\nu\rho\sigma} \partial \theta^{\mu\nu} \partial \theta_{\mu\nu} ijk \delta \\ &\quad / \delta ijk_{\mu\nu}^{abc}(xyz \dots n) ijk \delta \end{aligned}$$

$$\begin{aligned} &\quad \frac{ijk g \oint_v^\mu d\rho^\sigma \lambda xyz \dots n \frac{\delta \epsilon^{abc} \epsilon_{abc}(xyz \dots n)}{ijk \xi^{abcd} \xi_{abcd}(xyz \dots n) \partial \theta^{\mu\nu} \partial \theta_{\mu\nu} f \lambda_{\mu\nu\rho\sigma}^{abcd} f \mathcal{L}_{\mu\nu\rho\sigma}^{abcd} f \lambda_{abcd}^{\mu\nu\rho\sigma} f \mathcal{L}_{abcd}^{\mu\nu\rho\sigma} ijk \delta}}{\delta ijk_{\mu\nu}^{abc}(xyz \dots n) ijk \delta} \\ &= \delta ijk_{\mu\nu}^{abc}(xyz \dots n) x \exp \left(- \frac{\delta ijk_{\mu\nu}^{abc}(xyz \dots n) ijk \delta}{\delta ijk_{\mu\nu}^{abc}(xyz \dots n) \delta ijk_{\nu\mu}^{abc}(xyz \dots n)} \right) \end{aligned}$$

$$* Z_0[ijk, \epsilon, \xi]$$

$$\begin{aligned} &= \exp \left(- \oint_v^\mu d_{abcd}^{\mu\nu\rho\sigma} d_{\mu\nu\rho\sigma}^{abcd} \Delta \lambda \nabla \Omega (xyz \dots n) C^{abcd} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} C_{abcd} \theta \lambda \Omega (x \right. \\ &\quad \left. - y) \epsilon_{\mu\nu\rho\sigma}^{abcd} \epsilon_{abcd}^{\mu\nu\rho\sigma} (xyz \dots n) \right) \exp \left(\frac{1}{2 \oint_v^\mu d_{abcd}^{\mu\nu\rho\sigma} d_{\mu\nu\rho\sigma}^{abcd} \Delta \lambda \nabla \Omega} (xyz \dots n) \partial \Delta ijk_{\mu\nu\rho\sigma}^{abcd} \partial \nabla_{abcd}^{\mu\nu\rho\sigma} C^{abcd} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} C_{abcd} \theta \lambda \Omega (x \right. \\ &\quad \left. - y) \epsilon_{\mu\nu\rho\sigma}^{abcd} \epsilon_{abcd}^{\mu\nu\rho\sigma} (xyz \dots n) \right) \end{aligned}$$



b. Estructuras constantes antisimétricas.

$$\begin{aligned} \hat{H}|\psi\rangle &= E_\psi|\psi\rangle [T^a, T^b = if^{abc}T^c] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle [T^b, T^a = if^{abc}T^c] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle [T^c, T^a = if^{abc}T^b] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle [T^c, T^b = if^{abc}T^a] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle [T^a, T^c = if^{abc}T^b] + \\ \hat{H}|\psi\rangle &= E_\psi|\psi\rangle [T^b, T^c = if^{abc}T^a] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle [T^a, T^b = if^{bac}T^c] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle [T^b, T^a = if^{bac}T^c] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle [T^c, T^a = if^{bac}T^b] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle [T^c, T^b = if^{bac}T^a] + \\ \hat{H}|\psi\rangle &= E_\psi|\psi\rangle [T^a, T^c = if^{bac}T^b] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle [T^b, T^c = if^{bac}T^a] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle [T^a, T^b = if^{cba}T^c] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle [T^b, T^a = if^{cba}T^c] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle [T^c, T^a = if^{cba}T^b] + \\ \hat{H}|\psi\rangle &= E_\psi|\psi\rangle [T^c, T^b = if^{cba}T^a] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle [T^a, T^c = if^{cba}T^b] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle [T^b, T^c = if^{cba}T^a] = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{K} \mathfrak{Z} \mathbb{K} \mathbb{K} \mathfrak{h} \mathfrak{K} \mathbb{X} \zeta \pi m c \mathbb{R}^4 \end{aligned}$$

$$\begin{aligned} \hat{H}|\psi\rangle &= E_\psi|\psi\rangle \text{tr} T^a T^b = \frac{1}{2} \delta^{ab} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle \text{tr} T^b T^a = \frac{1}{2} \delta^{ba} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle \text{tr} T^a T^b = \\ \frac{1}{2} \delta^{ba} + \hat{H}|\psi\rangle &= E_\psi|\psi\rangle \text{tr} T^b T^a = \frac{1}{2} \delta^{ab} = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{K} \mathfrak{Z} \mathbb{K} \mathbb{K} \mathfrak{h} \mathfrak{K} \mathbb{X} \zeta \pi m c \mathbb{R}^4 \end{aligned}$$

c. Campo de Gauge.

$$\begin{aligned} \hat{H}|\psi\rangle &= E_\psi|\psi\rangle A_\mu = A_\mu^a T^a + \hat{H}|\psi\rangle = E_\psi|\psi\rangle A_\mu = A_\mu^b T^b + \hat{H}|\psi\rangle = E_\psi|\psi\rangle A_\mu = A_\mu^c T^c + \hat{H}|\psi\rangle \\ &= E_\psi|\psi\rangle A_\mu = A_\mu^{abc} T^{abc} = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{K} \mathfrak{Z} \mathbb{K} \mathbb{K} \mathfrak{h} \mathfrak{K} \mathbb{X} \zeta \pi m c \mathbb{R}^4 \end{aligned}$$

$$\begin{aligned} \hat{H}|\psi\rangle &= E_\psi|\psi\rangle F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle F_{\nu\mu} = \partial_\nu A_\mu - \partial_\mu A_\nu - i[A_\nu, A_\mu] = \\ \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \phi \mathbb{K} \mathfrak{Z} \mathbb{K} \mathbb{K} \mathfrak{h} \mathfrak{K} \mathbb{X} \zeta \pi m c \mathbb{R}^4 \end{aligned}$$

d. Covariante Derivada.

$$\begin{aligned} \hat{H}|\psi\rangle &= E_\psi|\psi\rangle D_\mu \psi = \partial_\mu \psi - iA_\mu \psi + \hat{H}|\psi\rangle = E_\psi|\psi\rangle D_\nu \psi = \partial_\nu \psi - iA_\nu \psi + \hat{H}|\psi\rangle = E_\psi|\psi\rangle D_{\mu\nu} \psi = \partial_{\mu\nu} \psi - iA_{\mu\nu} \psi + \hat{H}|\psi\rangle = E_\psi|\psi\rangle D_{\nu\mu} \psi = \partial_{\nu\mu} \psi - iA_{\nu\mu} \psi \\ &= \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \phi \mathbb{K} \mathfrak{Z} \mathbb{K} \mathbb{K} \mathfrak{h} \mathfrak{K} \mathbb{X} \zeta \pi m c \mathbb{R}^4 \end{aligned}$$

$$\begin{aligned} \hat{H}|\psi\rangle &= E_\psi|\psi\rangle D_\mu \psi^i = \partial_\nu \psi^i - iA_\mu^a T^a(R)_j^i \psi^j, j + \hat{H}|\psi\rangle = E_\psi|\psi\rangle D_\nu \psi^i \\ &= \partial_\mu \psi^i - iA_\nu^a T^a(R)_j^i \psi^j, j + \hat{H}|\psi\rangle = E_\psi|\psi\rangle D_{\mu\nu} \psi^{ij} \\ &= \partial_{\nu\mu} \psi^{ij} - ijA_{\mu\nu}^{abc} T^{abc}(R)_i^j \psi^j, i + \hat{H}|\psi\rangle = E_\psi|\psi\rangle D_{\nu\mu} \psi^{ji} \\ &= \partial_{\mu\nu} \psi^{ji} - jiA_{\nu\mu}^{abc} T^{abc}(R)_j^i \psi^i, j + \hat{H}|\psi\rangle = E_\psi|\psi\rangle D_{\mu\nu\nu\mu} \psi^{ijji} \\ &= \partial_{\nu\mu\mu\nu} \psi^{jiii} - jiiA_{\mu\nu}^{abcbcacabbac} T^{abcbcacabbac}(R)_{jii}^{jji} \psi^{jii} ijji, jii \\ &= \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{K} \mathfrak{Z} \mathbb{K} \mathbb{K} \mathfrak{h} \mathfrak{K} \mathbb{X} \zeta \pi m c \mathbb{R}^4 \end{aligned}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle D_\mu\phi = \partial_\mu\phi - i[A_\mu, \phi] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle D_\nu\phi = \partial_\nu\phi - i[A_\nu, \phi] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle D_{\mu\nu}\phi \\ &= \partial_{\mu\nu}\phi - ij[A_{\mu\nu}, \phi] + \hat{H}|\psi\rangle = E_\psi|\psi\rangle D_{v\mu}\phi = \partial_{v\mu}\phi - ji[A_{v\mu}, \phi] + \hat{H}|\psi\rangle \\ &= E_\psi|\psi\rangle D_{\mu\nu v\mu}\phi = \partial_{\mu\nu v\mu}\phi - ijji[A_{\mu\nu v\mu}, \phi] = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{H} \check{\mathbb{X}} \mathbb{K} \mathbb{H} \mathbb{K} \psi \check{\mathbb{X}} \mathbb{K} \mathbb{H} \mathbb{K} \zeta \pi m c \mathbb{R}^4\end{aligned}$$

e. Dinámicas de Yang – Mills.

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle S_{YM} = \frac{1}{2g^2} \int d^4x \operatorname{tr} F^{\mu\nu} F_{\mu\nu} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle S_{YM} = \frac{1}{2g^2} \int d^4x \operatorname{tr} F^{v\mu} F_{v\mu} + \hat{H}|\psi\rangle \\ \psi\rangle &= E_\psi|\psi\rangle S_{YM} = \frac{n^\infty}{ng^\infty} \int \int \int \int \int \mu v \nu \mu d^\infty x \operatorname{tr} F^{\mu\nu v\mu} F_{\mu\nu v} = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{H} \check{\mathbb{X}} \mathbb{K} \mathbb{H} \mathbb{K} \psi \check{\mathbb{X}} \mathbb{K} \mathbb{H} \mathbb{K} \zeta \pi m c \mathbb{R}^4\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle * F^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle * F^{v\mu} = \frac{1}{2} \epsilon^{v\mu\rho\sigma} F_{\rho\sigma} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle * \\ F^{\mu\nu} &= \frac{1}{2} \epsilon^{\mu\nu\sigma\rho} F_{\sigma\rho} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle * F^{v\mu} = \frac{1}{2} \epsilon^{v\mu\sigma\rho} F_{\sigma\rho} = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{H} \check{\mathbb{X}} \mathbb{K} \mathbb{H} \mathbb{K} \psi \check{\mathbb{X}} \mathbb{K} \mathbb{H} \mathbb{K} \zeta \pi m c \mathbb{R}^4\end{aligned}$$

f. Rescaling.

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle \tilde{A}_\mu = \frac{1}{g} A_\mu + \hat{H}|\psi\rangle = E_\psi|\psi\rangle \tilde{A}_v = \frac{1}{g} A_v + \hat{H}|\psi\rangle = E_\psi|\psi\rangle \tilde{A}_{\mu\nu} = \frac{1}{g} A_{\mu\nu} + \hat{H}|\psi\rangle \\ &= E_\psi|\psi\rangle \tilde{A}_{v\mu} = \frac{1}{g} A_{v\mu} + \mathbf{F}_{\mu\nu} = \partial_u \tilde{A}_v - \partial_v \tilde{A}_u - ig[\tilde{A}_u, \tilde{A}_v] + \mathbf{F}_{v\mu} \\ &= \partial_v \tilde{A}_\mu - \partial_\mu \tilde{A}_v - ig[\tilde{A}_v, \tilde{A}_u] = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{H} \check{\mathbb{X}} \mathbb{K} \mathbb{H} \mathbb{K} \psi \check{\mathbb{X}} \mathbb{K} \mathbb{H} \mathbb{K} \zeta \pi m c \mathbb{R}^4 \\ \hat{H}|\psi\rangle &= E_\psi|\psi\rangle S_{YM} = \frac{1}{2g^2} \int d^4x \operatorname{tr} F^{\mu\nu} F_{\mu\nu} = -\frac{1}{2} \int d^4x \operatorname{tr} F^{\mu\nu} F_{\mu\nu} \\ \hat{H}|\psi\rangle &= E_\psi|\psi\rangle S_{YM} = \frac{1}{2g^2} \int d^4x \operatorname{tr} F^{v\mu} F_{v\mu} = -\frac{1}{2} \int d^4x \operatorname{tr} F^{v\mu} F_{v\mu} \\ \hat{H}|\psi\rangle &= E_\psi|\psi\rangle S_{YM} = \frac{n}{ng^\infty} \int d^\infty x \operatorname{tr} F^{v\mu} F_{v\mu} = -\frac{n}{n-1} \int d^\infty x \operatorname{tr} F^{v\mu} F_{v\mu} \\ \hat{H}|\psi\rangle &= E_\psi|\psi\rangle S_{YM} = \frac{n}{ng^\infty} \int d^\infty x \operatorname{tr} F^{\mu\nu} F_{\mu\nu} = -\frac{n}{n-1} \int d^\infty x \operatorname{tr} F^{\mu\nu} F_{\mu\nu}\end{aligned}$$

g. Simetría de Gauge.

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle A_\mu \rightarrow \Omega(x) A_\mu \Omega^{-1}(x) + i\Omega(x) \partial_\mu \Omega^{-1}(x) \\ \hat{H}|\psi\rangle &= E_\psi|\psi\rangle A_v \rightarrow \Omega(x) A_v \Omega^{-1}(x) + i\Omega(x) \partial_v \Omega^{-1}(x) \\ \hat{H}|\psi\rangle &= E_\psi|\psi\rangle A_{\mu\nu} \rightarrow \Omega(x) A_{\mu\nu} \Omega^{-1}(x) + i\Omega(x) \partial_{\mu\nu} \Omega^{-1}(x) \\ \hat{H}|\psi\rangle &= E_\psi|\psi\rangle A_{v\mu} \rightarrow \Omega(x) A_{v\mu} \Omega^{-1}(x) + i\Omega(x) \partial_{v\mu} \Omega^{-1}(x)\end{aligned}$$



$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_\mu \rightarrow \Omega(y) A_\mu \Omega^{-1}(y) i\Omega(y) \partial_\mu \Omega^{-1}(y)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_v \rightarrow \Omega(y) A_v \Omega^{-1}(y) i\Omega(y) \partial_v \Omega^{-1}(y)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{\mu v} \rightarrow \Omega(y) A_{\mu v} \Omega^{-1}(y) i\Omega(y) \partial_{\mu v} \Omega^{-1}(y)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{v\mu} \rightarrow \Omega(y) A_{v\mu} \Omega^{-1}(y) i\Omega(y) \partial_{v\mu} \Omega^{-1}(y)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_\mu \rightarrow \Omega(z) A_\mu \Omega^{-1}(z) i\Omega(z) \partial_\mu \Omega^{-1}(z)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_v \rightarrow \Omega(z) A_v \Omega^{-1}(z) i\Omega(z) \partial_v \Omega^{-1}(z)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{\mu v} \rightarrow \Omega(z) A_{\mu v} \Omega^{-1}(z) i\Omega(z) \partial_{\mu v} \Omega^{-1}(z)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{v\mu} \rightarrow \Omega(z) A_{v\mu} \Omega^{-1}(z) i\Omega(z) \partial_{v\mu} \Omega^{-1}(z)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_\mu \rightarrow \Omega(\infty) A_\mu \Omega^{-1}(\infty) i\Omega(\infty) \partial_\mu \Omega^{-1}(\infty)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_v \rightarrow \Omega(\infty) A_v \Omega^{-1}(\infty) i\Omega(\infty) \partial_v \Omega^{-1}(\infty)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{\mu v} \rightarrow \Omega(\infty) A_{\mu v} \Omega^{-1}(\infty) i\Omega(\infty) \partial_{\mu v} \Omega^{-1}(\infty)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{v\mu} \rightarrow \Omega(\infty) A_{v\mu} \Omega^{-1}(\infty) i\Omega(\infty) \partial_{v\mu} \Omega^{-1}(\infty)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_\mu \rightarrow \Omega(x) A_\mu \Omega^{-1}(x) j\Omega(x) \partial_\mu \Omega^{-1}(x)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_v \rightarrow \Omega(x) A_v \Omega^{-1}(x) j\Omega(x) \partial_v \Omega^{-1}(x)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{\mu v} \rightarrow \Omega(x) A_{\mu v} \Omega^{-1}(x) j\Omega(x) \partial_{\mu v} \Omega^{-1}(x)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{v\mu} \rightarrow \Omega(x) A_{v\mu} \Omega^{-1}(x) j\Omega(x) \partial_{v\mu} \Omega^{-1}(x)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_\mu \rightarrow \Omega(y) A_\mu \Omega^{-1}(y) j\Omega(y) \partial_\mu \Omega^{-1}(y)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_v \rightarrow \Omega(y) A_v \Omega^{-1}(y) j\Omega(y) \partial_v \Omega^{-1}(y)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{\mu v} \rightarrow \Omega(y) A_{\mu v} \Omega^{-1}(y) j\Omega(y) \partial_{\mu v} \Omega^{-1}(y)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{v\mu} \rightarrow \Omega(y) A_{v\mu} \Omega^{-1}(y) j\Omega(y) \partial_{v\mu} \Omega^{-1}(y)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_\mu \rightarrow \Omega(z) A_\mu \Omega^{-1}(z) j\Omega(z) \partial_\mu \Omega^{-1}(z)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_v \rightarrow \Omega(z) A_v \Omega^{-1}(z) j\Omega(z) \partial_v \Omega^{-1}(z)$$



$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{\mu\nu} \rightarrow \Omega(z) A_{\mu\nu} \Omega^{-1}(z) j\Omega(z) \partial_{\mu\nu} \Omega^{-1}(z)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{v\mu} \rightarrow \Omega(z) A_{v\mu} \Omega^{-1}(z) j\Omega(z) \partial_{v\mu} \Omega^{-1}(z)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_\mu \rightarrow \Omega(\infty) A_\mu \Omega^{-1}(\infty) j\Omega(\infty) \partial_\mu \Omega^{-1}(\infty)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_v \rightarrow \Omega(\infty) A_v \Omega^{-1}(\infty) j\Omega(\infty) \partial_v \Omega^{-1}(\infty)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{\mu\nu} \rightarrow \Omega(\infty) A_{\mu\nu} \Omega^{-1}(\infty) j\Omega(\infty) \partial_{\mu\nu} \Omega^{-1}(\infty)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{v\mu} \rightarrow \Omega(\infty) A_{v\mu} \Omega^{-1}(\infty) j\Omega(\infty) \partial_{v\mu} \Omega^{-1}(\infty)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{\mu\nu} \rightarrow \Omega(x) F_{\mu\nu} \Omega^{-1}(x)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{v\mu} \rightarrow \Omega(x) F_{v\mu} \Omega^{-1}(x)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{\mu\nu} \rightarrow \Omega(y) F_{\mu\nu} \Omega^{-1}(y)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{v\mu} \rightarrow \Omega(y) F_{v\mu} \Omega^{-1}(y)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{\mu\nu} \rightarrow \Omega(z) F_{\mu\nu} \Omega^{-1}(z)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{v\mu} \rightarrow \Omega(z) F_{v\mu} \Omega^{-1}(z)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{\mu\nu} \rightarrow \Omega(\infty) F_{\mu\nu} \Omega^{-1}(\infty)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{v\mu} \rightarrow \Omega(\infty) F_{v\mu} \Omega^{-1}(\infty)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_\mu \rightarrow \mathcal{U}(x) A_\mu \mathcal{U}^{-1}(x) i\mathcal{U}(x) \partial_\mu \mathcal{U}^{-1}(x)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_v \rightarrow \mathcal{U}(x) A_v \mathcal{U}^{-1}(x) i\mathcal{U}(x) \partial_v \mathcal{U}^{-1}(x)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{\mu\nu} \rightarrow \mathcal{U}(x) A_{\mu\nu} \mathcal{U}^{-1}(x) i\mathcal{U}(x) \partial_{\mu\nu} \mathcal{U}^{-1}(x)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{v\mu} \rightarrow \mathcal{U}(x) A_{v\mu} \mathcal{U}^{-1}(x) i\mathcal{U}(x) \partial_{v\mu} \mathcal{U}^{-1}(x)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_\mu \rightarrow \mathcal{U}(y) A_\mu \mathcal{U}^{-1}(y) i\mathcal{U}(y) \partial_\mu \mathcal{U}^{-1}(y)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_v \rightarrow \mathcal{U}(y) A_v \mathcal{U}^{-1}(y) i\mathcal{U}(y) \partial_v \mathcal{U}^{-1}(y)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{\mu\nu} \rightarrow \mathcal{U}(y) A_{\mu\nu} \mathcal{U}^{-1}(y) i\mathcal{U}(y) \partial_{\mu\nu} \mathcal{U}^{-1}(y)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{v\mu} \rightarrow \mathcal{U}(y) A_{v\mu} \mathcal{U}^{-1}(y) i\mathcal{U}(y) \partial_{v\mu} \mathcal{U}^{-1}(y)$$



$$\begin{aligned}
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_\mu \rightarrow \mathcal{U}(z) A_\mu \mathcal{U}^{-1}(z) i\mathcal{U}(z) \partial_\mu \mathcal{U}^{-1}(z) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_v \rightarrow \mathcal{U}(z) A_v \mathcal{U}^{-1}(z) i\mathcal{U}(z) \partial_v \mathcal{U}^{-1}(z) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_{\mu v} \rightarrow \mathcal{U}(z) A_{\mu v} \mathcal{U}^{-1}(z) i\mathcal{U}(z) \partial_{\mu v} \mathcal{U}^{-1}(z) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_{v\mu} \rightarrow \mathcal{U}(z) A_{v\mu} \mathcal{U}^{-1}(z) i\mathcal{U}(z) \partial_{v\mu} \mathcal{U}^{-1}(z) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_\mu \rightarrow \mathcal{U}(\infty) A_\mu \mathcal{U}^{-1}(\infty) i\mathcal{U}(\infty) \partial_\mu \mathcal{U}^{-1}(\infty) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_v \rightarrow \mathcal{U}(\infty) A_v \mathcal{U}^{-1}(\infty) i\mathcal{U}(\infty) \partial_v \mathcal{U}^{-1}(\infty) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_{\mu v} \rightarrow \mathcal{U}(\infty) A_{\mu v} \mathcal{U}^{-1}(\infty) i\mathcal{U}(\infty) \partial_{\mu v} \mathcal{U}^{-1}(\infty) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_{v\mu} \rightarrow \mathcal{U}(\infty) A_{v\mu} \mathcal{U}^{-1}(\infty) i\mathcal{U}(\infty) \partial_{v\mu} \mathcal{U}^{-1}(\infty) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_\mu \rightarrow \mathcal{U}(x) A_\mu \mathcal{U}^{-1}(x) j\mathcal{U}(x) \partial_\mu \mathcal{U}^{-1}(x) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_v \rightarrow \mathcal{U}(x) A_v \mathcal{U}^{-1}(x) j\mathcal{U}(x) \partial_v \mathcal{U}^{-1}(x) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_{\mu v} \rightarrow \mathcal{U}(x) A_{\mu v} \mathcal{U}^{-1}(x) j\mathcal{U}(x) \partial_{\mu v} \mathcal{U}^{-1}(x) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_{v\mu} \rightarrow \mathcal{U}(x) A_{v\mu} \mathcal{U}^{-1}(x) j\mathcal{U}(x) \partial_{v\mu} \mathcal{U}^{-1}(x) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_\mu \rightarrow \mathcal{U}(y) A_\mu \mathcal{U}^{-1}(y) j\mathcal{U}(y) \partial_\mu \mathcal{U}^{-1}(y) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_v \rightarrow \mathcal{U}(y) A_v \mathcal{U}^{-1}(y) j\mathcal{U}(y) \partial_v \mathcal{U}^{-1}(y) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_{\mu v} \rightarrow \mathcal{U}(y) A_{\mu v} \mathcal{U}^{-1}(y) j\mathcal{U}(y) \partial_{\mu v} \mathcal{U}^{-1}(y) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_{v\mu} \rightarrow \mathcal{U}(y) A_{v\mu} \mathcal{U}^{-1}(y) j\mathcal{U}(y) \partial_{v\mu} \mathcal{U}^{-1}(y) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_\mu \rightarrow \mathcal{U}(z) A_\mu \mathcal{U}^{-1}(z) j\mathcal{U}(z) \partial_\mu \mathcal{U}^{-1}(z) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_v \rightarrow \mathcal{U}(z) A_v \mathcal{U}^{-1}(z) j\mathcal{U}(z) \partial_v \mathcal{U}^{-1}(z) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_{\mu v} \rightarrow \mathcal{U}(z) A_{\mu v} \mathcal{U}^{-1}(z) j\mathcal{U}(z) \partial_{\mu v} \mathcal{U}^{-1}(z) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_{v\mu} \rightarrow \mathcal{U}(z) A_{v\mu} \mathcal{U}^{-1}(z) j\mathcal{U}(z) \partial_{v\mu} \mathcal{U}^{-1}(z) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_\mu \rightarrow \mathcal{U}(\infty) A_\mu \mathcal{U}^{-1}(\infty) j\mathcal{U}(\infty) \partial_\mu \mathcal{U}^{-1}(\infty) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle A_v \rightarrow \mathcal{U}(\infty) A_v \mathcal{U}^{-1}(\infty) j\mathcal{U}(\infty) \partial_v \mathcal{U}^{-1}(\infty)
\end{aligned}$$



$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{\mu\nu} \rightarrow \mathcal{U}(\infty) A_{\mu\nu} \mathcal{U}^{-1}(\infty) j \mathcal{U}(\infty) \partial_{\mu\nu} \mathcal{U}^{-1}(\infty)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle A_{\nu\mu} \rightarrow \mathcal{U}(\infty) A_{\nu\mu} \mathcal{U}^{-1}(\infty) j \mathcal{U}(\infty) \partial_{\nu\mu} \mathcal{U}^{-1}(\infty)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{\mu\nu} \rightarrow \mathcal{U}(x) F_{\mu\nu} \mathcal{U}^{-1}(x)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{\nu\mu} \rightarrow \mathcal{U}(x) F_{\nu\mu} \mathcal{U}^{-1}(x)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{\mu\nu} \rightarrow \mathcal{U}(y) F_{\mu\nu} \mathcal{U}^{-1}(y)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{\nu\mu} \rightarrow \mathcal{U}(y) F_{\nu\mu} \mathcal{U}^{-1}(y)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{\mu\nu} \rightarrow \mathcal{U}(z) F_{\mu\nu} \mathcal{U}^{-1}(z)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{\nu\mu} \rightarrow \mathcal{U}(z) F_{\nu\mu} \mathcal{U}^{-1}(z)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{\mu\nu} \rightarrow \mathcal{U}(\infty) F_{\mu\nu} \mathcal{U}^{-1}(\infty)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle F_{\nu\mu} \rightarrow \mathcal{U}(\infty) F_{\nu\mu} \mathcal{U}^{-1}(\infty)$$

h. Ecuaciones de Transportación Paralela.

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle i \frac{d\omega}{d\tau} = \frac{dx^\mu}{d\tau} A_\mu(x) \omega + \hat{H} | \psi \rangle = E_\psi | \psi \rangle j \frac{d\omega}{d\tau} = \frac{dx^\mu}{d\tau} A_\mu(x) \omega$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle i \frac{d\omega}{d\tau} = \frac{dy^\mu}{d\tau} A_\mu(y) \omega + \hat{H} | \psi \rangle = E_\psi | \psi \rangle j \frac{d\omega}{d\tau} = \frac{dy^\mu}{d\tau} A_\mu(y) \omega$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle i \frac{d\omega}{d\tau} = \frac{dz^\mu}{d\tau} A_\mu(z) \omega + \hat{H} | \psi \rangle = E_\psi | \psi \rangle j \frac{d\omega}{d\tau} = \frac{dz^\mu}{d\tau} A_\mu(z) \omega$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle i \frac{d\omega}{d\tau} = \frac{d\omega^\mu}{d\tau} A_\mu(\infty) \omega + \hat{H} | \psi \rangle = E_\psi | \psi \rangle j \frac{d\omega}{d\tau} = \frac{d\omega^\mu}{d\tau} A_\mu(\infty) \omega$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle i \frac{d\omega}{d\tau} = \frac{dx^v}{d\tau} A_v(x) \omega + \hat{H} | \psi \rangle = E_\psi | \psi \rangle j \frac{d\omega}{d\tau} = \frac{dx^v}{d\tau} A_v(x) \omega$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle i \frac{d\omega}{d\tau} = \frac{dy^v}{d\tau} A_v(y) \omega + \hat{H} | \psi \rangle = E_\psi | \psi \rangle j \frac{d\omega}{d\tau} = \frac{dy^v}{d\tau} A_v(y) \omega$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle i \frac{d\omega}{d\tau} = \frac{dz^v}{d\tau} A_v(z) \omega + \hat{H} | \psi \rangle = E_\psi | \psi \rangle j \frac{d\omega}{d\tau} = \frac{dz^v}{d\tau} A_v(z) \omega$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle i \frac{d\omega}{d\tau} = \frac{d\omega^v}{d\tau} A_v(\infty) \omega + \hat{H} | \psi \rangle = E_\psi | \psi \rangle j \frac{d\omega}{d\tau} = \frac{d\omega^v}{d\tau} A_v(\infty) \omega$$



$$\begin{aligned}
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle i\frac{d\omega}{d\tau} = \frac{dx^{\mu\nu}}{d\tau}A_{\mu\nu}(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle j\frac{d\omega}{d\tau} = \frac{dx^{\mu\nu}}{d\tau}A_{\mu\nu}(x)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle i\frac{d\omega}{d\tau} = \frac{dy^{\mu\nu}}{d\tau}A_{\mu\nu}(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle j\frac{d\omega}{d\tau} = \frac{dy^{\mu\nu}}{d\tau}A_{\mu\nu}(y)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle i\frac{d\omega}{d\tau} = \frac{dz^{\mu\nu}}{d\tau}A_{\mu\nu}(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle j\frac{d\omega}{d\tau} = \frac{dz^{\mu\nu}}{d\tau}A_{\mu\nu}(z)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle i\frac{d\omega}{d\tau} = \frac{d\infty^{\mu\nu}}{d\tau}A_{\mu\nu}(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle j\frac{d\omega}{d\tau} = \frac{d\infty^{\mu\nu}}{d\tau}A_{\mu\nu}(\infty)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle i\frac{d\omega}{d\tau} = \frac{dx^{v\mu}}{d\tau}A_{v\mu}(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle j\frac{d\omega}{d\tau} = \frac{dx^{v\mu}}{d\tau}A_{v\mu}(x)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle i\frac{d\omega}{d\tau} = \frac{dy^{v\mu}}{d\tau}A_{v\mu}(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle j\frac{d\omega}{d\tau} = \frac{dy^{v\mu}}{d\tau}A_{v\mu}(y)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle i\frac{d\omega}{d\tau} = \frac{dz^{v\mu}}{d\tau}A_{v\mu}(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle j\frac{d\omega}{d\tau} = \frac{dz^{v\mu}}{d\tau}A_{v\mu}(z)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle i\frac{d\omega}{d\tau} = \frac{d\infty^{v\mu}}{d\tau}A_{v\mu}(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle j\frac{d\omega}{d\tau} = \frac{d\infty^{v\mu}}{d\tau}A_{v\mu}(\infty)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle j\frac{d\omega}{d\tau} = \frac{dx^\mu}{d\tau}A_\mu(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i\frac{d\omega}{d\tau} = \frac{dx^\mu}{d\tau}A_\mu(x)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle j\frac{d\omega}{d\tau} = \frac{dy^\mu}{d\tau}A_\mu(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i\frac{d\omega}{d\tau} = \frac{dy^\mu}{d\tau}A_\mu(y)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle j\frac{d\omega}{d\tau} = \frac{dz^\mu}{d\tau}A_\mu(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i\frac{d\omega}{d\tau} = \frac{dz^\mu}{d\tau}A_\mu(z)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle j\frac{d\omega}{d\tau} = \frac{d\infty^\mu}{d\tau}A_\mu(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i\frac{d\omega}{d\tau} = \frac{d\infty^\mu}{d\tau}A_\mu(\infty)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle j\frac{d\omega}{d\tau} = \frac{dx^v}{d\tau}A_v(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i\frac{d\omega}{d\tau} = \frac{dx^v}{d\tau}A_v(x)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle j\frac{d\omega}{d\tau} = \frac{dy^v}{d\tau}A_v(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i\frac{d\omega}{d\tau} = \frac{dy^v}{d\tau}A_v(y)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle j\frac{d\omega}{d\tau} = \frac{dz^v}{d\tau}A_v(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i\frac{d\omega}{d\tau} = \frac{dz^v}{d\tau}A_v(z)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle j\frac{d\omega}{d\tau} = \frac{d\infty^v}{d\tau}A_v(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i\frac{d\omega}{d\tau} = \frac{d\infty^v}{d\tau}A_v(\infty)\omega
\end{aligned}$$



$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, j\frac{d\omega}{d\tau} = \frac{d\omega^\nu}{d\tau}A_v(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, j\frac{d\omega}{d\tau} = \frac{d\omega^\nu}{d\tau}A_v(\infty)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle i,j \frac{d\omega}{d\tau} = \frac{dx^{\mu\nu}}{d\tau} A_{\mu\nu}(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i,j \frac{d\omega}{d\tau} = \frac{dx^{\mu\nu}}{d\tau} A_{\mu\nu}(x)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle i,j \frac{d\omega}{d\tau} = \frac{dy^{\mu\nu}}{d\tau} A_{\mu\nu}(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i,j \frac{d\omega}{d\tau} = \frac{dy^{\mu\nu}}{d\tau} A_{\mu\nu}(y)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle i,j \frac{d\omega}{d\tau} = \frac{dz^{\mu\nu}}{d\tau} A_{\mu\nu}(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i,j \frac{d\omega}{d\tau} = \frac{dz^{\mu\nu}}{d\tau} A_{\mu\nu}(z)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle i,j \frac{d\omega}{d\tau} = \frac{d\infty^{\mu\nu}}{d\tau} A_{\mu\nu}(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i,j \frac{d\omega}{d\tau} = \frac{d\infty^{\mu\nu}}{d\tau} A_{\mu\nu}(\infty)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i, j \frac{d\omega}{d\tau} = \frac{dx^{v_\mu}}{d\tau} A_{v_\mu}(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i, j \frac{d\omega}{d\tau} = \frac{dx^{v_\mu}}{d\tau} A_{v_\mu}(x)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle i,j \frac{d\omega}{d\tau} = \frac{dy^{v\mu}}{d\tau} A_{v\mu}(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i,j \frac{d\omega}{d\tau} = \frac{dy^{v\mu}}{d\tau} A_{v\mu}(y)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, j\frac{d\omega}{d\tau} = \frac{dz^{v\mu}}{d\tau}A_{v\mu}(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, j\frac{d\omega}{d\tau} = \frac{dz^{v\mu}}{d\tau}A_{v\mu}(z)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle i,j \frac{d\omega}{d\tau} = \frac{d\omega^{\nu\mu}}{d\tau} A_{\nu\mu}(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i,j \frac{d\omega}{d\tau} = \frac{d\omega^{\nu\mu}}{d\tau} A_{\nu\mu}(\infty)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dx^\mu}{d\tau}A_\mu(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dx^\mu}{d\tau}A_\mu(x)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dy^\mu}{d\tau}A_\mu(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dy^\mu}{d\tau}A_\mu(y)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle j, i\frac{d\omega}{d\tau} = \frac{dz^\mu}{d\tau}A_\mu(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle j, i\frac{d\omega}{d\tau} = \frac{dz^\mu}{d\tau}A_\mu(z)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{d\omega^\infty}{d\tau}A_\mu^{(\infty)}\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{d\omega^\infty}{d\tau}A_\mu^{(\infty)}\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dx^v}{d\tau}A_v(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dx^v}{d\tau}A_v(x)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dy^v}{d\tau}A_v(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dy^v}{d\tau}A_v(y)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dz^v}{d\tau}A_v(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dz^v}{d\tau}A_v(z)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{d\omega^\nu}{d\tau}A_\nu(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{d\omega^\nu}{d\tau}A_\nu(\infty)\omega$$



$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dx^{\mu\nu}}{d\tau}A_{\mu\nu}(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dx^{\mu\nu}}{d\tau}A_{\mu\nu}(x)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dy^{\mu\nu}}{d\tau}A_{\mu\nu}(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dy^{\mu\nu}}{d\tau}A_{\mu\nu}(y)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dz^{\mu\nu}}{d\tau}A_{\mu\nu}(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dz^{\mu\nu}}{d\tau}A_{\mu\nu}(z)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{d\infty^{\mu\nu}}{d\tau}A_{\mu\nu}(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{d\infty^{\mu\nu}}{d\tau}A_{\mu\nu}(\infty)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dx^{v\mu}}{d\tau}A_{v\mu}(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dx^{v\mu}}{d\tau}A_{v\mu}(x)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dy^{v\mu}}{d\tau}A_{v\mu}(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dy^{v\mu}}{d\tau}A_{v\mu}(y)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dz^{v\mu}}{d\tau}A_{v\mu}(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dz^{v\mu}}{d\tau}A_{v\mu}(z)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle j, i \frac{d\omega}{d\tau} = \frac{d\infty^{\nu\mu}}{d\tau} A_{\nu\mu}(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle j, i \frac{d\omega}{d\tau} = \frac{d\infty^{\nu\mu}}{d\tau} A_{\nu\mu}(\infty)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dx^\mu}{d\tau}A_\mu(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dx^\mu}{d\tau}A_\mu(x)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dy^\mu}{d\tau}A_\mu(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dy^\mu}{d\tau}A_\mu(y)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dz^\mu}{d\tau}A_\mu(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dz^\mu}{d\tau}A_\mu(z)\omega$$

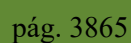
$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\hbar \frac{d\omega}{d\tau} = \frac{d\omega^\infty}{d\tau} A_\mu(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\hbar \frac{d\omega}{d\tau} = \frac{d\omega^\infty}{d\tau} A_\mu(\infty)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dx^v}{d\tau}A_v(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dx^v}{d\tau}A_v(x)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle i,j \frac{d\omega}{d\tau} = \frac{dy^v}{d\tau} A_v(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle j,i \frac{d\omega}{d\tau} = \frac{dy^v}{d\tau} A_v(y)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dz^v}{d\tau}A_v(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dz^v}{d\tau}A_v(z)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i,j \frac{d\omega}{d\tau} = \frac{d\omega^v}{d\tau} A_v(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, j,i \frac{d\omega}{d\tau} = \frac{d\omega^v}{d\tau} A_v(\infty)\omega$$



$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i, j \frac{d\omega}{d\tau} = \frac{dx^{\mu\nu}}{d\tau} A_{\mu\nu}(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, j, i \frac{d\omega}{d\tau} = \frac{dx^{\mu\nu}}{d\tau} A_{\mu\nu}(x)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle i,j \frac{d\omega}{d\tau} = \frac{dy^{\mu\nu}}{d\tau} A_{\mu\nu}(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle j,i \frac{d\omega}{d\tau} = \frac{dy^{\mu\nu}}{d\tau} A_{\mu\nu}(y)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dz^{\mu\nu}}{d\tau}A_{\mu\nu}(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dz^{\mu\nu}}{d\tau}A_{\mu\nu}(z)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i,j \frac{d\omega}{d\tau} = \frac{d\omega^{\mu\nu}}{d\tau} A_{\mu\nu}(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i \frac{d\omega}{d\tau} = \frac{d\omega^{\mu\nu}}{d\tau} A_{\mu\nu}(\infty)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dx^{v\mu}}{d\tau}A_{v\mu}(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dx^{v\mu}}{d\tau}A_{v\mu}(x)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dy^{v\mu}}{d\tau}A_{v\mu}(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dy^{v\mu}}{d\tau}A_{v\mu}(y)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dz^{v_\mu}}{d\tau}A_{v_\mu}(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dz^{v_\mu}}{d\tau}A_{v_\mu}(z)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle i, j \frac{d\omega}{d\tau} = \frac{d\omega^{\nu\mu}}{d\tau} A_{\nu\mu}(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle j, i \frac{d\omega}{d\tau} = \frac{d\omega^{\nu\mu}}{d\tau} A_{\nu\mu}(\infty)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dx^\mu}{d\tau}A_\mu(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dx^\mu}{d\tau}A_\mu(x)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, j, i \frac{d\omega}{d\tau} = \frac{dy^\mu}{d\tau} A_\mu(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, j, i \frac{d\omega}{d\tau} = \frac{dy^\mu}{d\tau} A_\mu(y)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle j, i \frac{d\omega}{d\tau} = \frac{dz^\mu}{d\tau} A_\mu(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i, j \frac{d\omega}{d\tau} = \frac{dz^\mu}{d\tau} A_\mu(z)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, j, i \frac{d\omega}{d\tau} = \frac{d^{\infty\mu}}{d\tau} A_\mu(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, j, i \frac{d\omega}{d\tau} = \frac{d^{\infty\mu}}{d\tau} A_\mu(\infty)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, j, i \frac{d\omega}{d\tau} = \frac{dx^v}{d\tau} A_v(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i, j \frac{d\omega}{d\tau} = \frac{dx^v}{d\tau} A_v(x)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, j, i \frac{d\omega}{d\tau} = \frac{dy^v}{d\tau} A_v(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i, j \frac{d\omega}{d\tau} = \frac{dy^v}{d\tau} A_v(y)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dz^v}{d\tau}A_v(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{dz^v}{d\tau}A_v(z)\omega$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle, i\frac{d\omega}{d\tau} = \frac{d^{\infty}v}{d\tau}A_v(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle, j\frac{d\omega}{d\tau} = \frac{d^{\infty}v}{d\tau}A_v(\infty)\omega$$



$$\begin{aligned}
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle j, i \frac{d\omega}{d\tau} = \frac{dx^{\mu\nu}}{d\tau} A_{\mu\nu}(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i, j \frac{d\omega}{d\tau} = \frac{dx^{\mu\nu}}{d\tau} A_{\mu\nu}(x)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle j, i \frac{d\omega}{d\tau} = \frac{dy^{\mu\nu}}{d\tau} A_{\mu\nu}(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i, j \frac{d\omega}{d\tau} = \frac{dy^{\mu\nu}}{d\tau} A_{\mu\nu}(y)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle j, i \frac{d\omega}{d\tau} = \frac{dz^{\mu\nu}}{d\tau} A_{\mu\nu}(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i, j \frac{d\omega}{d\tau} = \frac{dz^{\mu\nu}}{d\tau} A_{\mu\nu}(z)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle j, i \frac{d\omega}{d\tau} = \frac{d\infty^{\mu\nu}}{d\tau} A_{\mu\nu}(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i, j \frac{d\omega}{d\tau} = \frac{d\infty^{\mu\nu}}{d\tau} A_{\mu\nu}(\infty)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle j, i \frac{d\omega}{d\tau} = \frac{dx^{v\mu}}{d\tau} A_{v\mu}(x)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i, j \frac{d\omega}{d\tau} = \frac{dx^{v\mu}}{d\tau} A_{v\mu}(x)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle j, i \frac{d\omega}{d\tau} = \frac{dy^{v\mu}}{d\tau} A_{v\mu}(y)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i, j \frac{d\omega}{d\tau} = \frac{dy^{v\mu}}{d\tau} A_{v\mu}(y)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle j, i \frac{d\omega}{d\tau} = \frac{dz^{v\mu}}{d\tau} A_{v\mu}(z)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i, j \frac{d\omega}{d\tau} = \frac{dz^{v\mu}}{d\tau} A_{v\mu}(z)\omega \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle j, i \frac{d\omega}{d\tau} = \frac{d\infty^{v\mu}}{d\tau} A_{v\mu}(\infty)\omega + \hat{H}|\psi\rangle = E_\psi|\psi\rangle i, j \frac{d\omega}{d\tau} = \frac{d\infty^{v\mu}}{d\tau} A_{v\mu}(\infty)\omega
\end{aligned}$$

h. Movimiento de partículas.

$$\begin{aligned}
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle U[xyz_i, xyz_f; C] = Pexp(i \int_{\tau_i}^{\tau_f} d\tau \frac{dxyz^\mu}{d\tau} A_\mu(xyz(\tau))) = Pexp(i \int_{xyzi}^{xyzf} A) \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle U[xyz_f, xyz_i; C] = Pexp(i \int_{\tau_f}^{\tau_i} d\tau \frac{dxyz^\mu}{d\tau} A_\mu(xyz(\tau))) = Pexp(i \int_{xyzf}^{xyzi} A) \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle U[xyz_i, xyz_f; C] = Pexp(j \int_{\tau_i}^{\tau_f} d\tau \frac{dx^\mu}{d\tau} A_\mu(xyz(\tau))) = Pexp(j \int_{xyzi}^{xyzf} A) \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle U[xyz_f, xyz_i; C] = Pexp(j \int_{\tau_f}^{\tau_i} d\tau \frac{dxyz^\mu}{d\tau} A_\mu(xyz(\tau))) = Pexp(j \int_{xyzf}^{xyzi} A) \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle U[xyz_i, xyz_f; C] = Pexp(i \int_{\tau_i}^{\tau_f} d\tau \frac{dxyz^v}{d\tau} A_v(xyz(\tau))) = Pexp(i \int_{xyzi}^{xyzf} A) \\
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle U[xyz_f, xyz_i; C] = Pexp(i \int_{\tau_f}^{\tau_i} d\tau \frac{dxyz^v}{d\tau} A_v(xyz(\tau))) = Pexp(i \int_{xyzf}^{xyzi} A)
\end{aligned}$$



$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_i, xyz_f; C] = Pexp(j \int_{\tau_i}^{\tau_f} d\tau \frac{dxyz^v}{d\tau} A_v(xyz(\tau))) = Pexp(j \int_{xyzi}^{xyzf} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_f, xyz_i; C] = Pexp(j \int_{\tau_f}^{\tau_i} d\tau \frac{dxyz^v}{d\tau} A_v(xyz(\tau))) = Pexp(j \int_{xyzf}^{xyzi} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_i, xyz_f; C] = Pexp(i \int_{\tau_i}^{\tau_f} d\tau \frac{dxyz^{\mu\nu}}{d\tau} A_{\mu\nu}(xyz(\tau))) = Pexp(i \int_{xyzi}^{xyzf} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_f, xyz_i; C] = Pexp(i \int_{\tau_f}^{\tau_i} d\tau \frac{dxyz^{\mu\nu}}{d\tau} A_{\mu\nu}(xyz(\tau))) = Pexp(i \int_{xyzf}^{xyzi} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_i, xyz_f; C] = Pexp(j \int_{\tau_i}^{\tau_f} d\tau \frac{dxyz^{\mu\nu}}{d\tau} A_{\mu\nu}(xyz(\tau))) = Pexp(j \int_{xyzi}^{xyzf} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_f, xyz_i; C] = Pexp(j \int_{\tau_f}^{\tau_i} d\tau \frac{dxyz^{\mu\nu}}{d\tau} A_{\mu\nu}(xyz(\tau))) = Pexp(j \int_{xyzf}^{xyzi} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_i, xyz_f; C] = Pexp(i \int_{\tau_i}^{\tau_f} d\tau \frac{dxyz^{v\mu}}{d\tau} A_{v\mu}(xyz(\tau))) = Pexp(i \int_{xyzi}^{xyzf} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_f, xyz_i; C] = Pexp(i \int_{\tau_f}^{\tau_i} d\tau \frac{dxyz^{v\mu}}{d\tau} A_{v\mu}(xyz(\tau))) = Pexp(i \int_{xyzf}^{xyzi} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_i, xyz_f; C] = Pexp(j \int_{\tau_i}^{\tau_f} d\tau \frac{dxyz^{v\mu}}{d\tau} A_{v\mu}(xyz(\tau))) = Pexp(j \int_{xyzi}^{xyzf} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_f, xyz_i; C] = Pexp(j \int_{\tau_f}^{\tau_i} d\tau \frac{dxyz^{v\mu}}{d\tau} A_{v\mu}(xyz(\tau))) = Pexp(j \int_{xyzf}^{xyzi} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_i, xyz_f; C] = Pexp(i \int_{\tau_i}^{\tau_f} d\tau \frac{dxyz^\mu}{d\tau} A_\mu(xyz(\tau))) = Pexp(j \int_{xyzi}^{xyzf} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_f, xyz_i; C] = Pexp(i \int_{\tau_f}^{\tau_i} d\tau \frac{dxyz^\mu}{d\tau} A_\mu(xyz(\tau))) = Pexp(j \int_{xyzf}^{xyzi} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_i, xyz_f; C] = Pexp(j \int_{\tau_i}^{\tau_f} d\tau \frac{dxyz^\mu}{d\tau} A_\mu(xyz(\tau))) = Pexp(i \int_{xyzi}^{xyzf} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_f, xyz_i; C] = Pexp(j \int_{\tau_f}^{\tau_i} d\tau \frac{dxyz^\mu}{d\tau} A_\mu(xyz(\tau))) = Pexp(i \int_{xyzf}^{xyzi} A)$$



$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_i, xyz_f; C] = Pexp(i \int_{\tau_i}^{\tau_f} d\tau \frac{dxyz^v}{d\tau} A_v(xyz(\tau))) = Pexp(j \int_{xyzi}^{xyzf} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_f, xyz_i; C] = Pexp(i \int_{\tau_f}^{\tau_i} d\tau \frac{dxyz^v}{d\tau} A_v(xyz(\tau))) = Pexp(j \int_{xyzf}^{xyzi} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_i, xyz_f; C] = Pexp(j \int_{\tau_i}^{\tau_f} d\tau \frac{dxyz^v}{d\tau} A_v(xyz(\tau))) = Pexp(i \int_{xyzi}^{xyzf} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_f, xyz_i; C] = Pexp(j \int_{\tau_f}^{\tau_i} d\tau \frac{dxyz^v}{d\tau} A_v(xyz(\tau))) = Pexp(i \int_{xyzf}^{xyzi} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_i, xyz_f; C] = Pexp(i \int_{\tau_i}^{\tau_f} d\tau \frac{dxyz^{\mu\nu}}{d\tau} A_{\mu\nu}(xyz(\tau))) = Pexp(j \int_{xyzi}^{xyzf} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_f, xyz_i; C] = Pexp(i \int_{\tau_f}^{\tau_i} d\tau \frac{dxyz^{\mu\nu}}{d\tau} A_{\mu\nu}(xyz(\tau))) = Pexp(j \int_{xyzf}^{xyzi} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_i, xyz_f; C] = Pexp(j \int_{\tau_i}^{\tau_f} d\tau \frac{dxyz^{\mu\nu}}{d\tau} A_{\mu\nu}(xyz(\tau))) = Pexp(i \int_{xyzi}^{xyzf} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_f, xyz_i; C] = Pexp(j \int_{\tau_f}^{\tau_i} d\tau \frac{dxyz^{\mu\nu}}{d\tau} A_{\mu\nu}(xyz(\tau))) = Pexp(i \int_{xyzf}^{xyzi} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_i, xyz_f; C] = Pexp(i \int_{\tau_i}^{\tau_f} d\tau \frac{dxyz^{v\mu}}{d\tau} A_{v\mu}(xyz(\tau))) = Pexp(j \int_{xyzi}^{xyzf} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_f, xyz_i; C] = Pexp(i \int_{\tau_f}^{\tau_i} d\tau \frac{dxyz^{v\mu}}{d\tau} A_{v\mu}(xyz(\tau))) = Pexp(j \int_{xyzf}^{xyzi} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_i, xyz_f; C] = Pexp(j \int_{\tau_i}^{\tau_f} d\tau \frac{dxyz^{v\mu}}{d\tau} A_{v\mu}(xyz(\tau))) = Pexp(i \int_{xyzi}^{xyzf} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_f, xyz_i; C] = Pexp(j \int_{\tau_f}^{\tau_i} d\tau \frac{dxyz^{v\mu}}{d\tau} A_{v\mu}(xyz(\tau))) = Pexp(i \int_{xyzf}^{xyzi} A)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_i, xyz_f; C] \rightarrow \Omega(xyz_i) U[xyz_i, xyz_f; C] \Omega^\dagger(x_f)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle U[xyz_f, xyz_i; C] \rightarrow \Omega(xyz_j) U[xyz_f, xyz_i; C] \Omega^\dagger(x_f)$$



$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W[C] = \text{tr} P \exp(i \oint A) + \hat{H}|\psi\rangle = E_\psi|\psi\rangle W[C] = \text{tr} P \exp(f \oint A)$$

i. Cuantificación del grado de libertad.

$$\begin{aligned} \hat{H}|\psi\rangle &= E_\psi|\psi\rangle S_w = \int d\tau i w^\dagger \frac{dw}{dt} + \lambda(w^\dagger w - k) + w^\dagger A(xyz(\tau))w + [w_i, w_j^\dagger = \delta_{ij} + i i_1 \dots i_n] \\ &= w_{i_1}^\dagger \dots w_{i_n}^\dagger | \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \Phi \mathbb{K} \tilde{\mathbb{K}} \mathbb{K} \mathbb{K} \psi \mathbb{K} \mathbb{K} \zeta \pi m c \mathbb{R}^4 \rangle \end{aligned}$$

$$\begin{aligned} \hat{H}|\psi\rangle &= E_\psi|\psi\rangle S_w = \int d\tau j w^\dagger \frac{dw}{dt} + \lambda(w^\dagger w - k) + w^\dagger A(xyz(\tau))w + [w_j, w_i^\dagger = \delta_{ji} + i j_1 \dots j_n] \\ &= w_{j_1}^\dagger \dots w_{j_n}^\dagger | \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \Phi \mathbb{K} \tilde{\mathbb{K}} \mathbb{K} \mathbb{K} \psi \mathbb{K} \mathbb{K} \zeta \pi m c \mathbb{R}^4 \rangle \end{aligned}$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle Z_w[A] = \text{tr} P \exp(i \int d\tau A(\tau)) + \hat{H}|\psi\rangle = E_\psi|\psi\rangle Z_w[A] = \text{tr} P \exp(j \int d\tau A(\tau))$$

j. Término Theta.

$$\begin{aligned} \hat{H}|\psi\rangle &= E_\psi|\psi\rangle S_\theta = \frac{\theta}{16\pi^2} \int d^4 x \text{tr} * F^{\mu\nu} F_{\mu\nu} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle S_\theta \\ &= \frac{\theta}{16\pi^2} \int d^4 xyz \dots n \text{tr} * F^{v\mu} F_{\mu\nu} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle S_\theta \\ &= \frac{\theta}{16\pi^2} \int d^4 xyz \dots n \text{tr} * F^{v\mu} F_{v\mu} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle S_\theta \\ &= \frac{\theta}{16\pi^2} \int d^4 xyz \dots n \text{tr} * F^{\mu\nu} F_{v\mu} \end{aligned}$$

$$\begin{aligned} \hat{H}|\psi\rangle &= E_\psi|\psi\rangle S_\theta = \frac{\theta}{8\pi^2} \int d^4 x \partial_\mu K^\mu + \hat{H}|\psi\rangle = E_\psi|\psi\rangle S_\theta = \frac{\theta}{8\pi^2} \int d^4 xyz \dots n \partial_v K^v + \hat{H}|\psi\rangle \\ &= E_\psi|\psi\rangle S_\theta = \frac{\theta}{8\pi^2} \int d^4 xyz \dots n \partial_{\mu\nu} K^{\mu\nu} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle S_\theta \\ &= \frac{\theta}{8\pi^2} \int d^4 xyz \dots n \partial_{v\mu} K^{v\mu} \end{aligned}$$

$$\begin{aligned} \hat{H}|\psi\rangle &= E_\psi|\psi\rangle K^\mu = \epsilon^{\mu\nu\rho\sigma} \text{tr}(A_\nu \partial_\rho A_\sigma - \frac{2i}{3} A_\nu A_\rho A_\sigma) + \hat{H}|\psi\rangle = E_\psi|\psi\rangle K^\nu \\ &= \epsilon^{v\mu\sigma\rho} \text{tr}(A_\mu \partial_\sigma A_\rho A_\nu - \frac{2j}{3} A_\mu A_\sigma A_\rho A_\nu) \end{aligned}$$



k. Cuantificación Canónica de Yang – Mills.

$$\begin{aligned}
 \hat{H} | \psi \rangle &= E_\psi | \psi \rangle L = \frac{1}{2g^2} \text{tr} F^{\mu\nu} F_{\mu\nu} + \frac{\theta}{16\pi^2} \text{tr} * F^{\mu\nu} F_{\mu\nu} + \hat{H} | \psi \rangle = E_\psi | \psi \rangle L \\
 &= \frac{1}{2g^2} \text{tr} F^{v\mu} F_{v\mu} + \frac{\theta}{16\pi^2} \text{tr} * F^{v\mu} F_{v\mu} + L \\
 &= \frac{1}{2g^2} \text{tr} \left(\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n \prod_v^\mu \mu v v \mu \left(\sqrt{A^2 - B^2/B^2 - A^2} + \frac{\theta}{16\pi^2} \text{tr} \sqrt{A^2 \cdot \frac{B^2}{B^2} \cdot A^2} \right) \right) \\
 \hat{H} | \psi \rangle &= E_\psi | \psi \rangle H = g^{\mu\nu\rho\sigma} \text{tr} \left(\pi - \frac{\theta}{16\pi^{\mu\nu\rho\sigma}} B \right) e^{-\frac{i\omega t}{\mu\nu\rho\sigma}} + \frac{1}{g^{\mu\nu\rho\sigma}} \text{tr} B^{\mu\nu\rho\sigma} + \hat{H} | \psi \rangle = E_\psi | \psi \rangle H \\
 &= g^{\nu\mu\sigma\rho} \text{tr} \left(\pi - \frac{\theta}{16\pi^{\nu\mu\sigma\rho}} B \right) e^{-i\omega t/\nu\mu\sigma\rho} + \frac{1}{g^{\nu\mu\sigma\rho}} \text{tr} B^{\nu\mu\sigma\rho}
 \end{aligned}$$

l. Construcción de un Espacio de Hilbert.

$$\begin{aligned}
 \hat{H} | \psi \rangle &= E_\psi | \psi \rangle Q(w) = \oint d^n x \text{tr} (\pi \cdot \delta A) = \frac{1}{g^n} \oint d^n x \text{tr} \left(E_i + \frac{\theta g^{\mu\nu\rho\sigma}}{16\pi^{\mu\nu\rho\sigma}} B_i \right) D_{iw} \\
 &= -\frac{1}{g^{\mu\nu\rho\sigma}} \oint d^n x \text{tr} (D_i E_i w_{\mu\nu\rho\sigma}) \\
 \hat{H} | \psi \rangle &= E_\psi | \psi \rangle Q(w) = \oint d^n xyz \dots n \text{tr} (\pi \cdot \delta A \cdot B) = \frac{1}{g^n} \oint d^n xyz \dots n \text{tr} \left(E_i + \frac{\theta g^{\mu\nu\rho\sigma}}{16\pi^{\mu\nu\rho\sigma}} B_i \right) D_{iw} \\
 &= -\frac{1}{g^{\mu\nu\rho\sigma}} \oint d^n x \text{tr} (D_i E_i w_{\mu\nu\rho\sigma}) \\
 \hat{H} | \psi \rangle &= E_\psi | \psi \rangle Q(w) = \oint d^n xyz \dots n \text{tr} (\pi \cdot \delta A \cdot B) = \frac{1}{g^n} \oint d^n xyz \dots n \text{tr} \left(E_j + \frac{\theta g^{\mu\nu\rho\sigma}}{16\pi^{\mu\nu\rho\sigma}} B_j \right) D_{jw} \\
 &= -\frac{1}{g^{\mu\nu\rho\sigma}} \oint d^n x \text{tr} (D_j E_j w_{\mu\nu\rho\sigma}) \\
 \hat{H} | \psi \rangle &= E_\psi | \psi \rangle Q(w) = \oint d^n xyz \dots n \text{tr} (\pi \cdot \delta A \cdot B) = \frac{1}{g^n} \oint d^n xyz \dots n \text{tr} \left(E_{i,j} + \frac{\theta g^{\mu\nu\rho\sigma}}{16\pi^{\mu\nu\rho\sigma}} B_{i,j} \right) D_{i,jw} \\
 &= -\frac{1}{g^{\mu\nu\rho\sigma}} \oint d^n x \text{tr} (D_{i,j} E_{i,j} w_{\mu\nu\rho\sigma}) \\
 \hat{H} | \psi \rangle &= E_\psi | \psi \rangle Q(w) = \oint d^n xyz \dots n \text{tr} (\pi \cdot \delta A \cdot B) = \frac{1}{g^n} \oint d^n xyz \dots n \text{tr} \left(E_{j,i} + \frac{\theta g^{\mu\nu\rho\sigma}}{16\pi^{\mu\nu\rho\sigma}} B_{j,i} \right) D_{j,iw} \\
 &= -\frac{1}{g^{\mu\nu\rho\sigma}} \oint d^n x \text{tr} (D_{j,i} E_{j,i} w_{\mu\nu\rho\sigma})
 \end{aligned}$$



$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle H\psi = g^{\mu\nu\rho\sigma} tr(-j, i \delta/\delta A - \frac{\theta g^{\mu\nu\rho\sigma}}{16\pi^{\mu\nu\rho\sigma}} B) exp^{\mu\nu\rho\sigma} \psi + 1/g^{\mu\nu\rho\sigma} tr B^{\mu\nu\rho\sigma} \psi$$

$$= \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{K} \tilde{\mathbb{K}} \mathbb{K} \mathbb{K} \psi \mathbb{K} \mathbb{K} \zeta \pi m c \mathbb{R}^4$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle H\psi = g^{\nu\mu\sigma\rho} tr(-i \delta/\delta A - \frac{\theta g^{\nu\mu\sigma\rho}}{16\pi^{\nu\mu\sigma\rho}} B) exp^{\nu\mu\sigma\rho} \psi + 1/g^{\nu\mu\sigma\rho} tr B^{\nu\mu\sigma\rho} \psi$$

$$= \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{K} \tilde{\mathbb{K}} \mathbb{K} \mathbb{K} \psi \mathbb{K} \mathbb{K} \zeta \pi m c \mathbb{R}^4$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle H\psi = g^{\nu\mu\sigma\rho} tr(-j \delta/\delta A - \frac{\theta g^{\nu\mu\sigma\rho}}{16\pi^{\nu\mu\sigma\rho}} B) exp^{\nu\mu\sigma\rho} \psi + 1/g^{\nu\mu\sigma\rho} tr B^{\nu\mu\sigma\rho} \psi$$

$$= \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{K} \tilde{\mathbb{K}} \mathbb{K} \mathbb{K} \psi \mathbb{K} \mathbb{K} \zeta \pi m c \mathbb{R}^4$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle H\psi = g^{\nu\mu\sigma\rho} tr(-i, j \delta/\delta A - \frac{\theta g^{\nu\mu\sigma\rho}}{16\pi^{\nu\mu\sigma\rho}} B) exp^{\nu\mu\sigma\rho} \psi + 1/g^{\nu\mu\sigma\rho} tr B^{\nu\mu\sigma\rho} \psi$$

$$= \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{K} \tilde{\mathbb{K}} \mathbb{K} \mathbb{K} \psi \mathbb{K} \mathbb{K} \zeta \pi m c \mathbb{R}^4$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle H\psi = g^{\nu\mu\rho\sigma} tr(-j, i \delta/\delta A - \frac{\theta g^{\nu\mu\rho\sigma}}{16\pi^{\nu\mu\rho\sigma}} B) exp^{\nu\mu\rho\sigma} \psi + 1/g^{\nu\mu\rho\sigma} tr B^{\nu\mu\rho\sigma} \psi$$

$$= \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{K} \tilde{\mathbb{K}} \mathbb{K} \mathbb{K} \psi \mathbb{K} \mathbb{K} \zeta \pi m c \mathbb{R}^4$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle - g^{\mu\nu\rho\sigma} tr \delta^{\mu\nu\rho\sigma} \psi_{\mu\nu\rho\sigma} / \delta A + 1/g^{\mu\nu\rho\sigma} tr B^{\mu\nu\rho\sigma} \psi_{\mu\nu\rho\sigma}$$

$$= \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{K} \tilde{\mathbb{K}} \mathbb{K} \mathbb{K} \psi \mathbb{K} \mathbb{K} \zeta \pi m c \mathbb{R}^4$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle - g^{\nu\mu\sigma\rho} tr \delta^{\nu\mu\sigma\rho} \psi_{\nu\mu\sigma\rho} / \delta A + 1/g^{\nu\mu\sigma\rho} tr B^{\nu\mu\sigma\rho} \psi_{\nu\mu\sigma\rho}$$

$$= \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{K} \tilde{\mathbb{K}} \mathbb{K} \mathbb{K} \psi \mathbb{K} \mathbb{K} \zeta \pi m c \mathbb{R}^4$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W(A) = 1/16\pi^{\mu\nu\rho\sigma} \int d^{\mu\nu\rho\sigma} \epsilon^{ijk} tr (F_{ij} A_k + 2i/3 A_i A_j A_k)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W(A) = 1/16\pi^{\mu\nu\rho\sigma} \int d^{\mu\nu\rho\sigma} \epsilon^{jik} tr (F_{ji} A_k + 2i/3 A_j A_i A_k)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W(A) = 1/16\pi^{\mu\nu\rho\sigma} \int d^{\mu\nu\rho\sigma} \epsilon^{kij} tr (F_k A_{ij} + 2i/3 A_k A_i A_j)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W(A) = 1/16\pi^{\mu\nu\rho\sigma} \int d^{\mu\nu\rho\sigma} \epsilon^{kji} tr (F_k A_{ji} + 2i/3 A_k A_j A_i)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W(A) = 1/16\pi^{\mu\nu\rho\sigma} \int d^{\mu\nu\rho\sigma} \epsilon^{ikj} tr (F_{ik} A_j + 2i/3 A_i A_k A_j)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W(A) = 1/16\pi^{\mu\nu\rho\sigma} \int d^{\mu\nu\rho\sigma} \epsilon^{jki} tr (F_{jk} A_i + 2i/3 A_j A_k A_i)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W(A) = 1/16\pi^{\nu\mu\sigma\rho} \int d^{\nu\mu\sigma\rho} \epsilon^{ijk} tr (F_{ij} A_k + 2i/3 A_i A_j A_k)$$



$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W(A) = 1/16\pi^{\nu\mu\sigma\rho} \int d^{\nu\mu\sigma\rho} \epsilon^{jik} \text{tr} (F_{ji} A_k + 2i/3 A_j A_i A_k)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W(A) = 1/16\pi^{\nu\mu\sigma\rho} \int d^{\nu\mu\sigma\rho} \epsilon^{kij} \text{tr} (F_k A_{ij} + 2i/3 A_k A_i A_j)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W(A) = 1/16\pi^{\nu\mu\sigma\rho} \int d^{\nu\mu\sigma\rho} \epsilon^{kji} \text{tr} (F_k A_{ji} + 2i/3 A_k A_j A_i)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W(A) = 1/16\pi^{\nu\mu\sigma\rho} \int d^{\nu\mu\sigma\rho} \epsilon^{ikj} \text{tr} (F_{ik} A_j + 2i/3 A_i A_k A_j)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W(A) = 1/16\pi^{\nu\mu\sigma\rho} \int d^{\nu\mu\sigma\rho} \epsilon^{jki} \text{tr} (F_{jk} A_i + 2i/3 A_j A_k A_i)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle SW(A)/\delta A_i = 1/16\pi^{\mu\nu\rho\sigma} \epsilon^{ijk} F_{jk} = 1/16\pi^{\mu\nu\rho\sigma} B_i$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle SW(A)/\delta A_j = 1/16\pi^{\mu\nu\rho\sigma} \epsilon^{ijk} F_{jk} = 1/16\pi^{\mu\nu\rho\sigma} B_j$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle SW(A)/\delta A_{i,j} = 1/16\pi^{\mu\nu\rho\sigma} \epsilon^{ijk} F_{jk} = 1/16\pi^{\mu\nu\rho\sigma} B_{i,j}$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle SW(A)/\delta A_{j,i} = 1/16\pi^{\mu\nu\rho\sigma} \epsilon^{ijk} F_{jk} = 1/16\pi^{\mu\nu\rho\sigma} B_{j,i}$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle SW(A)/\delta A_i = 1/16\pi^{\mu\nu\rho\sigma} \epsilon^{ikj} F_{kj} = 1/16\pi^{\mu\nu\rho\sigma} B_i$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle SW(A)/\delta A_j = 1/16\pi^{\mu\nu\rho\sigma} \epsilon^{jki} F_{ki} = 1/16\pi^{\mu\nu\rho\sigma} B_j$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle SW(A)/\delta A_{i,j} = 1/16\pi^{\mu\nu\rho\sigma} \epsilon^{kij} F_{kj} = 1/16\pi^{\mu\nu\rho\sigma} B_{i,j}$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle SW(A)/\delta A_{j,i} = 1/16\pi^{\mu\nu\rho\sigma} \epsilon^{kji} F_{ki} = 1/16\pi^{\mu\nu\rho\sigma} B_{j,i}$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle SW(A)/\delta A_i = 1/16\pi^{\nu\mu\sigma\rho} \epsilon^{ijk} F_{jk} = 1/16\pi^{\nu\mu\sigma\rho} B_i$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle SW(A)/\delta A_j = 1/16\pi^{\nu\mu\sigma\rho} \epsilon^{ijk} F_{jk} = 1/16\pi^{\nu\mu\sigma\rho} B_j$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle SW(A)/\delta A_{i,j} = 1/16\pi^{\nu\mu\sigma\rho} \epsilon^{ijk} F_{jk} = 1/16\pi^{\nu\mu\sigma\rho} B_{i,j}$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle SW(A)/\delta A_{j,i} = 1/16\pi^{\nu\mu\sigma\rho} \epsilon^{ijk} F_{jk} = 1/16\pi^{\nu\mu\sigma\rho} B_{j,i}$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle SW(A)/\delta A_i = 1/16\pi^{\nu\mu\sigma\rho} \epsilon^{ikj} F_{kj} = 1/16\pi^{\nu\mu\sigma\rho} B_i$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle SW(A)/\delta A_j = 1/16\pi^{\nu\mu\sigma\rho} \epsilon^{jki} F_{ki} = 1/16\pi^{\nu\mu\sigma\rho} B_j$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle SW(A)/\delta A_{i,j} = 1/16\pi^{\nu\mu\sigma\rho} \epsilon^{kij} F_{kj} = 1/16\pi^{\nu\mu\sigma\rho} B_{i,j}$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle SW(A)/\delta A_{j,i} = 1/16\pi^{\nu\mu\sigma\rho} \epsilon^{kji} F_{ki} = 1/16\pi^{\nu\mu\sigma\rho} B_{j,i}$$



$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle - i\delta\psi(A)/\delta A_i = -ie^{i\theta W[A]} \frac{\delta\psi_{\mu\nu\sigma\rho}(A)}{\delta A_i} + \frac{\theta}{16\pi^{\mu\nu\sigma\rho}} B_i \psi(A)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle - j\delta\psi(A)/\delta A_j = -je^{j\theta W[A]} \frac{\delta\psi_{\mu\nu\sigma\rho}(A)}{\delta A_j} + \frac{\theta}{16\pi^{\mu\nu\sigma\rho}} B_j \psi(A)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle - i,j\delta\psi(A)/\delta A_{i,j} = -i,j e^{i,j\theta W[A]} \frac{\delta\psi_{\mu\nu\sigma\rho}(A)}{\delta A_{i,j}} + \frac{\theta}{16\pi^{\mu\nu\sigma\rho}} B_{i,j} \psi(A)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle - j,i\delta\psi(A)/\delta A_{j,i} = -j,i e^{j,i\theta W[A]} \frac{\delta\psi_{\mu\nu\sigma\rho}(A)}{\delta A_{j,i}} + \frac{\theta}{16\pi^{\mu\nu\sigma\rho}} B_{j,i} \psi(A)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle - i\delta\psi(A)/\delta A_i = -ie^{i\theta W[A]} \frac{\delta\psi_{\nu\mu\sigma\rho}(A)}{\delta A_i} + \frac{\theta}{16\pi^{\nu\mu\sigma\rho}} B_i \psi(A)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle - j\delta\psi(A)/\delta A_j = -je^{j\theta W[A]} \frac{\delta\psi_{\nu\mu\sigma\rho}(A)}{\delta A_j} + \frac{\theta}{16\pi^{\nu\mu\sigma\rho}} B_j \psi(A)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle - i,j\delta\psi(A)/\delta A_{i,j} = -i,j e^{i,j\theta W[A]} \frac{\delta\psi_{\nu\mu\sigma\rho}(A)}{\delta A_{i,j}} + \frac{\theta}{16\pi^{\nu\mu\sigma\rho}} B_{i,j} \psi(A)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle - j,i\delta\psi(A)/\delta A_{j,i} = -j,i e^{j,i\theta W[A]} \frac{\delta\psi_{\nu\mu\sigma\rho}(A)}{\delta A_{j,i}} + \frac{\theta}{16\pi^{\nu\mu\sigma\rho}} B_{j,i} \psi(A)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W[A]$$

$$\rightarrow W[B] + 1/16\pi^{\mu\nu\sigma\rho} \int d^{\mu\nu\sigma\rho} x [i\epsilon^{ijk} \partial_i \text{tr} (\partial_j \Omega \Omega \Delta \nabla \lambda_{k,i}) n^{-\Omega^{-\mu\nu\sigma\rho}} - 1/3\epsilon^{ijk} \text{tr} (\Omega^{-\mu\nu\sigma\rho} \partial_i \Omega \Omega^{-\mu\nu\sigma\rho} \partial_j \Omega \Omega^{-\mu\nu\sigma\rho} \partial_k \Omega \Omega^{-\mu\nu\sigma\rho})]$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W[A]$$

$$\rightarrow W[B] + 1/16\pi^{\mu\nu\sigma\rho} \int d^{\mu\nu\sigma\rho} x [j\epsilon^{ijk} \partial_j \text{tr} (\partial_i \Omega \Omega \Delta \nabla \lambda_{k,j}) n^{-\Omega^{-\mu\nu\sigma\rho}} - 1/3\epsilon^{ijk} \text{tr} (\Omega^{-\mu\nu\sigma\rho} \partial_j \Omega \Omega^{-\mu\nu\sigma\rho} \partial_i \Omega \Omega^{-\mu\nu\sigma\rho} \partial_k \Omega \Omega^{-\mu\nu\sigma\rho})]$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W[A]$$

$$\rightarrow W[B] + 1/16\pi^{\mu\nu\sigma\rho} \int d^{\mu\nu\sigma\rho} x [i\epsilon^{ijk} \partial_i \text{tr} (\partial_j \Omega \Omega \Delta \nabla \lambda_{i,k}) n^{-\Omega^{-\mu\nu\sigma\rho}} - 1/3\epsilon^{ijk} \text{tr} (\Omega^{-\mu\nu\sigma\rho} \partial_i \Omega \Omega^{-\mu\nu\sigma\rho} \partial_j \Omega \Omega^{-\mu\nu\sigma\rho} \partial_k \Omega \Omega^{-\mu\nu\sigma\rho})]$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle W[A]$$

$$\rightarrow W[B] + 1/16\pi^{\mu\nu\sigma\rho} \int d^{\mu\nu\sigma\rho} x [j\epsilon^{ijk} \partial_j \text{tr} (\partial_i \Omega \Omega \Delta \nabla \lambda_{j,k}) n^{-\Omega^{-\mu\nu\sigma\rho}} - 1/3\epsilon^{ijk} \text{tr} (\Omega^{-\mu\nu\sigma\rho} \partial_j \Omega \Omega^{-\mu\nu\sigma\rho} \partial_i \Omega \Omega^{-\mu\nu\sigma\rho} \partial_k \Omega \Omega^{-\mu\nu\sigma\rho})]$$



$$\begin{aligned} \hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{\mu\nu\sigma\rho} \int d^{\mu\nu\sigma\rho} x [i\epsilon^{jki} \partial_i \text{tr} (\partial_j \Omega \Omega \Delta \nabla \lambda_{k,i}) n^{-\Omega-\mu\nu\sigma\rho} \\ &\quad - 1/3 \epsilon^{jki} \text{tr} (\Omega^{-\mu\nu\sigma\rho} \partial_i \Omega \Omega^{-\mu\nu\sigma\rho} \partial_j \Omega \Omega^{-\mu\nu\sigma\rho} \partial_k \Omega \Omega^{-\mu\nu\sigma\rho})] \end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{\mu\nu\sigma\rho} \int d^{\mu\nu\sigma\rho} x[j\epsilon^{jki}\partial_j \operatorname{tr}(\partial_i\Omega\Omega\Delta\nabla\lambda_{j,k})n^{-\Omega-\mu\nu\sigma\rho} \\ &\quad - 1/3\epsilon^{jki} \operatorname{tr}(\Omega^{-\mu\nu\sigma\rho}\partial_j\Omega\Omega^{-\mu\nu\sigma\rho}\partial_i\Omega\Omega^{-\mu\nu\sigma\rho}\partial_k\Omega\Omega^{-\mu\nu\sigma\rho})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{\mu\nu\sigma\rho} \int d^{\mu\nu\sigma\rho} x[i\epsilon^{jki}\partial_i \operatorname{tr}(\partial_j\Omega\Omega\Delta\nabla\lambda_{i,k})n^{-\Omega-\mu\nu\sigma\rho} \\ &\quad - 1/3\epsilon^{jki} \operatorname{tr}(\Omega^{-\mu\nu\sigma\rho}\partial_i\Omega\Omega^{-\mu\nu\sigma\rho}\partial_j\Omega\Omega^{-\mu\nu\sigma\rho}\partial_k\Omega\Omega^{-\mu\nu\sigma\rho})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{\mu\nu\sigma\rho} \int d^{\mu\nu\sigma\rho} x[k\epsilon^{jki}\partial_k \operatorname{tr}(\partial_k\Omega\Omega\Delta\nabla\lambda_{j,i})n^{-\Omega-\mu\nu\sigma\rho} \\ &\quad - 1/3\epsilon^{jki} \operatorname{tr}(\Omega^{-\mu\nu\sigma\rho}\partial_k\Omega\Omega^{-\mu\nu\sigma\rho}\partial_j\Omega\Omega^{-\mu\nu\sigma\rho}\partial_i\Omega\Omega^{-\mu\nu\sigma\rho})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{\mu\nu\sigma\rho} \int d^{\mu\nu\sigma\rho} x[k\epsilon^{jki}\partial_k \operatorname{tr}(\partial_k\Omega\Omega\Delta\nabla\lambda_{i,j})n^{-\Omega-\mu\nu\sigma\rho} \\ &\quad - 1/3\epsilon^{jki} \operatorname{tr}(\Omega^{-\mu\nu\sigma\rho}\partial_k\Omega\Omega^{-\mu\nu\sigma\rho}\partial_i\Omega\Omega^{-\mu\nu\sigma\rho}\partial_j\Omega\Omega^{-\mu\nu\sigma\rho})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{\mu\nu\sigma\rho} \int d^{\mu\nu\sigma\rho} x[i\epsilon^{kij}\partial_i \operatorname{tr}(\partial_j\Omega\Omega\Delta\nabla\lambda_{k,i})n^{-\Omega-\mu\nu\sigma\rho} \\ &\quad - 1/3\epsilon^{kij} \operatorname{tr}(\Omega^{-\mu\nu\sigma\rho}\partial_i\Omega\Omega^{-\mu\nu\sigma\rho}\partial_j\Omega\Omega^{-\mu\nu\sigma\rho}\partial_k\Omega\Omega^{-\mu\nu\sigma\rho})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{\mu\nu\sigma\rho} \int d^{\mu\nu\sigma\rho} x[j\epsilon^{kij}\partial_j \operatorname{tr}(\partial_i\Omega\Omega\Delta\nabla\lambda_{k,j})n^{-\Omega-\mu\nu\sigma\rho} \\ &\quad - 1/3\epsilon^{kij} \operatorname{tr}(\Omega^{-\mu\nu\sigma\rho}\partial_j\Omega\Omega^{-\mu\nu\sigma\rho}\partial_i\Omega\Omega^{-\mu\nu\sigma\rho}\partial_k\Omega\Omega^{-\mu\nu\sigma\rho})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{\mu\nu\sigma\rho} \int d^{\mu\nu\sigma\rho} x[i\epsilon^{kij}\partial_i \operatorname{tr}(\partial_j\Omega\Omega\Delta\nabla\lambda_{i,k})n^{-\Omega-\mu\nu\sigma\rho} \\ &\quad - 1/3\epsilon^{kij} \operatorname{tr}(\Omega^{-\mu\nu\sigma\rho}\partial_i\Omega\Omega^{-\mu\nu\sigma\rho}\partial_j\Omega\Omega^{-\mu\nu\sigma\rho}\partial_k\Omega\Omega^{-\mu\nu\sigma\rho})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{\mu\nu\sigma\rho} \int d^{\mu\nu\sigma\rho} x[j\epsilon^{kij}\partial_j \operatorname{tr}(\partial_i\Omega\Omega\Delta\nabla\lambda_{j,k})n^{-\Omega-\mu\nu\sigma\rho} \\ &\quad - 1/3\epsilon^{kij} \operatorname{tr}(\Omega^{-\mu\nu\sigma\rho}\partial_j\Omega\Omega^{-\mu\nu\sigma\rho}\partial_i\Omega\Omega^{-\mu\nu\sigma\rho}\partial_k\Omega\Omega^{-\mu\nu\sigma\rho})]\end{aligned}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [i\epsilon^{ijk} \partial_i \text{tr}(\partial_j \Omega \Omega \Delta \nabla \lambda_{k,i}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{ijk} \text{tr}(\Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [j\epsilon^{ijk} \partial_j \text{tr}(\partial_i \Omega \Omega \Delta \nabla \lambda_{k,j}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{ijk} \text{tr}(\Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [i\epsilon^{ijk} \partial_i \text{tr}(\partial_j \Omega \Omega \Delta \nabla \lambda_{i,k}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{ijk} \text{tr}(\Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [j\epsilon^{ijk} \partial_j \text{tr}(\partial_i \Omega \Omega \Delta \nabla \lambda_{j,k}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{ijk} \text{tr}(\Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [k\epsilon^{ijk} \partial_k \text{tr}(\partial_k \Omega \Omega \Delta \nabla \lambda_{i,j}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{ijk} \text{tr}(\Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [k\epsilon^{ijk} \partial_k \text{tr}(\partial_k \Omega \Omega \Delta \nabla \lambda_{j,i}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{ijk} \text{tr}(\Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [j\epsilon^{jik} \partial_j \text{tr}(\partial_i \Omega \Omega \Delta \nabla \lambda_{k,j}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{jik} \text{tr}(\Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [i\epsilon^{jik} \partial_i \text{tr}(\partial_j \Omega \Omega \Delta \nabla \lambda_{k,i}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{jik} \text{tr}(\Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [j\epsilon^{jik} \partial_j \text{tr}(\partial_i \Omega \Omega \Delta \nabla \lambda_{i,k}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{jik} \text{tr}(\Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [i\epsilon^{jik} \partial_i \text{tr}(\partial_j \Omega \Omega \Delta \nabla \lambda_{j,k}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{jik} \text{tr}(\Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [k\epsilon^{jik} \partial_k \text{tr}(\partial_k \Omega \Omega \Delta \nabla \lambda_{j,i}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{jik} \text{tr}(\Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [k\epsilon^{jik} \partial_k \text{tr}(\partial_k \Omega \Omega \Delta \nabla \lambda_{i,j}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{jik} \text{tr}(\Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [i\epsilon^{ikj} \partial_i \text{tr}(\partial_j \Omega \Omega \Delta \nabla \lambda_{k,i}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{ikj} \text{tr}(\Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [j\epsilon^{ikj} \partial_j \text{tr}(\partial_i \Omega \Omega \Delta \nabla \lambda_{k,j}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{ikj} \text{tr}(\Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [i\epsilon^{ikj} \partial_i \text{tr}(\partial_j \Omega \Omega \Delta \nabla \lambda_{i,k}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{ikj} \text{tr}(\Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [j\epsilon^{ikj} \partial_j \text{tr}(\partial_i \Omega \Omega \Delta \nabla \lambda_{j,k}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{ikj} \text{tr}(\Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [k\epsilon^{ikj} \partial_k \text{tr}(\partial_k \Omega \Omega \Delta \nabla \lambda_{i,j}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{ikj} \text{tr}(\Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [k\epsilon^{ikj} \partial_k \text{tr}(\partial_k \Omega \Omega \Delta \nabla \lambda_{j,i}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{ikj} \text{tr}(\Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [j\epsilon^{jki} \partial_j \text{tr}(\partial_i \Omega \Omega \Delta \nabla \lambda_{k,j}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{jki} \text{tr}(\Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [i\epsilon^{jki} \partial_i \text{tr}(\partial_j \Omega \Omega \Delta \nabla \lambda_{k,i}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{jki} \text{tr}(\Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [j\epsilon^{jki} \partial_j \text{tr}(\partial_i \Omega \Omega \Delta \nabla \lambda_{j,k}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{jki} \text{tr}(\Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [i\epsilon^{jki} \partial_i \text{tr}(\partial_j \Omega \Omega \Delta \nabla \lambda_{i,k}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{jki} \text{tr}(\Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [k\epsilon^{jki} \partial_k \text{tr}(\partial_k \Omega \Omega \Delta \nabla \lambda_{j,i}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{jki} \text{tr}(\Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu p\sigma} \int d^{v\mu p\sigma} x [k\epsilon^{jki} \partial_k \text{tr}(\partial_k \Omega \Omega \Delta \nabla \lambda_{i,j}) n^{-\Omega-v\mu p\sigma} \\ &\quad - 1/3\epsilon^{jki} \text{tr}(\Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma})]\end{aligned}$$



$$\begin{aligned} \hat{H}|\psi\rangle &= E_\psi|\psi\rangle W[A] \\ &\rightarrow W[B] + 1/16\pi^{v\mu\rho\sigma} \int d^{v\mu\rho\sigma} x [i\epsilon^{kji} \partial_i \text{tr}(\partial_j \Omega \Omega \nabla \lambda_{k,i}) n^{-\Omega^{-v\mu\rho\sigma}} \\ &\quad - 1/3 \epsilon^{kji} \text{tr}(\Omega^{-v\mu\rho\sigma} \partial_i \Omega \Omega^{-v\mu\rho\sigma} \partial_j \Omega \Omega^{-v\mu\rho\sigma} \partial_k \Omega \Omega^{-v\mu\rho\sigma})] \end{aligned}$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle n(\Omega) = 1/32\pi^{v\mu p\sigma} \int_{S^3} d^{v\mu p\sigma} S \epsilon^{kji} \text{tr}(\Omega^{-v\mu p\sigma} \partial_k \Omega \Omega^{-v\mu p\sigma} \partial_j \Omega \Omega^{-v\mu p\sigma} \partial_i \Omega \Omega^{-v\mu p\sigma})$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle \psi(A) = e^{i\theta W[A]} \psi_0(A)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle \psi(B) = e^{i\theta W[B]} \psi_0(B)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle \psi(\partial\Delta\nabla\omega) = e^{i\theta W[\partial\Delta\nabla\omega]} \psi_\phi \psi \partial\Delta\nabla\vartheta\varphi\tau(A)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle \psi(\partial\Delta\nabla\omega) = e^{i\theta W[\partial\Delta\nabla\omega]} \psi_\phi \psi \partial\Delta\nabla\vartheta\varphi\tau(B)$$

n. Dinámica hamiltoniana de partículas según la teoría de Yang-Mills.

$$\begin{aligned} \hat{H}|\psi\rangle = E_\psi|\psi\rangle &= 1/\pi\epsilon^{ijk e^{i\theta W[\partial\Delta\nabla\omega]}}(-i\partial/\partial x + \Phi/\partial\pi R) \exp^{\mu\nu\sigma p} + \psi_\phi \psi \partial\Delta\nabla\vartheta\varphi\tau(A) & + & \psi = \\ \frac{1}{\sqrt{\pi\epsilon^{ijk} e^{\frac{i\theta W[\partial\Delta\nabla\omega]}{R}}}} &= E = \frac{1}{\pi e R^2} \left(n + \frac{\phi}{2\pi_\phi}\right) = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{H} \tilde{\mathbb{X}} \mathbb{K} \mathbb{D} \mathbb{K} \psi \mathbb{K} \mathbb{X} \zeta \pi m c \mathbb{R}^4 \end{aligned}$$

$$\begin{aligned} \hat{H}|\psi\rangle = E_\psi|\psi\rangle &= 1/\pi\epsilon^{ijk e^{i\theta W[\partial\Delta\nabla\omega]}}(-i\partial/\partial x + \Phi/\partial\pi R) \exp^{v\mu p\sigma} + \psi_\phi \psi \partial\Delta\nabla\vartheta\varphi\tau(A) & + & \psi = \\ \frac{1}{\sqrt{\pi\epsilon^{ijk} e^{\frac{i\theta W[\partial\Delta\nabla\omega]}{R}}}} &= E = \frac{1}{\pi e R^2} \left(n + \frac{\phi}{2\pi_\phi}\right) = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{H} \tilde{\mathbb{X}} \mathbb{K} \mathbb{D} \mathbb{K} \psi \mathbb{K} \mathbb{X} \zeta \pi m c \mathbb{R}^4 \end{aligned}$$

$$\begin{aligned} \hat{H}|\psi\rangle = E_\psi|\psi\rangle &= 1/\pi\epsilon^{ijk e^{i\theta W[\partial\Delta\nabla\omega]}}(-i\partial/\partial x + \Phi/\partial\pi R) \exp^{\mu\nu\sigma p} + \psi_\phi \psi \partial\Delta\nabla\vartheta\varphi\tau(B) & + & \psi = \\ \frac{1}{\sqrt{\pi\epsilon^{ijk} e^{\frac{i\theta W[\partial\Delta\nabla\omega]}{R}}}} &= E = \frac{1}{\pi e R^2} \left(n + \frac{\phi}{2\pi_\phi}\right) = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{H} \tilde{\mathbb{X}} \mathbb{K} \mathbb{D} \mathbb{K} \psi \mathbb{K} \mathbb{X} \zeta \pi m c \mathbb{R}^4 \end{aligned}$$

$$\begin{aligned} \hat{H}|\psi\rangle = E_\psi|\psi\rangle &= 1/\pi\epsilon^{ijk e^{i\theta W[\partial\Delta\nabla\omega]}}(-i\partial/\partial x + \Phi/\partial\pi R) \exp^{v\mu p\sigma} + \psi_\phi \psi \partial\Delta\nabla\vartheta\varphi\tau(B) & + & \psi = \\ \frac{1}{\sqrt{\pi\epsilon^{ijk} e^{\frac{i\theta W[\partial\Delta\nabla\omega]}{R}}}} &= E = \frac{1}{\pi e R^2} \left(n + \frac{\phi}{2\pi_\phi}\right) = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{H} \tilde{\mathbb{X}} \mathbb{K} \mathbb{D} \mathbb{K} \psi \mathbb{K} \mathbb{X} \zeta \pi m c \mathbb{R}^4 \end{aligned}$$

$$\begin{aligned} \hat{H}|\psi\rangle = E_\psi|\psi\rangle &= 1/\pi\epsilon^{ikj e^{i\theta W[\partial\Delta\nabla\omega]}}(-i\partial/\partial x + \Phi/\partial\pi R) \exp^{\mu\nu\sigma p} + \psi_\phi \psi \partial\Delta\nabla\vartheta\varphi\tau(A) & + & \psi = \\ \frac{1}{\sqrt{\pi\epsilon^{ikj} e^{\frac{i\theta W[\partial\Delta\nabla\omega]}{R}}}} &= E = \frac{1}{\pi e R^2} \left(n + \frac{\phi}{2\pi_\phi}\right) = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{H} \tilde{\mathbb{X}} \mathbb{K} \mathbb{D} \mathbb{K} \psi \mathbb{K} \mathbb{X} \zeta \pi m c \mathbb{R}^4 \end{aligned}$$

$$\begin{aligned} \hat{H}|\psi\rangle = E_\psi|\psi\rangle &= 1/\pi\epsilon^{ikj e^{i\theta W[\partial\Delta\nabla\omega]}}(-i\partial/\partial x + \Phi/\partial\pi R) \exp^{v\mu p\sigma} + \psi_\phi \psi \partial\Delta\nabla\vartheta\varphi\tau(A) & + & \psi = \\ \frac{1}{\sqrt{\pi\epsilon^{ikj} e^{\frac{i\theta W[\partial\Delta\nabla\omega]}{R}}}} &= E = \frac{1}{\pi e R^2} \left(n + \frac{\phi}{2\pi_\phi}\right) = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{H} \tilde{\mathbb{X}} \mathbb{K} \mathbb{D} \mathbb{K} \psi \mathbb{K} \mathbb{X} \zeta \pi m c \mathbb{R}^4 \end{aligned}$$

$$\begin{aligned} \hat{H}|\psi\rangle = E_\psi|\psi\rangle &= 1/\pi\epsilon^{ikj e^{i\theta W[\partial\Delta\nabla\omega]}}(-i\partial/\partial x + \Phi/\partial\pi R) \exp^{\mu\nu\sigma p} + \psi_\phi \psi \partial\Delta\nabla\vartheta\varphi\tau(B) & + & \psi = \\ \frac{1}{\sqrt{\pi\epsilon^{ikj} e^{\frac{i\theta W[\partial\Delta\nabla\omega]}{R}}}} &= E = \frac{1}{\pi e R^2} \left(n + \frac{\phi}{2\pi_\phi}\right) = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{H} \tilde{\mathbb{X}} \mathbb{K} \mathbb{D} \mathbb{K} \psi \mathbb{K} \mathbb{X} \zeta \pi m c \mathbb{R}^4 \end{aligned}$$



$$\hat{\mathbf{H}}\ket{\psi}=E_{\psi}\ket{\psi}=1/\pi \epsilon^{kij}e_{k\Theta W[\partial\Delta\nabla\omega]}(-k\partial/\partial x+\Phi/\partial\pi R)exp^{\mu\nu\sigma\rho+\psi}_{\phi}\psi\partial\Delta\nabla\vartheta\varphi\tau(A)+\psi=\frac{1}{\mu\nu\sigma\rho}\frac{1}{\sqrt{\frac{1}{\pi\epsilon^{kij}e_{k\Theta W[\partial\Delta\nabla\omega]}}\frac{k\Theta W[\partial\Delta\nabla\omega]}{R}}}=E=\frac{1}{\pi eR^2}(n+\frac{\phi}{2\pi_{\varphi}})=\xi_{\lambda\Omega\psi}^{\zeta\zeta}\Sigma\iiint\!\!\!\int h\phi\mathbb{K}\tilde{\mathcal{X}}\mathcal{M}\mathcal{H}\mathfrak{K}\mathfrak{L}\mathfrak{U}\mathfrak{Y}\mathfrak{Z}\pi mc\mathbb{R}^4$$

$$\hat{\mathbf{H}}\mid\psi\rangle=E_\psi\mid\psi\rangle=1/\pi\epsilon^{kij}e_{k\Theta W[\partial\Delta\nabla\omega]}(-k\partial/\partial x+\Phi/\partial\pi R)exp^{\nu\mu\rho\sigma}+\psi_\phi\psi\partial\Delta\nabla\vartheta\varphi\tau(A)+\psi=\\ \frac{1}{\nu\mu\rho\sigma}\frac{1}{\sqrt{\frac{1}{\pi\epsilon^{kij}}e^{\frac{k\Theta W[\partial\Delta\nabla\omega]}{R}}}}=E=\frac{1}{\pi eR^2}(n+\frac{\phi}{2\pi_\varphi})=\xi_{\lambda\Omega\psi}^{\varsigma\zeta}\Sigma\iiint\!\!\!\int\mathfrak{h}\mathfrak{d}\mathfrak{f}\mathfrak{I}\mathfrak{K}\mathfrak{J}\mathfrak{K}\mathfrak{L}\mathfrak{K}\mathfrak{h}\mathfrak{J}\mathfrak{K}\mathfrak{L}\mathfrak{Z}\pi mc\mathbb{R}^4$$

$$\hat{\mathbf{H}}\mid\psi\rangle=E_\psi\mid\psi\rangle=1/\pi\epsilon^{kij}e_{k\Theta W[\partial\Delta\nabla\omega]}(-k\partial/\partial x+\Phi/\partial\pi R)exp^{\mu\nu\sigma\rho+\psi_\phi}\psi\partial\Delta\nabla\vartheta\varphi\tau(B)\qquad+\qquad\psi=\frac{1}{\mu\nu\sigma\rho}\frac{1}{\sqrt{\frac{1}{\pi\epsilon^{kij}e_{k\Theta W[\partial\Delta\nabla\omega]}}\frac{1}{R}}}=E=\frac{1}{\pi eR^2}(n+\frac{\phi}{2\pi_\varphi})=\xi_{\lambda\Omega\psi}^{\sigma\zeta}\Sigma\int\int\int\int\hbar\mathfrak{c}\mathfrak{h}\mathfrak{f}\mathfrak{z}\mathfrak{K}\mathfrak{L}\mathfrak{H}\mathfrak{q}\mathfrak{h}\mathfrak{z}\mathfrak{K}\mathfrak{Z}\pi mc\mathbb{R}^4$$

$$\hat{\mathbf{H}}\mid\psi\rangle=E_\psi\mid\psi\rangle=1/\pi\epsilon^{kij}e_{k\Theta W[\partial\Delta\nabla\omega]}(-k\partial/\partial x+\Phi/\partial\pi R)exp^{\nu\mu\rho\sigma}+\psi_\phi\psi\partial\Delta\nabla\varphi\tau(B)\qquad+\qquad\psi=\frac{1}{\nu\mu\rho\sigma}\frac{1}{\sqrt{\frac{1}{\pi\epsilon^{kij}}e_{k\Theta W[\partial\Delta\nabla\omega]}}}=\frac{1}{\pi eR^2}(n+\frac{\phi}{2\pi_\varphi})=\xi_{\lambda\Omega\psi}^{\sigma\zeta}\Sigma\int\int\int\int\hbar\mathfrak{c}\mathfrak{h}\mathfrak{f}\mathfrak{z}\mathfrak{K}\mathfrak{L}\mathfrak{H}\mathfrak{q}\mathfrak{h}\mathfrak{z}\mathfrak{X}\zeta\pi mc\mathbb{R}^4$$

$$\hat{\mathbf{H}}\mid \psi\rangle = E_\psi\mid \psi\rangle = 1/\pi \epsilon^{kjie_{k\Theta W[\partial \Delta \nabla \omega]} }(-k\partial / \partial x + \Phi / \partial \pi R)exp^{\mu\nu\sigma\rho+\psi}_\phi \psi \partial \Delta \nabla \vartheta \varphi \tau(A)\qquad + \qquad \psi =$$

$$\frac{1}{\mu\nu\sigma\rho}\frac{1}{\frac{1}{\pi\epsilon^{kji}e^{\frac{k\Theta W[\partial \Delta \nabla \omega]}{R}}} }=E=\frac{1}{\pi eR^2}(n+\frac{\phi}{2\pi\varphi})=\xi_{\lambda\Omega\psi}^{\varsigma\zeta}\Sigma\int\int\int\int\hbar\phi\mathbb{K}\tilde{\mathfrak{Z}}\mathcal{K}_{\mathbb{A}}\mathbb{K}\psi\mathfrak{K}\mathbb{X}\zeta\pi mc\mathbb{R}^4$$

$$\hat{\text{H}}|\psi\rangle=E_{\psi}|\psi\rangle=1/\pi \epsilon^{kjie_k\Theta W[\partial\Delta\nabla\omega]}(-k\partial/\partial x+\Phi/\partial pR)\exp^{\nu\mu\rho\sigma}+\psi_\phi\psi\theta\Delta\nabla\vartheta\varphi\tau(A)~~~~~+~~~~~\psi=\frac{1}{\nu\mu\rho\sigma}\frac{1}{\sqrt{\frac{1}{\pi\epsilon^{kjie}}\frac{k\Theta W[\partial\Delta\nabla\omega]}{R}}} =E=\frac{1}{pe R^2}(n+\frac{\phi}{2\pi_{\varphi}})=\xi_{\lambda\Omega\psi}^\varsigma\xi\Sigma\int\!\! \int\!\! \int\!\! \int\!\! \int h\circledast f\circledast g\checkmark J_KDK\psi\breve KXZ\zeta\pi mc\mathbb{R}^4$$

$$\hat{\mathbf{H}}\,|\,\psi\rangle=E_\psi\,|\,\psi\rangle=1/\pi\epsilon^{kjie}k_{\Theta W[\partial\Delta\nabla\omega]}(-k\partial/\partial x+\Phi/\partial\pi R)exp^{\mu\nu\sigma\rho+\psi_\phi\psi\partial\Delta\nabla\vartheta\varphi\tau(B)}\qquad+\qquad\psi=\frac{1}{\mu\nu\sigma\rho}\frac{1}{\frac{1}{\pi\epsilon^{kjie}}\frac{k_{\Theta W[\partial\Delta\nabla\omega]}}{R}}=E=\frac{1}{\pi eR^2}(n+\frac{\phi}{2\pi\varphi})=\xi_{\lambda\Omega\psi}^{\sigma\zeta}\Sigma\int\int\int\int\hbar\phi\mathbb{K}\mathfrak{J}\mathcal{K}\mathbb{H}\mathfrak{L}\mathfrak{L}\mathfrak{L}\mathfrak{L}\pi mc\mathbb{R}^4$$

$$\hat{\mathbf{H}}\mid \psi\rangle=E_\psi\mid \psi\rangle=1/\pi\epsilon^{kjie_{k\Theta W[\partial\Delta\nabla\omega]} }(-k\partial/\partial x+\Phi/\partial\pi R)exp^{\nu\mu\rho\sigma}+\psi_\phi\psi\partial\Delta\nabla\vartheta\varphi\tau(B)\qquad +\qquad \psi=\frac{1}{\nu\mu\rho\sigma}\sqrt{\frac{1}{\pi\epsilon^{kjie_{k\Theta W[\partial\Delta\nabla\omega]} }}}\frac{k\Theta W[\partial\Delta\nabla\omega]}{R}=E=\frac{1}{\pi eR^2}(n+\frac{\phi}{2\pi_\varphi})=\xi_{\lambda\Omega\psi}^\varsigma\zeta\Sigma\text{f f f f f h}\mathfrak{c}\mathfrak{p}\mathfrak{l}\mathfrak{x}\mathfrak{z}\mathfrak{J}\mathfrak{L}\mathfrak{H}\mathfrak{K}\mathfrak{i}\mathfrak{h}\mathfrak{J}\mathfrak{K}\mathfrak{Z}\mathfrak{s}\pi mc\mathbb{R}^4$$

o. Ângulo Theta.

$$\begin{aligned}
\hat{H}|\psi\rangle &= E_\psi |\psi\rangle H = 1/2m\partial^2/\partial x^2 + \hat{H}|\psi\rangle = E_\psi |\psi\rangle H = 1/2m\partial^2/\partial y^2 + \hat{H}|\psi\rangle = E_\psi |\psi\rangle H \\
&= 1/2m\partial^2/\partial z^2 + \hat{H}|\psi\rangle = E_\psi |\psi\rangle H \\
&= 1/2m\partial^{ijk}/\partial n^{\mu\nu\sigma\rho}/\partial\psi\frac{i\theta W[\partial\Delta\nabla\omega]}{R} - \partial i\varphi\Phi\omega\Omega\Delta\nabla\phi\sigma\lambda\xi\Psi\mathbb{R}^{\alpha\beta}\psi_{\aleph\kappa\zeta\eta\pi\infty}\mathbb{R}^4 + \hat{H}|\psi\rangle \\
&= E_\psi |\psi\rangle H \\
&= 1/2m\partial^{ikj}/\partial n^{\mu\nu\sigma\rho}/\partial\psi\frac{i\theta W[\partial\Delta\nabla\omega]}{R} - \partial i\varphi\Phi\omega\Omega\Delta\nabla\phi\sigma\lambda\xi\Psi\mathbb{R}^{\alpha\beta}\psi_{\aleph\kappa\zeta\eta\pi\infty}\mathbb{R}^4 + \hat{H}|\psi\rangle \\
&= E_\psi |\psi\rangle H \\
&= 1/2m\partial^{jik}/\partial n^{\mu\nu\sigma\rho}/\partial\psi\frac{j\theta W[\partial\Delta\nabla\omega]}{R} - \partial j\varphi\Phi\omega\Omega\Delta\nabla\phi\sigma\lambda\xi\Psi\mathbb{R}^{\alpha\beta}\psi_{\aleph\kappa\zeta\eta\pi\infty}\mathbb{R}^4 + \hat{H}|\psi\rangle \\
&= E_\psi |\psi\rangle H \\
&= 1/2m\partial^{iki}/\partial n^{\mu\nu\sigma\rho}/\partial\psi\frac{j\theta W[\partial\Delta\nabla\omega]}{R} - \partial j\varphi\Phi\omega\Omega\Delta\nabla\phi\sigma\lambda\xi\Psi\mathbb{R}^{\alpha\beta}\psi_{\aleph\kappa\zeta\eta\pi\infty}\mathbb{R}^4 + \hat{H}|\psi\rangle \\
&= E_\psi |\psi\rangle H \\
&= 1/2m\partial^{kij}/\partial n^{\mu\nu\sigma\rho}/\partial\psi\frac{k\theta W[\partial\Delta\nabla\omega]}{R} - \partial k\varphi\Phi\omega\Omega\Delta\nabla\phi\sigma\lambda\xi\Psi\mathbb{R}^{\alpha\beta}\psi_{\aleph\kappa\zeta\eta\pi\infty}\mathbb{R}^4 + \hat{H}|\psi\rangle \\
&= E_\psi |\psi\rangle H = 1/2m\partial^{kji}/\partial n^{\mu\nu\sigma\rho}/\partial\psi\frac{k\theta W[\partial\Delta\nabla\omega]}{R} - \partial k\varphi\Phi\omega\Omega\Delta\nabla\phi\sigma\lambda\xi\Psi\mathbb{R}^{\alpha\beta}\psi_{\aleph\kappa\zeta\eta\pi\infty}\mathbb{R}^4
\end{aligned}$$

$$\begin{aligned}
\hat{H}|\psi\rangle &= E_\psi |\psi\rangle H = 1/2m\partial^2/\partial x^2 + \hat{H}|\psi\rangle = E_\psi |\psi\rangle H = 1/2m\partial^2/\partial y^2 + \hat{H}|\psi\rangle = E_\psi |\psi\rangle H \\
&= 1/2m\partial^2/\partial z^2 + \hat{H}|\psi\rangle = E_\psi |\psi\rangle H \\
&= 1/2m\partial^{ijk}/\partial n^{\nu\mu\rho\sigma}/\partial\psi \frac{i\theta W[\partial\Delta\nabla\omega]}{R} - \partial i\varphi\Phi\omega\Omega\Delta\nabla\phi\sigma\lambda\xi\Psi\mathbb{R}^{\alpha\beta\psi}_{\mathfrak{K}\mathfrak{C}\mathfrak{K}\mathfrak{I}\mathfrak{n}}\infty\mathbb{R}^4 + \hat{H}|\psi\rangle \\
&= E_\psi |\psi\rangle H \\
&= 1/2m\partial^{ikj}/\partial n^{\nu\mu\rho\sigma}/\partial\psi \frac{i\theta W[\partial\Delta\nabla\omega]}{R} - \partial i\varphi\Phi\omega\Omega\Delta\nabla\phi\sigma\lambda\xi\Psi\mathbb{R}^{\alpha\beta\psi}_{\mathfrak{K}\mathfrak{C}\mathfrak{K}\mathfrak{I}\mathfrak{n}}\infty\mathbb{R}^4 + \hat{H}|\psi\rangle \\
&= E_\psi |\psi\rangle H \\
&= 1/2m\partial^{jik}/\partial n^{\nu\mu\rho\sigma}/\partial\psi \frac{j\theta W[\partial\Delta\nabla\omega]}{R} - \partial j\varphi\Phi\omega\Omega\Delta\nabla\phi\sigma\lambda\xi\Psi\mathbb{R}^{\alpha\beta\psi}_{\mathfrak{K}\mathfrak{C}\mathfrak{K}\mathfrak{I}\mathfrak{n}}\infty\mathbb{R}^4 + \hat{H}|\psi\rangle \\
&= E_\psi |\psi\rangle H \\
&= 1/2m\partial^{jki}/\partial n^{\nu\mu\rho\sigma}/\partial\psi \frac{j\theta W[\partial\Delta\nabla\omega]}{R} - \partial j\varphi\Phi\omega\Omega\Delta\nabla\phi\sigma\lambda\xi\Psi\mathbb{R}^{\alpha\beta\psi}_{\mathfrak{K}\mathfrak{C}\mathfrak{K}\mathfrak{I}\mathfrak{n}}\infty\mathbb{R}^4 + \hat{H}|\psi\rangle \\
&= E_\psi |\psi\rangle H \\
&= 1/2m\partial^{kij}/\partial n^{\nu\mu\rho\sigma}/\partial\psi \frac{k\theta W[\partial\Delta\nabla\omega]}{R} - \partial k\varphi\Phi\omega\Omega\Delta\nabla\phi\sigma\lambda\xi\Psi\mathbb{R}^{\alpha\beta\psi}_{\mathfrak{K}\mathfrak{C}\mathfrak{K}\mathfrak{I}\mathfrak{n}}\infty\mathbb{R}^4 + \hat{H}|\psi\rangle \\
&= E_\psi |\psi\rangle H \\
&= 1/2m\partial^{kji}/\partial n^{\nu\mu\rho\sigma}/\partial\psi \frac{k\theta W[\partial\Delta\nabla\omega]}{R} - \partial k\varphi\Phi\omega\Omega\Delta\nabla\phi\sigma\lambda\xi\Psi\mathbb{R}^{\alpha\beta\psi}_{\mathfrak{K}\mathfrak{C}\mathfrak{K}\mathfrak{I}\mathfrak{n}}\infty\mathbb{R}^4
\end{aligned}$$



$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\mu\nu\sigma\rho}(A') = e^{j i k \theta n}\varphi_{\mu\nu\sigma\rho}(Bj, Bi, Bk)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\mu\nu\sigma\rho}(B') = e^{jik\theta n}\varphi_{\mu\nu\sigma\rho}(Aj, Ai, Ak)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\mu\nu\sigma\rho}(B') = e^{jik\theta n}\varphi_{\mu\nu\sigma\rho}(Bj, Bi, Bk)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\mu\nu\sigma\rho}(A') = e^{jki\theta n}\varphi_{\mu\nu\sigma\rho}(Aj, Ak, Ai)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\mu\nu\sigma\rho}(A') = e^{jki\theta n}\varphi_{\mu\nu\sigma\rho}(Bj, Bk, Bi)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\mu\nu\sigma\rho}(B') = e^{jki\theta n}\varphi_{\mu\nu\sigma\rho}(Aj, Ak, Ai)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\mu\nu\sigma\rho}(B') = e^{jki\theta n}\varphi_{\mu\nu\sigma\rho}(Bj, Bk, Bi)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\mu\nu\sigma\rho}(A') = e^{kij\theta n}\varphi_{\mu\nu\sigma\rho}(Ak, Ai, Aj)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\mu\nu\sigma\rho}(A') = e^{kij\theta n}\varphi_{\mu\nu\sigma\rho}(Bk, Bi, Bj)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\mu\nu\sigma\rho}(B') = e^{kij\theta n}\varphi_{\mu\nu\sigma\rho}(Ak, Ai, Aj)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\mu\nu\sigma\rho}(B') = e^{kij\theta n}\varphi_{\mu\nu\sigma\rho}(Bk, Bi, Bj)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\mu\nu\sigma\rho}(A') = e^{kji\theta n}\varphi_{\mu\nu\sigma\rho}(Ak, Aj, Ai)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\mu\nu\sigma\rho}(A') = e^{kji\theta n}\varphi_{\mu\nu\sigma\rho}(Bk, Bj, Bi)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\mu\nu\sigma\rho}(B') = e^{kji\theta n}\varphi_{\mu\nu\sigma\rho}(Ak, Aj, Ai)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\mu\nu\sigma\rho}(B') = e^{kji\theta n}\varphi_{\mu\nu\sigma\rho}(Bk, Bj, Bi)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\nu\mu\rho\sigma}(A') = e^{i\theta n}\varphi_{\nu\mu\rho\sigma}(Ai)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\nu\mu\rho\sigma}(A') = e^{i\theta n}\varphi_{\nu\mu\rho\sigma}(Bi)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\nu\mu\rho\sigma}(B') = e^{i\theta n}\varphi_{\nu\mu\rho\sigma}(Ai)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\nu\mu\rho\sigma}(B') = e^{i\theta n}\varphi_{\nu\mu\rho\sigma}(Bi)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\nu\mu\rho\sigma}(A') = e^{j\theta n}\varphi_{\nu\mu\rho\sigma}(Aj)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\nu\mu\rho\sigma}(A') = e^{j\theta n}\varphi_{\nu\mu\rho\sigma}(Bj)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\nu\mu\rho\sigma}(B') = e^{j\theta n}\varphi_{\nu\mu\rho\sigma}(Aj)$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle\varphi_{\nu\mu\rho\sigma}(B') = e^{j\theta n}\varphi_{\nu\mu\rho\sigma}(Bj)$$



$$\begin{aligned}
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{k\theta n} \varphi_{v\mu p\sigma}(Ak) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{k\theta n} \varphi_{v\mu p\sigma}(Bk) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{k\theta n} \varphi_{v\mu p\sigma}(Ak) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{k\theta n} \varphi_{v\mu p\sigma}(Bk) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{ijk\theta n} \varphi_{v\mu p\sigma}(Ai, Aj, Ak) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{ijk\theta n} \varphi_{v\mu p\sigma}(Bi, Bj, Bk) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{ijk\theta n} \varphi_{v\mu p\sigma}(Ai, Aj, Ak) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{ijk\theta n} \varphi_{v\mu p\sigma}(Bi, Bj, Bk) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{ikj\theta n} \varphi_{v\mu p\sigma}(Ai, Ak, Aj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{ikj\theta n} \varphi_{v\mu p\sigma}(Bi, Bk, Bj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{ikj\theta n} \varphi_{v\mu p\sigma}(Ai, Ak, Aj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{ikj\theta n} \varphi_{v\mu p\sigma}(Bi, Bk, Bj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{jik\theta n} \varphi_{v\mu p\sigma}(Aj, Ai, Ak) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{jik\theta n} \varphi_{v\mu p\sigma}(Bj, Bi, Bk) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{jik\theta n} \varphi_{v\mu p\sigma}(Aj, Ai, Ak) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{jik\theta n} \varphi_{v\mu p\sigma}(Bj, Bi, Bk) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{jki\theta n} \varphi_{v\mu p\sigma}(Aj, Ak, Ai) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{jki\theta n} \varphi_{v\mu p\sigma}(Bj, Bk, Bi) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{jki\theta n} \varphi_{v\mu p\sigma}(Aj, Ak, Ai) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{jki\theta n} \varphi_{v\mu p\sigma}(Bj, Bk, Bi) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{kij\theta n} \varphi_{v\mu p\sigma}(Ak, Ai, Aj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{kij\theta n} \varphi_{v\mu p\sigma}(Bk, Bi, Bj)
\end{aligned}$$



$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{kij\theta n} \varphi_{v\mu p\sigma}(Ak, Ai, Aj)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{kij\theta n} \varphi_{v\mu p\sigma}(Bk, Bi, Bj)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{kji\theta n} \varphi_{v\mu p\sigma}(Ak, Aj, Ai)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{kji\theta n} \varphi_{v\mu p\sigma}(Bk, Bj, Bi)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{kji\theta n} \varphi_{v\mu p\sigma}(Ak, Aj, Ai)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{kji\theta n} \varphi_{v\mu p\sigma}(Bk, Bj, Bi)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{\mu v \sigma p}(A') = e^{i\theta n} \varphi_{v\mu p\sigma}(Ai)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{\mu v \sigma p}(A') = e^{i\theta n} \varphi_{v\mu p\sigma}(Bi)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{\mu v \sigma p}(B') = e^{i\theta n} \varphi_{v\mu p\sigma}(Ai)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{\mu v \sigma p}(B') = e^{i\theta n} \varphi_{v\mu p\sigma}(Bi)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{\mu v \sigma p}(A') = e^{j\theta n} \varphi_{v\mu p\sigma}(Aj)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{\mu v \sigma p}(A') = e^{j\theta n} \varphi_{v\mu p\sigma}(Bj)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{\mu v \sigma p}(B') = e^{j\theta n} \varphi_{v\mu p\sigma}(Aj)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{\mu v \sigma p}(B') = e^{j\theta n} \varphi_{v\mu p\sigma}(Bj)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{\mu v \sigma p}(A') = e^{k\theta n} \varphi_{v\mu p\sigma}(Ak)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{\mu v \sigma p}(A') = e^{k\theta n} \varphi_{v\mu p\sigma}(Bk)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{\mu v \sigma p}(B') = e^{k\theta n} \varphi_{v\mu p\sigma}(Ak)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{\mu v \sigma p}(B') = e^{k\theta n} \varphi_{v\mu p\sigma}(Bk)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{\mu v \sigma p}(A') = e^{ijk\theta n} \varphi_{v\mu p\sigma}(Ai, Aj, Ak)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{\mu v \sigma p}(A') = e^{ijk\theta n} \varphi_{v\mu p\sigma}(Bi, Bj, Bk)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{\mu v \sigma p}(B') = e^{ijk\theta n} \varphi_{v\mu p\sigma}(Ai, Aj, Ak)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{\mu v \sigma p}(B') = e^{ijk\theta n} \varphi_{v\mu p\sigma}(Bi, Bj, Bk)$$



$$\begin{aligned}
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(A') = e^{ikj\theta n} \varphi_{\nu\mu p\sigma}(Ai, Ak, Aj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(A') = e^{ikj\theta n} \varphi_{\nu\mu p\sigma}(Bi, Bk, Bj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(B') = e^{ikj\theta n} \varphi_{\nu\mu p\sigma}(Ai, Ak, Aj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(B') = e^{ikj\theta n} \varphi_{\nu\mu p\sigma}(Bi, Bk, Bj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(A') = e^{jik\theta n} \varphi_{\nu\mu p\sigma}(Aj, Ai, Ak) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(A') = e^{jik\theta n} \varphi_{\nu\mu p\sigma}(Bj, Bi, Bk) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(B') = e^{jik\theta n} \varphi_{\nu\mu p\sigma}(Aj, Ai, Ak) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(B') = e^{jik\theta n} \varphi_{\nu\mu p\sigma}(Bj, Bi, Bk) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(A') = e^{jki\theta n} \varphi_{\nu\mu p\sigma}(Aj, Ak, Ai) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(A') = e^{jki\theta n} \varphi_{\nu\mu p\sigma}(Bj, Bk, Bi) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(B') = e^{jki\theta n} \varphi_{\nu\mu p\sigma}(Aj, Ak, Ai) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(B') = e^{jki\theta n} \varphi_{\nu\mu p\sigma}(Bj, Bk, Bi) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(A') = e^{kij\theta n} \varphi_{\nu\mu p\sigma}(Ak, Ai, Aj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(A') = e^{kij\theta n} \varphi_{\nu\mu p\sigma}(Bk, Bi, Bj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(B') = e^{kij\theta n} \varphi_{\nu\mu p\sigma}(Ak, Ai, Aj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(B') = e^{kij\theta n} \varphi_{\nu\mu p\sigma}(Bk, Bi, Bj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(A') = e^{kji\theta n} \varphi_{\nu\mu p\sigma}(Ak, Aj, Ai) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(A') = e^{kji\theta n} \varphi_{\nu\mu p\sigma}(Bk, Bj, Bi) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(B') = e^{kji\theta n} \varphi_{\nu\mu p\sigma}(Ak, Aj, Ai) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\mu\nu\sigma p}(B') = e^{kji\theta n} \varphi_{\nu\mu p\sigma}(Bk, Bj, Bi) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\nu\mu p\sigma}(A') = e^{i\theta n} \varphi_{\mu\nu\sigma p}(Ai) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{\nu\mu p\sigma}(A') = e^{i\theta n} \varphi_{\mu\nu\sigma p}(Bi)
\end{aligned}$$



$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{i\theta n} \varphi_{\mu\nu\sigma p}(Ai)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{i\theta n} \varphi_{\mu\nu\sigma p}(Bi)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{j\theta n} \varphi_{\mu\nu\sigma p}(Aj)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{j\theta n} \varphi_{\mu\nu\sigma p}(Bj)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{j\theta n} \varphi_{\mu\nu\sigma p}(Aj)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{j\theta n} \varphi_{\mu\nu\sigma p}(Bj)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{k\theta n} \varphi_{\mu\nu\sigma p}(Ak)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{k\theta n} \varphi_{\mu\nu\sigma p}(Bk)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{k\theta n} \varphi_{\mu\nu\sigma p}(Ak)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{k\theta n} \varphi_{\mu\nu\sigma p}(Bk)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{ijk\theta n} \varphi_{\mu\nu\sigma p}(Ai, Aj, Ak)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{ijk\theta n} \varphi_{\mu\nu\sigma p}(Bi, Bj, Bk)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{ijk\theta n} \varphi_{\mu\nu\sigma p}(Ai, Aj, Ak)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{ijk\theta n} \varphi_{\mu\nu\sigma p}(Bi, Bj, Bk)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{ikj\theta n} \varphi_{\mu\nu\sigma p}(Ai, Ak, Aj)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{ikj\theta n} \varphi_{\mu\nu\sigma p}(Bi, Bk, Bj)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{ikj\theta n} \varphi_{\mu\nu\sigma p}(Ai, Ak, Aj)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{ikj\theta n} \varphi_{\mu\nu\sigma p}(Bi, Bk, Bj)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{jik\theta n} \varphi_{\mu\nu\sigma p}(Aj, Ai, Ak)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(A') = e^{jik\theta n} \varphi_{\mu\nu\sigma p}(Bj, Bi, Bk)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{jik\theta n} \varphi_{\mu\nu\sigma p}(Aj, Ai, Ak)$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \varphi_{v\mu p\sigma}(B') = e^{jik\theta n} \varphi_{\mu\nu\sigma p}(Bj, Bi, Bk)$$



$$\begin{aligned}
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p \sigma}(A') = e^{jki\theta n} \varphi_{\mu\nu\sigma p}(Aj, Ak, Ai) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p \sigma}(A') = e^{jki\theta n} \varphi_{\mu\nu\sigma p}(Bj, Bk, Bi) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p \sigma}(B') = e^{jki\theta n} \varphi_{\mu\nu\sigma p}(Aj, Ak, Ai) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p \sigma}(B') = e^{jki\theta n} \varphi_{\mu\nu\sigma p}(Bj, Bk, Bi) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p \sigma}(A') = e^{kij\theta n} \varphi_{\mu\nu\sigma p}(Ak, Ai, Aj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p \sigma}(A') = e^{kij\theta n} \varphi_{\mu\nu\sigma p}(Bk, Bi, Bj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p \sigma}(B') = e^{kij\theta n} \varphi_{\mu\nu\sigma p}(Ak, Ai, Aj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p \sigma}(B') = e^{kij\theta n} \varphi_{\mu\nu\sigma p}(Bk, Bi, Bj) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p \sigma}(A') = e^{kji\theta n} \varphi_{\mu\nu\sigma p}(Ak, Aj, Ai) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p \sigma}(A') = e^{kji\theta n} \varphi_{\mu\nu\sigma p}(Bk, Bj, Bi) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p \sigma}(B') = e^{kji\theta n} \varphi_{\mu\nu\sigma p}(Ak, Aj, Ai) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \varphi_{v\mu p \sigma}(B') = e^{kji\theta n} \varphi_{\mu\nu\sigma p}(Bk, Bj, Bi)
\end{aligned}$$

p. Ondas de Bloch.

$$\begin{aligned}
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle iV\nabla V^{\mu\nu\sigma p} + iV\nabla V^{ijk} + iV\nabla V^{\mu\nu} + iV\nabla V_{vu} = \lambda\nabla\Delta_j^i k \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle iV\nabla V^{v\mu p \sigma} + iV\nabla V^{ijk} + iV\nabla V^{\mu\nu} + iV\nabla V_{vu} = \lambda\nabla\Delta_j^i k \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle iV\nabla V^{\mu\nu\sigma p} + iV\nabla V^{ikj} + iV\nabla V^{\mu\nu} + iV\nabla V_{vu} = \lambda\nabla\Delta_k^i j \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle iV\nabla V^{v\mu p \sigma} + iV\nabla V^{ikj} + iV\nabla V^{\mu\nu} + iV\nabla V_{vu} = \lambda\nabla\Delta_k^i j \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle jV\nabla V^{\mu\nu\sigma p} + jV\nabla V^{jik} + jV\nabla V^{\mu\nu} + jV\nabla V_{vu} = \lambda\nabla\Delta_i^j k \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle iV\nabla V^{v\mu p \sigma} + iV\nabla V^{jik} + iV\nabla V^{\mu\nu} + iV\nabla V_{vu} = \lambda\nabla\Delta_i^j k \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle iV\nabla V^{\mu\nu\sigma p} + iV\nabla V^{jki} + iV\nabla V^{\mu\nu} + iV\nabla V_{vu} = \lambda\nabla\Delta_k^j i \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle iV\nabla V^{v\mu p \sigma} + iV\nabla V^{jki} + iV\nabla V^{\mu\nu} + iV\nabla V_{vu} = \lambda\nabla\Delta_k^j i \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle kV\nabla V^{\mu\nu\sigma p} + kV\nabla V^{kij} + kV\nabla V^{\mu\nu} + kV\nabla V_{vu} = \lambda\nabla\Delta_i^k j
\end{aligned}$$



$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle iV\nabla V^{v\mu p\sigma} + iV\nabla V^{kij} + iV\nabla V^{\mu\nu} + iV\nabla V_{vu} = \lambda \nabla \Delta_i^k j$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle iV\nabla V^{\mu\nu\sigma p} + iV\nabla V^{kji} + iV\nabla V^{\mu\nu} + iV\nabla V_{vu} = \lambda \nabla \Delta_j^k i$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle iV\nabla V^{v\mu p\sigma} + iV\nabla V^{kji} + iV\nabla V^{\mu\nu} + iV\nabla V_{vu} = \lambda \nabla \Delta_j^k i$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega | \psi \rangle = e^{i\theta n'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega | \psi \rangle = e^{i\theta m'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega | \psi \rangle \\ &= e^{i\theta m', n'} | \psi \rangle \end{aligned}$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega | \psi \rangle = e^{j\theta n'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega | \psi \rangle = e^{j\theta m'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega | \psi \rangle \\ &= e^{j\theta m', n'} | \psi \rangle \end{aligned}$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega | \psi \rangle = e^{k\theta n'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega | \psi \rangle = e^{k\theta m'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega \\ &| \psi \rangle = e^{k\theta m', n'} | \psi \rangle \end{aligned}$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega | \psi \rangle = e^{ijk\theta n'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega | \psi \rangle = e^{ijk\theta m'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega \\ &| \psi \rangle = e^{ijk\theta m', n'} | \psi \rangle \end{aligned}$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega | \psi \rangle = e^{ikj\theta n'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega | \psi \rangle = e^{ikj\theta m'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega \\ &| \psi \rangle = e^{ikj\theta m', n'} | \psi \rangle \end{aligned}$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega | \psi \rangle = e^{jik\theta n'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega | \psi \rangle = e^{jik\theta m'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega \\ &| \psi \rangle = e^{jik\theta m', n'} | \psi \rangle \end{aligned}$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega | \psi \rangle = e^{jki\theta n'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega | \psi \rangle = e^{jki\theta m'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega \\ &| \psi \rangle = e^{jki\theta m', n'} | \psi \rangle \end{aligned}$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega | \psi \rangle = e^{kij\theta n'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega | \psi \rangle = e^{kij\theta m'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega \\ &| \psi \rangle = e^{kij\theta m', n'} | \psi \rangle \end{aligned}$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega | \psi \rangle = e^{kji\theta n'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega | \psi \rangle = e^{kji\theta m'} | \psi \rangle + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega \\ &| \psi \rangle = e^{kji\theta m', n'} | \psi \rangle \end{aligned}$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \Omega | \psi \rangle | \theta \rangle$$

$$\begin{aligned} &= \sum_{\nabla v \mu}^{\Delta \mu v} \lambda_{ijk} \iiint | d_{v\mu}^{\mu\nu} tr \rangle^{ijk} \sqrt{\partial e^{i\theta n'} \partial e^{i\theta m'} \partial e^{j\theta n'} \partial e^{j\theta m'} \partial e^{k\theta n'} \partial e^{k\theta m'} \partial e^{ijk\theta}}^\infty | \psi \rangle | n \rangle \\ &| m \rangle | \xi \sigma \mathbb{R}^4 \rangle \end{aligned}$$



q. Instantones.

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle S_\theta = \theta/16\pi^2 \int d^{ijk} x \operatorname{tr} * F^{\mu\nu} F_{\nu\mu} = \theta/8\pi^2 \int d^{ijk} x \partial_{\mu\nu} K^{\nu\mu} \\ &= e^{\mu\nu\rho\sigma} \operatorname{tr} (A_{\mu\nu} \partial_\rho A_\sigma - 2ijk/3 A_{\mu\nu} \partial_\rho A_\sigma) e^{\nu\mu\sigma\rho} \operatorname{tr} (A_{\nu\mu} \partial_\sigma A_\rho - 2ijk/3 A_{\nu\mu} \partial_\sigma A_\rho)\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle A_{\mu\nu} \mapsto i\Omega \partial_{\mu\nu} \Omega^{-ijk} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle A_{\mu\nu} \mapsto j\Omega \partial_{\mu\nu} \Omega^{-ijk} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle A_{\mu\nu} \\ &\mapsto k\Omega \partial_{\mu\nu} \Omega^{-ijk}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle A_{\nu\mu} \mapsto i\Omega \partial_{\nu\mu} \Omega^{-ijk} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle A_{\nu\mu} \mapsto j\Omega \partial_{\nu\mu} \Omega^{-ijk} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle A_{\nu\mu} \\ &\mapsto k\Omega \partial_{\nu\mu} \Omega^{-ijk}\end{aligned}$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle v(\Omega) = 1/24\pi^2 \oint_{v\mu}^{\mu\nu} S_\infty^{\mu\nu\nu\mu} d^{ijk\mu\nu\nu\mu} S \varepsilon^{ijk} \operatorname{tr} (\Omega \partial_i \Omega^{-i}) (\Omega \partial_j \Omega^{-j}) (\Omega \partial_k \Omega^{-k})$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle S_{YM} = 1/8g^2 \int d^{ijk} x \operatorname{tr} (F_{\mu\nu} \tilde{*} F_{\nu\mu}) \exp^2 \mp 1/4g^2 \int d^{ijk} x \operatorname{tr} F_{\mu\nu} * F^{\nu\mu} \\ &\approx 16\pi^2/g^{ijk} |\mu\nu| \pm |\nu\mu|\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle S_{YM} = 1/8g^2 \int d^{ijk} x \operatorname{tr} (F_{\nu\mu} \pm * F_{\mu\nu}) \exp^2 \pm 1/4g^2 \int d^{ijk} x \operatorname{tr} F_{\nu\mu} * F^{\mu\nu} \\ &\approx 16\pi^2/g^{ijk} |\nu\mu| \pm |\mu\nu|\end{aligned}$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle e^{-S_{\text{instanton}}} = e^{\partial^{\pi^2} |\mu\nu| \pm |\nu\mu|/g^{ijk}} e^{ijk\theta |\nu\mu| \pm |\mu\nu|}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle \Omega(x) = x_\mu \sigma^\mu / \sqrt{x_\mu^i + A_\mu} \mapsto i\Omega \partial_\mu \Omega^{ijk} = 1/x_{ijk}^\mu \eta_{\mu\nu}^i \sigma^i + A_\mu \\ &= 1/x^{ijk} + \rho^{ijk} \eta_{\mu\nu}^i x^\nu \sigma^i + F_{\mu\nu} = 2\rho^{ijk} / (x^{ijk} + \rho^{ijk}) \exp^{ijk} \eta_{\mu\nu}^i \sigma^i\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle \Omega(y) = y_\mu \sigma^\mu / \sqrt{y_\mu^i + A_\mu} \mapsto i\Omega \partial_\mu \Omega^{ijk} = 1/y_{ijk}^\mu \eta_{\mu\nu}^i \sigma^i + A_\mu \\ &= 1/y^{ijk} + \rho^{ijk} \eta_{\mu\nu}^i y^\nu \sigma^i + F_{\mu\nu} = 2\rho^{ijk} / (y^{ijk} + \rho^{ijk}) \exp^{ijk} \eta_{\mu\nu}^i \sigma^i\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle \Omega(z) = z_\mu \sigma^\mu / \sqrt{z_\mu^i + A_\mu} \mapsto i\Omega \partial_\mu \Omega^{ijk} = 1/z_{ijk}^\mu \eta_{\mu\nu}^i \sigma^i + A_\mu \\ &= 1/z^{ijk} + \rho^{ijk} \eta_{\mu\nu}^i z^\nu \sigma^i + F_{\mu\nu} = 2\rho^{ijk} / (z^{ijk} + \rho^{ijk}) \exp^{ijk} \eta_{\mu\nu}^i \sigma^i\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle \Omega(n) = n_\mu \sigma^\mu / \sqrt{n_\mu^i + A_\mu} \mapsto i\Omega \partial_\mu \Omega^{ijk} = 1/n_{ijk}^\mu \eta_{\mu\nu}^i \sigma^i + A_\mu \\ &= 1/n^{ijk} + \rho^{ijk} \eta_{\mu\nu}^i n^\nu \sigma^i + F_{\mu\nu} = 2\rho^{ijk} / (n^{ijk} + \rho^{ijk}) \exp^{ijk} \eta_{\mu\nu}^i \sigma^i\end{aligned}$$



$$\eta_{\mu i}^1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu i}^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu i}^3 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu i}^\infty = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega(x) = x_v \sigma^v / \sqrt{x_v^i} + A_v \mapsto i\Omega \partial_v \Omega^{ijk} = 1/x_{ijk}^v \eta_{v\mu}^i \sigma^i + A_v \\ &= 1/x^{ijk} + \rho^{ijk} \eta_{v\mu}^i x^\mu \sigma^i + F_{v\mu} = 2\rho^{ijk} / (x^{ijk} + \rho^{ijk}) \exp^{ijk} \eta_{v\mu}^i \sigma^i \end{aligned}$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega(y) = y_v \sigma^v / \sqrt{y_v^i} + A_v \mapsto i\Omega \partial_v \Omega^{ijk} = 1/y_{ijk}^v \eta_{v\mu}^i \sigma^i + A_v \\ &= 1/y^{ijk} + \rho^{ijk} \eta_{v\mu}^i y^\mu \sigma^i + F_{v\mu} = 2\rho^{ijk} / (y^{ijk} + \rho^{ijk}) \exp^{ijk} \eta_{v\mu}^i \sigma^i \end{aligned}$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega(z) = z_v \sigma^v / \sqrt{z_v^i} + A_v \mapsto i\Omega \partial_v \Omega^{ijk} = 1/z_{ijk}^v \eta_{v\mu}^i \sigma^i + A_v \\ &= 1/z^{ijk} + \rho^{ijk} \eta_{v\mu}^i z^\mu \sigma^i + F_{v\mu} = 2\rho^{ijk} / (z^{ijk} + \rho^{ijk}) \exp^{ijk} \eta_{v\mu}^i \sigma^i \end{aligned}$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega(n) = n_v \sigma^v / \sqrt{n_v^i} + A_v \mapsto i\Omega \partial_v \Omega^{ijk} = 1/n_{ijk}^v \eta_{v\mu}^i \sigma^i + A_v \\ &= 1/n^{ijk} + \rho^{ijk} \eta_{v\mu}^i n^\mu \sigma^i + F_{v\mu} = 2\rho^{ijk} / (n^{ijk} + \rho^{ijk}) \exp^{ijk} \eta_{v\mu}^i \sigma^i \end{aligned}$$

$$\eta_{vi}^1 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{vi}^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{vi}^3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{vi}^\infty = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega(x) = x_{\mu\nu} \sigma^{\mu\nu} / \sqrt{x_{\mu\nu}^{ijk}} + A_{\mu\nu} \mapsto ijk\Omega \partial_{\mu\nu} \Omega^{ijk} = 1/x_{ijk}^{\mu\nu} \eta_{\mu\nu}^{ijk} \sigma^{ijk} + A_{\mu\nu} \\ &= 1/x^{ijk} + \rho^{ijk} \eta_{\mu\nu}^{ijk} x^{\mu\nu} \sigma^{ijk} + F_{\mu\nu} = 2\rho^{ijk} / (x^{ijk} + \rho^{ijk}) \exp^{ijk} \eta_{\mu\nu}^{ijk} \sigma^{ijk} \end{aligned}$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega(y) = y_{\mu\nu} \sigma^{\mu\nu} / \sqrt{y_{\mu\nu}^{ijk}} + A_{\mu\nu} \mapsto ijk\Omega \partial_{\mu\nu} \Omega^{ijk} = 1/y_{ijk}^{\mu\nu} \eta_{\mu\nu}^{ijk} \sigma^{ijk} + A_{\mu\nu} \\ &= 1/y^{ijk} + \rho^{ijk} \eta_{\mu\nu}^{ijk} y^{\mu\nu} \sigma^{ijk} + F_{\mu\nu} = 2\rho^{ijk} / (y^{ijk} + \rho^{ijk}) \exp^{ijk} \eta_{\mu\nu}^{ijk} \sigma^{ijk} \end{aligned}$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega(z) = z_{\mu\nu} \sigma^{\mu\nu} / \sqrt{z_{\mu\nu}^{ijk}} + A_{\mu\nu} \mapsto ijk\Omega \partial_{\mu\nu} \Omega^{ijk} = 1/z_{ijk}^{\mu\nu} \eta_{\mu\nu}^{ijk} \sigma^{ijk} + A_{\mu\nu} \\ &= 1/z^{ijk} + \rho^{ijk} \eta_{\mu\nu}^{ijk} z^{\mu\nu} \sigma^{ijk} + F_{\mu\nu} = 2\rho^{ijk} / (z^{ijk} + \rho^{ijk}) \exp^{ijk} \eta_{\mu\nu}^{ijk} \sigma^{ijk} \end{aligned}$$

$$\begin{aligned} \hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Omega(n) = n_{\mu\nu} \sigma^{\mu\nu} / \sqrt{n_{\mu\nu}^{ijk}} + A_{\mu\nu} \mapsto ijk\Omega \partial_{\mu\nu} \Omega^{ijk} = 1/n_{ijk}^{\mu\nu} \eta_{\mu\nu}^{ijk} \sigma^{ijk} + A_{\mu\nu} \\ &= 1/n^{ijk} + \rho^{ijk} \eta_{\mu\nu}^{ijk} n^{\mu\nu} \sigma^{ijk} + F_{\mu\nu} = 2\rho^{ijk} / (n^{ijk} + \rho^{ijk}) \exp^{ijk} \eta_{\mu\nu}^{ijk} \sigma^{ijk} \end{aligned}$$

$$\eta_{\mu\nu i}^1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu i}^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu i}^3 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu i}^\infty = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_{v\mu}\sigma^{v\mu}/\sqrt{x_{v\mu}^{ijk}} + A_{v\mu} \mapsto ijk\Omega\partial_{v\mu}\Omega^{ijk} = 1/x_{ijk}^{v\mu}\eta_{v\mu}^{ijk}\sigma^{ijk} + A_{v\mu} \\ &= 1/x^{ijk} + \rho^{ijk}\eta_{v\mu}^{ijk}x^{v\mu}\sigma^{ijk} + F_{v\mu} = 2\rho^{ijk}/(x^{ijk} + \rho^{ijk})\exp^{ijk}\eta_{v\mu}^{ijk}\sigma^{ijk}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_{v\mu}\sigma^{v\mu}/\sqrt{y_{v\mu}^{ijk}} + A_{v\mu} \mapsto ijk\Omega\partial_{v\mu}\Omega^{ijk} = 1/y_{ijk}^{v\mu}\eta_{v\mu}^{ijk}\sigma^{ijk} + A_{v\mu} \\ &= 1/y^{ijk} + \rho^{ijk}\eta_{v\mu}^{ijk}y^{v\mu}\sigma^{ijk} + F_{v\mu} = 2\rho^{ijk}/(y^{ijk} + \rho^{ijk})\exp^{ijk}\eta_{v\mu}^{ijk}\sigma^{ijk}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_{v\mu}\sigma^{v\mu}/\sqrt{z_{v\mu}^{ijk}} + A_{v\mu} \mapsto ijk\Omega\partial_{v\mu}\Omega^{ijk} = 1/z_{ijk}^{v\mu}\eta_{v\mu}^{ijk}\sigma^{ijk} + A_{v\mu} \\ &= 1/z^{ijk} + \rho^{ijk}\eta_{v\mu}^{ijk}z^{v\mu}\sigma^{ijk} + F_{v\mu} = 2\rho^{ijk}/(z^{ijk} + \rho^{ijk})\exp^{ijk}\eta_{v\mu}^{ijk}\sigma^{ijk}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_{v\mu}\sigma^{v\mu}/\sqrt{n_{v\mu}^{ijk}} + A_{v\mu} \mapsto ijk\Omega\partial_{v\mu}\Omega^{ijk} = 1/n_{ijk}^{v\mu}\eta_{v\mu}^{ijk}\sigma^{ijk} + A_{v\mu} \\ &= 1/n^{ijk} + \rho^{ijk}\eta_{v\mu}^{ijk}n^{v\mu}\sigma^{ijk} + F_{v\mu} = 2\rho^{ijk}/(n^{ijk} + \rho^{ijk})\exp^{ijk}\eta_{v\mu}^{ijk}\sigma^{ijk}\end{aligned}$$

$$\eta_{v\mu i}^1 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{v\mu i}^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{v\mu i}^3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{v\mu i}^\infty = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_\mu\sigma^\mu/\sqrt{x_\mu^i} + A_\mu \mapsto i\Omega\partial_\mu\Omega^{ikj} = 1/x_{ikj}^\mu\eta_{\mu\nu}^i\sigma^i + A_\mu \\ &= 1/x^{ikj} + \rho^{ikj}\eta_{\mu\nu}^ix^\nu\sigma^i + F_{\mu\nu} = 2\rho^{ikj}/(x^{ikj} + \rho^{ikj})\exp^{ikj}\eta_{\mu\nu}^i\sigma^i\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_\mu\sigma^\mu/\sqrt{y_\mu^i} + A_\mu \mapsto i\Omega\partial_\mu\Omega^{ikj} = 1/y_{ikj}^\mu\eta_{\mu\nu}^i\sigma^i + A_\mu \\ &= 1/y^{ikj} + \rho^{ikj}\eta_{\mu\nu}^iy^\nu\sigma^i + F_{\mu\nu} = 2\rho^{ikj}/(y^{ikj} + \rho^{ikj})\exp^{ikj}\eta_{\mu\nu}^i\sigma^i\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_\mu\sigma^\mu/\sqrt{z_\mu^i} + A_\mu \mapsto i\Omega\partial_\mu\Omega^{ikj} = 1/z_{ikj}^\mu\eta_{\mu\nu}^i\sigma^i + A_\mu \\ &= 1/z^{ikj} + \rho^{ikj}\eta_{\mu\nu}^iz^\nu\sigma^i + F_{\mu\nu} = 2\rho^{ikj}/(z^{ikj} + \rho^{ikj})\exp^{ikj}\eta_{\mu\nu}^i\sigma^i\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_\mu\sigma^\mu/\sqrt{n_\mu^i} + A_\mu \mapsto i\Omega\partial_\mu\Omega^{ikj} = 1/n_{ikj}^\mu\eta_{\mu\nu}^i\sigma^i + A_\mu \\ &= 1/n^{ikj} + \rho^{ikj}\eta_{\mu\nu}^in^\nu\sigma^i + F_{\mu\nu} = 2\rho^{ikj}/(n^{ikj} + \rho^{ikj})\exp^{ikj}\eta_{\mu\nu}^i\sigma^i\end{aligned}$$

$$\eta_{\mu i}^1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu i}^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu i}^3 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu i}^\infty = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_v\sigma^v/\sqrt{x_v^i + A_v} \mapsto i\Omega\partial_v\Omega^{ikj} = 1/x_{ikj}^v\eta_{v\mu}^i\sigma^i + A_v \\ &= 1/x^{ikj} + \rho^{ikj}\eta_{v\mu}^i x^\mu\sigma^i + F_{v\mu} = 2\rho^{ikj}/(x^{ikj} + \rho^{ikj})\exp^{ikj}\eta_{v\mu}^i\sigma^i\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_v\sigma^v/\sqrt{y_v^i + A_v} \mapsto i\Omega\partial_v\Omega^{ikj} = 1/y_{ikj}^v\eta_{v\mu}^i\sigma^i + A_v \\ &= 1/y^{ikj} + \rho^{ikj}\eta_{v\mu}^i y^\mu\sigma^i + F_{v\mu} = 2\rho^{ikj}/(y^{ikj} + \rho^{ikj})\exp^{ikj}\eta_{v\mu}^i\sigma^i\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_v\sigma^v/\sqrt{z_v^i + A_v} \mapsto i\Omega\partial_v\Omega^{ikj} = 1/z_{ikj}^v\eta_{v\mu}^i\sigma^i + A_v \\ &= 1/z^{ikj} + \rho^{ikj}\eta_{v\mu}^i z^\mu\sigma^i + F_{v\mu} = 2\rho^{ikj}/(z^{ikj} + \rho^{ikj})\exp^{ikj}\eta_{v\mu}^i\sigma^i\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_v\sigma^v/\sqrt{n_v^i + A_v} \mapsto i\Omega\partial_v\Omega^{ikj} = 1/n_{ikj}^v\eta_{v\mu}^i\sigma^i + A_v \\ &= 1/n^{ikj} + \rho^{ikj}\eta_{v\mu}^i n^\mu\sigma^i + F_{v\mu} = 2\rho^{ikj}/(n^{ikj} + \rho^{ikj})\exp^{ikj}\eta_{v\mu}^i\sigma^i\end{aligned}$$

$$\eta_{vi}^1 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{vi}^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{vi}^3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{vi}^\infty = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_{\mu\nu}\sigma^{\mu\nu}/\sqrt{x_{\mu\nu}^{ikj} + A_{\mu\nu}} \mapsto ikj\Omega\partial_{\mu\nu}\Omega^{ikj} = 1/x_{ikj}^{\mu\nu}\eta_{\mu\nu}^{ikj}\sigma^{ikj} + A_{\mu\nu} \\ &= 1/x^{ikj} + \rho^{ikj}\eta_{\mu\nu}^{ikj} x^{\mu\nu}\sigma^{ikj} + F_{\mu\nu} = 2\rho^{ikj}/(x^{ikj} + \rho^{ikj})\exp^{ikj}\eta_{\mu\nu}^{ikj}\sigma^{ikj}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_{\mu\nu}\sigma^{\mu\nu}/\sqrt{y_{\mu\nu}^{ikj} + A_{\mu\nu}} \mapsto ikj\Omega\partial_{\mu\nu}\Omega^{ikj} = 1/y_{ikj}^{\mu\nu}\eta_{\mu\nu}^{ikj}\sigma^{ikj} + A_{\mu\nu} \\ &= 1/y^{ikj} + \rho^{ikj}\eta_{\mu\nu}^{ikj} y^{\mu\nu}\sigma^{ikj} + F_{\mu\nu} = 2\rho^{ikj}/(y^{ikj} + \rho^{ikj})\exp^{ikj}\eta_{\mu\nu}^{ikj}\sigma^{ikj}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_{\mu\nu}\sigma^{\mu\nu}/\sqrt{z_{\mu\nu}^{ikj} + A_{\mu\nu}} \mapsto ikj\Omega\partial_{\mu\nu}\Omega^{ikj} = 1/z_{ikj}^{\mu\nu}\eta_{\mu\nu}^{ikj}\sigma^{ikj} + A_{\mu\nu} \\ &= 1/z^{ikj} + \rho^{ikj}\eta_{\mu\nu}^{ikj} z^{\mu\nu}\sigma^{ikj} + F_{\mu\nu} = 2\rho^{ikj}/(z^{ikj} + \rho^{ikj})\exp^{ikj}\eta_{\mu\nu}^{ikj}\sigma^{ikj}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_{\mu\nu}\sigma^{\mu\nu}/\sqrt{n_{\mu\nu}^{ikj} + A_{\mu\nu}} \mapsto ikj\Omega\partial_{\mu\nu}\Omega^{ikj} = 1/n_{ikj}^{\mu\nu}\eta_{\mu\nu}^{ikj}\sigma^{ikj} + A_{\mu\nu} \\ &= 1/n^{ikj} + \rho^{ikj}\eta_{\mu\nu}^{ikj} n^{\mu\nu}\sigma^{ikj} + F_{\mu\nu} = 2\rho^{ikj}/(n^{ikj} + \rho^{ikj})\exp^{ikj}\eta_{\mu\nu}^{ikj}\sigma^{ikj}\end{aligned}$$

$$\eta_{\mu\nu i}^1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu i}^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu i}^3 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu i}^\infty = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_{v\mu}\sigma^{v\mu}/\sqrt{x_{v\mu}^{ikj}} + A_{v\mu} \mapsto ikj\Omega\partial_{v\mu}\Omega^{ikj} = 1/x_{ikj}^{v\mu}\eta_{v\mu}^{ikj}\sigma^{ikj} + A_{v\mu} \\ &= 1/x^{ikj} + \rho^{ikj}\eta_{v\mu}^{ikj}x^{v\mu}\sigma^{ikj} + F_{v\mu} = 2\rho^{ikj}/(x^{ikj} + \rho^{ikj})\exp^{ikj}\eta_{v\mu}^{ikj}\sigma^{ikj}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_{v\mu}\sigma^{v\mu}/\sqrt{y_{v\mu}^{ikj}} + A_{v\mu} \mapsto ikj\Omega\partial_{v\mu}\Omega^{ikj} = 1/y_{ikj}^{v\mu}\eta_{v\mu}^{ikj}\sigma^{ikj} + A_{v\mu} \\ &= 1/y^{ikj} + \rho^{ikj}\eta_{v\mu}^{ikj}y^{v\mu}\sigma^{ikj} + F_{v\mu} = 2\rho^{ikj}/(y^{ikj} + \rho^{ikj})\exp^{ikj}\eta_{v\mu}^{ikj}\sigma^{ikj}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_{v\mu}\sigma^{v\mu}/\sqrt{z_{v\mu}^{ikj}} + A_{v\mu} \mapsto ikj\Omega\partial_{v\mu}\Omega^{ikj} = 1/z_{ikj}^{v\mu}\eta_{v\mu}^{ikj}\sigma^{ikj} + A_{v\mu} \\ &= 1/z^{ikj} + \rho^{ikj}\eta_{v\mu}^{ikj}z^{v\mu}\sigma^{ikj} + F_{v\mu} = 2\rho^{ikj}/(z^{ikj} + \rho^{ikj})\exp^{ikj}\eta_{v\mu}^{ikj}\sigma^{ikj}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_{v\mu}\sigma^{v\mu}/\sqrt{n_{v\mu}^{ikj}} + A_{v\mu} \mapsto ikj\Omega\partial_{v\mu}\Omega^{ikj} = 1/n_{ikj}^{v\mu}\eta_{v\mu}^{ikj}\sigma^{ikj} + A_{v\mu} \\ &= 1/n^{ikj} + \rho^{ikj}\eta_{v\mu}^{ikj}n^{v\mu}\sigma^{ikj} + F_{v\mu} = 2\rho^{ikj}/(n^{ikj} + \rho^{ikj})\exp^{ikj}\eta_{v\mu}^{ikj}\sigma^{ikj}\end{aligned}$$

$$\eta_{v\mu i}^1 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{v\mu i}^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{v\mu i}^3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{v\mu i}^\infty = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_\mu\sigma^\mu/\sqrt{x_\mu^j} + A_\mu \mapsto j\Omega\partial_\mu\Omega^{jik} = 1/x_{jik}^\mu\eta_{\mu\nu}^j\sigma^j + A_\mu \\ &= 1/x^{jik} + \rho^{jik}\eta_{\mu\nu}^jx^\nu\sigma^j + F_{\mu\nu} = 2\rho^{jik}/(x^{jik} + \rho^{jik})\exp^{jik}\eta_{\mu\nu}^j\sigma^j\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_\mu\sigma^\mu/\sqrt{y_\mu^j} + A_\mu \mapsto j\Omega\partial_\mu\Omega^{jik} = 1/y_{jik}^\mu\eta_{\mu\nu}^j\sigma^j + A_\mu \\ &= 1/y^{jik} + \rho^{jik}\eta_{\mu\nu}^jy^\nu\sigma^j + F_{\mu\nu} = 2\rho^{jik}/(y^{jik} + \rho^{jik})\exp^{jik}\eta_{\mu\nu}^j\sigma^j\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_\mu\sigma^\mu/\sqrt{z_\mu^j} + A_\mu \mapsto j\Omega\partial_\mu\Omega^{jik} = 1/z_{jik}^\mu\eta_{\mu\nu}^j\sigma^j + A_\mu \\ &= 1/z^{jik} + \rho^{jik}\eta_{\mu\nu}^jz^\nu\sigma^j + F_{\mu\nu} = 2\rho^{jik}/(z^{jik} + \rho^{jik})\exp^{jik}\eta_{\mu\nu}^j\sigma^j\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_\mu\sigma^\mu/\sqrt{n_\mu^j} + A_\mu \mapsto j\Omega\partial_\mu\Omega^{jik} = 1/n_{jik}^\mu\eta_{\mu\nu}^j\sigma^j + A_\mu \\ &= 1/n^{jik} + \rho^{jik}\eta_{\mu\nu}^jn^\nu\sigma^j + F_{\mu\nu} = 2\rho^{jik}/(n^{jik} + \rho^{jik})\exp^{jik}\eta_{\mu\nu}^j\sigma^j\end{aligned}$$

$$\eta_{\mu j}^1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu j}^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu j}^3 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu j}^\infty = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_v\sigma^v/\sqrt{x_v^j} + A_v \mapsto j\Omega\partial_v\Omega^{jik} = 1/x_{jik}^v\eta_{v\mu}^j\sigma^j + A_v \\ &= 1/x^{jik} + \rho^{jik}\eta_{v\mu}^j x^\mu\sigma^j + F_{v\mu} = 2\rho^{jik}/(x^{jik} + \rho^{jik})\exp^{jik}\eta_{v\mu}^j\sigma^j\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_v\sigma^v/\sqrt{y_v^j} + A_v \mapsto j\Omega\partial_v\Omega^{jik} = 1/y_{jik}^v\eta_{v\mu}^j\sigma^j + A_v \\ &= 1/y^{jik} + \rho^{jik}\eta_{v\mu}^j y^\mu\sigma^j + F_{v\mu} = 2\rho^{jik}/(y^{jik} + \rho^{jik})\exp^{jik}\eta_{v\mu}^j\sigma^j\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_v\sigma^v/\sqrt{z_v^j} + A_v \mapsto j\Omega\partial_v\Omega^{jik} = 1/z_{jik}^v\eta_{v\mu}^j\sigma^j + A_v \\ &= 1/z^{jik} + \rho^{jik}\eta_{v\mu}^j z^\mu\sigma^j + F_{v\mu} = 2\rho^{jik}/(z^{jik} + \rho^{jik})\exp^{jik}\eta_{v\mu}^j\sigma^j\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_v\sigma^v/\sqrt{n_v^j} + A_v \mapsto j\Omega\partial_v\Omega^{jik} = 1/n_{jik}^v\eta_{v\mu}^j\sigma^j + A_v \\ &= 1/n^{jik} + \rho^{jik}\eta_{v\mu}^j n^\mu\sigma^j + F_{v\mu} = 2\rho^{jik}/(n^{jik} + \rho^{jik})\exp^{jik}\eta_{v\mu}^j\sigma^j\end{aligned}$$

$$\eta_{vj}^1 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{vj}^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{vj}^3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{vj}^\infty = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_{\mu\nu}\sigma^{\mu\nu}/\sqrt{x_{\mu\nu}^{jik}} + A_{\mu\nu} \mapsto jik\Omega\partial_{\mu\nu}\Omega^{jik} = 1/x_{jik}^{\mu\nu}\eta_{\mu\nu}^{jik}\sigma^{jik} + A_{\mu\nu} \\ &= 1/x^{jik} + \rho^{jik}\eta_{\mu\nu}^{jik} x^{\mu\nu}\sigma^{jik} + F_{\mu\nu} = 2\rho^{jik}/(x^{jik} + \rho^{jik})\exp^{jik}\eta_{\mu\nu}^{jik}\sigma^{jik}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_{\mu\nu}\sigma^{\mu\nu}/\sqrt{y_{\mu\nu}^{jik}} + A_{\mu\nu} \mapsto jik\Omega\partial_{\mu\nu}\Omega^{jik} = 1/y_{jik}^{\mu\nu}\eta_{\mu\nu}^{jik}\sigma^{jik} + A_{\mu\nu} \\ &= 1/y^{jik} + \rho^{jik}\eta_{\mu\nu}^{jik} y^{\mu\nu}\sigma^{jik} + F_{\mu\nu} = 2\rho^{jik}/(y^{jik} + \rho^{jik})\exp^{jik}\eta_{\mu\nu}^{jik}\sigma^{jik}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_{\mu\nu}\sigma^{\mu\nu}/\sqrt{z_{\mu\nu}^{jik}} + A_{\mu\nu} \mapsto jik\Omega\partial_{\mu\nu}\Omega^{jik} = 1/z_{jik}^{\mu\nu}\eta_{\mu\nu}^{jik}\sigma^{jik} + A_{\mu\nu} \\ &= 1/z^{jik} + \rho^{jik}\eta_{\mu\nu}^{jik} z^{\mu\nu}\sigma^{jik} + F_{\mu\nu} = 2\rho^{jik}/(z^{jik} + \rho^{jik})\exp^{jik}\eta_{\mu\nu}^{jik}\sigma^{jik}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_{\mu\nu}\sigma^{\mu\nu}/\sqrt{n_{\mu\nu}^{jik}} + A_{\mu\nu} \mapsto jik\Omega\partial_{\mu\nu}\Omega^{jik} = 1/n_{jik}^{\mu\nu}\eta_{\mu\nu}^{jik}\sigma^{jik} + A_{\mu\nu} \\ &= 1/n^{jik} + \rho^{jik}\eta_{\mu\nu}^{jik} n^{\mu\nu}\sigma^{jik} + F_{\mu\nu} = 2\rho^{jik}/(n^{jik} + \rho^{jik})\exp^{jik}\eta_{\mu\nu}^{jik}\sigma^{jik}\end{aligned}$$

$$\eta_{\mu\nu j}^1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu j}^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu j}^3 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu j}^\infty = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_{v\mu}\sigma^{v\mu}/\sqrt{x_{v\mu}^{jik}} + A_{v\mu} \mapsto jik\Omega\partial_{v\mu}\Omega^{jik} = 1/x_{jik}^{v\mu}\eta_{v\mu}^{jik}\sigma^{jik} + A_{v\mu} \\ &= 1/x^{jik} + \rho^{jik}\eta_{v\mu}^{jik}x^{v\mu}\sigma^{jik} + F_{v\mu} = 2\rho^{jik}/(x^{jik} + \rho^{jik})\exp^{jik}\eta_{v\mu}^{jik}\sigma^{jik}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_{v\mu}\sigma^{v\mu}/\sqrt{y_{v\mu}^{jik}} + A_{v\mu} \mapsto jik\Omega\partial_{v\mu}\Omega^{jik} = 1/y_{jik}^{v\mu}\eta_{v\mu}^{jik}\sigma^{jik} + A_{v\mu} \\ &= 1/y^{jik} + \rho^{jik}\eta_{v\mu}^{jik}y^{v\mu}\sigma^{jik} + F_{v\mu} = 2\rho^{jik}/(y^{jik} + \rho^{jik})\exp^{jik}\eta_{v\mu}^{jik}\sigma^{jik}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_{v\mu}\sigma^{v\mu}/\sqrt{z_{v\mu}^{jik}} + A_{v\mu} \mapsto jik\Omega\partial_{v\mu}\Omega^{jik} = 1/z_{jik}^{v\mu}\eta_{v\mu}^{jik}\sigma^{jik} + A_{v\mu} \\ &= 1/z^{jik} + \rho^{jik}\eta_{v\mu}^{jik}z^{v\mu}\sigma^{jik} + F_{v\mu} = 2\rho^{jik}/(z^{jik} + \rho^{jik})\exp^{jik}\eta_{v\mu}^{jik}\sigma^{jik}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_{v\mu}\sigma^{v\mu}/\sqrt{n_{v\mu}^{jik}} + A_{v\mu} \mapsto jik\Omega\partial_{v\mu}\Omega^{jik} = 1/n_{jik}^{v\mu}\eta_{v\mu}^{jik}\sigma^{jik} + A_{v\mu} \\ &= 1/n^{jik} + \rho^{jik}\eta_{v\mu}^{jik}n^{v\mu}\sigma^{jik} + F_{v\mu} = 2\rho^{jik}/(n^{jik} + \rho^{jik})\exp^{jik}\eta_{v\mu}^{jik}\sigma^{jik}\end{aligned}$$

$$\eta_{v\mu j}^1 = \begin{matrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{matrix} \quad \eta_{v\mu j}^2 = \begin{matrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{matrix} \quad \eta_{v\mu j}^3 = \begin{matrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{matrix} \quad \eta_{v\mu j}^\infty = \begin{matrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{matrix}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_\mu\sigma^\mu/\sqrt{x_\mu^j} + A_\mu \mapsto j\Omega\partial_\mu\Omega^{jki} = 1/x_{jki}^\mu\eta_{\mu\nu}^j\sigma^j + A_\mu \\ &= 1/x^{jki} + \rho^{jki}\eta_{\mu\nu}^jx^\nu\sigma^j + F_{\mu\nu} = 2\rho^{jki}/(x^{jki} + \rho^{jki})\exp^{jki}\eta_{\mu\nu}^j\sigma^j\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_\mu\sigma^\mu/\sqrt{y_\mu^j} + A_\mu \mapsto j\Omega\partial_\mu\Omega^{jki} = 1/y_{jki}^\mu\eta_{\mu\nu}^j\sigma^j + A_\mu \\ &= 1/y^{jki} + \rho^{jki}\eta_{\mu\nu}^jy^\nu\sigma^j + F_{\mu\nu} = 2\rho^{jki}/(y^{jki} + \rho^{jki})\exp^{jki}\eta_{\mu\nu}^j\sigma^j\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_\mu\sigma^\mu/\sqrt{z_\mu^j} + A_\mu \mapsto j\Omega\partial_\mu\Omega^{jki} = 1/z_{jki}^\mu\eta_{\mu\nu}^j\sigma^j + A_\mu \\ &= 1/z^{jki} + \rho^{jki}\eta_{\mu\nu}^jz^\nu\sigma^j + F_{\mu\nu} = 2\rho^{jki}/(z^{jki} + \rho^{jki})\exp^{jki}\eta_{\mu\nu}^j\sigma^j\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_\mu\sigma^\mu/\sqrt{n_\mu^j} + A_\mu \mapsto j\Omega\partial_\mu\Omega^{jki} = 1/n_{jki}^\mu\eta_{\mu\nu}^j\sigma^j + A_\mu \\ &= 1/n^{jki} + \rho^{jki}\eta_{\mu\nu}^jn^\nu\sigma^j + F_{\mu\nu} = 2\rho^{jki}/(n^{jki} + \rho^{jki})\exp^{jki}\eta_{\mu\nu}^j\sigma^j\end{aligned}$$

$$\eta_{\mu j}^1 = \begin{matrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{matrix} \quad \eta_{\mu j}^2 = \begin{matrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{matrix} \quad \eta_{\mu j}^3 = \begin{matrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{matrix} \quad \eta_{\mu j}^\infty = \begin{matrix} 0 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{matrix}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_v\sigma^v/\sqrt{x_v^j} + A_v \mapsto j\Omega\partial_v\Omega^{jki} = 1/x_{jki}^v\eta_{v\mu}^j\sigma^j + A_v \\ &= 1/x^{jki} + \rho^{jki}\eta_{v\mu}^jx^\mu\sigma^j + F_{v\mu} = 2\rho^{jki}/(x^{jki} + \rho^{jki})\exp^{jki}\eta_{v\mu}^j\sigma^j\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_v\sigma^v/\sqrt{y_v^j} + A_v \mapsto j\Omega\partial_v\Omega^{jki} = 1/y_{jki}^v\eta_{v\mu}^j\sigma^j + A_v \\ &= 1/y^{jki} + \rho^{jki}\eta_{v\mu}^jy^\mu\sigma^j + F_{v\mu} = 2\rho^{jki}/(y^{jki} + \rho^{jki})\exp^{jki}\eta_{v\mu}^j\sigma^j\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_v\sigma^v/\sqrt{z_v^j} + A_v \mapsto j\Omega\partial_v\Omega^{jki} = 1/z_{jki}^v\eta_{v\mu}^j\sigma^j + A_v \\ &= 1/z^{jki} + \rho^{jki}\eta_{v\mu}^jz^\mu\sigma^j + F_{v\mu} = 2\rho^{jki}/(z^{jki} + \rho^{jki})\exp^{jki}\eta_{v\mu}^j\sigma^j\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_v\sigma^v/\sqrt{n_v^j} + A_v \mapsto j\Omega\partial_v\Omega^{jki} = 1/n_{jki}^v\eta_{v\mu}^j\sigma^j + A_v \\ &= 1/n^{jki} + \rho^{jki}\eta_{v\mu}^jn^\mu\sigma^j + F_{v\mu} = 2\rho^{jki}/(n^{jki} + \rho^{jki})\exp^{jki}\eta_{v\mu}^j\sigma^j\end{aligned}$$

$$\eta_{vj}^1 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{vj}^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{vj}^3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{vj}^\infty = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_{\mu\nu}\sigma^{\mu\nu}/\sqrt{x_{\mu\nu}^{jki}} + A_{\mu\nu} \mapsto jki\Omega\partial_{\mu\nu}\Omega^{jki} = 1/x_{jki}^{\mu\nu}\eta_{\mu\nu}^{jki}\sigma^{jki} + A_{\mu\nu} \\ &= 1/x^{jki} + \rho^{jki}\eta_{\mu\nu}^{jki}x^{\mu\nu}\sigma^{jki} + F_{\mu\nu} = 2\rho^{jki}/(x^{jki} + \rho^{jki})\exp^{jki}\eta_{\mu\nu}^{jki}\sigma^{jki}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_{\mu\nu}\sigma^{\mu\nu}/\sqrt{y_{\mu\nu}^{jki}} + A_{\mu\nu} \mapsto jki\Omega\partial_{\mu\nu}\Omega^{jki} = 1/y_{jki}^{\mu\nu}\eta_{\mu\nu}^{jki}\sigma^{jki} + A_{\mu\nu} \\ &= 1/y^{jki} + \rho^{jki}\eta_{\mu\nu}^{jki}y^{\mu\nu}\sigma^{jki} + F_{\mu\nu} = 2\rho^{jki}/(y^{jki} + \rho^{jki})\exp^{jki}\eta_{\mu\nu}^{jki}\sigma^{jki}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_{\mu\nu}\sigma^{\mu\nu}/\sqrt{z_{\mu\nu}^{jki}} + A_{\mu\nu} \mapsto jki\Omega\partial_{\mu\nu}\Omega^{jki} = 1/z_{jki}^{\mu\nu}\eta_{\mu\nu}^{jki}\sigma^{jki} + A_{\mu\nu} \\ &= 1/z^{jki} + \rho^{jki}\eta_{\mu\nu}^{jki}z^{\mu\nu}\sigma^{jki} + F_{\mu\nu} = 2\rho^{jki}/(z^{jki} + \rho^{jki})\exp^{jki}\eta_{\mu\nu}^{jki}\sigma^{jki}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_{\mu\nu}\sigma^{\mu\nu}/\sqrt{n_{\mu\nu}^{jki}} + A_{\mu\nu} \mapsto jki\Omega\partial_{\mu\nu}\Omega^{jki} = 1/n_{jki}^{\mu\nu}\eta_{\mu\nu}^{jki}\sigma^{jki} + A_{\mu\nu} \\ &= 1/n^{jki} + \rho^{jki}\eta_{\mu\nu}^{jki}n^{\mu\nu}\sigma^j + F_{\mu\nu} = 2\rho^{jki}/(n^{jki} + \rho^{jki})\exp^{jki}\eta_{\mu\nu}^{jki}\sigma^{jki}\end{aligned}$$

$$\eta_{\mu\nu j}^1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu j}^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu j}^3 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu j}^\infty = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_{v\mu}\sigma^{v\mu}/\sqrt{x_{v\mu}^{jki}} + A_{v\mu} \mapsto jki\Omega\partial_{v\mu}\Omega^{jki} = 1/x_{jki}^{v\mu}\eta_{v\mu}^{jki}\sigma^{jki} + A_{v\mu} \\ &= 1/x^{jki} + \rho^{jki}\eta_{v\mu}^{jki}x^{v\mu}\sigma^{jki} + F_{v\mu} = 2\rho^{jki}/(x^{jki} + \rho^{jki})\exp^{jki}\eta_{v\mu}^{jki}\sigma^{jki}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_{v\mu}\sigma^{v\mu}/\sqrt{y_{v\mu}^{jki}} + A_{v\mu} \mapsto jki\Omega\partial_{v\mu}\Omega^{jki} = 1/y_{jki}^{v\mu}\eta_{v\mu}^{jki}\sigma^{jki} + A_{v\mu} \\ &= 1/y^{jki} + \rho^{jki}\eta_{v\mu}^{jki}y^{v\mu}\sigma^{jki} + F_{v\mu} = 2\rho^{jki}/(y^{jki} + \rho^{jki})\exp^{jki}\eta_{v\mu}^{jki}\sigma^{jki}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_{v\mu}\sigma^{v\mu}/\sqrt{z_{v\mu}^{jki}} + A_{v\mu} \mapsto jki\Omega\partial_{v\mu}\Omega^{jki} = 1/z_{jki}^{v\mu}\eta_{v\mu}^{jki}\sigma^{jki} + A_{v\mu} \\ &= 1/z^{jki} + \rho^{jki}\eta_{v\mu}^{jki}z^{v\mu}\sigma^{jki} + F_{v\mu} = 2\rho^{jki}/(z^{jki} + \rho^{jki})\exp^{jki}\eta_{v\mu}^{jki}\sigma^{jki}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_{v\mu}\sigma^{v\mu}/\sqrt{n_{v\mu}^{jki}} + A_{v\mu} \mapsto jki\Omega\partial_{v\mu}\Omega^{jki} = 1/n_{jki}^{v\mu}\eta_{v\mu}^{jki}\sigma^{jki} + A_{v\mu} \\ &= 1/n^{jki} + \rho^{jki}\eta_{v\mu}^{jki}n^{v\mu}\sigma^{jki} + F_{v\mu} = 2\rho^{jki}/(n^{jki} + \rho^{jki})\exp^{jki}\eta_{v\mu}^{jki}\sigma^{jki}\end{aligned}$$

$$\eta_{v\mu j}^1 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{v\mu j}^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{v\mu j}^3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{v\mu j}^\infty = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_\mu\sigma^\mu/\sqrt{x_\mu^k} + A_\mu \mapsto k\Omega\partial_\mu\Omega^{kij} = 1/x_{kij}^\mu\eta_{\mu v}^k\sigma^k + A_\mu \\ &= 1/x^{kij} + \rho^{kij}\eta_{\mu v}^kx^v\sigma^k + F_{\mu v} = 2\rho^{kij}/(x^{kij} + \rho^{kij})\exp^{kij}\eta_{\mu v}^k\sigma^k\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_\mu\sigma^\mu/\sqrt{y_\mu^k} + A_\mu \mapsto k\Omega\partial_\mu\Omega^{kij} = 1/y_{kij}^\mu\eta_{\mu v}^k\sigma^k + A_\mu \\ &= 1/y^{kij} + \rho^{kij}\eta_{\mu v}^ky^v\sigma^k + F_{\mu v} = 2\rho^{kij}/(y^{kij} + \rho^{kij})\exp^{kij}\eta_{\mu v}^k\sigma^k\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_\mu\sigma^\mu/\sqrt{z_\mu^k} + A_\mu \mapsto k\Omega\partial_\mu\Omega^{kij} = 1/z_{kij}^\mu\eta_{\mu v}^k\sigma^k + A_\mu \\ &= 1/z^{kij} + \rho^{kij}\eta_{\mu v}^kz^v\sigma^k + F_{\mu v} = 2\rho^{kij}/(z^{kij} + \rho^{kij})\exp^{kij}\eta_{\mu v}^k\sigma^k\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_\mu\sigma^\mu/\sqrt{n_\mu^k} + A_\mu \mapsto k\Omega\partial_\mu\Omega^{kij} = 1/n_{kij}^\mu\eta_{\mu v}^k\sigma^k + A_\mu \\ &= 1/n^{kij} + \rho^{kij}\eta_{\mu v}^kn^v\sigma^k + F_{\mu v} = 2\rho^{kij}/(n^{kij} + \rho^{kij})\exp^{kij}\eta_{\mu v}^k\sigma^k\end{aligned}$$

$$\eta_{\mu k}^1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu k}^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu k}^3 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu k}^\infty = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_v\sigma^v/\sqrt{x_v^k} + A_v \mapsto k\Omega\partial_v\Omega^{kij} = 1/x_{kij}^v\eta_{v\mu}^k\sigma^k + A_v \\ &= 1/x^{kij} + \rho^{kij}\eta_{v\mu}^k x^\mu\sigma^k + F_{v\mu} = 2\rho^{kij}/(x^{kij} + \rho^{kij})\exp^{kij}\eta_{v\mu}^k\sigma^k\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_v\sigma^v/\sqrt{y_v^k} + A_v \mapsto k\Omega\partial_v\Omega^{kij} = 1/y_{kij}^v\eta_{v\mu}^k\sigma^k + A_v \\ &= 1/y^{kij} + \rho^{kij}\eta_{v\mu}^k y^\mu\sigma^k + F_{v\mu} = 2\rho^{kij}/(y^{kij} + \rho^{kij})\exp^{kij}\eta_{v\mu}^k\sigma^k\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_v\sigma^v/\sqrt{z_v^k} + A_v \mapsto k\Omega\partial_v\Omega^{kij} = 1/z_{kij}^v\eta_{v\mu}^k\sigma^k + A_v \\ &= 1/z^{kij} + \rho^{kij}\eta_{v\mu}^k z^\mu\sigma^k + F_{v\mu} = 2\rho^{kij}/(z^{kij} + \rho^{kij})\exp^{kij}\eta_{v\mu}^k\sigma^k\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_v\sigma^v/\sqrt{n_v^k} + A_v \mapsto k\Omega\partial_v\Omega^{kij} = 1/n_{kij}^v\eta_{v\mu}^k\sigma^k + A_v \\ &= 1/n^{kij} + \rho^{kij}\eta_{v\mu}^k n^\mu\sigma^k + F_{v\mu} = 2\rho^{kij}/(n^{kij} + \rho^{kij})\exp^{kij}\eta_{v\mu}^k\sigma^k\end{aligned}$$

$$\eta_{vk}^1 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{vk}^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{vk}^3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{vk}^\infty = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_{\mu\nu}\sigma^{\mu\nu}/\sqrt{x_{\mu\nu}^{kij}} + A_{\mu\nu} \mapsto kij\Omega\partial_{\mu\nu}\Omega^{kij} = 1/x_{kij}^{\mu\nu}\eta_{\mu\nu}^{kij}\sigma^{kij} + A_{\mu\nu} \\ &= 1/x^{kij} + \rho^{kij}\eta_{\mu\nu}^{kij} x^{\mu\nu}\sigma^{kij} + F_{\mu\nu} = 2\rho^{kij}/(x^{kij} + \rho^{kij})\exp^{kij}\eta_{\mu\nu}^{kij}\sigma^{kij}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_{\mu\nu}\sigma^{\mu\nu}/\sqrt{y_{\mu\nu}^{kij}} + A_{\mu\nu} \mapsto kij\Omega\partial_{\mu\nu}\Omega^{kij} = 1/y_{kij}^{\mu\nu}\eta_{\mu\nu}^{kij}\sigma^{kij} + A_{\mu\nu} \\ &= 1/y^{kij} + \rho^{kij}\eta_{\mu\nu}^{kij} y^{\mu\nu}\sigma^{kij} + F_{\mu\nu} = 2\rho^{kij}/(y^{kij} + \rho^{kij})\exp^{kij}\eta_{\mu\nu}^{kij}\sigma^{kij}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_{\mu\nu}\sigma^{\mu\nu}/\sqrt{z_{\mu\nu}^{kij}} + A_{\mu\nu} \mapsto kij\Omega\partial_{\mu\nu}\Omega^{kij} = 1/z_{kij}^{\mu\nu}\eta_{\mu\nu}^{kij}\sigma^{kij} + A_{\mu\nu} \\ &= 1/z^{kij} + \rho^{kij}\eta_{\mu\nu}^{kij} z^{\mu\nu}\sigma^{kij} + F_{\mu\nu} = 2\rho^{kij}/(z^{kij} + \rho^{kij})\exp^{kij}\eta_{\mu\nu}^{kij}\sigma^{kij}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_{\mu\nu}\sigma^{\mu\nu}/\sqrt{n_{\mu\nu}^{kij}} + A_{\mu\nu} \mapsto kij\Omega\partial_{\mu\nu}\Omega^{kij} = 1/n_{kij}^{\mu\nu}\eta_{\mu\nu}^{kij}\sigma^{kij} + A_{\mu\nu} \\ &= 1/n^{kij} + \rho^{kij}\eta_{\mu\nu}^{kij} n^{\mu\nu}\sigma^{kij} + F_{\mu\nu} = 2\rho^{kij}/(n^{kij} + \rho^{kij})\exp^{kij}\eta_{\mu\nu}^{kij}\sigma^{kij}\end{aligned}$$

$$\eta_{\mu\nu k}^1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu k}^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu k}^3 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu k}^\infty = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_{v\mu}\sigma^{v\mu}/\sqrt{x_{v\mu}^{kij}} + A_{v\mu} \mapsto kij\Omega\partial_{v\mu}\Omega^{kij} = 1/x_{kij}^{v\mu}\eta_{v\mu}^{kij}\sigma^{kij} + A_{v\mu} \\ &= 1/x^{kij} + \rho^{kij}\eta_{v\mu}^{kij}x^{v\mu}\sigma^{kij} + F_{v\mu} = 2\rho^{kij}/(x^{kij} + \rho^{kij})\exp^{kij}\eta_{v\mu}^{kij}\sigma^{kij}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_{v\mu}\sigma^{v\mu}/\sqrt{y_{v\mu}^{kij}} + A_{v\mu} \mapsto kij\Omega\partial_{v\mu}\Omega^{kij} = 1/y_{kij}^{v\mu}\eta_{v\mu}^{kij}\sigma^{kij} + A_{v\mu} \\ &= 1/y^{kij} + \rho^{kij}\eta_{v\mu}^{kij}y^{v\mu}\sigma^{kij} + F_{v\mu} = 2\rho^{kij}/(y^{kij} + \rho^{kij})\exp^{kij}\eta_{v\mu}^{kij}\sigma^{kij}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_{v\mu}\sigma^{v\mu}/\sqrt{z_{v\mu}^{kij}} + A_{v\mu} \mapsto kij\Omega\partial_{v\mu}\Omega^{kij} = 1/z_{kij}^{v\mu}\eta_{v\mu}^{kij}\sigma^{kij} + A_{v\mu} \\ &= 1/z^{kij} + \rho^{kij}\eta_{v\mu}^{kij}z^{v\mu}\sigma^{kij} + F_{v\mu} = 2\rho^{kij}/(z^{kij} + \rho^{kij})\exp^{kij}\eta_{v\mu}^{kij}\sigma^{kij}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_{v\mu}\sigma^{v\mu}/\sqrt{n_{v\mu}^{kij}} + A_{v\mu} \mapsto kij\Omega\partial_{v\mu}\Omega^{kij} = 1/n_{kij}^{v\mu}\eta_{v\mu}^{kij}\sigma^{kij} + A_{v\mu} \\ &= 1/n^{kij} + \rho^{kij}\eta_{v\mu}^{kij}n^{v\mu}\sigma^{kij} + F_{v\mu} = 2\rho^{kij}/(n^{kij} + \rho^{kij})\exp^{kij}\eta_{v\mu}^{kij}\sigma^{kij}\end{aligned}$$

$$\eta_{v\mu k}^1 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{v\mu k}^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{v\mu k}^3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{v\mu k}^\infty = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_\mu\sigma^\mu/\sqrt{x_\mu^k} + A_\mu \mapsto k\Omega\partial_\mu\Omega^{kji} = 1/x_{kji}^\mu\eta_{\mu v}^k\sigma^k + A_\mu \\ &= 1/x^{kji} + \rho^{kji}\eta_{\mu v}^kx^v\sigma^k + F_{\mu v} = 2\rho^{kji}/(x^{kji} + \rho^{kji})\exp^{kji}\eta_{\mu v}^k\sigma^k\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_\mu\sigma^\mu/\sqrt{y_\mu^k} + A_\mu \mapsto k\Omega\partial_\mu\Omega^{kji} = 1/y_{kji}^\mu\eta_{\mu v}^k\sigma^k + A_\mu \\ &= 1/y^{kji} + \rho^{kji}\eta_{\mu v}^ky^v\sigma^k + F_{\mu v} = 2\rho^{kji}/(y^{kji} + \rho^{kji})\exp^{kji}\eta_{\mu v}^k\sigma^k\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_\mu\sigma^\mu/\sqrt{z_\mu^k} + A_\mu \mapsto k\Omega\partial_\mu\Omega^{kji} = 1/z_{kji}^\mu\eta_{\mu v}^k\sigma^k + A_\mu \\ &= 1/z^{kji} + \rho^{kji}\eta_{\mu v}^kz^v\sigma^k + F_{\mu v} = 2\rho^{kji}/(z^{kji} + \rho^{kji})\exp^{kji}\eta_{\mu v}^k\sigma^k\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_\mu\sigma^\mu/\sqrt{n_\mu^k} + A_\mu \mapsto k\Omega\partial_\mu\Omega^{kji} = 1/n_{kji}^\mu\eta_{\mu v}^k\sigma^k + A_\mu \\ &= 1/n^{kji} + \rho^{kji}\eta_{\mu v}^kn^v\sigma^k + F_{\mu v} = 2\rho^{kji}/(n^{kji} + \rho^{kji})\exp^{kji}\eta_{\mu v}^k\sigma^k\end{aligned}$$

$$\eta_{\mu k}^1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu k}^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu k}^3 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu k}^\infty = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_v\sigma^v/\sqrt{x_v^k} + A_v \mapsto k\Omega\partial_v\Omega^{kji} = 1/x_{kji}^v\eta_{v\mu}^k\sigma^k + A_v \\ &= 1/x^{kji} + \rho^{kji}\eta_{v\mu}^kx^\mu\sigma^k + F_{v\mu} = 2\rho^{kji}/(x^{kji} + \rho^{kji})\exp^{kji}\eta_{v\mu}^k\sigma^k\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_v\sigma^v/\sqrt{y_v^k} + A_v \mapsto k\Omega\partial_v\Omega^{kji} = 1/y_{kji}^v\eta_{v\mu}^k\sigma^k + A_v \\ &= 1/y^{kji} + \rho^{kji}\eta_{v\mu}^ky^\mu\sigma^k + F_{v\mu} = 2\rho^{kji}/(y^{kji} + \rho^{kji})\exp^{kji}\eta_{v\mu}^k\sigma^k\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_v\sigma^v/\sqrt{z_v^k} + A_v \mapsto k\Omega\partial_v\Omega^{kji} = 1/z_{kji}^v\eta_{v\mu}^k\sigma^k + A_v \\ &= 1/z^{kji} + \rho^{kji}\eta_{v\mu}^kz^\mu\sigma^k + F_{v\mu} = 2\rho^{kji}/(z^{kji} + \rho^{kji})\exp^{kji}\eta_{v\mu}^k\sigma^k\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_v\sigma^v/\sqrt{n_v^k} + A_v \mapsto k\Omega\partial_v\Omega^{kji} = 1/n_{kji}^v\eta_{v\mu}^k\sigma^k + A_v \\ &= 1/n^{kji} + \rho^{kji}\eta_{v\mu}^kn^\mu\sigma^k + F_{v\mu} = 2\rho^{kji}/(n^{kji} + \rho^{kji})\exp^{kji}\eta_{v\mu}^k\sigma^k\end{aligned}$$

$$\eta_{vk}^1 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{vk}^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{vk}^3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{vk}^\infty = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_{\mu\nu}\sigma^{\mu\nu}/\sqrt{x_{\mu\nu}^{kji}} + A_{\mu\nu} \mapsto kji\Omega\partial_{\mu\nu}\Omega^{kji} = 1/x_{kji}^{\mu\nu}\eta_{\mu\nu}^{kji}\sigma^{kji} + A_{\mu\nu} \\ &= 1/x^{kji} + \rho^{kji}\eta_{\mu\nu}^{kji}x^{\mu\nu}\sigma^{kji} + F_{\mu\nu} = 2\rho^{kji}/(x^{kji} + \rho^{kji})\exp^{kji}\eta_{\mu\nu}^{kji}\sigma^{kji}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_{\mu\nu}\sigma^{\mu\nu}/\sqrt{y_{\mu\nu}^{kji}} + A_{\mu\nu} \mapsto kji\Omega\partial_{\mu\nu}\Omega^{kji} = 1/y_{kji}^{\mu\nu}\eta_{\mu\nu}^{kji}\sigma^{kji} + A_{\mu\nu} \\ &= 1/y^{kji} + \rho^{kji}\eta_{\mu\nu}^{kji}y^{\mu\nu}\sigma^{kji} + F_{\mu\nu} = 2\rho^{kji}/(y^{kji} + \rho^{kji})\exp^{kji}\eta_{\mu\nu}^{kji}\sigma^{kji}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_{\mu\nu}\sigma^{\mu\nu}/\sqrt{z_{\mu\nu}^{kji}} + A_{\mu\nu} \mapsto kji\Omega\partial_{\mu\nu}\Omega^{kji} = 1/z_{kji}^{\mu\nu}\eta_{\mu\nu}^{kji}\sigma^{kji} + A_{\mu\nu} \\ &= 1/z^{kji} + \rho^{kji}\eta_{\mu\nu}^{kji}z^{\mu\nu}\sigma^{kji} + F_{\mu\nu} = 2\rho^{kji}/(z^{kji} + \rho^{kji})\exp^{kji}\eta_{\mu\nu}^{kji}\sigma^{kji}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_{\mu\nu}\sigma^{\mu\nu}/\sqrt{n_{\mu\nu}^{kji}} + A_{\mu\nu} \mapsto kji\Omega\partial_{\mu\nu}\Omega^{kji} = 1/n_{kji}^{\mu\nu}\eta_{\mu\nu}^{kji}\sigma^{kji} + A_{\mu\nu} \\ &= 1/n^{kji} + \rho^{kji}\eta_{\mu\nu}^{kji}n^{\mu\nu}\sigma^{kji} + F_{\mu\nu} = 2\rho^{kji}/(n^{kji} + \rho^{kji})\exp^{kji}\eta_{\mu\nu}^{kji}\sigma^{kji}\end{aligned}$$

$$\eta_{\mu\nu k}^1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu k}^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu k}^3 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{\mu\nu k}^\infty = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(x) = x_{v\mu}\sigma^{v\mu}/\sqrt{x_{v\mu}^{kji}} + A_{v\mu} \mapsto kji\Omega\partial_{v\mu}\Omega^{kji} = 1/x_{kji}^{v\mu}\eta_{v\mu}^{kji}\sigma^{kji} + A_{v\mu} \\ &= 1/x^{kji} + \rho^{kji}\eta_{v\mu}^{kji}x^{v\mu}\sigma^{kji} + F_{v\mu} = 2\rho^{kji}/(x^{kji} + \rho^{kji})\exp^{kji}\eta_{v\mu}^{kji}\sigma^{kji}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(y) = y_{v\mu}\sigma^{v\mu}/\sqrt{y_{v\mu}^{kji}} + A_{v\mu} \mapsto kji\Omega\partial_{v\mu}\Omega^{kji} = 1/y_{kji}^{v\mu}\eta_{v\mu}^{kji}\sigma^{kji} + A_{v\mu} \\ &= 1/y^{kji} + \rho^{kji}\eta_{v\mu}^{kji}y^{v\mu}\sigma^{kji} + F_{v\mu} = 2\rho^{kji}/(y^{kji} + \rho^{kji})\exp^{kji}\eta_{v\mu}^{kji}\sigma^{kji}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(z) = z_{v\mu}\sigma^{v\mu}/\sqrt{z_{v\mu}^{kji}} + A_{v\mu} \mapsto kji\Omega\partial_{v\mu}\Omega^{kji} = 1/z_{kji}^{v\mu}\eta_{v\mu}^{kji}\sigma^{kji} + A_{v\mu} \\ &= 1/z^{kji} + \rho^{kji}\eta_{v\mu}^{kji}z^{v\mu}\sigma^{kji} + F_{v\mu} = 2\rho^{kji}/(z^{kji} + \rho^{kji})\exp^{kji}\eta_{v\mu}^{kji}\sigma^{kji}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\Omega(n) = n_{v\mu}\sigma^{v\mu}/\sqrt{n_{v\mu}^{kji}} + A_{v\mu} \mapsto kji\Omega\partial_{v\mu}\Omega^{kji} = 1/n_{kji}^{v\mu}\eta_{v\mu}^{kji}\sigma^{kji} + A_{v\mu} \\ &= 1/n^{kji} + \rho^{kji}\eta_{v\mu}^{kji}n^{v\mu}\sigma^{kji} + F_{v\mu} = 2\rho^{kji}/(n^{kji} + \rho^{kji})\exp^{kji}\eta_{v\mu}^{kji}\sigma^{kji}\end{aligned}$$

$$\eta_{v\mu k}^1 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{v\mu k}^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \quad \eta_{v\mu k}^3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \quad \eta_{v\mu k}^\infty = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle V(x) = \lambda(x_{\mu\nu\rho\sigma}^{ijk} + a_{\mu\nu\rho\sigma}^{ijk})\exp_{\mu\nu\rho\sigma}^{ijk} \frac{\partial\theta}{\xi\mathbb{R}^4} = \delta\varphi\phi\phi\phi d\omega(\tau) \\ &= a \tanh/\cosh + \sinh(w/\pi^2(\tau - \tau_{\mu\nu\rho\sigma}^{ijk}))\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle V(x) = \lambda(x_{ijk}^{\mu\nu\rho\sigma} + a_{ijk}^{\mu\nu\rho\sigma})\exp_{ijk}^{\mu\nu\rho\sigma} \frac{\partial\theta}{\xi\mathbb{R}^4} = \delta\varphi\phi\phi\phi d\omega(\tau) \\ &= a \tanh/\cosh + \sinh(w/\pi^2(\tau - \tau_{ijk}^{\mu\nu\rho\sigma}))\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle V(x) = \lambda(x_{\mu\nu\rho\sigma}^{ijk} + a_{ijk}^{\mu\nu\rho\sigma})\exp_{\mu\nu\rho\sigma}^{ijk} \frac{\partial\theta}{\xi\mathbb{R}^4} = \delta\varphi\phi\phi\phi d\omega(\tau) \\ &= a \tanh/\cosh + \sinh(w/\pi^2(\tau - \tau_{\mu\nu\rho\sigma}^{ijk}))\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle V(x) = \lambda(x_{ijk}^{\mu\nu\rho\sigma} + a_{\mu\nu\rho\sigma}^{ijk})\exp_{ijk}^{\mu\nu\rho\sigma} \frac{\partial\theta}{\xi\mathbb{R}^4} = \delta\varphi\phi\phi\phi d\omega(\tau) \\ &= a \tanh/\cosh + \sinh(w/\pi^2(\tau - \tau_{ijk}^{\mu\nu\rho\sigma}))\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle a|e^{-HT}| -a\rangle = N \oint_{x(0)=-a}^{x(T)=+a} Dx(\tau) e^{S_E[x(\tau)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle a|e^{+HT}| +a\rangle \\ &= N \oint_{x(T)=+a}^{x(0)=-a} Dx(\tau) e^{S_E[x(\tau)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle b|e^{-HT}| -b\rangle = N \oint_{x(0)=-b}^{x(T)=+b} Dx(\tau) e^{S_E[x(\tau)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle b|e^{+HT}| +b\rangle \\ &= N \oint_{x(T)=+b}^{x(0)=-b} Dx(\tau) e^{S_E[x(\tau)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle c|e^{-HT}| -c\rangle = N \oint_{x(0)=-c}^{x(T)=+c} Dx(\tau) e^{S_E[x(\tau)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle c|e^{+HT}| +c\rangle \\ &= N \oint_{x(T)=+c}^{x(0)=-c} Dx(\tau) e^{S_E[x(\tau)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle a|e^{-HT}| -a\rangle = N \oint_{y(0)=-a}^{y(T)=+a} Dy(\tau) e^{S_E[y(\tau)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle a|e^{+HT}| +a\rangle \\ &= N \oint_{y(T)=+a}^{y(0)=-a} Dy(\tau) e^{S_E[y(\tau)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle b|e^{-HT}| -b\rangle = N \oint_{y(0)=-b}^{y(T)=+b} Dy(\tau) e^{S_E[y(\tau)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle b|e^{+HT}| +b\rangle \\ &= N \oint_{y(T)=+b}^{y(0)=-b} Dy(\tau) e^{S_E[y(\tau)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle c|e^{-HT}| -c\rangle = N \oint_{y(0)=-c}^{y(T)=+c} Dy(\tau) e^{S_E[y(\tau)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle c|e^{+HT}| +c\rangle \\ &= N \oint_{y(T)=+c}^{y(0)=-c} Dy(\tau) e^{S_E[y(\tau)]}\end{aligned}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle a|e^{-HT}|-a\rangle = N \oint_{z(0)=-a}^{z(T)=+a} Dz(\tau) e^{S_E[z(\tau)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle a|e^{+HT}|+a\rangle \\ &= N \oint_{z(T)=+a}^{z(0)=-a} Dz(\tau) e^{S_E[z(\tau)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle b|e^{-HT}|-b\rangle = N \oint_{z(0)=-b}^{z(T)=+b} Dz(\tau) e^{S_E[z(\tau)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle b|e^{+HT}|+b\rangle \\ &= N \oint_{z(T)=+b}^{z(0)=-b} Dz(\tau) e^{S_E[z(\tau)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle c|e^{-HT}|-c\rangle = N \oint_{z(0)=-c}^{z(T)=+c} Dz(\tau) e^{S_E[z(\tau)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle c|e^{+HT}|+c\rangle \\ &= N \oint_{z(T)=+c}^{z(0)=-c} Dz(\tau) e^{S_E[z(\tau)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle a|e^{-HT}|-a\rangle = N \oint_{x(0)=-a}^{x(T)=+a} Dx(\mu\nu\rho\sigma) e^{S_E[x(\mu\nu\rho\sigma)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle a|e^{+HT}|+a\rangle \\ &= N \oint_{x(T)=+a}^{x(0)=-a} Dx(\mu\nu\rho\sigma) e^{S_E[x(\mu\nu\rho\sigma)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle b|e^{-HT}|-b\rangle = N \oint_{x(0)=-b}^{x(T)=+b} Dx(\mu\nu\rho\sigma) e^{S_E[x(\mu\nu\rho\sigma)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle b|e^{+HT}|+b\rangle \\ &= N \oint_{x(T)=+b}^{x(0)=-b} Dx(\mu\nu\rho\sigma) e^{S_E[x(\mu\nu\rho\sigma)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle c|e^{-HT}|-c\rangle = N \oint_{x(0)=-c}^{x(T)=+c} Dx(\mu\nu\rho\sigma) e^{S_E[x(\mu\nu\rho\sigma)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle c|e^{+HT}|+c\rangle \\ &= N \oint_{x(T)=+c}^{x(0)=-c} Dx(\mu\nu\rho\sigma) e^{S_E[x(\mu\nu\rho\sigma)]}\end{aligned}$$



$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \langle a | e^{-HT} | -a \rangle = N \oint_{y(0)=-a}^{y(T)=+a} Dy(\mu\nu\rho\sigma) e^{S_E[y(\mu\nu\rho\sigma)]} + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \langle a | e^{+HT}$$

$$| +a \rangle = N \oint_{y(T)=+a}^{y(0)=-a} Dy(\mu\nu\rho\sigma) e^{S_E[y(\mu\nu\rho\sigma)]}$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \langle b | e^{-HT} | -b \rangle = N \oint_{y(0)=-b}^{y(T)=+b} Dy(\mu\nu\rho\sigma) e^{S_E[y(\mu\nu\rho\sigma)]} + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \langle b | e^{+HT}$$

$$| +b \rangle = N \oint_{y(T)=+b}^{y(0)=-b} Dy(\mu\nu\rho\sigma) e^{S_E[y(\mu\nu\rho\sigma)]}$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \langle c | e^{-HT} | -c \rangle = N \oint_{y(0)=-c}^{y(T)=+c} Dy(\mu\nu\rho\sigma) e^{S_E[y(\mu\nu\rho\sigma)]} + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \langle c | e^{+HT} | +c \rangle$$

$$= N \oint_{y(T)=+c}^{y(0)=-c} Dy(\mu\nu\rho\sigma) e^{S_E[y(\mu\nu\rho\sigma)]}$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \langle a | e^{-HT} | -a \rangle = N \oint_{z(0)=-a}^{z(T)=+a} Dz(\mu\nu\rho\sigma) e^{S_E[z(\mu\nu\rho\sigma)]} + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \langle a | e^{+HT}$$

$$| +a \rangle = N \oint_{z(T)=+a}^{z(0)=-a} Dz(\mu\nu\rho\sigma) e^{S_E[z(\mu\nu\rho\sigma)]}$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \langle b | e^{-HT} | -b \rangle = N \oint_{z(0)=-b}^{z(T)=+b} Dz(\mu\nu\rho\sigma) e^{S_E[z(\mu\nu\rho\sigma)]} + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \langle b | e^{+HT} | +b \rangle$$

$$= N \oint_{z(T)=+b}^{z(0)=-b} Dz(\mu\nu\rho\sigma) e^{S_E[z(\mu\nu\rho\sigma)]}$$

$$\hat{H} | \psi \rangle = E_\psi | \psi \rangle \langle c | e^{-HT} | -c \rangle = N \oint_{z(0)=-c}^{z(T)=+c} Dz(\mu\nu\rho\sigma) e^{S_E[z(\mu\nu\rho\sigma)]} + \hat{H} | \psi \rangle = E_\psi | \psi \rangle \langle c | e^{+HT} | +c \rangle$$

$$= N \oint_{z(T)=+c}^{z(0)=-c} Dz(\mu\nu\rho\sigma) e^{S_E[z(\mu\nu\rho\sigma)]}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle a|e^{-HT}|-a\rangle = N \oint_{x(0)=-a}^{x(T)=+a} Dx(ijk) e^{S_E[x(ijk)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle a|e^{+HT}|+a\rangle \\ &= N \oint_{x(T)=+a}^{x(0)=-a} Dx(ijk) e^{S_E[x(ijk)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle b|e^{-HT}|-b\rangle = N \oint_{x(0)=-b}^{x(T)=+b} Dx(ijk) e^{S_E[x(ijk)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle b|e^{+HT}|+b\rangle \\ &= N \oint_{x(T)=+b}^{x(0)=-b} Dx(ijk) e^{S_E[x(ijk)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle c|e^{-HT}|-c\rangle = N \oint_{x(0)=-c}^{x(T)=+c} Dx(ijk) e^{S_E[x(ijk)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle c|e^{+HT}|+c\rangle \\ &= N \oint_{x(T)=+c}^{x(0)=-c} Dx(ijk) e^{S_E[x(ijk)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle a|e^{-HT}|-a\rangle = N \oint_{y(0)=-a}^{y(T)=+a} Dy(ijk) e^{S_E[y(ijk)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle a|e^{+HT}|+a\rangle \\ &= N \oint_{y(T)=+a}^{y(0)=-a} Dy(ijk) e^{S_E[y(ijk)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle b|e^{-HT}|-b\rangle = N \oint_{y(0)=-b}^{y(T)=+b} Dy(ijk) e^{S_E[y(ijk)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle b|e^{+HT}|+b\rangle \\ &= N \oint_{y(T)=+b}^{y(0)=-b} Dy(ijk) e^{S_E[y(ijk)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle c|e^{-HT}|-c\rangle = N \oint_{y(0)=-c}^{y(T)=+c} Dy(ijk) e^{S_E[y(ijk)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle c|e^{+HT}|+c\rangle \\ &= N \oint_{y(T)=+c}^{y(0)=-c} Dy(ijk) e^{S_E[y(ijk)]}\end{aligned}$$



$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle a|e^{-HT}|-a\rangle = N \oint_{z(0)=-a}^{z(T)=+a} Dz(ijk) e^{S_E[z(ijk)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle a|e^{+HT}|+a\rangle \\ &= N \oint_{z(T)=+a}^{z(0)=-a} Dz(ijk) e^{S_E[z(ijk)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle b|e^{-HT}|-b\rangle = N \oint_{z(0)=-b}^{z(T)=+b} Dz(ijk) e^{S_E[z(ijk)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle b|e^{+HT}|+b\rangle \\ &= N \oint_{z(T)=+b}^{z(0)=-b} Dz(ijk) e^{S_E[z(ijk)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle\langle c|e^{-HT}|-c\rangle = N \oint_{z(0)=-c}^{z(T)=+c} Dz(ijk) e^{S_E[z(ijk)]} + \hat{H}|\psi\rangle = E_\psi|\psi\rangle\langle c|e^{+HT}|+c\rangle \\ &= N \oint_{z(T)=+c}^{z(0)=-c} Dz(ijk) e^{S_E[z(ijk)]}\end{aligned}$$

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle x(\tau) = \kappa(\tau) + \delta x(\tau) = \hat{H}|\psi\rangle = E_\psi|\psi\rangle S_E[x(\tau)] \\ &= S_{instanton} \\ &+ \oint_{ijk}^{\mu\nu\rho\sigma} d(\tau)\delta(\tau)\partial(\tau)\Omega(\tau)\varphi(\tau)\phi(\tau)\xi(\tau)\lambda(\tau)\frac{\omega}{p}\cdot\mathbb{R}^4/x^n \oint_{x(0)=-a}^{x(T)=+a} D\infty(\tau) e^{-S_E[x(\tau)]} \\ &= e^{-S_{instanton}} \oint_{\delta x(0)=0}^{\delta x(T)=0} D\delta(\tau) e^{d(\tau)\delta(\tau)\partial(\tau)\Omega(\tau)\varphi(\tau)\phi(\tau)\xi(\tau)\lambda(\tau)\frac{\omega}{p}\cdot\frac{\mathbb{R}^4}{x^n}} \\ &\approx e^{-S_{instanton}}/d\omega_{\lambda\nu/\lambda\sigma}^{\lambda\mu/\lambda\rho}\Delta\nabla\Omega\eta\varphi\phi\end{aligned}$$

r. Análisis de Campos Yang – Mills (Teorización Final).

$$\begin{aligned}\hat{H}|\psi\rangle &= E_\psi|\psi\rangle S_{YM} + S_{gf\mu\nu} \oint_{ijk=m}^{ijk=n} \lambda 1/g^{\mu\nu\rho\sigma} \oint_v^\mu d_\varphi^\omega \theta \Omega \operatorname{tr} \left[\frac{1}{2} \dot{F}_{\mu\nu} \dot{F}^{\mu\nu} + n \dot{F}^{\mu\nu} \dot{D}_\mu \lambda \partial / \Delta \nabla \delta A_\nu \right. \\ &+ \dot{D}^\mu \lambda \partial / \Delta \nabla \delta A^\nu \dot{D}_\mu \lambda \partial / \Delta \nabla \delta A_\nu - \dot{D}^\mu \lambda \partial / \Delta \nabla \delta A^\nu \dot{D}_\nu \lambda \partial / \Delta \nabla \delta A_\mu - i \dot{F}^{\mu\nu} \dot{F}_{\mu\nu} [\delta A_\mu, \delta A_\nu] \\ &\left. - 2ijk \dot{D}^\mu \lambda \partial / \Delta \nabla \delta A^\nu [\delta A_\mu, \delta A_\nu] - 1/2 [\delta A^\mu, \delta A^\nu] [\delta A_\mu, \delta A_\nu] \right]\end{aligned}$$



$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle S_{YM} + S_{gf v\mu} \oint_{ijk=m}^{ijk=n} \lambda 1/g^{\mu\nu\rho\sigma} \oint_{\mu}^v d\varphi^\omega \theta \Omega \operatorname{tr} [\frac{1}{2} \dot{F}_{v\mu} \dot{F}^{v\mu} + n \dot{F}^{v\mu} \dot{D}_v \lambda \partial / \Delta \nabla \delta A_\mu \\ + \dot{D}^v \lambda \partial / \Delta \nabla \delta A^\mu \dot{D}_v \lambda \partial / \Delta \nabla \delta A_\mu - \dot{D}^v \lambda \partial / \Delta \nabla \delta A^\mu \dot{D}_\mu \lambda \partial / \Delta \nabla \delta A_v - i \dot{F}^{v\mu} \dot{F}_{v\mu} [\delta A_v, \delta A_\mu] \\ - 2ijk \dot{D}^v \lambda \partial / \Delta \nabla \delta A^\mu [\delta A_v, \delta A_\mu] - 1/2 [\delta A^v, \delta A^\mu] [\delta A_v, \delta A_\mu]]$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle \oint_v^\mu d^{\mu\nu} n \operatorname{tr} \dot{D}^\mu \lambda \partial / \Delta \nabla \delta A^v \dot{D}_v \lambda \partial / \Delta \nabla \delta A_\mu = - \oint_v^\mu d^{\mu\nu} n \operatorname{tr} \delta A_v \dot{D}^\mu \dot{D}^v A_\mu \\ = - \oint_v^\mu d^{\mu\nu} n \operatorname{tr} \delta A_v ([\dot{D}^\mu \dot{D}^v] + \dot{D}^v \dot{D}^\mu) \delta A_\mu \\ = \oint_v^\mu d^{\mu\nu} [(\dot{D}^\mu \delta A_\mu) \exp^{\mu\nu} + ijk \delta A_v [\dot{F}^{\mu\nu}, \delta A_\mu]]$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle \oint_\mu^v d^{v\mu} n \operatorname{tr} \dot{D}^v \lambda \partial / \Delta \nabla \delta A^\mu \dot{D}_\mu \lambda \partial / \Delta \nabla \delta A_v = - \oint_\mu^v d^{v\mu} n \operatorname{tr} \delta A_\mu \dot{D}^v \dot{D}^\mu A_v \\ = - \oint_\mu^v d^{v\mu} n \operatorname{tr} \delta A_\mu ([\dot{D}^v \dot{D}^\mu] + \dot{D}^\mu \dot{D}^v) \delta A_v \\ = \oint_\mu^v d^{v\mu} [(\dot{D}^v \delta A_v) \exp^{v\mu} + ijk \delta A_\mu [\dot{F}^{v\mu}, \delta A_v]]$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle \oint_v^\mu D\omega \delta(G(\check{A}^\omega, \delta A^\omega)) \det (\partial G(\check{A}^\omega, \delta A^\omega) / \partial \omega) \\ = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \Sigma \int \int \int \int \int \hbar \Phi \mathbb{H} \check{\mathcal{K}} \mathcal{K} \mathbb{H} \psi \mathcal{K} \mathcal{K} \zeta \pi m c \mathbb{R}^4$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle \oint_\mu^v D\omega \delta(G(\check{A}^\omega, \delta A^\omega)) \det (\partial G(\check{A}^\omega, \delta A^\omega) / \partial \omega) \\ = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \Sigma \int \int \int \int \int \hbar \Phi \mathbb{H} \check{\mathcal{K}} \mathcal{K} \mathbb{H} \psi \mathcal{K} \mathcal{K} \zeta \pi m c \mathbb{R}^4$$

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle \det(\partial G(\check{A}^\omega, \delta A^\omega) / \partial \omega) \\ = \oint_v^\mu Dc Dc^\dagger \exp(-1/g^{\mu\nu} \oint_v^\mu d^{\mu\nu} \operatorname{tr} [-e^\dagger (\dot{D}^{\mu\nu\rho\sigma} c + ijk c^\dagger [(\dot{D}^{\mu\nu\rho\sigma} \delta A_{\mu\nu\rho\sigma}, c)])$$



$$\begin{aligned}
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \det(\partial G(\check{A}^\omega, \delta A^\omega) / \partial \omega) \\
&= \oint_{\mu}^v Dc Dc^\dagger \exp(-1/g^{v\mu} \oint_{\mu}^v d^{v\mu} \text{tr}[-e^\dagger (\dot{D}^{v\mu\rho\sigma} c + ijk c^\dagger [(\dot{D}^{v\mu\rho\sigma} \delta A_{v\mu\rho\sigma}, c)])]) \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle 1/g^{\mu\nu} \oint_{\nu}^{\mu} d^{\mu\nu} n \text{tr}[1/2 \dot{F}_{\mu\nu} \dot{F}^{\mu\nu} + n \dot{F}^{\mu\nu} \dot{D}_\mu \delta A_\nu + \dot{D}^\mu \delta A^\nu \dot{D}_\mu \delta A_\nu - nijk \dot{F}^{\mu\nu} [\delta A_\mu, \delta A_\nu] \\
&\quad + \dot{D}_\mu c^\dagger \dot{D}^\mu c - nijk \dot{D}^\mu \delta A^\nu [\delta A_\mu, \delta A_\nu] - 1/2 [\delta A^\mu, \delta A^\nu] [\delta A_\mu, \delta A_\nu] + ic^\dagger [\dot{D}^\mu \delta A_\mu, c]] \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle 1/g^{v\mu} \oint_{\mu}^v d^{v\mu} n \text{tr}[1/2 \dot{F}_{v\mu} \dot{F}^{v\mu} + n \dot{F}^{v\mu} \dot{D}_\nu \delta A_\mu + \dot{D}^\nu \delta A^\mu \dot{D}_\nu \delta A_\mu - nijk \dot{F}^{v\mu} [\delta A_\nu, \delta A_\mu] \\
&\quad + \dot{D}_\nu c^\dagger \dot{D}^\nu c - nijk \dot{D}^\nu \delta A^\mu [\delta A_\nu, \delta A_\mu] - 1/2 [\delta A^\nu, \delta A^\mu] [\delta A_\nu, \delta A_\mu] + ic^\dagger [\dot{D}^\nu \delta A_\nu, c]] \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle e^{-S_{eff}[\check{A}, \delta, A, c]} = \oint_{\nu}^{\mu} D\delta A Dc Dc^\dagger e^{-S[\check{A}, \delta, A, c]} = e^{-S_{eff}[\check{A}]} \\
&= \det^m \Delta_{gauge} \nabla_{gauge} \det^n \Delta_{ghost} \nabla_{ghost} e^{\frac{1}{2g^{\check{A}, \delta, A, c} \oint_{\nu}^{\mu} d_{\nu}^{\mu} n \text{tr} \dot{F}_{\mu\nu} \dot{F}^{\mu\nu}}} = \Delta_{gauge}^{\mu\nu} \nabla_{gauge}^{\mu\nu} \\
&= \dot{D}^{\mu\nu} \delta^{\mu\nu} + 2ijk [\dot{F}^{\mu\nu} \Delta_{ghost}^{\mu\nu} \nabla - c^\dagger = S_{eff}[\check{A}, \delta, A, c] \\
&= 1/2 g_{\nu}^{\mu} \oint_{\nu}^{\mu} d_{\nu}^{\mu} n \text{tr} \dot{F}_{\mu\nu} \dot{F}^{\mu\nu} + 1/2 \text{Trlog} \Delta_{gauge}^{\mu\nu} \nabla - \text{Trlog} \Delta_{ghost}^{\mu\nu} \nabla \\
&= \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{H} \check{\mathbb{X}} \check{\mathbb{X}} \mathbb{H} \psi \check{\mathbb{X}} \mathbb{X} \zeta \pi m c \mathbb{R}^4 \\
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle e^{-S_{eff}[\check{A}, \delta, A, c]} = \oint_{\mu}^v D\delta A Dc Dc^\dagger e^{-S[\check{A}, \delta, A, c]} = e^{-S_{eff}[\check{A}]} \\
&= \det^m \Delta_{gauge} \nabla_{gauge} \det^n \Delta_{ghost} \nabla_{ghost} e^{\frac{1}{2g^{\check{A}, \delta, A, c} \oint_{\mu}^v d_{\mu}^v n \text{tr} \dot{F}_{v\mu} \dot{F}^{v\mu}}} = \Delta_{gauge}^{v\mu} \nabla_{gauge}^{v\mu} \\
&= \dot{D}^{v\mu} \delta^{v\mu} + 2ijk [\dot{F}^{v\mu} \Delta_{ghost}^{v\mu} \nabla - c^\dagger = S_{eff}[\check{A}, \delta, A, c] \\
&= 1/2 g_{\mu}^v \oint_{\mu}^v d_{\mu}^v n \text{tr} \dot{F}_{v\mu} \dot{F}^{v\mu} + 1/2 \text{Trlog} \Delta_{gauge}^{v\mu} \nabla - \text{Trlog} \Delta_{ghost}^{v\mu} \nabla \\
&= \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \hbar \phi \mathbb{H} \check{\mathbb{X}} \check{\mathbb{X}} \mathbb{H} \psi \check{\mathbb{X}} \mathbb{X} \zeta \pi m c \mathbb{R}^4
\end{aligned}$$



$$\begin{aligned} \hat{H} \mid \psi \rangle &= E_\psi \mid \psi \rangle \Delta_{ghost\ ghost}^{v\mu} \nabla = -\partial_v^v + \Delta \nabla_v - \Delta \nabla_\mu, \Delta \nabla_v = ijk \partial^{v\mu} \check{\Delta}_{v\mu} + ijk \check{\Delta}_{v\mu} \partial^{v\mu}, \Delta \nabla_\mu = [\check{\Delta}^{v\mu}, [\check{\Delta}_{v\mu}]] \\ &= Tr log \Delta_{ghost\ ghost}^{v\mu} \nabla = Tr log (-\partial^{v\mu} + \Delta \nabla_v - \Delta \nabla_\mu) \\ &= Tr log (-\partial^{v\mu}) + Tr log (1 + (-\partial^{v\mu}) exp^{-n} (\Delta \nabla_v - \Delta \nabla_\mu)) \\ &= Tr \log (-\partial^{v\mu}) + Tr ((-\partial^{v\mu}) exp^{-n} (\Delta \nabla_v - \Delta \nabla_\mu)) \frac{1}{2} Tr ((-\partial^{v\mu}) exp^{-n} (\Delta \nabla_v \\ &\quad - \Delta \nabla_\mu)) exp^\infty \dots \omega \delta \lambda \varphi \theta \Omega \eta \xi \phi \prod_{\infty}^{\infty} \partial \pi_\infty^\infty = \xi_{\lambda \Omega \psi}^{\sigma \zeta \zeta} \Sigma \int \int \int \int \int \hbar \phi \mathbb{H} \check{\mathbb{X}} \mathbb{J} \mathbb{K} \mathbb{L} \mathbb{M} \mathbb{N} \mathbb{O} \mathbb{P} \mathbb{Q} \mathbb{R} \mathbb{S} \pi m c \mathbb{R}^4 \end{aligned}$$



$$\begin{aligned}
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle S_{quad} = 1/2 g_v^\mu \oint_\mu^v d_v^\mu n \operatorname{tr} (\partial_v \check{A}_\mu \partial^v \check{A}^\mu - \partial_v \check{A}^\mu \partial_\mu \check{A}^v) \\
&= 1/2 g^{v\mu} \oint_\mu^v d^{v\mu} k / (n\pi^{v\mu}) \operatorname{tr} [\check{A}_v(k) \check{A}_\mu(-k)] (k^v k^\mu - k_v k_\mu + k_\mu^v \delta_\mu^v) \\
&= \operatorname{Tr} \log \Delta_{ghost\ ghost}^{v\mu} \nabla \\
&= C(adj)/8\pi^2 (16\pi^2) \oint_\mu^v d_\mu^v k_\rho^\sigma \delta\Omega \lambda^\dagger / 8\pi^2 \operatorname{tr} [\check{A}_v(k) \check{A}_\mu(-k)] (k^v k^\mu - k_v k_\mu + k_\mu^v \delta_\mu^v) \\
&= \log \Lambda_{UV}^{v\mu} / k_\rho^\sigma \lambda \Omega \phi \xi \delta \varphi \eta c^\dagger \theta
\end{aligned}$$

$$\begin{aligned}
\hat{H} | \psi \rangle &= E_\psi | \psi \rangle \Delta \nabla_{gauge}^{\mu\nu} = \Delta \nabla_{ghost}^{\mu\nu} \partial \delta_{v\rho}^{\mu\sigma} + nijk [\dot{F}^{\mu\nu}, \infty] = \operatorname{Tr} \log \Delta \nabla_{gauge}^{\mu\nu} \\
&= 16 \operatorname{Tr} \log \Delta \nabla_{ghost}^{\mu\nu} + \dot{F}_{\mu\nu} \text{terms} \\
&= -1/2 (2ijk) \exp^{\mu\nu} \operatorname{Tr} ((-\partial^{\mu\nu}) \exp^{-\mu\nu} [\dot{F}_{\mu\nu}, [(-\partial^{\mu\nu}) \exp^{-\mu\nu} \dot{F}^{\mu\nu}, \infty]]) \\
&= 1/2 \oint_v^\mu d_\mu^\mu k_\rho^\sigma / (16\pi^2) \operatorname{tr}_{adj} [\check{A}_\mu(k) \check{A}_v(-k)] \oint_v^\mu d_\mu^\mu p_\rho^\sigma / (16\pi^2) - 16 (k^\rho \delta^{\mu\sigma} \\
&\quad - k^\sigma \delta^{\mu\rho}) (k_\sigma \delta_\rho^v - k_\rho \delta_\sigma^v) / \rho^{\mu\nu} \sigma^{\mu\nu} (\rho_v^\mu + k_v^\mu) = \dot{F}_{\mu\nu} \text{terms} \\
&= 16C(adj)/(32\pi^2) \oint_v^\mu \frac{d^{\mu\nu} d_{\mu\nu} k_v^\mu}{16\pi^2} \operatorname{tr} [\check{A}_\mu(k) \check{A}_v(-k)] (k^\mu k^v - k_\mu k_v \\
&\quad + k_v^\mu \delta_v^\mu) \log \Lambda_{UV}^{\mu\nu} / k_\sigma^\rho \lambda \Omega \phi \xi \delta \varphi \eta c^\dagger \theta = 1/2 \operatorname{Tr} \log \Delta \nabla_{gauge}^{\mu\nu} \\
&= 1/2 [16/4 - 8] C(adj)/16\pi^2 \oint_v^\mu d_\mu^\mu k_\rho^\sigma \delta\Omega \lambda^\dagger / 8\pi^2 \operatorname{tr} [\check{A}_\mu(k) \check{A}_v(-k)] (k^\mu k^v - k_\mu k_v \\
&\quad + k_v^\mu \delta_v^\mu) = \log \Lambda_{UV}^{\mu\nu} / k_\rho^\sigma \lambda \Omega \phi \xi \delta \varphi \eta c^\dagger \theta
\end{aligned}$$



$$\begin{aligned}
\hat{H}|\psi\rangle &= E_\psi|\psi\rangle \Delta \nabla_{gauge}^{v\mu} = \Delta \nabla_{ghost}^{v\mu} \partial \delta_{\mu\rho}^{v\sigma} + nijk[\dot{F}^{v\mu}, \infty] = Tr \log \Delta \nabla_{gauge}^{v\mu} \\
&= 16 Tr \log \Delta \nabla_{ghost}^{v\mu} + \dot{F}_{v\mu} terms \\
&= -1/2(2ijk) exp^{v\mu} Tr((- \partial^{v\mu}) exp^{-v\mu} [\dot{F}_{v\mu}, [(- \partial^{v\mu}) exp^{-v\mu} \dot{F}^{v\mu}, \infty]]) \\
&= 1/2 \oint_{\mu}^v d_{\mu}^v k_{\sigma}^{\rho} / (16\pi^2) tr_{adj}[\check{A}_v(k) \check{A}_{\mu}(-k)] \oint_{\mu}^v d_{\mu}^v p_{\sigma}^{\rho} / (16\pi^2) - 16(k^{\sigma} \delta^{v\rho} \\
&\quad - k^{\rho} \delta^{v\sigma})(k_{\rho} \delta_{\sigma}^{\mu} - k_{\sigma} \delta_{\rho}^{\mu}) / \rho^{v\mu} \sigma^{v\mu} (\rho_{\mu}^v + k_{\mu}^v) = \dot{F}_{v\mu} terms \\
&= 16C(adj)/(32\pi^2) \oint_{\mu}^v \frac{d^{v\mu} d_{v\mu} k_{\mu}^v}{16\pi^2} tr[\check{A}_v(k) \check{A}_{\mu}(-k)] (k^v k^{\mu} - k_v k_{\mu} \\
&\quad + k_{\mu}^v \delta_{\mu}^v) \log \Lambda_{vU}^{v\mu} / k_{\rho}^{\sigma} \lambda \Omega \phi \xi \delta \phi \eta c^{\dagger} \theta = 1/2 Tr \log \Delta \nabla_{gauge}^{v\mu} \\
&= 1/2[16/4 - 8] C(adj)/16\pi^2 \oint_{\mu}^v d_{\mu}^v k_{\sigma}^{\rho} \delta \Omega \lambda^{\dagger} / 8\pi^2 tr[\check{A}_v(k) \check{A}_{\mu}(-k)] (k^v k^{\mu} - k_v k_{\mu} \\
&\quad + k_{\mu}^v \delta_{\mu}^v) = \log \Lambda_{vU}^{v\mu} / k_{\sigma}^{\rho} \lambda \Omega \phi \xi \delta \phi \eta c^{\dagger} \theta
\end{aligned}$$

CONCLUSIONES

En mérito al análisis de campo antes descrito – marco praxeológico (campos de gauge y campos fantasma), bajo el marco metodológico de las teorías de Yang-Mills y bajo un esquema estrictamente hamiltoniano, sin perjuicio de las demás variables conjugadas (verbigracia, sistemas lagrangiano y algebra de Lie, etc), que conforman el sistema de campos de Yang-Mills, queda demostrado: **(i)** que, las excitaciones más bajas de una teoría pura de Yang-Mills (es decir, sin campos de materia) tienen una brecha de masa finita con respecto al estado de vacío; **(ii)** que, la propiedad de confinamiento en tratándose de física de partículas; y, **(iii)** que, para un hamiltoniano cuántico relativo a un campo de Yang-Mills no abeliano, en efecto existe un valor positivo mínimo de energía, calculado a través de la siguiente constante universal²:

$$\mu := \inf \text{Spec}(\hat{H}) \setminus 0 > 0 = \xi_{\lambda\Omega\psi}^{\sigma\zeta\zeta} \sum \int \int \int \int \int \hbar \phi \mathbb{H} \check{\mathcal{J}} \mathcal{K} \mathcal{H} \psi \check{\mathcal{K}} \mathcal{X} \pi m c \mathbb{R}^4$$

En consecuencia, este trabajo, demuestra que la teoría gauge no abeliana de Yang – Mills, describe otras fuerzas en la naturaleza, especialmente la fuerza débil (responsable, entre otras cosas, de ciertas formas de radiactividad) y la fuerza fuerte o nuclear (responsable, entre otras cosas, de la unión de protones y

² C. N. Yang & R. L. Mills, *Physical Review*, 96, 191 (1954).



neutrones en núcleos), sin perder las premisas esenciales de la teoría de campos de Yang – Mills, esto es, por fuera de la teoría electrodébil de Glashow-Salam-Weinberg o la teoría del “campo de Higgs”.

Si bien es cierto, constituyese en una propiedad notable de la teoría cuántica de Yang-Mills, la nominada "*libertad asintótica*", la misma que, permite determinar, que a distancias cortas el campo muestra un comportamiento cuántico muy similar a su comportamiento clásico; sin embargo, a largas distancias, la teoría de Yang – Mills, como queda demostrado, también aplica a largas distancias en el campo.

Finalmente, queda demostrado concluyentemente, que: **(i)** en los campos de Yang – Mills, existe una "brecha de masa", es decir, $\Delta > \text{constante}$, por lo que, cada excitación del vacío tiene energía de al menos Δ ; **(ii)** en los campos de Yang – Mills, existe un confinamiento de quarks, partiendo de la premisa de que, los estados físicos de las partículas, como el protón, el neutrón y el pión, son invariantes; y, **(iii)** en los campos de Yang – Mills, existe una ruptura de simetría quiral, lo que significa que el vacío es potencialmente invariante bajo un cierto subgrupo de simetría completa que actúa sobre los campos de quarks.

REFERENCIAS BIBLIOGRAFICAS

O’Raifeartaigh. L., and Straumann. N. (2000). Gauge theory: Historical origins and some modern developments. *Reviews of Modern Physics*. 72(1), 1-23.

<https://doi.org/10.1103/RevModPhys.72.1>

Tong, D. (2018). Gauge Theory. *Cambridge University*. United Kingdom.

<http://www.damtp.cam.ac.uk/user/tong/gaugetheory.html>

Baggott, J. (2017). La historia del cuanto: una historia en 40 momentos. *Ed. Intervención Cultural*. España.

Wilson. R., and Gray. J. (2000). Mathematical Conversations: Selections from The Mathematical Intelligencer. *Springer Ed.*

Straumann, N. (2000). On Pauli's invention of non-abelian Kaluza-Klein Theory in 1953.

<https://doi.org/10.48550/arXiv.gr-qc/0012054>



Hooft. G., and Veltman. M. (1972). Regularization and renormalization of gauge fields. *Nuclear Physics B*. 44(1). 189-213. [https://doi.org/10.1016/0550-3213\(72\)90279-9](https://doi.org/10.1016/0550-3213(72)90279-9)

Frampton, P. (2008). Gauge Field Theories. Weinheim, Alemania : Wiley-VCH Verlag GmbH & Co. KGaA. 3ra Edición.

Yang. C. N. and Mills, R. L., (1954). Conservation of Isotopic Spin and Isotopic Gauge Invariance. *Physical Review*. 96(1), 191-195. <https://doi.org/10.1103/PhysRev.96.191>



APÉNDICE A.

1. Formalización matemática relativa a gravedad cuántica desde la perspectiva de la teoría cuántica de campos curvos.

1.1. Análisis geométrico en espacios cuánticos curvos.

$$\begin{aligned}
 \mathcal{R}_{\beta\mu\nu}^{\alpha} &= \partial_{\mu}\Upsilon_{\beta\nu}^{\alpha} - \partial_{\nu}\Upsilon_{\mu\beta}^{\alpha} + \Upsilon_{\mu\lambda}^{\alpha}\Upsilon_{\nu\beta}^{\lambda} - \Upsilon_{\nu\lambda}^{\alpha}\Upsilon_{\mu\beta}^{\lambda}, \Upsilon_{\mu\nu}^{\alpha} = \Gamma_{\mu\nu}^{\alpha} + \kappa_{\mu\nu}^{\alpha} + \mathcal{L}_{\mu\nu}^{\alpha}, \Gamma_{\mu\nu}^{\alpha} \\
 &= \frac{1}{2}\mathcal{G}^{\alpha\lambda}(\partial_{\mu}\mathcal{G}_{\lambda\nu} + \partial_{\nu}\mathcal{G}_{\lambda\mu} - \partial_{\lambda}\mathcal{G}_{\mu\nu}), \kappa_{\mu\nu}^{\alpha} = \frac{1}{2}(\mathcal{T}_{\mu\nu}^{\alpha} + \mathcal{T}_{\mu}^{\alpha}{}_{\nu} + \mathcal{T}_{\nu}^{\alpha}{}_{\mu}), \mathcal{L}_{\mu\nu}^{\alpha} \\
 &= \frac{1}{2}(\mathcal{Q}_{\mu\nu}^{\alpha} + \mathcal{Q}_{\mu}^{\alpha}{}_{\nu} + \mathcal{Q}_{\nu}^{\alpha}{}_{\mu}), \mathcal{Q}_{\alpha\mu\nu} = \nabla_{\alpha}\mathcal{G}_{\mu\nu} = \partial_{\alpha}\mathcal{G}_{\mu\nu} - \Upsilon_{\alpha\mu}^{\beta}\mathcal{G}_{\beta\nu} - \Upsilon_{\alpha\nu}^{\beta}\mathcal{G}_{\mu\beta}, \mathcal{P}_{\mu\nu}^{\alpha} \\
 &= -\frac{1}{2}\mathcal{L}_{\mu\nu}^{\alpha} + \frac{1}{4}(\mathcal{Q}^{\alpha} - \tilde{\mathcal{Q}}^{\alpha})\mathcal{G}_{\mu\nu} - \frac{1}{4}\delta^{\alpha}{}_{\nu}(\mathcal{Q}_{\mu}^{\mu} - \tilde{\mathcal{Q}}_{\mu}^{\mu}), \mathcal{Q} = -\mathcal{Q}_{\lambda\mu\nu}\mathcal{P}^{\lambda\mu\nu}
 \end{aligned}$$

1.2. Principio variacional en espacios cuánticos curvos.

$$\begin{aligned}
 \delta &= \int f(\mathcal{Q}, \mathcal{L}_m) \sqrt{-\mathcal{G}} d^4\chi, \delta\mathcal{S} = \int [(f_{\mathcal{Q}}\delta\mathcal{Q} + f_{\mathcal{L}_m}\delta\mathcal{L}_m)\sqrt{-\mathcal{G}} + f\delta\sqrt{-\mathcal{G}}] d^4\chi, \delta\mathcal{Q} \\
 &= 2\mathcal{P}_{\alpha\nu\rho}\nabla^{\alpha}\delta\mathcal{G}^{\nu\rho} - (\mathcal{P}_{\mu\alpha\beta}\mathcal{Q}_{\nu}^{\alpha\beta} - 2\mathcal{Q}^{\alpha\beta}\xi_{\psi}\mathcal{P}_{\alpha\beta\nu})\delta\mathcal{G}^{\mu\nu}, \mathcal{T}_{\mu\nu} = -\frac{2}{\sqrt{-\mathcal{G}}}\frac{\delta(\sqrt{-\mathcal{G}}\mathcal{L}_m)}{\delta\mathcal{G}^{\mu\nu}} \\
 &= \mathcal{G}_{\mu\nu}\mathcal{L}_m - 2\frac{\partial\mathcal{L}_m}{\partial\mathcal{G}^{\mu\nu}}, \delta\sqrt{-\mathcal{G}} = -\frac{1}{2}\sqrt{-\mathcal{G}}\mathcal{G}_{\mu\nu}\delta\mathcal{G}^{\mu\nu}, \delta\mathcal{S} \\
 &= \int \left[(f_{\mathcal{Q}}(2\mathcal{P}_{\alpha\nu\rho}\nabla^{\alpha}\delta\mathcal{G}^{\nu\rho} - (\mathcal{P}_{\mu\alpha\beta}\mathcal{Q}_{\nu}^{\alpha\beta} - 2\mathcal{Q}_{\mu}^{\alpha\beta}\mathcal{P}_{\alpha\beta\nu})\delta\mathcal{G}^{\mu\nu}) \right. \\
 &\quad \left. + \frac{1}{2}f_{\mathcal{L}_m}(\mathcal{G}_{\mu\nu}\mathcal{L}_m - \mathcal{T}_{\mu\nu})\delta\mathcal{G}^{\mu\nu}) - \frac{1}{2}f\mathcal{G}_{\mu\nu}\delta\mathcal{G}^{\mu\nu} \right] \sqrt{-\mathcal{G}} d^4\chi \frac{2}{\sqrt{-\mathcal{G}}}\nabla_{\alpha}(f_{\mathcal{Q}}\sqrt{-\mathcal{G}}\mathcal{P}_{\mu\nu}^{\alpha}) \\
 &\quad + f_{\mathcal{Q}}(\mathcal{P}_{\mu\alpha\beta}\mathcal{Q}_{\nu}^{\alpha\beta} - 2\mathcal{Q}_{\mu}^{\alpha\beta}\mathcal{P}_{\alpha\beta\nu}) + \frac{1}{2}f\mathcal{G}_{\mu\nu} \\
 &= \frac{1}{2}f_{\mathcal{L}_m}(\mathcal{G}_{\mu\nu}\mathcal{L}_m - \mathcal{T}_{\mu\nu})\frac{2}{\sqrt{-\mathcal{G}}}\nabla_{\alpha}(\sqrt{-\mathcal{G}}f_{\mathcal{Q}}\mathcal{P}_{\mu\nu}^{\alpha}) + f_{\mathcal{Q}}\mathcal{G}_{\mu\nu} + \frac{1}{2}f(\mathcal{Q})\mathcal{G}_{\mu\nu} \\
 &= -\mathcal{T}_{\mu\nu}\frac{2}{\sqrt{-\mathcal{G}}}\nabla_{\alpha}(f_{\mathcal{Q}}\sqrt{-\mathcal{G}}\mathcal{P}_{\nu}^{\alpha\mu}) + f_{\mathcal{Q}}\mathcal{P}_{\alpha\beta}^{\mu}\mathcal{Q}_{\nu}^{\alpha\beta} + \frac{1}{2}\delta_{\nu}^{\mu}f = \frac{1}{2}f_{\mathcal{L}_m}(\delta_{\nu}^{\mu}\mathcal{L}_m - \mathcal{T}_{\nu}^{\mu}) \\
 \delta &= \int [f(\mathcal{Q}, \mathcal{L}_m)\sqrt{-\mathcal{G}} + \lambda_{\alpha}^{\beta\gamma}\mathcal{T}_{\beta\gamma}^{\alpha} + \xi_{\alpha}^{\beta\mu\nu}\mathcal{R}_{\beta\mu\nu}^{\alpha}] d^4\chi, \delta(\lambda_{\alpha}^{\beta\gamma}\mathcal{T}_{\beta\gamma}^{\alpha}) = 2\lambda_{\alpha}^{\beta\gamma}\delta\zeta_{\beta\gamma}^{\alpha}, \delta(\xi_{\alpha}^{\beta\mu\nu}\mathcal{R}_{\beta\mu\nu}^{\alpha}) \\
 &= \xi_{\alpha}^{\beta\mu\nu}[\nabla_{\mu}(\delta\zeta_{\nu\beta}^{\alpha}) - \nabla_{\nu}(\delta\zeta_{\mu\beta}^{\alpha})] = 2\zeta_{\alpha}^{\beta\mu\nu}\nabla_{\beta}(\delta\zeta_{\mu\nu}^{\alpha}) \simeq 2(\nabla_{\beta}\xi_{\alpha}^{\nu\beta\mu})\delta\zeta_{\mu\nu}^{\alpha}, \delta\mathcal{S} \\
 &= \int (4\sqrt{-\mathcal{G}}f_{\mathcal{Q}}\mathcal{P}_{\alpha}^{\mu\nu} + \mathcal{H}_{\alpha}^{\mu\nu} + 2\nabla_{\beta}\xi_{\alpha}^{\nu\beta\mu} + 2\lambda_{\alpha}^{\mu\nu}) d^4\chi \delta\zeta_{\mu\nu}^{\alpha}, \mathcal{H}_{\alpha}^{\mu\nu} \\
 &= \sqrt{-\mathcal{G}}f_{\mathcal{L}_m}\frac{\delta\mathcal{L}_m}{\delta\zeta_{\mu\nu}^{\alpha}}, \nabla_{\mu}\nabla_{\nu}(4\sqrt{-\mathcal{G}}f_{\mathcal{Q}}\mathcal{P}_{\alpha}^{\mu\nu} + \mathcal{H}_{\alpha}^{\mu\nu} + 2\nabla_{\beta}\xi_{\alpha}^{\nu\beta\mu} + 2\lambda_{\alpha}^{\mu\nu})
 \end{aligned}$$



1.3. Modelo Klein – Gordon en espacios cuánticos curvos.

$$\begin{aligned}
 \delta\phi &= -\frac{1}{2}\left(\nabla_\alpha\phi\nabla^\alpha\phi + \xi\mathcal{R}\phi^2 + \frac{m_0^2}{2}\phi^2\right)\sqrt{-g}d^4\chi(\Box + m_0^2 + \xi\mathcal{R})\phi f_Q\mathfrak{G}_{\mu\nu} + \frac{1}{2}\mathfrak{g}_{\mu\nu}(f - f_QQ) \\
 &\quad + 2f_QQ(\partial_\alpha Q)\mathcal{P}_{\mu\nu}^\alpha \\
 &= \frac{1}{2}f_{\mathcal{L}_m}(\mathfrak{g}_{\mu\nu}\mathcal{L}_m - \mathcal{T}_{\mu\nu})\int\Delta\Sigma\Psi_\alpha\int\mathfrak{G}_{\mu\nu} + \frac{1}{2}\mathfrak{g}_{\mu\nu}(\Sigma - Q) + \Psi_\alpha\mathcal{P}_{\mu\nu}^\alpha \\
 &= \frac{1}{2}\Delta(\mathfrak{g}_{\mu\nu}\mathcal{L}_m - \mathcal{T}_{\mu\nu})\int_{\substack{\otimes\triangleleft\alpha^* \\ \zeta^+\leftrightarrow\blacksquare\triangle}}\mathcal{R}^\circ\frac{1}{2}\Delta(\mathcal{T} - 4\mathcal{L}_m) + 2(\Sigma - Q) + \partial_\alpha Q(Q^\alpha \\
 &\quad - \tilde{Q}^\alpha)\frac{\partial^2\odot^*}{\partial Q}\log f_Q(\Box + m_{eff}^2)\phi, m_{eff}^2 \\
 &= m_0^2 + \xi\left[\frac{1}{2}\Delta(\mathcal{T} - 4\mathcal{L}_m) + 2(\Sigma - Q) + \partial_\alpha Q(Q^\alpha - \tilde{Q}^\alpha)\frac{\partial^2\odot^*}{\partial Q}\log f_Q\right] \\
 \delta &= \frac{e^2}{\hbar c}\epsilon^{\lambda\mu\nu}\int d^2\chi dt\left[\frac{1}{8\pi}a_\lambda^T\kappa\partial_\mu a_\nu - \frac{A_\lambda^1}{2\varpi}(t_1\oplus t_2)^T\partial_\mu a_\nu - \frac{A_\lambda^2}{2\varpi}(t'_1\oplus t'_2)^T\partial_\mu a_\nu\right] \\
 &\quad A_\mu^1t_1\oplus A_\mu^2t_2 = \kappa a_\mu \Rightarrow \kappa^{-1}(A_\mu^1t_1\oplus A_\mu^2t_2) = a_\mu
 \end{aligned}$$

1.4. Ecuaciones de Balance Energía – Momentum en espacios cuánticos curvos.

$$\begin{aligned}
 \nabla_\mu\omega_\nu^\mu &= \mathfrak{D}_\mu\omega_\nu^\mu - \frac{1}{2}Q_\rho\omega_\nu^\rho - \mathcal{L}_{\mu\nu}^\alpha\omega_\lambda^\mu, \mathfrak{D}_\mu\left[\frac{1}{2}f_{\mathcal{L}_m}(\delta_\nu^\mu\mathcal{L}_m - \mathcal{T}_\nu^\mu)\right] \\
 &= \frac{1}{2}\partial_\nu f + \mathfrak{D}_\mu(f_Q\mathcal{P}_{\alpha\beta}^\mu Q_\nu^{\alpha\beta}) + \mathfrak{D}_\mu\left[\frac{2}{\sqrt{-g}}\nabla_\alpha(f_Q\sqrt{-g}\mathcal{P}_\nu^{\alpha\mu})\right]\frac{1}{2\sqrt{-g}}\nabla_\alpha\nabla_\mu\mathcal{H}_\nu^{\alpha\mu} \\
 &\quad - \frac{1}{2}f_{\mathcal{L}_m}\mathfrak{D}_\mu\mathcal{T}_\nu^\mu \\
 &= \frac{1}{2}f_Q\partial_\nu Q + \nabla_\mu(f_Q\mathcal{P}_{\alpha\beta}^\mu Q_\nu^{\alpha\beta}) + \frac{1}{2}Q_\mu(f_Q\mathcal{P}_{\alpha\beta}^\mu Q_\nu^{\alpha\beta}) + \mathcal{L}_{\mu\nu}^\rho(f_Q\mathcal{P}_{\alpha\beta}^\mu Q_\rho^{\alpha\beta}) \\
 &\quad + \frac{2}{\sqrt{-g}}\mathcal{L}_{\mu\nu}^\rho\nabla_\alpha(\sqrt{-g}f_Q\mathcal{P}_\rho^{\alpha\mu}) + \frac{1}{\sqrt{-g}}Q_\mu\nabla_\alpha(\sqrt{-g}f_Q\mathcal{P}_\nu^{\alpha\mu}), \mathfrak{D}_\mu\mathcal{T}_\nu^\mu \\
 &= \frac{1}{f_{\mathcal{L}_m}\sqrt{-g}}[\nabla_\alpha\nabla_\mu\mathcal{H}_\nu^{\alpha\mu} - 2Q_\mu\nabla_\alpha(f_Q\sqrt{-g}\mathcal{P}_\nu^{\alpha\mu})], \nabla_\mu(4\sqrt{-g}f_Q\mathcal{P}_\alpha^{\mu\nu} + \mathcal{H}_\alpha^{\mu\nu} \\
 &\quad + 2\nabla_\beta\xi_\alpha^{\nu\beta\mu} + 2\lambda_\alpha^{\mu\nu}) = \sqrt{-g}A_\alpha^\nu, \nabla_\nu(\sqrt{-g}A_\alpha^\nu) = \sqrt{-g}\nabla_\nu A_\alpha^\nu + \frac{\sqrt{-g}}{2}Q_\nu A_\alpha^\nu, \mathfrak{D}_\mu\mathcal{T}_\nu^\mu \\
 &= \frac{1}{f_{\mathcal{L}_m}}\left[\frac{2}{\sqrt{-g}}\nabla_\alpha\nabla_\mu\mathcal{H}_\nu^{\alpha\mu} + \nabla_\mu A_\nu^\mu - \nabla_\mu\left(\frac{1}{\sqrt{-g}}\nabla_\alpha\mathcal{H}_\nu^{\alpha\mu}\right)\right], \mathcal{T}_\nu^\mu \\
 &= (p + \rho)\mu_\mu\mu^\nu + p\delta_\nu^\mu + 3\mathcal{H}B_\mu \\
 &\quad \frac{d^2\chi^\mu}{ds^2} + \Gamma_{\alpha\beta}^\mu\mu^\alpha\mu^\beta = \frac{\hbar^{\mu\nu}}{p + \rho}(B_\nu - \mathfrak{D}_\nu\rho) = \mathfrak{F}^\mu
 \end{aligned}$$



1.5. Método Friedmann-Lemaître-Robertson-Walker para espacios cuánticos curvos.

$$ds^2 = -dt^2 + \alpha^2(t)(d\chi^2 + d\gamma^2 + dz^2)3\mathcal{H}^2 \frac{1}{4f_Q} [f - f_{\mathcal{L}_m}(\rho + \mathcal{L}_m)]\mathcal{H} + \frac{3\mathcal{H}^2}{f_Q} + \frac{\dot{f}_Q}{f_Q}\mathcal{H} = \frac{1}{4f_Q} [f + f_{\mathcal{L}_m}(\rho - \mathcal{L}_m)] \frac{d}{dt}(f_Q, \mathcal{H}) = \frac{f_{\mathcal{L}_m}}{4}(p + \dot{\rho})$$

$$q = -1 - \frac{\mathcal{H}}{\mathcal{H}^2} = \frac{1}{4f_Q\mathcal{H}^2} (2Qf_Q + 4\hat{f}_Q\mathcal{H} - f - f_{\mathcal{L}_m}(\rho - \mathcal{L}_m)) 2\hat{\mathcal{H}} + 3\mathcal{H}^2 = \frac{1}{4f_Q} \left[f + f_{\mathcal{L}_m}(\rho + 2p - \mathcal{L}_m) - 2\frac{\dot{f}_Q}{f_Q}\mathcal{H}, 3\mathcal{H}^2 = \rho_{eff} + 2\hat{\mathcal{H}} + 3\mathcal{H}^2 - \rho_{eff} \right]$$

$$\begin{aligned} \overline{\rho_{eff}} &= \frac{1}{4\hat{f}_Q} [f - f_{\mathcal{L}_m}(\rho + \mathcal{L}_m)], \rho_{eff} \\ &= 2\frac{\hat{f}_Q}{f_Q}\mathcal{H} - \frac{1}{4f_Q} [f + f_{\mathcal{L}_m}(\rho + 2p - \mathcal{L}_m)], \hat{\rho}_{eff} + 3\mathcal{H}(\rho_{eff} + p_{eff}), \rho_{eff} \\ &= \frac{1}{4}\Delta[\delta - (\rho + \mathcal{L}_m)], p_{eff} = 2\frac{\hat{f}_Q}{f_Q}\mathcal{H} - \frac{1}{4}\Delta[\delta + (\rho + 2p - \mathcal{L}_m)] \end{aligned}$$

$$\tilde{\rho} + 3\mathcal{H}(p + \dot{\rho}) = \frac{1}{\Delta} \frac{d}{dt} [\Delta(\delta - \mathcal{L}_m)] + 3\mathcal{H} \left\{ 8\frac{\dot{f}_Q}{f_Q} \frac{\mathcal{H}}{\Delta} - \left[\left(1 + \frac{\hat{\Delta}}{\Delta} \rho + p \right) \right] \right\} = \Gamma$$

$$\begin{aligned} q &= \frac{1}{2} + \frac{3p_{eff}}{2\rho_{eff}} \\ &= \frac{1}{2} + 12 \frac{2\dot{f}_Q\mathcal{H} - \left(\frac{1}{4}\right) [f + f_{\mathcal{L}_m}(\rho + 2p - \mathcal{L}_m)]}{f - f_{\mathcal{L}_m}(\rho + \mathcal{L}_m)} \frac{d}{dt} (1+z)\mathcal{H}(z) \frac{d}{dz} q(z) - 1 \\ &\quad + (1+z) \frac{\mathcal{H}'(z)}{\mathcal{H}(z)} \end{aligned}$$

1.6. Solución de Sitter para espacios cuánticos curvos.

$$f(Q) = \mathcal{F}_0 Q + 4\Lambda, 6\mathcal{H}_0^2 = 12\mathcal{F}_0\mathcal{H}_0^2 + \frac{4\Lambda}{8\mathcal{F}_0}, \mathcal{H}_0 = \sqrt{\frac{\Lambda}{6\mathcal{F}_0}}$$



1.7. Modelos cosmológicos aplicables, bajo la óptica de la teoría cuántica de campos curvos.

17.1. Modelo A – Partícula o Antipartícula Masiva.

$$\begin{aligned} 3\mathcal{H}^2 &= -\frac{\beta}{2\alpha} + \frac{\rho}{\alpha} 2\hat{\mathcal{H}} + 3\mathcal{H}^2 - 3\mathcal{H}^2(\gamma - 1) - \frac{\beta\gamma}{2\alpha}, \rho_{eff} = \frac{\rho}{\alpha} - \frac{\beta}{2\alpha}, p_{eff} \\ &= 3\mathcal{H}^2(\gamma - 1) + \frac{\beta\gamma}{2\alpha}\ddot{\rho} \\ &\quad + 3\mathcal{H}''(p + \rho)\mathcal{H}(z)[(12\mathcal{H}_0^2\alpha + \beta)(1 + z)^{3\gamma} - \beta/12\alpha]^{1/2} \end{aligned}$$

17.2. Modelo B – Partícula o Antipartícula Supermasiva.

$$\begin{aligned} 3\mathcal{H}^2 &= -\frac{2}{\alpha}(1 - \gamma^2)\rho^2 - \frac{\beta}{2\alpha} 2\hat{\mathcal{H}} + 3\mathcal{H}^2 - \frac{\beta}{2\alpha} - \frac{(\gamma - 1)(\beta + 12\alpha\mathcal{H}^2)}{4\alpha(\gamma + 1)}, \rho_{eff} \\ &= -\frac{2}{\alpha}(1 - \gamma^2)\rho^2 - \frac{\beta}{2\alpha}, p_{eff} \\ &= \frac{\beta}{2\alpha} + \frac{(\gamma - 1)(\beta + 12\alpha\mathcal{H}^2)}{4\alpha(\gamma + 1)}\hat{\rho} + \frac{6\gamma}{\gamma + 1}\mathcal{H}\rho(\alpha)\rho_0\alpha^{-\frac{3\gamma}{(\gamma+1)}}\tilde{\rho} + 6\gamma\mathcal{H}\rho \\ &= 3\gamma^2/\gamma + 1\mathcal{H}\rho = \Gamma\mathcal{H}(z)[(12\mathcal{H}_0^2\alpha + \beta)(1 + z)^{12\gamma/1+\gamma} - \beta/12\alpha]^{1/2} \end{aligned}$$

$$\begin{aligned} \delta_{eff} &= -\frac{e^2}{4\pi\hbar c}\epsilon^{\lambda\mu\nu}\int d^2\chi dt[A_\lambda^1\partial_\mu A_\nu^1(t_1\oplus t_2)^T\kappa^{-1}(t_1\oplus t_2)^T \\ &\quad + A_\lambda^2\partial_\mu A_\nu^2(t'_1\oplus t'_2)^T\kappa^{-1}(t'_1\oplus t'_2)^T] \end{aligned}$$

$$\begin{aligned} \delta_{eff} &= -\frac{e^2}{4\pi\hbar c}\epsilon^{\lambda\mu\nu}\int d^2\chi dt[A_\lambda^0\partial_\mu A_\nu^0(t_1\oplus t_2)^T\kappa^{-1}(t_1\oplus t_2)^T \\ &\quad + (A_\lambda^0\partial_\mu\delta A_\nu^1 + \delta A_\nu^1\partial_\mu A_\lambda^0)(t_1\oplus t_2)^T\kappa^{-1}(t_1\oplus t_2)^T \\ &\quad + (A_\lambda^0\partial_\mu\delta A_\nu^2 + \delta A_\nu^2\partial_\mu A_\lambda^0)(t'_1\oplus t'_2)^T\kappa^{-1}(t'_1\oplus t'_2)^T + \mathcal{O}(\delta\Lambda)^2] \end{aligned}$$

$$\tilde{v}_1 = (t_1\oplus t_2)^T\kappa^{-1}(t_1\oplus t_2)^T, \tilde{v}_2 = (t'_1\oplus t'_2)^T\kappa^{-1}(t'_1\oplus t'_2)^T$$

$$\delta_\kappa = -\int d^2\chi dt \delta^{\mu\nu}j_\mu^{\mathfrak{S}}a_\nu$$

$$j^\lambda = \frac{e^2}{\hbar c}\epsilon^{\lambda\mu\nu}[-\partial_\mu(A_\nu^1t_1\oplus A_\nu^2t_2) + \kappa\partial_\mu a_\nu] \Rightarrow \kappa^{-1}j^\lambda = \frac{e^2}{\hbar c}\epsilon^{\lambda\mu\nu}[-\kappa^{-1}\partial_\mu(A_\nu^1t_1\oplus A_\nu^2t_2) + \partial_\mu a_\nu]$$

$$Q_1 = -e(t_1\oplus t_2)^T\kappa^{-1}\ell$$

$$Q_2 = -e(t'_1\oplus t'_2)^T\kappa^{-1}\ell$$

$$\theta_\delta = \pi\ell_e^{\mathfrak{S}}\kappa^{-1}\ell \beth 2\pi$$

1.8. Partículas y antipartículas deformantes en espacios curvos. Comportamiento y dinámica cuántica.



$$m_{eff}^2 = m_0^2 + \xi \left[3\mathcal{H}^2(8 - 6\gamma) - \frac{6}{4} \left(\frac{\beta\gamma}{\alpha} \right) \right] = m_0^2 + m_{\mathcal{Q}\mathcal{L}_m}^2, m_{eff}^2 \\ = m_0^2 + \xi \left[\left(\frac{2 - \gamma}{1 + \gamma} \right) \left(12\mathcal{H}^2 + \frac{\beta}{\alpha} - \frac{2\beta}{\alpha} \right) \right] m_0^2 + m_{\mathcal{Q}\mathcal{L}_m}^2 dt$$

2. Sistema alternativo al Modelo Estándar: Comportamiento de Hiperpartículas, Hipopartículas, Hiperantipartículas e Hipoantipartículas en un espacio cuántico curvo.

2.1. Deformación de campo.

$$\psi(\chi_2, \chi_1 \cdots \chi_\eta) = c\psi(\chi_2, \chi_1 \cdots \chi_\eta), \psi(\chi_2, \chi_1 \cdots \chi_\eta) = c\psi(\chi_2, \chi_1 \cdots \chi_\eta) = c^4\psi(\chi_2, \chi_1 \cdots \chi_\eta)$$

2.2. Transformación matricial.

$$\psi^j \left(\{\chi_j\}_{j=1}^\eta \right) \Big|_{\chi_j \leftrightarrow \chi_{j+1}} = \sum_j (\mathcal{R}_j)_j^j \psi^j \left(\{\chi_j\}_{j=1}^\eta \right)$$

2.3. Conmutaciones.

$$\hat{\psi}_{i,\alpha}^- \hat{\psi}_{j,\beta}^+ = \sum_{cd} \mathfrak{R}_{\beta d}^{\alpha c} \hat{\psi}_{j,c}^+ \hat{\psi}_{i,d}^- + \delta_{\alpha\beta} \delta_{ij}, \hat{\psi}_{i,\alpha}^+ \hat{\psi}_{j,\beta}^+ = \sum_{cd} \mathfrak{R}_{\alpha\beta}^{cd} \hat{\psi}_{j,c}^+ \hat{\psi}_{i,d}^+, \hat{\psi}_{i,\alpha}^- \hat{\psi}_{j,\beta}^- \\ = \sum_{\substack{cd \\ m}} \mathfrak{R}_{dc}^{\beta\alpha} \hat{\psi}_{j,c}^- \hat{\psi}_{i,d}^-, \hat{e}_{ij} \\ \equiv \sum_{\alpha=1} \hat{\psi}_{i,\alpha}^+ \hat{\psi}_{j,\alpha}^- [\hat{e}_{ij}, \hat{\psi}_{\kappa,\beta}^+] \delta_{j\kappa} \hat{\psi}_{i,\beta}^+ [\hat{e}_{ij}, \hat{\psi}_{\kappa,\beta}^-] - \delta_{j\kappa} \hat{\psi}_{j,\beta}^- [\hat{e}_{ij}, \hat{e}_{\kappa l}] \delta_{j\kappa} \hat{e}_{il} - \delta_{il} \hat{e}_{\kappa j}$$

$$\hat{\psi}_{i,\alpha}^+ \hat{\psi}_{j,\beta}^+ = -\hat{\psi}_{j,\alpha}^+ \hat{\psi}_{i,\beta}^+ \Big| \psi \rangle \hat{\psi}_{i_1,\alpha_1}^+ \hat{\psi}_{i_2,\alpha_2}^+ \bigotimes \hat{\psi}_{i_\eta,\alpha_\eta}^+ |0\rangle$$

$$z_{\mathcal{R}} = (e^{-\beta\epsilon}) \equiv Tr[e^{-\beta\epsilon\hat{\eta}}] = \sum_{\eta=0}^{\infty} d_{\eta} e^{-\hat{\eta}\beta\epsilon} \hat{\xi}_{ij} \hat{\psi}_{i,\alpha}^+ \hat{\xi}_{ij}^{\dagger} \hat{\psi}_{j,\alpha}^+ \otimes \hat{\xi}_{ij} \hat{\psi}_{j,\alpha}^+ \hat{\xi}_{ij}^{\dagger} \hat{\psi}_{i,\alpha}^+$$

$$\hat{\xi}_{ij}|0,i\alpha,j\beta\rangle = (\hat{\xi}_{ij}\hat{\psi}_{i,\alpha}^+\hat{\xi}_{ij}^{\dagger})(\hat{\xi}_{ij}\hat{\psi}_{j,\beta}^+\hat{\xi}_{ij}^{\dagger})\hat{\xi}_{ij}|0\rangle = \hat{\psi}_{j,\alpha}^+\hat{\psi}_{i,\beta}^+|0\rangle = \sum_{\alpha',\beta'} \mathfrak{R}_{\alpha\beta}^{\beta'\alpha'}|0,i\beta',j\alpha'\rangle$$

$$\hat{\mathcal{H}} = \sum_{1\leq i,j\leq N} \hbar_{ij} \hat{e}_{ij} = \sum_{\substack{1\leq i,j\leq N \\ 1\leq \alpha\leq m}} \hbar_{ij} \hat{\psi}_{i,\alpha}^+ \hat{\psi}_{j,\alpha}^-$$

$$\langle \bar{\eta}_{\kappa} \rangle_{\beta} \equiv \frac{Tr \left[\bar{\eta}_{\kappa} e^{-\beta \hat{\mathcal{H}}} \right]}{Tr \left[e^{-\beta \hat{\mathcal{H}}} \right]} = \frac{z'_{\mathcal{R}}(e^{-\beta \epsilon_{\kappa}}) e^{-\beta \epsilon_{\kappa}}}{z_{\mathcal{R}}(e^{-\beta \epsilon_{\kappa}})}$$

$$\hat{\mathcal{H}} = \sum_{i,\alpha} \mathcal{J}_i (\hat{\chi}_{i,\alpha}^+ \hat{\gamma}_{i+1,\alpha}^- + \hat{\chi}_{i,\alpha}^- \hat{\gamma}_{i+1,\alpha}^+) - \sum_{i,\alpha} \mu_i \hat{\gamma}_{i,\alpha}^+ \hat{\gamma}_{i,\alpha}^-$$



$$\hat{\mathcal{H}} = \sum_{i,\alpha} J_i (\hat{\psi}_{i,\alpha}^+ \hat{\psi}_{i+1,\alpha}^- + \hat{\psi}_{i,\alpha}^- \hat{\psi}_{i+1,\alpha}^+) - \sum_i \mu_i \hat{\eta}_i$$

2.4. Conmutadores.

$$\begin{aligned} [\hat{e}_{ij}, \hat{\psi}_{\kappa,\beta}^+] &= \sum_{\alpha} (\hat{\psi}_{i,\alpha}^+ \hat{\psi}_{j,\alpha}^- \hat{\psi}_{\kappa,\beta}^+ - \hat{\psi}_{\kappa,\beta}^+ \hat{\psi}_{i,\alpha}^+ \hat{\psi}_{j,\alpha}^-) \\ &= \sum_{\alpha} \hat{\psi}_{i,\alpha}^+ \left(\sum_{c,d} \Re_{\beta d}^{\alpha c} \hat{\psi}_{\kappa,c}^+ \hat{\psi}_{i,d}^- + |\delta_{j\kappa} \delta_{\alpha\beta}|^{\dagger} \right) - \sum_{\alpha} \hat{\psi}_{\kappa,\beta}^+ \hat{\psi}_{i,\alpha}^+ \hat{\psi}_{j,\alpha}^- \\ &= \sum_{\alpha,c,d} |\Re_{\beta d}^{\alpha c} \hat{\psi}_{i,\alpha}^+ \hat{\psi}_{\kappa,c}^+|^{\dagger} \hat{\psi}_{j,d}^- + \delta_{j\kappa} \hat{\psi}_{i,\beta}^+ - \sum_{\alpha} \hat{\psi}_{\kappa,\beta}^+ \hat{\psi}_{i,\alpha}^+ \hat{\psi}_{j,\alpha}^- = |\delta_{j\kappa} \hat{\psi}_{i,\beta}^+|^{\odot} \end{aligned}$$

$$\begin{aligned} [\hat{e}_{ij}, \hat{\psi}_{\kappa,\beta}^-] &= -\delta_{i\kappa} \hat{\psi}_{j,\beta}^-, [\hat{e}_{ij}, \hat{e}_{\kappa l}] = \sum_{\beta} [\hat{e}_{ij} \hat{\psi}_{\kappa,\beta}^+] \hat{\psi}_{l,\beta}^- + \sum_{\beta} \hat{\psi}_{\kappa,\beta}^+ [\hat{e}_{ij} \hat{\psi}_{l,\beta}^-] \\ &= \sum_{\beta} |\delta_{j\kappa} \hat{\psi}_{i,\beta}^+ \hat{\psi}_{l,\beta}^-|^{\otimes} - \sum_{\beta} |\delta_{j\kappa} \hat{\psi}_{i,\beta}^+ \hat{\psi}_{l,\beta}^-|^{\otimes} = \delta_{j\kappa} \hat{e}_{il} - \delta_{il} \hat{e}_{j\kappa} \end{aligned}$$

$$\hat{\psi}_{i,\alpha}^- = \sum_{\kappa=1}^{\eta} u_{\kappa i}^{\odot} \tilde{\psi}_{\kappa,\alpha}^-, \hat{\psi}_{i,\alpha}^+ = \sum_{\kappa=1}^{\eta} u_{\kappa i}^{\odot} \tilde{\psi}_{\kappa,\alpha}^+$$

$$\hat{\mathcal{H}} = \sum_{\substack{1 \leq \kappa, \rho \leq \mathcal{N} \\ 1 \leq i \leq m}} \hbar'_{\kappa \rho} \tilde{\psi}_{\kappa,\alpha}^+ \tilde{\psi}_{\rho,\alpha}^- \equiv \sum_{1 \leq \kappa, \rho \leq \mathcal{N}} \hbar'_{\kappa \rho} \tilde{e}_{\kappa \rho}$$

$$\mathcal{Z}(\beta) \equiv Tr[e^{-\beta \hat{\mathcal{H}}}] = \prod_{\kappa} z_{\mathcal{R}}(e^{-\beta \epsilon_{\kappa}}), \mathcal{F}(\beta) = -\frac{1}{\beta} \ln \mathcal{Z}(\beta) = -\frac{1}{\beta} \sum_{\kappa} \ln z_{\mathcal{R}}(e^{-\beta \epsilon_{\kappa}})$$

$$\langle \hat{\eta}_{\kappa}^l \rangle_{\beta} = \frac{Tr[\hat{\eta}_{\kappa}^l e^{-\beta \hat{\mathcal{H}}}]}{Tr[e^{-\beta \hat{\mathcal{H}}}] = \frac{(\chi \partial_{\chi})^l z_{\mathcal{R}}(\chi)}{z_{\mathcal{R}}(\chi)} \Big|_{\chi=e^{-\beta \epsilon_{\kappa}}} \quad \langle \tilde{e}_{\kappa \rho} \rangle_{\beta} = \delta_{\kappa \rho} \langle \bar{\eta}_{\kappa} \rangle_{\beta}$$

$$\langle \hat{e}_{ij} \rangle_{\beta} = \sum_{\kappa} u_{\kappa i} u_{\kappa j}^{\odot} \langle \bar{\eta}_{\kappa} \rangle_{\beta}$$

$$\hat{\mathcal{H}}_1 = -\sum_v \hat{A}_v - \sum_{\rho} \hat{B}_{\rho}, \hat{\mathcal{H}}_2 = -\sum_{\langle ij \rangle} \hat{h}_{ij} - \sum_l \mu_l \hat{\gamma}_{l,\alpha}^+ \hat{\gamma}_{l,\alpha}^-$$

$$\hat{\mathcal{H}}_2 = \sum_{\langle ij \rangle, 1 \leq \alpha \leq m} (J_{ij} \hat{\psi}_{j,\alpha}^+ \hat{\psi}_{i,\alpha}^- + \hbar \otimes c) - \sum_l \mu_l \hat{\eta}_l$$



2.5. Parapartículas y paraantipartículas en espacios cuánticos curvos. Partículas masivas y supermasivas y antipartículas masivas y supermasivas. Comportamiento y dinámica cuántica.

$$\begin{aligned}
 \hat{\psi}_i \hat{\psi}_j &= \omega \hat{\psi}_j \hat{\psi}_i \left[[\hat{\psi}_\kappa^\dagger \hat{\psi}_l]_\pm \hat{\psi}_m \right] = 2\delta_{\kappa m} \hat{\psi}_l \left[[\hat{\psi}_l^\dagger \hat{\psi}_\kappa]_\pm \hat{\psi}_m \right] \sum_{\alpha'_j, \alpha'_{j+1}} \mathfrak{R}_{\alpha'_j \alpha'_{j+1}}^{\alpha_j \alpha_{j+1}} \psi_{\alpha_1 \otimes \alpha'_j \alpha'_{j+1} \otimes \alpha_\eta} \\
 &= \psi_{\alpha_1 \otimes \alpha_j \alpha_{j+1} \otimes \alpha_\eta} \left| \begin{smallmatrix} \alpha_1 \alpha_2 \otimes \alpha_\eta \\ \eta_1 \eta_2 \otimes \eta_\eta \end{smallmatrix} \right\rangle = \hat{\psi}_{\eta_1, \alpha_1}^{(1)+} \hat{\psi}_{\eta_2, \alpha_2}^{(2)+} \bigotimes \hat{\psi}_{\eta_\eta, \alpha_\eta}^{(\eta)+} |0\rangle \hat{\psi}_{\eta, \alpha}^{(i)+} \\
 &\equiv \frac{1}{\sqrt{\eta!}} \sum_{\alpha_1 \alpha_2 \otimes \alpha_\eta} \psi_{\alpha_1 \alpha_2 \otimes \alpha_\eta}^\alpha \hat{\psi}_{i, \alpha_1}^+ \hat{\psi}_{i, \alpha_2}^+ \bigotimes \hat{\psi}_{i, \alpha_\eta}^+ \sum_{\alpha_1 \alpha_2 \otimes \alpha_\eta} \psi_{\alpha_1 \alpha_2 \otimes \alpha_\eta}^{\beta*} \psi_{\alpha_1 \alpha_2 \otimes \alpha_\eta}^\alpha \\
 &= \delta_{\alpha\beta} \left\langle \begin{smallmatrix} \beta_1 \beta_2 \otimes \beta_\eta \\ \eta'_1 \eta'_2 \otimes \eta'_\eta \end{smallmatrix} \middle| \begin{smallmatrix} \alpha_1 \alpha_2 \otimes \alpha_\eta \\ \eta_1 \eta_2 \otimes \eta_\eta \end{smallmatrix} \right\rangle = \prod_{j=1}^{\eta} \delta_{\eta_j, \eta'_j}, \delta_{\alpha_j \beta_j}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{U}_{\mathcal{R}_{j,j+1}} \mathcal{U}^\dagger &= -\chi_{j,j+1} \mathcal{U} \left| \chi_1 \cdots \chi_\eta \right\rangle \\
 &= \left| f_{\chi_\eta} f_{\chi_{\eta-1}} \cdots f_{\chi_2}(\chi_1) \cdots f_{\chi_\eta}(\chi_{\eta-1}), \chi_\eta \right\rangle \mathfrak{X}_{\mathcal{R}, \eta} \cong \mathfrak{X}_{\Pi \boxtimes \mathcal{R}, \eta} (\Pi \boxtimes \mathcal{R})_{\mathcal{CD}}^{\mathcal{AB}} \equiv \Pi_{kl}^{ij} \mathfrak{R}_{cd}^{\alpha\beta} \\
 \mathfrak{Y}_\eta &= \left\{ \hat{\psi}_{\alpha_1}^+ \hat{\psi}_{\alpha_2}^+ \boxtimes \hat{\psi}_{\alpha_\eta}^+ |0\rangle \middle| 1 \leq \alpha_j \leq m, j \right\}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{R}_{j,j+1} &= \frac{\mathbb{I}}{(1)} \otimes \frac{\mathbb{I}}{(j-1)} \otimes \frac{\mathfrak{R}}{(j, j+1)} \otimes \frac{\mathbb{I}}{(j+2)} \otimes \frac{\mathbb{I}}{(\eta)} \mathcal{R}_{j,j+1} \psi = \psi(\ln a^{\otimes \eta}) \mathbb{C}_{\mathcal{R}}^{(\eta)} \langle \hat{\psi}_\alpha^+ \rangle \\
 &\cong \frac{\mathbb{I}^{\otimes \eta}}{\sum_{j=1}^{\eta-1} [(\mathbb{I} - \mathcal{R}_{j,j+1}) \mathbb{I}^{\otimes \eta}]} \{ |\psi\rangle \in \mathcal{V} | \mathcal{H}_j | \psi\rangle = |\psi\rangle, 1 \leq j \leq \kappa \} \\
 &\cong \frac{\mathcal{V}}{\sum_{j=1}^{\kappa} [(\mathbb{I} - \mathcal{H}_j) \mathcal{V}]} \cong \left\{ \sum_{j=1}^{\kappa} [(\mathbb{I} - \mathcal{H}_j) \mathcal{V}] \right\}^\perp = \bigcap_{j=1}^{\kappa} [(\mathbb{I} - \mathcal{H}_j) \mathcal{V}]^\perp \\
 \langle \Lambda_1 | \Lambda_2 \rangle &= \prod_{\alpha, \beta} \left\langle \hat{e}_{-\beta}^\dagger | \hat{e}_{-\alpha} | \Lambda \right\rangle = \langle \Lambda | \Pi_{\alpha, \beta} \hat{e}_\beta \hat{e}_{-\alpha} | \Lambda \rangle \psi^{\alpha_1 \alpha_2 \alpha_3}(\chi_2, \chi_1, \chi_3)
 \end{aligned}$$

$$\begin{aligned}
 &= \sum_{\beta_1, \beta_2} \mathfrak{R}_{\beta_1 \beta_2}^{\alpha_1 \alpha_2} \psi^{\beta_1 \beta_2 \beta_3}(\chi_1, \chi_2, \chi_3) \\
 |\psi\rangle &= \frac{1}{\sqrt{\eta!}} \sum_{j, \chi_1 \boxtimes \chi_\eta} \psi^j(\chi_1 \cdots \chi_\eta) \hat{\psi}_{\chi_1, \alpha_1}^+ \cdots \hat{\psi}_{\chi_\eta, \alpha_\eta}^+ |0\rangle \\
 \hat{\gamma}_\alpha^- \hat{\gamma}_\beta^+ &= \sum_{c,d} \mathcal{R}_{\beta d}^{ac} \hat{\gamma}_c^+ \hat{\gamma}_d^- + \delta_{\alpha\beta} \hat{\gamma}_\alpha^+ \hat{\gamma}_\beta^+ = \sum_{c,d} \mathcal{R}_{\alpha\beta}^{cd} \hat{\gamma}_c^+ \hat{\gamma}_d^+, \hat{\gamma}_\alpha^- \hat{\gamma}_\beta^- = \sum_{c,d} \mathcal{R}_{dc}^{\beta\alpha} \hat{\gamma}_c^- \hat{\gamma}_d^-, \hat{\chi}_\alpha^- \hat{\chi}_\beta^+ \\
 &= \sum_{c,d} \mathcal{R}_{d\beta}^{c\alpha} \hat{\chi}_c^+ \hat{\chi}_d^- + \delta_{\alpha\beta} \hat{\chi}_\alpha^+ \hat{\chi}_\beta^+ = \sum_{c,d} \mathcal{R}_{\beta\alpha}^{dc} \hat{\chi}_c^+ \hat{\chi}_d^+, \hat{\chi}_\alpha^- \hat{\chi}_\beta^- \\
 &= \sum_{c,d} \mathcal{R}_{cd}^{\alpha\beta} \hat{\chi}_c^- \hat{\chi}_d^- [\hat{\chi}_\alpha^+, \hat{\gamma}_\beta^+] \otimes [\hat{\chi}_\alpha^-, \hat{\gamma}_\beta^-] \sum_\alpha \hat{\chi}_\alpha^+ \hat{\chi}_\alpha^- = \sum_\alpha \hat{\gamma}_\alpha^+ \hat{\gamma}_\alpha^-
 \end{aligned}$$



$$|\eta, \alpha\rangle \equiv \frac{1}{\sqrt{\eta!}} \sum_{\alpha_1 \alpha_2 \otimes \alpha_\eta} \psi_{\alpha_1 \alpha_2 \otimes \alpha_\eta}^\alpha \hat{\gamma}_{\alpha_1}^+ \hat{\gamma}_{\alpha_2}^+ \cdots \hat{\gamma}_{\alpha_\eta}^+ |0\rangle$$

$$\begin{aligned} \hat{\gamma}_\alpha^\pm |\eta, \alpha\rangle &= \sum_{\beta=1}^{d_{\eta\pm 1}} \hat{\Upsilon}_{\alpha, \beta\alpha}^\pm \hat{\Upsilon}_{\alpha, \beta\alpha}^\pm |\eta \pm 1, \beta\rangle, \hat{\chi}_\alpha^\pm |\eta, \alpha\rangle = \sum_{\beta=1}^{d_{\eta\pm 1}} \hat{X}_{\alpha, \beta\alpha}^\pm \hat{X}_{\alpha, \beta\alpha}^\pm |\eta \pm 1, \beta\rangle \sum_{\beta} \hat{\Upsilon}_{\alpha, \beta\alpha}^\pm \psi_{\alpha_0 \alpha_1 \otimes \alpha_\eta}^\beta \\ &= \frac{1}{\sqrt{\eta+1}} \bar{\Upsilon}_{\alpha, \beta_1 \otimes \beta_\eta}^{\alpha_0 \alpha_1 \otimes \alpha_\eta} \sum_{\beta} \hat{X}_{\alpha, \beta\alpha}^\pm \psi_{\alpha_\eta \alpha_1 \otimes \alpha_0}^\beta \\ &= \frac{1}{\sqrt{\eta+1}} \bar{X}_{\beta_\eta \otimes \beta_1, \alpha}^{\alpha_\eta \otimes \alpha_1 \alpha_0} \psi_{\beta_1 \otimes \beta_\eta}^\alpha \sum_{\beta} \hat{\Upsilon}_{\alpha, \beta\alpha}^\pm \psi_{\alpha_2 \otimes \alpha_\eta}^\beta = \sqrt{\eta} \psi_{\alpha_2 \otimes \alpha_\eta}^\alpha \sum_{\beta} \hat{X}_{\alpha, \beta\alpha}^\pm \psi_{\alpha_\eta \otimes \alpha_2}^\beta \\ &= \sqrt{\eta} \psi_{\alpha_\eta \otimes \alpha_2 \alpha}^\alpha + \langle \bar{\mathcal{R}}_{\eta-1, \eta} + \bar{\mathcal{R}}_{\eta, \eta-1} \rangle [\hat{\eta}, \hat{\chi}_\alpha^\pm] = \pm \hat{\chi}_\alpha^\pm, [\hat{\eta}, \hat{\gamma}_\alpha^\pm] = \pm \hat{\gamma}_\alpha^\pm \end{aligned}$$

$$\begin{aligned} \hat{\mathcal{T}}_{\alpha\beta}^\pm |\eta, \alpha\rangle &= \sum_{\beta=1}^{d_\eta} \hat{\mathcal{T}}_{\alpha\beta, \beta\alpha}^\pm |\eta, \beta\rangle \sum_{\beta} \hat{\mathcal{T}}_{\alpha\beta, \beta\alpha}^+ \psi_{\alpha_1 \alpha_2 \otimes \alpha_\eta}^{\pm\beta} \sum_{\alpha'_1 \otimes \alpha'_\eta} \psi_{\alpha'_1 \alpha'_2 \otimes \alpha'_\eta}^\alpha \\ &\quad \boxtimes \left\| \begin{array}{c} \alpha_1 \\ \alpha_2 \\ \alpha'_1 \end{array} \right\| \left\| \begin{array}{c} \alpha_2 \\ \alpha_2 \\ \alpha'_2 \end{array} \right\| \otimes \langle \alpha_1 \alpha_2 \rangle_{\alpha'_1} \left| \frac{\partial\beta}{\partial t} + \frac{\partial\varphi}{\partial\omega} - \frac{\partial\lambda}{\partial\delta} + \frac{\partial\xi}{\partial\Gamma} - \frac{\partial\Delta}{\partial\phi} \right| \otimes \left| \frac{\partial\epsilon}{\partial\zeta} - \frac{\partial\epsilon}{\partial\theta} + \frac{\partial\kappa}{\partial\varpi} - \frac{\partial\sigma}{\partial\rho} \right. \\ &\quad \left. + \frac{\partial\tau}{\partial\Lambda} \right| d\Psi d\varsigma \sum_{\beta} \hat{\mathcal{T}}_{\alpha\beta, \beta\alpha}^- \psi_{\alpha_1 \alpha_2 \otimes \alpha_\eta}^{*\beta} \sum_{\alpha'_1 \otimes \alpha'_\eta} \psi_{\alpha'_1 \alpha'_2 \otimes \alpha'_\eta}^\alpha \\ &\quad \boxtimes \left\| \begin{array}{c} \alpha_1 \\ \alpha_2 \\ \alpha'_1 \end{array} \right\| \left\| \begin{array}{c} \alpha_2 \\ \alpha_2 \\ \alpha'_2 \end{array} \right\| \otimes \langle \alpha_1 \alpha_2 \rangle_{\alpha'_1} \left| \frac{\partial\beta}{\partial t} + \frac{\partial\varphi}{\partial\omega} - \frac{\partial\lambda}{\partial\delta} + \frac{\partial\xi}{\partial\Gamma} - \frac{\partial\Delta}{\partial\phi} \right| \otimes \left| \frac{\partial\epsilon}{\partial\zeta} - \frac{\partial\epsilon}{\partial\theta} + \frac{\partial\kappa}{\partial\varpi} - \frac{\partial\sigma}{\partial\rho} + \frac{\partial\tau}{\partial\Lambda} \right| d\Psi d\varsigma \end{aligned}$$

$$\begin{aligned} \hat{\mathcal{I}}_\Lambda \hat{\chi}_\alpha^+ &= \sum_{\beta, \text{B}} \omega_{\alpha\Lambda}^{\beta\text{B}} \hat{\chi}_\beta^+ \hat{\mathcal{I}}_\beta, \hat{\mathcal{R}}_\Lambda \hat{\chi}_\alpha^+ \\ &= \sum_{\beta, \text{B}} \omega_{\alpha\Lambda}^{\beta\text{B}} \hat{\chi}_\beta^+ \hat{\mathcal{R}}_\beta \langle \mu^- \rangle_{\mu^+} \langle \mu^- \rangle_{\mu^+} [\mu^+]_{\alpha\Lambda}^{\text{B}\beta} \begin{pmatrix} \alpha & \cdots & \beta \\ \vdots & \ddots & \vdots \\ \beta & \cdots & \alpha \end{pmatrix} [\mu^-]_{\beta\text{B}}^{\Lambda\alpha} \begin{pmatrix} \beta & \cdots & \alpha \\ \vdots & \ddots & \vdots \\ \alpha & \cdots & \beta \end{pmatrix} \left| \begin{array}{cc} \mu_{1q}^+ & \mu_{2q}^+ \\ \nu_{1q}^+ & \nu_{2q}^+ \end{array} \right| \oint \mathcal{R}' \end{aligned}$$

$$\left\| \begin{array}{c} - \\ i \end{array} \quad \begin{array}{c} \omega^+ \\ \kappa \end{array} \quad \begin{array}{c} + \\ j \end{array} \right\| = \sum_{\alpha=1}^m \hat{\psi}_{j,\alpha}^+ \hat{\psi}_{i,\alpha}^- \left\| \begin{array}{c} + \\ i \end{array} \quad \begin{array}{c} \omega^- \\ \kappa \end{array} \quad \begin{array}{c} - \\ j \end{array} \right\| = \sum_{\alpha=1}^m \hat{\psi}_{i,\alpha}^+ \hat{\psi}_{j,\alpha}^-$$

$$\hat{\eta}_i = \sum_{\alpha=1}^m \hat{\psi}_{i,\alpha}^+ \hat{\psi}_{i,\alpha}^- = \sum_{\alpha=1}^m \hat{\gamma}_{i,\alpha}^+ \hat{\gamma}_{i,\alpha}^- = \sum_{\alpha=1}^m \hat{\psi}_{i,\alpha}^+ \hat{\psi}_{i,\alpha}^{'-} \hat{\psi}_{i_1,\alpha_1}^+ \hat{\psi}_{i_2,\alpha_2}^+ \otimes \hat{\psi}_{i_\eta,\alpha_\eta}^+ |\Omega\rangle$$

$$\left| \psi \right\rangle = \frac{1}{\eta_j} \hat{\eta}_j \left| \psi \right\rangle = \frac{1}{\eta_j} \sum_{\alpha=1}^m \hat{\psi}_{j,\alpha}^+ \hat{\psi}_{j,\alpha}^- \left| \psi \right\rangle \hat{\psi}_{i,\alpha}^+ (\mathcal{P}_1) \hat{\psi}_{j,\beta}^+ (\mathcal{P}_2) = \sum_{cd} \Re_{\alpha\beta}^{cd} \hat{\psi}_{j,c}^+ (\mathcal{P}_2) \hat{\psi}_{i,d}^+ (\mathcal{P}_1)$$

$$\hat{\gamma}_{i,\alpha}^+ \hat{\gamma}_{j,\beta}^+ |\mathfrak{G}\rangle = \hat{\psi}_{i,\alpha}^+ \hat{\psi}_{j,\beta}^+ |\mathfrak{G}\rangle \equiv |\mathfrak{G}, i\alpha, j\beta\rangle$$



$$\hat{c}_l = \sum_{c=1}^4 c \|\hat{\gamma}_{l,c}^+ \hat{\gamma}_{l,c}^-\|, \hat{c}_i = |\mathfrak{G}, i\alpha', j\beta'\rangle = \sum_c \hat{\gamma}_{i,c}^+ \hat{\gamma}_{i,c}^- \hat{\gamma}_{i,\alpha'}^+ \hat{\gamma}_{j,\beta'}^+ |\mathfrak{G}\rangle = \beta' \hat{\gamma}_{i,\beta'}^+ \hat{\gamma}_{j,\alpha'}^+ |\mathfrak{G}\rangle \\ = \beta' |\mathfrak{G}, i\beta', j\alpha'\rangle$$

2.6. Efecto Hall en campos cuánticos curvos.

$${}^\alpha_\beta \hat{v}_1^+ = \frac{\Delta \boxtimes}{\nabla \Delta} \left\| \frac{\hat{v}_1^\ominus}{1 - \alpha v_1 - \beta v_2} \right\|_{\Delta \star}^{\nabla \circ}, {}^\alpha_\beta \hat{v}_2^\triangle = \frac{\Delta \wr}{\nabla \mathfrak{d}} \left\| \frac{\hat{v}_2^\ominus}{1 - \alpha v_2 - \beta v_1} \right\|_{\Delta \odot}^{\nabla \odot}$$

$$\delta_z = \langle \frac{\bar{v}_1 - \bar{v}_2}{\bar{v}_1 + \bar{v}_2} \rangle = \left| \frac{\Delta v}{\tilde{v}_\zeta} \right|^\ddagger \int \frac{\partial^2 \bowtie}{\partial \triangleq} d \S$$

$$\xi_{eff} = -(j_\varsigma + 2j_\varrho) \frac{\psi_0}{ec_\phi} d\varphi, j_\varrho = \frac{\hat{v}_2^\ominus}{\hbar} \xi_{eff} = -\frac{\hat{v}_2^\ominus}{1 + 2\hat{v}_2^\ominus} j_\varsigma \\ \kappa = \left(\begin{array}{ccc} \kappa_1 & \cdots & \beta \mathcal{J} \\ \vdots & \ddots & \vdots \\ \beta \mathcal{J}^\mathcal{T} & \cdots & \kappa_2 \end{array} \right) t_1 \oplus t_{21}^2 (1,2) \kappa = \left\| \begin{array}{ccc} 3 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{array} \right\|, t = \left\| \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right\|$$

$$Q_1 = -e(t_1 \oplus t_2)^{\mathfrak{J}} \kappa^{-1} \mathbb{I}, Q_2 = e(t'_1 \oplus t'_2)^{\mathfrak{J}} \kappa^{+1} \mathbb{J}, \theta_\delta = \pi \mathbb{I}^{\mathfrak{J}} \kappa^{-1} \mathbb{I}, \ell_1 \equiv \sum_{i=1}^{\dim t_1} \ell_i, \ell_2 \\ \equiv \sum_{i=1}^{\dim t_2} \ell_{i+\dim t_1}$$

$$\delta \mathfrak{E}_{1,\chi} = -\frac{2\Phi_0}{ec} j_{1,\gamma} - \frac{\Phi_0}{ec} j_{2,\gamma}$$

$$\delta \mathfrak{E}_{1,\gamma} = -\frac{2\Phi_0}{ec} j_{1,\chi} - \frac{\Phi_0}{ec} j_{2,\chi}$$

$$\delta \mathfrak{E}_{2,\chi} = -\frac{\Phi_0}{ec} j_{1,\gamma} - \frac{2\Phi_0}{ec} j_{2,\gamma}$$

$$\delta \mathfrak{E}_{2,\gamma} = -\frac{\Phi_0}{ec} j_{1,\chi} + \frac{2\Phi_0}{ec} j_{2,\chi}$$

$$j_{1,\chi} = \sigma_{\chi\chi}^1 \left[\xi - \frac{2\Phi_0}{ec} j_{1,\gamma} - \frac{\Phi_0}{ec} j_{2,\gamma} \right] - \sigma_{\chi\gamma}^1 \left[\frac{2\Phi_0}{ec} j_{1,\chi} + \frac{\Phi_0}{ec} j_{2,\chi} \right]$$

$$j_{2,\chi} = \sigma_{\chi\chi}^2 \left[-\frac{\Phi_0}{ec} j_{1,\gamma} - \frac{2\Phi_0}{ec} j_{2,\gamma} \right] - \sigma_{\chi\gamma}^2 \left[\frac{\Phi_0}{ec} j_{1,\chi} + \frac{2\Phi_0}{ec} j_{2,\chi} \right]$$



$$j_{1,\gamma} = \sigma_{\chi\chi}^1 \left[\frac{2\Phi_0}{ec} j_{1,\chi} + \frac{\Phi_0}{ec} j_{2,\chi} \right] + \sigma_{\chi\gamma}^1 \left[\xi - \frac{2\Phi_0}{ec} j_{1,\gamma} - \frac{\Phi_0}{ec} j_{2,\gamma} \right]$$

$$j_{2,\gamma} = \sigma_{\chi\chi}^2 \left[\frac{\Phi_0}{ec} j_{1,\chi} + \frac{2\Phi_0}{ec} j_{2,\chi} \right] + \sigma_{\chi\gamma}^2 \left[-\frac{\Phi_0}{ec} j_{1,\gamma} - \frac{2\Phi_0}{ec} j_{2,\gamma} \right]$$

$$j_{1,\gamma} = \frac{\hat{v}_1^{\odot} (1 + 2\hat{v}_2^{\odot})}{1 + 2(\hat{v}_1^{\odot} + \hat{v}_2^{\odot}) + 3\hat{v}_1^{\odot} \hat{v}_2^{\odot}} \frac{e^2}{\hbar} \mathfrak{E}$$

$$j_{2,\gamma} = \frac{\hat{v}_1^{\odot} \hat{v}_2^{\odot}}{1 + 2(\hat{v}_1^{\odot} + \hat{v}_2^{\odot}) + 3\hat{v}_1^{\odot} \hat{v}_2^{\odot}} \frac{e^2}{\hbar} \mathfrak{E}$$

$$j_{1,\chi} = \frac{\hat{v}_1^{\odot 2} + (1 + 2\hat{v}_2^{\odot})^2}{[1 + 2(\hat{v}_1^{\odot} + \hat{v}_2^{\odot}) + 3\hat{v}_1^{\odot} \hat{v}_2^{\odot}]^2} \sigma_{\chi\chi} \mathfrak{E}$$

$$j_{2,\chi} = \frac{\hat{v}_1^{\odot} + \hat{v}_2^{\odot} + 2(\hat{v}_1^{\odot 2} + \hat{v}_2^{\odot 2})}{[1 + 2(\hat{v}_1^{\odot} + \hat{v}_2^{\odot}) + 3\hat{v}_1^{\odot} \hat{v}_2^{\odot}]^2} \sigma_{\chi\chi} \mathfrak{E}$$

$$\hat{v}_1^{\odot 2} + (1 + 2\hat{v}_2^{\odot})^2 = \hat{v}_1^{\odot} + \hat{v}_2^{\odot} + 2(\hat{v}_1^{\odot 2} + \hat{v}_2^{\odot 2})$$

$$\hat{v}_2^{\odot 2} + (1 + 2\hat{v}_1^{\odot})^2 = \hat{v}_1^{\odot} + \hat{v}_2^{\odot} + 2(\hat{v}_1^{\odot 2} + \hat{v}_2^{\odot 2})$$

$$(1 + \hat{v}_1^{\odot})^2 + (1 + \hat{v}_2^{\odot})^2$$

2.7. Análisis matricial para espacios cuánticos curvos.

$$\kappa_i = \sigma_i \mathbb{I} + \rho_i \mathbb{J}, \kappa^{-1} = \sigma_i \mathbb{I} - \frac{\sigma_i \rho_i}{\rho_i \dim \kappa_i + \sigma_i} \mathcal{J}$$

$$\kappa^{-1} = \left\| \begin{array}{cc} \sigma_1 \mathbb{I} + \lambda_1 \mathcal{J}_1 & \lambda' \mathcal{J} \\ \lambda' \mathcal{J}^T & \sigma_2 \mathbb{I} + \lambda_2 \mathcal{J}_2 \end{array} \right\|$$

$$\begin{aligned} \mathcal{Q}_1 &= -e(t_1 \oplus t_2)^{\mathfrak{S}} \kappa^{-1} \mathbb{I} = -e \sum_{i,j=1}^{\eta} \kappa_{ij}^{-1} \mathbb{I}_j - e \sum_{i=1}^{\eta} \sum_{j=1+\eta}^{\eta+m} \kappa_{ij}^{-1} \mathbb{I}_j \\ &= -e \sum_{i,j=1}^{\eta} (\sigma_1 \delta_{ij} + \lambda_1) \mathbb{I}_j - e \eta \lambda' \sum_{j=1}^m \mathbb{I}_j = -e[(\sigma_1 + \eta \lambda_1) \ell_1 + \eta \lambda' \ell_2] \end{aligned}$$



$$\begin{aligned}
\mathbb{I}^T \kappa^{-1} \mathbb{I} &= \sum_{i,j=1}^{\eta} \mathbb{I}_i \mathbb{I}_j \kappa_{ij}^{-1} + \sum_{i,j=1+\eta}^{\eta+m} \mathbb{I}_i \mathbb{I}_j \kappa_{ij}^{-1} + 2 \sum_{i=1}^{\eta} \sum_{j=1+\eta}^{\eta+m} \mathbb{I}_i \mathbb{I}_j \kappa_{ij}^{-1} \\
&= \sum_{i,j=1}^{\eta} \mathbb{I}_i \mathbb{I}_j (\sigma_1 \delta_{ij} + \lambda_1) + \sum_{i,j=1+\eta}^{\eta+m} \mathbb{I}_i \mathbb{I}_j (\sigma_2 \delta_{ij} + \lambda_2) + 2\lambda' \sum_{i=1}^{\eta} \mathbb{I}_i \sum_{j=1+\eta}^{\eta+m} \mathbb{I}_j \\
&= \sigma_1 \sum_{i=1}^{\eta} \mathbb{I}_i^2 + \sigma_2 \sum_{i=1}^m \mathbb{I}_{i+\eta}^2 + \lambda_1 \ell_1^2 + \lambda_2 \ell_2^2 + 2\lambda' \ell_1 \ell_2 \\
\sigma_1 \sum_{i=1}^{\eta} \mathbb{I}_i^2 (\lambda_2) &\rightarrow \sigma_1 (\mathcal{I}_\kappa + \delta_1)^2 + \sigma_2 (\mathcal{I}_j - \delta_1)^2 + \sigma_1 (72) \\
&= 2\sigma_1 \delta_1 (\mathcal{I}_\kappa + \mathcal{I}_j) + 2\sigma_1 \delta_1^2 + \sigma_1 \sum_{i=1}^{\eta} \mathbb{I}_i^2 (\lambda_2) = \sigma_1 \sum_{i=1}^{\eta} \mathbb{I}_i^2 (\aleph_2)
\end{aligned}$$

REFERENCIAS BIBLIOGRÁFICAS ADICIONALES.

Ayush Hazarika, Simran Arora, P.K. Sahoo y Tiberiu Harko, $f(Q, \mathcal{L}_m)$ gravity, and its cosmological implications, arXiv:2407.00989v3 [gr-qc] 10 Dec 2024.

Zhiyuan Wang y Kaden R. A. Hazzard, Particle exchange statistics beyond fermions and bosons, arXiv:2308.05203v2 [quant-ph] 21 Jul 2024.

Naiyuan J. Zhang, Ron Q. Nguyen, Navketan Batra, Xiaoxue Liu, Kenji Watanabe, Takashi Taniguchi, D. E. Feldman y J.I.A. Li, Excitons in the Fractional Quantum Hall Effect, arXiv:2407.18224v4 [cond-mat.mes-hall] 20 Sep 2024.

FE DE ERRATAS – 10 de enero del 2025.

En todas las ecuaciones contenidas en todos los artículos publicados por este autor, hasta la actualidad (10 de enero del 2025), en tanto corresponda por rigor matemático, se incorporará cualquiera de estos formatos de fracción $\frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}$

, $\frac{\square}{\square}, \frac{\square}{\square} / \frac{\square}{\square}, \frac{\square}{\square} / \frac{\square}{\square}$ o $\frac{\square}{\square}$, según sea el caso.



APÉNDICE B.

1. Partículas y antipartículas relativistas.

1.1. Gravedad relativista en espacios cuánticos curvos.

$$\langle \mu, \nu \rangle = g(\mu, \nu)$$

$$A_{\nu\rho}^{\mu} B_{\mu}^{\rho} = \sum_{\mu=0}^3 \sum_{\rho=0}^3 A_{\nu\rho}^{\mu} B_{\mu}^{\rho}$$

$$\langle \mu, \nu \rangle = u^{\mu} v^{\nu} g(\partial_{\mu}, \partial_{\nu}) = g_{\mu\nu} u^{\mu} v^{\nu}$$

$$(\nabla_{\mu} - \partial_{\mu}) v^{\nu} = \Gamma_{\mu\rho}^{\nu} v^{\rho}$$

$$\nabla_{\mu} \nabla_{\nu} f = \nabla_{\nu} \nabla_{\mu} f \forall f \in \mathfrak{C}^1(\mathcal{M}) \nabla_{\mu} g_{\nu\rho}$$

$$\Gamma_{\mu\nu}^{\lambda} = \frac{1}{2} g^{\lambda\kappa} (\partial_{\mu} g_{\kappa\nu} + \partial_{\nu} g_{\kappa\mu} - \partial_{\kappa} g_{\mu\nu}) [\nabla_{\mu} \nabla_{\nu} - \nabla_{\nu} \nabla_{\mu}] v^{\rho} = \mathcal{R}_{\lambda\mu\nu}^{\rho} v^{\lambda}$$

$$\mathcal{S} = \frac{c^4}{32\pi\mathcal{G}} \int \sqrt{-g} \mathcal{R} d^4\chi + \int \sqrt{-g} \mathcal{L}_m d^4\chi$$

$$\mathcal{G} := \mathcal{R}_{\mu\nu} - \frac{1}{2} \mathcal{R} g_{\mu\nu} = \frac{16\pi\mathcal{G}}{c^4} T_{\mu\nu}$$

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}_m)}{\delta g_{\mu\nu}}$$

$$\mathcal{R}_{\sigma\mu\nu}^{\rho} = \partial_{\mu} \Gamma_{\sigma\nu}^{\rho} + \Gamma_{\sigma\nu}^{\alpha} \Gamma_{\alpha\mu}^{\rho} - \partial_{\nu} \Gamma_{\sigma\mu}^{\rho} + \Gamma_{\sigma\mu}^{\alpha} \Gamma_{\alpha\nu}^{\rho}$$

1.2. Presión y densidad (Modelo Tolman-Oppenheimer-Volkoff - TOV).

$$ds^2 = e^{2\nu(r)} dt^2 - e^{2\phi(r)} dr^2 - r^2 d\Omega^2$$

$$\mathcal{T}^{\mu\nu} = (\rho + p) u^{\mu} u^{\nu} - p g^{\mu\nu}$$

$$\mathcal{G}_{tt} = -e^{2(\nu-\phi)} \left(\frac{1}{r^2} - 2 \frac{\phi'}{r} \right) + \frac{e^{2\nu}}{r^2}$$

$$\mathcal{G}_{rr} = - \left(\frac{1}{r^2} - 2 \frac{\nu'}{r} \right) + \frac{e^{2\phi}}{r}$$

$$\mathcal{G}_{\theta\theta} = -r^2 e^{-2\phi} \left(\nu'' + \nu'^2 - \phi' \nu' + \frac{\nu' - \phi'}{r} \right)$$

$$\mathcal{T}_{\mu\nu} = (\rho + p) u_{\mu} u_{\nu} - p g_{\mu\nu}$$

$$\mathcal{T}_{tt} = \rho e^{2\nu}$$



$$\mathcal{T}_{rr} = -pe^{2\phi}$$

$$\mathcal{T}_{\theta\theta} = -pr^2$$

$$\begin{aligned} e^{2(\nu-\phi)} \left(\frac{1}{r^2} - 2 \frac{\phi'}{r} \right) - \frac{e^{2\nu}}{r^2} &= -16\pi\rho e^{2\nu} - \left(\frac{1}{r^2} - 2 \frac{\nu'}{r} \right) + \frac{e^{2\phi}}{r} \\ &= -16\pi p e^{2\phi} e^{-2\phi} \left(\nu'' + \nu'^2 - \phi' \nu' + \frac{\nu' - \phi'}{r} \right) = 16\pi p \end{aligned}$$

$$\frac{1}{r^2} \frac{d}{dr} [r(1 - e^{-2\phi(r)})] = 16\pi\rho$$

$$e^{-2\phi(r)} = 1 - \frac{16\pi}{r} \int_0^r \rho(s) s^2 ds = 1 - \frac{2m(r)}{r}$$

$$m(r) = 4\pi \int_0^r s^2 \rho(s) ds$$

$$\frac{dp}{dr} = - \frac{[m(r) + 8\pi r^3 p(r)][\rho(r) + p(r)]}{r^2 \left(1 - \frac{2m(r)}{r} \right)}$$

$$\frac{dm}{dr} = 8\pi r^2 \rho(r)$$

1.3. Termodinámica (Modelo Tolman-Oppenheimer-Volkoff – TOV).

$$\mathcal{T}d\mathcal{S} = d\mathfrak{E} + pd\mathcal{V} - \mu d\mathcal{N}$$

$$\mathcal{T}d(s\mathcal{V}) = d(\rho\mathcal{V}) + pd\mathcal{V} - \mu d(n\mathcal{V})$$

$$\mathcal{T}sd\mathcal{V} + \mathcal{T}\mathcal{V}ds = \rho d\mathcal{V} + \mathcal{V}d\rho + pd\mathcal{V} - \mu nd\mathcal{V} - \mu\mathcal{V}dn$$

$$\mathcal{T}ds = d\rho - \mu dn$$

$$s = \frac{1}{\mathcal{T}}(p + \rho - un)$$

$$\mathcal{S} = \int_0^{\mathcal{R}} s(r) d\mathcal{V}^{(3)} = \int_0^{\mathcal{R}} s(r) e^{\phi(r)} r^2 dr d\Omega = 4\pi \int_0^{\mathcal{R}} \frac{r^2 s(r)}{\sqrt{1 - \frac{2m(r)}{r}}} dr$$

$$\mathcal{N} = 4\pi \int_0^{\mathcal{R}} \frac{r^2 n(r)}{\sqrt{1 - \frac{2m(r)}{r}}} dr$$

$$\mathcal{L}(m, m', n) = [s(\rho(m')), n + \lambda n(r)] \left[1 - \frac{2m(r)}{r} \right]^{-\frac{1}{2}} r^2 \left\langle \frac{\partial \mathcal{L}}{\partial n} \frac{\partial \mathcal{L}}{\partial m} \right\rangle + \left\langle \frac{d}{dr} \frac{\partial \mathcal{L}}{\partial m'} \frac{\partial s}{\partial n} \frac{\mu}{\mathcal{T}} \right\rangle + |\lambda|$$



$$\frac{\partial \mathcal{L}}{\partial m} = [s(r) + \lambda n(r)] \left[1 - \frac{2m(r)}{r} \right]^{-\frac{3}{2}} r$$

$$\frac{\partial \mathcal{L}}{\partial m'} = \frac{\partial s}{\partial m'} \left[1 - \frac{2m(r)}{r} \right]^{-\frac{1}{2}} r^2 = \frac{\partial s}{\partial \rho} \frac{\partial \rho}{\partial m'} \left[1 - \frac{2m(r)}{r} \right]^{-\frac{1}{2}} r^2$$

$$\frac{\partial \mathcal{L}}{\partial m'} = \frac{1}{\mathcal{T}} \frac{1}{8\pi r^2} \left[1 - \frac{2m(r)}{r} \right]^{-\frac{1}{2}} r^2 = \frac{1}{4\pi \mathcal{T}} \left[1 - \frac{2m(r)}{r} \right]^{-\frac{1}{2}}$$

$$\frac{d}{dr} \frac{\partial \mathcal{L}}{\partial m'} = \frac{\left(\frac{m'}{r} - \frac{m}{r^2} \right) - \left(1 - \frac{2m(r)}{r} \right) \frac{\mathcal{T}'}{\mathcal{T}}}{8\pi \mathcal{T} (r - 2m)^{3/2}}$$

$$\frac{\partial \mathcal{L}}{\partial m} = \frac{p + \rho}{\mathcal{T}} \left[1 - \frac{2m(r)}{r} \right]^{-\frac{3}{2}} r$$

$$\frac{\mathcal{T}'}{\mathcal{T}} r^2 \left[1 - \frac{2m}{r} \right] = -(m + 8\pi r^3 p)$$

$$dp = d(\mathcal{T}s) - d\rho + (\mu n) = s d\mathcal{T} + \mathcal{T} ds - d\rho + n d\mu + \mu dn$$

$$dp = s d\mathcal{T} + n d\mu$$

$$\frac{dp}{dr} = s \mathcal{T}' + n \mu' = s \mathcal{T}' + n \lambda \mathcal{T}' = \frac{\mathcal{T}'}{\mathcal{T}} (\rho + p)$$

$$\frac{dp}{dr} = - \frac{[m(r) + 8\pi r^3 p(r)][\rho(r) + p(r)]}{r^2 \left(1 - \frac{2m(r)}{r} \right)}$$

$$\mathcal{T}s = \rho(m, m') + p(\rho(m, m')) - \mu n$$

$$\mathcal{L}(m, m', n) = [s(\rho(m, m')), n + \lambda n(r)] \left[1 - \frac{2m(r)}{r} \right]^{-\frac{1}{2}} r^2$$

$$\mu = \lambda \mathcal{T}$$

$$\frac{\partial \mathcal{L}}{\partial m} = (s + \lambda n) r \left[1 - \frac{2m(r)}{r} \right]^{-\frac{3}{2}} + \left[1 - \frac{2m(r)}{r} \right]^{-\frac{1}{2}} r^2 \frac{\partial s}{\partial m}$$

$$\frac{\partial \mathcal{L}}{\partial m'} = \left[1 - \frac{2m(r)}{r} \right]^{-\frac{1}{2}} r^2 \frac{\partial s}{\partial m'}$$

$$\frac{\partial s}{\partial m'} = \frac{\partial s}{\partial \rho} \frac{\partial \rho}{\partial \phi'} \frac{\partial \phi'}{\partial m'} = \frac{1}{\mathcal{T}} \frac{\partial \rho}{\partial \phi'} \frac{1}{(r - 2m)} = \frac{1}{16\pi \mathcal{T}} \left[\frac{2}{r^2} \frac{df}{d\mathcal{R}} + \frac{1}{r} \frac{df'}{d\mathcal{R}} \right]$$



$$\frac{\partial s}{\partial m} = \frac{\partial s}{\partial \rho} \left[\frac{\partial \rho}{\partial \phi} \frac{\partial \phi}{\partial m} + \frac{\partial \rho}{\partial \phi'} \frac{\partial \phi'}{\partial m} \right] = \frac{1}{\mathcal{T}} \frac{1}{16\pi} \left[\frac{-2}{r} \left(\frac{2r\phi' - 1}{r^2} \frac{df}{d\mathcal{R}} - \frac{df''}{d\mathcal{R}} + \left(\phi' - \frac{2}{r} \right) \frac{df'}{d\mathcal{R}} \right) + \frac{1}{r} \left(\frac{2}{r} \frac{df}{d\mathcal{R}} + \frac{df'}{d\mathcal{R}} \right) \right]$$

$$\frac{\partial \mathcal{L}}{\partial m} = (s + \lambda n) r \left[1 - \frac{2m(r)}{r} \right]^{-\frac{3}{2}} + \frac{1}{8\pi\mathcal{T}} \left[1 - \frac{2m(r)}{r} \right]^{-\frac{1}{2}} \left(3 \frac{df''}{d\mathcal{R}} + 2r \frac{df}{d\mathcal{R}} \right)$$

$$\frac{\partial \mathcal{L}}{\partial m'} = \frac{1}{8\pi\mathcal{T}} \left[1 - \frac{2m}{r} \right]^{\frac{1}{2}} \left(2 \frac{df}{d\mathcal{R}} + r \frac{df'}{d\mathcal{R}} \right)$$

$$\begin{aligned} & \frac{\mathcal{T}'}{8\pi\mathcal{T}^2} \left(\frac{2}{r^2} \frac{df}{d\mathcal{R}} + \frac{1}{r} \frac{df'}{d\mathcal{R}} \right) r^2 \\ &= \frac{1}{8\pi\mathcal{T}} \left(-\frac{4}{r^3} \frac{df}{d\mathcal{R}} - \frac{1}{r^2} \frac{df'}{d\mathcal{R}} + \frac{2}{r^2} \frac{df'}{d\mathcal{R}} + \frac{1}{r} \frac{df''}{d\mathcal{R}} \right) r^2 \\ &+ \frac{1}{8\pi\mathcal{T}} \left(\frac{2}{r^2} \frac{df}{d\mathcal{R}} + \frac{1}{r} \frac{df'}{d\mathcal{R}} \right) (2r + rm' - 5m) \left(1 - \frac{2m}{r} \right)^{-1} - \frac{1}{8\pi\mathcal{T}} \left(\frac{2}{r} \frac{df''}{d\mathcal{R}} + \frac{3}{r^2} \frac{df'}{d\mathcal{R}} \right) r^2 \\ &- \frac{\rho + p}{\mathcal{T}} r \left(1 - \frac{2m}{r} \right)^{-1} \end{aligned}$$

$$\begin{aligned} \frac{dp}{dr} = & -(\rho + p) \left(1 - \frac{2m}{r} \right)^{-1} \left(\frac{2}{r} \frac{df}{d\mathcal{R}} + \frac{df'}{d\mathcal{R}} \right)^{-1} \left[16\pi p + \frac{2m}{r^3} \frac{df}{d\mathcal{R}} + \frac{1}{2} f(\mathcal{R}) - \frac{\mathcal{R}}{2} \frac{df}{d\mathcal{R}} \right. \\ & \left. - \frac{2}{r} \left(1 - \frac{2m}{r} \right) \frac{df'}{d\mathcal{R}} \right] \end{aligned}$$

1.4. Ecuaciones de campo (Modelo Tolman-Oppenheimer-Volkoff - TOV).

$$\mathcal{S} = \frac{1}{32\pi} \int d^4\chi \sqrt{-g} f(\mathcal{R}) \mathcal{L} + \mathcal{S}_m$$

$$\frac{df(\mathcal{R})}{d\mathcal{R}} \mathcal{R}_{\mu\nu} - \frac{1}{2} f(\mathcal{R}) g_{\mu\nu} - [\nabla_\mu \nabla_\nu - g_{\mu\nu} \square] \frac{df(\mathcal{R})}{d\mathcal{R}} = 16\pi \mathcal{T}_{\mu\nu}$$

$$\nabla_\mu \mathcal{G}^{\mu\nu} = 0 = \nabla_\mu \mathcal{T}^{\mu\nu}$$

$$16\pi \nabla_\mu \mathcal{T}^{\mu\nu} = (\nabla^\mu f') \mathcal{R}_{\mu\nu} + f' \nabla^\mu \mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \nabla^\mu f - [\square \nabla_\nu - \nabla_\nu \square] f'$$

$$(\nabla^\mu f') \mathcal{R}_{\mu\nu} + f' \nabla^\mu \left(\mathcal{R}_{\mu\nu} - \frac{1}{2} \mathcal{R} g_{\mu\nu} \right) - \mathcal{R}_{\mu\nu} \nabla^\mu f'$$

$$\mathcal{R} f(\mathcal{R}) - 2 \frac{df(\mathcal{R})}{d\mathcal{R}} + 3 \square \frac{df(\mathcal{R})}{d\mathcal{R}} = 16\pi \mathcal{T}$$

$$\mathcal{S} = \frac{1}{32\pi} \int d^4\chi \sqrt{-g} f(\mathcal{R}, \mathcal{T}) + \int d^4\chi \mathcal{L} \sqrt{-g} \mathcal{L}_m$$

$$\begin{aligned} \partial_{\mathcal{R}} f(\mathcal{R}, \mathcal{T}) \mathcal{R}_{\mu\nu} - \frac{1}{2} f(\mathcal{R}, \mathcal{T}) g_{\mu\nu} + (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) \partial_{\mathcal{R}} f(\mathcal{R}, \mathcal{T}) \\ = 16\pi \mathcal{T}_{\mu\nu} - \partial_{\mathcal{T}} f(\mathcal{R}, \mathcal{T}) \mathcal{T}_{\mu\nu} - \partial_{\mathcal{T}} f(\mathcal{R}, \mathcal{T}) \Theta_{\mu\nu} \end{aligned}$$



$$\nabla^\mu \mathcal{T}_{\mu\nu} = \frac{\partial_{\mathcal{T}} f}{8\pi - \partial_{\mathcal{T}} f} \left[(\mathcal{T}_{\mu\nu} + \Theta_{\mu\nu}) \nabla^\mu \ln(\partial_{\mathcal{T}} f) + \nabla^\mu \Theta_{\mu\nu} - \frac{1}{2} \nabla_\nu \mathcal{T} \right]$$

$$f(\mathcal{R}, \mathcal{T}) = \mathcal{R} + 2\chi\mathcal{T}$$

$$\mathcal{G}_{\mu\nu} = 16\pi\mathcal{T}_{\mu\nu} + \chi\mathcal{T}g_{\mu\nu} + 2\chi(\mathcal{T}_{\mu\nu} + pg_{\mu\nu})$$

$$(8\pi + 3\chi)\rho = \chi p - e^{-2\phi} \left(\frac{1}{r^2} - \frac{2\phi'}{r} \right) + \frac{1}{r^2}$$

$$(8\pi + 3\chi)p = \chi\rho + e^{-2\phi} \left(\frac{1}{r^2} - \frac{2\nu'}{r} \right) - \frac{1}{r^2}$$

$$8\pi\rho_{eff} = (8\pi + 3\chi)\rho - \chi p$$

$$8\pi p_{eff} = (8\pi + 3\chi)p - \chi\rho$$

$$\rho = \frac{8\pi}{\alpha^2 - \chi^2} (\alpha\rho_{eff} + \chi p_{eff})$$

$$p = \frac{8\pi}{\alpha^2 - \chi^2} (\chi\rho_{eff} + \alpha p_{eff})$$

$$8\pi\rho_{eff} = e^{-2\phi} \left(\frac{1}{r^2} - \frac{2\phi'}{r} \right) + \frac{1}{r^2}$$

$$8\pi p_{eff} = -e^{-2\phi} \left(\frac{1}{r^2} - \frac{2\nu'}{r} \right) - \frac{1}{r^2}$$

$$e^{-2\phi} = 1 - \frac{2m(r)}{r}$$

$$m(r) = \int_0^r 4\pi\rho_{eff}(s)s^2 ds$$

$$(4\pi + \chi)\nabla^\mu \mathcal{T}_{\mu\nu} = \frac{1}{2}\chi\nabla_\nu(\mathcal{T} + 2p) - \frac{dp}{dr} - \frac{d\nu}{dr}(\rho + p) + \frac{\chi}{8\pi + 2\chi}(\rho' - p')$$

1.4.1. Ecuaciones de Campo Oppenheimer – Snyder.

$$\begin{aligned} ds^2 &= e^\nu dt^2 - e^\lambda dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) - 8\pi\mathcal{T}_1^1 = e^{-\lambda} \left(\frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} + 8\pi\mathcal{T}_4^4 \\ &= e^{-\lambda} \left(\frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} - 8\pi\mathcal{T}_2^2 = -8\pi\mathcal{T}_3^3 \\ &= e^{-\lambda} \left(\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'\lambda'}{4} + \frac{\nu' - \lambda'}{2r} \right) - e^\nu \left(\frac{\ddot{\lambda}}{2} + \frac{\dot{\lambda}^2}{4} - \frac{\dot{\lambda}\dot{\nu}}{4} \right) 8\pi\mathcal{T}_1^4 = 8\pi e^{\nu-\lambda}\mathcal{T}_1^4 \\ &= -e^{-\lambda} \frac{\dot{\lambda}}{r} \end{aligned}$$



$$\lambda = -\ln \left\{ 1 - \frac{8\pi}{r} \int_0^r \mathcal{T}_4^4 r^2 dr \right\} 8\pi(\mathcal{T}_4^4 - \mathcal{T}_1^1) = \frac{e^\lambda(\lambda' - \nu')}{r}$$

$$\mathcal{V} = 4\pi \int_0^{r_b} e^{\frac{\lambda}{2}} r^2 dr$$

$$ds^2 = d\tau^2 - e^{\bar{\omega}} d\mathcal{R}^2 - e^{\omega} (d\theta^2 + \sin^2 \theta d\varphi^2) 8\pi \mathcal{T}_1^1 = 0 = e^{-\omega} - e^{-\bar{\omega}} \frac{\omega'^2}{4} + \ddot{\omega} + \frac{3}{4} \dot{\omega}^2 = 0$$

$$8\pi \mathcal{T}_2^2 = -8\pi \mathcal{T}_3^3 = 0 = -e^{-\bar{\omega}} \left(\frac{\omega''}{2} + \frac{\omega'^2}{4} - \frac{\bar{\omega}' \omega'}{4} \right) + \frac{\ddot{\omega}}{2} + \frac{\dot{\omega}^2}{4} + \frac{\ddot{\bar{\omega}}}{2} + \frac{\dot{\bar{\omega}}^2}{4} + \frac{\ddot{\omega}}{4}$$

$$8\pi \mathcal{T}_4^4 = 8\pi \rho = e^{-\omega} - e^{-\bar{\omega}} \left(\omega'' + \frac{3}{4} \omega'^2 - \frac{\bar{\omega}' \omega'}{2} \right) + \frac{\dot{\omega}^2}{4} + \frac{\dot{\bar{\omega}} \dot{\omega}}{4}$$

$$8\pi e^{\dot{\omega}} \mathcal{T}_4^1 = -8\pi \mathcal{T}_1^4 = 0 = \frac{\omega' \dot{\omega}}{2} - \frac{\dot{\bar{\omega}} \omega'}{2} + \dot{\omega}'$$

$$e^{\bar{\omega}} = e^{\omega} \omega'^2 / 4 f^2(\mathcal{R})$$

$$e^{\omega} = (\mathcal{F}r + \mathcal{G})^{\frac{4}{3}}$$

$$8\pi \rho = \frac{4}{3} \left(\frac{r + \mathcal{G}}{\mathcal{F}} \right)^{-1} \left(\frac{r + \mathcal{G}'}{\mathcal{F}'} \right)^{-1}$$

$$\mathcal{F}\mathcal{F}' = 9\pi \mathcal{R}^2 \rho_0(\mathcal{R})$$

$$\mathcal{F} = \langle \begin{array}{ll} -\frac{3}{2} r_0^{1/2} (\mathcal{R}/\mathcal{R}_b)^{3/2} & \mathcal{R} < \mathcal{R}_b \\ -\frac{3}{2} r_0^{1/2} & \mathcal{R} > \mathcal{R}_b \end{array} \rangle$$

$$e^{\omega/2} = (\mathcal{F}r + \mathcal{G})^{2/3} = r$$

$$g^{44} = e^{-\nu} = \dot{t}^2 - t'^2/r'^2 = \dot{t}^2(1 - \dot{r}^2)$$

$$g^{11} = e^{-\lambda} = -(1 - \dot{r}^2)$$

$$g^{14} = 0 = \dot{t}\dot{r} - t'/r'$$

$$\frac{t'}{\dot{t}} = \dot{r}r' = \langle \begin{array}{ll} -(r_0\mathcal{R})^{1/2} \left[\mathcal{R}^{3/2} - \frac{3}{2} r_0^{1/2} \tau \right]^{-2/3} & \mathcal{R} > \mathcal{R}_b \\ r_0^{1/2} \mathcal{R} \mathcal{R}_b^{-3/2} \left[1 - \frac{3}{2} r_0^{1/2} \tau \mathcal{R}_b^{-3/2} \right]^{1/3} & \mathcal{R} < \mathcal{R}_b \end{array} \rangle$$



$$\chi = \frac{2}{3}r_0^{1/2}(\mathcal{R}^{3/2} - r^{3/2}) - 2(rr_0)^{\frac{1}{2}} + r_0 + \ln \frac{r^{1/2} + r_0^{1/2}}{r^{1/2} - r_0^{1/2}}$$

$$\gamma = \frac{1}{2}[(\mathcal{R}/\mathcal{R}_b)^2 - 1] + \mathcal{R}_b r/r_0 \mathcal{R}$$

$$e^\lambda = (1 - r_0/r)^{-1}$$

$$e^\nu = (1 - r_0/r)$$

$$t = \mathcal{M}(\gamma) = \frac{2}{3}r_0^{1/2}(\mathcal{R}_b^{3/2} - r_0^{3/2}\gamma^{3/2}) - 2r_0\gamma^{\frac{1}{2}} + r_0 \ln \frac{\gamma^{\frac{1}{2}} + 1}{\gamma^{\frac{1}{2}} - 1}$$

$$t \sim -r_0 \ln \left\{ \frac{1}{2} [(\mathcal{R}/\mathcal{R}_b)^2 - 3] + \mathcal{R}_b/r_0 (1 - 3r_0^{1/2}/2\mathcal{R}_b^2)^{2/3} \right\}$$

$$e^{-\lambda} \simeq 1 - (\mathcal{R}/\mathcal{R}_b)^2 \left\{ e^{t/r_0} + \frac{1}{2} [3 - (\mathcal{R}/\mathcal{R}_b)^2] \right\}^{-1}$$

$$e^\nu \simeq e^{\lambda-2t/r_0} \left\{ e^{-t/r_0} + \frac{1}{2} [3 - (\mathcal{R}/\mathcal{R}_b)^2] \right\}$$

1.5. Dinámica relativista (Modelo Tolman-Oppenheimer-Volkoff - TOV).

$$16\pi\rho = \frac{1}{2}f(\mathcal{R}) + \left[\frac{1}{r^2} + \frac{e^{-2\Phi}(2r\phi' - 1)}{r^2} - \frac{\mathcal{R}}{2} \right] \frac{df}{d\mathcal{R}} + e^{-2\Phi} \left(\phi' - \frac{2}{r} \right) \frac{df'}{d\mathcal{R}} - e^{-2\Phi} \frac{df''}{d\mathcal{R}}$$

$$16\pi\rho = -\frac{1}{2}f(\mathcal{R}) + \left[-\frac{1}{r^2} + \frac{\mathcal{R}}{2} + \frac{e^{-2\Phi}(2rv' + 1)}{r^2} \right] \frac{df}{d\mathcal{R}} + e^{-2\Phi} \left(v' - \frac{2}{r} \right) \frac{df'}{d\mathcal{R}}$$

$$\nabla^\mu \mathcal{T}_{\mu\nu} = 0 \mapsto p' = -(\rho + p)v'$$

$$\frac{dp}{dr} = -(\rho + p) \left(1 - \frac{2m}{r} \right)^{-1} \left(\frac{2}{r} \frac{df}{d\mathcal{R}} + \frac{df'}{d\mathcal{R}} \right)^{-1} \left[16\pi p + \frac{2m}{r^3} \frac{df}{d\mathcal{R}} + \frac{1}{2} f(\mathcal{R}) - \frac{\mathcal{R}}{2} \frac{df}{d\mathcal{R}} - \frac{2}{r} \left(1 - \frac{2m}{r} \right) \frac{df'}{d\mathcal{R}} \right]$$

$$\phi' = \frac{1}{r} \left(m' - \frac{m}{r} \right) \left(1 - \frac{2m}{r} \right)^{-1}$$

$$\frac{dp}{dr} = -(\rho + p) \frac{8\pi pr + \frac{m}{r^2} - \frac{\chi(\rho - 3p)r}{2}}{\left(1 - \frac{2m}{r} \right) \left[1 - \frac{\chi}{8\pi + 2\chi} \left(1 - \frac{d\rho}{dp} \right) \right]}$$

$$\mathcal{L}(m,m',n) = \left[s\left(\rho_{eff}(m')\right), n + \lambda n(r) \right] \left[1 - \frac{2m(r)}{r} \right]^{-\frac{1}{2}} r^2$$



$$\mathcal{L}(m, m', n) = k \frac{\rho_{eff} + p_{eff}}{\mathcal{T}} \left[1 - \frac{2m(r)}{r} \right]^{-\frac{1}{2}} r^2$$

$$\frac{\partial \mathcal{L}}{\partial m} = k \frac{\rho_{eff} + p_{eff}}{\mathcal{T}} \left[1 - \frac{2m(r)}{r} \right]^{-\frac{3}{2}} r$$

$$\frac{\partial \mathcal{L}}{\partial m'} = \frac{k}{\mathcal{T}} \left[1 - \frac{2m(r)}{r} \right]^{-\frac{1}{2}} r^2 \frac{\partial \rho_{eff}}{\partial m'}$$

$$\frac{\partial \mathcal{L}}{\partial m'} = \frac{k}{4\pi \mathcal{T}} \left[1 - \frac{2m(r)}{r} \right]^{-\frac{1}{2}}$$

$$\frac{d}{dr} \frac{\partial \mathcal{L}}{\partial m'} = \frac{k}{4\pi r^2 \mathcal{T}} (m'r - m) \left[1 - \frac{2m}{r} \right]^{-\frac{3}{2}} - \frac{k}{4\pi \mathcal{T} \mathcal{T}'} \left[1 - \frac{2m}{r} \right]^{-\frac{1}{2}} = k \frac{\left(\frac{m'}{r} - \frac{m}{r^2} \right) - \left(1 - \frac{2m}{r} \right) \frac{\mathcal{T}'}{\mathcal{T}}}{8\pi \mathcal{T} \left(1 - \frac{2m}{r} \right)^{\frac{3}{2}}}$$

$$8\pi \rho_{eff} + m = -r^2 \left(1 - \frac{2m}{r} \right) \frac{\mathcal{T}'}{\mathcal{T}}$$

$$\frac{\mathcal{T}'}{\mathcal{T}} = \frac{p'}{p + \rho} = \frac{1}{8\pi + 4\chi} \left(\frac{\alpha p'_{eff} + \chi \rho'_{eff}}{p_{eff} + \rho_{eff}} \right)$$

$$\frac{dp_{eff}}{dr} = -(8\pi + 4\chi) \frac{(4\pi p_{eff} r^3 + m)(p_{eff} + \rho_{eff})}{r^2 \left(1 - \frac{2m}{r} \right) \left(\alpha + \chi \frac{\partial \rho_{eff}}{\partial p_{eff}} \right)}$$

2. Geometría diferencial y características topológicas tanto de las partículas y antipartículas supermasivas como de las partículas y antipartículas masivas, en espacios cuánticos curvos.

2.1. Métrica tensorial por curvatura de Perelman – Hamilton – Ricci (PHR) en dimensión $\mathbb{R}^4$.

$$\nabla_i \nabla_j f + \mathcal{R}_{ij} + \frac{1}{2t} g_{ij}$$

$$\nabla_i \mathcal{R} = 2\mathcal{R}_{ij} \nabla_j f$$

$$\mathcal{R} - 2\text{Ric}(\mathcal{X}, \mathcal{X}) + 2 \frac{\det \text{Hess} f}{|\nabla f|^2} \leq \mathcal{R} - 2\text{Ric}(\mathcal{X}, \mathcal{X}) + \frac{(1 - \mathcal{R} + \text{Ric}(\mathcal{X}, \mathcal{X}))^2}{2|\nabla f|^2} \leq 1$$

$$|\nabla \mathcal{R}| \leq \eta \mathcal{R}^{\frac{3}{2}}, |\mathcal{R}_t| \leq \eta \mathcal{R}^2$$

$$f_t = f'' + a_1 f' + b_1 g' + c_1 f + d_1 g$$

$$g_t = f'' + a_2 f' + b_2 g' + c_2 f + d_2 g$$



$$\begin{aligned} \mathcal{R}m &\geq -\phi(\mathcal{R}(t+1))\mathcal{R} \\ 2\sqrt{t_0-t_\gamma} \int\limits_{t_\gamma}^{t_0} \sqrt{t_0-t} \left(\mathcal{R}(\gamma(t),t)+|\dot{\gamma}(t)|^2\right)dt &\geq \mathfrak{C}(\mathcal{A})r_0^2 \\ \square \hbar &\geq -\mathfrak{C}(\mathcal{A})\hbar - \left(6-1/\sqrt{\tau}\right)\phi \end{aligned}$$

$$\frac{d}{d\tau} \big(\log(\hbar_0(\tau)/\sqrt{\tau})\big) \leq \mathfrak{C}(\mathcal{A}) + \frac{6\sqrt{\tau}+1}{2\tau-4r^2\sqrt{\tau}} - \frac{1}{2\tau} \leq \mathfrak{C}(\mathcal{A}) + \frac{50}{\sqrt{\tau}}$$

$$\frac{d}{dt}\mathcal{R}=\triangle\mathcal{R}+2|\mathit{Ric}|^2=\triangle\mathcal{R}+2|\mathit{Ric}^\circ|^2+\frac{2}{3}\mathcal{R}^2$$

$$\mathcal{R}_{mint}(t) \geq -\frac{3}{2} \frac{1}{t+1/4}$$

$$\frac{d}{dt}\mathcal{V}\leq -\mathcal{R}_{min}\mathcal{V}$$

$$\frac{d}{dt}\hat{\mathcal{R}}(t)\geq \frac{2}{3}\hat{\mathcal{R}}\mathcal{V}^{-1}\int (\mathcal{R}_{min}-\mathcal{R})\,d\mathcal{V}$$

$$|2t\mathcal{R}_{ij}+g_{ij}|<\xi$$

$$Vol\,B(x,t,\rho(x,t))\leq \omega\rho^3(x,t)\int (4|\nabla_\alpha|^2+\mathcal{R}\alpha^2)=4\,\triangle\,\alpha-\lambda^-\alpha$$

$$\begin{aligned} \int_{\mathcal{M}_z} -4\alpha\alpha_z &= \int_{\mathcal{M}_z^+} (4|\nabla_\alpha|^2+\mathcal{R}\alpha^2-\lambda^-\alpha^2) \geq \frac{r_0^{-2}}{2} \int_{\mathcal{M}_z^+} \alpha^2 \left| \int_{\mathcal{M}_z} 2\alpha\alpha_z - \left(\int_{\mathcal{M}_z} \alpha^2 \right)_z \right| \\ &\leq const \bigotimes \int_{\mathcal{M}_z} \epsilon r_0^{-1} \alpha^2 \int_{\mathcal{M}_0^+} \alpha^2 \geq \exp\left(\frac{\epsilon^{-1}}{10}\right) \int_{\mathcal{M}_{\epsilon^{-1}r_0}^+} \alpha^2 \end{aligned}$$

$$\begin{aligned} \square \\ \delta\mathcal{F}(v_{ij},\hbar) &= \int\limits_{\mathcal{M}} e^{-f} \Big[-\triangle v + \nabla_i\nabla_j v_{ij} - \mathcal{R}_{ij}v_{ij} - v_{ij}\nabla_i f\nabla_j f + 2\langle\nabla f,\nabla\hbar\rangle + (\mathcal{R}+|\nabla f|^2)\Big(\frac{v}{2}-\hbar\Big) \Big] \\ \square \\ &= \int\limits_{\mathcal{M}} e^{-f} \Big[-v_{ij}(\mathcal{R}_{ij}+\nabla_i\nabla_j f) + \Big(\frac{v}{2}-\hbar\Big)(2\triangle f-|\nabla f|^2+\mathcal{R}) \Big] \end{aligned}$$

$$(g_{ij})_t=-2(\mathcal{R}_{ij}+\nabla_i\nabla_j f)$$

$$f_t=-\mathcal{R}-\triangle f$$

$$\mathcal{F}_t^m=2\int |\mathcal{R}_{ij}+\nabla_i\nabla_j f|^2\,dm$$



$$(g_{ij})_t = -2\mathcal{R}_{ij}, f_t = -\Delta f + |\nabla f|^2 - \mathcal{R}$$

$$\mathcal{F}_t = 2 \int |\mathcal{R}_{ij} + \nabla_i \nabla_j f|^2 e^{-f} d\mathcal{V}$$

$$\mathcal{F}_t \geq \frac{2}{n} \int (\mathcal{R} + \Delta f)^2 e^{-f} d\mathcal{V} \geq \frac{2}{n} \left(\int (\mathcal{R} + \Delta f) e^{-f} d\mathcal{V} \right)^2 = \frac{2}{n} \mathcal{F}^2$$

$$\begin{aligned} \frac{d\bar{\lambda}(t)}{dt} &\geq 2\mathcal{V}^{2/n} \int |\mathcal{R}_{ij} + \nabla_i \nabla_j f|^2 e^{-f} d\mathcal{V} \\ &\quad + \frac{2}{n} \mathcal{V}^{(2-n)/n} \lambda \int -\mathcal{R} d\mathcal{V} \\ &\geq 2\mathcal{V}^{2/n} \left[\int \left| \mathcal{R}_{ij} + \nabla_i \nabla_j f - \frac{1}{n} (\mathcal{R} + \Delta f) g_{ij} \right|^2 e^{-f} d\mathcal{V} \right. \\ &\quad \left. + \frac{1}{n} \left(\int (\mathcal{R} + \Delta f)^2 e^{-f} d\mathcal{V} - \left(\int (\mathcal{R} + \Delta f) e^{-f} d\mathcal{V} \right)^2 \right) \right] \end{aligned}$$

$$\begin{aligned} \mathcal{W}(g_{ij}, f, \tau) &= \int_{\mathcal{M}} [\tau(|\nabla f|^2 + \mathcal{R}) + f + n] (8\pi\tau)^{-n/2} e^{-f} d\mathcal{V} \\ &\int_{\mathcal{M}} (8\pi\tau)^{-n/2} e^{-f} d\mathcal{V} = 1 \end{aligned}$$

$$(g_{ij})_t = -2\mathcal{R}_{ij}, f_t = -\Delta f + |\nabla f|^2 - \mathcal{R} + \frac{n}{2\tau}, \tau_t = -1$$

$$\begin{aligned} \frac{d\mathcal{W}}{dt} &= \int_{\mathcal{M}} 2\tau \left| \mathcal{R}_{ij} + \nabla_i \nabla_j f - \frac{1}{2\tau} g_{ij} \right|^2 (8\pi\tau)^{-n/2} e^{-f} d\mathcal{V} \\ \mathcal{W}\left(\frac{1}{2}\tau^{-1}g_{ij}, f^\tau, \frac{1}{2}\right) &= \mathcal{W}(g_{ij}, f^\tau, \tau) = \mu(g_{ij}, \tau) \leq c \end{aligned}$$

$$\begin{aligned} &\int_{\mathbb{R}^n} \left[\frac{1}{2} |\nabla f|^2 + f - n \right] (2\pi)^{-\frac{n}{2}} e^{-f} d\chi \\ \langle \mathfrak{E} \rangle &= -\tau^2 \int_{\mathcal{M}} \left(\mathcal{R} + |\nabla f|^2 - \frac{n}{2\tau} \right) dm \frac{\partial^2}{(\partial \beta)^2} \log Z \end{aligned}$$

$$\mathcal{S} = - \int_{\mathcal{M}} (\tau(\mathcal{R} + |\nabla f|^2) + f - n) dm$$

$$\sigma = 2\tau^4 \int_{\mathcal{M}} \left| \mathcal{R}_{ij} + \nabla_i \nabla_j f - \frac{1}{2\tau} g_{ij} \right|^2 dm$$

$$\tilde{g}_{ij} = g_{ij}, \tilde{g}_{\alpha\beta} = \tau g_{\alpha\beta}, \tilde{g}_{00} = \frac{\mathcal{N}}{2\tau} + \mathcal{R}, \tilde{g}_{i\alpha} = \tilde{g}_{i0} = \tilde{g}_{\alpha 0}$$



$$\tilde{g}_{ij}^m = \tilde{g}_{ij}, \tilde{g}_{\alpha\beta}^m = \left(1 - \frac{2f}{\mathcal{N}}\right) \tilde{g}_{\alpha\beta}, \tilde{g}_{00}^m = \tilde{g}_{00} - 2f_\tau - \frac{f}{\tau}, \tilde{g}_{i0}^m = -\nabla_i f, \tilde{g}_{i\alpha}^m = \tilde{g}_{\alpha 0}^m$$

$$g_{\alpha\beta}^m = \left(1 - \frac{2f}{\mathcal{N}}\right) \tilde{g}_{\alpha\beta}, g_{00}^m = \tilde{g}_{00} - |\nabla f|^2 = \frac{1}{\tau} \left(\frac{\mathcal{N}}{2} - [\tau(2 \triangle f - |\nabla f|^2 + \mathcal{R}) + f - n] \right) g_{i0}^m = g_{\alpha 0}^m = g_{i\alpha}^m = 0$$

$$\int_0^{\tau(q)} \sqrt{\frac{\mathcal{N}}{2\tau}} + \mathcal{R} + |\dot{\gamma}_{\mathcal{M}}(\tau)|^2 d\tau = \sqrt{2\mathcal{N}\tau(q)} + \frac{1}{\sqrt{2\mathcal{N}}} \int_0^{\tau(q)} \sqrt{\tau} (\mathcal{R} + |\dot{\gamma}_{\mathcal{M}}(\tau)|^2) d\tau + \mathcal{O}(\mathcal{N}^{-3/2})$$

$$\int_{\mathcal{M}}^{\square} \tau(q)^{-n/2} \exp\left(-\frac{1}{\sqrt{2\tau(q)}} \mathcal{L}(\chi)\right) d\chi + \mathcal{O}(\mathcal{N}^{-1})$$

$$\mathcal{L}(\gamma) = \int_{\tau_2}^{\tau_1} \sqrt{\tau} \left(\mathcal{R}(\gamma(\tau)) + |\gamma(\tau)|^2 \right) d\tau$$

$$\begin{aligned} \int_{\tau_1}^{\tau_2} \sqrt{\tau} (\langle \mathcal{Y}, \nabla \mathcal{R} \rangle + 2 \langle \nabla_{\mathcal{Y}} \mathcal{X}, \mathcal{X} \rangle) d\tau &= \int_{\tau_1}^{\tau_2} \sqrt{\tau} (\langle \mathcal{Y}, \nabla \mathcal{R} \rangle + 2 \langle \nabla_{\mathcal{X}} \mathcal{Y}, \mathcal{X} \rangle) d\tau \\ &= \int_{\tau_1}^{\tau_2} \sqrt{\tau} \left(\langle \mathcal{Y}, \nabla \mathcal{R} \rangle + 2 \frac{d}{d\tau} \langle \mathcal{Y}, \mathcal{X} \rangle - 2 \langle \mathcal{Y}, \nabla_{\mathcal{X}} \mathcal{X} \rangle - 4 Ric(\mathcal{Y}, \mathcal{X}) \right) d\tau \\ &= 2\sqrt{\tau} \mathcal{X}, \mathcal{Y} \Big|_{\tau_1}^{\tau_2} + \int_{\tau_1}^{\tau_2} \sqrt{\tau} \langle \mathcal{Y}, \nabla \mathcal{R} - 2 \nabla_{\mathcal{X}} \mathcal{X} - 4 Ric(\mathcal{X}, \odot) - \frac{1}{\tau} \mathcal{X} \rangle d\tau \end{aligned}$$

$$\nabla_{\mathcal{X}} \mathcal{X} - \frac{1}{2} \nabla \mathcal{R} + \frac{1}{2\tau} \mathcal{X} + 2 Ric(\mathcal{X}, \odot)$$

$$\mathcal{L}_{\bar{\tau}}(q, \bar{\tau}) = \sqrt{\bar{\tau}}(\mathcal{R} + |\mathcal{X}|^2) - \langle \mathcal{X}, \nabla \mathcal{L} \rangle = 2\sqrt{\bar{\tau}}\mathcal{R} - \sqrt{\bar{\tau}}(\mathcal{R} + |\mathcal{X}|^2)$$

$$\begin{aligned} \frac{d}{d\tau} (\mathcal{R}(\gamma(\tau)) + |\mathcal{X}(\tau)|^2) &= \mathcal{R}_\tau + \langle \nabla \mathcal{R}, \mathcal{X} \rangle + 2 \langle \nabla_{\mathcal{X}} \mathcal{X}, \mathcal{X} \rangle + 2 Ric(\mathcal{X}, \mathcal{X}) \\ &= \mathcal{R}_\tau + \frac{1}{\tau} \mathcal{R} + 2 \langle \nabla \mathcal{R}, \mathcal{X} \rangle - 2 Ric(\mathcal{X}, \mathcal{X}) - \frac{1}{\tau} (\mathcal{R} + |\mathcal{X}|^2) = -\mathcal{H}(\mathcal{X}) - \frac{1}{\tau} (\mathcal{R} + |\mathcal{X}|^2) \end{aligned}$$

$$\bar{\tau}^{3/2}(\mathcal{R} + |\mathcal{X}|^2)(\bar{\tau}) = -\mathcal{K} + \frac{1}{2} \mathcal{L}(q, \bar{\tau})$$

$$\mathcal{L}_{\bar{\tau}} = 2\sqrt{\bar{\tau}}\mathcal{R} - \frac{1}{2\bar{\tau}}\mathcal{L} + \frac{1}{\bar{\tau}}\mathcal{K}$$



$$\begin{aligned}\delta_{\mathcal{Y}}^2(\mathcal{L}) &= \int_0^{\bar{\tau}} \sqrt{\tau} \left(\mathcal{Y} \otimes \mathcal{Y} \otimes \mathcal{R} + 2\langle \nabla_{\mathcal{Y}} \nabla_{\mathcal{Y}} \mathcal{X}, \mathcal{X} \rangle + 2|\nabla_{\mathcal{Y}} \mathcal{X}|^2 \right) d\tau \\ &= \int_0^{\bar{\tau}} \sqrt{\tau} \left(\mathcal{Y} \otimes \mathcal{Y} \otimes \mathcal{R} + 2\langle \nabla_{\mathcal{Y}} \nabla_{\mathcal{Y}} \mathcal{Y}, \mathcal{X} \rangle + 2\langle \mathcal{R}(\mathcal{Y}, \mathcal{X}) \mathcal{Y}, \mathcal{X} \rangle + 2|\nabla_{\mathcal{X}} \mathcal{Y}|^2 \right) d\tau\end{aligned}$$

$$\frac{d}{d\tau} \langle \nabla_{\mathcal{Y}} \mathcal{Y}, \mathcal{X} \rangle = \langle \nabla_{\mathcal{X}} \nabla_{\mathcal{Y}} \mathcal{Y}, \mathcal{X} \rangle + \langle \nabla_{\mathcal{Y}} \mathcal{Y}, \nabla_{\mathcal{X}} \mathcal{X} \rangle + 2\mathcal{Y} \otimes Ric(\mathcal{Y}, \mathcal{X}) - \mathcal{X} \otimes Ric(\mathcal{Y}, \mathcal{Y})$$

$$\begin{aligned}\delta_{\mathcal{Y}}^2(\mathcal{L}) &= 2\langle \nabla_{\mathcal{Y}} \mathcal{Y}, \mathcal{X} \rangle \sqrt{\bar{\tau}} \\ &\quad + \int_0^{\bar{\tau}} \sqrt{\tau} \left(\nabla_{\mathcal{Y}} \nabla_{\mathcal{Y}} \mathcal{R} + 2\langle \mathcal{R}(\mathcal{Y}, \mathcal{X}), \mathcal{Y}, \mathcal{X} \rangle + 2|\nabla_{\mathcal{X}} \mathcal{Y}|^2 + 2\nabla_{\mathcal{X}} Ric(\mathcal{Y}, \mathcal{Y}) \right. \\ &\quad \left. - 4\nabla_{\mathcal{Y}} Ric(\mathcal{Y}, \mathcal{X}) \right) d\tau\end{aligned}$$

$$\nabla_{\mathcal{X}} \mathcal{Y} = -Ric(\mathcal{Y}, \odot) + \frac{1}{2\tau} \mathcal{Y}$$

$$\begin{aligned}\frac{d}{d\tau} \langle \mathcal{Y}, \mathcal{Y} \rangle &= 2Ric(\mathcal{Y}, \mathcal{Y}) + 2\langle \nabla_{\mathcal{X}} \mathcal{Y}, \mathcal{Y} \rangle = \frac{1}{\tau} \langle \mathcal{Y}, \mathcal{Y} \rangle \mathcal{H}_{ess_{\mathcal{L}}}(\mathcal{Y}, \mathcal{Y}) \\ &\leq \int_0^{\bar{\tau}} \sqrt{\tau} \left(\nabla_{\mathcal{Y}} \nabla_{\mathcal{Y}} \mathcal{R} + 2\langle \mathcal{R}(\mathcal{Y}, \mathcal{X}), \mathcal{Y}, \mathcal{X} \rangle + 2\nabla_{\mathcal{X}} Ric(\mathcal{Y}, \mathcal{Y}) - 4\nabla_{\mathcal{Y}} Ric(\mathcal{Y}, \mathcal{X}) \right. \\ &\quad \left. + 2|Ric(\mathcal{Y}, \odot)|^2 - \frac{2}{\tau} Ric(\mathcal{Y}, \mathcal{Y}) + \frac{1}{2\tau\bar{\tau}} \right) d\tau\end{aligned}$$

$$\begin{aligned}\frac{d}{d\tau} Ric(\mathcal{Y}(\tau), \mathcal{Y}(\tau)) &= Ric_{\tau}(\mathcal{Y}, \mathcal{Y}) + \nabla_{\mathcal{X}} Ric(\mathcal{Y}, \mathcal{Y}) + 2Ric(\nabla_{\mathcal{X}} \mathcal{Y}, \mathcal{Y}) \\ &= Ric_{\tau}(\mathcal{Y}, \mathcal{Y}) + \nabla_{\mathcal{X}} Ric(\mathcal{Y}, \mathcal{Y}) + \frac{1}{\tau} Ric(\mathcal{Y}, \mathcal{Y}) - 2|Ric(\mathcal{Y}, \odot)|^2\end{aligned}$$

$$\mathcal{H}_{ess_{\mathcal{L}}}(\mathcal{Y}, \mathcal{Y}) \leq \frac{1}{\sqrt{\bar{\tau}}} - 2\sqrt{\bar{\tau}} Ric(\mathcal{Y}, \mathcal{Y}) - \int_0^{\bar{\tau}} \sqrt{\tau} \mathcal{H}(\mathcal{X}, \mathcal{Y}) d\tau$$

$$\begin{aligned}\mathcal{H}(\mathcal{X}, \mathcal{Y}) &= -\nabla_{\mathcal{Y}} \nabla_{\mathcal{Y}} \mathcal{R} - 2\langle \mathcal{R}(\mathcal{Y}, \mathcal{X}), \mathcal{Y}, \mathcal{X} \rangle - 4 \left(\nabla_{\mathcal{X}} Ric(\mathcal{Y}, \mathcal{Y}) - \nabla_{\mathcal{Y}} Ric(\mathcal{Y}, \mathcal{X}) \right) - 2Ric_{\tau}(\mathcal{Y}, \mathcal{Y}) \\ &\quad + 2|Ric(\mathcal{Y}, \odot)|^2 - \frac{1}{\tau} Ric(\mathcal{Y}, \mathcal{Y})\end{aligned}$$

$$\Delta \mathcal{L} \leq -2\sqrt{\tau} \mathcal{R} + \frac{n}{\sqrt{\tau}} - \frac{1}{\tau} \mathcal{K}$$

$$\begin{aligned}\frac{d}{d\tau} |\mathcal{Y}|^2 &= 2Ric(\mathcal{Y}, \mathcal{Y}) + 2\langle \nabla_{\mathcal{X}} \mathcal{Y}, \mathcal{Y} \rangle = 2Ric(\mathcal{Y}, \mathcal{Y}) + 2\langle \nabla_{\mathcal{Y}} \mathcal{X}, \mathcal{Y} \rangle = 2Ric(\mathcal{Y}, \mathcal{Y}) + \frac{1}{\sqrt{\bar{\tau}}} \mathcal{H}_{ess_{\mathcal{L}}}(\mathcal{Y}, \mathcal{Y}) \\ &\leq \frac{1}{\bar{\tau}} - \frac{1}{\sqrt{\bar{\tau}}} \int_0^{\bar{\tau}} \tau^{1/2} \mathcal{H}(\mathcal{X}, \mathcal{Y}) d\tau\end{aligned}$$



$$\frac{d}{d\tau}\log\mathcal{J}(\tau)\leq\frac{n}{2\bar{\tau}}-\frac{1}{2}\bar{\tau}^{-3/2}\mathcal{K}$$

$$\frac{d}{d\tau}l(\tau)=\frac{1}{2\bar{\tau}}l+\frac{1}{2}(\mathcal{R}+|\mathcal{X}|^2)=-\frac{1}{2}\bar{\tau}^{-\frac{3}{2}}\mathcal{K}$$

$$l_{\bar{\tau}}-\triangle l+|\nabla l|^2-\mathcal{R}+\frac{n}{2\bar{\tau}}\geq 2\triangle l-|\nabla l|^2+\mathcal{R}+\frac{l-n}{\bar{\tau}}\leq \bar{\mathcal{L}}_{\bar{\tau}}+\triangle \bar{\mathcal{L}}\leq 2n$$

$$|\nabla l|^2+\mathcal{R}\leq \frac{\mathfrak{C}l}{\tau}$$

$$\frac{d}{d\tau}\log\,|\mathcal{Y}|^2\leq\frac{1}{\tau}(\mathfrak{C}l+1)$$

$$d_t-\triangle d\geq -(n-1)\left(\frac{2}{3}\mathcal{K}r_0+r_0^{-1}\right)$$

$$\frac{d}{dt}dist_t(x_0,x_1)\geq -2(n-1)\left(\frac{2}{3}\mathcal{K}r_0+r_0^{-1}\right)at$$

$$\begin{aligned}\triangle d\leq \sum_{k=1}^{n-1}s''_{y_k}(\gamma)&= \int_{r_0}^{d(x,t_0)}-Ric(\mathcal{X},\mathcal{X})\,ds+\int_0^{r_0}\left(\frac{s^2}{r_0^2}(-Ric(\mathcal{X},\mathcal{X}))+\frac{n-1}{r_0^2}\right)ds\\&\stackrel{\square}{=} \int_{\gamma}-Ric(\mathcal{X},\mathcal{X})+\int_0^{r_0}\left(Ric(\mathcal{X},\mathcal{X})\left(1-\frac{s^2}{r_0^2}\right)+\frac{n-1}{r_0^2}\right)ds\\&\leq d_t+(n-1)\left(\frac{2}{3}\mathcal{K}r_0+r_0^{-1}\right)\end{aligned}$$

$$\frac{2(\phi')^2}{\phi}-\phi''\geq (2A+100n)\phi'-\mathfrak{C}(A)\phi$$

$$\square \hbar = (\bar{\mathcal{L}}+2n+1)(-\phi''+(d_t-\triangle d-2A)\phi')-2<\nabla \phi \nabla \bar{\mathcal{L}}>+(\bar{\mathcal{L}}_t-\triangle \bar{\mathcal{L}})\phi$$

$$\nabla \hbar = (\bar{\mathcal{L}}+2n+1)\nabla \phi + \phi \nabla \bar{\mathcal{L}}$$

$$\square \hbar = (\bar{\mathcal{L}}+2n+1)\left(-\phi''+(d_t-\triangle d-2A)\phi'+\frac{2(\phi')^2}{\phi}\right)+(\bar{\mathcal{L}}_t-\triangle \bar{\mathcal{L}})\phi$$

$$\square \hbar \geq -(\bar{\mathcal{L}}+2n+1)\mathfrak{C}(A)\phi-2n\phi\geq -(2n+\mathfrak{C}(A))\hbar$$

$$\square^\dagger v=-2(\mathcal{T}-t)\left|\mathcal{R}_{ij}+\nabla_i\nabla_jf-\frac{1}{2(\mathcal{T}-t)}g_{ij}\right|^2$$

$$-\frac{d}{dt}f(\gamma(t),t)\leq \frac{1}{2}(\mathcal{R}(\gamma(t),t)+|\dot{\gamma}(t)|^2)-\frac{1}{2(\mathcal{T}-t)}f(\gamma(t),t)$$



$$|\mathcal{R}m|(x,t)\leq 4|\mathcal{R}m|(\overline{x},t)$$

$$(x,t)\in \mathcal{M}_\alpha, 0< t\leq \bar{t}, d(x,t)\leq d(\overline{x},t)+\mathcal{A}|\mathcal{R}m|^{-1/2}(\overline{x},t)$$

$$\bar{t}-\frac{1}{2}\alpha Q^{-1}\leq t\leq \bar{t}, dist_{\bar{t}}(x,\bar{x})\leq \frac{1}{10}\mathcal{A}Q^{-\frac{1}{2}}\leq \alpha t^{-1}+2\epsilon^{-2}$$

$$\begin{aligned}\beta(1-A^{-2}) &\leq -\int_{\mathcal{M}} \hbar v \\ &= \int_{\mathcal{M}} [(-2\triangle f + |\nabla f|^2 - \mathcal{R})\bar{t} - f + n]\hbar v = \int_{\mathcal{M}} \left[-\bar{t}|\nabla \tilde{f}|^2 - \tilde{f} + n\right]\tilde{u} \\ &\quad + \int_{\mathcal{M}} \left[\bar{t}\left(\frac{|\nabla \hbar|^2}{\hbar} - \mathcal{R}\hbar\right) - \hbar \log \hbar\right]u \leq \int_{\mathcal{M}} \left[-\bar{t}|\nabla \tilde{f}|^2 - \tilde{f} + n\right]\tilde{u} + \mathcal{A}^{-2} + 100\epsilon^2\end{aligned}$$

$$\mathcal{R}_t+2\langle \mathcal{X},\nabla \mathcal{R}\rangle+2Ric(\mathcal{X},\mathcal{X})$$

$$\begin{aligned}\frac{d}{dt}A^t&\leq -2\pi-\frac{1}{2}\mathcal{R}_{mint}^tA^t\int_{\mathcal{D}_c}\Big(-Tr(Ric^T)\Big)+\int_c(-k_g)-\int_{\mathcal{D}_c}\Big(-\frac{1}{2}\mathcal{R}-\mathcal{K}\Big)+\int_c(-k_g)\\&=\int_{\mathcal{D}_c}\Big(-\frac{1}{2}\mathcal{R}\Big)-2\pi\end{aligned}$$

$$\frac{d}{dt}\mathcal{R}=\triangle \mathcal{R}+2|Ric|^2=\triangle \mathcal{R}+\frac{2}{3}\mathfrak{R}^2+2|Ric^\circ|^2$$

$$\frac{d}{dt}g(\mathcal{X},\mathcal{X})=2Ric(\mathcal{X},\mathcal{X})-2g(\mathcal{X},\mathcal{X})k^2$$

$$|\mathcal{H},\mathcal{S}|=\big(k^2+Ric(\mathcal{S},\mathcal{S})\big)\mathcal{S}$$

$$\frac{d}{dt}k^2=(k^2)''-2g\left((\nabla_{\mathcal{S}}\mathcal{H})^\perp,(\nabla_{\mathcal{S}}\mathcal{H})^\perp\right)+2k^4$$

$$\frac{d}{dt}k\leq k''+k^3+const\bigotimes(k+1)$$

$$\frac{d}{dt}\mathcal{L}\leq \int (const-k^2)\,ds$$

$$\frac{d}{dt}\Theta\leq \int const\bigotimes(k+1)\,ds$$

$$\frac{d}{dt}u = u'' + (k^2 + Ric(\mathcal{S}, \mathcal{S}))u$$

$$\begin{aligned} \frac{d}{dt}\mu^t &\leq (2n-1)|\mathcal{R}m^t|\mu^t \int_A (-Tr(Ric^T)) + \int_{\partial A} (-k_g) \leq \int_A (-Tr(Ric^T) + \mathcal{K}) \\ &\leq \int_A (-Tr(Ric^T) + \mathcal{R}m^T) \leq (2n-1)|\mathcal{R}m|\mu \end{aligned}$$

REFERENCIAS BIBLIOGRÁFICAS ADICIONALES

Rodrigo Garrido, ESTRUCTURA INTERNA DE ESTRELLAS RELATIVISTAS EN EXTENSIONES DE RELATIVIDAD GENERAL, Universidad de Valladolid, 2024.

Grisha Perelman, The entropy formula for the Ricci flow and its geometric applications, arXiv:math/0211159v1 [math.DG] 11 Nov 2002.

Grisha Perelman, Ricci flow with surgery on three-manifolds, arXiv:math/0303109v1 [math.DG] 10 Mar 2003.

Grisha Perelman, Finite extinction time for the solutions to the Ricci flow on certain three-manifolds, arXiv:math/0307245v1 [math.DG] 17 Jul 2003.

J. R. OPPENHEIMER y H. SNYDER, On Continued Gravitational Contraction, PHYSICAL REVIEW, 1939, Vol. 56.



APÉNDICE C.

Modelo Brout-Englert-Higgs para espacios cuánticos curvos, a propósito de la fenomenología de las partículas y antipartículas supermasivas e hiperpartículas.

1. Nociones preliminares.

$$\mathcal{L} = -\frac{1}{4}\mathcal{W}_{\mu\nu}^a\mathcal{W}_a^{\mu\nu} + (\mathcal{D}_\mu\phi_\alpha)^\dagger\mathcal{D}^\mu\phi_\alpha - \mathcal{V}(\phi) + \mathcal{L}_r$$

$$\mathcal{W}_{\mu\nu}^\alpha = \partial_\mu\mathcal{W}_\nu^\alpha - \partial_\nu\mathcal{W}_\mu^\alpha - gf^{abc}\mathcal{W}_\mu^b\mathcal{W}_\nu^c$$

$$\mathcal{D}_\mu^{ij} = \partial_\mu\delta^{ij} - ig\mathcal{W}_\mu^\alpha T_a^{\mathcal{R}\alpha ij}$$

$$\mathcal{L} = -\frac{1}{4}\mathcal{W}_{\mu\nu}^a\mathcal{W}_a^{\mu\nu} + (\mathcal{D}_\mu\phi)^\dagger\mathcal{D}^\mu\phi - \lambda((\phi\phi^\dagger)^2 - f^2)^2$$

$$\phi(\chi) = v + \eta(\chi)$$

$$\mathcal{L} = \partial_\mu\phi_i^\dagger\partial^\mu\phi_i - \mathcal{V}(\phi)$$

$$\mathcal{V}(\phi) = -\frac{\mu^2}{2}\phi^2 + \lambda\phi^4$$

$$\mathcal{V}(\phi) = -\frac{\mu^2}{2}r^2 + \lambda r^4$$

$$\mathcal{O}_\Phi = \mathcal{O}_\Phi^{\mathcal{G}} \Rightarrow g(\bar{g})\Phi = \Phi\forall\bar{g} \in \mathcal{G}$$

$$\mathcal{O}_\Phi = \mathcal{O}_\Phi^{\mathcal{H}_i} \Rightarrow \hbar(\bar{\hbar})\Phi = \Phi\forall\bar{\hbar} \in \mathcal{H}_i$$

$$\mathcal{Z} = \int d^{\mathcal{N}}\phi e^{-i\left(\frac{\mu^2}{2}\phi^2 + \lambda\phi^4\right)} = \Omega(\mathcal{N}) \int d^{\mathcal{N}-1}dr e^{-i\left(\frac{\mu^2}{2}r^2 + \lambda r^4\right)} = \Omega(\mathcal{N})f(\mu, \lambda)$$

$$\langle\phi_jg(\phi^2)\rangle = \int d^{\mathcal{N}}\phi\phi_jg(\phi^2)e^{-i\left(\frac{\mu^2}{2}\phi^2 + \lambda\phi^4\right)} = 0$$

$$\langle g(\phi^2)\rangle = \int d^{\mathcal{N}}\phi g(\phi^2)e^{-i\left(\frac{\mu^2}{2}\phi^2 + \lambda\phi^4\right)} = \Omega(\mathcal{N})\hbar(\mu, \lambda)$$

$$\mathcal{Z}[j] = \int d^{\mathcal{N}}\phi_i e^{-i\left(\frac{\mu^2}{2}\phi^2 + \lambda\phi^4 + j\phi\right)}, \langle\phi_i g(\phi^2)\rangle_{j\neq 0} \neq 0$$

$$\langle\phi_i\rangle_{j\neq 0} = \delta_{i_1}f(j)\lim_{j\rightarrow 0}f(j) = v \neq 0, \lim_{j\rightarrow 0}\langle\phi_i\rangle_j = v\delta_{i_1}0 = f(0) \neq \lim_{j\rightarrow 0}f(j)$$

$$m_r = \left\langle \left(\int d^d\chi \phi_i(\chi) \right)^2 \right\rangle \lim_{j\rightarrow 0} \lim_{V\rightarrow\infty} f(j)$$

2. Simetría de campo en espacios cuánticos curvos.



$$\mathcal{V}(\phi, \phi^\dagger) = -\frac{1}{2}\mu^2\phi^\dagger\phi + \frac{1}{2}\frac{\mu^2}{f^2}(\phi^\dagger\phi)^2\phi \mapsto e^{-i\theta}\phi \approx \phi - i\theta\phi$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\chi\partial^\mu\chi) + \frac{\mu^2}{2}(\sigma^2 + \chi^2) - \frac{1}{2}\frac{\mu^2}{f^2}(\sigma^2 + \chi^2)^2\sigma \mapsto \sigma + \theta_\chi\chi \mapsto \chi + \theta_\sigma$$

$$\sigma^2 + \chi^2 = \frac{f^2}{\sqrt{2}} = \phi^\dagger\phi\sigma \mapsto \sigma + \frac{v}{\sqrt{2}}\chi \mapsto \chi$$

$$\mathcal{V} = \frac{\mu^2}{f^2}\left(\phi\phi^\dagger - \frac{f^2}{\sqrt{2}}\right)^2$$

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\chi\partial^\mu\chi) + \frac{\mu^2 v}{\sqrt{2}}\left(\frac{v^2}{f^2} - 1\right)\sigma + \frac{\mu^2 v^2}{4}\left(\frac{v^2}{2f^2} - 1\right) - \mu^2\left(\frac{3v^2}{2f^2} - \frac{1}{2}\right)\sigma^2 \\ &\quad - \mu^2\left(\frac{1v^2}{2f^2} - \frac{1}{2}\right)\chi^2 + \frac{\sqrt{2}\mu^2 v}{f}\sigma(\sigma^2 + \chi^2) + \frac{\mu^2}{2f^2}(\sigma^2 + \chi^2)^2 \\ &\triangleq \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\chi\partial^\mu\chi) - \mu^2\sigma^2 + \frac{\sqrt{2}\mu^2}{f}\sigma(\sigma^2 + \chi^2) + \frac{1}{2}\frac{\mu^2}{f^2}(\sigma^2 + \chi^2)^2 \\ &\quad - \frac{\mu^2 f^2}{8}\sigma \mapsto \sigma + \theta_\chi\chi \mapsto \chi - \theta(\sigma - v)v \mapsto v + \theta_\chi\end{aligned}$$

3. Teorema de Goldstone para espacios cuánticos curvos.

$$\delta\phi_i = iT_a^{ij}\phi_j\theta^a$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi^i - \mathcal{V}(\phi)$$

$$0 = \frac{\partial\mathcal{V}}{\partial\phi_i}\delta\phi^i = i\frac{\partial\mathcal{V}}{\partial\phi_i}T_{ij}^a\phi^j\theta_a\frac{\partial^2\mathcal{V}}{\partial\phi_\kappa\partial\phi_i}\bigg|_{\phi=v} = \left((\mathcal{M}(v))^2\right)_{\kappa i}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\psi_i\partial^\mu\psi^i - \frac{1}{2}(\mathcal{M}^2)_{\kappa i}\mathcal{V}(\phi)\psi_\kappa\psi_i(\mathcal{M}^2)^{\kappa i}T_{ij}^av^jt_{ij}^av^j\tau_{ij}^av^j \neq 0$$

$$\phi^i\phi_i = \frac{v^iv_i}{\sqrt{2}} \triangleq \frac{f^2}{\sqrt{2}} > 0$$

3.1. Cuantización.

$$\Omega[j] = \frac{Z[j]}{Z[0]} = \frac{1}{Z[0]} \int \mathcal{D}\phi \exp\left(i \int d^4\chi(\mathcal{L} + j_i\phi_i)\right)$$

$$\begin{aligned}0 = \delta\Omega[j] &= \int \mathcal{D}\phi e^{i\delta + i \int d^4\chi j_i\phi_i} \left(\frac{\partial\delta\phi_i}{\partial\phi_j} + \delta\left(i\delta + i \int d^4\chi j_i\phi_i\right) \right) \int d^4\chi j_i T_{ik}^a \frac{\delta\Omega[\mathfrak{J}]}{i\delta j_\kappa} \delta\Omega \\ &\equiv \delta(e^{\Omega_c}) = e^{\Omega_c} \delta\Omega_c\end{aligned}$$



$$\begin{aligned}
0 &= \int d^4\chi j_i \mathcal{T}_{i\kappa}^a \frac{\delta\Omega_c[\mathfrak{S}]}{i\delta j_\kappa} \frac{\delta\Omega_c[\mathfrak{S}]}{i\delta j_i} \langle\phi_i\rangle \\
i\Gamma[\phi] &= i \int d^4\chi j_i \phi_i + \Omega_c[\mathfrak{S}] j_i \frac{\delta\Gamma[\phi]}{i\delta\phi_i} \int d^4\chi \frac{\delta\Gamma[\phi]}{\delta\phi_i} \mathcal{T}_{i\kappa}^a \langle\phi_\kappa\rangle \frac{i\delta^2\Gamma}{\delta\phi_i(\chi)\delta\phi_j(\gamma)} \\
&= \left((\mathcal{D}(\chi-\gamma))^{-1}\right)_{i\kappa} \int d^4\chi \left(\frac{i\delta^2\Gamma}{\delta\phi_i(\chi)\delta\phi_j(\gamma)} \mathcal{T}_{i\kappa}^a \langle\phi_\kappa\rangle\right) \left((\mathcal{G}(\wp=0))^{-1}\right)_{ij} \mathcal{T}_{i\kappa}^a \langle\phi_\kappa\rangle
\end{aligned}$$

4. Sistema de simetrías Brout-Englert-Higgs.

$$\begin{aligned}
\mathcal{L} &= \partial_\mu\phi^\dagger\partial^\mu\phi - \mathcal{V}(\phi) \\
\mathcal{L} &= -\frac{1}{4}\mathcal{W}_{\mu\nu}^a\mathcal{W}_a^{\mu\nu} + (\mathcal{D}_\mu\phi_\alpha)^\dagger\mathcal{D}^\mu\phi_\alpha - \mathcal{V}(\phi) \\
\mathcal{W}_{\mu\nu}^\alpha &= \partial_\mu\mathcal{W}_\nu^\alpha - \partial_\nu\mathcal{W}_\mu^\alpha - gf^{abc}\mathcal{W}_\mu^b\mathcal{W}_\nu^c \\
\mathcal{D}_\mu &= \partial_\mu + g\mathcal{T}^\mathcal{R} \\
\mathcal{X} &= \begin{pmatrix} \phi_1 & -\phi_2^\dagger \\ \phi_1 & \phi_1^\dagger \end{pmatrix} \\
\mathcal{L} &= -\frac{1}{4}\mathcal{W}_{\mu\nu}^a\mathcal{W}_a^{\mu\nu} + \frac{1}{2}tr\left((\mathcal{D}_\mu\mathcal{X})^\dagger\mathcal{D}_\mu\mathcal{X}\right) - \lambda\left(\frac{1}{2}tr(\mathcal{X}^\dagger\mathcal{X}) - f^2\right)^2 \\
\mathcal{X} &= \rho\alpha \\
\phi^\dagger\phi &= f^2 \\
\mathcal{X} &= v\alpha \\
\phi_i^c &= v_j\eta_i^j \\
\mathcal{X}^c &= v1 \\
\mathfrak{G}\times\mathfrak{C} &\mapsto\mathfrak{G}'\times\mathfrak{C}'=\mathcal{H}' \\
g(\overline{h}')\phi_i^c &= \phi_i^c \\
\mathcal{A}^\dagger\mathcal{X}^c\mathcal{A} &= v\mathcal{A}^\dagger 1\mathcal{A} = v1
\end{aligned}$$

5. Campos de Gauge en espacios cuánticos curvos.

$$\begin{aligned}
Z &= \int \mathcal{D}\phi_i \mathcal{D}\mathcal{W}_\mu^a e^{i\int d^d\chi \mathcal{L}} \\
\mathcal{W}_\mu &\mapsto g\mathcal{W}_\mu g^{-1} - i(\partial_\mu g)g^{-1} \approx \mathcal{T}^a(\mathcal{W}_\mu^a + \mathcal{D}_\mu^{ab}\theta^b) + \mathcal{O}(\theta^2) \\
\mathcal{D}_\mu^{ab} &= \delta^{ab}\partial_\mu + gf^{abc}\mathcal{W}_\mu^c
\end{aligned}$$



$$\mathfrak{E}^a = \partial^\mu \mathcal{W}_\mu^a$$

$$\Delta^{-1}[\mathcal{W}_\mu^a] = \int \mathcal{D}g \delta(\mathfrak{E}^a[\mathcal{W}_\mu^{ag}])$$

$$1 = \Delta[\mathcal{W}_\mu^a] = \int \mathcal{D}g \delta(\mathfrak{E}^a[\mathcal{W}_\mu^{ag}])$$

$$\begin{aligned} Z &= \int \mathcal{D}\mathcal{W}_\mu^a \Delta[\mathcal{W}_\mu^a] \int \mathcal{D}g \delta(\mathfrak{E}^a[\mathcal{W}_\mu^{ag}]) \exp(i\delta[\mathcal{W}_\mu^a]) \\ &= \left(\int \mathcal{D}g \right) \int \mathcal{D}\mathcal{W}_\mu^a \Delta[\mathcal{W}_\mu^a] \delta(\mathfrak{E}^a[\mathcal{W}_\mu^{ag}]) \exp(i\delta[\mathcal{W}_\mu^a]) \end{aligned}$$

$$\Delta[\mathcal{W}_\mu^a] = \left(\det \frac{\delta \mathfrak{E}^a(\chi)}{\delta \theta^b(\gamma)} \right)_{\mathfrak{E}^a=0} = \det \mathcal{M}^{ab}(\chi, \gamma)$$

$$\mathcal{M}^{ab}(\chi, \gamma) = \frac{\delta \mathfrak{E}^a(\chi)}{\delta \theta^b(\gamma)} = \int d^4z \sum_{ij} \frac{\delta \mathfrak{E}^a(\chi)}{\delta \omega_j^i(z)} \frac{\delta \omega_j^i(z)}{\delta \theta^b(\gamma)}$$

$$\mathcal{M}^{ab}(\chi, \gamma) = \frac{\delta \mathfrak{E}^a(\chi)}{\delta \mathcal{W}_\mu^c(\gamma)} \mathcal{D}_\mu^{cb}(\gamma)$$

$$\det \mathcal{M} \sim \int \mathcal{D}c^a \mathcal{D}\bar{c}^a \exp \left(-i \int d^4\chi d^4\gamma \bar{c}^a(\chi) \mathcal{M}^{ab}(\chi, \gamma) c^b(\gamma) \right)$$

$$\mathcal{M}^{ab}(\chi, \gamma) = \partial^\mu \mathcal{D}_\mu^{ab} \delta(\chi - \gamma)$$

$$Z = \int \mathcal{D}\mathcal{W}_\mu^a \mathcal{D}c^a \mathcal{D}\bar{c}^a \exp \left(i\delta - \frac{i}{2\xi} \int d^4\chi (\partial^\mu \mathcal{W}_a^\mu)^2 - i \int d^4\chi \bar{c}^a \partial^\mu \mathcal{D}_\mu^{ab} c^b \right)$$

$$\mathfrak{E}^a = \partial^\mu \mathcal{W}_\mu^a + ig\zeta \phi_i \mathcal{T}_{ij}^a v_j + \Lambda^a$$

$$\begin{aligned} Z &= \int \mathcal{D}\mathcal{W}_\mu^a \mathcal{D}c^a \mathcal{D}\bar{c}^a \exp \left(i\delta \right. \\ &\quad \left. - \frac{i}{2\xi} \int d^4\chi |\partial^\mu \mathcal{W}_a^\mu + ig\zeta \phi_i \mathcal{T}_{ij}^a v_j|^2 - i \int d^4\chi \bar{c}^a (\partial^\mu \mathcal{D}_\mu^{ab} - \zeta g^2 v_i \mathcal{T}_{ij}^a \mathcal{T}_{jk}^b \phi_k) c^b \right) \end{aligned}$$

$$\phi(\chi) = v + \varphi(\chi)$$

$$\mathcal{L} = -\frac{1}{4} \mathcal{W}_{\mu\nu}^a \mathcal{W}_a^{\mu\nu} + \frac{1}{2} \left((\partial_\mu \delta_{ij} - ig \mathcal{W}_\mu^a \mathcal{T}_{ij}^a) \phi_j \right)^\dagger - (\partial_\mu \delta_{ik} - ig \mathcal{W}_\mu^b \mathcal{T}_{jk}^b) \phi - \lambda (\phi^2 - f^2)^2$$



$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}\mathcal{W}_{\mu\nu}^a\mathcal{W}_a^{\mu\nu} + \frac{1}{2}(\mathcal{D}_\mu^{ij}\varphi_j)^\dagger(\mathcal{D}_{i\kappa}^\mu\varphi_\kappa) + \frac{g^2f^2}{2}\mathcal{T}_{ij}^a\mathcal{T}_{j\kappa}^b\eta_j^\dagger\eta_\kappa\mathcal{W}_\mu^a\mathcal{W}_\mu^b - \lambda f^2|\eta\varphi|^2 \\ & + \frac{ig^2\xi f^2}{2}|\varphi\mathcal{T}^a\eta|^2 + i\xi g^2f^2\eta_i^\dagger\mathcal{T}_{ij}^a\mathcal{T}_{j\kappa}^b\eta_\kappa\bar{c}^ac^b - 2\lambda f\left((\eta\varphi) + (\eta\varphi)^\dagger\right)\varphi^2 \\ & - \lambda(\varphi^2)^2 + i\bar{c}^a\xi g^2f(\eta_i\mathcal{T}_{ij}^a\mathcal{T}_{j\kappa}^b\varphi_\kappa^\dagger + \eta_i^\dagger\mathcal{T}_{ij}^a\mathcal{T}_{j\kappa}^b\varphi_\kappa)c^b - \frac{i}{2\xi}(\partial^\mu\mathcal{W}_\mu^a)^2 \\ & - i\bar{c}^a\partial^\mu\mathcal{D}_\mu^{ab}c^b\end{aligned}$$

$$\mathcal{D}_{\mu\nu}^{ab}=\delta^{ab}\left(\left(g_{\mu\nu}-\frac{\kappa_\mu\kappa_\nu}{\kappa^2}\right)\frac{-i}{\kappa^2-m_{\mathcal{W}}^2}\right)-\frac{i\xi\kappa_\mu\kappa_\nu}{\kappa^2(\kappa^2-\xi m_{\mathcal{W}}^2)}\Bigg)$$

$$\mathcal{F}[\mathcal{W}_\mu^a]=\int d^d\chi\,\mathcal{W}_\mu^a\mathcal{W}_\mu^a$$

$$\mathcal{W}_\mu^a\mapsto \mathcal{W}_\mu^a+\partial_\mu\omega^a+igf^{abc}\mathcal{W}_\mu^b\omega^c$$

$$\begin{aligned}\delta = \sum_{\chi} \bigg(& \phi^\dagger(\chi)\phi(\chi) + \gamma(\phi^\dagger(\chi)\phi(\chi) - 1)^2 - \kappa \sum_{\pm\mu} \phi(\chi)^\dagger \mathcal{U}_\mu^{\mathcal{R}}(\chi) \phi(\chi + \mu) \\ & + \frac{\beta}{d_{\mathcal{F}}} \sum_{\mu < \nu} \Re tr(1 - \mathcal{U}(\chi)_{\mu\nu}) \bigg)\end{aligned}$$

$$\mathcal{U}(\chi)_{\mu\nu}=\mathcal{U}_\mu(\chi)\mathcal{U}_\nu(\chi+\mu)\mathcal{U}_\mu(\chi+\nu)^\dagger\mathcal{U}_\nu(\chi)^\dagger$$

$$\mathcal{U}_\mu(\chi)=\exp(i\mathcal{W}_\mu^a\sigma_a)$$

$$\beta=\frac{2\mathfrak{G}_{\mathfrak{F}}}{g^2}$$

$$-a^2(2\lambda v^2)=\frac{1-2\gamma}{\kappa}-2d$$

$$\frac{1}{2\lambda}=\frac{\kappa^2}{2\gamma}$$

6. Métrica de Elitzur – Higgs para espacios cuánticos curvos.

$$\langle f(\mathcal{W})\rangle=\int \mathcal{D}\mu f(\mathcal{W})=\int \mathcal{D}\mu^{g^{-1}}f(\mathcal{W})=\int \mathcal{D}\mu f\left(\mathcal{W}^g\right)=\int \mathcal{D}\mu f\left(\mathcal{W}\right)^g=\langle f(\mathcal{W})^g\rangle$$

7. Construcción de Osterwalder-Seiler-Fradkin-Shenker para espacios cuánticos curvos.

$$g=\alpha^{-1}$$

$$\delta=-\kappa\mathcal{H}(\mathcal{V}_\mu)-\beta\delta_{y\mathcal{M}}$$



$$\begin{aligned}\langle \mathcal{O} \rangle &= \frac{\int \Pi_{\chi,\mu} d\mathcal{V}_\mu(\chi) \mathcal{O} e^{-\kappa \mathcal{H}(\mathcal{V}_\mu) - \beta \delta y_{\mathcal{M}}}}{\int \Pi_{\chi,\mu} d\mathcal{V}_\mu(\chi) e^{-\kappa \mathcal{H}(\mathcal{V}_\mu) - \beta \delta y_{\mathcal{M}}}} = \frac{\int \Pi_{\chi,\mu} d\mathcal{V}_\mu(\chi) e^{-\kappa \mathcal{H}(\mathcal{V}_\mu)} \mathcal{O} e^{-\beta \delta y_{\mathcal{M}}}}{\int \Pi_{\chi,\mu} d\mathcal{V}_\mu(\chi) e^{-\kappa \mathcal{H}(\mathcal{V}_\mu)} e^{-\beta \delta y_{\mathcal{M}}}} \\ e^{-\beta \delta y_{\mathcal{M}}} &= \Pi_{\chi,\mu\nu} (1 + \rho_{\mu\nu}) \\ \langle \mathcal{O} \rangle &= \frac{\int \Pi_{\chi,\mu} d\mathcal{V}_\mu(\chi) e^{-\kappa \mathcal{H}(\mathcal{V}_\mu)} \mathcal{O} \Pi_{\chi,\mu\nu} (1 + \rho_{\mu\nu})}{\int \Pi_{\chi,\mu\nu} d\mathcal{V}_\mu(\chi) e^{-\kappa \mathcal{H}(\mathcal{V}_\mu)} \mathcal{O} \Pi_{\chi,\mu\nu} (1 + \rho_{\mu\nu})} \\ \int \Pi_{\chi,\mu\nu \in \mathcal{Q}} d\mathcal{V}_\mu(\chi) e^{-\kappa \mathcal{H}(\mathcal{V}_\mu)} \mathcal{O} \Pi_{\chi,\mu\nu} \rho_{\mu\nu} &< c_1 c_2^\eta\end{aligned}$$

8. Estructuras cuánticas diagramáticas en espacios cuánticos curvos.

$$\begin{aligned}\mathcal{Q}_{\mathcal{L}} &= \frac{1}{2} \left\langle \left| \sum_{\chi} \phi(\chi) \right|^2 \right\rangle \\ \mathcal{Q}_c &= \frac{1}{2\mathcal{N}_t \mathcal{V}_{d-1}^2} \sum_t \left\langle \text{tr} \left| \sum_{\vec{\chi}, \vec{\gamma}} \mathcal{U}_0(t, \vec{\chi})^\dagger \mathcal{U}_0(t, \vec{\gamma}) \right| \right\rangle \\ &\quad \text{tr}(\Sigma^3) \\ \sum &= \mathcal{T}^a \phi^a\end{aligned}$$

$$\langle \mathcal{O} \rangle = \int \mathcal{DW} \left(\int_{\text{Selected stratum}} \mathcal{D}\phi \mathcal{O} e^{i\delta} + \int_{\text{Other strata}} \mathcal{D}\phi \mathcal{O} e^{i\delta} \right) = \langle \mathcal{O} \rangle_\delta + \langle \mathcal{O} \rangle_o$$

9. Espectro de Masa y Brecha de Masa según las teorías cuánticas de campo de Yang – Mills - Higgs.

$$\begin{aligned}\mathcal{H}(\gamma) &= \phi^\dagger(\gamma)\phi(\gamma) = \det \chi(\gamma) \\ \mathcal{V}_\mu^c(\gamma) &= \frac{1}{\det \chi(\gamma)} \text{tr} \left(\mathcal{T}^c \chi^\dagger(\gamma) \exp \left(i \mathcal{T}_b \omega_\mu^b(\gamma) \right) \chi(\gamma + \epsilon_\mu) \right) \\ &= \frac{1}{\mathcal{H}(\gamma)} \text{tr} \left(\tau^c \chi^\dagger(\gamma) \mathcal{D}_\mu \chi(\gamma) \right) + \mathcal{O}(a^2) \\ \mathcal{V}_\mu(\gamma) &= \mathcal{T}^c \mathcal{V}_\mu^c(\gamma)\end{aligned}$$



$$\delta = \beta \sum_{\gamma\mu < \nu} \left(1 - \frac{1}{2} \Re \text{tr} \mathcal{V}_{\mu\nu}(\gamma) \right) + \sum_{\gamma} \left(\mathcal{H}^2(\gamma) - 3 \log \mathcal{H}(\gamma) + \lambda(\mathcal{H}(\gamma) - 1)^2 - \kappa \sum_{\mu > 0} \mathcal{H}(\gamma + \mu) \mathcal{H}(\gamma) \text{tr} \mathcal{V}_{\mu}(\gamma) \right)$$

10. Dinámica perturbativa de la invariante de gauge en espacios cuánticos curvos.

$$\langle \mathcal{O}^{\dagger} \mathcal{O} \rangle = v^2 \eta_i \eta_j \langle \mathcal{O}_i^{\prime \dagger} \mathcal{O}_j' \rangle + v \eta_i \langle (\mathcal{O}_i^{\prime \dagger} \mathcal{O}'' + \mathcal{O}''^{\dagger} \mathcal{O}_i') \rangle + \langle \mathcal{O}'' \mathcal{O}'' \rangle$$

$$\mathcal{H}(\chi) = \langle \phi^{\dagger} \phi \rangle \langle \chi \rangle$$

$$\begin{aligned} & \langle (\phi^{\dagger} \phi)(\chi)(\phi^{\dagger} \phi)(\gamma) \rangle \\ &= dv^4 + 4v^2 \langle \Re[\eta_i^{\dagger} \eta_i]^{\dagger}(\chi) \Re[\eta_j^{\dagger} \eta_j](\gamma) \rangle \\ &+ 2v \left(\langle (\eta_i^{\dagger} \eta_i)(\chi) \Re[\eta_j^{\dagger} \eta_j](\gamma) \rangle + (\chi \leftrightarrow \gamma) \right) + \langle (\eta_i^{\dagger} \eta_i)(\chi)(\eta_j^{\dagger} \eta_j)(\gamma) \rangle \end{aligned}$$

$$\begin{aligned} & \langle (\phi^{\dagger} \phi)(\chi)(\phi^{\dagger} \phi)(\gamma) \rangle \\ &= d'v^4 + 4v^2 \langle \Re[\eta_i^{\dagger} \eta_i]^{\dagger}(\chi) \Re[\eta_j^{\dagger} \eta_j](\gamma) \rangle_{\mathfrak{S}^{\ell}} + \langle \Re[\eta_i^{\dagger} \eta_i]^{\dagger}(\chi) \Re[\eta_j^{\dagger} \eta_j](\gamma) \rangle_{\mathfrak{S}^{\ell}}^2 \\ &+ \mathcal{O}(g^2, \lambda) \end{aligned}$$

$$\mathcal{D}_{\mathcal{H}}(\mathcal{P}^2) = 4v^2 \mathcal{D}_{\eta}(\mathcal{P}^2)^{\mathfrak{S}^{\ell}} + 2 \int d^4 q \mathcal{D}_{\eta}^{\mathfrak{S}^{\ell}}((\mathcal{P} - q)^2) \mathcal{D}_{\eta}^{\mathfrak{S}^{\ell}}(q^2) = \frac{4v^2}{\mathcal{P}^2 - m_h^2 + i\epsilon} + \prod(\mathcal{P}^2)$$

$$\approx \frac{4v^2}{(\mathcal{P}^2 - m_h^2) \left(1 + \frac{\mathcal{P}^2 - m_h^2}{4v^2} \prod(\mathcal{P}^2) \right) + i\epsilon}$$

$$\prod(\mathcal{P}^2) = -2i\pi^2 \left(\ln \frac{m_h^2}{\mu^2} + \frac{m_h^2}{\wp^2} \left(\frac{1}{r(\wp^2)} - r(\wp^2) \right) \ln r(\wp^2) \right)$$

$$r(\wp^2) = \frac{2m_h^2 - \wp^2 - i\epsilon \pm \sqrt{(\wp^2 - 2m_h^2 + i\epsilon)^2 - 4m_h^2}}{2m_h^2}$$

$$\langle (\phi^{\dagger} \phi)(\chi)(\phi^{\dagger} \phi)(\gamma) \rangle_c = 4v^2 \langle \Re[\eta_i^{\dagger} \eta_i]^{\dagger}(\chi) \Re[\eta_j^{\dagger} \eta_j](\gamma) \rangle_c + \mathcal{O}(v) = 4v^2 \mathcal{D}_{\eta}(\chi - \gamma) + \mathcal{O}(v)$$

$$\mathcal{O}_{2\mathcal{H}} = \mathcal{H}(\gamma) \mathcal{H}(\gamma)$$

$$\mathcal{O}_{\omega -ball} = (\omega_{\mu\nu}^a \omega_a^{\mu\nu})(\gamma)$$

$$\mathcal{W}(\chi) = tr(\mathcal{T}^a \chi^{\dagger} \mathcal{D}_{\mu} \chi)(\gamma)$$



$$\langle tr(\mathcal{T}^a \chi^\dagger \mathcal{D}_\mu \chi)(z) tr(\mathcal{T}^b \chi^\dagger \mathcal{D}_\mu \chi)(\gamma) \rangle = v^2 c_{\kappa\ell}^{ab} \langle \omega^{\kappa\mu}(z) \omega_\mu^\ell(\gamma) \rangle + \mathcal{O}(v)$$

$$\mathcal{D}(t) = \langle \mathcal{O}(t) \mathcal{O}(0) \rangle = \sum_{\eta} |\langle \eta | \mathcal{O} \rangle|^2 e^{-\mathfrak{E}_{\eta} t} \langle t \gg \underset{\sim}{\mathfrak{E}_1^{-1}} \rangle e^{-\mathfrak{E}_0 t}$$

$$\mathcal{O}(t) = \sum_{\vec{\chi}} \mathcal{O}(\vec{\chi}, t)$$

$$m(t)=\ln-\frac{\mathcal{D}(t)}{\mathcal{D}(t+a)}$$

$$\mathcal{D}^{ij}(\wp^2)=\frac{\delta^{ij}}{\mathcal{Z}(\wp^2+m_r^2+\Pi(\wp^2)+\delta m^2)}$$

$$\mathcal{D}^{ij}(\mu^2)=\frac{\delta^{ij}}{\mu^2+m_r^2}$$

$$\frac{\partial \mathcal{D}^{ij}}{\partial \wp}(\mu^2)=-\frac{2\mu \delta^{ij}}{(\mu^2+m_r^2)^2}$$

$$\mathcal{Z}=\frac{2\mu-\frac{d\,\Pi(\wp^2)}{d\wp}(\mu^2)}{2\mu}$$

$$\delta m^2=\frac{(\mu^2+m_r^2)\frac{d\,\Pi(\wp^2)}{d\wp}(\mu^2)-2\mu\,\Pi(\mu^2)}{2\mu}$$

10.1. Simetría QED.

$$\frac{1}{2}tr\left((\mathcal{D}_\mu\chi)^\dagger\mathcal{D}^\mu\chi\right)$$

$$\mathcal{O}_Z = \sin \theta_{\mathcal{W}} d\mathcal{O}_{13}^3 + \cos \theta_{\mathcal{W}} \mathcal{D}\mathcal{L} \approx \sin \theta_{\mathcal{W}} \omega^3 + \cos \theta_{\mathcal{W}} \mathcal{L} + \mathcal{O}(v^{-1}) = \mathcal{Z} + \mathcal{O}(v^{-1})$$

$$\mathcal{O}_\Gamma = \sin \theta_{\mathcal{W}} d\mathcal{O}_{13}^3 - \cos \theta_{\mathcal{W}} \mathcal{D}\mathcal{L} \approx \sin \theta_{\mathcal{W}} \omega^3 - \cos \theta_{\mathcal{W}} \mathcal{L} + \mathcal{O}(v^{-1}) = \gamma + \mathcal{O}(v^{-1})$$

$$\begin{aligned} \left\langle \left(\mathcal{O}_{13}^3\right)_\mu \left| \left(\mathcal{O}_{13}^3\right)_\nu \right| \left(\mathcal{O}_\Gamma\right)_\rho \right\rangle &\approx v^4 \langle \omega_\mu^+ | \omega_\nu^- | \gamma_\rho \rangle + \mathcal{O}(v^3) \\ &= v^4 (\sin \theta_{\mathcal{W}} \langle \omega_\mu^+ | \omega_\nu^- | \omega_\rho^3 \rangle + \cos \theta_{\mathcal{W}} \langle \omega_\mu^+ | \omega_\nu^- | \mathcal{L}_\rho \rangle) \\ &\quad + \mathcal{O}(v^3) \overset{tree-level}{\approx} v^4 \sin \theta_{\mathcal{W}} \langle \omega_\mu^+ | \omega_\nu^- | \omega_\rho^3 \rangle_{\mathfrak{S}\ell} \end{aligned}$$

$$tr \tau^a \chi^\dagger \mathcal{D}_\mu \chi = v^2 \omega_\mu^\mathcal{L} tr \tau^a \tau^\mathcal{L} + \mathcal{O}(v) = \omega_\mu^a + \mathcal{O}(v)$$

10.2. Otras simetrías de gauge.

$$\mathcal{O}_\psi(\gamma) = (\chi \epsilon)^\dagger(\gamma) \psi(\gamma)$$



$$\mathcal{O}_{\mathcal{N}\mathfrak{E}} = \binom{\mathcal{N}}{\mathfrak{E}} = (\chi\epsilon)^\dagger \begin{pmatrix} \nu \\ e \end{pmatrix} = \begin{pmatrix} \phi_2\nu - \phi_1e \\ \phi_1^*\nu + \phi_2^*e \end{pmatrix} \approx \nu \begin{pmatrix} \nu \\ e \end{pmatrix} + \mathcal{O}(v^0)$$

$$\mathcal{L}_Y = g_Y(\bar{\mathcal{O}}_{\mathcal{N}\mathfrak{E}}\psi_{\mathfrak{R}} - \bar{\psi}_{\mathfrak{R}}\mathcal{O}_{\mathcal{N}\mathfrak{E}})$$

$$\mathcal{L}_Y = g_Y(\bar{\mathcal{O}}_{\mathcal{N}\mathfrak{E}}\Omega\psi_{\mathfrak{R}} - \bar{\psi}_{\mathfrak{R}}\Omega^\dagger\mathcal{O}_{\mathcal{N}\mathfrak{E}})$$

$$\mathcal{L}_Y = g_\nu(\phi_2\bar{\nu}_L - \phi_1\bar{e}_L)\mathcal{N}_{\mathcal{R}} + g_\varepsilon(\phi_1^*\bar{\nu}_L - \phi_2^*\bar{e}_L)\mathfrak{E}_{\mathfrak{R}} + \hbar.c$$

10.3. Simetría QCD.

$$\mathcal{O}_\sigma = \bar{\mu}_L\sigma\mu_L + \bar{d}_L\sigma d_L + \bar{\mu}_{\mathfrak{R}}\sigma\mu_{\mathfrak{R}} + \bar{d}_{\mathfrak{R}}\sigma d_{\mathfrak{R}}$$

$$\psi = \begin{pmatrix} (\chi\epsilon)^\dagger\psi_L \\ \mathcal{U}_{\mathcal{R}} \\ \mathcal{D}_{\mathcal{R}} \end{pmatrix}$$

$$\pi^a = \bar{\psi}\tau^a\gamma_5\psi$$

$$\pi^+ = \bar{\mathcal{D}}_{\mathcal{R}}((\chi\epsilon)^\dagger\psi_L)^\mu + \overline{((\chi\epsilon)^\dagger\psi_L)^d}\mathcal{U}_{\mathcal{R}}$$

$$\pi^- = \bar{\mathcal{U}}_{\mathcal{R}}((\chi\epsilon)^\dagger\psi_L)^d + \overline{((\chi\epsilon)^\dagger\psi_L)^\mu}\mathcal{D}_{\mathcal{R}}$$

$$\pi^0 = \bar{\mathcal{D}}_{\mathcal{R}}((\chi\epsilon)^\dagger\psi_L)^d + \overline{((\chi\epsilon)^\dagger\psi_L)^d}\mathcal{U}_{\mathcal{R}} - \bar{\mathcal{U}}_{\mathcal{R}}((\chi\epsilon)^\dagger\psi_L)^\mu + \overline{((\chi\epsilon)^\dagger\psi_L)^\mu}\mathcal{D}_{\mathcal{R}}$$

$$\mathcal{N}_L = \epsilon^{JJ\mathcal{K}}c_{ijk\ell}q_i^Jq_j^Jq_k^K(\chi\epsilon)_{il}^\dagger$$

$$\Delta_L = \epsilon^{JJ\mathcal{K}}q_i^Jq_j^Jq_k^K(\chi\epsilon)_{ii}^\dagger(\chi\epsilon)_{jj}^\dagger(\chi\epsilon)_{\tilde{k}k}^\dagger$$

$$c_{ij\kappa\ell}^{\wp} = \alpha_1\epsilon_{ij}\delta_{\kappa\ell} + \alpha_2\epsilon_{i\kappa}\delta_{j\ell} + \alpha_3\epsilon_{j\kappa}\delta_{i\ell}$$

$$\mathcal{N}^{rst} = \epsilon^{JJ\mathcal{K}}\mathcal{F}_{uvw}^{rst}\psi_{\mathcal{I}\mu}(\psi_{\mathcal{I}v}^T\mathcal{C}\gamma_5\psi_{\kappa\omega})$$

$$\begin{aligned} \langle\phi^\dagger\phi\rangle &= \mathcal{W}^a tr(\mathcal{T}_{ij}^a\chi_j^\dagger\mathcal{D}_\mu\chi_i)\sin\theta_{\mathcal{W}}\,d\mathcal{O}_{1\overline{3}}^3 + \cos\theta_{\mathcal{W}}\,\mathcal{D}\mathcal{L}\sin\theta_{\mathcal{W}}\,d\mathcal{O}_{1\overline{3}}^3 \\ &\quad - \cos\theta_{\mathcal{W}}\,\mathcal{D}\mathcal{L}\left|d_{e_{\mathcal{R}}}^{\nu_{\mathcal{R}}}\right|d(\chi\epsilon)^\dagger\left|e_{\mathcal{L}}^{\nu_{\mathcal{L}}}\right|d\epsilon^{JJ\mathcal{K}}\mathcal{F}_{uvw}^{rst}\psi_{\mathcal{I}\mu}(\psi_{\mathcal{I}v}^T\mathcal{C}\gamma_5\psi_{\kappa\omega}) \end{aligned}$$

$$\mathcal{O}_{0^+}^{ab} = tr(\chi\tau^a\tau^b\gamma)$$

$$\begin{aligned} \sigma_{\xi^+\xi^-\mapsto\bar{f}\mathcal{F}}(\delta) &= \sum_i\int_0^1d\chi\int_0^1d\gamma f_i(\chi)f_i(\gamma)\,\sigma_{\bar{u}i\mapsto\bar{f}f}(\chi\wp_1,\gamma\wp_2) \\ &= \theta(4m_h^2-\delta)\sigma_{e^+e^-\mapsto\bar{f}f} \\ &\quad + \theta(\delta-4m_h^2)\bigotimes\sum_i\int_0^1d\chi\int_0^1d\gamma f_i(\chi)f_i(\gamma)\sigma_{\bar{u}i\mapsto\bar{f}f}(\chi\wp_1,\gamma\wp_2) \end{aligned}$$

$$f_i(\chi) = \alpha_i\delta(\chi) + c_i\delta(\chi-1)$$



10.4. Simetrías amplias de gauge.

$$\langle \mathcal{O}_\mu^{1\bar{0}}(\chi) | \mathcal{O}_\mu^{\dagger 1\bar{0}}(\gamma) \rangle = \frac{v^4 g^2}{4} \langle \omega_\mu^8(\chi) | \omega_\mu^8(\gamma) \rangle + \mathcal{O}(\eta)$$

$$\mathcal{O}_\mu^{1\bar{0}}(\chi) = i(\phi^\dagger \mathcal{D}_\mu \phi)(\chi)$$

$$\mathcal{O}^{0\bar{1}} = \epsilon_{abc} \phi_a \mathcal{D}_\mu \phi_b \mathcal{D}_\mu \mathcal{D}_\nu \mathcal{D}_\nu \phi_c$$

$$\mathcal{O}_\mu^{1\bar{1}} = \epsilon_{abc} \phi_a \mathcal{D}_\nu \phi_b \mathcal{D}_\mu \mathcal{D}_\nu \phi_c$$

$$\mathcal{O}_\mu^{1\bar{0}} = -\frac{v^2 g}{2} \omega_\mu^8 + \frac{v}{\sqrt{2}} \partial_\mu \eta - \sqrt{2} g v \omega_\mu^8 \eta + \mathcal{O}(\eta^2)$$

11. Representación de Higgs en espacios cuánticos curvos.

$$\mathcal{O}_\mu^\Gamma = \frac{\partial^\nu}{\partial^2} \text{tr}(\phi^a \mathcal{T}^a \omega_{\mu\nu}) \approx -v \text{tr} \left(\mathcal{T}^3 \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) \omega^\nu \right) + \mathcal{O}(\eta)$$

$$\begin{aligned} \langle \mathcal{O}^\dagger(\chi) | \mathcal{O}(\gamma) \rangle &= \int \mathcal{D}\omega \left(\int_{\text{Selected stratum}} \mathcal{D}\phi \mathcal{O}^\dagger(\chi) \mathcal{O}(\gamma) e^{i\delta} + \int_{\text{Other strata}} \mathcal{D}\phi \mathcal{O}^\dagger(\chi) \mathcal{O}(\gamma) e^{i\delta} \right) \\ &= \langle \mathcal{O}^\dagger(\chi) | \mathcal{O}(\gamma) \rangle_\varepsilon + \langle \mathcal{O}^\dagger(\chi) | \mathcal{O}(\gamma) \rangle_\eta \end{aligned}$$

12. Procesos de dispersión en espacios cuánticos curvos.

$$\langle \mathcal{O}(\chi)^\dagger | \varphi(\gamma) | \varphi(z) \rangle$$

$$\langle \mathcal{O}'(\chi) | \mathcal{O}'(\gamma) | \mathcal{O}'(z) \rangle = v^2 \langle \mathcal{O}(\chi) | \varphi(\gamma) | \varphi(z) \rangle + \mathcal{O}(v)$$

$$\mathcal{M} = \langle \mathcal{O}_2^{\mathcal{N}\mathcal{E}}(\wp_1) \bar{\mathcal{O}}_2^{\mathcal{N}\mathcal{E}}(\wp_2) \mathcal{O}_i^{\mathfrak{F}}(q_1) \bar{\mathcal{O}}_i^{\mathfrak{F}}(q_2) \rangle$$

$$\mathcal{M} \sim v^4 \langle e^+ e^- \hat{f} f \rangle + \mathcal{O}(v^2)$$

$$\mathcal{M} \approx v^4 \langle e^+ e^- \hat{f} f \rangle + v^2 \langle \eta^\dagger \eta e^+ e^- \hat{f} f \rangle + \langle \eta^\dagger \eta \eta^\dagger \eta e^+ e^- \hat{f} f \rangle + \text{rest}$$

$$\langle \eta^\dagger \eta e^+ e^- \hat{f} f \rangle \approx \langle \eta^\dagger | \eta \rangle \langle e^+ e^- \hat{f} f \rangle + \langle e^+ | e^- \rangle \langle \eta^\dagger \eta \hat{f} f \rangle + \langle \hat{f} | f \rangle \langle e^+ e^- \eta^\dagger \eta \rangle$$

13. Métrica SU(3) para espacios cuánticos curvos.

$$\langle (\phi^\dagger \psi)^\dagger(\chi) \gamma_0 (\phi^\dagger \psi)^\dagger(\gamma) \rangle = v^2 \langle \bar{\psi}_3(\chi) | \bar{\psi}_3(\gamma) \rangle \Big|_{\mathfrak{S}\ell}$$

$$\begin{aligned} d\sigma &= \frac{(2\pi)^4}{4\sqrt{(\wp_1\wp_2)-m^4}} |\mathcal{M}_{\bar{\psi}\psi\mapsto\bar{\psi}\psi}|^2 \delta(\wp_1+\wp_2-q_1 \\ &\quad -q_2) \frac{d^3\vec{q}_1}{(2\pi)^3 2\xi_{q_1}} \frac{d^3\vec{q}_2}{(2\pi)^3 2\xi_{q_2}} \frac{d^3\vec{q}_1}{(2\pi)^3 2\zeta_{q_1}} \frac{d^3\vec{q}_2}{(2\pi)^3 2\zeta_{q_2}} \end{aligned}$$



$$\begin{aligned}\mathcal{M}_{\bar{\psi}\psi\rightarrow\bar{\psi}\psi} &= \langle \bar{\psi}(\not{s}_1)\bar{\psi}(q_1)\bar{\psi}(\not{s}_2)\bar{\psi}(q_2) \rangle = v^4 \langle \bar{\psi}_3(\not{s}_1)\bar{\psi}_3(q_1)\bar{\psi}_3(\not{s}_2)\bar{\psi}_3(q_2) \rangle_{\mathfrak{S}\ell} \\ |\mathcal{M}_{\bar{\psi}\psi\rightarrow\bar{\psi}\psi}|^2 &= \frac{32g^4}{9} \left(\frac{\delta^2 + \mu^2}{(t - \mathcal{M}_\Lambda^2)^2} + \frac{t^2 + \Lambda^2}{(\delta - \mathcal{M}_\Lambda^2)^2} + \frac{2\mu^2}{(\delta - \mathcal{M}_\Lambda^2)(t - \mathcal{M}_\Lambda^2)(\Lambda - \mathcal{M}_\Lambda^2)} \right) \\ \langle (\phi^\dagger \phi)^\dagger(\chi) | (\phi^\dagger \phi)^\dagger(\gamma) \rangle & \\ &= v^\dagger v \langle (\phi^\dagger \phi)(\chi) \rangle + v^{a^\dagger} \langle (\phi^\dagger \phi)(\chi) | \eta^a(\gamma) \rangle + v^a \langle (\phi^\dagger \phi)(\chi) | \eta^{a^\dagger}(\gamma) \rangle \\ &+ \langle (\phi^\dagger \phi)(\chi) | (\eta^\dagger \eta)(\gamma) \rangle\end{aligned}$$

REFERENCIAS BIBLIOGRÁFICAS ADICIONALES.

Axel Maas, Brout-Englert-Higgs physics: From foundations to phenomenology, arXiv:1712.04721v4 [hep-ph] 11 Feb 2019.

